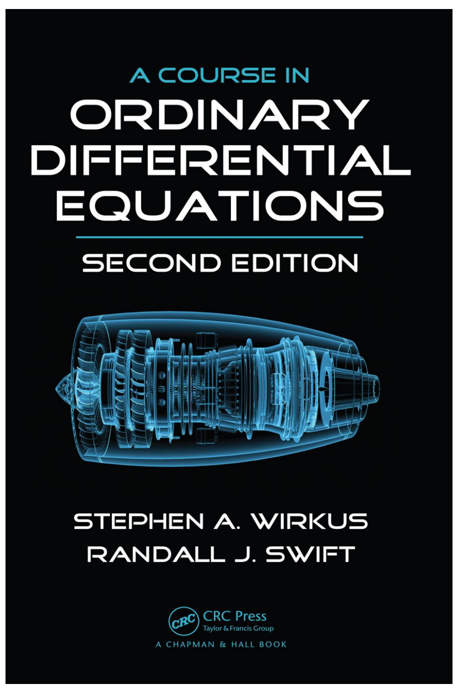
A Solution Manual For

A course in Ordinary Differential Equations. by Stephen A. Wirkus, Randall J. Swift. CRC Press NY. 2015. 2nd Edition



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1.1 problem 1. Using series method

Internal problem ID [6544]

Book: A course in Ordinary Differential Equations. by Stephen A. Wirkus, Randall J. Swift. CRC Press NY. 2015. 2nd Edition
Section: Chapter 8. Series Methods. section 8.2. The Power Series Method. Problems Page 603
Problem number: 1. Using series method.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{-y^2 + y' = -x}$$

With initial conditions

$$[y(0) = 1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=8;
dsolve([diff(y(x),x)=y(x)^2-x,y(0) = 1],y(x),type='series',x=0);
```

$$y(x) = 1 + x + \frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{7}{12}x^4 + \frac{11}{20}x^5 + \frac{22}{45}x^6 + \frac{559}{1260}x^7 + \mathrm{O}\left(x^8\right)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 48

```
AsymptoticDSolveValue[{y'[x]==y[x]^2-x,{y[0]==1}},y[x],{x,0,7}]
```

$$y(x) \to \frac{559x^7}{1260} + \frac{22x^6}{45} + \frac{11x^5}{20} + \frac{7x^4}{12} + \frac{2x^3}{3} + \frac{x^2}{2} + x + 1$$

1.2 problem 1. direct method

Internal problem ID [6545]

Book: A course in Ordinary Differential Equations. by Stephen A. Wirkus, Randall J. Swift. CRC Press NY. 2015. 2nd Edition
Section: Chapter 8. Series Methods. section 8.2. The Power Series Method. Problems Page 603
Problem number: 1. direct method.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{-y^2 + y' = -x}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.171 (sec). Leaf size: 90

```
dsolve([diff(y(x),x)=y(x)^2-x,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{-2\,\mathrm{AiryAi}(1,x)\,3^{\frac{5}{6}}\pi - 3\,\mathrm{AiryAi}(1,x)\,\Gamma\left(\frac{2}{3}\right)^2 3^{\frac{2}{3}} - 3\,\mathrm{AiryBi}(1,x)\,3^{\frac{1}{6}}\Gamma\left(\frac{2}{3}\right)^2 + 2\,\mathrm{AiryBi}(1,x)\,3^{\frac{1}{3}}\pi}{2\,\mathrm{AiryAi}(x)\,3^{\frac{5}{6}}\pi + 3\,\mathrm{AiryAi}(x)\,\Gamma\left(\frac{2}{3}\right)^2 3^{\frac{2}{3}} + 3\,\mathrm{AiryBi}(x)\,3^{\frac{1}{6}}\Gamma\left(\frac{2}{3}\right)^2 - 2\,\mathrm{AiryBi}(x)\,3^{\frac{1}{3}}\pi}$$

✓ Solution by Mathematica

Time used: 7.282 (sec). Leaf size: 164

```
DSolve[{y'[x]==y[x]^2-x,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{-3}\,\mathrm{Gamma}\left(\frac{2}{3}\right)\left(ix^{3/2}\,\mathrm{BesselJ}\left(-\frac{4}{3},\frac{2}{3}ix^{3/2}\right) - ix^{3/2}\,\mathrm{BesselJ}\left(\frac{2}{3},\frac{2}{3}ix^{3/2}\right) + \mathrm{BesselJ}\left(-\frac{1}{3},\frac{2}{3}ix^{3/2}\right)\right) - 2ix^{3}\cdots}{2x\left(\mathrm{Gamma}\left(\frac{1}{3}\right)\mathrm{BesselJ}\left(\frac{1}{3},\frac{2}{3}ix^{3/2}\right) - \sqrt[3]{-3}\,\mathrm{Gamma}\left(\frac{2}{3}\right)\mathrm{BesselJ}\left(-\frac{1}{3},\frac{2}{3}ix\cdots\right.\right.}$$

1.3 problem 2. Using series method

Internal problem ID [6546]

Book: A course in Ordinary Differential Equations. by Stephen A. Wirkus, Randall J. Swift. CRC Press NY. 2015. 2nd Edition
Section: Chapter 8. Series Methods. section 8.2. The Power Series Method. Problems Page 603
Problem number: 2. Using series method.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{-2y + y' = x^2}$$

With initial conditions

$$[y(1) = 1]$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=8;
dsolve([diff(y(x),x)-2*y(x)=x^2,y(1) = 1],y(x),type='series',x=1);
```

$$y(x) = 1 + 3(x-1) + 4(x-1)^2 + 3(x-1)^3 + \frac{3}{2}(x-1)^4 + \frac{3}{5}(x-1)^5 + \frac{1}{5}(x-1)^6 + \frac{2}{35}(x-1)^7 + \mathrm{O}\left((x-1)^8\right)$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 60

```
AsymptoticDSolveValue[{y'[x]-2*y[x]==x^2,{y[1]==1}},y[x],{x,1,7}]
```

$$y(x) \to \frac{2}{35}(x-1)^7 + \frac{1}{5}(x-1)^6 + \frac{3}{5}(x-1)^5 + \frac{3}{2}(x-1)^4 + 3(x-1)^3 + 4(x-1)^2 + 3(x-1) + 1$$

1.4 problem 2. direct method

Internal problem ID [6547]

Book: A course in Ordinary Differential Equations. by Stephen A. Wirkus, Randall J. Swift. CRC Press NY. 2015. 2nd Edition
Section: Chapter 8. Series Methods. section 8.2. The Power Series Method. Problems Page 603
Problem number: 2. direct method.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{-2y + y' = x^2}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve([diff(y(x),x)-2*y(x)=x^2,y(1) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{2} - \frac{x}{2} - \frac{1}{4} + \frac{9\,e^{2x-2}}{4}$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 28

```
DSolve[{y'[x]-2*y[x]==x^2,{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-2x^2 - 2x + 9e^{2x-2} - 1\right)$$

1.5 problem 3. series method

Internal problem ID [6548]

Book: A course in Ordinary Differential Equations. by Stephen A. Wirkus, Randall J. Swift. CRC Press NY. 2015. 2nd Edition
Section: Chapter 8. Series Methods. section 8.2. The Power Series Method. Problems Page 603
Problem number: 3. series method.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - y - e^{y}x = 0}$$

With initial conditions

$$[y(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=8;
dsolve([diff(y(x),x)=y(x)+x*exp(y(x)),y(0) = 0],y(x),type='series',x=0);
```

$$y(x) = \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{6}x^4 + \frac{1}{15}x^5 + \frac{43}{720}x^6 + \frac{151}{5040}x^7 + O\left(x^8\right)$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 46

```
AsymptoticDSolveValue[{y'[x]==y[x]+x*Exp[y[x]],{y[0]==0}},y[x],{x,0,7}]
```

$$y(x) \to \frac{151x^7}{5040} + \frac{43x^6}{720} + \frac{x^5}{15} + \frac{x^4}{6} + \frac{x^3}{6} + \frac{x^2}{2}$$

1.6 problem 3. direct method

Internal problem ID [6549]

Book: A course in Ordinary Differential Equations. by Stephen A. Wirkus, Randall J. Swift. CRC Press NY. 2015. 2nd Edition
Section: Chapter 8. Series Methods. section 8.2. The Power Series Method. Problems Page 603
Problem number: 3. direct method.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - y - e^y x = 0}$$

With initial conditions

$$[y(0) = 0]$$

✗ Solution by Maple

```
dsolve([diff(y(x),x)=y(x)+x*exp(y(x)),y(0) = 0],y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'[x]==y[x]+x*Exp[y[x]],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

Not solved