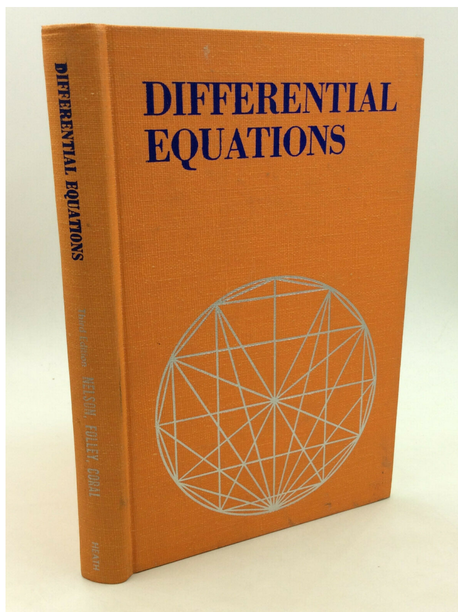
A Solution Manual For

Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964



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1 Exercise 5, page 21

1.1 problem 1

Internal problem ID [1870]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yx + (x^2 + 1) y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x*y(x)+(x^2+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{x^2 + 1}}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 22

```
DSolve[x*y[x]+(x^2+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{\sqrt{x^2 + 1}}$$
$$y(x) \to 0$$

1.2 problem 2

Internal problem ID [1871]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy^2 + \left(y - yx^2\right) y' = -x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
dsolve((x*y(x)^2+x)+(y(x)-x^2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{c_1 x^2 - c_1 - 1}$$
$$y(x) = -\sqrt{c_1 x^2 - c_1 - 1}$$

✓ Solution by Mathematica

Time used: 1.353 (sec). Leaf size: 61

```
DSolve[(x*y[x]^2+x)+(y[x]-x^2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-1 + e^{2c_1} \left(x^2 - 1\right)}$$
$$y(x) \to \sqrt{-1 + e^{2c_1} \left(x^2 - 1\right)}$$
$$y(x) \to -i$$
$$y(x) \to i$$

1.3 problem 3

Internal problem ID [1872]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^2 + (x^2 + 1)\, y' = -1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve((1+y(x)^2)+(1+x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\tan(\arctan(x) + c_1)$$

✓ Solution by Mathematica

Time used: 0.233 (sec). Leaf size: 29

```
DSolve[(1+y[x]^2)+(1+x^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\tan(\arctan(x) - c_1)$$
$$y(x) \to -i$$
$$y(x) \to i$$

1.4 problem 4

Internal problem ID [1873]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y + xy' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 9

```
dsolve(y(x)+x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 16

```
DSolve[y[x]+x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x}$$
$$y(x) \to 0$$

1.5 problem 5

Internal problem ID [1874]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 2yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=2*x*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x^2}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 18

```
DSolve[y'[x]==2*x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x^2}$$
$$y(x) \to 0$$

1.6 problem 6

Internal problem ID [1875]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy^2 + (yx^2 - y)\,y' = -x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve((x*y(x)^2+x)+(x^2*y(x)-y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(x^2-1)(-x^2+c_1)}}{x^2-1}$$
$$y(x) = -\frac{\sqrt{(x^2-1)(-x^2+c_1)}}{x^2-1}$$

✓ Solution by Mathematica

Time used: 0.369 (sec). Leaf size: 133

```
DSolve[(x*y[x]^2+x)+(x^2*y[x]-y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-x^2+1-e^{2c_1}}}{\sqrt{x^2-1}}$$
$$y(x) \to \frac{\sqrt{-x^2+1-e^{2c_1}}}{\sqrt{x^2-1}}$$
$$y(x) \to -i$$
$$y(x) \to i$$
$$y(x) \to -\frac{\sqrt{1-x^2}}{\sqrt{x^2-1}}$$
$$y(x) \to \frac{\sqrt{1-x^2}}{\sqrt{x^2-1}}$$

1.7 problem 7

Internal problem ID [1876]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sqrt{1-y^2}\,y' = -\sqrt{1-x^2}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(sqrt(1-x^2)+sqrt(1-y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$c_1 + x\sqrt{-x^2+1} + \arcsin(x) + y(x)\sqrt{1-y(x)^2} + \arcsin(y(x)) = 0$$

✓ Solution by Mathematica

Time used: 0.626 (sec). Leaf size: 85

```
DSolve[Sqrt[1-x^2]+Sqrt[1-y[x]^2]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{1}{2}\#1\sqrt{1-\#1^2} - \arctan\left(\frac{\sqrt{1-\#1^2}}{\#1+1}\right)\&\right]\left[\arctan\left(\frac{\sqrt{1-x^2}}{x+1}\right) - \frac{1}{2}\sqrt{1-x^2}x + c_1\right]$$

1.8 problem 8

Internal problem ID [1877]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x+1)\,y'+y=1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve((1+x)*diff(y(x),x)-(1-y(x))=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1+x}{x+1}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 20

```
DSolve[(1+x)*y'[x]-(1-y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x+c_1}{x+1}$$
$$y(x)\to 1$$

1.9 problem 9

Internal problem ID [1878]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' \tan(x) - y = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)*tan(x)-y(x)=1,y(x), singsol=all)
```

$$y(x) = (-\csc(x) + c_1)\sin(x)$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 17

```
DSolve[y'[x]*Tan[x]-y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -1 + c_1 \sin(x)$$
$$y(x) \to -1$$

1.10 problem 10

Internal problem ID [1879]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y + \cot(x)\, y' = -3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve((y(x)+3)+cot(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = (-3\sec(x) + c_1)\cos(x)$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 17

```
DSolve[(y[x]+3)+Cot[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -3 + c_1 \cos(x)$$
$$y(x) \to -3$$

1.11 problem 11

Internal problem ID [1880]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{x}{y} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)=x/y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{x^2 + c_1}$$
$$y(x) = -\sqrt{x^2 + c_1}$$

✓ Solution by Mathematica

Time used: 0.087 (sec). Leaf size: 35

```
DSolve[y'[x]==x/y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x^2 + 2c_1}$$
$$y(x) \to \sqrt{x^2 + 2c_1}$$

1.12 problem 12

Internal problem ID [1881]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' = 1 - \sin(2t)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(x(t),t)=1-sin(2*t),x(t), singsol=all)
```

$$x(t) = \frac{\cos(2t)}{2} + t + c_1$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 17

```
DSolve[x'[t]==1-Sin[2*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to t + \frac{1}{2}\cos(2t) + c_1$$

1.13 problem 13

Internal problem ID [1882]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y + xy' - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x*diff(y(x),x)+y(x)=y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{1}{c_1 x + 1}$$

✓ Solution by Mathematica

Time used: 0.274 (sec). Leaf size: 25

```
DSolve[x*y'[x]+y[x]==y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{1 + e^{c_1} x}$$
$$y(x) \to 0$$
$$y(x) \to 1$$

1.14 problem 14

Internal problem ID [1883]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sin(x)\cos(y)^{2}+\cos(x)^{2} y'=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve(sin(x)*cos(y(x))^2+cos(x)^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=-\arctan(\sec(x)+c_1)$$

✓ Solution by Mathematica

Time used: 1.616 (sec). Leaf size: 31

```
DSolve[Sin[x]*Cos[y[x]]^2+Cos[x]^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \arctan(-\sec(x)+c_1)$$
$$y(x)\to -\frac{\pi}{2}$$
$$y(x)\to \frac{\pi}{2}$$

1.15 problem 15

Internal problem ID [1884]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sec(x)\cos(y)^2 - \cos(x)\sin(y)\,y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve(sec(x)*cos(y(x))^2=cos(x)*sin(y(x))*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{1}{\tan(x) + c_1}\right)$$

✓ Solution by Mathematica

Time used: 0.83 (sec). Leaf size: 45

```
DSolve[Sec[x]*Cos[y[x]]^2==Cos[x]*Sin[y[x]]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sec^{-1}(\tan(x) + 2c_1)$$
$$y(x) \to \sec^{-1}(\tan(x) + 2c_1)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

1.16 problem 16

Internal problem ID [1885]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y + xy' - xy(y' - 1) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(y(x)+x*diff(y(x),x)=x*y(x)*(diff(y(x),x)-1),y(x), singsol=all)
```

$$y(x) = -\operatorname{LambertW}\left(-\frac{\mathrm{e}^{-x}}{c_1 x}\right)$$

✓ Solution by Mathematica

Time used: 3.823 (sec). Leaf size: 28

```
DSolve[y[x]+x*y'[x]==x*y[x]*(y'[x]-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -W\left(-\frac{e^{-x-c_1}}{x}\right)$$
$$y(x) \to 0$$

1.17 problem 17

Internal problem ID [1886]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yx + \sqrt{x^2+1}\, y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*y(x)+sqrt(1+x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\sqrt{x^2+1}}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 26

```
DSolve[x*y[x]+Sqrt[1+x^2]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\sqrt{x^2+1}}$$
$$y(x) \to 0$$

1.18 problem 18

Internal problem ID [1887]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y - yx - y'x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(y(x)=x*y(x)+x^2*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{-\frac{1}{x}}}{x}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 23

```
DSolve[y[x]==x*y[x]+x^2*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 e^{-1/x}}{x}$$
$$y(x) \to 0$$

1.19 problem 19

Internal problem ID [1888]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\cos(x)^2 \cot(y) y' = -\tan(x) \sin(x)^2}$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 40

```
dsolve(tan(x)*sin(x)^2+cos(x)^2*cot(y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \arcsin\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{1+\cos(2x)}}\,\mathrm{e}^{\frac{-1+\cos(2x)}{2\cos(2x)+2}}}{c_1}\right)$$

✓ Solution by Mathematica

Time used: 17.888 (sec). Leaf size: 24

```
DSolve[Tan[x]*Sin[x]^2+Cos[x]^2*Cot[y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin\left(\frac{1}{8} c_1 e^{-\frac{1}{2}\sec^2(x)} \sec(x)\right)$$

1.20 problem 20

Internal problem ID [1889]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^2 + y'y + y'yx^2 = 1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(y(x)^2+y(x)*diff(y(x),x)+x^2*y(x)*diff(y(x),x)-1=0,y(x), singsol=all)
```

$$y(x) = \sqrt{\mathrm{e}^{-2\arctan(x)}c_1 + 1}$$
$$y(x) = -\sqrt{\mathrm{e}^{-2\arctan(x)}c_1 + 1}$$

✓ Solution by Mathematica

Time used: 0.923 (sec). Leaf size: 55

```
DSolve[y[x]^2+y[x]*y'[x]+x^2*y[x]*y'[x]-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{1 + e^{-2\arctan(x)+2c_1}}$$
$$y(x) \to \sqrt{1 + e^{-2\arctan(x)+2c_1}}$$
$$y(x) \to -1$$
$$y(x) \to 1$$

1.21 problem 21

Internal problem ID [1890]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{y}{x} = 0}$$

With initial conditions

$$[y(1) = 3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 7

```
dsolve([diff(y(x),x)=y(x)/x,y(1) = 3],y(x), singsol=all)
```

$$y(x) = 3x$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 8

```
DSolve[{y'[x]==y[x]/x,y[1]==3},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3x$$

1.22 problem 22

Internal problem ID [1891]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy' + 2y = 0}$$

With initial conditions

$$[y(2) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 9

```
dsolve([x*diff(y(x),x)+2*y(x)=0,y(2) = 1],y(x), singsol=all)
```

$$y(x) = \frac{4}{x^2}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 10

```
DSolve[{x*y'[x]+2*y[x]==0,y[2]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4}{x^2}$$

1.23 problem 23

Internal problem ID [1892]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sin(x)\cos(y) + \cos(x)\sin(y)\,y' = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.297 (sec). Leaf size: 13

```
dsolve([sin(x)*cos(y(x))+cos(x)*sin(y(x))*diff(y(x),x)=0,y(0) = 0],y(x), singsol=all)
```

$$y(x) = (1 - 2\_B5)\arccos(\sec(x))$$

✓ Solution by Mathematica

Time used: 6.227 (sec). Leaf size: 17

```
DSolve[{Sin[x]*Cos[y[x]]+Cos[x]*Sin[y[x]]*y'[x]==0,y[0]==0},y[x],x,IncludeSingularSolutions
```

$$y(x) \to -\arccos(\sec(x))$$
$$y(x) \to \arccos(\sec(x))$$

1.24 problem 24

Internal problem ID [1893]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x^2 + y^2 = 0}$$

With initial conditions

$$[y(3) = 1]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 14

```
dsolve([x^2*diff(y(x),x)+y(x)^2=0,y(3) = 1],y(x), singsol=all)
```

$$y(x) = \frac{3x}{4x - 3}$$

✓ Solution by Mathematica

Time used: 0.138 (sec). Leaf size: 15

```
DSolve[{x^2*y'[x]+y[x]^2==0,y[3]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3x}{4x - 3}$$

1.25 problem 25

Internal problem ID [1894]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - e^{y} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)=exp(y(x)),y(0) = 0],y(x), singsol=all)
```

$$y(x) = -\ln(1-x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 13

```
DSolve[{y'[x]==Exp[y[x]],y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\log(1-x)$$

1.26 problem 26

Internal problem ID [1895]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_quadrature]**

$$\boxed{\mathrm{e}^{y}(y'+1)=1}$$

With initial conditions

$$[y(0)=1]$$

✓ Solution by Maple

Time used: 0.438 (sec). Leaf size: 32

```
dsolve([exp(y(x))*(diff(y(x),x)+1)=1,y(0) = 1],y(x), singsol=all)
```

$$y(x)=-x+\ln\left(-\mathrm{e}^{x}-\mathrm{e}+1\right)-i\pi$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 18

```
DSolve[{Exp[y[x]]*(y'[x]+1)==1,y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\log\left(e^{-x}(e^{x}-1+e)\right)$$

1.27 problem 27

Internal problem ID [1896]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^2 - \frac{y'}{x^3(x-1)} = -1}$$

With initial conditions

$$[y(2) = 0]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 17

```
dsolve([(1+y(x)^2)=diff(y(x),x)/(x^3*(x-1)),y(2) = 0],y(x), singsol=all)
```

$$y(x) = \tan\left(\frac{1}{5}x^5 - \frac{1}{4}x^4 - \frac{12}{5}\right)$$

✓ Solution by Mathematica

Time used: 0.353 (sec). Leaf size: 22

```
DSolve[{(1+y[x]^2)==y'[x]/(x^3*(x-1)),y[2]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan\left(\frac{1}{20}(4x^5 - 5x^4 - 48)\right)$$

1.28 problem 28

Internal problem ID [1897]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Abel]`

$$\boxed{3y'x - y^3 - 2y = -x^2}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 7

```
dsolve([x^2+3*x*diff(y(x),x)=y(x)^3+2*y(x),y(1) = 1],y(x), singsol=all)
```

$$y(x) = x^{\frac{2}{3}}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x^2+3*x*y'[x]==y[x]^3+2*y[x],y[1]==1},y[x],x,IncludeSingularSolutions -> True]
```

{}

1.29 problem 29

Internal problem ID [1898]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2+x+1)\,y'-y^2-2y=5}$$

With initial conditions

$$[y(1)=1]$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 35

```
dsolve([(x^2+x+1)*diff(y(x),x)=y(x)^2+2*y(x)+5,y(1) = 1],y(x), singsol=all)
```

$$y(x)=-1+2\cot\left(\frac{4\sqrt{3}\,\pi}{9}-\frac{4\sqrt{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)}{3}+\frac{\pi}{4}\right)$$

✓ Solution by Mathematica

Time used: 0.905 (sec). Leaf size: 44

```
DSolve[{(x^2+x+1)*y'[x]==y[x]^2+2*y[x]+5,y[1]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to 2\tan\left(\frac{4\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{36}\left(9-16\sqrt{3}\right)\pi\right)-1$$

1.30 problem 30

Internal problem ID [1899]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 5, page 21
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2 - 2x - 8)\, y' - y^2 - y = -2}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.328 (sec). Leaf size: 31

```
dsolve([(x^2-2*x-8)*diff(y(x),x)=y(x)^2+y(x)-2,y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{2x + 8 - 2\sqrt{-2x^2 + 4x + 16}}{4 + 3x}$$

✓ Solution by Mathematica

Time used: 3.813 (sec). Leaf size: 48

```
DSolve[{(x^2-2*x-8)*y'[x]==y[x]^2+y[x]-2,y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4\sqrt{x+2} - 2\sqrt{8-2x}}{\sqrt{8-2x} + 4\sqrt{x+2}}$$

2 Exercise 6, page 25

2.1 problem 1

Internal problem ID [1900]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 6, page 25
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y - xy' = -x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(x+y(x)=x*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = (\ln(x) + c_1)\, x$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 12

```
DSolve[x+y[x]==x*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(\log(x) + c_1)$$

2.2 problem 2

Internal problem ID [1901]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 6, page 25
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(y+x)\,y'-y=-x}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 24

```
dsolve((x+y(x))*diff(y(x),x)+x=y(x),y(x), singsol=all)
```

$$y(x)=\tan\left(\operatorname{RootOf}\left(2\_Z+\ln\left(\frac{1}{\cos(\_Z)^{2}}\right)+2\ln(x)+2c_1\right)\right)x$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 34

```
DSolve[(x+y[x])*y'[x]+x==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\arctan\left(\frac{y(x)}{x}\right)+\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right)=-\log(x)+c_1,y(x)\right]$$

2.3 problem 3

Internal problem ID [1902]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 6, page 25
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{xy' - y - \sqrt{yx} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(x*diff(y(x),x)-y(x)=sqrt(x*y(x)),y(x), singsol=all)
```

$$-\frac{y(x)}{\sqrt{xy(x)}} + \frac{\ln(x)}{2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.162 (sec). Leaf size: 17

```
DSolve[x*y'[x]-y[x]==Sqrt[x*y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}x(\log(x) + c_1)^2$$

2.4 problem 4

Internal problem ID [1903]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 6, page 25
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{2x - y}{x + 4y} = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 53

```
dsolve(diff(y(x),x)=(2*x-y(x))/(x+4*y(x)),y(x), singsol=all)
```

$$y(x) = \frac{-\frac{c_1 x}{4} - \frac{\sqrt{9c_1^2x^2+8}}{4}}{c_1}$$
$$y(x) = \frac{-\frac{c_1 x}{4} + \frac{\sqrt{9c_1^2x^2+8}}{4}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.472 (sec). Leaf size: 101

```
DSolve[y'[x]==(2*x-y[x])/(x+4*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-x-\sqrt{9x^2+8e^{c_1}}\right)$$
$$y(x) \to \frac{1}{4}\left(-x+\sqrt{9x^2+8e^{c_1}}\right)$$
$$y(x) \to \frac{1}{4}\left(-3\sqrt{x^2}-x\right)$$
$$y(x) \to \frac{1}{4}\left(3\sqrt{x^2}-x\right)$$

2.5 problem 5

Internal problem ID [1904]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 6, page 25
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{xy' - y - \sqrt{x^2 - y^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x)-y(x)=sqrt(x^2-y(x)^2),y(x), singsol=all)
```

$$-\arctan\left(\frac{y(x)}{\sqrt{x^2 - y(x)^2}}\right) + \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.269 (sec). Leaf size: 18

```
DSolve[x*y'[x]-y[x]==Sqrt[x^2-y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x \cosh(i \log(x) + c_1)$$

2.6 problem 6

Internal problem ID [1905]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 6, page 25
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y'y - 2y = -x}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 18

```
dsolve(x+y(x)*diff(y(x),x)=2*y(x),y(x), singsol=all)
```

$$y(x) = \frac{x(\text{LambertW}(c_1 x) + 1)}{\text{LambertW}(c_1 x)}$$

✓ Solution by Mathematica

Time used: 0.118 (sec). Leaf size: 33

```
DSolve[x+y[x]*y'[x]==2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\log\left(\frac{y(x)}{x} - 1\right) - \frac{1}{\frac{y(x)}{x} - 1} = -\log(x) + c_1, y(x)\right]$$

2.7 problem 7

Internal problem ID [1906]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 6, page 25
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{xy' - y + \sqrt{y^2 - x^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(x*diff(y(x),x)-y(x)+sqrt(y(x)^2-x^2)=0,y(x), singsol=all)
```

$$y(x) + \sqrt{y(x)^2 - x^2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.362 (sec). Leaf size: 16

```
DSolve[x*y'[x]-y[x]+Sqrt[y[x]^2-x^2]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\cosh(-\log(x) + c_1)$$

2.8 problem 8

Internal problem ID [1907]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 6, page 25
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y^2 - y'yx = -x^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve((x^2+y(x)^2)=x*y(x)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \sqrt{2\ln(x) + c_1}\, x$$
$$y(x) = -\sqrt{2\ln(x) + c_1}\, x$$

✓ Solution by Mathematica

Time used: 0.17 (sec). Leaf size: 36

```
DSolve[(x^2+y[x]^2)==x*y[x]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\sqrt{2\log(x) + c_1}$$
$$y(x) \to x\sqrt{2\log(x) + c_1}$$

2.9 problem 9

Internal problem ID [1908]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 6, page 25
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(yx - x^2)\, y' - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 21

```
dsolve((x*y(x)-x^2)*diff(y(x),x)-y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\operatorname{LambertW}\left(-\frac{\mathrm{e}^{-c_1}}{x}\right)-c_1}$$

✓ Solution by Mathematica

Time used: 2.229 (sec). Leaf size: 25

```
DSolve[(x*y[x]-x^2)*y'[x]-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -xW\left(-\frac{e^{-c_1}}{x}\right)$$

$$y(x) \to 0$$

2.10 problem 10

Internal problem ID [1909]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 6, page 25
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{xy' + y - 2\sqrt{yx} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 58

```
dsolve(x*diff(y(x),x)+y(x)=2*sqrt(x*y(x)),y(x), singsol=all)
```

$$\frac{\sqrt{xy(x)}}{(-x+y(x))\left(-x+\sqrt{xy(x)}\right)x} + \frac{1}{(-x+y(x))\left(-x+\sqrt{xy(x)}\right)} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.209 (sec). Leaf size: 26

```
DSolve[x*y'[x]+y[x]==2*Sqrt[x*y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\left(x + e^{\frac{c_1}{2}}\right)^2}{x}$$
$$y(x) \to x$$

2.11 problem 11

Internal problem ID [1910]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 6, page 25
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{y + (-y + x)\, y' = -x}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 49

```
dsolve((x+y(x))+(x-y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x - \sqrt{2c_1^2 x^2 + 1}}{c_1}$$
$$y(x) = \frac{c_1 x + \sqrt{2c_1^2 x^2 + 1}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.489 (sec). Leaf size: 86

```
DSolve[(x+y[x])+(x-y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - \sqrt{2x^2 + e^{2c_1}}$$
$$y(x) \to x + \sqrt{2x^2 + e^{2c_1}}$$
$$y(x) \to x - \sqrt{2}\sqrt{x^2}$$
$$y(x) \to \sqrt{2}\sqrt{x^2} + x$$

2.12 problem 12

Internal problem ID [1911]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 6, page 25
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{y(x^2 - yx + y^2) + xy'(x^2 + yx + y^2) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(y(x)*(x^2-x*y(x)+y(x)^2)+x*diff(y(x),x)*(x^2+x*y(x)+y(x)^2)=0,y(x), singsol=all)
```

$$y(x) = \tan\left(\text{RootOf}\left(\ln\left(\tan\left(\_Z\right)\right) + \_Z + 2\ln\left(x\right) + 2c_1\right)\right) x$$

✓ Solution by Mathematica

Time used: 0.115 (sec). Leaf size: 26

```
DSolve[y[x]*(x^2-x*y[x]+y[x]^2)+x*y'[x]*(x^2+x*y[x]+y[x]^2)==0,y[x],x,IncludeSingularSolutio
```

$$\text{Solve}\left[\arctan\left(\frac{y(x)}{x}\right) + \log\left(\frac{y(x)}{x}\right) = -2\log(x) + c_1, y(x)\right]$$

2.13 problem 13

Internal problem ID [1912]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 6, page 25
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{xy' - y - x\sin\left(\frac{y}{x}\right) = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 44

```
dsolve(diff(y(x),x)*x-y(x)-x*sin(y(x)/x)=0,y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{2c_1x}{c_1^2x^2+1}, -\frac{c_1^2x^2-1}{c_1^2x^2+1}\right)x$$

✓ Solution by Mathematica

Time used: 0.341 (sec). Leaf size: 52

```
DSolve[y'[x]*x-y[x]-x*Sin[y[x]/x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\arccos(-\tanh(\log(x)+c_1))$$
$$y(x) \to x\arccos(-\tanh(\log(x)+c_1))$$
$$y(x) \to 0$$
$$y(x) \to -\pi x$$
$$y(x) \to \pi x$$

2.14 problem 14

Internal problem ID [1913]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 6, page 25
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y' - \frac{y}{x} - \cosh\left(\frac{y}{x}\right) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=y(x)/x+cosh(y(x)/x),y(x), singsol=all)
```

$$y(x) = \ln\left(\tan\left(\frac{\ln(x)}{2} + \frac{c_1}{2}\right)\right) x$$

✓ Solution by Mathematica

Time used: 1.62 (sec). Leaf size: 15

```
DSolve[y'[x]==y[x]/x+Cosh[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\operatorname{arcsinh}(\cot(\log(x) + c_1))$$

2.15 problem 15

Internal problem ID [1914]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 6, page 25
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y^2 - 2y'yx = -x^2}$$

With initial conditions

$$[y(-1) = 0]$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 23

```
dsolve([(x^2+y(x)^2)=2*x*y(x)*diff(y(x),x),y(-1) = 0],y(x), singsol=all)
```

$$y(x) = \sqrt{(x+1)\,x}$$
$$y(x) = -\sqrt{(x+1)\,x}$$

✓ Solution by Mathematica

Time used: 0.197 (sec). Leaf size: 36

```
DSolve[{(x^2+y[x]^2)==2*x*y[x]*y'[x],y[-1]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x}\sqrt{x+1}$$
$$y(x) \to \sqrt{x}\sqrt{x+1}$$

2.16 problem 16

Internal problem ID [1915]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 6, page 25
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{\left(\frac{x}{y}+\frac{y}{x}\right)y'=-1}$$

✓ Solution by Maple

Time used: 0.703 (sec). Leaf size: 223

```
dsolve((x/y(x)+y(x)/x)*diff(y(x),x)+1=0,y(x), singsol=all)
```

$$y(x)=\frac{\sqrt{x^2c_1\left(c_1x^2+\sqrt{c_1^2x^4+1}\right)}}{x\left(c_1x^2+\sqrt{c_1^2x^4+1}\right)c_1}$$

$$y(x)=\frac{\sqrt{-x^2c_1\left(-c_1x^2+\sqrt{c_1^2x^4+1}\right)}}{x\left(c_1x^2-\sqrt{c_1^2x^4+1}\right)c_1}$$

$$y(x)=-\frac{\sqrt{x^2c_1\left(c_1x^2+\sqrt{c_1^2x^4+1}\right)}}{x\left(c_1x^2+\sqrt{c_1^2x^4+1}\right)c_1}$$

$$y(x)=-\frac{\sqrt{-x^2c_1\left(-c_1x^2+\sqrt{c_1^2x^4+1}\right)}}{x\left(c_1x^2-\sqrt{c_1^2x^4+1}\right)c_1}$$

✓ Solution by Mathematica

Time used: 0.103 (sec). Leaf size: 121

```
DSolve[(x/y[x]+y[x]/x)*y'[x]+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^2 - \sqrt{x^4 + e^{4c_1}}}$$
$$y(x) \to \sqrt{-x^2 - \sqrt{x^4 + e^{4c_1}}}$$
$$y(x) \to -\sqrt{-x^2 + \sqrt{x^4 + e^{4c_1}}}$$
$$y(x) \to \sqrt{-x^2 + \sqrt{x^4 + e^{4c_1}}}$$

2.17 problem 17

Internal problem ID [1916]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 6, page 25
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{x\,\mathrm{e}^{\frac{y}{x}}+y-xy'=0}$$

With initial conditions

$$[y(1)=0]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 15

```
dsolve([x*exp(y(x)/x)+y(x)=x*diff(y(x),x),y(1) = 0],y(x), singsol=all)
```

$$y(x)=\ln\left(-\frac{1}{-1+\ln(x)}\right)x$$

✓ Solution by Mathematica

Time used: 0.319 (sec). Leaf size: 15

```
DSolve[{x*Exp[y[x]/x]+y[x]==x*y'[x],y[1]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -x\log(1-\log(x))$$

2.18 problem 18

Internal problem ID [1917]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 6, page 25
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{y+x}{-y+x} = 0}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 23

```
dsolve([diff(y(x),x)=(x+y(x))/(x-y(x)),y(1) = 0],y(x), singsol=all)
```

$$y(x) = \tan\left(\operatorname{RootOf}\left(2\_Z - \ln\left(\sec\left(\_Z\right)^2\right) - 2\ln\left(x\right)\right)\right) x$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 33

```
DSolve[{y'[x]==(x+y[x])/(x-y[x]),y[1]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right) - \arctan\left(\frac{y(x)}{x}\right) = -\log(x), y(x)\right]$$

2.19 problem 19

Internal problem ID [1918]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 6, page 25
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y' - \frac{y}{x} - \tan\left(\frac{y}{x}\right) = 0}$$

With initial conditions

$$[y(6) = \pi]$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 10

```
dsolve([diff(y(x),x)=y(x)/x+tan(y(x)/x),y(6) = Pi],y(x), singsol=all)
```

$$y(x) = \arcsin\left(\frac{x}{12}\right)x$$

✓ Solution by Mathematica

Time used: 4.285 (sec). Leaf size: 13

```
DSolve[{y'[x]==y[x]/x+Tan[y[x]/x],y[6]==Pi},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \arcsin\left(\frac{x}{12}\right)$$

2.20 problem 20

Internal problem ID [1919]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 6, page 25
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{(3yx-2x^2)\,y'-2y^2+yx=0}$$

With initial conditions

$$[y(1)=-1]$$

✓ Solution by Maple

Time used: 0.672 (sec). Leaf size: 114

```
dsolve([(3*x*y(x)-2*x^2)*diff(y(x),x)=2*y(x)^2-x*y(x),y(1) = -1],y(x), singsol=all)
```

$$y(x) = \frac{i\left(\left(-27x^2+x^3+3\sqrt{3}\,\sqrt{-2x^5+27x^4}\right)^{\frac{2}{3}}-x^2\right)\sqrt{3}-\left(\left(-27x^2+x^3+3\sqrt{3}\,\sqrt{-2x^5+27x^4}\right)^{\frac{1}{3}}-x\right)^2}{6\left(-27x^2+x^3+3\sqrt{3}\,\sqrt{-2x^5+27x^4}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 60.327 (sec). Leaf size: 140

```
DSolve[{(3*x*y[x]-2*x^2)*y'[x]==2*y[x]^2-x*y[x],y[1]==-1},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{\left(\sqrt[3]{3\sqrt{3}\sqrt{-x^4(2x-27)}+x^3-27x^2}-x\right)\left(i\left(\sqrt{3}+i\right)\sqrt[3]{3\sqrt{3}\sqrt{-x^4(2x-27)}+x^3-27x^2}+i\sqrt{3}x+\right.}{6\sqrt[3]{3\sqrt{3}\sqrt{-x^4(2x-27)}+x^3-27x^2}}$$

2.21 problem 21

Internal problem ID [1920]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 6, page 25
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{y' - \frac{y}{x - k\sqrt{y^2 + x^2}} = 0}$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 32

```
dsolve(diff(y(x),x)=y(x)/(x-k*sqrt(x^2+y(x)^2)),y(x), singsol=all)
```

$$-c_1 + y(x)^{k-1}\sqrt{x^2 + y(x)^2} + y(x)^{k-1}x = 0$$

✓ Solution by Mathematica

Time used: 0.22 (sec). Leaf size: 59

```
DSolve[y'[x]==y[x]/(x-k*Sqrt[x^2+y[x]^2]),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\left((k-1)\log\left(\sqrt{\frac{y(x)^2}{x^2}+1}-1\right)+(k+1)\log\left(\sqrt{\frac{y(x)^2}{x^2}+1}+1\right)\right)= -k\log(x)+c_1, y(x)\right]$$

2.22 problem 22

Internal problem ID [1921]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 6, page 25
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{y^2(y'y - x) = -x^3}$$

✓ Solution by Maple

Time used: 0.328 (sec). Leaf size: 50

```
dsolve(y(x)^2*(y(x)*diff(y(x),x)-x)+x^3=0,y(x), singsol=all)
```

$$y(x) = \mathrm{RootOf}\left(2\_Z^2 + \sqrt{3}\,\tan\left(\mathrm{RootOf}\left(\sqrt{3}\,\ln\left(\frac{3\tan(\_Z)^2 x^4}{4} + \frac{3x^4}{4}\right) + 4\sqrt{3}\,c_1 - 2\_Z\right)\right) - 1\right) x$$

✓ Solution by Mathematica

Time used: 0.113 (sec). Leaf size: 63

```
DSolve[y[x]^2*(y[x]*y'[x]-x)+x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\frac{\arctan\left(\frac{\frac{2y(x)^2}{x^2}-1}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{4}\log\left(\frac{y(x)^4}{x^4} - \frac{y(x)^2}{x^2} + 1\right) = -\log(x) + c_1, y(x)\right]$$

2.23 problem 23

Internal problem ID [1922]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 6, page 25
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y' - \frac{y}{x} - \tanh\left(\frac{y}{x}\right) = 0}$$

✓ Solution by Maple

Time used: 0.578 (sec). Leaf size: 113

```
dsolve(diff(y(x),x)=y(x)/x+tanh(y(x)/x),y(x), singsol=all)
```

$$y(x) = \operatorname{arctanh}\left(\frac{c_1 x^2 - \sqrt{c_1^2 x^4 - c_1 x^2}}{c_1 x^2 - 1 - \sqrt{c_1^2 x^4 - c_1 x^2}}\right) x$$
$$y(x) = \operatorname{arctanh}\left(\frac{c_1 x^2 + \sqrt{c_1^2 x^4 - c_1 x^2}}{c_1 x^2 - 1 + \sqrt{c_1^2 x^4 - c_1 x^2}}\right) x$$

✓ Solution by Mathematica

Time used: 2.284 (sec). Leaf size: 19

```
DSolve[y'[x]==y[x]/x+Tanh[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\operatorname{arcsinh}(e^{c_1} x)$$
$$y(x) \to 0$$

3 Exercise 7, page 28

3.1 problem 1

Internal problem ID [1923]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 7, page 28
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y-(x-y+2)\,y'=-x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve((x+y(x))-(x-y(x)+2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y=1-\tan\left(\mathrm{RootOf}\left(2\_Z+\ln\left(\frac{1}{\cos\left(\_Z\right)^{2}}\right)+2\ln\left(x+1\right)+2c_{1}\right)\right)\left(x+1\right)$$

✓ Solution by Mathematica

Time used: 0.16 (sec). Leaf size: 56

```
DSolve[(x+y[x])-(x-y[x]+2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}\text{Solve}\Bigg[&2\arctan\left(\frac{y(x)+x}{-y(x)+x+2}\right)=\log\left(\frac{x^2+y(x)^2-2y(x)+2x+2}{2(x+1)^2}\right)\\&+2\log(x+1)+c_1,y(x)\Bigg]\end{aligned}$$

3.2 problem 2

Internal problem ID [1924]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 7, page 28
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(x-2y+2)\,y'=-x}$$

✓ Solution by Maple

Time used: 0.39 (sec). Leaf size: 174

```
dsolve(x+(x-2*y(x)+2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = 1 - \frac{\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{6}\left(-2\left(2c_1x^3+2\sqrt{-2c_1^3x^9+c_1^2x^6}\right)^{\frac{1}{3}}-\frac{4x^3c_1}{\left(2c_1x^3+2\sqrt{-2c_1^3x^9+c_1^2x^6}\right)^{\frac{1}{3}}}+4i\sqrt{3}\left(\frac{\left(2c_1x^3+2\sqrt{-2c_1^3x^9+c_1^2x^6}\right)^{\frac{1}{3}}}{2}-\frac{x^3c_1}{\left(2c_1x^3+2\sqrt{-2c_1^3x^9+c\cdots\right.}\right.\right.}{64c_1}}{x^2}$$

✓ Solution by Mathematica

Time used: 60.036 (sec). Leaf size: 445

```
DSolve[x+(x-2*y[x]+2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x+2}{2} - \frac{1}{2\text{Root}\left[\#1^6\left(16x^6+16e^{12c_1}\right)-24\#1^4x^4+8\#1^3x^3+9\#1^2x^2-6\#1x+1\&,1\right]}$$

$$y(x) \to \frac{x+2}{2} - \frac{1}{2\text{Root}\left[\#1^6\left(16x^6+16e^{12c_1}\right)-24\#1^4x^4+8\#1^3x^3+9\#1^2x^2-6\#1x+1\&,2\right]}$$

$$y(x) \to \frac{x+2}{2} - \frac{1}{2\text{Root}\left[\#1^6\left(16x^6+16e^{12c_1}\right)-24\#1^4x^4+8\#1^3x^3+9\#1^2x^2-6\#1x+1\&,3\right]}$$

$$y(x) \to \frac{x+2}{2} - \frac{1}{2\text{Root}\left[\#1^6\left(16x^6+16e^{12c_1}\right)-24\#1^4x^4+8\#1^3x^3+9\#1^2x^2-6\#1x+1\&,4\right]}$$

$$y(x) \to \frac{x+2}{2} - \frac{1}{2\text{Root}\left[\#1^6\left(16x^6+16e^{12c_1}\right)-24\#1^4x^4+8\#1^3x^3+9\#1^2x^2-6\#1x+1\&,5\right]}$$

$$y(x) \to \frac{x+2}{2} - \frac{1}{2\text{Root}\left[\#1^6\left(16x^6+16e^{12c_1}\right)-24\#1^4x^4+8\#1^3x^3+9\#1^2x^2-6\#1x+1\&,6\right]}$$

3.3 problem 3

Internal problem ID [1925]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 7, page 28
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{-y + (x + y)\, y' = -2x - 1}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 56

```
dsolve((2*x-y(x)+1)+(x+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{1}{3} - \frac{\tan\left(\mathrm{RootOf}\left(\sqrt{2}\,\ln\left(2\tan\left(\_Z\right)^2 (3x+1)^2 + 2(3x+1)^2\right) + 2\sqrt{2}\,c_1 - 2\_Z\right)\right)(3x+1)\sqrt{2}}{3}$$

✓ Solution by Mathematica

Time used: 0.097 (sec). Leaf size: 77

```
DSolve[(2*x-y[x]+1)+(x+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[2\sqrt{2}\arctan\left(\frac{-y(x)+2x+1}{\sqrt{2}(y(x)+x)}\right) = 2\log\left(\frac{6x^2+3y(x)^2-2y(x)+4x+1}{(3x+1)^2}\right) + 4\log(3x+1) + 3c_1, y(x)\right]$$

3.4 problem 4

Internal problem ID [1926]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 7, page 28
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{-y + (x + y - 1)\, y' = -x - 2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve((x-y(x)+2)+(x+y(x)-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{3}{2} - \frac{\tan\left(\operatorname{RootOf}\left(-2\_Z + \ln\left(\frac{1}{\cos\left(\_Z\right)^2}\right) + 2\ln\left(2x+1\right) + 2c_1\right)\right)(2x+1)}{2}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 63

```
DSolve[(x-y[x]+2)+(x+y[x]-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2\arctan\left(\frac{y(x)-x-2}{y(x)+x-1}\right) + \log\left(\frac{2x^2+2y(x)^2-6y(x)+2x+5}{(2x+1)^2}\right) + 2\log(2x+1) + c_1 = 0, y(x)\right]$$

3.5 problem 5

Internal problem ID [1927]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 7, page 28
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{-y + (y - x + 1)\, y' = -x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
dsolve((x-y(x))+(y(x)-x+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = x - 1 - \sqrt{2c_1 - 2x + 1}$$
$$y = x - 1 + \sqrt{2c_1 - 2x + 1}$$

✓ Solution by Mathematica

Time used: 0.092 (sec). Leaf size: 49

```
DSolve[(x-y[x])+(y[x]-x+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - i\sqrt{2x - 1 - c_1} - 1$$
$$y(x) \to x + i\sqrt{2x - 1 - c_1} - 1$$

3.6 problem 6

Internal problem ID [1928]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 7, page 28
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{x+y-1}{-y+x-1} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)=(x+y(x)-1)/(x-y(x)-1),y(x), singsol=all)
```

$$y = -\tan\left(\operatorname{RootOf}\left(2\_Z + \ln\left(\frac{1}{\cos(\_Z)^2}\right) + 2\ln(x-1) + 2c_1\right)\right)(x-1)$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 48

```
DSolve[y'[x]==(x+y[x]-1)/(x-y[x]-1),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2\arctan\left(\frac{y(x)+x-1}{-y(x)+x-1}\right) = \log\left(\frac{1}{2}\left(\frac{y(x)^2}{(x-1)^2}+1\right)\right) + 2\log(x-1) + c_1, y(x)\right]$$

3.7 problem 7

Internal problem ID [1929]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 7, page 28
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y + (2x + 2y - 1)\, y' = -x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve((x+y(x))+(2*x+2*y(x)-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \mathrm{e}^{-\operatorname{LambertW}\left(2\,\mathrm{e}^{x}\mathrm{e}^{-2}\mathrm{e}^{-c_1}\right)+x-2-c_1} - x + 1$$

✓ Solution by Mathematica

Time used: 4.381 (sec). Leaf size: 33

```
DSolve[(x+y[x])+(2*x+2*y[x]-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(W\left(-e^{x-1+c_1}\right) - 2x + 2\right)$$
$$y(x) \to 1 - x$$

3.8 problem 8

Internal problem ID [1930]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 7, page 28
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{-y + (-y + x - 1)\, y' = -x - 1}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 13

```
dsolve((x-y(x)+1)+(x-y(x)-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \mathrm{LambertW}\left(\mathrm{e}^{-2x} c_1\right) + x$$

✓ Solution by Mathematica

Time used: 3.902 (sec). Leaf size: 24

```
DSolve[(x-y[x]+1)+(x-y[x]-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + W\left(-e^{-2x-1+c_1}\right)$$
$$y(x) \to x$$

3.9 problem 9

Internal problem ID [1931]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 7, page 28
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{2y + (3x + 6y + 3)\, y' = -x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
dsolve((x+2*y(x))+(3*x+6*y(x)+3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{\mathrm{e}^{-\operatorname{LambertW}\left(-\frac{\mathrm{e}^{-\frac{3}{2}}\mathrm{e}^{-\frac{x}{6}}\mathrm{e}^{\frac{c_1}{6}}}{2}\right)-\frac{3}{2}-\frac{x}{6}+\frac{c_1}{6}}}{2} - \frac{3}{2} - \frac{x}{2}$$

✓ Solution by Mathematica

Time used: 4.116 (sec). Leaf size: 43

```
DSolve[(x+2*y[x])+(3*x+6*y[x]+3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-2W\left(-e^{-\frac{x}{6}-1+c_1}\right) - x - 3\right)$$

$$y(x) \to \frac{1}{2}(-x - 3)$$

3.10 problem 10

Internal problem ID [1932]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 7, page 28
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{2y - (2x + y - 1)\, y' = -x - 2}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 377

```
dsolve((x+2*y(x)+2)=(2*x+y(x)-1)*diff(y(x),x),y(x), singsol=all)
```

$$y = -\frac{5}{3} + \frac{12(3x-4)\left(-\frac{\left(1-54(3x-4)^2c_1+6\sqrt{81(3x-4)^4c_1^2-3(3x-4)^2c_1}\right)^{\frac{1}{3}}}{12} - \frac{1}{12\left(1-54(3x-4)^2c_1+6\sqrt{81(3x-4)^4c_1^2-3(3x\cdots}\right.}\right.}{-3\left(1-54\left(3x-4\right)^2c_1+6\sqrt{81\left(3x-4\right)^4c_1^2-3\left(3x-4\right)^2c_1}\right)^{\frac{1}{3}} - \frac{3}{\left(1-54(3x-4)^2c_1+6\sqrt{81(3x-4)^4c_1^2-3(3x-4)\cdots}\right.}}$$

✓ Solution by Mathematica

Time used: 60.167 (sec). Leaf size: 1687

```
DSolve[(x+2*y[x]+2)==(2*x+y[x]-1)*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

3.11 problem 11

Internal problem ID [1933]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 7, page 28
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{-y + (x - 3y - 5)\, y' = -3x - 1}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 1.531 (sec). Leaf size: 84

```
dsolve([(3*x-y(x)+1)+(x-3*y(x)-5)*diff(y(x),x)=0,y(0) = 0],y(x), singsol=all)
```

$$y = \frac{\left(-324 + 12\sqrt{96x^3 + 288x^2 + 288x + 825}\right)^{\frac{4}{3}} - 12\left(-324 + 12\sqrt{96x^3 + 288x^2 + 288x + 825}\right)^{\frac{2}{3}} x - 84\left(-3\right.}{36\left(-324 + 12\sqrt{96x^3 + 288x^2 + 288x + 82\right.}}$$

✓ Solution by Mathematica

Time used: 60.756 (sec). Leaf size: 341

```
DSolve[{(3*x-y[x]+1)+(x-3*y[x]-5)*y'[x]==0,{y[0]==0}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{x\text{Root}\left[\#1^6(1024x^6 + 6144x^5 + 15360x^4 + 20480x^3 + 15360x^2 + 6144x - 58025) + \#1^4(-384x^4 - 153\right.}{}$$

3.12 problem 12

Internal problem ID [1934]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 7, page 28
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{-3y + (2x - y + 5)\, y' = -6x - 6}$$

With initial conditions

$$[y(-1) = 1]$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 20

```
dsolve([3*(2*x-y(x)+2)+(2*x-y(x)+5)*diff(y(x),x)=0,y(-1) = 1],y(x), singsol=all)
```

$$y = \frac{16}{5} - \frac{9\,\mathrm{LambertW}\left(\frac{\mathrm{e}^{\frac{25x}{9}+\frac{26}{9}}}{9}\right)}{5} + 2x$$

✓ Solution by Mathematica

Time used: 3.718 (sec). Leaf size: 32

```
DSolve[{3*(2*x-y[x]+2)+(2*x-y[x]+5)*y'[x]==0,{y[-1]==1}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{9}{5}W\left(\frac{1}{9}e^{\frac{25x}{9}+\frac{26}{9}}\right) + 2x + \frac{16}{5}$$

3.13 problem 13

Internal problem ID [1935]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 7, page 28
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{3y + (-x + y)\, y' = -2 - 2x}$$

With initial conditions

$$[y(0) = -2]$$

✓ Solution by Maple

Time used: 2.484 (sec). Leaf size: 75

```
dsolve([(2*x+3*y(x)+2)+(y(x)-x)*diff(y(x),x)=0,y(0) = -2],y(x), singsol=all)
```

$$y = \tan\left(\mathrm{RootOf}\left(4\_Z - \ln\left(\sec\left(\_Z\right)^2\right) - 2\ln\left(5x+2\right) + 3\ln\left(2\right) + \ln\left(5\right) + 4\arctan\left(3\right)\right)\right) x + \frac{2\tan\left(\mathrm{RootOf}\left(4\_Z - \ln\left(\sec\left(\_Z\right)^2\right) - 2\ln\left(5x+2\right) + 3\ln\left(2\right) + \ln\left(5\right) + 4\arctan\left(3\right)\right)\right)}{5} - x - \frac{4}{5}$$

✓ Solution by Mathematica

Time used: 0.075 (sec). Leaf size: 78

```
DSolve[{(2*x+3*y[x]+2)+(y[x]-x)*y'[x]==0,{y[0]==-2}},y[x],x,IncludeSingularSolutions -> True
```

$$\mathrm{Solve}\left[32\arctan\left(\frac{2y(x)+3x+2}{x-y(x)}\right) = 8\log\left(\frac{10x^2+10xy(x)+5y(x)^2+8y(x)+12x+4}{(5x+2)^2}\right) + 16\log(5x+2) - 8(\pi + 3\log(2)), y(x)\right]$$

3.14 problem 14

Internal problem ID [1936]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 7, page 28
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y-(2x+2y-1)\,y'=-x-4}$$

With initial conditions

$$[y(0)=0]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 20

```
dsolve([(x+y(x)+4)=(2*x+2*y(x)-1)*diff(y(x),x),y(0) = 0],y(x), singsol=all)
```

$$y=-x-\frac{3\,\mathrm{LambertW}\left(-\frac{2\,\mathrm{e}^{-x-\frac{2}{3}}}{3}\right)}{2}-1$$

✓ Solution by Mathematica

Time used: 3.156 (sec). Leaf size: 28

```
DSolve[{(x+y[x]+4)==(2*x+2*y[x]-1)*y'[x],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{3}{2}W\left(-\frac{2}{3}e^{-x-\frac{2}{3}}\right)-x-1$$

3.15 problem 15

Internal problem ID [1937]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 7, page 28
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{3y + (2x + 3y + 2)\, y' = -2x + 1}$$

With initial conditions

$$[y(3) = 1]$$

✓ Solution by Maple

Time used: 0.218 (sec). Leaf size: 20

```
dsolve([(2*x+3*y(x)-1)+(2*x+3*y(x)+2)*diff(y(x),x)=0,y(3) = 1],y(x), singsol=all)
```

$$y = \frac{7}{3} - \frac{2x}{3} + 3\,\mathrm{LambertW}\left(\frac{2\,\mathrm{e}^{\frac{5}{9}-\frac{x}{9}}}{9}\right)$$

✓ Solution by Mathematica

Time used: 4.691 (sec). Leaf size: 32

```
DSolve[{(2*x+3*y[x]-1)+(2*x+3*y[x]+2)*y'[x]==0,{y[3]==1}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{3}\left(9W\left(\frac{2}{9}e^{\frac{5}{9}-\frac{x}{9}}\right) - 2x + 7\right)$$

3.16 problem 16

Internal problem ID [1938]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 7, page 28
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{-y + (x + 2y + 1)\, y' = -2 - 3x}$$

With initial conditions

$$[y(0) = 0]$$

✗ Solution by Maple

```
dsolve([(3*x-y(x)+2)+(x+2*y(x)+1)*diff(y(x),x)=0,y(0) = 0],y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 0.186 (sec). Leaf size: 111

```
DSolve[{(3*x-y[x]+2)+(x+2*y[x]+1)*y'[x]==0,{y[0]==0}},y[x],x,IncludeSingularSolutions -> Tru
```

$$\text{Solve}\left[2\sqrt{6}\arctan\left(\frac{\sqrt{\frac{2}{3}}(-y(x)+3x+2)}{2y(x)+x+1}\right)=3\left(\frac{2}{3}\left(\sqrt{6}\arctan\left(2\sqrt{\frac{2}{3}}\right)+3\log\left(\frac{25}{22}\right)-6\log(5)\right)\right.\right.$$
$$\left.\left.+2\log\left(\frac{42x^2+28y(x)^2+8y(x)+60x+22}{(7x+5)^2}\right)+4\log(7x+5)\right),y(x)\right]$$

3.17 problem 17

Internal problem ID [1939]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 7, page 28
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{2y-(x+2y-1)\,y'=-3x-3}$$

With initial conditions

$$[y(-2)=1]$$

✓ Solution by Maple

Time used: 1.282 (sec). Leaf size: 136

```
dsolve([(3*x+2*y(x)+3)-(x+2*y(x)-1)*diff(y(x),x)=0,y(-2) = 1],y(x), singsol=all)
```

$$y = \frac{(-x-2)\operatorname{RootOf}\left(-1+(x^5+10x^4+40x^3+80x^2+80x+32)\_Z^{25}+(-5x^5-50x^4-200x^3-400x^2}{2} + \frac{3x}{2}+\frac{9}{2}$$

✓ Solution by Mathematica

Time used: 63.287 (sec). Leaf size: 850

```
DSolve[{(3*x+2*y[x]+3)-(x+2*y[x]-1)*y'[x]==0,{y[-2]==1}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{-x\operatorname{Root}\left[(65536x^{10}+1310720x^9+11796480x^8+62914560x^7+220200960x^6+528482304x^5+880803}{}$$

3.18 problem 18

Internal problem ID [1940]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 7, page 28
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{-2y + (1 - x + 2y)\, y' = -3 - x}$$

With initial conditions

$$[y(-4) = 2]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 20

```
dsolve([(x-2*y(x)+3)+(1-x+2*y(x))*diff(y(x),x)=0,y(-4) = 2],y(x), singsol=all)
```

$$y = \frac{7}{2} + \frac{x}{2} + 4\,\mathrm{LambertW}\left(\frac{\mathrm{e}^{\frac{5}{8}+\frac{x}{8}}}{8}\right)$$

✓ Solution by Mathematica

Time used: 4.645 (sec). Leaf size: 28

```
DSolve[{(x-2*y[x]+3)+(1-x+2*y[x])*y'[x]==0,{y[-4]==2}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{1}{2}\left(8W\left(\frac{1}{8}e^{\frac{x+5}{8}}\right) + x + 7\right)$$

3.19 problem 19

Internal problem ID [1941]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 7, page 28
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y + (4x + 2y + 1)\, y' = -2x}$$

With initial conditions

$$\left[y\left(-\frac{1}{6}\right) = 0\right]$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 21

```
dsolve([(2*x+y(x))+(4*x+2*y(x)+1)*diff(y(x),x)=0,y(-1/6) = 0],y(x), singsol=all)
```

$$y = -\frac{\text{LambertW}\left(-1, -2\,\mathrm{e}^{-\frac{7}{2}-9x}\right)}{6} - \frac{2}{3} - 2x$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{(2*x+y[x])+(4*x+2*y[x]+1)*y'[x]==0,{y[-1/6]==0}},y[x],x,IncludeSingularSolutions ->
```

{}

3.20 problem 20

Internal problem ID [1942]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 7, page 28
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y + (4x - 2y + 1)\, y' = -2x}$$

With initial conditions

$$\left[y\left(\frac{1}{2}\right) = 0\right]$$

✓ Solution by Maple

Time used: 14.453 (sec). Leaf size: 93

```
dsolve([(2*x+y(x))+(4*x-2*y(x)+1)*diff(y(x),x)=0,y(1/2) = 0],y(x), singsol=all)
```

$$y = \operatorname{RootOf}\left(6\sqrt{41}\,\operatorname{arctanh}\left(\frac{(32\_Z - 13 - 40x)\sqrt{41}}{328x + 41}\right) + 41\ln\left(\frac{16\_Z^2 - 40x\_Z - 16x^2 - 13\_Z + 6x + 2}{(8x+1)^2}\right) + 82\ln(8x+1) + 6\sqrt{41}\,\operatorname{arctanh}\left(\frac{33\sqrt{41}}{205}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.172 (sec). Leaf size: 128

```
DSolve[{(2*x+y[x])+(4*x-2*y[x]+1)*y'[x]==0,{y[1/2]==0}},y[x],x,IncludeSingularSolutions -> T
```

$$\text{Solve}\left[\frac{9}{656}\left(6\sqrt{41}\operatorname{arctanh}\left(\frac{-\frac{2(8x+1)}{-2y(x)+4x+1}-3}{\sqrt{41}}\right)\right.\right.$$
$$\left.+41\left(\log\left(\frac{2(16x^2-16y(x)^2+(40x+13)y(x)-6x-2)}{(8x+1)^2}\right)+2\log(8x+1)\right)\right)=\frac{1}{656}\left(-9\left(6\sqrt{41}\operatorname{arctanh}\left(\right.\right.\right.$$
$$\left.\left.+369i\pi\right), y(x)\right]$$

4 Exercise 8, page 34

4.1 problem 1

Internal problem ID [1943]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 8, page 34
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{y + (x - 2y)\, y' = -x}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 53

```
dsolve((x+y(x))+(x-2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{\frac{c_1 x}{2} - \frac{\sqrt{3c_1^2x^2+2}}{2}}{c_1}$$

$$y = \frac{\frac{c_1 x}{2} + \frac{\sqrt{3c_1^2x^2+2}}{2}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.535 (sec). Leaf size: 106

```
DSolve[(x+y[x])+(x-2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(x - \sqrt{3x^2 - 2e^{2c_1}}\right)$$
$$y(x) \to \frac{1}{2}\left(x + \sqrt{3x^2 - 2e^{2c_1}}\right)$$
$$y(x) \to \frac{1}{2}\left(x - \sqrt{3}\sqrt{x^2}\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{3}\sqrt{x^2} + x\right)$$

4.2 problem 2

Internal problem ID [1944]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 8, page 34
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{y + (x + 3y)\, y' = -3x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 53

```
dsolve((3*x+y(x))+(x+3*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{-\frac{c_1 x}{3} - \frac{\sqrt{-8c_1^2x^2+3}}{3}}{c_1}$$
$$y = \frac{-\frac{c_1 x}{3} + \frac{\sqrt{-8c_1^2x^2+3}}{3}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.487 (sec). Leaf size: 119

```
DSolve[(3*x+y[x])+(x+3*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(-x - \sqrt{-8x^2 + 3e^{2c_1}}\right)$$
$$y(x) \to \frac{1}{3}\left(-x + \sqrt{-8x^2 + 3e^{2c_1}}\right)$$
$$y(x) \to \frac{1}{3}\left(-2\sqrt{2}\sqrt{-x^2} - x\right)$$
$$y(x) \to \frac{1}{3}\left(2\sqrt{2}\sqrt{-x^2} - x\right)$$

4.3 problem 3

Internal problem ID [1945]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 8, page 34
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _exact, _rational, [_Abel, '2nd ty`

$$\boxed{b_1y + (b_1x + b_2y + c_2)\, y' = -a_1x - c_1}$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 119

```
dsolve((a__1*x+b__1*y(x)+c__1)+(b__1*x+b__2*y(x)+c__2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{-c_1b_1 + c_2a_1 - \frac{\left(-b_1\left(x\left(a_1b_2-b_1^2\right)+c_1b_2-c_2b_1\right)e^{c_1}+\sqrt{-e^{2c_1}a_1b_2\left(x\left(a_1b_2-b_1^2\right)+c_1b_2-c_2b_1\right)^2+b_1^2\left(x\left(a_1b_2-b_1^2\right)+c_1b_2-c_2b_1\right)^2e^{2c_1}+b_2}\right)e}{b_2}}{-a_1b_2+b_1^2}$$

✓ Solution by Mathematica

Time used: 16.604 (sec). Leaf size: 106

```
DSolve[(a1*x+b1*y[x]+c1)+(b1*x+b2*y[x]+c2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\frac{\sqrt{-x(\mathrm{a1}x+2\mathrm{c1})+\frac{(\mathrm{b1}x+\mathrm{c2})^2}{\mathrm{b2}}+\mathrm{b2}c_1}}{\sqrt{\frac{1}{\mathrm{b2}}}}+\mathrm{b1}x+\mathrm{c2}}{\mathrm{b2}}$$

$$y(x) \to -\frac{\mathrm{b1}x+\mathrm{c2}}{\mathrm{b2}}+\sqrt{\frac{1}{\mathrm{b2}}}\sqrt{-x(\mathrm{a1}x+2\mathrm{c1})+\frac{(\mathrm{b1}x+\mathrm{c2})^2}{\mathrm{b2}}+\mathrm{b2}c_1}$$

4.4 problem 4

Internal problem ID [1946]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 8, page 34
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x),G(x)]']`

$$\boxed{x(6yx+5)+\left(2x^3+3y\right)y'=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
dsolve(x*(6*x*y(x)+5)+(2*x^3+3*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y=-\frac{2x^3}{3}-\frac{\sqrt{4x^6-15x^2-6c_1}}{3}$$
$$y=-\frac{2x^3}{3}+\frac{\sqrt{4x^6-15x^2-6c_1}}{3}$$

✓ Solution by Mathematica

Time used: 0.145 (sec). Leaf size: 69

```
DSolve[x*(6*x*y[x]+5)+(2*x^3+3*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{3}\left(-2x^3-\sqrt{4x^6-15x^2+9c_1}\right)$$
$$y(x)\to\frac{1}{3}\left(-2x^3+\sqrt{4x^6-15x^2+9c_1}\right)$$

4.5 problem 5

Internal problem ID [1947]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 8, page 34
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{3yx^2 + y^2x + \left(x^3 + yx^2 + \sin(y)\right)y' = -\mathrm{e}^x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve((3*x^2*y(x)+x*y(x)^2+exp(x))+(x^3+x^2*y(x)+sin(y(x)))*diff(y(x),x)=0,y(x), singsol=al
```

$$\frac{x^2y^2}{2} + x^3y + \mathrm{e}^x - \cos(y) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.377 (sec). Leaf size: 32

```
DSolve[(3*x^2*y[x]+x*y[x]^2+Exp[x])+(x^3+x^2*y[x]+Sin[y[x]])*y'[x]==0,y[x],x,IncludeSingular
```

$$\text{Solve}\left[x^3y(x) + \frac{1}{2}x^2y(x)^2 - \cos(y(x)) + e^x = c_1, y(x)\right]$$

4.6 problem 6

Internal problem ID [1948]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 8, page 34
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{2yx - (x^2 + y^2)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
dsolve(2*x*y(x)-(x^2+y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = -\frac{-1 + \sqrt{4c_1^2x^2 + 1}}{2c_1}$$
$$y = \frac{1 + \sqrt{4c_1^2x^2 + 1}}{2c_1}$$

✓ Solution by Mathematica

Time used: 0.923 (sec). Leaf size: 70

```
DSolve[2*x*y[x]-(x^2+y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-\sqrt{4x^2 + e^{2c_1}} - e^{c_1}\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{4x^2 + e^{2c_1}} - e^{c_1}\right)$$
$$y(x) \to 0$$

4.7 problem 7

Internal problem ID [1949]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 8, page 34
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{y\cos(x) - 2\sin(y) - (2x\cos(y) - \sin(x))\,y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve((y(x)*cos(x)-2*sin(y(x)))=(2*x*cos(y(x))-sin(x))*diff(y(x),x),y(x), singsol=all)
```

$$\sin(x)\,y - 2x\sin(y) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.151 (sec). Leaf size: 19

```
DSolve[(y[x]*Cos[x]-2*Sin[y[x]])==(2*x*Cos[y[x]]-Sin[x])*y'[x],y[x],x,IncludeSingularSolutio
```

$$\text{Solve}[2x\sin(y(x)) - y(x)\sin(x) = c_1, y(x)]$$

4.8 problem 8

Internal problem ID [1950]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 8, page 34
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{\frac{2yx-1}{y}+\frac{(x+3y)\,y'}{y^2}=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve((2*x*y(x)-1)/y(x)+(x+3*y(x))/y(x)^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y=\frac{x}{3\,\mathrm{LambertW}\left(\frac{\mathrm{e}^{\frac{x^2}{3}}c_1x}{3}\right)}$$

✓ Solution by Mathematica

Time used: 3.032 (sec). Leaf size: 37

```
DSolve[(2*x*y[x]-1)/y[x]+(x+3*y[x])/y[x]^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x}{3W\left(\frac{1}{3}xe^{\frac{1}{3}(x^2-c_1)}\right)}$$

$$y(x)\to 0$$

4.9 problem 10

Internal problem ID [1951]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 8, page 34
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y\,\mathrm{e}^{x}+\mathrm{e}^{x}y'=2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve((y(x)*exp(x)-2*x)+exp(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y=\left(x^{2}+c_{1}\right)\mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 17

```
DSolve[(y[x]*Exp[x]-2*x)+Exp[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{-x}\left(x^{2}+c_{1}\right)$$

4.10 problem 11

Internal problem ID [1952]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 8, page 34
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{3y\sin(x) - \cos(y) + (\sin(y)\,x - 3\cos(x))\,y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve((3*y(x)*sin(x)-cos(y(x)))+(x*sin(y(x))-3*cos(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$-3y\cos(x) - x\cos(y) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.162 (sec). Leaf size: 18

```
DSolve[(3*y[x]*Sin[x]-Cos[y[x]])+(x*Sin[y[x]]-3*Cos[x])*y'[x]==0,y[x],x,IncludeSingularSolut
```

$$\text{Solve}[x\cos(y(x)) + 3y(x)\cos(x) = c_1, y(x)]$$

4.11 problem 12

Internal problem ID [1953]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 8, page 34
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{y^2x + 2y + (2y^3 - yx^2 + 2x)\,y' = 0}$$

✗ Solution by Maple

```
dsolve((x*y(x)^2+2*y(x))+(2*y(x)^3-x^2*y(x)+2*x)*diff(y(x),x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(x*y[x]^2+2*y[x])+(2*y[x]^3-x^2*y[x]+2*x)*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

Not solved

4.12 problem 13

Internal problem ID [1954]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 8, page 34
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\frac{2}{y}-\frac{y}{x^2}+\left(\frac{1}{x}-\frac{2x}{y^2}\right)y'=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve((2/y(x)-y(x)/x^2)+(1/x-2*x/y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y=\sqrt{2}\,x$$
$$y=-\sqrt{2}\,x$$
$$y=\left(-\frac{c_1}{2}-\frac{\sqrt{c_1^2-8}}{2}\right)x$$
$$y=\left(-\frac{c_1}{2}+\frac{\sqrt{c_1^2-8}}{2}\right)x$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 55

```
DSolve[(2/y[x]-y[x]/x^2)+(1/x-2*x/y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}x$$

$$y(x) \to \sqrt{2}x$$

$$y(x) \to c_1 x$$

$$y(x) \to -\sqrt{2}x$$

$$y(x) \to \sqrt{2}x$$

4.13 problem 14

Internal problem ID [1955]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 8, page 34
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{\frac{yx+1}{y}+\frac{(-x+2y)\,y'}{y^2}=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve((x*y(x)+1)/y(x)+(2*y(x)-x)/y(x)^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y=-\frac{x}{2\,\mathrm{LambertW}\left(-\frac{\mathrm{e}^{\frac{x^2}{4}}c_1x}{2}\right)}$$

✓ Solution by Mathematica

Time used: 3.516 (sec). Leaf size: 37

```
DSolve[(x*y[x]+1)/y[x]+(2*y[x]-x)/y[x]^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{x}{2W\left(-\frac{1}{2}xe^{\frac{1}{4}(x^2-2c_1)}\right)}$$

$$y(x)\to 0$$

4.14 problem 15

Internal problem ID [1956]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 8, page 34
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{\frac{y(2+x^3y)}{x^3}-\frac{(1-2x^3y)\,y'}{x^2}=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 57

```
dsolve(y(x)*(2+x^3*y(x))/x^3=(1-2*x^3*y(x))/x^2*diff(y(x),x),y(x), singsol=all)
```

$$y=\frac{c_1^3+\sqrt{c_1^6+4c_1x^5}}{2x^3c_1^3}$$

$$y=-\frac{-c_1^3+\sqrt{c_1^6+4c_1x^5}}{2x^3c_1^3}$$

✓ Solution by Mathematica

Time used: 0.861 (sec). Leaf size: 80

```
DSolve[y[x]*(2+x^3*y[x])/x^3==(1-2*x^3*y[x])/x^2*y'[x],y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x)\to\frac{1-\sqrt{\frac{1}{x^3}}x^2\sqrt{\frac{1}{x}+4c_1x^4}}{2x^3}$$

$$y(x)\to\frac{1+\sqrt{\frac{1}{x^3}}x^2\sqrt{\frac{1}{x}+4c_1x^4}}{2x^3}$$

4.15 problem 16

Internal problem ID [1957]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 8, page 34
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_Abel, '2nd type', 'class B']]`

$$\boxed{y^2 \csc(x)^2 + 6yx - (2y\cot(x) - 3x^2)\, y' = 2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 71

```
dsolve(y(x)^2*csc(x)^2+6*x*y(x)-2=(2*y(x)*cot(x)-3*x^2)*diff(y(x),x),y(x), singsol=all)
```

$$y = \frac{3\tan(x)\, x^2}{2} - \frac{\sqrt{9\tan(x)^2 x^4 + 4c_1 \tan(x) - 8x\tan(x)}}{2}$$

$$y = \frac{3\tan(x)\, x^2}{2} + \frac{\sqrt{9\tan(x)^2 x^4 + 4c_1 \tan(x) - 8x\tan(x)}}{2}$$

✓ Solution by Mathematica

Time used: 31.692 (sec). Leaf size: 201

```
DSolve[y[x]^2*Csc[x]^2+6*x*y[x]-2==(2*y[x]*Cot[x]-3*x^2)*y'[x],y[x],x,IncludeSingularSolutio
```

$$y(x) \to \frac{3}{2}x^2\tan(x) - \frac{\csc(2x)\sqrt{-\left(\tan(x)\left(16\cos^2(x)\arcsin\left(\sqrt{\sin^2(x)}\right) - 9x^4e^{\operatorname{arctanh}(\cos(2x))} + \cos(2x)\left(9x^4e^{\operatorname{arctanh}(\cos(2x))}\right.\right.\right.}}{2\sqrt{\csc(2x)e^{\operatorname{arctanh}(\cos(2x))}}}$$

$$y(x) \to \frac{3}{2}x^2\tan(x) + \frac{\csc(2x)\sqrt{-\left(\tan(x)\left(16\cos^2(x)\arcsin\left(\sqrt{\sin^2(x)}\right) - 9x^4e^{\operatorname{arctanh}(\cos(2x))} + \cos(2x)\left(9x^4e^{\operatorname{arctanh}(\cos(2x))}\right.\right.\right.}}{2\sqrt{\csc(2x)e^{\operatorname{arctanh}(\cos(2x))}}}$$

4.16 problem 17

Internal problem ID [1958]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 8, page 34
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _exact, _rational]`

$$\boxed{\frac{2y}{x^3}+\frac{2x}{y^2}-\left(\frac{1}{x^2}+\frac{2x^2}{y^3}\right)y'=0}$$

✓ Solution by Maple

Time used: 0.312 (sec). Leaf size: 231

```
dsolve(2*(y(x)/x^3+x/y(x)^2)=(1/x^2+2*x^2/y(x)^3)*diff(y(x),x),y(x), singsol=all)
```

$$y = \frac{x^{\frac{4}{3}} \operatorname{RootOf}\left(x^{14}\_Z^{18}+6x^{\frac{38}{3}}\_Z^{16}+15x^{\frac{34}{3}}\_Z^{14}+\left(20x^{10}-27c_1x^8\right)\_Z^{12}+\left(15x^{\frac{26}{3}}-81x^{\frac{20}{3}}c_1\right)\_Z^{10}+\right.}{\operatorname{RootOf}\left(x^{14}\_Z^{18}+6x^{\frac{38}{3}}\_Z^{16}+15x^{\frac{34}{3}}\_Z^{14}+\left(20x^{10}-27c_1x^8\right)\_Z^{12}+\left(15x^{\frac{26}{3}}-81x^{\frac{20}{3}}c_1\right)\_Z^{10}+\right.}$$

✓ Solution by Mathematica

Time used: 10.8 (sec). Leaf size: 414

```
DSolve[2*(y[x]/x^3+x/y[x]^2)==(1/x^2+2*x^2/y[x]^3)*y'[x],y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{3}\left(c_1x^2 + \frac{c_1{}^2x^4}{\sqrt[3]{c_1{}^3x^6 + \frac{27x^4}{2} + \frac{3}{2}\sqrt{3}\sqrt{x^8\left(27+4c_1{}^3x^2\right)}}} + \sqrt[3]{c_1{}^3x^6 + \frac{27x^4}{2} + \frac{3}{2}\sqrt{3}\sqrt{x^8\left(27+4c_1{}^3x^2\right)}}\right)$$

$$y(x) \to \frac{1}{12}\left(4c_1x^2 - \frac{2\left(1+i\sqrt{3}\right)c_1{}^2x^4}{\sqrt[3]{c_1{}^3x^6 + \frac{27x^4}{2} + \frac{3}{2}\sqrt{3}\sqrt{x^8\left(27+4c_1{}^3x^2\right)}}} + i2^{2/3}\left(\sqrt{3}+i\right)\sqrt[3]{2c_1{}^3x^6 + 27x^4 + 3\sqrt{3}\sqrt{x^8\left(27+4c_1{}^3x^2\right)}}\right)$$

$$y(x) \to \frac{1}{12}\left(4c_1x^2 + \frac{2i\left(\sqrt{3}+i\right)c_1{}^2x^4}{\sqrt[3]{c_1{}^3x^6 + \frac{27x^4}{2} + \frac{3}{2}\sqrt{3}\sqrt{x^8\left(27+4c_1{}^3x^2\right)}}} - 2^{2/3}\left(1+i\sqrt{3}\right)\sqrt[3]{2c_1{}^3x^6 + 27x^4 + 3\sqrt{3}\sqrt{x^8\left(27+4c_1{}^3x^2\right)}}\right)$$

4.17 problem 18

Internal problem ID [1959]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 8, page 34
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{\cos(y) - (\sin(y)\, x - y^2)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(cos(y(x))-(x*sin(y(x))-y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$x - \frac{-\frac{y^3}{3} + c_1}{\cos(y)} = 0$$

✓ Solution by Mathematica

Time used: 0.124 (sec). Leaf size: 23

```
DSolve[Cos[y[x]]-(x*Sin[y[x]]-y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x = -\frac{1}{3}y(x)^3 \sec(y(x)) + c_1 \sec(y(x)), y(x)\right]$$

4.18 problem 19

Internal problem ID [1960]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 8, page 34
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{2y\sin(yx) + \left(2x\sin(yx) + y^3\right)y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 26

```
dsolve(2*y(x)*sin(x*y(x))+(2*x*sin(x*y(x))+y(x)^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{\mathrm{RootOf}\left(-8\cos(\_Z)\,x^4 + 4c_1x^4 + \_Z^4\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.158 (sec). Leaf size: 22

```
DSolve[2*y[x]*Sin[x*y[x]]+(2*x*Sin[x*y[x]]+y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions
```

$$\mathrm{Solve}\left[\frac{y(x)^4}{4} - 2\cos(xy(x)) = c_1, y(x)\right]$$

4.19 problem 20

Internal problem ID [1961]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 8, page 34
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{\frac{x\cos\left(\frac{x}{y}\right)}{y}+\sin\left(\frac{x}{y}\right)-\frac{x^{2}\cos\left(\frac{x}{y}\right)y'}{y^{2}}=-\cos\left(x\right)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((x/y(x)*cos(x/y(x))+sin(x/y(x))+cos(x) )-x^2/y(x)^2*cos(x/y(x))*diff(y(x),x)=0,y(x),
```

$$y=-\frac{x}{\arcsin\left(\frac{\sin(x)+c_{1}}{x}\right)}$$

✓ Solution by Mathematica

Time used: 25.647 (sec). Leaf size: 25

```
DSolve[(x/y[x]*Cos[x/y[x]]+Sin[x/y[x]]+Cos[x] )-x^2/y[x]^2*Cos[x/y[x]]*y'[x]==0,y[x],x,Inclu
```

$$y(x)\to -\frac{x}{\arcsin\left(\frac{\sin(x)+c_{1}}{x}\right)}$$

$$y(x)\to 0$$

4.20 problem 21

Internal problem ID [1962]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 8, page 34
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{y\,e^{yx} + 2yx + \left(x\,e^{yx} + x^2\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve((y(x)*exp(x*y(x))+2*x*y(x))+(x*exp(x*y(x))+x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{-\operatorname{LambertW}\left(\frac{e^{-\frac{c_1}{x}}}{x}\right) - \frac{c_1}{x}}{x}$$

✓ Solution by Mathematica

Time used: 3.117 (sec). Leaf size: 28

```
DSolve[(y[x]*Exp[x*y[x]]+2*x*y[x])+(x*Exp[x*y[x]]+x^2)*y'[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{c_1 - xW\left(\frac{e^{\frac{c_1}{x}}}{x}\right)}{x^2}$$

4.21 problem 22

Internal problem ID [1963]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 8, page 34
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$\boxed{\frac{x^2+3y^2}{x\,(3x^2+4y^2)}+\frac{(2x^2+y^2)\,y'}{y\,(3x^2+4y^2)}=0}$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 45

```
dsolve((x^2+3*y(x)^2)/(x*(3*x^2+4*y(x)^2))+(2*x^2+y(x)^2)/(y(x)*(3*x^2+4*y(x)^2))*diff(y(x),
```

$$y=\left(\frac{\operatorname{RootOf}\left(x^3\mathrm{e}^{3c_1}\_Z^{24}-4\_Z^{15}-3x^3\mathrm{e}^{3c_1}\right)^5}{x}\right)^{\frac{3}{2}}\mathrm{e}^{-\frac{3c_1}{2}}x$$

✓ Solution by Mathematica

Time used: 60.136 (sec). Leaf size: 1649

```
DSolve[(x^2+3*y[x]^2)/(x*(3*x^2+4*y[x]^2))+(2*x^2+y[x]^2)/(y[x]*(3*x^2+4*y[x]^2))*y'[x]==0,y
```

Too large to display

4.22 problem 23

Internal problem ID [1964]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 8, page 34
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$\boxed{\frac{x^2-y^2}{x\left(2x^2+y^2\right)}+\frac{\left(x^2+2y^2\right)y'}{y\left(2x^2+y^2\right)}=0}$$

✓ Solution by Maple

Time used: 0.422 (sec). Leaf size: 35

```
dsolve((x^2-y(x)^2)/(x*(2*x^2+y(x)^2))+(x^2+2*y(x)^2)/(y(x)*(2*x^2+y(x)^2))*diff(y(x),x)=0,y
```

$$y=\frac{c_1\,\mathrm{RootOf}\left(\_Z^{16}+\frac{2x^4\_Z^4}{c_1^2}-\frac{x^4}{c_1^2}\right)^6}{x}$$

✓ Solution by Mathematica

Time used: 60.251 (sec). Leaf size: 3381

```
DSolve[(x^2-y[x]^2)/(x*(2*x^2+y[x]^2))+(x^2+2*y[x]^2)/(y[x]*(2*x^2+y[x]^2))*y'[x]==0,y[x],x,
```

Too large to display

4.23 problem 24

Internal problem ID [1965]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 8, page 34
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{\frac{2x^2}{x^2+y^2}+\ln\left(x^2+y^2\right)+\frac{2xyy'}{x^2+y^2}=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
dsolve((2*x^2/(x^2+y(x)^2)+ln(x^2+y(x)^2))+(2*x*y(x))/(x^2+y(x)^2)*diff(y(x),x)=0,y(x), sing
```

$$y=\sqrt{-x^2+\mathrm{e}^{-\frac{c_1}{x}}}$$
$$y=-\sqrt{-x^2+\mathrm{e}^{-\frac{c_1}{x}}}$$

✓ Solution by Mathematica

Time used: 0.823 (sec). Leaf size: 47

```
DSolve[(2*x^2/(x^2+y[x]^2)+Log[x^2+y[x]^2])+(2*x*y[x])/(x^2+y[x]^2)*y'[x]==0,y[x],x,IncludeS
```

$$y(x)\to -\sqrt{-x^2+e^{\frac{c_1}{x}}}$$
$$y(x)\to \sqrt{-x^2+e^{\frac{c_1}{x}}}$$

5 Exercise 9, page 38

5.1 problem 1

Internal problem ID [1966]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 9, page 38
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - y = -\ln(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x*diff(y(x),x)+(ln(x)-y(x))=0,y(x), singsol=all)
```

$$y = c_1 x + \ln(x) + 1$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 13

```
DSolve[x*y'[x]+(Log[x]-y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(x) + c_1 x + 1$$

5.2 problem 2

Internal problem ID [1967]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 9, page 38
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{yx + \left(x^2 + y\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 1153

```
dsolve(x*y(x)+(x^2+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{\left(\frac{4\left(2c_1^2-x^6+2c_1\sqrt{-x^6+c_1^2}\right)^{\frac{1}{3}}}{x^2} + \frac{4x^2}{\left(2c_1^2-x^6+2c_1\sqrt{-x^6+c_1^2}\right)^{\frac{1}{3}}} - 4\right)x^2}{8}$$

$$y = \frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)^3\left(\frac{4\left(2c_1^2-x^6+2c_1\sqrt{-x^6+c_1^2}\right)^{\frac{1}{3}}}{x^2} + \frac{4x^2}{\left(2c_1^2-x^6+2c_1\sqrt{-x^6+c_1^2}\right)^{\frac{1}{3}}} - 4\right)x^2}{8}$$

$$y = \frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^3\left(\frac{4\left(2c_1^2-x^6+2c_1\sqrt{-x^6+c_1^2}\right)^{\frac{1}{3}}}{x^2} + \frac{4x^2}{\left(2c_1^2-x^6+2c_1\sqrt{-x^6+c_1^2}\right)^{\frac{1}{3}}} - 4\right)x^2}{8}$$

$$y = \frac{\left(-\frac{2\left(2c_1^2-x^6+2c_1\sqrt{-x^6+c_1^2}\right)^{\frac{1}{3}}}{x^2} - \frac{2x^2}{\left(2c_1^2-x^6+2c_1\sqrt{-x^6+c_1^2}\right)^{\frac{1}{3}}} - 4 - 4i\sqrt{3}\left(\frac{\left(2c_1^2-x^6+2c_1\sqrt{-x^6+c_1^2}\right)^{\frac{1}{3}}}{2x^2} - \frac{x^2}{2\left(2c_1^2-x^6+2c_1\sqrt{-\cdots}\right.}\right.\right.}{8}$$

$$y = \frac{\left(-\frac{2\left(2c_1^2-x^6+2c_1\sqrt{-x^6+c_1^2}\right)^{\frac{1}{3}}}{x^2} - \frac{2x^2}{\left(2c_1^2-x^6+2c_1\sqrt{-x^6+c_1^2}\right)^{\frac{1}{3}}} - 4 + 4i\sqrt{3}\left(\frac{\left(2c_1^2-x^6+2c_1\sqrt{-x^6+c_1^2}\right)^{\frac{1}{3}}}{2x^2} - \frac{x^2}{2\left(2c_1^2-x^6+2c_1\sqrt{-\cdots}\right.}\right.\right.}{8}$$

$$y = \frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)^3\left(-\frac{2\left(2c_1^2-x^6+2c_1\sqrt{-x^6+c_1^2}\right)^{\frac{1}{3}}}{x^2} - \frac{2x^2}{\left(2c_1^2-x^6+2c_1\sqrt{-x^6+c_1^2}\right)^{\frac{1}{3}}} - 4 - 4i\sqrt{3}\left(\frac{\left(2c_1^2-x^6+2c_1\sqrt{-x^6+c_1^2}\right)^{\frac{1}{3}}}{2x^2} - \cdots\right.\right.}{8}$$

$$y = \frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)^3\left(-\frac{2\left(2c_1^2-x^6+2c_1\sqrt{-x^6+c_1^2}\right)^{\frac{1}{3}}}{x^2} - \frac{2x^2}{\left(2c_1^2-x^6+2c_1\sqrt{-x^6+c_1^2}\right)^{\frac{1}{3}}} - 4 + 4i\sqrt{3}\left(\frac{\left(2c_1^2-x^6+2c_1\sqrt{-x^6+c_1^2}\right)^{\frac{1}{3}}}{2x^2} - \cdots\right.\right.}{8}$$

$$y = \frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^3\left(-\frac{2\left(2c_1^2-x^6+2c_1\sqrt{-x^6+c_1^2}\right)^{\frac{1}{3}}}{x^2} - \frac{2x^2}{\left(2c_1^2-x^6+2c_1\sqrt{-x^6+c_1^2}\right)^{\frac{1}{3}}} - 4 - 4i\sqrt{3}\left(\frac{\left(2c_1^2-x^6+2c_1\sqrt{-x^6+c_1^2}\right)^{\frac{1}{3}}}{2x^2} - \cdots\right.\right.}{8}$$

✓ Solution by Mathematica

Time used: 60.038 (sec). Leaf size: 397

```
DSolve[x*y[x]+(x^2+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^2 + \frac{1}{\text{Root}\left[\#1^6\left(x^{12}+e^{12c_1}\right)-6\#1^4x^8+4\#1^3x^6+9\#1^2x^4-12\#1x^2+4\&,1\right]}$$

$$y(x) \to -x^2 + \frac{1}{\text{Root}\left[\#1^6\left(x^{12}+e^{12c_1}\right)-6\#1^4x^8+4\#1^3x^6+9\#1^2x^4-12\#1x^2+4\&,2\right]}$$

$$y(x) \to -x^2 + \frac{1}{\text{Root}\left[\#1^6\left(x^{12}+e^{12c_1}\right)-6\#1^4x^8+4\#1^3x^6+9\#1^2x^4-12\#1x^2+4\&,3\right]}$$

$$y(x) \to -x^2 + \frac{1}{\text{Root}\left[\#1^6\left(x^{12}+e^{12c_1}\right)-6\#1^4x^8+4\#1^3x^6+9\#1^2x^4-12\#1x^2+4\&,4\right]}$$

$$y(x) \to -x^2 + \frac{1}{\text{Root}\left[\#1^6\left(x^{12}+e^{12c_1}\right)-6\#1^4x^8+4\#1^3x^6+9\#1^2x^4-12\#1x^2+4\&,5\right]}$$

$$y(x) \to -x^2 + \frac{1}{\text{Root}\left[\#1^6\left(x^{12}+e^{12c_1}\right)-6\#1^4x^8+4\#1^3x^6+9\#1^2x^4-12\#1x^2+4\&,6\right]}$$

5.3 problem 3

Internal problem ID [1968]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 9, page 38
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x-2yx)\,y'+2y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve((x-2*x*y(x))*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y=-\frac{\operatorname{LambertW}\left(-\frac{2c_1}{x^2}\right)}{2}$$

✓ Solution by Mathematica

Time used: 2.94 (sec). Leaf size: 26

```
DSolve[(x-2*x*y[x])*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{1}{2}W\left(-\frac{2e^{-c_1}}{x^2}\right)$$
$$y(x)\to 0$$

5.4 problem 4

Internal problem ID [1969]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 9, page 38
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{yx^2 + y^2 + x^3y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve((x^2*y(x)+y(x)^2)+x^3*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{3x^2}{3c_1x^3 - 1}$$

✓ Solution by Mathematica

Time used: 0.135 (sec). Leaf size: 26

```
DSolve[(x^2*y[x]+y[x]^2)+x^3*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3x^2}{-1 + 3c_1x^3}$$

$$y(x) \to 0$$

5.5 problem 5

Internal problem ID [1970]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 9, page 38
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$\boxed{xy^3 + x^2y^2y' = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 96

```
dsolve((x*y(x)^3-1)+x^2*y(x)^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{(12x^2 + 8c_1)^{\frac{1}{3}}}{2x}$$
$$y = \frac{-\frac{(12x^2+8c_1)^{\frac{1}{3}}}{4} - \frac{i\sqrt{3}\,(12x^2+8c_1)^{\frac{1}{3}}}{4}}{x}$$
$$y = \frac{-\frac{(12x^2+8c_1)^{\frac{1}{3}}}{4} + \frac{i\sqrt{3}\,(12x^2+8c_1)^{\frac{1}{3}}}{4}}{x}$$

✓ Solution by Mathematica

Time used: 0.208 (sec). Leaf size: 80

```
DSolve[(x*y[x]^3-1)+x^2*y[x]^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[3]{-\frac{1}{2}}\sqrt[3]{3x^2+2c_1}}{x}$$
$$y(x) \to \frac{\sqrt[3]{\frac{3x^2}{2}+c_1}}{x}$$
$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{\frac{3x^2}{2}+c_1}}{x}$$

5.6 problem 6

Internal problem ID [1971]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 9, page 38
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{\left(y^3x^3-1\right)y'+x^2y^4=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve((x^3*y(x)^3-1)*diff(y(x),x)+x^2*y(x)^4=0,y(x), singsol=all)
```

$$y=\mathrm{e}^{-\frac{\mathrm{LambertW}\left(-x^3\mathrm{e}^{-3c_1}\right)}{3}-c_1}$$

✓ Solution by Mathematica

Time used: 4.808 (sec). Leaf size: 90

```
DSolve[(x^3*y[x]^3-1)*y'[x]+x^2*y[x]^4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\frac{\sqrt[3]{W\left(-e^{-3c_1}x^3\right)}}{x}$$
$$y(x)\to\frac{\sqrt[3]{-1}\sqrt[3]{W\left(-e^{-3c_1}x^3\right)}}{x}$$
$$y(x)\to-\frac{(-1)^{2/3}\sqrt[3]{W\left(-e^{-3c_1}x^3\right)}}{x}$$
$$y(x)\to 0$$

5.7 problem 7

Internal problem ID [1972]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 9, page 38
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{y(y - x^2) + x^3 y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(y(x)*(y(x)-x^2)+x^3*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{x^2}{c_1 x - 1}$$

✓ Solution by Mathematica

Time used: 0.141 (sec). Leaf size: 22

```
DSolve[y[x]*(y[x]-x^2)+x^3*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{-1 + c_1 x}$$
$$y(x) \to 0$$

5.8 problem 8

Internal problem ID [1973]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 9, page 38
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y + y^2 x + (x - y x^2)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((y(x)+x*y(x)^2)+(x-x^2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = -\frac{1}{\mathrm{LambertW}\left(-\frac{c_1}{x^2}\right) x}$$

✓ Solution by Mathematica

Time used: 5.848 (sec). Leaf size: 35

```
DSolve[(y[x]+x*y[x]^2)+(x-x^2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{xW\left(\frac{e^{-1+\frac{9c_1}{2^{2/3}}}}{x^2}\right)}$$

$$y(x) \to 0$$

5.9 problem 9

Internal problem ID [1974]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 9, page 38
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{\left(x - x\sqrt{x^2 - y^2}\right) y' - y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 27

```
dsolve((x-x*sqrt(x^2-y(x)^2))*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y - \arctan\left(\frac{y}{\sqrt{x^2 - y^2}}\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.529 (sec). Leaf size: 29

```
DSolve[(x-x*Sqrt[x^2-y[x]^2])*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\arctan\left(\frac{\sqrt{x^2 - y(x)^2}}{y(x)}\right) + y(x) = c_1, y(x)\right]$$

5.10 problem 10

Internal problem ID [1975]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 9, page 38
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{2yx + (y - x^2)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve(2*x*y(x)+(y(x)-x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = -\frac{x^2}{\operatorname{LambertW}(-c_1 x^2)}$$

✓ Solution by Mathematica

Time used: 3.007 (sec). Leaf size: 285

```
DSolve[2*x*y[x]+(y[x]-x^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\left(2-\frac{2(x^3+2xy(x))}{\sqrt[3]{x^3}(x^2-y(x))}\right)\left(\frac{x^3+2xy(x)}{\sqrt[3]{x^3}(x^2-y(x))}+2\right)\left(\left(1-\frac{x(x^2+2y(x))}{\sqrt[3]{x^3}(x^2-y(x))}\right)\log\left(\frac{2-\frac{2(x^3+2xy(x))}{\sqrt[3]{x^3}(x^2-y(x))}}{\sqrt[3]{2}}\right)+\left(\frac{x^3+2x}{\sqrt[3]{x^3}(x^2}\right.\right.}{9\sqrt[3]{2}\left(-\frac{(x^2+2y(x))^3}{(x^2-y(x))^3}+\frac{3(x^3+2xy(x))}{\sqrt[3]{x^3}(x^2-y(x))}-2\right)}\right.$$

$$\left.+c_1, y(x)\right]$$

5.11 problem 11

Internal problem ID [1976]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 9, page 38
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y - x(y x^2 - 1)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 46

```
dsolve(y(x)=x*(x^2*y(x)-1)*diff(y(x),x),y(x), singsol=all)
```

$$y = \frac{x + \sqrt{x^2 - c_1}}{c_1 x}$$
$$y = -\frac{-x + \sqrt{x^2 - c_1}}{c_1 x}$$

✓ Solution by Mathematica

Time used: 1.509 (sec). Leaf size: 77

```
DSolve[y[x]==x*(x^2*y[x]-1)*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{x^2 + \sqrt{-\frac{1}{x^3}} x^2 \sqrt{-x\,(x^2 + c_1)}}$$
$$y(x) \to \frac{x}{x^3 + \frac{\sqrt{-x(x^2+c_1)}}{\sqrt{-\frac{1}{x^3}}}}$$
$$y(x) \to 0$$

5.12 problem 12

Internal problem ID [1977]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 9, page 38
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{e^{x} y' - 2y^{2} x - y\, e^{x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(exp(x)*diff(y(x),x)=2*x*y(x)^2+exp(x)*y(x),y(x), singsol=all)
```

$$y = \frac{e^{x}}{-x^{2} + c_1}$$

✓ Solution by Mathematica

Time used: 0.286 (sec). Leaf size: 25

```
DSolve[Exp[x]*y'[x]==2*x*y[x]^2+Exp[x]*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{x}}{x^{2} - c_1}$$
$$y(x) \to 0$$

5.13 problem 13

Internal problem ID [1978]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 9, page 38
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{(x^2 + y^2 + x)\, y' - y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 30

```
dsolve((x^2+y(x)^2+x)*diff(y(x),x)=y(x),y(x), singsol=all)
```

$$c_1 + \frac{\mathrm{e}^{-2iy}(ix + y)}{2iy + 2x} = 0$$

✓ Solution by Mathematica

Time used: 0.099 (sec). Leaf size: 18

```
DSolve[(x^2+y[x]^2+x)*y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[y(x) - \arctan\left(\frac{x}{y(x)}\right) = c_1, y(x)\right]$$

5.14 problem 14

Internal problem ID [1979]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 9, page 38
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(2x+3yx^2)\,y'+y+2y^2x=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 412

```
dsolve((2*x+3*x^2*y(x))*diff(y(x),x)+(y(x)+2*y(x)^2*x)=0,y(x), singsol=all)
```

$$y = \frac{\left(\left(12\sqrt{-\frac{3(4c_1-27x)}{x}}\, x - 8c_1 + 108x\right) c_1^2\right)^{\frac{1}{3}}}{6c_1 x} + \frac{2c_1}{3x\left(\left(12\sqrt{-\frac{3(4c_1-27x)}{x}}\, x - 8c_1 + 108x\right) c_1^2\right)^{\frac{1}{3}}} - \frac{1}{3x}$$

$$y = -\frac{\left(\left(12\sqrt{-\frac{3(4c_1-27x)}{x}}\, x - 8c_1 + 108x\right) c_1^2\right)^{\frac{1}{3}}}{12c_1 x} - \frac{c_1}{3x\left(\left(12\sqrt{-\frac{3(4c_1-27x)}{x}}\, x - 8c_1 + 108x\right) c_1^2\right)^{\frac{1}{3}}} - \frac{1}{3x} - \frac{i\sqrt{3}\left(\frac{\left(\left(12\sqrt{-\frac{3(4c_1-27x)}{x}}\, x-8c_1+108x\right) c_1^2\right)^{\frac{1}{3}}}{6c_1 x} - \frac{2c_1}{3x\left(\left(12\sqrt{-\frac{3(4c_1-27x)}{x}}\, x-8c_1+108x\right) c_1^2\right)^{\frac{1}{3}}}\right)}{2}$$

$$y = -\frac{\left(\left(12\sqrt{-\frac{3(4c_1-27x)}{x}}\, x - 8c_1 + 108x\right) c_1^2\right)^{\frac{1}{3}}}{12c_1 x} - \frac{c_1}{3x\left(\left(12\sqrt{-\frac{3(4c_1-27x)}{x}}\, x - 8c_1 + 108x\right) c_1^2\right)^{\frac{1}{3}}} - \frac{1}{3x} + \frac{i\sqrt{3}\left(\frac{\left(\left(12\sqrt{-\frac{3(4c_1-27x)}{x}}\, x-8c_1+108x\right) c_1^2\right)^{\frac{1}{3}}}{6c_1 x} - \frac{2c_1}{3x\left(\left(12\sqrt{-\frac{3(4c_1-27x)}{x}}\, x-8c_1+108x\right) c_1^2\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 21.875 (sec). Leaf size: 380

```
DSolve[(2*x+3*x^2*y[x])*y'[x]+(y[x]+2*y[x]^2*x)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}\left(\frac{2}{\sqrt[3]{\frac{3}{2}\sqrt{3}\sqrt{c_1x^7(-4+27c_1x)}+\frac{27c_1x^4}{2}-x^3}} + \frac{2^{2/3}\sqrt[3]{3\sqrt{3}\sqrt{c_1x^7(-4+27c_1x)}+27c_1x^4-2x^3}}{x^2}-\frac{2}{x}\right)$$

$$y(x) \to \frac{1}{12}\left(-\frac{2(1+i\sqrt{3})}{\sqrt[3]{\frac{3}{2}\sqrt{3}\sqrt{c_1x^7(-4+27c_1x)}+\frac{27c_1x^4}{2}-x^3}} + \frac{i2^{2/3}(\sqrt{3}+i)\sqrt[3]{3\sqrt{3}\sqrt{c_1x^7(-4+27c_1x)}+27c_1x^4-2x^3}}{x^2}-\frac{4}{x}\right)$$

$$y(x) \to \frac{1}{12}\left(\frac{2i(\sqrt{3}+i)}{\sqrt[3]{\frac{3}{2}\sqrt{3}\sqrt{c_1x^7(-4+27c_1x)}+\frac{27c_1x^4}{2}-x^3}} - \frac{2^{2/3}(1+i\sqrt{3})\sqrt[3]{3\sqrt{3}\sqrt{c_1x^7(-4+27c_1x)}+27c_1x^4-2x^3}}{x^2}-\frac{4}{x}\right)$$

5.15 problem 15

Internal problem ID [1980]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 9, page 38
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _Bernoulli]`

$$\boxed{2x^2yy'-2y^2x=-\mathrm{e}^x x^4}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve(2*x^2*y(x)*diff(y(x),x)+(x^4*exp(x)-2*x*y(x)^2)=0,y(x), singsol=all)
```

$$y=\sqrt{-\mathrm{e}^x+c_1}\,x$$
$$y=-\sqrt{-\mathrm{e}^x+c_1}\,x$$

✓ Solution by Mathematica

Time used: 7.23 (sec). Leaf size: 45

```
DSolve[2*x^2*y[x]*y'[x]+(x^4*Exp[x]-2*x*y[x]^2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\sqrt{-x^2\left(e^x-c_1\right)}$$
$$y(x)\to\sqrt{-x^2\left(e^x-c_1\right)}$$

5.16 problem 20

Internal problem ID [1981]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 9, page 38
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y(1 - y^2x^4) + y'x = 0}$$

With initial conditions

$$[y(1) = -1]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 18

```
dsolve([y(x)*(1-x^4*y(x)^2)+x*diff(y(x),x)=0,y(1) = -1],y(x), singsol=all)
```

$$y = -\frac{1}{\sqrt{-x^2 + 2x}}$$

✓ Solution by Mathematica

Time used: 0.39 (sec). Leaf size: 21

```
DSolve[{y[x]*(1-x^4*y[x]^2)+x*y'[x]==0,{y[1]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{-x^2 (x^2 - 2)}}$$

5.17 problem 21

Internal problem ID [1982]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 9, page 38
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y(x^2-1)+x(x^2+1)\,y'=0}$$

With initial conditions

$$[y(1)=2]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve([y(x)*(x^2-1)+x*(x^2+1)*diff(y(x),x)=0,y(1) = 2],y(x), singsol=all)
```

$$y=\frac{4x}{x^2+1}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 15

```
DSolve[{y[x]*(x^2-1)+x*(x^2+1)*y'[x]==0,{y[1]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{4x}{x^2+1}$$

5.18 problem 22

Internal problem ID [1983]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 9, page 38
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{x^2y^2 - y + (2x^3y + x)\, y' = 0}$$

With initial conditions

$$[y(2) = -2]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 20

```
dsolve([(x^2*y(x)^2-y(x))+(2*x^3*y(x)+x)*diff(y(x),x)=0,y(2) = -2],y(x), singsol=all)
```

$$y = \frac{-1 - \sqrt{28x^3 + 1}}{2x^2}$$

✓ Solution by Mathematica

Time used: 0.648 (sec). Leaf size: 34

```
DSolve[{(x^2*y[x]^2-y[x])+(2*x^3*y[x]+x)*y'[x]==0,{y[2]==-2}},y[x],x,IncludeSingularSolution
```

$$y(x) \to -\frac{\sqrt{\frac{1}{x^2}}\sqrt{28x^3 + 1}x + 1}{2x^2}$$

5.19 problem 23

Internal problem ID [1984]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 9, page 38
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{(x^2 + y^2 - 2y)\, y' = 2x}$$

With initial conditions

$$[y(1) = 0]$$

✗ Solution by Maple

```
dsolve([(x^2+y(x)^2-2*y(x))*diff(y(x),x)=2*x,y(1) = 0],y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 0.159 (sec). Leaf size: 29

```
DSolve[{(x^2+y[x]^2-2*y[x])*y'[x]==2*x,{y[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x^2\left(-e^{-y(x)}\right) - e^{-y(x)}y(x)^2 = -1, y(x)\right]$$

5.20 problem 24

Internal problem ID [1985]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 9, page 38
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y - x^2\sqrt{x^2 - y^2} - y'x = 0}$$

With initial conditions

$$[y(1) = 1]$$

✗ Solution by Maple

```
dsolve([(y(x)-x^2*sqrt(x^2-y(x)^2))-x*diff(y(x),x)=0,y(1) = 1],y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{(y[x]-x^2*Sqrt[x^2-y[x]^2])-x*y'[x]==0,{y[1]==1}},y[x],x,IncludeSingularSolutions ->
```

{}

5.21 problem 25

Internal problem ID [1986]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 9, page 38
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{y(x+y^2)+x(x-y^2)\,y'=0}$$

With initial conditions

$$[y(2)=2]$$

✓ Solution by Maple

Time used: 0.36 (sec). Leaf size: 40

```
dsolve([y(x)*(x+y(x)^2)+x*(x-y(x)^2)*diff(y(x),x)=0,y(2) = 2],y(x), singsol=all)
```

$$y=\operatorname{RootOf}\left(-3\ln(x)+4\ln(2)-4\ln(5)+4\ln\left(\frac{\_Z^2+3x}{x}\right)-2\ln\left(\frac{\_Z}{\sqrt{x}}\right)\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y[x]*(x+y[x]^2)+x*(x-y[x]^2)*y'[x]==0,{y[2]==2}},y[x],x,IncludeSingularSolutions ->
```

{}

6 Exercise 10, page 41

6.1 problem 1

Internal problem ID [1987]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 10, page 41
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + 2y = x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x)+2*y(x)=x^2,y(x), singsol=all)
```

$$y = \frac{\frac{x^4}{4} + c_1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 19

```
DSolve[x*y'[x]+2*y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{4} + \frac{c_1}{x^2}$$

6.2 problem 2

Internal problem ID [1988]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 10, page 41
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - yx = e^{\frac{x^2}{2}} \cos(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)-x*y(x)=exp(x^2/2)*cos(x),y(x), singsol=all)
```

$$y = (\sin(x) + c_1) e^{\frac{x^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 20

```
DSolve[y'[x]-x*y[x]==Exp[x^2/2]*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{x^2}{2}}(\sin(x) + c_1)$$

6.3 problem 3

Internal problem ID [1989]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 10, page 41
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + 2yx = 2x\,\mathrm{e}^{-x^2}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)+2*x*y(x)=2*x*exp(-x^2),y(x), singsol=all)
```

$$y = \left(x^2 + c_1\right)\mathrm{e}^{-x^2}$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 19

```
DSolve[y'[x]+2*x*y[x]==2*x*Exp[-x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x^2}\left(x^2 + c_1\right)$$

6.4 problem 4

Internal problem ID [1990]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 10, page 41
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - y = 3\,\mathrm{e}^{x} x^{2}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=y(x)+3*x^2*exp(x),y(x), singsol=all)
```

$$y = \left(x^{3} + c_{1}\right) \mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 15

```
DSolve[y'[x]==y[x]+3*x^2*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}\left(x^{3} + c_{1}\right)$$

6.5 problem 5

Internal problem ID [1991]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 10, page 41
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{x' + x = e^{-y}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(x(y),y)+x(y)=exp(-y),x(y), singsol=all)
```

$$x(y) = e^{-y}(y + c_1)$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 15

```
DSolve[x'[y]+x[y]==Exp[-y],x[y],y,IncludeSingularSolutions -> True]
```

$$x(y) \to e^{-y}(y + c_1)$$

6.6 problem 6

Internal problem ID [1992]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 10, page 41
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{yx' + (y+1)\,x = \mathrm{e}^{y}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(y*diff(x(y),y)+(1+y)*x(y)=exp(y),x(y), singsol=all)
```

$$x(y) = \frac{\left(\frac{\mathrm{e}^{2y}}{2} + c_1\right)\mathrm{e}^{-y}}{y}$$

✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 18

```
DSolve[y*x'[y]+(1+y)*x[y]==Exp[-y],x[y],y,IncludeSingularSolutions -> True]
```

$$x(y) \to \frac{e^{-y}(y + c_1)}{y}$$

6.7 problem 7

Internal problem ID [1993]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 10, page 41
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y + (2x - 3y)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 341

```
dsolve(y(x)+(2*x-3*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{\left(-108c_1 + 8x^3 + 12\sqrt{-12c_1x^3 + 81c_1^2}\right)^{\frac{1}{3}}}{6} + \frac{2x^2}{3\left(-108c_1 + 8x^3 + 12\sqrt{-12c_1x^3 + 81c_1^2}\right)^{\frac{1}{3}}} + \frac{x}{3}$$

$$y = -\frac{\left(-108c_1 + 8x^3 + 12\sqrt{-12c_1x^3 + 81c_1^2}\right)^{\frac{1}{3}}}{12} - \frac{x^2}{3\left(-108c_1 + 8x^3 + 12\sqrt{-12c_1x^3 + 81c_1^2}\right)^{\frac{1}{3}}} + \frac{x}{3} - \frac{i\sqrt{3}\left(\frac{\left(-108c_1+8x^3+12\sqrt{-12c_1x^3+81c_1^2}\right)^{\frac{1}{3}}}{6} - \frac{2x^2}{3\left(-108c_1+8x^3+12\sqrt{-12c_1x^3+81c_1^2}\right)^{\frac{1}{3}}}\right)}{2}$$

$$y = -\frac{\left(-108c_1 + 8x^3 + 12\sqrt{-12c_1x^3 + 81c_1^2}\right)^{\frac{1}{3}}}{12} - \frac{x^2}{3\left(-108c_1 + 8x^3 + 12\sqrt{-12c_1x^3 + 81c_1^2}\right)^{\frac{1}{3}}} + \frac{x}{3} + \frac{i\sqrt{3}\left(\frac{\left(-108c_1+8x^3+12\sqrt{-12c_1x^3+81c_1^2}\right)^{\frac{1}{3}}}{6} - \frac{2x^2}{3\left(-108c_1+8x^3+12\sqrt{-12c_1x^3+81c_1^2}\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 60.066 (sec). Leaf size: 379

```
DSolve[y[x]+(2*x-3*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(\sqrt[3]{x^3+\frac{3}{2}\sqrt{3}\sqrt{e^{c_1}\left(-4x^3+27e^{c_1}\right)}-\frac{27e^{c_1}}{2}}+\frac{x^2}{\sqrt[3]{x^3+\frac{3}{2}\sqrt{3}\sqrt{e^{c_1}\left(-4x^3+27e^{c_1}\right)}-\frac{27e^{c_1}}{2}}}+x\right)$$

$$y(x) \to \frac{1}{12}\left(i2^{2/3}\left(\sqrt{3}+i\right)\sqrt[3]{2x^3+3\sqrt{3}\sqrt{e^{c_1}\left(-4x^3+27e^{c_1}\right)}-27e^{c_1}}-\frac{2\left(1+i\sqrt{3}\right)x^2}{\sqrt[3]{x^3+\frac{3}{2}\sqrt{3}\sqrt{e^{c_1}\left(-4x^3+27e^{c_1}\right)}-\frac{27e^{c_1}}{2}}}+4x\right)$$

$$y(x) \to \frac{1}{12}\left(-2^{2/3}\left(1+i\sqrt{3}\right)\sqrt[3]{2x^3+3\sqrt{3}\sqrt{e^{c_1}\left(-4x^3+27e^{c_1}\right)}-27e^{c_1}}+\frac{2i\left(\sqrt{3}+i\right)x^2}{\sqrt[3]{x^3+\frac{3}{2}\sqrt{3}\sqrt{e^{c_1}\left(-4x^3+27e^{c_1}\right)}-\frac{27e^{c_1}}{2}}}+4x\right)$$

6.8 problem 8

Internal problem ID [1994]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 10, page 41
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - 2y = 2x^4}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x*diff(y(x),x)-2*(x^4+y(x))=0,y(x), singsol=all)
```

$$y = (x^2 + c_1)\, x^2$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 15

```
DSolve[x*y'[x]-2*(x^4+y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(x^2 + c_1)$$

6.9 problem 9

Internal problem ID [1995]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 10, page 41
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_exponential_symmetries]]`

$$\boxed{-(x+e^{y})\,y' = -1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(1=(x+exp(y(x)))*diff(y(x),x),y(x), singsol=all)
```

$$y = \mathrm{LambertW}\left(x\,e^{c_1}\right) - c_1$$

✓ Solution by Mathematica

Time used: 0.138 (sec). Leaf size: 17

```
DSolve[1==(x+Exp[y[x]])*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to W(e^{c_1}x) - c_1$$

6.10 problem 10

Internal problem ID [1996]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 10, page 41
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y^2x' + (y^2 + 2y)\, x = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(y^2*diff(x(y),y)+(y^2+2*y)*x(y)=1,x(y), singsol=all)
```

$$x(y) = \frac{1}{y^2} + \frac{\mathrm{e}^{-y} c_1}{y^2}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 19

```
DSolve[y^2*x'[y]+(y^2+2*y)*x[y]==1,x[y],y,IncludeSingularSolutions -> True]
```

$$x(y) \to \frac{1 + c_1 e^{-y}}{y^2}$$

6.11 problem 11

Internal problem ID [1997]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 10, page 41
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - 5y = 1 + x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x*diff(y(x),x)=5*y(x)+x+1,y(x), singsol=all)
```

$$y = -\frac{1}{4}x - \frac{1}{5} + c_1 x^5$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 20

```
DSolve[x*y'[x]==5*y[x]+x+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x^5 - \frac{x}{4} - \frac{1}{5}$$

6.12 problem 12

Internal problem ID [1998]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 10, page 41
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x^2 + y - 2yx = 2x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x)+(y(x)-2*x*y(x)-2*x^2)=0,y(x), singsol=all)
```

$$y = 2x^2 + e^{\frac{1}{x}} c_1 x^2$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 19

```
DSolve[x^2*y'[x]+(y[x]-2*x*y[x]-2*x^2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 \left(2 + c_1 e^{\frac{1}{x}}\right)$$

6.13 problem 13

Internal problem ID [1999]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 10, page 41
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(1+x)\,y'+2y=\frac{e^x}{1+x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve((x+1)*diff(y(x),x)+2*y(x)=exp(x)/(1+x),y(x), singsol=all)
```

$$y=\frac{e^x+c_1}{(x+1)^2}$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 17

```
DSolve[(x+1)*y'[x]+2*y[x]==Exp[x]/(1+x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{e^x+c_1}{(x+1)^2}$$

6.14 problem 14

Internal problem ID [2000]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 10, page 41
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{\cos(y)^2 + (x - \tan(y))\, y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve(cos(y(x))^2+(x-tan(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \arctan\left(\operatorname{LambertW}\left(-c_1 \mathrm{e}^{-1-x}\right) + x + 1\right)$$

✓ Solution by Mathematica

Time used: 60.291 (sec). Leaf size: 21

```
DSolve[Cos[y[x]]^2+(x-Tan[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arctan\left(W\left(c_1\left(-e^{-x-1}\right)\right) + x + 1\right)$$

6.15 problem 15

Internal problem ID [2001]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 10, page 41
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{2y - \left(y^4 + x\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(2*y(x)=(y(x)^4+x)*diff(y(x),x),y(x), singsol=all)
```

$$x - \frac{y^4}{7} - \sqrt{y}\, c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.102 (sec). Leaf size: 257

```
DSolve[2*y[x]==(y[x]^4+x)*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{Root}\left[\#1^8 - 14\#1^4x - 49\#1c_1{}^2 + 49x^2\&, 1\right]$$
$$y(x) \to \text{Root}\left[\#1^8 - 14\#1^4x - 49\#1c_1{}^2 + 49x^2\&, 2\right]$$
$$y(x) \to \text{Root}\left[\#1^8 - 14\#1^4x - 49\#1c_1{}^2 + 49x^2\&, 3\right]$$
$$y(x) \to \text{Root}\left[\#1^8 - 14\#1^4x - 49\#1c_1{}^2 + 49x^2\&, 4\right]$$
$$y(x) \to \text{Root}\left[\#1^8 - 14\#1^4x - 49\#1c_1{}^2 + 49x^2\&, 5\right]$$
$$y(x) \to \text{Root}\left[\#1^8 - 14\#1^4x - 49\#1c_1{}^2 + 49x^2\&, 6\right]$$
$$y(x) \to \text{Root}\left[\#1^8 - 14\#1^4x - 49\#1c_1{}^2 + 49x^2\&, 7\right]$$
$$y(x) \to \text{Root}\left[\#1^8 - 14\#1^4x - 49\#1c_1{}^2 + 49x^2\&, 8\right]$$

6.16 problem 16

Internal problem ID [2002]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 10, page 41
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\cos(\theta) r' - 2r\sin(\theta) = 2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(cos(theta)*diff(r(theta),theta)=2+2*r(theta)*sin(theta),r(theta), singsol=all)
```

$$r(\theta) = \frac{4\sin(\theta) + 2c_1}{\cos(2\theta) + 1}$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 17

```
DSolve[Cos[\[Theta]]*r'[\[Theta]]==2+2*r[\[Theta]]*Sin[\[Theta]],r[\[Theta]],\[Theta],Includ
```

$$r(\theta) \to \sec^2(\theta)(2\sin(\theta) + c_1)$$

6.17 problem 17

Internal problem ID [2003]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 10, page 41
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{r' \sin(\theta) + \tan(\theta)\, r = \cos(\theta) - 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(sin(theta)*diff(r(theta),theta)+1+r(theta)*tan(theta)=cos(theta),r(theta), singsol=al
```

$$r(\theta) = \frac{2\ln\left(\tan\left(\frac{\theta}{2}\right) - 1\right) + \theta + c_1}{\sec(\theta) + \tan(\theta)}$$

✓ Solution by Mathematica

Time used: 6.359 (sec). Leaf size: 171

```
DSolve[Sin[\[Theta]]*r'[\[Theta]]+1+r[\[Theta]]*Tan[\[Theta]]==Cos[\[Theta]],r[\[Theta]],\[T
```

$$r(\theta) \to \frac{1}{4} e^{-\coth^{-1}(\sin(\theta))} \left(- \frac{\sqrt{2}\sqrt{-\cot^2(\theta)} \left(\frac{2\left(\sqrt{\sin^2(\theta)} - 1\right)\left(\sqrt{\cos(2\theta) - 1}\,\text{arctanh}\left(\sqrt{\cos^2(\theta)}\right) + \sqrt{2}\sqrt{\sin^2(\theta)}\log\left(\sqrt{\cos(2\theta)+1} - \sqrt{\text{co}\cdots}\right.\right.}{\sqrt{-\sin^2(\theta)}(\csc(\theta) - 1)} \cdots}{\sqrt{\cos^2(\theta)} \cdots} + 4c_1 \right)$$

6.18 problem 18

Internal problem ID [2004]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 10, page 41
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{yx' - x(3y+2) = 2y\,\mathrm{e}^{3y}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(y*diff(x(y),y)=2*y*exp(3*y)+x(y)*(3*y+2),x(y), singsol=all)
```

$$x(y) = \left(-\frac{2}{y} + c_1\right)\mathrm{e}^{3y}y^2$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 18

```
DSolve[y*x'[y]==2*y*Exp[3*y]+x[y]*(3*y+2),x[y],y,IncludeSingularSolutions -> True]
```

$$x(y) \to e^{3y}y(-2 + c_1 y)$$

6.19 problem 19

Internal problem ID [2005]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 10, page 41
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]`

$$\boxed{y^2 + (2yx - y^2)\, y' = -1}$$

With initial conditions

$$[y(0) = -1]$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 137

```
dsolve([(y(x)^2+1)+(2*x*y(x)-y(x)^2)*diff(y(x),x)=0,y(0) = -1],y(x), singsol=all)
```

$$y = \frac{\left(-4+12x+8x^3+4\sqrt{12x^4-4x^3+9x^2-6x+1}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)}{4} - \frac{\left(ix\sqrt{3}+x-\left(-4+12x+8x^3+4\sqrt{12x^4-4x^3+9x^2-6x+1}\right)^{\frac{1}{3}}\right)x}{\left(-4+12x+8x^3+4\sqrt{12x^4-4x^3+9x^2-6x+1}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 4.803 (sec). Leaf size: 100

```
DSolve[{(y[x]^2+1)+(2*x*y[x]-y[x]^2)*y'[x]==0,{y[0]==-1}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{\sqrt[3]{2}x^2}{\sqrt[3]{-2x^3 + \sqrt{12x^4 - 4x^3 + 9x^2 - 6x + 1} - 3x + 1}} - \frac{\sqrt[3]{-2x^3 + \sqrt{12x^4 - 4x^3 + 9x^2 - 6x + 1} - 3x + 1}}{\sqrt[3]{2}} + x$$

6.20 problem 20

Internal problem ID [2006]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 10, page 41
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\cot(x) = \sec(x)}$$

With initial conditions

$$[y(0) = 1]$$

✗ Solution by Maple

```
dsolve([diff(y(x),x)+(y(x)*cot(x)-sec(x))=0,y(0) = 1],y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'[x]+(y[x]*Cot[x]-Sec[x])==0,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

Not solved

6.21 problem 21

Internal problem ID [2007]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 10, page 41
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{y + y^3 + 4(y^2x - 1)\, y' = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 34

```
dsolve([(y(x)+y(x)^3)+4*(x*y(x)^2-1)*diff(y(x),x)=0,y(0) = 1],y(x), singsol=all)
```

$$y = e^{\mathrm{RootOf}\left(-x\,e^{4\_Z} - 2x\,e^{2\_Z} + 2\,e^{2\_Z} + 4\_Z - x - 2\right)}$$

✓ Solution by Mathematica

Time used: 0.207 (sec). Leaf size: 37

```
DSolve[{(y[x]+y[x]^3)+4*(x*y[x]^2-1)*y'[x]==0,{y[0]==1}},y[x],x,IncludeSingularSolutions ->
```

$$\mathrm{Solve}\left[x = \frac{2y(x)^2 + 4\log(y(x))}{(y(x)^2+1)^2} - \frac{2}{(y(x)^2+1)^2}, y(x)\right]$$

6.22 problem 22

Internal problem ID [2008]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 10, page 41
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2y - yx + y'x = 3}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve([(2*y(x)-x*y(x)-3)+x*diff(y(x),x)=0,y(1) = 1],y(x), singsol=all)
```

$$y = \frac{-3x - 3 + 7\,\mathrm{e}^{x-1}}{x^2}$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 24

```
DSolve[{(2*y[x]-x*y[x]-3)+x*y'[x]==0,{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{7e^x - 3e(x+1)}{ex^2}$$

6.23 problem 23

Internal problem ID [2009]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 10, page 41
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{y + 2(x - 2y^2)\, y' = 0}$$

With initial conditions

$$[y(2) = -1]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 41

```
dsolve([y(x)+2*(x-2*y(x)^2)*diff(y(x),x)=0,y(2) = -1],y(x), singsol=all)
```

$$y = -\frac{\sqrt{-2\sqrt{x^2-4}+2x}}{2}$$
$$y = -\frac{\sqrt{2\sqrt{x^2-4}+2x}}{2}$$

✓ Solution by Mathematica

Time used: 2.237 (sec). Leaf size: 55

```
DSolve[{y[x]+2*(x-2*y[x]^2)*y'[x]==0,{y[2]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{x-\sqrt{x^2-4}}}{\sqrt{2}}$$
$$y(x) \to -\frac{\sqrt{\sqrt{x^2-4}+x}}{\sqrt{2}}$$

6.24 problem 24

Internal problem ID [2010]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 10, page 41
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2-1)\,y'+4y=-(x^2-1)^2}$$

With initial conditions

$$[y(0)=-6]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 38

```
dsolve([(x^2-1)*diff(y(x),x)+(x^2-1)^2+4*y(x)=0,y(0) = -6],y(x), singsol=all)
```

$$y=\frac{\left(-\frac{x^3}{3}+2x^2-7x+8\ln(x+1)-6\right)(x+1)^4}{(x^2-1)^2}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 38

```
DSolve[{(x^2-1)*y'[x]+(x^2-1)^2+4*y[x]==0,{y[0]==-6}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to-\frac{(x+1)^2\,(x^3-6x^2+21x-24\log(x+1)+18)}{3(x-1)^2}$$

7 Exercise 11, page 45

7.1 problem 1

Internal problem ID [2011]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 11, page 45
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{3y^2y' - xy^3 = e^{\frac{x^2}{2}} \cos(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 144

```
dsolve(3*y(x)^2*diff(y(x),x)-x*y(x)^3=exp(x^2/2)*cos(x),y(x), singsol=all)
```

$$y = e^{\frac{x^2}{2}} \left((\sin(x) + c_1) e^{-x^2}\right)^{\frac{1}{3}}$$

$$y = -\frac{e^{\frac{x^2}{2}} \left((\sin(x) + c_1) e^{-x^2}\right)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}\, e^{\frac{x^2}{2}} \left((\sin(x) + c_1) e^{-x^2}\right)^{\frac{1}{3}}}{2}$$

$$y = -\frac{e^{\frac{x^2}{2}} \left((\sin(x) + c_1) e^{-x^2}\right)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}\, e^{\frac{x^2}{2}} \left((\sin(x) + c_1) e^{-x^2}\right)^{\frac{1}{3}}}{2}$$

✓ Solution by Mathematica

Time used: 0.426 (sec). Leaf size: 81

```
DSolve[3*y[x]^2*y'[x]-x*y[x]^3==Exp[x^2/2]*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{x^2}{6}} \sqrt[3]{\sin(x) + c_1}$$
$$y(x) \to -\sqrt[3]{-1} e^{\frac{x^2}{6}} \sqrt[3]{\sin(x) + c_1}$$
$$y(x) \to (-1)^{2/3} e^{\frac{x^2}{6}} \sqrt[3]{\sin(x) + c_1}$$

7.2 problem 2

Internal problem ID [2012]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 11, page 45
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y^3 y' + y^4 x = x\,e^{-x^2}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 114

```
dsolve(y(x)^3*diff(y(x),x)+x*y(x)^4=x*exp(-x^2),y(x), singsol=all)
```

$$y = e^{-x^2}\left(\left(2\,e^{x^2} + c_1\right)e^{2x^2}\right)^{\frac{1}{4}}$$
$$y = -e^{-x^2}\left(\left(2\,e^{x^2} + c_1\right)e^{2x^2}\right)^{\frac{1}{4}}$$
$$y = -ie^{-x^2}\left(\left(2\,e^{x^2} + c_1\right)e^{2x^2}\right)^{\frac{1}{4}}$$
$$y = ie^{-x^2}\left(\left(2\,e^{x^2} + c_1\right)e^{2x^2}\right)^{\frac{1}{4}}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]^4*y'[x]+x*y[x]^4==x*Exp[-x^2],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.3 problem 3

Internal problem ID [2013]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 11, page 45
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{\cosh(y)\, y' + \sinh(y) = e^{-x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(cosh(y(x))*diff(y(x),x)+(sinh(y(x))-exp(-x) )=0,y(x), singsol=all)
```

$$y = -\operatorname{arcsinh}\left((c_1 - x)\, e^{-x}\right)$$

✓ Solution by Mathematica

Time used: 14.919 (sec). Leaf size: 16

```
DSolve[Cosh[y[x]]*y'[x]+(Sinh[y[x]]-Exp[-x] )==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \operatorname{arcsinh}\left(e^{-x}(x + c_1)\right)$$

7.4 problem 4

Internal problem ID [2014]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 11, page 45
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{\sin(\theta)\,\theta' + \cos(\theta) = t\,\mathrm{e}^{-t}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(sin(theta(t))*diff(theta(t),t)+(cos(theta(t))-t*exp(-t) )=0,theta(t), singsol=all)
```

$$\theta(t) = \arccos\left(\frac{(4c_1\mathrm{e}^{2t} + 2t + 1)\,\mathrm{e}^{-t}}{4}\right)$$

✓ Solution by Mathematica

Time used: 21.418 (sec). Leaf size: 59

```
DSolve[Sin[\[Theta][t]]*\[Theta]'[t]+(Cos[\[Theta][t]]-t*Exp[-t] )==0,\[Theta][t],t,IncludeS
```

$$\theta(t) \to -\arccos\left(\frac{1}{4}e^{-t}(2t + 4c_1e^{2t} + 1)\right)$$

$$\theta(t) \to \arccos\left(\frac{1}{4}e^{-t}(2t + 4c_1e^{2t} + 1)\right)$$

7.5 problem 5

Internal problem ID [2015]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 11, page 45
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y'yx + y^2 = x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(x*y(x)*diff(y(x),x)=(x^2-y(x)^2),y(x), singsol=all)
```

$$y = -\frac{\sqrt{2x^4 + 4c_1}}{2x}$$
$$y = \frac{\sqrt{2x^4 + 4c_1}}{2x}$$

✓ Solution by Mathematica

Time used: 0.21 (sec). Leaf size: 46

```
DSolve[x*y[x]*y'[x]==(x^2-y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{x^4}{2} + c_1}}{x}$$
$$y(x) \to \frac{\sqrt{\frac{x^4}{2} + c_1}}{x}$$

7.6 problem 6

Internal problem ID [2016]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 11, page 45
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' - yx - \sqrt{y}\, x\, \mathrm{e}^{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)-x*y(x)=sqrt(y(x))*x*exp(x^2),y(x), singsol=all)
```

$$\sqrt{y} - \frac{\mathrm{e}^{x^2}}{3} - \mathrm{e}^{\frac{x^2}{4}} c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.307 (sec). Leaf size: 34

```
DSolve[y'[x]-x*y[x]==Sqrt[y[x]]*x*Exp[x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{9} e^{\frac{x^2}{2}} \left(e^{\frac{3x^2}{4}} + 3c_1\right)^2$$

7.7 problem 7

Internal problem ID [2017]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 11, page 45
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{tx' + x(1 - x^2t^4) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(t*diff(x(t),t)+x(t)*(1-x(t)^2*t^4)=0,x(t), singsol=all)
```

$$x(t) = \frac{1}{\sqrt{-t^2 + c_1}\, t}$$
$$x(t) = -\frac{1}{\sqrt{-t^2 + c_1}\, t}$$

✓ Solution by Mathematica

Time used: 0.37 (sec). Leaf size: 48

```
DSolve[t*x'[t]+x[t]*(1-x[t]^2*t^4)==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{1}{\sqrt{-t^4 + c_1 t^2}}$$
$$x(t) \to \frac{1}{\sqrt{-t^4 + c_1 t^2}}$$
$$x(t) \to 0$$

7.8 problem 8

Internal problem ID [2018]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 11, page 45
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y'x^2 + y^2 - yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve(x^2*diff(y(x),x)+y(x)^2=x*y(x),y(x), singsol=all)
```

$$y = \frac{x}{\ln(x) + c_1}$$

✓ Solution by Mathematica

Time used: 0.134 (sec). Leaf size: 19

```
DSolve[x^2*y'[x]+y[x]^2==x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{\log(x) + c_1}$$
$$y(x) \to 0$$

7.9 problem 9

Internal problem ID [2019]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 11, page 45
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{\csc(y)\cot(y)\,y' - \csc(y) = e^{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(csc(y(x))*cot(y(x))*diff(y(x),x)=(csc(y(x))+exp(x)),y(x), singsol=all)
```

$$y = -\operatorname{arccsc}\left(\frac{(e^{2x} - 2c_1)\,e^{-x}}{2}\right)$$

✓ Solution by Mathematica

Time used: 1.115 (sec). Leaf size: 30

```
DSolve[Csc[y[x]]*Cot[y[x]]*y'[x]==(Csc[y[x]]+Exp[x]),y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -\csc^{-1}\left(\frac{e^{x}}{2} - c_1e^{-x}\right)$$

$$y(x) \to 0$$

7.10 problem 10

Internal problem ID [2020]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 11, page 45
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - yx - \frac{x}{y} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)-x*y(x)=x/y(x),y(x), singsol=all)
```

$$y = \sqrt{\mathrm{e}^{x^2} c_1 - 1}$$
$$y = -\sqrt{\mathrm{e}^{x^2} c_1 - 1}$$

✓ Solution by Mathematica

Time used: 6.967 (sec). Leaf size: 57

```
DSolve[y'[x]-x*y[x]==x/y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-1 + e^{x^2+2c_1}}$$
$$y(x) \to \sqrt{-1 + e^{x^2+2c_1}}$$
$$y(x) \to -i$$
$$y(x) \to i$$

7.11 problem 11

Internal problem ID [2021]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 11, page 45
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y'x + y - y^2x^2 \cos(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x)+y(x)=y(x)^2*x^2*cos(x),y(x), singsol=all)
```

$$y = -\frac{1}{(\sin(x) - c_1)\, x}$$

✓ Solution by Mathematica

Time used: 0.177 (sec). Leaf size: 22

```
DSolve[x*y'[x]+y[x]==y[x]^2*x^2*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{-x\sin(x) + c_1 x}$$
$$y(x) \to 0$$

7.12 problem 12

Internal problem ID [2022]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 11, page 45
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{r' + \left(r - \frac{1}{r}\right)\theta = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(diff(r(theta),theta)+(r(theta)-1/r(theta))*theta=0,r(theta), singsol=all)
```

$$r(\theta) = \sqrt{\mathrm{e}^{-\theta^2} c_1 + 1}$$
$$r(\theta) = -\sqrt{\mathrm{e}^{-\theta^2} c_1 + 1}$$

✓ Solution by Mathematica

Time used: 1.969 (sec). Leaf size: 57

```
DSolve[r'[\[Theta]]+(r[\[Theta]]-1/r[\[Theta]])*\[Theta]==0,r[\[Theta]],\[Theta],IncludeSing
```

$$r(\theta) \to -\sqrt{1 + e^{-\theta^2 + 2c_1}}$$
$$r(\theta) \to \sqrt{1 + e^{-\theta^2 + 2c_1}}$$
$$r(\theta) \to -1$$
$$r(\theta) \to 1$$

7.13 problem 13

Internal problem ID [2023]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 11, page 45
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y'x + 2y - 3x^3 y^{\frac{4}{3}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x)+2*y(x)=3*x^3*y(x)^(4/3),y(x), singsol=all)
```

$$\frac{1}{y^{\frac{1}{3}}} + \frac{3x^3}{7} - x^{\frac{2}{3}} c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.159 (sec). Leaf size: 29

```
DSolve[x*y'[x]+2*y[x]==3*x^3*y[x]^(4/3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{343}{x^2 \left(3x^{7/3} - 7c_1\right)^3}$$

$$y(x) \to 0$$

7.14 problem 14

Internal problem ID [2024]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 11, page 45
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{3y' + \frac{2y}{1+x} - \frac{x}{y^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 240

```
dsolve(3*diff(y(x),x)+2*y(x)/(x+1)=x/y(x)^2,y(x), singsol=all)
```

$$y = \frac{\left((54x^4 + 144x^3 + 108x^2 + 216c_1)(x^2 + 2x + 1)^2\right)^{\frac{1}{3}}}{6x^2 + 12x + 6}$$

$$\begin{aligned} y = &-\frac{\left((54x^4 + 144x^3 + 108x^2 + 216c_1)(x^2 + 2x + 1)^2\right)^{\frac{1}{3}}}{12(x^2 + 2x + 1)} \\ &- \frac{i\sqrt{3}\left((54x^4 + 144x^3 + 108x^2 + 216c_1)(x^2 + 2x + 1)^2\right)^{\frac{1}{3}}}{12(x^2 + 2x + 1)} \end{aligned}$$

$$\begin{aligned} y = &-\frac{\left((54x^4 + 144x^3 + 108x^2 + 216c_1)(x^2 + 2x + 1)^2\right)^{\frac{1}{3}}}{12(x^2 + 2x + 1)} \\ &+ \frac{i\sqrt{3}\left((54x^4 + 144x^3 + 108x^2 + 216c_1)(x^2 + 2x + 1)^2\right)^{\frac{1}{3}}}{12x^2 + 24x + 12} \end{aligned}$$

✓ Solution by Mathematica

Time used: 3.9 (sec). Leaf size: 144

```
DSolve[3*y'[x]+2*y[x]/(x+1)==x/y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{\frac{3x^4+8x^3+6x^2+12c_1}{(x+1)^2}}}{2^{2/3}\sqrt[3]{3}}$$

$$y(x) \to -\frac{\sqrt[3]{-\frac{1}{3}}\sqrt[3]{\frac{3x^4+8x^3+6x^2+12c_1}{(x+1)^2}}}{2^{2/3}}$$

$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{\frac{3x^4+8x^3+6x^2+12c_1}{(x+1)^2}}}{2^{2/3}\sqrt[3]{3}}$$

7.15 problem 15

Internal problem ID [2025]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 11, page 45
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\cos(y) y' + (\sin(y) - 1)\cos(x) = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 16

```
dsolve(cos(y(x))*diff(y(x),x)+(sin(y(x))-1)*cos(x)=0,y(x), singsol=all)
```

$$y = \arcsin\left(\frac{e^{-\sin(x)} + c_1}{c_1}\right)$$

✓ Solution by Mathematica

Time used: 60.309 (sec). Leaf size: 225

```
DSolve[Cos[y[x]]*y'[x]+(Sin[y[x]]-1)*Cos[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{3\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$
$$y(x) \to -2\arccos\left(-\frac{1}{8}e^{-\sin(x)}\left(c_1 e^{\frac{\sin(x)}{2}} + \sqrt{e^{\sin(x)}\left(32e^{\sin(x)} - c_1{}^2\right)}\right)\right)$$
$$y(x) \to 2\arccos\left(-\frac{1}{8}e^{-\sin(x)}\left(c_1 e^{\frac{\sin(x)}{2}} + \sqrt{e^{\sin(x)}\left(32e^{\sin(x)} - c_1{}^2\right)}\right)\right)$$
$$y(x) \to -2\arccos\left(\frac{1}{8}e^{-\sin(x)}\left(\sqrt{e^{\sin(x)}\left(32e^{\sin(x)} - c_1{}^2\right)} - c_1 e^{\frac{\sin(x)}{2}}\right)\right)$$
$$y(x) \to 2\arccos\left(\frac{1}{8}e^{-\sin(x)}\left(\sqrt{e^{\sin(x)}\left(32e^{\sin(x)} - c_1{}^2\right)} - c_1 e^{\frac{\sin(x)}{2}}\right)\right)$$

7.16 problem 16

Internal problem ID [2026]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 11, page 45
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{\left(x \tan \left(y\right)^{2}-x\right) y'-\tan \left(y\right)=2 x^{2}}$$

✗ Solution by Maple

```
dsolve((x*tan(y(x))^2-x)*diff(y(x),x)=(2*x^2+tan(y(x))),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(x*Tan[y[x]]^2-x)*y'[x]==(2*x^2+Tan[y[x]]),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.17 problem 17

Internal problem ID [2027]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 11, page 45
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + y\cos(x) - y^3 \sin(x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 96

```
dsolve(diff(y(x),x)+y(x)*cos(x)=y(x)^3*sin(x),y(x), singsol=all)
```

$$y = \frac{\sqrt{-\left(-c_1 + 2\left(\int \mathrm{e}^{-2\sin(x)} \sin(x)\, dx\right)\right) \mathrm{e}^{-2\sin(x)}}}{-c_1 + 2\left(\int \mathrm{e}^{-2\sin(x)} \sin(x)\, dx\right)}$$

$$y = -\frac{\sqrt{-\left(-c_1 + 2\left(\int \mathrm{e}^{-2\sin(x)} \sin(x)\, dx\right)\right) \mathrm{e}^{-2\sin(x)}}}{-c_1 + 2\left(\int \mathrm{e}^{-2\sin(x)} \sin(x)\, dx\right)}$$

✓ Solution by Mathematica

Time used: 10.83 (sec). Leaf size: 84

```
DSolve[y'[x]+y[x]*Cos[x]==y[x]^3*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{e^{2\sin(x)}\left(-2\int_1^x e^{-2\sin(K[1])}\sin(K[1])dK[1]+c_1\right)}}$$

$$y(x) \to \frac{1}{\sqrt{e^{2\sin(x)}\left(-2\int_1^x e^{-2\sin(K[1])}\sin(K[1])dK[1]+c_1\right)}}$$

$$y(x) \to 0$$

7.18 problem 18

Internal problem ID [2028]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 11, page 45
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Bernoulli]`

$$\boxed{y' + y - y^2 \mathrm{e}^{-t} = 0}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 8

```
dsolve([diff(y(t),t)+y(t)=y(t)^2*exp(-t),y(0) = 2],y(t), singsol=all)
```

$$y(t) = 2\,\mathrm{e}^{t}$$

✓ Solution by Mathematica

Time used: 0.281 (sec). Leaf size: 10

```
DSolve[{y'[t]+y[t]==y[t]^2*Exp[-t],{y[0]==2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 2e^t$$

7.19 problem 19

Internal problem ID [2029]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 11, page 45
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{y' - x\left(1 - e^{-x^2+2y}\right) = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 18

```
dsolve([diff(y(x),x)=x*(1-exp(2*y(x)-x^2)),y(0) = 0],y(x), singsol=all)
```

$$y = \frac{x^2}{2} - \frac{\ln(x^2+1)}{2}$$

✓ Solution by Mathematica

Time used: 0.5 (sec). Leaf size: 21

```
DSolve[{y'[x]==x*(1-Exp[2*y[x]-x^2]),{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(x^2 - \log\left(x^2+1\right)\right)$$

7.20 problem 20

Internal problem ID [2030]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 11, page 45
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{2y - (x^2y^4 + x)\, y' = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.906 (sec). Leaf size: 26

```
dsolve([2*y(x)=(x^2*y(x)^4+x)*diff(y(x),x),y(1) = 1],y(x), singsol=all)
```

$$y = \frac{100x^2}{\mathrm{RootOf}\left(\_Z^9 - 100000000x^9 - 9\_Z^8\right)^2}$$

✓ Solution by Mathematica

Time used: 4.207 (sec). Leaf size: 33

```
DSolve[{2*y[x]==(x^2*y[x]^4+x)*y'[x],{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \mathrm{Root}\left[\#1^9x^2 + 18\#1^5x + 81\#1 - 100x^2\&, 1\right]$$

7.21 problem 21

Internal problem ID [2031]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 11, page 45
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{xy\left(1+y^{2}x\right)y'=-1}$$

With initial conditions

$$[y(1)=0]$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 66

```
dsolve([1+x*y(x)*(1+x*y(x)^2)*diff(y(x),x)=0,y(1) = 0],y(x), singsol=all)
```

$$y=\frac{\sqrt{-2\left(\operatorname{LambertW}\left(-1,-\frac{3\,\mathrm{e}^{-\frac{2x+1}{2x}}}{2}\right)x+x+\frac{1}{2}\right)x}}{x}$$
$$y=-\frac{\sqrt{-2\left(\operatorname{LambertW}\left(-1,-\frac{3\,\mathrm{e}^{-\frac{2x+1}{2x}}}{2}\right)x+x+\frac{1}{2}\right)x}}{x}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{1+x*y[x]*(1+x*y[x]^2)*y'[x]==0,{y[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

{}

7.22 problem 22

Internal problem ID [2032]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 11, page 45
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{(-x^2+1)\,y' + yx - x(-x^2+1)\sqrt{y} = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.343 (sec). Leaf size: 46

```
dsolve([(1-x^2)*diff(y(x),x)+x*y(x)=x*(1-x^2)*sqrt(y(x)),y(0) = 1],y(x), singsol=all)
```

$$y = \left(\frac{4}{9} - \frac{4i}{9}\right)(x-1)^{\frac{5}{4}}(x+1)^{\frac{5}{4}}\sqrt{2} + \frac{x^4}{9} - \frac{16i\sqrt{x-1}\,\sqrt{x+1}}{9} - \frac{2x^2}{9} + \frac{1}{9}$$

✓ Solution by Mathematica

Time used: 0.228 (sec). Leaf size: 130

```
DSolve[{(1-x^2)*y'[x]+x*y[x]==x*(1-x^2)*Sqrt[y[x]],{y[0]==1}},y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{1}{9}\left(x^4 + \left(4(-1)^{3/4}\sqrt[4]{x^2-1} - 2\right)x^2 - 4i\sqrt{x^2-1} - 4(-1)^{3/4}\sqrt[4]{x^2-1} + 1\right)$$

$$y(x) \to \frac{1}{9}\left(x^4 - 2\left(4(-1)^{3/4}\sqrt[4]{x^2-1} + 1\right)x^2 - 16i\sqrt{x^2-1} + 8(-1)^{3/4}\sqrt[4]{x^2-1} + 1\right)$$

8 Exercise 12, page 46

8.1 problem 1

Internal problem ID [2033]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(1-x)\,y'-y=1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((1-x)*diff(y(x),x)-(1+y(x))=0,y(x), singsol=all)
```

$$y=\frac{c_1-x}{x-1}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 22

```
DSolve[(1-x)*y'[x]-(1+y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\rightarrow\frac{x+c_1}{1-x}$$
$$y(x)\rightarrow -1$$

8.2 problem 2

Internal problem ID [2034]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y^2 + (yx + x^2)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 44

```
dsolve(y(x)^2+(x*y(x)+x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{1 + \sqrt{c_1 x^2 + 1}}{c_1 x}$$
$$y = -\frac{-1 + \sqrt{c_1 x^2 + 1}}{c_1 x}$$

✓ Solution by Mathematica

Time used: 2.48 (sec). Leaf size: 80

```
DSolve[y[x]^2+(x*y[x]+x^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{2c_1} - \sqrt{e^{2c_1} (x^2 + e^{2c_1})}}{x}$$
$$y(x) \to \frac{\sqrt{e^{2c_1} (x^2 + e^{2c_1})} + e^{2c_1}}{x}$$
$$y(x) \to 0$$

8.3 problem 3

Internal problem ID [2035]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y - (x - 2y)\, y' = -2x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve((2*x+y(x))-(x-2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \tan\left(\operatorname{RootOf}\left(\ln\left(\frac{1}{\cos(\_Z)^2}\right) - \_Z + 2\ln(x) + 2c_1\right)\right) x$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 32

```
DSolve[(2*x+y[x])-(x-2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\log\left(\frac{y(x)^2}{x^2} + 1\right) - \arctan\left(\frac{y(x)}{x}\right) = -2\log(x) + c_1, y(x)\right]$$

8.4 problem 4

Internal problem ID [2036]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x \ln(x) y' + y = x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x*ln(x)*diff(y(x),x)+(y(x)-x)=0,y(x), singsol=all)
```

$$y = \frac{x + c_1}{\ln(x)}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 14

```
DSolve[x*Log[x]*y'[x]+(y[x]-x)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x + c_1}{\log(x)}$$

8.5 problem 5

Internal problem ID [2037]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{-2y+(y-2)\,y'=-x-1}$$

✓ Solution by Maple

Time used: 0.578 (sec). Leaf size: 28

```
dsolve((x-2*y(x)+1)+(y(x)-2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y=2+\frac{(-3+x)\left(\operatorname{LambertW}\left(-c_1(-3+x)\right)+1\right)}{\operatorname{LambertW}\left(-c_1\left(-3+x\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.773 (sec). Leaf size: 135

```
DSolve[(x-2*y[x]+1)+(y[x]-2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2^{2/3}\left(x\log\left(\frac{x-3}{y(x)-2}\right)-\log\left(\frac{3(x-3)}{y(x)-2}\right)-x\log\left(\frac{y(x)-x+1}{y(x)-2}\right)+\log\left(\frac{y(x)-x+1}{y(x)-2}\right)+y(x)\left(-\log\left(\frac{x-3}{y(x)-2}\right)+\text{lo}\right.\right.}{9(-y(x)+x-1)}\right.$$

8.6 problem 6

Internal problem ID [2038]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{2yx - 2xy^3 + (x^2 + y^2 - 3x^2y^2)\, y' = -x^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 847

```
dsolve((2*x*y(x)-2*x*y(x)^3+x^3)+(x^2+y(x)^2-3*x^2*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{\left(\left(3x^4 + \sqrt{\frac{27x^{10}-9x^8+216c_1x^6-64x^6-72c_1x^4+432c_1^2x^2-144c_1^2}{3x^2-1}} + 12c_1\right)(3x^2-1)^2\right)^{\frac{1}{3}}}{6x^2-2} + \frac{2x^2}{\left(\left(3x^4 + \sqrt{\frac{27x^{10}-9x^8+216c_1x^6-64x^6-72c_1x^4+432c_1^2x^2-144c_1^2}{3x^2-1}} + 12c_1\right)(3x^2-1)^2\right)^{\frac{1}{3}}}$$

$$y = -\frac{\left(\left(3x^4 + \sqrt{\frac{27x^{10}-9x^8+216c_1x^6-64x^6-72c_1x^4+432c_1^2x^2-144c_1^2}{3x^2-1}} + 12c_1\right)(3x^2-1)^2\right)^{\frac{1}{3}}}{4(3x^2-1)} - \frac{x^2}{\left(\left(3x^4 + \sqrt{\frac{27x^{10}-9x^8+216c_1x^6-64x^6-72c_1x^4+432c_1^2x^2-144c_1^2}{3x^2-1}} + 12c_1\right)(3x^2-1)^2\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(\left(3x^4+\sqrt{\frac{27x^{10}-9x^8+216c_1x^6-64x^6-72c_1x^4+432c_1^2x^2-144c_1^2}{3x^2-1}}+12c_1\right)(3x^2-1)^2\right)^{\frac{1}{3}}}{6x^2-2} - \frac{2x^2}{\left(\left(3x^4+\sqrt{\frac{27x^{10}-9x^8+216c_1x^6-64x^6-72c_1}{3x^2-1}}\right.\right.}\right.}{2}$$

$$y = -\frac{\left(\left(3x^4 + \sqrt{\frac{27x^{10}-9x^8+216c_1x^6-64x^6-72c_1x^4+432c_1^2x^2-144c_1^2}{3x^2-1}} + 12c_1\right)(3x^2-1)^2\right)^{\frac{1}{3}}}{4(3x^2-1)} - \frac{x^2}{\left(\left(3x^4 + \sqrt{\frac{27x^{10}-9x^8+216c_1x^6-64x^6-72c_1x^4+432c_1^2x^2-144c_1^2}{3x^2-1}} + 12c_1\right)(3x^2-1)^2\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(\left(3x^4+\sqrt{\frac{27x^{10}-9x^8+216c_1x^6-64x^6-72c_1x^4+432c_1^2x^2-144c_1^2}{3x^2-1}}+12c_1\right)(3x^2-1)^2\right)^{\frac{1}{3}}}{6x^2-2} - \frac{2x^2}{\left(\left(3x^4+\sqrt{\frac{27x^{10}-9x^8+216c_1x^6-64x^6-72c_1}{3x^2-1}}\right.\right.}\right.}{2}$$

✓ Solution by Mathematica

Time used: 36.799 (sec). Leaf size: 723

```
DSolve[(2*x*y[x]-2*x*y[x]^3+x^3)+(x^2+y[x]^2-3*x^2*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSo
```

$$y(x) \to \frac{12x^4 - 4x^2 + \left(27x^8 - 18x^6 + 3(1+36c_1)x^4 - 72c_1x^2 + \sqrt{(3x^2-1)^3\,(27x^{10} - 9x^8 + 8(-8+27c_1)x^6 \cdots}}{2\,(3x^2-1)\sqrt[3]{27x^8 - 18x^6 + 3(1+36c_1)x^4 - 72c_1x^2 + \sqrt{(3x^2-1)^3\,(27x^{10} - 9x^8 + 8(-8+27c_1)x^6 \cdots}}}$$

$$y(x) \to \frac{-12i\left(\sqrt{3}-i\right)x^4 + \left(4+4i\sqrt{3}\right)x^2 + i\left(\sqrt{3}+i\right)\left(27x^8 - 18x^6 + 3(1+36c_1)x^4 - 72c_1x^2 + \sqrt{(3x^2-1)^3 \cdots}}{4\,(3x^2-1)\sqrt[3]{27x^8 - 18x^6 + 3(1+36c_1)x^4 - 72c_1x^2 + \sqrt{(3x^2-1)^3\,(27x^{10} - 9x^8 + \cdots}}}$$

$$y(x) \to \frac{12i\left(\sqrt{3}+i\right)x^4 + \left(4-4i\sqrt{3}\right)x^2 - i\left(\sqrt{3}-i\right)\left(27x^8 - 18x^6 + 3(1+36c_1)x^4 - 72c_1x^2 + \sqrt{(3x^2-1)^3 \cdots}}{4\,(3x^2-1)\sqrt[3]{27x^8 - 18x^6 + 3(1+36c_1)x^4 - 72c_1x^2 + \sqrt{(3x^2-1)^3\,(27x^{10} - 9x^8 + \cdots}}}$$

8.7 problem 7

Internal problem ID [2039]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{2\,\mathrm{e}^{x}+t\,\mathrm{e}^{x}x'=t^{2}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((2*exp(x(t))-t^2)+(t*exp(x(t)))*diff(x(t),t)=0,x(t), singsol=all)
```

$$x(t)=\ln\left(\frac{t^{4}+c_{1}}{4t^{2}}\right)$$

✓ Solution by Mathematica

Time used: 1.926 (sec). Leaf size: 20

```
DSolve[(2*Exp[x[t]]-t^2)+(t*Exp[x[t]])*x'[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t)\to\log\left(\frac{t^{2}}{4}+\frac{c_{1}}{t^{2}}\right)$$

8.8 problem 8

Internal problem ID [2040]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2y - y'yx = -6}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(2*(y(x)+3)=x*y(x)*diff(y(x),x),y(x), singsol=all)
```

$$y = \mathrm{e}^{-\operatorname{LambertW}\left(-\frac{\mathrm{e}^{-1-\frac{2c_1}{3}}}{3x^{\frac{2}{3}}}\right)-1-\frac{2c_1}{3}-\frac{2\ln(x)}{3}} - 3$$

✓ Solution by Mathematica

Time used: 20.439 (sec). Leaf size: 106

```
DSolve[2*(y[x]+3)==x*y[x]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -3\left(1 + W\left(\frac{1}{3}\sqrt[3]{-\frac{e^{-3-c_1}}{x^2}}\right)\right)$$

$$y(x) \to -3\left(1 + W\left(-\frac{1}{3}\sqrt[3]{-1}\sqrt[3]{-\frac{e^{-3-c_1}}{x^2}}\right)\right)$$

$$y(x) \to -3\left(1 + W\left(\frac{1}{3}(-1)^{2/3}\sqrt[3]{-\frac{e^{-3-c_1}}{x^2}}\right)\right)$$

$$y(x) \to -3$$

8.9 problem 9

Internal problem ID [2041]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{-3y-(3y-x+2)\,y'=-x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve((x-3*y(x))=(3*y(x)-x+2)*diff(y(x),x),y(x), singsol=all)
```

$$y=\frac{x}{3}+\frac{\mathrm{LambertW}\left(\frac{\mathrm{e}^{-\frac{8x}{3}}\mathrm{e}^{\frac{1}{3}c_1}}{3}\right)}{2}-\frac{1}{6}$$

✓ Solution by Mathematica

Time used: 4.689 (sec). Leaf size: 43

```
DSolve[(x-3*y[x])==(3*y[x]-x+2)*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{6}\left(3W\left(-e^{-\frac{8x}{3}-1+c_1}\right)+2x-1\right)$$
$$y(x)\to\frac{1}{6}(2x-1)$$

8.10 problem 10

Internal problem ID [2042]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{y \sin(x) - 2\cos(y) - (\cos(x) - 2\sin(y)\, x + \sin(y))\, y' = -\tan(x)}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 25

```
dsolve((y(x)*sin(x)-2*cos(y(x))+tan(x) )-(cos(x)-2*x*sin(y(x))+sin(y(x)) )*diff(y(x),x)=0,y(
```

$$-y\cos(x) - 2x\cos(y) - \ln(\cos(x)) + \cos(y) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.673 (sec). Leaf size: 29

```
DSolve[(y[x]*Sin[x]-2*Cos[y[x]]+Tan[x] )-(Cos[x]-2*x*Sin[y[x]]+Sin[y[x]] )*y'[x]==0,y[x],x,I
```

$$\text{Solve}[4x\cos(y(x)) - 2\cos(y(x)) + 2y(x)\cos(x) + 2\log(\cos(x)) = c_1, y(x)]$$

8.11 problem 11

Internal problem ID [2043]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{yx^2 - (x^3 + y^3)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve((x^2*y(x))-(x^3+y(x)^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \left(\frac{1}{\operatorname{LambertW}(c_1 x^3)}\right)^{\frac{1}{3}} x$$

✓ Solution by Mathematica

Time used: 7.211 (sec). Leaf size: 80

```
DSolve[(x^2*y[x])-(x^3+y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{\sqrt[3]{W(e^{-3c_1}x^3)}}$$
$$y(x) \to -\frac{\sqrt[3]{-1}x}{\sqrt[3]{W(e^{-3c_1}x^3)}}$$
$$y(x) \to \frac{(-1)^{2/3}x}{\sqrt[3]{W(e^{-3c_1}x^3)}}$$
$$y(x) \to 0$$

8.12 problem 12

Internal problem ID [2044]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{-y'x + y - 2y^2 - 2y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(y(x)-x*diff(y(x),x)=2*(y(x)^2+diff(y(x),x)),y(x), singsol=all)
```

$$y = \frac{x+2}{2x+c_1}$$

✓ Solution by Mathematica

Time used: 0.39 (sec). Leaf size: 32

```
DSolve[y[x]-x*y'[x]==2*(y[x]^2+y'[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x+2}{2x+4+e^{c_1}}$$
$$y(x) \to 0$$
$$y(x) \to \frac{1}{2}$$

8.13 problem 13

Internal problem ID [2045]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\tan(y) - (3x + 4)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(tan(y(x))=(3*x+4)*diff(y(x),x),y(x), singsol=all)
```

$$y = \arcsin\left((3x + 4)^{\frac{1}{3}} c_1\right)$$

✓ Solution by Mathematica

Time used: 23.059 (sec). Leaf size: 25

```
DSolve[Tan[y[x]]==(3*x+4)*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin\left(e^{c_1} \sqrt[3]{3x + 4}\right)$$
$$y(x) \to 0$$

8.14 problem 14

Internal problem ID [2046]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y' + y\ln(y)\tan(x) - 2y = 0}$$

✓ Solution by Maple

Time used: 1.11 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)+y(x)*ln(y(x))*tan(x)=2*y(x),y(x), singsol=all)
```

$$y = \left(-\frac{\cos(x)}{\sin(x)-1}\right)^{2\cos(x)} e^{c_1 \cos(x)}$$

✓ Solution by Mathematica

Time used: 2.002 (sec). Leaf size: 17

```
DSolve[y'[x]+y[x]*Log[y[x]]*Tan[x]==2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2\cos(x)(\coth^{-1}(\sin(x))+c_1)}$$

8.15 problem 15

Internal problem ID [2047]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{2yx + y^4 + \left(x y^3 - 2x^2\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 420

```
dsolve((2*x*y(x)+y(x)^4)+(x*y(x)^3-2*x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{\left(-108x^4+8c_1^3+12\sqrt{81x^4-12c_1^3\,x^2}\right)^{\frac{1}{3}}}{6x} + \frac{2c_1^2}{3x\left(-108x^4+8c_1^3+12\sqrt{81x^4-12c_1^3\,x^2}\right)^{\frac{1}{3}}} + \frac{c_1}{3x}$$

$$y = -\frac{\left(-108x^4+8c_1^3+12\sqrt{81x^4-12c_1^3\,x^2}\right)^{\frac{1}{3}}}{12x} - \frac{c_1^2}{3x\left(-108x^4+8c_1^3+12\sqrt{81x^4-12c_1^3\,x^2}\right)^{\frac{1}{3}}} + \frac{c_1}{3x} - \frac{i\sqrt{3}\left(\frac{\left(-108x^4+8c_1^3+12\sqrt{81x^4-12c_1^3\,x^2}\right)^{\frac{1}{3}}}{6x} - \frac{2c_1^2}{3x\left(-108x^4+8c_1^3+12\sqrt{81x^4-12c_1^3\,x^2}\right)^{\frac{1}{3}}}\right)}{2}$$

$$y = -\frac{\left(-108x^4+8c_1^3+12\sqrt{81x^4-12c_1^3\,x^2}\right)^{\frac{1}{3}}}{12x} - \frac{c_1^2}{3x\left(-108x^4+8c_1^3+12\sqrt{81x^4-12c_1^3\,x^2}\right)^{\frac{1}{3}}} + \frac{c_1}{3x} + \frac{i\sqrt{3}\left(\frac{\left(-108x^4+8c_1^3+12\sqrt{81x^4-12c_1^3\,x^2}\right)^{\frac{1}{3}}}{6x} - \frac{2c_1^2}{3x\left(-108x^4+8c_1^3+12\sqrt{81x^4-12c_1^3\,x^2}\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 12.491 (sec). Leaf size: 371

```
DSolve[(2*x*y[x]+y[x]^4)+(x*y[x]^3-2*x^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\frac{2\sqrt[3]{2}c_1^2}{\sqrt[3]{27x^4+3\sqrt{81x^8+12c_1^3x^4}+2c_1^3}}+2^{2/3}\sqrt[3]{27x^4+3\sqrt{81x^8+12c_1^3x^4}+2c_1^3}+2c_1}{6x}$$

$$y(x) \to \frac{\frac{2\sqrt[3]{2}\left(1+i\sqrt{3}\right)c_1^2}{\sqrt[3]{27x^4+3\sqrt{81x^8+12c_1^3x^4}+2c_1^3}}+2^{2/3}\left(1-i\sqrt{3}\right)\sqrt[3]{27x^4+3\sqrt{81x^8+12c_1^3x^4}+2c_1^3}-4c_1}{12x}$$

$$y(x) \to \frac{\frac{2\sqrt[3]{2}\left(1-i\sqrt{3}\right)c_1^2}{\sqrt[3]{27x^4+3\sqrt{81x^8+12c_1^3x^4}+2c_1^3}}+2^{2/3}\left(1+i\sqrt{3}\right)\sqrt[3]{27x^4+3\sqrt{81x^8+12c_1^3x^4}+2c_1^3}-4c_1}{12x}$$

$$y(x) \to 0$$

8.16 problem 16

Internal problem ID [2048]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y + (3x - 2y)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve((y(x))+(3*x-2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$x - \frac{y}{2} - \frac{c_1}{y^3} = 0$$

✓ Solution by Mathematica

Time used: 60.08 (sec). Leaf size: 1509

```
DSolve[(y[x])+(3*x-2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}\sqrt{x^2+\frac{\sqrt[3]{2}\sqrt[3]{\sqrt{81e^{4c_1}x^4-48e^{6c_1}}+9e^{2c_1}x^2}}{3^{2/3}}+\frac{2\ 2^{2/3}e^{2c_1}}{\sqrt[3]{3}\sqrt[3]{\sqrt{81e^{4c_1}x^4-48e^{6c_1}}+9e^{2c_1}x^2}}}$$
$$-\frac{1}{2}\sqrt{2x^2-\frac{\sqrt[3]{2}\sqrt[3]{\sqrt{81e^{4c_1}x^4-48e^{6c_1}}+9e^{2c_1}x^2}}{3^{2/3}}-\frac{2\ 2^{2/3}e^{2c_1}}{\sqrt[3]{3}\sqrt[3]{\sqrt{81e^{4c_1}x^4-48e^{6c_1}}+9e^{2c_1}x^2}}-\frac{\cdots}{\sqrt{x^2+\frac{\sqrt[3]{2}\sqrt[3]{\cdots}}{\cdots}}}}$$
$$+\frac{x}{2}$$

$$y(x) \to -\frac{1}{2}\sqrt{x^2+\frac{\sqrt[3]{2}\sqrt[3]{\sqrt{81e^{4c_1}x^4-48e^{6c_1}}+9e^{2c_1}x^2}}{3^{2/3}}+\frac{2\ 2^{2/3}e^{2c_1}}{\sqrt[3]{3}\sqrt[3]{\sqrt{81e^{4c_1}x^4-48e^{6c_1}}+9e^{2c_1}x^2}}}$$
$$+\frac{1}{2}\sqrt{2x^2-\frac{\sqrt[3]{2}\sqrt[3]{\sqrt{81e^{4c_1}x^4-48e^{6c_1}}+9e^{2c_1}x^2}}{3^{2/3}}-\frac{2\ 2^{2/3}e^{2c_1}}{\sqrt[3]{3}\sqrt[3]{\sqrt{81e^{4c_1}x^4-48e^{6c_1}}+9e^{2c_1}x^2}}-\frac{\cdots}{\sqrt{x^2+\frac{\sqrt[3]{2}\sqrt[3]{\cdots}}{\cdots}}}}$$
$$+\frac{x}{2}$$

$$y(x) \to \frac{1}{2}\sqrt{x^2+\frac{\sqrt[3]{2}\sqrt[3]{\sqrt{81e^{4c_1}x^4-48e^{6c_1}}+9e^{2c_1}x^2}}{3^{2/3}}+\frac{2\ 2^{2/3}e^{2c_1}}{\sqrt[3]{3}\sqrt[3]{\sqrt{81e^{4c_1}x^4-48e^{6c_1}}+9e^{2c_1}x^2}}}$$
$$-\frac{1}{2}\sqrt{2x^2-\frac{\sqrt[3]{2}\sqrt[3]{\sqrt{81e^{4c_1}x^4-48e^{6c_1}}+9e^{2c_1}x^2}}{3^{2/3}}-\frac{2\ 2^{2/3}e^{2c_1}}{\sqrt[3]{3}\sqrt[3]{\sqrt{81e^{4c_1}x^4-48e^{6c_1}}+9e^{2c_1}x^2}}+\frac{\cdots}{\sqrt{x^2+\frac{\sqrt[3]{2}\sqrt[3]{\cdots}}{\cdots}}}}$$
$$+\frac{x}{2}$$

$$y(x) \to \frac{1}{2}\sqrt{x^2+\frac{\sqrt[3]{2}\sqrt[3]{\sqrt{81e^{4c_1}x^4-48e^{6c_1}}+9e^{2c_1}x^2}}{3^{2/3}}+\frac{2\ 2^{2/3}e^{2c_1}}{\cdots}}$$

8.17 problem 17

Internal problem ID [2049]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{r' - r\cot(\theta) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(r(theta),theta)=r(theta)*cot(theta),r(theta), singsol=all)
```

$$r(\theta) = c_1 \sin(\theta)$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 15

```
DSolve[r'[\[Theta]]==r[\[Theta]]*Cot[\[Theta]],r[\[Theta]],\[Theta],IncludeSingularSolutions
```

$$r(\theta) \to c_1 \sin(\theta)$$
$$r(\theta) \to 0$$

8.18 problem 18

Internal problem ID [2050]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(3x+4y)\,y'+y=-2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve((3*x+4*y(x))*diff(y(x),x)+(y(x)+2*x)=0,y(x), singsol=all)
```

$$y=\frac{x\left(\tan\left(\operatorname{RootOf}\left(\ln\left(\frac{1}{2\cos\left(\_Z\right)^{2}}\right)+\_Z+2\ln\left(x\right)+2c_1\right)\right)-1\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 41

```
DSolve[(3*x+4*y[x])*y'[x]+(y[x]+2*x)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\arctan\left(\frac{2y(x)}{x}+1\right)+\log\left(\frac{2y(x)^2}{x^2}+\frac{2y(x)}{x}+1\right)=-2\log(x)+c_1,y(x)\right]$$

8.19 problem 19

Internal problem ID [2051]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{-y^3 + 3y'y^2x = -2x^3 + 3x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 108

```
dsolve((2*x^3-y(x)^3-3*x)+(3*x*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \left(-x^3 + 3x\ln(x) + c_1x\right)^{\frac{1}{3}}$$
$$y = -\frac{\left(-x^3 + 3x\ln(x) + c_1x\right)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}\left(-x^3 + 3x\ln(x) + c_1x\right)^{\frac{1}{3}}}{2}$$
$$y = -\frac{\left(-x^3 + 3x\ln(x) + c_1x\right)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}\left(-x^3 + 3x\ln(x) + c_1x\right)^{\frac{1}{3}}}{2}$$

✓ Solution by Mathematica

Time used: 0.354 (sec). Leaf size: 80

```
DSolve[(2*x^3-y[x]^3-3*x)+(3*x*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[3]{x\left(-x^2 + 3\log(x) + c_1\right)}$$
$$y(x) \to -\sqrt[3]{-1}\sqrt[3]{x\left(-x^2 + 3\log(x) + c_1\right)}$$
$$y(x) \to (-1)^{2/3}\sqrt[3]{x\left(-x^2 + 3\log(x) + c_1\right)}$$

8.20 problem 20

Internal problem ID [2052]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{y'x - y - \sqrt{x^2 + y^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x)-y(x)-sqrt(x^2+y(x)^2)=0,y(x), singsol=all)
```

$$\frac{y}{x^2} + \frac{\sqrt{x^2 + y^2}}{x^2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.349 (sec). Leaf size: 27

```
DSolve[x*y'[x]-y[x]-Sqrt[x^2+y[x]^2]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-c_1} \left(-1 + e^{2c_1} x^2\right)$$

8.21 problem 22

Internal problem ID [2053]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \cos(x)^2 \cos(y) = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 69

```
dsolve(diff(y(x),x)=cos(y(x))*cos(x)^2,y(x), singsol=all)
```

$$y = \arctan\left(\frac{c_1^2 e^{x+\frac{\sin(2x)}{2}} - 1}{c_1^2 e^{x+\frac{\sin(2x)}{2}} + 1}, \frac{2c_1 e^{\frac{x}{2}+\frac{\sin(2x)}{4}}}{c_1^2 e^{x+\frac{\sin(2x)}{2}} + 1}\right)$$

✓ Solution by Mathematica

Time used: 0.942 (sec). Leaf size: 41

```
DSolve[y'[x]==Cos[y[x]]*Cos[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\arctan\left(\tanh\left(\frac{1}{8}(2x + \sin(2x) + c_1)\right)\right)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

8.22 problem 23

Internal problem ID [2054]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{y + (2x + 3y - 1)\, y' = -x}$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 53

```
dsolve((x+y(x))+(2*x+3*y(x)-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{1}{2} + \frac{\sqrt{3}\,(x+1)\tan\left(\operatorname{RootOf}\left(\sqrt{3}\,\ln\left(\frac{(x+1)^2}{4} + \frac{\tan(\_Z)^2 (x+1)^2}{4}\right) + 2\sqrt{3}\,c_1 + 2\_Z\right)\right)}{6} - \frac{x}{2}$$

✓ Solution by Mathematica

Time used: 0.104 (sec). Leaf size: 73

```
DSolve[(x+y[x])+(2*x+3*y[x]-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\arctan\left(\frac{\sqrt{3}(y(x)-1)}{3y(x)+2x-1}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(\frac{3(x^2+3y(x)^2+3(x-1)y(x)-x+1)}{(x+1)^2}\right)\right.$$
$$\left.+\log(x+1)+c_1=0,y(x)\right]$$

8.23 problem 24

Internal problem ID [2055]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _dAlembert]`

$$\boxed{e^{\frac{x}{y}} + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right) y' = -1}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 20

```
dsolve((1+exp(x/y(x)))+( exp(x/y(x))*(1-x/y(x)) )*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = -\frac{x}{\operatorname{LambertW}\left(\frac{x c_1}{c_1 x - 1}\right)}$$

✓ Solution by Mathematica

Time used: 1.251 (sec). Leaf size: 34

```
DSolve[(1+Exp[x/y[x]])+( Exp[x/y[x]]*(1-x/y[x]) )*y'[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to -\frac{x}{W\left(\frac{x}{x - e^{c_1}}\right)}$$

$$y(x) \to -\frac{x}{W(1)}$$

8.24 problem 25

Internal problem ID [2056]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\cot(x) = -x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)+x+y(x)*cot(x)=0,y(x), singsol=all)
```

$$y = \frac{-\sin(x) + x\cos(x) + c_1}{\sin(x)}$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 16

```
DSolve[y'[x]+x+y[x]*Cot[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\cot(x) + c_1\csc(x) - 1$$

8.25 problem 26

Internal problem ID [2057]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{-y'yx = -3x + 6}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(3*(x-2)=x*y(x)*diff(y(x),x),y(x), singsol=all)
```

$$y = \sqrt{-12\ln(x) + c_1 + 6x}$$
$$y = -\sqrt{-12\ln(x) + c_1 + 6x}$$

✓ Solution by Mathematica

Time used: 0.073 (sec). Leaf size: 50

```
DSolve[3*(x-2)==x*y[x]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}\sqrt{3x - 6\log(x) + c_1}$$
$$y(x) \to \sqrt{2}\sqrt{3x - 6\log(x) + c_1}$$

8.26 problem 27

Internal problem ID [2058]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{-2yx + e^{y} + \left(y - x^{2} + x\,e^{y}\right)y' = -x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve((x-2*x*y(x)+exp(y(x)) )+(y(x)-x^2+x*exp(y(x)) )*diff(y(x),x)=0,y(x), singsol=all)
```

$$-yx^{2} + x\,e^{y} + \frac{x^{2}}{2} + \frac{y^{2}}{2} + c_{1} = 0$$

✓ Solution by Mathematica

Time used: 0.306 (sec). Leaf size: 35

```
DSolve[(x-2*x*y[x]+Exp[y[x]] )+(y[x]-x^2+x*Exp[y[x]] )*y'[x]==0,y[x],x,IncludeSingularSoluti
```

$$\text{Solve}\left[x^{2}(-y(x)) + \frac{x^{2}}{2} + xe^{y(x)} + \frac{y(x)^{2}}{2} = c_{1}, y(x)\right]$$

8.27 problem 28

Internal problem ID [2059]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{2y'x - y + \frac{x^2}{y^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 93

```
dsolve(2*x*diff(y(x),x)-y(x)+x^2/y(x)^2=0,y(x), singsol=all)
```

$$y = \left(x^{\frac{3}{2}} c_1 - 3x^2\right)^{\frac{1}{3}}$$
$$y = -\frac{\left(x^{\frac{3}{2}} c_1 - 3x^2\right)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}\left(x^{\frac{3}{2}} c_1 - 3x^2\right)^{\frac{1}{3}}}{2}$$
$$y = -\frac{\left(x^{\frac{3}{2}} c_1 - 3x^2\right)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}\left(x^{\frac{3}{2}} c_1 - 3x^2\right)^{\frac{1}{3}}}{2}$$

✓ Solution by Mathematica

Time used: 3.57 (sec). Leaf size: 80

```
DSolve[2*x*y'[x]-y[x]+x^2/y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[3]{-3x^2 + c_1 x^{3/2}}$$

$$y(x) \to -\sqrt[3]{-1}\sqrt[3]{-3x^2 + c_1 x^{3/2}}$$

$$y(x) \to (-1)^{2/3}\sqrt[3]{-3x^2 + c_1 x^{3/2}}$$

8.28 problem 29

Internal problem ID [2060]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x + y(y^2 + 1) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x)+y(x)*(y(x)^2+1)=0,y(x), singsol=all)
```

$$y = \frac{1}{\sqrt{c_1 x^2 - 1}}$$
$$y = -\frac{1}{\sqrt{c_1 x^2 - 1}}$$

✓ Solution by Mathematica

Time used: 0.438 (sec). Leaf size: 76

```
DSolve[x*y'[x]+y[x]*(y[x]^2+1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{ie^{c_1}}{\sqrt{-x^2+e^{2c_1}}}$$
$$y(x) \to \frac{ie^{c_1}}{\sqrt{-x^2+e^{2c_1}}}$$
$$y(x) \to 0$$
$$y(x) \to -i$$
$$y(x) \to i$$

8.29 problem 30

Internal problem ID [2061]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{\sqrt{x^2+y^2}\,y+yx-y'x^2=0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 25

```
dsolve(y(x)*sqrt(x^2+y(x)^2)+x*y(x)=x^2*diff(y(x),x),y(x), singsol=all)
```

$$-c_1+\frac{x\left(\sqrt{x^2+y^2}+x\right)}{y}=0$$

✓ Solution by Mathematica

Time used: 0.295 (sec). Leaf size: 47

```
DSolve[y[x]*Sqrt[x^2+y[x]^2]+x*y[x]==x^2*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -x\sqrt{-\operatorname{sech}^2(\log(x)+c_1)}$$
$$y(x)\to x\sqrt{-\operatorname{sech}^2(\log(x)+c_1)}$$
$$y(x)\to 0$$

8.30 problem 31

Internal problem ID [2062]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{3\,\mathrm{e}^{x}\tan\left(y\right)-\left(1-\mathrm{e}^{x}\right)\sec\left(y\right)^{2}y'=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 144

```
dsolve(3*exp(x)*tan(y(x))=(1-exp(x))*sec(y(x))^2*diff(y(x),x),y(x), singsol=all)
```

$$y=\frac{\arctan\left(\frac{2c_1\left(\mathrm{e}^{3x}-3\,\mathrm{e}^{2x}+3\,\mathrm{e}^{x}-1\right)}{\mathrm{e}^{6x}-6\,\mathrm{e}^{5x}+15\,\mathrm{e}^{4x}-20\,\mathrm{e}^{3x}+15\,\mathrm{e}^{2x}+c_1^2-6\,\mathrm{e}^{x}+1},\frac{\mathrm{e}^{6x}-6\,\mathrm{e}^{5x}+15\,\mathrm{e}^{4x}-20\,\mathrm{e}^{3x}+15\,\mathrm{e}^{2x}-c_1^2-6\,\mathrm{e}^{x}+1}{\mathrm{e}^{6x}-6\,\mathrm{e}^{5x}+15\,\mathrm{e}^{4x}-20\,\mathrm{e}^{3x}+15\,\mathrm{e}^{2x}+c_1^2-6\,\mathrm{e}^{x}+1}\right)}{2}$$

✓ Solution by Mathematica

Time used: 1.19 (sec). Leaf size: 78

```
DSolve[3*Exp[x]*Tan[y[x]]==(1-Exp[x])*Sec[y[x]]^2*y'[x],y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\to -\frac{1}{2}\arccos\left(-\tanh\left(-3\log\left(2-2e^{x}\right)+2c_1\right)\right)$$
$$y(x)\to \frac{1}{2}\arccos\left(-\tanh\left(-3\log\left(2-2e^{x}\right)+2c_1\right)\right)$$
$$y(x)\to 0$$
$$y(x)\to -\frac{\pi}{2}$$
$$y(x)\to \frac{\pi}{2}$$

8.31 problem 32

Internal problem ID [2063]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{\sec(y)^2 y' - \tan(y) = 2\,\mathrm{e}^x x}$$

✗ Solution by Maple

```
dsolve(sec(y(x))^2*diff(y(x),x)=tan(y(x))+2*x*exp(x),y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 10.872 (sec). Leaf size: 64

```
DSolve[Sec[y[x]]^2*y'[x]==Tan[y[x]]+2*x*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arctan\left(e^x\left(x^2+2c_1\right)\right)$$
$$y(x) \to -\frac{1}{2}\pi e^{-x}\sqrt{e^{2x}}$$
$$y(x) \to \frac{1}{2}\pi e^{-x}\sqrt{e^{2x}}$$

8.32 problem 33

Internal problem ID [2064]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{2x\tan(y) + 3y^2 + \left(x^2\sec(y)^2 + 6yx - y^2\right)y' = -x^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve((2*x*tan(y(x))+3*y(x)^2+x^2)+(x^2*sec(y(x))^2+6*x*y(x)-y(x)^2)*diff(y(x),x)=0,y(x), s
```

$$\tan(y)\,x^2 + \frac{x^3}{3} + 3xy^2 - \frac{y^3}{3} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.443 (sec). Leaf size: 87

```
DSolve[(2*x*Tan[y[x]]+3*y[x]^2+x^2)+(x^2*Sec[y[x]]^2+6*x*y[x]-y[x]^2)*y'[x]==0,y[x],x,Includ
```

$$\begin{aligned}\text{Solve}\Big[&\frac{1}{3}x^3\sec^2(y(x)) + \frac{1}{3}x^3\cos(2y(x))\sec^2(y(x)) + x^2\sin(2y(x))\sec^2(y(x))\\&- \frac{2y(x)^3}{3} + 3xy(x)^2\sec^2(y(x)) + 3xy(x)^2\cos(2y(x))\sec^2(y(x)) = c_1, y(x)\Big]\end{aligned}$$

8.33 problem 35

Internal problem ID [2065]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y\cos\left(\frac{x}{y}\right)-\left(y+x\cos\left(\frac{x}{y}\right)\right)y'=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve((y(x)*cos(x/y(x)))-(y(x)+x*cos(x/y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y=\frac{x}{\operatorname{RootOf}\left(\frac{\_Z\,\mathrm{e}^{\sin(\_Z)}}{c_1}-x\right)}$$

✓ Solution by Mathematica

Time used: 0.185 (sec). Leaf size: 28

```
DSolve[(y[x]*Cos[x/y[x]])-(y[x]+x*Cos[x/y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$\text{Solve}\left[\log\left(\frac{y(x)}{x}\right)-\sin\left(\frac{x}{y(x)}\right)=-\log(x)+c_1,y(x)\right]$$

8.34 problem 36

Internal problem ID [2066]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y(3x^2+y)-x(x^2-y)\,y'=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 43

```
dsolve(y(x)*(3*x^2+y(x))-x*(x^2-y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y=-\frac{x^2\left(\operatorname{RootOf}\left(\_Z^4c_1-\_Zc_1+3x\right)^3-1\right)}{3\operatorname{RootOf}\left(\_Z^4c_1-\_Zc_1+3x\right)^3}$$

✓ Solution by Mathematica

Time used: 60.121 (sec). Leaf size: 1665

```
DSolve[y[x]*(3*x^2+y[x])-x*(x^2-y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

8.35 problem 37

Internal problem ID [2067]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(2x + 3y + 2)\, y' = -x}$$

✓ Solution by Maple

Time used: 0.657 (sec). Leaf size: 47

```
dsolve(x+(2*x+3*y(x)+2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = -\frac{2}{3} + \frac{\sqrt{2}\, x \tan\left(\operatorname{RootOf}\left(\sqrt{2}\, \ln\left(6x^2 + 6\tan\left(\_Z\right)^2 x^2\right) + 2\sqrt{2}\, c_1 + 2\_Z\right)\right)}{3} - \frac{x}{3}$$

✓ Solution by Mathematica

Time used: 0.118 (sec). Leaf size: 78

```
DSolve[x+(2*x+3*y[x]+2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2\sqrt{2}\arctan\left(\frac{-3y(x) + x - 2}{\sqrt{2}(3y(x) + 2x + 2)}\right) = 2\log\left(\frac{3x^2 + 9y(x)^2 + 6(x+2)y(x) + 4x + 4}{3x^2}\right) + 4\log(x) + 3c_1, y(x)\right]$$

8.36 problem 38

Internal problem ID [2068]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y'x - 5y - x\sqrt{y} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x)-(5*y(x)+x*sqrt(y(x)))=0,y(x), singsol=all)
```

$$\sqrt{y} + \frac{x}{3} - x^{\frac{5}{2}} c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.153 (sec). Leaf size: 25

```
DSolve[x*y'[x]-(5*y[x]+x*Sqrt[y[x]])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{9} x^2 \left(1 - 3c_1 x^{3/2}\right)^2$$

8.37 problem 39

Internal problem ID [2069]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x\sqrt{1-y}-\sqrt{-x^2+1}\,y'=0}$$

With initial conditions

$$[y(0)=0]$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 22

```
dsolve([x*sqrt(1-y(x))-sqrt(1-x^2)*diff(y(x),x)=0,y(0) = 0],y(x), singsol=all)
```

$$y=\frac{x^2}{4}-\frac{\sqrt{-x^2+1}}{2}+\frac{1}{2}$$

✓ Solution by Mathematica

Time used: 0.231 (sec). Leaf size: 53

```
DSolve[{x*Sqrt[1-y[x]]-Sqrt[1-x^2]*y'[x]==0,{y[0]==0}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x)\to\frac{1}{4}\left(x^2-2\sqrt{1-x^2}+2\right)$$
$$y(x)\to\frac{1}{4}\left(x^2+6\sqrt{1-x^2}-6\right)$$

8.38 problem 40

Internal problem ID [2070]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{yx - y^2 - y'x^2 = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve([(x*y(x)-y(x)^2)-x^2*diff(y(x),x)=0,y(1) = 1],y(x), singsol=all)
```

$$y = \frac{x}{\ln(x) + 1}$$

✓ Solution by Mathematica

Time used: 0.137 (sec). Leaf size: 13

```
DSolve[{(x*y[x]-y[x]^2)-x^2*y'[x]==0,{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{\log(x) + 1}$$

8.39 problem 41

Internal problem ID [2071]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x\,\mathrm{e}^{-y^2} + yy' = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 29

```
dsolve([(x*exp(-y(x)^2))+y(x)*diff(y(x),x)=0,y(0) = 0],y(x), singsol=all)
```

$$y = \sqrt{\ln(-x^2+1)}$$
$$y = -\sqrt{\ln(-x^2+1)}$$

✓ Solution by Mathematica

Time used: 2.245 (sec). Leaf size: 35

```
DSolve[{(x*Exp[-y[x]^2])+y[x]*y'[x]==0,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\log(1-x^2)}$$
$$y(x) \to \sqrt{\log(1-x^2)}$$

8.40 problem 42

Internal problem ID [2072]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{\frac{2y^3 - 2y^3x^2 - x + xy^2\ln(y)}{y^2x} + \frac{(2\ln(x)\,y^3 - y^3x^2 + 2x + y^2x)\,y'}{y^3} = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.297 (sec). Leaf size: 40

```
dsolve([((2*y(x)^3-2*x^2*y(x)^3-x+x*y(x)^2*ln(y(x)))/(x*y(x)^2))+( (2*y(x)^3*ln(x)-x^2*y(x)^
```

$$y = \mathrm{e}^{\mathrm{RootOf}\left(-x^2\mathrm{e}^{3\_Z}+2\ln(x)\mathrm{e}^{3\_Z}+\_Zx\,\mathrm{e}^{2\_Z}+2\,\mathrm{e}^{2\_Z}-x\right)}$$

✓ Solution by Mathematica

Time used: 0.685 (sec). Leaf size: 30

```
DSolve[{((2*y[x]^3-2*x^2*y[x]^3-x+x*y[x]^2*Log[y[x]])/(x*y[x]^2))+( (2*y[x]^3*Log[x]-x^2*y[x
```

$$\mathrm{Solve}\left[x^2y(x) + \frac{x}{y(x)^2} - x\log(y(x)) - 2y(x)\log(x) = 2, y(x)\right]$$

8.41 problem 43

Internal problem ID [2073]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y'x - 2y - 2y^3x^4 = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 18

```
dsolve([x*diff(y(x),x)+(-2*y(x)-2*x^4*y(x)^3)=0,y(1) = 1],y(x), singsol=all)
```

$$y = \frac{2x^2}{\sqrt{-2x^8 + 6}}$$

✓ Solution by Mathematica

Time used: 0.225 (sec). Leaf size: 25

```
DSolve[{x*y'[x]+(-2*y[x]-2*x^4*y[x]^3)==0,{y[1]==1}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{\sqrt{2}x^2}{\sqrt{3 - x^8}}$$

8.42 problem 44

Internal problem ID [2074]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(-2x^2 - 3yx)\, y' + y^2 = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.359 (sec). Leaf size: 86

```
dsolve([(-2*x^2-3*x*y(x))*diff(y(x),x)+y(x)^2=0,y(1) = 1],y(x), singsol=all)
```

$$y = \frac{\left(-x^3 + 3\sqrt{-2x^4 + 27x^2}\,\sqrt{3} + 27x\right)^{\frac{2}{3}} - \left(-x^3 + 3\sqrt{-2x^4 + 27x^2}\,\sqrt{3} + 27x\right)^{\frac{1}{3}} x + x^2}{3\left(-x^3 + 3\sqrt{-2x^4 + 27x^2}\,\sqrt{3} + 27x\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 60.281 (sec). Leaf size: 77

```
DSolve[{(-2*x^2-3*x*y[x])*y'[x]+y[x]^2==0,{y[1]==1}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{1}{3}\left(\frac{x^2}{\sqrt[3]{-x^3 + 3\sqrt{81x^2 - 6x^4} + 27x}} + \sqrt[3]{-x^3 + 3\sqrt{81x^2 - 6x^4} + 27x} - x\right)$$

8.43 problem 45

Internal problem ID [2075]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - 4y = x^4}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 10

```
dsolve([x*diff(y(x),x)=x^4+4*y(x),y(1) = 0],y(x), singsol=all)
```

$$y = \ln(x)\, x^4$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 11

```
DSolve[{x*y'[x]==x^4+4*y[x],{y[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^4 \log(x)$$

8.44 problem 46

Internal problem ID [2076]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y'x + y - y^6x^3 = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.515 (sec). Leaf size: 54

```
dsolve([x*diff(y(x),x)+y(x)=x^3*y(x)^6,y(1) = 1],y(x), singsol=all)
```

$$y = \frac{\left(-\sqrt{5} - 1 + i\sqrt{10 - 2\sqrt{5}}\right) 2^{\frac{1}{5}} \left(-x^2(3x^2 - 5)^4\right)^{\frac{1}{5}}}{12x^3 - 20x}$$

✓ Solution by Mathematica

Time used: 0.473 (sec). Leaf size: 26

```
DSolve[{x*y'[x]+y[x]==x^3*y[x]^6,{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[5]{2}}{\sqrt[5]{5x^3 - 3x^5}}$$

8.45 problem 48

Internal problem ID [2077]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Bernoulli]`

$$\boxed{x' - x - x^2 e^{\theta} = 0}$$

With initial conditions

$$[x(0) = 2]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 16

```
dsolve([diff(x(theta),theta)=x(theta)+x(theta)^2*exp(theta),x(0) = 2],x(theta), singsol=all)
```

$$x(\theta) = -\frac{2\,e^{\theta}}{e^{2\theta} - 2}$$

✓ Solution by Mathematica

Time used: 0.217 (sec). Leaf size: 19

```
DSolve[{x'[\[Theta]]==x[\[Theta]]+x[\[Theta]]^2*Exp[\[Theta]],{x[0]==2}},x[\[Theta]],\[Theta
```

$$x(\theta) \to -\frac{2e^{\theta}}{e^{2\theta} - 2}$$

8.46 problem 49

Internal problem ID [2078]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y^2 - 2y'yx = -x^2}$$

With initial conditions

$$[y(2) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 27

```
dsolve([(x^2+y(x)^2)=2*x*y(x)*diff(y(x),x),y(2) = 0],y(x), singsol=all)
```

$$y = \sqrt{(x-2)\,x}$$
$$y = -\sqrt{(x-2)\,x}$$

✓ Solution by Mathematica

Time used: 0.182 (sec). Leaf size: 36

```
DSolve[{(x^2+y[x]^2)==2*x*y[x]*y'[x],{y[2]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x-2}\sqrt{x}$$
$$y(x) \to \sqrt{x-2}\sqrt{x}$$

8.47 problem 50

Internal problem ID [2079]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{3yx + (3x^2 + y^2)\, y' = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 3.485 (sec). Leaf size: 21

```
dsolve([(3*x*y(x))+(3*x^2+y(x)^2)*diff(y(x),x)=0,y(0) = 1],y(x), singsol=all)
```

$$y = \sqrt{-3x^2 + \sqrt{9x^4 + 1}}$$

✓ Solution by Mathematica

Time used: 8.701 (sec). Leaf size: 26

```
DSolve[{(3*x*y[x])+(3*x^2+y[x]^2)*y'[x]==0,{y[0]==1}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \sqrt{\sqrt{9x^4 + 1} - 3x^2}$$

8.48 problem 51

Internal problem ID [2080]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 51.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + 2y = 3\,\mathrm{e}^{2x}}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve([diff(y(x),x)+2*y(x)=3*exp(2*x),y(0) = 1],y(x), singsol=all)
```

$$y = \frac{3\,\mathrm{e}^{2x}}{4} + \frac{\mathrm{e}^{-2x}}{4}$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 23

```
DSolve[{y'[x]+2*y[x]==3*Exp[2*x],{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{-2x}\left(3e^{4x} + 1\right)$$

8.49 problem 52

Internal problem ID [2081]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{4y^2x + (x^2+1)\,y' = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve([4*x*y(x)^2+(x^2+1)*diff(y(x),x)=0,y(0) = 1],y(x), singsol=all)
```

$$y = \frac{1}{1 + 2\ln(x^2+1)}$$

✓ Solution by Mathematica

Time used: 0.156 (sec). Leaf size: 17

```
DSolve[{4*x*y[x]^2+(x^2+1)*y'[x]==0,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2\log(x^2+1)+1}$$

8.50 problem 53

Internal problem ID [2082]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 53.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{-2y-(x-2y+1)\,y'=-3-x}$$

With initial conditions

$$[y(0)=2]$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 20

```
dsolve([(x-2*y(x)+3)=(x-2*y(x)+1)*diff(y(x),x),y(0) = 2],y(x), singsol=all)
```

$$y=\frac{5}{2}+\frac{x}{2}+2\,\mathrm{LambertW}\left(-\frac{\mathrm{e}^{\frac{x}{4}-\frac{1}{4}}}{4}\right)$$

✓ Solution by Mathematica

Time used: 4.657 (sec). Leaf size: 28

```
DSolve[{(x-2*y[x]+3)==(x-2*y[x]+1)*y'[x],{y[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}\left(4W\left(-\frac{1}{4}e^{\frac{x-1}{4}}\right)+x+5\right)$$

8.51 problem 54

Internal problem ID [2083]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 54.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y^2 + (x^3 - 2yx)\, y' = 0}$$

With initial conditions

$$[y(2) = 1]$$

✓ Solution by Maple

Time used: 0.969 (sec). Leaf size: 29

```
dsolve([y(x)^2+(x^3-2*x*y(x))*diff(y(x),x)=0,y(2) = 1],y(x), singsol=all)
```

$$y = \operatorname{RootOf}\left(12x^6\_Z^8 - 8x^6\_Z^6 + 5\right)^2 x^2$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y[x]^2+(x^3-2*x*y[x])*y'[x]==0,{y[2]==1}},y[x],x,IncludeSingularSolutions -> True]
```

Timed out

8.52 problem 55

Internal problem ID [2084]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 55.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2yx - 2y + x(x-1)\,y' = -1}$$

With initial conditions

$$[y(2) = 2]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve([(2*x*y(x)-2*y(x)+1)+x*(x-1)*diff(y(x),x)=0,y(2) = 2],y(x), singsol=all)
```

$$y = \frac{-x - \ln(x-1) + 10}{x^2}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 17

```
DSolve[{(2*x*y[x]-2*y[x]+1)+x*(x-1)*y'[x]==0,{y[2]==2}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to -\frac{x + \log(x-1) - 10}{x^2}$$

8.53 problem 56

Internal problem ID [2085]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 56.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{y^3 + 2yx^2 + (-3x^3 - 2y^2x)\, y' = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 1.312 (sec). Leaf size: 62

```
dsolve([(y(x)^3+2*x^2*y(x))+(-3*x^3-2*x*y(x)^2)*diff(y(x),x)=0,y(1) = 1],y(x), singsol=all)
```

$$y = \frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\frac{\left(54x^4+6\sqrt{3}\,\sqrt{27x^8-2x^6}\right)^{\frac{2}{3}}+6x^2}{\left(54x^4+6\sqrt{3}\,\sqrt{27x^8-2x^6}\right)^{\frac{1}{3}}}}}{6}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{(y[x]^3+2*x^2*y[x])+(-3*x^3-2*x*y[x]^2)*y'[x]==0,{y[1]==1}},y[x],x,IncludeSingularSo
```

Timed out

8.54 problem 57

Internal problem ID [2086]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 12, page 46
Problem number: 57.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2(x^2+1)\,y' - (2y^2-1)\,xy = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 17

```
dsolve([2*(1+x^2)*diff(y(x),x)=(2*y(x)^2-1)*x*y(x),y(0) = 1],y(x), singsol=all)
```

$$y = \frac{1}{\sqrt{2-\sqrt{x^2+1}}}$$

✓ Solution by Mathematica

Time used: 4.797 (sec). Leaf size: 32

```
DSolve[{2*(1+x^2)*y'[x]==(2*y[x]^2-1)*x*y[x],{y[0]==1}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{\sqrt{-\sqrt{x^2+1}-2}}{\sqrt{x^2-3}}$$

9 Exercise 17, page 78

9.1 problem 15

Internal problem ID [2087]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y = c_1 \mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 16

```
DSolve[y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x$$
$$y(x) \to 0$$

9.2 problem 16

Internal problem ID [2088]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-4*y(x)=0,y(x), singsol=all)
```

$$y = \mathrm{e}^{-2x} c_1 + c_2 \mathrm{e}^{2x}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 22

```
DSolve[y''[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x} \left(c_1 e^{4x} + c_2\right)$$

9.3 problem 17

Internal problem ID [2089]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 7y' + 12y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+7*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^{-3x} + c_2 e^{-4x}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 20

```
DSolve[y''[x]+7*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-4x}(c_2 e^x + c_1)$$

9.4 problem 18

Internal problem ID [2090]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 3y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y = c_1 \mathrm{e}^{x} + c_2 \mathrm{e}^{2x}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 18

```
DSolve[y''[x]-3*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x (c_2 e^x + c_1)$$

9.5 problem 19

Internal problem ID [2091]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 7y' + 6y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)-7*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y = c_1 \mathrm{e}^{6x} + c_2 \mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 20

```
DSolve[y''[x]-7*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x \left(c_2 e^{5x} + c_1\right)$$

9.6 problem 20

Internal problem ID [2092]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2y'' + 3y' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(2*diff(y(x),x$2)+3*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^{\frac{x}{2}} + c_2 e^{-2x}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 24

```
DSolve[2*y''[x]+3*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(c_1 e^{5x/2} + c_2\right)$$

9.7 problem 21

Internal problem ID [2093]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y = c_1 \mathrm{e}^{\left(1+\sqrt{2}\right)x} + c_2 \mathrm{e}^{-\left(\sqrt{2}-1\right)x}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 34

```
DSolve[y''[x]-2*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x-\sqrt{2}x}\left(c_2 e^{2\sqrt{2}x} + c_1\right)$$

9.8 problem 22

Internal problem ID [2094]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y = c_1 \mathrm{e}^{\left(1+\sqrt{3}\right)x} + c_2 \mathrm{e}^{-\left(\sqrt{3}-1\right)x}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 34

```
DSolve[y''[x]-2*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x-\sqrt{3}x}\left(c_2 e^{2\sqrt{3}x} + c_1\right)$$

9.9 problem 23

Internal problem ID [2095]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 3y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y = c_1 \mathrm{e}^{\frac{(3+\sqrt{5})x}{2}} + c_2 \mathrm{e}^{-\frac{(\sqrt{5}-3)x}{2}}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 35

```
DSolve[y''[x]-3*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{1}{2}(\sqrt{5}-3)x}\left(c_2 e^{\sqrt{5}x} + c_1\right)$$

9.10 problem 24

Internal problem ID [2096]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2y'' + 2y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(2*diff(y(x),x$2)+2*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y = c_1 \mathrm{e}^{\frac{(\sqrt{3}-1)x}{2}} + c_2 \mathrm{e}^{-\frac{(1+\sqrt{3})x}{2}}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 35

```
DSolve[2*y''[x]+2*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{1}{2}(1+\sqrt{3})x}\left(c_2 e^{\sqrt{3}x} + c_1\right)$$

9.11 problem 25

Internal problem ID [2097]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 25.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{2y''' - y'' - 2y' + y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(2*diff(y(x),x$3)-diff(y(x),x$2)-2*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^{\frac{x}{2}} + c_2 e^{x} + c_3 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 30

```
DSolve[2*y'''[x]-y''[x]-2*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x/2} + c_2 e^{-x} + c_3 e^{x}$$

9.12 problem 26

Internal problem ID [2098]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 26.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 3y'' - 4y' + 12y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)-4*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y = e^{-2x}c_1 + c_2e^{2x} + c_3e^{3x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 29

```
DSolve[y'''[x]-3*y''[x]-4*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(e^{4x}(c_3e^x + c_2) + c_1\right)$$

9.13 problem 27

Internal problem ID [2099]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 27.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 4y'' + y' + 6y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$3)-4*diff(y(x),x$2)+diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y = c_1 \mathrm{e}^{-x} + c_2 \mathrm{e}^{2x} + c_3 \mathrm{e}^{3x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 29

```
DSolve[y'''[x]-4*y''[x]+y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(e^{3x}(c_3 e^x + c_2) + c_1\right)$$

9.14 problem 28

Internal problem ID [2100]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 28.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 6y'' + 8y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$4)-6*diff(y(x),x$2)+8*y(x)=0,y(x), singsol=all)
```

$$y = \mathrm{e}^{-2x}c_1 + c_2\mathrm{e}^{2x} + c_3\mathrm{e}^{\sqrt{2}\,x} + c_4\mathrm{e}^{-\sqrt{2}\,x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 47

```
DSolve[y''''[x]-6*y''[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1e^{\sqrt{2}x} + c_2e^{-\sqrt{2}x} + c_3e^{-2x} + c_4e^{2x}$$

9.15 problem 29

Internal problem ID [2101]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 29.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 7y' + 6y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)-7*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^{-3x} + c_2 e^{x} + c_3 e^{2x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 28

```
DSolve[y'''[x]-7*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-3x} + c_2 e^{x} + c_3 e^{2x}$$

9.16 problem 30

Internal problem ID [2102]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 30.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 6y'' + 11y' - 6y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)-6*diff(y(x),x$2)+11*diff(y(x),x)-6*y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^x + c_2 e^{2x} + c_3 e^{3x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 25

```
DSolve[y'''[x]-6*y''[x]+11*y'[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x (e^x (c_3 e^x + c_2) + c_1)$$

9.17 problem 31

Internal problem ID [2103]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 31.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 4y'' - 17y' + 60y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$3)-4*diff(y(x),x$2)-17*diff(y(x),x)+60*y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^{5x} + c_2 e^{-4x} + c_3 e^{3x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[y'''[x]-4*y''[x]-17*y'[x]+60*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-4x}\left(c_2 e^{7x} + c_3 e^{9x} + c_1\right)$$

9.18 problem 32

Internal problem ID [2104]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 32.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 9y'' + 23y' - 15y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)-9*diff(y(x),x$2)+23*diff(y(x),x)-15*y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^{5x} + c_2 e^{x} + c_3 e^{3x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 28

```
DSolve[y'''[x]-9*y''[x]+23*y'[x]-15*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}\left(c_2 e^{2x} + c_3 e^{4x} + c_1\right)$$

9.19 problem 33

Internal problem ID [2105]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 33.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + y''' - 7y'' - y' + 6y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$4)+diff(y(x),x$3)-7*diff(y(x),x$2)-diff(y(x),x)+6*y(x)=0,y(x), singsol=al
```

$$y = c_1 e^{-3x} + c_2 e^{x} + c_3 e^{-x} + c_4 e^{2x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 36

```
DSolve[y''''[x]+y'''[x]-7*y''[x]-y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-3x} + c_2 e^{-x} + c_3 e^{x} + c_4 e^{2x}$$

9.20 problem 34

Internal problem ID [2106]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 34.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{2y'''' - 3y''' - 20y'' + 27y' + 18y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(2*diff(y(x),x$4)-3*diff(y(x),x$3)-20*diff(y(x),x$2)+27*diff(y(x),x)+18*y(x)=0,y(x), s
```

$$y = c_1 \mathrm{e}^{-3x} + c_2 \mathrm{e}^{-\frac{x}{2}} + c_3 \mathrm{e}^{2x} + c_4 \mathrm{e}^{3x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 40

```
DSolve[2*y''''[x]-3*y'''[x]-20*y''[x]+27*y'[x]+18*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to e^{-3x} \left(c_1 e^{5x/2} + c_3 e^{5x} + c_4 e^{6x} + c_2\right)$$

9.21 problem 35

Internal problem ID [2107]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 35.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{12y'''' - 4y''' - 3y'' + y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(12*diff(y(x),x$4)-4*diff(y(x),x$3)-3*diff(y(x),x$2)+diff(y(x),x)=0,y(x), singsol=all)
```

$$y = c_1 + c_2 e^{-\frac{x}{2}} + c_3 e^{\frac{x}{2}} + c_4 e^{\frac{x}{3}}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 39

```
DSolve[12*y''''[x]-4*y'''[x]-3*y''[x]+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}\left(3c_1 e^{5x/6} + 2c_3 e^{x} - 2c_2\right) + c_4$$

9.22 problem 36

Internal problem ID [2108]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 36.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 4y'' + 3y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$3)-4*diff(y(x),x$2)+3*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = c_1 + c_2 e^x + c_3 e^{3x}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 25

```
DSolve[y'''[x]-4*y''[x]+3*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x + \frac{1}{3} c_2 e^{3x} + c_3$$

9.23 problem 37

Internal problem ID [2109]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 37.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{4y''' + 2y'' - 4y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(4*diff(y(x),x$3)+2*diff(y(x),x$2)-4*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^{\frac{x}{2}} + c_2 e^{\frac{(\sqrt{3}-1)x}{2}} + c_3 e^{-\frac{(1+\sqrt{3})x}{2}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 50

```
DSolve[4*y'''[x]+2*y''[x]-4*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\frac{1}{2}(1+\sqrt{3})x} + c_2 e^{\frac{1}{2}(\sqrt{3}-1)x} + c_3 e^{x/2}$$

9.24 problem 38

Internal problem ID [2110]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 38.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 5y'' - 2y' + 24y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$3)-5*diff(y(x),x$2)-2*diff(y(x),x)+24*y(x)=0,y(x), singsol=all)
```

$$y = e^{-2x}c_1 + c_2e^{4x} + c_3e^{3x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 29

```
DSolve[y'''[x]-5*y''[x]-2*y'[x]+24*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(e^{5x}(c_3e^x + c_2) + c_1\right)$$

9.25 problem 39

Internal problem ID [2111]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 39.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 2y''' - 7y'' - 8y' + 12y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$3)-7*diff(y(x),x$2)-8*diff(y(x),x)+12*y(x)=0,y(x), sings
```

$$y = c_1 e^{-3x} + c_2 e^{-2x} + c_3 e^{x} + c_4 e^{2x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 35

```
DSolve[y''''[x]+2*y'''[x]-7*y''[x]-8*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^{-3x}\left(c_2 e^{x} + e^{4x}(c_4 e^{x} + c_3) + c_1\right)$$

9.26 problem 40

Internal problem ID [2112]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 40.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(5)} - 3y'''' - 5y''' + 15y'' + 4y' - 12y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$5)-3*diff(y(x),x$4)-5*diff(y(x),x$3)+15*diff(y(x),x$2)+4*diff(y(x),x)-12*
```

$$y = \mathrm{e}^{-2x} c_1 + c_2 \mathrm{e}^{x} + c_3 \mathrm{e}^{-x} + c_4 \mathrm{e}^{2x} + c_5 \mathrm{e}^{3x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 42

```
DSolve[y'''''[x]-3*y''''[x]-5*y'''[x]+15*y''[x]+4*y'[x]-12*y[x]==0,y[x],x,IncludeSingularSol
```

$$y(x) \to e^{-2x} \left(c_2 e^x + e^{3x} \left(e^x \left(c_5 e^x + c_4\right) + c_3\right) + c_1\right)$$

9.27 problem 41

Internal problem ID [2113]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 41.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(5)} + y'''' - 13y''' - 13y'' + 36y' + 36y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$5)+diff(y(x),x$4)-13*diff(y(x),x$3)-13*diff(y(x),x$2)+36*diff(y(x),x)+36*
```

$$y = c_1 e^{-3x} + c_2 e^{-2x} + c_3 e^{-x} + c_4 e^{2x} + c_5 e^{3x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 42

```
DSolve[y'''''[x]+y''''[x]-13*y'''[x]-13*y''[x]+36*y'[x]+36*y[x]==0,y[x],x,IncludeSingularSol
```

$$y(x) \to e^{-3x}\left(e^x\left(c_3 e^x + e^{4x}(c_5 e^x + c_4) + c_2\right) + c_1\right)$$

9.28 problem 42

Internal problem ID [2114]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 42.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(5)} + 3y'''' - 15y''' - 19y'' + 30y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$5)+3*diff(y(x),x$4)-15*diff(y(x),x$3)-19*diff(y(x),x$2)+30*diff(y(x),x)=0
```

$$y = c_1 + c_2 e^{-5x} + c_3 e^{-2x} + c_4 e^{x} + c_5 e^{3x}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 47

```
DSolve[y'''''[x]+3*y''''[x]-15*y'''[x]-19*y''[x]+30*y'[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to -\frac{1}{5}c_1 e^{-5x} - \frac{1}{2}c_2 e^{-2x} + c_3 e^{x} + \frac{1}{3}c_4 e^{3x} + c_5$$

9.29 problem 43

Internal problem ID [2115]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 43.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 3y'' - 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$4)+3*diff(y(x),x$2)-4*y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^x + c_2 e^{-x} + c_3 \sin(2x) + c_4 \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 34

```
DSolve[y''''[x]+3*y''[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3 e^{-x} + c_4 e^x + c_1 \cos(2x) + c_2 \sin(2x)$$

9.30 problem 44

Internal problem ID [2116]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 17, page 78
Problem number: 44.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(5)} + 3y''' + 2y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$5)+3*diff(y(x),x$3)+2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = c_1 + c_2 \sin(x) + c_3 \cos(x) + c_4 \sin\left(\sqrt{2}\, x\right) + c_5 \cos\left(\sqrt{2}\, x\right)$$

✓ Solution by Mathematica

Time used: 0.083 (sec). Leaf size: 52

```
DSolve[y'''''[x]+3*y'''[x]+2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -c_4 \cos(x) - \frac{c_2 \cos\left(\sqrt{2}x\right)}{\sqrt{2}} + c_3 \sin(x) + \frac{c_1 \sin\left(\sqrt{2}x\right)}{\sqrt{2}} + c_5$$

10 Exercise 18, page 82

10.1 problem 1

Internal problem ID [2117]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 18, page 82
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^x + c_2 e^x x$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 16

```
DSolve[y''[x]-2*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x(c_2 x + c_1)$$

10.2 problem 2

Internal problem ID [2118]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 18, page 82
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y''=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y = c_1 x + c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 12

```
DSolve[y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_2 x + c_1$$

10.3 problem 3

Internal problem ID [2119]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 18, page 82
Problem number: 3.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{2y''' + y'' - 4y' - 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(2*diff(y(x),x$3)+diff(y(x),x$2)-4*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y = c_1 \mathrm{e}^{\frac{3x}{2}} + c_2 \mathrm{e}^{-x} + c_3 \mathrm{e}^{-x} x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 28

```
DSolve[2*y'''[x]+y''[x]-4*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x} \left(c_1 e^{5x/2} + c_3 x + c_2\right)$$

10.4 problem 4

Internal problem ID [2120]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 18, page 82
Problem number: 4.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 3y'' + 3y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)+3*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^x + c_2 e^x x + c_3 e^x x^2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 21

```
DSolve[y'''[x]-3*y''[x]+3*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x (x(c_3 x + c_2) + c_1)$$

10.5 problem 5

Internal problem ID [2121]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 18, page 82
Problem number: 5.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _quadrature]]`

$$\boxed{y'''' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$4)=0,y(x), singsol=all)
```

$$y = \frac{1}{6}c_1x^3 + \frac{1}{2}c_2x^2 + c_3x + c_4$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 22

```
DSolve[y''''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(x(c_4x + c_3) + c_2) + c_1$$

10.6 problem 6

Internal problem ID [2122]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 18, page 82
Problem number: 6.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + y'' - y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$3)+diff(y(x),x$2)-diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^x + c_2 e^{-x} + c_3 e^{-x} x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 26

```
DSolve[y'''[x]+y''[x]-y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_2 x + c_3 e^{2x} + c_1\right)$$

10.7 problem 7

Internal problem ID [2123]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 18, page 82
Problem number: 7.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{4y''' - 3y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(4*diff(y(x),x$3)-3*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y = c_1 \mathrm{e}^{-x} + c_2 \mathrm{e}^{\frac{x}{2}} + c_3 \mathrm{e}^{\frac{x}{2}} x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 29

```
DSolve[4*y'''[x]-3*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x} \left(e^{3x/2}(c_2 x + c_1) + c_3\right)$$

10.8 problem 8

Internal problem ID [2124]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 18, page 82
Problem number: 8.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{4y^{(5)} - 3y''' - y'' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(4*diff(y(x),x$5)-3*diff(y(x),x$3)-diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y = c_1 + c_2x + c_3\mathrm{e}^{x} + c_4\mathrm{e}^{-\frac{x}{2}} + c_5\mathrm{e}^{-\frac{x}{2}}x$$

✓ Solution by Mathematica

Time used: 0.196 (sec). Leaf size: 36

```
DSolve[4*y'''''[x]-3*y'''[x]-y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 4e^{-x/2}(c_2(x+4)+c_1) + c_3e^{x} + c_5x + c_4$$

10.9 problem 9

Internal problem ID [2125]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 18, page 82
Problem number: 9.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 7y'' + 16y' - 12y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$3)-7*diff(y(x),x$2)+16*diff(y(x),x)-12*y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^{3x} + c_2 e^{2x} + c_3 e^{2x} x$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 24

```
DSolve[y'''[x]-7*y''[x]+16*y'[x]-12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}(c_2 x + c_3 e^x + c_1)$$

10.10 problem 10

Internal problem ID [2126]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 18, page 82
Problem number: 10.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{4y''' - 8y'' + 5y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(4*diff(y(x),x$3)-8*diff(y(x),x$2)+5*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^x + c_2 e^{\frac{x}{2}} + c_3 e^{\frac{x}{2}} x$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 30

```
DSolve[4*y'''[x]-8*y''[x]+5*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x/2}\left(c_2 x + c_3 e^{x/2} + c_1\right)$$

10.11 problem 11

Internal problem ID [2127]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 18, page 82
Problem number: 11.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$4)-y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^{-x} + c_2 e^{x} + c_3 \sin(x) + c_4 \cos(x)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 30

```
DSolve[y''''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x} + c_3 e^{-x} + c_2 \cos(x) + c_4 \sin(x)$$

10.12 problem 12

Internal problem ID [2128]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 18, page 82
Problem number: 12.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 8y = 0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$3)-8*y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^{2x} + c_2 e^{-x} \sin\left(\sqrt{3}\, x\right) + c_3 e^{-x} \cos\left(\sqrt{3}\, x\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 42

```
DSolve[y'''[x]-8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_1 e^{3x} + c_2 \cos\left(\sqrt{3}x\right) + c_3 \sin\left(\sqrt{3}x\right)\right)$$

10.13 problem 13

Internal problem ID [2129]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 18, page 82
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^x \sin\left(\sqrt{2}\,x\right) + c_2 e^x \cos\left(\sqrt{2}\,x\right)$$

✓ Solution by Mathematica

Time used: 0.13 (sec). Leaf size: 32

```
DSolve[y''[x]-2*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x\left(c_2 \cos\left(\sqrt{2}x\right) + c_1 \sin\left(\sqrt{2}x\right)\right)$$

10.14 problem 14

Internal problem ID [2130]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 18, page 82
Problem number: 14.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + y'' - 20y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$4)+diff(y(x),x$2)-20*y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^{-2x} + c_2 e^{2x} + c_3 \sin\left(\sqrt{5}\, x\right) + c_4 \cos\left(\sqrt{5}\, x\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 44

```
DSolve[y''''[x]+y''[x]-20*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3 e^{-2x} + c_4 e^{2x} + c_1 \cos\left(\sqrt{5}x\right) + c_2 \sin\left(\sqrt{5}x\right)$$

10.15 problem 15

Internal problem ID [2131]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 18, page 82
Problem number: 15.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 5y'' + 6y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$4)+5*diff(y(x),x$2)+6*y(x)=0,y(x), singsol=all)
```

$$y = c_1 \sin\left(\sqrt{3}\, x\right) + c_2 \cos\left(\sqrt{3}\, x\right) + c_3 \sin\left(\sqrt{2}\, x\right) + c_4 \cos\left(\sqrt{2}\, x\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 50

```
DSolve[y''''[x]+5*y''[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3 \cos\left(\sqrt{2}x\right) + c_1 \cos\left(\sqrt{3}x\right) + c_4 \sin\left(\sqrt{2}x\right) + c_2 \sin\left(\sqrt{3}x\right)$$

10.16 problem 16

Internal problem ID [2132]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 18, page 82
Problem number: 16.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 4y''' + 6y'' - 8y' + 8y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$4)-4*diff(y(x),x$3)+6*diff(y(x),x$2)-8*diff(y(x),x)+8*y(x)=0,y(x), singso
```

$$y = c_1 e^{2x} + c_2 e^{2x} x + c_3 \sin\left(\sqrt{2}\, x\right) + c_4 \cos\left(\sqrt{2}\, x\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 41

```
DSolve[y''''[x]-4*y'''[x]+6*y''[x]-8*y'[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to e^{2x}(c_4 x + c_3) + c_1 \cos\left(\sqrt{2}x\right) + c_2 \sin\left(\sqrt{2}x\right)$$

10.17 problem 17

Internal problem ID [2133]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 18, page 82
Problem number: 17.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 2y''' - 6y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$4)-2*diff(y(x),x$3)-6*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y = \sum_{\_a=1}^{4} \mathrm{e}^{\mathrm{RootOf}\left(\_Z^4-2\_Z^3-6\_Z+2,\mathrm{index}=\_a\right)x} \_C\_a$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 114

```
DSolve[y''''[x]-2*y'''[x]-6*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \to\ & c_1 \exp\left(x\mathrm{Root}\left[\#1^4 - 2\#1^3 - 6\#1 + 2\&, 1\right]\right) \\ & + c_3 \exp\left(x\mathrm{Root}\left[\#1^4 - 2\#1^3 - 6\#1 + 2\&, 3\right]\right) \\ & + c_4 \exp\left(x\mathrm{Root}\left[\#1^4 - 2\#1^3 - 6\#1 + 2\&, 4\right]\right) \\ & + c_2 \exp\left(x\mathrm{Root}\left[\#1^4 - 2\#1^3 - 6\#1 + 2\&, 2\right]\right) \end{aligned}$$

10.18 problem 18

Internal problem ID [2134]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 18, page 82
Problem number: 18.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + y''' - 3y'' - 4y' - 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x$4)+diff(y(x),x$3)-3*diff(y(x),x$2)-4*diff(y(x),x)-4*y(x)=0,y(x), singsol=
```

$$y = c_1 e^{-2x} + c_2 e^{2x} + c_3 e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right) + c_4 e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 64

```
DSolve[y''''[x]+y'''[x]-3*y''[x]-4*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(c_4 e^{4x} + c_2 e^{3x/2} \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 e^{3x/2} \sin\left(\frac{\sqrt{3}x}{2}\right) + c_3\right)$$

10.19 problem 19

Internal problem ID [2135]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 18, page 82
Problem number: 19.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{2y''' - 3y'' + 10y' - 15y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(2*diff(y(x),x$3)-3*diff(y(x),x$2)+10*diff(y(x),x)-15*y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^{\frac{3x}{2}} + c_2 \sin\left(\sqrt{5}\, x\right) + c_3 \cos\left(\sqrt{5}\, x\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 38

```
DSolve[2*y'''[x]-3*y''[x]+10*y'[x]-15*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3 e^{3x/2} + c_1 \cos\left(\sqrt{5}x\right) + c_2 \sin\left(\sqrt{5}x\right)$$

10.20 problem 20

Internal problem ID [2136]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 18, page 82
Problem number: 20.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{2y''' - 3y'' + 11y' - 40y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(2*diff(y(x),x$3)-3*diff(y(x),x$2)+11*diff(y(x),x)-40*y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^{\frac{5x}{2}} + c_2 e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{31}\,x}{2}\right) + c_3 e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{31}\,x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 50

```
DSolve[2*y'''[x]-3*y''[x]+11*y'[x]-40*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}\left(c_3 e^{3x} + c_2 \cos\left(\frac{\sqrt{31}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{31}x}{2}\right)\right)$$

10.21 problem 21

Internal problem ID [2137]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 18, page 82
Problem number: 21.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 3y''' + 4y'' - 12y' + 16y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
dsolve(diff(y(x),x$4)-3*diff(y(x),x$3)+4*diff(y(x),x$2)-12*diff(y(x),x)+16*y(x)=0,y(x), sing
```

$$y = c_1 e^{2x} + c_2 e^{2x} x + c_3 e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{15}\, x}{2}\right) + c_4 e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{15}\, x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 57

```
DSolve[y''''[x]-3*y'''[x]+4*y''[x]-12*y'[x]+16*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to e^{-x/2}\left(e^{5x/2}(c_4 x + c_3) + c_2 \cos\left(\frac{\sqrt{15}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{15}x}{2}\right)\right)$$

10.22 problem 22

Internal problem ID [2138]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 18, page 82
Problem number: 22.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{4y''' + 12y'' - 3y' + 14y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(4*diff(y(x),x$3)+12*diff(y(x),x$2)-3*diff(y(x),x)+14*y(x)=0,y(x), singsol=all)
```

$$y = c_1 \mathrm{e}^{-\frac{7x}{2}} + c_2 \mathrm{e}^{\frac{x}{4}} \sin\left(\frac{\sqrt{15}\, x}{4}\right) + c_3 \mathrm{e}^{\frac{x}{4}} \cos\left(\frac{\sqrt{15}\, x}{4}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 58

```
DSolve[4*y'''[x]+12*y''[x]-3*y'[x]+14*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3 e^{-7x/2} + c_2 e^{x/4} \cos\left(\frac{\sqrt{15}x}{4}\right) + c_1 e^{x/4} \sin\left(\frac{\sqrt{15}x}{4}\right)$$

10.23 problem 23

Internal problem ID [2139]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 18, page 82
Problem number: 23.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(5)} - y'''' + 6y''' - 6y'' + 8y' - 8y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$5)-diff(y(x),x$4)+6*diff(y(x),x$3)-6*diff(y(x),x$2)+8*diff(y(x),x)-8*y(x)
```

$$y = c_1 e^x + c_2 \sin(2x) + c_3 \cos(2x) + c_4 \sin\left(\sqrt{2}\, x\right) + c_5 \cos\left(\sqrt{2}\, x\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 48

```
DSolve[y'''''[x]-y''''[x]+6*y'''[x]-6*y''[x]+8*y'[x]-8*y[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to c_5 e^x + c_1 \cos(2x) + c_3 \cos\left(\sqrt{2}x\right) + c_2 \sin(2x) + c_4 \sin\left(\sqrt{2}x\right)$$

11 Exercise 19, page 86

11.1 problem 1

Internal problem ID [2140]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y = 3\cos(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-4*y(x)=3*cos(x),y(x), singsol=all)
```

$$y = e^{-2x}c_2 + c_1 e^{2x} - \frac{3\cos(x)}{5}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 28

```
DSolve[y''[x]-4*y[x]==3*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{3\cos(x)}{5} + c_1 e^{2x} + c_2 e^{-2x}$$

11.2 problem 2

Internal problem ID [2141]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sin(2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)+y(x)=sin(2*x),y(x), singsol=all)
```

$$y = \sin(x)\, c_2 + \cos(x)\, c_1 - \frac{\sin(2x)}{3}$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 23

```
DSolve[y''[x]+y[x]==Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \sin(x) + \cos(x)\left(-\frac{2\sin(x)}{3} + c_1\right)$$

11.3 problem 3

Internal problem ID [2142]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' - 2y = \mathrm{e}^{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-2*y(x)=exp(x),y(x), singsol=all)
```

$$y = \mathrm{e}^{-2x} c_2 + c_1 \mathrm{e}^{x} + \frac{x\,\mathrm{e}^{x}}{3}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 29

```
DSolve[y''[x]+y'[x]-2*y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-2x} + e^{x}\left(\frac{x}{3} - \frac{1}{9} + c_2\right)$$

11.4 problem 4

Internal problem ID [2143]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 3y' + 2y = e^{-2x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=exp(-2*x),y(x), singsol=all)
```

$$y = \left(-e^{-x}(x + c_1 + 1) + c_2\right) e^{-x}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 24

```
DSolve[y''[x]+3*y'[x]+2*y[x]==Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}(-x + c_2 e^x - 1 + c_1)$$

11.5 problem 5

Internal problem ID [2144]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' + y = \sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=sin(x),y(x), singsol=all)
```

$$y = e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\, x}{2}\right) c_2 + e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\, x}{2}\right) c_1 - \cos(x)$$

✓ Solution by Mathematica

Time used: 1.174 (sec). Leaf size: 53

```
DSolve[y''[x]+y'[x]+y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}\left(-e^{x/2}\cos(x) + c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

11.6 problem 6

Internal problem ID [2145]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' + y = x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=x^2,y(x), singsol=all)
```

$$y = \mathrm{e}^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\, x}{2}\right) c_2 + \mathrm{e}^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\, x}{2}\right) c_1 + x^2 - 2x$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 54

```
DSolve[y''[x]+y'[x]+y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}\left(e^{x/2}(x-2)x + c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

11.7 problem 7

Internal problem ID [2146]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 3y' + 2y = x\,\mathrm{e}^{-x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=x*exp(-x),y(x), singsol=all)
```

$$y = \left(-x + \frac{x^2}{2} - c_1\mathrm{e}^{-x} + c_2\right)\mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 35

```
DSolve[y''[x]+3*y'[x]+2*y[x]==x*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-2x}\left(e^x\left(x^2 - 2x + 2 + 2c_2\right) + 2c_1\right)$$

11.8 problem 8

Internal problem ID [2147]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 8.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$\boxed{y'''' - y = e^{x}}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$4)-y(x)=exp(x),y(x), singsol=all)
```

$$y = \frac{x\,e^{x}}{4} + \cos\left(x\right)c_1 + c_2 e^{x} + c_3\sin\left(x\right) + c_4 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 45

```
DSolve[y''''[x]-y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x x}{4} - \frac{3e^x}{8} + c_1 e^x + c_3 e^{-x} + c_2\cos(x) + c_4\sin(x)$$

11.9 problem 9

Internal problem ID [2148]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y = x + \mathrm{e}^{2x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-4*y(x)=x+exp(2*x),y(x), singsol=all)
```

$$y = \mathrm{e}^{-2x} c_2 + c_1 \mathrm{e}^{2x} + \frac{(4x-1)\,\mathrm{e}^{2x}}{16} - \frac{x}{4}$$

✓ Solution by Mathematica

Time used: 0.082 (sec). Leaf size: 36

```
DSolve[y''[x]-4*y[x]==x+Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{4} + e^{2x}\left(\frac{x}{4} - \frac{1}{16} + c_1\right) + c_2 e^{-2x}$$

11.10 problem 10

Internal problem ID [2149]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 9y = e^{3x} + \sin(3x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)-9*y(x)=exp(3*x)+sin(3*x),y(x), singsol=all)
```

$$y = e^{3x} c_2 + e^{-3x} c_1 + \frac{(6x - 1) e^{3x}}{36} - \frac{\sin(3x)}{18}$$

✓ Solution by Mathematica

Time used: 0.247 (sec). Leaf size: 39

```
DSolve[y''[x]-9*y[x]==Exp[3*x]+Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{18} \sin(3x) + e^{3x} \left(\frac{x}{6} - \frac{1}{36} + c_1\right) + c_2 e^{-3x}$$

11.11 problem 11

Internal problem ID [2150]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y' - 6y = x^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-6*y(x)=x^3,y(x), singsol=all)
```

$$y(x) = c_1 e^{3x} + c_2 e^{-2x} - \frac{x^3}{6} + \frac{x^2}{12} - \frac{7x}{36} + \frac{13}{216}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 41

```
DSolve[y''[x]-y'[x]-6*y[x]==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{216}\left(-36x^3 + 18x^2 - 42x + 13\right) + c_1 e^{-2x} + c_2 e^{3x}$$

11.12 problem 12

Internal problem ID [2151]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{-2y'' + 3y = x\,\mathrm{e}^{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x$2)+3*y(x)=x*exp(x),y(x), singsol=all)
```

$$y = \mathrm{e}^{\frac{\sqrt{6}\,x}{2}} c_2 + \mathrm{e}^{-\frac{\sqrt{6}\,x}{2}} c_1 + (x+4)\,\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 54

```
DSolve[y''[x]-3*y'[x]+3*y[x]==x*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}\left(x + c_2 e^{x/2}\cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 e^{x/2}\sin\left(\frac{\sqrt{3}x}{2}\right) + 1\right)$$

11.13 problem 13

Internal problem ID [2152]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = x \sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+4*y(x)=x*sin(x),y(x), singsol=all)
```

$$y = c_2 \sin(2x) + \cos(2x) c_1 - \frac{2\cos(x)}{9} + \frac{x \sin(x)}{3}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 33

```
DSolve[y''[x]+4*y[x]==x*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} x \sin(x) + c_1 \cos(2x) + \cos(x) \left(-\frac{2}{9} + 2c_2 \sin(x)\right)$$

11.14 problem 14

Internal problem ID [2153]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 14.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - 4y'' = x^2 + 8}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$3)-4*diff(y(x),x$2)=x^2+8,y(x), singsol=all)
```

$$y = -\frac{65x^2}{64} - \frac{x^3}{48} - \frac{x^4}{48} + \frac{e^{4x}c_1}{16} + c_2x + c_3$$

✓ Solution by Mathematica

Time used: 0.143 (sec). Leaf size: 41

```
DSolve[y'''[x]-4*y''[x]==x^2+8,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{192}\left(-4x^4 - 4x^3 - 195x^2 + 12c_1e^{4x}\right) + c_3x + c_2$$

11.15 problem 15

Internal problem ID [2154]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' + y = \mathrm{e}^{x} \sin(3x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=exp(x)*sin(3*x),y(x), singsol=all)
```

$$y = \mathrm{e}^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\, x}{2}\right) c_2 + \mathrm{e}^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\, x}{2}\right) c_1 - \frac{\mathrm{e}^{x}(2\sin(3x) + 3\cos(3x))}{39}$$

✓ Solution by Mathematica

Time used: 1.715 (sec). Leaf size: 70

```
DSolve[y''[x]+y'[x]+y[x]==Exp[x]*Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{39} e^{x} \sin(3x) - \frac{1}{13} e^{x} \cos(3x) + c_2 e^{-x/2} \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 e^{-x/2} \sin\left(\frac{\sqrt{3}x}{2}\right)$$

11.16 problem 16

Internal problem ID [2155]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 16.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - 3y'' + 4y' - 12y = x + e^{2x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)+4*diff(y(x),x)-12*y(x)=x+exp(2*x),y(x), singsol=all)
```

$$y = \frac{e^{3x}e^{-3x}\cos(2x)}{52} + \frac{3\,e^{3x}e^{-3x}\sin(2x)}{104} - \frac{e^{-3x}(6\,e^{3x}x + 9\,e^{5x} + 2\,e^{3x})}{72} + \cos(2x)\,c_1 + e^{3x}c_2 + c_3\sin(2x)$$

✓ Solution by Mathematica

Time used: 0.337 (sec). Leaf size: 45

```
DSolve[y'''[x]-3*y''[x]+4*y'[x]-12*y[x]==x+Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{72}\left(-6x - 9e^{2x} + 72c_3e^{3x} - 2\right) + c_1\cos(2x) + c_2\sin(2x)$$

11.17 problem 17

Internal problem ID [2156]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 17.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 4y'' + y' - 4y = e^{4x} \sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$3)-4*diff(y(x),x$2)+diff(y(x),x)-4*y(x)=exp(4*x)*sin(x),y(x), singsol=all
```

$$y = -\frac{\cos(x) e^{4x}}{20} - \frac{e^{4x} \sin(x)}{40} + \cos(x) c_1 + \sin(x) c_2 + c_3 e^{4x}$$

✓ Solution by Mathematica

Time used: 0.098 (sec). Leaf size: 44

```
DSolve[y'''[x]-4*y''[x]+y'[x]-4*y[x]==Exp[4*x]*Sin[x],y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to c_3 e^{4x} + \left(-\frac{e^{4x}}{20} + c_1\right) \cos(x) + \left(-\frac{e^{4x}}{40} + c_2\right) \sin(x)$$

11.18 problem 18

Internal problem ID [2157]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 4y = x^3 e^{2x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+4*y(x)=x^3*exp(2*x),y(x), singsol=all)
```

$$y = e^{-2x} c_2 + e^{-2x} x c_1 + \frac{(8x^3 - 12x^2 + 9x - 3) e^{2x}}{128}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 43

```
DSolve[y''[x]+4*y'[x]+4*y[x]==x^3*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{128} e^{2x} (8x^3 - 12x^2 + 9x - 3) + e^{-2x}(c_2 x + c_1)$$

11.19 problem 19

Internal problem ID [2158]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 19.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 2y'' + y' - 2y = x\,\mathrm{e}^{2x}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$3)-2*diff(y(x),x$2)+diff(y(x),x)-2*y(x)=x*exp(2*x),y(x), singsol=all)
```

$$y = \left(\frac{x}{10} - \frac{4}{25}\right) x\,\mathrm{e}^{2x} + \cos(x)\, c_1 + \sin(x)\, c_2 + c_3 \mathrm{e}^{2x}$$

✓ Solution by Mathematica

Time used: 0.082 (sec). Leaf size: 39

```
DSolve[y'''[x]-2*y''[x]+y'[x]-2*y[x]==x*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{250} e^{2x} \left(25x^2 - 40x + 22 + 250c_3\right) + c_1 \cos(x) + c_2 \sin(x)$$

11.20 problem 20

Internal problem ID [2159]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 20.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' + 2n^2y'' + n^4y = \sin(kx)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 48

```
dsolve(diff(y(x),x$4)+2*n^2*diff(y(x),x$2)+n^4*y(x)=sin(k*x),y(x), singsol=all)
```

$$y = \frac{\sin(kx)}{(k-n)^2(k+n)^2} + c_1\cos(nx) + c_2\sin(nx) + c_3\cos(nx)x + c_4\sin(nx)x$$

✓ Solution by Mathematica

Time used: 0.457 (sec). Leaf size: 69

```
DSolve[y''''[x]+2*n^2*y''[x]+n^4*y[x]==Sin[k*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(c_2x+c_1)(k^2-n^2)^2\cos(nx) + (c_4x+c_3)(k^2-n^2)^2\sin(nx) + \sin(kx)}{(k-n)^2(k+n)^2}$$

11.21 problem 21

Internal problem ID [2160]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2ny' + n^2y = 5\cos(6x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
dsolve(diff(y(x),x$2)+2*n*diff(y(x),x)+n^2*y(x)=5*cos(6*x),y(x), singsol=all)
```

$$y = e^{-nx}c_2 + e^{-nx}xc_1 + \frac{5\cos(6x)\,n^2 + 60\sin(6x)\,n - 180\cos(6x)}{(n^2+36)^2}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 57

```
DSolve[y''[x]+2*n*y''[x]+n^2*y[x]==5*Cos[6*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{5\cos(6x)}{n^2 - 72n - 36} + c_1 e^{\frac{nx}{\sqrt{-2n-1}}} + c_2 e^{-\frac{nx}{\sqrt{-2n-1}}}$$

11.22 problem 22

Internal problem ID [2161]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = (1 + \sin(3x)) \cos(2x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)+9*y(x)=(1+sin(3*x))*cos(2*x),y(x), singsol=all)
```

$$y = \sin(3x) c_2 + \cos(3x) c_1 + \frac{\cos(2x)}{5} + \frac{\sin(x)}{16} - \frac{\sin(5x)}{32}$$

✓ Solution by Mathematica

Time used: 0.266 (sec). Leaf size: 42

```
DSolve[y''[x]+9*y[x]==(1+Sin[3*x])*Cos[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sin(x)}{16} - \frac{1}{32} \sin(5x) + \frac{1}{5} \cos(2x) + c_1 \cos(3x) + c_2 \sin(3x)$$

11.23 problem 23

Internal problem ID [2162]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 5y = 2x - e^{-4x} + \sin(2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+5*y(x)=2*x-exp(-4*x)+sin(2*x),y(x), singsol=all)
```

$$y = e^{-2x}\sin(x)\,c_2 + e^{-2x}\cos(x)\,c_1 + \frac{\sin(2x)}{65} + \frac{2x}{5} - \frac{8}{25} - \frac{8\cos(2x)}{65} - \frac{e^{-4x}}{5}$$

✓ Solution by Mathematica

Time used: 0.784 (sec). Leaf size: 59

```
DSolve[y''[x]+4*y'[x]+5*y[x]==2*x-Exp[-4*x]+Sin[2*x],y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{2x}{5} - \frac{e^{-4x}}{5} + \frac{1}{65}\sin(2x) - \frac{8}{65}\cos(2x) + c_2 e^{-2x}\cos(x) + c_1 e^{-2x}\sin(x) - \frac{8}{25}$$

11.24 problem 24

Internal problem ID [2163]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 24.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + 2y'' = (2x^2 + x)\,\mathrm{e}^{-2x} + 5\cos(3x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 58

```
dsolve(diff(y(x),x$3)+2*diff(y(x),x$2)=(2*x^2+x)*exp(-2*x)+5*cos(3*x),y(x), singsol=all)
```

$$y = \mathrm{e}^{-2x}x + \frac{11\,\mathrm{e}^{-2x}}{16} + \frac{5\,\mathrm{e}^{-2x}x^2}{8} + \frac{\mathrm{e}^{-2x}x^3}{6} + \frac{c_1\mathrm{e}^{-2x}}{4} - \frac{5\sin(3x)}{39} - \frac{10\cos(3x)}{117} + c_2x + c_3$$

✓ Solution by Mathematica

Time used: 0.974 (sec). Leaf size: 56

```
DSolve[y'''[x]+2*y''[x]==(2*x^2+x)*Exp[-2*x]+5*Cos[3*x],y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{1}{48}e^{-2x}\left(8x^3 + 30x^2 + 48x + 33 + 12c_1\right) - \frac{5}{39}\sin(3x) - \frac{10}{117}\cos(3x) + c_3x + c_2$$

11.25 problem 25

Internal problem ID [2164]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = 8\sin(x)^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+4*y(x)=8*sin(x)^2,y(x), singsol=all)
```

$$y = c_2 \sin(2x) + \cos(2x) c_1 - \sin(2x) x + 1 - \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 27

```
DSolve[y''[x]+4*y[x]==8*Sin[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (-1 + c_1)\cos(2x) + (-x + c_2)\sin(2x) + 1$$

11.26 problem 26

Internal problem ID [2165]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 26.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' + 4y = 5\,e^{2x}\sin(3x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 53

```
dsolve(diff(y(x),x$4)+4*y(x)=5*exp(2*x)*sin(3*x),y(x), singsol=all)
```

$$y = \frac{24\,e^{2x}\cos(3x)}{1105} - \frac{23\,e^{2x}\sin(3x)}{1105} + c_1 e^{x}\cos(x) + c_2 e^{x}\sin(x) + c_3 e^{-x}\cos(x) + c_4 e^{-x}\sin(x)$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 64

```
DSolve[y''''[x]+4*y[x]==5*Exp[2*x]*Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{2x}(24\cos(3x) - 23\sin(3x))}{1105} + c_1 e^{-x}\cos(x) + c_4 e^{x}\cos(x) + c_2 e^{-x}\sin(x) + c_3 e^{x}\sin(x)$$

11.27 problem 27

Internal problem ID [2166]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 5y' - 6y = \mathrm{e}^{3x}}$$

With initial conditions

$$[y(0) = 2, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$2)-5*diff(y(x),x)-6*y(x)=exp(3*x),y(0) = 2, D(y)(0) = 1],y(x), singsol=a
```

$$y = \frac{45\,\mathrm{e}^{-x}}{28} + \frac{10\,\mathrm{e}^{6x}}{21} - \frac{\mathrm{e}^{3x}}{12}$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 30

```
DSolve[{y''[x]-5*y'[x]-6*y[x]==Exp[3*x],{y[0]==2,y'[0]==1}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{84}e^{-x}\left(-7e^{4x} + 40e^{7x} + 135\right)$$

11.28 problem 28

Internal problem ID [2167]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = 12\cos(x)^2}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = 0, y'\left(\frac{\pi}{2}\right) = \frac{\pi}{2}\right]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve([diff(y(x),x$2)+4*y(x)=12*cos(x)^2,y(1/2*Pi) = 0, D(y)(1/2*Pi) = 1/2*Pi],y(x), singso
```

$$y = \frac{(3x - 2\pi)\sin(2x)}{2} + \frac{3\cos(2x)}{2} + \frac{3}{2}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 23

```
DSolve[{y''[x]+4*y[x]==12*Cos[x]^2,{y[Pi/2]==0,y'[Pi/2]==Pi/2}},y[x],x,IncludeSingularSoluti
```

$$y(x) \to \cos(x)((3x - 2\pi)\sin(x) + 3\cos(x))$$

11.29 problem 29

Internal problem ID [2168]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 3y' + 2y = x\,\mathrm{e}^{-x}}$$

With initial conditions

$$\left[y(0) = \frac{1}{9}, y'(0) = 0\right]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=x*exp(-x),y(0) = 1/9, D(y)(0) = 0],y(x), singsol
```

$$y = \frac{(6x+5)\,\mathrm{e}^{-x}}{36} - \frac{\mathrm{e}^{x}}{36}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 24

```
DSolve[{y''[x]-3*y'[x]+2*y[x]==x*Exp[-x],{y[0]==1/9,y'[0]==0}},y[x],x,IncludeSingularSolutio
```

$$y(x) \to -\frac{1}{36}e^{-x}\left(-6x + e^{2x} - 5\right)$$

11.30 problem 30

Internal problem ID [2169]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = e^{x} \sin(x)}$$

With initial conditions

$$[y(0) = 3, y'(0) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$2)+y(x)=exp(x)*sin(x),y(0) = 3, D(y)(0) = 2],y(x), singsol=all)
```

$$y = \frac{(-2\,e^{x} + 17)\cos(x)}{5} + \frac{\sin(x)\,(e^{x} + 11)}{5}$$

✓ Solution by Mathematica

Time used: 0.068 (sec). Leaf size: 28

```
DSolve[{y''[x]+y[x]==Exp[x]*Sin[x],{y[0]==3,y'[0]==2}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{1}{5}((e^{x} + 11)\sin(x) + (17 - 2e^{x})\cos(x))$$

11.31 problem 31

Internal problem ID [2170]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{2y'' + y' = 8\sin(2x) + e^{-x}}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve([2*diff(y(x),x$2)+diff(y(x),x)=8*sin(2*x)+exp(-x),y(0) = 1, D(y)(0) = 0],y(x), singso
```

$$y = -\frac{98\,e^{-\frac{x}{2}}}{17} - \frac{16\sin(2x)}{17} + e^{-x} - \frac{4\cos(2x)}{17} + 6$$

✓ Solution by Mathematica

Time used: 0.384 (sec). Leaf size: 39

```
DSolve[{2*y''[x]+y'[x]==8*Sin[2*x]+Exp[-x],{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutio
```

$$y(x) \to e^{-x} - \frac{98e^{-x/2}}{17} - \frac{16}{17}\sin(2x) - \frac{4}{17}\cos(2x) + 6$$

11.32 problem 32

Internal problem ID [2171]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = 3x \sin(x)}$$

With initial conditions

$$[y(0) = 2, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$2)+y(x)=3*x*sin(x),y(0) = 2, D(y)(0) = 1],y(x), singsol=all)
```

$$y = \sin(x) + 2\cos(x) - \frac{3\cos(x)\, x^2}{4} + \frac{3x \sin(x)}{4}$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 28

```
DSolve[{y''[x]+y[x]==3*x*Sin[x],{y[0]==2,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(2 - \frac{3x^2}{4}\right)\cos(x) + \left(\frac{3x}{4} + 1\right)\sin(x)$$

11.33 problem 33

Internal problem ID [2172]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{2y'' + 5y' - 3y = \sin(x) - 8x}$$

With initial conditions

$$\left[y(0) = \frac{1}{2}, y'(0) = \frac{1}{2}\right]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve([2*diff(y(x),x$2)+5*diff(y(x),x)-3*y(x)=sin(x)-8*x,y(0) = 1/2, D(y)(0) = 1/2],y(x), s
```

$$y = \frac{8\,\mathrm{e}^{-3x}\left(-\frac{51\,\mathrm{e}^{\frac{7x}{2}}}{35} + \frac{13}{840} + \left(x - \frac{3\cos(x)}{80} - \frac{3\sin(x)}{80} + \frac{5}{3}\right)\mathrm{e}^{3x}\right)}{3}$$

✓ Solution by Mathematica

Time used: 0.208 (sec). Leaf size: 38

```
DSolve[{2*y''[x]+5*y'[x]-3*y[x]==Sin[x]-8*x,{y[0]==1/2,y'[0]==1/2}},y[x],x,IncludeSingularSo
```

$$y(x) \to \frac{1}{630}\left(1680x + 26e^{-3x} - 2448e^{x/2} - 63\sin(x) - 63\cos(x) + 2800\right)$$

11.34 problem 34

Internal problem ID [2173]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 19, page 86
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{8y'' - y = x\,\mathrm{e}^{-\frac{x}{2}}}$$

With initial conditions

$$[y(0) = 3, y'(0) = 5]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 43

```
dsolve([8*diff(y(x),x$2)-y(x)=x*exp(-x/2),y(0) = 3, D(y)(0) = 5],y(x), singsol=all)
```

$$y = \frac{\left(-5-16\sqrt{2}\right)\mathrm{e}^{-\frac{\sqrt{2}\,x}{4}}}{2} + \frac{\left(-5+16\sqrt{2}\right)\mathrm{e}^{\frac{\sqrt{2}\,x}{4}}}{2} + (x+8)\,\mathrm{e}^{-\frac{x}{2}}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 83

```
DSolve[{8*y''[x]-y[x]==x*Exp[-x/2],{y[0]==3,y'[0]==5}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{1}{2}e^{-\frac{1}{4}\left(2+\sqrt{2}\right)x}\left(2e^{\frac{x}{2\sqrt{2}}}(x+8) - \left(5+16\sqrt{2}\right)e^{x/2} + \left(16\sqrt{2}-5\right)e^{\frac{1}{2}\left(1+\sqrt{2}\right)x}\right)$$

12 Exercise 20, page 90

12.1 problem 1

Internal problem ID [2174]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 20, page 90
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sec(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+y(x)=sec(x),y(x), singsol=all)
```

$$y = \sin(x) c_2 + \cos(x) c_1 + x \sin(x) - \ln(\sec(x)) \cos(x)$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 22

```
DSolve[y''[x]+y[x]==Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_2) \sin(x) + \cos(x)(\log(\cos(x)) + c_1)$$

12.2 problem 2

Internal problem ID [2175]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 20, page 90
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y' + 4y = e^{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+4*y(x)=exp(x),y(x), singsol=all)
```

$$y = e^{-2x} c_2 + e^{-2x} x c_1 + \frac{e^{x}}{9}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 26

```
DSolve[y''[x]+4*y'[x]+4*y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{x}}{9} + e^{-2x}(c_2 x + c_1)$$

12.3 problem 3

Internal problem ID [2176]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 20, page 90
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y = x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+4*y(x)=x^2,y(x), singsol=all)
```

$$y = c_2 \sin(2x) + \cos(2x) c_1 + \frac{x^2}{4} - \frac{1}{8}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 30

```
DSolve[y''[x]+4*y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{4} + c_1 \cos(2x) + c_2 \sin(2x) - \frac{1}{8}$$

12.4 problem 4

Internal problem ID [2177]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 20, page 90
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y' + y = e^{2x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=exp(2*x),y(x), singsol=all)
```

$$y = c_2 e^x + x\, e^x c_1 + e^{2x}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 19

```
DSolve[y''[x]-2*y'[x]+y[x]==Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x(e^x + c_2 x + c_1)$$

12.5 problem 5

Internal problem ID [2178]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 20, page 90
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = 4\sin(2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)+y(x)=4*sin(2*x),y(x), singsol=all)
```

$$y = \sin(x)\, c_2 + \cos(x)\, c_1 - \frac{4\sin(2x)}{3}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 24

```
DSolve[y''[x]+y[x]==4*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{4}{3}\sin(2x) + c_1\cos(x) + c_2\sin(x)$$

12.6 problem 6

Internal problem ID [2179]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 20, page 90
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = 2x - 2\sin(2x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+4*y(x)=2*(x-sin(2*x)),y(x), singsol=all)
```

$$y = c_2 \sin(2x) + \cos(2x) c_1 + \frac{x(1 + \cos(2x))}{2}$$

✓ Solution by Mathematica

Time used: 0.166 (sec). Leaf size: 30

```
DSolve[y''[x]+4*y[x]==2*(x-Sin[2*x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(x + (x + 2c_1)\cos(2x) + 2c_2 \sin(2x))$$

12.7 problem 7

Internal problem ID [2180]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 20, page 90
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y = 3x + 5\,\mathrm{e}^{x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-y(x)=3*x+5*exp(x),y(x), singsol=all)
```

$$y = c_2\mathrm{e}^{x} + c_1\mathrm{e}^{-x} + \frac{5(2x-1)\,\mathrm{e}^{x}}{4} - 3x$$

✓ Solution by Mathematica

Time used: 0.095 (sec). Leaf size: 32

```
DSolve[y''[x]-y[x]==3*x+5*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -3x + e^{x}\left(\frac{5x}{2} - \frac{5}{4} + c_1\right) + c_2e^{-x}$$

12.8 problem 8

Internal problem ID [2181]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 20, page 90
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = e^{x} + \sin(4x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+9*y(x)=exp(x)+sin(4*x),y(x), singsol=all)
```

$$y = \sin(3x)\, c_2 + \cos(3x)\, c_1 + \frac{e^{x}}{10} - \frac{\sin(4x)}{7}$$

✓ Solution by Mathematica

Time used: 0.194 (sec). Leaf size: 35

```
DSolve[y''[x]+9*y[x]==Exp[x]+Sin[4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{x}}{10} - \frac{1}{7}\sin(4x) + c_1\cos(3x) + c_2\sin(3x)$$

12.9 problem 9

Internal problem ID [2182]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 20, page 90
Problem number: 9.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + 3y'' - 4y' = \cos(2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$3)+3*diff(y(x),x$2)-4*diff(y(x),x)=cos(2*x),y(x), singsol=all)
```

$$y = -\frac{2\sin(x)\cos(x)}{25} - \frac{3\cos(x)^2}{50} + c_1 e^x - \frac{e^{-4x} c_2}{4} + c_3$$

✓ Solution by Mathematica

Time used: 0.306 (sec). Leaf size: 41

```
DSolve[y'''[x]+3*y''[x]-4*y'[x]==Cos[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{25}\sin(2x) - \frac{3}{100}\cos(2x) - \frac{1}{4}c_1 e^{-4x} + c_2 e^x + c_3$$

12.10 problem 10

Internal problem ID [2183]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 20, page 90
Problem number: 10.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + 4y'' - 5y' = e^{3x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$3)+4*diff(y(x),x$2)-5*diff(y(x),x)=exp(3*x),y(x), singsol=all)
```

$$y = \frac{e^{3x}}{48} - \frac{e^{-5x}c_1}{5} + c_2e^x + c_3$$

✓ Solution by Mathematica

Time used: 0.061 (sec). Leaf size: 34

```
DSolve[y'''[x]+4*y''[x]-5*y'[x]==Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{3x}}{48} - \frac{1}{5}c_1e^{-5x} + c_2e^x + c_3$$

12.11 problem 11

Internal problem ID [2184]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 20, page 90
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \tan(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+y(x)=tan(x),y(x), singsol=all)
```

$$y = \sin(x) c_2 + \cos(x) c_1 - \cos(x) \ln(\sec(x) + \tan(x))$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 23

```
DSolve[y''[x]+y[x]==Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x)(-\text{arctanh}(\sin(x))) + c_1 \cos(x) + c_2 \sin(x)$$

12.12 problem 12

Internal problem ID [2185]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 20, page 90
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + a^2 y = \sec(ax)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$2)+a^2*y(x)=sec(a*x),y(x), singsol=all)
```

$$y = \sin(ax) c_2 + \cos(ax) c_1 + \frac{x \sin(ax) a - \ln(\sec(ax)) \cos(ax)}{a^2}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 39

```
DSolve[y''[x]+a^2*y[x]==Sec[a*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\cos(ax)\left(\log(\cos(ax)) + a^2 c_1\right) + a(x + a c_2)\sin(ax)}{a^2}$$

12.13 problem 13

Internal problem ID [2186]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 20, page 90
Problem number: 13.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - 2y'' + y' = e^{2x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$3)-2*diff(y(x),x$2)+diff(y(x),x)=exp(2*x),y(x), singsol=all)
```

$$y = c_2 e^x + c_1(x e^x - e^x) + \frac{e^{2x}}{2} + c_3$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 30

```
DSolve[y'''[x]-2*y''[x]+y'[x]==Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{2x}}{2} + e^x(c_2(x-1) + c_1) + c_3$$

12.14 problem 14

Internal problem ID [2187]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 20, page 90
Problem number: 14.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y'''' - 2y''' + y'' = x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$4)-2*diff(y(x),x$3)+diff(y(x),x$2)=x^2,y(x), singsol=all)
```

$$y = \frac{x^4}{12} + 3x^2 + \frac{2x^3}{3} + c_1(x\,\mathrm{e}^x - 2\,\mathrm{e}^x) + c_2\mathrm{e}^x + c_3x + c_4$$

✓ Solution by Mathematica

Time used: 0.092 (sec). Leaf size: 46

```
DSolve[y''''[x]-2*y'''[x]+y''[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^4}{12} + \frac{2x^3}{3} + 3x^2 + c_4x + c_1e^x + c_2e^x(x-2) + c_3$$

12.15 problem 15

Internal problem ID [2188]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 20, page 90
Problem number: 15.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - 3y'' - 4y' = e^{2x} + \sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)-4*diff(y(x),x)=exp(2*x)+sin(x),y(x), singsol=all)
```

$$y = -c_1 e^{-x} + \frac{e^{4x} c_2}{4} + \frac{3 \sin(x)}{34} - \frac{e^{2x}}{12} + \frac{5 \cos(x)}{34} + c_3$$

✓ Solution by Mathematica

Time used: 0.267 (sec). Leaf size: 49

```
DSolve[y'''[x]-3*y''[x]-4*y'[x]==Exp[2*x]+Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{2x}}{12} + \frac{3\sin(x)}{34} + \frac{5\cos(x)}{34} + c_1 \left(-e^{-x}\right) + \frac{1}{4} c_2 e^{4x} + c_3$$

12.16 problem 16

Internal problem ID [2189]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 20, page 90
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + y = \frac{e^x}{(1-x)^2}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=exp(x)/(1-x)^2,y(x), singsol=all)
```

$$y = c_2 e^x + x e^x c_1 + e^x(-1 - \ln(x-1))$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 23

```
DSolve[y''[x]-2*y'[x]+y[x]==Exp[x]/(1-x)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x(-\log(x-1) + c_2 x - 1 + c_1)$$

12.17 problem 17

Internal problem ID [2190]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 20, page 90
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 3y' + 2y = \sin\left(e^{-x}\right)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=sin(exp(-x)),y(x), singsol=all)
```

$$y = \left(c_1 e^x - e^x \sin\left(e^{-x}\right) + c_2\right) e^x$$

✓ Solution by Mathematica

Time used: 0.086 (sec). Leaf size: 29

```
DSolve[y''[x]-3*y'[x]+2*y[x]==Sin[Exp[-x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x \left(-e^x \sin\left(e^{-x}\right) + c_2 e^x + c_1\right)$$

12.18 problem 18

Internal problem ID [2191]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 20, page 90
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \sec(x)\tan(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)+4*y(x)=sec(x)*tan(x),y(x), singsol=all)
```

$$y = c_2 \sin(2x) + \cos(2x) c_1 + \left(1 - 2\cos(x)^2\right) \ln(\sec(x) + \tan(x)) - 2\sin(x)$$

✓ Solution by Mathematica

Time used: 0.081 (sec). Leaf size: 33

```
DSolve[y''[x]+4*y[x]==Sec[x]*Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(2x)(-\operatorname{arctanh}(\sin(x))) + c_1\cos(2x) + 2\sin(x)(-1 + c_2\cos(x))$$

12.19 problem 19

Internal problem ID [2192]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 20, page 90
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y = \mathrm{e}^{-x} \sin(2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
dsolve(diff(y(x),x$2)-2*y(x)=exp(-x)*sin(2*x),y(x), singsol=all)
```

$$y = \mathrm{e}^{\sqrt{2}\,x} c_2 + \mathrm{e}^{-\sqrt{2}\,x} c_1 + \frac{(4\cos(2x) - 5\sin(2x))\,\mathrm{e}^{-x}}{41}$$

✓ Solution by Mathematica

Time used: 0.235 (sec). Leaf size: 57

```
DSolve[y''[x]-2*y[x]==Exp[-x]*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{5}{41} e^{-x} \sin(2x) + \frac{4}{41} e^{-x} \cos(2x) + c_1 e^{\sqrt{2}x} + c_2 e^{-\sqrt{2}x}$$

12.20 problem 20

Internal problem ID [2193]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 20, page 90
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = \sec(x)\csc(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(diff(y(x),x$2)+9*y(x)=sec(x)*csc(x),y(x), singsol=all)
```

$$y = \sin(3x)\,c_2 + \cos(3x)\,c_1 + \frac{\sin(x)\left(4\cos(x)^2 - 1\right)\ln(\csc(x) - \cot(x))}{3} + \frac{4\left(\left(\cos(x)^2 - \frac{3}{4}\right)\ln(\sec(x) + \tan(x)) + 2\sin(x)\right)\cos(x)}{3}$$

✓ Solution by Mathematica

Time used: 0.076 (sec). Leaf size: 65

```
DSolve[y''[x]+9*y[x]==Sec[x]*Csc[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(\cos(3x)\text{arctanh}(\sin(x)) + 4\sin(2x) + \sin(3x)\log\left(\sin\left(\frac{x}{2}\right)\right) + 3c_1\cos(3x) + 3c_2\sin(3x) - \sin(3x)\log\left(\cos\left(\frac{x}{2}\right)\right)\right)$$

12.21 problem 21

Internal problem ID [2194]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 20, page 90
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = \csc(2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(diff(y(x),x$2)+9*y(x)=csc(2*x),y(x), singsol=all)
```

$$y = \sin(3x)\, c_2 + \cos(3x)\, c_1 + \frac{\sin(x)\left(4\cos(x)^2 - 1\right)\ln(\csc(x) - \cot(x))}{6} + \frac{2\left(\left(\cos(x)^2 - \frac{3}{4}\right)\ln(\sec(x) + \tan(x)) + 2\sin(x)\right)\cos(x)}{3}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 65

```
DSolve[y''[x]+9*y[x]==Csc[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}\left(\cos(3x)\text{arctanh}(\sin(x)) + 4\sin(2x) + \sin(3x)\log\left(\sin\left(\frac{x}{2}\right)\right) + 6c_1\cos(3x) + 6c_2\sin(3x) - \sin(3x)\log\left(\cos\left(\frac{x}{2}\right)\right)\right)$$

12.22 problem 22

Internal problem ID [2195]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 20, page 90
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \tan\left(\frac{x}{3}\right)^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 48

```
dsolve(diff(y(x),x$2)+y(x)=tan(x/3)^2,y(x), singsol=all)
```

$$y = \sin(x) c_2 + \cos(x) c_1 - 22 + 9\left(1 - 4\cos\left(\frac{x}{3}\right)^2\right)\sin\left(\frac{x}{3}\right)\ln\left(\sec\left(\frac{x}{3}\right) + \tan\left(\frac{x}{3}\right)\right) + 36\cos\left(\frac{x}{3}\right)^2$$

✓ Solution by Mathematica

Time used: 0.283 (sec). Leaf size: 36

```
DSolve[y''[x]+y[x]==Tan[x/3]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -9\sin(x)\operatorname{arctanh}\left(\sin\left(\frac{x}{3}\right)\right) + 18\cos\left(\frac{2x}{3}\right) + c_1\cos(x) + c_2\sin(x) - 4$$

12.23 problem 23

Internal problem ID [2196]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 20, page 90
Problem number: 23.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + y' = \tan(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 117

```
dsolve(diff(y(x),x$3)+diff(y(x),x)=tan(x),y(x), singsol=all)
```

$$y = c_1 \sin(x) - c_2 \cos(x) - \ln\left(e^{ix} - i\right) - \ln\left(i + e^{ix}\right) + \frac{i e^{ix} \ln\left(\frac{-i e^{2ix} + i + 2\,e^{ix}}{e^{2ix} + 1}\right)}{2} + \ln\left(e^{ix}\right) - \frac{i \ln\left(\frac{-i e^{2ix} + i + 2\,e^{ix}}{e^{2ix} + 1}\right) e^{-ix}}{2} + c_3$$

✓ Solution by Mathematica

Time used: 0.101 (sec). Leaf size: 35

```
DSolve[y'''[x]+y'[x]==Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sin(x)\text{arctanh}(\sin(x)) - \frac{1}{2}\log\left(\cos^2(x)\right) - c_2 \cos(x) + c_1 \sin(x) + c_3$$

12.24 problem 24

Internal problem ID [2197]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 20, page 90
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{4y'' - 4y' + y = e^{\frac{x}{2}} \ln(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(4*diff(y(x),x$2)-4*diff(y(x),x)+y(x)=exp(x/2)*ln(x),y(x), singsol=all)
```

$$y = c_2 e^{\frac{x}{2}} + e^{\frac{x}{2}} x c_1 + \frac{e^{\frac{x}{2}} x^2 (2\ln(x) - 3)}{16}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 38

```
DSolve[4*y''[x]-4*y'[x]+y[x]==Exp[x/2]*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{16} e^{x/2} \left(-3x^2 + 2x^2 \log(x) + 16c_2 x + 16c_1\right)$$

12.25 problem 25

Internal problem ID [2198]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 20, page 90
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + P(x)\, y = Q(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)+P(x)*y(x)=Q(x),y(x), singsol=all)
```

$$y = \left(\int Q(x)\, \mathrm{e}^{\int P(x)dx}dx + c_1\right) \mathrm{e}^{\int -P(x)dx}$$

✓ Solution by Mathematica

Time used: 0.105 (sec). Leaf size: 51

```
DSolve[y'[x]+p[x]*y[x]==q[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \exp\left(\int_1^x -p(K[1])dK[1]\right) \left(\int_1^x \exp\left(-\int_1^{K[2]} -p(K[1])dK[1]\right) q(K[2])dK[2] + c_1\right)$$

13 Exercise 22, page 99

13.1 problem 1

Internal problem ID [2199]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 22, page 99
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 6y' + 9y = \mathrm{e}^{3x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+9*y(x)=exp(3*x),y(x), singsol=all)
```

$$y = \mathrm{e}^{3x}c_2 + \mathrm{e}^{3x}xc_1 + \frac{x^2\mathrm{e}^{3x}}{2}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 27

```
DSolve[y''[x]-6*y'[x]+9*y[x]==Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{3x}\left(x^2 + 2c_2x + 2c_1\right)$$

13.2 problem 2

Internal problem ID [2200]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 22, page 99
Problem number: 2.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - 3y'' + 3y' - y = \mathrm{e}^{x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)+3*diff(y(x),x)-y(x)=exp(x),y(x), singsol=all)
```

$$y = \frac{x^3 \mathrm{e}^{x}}{6} + c_1 \mathrm{e}^{x} + c_2 x\, \mathrm{e}^{x} + c_3 \mathrm{e}^{x} x^2$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 32

```
DSolve[y'''[x]-3*y''[x]+3*y'[x]-y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6} e^{x} \left(x^3 + 6c_3 x^2 + 6c_2 x + 6c_1\right)$$

13.3 problem 3

Internal problem ID [2201]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 22, page 99
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 5y' + 6y = x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=x^2,y(x), singsol=all)
```

$$y = c_2 e^{2x} + c_1 e^{3x} + \frac{x^2}{6} + \frac{5x}{18} + \frac{19}{108}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 37

```
DSolve[y''[x]-5*y'[x]+6*y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{6} + \frac{5x}{18} + c_1 e^{2x} + c_2 e^{3x} + \frac{19}{108}$$

14 Exercise 23, page 106

14.1 problem 1

Internal problem ID [2202]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y = 2\,\mathrm{e}^{x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+4*y(x)=2*exp(x),y(x), singsol=all)
```

$$y = c_2 \sin(2x) + \cos(2x)\, c_1 + \frac{2\,\mathrm{e}^{x}}{5}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 27

```
DSolve[y''[x]+4*y[x]==2*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2e^x}{5} + c_1 \cos(2x) + c_2 \sin(2x)$$

14.2 problem 2

Internal problem ID [2203]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 3y = 3\,\mathrm{e}^{-4x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+3*y(x)=3*exp(-4*x),y(x), singsol=all)
```

$$y = \sin\left(\sqrt{3}\,x\right) c_2 + \cos\left(\sqrt{3}\,x\right) c_1 + \frac{3\,\mathrm{e}^{-4x}}{19}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 37

```
DSolve[y''[x]+3*y[x]==3*Exp[-4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3e^{-4x}}{19} + c_1 \cos\left(\sqrt{3}x\right) + c_2 \sin\left(\sqrt{3}x\right)$$

14.3 problem 3

Internal problem ID [2204]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 4y = \frac{e^{x}}{2} + \frac{e^{-x}}{2}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+4*y(x)=1/2*(exp(x)+exp(-x)),y(x), singsol=all)
```

$$y = e^{-2x}c_2 + e^{-2x}xc_1 + \frac{e^{-x}}{2} + \frac{e^{x}}{18}$$

✓ Solution by Mathematica

Time used: 0.097 (sec). Leaf size: 34

```
DSolve[y''[x]+4*y'[x]+4*y[x]==1/2*(Exp[x]+Exp[-x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{18}e^{-2x}\left(9e^{x} + e^{3x} + 18(c_2x + c_1)\right)$$

14.4 problem 4

Internal problem ID [2205]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' - 2y = e^{-2x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-2*y(x)=exp(-2*x),y(x), singsol=all)
```

$$y = c_2 e^{x} + c_1 e^{-2x} - \frac{e^{-2x} x}{3}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 32

```
DSolve[y''[x]+y'[x]-2*y[x]==Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{9} e^{-2x} \left(-3x + 9c_2 e^{3x} - 1 + 9c_1\right)$$

14.5 problem 5

Internal problem ID [2206]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y = \sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+2*y(x)=sin(x),y(x), singsol=all)
```

$$y = \sin\left(\sqrt{2}\,x\right) c_2 + \cos\left(\sqrt{2}\,x\right) c_1 + \sin(x)$$

✓ Solution by Mathematica

Time used: 0.185 (sec). Leaf size: 30

```
DSolve[y''[x]+2*y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x) + c_1 \cos\left(\sqrt{2}x\right) + c_2 \sin\left(\sqrt{2}x\right)$$

14.6 problem 6

Internal problem ID [2207]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 4y = \frac{e^{3x}}{2} - \frac{e^{-3x}}{2}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+4*y(x)=1/2*(exp(3*x)-exp(-3*x)),y(x), singsol=all)
```

$$y = e^{-2x}c_2 + e^{-2x}xc_1 + \frac{e^{3x}}{50} - \frac{e^{-3x}}{2}$$

✓ Solution by Mathematica

Time used: 0.217 (sec). Leaf size: 33

```
DSolve[y''[x]+4*y'[x]+4*y[x]==1/2*(Exp[3*x]-Exp[-3*x]),y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{1}{50}e^{-3x}\left(e^{6x} + 50e^{x}(c_2x + c_1) - 25\right)$$

14.7 problem 7

Internal problem ID [2208]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 3y' - 2y = \sin(2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)-2*y(x)=sin(2*x),y(x), singsol=all)
```

$$y = \mathrm{e}^{\frac{\left(-3+\sqrt{17}\right)x}{2}} c_2 + \mathrm{e}^{-\frac{\left(3+\sqrt{17}\right)x}{2}} c_1 - \frac{\cos(2x)}{12} - \frac{\sin(2x)}{12}$$

✓ Solution by Mathematica

Time used: 0.346 (sec). Leaf size: 52

```
DSolve[y''[x]+3*y'[x]-2*y[x]==Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{12}\cos(2x) + e^{-\frac{1}{2}\left(3+\sqrt{17}\right)x}\left(c_2 e^{\sqrt{17}x} + c_1\right) - \frac{1}{6}\sin(x)\cos(x)$$

14.8 problem 8

Internal problem ID [2209]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 3y' + 2y = e^{x} \sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=exp(x)*sin(x),y(x), singsol=all)
```

$$y = -\frac{e^{x} \cos(x)}{10} + \frac{e^{x} \sin(x)}{10} - c_1 e^{-2x} + e^{-x} c_2$$

✓ Solution by Mathematica

Time used: 0.061 (sec). Leaf size: 36

```
DSolve[y''[x]+3*y'[x]+2*y[x]==Exp[x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-2x} + c_2 e^{-x} + \frac{1}{10} e^{x} (\sin(x) - \cos(x))$$

14.9 problem 9

Internal problem ID [2210]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 9.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - y = \mathrm{e}^{x}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$3)-y(x)=exp(x),y(x), singsol=all)
```

$$y = \frac{x\,\mathrm{e}^{x}}{3} + c_1\mathrm{e}^{x} + c_2\mathrm{e}^{-\frac{x}{2}}\cos\left(\frac{\sqrt{3}\,x}{2}\right) + c_3\mathrm{e}^{-\frac{x}{2}}\sin\left(\frac{\sqrt{3}\,x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.408 (sec). Leaf size: 62

```
DSolve[y'''[x]-y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}e^{-x/2}\left(e^{3x/2}(x-1+3c_1) + 3c_2\cos\left(\frac{\sqrt{3}x}{2}\right) + 3c_3\sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

14.10 problem 10

Internal problem ID [2211]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 10.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 4y'' + y' - 4y = \sin(x) - e^{4x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 68

```
dsolve(diff(y(x),x$3)-4*diff(y(x),x$2)+diff(y(x),x)-4*y(x)=sin(x)-exp(4*x),y(x), singsol=all
```

$$y = \frac{e^{-4x} e^{4x} (68x + 15) \cos(x)}{578} - \frac{e^{-4x} e^{4x} (17x + 8) \sin(x)}{578} - \frac{e^{-4x} e^{8x} (17x - 8)}{289} + \cos(x) c_1 + \sin(x) c_2 + c_3 e^{4x}$$

✓ Solution by Mathematica

Time used: 0.149 (sec). Leaf size: 52

```
DSolve[y'''[x]-4*y''[x]+y'[x]-4*y[x]==Sin[x]-Exp[4*x],y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{289} e^{4x} (-17x + 8 + 289c_3) + \left(\frac{2x}{17} + \frac{13}{1156} + c_1\right) \cos(x) + \left(-\frac{x}{34} - \frac{21}{289} + c_2\right) \sin(x)$$

14.11 problem 11

Internal problem ID [2212]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 11.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' + 3y'' - 4y = 4\,e^{x} + 3\cos(2x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 49

```
dsolve(diff(y(x),x$4)+3*diff(y(x),x$2)-4*y(x)=4*exp(x)+3*cos(2*x),y(x), singsol=all)
```

$$y = \frac{2x\,e^{x}}{5} - \frac{9\,e^{x}}{25} - \frac{3\cos(2x)}{25} - \frac{3\sin(2x)\,x}{20} + c_1 e^{x} + c_2\cos(2x) + c_3 e^{-x} + c_4\sin(2x)$$

✓ Solution by Mathematica

Time used: 0.17 (sec). Leaf size: 55

```
DSolve[y''''[x]+3*y''[x]-4*y[x]==4*Exp[x]+3*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2e^{x}x}{5} - \frac{9e^{x}}{25} - \frac{\cos(x)}{2} + c_3 e^{-x} + c_4 e^{x} + c_1\cos(2x) + c_2\sin(2x)$$

14.12 problem 12

Internal problem ID [2213]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = e^{3x}(1 + \sin(2x))}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)+y(x)=exp(3*x)*(1+sin(2*x)),y(x), singsol=all)
```

$$y = \sin(x) c_2 + \cos(x) c_1 - \frac{e^{3x}(-\sin(2x) - 3 + 2\cos(2x))}{30}$$

✓ Solution by Mathematica

Time used: 0.288 (sec). Leaf size: 50

```
DSolve[y''[x]+y[x]==Exp[3*x]*(1+Sin[2*x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{30}\left(3e^{3x} + e^{3x}\sin(2x) - 2e^{3x}\cos(2x) + 30c_1\cos(x) + 30c_2\sin(x)\right)$$

14.13 problem 13

Internal problem ID [2214]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2n^2y' + n^4y = \sin(kx)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(diff(y(x),x$2)+2*n^2*diff(y(x),x)+n^4*y(x)=sin(k*x),y(x), singsol=all)
```

$$y = \mathrm{e}^{-n^2x}c_2 + \mathrm{e}^{-n^2x}xc_1 + \frac{(n^4 - k^2)\sin(kx) - 2\cos(kx)\,k\,n^2}{(n^4 + k^2)^2}$$

✓ Solution by Mathematica

Time used: 0.198 (sec). Leaf size: 64

```
DSolve[y''[x]+2*n^2*y'[x]+n^4*y[x]==Sin[k*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(n^4 - k^2)\sin(kx)}{(k^2 + n^4)^2} - \frac{2kn^2\cos(kx)}{(k^2 + n^4)^2} + (c_2x + c_1)e^{-n^2x}$$

14.14 problem 14

Internal problem ID [2215]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''+4y'+5y=\frac{e^{x}}{2}+\frac{e^{-x}}{2}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+5*y(x)=1/2*(exp(x)+exp(-x)),y(x), singsol=all)
```

$$y=e^{-2x}\sin(x)\,c_2+e^{-2x}\cos(x)\,c_1+\frac{e^{x}}{20}+\frac{e^{-x}}{4}$$

✓ Solution by Mathematica

Time used: 0.203 (sec). Leaf size: 37

```
DSolve[y''[x]+4*y'[x]+5*y[x]==1/2*(Exp[x]+Exp[-x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{20}e^{-2x}\left(5e^{x}+e^{3x}+20c_2\cos(x)+20c_1\sin(x)\right)$$

14.15 problem 15

Internal problem ID [2216]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' - 2y = x\,\mathrm{e}^{-x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-2*y(x)=x*exp(-x),y(x), singsol=all)
```

$$y = c_2\mathrm{e}^{x} + c_1\mathrm{e}^{-2x} - \frac{(2x-1)\,\mathrm{e}^{-x}}{4}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 34

```
DSolve[y''[x]+y'[x]-2*y[x]==x*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{-x}(1-2x) + c_1e^{-2x} + c_2e^{x}$$

14.16 problem 16

Internal problem ID [2217]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = x\,\mathrm{e}^{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+4*y(x)=x*exp(x),y(x), singsol=all)
```

$$y = c_2 \sin(2x) + \cos(2x)\, c_1 + \frac{(5x-2)\,\mathrm{e}^{x}}{25}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 32

```
DSolve[y''[x]+4*y[x]==x*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{25}e^{x}(5x-2) + c_1 \cos(2x) + c_2 \sin(2x)$$

14.17 problem 17

Internal problem ID [2218]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y = x^2 \mathrm{e}^{-x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)+2*y(x)=x^2*exp(-x),y(x), singsol=all)
```

$$y = \sin\left(\sqrt{2}\, x\right) c_2 + \cos\left(\sqrt{2}\, x\right) c_1 + \frac{(9x^2 + 12x + 2)\, \mathrm{e}^{-x}}{27}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 47

```
DSolve[y''[x]+2*y[x]==x^2*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{27} e^{-x} (9x^2 + 12x + 2) + c_1 \cos\left(\sqrt{2}x\right) + c_2 \sin\left(\sqrt{2}x\right)$$

14.18 problem 18

Internal problem ID [2219]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y' - 2y = x^2 - 8}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-2*y(x)=x^2-8,y(x), singsol=all)
```

$$y = \mathrm{e}^{-x} c_2 + c_1 \mathrm{e}^{2x} - \frac{x^2}{2} + \frac{x}{2} + \frac{13}{4}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 36

```
DSolve[y''[x]-y'[x]-2*y[x]==x^2-8,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-2x^2 + 2x + 13\right) + c_1 e^{-x} + c_2 e^{2x}$$

14.19 problem 19

Internal problem ID [2220]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 19.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - y = x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$3)-y(x)=x^2,y(x), singsol=all)
```

$$y = -x^2 + c_1 e^x + c_2 e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right) + c_3 e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 59

```
DSolve[y'''[x]-y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^2 + c_1 e^x + c_2 e^{-x/2} \cos\left(\frac{\sqrt{3}x}{2}\right) + c_3 e^{-x/2} \sin\left(\frac{\sqrt{3}x}{2}\right)$$

14.20 problem 20

Internal problem ID [2221]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 20.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + 4y'' - 5y' = x^2 e^{-x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$3)+4*diff(y(x),x$2)-5*diff(y(x),x)=x^2*exp(-x),y(x), singsol=all)
```

$$y = \frac{23\,e^{-x}}{64} + c_1 e^x - \frac{e^{-5x} c_2}{5} + \frac{5\,e^{-x} x}{16} + \frac{x^2 e^{-x}}{8} + c_3$$

✓ Solution by Mathematica

Time used: 0.192 (sec). Leaf size: 44

```
DSolve[y'''[x]+4*y''[x]-5*y'[x]==x^2*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{64} e^{-x} \left(8x^2 + 20x + 23\right) - \frac{1}{5} c_1 e^{-5x} + c_2 e^x + c_3$$

14.21 problem 21

Internal problem ID [2222]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 21.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y'''' - 2y''' + y'' = x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$4)-2*diff(y(x),x$3)+diff(y(x),x$2)=x^2,y(x), singsol=all)
```

$$y = \frac{x^4}{12} + 3x^2 + \frac{2x^3}{3} + c_1(x\,e^x - 2\,e^x) + c_2e^x + c_3x + c_4$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 46

```
DSolve[y''''[x]-2*y'''[x]+y''[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^4}{12} + \frac{2x^3}{3} + 3x^2 + c_4x + c_1e^x + c_2e^x(x-2) + c_3$$

14.22 problem 22

Internal problem ID [2223]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 22.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - y' = e^{x}\left(\sin\left(x\right) - x^{2}\right)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 52

```
dsolve(diff(y(x),x$3)-diff(y(x),x)=exp(x)*(sin(x)-x^2),y(x), singsol=all)
```

$$y = c_1 e^{x} - \frac{7x\,e^{x}}{4} + \frac{15\,e^{x}}{8} + \frac{3\,e^{x}x^{2}}{4} - \frac{x^{3}e^{x}}{6} - \frac{e^{x}\cos\left(x\right)}{10} - \frac{3\,e^{x}\sin\left(x\right)}{10} - e^{-x}c_2 + c_3$$

✓ Solution by Mathematica

Time used: 1.016 (sec). Leaf size: 63

```
DSolve[y'''[x]-y'[x]==Exp[x]*(Sin[x]-x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{24}e^{x}\left(-4x^{3} + 18x^{2} - 42x + 45\right) - \frac{3}{10}e^{x}\sin(x) - \frac{1}{10}e^{x}\cos(x) + c_1e^{x} - c_2e^{-x} + c_3$$

14.23 problem 23

Internal problem ID [2224]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 23.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - 4y'' = e^{2x}(x-3)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$3)-4*diff(y(x),x$2)=exp(2*x)*(x-3),y(x), singsol=all)
```

$$y = \frac{e^{4x} c_1}{16} - \frac{e^{2x} x}{8} + \frac{7 e^{2x}}{16} + c_2 x + c_3$$

✓ Solution by Mathematica

Time used: 0.268 (sec). Leaf size: 34

```
DSolve[y'''[x]-4*y''[x]==Exp[2*x]*(x-3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{16} e^{2x} \left(-2x + c_1 e^{2x} + 7\right) + c_3 x + c_2$$

14.24 problem 24

Internal problem ID [2225]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 24.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y'''' - 6y''' + 9y'' = \sin(3x) + x\,e^{x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
dsolve(diff(y(x),x$4)-6*diff(y(x),x$3)+9*diff(y(x),x$2)=sin(3*x)+x*exp(x),y(x), singsol=all)
```

$$y = \frac{c_1\left(e^{3x}x - \frac{2\,e^{3x}}{3}\right)}{9} + \frac{x\,e^{x}}{4} - \frac{e^{x}}{4} + \frac{e^{3x}c_2}{9} - \frac{\cos(3x)}{162} + c_3x + c_4$$

✓ Solution by Mathematica

Time used: 1.069 (sec). Leaf size: 52

```
DSolve[y''''[x]-6*y'''[x]+9*y''[x]==Sin[3*x]+x*Exp[x],y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{4}e^{x}(x-1) - \frac{1}{162}\cos(3x) + \frac{1}{27}e^{3x}(c_2(3x-2)+3c_1) + c_4x + c_3$$

14.25 problem 25

Internal problem ID [2226]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 25.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 6y'' + 11y' - 6y = x^2 e^{2x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$3)-6*diff(y(x),x$2)+11*diff(y(x),x)-6*y(x)=x^2*exp(2*x),y(x), singsol=all
```

$$y = -\frac{x(x^2+6)\,e^{2x}}{3} + c_1 e^x + c_2 e^{2x} + c_3 e^{3x}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 37

```
DSolve[y'''[x]-6*y''[x]+11*y'[x]-6*y[x]==x^2*Exp[2*x],y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^x \left(e^x \left(-\frac{x^3}{3} - 2x + c_2\right) + c_3 e^{2x} + c_1\right)$$

14.26 problem 26

Internal problem ID [2227]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 26.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + 2y' = x^2 + \cos(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$3)+2*diff(y(x),x)=x^2+cos(x),y(x), singsol=all)
```

$$y = \frac{x^3}{6} + \frac{\sqrt{2}\sin\left(\sqrt{2}\,x\right)c_1}{2} - \frac{\sqrt{2}\cos\left(\sqrt{2}\,x\right)c_2}{2} + \sin(x) - \frac{x}{2} + c_3$$

✓ Solution by Mathematica

Time used: 0.417 (sec). Leaf size: 55

```
DSolve[y'''[x]+2*y'[x]==x^2+Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{6} - \frac{x}{2} + \sin(x) - \frac{c_2\cos\left(\sqrt{2}x\right)}{\sqrt{2}} + \frac{c_1\sin\left(\sqrt{2}x\right)}{\sqrt{2}} + c_3$$

14.27 problem 27

Internal problem ID [2228]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 27.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' + 3y'' - y' + 2y = \sin(2x)}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 1673

```
dsolve(diff(y(x),x$4)+3*diff(y(x),x$2)-diff(y(x),x)+2*y(x)=sin(2*x),y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 1.487 (sec). Leaf size: 1124

```
DSolve[y''''[x]+3*y''[x]-y'[x]+2*y[x]==Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x\text{Root}[\#1^4+3\#1^2-\#1+2\&,1]}c_1 + e^{x\text{Root}[\#1^4+3\#1^2-\#1+2\&,2]}c_2$$
$$+ e^{x\text{Root}[\#1^4+3\#1^2-\#1+2\&,3]}c_3 + e^{x\text{Root}[\#1^4+3\#1^2-\#1+2\&,4]}c_4$$
$$- \frac{(\text{Root}[\#1^4+3\#1^2-\#1+2\&,1] - \text{Root}[\#1^4+3\#1^2-\#1+2\&,3])\,(\text{Root}[\#1^4+3\#1^2-\#1+}{}$$
$$+ \frac{(\text{Root}[\#1^4+3\#1^2-\#1+2\&,1] - \text{Root}[\#1^4+3\#1^2-\#1+2\&,2])\,(\text{Root}[\#1^4+3\#1^2-\#1+}{}$$
$$- \frac{(\text{Root}[\#1^4+3\#1^2-\#1+2\&,1] - \text{Root}[\#1^4+3\#1^2-\#1+2\&,2])\,(\text{Root}[\#1^4+3\#1^2-\#1+}{}$$
$$- \frac{e^{(\text{Root}[\#1^4+3\#1^2-\#1+2\&,2]+\text{Root}[\#1^4+3\#1^2-\#1+2\&,3]+\text{Root}[\#1^4+3\#1^2-\#1+2\&,4])x+\text{Root}[\#1^4+3\#1^2-\#1+2\&,1]x}(\text{Roo}}{}$$

14.28 problem 30

Internal problem ID [2229]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 30.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y'''' + 2y'' + y' = x^3 - \frac{\cos(2x)}{2}}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 1694

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$2)+diff(y(x),x)=x^3-1/2*cos(2*x),y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 5.87 (sec). Leaf size: 1293

```
DSolve[y''''[x]+2*y''[x]+y'[x]==x^3-1/2*Cos[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{x\text{Root}[\#1^3+2\#1+1\&,1]}c_1}{\text{Root}\left[\#1^3+2\#1+1\&,1\right]} + c_4$$
$$+ \frac{ix(\text{Root}[\#1^3+2\#1+1\&,1]-\text{Root}[\#1^3+2\#1+1\&,2])\left(4+\text{Root}[\#1^3+2\#1+1\&,3]^2\right)\left(\text{Roo}\right.}{4\sqrt{59}\left(-1+4\text{Root}\left[\#\right.\right.}$$
$$+ \frac{ix(\text{Root}[\#1^3+2\#1+1\&,2]-\text{Root}[\#1^3+2\#1+1\&,3])\left(\text{Root}[\#1^3+2\#1+1\&,3]^3x^3+3\text{Root}\right.}{}$$
$$- \frac{i(\text{Root}[\#1^3+2\#1+1\&,1]-\text{Root}[\#1^3+2\#1+1\&,2])\,\text{Root}[\#1^3+2\#1+1\&,3]^5\sin(2x)}{4\sqrt{59}\left(-1+4\text{Root}\left[\#1^3+2\#1+1\&,3\right]^2\right)}$$
$$- \frac{i(\text{Root}[\#1^3+2\#1+1\&,2]-\text{Root}[\#1^3+2\#1+1\&,3])\,(\text{Root}[\#1^3+2\#1+1\&,2]+\text{Root}[}{4\sqrt{59}\left(-2i+\text{Root}\left[\#1^3+2\#1+1\&,2\right]+\text{Root}\left[\#1^3+2\#1+1\&,3\right]\right)\left(2i+\text{Root}\left[\#1^3+2\#1+1\&\right.\right.}$$
$$- \frac{i\text{Root}[\#1^3+2\#1+1\&,2]^5\,(\text{Root}[\#1^3+2\#1+1\&,1]-\text{Root}[\#1^3+2\#1+1\&,3])\sin(2x)}{4\sqrt{59}\left(1-4\text{Root}\left[\#1^3+2\#1+1\&,2\right]^2\right)}$$
$$- \frac{i\cos(2x)\,(\text{Root}[\#1^3+2\#1+1\&,1]-\text{Root}[\#1^3+2\#1+1\&,2])\,\text{Root}[\#1^3+2\#1+1\&,3]^4}{2\sqrt{59}\left(-1+4\text{Root}\left[\#1^3+2\#1+1\&,3\right]^2\right)}$$
$$+ \frac{i\cos(2x)\,(\text{Root}[\#1^3+2\#1+1\&,2]-\text{Root}[\#1^3+2\#1+1\&,3])}{2\sqrt{59}\left(-2i+\text{Root}\left[\#1^3+2\#1+1\&,2\right]+\text{Root}\left[\#1^3+2\#1+1\&,3\right]\right)\left(2i+\text{Root}\left[\#1^3+2\#1+1\&\right.\right.}$$
$$+ \frac{e^{x\text{Root}[\#1^3+2\#1+1\&,3]}c_3}{\text{Root}\left[\#1^3+2\#1+1\&,3\right]}$$
$$+ \frac{ix\left(4+\text{Root}[\#1^3+2\#1+1\&,2]^2\right)\left(\text{Root}[\#1^3+2\#1+1\&,2]^3x^3+4\text{Root}[\#1^3+2\#1+1\&,2]^2\right.}{4\sqrt{59}\left(1-4\text{Root}\left[\#\right.\right.}$$
$$- \frac{i\cos(2x)\text{Root}[\#1^3+2\#1+1\&,2]^4\,(\text{Root}[\#1^3+2\#1+1\&,1]-\text{Root}[\#1^3+2\#1+1\&,3])}{2\sqrt{59}\left(1-4\text{Root}\left[\#1^3+2\#1+1\&,2\right]^2\right)}$$
$$+ \frac{e^{x\text{Root}[\#1^3+2\#1+1\&,2]}c_2}{\text{Root}\left[\#1^3+2\#1+1\&,2\right]}$$

14.29 problem 31

Internal problem ID [2230]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 31.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + 4y'' + 5y' = \mathrm{e}^{-2x} \cos(x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 70

```
dsolve(diff(y(x),x$3)+4*diff(y(x),x$2)+5*diff(y(x),x)=exp(-2*x)*cos(x),y(x), singsol=all)
```

$$y = c_2\left(-\frac{\mathrm{e}^{-2x}\cos(x)}{5} - \frac{2\,\mathrm{e}^{-2x}\sin(x)}{5}\right) + \frac{\left(-\frac{x}{5} - \frac{4}{25}\right)\mathrm{e}^{-2x}\cos(x)}{2} + \frac{\left(-\frac{2x}{5} - \frac{3}{25}\right)\mathrm{e}^{-2x}\sin(x)}{2} + c_1\left(-\frac{2\,\mathrm{e}^{-2x}\cos(x)}{5} + \frac{\mathrm{e}^{-2x}\sin(x)}{5}\right) + c_3$$

✓ Solution by Mathematica

Time used: 0.337 (sec). Leaf size: 52

```
DSolve[y'''[x]+4*y''[x]+5*y'[x]==Exp[-2*x]*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{50}e^{-2x}(2(-5x + 1 - 10c_1 + 5c_2)\sin(x) - (5x + 14 + 10c_1 + 20c_2)\cos(x)) + c_3$$

14.30 problem 32

Internal problem ID [2231]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 32.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + y'' - 2y' = \mathrm{e}^{-2x} \cos (2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$3)+diff(y(x),x$2)-2*diff(y(x),x)=exp(-2*x)*cos(2*x),y(x), singsol=all)
```

$$y = \frac{\mathrm{e}^{-2x} \sin (2x)}{104} + \frac{5\,\mathrm{e}^{-2x} \cos (2x)}{104} - \frac{\mathrm{e}^{-2x} c_2}{2} + c_1 \mathrm{e}^{x} + c_3$$

✓ Solution by Mathematica

Time used: 0.872 (sec). Leaf size: 42

```
DSolve[y'''[x]+y''[x]-2*y'[x]==Exp[-2*x]*Cos[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{104} e^{-2x} \left(\sin(2x) + 5\cos(2x) - 52\left(c_1 - 2c_2 e^{3x}\right)\right) + c_3$$

14.31 problem 33

Internal problem ID [2232]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 33.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + 2y' = \sin(x)\, x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 46

```
dsolve(diff(y(x),x$3)+2*diff(y(x),x)=x^2*sin(x),y(x), singsol=all)
```

$$y = -\cos(x)\, x^2 + 8\cos(x) - 2x\sin(x) + \frac{\sqrt{2}\,\sin\left(\sqrt{2}\,x\right) c_1}{2} - \frac{\sqrt{2}\,\cos\left(\sqrt{2}\,x\right) c_2}{2} + c_3$$

✓ Solution by Mathematica

Time used: 0.164 (sec). Leaf size: 55

```
DSolve[y'''[x]+2*y'[x]==x^2*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\left(x^2 - 8\right)\cos(x) - 2x\sin(x) - \frac{c_2\cos\left(\sqrt{2}x\right)}{\sqrt{2}} + \frac{c_1\sin\left(\sqrt{2}x\right)}{\sqrt{2}} + c_3$$

14.32 problem 34

Internal problem ID [2233]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 23, page 106
Problem number: 34.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' - y = \cos(x)\, x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x$4)-y(x)=x^2*cos(x),y(x), singsol=all)
```

$$y = \left(-\frac{3x^2}{8} + \frac{1}{4}\right)\cos(x) + \left(\frac{5}{8}x - \frac{1}{12}x^3\right)\sin(x) + \cos(x)\, c_1 + c_2 e^x + c_3 \sin(x) + c_4 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.092 (sec). Leaf size: 54

```
DSolve[y''''[x]-y[x]==x^2*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(-\frac{x^3}{12} + \frac{5x}{8} + c_4\right)\sin(x) + \left(-\frac{3x^2}{8} + \frac{5}{16} + c_2\right)\cos(x) + c_1 e^x + c_3 e^{-x}$$

15 Exercise 24, page 109

15.1 problem 1

Internal problem ID [2234]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 24, page 109
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = x \sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+4*y(x)=x*sin(x),y(x), singsol=all)
```

$$y = c_2 \sin(2x) + \cos(2x)\, c_1 - \frac{2\cos(x)}{9} + \frac{x \sin(x)}{3}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 33

```
DSolve[y''[x]+4*y[x]==x*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} x \sin(x) + c_1 \cos(2x) + \cos(x)\left(-\frac{2}{9} + 2c_2 \sin(x)\right)$$

15.2 problem 2

Internal problem ID [2235]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 24, page 109
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \cos(x)\, x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)+y(x)=x^2*cos(x),y(x), singsol=all)
```

$$y = \sin(x)\, c_2 + \cos(x)\, c_1 + \frac{(2x^3 - 3x)\sin(x)}{12} + \frac{\cos(x)\, x^2}{4}$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 41

```
DSolve[y''[x]+y[x]==x^2*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}(2x^3 - 3x + 12c_2)\sin(x) + \left(\frac{x^2}{4} - \frac{1}{8} + c_1\right)\cos(x)$$

15.3 problem 3

Internal problem ID [2236]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 24, page 109
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y = \cos(x)\, x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)-y(x)=x^2*cos(x),y(x), singsol=all)
```

$$y = e^{-x} c_2 + c_1 e^x - \frac{\cos(x)\, x^2}{2} + \frac{\cos(x)}{2} + x \sin(x)$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 35

```
DSolve[y''[x]-y[x]==x^2*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}(x^2 - 1)\cos(x) + x\sin(x) + c_1 e^x + c_2 e^{-x}$$

15.4 problem 4

Internal problem ID [2237]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 24, page 109
Problem number: 4.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + 4y' = e^{x} + \sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$3)+4*diff(y(x),x)=exp(x)+sin(x),y(x), singsol=all)
```

$$y = \frac{\sin(2x)\, c_1}{2} - \frac{c_2 \cos(2x)}{2} + \frac{e^{x}}{5} - \frac{\cos(x)}{3} + c_3$$

✓ Solution by Mathematica

Time used: 0.241 (sec). Leaf size: 37

```
DSolve[y'''[x]+4*y'[x]==Exp[x]+Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{x}}{5} - \frac{1}{2}c_2 \cos(2x) + \cos(x)\left(-\frac{1}{3} + c_1 \sin(x)\right) + c_3$$

15.5 problem 5

Internal problem ID [2238]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 24, page 109
Problem number: 5.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y^{(5)} + y'''' = x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$5)+diff(y(x),x$4)=x^2,y(x), singsol=all)
```

$$y = \frac{x^6}{360} + \frac{x^4}{12} - \frac{x^5}{60} + c_1 e^{-x} + \frac{c_2 x^3}{6} + \frac{c_3 x^2}{2} + c_4 x + c_5$$

✓ Solution by Mathematica

Time used: 0.134 (sec). Leaf size: 53

```
DSolve[y'''''[x]+y''''[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^6}{360} - \frac{x^5}{60} + \frac{x^4}{12} + c_5 x^3 + c_4 x^2 + c_3 x + c_1 e^{-x} + c_2$$

15.6 problem 6

Internal problem ID [2239]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 24, page 109
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{2y'' + 3y' - 2y = \mathrm{e}^{x} x^{2}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(2*diff(y(x),x$2)+3*diff(y(x),x)-2*y(x)=x^2*exp(x),y(x), singsol=all)
```

$$y = c_2 \mathrm{e}^{\frac{x}{2}} + c_1 \mathrm{e}^{-2x} + \frac{(9x^2 - 42x + 86)\,\mathrm{e}^{x}}{27}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 41

```
DSolve[2*y''[x]+3*y'[x]-2*y[x]==x^2*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{27} e^{x} \left(9x^2 - 42x + 86\right) + c_1 e^{x/2} + c_2 e^{-2x}$$

15.7 problem 8

Internal problem ID [2240]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 24, page 109
Problem number: 8.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + y' = \sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$3)+diff(y(x),x)=sin(x),y(x), singsol=all)
```

$$y = -\cos(x) - \frac{x\sin(x)}{2} + c_1\sin(x) - c_2\cos(x) + c_3$$

✓ Solution by Mathematica

Time used: 0.074 (sec). Leaf size: 31

```
DSolve[y'''[x]+y'[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}(1+2c_2)\cos(x) + \left(-\frac{x}{2}+c_1\right)\sin(x) + c_3$$

15.8 problem 9

Internal problem ID [2241]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 24, page 109
Problem number: 9.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - y' = x \sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$3)-diff(y(x),x)=x*sin(x),y(x), singsol=all)
```

$$y = \frac{x \cos(x)}{2} - \sin(x) + c_1 e^x - e^{-x} c_2 + c_3$$

✓ Solution by Mathematica

Time used: 0.142 (sec). Leaf size: 34

```
DSolve[y'''[x]-y'[x]==x*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sin(x) + \frac{1}{2} x \cos(x) + c_1 e^x - c_2 e^{-x} + c_3$$

15.9 problem 10

Internal problem ID [2242]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 24, page 109
Problem number: 10.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + 2y'' = x\cos(2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$3)+2*diff(y(x),x$2)=x*cos(2*x),y(x), singsol=all)
```

$$y = -\frac{\cos(2x)}{16} - \frac{\sin(2x)\,x}{16} + \frac{c_1 e^{-2x}}{4} + \frac{3\sin(2x)}{32} - \frac{\cos(2x)\,x}{16} + c_2 x + c_3$$

✓ Solution by Mathematica

Time used: 1.447 (sec). Leaf size: 48

```
DSolve[y'''[x]+2*y''[x]==x*Cos[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3 x + \frac{1}{16}\left(-x\sin(2x) - (x+1)\cos(2x) + 4c_1 e^{-2x} + 3\sin(x)\cos(x)\right) + c_2$$

15.10 problem 11

Internal problem ID [2243]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 24, page 109
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 3y' + 2y = \cos(x)\, x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=x^2*cos(x),y(x), singsol=all)
```

$$y = -c_1 e^{-2x} + \frac{\cos(x)\, x^2}{10} - \frac{133\cos(x)}{250} + \frac{3x^2 \sin(x)}{10} - \frac{17x\sin(x)}{25} + \frac{81\sin(x)}{250} + \frac{6x\cos(x)}{25} + e^{-x} c_2$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 53

```
DSolve[y''[x]+3*y'[x]+2*y[x]==x^2*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{250}\left(\left(75x^2 - 170x + 81\right)\sin(x) + \left(25x^2 + 60x - 133\right)\cos(x)\right) + c_1 e^{-2x} + c_2 e^{-x}$$

15.11 problem 12

Internal problem ID [2244]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 24, page 109
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 3y = \sin(x)\, x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+3*y(x)=x^2*sin(x),y(x), singsol=all)
```

$$y = e^{3x} c_2 + c_1 e^x + \frac{(25x^2 + 55x + 28)\cos(x)}{125} + \frac{\left(x^2 - \frac{4}{5}x - \frac{67}{25}\right)\sin(x)}{10}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 52

```
DSolve[y''[x]-4*y'[x]+3*y[x]==x^2*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{250}\left((25x^2 - 20x - 67)\sin(x) + 2(25x^2 + 55x + 28)\cos(x)\right) + c_1 e^x + c_2 e^{3x}$$

15.12 problem 13

Internal problem ID [2245]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 24, page 109
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y = x \sin(2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)-y(x)=x*sin(2*x),y(x), singsol=all)
```

$$y = \mathrm{e}^{-x} c_2 + c_1 \mathrm{e}^{x} - \frac{4\cos(2x)}{25} - \frac{\sin(2x)\, x}{5}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 37

```
DSolve[y''[x]-y[x]==x*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{5} x \sin(2x) - \frac{4}{25} \cos(2x) + c_1 e^{x} + c_2 e^{-x}$$

15.13 problem 14

Internal problem ID [2246]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 24, page 109
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + 2y' = x^3 \sin(2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 69

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)=x^3*sin(2*x),y(x), singsol=all)
```

$$\begin{aligned} y = &-\frac{\cos(2x)\, x^3}{8} + \frac{3x^2 \sin(2x)}{8} - \frac{15\sin(2x)}{64} + \frac{15\cos(2x)\, x}{32} \\ &+ \frac{3\sin(2x)\, x}{32} - \frac{x^3\sin(2x)}{8} - \frac{3\cos(2x)\, x^2}{16} - \frac{c_1 \mathrm{e}^{-2x}}{2} + c_2 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.435 (sec). Leaf size: 61

```
DSolve[y''[x]+2*y'[x]==x^3*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{32}x(4x^2+6x-15)\cos(2x) + \frac{1}{64}(-8x^3+24x^2+6x-15)\sin(2x) - \frac{1}{2}c_1 e^{-2x} + c_2$$

15.14 problem 15

Internal problem ID [2247]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 24, page 109
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - y' = x\,e^{2x} \sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(diff(y(x),x$2)-diff(y(x),x)=x*exp(2*x)*sin(x),y(x), singsol=all)
```

$$y = -\frac{3\,e^{2x} \cos(x)\,x}{10} + \frac{17\,e^{2x} \cos(x)}{50} + \frac{e^{2x} \sin(x)\,x}{10} + \frac{3\,e^{2x} \sin(x)}{25} + c_1 e^{x} + c_2$$

✓ Solution by Mathematica

Time used: 0.486 (sec). Leaf size: 46

```
DSolve[y''[x]-y'[x]==x*Exp[2*x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{50} e^{2x} (5x + 6) \sin(x) - \frac{1}{50} e^{2x} (15x - 17) \cos(x) + c_1 e^{x} + c_2$$

15.15 problem 16

Internal problem ID [2248]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 24, page 109
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y = x\,e^{2x}\cos(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$2)-4*y(x)=x*exp(2*x)*cos(x),y(x), singsol=all)
```

$$y = c_2 e^{2x} + c_1 e^{-2x} + \frac{((-17x + 76)\cos(x) + (68x + 2)\sin(x))\,e^{2x}}{289}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 49

```
DSolve[y''[x]-4*y[x]==x*Exp[2*x]*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{2x} + c_2 e^{-2x} + \frac{1}{289} e^{2x}(2(34x + 1)\sin(x) + (76 - 17x)\cos(x))$$

15.16 problem 17

Internal problem ID [2249]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 24, page 109
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + 2y' = x^2 e^{-x} \sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)=x^2*exp(-x)*sin(x),y(x), singsol=all)
```

$$y = -\frac{x^2 \sin(x) e^{-x}}{2} + \frac{e^{-x} \sin(x)}{2} - x e^{-x} \cos(x) - \frac{c_1 e^{-2x}}{2} + c_2$$

✓ Solution by Mathematica

Time used: 1.508 (sec). Leaf size: 39

```
DSolve[y''[x]+2*y'[x]==x^2*Exp[-x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{1}{2} e^{-2x} \left(e^x \left(x^2 - 1\right) \sin(x) + 2e^x x \cos(x) + c_1\right)$$

16 Exercise 25, page 112

16.1 problem 1

Internal problem ID [2250]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 25, page 112
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 4y'x + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y = c_1 x^{\frac{5}{2}+\frac{\sqrt{21}}{2}} + c_2 x^{\frac{5}{2}-\frac{\sqrt{21}}{2}}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 34

```
DSolve[x^2*y''[x]-4*x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^{\frac{5}{2}-\frac{\sqrt{21}}{2}}\left(c_2 x^{\sqrt{21}} + c_1\right)$$

16.2 problem 2

Internal problem ID [2251]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 25, page 112
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' + y'x + 16y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+16*y(x)=0,y(x), singsol=all)
```

$$y = c_1 \sin(4\ln(x)) + c_2 \cos(4\ln(x))$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]+x*y'[x]+16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(4\log(x)) + c_2 \sin(4\log(x))$$

16.3 problem 3

Internal problem ID [2252]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 25, page 112
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{4x^2y'' - 16y'x + 25y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(4*x^2*diff(y(x),x$2)-16*x*diff(y(x),x)+25*y(x)=0,y(x), singsol=all)
```

$$y = c_1 x^{\frac{5}{2}} + c_2 x^{\frac{5}{2}} \ln (x)$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 25

```
DSolve[4*x^2*y''[x]-16*x*y'[x]+25*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} x^{5/2} (5c_2 \log(x) + 2c_1)$$

16.4 problem 4

Internal problem ID [2253]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 25, page 112
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 5y'x + 10y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+10*y(x)=0,y(x), singsol=all)
```

$$y = \frac{c_1 \sin\left(\sqrt{6}\,\ln(x)\right)}{x^2} + \frac{c_2 \cos\left(\sqrt{6}\,\ln(x)\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 34

```
DSolve[x^2*y''[x]+5*x*y'[x]+10*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 \cos\left(\sqrt{6}\log(x)\right) + c_1 \sin\left(\sqrt{6}\log(x)\right)}{x^2}$$

16.5 problem 5

Internal problem ID [2254]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 25, page 112
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' - 3y'x - 18y = \ln(x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(2*x^2*diff(y(x),x$2)-3*x*diff(y(x),x)-18*y(x)=ln(x),y(x), singsol=all)
```

$$y = x^{\frac{9}{2}}c_2 + \frac{c_1}{x^2} - \frac{\ln(x)}{18} + \frac{5}{324}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 29

```
DSolve[2*x^2*y''[x]-3*x*y'[x]-18*y[x]==Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x^{9/2} + \frac{c_1}{x^2} - \frac{\log(x)}{18} + \frac{5}{324}$$

16.6 problem 6

Internal problem ID [2255]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 25, page 112
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' - 3y'x + 2y = \ln(x^2)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(2*x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+2*y(x)=ln(x^2),y(x), singsol=all)
```

$$y = \frac{2c_1x^2}{3} + \frac{5}{2} + \frac{\ln(x^2)}{2} + \sqrt{x}\, c_2$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 30

```
DSolve[2*x^2*y''[x]-3*x*y'[x]+2*y[x]==Log[x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(\log(x^2) + 5) + c_2x^2 + c_1\sqrt{x}$$

16.7 problem 7

Internal problem ID [2256]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 25, page 112
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' - 3y'x + 4y = x^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*y(x)=x^3,y(x), singsol=all)
```

$$y = c_2 x^2 + \ln(x) x^2 c_1 + x^3$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 19

```
DSolve[x^2*y''[x]-3*x*y'[x]+4*y[x]==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(x + 2c_2 \log(x) + c_1)$$

16.8 problem 8

Internal problem ID [2257]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 25, page 112
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + 3y'x + y = 1 - x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+y(x)=1-x,y(x), singsol=all)
```

$$y = \frac{c_2}{x} - \frac{x}{4} + 1 + \frac{c_1 \ln(x)}{x}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 26

```
DSolve[x^2*y''[x]+3*x*y'[x]+y[x]==1-x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{4} + \frac{c_1}{x} + \frac{c_2 \log(x)}{x} + 1$$

16.9 problem 9

Internal problem ID [2258]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 25, page 112
Problem number: 9.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^3y''' + 2x^2y'' - y'x + y = \frac{1}{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^3*diff(y(x),x$3)+2*x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=1/x,y(x), singsol=all)
```

$$y = xc_2 \ln(x) + c_3 x + \frac{\ln(x) + 1 + c_1}{4x}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 33

```
DSolve[x^3*y'''[x]+2*x^2*y''[x]-x*y'[x]+y[x]==1/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\log(x) + 1}{4x} + \frac{c_1}{x} + c_2 x + c_3 x \log(x)$$

16.10 problem 10

Internal problem ID [2259]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 25, page 112
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2y'x + 2y = 4x + \sin(\ln(x))}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 43

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=4*x+sin(ln(x)),y(x), singsol=all)
```

$$y = c_2x + c_1x^2 + \frac{\cos\left(\frac{\ln(x)}{2}\right)\sin\left(\frac{\ln(x)}{2}\right)}{5} - 4\ln(x)\,x + \frac{3\cos\left(\frac{\ln(x)}{2}\right)^2}{5} - 4x - \frac{3}{10}$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]-2*x*y'[x]+2*y[x]==4*x+Sin[Log[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{10}(\sin(\log(x)) + 3\cos(\log(x)) + 10x(-4\log(x) + c_2x - 4 + c_1))$$

16.11 problem 11

Internal problem ID [2260]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 25, page 112
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' - y'x + 2y = x^2 \ln(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)+2*y(x)=x^2*ln(x),y(x), singsol=all)
```

$$y = \sin(\ln(x))\, x c_2 + \cos(\ln(x))\, x c_1 + \frac{x^2(\ln(x) - 1)}{2}$$

✓ Solution by Mathematica

Time used: 0.078 (sec). Leaf size: 31

```
DSolve[x^2*y''[x]-x*y'[x]+2*y[x]==x^2*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} x(x(\log(x) - 1) + 2c_2 \cos(\log(x)) + 2c_1 \sin(\log(x)))$$

16.12 problem 12

Internal problem ID [2261]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 25, page 112
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + 4xy' + 3y = (x-1)\ln(x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
dsolve(x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+3*y(x)=(x-1)*ln(x),y(x), singsol=all)
```

$$y = \frac{\sin\left(\frac{\sqrt{3}\,\ln(x)}{2}\right)c_2}{x^{\frac{3}{2}}} + \frac{\cos\left(\frac{\sqrt{3}\,\ln(x)}{2}\right)c_1}{x^{\frac{3}{2}}} + \frac{1}{3} + \frac{(3x-7)\ln(x)}{21} - \frac{5x}{49}$$

✓ Solution by Mathematica

Time used: 0.516 (sec). Leaf size: 67

```
DSolve[x^2*y''[x]+4*x*y'[x]+3*y[x]==(x-1)*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2\cos\left(\frac{1}{2}\sqrt{3}\log(x)\right)}{x^{3/2}} + \frac{c_1\sin\left(\frac{1}{2}\sqrt{3}\log(x)\right)}{x^{3/2}} - \frac{5x}{49} + \frac{1}{7}x\log(x) - \frac{\log(x)}{3} + \frac{1}{3}$$

16.13 problem 13

Internal problem ID [2262]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 25, page 112
Problem number: 13.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{4x^3y''' + 8x^2y'' - xy' + y = x + \ln(x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(4*x^3*diff(y(x),x$3)+8*x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=x+ln(x),y(x), singsol=a
```

$$y = \frac{\ln(x)\,x}{3} + \ln(x) + 1 - \frac{8x}{9} + c_1x + \frac{c_2}{\sqrt{x}} + c_3\sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 40

```
DSolve[4*x^3*y'''[x]+8*x^2*y''[x]-x*y'[x]+y[x]==x+Log[x],y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{3}(x+3)\log(x) + \frac{c_1}{\sqrt{x}} + c_2\sqrt{x} + \left(-\frac{8}{9} + c_3\right)x + 1$$

16.14 problem 14

Internal problem ID [2263]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 25, page 112
Problem number: 14.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{3x^3y''' + 4x^2y'' - 10xy' + 10y = \frac{4}{x^2}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(3*x^3*diff(y(x),x$3)+4*x^2*diff(y(x),x$2)-10*x*diff(y(x),x)+10*y(x)=4/x^2,y(x), sings
```

$$y = -\frac{2}{9x^2} + c_1x + c_2x^{\frac{1}{3}-\frac{\sqrt{31}}{3}} + c_3x^{\frac{1}{3}+\frac{\sqrt{31}}{3}}$$

✓ Solution by Mathematica

Time used: 0.074 (sec). Leaf size: 51

```
DSolve[3*x^3*y'''[x]+4*x^2*y''[x]-10*x*y'[x]+10*y[x]==4/x^2,y[x],x,IncludeSingularSolutions
```

$$y(x) \to c_2x^{\frac{1}{3}\left(1+\sqrt{31}\right)} + c_1x^{\frac{1}{3}-\frac{\sqrt{31}}{3}} - \frac{2}{9x^2} + c_3x$$

16.15 problem 15

Internal problem ID [2264]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 25, page 112
Problem number: 15.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^4y'''' + 7x^3y''' + 9x^2y'' - 6xy' - 6y = \cos(\ln(x))}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 394

```
dsolve(x^4*diff(y(x),x$4)+7*x^3*diff(y(x),x$3)+9*x^2*diff(y(x),x$2)-6*x*diff(y(x),x)-6*y(x)=
```

$$y = \frac{\sin\left(\sqrt{2}\,\ln(x)\right)c_4}{x} + \frac{\cos\left(\sqrt{2}\,\ln(x)\right)c_3}{x}$$
$$+ \frac{33\left(\left(-2-4i+(-1+3i)\sqrt{2}\right)\cos\left(\sqrt{2}\,\ln(x)\right) - \left(-4+2i+(3+i)\sqrt{2}\right)\sin\left(\sqrt{2}\,\ln(x)\right)\right)x^{-i\sqrt{2}+1-i} +}{}$$

✓ Solution by Mathematica

Time used: 0.318 (sec). Leaf size: 62

```
DSolve[x^4*y''''[x]+7*x^3*y'''[x]+9*x^2*y''[x]-6*x*y'[x]-6*y[x]==Cos[Log[x]],y[x],x,IncludeS
```

$$y(x) \to c_4x^2 + \frac{c_3}{x} - \frac{1}{10}\sin(\log(x)) - \frac{1}{20}\cos(\log(x)) + \frac{c_2\cos\left(\sqrt{2}\log(x)\right)}{x} + \frac{c_1\sin\left(\sqrt{2}\log(x)\right)}{x}$$

16.16 problem 16

Internal problem ID [2265]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 25, page 112
Problem number: 16.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^3y''' - 2x^2y'' - xy' + 4y = \sin(\ln(x))}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 164

```
dsolve(x^3*diff(y(x),x$3)-2*x^2*diff(y(x),x$2)-x*diff(y(x),x)+4*y(x)=sin(ln(x)),y(x), singso
```

$$y = -\frac{\left(119ix^{2i}x^{-\frac{\sqrt{5}}{2}+\frac{1}{2}}x^{\frac{\sqrt{5}}{2}+\frac{1}{2}} - 20ix^{2i}x + 17x^{2i}x^{-\frac{\sqrt{5}}{2}+\frac{1}{2}}x^{\frac{\sqrt{5}}{2}+\frac{1}{2}} - 119ix^{-\frac{\sqrt{5}}{2}+\frac{1}{2}}x^{\frac{\sqrt{5}}{2}+\frac{1}{2}} + 5x^{2i}x + 20ix + 17x^{-}\right.}{1870x}$$
$$+ c_1x^4 + c_2x^{-\frac{\sqrt{5}}{2}+\frac{1}{2}} + c_3x^{\frac{\sqrt{5}}{2}+\frac{1}{2}}$$

✓ Solution by Mathematica

Time used: 0.212 (sec). Leaf size: 60

```
DSolve[x^3*y'''[x]-2*x^2*y''[x]-x*y'[x]+4*y[x]==Sin[Log[x]],y[x],x,IncludeSingularSolutions
```

$$y(x) \to c_2x^{\frac{1}{2}\left(1+\sqrt{5}\right)} + c_1x^{\frac{1}{2}-\frac{\sqrt{5}}{2}} + c_3x^4 + \frac{9}{85}\sin(\log(x)) - \frac{2}{85}\cos(\log(x))$$

17 Exercise 26, page 115

17.1 problem 1

Internal problem ID [2266]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 26, page 115
Problem number: 1.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) + \cos(t)$$
$$y'(t) = -y(t) + 4t$$

✓ Solution by Maple

Time used: 0.234 (sec). Leaf size: 32

```
dsolve([diff(x(t),t)-x(t)=cos(t),diff(y(t),t)+y(t)=4*t],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{\cos(t)}{2} + \frac{\sin(t)}{2} + c_1 e^t$$
$$y(t) = 4t - 4 + c_2 e^{-t}$$

✓ Solution by Mathematica

Time used: 0.076 (sec). Leaf size: 39

```
DSolve[{x'[t]-x[t]==Cos[t],y'[t]+y[t]==4*t},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{2}\left(\sin(t) - \cos(t) + 2c_1 e^t\right)$$
$$y(t) \to 4t + c_2 e^{-t} - 4$$

17.2 problem 2

Internal problem ID [2267]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 26, page 115
Problem number: 2.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= 3t^2 - 5x(t) \\
y'(t) &= -y(t) + e^{3t}
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 37

```
dsolve([diff(x(t),t)+5*x(t)=3*t^2,diff(y(t),t)+y(t)=exp(3*t)],[x(t), y(t)], singsol=all)
```

$$
\begin{aligned}
x(t) &= \frac{3t^2}{5} - \frac{6t}{25} + \frac{6}{125} + e^{-5t}c_1 \\
y(t) &= \frac{e^{3t}}{4} + e^{-t}c_2
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.216 (sec). Leaf size: 50

```
DSolve[{x'[t]+5*x[t]==3*t^2,y'[t]+y[t]==Exp[3*t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$
\begin{aligned}
x(t) &\to \frac{3t^2}{5} - \frac{6t}{25} + c_1e^{-5t} + \frac{6}{125} \\
y(t) &\to \frac{e^{3t}}{4} + c_2e^{-t}
\end{aligned}
$$

17.3 problem 3

Internal problem ID [2268]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 26, page 115
Problem number: 3.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -2x(t) + 3t$$
$$y'(t) = x(t) - \frac{3t}{2} - \frac{y(t)}{2} + \cos(t)^2 - \frac{1}{2}$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 46

```
dsolve([diff(x(t),t)+2*x(t)=3*t,diff(x(t),t)+2*diff(y(t),t)+y(t)=cos(2*t)],[x(t), y(t)], sin
```

$$x(t) = \frac{3t}{2} - \frac{3\,\mathrm{e}^{-2t}c_1}{2} - \frac{3}{4}$$
$$y(t) = \mathrm{e}^{-\frac{t}{2}}c_2 + \mathrm{e}^{-2t}c_1 - \frac{3}{2} + \frac{\cos(2t)}{17} + \frac{4\sin(2t)}{17}$$

✓ Solution by Mathematica

Time used: 0.348 (sec). Leaf size: 74

```
DSolve[{x'[t]+2*x[t]==3*t,x'[t]+2*y'[t]+y[t]==Cos[2*t]},{x[t],y[t]},t,IncludeSingularSolutio
```

$$x(t) \to \frac{3t}{2} + c_1e^{-2t} - \frac{3}{4}$$
$$y(t) \to \frac{4}{17}\sin(2t) + \frac{1}{17}\cos(2t) + \frac{1}{6}\left(-4c_1e^{-2t} + (4c_1 + 6c_2)e^{-t/2} - 9\right)$$

17.4 problem 4

Internal problem ID [2269]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 26, page 115
Problem number: 4.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= -y(t) + x(t) + 2\sin(t) \\
y'(t) &= 4y(t) - 4x(t) - 2\sin(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 42

```
dsolve([diff(x(t),t)-x(t)+y(t)=2*sin(t),diff(x(t),t)+diff(y(t),t)=3*y(t)-3*x(t)],[x(t), y(t)
```

$$
\begin{aligned}
x(t) &= -\frac{e^{5t}c_1}{20} - \frac{16\cos(t)}{13} - \frac{2\sin(t)}{13} + c_2 \\
y(t) &= \frac{e^{5t}c_1}{5} + \frac{8\sin(t)}{13} - \frac{14\cos(t)}{13} + c_2
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 80

```
DSolve[{x'[t]-x[t]+y[t]==2*Sin[t],x'[t]+y'[t]==3*y[t]-3*x[t]},{x[t],y[t]},t,IncludeSingularS
```

$$
\begin{aligned}
x(t) &\to \frac{1}{65}\left(-10\sin(t) - 80\cos(t) + 13c_1\left(e^{5t}+4\right) - 13c_2\left(e^{5t}-1\right)\right) \\
y(t) &\to \frac{1}{65}\left(40\sin(t) - 70\cos(t) - 52c_1\left(e^{5t}-1\right) + 13c_2\left(4e^{5t}+1\right)\right)
\end{aligned}
$$

17.5 problem 5

Internal problem ID [2270]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 26, page 115
Problem number: 5.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= -\frac{3x(t)}{2} + \frac{y(t)}{2} + \frac{e^t}{2} \\
y'(t) &= \frac{5x(t)}{3} - \frac{y(t)}{3} - \frac{2t}{3}
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 50

```
dsolve([2*diff(x(t),t)+3*x(t)-y(t)=exp(t),5*x(t)-3*diff(y(t),t)=y(t)+2*t],[x(t), y(t)], sing
```

$$
\begin{aligned}
x(t) &= \frac{3\,e^{\frac{t}{6}} c_2}{10} - e^{-2t} c_1 + \frac{11}{2} + \frac{4\,e^t}{15} + t \\
y(t) &= e^{\frac{t}{6}} c_2 + e^{-2t} c_1 + 3t + \frac{37}{2} + \frac{e^t}{3}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.685 (sec). Leaf size: 105

```
DSolve[{2*x'[t]+3*x[t]-y[t]==Exp[t],5*x[t]-3*y'[t]==y[t]+2*t},{x[t],y[t]},t,IncludeSingularS
```

$$
\begin{aligned}
x(t) &\to t + \frac{4e^t}{15} + \frac{1}{13}(10c_1 - 3c_2)e^{-2t} + \frac{3}{13}(c_1 + c_2)e^{t/6} + \frac{11}{2} \\
y(t) &\to \frac{1}{78}e^{-2t}\left(39e^{2t}(6t + 37) + 26e^{3t} + 60(c_1 + c_2)e^{13t/6} - 60c_1 + 18c_2\right)
\end{aligned}
$$

17.6 problem 6

Internal problem ID [2271]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 26, page 115
Problem number: 6.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
0 &= -5y'(t) + 3x'(t) + 5y(t) + 5t \\
0 &= -3x'(t) + 5y'(t) + 2x(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve([5*diff(y(t),t)-3*diff(x(t),t)-5*y(t)=5*t,3*diff(x(t),t)-5*diff(y(t),t)-2*x(t)=0],[x(
```

$$
\begin{aligned}
x(t) &= \frac{5}{2} - \frac{5\,\mathrm{e}^{\frac{2t}{5}} c_1}{2} \\
y(t) &= -t - 1 + \mathrm{e}^{\frac{2t}{5}} c_1
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.137 (sec). Leaf size: 43

```
DSolve[{5*y'[t]-3*x'[t]-5*y[t]==5*t,3*x'[t]-5*y'[t]-2*x[t]==0},{x[t],y[t]},t,IncludeSingular
```

$$
\begin{aligned}
x(t) &\to \frac{5}{6}\left(3 + 2c_1 e^{2t/5}\right) \\
y(t) &\to -t - \frac{2}{3}c_1 e^{2t/5} - 1
\end{aligned}
$$

17.7 problem 12

Internal problem ID [2272]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 26, page 115
Problem number: 12.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= 3x(t) \\
y'(t) &= 2x(t) + 3y(t) \\
z'(t) &= 3y(t) - 2z(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 53

```
dsolve([diff(x(t),t)=3*x(t),diff(y(t),t)=2*x(t)+3*y(t),diff(z(t),t)=3*y(t)-2*z(t)],[x(t), y(
```

$$
\begin{aligned}
x(t) &= \frac{5c_3 e^{3t}}{6} \\
y(t) &= \frac{e^{3t}(5c_3 t + 5c_2 + c_3)}{3} \\
z(t) &= e^{-2t}c_1 + c_2 e^{3t} + c_3 e^{3t} t
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 78

```
DSolve[{x'[t]==3*x[t],y'[t]==2*x[t]+3*y[t],z'[t]==3*y[t]-2*z[t]},{x[t],y[t],z[t]},t,IncludeS
```

$$
\begin{aligned}
x(t) &\to c_1 e^{3t} \\
y(t) &\to e^{3t}(2c_1 t + c_2) \\
z(t) &\to \frac{1}{25} e^{-2t}\left(6c_1\left(e^{5t}(5t-1)+1\right) + 5\left(3c_2\left(e^{5t}-1\right) + 5c_3\right)\right)
\end{aligned}
$$

18 Exercise 35, page 157

18.1 problem 1

Internal problem ID [2273]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = \cos(t)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(t),t$2)=cos(t),y(t), singsol=all)
```

$$y(t) = -\cos(t) + c_1 t + c_2$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 16

```
DSolve[y''[t]==Cos[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\cos(t) + c_2 t + c_1$$

18.2 problem 2

Internal problem ID [2274]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - k^2 y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(t),t$2)=k^2*y(t),y(t), singsol=all)
```

$$y(t) = c_1 e^{-kt} + c_2 e^{kt}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 23

```
DSolve[y''[t]==k^2*y[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^{kt} + c_2 e^{-kt}$$

18.3 problem 3

Internal problem ID [2275]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + k^2 x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(x(t),t$2)+k^2*x(t)=0,x(t), singsol=all)
```

$$x(t) = c_1 \sin(kt) + c_2 \cos(kt)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 20

```
DSolve[x''[t]+k^2*x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 \cos(kt) + c_2 \sin(kt)$$

18.4 problem 4

Internal problem ID [2276]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y^3 y'' = -4}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 70

```
dsolve(y(x)^3*diff(y(x),x$2)+4=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{c_1 \left(c_1^2 c_2^2 + 2 c_1^2 c_2 x + c_1^2 x^2 - 4\right)}}{c_1}$$
$$y(x) = -\frac{\sqrt{c_1 \left(c_1^2 c_2^2 + 2 c_1^2 c_2 x + c_1^2 x^2 - 4\right)}}{c_1}$$

✓ Solution by Mathematica

Time used: 3.404 (sec). Leaf size: 93

```
DSolve[y[x]^3*y''[x]+4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{c_1{}^2 x^2 + 2 c_2 c_1{}^2 x - 4 + c_2{}^2 c_1{}^2}}{\sqrt{c_1}}$$
$$y(x) \to \frac{\sqrt{c_1{}^2 x^2 + 2 c_2 c_1{}^2 x - 4 + c_2{}^2 c_1{}^2}}{\sqrt{c_1}}$$
$$y(x) \to \text{Indeterminate}$$

18.5 problem 5

Internal problem ID [2277]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{x'' - \frac{k^2}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 385

```
dsolve(diff(x(t),t$2)=k^2/x(t)^2,x(t), singsol=all)
```

$$x(t) = \frac{c_1\left(c_1^2k^4 + 2k^2c_1\mathrm{e}^{\mathrm{RootOf}\left(\mathrm{csgn}\left(\frac{1}{c_1}\right)c_1^4k^4 - 2\_Z c_1^3k^2\mathrm{e}^{\_Z} - \mathrm{e}^{2\_Z}\mathrm{csgn}\left(\frac{1}{c_1}\right)c_1^2 - 2\,\mathrm{e}^{\_Z}\mathrm{csgn}\left(\frac{1}{c_1}\right)c_2 - 2\,\mathrm{e}^{\_Z}\mathrm{csgn}\left(\frac{1}{c_1}\right)t\right)} + \mathrm{e}^{2\,\mathrm{RootOf}\left(\mathrm{csgn}\right.}\right.}{}$$

$$x(t) = \frac{c_1\left(c_1^2k^4 + 2k^2c_1\mathrm{e}^{\mathrm{RootOf}\left(\mathrm{csgn}\left(\frac{1}{c_1}\right)c_1^4k^4 - 2\_Z c_1^3k^2\mathrm{e}^{\_Z} - \mathrm{e}^{2\_Z}\mathrm{csgn}\left(\frac{1}{c_1}\right)c_1^2 + 2\,\mathrm{e}^{\_Z}\mathrm{csgn}\left(\frac{1}{c_1}\right)c_2 + 2\,\mathrm{e}^{\_Z}\mathrm{csgn}\left(\frac{1}{c_1}\right)t\right)} + \mathrm{e}^{2\,\mathrm{RootOf}\left(\mathrm{csgn}\right.}\right.}{}$$

✓ Solution by Mathematica

Time used: 0.188 (sec). Leaf size: 71

```
DSolve[x''[t]==k^2/x[t]^2,x[t],t,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left(\frac{2k^2 \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2k^2}{x(t)}+c_1}}{\sqrt{c_1}}\right)}{c_1{}^{3/2}}+\frac{x(t)\sqrt{-\frac{2k^2}{x(t)}+c_1}}{c_1}\right)^2=(t+c_2)^2, x(t)\right]$$

18.6 problem 6

Internal problem ID [2278]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{xy'' = x^2 + 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x*diff(y(x),x$2)=1+x^2,y(x), singsol=all)
```

$$y(x) = \frac{x^3}{6} + x\ln(x) - x + c_1x + c_2$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 25

```
DSolve[x*y''[x]==1+x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{6} + x\log(x) + (-1 + c_2)x + c_1$$

18.7 problem 7

Internal problem ID [2279]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{(1-x)\,y''-y'=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((1-x)*diff(y(x),x$2)=diff(y(x),x),y(x), singsol=all)
```

$$y(x)=c_2\ln(x-1)+c_1$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 18

```
DSolve[(1-x)*y''[x]==y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_2-c_1\log(1-x)$$

18.8 problem 8

Internal problem ID [2280]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{(x^2+1)\,y''+2x(y'+1)=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((1+x^2)*diff(y(x),x$2)+2*x*(diff(y(x),x)+1)=0,y(x), singsol=all)
```

$$y(x)=-x+(c_1+1)\arctan(x)+c_2$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 18

```
DSolve[(1+x^2)*y''[x]+2*x*(y'[x]+1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to(1+c_1)\arctan(x)-x+c_2$$

18.9 problem 9

Internal problem ID [2281]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - {y'}^3 - y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 251

```
dsolve(diff(y(x),x$2)=diff(y(x),x)^3+diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-\left(e^{2x}c_1 - 1\right)e^{2x}c_1}}{2} - \frac{c_1 \arctan\left(\frac{\sqrt{c_1^2}\left(e^{2x} - \frac{1}{2c_1}\right)}{\sqrt{-(e^{2x}c_1 - 1)e^{2x}c_1}}\right)}{4\sqrt{c_1^2}}$$
$$- \frac{\sqrt{c_1\left(-c_1 e^{4x} + e^{2x}\right)}}{2} - \frac{c_1 \arctan\left(\frac{\sqrt{c_1^2}\left(e^{2x} - \frac{1}{2c_1}\right)}{\sqrt{c_1(-c_1 e^{4x} + e^{2x})}}\right)}{4\sqrt{c_1^2}} + c_2$$

$$y(x) = -\frac{\sqrt{-\left(e^{2x}c_1 - 1\right)e^{2x}c_1}}{2} + \frac{c_1 \arctan\left(\frac{\sqrt{c_1^2}\left(e^{2x} - \frac{1}{2c_1}\right)}{\sqrt{-(e^{2x}c_1 - 1)e^{2x}c_1}}\right)}{4\sqrt{c_1^2}}$$
$$+ \frac{\sqrt{c_1\left(-c_1 e^{4x} + e^{2x}\right)}}{2} + \frac{c_1 \arctan\left(\frac{\sqrt{c_1^2}\left(e^{2x} - \frac{1}{2c_1}\right)}{\sqrt{c_1(-c_1 e^{4x} + e^{2x})}}\right)}{4\sqrt{c_1^2}} + c_2$$

✓ Solution by Mathematica

Time used: 60.102 (sec). Leaf size: 71

```
DSolve[y''[x]==y'[x]^3+y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - i \log \left(\sqrt{-1+e^{2(x+c_1)}} - e^{x+c_1}\right)$$

$$y(x) \to i \log \left(\sqrt{-1+e^{2(x+c_1)}} - e^{x+c_1}\right) + c_2$$

18.10 problem 10

Internal problem ID [2282]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' - y' = -x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x*diff(y(x),x$2)+x=diff(y(x),x),y(x), singsol=all)
```

$$y(x) = -\frac{x^2 \ln(x)}{2} + \frac{x^2}{4} + \frac{c_1 x^2}{2} + c_2$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 30

```
DSolve[x*y''[x]+x==y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}x^2 \log(x) + \frac{1}{4}(1 + 2c_1)x^2 + c_2$$

18.11 problem 11

Internal problem ID [2283]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x'' + x't = t^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(x(t),t$2)+t*diff(x(t),t)=t^3,x(t), singsol=all)
```

$$x(t) = \frac{t^3}{3} + \frac{c_1 \sqrt{\pi}\, \sqrt{2}\, \operatorname{erf}\left(\frac{\sqrt{2}\, t}{2}\right)}{2} - 2t + c_2$$

✓ Solution by Mathematica

Time used: 0.108 (sec). Leaf size: 38

```
DSolve[x''[t]+t*x'[t]==t^3,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \sqrt{\frac{\pi}{2}} c_1 \operatorname{erf}\left(\frac{t}{\sqrt{2}}\right) + \frac{t^3}{3} - 2t + c_2$$

18.12 problem 12

Internal problem ID [2284]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x^2y'' - y'x = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x^2*diff(y(x),x$2)=x*diff(y(x),x)+1,y(x), singsol=all)
```

$$y(x) = \frac{c_1x^2}{2} - \frac{\ln(x)}{2} + c_2$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 23

```
DSolve[x^2*y''[x]==x*y'[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1x^2}{2} - \frac{\log(x)}{2} + c_2$$

18.13 problem 13

Internal problem ID [2285]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{y'' - y'^2 = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)=1+diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x) = -\ln\left(\frac{c_1 \tan(x) - c_2}{\sec(x)}\right)$$

✓ Solution by Mathematica

Time used: 1.736 (sec). Leaf size: 16

```
DSolve[y''[x]==1+y'[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \log(\cos(x + c_1))$$

18.14 problem 14

Internal problem ID [2286]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{(-x^2+1)\,y''+y'x=1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 69

```
dsolve((1-x^2)*diff(y(x),x$2)+x*diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x)=\frac{x^2}{2}+c_1\left(\frac{\sqrt{x-1}\,(x+1)^{\frac{3}{2}}}{2}-\frac{\sqrt{x-1}\,\sqrt{x+1}}{2}-\frac{\sqrt{(x+1)\,(x-1)}\,\ln\left(x+\sqrt{x^2-1}\right)}{2\sqrt{x+1}\,\sqrt{x-1}}\right)+c_2$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 50

```
DSolve[(1-x^2)*y''[x]+x*y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}\left(-2c_1\operatorname{arctanh}\left(\frac{\sqrt{x^2-1}}{x-1}\right)+x^2+c_1\sqrt{x^2-1}x+2c_2\right)$$

18.15 problem 15

Internal problem ID [2287]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 15.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - \sqrt{y'^2 + 1} = 0}$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)=sqrt(1+diff(y(x),x)^2),y(x), singsol=all)
```

$$y(x) = -ix + c_1$$
$$y(x) = ix + c_1$$
$$y(x) = \cosh(x + c_1) + c_2$$

✓ Solution by Mathematica

Time used: 0.34 (sec). Leaf size: 29

```
DSolve[y''[x]==Sqrt[1+y'[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(e^{-x-c_1} + e^{x+c_1}\right) + c_2$$

18.16 problem 16

Internal problem ID [2288]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{y'' - y'^2 - y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$2)=diff(y(x),x)^2+diff(y(x),x),y(x), singsol=all)
```

$$y(x) = -\ln\left(-c_1 e^{x} - c_2\right)$$

✓ Solution by Mathematica

Time used: 1.815 (sec). Leaf size: 31

```
DSolve[y''[x]==y'[x]^2+y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \log\left(-1 + e^{x+c_1}\right)$$
$$y(x) \to c_2 - i\pi$$

18.17 problem 17

Internal problem ID [2289]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _`

$$\boxed{y'' - y'y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)=y(x)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \frac{\tan\left(\frac{(c_2+x)\sqrt{2}}{2c_1}\right)\sqrt{2}}{c_1}$$

✓ Solution by Mathematica

Time used: 15.535 (sec). Leaf size: 34

```
DSolve[y''[x]==y[x]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{2}\sqrt{c_1}\tan\left(\frac{\sqrt{c_1}(x+c_2)}{\sqrt{2}}\right)$$

18.18 problem 18

Internal problem ID [2290]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$\boxed{(x^2+1)\,y''+y'^2=-1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve((1+x^2)*diff(y(x),x$2)+1+diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x)=\frac{x}{c_1}-\frac{(-c_1^2-1)\ln(c_1x-1)}{c_1^2}+c_2$$

✓ Solution by Mathematica

Time used: 7.025 (sec). Leaf size: 33

```
DSolve[(1+x^2)*y''[x]+1+y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -x\cot(c_1)+\csc^2(c_1)\log(-x\sin(c_1)-\cos(c_1))+c_2$$

18.19 problem 19

Internal problem ID [2291]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _`

$$\boxed{y'' + y'y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\tanh\left(\frac{(c_2+x)\sqrt{2}}{2c_1}\right)\sqrt{2}}{c_1}$$

✓ Solution by Mathematica

Time used: 7.104 (sec). Leaf size: 34

```
DSolve[y''[x]+y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{2}\sqrt{c_1}\tanh\left(\frac{\sqrt{c_1}(x+c_2)}{\sqrt{2}}\right)$$

18.20 problem 20

Internal problem ID [2292]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{y'' + 2y'^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{\ln(2c_1x + 2c_2)}{2}$$

✓ Solution by Mathematica

Time used: 0.183 (sec). Leaf size: 21

```
DSolve[y''[x]+2*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\log(2x - c_1) + c_2$$

18.21 problem 21

Internal problem ID [2293]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _`

$$\boxed{yy'' + y'^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(y(x)*diff(y(x),x$2)+diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \sqrt{2c_1x + 2c_2}$$
$$y(x) = -\sqrt{2c_1x + 2c_2}$$

✓ Solution by Mathematica

Time used: 0.161 (sec). Leaf size: 20

```
DSolve[y[x]*y''[x]+y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2\sqrt{2x - c_1}$$

18.22 problem 22

Internal problem ID [2294]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{yy'' - y'^2 = -1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 79

```
dsolve(y(x)*diff(y(x),x$2)+1=diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \left(\mathrm{e}^{-\frac{2c_2}{c_1}} \mathrm{e}^{-\frac{2x}{c_1}} - 1\right) \mathrm{e}^{\frac{c_2}{c_1}} \mathrm{e}^{\frac{x}{c_1}}}{2}$$

$$y(x) = \frac{c_1 \left(\mathrm{e}^{\frac{2c_2}{c_1}} \mathrm{e}^{\frac{2x}{c_1}} - 1\right) \mathrm{e}^{-\frac{c_2}{c_1}} \mathrm{e}^{-\frac{x}{c_1}}}{2}$$

✓ Solution by Mathematica

Time used: 60.25 (sec). Leaf size: 85

```
DSolve[y[x]*y''[x]+1==y'[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{ie^{-c_1} \tanh\left(e^{c_1}(x + c_2)\right)}{\sqrt{-\operatorname{sech}^2\left(e^{c_1}(x + c_2)\right)}}$$

$$y(x) \to \frac{ie^{-c_1} \tanh\left(e^{c_1}(x + c_2)\right)}{\sqrt{-\operatorname{sech}^2\left(e^{c_1}(x + c_2)\right)}}$$

18.23 problem 23

Internal problem ID [2295]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)=y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 20

```
DSolve[y''[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^x + c_2 e^{-x}$$

18.24 problem 24

Internal problem ID [2296]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _`

$$\boxed{yy'' + y'^2 - y'y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(y(x)*diff(y(x),x$2)+diff(y(x),x)^2=y(x)*diff(y(x),x),y(x), singsol=all)
```

$$
\begin{aligned}
y(x) &= 0 \\
y(x) &= \sqrt{2c_1 \mathrm{e}^x + 2c_2} \\
y(x) &= -\sqrt{2c_1 \mathrm{e}^x + 2c_2}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 1.093 (sec). Leaf size: 41

```
DSolve[y[x]*y''[x]+y'[x]^2==y[x]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) &\to c_2 \sqrt{2e^x + e^{c_1}} \\
y(x) &\to \sqrt{2} c_2 \sqrt{e^x}
\end{aligned}
$$

18.25 problem 25

Internal problem ID [2297]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{2yy'' - {y'}^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(2*y(x)*diff(y(x),x$2)-diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{1}{4}c_1^2x^2 + \frac{1}{2}c_1xc_2 + \frac{1}{4}c_2^2$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 29

```
DSolve[2*y[x]*y''[x]-y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(c_1x + 2c_2)^2}{4c_2}$$
$$y(x) \to \text{Indeterminate}$$

18.26 problem 26

Internal problem ID [2298]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{y'' + 2{y'}^2 = 2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)^2=2,y(x), singsol=all)
```

$$y(x) = x + \frac{\ln\left(\frac{e^{-4x}c_1}{2} - \frac{c_2}{2}\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.389 (sec). Leaf size: 62

```
DSolve[y''[x]+2*y'[x]^2==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-\log\left(e^{2x}\right) + \log\left(e^{4x} + e^{2c_1}\right) + 2c_2\right)$$
$$y(x) \to \frac{1}{2}\left(-\log\left(e^{2x}\right) + \log\left(e^{4x}\right) + 2c_2\right)$$

18.27 problem 27

Internal problem ID [2299]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - {y'}^3 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=diff(y(x),x)^3,y(x), singsol=all)
```

$$y(x) = -\operatorname{arctanh}\left(\sqrt{-e^{2x}c_1 + 1}\right) + c_2$$
$$y(x) = \operatorname{arctanh}\left(\sqrt{-e^{2x}c_1 + 1}\right) + c_2$$

✓ Solution by Mathematica

Time used: 60.1 (sec). Leaf size: 47

```
DSolve[y''[x]+y'[x]==y'[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \operatorname{arctanh}\left(\sqrt{1 + e^{2(x+c_1)}}\right)$$
$$y(x) \to \operatorname{arctanh}\left(\sqrt{1 + e^{2(x+c_1)}}\right) + c_2$$

18.28 problem 28

Internal problem ID [2300]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{(y+1)\, y'' - 3{y'}^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 59

```
dsolve((y(x)+1)*diff(y(x),x$2)=3*diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x) = -1$$
$$y(x) = -\frac{\sqrt{-2c_1x - 2c_2} - 1}{\sqrt{-2c_1x - 2c_2}}$$
$$y(x) = -\frac{\sqrt{-2c_1x - 2c_2} + 1}{\sqrt{-2c_1x - 2c_2}}$$

✓ Solution by Mathematica

Time used: 1.378 (sec). Leaf size: 107

```
DSolve[(y[x]+1)*y''[x]==3*y'[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2c_1x + \sqrt{2}\sqrt{-c_1(x+c_2)} + 2c_2c_1}{2c_1(x+c_2)}$$

$$y(x) \to \frac{-2c_1x + \sqrt{2}\sqrt{-c_1(x+c_2)} - 2c_2c_1}{2c_1(x+c_2)}$$

$$y(x) \to -1$$

$$y(x) \to \text{Indeterminate}$$

18.29 problem 29

Internal problem ID [2301]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = \sec(x)\tan(x)}$$

With initial conditions

$$\left[y(0) = \frac{\pi}{4}, y'(0) = 1\right]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 14

```
dsolve([diff(y(x),x$2)=sec(x)*tan(x),y(0) = 1/4*Pi, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = \ln(\sec(x) + \tan(x)) + \frac{\pi}{4}$$

✓ Solution by Mathematica

Time used: 0.09 (sec). Leaf size: 20

```
DSolve[{y''[x]==Sec[x]*Tan[x],{y[0]==Pi/4,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{1}{4}\left(8\text{arctanh}\left(\tan\left(\frac{x}{2}\right)\right) + \pi\right)$$

18.30 problem 30

Internal problem ID [2302]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{2y'' - e^{y} = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 15

```
dsolve([2*diff(y(x),x$2)=exp(y(x)),y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = 2\ln(2) + \ln\left(\frac{1}{(x-2)^2}\right)$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 15

```
DSolve[{2*y''[x]==Exp[y[x]],{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2\log\left(1 - \frac{x}{2}\right)$$

18.31 problem 31

Internal problem ID [2303]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' - y^3 = 0}$$

With initial conditions

$$\left[y(0) = -1, y'(0) = \frac{\sqrt{2}}{2}\right]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 16

```
dsolve([diff(y(x),x$2)=y(x)^3,y(0) = -1, D(y)(0) = 1/2*2^(1/2)],y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{2}}{x + \sqrt{2}}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 18

```
DSolve[{y''[x]==y[x]^3,{y[0]==1,y'[0]==Sqrt[2]/2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{\sqrt{2}x - 2}$$

18.32 problem 32

Internal problem ID [2304]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$\boxed{y'' - {y'}^2 \cos(x) = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 1]$$

✗ Solution by Maple

```
dsolve([diff(y(x),x$2)=diff(y(x),x)^2*cos(x),y(0) = 2, D(y)(0) = 1],y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y''[x]==y'[x]^2*Cos[x],{y[0]==2,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

{}

18.33 problem 33

Internal problem ID [2305]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _with_potential_symmet`

$$\boxed{yy'' - y^2y' - y'^2 = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 14

```
dsolve([y(x)*diff(y(x),x$2)-y(x)^2*diff(y(x),x)=diff(y(x),x)^2,y(0) = 2, D(y)(0) = 1],y(x),
```

$$y(x) = -\frac{6}{\mathrm{e}^{\frac{3x}{2}} - 4}$$

✓ Solution by Mathematica

Time used: 1.908 (sec). Leaf size: 18

```
DSolve[{y[x]*y''[x]-y[x]^2*y'[x]==y'[x]^2,{y[0]==2,y'[0]==1}},y[x],x,IncludeSingularSolution
```

$$y(x) \to -\frac{6}{e^{3x/2} - 4}$$

18.34 problem 34

Internal problem ID [2306]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$\boxed{(x^2+1)\, y'' + {y'}^2 = -1}$$

With initial conditions

$$[y(0) = 1, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 21

```
dsolve([(1+x^2)*diff(y(x),x$2)+1+diff(y(x),x)^2=0,y(0) = 1, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = -x + 2\ln(-x-1) - 2i\pi + 1$$

✓ Solution by Mathematica

Time used: 6.806 (sec). Leaf size: 23

```
DSolve[{(1+x^2)*y''[x]+1+y'[x]^2==0,{y[0]==1,y'[0]==1}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to -x + 2\log(-x-1) - 2i\pi + 1$$

18.35 problem 35

Internal problem ID [2307]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{yy'' - y^3 - {y'}^2 = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 2]$$

✓ Solution by Maple

Time used: 12.234 (sec). Leaf size: 25

```
dsolve([y(x)*diff(y(x),x$2)=y(x)^3+diff(y(x),x)^2,y(0) = 1, D(y)(0) = 2],y(x), singsol=all)
```

$$y(x) = -\operatorname{sech}\left(\frac{\sqrt{2}\,(x - \sqrt{2}\operatorname{arctanh}(\sqrt{2}))}{2}\right)^2$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y[x]*y''[x]==y[x]^3+y'[x]^2,{y[0]==1,y'[0]==2}},y[x],x,IncludeSingularSolutions -> T
```

{}

18.36 problem 36

Internal problem ID [2308]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{\left(y'^2+1\right)^2-y^2y''=0}$$

With initial conditions

$$\left[y(0)=3, y'(0)=\sqrt{2}\right]$$

✓ Solution by Maple

Time used: 29.704 (sec). Leaf size: 221

```
dsolve([(1+diff(y(x),x)^2)^2=y(x)^2*diff(y(x),x$2),y(0) = 3, D(y)(0) = 2^(1/2)],y(x), singso
```

$$y(x) = \operatorname{RootOf}\left(\sqrt{2}\left(\int_{\_Z}^{3}\frac{\operatorname{RootOf}\left(\left(-\sqrt{-(3\_Z-1)(6\_Z+1)}+6\right.\right.}{\sqrt{-\left(\operatorname{RootOf}\left(\left(-\sqrt{-(3\_Z-1)(6\_Z+1)}+6\_Z-2\right)\sqrt{2}\right)\_a-1\right)\left(2\operatorname{RootOf}\right.}}\right.\right. + x\Bigg)$$

$$y(x) = \operatorname{RootOf}\left(\sqrt{2}\left(\int_{3}^{\_Z}\frac{\operatorname{RootOf}\left(\left(\sqrt{-(3\_Z-1)(6\_Z+1)}+6\_\right.\right.}{\sqrt{-\left(\operatorname{RootOf}\left(\left(\sqrt{-(3\_Z-1)(6\_Z+1)}+6\_Z-2\right)\sqrt{2}\right)\_a-1\right)\left(2\operatorname{RootOf}\left(\right.\right.}}\right.\right. + x\Bigg)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{(1+y'[x]^2)^2==y[x]^2+y''[x],{y[0]==3,y'[0]==Sqrt[2]}},y[x],x,IncludeSingularSolutio
```

Not solved

18.37 problem 37

Internal problem ID [2309]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 37.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$\boxed{y'' - {y'}^2 \sin(x) = 0}$$

With initial conditions

$$\left[y(0) = 0, y'(0) = \frac{1}{2}\right]$$

✗ Solution by Maple

```
dsolve([diff(y(x),x$2)=diff(y(x),x)^2*sin(x),y(0) = 0, D(y)(0) = 1/2],y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 1.773 (sec). Leaf size: 6

```
DSolve[{y''[x]==y'[x]^2*Sin[x],{y[0]==0,y'[0]==1/2}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \text{Indeterminate}$$

18.38 problem 38

Internal problem ID [2310]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2yy'' - y^3 - 2y'^2 = 0}$$

With initial conditions

$$[y(0) = -1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.39 (sec). Leaf size: 15

```
dsolve([2*y(x)*diff(y(x),x$2)=y(x)^3+2*diff(y(x),x)^2,y(0) = -1, D(y)(0) = 0],y(x), singsol=
```

$$y(x) = \mathrm{RootOf}\left(2\ \mathrm{arctanh}\left(\sqrt{\_Z+1}\right)+x\right)$$

✓ Solution by Mathematica

Time used: 60.205 (sec). Leaf size: 15

```
DSolve[{2*y[x]*y''[x]==y[x]^3+2*y'[x]^2,{y[0]==-1,y'[0]==0}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to -\mathrm{sech}^2\left(\frac{x}{2}\right)$$

18.39 problem 39

Internal problem ID [2311]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' - k^2 x = 0}$$

With initial conditions

$$[x(0) = 0, x'(0) = v_0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve([diff(x(t),t$2)-k^2*x(t)=0,x(0) = 0, D(x)(0) = v__0],x(t), singsol=all)
```

$$x(t) = \frac{v_0 (e^{kt} - e^{-kt})}{2k}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 27

```
DSolve[{x''[t]-k^2*x[t]==0,{x[0]==0,x'[0]==v0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{\mathrm{v0} e^{-kt} (e^{2kt} - 1)}{2k}$$

18.40 problem 40

Internal problem ID [2312]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{yy'' - 2y'^2 - y^2 = 0}$$

With initial conditions

$$\left[y(0) = 1, y'(0) = \sqrt{3}\right]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 16

```
dsolve([y(x)*diff(y(x),x$2)=2*diff(y(x),x)^2+y(x)^2,y(0) = 1, D(y)(0) = 3^(1/2)],y(x), sings
```

$$y(x) = \frac{1}{-\sqrt{3}\,\sin(x) + \cos(x)}$$

✓ Solution by Mathematica

Time used: 0.175 (sec). Leaf size: 19

```
DSolve[{y[x]*y''[x]==2*y'[x]^2+y[x]^2,{y[0]==1,y'[0]==Sqrt[3]}},y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{1}{2}\csc\left(\frac{1}{6}(\pi - 6x)\right)$$

18.41 problem 41

Internal problem ID [2313]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 35, page 157
Problem number: 41.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{(1-\mathrm{e}^{x})\, y''-\mathrm{e}^{x} y'=0}$$

With initial conditions

$$[y(1)=0, y'(1)=1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 27

```
dsolve([(1-exp(x))*diff(y(x),x$2)=exp(x)*diff(y(x),x),y(1) = 0, D(y)(1) = 1],y(x), singsol=a
```

$$y(x)=-(\ln(\mathrm{e}^{x})+\ln(-1+\mathrm{e})-\ln(-1+\mathrm{e}^{x})-1)\,(-1+\mathrm{e})$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 27

```
DSolve[{(1-Exp[x])*y''[x]==Exp[x]*y'[x],{y[1]==0,y'[1]==1}},y[x],x,IncludeSingularSolutions
```

$$y(x)\to -2(e-1)\,(\operatorname{arctanh}(1-2e)-\operatorname{arctanh}(1-2e^{x}))$$

19 Exercise 37, page 171

19.1 problem 1

Internal problem ID [2314]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 37, page 171
Problem number: 1.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{4y^2 - {y'}^2 x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(4*y(x)^2=diff(y(x),x)^2*x^2,y(x), singsol=all)
```

$$y(x) = c_1 x^2$$
$$y(x) = \frac{c_1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 26

```
DSolve[4*y[x]^2==y'[x]^2*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x^2}$$
$$y(x) \to c_1 x^2$$
$$y(x) \to 0$$

19.2 problem 2

Internal problem ID [2315]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 37, page 171
Problem number: 2.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xyy'^2 + (y + x)\, y' = -1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(x*y(x)*diff(y(x),x)^2+(x+y(x))*diff(y(x),x)+1=0,y(x), singsol=all)
```

$$y(x) = -\ln(x) + c_1$$
$$y(x) = \sqrt{-2x + c_1}$$
$$y(x) = -\sqrt{-2x + c_1}$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 53

```
DSolve[x*y[x]*y'[x]^2+(x+y[x])*y'[x]+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}\sqrt{-x + c_1}$$
$$y(x) \to \sqrt{2}\sqrt{-x + c_1}$$
$$y(x) \to -\log(x) + c_1$$

19.3 problem 3

Internal problem ID [2316]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 37, page 171
Problem number: 3.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{(2y - x^2)\, y'^2 - 2x^2 y y'^2 = -1}$$

✗ Solution by Maple

```
dsolve(1+(2*y(x)-x^2)*diff(y(x),x)^2-2*x^2*y(x)*diff(y(x),x)^2=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[1+(2*y[x]-x^2)*y'[x]^2-2*x^2*y[x]*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

19.4 problem 4

Internal problem ID [2317]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 37, page 171
Problem number: 4.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{x\left(-1+y'^2\right)-2y'y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 23

```
dsolve(x*(diff(y(x),x)^2-1)=2*y(x)*diff(y(x),x),y(x), singsol=all)
```

$$y(x)=-\frac{\left(-\frac{x^2}{c_1^2}+1\right)c_1}{2}$$
$$y(x)=c_1x$$

✓ Solution by Mathematica

Time used: 0.138 (sec). Leaf size: 71

```
DSolve[x*(y'[x]^2-1)==2*y[x]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}e^{-c_1}\left(-x^2+e^{2c_1}\right)$$
$$y(x)\to\frac{1}{2}e^{-c_1}\left(-1+e^{2c_1}x^2\right)$$
$$y(x)\to-ix$$
$$y(x)\to ix$$

19.5 problem 5

Internal problem ID [2318]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 37, page 171
Problem number: 5.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(1-y^2)\, y'^2 = 1}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 48

```
dsolve((1-y(x)^2)*diff(y(x),x)^2=1,y(x), singsol=all)
```

$$y(x) = \sin\left(\operatorname{RootOf}\left(\sin(\_Z)\sqrt{\frac{\cos(2\_Z)}{2}+\frac{1}{2}}+\_Z+2c_1-2x\right)\right)$$

$$y(x) = \sin\left(\operatorname{RootOf}\left(-\sin(\_Z)\sqrt{\frac{\cos(2\_Z)}{2}+\frac{1}{2}}-\_Z+2c_1-2x\right)\right)$$

✓ Solution by Mathematica

Time used: 0.089 (sec). Leaf size: 105

```
DSolve[(1-y[x]^2)*y'[x]^2==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{1}{2}\#1\sqrt{1-\#1^2}-\arctan\left(\frac{\sqrt{1-\#1^2}}{\#1+1}\right)\&\right][-x+c_1]$$

$$y(x) \to \text{InverseFunction}\left[\frac{1}{2}\#1\sqrt{1-\#1^2}-\arctan\left(\frac{\sqrt{1-\#1^2}}{\#1+1}\right)\&\right][x+c_1]$$

19.6 problem 6

Internal problem ID [2319]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 37, page 171
Problem number: 6.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational]`

$$\boxed{xyy'^2 + (xy - 1)\, y' - y = 0}$$

✗ Solution by Maple

```
dsolve(x*y(x)*diff(y(x),x)^2+(x*y(x)-1)*diff(y(x),x)=y(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y[x]*y'[x]^2+(x*y[x]-1)*y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

19.7 problem 7

Internal problem ID [2320]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 37, page 171
Problem number: 7.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^2 {y'}^2 + xyy' = 2x^2}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 49

```
dsolve(y(x)^2*diff(y(x),x)^2+x*y(x)*diff(y(x),x)-2*x^2=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x^2 + c_1}$$
$$y(x) = -\sqrt{x^2 + c_1}$$
$$y(x) = \sqrt{-2x^2 + c_1}$$
$$y(x) = -\sqrt{-2x^2 + c_1}$$

✓ Solution by Mathematica

Time used: 0.138 (sec). Leaf size: 80

```
DSolve[y[x]^2*y'[x]^2+x*y[x]*y'[x]-2*x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}\sqrt{-x^2 + c_1}$$
$$y(x) \to \sqrt{2}\sqrt{-x^2 + c_1}$$
$$y(x) \to -\sqrt{x^2 + 2c_1}$$
$$y(x) \to \sqrt{x^2 + 2c_1}$$

19.8 problem 8

Internal problem ID [2321]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 37, page 171
Problem number: 8.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y^2 {y'}^2 - 2xyy' + 2y^2 = x^2}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 107

```
dsolve(y(x)^2*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x)+2*y(x)^2=x^2,y(x), singsol=all)
```

$$y(x) = -x$$
$$y(x) = x$$
$$y(x) = \sqrt{-2c_1\sqrt{2}\,x - c_1^2 - x^2}$$
$$y(x) = \sqrt{2c_1\sqrt{2}\,x - c_1^2 - x^2}$$
$$y(x) = -\sqrt{-2c_1\sqrt{2}\,x - c_1^2 - x^2}$$
$$y(x) = -\sqrt{2c_1\sqrt{2}\,x - c_1^2 - x^2}$$

✓ Solution by Mathematica

Time used: 6.072 (sec). Leaf size: 233

```
DSolve[y[x]^2*y'[x]^2-2*x*y[x]*y'[x]+2*y[x]^2==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^2 - 4\sqrt{2}x\cosh(c_1) - 4\sqrt{2}x\sinh(c_1) - 4\cosh(2c_1) - 4\sinh(2c_1)}$$
$$y(x) \to \sqrt{-x^2 - 4\sqrt{2}x\cosh(c_1) - 4\sqrt{2}x\sinh(c_1) - 4\cosh(2c_1) - 4\sinh(2c_1)}$$
$$y(x) \to -\sqrt{-x^2 + 4\sqrt{2}x\cosh(c_1) + 4\sqrt{2}x\sinh(c_1) - 4\cosh(2c_1) - 4\sinh(2c_1)}$$
$$y(x) \to \sqrt{-x^2 + 4\sqrt{2}x\cosh(c_1) + 4\sqrt{2}x\sinh(c_1) - 4\cosh(2c_1) - 4\sinh(2c_1)}$$
$$y(x) \to -\sqrt{-x^2}$$
$$y(x) \to \sqrt{-x^2}$$

19.9 problem 9

Internal problem ID [2322]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 37, page 171
Problem number: 9.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^3 + (x + y - 2yx)\, y'^2 - 2y'xy(y + x) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 28

```
dsolve(diff(y(x),x)^3+(x+y(x)-2*x*y(x))*diff(y(x),x)^2-2*diff(y(x),x)*x*y(x)*(x+y(x))=0,y(x)
```

$$y(x) = c_1 \mathrm{e}^{x^2}$$
$$y(x) = -x + 1 + \mathrm{e}^{-x} c_1$$
$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 36

```
DSolve[y'[x]^3+(x+y[x]-2*x*y[x])*y'[x]^2-2*y'[x]*x*y[x]*(x+y[x])==0,y[x],x,IncludeSingularSo
```

$$y(x) \to c_1$$
$$y(x) \to c_1 e^{x^2}$$
$$y(x) \to -x + c_1 e^{-x} + 1$$

19.10 problem 10

Internal problem ID [2323]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 37, page 171
Problem number: 10.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{yy'^2 + (y^2 - x^3 - y^2x)\, y' - xy(x^2 + y^2) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 48

```
dsolve(y(x)*diff(y(x),x)^2+(y(x)^2-x^3-x*y(x)^2)*diff(y(x),x)-x*y(x)*(x^2+y(x)^2)=0,y(x), si
```

$$y(x) = e^{-x}c_1$$
$$y(x) = \sqrt{c_1 e^{x^2} - x^2 - 1}$$
$$y(x) = -\sqrt{c_1 e^{x^2} - x^2 - 1}$$

✓ Solution by Mathematica

Time used: 14.233 (sec). Leaf size: 61

```
DSolve[y[x]*y'[x]^2+(y[x]^2-x^3-x*y[x]^2)*y'[x]-x*y[x]*(x^2+y[x]^2)==0,y[x],x,IncludeSingula
```

$$y(x) \to c_1 e^{-x}$$
$$y(x) \to -\sqrt{-x^2 + c_1 e^{x^2} - 1}$$
$$y(x) \to \sqrt{-x^2 + c_1 e^{x^2} - 1}$$

19.11 problem 11

Internal problem ID [2324]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 37, page 171
Problem number: 11.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y - y'x(1 + y') = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 69

```
dsolve(y(x)=diff(y(x),x)*x*(1+diff(y(x),x)),y(x), singsol=all)
```

$$y(x) = \frac{x\left(\frac{1}{2\,\mathrm{LambertW}\left(-\frac{1}{2\sqrt{\frac{c_1}{x}}}\right)} + 1\right)}{2\,\mathrm{LambertW}\left(-\frac{1}{2\sqrt{\frac{c_1}{x}}}\right)}$$

$$y(x) = \frac{x\left(\frac{1}{2\,\mathrm{LambertW}\left(\frac{1}{2\sqrt{\frac{c_1}{x}}}\right)} + 1\right)}{2\,\mathrm{LambertW}\left(\frac{1}{2\sqrt{\frac{c_1}{x}}}\right)}$$

✓ Solution by Mathematica

Time used: 0.566 (sec). Leaf size: 102

```
DSolve[y[x]==y'[x]*x*(1+y'[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{\sqrt{\frac{4y(x)}{x}+1}-1}-\log\left(\sqrt{\frac{4y(x)}{x}+1}-1\right)=\frac{\log(x)}{2}+c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{\sqrt{\frac{4y(x)}{x}+1}+1}+\log\left(\sqrt{\frac{4y(x)}{x}+1}+1\right)=-\frac{\log(x)}{2}+c_1,y(x)\right]$$

$$y(x)\to 0$$

19.12 problem 12

Internal problem ID [2325]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 37, page 171
Problem number: 12.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_separable]`

$$\boxed{y - 3\ln(y') = x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(y(x)=x+3*ln(diff(y(x),x)),y(x), singsol=all)
```

$$y(x) = x$$
$$y(x) = x + 3\ln\left(\frac{\mathrm{e}^{-\frac{x}{3}}c_1}{-1 + \mathrm{e}^{-\frac{x}{3}}c_1}\right)$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 22

```
DSolve[y[x]==x+3*Log[y'[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -3\log\left(e^{-x/3} - \frac{c_1}{3}\right)$$

19.13 problem 13

Internal problem ID [2326]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 37, page 171
Problem number: 13.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y\left(y'^2+1\right)=2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
dsolve(y(x)*(1+diff(y(x),x)^2)=2,y(x), singsol=all)
```

$$y(x)=2$$
$$y(x)=-\sin\left(\operatorname{RootOf}\left(-\_Z-x-\sqrt{\frac{\cos(2\_Z)}{2}+\frac{1}{2}}+c_1\right)\right)+1$$
$$y(x)=\sin\left(\operatorname{RootOf}\left(-\_Z-x+\sqrt{\frac{\cos(2\_Z)}{2}+\frac{1}{2}}+c_1\right)\right)+1$$

✓ Solution by Mathematica

Time used: 0.359 (sec). Leaf size: 118

```
DSolve[y[x]*(1+y'[x]^2)==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-4\arctan\left(\frac{\sqrt{\#1}}{\sqrt{2}-\sqrt{2-\#1}}\right)-\sqrt{-((\#1-2)\#1)}\&\right][-x+c_1]$$

$$y(x) \to \text{InverseFunction}\left[-4\arctan\left(\frac{\sqrt{\#1}}{\sqrt{2}-\sqrt{2-\#1}}\right)-\sqrt{-((\#1-2)\#1)}\&\right][x+c_1]$$

$$y(x) \to 2$$

19.14 problem 14

Internal problem ID [2327]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 37, page 171
Problem number: 14.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{yy'^2 - 2y'x + y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 75

```
dsolve(y(x)*diff(y(x),x)^2-2*x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$
\begin{aligned}
y(x) &= -x \\
y(x) &= x \\
y(x) &= 0 \\
y(x) &= \sqrt{-2c_1xi + c_1^2} \\
y(x) &= \sqrt{2c_1xi + c_1^2} \\
y(x) &= -\sqrt{-2c_1xi + c_1^2} \\
y(x) &= -\sqrt{2c_1xi + c_1^2}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 2.388 (sec). Leaf size: 174

```
DSolve[y[x]*y'[x]^2-2*x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{4}\left(\cosh\left(\frac{c_1}{2}\right) + \sinh\left(\frac{c_1}{2}\right)\right)\sqrt{-8ix + \cosh(c_1) + \sinh(c_1)}$$
$$y(x) \to \frac{1}{4}\left(\cosh\left(\frac{c_1}{2}\right) + \sinh\left(\frac{c_1}{2}\right)\right)\sqrt{-8ix + \cosh(c_1) + \sinh(c_1)}$$
$$y(x) \to -\frac{1}{4}\left(\cosh\left(\frac{c_1}{2}\right) + \sinh\left(\frac{c_1}{2}\right)\right)\sqrt{8ix + \cosh(c_1) + \sinh(c_1)}$$
$$y(x) \to \frac{1}{4}\left(\cosh\left(\frac{c_1}{2}\right) + \sinh\left(\frac{c_1}{2}\right)\right)\sqrt{8ix + \cosh(c_1) + \sinh(c_1)}$$
$$y(x) \to 0$$
$$y(x) \to -x$$
$$y(x) \to x$$

19.15 problem 15

Internal problem ID [2328]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 37, page 171
Problem number: 15.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 + y^2 = 1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)^2+y(x)^2=1,y(x), singsol=all)
```

$$y(x) = -1$$
$$y(x) = 1$$
$$y(x) = -\sin(-x + c_1)$$
$$y(x) = \sin(-x + c_1)$$

✓ Solution by Mathematica

Time used: 0.175 (sec). Leaf size: 39

```
DSolve[y'[x]^2+y[x]^2==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x + c_1)$$
$$y(x) \to \cos(x - c_1)$$
$$y(x) \to -1$$
$$y(x) \to 1$$
$$y(x) \to \text{Interval}[\{-1, 1\}]$$

19.16 problem 16

Internal problem ID [2329]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 37, page 171
Problem number: 16.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{x\left(-1+y'^2\right)-2y'y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve((diff(y(x),x)^2-1)*x=2*diff(y(x),x)*y(x),y(x), singsol=all)
```

$$y(x)=-\frac{\left(-\frac{x^2}{c_1^2}+1\right)c_1}{2}$$
$$y(x)=c_1x$$

✓ Solution by Mathematica

Time used: 0.14 (sec). Leaf size: 71

```
DSolve[(y'[x]^2-1)*x==2*y'[x]*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}e^{-c_1}\left(-x^2+e^{2c_1}\right)$$
$$y(x)\to\frac{1}{2}e^{-c_1}\left(-1+e^{2c_1}x^2\right)$$
$$y(x)\to -ix$$
$$y(x)\to ix$$

19.17 problem 17

Internal problem ID [2330]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 37, page 171
Problem number: 17.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{-2y'y + {y'}^2 x = -4x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(4*x-2*diff(y(x),x)*y(x)+diff(y(x),x)^2*x=0,y(x), singsol=all)
```

$$y(x) = -2x$$
$$y(x) = 2x$$
$$y(x) = -\frac{\left(-\frac{x^2}{c_1^2} - 4\right) c_1}{2}$$

✓ Solution by Mathematica

Time used: 0.295 (sec). Leaf size: 43

```
DSolve[4*x-2*y'[x]*y[x]+y'[x]^2*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2x\cosh(-\log(x) + c_1)$$
$$y(x) \to -2x\cosh(\log(x) + c_1)$$
$$y(x) \to -2x$$
$$y(x) \to 2x$$

19.18 problem 18

Internal problem ID [2331]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 37, page 171
Problem number: 18.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{2x^2y + y'^2 - y'x^3 = 0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 23

```
dsolve(2*x^2*y(x)+diff(y(x),x)^2=diff(y(x),x)*x^3,y(x), singsol=all)
```

$$y(x) = \frac{x^4}{8}$$
$$y(x) = c_1x^2 - 2c_1^2$$

✓ Solution by Mathematica

Time used: 2.642 (sec). Leaf size: 216

```
DSolve[2*x^2*y[x]+y'[x]^2==y'[x]*x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\sqrt{x^6-8x^2y(x)}\log\left(\sqrt{x^4-8y(x)}+x^2\right)}{2x\sqrt{x^4-8y(x)}} - \frac{\sqrt{x^6-8x^2y(x)}\log(y(x))}{4x\sqrt{x^4-8y(x)}}+\frac{1}{4}\log(y(x))=c_1,y(x)\right]$$

$$\text{Solve}\left[-\frac{\sqrt{x^6-8x^2y(x)}\log\left(\sqrt{x^4-8y(x)}+x^2\right)}{2x\sqrt{x^4-8y(x)}} + \frac{\sqrt{x^6-8x^2y(x)}\log(y(x))}{4x\sqrt{x^4-8y(x)}}+\frac{1}{4}\log(y(x))=c_1,y(x)\right]$$

$$y(x)\to\frac{x^4}{8}$$

19.19 problem 19

Internal problem ID [2332]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 37, page 171
Problem number: 19.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{yy'^2 - 3y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.703 (sec). Leaf size: 279

```
dsolve(diff(y(x),x)^2*y(x)=3*diff(y(x),x)*x+y(x),y(x), singsol=all)
```

$$y(x) = 0$$

$$\ln(x) - \frac{3\,\mathrm{arctanh}\left(\frac{3}{\sqrt{\frac{9x^2+4y(x)^2}{x^2}}}\right)}{8} + \frac{5\,\mathrm{arctanh}\left(\frac{9x+8y(x)}{5x\sqrt{\frac{9x^2+4y(x)^2}{x^2}}}\right)}{16} + \frac{5\,\mathrm{arctanh}\left(\frac{9x-8y(x)}{5x\sqrt{\frac{9x^2+4y(x)^2}{x^2}}}\right)}{16} + \frac{3\ln\left(\frac{y(x)}{x}\right)}{8} + \frac{5\ln\left(-\frac{2x-y(x)}{x}\right)}{16} + \frac{5\ln\left(\frac{y(x)+2x}{x}\right)}{16} - c_1 = 0$$

$$\ln(x) + \frac{3\,\mathrm{arctanh}\left(\frac{3}{\sqrt{\frac{9x^2+4y(x)^2}{x^2}}}\right)}{8} - \frac{5\,\mathrm{arctanh}\left(\frac{9x+8y(x)}{5x\sqrt{\frac{9x^2+4y(x)^2}{x^2}}}\right)}{16} - \frac{5\,\mathrm{arctanh}\left(\frac{9x-8y(x)}{5x\sqrt{\frac{9x^2+4y(x)^2}{x^2}}}\right)}{16} + \frac{3\ln\left(\frac{y(x)}{x}\right)}{8} + \frac{5\ln\left(-\frac{2x-y(x)}{x}\right)}{16} + \frac{5\ln\left(\frac{y(x)+2x}{x}\right)}{16} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 75.255 (sec). Leaf size: 2113

```
DSolve[y'[x]^2*y[x]==3*y'[x]*x+y[x],y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

19.20 problem 20

Internal problem ID [2333]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 37, page 171
Problem number: 20.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, _dAlembert]`

$$\boxed{-y{y'}^2=-8x-1}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 135

```
dsolve(8*x+1=diff(y(x),x)^2*y(x),y(x), singsol=all)
```

$$\frac{(-64x-8)\,c_1}{\left(\frac{8x+1-2\sqrt{y(x)(8x+1)}+4y(x)}{y(x)}\right)^{\frac{2}{3}}y\left(x\right)\left(\frac{-\sqrt{y(x)(8x+1)}-2y(x)}{y(x)}\right)^{\frac{2}{3}}}+x+\frac{1}{8}=0$$
$$\frac{(-64x-8)\,c_1}{\left(\frac{8x+1+2\sqrt{y(x)(8x+1)}+4y(x)}{y(x)}\right)^{\frac{2}{3}}y\left(x\right)\left(\frac{\sqrt{y(x)(8x+1)}-2y(x)}{y(x)}\right)^{\frac{2}{3}}}+x+\frac{1}{8}=0$$

✓ Solution by Mathematica

Time used: 3.669 (sec). Leaf size: 79

```
DSolve[8*x+1==y'[x]^2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{4}\left(-8\sqrt{8x+1}x-\sqrt{8x+1}+12c_1\right)^{2/3}$$
$$y(x)\to\frac{1}{4}\left(8\sqrt{8x+1}x+\sqrt{8x+1}+12c_1\right)^{2/3}$$

19.21 problem 21

Internal problem ID [2334]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 37, page 171
Problem number: 21.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{yy'^2 + 2y' = -1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
dsolve(diff(y(x),x)^2*y(x)+2*diff(y(x),x)+1=0,y(x), singsol=all)
```

$$x - \frac{2(1-y(x))^{\frac{3}{2}}}{3} + y(x) - 1 - c_1 = 0$$
$$x + \frac{2(1-y(x))^{\frac{3}{2}}}{3} + y(x) - 1 - c_1 = 0$$

✓ Solution by Mathematica

Time used: 23.957 (sec). Leaf size: 1098

```
DSolve[y'[x]^2*y[x]+2*y'[x]+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{\sqrt[3]{3}\left(-24x^2+36x-48c_1x+8\sqrt{3}\sqrt{(x+c_1)^3(3x-1+3c_1)}-9-24c_1{}^2+36c_1\right)^{2/3}+\sqrt[3]{-24x^2+36x}}{4\sqrt[3]{-24x^2+(36-48c_1)x+8\sqrt{3}\sqrt{(x+c_1)^3(}}}$$

$$y(x) \rightarrow \frac{1}{8}\left(\frac{3^{2/3}\left(1+i\sqrt{3}\right)(8x-3+8c_1)}{\sqrt[3]{-24x^2+(36-48c_1)x+8\sqrt{3}\sqrt{(x+c_1)^3(3x-1+3c_1)}-9-24c_1{}^2+36c_1}} + i\sqrt[3]{3}\left(\sqrt{3}+i\right)\sqrt[3]{-24x^2+36x-48c_1x+8\sqrt{3}\sqrt{(x+c_1)^3(3x-1+3c_1)}-9-24c_1{}^2+36c_1} + 2\right)$$

$$y(x) \rightarrow \frac{1}{8}\left(\frac{3^{2/3}\left(1-i\sqrt{3}\right)(8x-3+8c_1)}{\sqrt[3]{-24x^2+(36-48c_1)x+8\sqrt{3}\sqrt{(x+c_1)^3(3x-1+3c_1)}-9-24c_1{}^2+36c_1}} - \sqrt[3]{3}\left(1+i\sqrt{3}\right)\sqrt[3]{-24x^2+36x-48c_1x+8\sqrt{3}\sqrt{(x+c_1)^3(3x-1+3c_1)}-9-24c_1{}^2+36c_1} + 2\right)$$

$$y(x) \rightarrow \frac{\sqrt[3]{3}\left(-24x^2+36x+48c_1x+8\sqrt{3}\sqrt{(-x+c_1)^3(-3x+1+3c_1)}-9-24c_1{}^2-36c_1\right)^{2/3}+\sqrt[3]{-24x^2+3}}{4\sqrt[3]{-24x^2+12(3+4c_1)x+8\sqrt{3}\sqrt{(-x+c}}}$$

$$y(x) \rightarrow \frac{1}{8}\left(\frac{3^{2/3}\left(1+i\sqrt{3}\right)(8x-3-8c_1)}{\sqrt[3]{-24x^2+12(3+4c_1)x+8\sqrt{3}\sqrt{(-x+c_1)^3(-3x+1+3c_1)}-9-24c_1{}^2-36c_1}} + i\sqrt[3]{3}\left(\sqrt{3}+i\right)\sqrt[3]{-24x^2+12(3+4c_1)x+8\sqrt{3}\sqrt{(-x+c_1)^3(-3x+1+3c_1)}-9-24c_1{}^2-36c_1} + 2\right)$$

$y(x)$

19.22 problem 22

Internal problem ID [2335]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 37, page 171
Problem number: 22.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{\left(y'^2+1\right)x-(y+x)\,y'=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 36

```
dsolve((diff(y(x),x)^2+1)*x=diff(y(x),x)*(x+y(x)),y(x), singsol=all)
```

$$y(x)=x$$

$$y(x)=\frac{x\left(\mathrm{LambertW}\left(\frac{x}{c_1}\right)^2-\mathrm{LambertW}\left(\frac{x}{c_1}\right)+1\right)}{\mathrm{LambertW}\left(\frac{x}{c_1}\right)}$$

✓ Solution by Mathematica

Time used: 3.512 (sec). Leaf size: 162

```
DSolve[(y'[x]^2+1)*x==y'[x]*(x+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{4}\left(-\frac{y(x)}{x}+\sqrt{\frac{y(x)}{x}-1}\sqrt{\frac{y(x)}{x}+3}-4\log\left(\sqrt{\frac{y(x)}{x}-1}-\sqrt{\frac{y(x)}{x}+3}\right)-3\right)=-\frac{\log(x)}{2}+c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{4}\left(\frac{y(x)}{x}+\sqrt{\frac{y(x)}{x}-1}\sqrt{\frac{y(x)}{x}+3}-4\log\left(\sqrt{\frac{y(x)}{x}-1}-\sqrt{\frac{y(x)}{x}+3}\right)+3\right)=\frac{\log(x)}{2}+c_1,y(x)\right]$$

19.23 problem 23

Internal problem ID [2336]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 37, page 171
Problem number: 23.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{-3y'y + {y'}^2 x = -x^2}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 43

```
dsolve(x^2-3*diff(y(x),x)*y(x)+x*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{4x^3 + c_1^2}{6c_1}$$
$$y(x) = \frac{c_1^2 x^3 + 4}{6c_1}$$
$$y(x) = c_1 x^{\frac{3}{2}}$$

✓ Solution by Mathematica

Time used: 0.293 (sec). Leaf size: 83

```
DSolve[x^2-3*y'[x]*y[x]+x*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{6}e^{-\frac{3c_1}{2}}\left(4x^3 + e^{3c_1}\right)$$
$$y(x) \to \frac{1}{6}e^{-\frac{3c_1}{2}}\left(4x^3 + e^{3c_1}\right)$$
$$y(x) \to -\frac{2x^{3/2}}{3}$$
$$y(x) \to \frac{2x^{3/2}}{3}$$

19.24 problem 24

Internal problem ID [2337]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 37, page 171
Problem number: 24.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y + 2y'x - {y'}^2 x = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 116

```
dsolve(y(x)+2*diff(y(x),x)*x=diff(y(x),x)^2*x,y(x), singsol=all)
```

$$x - \frac{c_1 x}{\left(-2x + \sqrt{x\left(x + y\left(x\right)\right)}\right)\left(\frac{-2x+\sqrt{x(x+y(x))}}{x}\right)^{\frac{1}{3}}\left(\frac{x+\sqrt{x(x+y(x))}}{x}\right)^{\frac{2}{3}}} = 0$$

$$x + \frac{c_1 x}{\left(2x + \sqrt{x\left(x + y\left(x\right)\right)}\right)\left(\frac{-2x-\sqrt{x(x+y(x))}}{x}\right)^{\frac{1}{3}}\left(\frac{x-\sqrt{x(x+y(x))}}{x}\right)^{\frac{2}{3}}} = 0$$

✓ Solution by Mathematica

Time used: 60.106 (sec). Leaf size: 1178

```
DSolve[y[x]+2*y'[x]*x==y'[x]^2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{2}\sqrt{x}\left(x^{3/2}-2e^{\frac{3c_1}{2}}\right)}{\sqrt[3]{-10e^{\frac{3c_1}{2}}x^{3/2}+\sqrt{e^{\frac{3c_1}{2}}\left(4x^{3/2}+e^{\frac{3c_1}{2}}\right)^3}-2x^3+e^{3c_1}}} + \frac{\sqrt[3]{-10e^{\frac{3c_1}{2}}x^{3/2}+\sqrt{e^{\frac{3c_1}{2}}\left(4x^{3/2}+e^{\frac{3c_1}{2}}\right)^3}-2x^3+e^{3c_1}}}{\sqrt[3]{2}} + 2x$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)\sqrt{x}\left(-x^{3/2}+2e^{\frac{3c_1}{2}}\right)}{2^{2/3}\sqrt[3]{-10e^{\frac{3c_1}{2}}x^{3/2}+\sqrt{e^{\frac{3c_1}{2}}\left(4x^{3/2}+e^{\frac{3c_1}{2}}\right)^3}-2x^3+e^{3c_1}}} + \frac{i\left(\sqrt{3}+i\right)\sqrt[3]{-10e^{\frac{3c_1}{2}}x^{3/2}+\sqrt{e^{\frac{3c_1}{2}}\left(4x^{3/2}+e^{\frac{3c_1}{2}}\right)^3}-2x^3+e^{3c_1}}}{2\sqrt[3]{2}} + 2x$$

$$y(x) \to \frac{i\left(\sqrt{3}+i\right)\sqrt{x}\left(x^{3/2}-2e^{\frac{3c_1}{2}}\right)}{2^{2/3}\sqrt[3]{-10e^{\frac{3c_1}{2}}x^{3/2}+\sqrt{e^{\frac{3c_1}{2}}\left(4x^{3/2}+e^{\frac{3c_1}{2}}\right)^3}-2x^3+e^{3c_1}}} - \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{-10e^{\frac{3c_1}{2}}x^{3/2}+\sqrt{e^{\frac{3c_1}{2}}\left(4x^{3/2}+e^{\frac{3c_1}{2}}\right)^3}-2x^3+e^{3c_1}}}{2\sqrt[3]{2}} + 2x$$

$$y(x) \to \frac{\sqrt[3]{2}e^{\frac{3c_1}{2}}\sqrt{x}\left(2+e^{\frac{3c_1}{2}}x^{3/2}\right)}{\sqrt[3]{10e^{\frac{15c_1}{2}}x^{3/2}+\sqrt{-e^{12c_1}\left(-1+4e^{\frac{3c_1}{2}}x^{3/2}\right)^3}-2e^{9c_1}x^3+e^{6c_1}}} + \frac{e^{-3c_1}\sqrt[3]{10e^{\frac{15c_1}{2}}x^{3/2}+\sqrt{-e^{12c_1}\left(-1+4e^{\frac{3c_1}{2}}x^{3/2}\right)^3}-2e^{9c_1}x^3+e^{6c_1}}}{\sqrt[3]{2}} + 2x$$

$$y(x) \to \frac{1}{4}\left(-\frac{2\sqrt[3]{2}\left(1+i\sqrt{3}\right)e^{\frac{3c_1}{2}}\sqrt{x}\left(2+e^{\frac{3c_1}{2}}x^{3/2}\right)}{\sqrt[3]{10e^{\frac{15c_1}{2}}x^{3/2}+\sqrt{-e^{12c_1}\left(-1+4e^{\frac{3c_1}{2}}x^{3/2}\right)^3}-2e^{9c_1}x^3+e^{6c_1}}}\right.$$

20 Exercise 38, page 173

20.1 problem 1

Internal problem ID [2338]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 38, page 173
Problem number: 1.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{-y'^2 - y' = -x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
dsolve(x=diff(y(x),x)^2+diff(y(x),x),y(x), singsol=all)
```

$$y(x) = -\frac{x}{2} - \frac{(1+4x)^{\frac{3}{2}}}{12} + c_1$$
$$y(x) = -\frac{x}{2} + \frac{(1+4x)^{\frac{3}{2}}}{12} + c_1$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 54

```
DSolve[x==y'[x]^2+y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-\frac{1}{6}(4x+1)^{3/2} - x\right) + c_1$$
$$y(x) \to \frac{1}{12}\left((4x+1)^{3/2} - 6x + 12c_1\right)$$

20.2 problem 2

Internal problem ID [2339]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 38, page 173
Problem number: 2.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{-y + {y'}^3 = -x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 209

```
dsolve(x=y(x)-diff(y(x),x)^3,y(x), singsol=all)
```

$$x - \frac{3(y(x)-x)^{\frac{2}{3}}}{2} - 3(y(x)-x)^{\frac{1}{3}} - 3\ln\left((y(x)-x)^{\frac{1}{3}} - 1\right) - c_1 = 0$$

$$x + \frac{3(y(x)-x)^{\frac{2}{3}}}{4} - \frac{3i\sqrt{3}\,(y(x)-x)^{\frac{2}{3}}}{4} + \frac{3(y(x)-x)^{\frac{1}{3}}}{2} + \frac{3i\sqrt{3}\,(y(x)-x)^{\frac{1}{3}}}{2} - 3\ln\left(-\frac{(y(x)-x)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}\,(y(x)-x)^{\frac{1}{3}}}{2} - 1\right) - c_1 = 0$$

$$x + \frac{3(y(x)-x)^{\frac{2}{3}}}{4} + \frac{3i\sqrt{3}\,(y(x)-x)^{\frac{2}{3}}}{4} + \frac{3(y(x)-x)^{\frac{1}{3}}}{2} - \frac{3i\sqrt{3}\,(y(x)-x)^{\frac{1}{3}}}{2} - 3\ln\left(-\frac{(y(x)-x)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}\,(y(x)-x)^{\frac{1}{3}}}{2} - 1\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 11.095 (sec). Leaf size: 298

```
DSolve[x==y[x]-y'[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{3}{2}(y(x)-x)^{2/3}+3\sqrt[3]{y(x)-x}+3\log\left(\sqrt[3]{y(x)-x}-1\right)-x=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{2}\left(\frac{1}{2}\sqrt[3]{y(x)-x}\left(4i(y(x)-x)^{2/3}+3\sqrt{3}\sqrt[3]{y(x)-x}-3i\sqrt[3]{y(x)-x}-6\sqrt{3}-6i\right)+6i\log\left(\sqrt{2-2i\sqrt{3}}\right.\right.\right.$$

$$\left.-i(y(x)-x)=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{y(x)}{2}\right.$$

$$+\frac{1}{4}\left(-\frac{1}{2}\sqrt[3]{y(x)-x}\left(4(y(x)-x)^{2/3}+3i\sqrt{3}\sqrt[3]{y(x)-x}-3\sqrt[3]{y(x)-x}-6i\sqrt{3}-6\right)-6\log\left(2i\sqrt[3]{y(x)-x}+\sqrt{}\right.\right.$$

20.3 problem 3

Internal problem ID [2340]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 38, page 173
Problem number: 3.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{2y'y - {y'}^2x = -x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 22

```
dsolve(x+2*diff(y(x),x)*y(x)=diff(y(x),x)^2*x,y(x), singsol=all)
```

$$y(x) = \frac{\left(\frac{x^2}{c_1^2} - 1\right) c_1}{2}$$
$$y(x) = c_1 x$$

✓ Solution by Mathematica

Time used: 0.133 (sec). Leaf size: 71

```
DSolve[x+2*y'[x]*y[x]==y'[x]^2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-c_1} \left(-x^2 + e^{2c_1}\right)$$
$$y(x) \to \frac{1}{2} e^{-c_1} \left(-1 + e^{2c_1} x^2\right)$$
$$y(x) \to -ix$$
$$y(x) \to ix$$

20.4 problem 4

Internal problem ID [2341]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 38, page 173
Problem number: 4.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{-2y'y + {y'}^2 x = -4x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(4*x-2*diff(y(x),x)*y(x)+x*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$\begin{aligned} y(x) &= -2x \\ y(x) &= 2x \\ y(x) &= -\frac{\left(-\frac{x^2}{c_1^2} - 4\right) c_1}{2} \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.172 (sec). Leaf size: 43

```
DSolve[4*x-2*y'[x]*y[x]+x*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to -2x\cosh(-\log(x) + c_1) \\ y(x) &\to -2x\cosh(\log(x) + c_1) \\ y(x) &\to -2x \\ y(x) &\to 2x \end{aligned}$$

20.5 problem 5

Internal problem ID [2342]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 38, page 173
Problem number: 5.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_dAlembert]`

$$\boxed{xy'^3 - y'y = 1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 2249

```
dsolve(x*diff(y(x),x)^3=y(x)*diff(y(x),x)+1,y(x), singsol=all)
```

$$\frac{c_1 x^2 \left(\left(\sqrt{3}\,\sqrt{\frac{-4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{2}{3}} \left(2\left(\sqrt{3}\,\sqrt{\frac{-4y(x)^3+27x}{x}}+9\right)x^2 18^{\frac{1}{3}}\left(\left(\sqrt{3}\,\sqrt{\frac{-4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{1}{3}}+1\cdots\right.}{\left(y(x)\,12^{\frac{2}{3}}x+12^{\frac{1}{3}}\left(\left(\sqrt{3}\,\sqrt{\frac{-4y(x)^3+27x}{x}}+9\right)^2 x^4\right)^{\frac{1}{3}}-6x\left(\left(\sqrt{3}\,\sqrt{\frac{-4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{1}{3}}\right)^2 \left(y\cdots\right.}$$
$$+\,x$$
$$-\frac{3x^2\left(6\sqrt{3}\,\sqrt{\frac{-4y(x)^3+27x}{x}}\,12^{\frac{1}{3}}x^3+12x^2y(x)\,18^{\frac{1}{3}}\left(\left(\sqrt{3}\,\sqrt{\frac{-4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{1}{3}}+54\,12^{\frac{1}{3}}x^3-18x^2\left(\left(\sqrt{3}\cdots\right.\right.\right.}{\left(y(x)\,12^{\frac{2}{3}}x+12^{\frac{1}{3}}\left(\left(\sqrt{3}\,\sqrt{\frac{-4y(x)^3+27x}{x}}+9\right)^2 x^4\right)^{\frac{1}{3}}-6x\left(\left(\sqrt{3}\,\sqrt{\frac{-4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{1}{3}}\right)\cdots}$$
$$=0$$

Expression too large to display

Expression too large to display

✓ Solution by Mathematica

Time used: 144.072 (sec). Leaf size: 21579

```
DSolve[x*y'[x]^3==y[x]*y'[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

20.6 problem 6

Internal problem ID [2343]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 38, page 173
Problem number: 6.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y\left(y'^2+1\right)-2y'x=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 75

```
dsolve((diff(y(x),x)^2+1)*y(x)=2*diff(y(x),x)*x,y(x), singsol=all)
```

$$y(x)=-x$$
$$y(x)=x$$
$$y(x)=0$$
$$y(x)=\sqrt{-2c_1xi+c_1^2}$$
$$y(x)=\sqrt{2c_1xi+c_1^2}$$
$$y(x)=-\sqrt{-2c_1xi+c_1^2}$$
$$y(x)=-\sqrt{2c_1xi+c_1^2}$$

✓ Solution by Mathematica

Time used: 1.091 (sec). Leaf size: 174

```
DSolve[(y'[x]^2+1)*y[x]==2*y'[x]*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{4}\left(\cosh\left(\frac{c_1}{2}\right) + \sinh\left(\frac{c_1}{2}\right)\right)\sqrt{-8ix + \cosh(c_1) + \sinh(c_1)}$$
$$y(x) \to \frac{1}{4}\left(\cosh\left(\frac{c_1}{2}\right) + \sinh\left(\frac{c_1}{2}\right)\right)\sqrt{-8ix + \cosh(c_1) + \sinh(c_1)}$$
$$y(x) \to -\frac{1}{4}\left(\cosh\left(\frac{c_1}{2}\right) + \sinh\left(\frac{c_1}{2}\right)\right)\sqrt{8ix + \cosh(c_1) + \sinh(c_1)}$$
$$y(x) \to \frac{1}{4}\left(\cosh\left(\frac{c_1}{2}\right) + \sinh\left(\frac{c_1}{2}\right)\right)\sqrt{8ix + \cosh(c_1) + \sinh(c_1)}$$
$$y(x) \to 0$$
$$y(x) \to -x$$
$$y(x) \to x$$

20.7 problem 7

Internal problem ID [2344]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 38, page 173
Problem number: 7.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{-2y'y + {y'}^2 x = -2x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve(2*x+diff(y(x),x)^2*x=2*diff(y(x),x)*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{\left(-\frac{x^2}{c_1^2} - 2\right) c_1}{2}$$
$$y(x) = c_1 x$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 63

```
DSolve[2*x+y'[x]*x==2*y'[x]*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{34}\left(\left(17+\sqrt{17}\right)\log\left(\frac{4y(x)}{x}+\sqrt{17}-1\right) - \left(\sqrt{17}-17\right)\log\left(-\frac{4y(x)}{x}+\sqrt{17}+1\right)\right) = -\log(x)+c_1, y(x)\right]$$

20.8 problem 8

Internal problem ID [2345]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 38, page 173
Problem number: 8.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_dAlembert]`

$$\boxed{-y'y - y'^2 = -x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 300

```
dsolve(x=diff(y(x),x)*y(x)+diff(y(x),x)^2,y(x), singsol=all)
```

$$\frac{\left(-y(x)+\sqrt{y(x)^2+4x}\right)c_1}{\sqrt{-2y(x)+2\sqrt{y(x)^2+4x}-4}\,\sqrt{-2y(x)+2\sqrt{y(x)^2+4x}+4}}+x$$
$$+\frac{\left(-y(x)+\sqrt{y(x)^2+4x}\right)\ln\left(-\frac{y(x)}{2}+\frac{\sqrt{y(x)^2+4x}}{2}+\frac{\sqrt{2y(x)^2-2y(x)\sqrt{y(x)^2+4x}+4x-4}}{2}\right)}{\sqrt{2y(x)^2-2y(x)\sqrt{y(x)^2+4x}+4x-4}}$$
$$=0$$

$$\frac{\left(y(x)+\sqrt{y(x)^2+4x}\right)c_1}{\sqrt{-2y(x)-2\sqrt{y(x)^2+4x}-4}\,\sqrt{-2y(x)-2\sqrt{y(x)^2+4x}+4}}+x$$
$$-\frac{\left(y(x)+\sqrt{y(x)^2+4x}\right)\ln\left(-\frac{y(x)}{2}-\frac{\sqrt{y(x)^2+4x}}{2}+\frac{\sqrt{2y(x)^2+2y(x)\sqrt{y(x)^2+4x}+4x-4}}{2}\right)}{\sqrt{2y(x)^2+2y(x)\sqrt{y(x)^2+4x}+4x-4}}$$
$$=0$$

✓ Solution by Mathematica

Time used: 0.497 (sec). Leaf size: 77

```
DSolve[x==y'[x]*y[x]+y'[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x=-\frac{2K[1]\arctan\left(\frac{\sqrt{1-K[1]^2}}{K[1]+1}\right)}{\sqrt{1-K[1]^2}}+\frac{c_1K[1]}{\sqrt{1-K[1]^2}},y(x)=\frac{x}{K[1]}-K[1]\right\},\{y(x),K[1]\}\right]$$

20.9 problem 9

Internal problem ID [2346]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 38, page 173
Problem number: 9.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{4{y'}^2 x + 2y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
dsolve(4*diff(y(x),x)^2*x+2*diff(y(x),x)*x=y(x),y(x), singsol=all)
```

$$y(x) = -\frac{x}{4}$$
$$y(x) = \left(\frac{4c_1}{x} + \frac{2\sqrt{c_1 x}}{x}\right) x$$
$$y(x) = \left(\frac{4c_1}{x} - \frac{2\sqrt{c_1 x}}{x}\right) x$$

✓ Solution by Mathematica

Time used: 0.152 (sec). Leaf size: 72

```
DSolve[4*y'[x]^2*x+2*y'[x]*x==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{2c_1}\left(-2\sqrt{x}+e^{2c_1}\right)$$
$$y(x) \to \frac{1}{4}e^{-4c_1}\left(1+2e^{2c_1}\sqrt{x}\right)$$
$$y(x) \to 0$$
$$y(x) \to -\frac{x}{4}$$

20.10 problem 10

Internal problem ID [2347]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 38, page 173
Problem number: 10.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y - y'x(y' + 1) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 69

```
dsolve(y(x)=diff(y(x),x)*x*(diff(y(x),x)+1),y(x), singsol=all)
```

$$y(x) = \frac{x\left(\frac{1}{2\,\mathrm{LambertW}\left(-\frac{1}{2\sqrt{\frac{c_1}{x}}}\right)}+1\right)}{2\,\mathrm{LambertW}\left(-\frac{1}{2\sqrt{\frac{c_1}{x}}}\right)}$$

$$y(x) = \frac{x\left(\frac{1}{2\,\mathrm{LambertW}\left(\frac{1}{2\sqrt{\frac{c_1}{x}}}\right)}+1\right)}{2\,\mathrm{LambertW}\left(\frac{1}{2\sqrt{\frac{c_1}{x}}}\right)}$$

✓ Solution by Mathematica

Time used: 0.523 (sec). Leaf size: 102

```
DSolve[y[x]==y'[x]*x*(y'[x]+1),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{\sqrt{\frac{4y(x)}{x}+1}-1}-\log\left(\sqrt{\frac{4y(x)}{x}+1}-1\right)=\frac{\log(x)}{2}+c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{\sqrt{\frac{4y(x)}{x}+1}+1}+\log\left(\sqrt{\frac{4y(x)}{x}+1}+1\right)=-\frac{\log(x)}{2}+c_1,y(x)\right]$$

$$y(x)\to 0$$

20.11 problem 11

Internal problem ID [2348]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 38, page 173
Problem number: 11.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{2xy'^3 - yy'^2 = -1}$$

✓ Solution by Maple

Time used: 0.234 (sec). Leaf size: 620

```
dsolve(2*diff(y(x),x)^3*x+1=diff(y(x),x)^2*y(x),y(x), singsol=all)
```

$$
y(x) = 2x\left(\frac{\left(\left(-27+3\sqrt{-\frac{3(c_1^3-27x)}{x}}\right)x^2\right)^{\frac{1}{3}}}{3x} + \frac{c_1}{\left(\left(-27+3\sqrt{-\frac{3(c_1^3-27x)}{x}}\right)x^2\right)^{\frac{1}{3}}}\right) + \frac{1}{\left(\frac{\left(\left(-27+3\sqrt{-\frac{3(c_1^3-27x)}{x}}\right)x^2\right)^{\frac{1}{3}}}{3x} + \frac{c_1}{\left(\left(-27+3\sqrt{-\frac{3(c_1^3-27x)}{x}}\right)x^2\right)^{\frac{1}{3}}}\right)^2}
$$

$$
y(x) = 2x\left(-\frac{\left(\left(-27+3\sqrt{-\frac{3(c_1^3-27x)}{x}}\right)x^2\right)^{\frac{1}{3}}}{6x} - \frac{c_1}{2\left(\left(-27+3\sqrt{-\frac{3(c_1^3-27x)}{x}}\right)x^2\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(\left(-27+3\sqrt{-\frac{3(c_1^3-27x)}{x}}\right)x^2\right)^{\frac{1}{3}}}{3x} - \frac{c_1}{\left(\left(-27+3\sqrt{-\frac{3(c_1^3-27x)}{x}}\right)x^2\right)^{\frac{1}{3}}}\right)}{2}\right) + \frac{1}{\left(-\frac{\left(\left(-27+3\sqrt{-\frac{3(c_1^3-27x)}{x}}\right)x^2\right)^{\frac{1}{3}}}{6x} - \frac{c_1}{2\left(\left(-27+3\sqrt{-\frac{3(c_1^3-27x)}{x}}\right)x^2\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(\left(-27+3\sqrt{-\frac{3(c_1^3-27x)}{x}}\right)x^2\right)^{\frac{1}{3}}}{3x} - \cdots\right.}{2}\right.}
$$

✓ Solution by Mathematica

Time used: 151.15 (sec). Leaf size: 17695

```
DSolve[2*y'[x]^3*x+1==y'[x]^2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

20.12 problem 12

Internal problem ID [2349]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 38, page 173
Problem number: 12.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y'^3 + xyy' - 2y^2 = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 36

```
dsolve(diff(y(x),x)^3+diff(y(x),x)*x*y(x)=2*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{x^3}{27}$$
$$y(x) = 0$$
$$y(x) = \frac{x^2}{4c_1} + \frac{x}{2c_1^2} + \frac{1}{4c_1^3}$$

✓ Solution by Mathematica

Time used: 141.328 (sec). Leaf size: 10666

```
DSolve[y'[x]^3+y'[x]*x*y[x]==2*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

20.13 problem 13

Internal problem ID [2350]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 38, page 173
Problem number: 13.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{3{y'}^4x-{y'}^3y=1}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 37

```
dsolve(3*diff(y(x),x)^4*x=diff(y(x),x)^3*y(x)+1,y(x), singsol=all)
```

$$\left[x(\_T)=\frac{\frac{3}{5\_T^{\frac{5}{2}}}+c_1}{\_T^{\frac{3}{2}}},y(\_T)=\frac{\frac{9}{5\_T^{\frac{5}{2}}}+3c_1}{\sqrt{\_T}}-\frac{1}{\_T^3}\right]$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[3*y'[x]^4*x==y'[x]^3*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

20.14 problem 14

Internal problem ID [2351]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 38, page 173
Problem number: 14.
ODE order: 1.
ODE degree: 5.

CAS Maple gives this as type `[_dAlembert]`

$$\boxed{2y'^5 + 2y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 1505

```
dsolve(2*diff(y(x),x)^5+2*diff(y(x),x)*x=y(x),y(x), singsol=all)
```

$$y(x) = -\frac{\left(i\left(300c_1+20\sqrt{20x^3+225c_1^2}\right)^{\frac{2}{3}}\sqrt{3}+20i\sqrt{3}\,x+\left(300c_1+20\sqrt{20x^3+225c_1^2}\right)^{\frac{2}{3}}-20x\right)^2\sqrt{-5\,(300 \cdots}}{2000\,(300 \cdots} - \frac{x\sqrt{-5\left(300c_1+20\sqrt{20x^3+225c_1^2}\right)^{\frac{1}{3}}\left(i\left(300c_1+20\sqrt{20x^3+225c_1^2}\right)^{\frac{2}{3}}\sqrt{3}+20i\sqrt{3}\,x+(300c_1+20 \cdots}}{5\left(300c_1+20\sqrt{20x^3+225c_1^2}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{\left(i\left(300c_1+20\sqrt{20x^3+225c_1^2}\right)^{\frac{2}{3}}\sqrt{3}+20i\sqrt{3}\,x+\left(300c_1+20\sqrt{20x^3+225c_1^2}\right)^{\frac{2}{3}}-20x\right)^2\sqrt{-5\,(300c_1 \cdots}}{600000c_1+\cdots} + \frac{x\sqrt{-5\left(300c_1+20\sqrt{20x^3+225c_1^2}\right)^{\frac{1}{3}}\left(i\left(300c_1+20\sqrt{20x^3+225c_1^2}\right)^{\frac{2}{3}}\sqrt{3}+20i\sqrt{3}\,x+(300c_1+20 \cdots}}{5\left(300c_1+20\sqrt{20x^3+225c_1^2}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(i\left(300c_1+20\sqrt{20x^3+225c_1^2}\right)^{\frac{2}{3}}\sqrt{3}+20i\sqrt{3}\,x-\left(300c_1+20\sqrt{20x^3+225c_1^2}\right)^{\frac{2}{3}}+20x\right)^2\sqrt{5}\sqrt{(300 \cdots}}{2000\,(300 \cdots} - \frac{x\sqrt{5}\sqrt{\left(300c_1+20\sqrt{20x^3+225c_1^2}\right)^{\frac{1}{3}}\left(i\left(300c_1+20\sqrt{20x^3+225c_1^2}\right)^{\frac{2}{3}}\sqrt{3}+20i\sqrt{3}\,x-(300c_1+20 \cdots}}{5\left(300c_1+20\sqrt{20x^3+225c_1^2}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{\left(i\left(300c_1+20\sqrt{20x^3+225c_1^2}\right)^{\frac{2}{3}}\sqrt{3}+20i\sqrt{3}\,x-\left(300c_1+20\sqrt{20x^3+225c_1^2}\right)^{\frac{2}{3}}+20x\right)^2\sqrt{5}\sqrt{(300c_1 \cdots}}{600000c_1+\cdots} + \frac{x\sqrt{5}\sqrt{\left(300c_1+20\sqrt{20x^3+225c_1^2}\right)^{\frac{1}{3}}\left(i\left(300c_1+20\sqrt{20x^3+225c_1^2}\right)^{\frac{2}{3}}\sqrt{3}+20i\sqrt{3}\,x-(300c_1+20 \cdots}}{5\left(300c_1+20\sqrt{20x^3+225c_1^2}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 2.303 (sec). Leaf size: 2226

```
DSolve[2*y'[x]^5+2*y'[x]*x==y[x],y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

20.15 problem 15

Internal problem ID [2352]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 38, page 173
Problem number: 15.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{\frac{1}{y'^2} + y'x - 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 2120

```
dsolve(1/diff(y(x),x)^2+diff(y(x),x)*x=2*y(x),y(x), singsol=all)
```

$$\frac{\left(-442368xy(x)^5\sqrt{-96y(x)^3+81x^2}-\left(12x\sqrt{-96y(x)^3+81x^2}+64y(x)^3-108x^2\right)^{\frac{8}{3}}-16\left(12x\sqrt{-96\right.\right.}{}$$

$$+x$$

$$-\frac{108x^3\left(12x\sqrt{-96y(x)^3+81x^2}+64y(x)^3-108x^2\right)}{\left(\left(\left(12x\sqrt{-96y(x)^3+81x^2}+64y(x)^3-108x^2\right)^{\frac{2}{3}}+4y(x)\left(12x\sqrt{-96y(x)^3+81x^2}+64y(x)^3-108x\right.\right.\right.}$$

$$=0$$

$$\frac{\left(-1769472i\sqrt{3}\,y(x)^8+1024iy(x)^3\left(12x\sqrt{-96y(x)^3+81x^2}+64y(x)^3-108x^2\right)^{\frac{5}{3}}\sqrt{3}+1244160i\sqrt{-32\right.}{}$$

$$+x$$

$$-\frac{864x^3\left(12x\sqrt{-96y(x)^3+81x^2}+\right.}{\left(i\left(12x\sqrt{-96y(x)^3+81x^2}+64y(x)^3-108x^2\right)^{\frac{2}{3}}\sqrt{3}-16i\sqrt{3}\,y(x)^2-\left(12x\sqrt{-96y(x)^3+81x^2}+64\right.\right.}$$

$$=0$$

$$\frac{\left(-1769472i\sqrt{3}\,y(x)^8+1024iy(x)^3\left(12x\sqrt{-96y(x)^3+81x^2}+64y(x)^3-108x^2\right)^{\frac{5}{3}}\sqrt{3}+1244160i\sqrt{-32\right.}{}$$

$$+x$$

$$+\frac{864x^3\left(12x\sqrt{-96y(x)^3+81x^2}+\right.}{\left(i\left(12x\sqrt{-96y(x)^3+81x^2}+64y(x)^3-108x^2\right)^{\frac{2}{3}}\sqrt{3}-16i\sqrt{3}\,y(x)^2+\left(12x\sqrt{-96y(x)^3+81x^2}+64\right.\right.}$$

$$=0$$

✓ Solution by Mathematica

Time used: 149.881 (sec). Leaf size: 10773

```
DSolve[1/(y'[x]^2)+y'[x]*x==2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

20.16 problem 16

Internal problem ID [2353]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 38, page 173
Problem number: 16.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{2y - 3y'x - 2\ln(y') = 4}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 752

```
dsolve(2*y(x)=3*diff(y(x),x)*x+4+2*ln(diff(y(x),x)),y(x), singsol=all)
```

$$y(x) = \frac{3x\left(\frac{\left(12\sqrt{3}\,\sqrt{c_1(27c_1x^2-4)}\,x+108c_1x^2-8\right)^{\frac{1}{3}}}{6x}+\frac{2}{3x\left(12\sqrt{3}\,\sqrt{c_1(27c_1x^2-4)}\,x+108c_1x^2-8\right)^{\frac{1}{3}}}-\frac{1}{3x}\right)}{2} + 2 + \ln\left(\frac{\left(12\sqrt{3}\,\sqrt{c_1\,(27c_1x^2-4)}\,x+108c_1x^2-8\right)^{\frac{1}{3}}}{6x} + \frac{2}{3x\left(12\sqrt{3}\,\sqrt{c_1\,(27c_1x^2-4)}\,x+108c_1x^2-8\right)^{\frac{1}{3}}}-\frac{1}{3x}\right)$$

$$y(x) = \frac{3x\left(-\frac{\left(12\sqrt{3}\,\sqrt{c_1(27c_1x^2-4)}\,x+108c_1x^2-8\right)^{\frac{1}{3}}}{12x}-\frac{1}{3x\left(12\sqrt{3}\,\sqrt{c_1(27c_1x^2-4)}\,x+108c_1x^2-8\right)^{\frac{1}{3}}}-\frac{1}{3x}-\frac{i\sqrt{3}\left(\frac{\left(12\sqrt{3}\,\sqrt{c_1\left(27c_1x^2-4\right)}\,x+\cdots\right.}{6x}\cdots\right.}{\cdots}\cdots\right.}{2} + 2 + \ln\left(-\frac{\left(12\sqrt{3}\,\sqrt{c_1\,(27c_1x^2-4)}\,x+108c_1x^2-8\right)^{\frac{1}{3}}}{12x} - \frac{1}{3x\left(12\sqrt{3}\,\sqrt{c_1\,(27c_1x^2-4)}\,x+108c_1x^2-8\right)^{\frac{1}{3}}}-\frac{1}{3x} - \frac{i\sqrt{3}\left(\frac{\left(12\sqrt{3}\,\sqrt{c_1(27c_1x^2-4)}\,x+108c_1x^2-8\right)^{\frac{1}{3}}}{6x}-\frac{2}{3x\left(12\sqrt{3}\,\sqrt{c_1(27c_1x^2-4)}\,x+108c_1x^2-8\right)^{\frac{1}{3}}}\right)}{2}\right)$$

$$y(x) = \frac{3x\left(-\frac{\left(12\sqrt{3}\,\sqrt{c_1(27c_1x^2-4)}\,x+108c_1x^2-8\right)^{\frac{1}{3}}}{12x}-\frac{1}{3x\left(12\sqrt{3}\,\sqrt{c_1(27c_1x^2-4)}\,x+108c_1x^2-8\right)^{\frac{1}{3}}}-\frac{1}{3x}+\frac{i\sqrt{3}\left(\frac{\left(12\sqrt{3}\,\sqrt{c_1\left(27c_1x^2-4\right)}\,x+\cdots\right.}{6x}\cdots\right.}{\cdots}\cdots\right.}{2}\cdots$$

✓ Solution by Mathematica

Time used: 0.929 (sec). Leaf size: 137

```
DSolve[2*y[x]==3*y'[x]*x+4+2*Log[y'[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\left(2W\left(-\frac{3}{2}\sqrt{x^2e^{2y(x)-4}}\right)-\log\left(2W\left(-\frac{3}{2}\sqrt{x^2e^{2y(x)-4}}\right)+3\right)+3\right)-y(x)=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{2}\left(2W\left(\frac{3}{2}\sqrt{x^2e^{2y(x)-4}}\right)-\log\left(2W\left(\frac{3}{2}\sqrt{x^2e^{2y(x)-4}}\right)+3\right)+3\right)-y(x)=c_1,y(x)\right]$$

21 Exercise 39, page 179

21.1 problem 23

Internal problem ID [2354]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 39, page 179
Problem number: 23.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y - y'x - y'^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(y(x)=diff(y(x),x)*x+diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{4}$$
$$y(x) = c_1^2 + c_1 x$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 23

```
DSolve[y[x]==y'[x]*x+y'[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1(x + c_1)$$
$$y(x) \rightarrow -\frac{x^2}{4}$$

21.2 problem 24

Internal problem ID [2355]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 39, page 179
Problem number: 24.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Clairaut]`

$$\boxed{y - y'x - \frac{1}{y'} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 27

```
dsolve(y(x)=diff(y(x),x)*x+1/diff(y(x),x),y(x), singsol=all)
```

$$y(x) = -2\sqrt{x}$$
$$y(x) = 2\sqrt{x}$$
$$y(x) = c_1 x + \frac{1}{c_1}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 41

```
DSolve[y[x]==y'[x]*x+1/y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x + \frac{1}{c_1}$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to -2\sqrt{x}$$
$$y(x) \to 2\sqrt{x}$$

21.3 problem 25

Internal problem ID [2356]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 39, page 179
Problem number: 25.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _Clairaut]`

$$\boxed{y - y'x + \sqrt{y'} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve(y(x)=diff(y(x),x)*x-sqrt(diff(y(x),x)),y(x), singsol=all)
```

$$y(x) = -\frac{1}{4x}$$
$$y(x) = c_1 x - \sqrt{c_1}$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 23

```
DSolve[y[x]==y'[x]*x-Sqrt[y'[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x - \sqrt{c_1}$$
$$y(x) \to 0$$

21.4 problem 26

Internal problem ID [2357]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 39, page 179
Problem number: 26.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y - y'x - \ln(y') = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(y(x)=diff(y(x),x)*x+ln(diff(y(x),x)),y(x), singsol=all)
```

$$y(x) = \ln\left(-\frac{1}{x}\right) - 1$$
$$y(x) = c_1 x + \ln(c_1)$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 25

```
DSolve[y[x]==y'[x]*x+Log[y'[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x + \log(c_1)$$
$$y(x) \to \log\left(-\frac{1}{x}\right) - 1$$

21.5 problem 27

Internal problem ID [2358]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 39, page 179
Problem number: 27.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y - y'x - \frac{3}{y'^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 82

```
dsolve(y(x)=diff(y(x),x)*x+3/diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x) = \frac{3\,6^{\frac{1}{3}}(x^2)^{\frac{1}{3}}}{2}$$
$$y(x) = -\frac{3\,6^{\frac{1}{3}}(x^2)^{\frac{1}{3}}}{4} - \frac{3i\sqrt{3}\,6^{\frac{1}{3}}(x^2)^{\frac{1}{3}}}{4}$$
$$y(x) = -\frac{3\,6^{\frac{1}{3}}(x^2)^{\frac{1}{3}}}{4} + \frac{3i\sqrt{3}\,6^{\frac{1}{3}}(x^2)^{\frac{1}{3}}}{4}$$
$$y(x) = c_1x + \frac{3}{c_1^2}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 84

```
DSolve[y[x]==y'[x]*x+3/y'[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x + \frac{3}{{c_1}^2}$$

$$y(x) \to -\frac{3\sqrt[3]{-3}x^{2/3}}{2^{2/3}}$$

$$y(x) \to \frac{3\sqrt[3]{3}x^{2/3}}{2^{2/3}}$$

$$y(x) \to \frac{3(-1)^{2/3}\sqrt[3]{3}x^{2/3}}{2^{2/3}}$$

21.6 problem 28

Internal problem ID [2359]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 39, page 179
Problem number: 28.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y - y' x + y'^{\frac{2}{3}} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 25

```
dsolve(y(x)=diff(y(x),x)*x-diff(y(x),x)^(2/3),y(x), singsol=all)
```

$$y(x) = -\frac{4}{27x^2}$$
$$y(x) = 0$$
$$y(x) = c_1 x - c_1^{\frac{2}{3}}$$

✓ Solution by Mathematica

Time used: 0.061 (sec). Leaf size: 23

```
DSolve[y[x]==y'[x]*x-y'[x]^(2/3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x - c_1{}^{2/3}$$
$$y(x) \to 0$$

21.7 problem 29

Internal problem ID [2360]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 39, page 179
Problem number: 29.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y - y'x - e^{y'} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(y(x)=diff(y(x),x)*x+exp(diff(y(x),x)),y(x), singsol=all)
```

$$y(x) = x \ln(-x) - x$$
$$y(x) = c_1 x + e^{c_1}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 26

```
DSolve[y[x]==y'[x]*x+Exp[y'[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x + e^{c_1}$$
$$y(x) \to x(\log(-x) - 1)$$

21.8 problem 30

Internal problem ID [2361]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 39, page 179
Problem number: 30.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Clairaut]`

$$\boxed{(y - y'x)^2 - y'^2 = 1}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 46

```
dsolve((y(x)-diff(y(x),x)*x)^2=diff(y(x),x)^2+1,y(x), singsol=all)
```

$$y(x) = c_1x - \sqrt{c_1^2 + 1}$$
$$y(x) = c_1x + \sqrt{c_1^2 + 1}$$
$$y(x) = \sqrt{x - 1}\,\sqrt{x + 1}\,c_1$$

✓ Solution by Mathematica

Time used: 0.141 (sec). Leaf size: 73

```
DSolve[(y[x]-y'[x]*x)^2==y'[x]^2+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1x - \sqrt{1 + c_1{}^2}$$
$$y(x) \to c_1x + \sqrt{1 + c_1{}^2}$$
$$y(x) \to -\sqrt{1 - x^2}$$
$$y(x) \to \sqrt{1 - x^2}$$

21.9 problem 31

Internal problem ID [2362]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 39, page 179
Problem number: 31.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Clairaut]`

$$\boxed{{y'}^2 x - y' y = 2}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)^2*x-diff(y(x),x)*y(x)-2=0,y(x), singsol=all)
```

$$y(x) = c_1 x - \frac{2}{c_1}$$
$$y(x) = c_1 \sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 57

```
DSolve[y'[x]^2*x-y'[x]*y[x]-2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x - \frac{2}{c_1}$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to -2i\sqrt{2}\sqrt{x}$$
$$y(x) \to 2i\sqrt{2}\sqrt{x}$$

21.10 problem 32

Internal problem ID [2363]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 39, page 179
Problem number: 32.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2-1)\,y'^2-2xyy'+y^2=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(y(x)^2-2*diff(y(x),x)*x*y(x)+diff(y(x),x)^2*(x^2-1)=0,y(x), singsol=all)
```

$$y(x)=c_1(x+1)$$
$$y(x)=c_1(x-1)$$

✓ Solution by Mathematica

Time used: 0.106 (sec). Leaf size: 26

```
DSolve[y[x]^2-2*y'[x]*x*y[x]+y'[x]^2*(x^2-1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1(x-1)$$
$$y(x)\to c_1(x+1)$$
$$y(x)\to 0$$

22 Exercise 40, page 186

22.1 problem 1

Internal problem ID [2364]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 40, page 186
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \sqrt{1-y} = 0}$$

With initial conditions

$$[y(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
Order:=5;
dsolve([diff(y(x),x)=(1-y(x))^(1/2),y(0) = 0],y(x),type='series',x=0);
```

$$y(x) = x - \frac{1}{4}x^2$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 12

```
AsymptoticDSolveValue[{y'[x]==(1-y[x])^(1/2),{y[0]==0}},y[x],{x,0,4}]
```

$$y(x) \to x - \frac{x^2}{4}$$

22.2 problem 2

Internal problem ID [2365]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 40, page 186
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - yx = -x^2}$$

With initial conditions

$$[y(0) = 2]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
Order:=5;
dsolve([diff(y(x),x)=x*y(x)-x^2,y(0) = 2],y(x),type='series',x=0);
```

$$y(x) = 2 + x^2 - \frac{1}{3}x^3 + \frac{1}{4}x^4 + \mathrm{O}\left(x^5\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 22

```
AsymptoticDSolveValue[{y'[x]==x*y[x]-x^2,{y[0]==2}},y[x],{x,0,4}]
```

$$y(x) \to \frac{x^4}{4} - \frac{x^3}{3} + x^2 + 2$$

22.3 problem 3

Internal problem ID [2366]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 40, page 186
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y^2x^2 = 0}$$

With initial conditions

$$[y(1) = 0]$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
Order:=5;
dsolve([diff(y(x),x)=x^2*y(x)^2,y(1) = 0],y(x),type='series',x=1);
```

$$y(x) = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
AsymptoticDSolveValue[{y'[x]==x^2*y[x]^2,{y[1]==0}},y[x],{x,1,4}]
```

Not solved

22.4 problem 4

Internal problem ID [2367]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 40, page 186
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{y}{x} = 3x}$$

With initial conditions

$$[y(1) = 3]$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
Order:=5;
dsolve([diff(y(x),x)=3*x+y(x)/x,y(1) = 3],y(x),type='series',x=1);
```

$$y(x) = 3x^2$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 17

```
AsymptoticDSolveValue[{y'[x]==3*x+y[x]/x,{y[1]==3}},y[x],{x,1,4}]
```

$$y(x) \to 3(x-1)^2 + 6(x-1) + 3$$

22.5 problem 5

Internal problem ID [2368]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 40, page 186
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - \ln(yx) = 0}$$

With initial conditions

$$[y(1) = 1]$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=5;
dsolve([diff(y(x),x)=ln(x*y(x)),y(1) = 1],y(x),type='series',x=1);
```

$$y(x) = 1 + \frac{1}{2}(x-1)^2 + \frac{1}{12}(x-1)^4 + \mathrm{O}\left((x-1)^5\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
AsymptoticDSolveValue[{y'[x]==Log[x*y[x]],{y[1]==1}},y[x],{x,1,4}]
```

Not solved

22.6 problem 6

Internal problem ID [2369]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 40, page 186
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^2 = 1}$$

With initial conditions

$$[y(1) = -1]$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
Order:=5;
dsolve([diff(y(x),x)=1+y(x)^2,y(1) = -1],y(x),type='series',x=1);
```

$$y(x) = -1 + 2(x-1) - 2(x-1)^2 + \frac{8}{3}(x-1)^3 - \frac{10}{3}(x-1)^4 + \mathrm{O}\left((x-1)^5\right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 35

```
AsymptoticDSolveValue[{y'[x]==1+y[x]^2,{y[1]==-1}},y[x],{x,1,4}]
```

$$y(x) \to -\frac{10}{3}(x-1)^4 + \frac{8}{3}(x-1)^3 - 2(x-1)^2 + 2(x-1) - 1$$

22.7 problem 7

Internal problem ID [2370]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 40, page 186
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{y' - y^2 = x^2}$$

With initial conditions

$$[y(2) = 0]$$

With the expansion point for the power series method at $x = 2$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
Order:=6;
dsolve([diff(y(x),x)=x^2+y(x)^2,y(2) = 0],y(x),type='series',x=2);
```

$$y(x) = 4(x-2) + 2(x-2)^2 + \frac{17}{3}(x-2)^3 + 4(x-2)^4 + \frac{148}{15}(x-2)^5 + \mathrm{O}\left((x-2)^6\right)$$

✓ Solution by Mathematica

Time used: 0.154 (sec). Leaf size: 9983

```
AsymptoticDSolveValue[{y'[x]==x^2+y[x]^2,{y[2]==0}},y[x],{x,2,5}]
```

Too large to display

22.8 problem 8

Internal problem ID [2371]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 40, page 186
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - \sqrt{1 + yx} = 0}$$

With initial conditions

$$[y(0) = 1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
Order:=4;
dsolve([diff(y(x),x)=sqrt(1+x*y(x)),y(0) = 1],y(x),type='series',x=0);
```

$$y(x) = 1 + x + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \mathrm{O}\left(x^4\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
AsymptoticDSolveValue[{y'[x]==Sqrt[1+x*y[x]],{y[0]==1}},y[x],{x,0,3}]
```

Not solved

22.9 problem 9

Internal problem ID [2372]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 40, page 186
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - \sin(y) = \cos(x)}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}\right]$$

With the expansion point for the power series method at $x = \frac{\pi}{2}$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
Order:=4;
dsolve([diff(y(x),x)=cos(x)+sin(y(x)),y(1/2*Pi) = 1/2*Pi],y(x),type='series',x=Pi/2);
```

$$y(x) = \frac{\pi}{2} + \left(x - \frac{\pi}{2}\right) - \frac{1}{2}\left(x - \frac{\pi}{2}\right)^2 - \frac{1}{6}\left(x - \frac{\pi}{2}\right)^3 + \mathrm{O}\left(\left(x - \frac{\pi}{2}\right)^4\right)$$

✓ Solution by Mathematica

Time used: 0.088 (sec). Leaf size: 22

```
AsymptoticDSolveValue[{y'[x]==Cos[x]*Sin[y[x]],{y[Pi/2]==Pi/2}},y[x],{x,Pi/2,3}]
```

$$y(x) \to \frac{\pi}{2} - \frac{1}{2}\left(x - \frac{\pi}{2}\right)^2$$

22.10 problem 10

Internal problem ID [2373]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 40, page 186
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y = \sin(x)}$$

With initial conditions

$$[y(0) = 1, y'(0) = 2]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
Order:=7;
dsolve([diff(y(x),x$2)-y(x)=sin(x),y(0) = 1, D(y)(0) = 2],y(x),type='series',x=0);
```

$$y(x) = 1 + 2x + \frac{1}{2}x^2 + \frac{1}{2}x^3 + \frac{1}{24}x^4 + \frac{1}{60}x^5 + \frac{1}{720}x^6 + \mathrm{O}\left(x^7\right)$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 43

```
AsymptoticDSolveValue[{y''[x]-y[x]==Sin[x],{y[0]==1,y'[0]==2}},y[x],{x,0,6}]
```

$$y(x) \to \frac{x^6}{720} + \frac{x^5}{60} + \frac{x^4}{24} + \frac{x^3}{2} + \frac{x^2}{2} + 2x + 1$$

22.11 problem 11

Internal problem ID [2374]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 40, page 186
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y = \mathrm{e}^{2x}}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
Order:=7;
dsolve([diff(y(x),x$2)-2*y(x)=exp(2*x),y(0) = 0, D(y)(0) = 0],y(x),type='series',x=0);
```

$$y(x) = \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \frac{1}{10}x^5 + \frac{7}{180}x^6 + \mathrm{O}\left(x^7\right)$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 39

```
AsymptoticDSolveValue[{y''[x]-2*y[x]==Exp[2*x],{y[0]==0,y'[0]==0}},y[x],{x,0,6}]
```

$$y(x) \to \frac{7x^6}{180} + \frac{x^5}{10} + \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2}$$

22.12 problem 12

Internal problem ID [2375]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 40, page 186
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _`

$$\boxed{y'' + 2y'y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=7;
dsolve([diff(y(x),x$2)+2*y(x)*diff(y(x),x)=0,y(0) = 0, D(y)(0) = 1],y(x),type='series',x=0);
```

$$y(x) = x - \frac{1}{3}x^3 + \frac{2}{15}x^5 + \mathrm{O}\left(x^7\right)$$

✓ Solution by Mathematica

Time used: 0.082 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{y''[x]+2*y[x]*y'[x]==0,{y[0]==0,y'[0]==1}},y[x],{x,0,6}]
```

$$y(x) \to \frac{2x^5}{15} - \frac{x^3}{3} + x$$

22.13 problem 13

Internal problem ID [2376]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 40, page 186
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' - \sin(y) = 0}$$

With initial conditions

$$\left[y(0) = \frac{\pi}{4}, y'(0) = 0\right]$$

With the expansion point for the power series method at $x = \frac{\pi}{4}$.

✗ Solution by Maple

```
Order:=7;
dsolve([diff(y(x),x$2)=sin(y(x)),y(0) = 1/4*Pi, D(y)(0) = 0],y(x),type='series',x=Pi/4);
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
AsymptoticDSolveValue[{y''[x]==Sin[y[x]],{y[0]==Pi/4,y'[0]==0}},y[x],{x,Pi/4,6}]
```

Not solved

22.14 problem 14

Internal problem ID [2377]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 40, page 186
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + \frac{y'^2}{2} - y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=7;
dsolve([diff(y(x),x$2)+1/2*diff(y(x),x)^2-y(x)=0,y(0) = 0, D(y)(0) = 1],y(x),type='series',x
```

$$y(x) = x - \frac{1}{4}x^2 + \frac{1}{4}x^3 - \frac{3}{32}x^4 + \frac{1}{20}x^5 - \frac{13}{480}x^6 + \mathrm{O}\left(x^7\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
AsymptoticDSolveValue[{y''[x]+1/2*y'[x]^2-y[x]==0,{y[0]==0,y'[0]==1}},y[x],{x,0,6}]
```

Not solved

22.15 problem 15

Internal problem ID [2378]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 40, page 186
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$\boxed{y'' - \sin(yx) = 0}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = 1, y'\left(\frac{\pi}{2}\right) = 1\right]$$

With the expansion point for the power series method at $x = \frac{\pi}{2}$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
Order:=5;
dsolve([diff(y(x),x$2)=sin(x*y(x)),y(1/2*Pi) = 1, D(y)(1/2*Pi) = 1],y(x),type='series',x=Pi/
```

$$y(x) = 1 + \left(x - \frac{\pi}{2}\right) + \frac{1}{2}\left(x - \frac{\pi}{2}\right)^2 - \frac{1}{96}(2+\pi)^2\left(x - \frac{\pi}{2}\right)^4 + \mathrm{O}\left(\left(x - \frac{\pi}{2}\right)^5\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
AsymptoticDSolveValue[{y''[x]==Sin[x*y[x]],{y[Pi/2]==1,y'[Pi/2]==1}},y[x],{x,Pi/2,4}]
```

Not solved

22.16 problem 16

Internal problem ID [2379]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 40, page 186
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$\boxed{y'' - \cos(yx) = 0}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = 1, y'\left(\frac{\pi}{2}\right) = 1\right]$$

With the expansion point for the power series method at $x = \frac{\pi}{2}$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=5;
dsolve([diff(y(x),x$2)=cos(x*y(x)),y(1/2*Pi) = 1, D(y)(1/2*Pi) = 1],y(x),type='series',x=Pi/
```

$$y(x) = 1 + \left(x - \frac{\pi}{2}\right) + \left(-\frac{1}{6} - \frac{\pi}{12}\right)\left(x - \frac{\pi}{2}\right)^3 - \frac{1}{12}\left(x - \frac{\pi}{2}\right)^4 + \mathrm{O}\left(\left(x - \frac{\pi}{2}\right)^5\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
AsymptoticDSolveValue[{y''[x]==Cos[x*y[x]],{y[Pi/2]==1,y'[Pi/2]==1}},y[x],{x,Pi/2,4}]
```

Not solved

23 Exercise 41, page 195

23.1 problem 1

Internal problem ID [2380]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 41, page 195
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + 5y' + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
Order:=6;
dsolve(2*x*diff(y(x),x$2)+5*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2\left(1 - \frac{1}{14}x^2 + \frac{1}{616}x^4 + \mathrm{O}\left(x^6\right)\right)x^{\frac{3}{2}} + c_1\left(1 - \frac{1}{2}x^2 + \frac{1}{40}x^4 + \mathrm{O}\left(x^6\right)\right)}{x^{\frac{3}{2}}}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
AsymptoticDSolveValue[2*x*y''[x]+5*y'x[]+x*y[x]==0,y[x],{x,0,5}]
```

Timed out

23.2 problem 2

Internal problem ID [2381]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 41, page 195
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3x(2+3x)\,y''-4y'+4y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 44

```
Order:=6;
dsolve(3*x*(2+3*x)*diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1x^{\frac{5}{3}}\left(1-\frac{7}{8}x+\frac{7}{8}x^2-\frac{23}{24}x^3+\frac{1817}{1632}x^4-\frac{219857}{163200}x^5+\mathrm{O}\left(x^6\right)\right)+c_2\left(1+x-x^2+\frac{11}{12}x^3-\frac{319}{336}x^4+\frac{319}{300}x^5+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 79

```
AsymptoticDSolveValue[3*x*(2+3*x)*y''[x]-4*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{319x^5}{300}-\frac{319x^4}{336}+\frac{11x^3}{12}-x^2+x+1\right)+c_1\left(-\frac{219857x^5}{163200}+\frac{1817x^4}{1632}-\frac{23x^3}{24}+\frac{7x^2}{8}-\frac{7x}{8}+1\right)x^{5/3}$$

23.3 problem 3

Internal problem ID [2382]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 41, page 195
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(4+x)\,y''+7y'x-y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 39

```
Order:=6;
dsolve(x^2*(4+x)*diff(y(x),x$2)+7*x*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{5}{4}}\left(1+\frac{1}{48}x-\frac{5}{19968}x^2+\frac{25}{1810432}x^3-\frac{75}{62390272}x^4+\frac{39}{293601280}x^5+\mathrm{O}\left(x^6\right)\right)+c_1\left(1+2x+\mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 58

```
AsymptoticDSolveValue[x^2*(4+x)*y''[x]+7*x*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\sqrt[4]{x}\left(\frac{39x^5}{293601280}-\frac{75x^4}{62390272}+\frac{25x^3}{1810432}-\frac{5x^2}{19968}+\frac{x}{48}+1\right)+\frac{c_2(2x+1)}{x}$$

23.4 problem 4

Internal problem ID [2383]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 41, page 195
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' + \left(-x^2+x\right)y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)+(x-x^2)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{3}{2}}\left(1+\frac{1}{5}x+\frac{1}{35}x^2+\frac{1}{315}x^3+\frac{1}{3465}x^4+\frac{1}{45045}x^5+\mathrm{O}\left(x^6\right)\right)+c_1\left(1+\frac{1}{2}x+\frac{1}{8}x^2+\frac{1}{48}x^3+\frac{1}{384}x^4+\frac{1}{3840}x^5+\right.}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 86

```
AsymptoticDSolveValue[2*x^2*y''[x]+(x-x^2)*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 x\left(\frac{x^5}{45045}+\frac{x^4}{3465}+\frac{x^3}{315}+\frac{x^2}{35}+\frac{x}{5}+1\right)+\frac{c_2\left(\frac{x^5}{3840}+\frac{x^4}{384}+\frac{x^3}{48}+\frac{x^2}{8}+\frac{x}{2}+1\right)}{\sqrt{x}}$$

23.5 problem 5

Internal problem ID [2384]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 41, page 195
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' + 5y'x + y(x+1) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 - x + \frac{1}{6}x^2 - \frac{1}{90}x^3 + \frac{1}{2520}x^4 - \frac{1}{113400}x^5 + \mathrm{O}\left(x^6\right)\right)\sqrt{x} + c_2\left(1 - \frac{1}{3}x + \frac{1}{30}x^2 - \frac{1}{630}x^3 + \frac{1}{22680}x^4 - \frac{1}{124740\cdots}\right.}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 86

```
AsymptoticDSolveValue[2*x^2*y''[x]+5*x*y'[x]+(1+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \frac{c_1\left(-\frac{x^5}{1247400} + \frac{x^4}{22680} - \frac{x^3}{630} + \frac{x^2}{30} - \frac{x}{3} + 1\right)}{\sqrt{x}} + \frac{c_2\left(-\frac{x^5}{113400} + \frac{x^4}{2520} - \frac{x^3}{90} + \frac{x^2}{6} - x + 1\right)}{x}$$

23.6 problem 6

Internal problem ID [2385]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 41, page 195
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9x^2y'' + (2+3x)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
Order:=6;
dsolve(9*x^2*diff(y(x),x$2)+(2+3*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^{\frac{1}{3}}\left(1 - \frac{1}{2}x + \frac{1}{20}x^2 - \frac{1}{480}x^3 + \frac{1}{21120}x^4 - \frac{1}{1478400}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2x^{\frac{2}{3}}\left(1 - \frac{1}{4}x + \frac{1}{56}x^2 - \frac{1}{1680}x^3 + \frac{1}{87360}x^4 - \frac{1}{6988800}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 90

```
AsymptoticDSolveValue[9*x^2*y''[x]+(2+3*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\sqrt[3]{x}\left(-\frac{x^5}{1478400} + \frac{x^4}{21120} - \frac{x^3}{480} + \frac{x^2}{20} - \frac{x}{2} + 1\right) + c_1x^{2/3}\left(-\frac{x^5}{6988800} + \frac{x^4}{87360} - \frac{x^3}{1680} + \frac{x^2}{56} - \frac{x}{4} + 1\right)$$

23.7 problem 7

Internal problem ID [2386]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 41, page 195
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^3 + 2x^2)\, y'' - y'x + (1 - x)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
Order:=6;
dsolve((2*x^2+x^3)*diff(y(x),x$2)-x*diff(y(x),x)+(1-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1 + \frac{5}{4}x + \frac{5}{96}x^2 - \frac{11}{1152}x^3 + \frac{341}{129024}x^4 - \frac{20119}{23224320}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2 x\left(1 + \frac{1}{3}x - \frac{1}{30}x^2 + \frac{1}{126}x^3 - \frac{11}{4536}x^4 + \frac{19}{22680}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 86

```
AsymptoticDSolveValue[(2*x^2+x^3)*y''[x]-x*y'[x]+(1-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 x\left(\frac{19x^5}{22680} - \frac{11x^4}{4536} + \frac{x^3}{126} - \frac{x^2}{30} + \frac{x}{3} + 1\right) + c_2\sqrt{x}\left(-\frac{20119x^5}{23224320} + \frac{341x^4}{129024} - \frac{11x^3}{1152} + \frac{5x^2}{96} + \frac{5x}{4} + 1\right)$$

23.8 problem 8

Internal problem ID [2387]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 41, page 195
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' - 3(x^2 + x)\, y' + (2 + 3x)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)-3*(x+x^2)*diff(y(x),x)+(2+3*x)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = {} & c_1\sqrt{x}\left(1 + \frac{3}{2}x + \frac{9}{8}x^2 + \frac{9}{16}x^3 + \frac{27}{128}x^4 + \frac{81}{1280}x^5 + \mathrm{O}\left(x^6\right)\right) \\ & + c_2x^2\left(1 + \frac{3}{5}x + \frac{9}{35}x^2 + \frac{3}{35}x^3 + \frac{9}{385}x^4 + \frac{27}{5005}x^5 + \mathrm{O}\left(x^6\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 88

```
AsymptoticDSolveValue[2*x^2*y''[x]-3*(x+x^2)*y'[x]+(2+3*x)*y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned} y(x) \to {} & c_1\left(\frac{27x^5}{5005} + \frac{9x^4}{385} + \frac{3x^3}{35} + \frac{9x^2}{35} + \frac{3x}{5} + 1\right)x^2 \\ & + c_2\left(\frac{81x^5}{1280} + \frac{27x^4}{128} + \frac{9x^3}{16} + \frac{9x^2}{8} + \frac{3x}{2} + 1\right)\sqrt{x} \end{aligned}$$

23.9 problem 9

Internal problem ID [2388]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 41, page 195
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3x^2y'' + (-x^2 + 5x)\,y' + (2x^2 - 1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
Order:=6;
dsolve(3*x^2*diff(y(x),x$2)+(5*x-x^2)*diff(y(x),x)+(2*x^2-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2x^{\frac{4}{3}}\left(1+\frac{1}{21}x-\frac{61}{630}x^2-\frac{607}{73710}x^3+\frac{2297}{884520}x^4+\frac{14713}{50417640}x^5+\mathrm{O}\left(x^6\right)\right)+c_1\left(1+x-\frac{1}{2}x^2-\frac{1}{6}x^3+\frac{1}{48}x^4+\frac{19}{2640}\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 84

```
AsymptoticDSolveValue[3*x^2*y''[x]+(5*x-x^2)*y'[x]+(2*x^2-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt[3]{x}\left(\frac{14713x^5}{50417640}+\frac{2297x^4}{884520}-\frac{607x^3}{73710}-\frac{61x^2}{630}+\frac{x}{21}+1\right) + \frac{c_2\left(\frac{19x^5}{2640}+\frac{x^4}{48}-\frac{x^3}{6}-\frac{x^2}{2}+x+1\right)}{x}$$

23.10 problem 10

Internal problem ID [2389]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 41, page 195
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + x(x^2-4)\,y' + 3y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)+x*(x^2-4)*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x}\left(x\left(1-\frac{1}{16}x^2+\frac{7}{2560}x^4+\mathrm{O}\left(x^6\right)\right)c_1+\left(1-\frac{1}{16}x^2+\frac{5}{1536}x^4+\mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 58

```
AsymptoticDSolveValue[4*x^2*y''[x]+x*(x^2-4)*y'[x]+3*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{5x^{9/2}}{1536}-\frac{x^{5/2}}{16}+\sqrt{x}\right)+c_2\left(\frac{7x^{11/2}}{2560}-\frac{x^{7/2}}{16}+x^{3/2}\right)$$

23.11 problem 15

Internal problem ID [2390]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 41, page 195
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' - 3(x^2 + x)\, y' + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 879

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)-3*(x+x^2)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$
\begin{aligned}
y(x) &= x^{\frac{7}{8}}\Bigg(c_2 x^{\frac{\sqrt{17}}{8}}\Bigg(1+\frac{21+3\sqrt{17}}{32+8\sqrt{17}}x+\frac{9}{128}\frac{(15+\sqrt{17})(7+\sqrt{17})}{(4+\sqrt{17})(8+\sqrt{17})}x^2 \\
&\quad+\frac{9}{1024}\frac{(23+\sqrt{17})(15+\sqrt{17})(7+\sqrt{17})}{(4+\sqrt{17})(8+\sqrt{17})(12+\sqrt{17})}x^3 \\
&\quad+\frac{27}{32768}\frac{(31+\sqrt{17})(23+\sqrt{17})(15+\sqrt{17})(7+\sqrt{17})}{(4+\sqrt{17})(8+\sqrt{17})(12+\sqrt{17})(16+\sqrt{17})}x^4 \\
&\quad+\frac{81}{1310720}\frac{(39+\sqrt{17})(31+\sqrt{17})(23+\sqrt{17})(15+\sqrt{17})(7+\sqrt{17})}{(4+\sqrt{17})(8+\sqrt{17})(12+\sqrt{17})(16+\sqrt{17})(20+\sqrt{17})}x^5 \\
&\quad+\mathrm{O}\left(x^6\right)\Bigg)+c_1 x^{-\frac{\sqrt{17}}{8}}\Bigg(1+\frac{-21+3\sqrt{17}}{-32+8\sqrt{17}}x+\frac{9}{128}\frac{(-15+\sqrt{17})(-7+\sqrt{17})}{(-4+\sqrt{17})(-8+\sqrt{17})}x^2 \\
&\quad+\frac{9}{1024}\frac{(-23+\sqrt{17})(-15+\sqrt{17})(-7+\sqrt{17})}{(-4+\sqrt{17})(-8+\sqrt{17})(-12+\sqrt{17})}x^3 \\
&\quad+\frac{27}{32768}\frac{(-31+\sqrt{17})(-23+\sqrt{17})(-15+\sqrt{17})(-7+\sqrt{17})}{(-4+\sqrt{17})(-8+\sqrt{17})(-12+\sqrt{17})(-16+\sqrt{17})}x^4 \\
&\quad+\frac{81}{1310720}\frac{(-39+\sqrt{17})(-31+\sqrt{17})(-23+\sqrt{17})(-15+\sqrt{17})(-7+\sqrt{17})}{(-4+\sqrt{17})(-8+\sqrt{17})(-12+\sqrt{17})(-16+\sqrt{17})(-20+\sqrt{17})}x^5 \\
&\quad+\mathrm{O}\left(x^6\right)\Bigg)\Bigg)
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 2028

```
AsymptoticDSolveValue[4*x^2*y''[x]-3*(x+x^2)*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

Too large to display

23.12 problem 16

Internal problem ID [2391]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 41, page 195
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9x^2y'' + 9(-x^2 + x)\,y' + y(x - 1) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
dsolve(9*x^2*diff(y(x),x$2)+9*(x-x^2)*diff(y(x),x)+(x-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{2}{3}}\left(1 + \frac{2}{15}x + \frac{11}{360}x^2 + \frac{1}{162}x^3 + \frac{29}{27216}x^4 + \frac{551}{3470040}x^5 + \mathrm{O}\left(x^6\right)\right) + c_1\left(1 - \frac{4}{3}x - \frac{5}{18}x^2 - \frac{5}{81}x^3 - \frac{23}{1944}x^4 - \cdots\right.}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 90

```
AsymptoticDSolveValue[9*x^2*y''[x]+9*(x-x^2)*y'[x]+(x-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt[3]{x}\left(\frac{551x^5}{3470040} + \frac{29x^4}{27216} + \frac{x^3}{162} + \frac{11x^2}{360} + \frac{2x}{15} + 1\right) + \frac{c_2\left(-\frac{92x^5}{47385} - \frac{23x^4}{1944} - \frac{5x^3}{81} - \frac{5x^2}{18} - \frac{4x}{3} + 1\right)}{\sqrt[3]{x}}$$

23.13 problem 17

Internal problem ID [2392]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 41, page 195
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2(1-x)\,y''+3x(1+2x)\,y'-3y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

```
Order:=6;
dsolve(4*x^2*(1-x)*diff(y(x),x$2)+3*x*(1+2*x)*diff(y(x),x)-3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{7}{4}}\left(1-\frac{6}{11}x+\frac{4}{55}x^2+\frac{8}{1045}x^3+\frac{48}{24035}x^4+\frac{32}{43263}x^5+\mathrm{O}\left(x^6\right)\right)+c_1\left(1-\frac{13}{4}x+\frac{117}{32}x^2-\frac{195}{128}x^3+\frac{195}{2048}x^4+\cdots\right.}{x^{\frac{3}{4}}}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 86

```
AsymptoticDSolveValue[4*x^2*(1-x)*y''[x]+3*x*(1+2*x)*y'[x]-3*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1 x\left(\frac{32x^5}{43263}+\frac{48x^4}{24035}+\frac{8x^3}{1045}+\frac{4x^2}{55}-\frac{6x}{11}+1\right)+\frac{c_2\left(\frac{117x^5}{8192}+\frac{195x^4}{2048}-\frac{195x^3}{128}+\frac{117x^2}{32}-\frac{13x}{4}+1\right)}{x^{3/4}}$$

23.14 problem 18

Internal problem ID [2393]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 41, page 195
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(1-3x)\,y''+5y'x-2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
Order:=6;
dsolve(2*x^2*(1-3*x)*diff(y(x),x$2)+5*x*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{5}{2}}\left(1-\frac{3}{14}x-\frac{3}{56}x^2-\frac{45}{1232}x^3-\frac{675}{18304}x^4-\frac{1701}{36608}x^5+\mathrm{O}\left(x^6\right)\right)+c_1\left(1-12x+72x^2+\mathrm{O}\left(x^6\right)\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 63

```
AsymptoticDSolveValue[2*x^2*(1-3*x)*y''[x]+5*x*y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to \frac{c_2(72x^2-12x+1)}{x^2}+c_1\sqrt{x}\left(-\frac{1701x^5}{36608}-\frac{675x^4}{18304}-\frac{45x^3}{1232}-\frac{3x^2}{56}-\frac{3x}{14}+1\right)$$

23.15 problem 19

Internal problem ID [2394]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 41, page 195
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2(x+1)\,y''-5y'x+2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
dsolve(4*x^2*(1+x)*diff(y(x),x$2)-5*x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1x^{\frac{1}{4}}\left(1-\frac{1}{4}x+\frac{5}{32}x^2-\frac{15}{128}x^3+\frac{195}{2048}x^4-\frac{663}{8192}x^5+\mathrm{O}\left(x^6\right)\right)+c_2x^2\left(1-\frac{8}{11}x+\frac{32}{55}x^2-\frac{512}{1045}x^3+\frac{2048}{4807}x^4-\frac{16384}{43263}x^5+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 88

```
AsymptoticDSolveValue[4*x^2*(1+x)*y''[x]-5*x*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(-\frac{16384x^5}{43263}+\frac{2048x^4}{4807}-\frac{512x^3}{1045}+\frac{32x^2}{55}-\frac{8x}{11}+1\right)x^2+c_2\left(-\frac{663x^5}{8192}+\frac{195x^4}{2048}-\frac{15x^3}{128}+\frac{5x^2}{32}-\frac{x}{4}+1\right)\sqrt[4]{x}$$

23.16 problem 20

Internal problem ID [2395]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 41, page 195
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(4+x)\,x^2y''+x(x-1)\,y'+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
Order:=6;
dsolve((4+x)*x^2*diff(y(x),x$2)+x*(x-1)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1x^{\frac{1}{4}}\left(1-\frac{1}{16}x+\frac{5}{512}x^2-\frac{15}{8192}x^3+\frac{195}{524288}x^4-\frac{663}{8388608}x^5+\mathrm{O}\left(x^6\right)\right)+c_2x\left(1-\frac{1}{7}x+\frac{2}{77}x^2-\frac{2}{385}x^3+\frac{8}{7315}x^4-\frac{8}{33649}x^5+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 86

```
AsymptoticDSolveValue[(4+x)*x^2*y''[x]+x*(x-1)*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1x\left(-\frac{8x^5}{33649}+\frac{8x^4}{7315}-\frac{2x^3}{385}+\frac{2x^2}{77}-\frac{x}{7}+1\right)+c_2\sqrt[4]{x}\left(-\frac{663x^5}{8388608}+\frac{195x^4}{524288}-\frac{15x^3}{8192}+\frac{5x^2}{512}-\frac{x}{16}+1\right)$$

23.17 problem 21

Internal problem ID [2396]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 41, page 195
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(8-x)\,x^2y''+6y'x-y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
dsolve((8-x)*x^2*diff(y(x),x$2)+6*x*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2x^{\frac{3}{4}}\left(1-\frac{1}{56}x-\frac{3}{9856}x^2-\frac{1}{78848}x^3-\frac{5}{6848512}x^4-\frac{63}{1260126208}x^5+\mathrm{O}\left(x^6\right)\right)+c_1\left(1+\frac{5}{32}x-\frac{3}{2048}x^2-\frac{7}{196608}x^3\right.}{x^{\frac{1}{4}}}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 90

```
AsymptoticDSolveValue[(8-x)*x^2*y''[x]+6*x*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\sqrt{x}\left(-\frac{63x^5}{1260126208}-\frac{5x^4}{6848512}-\frac{x^3}{78848}-\frac{3x^2}{9856}-\frac{x}{56}+1\right)+\frac{c_2\left(-\frac{5929x^5}{59324235776}-\frac{539x^4}{327155712}-\frac{7x^3}{196608}-\frac{3x^2}{2048}+\frac{5x}{32}+1\right)}{\sqrt[4]{x}}$$

23.18 problem 22

Internal problem ID [2397]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 41, page 195
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' + x(x^2+1)\,y' - y(x+1) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)+x*(1+x^2)*diff(y(x),x)-(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{3}{2}}\left(1+\frac{1}{5}x-\frac{2}{35}x^2-\frac{16}{945}x^3+\frac{73}{20790}x^4+\frac{1481}{1351350}x^5+\mathrm{O}\left(x^6\right)\right)+c_1\left(1-x-\frac{1}{4}x^2+\frac{1}{36}x^3+\frac{29}{1440}x^4-\frac{71}{50400}x\right.}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 84

```
AsymptoticDSolveValue[2*x^2*y''[x]+x*(1+x^2)*y'[x]-(1+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 x\left(\frac{1481x^5}{1351350}+\frac{73x^4}{20790}-\frac{16x^3}{945}-\frac{2x^2}{35}+\frac{x}{5}+1\right) + \frac{c_2\left(-\frac{71x^5}{50400}+\frac{29x^4}{1440}+\frac{x^3}{36}-\frac{x^2}{4}-x+1\right)}{\sqrt{x}}$$

23.19 problem 23

Internal problem ID [2398]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 41, page 195
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' - y'x + (x^2+1)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)-x*diff(y(x),x)+(1+x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1 - \frac{1}{6}x^2 + \frac{1}{168}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2x\left(1 - \frac{1}{10}x^2 + \frac{1}{360}x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 48

```
AsymptoticDSolveValue[2*x^2*y''[x]-x*y'[x]+(1+x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1x\left(\frac{x^4}{360} - \frac{x^2}{10} + 1\right) + c_2\sqrt{x}\left(\frac{x^4}{168} - \frac{x^2}{6} + 1\right)$$

23.20 problem 24

Internal problem ID [2399]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 41, page 195
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3x^2y'' + 2y'x + (x^2 - 2)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
Order:=6;
dsolve(3*x^2*diff(y(x),x$2)+2*x*diff(y(x),x)+(x^2-2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{5}{3}} \left(1 - \frac{1}{22}x^2 + \frac{1}{1496}x^4 + \mathrm{O}\left(x^6\right)\right) + c_1 \left(1 - \frac{1}{2}x^2 + \frac{1}{56}x^4 + \mathrm{O}\left(x^6\right)\right)}{x^{\frac{2}{3}}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 48

```
AsymptoticDSolveValue[3*x^2*y''[x]+2*x*y'[x]+(x^2-2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 x \left(\frac{x^4}{1496} - \frac{x^2}{22} + 1\right) + \frac{c_2 \left(\frac{x^4}{56} - \frac{x^2}{2} + 1\right)}{x^{2/3}}$$

23.21 problem 25

Internal problem ID [2400]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 41, page 195
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^3(x^2+3)\,y''+5y'x-(1+x)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✗ Solution by Maple

```
Order:=6;
dsolve(x^3*(3+x^2)*diff(y(x),x$2)+5*x*diff(y(x),x)-(1+x)*y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 99

```
AsymptoticDSolveValue[x^3*(3+x^2)*y''[x]+5*x*y'[x]-(1+x)*y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned}y(x)\to{}& c_1\left(\frac{18968303719x^5}{1220703125000}-\frac{20383193x^4}{1953125000}+\frac{26731x^3}{3906250}+\frac{259x^2}{31250}+\frac{37x}{125}+1\right)\sqrt[5]{x}\\&+c_2e^{\frac{5}{3}/x}\left(\frac{869909160612721304x^5}{27030487060546875}+\frac{468477888792624x^4}{4805419921875}+\frac{15542572604x^3}{4271484375}\right.\\&\left.+\frac{2270672x^2}{1265625}+\frac{1372x}{1125}+1\right)x^{9/5}\end{aligned}$$

23.22 problem 26

Internal problem ID [2401]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 41, page 195
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' - (x^3+1)\,y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
Order:=6;
dsolve(2*x*diff(y(x),x$2)-(1+x^3)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{3}{2}}\left(1 - \frac{1}{5}x + \frac{1}{70}x^2 + \frac{52}{945}x^3 - \frac{1049}{83160}x^4 + \frac{5207}{5405400}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(1 + x - \frac{1}{2}x^2 + \frac{1}{18}x^3 + \frac{17}{360}x^4 - \frac{377}{12600}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 81

```
AsymptoticDSolveValue[2*x*y''[x]-(1+x^3)*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(-\frac{377x^5}{12600} + \frac{17x^4}{360} + \frac{x^3}{18} - \frac{x^2}{2} + x + 1\right) + c_1\left(\frac{5207x^5}{5405400} - \frac{1049x^4}{83160} + \frac{52x^3}{945} + \frac{x^2}{70} - \frac{x}{5} + 1\right)x^{3/2}$$

24 Exercise 42, page 206

24.1 problem 1

Internal problem ID [2402]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 42, page 206
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + y' + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
Order:=6;
dsolve(x*diff(y(x),x$2)+diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1)\left(1 - 2x + x^2 - \frac{2}{9}x^3 + \frac{1}{36}x^4 - \frac{1}{450}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(4x - 3x^2 + \frac{22}{27}x^3 - \frac{25}{216}x^4 + \frac{137}{13500}x^5 + \mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 101

```
AsymptoticDSolveValue[x*y''[x]+y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{x^5}{450} + \frac{x^4}{36} - \frac{2x^3}{9} + x^2 - 2x + 1\right) + c_2\left(\frac{137x^5}{13500} - \frac{25x^4}{216} + \frac{22x^3}{27} - 3x^2 + \left(-\frac{x^5}{450} + \frac{x^4}{36} - \frac{2x^3}{9} + x^2 - 2x + 1\right)\log(x) + 4x\right)$$

24.2 problem 2

Internal problem ID [2403]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 42, page 206
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + y' + 2yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 41

```
Order:=6;
dsolve(x*diff(y(x),x$2)+diff(y(x),x)+2*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1)\left(1 - \frac{1}{2}x^2 + \frac{1}{16}x^4 + \mathrm{O}(x^6)\right) + \left(\frac{1}{2}x^2 - \frac{3}{32}x^4 + \mathrm{O}(x^6)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 60

```
AsymptoticDSolveValue[x*y''[x]+y'[x]+2*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^4}{16} - \frac{x^2}{2} + 1\right) + c_2\left(-\frac{3x^4}{32} + \frac{x^2}{2} + \left(\frac{x^4}{16} - \frac{x^2}{2} + 1\right)\log(x)\right)$$

24.3 problem 3

Internal problem ID [2404]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964

Section: Exercise 42, page 206

Problem number: 3.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 3y'x + 4y(x+1) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 69

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((c_2 \ln(x) + c_1)\left(1 - 4x + 4x^2 - \frac{16}{9}x^3 + \frac{4}{9}x^4 - \frac{16}{225}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(8x - 12x^2 + \frac{176}{27}x^3 - \frac{50}{27}x^4 + \frac{1096}{3375}x^5 + \mathrm{O}\left(x^6\right)\right)c_2\right)x^2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 116

```
AsymptoticDSolveValue[x^2*y''[x]-3*x*y'[x]+4*(1+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{16x^5}{225} + \frac{4x^4}{9} - \frac{16x^3}{9} + 4x^2 - 4x + 1\right)x^2 + c_2\left(\left(\frac{1096x^5}{3375} - \frac{50x^4}{27} + \frac{176x^3}{27} - 12x^2 + 8x\right)x^2 + \left(-\frac{16x^5}{225} + \frac{4x^4}{9} - \frac{16x^3}{9} + 4x^2 - 4x + 1\right)x^2\log(x)\right)$$

24.4 problem 4

Internal problem ID [2405]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 42, page 206
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(x+1)\,y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 63

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*(1+x)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((c_2\ln(x) + c_1)\left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(-x - \frac{3}{4}x^2 - \frac{11}{36}x^3 - \frac{25}{288}x^4 - \frac{137}{7200}x^5 + \mathrm{O}\left(x^6\right)\right)c_2\right)x$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 112

```
AsymptoticDSolveValue[x^2*y''[x]-x*(1+x)*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1x\left(\frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1\right) + c_2\left(x\left(-\frac{137x^5}{7200} - \frac{25x^4}{288} - \frac{11x^3}{36} - \frac{3x^2}{4} - x\right) + x\left(\frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1\right)\log(x)\right)$$

24.5 problem 5

Internal problem ID [2406]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 42, page 206
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(2x+3)\,y' + 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 69

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*(2*x+3)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((c_2 \ln(x) + c_1)\left(1 + 4x + 6x^2 + \frac{16}{3}x^3 + \frac{10}{3}x^4 + \frac{8}{5}x^5 + \mathrm{O}\left(x^6\right)\right) + \left((-6)\,x - 13x^2 - \frac{124}{9}x^3 - \frac{173}{18}x^4 - \frac{374}{75}x^5 + \mathrm{O}\left(x^6\right)\right)c_2\right)x^2$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 116

```
AsymptoticDSolveValue[x^2*y''[x]-x*(2*x+3)*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{8x^5}{5} + \frac{10x^4}{3} + \frac{16x^3}{3} + 6x^2 + 4x + 1\right)x^2 + c_2\left(\left(-\frac{374x^5}{75} - \frac{173x^4}{18} - \frac{124x^3}{9} - 13x^2 - 6x\right)x^2 + \left(\frac{8x^5}{5} + \frac{10x^4}{3} + \frac{16x^3}{3} + 6x^2 + 4x + 1\right)x^2 \log(x)\right)$$

24.6 problem 6

Internal problem ID [2407]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 42, page 206
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(-x^2+1)\,y''-5y'x+9y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 51

```
Order:=6;
dsolve(x^2*(1-x^2)*diff(y(x),x$2)-5*x*diff(y(x),x)+9*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left((c_2\ln(x)+c_1)\left(1+\frac{3}{2}x^2+\frac{15}{8}x^4+\mathrm{O}\left(x^6\right)\right)+\left(-\frac{1}{4}x^2-\frac{13}{32}x^4+\mathrm{O}\left(x^6\right)\right)c_2\right)x^3$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 71

```
AsymptoticDSolveValue[x^2*(1-x^2)*y''[x]-5*x*y'[x]+9*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{15x^4}{8}+\frac{3x^2}{2}+1\right)x^3+c_2\left(\left(-\frac{13x^4}{32}-\frac{x^2}{4}\right)x^3+\left(\frac{15x^4}{8}+\frac{3x^2}{2}+1\right)x^3\log(x)\right)$$

24.7 problem 7

Internal problem ID [2408]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 42, page 206
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(x^2-1)\,y' + (-x^2+1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(x^2-1)*diff(y(x),x)+(1-x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((c_2\ln(x)+c_1)\left(1+\mathrm{O}\left(x^6\right)\right)+\left(-\frac{1}{4}x^2+\frac{1}{32}x^4+\mathrm{O}\left(x^6\right)\right)c_2\right)x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 33

```
AsymptoticDSolveValue[x^2*y''[x]+x*(x^2-1)*y'[x]+(1-x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x\left(\frac{x^4}{32}-\frac{x^2}{4}\right)+x\log(x)\right)+c_1x$$

24.8 problem 8

Internal problem ID [2409]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 42, page 206
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(2x-1)\,y' + x(x-1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(2*x-1)*diff(y(x),x)+x*(x-1)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = {} & c_1x^2\left(1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + \mathrm{O}\left(x^6\right)\right) \\ & + c_2\left(-2 + 2x - \frac{2}{3}x^3 + \frac{5}{12}x^4 - \frac{3}{20}x^5 + \mathrm{O}\left(x^6\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 59

```
AsymptoticDSolveValue[x^2*y''[x]+x*(2*x-1)*y'[x]+x*(x-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{5x^4}{24} + \frac{x^3}{3} - x + 1\right) + c_2\left(\frac{x^6}{24} - \frac{x^5}{6} + \frac{x^4}{2} - x^3 + x^2\right)$$

24.9 problem 9

Internal problem ID [2410]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 42, page 206
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x^2y' + (x^2 - 2)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x^2*diff(y(x),x)+(x^2-2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^2 \left(1 + \frac{1}{2}x + \frac{1}{20}x^2 - \frac{1}{60}x^3 - \frac{1}{210}x^4 - \frac{1}{3360}x^5 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(12 + 6x + 6x^2 + 5x^3 + x^4 - \frac{1}{5}x^5 + \mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 68

```
AsymptoticDSolveValue[x^2*y''[x]-x^2*y'[x]+(x^2-2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^3}{12} + \frac{5x^2}{12} + \frac{x}{2} + \frac{1}{x} + \frac{1}{2}\right) + c_2\left(-\frac{x^6}{210} - \frac{x^5}{60} + \frac{x^4}{20} + \frac{x^3}{2} + x^2\right)$$

24.10 problem 10

Internal problem ID [2411]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 42, page 206
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 2x^2y' - (3x^2 + 2)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+2*x^2*diff(y(x),x)-(3*x^2+2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^2 \left(1 - x + \frac{9}{10}x^2 - \frac{17}{30}x^3 + \frac{251}{840}x^4 - \frac{37}{280}x^5 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(12 - 12x - 18x^2 + 44x^3 - \frac{115}{2}x^4 + \frac{477}{10}x^5 + \mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 64

```
AsymptoticDSolveValue[x^2*y''[x]+2*x^2*y'[x]-(3*x^2+2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{115x^3}{24} + \frac{11x^2}{3} - \frac{3x}{2} + \frac{1}{x} - 1\right) + c_2\left(\frac{251x^6}{840} - \frac{17x^5}{30} + \frac{9x^4}{10} - x^3 + x^2\right)$$

24.11 problem 11

Internal problem ID [2412]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 42, page 206
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1-x)\,y'' + x(x+1)\,y' - 9y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
Order:=6;
dsolve(x^2*(1-x)*diff(y(x),x$2)+x*(1+x)*diff(y(x),x)-9*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^3 \left(1 + \frac{3}{7}x + \frac{3}{14}x^2 + \frac{5}{42}x^3 + \frac{1}{14}x^4 + \frac{1}{22}x^5 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2(-86400 + 259200x - 259200x^2 + 86400x^3 + \mathrm{O}\left(x^6\right))}{x^3}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 57

```
AsymptoticDSolveValue[x^2*(1-x)*y''[x]+x*(1+x)*y'[x]-9*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{1}{x^3} - \frac{3}{x^2} + \frac{3}{x} - 1\right) + c_2 \left(\frac{x^7}{14} + \frac{5x^6}{42} + \frac{3x^5}{14} + \frac{3x^4}{7} + x^3\right)$$

24.12 problem 12

Internal problem ID [2413]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 42, page 206
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(-x^2+x)\,y''-3y'+2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
Order:=6;
dsolve((x-x^2)*diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = {} & c_1x^4\left(1+2x+3x^2+4x^3+5x^4+6x^5+\mathrm{O}\left(x^6\right)\right) \\ & + c_2\left(-144-96x-48x^2+48x^4+96x^5+\mathrm{O}\left(x^6\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 55

```
AsymptoticDSolveValue[(x-x^2)*y''[x]-3*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(-\frac{x^4}{3}+\frac{x^2}{3}+\frac{2x}{3}+1\right)+c_2\left(5x^8+4x^7+3x^6+2x^5+x^4\right)$$

24.13 problem 13

Internal problem ID [2414]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 42, page 206
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(x-7)\,y' + (x+12)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 63

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(x-7)*diff(y(x),x)+(x+12)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(c_1x^4\left(1-\frac{7}{5}x+\frac{14}{15}x^2-\frac{2}{5}x^3+\frac{1}{8}x^4-\frac{11}{360}x^5+\mathrm{O}\left(x^6\right)\right)\right.$$
$$+ c_2\left(\ln\left(x\right)\left(360x^4-504x^5+\mathrm{O}\left(x^6\right)\right)\right.$$
$$\left.\left.+\left(-144-144x-144x^2-240x^3+342x^4+54x^5+\mathrm{O}\left(x^6\right)\right)\right)\right)x^2$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 79

```
AsymptoticDSolveValue[x^2*y''[x]+x*(x-7)*y'[x]+(x+12)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{5}{2}x^6\log(x)-\frac{1}{12}\left(21x^4-20x^3-12x^2-12x-12\right)x^2\right)$$
$$+ c_2\left(\frac{x^{10}}{8}-\frac{2x^9}{5}+\frac{14x^8}{15}-\frac{7x^7}{5}+x^6\right)$$

24.14 problem 14

Internal problem ID [2415]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 42, page 206
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x+1)\,y''+x(x-4)\,y'+4y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 61

```
Order:=6;
dsolve(x^2*(x+1)*diff(y(x),x$2)+x*(x-4)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = x\big(c_1x^3\left(1-4x+10x^2-20x^3+35x^4-56x^5+\mathrm{O}\left(x^6\right)\right) \\ + c_2\big(\ln\left(x\right)\left(\left(-36\right)x^3+144x^4-360x^5+\mathrm{O}\left(x^6\right)\right) \\ + \left(12+6x+12x^2-240x^3+852x^4-2022x^5+\mathrm{O}\left(x^6\right)\right)\big)\big) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 70

```
AsymptoticDSolveValue[x^2*(x+1)*y''[x]+x*(x-4)*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned} y(x) \to c_1\left(3(4x-1)x^4\log(x)+\frac{1}{2}\left(62x^4-20x^3+2x^2+x+2\right)x\right) \\ + c_2\left(35x^8-20x^7+10x^6-4x^5+x^4\right) \end{aligned}$$

24.15 problem 15

Internal problem ID [2416]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 42, page 206
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(-x^2+3)\, y' - 3y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(3-x^2)*diff(y(x),x)-3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^4 \left(1 + \frac{1}{12}x^2 + \frac{1}{128}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2\left(\ln\left(x\right)\left(27x^4 + \mathrm{O}\left(x^6\right)\right) + \left(-144 - 108x^2 - 36x^4 + \mathrm{O}\left(x^6\right)\right)\right)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 53

```
AsymptoticDSolveValue[x^2*y''[x]+x*(3-x^2)*y'[x]-3*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{128} + \frac{x^3}{12} + x\right) + c_1\left(\frac{19x^4 + 48x^2 + 64}{64x^3} - \frac{3}{16}x\log(x)\right)$$

25 Exercise 43, page 209

25.1 problem 1

Internal problem ID [2417]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 43, page 209
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + 3y' - y = x}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 78

```
Order:=6;
dsolve(x*diff(y(x),x$2)+3*diff(y(x),x)-y(x)=x,y(x),type='series',x=0);
```

$$y(x) = c_1\left(1 + \frac{1}{3}x + \frac{1}{24}x^2 + \frac{1}{360}x^3 + \frac{1}{8640}x^4 + \frac{1}{302400}x^5 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(\ln\left(x\right)\left(x^2 + \frac{1}{3}x^3 + \frac{1}{24}x^4 + \frac{1}{360}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(-2 + 2x - \frac{4}{9}x^3 - \frac{25}{288}x^4 - \frac{157}{21600}x^5 + \mathrm{O}\left(x^6\right)\right)\right)}{x^2} + x^2\left(\frac{1}{8} + \frac{1}{120}x + \frac{1}{2880}x^2 + \frac{1}{100800}x^3 + \mathrm{O}\left(x^4\right)\right)$$

✓ Solution by Mathematica

Time used: 0.083 (sec). Leaf size: 248

```
AsymptoticDSolveValue[x*y''[x]+3*y'[x]-y[x]==x,y[x],{x,0,5}]
```

$$
\begin{aligned}
y(x) \to\ & \frac{c_2\left(x^4\left(\frac{25}{576}-\frac{\log(x)}{48}\right)+x^3\left(\frac{2}{9}-\frac{\log(x)}{6}\right)-\frac{1}{2}x^2\log(x)-x+1\right)}{x^2} \\
& + c_1\left(\frac{x^5}{302400}+\frac{x^4}{8640}+\frac{x^3}{360}+\frac{x^2}{24}+\frac{x}{3}+1\right)+\left(\frac{x^5}{302400}+\frac{x^4}{8640}+\frac{x^3}{360}+\frac{x^2}{24}+\frac{x}{3}\right. \\
& \left.+1\right)\left(\frac{x^6(9-4\log(x))}{2304}+\frac{1}{900}x^5(23-15\log(x))+\frac{1}{64}x^4(1-4\log(x))-\frac{x^3}{6}+\frac{x^2}{4}\right) \\
& + \frac{\left(-\frac{x^6}{288}-\frac{x^5}{30}-\frac{x^4}{8}\right)\left(x^4\left(\frac{25}{576}-\frac{\log(x)}{48}\right)+x^3\left(\frac{2}{9}-\frac{\log(x)}{6}\right)-\frac{1}{2}x^2\log(x)-x+1\right)}{x^2}
\end{aligned}
$$

25.2 problem 1

Internal problem ID [2418]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 43, page 209
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + 3y' - y = x}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 78

```
Order:=6;
dsolve(x*diff(y(x),x$2)+3*diff(y(x),x)-y(x)=x,y(x),type='series',x=0);
```

$$
\begin{aligned}
y(x) = {} & c_1 \left(1 + \frac{1}{3}x + \frac{1}{24}x^2 + \frac{1}{360}x^3 + \frac{1}{8640}x^4 + \frac{1}{302400}x^5 + \mathrm{O}\left(x^6\right)\right) \\
& + \frac{c_2 \left(\ln(x) \left(x^2 + \frac{1}{3}x^3 + \frac{1}{24}x^4 + \frac{1}{360}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(-2 + 2x - \frac{4}{9}x^3 - \frac{25}{288}x^4 - \frac{157}{21600}x^5 + \mathrm{O}\left(x^6\right)\right)\right)}{x^2} \\
& + x^2 \left(\frac{1}{8} + \frac{1}{120}x + \frac{1}{2880}x^2 + \frac{1}{100800}x^3 + \mathrm{O}\left(x^4\right)\right)
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.084 (sec). Leaf size: 248

```
AsymptoticDSolveValue[x*y''[x]+3*y'[x]-y[x]==x,y[x],{x,0,5}]
```

$$
\begin{aligned}
y(x) \rightarrow & \frac{c_2\left(x^4\left(\frac{25}{576}-\frac{\log(x)}{48}\right)+x^3\left(\frac{2}{9}-\frac{\log(x)}{6}\right)-\frac{1}{2}x^2\log(x)-x+1\right)}{x^2} \\
& +c_1\left(\frac{x^5}{302400}+\frac{x^4}{8640}+\frac{x^3}{360}+\frac{x^2}{24}+\frac{x}{3}+1\right)+\left(\frac{x^5}{302400}+\frac{x^4}{8640}+\frac{x^3}{360}+\frac{x^2}{24}+\frac{x}{3}\right. \\
& \left.+1\right)\left(\frac{x^6(9-4\log(x))}{2304}+\frac{1}{900}x^5(23-15\log(x))+\frac{1}{64}x^4(1-4\log(x))-\frac{x^3}{6}+\frac{x^2}{4}\right) \\
& +\frac{\left(-\frac{x^6}{288}-\frac{x^5}{30}-\frac{x^4}{8}\right)\left(x^4\left(\frac{25}{576}-\frac{\log(x)}{48}\right)+x^3\left(\frac{2}{9}-\frac{\log(x)}{6}\right)-\frac{1}{2}x^2\log(x)-x+1\right)}{x^2}
\end{aligned}
$$

25.3 problem 2

Internal problem ID [2419]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 43, page 209
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{xy'' + y' - 2yx = x^2}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
Order:=6;
dsolve(x*diff(y(x),x$2)+diff(y(x),x)-2*x*y(x)=x^2,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = &(c_2 \ln(x) + c_1)\left(1 + \frac{1}{2}x^2 + \frac{1}{16}x^4 + \mathrm{O}\left(x^6\right)\right) \\ &+ x^3\left(\frac{1}{9} + \frac{2}{225}x^2 + \mathrm{O}\left(x^3\right)\right) + \left(-\frac{1}{2}x^2 - \frac{3}{32}x^4 + \mathrm{O}\left(x^6\right)\right)c_2 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 188

```
AsymptoticDSolveValue[x*y''[x]+y'[x]-2*x*y[x]==x^2,y[x],{x,0,5}]
```

$$
\begin{aligned}
y(x) \to\ & c_2\left(\frac{x^6}{288}+\frac{x^4}{16}+\frac{x^2}{2}+1\right) \\
& + c_1\left(x^6\left(\frac{\log(x)}{144}-\frac{1}{108}\right)+x^4\left(\frac{\log(x)}{8}-\frac{1}{8}\right)+x^2\left(\log(x)-\frac{1}{2}\right)+2\log(x)+1\right) \\
& + \left(\frac{x^6}{288}+\frac{x^4}{16}+\frac{x^2}{2}+1\right)\left(\frac{1}{100}x^5(7-10\log(x))+\frac{1}{18}x^3(-6\log(x)-1)\right)+\left(\frac{x^5}{20}\right. \\
& \left.+\frac{x^3}{6}\right)\left(x^6\left(\frac{\log(x)}{144}-\frac{1}{108}\right)+x^4\left(\frac{\log(x)}{8}-\frac{1}{8}\right)+x^2\left(\log(x)-\frac{1}{2}\right)+2\log(x)+1\right)
\end{aligned}
$$

25.4 problem 3

Internal problem ID [2420]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964

Section: Exercise 43, page 209

Problem number: 3.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - y'x + y = x^3}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve(x*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=x^3,y(x),type='series',x=0);
```

$$y(x) = x^4\left(\frac{1}{12} + \frac{1}{80}x + \mathrm{O}\left(x^2\right)\right) + \ln(x)\left(-x + \mathrm{O}\left(x^6\right)\right)c_2 + c_1x\left(1 + \mathrm{O}\left(x^6\right)\right) + \left(1 + x - \frac{1}{2}x^2 - \frac{1}{12}x^3 - \frac{1}{72}x^4 - \frac{1}{480}x^5 + \mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.072 (sec). Leaf size: 139

```
AsymptoticDSolveValue[x*y''[x]-x*y'[x]+y[x]==x^3,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{x^4}{72} - \frac{x^3}{12} - \frac{x^2}{2} - x\log(x) + 1\right) + x\left(\frac{1}{36}x^6(2 - 3\log(x)) + \frac{1}{25}x^5(5\log(x) - 1) + \frac{1}{16}x^4(-4\log(x) - 3) + \frac{x^3}{3}\right) + \left(-\frac{x^6}{12} + \frac{x^5}{5} - \frac{x^4}{4}\right)\left(-\frac{x^4}{72} - \frac{x^3}{12} - \frac{x^2}{2} - x\log(x) + 1\right) + c_2x$$

25.5 problem 4

Internal problem ID [2421]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 43, page 209
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-2x+1)\,y''+4y'x-4y=x^2-x}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
Order:=6;
dsolve((1-2*x)*diff(y(x),x$2)+4*x*diff(y(x),x)-4*y(x)=x^2-x,y(x),type='series',x=0);
```

$$y(x)=\left(1+2x^2+\frac{4}{3}x^3+\frac{2}{3}x^4+\frac{4}{15}x^5\right)y(0)+D(y)\,(0)\,x-\frac{x^3}{6}-\frac{x^4}{12}-\frac{x^5}{30}+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 60

```
AsymptoticDSolveValue[(1-2*x)*y''[x]+4*x*y'[x]-4*y[x]==x^2-x,y[x],{x,0,5}]
```

$$y(x)\to -\frac{x^5}{30}-\frac{x^4}{12}-\frac{x^3}{6}+c_1\left(\frac{4x^5}{15}+\frac{2x^4}{3}+\frac{4x^3}{3}+2x^2+1\right)+c_2x$$

25.6 problem 5

Internal problem ID [2422]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 43, page 209
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + y'x + (x+12)\,y = x^2 + x}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 379

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x+12)*y(x)=x^2+x,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = {} & c_2 x^{2i\sqrt{3}} \left(1 - \frac{1}{4i\sqrt{3}+1}x + \frac{1}{-92+24i\sqrt{3}}x^2 + \frac{1}{888i\sqrt{3}+1692}x^3 \right. \\ & \left. + \frac{1}{15552-41280i\sqrt{3}}x^4 + \frac{1}{720960i\sqrt{3}-2865600}x^5 + \mathrm{O}\left(x^6\right)\right) \\ & + c_1 x^{-2i\sqrt{3}} \left(1 + \frac{1}{4i\sqrt{3}-1}x + \frac{1}{-92-24i\sqrt{3}}x^2 + \frac{1}{-888i\sqrt{3}+1692}x^3 \right. \\ & \left. + \frac{1}{15552+41280i\sqrt{3}}x^4 + \frac{1}{-720960i\sqrt{3}-2865600}x^5 + \mathrm{O}\left(x^6\right)\right) \\ & + x\left(\frac{1}{13} + \frac{3}{52}x - \frac{1}{364}x^2 + \frac{1}{10192}x^3 - \frac{1}{377104}x^4 + \mathrm{O}\left(x^5\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 1.224 (sec). Leaf size: 704

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x+12)*y[x]==x^2+x,y[x],{x,0,5}]
```

$$y(x) \to$$

$$-\frac{\left(518\left(139\sqrt{3}-100i\right)x^5-2555\left(929\sqrt{3}+1053i\right)x^4-46720\left(121\sqrt{3}-2726i\right)x^3+9320640\left(125\sqrt{3}-\right.\right.}{}$$

$$+\frac{\left(141+74i\sqrt{3}\right)\left(4\sqrt{3}-i\right)\left(6\sqrt{3}+23i\right)\left(215\sqrt{3}+81i\right)\left(751\sqrt{3}+2985i\right)\left(-\frac{ix^5}{720960\sqrt{3}+2865600i}+\frac{ix}{41280\sqrt{3}}\right.}{}$$

$$+c_2\left(-\frac{ix^5}{720960\sqrt{3}+2865600i}+\frac{ix^4}{41280\sqrt{3}+15552i}-\frac{ix^3}{888\sqrt{3}-1692i}-\frac{ix^2}{24\sqrt{3}+92i}-\frac{x}{1+4i\sqrt{3}}+1\right)x^{2i\sqrt{3}}+c_1\left(\frac{ix^5}{720960\sqrt{3}-2865600i}-\frac{ix^4}{41280\sqrt{3}-15552i}+\frac{ix^3}{888\sqrt{3}+1692i}+\frac{ix^2}{24\sqrt{3}-92i}-\frac{x}{1-4i\sqrt{3}}+1\right)x^{-2i\sqrt{3}}$$

25.7 problem 6

Internal problem ID [2423]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 43, page 209
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2(x+1)\,y''+x\left(x^2+3\right)y'+y=-2x^2+x}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 81

```
Order:=6;
dsolve(x^2*(x+1)*diff(y(x),x$2)+x*(x^2+3)*diff(y(x),x)+y(x)=x-2*x^2,y(x),type='series',x=0);
```

$$y(x) = \frac{x^2\left(\frac{1}{4}-\frac{2}{9}x+\frac{7}{576}x^2+\frac{107}{7200}x^3-\frac{1031}{172800}x^4+\mathrm{O}\left(x^5\right)\right)+\left(c_2\ln\left(x\right)+c_1\right)\left(1-2x+\frac{1}{4}x^2-\frac{1}{64}x^4+\frac{3}{800}x^5+\mathrm{O}\left(x\right.\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.138 (sec). Leaf size: 300

```
AsymptoticDSolveValue[x^2*(x+1)*y''[x]+x*(x^2+3)*y'[x]+y[x]==x-2*x^2,y[x],{x,0,5}]
```

$$y(x) \to \frac{c_2\left(\frac{3x^5}{800} - \frac{x^4}{64} + \frac{x^2}{4} - 2x + 1\right)}{x} + \frac{c_1\left(x^5\left(\frac{3\log(x)}{800} - \frac{629}{36000}\right) + x^4\left(\frac{17}{1152} - \frac{\log(x)}{64}\right) + \frac{7x^3}{36} + x^2\left(\frac{\log(x)}{4} - \frac{3}{4}\right) + x(5 - 2\log(x)) + \log(x) + 1\right)}{x} + \frac{\left(\frac{34109x^6}{1152} - \frac{491x^5}{30} + \frac{123x^4}{16} - \frac{8x^3}{3} + \frac{x^2}{2}\right)\left(x^5\left(\frac{3\log(x)}{800} - \frac{629}{36000}\right) + x^4\left(\frac{17}{1152} - \frac{\log(x)}{64}\right) + \frac{7x^3}{36} + x^2\left(\frac{\log(x)}{4} - \frac{3}{4}\right) \cdots}{x} + \frac{\left(\frac{3x^5}{800} - \frac{x^4}{64} + \frac{x^2}{4} - 2x + 1\right)\left(\frac{x^6(33586 - 34109\log(x))}{1152} + \frac{1}{900}x^5(14730\log(x) - 12641) + \frac{1}{64}x^4(319 - 492\log(x \cdots}{x}$$

25.8 problem 7

Internal problem ID [2424]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 43, page 209
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{3x^2(x+1)\,y''+x(5-x)\,y'+\left(2x^2-1\right)y=-x^3+x}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 63

```
Order:=6;
dsolve(3*x^2*(x+1)*diff(y(x),x$2)+x*(5-x)*diff(y(x),x)+(2*x^2-1)*y(x)=x-x^3,y(x),type='serie
```

$$y(x) = \frac{c_2x^{\frac{4}{3}}\left(1+\frac{1}{7}x-\frac{1}{10}x^2+\frac{29}{2730}x^3-\frac{17}{87360}x^4-\frac{1193}{8299200}x^5+\mathrm{O}\left(x^6\right)\right)+x^2\left(\frac{1}{4}+\frac{1}{60}x-\frac{47}{960}x^2+\frac{673}{52800}x^3-\frac{1169}{316800}x^{}}{x}$$

✓ Solution by Mathematica

Time used: 0.057 (sec). Leaf size: 255

```
AsymptoticDSolveValue[3*x^2*(x+1)*y''[x]+x*(5-x)*y'[x]+(2*x^2-1)*y[x]==x-x^3,y[x],{x,0,5}]
```

$$
\begin{aligned}
y(x) \to\; & \frac{c_1\left(-\frac{167x^5}{26400}+\frac{73x^4}{480}-\frac{29x^3}{30}-\frac{x^2}{2}+7x+1\right)}{x} \\
& + c_2\sqrt[3]{x}\left(-\frac{1193x^5}{8299200}-\frac{17x^4}{87360}+\frac{29x^3}{2730}-\frac{x^2}{10}+\frac{x}{7}+1\right)+\sqrt[3]{x}\left(-\frac{1193x^5}{8299200}-\frac{17x^4}{87360}\right. \\
& \left.+\frac{29x^3}{2730}-\frac{x^2}{10}+\frac{x}{7}+1\right)\left(\frac{19491x^{17/3}}{8800}-\frac{541x^{14/3}}{256}+\frac{107x^{11/3}}{55}-\frac{99x^{8/3}}{64}+\frac{3x^{5/3}}{5}\right. \\
& \left.+\frac{3x^{2/3}}{8}\right)+\frac{\left(-\frac{167x^5}{26400}+\frac{73x^4}{480}-\frac{29x^3}{30}-\frac{x^2}{2}+7x+1\right)\left(-\frac{652399x^6}{2096640}+\frac{2039x^5}{6825}-\frac{313x^4}{1120}+\frac{5x^3}{21}-\frac{x^2}{8}\right)}{x}
\end{aligned}
$$

25.9 problem 8

Internal problem ID [2425]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 43, page 209
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{9x^2y'' + (2+3x)\,y = x^4 + x^2}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 63

```
Order:=6;
dsolve(9*x^2*diff(y(x),x$2)+(2+3*x)*y(x)=x^2+x^4,y(x),type='series',x=0);
```

$$\begin{aligned}y(x) = {} & c_1 x^{\frac{1}{3}}\left(1-\frac{1}{2}x+\frac{1}{20}x^2-\frac{1}{480}x^3+\frac{1}{21120}x^4-\frac{1}{1478400}x^5+\mathrm{O}\left(x^6\right)\right)\\ & + c_2 x^{\frac{2}{3}}\left(1-\frac{1}{4}x+\frac{1}{56}x^2-\frac{1}{1680}x^3+\frac{1}{87360}x^4-\frac{1}{6988800}x^5+\mathrm{O}\left(x^6\right)\right)\\ & + x^2\left(\frac{1}{20}-\frac{3}{1120}x+\frac{1129}{123200}x^2-\frac{3387}{22422400}x^3+\mathrm{O}\left(x^4\right)\right)\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.182 (sec). Leaf size: 264

```
AsymptoticDSolveValue[9*x^2*y''[x]+(2+3*x)*y[x]==x^2+x^4,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt[3]{x}\left(-\frac{x^5}{1478400}+\frac{x^4}{21120}-\frac{x^3}{480}+\frac{x^2}{20}-\frac{x}{2}+1\right)+x^{2/3}\left(-\frac{x^5}{6988800}+\frac{x^4}{87360}-\frac{x^3}{1680}+\frac{x^2}{56}-\frac{x}{4}+1\right)\left(\frac{1057x^{16/3}}{337920}-\frac{241x^{13/3}}{6240}+\frac{21x^{10/3}}{200}-\frac{x^{7/3}}{14}+\frac{x^{4/3}}{4}\right)+\sqrt[3]{x}\left(-\frac{x^5}{1478400}+\frac{x^4}{21120}-\frac{x^3}{480}+\frac{x^2}{20}-\frac{x}{2}+1\right)\left(-\frac{223x^{17/3}}{212160}+\frac{421x^{14/3}}{23520}-\frac{57x^{11/3}}{616}+\frac{x^{8/3}}{32}-\frac{x^{5/3}}{5}\right)$$

25.10 problem 9

Internal problem ID [2426]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 43, page 209
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9x^2y'' + 10y'x + y = x - 1}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 49

```
Order:=6;
dsolve(9*x^2*diff(y(x),x$2)+10*x*diff(y(x),x)+y(x)=x-1,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{-\frac{1}{18}-\frac{i\sqrt{35}}{18}}\left(1+\mathrm{O}\left(x^6\right)\right) + c_2 x^{-\frac{1}{18}+\frac{i\sqrt{35}}{18}}\left(1+\mathrm{O}\left(x^6\right)\right) + \left(-1+\frac{1}{11}x+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.294 (sec). Leaf size: 198

```
AsymptoticDSolveValue[9*x^2*y''[x]+10*x*y'[x]+y[x]==x-1,y[x],{x,0,5}]
```

$$\begin{aligned} y(x) \to\ & \frac{9i\left(\left(\sqrt{35}-i\right)x-\sqrt{35}+19i\right)x^{\frac{1}{18}\left(-1-i\sqrt{35}\right)+\frac{1}{18}\left(1+i\sqrt{35}\right)}}{\sqrt{35}\left(10\sqrt{35}+8i\right)} \\ & - \frac{9i\left(\left(\sqrt{35}+i\right)x-\sqrt{35}-19i\right)x^{\frac{1}{18}\left(1-i\sqrt{35}\right)+\frac{1}{18}\left(-1+i\sqrt{35}\right)}}{\sqrt{35}\left(10\sqrt{35}-8i\right)} \\ & + c_2 x^{\frac{1}{18}\left(-1+i\sqrt{35}\right)} + c_1 x^{\frac{1}{18}\left(-1-i\sqrt{35}\right)} \end{aligned}$$

25.11 problem 10

Internal problem ID [2427]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 43, page 209
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{2x^2y'' + \left(-x^2 + x\right)y' - y = x^3 + 1}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 57

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)+(x-x^2)*diff(y(x),x)-y(x)=1+x^3,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{3}{2}}\left(1 + \frac{1}{5}x + \frac{1}{35}x^2 + \frac{1}{315}x^3 + \frac{1}{3465}x^4 + \frac{1}{45045}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(-1 + \frac{1}{14}x^3 + \frac{1}{126}x^4 + \frac{1}{1386}x^5 + \mathrm{O}\left(x^6\right)\right)\sqrt{x}}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 248

```
AsymptoticDSolveValue[2*x^2*y''[x]+(x-x^2)*y'[x]-y[x]==1+x^3,y[x],{x,0,5}]
```

$$
\begin{aligned}
y(x) \to\ & \frac{c_1\left(\frac{x^5}{3840}+\frac{x^4}{384}+\frac{x^3}{48}+\frac{x^2}{8}+\frac{x}{2}+1\right)}{\sqrt{x}}+c_2 x\left(\frac{x^5}{45045}+\frac{x^4}{3465}+\frac{x^3}{315}+\frac{x^2}{35}+\frac{x}{5}+1\right) \\
& +\frac{\left(\frac{x^5}{3840}+\frac{x^4}{384}+\frac{x^3}{48}+\frac{x^2}{8}+\frac{x}{2}+1\right)\left(-\frac{6233x^{11/2}}{1921920}+\frac{2107x^{9/2}}{95040}-\frac{143x^{7/2}}{1512}-\frac{x^{5/2}}{140}+\frac{x^{3/2}}{15}-\frac{2\sqrt{x}}{3}\right)}{\sqrt{x}} \\
& +x\left(\frac{x^5}{45045}+\frac{x^4}{3465}+\frac{x^3}{315}+\frac{x^2}{35}+\frac{x}{5}+1\right)\left(\frac{x^6}{691200}-\frac{2747x^5}{518918400}+\frac{x^2}{6}-\frac{1}{3x}\right)
\end{aligned}
$$

25.12 problem 11

Internal problem ID [2428]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 43, page 209
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-x^2+1)\,y''+2y'x-2y=6(-x^2+1)^2}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
Order:=6;
dsolve((1-x^2)*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=6*(1-x^2)^2,y(x),type='series',x=0);
```

$$y(x)=(x^2+1)\,y(0)-x^4+D(y)\,(0)\,x+3x^2+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 26

```
AsymptoticDSolveValue[(1-x^2)*y''[x]+2*x*y'[x]-2*y[x]==6*(1-x^2)^2,y[x],{x,0,5}]
```

$$y(x)\to -x^4+3x^2+c_1(x^2+1)+c_2x$$

25.13 problem 12

Internal problem ID [2429]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 43, page 209
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+2x)\,y''-(2x+2)\,y'+2y=x^2(2+x)^2}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
Order:=6;
dsolve((x^2+2*x)*diff(y(x),x$2)-(2+2*x)*diff(y(x),x)+2*y(x)=x^2*(x+2)^2,y(x),type='series',x
```

$$y(x)=c_1x^2\left(1+\mathrm{O}\left(x^6\right)\right)+c_2\left(-2-2x-\frac{1}{2}x^2+\mathrm{O}\left(x^6\right)\right)+x^3\left(\frac{2}{3}+\frac{1}{6}x+\mathrm{O}\left(x^3\right)\right)$$

✓ Solution by Mathematica

Time used: 0.317 (sec). Leaf size: 39

```
AsymptoticDSolveValue[(x^2+2*x)*y''[x]-(2+2*x)*y'[x]+2*y[x]==x^2*(x+2)^2,y[x],{x,0,5}]
```

$$y(x)\to-\frac{1}{3}(x+1)x^3+\left(\frac{x^2}{2}+x\right)x^2+c_2x^2+c_1(x+1)$$

25.14 problem 13

Internal problem ID [2430]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 43, page 209
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{2x^2y'' + 5y'x + y(x+1) = x(x^2+x+1)}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 63

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+(1+x)*y(x)=x*(1+x+x^2),y(x),type='series',x=0);
```

$$y(x) = \frac{x^{\frac{5}{2}}\left(\frac{1}{6} + \frac{1}{18}x + \frac{17}{504}x^2 - \frac{17}{22680}x^3 + \frac{17}{1496880}x^4 + \mathrm{O}\left(x^5\right)\right) + c_1\left(1 - x + \frac{1}{6}x^2 - \frac{1}{90}x^3 + \frac{1}{2520}x^4 - \frac{1}{113400}x^5 + \mathrm{O}\left(\right.\right.}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 237

```
AsymptoticDSolveValue[2*x^2*y''[x]+5*x*y'[x]+(1+x)*y[x]==x*(1+x+x^2),y[x],{x,0,5}]
```

$$
\begin{aligned}
y(x) \to & \frac{c_1\left(-\frac{x^5}{113400}+\frac{x^4}{2520}-\frac{x^3}{90}+\frac{x^2}{6}-x+1\right)}{x} \\
& +\frac{c_2\left(-\frac{x^5}{1247400}+\frac{x^4}{22680}-\frac{x^3}{630}+\frac{x^2}{30}-\frac{x}{3}+1\right)}{\sqrt{x}} \\
& +\frac{\left(-\frac{x^5}{1247400}+\frac{x^4}{22680}-\frac{x^3}{630}+\frac{x^2}{30}-\frac{x}{3}+1\right)\left(\frac{131x^{11/2}}{4620}-\frac{76x^{9/2}}{405}+\frac{x^{7/2}}{21}+\frac{2x^{3/2}}{3}\right)}{\sqrt{x}} \\
& +\frac{\left(-\frac{x^5}{113400}+\frac{x^4}{2520}-\frac{x^3}{90}+\frac{x^2}{6}-x+1\right)\left(-\frac{103x^6}{19440}+\frac{19x^5}{315}-\frac{7x^4}{40}-\frac{2x^3}{9}-\frac{x^2}{2}\right)}{x}
\end{aligned}
$$

25.15 problem 14

Internal problem ID [2431]

Book: Differential Equations by Alfred L. Nelson, Karl W. Folley, Max Coral. 3rd ed. DC heath. Boston. 1964
Section: Exercise 43, page 209
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{(x^3+2x^2)\,y''-y'x+(1-x)\,y=x^2(x+1)^2}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 61

```
Order:=6;
dsolve((2*x^2+x^3)*diff(y(x),x$2)-x*diff(y(x),x)+(1-x)*y(x)=x^2*(1+x)^2,y(x),type='series',x
```

$$y(x)=c_1\sqrt{x}\left(1+\frac{5}{4}x+\frac{5}{96}x^2-\frac{11}{1152}x^3+\frac{341}{129024}x^4-\frac{20119}{23224320}x^5+\mathrm{O}\left(x^6\right)\right)$$
$$+c_2x\left(1+\frac{1}{3}x-\frac{1}{30}x^2+\frac{1}{126}x^3-\frac{11}{4536}x^4+\frac{19}{22680}x^5+\mathrm{O}\left(x^6\right)\right)$$
$$+x^2\left(\frac{1}{3}+\frac{1}{6}x+\frac{1}{126}x^2-\frac{11}{4536}x^3+\mathrm{O}\left(x^4\right)\right)$$

✓ Solution by Mathematica

Time used: 0.273 (sec). Leaf size: 247

```
AsymptoticDSolveValue[(2*x^2+x^3)*y''[x]-x*y'[x]+(1-x)*y[x]==x^2*(1+x)^2,y[x],{x,0,5}]
```

$$
\begin{aligned}
y(x) \to{} & c_1\sqrt{x}\left(-\frac{20119x^5}{23224320}+\frac{341x^4}{129024}-\frac{11x^3}{1152}+\frac{5x^2}{96}+\frac{5x}{4}+1\right) \\
& + c_2 x\left(\frac{19x^5}{22680}-\frac{11x^4}{4536}+\frac{x^3}{126}-\frac{x^2}{30}+\frac{x}{3}+1\right)+\sqrt{x}\left(-\frac{20119x^5}{23224320}+\frac{341x^4}{129024}-\frac{11x^3}{1152}\right. \\
& \left.+\frac{5x^2}{96}+\frac{5x}{4}+1\right)\left(\frac{4997x^{11/2}}{2903040}-\frac{1853x^{9/2}}{181440}-\frac{183x^{7/2}}{560}-\frac{5x^{5/2}}{6}\right. \\
& \left.-\frac{2x^{3/2}}{3}\right)+x\left(\frac{19x^5}{22680}-\frac{11x^4}{4536}+\frac{x^3}{126}-\frac{x^2}{30}+\frac{x}{3}+1\right)\left(\frac{479x^6}{136080}-\frac{13x^5}{840}+\frac{13x^4}{72}+\frac{17x^3}{18}+\frac{3x^2}{2}+x\right)
\end{aligned}
$$