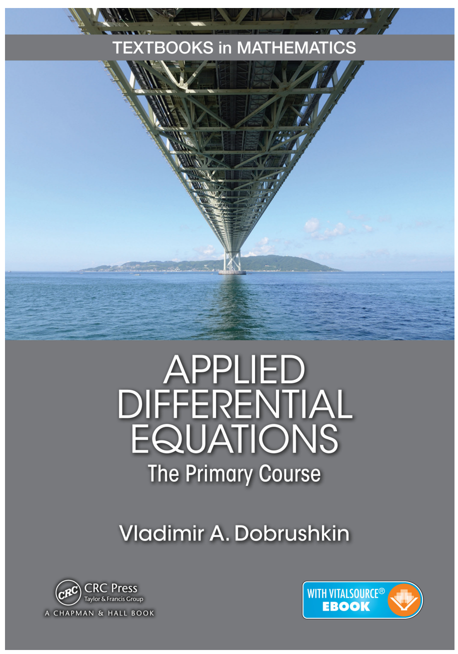
A Solution Manual For

APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015



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1 Chapter 2, First Order Equations. Problems page 149

1.1 problem Problem 1(a)

Internal problem ID [11892]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 2, First Order Equations. Problems page 149
Problem number: Problem 1(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y\,\mathrm{e}^{x+y}(x^2+1) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)=y(x)*exp(x+y(x))*(x^2+1),y(x), singsol=all)
```

$$(x^2-2x+3)\,\mathrm{e}^{x} + \mathrm{Ei}_1(y(x)) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.859 (sec). Leaf size: 32

```
DSolve[y'[x]==y[x]*Exp[x+y[x]]*(x^2+1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction[ExpIntegralEi}(-\#1)\&]\left[e^x(x^2-2x+3)+c_1\right]$$
$$y(x) \to 0$$

1.2 problem Problem 1(b)

Internal problem ID [11893]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 2, First Order Equations. Problems page 149
Problem number: Problem 1(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2y' - y^2 = 1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve(x^2*diff(y(x),x)=1+y(x)^2,y(x), singsol=all)
```

$$y(x) = \tan\left(\frac{c_1x - 1}{x}\right)$$

✓ Solution by Mathematica

Time used: 0.354 (sec). Leaf size: 30

```
DSolve[x^2*y'[x]==1+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan\left(\frac{-1 + c_1x}{x}\right)$$
$$y(x) \to -i$$
$$y(x) \to i$$

1.3 problem Problem 1(c)

Internal problem ID [11894]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 2, First Order Equations. Problems page 149
Problem number: Problem 1(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - \sin(yx) = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=sin(x*y(x)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==Sin[x*y[x]],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.4 problem Problem 1(d)

Internal problem ID [11895]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 2, First Order Equations. Problems page 149
Problem number: Problem 1(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x(\mathrm{e}^{y}+4)-\mathrm{e}^{x+y}y'=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 24

```
dsolve(x*(exp(y(x))+4)=exp(x+y(x))*diff(y(x),x),y(x), singsol=all)
```

$$y(x)=\ln\left(-4+c_1\mathrm{e}^{-x\,\mathrm{e}^{-x}-\mathrm{e}^{-x}}\right)$$

✓ Solution by Mathematica

Time used: 4.746 (sec). Leaf size: 51

```
DSolve[x*(Exp[y[x]]+4)==Exp[x+y[x]]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\log\left(-4+e^{e^{-x}(-x+c_1e^{x}-1)}\right)$$
$$y(x)\to\log(4)+i\pi$$
$$y(x)\to\log(4)+i\pi$$

1.5 problem Problem 1(e)

Internal problem ID [11896]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 2, First Order Equations. Problems page 149
Problem number: Problem 1(e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - \cos(x + y) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=cos(x+y(x)),y(x), singsol=all)
```

$$y(x) = -x - 2\arctan(-x + c_1)$$

✓ Solution by Mathematica

Time used: 1.551 (sec). Leaf size: 59

```
DSolve[y'[x]==Cos[x+y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + 2\arctan\left(x + \frac{c_1}{2}\right)$$
$$y(x) \to -x + 2\arctan\left(x + \frac{c_1}{2}\right)$$
$$y(x) \to -x - \pi$$
$$y(x) \to \pi - x$$

1.6 problem Problem 1(f)

Internal problem ID [11897]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 2, First Order Equations. Problems page 149
Problem number: Problem 1(f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{xy' + y - y^2x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x)+y(x)=x*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{1}{(\ln(x) - c_1)\,x}$$

✓ Solution by Mathematica

Time used: 0.228 (sec). Leaf size: 22

```
DSolve[x*y'[x]+y[x]==x*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{-x\log(x) + c_1x}$$

$$y(x) \to 0$$

1.7 problem Problem 1(g)

Internal problem ID [11898]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 2, First Order Equations. Problems page 149
Problem number: Problem 1(g).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - t\ln\left(y^{2t}\right) = t^2}$$

✗ Solution by Maple

```
dsolve(diff(y(t),t)=t*ln(y(t)^(2*t))+t^2,y(t), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 0.47 (sec). Leaf size: 43

```
DSolve[y'[t]==t*Log[y[t]^(2*t)]+t^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \text{InverseFunction}\left[\frac{\text{ExpIntegralEi}\left(\log(\#1) + \frac{1}{2}\right)}{2\sqrt{e}}\&\right]\left[\frac{t^3}{3} + c_1\right]$$

$$y(t) \to \frac{1}{\sqrt{e}}$$

1.8 problem Problem 1(h)

Internal problem ID [11899]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 2, First Order Equations. Problems page 149
Problem number: Problem 1(h).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - x\,\mathrm{e}^{-x+y^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(diff(y(x),x)=x*exp(y(x)^2-x),y(x), singsol=all)
```

$$-(x+1)\,\mathrm{e}^{-x} - \frac{\sqrt{\pi}\,\operatorname{erf}\left(y(x)\right)}{2} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.288 (sec). Leaf size: 28

```
DSolve[y'[x]==x*Exp[y[x]^2-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \operatorname{erf}^{-1}\left(-\frac{2e^{-x}(x - c_1e^x + 1)}{\sqrt{\pi}}\right)$$

1.9 problem Problem 1(i)

Internal problem ID [11900]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 2, First Order Equations. Problems page 149
Problem number: Problem 1(i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - \ln (yx) = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=ln(x*y(x)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==Log[x*y[x]],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.10 problem Problem 2(a)

Internal problem ID [11901]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 2, First Order Equations. Problems page 149
Problem number: Problem 2(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$x(y+1)^2 - (x^2+1)\, y\, e^y y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(x*(y(x)+1)^2=(x^2+1)*y(x)*exp(y(x))*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = -\operatorname{LambertW}\left(-\frac{e^{-1}}{\frac{\ln(x^2+1)}{2} + c_1}\right) - 1$$

✓ Solution by Mathematica

Time used: 1.003 (sec). Leaf size: 33

```
DSolve[x*(y[x]+1)^2==(x^2+1)*y[x]*Exp[y[x]]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -1 - W\left(-\frac{2}{e \log(x^2+1) + 2ec_1}\right)$$
$$y(x) \to -1$$

2 Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221

2.1 problem Problem 1(a)

Internal problem ID [11902]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + yx^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\mathrm{BesselJ}\left(\frac{1}{4}, \frac{x^2}{2}\right) + c_2\sqrt{x}\,\mathrm{BesselY}\left(\frac{1}{4}, \frac{x^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 30

```
DSolve[y''[x]+x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2\,\mathrm{ParabolicCylinderD}\left(-\frac{1}{2}, (-1+i)x\right) + c_1\,\mathrm{ParabolicCylinderD}\left(-\frac{1}{2}, (1+i)x\right)$$

2.2 problem Problem 1(b)

Internal problem ID [11903]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 1(b).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' + xy = \sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 2002

```
dsolve(diff(y(x),x$3)+x*y(x)=sin(x),y(x), singsol=all)
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'''[x]+x*y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

Timed out

2.3 problem Problem 1(c)

Internal problem ID [11904]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 1(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], [`

$$\boxed{y'' + yy' = 1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 60

```
dsolve(diff(y(x),x$2)+y(x)*diff(y(x),x)=1,y(x), singsol=all)
```

$$\int^{y(x)} -\frac{2\,2^{\frac{2}{3}}}{2^{\frac{2}{3}}\_a^2 - 4\,\mathrm{RootOf}\left(2^{\frac{1}{3}}\,\mathrm{AiryBi}\left(\_Z\right)c_1\_a + 2^{\frac{1}{3}}\_a\,\mathrm{AiryAi}\left(\_Z\right) - 2\,\mathrm{AiryBi}\left(1,\_Z\right)c_1 - 2\,\mathrm{AiryAi}\left(1,\_Z\right)\right.}$$
$$- x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 71.741 (sec). Leaf size: 73

```
DSolve[y''[x]+y[x]*y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2^{2/3}\left(c_2\,\mathrm{AiryAiPrime}\left(\frac{x-c_1}{\sqrt[3]{2}}\right) + \mathrm{AiryBiPrime}\left(\frac{x-c_1}{\sqrt[3]{2}}\right)\right)}{c_2\,\mathrm{AiryAi}\left(\frac{x-c_1}{\sqrt[3]{2}}\right) + \mathrm{AiryBi}\left(\frac{x-c_1}{\sqrt[3]{2}}\right)}$$

2.4 problem Problem 1(d)

Internal problem ID [11905]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 1(d).
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type **[[_high_order, _missing_y]]**

$$\boxed{y^{(5)} - y'''' + y' = 2x^2 + 3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 154

```
dsolve(diff(y(x),x$5)-diff(y(x),x$4) +diff(y(x),x)=2*x^2+3,y(x), singsol=all)
```

$$\begin{aligned} y(x) = {} & \frac{c_1 e^{\mathrm{RootOf}(\_Z^4 - \_Z^3 + 1, \mathrm{index}=1)x}}{\mathrm{RootOf}(\_Z^4 - \_Z^3 + 1, \mathrm{index} = 1)} + \frac{c_2 e^{\mathrm{RootOf}(\_Z^4 - \_Z^3 + 1, \mathrm{index}=2)x}}{\mathrm{RootOf}(\_Z^4 - \_Z^3 + 1, \mathrm{index} = 2)} \\ & + \frac{c_3 e^{\mathrm{RootOf}(\_Z^4 - \_Z^3 + 1, \mathrm{index}=3)x}}{\mathrm{RootOf}(\_Z^4 - \_Z^3 + 1, \mathrm{index} = 3)} \\ & + \frac{c_4 e^{\mathrm{RootOf}(\_Z^4 - \_Z^3 + 1, \mathrm{index}=4)x}}{\mathrm{RootOf}(\_Z^4 - \_Z^3 + 1, \mathrm{index} = 4)} + \frac{2x^3}{3} + 3x + c_5 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.093 (sec). Leaf size: 182

```
DSolve[y'''''[x]-y''''[x] +y'[x]==2*x^2+3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 \exp\left(x\text{Root}\left[\#1^4 - \#1^3 + 1\&, 2\right]\right)}{\text{Root}\left[\#1^4 - \#1^3 + 1\&, 2\right]} + \frac{c_1 \exp\left(x\text{Root}\left[\#1^4 - \#1^3 + 1\&, 1\right]\right)}{\text{Root}\left[\#1^4 - \#1^3 + 1\&, 1\right]} + \frac{c_4 \exp\left(x\text{Root}\left[\#1^4 - \#1^3 + 1\&, 4\right]\right)}{\text{Root}\left[\#1^4 - \#1^3 + 1\&, 4\right]} + \frac{c_3 \exp\left(x\text{Root}\left[\#1^4 - \#1^3 + 1\&, 3\right]\right)}{\text{Root}\left[\#1^4 - \#1^3 + 1\&, 3\right]} + \frac{2x^3}{3} + 3x + c_5$$

2.5 problem Problem 1(e)

Internal problem ID [11906]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 1(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x], [_high_order, _with_linear_symmetr`

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+y(x)*diff(y(x),x$4)=1,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+y[x]*y''''[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.6 problem Problem 1(f)

Internal problem ID [11907]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 1(f).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' + xy = \cosh(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 2003

```
dsolve(diff(y(x),x$3)+x*y(x)=cosh(x),y(x), singsol=all)
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'''[x]+x*y[x]==Cosh[x],y[x],x,IncludeSingularSolutions -> True]
```

Timed out

2.7 problem Problem 1(g)

Internal problem ID [11908]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 1(g).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' \cos(x) + y\,\mathrm{e}^{x^2} = \sinh(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(cos(x)*diff(y(x),x)+y(x)*exp(x^2)=sinh(x),y(x), singsol=all)
```

$$y(x) = \left(\int \mathrm{e}^{\int \mathrm{e}^{x^2} \sec(x)dx} \sinh(x) \sec(x)\,dx + c_1\right) \mathrm{e}^{\int -\mathrm{e}^{x^2} \sec(x)dx}$$

✓ Solution by Mathematica

Time used: 1.562 (sec). Leaf size: 66

```
DSolve[Cos[x]*y'[x]+y[x]*Exp[x^2]==Sinh[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \exp\left(\int_1^x -e^{K[1]^2} \sec(K[1])dK[1]\right) \left(\int_1^x \exp\left(-\int_1^{K[2]} -e^{K[1]^2} \sec(K[1])dK[1]\right) \sec(K[2]) \sinh(K[2])dK[2] + c_1\right)$$

2.8 problem Problem 1(h)

Internal problem ID [11909]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 1(h).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' + xy = \cosh(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 2003

```
dsolve(diff(y(x),x$3)+x*y(x)=cosh(x),y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 91.544 (sec). Leaf size: 2230

```
DSolve[y'''[x]+x*y[x]==Cosh[x],y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

2.9 problem Problem 1(i)

Internal problem ID [11910]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 1(i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{yy' = 1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(y(x)*diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x) = \sqrt{2x + c_1}$$
$$y(x) = -\sqrt{2x + c_1}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 38

```
DSolve[y[x]*y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}\sqrt{x + c_1}$$
$$y(x) \to \sqrt{2}\sqrt{x + c_1}$$

2.10 problem Problem 1(j)

Internal problem ID [11911]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 1(j).
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{\sinh(x)\, {y'}^2 + 3y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 799

```
dsolve(sinh(x)*diff(y(x),x)^2+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = -\frac{e^{-x}\text{RootOf}\left(-\text{JacobiSN}\left(\frac{\left(-\frac{3\,e^{3x}c_1}{\sqrt{-6\,e^{3x}+6\,e^{x}}}+\frac{3\,e^{x}c_1}{\sqrt{-6\,e^{3x}+6\,e^{x}}}-\_Z\right)\sqrt{-e^{x}+1}\;\text{RootOf}\left(\_Z^2-2\,e^{x}-2,\text{index}=1\right)\text{RootOf}\left(\_Z^2 \cdots\right.}{6(e^{x}-1)(e^{x}+1)}\right.\right.}{6\left(e^{2x}-1\right)}$$

$$y(x) = -\frac{e^{-x}\text{RootOf}\left(-\text{JacobiSN}\left(\frac{\left(\frac{3\,e^{3x}c_1}{\sqrt{-6\,e^{3x}+6\,e^{x}}}-\frac{3\,e^{x}c_1}{\sqrt{-6\,e^{3x}+6\,e^{x}}}-\_Z\right)\sqrt{-e^{x}+1}\;\text{RootOf}\left(\_Z^2-2\,e^{x}-2,\text{index}=1\right)\text{RootOf}\left(\_Z^2- \cdots\right.}{6(e^{x}-1)(e^{x}+1)}\right.\right.}{6\left(e^{2x}-1\right)}$$

$$y(x) = -\frac{e^{-x}\text{RootOf}\left(-\text{JacobiSN}\left(\frac{\left(3\,e^{3x}\,\text{RootOf}\left(\left(6\,e^{3x}-6\,e^{x}\right)\_Z^2+1\right)c_1-3\,e^{x}\,\text{RootOf}\left(\left(6\,e^{3x}-6\,e^{x}\right)\_Z^2+1\right)c_1-\_Z\right)\sqrt{-e^{x}+1}\;\text{Ro}\cdots}{6(e^{x}-1)(e^{x}+1)}\right.\right.}{6\left(e^{2x}-1\right)}$$

$$y(x) = -\frac{e^{-x}\text{RootOf}\left(\text{JacobiSN}\left(\frac{\left(-\frac{3\,e^{3x}c_1}{\sqrt{-6\,e^{3x}+6\,e^{x}}}+\frac{3\,e^{x}c_1}{\sqrt{-6\,e^{3x}+6\,e^{x}}}-\_Z\right)\sqrt{-e^{x}+1}\;\text{RootOf}\left(\_Z^2-2\,e^{x}-2,\text{index}=1\right)\text{RootOf}\left(\_Z^2-e^{x} \cdots\right.}{6(e^{x}-1)(e^{x}+1)}\right.\right.}{6\left(e^{2x}-1\right)}$$

$$y(x) = -\frac{e^{-x}\text{RootOf}\left(\text{JacobiSN}\left(\frac{\left(\frac{3\,e^{3x}c_1}{\sqrt{-6\,e^{3x}+6\,e^{x}}}-\frac{3\,e^{x}c_1}{\sqrt{-6\,e^{3x}+6\,e^{x}}}-\_Z\right)\sqrt{-e^{x}+1}\;\text{RootOf}\left(\_Z^2-2\,e^{x}-2,\text{index}=1\right)\text{RootOf}\left(\_Z^2-e^{x},\text{i}\cdots\right.}{6(e^{x}-1)(e^{x}+1)}\right.\right.}{6\left(e^{2x}-1\right)}$$

$$y(x) = -\frac{e^{-x}\text{RootOf}\left(\text{JacobiSN}\left(\frac{\left(3\,e^{3x}\,\text{RootOf}\left(\left(6\,e^{3x}-6\,e^{x}\right)\_Z^2+1\right)c_1-3\,e^{x}\,\text{RootOf}\left(\left(6\,e^{3x}-6\,e^{x}\right)\_Z^2+1\right)c_1-\_Z\right)\sqrt{-e^{x}+1}\;\text{Root}\cdots}{6(e^{x}-1)(e^{x}+1)}\right.\right.}{6\left(e^{2x}-1\right)}$$

✓ Solution by Mathematica

Time used: 0.648 (sec). Leaf size: 145

```
DSolve[Sinh[x]*y'[x]^2+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3i \operatorname{EllipticF}\left(\frac{1}{4}(\pi - 2ix), 2\right)^2 - \sqrt{3}c_1\sqrt{i\sinh(x)}\sqrt{\operatorname{csch}(x)} \operatorname{EllipticF}\left(\frac{1}{4}(\pi - 2ix), 2\right) + \frac{{c_1}^2}{4}$$

$$y(x) \to 3i \operatorname{EllipticF}\left(\frac{1}{4}(\pi - 2ix), 2\right)^2 + \sqrt{3}c_1\sqrt{i\sinh(x)}\sqrt{\operatorname{csch}(x)} \operatorname{EllipticF}\left(\frac{1}{4}(\pi - 2ix), 2\right) + \frac{{c_1}^2}{4}$$

$$y(x) \to 0$$

2.11 problem Problem 1(k)

Internal problem ID [11912]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 1(k).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{5y' - yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(5*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{x^2}{10}}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 22

```
DSolve[5*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{\frac{x^2}{10}}$$
$$y(x) \to 0$$

2.12 problem Problem 1(L)

Internal problem ID [11913]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 1(L).
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y'^2\sqrt{y} = \sin(x)}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 58

```
dsolve(diff(y(x),x)^2*sqrt(y(x))=sin(x),y(x), singsol=all)
```

$$\frac{4y(x)^{\frac{5}{4}}}{5} + \int^x -\frac{\sqrt{\sqrt{y(x)}\,\sin(\_a)}}{y(x)^{\frac{1}{4}}} d\_a + c_1 = 0$$
$$\frac{4y(x)^{\frac{5}{4}}}{5} + \int^x \frac{\sqrt{\sqrt{y(x)}\,\sin(\_a)}}{y(x)^{\frac{1}{4}}} d\_a + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.436 (sec). Leaf size: 77

```
DSolve[y'[x]^2*Sqrt[y[x]]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{5^{4/5}\left(-2E\left(\frac{1}{4}(\pi-2x)\middle|2\right)+c_1\right)^{4/5}}{2\ 2^{3/5}}$$
$$y(x) \to \frac{5^{4/5}\left(2E\left(\frac{1}{4}(\pi-2x)\middle|2\right)+c_1\right)^{4/5}}{2\ 2^{3/5}}$$

2.13 problem Problem 1(m)

Internal problem ID [11914]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 1(m).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{2y'' + 3y' + 4x^2 y = 1}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 385

```
dsolve(2*diff(y(x),x$2)+3*diff(y(x),x)+4*x^2*y(x)=1,y(x), singsol=all)
```

$$y(x) = x\,\mathrm{KummerM}\left(\frac{3}{4} - \frac{9i\sqrt{2}}{128}, \frac{3}{2}, i\sqrt{2}\,x^2\right) \mathrm{e}^{-\frac{x\left(i\sqrt{2}\,x+\frac{3}{2}\right)}{2}} c_2$$
$$+ x\,\mathrm{KummerU}\left(\frac{3}{4} - \frac{9i\sqrt{2}}{128}, \frac{3}{2}, i\sqrt{2}\,x^2\right) \mathrm{e}^{-\frac{x\left(i\sqrt{2}\,x+\frac{3}{2}\right)}{2}} c_1 - 32x\Bigg(\mathrm{KummerU}\Bigg(\frac{3}{4}$$
$$- \frac{9i\sqrt{2}}{128}, \frac{3}{2}, i\sqrt{2}\,x^2\Bigg)\left(\int \frac{\mathrm{e}^{\frac{i\sqrt{2}\,x^2}{2}+\frac{3x}{4}}\,\mathrm{KummerM}}{\left(9i\sqrt{2}+96\right)\mathrm{KummerU}\left(\frac{3}{4} - \frac{9i\sqrt{2}}{128}, \frac{3}{2}, i\sqrt{2}\,x^2\right)\mathrm{KummerM}\left(-\frac{9i\sqrt{2}}{128} - \frac{1}{4}, \frac{3}{2}, i\sqrt{2}\right.}\right.$$
$$-\left(\int \frac{\mathrm{e}^{\frac{i\sqrt{2}\,x^2}{2}+\frac{3x}{4}}\,\mathrm{KummerU}\left(\frac{3}{4} - \frac{9i\sqrt{2}}{128}, \frac{3}{2}, i\sqrt{2}\right.}{\left(9i\sqrt{2}+96\right)\mathrm{KummerU}\left(\frac{3}{4} - \frac{9i\sqrt{2}}{128}, \frac{3}{2}, i\sqrt{2}\,x^2\right)\mathrm{KummerM}\left(-\frac{9i\sqrt{2}}{128} - \frac{1}{4}, \frac{3}{2}, i\sqrt{2}\,x^2\right) + 128\,\mathrm{Kumme}}\right.$$
$$\left.\left.- \frac{9i\sqrt{2}}{128}, \frac{3}{2}, i\sqrt{2}\,x^2\right)\right)\Bigg)\,\mathrm{e}^{-\frac{x\left(i\sqrt{2}\,x+\frac{3}{2}\right)}{2}}$$

✓ Solution by Mathematica

Time used: 11.093 (sec). Leaf size: 553

```
DSolve[2*y''[x]+3*y'[x]+4*x^2*y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{1}{4}x\left(-3-2i\sqrt{2}x\right)}\left(\text{Hypergeometric1F1}\left(\frac{1}{4}\right.\right.$$

$$\left.-\frac{9i}{64\sqrt{2}},\frac{1}{2},i\sqrt{2}x^2\right)\int_1^x \frac{(8+8i)e^{\frac{1}{4}K}}{\left(9+16i\sqrt{2}\right)\left(\sqrt[4]{2}\,\text{HermiteH}\left(-\frac{3}{2}+\frac{9i}{32\sqrt{2}},\frac{(1+i)K[2]}{\sqrt[4]{2}}\right)\text{Hypergeometric1F1}\left(\frac{1}{4}-\right.\right.}$$

$$+\text{HermiteH}\left(-\frac{1}{2}\right.$$

$$\left.+\frac{9i}{32\sqrt{2}},\sqrt[4]{-2}x\right)\int_1^x \frac{16e^{\frac{1}{4}K[1]\left(2i\right.}}{\sqrt[4]{-2}\left(-32+9i\sqrt{2}\right)\text{HermiteH}\left(-\frac{3}{2}+\frac{9i}{32\sqrt{2}},\sqrt[4]{-2}K[1]\right)\text{Hypergeometric1F1}\left(\frac{1}{4}-\right.}$$

$$+c_1\,\text{HermiteH}\left(-\frac{1}{2}+\frac{9i}{32\sqrt{2}},\sqrt[4]{-2}x\right)$$

$$\left.+c_2\,\text{Hypergeometric1F1}\left(\frac{1}{4}-\frac{9i}{64\sqrt{2}},\frac{1}{2},i\sqrt{2}x^2\right)\right)$$

2.14 problem Problem 1(n)

Internal problem ID [11915]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 1(n).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _quadrature]]`

$$\boxed{y''' = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$3)=1,y(x), singsol=all)
```

$$y(x) = \frac{1}{6}x^3 + \frac{1}{2}c_1x^2 + xc_2 + c_3$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 25

```
DSolve[y'''[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{6} + c_3x^2 + c_2x + c_1$$

2.15 problem Problem 1(o)

Internal problem ID [11916]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 1(o).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' - y = \sin(x)^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 147

```
dsolve(x^2*diff(y(x),x$2)-y(x)=sin(x)^2,y(x), singsol=all)
```

$$y(x) = c_2 x^{\frac{\sqrt{5}}{2}+\frac{1}{2}} + c_1 x^{-\frac{\sqrt{5}}{2}+\frac{1}{2}} + \frac{x^2\left(3\,\text{hypergeom}\left(\left[1, -\frac{\sqrt{5}}{4}+\frac{3}{4}\right], \left[\frac{3}{2}, 2, \frac{7}{4}-\frac{\sqrt{5}}{4}\right], -x^2\right)\sqrt{5} - 3\,\text{hypergeom}\left(\left[1, \frac{\sqrt{5}}{4}+\frac{3}{4}\right], \left[\frac{3}{2}, 2, \frac{7}{4}+\frac{\sqrt{5}}{4}\right.\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 1.679 (sec). Leaf size: 445

```
DSolve[x^2*y''[x]-y[x]==Sin[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{10\sqrt{5}c_1 x^{\frac{1}{2}-\frac{\sqrt{5}}{2}} + 10c_1 x^{\frac{1}{2}-\frac{\sqrt{5}}{2}} + 10\sqrt{5}c_2 x^{\frac{1}{2}(1+\sqrt{5})} + 10c_2 x^{\frac{1}{2}(1+\sqrt{5})} + 2^{\frac{1}{2}(\sqrt{5}-1)}(5+\sqrt{5})(-ix)^{\frac{1}{2}(1+\sqrt{5})}\Gamma\left(-\right.}{}$$

2.16 problem Problem 2(a)

Internal problem ID [11917]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 2(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y = x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)=x^2+y(x),y(x), singsol=all)
```

$$y(x) = c_2 e^x + c_1 e^{-x} - x^2 - 2$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 26

```
DSolve[y''[x]==x^2+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^2 + c_1 e^x + c_2 e^{-x} - 2$$

2.17 problem Problem 2(b)

Internal problem ID [11918]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 2(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

✗ Solution by Maple

```
dsolve(diff(y(x),x$3)+x*diff(y(x),x$2)-y(x)^2=sin(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'''[x]+x*y''[x]-y[x]^2==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.18 problem Problem 2(c)

Internal problem ID [11919]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 2(c).
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y'^2 + yy'^2x = \ln(x)}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)^2+y(x)*diff(y(x),x)^2*x=ln(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]^2+y[x]*y'[x]^2*x==Log[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.19 problem Problem 2(d)

Internal problem ID [11920]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 2(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x], [_high_order, _with_linear_symmetr`

✗ Solution by Maple

```
dsolve(sin(diff(y(x),x$2))+y(x)*diff(y(x),x$4)=1,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[Sin[y''[x]]+y[x]*y''''[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.20 problem Problem 2(e)

Internal problem ID [11921]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 2(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$\sinh(x)\, {y'}^2 + y'' - yx = 0$$

✗ Solution by Maple

```
dsolve(sinh(x)*diff(y(x),x)^2+diff(y(x),x$2)=x*y(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[Sinh[x]*y'[x]^2+y''[x]==x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.21 problem Problem 2(f)

Internal problem ID [11922]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 2(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{yy'' = 1}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 51

```
dsolve(y(x)*diff(y(x),x$2)=1,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{\sqrt{2\ln(\_a) - c_1}} d\_a - x - c_2 = 0$$
$$\int^{y(x)} -\frac{1}{\sqrt{2\ln(\_a) - c_1}} d\_a - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 60.104 (sec). Leaf size: 93

```
DSolve[y[x]*y''[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \exp\left(-\operatorname{erf}^{-1}\left(-i\sqrt{\frac{2}{\pi}}\sqrt{e^{c_1}(x+c_2)^2}\right)^2 - \frac{c_1}{2}\right)$$
$$y(x) \to \exp\left(-\operatorname{erf}^{-1}\left(i\sqrt{\frac{2}{\pi}}\sqrt{e^{c_1}(x+c_2)^2}\right)^2 - \frac{c_1}{2}\right)$$

2.22 problem Problem 2(h)

Internal problem ID [11923]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 2(h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

✗ Solution by Maple

```
dsolve(diff(y(x),x$3)^2+sqrt(y(x))=sin(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'''[x]^2+Sqrt[y[x]]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.23 problem Problem 3(a)

Internal problem ID [11924]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 3(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\left(-2+\sqrt{3}\right)x} + c_2 \mathrm{e}^{-\left(2+\sqrt{3}\right)x}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 34

```
DSolve[y''[x]+4*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\left(\left(2+\sqrt{3}\right)x\right)} \left(c_2 e^{2\sqrt{3}x} + c_1\right)$$

2.24 problem Problem 3(b)

Internal problem ID [11925]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 3(b).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' - 5y'' + y' - y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 181

```
dsolve(diff(y(x),x$3)-5*diff(y(x),x$2)+diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$\begin{aligned} y(x) = {} & c_1 \mathrm{e}^{\frac{\left(\left(116+6\sqrt{78}\right)^{\frac{2}{3}}+5\left(116+6\sqrt{78}\right)^{\frac{1}{3}}+22\right)x}{3\left(116+6\sqrt{78}\right)^{\frac{1}{3}}}} \\ & - c_2 \mathrm{e}^{-\frac{\left(22+\left(116+6\sqrt{78}\right)^{\frac{2}{3}}-10\left(116+6\sqrt{78}\right)^{\frac{1}{3}}\right)x}{6\left(116+6\sqrt{78}\right)^{\frac{1}{3}}}} \sin\left(\frac{\left(\sqrt{3}\left(116+6\sqrt{78}\right)^{\frac{2}{3}}-22\sqrt{3}\right)x}{6\left(116+6\sqrt{78}\right)^{\frac{1}{3}}}\right) \\ & + c_3 \mathrm{e}^{-\frac{\left(22+\left(116+6\sqrt{78}\right)^{\frac{2}{3}}-10\left(116+6\sqrt{78}\right)^{\frac{1}{3}}\right)x}{6\left(116+6\sqrt{78}\right)^{\frac{1}{3}}}} \cos\left(\frac{\left(\sqrt{3}\left(116+6\sqrt{78}\right)^{\frac{2}{3}}-22\sqrt{3}\right)x}{6\left(116+6\sqrt{78}\right)^{\frac{1}{3}}}\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 81

```
DSolve[y'''[x]-5*y''[x]+y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) \to\ & c_2 \exp\left(x\text{Root}\left[\#1^3 - 5\#1^2 + \#1 - 1\&, 2\right]\right) \\
& + c_3 \exp\left(x\text{Root}\left[\#1^3 - 5\#1^2 + \#1 - 1\&, 3\right]\right) \\
& + c_1 \exp\left(x\text{Root}\left[\#1^3 - 5\#1^2 + \#1 - 1\&, 1\right]\right)
\end{aligned}
$$

2.25 problem Problem 3(c)

Internal problem ID [11926]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 3(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2y'' - 3y' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(2*diff(y(x),x$2)-3*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{2x} + c_2 e^{-\frac{x}{2}}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 24

```
DSolve[2*y''[x]-3*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x/2} + c_2 e^{2x}$$

2.26 problem Problem 3(d)

Internal problem ID [11927]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 3(d).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{3y'''' - 2y'' + y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(3*diff(y(x),x$4)-2*diff(y(x),x$2)+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 e^{-x} + c_3 e^{\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{6}\right) + c_4 e^{\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{6}\right)$$

✓ Solution by Mathematica

Time used: 1.175 (sec). Leaf size: 87

```
DSolve[3*y''''[x]-2*y''[x]+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3(-e^{-x}) - \frac{1}{2}\left(\sqrt{3}c_1 - 3c_2\right) e^{x/2} \cos\left(\frac{x}{2\sqrt{3}}\right) + \frac{1}{2}\left(3c_1 + \sqrt{3}c_2\right) e^{x/2} \sin\left(\frac{x}{2\sqrt{3}}\right) + c_4$$

2.27 problem Problem 5(a)

Internal problem ID [11928]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 5(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{(x-3)\,y''+y\ln(x)=x^2}$$

With initial conditions

$$[y(1)=1, y'(1)=2]$$

✗ Solution by Maple

```
dsolve([(x-3)*diff(y(x),x$2)+ln(x)*y(x)=x^2,y(1) = 1, D(y)(1) = 2],y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{(x-3)*y''[x]+log[x]*y[x]==x^2,{y[1]==1,y'[1]==2}},y[x],x,IncludeSingularSolutions ->
```

Not solved

2.28 problem Problem 5(b)

Internal problem ID [11929]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 5(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' \tan(x) + \cot(x) y = 0}$$

With initial conditions

$$\left[y\left(\frac{\pi}{4}\right) = 1, y'\left(\frac{\pi}{4}\right) = 0\right]$$

✓ Solution by Maple

Time used: 3.828 (sec). Leaf size: 46436

```
dsolve([diff(y(x),x$2)+tan(x)*diff(y(x),x)+cot(x)*y(x)=0,y(1/4*Pi) = 1, D(y)(1/4*Pi) = 0],y(
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y''[x]+Tan[x]*y'[x]+Cot[x]*y[x]==0,{y[Pi/4]==1,y'[Pi/4]==0}},y[x],x,IncludeSingularS
```

Not solved

2.29 problem Problem 5(c)

Internal problem ID [11930]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 5(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\, y'' + (x-1)\, y' + y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.375 (sec). Leaf size: 157

```
dsolve([(x^2+1)*diff(y(x),x$2)+(x-1)*diff(y(x),x)+y(x)=0,y(0) = 0, D(y)(0) = 1],y(x), singso
```

$$y(x) = \frac{-20\, e^{\left(\frac{1}{4}-\frac{i}{4}\right)\pi} \operatorname{hypergeom}\left([i,-i], \left[\frac{1}{2}-\frac{i}{2}\right], \frac{1}{2}\right) (i+x)^{\frac{1}{2}+\frac{i}{2}} \operatorname{hypergeom}\left(\left[\frac{1}{2}-\frac{i}{2}, \frac{1}{2}+\frac{3i}{2}\right], \left[\frac{3}{2}+\right.\right.}{(10-10i)\left(\operatorname{hypergeom}\left([1-i,1+i], \left[\frac{3}{2}-\frac{i}{2}\right], \frac{1}{2}\right) - \operatorname{hypergeom}\left([i,-i], \left[\frac{1}{2}-\frac{i}{2}\right], \frac{1}{2}\right)\right) \operatorname{hypergeom}\left(\left[\frac{1}{2}-\right.\right.}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{(x^2+1)*y''[x]+(x-1)*y'[x]+y[x]==0,{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSolutio
```

Not solved

2.30 problem Problem 5(d)

Internal problem ID [11931]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 5(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{xy'' + 2x^2y' + \sin(x)\, y = \sinh(x)}$$

With initial conditions

$$[y(0) = 1, y'(0) = 1]$$

✗ Solution by Maple

```
dsolve([x*diff(y(x),x$2)+2*x^2*diff(y(x),x)+y(x)*sin(x)=sinh(x),y(0) = 1, D(y)(0) = 1],y(x),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x^2*y''[x]+2*x^2*y'[x]+y[x]*Sin[x]==Sinh[x],{y[0]==1,y'[0]==1}},y[x],x,IncludeSingul
```

Not solved

2.31 problem Problem 5(e)

Internal problem ID [11932]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 5(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\sin(x)\, y'' + xy' + 7y = 1}$$

With initial conditions

$$[y(1) = 1, y'(1) = 0]$$

✗ Solution by Maple

```
dsolve([sin(x)*diff(y(x),x$2)+x*diff(y(x),x)+7*y(x)=1,y(1) = 1, D(y)(1) = 0],y(x), singsol=a
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{Sin[x]*y''[x]+x*y'[x]+7*y[x]==1,{y[1]==1,y'[1]==0}},y[x],x,IncludeSingularSolutions
```

Not solved

2.32 problem Problem 5(f)

Internal problem ID [11933]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 5(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - (x-1)\,y' + x^2 y = \tan(x)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.328 (sec). Leaf size: 528

```
dsolve([diff(y(x),x$2)-(x-1)*diff(y(x),x)+x^2*y(x)=tan(x),y(0) = 0, D(y)(0) = 0],y(x), sings
```

Expression too large to display

✓ Solution by Mathematica

Time used: 90.104 (sec). Leaf size: 4228

```
DSolve[{y''[x]-(x-1)*y'[x]+x^2*y[x]==Tan[x],{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSoluti
```

Too large to display

2.33 problem Problem 10

Internal problem ID [11934]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-1)\,y''-xy'+y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+c_2\mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 17

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1e^x-c_2x$$

2.34 problem Problem 13

Internal problem ID [11935]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 4x^2y' + (x^2+1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(x^2*diff(y(x),x$2)-4*x^2*diff(y(x),x)+(x^2+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{2x}\sqrt{x}\,\mathrm{BesselI}\left(\frac{i\sqrt{3}}{2}, \sqrt{3}\,x\right) + c_2 e^{2x}\sqrt{x}\,\mathrm{BesselK}\left(\frac{i\sqrt{3}}{2}, \sqrt{3}\,x\right)$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 67

```
DSolve[x^2*y''[x]-4*x^2*y'[x]+(x^2+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}\sqrt{x}\left(c_1\,\mathrm{BesselJ}\left(\frac{i\sqrt{3}}{2}, -i\sqrt{3}x\right) + c_2\,\mathrm{BesselY}\left(\frac{i\sqrt{3}}{2}, -i\sqrt{3}x\right)\right)$$

2.35 problem Problem 15

Internal problem ID [11936]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{kx}{y^4} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 91

```
dsolve(diff(y(x),x$2)+k*x/(y(x)^4)=0,y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(-15\left(\int^{\_Z} \frac{\sqrt{-3c_1\_f^4 + 150\_fk}\,\_f}{c_1\_f^3 - 50k} d\_f\right) x + 5xc_2 + 3\right) x$$

$$y(x) = \operatorname{RootOf}\left(15\left(\int^{\_Z} \frac{\sqrt{-3c_1\_f^4 + 150\_fk}\,\_f}{c_1\_f^3 - 50k} d\_f\right) x + 5xc_2 + 3\right) x$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+k*x/(y[x]^4)==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.36 problem Problem 18(a)

Internal problem ID [11937]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 18(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + 2xy' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \operatorname{erfi}(x)\,\mathrm{e}^{-x^2} c_1 + c_2 \mathrm{e}^{-x^2}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 31

```
DSolve[y''[x]+2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-x^2} \left(\sqrt{\pi} c_1 \operatorname{erfi}(x) + 2c_2\right)$$

2.37 problem Problem 18(b)

Internal problem ID [11938]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 18(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{xy'' + \sin(x) y' + y \cos(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x*diff(y(x),x$2)+sin(x)*diff(y(x),x)+cos(x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_1 \left(\int \frac{e^{\mathrm{Si}(x)}}{x^2} dx\right) + c_2\right) x\, e^{-\mathrm{Si}(x)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]+Sin[x]*y'[x]+Cos[x]*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.38 problem Problem 18(c)

Internal problem ID [11939]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 18(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2x^2y' + 4yx = 2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 63

```
dsolve(diff(y(x),x$2)+2*x^2*diff(y(x),x)+4*x*y(x)=2*x,y(x), singsol=all)
```

$$y(x) = \frac{e^{-\frac{2x^3}{3}} x \left(2\sqrt{3}\,\pi - 3\Gamma\left(\frac{1}{3}, -\frac{2x^3}{3}\right)\Gamma\left(\frac{2}{3}\right)\right) c_1}{(-x^3)^{\frac{1}{3}}} + e^{-\frac{2x^3}{3}} c_2 + \frac{\left(-1 + e^{\frac{2x^3}{3}}\right) e^{-\frac{2x^3}{3}}}{2}$$

✓ Solution by Mathematica

Time used: 0.063 (sec). Leaf size: 66

```
DSolve[y''[x]+2*x^2*y'[x]+4*x*y[x]==2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 e^{-\frac{2x^3}{3}} + \frac{c_1 e^{-\frac{2x^3}{3}} (-x^3)^{2/3}\,\Gamma\left(\frac{1}{3}, -\frac{2x^3}{3}\right)}{\sqrt[3]{2}3^{2/3}x^2} + \frac{1}{2}$$

2.39 problem Problem 18(d)

Internal problem ID [11940]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 18(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{(-x^2+1)\,y''+(1-x)\,y'+y=-2x+1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve((1-x^2)*diff(y(x),x$2)+(1-x)*diff(y(x),x)+y(x)=1-2*x,y(x), singsol=all)
```

$$y(x)=\left(-\frac{\ln(x+1)\,x}{4}+\frac{\ln(x+1)}{4}+\frac{1}{2}+\frac{\ln(x-1)\,x}{4}-\frac{\ln(x-1)}{4}\right)c_1+(x-1)\,c_2+\frac{(\ln(x+1)+\ln(x-1))\,(x-1)}{2}$$

✓ Solution by Mathematica

Time used: 0.198 (sec). Leaf size: 74

```
DSolve[(1-x^2)*y''[x]+(1-x)*y'[x]+y[x]==1-2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{4}((x-1)\log(1-x)+2x\log(x+1)-2\log(x+1)-4c_1x+(1+c_2)(x-1)\log(x-1)-c_2x\log(x+1)+c_2\log(x+1)+4c_1+2c_2)$$

2.40 problem Problem 18(e)

Internal problem ID [11941]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 18(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4xy' + (4x^2 + 2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+4*x*diff(y(x),x)+(2+4*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x^2} + c_2 x\, e^{-x^2}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 20

```
DSolve[y''[x]+4*x*y'[x]+(2+4*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x^2}(c_2 x + c_1)$$

2.41 problem Problem 18(f)

Internal problem ID [11942]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 18(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x^2y' + 2(1-x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 123

```
dsolve(x^2*diff(y(x),x$2)+x^2*diff(y(x),x)+2*(1-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\mathrm{e}^{-\frac{x}{2}}\left(\left(x^2+2x\right)\mathrm{BesselI}\left(\frac{i\sqrt{7}}{2}+1, \frac{x}{2}\right) + \left(-2+i(x+2)\sqrt{7}+x^2+3x\right)\mathrm{BesselI}\left(\frac{i\sqrt{7}}{2}, \frac{x}{2}\right)\right) + c_2\left(\left(-x^2-2x\right)\mathrm{BesselK}\left(\frac{i\sqrt{7}}{2}+1, \frac{x}{2}\right) + \left(-2+i(x+2)\sqrt{7}+x^2+3x\right)\mathrm{BesselK}\left(\frac{i\sqrt{7}}{2}, \frac{x}{2}\right)\right)\sqrt{x}\,\mathrm{e}^{-\frac{x}{2}}$$

✓ Solution by Mathematica

Time used: 0.085 (sec). Leaf size: 89

```
DSolve[x^2*y''[x]+x^2*y'[x]+2*(1-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}x^{\frac{1}{2}+\frac{i\sqrt{7}}{2}}\left(c_1 \operatorname{HypergeometricU}\left(\frac{5}{2}+\frac{i\sqrt{7}}{2},1+i\sqrt{7},x\right)+c_2 L_{-\frac{1}{2}i\left(-5i+\sqrt{7}\right)}^{i\sqrt{7}}(x)\right)$$

2.42 problem Problem 18(g)

Internal problem ID [11943]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 18(g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{y'' + x^2 y' + 2yx = 2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 62

```
dsolve(diff(y(x),x$2)+x^2*diff(y(x),x)+2*x*y(x)=2*x,y(x), singsol=all)
```

$$y(x) = \frac{x\left(2\sqrt{3}\,\pi - 3\Gamma\left(\frac{1}{3}, -\frac{x^3}{3}\right)\Gamma\left(\frac{2}{3}\right)\right)\mathrm{e}^{-\frac{x^3}{3}} c_1}{\left(-x^3\right)^{\frac{1}{3}}} + \mathrm{e}^{-\frac{x^3}{3}} c_2 + \left(-1 + \mathrm{e}^{\frac{x^3}{3}}\right)\mathrm{e}^{-\frac{x^3}{3}}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 59

```
DSolve[y''[x]+x^2*y'[x]+2*x*y[x]==2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 e^{-\frac{x^3}{3}} + \frac{c_1 e^{-\frac{x^3}{3}}\left(-x^3\right)^{2/3}\Gamma\left(\frac{1}{3}, -\frac{x^3}{3}\right)}{3^{2/3}x^2} + 1$$

2.43 problem Problem 18(h)

Internal problem ID [11944]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 18(h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\ln\left(x^2+1\right) y'' + \frac{4xy'}{x^2+1} + \frac{\left(-x^2+1\right) y}{\left(x^2+1\right)^2} = 0$$

✗ Solution by Maple

```
dsolve(ln(1+x^2)*diff(y(x),x$2)+4*x/(1+x^2)*diff(y(x),x)+(1-x^2)/(1+x^2)^2*y(x)=0,y(x), sing
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[Log[1+x^2]*y''[x]+4*x/(1+x^2)*y'[x]+(1-x^2)/(1+x^2)^2*y[x]==0,y[x],x,IncludeSingularS
```

Not solved

2.44 problem Problem 18(i)

Internal problem ID [11945]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 18(i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{xy'' + x^2y' + 2yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve(x*diff(y(x),x$2)+x^2*diff(y(x),x)+2*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \,\mathrm{e}^{-\frac{x^2}{2}} + c_2 \left(i \mathrm{e}^{-\frac{x^2}{2}} \operatorname{erf}\left(\frac{i\sqrt{2}\,x}{2}\right) \sqrt{2}\,\sqrt{\pi}\,x + 2 \right)$$

✓ Solution by Mathematica

Time used: 0.118 (sec). Leaf size: 69

```
DSolve[x*y''[x]+x^2*y'[x]+2*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\frac{\pi}{2}} c_2 e^{-\frac{x^2}{2}} \sqrt{x^2} \operatorname{erfi}\left(\frac{\sqrt{x^2}}{\sqrt{2}}\right) + \sqrt{2} c_1 e^{-\frac{x^2}{2}} x + c_2$$

2.45 problem Problem 18(j)

Internal problem ID [11946]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 18(j).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{y'' + \sin(x) y' + y \cos(x) = \cos(x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+sin(x)*diff(y(x),x)+cos(x)*y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = \left(c_2 + \int (c_1 + \sin(x))\, \mathrm{e}^{-\cos(x)} dx\right) \mathrm{e}^{\cos(x)}$$

✓ Solution by Mathematica

Time used: 1.199 (sec). Leaf size: 34

```
DSolve[y''[x]+Sin[x]*y'[x]+Cos[x]*y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\cos(x)} \left(\int_1^x e^{-\cos(K[1])} (c_1 + \sin(K[1])) dK[1] + c_2\right)$$

2.46 problem Problem 18(k)

Internal problem ID [11947]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 18(k).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + \cot(x)\, y' - \csc(x)^2\, y = \cos(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+cot(x)*diff(y(x),x)-csc(x)^2*y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = (\cot(x) + \csc(x))\, c_2 + \frac{c_1}{\cot(x) + \csc(x)} - \frac{\cos(x)}{2} + \frac{\csc(x)\, x}{2}$$

✓ Solution by Mathematica

Time used: 0.349 (sec). Leaf size: 45

```
DSolve[y''[x]+Cot[x]*y'[x]-Csc[x]^2*y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(x\csc(x) + \frac{2c_1}{\sqrt{\sin^2(x)}} + \cos(x)\left(-1 - \frac{2ic_2}{\sqrt{\sin^2(x)}}\right)\right)$$

2.47 problem Problem 18(L)

Internal problem ID [11948]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 18(L).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x \ln(x) y'' + 2y' - \frac{y}{x} = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*ln(x)*diff(y(x),x$2)+2*diff(y(x),x)-y(x)/x=1,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\ln(x)} + x + \frac{c_2 x}{\ln(x)}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 23

```
DSolve[x*Log[x]*y''[x]+2*y'[x]-y[x]/x==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x \log(x) + (-1 + c_2)x + c_1}{\log(x)}$$

2.48 problem Problem 19(a)

Internal problem ID [11949]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 19(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _nonlinear], [_2nd_order, _reducible, _m`

$$xy'' + (6y^2x + 1)\, y' + 2y^3 = -1$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x$2)+(6*x*y(x)^2+1)*diff(y(x),x)+2*y(x)^3+1=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]+(6*x*y[x]^2+1)*y'[x]+2*y[x]^3+1==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.49 problem Problem 19(b)

Internal problem ID [11950]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 19(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _nonlinear], [_2nd_order, _with_linear_s`

$$\frac{xy''}{y+1} + \frac{yy' - xy'^2 + y'}{(y+1)^2} = x\sin(x)$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x$2)/(1+y(x))+( y(x)*diff(y(x),x)-x* diff(y(x),x)^2+diff(y(x),x))/( 1+y(x
```

$$y(x) = \mathrm{e}^{-\frac{\pi\ \mathrm{csgn}(x)}{2}} x^{-c_2} \mathrm{e}^{-\sin(x)} \mathrm{e}^{\mathrm{Si}(x)} c_1 - 1$$

✓ Solution by Mathematica

Time used: 1.681 (sec). Leaf size: 28

```
DSolve[x*y''[x]/(1+y[x])+( y[x]*y'[x]-x* y'[x]^2+y'[x])/( 1+y[x])^2==x*Sin[x],y[x],x,Include
```

$$y(x) \to -1 + x^{c_2} e^{\mathrm{Si}(x) - \sin(x) + c_1}$$
$$y(x) \to -1$$

2.50 problem Problem 19(c)

Internal problem ID [11951]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 19(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _nonlinear], [_2nd_order, _reducible, _m`

$$(x\cos(y)+\sin(x))\,y''-xy'^2\sin(y)+2(\cos(y)+\cos(x))\,y'-\sin(x)\,y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve((x*cos(y(x))+sin(x))*diff(y(x),x$2)- x*diff(y(x),x)^2*sin(y(x)) + 2*(cos(y(x))+cos(x)
```

$$-y(x)\sin(x)-x\sin(y(x))-c_1x+c_2=0$$

✓ Solution by Mathematica

Time used: 0.292 (sec). Leaf size: 25

```
DSolve[(x*Cos[y[x]]+Sin[x])*y''[x]- x*y'[x]^2*Sin[y[x]] + 2*(Cos[y[x]]+Cos[x])*y'[x]==y[x]*S
```

$$\text{Solve}\left[\sin(y(x))+\frac{y(x)\sin(x)}{x}-\frac{c_1}{x}=c_2,y(x)\right]$$

2.51 problem Problem 19(d)

Internal problem ID [11952]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 19(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _nonlinear], [_2nd_order, _reducible, _m`

$$\boxed{yy'' \sin (x)+(y \cos (x)+\sin (x) y') y'=\cos (x)}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 119

```
dsolve(y(x)*diff(y(x),x$2)*sin(x)+ ( diff(y(x),x)*sin(x)+y(x)*cos(x) )*diff(y(x),x)=cos(x),y
```

$$y(x) = \sqrt{\sqrt{2}\, \operatorname{csgn}(\sin(x)) \operatorname{arctanh}(\cos(x))\, c_2 - \sqrt{2}\, \operatorname{csgn}(\sin(x)) \operatorname{csgn}(\cos(x))\, c_1 + 2 \operatorname{csgn}(\sin(x)) \left(\int \operatorname{csgn}\right.}$$

$$y(x) = -\sqrt{\sqrt{2}\, \operatorname{csgn}(\sin(x)) \operatorname{arctanh}(\cos(x))\, c_2 - \sqrt{2}\, \operatorname{csgn}(\sin(x)) \operatorname{csgn}(\cos(x))\, c_1 + 2 \operatorname{csgn}(\sin(x)) \left(\int \operatorname{csg}\right.}$$

✓ Solution by Mathematica

Time used: 0.159 (sec). Leaf size: 50

```
DSolve[y[x]*y''[x]*Sin[x]+ ( y'[x]*Sin[x]+y[x]*Cos[x] )*y'[x]==Cos[x],y[x],x,IncludeSingular
```

$$y(x) \to -\sqrt{2}\sqrt{c_1 \operatorname{arctanh}(\cos(x)) + x + c_2}$$
$$y(x) \to \sqrt{2}\sqrt{c_1 \operatorname{arctanh}(\cos(x)) + x + c_2}$$

2.52 problem Problem 19(e)

Internal problem ID [11953]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 19(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _`

$$\boxed{(1-y)\, y''-{y'}^{2}=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve((1-y(x))*diff(y(x),x$2)-diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x)=1$$
$$y(x)=1-\sqrt{2c_1x+2c_2+1}$$
$$y(x)=1+\sqrt{2c_1x+2c_2+1}$$

✓ Solution by Mathematica

Time used: 0.881 (sec). Leaf size: 49

```
DSolve[(1-y[x])*y''[x]- y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to 1-\sqrt{-2c_1x+1-2c_2c_1}$$
$$y(x)\to 1+\sqrt{-2c_1x+1-2c_2c_1}$$

2.53 problem Problem 19(f)

Internal problem ID [11954]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 19(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _nonlinear], [_2nd_order, _reducible, _m`

$$(\cos(y) - y\sin(y))\,y'' - {y'}^2(2\sin(y) + \cos(y)\,y) = \sin(x)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve((cos(y(x))-y(x)*sin(y(x)))*diff(y(x),x$2)- diff(y(x),x)^2* (2*sin(y(x))+y(x)*cos(y(x)
```

$$-y(x)\cos(y(x)) - c_1x - \sin(x) + c_2 = 0$$

✓ Solution by Mathematica

Time used: 0.531 (sec). Leaf size: 28

```
DSolve[(Cos[y[x]]-y[x]*Sin[y[x]])*y''[x]- y'[x]^2* (2*Sin[y[x]]+y[x]*Cos[y[x]])==Sin[x],y[x]
```

$$\text{Solve}\left[\frac{y(x)\cos(y(x))}{x} + \frac{\sin(x)}{x} + \frac{c_1}{x} = c_2, y(x)\right]$$

2.54 problem Problem 20(a)

Internal problem ID [11955]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 20(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{2xy'}{2x-1} - \frac{4xy}{(2x-1)^2} = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 43

```
dsolve(diff(y(x),x$2)+ (2*x)/(2*x-1)*diff(y(x),x)- 4*x/( (2*x-1)^2)*y(x)=0,y(x), singsol=all
```

$$y(x) = \frac{c_1 \operatorname{WhittakerM}\left(-\frac{5}{4}, -\frac{3}{4}, x - \frac{1}{2}\right) e^{-\frac{x}{2}}}{(2x-1)^{\frac{1}{4}}} + \frac{c_2 \operatorname{WhittakerW}\left(-\frac{5}{4}, -\frac{3}{4}, x - \frac{1}{2}\right) e^{-\frac{x}{2}}}{(2x-1)^{\frac{1}{4}}}$$

✓ Solution by Mathematica

Time used: 0.508 (sec). Leaf size: 64

```
DSolve[y''[x]+ (2*x)/(2*x-1)*y'[x]- 4*x/( (2*x-1)^2)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to c_1(2x-1) + \frac{1}{6}c_2\left(\frac{4e^{\frac{1}{2}-x}(x-1)}{\sqrt{2x-1}} + \sqrt{2}(1-2x)\Gamma\left(\frac{1}{2}, x - \frac{1}{2}\right)\right)$$

2.55 problem Problem 20(b)

Internal problem ID [11956]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 20(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(x^2+2x\right)y''+\left(x^2+x+10\right)y'-(25-6x)\,y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 113

```
dsolve((2*x+x^2)*diff(y(x),x$2)+ (10+x+x^2)*diff(y(x),x)=(25-6*x)*y(x),y(x), singsol=all)
```

$$y(x)=c_1(x+2)^7\,\mathrm{e}^{-x}+\frac{c_2\left(88447(x+2)^7\,x^4\mathrm{e}^{-x-2}\,\mathrm{Ei}_1\left(-x-2\right)-11970\,\mathrm{e}^{-x}x^4(x+2)^7\,\mathrm{Ei}_1\left(-x\right)+76477x^{10}+970261x^9+517}{}$$

✓ Solution by Mathematica

Time used: 1.158 (sec). Leaf size: 217

```
DSolve[(2*x+x^2)*y''[x]+ (10+x+x^2)*y'[x]==(25-6*x)*y[x],y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{e^{-x-2}(11970e^2c_2x^4(x+2)^7\,\mathrm{ExpIntegralEi}(x)-88447c_2x^4(x+2)^7\,\mathrm{ExpIntegralEi}(x+2)+e^2(322560c_1}{}$$

2.56 problem Problem 20(c)

Internal problem ID [11957]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 20(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{y'}{x+1} - \frac{(x+2)\,y}{x^2\,(x+1)} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)+diff(y(x),x)/(1+x)-(2+x)/(x^2*(1+x))*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + \frac{c_2(x^2 + 2\ln(x+1) - 2x)}{x}$$

✓ Solution by Mathematica

Time used: 0.064 (sec). Leaf size: 34

```
DSolve[y''[x]+y'[x]/(1+x)-(2+x)/(x^2*(1+x))*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2(x^2 - 2x + 2\log(x+1) - 3) + 2c_1}{2x}$$

2.57 problem Problem 20(d)

Internal problem ID [11958]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 20(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-x)\, y''+\left(2x^2+4x-3\right) y'+8yx=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve((x^2-x)*diff(y(x),x$2)+(2*x^2+4*x-3)*diff(y(x),x)+8*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1}{x^2\,(x-1)^2}+\frac{c_2\mathrm{e}^{-2x}}{(x-1)^2}$$

✓ Solution by Mathematica

Time used: 0.148 (sec). Leaf size: 30

```
DSolve[(x^2-x)*y''[x]+(2*x^2+4*x-3)*y'[x]+8*x*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to\frac{\frac{2c_1}{x^2}+c_2e^{-2x}}{2(x-1)^2}$$

2.58 problem Problem 20(e)

Internal problem ID [11959]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 20(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\frac{(x^2-x)\,y''}{x}+\frac{(3x+1)\,y'}{x}+\frac{y}{x}=3x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 52

```
dsolve((x^2-x)/x*diff(y(x),x$2)+(3*x+1)/x*diff(y(x),x)+y(x)/x=3*x,y(x), singsol=all)
```

$$y(x)=\frac{c_2(2\ln(x)\,x^2+4x-1)}{(x-1)^3}+\frac{c_1x^2}{(x-1)^3}+\frac{x^3(x^2-3x+3)}{3\,(x-1)^3}$$

✓ Solution by Mathematica

Time used: 0.086 (sec). Leaf size: 55

```
DSolve[(x^2-x)/x*y''[x]+(3*x+1)/x*y'[x]+y[x]/x==3*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{2x^5-6x^4+6x^3-6c_1x^2-6c_2x^2\log(x)-12c_2x+3c_2}{6(x-1)^3}$$

2.59 problem Problem 20(f)

Internal problem ID [11960]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 20(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(2\sin(x)-\cos(x))\,y''+(7\sin(x)+4\cos(x))\,y'+10\cos(x)\,y=0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 100

```
dsolve((2*sin(x)-cos(x))*diff(y(x),x$2)+(7*sin(x)+4*cos(x))*diff(y(x),x)+10*y(x)*cos(x)=0,y(
```

$$y(x)=c_1\mathrm{e}^{-\left(\int\frac{5\cos(x)\cot(x)-6\csc(x)}{-2\sin(x)+\cos(x)}dx\right)}+c_2\mathrm{e}^{-\left(\int\frac{5\cos(x)\cot(x)-6\csc(x)}{-2\sin(x)+\cos(x)}dx\right)}\left(\int-\frac{\csc(x)\,\mathrm{e}^{\int\frac{5\cos(x)\cot(x)-6\csc(x)}{-2\sin(x)+\cos(x)}dx}}{-2\sin(x)+\cos(x)}dx\right)$$

✓ Solution by Mathematica

Time used: 3.823 (sec). Leaf size: 112

```
DSolve[(2*Sin[x]-Cos[x])*y''[x]+(7*Sin[x]+4*Cos[x])*y'[x]+10*y[x]*Cos[x]==0,y[x],x,IncludeSi
```

$$y(x)\to\frac{e^{2ix}\left(c_2\int_1^{e^{ix}}\frac{e^{3i\arctan\left(\frac{2-2K[1]^2}{K[1]^2+1}\right)}K[1]^{-2+2i}\left((1+2i)K[1]^2+(1-2i)\right)^4}{\left(5K[1]^4-6K[1]^2+5\right)^{3/2}}dK[1]+c_1\right)}{\left((1+2i)e^{2ix}+(1-2i)\right)^2}$$

2.60 problem Problem 20(g)

Internal problem ID [11961]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 20(g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + \frac{(x-1)\, y'}{x} + \frac{y}{x^3} = \frac{e^{-\frac{1}{x}}}{x^3}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+(x-1)/x*diff(y(x),x)+y(x)/x^3=1/x^3*exp(-1/x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+(x-1)/x*y'[x]+y[x]/x^3==1/x^3*Exp[-1/x],y[x],x,IncludeSingularSolutions -> Tru
```

Not solved

2.61 problem Problem 20(h)

Internal problem ID [11962]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 4, Second and Higher Order Linear Differential Equations. Problems page 221
Problem number: Problem 20(h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + (2x+5)\,y' + (4x+8)\,y = \mathrm{e}^{-2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)+(2*x+5)*diff(y(x),x)+(4*x+8)*y(x)=exp(-2*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x(x+3)} c_2 + \mathrm{e}^{-x(x+3)} \operatorname{erf}\left(ix + \frac{1}{2}i\right) c_1 + \frac{\mathrm{e}^{-2x}}{2}$$

✓ Solution by Mathematica

Time used: 0.333 (sec). Leaf size: 61

```
DSolve[y''[x]+(2*x+5)*y'[x]+(4*x+8)*y[x]==Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} e^{-x(x+3)-\frac{1}{4}} \left(\sqrt{\pi}(-1+2c_2)\operatorname{erfi}\left(x+\frac{1}{2}\right) + 2\left(e^{\left(x+\frac{1}{2}\right)^2} + 2\sqrt[4]{e}c_1\right)\right)$$

3 Chapter 5.5 Laplace transform. Homogeneous equations. Problems page 357

3.1 problem Problem 2

Internal problem ID [11963]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.5 Laplace transform. Homogeneous equations. Problems page 357
Problem number: Problem 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 9y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 10

```
dsolve([diff(y(t),t$2)+9*y(t)=0,y(0) = 2, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = 2\cos(3t)$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 11

```
DSolve[{y''[t]+9*y[t]==0,{y[0]==2,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 2\cos(3t)$$

3.2 problem Problem 3

Internal problem ID [11964]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.5 Laplace transform. Homogeneous equations. Problems page 357
Problem number: Problem 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' - 4y' + 5y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 3]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve([4*diff(y(t),t$2)-4*diff(y(t),t)+5*y(t)=0,y(0) = 2, D(y)(0) = 3],y(t), singsol=all)
```

$$y(t) = 2\,\mathrm{e}^{\frac{t}{2}}(\cos(t) + \sin(t))$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 19

```
DSolve[{4*y''[t]-4*y'[t]+5*y[t]==0,{y[0]==2,y'[0]==3}},y[t],t,IncludeSingularSolutions -> Tr
```

$$y(t) \to 2e^{t/2}(\sin(t) + \cos(t))$$

3.3 problem Problem 4

Internal problem ID [11965]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.5 Laplace transform. Homogeneous equations. Problems page 357
Problem number: Problem 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + y = 0}$$

With initial conditions

$$[y(0) = -1, y'(0) = 2]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+y(t)=0,y(0) = -1, D(y)(0) = 2],y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-t}(t - 1)$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 14

```
DSolve[{y''[t]+2*y'[t]+y[t]==0,{y[0]==-1,y'[0]==2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t}(t - 1)$$

3.4 problem Problem 5

Internal problem ID [11966]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.5 Laplace transform. Homogeneous equations. Problems page 357
Problem number: Problem 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y' + 5y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(t),t$2)-4*diff(y(t),t)+5*y(t)=0,y(0) = 0, D(y)(0) = 3],y(t), singsol=all)
```

$$y(t) = 3\,e^{2t}\sin(t)$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 14

```
DSolve[{y''[t]-4*y'[t]+5*y[t]==0,{y[0]==0,y'[0]==3}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to 3e^{2t}\sin(t)$$

3.5 problem Problem 6

Internal problem ID [11967]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.5 Laplace transform. Homogeneous equations. Problems page 357
Problem number: Problem 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' - 6y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 13

```
dsolve([diff(y(t),t$2)-diff(y(t),t)-6*y(t)=0,y(0) = 2, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = \left(e^{5t} + 1\right) e^{-2t}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 16

```
DSolve[{y''[t]-y'[t]-6*y[t]==0,{y[0]==2,y'[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-2t} + e^{3t}$$

3.6 problem Problem 7

Internal problem ID [11968]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.5 Laplace transform. Homogeneous equations. Problems page 357
Problem number: Problem 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' - 4y' + 37y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = -3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve([4*diff(y(t),t$2)-4*diff(y(t),t)+37*y(t)=0,y(0) = 2, D(y)(0) = -3],y(t), singsol=all)
```

$$y(t) = -\frac{2\,e^{\frac{t}{2}}(2\sin(3t) - 3\cos(3t))}{3}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 29

```
DSolve[{4*y''[t]-4*y'[t]+37*y[t]==0,{y[0]==2,y'[0]==-3}},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \to \frac{2}{3}e^{t/2}(3\cos(3t) - 2\sin(3t))$$

3.7 problem Problem 8

Internal problem ID [11969]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.5 Laplace transform. Homogeneous equations. Problems page 357
Problem number: Problem 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 3y' + 2y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)+3*diff(y(t),t)+2*y(t)=0,y(0) = 2, D(y)(0) = 3],y(t), singsol=all)
```

$$y(t) = -5\,\mathrm{e}^{-2t} + 7\,\mathrm{e}^{-t}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 18

```
DSolve[{y''[t]+3*y'[t]+2*y[t]==0,{y[0]==2,y'[0]==3}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to e^{-2t}(7e^t - 5)$$

3.8 problem Problem 9

Internal problem ID [11970]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.5 Laplace transform. Homogeneous equations. Problems page 357
Problem number: Problem 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + 5y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 13

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+5*y(t)=0,y(0) = 1, D(y)(0) = -1],y(t), singsol=all)
```

$$y(t) = \cos(2t)\,\mathrm{e}^{-t}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 15

```
DSolve[{y''[t]+2*y'[t]+5*y[t]==0,{y[0]==1,y'[0]==-1}},y[t],t,IncludeSingularSolutions -> Tru
```

$$y(t) \to e^{-t}\cos(2t)$$

3.9 problem Problem 10

Internal problem ID [11971]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.5 Laplace transform. Homogeneous equations. Problems page 357
Problem number: Problem 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' - 12y' + 13y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 3]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve([4*diff(y(t),t$2)-12*diff(y(t),t)+13*y(t)=0,y(0) = 2, D(y)(0) = 3],y(t), singsol=all)
```

$$y(t) = 2\,\mathrm{e}^{\frac{3t}{2}} \cos(t)$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 16

```
DSolve[{4*y''[t]-12*y'[t]+13*y[t]==0,{y[0]==2,y'[0]==3}},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \to 2e^{3t/2} \cos(t)$$

3.10 problem Problem 11

Internal problem ID [11972]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.5 Laplace transform. Homogeneous equations. Problems page 357
Problem number: Problem 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y' + 13y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -6]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve([diff(y(t),t$2)+4*diff(y(t),t)+13*y(t)=0,y(0) = 1, D(y)(0) = -6],y(t), singsol=all)
```

$$y(t) = -\frac{e^{-2t}(4\sin(3t) - 3\cos(3t))}{3}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 27

```
DSolve[{y''[t]+4*y'[t]+13*y[t]==0,{y[0]==1,y'[0]==-6}},y[t],t,IncludeSingularSolutions -> Tr
```

$$y(t) \to \frac{1}{3}e^{-2t}(3\cos(3t) - 4\sin(3t))$$

3.11 problem Problem 12

Internal problem ID [11973]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.5 Laplace transform. Homogeneous equations. Problems page 357
Problem number: Problem 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 6y' + 9y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 8

```
dsolve([diff(y(t),t$2)+6*diff(y(t),t)+9*y(t)=0,y(0) = 1, D(y)(0) = -3],y(t), singsol=all)
```

$$y(t) = e^{-3t}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 10

```
DSolve[{y''[t]+6*y'[t]+9*y[t]==0,{y[0]==1,y'[0]==-3}},y[t],t,IncludeSingularSolutions -> Tru
```

$$y(t) \to e^{-3t}$$

3.12 problem Problem 13

Internal problem ID [11974]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.5 Laplace transform. Homogeneous equations. Problems page 357
Problem number: Problem 13.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + y = 0}$$

With initial conditions

$$\left[y(0) = 1, y'(0) = 0, y''(0) = 0, y'''(0) = \frac{\sqrt{2}}{2}\right]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 55

```
dsolve([diff(y(t),t$4)+y(t)=0,y(0) = 1, D(y)(0) = 0, (D@@2)(y)(0) = 0, (D@@3)(y)(0) = 1/2*2^
```

$$y(t) = \frac{\left(3\,\mathrm{e}^{-\frac{\sqrt{2}\,t}{2}} + \mathrm{e}^{\frac{\sqrt{2}\,t}{2}}\right)\cos\left(\frac{\sqrt{2}\,t}{2}\right)}{4} + \frac{\sin\left(\frac{\sqrt{2}\,t}{2}\right)\left(\mathrm{e}^{-\frac{\sqrt{2}\,t}{2}} + \mathrm{e}^{\frac{\sqrt{2}\,t}{2}}\right)}{4}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 61

```
DSolve[{y''''[t]+y[t]==0,{y[0]==0,y'[0]==0,y''[0]==0,y'''[0]==1/Sqrt[2]}},y[t],t,IncludeSing
```

$$y(t) \to \frac{1}{4}e^{-\frac{t}{\sqrt{2}}}\left(\left(e^{\sqrt{2}t} + 1\right)\sin\left(\frac{t}{\sqrt{2}}\right) - \left(e^{\sqrt{2}t} - 1\right)\cos\left(\frac{t}{\sqrt{2}}\right)\right)$$

3.13 problem Problem 14

Internal problem ID [11975]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.5 Laplace transform. Homogeneous equations. Problems page 357
Problem number: Problem 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + 5y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(t),t$2)-2*diff(y(t),t)+5*y(t)=0,y(0) = 0, D(y)(0) = -1],y(t), singsol=all)
```

$$y(t) = -\frac{e^t \sin(2t)}{2}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 14

```
DSolve[{y''[t]-2*y'[t]+5*+y[t]==0,{y[0]==0,y'[0]==-1}},y[t],t,IncludeSingularSolutions -> Tr
```

$$y(t) \to -e^t \sin(t) \cos(t)$$

3.14 problem Problem 15

Internal problem ID [11976]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.5 Laplace transform. Homogeneous equations. Problems page 357
Problem number: Problem 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 20y' + 51y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = -14]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(t),t$2)-20*diff(y(t),t)+51*y(t)=0,y(0) = 0, D(y)(0) = -14],y(t), singsol=all)
```

$$y(t) = e^{3t} - e^{17t}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 18

```
DSolve[{y''[t]-20*y'[t]+51*+y[t]==0,{y[0]==0,y'[0]==-14}},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \to e^{3t} - e^{17t}$$

3.15 problem Problem 16

Internal problem ID [11977]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.5 Laplace transform. Homogeneous equations. Problems page 357
Problem number: Problem 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2y'' + 3y' + y = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve([2*diff(y(t),t$2)+3*diff(y(t),t)+y(t)=0,y(0) = 3, D(y)(0) = -1],y(t), singsol=all)
```

$$y(t) = 4\,\mathrm{e}^{-\frac{t}{2}} - \mathrm{e}^{-t}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 22

```
DSolve[{2*y''[t]+3*y'[t]+y[t]==0,{y[0]==3,y'[0]==-1}},y[t],t,IncludeSingularSolutions -> Tru
```

$$y(t) \to e^{-t}\left(4e^{t/2} - 1\right)$$

3.16 problem Problem 17

Internal problem ID [11978]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.5 Laplace transform. Homogeneous equations. Problems page 357
Problem number: Problem 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{3y'' + 8y' - 3y = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = -4]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve([3*diff(y(t),t$2)+8*diff(y(t),t)-3*y(t)=0,y(0) = 3, D(y)(0) = -4],y(t), singsol=all)
```

$$y(t) = \frac{3\left(e^{\frac{10t}{3}} + 1\right) e^{-3t}}{2}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 23

```
DSolve[{3*y''[t]+8*y'[t]-3*y[t]==0,{y[0]==3,y'[0]==-4}},y[t],t,IncludeSingularSolutions -> T
```

$$y(t) \to \frac{3}{2} e^{-3t} \left(e^{10t/3} + 1\right)$$

3.17 problem Problem 18

Internal problem ID [11979]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.5 Laplace transform. Homogeneous equations. Problems page 357
Problem number: Problem 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2y'' + 20y' + 51y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -5]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve([2*diff(y(t),t$2)+20*diff(y(t),t)+51*y(t)=0,y(0) = 1, D(y)(0) = -5],y(t), singsol=all
```

$$y(t) = \mathrm{e}^{-5t} \cos\left(\frac{\sqrt{2}\, t}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 19

```
DSolve[{2*y''[t]+20*y'[t]+51*y[t]==0,{y[0]==1,y'[0]==-5}},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \to e^{-5t} \cos\left(\frac{t}{\sqrt{2}}\right)$$

3.18 problem Problem 19

Internal problem ID [11980]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.5 Laplace transform. Homogeneous equations. Problems page 357
Problem number: Problem 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' + 40y' + 101y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -5]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve([4*diff(y(t),t$2)+40*diff(y(t),t)+101*y(t)=0,y(0) = 1, D(y)(0) = -5],y(t), singsol=al
```

$$y(t) = \mathrm{e}^{-5t} \cos\left(\frac{t}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 17

```
DSolve[{4*y''[t]+40*y'[t]+101*y[t]==0,{y[0]==1,y'[0]==-5}},y[t],t,IncludeSingularSolutions -
```

$$y(t) \to e^{-5t} \cos\left(\frac{t}{2}\right)$$

3.19 problem Problem 20

Internal problem ID [11981]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.5 Laplace transform. Homogeneous equations. Problems page 357
Problem number: Problem 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 6y' + 34y = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 22

```
dsolve([diff(y(t),t$2)+6*diff(y(t),t)+34*y(t)=0,y(0) = 3, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = e^{-3t}(3\cos(5t) + 2\sin(5t))$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 24

```
DSolve[{y''[t]+6*y'[t]+34*y[t]==0,{y[0]==3,y'[0]==1}},y[t],t,IncludeSingularSolutions -> Tru
```

$$y(t) \to e^{-3t}(2\sin(5t) + 3\cos(5t))$$

3.20 problem Problem 21

Internal problem ID [11982]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.5 Laplace transform. Homogeneous equations. Problems page 357
Problem number: Problem 21.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 8y'' + 16y' = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 1, y''(0) = -8]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(t),t$3)+8*diff(y(t),t$2)+16*diff(y(t),t)=0,y(0) = 1, D(y)(0) = 1, (D@@2)(y)(0
```

$$y(t) = t\,\mathrm{e}^{-4t} + 1$$

✓ Solution by Mathematica

Time used: 0.083 (sec). Leaf size: 14

```
DSolve[{y'''[t]+8*y''[t]+16*y'[t]==0,{y[0]==1,y'[0]==1,y''[0]==-8}},y[t],t,IncludeSingularSo
```

$$y(t) \to e^{-4t}t + 1$$

3.21 problem Problem 22

Internal problem ID [11983]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.5 Laplace transform. Homogeneous equations. Problems page 357
Problem number: Problem 22.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 6y'' + 13y' = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 1, y''(0) = -6]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve([diff(y(t),t$3)+6*diff(y(t),t$2)+13*diff(y(t),t)=0,y(0) = 1, D(y)(0) = 1, (D@@2)(y)(0
```

$$y(t) = \frac{e^{-3t} \sin(2t)}{2} + 1$$

✓ Solution by Mathematica

Time used: 0.456 (sec). Leaf size: 17

```
DSolve[{y'''[t]+6*y''[t]+13*y'[t]==0,{y[0]==1,y'[0]==1,y''[0]==-6}},y[t],t,IncludeSingularSo
```

$$y(t) \to e^{-3t} \sin(t) \cos(t) + 1$$

3.22 problem Problem 23

Internal problem ID [11984]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.5 Laplace transform. Homogeneous equations. Problems page 357
Problem number: Problem 23.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 6y'' + 13y' = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 1, y''(0) = 6]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve([diff(y(t),t$3)-6*diff(y(t),t$2)+13*diff(y(t),t)=0,y(0) = 1, D(y)(0) = 1, (D@@2)(y)(0
```

$$y(t) = \frac{e^{3t} \sin(2t)}{2} + 1$$

✓ Solution by Mathematica

Time used: 0.443 (sec). Leaf size: 17

```
DSolve[{y'''[t]-6*y''[t]+13*y'[t]==0,{y[0]==1,y'[0]==1,y''[0]==6}},y[t],t,IncludeSingularSol
```

$$y(t) \to e^{3t} \sin(t) \cos(t) + 1$$

3.23 problem Problem 24

Internal problem ID [11985]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.5 Laplace transform. Homogeneous equations. Problems page 357
Problem number: Problem 24.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 4y'' + 29y' = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 5, y''(0) = -20]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve([diff(y(t),t$3)+4*diff(y(t),t$2)+29*diff(y(t),t)=0,y(0) = 1, D(y)(0) = 5, (D@@2)(y)(0
```

$$y(t) = e^{-2t} \sin(5t) + 1$$

✓ Solution by Mathematica

Time used: 0.58 (sec). Leaf size: 49

```
DSolve[{y'''[t]+4*y''[t]-20*y'[t]==0,{y[0]==1,y'[0]==5,y''[0]==-20}},y[t],t,IncludeSingularS
```

$$y(t) \to \frac{5e^{2(\sqrt{6}-1)t}}{4\sqrt{6}} - \frac{5e^{-2(1+\sqrt{6})t}}{4\sqrt{6}} + 1$$

3.24 problem Problem 25

Internal problem ID [11986]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.5 Laplace transform. Homogeneous equations. Problems page 357
Problem number: Problem 25.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 6y'' + 25y' = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 4, y''(0) = -24]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(t),t$3)+6*diff(y(t),t$2)+25*diff(y(t),t)=0,y(0) = 1, D(y)(0) = 4, (D@@2)(y)(0
```

$$y(t) = e^{-3t} \sin(4t) + 1$$

✓ Solution by Mathematica

Time used: 0.467 (sec). Leaf size: 17

```
DSolve[{y'''[t]+6*y''[t]+25*y'[t]==0,{y[0]==1,y'[0]==4,y''[0]==-24}},y[t],t,IncludeSingularS
```

$$y(t) \to e^{-3t} \sin(4t) + 1$$

3.25 problem Problem 26

Internal problem ID [11987]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.5 Laplace transform. Homogeneous equations. Problems page 357
Problem number: Problem 26.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 6y'' + 10y' = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 3, y''(0) = 8]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 11

```
dsolve([diff(y(t),t$3)-6*diff(y(t),t$2)+10*diff(y(t),t)=0,y(0) = 1, D(y)(0) = 3, (D@@2)(y)(0
```

$$y(t) = e^{3t} \cos(t)$$

✓ Solution by Mathematica

Time used: 0.212 (sec). Leaf size: 13

```
DSolve[{y'''[t]-6*y''[t]+10*y'[t]==0,{y[0]==1,y'[0]==3,y''[0]==8}},y[t],t,IncludeSingularSol
```

$$y(t) \to e^{3t} \cos(t)$$

3.26 problem Problem 27

Internal problem ID [11988]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.5 Laplace transform. Homogeneous equations. Problems page 357
Problem number: Problem 27.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 13y'' + 36y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = -1, y''(0) = 5, y'''(0) = 19]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve([diff(y(t),t$4)+13*diff(y(t),t$2)+36*y(t)=0,y(0) = 0, D(y)(0) = -1, (D@@2)(y)(0) = 5,
```

$$y(t) = \cos(2t) + \sin(2t) - \cos(3t) - \sin(3t)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 26

```
DSolve[{y''''[t]+13*y''[t]+36*y[t]==0,{y[0]==0,y'[0]==-1,y''[0]==5,y'''[0]==19}},y[t],t,Incl
```

$$y(t) \to \sin(2t) - \sin(3t) + \cos(2t) - \cos(3t)$$

4 Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368

4.1 problem Problem 2(a)

Internal problem ID [11989]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 2(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''+2y'+3y=9t}$$

With initial conditions

$$[y(0)=0, y'(0)=1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+3*y(t)=9*t,y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t)=3t+2\,\mathrm{e}^{-t}\cos\left(\sqrt{2}\,t\right)-2$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 13

```
DSolve[{y''[t]+2*y''[t]+3*y[t]==9*t,{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions -> T
```

$$y(t)\to 3t-2\sin(t)$$

4.2 problem Problem 2(b)

Internal problem ID [11990]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 2(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4y'' + 16y' + 17y = 17t - 1}$$

With initial conditions

$$[y(0) = -1, y'(0) = 2]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve([4*diff(y(t),t$2)+16*diff(y(t),t)+17*y(t)=17*t-1,y(0) = -1, D(y)(0) = 2],y(t), singso
```

$$y(t) = t + 2\,\mathrm{e}^{-2t} \sin\left(\frac{t}{2}\right) - 1$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 21

```
DSolve[{4*y''[t]+16*y'[t]+17*y[t]==17*t-1,{y[0]==-1,y'[0]==2}},y[t],t,IncludeSingularSolutio
```

$$y(t) \to t + 2e^{-2t} \sin\left(\frac{t}{2}\right) - 1$$

4.3 problem Problem 2(c)

Internal problem ID [11991]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 2(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4y'' + 5y' + 4y = 3\,\mathrm{e}^{-t}}$$

With initial conditions

$$[y(0) = -1, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 38

```
dsolve([4*diff(y(t),t$2)+5*diff(y(t),t)+4*y(t)=3*exp(-t),y(0) = -1, D(y)(0) = 1],y(t), sings
```

$$y(t) = \frac{2\,\mathrm{e}^{-\frac{5t}{8}}\sqrt{39}\,\sin\left(\frac{\sqrt{39}\,t}{8}\right)}{13} - 2\,\mathrm{e}^{-\frac{5t}{8}}\cos\left(\frac{\sqrt{39}\,t}{8}\right) + \mathrm{e}^{-t}$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 58

```
DSolve[{4*y''[t]+5*y'[t]+4*y[t]==3*Exp[-t],{y[0]==-1,y'[0]==1}},y[t],t,IncludeSingularSoluti
```

$$y(t) \to e^{-t} + 2\sqrt{\frac{3}{13}}e^{-5t/8}\sin\left(\frac{\sqrt{39}t}{8}\right) - 2e^{-5t/8}\cos\left(\frac{\sqrt{39}t}{8}\right)$$

4.4 problem Problem 2(d)

Internal problem ID [11992]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 2(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 4y = e^{2t}t^2}$$

With initial conditions

$$[y(0) = 1, y'(0) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve([diff(y(t),t$2)-4*diff(y(t),t)+4*y(t)=t^2*exp(2*t),y(0) = 1, D(y)(0) = 2],y(t), sings
```

$$y(t) = e^{2t}\left(1 + \frac{t^4}{12}\right)$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 19

```
DSolve[{y''[t]-4*y'[t]+4*y[t]==t^2*Exp[2*t],{y[0]==1,y'[0]==2}},y[t],t,IncludeSingularSoluti
```

$$y(t) \to \frac{1}{12}e^{2t}(t^4 + 12)$$

4.5 problem Problem 2(e)

Internal problem ID [11993]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 2(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 9y = e^{-2t}}$$

With initial conditions

$$\left[y(0) = -\frac{2}{13}, y'(0) = \frac{1}{13}\right]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$2)+9*y(t)=exp(-2*t),y(0) = -2/13, D(y)(0) = 1/13],y(t), singsol=all)
```

$$y(t) = \frac{\sin(3t)}{13} - \frac{3\cos(3t)}{13} + \frac{e^{-2t}}{13}$$

✓ Solution by Mathematica

Time used: 0.242 (sec). Leaf size: 25

```
DSolve[{y''[t]+9*y[t]==Exp[-2*t],{y[0]==-2/13,y'[0]==1/13}},y[t],t,IncludeSingularSolutions
```

$$y(t) \to \frac{1}{13}\left(e^{-2t} + \sin(3t) - 3\cos(3t)\right)$$

4.6 problem Problem 2(f)

Internal problem ID [11994]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 2(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2y'' - 3y' + 17y = 17t - 1}$$

With initial conditions

$$[y(0) = -1, y'(0) = 2]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 36

```
dsolve([2*diff(y(t),t$2)-3*diff(y(t),t)+17*y(t)=17*t-1,y(0) = -1, D(y)(0) = 2],y(t), singsol
```

$$y(t) = \frac{125\,e^{\frac{3t}{4}} \sin\left(\frac{\sqrt{127}\,t}{4}\right)\sqrt{127}}{2159} - \frac{19\,e^{\frac{3t}{4}} \cos\left(\frac{\sqrt{127}\,t}{4}\right)}{17} + t + \frac{2}{17}$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 59

```
DSolve[{2*y''[t]-3*y'[t]+17*y[t]==17*t-1,{y[0]==-1,y'[0]==2}},y[t],t,IncludeSingularSolution
```

$$y(t) \to t + \frac{125e^{3t/4}\sin\left(\frac{\sqrt{127}t}{4}\right)}{17\sqrt{127}} - \frac{19}{17}e^{3t/4}\cos\left(\frac{\sqrt{127}t}{4}\right) + \frac{2}{17}$$

4.7 problem Problem 2(g)

Internal problem ID [11995]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 2(g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y' + y = e^{-t}}$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+y(t)=exp(-t),y(0) = 1, D(y)(0) = -1],y(t), singsol=all
```

$$y(t) = e^{-t}\left(1 + \frac{t^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 19

```
DSolve[{y''[t]+2*y'[t]+y[t]==Exp[-t],{y[0]==1,y'[0]==-1}},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \to \frac{1}{2}e^{-t}(t^2 + 2)$$

4.8 problem Problem 2(h)

Internal problem ID [11996]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 2(h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y' + 5y = t + 2}$$

With initial conditions

$$[y(0) = 4, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve([diff(y(t),t$2)-2*diff(y(t),t)+5*y(t)=2+t,y(0) = 4, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = -\frac{34\,\mathrm{e}^{t}\sin(2t)}{25} + \frac{88\,\mathrm{e}^{t}\cos(2t)}{25} + \frac{t}{5} + \frac{12}{25}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 32

```
DSolve[{y''[t]-2*y'[t]+5*y[t]==2+t,{y[0]==4,y'[0]==1}},y[t],t,IncludeSingularSolutions -> Tr
```

$$y(t) \to \frac{1}{25}\left(5t - 34e^{t}\sin(2t) + 88e^{t}\cos(2t) + 12\right)$$

4.9 problem Problem 2(i)

Internal problem ID [11997]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 2(i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{2y' + y = \mathrm{e}^{-\frac{t}{2}}}$$

With initial conditions

$$[y(0) = -1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve([2*diff(y(t),t)+y(t)=exp(-t/2),y(0) = -1],y(t), singsol=all)
```

$$y(t) = \frac{(t-2)\,\mathrm{e}^{-\frac{t}{2}}}{2}$$

✓ Solution by Mathematica

Time used: 0.088 (sec). Leaf size: 19

```
DSolve[{2*y'[t]+y[t]==Exp[-t/2],{y[0]==-1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}e^{-t/2}(t-2)$$

4.10 problem Problem 2(i)[j]

Internal problem ID [11998]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 2(i)[j].
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 8y' + 20y = \sin(2t)}$$

With initial conditions

$$[y(0) = 1, y'(0) = -4]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve([diff(y(t),t$2)+8*diff(y(t),t)+20*y(t)=sin(2*t),y(0) = 1, D(y)(0) = -4],y(t), singsol
```

$$y(t) = \frac{(33\,\mathrm{e}^{-4t} - 1)\cos(2t)}{32} + \frac{\sin(2t)\,(\mathrm{e}^{-4t} + 1)}{32}$$

✓ Solution by Mathematica

Time used: 0.292 (sec). Leaf size: 40

```
DSolve[{y''[t]+8*y'[t]+20*y[t]==Sin[2*t],{y[0]==1,y'[0]==-4}},y[t],t,IncludeSingularSolution
```

$$y(t) \to \frac{1}{32}e^{-4t}\left(\left(e^{4t} + 1\right)\sin(2t) - \left(e^{4t} - 33\right)\cos(2t)\right)$$

4.11 problem Problem 2(j)[k]

Internal problem ID [11999]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 2(j)[k].
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4y'' - 4y' + y = t^2}$$

With initial conditions

$$[y(0) = -12, y'(0) = 7]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve([4*diff(y(t),t$2)-4*diff(y(t),t)+y(t)=t^2,y(0) = -12, D(y)(0) = 7],y(t), singsol=all)
```

$$y(t) = (17t - 36)\,e^{\frac{t}{2}} + t^2 + 8t + 24$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 26

```
DSolve[{4*y''[t]-4*y'[t]+y[t]==t^2,{y[0]==-12,y'[0]==7}},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \to t^2 + 8t + e^{t/2}(17t - 36) + 24$$

4.12 problem Problem 2(k)[l]

Internal problem ID [12000]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 2(k)[l].
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{2y'' + y' - y = 4\sin(t)}$$

With initial conditions

$$[y(0) = 0, y'(0) = -4]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 25

```
dsolve([2*diff(y(t),t$2)+diff(y(t),t)-y(t)=4*sin(t),y(0) = 0, D(y)(0) = -4],y(t), singsol=al
```

$$y(t) = -\frac{2\,e^{-t}\left(4\,e^{\frac{3t}{2}} - 5 + (\cos(t) + 3\sin(t))\,e^{t}\right)}{5}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 34

```
DSolve[{2*y''[t]+y'[t]-y[t]==4*Sin[t],{y[0]==0,y'[0]==-4}},y[t],t,IncludeSingularSolutions -
```

$$y(t) \to \frac{2}{5}\left(5e^{-t} - 4e^{t/2} - 3\sin(t) - \cos(t)\right)$$

4.13 problem Problem 2(m)

Internal problem ID [12001]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 2(m).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - y = \mathrm{e}^{2t}}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve([diff(y(t),t)-y(t)=exp(2*t),y(0) = 1],y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{2t}$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 10

```
DSolve[{y'[t]-y[t]==Exp[2*t],{y[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{2t}$$

4.14 problem Problem 2(l)[n]

Internal problem ID [12002]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 2(l)[n].
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3y'' + 5y' - 2y = 7\,\mathrm{e}^{-2t}}$$

With initial conditions

$$[y(0) = 3, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve([3*diff(y(t),t$2)+5*diff(y(t),t)-2*y(t)=7*exp(-2*t),y(0) = 3, D(y)(0) = 0],y(t), sing
```

$$y(t) = -\left(-3\,\mathrm{e}^{\frac{7t}{3}} + t\right)\mathrm{e}^{-2t}$$

✓ Solution by Mathematica

Time used: 0.072 (sec). Leaf size: 23

```
DSolve[{3*y''[t]+5*y'[t]-2*y[t]==7*Exp[-2*t],{y[0]==3,y'[0]==0}},y[t],t,IncludeSingularSolut
```

$$y(t) \to 3e^{t/3} - e^{-2t}t$$

4.15 problem Problem 3(a)

Internal problem ID [12003]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 3(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + y = \text{Heaviside}\,(t) - \text{Heaviside}\,(-2 + t)}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
dsolve([diff(y(t),t)+y(t)=Heaviside(t)-Heaviside(t-2),y(0) = 1],y(t), singsol=all)
```

$$y(t) = \text{Heaviside}\,(t) - \text{Heaviside}\,(t-2) + \text{Heaviside}\,(t-2)\,\mathrm{e}^{-t+2} - \mathrm{e}^{-t}\,\text{Heaviside}\,(t) + \mathrm{e}^{-t}$$

✓ Solution by Mathematica

Time used: 0.108 (sec). Leaf size: 31

```
DSolve[{y'[t]+y[t]==UnitStep[t]-UnitStep[t-2],{y[0]==1}},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \to \left\{ \begin{array}{cc} 1 & 0 \leq t \leq 2 \\ e^{2-t} & t > 2 \\ e^{-t} & \text{True} \end{array} \right.$$

4.16 problem Problem 3(b)

Internal problem ID [12004]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 3(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - 2y = 4t(\text{Heaviside}(t) - \text{Heaviside}(-2+t))}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 48

```
dsolve([diff(y(t),t)-2*y(t)=4*t*(Heaviside(t)-Heaviside(t-2)),y(0) = 1],y(t), singsol=all)
```

$$\begin{aligned} y(t) &= 2t\,\text{Heaviside}(t-2) - 2t\,\text{Heaviside}(t) + \text{Heaviside}(t-2) \\ &\quad - \text{Heaviside}(t) - 5\,\text{Heaviside}(t-2)\,\mathrm{e}^{-4+2t} + \text{Heaviside}(t)\,\mathrm{e}^{2t} + \mathrm{e}^{2t} \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.121 (sec). Leaf size: 47

```
DSolve[{y'[t]-2*y[t]==4*t*(UnitStep[t]-UnitStep[t-2]),{y[0]==1}},y[t],t,IncludeSingularSolut
```

$$y(t) \to \left\{ \begin{array}{ll} e^{2t} & t < 0 \\ e^{2t-4}(-5 + 2e^4) & t > 2 \\ -2t + 2e^{2t} - 1 & \text{True} \end{array} \right.$$

4.17 problem Problem 3(c)

Internal problem ID [12005]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 3(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = 24\sin(t)\,(\text{Heaviside}(t) + \text{Heaviside}(t - \pi))}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve([diff(y(t),t$2)+9*y(t)=24*sin(t)*(Heaviside(t)+Heaviside(t-Pi)),y(0) = 0, D(y)(0) = 0
```

$$y(t) = 4\sin(t)^3\,(\text{Heaviside}(t) + \text{Heaviside}(t - \pi))$$

✓ Solution by Mathematica

Time used: 0.089 (sec). Leaf size: 24

```
DSolve[{y''[t]+9*y[t]==24*Sin[t]*(UnitStep[t]+UnitStep[t-Pi]),{y[0]==0,y'[0]==0}},y[t],t,Inc
```

$$y(t) \to 4(\theta(\pi - t)(\theta(t) - 2) + 2)\sin^3(t)$$

4.18 problem Problem 3(d)

Internal problem ID [12006]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 3(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + y = \text{Heaviside}(t) - \text{Heaviside}(t-1)}$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+y(t)=Heaviside(t)-Heaviside(t-1),y(0) = 1, D(y)(0) = -
```

$$y(t) = t\,\text{Heaviside}(t-1)\,\mathrm{e}^{-t+1} + (1 + \text{Heaviside}(t)(-t-1))\,\mathrm{e}^{-t} + \text{Heaviside}(t) - \text{Heaviside}(t-1)$$

✓ Solution by Mathematica

Time used: 0.072 (sec). Leaf size: 43

```
DSolve[{y''[t]+2*y'[t]+y[t]==UnitStep[t]-UnitStep[t-1],{y[0]==1,y'[0]==-1}},y[t],t,IncludeSi
```

$$y(t) \to \begin{cases} e^{-t} & t < 0 \\ 1 - e^{-t}t & 0 \leq t \leq 1 \\ (-1+e)e^{-t}t & \text{True} \end{cases}$$

4.19 problem Problem 3(e)

Internal problem ID [12007]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 3(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' + 2y = 5\cos(t)\left(\text{Heaviside}(t) - \text{Heaviside}\left(t - \frac{\pi}{2}\right)\right)$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 76

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+2*y(t)=5*cos(t)*(Heaviside(t)-Heaviside(t-Pi/2)),y(0)
```

$$\begin{aligned} y(t) = &-\text{Heaviside}\left(t - \frac{\pi}{2}\right)(\cos(t) - 2\sin(t))\,e^{\frac{\pi}{2}-t} \\ &+ (-\cos(t) - 2\sin(t))\,\text{Heaviside}\left(t - \frac{\pi}{2}\right) \\ &+ ((1 - \text{Heaviside}(t))\cos(t) - 3\sin(t)\,\text{Heaviside}(t))\,e^{-t} \\ &+ \text{Heaviside}(t)(\cos(t) + 2\sin(t)) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.095 (sec). Leaf size: 72

```
DSolve[{y''[t]+2*y'[t]+2*y[t]==5*Cos[t]*(UnitStep[t]-UnitStep[t-Pi/2]),{y[0]==1,y'[0]==-1}},
```

$$y(t) \rightarrow \begin{cases} e^{-t}\cos(t) & t < 0 \\ e^{-t}\left(\left(-3 + 2e^{\pi/2}\right)\sin(t) - e^{\pi/2}\cos(t)\right) & 2t > \pi \\ \cos(t) + (2 - 3e^{-t})\sin(t) & \text{True} \end{cases}$$

4.20 problem Problem 3(f)

Internal problem ID [12008]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 3(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 5y' + 6y = 36t(\text{Heaviside}(t) - \text{Heaviside}(t-1))$$

With initial conditions

$$[y(0) = -1, y'(0) = -2]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 67

```
dsolve([diff(y(t),t$2)+5*diff(y(t),t)+6*y(t)=36*t*(Heaviside(t)-Heaviside(t-1)),y(0) = -1, D
```

$$y(t) = 6\left(\left(\left(\left(-t+\frac{5}{6}\right)e^{3t} - \frac{4\,e^3}{3} + \frac{3\,e^{t+2}}{2}\right)\text{Heaviside}(t-1) + \text{Heaviside}(t)\left(t-\frac{5}{6}\right)e^{3t} + \left(\frac{3\,e^t}{2} - \frac{2}{3}\right)\text{Heaviside}(t) - \frac{5\,e^t}{6} + \frac{2}{3}\right)e^{-3t}\right.$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 64

```
DSolve[{y''[t]+5*y'[t]+6*y[t]==36*t*(UnitStep[t]-UnitStep[t-1]),{y[0]==-1,y'[0]==-2}},y[t],t
```

$$y(t) \to \begin{cases} e^{-3t}(4-5e^t) & t < 0 \\ e^{-3t}(-8e^3+4e^t+9e^{t+2}) & t > 1 \\ 6t+4e^{-2t}-5 & \text{True} \end{cases}$$

4.21 problem Problem 3(g)

Internal problem ID [12009]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 3(g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y' + 13y = 39\,\text{Heaviside}(t) - 507(-2+t)\,\text{Heaviside}(-2+t)$$

With initial conditions

$$[y(0) = 3, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 84

```
dsolve([diff(y(t),t$2)+4*diff(y(t),t)+13*y(t)=39*Heaviside(t)-507*(t-2)*Heaviside(t-2),y(0)
```

$$y(t) = -12\,\text{Heaviside}(t-2)\left(\left(\cos(6) + \frac{5\sin(6)}{12}\right)\cos(3t) - \frac{5\sin(3t)\left(\cos(6) - \frac{12\sin(6)}{5}\right)}{12}\right)\mathrm{e}^{-2t+4} + 3(30-13t)\,\text{Heaviside}(t-2) - 3\,\mathrm{e}^{-2t}(\text{Heaviside}(t)-1)\cos(3t) + \frac{(-6\,\text{Heaviside}(t)+7)\sin(3t)\,\mathrm{e}^{-2t}}{3} + 3\,\text{Heaviside}(t)$$

✓ Solution by Mathematica

Time used: 0.097 (sec). Leaf size: 103

```
DSolve[{y''[t]+4*y'[t]+13*y[t]==39*UnitStep[t]-507*(t-2)*UnitStep[t-2],{y[0]==3,y'[0]==1}},y
```

$y(t)$

$$\to \begin{cases} -39t - 12e^{4-2t}\cos(6-3t) - 5e^{4-2t}\sin(6-3t) + \frac{1}{3}e^{-2t}\sin(3t) + 93 & t > 2 \\ \frac{1}{3}e^{-2t}\sin(3t) + 3 & 0 \leq t \leq 2 \\ \frac{1}{3}e^{-2t}(9\cos(3t) + 7\sin(3t)) & \text{True} \end{cases}$$

4.22 problem Problem 3(h)

Internal problem ID [12010]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 3(h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y = 3\,\text{Heaviside}\,(t) - 3\,\text{Heaviside}\,(t-4) + (2t-5)\,\text{Heaviside}\,(t-4)$$

With initial conditions

$$\left[y(0) = \frac{3}{4}, y'(0) = 2\right]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 48

```
dsolve([diff(y(t),t$2)+4*y(t)=3*(Heaviside(t)-Heaviside(t-4))+(2*t-5)*Heaviside(t-4),y(0) =
```

$$y(t) = \sin\,(2t) + \frac{3\cos\,(2t)}{4} - \frac{\text{Heaviside}\,(t-4)\sin\,(2t-8)}{4} + \frac{\text{Heaviside}\,(t-4)\,t}{2} - 2\,\text{Heaviside}\,(t-4) - \frac{3\,\text{Heaviside}\,(t)\cos\,(2t)}{4} + \frac{3\,\text{Heaviside}\,(t)}{4}$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 60

```
DSolve[{y''[t]+4*y[t]==3*(UnitStep[t]-UnitStep[t-4])+(2*t-5)*UnitStep[t-4],{y[0]==3/4,y'[0]=
```

$$y(t) \to \begin{cases} \sin(2t) + \frac{3}{4} & 0 \leq t \leq 4 \\ \frac{3}{4}\cos(2t) + \sin(2t) & t < 0 \\ \frac{1}{4}(2t + \sin(8-2t) + 4\sin(2t) - 5) & \text{True} \end{cases}$$

4.23 problem Problem 3(i)

Internal problem ID [12011]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 3(i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$4y'' + 4y' + 5y = 25t\left(\text{Heaviside}\,(t) - \text{Heaviside}\left(t - \frac{\pi}{2}\right)\right)$$

With initial conditions

$$[y(0) = 2, y'(0) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 85

```
dsolve([4*diff(y(t),t$2)+4*diff(y(t),t)+5*y(t)=25*t*(Heaviside(t)-Heaviside(t-Pi/2)),y(0) =
```

$$\begin{aligned} y(t) = &-\frac{5\left(\left(\pi + \frac{12}{5}\right)\cos(t) - 2\left(\pi - \frac{8}{5}\right)\sin(t)\right)\text{Heaviside}\left(t - \frac{\pi}{2}\right)e^{-\frac{t}{2}+\frac{\pi}{4}}}{4} \\ &+ (4 - 5t)\,\text{Heaviside}\left(t - \frac{\pi}{2}\right) \\ &+ ((4\cos(t) - 3\sin(t))\,\text{Heaviside}\,(t) + 2\cos(t) + 3\sin(t))\,e^{-\frac{t}{2}} \\ &+ \text{Heaviside}\,(t)\,(-4 + 5t) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.085 (sec). Leaf size: 101

```
DSolve[{4*y''[t]+4*y'[t]+5*y[t]==25*t*(UnitStep[t]-UnitStep[t-Pi/2]),{y[0]==2,y'[0]==2}},y[t
```

$y(t)$

$$\to \begin{cases} 5t + 6e^{-t/2}\cos(t) - 4 & t \geq 0 \land 2t \leq \pi \\ e^{-t/2}(2\cos(t) + 3\sin(t)) & t < 0 \\ \frac{1}{4}e^{-t/2}\left(\left(24 - e^{\pi/4}(12 + 5\pi)\right)\cos(t) + 2e^{\pi/4}(-8 + 5\pi)\sin(t)\right) & \text{True} \end{cases}$$

4.24 problem Problem 3(j)

Internal problem ID [12012]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 3(j).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y' + 3y = \text{Heaviside}(t) - \text{Heaviside}(t-1) + \text{Heaviside}(-2+t) - \text{Heaviside}(-3+t)$$

With initial conditions

$$\left[y(0) = -\frac{2}{3}, y'(0) = 1\right]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 117

```
dsolve([diff(y(t),t$2)+4*diff(y(t),t)+3*y(t)=Heaviside(t)-Heaviside(t-1)+Heaviside(t-2)-Heav
```

$$y(t) = \frac{\left(-\frac{1}{3} - e^{2+2t}\,\text{Heaviside}(t-2) + e^{3+2t}\,\text{Heaviside}(t-3) + e^{2t+1}\,\text{Heaviside}(t-1) + \frac{2(\text{Heaviside}(t)-\text{Heaviside}(t}{}\right.}{}$$

✓ Solution by Mathematica

Time used: 0.115 (sec). Leaf size: 199

```
DSolve[{y''[t]+4*y'[t]+3*y[t]==UnitStep[t]-UnitStep[t-1]+UnitStep[t-2]-UnitStep[t-3],{y[0]==
```

$$
y(t) \to \begin{cases} \frac{1}{3}-e^{-t} & 0 \leq t \leq 1 \\ -\frac{1}{6}e^{-3t}(1+3e^{2t}) & t<0 \\ \frac{1}{6}e^{-3t}(-e^3-6e^{2t}+3e^{2t+1}) & 1<t\leq 2 \\ \frac{1}{6}e^{-3t}(-e^3+e^6-6e^{2t}+2e^{3t}+3e^{2t+1}-3e^{2t+2}) & 2<t\leq 3 \\ \frac{1}{6}e^{-3t}(-e^3+e^6-e^9-6e^{2t}+3e^{2t+1}-3e^{2t+2}+3e^{2t+3}) & \text{True} \end{cases}
$$

4.25 problem Problem 4(a)

Internal problem ID [12013]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 4(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$y'' - 2y' = \begin{cases} 4 & 0 \le t < 1 \\ 6 & 1 \le t \end{cases}$$

With initial conditions

$$[y(0) = -6, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 50

```
dsolve([diff(y(t),t$2)-2*diff(y(t),t)=piecewise(0<=t and t<1,4,t>=1,6),y(0) = -6, D(y)(0) =
```

$$y(t) = \frac{\left(\begin{cases} -13 + e^{2t} & t < 0 \\ 3\,e^{2t} - 15 - 4t & t < 1 \\ 3\,e^{2t} - 14 + e^{2t-2} - 6t & 1 \le t \end{cases}\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 68

```
DSolve[{y''[t]-2*y'[t]==Piecewise[{{4,0<=t<1},{6,t>=1}}],{y[0]==-6,y'[0]==1}},y[t],t,Include
```

$$y(t) \to \begin{cases} \frac{1}{2}(-13 + e^{2t}) & t \leq 0 \\ \frac{1}{2}(-4t + 3e^{2t} - 15) & 0 < t \leq 1 \\ \frac{1}{2}(-6t + 3e^{2t} + e^{2t-2} - 14) & \text{True} \end{cases}$$

4.26 problem Problem 4(b)

Internal problem ID [12014]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 4(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 3y' + 2y = \begin{cases} 0 & 0 \leq t < 1 \\ 1 & 1 \leq t < 2 \\ -1 & 2 \leq t \end{cases}$$

With initial conditions

$$[y(0) = 3, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 74

```
dsolve([diff(y(t),t$2)-3*diff(y(t),t)+2*y(t)=piecewise(0<=t and t<1,0,t>=1 and t<2,1,t>=2,-1
```

$$y(t) = -4\,e^{2t} + 7\,e^{t} - \frac{\left(\begin{cases} 0 & t < 1 \\ -1 + 2\,e^{t-1} - e^{2t-2} & t < 2 \\ 1 + 2\,e^{t-1} - e^{2t-2} - 4\,e^{t-2} + 2\,e^{-4+2t} & 2 \leq t \end{cases}\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.068 (sec). Leaf size: 109

```
DSolve[{y''[t]-3*y'[t]+2*y[t]==Piecewise[{{0,0<=t<1},{1,1<=t<2},{-1,t>=2}}],{y[0]==3,y'[0]==
```

$$y(t) \to \begin{cases} e^t(7-4e^t) & t \leq 1 \\ \frac{1}{2}(1-2e^{t-1}+14e^t-8e^{2t}+e^{2t-2}) & 1 < t \leq 2 \\ \frac{1}{2}(-1+4e^{t-2}-2e^{t-1}+14e^t-8e^{2t}-2e^{2t-4}+e^{2t-2}) & \text{True} \end{cases}$$

4.27 problem Problem 4(c)

Internal problem ID [12015]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 4(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 3y' + 2y = \begin{cases} 1 & 0 \leq t < 2 \\ -1 & 2 \leq t \end{cases}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 61

```
dsolve([diff(y(t),t$2)+3*diff(y(t),t)+2*y(t)=piecewise(0<=t and t<2,1,t>=2,-1),y(0) = 0, D(y
```

$$y(t) = -\frac{\left(\begin{cases} 0 & t < 0 \\ -1 + 2\,\mathrm{e}^{-t} - \mathrm{e}^{-2t} & t < 2 \\ 1 + 2\,\mathrm{e}^{-t} - \mathrm{e}^{-2t} - 4\,\mathrm{e}^{-t+2} + 2\,\mathrm{e}^{-2t+4} & 2 \leq t \end{cases}\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.072 (sec). Leaf size: 68

```
DSolve[{y''[t]+3*y'[t]+2*y[t]==Piecewise[{{1,0<=t<2},{-1,t>=2}}],{y[0]==0,y'[0]==0}},y[t],t,
```

$$y(t) \to \begin{cases} 0 & t \leq 0 \\ \frac{1}{2}e^{-2t}(-1+e^t)^2 & 0 < t \leq 2 \\ -\frac{1}{2}e^{-2t}(-1+2e^4+2e^t+e^{2t}-4e^{t+2}) & \text{True} \end{cases}$$

4.28 problem Problem 4(d)

Internal problem ID [12016]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 4(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y = \begin{cases} t & 0 \leq t < \pi \\ -t & \pi \leq t \end{cases}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 34

```
dsolve([diff(y(t),t$2)+y(t)=piecewise(0<=t and t<Pi,t,t>=Pi,-t),y(0) = 0, D(y)(0) = 0],y(t),
```

$$y(t) = \begin{cases} 0 & t < 0 \\ t - \sin(t) & t < \pi \\ -2\cos(t)\pi - 3\sin(t) - t & \pi \leq t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 38

```
DSolve[{y''[t]+y[t]==Piecewise[{{t,0<=t<Pi},{-t,t>=Pi}}],{y[0]==0,y'[0]==0}},y[t],t,IncludeS
```

$$y(t) \to \begin{cases} 0 & t \leq 0 \\ t - \sin(t) & 0 < t \leq \pi \\ -t - 2\pi\cos(t) - 3\sin(t) & \text{True} \end{cases}$$

4.29 problem Problem 4(e)

Internal problem ID [12017]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 4(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y = \begin{cases} 8t & 0 \leq t < \frac{\pi}{2} \\ 8\pi & \frac{\pi}{2} \leq t \end{cases}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 45

```
dsolve([diff(y(t),t$2)+4*y(t)=piecewise(0<=t and t<Pi/2,8*t,t>=Pi/2,8*Pi),y(0) = 0, D(y)(0)
```

$$y(t) = \begin{cases} 0 & t < 0 \\ -\sin(2t) + 2t & t < \frac{\pi}{2} \\ \cos(2t)\pi - 2\sin(2t) + 2\pi & \frac{\pi}{2} \leq t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.068 (sec). Leaf size: 48

```
DSolve[{y''[t]+4*y[t]==Piecewise[{{8*t,0<=t<Pi/2},{8*Pi,t>=Pi/2}}],{y[0]==0,y'[0]==0}},y[t],
```

$$y(t) \to \begin{cases} 0 & t \leq 0 \\ 2t - \sin(2t) & t > 0 \land 2t \leq \pi \\ \pi \cos(2t) - 2\sin(2t) + 2\pi & \text{True} \end{cases}$$

4.30 problem Problem 5(a)

Internal problem ID [12018]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 5(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4\pi^2 y = 3\left(\delta\left(t - \frac{1}{3}\right)\right) - (\delta(t-1))$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
dsolve([diff(y(t),t$2)+(2*Pi)^2*y(t)=3*Dirac(t-1/3)-Dirac(t-1),y(0) = 0, D(y)(0) = 0],y(t),
```

$$y(t) = \frac{\left(-3\sqrt{3}\,\cos(2\pi t) - 3\sin(2\pi t)\right)\text{Heaviside}\left(t - \frac{1}{3}\right) - 2\sin(2\pi t)\,\text{Heaviside}\,(t-1)}{4\pi}$$

✓ Solution by Mathematica

Time used: 0.125 (sec). Leaf size: 49

```
DSolve[{y''[t]+(2*Pi)^2*y[t]==3*DiracDelta[t-1/3]-DiracDelta[t-1],{y[0]==0,y'[0]==0}},y[t],t
```

$$y(t) \to -\frac{2\theta(t-1)\sin(2\pi t) + 3\theta(3t-1)\left(\sin(2\pi t) + \sqrt{3}\cos(2\pi t)\right)}{4\pi}$$

4.31 problem Problem 5(b)

Internal problem ID [12019]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 5(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + 2y = 3(\delta(t-1))}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+2*y(t)=3*Dirac(t-1),y(0) = 0, D(y)(0) = 0],y(t), sings
```

$$y(t) = 3\,\mathrm{e}^{-t+1}\,\mathrm{Heaviside}\,(t-1)\sin\,(t-1)$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 24

```
DSolve[{y''[t]+2*y'[t]+2*y[t]==3*DiracDelta[t-1],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularS
```

$$y(t) \to -3e^{1-t}\theta(t-1)\sin(1-t)$$

4.32 problem Problem 5(c)

Internal problem ID [12020]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 5(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 29y = 5(\delta(t-\pi)) - 5(\delta(-2\pi + t))}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
dsolve([diff(y(t),t$2)+4*diff(y(t),t)+29*y(t)=5*Dirac(t-Pi)-5*Dirac(t-2*Pi),y(0) = 0, D(y)(0
```

$$y(t) = -\mathrm{e}^{-2t+2\pi} \sin(5t) \left(\mathrm{e}^{2\pi} \mathrm{Heaviside}(t-2\pi) + \mathrm{Heaviside}(t-\pi)\right)$$

✓ Solution by Mathematica

Time used: 0.143 (sec). Leaf size: 39

```
DSolve[{y''[t]+4*y'[t]+29*y[t]==5*DiracDelta[t-Pi]-5*DiracDelta[t-2*Pi],{y[0]==0,y'[0]==0}},
```

$$y(t) \to -e^{2\pi-2t}\left(e^{2\pi}\theta(t-2\pi) + \theta(t-\pi)\right)\sin(5t)$$

4.33 problem Problem 5(d)

Internal problem ID [12021]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 5(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 3y' + 2y = 1 - (\delta(t-1))}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
dsolve([diff(y(t),t$2)+3*diff(y(t),t)+2*y(t)=1-Dirac(t-1),y(0) = 0, D(y)(0) = 0],y(t), sings
```

$$y(t) = \frac{e^{-2t}}{2} + \text{Heaviside}\,(t-1)\,e^{-2t+2} - e^{-t+1}\,\text{Heaviside}\,(t-1) + \frac{1}{2} - e^{-t}$$

✓ Solution by Mathematica

Time used: 0.099 (sec). Leaf size: 36

```
DSolve[{y''[t]+3*y'[t]+2*y[t]==1-DiracDelta[t-1],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularS
```

$$y(t) \to \frac{1}{2}e^{-2t}\left(\left(e^t - 1\right)^2 - 2e\left(e^t - e\right)\theta(t-1)\right)$$

4.34 problem Problem 5(e)

Internal problem ID [12022]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 5(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{4y'' + 4y' + y = \mathrm{e}^{-\frac{t}{2}}(\delta(t-1))}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve([4*diff(y(t),t$2)+4*diff(y(t),t)+y(t)=exp(-t/2)*Dirac(t-1),y(0) = 0, D(y)(0) = 0],y(t
```

$$y(t) = \frac{\text{Heaviside}\,(t-1)\,(t-1)\,\mathrm{e}^{-\frac{t}{2}}}{4}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 23

```
DSolve[{4*y''[t]+4*y'[t]+y[t]==Exp[-t/2]*DiracDelta[t-1],{y[0]==0,y'[0]==0}},y[t],t,IncludeS
```

$$y(t) \to \frac{1}{4}e^{-t/2}(t-1)\theta(t-1)$$

4.35 problem Problem 5(f)

Internal problem ID [12023]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 5(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 7y' + 6y = \delta(t-1)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$2)-7*diff(y(t),t)+6*y(t)=Dirac(t-1),y(0) = 0, D(y)(0) = 0],y(t), singsol
```

$$y(t) = \frac{\text{Heaviside}\,(t-1)\left(\mathrm{e}^{-6+6t} - \mathrm{e}^{t-1}\right)}{5}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 29

```
DSolve[{y''[t]-7*y'[t]+6*y[t]==DiracDelta[t-1],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSol
```

$$y(t) \to \frac{1}{5}e^{t-6}\left(e^{5t} - e^5\right)\theta(t-1)$$

4.36 problem Problem 6(a)

Internal problem ID [12024]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 6(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$10Q' + 100Q = \text{Heaviside}\,(t-1) - \text{Heaviside}\,(-2+t)$$

With initial conditions

$$[Q(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
dsolve([10*diff(Q(t),t)+100*Q(t)=Heaviside(t-1)-Heaviside(t-2),Q(0) = 0],Q(t), singsol=all)
```

$$Q(t) = -\frac{\text{Heaviside}\,(t-2)}{100} + \frac{\text{Heaviside}\,(t-2)\,\mathrm{e}^{-10t+20}}{100} + \frac{\text{Heaviside}\,(t-1)}{100} - \frac{\text{Heaviside}\,(t-1)\,\mathrm{e}^{-10t+10}}{100}$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 22

```
DSolve[{10*q'[t]+100*q[t]==UnitStep(t-1)-UnitStep(t-2),{q[0]==0}},q[t],t,IncludeSingularSolu
```

$$q(t) \to \frac{1}{100} e^{-10t} \left(e^{10t} - 1\right) \text{UnitStep}$$

4.37 problem Problem 13(a)

Internal problem ID [12025]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 13(a).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + y'' + 4y' + 4y = 8}$$

With initial conditions

$$[y(0) = 4, y'(0) = -3, y''(0) = -3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve([diff(y(t),t$3)+diff(y(t),t$2)+4*diff(y(t),t)+4*y(t)=8,y(0) = 4, D(y)(0) = -3, (D@@2)
```

$$y(t) = 2 + \cos(2t) + \mathrm{e}^{-t} - \sin(2t)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 22

```
DSolve[{y'''[t]+y''[t]+4*y'[t]+4*y[t]==8,{y[0]==4,y'[0]==-3,y''[0]==-3}},y[t],t,IncludeSingu
```

$$y(t) \to e^{-t} - \sin(2t) + \cos(2t) + 2$$

4.38 problem Problem 13(b)

Internal problem ID [12026]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 13(b).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - 2y'' - y' + 2y = 4t}$$

With initial conditions

$$[y(0) = 2, y'(0) = -2, y''(0) = 4]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$3)-2*diff(y(t),t$2)-diff(y(t),t)+2*y(t)=4*t,y(0) = 2, D(y)(0) = -2, (D@@
```

$$y(t) = 2t + 1 - 3\,e^{t} + 3\,e^{-t} + e^{2t}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 27

```
DSolve[{y'''[t]-2*y''[t]-y'[t]+2*y[t]==4*t,{y[0]==2,y'[0]==-2,y''[0]==4}},y[t],t,IncludeSing
```

$$y(t) \to 2t + 3e^{-t} - 3e^{t} + e^{2t} + 1$$

4.39 problem Problem 13(c)

Internal problem ID [12027]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 13(c).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - y'' + 4y' - 4y = 8\,e^{2t} - 5\,e^{t}}$$

With initial conditions

$$[y(0) = 2, y'(0) = 0, y''(0) = 3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve([diff(y(t),t$3)-diff(y(t),t$2)+4*diff(y(t),t)-4*y(t)=8*exp(2*t)-5*exp(t),y(0) = 2, D(
```

$$y(t) = e^{2t} - e^{t}t + e^{t} - \sin(2t)$$

✓ Solution by Mathematica

Time used: 0.561 (sec). Leaf size: 24

```
DSolve[{y'''[t]-y''[t]+4*y'[t]-4*y[t]==8*Exp[2*t]-5*Exp[t],{y[0]==2,y'[0]==0,y''[0]==3}},y[t
```

$$y(t) \to e^{t}\left(-t + e^{t} + 1\right) - \sin(2t)$$

4.40 problem Problem 13(d)

Internal problem ID [12028]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 13(d).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - 5y'' + y' - y = -t^2 + 2t - 10}$$

With initial conditions

$$[y(0) = 2, y'(0) = 0, y''(0) = 0]$$

✓ Solution by Maple

Time used: 0.453 (sec). Leaf size: 369

```
dsolve([diff(y(t),t$3)-5*diff(y(t),t$2)+diff(y(t),t)-y(t)=2*t-10-t^2,y(0) = 2, D(y)(0) = 0,
```

$$y(t) = \frac{154\left(\left(116 + 6\sqrt{3}\sqrt{26}\right)^{\frac{1}{3}}\sqrt{3}\sqrt{26} + \frac{58\left(116+6\sqrt{3}\sqrt{26}\right)^{\frac{2}{3}}\sqrt{26}\sqrt{3}}{77} + \frac{55\sqrt{3}\sqrt{26}}{14} - \frac{69\left(116+6\sqrt{3}\sqrt{26}\right)^{\frac{1}{3}}}{14} - \frac{234\left(116+6\sqrt{3}\sqrt{2}\right.}{77}\right.}{}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 1009

```
DSolve[{y'''[t]-5*y''[t]+y'[t]-y[t]==2*t-10-t^2,{y[0]==2,y'[0]==0,y''[0]==0}},y[t],t,Include
```

$$y(t) \rightarrow \frac{-\text{Root}\left[\#1^3 - 5\#1^2 + \#1 - 1\&, 2\right]\text{Root}\left[\#1^3 - 5\#1^2 + \#1 - 1\&, 3\right]^2 t^2 + \text{Root}\left[\#1^3 - 5\#1^2 + \#1 -\right.}{}$$

4.41 problem Problem 14(a)

Internal problem ID [12029]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 14(a).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 5y'' + 4y = 12\,\mathrm{Heaviside}\,(t) - 12\,\mathrm{Heaviside}\,(t-1)$$

With initial conditions

$$[y(0) = 0, y'(0) = 0, y''(0) = 0, y'''(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 91

```
dsolve([diff(y(t),t$4)-5*diff(y(t),t$2)+4*y(t)=12*(Heaviside(t)-Heaviside(t-1)),y(0) = 0, D(
```

$$y(t) = 2\,\mathrm{e}^{-2t}\left(\mathrm{e}^{3t-1}\,\mathrm{Heaviside}\,(t-1) - \frac{\mathrm{e}^{4t-2}\,\mathrm{Heaviside}\,(t-1)}{4} + \left(-\frac{\mathrm{e}^{2}}{4} - \frac{3\,\mathrm{e}^{2t}}{2} + \mathrm{e}^{1+t}\right)\mathrm{Heaviside}\,(t-1) - \left(\mathrm{e}^{t} - \frac{3\,\mathrm{e}^{2t}}{2} + \mathrm{e}^{3t} - \frac{\mathrm{e}^{4t}}{4} - \frac{1}{4}\right)\mathrm{Heaviside}\,(t)\right)$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 88

```
DSolve[{y''''[t]-5*y''[t]+4*y[t]==12*(UnitStep[t]-UnitStep[t-1]),{y[0]==0,y'[0]==0,y''[0]==0
```

$$y(t) \to \begin{cases} \frac{1}{2}e^{-2t}(-1+e^t)^4 & 0 \leq t \leq 1 \\ \frac{1}{2}(-1+e)e^{-2(t+1)}(-e^2-e^3+e^{4t}+4e^{t+2}-4e^{3t+1}+e^{4t+1}) & t > 1 \end{cases}$$

4.42 problem Problem 14(b)

Internal problem ID [12030]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 5.6 Laplace transform. Nonhomogeneous equations. Problems page 368
Problem number: Problem 14(b).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' - 16y = 32\,\text{Heaviside}\,(t) - 32\,\text{Heaviside}\,(t-\pi)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0, y''(0) = 0, y'''(0) = 0]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 119

```
dsolve([diff(y(t),t$4)-16*y(t)=32*(Heaviside(t)-Heaviside(t-Pi)),y(0) = 0, D(y)(0) = 0, (D@@
```

$$y(t) = -\frac{\text{Heaviside}\,(t-\pi)\,\mathrm{e}^{-2t+2\pi}}{2} - \frac{\text{Heaviside}\,(t-\pi)\,\mathrm{e}^{2t-2\pi}}{2} + (2-\cos(2t))\,\text{Heaviside}\,(t-\pi) + \left(\cos(2t) + \frac{\mathrm{e}^{-2t}}{2} + \frac{\mathrm{e}^{2t}}{2} - 2\right)\text{Heaviside}\,(t)$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 72

```
DSolve[{y''''[t]-16*y[t]==32*(UnitStep[t]-UnitStep[t-Pi]),{y[0]==0,y'[0]==0,y''[0]==0,y'''[0
```

$$y(t) \to \begin{cases} \frac{1}{2}e^{-2(t+\pi)}(-1+e^{2\pi})(-e^{2\pi}+e^{4t}) & t > \pi \\ \frac{1}{2}(2\cos(2t)+e^{-2t}+e^{2t}-4) & 0 \leq t \leq \pi \end{cases}$$

5 Chapter 6. Introduction to Systems of ODEs. Problems page 408

5.1 problem Problem 1(a)

Internal problem ID [12031]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 6. Introduction to Systems of ODEs. Problems page 408
Problem number: Problem 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{t^2y'' + 3y't + y = t^7}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(t^2*diff(y(t),t$2)+3*t*diff(y(t),t)+y(t)=t^7,y(t), singsol=all)
```

$$y(t) = \frac{c_2}{t} + \frac{t^7}{64} + \frac{c_1 \ln(t)}{t}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 26

```
DSolve[t^2*y''[t]+3*t*y'[t]+y[t]==t^7,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{t^8 + 64c_2 \log(t) + 64c_1}{64t}$$

5.2 problem Problem 1(b)

Internal problem ID [12032]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 6. Introduction to Systems of ODEs. Problems page 408
Problem number: Problem 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{t^2y'' - 6y't + \sin(2t)\, y = \ln(t)}$$

✗ Solution by Maple

```
dsolve(t^2*diff(y(t),t$2)-6*t*diff(y(t),t)+sin(2*t)*y(t)=ln(t),y(t), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[t^2*y''[t]-6*t*y'[t]+Sin[2*t]*y[t]==Log[t],y[t],t,IncludeSingularSolutions -> True]
```

Not solved

5.3 problem Problem 1(c)

Internal problem ID [12033]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 6. Introduction to Systems of ODEs. Problems page 408
Problem number: Problem 1(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 3y' + \frac{y}{t} = t}$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 39

```
dsolve(diff(y(t),t$2)+3*diff(y(t),t)+y(t)/t=t,y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-3t} t \,\mathrm{KummerM}\left(\frac{2}{3}, 2, 3t\right) c_2 + \mathrm{e}^{-3t} t \,\mathrm{KummerU}\left(\frac{2}{3}, 2, 3t\right) c_1 + \frac{t^2}{7} - \frac{t}{14}$$

✓ Solution by Mathematica

Time used: 23.552 (sec). Leaf size: 253

```
DSolve[y''[t]+3*y'[t]+y[t]/t==t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to G_{1,2}^{2,0}\left(3t \,\middle|\, \begin{matrix} \frac{2}{3} \\ 0,1 \end{matrix}\right)\left(\int_1^t -\frac{3\,\text{Hypergeometric1F1}\left(\frac{4}{3},2,-3\right.}{3\,\text{Hypergeometric1F1}\left(\frac{4}{3},2,-3K[2]\right)G_{1,2}^{2,0}\left(3K[2] \,\middle|\, \begin{matrix} \frac{2}{3} \\ 0,1 \end{matrix}\right)+3\,\text{Hypergeometric1F1}\left(\frac{4}{3},2,-3K[2]\right)G} + c_2\right) - 3t\,\text{Hypergeometric1F1}\left(\frac{4}{3},2,-3t\right)\left(\int_1^t \frac{G_{1,2}^{2,0}\left(3K\right.}{-9\,\text{Hypergeometric1F1}\left(\frac{4}{3},2,-3K[1]\right)G_{1,2}^{2,0}\left(3K[1] \,\middle|\, \begin{matrix} \frac{2}{3} \\ 0,1 \end{matrix}\right)-9\,\text{Hypergeometric1F1}\left(\frac{4}{3},2,\right.} + c_1\right)$$

5.4 problem Problem 1(d)

Internal problem ID [12034]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 6. Introduction to Systems of ODEs. Problems page 408
Problem number: Problem 1(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y't - y\ln(t) = \cos(2t)}$$

✗ Solution by Maple

```
dsolve(diff(y(t),t$2)+t*diff(y(t),t)-y(t)*ln(t)=cos(2*t),y(t), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[t]+t*y'[t]-y[t]*Log[t]==Cos[2*t],y[t],t,IncludeSingularSolutions -> True]
```

Not solved

5.5 problem Problem 1(e)

Internal problem ID [12035]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 6. Introduction to Systems of ODEs. Problems page 408
Problem number: Problem 1(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{t^3 y'' - 2ty' + y = t^4}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 120

```
dsolve(t^3*diff(y(t),t$2)-2*t*diff(y(t),t)+y(t)=t^4,y(t), singsol=all)
```

$$\begin{aligned}
y(t) = {} & \mathrm{e}^{-\frac{1}{t}} \left(\mathrm{BesselI}\left(0, \frac{1}{t}\right) + \mathrm{BesselI}\left(1, \frac{1}{t}\right)\right) c_2 \\
& + \mathrm{e}^{-\frac{1}{t}} \left(-\mathrm{BesselK}\left(0, \frac{1}{t}\right) + \mathrm{BesselK}\left(1, \frac{1}{t}\right)\right) c_1 - \left(\left(\mathrm{BesselI}\left(0, \frac{1}{t}\right)\right.\right. \\
& \left. + \mathrm{BesselI}\left(1, \frac{1}{t}\right)\right) \left(\int t \left(-\mathrm{BesselK}\left(0, \frac{1}{t}\right) + \mathrm{BesselK}\left(1, \frac{1}{t}\right)\right) \mathrm{e}^{\frac{1}{t}} dt\right) \\
& + \left(\int t \left(\mathrm{BesselI}\left(0, \frac{1}{t}\right) + \mathrm{BesselI}\left(1, \frac{1}{t}\right)\right) \mathrm{e}^{\frac{1}{t}} dt\right) \left(\mathrm{BesselK}\left(0, \frac{1}{t}\right)\right. \\
& \left.\left. - \mathrm{BesselK}\left(1, \frac{1}{t}\right)\right)\right) \mathrm{e}^{-\frac{1}{t}}
\end{aligned}$$

✓ Solution by Mathematica

Time used: 27.071 (sec). Leaf size: 272

```
DSolve[t^3*y''[t]-2*t*y'[t]+y[t]==t^4,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-1/t}\left(\text{BesselI}\left(0,\frac{1}{t}\right) + \text{BesselI}\left(1,\frac{1}{t}\right)\right)\left(\int_1^t \frac{2e^{\frac{2}{K[1]}}\sqrt{\pi}K[1]^3 G_{1,2}^{2,0}\left(\frac{2}{K[1]}\middle| \begin{array}{c} \frac{1}{2} \\ -1,0 \end{array}\right)}{e^{\frac{1}{K[1]}}\sqrt{\pi}\left(\text{BesselI}\left(0,\frac{1}{K[1]}\right) - \text{BesselI}\left(2,\frac{1}{K[1]}\right)\right)G_{1,2}^{2,0}\left(\frac{2}{K[1]}\middle| \begin{array}{c} \frac{1}{2} \\ -1,0 \end{array}\right) - 2\left(\text{Bes}\right.}\right.$$

5.6 problem Problem 2(a)

Internal problem ID [12036]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 6. Introduction to Systems of ODEs. Problems page 408
Problem number: Problem 2(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + y = 1}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(diff(y(t),t$2)+2*diff(y(t),t)+y(t)=1,y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-t}c_2 + \mathrm{e}^{-t}tc_1 + 1$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 21

```
DSolve[y''[t]+2*y'[t]+y[t]==1,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t}(e^t + c_2t + c_1)$$

5.7 problem Problem 2(b)

Internal problem ID [12037]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 6. Introduction to Systems of ODEs. Problems page 408
Problem number: Problem 2(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y' + 5y = e^t}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(diff(y(t),t$2)-2*diff(y(t),t)+5*y(t)=exp(t),y(t), singsol=all)
```

$$y(t) = e^t \sin(2t)\, c_2 + e^t \cos(2t)\, c_1 + \frac{e^t}{4}$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 33

```
DSolve[y''[t]-2*y'[t]+5*y[t]==Exp[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{4} e^t((1 + 4c_2)\cos(2t) + 4c_1 \sin(2t) + 1)$$

5.8 problem Problem 2(c)

Internal problem ID [12038]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 6. Introduction to Systems of ODEs. Problems page 408
Problem number: Problem 2(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 3y' - 7y = 4}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(t),t$2)-3*diff(y(t),t)-7*y(t)=4,y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{\frac{\left(3+\sqrt{37}\right)t}{2}} c_2 + \mathrm{e}^{-\frac{\left(-3+\sqrt{37}\right)t}{2}} c_1 - \frac{4}{7}$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 43

```
DSolve[y''[t]-3*y'[t]-7*y[t]==4,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^{-\frac{1}{2}\left(\sqrt{37}-3\right)t} + c_2 e^{\frac{1}{2}\left(3+\sqrt{37}\right)t} - \frac{4}{7}$$

5.9 problem Problem 2(d)

Internal problem ID [12039]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 6. Introduction to Systems of ODEs. Problems page 408
Problem number: Problem 2(d).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 3y'' + 3y' + y = 5}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(t),t$3)+3*diff(y(t),t$2)+3*diff(y(t),t)+y(t)=5,y(t), singsol=all)
```

$$y(t) = 5 + c_1 e^{-t} + c_2 t^2 e^{-t} + c_3 e^{-t} t$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 28

```
DSolve[y'''[t]+3*y''[t]+3*y'[t]+y[t]==5,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t}(5e^t + t(c_3 t + c_2) + c_1)$$

5.10 problem Problem 2(e)

Internal problem ID [12040]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 6. Introduction to Systems of ODEs. Problems page 408
Problem number: Problem 2(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3y'' + 5y' - 2y = 3t^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(3*diff(y(t),t$2)+5*diff(y(t),t)-2*y(t)=3*t^2,y(t), singsol=all)
```

$$y(t) = c_2 e^{-2t} + e^{\frac{t}{3}} c_1 - \frac{3t^2}{2} - \frac{15t}{2} - \frac{93}{4}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 38

```
DSolve[3*y''[t]+5*y'[t]-2*y[t]==3*t^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{3}{4}\left(2t^2 + 10t + 31\right) + c_1 e^{t/3} + c_2 e^{-2t}$$

5.11 problem Problem 2(f)

Internal problem ID [12041]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 6. Introduction to Systems of ODEs. Problems page 408
Problem number: Problem 2(f).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - 2y'' + 4y' = \sin(t)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 64

```
dsolve(diff(y(t),t$3)=2*diff(y(t),t$2)-4*diff(y(t),t)+sin(t),y(t), singsol=all)
```

$$y(t) = \frac{e^{t} \cos\left(\sqrt{3}\, t\right) c_1}{4} + \frac{c_1 \sqrt{3}\, e^{t} \sin\left(\sqrt{3}\, t\right)}{4} - \frac{c_2 \sqrt{3}\, e^{t} \cos\left(\sqrt{3}\, t\right)}{4} + \frac{e^{t} \sin\left(\sqrt{3}\, t\right) c_2}{4} + \frac{2\sin(t)}{13} - \frac{3\cos(t)}{13} + c_3$$

✓ Solution by Mathematica

Time used: 1.636 (sec). Leaf size: 82

```
DSolve[y'''[t]==2*y''[t]-4*y'[t]+Sin[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{52}\left(8\sin(t) - 12\cos(t) - 13\left(\sqrt{3}c_1 - c_2\right) e^t \cos\left(\sqrt{3}t\right) + 13c_1 e^t \sin\left(\sqrt{3}t\right) + 13\sqrt{3}c_2 e^t \sin\left(\sqrt{3}t\right)\right) + c_3$$

5.12 problem Problem 3(a)

Internal problem ID [12042]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 6. Introduction to Systems of ODEs. Problems page 408
Problem number: Problem 3(a).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) - 2y$$
$$y' = 3x(t) - 4y$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 35

```
dsolve([diff(x(t),t)=x(t)-2*y(t),diff(y(t),t)=3*x(t)-4*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = \frac{2c_1 e^{-2t}}{3} + e^{-t}c_2$$

$$y(t) = c_1 e^{-2t} + e^{-t}c_2$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 60

```
DSolve[{x'[t]==x[t]-2*y[t],y'[t]==3*x[t]-4*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> T
```

$$x(t) \to e^{-2t}(c_1(3e^t - 2) - 2c_2(e^t - 1))$$
$$y(t) \to e^{-2t}(3c_1(e^t - 1) + c_2(3 - 2e^t))$$

5.13 problem Problem 3(b)

Internal problem ID [12043]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 6. Introduction to Systems of ODEs. Problems page 408
Problem number: Problem 3(b).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = \frac{5x(t)}{4} + \frac{3y}{4}$$
$$y' = \frac{x(t)}{2} - \frac{3y}{2}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 86

```
dsolve([diff(x(t),t)=5/4*x(t)+3/4*y(t),diff(y(t),t)=1/2*x(t)-3/2*y(t)],[x(t), y(t)], singsol
```

$$x(t) = \frac{c_1 \mathrm{e}^{\frac{\left(-1+\sqrt{145}\right)t}{8}}\sqrt{145}}{4} - \frac{c_2 \mathrm{e}^{-\frac{\left(1+\sqrt{145}\right)t}{8}}\sqrt{145}}{4} + \frac{11 c_1 \mathrm{e}^{\frac{\left(-1+\sqrt{145}\right)t}{8}}}{4} + \frac{11 c_2 \mathrm{e}^{-\frac{\left(1+\sqrt{145}\right)t}{8}}}{4}$$

$$y(t) = c_1 \mathrm{e}^{\frac{\left(-1+\sqrt{145}\right)t}{8}} + c_2 \mathrm{e}^{-\frac{\left(1+\sqrt{145}\right)t}{8}}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 161

```
DSolve[{x'[t]==5/4*x[t]+3/4*y[t],y'[t]==1/2*x[t]-3/2*y[t]},{x[t],y[t]},t,IncludeSingularSolu
```

$$x(t) \to \frac{1}{290} e^{-\frac{1}{8}\left(1+\sqrt{145}\right)t} \left(c_1 \left(\left(145 + 11\sqrt{145}\right) e^{\frac{\sqrt{145}t}{4}} + 145 - 11\sqrt{145}\right) + 6\sqrt{145} c_2 \left(e^{\frac{\sqrt{145}t}{4}} - 1\right)\right)$$

$$y(t) \to \frac{1}{290} e^{-\frac{1}{8}\left(1+\sqrt{145}\right)t} \left(4\sqrt{145} c_1 \left(e^{\frac{\sqrt{145}t}{4}} - 1\right) - c_2 \left(\left(11\sqrt{145} - 145\right) e^{\frac{\sqrt{145}t}{4}} - 145 - 11\sqrt{145}\right)\right)$$

5.14 problem Problem 3(c)

Internal problem ID [12044]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 6. Introduction to Systems of ODEs. Problems page 408
Problem number: Problem 3(c).
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= x(t) - 2y \\
y' &= -y + x(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve([diff(x(t),t)-x(t)+2*y(t)=0,diff(y(t),t)+y(t)-x(t)=0],[x(t), y(t)], singsol=all)
```

$$x(t) = c_1 \cos(t) - c_2 \sin(t) + c_1 \sin(t) + c_2 \cos(t)$$

$$y(t) = c_1 \sin(t) + c_2 \cos(t)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 39

```
DSolve[{x'[t]-x[t]+2*y[t]==0,y'[t]+y[t]-x[t]==0},{x[t],y[t]},t,IncludeSingularSolutions -> T
```

$$
\begin{aligned}
x(t) &\to c_1(\sin(t) + \cos(t)) - 2c_2 \sin(t) \\
y(t) &\to c_2 \cos(t) + (c_1 - c_2) \sin(t)
\end{aligned}
$$

5.15 problem Problem 3(d)

Internal problem ID [12045]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 6. Introduction to Systems of ODEs. Problems page 408
Problem number: Problem 3(d).
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= -5x(t) + 2y \\
y' &= -2x(t) + y
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 83

```
dsolve([diff(x(t),t)+5*x(t)-2*y(t)=0,diff(y(t),t)+2*x(t)-y(t)=0],[x(t), y(t)], singsol=all)
```

$$
x(t) = -\frac{c_1 e^{\left(-2+\sqrt{5}\right)t}\sqrt{5}}{2} + \frac{c_2 e^{-\left(2+\sqrt{5}\right)t}\sqrt{5}}{2} + \frac{3c_1 e^{\left(-2+\sqrt{5}\right)t}}{2} + \frac{3c_2 e^{-\left(2+\sqrt{5}\right)t}}{2}
$$

$$
y(t) = c_1 e^{\left(-2+\sqrt{5}\right)t} + c_2 e^{-\left(2+\sqrt{5}\right)t}
$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 148

```
DSolve[{x'[t]+5*x[t]-2*y[t]==0,y'[t]+2*x[t]-y[t]==0},{x[t],y[t]},t,IncludeSingularSolutions
```

$$
\begin{aligned}
x(t) &\to \frac{1}{10}e^{-\left(\left(2+\sqrt{5}\right)t\right)}\left(c_1\left(\left(5-3\sqrt{5}\right)e^{2\sqrt{5}t}+5+3\sqrt{5}\right)+2\sqrt{5}c_2\left(e^{2\sqrt{5}t}-1\right)\right) \\
y(t) &\to \frac{1}{10}e^{-\left(\left(2+\sqrt{5}\right)t\right)}\left(c_2\left(\left(5+3\sqrt{5}\right)e^{2\sqrt{5}t}+5-3\sqrt{5}\right)-2\sqrt{5}c_1\left(e^{2\sqrt{5}t}-1\right)\right)
\end{aligned}
$$

5.16 problem Problem 3(e)

Internal problem ID [12046]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 6. Introduction to Systems of ODEs. Problems page 408
Problem number: Problem 3(e).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 3x(t) - 2y$$
$$y' = x(t) - 3y$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 70

```
dsolve([diff(x(t),t)-3*x(t)+2*y(t)=0,diff(y(t),t)-x(t)+3*y(t)=0],[x(t), y(t)], singsol=all)
```

$$x(t) = c_1\sqrt{7}\,\mathrm{e}^{\sqrt{7}\,t} - c_2\sqrt{7}\,\mathrm{e}^{-\sqrt{7}\,t} + 3c_1\mathrm{e}^{\sqrt{7}\,t} + 3c_2\mathrm{e}^{-\sqrt{7}\,t}$$

$$y(t) = c_1\mathrm{e}^{\sqrt{7}\,t} + c_2\mathrm{e}^{-\sqrt{7}\,t}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 144

```
DSolve[{x'[t]-3*x[t]+2*y[t]==0,y'[t]-x[t]+3*y[t]==0},{x[t],y[t]},t,IncludeSingularSolutions
```

$$x(t) \to \frac{1}{14}e^{-\sqrt{7}t}\left(c_1\left(\left(7+3\sqrt{7}\right)e^{2\sqrt{7}t} + 7 - 3\sqrt{7}\right) - 2\sqrt{7}c_2\left(e^{2\sqrt{7}t} - 1\right)\right)$$
$$y(t) \to \frac{1}{14}e^{-\sqrt{7}t}\left(\sqrt{7}c_1\left(e^{2\sqrt{7}t} - 1\right) - c_2\left(\left(3\sqrt{7} - 7\right)e^{2\sqrt{7}t} - 7 - 3\sqrt{7}\right)\right)$$

5.17 problem Problem 3(f)

Internal problem ID [12047]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 6. Introduction to Systems of ODEs. Problems page 408
Problem number: Problem 3(f).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -x(t) + z(t)$$
$$y' = y - x(t)$$
$$z'(t) = -x(t) - 2y + 3z(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 49

```
dsolve([diff(x(t),t)+x(t)-z(t)=0,diff(y(t),t)-y(t)+x(t)=0,diff(z(t),t)+x(t)+2*y(t)-3*z(t)=0]
```

$$x(t) = \frac{c_3 e^{3t}}{4} - c_2 + c_1 + c_2 t$$

$$y(t) = -\frac{c_3 e^{3t}}{8} + c_1 + c_2 t$$

$$z(t) = c_1 + c_2 t + c_3 e^{3t}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 132

```
DSolve[{x'[t]+x[t]-z[t]==0,y'[t]-y[t]+x[t]==0,z'[t]+x[t]+2*y[t]-3*z[t]==0},{x[t],y[t],z[t]},
```

$$x(t) \to \frac{1}{9}\left(-9c_1(t-1) + c_2\left(6t - 2e^{3t} + 2\right) + c_3\left(3t + 2e^{3t} - 2\right)\right)$$

$$y(t) \to \frac{1}{9}\left(-9c_1t + c_2\left(6t + e^{3t} + 8\right) + c_3\left(3t - e^{3t} + 1\right)\right)$$

$$z(t) \to \frac{1}{9}\left(-9c_1t - 2c_2\left(-3t + 4e^{3t} - 4\right) + c_3\left(3t + 8e^{3t} + 1\right)\right)$$

5.18 problem Problem 3(g)

Internal problem ID [12048]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 6. Introduction to Systems of ODEs. Problems page 408
Problem number: Problem 3(g).
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= -\frac{x(t)}{2} + 2y - 3z(t) \\
y' &= y - \frac{z(t)}{2} \\
z'(t) &= -2x(t) + z(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 164

```
dsolve([diff(x(t),t)=-1/2*x(t)+2*y(t)-3*z(t),diff(y(t),t)=y(t)-1/2*z(t),diff(z(t),t)=-2*x(t)
```

$$
x(t) = -\frac{c_2 \mathrm{e}^{\frac{(-3+\sqrt{33})t}{4}} \sqrt{33}}{8} + \frac{c_3 \mathrm{e}^{-\frac{(3+\sqrt{33})t}{4}} \sqrt{33}}{8} + \frac{7c_2 \mathrm{e}^{\frac{(-3+\sqrt{33})t}{4}}}{8} + \frac{7c_3 \mathrm{e}^{-\frac{(3+\sqrt{33})t}{4}}}{8} - c_1 \mathrm{e}^{3t}
$$

$$
y(t) = \frac{c_2 \mathrm{e}^{\frac{(-3+\sqrt{33})t}{4}} \sqrt{33}}{8} - \frac{c_3 \mathrm{e}^{-\frac{(3+\sqrt{33})t}{4}} \sqrt{33}}{8} + \frac{7c_2 \mathrm{e}^{\frac{(-3+\sqrt{33})t}{4}}}{8} + \frac{7c_3 \mathrm{e}^{-\frac{(3+\sqrt{33})t}{4}}}{8} - \frac{c_1 \mathrm{e}^{3t}}{4}
$$

$$
z(t) = c_1 \mathrm{e}^{3t} + c_2 \mathrm{e}^{\frac{(-3+\sqrt{33})t}{4}} + c_3 \mathrm{e}^{-\frac{(3+\sqrt{33})t}{4}}
$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 523

```
DSolve[{x'[t]==-1/2*x[t]+2*y[t]-3*z[t],y'[t]==y[t]-1/2*z[t],z'[t]==-2*x[t]+z[t]},{x[t],y[t],
```

$$
\begin{aligned}
x(t) \rightarrow \frac{1}{264} e^{-\frac{1}{4}\left(3+\sqrt{33}\right)t}\Bigg(c_1\left(\left(88-16\sqrt{33}\right)e^{\frac{\sqrt{33}t}{2}}+88e^{\frac{1}{4}\left(15+\sqrt{33}\right)t}+88+16\sqrt{33}\right)\\
+4c_2\left(\left(3\sqrt{33}-11\right)e^{\frac{\sqrt{33}t}{2}}+22e^{\frac{1}{4}\left(15+\sqrt{33}\right)t}-11-3\sqrt{33}\right)\\
-c_3\left(\left(13\sqrt{33}-77\right)e^{\frac{\sqrt{33}t}{2}}+154e^{\frac{1}{4}\left(15+\sqrt{33}\right)t}-77-13\sqrt{33}\right)\Bigg)
\end{aligned}
$$

$$
y(t) \rightarrow \frac{e^{-\frac{1}{4}\left(3+\sqrt{33}\right)t}\left(-4c_1\left(\left(11+5\sqrt{33}\right)e^{\frac{\sqrt{33}t}{2}}-22e^{\frac{1}{4}\left(15+\sqrt{33}\right)t}+11-5\sqrt{33}\right)+c_2\left(\left(484+92\sqrt{33}\right)e^{\frac{\sqrt{33}t}{2}}+88\right.\right.}{1056}
$$

$$
\begin{aligned}
z(t) \rightarrow -\frac{1}{264} e^{-\frac{1}{4}\left(3+\sqrt{33}\right)t}\Bigg(4c_1\left(\left(3\sqrt{33}-11\right)e^{\frac{\sqrt{33}t}{2}}+22e^{\frac{1}{4}\left(15+\sqrt{33}\right)t}-11-3\sqrt{33}\right)\\
-4c_2\left(\left(11+5\sqrt{33}\right)e^{\frac{\sqrt{33}t}{2}}-22e^{\frac{1}{4}\left(15+\sqrt{33}\right)t}+11-5\sqrt{33}\right)\\
+c_3\left(\left(7\sqrt{33}-55\right)e^{\frac{\sqrt{33}t}{2}}-154e^{\frac{1}{4}\left(15+\sqrt{33}\right)t}-55-7\sqrt{33}\right)\Bigg)
\end{aligned}
$$

6 Chapter 6.4 Reduction to a single ODE. Problems page 415

6.1 problem Problem 4(a)

Internal problem ID [12049]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 6.4 Reduction to a single ODE. Problems page 415
Problem number: Problem 4(a).
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= \frac{y}{2} + \frac{x(t)}{2} \\
y' &= \frac{y}{2} - \frac{x(t)}{2}
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 46

```
dsolve([diff(x(t),t)+diff(y(t),t)=y(t),diff(x(t),t)-diff(y(t),t)=x(t)],[x(t), y(t)], singsol
```

$$
x(t) = -e^{\frac{t}{2}} \left(\cos\left(\frac{t}{2}\right) c_1 - \sin\left(\frac{t}{2}\right) c_2 \right)
$$

$$
y(t) = e^{\frac{t}{2}} \left(c_2 \cos\left(\frac{t}{2}\right) + c_1 \sin\left(\frac{t}{2}\right) \right)
$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 63

```
DSolve[{x'[t]+y'[t]==y[t],x'[t]-y'[t]==x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
x(t) &\to e^{t/2} \left(c_1 \cos\left(\frac{t}{2}\right) + c_2 \sin\left(\frac{t}{2}\right) \right) \\
y(t) &\to e^{t/2} \left(c_2 \cos\left(\frac{t}{2}\right) - c_1 \sin\left(\frac{t}{2}\right) \right)
\end{aligned}
$$

6.2 problem Problem 4(b)

Internal problem ID [12050]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 6.4 Reduction to a single ODE. Problems page 415
Problem number: Problem 4(b).
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= \frac{t}{3} + \frac{2x(t)}{3} + \frac{2y}{3} \\
y' &= \frac{t}{3} - \frac{x(t)}{3} - \frac{y}{3}
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 45

```
dsolve([diff(x(t),t)+2*diff(y(t),t)=t,diff(x(t),t)-diff(y(t),t)=x(t)+y(t)],[x(t), y(t)], sin
```

$$
x(t) = -4t - 6\,\mathrm{e}^{\frac{t}{3}} c_1 - 6 - \frac{t^2}{2} - c_2
$$

$$
y(t) = \frac{t^2}{2} + 3\,\mathrm{e}^{\frac{t}{3}} c_1 + 2t + c_2
$$

✓ Solution by Mathematica

Time used: 0.178 (sec). Leaf size: 87

```
DSolve[{x'[t]+2*y'[t]==t,x'[t]-y'[t]==x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> T
```

$$
\begin{aligned}
x(t) &\to -\frac{t^2}{2} - 4t + c_1\left(2e^{t/3} - 1\right) + 2c_2\left(e^{t/3} - 1\right) - 12 \\
y(t) &\to \frac{t^2}{2} + 2t - c_1e^{t/3} - c_2e^{t/3} + 6 + c_1 + 2c_2
\end{aligned}
$$

6.3 problem Problem 4(c)

Internal problem ID [12051]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 6.4 Reduction to a single ODE. Problems page 415
Problem number: Problem 4(c).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = \frac{6}{5} + \frac{3y}{5} - \frac{3t}{5} + x(t)$$
$$y' = \frac{6}{5} - \frac{2y}{5} + \frac{2t}{5}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 30

```
dsolve([diff(x(t),t)-diff(y(t),t)=x(t)+y(t)-t,2*diff(x(t),t)+3*diff(y(t),t)=2*x(t)+6],[x(t),
```

$$x(t) = -\frac{3}{2} - \frac{3\,\mathrm{e}^{-\frac{2t}{5}}c_2}{7} + c_1\mathrm{e}^{t}$$

$$y(t) = t + \frac{1}{2} + \mathrm{e}^{-\frac{2t}{5}}c_2$$

✓ Solution by Mathematica

Time used: 0.438 (sec). Leaf size: 53

```
DSolve[{x'[t]-y'[t]==x[t]+y[t]-t,2*x'[t]+3*y'[t]==2*x[t]+6},{x[t],y[t]},t,IncludeSingularSol
```

$$x(t) \to \left(c_1 + \frac{3c_2}{7}\right)e^t - \frac{3}{7}c_2e^{-2t/5} - \frac{3}{2}$$
$$y(t) \to t + c_2e^{-2t/5} + \frac{1}{2}$$

6.4 problem Problem 4(d)

Internal problem ID [12052]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 6.4 Reduction to a single ODE. Problems page 415
Problem number: Problem 4(d).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = \frac{2t}{7} + \frac{y}{7}$$
$$y' = -\frac{3t}{7} + \frac{2y}{7}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
dsolve([2*diff(x(t),t)-diff(y(t),t)=t,3*diff(x(t),t)+2*diff(y(t),t)=y(t)],[x(t), y(t)], sing
```

$$x(t) = \frac{t^2}{4} + \frac{3t}{4} + \frac{e^{\frac{2t}{7}} c_2}{2} + c_1$$

$$y(t) = \frac{3t}{2} + \frac{21}{4} + e^{\frac{2t}{7}} c_2$$

✓ Solution by Mathematica

Time used: 0.129 (sec). Leaf size: 60

```
DSolve[{2*x'[t]-y'[t]==t,3*x'[t]+2*y'[t]==y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tr
```

$$x(t) \to \frac{1}{8}\left(2t^2 + 6t + 4c_2 e^{2t/7} + 21 + 8c_1 - 4c_2\right)$$
$$y(t) \to \frac{3t}{2} + c_2 e^{2t/7} + \frac{21}{4}$$

6.5 problem Problem 4(e)

Internal problem ID [12053]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 6.4 Reduction to a single ODE. Problems page 415
Problem number: Problem 4(e).
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= \frac{3t}{4} - \frac{x(t)}{4} - \frac{y}{4} \\
y' &= \frac{5t}{4} - \frac{3x(t)}{4} - \frac{3y}{4}
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
dsolve([5*diff(x(t),t)-3*diff(y(t),t)=x(t)+y(t),3*diff(x(t),t)-diff(y(t),t)=t],[x(t), y(t)],
```

$$
x(t) = \frac{t}{2} - \frac{e^{-t}c_1}{3} - 2 + \frac{t^2}{8} - c_2
$$

$$
y(t) = -\frac{t^2}{8} - e^{-t}c_1 + \frac{3t}{2} + c_2
$$

✓ Solution by Mathematica

Time used: 0.075 (sec). Leaf size: 75

```
DSolve[{5*x'[t]-3*y'[t]==x[t]+y[t],3*x'[t]-y'[t]==t},{x[t],y[t]},t,IncludeSingularSolutions
```

$$
\begin{aligned}
x(t) &\to \frac{1}{8}\left(t^2 + 4t + 2(c_1 + c_2)e^{-t} - 4 + 6c_1 - 2c_2\right) \\
y(t) &\to \frac{1}{8}\left(-t^2 + 12t + 2\left(3(c_1 + c_2)e^{-t} - 6 - 3c_1 + c_2\right)\right)
\end{aligned}
$$

6.6 problem Problem 4(f)

Internal problem ID [12054]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 6.4 Reduction to a single ODE. Problems page 415
Problem number: Problem 4(f).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = \frac{4t}{5} + \frac{4y}{5}$$
$$y' = \frac{t}{5} + \frac{y}{5}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve([diff(x(t),t)-4*diff(y(t),t)=0,2*diff(x(t),t)-3*diff(y(t),t)=y(t)+t],[x(t), y(t)], si
```

$$x(t) = -4t + 4\,\mathrm{e}^{\frac{t}{5}} c_2 + c_1$$

$$y(t) = -t - 5 + \mathrm{e}^{\frac{t}{5}} c_2$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 45

```
DSolve[{x'[t]-4*y'[t]==0,2*x'[t]-3*y'[t]==y[t]+t},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to -4t + 4c_2 e^{t/5} - 20 + c_1 - 4c_2$$
$$y(t) \to -t + c_2 e^{t/5} - 5$$

6.7 problem Problem 4(g)

Internal problem ID [12055]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 6.4 Reduction to a single ODE. Problems page 415
Problem number: Problem 4(g).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = \frac{\sin(t)}{4} + \frac{x(t)}{4} + \frac{y}{4} + \frac{t}{4}$$
$$y' = \frac{\sin(t)}{8} - \frac{3x(t)}{8} - \frac{3y}{8} - \frac{3t}{8}$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 51

```
dsolve([3*diff(x(t),t)+2*diff(y(t),t)=sin(t),diff(x(t),t)-2*diff(y(t),t)=x(t)+y(t)+t],[x(t),
```

$$x(t) = \frac{16\,e^{-\frac{t}{8}}c_1}{3} - \frac{17\cos(t)}{65} - \frac{6\sin(t)}{65} + 8 + 2t - c_2$$

$$y(t) = -8\,e^{-\frac{t}{8}}c_1 + \frac{9\sin(t)}{65} - \frac{7\cos(t)}{65} - 3t + c_2$$

✓ Solution by Mathematica

Time used: 0.358 (sec). Leaf size: 98

```
DSolve[{x'[t]+2*y'[t]==Sin[t],x'[t]-2*y'[t]==x[t]+y[t]+t},{x[t],y[t]},t,IncludeSingularSolut
```

$$x(t) \to -2t - \frac{6\sin(t)}{17} - \frac{7\cos(t)}{17} + 2c_1e^{t/4} + 2c_2e^{t/4} - 8 - c_1 - 2c_2$$
$$y(t) \to t + \frac{3\sin(t)}{17} - \frac{5\cos(t)}{17} - c_1e^{t/4} - c_2e^{t/4} + 4 + c_1 + 2c_2$$

7 Chapter 8.3 Systems of Linear Differential Equations (Variation of Parameters). Problems page 514

7.1 problem Problem 3(a)

Internal problem ID [12056]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 8.3 Systems of Linear Differential Equations (Variation of Parameters). Problems page 514
Problem number: Problem 3(a).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -4x(t) + 9y + 12\,\mathrm{e}^{-t}$$
$$y' = -5x(t) + 2y$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 66

```
dsolve([diff(x(t),t)=-4*x(t)+9*y(t)+12*exp(-t),diff(y(t),t)=-5*x(t)+2*y(t)],[x(t), y(t)], si
```

$$x(t) = \frac{\mathrm{e}^{-t}(6\sin(6t)\,c_1 + 3\sin(6t)\,c_2 + 3\cos(6t)\,c_1 - 6\cos(6t)\,c_2 - 5)}{5}$$

$$y(t) = \frac{\mathrm{e}^{-t}(3\sin(6t)\,c_2 + 3\cos(6t)\,c_1 - 5)}{3}$$

✓ Solution by Mathematica

Time used: 0.095 (sec). Leaf size: 73

```
DSolve[{x'[t]==-4*x[t]+9*y[t]+12*Exp[-t],y'[t]==-5*x[t]+2*y[t]},{x[t],y[t]},t,IncludeSingula
```

$$x(t) \to \frac{1}{2}e^{-t}(2c_1\cos(6t) - (c_1 - 3c_2)\sin(6t) - 2)$$
$$y(t) \to \frac{1}{6}e^{-t}(6c_2\cos(6t) + (3c_2 - 5c_1)\sin(6t) - 10)$$

7.2 problem Problem 3(b)

Internal problem ID [12057]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 8.3 Systems of Linear Differential Equations (Variation of Parameters). Problems page 514
Problem number: Problem 3(b).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -7x(t) + 6y + 6\,\mathrm{e}^{-t}$$
$$y' = -12x(t) + 5y + 37$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 82

```
dsolve([diff(x(t),t)=-7*x(t)+6*y(t)+6*exp(-t),diff(y(t),t)=-12*x(t)+5*y(t)+37],[x(t), y(t)],
```

$$x(t) = 6 + \frac{\mathrm{e}^{-t}(\sin(6t)\,c_1 + \sin(6t)\,c_2 + \cos(6t)\,c_1 - \cos(6t)\,c_2 - 2\sin(6t) - 2\cos(6t) - 2)}{2}$$

$$y(t) = 7 + \mathrm{e}^{-t}(\sin(6t)\,c_2 + \cos(6t)\,c_1 - 2\cos(6t) - 2)$$

✓ Solution by Mathematica

Time used: 0.387 (sec). Leaf size: 72

```
DSolve[{x'[t]==-7*x[t]+6*y[t]+6*Exp[-t],y'[t]==-12*x[t]+5*y[t]+37},{x[t],y[t]},t,IncludeSing
```

$$x(t) \to e^{-t}\left(6e^t + c_1\cos(6t) + (c_2 - c_1)\sin(6t) - 1\right)$$
$$y(t) \to e^{-t}\left(7e^t + c_2\cos(6t) + (c_2 - 2c_1)\sin(6t) - 2\right)$$

7.3 problem Problem 3(c)

Internal problem ID [12058]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 8.3 Systems of Linear Differential Equations (Variation of Parameters). Problems page 514
Problem number: Problem 3(c).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -7x(t) + 10y + 18\,\mathrm{e}^t$$
$$y' = -10x(t) + 9y + 37$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 81

```
dsolve([diff(x(t),t)=-7*x(t)+10*y(t)+18*exp(t),diff(y(t),t)=-10*x(t)+9*y(t)+37],[x(t), y(t)]
```

$$x(t) = 10 + \frac{\mathrm{e}^t(3\sin(6t)\,c_1 + 4\sin(6t)\,c_2 + 4\cos(6t)\,c_1 - 3\cos(6t)\,c_2 - 15\sin(6t) - 20\cos(6t) - 20)}{5}$$

$$y(t) = 7 + \mathrm{e}^t(\sin(6t)\,c_2 + \cos(6t)\,c_1 - 5\cos(6t) - 5)$$

✓ Solution by Mathematica

Time used: 0.622 (sec). Leaf size: 82

```
DSolve[{x'[t]==-7*x[t]+10*y[t]+18*Exp[t],y'[t]==-10*x[t]+9*y[t]+37},{x[t],y[t]},t,IncludeSin
```

$$x(t) \to -4e^t + c_1e^t\cos(6t) - \frac{1}{3}(4c_1 - 5c_2)e^t\sin(6t) + 10$$
$$y(t) \to -5e^t + c_2e^t\cos(6t) - \frac{1}{3}(5c_1 - 4c_2)e^t\sin(6t) + 7$$

7.4 problem Problem 3(d)

Internal problem ID [12059]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 8.3 Systems of Linear Differential Equations (Variation of Parameters). Problems page 514
Problem number: Problem 3(d).
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= -14x(t) + 39y + 78\sinh(t) \\
y' &= -6x(t) + 16y + 6\cosh(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.329 (sec). Leaf size: 84

```
dsolve([diff(x(t),t)=-14*x(t)+39*y(t)+78*sinh(t),diff(y(t),t)=-6*x(t)+16*y(t)+6*cosh(t)],[x(
```

$$
\begin{aligned}
x(t) = {} & \frac{5\,\mathrm{e}^{t}\sin(3t)\,c_2}{2} - \frac{\mathrm{e}^{t}\cos(3t)\,c_2}{2} + \frac{5\,\mathrm{e}^{t}\cos(3t)\,c_1}{2} \\
& + \frac{\mathrm{e}^{t}\sin(3t)\,c_1}{2} + \frac{119\,\mathrm{e}^{-t}}{2} - \frac{105\,\mathrm{e}^{t}}{2} + \cosh(t)
\end{aligned}
$$

$$
y(t) = \mathrm{e}^{t}\sin(3t)\,c_2 + \mathrm{e}^{t}\cos(3t)\,c_1 + 21\,\mathrm{e}^{-t} - 21\,\mathrm{e}^{t}
$$

✓ Solution by Mathematica

Time used: 0.623 (sec). Leaf size: 90

```
DSolve[{x'[t]==-14*x[t]+39*y[t]+78*Sinh[t],y'[t]==-6*x[t]+16*y[t]+6*Cosh[t]},{x[t],y[t]},t,I
```

$$
\begin{aligned}
x(t) &\to 60e^{-t} - 52e^{t} + c_1e^{t}\cos(3t) - (5c_1 - 13c_2)e^{t}\sin(3t) \\
y(t) &\to 21e^{-t} - 21e^{t} + c_2e^{t}\cos(3t) - (2c_1 - 5c_2)e^{t}\sin(3t)
\end{aligned}
$$

7.5 problem Problem 4(a)

Internal problem ID [12060]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 8.3 Systems of Linear Differential Equations (Variation of Parameters). Problems page 514
Problem number: Problem 4(a).
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= 2x(t) + 4y - 2z(t) - 2\sinh(t) \\
y' &= 4x(t) + 2y - 2z(t) + 10\cosh(t) \\
z'(t) &= -x(t) + 3y + z(t) + 5
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 429

```
dsolve([diff(x(t),t)=2*x(t)+4*y(t)-2*z(t)-2*sinh(t),diff(y(t),t)=4*x(t)+2*y(t)-2*z(t)+10*cos
```

$$x(t) = -1 - \frac{3\sinh(4t)\,e^{5t}}{14} - \frac{275\sinh(6t)\,e^{5t}}{1008} + \frac{3\cosh(4t)\,e^{5t}}{14} + \frac{275\cosh(6t)\,e^{5t}}{1008} - \frac{3\sinh(t)}{16} - \frac{45\cosh(t)}{16} - \frac{275\,e^{-2t}\sinh(t)}{224} + \frac{9c_1e^{-2t}}{8} + \frac{c_2e^{2t}}{2} + 2c_3e^{5t} - \frac{275\,e^{-2t}\cosh(t)}{224} + \frac{3\,e^{2t}\sinh(t)}{2} - \frac{3\,e^{2t}\cosh(t)}{2} + \frac{275\,e^{2t}\sinh(3t)}{288} - \frac{3\,e^{-2t}\sinh(3t)}{14} - \frac{275\,e^{2t}\cosh(3t)}{288} - \frac{3\,e^{-2t}\cosh(3t)}{14}$$

$$y(t) = -1 - \frac{\sinh(4t)\,e^{5t}}{14} + \frac{25\sinh(6t)\,e^{5t}}{144} + \frac{\cosh(4t)\,e^{5t}}{14} - \frac{25\cosh(6t)\,e^{5t}}{144} - \frac{\sinh(t)}{16} - \frac{15\cosh(t)}{16} + \frac{25\,e^{-2t}\sinh(t)}{32} - \frac{5c_1e^{-2t}}{8} + \frac{c_2e^{2t}}{2} + 2c_3e^{5t} + \frac{25\,e^{-2t}\cosh(t)}{32} + \frac{e^{2t}\sinh(t)}{2} - \frac{e^{2t}\cosh(t)}{2} - \frac{175\,e^{2t}\sinh(3t)}{288} - \frac{e^{-2t}\sinh(3t)}{14} + \frac{175\,e^{2t}\cosh(3t)}{288} - \frac{e^{-2t}\cosh(3t)}{14}$$

$$z(t) = -\frac{25\,e^{-2t}\sinh(t)}{14} - 3 - \frac{4\,e^{-2t}\sinh(3t)}{7} - \frac{25\,e^{-2t}\cosh(t)}{14} - \frac{4\,e^{-2t}\cosh(3t)}{7} + 4\,e^{2t}\sinh(t) + \frac{25\,e^{2t}\sinh(3t)}{18} - 4\,e^{2t}\cosh(t) - \frac{25\,e^{2t}\cosh(3t)}{18} - \frac{4\sinh(4t)\,e^{5t}}{7} - \frac{25\sinh(6t)\,e^{5t}}{63} + \frac{4\cosh(4t)\,e^{5t}}{7} + \frac{25\cosh(6t)\,e^{5t}}{63} + c_1e^{-2t} + c_2e^{2t} + c_3e^{5t}$$

✓ Solution by Mathematica

Time used: 0.292 (sec). Leaf size: 233

```
DSolve[{x'[t]==2*x[t]+4*y[t]-2*z[t]-2*Sinh[t],y'[t]==4*x[t]+2*y[t]-2*z[t]+10*Cosh[t],z'[t]==
```

$$x(t) \to -\frac{29e^{-t}}{9} - 3e^t + \frac{9}{14}(c_1 - c_2)e^{-2t} + \frac{2}{21}(9c_1 + 5c_2 - 7c_3)e^{5t} + \frac{1}{6}(-3c_1 + c_2 + 4c_3)e^{2t} - 1$$
$$y(t) \to \frac{7e^{-t}}{9} - e^t + \frac{5}{14}(c_2 - c_1)e^{-2t} + \frac{2}{21}(9c_1 + 5c_2 - 7c_3)e^{5t} + \frac{1}{6}(-3c_1 + c_2 + 4c_3)e^{2t} - 1$$
$$z(t) \to -\frac{25e^{-t}}{9} - 4e^t + \frac{4}{7}(c_1 - c_2)e^{-2t} + \frac{1}{21}(9c_1 + 5c_2 - 7c_3)e^{5t} + \frac{1}{3}(-3c_1 + c_2 + 4c_3)e^{2t} - 3$$

7.6 problem Problem 4(b)

Internal problem ID [12061]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 8.3 Systems of Linear Differential Equations (Variation of Parameters). Problems page 514
Problem number: Problem 4(b).
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= 2x(t) + 6y - 2z(t) + 50\,\mathrm{e}^{t} \\
y' &= 6x(t) + 2y - 2z(t) + 21\,\mathrm{e}^{-t} \\
z'(t) &= -x(t) + 6y + z(t) + 9
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 102

```
dsolve([diff(x(t),t)=2*x(t)+6*y(t)-2*z(t)+50*exp(t),diff(y(t),t)=6*x(t)+2*y(t)-2*z(t)+21*exp
```

$$x(t) = 12\,\mathrm{e}^{t} - 1 - 6\,\mathrm{e}^{-t} + c_3\mathrm{e}^{6t} + c_1\mathrm{e}^{-4t} + \frac{2c_2\mathrm{e}^{3t}}{5}$$

$$y(t) = 2\,\mathrm{e}^{t} - 1 + \mathrm{e}^{-t} + c_3\mathrm{e}^{6t} - \frac{2c_1\mathrm{e}^{-4t}}{3} + \frac{2c_2\mathrm{e}^{3t}}{5}$$

$$z(t) = 37\,\mathrm{e}^{t} - 4 - 6\,\mathrm{e}^{-t} + c_3\mathrm{e}^{6t} + c_2\mathrm{e}^{3t} + c_1\mathrm{e}^{-4t}$$

✓ Solution by Mathematica

Time used: 0.2 (sec). Leaf size: 213

```
DSolve[{x'[t]==2*x[t]+6*y[t]-2*z[t]+50*Exp[t],y'[t]==6*x[t]+2*y[t]-2*z[t]+21*Exp[-t],z'[t]==
```

$$x(t) \to -6e^{-t} + 12e^{t} + \frac{3}{5}(c_1 - c_2)e^{-4t} + \frac{1}{15}(16c_1 + 9c_2 - 10c_3)e^{6t} - \frac{2}{3}(c_1 - c_3)e^{3t} - 1$$

$$y(t) \to e^{-t} + 2e^{t} - \frac{2}{5}(c_1 - c_2)e^{-4t} + \frac{1}{15}(16c_1 + 9c_2 - 10c_3)e^{6t} - \frac{2}{3}(c_1 - c_3)e^{3t} - 1$$

$$z(t) \to -6e^{-t} + 37e^{t} + \frac{3}{5}(c_1 - c_2)e^{-4t} + \frac{1}{15}(16c_1 + 9c_2 - 10c_3)e^{6t} - \frac{5}{3}(c_1 - c_3)e^{3t} - 4$$

7.7 problem Problem 4(c)

Internal problem ID [12062]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 8.3 Systems of Linear Differential Equations (Variation of Parameters). Problems page 514
Problem number: Problem 4(c).
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= -2x(t) - 2y + 4z(t) \\
y' &= -2x(t) + y + 2z(t) \\
z'(t) &= -4x(t) - 2y + 6z(t) + e^{2t}
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 89

```
dsolve([diff(x(t),t)=-2*x(t)-2*y(t)+4*z(t),diff(y(t),t)=-2*x(t)+1*y(t)+2*z(t),diff(z(t),t)=-
```

$$x(t) = \frac{3c_2 e^{2t}}{4} + 4\,e^{2t}t - \frac{19\,e^{2t}}{4} + e^t c_3 - \frac{e^{2t}c_1}{2}$$

$$y(t) = \frac{c_2 e^{2t}}{2} + 2\,e^{2t}t - \frac{5\,e^{2t}}{2} + \frac{e^t c_3}{2} + e^{2t}c_1$$

$$z(t) = \left(e^t(5t + c_2 - 5) + c_3\right)e^t$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 118

```
DSolve[{x'[t]==-2*x[t]-2*y[t]+4*z[t],y'[t]==-2*x[t]+y[t]+2*z[t],z'[t]==-4*x[t]-2*y[t]+6*z[t]
```

$$x(t) \to e^t \left(e^t(4t - 4 - 3c_1 - 2c_2 + 4c_3) + 2(2c_1 + c_2 - 2c_3)\right)$$
$$y(t) \to e^t \left(2e^t(t - 1 - c_1 + c_3) + 2c_1 + c_2 - 2c_3\right)$$
$$z(t) \to e^t \left(e^t(5t - 4 - 4c_1 - 2c_2 + 5c_3) + 2(2c_1 + c_2 - 2c_3)\right)$$

7.8 problem Problem 4(d)

Internal problem ID [12063]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 8.3 Systems of Linear Differential Equations (Variation of Parameters). Problems page 514
Problem number: Problem 4(d).
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= 3x(t) - 2y + 3z(t) \\
y' &= x(t) - y + 2z(t) + 2\,\mathrm{e}^{-t} \\
z'(t) &= -2x(t) + 2y - 2z(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 91

```
dsolve([diff(x(t),t)=3*x(t)-2*y(t)+3*z(t),diff(y(t),t)=x(t)-y(t)+2*z(t)+2*exp(-t),diff(z(t),
```

$$
x(t) = -\mathrm{e}^{t} c_1 - c_2 \mathrm{e}^{-2t} - c_3 \mathrm{e}^{t} t - \frac{3\,\mathrm{e}^{t} c_3}{2} + 2\,\mathrm{e}^{-t}
$$

$$
y(t) = \mathrm{e}^{-t} + \frac{\mathrm{e}^{t} c_1}{2} - c_2 \mathrm{e}^{-2t} + \frac{c_3 \mathrm{e}^{t} t}{2} - \mathrm{e}^{t} c_3
$$

$$
z(t) = -2\,\mathrm{e}^{-t} + \mathrm{e}^{t} c_1 + c_2 \mathrm{e}^{-2t} + c_3 \mathrm{e}^{t} t
$$

✓ Solution by Mathematica

Time used: 0.106 (sec). Leaf size: 174

```
DSolve[{x'[t]==3*x[t]-2*y[t]+3*z[t],y'[t]==x[t]-y[t]+2*z[t]+2*Exp[-t],z'[t]==-2*x[t]+2*y[t]-
```

$$x(t) \to \frac{1}{9}e^{-2t}\left(18e^t + e^{3t}(c_1(6t+13) + c_3(6t+7) - 6c_2) - 4c_1 + 6c_2 - 7c_3\right)$$
$$y(t) \to \frac{1}{9}e^{-2t}\left(9e^t + e^{3t}(c_1(4-3t) + c_3(7-3t) + 3c_2) - 4c_1 + 6c_2 - 7c_3\right)$$
$$z(t) \to \frac{1}{9}e^{-2t}\left(-18e^t + 2e^{3t}(-(c_1(3t+2)) - 3c_3t + 3c_2 + c_3) + 4c_1 - 6c_2 + 7c_3\right)$$

7.9 problem Problem 5(a)

Internal problem ID [12064]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 8.3 Systems of Linear Differential Equations (Variation of Parameters). Problems page 514
Problem number: Problem 5(a).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 7x(t) + y - 1 - 6\,\mathrm{e}^t$$
$$y' = -4x(t) + 3y + 4\,\mathrm{e}^t - 3$$

With initial conditions

$$[x(0) = 1, y(0) = -1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 30

```
dsolve([diff(x(t),t) = 7*x(t)+y(t)-1-6*exp(t), diff(y(t),t) = -4*x(t)+3*y(t)+4*exp(t)-3, x(0
```

$$x(t) = -2t\,\mathrm{e}^{5t} + \mathrm{e}^t$$

$$y(t) = 1 + (4t - 2)\,\mathrm{e}^{5t}$$

✓ Solution by Mathematica

Time used: 0.325 (sec). Leaf size: 51

```
DSolve[{x'[t]==7*x[t]+y[t]-1-Exp[t],y'[t]==-4*x[t]+3*y[t]+4*Exp[t]-3},{x[0]==1,y[0]==-1},{x[
```

$$x(t) \to \frac{1}{8}e^t\left(e^{4t}(4t+5)+3\right)$$
$$y(t) \to \frac{1}{4}\left(-e^{5t}(4t+3)-5e^t+4\right)$$

7.10 problem Problem 5(b)

Internal problem ID [12065]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 8.3 Systems of Linear Differential Equations (Variation of Parameters). Problems page 514
Problem number: Problem 5(b).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 3x(t) - 2y + 24\sin(t)$$
$$y' = 9x(t) - 3y + 12\cos(t)$$

With initial conditions

$$[x(0) = 1, y(0) = -1]$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 44

```
dsolve([diff(x(t),t) = 3*x(t)-2*y(t)+24*sin(t), diff(y(t),t) = 9*x(t)-3*y(t)+12*cos(t), x(0)
```

$$x(t) = \cos(3t) - \frac{4\sin(3t)}{3} + 9\sin(t)$$

$$y(t) = \frac{7\cos(3t)}{2} - \frac{\sin(3t)}{2} - \frac{9\cos(t)}{2} + \frac{51\sin(t)}{2}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 50

```
DSolve[{x'[t]==3*x[t]-2*y[t]+24*Sin[t],y'[t]==9*x[t]-3*y[t]+12*Cos[t]},{x[0]==1,y[0]==-1},{x
```

$$x(t) \to 9\sin(t) - \frac{4}{3}\sin(3t) + \cos(3t)$$

$$y(t) \to \frac{1}{2}(51\sin(t) - \sin(3t) - 9\cos(t) + 7\cos(3t))$$

7.11 problem Problem 5(c)

Internal problem ID [12066]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 8.3 Systems of Linear Differential Equations (Variation of Parameters). Problems page 514
Problem number: Problem 5(c).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 7x(t) - 4y + 10\,e^{t}$$
$$y' = 3x(t) + 14y + 6\,e^{2t}$$

With initial conditions

$$[x(0) = 1, y(0) = -1]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 54

```
dsolve([diff(x(t),t) = 7*x(t)-4*y(t)+10*exp(t), diff(y(t),t) = 3*x(t)+14*y(t)+6*exp(2*t), x(
```

$$x(t) = \frac{67\,e^{10t}}{9} - \frac{14\,e^{11t}}{3} - \frac{e^{2t}}{3} - \frac{13\,e^{t}}{9}$$

$$y(t) = -\frac{67\,e^{10t}}{12} + \frac{14\,e^{11t}}{3} - \frac{5\,e^{2t}}{12} + \frac{e^{t}}{3}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 54

```
DSolve[{x'[t]==7*x[t]-4*y[t]+10*Exp[t],y'[t]==3*x[t]+14*y[t]+6*Exp[2*t]},{x[0]==1,y[0]==-1},
```

$$x(t) \to -\frac{1}{9}e^t\left(-40e^{9t} + 18e^{10t} + 13\right)$$

$$y(t) \to \frac{1}{3}e^t\left(-10e^{9t} + 6e^{10t} + 1\right)$$

7.12 problem Problem 5(d)

Internal problem ID [12067]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 8.3 Systems of Linear Differential Equations (Variation of Parameters). Problems page 514
Problem number: Problem 5(d).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -7x(t) + 4y + 6\,\mathrm{e}^{3t}$$
$$y' = -5x(t) + 2y + 6\,\mathrm{e}^{2t}$$

With initial conditions

$$[x(0) = 1, y(0) = -1]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 58

```
dsolve([diff(x(t),t) = -7*x(t)+4*y(t)+6*exp(3*t), diff(y(t),t) = -5*x(t)+2*y(t)+6*exp(2*t),
```

$$x(t) = \frac{6\,\mathrm{e}^{2t}}{5} + \frac{44\,\mathrm{e}^{-3t}}{5} - \frac{46\,\mathrm{e}^{-2t}}{5} + \frac{\mathrm{e}^{3t}}{5}$$

$$y(t) = \frac{44\,\mathrm{e}^{-3t}}{5} - \frac{23\,\mathrm{e}^{-2t}}{2} + \frac{27\,\mathrm{e}^{2t}}{10} - \mathrm{e}^{3t}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 48

```
DSolve[{x'[t]==-7*x[t]+4*y[t]+6*Exp[3*t],y'[t]==-5*x[t]+2*y[t]+6*Exp[2*t]},{x[0]==1,y[0]==-1
```

$$x(t) \to \frac{1}{5} e^{-3t} \left(-16 e^{t} + e^{6t} + 20\right)$$
$$y(t) \to -e^{-3t} \left(4 e^{t} + e^{6t} - 4\right)$$

7.13 problem Problem 6(a)

Internal problem ID [12068]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 8.3 Systems of Linear Differential Equations (Variation of Parameters). Problems page 514
Problem number: Problem 6(a).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -3x(t) - 3y + z(t)$$
$$y' = 2y + 2z(t) + 29\,\mathrm{e}^{-t}$$
$$z'(t) = 5x(t) + y + z(t) + 39\,\mathrm{e}^{t}$$

With initial conditions

$$[x(0) = 1, y(0) = 2, z(0) = 3]$$

✓ Solution by Maple

Time used: 7.391 (sec). Leaf size: 949416

```
dsolve([diff(x(t),t) = -3*x(t)-3*y(t)+z(t), diff(y(t),t) = 2*y(t)+2*z(t)+29*exp(-t), diff(z(
```

Expression too large to display

Expression too large to display

Expression too large to display

✓ Solution by Mathematica

Time used: 0.228 (sec). Leaf size: 3462

```
DSolve[{x'[t]==-3*x[t]-3*y[t]+z[t],y'[t]==2*y[t]+2*z[t]+29*Exp[-t],z'[t]==5*x[t]+y[t]+z[t]+3
```

Too large to display

7.14 problem Problem 6(b)

Internal problem ID [12069]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 8.3 Systems of Linear Differential Equations (Variation of Parameters). Problems page 514
Problem number: Problem 6(b).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 2x(t) + y - z(t) + 5\sin(t)$$
$$y' = y + z(t) - 10\cos(t)$$
$$z'(t) = x(t) + z(t) + 2$$

With initial conditions

$$[x(0) = 1, y(0) = 2, z(0) = 3]$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 71

```
dsolve([diff(x(t),t) = 2*x(t)+y(t)-z(t)+5*sin(t), diff(y(t),t) = y(t)+z(t)-10*cos(t), diff(z
```

$$x(t) = -3\,e^{t}\sin(t) + 4\,e^{t}\cos(t) - 1 - 2\cos(t)$$

$$y(t) = -4\sin(t) + 5\cos(t) + 1 + 3\,e^{t}\sin(t) - 4\,e^{t}\cos(t)$$

$$z(t) = 3\,e^{t}\cos(t) + 4\,e^{t}\sin(t) - 1 + \cos(t) - \sin(t)$$

✓ Solution by Mathematica

Time used: 4.398 (sec). Leaf size: 74

```
DSolve[{x'[t]==2*x[t]+y[t]-z[t]+5*Sin[t],y'[t]==y[t]+z[t]-10*Cos[t],z'[t]==x[t]+z[t]+2},{x[0
```

$$x(t) \to -3e^t \sin(t) + \left(4e^t - 2\right)\cos(t) - 1$$
$$y(t) \to \left(3e^t - 4\right)\sin(t) + \left(5 - 4e^t\right)\cos(t) + 1$$
$$z(t) \to \left(4e^t - 1\right)\sin(t) + \left(3e^t + 1\right)\cos(t) - 1$$

7.15 problem Problem 6(c)

Internal problem ID [12070]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 8.3 Systems of Linear Differential Equations (Variation of Parameters). Problems page 514
Problem number: Problem 6(c).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -3x(t) + 3y + z(t) + 10\cos(t)\sin(t)$$
$$y' = x(t) - 5y - 3z(t) + 10\cos(t)^2 - 5$$
$$z'(t) = -3x(t) + 7y + 3z(t) + 23\,e^t$$

With initial conditions

$$[x(0) = 1, y(0) = 2, z(0) = 3]$$

✓ Solution by Maple

Time used: 0.828 (sec). Leaf size: 132

```
dsolve([diff(x(t),t) = -3*x(t)+3*y(t)+z(t)+5*sin(2*t), diff(y(t),t) = x(t)-5*y(t)-3*z(t)+5*c
```

$$x(t) = -\frac{69\,e^t}{26} + \sin(2t) + \frac{\cos(2t)}{2} + \frac{21\,e^{-t}}{2} - \frac{191\,e^{-2t}\cos(2t)}{26} + \frac{16\,e^{-2t}\sin(2t)}{13}$$

$$y(t) = -\frac{253\,e^t}{26} - \frac{5\sin(2t)}{2} + \frac{21\,e^{-t}}{2} + \frac{16\,e^{-2t}\cos(2t)}{13} + \frac{191\,e^{-2t}\sin(2t)}{26}$$

$$z(t) = \frac{483\,e^t}{26} + \frac{7\cos(2t)}{2} + \frac{9\sin(2t)}{2} - \frac{21\,e^{-t}}{2} - \frac{223\,e^{-2t}\cos(2t)}{26} - \frac{159\,e^{-2t}\sin(2t)}{26}$$

✓ Solution by Mathematica

Time used: 14.393 (sec). Leaf size: 197

```
DSolve[{x'[t]==-3*x[t]+3*y[t]+z[t]+5*Sin[3*t],y'[t]==x[t]-5*y[t]-3*z[t]+5*Cos[2*t],z'[t]==-3
```

$$x(t) \to \frac{1}{754}\left(7540e^{-t} - 2001e^{t} + 603e^{-2t}\sin(2t) + 377\sin(2t) + 429\sin(3t) + \left(1131 - 5409e^{-2t}\right)\cos(2t) - 507\cos(3t)\right)$$

$$y(t) \to \frac{1}{754}\left(7540e^{-t} - 7337e^{t} + 5409e^{-2t}\sin(2t) - 1508\sin(2t) - 507\sin(3t) + \left(603e^{-2t} + 1131\right)\cos(2t) - 429\cos(3t)\right)$$

$$z(t) \to -10e^{-t} + \frac{483e^{t}}{26} - \frac{2403}{377}e^{-2t}\sin(2t) + \frac{43}{58}\sin(3t) + \left(1 - \frac{3006e^{-2t}}{377}\right)\cos(2t) + \frac{81}{58}\cos(3t) + 9\sin(t)\cos(t)$$

7.16 problem Problem 6(d)

Internal problem ID [12071]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 8.3 Systems of Linear Differential Equations (Variation of Parameters). Problems page 514
Problem number: Problem 6(d).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -3x(t) + y - 3z(t) + 2\,e^{t}$$
$$y' = 4x(t) - y + 2z(t) + 4\,e^{t}$$
$$z'(t) = 4x(t) - 2y + 3z(t) + 4\,e^{t}$$

With initial conditions

$$[x(0) = 1, y(0) = 2, z(0) = 3]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 86

```
dsolve([diff(x(t),t) = -3*x(t)+y(t)-3*z(t)+2*exp(t), diff(y(t),t) = 4*x(t)-y(t)+2*z(t)+4*exp
```

$$x(t) = -\frac{3\,e^{t}}{2} - 2\,e^{-t}\sin(2t) + \frac{5\,e^{-t}\cos(2t)}{2}$$

$$y(t) = \frac{5\,e^{t}}{2} + \frac{9\,e^{-t}\sin(2t)}{2} - \frac{e^{-t}\cos(2t)}{2}$$

$$z(t) = \frac{7\,e^{t}}{2} + \frac{9\,e^{-t}\sin(2t)}{2} - \frac{e^{-t}\cos(2t)}{2}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 98

```
DSolve[{x'[t]==-3*x[t]+y[t]-3*z[t]+2*Exp[t],y'[t]==4*x[t]-y[t]+2*z[t]+4*Exp[t],z'[t]==4*x[t]
```

$$
\begin{aligned}
x(t) &\to -\frac{1}{2}e^{-t}\left(3e^{2t}+4\sin(2t)-5\cos(2t)\right)\\
y(t) &\to \frac{1}{2}e^{-t}\left(5e^{2t}+9\sin(2t)-\cos(2t)\right)\\
z(t) &\to \frac{1}{2}e^{-t}\left(7e^{2t}+9\sin(2t)-\cos(2t)\right)
\end{aligned}
$$

8 Chapter 8.4 Systems of Linear Differential Equations (Method of Undetermined Coefficients). Problems page 520

8.1 problem Problem 1(a)

Internal problem ID [12072]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 8.4 Systems of Linear Differential Equations (Method of Undetermined Coefficients). Problems page 520
Problem number: Problem 1(a).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) + 5y + 10\sinh(t)$$
$$y' = 19x(t) - 13y + 24\sinh(t)$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 176

```
dsolve([diff(x(t),t)=x(t)+5*y(t)+10*sinh(t),diff(y(t),t)=19*x(t)-13*y(t)+24*sinh(t)],[x(t),
```

$$x(t) = -\frac{71\sinh(7t)\,e^{6t}}{266} - \frac{7\cosh(5t)\,e^{6t}}{12} + \frac{71\cosh(7t)\,e^{6t}}{266} + \frac{7\sinh(5t)\,e^{6t}}{12} + \frac{71\,e^{-18t}\cosh(17t)}{646} - \frac{35\,e^{-18t}\cosh(19t)}{228} + \frac{71\,e^{-18t}\sinh(17t)}{646} - \frac{35\,e^{-18t}\sinh(19t)}{228} + c_2e^{6t} - \frac{5c_1e^{-18t}}{19} - \frac{24\sinh(t)}{19}$$

$$y(t) = c_2e^{6t} + c_1e^{-18t} + \frac{71\left(\left(-\frac{323\cosh(5t)}{71} + \frac{17\cosh(7t)}{7} + \frac{323\sinh(5t)}{71} - \frac{17\sinh(7t)}{7}\right)e^{24t} + \sinh(17t) - \frac{85\sinh(19t)}{71} + \cosh(17t) - \frac{85\cos}{7}\right.}{408}$$

✓ Solution by Mathematica

Time used: 0.072 (sec). Leaf size: 108

```
DSolve[{x'[t]==x[t]+5*y[t]+10*Sinh[t],y'[t]==19*x[t]-13*y[t]+24*Sinh[t]},{x[t],y[t]},t,Inclu
```

$$x(t) \to \frac{120e^{-t}}{119} - \frac{26e^{t}}{19} + \frac{5}{24}(c_1 - c_2)e^{-18t} + \frac{1}{24}(19c_1 + 5c_2)e^{6t}$$

$$y(t) \to \frac{71e^{-t}}{119} - e^{t} - \frac{19}{24}(c_1 - c_2)e^{-18t} + \frac{1}{24}(19c_1 + 5c_2)e^{6t}$$

8.2 problem Problem 1(b)

Internal problem ID [12073]

Book: APPLIED DIFFERENTIAL EQUATIONS The Primary Course by Vladimir A. Dobrushkin. CRC Press 2015
Section: Chapter 8.4 Systems of Linear Differential Equations (Method of Undetermined Coefficients). Problems page 520
Problem number: Problem 1(b).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 9x(t) - 3y - 6t$$
$$y' = -x(t) + 11y + 10t$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 44

```
dsolve([diff(x(t),t)=9*x(t)-3*y(t)-6*t,diff(y(t),t)=-x(t)+11*y(t)+10*t],[x(t), y(t)], singso
```

$$x(t) = 3\,e^{8t}c_2 - e^{12t}c_1 + \frac{1}{64} + \frac{3t}{8}$$

$$y(t) = e^{8t}c_2 + e^{12t}c_1 - \frac{7t}{8} - \frac{5}{64}$$

✓ Solution by Mathematica

Time used: 0.113 (sec). Leaf size: 78

```
DSolve[{x'[t]==9*x[t]-3*y[t]-6*t,y'[t]==-x[t]+11*y[t]+10*t},{x[t],y[t]},t,IncludeSingularSol
```

$$x(t) \to \frac{1}{64}\left(24t + 16(c_1 - 3c_2)e^{12t} + 48(c_1 + c_2)e^{8t} + 1\right)$$
$$y(t) \to \frac{1}{64}\left(-56t - 16(c_1 - 3c_2)e^{12t} + 16(c_1 + c_2)e^{8t} - 5\right)$$