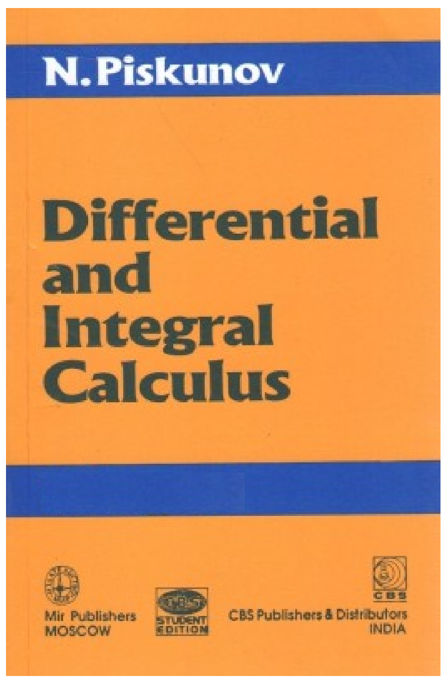
A Solution Manual For

DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.



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1.1 problem 1

Internal problem ID [12098]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\cos(x) = \frac{\sin(2x)}{2}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+y(x)*cos(x)=1/2*sin(2*x),y(x), singsol=all)
```

$$y(x) = \sin(x) - 1 + e^{-\sin(x)} c_1$$

✓ Solution by Mathematica

Time used: 0.082 (sec). Leaf size: 18

```
DSolve[y'[x]+y[x]*Cos[x]==1/2*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x) + c_1 e^{-\sin(x)} - 1$$

1.2 problem 2

Internal problem ID [12099]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 2.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$y'^2 - y' - xy' + y = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 27

```
dsolve(diff(y(x),x)^2-diff(y(x),x)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{4}x^2 + \frac{1}{2}x + \frac{1}{4}$$
$$y(x) = -c_1^2 + c_1 x + c_1$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 28

```
DSolve[(y'[x])^2-y'[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x + 1 - c_1)$$
$$y(x) \to \frac{1}{4}(x+1)^2$$

1.3 problem 3

Internal problem ID [12100]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 3.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{yy'^2 + 2xy' - y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 75

```
dsolve(y(x)*diff(y(x),x)^2+2*x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = -ix$$
$$y(x) = ix$$
$$y(x) = 0$$
$$y(x) = \sqrt{c_1^2 - 2c_1x}$$
$$y(x) = \sqrt{c_1^2 + 2c_1x}$$
$$y(x) = -\sqrt{c_1^2 - 2c_1x}$$
$$y(x) = -\sqrt{c_1^2 + 2c_1x}$$

✓ Solution by Mathematica

Time used: 0.759 (sec). Leaf size: 126

```
DSolve[y[x]*y'[x]^2+2*x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{\frac{c_1}{2}}\sqrt{-2x + e^{c_1}}$$
$$y(x) \to e^{\frac{c_1}{2}}\sqrt{-2x + e^{c_1}}$$
$$y(x) \to -e^{\frac{c_1}{2}}\sqrt{2x + e^{c_1}}$$
$$y(x) \to e^{\frac{c_1}{2}}\sqrt{2x + e^{c_1}}$$
$$y(x) \to 0$$
$$y(x) \to -ix$$
$$y(x) \to ix$$

1.4 problem 4

Internal problem ID [12101]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 4.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational]`

$$xy\left(1-y'^2\right)-\left(x^2-y^2-a^2\right)y'=0$$

✗ Solution by Maple

```
dsolve(x*y(x)*(1-diff(y(x),x)^2)=(x^2-y(x)^2-a^2)*diff(y(x),x),y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 0.612 (sec). Leaf size: 75

```
DSolve[x*y[x]*(1-y'[x]^2)==(x^2-y[x]^2-a^2)*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\sqrt{c_1\left(x^2-\frac{a^2}{1+c_1}\right)}$$
$$y(x)\to -i(a-x)$$
$$y(x)\to i(a-x)$$
$$y(x)\to -i(a+x)$$
$$y(x)\to i(a+x)$$

1.5 problem 5

Internal problem ID [12102]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 5.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + \frac{3y''}{x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$3)+3/x*diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = c_1 + \frac{c_2}{x} + c_3 x$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 21

```
DSolve[y'''[x]+3/x*y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{2x} + c_3 x + c_2$$

1.6 problem 6

Internal problem ID [12103]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2ky' + k^2 y = \mathrm{e}^x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)-2*k*diff(y(x),x)+k^2*y(x)=exp(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{kx} c_2 + \mathrm{e}^{kx} x c_1 + \frac{\mathrm{e}^x}{(k-1)^2}$$

✓ Solution by Mathematica

Time used: 0.221 (sec). Leaf size: 28

```
DSolve[y''[x]-2*k*y'[x]+k^2*y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x}{(k-1)^2} + (c_2 x + c_1) e^{kx}$$

1.7 problem 7

Internal problem ID [12104]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']]`

$$\boxed{(-x^2+1)\,y''-xy'-ya^2=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve((1-x^2)*diff(y(x),x$2)-x*diff(y(x),x)-a^2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x+\sqrt{x^2-1}\right)^{ia}+c_2\left(x+\sqrt{x^2-1}\right)^{-ia}$$

✓ Solution by Mathematica

Time used: 0.151 (sec). Leaf size: 89

```
DSolve[(1-x^2)*y''[x]-x*y'[x]-a^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\cos\left(\frac{1}{2}a\left(\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)-\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)\right)\right)-c_2\sin\left(\frac{1}{2}a\left(\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)-\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)\right)\right)$$

1.8 problem 8

Internal problem ID [12105]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$y'' + \frac{2y'}{x} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve(diff(y(x),x$2)+2/x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + \frac{c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 15

```
DSolve[y''[x]+2/x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{c_1}{x}$$

1.9 problem 9

Internal problem ID [12106]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y - xy' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 7

```
dsolve(y(x)-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 14

```
DSolve[y[x]-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x$$
$$y(x) \to 0$$

1.10 problem 10

Internal problem ID [12107]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(1+u)\, v+(1-v)\, u v'=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve((1+u)*v(u)+(1-v(u))*u*diff(v(u),u)=0,v(u), singsol=all)
```

$$v(u)=-\operatorname{LambertW}\left(-\frac{\mathrm{e}^{-u}}{c_1 u}\right)$$

✓ Solution by Mathematica

Time used: 4.942 (sec). Leaf size: 28

```
DSolve[(1+u)*v[u]+(1-v[u])*u*v'[u]==0,v[u],u,IncludeSingularSolutions -> True]
```

$$v(u)\to -W\left(-\frac{e^{-u-c_1}}{u}\right)$$
$$v(u)\to 0$$

1.11 problem 11

Internal problem ID [12108]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y-(1-x)\,y'=-1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((1+y(x))-(1-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\frac{-x+c_1}{x-1}$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 22

```
DSolve[(1+y[x])-(1-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\rightarrow\frac{x+c_1}{1-x}$$
$$y(x)\rightarrow -1$$

1.12 problem 12

Internal problem ID [12109]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(t^2 + t^2 x)\, x' + x^2 + t x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve((t^2+x(t)*t^2)*diff(x(t),t)+x(t)^2+t*x(t)^2=0,x(t), singsol=all)
```

$$x(t) = \frac{1}{\text{LambertW}\left(t c_1 \mathrm{e}^{-\frac{1}{t}}\right)}$$

✓ Solution by Mathematica

Time used: 5.02 (sec). Leaf size: 27

```
DSolve[(t^2+x[t]*t^2)*x'[t]+x[t]^2+t*x[t]^2==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{W\left(t e^{-\frac{1}{t}-c_1}\right)}$$

$$x(t) \to 0$$

1.13 problem 13

Internal problem ID [12110]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2y' + y = a}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((y(x)-a)+x^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = a + e^{\frac{1}{x}} c_1$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 20

```
DSolve[(y[x]-a)+x^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to a + c_1 e^{\frac{1}{x}}$$
$$y(x) \to a$$

1.14 problem 14

Internal problem ID [12111]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{z - \left(-a^2 + t^2\right) z' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(z(t)-(t^2-a^2)*diff(z(t),t)=0,z(t), singsol=all)
```

$$z(t) = c_1(-a+t)^{\frac{1}{2a}} (a+t)^{-\frac{1}{2a}}$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 26

```
DSolve[z[t]-(t^2-a^2)*z'[t]==0,z[t],t,IncludeSingularSolutions -> True]
```

$$z(t) \to c_1 e^{-\frac{\operatorname{arctanh}\left(\frac{t}{a}\right)}{a}}$$
$$z(t) \to 0$$

1.15 problem 15

Internal problem ID [12112]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{y^2+1}{x^2+1} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)=(1+y(x)^2)/(1+x^2),y(x), singsol=all)
```

$$y(x) = \tan(\arctan(x) + c_1)$$

✓ Solution by Mathematica

Time used: 0.408 (sec). Leaf size: 25

```
DSolve[y'[x]==(1+y[x]^2)/(1+x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan(\arctan(x) + c_1)$$
$$y(x) \to -i$$
$$y(x) \to i$$

1.16 problem 16

Internal problem ID [12113]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{s^2 - \sqrt{t}\, s' = -1}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 12

```
dsolve((1+s(t)^2)-sqrt(t)*diff(s(t),t)=0,s(t), singsol=all)
```

$$s(t) = \tan\left(2\sqrt{t} + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.332 (sec). Leaf size: 30

```
DSolve[(1+s[t]^2)-Sqrt[t]*s'[t]==0,s[t],t,IncludeSingularSolutions -> True]
```

$$s(t) \to \tan\left(2\sqrt{t} + c_1\right)$$
$$s(t) \to -i$$
$$s(t) \to i$$

1.17 problem 17

Internal problem ID [12114]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{r' + r\tan(t) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(r(t),t)+r(t)*tan(t)=0,r(t), singsol=all)
```

$$r(t) = c_1 \cos(t)$$

✓ Solution by Mathematica

Time used: 0.068 (sec). Leaf size: 15

```
DSolve[r'[t]+r[t]*Tan[t]==0,r[t],t,IncludeSingularSolutions -> True]
```

$$r(t) \to c_1 \cos(t)$$
$$r(t) \to 0$$

1.18 problem 21

Internal problem ID [12115]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\left(x^2+1\right) y' - \sqrt{-y^2+1} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve((1+x^2)*diff(y(x),x)-sqrt(1-y(x)^2)=0,y(x), singsol=all)
```

$$y(x) = \sin\left(\arctan\left(x\right) + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.479 (sec). Leaf size: 29

```
DSolve[(1+x^2)*y'[x]-Sqrt[1-y[x]^2]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(\arctan(x) + c_1)$$
$$y(x) \to -1$$
$$y(x) \to 1$$
$$y(x) \to \text{Interval}[\{-1, 1\}]$$

1.19 problem 22

Internal problem ID [12116]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sqrt{-x^2+1}\,y' - \sqrt{-y^2+1} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 9

```
dsolve(sqrt(1-x^2)*diff(y(x),x)-sqrt(1-y(x)^2)=0,y(x), singsol=all)
```

$$y(x) = \sin(\arcsin(x) + c_1)$$

✓ Solution by Mathematica

Time used: 0.488 (sec). Leaf size: 49

```
DSolve[Sqrt[1-x^2]*y'[x]-Sqrt[1-y[x]^2]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos\left(2\arctan\left(\frac{\sqrt{1-x^2}}{x+1}\right) - c_1\right)$$
$$y(x) \to -1$$
$$y(x) \to 1$$
$$y(x) \to \text{Interval}[\{-1,1\}]$$

1.20 problem 23

Internal problem ID [12117]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$3\,\mathrm{e}^{x} \tan\left(y\right)+\left(1-\mathrm{e}^{x}\right) \sec\left(y\right)^{2} y' = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 200

```
dsolve(3*exp(x)*tan(y(x))+(1-exp(x))*sec(y(x))^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\arctan\left(\frac{2c_1\left(\mathrm{e}^{3x}-3\,\mathrm{e}^{2x}+3\,\mathrm{e}^{x}-1\right)}{\mathrm{e}^{6x}c_1^2-6\,\mathrm{e}^{5x}c_1^2+15\,\mathrm{e}^{4x}c_1^2-20\,\mathrm{e}^{3x}c_1^2+15\,\mathrm{e}^{2x}c_1^2-6\,\mathrm{e}^{x}c_1^2+c_1^2+1}, -\frac{\mathrm{e}^{6x}c_1^2-6\,\mathrm{e}^{5x}c_1^2+15\,\mathrm{e}^{4x}c_1^2-20\,\mathrm{e}^{3x}c_1^2+15\,\mathrm{e}^{2x}c_1^2-6\,\mathrm{e}^{x}c_1^2+c_1^2-1}{\mathrm{e}^{6x}c_1^2-6\,\mathrm{e}^{5x}c_1^2+15\,\mathrm{e}^{4x}c_1^2-20\,\mathrm{e}^{3x}c_1^2+15\,\mathrm{e}^{2x}c_1^2-6\,\mathrm{e}^{x}c_1^2+c_1^2+1}\right)}{2}$$

✓ Solution by Mathematica

Time used: 1.829 (sec). Leaf size: 74

```
DSolve[3*Exp[x]*Tan[y[x]]+(1-Exp[x])*Sec[y[x]]^2*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{1}{2} \arccos \left(-\tanh \left(3 \log \left(e^x-1\right)+2 c_1\right)\right)$$

$$y(x) \to \frac{1}{2} \arccos \left(-\tanh \left(3 \log \left(e^x-1\right)+2 c_1\right)\right)$$

$$y(x) \to 0$$

$$y(x) \to -\frac{\pi}{2}$$

$$y(x) \to \frac{\pi}{2}$$

1.21 problem 24

Internal problem ID [12118]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$-y^2x + (y - yx^2)\, y' = -x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve((x-y(x)^2*x)+(y(x)-x^2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(x^2-1)(x^2+c_1)}}{x^2-1}$$

$$y(x) = -\frac{\sqrt{(x^2-1)(x^2+c_1)}}{x^2-1}$$

✓ Solution by Mathematica

Time used: 0.652 (sec). Leaf size: 74

```
DSolve[(x-y[x]^2*x)+(y[x]-x^2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{x^2-1-e^{2c_1}}}{\sqrt{x^2-1}}$$

$$y(x) \to \frac{\sqrt{x^2-1-e^{2c_1}}}{\sqrt{x^2-1}}$$

$$y(x) \to -1$$

$$y(x) \to 1$$

1.22 problem 39

Internal problem ID [12119]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{y + (x + y)\, y' = x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
dsolve((y(x)-x)+(y(x)+x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-c_1 x - \sqrt{2c_1^2 x^2 + 1}}{c_1}$$

$$y(x) = \frac{-c_1 x + \sqrt{2c_1^2 x^2 + 1}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.763 (sec). Leaf size: 94

```
DSolve[(y[x]-x)+(y[x]+x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x - \sqrt{2x^2 + e^{2c_1}}$$

$$y(x) \to -x + \sqrt{2x^2 + e^{2c_1}}$$

$$y(x) \to -\sqrt{2}\sqrt{x^2} - x$$

$$y(x) \to \sqrt{2}\sqrt{x^2} - x$$

1.23 problem 40

Internal problem ID [12120]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' + y = -x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve((x+y(x))+x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{x}{2} + \frac{c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 17

```
DSolve[(x+y[x])+x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{2} + \frac{c_1}{x}$$

1.24 problem 41

Internal problem ID [12121]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y + (y - x)\, y' = -x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve((x+y(x))+(y(x)-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \tan\left(\operatorname{RootOf}\left(-2\_Z + \ln\left(\frac{1}{\cos(\_Z)^2}\right) + 2\ln(x) + 2c_1\right)\right) x$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 36

```
DSolve[(x+y[x])+(y[x]-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right) - \arctan\left(\frac{y(x)}{x}\right) = -\log(x) + c_1, y(x)\right]$$

1.25 problem 42

Internal problem ID [12122]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{xy' - y - \sqrt{x^2 + y^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x)-y(x)=sqrt(x^2+y(x)^2),y(x), singsol=all)
```

$$\frac{y(x)}{x^2} + \frac{\sqrt{y(x)^2 + x^2}}{x^2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.575 (sec). Leaf size: 27

```
DSolve[x*y'[x]-y[x]==Sqrt[x^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-c_1} \left(-1 + e^{2c_1} x^2\right)$$

1.26 problem 43

Internal problem ID [12123]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{8y + (5y + 7x)\, y' = -10x}$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 49

```
dsolve((8*y(x)+10*x)+(5*y(x)+7*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x\left(-2c_1^2 + c_1^2 \operatorname{RootOf}\left(\_Z^{25}c_1x^5 - 2\_Z^{20}c_1x^5 + \_Z^{15}c_1x^5 - 1\right)^5\right)}{c_1^2}$$

✓ Solution by Mathematica

Time used: 3.57 (sec). Leaf size: 276

```
DSolve[(8*y[x]+10*x)+(5*y[x]+7*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \operatorname{Root}\left[\#1^5 + 8\#1^4x + 25\#1^3x^2 + 38\#1^2x^3 + 28\#1x^4 + 8x^5 - e^{c_1}\&, 1\right]$$
$$y(x) \to \operatorname{Root}\left[\#1^5 + 8\#1^4x + 25\#1^3x^2 + 38\#1^2x^3 + 28\#1x^4 + 8x^5 - e^{c_1}\&, 2\right]$$
$$y(x) \to \operatorname{Root}\left[\#1^5 + 8\#1^4x + 25\#1^3x^2 + 38\#1^2x^3 + 28\#1x^4 + 8x^5 - e^{c_1}\&, 3\right]$$
$$y(x) \to \operatorname{Root}\left[\#1^5 + 8\#1^4x + 25\#1^3x^2 + 38\#1^2x^3 + 28\#1x^4 + 8x^5 - e^{c_1}\&, 4\right]$$
$$y(x) \to \operatorname{Root}\left[\#1^5 + 8\#1^4x + 25\#1^3x^2 + 38\#1^2x^3 + 28\#1x^4 + 8x^5 - e^{c_1}\&, 5\right]$$

1.27 problem 44

Internal problem ID [12124]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{2\sqrt{st} - s + ts' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((2*sqrt(s(t)*t)-s(t))+t*diff(s(t),t)=0,s(t), singsol=all)
```

$$\frac{s(t)}{\sqrt{s(t)\,t}} + \ln(t) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.315 (sec). Leaf size: 19

```
DSolve[(2*Sqrt[s[t]*t]-s[t])+t*s'[t]==0,s[t],t,IncludeSingularSolutions -> True]
```

$$s(t) \to \frac{1}{4} t(-2\log(t) + c_1)^2$$

1.28 problem 45

Internal problem ID [12125]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{-s + ts' = -t}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((t-s(t))+t*diff(s(t),t)=0,s(t), singsol=all)
```

$$s(t) = (-\ln(t) + c_1)\, t$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 14

```
DSolve[(t-s[t])+t*s'[t]==0,s[t],t,IncludeSingularSolutions -> True]
```

$$s(t) \to t(-\log(t) + c_1)$$

1.29 problem 46

Internal problem ID [12126]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{xy^2y' - y^3 = x^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 74

```
dsolve(x*y(x)^2*diff(y(x),x)=(x^3+y(x)^3),y(x), singsol=all)
```

$$y(x) = (3\ln(x) + c_1)^{\frac{1}{3}} x$$
$$y(x) = \left(-\frac{(3\ln(x) + c_1)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}\,(3\ln(x) + c_1)^{\frac{1}{3}}}{2}\right) x$$
$$y(x) = \left(-\frac{(3\ln(x) + c_1)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}\,(3\ln(x) + c_1)^{\frac{1}{3}}}{2}\right) x$$

✓ Solution by Mathematica

Time used: 0.331 (sec). Leaf size: 63

```
DSolve[x*y[x]^2*y'[x]==(x^3+y[x]^3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\sqrt[3]{3\log(x) + c_1}$$
$$y(x) \to -\sqrt[3]{-1}x\sqrt[3]{3\log(x) + c_1}$$
$$y(x) \to (-1)^{2/3}x\sqrt[3]{3\log(x) + c_1}$$

1.30 problem 47

Internal problem ID [12127]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{x\cos\left(\frac{y}{x}\right)(xy'+y)-y\sin\left(\frac{y}{x}\right)(xy'-y)=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 24

```
dsolve(x*cos(y(x)/x)*(y(x)+x*diff(y(x),x))=y(x)*sin(y(x)/x)*(x*diff(y(x),x)-y(x)),y(x), sing
```

$$y(x)=\frac{c_1}{\cos\left(\operatorname{RootOf}\left(\_Z\cos\left(\_Z\right)x^2-c_1\right)\right)x}$$

✓ Solution by Mathematica

Time used: 0.569 (sec). Leaf size: 31

```
DSolve[x*Cos[y[x]/x]*(y[x]+x*y'[x])==y[x]*Sin[y[x]/x]*(x*y'[x]-y[x]),y[x],x,IncludeSingularS
```

$$\text{Solve}\left[-\log\left(\frac{y(x)}{x}\right)-\log\left(\cos\left(\frac{y(x)}{x}\right)\right)=2\log(x)+c_1,y(x)\right]$$

1.31 problem 48

Internal problem ID [12128]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$3y - (3x - 7y - 3)\, y' = 7x - 7$$

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 705

```
dsolve((3*y(x)-7*x+7)-(3*x-7*y(x)-3)*diff(y(x),x)=0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 61.254 (sec). Leaf size: 7785

```
DSolve[(3*y[x]-7*x+7)-(3*x-7*y[x]-3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

1.32 problem 49

Internal problem ID [12129]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{2y - (4y + 2x + 3)\, y' = -x - 1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve((x+2*y(x)+1)-(2*x+4*y(x)+3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{x}{2} + \frac{\mathrm{LambertW}\left(\mathrm{e}^{5}\mathrm{e}^{8x}c_1\right)}{8} - \frac{5}{8}$$

✓ Solution by Mathematica

Time used: 6.228 (sec). Leaf size: 39

```
DSolve[(x+2*y[x]+1)-(2*x+4*y[x]+3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}\left(W\left(-e^{8x-1+c_1}\right) - 4x - 5\right)$$
$$y(x) \to \frac{1}{8}(-4x - 5)$$

1.33 problem 50

Internal problem ID [12130]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2y - (2x - 3)\, y' = -x - 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve((x+2*y(x)+1)-(2*x-3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \left(\frac{\ln(2x-3)}{4} - \frac{5}{4(2x-3)} + c_1\right)(2x-3)$$

✓ Solution by Mathematica

Time used: 0.064 (sec). Leaf size: 32

```
DSolve[(x+2*y[x]+1)-(2*x-3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}((2x-3)\log(3-2x) + 4c_1(2x-3) - 5)$$

1.34 problem 52

Internal problem ID [12131]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{\frac{y - xy'}{\sqrt{x^2+y^2}} = m}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve((y(x)-x*diff(y(x),x))/sqrt(x^2+y(x)^2)=m,y(x), singsol=all)
```

$$\frac{x^m y(x)}{x} + \frac{x^m \sqrt{y(x)^2 + x^2}}{x} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.442 (sec). Leaf size: 36

```
DSolve[(y[x]-x*y'[x])/Sqrt[x^2+y[x]^2]==m,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-c_1} x^{1-m} \left(-x^{2m} + e^{2c_1}\right)$$

1.35 problem 53

Internal problem ID [12132]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 53.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _dAlembert]`

$$\boxed{\frac{x+yy'}{\sqrt{x^2+y^2}}=m}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 194

```
dsolve((x+y(x)*diff(y(x),x))/sqrt(x^2+y(x)^2)=m,y(x), singsol=all)
```

$$\int_{\_b}^{x}-\frac{m\sqrt{\_a^2+y(x)^2}-\_a}{m\sqrt{\_a^2+y(x)^2}\,\_a-\_a^2-y(x)^2}d\_a+\int^{y(x)}\left(\frac{\_f}{m\sqrt{\_f^2+x^2}\,x-x^2-\_f^2}-\left(\int_{\_b}^{x}\left(\frac{\left(m\sqrt{\_a^2+\_f^2}-\_a\right)\left(\frac{m\_a\_f}{\sqrt{\_a^2+\_f^2}}-2\_f\right)}{\left(m\sqrt{\_a^2+\_f^2}\,\_a-\_a^2-\_f^2\right)^2}-\frac{m\_f}{\left(m\sqrt{\_a^2+\_f^2}\,\_a-\_a^2-\_f^2\right)\sqrt{\_a^2+\_f^2}}\right)\right)\right)+c_1=0$$

✓ Solution by Mathematica

Time used: 2.496 (sec). Leaf size: 103

```
DSolve[(x+y[x]*y'[x])/Sqrt[x^2+y[x]^2]==m,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{(m^2-1)\,x^2 - 2e^{c_1}mx + e^{2c_1}}$$
$$y(x) \to \sqrt{(m^2-1)\,x^2 - 2e^{c_1}mx + e^{2c_1}}$$
$$y(x) \to -\sqrt{(m^2-1)\,x^2}$$
$$y(x) \to \sqrt{(m^2-1)\,x^2}$$

1.36 problem 55

Internal problem ID [12133]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 55.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$y+\frac{x}{y'}-\sqrt{x^2+y^2}=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(y(x)+x/diff(y(x),x)=sqrt(x^2+y(x)^2),y(x), singsol=all)
```

$$\frac{y(x)}{x^2}+\frac{\sqrt{y(x)^2+x^2}}{x^2}-c_1=0$$

✓ Solution by Mathematica

Time used: 0.527 (sec). Leaf size: 27

```
DSolve[y[x]+x/y'[x]==Sqrt[x^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}e^{-c_1}\left(x^2-e^{2c_1}\right)$$

1.37 problem 56

Internal problem ID [12134]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 56.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{yy' - \sqrt{x^2+y^2} = -x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve(y(x)*diff(y(x),x)=-x+sqrt(x^2+y(x)^2),y(x), singsol=all)
```

$$-c_1 + \frac{x}{y(x)^2} + \frac{\sqrt{y(x)^2+x^2}}{y(x)^2} = 0$$

✓ Solution by Mathematica

Time used: 0.656 (sec). Leaf size: 57

```
DSolve[y[x]*y'[x]==-x+Sqrt[x^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{\frac{c_1}{2}}\sqrt{2x+e^{c_1}}$$
$$y(x) \to e^{\frac{c_1}{2}}\sqrt{2x+e^{c_1}}$$
$$y(x) \to 0$$

1.38 problem 57

Internal problem ID [12135]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 57.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{2y}{x+1} = (x+1)^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)-2*y(x)/(x+1)=(x+1)^3,y(x), singsol=all)
```

$$y(x) = \left(\frac{1}{2}x^2 + x + c_1\right)(x+1)^2$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 22

```
DSolve[y'[x]-2*y[x]/(x+1)==(x+1)^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x+1)^2 \left(\frac{x^2}{2} + x + c_1\right)$$

1.39 problem 58

Internal problem ID [12136]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 58.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{ay}{x} = \frac{x+1}{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x)-a*y(x)/x=(x+1)/x,y(x), singsol=all)
```

$$y(x) = \left(-\frac{x^{-a}(ax+a-1)}{a\,(a-1)} + c_1\right) x^a$$

✓ Solution by Mathematica

Time used: 0.136 (sec). Leaf size: 28

```
DSolve[y'[x]-a*y[x]/x==(x+1)/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{ax+a-1}{(a-1)a} + c_1 x^a$$

1.40 problem 59

Internal problem ID [12137]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 59.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$
\boxed{\left(-x^{2}+x\right) y^{\prime}+\left(2 x^{2}-1\right) y=a x^{3}}
$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve((x-x^2)*diff(y(x),x)+(2*x^2-1)*y(x)-a*x^3=0,y(x), singsol=all)
```

$$
y(x)=\left(-a\left(\mathrm{e}^{-2} \mathrm{Ei}_{1}(2 x-2)+\frac{2 \mathrm{e}^{-2 x}}{-2 x+2}\right)+c_{1}\right)\left(\mathrm{e}^{2 x} x^{2}-\mathrm{e}^{2 x} x\right)
$$

✓ Solution by Mathematica

Time used: 0.473 (sec). Leaf size: 39

```
DSolve[(x-x^2)*y'[x]+(2*x^2-1)*y[x]-a*x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x) \rightarrow x\left(a e^{2 x-2}(x-1) \operatorname{ExpIntegralEi}(2-2 x)+a-c_{1} e^{2 x}(x-1)\right)
$$

1.41 problem 60

Internal problem ID [12138]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 60.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{s' \cos(t) + s \sin(t) = 1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve(diff(s(t),t)*cos(t)+s(t)*sin(t)=1,s(t), singsol=all)
```

$$s(t) = (\tan(t) + c_1)\cos(t)$$

✓ Solution by Mathematica

Time used: 0.068 (sec). Leaf size: 13

```
DSolve[s'[t]*Cos[t]+s[t]*Sin[t]==1,s[t],t,IncludeSingularSolutions -> True]
```

$$s(t) \to \sin(t) + c_1 \cos(t)$$

1.42 problem 61

Internal problem ID [12139]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{s' + s\cos(t) = \frac{\sin(2t)}{2}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(s(t),t)+s(t)*cos(t)=1/2*sin(2*t),s(t), singsol=all)
```

$$s(t) = \sin(t) - 1 + e^{-\sin(t)} c_1$$

✓ Solution by Mathematica

Time used: 0.087 (sec). Leaf size: 18

```
DSolve[s'[t]+s[t]*Cos[t]==1/2*Sin[2*t],s[t],t,IncludeSingularSolutions -> True]
```

$$s(t) \to \sin(t) + c_1 e^{-\sin(t)} - 1$$

1.43 problem 62

Internal problem ID [12140]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 62.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' - \frac{ny}{x} = e^x x^n$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)-n/x*y(x)=exp(x)*x^n,y(x), singsol=all)
```

$$y(x) = (e^x + c_1)\, x^n$$

✓ Solution by Mathematica

Time used: 0.095 (sec). Leaf size: 15

```
DSolve[y'[x]-n/x*y[x]==Exp[x]*x^n,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (e^x + c_1)\, x^n$$

1.44 problem 63

Internal problem ID [12141]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 63.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{ny}{x} = a\,x^{-n}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+n/x*y(x)=a/x^n,y(x), singsol=all)
```

$$y(x) = (ax + c_1)\,x^{-n}$$

✓ Solution by Mathematica

Time used: 0.098 (sec). Leaf size: 17

```
DSolve[y'[x]+n/x*y[x]==a/x^n,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^{-n}(ax + c_1)$$

1.45 problem 64

Internal problem ID [12142]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 64.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = e^{-x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)+y(x)=exp(-x),y(x), singsol=all)
```

$$y(x) = (x + c_1) e^{-x}$$

✓ Solution by Mathematica

Time used: 0.079 (sec). Leaf size: 15

```
DSolve[y'[x]+y[x]==Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(x + c_1)$$

1.46 problem 65

Internal problem ID [12143]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 65.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{(-2x+1)\, y}{x^2} = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+(1-2*x)/x^2*y(x)-1=0,y(x), singsol=all)
```

$$y(x) = x^2 + \mathrm{e}^{\frac{1}{x}} c_1 x^2$$

✓ Solution by Mathematica

Time used: 0.057 (sec). Leaf size: 19

```
DSolve[y'[x]+(1-2*x)/x^2*y[x]-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 \left(1 + c_1 e^{\frac{1}{x}}\right)$$

1.47 problem 66

Internal problem ID [12144]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 66.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + yx - y^3x^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x)+x*y(x)=x^3*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{\mathrm{e}^{x^2}c_1 + x^2 + 1}}$$
$$y(x) = -\frac{1}{\sqrt{\mathrm{e}^{x^2}c_1 + x^2 + 1}}$$

✓ Solution by Mathematica

Time used: 7.379 (sec). Leaf size: 50

```
DSolve[y'[x]+x*y[x]==x^3*y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{x^2 + c_1e^{x^2} + 1}}$$
$$y(x) \to \frac{1}{\sqrt{x^2 + c_1e^{x^2} + 1}}$$
$$y(x) \to 0$$

1.48 problem 67

Internal problem ID [12145]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 67.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(-x^2+1)\, y' - yx + axy^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve((1-x^2)*diff(y(x),x)-x*y(x)+a*x*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{x-1}\,\sqrt{x+1}\,c_1 + a}$$

✓ Solution by Mathematica

Time used: 4.118 (sec). Leaf size: 35

```
DSolve[(1-x^2)*y'[x]-x*y[x]+a*x*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{a + e^{c_1}\sqrt{x^2-1}}$$
$$y(x) \to 0$$
$$y(x) \to \frac{1}{a}$$

1.49 problem 68

Internal problem ID [12146]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 68.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{3y^2y' - ay^3 = x + 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 154

```
dsolve(3*y(x)^2*diff(y(x),x)-a*y(x)^3-x-1=0,y(x), singsol=all)
```

$$y(x) = \frac{((e^{ax}c_1a^2 - ax - a - 1)\,a)^{\frac{1}{3}}}{a}$$

$$y(x) = -\frac{((e^{ax}c_1a^2 - ax - a - 1)\,a)^{\frac{1}{3}}}{2a} - \frac{i\sqrt{3}\,((e^{ax}c_1a^2 - ax - a - 1)\,a)^{\frac{1}{3}}}{2a}$$

$$y(x) = -\frac{((e^{ax}c_1a^2 - ax - a - 1)\,a)^{\frac{1}{3}}}{2a} + \frac{i\sqrt{3}\,((e^{ax}c_1a^2 - ax - a - 1)\,a)^{\frac{1}{3}}}{2a}$$

✓ Solution by Mathematica

Time used: 17.994 (sec). Leaf size: 111

```
DSolve[3*y[x]^2*y'[x]-a*y[x]^3-x-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{a^2 c_1 e^{ax} - a(x+1) - 1}}{a^{2/3}}$$

$$y(x) \to -\frac{\sqrt[3]{-1}\sqrt[3]{a^2 c_1 e^{ax} - a(x+1) - 1}}{a^{2/3}}$$

$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{a^2 c_1 e^{ax} - a(x+1) - 1}}{a^{2/3}}$$

1.50 problem 69

Internal problem ID [12147]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{y'(x^2y^3 + xy) = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 70

```
dsolve(diff(y(x),x)*(x^2*y(x)^3+x*y(x))=1,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{x\left(2\,\mathrm{LambertW}\left(\frac{c_1 \mathrm{e}^{-\frac{2x-1}{2x}}}{2}\right)x + 2x - 1\right)}}{x}$$

$$y(x) = -\frac{\sqrt{x\left(2\,\mathrm{LambertW}\left(\frac{c_1 \mathrm{e}^{-\frac{2x-1}{2x}}}{2}\right)x + 2x - 1\right)}}{x}$$

✓ Solution by Mathematica

Time used: 0.18 (sec). Leaf size: 76

```
DSolve[y'[x]*(x^2*y[x]^3+x*y[x])==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{2xW\left(c_1e^{\frac{1}{2x}-1}\right)+2x-1}}{\sqrt{x}}$$

$$y(x) \to \frac{\sqrt{2xW\left(c_1e^{\frac{1}{2x}-1}\right)+2x-1}}{\sqrt{x}}$$

1.51 problem 70

Internal problem ID [12148]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 70.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{xy' - (y \ln(x) - 2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve(x*diff(y(x),x)=(y(x)*ln(x)-2)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{4}{1 + 4c_1x^2 + 2\ln(x)}$$

✓ Solution by Mathematica

Time used: 0.254 (sec). Leaf size: 27

```
DSolve[x*y'[x]==(y[x]*Log[x]-2)*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4}{4c_1x^2 + 2\log(x) + 1}$$

$$y(x) \to 0$$

1.52 problem 71

Internal problem ID [12149]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 71.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y - y' \cos(x) - y^2 \cos(x) (-\sin(x) + 1) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(y(x)-diff(y(x),x)*cos(x)=y(x)^2*cos(x)*(1-sin(x)),y(x), singsol=all)
```

$$y(x) = \frac{\sec(x) + \tan(x)}{\sin(x) + c_1}$$

✓ Solution by Mathematica

Time used: 0.778 (sec). Leaf size: 41

```
DSolve[y[x]-y'[x]*Cos[x]==y[x]^2*Cos[x]*(1-Sin[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{2\operatorname{arctanh}\left(\tan\left(\frac{x}{2}\right)\right)}}{\cos(x)e^{2\operatorname{arctanh}\left(\tan\left(\frac{x}{2}\right)\right)} + c_1}$$

$$y(x) \to 0$$

1.53 problem 72

Internal problem ID [12150]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 72.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x),G(x)]']`

$$y + (x - 2y)\, y' = -x^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 51

```
dsolve((x^2+y(x))+(x-2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x}{2} - \frac{\sqrt{12x^3 + 9x^2 + 36c_1}}{6}$$
$$y(x) = \frac{x}{2} + \frac{\sqrt{12x^3 + 9x^2 + 36c_1}}{6}$$

✓ Solution by Mathematica

Time used: 0.252 (sec). Leaf size: 81

```
DSolve[(x^2+y[x])+(x-2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}\left(3x - i\sqrt{3}\sqrt{-4x^3 - 3x^2 - 12c_1}\right)$$
$$y(x) \to \frac{1}{6}\left(3x + i\sqrt{3}\sqrt{-4x^3 - 3x^2 - 12c_1}\right)$$

1.54 problem 73

Internal problem ID [12151]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 73.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x),G(x)]']`

$$\boxed{y - (4y - x)\, y' = 3x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve((y(x)-3*x^2)-(4*y(x)-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x}{4} - \frac{\sqrt{-8x^3 + x^2 + 8c_1}}{4}$$
$$y(x) = \frac{x}{4} + \frac{\sqrt{-8x^3 + x^2 + 8c_1}}{4}$$

✓ Solution by Mathematica

Time used: 0.229 (sec). Leaf size: 67

```
DSolve[(y[x]-3*x^2)-(4*y[x]-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(x - i\sqrt{8x^3 - x^2 - 16c_1}\right)$$
$$y(x) \to \frac{1}{4}\left(x + i\sqrt{8x^3 - x^2 - 16c_1}\right)$$

1.55 problem 74

Internal problem ID [12152]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 74.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact, _rational]`

$$\boxed{(y^3 - x)\, y' - y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve((y(x)^3-x)*diff(y(x),x)=y(x),y(x), singsol=all)
```

$$-\frac{c_1}{y(x)} + x - \frac{y(x)^3}{4} = 0$$

✓ Solution by Mathematica

Time used: 57.499 (sec). Leaf size: 996

```
DSolve[(y[x]^3-x)*y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{\sqrt{\frac{\left(9x^2-\sqrt{81x^4-192c_1{}^3}\right)^{2/3}+4\sqrt[3]{3}c_1}{\sqrt[3]{9x^2-\sqrt{81x^4-192c_1{}^3}}}}}{\sqrt{2}\sqrt[3]{3}}-\frac{1}{2}\sqrt{-\frac{4\sqrt{2}\sqrt[3]{3}x}{\sqrt{\frac{\left(9x^2-\sqrt{81x^4-192c_1{}^3}\right)^{2/3}+4\sqrt[3]{3}c_1}{\sqrt[3]{9x^2-\sqrt{81x^4-192c_1{}^3}}}}}-\frac{2\sqrt[3]{9x^2-\sqrt{81x^4-192c_1{}^3}}}{3^{2/3}}-\frac{8c_1}{\sqrt[3]{27x^2-3\sqrt{81x^4-192c_1{}^3}}}}$$

$$y(x)\to \frac{1}{2}\left(\sqrt{-\frac{4\sqrt{2}\sqrt[3]{3}x}{\sqrt{\frac{\left(9x^2-\sqrt{81x^4-192c_1{}^3}\right)^{2/3}+4\sqrt[3]{3}c_1}{\sqrt[3]{9x^2-\sqrt{81x^4-192c_1{}^3}}}}}-\frac{2\sqrt[3]{9x^2-\sqrt{81x^4-192c_1{}^3}}}{3^{2/3}}-\frac{8c_1}{\sqrt[3]{27x^2-3\sqrt{81x^4-192c_1{}^3}}}}-\frac{\sqrt{2}\sqrt{\frac{\left(9x^2-\sqrt{81x^4-192c_1{}^3}\right)^{2/3}+4\sqrt[3]{3}c_1}{\sqrt[3]{9x^2-\sqrt{81x^4-192c_1{}^3}}}}}{\sqrt[3]{3}}\right)$$

$$y(x)\to \frac{\sqrt{\frac{\left(9x^2-\sqrt{81x^4-192c_1{}^3}\right)^{2/3}+4\sqrt[3]{3}c_1}{\sqrt[3]{9x^2-\sqrt{81x^4-192c_1{}^3}}}}}{\sqrt{2}\sqrt[3]{3}}-\frac{1}{2}\sqrt{\frac{4\sqrt{2}\sqrt[3]{3}x}{\sqrt{\frac{\left(9x^2-\sqrt{81x^4-192c_1{}^3}\right)^{2/3}+4\sqrt[3]{3}c_1}{\sqrt[3]{9x^2-\sqrt{81x^4-192c_1{}^3}}}}}-\frac{2\sqrt[3]{9x^2-\sqrt{81x^4-192c_1{}^3}}}{3^{2/3}}-\frac{8c_1}{\sqrt[3]{27x^2-3\sqrt{81x^4-192c_1{}^3}}}}$$

$$y(x)\to \frac{1}{2}\left(\frac{\sqrt{2}\sqrt{\frac{\left(9x^2-\sqrt{81x^4-192c_1{}^3}\right)^{2/3}+4\sqrt[3]{3}c_1}{\sqrt[3]{9x^2-\sqrt{81x^4-192c_1{}^3}}}}}{\sqrt[3]{3}}\right.$$

1.56 problem 75

Internal problem ID [12153]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 75.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{\frac{y^2}{(-y+x)^2}+\left(\frac{1}{y}-\frac{x^2}{(-y+x)^2}\right)y'=\frac{1}{x}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

```
dsolve((y(x)^2/(x-y(x))^2-1/x )+(1/y(x)-x^2/(x-y(x))^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{\mathrm{RootOf}\left(-\ln(x)\mathrm{e}^{-Z}+x\ln(x)+\mathrm{e}^{-Z}c_1+\_Z\mathrm{e}^{-Z}+\mathrm{e}^{-Z}x-c_1x-x\_Z\right)}$$

✓ Solution by Mathematica

Time used: 0.721 (sec). Leaf size: 29

```
DSolve[(y[x]^2/(x-y[x])^2-1/x )+(1/y[x]-x^2/(x-y[x])^2)*y'[x]==0,y[x],x,IncludeSingularSolut
```

$$\text{Solve}\left[\frac{y(x)^2}{x-y(x)}+y(x)-\log(y(x))+\log(x)=c_1,y(x)\right]$$

1.57 problem 76

Internal problem ID [12154]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 76.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{6y^2x+3(2yx^2+y^2)\,y'=-4x^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 517

```
dsolve(2*(3*x*y(x)^2+2*x^3)+3*(2*x^2*y(x)+y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(-4x^4-4c_1-8x^6+4\sqrt{4x^{10}+x^8+4c_1x^6+2c_1x^4+c_1^2}\right)^{\frac{1}{3}}}{2} + \frac{2x^4}{\left(-4x^4-4c_1-8x^6+4\sqrt{4x^{10}+x^8+4c_1x^6+2c_1x^4+c_1^2}\right)^{\frac{1}{3}}} - x^2$$

$$y(x) = -\frac{\left(-4x^4-4c_1-8x^6+4\sqrt{4x^{10}+x^8+4c_1x^6+2c_1x^4+c_1^2}\right)^{\frac{1}{3}}}{4} - \frac{x^4}{\left(-4x^4-4c_1-8x^6+4\sqrt{4x^{10}+x^8+4c_1x^6+2c_1x^4+c_1^2}\right)^{\frac{1}{3}}} - x^2 - \frac{i\sqrt{3}\left(\frac{\left(-4x^4-4c_1-8x^6+4\sqrt{4x^{10}+x^8+4c_1x^6+2c_1x^4+c_1^2}\right)^{\frac{1}{3}}}{2} - \frac{2x^4}{\left(-4x^4-4c_1-8x^6+4\sqrt{4x^{10}+x^8+4c_1x^6+2c_1x^4+c_1^2}\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(-4x^4-4c_1-8x^6+4\sqrt{4x^{10}+x^8+4c_1x^6+2c_1x^4+c_1^2}\right)^{\frac{1}{3}}}{4} - \frac{x^4}{\left(-4x^4-4c_1-8x^6+4\sqrt{4x^{10}+x^8+4c_1x^6+2c_1x^4+c_1^2}\right)^{\frac{1}{3}}} - x^2 + \frac{i\sqrt{3}\left(\frac{\left(-4x^4-4c_1-8x^6+4\sqrt{4x^{10}+x^8+4c_1x^6+2c_1x^4+c_1^2}\right)^{\frac{1}{3}}}{2} - \frac{2x^4}{\left(-4x^4-4c_1-8x^6+4\sqrt{4x^{10}+x^8+4c_1x^6+2c_1x^4+c_1^2}\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 25.227 (sec). Leaf size: 419

```
DSolve[2*(3*x*y[x]^2+2*x^3)+3*(2*x^2*y[x]+y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to -x^2 + \frac{\sqrt[3]{2}x^4}{\sqrt[3]{-2x^6 - x^4 + \sqrt{4x^{10} + x^8 - 4c_1x^6 - 2c_1x^4 + c_1^2} + c_1}} + \frac{\sqrt[3]{-2x^6 - x^4 + \sqrt{4x^{10} + x^8 - 4c_1x^6 - 2c_1x^4 + c_1^2} + c_1}}{\sqrt[3]{2}}$$

$$y(x) \to \frac{1}{4}\left(-4x^2 - \frac{2\sqrt[3]{2}\left(1 + i\sqrt{3}\right)x^4}{\sqrt[3]{-2x^6 - x^4 + \sqrt{4x^{10} + x^8 - 4c_1x^6 - 2c_1x^4 + c_1^2} + c_1}} + i2^{2/3}\left(\sqrt{3} + i\right)\sqrt[3]{-2x^6 - x^4 + \sqrt{4x^{10} + x^8 - 4c_1x^6 - 2c_1x^4 + c_1^2} + c_1}\right)$$

$$y(x) \to \frac{1}{4}\left(-4x^2 + \frac{2i\sqrt[3]{2}\left(\sqrt{3} + i\right)x^4}{\sqrt[3]{-2x^6 - x^4 + \sqrt{4x^{10} + x^8 - 4c_1x^6 - 2c_1x^4 + c_1^2} + c_1}} + 2^{2/3}\left(-1 - i\sqrt{3}\right)\sqrt[3]{-2x^6 - x^4 + \sqrt{4x^{10} + x^8 - 4c_1x^6 - 2c_1x^4 + c_1^2} + c_1}\right)$$

1.58 problem 77

Internal problem ID [12155]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 77.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{\frac{x}{(x+y)^2}+\frac{(2x+y)\,y'}{(x+y)^2}=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(x/(x+y(x))^2+(2*x+y(x))/(x+y(x))^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=-\frac{x(\mathrm{LambertW}\,(c_1x)-1)}{\mathrm{LambertW}\,(c_1x)}$$

✓ Solution by Mathematica

Time used: 0.192 (sec). Leaf size: 33

```
DSolve[x/(x+y[x])^2+(2*x+y[x])/(x+y[x])^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\log\left(\frac{y(x)}{x}+1\right)-\frac{1}{\frac{y(x)}{x}+1}=-\log(x)+c_1,y(x)\right]$$

1.59 problem 78

Internal problem ID [12156]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 78.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _Bernoulli]`

$$\boxed{\frac{3y^2}{x^4} - \frac{2yy'}{x^3} = -\frac{1}{x^2}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve(1/x^2+ 3*y(x)^2/x^4=2*y(x)/x^3*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \sqrt{c_1 x - 1}\, x$$
$$y(x) = -\sqrt{c_1 x - 1}\, x$$

✓ Solution by Mathematica

Time used: 0.465 (sec). Leaf size: 34

```
DSolve[1/x^2+ 3*y[x]^2/x^4==2*y[x]/x^3*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\sqrt{-1 + c_1 x}$$
$$y(x) \to x\sqrt{-1 + c_1 x}$$

1.60 problem 79

Internal problem ID [12157]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 79.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\frac{x^2 y'}{(-y+x)^2} - \frac{y^2}{(-y+x)^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x^2/(x-y(x))^2*diff(y(x),x)- y(x)^2/(x-y(x))^2=0,y(x), singsol=all)
```

$$y(x) = \frac{x}{c_1 x + 1}$$

✓ Solution by Mathematica

Time used: 0.193 (sec). Leaf size: 21

```
DSolve[x^2/(x-y[x])^2*y'[x]- y[x]^2/(x-y[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{1 - c_1 x}$$

$$y(x) \to 0$$

1.61 problem 80

Internal problem ID [12158]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 80.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _exact, _rational]`

$$yy' - \frac{y}{x^2+y^2} + \frac{xy'}{x^2+y^2} = -x$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 29

```
dsolve(x+y(x)*diff(y(x),x)= y(x)/(x^2+y(x)^2)- x/(x^2+y(x)^2)*diff(y(x),x),y(x), singsol=all
```

$$y(x) = \tan\left(\operatorname{RootOf}\left(-\tan\left(\_Z\right)^2 x^2 - x^2 + 2c_1 - 2\_Z\right)\right) x$$

✓ Solution by Mathematica

Time used: 0.18 (sec). Leaf size: 31

```
DSolve[x+y[x]*y'[x]== y[x]/(x^2+y[x]^2)- x/(x^2+y[x]^2)*y'[x],y[x],x,IncludeSingularSolution
```

$$\text{Solve}\left[-\arctan\left(\frac{x}{y(x)}\right) + \frac{x^2}{2} + \frac{y(x)^2}{2} = c_1, y(x)\right]$$

1.62 problem 89

Internal problem ID [12159]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 89.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y - 2xy' - {y'}^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 690

```
dsolve(y(x)=2*x*diff(y(x),x)+diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x)=\left(\frac{\left(6c_1-x^3+2\sqrt{-3c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}{2}+\frac{x^2}{2\left(6c_1-x^3+2\sqrt{-3c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}-\frac{x}{2}\right)^2+2\left(\frac{\left(6c_1-x^3+2\sqrt{-3c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}{2}+\frac{x^2}{2\left(6c_1-x^3+2\sqrt{-3c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}-\frac{x}{2}\right)x$$

$$y(x)=\left(-\frac{\left(6c_1-x^3+2\sqrt{-3c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}{4}-\frac{x^2}{4\left(6c_1-x^3+2\sqrt{-3c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}-\frac{x}{2}-\frac{i\sqrt{3}\left(\frac{\left(6c_1-x^3+2\sqrt{-3c_1\cdots}\right.}{2}\cdots\right.}{\cdots}\cdots\right.$$

$$+2\left(-\frac{\left(6c_1-x^3+2\sqrt{-3c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}{4}-\frac{x^2}{4\left(6c_1-x^3+2\sqrt{-3c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}-\frac{x}{2}-\frac{i\sqrt{3}\left(\frac{\left(6c_1-x^3+2\sqrt{-3c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}{2}-\frac{x^2}{2\left(6c_1-x^3+2\sqrt{-3c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}\right)}{2}\right)x$$

$$y(x)=\left(-\frac{\left(6c_1-x^3+2\sqrt{-3c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}{4}-\frac{x^2}{4\left(6c_1-x^3+2\sqrt{-3c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}-\frac{x}{2}+\frac{i\sqrt{3}\left(\frac{\left(6c_1-x^3+2\sqrt{-3c_1\cdots}\right.}{2}\cdots\right.}{\cdots}\cdots\right.$$

✓ Solution by Mathematica

Time used: 60.162 (sec). Leaf size: 931

```
DSolve[y[x]==2*x*y'[x]+(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-x^2+\frac{x(x^3+8e^{3c_1})}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}+\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(-18x^2-\frac{9i(\sqrt{3}-i)\,x(x^3+8e^{3c_1})}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}+9i\left(\sqrt{3}+i\right)\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(-18x^2+\frac{9i(\sqrt{3}+i)\,x(x^3+8e^{3c_1})}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}-9\left(1+i\sqrt{3}\right)\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{4}\left(-x^2+\frac{x(x^3-8e^{3c_1})}{\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}+\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(-18x^2+\frac{9(1+i\sqrt{3})\,x(-x^3+8e^{3c_1})}{\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}+9i\left(\sqrt{3}+i\right)\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(-18x^2+\frac{9i(\sqrt{3}+i)\,x(x^3-8e^{3c_1})}{\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}-9\left(1+i\sqrt{3}\right)\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

1.63 problem 90

Internal problem ID [12160]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 90.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, _dAlembert]`

$$y - x{y'}^2 - {y'}^2 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 99

```
dsolve(y(x)=x*diff(y(x),x)^2+diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \frac{x\left(x+1+\sqrt{c_1 x + c_1 + x + 1}\right)^2}{(x+1)^2} + \frac{\left(x+1+\sqrt{c_1 x + c_1 + x + 1}\right)^2}{(x+1)^2}$$

$$y(x) = \frac{x\left(-x-1+\sqrt{c_1 x + c_1 + x + 1}\right)^2}{(x+1)^2} + \frac{\left(-x-1+\sqrt{c_1 x + c_1 + x + 1}\right)^2}{(x+1)^2}$$

✓ Solution by Mathematica

Time used: 0.111 (sec). Leaf size: 57

```
DSolve[y[x]==x*(y'[x])^2+(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - c_1\sqrt{x+1} + 1 + \frac{{c_1}^2}{4}$$

$$y(x) \to x + c_1\sqrt{x+1} + 1 + \frac{{c_1}^2}{4}$$

$$y(x) \to 0$$

1.64 problem 91

Internal problem ID [12161]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 91.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y - x(1 + y') - y'^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve(y(x)=x*(1+diff(y(x),x))+diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x) = x\left(2 + \text{LambertW}\left(\frac{c_1 e^{\frac{x}{2}-1}}{2}\right) - \frac{x}{2}\right) + \left(\text{LambertW}\left(\frac{c_1 e^{\frac{x}{2}-1}}{2}\right) - \frac{x}{2} + 1\right)^2$$

✓ Solution by Mathematica

Time used: 3.313 (sec). Leaf size: 177

```
DSolve[y[x]==x*(1+y'[x])+(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\sqrt{x^2+4y(x)-4x} + 2\log\left(\sqrt{x^2+4y(x)-4x} - x + 2\right) - 2\log\left(-x\sqrt{x^2+4y(x)-4x} + x^2 + 4y(x) - 2x - 4\right) + x = c_1, y(x)\right]$$

$$\text{Solve}\left[-4\text{arctanh}\left(\frac{(x-5)\sqrt{x^2+4y(x)-4x} - x^2 - 4y(x) + 7x - 6}{(x-3)\sqrt{x^2+4y(x)-4x} - x^2 - 4y(x) + 5x - 2}\right) + \sqrt{x^2+4y(x)-4x} + x = c_1, y(x)\right]$$

1.65 problem 92

Internal problem ID [12162]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 92.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y - y{y'}^2 - 2xy' = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 75

```
dsolve(y(x)=y(x)*diff(y(x),x)^2+2*x*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = -ix$$
$$y(x) = ix$$
$$y(x) = 0$$
$$y(x) = \sqrt{c_1^2 - 2c_1x}$$
$$y(x) = \sqrt{c_1^2 + 2c_1x}$$
$$y(x) = -\sqrt{c_1^2 - 2c_1x}$$
$$y(x) = -\sqrt{c_1^2 + 2c_1x}$$

✓ Solution by Mathematica

Time used: 0.788 (sec). Leaf size: 126

```
DSolve[y[x]==y[x]*(y'[x])^2+2*x*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) &\to -e^{\frac{c_1}{2}}\sqrt{-2x+e^{c_1}} \\
y(x) &\to e^{\frac{c_1}{2}}\sqrt{-2x+e^{c_1}} \\
y(x) &\to -e^{\frac{c_1}{2}}\sqrt{2x+e^{c_1}} \\
y(x) &\to e^{\frac{c_1}{2}}\sqrt{2x+e^{c_1}} \\
y(x) &\to 0 \\
y(x) &\to -ix \\
y(x) &\to ix
\end{aligned}
$$

1.66 problem 94

Internal problem ID [12163]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 94.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type **[_quadrature]**

$$\boxed{y - yy' - y' + {y'}^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(y(x)=y(x)*diff(y(x),x)+diff(y(x),x)-diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x) = c_1 e^x$$
$$y(x) = x + c_1$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 19

```
DSolve[y[x]==y[x]*y'[x]+y'[x]-(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x$$
$$y(x) \to x + c_1$$

1.67 problem 95

Internal problem ID [12164]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 95.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Clairaut]`

$$y - xy' - \sqrt{1 - y'^2} = 0$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 17

```
dsolve(y(x)=x*diff(y(x),x)+sqrt(1-diff(y(x),x)^2),y(x), singsol=all)
```

$$y(x) = c_1 x + \sqrt{-c_1^2 + 1}$$

✓ Solution by Mathematica

Time used: 0.08 (sec). Leaf size: 27

```
DSolve[y[x]==x*y'[x]+Sqrt[1-y'[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 x + \sqrt{1 - c_1{}^2}$$
$$y(x) \rightarrow 1$$

1.68 problem 96

Internal problem ID [12165]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 96.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y - xy' - y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(y(x)=x*diff(y(x),x)+diff(y(x),x),y(x), singsol=all)
```

$$y(x) = c_1(x + 1)$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 16

```
DSolve[y[x]==x*y'[x]+y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x + 1)$$
$$y(x) \to 0$$

1.69 problem 97

Internal problem ID [12166]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 97.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Clairaut]`

$$y - xy' - \frac{1}{y'} = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 27

```
dsolve(y(x)=x*diff(y(x),x)+1/diff(y(x),x),y(x), singsol=all)
```

$$y(x) = -2\sqrt{x}$$
$$y(x) = 2\sqrt{x}$$
$$y(x) = c_1 x + \frac{1}{c_1}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 41

```
DSolve[y[x]==x*y'[x]+1/y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x + \frac{1}{c_1}$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to -2\sqrt{x}$$
$$y(x) \to 2\sqrt{x}$$

1.70 problem 98

Internal problem ID [12167]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 98.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y - xy' + \frac{1}{y'^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 77

```
dsolve(y(x)=x*diff(y(x),x)-1/diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x) = \frac{3(-2x^2)^{\frac{1}{3}}}{2}$$
$$y(x) = -\frac{3(-2x^2)^{\frac{1}{3}}}{4} - \frac{3i\sqrt{3}\,(-2x^2)^{\frac{1}{3}}}{4}$$
$$y(x) = -\frac{3(-2x^2)^{\frac{1}{3}}}{4} + \frac{3i\sqrt{3}\,(-2x^2)^{\frac{1}{3}}}{4}$$
$$y(x) = c_1 x - \frac{1}{c_1^2}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 71

```
DSolve[y[x]==x*y'[x]-1/(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x - \frac{1}{{c_1}^2}$$

$$y(x) \to -3\left(-\frac{1}{2}\right)^{2/3} x^{2/3}$$

$$y(x) \to -\frac{3x^{2/3}}{2^{2/3}}$$

$$y(x) \to \frac{3\sqrt[3]{-1}x^{2/3}}{2^{2/3}}$$

1.71 problem 110

Internal problem ID [12168]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 110.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{2y}{x} = -\sqrt{3}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=2*y(x)/x-sqrt(3),y(x), singsol=all)
```

$$y(x) = \left(\frac{\sqrt{3}}{x} + c_1\right) x^2$$

✓ Solution by Mathematica

Time used: 0.066 (sec). Leaf size: 17

```
DSolve[y'[x]==2*y[x]/x-Sqrt[3],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(\sqrt{3} + c_1 x\right)$$

1.72 problem 116

Internal problem ID [12169]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 116.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 2y'' - y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)-2*diff(y(x),x$2)-diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x} + c_2 e^{2x} + c_3 e^{x}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 28

```
DSolve[y'''[x]-2*y''[x]-y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x} + c_2 e^{x} + c_3 e^{2x}$$

1.73 problem 117

Internal problem ID [12170]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 117.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_poly_y`

$$\boxed{y'' - \frac{1}{2y'} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)=1/(2*diff(y(x),x)),y(x), singsol=all)
```

$$y(x) = \frac{2(x + c_1)^{\frac{3}{2}}}{3} + c_2$$
$$y(x) = -\frac{2(x + c_1)^{\frac{3}{2}}}{3} + c_2$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 43

```
DSolve[y''[x]==1/(2*y'[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{2}{3}(x + 2c_1)^{3/2}$$
$$y(x) \to \frac{2}{3}(x + 2c_1)^{3/2} + c_2$$

1.74 problem 118

Internal problem ID [12171]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 118.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _quadrature]]`

$$\boxed{xy''' = 2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(x*diff(y(x),x$3)=2,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^2}{2} - \frac{3x^2}{2} + x^2 \ln(x) + c_2 x + c_3$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 28

```
DSolve[x*y'''[x]==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 \log(x) + \left(-\frac{3}{2} + c_3\right) x^2 + c_2 x + c_1$$

1.75 problem 120

Internal problem ID [12172]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 120.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - ya^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)=a^2*y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^{-ax} + c_2 e^{ax}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 23

```
DSolve[y''[x]==a^2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{ax} + c_2 e^{-ax}$$

1.76 problem 121

Internal problem ID [12173]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 121.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' - \frac{a}{y^3} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 70

```
dsolve(diff(y(x),x$2)=a/y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{c_1 \left(c_1^2 c_2^2 + 2c_1^2 c_2 x + c_1^2 x^2 + a\right)}}{c_1}$$
$$y(x) = -\frac{\sqrt{c_1 \left(c_1^2 c_2^2 + 2c_1^2 c_2 x + c_1^2 x^2 + a\right)}}{c_1}$$

✓ Solution by Mathematica

Time used: 4.493 (sec). Leaf size: 63

```
DSolve[y''[x]==a/y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{a + c_1{}^2 (x + c_2)^2}}{\sqrt{c_1}}$$
$$y(x) \to \frac{\sqrt{a + c_1{}^2 (x + c_2)^2}}{\sqrt{c_1}}$$
$$y(x) \to \text{Indeterminate}$$

1.77 problem 122

Internal problem ID [12174]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 122.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$xy'' - y' = x^2 e^x$$

With initial conditions

$$[y(0) = -1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 17

```
dsolve([x*diff(y(x),x$2)-diff(y(x),x)=x^2*exp(x),y(0) = -1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = (x-1)\,e^x + \frac{c_1 x^2}{2}$$

✓ Solution by Mathematica

Time used: 0.114 (sec). Leaf size: 22

```
DSolve[{x*y''[x]-y'[x]==x^2*Exp[x],{y[0]==-1,y'[0]==0}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to e^x(x-1) + \frac{c_1 x^2}{2}$$

1.78 problem 123

Internal problem ID [12175]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 123.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1],`

$$\boxed{yy'' - y'^2 + y'^3 = 0}$$

With initial conditions

$$[y(0) = -1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 5

```
dsolve([y(x)*diff(y(x),x$2)-diff(y(x),x)^2+diff(y(x),x)^3=0,y(0) = -1, D(y)(0) = 0],y(x), si
```

$$y(x) = -1$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y[x]*y''[x]-(y'[x])^2+(y'[x])^3==0,{y[0]==-1,y'[0]==0}},y[x],x,IncludeSingularSoluti
```

{}

1.79 problem 124

Internal problem ID [12176]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 124.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + y' \tan(x) = \sin(2x)}$$

With initial conditions

$$[y(0) = -1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$2)+tan(x)*diff(y(x),x)=sin(2*x),y(0) = -1, D(y)(0) = 0],y(x), singsol=al
```

$$y(x) = -x - 1 + 2\sin(x) - \frac{\sin(2x)}{2}$$

✓ Solution by Mathematica

Time used: 0.154 (sec). Leaf size: 18

```
DSolve[{y''[x]+Tan[x]*y'[x]==Sin[2*x],{y[0]==-1,y'[0]==0}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to -x - \sin(x)(\cos(x) - 2) - 1$$

1.80 problem 125

Internal problem ID [12177]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 125.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y''^2 + y'^2 = a^2}$$

With initial conditions

$$[y(0) = -1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 24

```
dsolve([diff(y(x),x$2)^2+diff(y(x),x)^2=a^2,y(0) = -1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = -1 - a + a\cos(x)$$
$$y(x) = a - 1 - a\cos(x)$$

✓ Solution by Mathematica

Time used: 15.637 (sec). Leaf size: 37

```
DSolve[{(y''[x])^2+(y'[x])^2==a^2,{y[0]==-1,y'[0]==0}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to a\left(\frac{1}{\sqrt{\sec^2(x)}} - 1\right) - 1$$
$$y(x) \to -\frac{a}{\sqrt{\sec^2(x)}} + a - 1$$

1.81 problem 126

Internal problem ID [12178]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 126.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_poly_y`

$$y'' - \frac{1}{2y'} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)=1/(2*diff(y(x),x)),y(x), singsol=all)
```

$$y(x) = \frac{2(x+c_1)^{\frac{3}{2}}}{3} + c_2$$

$$y(x) = -\frac{2(x+c_1)^{\frac{3}{2}}}{3} + c_2$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 43

```
DSolve[y''[x]==1/(2*y'[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{2}{3}(x+2c_1)^{3/2}$$

$$y(x) \to \frac{2}{3}(x+2c_1)^{3/2} + c_2$$

1.82 problem 127

Internal problem ID [12179]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 127.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x], [_3rd_order, _missing_y], [_3rd_ord`

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$3)=diff(y(x),x$2)^2,y(x), singsol=all)
```

$$y(x) = -\ln(x + c_1)(x + c_1) + c_1 + x + c_2x + c_3$$

✓ Solution by Mathematica

Time used: 0.607 (sec). Leaf size: 24

```
DSolve[y'''[x]==(y''[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + c_3x - (x + c_1)\log(x + c_1) + c_2$$

1.83 problem 128

Internal problem ID [12180]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 128.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x], [_3rd_order, _missing_y], [_3rd_ord`

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 65

```
dsolve(diff(y(x),x)*diff(y(x),x$3)-3*diff(y(x),x$2)^2=0,y(x), singsol=all)
```

$$y(x) = c_1$$

$$y(x) = \frac{-c_2c_1 + \sqrt{c_1^2c_2^2 - 2c_1c_3 - 2c_1x}}{c_1}$$

$$y(x) = -\frac{c_2c_1 + \sqrt{c_1^2c_2^2 - 2c_1c_3 - 2c_1x}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.307 (sec). Leaf size: 21

```
DSolve[y'[x]*y'''[x]-3*(y''[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2\sqrt{2x + c_1} + c_3$$

1.84 problem 129

Internal problem ID [12181]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 129.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 9y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)=9*y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^{-3x} + c_2 e^{3x}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 22

```
DSolve[y''[x]==9*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(c_1 e^{6x} + c_2\right)$$

1.85 problem 130

Internal problem ID [12182]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 130.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x$2)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(x) + c_2 \cos(x)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 16

```
DSolve[y''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(x) + c_2 \sin(x)$$

1.86 problem 131

Internal problem ID [12183]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 131.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x} + c_2 e^{x}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 20

```
DSolve[y''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x} + c_2 e^{-x}$$

1.87 problem 132

Internal problem ID [12184]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 132.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 7y' + 12y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+12*y(x)=7*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = c_1 e^{3x} + c_2 e^{4x}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 20

```
DSolve[y''[x]+12*y[x]==7*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{3x}(c_2 e^x + c_1)$$

1.88 problem 133

Internal problem ID [12185]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 133.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{2x} + c_2 e^{2x} x$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 18

```
DSolve[y''[x]-4*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}(c_2 x + c_1)$$

1.89 problem 134

Internal problem ID [12186]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 134.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 2y' + 10y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+10*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x} \sin(3x) + c_2 e^{-x} \cos(3x)$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 26

```
DSolve[y''[x]+2*y'[x]+10*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_2 \cos(3x) + c_1 \sin(3x))$$

1.90 problem 135

Internal problem ID [12187]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 135.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 3y' - 2y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{\left(-3+\sqrt{17}\right)x}{2}} + c_2 e^{-\frac{\left(3+\sqrt{17}\right)x}{2}}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 35

```
DSolve[y''[x]+3*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{1}{2}\left(3+\sqrt{17}\right)x}\left(c_2 e^{\sqrt{17}x} + c_1\right)$$

1.91 problem 136

Internal problem ID [12188]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 136.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' - 12y' + 9y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(4*diff(y(x),x$2)-12*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{3x}{2}} + c_2 e^{\frac{3x}{2}} x$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 20

```
DSolve[4*y''[x]-12*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{3x/2}(c_2 x + c_1)$$

1.92 problem 137

Internal problem ID [12189]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 137.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\, x}{2}\right) + c_2 e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\, x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 42

```
DSolve[y''[x]+y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}\left(c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

1.93 problem 140

Internal problem ID [12190]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 140.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 5y'' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$4)-5*diff(y(x),x$2)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x} + c_2 e^{-2x} + c_3 e^{2x} + c_4 e^{x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 35

```
DSolve[y''''[x]-5*y''[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(c_2 e^{x} + e^{3x}(c_4 e^{x} + c_3) + c_1\right)$$

1.94 problem 141

Internal problem ID [12191]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 141.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 2y'' - y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)-2*diff(y(x),x$2)-diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x} + c_2 e^{2x} + c_3 e^{x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 28

```
DSolve[y'''[x]-2*y''[x]-y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x} + c_2 e^{x} + c_3 e^{2x}$$

1.95 problem 142

Internal problem ID [12192]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 142.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 3ay'' + 3a^2y' - a^3y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$3)-3*a*diff(y(x),x$2)+3*a^2*diff(y(x),x)-a^3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{ax} + c_2 e^{ax} x + c_3 e^{ax} x^2$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 23

```
DSolve[y'''[x]-3*a*y''[x]+3*a^2*y'[x]-a^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{ax}(x(c_3x + c_2) + c_1)$$

1.96 problem 143

Internal problem ID [12193]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 143.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(5)} - 4y''' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$5)-4*diff(y(x),x$3)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2x + c_3x^2 + c_4\mathrm{e}^{-2x} + c_5\mathrm{e}^{2x}$$

✓ Solution by Mathematica

Time used: 0.073 (sec). Leaf size: 39

```
DSolve[y'''''[x]-4*y'''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}c_1e^{2x} - \frac{1}{8}c_2e^{-2x} + x(c_5x + c_4) + c_3$$

1.97 problem 144

Internal problem ID [12194]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 144.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 2y'' + 9y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$2)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x} \sin\left(\sqrt{2}\, x\right) + c_2 e^{-x} \cos\left(\sqrt{2}\, x\right) + c_3 e^{x} \sin\left(\sqrt{2}\, x\right) + c_4 e^{x} \cos\left(\sqrt{2}\, x\right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 52

```
DSolve[y''''[x]+2*y''[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(\left(c_4 e^{2x} + c_2\right) \cos\left(\sqrt{2}x\right) + \left(c_3 e^{2x} + c_1\right) \sin\left(\sqrt{2}x\right)\right)$$

1.98 problem 145

Internal problem ID [12195]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 145.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 8y'' + 16y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$4)-8*diff(y(x),x$2)+16*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-2x} + c_2 e^{-2x} x + c_3 e^{2x} + c_4 e^{2x} x$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 35

```
DSolve[y''''[x]-8*y''[x]+16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x} \left(c_3 e^{4x} + x \left(c_4 e^{4x} + c_2\right) + c_1\right)$$

1.99 problem 146

Internal problem ID [12196]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 146.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 71

```
dsolve(diff(y(x),x$4)+y(x)=0,y(x), singsol=all)
```

$$y(x) = -c_1 e^{-\frac{\sqrt{2}\,x}{2}} \sin\left(\frac{\sqrt{2}\,x}{2}\right) - c_2 e^{\frac{\sqrt{2}\,x}{2}} \sin\left(\frac{\sqrt{2}\,x}{2}\right) + c_3 e^{-\frac{\sqrt{2}\,x}{2}} \cos\left(\frac{\sqrt{2}\,x}{2}\right) + c_4 e^{\frac{\sqrt{2}\,x}{2}} \cos\left(\frac{\sqrt{2}\,x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 65

```
DSolve[y''''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{x}{\sqrt{2}}} \left(\left(c_1 e^{\sqrt{2}x} + c_2\right) \cos\left(\frac{x}{\sqrt{2}}\right) + \left(c_4 e^{\sqrt{2}x} + c_3\right) \sin\left(\frac{x}{\sqrt{2}}\right)\right)$$

1.100 problem 147

Internal problem ID [12197]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 147.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - a^4 y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$4)-a^4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{ax} + c_2 \mathrm{e}^{-ax} + c_3 \sin(ax) + c_4 \cos(ax)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 37

```
DSolve[y''''[x]-a^4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 e^{-ax} + c_4 e^{ax} + c_1 \cos(ax) + c_3 \sin(ax)$$

1.101 problem 148

Internal problem ID [12198]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 148.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 7y' + 12y = x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-7*diff(y(x),x)+12*y(x)=x,y(x), singsol=all)
```

$$y(x) = c_2 \mathrm{e}^{3x} + \mathrm{e}^{4x} c_1 + \frac{x}{12} + \frac{7}{144}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 30

```
DSolve[y''[x]-7*y'[x]+12*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{12} + c_1 e^{3x} + c_2 e^{4x} + \frac{7}{144}$$

1.102 problem 149

Internal problem ID [12199]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 149.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{s'' - a^2 s = t + 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(s(t),t$2)-a^2*s(t)=t+1,s(t), singsol=all)
```

$$s(t) = e^{at} c_2 + e^{-at} c_1 + \frac{-t-1}{a^2}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 31

```
DSolve[s''[t]-a^2*s[t]==1+t,s[t],t,IncludeSingularSolutions -> True]
```

$$s(t) \to -\frac{t+1}{a^2} + c_1 e^{at} + c_2 e^{-at}$$

1.103 problem 150

Internal problem ID [12200]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 150.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' - 2y = 8 \sin(2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-2*y(x)=8*sin(2*x),y(x), singsol=all)
```

$$y(x) = c_2 e^{-2x} + e^x c_1 - \frac{2\cos(2x)}{5} - \frac{6\sin(2x)}{5}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 35

```
DSolve[y''[x]+y'[x]-2*y[x]==8*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-2x} + c_2 e^x - \frac{2}{5}(3\sin(2x) + \cos(2x))$$

1.104 problem 151

Internal problem ID [12201]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 151.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y = 5x + 2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)-y(x)=5*x+2,y(x), singsol=all)
```

$$y(x) = c_2 \mathrm{e}^{x} + c_1 \mathrm{e}^{-x} - 2 - 5x$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 24

```
DSolve[y''[x]-y[x]==5*x+2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -5x + c_1 e^{x} + c_2 e^{-x} - 2$$

1.105 problem 152

Internal problem ID [12202]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 152.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2ay' + ya^2 = e^x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)-2*a*diff(y(x),x)+a^2*y(x)=exp(x),y(x), singsol=all)
```

$$y(x) = c_2 e^{ax} + x\,e^{ax} c_1 + \frac{e^x}{(a-1)^2}$$

✓ Solution by Mathematica

Time used: 0.224 (sec). Leaf size: 28

```
DSolve[y''[x]-2*a*y'[x]+a^2*y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x}{(a-1)^2} + e^{ax}(c_2 x + c_1)$$

1.106 problem 153

Internal problem ID [12203]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 153.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 6y' + 5y = e^{2x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+6*diff(y(x),x)+5*y(x)=exp(2*x),y(x), singsol=all)
```

$$y(x) = c_1 e^{-x} + c_2 e^{-5x} + \frac{e^{2x}}{21}$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 31

```
DSolve[y''[x]+6*y'[x]+5*y[x]==Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{2x}}{21} + c_1 e^{-5x} + c_2 e^{-x}$$

1.107 problem 154

Internal problem ID [12204]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 154.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 9y = 6\,e^{3x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+9*y(x)=6*exp(3*x),y(x), singsol=all)
```

$$y(x) = c_1 \cos(3x) + c_2 \sin(3x) + \frac{e^{3x}}{3}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 29

```
DSolve[y''[x]+9*y[x]==6*Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{3x}}{3} + c_1 \cos(3x) + c_2 \sin(3x)$$

1.108 problem 155

Internal problem ID [12205]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 155.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - 3y' = 2 - 6x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)=2-6*x,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{3x}}{3} + x^2 + c_2$$

✓ Solution by Mathematica

Time used: 0.121 (sec). Leaf size: 22

```
DSolve[y''[x]-3*y'[x]==2-6*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + \frac{1}{3} c_1 e^{3x} + c_2$$

1.109 problem 156

Internal problem ID [12206]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 156.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + 3y = e^{-x} \cos(x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+3*y(x)=exp(-x)*cos(x),y(x), singsol=all)
```

$$y(x) = \sin\left(\sqrt{2}\, x\right) e^{x} c_2 + \cos\left(\sqrt{2}\, x\right) e^{x} c_1 - \frac{e^{-x}(4 \sin(x) - 5 \cos(x))}{41}$$

✓ Solution by Mathematica

Time used: 1.089 (sec). Leaf size: 56

```
DSolve[y''[x]-2*y'[x]+3*y[x]==Exp[-x]*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{4}{41} e^{-x} \sin(x) + \frac{5}{41} e^{-x} \cos(x) + c_2 e^{x} \cos\left(\sqrt{2}x\right) + c_1 e^{x} \sin\left(\sqrt{2}x\right)$$

1.110 problem 157

Internal problem ID [12207]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 157.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = 2\sin(2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+4*y(x)=2*sin(2*x),y(x), singsol=all)
```

$$y(x) = c_2 \sin(2x) + c_1 \cos(2x) - \frac{x\cos(2x)}{2}$$

✓ Solution by Mathematica

Time used: 0.17 (sec). Leaf size: 33

```
DSolve[y''[x]+4*y[x]==2*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}((1 + 8c_2)\sin(2x) - 4(x - 2c_1)\cos(2x))$$

1.111 problem 158

Internal problem ID [12208]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 158.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - 4y'' + 5y' - 2y = 2x + 3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$3)-4*diff(y(x),x$2)+5*diff(y(x),x)-2*y(x)=2*x+3,y(x), singsol=all)
```

$$y(x) = -x - 4 + c_1 e^x + c_2 e^{2x} + c_3 e^x x$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 31

```
DSolve[y'''[x]-4*y''[x]+5*y'[x]-2*y[x]==2*x+3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x + x(-1 + c_2 e^x) + c_3 e^{2x} - 4$$

1.112 problem 159

Internal problem ID [12209]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 159.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - a^4 y = 5a^4 e^{ax} \sin(ax)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$4)-a^4*y(x)=5*a^4*exp(a*x)*sin(a*x),y(x), singsol=all)
```

$$y(x) = -e^{ax} \sin(ax) + c_1 \cos(ax) + c_2 e^{ax} + c_3 \sin(ax) + c_4 e^{-ax}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 45

```
DSolve[y''''[x]-a^4*y[x]==5*a^4*Exp[a*x]*Sin[a*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 e^{-ax} + c_4 e^{ax} + c_1 \cos(ax) + (-e^{ax} + c_3) \sin(ax)$$

1.113 problem 160

Internal problem ID [12210]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 160.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' + 2a^2y'' + a^4y = 8\cos(ax)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(diff(y(x),x$4)+2*a^2*diff(y(x),x$2)+a^4*y(x)=8*cos(a*x),y(x), singsol=all)
```

$$y(x) = -\frac{(a^2x^2-2)\cos(ax)}{a^4} + \frac{3\sin(ax)x}{a^3} + c_1\cos(ax) + c_2\sin(ax) + c_3\cos(ax)x + c_4\sin(ax)x$$

✓ Solution by Mathematica

Time used: 0.187 (sec). Leaf size: 64

```
DSolve[y''''[x]+2*a^2*y''[x]+a^4*y[x]==8*Cos[a*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2a(x(2+a^3c_4)+a^3c_3)\sin(ax) + (2a^4(c_2x+c_1)-2a^2x^2+5)\cos(ax)}{2a^4}$$

1.114 problem 162

Internal problem ID [12211]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 162.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2hy' + yn^2 = 0}$$

With initial conditions

$$[y(0) = a, y'(0) = c]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 93

```
dsolve([diff(y(x),x$2)+2*h*diff(y(x),x)+n^2*y(x)=0,y(0) = a, D(y)(0) = c],y(x), singsol=all)
```

$$y(x) = \frac{\left(\sqrt{h^2-n^2}\, a + ha + c\right) e^{\left(-h+\sqrt{h^2-n^2}\right)x} - e^{-\left(h+\sqrt{h^2-n^2}\right)x} \left(-\sqrt{h^2-n^2}\, a + ha + c\right)}{2\sqrt{h^2-n^2}}$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 123

```
DSolve[{y''[x]+2*h*y'[x]+n^2*y[x]==0,{y[0]==a,y'[0]==c}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{e^{-\left(x\left(\sqrt{h^2-n^2}+h\right)\right)} \left(ah\left(e^{2x\sqrt{h^2-n^2}}-1\right) + a\sqrt{h^2-n^2}\left(e^{2x\sqrt{h^2-n^2}}+1\right) + c\left(e^{2x\sqrt{h^2-n^2}}-1\right)\right)}{2\sqrt{h^2-n^2}}$$

1.115 problem 163

Internal problem ID [12212]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 163.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y n^2 = h \sin(rx)}$$

With initial conditions

$$[y(0) = a, y'(0) = c]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 60

```
dsolve([diff(y(x),x$2)+n^2*y(x)=h*sin(r*x),y(0) = a, D(y)(0) = c],y(x), singsol=all)
```

$$y(x) = -\frac{(-c n^2 + c r^2 + hr)\sin(nx)}{n^3 - n r^2} + \cos(nx)\, a + \frac{h \sin(rx)}{n^2 - r^2}$$

✓ Solution by Mathematica

Time used: 0.066 (sec). Leaf size: 63

```
DSolve[{y''[x]+n^2*y[x]==h*Sin[r*x],{y[0]==a,y'[0]==c}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{an(n^2 - r^2)\cos(nx) + \sin(nx)\,(cn^2 - cr^2 - hr) + hn\sin(rx)}{n^3 - nr^2}$$

1.116 problem 167

Internal problem ID [12213]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 167.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 7y' + 6y = \sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)-7*diff(y(x),x)+6*y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = c_1 e^{6x} + c_2 e^{x} + \frac{7\cos(x)}{74} + \frac{5\sin(x)}{74}$$

✓ Solution by Mathematica

Time used: 0.104 (sec). Leaf size: 32

```
DSolve[y''[x]-7*y'[x]+6*y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{5\sin(x)}{74} + \frac{7\cos(x)}{74} + c_1 e^{x} + c_2 e^{6x}$$

1.117 problem 168

Internal problem ID [12214]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 168.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sec(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+y(x)=sec(x),y(x), singsol=all)
```

$$y(x) = c_2 \sin(x) + c_1 \cos(x) + x \sin(x) - \ln(\sec(x)) \cos(x)$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 22

```
DSolve[y''[x]+y[x]==Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_2) \sin(x) + \cos(x)(\log(\cos(x)) + c_1)$$

1.118 problem 169

Internal problem ID [12215]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 169.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y = \frac{1}{\cos(2x)^{\frac{3}{2}}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)+y(x)=1/(cos(2*x)*sqrt(cos(2*x))),y(x), singsol=all)
```

$$y(x) = c_1 \cos(x) + c_2 \sin(x) + \frac{-\cos(x)^2 \sqrt{-2\sin(x)^4 + \sin(x)^2} + \sqrt{(2\cos(x)^2 - 1)\sin(x)^2}\,\sin(x)^2}{\sqrt{-2\sin(x)^4 + \sin(x)^2}\,\sqrt{2\cos(x)^2 - 1}}$$

✓ Solution by Mathematica

Time used: 0.123 (sec). Leaf size: 26

```
DSolve[y''[x]+y[x]==1/(Cos[2*x]*Sqrt[Cos[2*x]]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\cos(2x)} + c_1 \cos(x) + c_2 \sin(x)$$

1.119 problem 170

Internal problem ID [12216]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 170.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = y(t) + 1$$
$$y'(t) = x(t) + 1$$

With initial conditions

$$[x(0) = -2, y(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 22

```
dsolve([diff(x(t),t) = y(t)+1, diff(y(t),t) = x(t)+1, x(0) = -2, y(0) = 0],[x(t), y(t)], sin
```

$$x(t) = -1 - e^{-t}$$

$$y(t) = -1 + e^{-t}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 24

```
DSolve[{x'[t]==y[t]+1,y'[t]==x[t]+1},{x[0]==-2,y[0]==0},{x[t],y[t]},t,IncludeSingularSolutio
```

$$x(t) \to -e^{-t} - 1$$
$$y(t) \to e^{-t} - 1$$

1.120 problem 171

Internal problem ID [12217]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 171.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) - 2y(t)$$
$$y'(t) = x(t) - y(t)$$

With initial conditions

$$[x(0) = 1, y(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(x(t),t) = x(t)-2*y(t), diff(y(t),t) = x(t)-y(t), x(0) = 1, y(0) = 1],[x(t), y(t
```

$$x(t) = \cos(t) - \sin(t)$$

$$y(t) = \cos(t)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 17

```
DSolve[{x'[t]==x[t]-2*y[t],y'[t]==x[t]-y[t]},{x[0]==1,y[0]==1},{x[t],y[t]},t,IncludeSingular
```

$$x(t) \to \cos(t) - \sin(t)$$
$$y(t) \to \cos(t)$$

1.121 problem 172

Internal problem ID [12218]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 172.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -y(t) + \cos(t)$$
$$y'(t) = -4y(t) + 4\cos(t) + 3x(t) - \sin(t)$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 37

```
dsolve([4*diff(x(t),t)-diff(y(t),t)+3*x(t)=sin(t),diff(x(t),t)+y(t)=cos(t)],[x(t), y(t)], si
```

$$x(t) = \frac{c_2 e^{-3t}}{3} + c_1 e^{-t}$$

$$y(t) = c_2 e^{-3t} + c_1 e^{-t} + \cos(t)$$

✓ Solution by Mathematica

Time used: 0.352 (sec). Leaf size: 76

```
DSolve[{4*x'[t]-y'[t]+3*x[t]==Sin[t],x'[t]+y[t]==Cos[t]},{x[t],y[t]},t,IncludeSingularSoluti
```

$$x(t) \to \frac{1}{2}e^{-3t}\left(c_1\left(3e^{2t}-1\right) - c_2\left(e^{2t}-1\right)\right)$$
$$y(t) \to \cos(t) + \frac{1}{2}e^{-3t}\left(3c_1\left(e^{2t}-1\right) - c_2\left(e^{2t}-3\right)\right)$$

1.122 problem 181

Internal problem ID [12219]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 181.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{yy'' - y'^2 = 1}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 79

```
dsolve(y(x)*diff(y(x),x$2)=1+diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x) = \frac{c_1\left(\mathrm{e}^{-\frac{2x}{c_1}}\,\mathrm{e}^{-\frac{2c_2}{c_1}} + 1\right)\mathrm{e}^{\frac{x}{c_1}}\,\mathrm{e}^{\frac{c_2}{c_1}}}{2}$$
$$y(x) = \frac{c_1\left(\mathrm{e}^{\frac{2x}{c_1}}\,\mathrm{e}^{\frac{2c_2}{c_1}} + 1\right)\mathrm{e}^{-\frac{x}{c_1}}\,\mathrm{e}^{-\frac{c_2}{c_1}}}{2}$$

✓ Solution by Mathematica

Time used: 60.235 (sec). Leaf size: 80

```
DSolve[y[x]*y''[x]==1+(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{-c_1}\tanh\left(e^{c_1}(x+c_2)\right)}{\sqrt{-\operatorname{sech}^2\left(e^{c_1}(x+c_2)\right)}}$$
$$y(x) \to \frac{e^{-c_1}\tanh\left(e^{c_1}(x+c_2)\right)}{\sqrt{-\operatorname{sech}^2\left(e^{c_1}(x+c_2)\right)}}$$

1.123 problem 182

Internal problem ID [12220]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 182.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\frac{x^2y'}{(-y+x)^2}-\frac{y^2}{(-y+x)^2}=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x^2*diff(y(x),x)/(x-y(x))^2-y(x)^2/(x-y(x))^2=0,y(x), singsol=all)
```

$$y(x)=\frac{x}{c_1x+1}$$

✓ Solution by Mathematica

Time used: 0.225 (sec). Leaf size: 21

```
DSolve[x^2*y'[x]/(x-y[x])^2-y[x]^2/(x-y[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x}{1-c_1x}$$
$$y(x)\to 0$$

1.124 problem 183

Internal problem ID [12221]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 183.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, _dAlembert]`

$$\boxed{y - x{y'}^2 - {y'}^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 99

```
dsolve(y(x)=x*diff(y(x),x)^2+diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{x\left(x+1+\sqrt{c_1x+c_1+x+1}\right)^2}{\left(x+1\right)^2} + \frac{\left(x+1+\sqrt{c_1x+c_1+x+1}\right)^2}{\left(x+1\right)^2}$$
$$y(x) = \frac{x\left(-x-1+\sqrt{c_1x+c_1+x+1}\right)^2}{\left(x+1\right)^2} + \frac{\left(-x-1+\sqrt{c_1x+c_1+x+1}\right)^2}{\left(x+1\right)^2}$$

✓ Solution by Mathematica

Time used: 0.108 (sec). Leaf size: 57

```
DSolve[y[x]==x*(y'[x])^2+(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - c_1\sqrt{x+1} + 1 + \frac{{c_1}^2}{4}$$
$$y(x) \to x + c_1\sqrt{x+1} + 1 + \frac{{c_1}^2}{4}$$
$$y(x) \to 0$$

1.125 problem 184

Internal problem ID [12222]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 184.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sec(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+y(x)=sec(x),y(x), singsol=all)
```

$$y(x) = c_2 \sin(x) + c_1 \cos(x) + x \sin(x) - \ln(\sec(x)) \cos(x)$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 22

```
DSolve[y''[x]+y[x]==Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_2) \sin(x) + \cos(x)(\log(\cos(x)) + c_1)$$

1.126 problem 185

Internal problem ID [12223]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 185.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2+1)\,y'-yx=\alpha}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve((1+x^2)*diff(y(x),x)-x*y(x)-alpha=0,y(x), singsol=all)
```

$$y(x)=\alpha x+c_1\sqrt{x^2+1}$$

✓ Solution by Mathematica

Time used: 0.114 (sec). Leaf size: 21

```
DSolve[(1+x^2)*y'[x]-x*y[x]-a==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to ax+c_1\sqrt{x^2+1}$$

1.127 problem 186

Internal problem ID [12224]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 186.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{x \cos\left(\frac{y}{x}\right) y' - y \cos\left(\frac{y}{x}\right) = -x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x*cos( y(x)/x)*diff(y(x),x)=y(x)*cos( y(x)/x) - x,y(x), singsol=all)
```

$$y(x) = -\arcsin\left(\ln\left(x\right) + c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.628 (sec). Leaf size: 15

```
DSolve[x*Cos[ y[x]/x]*y'[x]==y[x]*Cos[ y[x]/x] - x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \arcsin(-\log(x) + c_1)$$

1.128 problem 187

Internal problem ID [12225]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 187.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y = \sin(2x)\,e^{2x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)-4*y(x)=exp(2*x)*sin(2*x),y(x), singsol=all)
```

$$y(x) = c_2 e^{2x} + c_1 e^{-2x} - \frac{e^{2x}(2\cos(2x) + \sin(2x))}{20}$$

✓ Solution by Mathematica

Time used: 0.152 (sec). Leaf size: 42

```
DSolve[y''[x]-4*y[x]==Exp[2*x]*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{2x} + c_2 e^{-2x} - \frac{1}{20}e^{2x}(\sin(2x) + 2\cos(2x))$$

1.129 problem 188

Internal problem ID [12226]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 188.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{xy' + y - y^2 \ln(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x*diff(y(x),x)+y(x)-y(x)^2*ln(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{c_1 x + \ln(x) + 1}$$

✓ Solution by Mathematica

Time used: 0.237 (sec). Leaf size: 20

```
DSolve[x*y'[x]+y[x]-y[x]^2*Log[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{\log(x) + c_1 x + 1}$$

$$y(x) \to 0$$

1.130 problem 189

Internal problem ID [12227]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 189.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{2y + (x + y - 2)\, y' = -2x + 1}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve((2*x+2*y(x)-1)+(x+y(x)-2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -x - 3\,\mathrm{LambertW}\left(-\frac{\mathrm{e}^{\frac{x}{3}} c_1 \mathrm{e}^{-\frac{1}{3}}}{3}\right) - 1$$

✓ Solution by Mathematica

Time used: 5.545 (sec). Leaf size: 35

```
DSolve[(2*x+2*y[x]-1)+(x+y[x]-2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -3W\left(-e^{\frac{x}{3}-1+c_1}\right) - x - 1$$
$$y(x) \to -x - 1$$

1.131 problem 190

Internal problem ID [12228]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 190.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$3\,\mathrm{e}^{x}\tan(y)+(1-\mathrm{e}^{x})\sec(y)^{2}y'=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 200

```
dsolve(3*exp(x)*tan(y(x))+(1-exp(x))*sec(y(x))^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\frac{\arctan\left(\frac{2c_1\left(-1+\mathrm{e}^{3x}-3\,\mathrm{e}^{2x}+3\,\mathrm{e}^{x}\right)}{\mathrm{e}^{6x}c_1^2-6\,\mathrm{e}^{5x}c_1^2+15\,\mathrm{e}^{4x}c_1^2-20\,\mathrm{e}^{3x}c_1^2+15\,\mathrm{e}^{2x}c_1^2-6\,\mathrm{e}^{x}c_1^2+c_1^2+1},-\frac{\mathrm{e}^{6x}c_1^2-6\,\mathrm{e}^{5x}c_1^2+15\,\mathrm{e}^{4x}c_1^2-20\,\mathrm{e}^{3x}c_1^2+15\,\mathrm{e}^{2x}c_1^2-6\,\mathrm{e}^{x}c_1^2+c_1^2-1}{\mathrm{e}^{6x}c_1^2-6\,\mathrm{e}^{5x}c_1^2+15\,\mathrm{e}^{4x}c_1^2-20\,\mathrm{e}^{3x}c_1^2+15\,\mathrm{e}^{2x}c_1^2-6\,\mathrm{e}^{x}c_1^2+c_1^2+1}\right)}{2}$$

✓ Solution by Mathematica

Time used: 1.847 (sec). Leaf size: 74

```
DSolve[3*Exp[x]*Tan[y[x]]+(1-Exp[x])*Sec[y[x]]^2*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{1}{2} \arccos \left(-\tanh \left(3 \log \left(e^x-1\right)+2 c_1\right)\right)$$
$$y(x) \to \frac{1}{2} \arccos \left(-\tanh \left(3 \log \left(e^x-1\right)+2 c_1\right)\right)$$
$$y(x) \to 0$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

1.132 problem 191

Internal problem ID [12229]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 191.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 2x(t) - 3y(t)$$
$$y'(t) = 5x(t) + 6y(t)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 78

```
dsolve([diff(x(t),t)=2*x(t)-3*y(t),diff(y(t),t)=5*x(t)+6*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{e^{4t}\left(\sin\left(\sqrt{11}\,t\right)\sqrt{11}\,c_2 - \cos\left(\sqrt{11}\,t\right)\sqrt{11}\,c_1 + 2\sin\left(\sqrt{11}\,t\right)c_1 + 2\cos\left(\sqrt{11}\,t\right)c_2\right)}{5}$$

$$y(t) = e^{4t}\left(\sin\left(\sqrt{11}\,t\right)c_1 + \cos\left(\sqrt{11}\,t\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 99

```
DSolve[{x'[t]==2*x[t]-3*y[t],y'[t]==5*x[t]+6*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to c_1 e^{4t}\cos\left(\sqrt{11}t\right) - \frac{(2c_1 + 3c_2)e^{4t}\sin\left(\sqrt{11}t\right)}{\sqrt{11}}$$
$$y(t) \to c_2 e^{4t}\cos\left(\sqrt{11}t\right) + \frac{(5c_1 + 2c_2)e^{4t}\sin\left(\sqrt{11}t\right)}{\sqrt{11}}$$

1.133 problem 192

Internal problem ID [12230]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 192.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -4x(t) - 10y(t)$$
$$y'(t) = x(t) - 2y(t)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 59

```
dsolve([diff(x(t),t)=-4*x(t)-10*y(t),diff(y(t),t)=x(t)-2*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -e^{-3t}(\sin(3t)\, c_1 + 3\sin(3t)\, c_2 - 3\cos(3t)\, c_1 + \cos(3t)\, c_2)$$

$$y(t) = e^{-3t}(\sin(3t)\, c_1 + \cos(3t)\, c_2)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 67

```
DSolve[{x'[t]==-4*x[t]-10*y[t],y'[t]==x[t]-2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{1}{3}e^{-3t}(3c_1\cos(3t) - (c_1 + 10c_2)\sin(3t))$$
$$y(t) \to \frac{1}{3}e^{-3t}(3c_2\cos(3t) + (c_1 + c_2)\sin(3t))$$

1.134 problem 193

Internal problem ID [12231]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 193.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 12x(t) + 18y(t)$$
$$y'(t) = -8x(t) - 12y(t)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve([diff(x(t),t)=12*x(t)+18*y(t),diff(y(t),t)=-8*x(t)-12*y(t)],[x(t), y(t)], singsol=all
```

$$x(t) = -\frac{1}{8}c_1 - \frac{3}{2}c_1t - \frac{3}{2}c_2$$

$$y(t) = c_1t + c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 36

```
DSolve[{x'[t]==12*x[t]+18*y[t],y'[t]==-8*x[t]-12*y[t]},{x[t],y[t]},t,IncludeSingularSolution
```

$$x(t) \to 12c_1t + 18c_2t + c_1$$
$$y(t) \to c_2 - 4(2c_1 + 3c_2)t$$

1.135 problem 194

Internal problem ID [12232]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 194.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{y' - y^2 = x}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 97

```
dsolve([diff(y(x),x)=y(x)^2+x,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\left(-2\pi 3^{\frac{5}{6}} + 3\,3^{\frac{2}{3}}\Gamma\left(\frac{2}{3}\right)^2\right)\mathrm{AiryAi}\left(1, -x\right) + \mathrm{AiryBi}\left(1, -x\right)\left(3\,3^{\frac{1}{6}}\Gamma\left(\frac{2}{3}\right)^2 + 2\pi 3^{\frac{1}{3}}\right)}{\left(-2\pi 3^{\frac{5}{6}} + 3\,3^{\frac{2}{3}}\Gamma\left(\frac{2}{3}\right)^2\right)\mathrm{AiryAi}\left(-x\right) + \mathrm{AiryBi}\left(-x\right)\left(3\,3^{\frac{1}{6}}\Gamma\left(\frac{2}{3}\right)^2 + 2\pi 3^{\frac{1}{3}}\right)}$$

✓ Solution by Mathematica

Time used: 1.986 (sec). Leaf size: 145

```
DSolve[{y'[x]==y[x]^2+x,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{3}\,\mathrm{Gamma}\left(\frac{2}{3}\right)\left(x^{3/2}\,\mathrm{BesselJ}\left(-\frac{4}{3}, \frac{2x^{3/2}}{3}\right) - x^{3/2}\,\mathrm{BesselJ}\left(\frac{2}{3}, \frac{2x^{3/2}}{3}\right) + \mathrm{BesselJ}\left(-\frac{1}{3}, \frac{2x^{3/2}}{3}\right)\right) - 2x^{3/2}\,\mathrm{Ga}}{2x\left(\mathrm{Gamma}\left(\frac{1}{3}\right)\mathrm{BesselJ}\left(\frac{1}{3}, \frac{2x^{3/2}}{3}\right) - \sqrt[3]{3}\,\mathrm{Gamma}\left(\frac{2}{3}\right)\mathrm{BesselJ}\left(-\frac{1}{3}, \frac{2x^{3/2}}{3}\right)\right)}$$

1.136 problem 195

Internal problem ID [12233]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 195.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' + \frac{y}{x} = e^x$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve([diff(y(x),x)+1/x*y(x)=exp(x),y(1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{1 + e^x(x - 1)}{x}$$

✓ Solution by Mathematica

Time used: 0.082 (sec). Leaf size: 18

```
DSolve[{y'[x]+1/x*y[x]==Exp[x],{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x(x - 1) + 1}{x}$$

1.137 problem 196

Internal problem ID [12234]

Book: DIFFERENTIAL and INTEGRAL CALCULUS. VOL I. by N. PISKUNOV. MIR PUBLISHERS, Moscow 1969.
Section: Chapter 8. Differential equations. Exercises page 595
Problem number: 196.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = y(t) - x(t)$$
$$y'(t) = -x(t) - 3y(t)$$

With initial conditions

$$[x(1) = 0, y(1) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

```
dsolve([diff(x(t),t) = y(t)-x(t), diff(y(t),t) = -x(t)-3*y(t), x(1) = 0, y(1) = 1],[x(t), y(
```

$$x(t) = -\mathrm{e}^{-2t}\left(-\mathrm{e}^{2}t + \mathrm{e}^{2}\right)$$

$$y(t) = \mathrm{e}^{-2t}\left(-\mathrm{e}^{2}t + 2\,\mathrm{e}^{2}\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 31

```
DSolve[{x'[t]==y[t]-x[t],y'[t]==-x[t]-3*y[t]},{x[1]==0,y[1]==1},{x[t],y[t]},t,IncludeSingula
```

$$x(t) \to e^{2-2t}(t-1)$$
$$y(t) \to -e^{2-2t}(t-2)$$