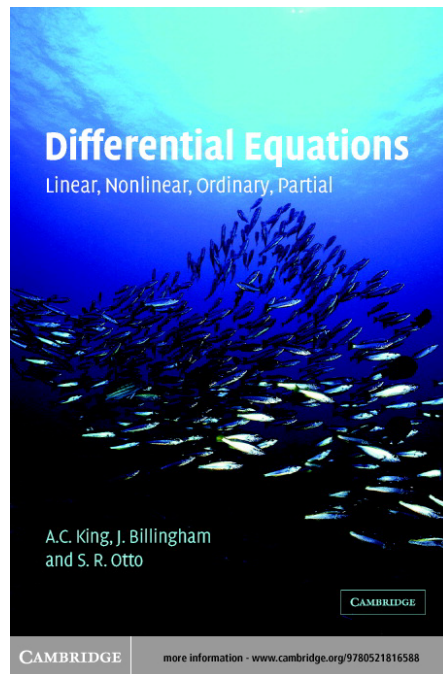


A Solution Manual For

**Differential Equations, Linear,
Nonlinear, Ordinary, Partial.**

**A.C. King, J. Billingham,
S.R. Otto. Cambridge Univ.
Press 2003**



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1.1 problem Problem 1.1(a)

Internal problem ID [12074]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003

Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28

Problem number: Problem 1.1(a).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x - 1)y'' - xy' + y = 0$$

Given that one solution of the ode is

$$y_1 = e^x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve([(x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,exp(x)],y(x), singsol=all)
```

$$y(x) = c_1x + c_2e^x$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 17

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1e^x - c_2x$$

1.2 problem Problem 1.1(b)

Internal problem ID [12075]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003

Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28

Problem number: Problem 1.1(b).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [Lienard]

$$xy'' + 2y' + yx = 0$$

Given that one solution of the ode is

$$y_1 = \frac{\sin(x)}{x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve([x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,sin(x)/x],y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{x} + \frac{c_2 \cos(x)}{x}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 37

```
DSolve[x*y''[x]+2*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x}$$

1.3 problem Problem 1.3(a)

Internal problem ID [12076]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003

Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28

Problem number: Problem 1.3(a).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2y' + y = x^{\frac{3}{2}}e^x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=x^(3/2)*exp(x),y(x), singsol=all)
```

$$y(x) = \frac{4x^{\frac{7}{2}}e^x}{35} + c_1x e^x + c_2e^x$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 29

```
DSolve[y''[x]-2*y'[x]+y[x]==x^(3/2)*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{35}e^x(4x^{7/2} + 35c_2x + 35c_1)$$

1.4 problem Problem 1.3(b)

Internal problem ID [12077]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003

Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28

Problem number: Problem 1.3(b).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y = 2 \sec(2x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)+4*y(x)=2*sec(2*x),y(x), singsol=all)
```

$$y(x) = c_2 \sin(2x) + c_1 \cos(2x) + x \sin(2x) - \frac{\ln(\sec(2x)) \cos(2x)}{2}$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 32

```
DSolve[y''[x]+4*y[x]==2*Sec[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow (x + c_2) \sin(2x) + \cos(2x) \left(\frac{1}{2} \log(\cos(2x)) + c_1 \right)$$

1.5 problem Problem 1.3(c)

Internal problem ID [12078]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003

Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28

Problem number: Problem 1.3(c).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + \frac{y'}{x} + \left(1 - \frac{1}{4x^2}\right) y = x$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 71

```
dsolve(diff(y(x),x$2)+1/x*diff(y(x),x)+(1-1/(4*x^2))*y(x)=x,y(x), singsol=all)
```

$$y(x) = \frac{c_2 \sin(x)}{\sqrt{x}} + \frac{\cos(x) c_1}{\sqrt{x}} - \frac{3 \left(\sin(x) \sqrt{2} \sqrt{\pi} \operatorname{FresnelC}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right) - \cos(x) \sqrt{2} \sqrt{\pi} \operatorname{FresnelS}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right) - \frac{4x^{\frac{3}{2}}}{3} \right)}{4\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.443 (sec). Leaf size: 111

```
DSolve[y''[x]+1/x*y'[x]+(1-1/(4*x^2))*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^{-ix} \left(-\frac{e^{2ix} x^{3/2} \Gamma\left(\frac{5}{2}, ix\right)}{\sqrt{-ix}} + \sqrt{x^2} (2c_1 - ic_2 e^{2ix}) + \frac{(ix)^{3/2} \Gamma\left(\frac{5}{2}, -ix\right)}{\sqrt{x}} \right)}{2\sqrt{x}\sqrt{x^2}}$$

1.6 problem Problem 1.3(d)

Internal problem ID [12079]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003

Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28

Problem number: Problem 1.3(d).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y = f(x)$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve([diff(y(x),x$2)+y(x)=f(x),y(0) = 0, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \left(\int_0^x \cos(_z1) f(_z1) d_z1 \right) \sin(x) - \left(\int_0^x \sin(_z1) f(_z1) d_z1 \right) \cos(x)$$

✓ Solution by Mathematica

Time used: 0.105 (sec). Leaf size: 77

```
DSolve[{y'[x]+y[x]==f[x],{y[0]==0,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\sin(x) \int_1^0 \cos(K[2])f(K[2])dK[2] + \sin(x) \int_1^x \cos(K[2])f(K[2])dK[2] \\ + \cos(x) \left(\int_1^x -f(K[1]) \sin(K[1])dK[1] - \int_1^0 -f(K[1]) \sin(K[1])dK[1] \right)$$

1.7 problem Problem 1.6(a)

Internal problem ID [12080]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003

Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28

Problem number: Problem 1.6(a).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y''x^2 + x\left(x - \frac{1}{2}\right)y' + \frac{y}{2} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(x^2*diff(y(x),x$2)+x*(x-1/2)*diff(y(x),x)+1/2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \text{WhittakerM}\left(\frac{1}{4}, \frac{1}{4}, x\right) x^{\frac{1}{4}} e^{-\frac{x}{2}} + c_2 \sqrt{x} e^{-x}$$

✓ Solution by Mathematica

Time used: 0.084 (sec). Leaf size: 37

```
DSolve[x^2*y'[x]+x*(x-1/2)*y'[x]+1/2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x} \left(c_2 \sqrt{x} + c_1 \sqrt{-x} \Gamma\left(\frac{1}{2}, -x\right) \right)$$

1.8 problem Problem 1.6(b)

Internal problem ID [12081]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003

Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28

Problem number: Problem 1.6(b).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y''x^2 + x(x+1)y' - y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x$2)+x*(1+x)*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x-1)}{x} + \frac{c_2e^{-x}}{x}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 26

```
DSolve[x^2*y''[x]+x*(1+x)*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^{-x}(c_1e^x(x-1) + c_2)}{x}$$

1.9 problem Problem 1.7

Internal problem ID [12082]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003

Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28

Problem number: Problem 1.7.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [Jacobi]

$$x(1-x)y'' + (-5x+1)y' - 4y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
Order:=6;  
dsolve(x*(1-x)*diff(y(x),x$2)+(1-5*x)*diff(y(x),x)-4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1) (1 + 4x + 9x^2 + 16x^3 + 25x^4 + 36x^5 + O(x^6)) \\ + ((-4)x - 12x^2 - 24x^3 - 40x^4 - 60x^5 + O(x^6)) c_2$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 87

```
AsymptoticDSolveValue[x*(1-x)*y''[x]+(1-5*x)*y'[x]-4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1(36x^5 + 25x^4 + 16x^3 + 9x^2 + 4x + 1) \\ + c_2(-60x^5 - 40x^4 - 24x^3 - 12x^2 + (36x^5 + 25x^4 + 16x^3 + 9x^2 + 4x + 1) \log(x) - 4x)$$

1.10 problem Problem 1.8(a)

Internal problem ID [12083]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003

Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28

Problem number: Problem 1.8(a).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 - 1)^2 y'' + (x + 1) y' - y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;  
dsolve((x^2-1)^2*diff(y(x),x$2)+(x+1)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{6}x^4 - \frac{7}{60}x^5\right) y(0) \\ + \left(x - \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{6}x^4 + \frac{7}{60}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 70

```
AsymptoticDSolveValue[(x^2-1)^2*y''[x]+(x+1)*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(-\frac{7x^5}{60} + \frac{x^4}{6} - \frac{x^3}{6} + \frac{x^2}{2} + 1 \right) + c_2 \left(\frac{7x^5}{60} - \frac{x^4}{6} + \frac{x^3}{6} - \frac{x^2}{2} + x \right)$$

1.11 problem Problem 1.8(b)

Internal problem ID [12084]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003

Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28

Problem number: Problem 1.8(b).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' + 4y' - yx = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
Order:=6;  
dsolve(x*diff(y(x),x$2)+4*diff(y(x),x)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 \left(1 + \frac{1}{10}x^2 + \frac{1}{280}x^4 + O(x^6) \right) + \frac{c_2(12 - 6x^2 - \frac{3}{2}x^4 + O(x^6))}{x^3}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 42

```
AsymptoticDSolveValue[x*y''[x]+4*y'[x]-x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{1}{x^3} - \frac{x}{8} - \frac{1}{2x} \right) + c_2 \left(\frac{x^4}{280} + \frac{x^2}{10} + 1 \right)$$

1.12 problem Problem 1.9

Internal problem ID [12085]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003

Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28

Problem number: Problem 1.9.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2xy'' + (x + 1)y' - ky = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 132

```
Order:=6;
dsolve(2*x*diff(y(x),x$2)+(1+x)*diff(y(x),x)-k*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = & c_1 \sqrt{x} \left(1 + \left(\frac{k}{3} - \frac{1}{6} \right) x + \left(\frac{1}{30} k^2 - \frac{1}{15} k + \frac{1}{40} \right) x^2 + \frac{1}{5040} (2k-5)(2k-3)(2k-1) x^3 \right. \\ & \left. + \frac{1}{362880} (2k-7)(2k-5)(2k-3)(2k-1) x^4 \right. \\ & \left. + \frac{1}{39916800} (2k-9)(2k-7)(2k-5)(2k-3)(2k-1) x^5 + O(x^6) \right) \\ & + c_2 \left(1 + kx + \frac{1}{6} (k-1) kx^2 + \frac{1}{90} (k-2)(k-1) kx^3 + \frac{1}{2520} (k-3)(k-2)(k-1) kx^4 \right. \\ & \left. + \frac{1}{113400} (k-4)(k-3)(k-2)(k-1) kx^5 + O(x^6) \right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 304

```
AsymptoticDSolveValue[2*x*y'[x]+(1+x)*y'[x]-k*y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned} y(x) \rightarrow & c_1 \sqrt{x} \left(\frac{4 \left(\frac{3}{4} - \frac{k}{2}\right) \left(\frac{5}{4} - \frac{k}{2}\right) \left(\frac{7}{4} - \frac{k}{2}\right) \left(\frac{9}{4} - \frac{k}{2}\right) \left(\frac{k}{2} - \frac{1}{4}\right) x^5}{155925} \right. \\ & - \frac{2 \left(\frac{3}{4} - \frac{k}{2}\right) \left(\frac{5}{4} - \frac{k}{2}\right) \left(\frac{7}{4} - \frac{k}{2}\right) \left(\frac{k}{2} - \frac{1}{4}\right) x^4}{2835} + \frac{4 \left(\frac{3}{4} - \frac{k}{2}\right) \left(\frac{5}{4} - \frac{k}{2}\right) \left(\frac{k}{2} - \frac{1}{4}\right) x^3}{315} \\ & \left. - \frac{2 \left(\frac{3}{4} - \frac{k}{2}\right) \left(\frac{k}{2} - \frac{1}{4}\right) x^2}{15} + \frac{2 \left(\frac{k}{2} - \frac{1}{4}\right) x}{3} + 1 \right) \\ & + c_2 \left(\frac{2 \left(\frac{1}{2} - \frac{k}{2}\right) \left(1 - \frac{k}{2}\right) \left(\frac{3}{2} - \frac{k}{2}\right) \left(2 - \frac{k}{2}\right) k x^5}{14175} \right. \\ & - \frac{1}{315} \left(\frac{1}{2} - \frac{k}{2}\right) \left(1 - \frac{k}{2}\right) \left(\frac{3}{2} - \frac{k}{2}\right) k x^4 + \frac{2}{45} \left(\frac{1}{2} - \frac{k}{2}\right) \left(1 - \frac{k}{2}\right) k x^3 \\ & \left. - \frac{1}{3} \left(\frac{1}{2} - \frac{k}{2}\right) k x^2 + k x + 1 \right) \end{aligned}$$

1.13 problem Problem 1.11(a)

Internal problem ID [12086]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003

Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28

Problem number: Problem 1.11(a).


ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [[_Emden, _Fowler]]

$$x^3 y'' + x^2 y' + y = 0$$

With the expansion point for the power series method at $x = 0$.

 Solution by Maple

```
Order:=6;
dsolve(x^3*dif(y(x),x$2)+x^2*dif(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

No solution found

 Solution by Mathematica

Time used: 0.057 (sec). Leaf size: 222

```
AsymptoticDSolveValue[x^3*y'[x]+x^2*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 e^{-\frac{2i}{\sqrt{x}} \sqrt[4]{x}} \left(\frac{418854310875ix^{9/2}}{8796093022208} - \frac{57972915ix^{7/2}}{4294967296} + \frac{59535ix^{5/2}}{8388608} - \frac{75ix^{3/2}}{8192} \right. \\ \left. - \frac{30241281245175x^5}{281474976710656} + \frac{13043905875x^4}{549755813888} - \frac{2401245x^3}{268435456} + \frac{3675x^2}{524288} - \frac{9x}{512} + \frac{i\sqrt{x}}{16} \right. \\ \left. + 1 \right) + c_2 e^{\frac{2i}{\sqrt{x}} \sqrt[4]{x}} \left(-\frac{418854310875ix^{9/2}}{8796093022208} + \frac{57972915ix^{7/2}}{4294967296} - \frac{59535ix^{5/2}}{8388608} + \frac{75ix^{3/2}}{8192} - \frac{30241281245175x^5}{281474976710656} + \right.$$

1.14 problem Problem 1.11(b)

Internal problem ID [12087]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003

Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28

Problem number: Problem 1.11(b).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$y''x^2 + y' - 2y = 0$$

With the expansion point for the power series method at $x = 0$.

 Solution by Maple

```
Order:=6;  
dsolve(x^2*diff(y(x),x$2)+diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

No solution found

 Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 28

```
AsymptoticDSolveValue[x^2*y'[x]+y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 e^{\frac{1}{x}x^2} + c_1(2x^2 + 2x + 1)$$

1.15 problem Problem 1.12

Internal problem ID [12088]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003

Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28

Problem number: Problem 1.12.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2y''x^2 + x(1-x)y' - y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

Order:=6;

```
dsolve(2*x^2*diff(y(x),x$2)+x*(1-x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{3}{2}} \left(1 + \frac{1}{5}x + \frac{1}{35}x^2 + \frac{1}{315}x^3 + \frac{1}{3465}x^4 + \frac{1}{45045}x^5 + O(x^6)\right) + c_1 \left(1 + \frac{1}{2}x + \frac{1}{8}x^2 + \frac{1}{48}x^3 + \frac{1}{384}x^4 + \frac{1}{3840}x^5 + \dots\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 86

```
AsymptoticDSolveValue[2*x^2*y''[x]+x*(1-x)*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 x \left(\frac{x^5}{45045} + \frac{x^4}{3465} + \frac{x^3}{315} + \frac{x^2}{35} + \frac{x}{5} + 1 \right) + \frac{c_2 \left(\frac{x^5}{3840} + \frac{x^4}{384} + \frac{x^3}{48} + \frac{x^2}{8} + \frac{x}{2} + 1 \right)}{\sqrt{x}}$$

1.16 problem Problem 1.13

Internal problem ID [12089]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003

Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28

Problem number: Problem 1.13.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$y''x(x-1) + 3xy' + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 60

```
Order:=6;  
dsolve(x*(x-1)*diff(y(x),x$2)+3*x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = & \ln(x) (x + 2x^2 + 3x^3 + 4x^4 + 5x^5 + O(x^6)) c_2 \\ & + c_1 x (1 + 2x + 3x^2 + 4x^3 + 5x^4 + 6x^5 + O(x^6)) \\ & + (1 + 3x + 5x^2 + 7x^3 + 9x^4 + 11x^5 + O(x^6)) c_2 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 63

```
AsymptoticDSolveValue[x*(x-1)*y''[x]+3*x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned} y(x) \rightarrow & c_1 (x^4 + x^3 + x^2 + (4x^3 + 3x^2 + 2x + 1) x \log(x) + x + 1) \\ & + c_2 (5x^5 + 4x^4 + 3x^3 + 2x^2 + x) \end{aligned}$$

2 Chapter 3 Bessel functions. Problems page 89

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2.1 problem Problem 3.7(a)

Internal problem ID [12090]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003

Section: Chapter 3 Bessel functions. Problems page 89

Problem number: Problem 3.7(a).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$y'' - yx^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)-x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sqrt{x} \text{BesselI}\left(\frac{1}{4}, \frac{x^2}{2}\right) + c_2 \sqrt{x} \text{BesselK}\left(\frac{1}{4}, \frac{x^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 37

```
DSolve[y''[x]-x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_2 \text{ParabolicCylinderD}\left(-\frac{1}{2}, i\sqrt{2}x\right) + c_1 \text{ParabolicCylinderD}\left(-\frac{1}{2}, \sqrt{2}x\right)$$

2.2 problem Problem 3.7(b)

Internal problem ID [12091]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003

Section: Chapter 3 Bessel functions. Problems page 89

Problem number: Problem 3.7(b).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$xy'' + y' + y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x$2)+diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \text{BesselJ}(0, 2\sqrt{x}) + c_2 \text{BesselY}(0, 2\sqrt{x})$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 31

```
DSolve[x*y''[x]+y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \text{BesselJ}(0, 2\sqrt{x}) + 2c_2 \text{BesselY}(0, 2\sqrt{x})$$

2.3 problem Problem 3.7(c)

Internal problem ID [12092]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003

Section: Chapter 3 Bessel functions. Problems page 89

Problem number: Problem 3.7(c).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' + (x + 1)^2 y = 0$$

X Solution by Maple

```
dsolve(x*diff(y(x),x$2)+(x+1)^2*y(x)=0,y(x), singsol=all)
```

No solution found

X Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]+(x+1)^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.4 problem Problem 3.7(d)

Internal problem ID [12093]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003

Section: Chapter 3 Bessel functions. Problems page 89

Problem number: Problem 3.7(d).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + \alpha^2 y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+alpha^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(\alpha x) + c_2 \cos(\alpha x)$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 20

```
DSolve[y''[x]+a^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \cos(ax) + c_2 \sin(ax)$$

2.5 problem Problem 3.7(e)

Internal problem ID [12094]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003

Section: Chapter 3 Bessel functions. Problems page 89

Problem number: Problem 3.7(e).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - \alpha^2 y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-alpha^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\alpha x} + c_2 e^{-\alpha x}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 23

```
DSolve[y''[x]-a^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{\alpha x} + c_2 e^{-\alpha x}$$

2.6 problem Problem 3.7(f)

Internal problem ID [12095]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003

Section: Chapter 3 Bessel functions. Problems page 89

Problem number: Problem 3.7(f).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + \beta y' + \gamma y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x$2)+beta*diff(y(x),x)+gamma*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\left(-\frac{\beta}{2} + \frac{\sqrt{\beta^2 - 4\gamma}}{2}\right)x} + c_2 e^{\left(-\frac{\beta}{2} - \frac{\sqrt{\beta^2 - 4\gamma}}{2}\right)x}$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 47

```
DSolve[y''[x]+\[Beta]*y'[x]+\[Gamma]*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-\frac{1}{2}x(\sqrt{\beta^2 - 4\gamma} + \beta)} \left(c_2 e^{x\sqrt{\beta^2 - 4\gamma}} + c_1 \right)$$

2.7 problem Problem 3.7(g)

Internal problem ID [12096]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003

Section: Chapter 3 Bessel functions. Problems page 89

Problem number: Problem 3.7(g).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [Gegenbauer]

$$(-x^2 + 1)y'' - 2y'x + n(1 + n)y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+n*(n+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \text{LegendreP}(n, x) + c_2 \text{LegendreQ}(n, x)$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 18

```
DSolve[(1-x^2)*y'[x]-2*x*y'[x]+n*(n+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \text{LegendreP}(n, x) + c_2 \text{LegendreQ}(n, x)$$

2.8 problem Problem 3.12

Internal problem ID [12097]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003

Section: Chapter 3 Bessel functions. Problems page 89

Problem number: Problem 3.12.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y''x^2 + xy' + (-\nu^2 + x^2)y = \sin(x)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 161

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-nu^2)*y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = \text{BesselJ}(\nu, x) c_2 + \text{BesselY}(\nu, x) c_1$$

$$- \frac{x(\text{hypergeom}([\frac{1}{2} - \frac{\nu}{2}, \frac{5}{4} - \frac{\nu}{2}, \frac{3}{4} - \frac{\nu}{2}], [\frac{3}{2}, 1 - \nu, \frac{3}{2} - \nu, \frac{3}{2} - \frac{\nu}{2}], -x^2) \text{BesselJ}(\nu, x) \Gamma(\nu + 2)^2 2^\nu x^{-\nu} + \dots}{\dots}$$

✓ Solution by Mathematica

Time used: 1.228 (sec). Leaf size: 205

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-\[Nu]^2)*y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \rightarrow$$

$$- \frac{\pi 2^{\nu-1} \csc(\pi \nu) x^{1-\nu} \text{BesselJ}(\nu, x) {}_3F_4(\frac{1}{2} - \frac{\nu}{2}, \frac{3}{4} - \frac{\nu}{2}, \frac{5}{4} - \frac{\nu}{2}; \frac{3}{2}, 1 - \nu, \frac{3}{2} - \nu, \frac{3}{2} - \frac{\nu}{2}; -x^2)}{(\nu - 1) \Gamma(1 - \nu)}$$

$$+ \frac{\pi 2^{-\nu-1} x^{\nu+1} (\text{BesselY}(\nu, x) - \cot(\pi \nu) \text{BesselJ}(\nu, x)) {}_3F_4(\frac{\nu}{2} + \frac{1}{2}, \frac{\nu}{2} + \frac{3}{4}, \frac{\nu}{2} + \frac{5}{4}; \frac{3}{2}, \frac{\nu}{2} + \frac{3}{2}, \nu + 1, \nu + \frac{3}{2}; -x^2)}{(\nu + 1) \Gamma(\nu + 1)}$$

$$+ c_1 \text{BesselJ}(\nu, x) + c_2 \text{BesselY}(\nu, x)$$