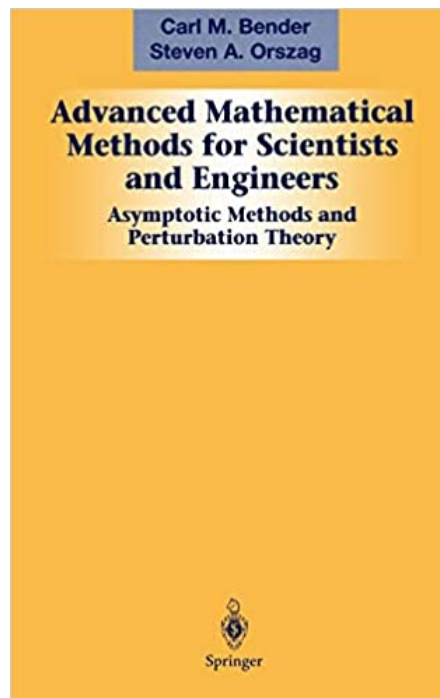


A Solution Manual For

**Advanced Mathematical
Methods for Scientists and
Engineers, Bender and Orszag.
Springer October 29, 1999**



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March 3, 2024

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1.1 problem 3.5

Internal problem ID [5480]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag.
Springer October 29, 1999

Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136

Problem number: 3.5.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(x - 1)(x - 2)y'' + (4x - 6)y' + 2y = 0$$

With initial conditions

$$[y(0) = 2, y'(0) = 1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
```

```
dsolve([(x-1)*(x-2)*diff(y(x),x$2)+(4*x-6)*diff(y(x),x)+2*y(x)=0,y(0) = 2, D(y)(0) = 1],y(x),{
```

$$y(x) = 2 + x + \frac{1}{2}x^2 + \frac{1}{4}x^3 + \frac{1}{8}x^4 + \frac{1}{16}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[{{(x-1)*(x-2)*y'[x]+(4*x-6)*y'[x]+2*y[x]==0,{y[0]==2,y'[0]==1}},y[x],{
```

$$y(x) \rightarrow \frac{x^5}{16} + \frac{x^4}{8} + \frac{x^3}{4} + \frac{x^2}{2} + x + 2$$

1.2 problem 3.6 (a)

Internal problem ID [5481]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag.
Springer October 29, 1999

Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136

Problem number: 3.6 (a).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 2y'x + 8y = 0$$

With initial conditions

$$[y(0) = 4, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;  
dsolve([diff(y(x),x$2)-2*x*diff(y(x),x)+8*y(x)=0,y(0) = 4, D(y)(0) = 0],y(x),type='series',x
```

$$y(x) = 4 - 16x^2 + \frac{16}{3}x^4 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 17

```
AsymptoticDSolveValue[{y''[x]-2*x*y'[x]+8*y[x]==0,{y[0]==4,y'[0]==0}},y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{16x^4}{3} - 16x^2 + 4$$

1.3 problem 3.6 (b)

Internal problem ID [5482]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag.
Springer October 29, 1999

Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136

Problem number: 3.6 (b).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 2y'x + 8y = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 4]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;  
dsolve([diff(y(x),x$2)-2*x*diff(y(x),x)+8*y(x)=0,y(0) = 0, D(y)(0) = 4],y(x),type='series',x
```

$$y(x) = 4x - 4x^3 + \frac{2}{5}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{y''[x]-2*x*y'[x]+8*y[x]==0,{y[0]==0,y'[0]==4}},y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{2x^5}{5} - 4x^3 + 4x$$

1.4 problem 3.6 (c)

Internal problem ID [5483]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag.
Springer October 29, 1999

Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136

Problem number: 3.6 (c).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [Gegenbauer]

$$(-x^2 + 1)y'' - 2y'x + 12y = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 3]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 13

```
Order:=6;
```

```
dsolve([(1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+12*y(x)=0,y(0) = 0, D(y)(0) = 3],y(x),type=''
```

$$y(x) = -5x^3 + 3x$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 12

```
AsymptoticDSolveValue[{{(1-x^2)*y''[x]-2*x*y'[x]+12*y[x]==0,{y[0]==0,y'[0]==3}},y[x],{x,0,5}]
```

$$y(x) \rightarrow 3x - 5x^3$$

1.5 problem 3.6 (d)

Internal problem ID [5484]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag.
Springer October 29, 1999

Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136

Problem number: 3.6 (d).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - (x - 1)y = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
Order:=6;  
dsolve([diff(y(x),x$2)=(x-1)*y(x),y(0) = 1, D(y)(0) = 0],y(x),type='series',x=0);
```

$$y(x) = 1 - \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{30}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 33

```
AsymptoticDSolveValue[{y'[x]==(x-1)*y[x],{y[0]==1,y'[0]==0}},y[x],{x,0,5}]
```

$$y(x) \rightarrow -\frac{x^5}{30} + \frac{x^4}{24} + \frac{x^3}{6} - \frac{x^2}{2} + 1$$

1.6 problem 3.24 (a)

Internal problem ID [5485]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag.
Springer October 29, 1999

Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136

Problem number: 3.24 (a).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(x+2)y'' + 2y'(1+x) - 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
Order:=6;  
dsolve(x*(x+2)*diff(y(x),x$2)+2*(x+1)*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x) c_2 + c_1) (1 + x + O(x^6)) \\ + \left(-\frac{5}{2}x - \frac{3}{8}x^2 + \frac{1}{12}x^3 - \frac{5}{192}x^4 + \frac{3}{320}x^5 + O(x^6) \right) c_2$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 53

```
AsymptoticDSolveValue[x*(x+2)*y''[x]+2*(x+1)*y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{3x^5}{320} - \frac{5x^4}{192} + \frac{x^3}{12} - \frac{3x^2}{8} - \frac{5x}{2} + (x+1)\log(x) \right) + c_1(x+1)$$

1.7 problem 3.24 (b)

Internal problem ID [5486]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag.
Springer October 29, 1999

Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136

Problem number: 3.24 (b).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [[_Emden, _Fowler]]

$$xy'' + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 58

```
Order:=6;  
dsolve(x*diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = & c_1 x \left(1 - \frac{1}{2}x + \frac{1}{12}x^2 - \frac{1}{144}x^3 + \frac{1}{2880}x^4 - \frac{1}{86400}x^5 + O(x^6) \right) \\ & + c_2 \left(\ln(x) \left(-x + \frac{1}{2}x^2 - \frac{1}{12}x^3 + \frac{1}{144}x^4 - \frac{1}{2880}x^5 + O(x^6) \right) \right. \\ & \left. + \left(1 - \frac{3}{4}x^2 + \frac{7}{36}x^3 - \frac{35}{1728}x^4 + \frac{101}{86400}x^5 + O(x^6) \right) \right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 85

```
AsymptoticDSolveValue[x*y''[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{1}{144} x(x^3 - 12x^2 + 72x - 144) \log(x) + \frac{-47x^4 + 480x^3 - 2160x^2 + 1728x + 1728}{1728} \right) + c_2 \left(\frac{x^5}{2880} - \frac{x^4}{144} + \frac{x^3}{12} - \frac{x^2}{2} + x \right)$$

1.8 problem 3.24 (c)

Internal problem ID [5487]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag.
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Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136

Problem number: 3.24 (c).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + (e^x - 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 39

```
Order:=6;  
dsolve(diff(y(x),x$2)+(exp(x)-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{6}x^3 - \frac{1}{24}x^4 - \frac{1}{120}x^5\right) y(0) + \left(x - \frac{1}{12}x^4 - \frac{1}{40}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 49

```
AsymptoticDSolveValue[y''[x]+(Exp[x]-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(-\frac{x^5}{40} - \frac{x^4}{12} + x \right) + c_1 \left(-\frac{x^5}{120} - \frac{x^4}{24} - \frac{x^3}{6} + 1 \right)$$

1.9 problem 3.24 (d)

Internal problem ID [5488]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag.
Springer October 29, 1999

Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136

Problem number: 3.24 (d).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x(1-x)y'' - 3y'x - y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 60

```
Order:=6;  
dsolve(x*(1-x)*diff(y(x),x$2)-3*x*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = & c_1x(1 + 2x + 3x^2 + 4x^3 + 5x^4 + 6x^5 + O(x^6)) \\ & + (x + 2x^2 + 3x^3 + 4x^4 + 5x^5 + O(x^6)) \ln(x) c_2 \\ & + (1 + 3x + 5x^2 + 7x^3 + 9x^4 + 11x^5 + O(x^6)) c_2 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 63

```
AsymptoticDSolveValue[x*(1-x)*y''[x]-3*x*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned} y(x) \rightarrow & c_1(x^4 + x^3 + x^2 + (4x^3 + 3x^2 + 2x + 1)x \log(x) + x + 1) \\ & + c_2(5x^5 + 4x^4 + 3x^3 + 2x^2 + x) \end{aligned}$$

1.10 problem 3.24 (e)

Internal problem ID [5489]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag.
Springer October 29, 1999

Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136

Problem number: 3.24 (e).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$2xy'' - y' + yx^2 = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 28

```
Order:=6;  
dsolve(2*x*diff(y(x),x^2)-diff(y(x),x)+x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{3}{2}} \left(1 - \frac{1}{27} x^3 + O(x^6) \right) + c_2 \left(1 - \frac{1}{9} x^3 + O(x^6) \right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 33

```
AsymptoticDSolveValue[2*x*y'[x]-y'[x]+x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(1 - \frac{x^3}{9} \right) + c_1 \left(1 - \frac{x^3}{27} \right) x^{3/2}$$

1.11 problem 3.24 (f)

Internal problem ID [5490]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag.
Springer October 29, 1999

Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136

Problem number: 3.24 (f).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\sin(x)y'' - 2y'\cos(x) - \sin(x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 32

```
Order:=6;
```

```
dsolve(sin(x)*diff(y(x),x$2)-2*cos(x)*diff(y(x),x)-sin(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^3 \left(1 - \frac{1}{10}x^2 + \frac{1}{280}x^4 + O(x^6) \right) + c_2 \left(12 - 6x^2 + \frac{1}{2}x^4 + O(x^6) \right)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 44

```
AsymptoticDSolveValue[Sin[x]*y''[x]-2*Cos[x]*y'[x]-Sin[x]*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{x^4}{24} - \frac{x^2}{2} + 1 \right) + c_2 \left(\frac{x^7}{280} - \frac{x^5}{10} + x^3 \right)$$

1.12 problem 3.24 (g)

Internal problem ID [5491]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag.
Springer October 29, 1999

Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136

Problem number: 3.24 (g).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$y'' - yx^2 = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;  
dsolve(diff(y(x),x$2)-x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{x^4}{12}\right) y(0) + \left(x + \frac{1}{20}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y'[x]-x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{x^5}{20} + x \right) + c_1 \left(\frac{x^4}{12} + 1 \right)$$

1.13 problem 3.24 (h)

Internal problem ID [5492]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag.
Springer October 29, 1999

Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136

Problem number: 3.24 (h).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`, `[_2nd_order, _linear,`

$$x(x+2)y'' + y'(1+x) - 4y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 38

```
Order:=6;
```

```
dsolve(x*(x+2)*diff(y(x),x$2)+(x+1)*diff(y(x),x)-4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x} \left(1 + \frac{5}{4}x + \frac{7}{32}x^2 - \frac{3}{128}x^3 + \frac{11}{2048}x^4 - \frac{13}{8192}x^5 + O(x^6) \right) \\ + c_2(1 + 4x + 2x^2 + O(x^6))$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 60

```
AsymptoticDSolveValue[x*(x+2)*y''[x]+(x+1)*y'[x]-4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2(2x^2 + 4x + 1) + c_1\sqrt{x} \left(-\frac{13x^5}{8192} + \frac{11x^4}{2048} - \frac{3x^3}{128} + \frac{7x^2}{32} + \frac{5x}{4} + 1 \right)$$

1.14 problem 3.24 (i)

Internal problem ID [5493]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag.
Springer October 29, 1999

Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136

Problem number: 3.24 (i).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$xy'' + \left(\frac{1}{2} - x\right)y' - y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=6;  
dsolve(x*diff(y(x),x$2)+(1/2-x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x} \left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + O(x^6)\right) \\ + c_2 \left(1 + 2x + \frac{4}{3}x^2 + \frac{8}{15}x^3 + \frac{16}{105}x^4 + \frac{32}{945}x^5 + O(x^6)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 79

```
AsymptoticDSolveValue[x*y''[x]+(1/2-x)*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1\sqrt{x} \left(\frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1\right) + c_2 \left(\frac{32x^5}{945} + \frac{16x^4}{105} + \frac{8x^3}{15} + \frac{4x^2}{3} + 2x + 1\right)$$

1.15 problem 3.25 $v=1/2$

Internal problem ID [5494]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag.
Springer October 29, 1999

Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136

Problem number: 3.25 $v=1/2$.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + y' x + \left(x^2 + \frac{1}{4}\right) y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
Order:=6;
```

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2+(1/2)^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{-\frac{i}{2}} \left(1 + \left(-\frac{1}{5} - \frac{i}{10} \right) x^2 + \left(\frac{7}{680} + \frac{3i}{340} \right) x^4 + O(x^6) \right) \\ + c_2 x^{\frac{i}{2}} \left(1 + \left(-\frac{1}{5} + \frac{i}{10} \right) x^2 + \left(\frac{7}{680} - \frac{3i}{340} \right) x^4 + O(x^6) \right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
AsymptoticDSolveValue[x^2*y'[x]+x*y'[x]+(x^2+(1/2)^2)*y[x]==0,y[x],{x,0,5}]
```

Not solved

1.16 problem 3.25 $v=3/2$

Internal problem ID [5495]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag.
Springer October 29, 1999

Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136

Problem number: 3.25 $v=3/2$.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + y' x + \left(x^2 + \frac{9}{4}\right) y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
Order:=6;  
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2+(3/2)^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{-\frac{3i}{2}} \left(1 + \left(-\frac{1}{13} - \frac{3i}{26} \right) x^2 + \left(-\frac{1}{2600} + \frac{9i}{1300} \right) x^4 + O(x^6) \right) \\ + c_2 x^{\frac{3i}{2}} \left(1 + \left(-\frac{1}{13} + \frac{3i}{26} \right) x^2 + \left(-\frac{1}{2600} - \frac{9i}{1300} \right) x^4 + O(x^6) \right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
AsymptoticDSolveValue[x^2*y'[x]+x*y'[x]+(x^2+(3/2)^2)*y[x]==0,y[x],{x,0,5}]
```

Not solved

1.17 problem 3.25 $v=5/2$

Internal problem ID [5496]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag.
Springer October 29, 1999

Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136

Problem number: 3.25 $v=5/2$.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + y' x + \left(x^2 + \frac{25}{4}\right) y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;  
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2+(5/2)^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{-\frac{5i}{2}} \left(1 + \left(-\frac{1}{29} - \frac{5i}{58} \right) x^2 + \left(-\frac{17}{9512} + \frac{15i}{4756} \right) x^4 + O(x^6) \right) \\ + c_2 x^{\frac{5i}{2}} \left(1 + \left(-\frac{1}{29} + \frac{5i}{58} \right) x^2 + \left(-\frac{17}{9512} - \frac{15i}{4756} \right) x^4 + O(x^6) \right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
AsymptoticDSolveValue[x^2*y'[x]+x*y'[x]+(x^2+(5/2)^2)*y[x]==0,y[x],{x,0,5}]
```

Not solved

1.18 problem 3.26

Internal problem ID [5497]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag.
Springer October 29, 1999

Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136

Problem number: 3.26.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x - 1)y'' - y'x + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;  
dsolve((x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5\right)y(0) + D(y)(0)x + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 41

```
AsymptoticDSolveValue[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + 1 \right) + c_2 x$$

1.19 problem 3.48 (a)

Internal problem ID [5498]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag.
Springer October 29, 1999

Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136

Problem number: 3.48 (a).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [`_linear`]

$$yx + y' = \cos(x)$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
Order:=6;  
dsolve(diff(y(x),x)+x*y(x)=cos(x),y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{8}x^4\right) y(0) + x - \frac{x^3}{2} + \frac{13x^5}{120} + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 38

```
AsymptoticDSolveValue[y'[x]+x*y[x]==Cos[x],y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{13x^5}{120} - \frac{x^3}{2} + c_1 \left(\frac{x^4}{8} - \frac{x^2}{2} + 1 \right) + x$$

1.20 problem 3.48 (b)

Internal problem ID [5499]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag.
Springer October 29, 1999

Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136

Problem number: 3.48 (b).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$yx + y' = \frac{1}{x^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x)+x*y(x)=1/x^3,y(x), singsol=all)
```

$$y(x) = \left(-\frac{e^{\frac{x^2}{2}}}{2x^2} - \frac{\text{Ei}_1\left(-\frac{x^2}{2}\right)}{4} + c_1 \right) e^{-\frac{x^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 46

```
DSolve[y'[x]+x*y[x]==1/x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{4} e^{-\frac{x^2}{2}} \text{ExpIntegralEi}\left(\frac{x^2}{2}\right) - \frac{1}{2x^2} + c_1 e^{-\frac{x^2}{2}}$$

1.21 problem 3.48 (c)

Internal problem ID [5500]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag.
Springer October 29, 1999

Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136

Problem number: 3.48 (c).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x^3 y'' + y = \frac{1}{x^4}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;  
dsolve(x^3*diff(y(x),x$2)+y(x)=1/x^4,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.364 (sec). Leaf size: 800

```
AsymptoticDSolveValue[x^3*y''[x]+y[x]==1/x^4,y[x],{x,0,5}]
```

$$y(x) \rightarrow e^{-\frac{2i}{\sqrt{x}}x^{3/4}} \left(\frac{33424574007825x^5}{281474976710656} - \frac{468131288625ix^{9/2}}{8796093022208} - \frac{14783093325x^4}{549755813888} \right. \\ \left. + \frac{66891825ix^{7/2}}{4294967296} + \frac{2837835x^3}{268435456} - \frac{72765ix^{5/2}}{8388608} - \frac{4725x^2}{524288} + \frac{105ix^{3/2}}{8192} + \frac{15x}{512} - \frac{3i\sqrt{x}}{16} \right. \\ \left. + 1 \right) c_1 + e^{\frac{2i}{\sqrt{x}}x^{3/4}} \left(\frac{33424574007825x^5}{281474976710656} + \frac{468131288625ix^{9/2}}{8796093022208} - \frac{14783093325x^4}{549755813888} - \frac{66891825ix^{7/2}}{4294967296} + \frac{2837835x^3}{268435456} \right.$$

1.22 problem 3.48 (d)

Internal problem ID [5501]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag.
Springer October 29, 1999

Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136

Problem number: 3.48 (d).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$xy'' - 2y' + y = \cos(x)$$

With the expansion point for the power series method at $x = 0$.

X Solution by Maple

```
Order:=6;  
dsolve(x*diff(y(x),x$2)-2*diff(y(x),x)+y(x)=cos(x),y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.161 (sec). Leaf size: 312

AsymptoticDSolveValue[x*y'[x]-2*y'[x]+y[x]==Cos[x],y[x],{x,0,5}]

$$\begin{aligned}
 y(x) \rightarrow & c_1 \left(x^4 \left(\frac{\log(x)}{48} - \frac{5}{192} \right) - \frac{1}{12} x^3 \log(x) + \frac{x^2}{4} + \frac{x}{2} + 1 \right) \\
 & + c_2 \left(-\frac{x^5}{806400} + \frac{x^4}{20160} - \frac{x^3}{720} + \frac{x^2}{40} - \frac{x}{4} + 1 \right) x^3 + \left(-\frac{x^5}{806400} + \frac{x^4}{20160} - \frac{x^3}{720} \right. \\
 & \quad \left. + \frac{x^2}{40} - \frac{x}{4} + 1 \right) x^3 \left(\frac{x^6(-20160 \log^2(x) + 141222 \log(x) - 201569)}{3135283200} \right. \\
 & \quad \left. + \frac{x^5(22277 - 114360 \log(x))}{435456000} + \frac{x^4(69541 - 29064 \log(x))}{34836480} \right. \\
 & \quad \left. + \frac{x^3(1860 \log(x) + 193)}{388800} - \frac{1}{6x^2} + \frac{x^2(4 \log(x) - 23)}{1152} - \frac{1}{6x} + \frac{1}{36} x(-\log(x) - 2) \right. \\
 & \quad \left. - \frac{\log(x)}{12} \right) + \left(\frac{x^6(5791 - 672 \log(x))}{8709120} - \frac{589x^5}{302400} - \frac{89x^4}{8640} + \frac{19x^3}{360} + \frac{x^2}{24} \right. \\
 & \quad \left. - \frac{x}{3} \right) \left(x^4 \left(\frac{\log(x)}{48} - \frac{5}{192} \right) - \frac{1}{12} x^3 \log(x) + \frac{x^2}{4} + \frac{x}{2} + 1 \right)
 \end{aligned}$$

1.23 problem 3.50

Internal problem ID [5502]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag.
Springer October 29, 1999

Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136

Problem number: 3.50.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y' - \frac{y}{x} = \cos(x)$$

With the expansion point for the power series method at $x = 0$.

 Solution by Maple

```
Order:=6;  
dsolve(diff(y(x),x)-y(x)/x=cos(x),y(x),type='series',x=0);
```

No solution found

 Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 34

```
AsymptoticDSolveValue[y'[x]-y[x]/x==Cos[x],y[x],{x,0,5}]
```

$$y(x) \rightarrow x \left(-\frac{x^6}{4320} + \frac{x^4}{96} - \frac{x^2}{4} + \log(x) \right) + c_1 x$$