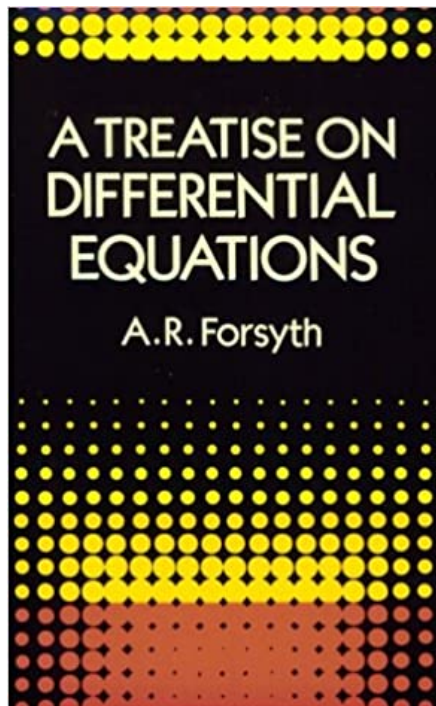


A Solution Manual For

**A treatise on Differential
Equations by A. R. Forsyth. 6th
edition. 1929. Macmillan Co.
ltd. New York, reprinted 1956**



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1.1 problem Ex. 5, page 256

Internal problem ID [5471]

Book: A treatise on Differential Equations by A. R. Forsyth. 6th edition. 1929. Macmillan Co. ltd. New York, reprinted 1956

Section: Chapter VI. Note I. Integration of linear equations in series by the method of Frobenius. page 243

Problem number: Ex. 5, page 256.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(-x^2 + 2)y'' - (x^2 + 4x + 2)((1 - x)y' + y) = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 44

Order:=6;

```
dsolve(x*(2-x^2)*diff(y(x),x$2)-(x^2+4*x+2)*((1-x)*diff(y(x),x)+y(x))=0,y(x),type='series',x
```

$$y(x) = c_1 x^2 \left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + O(x^6) \right) \\ + c_2 \left(-2 + 2x + 4x^2 + 4x^3 + 2x^4 + \frac{2}{3}x^5 + O(x^6) \right)$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 64

```
AsymptoticDSolveValue[x*(2-x^2)*y'[x]-(x^2+4*x+2)*((1-x)*y'[x]+y[x])==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(-\frac{5x^4}{4} - \frac{5x^3}{2} - \frac{5x^2}{2} - x + 1 \right) + c_2 \left(\frac{x^6}{24} + \frac{x^5}{6} + \frac{x^4}{2} + x^3 + x^2 \right)$$

1.2 problem Ex. 6(i), page 257

Internal problem ID [5472]

Book: A treatise on Differential Equations by A. R. Forsyth. 6th edition. 1929. Macmillan Co. ltd. New York, reprinted 1956

Section: Chapter VI. Note I. Integration of linear equations in series by the method of Frobenius. page 243

Problem number: Ex. 6(i), page 257.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1+x)y'' - (1+2x)(y'x - y) = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

Order:=6;

```
dsolve(x^2*(1+x)*diff(y(x),x$2)-(1+2*x)*(x*diff(y(x),x)-y(x))=0,y(x),type='series',x=0);
```

$$y(x) = ((\ln(x) c_2 + c_1) (1 + O(x^6)) + (x + O(x^6)) c_2) x$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 2760

```
AsymptoticDSolveValue[x^2*(1+x)*y'[x]-(1+2*x)*(x*y'[x]+y[x])==0,y[x],{x,0,5}]
```

Too large to display

1.3 problem Ex. 6(ii), page 257

Internal problem ID [5473]

Book: A treatise on Differential Equations by A. R. Forsyth. 6th edition. 1929. Macmillan Co. ltd. New York, reprinted 1956

Section: Chapter VI. Note I. Integration of linear equations in series by the method of Frobenius. page 243

Problem number: Ex. 6(ii), page 257.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

Solve

$$x^3(1+x)y''' - (2+4x)x^2y'' + (4+10x)xy' - (4+12x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 79

Order:=6;

```
dsolve(x^3*(1+x)*diff(y(x),x$3)-(2+4*x)*x^2*diff(y(x),x$2)+(4+10*x)*x*diff(y(x),x)-(4+12*x)*
```

$$y(x) = x((2 + O(x^6)) \ln(x) c_2 x + (2x + O(x^6)) \ln(x)^2 c_3 + (5 + O(x^6)) c_2 x + 2((-4)x + O(x^6)) \ln(x) c_3 + c_1 x(1 + O(x^6)) + (2 + 4x + 2x^2 + O(x^6)) c_3)$$

✓ Solution by Mathematica

Time used: 0.514 (sec). Leaf size: 49

```
AsymptoticDSolveValue[x^3*(1+x)*y'''[x]-(2+4*x)*x^2*y''[x]+(4+10*x)*x*y'[x]-(4+12*x)*y[x]==0
```

$$y(x) \rightarrow c_2 x^2 + c_1(2(x^2 + 11x + 1)x + 2x^2 \log^2(x) - 14x^2 \log(x)) + c_3 x^2 \log(x)$$

1.4 problem Ex. 6(iii), page 257

Internal problem ID [5474]

Book: A treatise on Differential Equations by A. R. Forsyth. 6th edition. 1929. Macmillan Co. ltd. New York, reprinted 1956

Section: Chapter VI. Note I. Integration of linear equations in series by the method of Frobenius. page 243

Problem number: Ex. 6(iii), page 257.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

Solve

$$x^3(x^2 + 1)y''' - (4x^2 + 2)x^2y'' + (10x^2 + 4)xy' - (12x^2 + 4)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 52

```
Order:=6;
```

```
dsolve(x^3*(1+x^2)*diff(y(x),x$3)-(2+4*x^2)*x^2*diff(y(x),x$2)+(4+10*x^2)*x*diff(y(x),x)-(4+12*x^2)*y(x))=0,x)
```

$$y(x) = x(c_3(2 + 2x^2 + O(x^6)) + ((1 + O(x^6))c_1 + c_2(\ln(x)(2 + O(x^6)) + (5 + O(x^6))))x)$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 30

```
AsymptoticDSolveValue[x^3*(1+x^2)*y'''[x]-(2+4*x^2)*x^2*y''[x]+(4+10*x^2)*x*y'[x]-(4+12*x^2)*y[x]=0,x,Order[6]]
```

$$y(x) \rightarrow c_1(2x^3 + 2x) + c_2x^2 + c_3x^2 \log(x)$$

1.5 problem Ex. 6(iv), page 257

Internal problem ID [5475]

Book: A treatise on Differential Equations by A. R. Forsyth. 6th edition. 1929. Macmillan Co. ltd. New York, reprinted 1956

Section: Chapter VI. Note I. Integration of linear equations in series by the method of Frobenius. page 243

Problem number: Ex. 6(iv), page 257.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2(-x + 2)x^2y'' - (4 - x)xy' + (-x + 3)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

`Order:=6;`

`dsolve(2*(2-x)*x^2*diff(y(x),x$2)-(4-x)*x*diff(y(x),x)+(3-x)*y(x)=0,y(x),type='series',x=0);`

$$y(x) = \sqrt{x} \left(x \left(1 + \frac{1}{8}x + \frac{1}{32}x^2 + \frac{5}{512}x^3 + \frac{7}{2048}x^4 + \frac{21}{16384}x^5 + O(x^6) \right) c_1 \right. \\ \left. + \left(1 + \frac{1}{4}x + \frac{1}{32}x^2 + \frac{1}{128}x^3 + \frac{5}{2048}x^4 + \frac{7}{8192}x^5 + O(x^6) \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 94

`AsymptoticDSolveValue[2*(2-x)*x^2*y'[x]-(4-x)*x*y'[x]+(3-x)*y[x]==0,y[x],{x,0,5}]`

$$y(x) \rightarrow c_1 \left(-\frac{5x^{9/2}}{2048} - \frac{x^{7/2}}{128} - \frac{x^{5/2}}{32} - \frac{x^{3/2}}{4} + \sqrt{x} \right) + c_2 \left(\frac{7x^{11/2}}{2048} + \frac{5x^{9/2}}{512} + \frac{x^{7/2}}{32} + \frac{x^{5/2}}{8} + x^{3/2} \right)$$

1.6 problem Ex. 6(v), page 257

Internal problem ID [5476]

Book: A treatise on Differential Equations by A. R. Forsyth. 6th edition. 1929. Macmillan Co. Ltd. New York, reprinted 1956

Section: Chapter VI. Note I. Integration of linear equations in series by the method of Frobenius. page 243

Problem number: Ex. 6(v), page 257.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(1-x)x^2y'' + (5x-4)xy' + (6-9x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
Order:=6;
```

```
dsolve((1-x)*x^2*diff(y(x),x$2)+(5*x-4)*x*diff(y(x),x)+(6-9*x)*y(x)=0,y(x),type='series',x=0
```

$$y(x) = x^2(c_1x(1 + O(x^6)) + (x + O(x^6)) \ln(x) c_2 + (1 - x + O(x^6)) c_2)$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 30

```
AsymptoticDSolveValue[(1-x)*x^2*y'[x]+(5*x-4)*x*y'[x]+(6-9*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2x^3 + c_1(x^3 \log(x) - x^2(3x - 1))$$

1.7 problem Ex. 6(vi), page 257

Internal problem ID [5477]

Book: A treatise on Differential Equations by A. R. Forsyth. 6th edition. 1929. Macmillan Co. ltd. New York, reprinted 1956

Section: Chapter VI. Note I. Integration of linear equations in series by the method of Frobenius. page 243

Problem number: Ex. 6(vi), page 257.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' + (4x^2 + 1)y' + 4xy(x^2 + 1) = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
Order:=6;
dsolve(x*diff(y(x),x$2)+(4*x^2+1)*diff(y(x),x)+4*x*(x^2+1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x^2 + \frac{1}{2}x^4\right) (\ln(x)c_2 + c_1) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 40

```
AsymptoticDSolveValue[x*y''[x]+(4*x^2+1)*y'[x]+4*x*(x^2+1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{x^4}{2} - x^2 + 1\right) + c_2 \left(\frac{x^4}{2} - x^2 + 1\right) \log(x)$$

1.8 problem Ex. 8(i), page 258

Internal problem ID [5478]

Book: A treatise on Differential Equations by A. R. Forsyth. 6th edition. 1929. Macmillan Co. ltd. New York, reprinted 1956

Section: Chapter VI. Note I. Integration of linear equations in series by the method of Frobenius. page 243

Problem number: Ex. 8(i), page 258.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + 4(a + x) y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 947

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+4*(x+a)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned}
 & y(x) \\
 &= \sqrt{x} \left(c_1 x^{-\frac{\sqrt{1-16a}}{2}} \left(1 + 4 \frac{1}{-1 + \sqrt{1-16a}} x + 8 \frac{1}{(-1 + \sqrt{1-16a})(-2 + \sqrt{1-16a})} x^2 \right. \right. \\
 &\quad \left. \left. + \frac{32}{3} \frac{1}{(-1 + \sqrt{1-16a})(-2 + \sqrt{1-16a})(-3 + \sqrt{1-16a})} x^3 \right. \right. \\
 &\quad \left. \left. + \frac{32}{3} \frac{1}{(-1 + \sqrt{1-16a})(-2 + \sqrt{1-16a})(-3 + \sqrt{1-16a})(-4 + \sqrt{1-16a})} x^4 \right. \right. \\
 &\quad \left. \left. + \frac{128}{15} \frac{1}{(-1 + \sqrt{1-16a})(-2 + \sqrt{1-16a})(-3 + \sqrt{1-16a})(-4 + \sqrt{1-16a})(-5 + \sqrt{1-16a})} x^5 \right. \right. \\
 &\quad \left. \left. + O(x^6) \right) + c_2 x^{\frac{\sqrt{1-16a}}{2}} \left(1 - 4 \frac{1}{1 + \sqrt{1-16a}} x + 8 \frac{1}{(1 + \sqrt{1-16a})(2 + \sqrt{1-16a})} x^2 \right. \right. \\
 &\quad \left. \left. - \frac{32}{3} \frac{1}{(1 + \sqrt{1-16a})(2 + \sqrt{1-16a})(3 + \sqrt{1-16a})} x^3 \right. \right. \\
 &\quad \left. \left. + \frac{32}{3} \frac{1}{(1 + \sqrt{1-16a})(2 + \sqrt{1-16a})(3 + \sqrt{1-16a})(4 + \sqrt{1-16a})} x^4 \right. \right. \\
 &\quad \left. \left. - \frac{128}{15} \frac{1}{(1 + \sqrt{1-16a})(2 + \sqrt{1-16a})(3 + \sqrt{1-16a})(4 + \sqrt{1-16a})(5 + \sqrt{1-16a})} x^5 \right. \right. \\
 &\quad \left. \left. + O(x^6) \right) \right)
 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 1356

AsymptoticDSolveValue[x^2*y'[x]+4*(x+a)*y[x]==0,y[x],{x,0,5}]

$$\begin{aligned}
 & y(x) \\
 \rightarrow & \left(-\frac{\left(\frac{1}{2}(1-\sqrt{1-16a})+1\right)\left(\frac{1}{2}(1-\sqrt{1-16a})+2\right)+4a}{\left(\frac{1}{2}(1-\sqrt{1-16a})+2\right)\left(\frac{1}{2}(1-\sqrt{1-16a})+1\right)} \right. \\
 & + \frac{\left(\frac{1}{2}(1-\sqrt{1-16a})+1\right)\left(\frac{1}{2}(1-\sqrt{1-16a})+2\right)+4a}{\left(\frac{1}{2}(1-\sqrt{1-16a})+2\right)\left(\frac{1}{2}(1-\sqrt{1-16a})+1\right)} \frac{64x^3}{16x^2} \\
 & - \frac{\left(\frac{1}{2}(1-\sqrt{1-16a})+1\right)\left(\frac{1}{2}(1-\sqrt{1-16a})+2\right)+4a}{\left(\frac{1}{2}(1-\sqrt{1-16a})+2\right)\left(\frac{1}{2}(1-\sqrt{1-16a})+1\right)} \frac{16x^2}{16x^2} \\
 & \left. + \frac{\left(\frac{1}{2}(1-\sqrt{1-16a})+1\right)\left(\frac{1}{2}(1-\sqrt{1-16a})+2\right)+4a}{\left(\frac{1}{2}(1-\sqrt{1-16a})+2\right)\left(\frac{1}{2}(1-\sqrt{1-16a})+1\right)} \frac{4x}{\left(\frac{1}{2}(1-\sqrt{1-16a})+1\right)(1-\sqrt{1-16a})} + 1 \right) c_2 x^{\frac{1}{2}(1-\sqrt{1-16a})} \\
 & + \left(-\frac{\left(\frac{1}{2}(\sqrt{1-16a}+1)+1\right)\left(\frac{1}{2}(\sqrt{1-16a}+1)+2\right)+4a}{\left(\frac{1}{2}(\sqrt{1-16a}+1)+2\right)\left(\frac{1}{2}(\sqrt{1-16a}+1)+1\right)} \right. \\
 & + \frac{\left(\frac{1}{2}(\sqrt{1-16a}+1)+1\right)\left(\frac{1}{2}(\sqrt{1-16a}+1)+2\right)+4a}{\left(\frac{1}{2}(\sqrt{1-16a}+1)+2\right)\left(\frac{1}{2}(\sqrt{1-16a}+1)+1\right)} \frac{64x^3}{16x^2} \\
 & - \frac{\left(\frac{1}{2}(\sqrt{1-16a}+1)+1\right)\left(\frac{1}{2}(\sqrt{1-16a}+1)+2\right)+4a}{\left(\frac{1}{2}(\sqrt{1-16a}+1)+2\right)\left(\frac{1}{2}(\sqrt{1-16a}+1)+1\right)} \frac{16x^2}{16x^2} \\
 & \left. + \frac{\left(\frac{1}{2}(\sqrt{1-16a}+1)+1\right)\left(\frac{1}{2}(\sqrt{1-16a}+1)+2\right)+4a}{\left(\frac{1}{2}(\sqrt{1-16a}+1)+2\right)\left(\frac{1}{2}(\sqrt{1-16a}+1)+1\right)} \frac{4x}{\left(\frac{1}{2}(\sqrt{1-16a}+1)+1\right)(\sqrt{1-16a}+1)} + 1 \right) c_1 x^{\frac{1}{2}(\sqrt{1-16a}+1)}
 \end{aligned}$$

1.9 problem Ex. 8(ii), page 258

Internal problem ID [5479]

Book: A treatise on Differential Equations by A. R. Forsyth. 6th edition. 1929. Macmillan Co. ltd. New York, reprinted 1956

Section: Chapter VI. Note I. Integration of linear equations in series by the method of Frobenius. page 243

Problem number: Ex. 8(ii), page 258.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' + (x^3 + 1)y' + bxy = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 73

```
Order:=6;  
dsolve(x*dif(y(x),x$2)+(1+x*x^2)*dif(y(x),x)+b*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x) c_2 + c_1) \left(1 - \frac{1}{4}bx^2 + \frac{1}{64}b^2x^4 + \frac{1}{50}bx^5 + O(x^6) \right) \\ + \left(\frac{b}{4}x^2 - \frac{1}{9}x^3 - \frac{3}{128}b^2x^4 - \frac{61}{4500}bx^5 + O(x^6) \right) c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 103

```
AsymptoticDSolveValue[x*y''[x]+(1+x*x^2)*y'[x]+b*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{b^2x^4}{64} + \frac{bx^5}{50} - \frac{bx^2}{4} + 1 \right) \\ + c_2 \left(-\frac{3b^2x^4}{128} + \left(\frac{b^2x^4}{64} + \frac{bx^5}{50} - \frac{bx^2}{4} + 1 \right) \log(x) - \frac{61bx^5}{4500} + \frac{bx^2}{4} - \frac{x^3}{9} \right)$$