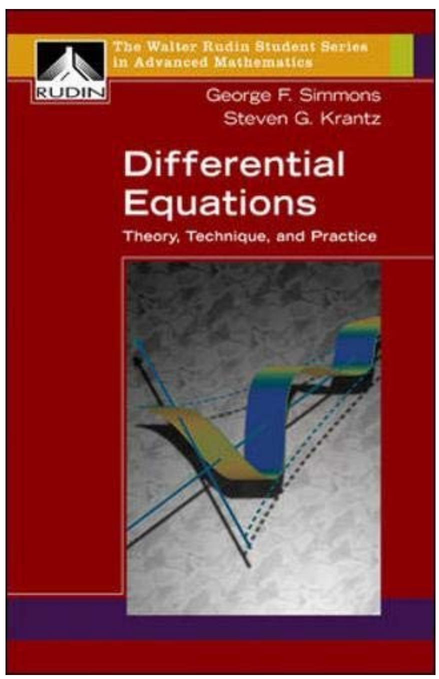
A Solution Manual For

# Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.



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March 3, 2024

# Contents

# 1 Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

## 1.1 problem 1(a)

Internal problem ID [6105]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 1(a).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = 2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)=2*x,y(x), singsol=all)
```

$$y(x) = x^2 + c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 11

```
DSolve[y'[x]==2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + c_1$$

## 1.2 problem 1(b)

Internal problem ID [6106]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 1(b).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{y'x - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(x*diff(y(x),x)=2*y(x),y(x), singsol=all)
```

$$y(x) = c_1 x^2$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 16

```
DSolve[x*y'[x]==2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x^2$$
$$y(x) \to 0$$

## 1.3 problem 1(c)

Internal problem ID [6107]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 1(c).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yy' = e^{2x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(y(x)*diff(y(x),x)=exp(2*x),y(x), singsol=all)
```

$$y(x) = \sqrt{e^{2x} + c_1}$$
$$y(x) = -\sqrt{e^{2x} + c_1}$$

✓ Solution by Mathematica

Time used: 0.658 (sec). Leaf size: 39

```
DSolve[y[x]*y'[x]==Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{e^{2x} + 2c_1}$$
$$y(x) \to \sqrt{e^{2x} + 2c_1}$$

## 1.4 problem 1(d)

Internal problem ID [6108]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 1(d).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - yk = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=k*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{kx}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 18

```
DSolve[y'[x]==k*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{kx}$$
$$y(x) \to 0$$

## 1.5 problem 1(e)

Internal problem ID [6109]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 1(e).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(2x) + c_2 \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 20

```
DSolve[y''[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(2x) + c_2 \sin(2x)$$

## 1.6 problem 1(f)

Internal problem ID [6110]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 1(f).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-2x}c_1 + \mathrm{e}^{2x}c_2$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 22

```
DSolve[y''[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(c_1 e^{4x} + c_2\right)$$

## 1.7 problem 1(h)

Internal problem ID [6111]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 1(h).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_rational]`

$$y + y'x - y'\sqrt{1 - yx^2} = 0$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)+y(x)=diff(y(x),x)*sqrt(1-x^2*y(x)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]+y[x]==y'[x]*Sqrt[1-x^2*y[x]],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

## 1.8 problem 1(i)

Internal problem ID [6112]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 1(i).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Riccati]`

$$\boxed{y'x - y - y^2 = x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(x*diff(y(x),x)=y(x)+x^2+y(x)^2,y(x), singsol=all)
```

$$y(x) = \tan(x + c_1)\, x$$

✓ Solution by Mathematica

Time used: 0.184 (sec). Leaf size: 12

```
DSolve[x*y'[x]==y[x]+x^2+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \tan(x + c_1)$$

## 1.9 problem 1(j)

Internal problem ID [6113]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 1(j).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y' - \frac{xy}{x^2 + y^2} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=(x*y(x))/(x^2+y(x)^2),y(x), singsol=all)
```

$$y(x) = \sqrt{\frac{1}{\operatorname{LambertW}(c_1 x^2)}}\, x$$

✓ Solution by Mathematica

Time used: 7.664 (sec). Leaf size: 49

```
DSolve[y'[x]==(x*y[x])/(x^2+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{\sqrt{W(e^{-2c_1} x^2)}}$$

$$y(x) \to \frac{x}{\sqrt{W(e^{-2c_1} x^2)}}$$

$$y(x) \to 0$$

## 1.10 problem 1(k)

Internal problem ID [6114]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 1(k).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{2xyy' - y^2 = x^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(2*x*y(x)*diff(y(x),x)=x^2+y(x)^2,y(x), singsol=all)
```

$$y(x) = \sqrt{c_1 x + x^2}$$
$$y(x) = -\sqrt{c_1 x + x^2}$$

✓ Solution by Mathematica

Time used: 0.182 (sec). Leaf size: 38

```
DSolve[2*x*y[x]*y'[x]==x^2+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x}\sqrt{x + c_1}$$
$$y(x) \to \sqrt{x}\sqrt{x + c_1}$$

## 1.11 problem 1(L)

Internal problem ID [6115]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 1(L).
**ODE order**: 1.
**ODE degree**: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{y + y'x - x^4{y'}^2 = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 135

```
dsolve(y(x)+x*diff(y(x),x)=x^4*(diff(y(x),x))^2,y(x), singsol=all)
```

$$y(x) = -\frac{1}{4x^2}$$
$$y(x) = \frac{-c_1^2 - c_1(2ix - c_1) - 2x^2}{2x^2c_1^2}$$
$$y(x) = \frac{-c_1^2 - c_1(-2ix - c_1) - 2x^2}{2x^2c_1^2}$$
$$y(x) = \frac{c_1(2ix + c_1) - 2x^2 - c_1^2}{2c_1^2x^2}$$
$$y(x) = \frac{c_1(-2ix + c_1) - 2x^2 - c_1^2}{2c_1^2x^2}$$

✓ Solution by Mathematica

Time used: 0.5 (sec). Leaf size: 123

```
DSolve[y[x]+x*y'[x]==x^4*(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{x\sqrt{4x^2y(x)+1}\operatorname{arctanh}\left(\sqrt{4x^2y(x)+1}\right)}{\sqrt{4x^4y(x)+x^2}}-\frac{1}{2}\log(y(x))=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{x\sqrt{4x^2y(x)+1}\operatorname{arctanh}\left(\sqrt{4x^2y(x)+1}\right)}{\sqrt{4x^4y(x)+x^2}}-\frac{1}{2}\log(y(x))=c_1,y(x)\right]$$

$$y(x)\to 0$$

## 1.12 problem 1(m)

Internal problem ID [6116]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 1(m).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{y^2}{yx - x^2} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)=y(x)^2/(x*y(x)-x^2),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\operatorname{LambertW}\left(-\frac{\mathrm{e}^{-c_1}}{x}\right)-c_1}$$

✓ Solution by Mathematica

Time used: 2.335 (sec). Leaf size: 25

```
DSolve[y'[x]==y[x]^2/(x*y[x]-x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -xW\left(-\frac{e^{-c_1}}{x}\right)$$
$$y(x) \to 0$$

## 1.13 problem 1(n)

Internal problem ID [6117]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 1(n).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{(y\cos(y)-\sin(y)+x)\,y'-y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve((y(x)*cos(y(x))-sin(y(x))+x)*diff(y(x),x)=y(x),y(x), singsol=all)
```

$$x-c_1y(x)-\sin(y(x))=0$$

✓ Solution by Mathematica

Time used: 0.245 (sec). Leaf size: 14

```
DSolve[(y[x]*Cos[y[x]]-Sin[y[x]]+x)*y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}[x=\sin(y(x))+c_1y(x),y(x)]$$

## 1.14 problem 1(o)

Internal problem ID [6118]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 1(o).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y^2 + y^2 y' = -1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve(1+y(x)^2+y(x)^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \tan\left(\mathrm{RootOf}\left(-\_Z + x + c_1 + \tan\left(\_Z\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.186 (sec). Leaf size: 35

```
DSolve[1+y[x]^2+y[x]^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}[\#1 - \arctan(\#1)\&][-x + c_1]$$
$$y(x) \to -i$$
$$y(x) \to i$$

## 1.15 problem 2(a)

Internal problem ID [6119]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 2(a).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \mathrm{e}^{3x} - x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=exp(3*x)-x,y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{2} + \frac{\mathrm{e}^{3x}}{3} + c_1$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 24

```
DSolve[y'[x]==Exp[3*x]-x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2}{2} + \frac{e^{3x}}{3} + c_1$$

## 1.16 problem 2(b)

Internal problem ID [6120]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 2(b).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type **[_quadrature]**

$$\boxed{y' = \mathrm{e}^{x^2} x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=x*exp(x^2),y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{x^2}}{2} + c_1$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 17

```
DSolve[y'[x]==x*Exp[x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{x^2}}{2} + c_1$$

## 1.17 problem 2(c)

Internal problem ID [6121]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 2(c).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'(1+x) = x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve((1+x)*diff(y(x),x)=x,y(x), singsol=all)
```

$$y(x) = x - \ln(x+1) + c_1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 15

```
DSolve[(1+x)*y'[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - \log(x+1) + c_1$$

## 1.18 problem 2(d)

Internal problem ID [6122]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 2(d).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(x^2+1)\, y' = x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve((1+x^2)*diff(y(x),x)=x,y(x), singsol=all)
```

$$y(x) = \frac{\ln(x^2+1)}{2} + c_1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[(1+x^2)*y'[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\log(x^2+1) + c_1$$

## 1.19 problem 2(e)

Internal problem ID [6123]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 2(e).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(x^2+1)\,y' = \arctan(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((1+x^2)*diff(y(x),x)=arctan(x),y(x), singsol=all)
```

$$y(x) = \frac{\arctan(x)^2}{2} + c_1$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 16

```
DSolve[(1+x^2)*y'[x]==ArcTan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\arctan(x)^2}{2} + c_1$$

## 1.20 problem 2(f)

Internal problem ID [6124]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 2(f).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'x = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(x*diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x) = \ln(x) + c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 10

```
DSolve[x*y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(x) + c_1$$

## 1.21 problem 2(g)

Internal problem ID [6125]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 2(g).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \arcsin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)=arcsin(x),y(x), singsol=all)
```

$$y(x) = x \arcsin(x) + \sqrt{-x^2+1} + c_1$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 23

```
DSolve[y'[x]==ArcSin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \arcsin(x) + \sqrt{1-x^2} + c_1$$

## 1.22 problem 2(h)

Internal problem ID [6126]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 2(h).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' \sin(x) = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(sin(x)*diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x) = -\ln(\csc(x) + \cot(x)) + c_1$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 13

```
DSolve[Sin[x]*y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\text{arctanh}(\cos(x)) + c_1$$

## 1.23 problem 2(i)

Internal problem ID [6127]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 2(i).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(x^3+1)\,y'=x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve((1+x^3)*diff(y(x),x)=x,y(x), singsol=all)
```

$$y(x)=-\frac{\ln(x+1)}{3}+\frac{\ln(x^2-x+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}+c_1$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 48

```
DSolve[(1+x^3)*y'[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{6}\left(2\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)+\log\left(x^2-x+1\right)-2\log(x+1)+6c_1\right)$$

## 1.24 problem 2(j)

Internal problem ID [6128]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 2(j).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(x^2 - 3x + 2)\, y' = x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((x^2-3*x+2)*diff(y(x),x)=x,y(x), singsol=all)
```

$$y(x) = -\ln(x-1) + 2\ln(x-2) + c_1$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 24

```
DSolve[(x^2-3*x+2)*y'[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\log(1-x) + 2\log(2-x) + c_1$$

## 1.25 problem 3(a)

Internal problem ID [6129]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 3(a).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = x\,\mathrm{e}^{x}}$$

With initial conditions

$$[y(1) = 3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)=x*exp(x),y(1) = 3],y(x), singsol=all)
```

$$y(x) = (x-1)\,\mathrm{e}^{x} + 3$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 14

```
DSolve[{y'[x]==x*Exp[x],{y[1]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(x-1) + 3$$

## 1.26 problem 3(b)

Internal problem ID [6130]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 3(b).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type **[_quadrature]**

$$\boxed{y' = 2\cos(x)\sin(x)}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)=2*sin(x)*cos(x),y(0) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{\cos(2x)}{2} + \frac{3}{2}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 17

```
DSolve[{y'[x]==2*Sin[x]*Cos[x],{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(3 - \cos(2x))$$

## 1.27 problem 3(c)

Internal problem ID [6131]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 3(c).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \ln(x)}$$

With initial conditions

$$[y(e) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve([diff(y(x),x)=ln(x),y(exp(1)) = 0],y(x), singsol=all)
```

$$y(x) = x(\ln(x) - 1)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 11

```
DSolve[{y'[x]==Log[x],{y[Exp[1]]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(\log(x) - 1)$$

## 1.28 problem 3(d)

Internal problem ID [6132]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 3(d).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(x^2-1)\, y' = 1}$$

With initial conditions

$$[y(2) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([(x^2-1)*diff(y(x),x)=1,y(2) = 0],y(x), singsol=all)
```

$$y(x) = -\operatorname{arctanh}(x) + \operatorname{arctanh}\left(\frac{1}{2}\right) - \frac{i\pi}{2}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 27

```
DSolve[{(x^2-1)*y'[x]==1,{y[2]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(\log(3-3x) - \log(x+1) - i\pi)$$

## 1.29 problem 3(e)

Internal problem ID [6133]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 3(e).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x\left(x^{2}-4\right) y' = 1}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve([x*(x^2-4)*diff(y(x),x)=1,y(1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{\ln\left(x+2\right)}{8}+\frac{\ln\left(x-2\right)}{8}-\frac{\ln\left(x\right)}{4}-\frac{\ln\left(3\right)}{8}-\frac{i\pi}{8}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 26

```
DSolve[{x*(x^2-4)*y'[x]==1,{y[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}\left(\log\left(\frac{1}{3}(4-x^{2})\right)-2\log(x)\right)$$

## 1.30 problem 3(f)

Internal problem ID [6134]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 3(f).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(1+x)\left(x^2+1\right)y' = 2x^2+x}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve([(x+1)*(x^2+1)*diff(y(x),x)=2*x^2+x,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\ln(x+1)}{2} + \frac{3\ln(x^2+1)}{4} - \frac{\arctan(x)}{2} + 1$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 29

```
DSolve[{(x+1)*(x^2+1)*y'[x]==2*x^2+x,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-2\arctan(x) + 3\log\left(x^2+1\right) + 2\log(x+1) + 4\right)$$

## 1.31 problem 4

Internal problem ID [6135]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 4.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{-2yx + y' = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)=2*x*y(x)+1,y(x), singsol=all)
```

$$y(x) = \left(\frac{\sqrt{\pi}\,\operatorname{erf}(x)}{2} + c_1\right) e^{x^2}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 27

```
DSolve[y'[x]==2*x*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{x^2} \left(\sqrt{\pi}\operatorname{erf}(x) + 2c_1\right)$$

## 1.32 problem 5

Internal problem ID [6136]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 5.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 5y' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)-5*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^x c_1 + c_2 e^{4x}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 20

```
DSolve[y''[x]-5*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x (c_2 e^{3x} + c_1)$$

## 1.33 problem 6

Internal problem ID [6137]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 6.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{2xy^2}{1 - yx^2} = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)=2*x*y(x)^2/(1-x^2*y(x)),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\operatorname{LambertW}\left(-x^2 \mathrm{e}^{-2c_1}\right) - 2c_1}$$

✓ Solution by Mathematica

Time used: 4.457 (sec). Leaf size: 27

```
DSolve[y'[x]==2*x*y[x]^2/(1-x^2*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{W(-e^{-1+c_1}x^2)}{x^2}$$
$$y(x) \to 0$$

## 1.34 problem 7

Internal problem ID [6138]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9
**Problem number**: 7.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{2y''' + y'' - 5y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(2*diff(y(x),x$3)+diff(y(x),x$2)-5*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{x}{2}} + c_2 e^{-2x} + c_3 e^{x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[2*y'''[x]+y''[x]-5*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x/2} + c_2 e^{-2x} + c_3 e^{x}$$

# 2 Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12

## 2.1 problem 1(a)

Internal problem ID [6139]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12
**Problem number**: 1(a).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^5y' + y^5 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 67

```
dsolve(x^5*diff(y(x),x)+y(x)^5=0,y(x), singsol=all)
```

$$y(x) = \frac{x}{(c_1x^4 - 1)^{\frac{1}{4}}}$$
$$y(x) = -\frac{x}{(c_1x^4 - 1)^{\frac{1}{4}}}$$
$$y(x) = \frac{x}{\sqrt{-\sqrt{c_1x^4 - 1}}}$$
$$y(x) = -\frac{x}{\sqrt{-\sqrt{c_1x^4 - 1}}}$$

✓ Solution by Mathematica

Time used: 0.489 (sec). Leaf size: 145

```
DSolve[x^5*y'[x]+y[x]^5==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{\sqrt[4]{-1 - 4c_1x^4}}$$

$$y(x) \to -\frac{ix}{\sqrt[4]{-1 - 4c_1x^4}}$$

$$y(x) \to \frac{ix}{\sqrt[4]{-1 - 4c_1x^4}}$$

$$y(x) \to \frac{x}{\sqrt[4]{-1 - 4c_1x^4}}$$

$$y(x) \to 0$$

$$y(x) \to -\frac{(1+i)x}{\sqrt{2}}$$

$$y(x) \to -\frac{(1-i)x}{\sqrt{2}}$$

$$y(x) \to \frac{(1-i)x}{\sqrt{2}}$$

$$y(x) \to \frac{(1+i)x}{\sqrt{2}}$$

## 2.2 problem 1(b)

Internal problem ID [6140]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12
**Problem number**: 1(b).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 4yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=4*x*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{2x^2}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 20

```
DSolve[y'[x]==4*x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{2x^2}$$
$$y(x) \to 0$$

## 2.3 problem 1(c)

Internal problem ID [6141]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12
**Problem number**: 1(c).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + y\tan(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x)+y(x)*tan(x)=0,y(x), singsol=all)
```

$$y(x) = \cos(x) c_1$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 15

```
DSolve[y'[x]+y[x]*Tan[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(x)$$
$$y(x) \to 0$$

## 2.4 problem 1(d)

Internal problem ID [6142]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12
**Problem number**: 1(d).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2+1)\,y'+y^2=-1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve((1+x^2)*diff(y(x),x)+1+y(x)^2=0,y(x), singsol=all)
```

$$y(x) = -\tan\left(\arctan\left(x\right)+c_1\right)$$

✓ Solution by Mathematica

Time used: 0.233 (sec). Leaf size: 29

```
DSolve[(1+x^2)*y'[x]+1+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\tan(\arctan(x)-c_1)$$
$$y(x) \to -i$$
$$y(x) \to i$$

## 2.5 problem 1(e)

Internal problem ID [6143]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12
**Problem number**: 1(e).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y \ln(y) - y'x = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 8

```
dsolve(y(x)*ln(y(x))-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{c_1 x}$$

✓ Solution by Mathematica

Time used: 0.172 (sec). Leaf size: 18

```
DSolve[y[x]*Log[y[x]]-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{e^{c_1} x}$$
$$y(x) \rightarrow 1$$

## 2.6 problem 1(f)

Internal problem ID [6144]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12
**Problem number**: 1(f).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x - (-4x^2 + 1)\tan(y) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x)=(1-4*x^2)*tan(y(x)),y(x), singsol=all)
```

$$y(x) = \arcsin\left(\frac{x\,\mathrm{e}^{-2x^2}}{c_1}\right)$$

✓ Solution by Mathematica

Time used: 53.453 (sec). Leaf size: 23

```
DSolve[x*y'[x]==(1-4*x^2)*Tan[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin\left(xe^{-2x^2+c_1}\right)$$
$$y(x) \to 0$$

## 2.7 problem 1(g)

Internal problem ID [6145]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12
**Problem number**: 1(g).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' \sin(y) = x^2}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)*sin(y(x))=x^2,y(x), singsol=all)
```

$$y(x) = \pi - \arccos\left(\frac{x^3}{3} + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.499 (sec). Leaf size: 37

```
DSolve[y'[x]*Sin[y[x]]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arccos\left(-\frac{x^3}{3} - c_1\right)$$
$$y(x) \to \arccos\left(-\frac{x^3}{3} - c_1\right)$$

## 2.8 problem 1(h)

Internal problem ID [6146]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12
**Problem number**: 1(h).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y\tan(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)-y(x)*tan(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\cos(x)}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 15

```
DSolve[y'[x]-y[x]*Tan[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \sec(x)$$
$$y(x) \to 0$$

## 2.9 problem 1(i)

Internal problem ID [6147]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12
**Problem number**: 1(i).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xyy' - y = -1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve(x*y(x)*diff(y(x),x)=y(x)-1,y(x), singsol=all)
```

$$y(x) = \text{LambertW}\left(x c_1 \mathrm{e}^{-1}\right) + 1$$

✓ Solution by Mathematica

Time used: 3.215 (sec). Leaf size: 21

```
DSolve[x*y[x]*y'[x]==y[x]-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 + W\left(e^{-1+c_1} x\right)$$
$$y(x) \to 1$$

## 2.10 problem 1(j)

Internal problem ID [6148]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12
**Problem number**: 1(j).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy^2 - x^2y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x*y(x)^2-diff(y(x),x)*x^2=0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{\ln(x) - c_1}$$

✓ Solution by Mathematica

Time used: 0.107 (sec). Leaf size: 19

```
DSolve[x*y[x]^2-y'[x]*x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\log(x) + c_1}$$
$$y(x) \to 0$$

## 2.11 problem 2(a)

Internal problem ID [6149]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12
**Problem number**: 2(a).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yy' = 1 + x}$$

With initial conditions

$$[y(1) = 3]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)*y(x)=x+1,y(1) = 3],y(x), singsol=all)
```

$$y(x) = \sqrt{x^2 + 2x + 6}$$

✓ Solution by Mathematica

Time used: 0.097 (sec). Leaf size: 17

```
DSolve[{y'[x]*y[x]==x+1,{y[1]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x^2 + 2x + 6}$$

## 2.12 problem 2(b)

Internal problem ID [6150]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12
**Problem number**: 2(b).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2y' - y = 0}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)*x^2=y(x),y(1) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y'[x]*x^2==y[x],{y[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

## 2.13 problem 2(c)

Internal problem ID [6151]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12
**Problem number**: 2(c).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\frac{y'}{x^2+1} - \frac{x}{y} = 0}$$

With initial conditions

$$[y(1) = 3]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 20

```
dsolve([diff(y(x),x)/(1+x^2)=x/y(x),y(1) = 3],y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{2x^4+4x^2+30}}{2}$$

✓ Solution by Mathematica

Time used: 0.104 (sec). Leaf size: 25

```
DSolve[{y'[x]/(1+x^2)==x/y[x],{y[1]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{x^4+2x^2+15}}{\sqrt{2}}$$

## 2.14 problem 2(d)

Internal problem ID [6152]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12
**Problem number**: 2(d).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^2y' = x + 2}$$

With initial conditions

$$[y(0) = 4]$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 18

```
dsolve([y(x)^2*diff(y(x),x)=x+2,y(0) = 4],y(x), singsol=all)
```

$$y(x) = \frac{(12x^2 + 48x + 512)^{\frac{1}{3}}}{2}$$

✓ Solution by Mathematica

Time used: 0.211 (sec). Leaf size: 21

```
DSolve[{y[x]^2*y'[x]==x+2,{y[0]==4}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[3]{\frac{3x^2}{2} + 6x + 64}$$

## 2.15 problem 2(e)

Internal problem ID [6153]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12
**Problem number**: 2(e).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{y' - x^2y^2 = 0}$$

With initial conditions

$$[y(-1) = 2]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 15

```
dsolve([diff(y(x),x)=x^2*y(x)^2,y(-1) = 2],y(x), singsol=all)
```

$$y(x) = -\frac{6}{2x^3 - 1}$$

✓ Solution by Mathematica

Time used: 0.106 (sec). Leaf size: 16

```
DSolve[{y'[x]==x^2*y[x]^2,{y[-1]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{6}{1 - 2x^3}$$

## 2.16 problem 2(e)

Internal problem ID [6154]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12
**Problem number**: 2(e).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(1+y)\,y' = -x^2+1}$$

With initial conditions

$$[y(-1) = -2]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 20

```
dsolve([diff(y(x),x)*(1+y(x))=1-x^2,y(-1) = -2],y(x), singsol=all)
```

$$y(x) = -1 - \frac{\sqrt{-6x^3+18x+21}}{3}$$

✓ Solution by Mathematica

Time used: 0.151 (sec). Leaf size: 28

```
DSolve[{y'[x]*(1+y[x])==1-x^2,{y[-1]==-2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-2x^3+6x+7}}{\sqrt{3}} - 1$$

## 2.17 problem 3

Internal problem ID [6155]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{0 = -\frac{y''}{y'} + x^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)/diff(y(x),x)=x^2,y(x), singsol=all)
```

$$y(x) = c_1 + \left(2\sqrt{3}\,\pi - 3\Gamma\left(\frac{1}{3}, -\frac{x^3}{3}\right)\Gamma\left(\frac{2}{3}\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 39

```
DSolve[y''[x]/y'[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1(-x^3)^{2/3}\,\Gamma\left(\frac{1}{3}, -\frac{x^3}{3}\right)}{3^{2/3}x^2} + c_2$$

## 2.18 problem 4

Internal problem ID [6156]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _exact, _nonlinear], [`

$$\boxed{y''y' = x(1+x)}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 51

```
dsolve(diff(y(x),x$2)*diff(y(x),x)=x*(1+x),y(x), singsol=all)
```

$$y(x) = \int -\frac{\sqrt{6x^3 + 9x^2 + 9c_1}}{3}dx + c_2$$
$$y(x) = \int \frac{\sqrt{6x^3 + 9x^2 + 9c_1}}{3}dx + c_2$$

✓ Solution by Mathematica

Time used: 61.466 (sec). Leaf size: 12885

```
DSolve[y''[x]*y'[x]==x*(1+x),y[x],x,IncludeSingularSolutions -> True]
```

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# 3 Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15

## 3.1 problem 1(a)

Internal problem ID [6157]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15
**Problem number**: 1(a).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{x^2}{2}} c_1$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 22

```
DSolve[y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{\frac{x^2}{2}}$$
$$y(x) \to 0$$

## 3.2 problem 1(b)

Internal problem ID [6158]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15
**Problem number**: 1(b).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yx + y' = x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+x*y(x)=x,y(x), singsol=all)
```

$$y(x) = 1 + \mathrm{e}^{-\frac{x^2}{2}} c_1$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 24

```
DSolve[y'[x]+x*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 1 + c_1 e^{-\frac{x^2}{2}}$$
$$y(x) \rightarrow 1$$

## 3.3 problem 1(c)

Internal problem ID [6159]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15
**Problem number**: 1(c).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y = \frac{1}{e^{2x} + 1}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+y(x)=1/(1+exp(2*x)),y(x), singsol=all)
```

$$y(x) = (\arctan(e^x) + c_1) e^{-x}$$

✓ Solution by Mathematica

Time used: 0.061 (sec). Leaf size: 18

```
DSolve[y'[x]+y[x]==1/(1+Exp[2*x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(\arctan(e^x) + c_1)$$

## 3.4 problem 1(d)

Internal problem ID [6160]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15
**Problem number**: 1(d).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = 2x\,\mathrm{e}^{-x} + x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x)+y(x)=2*x*exp(-x)+x^2,y(x), singsol=all)
```

$$y(x) = x^2 - 2x + \mathrm{e}^{-x}x^2 + 2 + \mathrm{e}^{-x}c_1$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 29

```
DSolve[y'[x]+y[x]==2*x*Exp[-x]+x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(x^2 + e^x\left(x^2 - 2x + 2\right) + c_1\right)$$

## 3.5 problem 1(e)

Internal problem ID [6161]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15
**Problem number**: 1(e).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2y - y'x = x^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(2*y(x)-x^3=x*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = (-x + c_1)\, x^2$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 15

```
DSolve[2*y[x]-x^3==x*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(-x + c_1)$$

## 3.6 problem 1(f)

Internal problem ID [6162]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15
**Problem number**: 1(f).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2yx + y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)+2*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^2}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 20

```
DSolve[y'[x]+2*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x^2}$$
$$y(x) \to 0$$

## 3.7 problem 1(g)

Internal problem ID [6163]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15
**Problem number**: 1(g).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{-3y + y'x = x^4}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve(x*diff(y(x),x)-3*y(x)=x^4,y(x), singsol=all)
```

$$y(x) = (x + c_1)\, x^3$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 13

```
DSolve[x*y'[x]-3*y[x]==x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^3(x + c_1)$$

## 3.8 problem 1(h)

Internal problem ID [6164]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15
**Problem number**: 1(h).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2 + 1)\, y' + 2yx = \cot(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve((1+x^2)*diff(y(x),x)+2*x*y(x)=cot(x),y(x), singsol=all)
```

$$y(x) = \frac{\ln(\sin(x)) + c_1}{x^2 + 1}$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 19

```
DSolve[(1+x^2)*y'[x]+2*x*y[x]==Cot[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\log(\sin(x)) + c_1}{x^2 + 1}$$

## 3.9 problem 1(i)

Internal problem ID [6165]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15
**Problem number**: 1(i).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\cot(x) = 2x\csc(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+y(x)*cot(x)=2*x*csc(x),y(x), singsol=all)
```

$$y(x) = \frac{x^2 + c_1}{\sin(x)}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 14

```
DSolve[y'[x]+y[x]*Cot[x]==2*x*Csc[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x^2 + c_1)\csc(x)$$

## 3.10 problem 1(j)

Internal problem ID [6166]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15
**Problem number**: 1(j).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y + xy\cot(x) + y'x = x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(y(x)-x+x*y(x)*cot(x)+x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sin(x) - \cos(x)\,x + c_1}{\sin(x)\,x}$$

✓ Solution by Mathematica

Time used: 0.072 (sec). Leaf size: 21

```
DSolve[y[x]-x+x*y[x]*Cot[x]+x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-x\cot(x) + c_1\csc(x) + 1}{x}$$

## 3.11 problem 2(a)

Internal problem ID [6167]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15
**Problem number**: 2(a).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{y' - yx = 0}$$

With initial conditions

$$[y(1) = 3]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 15

```
dsolve([diff(y(x),x)-x*y(x)=0,y(1) = 3],y(x), singsol=all)
```

$$y(x) = 3\,e^{\frac{(x-1)(x+1)}{2}}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 18

```
DSolve[{y'[x]-x*y[x]==0,{y[1]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3e^{\frac{1}{2}(x^2-1)}$$

## 3.12 problem 2(b)

Internal problem ID [6168]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15
**Problem number**: 2(b).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{-2yx + y' = 6\,\mathrm{e}^{x^2} x}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 18

```
dsolve([diff(y(x),x)-2*x*y(x)=6*x*exp(x^2),y(1) = 1],y(x), singsol=all)
```

$$y(x) = \left(3x^2 - 3 + \mathrm{e}^{-1}\right) \mathrm{e}^{x^2}$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 23

```
DSolve[{y'[x]-2*x*y[x]==6*x*Exp[x^2],{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x^2-1}\left(3e\left(x^2-1\right)+1\right)$$

## 3.13 problem 2(c)

Internal problem ID [6169]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15
**Problem number**: 2(c).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x \ln(x) y' + y = 3x^3}$$

With initial conditions

$$[y(1) = 0]$$

✗ Solution by Maple

```
dsolve([(x*ln(x))*diff(y(x),x)+y(x)=3*x^3,y(1) = 0],y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{(x*Log[x])*y'[x]+y[x]==3*x^3,{y[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

Not solved

## 3.14 problem 2(d)

Internal problem ID [6170]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15
**Problem number**: 2(d).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{y}{x} = x^2}$$

With initial conditions

$$[y(1) = 3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)-y(x)/x=x^2,y(1) = 3],y(x), singsol=all)
```

$$y(x) = \frac{(x^2 + 5)\, x}{2}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 15

```
DSolve[{y'[x]-y[x]/x==x^2,{y[1]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} x\left(x^2 + 5\right)$$

## 3.15 problem 2(e)

Internal problem ID [6171]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15
**Problem number**: 2(e).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{4y + y' = e^{-x}}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve([diff(y(x),x)+4*y(x)=exp(-x),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{(e^{3x} - 1)\, e^{-4x}}{3}$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 21

```
DSolve[{y'[x]+4*y[x]==Exp[-x],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} e^{-4x} (e^{3x} - 1)$$

## 3.16 problem 2(f)

Internal problem ID [6172]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15
**Problem number**: 2(f).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2y' + yx = 2x}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve([x^2*diff(y(x),x)+x*y(x)=2*x,y(1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{2x - 1}{x}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 12

```
DSolve[{x^2*y'[x]+x*y[x]==2*x,{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2 - \frac{1}{x}$$

## 3.17 problem 3(a)

Internal problem ID [6173]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15
**Problem number**: 3(a).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y + y'x - y^3x^4 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve(x*diff(y(x),x)+y(x)=x^4*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{-x^2 + c_1 x}}$$
$$y(x) = -\frac{1}{\sqrt{-x^2 + c_1 x}}$$

✓ Solution by Mathematica

Time used: 0.414 (sec). Leaf size: 48

```
DSolve[x*y'[x]+y[x]==x^4*y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{-x^4 + c_1 x^2}}$$
$$y(x) \to \frac{1}{\sqrt{-x^4 + c_1 x^2}}$$
$$y(x) \to 0$$

## 3.18 problem 3(b)

Internal problem ID [6174]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15
**Problem number**: 3(b).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{xy^2y' + y^3 = \cos(x)\, x}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 175

```
dsolve(x*y(x)^2*diff(y(x),x)+y(x)^3=x*cos(x),y(x), singsol=all)
```

$$y(x) = \frac{\left(3\sin(x)\, x^3 + 9x^2\cos(x) - 18\cos(x) - 18\sin(x)\, x + c_1\right)^{\frac{1}{3}}}{x}$$

$$y(x) = \frac{-\frac{\left(3\sin(x)x^3+9x^2\cos(x)-18\cos(x)-18\sin(x)x+c_1\right)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}\left(3\sin(x)x^3+9x^2\cos(x)-18\cos(x)-18\sin(x)x+c_1\right)^{\frac{1}{3}}}{2}}{x}$$

$$y(x) = \frac{-\frac{\left(3\sin(x)x^3+9x^2\cos(x)-18\cos(x)-18\sin(x)x+c_1\right)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}\left(3\sin(x)x^3+9x^2\cos(x)-18\cos(x)-18\sin(x)x+c_1\right)^{\frac{1}{3}}}{2}}{x}$$

✓ Solution by Mathematica

Time used: 0.513 (sec). Leaf size: 114

```
DSolve[x*y[x]^2*y'[x]+y[x]^3==x*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{3x\,(x^2-6)\sin(x)+9\,(x^2-2)\cos(x)+c_1}}{x}$$

$$y(x) \to -\frac{\sqrt[3]{-1}\,\sqrt[3]{3x\,(x^2-6)\sin(x)+9\,(x^2-2)\cos(x)+c_1}}{x}$$

$$y(x) \to \frac{(-1)^{2/3}\,\sqrt[3]{3x\,(x^2-6)\sin(x)+9\,(x^2-2)\cos(x)+c_1}}{x}$$

## 3.19 problem 3(c)

Internal problem ID [6175]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15
**Problem number**: 3(c).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y + y'x - xy^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x)+y(x)=x*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{1}{(\ln(x) - c_1)\, x}$$

✓ Solution by Mathematica

Time used: 0.128 (sec). Leaf size: 22

```
DSolve[x*y'[x]+y[x]==x*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{-x\log(x) + c_1 x}$$
$$y(x) \to 0$$

## 3.20 problem 3(d)

Internal problem ID [6176]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15
**Problem number**: 3(d).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{yx + y' - y^4x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 88

```
dsolve(diff(y(x),x)+x*y(x)=x*y(x)^4,y(x), singsol=all)
```

$$y(x) = \frac{1}{\left(\mathrm{e}^{\frac{3x^2}{2}}c_1 + 1\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{1}{2\left(\mathrm{e}^{\frac{3x^2}{2}}c_1 + 1\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}}{2\left(\mathrm{e}^{\frac{3x^2}{2}}c_1 + 1\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{1}{2\left(\mathrm{e}^{\frac{3x^2}{2}}c_1 + 1\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}}{2\left(\mathrm{e}^{\frac{3x^2}{2}}c_1 + 1\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 1.97 (sec). Leaf size: 116

```
DSolve[y'[x]+x*y[x]==x*y[x]^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{\sqrt[3]{1 + e^{\frac{3x^2}{2} + 3c_1}}}$$

$$y(x) \to -\frac{\sqrt[3]{-1}}{\sqrt[3]{1 + e^{\frac{3x^2}{2} + 3c_1}}}$$

$$y(x) \to \frac{(-1)^{2/3}}{\sqrt[3]{1 + e^{\frac{3x^2}{2} + 3c_1}}}$$

$$y(x) \to 0$$

$$y(x) \to 1$$

$$y(x) \to -\sqrt[3]{-1}$$

$$y(x) \to (-1)^{2/3}$$

## 3.21 problem 4(a)

Internal problem ID [6177]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15
**Problem number**: 4(a).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{(\mathrm{e}^{y} - 2yx)\, y' - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve((exp(y(x))-2*x*y(x))*diff(y(x),x)=y(x)^2,y(x), singsol=all)
```

$$x - \frac{\mathrm{e}^{y(x)} + c_1}{y(x)^2} = 0$$

✓ Solution by Mathematica

Time used: 0.236 (sec). Leaf size: 22

```
DSolve[(Exp[y[x]]-2*x*y[x])*y'[x]==y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x = \frac{e^{y(x)}}{y(x)^2} + \frac{c_1}{y(x)^2}, y(x)\right]$$

## 3.22 problem 4(b)

Internal problem ID [6178]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15
**Problem number**: 4(b).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{-y'x + y - y'y^2 e^y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 14

```
dsolve(y(x)-x*diff(y(x),x)=diff(y(x),x)*y(x)^2*exp(y(x)),y(x), singsol=all)
```

$$x - \left(e^{y(x)} + c_1\right) y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.203 (sec). Leaf size: 18

```
DSolve[y[x]-x*y'[x]==y'[x]*y[x]^2*Exp[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x = e^{y(x)} y(x) + c_1 y(x), y(x)\right]$$

## 3.23 problem 4(c)

Internal problem ID [6179]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15
**Problem number**: 4(c).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]'], [_Abel, '2nd ty`

$$y'x - x^3(-1+y)\,y' = -2$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 22

```
dsolve(x*diff(y(x),x)+2=x^3*(y(x)-1)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = -\frac{\operatorname{LambertW}\left(c_1 \mathrm{e}^{\frac{1}{x^2}}\right) x^2 - 1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.405 (sec). Leaf size: 33

```
DSolve[x*y'[x]+2==x^3*(y[x]-1)*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{x^2} - W\left(e^{\frac{1}{x^2}+\frac{1}{2}\left(-2-9\sqrt[3]{-2}c_1\right)}\right)$$

## 3.24 problem 6

Internal problem ID [6180]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15
**Problem number**: 6.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x - 2yx^2 - \ln(x)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x)=2*x^2*y(x)+y(x)*ln(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{\ln(x)^2}{2}+x^2}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 27

```
DSolve[x*y'[x]==2*x^2*y[x]+y[x]*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x^2+\frac{\log^2(x)}{2}}$$
$$y(x) \to 0$$

## 3.25 problem 7

Internal problem ID [6181]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15
**Problem number**: 7.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' \sin(2x) - 2y = 2\cos(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)*sin(2*x)=2*y(x)+2*cos(x),y(x), singsol=all)
```

$$y(x) = -\left(-\frac{1}{\sin(x)} + c_1\right)(-\csc(2x) + \cot(2x))$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 15

```
DSolve[y'[x]*Sin[2*x]==2*y[x]+2*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sec(x)(-1 + c_1 \sin(x))$$

# 4 Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20

## 4.1 problem 1

Internal problem ID [6182]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20
**Problem number**: 1.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _exact, _rational, [_Abel, '2nd ty`

$$\left(x+\frac{2}{y}\right)y'+y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve((x+2/y(x))*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=e^{-\operatorname{LambertW}\left(\frac{x\,e^{\frac{c_1}{2}}}{2}\right)+\frac{c_1}{2}}$$

✓ Solution by Mathematica

Time used: 17.046 (sec). Leaf size: 58

```
DSolve[(x+2/y[x])*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{2W\left(-\frac{1}{2}\sqrt{e^{c_1}x^2}\right)}{x}$$
$$y(x)\to \frac{2W\left(\frac{1}{2}\sqrt{e^{c_1}x^2}\right)}{x}$$
$$y(x)\to 0$$

## 4.2 problem 2

Internal problem ID [6183]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20
**Problem number**: 2.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[‘x=_G(y,y’)‘]`

$$\sin(x)\tan(y) + \cos(x)\sec(x)^2 yy' = -1$$

✗ Solution by Maple

```
dsolve((sin(x)*tan(y(x))+1)+(cos(x)*sec(x)^2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(Sin[x]*Tan[y[x]]+1)+(Cos[x]*Sec[x]^2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions
```

Not solved

## 4.3 problem 3

Internal problem ID [6184]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20
**Problem number**: 3.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{y + (x + y^3)\, y' = x^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve((y(x)-x^3)+(x+y(x)^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$-\frac{x^4}{4} + y(x)\, x + \frac{y(x)^4}{4} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.218 (sec). Leaf size: 1210

```
DSolve[(y[x]-x^3)+(x+y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4+(x^4+4c_1)^3}} - \frac{\sqrt[3]{3}(x^4+4c_1)}{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4+(x^4+4c_1)^3}}}} + \sqrt{\frac{\quad}{\sqrt{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4 \cdots}}}}}}{\quad}$$

$$y(x) \to \frac{\sqrt{\frac{6\sqrt{2}x}{\sqrt{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4+(x^4+4c_1)^3}} - \frac{\sqrt[3]{3}(x^4+4c_1)}{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4+(x^4+4c_1)^3}}}}} - \sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4+(x^4 \cdots}}}}{\quad}$$

$$y(x) \to \frac{\sqrt{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4+(x^4+4c_1)^3}} - \frac{\sqrt[3]{3}(x^4+4c_1)}{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4+(x^4+4c_1)^3}}}} - \sqrt{-\frac{\quad}{\sqrt{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4 \cdots}}}}}}{\quad}$$

$$y(x) \to \frac{\sqrt{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4+(x^4+4c_1)^3}} - \frac{\sqrt[3]{3}(x^4+4c_1)}{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4+(x^4+4c_1)^3}}}} + \sqrt{-\frac{\quad}{\sqrt{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4 \cdots}}}}}}{\quad}$$

## 4.4 problem 4

Internal problem ID [6185]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20
**Problem number**: 4.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{2y^2 - (4 - 2y + 4yx)\, y' = 4x - 5}$$

✗ Solution by Maple

```
dsolve((2*y(x)^2-4*x+5)=(4-2*y(x)+4*x*y(x))*diff(y(x),x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(2*y[x]^2-4*x+5)==(4-2*y[x]+4*x*y[x])*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

## 4.5 problem 5

Internal problem ID [6186]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20
**Problem number**: 5.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y + y\cos(yx) + (x + x\cos(yx))\, y' = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 17

```
dsolve((y(x)+y(x)*cos(x*y(x)))+(x+x*cos(x*y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\pi}{x}$$
$$y(x) = \frac{c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 49

```
DSolve[(y[x]+y[x]*Cos[x*y[x]])+(x+x*Cos[x*y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{\pi}{x}$$
$$y(x) \to \frac{\pi}{x}$$
$$y(x) \to \frac{c_1}{x}$$
$$y(x) \to -\frac{\pi}{x}$$
$$y(x) \to \frac{\pi}{x}$$

## 4.6 problem 6

Internal problem ID [6187]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20
**Problem number**: 6.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\cos(x)\cos(y)^2 + 2\sin(x)\sin(y)\cos(y)\,y' = 0}$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 31

```
dsolve((cos(x)*cos(y(x))^2)+(2*sin(x)*sin(y(x))*cos(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\pi}{2}$$
$$y(x) = \arccos\left(\sqrt{c_1 \sin(x)}\right)$$
$$y(x) = \pi - \arccos\left(\sqrt{c_1 \sin(x)}\right)$$

✓ Solution by Mathematica

Time used: 5.453 (sec). Leaf size: 73

```
DSolve[(Cos[x]*Cos[y[x]]^2)+(2*Sin[x]*Sin[y[x]]*Cos[y[x]])*y'[x]==0,y[x],x,IncludeSingularSo
```

$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$
$$y(x) \to -\arccos\left(-\frac{1}{4}c_1\sqrt{\sin(x)}\right)$$
$$y(x) \to \arccos\left(-\frac{1}{4}c_1\sqrt{\sin(x)}\right)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

## 4.7 problem 7

Internal problem ID [6188]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20
**Problem number**: 7.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{(\sin(x)\sin(y) - \mathrm{e}^{y}x)\,y' - \mathrm{e}^{y} - \cos(x)\cos(y) = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 16

```
dsolve((sin(x)*sin(y(x))-x*exp(y(x)))*diff(y(x),x)=exp(y(x))+cos(x)*cos(y(x)),y(x), singsol=
```

$$c_1 + \sin(x)\cos(y(x)) + x\,\mathrm{e}^{y(x)} = 0$$

✓ Solution by Mathematica

Time used: 0.594 (sec). Leaf size: 21

```
DSolve[(Sin[x]*Sin[y[x]]-x*Exp[y[x]])*y'[x]==Exp[y[x]]+Cos[x]*Cos[y[x]],y[x],x,IncludeSingul
```

$$\text{Solve}\left[2\left(xe^{y(x)} + \sin(x)\cos(y(x))\right) = c_1, y(x)\right]$$

## 4.8 problem 8

Internal problem ID [6189]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20
**Problem number**: 8.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$-\frac{\sin\left(\frac{x}{y}\right)}{y}+\frac{x\sin\left(\frac{x}{y}\right)y'}{y^2}=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 13

```
dsolve(-1/y(x)*sin(x/y(x))+(x/y(x)^2*sin(x/y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\frac{x}{\pi-c_1}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 19

```
DSolve[-1/y[x]*Sin[x/y[x]]+(x/y[x]^2*Sin[x/y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x)\to c_1x$$
$$y(x)\to \text{ComplexInfinity}$$
$$y(x)\to \text{ComplexInfinity}$$

## 4.9 problem 9

Internal problem ID [6190]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20
**Problem number**: 9.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y + (1 - x)\, y' = -1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve((1+y(x))+(1-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -1 + c_1(x - 1)$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 18

```
DSolve[(1+y[x])+(1-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -1 + c_1(x - 1)$$
$$y(x) \to -1$$

## 4.10 problem 10

Internal problem ID [6191]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20
**Problem number**: 10.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_exact, [_1st_order, `_with_symmetry_[F(x),G(x)*y+H(x)]`]]`

$$\boxed{2xy^3+\cos(x)\,y+\left(3x^2y^2+\sin(x)\right)y'=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 375

```
dsolve((2*x*y(x)^3+y(x)*cos(x))+(3*x^2*y(x)^2+sin(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin(x)^3}-108c_1x\right)^{\frac{1}{3}}}{6x} - \frac{2\sin(x)}{x\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin(x)^3}-108c_1x\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin(x)^3}-108c_1x\right)^{\frac{1}{3}}}{12x} + \frac{\sin(x)}{x\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin(x)^3}-108c_1x\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin(x)^3}-108c_1x\right)^{\frac{1}{3}}}{6x}+\frac{2\sin(x)}{x\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin(x)^3}-108c_1x\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin(x)^3}-108c_1x\right)^{\frac{1}{3}}}{12x} + \frac{\sin(x)}{x\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin(x)^3}-108c_1x\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin(x)^3}-108c_1x\right)^{\frac{1}{3}}}{6x}+\frac{2\sin(x)}{x\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin(x)^3}-108c_1x\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 34.571 (sec). Leaf size: 339

```
DSolve[(2*x*y[x]^3+y[x]*Cos[x])+(3*x^2*y[x]^2+Sin[x])*y'[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to \frac{\sqrt[3]{9c_1x^4 + \sqrt{12x^6\sin^3(x) + 81c_1{}^2x^8}}}{\sqrt[3]{2}3^{2/3}x^2} - \frac{\sqrt[3]{\frac{2}{3}}\sin(x)}{\sqrt[3]{9c_1x^4 + \sqrt{12x^6\sin^3(x) + 81c_1{}^2x^8}}}$$

$$y(x) \to \frac{(1 + i\sqrt{3})\sin(x)}{2^{2/3}\sqrt[3]{27c_1x^4 + 3\sqrt{12x^6\sin^3(x) + 81c_1{}^2x^8}}} - \frac{(1 - i\sqrt{3})\sqrt[3]{27c_1x^4 + \sqrt{108x^6\sin^3(x) + 729c_1{}^2x^8}}}{6\sqrt[3]{2}x^2}$$

$$y(x) \to \frac{(1 - i\sqrt{3})\sin(x)}{2^{2/3}\sqrt[3]{27c_1x^4 + 3\sqrt{12x^6\sin^3(x) + 81c_1{}^2x^8}}} - \frac{(1 + i\sqrt{3})\sqrt[3]{27c_1x^4 + \sqrt{108x^6\sin^3(x) + 729c_1{}^2x^8}}}{6\sqrt[3]{2}x^2}$$

## 4.11 problem 11

Internal problem ID [6192]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20
**Problem number**: 11.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_exact, _rational, _Riccati]`

$$\boxed{\frac{y}{1-x^2y^2}+\frac{xy'}{1-x^2y^2}=1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(y(x)/(1-x^2*y(x)^2)+x/(1-x^2*y(x)^2)*diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x)=-\frac{e^{-2x}c_1+1}{x\left(e^{-2x}c_1-1\right)}$$

✓ Solution by Mathematica

Time used: 0.16 (sec). Leaf size: 18

```
DSolve[y[x]/(1-x^2*y[x]^2)+x/(1-x^2*y[x]^2)*y'[x]==1,y[x],x,IncludeSingularSolutions -> True
```

$$y(x)\to\frac{\tanh(x+ic_1)}{x}$$

## 4.12 problem 12

Internal problem ID [6193]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20
**Problem number**: 12.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{2y^4x + \sin(y) + (4y^3x^2 + x\cos(y))\, y' = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 18

```
dsolve((2*x*y(x)^4+sin(y(x)))+(4*x^2*y(x)^3+x*cos(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$x^2y(x)^4 + x\sin(y(x)) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.227 (sec). Leaf size: 20

```
DSolve[(2*x*y[x]^4+Sin[y[x]])+(4*x^2*y[x]^3+x*Cos[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolu
```

$$\text{Solve}\left[x^2y(x)^4 + x\sin(y(x)) = c_1, y(x)\right]$$

## 4.13 problem 13

Internal problem ID [6194]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20
**Problem number**: 13.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_exact, _rational, _Riccati]`

$$\boxed{\frac{y + y'x}{1 - x^2y^2} = -x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve((y(x)+x*diff(y(x),x))/(1-x^2*y(x)^2)+x=0,y(x), singsol=all)
```

$$y(x) = \frac{i \tan\left(\frac{ix^2}{2} + c_1\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.142 (sec). Leaf size: 25

```
DSolve[(y[x]+x*y'[x])/(1-x^2*y[x]^2)+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\tanh\left(\frac{1}{2}(x^2 - 2ic_1)\right)}{x}$$

## 4.14 problem 14

Internal problem ID [6195]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20
**Problem number**: 14.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_exact, [_1st_order, ‘_with_symmetry_[F(x),G(y)]‘]]`

$$\boxed{2x\left(1+\sqrt{x^2-y}\right)-\sqrt{x^2-y}\, y'=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve((2*x*(1+sqrt(x^2-y(x))))=sqrt(x^2-y(x))*diff(y(x),x),y(x), singsol=all)
```

$$x^2+\frac{2(x^2-y(x))^{\frac{3}{2}}}{3}+c_1=0$$

✓ Solution by Mathematica

Time used: 0.898 (sec). Leaf size: 121

```
DSolve[2*x*(1+Sqrt[x^2-y[x]])==Sqrt[x^2-y[x]]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x^2+\left(\frac{3}{2}\right)^{2/3}\sqrt[3]{-\left(x^2+c_1\right)^2}$$
$$y(x)\to x^2-\frac{\sqrt[6]{3}\left(\sqrt{3}-3i\right)\sqrt[3]{-\left(x^2+c_1\right)^2}}{2\ 2^{2/3}}$$
$$y(x)\to x^2-\frac{\sqrt[6]{3}\left(\sqrt{3}+3i\right)\sqrt[3]{-\left(x^2+c_1\right)^2}}{2\ 2^{2/3}}$$

## 4.15 problem 15

Internal problem ID [6196]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20
**Problem number**: 15.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x \ln(y) + yx + (\ln(x) y + yx) y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve((x*ln(y(x))+x*y(x))+(y(x)*ln(x)+x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$\int \frac{x}{\ln(x)+x} dx + \int^{y(x)} \frac{\_a}{\_a+\ln(\_a)} d\_a + c_1 = 0$$

✓ Solution by Mathematica

Time used: 36.692 (sec). Leaf size: 54

```
DSolve[(x*Log[y[x]]+x*y[x])+(y[x]*Log[x]+x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{K[1]}{K[1]+\log(K[1])} dK[1]\&\right]\left[\int_1^x -\frac{K[2]}{K[2]+\log(K[2])} dK[2] + c_1\right]$$

$$y(x) \to W(1)$$

## 4.16 problem 16

Internal problem ID [6197]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20
**Problem number**: 16.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{e^{y^2} - \csc(y)\csc(x)^2 + \left(2xy\,e^{y^2} - \csc(y)\cot(y)\cot(x)\right)y' = 0}$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 18

```
dsolve((exp(y(x)^2)-csc(y(x))*csc(x)^2)+(2*x*y(x)*exp(y(x)^2)-csc(y(x))*cot(y(x))*cot(x))*di
```

$$\csc(y(x))\cot(x) + x\,e^{y(x)^2} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 175.525 (sec). Leaf size: 23

```
DSolve[(Exp[y[x]^2]-Csc[y[x]]*Csc[x]^2)+(2*x*y[x]*Exp[y[x]^2]-Csc[y[x]]*Cot[y[x]]*Cot[x])*y'
```

$$\text{Solve}\left[-2xe^{y(x)^2} - 2\cot(x)\csc(y(x)) = c_1, y(x)\right]$$

## 4.17 problem 17

Internal problem ID [6198]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20
**Problem number**: 17.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_exact, _Bernoulli]`

$$\boxed{y^2 \sin(2x) - 2y \cos(x)^2 y' = -1}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 28

```
dsolve((1+y(x)^2*sin(2*x))-(2*y(x)*cos(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{x + c_1}}{\cos(x)}$$
$$y(x) = -\frac{\sqrt{x + c_1}}{\cos(x)}$$

✓ Solution by Mathematica

Time used: 0.294 (sec). Leaf size: 32

```
DSolve[(1+y[x]^2*Sin[2*x])-(2*y[x]*Cos[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to -\sqrt{x + c_1} \sec(x)$$
$$y(x) \to \sqrt{x + c_1} \sec(x)$$

## 4.18 problem 18

Internal problem ID [6199]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20
**Problem number**: 18.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\frac{x}{(x^2+y^2)^{\frac{3}{2}}}+\frac{yy'}{(x^2+y^2)^{\frac{3}{2}}}=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve((x/(x^2+y(x)^2)^(3/2))+(y(x)/(x^2+y(x)^2)^(3/2))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\sqrt{-x^2+c_1}$$
$$y(x)=-\sqrt{-x^2+c_1}$$

✓ Solution by Mathematica

Time used: 0.086 (sec). Leaf size: 39

```
DSolve[(x/(x^2+y[x]^2)^(3/2))+(y[x]/(x^2+y[x]^2)^(3/2))*y'[x]==0,y[x],x,IncludeSingularSolut
```

$$y(x)\to-\sqrt{-x^2+2c_1}$$
$$y(x)\to\sqrt{-x^2+2c_1}$$

## 4.19 problem 19

Internal problem ID [6200]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20
**Problem number**: 19.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{3x^2(\ln(y)+1)+\left(\frac{x^3}{y}-2y\right)y'=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 39

```
dsolve((3*x^2*(1+ln(y(x))))+(x^3/y(x)-2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-\frac{x^3 \operatorname{LambertW}\left(-\frac{2\,\mathrm{e}^{-2}\mathrm{e}^{-\frac{2c_1}{x^3}}}{x^3}\right)+2x^3+2c_1}{2x^3}}$$

✓ Solution by Mathematica

Time used: 60.17 (sec). Leaf size: 79

```
DSolve[(3*x^2*(1+Log[y[x]]))+(x^3/y[x]-2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\to -\frac{ix^{3/2}\sqrt{W\left(-\frac{2e^{-2+\frac{2c_1}{x^3}}}{x^3}\right)}}{\sqrt{2}}$$
$$y(x)\to \frac{ix^{3/2}\sqrt{W\left(-\frac{2e^{-2+\frac{2c_1}{x^3}}}{x^3}\right)}}{\sqrt{2}}$$

## 4.20 problem 20

Internal problem ID [6201]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20
**Problem number**: 20.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _exact, _rational]`

$$\boxed{\frac{-y'x+y}{(x+y)^2}+y'=1}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 65

```
dsolve((y(x)-x*diff(y(x),x))/(x+y(x))^2+diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x)=\frac{1}{4}+\frac{c_1}{4}-\frac{\sqrt{c_1^2+8c_1x+16x^2+2c_1-8x+1}}{4}$$
$$y(x)=\frac{1}{4}+\frac{c_1}{4}+\frac{\sqrt{c_1^2+8c_1x+16x^2+2c_1-8x+1}}{4}$$

✓ Solution by Mathematica

Time used: 0.468 (sec). Leaf size: 76

```
DSolve[(y[x]-x*y'[x])/(x+y[x])^2+y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}\left(-\sqrt{4x^2+4c_1x+(1+c_1)^2}+1+c_1\right)$$
$$y(x)\to\frac{1}{2}\left(\sqrt{4x^2+4c_1x+(1+c_1)^2}+1+c_1\right)$$
$$y(x)\to -x$$

## 4.21 problem 21

Internal problem ID [6202]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20
**Problem number**: 21.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$\boxed{\frac{4y^2-2x^2}{4xy^2-x^3}+\frac{(8y^2-x^2)\,y'}{4y^3-yx^2}=0}$$

✓ Solution by Maple

Time used: 0.438 (sec). Leaf size: 225

```
dsolve(( (4*y(x)^2-2*x^2)/(4*x*y(x)^2-x^3))+( (8*y(x)^2-x^2)/(4*y(x)^3-x^2*y(x)) )*diff(y(x)
```

$$y(x)=\frac{-c_1x-\frac{-2c_1^2x^2+\sqrt{2x^4c_1^4-2\sqrt{c_1^6x^6+16}\,c_1x}}{2c_1x}}{2c_1}$$

$$y(x)=\frac{-c_1x-\frac{-2c_1^2x^2+\sqrt{2x^4c_1^4+2\sqrt{c_1^6x^6+16}\,c_1x}}{2c_1x}}{2c_1}$$

$$y(x)=\frac{-c_1x+\frac{2c_1^2x^2+\sqrt{2x^4c_1^4-2\sqrt{c_1^6x^6+16}\,c_1x}}{2c_1x}}{2c_1}$$

$$y(x)=\frac{-c_1x+\frac{2c_1^2x^2+\sqrt{2x^4c_1^4+2\sqrt{c_1^6x^6+16}\,c_1x}}{2c_1x}}{2c_1}$$

## ✓ Solution by Mathematica

Time used: 12.331 (sec). Leaf size: 297

```
DSolve[( (4*y[x]^2-2*x^2)/(4*x*y[x]^2-x^3))+( (8*y[x]^2-x^2)/(4*y[x]^3-x^2*y[x]) )*y'[x]==0,
```

$$y(x) \to -\frac{\sqrt{x^2 - \frac{\sqrt{x^6-16e^{2c_1}}}{x}}}{2\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{x^2 - \frac{\sqrt{x^6-16e^{2c_1}}}{x}}}{2\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{\frac{x^3+\sqrt{x^6-16e^{2c_1}}}{x}}}{2\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{\frac{x^3+\sqrt{x^6-16e^{2c_1}}}{x}}}{2\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{x^2 - \frac{\sqrt{x^6}}{x}}}{2\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{x^2 - \frac{\sqrt{x^6}}{x}}}{2\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{\frac{\sqrt{x^6}+x^3}{x}}}{2\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{\frac{\sqrt{x^6}+x^3}{x}}}{2\sqrt{2}}$$

# 5 Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28

## 5.1 problem 1(a)

Internal problem ID [6203]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28
**Problem number**: 1(a).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{-2y^2 + xyy' = -x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve((x^2-2*y(x)^2)+(x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{c_1x^2 + 1}\,x$$
$$y(x) = -\sqrt{c_1x^2 + 1}\,x$$

✓ Solution by Mathematica

Time used: 0.451 (sec). Leaf size: 39

```
DSolve[(x^2-2*y[x]^2)+(x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x^2 + c_1x^4}$$
$$y(x) \to \sqrt{x^2 + c_1x^4}$$

## 5.2 problem 1(b)

Internal problem ID [6204]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28
**Problem number**: 1(b).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{x^2y' - 3yx - 2y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x)-3*x*y(x)-2*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{x^3}{-x^2 + c_1}$$

✓ Solution by Mathematica

Time used: 0.141 (sec). Leaf size: 25

```
DSolve[x^2*y'[x]-3*x*y[x]-2*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^3}{x^2 - c_1}$$
$$y(x) \to 0$$

## 5.3 problem 1(c)

Internal problem ID [6205]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28
**Problem number**: 1(c).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$x^2y' - 3(x^2+y^2)\arctan\left(\frac{y}{x}\right) - yx = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 12

```
dsolve(x^2*diff(y(x),x)=3*(x^2+y(x)^2)*arctan(y(x)/x)+x*y(x),y(x), singsol=all)
```

$$y(x) = \tan\left(c_1x^3\right)x$$

✓ Solution by Mathematica

Time used: 5.758 (sec). Leaf size: 30

```
DSolve[x^2*y'[x]==3*(x^2+y[x]^2)*ArcTan[x,y[x]]+x*y[x],y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to x\tan\left(x^3(\cosh(3c_1) - \sinh(3c_1))\right)$$
$$y(x) \to 0$$

## 5.4 problem 1(d)

Internal problem ID [6206]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28
**Problem number**: 1(d).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$x \sin\left(\frac{y}{x}\right) y' - y \sin\left(\frac{y}{x}\right) = x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*sin(y(x)/x)*diff(y(x),x)=y(x)*sin(y(x)/x)+x,y(x), singsol=all)
```

$$y(x) = (\pi - \arccos(\ln(x) + c_1))\, x$$

✓ Solution by Mathematica

Time used: 0.461 (sec). Leaf size: 34

```
DSolve[x*Sin[y[x]/x]*y'[x]==y[x]*Sin[y[x]/x]+x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x \arccos(-\log(x) - c_1)$$
$$y(x) \to x \arccos(-\log(x) - c_1)$$

## 5.5 problem 1(e)

Internal problem ID [6207]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28
**Problem number**: 1(e).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y'x - y - 2x\,\mathrm{e}^{-\frac{y}{x}} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x)=y(x)+2*x*exp(-y(x)/x),y(x), singsol=all)
```

$$y(x) = \ln\left(2\ln\left(x\right) + 2c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.42 (sec). Leaf size: 15

```
DSolve[x*y'[x]==y[x]+2*x*Exp[-y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \log(2\log(x) + c_1)$$

## 5.6 problem 1(f)

Internal problem ID [6208]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28
**Problem number**: 1(f).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{-y - y'(x+y) = -x}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 51

```
dsolve((x-y(x))-(x+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-c_1x - \sqrt{2c_1^2x^2 + 1}}{c_1}$$
$$y(x) = \frac{-c_1x + \sqrt{2c_1^2x^2 + 1}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.493 (sec). Leaf size: 94

```
DSolve[(x-y[x])-(x+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x - \sqrt{2x^2 + e^{2c_1}}$$
$$y(x) \to -x + \sqrt{2x^2 + e^{2c_1}}$$
$$y(x) \to -\sqrt{2}\sqrt{x^2} - x$$
$$y(x) \to \sqrt{2}\sqrt{x^2} - x$$

## 5.7 problem 1(g)

Internal problem ID [6209]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28
**Problem number**: 1(g).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + 6y = 2x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve(x*diff(y(x),x)=2*x-6*y(x),y(x), singsol=all)
```

$$y(x) = \frac{2x}{7} + \frac{c_1}{x^6}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 17

```
DSolve[x*y'[x]==2*x-6*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x}{7} + \frac{c_1}{x^6}$$

## 5.8 problem 1(h)

Internal problem ID [6210]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28
**Problem number**: 1(h).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y'x - \sqrt{x^2 + y^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve(x*diff(y(x),x)=sqrt(x^2+y(x)^2),y(x), singsol=all)
```

$$\frac{y(x)^2}{x^2} + \frac{y(x)\sqrt{x^2 + y(x)^2}}{x^2} + \ln\left(y(x) + \sqrt{x^2 + y(x)^2}\right) - 3\ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.296 (sec). Leaf size: 66

```
DSolve[x*y'[x]==Sqrt[x^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\left(\frac{y(x)\left(\sqrt{\frac{y(x)^2}{x^2}+1}+\frac{y(x)}{x}\right)}{x} - \log\left(\sqrt{\frac{y(x)^2}{x^2}+1} - \frac{y(x)}{x}\right)\right) = \log(x) + c_1, y(x)\right]$$

## 5.9 problem 1(i)

Internal problem ID [6211]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28
**Problem number**: 1(i).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{x^2y' - 2yx - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x)=y(x)^2+2*x*y(x),y(x), singsol=all)
```

$$y(x) = \frac{x^2}{-x + c_1}$$

✓ Solution by Mathematica

Time used: 0.141 (sec). Leaf size: 23

```
DSolve[x^2*y'[x]==y[x]^2+2*x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2}{x - c_1}$$
$$y(x) \to 0$$

## 5.10 problem 1(j)

Internal problem ID [6212]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28
**Problem number**: 1(j).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{y^3 - xy^2y' = -x^3}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 74

```
dsolve((x^3+y(x)^3)-(x*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = (3\ln(x) + c_1)^{\frac{1}{3}} x$$
$$y(x) = \left(-\frac{(3\ln(x) + c_1)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}\,(3\ln(x) + c_1)^{\frac{1}{3}}}{2}\right) x$$
$$y(x) = \left(-\frac{(3\ln(x) + c_1)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}\,(3\ln(x) + c_1)^{\frac{1}{3}}}{2}\right) x$$

✓ Solution by Mathematica

Time used: 0.197 (sec). Leaf size: 63

```
DSolve[(x^3+y[x]^3)-(x*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\sqrt[3]{3\log(x) + c_1}$$
$$y(x) \to -\sqrt[3]{-1}x\sqrt[3]{3\log(x) + c_1}$$
$$y(x) \to (-1)^{2/3}x\sqrt[3]{3\log(x) + c_1}$$

## 5.11 problem 4(a)

Internal problem ID [6213]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28
**Problem number**: 4(a).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{x+y+4}{x-y-6} = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 31

```
dsolve(diff(y(x),x)=(x+y(x)+4)/(x-y(x)-6),y(x), singsol=all)
```

$$y(x) = -5 - \tan\left(\operatorname{RootOf}\left(2\_Z + \ln\left(\frac{1}{\cos(\_Z)^2}\right) + 2\ln(x-1) + 2c_1\right)\right)(x-1)$$

✓ Solution by Mathematica

Time used: 0.063 (sec). Leaf size: 58

```
DSolve[y'[x]==(x+y[x]+4)/(x-y[x]-6),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2\arctan\left(\frac{y(x)+x+4}{y(x)-x+6}\right) + \log\left(\frac{x^2+y(x)^2+10y(x)-2x+26}{2(x-1)^2}\right) + 2\log(x-1) + c_1 = 0, y(x)\right]$$

## 5.12 problem 4(b)

Internal problem ID [6214]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28
**Problem number**: 4(b).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{x + y + 4}{x + y - 6} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)=(x+y(x)+4)/(x+y(x)-6),y(x), singsol=all)
```

$$y(x) = -x - 5\,\mathrm{LambertW}\left(-\frac{\mathrm{e}^{-\frac{2x}{5}} c_1 \mathrm{e}^{\frac{1}{5}}}{5}\right) + 1$$

✓ Solution by Mathematica

Time used: 4.043 (sec). Leaf size: 35

```
DSolve[y'[x]==(x+y[x]+4)/(x+y[x]-6),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -5W\left(-e^{-\frac{2x}{5}-1+c_1}\right) - x + 1$$

$$y(x) \to 1 - x$$

## 5.13 problem 4(c)

Internal problem ID [6215]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28
**Problem number**: 4(c).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{-2y+(-1+y)\,y'=-2x}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 31

```
dsolve((2*x-2*y(x))+(y(x)-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=-\tan\left(\operatorname{RootOf}\left(-2\_Z+\ln\left(\frac{1}{\cos\left(\_Z\right)^2}\right)+2\ln\left(x-1\right)+2c_1\right)\right)(x-1)+x$$

✓ Solution by Mathematica

Time used: 0.057 (sec). Leaf size: 60

```
DSolve[(2*x-2*y[x])+(y[x]-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2\arctan\left(\frac{y(x)-2x+1}{y(x)-1}\right)+\log\left(\frac{2x^2-2xy(x)+y(x)^2-2x+1}{2(x-1)^2}\right)+2\log(x-1)+c_1=0,y(x)\right]$$

## 5.14 problem 4(d)

Internal problem ID [6216]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28
**Problem number**: 4(d).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{y + x - 1}{x + 4y + 2} = 0}$$

✓ Solution by Maple

Time used: 1.719 (sec). Leaf size: 65

```
dsolve(diff(y(x),x)=(x+y(x)-1)/(x+4*y(x)+2),y(x), singsol=all)
```

$$y(x) = -1 + \frac{(x-2)\left(\operatorname{RootOf}\left(\_Z^{16} + 2(x-2)^4 c_1\_Z^4 - (x-2)^4 c_1\right)^4 - 1\right)}{2\operatorname{RootOf}\left(\_Z^{16} + 2(x-2)^4 c_1\_Z^4 - (x-2)^4 c_1\right)^4}$$

✓ Solution by Mathematica

Time used: 60.343 (sec). Leaf size: 8141

```
DSolve[y'[x]==(x+y[x]-1)/(x+4*y[x]+2),y[x],x,IncludeSingularSolutions -> True]
```

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## 5.15 problem 4(e)

Internal problem ID [6217]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28
**Problem number**: 4(e).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{3y - 4y'(1 + x) = -2x + 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve((2*x+3*y(x)-1)-4*(x+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = (x + 1)^{\frac{3}{4}} c_1 + 2x + 3$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 20

```
DSolve[(2*x+3*y[x]-1)-4*(x+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x + c_1(x + 1)^{3/4} + 3$$

## 5.16 problem 5(a)

Internal problem ID [6218]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28
**Problem number**: 5(a).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y' - \frac{1 - x y^2}{2 y x^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
dsolve(diff(y(x),x)=(1-x*y(x)^2)/(2*x^2*y(x)),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{x\left(\ln\left(x\right) + c_1\right)}}{x}$$
$$y(x) = -\frac{\sqrt{x\left(\ln\left(x\right) + c_1\right)}}{x}$$

✓ Solution by Mathematica

Time used: 0.184 (sec). Leaf size: 40

```
DSolve[y'[x]==(1-x*y[x]^2)/(2*x^2*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\log(x) + c_1}}{\sqrt{x}}$$
$$y(x) \to \frac{\sqrt{\log(x) + c_1}}{\sqrt{x}}$$

## 5.17 problem 5(b)

Internal problem ID [6219]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28
**Problem number**: 5(b).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y' - \frac{2 + 3xy^2}{4yx^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(diff(y(x),x)=(2+3*x*y(x)^2)/(4*x^2*y(x)),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{5}\,\sqrt{x\left(5x^{\frac{5}{2}}c_1 - 2\right)}}{5x}$$

$$y(x) = \frac{\sqrt{5}\,\sqrt{x\left(5x^{\frac{5}{2}}c_1 - 2\right)}}{5x}$$

✓ Solution by Mathematica

Time used: 3.667 (sec). Leaf size: 51

```
DSolve[y'[x]==(2+3*x*y[x]^2)/(4*x^2*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-\frac{2}{5x} + c_1 x^{3/2}}$$

$$y(x) \to \sqrt{-\frac{2}{5x} + c_1 x^{3/2}}$$

## 5.18 problem 5(c)

Internal problem ID [6220]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28
**Problem number**: 5(c).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{y - x y^2}{x + y x^2} = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 22

```
dsolve(diff(y(x),x)=(y(x)-x*y(x)^2)/(x+x^2*y(x)),y(x), singsol=all)
```

$$y(x) = x\,\mathrm{e}^{-\operatorname{LambertW}\left(x^2 \mathrm{e}^{-2c_1}\right) - 2c_1}$$

✓ Solution by Mathematica

Time used: 60.444 (sec). Leaf size: 31

```
DSolve[y'[x]==(y[x]-x*y[x]^2)/(x+x^2*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{W\left(e^{\frac{1}{2}\left(-2-9\sqrt[3]{-2}c_1\right)} x^2\right)}{x}$$

## 5.19 problem 7(a)

Internal problem ID [6221]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28
**Problem number**: 7(a).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$y' - \sin\left(\frac{y}{x}\right) + \cos\left(\frac{y}{x}\right) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)=sin(y(x)/x)-cos(y(x)/x),y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(\int^{\_Z} \frac{1}{-\sin(\_a) + \cos(\_a) + \_a} d\_a + \ln(x) + c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.367 (sec). Leaf size: 36

```
DSolve[y'[x]==Sin[y[x]/x]-Cos[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$\operatorname{Solve}\left[\int_1^{\frac{y(x)}{x}} \frac{1}{\cos(K[1]) + K[1] - \sin(K[1])} dK[1] = -\log(x) + c_1, y(x)\right]$$

## 5.20 problem 7(b)

Internal problem ID [6222]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28
**Problem number**: 7(b).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{e^{\frac{x}{y}}-\frac{y'y}{x}=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(exp(x/y(x))-y(x)/x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\operatorname{RootOf}\left(-\left(\int^{\_Z}\frac{\_a}{-\_a^2+e^{\frac{1}{\_a}}}d\_a\right)+\ln(x)+c_1\right)x$$

✓ Solution by Mathematica

Time used: 0.192 (sec). Leaf size: 41

```
DSolve[Exp[x/y[x]]-y[x]/x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{\frac{y(x)}{x}}\frac{K[1]}{K[1]^2-e^{\frac{1}{K[1]}}}dK[1]=-\log(x)+c_1,y(x)\right]$$

## 5.21 problem 7(c)

Internal problem ID [6223]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28
**Problem number**: 7(c).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$y' - \frac{x^2 - yx}{y^2 \cos\left(\frac{x}{y}\right)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x)=(x^2-x*y(x))/(y(x)^2*cos(x/y(x))),y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(\int^{\_Z} \frac{\_a^2 \cos\left(\frac{1}{\_a}\right)}{\_a^3 \cos\left(\frac{1}{\_a}\right) + \_a - 1} d\_a + \ln(x) + c_1\right) x$$

✓ Solution by Mathematica

Time used: 1.114 (sec). Leaf size: 49

```
DSolve[y'[x]==(x^2-x*y[x])/(y[x]^2*Cos[x/y[x]]),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{\frac{y(x)}{x}} \frac{\cos\left(\frac{1}{K[1]}\right) K[1]^2}{\cos\left(\frac{1}{K[1]}\right) K[1]^3 + K[1] - 1} dK[1] = -\log(x) + c_1, y(x)\right]$$

## 5.22 problem 7(d)

Internal problem ID [6224]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28
**Problem number**: 7(d).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$y' - \frac{y \tan\left(\frac{y}{x}\right)}{x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x)=y(x)/x*tan(y(x)/x),y(x), singsol=all)
```

$$y(x) = \mathrm{RootOf}\left(\ln(x) + c_1 - \left(\int^{\_Z} \frac{1}{\_a\left(-1+\tan(\_a)\right)}d\_a\right)\right)x$$

✓ Solution by Mathematica

Time used: 1.796 (sec). Leaf size: 33

```
DSolve[y'[x]==y[x]/x*Tan[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\int_1^{\frac{y(x)}{x}} \frac{1}{K[1](\tan(K[1])-1)}dK[1] = \log(x) + c_1, y(x)\right]$$

# 6 Chapter 1. What is a differential equation. Section 1.8. Integrating Factors. Page 32

## 6.1 problem 1(a)

Internal problem ID [6225]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.8. Integrating Factors. Page 32
**Problem number**: 1(a).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{(3x^2 - y^2)\, y' - 2yx = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 402

```
dsolve((3*x^2-y(x)^2)*diff(y(x),x)-2*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}}{6c_1} + \frac{2}{3c_1\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}} + \frac{1}{3c_1}$$

$$y(x) = -\frac{\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}}{12c_1} - \frac{1}{3c_1\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}} + \frac{1}{3c_1} - \frac{i\sqrt{3}\left(\frac{\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}}{6c_1} - \frac{2}{3c_1\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}}{12c_1} - \frac{1}{3c_1\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}} + \frac{1}{3c_1} + \frac{i\sqrt{3}\left(\frac{\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}}{6c_1} - \frac{2}{3c_1\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 60.184 (sec). Leaf size: 458

```
DSolve[(3*x^2-y[x]^2)*y'[x]-2*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(\frac{\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}}{\sqrt[3]{2}} + \frac{\sqrt[3]{2}e^{2c_1}}{\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}} - e^{c_1}\right)$$

$$y(x) \to \frac{i\left(\sqrt{3}+i\right)\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}}{6\sqrt[3]{2}} - \frac{i\left(\sqrt{3}-i\right)e^{2c_1}}{3\ 2^{2/3}\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}} - \frac{e^{c_1}}{3}$$

$$y(x) \to -\frac{i\left(\sqrt{3}-i\right)\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}}{6\sqrt[3]{2}} + \frac{i\left(\sqrt{3}+i\right)e^{2c_1}}{3\ 2^{2/3}\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}} - \frac{e^{c_1}}{3}$$

## 6.2 problem 1(b)

Internal problem ID [6226]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.8. Integrating Factors. Page 32
**Problem number**: 1(b).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], [_Abel`

$$\boxed{yx + (x^2 - yx)\, y' = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve((x*y(x)-1)+(x^2-x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = x - \sqrt{x^2 - 2\ln(x) + 2c_1}$$
$$y(x) = x + \sqrt{x^2 - 2\ln(x) + 2c_1}$$

✓ Solution by Mathematica

Time used: 0.393 (sec). Leaf size: 68

```
DSolve[(x*y[x]-1)+(x^2-x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + \sqrt{-\frac{1}{x}}\sqrt{-x\,(x^2 - 2\log(x) + c_1)}$$
$$y(x) \to x + x\left(-\frac{1}{x}\right)^{3/2}\sqrt{-x\,(x^2 - 2\log(x) + c_1)}$$

## 6.3 problem 1(c)

Internal problem ID [6227]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.8. Integrating Factors. Page 32
**Problem number**: 1(c).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{y'x + y + 3y^4y'x^3 = 0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 129

```
dsolve(x*diff(y(x),x)+y(x)+3*x^3*y(x)^4*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-6xc_1\left(-x+\sqrt{12c_1^2+x^2}\right)}}{6xc_1}$$

$$y(x) = \frac{\sqrt{-6xc_1\left(-x+\sqrt{12c_1^2+x^2}\right)}}{6xc_1}$$

$$y(x) = -\frac{\sqrt{6}\sqrt{xc_1\left(x+\sqrt{12c_1^2+x^2}\right)}}{6xc_1}$$

$$y(x) = \frac{\sqrt{6}\sqrt{xc_1\left(x+\sqrt{12c_1^2+x^2}\right)}}{6xc_1}$$

✓ Solution by Mathematica

Time used: 9.711 (sec). Leaf size: 166

```
DSolve[x*y'[x]+y[x]+3*x^3*y[x]^4*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{c_1 - \frac{\sqrt{x^2(3+c_1{}^2x^2)}}{x^2}}}{\sqrt{3}}$$

$$y(x) \to \frac{\sqrt{c_1 - \frac{\sqrt{x^2(3+c_1{}^2x^2)}}{x^2}}}{\sqrt{3}}$$

$$y(x) \to -\frac{\sqrt{\frac{\sqrt{x^2(3+c_1{}^2x^2)}}{x^2} + c_1}}{\sqrt{3}}$$

$$y(x) \to \frac{\sqrt{\frac{\sqrt{x^2(3+c_1{}^2x^2)}}{x^2} + c_1}}{\sqrt{3}}$$

$$y(x) \to 0$$

## 6.4 problem 1(d)

Internal problem ID [6228]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.8. Integrating Factors. Page 32
**Problem number**: 1(d).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{(e^{x} \cot (y)+2 y \csc (y)) y^{\prime}=-e^{x}}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 15

```
dsolve(exp(x)+(exp(x)*cot(y(x))+2*y(x)*csc(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$e^{x} \sin (y(x))+y(x)^{2}+c_{1}=0$$

✓ Solution by Mathematica

Time used: 0.322 (sec). Leaf size: 18

```
DSolve[Exp[x]+(Exp[x]*Cot[y[x]]+2*y[x]*Csc[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -
```

$$\text{Solve}\left[y(x)^{2}+e^{x} \sin (y(x))=c_{1}, y(x)\right]$$

## 6.5 problem 1(e)

Internal problem ID [6229]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.8. Integrating Factors. Page 32
**Problem number**: 1(e).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x+2)\sin(y)+x\cos(y)\,y'=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve((x+2)*sin(y(x))+x*cos(y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\arcsin\left(\frac{e^{-x}}{c_1x^2}\right)$$

✓ Solution by Mathematica

Time used: 51.335 (sec). Leaf size: 23

```
DSolve[(x+2)*Sin[y[x]]+x*Cos[y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\arcsin\left(\frac{e^{-x+c_1}}{x^2}\right)$$
$$y(x)\to 0$$

## 6.6 problem 1(f)

Internal problem ID [6230]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.8. Integrating Factors. Page 32
**Problem number**: 1(f).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{y + (x - 2y^3x^2)\,y' = 0}$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 432

```
dsolve(y(x)+(x-2*x^2*y(x)^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(12\sqrt{3}\,\sqrt{\frac{27c_1^3-4x^2}{c_1}}-108c_1\right)c_1^2x^2\right)^{\frac{1}{3}}}{6c_1x}+\frac{2x}{\left(\left(12\sqrt{3}\,\sqrt{\frac{27c_1^3-4x^2}{c_1}}-108c_1\right)c_1^2x^2\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(\left(12\sqrt{3}\,\sqrt{\frac{27c_1^3-4x^2}{c_1}}-108c_1\right)c_1^2x^2\right)^{\frac{1}{3}}}{12c_1x}-\frac{x}{\left(\left(12\sqrt{3}\,\sqrt{\frac{27c_1^3-4x^2}{c_1}}-108c_1\right)c_1^2x^2\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(\left(12\sqrt{3}\,\sqrt{\frac{27c_1^3-4x^2}{c_1}}-108c_1\right)c_1^2x^2\right)^{\frac{1}{3}}}{6c_1x}-\frac{2x}{\left(\left(12\sqrt{3}\,\sqrt{\frac{27c_1^3-4x^2}{c_1}}-108c_1\right)c_1^2x^2\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(\left(12\sqrt{3}\,\sqrt{\frac{27c_1^3-4x^2}{c_1}}-108c_1\right)c_1^2x^2\right)^{\frac{1}{3}}}{12c_1x}-\frac{x}{\left(\left(12\sqrt{3}\,\sqrt{\frac{27c_1^3-4x^2}{c_1}}-108c_1\right)c_1^2x^2\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(\left(12\sqrt{3}\,\sqrt{\frac{27c_1^3-4x^2}{c_1}}-108c_1\right)c_1^2x^2\right)^{\frac{1}{3}}}{6c_1x}-\frac{2x}{\left(\left(12\sqrt{3}\,\sqrt{\frac{27c_1^3-4x^2}{c_1}}-108c_1\right)c_1^2x^2\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 28.221 (sec). Leaf size: 327

```
DSolve[y[x]+(x-2*x^2*y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2\sqrt[3]{3}c_1x^2 + \sqrt[3]{2}\left(-9x^2 + \sqrt{81x^4 - 12c_1{}^3x^6}\right)^{2/3}}{6^{2/3}x\sqrt[3]{-9x^2 + \sqrt{81x^4 - 12c_1{}^3x^6}}}$$

$$y(x) \to \frac{i\sqrt[3]{3}\left(\sqrt{3}+i\right)\left(-18x^2 + 2\sqrt{81x^4 - 12c_1{}^3x^6}\right)^{2/3} - 2\sqrt[3]{2}\sqrt[6]{3}\left(\sqrt{3}+3i\right)c_1x^2}{12x\sqrt[3]{-9x^2 + \sqrt{81x^4 - 12c_1{}^3x^6}}}$$

$$y(x) \to \frac{\sqrt[3]{3}\left(-1-i\sqrt{3}\right)\left(-18x^2 + 2\sqrt{81x^4 - 12c_1{}^3x^6}\right)^{2/3} - 2\sqrt[3]{2}\sqrt[6]{3}\left(\sqrt{3}-3i\right)c_1x^2}{12x\sqrt[3]{-9x^2 + \sqrt{81x^4 - 12c_1{}^3x^6}}}$$

$$y(x) \to 0$$

## 6.7 problem 1(g)

Internal problem ID [6231]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.8. Integrating Factors. Page 32
**Problem number**: 1(g).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{3y^2 + 2xyy' = -x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve((x+3*y(x)^2)+(2*x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{x\left(-x^4+4c_1\right)}}{2x^2}$$
$$y(x) = \frac{\sqrt{x\left(-x^4+4c_1\right)}}{2x^2}$$

✓ Solution by Mathematica

Time used: 0.206 (sec). Leaf size: 55

```
DSolve[(x+3*y[x]^2)+(2*x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-x^4+4c_1}}{2x^{3/2}}$$
$$y(x) \to \frac{\sqrt{-x^4+4c_1}}{2x^{3/2}}$$

## 6.8 problem 1(h)

Internal problem ID [6232]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.8. Integrating Factors. Page 32
**Problem number**: 1(h).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_1st_order, ‘_with_symmetry_[F(x)*G(y),0]‘]]`

$$\boxed{y + (2x - e^y y)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 27

```
dsolve(y(x)+(2*x-y(x)*exp(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$x - \frac{\left(y(x)^2 - 2y(x) + 2\right) e^{y(x)} + c_1}{y\left(x\right)^2} = 0$$

✓ Solution by Mathematica

Time used: 0.259 (sec). Leaf size: 32

```
DSolve[y[x]+(2*x-y[x]*Exp[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x = \frac{e^{y(x)}(y(x)^2 - 2y(x) + 2)}{y(x)^2} + \frac{c_1}{y(x)^2}, y(x)\right]$$

## 6.9 problem 1(i)

Internal problem ID [6233]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.8. Integrating Factors. Page 32
**Problem number**: 1(i).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y \ln(y) - 2yx + y'(x+y) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve((y(x)*ln(y(x))-2*x*y(x))+(x+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x \,\mathrm{LambertW}\left(\frac{\mathrm{e}^{x} \mathrm{e}^{-\frac{c_1}{x}}}{x}\right) - x^2 + c_1}{x}}$$

✓ Solution by Mathematica

Time used: 1.073 (sec). Leaf size: 22

```
DSolve[(y[x]*Log[y[x]]-2*x*y[x])+(x+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to xW\left(\frac{e^{x+\frac{c_1}{x}}}{x}\right)$$

## 6.10 problem 1(j)

Internal problem ID [6234]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.8. Integrating Factors. Page 32
**Problem number**: 1(j).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$\boxed{y^2 + yx + (x^2 + yx + 1)\, y' = -1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve((y(x)^2+x*y(x)+1)+(x^2+x*y(x)+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-x^2 + \mathrm{LambertW}\left(-2x\, \mathrm{e}^{x^2} c_1 \mathrm{e}^{-1}\right)}{x}$$

✓ Solution by Mathematica

Time used: 6.606 (sec). Leaf size: 56

```
DSolve[(y[x]^2+x*y[x]+1)+(x^2+x*y[x]+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + \frac{W\left(x\left(-e^{x^2-1+c_1}\right)\right)}{x}$$

$$y(x) \to -x$$

$$y(x) \to \frac{W\left(-e^{x^2-1}x\right)}{x} - x$$

## 6.11 problem 1(k)

Internal problem ID [6235]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.8. Integrating Factors. Page 32
**Problem number**: 1(k).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{x y^{3}+3 y^{2} y^{\prime}=-x^{3}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 113

```
dsolve((x^3+x*y(x)^3)+(3*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \left(\mathrm{e}^{-\frac{x^{2}}{2}} c_{1}-x^{2}+2\right)^{\frac{1}{3}}$$

$$y(x) = -\frac{\left(\mathrm{e}^{-\frac{x^{2}}{2}} c_{1}-x^{2}+2\right)^{\frac{1}{3}}}{2}-\frac{i \sqrt{3}\, \left(\mathrm{e}^{-\frac{x^{2}}{2}} c_{1}-x^{2}+2\right)^{\frac{1}{3}}}{2}$$

$$y(x) = -\frac{\left(\mathrm{e}^{-\frac{x^{2}}{2}} c_{1}-x^{2}+2\right)^{\frac{1}{3}}}{2}+\frac{i \sqrt{3}\, \left(\mathrm{e}^{-\frac{x^{2}}{2}} c_{1}-x^{2}+2\right)^{\frac{1}{3}}}{2}$$

✓ Solution by Mathematica

Time used: 10.689 (sec). Leaf size: 95

```
DSolve[(x^3+x*y[x]^3)+(3*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[3]{-x^2 + c_1 e^{-\frac{x^2}{2}} + 2}$$
$$y(x) \to -\sqrt[3]{-1}\sqrt[3]{-x^2 + c_1 e^{-\frac{x^2}{2}} + 2}$$
$$y(x) \to (-1)^{2/3}\sqrt[3]{-x^2 + c_1 e^{-\frac{x^2}{2}} + 2}$$

## 6.12 problem 4

Internal problem ID [6236]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.8. Integrating Factors. Page 32
**Problem number**: 4.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{y' - \frac{2y}{x} - \frac{x^3}{y} - x\tan\left(\frac{y}{x^2}\right) = 0}$$

✓ Solution by Maple

Time used: 0.89 (sec). Leaf size: 216

```
dsolve(diff(y(x),x)=2*y(x)/x+x^3/y(x)+x*tan(y(x)/x^2),y(x), singsol=all)
```

$$y(x) = -\frac{x^2\left(c_1\cos\left(\operatorname{RootOf}\left(c_1^2\_Z^2\cos(2\_Z) + 4c_1\sin(\_Z)\,x\_Z - c_1^2\_Z^2 + c_1^2\cos(2\_Z) + c_1^2 - 2x^2\right)\right) - x\right)}{c_1\sin\left(\operatorname{RootOf}\left(c_1^2\_Z^2\cos(2\_Z) + 4c_1\sin(\_Z)\,x\_Z - c_1^2\_Z^2 + c_1^2\cos(2\_Z) + c_1^2 - 2x^2\right)\right)}$$

$$y(x) = \frac{x^2\left(c_1\cos\left(\operatorname{RootOf}\left(c_1^2\_Z^2\cos(2\_Z) + 4c_1\sin(\_Z)\,x\_Z - c_1^2\_Z^2 + c_1^2\cos(2\_Z) + c_1^2 - 2x^2\right)\right) + x\right)}{c_1\sin\left(\operatorname{RootOf}\left(c_1^2\_Z^2\cos(2\_Z) + 4c_1\sin(\_Z)\,x\_Z - c_1^2\_Z^2 + c_1^2\cos(2\_Z) + c_1^2 - 2x^2\right)\right)}$$

✓ Solution by Mathematica

Time used: 1.103 (sec). Leaf size: 36

```
DSolve[y'[x]==2*y[x]/x+x^3/y[x]+x*Tan[y[x]/x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[3\log(x) - \log\left(y(x)\sin\left(\frac{y(x)}{x^2}\right) + x^2\cos\left(\frac{y(x)}{x^2}\right)\right) = c_1, y(x)\right]$$

# 7 Chapter 1. What is a differential equation. Section 1.9. Reduction of Order. Page 38

## 7.1 problem 1(a)

Internal problem ID [6237]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.9. Reduction of Order. Page 38
**Problem number**: 1(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _`

$$\boxed{yy'' + y'^2 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 33

```
dsolve(y(x)*diff(y(x),x$2)+(diff(y(x),x))^2=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \sqrt{2c_1x + 2c_2}$$
$$y(x) = -\sqrt{2c_1x + 2c_2}$$

✓ Solution by Mathematica

Time used: 0.166 (sec). Leaf size: 20

```
DSolve[y[x]*y''[x]+(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2\sqrt{2x - c_1}$$

## 7.2 problem 1(b)

Internal problem ID [6238]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.9. Reduction of Order. Page 38
**Problem number**: 1(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[NONE]`

$$\boxed{xyy'' - y' - y'^3 = 0}$$

✗ Solution by Maple

```
dsolve(x*y(x)*diff(y(x),x$2)=diff(y(x),x)+(diff(y(x),x))^3,y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 1.417 (sec). Leaf size: 103

```
DSolve[x*y''[x]==y'[x]+(y'[x])^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - ie^{-c_1}\sqrt{-1 + e^{2c_1}x^2}$$
$$y(x) \to ie^{-c_1}\sqrt{-1 + e^{2c_1}x^2} + c_2$$
$$y(x) \to c_2 - i\sqrt{x^2}$$
$$y(x) \to i\sqrt{x^2} + c_2$$

## 7.3 problem 1(c)

Internal problem ID [6239]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.9. Reduction of Order. Page 38
**Problem number**: 1(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - k^2 y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-k^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-kx} + c_2 e^{kx}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 23

```
DSolve[y''[x]-k^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{kx} + c_2 e^{-kx}$$

## 7.4 problem 1(d)

Internal problem ID [6240]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.9. Reduction of Order. Page 38
**Problem number**: 1(d).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x^2y'' - 2y'x - y'^2 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 26

```
dsolve(x^2*diff(y(x),x$2)=2*x*diff(y(x),x)+(diff(y(x),x))^2,y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{2} - c_1x - c_1^2 \ln(x - c_1) + c_2$$

✓ Solution by Mathematica

Time used: 0.435 (sec). Leaf size: 41

```
DSolve[x^2*y''[x]==2*x*y'[x]+(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2}{2} - c_1x - c_1{}^2 \log(x - c_1) + \frac{3c_1{}^2}{2} + c_2$$

## 7.5 problem 1(e)

Internal problem ID [6241]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.9. Reduction of Order. Page 38
**Problem number**: 1(e).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{2yy'' - y'^2 = 1}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 22

```
dsolve(2*y(x)*diff(y(x),x$2)=1+(diff(y(x),x))^2,y(x), singsol=all)
```

$$y(x) = \frac{(c_1^2 + 1)\, x^2}{4c_2} + c_1 x + c_2$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 34

```
DSolve[2*y[x]*y''[x]==1+(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(1 + c_1{}^2)\, x^2}{4c_2} + c_1 x + c_2$$

$$y(x) \to \text{Indeterminate}$$

## 7.6 problem 1(f)

Internal problem ID [6242]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.9. Reduction of Order. Page 38
**Problem number**: 1(f).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{yy'' - y'^2 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 14

```
dsolve(y(x)*diff(y(x),x$2)-(diff(y(x),x))^2=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = e^{c_1 x} c_2$$

✓ Solution by Mathematica

Time used: 0.101 (sec). Leaf size: 14

```
DSolve[y[x]*y''[x]-(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 e^{c_1 x}$$

## 7.7 problem 1(g)

Internal problem ID [6243]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.9. Reduction of Order. Page 38
**Problem number**: 1(g).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' + y' = 4x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x*diff(y(x),x$2)+diff(y(x),x)=4*x,y(x), singsol=all)
```

$$y(x) = x^2 + c_1 \ln(x) + c_2$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 16

```
DSolve[x*y''[x]+y'[x]==4*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + c_1 \log(x) + c_2$$

## 7.8 problem 2(a)

Internal problem ID [6244]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.9. Reduction of Order. Page 38
**Problem number**: 2(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _exact, _nonlinear], [`

$$\boxed{(x^2 + 2y')\, y'' + 2y'x = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 15

```
dsolve([(x^2+2*diff(y(x),x))*diff(y(x),x$2)+2*x*diff(y(x),x)=0,y(0) = 1, D(y)(0) = 0],y(x),
```

$$y(x) = 1$$
$$y(x) = -\frac{x^3}{3} + 1$$

✓ Solution by Mathematica

Time used: 0.252 (sec). Leaf size: 32

```
DSolve[{(x^2+2*y'[x])*y''[x]+2*x*y'[x]==0,{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolution
```

$$y(x) \to \text{Indeterminate}$$
$$y(x) \to 0\,\text{Hypergeometric2F1}\left(-\frac{1}{2}, \frac{1}{4}, \frac{5}{4}, \text{ComplexInfinity}\right) - \frac{x^3}{6} + 1$$

## 7.9 problem 2(b)

Internal problem ID [6245]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.9. Reduction of Order. Page 38
**Problem number**: 2(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _with_potential_symmet`

$$yy'' - y^2y' - y'^2 = 0$$

With initial conditions

$$\left[y(0) = -\frac{1}{2}, y'(0) = 1\right]$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 16

```
dsolve([y(x)*diff(y(x),x$2)=y(x)^2*diff(y(x),x)+(diff(y(x),x))^2,y(0) = -1/2, D(y)(0) = 1],y
```

$$y(x) = -\frac{3}{8\,\mathrm{e}^{\frac{3x}{2}} - 2}$$

✓ Solution by Mathematica

Time used: 1.982 (sec). Leaf size: 20

```
DSolve[{y[x]*y''[x]==y[x]^2*y'[x]+(y'[x])^2,{y[0]==-1/2,y'[0]==1}},y[x],x,IncludeSingularSol
```

$$y(x) \to \frac{3}{2 - 8e^{3x/2}}$$

## 7.10 problem 2(c)

Internal problem ID [6246]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.9. Reduction of Order. Page 38
**Problem number**: 2(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], [`

$$\boxed{y'' - y' e^y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 2]$$

✓ Solution by Maple

Time used: 0.516 (sec). Leaf size: 21

```
dsolve([diff(y(x),x$2)=diff(y(x),x)*exp(y(x)),y(0) = 0, D(y)(0) = 2],y(x), singsol=all)
```

$$y(x) = x + \ln\left(-\frac{1}{e^x - 2}\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y''[x]==y'[x]*Exp[y[x]],{y[0]==0,y'[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

{}

## 7.11 problem 3(a)

Internal problem ID [6247]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.9. Reduction of Order. Page 38
**Problem number**: 3(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{y'' - y'^2 = 1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)=1+(diff(y(x),x))^2,y(x), singsol=all)
```

$$y(x) = -\ln\left(\frac{-c_2 + \tan(x) c_1}{\sec(x)}\right)$$

✓ Solution by Mathematica

Time used: 1.938 (sec). Leaf size: 16

```
DSolve[y''[x]==1+(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \log(\cos(x + c_1))$$

## 7.12 problem 3(b)

Internal problem ID [6248]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Section 1.9. Reduction of Order. Page 38
**Problem number**: 3(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{y'' + {y'}^2 = 1}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)+(diff(y(x),x))^2=1,y(x), singsol=all)
```

$$y(x) = x + \ln\left(\frac{\mathrm{e}^{-2x} c_1}{2} - \frac{c_2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.388 (sec). Leaf size: 46

```
DSolve[y''[x]+(y'[x])^2==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\log\left(e^x\right) + \log\left(e^{2x} + e^{2c_1}\right) + c_2$$
$$y(x) \to -\log\left(e^x\right) + \log\left(e^{2x}\right) + c_2$$

# 8 Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53

## 8.1 problem 1(a)

Internal problem ID [6249]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53
**Problem number**: 1(a).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y + y'x = x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x*diff(y(x),x)+y(x)=x,y(x), singsol=all)
```

$$y(x) = \frac{x}{2} + \frac{c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 17

```
DSolve[x*y'[x]+y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{2} + \frac{c_1}{x}$$

## 8.2 problem 1(b)

Internal problem ID [6250]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53
**Problem number**: 1(b).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x^2y' + y = x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x)+y(x)=x^2,y(x), singsol=all)
```

$$y(x) = x - \mathrm{Ei}_1\left(\frac{1}{x}\right) \mathrm{e}^{\frac{1}{x}} + \mathrm{e}^{\frac{1}{x}} c_1$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 27

```
DSolve[x^2*y'[x]+y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{1}{x}} \operatorname{ExpIntegralEi}\left(-\frac{1}{x}\right) + x + c_1 e^{\frac{1}{x}}$$

## 8.3 problem 1(c)

Internal problem ID [6251]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53
**Problem number**: 1(c).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x^2*diff(y(x),x)=y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{1}{x}}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 20

```
DSolve[x^2*y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-1/x}$$
$$y(x) \to 0$$

## 8.4 problem 1(d)

Internal problem ID [6252]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53
**Problem number**: 1(d).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' \sec(x) - \sec(y) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(sec(x)*diff(y(x),x)=sec(y(x)),y(x), singsol=all)
```

$$y(x) = \arcsin(\sin(x) + c_1)$$

✓ Solution by Mathematica

Time used: 0.335 (sec). Leaf size: 11

```
DSolve[Sec[x]*y'[x]==Sec[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin(\sin(x) + c_1)$$

## 8.5 problem 1(e)

Internal problem ID [6253]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53
**Problem number**: 1(e).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{y' - \frac{x^2+y^2}{-y^2+x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x)=(x^2+y(x)^2)/(x^2-y(x)^2),y(x), singsol=all)
```

$$y(x) = \text{RootOf}\left(\int^{\_Z} \frac{\_a^2-1}{\_a^3+\_a^2-\_a+1} d\_a + \ln(x) + c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.115 (sec). Leaf size: 67

```
DSolve[y'[x]==(x^2+y[x]^2)/(x^2-y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\text{RootSum}\left[\#1^3+\#1^2-\#1 + 1\&, \frac{\#1\log\left(\frac{y(x)}{x}-\#1\right)-\log\left(\frac{y(x)}{x}-\#1\right)}{3\#1-1}\&\right] = -\log(x)+c_1, y(x)\right]$$

## 8.6 problem 1(f)

Internal problem ID [6254]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53
**Problem number**: 1(f).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$y' - \frac{x + 2y}{2x - y} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)=(x+2*y(x))/(2*x-y(x)),y(x), singsol=all)
```

$$y(x) = \tan\left(\operatorname{RootOf}\left(-4\_Z + \ln\left(\frac{1}{\cos\left(\_Z\right)^2}\right) + 2\ln\left(x\right) + 2c_1\right)\right)x$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==(x+2*y[x]^2)/(2*x-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

## 8.7 problem 1(g)

Internal problem ID [6255]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53
**Problem number**: 1(g).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2yx + x^2y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 9

```
dsolve(2*x*y(x)+x^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 16

```
DSolve[2*x*y[x]+x^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x^2}$$
$$y(x) \to 0$$

## 8.8 problem 1(h)

Internal problem ID [6256]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53
**Problem number**: 1(h).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{-\sin(x)\sin(y)+\cos(x)\cos(y)\,y'=0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 11

```
dsolve(-sin(x)*sin(y(x))+cos(x)*cos(y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\arcsin\left(\frac{c_1}{\cos(x)}\right)$$

✓ Solution by Mathematica

Time used: 3.473 (sec). Leaf size: 19

```
DSolve[-Sin[x]*Sin[y[x]]+Cos[x]*Cos[y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\arcsin\left(\frac{1}{2}c_1\sec(x)\right)$$
$$y(x)\to 0$$

## 8.9 problem 2(a)

Internal problem ID [6257]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53
**Problem number**: 2(a).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - y = 2x}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve([x*diff(y(x),x)-y(x)=2*x,y(1) = 0],y(x), singsol=all)
```

$$y(x) = 2\ln(x)\,x$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 10

```
DSolve[{x*y'[x]-y[x]==2*x,{y[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x\log(x)$$

## 8.10 problem 2(b)

Internal problem ID [6258]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53
**Problem number**: 2(b).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x^2y' - 2y = 3x^2}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 39

```
dsolve([x^2*diff(y(x),x)-2*y(x)=3*x^2,y(1) = 2],y(x), singsol=all)
```

$$y(x) = 3x - e^{2-\frac{2}{x}} + 6\,e^{-\frac{2}{x}}\,\mathrm{Ei}_1\left(-\frac{2}{x}\right) - 6\;\mathrm{Ei}_1\left(-2\right)e^{-\frac{2}{x}}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 41

```
DSolve[{x^2*y'[x]-2*y[x]==3*x^2,{y[1]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2/x}\left(-6\,\mathrm{ExpIntegralEi}\left(\frac{2}{x}\right) + 6\,\mathrm{ExpIntegralEi}(2) + 3e^{2/x}x - e^2\right)$$

## 8.11 problem 2(c)

Internal problem ID [6259]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53
**Problem number**: 2(c).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{y^2y' = x}$$

With initial conditions

$$[y(-1) = 3]$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 15

```
dsolve([y(x)^2*diff(y(x),x)=x,y(-1) = 3],y(x), singsol=all)
```

$$y(x) = \frac{(12x^2 + 204)^{\frac{1}{3}}}{2}$$

✓ Solution by Mathematica

Time used: 0.191 (sec). Leaf size: 22

```
DSolve[{y[x]^2*y'[x]==x,{y[-1]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[3]{\frac{3}{2}}\sqrt[3]{x^2 + 17}$$

## 8.12 problem 2(d)

Internal problem ID [6260]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53
**Problem number**: 2(d).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' \csc(x) - \csc(y) = 0}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = 1\right]$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 10

```
dsolve([csc(x)*diff(y(x),x)=csc(y(x)),y(1/2*Pi) = 1],y(x), singsol=all)
```

$$y(x) = \arccos(\cos(x) + \cos(1))$$

✓ Solution by Mathematica

Time used: 0.398 (sec). Leaf size: 11

```
DSolve[{Csc[x]*y'[x]==Csc[y[x]],{y[Pi/2]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arccos(\cos(x) + \cos(1))$$

## 8.13 problem 2(e)

Internal problem ID [6261]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53
**Problem number**: 2(e).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{x+y}{x-y} = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 30

```
dsolve([diff(y(x),x)=(x+y(x))/(x-y(x)),y(1) = 1],y(x), singsol=all)
```

$$y(x) = \tan\left(\mathrm{RootOf}\left(4\_Z - 2\ln\left(\sec\left(\_Z\right)^2\right) - 4\ln\left(x\right) + 2\ln\left(2\right) - \pi\right)\right) x$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 46

```
DSolve[{y'[x]==(x+y[x])/(x-y[x]),{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right) - \arctan\left(\frac{y(x)}{x}\right) = \frac{1}{4}(2\log(2)-\pi) - \log(x), y(x)\right]$$

## 8.14 problem 2(f)

Internal problem ID [6262]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53
**Problem number**: 2(f).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$y' - \frac{x^2 + 2y^2}{x^2 - 2y^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(diff(y(x),x)=(x^2+2*y(x)^2)/(x^2-2*y(x)^2),y(x), singsol=all)
```

$$y = \text{RootOf}\left(\int^{\_Z} \frac{2\_a^2 - 1}{2\_a^3 + 2\_a^2 - \_a + 1} d\_a + \ln(x) + c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.106 (sec). Leaf size: 80

```
DSolve[y'[x]==(x^2+2*y[x]^2)/(x^2-2*y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\text{RootSum}\left[2\#1^3 + 2\#1^2 - \#1 + 1\&, \frac{2\#1^2 \log\left(\frac{y(x)}{x} - \#1\right) - \log\left(\frac{y(x)}{x} - \#1\right)}{6\#1^2 + 4\#1 - 1}\&\right] = -\log(x) + c_1, y(x)\right]$$

## 8.15 problem 2(g)

Internal problem ID [6263]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53
**Problem number**: 2(g).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2x\cos(y) - x^2\sin(y)\,y' = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.766 (sec). Leaf size: 11

```
dsolve([2*x*cos(y(x))-x^2*sin(y(x))*diff(y(x),x)=0,y(1) = 1],y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{\cos(1)}{x^2}\right)$$

✓ Solution by Mathematica

Time used: 29.379 (sec). Leaf size: 12

```
DSolve[{2*x*Cos[y[x]]-x^2*Sin[y[x]]*y'[x]==0,{y[1]==1}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \arccos\left(\frac{\cos(1)}{x^2}\right)$$

## 8.16 problem 2(h)

Internal problem ID [6264]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53
**Problem number**: 2(h).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\frac{1}{y} - \frac{xy'}{y^2} = 0}$$

With initial conditions

$$[y(0) = 2]$$

✗ Solution by Maple

```
dsolve([1/y(x)-x/y(x)^2*diff(y(x),x)=0,y(0) = 2],y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{1/y[x]-x/y[x]^2*y'[x]==0,{y[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$\{\}$$

## 8.17 problem 4(a)

Internal problem ID [6265]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53
**Problem number**: 4(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{yy'' - y'^2 = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 14

```
dsolve(y(x)*diff(y(x),x$2)-(diff(y(x),x))^2=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \mathrm{e}^{c_1 x} c_2$$

✓ Solution by Mathematica

Time used: 0.113 (sec). Leaf size: 14

```
DSolve[y[x]*y''[x]-(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 e^{c_1 x}$$

## 8.18 problem 4(b)

Internal problem ID [6266]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53
**Problem number**: 4(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$\boxed{xy'' - y' + 2{y'}^3 = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 37

```
dsolve(x*diff(y(x),x$2)=diff(y(x),x)-2*(diff(y(x),x))^3,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{2x^2 - c_1}}{2} + c_2$$
$$y(x) = -\frac{\sqrt{2x^2 - c_1}}{2} + c_2$$

✓ Solution by Mathematica

Time used: 0.628 (sec). Leaf size: 96

```
DSolve[x*y''[x]==y'[x]-2*(y'[x])^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{1}{2}\sqrt{2x^2 + e^{2c_1}}$$
$$y(x) \to \frac{1}{2}\sqrt{2x^2 + e^{2c_1}} + c_2$$
$$y(x) \to -\frac{\sqrt{x^2}}{\sqrt{2}} + c_2$$
$$y(x) \to \frac{\sqrt{x^2}}{\sqrt{2}} + c_2$$

## 8.19 problem 4(c)

Internal problem ID [6267]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53
**Problem number**: 4(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1],`

$$\boxed{yy'' + y' = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 24

```
dsolve(y(x)*diff(y(x),x$2)+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \mathrm{e}^{\mathrm{RootOf}(-\mathrm{e}^{c_1}\,\mathrm{Ei}_1(-\_Z+c_1)+x+c_2)}$$

✓ Solution by Mathematica

Time used: 0.248 (sec). Leaf size: 80

```
DSolve[y[x]*y''[x]+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}[-e^{c_1}\,\text{ExpIntegralEi}(\log(\#1) - c_1)\&]\,[x + c_2]$$
$$y(x) \to \text{InverseFunction}\left[-e^{-c_1}\,\text{ExpIntegralEi}(\log(\#1) - -c_1)\&\right]\,[x + c_2]$$
$$y(x) \to \text{InverseFunction}[-e^{c_1}\,\text{ExpIntegralEi}(\log(\#1) - c_1)\&]\,[x + c_2]$$

## 8.20 problem 4(d)

Internal problem ID [6268]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53
**Problem number**: 4(d).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' - 3y' = 5x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x*diff(y(x),x$2)-3*diff(y(x),x)=5*x,y(x), singsol=all)
```

$$y(x) = \frac{(2c_1x^2 - 5)^2}{16c_1} + c_2$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 24

```
DSolve[x*y''[x]-3*y'[x]==5*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1x^4}{4} - \frac{5x^2}{4} + c_2$$

# 9 Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

## 9.1 problem 1(a)

Internal problem ID [6269]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 1(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - 6y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-6*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-3x} + e^{2x} c_2$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 22

```
DSolve[y''[x]+y'[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(c_2 e^{5x} + c_1\right)$$

## 9.2 problem 1(b)

Internal problem ID [6270]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 1(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}c_1 + c_2\mathrm{e}^{-x}x$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 18

```
DSolve[y''[x]+2*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_2x + c_1)$$

## 9.3 problem 1(c)

Internal problem ID [6271]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 1(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 8y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(2\sqrt{2}\, x\right) + c_2 \cos\left(2\sqrt{2}\, x\right)$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 30

```
DSolve[y''[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos\left(2\sqrt{2}x\right) + c_2 \sin\left(2\sqrt{2}x\right)$$

## 9.4 problem 1(d)

Internal problem ID [6272]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 1(d).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2y'' - 4y' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(2*diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x \sin(x) + c_2 e^x \cos(x)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 20

```
DSolve[2*y''[x]-4*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x(c_2 \cos(x) + c_1 \sin(x))$$

## 9.5 problem 1(e)

Internal problem ID [6273]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 1(e).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2x}c_1 + c_2\mathrm{e}^{2x}x$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 18

```
DSolve[y''[x]-4*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}(c_2x + c_1)$$

## 9.6 problem 1(f)

Internal problem ID [6274]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 1(f).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 9y' + 20y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-9*diff(y(x),x)+20*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{5x} + c_2 e^{4x}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 20

```
DSolve[y''[x]-9*y'[x]+20*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{4x}(c_2 e^x + c_1)$$

## 9.7 problem 1(g)

Internal problem ID [6275]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 1(g).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2y'' + 2y' + 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(2*diff(y(x),x$2)+2*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{5}\,x}{2}\right) + c_2 e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{5}\,x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 42

```
DSolve[2*y''[x]+2*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}\left(c_2 \cos\left(\frac{\sqrt{5}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{5}x}{2}\right)\right)$$

## 9.8 problem 1(h)

Internal problem ID [6276]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 1(h).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' - 12y' + 9y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(4*diff(y(x),x$2)-12*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{3x}{2}} + c_2 \mathrm{e}^{\frac{3x}{2}} x$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 20

```
DSolve[4*y''[x]-12*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{3x/2}(c_2 x + c_1)$$

## 9.9 problem 1(i)

Internal problem ID [6277]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 1(i).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve(diff(y(x),x$2)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(x) + c_2 \cos(x)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 16

```
DSolve[y''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(x) + c_2 \sin(x)$$

## 9.10 problem 1(j)

Internal problem ID [6278]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 1(j).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 6y' + 25y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+25*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{3x} \sin(4x) + c_2 e^{3x} \cos(4x)$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 26

```
DSolve[y''[x]-6*y'[x]+25*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{3x}(c_2 \cos(4x) + c_1 \sin(4x))$$

## 9.11 problem 1(k)

Internal problem ID [6279]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 1(k).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' + 20y' + 25y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(4*diff(y(x),x$2)+20*diff(y(x),x)+25*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{5x}{2}} + c_2 e^{-\frac{5x}{2}} x$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 20

```
DSolve[4*y''[x]+20*y'[x]+25*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-5x/2}(c_2 x + c_1)$$

## 9.12 problem 1(l)

Internal problem ID [6280]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 1(l).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x} \sin\left(\sqrt{2}\, x\right) + c_2 e^{-x} \cos\left(\sqrt{2}\, x\right)$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 34

```
DSolve[y''[x]+2*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_2 \cos\left(\sqrt{2}x\right) + c_1 \sin\left(\sqrt{2}x\right)\right)$$

## 9.13 problem 1(m)

Internal problem ID [6281]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 1(m).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)=4*y(x),y(x), singsol=all)
```

$$y(x) = e^{-2x}c_1 + e^{2x}c_2$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 22

```
DSolve[y''[x]==4*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(c_1e^{4x} + c_2\right)$$

## 9.14 problem 1(n)

Internal problem ID [6282]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 1(n).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' - 8y' + 7y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(4*diff(y(x),x$2)-8*diff(y(x),x)+7*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x \sin\left(\frac{\sqrt{3}\,x}{2}\right) + c_2 e^x \cos\left(\frac{\sqrt{3}\,x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 38

```
DSolve[4*y''[x]-8*y'[x]+7*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x \left(c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

## 9.15 problem 1(o)

Internal problem ID [6283]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 1(o).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2y'' + y' - y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(2*diff(y(x),x$2)+diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{x}{2}} + c_2 \mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 24

```
DSolve[2*y''[x]+y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_1 e^{3x/2} + c_2\right)$$

## 9.16 problem 1(p)

Internal problem ID [6284]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 1(p).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y' + 5y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+5*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-2x} \sin(x) + c_2 e^{-2x} \cos(x)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 22

```
DSolve[y''[x]+4*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}(c_2 \cos(x) + c_1 \sin(x))$$

## 9.17 problem 1(q)

Internal problem ID [6285]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 1(q).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y' + 5y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+5*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-2x} \sin(x) + c_2 e^{-2x} \cos(x)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 22

```
DSolve[y''[x]+4*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}(c_2 \cos(x) + c_1 \sin(x))$$

## 9.18 problem 1(r)

Internal problem ID [6286]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 1(r).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y' - 5y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)-5*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-5x} + \mathrm{e}^{x} c_2$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 20

```
DSolve[y''[x]+4*y'[x]-5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-5x} + c_2 e^{x}$$

## 9.19 problem 2(a)

Internal problem ID [6287]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 2(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 5y' + 6y = 0}$$

With initial conditions

$$[y(1) = \mathrm{e}^2, y'(1) = 3\,\mathrm{e}^2]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 11

```
dsolve([diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=0,y(1) = exp(2), D(y)(1) = 3*exp(2)],y(x), sing
```

$$y(x) = \mathrm{e}^{3x-1}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 12

```
DSolve[{y''[x]-5*y'[x]+6*y[x]==0,{y[1]==Exp[2],y'[1]==3*Exp[2]}},y[x],x,IncludeSingularSolut
```

$$y(x) \to e^{3x-1}$$

## 9.20 problem 2(b)

Internal problem ID [6288]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 2(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 6y' + 5y = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = 11]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve([diff(y(x),x$2)-6*diff(y(x),x)+5*y(x)=0,y(0) = 3, D(y)(0) = 11],y(x), singsol=all)
```

$$y(x) = 2\,e^{5x} + e^{x}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 18

```
DSolve[{y''[x]-5*y'[x]+6*y[x]==0,{y[0]==3,y'[0]==11}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^{2x}(5e^{x} - 2)$$

## 9.21 problem 2(c)

Internal problem ID [6289]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 2(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 6y' + 9y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 5]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 11

```
dsolve([diff(y(x),x$2)-6*diff(y(x),x)+9*y(x)=0,y(0) = 0, D(y)(0) = 5],y(x), singsol=all)
```

$$y(x) = 5\,\mathrm{e}^{3x}x$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 13

```
DSolve[{y''[x]-6*y'[x]+9*y[x]==0,{y[0]==0,y'[0]==5}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to 5e^{3x}x$$

## 9.22 problem 2(d)

Internal problem ID [6290]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 2(d).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y' + 5y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve([diff(y(x),x$2)+4*diff(y(x),x)+5*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = e^{-2x}(2\sin(x) + \cos(x))$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 18

```
DSolve[{y''[x]+4*y'[x]+5*y[x]==0,{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to e^{-2x}(2\sin(x) + \cos(x))$$

## 9.23 problem 2(e)

Internal problem ID [6291]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 2(e).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y' + 2y = 0}$$

With initial conditions

$$\left[y(0) = -1, y'(0) = 2 + 3\sqrt{2}\right]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 24

```
dsolve([diff(y(x),x$2)+4*diff(y(x),x)+2*y(x)=0,y(0) = -1, D(y)(0) = 2+3*2^(1/2)],y(x), sings
```

$$y(x) = \mathrm{e}^{\left(-2+\sqrt{2}\right)x} - 2\,\mathrm{e}^{-\left(2+\sqrt{2}\right)x}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 30

```
DSolve[{y''[x]+4*y'[x]+2*y[x]==0,{y[0]==-1,y'[0]==2+3*Sqrt[2]}},y[x],x,IncludeSingularSoluti
```

$$y(x) \to e^{-\left(\left(2+\sqrt{2}\right)x\right)}\left(e^{2\sqrt{2}x} - 2\right)$$

## 9.24 problem 2(f)

Internal problem ID [6292]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 2(f).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 8y' - 9y = 0}$$

With initial conditions

$$[y(1) = 2, y'(1) = 0]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$2)+8*diff(y(x),x)-9*y(x)=0,y(1) = 2, D(y)(1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{e^{9-9x}}{5} + \frac{9\,e^{x-1}}{5}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 26

```
DSolve[{y''[x]+8*y'[x]-9*y[x]==0,{y[1]==2,y'[1]==0}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{1}{5}e^{9-9x} + \frac{9e^{x-1}}{5}$$

## 9.25 problem 5(a)

Internal problem ID [6293]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 5(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 3y'x + 10y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 25

```
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+10*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(3\ln(x))}{x} + \frac{c_2 \cos(3\ln(x))}{x}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 26

```
DSolve[x^2*y''[x]+3*x*y'[x]+10*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 \cos(3\log(x)) + c_1 \sin(3\log(x))}{x}$$

## 9.26 problem 5(b)

Internal problem ID [6294]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 5(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2x^2y'' + 10y'x + 8y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(2*x^2*diff(y(x),x$2)+10*x*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^2} + \frac{c_2 \ln(x)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 18

```
DSolve[2*x^2*y''[x]+10*x*y'[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_2 \log(x) + c_1}{x^2}$$

## 9.27 problem 5(c)

Internal problem ID [6295]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 5(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 2y'x - 12y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-12*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^3 + \frac{c_2}{x^4}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+2*x*y'[x]-12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^7 + c_1}{x^4}$$

## 9.28 problem 5(d)

Internal problem ID [6296]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 5(d).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{4x^2y'' - 3y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(4*x^2*diff(y(x),x$2)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^{\frac{3}{2}}c_1 + \frac{c_2}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 20

```
DSolve[4*x^2*y''[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^2 + c_1}{\sqrt{x}}$$

## 9.29 problem 5(e)

Internal problem ID [6297]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 5(e).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, ‘_with_symmetry_[0,F`

$$x^2y'' - 3y'x + 4y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^2 + c_2x^2 \ln(x)$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]-3*x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(2c_2 \log(x) + c_1)$$

## 9.30 problem 5(f)

Internal problem ID [6298]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 5(f).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' + 2y'x - 6y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-6*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^3} + c_2x^2$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+2*x*y'[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^5 + c_1}{x^3}$$

## 9.31 problem 5(g)

Internal problem ID [6299]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 5(g).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 2y'x + 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(x^2*diff(y(x),x$2)+2*x*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin\left(\frac{\sqrt{11}\,\ln(x)}{2}\right)}{\sqrt{x}} + \frac{c_2 \cos\left(\frac{\sqrt{11}\,\ln(x)}{2}\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 42

```
DSolve[x^2*y''[x]+2*x*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 \cos\left(\frac{1}{2}\sqrt{11}\log(x)\right) + c_1 \sin\left(\frac{1}{2}\sqrt{11}\log(x)\right)}{\sqrt{x}}$$

## 9.32 problem 5(h)

Internal problem ID [6300]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 5(h).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' + y'x - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{\sqrt{2}} + c_2 x^{-\sqrt{2}}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 28

```
DSolve[x^2*y''[x]+x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x^{-\sqrt{2}} + c_2 x^{\sqrt{2}}$$

## 9.33 problem 5(i)

Internal problem ID [6301]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62
**Problem number**: 5(i).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' + y'x - 16y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-16*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^4} + c_2x^4$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+x*y'[x]-16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^8 + c_1}{x^4}$$

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## 10.1 problem 1(a)

Internal problem ID [6302]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67
**Problem number**: 1(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 3y' - 10y = 6\,\mathrm{e}^{4x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)-10*y(x)=6*exp(4*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-5x}c_2 + \mathrm{e}^{2x}c_1 + \frac{\mathrm{e}^{4x}}{3}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 31

```
DSolve[y''[x]+3*y'[x]-10*y[x]==6*Exp[4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{4x}}{3} + c_1e^{-5x} + c_2e^{2x}$$

## 10.2 problem 1(b)

Internal problem ID [6303]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67
**Problem number**: 1(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = 3\sin(x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)+4*y(x)=3*sin(x),y(x), singsol=all)
```

$$y(x) = \sin(2x)\, c_2 + \cos(2x)\, c_1 + \sin(x)$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 22

```
DSolve[y''[x]+4*y[x]==3*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x) + c_1 \cos(2x) + c_2 \sin(2x)$$

## 10.3 problem 1(c)

Internal problem ID [6304]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67
**Problem number**: 1(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 10y' + 25y = 14\,\mathrm{e}^{-5x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+10*diff(y(x),x)+25*y(x)=14*exp(-5*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-5x}c_2 + \mathrm{e}^{-5x}xc_1 + 7x^2\mathrm{e}^{-5x}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 23

```
DSolve[y''[x]+10*y'[x]+25*y[x]==14*Exp[-5*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-5x}\left(7x^2 + c_2x + c_1\right)$$

## 10.4 problem 1(d)

Internal problem ID [6305]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67
**Problem number**: 1(d).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y' + 5y = 25x^2 + 12}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+5*y(x)=25*x^2+12,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x} \sin(2x) c_2 + \mathrm{e}^{x} \cos(2x) c_1 + 5x^2 + 4x + 2$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 35

```
DSolve[y''[x]-2*y'[x]+5*y[x]==25*x^2+12,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 5x^2 + 4x + c_2 e^x \cos(2x) + c_1 e^x \sin(2x) + 2$$

## 10.5 problem 1(e)

Internal problem ID [6306]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67
**Problem number**: 1(e).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y' - 6y = 20\,\mathrm{e}^{-2x}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-6*y(x)=20*exp(-2*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{3x} c_2 + \mathrm{e}^{-2x} c_1 - 4\,\mathrm{e}^{-2x} x$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 32

```
DSolve[y''[x]-y'[x]-6*y[x]==20*Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{5} e^{-2x} \left(-20x + 5c_2 e^{5x} - 4 + 5c_1\right)$$

## 10.6 problem 1(f)

Internal problem ID [6307]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67
**Problem number**: 1(f).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 3y' + 2y = 14\sin(2x) - 18\cos(2x)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=14*sin(2*x)-18*cos(2*x),y(x), singsol=all)
```

$$y(x) = e^{2x}c_1 + e^{x}c_2 + 2\sin(2x) + 3\cos(2x)$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 31

```
DSolve[y''[x]-3*y'[x]+2*y[x]==14*Sin[2*x]-18*Cos[2*x],y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to 2\sin(2x) + 3\cos(2x) + e^{x}(c_2e^{x} + c_1)$$

## 10.7 problem 1(g)

Internal problem ID [6308]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67
**Problem number**: 1(g).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = 2\cos(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+y(x)=2*cos(x),y(x), singsol=all)
```

$$y(x) = \sin(x)\, c_2 + \cos(x)\, c_1 + \sin(x)\, x$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 20

```
DSolve[y''[x]+y[x]==2*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (1 + c_1)\cos(x) + (x + c_2)\sin(x)$$

## 10.8 problem 1(h)

Internal problem ID [6309]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67
**Problem number**: 1(h).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - 2y' = 12x - 10}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)=12*x-10,y(x), singsol=all)
```

$$y(x) = \frac{e^{2x}c_1}{2} - 3x^2 + 2x + c_2$$

✓ Solution by Mathematica

Time used: 0.103 (sec). Leaf size: 27

```
DSolve[y''[x]-2*y'[x]==12*x-10,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -3x^2 + 2x + \frac{1}{2}c_1e^{2x} + c_2$$

## 10.9 problem 1(i)

Internal problem ID [6310]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67
**Problem number**: 1(i).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y' + y = 6\,\mathrm{e}^{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=6*exp(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x} c_2 + \mathrm{e}^{x} x c_1 + 3\,\mathrm{e}^{x} x^2$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 21

```
DSolve[y''[x]-2*y'[x]+y[x]==6*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x \left(3x^2 + c_2 x + c_1\right)$$

## 10.10 problem 1(j)

Internal problem ID [6311]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67
**Problem number**: 1(j).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + 2y = \sin(x)\,e^x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+2*y(x)=exp(x)*sin(x),y(x), singsol=all)
```

$$y(x) = e^x \sin(x)\,c_2 + e^x \cos(x)\,c_1 + \frac{e^x(\sin(x) - \cos(x)\,x)}{2}$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 28

```
DSolve[y''[x]-2*y'[x]+2*y[x]==Exp[x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}e^x((x - 2c_2)\cos(x) - 2c_1\sin(x))$$

## 10.11 problem 1(k)

Internal problem ID [6312]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67
**Problem number**: 1(k).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + y' = 10x^4 + 2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=10*x^4+2,y(x), singsol=all)
```

$$y(x) = -\mathrm{e}^{-x}c_1 - 120x^2 + 40x^3 - 10x^4 + 2x^5 + 242x + c_2$$

✓ Solution by Mathematica

Time used: 0.106 (sec). Leaf size: 40

```
DSolve[y''[x]+y'[x]==10*x^4+2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x^5 - 10x^4 + 40x^3 - 120x^2 + 242x - c_1e^{-x} + c_2$$

## 10.12 problem 3(a)

Internal problem ID [6313]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67
**Problem number**: 3(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = 4\cos(2x) + 6\cos(x) + 8x^2 - 4x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$2)+4*y(x)=4*cos(2*x)+6*cos(x)+8*x^2-4*x,y(x), singsol=all)
```

$$y(x) = \sin(2x)\, c_2 + \cos(2x)\, c_1 + x\sin(2x) + 2x^2 - 1 - x + 2\cos(x) + \frac{\cos(2x)}{4}$$

✓ Solution by Mathematica

Time used: 0.515 (sec). Leaf size: 43

```
DSolve[y''[x]+4*y[x]==4*Cos[2*x]+6*Cos[x]+8*x^2-4*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x^2 - x + x\sin(2x) + 2\cos(x) + \left(\frac{1}{2} + c_1\right)\cos(2x) + c_2\sin(2x) - 1$$

## 10.13 problem 3(b)

Internal problem ID [6314]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67
**Problem number**: 3(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 9y = 2\sin(3x) + 4\sin(x) - 26\,\mathrm{e}^{-2x} + 27x^3$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve(diff(y(x),x$2)+9*y(x)=2*sin(3*x)+4*sin(x)-26*exp(-2*x)+27*x^3,y(x), singsol=all)
```

$$y(x) = \sin(3x)\,c_2 + \cos(3x)\,c_1 - \frac{4x\cos(x)^3}{3} + 3x^3 + 2\sin(x)\cos(x)^2 + \cos(x)\,x - 2x - 2\,\mathrm{e}^{-2x}$$

✓ Solution by Mathematica

Time used: 2.215 (sec). Leaf size: 55

```
DSolve[y''[x]+9*y[x]==2*Sin[3*x]+4*Sin[x]-26*Exp[-2*x]+27*x^3,y[x],x,IncludeSingularSolution
```

$$y(x) \to 3x^3 - 2x - 2e^{-2x} + \frac{\sin(x)}{2} + \frac{1}{18}\sin(3x) + \left(-\frac{x}{3} + c_1\right)\cos(3x) + c_2\sin(3x)$$

## 10.14 problem 4(a)

Internal problem ID [6315]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67
**Problem number**: 4(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 3y = e^{2x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)-3*y(x)=exp(2*x),y(x), singsol=all)
```

$$y(x) = e^{\sqrt{3}\,x} c_2 + e^{-\sqrt{3}\,x} c_1 + e^{2x}$$

✓ Solution by Mathematica

Time used: 0.103 (sec). Leaf size: 36

```
DSolve[y''[x]-3*y[x]==Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x} + c_1 e^{\sqrt{3}x} + c_2 e^{-\sqrt{3}x}$$

## 10.15 problem 4(b)

Internal problem ID [6316]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67
**Problem number**: 4(b).
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + y' = \sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$3)+diff(y(x),x)=sin(x),y(x), singsol=all)
```

$$y(x) = -\cos(x) - \frac{\sin(x)\, x}{2} + c_1 \sin(x) - c_2 \cos(x) + c_3$$

✓ Solution by Mathematica

Time used: 0.118 (sec). Leaf size: 31

```
DSolve[y'''[x]+y'[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}(1 + 2c_2)\cos(x) + \left(-\frac{x}{2} + c_1\right)\sin(x) + c_3$$

# 11 Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71

## 11.1 problem 1(a)

Internal problem ID [6317]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71
**Problem number**: 1(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \tan(2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)+4*y(x)=tan(2*x),y(x), singsol=all)
```

$$y(x) = \sin(2x)\, c_2 + \cos(2x)\, c_1 - \frac{\cos(2x) \ln(\sec(2x) + \tan(2x))}{4}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 40

```
DSolve[y''[x]+4*y[x]==Tan[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{4} \cos(2x) \operatorname{arctanh}(\sin(2x)) + c_1 \cos(2x) + \frac{1}{4}(-1 + 4c_2) \sin(2x)$$

## 11.2 problem 1(b)

Internal problem ID [6318]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71
**Problem number**: 1(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + y = e^{-x} \ln(x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=exp(-x)*ln(x),y(x), singsol=all)
```

$$y(x) = c_2 e^{-x} + e^{-x} x c_1 + \frac{x^2(2\ln(x) - 3) e^{-x}}{4}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 36

```
DSolve[y''[x]+2*y'[x]+y[x]==Exp[-x]*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} e^{-x} \left(-3x^2 + 2x^2 \log(x) + 4c_2 x + 4c_1\right)$$

## 11.3 problem 1(c)

Internal problem ID [6319]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71
**Problem number**: 1(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' - 3y = 64x\,\mathrm{e}^{-x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-3*y(x)=64*x*exp(-x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{3x}c_2 + \mathrm{e}^{-x}c_1 + \left(-8x^2 - 4x\right)\mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 31

```
DSolve[y''[x]-2*y'[x]-3*y[x]==64*x*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(-8x^2 - 4x + c_2e^{4x} - 1 + c_1\right)$$

## 11.4 problem 1(d)

Internal problem ID [6320]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71
**Problem number**: 1(d).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + 5y = e^{-x} \sec(2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+5*y(x)=exp(-x)*sec(2*x),y(x), singsol=all)
```

$$y(x) = e^{-x} \sin(2x) c_2 + e^{-x} \cos(2x) c_1 - \frac{(\cos(2x) \ln(\sec(2x)) - 2x \sin(2x)) e^{-x}}{4}$$

✓ Solution by Mathematica

Time used: 0.068 (sec). Leaf size: 42

```
DSolve[y''[x]+2*y'[x]+5*y[x]==Exp[-x]*Sec[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} e^{-x} (2(x + 2c_1) \sin(2x) + \cos(2x)(\log(\cos(2x)) + 4c_2))$$

## 11.5 problem 1(e)

Internal problem ID [6321]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71
**Problem number**: 1(e).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2y'' + 3y' + y = e^{-3x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(2*diff(y(x),x$2)+3*diff(y(x),x)+y(x)=exp(-3*x),y(x), singsol=all)
```

$$y(x) = \frac{e^{-3x}}{10} - 2\,e^{-x}c_1 + c_2 e^{-\frac{x}{2}}$$

✓ Solution by Mathematica

Time used: 0.11 (sec). Leaf size: 45

```
DSolve[y''[x]+3*y'[x]+y[x]==Exp[-3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x} + c_1 e^{-\frac{1}{2}\left(3+\sqrt{5}\right)x} + c_2 e^{\frac{1}{2}\left(\sqrt{5}-3\right)x}$$

## 11.6 problem 1(f)

Internal problem ID [6322]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71
**Problem number**: 1(f).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 3y' + 2y = \frac{1}{1 + \mathrm{e}^{-x}}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=1/(1+exp(-x)),y(x), singsol=all)
```

$$y(x) = (\mathrm{e}^x c_1 - \ln(\mathrm{e}^x) - \ln(\mathrm{e}^x)\,\mathrm{e}^x + \ln(\mathrm{e}^x + 1)(\mathrm{e}^x + 1) - 1 + c_2)\,\mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.086 (sec). Leaf size: 34

```
DSolve[y''[x]-3*y'[x]+2*y[x]==1/(1+Exp[-x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x(2(e^x + 1)\,\text{arctanh}(2e^x + 1) + c_2 e^x - 1 + c_1)$$

## 11.7 problem 2(a)

Internal problem ID [6323]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71
**Problem number**: 2(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sec(x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+y(x)=sec(x),y(x), singsol=all)
```

$$y(x) = \sin(x)\, c_2 + \cos(x)\, c_1 + \sin(x)\, x - \ln(\sec(x))\cos(x)$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 22

```
DSolve[y''[x]+y[x]==Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_2)\sin(x) + \cos(x)(\log(\cos(x)) + c_1)$$

## 11.8 problem 2(b)

Internal problem ID [6324]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71
**Problem number**: 2(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \cot(x)^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+y(x)=cot(x)^2,y(x), singsol=all)
```

$$y(x) = \sin(x)\, c_2 + \cos(x)\, c_1 - 2 - \cos(x)\ln(\csc(x) - \cot(x))$$

✓ Solution by Mathematica

Time used: 0.075 (sec). Leaf size: 34

```
DSolve[y''[x]+y[x]==Cot[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \sin(x) + \cos(x)\left(-\log\left(\sin\left(\frac{x}{2}\right)\right) + \log\left(\cos\left(\frac{x}{2}\right)\right) + c_1\right) - 2$$

## 11.9 problem 2(c)

Internal problem ID [6325]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71
**Problem number**: 2(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \cot(2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)+y(x)=cot(2*x),y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 + \frac{\sin(x)\ln(\csc(x) - \cot(x))}{2} + \frac{\cos(x)\ln(\sec(x) + \tan(x))}{2}$$

✓ Solution by Mathematica

Time used: 0.082 (sec). Leaf size: 46

```
DSolve[y''[x]+y[x]==Cot[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\Big(\cos(x)\text{arctanh}(\sin(x)) + 2c_1\cos(x) + \sin(x)\left(\log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right) + 2c_2\right)\Big)$$

## 11.10 problem 2(d)

Internal problem ID [6326]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71
**Problem number**: 2(d).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \cos(x)\, x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+y(x)=x*cos(x),y(x), singsol=all)
```

$$y(x) = \sin(x)\, c_2 + \cos(x)\, c_1 + \frac{\cos(x)\, x}{4} + \frac{\sin(x)\, x^2}{4} - \frac{\sin(x)}{4}$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 34

```
DSolve[y''[x]+y[x]==x*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}\left(\left(2x^2 - 1 + 8c_2\right)\sin(x) + 2(x + 4c_1)\cos(x)\right)$$

## 11.11 problem 2(e)

Internal problem ID [6327]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71
**Problem number**: 2(e).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \tan(x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+y(x)=tan(x),y(x), singsol=all)
```

$$y(x) = \sin(x)\, c_2 + \cos(x)\, c_1 - \cos(x) \ln(\sec(x) + \tan(x))$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 23

```
DSolve[y''[x]+y[x]==Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x)(-\text{arctanh}(\sin(x))) + c_1 \cos(x) + c_2 \sin(x)$$

## 11.12 problem 2(f)

Internal problem ID [6328]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71
**Problem number**: 2(f).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sec(x)\tan(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+y(x)=sec(x)*tan(x),y(x), singsol=all)
```

$$y(x) = \sin(x)\,c_2 + \cos(x)\,c_1 + \ln(\sec(x))\sin(x) - \sin(x) + \cos(x)\,x$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 29

```
DSolve[y''[x]+y[x]==Sec[x]*Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x)\arctan(\tan(x)) + c_1\cos(x) + \sin(x)(-\log(\cos(x)) - 1 + c_2)$$

## 11.13 problem 2(g)

Internal problem ID [6329]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71
**Problem number**: 2(g).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sec(x)\csc(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)+y(x)=sec(x)*csc(x),y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 + \sin(x)\ln(\csc(x) - \cot(x)) - \cos(x)\ln(\sec(x) + \tan(x))$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 30

```
DSolve[y''[x]+y[x]==Sec[x]*Csc[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sin(x)\text{arctanh}(\cos(x)) + c_1\cos(x) + c_2\sin(x) + \cos(x)\left(-\coth^{-1}(\sin(x))\right)$$

## 11.14 problem 3

Internal problem ID [6330]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y' + y = 2x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=2*x,y(x), singsol=all)
```

$$y(x) = e^x c_2 + e^x x c_1 + 2x + 4$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 26

```
DSolve[y''[x]+2*y'[x]+y[x]==2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(2e^x(x-2) + c_2 x + c_1)$$

## 11.15 problem 4

Internal problem ID [6331]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y' - 6y = \mathrm{e}^{-x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-6*y(x)=exp(-x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{3x} c_2 + \mathrm{e}^{-2x} c_1 - \frac{\mathrm{e}^{-x}}{4}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 31

```
DSolve[y''[x]-y'[x]-6*y[x]==Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{-x}}{4} + c_1 e^{-2x} + c_2 e^{3x}$$

## 11.16 problem 5(a)

Internal problem ID [6332]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71
**Problem number**: 5(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-1)\,y''-2y'x+2y=(x^2-1)^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve((x^2-1)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=(x^2-1)^2,y(x), singsol=all)
```

$$y(x)=xc_2+(x^2+1)\,c_1+\frac{1}{2}+\frac{x^4}{6}$$

✓ Solution by Mathematica

Time used: 0.375 (sec). Leaf size: 111

```
DSolve[(x^2-1)*y''[x]-2*x*y'[x]+2*y[x]==(x^2-1)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\\ \to \frac{-x^6+4x^4-2x^3+3x^2\left(-1+2c_1\sqrt{-(x^2-1)^2}\right)+2x\left(-6c_1\sqrt{-(x^2-1)^2}+3c_2\sqrt{-(x^2-1)^2}+1\right)}{6-6x^2}$$

## 11.17 problem 5(b)

Internal problem ID [6333]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71
**Problem number**: 5(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{(x^2 + x)\, y'' + (-x^2 + 2)\, y' - (x + 2)\, y = x(1 + x)^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve((x^2+x)*diff(y(x),x$2)+(2-x^2)*diff(y(x),x)-(2+x)*y(x)=x*(x+1)^2,y(x), singsol=all)
```

$$y(x) = \frac{c_2}{x} + \mathrm{e}^x c_1 - \frac{x^2}{3} - x - 1$$

✓ Solution by Mathematica

Time used: 0.143 (sec). Leaf size: 45

```
DSolve[(x^2+x)*y''[x]+(2-x^2)*y'[x]-(2+x)*y[x]==x*(x+1)^2,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{x^2}{3} - x + \sqrt{2}c_2 e^{x+\frac{1}{2}} + \frac{c_1}{\sqrt{2e}x} - 1$$

## 11.18 problem 5(c)

Internal problem ID [6334]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71
**Problem number**: 5(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1-x)\,y''+y'x-y=(1-x)^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve((1-x)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=(1-x)^2,y(x), singsol=all)
```

$$y(x)=xc_2+e^x c_1+x^2+1$$

✓ Solution by Mathematica

Time used: 0.087 (sec). Leaf size: 22

```
DSolve[(1-x)*y''[x]+x*y'[x]-y[x]==(1-x)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x^2+x-c_2x+c_1e^x+1$$

## 11.19 problem 5(d)

Internal problem ID [6335]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71
**Problem number**: 5(d).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - y'(1+x) + y = x^2 \mathrm{e}^{2x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x$2)-(1+x)*diff(y(x),x)+y(x)=x^2*exp(2*x),y(x), singsol=all)
```

$$y(x) = (x+1)\, c_2 + \mathrm{e}^x c_1 + \frac{(x-1)\, \mathrm{e}^{2x}}{2}$$

✓ Solution by Mathematica

Time used: 0.066 (sec). Leaf size: 31

```
DSolve[x*y''[x]-(1+x)*y'[x]+y[x]==x^2*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{2x}(x-1) + c_1 e^x - c_2(x+1)$$

## 11.20 problem 5(e)

Internal problem ID [6336]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71
**Problem number**: 5(e).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2y'x + 2y = x\,\mathrm{e}^{-x}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=x*exp(-x),y(x), singsol=all)
```

$$y(x) = xc_2 + c_1x^2 + \left(-\mathrm{e}^{-x} + \mathrm{Ei}_1\left(x\right)(x+1)\right)x$$

✓ Solution by Mathematica

Time used: 0.073 (sec). Leaf size: 30

```
DSolve[x^2*y''[x]-2*x*y'[x]+2*y[x]==x*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(-(x+1)\,\mathrm{ExpIntegralEi}(-x) - e^{-x} + c_2x + c_1\right)$$

# 12 Chapter 2. Second-Order Linear Equations. Section 2.4. THE USE OF A KNOWN SOLUTION TO FIND ANOTHER. Page 74

## 12.1 problem 1(a)

Internal problem ID [6337]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.4. THE USE OF A KNOWN SOLUTION TO FIND ANOTHER. Page 74
**Problem number**: 1(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

Given that one solution of the ode is

$$y_1 = \sin(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve([diff(y(x),x$2)+y(x)=0,sin(x)],y(x), singsol=all)
```

$$y(x) = c_1 \sin(x) + c_2 \cos(x)$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 16

```
DSolve[y''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(x) + c_2 \sin(x)$$

## 12.2 problem 1(b)

Internal problem ID [6338]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.4. THE USE OF A KNOWN SOLUTION TO FIND ANOTHER. Page 74
**Problem number**: 1(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

Given that one solution of the ode is

$$y_1 = e^x$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve([diff(y(x),x$2)-y(x)=0,exp(x)],y(x), singsol=all)
```

$$y(x) = e^{-x}c_1 + e^x c_2$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 20

```
DSolve[y''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x + c_2 e^{-x}$$

## 12.3 problem 2

Internal problem ID [6339]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.4. THE USE OF A KNOWN SOLUTION TO FIND ANOTHER. Page 74
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' + 3y' = 0}$$

Given that one solution of the ode is

$$y_1 = 1$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve([x*diff(y(x),x$2)+3*diff(y(x),x)=0,1],y(x), singsol=all)
```

$$y(x) = c_1 + \frac{c_2}{x^2}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 17

```
DSolve[x*y''[x]+3*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{c_1}{2x^2}$$

## 12.4 problem 3

Internal problem ID [6340]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.4. THE USE OF A KNOWN SOLUTION TO FIND ANOTHER. Page 74
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' + y'x - 4y = 0}$$

Given that one solution of the ode is

$$y_1 = x^2$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=0,x^2],y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^2} + c_2x^2$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+x*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^4 + c_1}{x^2}$$

## 12.5 problem 4

Internal problem ID [6341]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.4. THE USE OF A KNOWN SOLUTION TO FIND ANOTHER. Page 74
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-x^2+1)\, y'' - 2y'x + 2y = 0}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 26

```
dsolve([(1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,x],y(x), singsol=all)
```

$$y(x) = c_1 x + c_2 \left(\frac{\ln(x-1)\, x}{2} - \frac{\ln(x+1)\, x}{2} + 1\right)$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 33

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x - \frac{1}{2} c_2 (x \log(1-x) - x \log(x+1) + 2)$$

## 12.6 problem 5

Internal problem ID [6342]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.4. THE USE OF A KNOWN SOLUTION TO FIND ANOTHER. Page 74
**Problem number**: 5.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(-\frac{1}{4} + x^2\right)y = 0}$$

Given that one solution of the ode is

$$y_1 = \frac{\sin(x)}{\sqrt{x}}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve([x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x)=0,x^(-1/2)*sin(x)],y(x), singsol=al
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2e^{2ix})}{2\sqrt{x}}$$

## 12.7 problem 6(a)

Internal problem ID [6343]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.4. THE USE OF A KNOWN SOLUTION TO FIND ANOTHER. Page 74
**Problem number**: 6(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{xy'}{x-1} + \frac{y}{x-1} = 0$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve([diff(y(x),x$2)-x/(x-1)*diff(y(x),x)+1/(x-1)*y(x)=0,x],y(x), singsol=all)
```

$$y(x) = c_1 x + e^x c_2$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]-x/(x-1)*x*y'[x]+1/(x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

## 12.8 problem 6(b)

Internal problem ID [6344]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.4. THE USE OF A KNOWN SOLUTION TO FIND ANOTHER. Page 74
**Problem number**: 6(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 2y'x - 2y = 0}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve([x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=0,x],y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^2} + xc_2$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]-x/(x-1)*x*y'[x]+1/(x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

## 12.9 problem 6(c)

Internal problem ID [6345]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.4. THE USE OF A KNOWN SOLUTION TO FIND ANOTHER. Page 74
**Problem number**: 6(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(x+2)\,y' + (x+2)\,y = 0}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve([x^2*diff(y(x),x$2)-x*(x+2)*diff(y(x),x)+(x+2)*y(x)=0,x],y(x), singsol=all)
```

$$y(x) = c_1x + c_2\mathrm{e}^x x$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 16

```
DSolve[x^2*y''[x]-x*(x+2)*y'[x]+(x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(c_2e^x + c_1)$$

## 12.10 problem 7

Internal problem ID [6346]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.4. THE USE OF A KNOWN SOLUTION TO FIND ANOTHER. Page 74
**Problem number**: 7.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - xf(x)\, y' + f(x)\, y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)-x*f(x)*diff(y(x),x)+f(x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \left( \int \mathrm{e}^{\int \frac{-2+f(x)x^2}{x}dx} dx \right) x + xc_2$$

✓ Solution by Mathematica

Time used: 0.287 (sec). Leaf size: 44

```
DSolve[y''[x]-x*f[x]*y'[x]+f[x]*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \left( c_2 \int_1^x \frac{\exp\left(-\int_1^{K[2]} -f(K[1])K[1]dK[1]\right)}{K[2]^2} dK[2] + c_1 \right)$$

## 12.11 problem 8

Internal problem ID [6347]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Second-Order Linear Equations. Section 2.4. THE USE OF A KNOWN SOLUTION TO FIND ANOTHER. Page 74
**Problem number**: 8.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (1+2x)\,y' + (1+x)\,y = 0}$$

Given that one solution of the ode is

$$y_1 = e^x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve([x*diff(y(x),x$2)-(2*x+1)*diff(y(x),x)+(x+1)*y(x)=0,exp(x)],y(x), singsol=all)
```

$$y(x) = e^x c_1 + c_2 x^2 e^x$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 23

```
DSolve[x*y''[x]-(2*x+1)*y'[x]+(x+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^x \left(c_2 x^2 + 2c_1\right)$$

# 13 Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98

## 13.1 problem 1

Internal problem ID [6348]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98
**Problem number**: 1.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 3y'' + 2y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)+2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + \mathrm{e}^x c_2 + c_3 \mathrm{e}^{2x}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 25

```
DSolve[y'''[x]-3*y''[x]+2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x + \frac{1}{2} c_2 e^{2x} + c_3$$

## 13.2 problem 2

Internal problem ID [6349]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98
**Problem number**: 2.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 3y'' + 4y' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)+4*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^x c_1 + e^x \sin(x) c_2 + c_3 e^x \cos(x)$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 22

```
DSolve[y'''[x]-3*y''[x]+4*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x(c_2 \cos(x) + c_1 \sin(x) + c_3)$$

## 13.3 problem 3

Internal problem ID [6350]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98
**Problem number**: 3.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$3)-y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{x}c_1 + c_2 e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right) + c_3 e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 52

```
DSolve[y'''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}\left(c_1 e^{3x/2} + c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_3 \sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

## 13.4 problem 4

Internal problem ID [6351]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98
**Problem number**: 4.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$3)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} c_1 + c_2 \mathrm{e}^{\frac{x}{2}} \sin\left(\frac{\sqrt{3}\, x}{2}\right) + c_3 \mathrm{e}^{\frac{x}{2}} \cos\left(\frac{\sqrt{3}\, x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 56

```
DSolve[y'''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_3 e^{3x/2} \cos\left(\frac{\sqrt{3}x}{2}\right) + c_2 e^{3x/2} \sin\left(\frac{\sqrt{3}x}{2}\right) + c_1\right)$$

## 13.5 problem 5

Internal problem ID [6352]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98
**Problem number**: 5.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 3y'' + 3y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$3)+3*diff(y(x),x$2)+3*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} c_1 + c_2 \mathrm{e}^{-x} x + c_3 \mathrm{e}^{-x} x^2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 23

```
DSolve[y'''[x]+3*y''[x]+3*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(x(c_3 x + c_2) + c_1)$$

## 13.6 problem 6

Internal problem ID [6353]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98
**Problem number**: 6.
**ODE order**: 4.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' + 4y''' + 6y'' + 4y' + y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$4)+4*diff(y(x),x$3)+6*diff(y(x),x$2)+4*diff(y(x),x)+y(x)=0,y(x), singsol=
```

$$y(x) = e^{-x}c_1 + c_2e^{-x}x + c_3e^{-x}x^2 + c_4e^{-x}x^3$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 28

```
DSolve[y''''[x]+4*y'''[x]+6*y''[x]+4*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(x(x(c_4x + c_3) + c_2) + c_1)$$

## 13.7 problem 7

Internal problem ID [6354]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98
**Problem number**: 7.
**ODE order**: 4.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$4)-y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-x} c_1 + e^x c_2 + c_3 \sin(x) + c_4 \cos(x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[y''''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x + c_3 e^{-x} + c_2 \cos(x) + c_4 \sin(x)$$

## 13.8 problem 8

Internal problem ID [6355]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98
**Problem number**: 8.
**ODE order**: 4.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 5y'' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$4)+5*diff(y(x),x$2)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(2x) + c_2 \cos(2x) + c_3 \sin(x) + c_4 \cos(x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[y''''[x]+5*y''[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(2x) + c_4 \sin(x) + \cos(x)(2c_2 \sin(x) + c_3)$$

## 13.9 problem 9

Internal problem ID [6356]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98
**Problem number**: 9.
**ODE order**: 4.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 2a^2y'' + a^4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$4)-2*a^2*diff(y(x),x$2)+a^4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{ax} + c_2 e^{ax} x + c_3 e^{-ax} + c_4 e^{-ax} x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 38

```
DSolve[y''''[x]-2*a^2*y''[x]+a^4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-ax}\left(c_3 e^{2ax} + x\left(c_4 e^{2ax} + c_2\right) + c_1\right)$$

## 13.10 problem 10

Internal problem ID [6357]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98
**Problem number**: 10.
**ODE order**: 4.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 2a^2y'' + a^4y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$4)+2*a^2*diff(y(x),x$2)+a^4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(ax) + c_2 \cos(ax) + c_3 \sin(ax)\, x + c_4 \cos(ax)\, x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[y''''[x]+2*a^2*y''[x]+a^4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (c_2x + c_1)\cos(ax) + (c_4x + c_3)\sin(ax)$$

## 13.11 problem 11

Internal problem ID [6358]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98
**Problem number**: 11.
**ODE order**: 4.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' + 2y''' + 2y'' + 2y' + y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$3)+2*diff(y(x),x$2)+2*diff(y(x),x)+y(x)=0,y(x), singsol=
```

$$y(x) = \mathrm{e}^{-x}c_1 + c_2\mathrm{e}^{-x}x + c_3 \sin(x) + c_4 \cos(x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 34

```
DSolve[y''''[x]+2*y'''[x]+2*y''[x]+2*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_4x + c_1e^x \cos(x) + c_2e^x \sin(x) + c_3)$$

## 13.12 problem 12

Internal problem ID [6359]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98
**Problem number**: 12.
**ODE order**: 4.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 2y''' - 2y'' - 6y' + 5y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$3)-2*diff(y(x),x$2)-6*diff(y(x),x)+5*y(x)=0,y(x), singso
```

$$y(x) = e^x c_1 + c_2 e^x x + c_3 e^{-2x} \sin(x) + c_4 e^{-2x} \cos(x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 35

```
DSolve[y''''[x]+2*y'''[x]-2*y''[x]-6*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to e^{-2x}\left(e^{3x}(c_4 x + c_3) + c_2 \cos(x) + c_1 \sin(x)\right)$$

## 13.13 problem 13

Internal problem ID [6360]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98
**Problem number**: 13.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 6y'' + 11y' - 6y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)-6*diff(y(x),x$2)+11*diff(y(x),x)-6*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{3x} + e^x c_2 + c_3 e^{2x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 25

```
DSolve[y'''[x]-6*y''[x]+11*y'[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x(e^x(c_3 e^x + c_2) + c_1)$$

## 13.14 problem 14

Internal problem ID [6361]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98
**Problem number**: 14.
**ODE order**: 4.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + y''' - 3y'' - 5y' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$4)+diff(y(x),x$3)-3*diff(y(x),x$2)-5*diff(y(x),x)-2*y(x)=0,y(x), singsol=
```

$$y(x) = \mathrm{e}^{2x}c_1 + c_2\mathrm{e}^{-x} + c_3\mathrm{e}^{-x}x + c_4\mathrm{e}^{-x}x^2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 32

```
DSolve[y''''[x]+y'''[x]-3*y''[x]-5*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_3x^2 + c_2x + c_4e^{3x} + c_1\right)$$

## 13.15 problem 15

Internal problem ID [6362]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98
**Problem number**: 15.
**ODE order**: 5.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(5)} - 6y'''' - 8y''' + 48y'' + 16y' - 96y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$5)-6*diff(y(x),x$4)-8*diff(y(x),x$3)+48*diff(y(x),x$2)+16*diff(y(x),x)-96
```

$$y(x) = c_1 e^{6x} + c_2 e^{-2x} + c_3 e^{-2x} x + c_4 e^{2x} + c_5 e^{2x} x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 43

```
DSolve[y'''''[x]-6*y''''[x]-8*y'''[x]+48*y''[x]+16*y'[x]-96*y[x]==0,y[x],x,IncludeSingularSo
```

$$y(x) \to e^{-2x}\left(c_2 x + c_3 e^{4x} + c_4 e^{4x} x + c_5 e^{8x} + c_1\right)$$

## 13.16 problem 16(a)

Internal problem ID [6363]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98
**Problem number**: 16(a).
**ODE order**: 4.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _quadrature]]`

$$\boxed{y'''' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$4)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{6}c_1 x^3 + \frac{1}{2}c_2 x^2 + c_3 x + c_4$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 22

```
DSolve[y''''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(x(c_4 x + c_3) + c_2) + c_1$$

## 13.17 problem 16(b)

Internal problem ID [6364]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98
**Problem number**: 16(b).
**ODE order**: 4.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _quadrature]]`

$$\boxed{y'''' = \sin(x) + 24}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$4)=sin(x)+24,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^3}{6} + x^4 + \frac{c_2 x^2}{2} + \sin(x) + c_3 x + c_4$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 29

```
DSolve[y''''[x]==Sin[x]+24,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^4 + c_4 x^3 + c_3 x^2 + \sin(x) + c_2 x + c_1$$

## 13.18 problem 17

Internal problem ID [6365]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98
**Problem number**: 17.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - 3y'' + 2y' = 10 + 42\,\mathrm{e}^{3x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)+2*diff(y(x),x)=10+42*exp(3*x),y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{2x} c_1}{2} + \mathrm{e}^{x} c_2 + 7\,\mathrm{e}^{3x} + 5x + c_3$$

✓ Solution by Mathematica

Time used: 0.102 (sec). Leaf size: 35

```
DSolve[y'''[x]-3*y''[x]+2*y'[x]==10+42*Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 5x + 7e^{3x} + c_1 e^{x} + \frac{1}{2} c_2 e^{2x} + c_3$$

## 13.19 problem 18

Internal problem ID [6366]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98
**Problem number**: 18.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - y' = 1}$$

With initial conditions

$$[y(0) = 4, y'(0) = 4, y''(0) = 4]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$3)-diff(y(x),x)=1,y(0) = 4, D(y)(0) = 4, (D@@2)(y)(0) = 4],y(x), singsol
```

$$y(x) = -\frac{e^{-x}}{2} + \frac{9\,e^{x}}{2} - x$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 25

```
DSolve[{y'''[x]-y'[x]==1,{y[0]==4,y'[0]==4,y''[0]==4}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -x - \frac{e^{-x}}{2} + \frac{9e^{x}}{2}$$

## 13.20 problem 19(a)

Internal problem ID [6367]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98
**Problem number**: 19(a).
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{x^3 y''' + 3x^2 y'' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x^3*diff(y(x),x$3)+3*x^2*diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = c_1 + \frac{c_2}{x} + c_3 x$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 21

```
DSolve[x^3*y'''[x]+3*x^2*y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{2x} + c_3 x + c_2$$

## 13.21 problem 19(b)

Internal problem ID [6368]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98
**Problem number**: 19(b).
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^3y''' + x^2y'' - 2y'x + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^3*diff(y(x),x$3)+x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + xc_2 + c_3x^2$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 22

```
DSolve[x^3*y'''[x]+x^2*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3x^2 + c_2x + \frac{c_1}{x}$$

## 13.22 problem 19(c)

Internal problem ID [6369]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98
**Problem number**: 19(c).
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3 y''' + 2x^2 y'' + y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^3*diff(y(x),x$3)+2*x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x + \sin(\ln(x))\, c_2 + c_3 \cos(\ln(x))$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 22

```
DSolve[x^3*y'''[x]+2*x^2*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3 x + c_1 \cos(\log(x)) + c_2 \sin(\log(x))$$

## 13.23 problem 20

Internal problem ID [6370]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98
**Problem number**: 20.
**ODE order**: 4.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{x^3y'''' + 8x^2y''' + 8xy'' - 8y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x^3*diff(y(x),x$4)+8*x^2*diff(y(x),x$3)+8*x*diff(y(x),x$2)-8*diff(y(x),x)=0,y(x), sin
```

$$y(x) = c_1 + \frac{c_2}{x} + \frac{c_3}{x^3} + c_4x^2$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 33

```
DSolve[x^3*y''''[x]+8*x^2*y'''[x]+8*x*y''[x]-8*y'[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to -\frac{c_1}{3x^3} + \frac{c_3x^2}{2} - \frac{c_2}{x} + c_4$$

# 14 Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105

## 14.1 problem 1(a)

Internal problem ID [6371]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105
**Problem number**: 1(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 3y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{(3+\sqrt{5})x}{2}} + c_2 \mathrm{e}^{-\frac{(\sqrt{5}-3)x}{2}}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 35

```
DSolve[y''[x]-3*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{1}{2}(\sqrt{5}-3)x} \left(c_2 e^{\sqrt{5}x} + c_1\right)$$

## 14.2 problem 1(b)

Internal problem ID [6372]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105
**Problem number**: 1(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' + y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\, x}{2}\right) + c_2 \mathrm{e}^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\, x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 42

```
DSolve[y''[x]+y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}\left(c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

## 14.3 problem 1(c)

Internal problem ID [6373]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105
**Problem number**: 1(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 6y' + 9y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)+6*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-3x} + c_2 e^{-3x} x$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 18

```
DSolve[y''[x]+6*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}(c_2 x + c_1)$$

## 14.4 problem 1(d)

Internal problem ID [6374]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105
**Problem number**: 1(d).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' + 6y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{x}{2}} \sin\left(\frac{\sqrt{23}\,x}{2}\right) + c_2 e^{\frac{x}{2}} \cos\left(\frac{\sqrt{23}\,x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 42

```
DSolve[y''[x]-y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x/2}\left(c_2 \cos\left(\frac{\sqrt{23}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{23}x}{2}\right)\right)$$

## 14.5 problem 1(e)

Internal problem ID [6375]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105
**Problem number**: 1(e).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y' - 5y = x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-5*y(x)=x,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\left(1+\sqrt{6}\right)x} c_2 + \mathrm{e}^{-\left(-1+\sqrt{6}\right)x} c_1 - \frac{x}{5} + \frac{2}{25}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 43

```
DSolve[y''[x]-2*y'[x]-5*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{5} + c_1 e^{x-\sqrt{6}x} + c_2 e^{\left(1+\sqrt{6}\right)x} + \frac{2}{25}$$

## 14.6 problem 1(f)

Internal problem ID [6376]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105
**Problem number**: 1(f).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y = e^x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+y(x)=exp(x),y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 + \frac{e^x}{2}$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 23

```
DSolve[y''[x]+y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x}{2} + c_1 \cos(x) + c_2 \sin(x)$$

## 14.7 problem 1(g)

Internal problem ID [6377]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105
**Problem number**: 1(g).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' + y = \sin(x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = c_2 e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right) + e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right) c_1 - \cos(x)$$

✓ Solution by Mathematica

Time used: 1.371 (sec). Leaf size: 53

```
DSolve[y''[x]+y'[x]+y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}\left(-e^{x/2}\cos(x) + c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

## 14.8 problem 1(h)

Internal problem ID [6378]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105
**Problem number**: 1(h).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y = e^{3x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-y(x)=exp(3*x),y(x), singsol=all)
```

$$y(x) = c_2 e^{-x} + e^{x} c_1 + \frac{e^{3x}}{8}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 29

```
DSolve[y''[x]-y[x]==Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{3x}}{8} + c_1 e^{x} + c_2 e^{-x}$$

## 14.9 problem 2(a)

Internal problem ID [6379]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105
**Problem number**: 2(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 9y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(x),x$2)+9*y(x)=0,y(0) = 1, D(y)(0) = 2],y(x), singsol=all)
```

$$y(x) = \frac{2\sin(3x)}{3} + \cos(3x)$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 18

```
DSolve[{y''[x]+9*y[x]==0,{y[0]==1,y'[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{3}\sin(3x) + \cos(3x)$$

## 14.10 problem 2(b)

Internal problem ID [6380]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105
**Problem number**: 2(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y' + 4y = x}$$

With initial conditions

$$[y(1) = 2, y'(1) = 1]$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 73

```
dsolve([diff(y(x),x$2)-diff(y(x),x)+4*y(x)=x,y(1) = 2, D(y)(1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(\sin\left(\frac{\sqrt{15}}{2}\right)\sqrt{15} + 135\cos\left(\frac{\sqrt{15}}{2}\right)\right)\cos\left(\frac{\sqrt{15}\,x}{2}\right) - \sin\left(\frac{\sqrt{15}\,x}{2}\right)\left(\sqrt{15}\cos\left(\frac{\sqrt{15}}{2}\right) - 135\sin\left(\frac{\sqrt{15}}{2}\right)\right)\right)e^{\frac{x}{2}-\frac{1}{2}}}{80} + \frac{x}{4} + \frac{1}{16}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 67

```
DSolve[{y''[x]-y'[x]+4*y[x]==x,{y[1]==2,y'[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{80}\left(20x - \sqrt{15}e^{\frac{x-1}{2}}\sin\left(\frac{1}{2}\sqrt{15}(x-1)\right) + 135e^{\frac{x-1}{2}}\cos\left(\frac{1}{2}\sqrt{15}(x-1)\right) + 5\right)$$

## 14.11 problem 2(c)

Internal problem ID [6381]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105
**Problem number**: 2(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y' + 5y = \mathrm{e}^{x}}$$

With initial conditions

$$[y(0) = -1, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve([diff(y(x),x$2)+2*diff(y(x),x)+5*y(x)=exp(x),y(0) = -1, D(y)(0) = 1],y(x), singsol=al
```

$$y(x) = \frac{(-9\cos(2x) - \sin(2x))\,\mathrm{e}^{-x}}{8} + \frac{\mathrm{e}^{x}}{8}$$

✓ Solution by Mathematica

Time used: 0.108 (sec). Leaf size: 32

```
DSolve[{y''[x]+2*y'[x]+5*y[x]==Exp[x],{y[0]==-1,y'[0]==1}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{1}{8}e^{-x}\left(e^{2x} - \sin(2x) - 9\cos(2x)\right)$$

## 14.12 problem 2(d)

Internal problem ID [6382]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105
**Problem number**: 2(d).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 3y' + 4y = \sin(x)}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = 1, y'\left(\frac{\pi}{2}\right) = -1\right]$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 95

```
dsolve([diff(y(x),x$2)+3*diff(y(x),x)+4*y(x)=sin(x),y(1/2*Pi) = 1, D(y)(1/2*Pi) = -1],y(x),
```

$$y(x) = \frac{\left(\left(\sqrt{7}\,\sin\left(\frac{\sqrt{7}\,x}{2}\right) + 35\cos\left(\frac{\sqrt{7}\,x}{2}\right)\right)\cos\left(\frac{\sqrt{7}\,\pi}{4}\right) - \sin\left(\frac{\sqrt{7}\,\pi}{4}\right)\left(\sqrt{7}\,\cos\left(\frac{\sqrt{7}\,x}{2}\right) - 35\sin\left(\frac{\sqrt{7}\,x}{2}\right)\right)\right)e^{-\frac{3x}{2}+\frac{3\pi}{4}}}{42} - \frac{\cos(x)}{6} + \frac{\sin(x)}{6}$$

✓ Solution by Mathematica

Time used: 1.46 (sec). Leaf size: 79

```
DSolve[{y''[x]+3*y'[x]+4*y[x]==Sin[x],{y[Pi/2]==1,y'[Pi/2]==-1}},y[x],x,IncludeSingularSolut
```

$$y(x) \to \frac{1}{42}\left(-\sqrt{7}e^{\frac{3}{4}(\pi-2x)}\sin\left(\frac{1}{4}\sqrt{7}(\pi-2x)\right)+7\sin(x)\right.$$
$$\left.+35e^{\frac{3}{4}(\pi-2x)}\cos\left(\frac{1}{4}\sqrt{7}(\pi-2x)\right)-7\cos(x)\right)$$

## 14.13 problem 2(e)

Internal problem ID [6383]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105
**Problem number**: 2(e).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y = \mathrm{e}^{-x}}$$

With initial conditions

$$[y(2) = 0, y'(2) = -2]$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 50

```
dsolve([diff(y(x),x$2)+y(x)=exp(-x),y(2) = 0, D(y)(2) = -2],y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{-x}}{2} + \frac{((-\cos(x) + \sin(x))\cos(2) - \cos(x)\sin(2) - \sin(x)\sin(2))\,\mathrm{e}^{-2}}{2} - 2\sin(x)\cos(2) + 2\cos(x)\sin(2)$$

✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 48

```
DSolve[{y''[x]+y[x]==Exp[-x],{y[2]==0,y'[2]==-2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x-2}\left(\left(4e^2 - 1\right)e^x\sin(2-x) - e^x\cos(2-x) + e^2\right)$$

## 14.14 problem 2(f)

Internal problem ID [6384]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105
**Problem number**: 2(f).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y = \cos(x)}$$

With initial conditions

$$[y(0) = 3, y'(2) = 2]$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 54

```
dsolve([diff(y(x),x$2)-y(x)=cos(x),y(0) = 3, D(y)(2) = 2],y(x), singsol=all)
```

$$y(x) = \frac{(\sin(2) - 4)\,\mathrm{e}^{-x+2} + 7\,\mathrm{e}^{-x+4} + (-\sin(2) + 4)\,\mathrm{e}^{x+2} + (-\mathrm{e}^4 - 1)\cos(x) + 7\,\mathrm{e}^x}{2\,\mathrm{e}^4 + 2}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 67

```
DSolve[{y''[x]-y[x]==Cos[x],{y[0]==3,y'[2]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}(7e^{2x} - e^{2x+2}(\sin(2) - 4) + (1 + e^4)(-e^x)\cos(x) + 7e^4 + e^2(\sin(2) - 4))}{2(1 + e^4)}$$

## 14.15 problem 2(g)

Internal problem ID [6385]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105
**Problem number**: 2(g).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = \tan(x)}$$

With initial conditions

$$[y(1) = 1, y'(1) = -1]$$

✓ Solution by Maple

Time used: 0.579 (sec). Leaf size: 135

```
dsolve([diff(y(x),x$2)=tan(x),y(1) = 1, D(y)(1) = -1],y(x), singsol=all)
```

$$y(x) = \frac{(-ie^{2i} - i)\,\mathrm{polylog}(2, -e^{2ix}) + 2x(1 + e^{2i})\ln(1 + e^{2ix}) + (ie^{2i} + i)\,\mathrm{polylog}(2, -e^{2i}) + (-2e^{2i} - 2)\ln(1}{}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 86

```
DSolve[{y''[x]==Tan[x],{y[1]==1,y'[1]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-i\,\mathrm{PolyLog}(2, -e^{2ix}) + i\,\mathrm{PolyLog}(2, -e^{2i}) - ix^2 - 2x + 2x\log(1 + e^{2ix}) - 2x\log(\cos(x)) + 2x\log(\cos(1)) + (4 + i) - 2\log(1 + e^{2i})\right)$$

## 14.16 problem 2(h)

Internal problem ID [6386]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105
**Problem number**: 2(h).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - 2y' = \ln(x)}$$

With initial conditions

$$[y(1) = \mathrm{e}, y'(1) = \mathrm{e}^{-1}]$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 42

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)=ln(x),y(1) = exp(1), D(y)(1) = 1/exp(1)],y(x), singsol
```

$$y(x) = \frac{\left(\int_1^x \left(-\mathrm{e}^{2\_z1}\,\mathrm{Ei}_1(2\_z1) + \mathrm{e}^{2\_z1}\,\mathrm{Ei}_1(2) - \ln(\_z1) + 2\,\mathrm{e}^{2\_z1-3}\right) d\_z1\right)}{2} + \mathrm{e}$$

✓ Solution by Mathematica

Time used: 0.101 (sec). Leaf size: 66

```
DSolve[{y''[x]-2*y'[x]==Log[x],{y[1]==Exp[1],y'[1]==1/Exp[1]}},y[x],x,IncludeSingularSolutio
```

$$y(x) \to \frac{1}{4}\left(e^{2x}\,\text{ExpIntegralEi}(-2x) - \text{ExpIntegralEi}(-2)e^{2x} + 2x + 2e^{2x-3} - 2x\log(x) - \log(-x) + i\pi + 4e - \frac{2}{e} - 2\right)$$

## 14.17 problem 3(a)

Internal problem ID [6387]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105
**Problem number**: 3(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 3y' + 2y = 2x - 1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=2*x-1,y(x), singsol=all)
```

$$y(x) = -\mathrm{e}^{-2x}c_1 + c_2\mathrm{e}^{-x} + x - 2$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 24

```
DSolve[y''[x]+3*y'[x]+2*y[x]==2*x-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + c_1e^{-2x} + c_2e^{-x} - 2$$

## 14.18 problem 3(b)

Internal problem ID [6388]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105
**Problem number**: 3(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 3y' + 2y = \mathrm{e}^{-x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=exp(-x),y(x), singsol=all)
```

$$y(x) = \left(\mathrm{e}^{x} c_1 + \frac{\mathrm{e}^{-2x}}{6} + c_2\right) \mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 29

```
DSolve[y''[x]-3*y'[x]+2*y[x]==Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}}{6} + c_1 e^{x} + c_2 e^{2x}$$

## 14.19 problem 3(c)

Internal problem ID [6389]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105
**Problem number**: 3(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y' - 2y = \cos(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-2*y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = c_2 \mathrm{e}^{-x} + \mathrm{e}^{2x} c_1 - \frac{3\cos(x)}{10} - \frac{\sin(x)}{10}$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 34

```
DSolve[y''[x]-y'[x]-2*y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sin(x)}{10} - \frac{3\cos(x)}{10} + c_1 e^{-x} + c_2 e^{2x}$$

## 14.20 problem 3(d)

Internal problem ID [6390]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105
**Problem number**: 3(d).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' - y = \sin(x)\,e^{x} x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 49

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)-y(x)=x*exp(x)*sin(x),y(x), singsol=all)
```

$$y(x) = e^{\left(\sqrt{2}-1\right)x} c_2 + e^{-\left(1+\sqrt{2}\right)x} c_1 + \frac{e^{x}\left(17\sin(x)\,x - 68\cos(x)\,x + 44\sin(x) + 62\cos(x)\right)}{289}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 59

```
DSolve[y''[x]+2*y'[x]-y[x]==x*Exp[x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\left(\left(1+\sqrt{2}\right)x\right)} + c_2 e^{\left(\sqrt{2}-1\right)x} + \frac{1}{289} e^{x}\left((17x+44)\sin(x) + (62-68x)\cos(x)\right)$$

## 14.21 problem 3(e)

Internal problem ID [6391]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105
**Problem number**: 3(e).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 9y = \sec(2x)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

```
dsolve(diff(y(x),x$2)+9*y(x)=sec(2*x),y(x), singsol=all)
```

$$y(x) = \sin(3x)\, c_2 + \cos(3x)\, c_1 - \frac{2}{3} + \frac{\sin(x)\left(-4\cos(x)^2 + 1\right)\sqrt{2}\,\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)}{6} + \frac{\left(-4\cos(x)^3 + 3\cos(x)\right)\sqrt{2}\,\operatorname{arctanh}\left(\cos(x)\sqrt{2}\right)}{6} + \frac{4\cos(x)^2}{3}$$

✓ Solution by Mathematica

Time used: 0.25 (sec). Leaf size: 102

```
DSolve[y''[x]+9*y[x]==Sec[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}\left(-\sqrt{2}\sin(3x)\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right) - \sqrt{2}\cos(3x)\operatorname{arctanh}\left(\sqrt{2} - \tan\left(\frac{x}{2}\right)\right) - \sqrt{2}\cos(3x)\operatorname{arctanh}\left(\tan\left(\frac{x}{2}\right) + \sqrt{2}\right) + 4\cos(2x) + 6c_1\cos(3x) + 6c_2\sin(3x)\right)$$

## 14.22 problem 3(f)

Internal problem ID [6392]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105
**Problem number**: 3(f).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 4y = \ln(x)\, x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+4*y(x)=x*ln(x),y(x), singsol=all)
```

$$y(x) = c_2 \mathrm{e}^{-2x} + \mathrm{e}^{-2x} x c_1 - \frac{\mathrm{e}^{-2x}(x+1)\,\mathrm{Ei}_1(-2x)}{4} - \frac{3}{8} + \frac{(2x-2)\ln(x)}{8}$$

✓ Solution by Mathematica

Time used: 0.09 (sec). Leaf size: 52

```
DSolve[y''[x]+4*y'[x]+4*y[x]==x*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8} e^{-2x} \left(2(x+1)\,\mathrm{ExpIntegralEi}(2x) - 3e^{2x} + 2e^{2x}(x-1)\log(x) + 8c_2 x + 8c_1\right)$$

## 14.23 problem 3(g)

Internal problem ID [6393]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105
**Problem number**: 3(g).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + 3y'x + y = \frac{2}{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+y(x)=2/x,y(x), singsol=all)
```

$$y(x) = \frac{\ln(x)\, c_1}{x} + \frac{\ln(x)^2}{x} + \frac{c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 21

```
DSolve[x^2*y''[x]+3*x*y'[x]+y[x]==2/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\log^2(x) + c_2 \log(x) + c_1}{x}$$

## 14.24 problem 3(h)

Internal problem ID [6394]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105
**Problem number**: 3(h).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \tan(x)^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$2)+4*y(x)=tan(x)^2,y(x), singsol=all)
```

$$y(x) = \sin(2x)\, c_2 + \cos(2x)\, c_1 + \left(2\cos(x)^2 - 1\right)\ln(\cos(x)) + 2\cos(x)\sin(x)\, x - \frac{3\sin(x)^2}{2}$$

✓ Solution by Mathematica

Time used: 0.091 (sec). Leaf size: 32

```
DSolve[y''[x]+4*y[x]==Tan[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_2)\sin(2x) + \cos(2x)\left(\log(\cos(x)) + \frac{1}{4} + c_1\right) - \frac{3}{4}$$

## 14.25 problem 4(a)

Internal problem ID [6395]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105
**Problem number**: 4(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y = 3\,e^{2x}}$$

Given that one solution of the ode is

$$y_1 = e^{2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$2)-y(x)=3*exp(2*x),exp(2*x)],y(x), singsol=all)
```

$$y(x) = c_2 e^{-x} + e^{x} c_1 + e^{2x}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 25

```
DSolve[y''[x]-y[x]==3*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x} + c_1 e^{x} + c_2 e^{-x}$$

## 14.26 problem 4(b)

Internal problem ID [6396]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105
**Problem number**: 4(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = -8\sin(3x)}$$

Given that one solution of the ode is

$$y_1 = \sin(3x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)+y(x)=-8*sin(3*x),sin(3*x)],y(x), singsol=all)
```

$$y(x) = \sin(x)\, c_2 + \cos(x)\, c_1 + \sin(3x)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 20

```
DSolve[y''[x]+y[x]==-8*Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(3x) + c_1 \cos(x) + c_2 \sin(x)$$

## 14.27 problem 4(c)

Internal problem ID [6397]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105
**Problem number**: 4(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' + y = x^2 + 2x + 2}$$

Given that one solution of the ode is

$$y_1 = x^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve([diff(y(x),x$2)+diff(y(x),x)+y(x)=x^2+2*x+2,x^2],y(x), singsol=all)
```

$$y(x) = c_2 \mathrm{e}^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\, x}{2}\right) + \mathrm{e}^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\, x}{2}\right) c_1 + x^2$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 51

```
DSolve[y''[x]+y'[x]+y[x]==x^2+2*x+2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + c_2 e^{-x/2} \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 e^{-x/2} \sin\left(\frac{\sqrt{3}x}{2}\right)$$

## 14.28 problem 4(d)

Internal problem ID [6398]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Drill excercises. Page 105
**Problem number**: 4(d).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + y' = \frac{x-1}{x}}$$

Given that one solution of the ode is

$$y_1 = \ln(x)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve([diff(y(x),x$2)+diff(y(x),x)=(x-1)/x,ln(x)],y(x), singsol=all)
```

$$y(x) = \int \left(1 + \mathrm{e}^{-x} \operatorname{Ei}_1(-x) + \mathrm{e}^{-x} c_1\right) dx + c_2$$

✓ Solution by Mathematica

Time used: 0.12 (sec). Leaf size: 30

```
DSolve[y''[x]+y'[x]==(x-1)/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x} \text{ExpIntegralEi}(x) + x - \log(x) - c_1 e^{-x} + c_2$$

# 15 Chapter 2. Problems for Review and Discovery. Challenge excercises. Page 105

## 15.1 problem 3

Internal problem ID [6399]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Challenge excercises. Page 105
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' - 2y'x + 2y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 9

```
dsolve([x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(0) = 0, D(y)(0) = 0],y(x), singsol=al
```

$$y(x) = c_2x^2$$

✓ Solution by Mathematica

Time used: 0.077 (sec). Leaf size: 11

```
DSolve[{x^2*y''[x]-2*x*y'[x]+2*y[x]==0,{y[0]==0,y'[0]==0}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to c_2x^2$$

## 15.2 problem 4

Internal problem ID [6400]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Challenge excercises. Page 105
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = -3\cos(2x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+9*y(x)=-3*cos(2*x),y(x), singsol=all)
```

$$y(x) = \sin(3x) c_2 + \cos(3x) c_1 - \frac{3\cos(2x)}{5}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 28

```
DSolve[y''[x]+9*y[x]==-3*Cos[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{3}{5}\cos(2x) + c_1\cos(3x) + c_2\sin(3x)$$

# 16 Chapter 2. Problems for Review and Discovery. Problems for Discussion and Exploration. Page 105

## 16.1 problem 1

Internal problem ID [6401]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Problems for Discussion and Exploration. Page 105
**Problem number**: 1.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = \cos(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)+y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = \frac{\cos(x)}{2} + \frac{\sin(x)}{2} + \mathrm{e}^{-x} c_1$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 23

```
DSolve[y'[x]+y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(\sin(x) + \cos(x) + 2c_1 e^{-x}\right)$$

## 16.2 problem 2

Internal problem ID [6402]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Problems for Discussion and Exploration. Page 105
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 3y = 0}$$

With initial conditions

$$[y(0) = -1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve([diff(y(x),x$2)=-3*y(x),y(0) = -1],y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(\sqrt{3}\, x\right) - \cos\left(\sqrt{3}\, x\right)$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 27

```
DSolve[{y''[x]==-3*y[x],{y[0]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\cos\left(\sqrt{3}x\right) + c_2 \sin\left(\sqrt{3}x\right)$$

## 16.3 problem 4

Internal problem ID [6403]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 2. Problems for Review and Discovery. Problems for Discussion and Exploration. Page 105
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' + \sin(y) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
dsolve(diff(y(x),x$2)+sin(y(x))=0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{\sqrt{2\cos(\_a) + c_1}} d\_a - x - c_2 = 0$$
$$\int^{y(x)} -\frac{1}{\sqrt{2\cos(\_a) + c_1}} d\_a - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 3.86 (sec). Leaf size: 69

```
DSolve[y''[x]+Sin[y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2\,\text{JacobiAmplitude}\left(\frac{1}{2}\sqrt{(c_1+2)(x+c_2)^2}, \frac{4}{c_1+2}\right)$$
$$y(x) \to 2\,\text{JacobiAmplitude}\left(\frac{1}{2}\sqrt{(c_1+2)(x+c_2)^2}, \frac{4}{c_1+2}\right)$$

# 17 Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162

## 17.1 problem 1(a) solving using series

Internal problem ID [6404]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162
**Problem number**: 1(a) solving using series.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{-2yx + y' = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
Order:=8;
dsolve(diff(y(x),x)=2*x*y(x),y(x),type='series',x=0);
```

$$y(x) = \left(1 + x^2 + \frac{1}{2}x^4 + \frac{1}{6}x^6\right) y(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 25

```
AsymptoticDSolveValue[y'[x]==2*x*y[x],y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{x^6}{6} + \frac{x^4}{2} + x^2 + 1\right)$$

## 17.2 problem 1(a) solving directly

Internal problem ID [6405]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162
**Problem number**: 1(a) solving directly.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{-2yx + y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=2*x*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x^2}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 18

```
DSolve[y'[x]==2*x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x^2}$$
$$y(x) \to 0$$

## 17.3 problem 1(b) solving using series

Internal problem ID [6406]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162
**Problem number**: 1(b) solving using series.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + y = 1}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 60

```
Order:=8;
dsolve(diff(y(x),x)+y(x)=1,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + \frac{1}{720}x^6 - \frac{1}{5040}x^7\right) y(0) + x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{24} + \frac{x^5}{120} - \frac{x^6}{720} + \frac{x^7}{5040} + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 97

```
AsymptoticDSolveValue[y'[x]+y[x]==1,y[x],{x,0,7}]
```

$$y(x) \to \frac{x^7}{5040} - \frac{x^6}{720} + \frac{x^5}{120} - \frac{x^4}{24} + \frac{x^3}{6} - \frac{x^2}{2} + c_1\left(-\frac{x^7}{5040} + \frac{x^6}{720} - \frac{x^5}{120} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} - x + 1\right) + x$$

## 17.4 problem 1(b) solving directly

Internal problem ID [6407]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162
**Problem number**: 1(b) solving directly.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + y = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)+y(x)=1,y(x), singsol=all)
```

$$y(x) = 1 + \mathrm{e}^{-x} c_1$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 20

```
DSolve[y'[x]+y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 + c_1 e^{-x}$$
$$y(x) \to 1$$

## 17.5 problem 1(c) solving using series

Internal problem ID [6408]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162
**Problem number**: 1(c) solving using series.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y = 2}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 58

```
Order:=8;
dsolve(diff(y(x),x)-y(x)=2,y(x),type='series',x=0);
```

$$y(x) = \left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \frac{1}{5040}x^7\right) y(0) + 2x + x^2 + \frac{x^3}{3} + \frac{x^4}{12} + \frac{x^5}{60} + \frac{x^6}{360} + \frac{x^7}{2520} + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 93

```
AsymptoticDSolveValue[y'[x]-y[x]==2,y[x],{x,0,7}]
```

$$y(x) \to \frac{x^7}{2520} + \frac{x^6}{360} + \frac{x^5}{60} + \frac{x^4}{12} + \frac{x^3}{3} + x^2 + c_1\left(\frac{x^7}{5040} + \frac{x^6}{720} + \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1\right) + 2x$$

## 17.6 problem 1(c) solving directly

Internal problem ID [6409]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162
**Problem number**: 1(c) solving directly.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y = 2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)-y(x)=2,y(x), singsol=all)
```

$$y(x) = -2 + e^{x} c_1$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 18

```
DSolve[y'[x]-y[x]==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2 + c_1 e^{x}$$
$$y(x) \to -2$$

## 17.7 problem 1(d) solving using series

Internal problem ID [6410]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162
**Problem number**: 1(d) solving using series.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 46

```
Order:=8;
dsolve(diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + \frac{1}{720}x^6 - \frac{1}{5040}x^7\right) y(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 53

```
AsymptoticDSolveValue[y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{x^7}{5040} + \frac{x^6}{720} - \frac{x^5}{120} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} - x + 1\right)$$

## 17.8 problem 1(d) solving directly

Internal problem ID [6411]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162
**Problem number**: 1(d) solving directly.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} c_1$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 18

```
DSolve[y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x}$$
$$y(x) \to 0$$

## 17.9 problem 1(e) solving using series

Internal problem ID [6412]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162
**Problem number**: 1(e) solving using series.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;
dsolve(diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \frac{1}{5040}x^7\right) y(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 51

```
AsymptoticDSolveValue[y'[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{x^7}{5040} + \frac{x^6}{720} + \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1\right)$$

## 17.10 problem 1(e) solving directly

Internal problem ID [6413]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162
**Problem number**: 1(e) solving directly.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = e^x c_1$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 16

```
DSolve[y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x$$
$$y(x) \to 0$$

## 17.11 problem 1(f) solving using series

Internal problem ID [6414]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162
**Problem number**: 1(f) solving using series.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - y = x^2}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=8;
dsolve(diff(y(x),x)-y(x)=x^2,y(x),type='series',x=0);
```

$$y(x) = \left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \frac{1}{5040}x^7\right) y(0) + \frac{x^3}{3} + \frac{x^4}{12} + \frac{x^5}{60} + \frac{x^6}{360} + \frac{x^7}{2520} + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 87

```
AsymptoticDSolveValue[y'[x]-y[x]==x^2,y[x],{x,0,7}]
```

$$y(x) \to \frac{x^7}{2520} + \frac{x^6}{360} + \frac{x^5}{60} + \frac{x^4}{12} + \frac{x^3}{3} + c_1\left(\frac{x^7}{5040} + \frac{x^6}{720} + \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1\right)$$

## 17.12 problem 1(f) solving directly

Internal problem ID [6415]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162
**Problem number**: 1(f) solving directly.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - y = x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)-y(x)=x^2,y(x), singsol=all)
```

$$y(x) = -x^2 - 2x - 2 + e^x c_1$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 21

```
DSolve[y'[x]-y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^2 - 2x + c_1 e^x - 2$$

## 17.13 problem 2(a) solving using series

Internal problem ID [6416]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162
**Problem number**: 2(a) solving using series.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
Order:=8;
dsolve(x*diff(y(x),x)=y(x),y(x),type='series',x=0);
```

$$y(x) = c_1x + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 7

```
AsymptoticDSolveValue[x*y'[x]==y[x],y[x],{x,0,7}]
```

$$y(x) \to c_1x$$

## 17.14 problem 2(a) solving directly

Internal problem ID [6417]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162
**Problem number**: 2(a) solving directly.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 7

```
dsolve(x*diff(y(x),x)=y(x),y(x), singsol=all)
```

$$y(x) = c_1 x$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 14

```
DSolve[x*y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x$$
$$y(x) \to 0$$

## 17.15 problem 2(b) solving using series

Internal problem ID [6418]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162
**Problem number**: 2(b) solving using series.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=8;
dsolve(x^2*diff(y(x),x)=y(x),y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 13

```
AsymptoticDSolveValue[x^2*y'[x]==y[x],y[x],{x,0,7}]
```

$$y(x) \to c_1 e^{-1/x}$$

## 17.16 problem 2(b) solving directly

Internal problem ID [6419]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162
**Problem number**: 2(b) solving directly.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x^2*diff(y(x),x)=y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{1}{x}}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 20

```
DSolve[x^2*y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-1/x}$$
$$y(x) \to 0$$

## 17.17 problem 2(c) solving using series

Internal problem ID [6420]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162
**Problem number**: 2(c) solving using series.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{y}{x} = x^2}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
Order:=8;
dsolve(diff(y(x),x)-(1/x)*y(x)=x^2,y(x),type='series',x=0);
```

$$y(x) = c_1 x \left(1 + \mathrm{O}\left(x^8\right)\right) + x^3 \left(\frac{1}{2} + \mathrm{O}\left(x^5\right)\right)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 15

```
AsymptoticDSolveValue[y'[x]-1/x*y[x]==x^2,y[x],{x,0,7}]
```

$$y(x) \to \frac{x^3}{2} + c_1 x$$

## 17.18 problem 2(c) solving directly

Internal problem ID [6421]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162
**Problem number**: 2(c) solving directly.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{y}{x} = x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)-(1/x)*y(x)=x^2,y(x), singsol=all)
```

$$y(x) = \left(\frac{x^2}{2} + c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 17

```
DSolve[y'[x]-1/x*y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{2} + c_1 x$$

## 17.19 problem 2(d) solving using series

Internal problem ID [6422]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162
**Problem number**: 2(d) solving using series.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{y}{x} = x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+(1/x)*y(x)=x,y(x), singsol=all)
```

$$y(x) = \frac{\frac{x^3}{3} + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 19

```
DSolve[y'[x]+1/x*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{3} + \frac{c_1}{x}$$

## 17.20 problem 3

Internal problem ID [6423]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162
**Problem number**: 3.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{1}{\sqrt{-x^2+1}}}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
Order:=8;
dsolve(diff(y(x),x)=(1-x^2)^(-1/2),y(x),type='series',x=0);
```

$$y(x) = y(0) + x + \frac{x^3}{6} + \frac{3x^5}{40} + \frac{5x^7}{112} + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y'[x]==(1-x^2)^(-1/2),y[x],{x,0,7}]
```

$$y(x) \to \frac{5x^7}{112} + \frac{3x^5}{40} + \frac{x^3}{6} + x + c_1$$

## 17.21 problem 4

Internal problem ID [6424]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162
**Problem number**: 4.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y = 1}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 58

```
Order:=8;
dsolve(diff(y(x),x)=1+y(x),y(x),type='series',x=0);
```

$$y(x) = \left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \frac{1}{5040}x^7\right) y(0) + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{5040} + O\left(x^8\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 95

```
AsymptoticDSolveValue[y'[x]==1+y[x],y[x],{x,0,7}]
```

$$y(x) \to \frac{x^7}{5040} + \frac{x^6}{720} + \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + c_1\left(\frac{x^7}{5040} + \frac{x^6}{720} + \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1\right) + x$$

## 17.22 problem 5 solved using series

Internal problem ID [6425]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162
**Problem number**: 5 solved using series.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + y = x}$$

With initial conditions

$$[y(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
Order:=8;
dsolve([diff(y(x),x)=x-y(x),y(0) = 0],y(x),type='series',x=0);
```

$$y(x) = \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + \frac{1}{720}x^6 - \frac{1}{5040}x^7 + \mathrm{O}\left(x^8\right)$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 46

```
AsymptoticDSolveValue[{y'[x]==x-y[x],{y[0]==0}},y[x],{x,0,7}]
```

$$y(x) \to -\frac{x^7}{5040} + \frac{x^6}{720} - \frac{x^5}{120} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2}$$

## 17.23 problem 5 solved directly

Internal problem ID [6426]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162
**Problem number**: 5 solved directly.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = x}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve([diff(y(x),x)=x-y(x),y(0) = 0],y(x), singsol=all)
```

$$y(x) = x - 1 + e^{-x}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 13

```
DSolve[{y'[x]==x-y[x],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + e^{-x} - 1$$

# 18 Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169

## 18.1 problem 1(a)

Internal problem ID [6427]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169
**Problem number**: 1(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + y'x + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{8}x^4 - \frac{1}{48}x^6\right) y(0) + \left(x - \frac{1}{3}x^3 + \frac{1}{15}x^5 - \frac{1}{105}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[y''[x]+x*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(-\frac{x^7}{105} + \frac{x^5}{15} - \frac{x^3}{3} + x\right) + c_1\left(-\frac{x^6}{48} + \frac{x^4}{8} - \frac{x^2}{2} + 1\right)$$

## 18.2 problem 1(b)

Internal problem ID [6428]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169
**Problem number**: 1(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y' + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 69

```
Order:=8;
dsolve(diff(y(x),x$2)-diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{6}x^3 - \frac{1}{24}x^4 - \frac{1}{120}x^5 + \frac{1}{240}x^6 + \frac{1}{630}x^7\right)y(0) + \left(x + \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{24}x^4 - \frac{1}{30}x^5 - \frac{1}{90}x^6 - \frac{1}{1680}x^7\right)D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 91

```
AsymptoticDSolveValue[y''[x]-y'[x]+x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{x^7}{630} + \frac{x^6}{240} - \frac{x^5}{120} - \frac{x^4}{24} - \frac{x^3}{6} + 1\right) + c_2\left(-\frac{x^7}{1680} - \frac{x^6}{90} - \frac{x^5}{30} - \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x\right)$$

## 18.3 problem 1(c)

Internal problem ID [6429]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169
**Problem number**: 1(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y'x - y = x}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
Order:=8;
dsolve(diff(y(x),x$2)+2*x*diff(y(x),x)-y(x)=x,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 - \frac{1}{8}x^4 + \frac{7}{240}x^6\right) y(0) + \left(x - \frac{1}{6}x^3 + \frac{1}{24}x^5 - \frac{1}{112}x^7\right) D(y)(0) + \frac{x^3}{6} - \frac{x^5}{24} + \frac{x^7}{112} + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 77

```
AsymptoticDSolveValue[y''[x]+2*x*y'[x]-y[x]==x,y[x],{x,0,7}]
```

$$y(x) \to \frac{x^7}{112} - \frac{x^5}{24} + \frac{x^3}{6} + c_2\left(-\frac{x^7}{112} + \frac{x^5}{24} - \frac{x^3}{6} + x\right) + c_1\left(\frac{7x^6}{240} - \frac{x^4}{8} + \frac{x^2}{2} + 1\right)$$

## 18.4 problem 1(d)

Internal problem ID [6430]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169
**Problem number**: 1(d).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' - yx^2 = 1}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 72

```
Order:=8;
dsolve(diff(y(x),x$2)+diff(y(x),x)-x^2*y(x)=1,y(x),type='series',x=0);
```

$$\begin{aligned}y(x) = &\left(1 + \frac{1}{12}x^4 - \frac{1}{60}x^5 + \frac{1}{360}x^6 - \frac{1}{2520}x^7\right)y(0)\\&+\left(x - \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{24}x^4 + \frac{7}{120}x^5 - \frac{19}{720}x^6 + \frac{13}{1680}x^7\right)D(y)(0)\\&+\frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} - \frac{x^5}{120} + \frac{13x^6}{720} - \frac{11x^7}{1680} + O(x^8)\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 126

```
AsymptoticDSolveValue[y''[x]+y'[x]+x^2*y[x]==1,y[x],{x,0,7}]
```

$$y(x) \to \frac{31x^7}{5040} - \frac{11x^6}{720} - \frac{x^5}{120} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} + c_1\left(\frac{x^7}{2520} - \frac{x^6}{360} + \frac{x^5}{60} - \frac{x^4}{12} + 1\right) + c_2\left(-\frac{37x^7}{5040} + \frac{17x^6}{720} - \frac{x^5}{24} - \frac{x^4}{24} + \frac{x^3}{6} - \frac{x^2}{2} + x\right)$$

## 18.5 problem 1(e)

Internal problem ID [6431]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169
**Problem number**: 1(e).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{(x^2+1)\,y''+y'x+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
Order:=8;
dsolve((1+x^2)*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{1}{2}x^2+\frac{5}{24}x^4-\frac{17}{144}x^6\right)y(0)+\left(x-\frac{1}{3}x^3+\frac{1}{6}x^5-\frac{13}{126}x^7\right)D(y)(0)+O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[(1+x^2)*y''[x]+x*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_2\left(-\frac{13x^7}{126}+\frac{x^5}{6}-\frac{x^3}{3}+x\right)+c_1\left(-\frac{17x^6}{144}+\frac{5x^4}{24}-\frac{x^2}{2}+1\right)$$

## 18.6 problem 1(f)

Internal problem ID [6432]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169
**Problem number**: 1(f).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'(1 + x) - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 64

```
Order:=8;
dsolve(diff(y(x),x$2)+(1+x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{60}x^5 - \frac{1}{360}x^6 - \frac{1}{840}x^7\right) y(0) + \left(x - \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{60}x^5 + \frac{1}{360}x^6 + \frac{1}{840}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 84

```
AsymptoticDSolveValue[y''[x]+(1+x)*y'[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{x^7}{840} - \frac{x^6}{360} + \frac{x^5}{60} - \frac{x^3}{6} + \frac{x^2}{2} + 1\right) + c_2\left(\frac{x^7}{840} + \frac{x^6}{360} - \frac{x^5}{60} + \frac{x^3}{6} - \frac{x^2}{2} + x\right)$$

## 18.7 problem 2

Internal problem ID [6433]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''+2y'x-2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
Order:=8;
dsolve((1+x^2)*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+x^2-\frac{1}{3}x^4+\frac{1}{5}x^6\right)y(0)+D(y)(0)\,x+O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 30

```
AsymptoticDSolveValue[(1+x^2)*y''[x]+2*x*y'[x]-2*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_1\left(\frac{x^6}{5}-\frac{x^4}{3}+x^2+1\right)+c_2x$$

## 18.8 problem 3

Internal problem ID [6434]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + y'x + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{8}x^4 - \frac{1}{48}x^6\right) y(0) + \left(x - \frac{1}{3}x^3 + \frac{1}{15}x^5 - \frac{1}{105}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[y''[x]+x*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(-\frac{x^7}{105} + \frac{x^5}{15} - \frac{x^3}{3} + x\right) + c_1\left(-\frac{x^6}{48} + \frac{x^4}{8} - \frac{x^2}{2} + 1\right)$$

## 18.9 problem 4(a)

Internal problem ID [6435]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169
**Problem number**: 4(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' - yx = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
Order:=8;
dsolve([diff(y(x),x$2)+diff(y(x),x)-x*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x),type='series',x=0);
```

$$y(x) = 1 + \frac{1}{6}x^3 - \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{240}x^6 - \frac{1}{630}x^7 + \mathrm{O}\left(x^8\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[{y''[x]+y'[x]-x*y[x]==0,{y[0]==1,y'[0]==0}},y[x],{x,0,7}]
```

$$y(x) \to -\frac{x^7}{630} + \frac{x^6}{240} + \frac{x^5}{120} - \frac{x^4}{24} + \frac{x^3}{6} + 1$$

## 18.10 problem 4(b)

Internal problem ID [6436]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169
**Problem number**: 4(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' - yx = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
Order:=8;
dsolve([diff(y(x),x$2)+diff(y(x),x)-x*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x),type='series',x=0);
```

$$y(x) = x - \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{30}x^5 + \frac{1}{90}x^6 - \frac{1}{1680}x^7 + \mathrm{O}\left(x^8\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 47

```
AsymptoticDSolveValue[{y''[x]+y'[x]-x*y[x]==0,{y[0]==0,y'[0]==1}},y[x],{x,0,7}]
```

$$y(x) \to -\frac{x^7}{1680} + \frac{x^6}{90} - \frac{x^5}{30} + \frac{x^4}{24} + \frac{x^3}{6} - \frac{x^2}{2} + x$$

## 18.11 problem 5

Internal problem ID [6437]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169
**Problem number**: 5.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(p + \frac{1}{2} - \frac{x^2}{4}\right) y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 120

```
Order:=8;
dsolve(diff(y(x),x$2)+(p+1/2-x^2/4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{(2p+1)\,x^2}{4} + \frac{(4p^2+4p+3)\,x^4}{96} - \frac{(8p^3+12p^2+34p+15)\,x^6}{5760}\right) y(0) + \left(x - \frac{(2p+1)\,x^3}{12} + \frac{(4p^2+4p+7)\,x^5}{480} - \frac{(8p^3+12p^2+58p+27)\,x^7}{40320}\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 142

```
AsymptoticDSolveValue[y''[x]+(p+1/2-x^2/4)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(\frac{(-4p-2)(4p+2)^2x^7}{322560} + \frac{13(-4p-2)x^7}{40320} + \frac{(4p+2)^2x^5}{1920} + \frac{1}{24}(-4p-2)x^3 + \frac{x^5}{80} + x\right) + c_1\left(\frac{(-4p-2)(4p+2)^2x^6}{46080} + \frac{7(-4p-2)x^6}{5760} + \frac{1}{384}(4p+2)^2x^4 + \frac{1}{8}(-4p-2)x^2 + \frac{x^4}{48} + 1\right)$$

## 18.12 problem 6

Internal problem ID [6438]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169
**Problem number**: 6.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
Order:=8;
dsolve(diff(y(x),x$2)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{6}x^3 + \frac{1}{180}x^6\right) y(0) + \left(x - \frac{1}{12}x^4 + \frac{1}{504}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]+x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(\frac{x^7}{504} - \frac{x^4}{12} + x\right) + c_1\left(\frac{x^6}{180} - \frac{x^3}{6} + 1\right)$$

## 18.13 problem 7

Internal problem ID [6439]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169
**Problem number**: 7.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Gegenbauer, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']]`

$$\boxed{(-x^2+1)\,y''-y'x+p^2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 121

```
Order:=8;
dsolve((1-x^2)*diff(y(x),x$2)-x*diff(y(x),x)+p^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{p^2x^2}{2}+\frac{p^2(p^2-4)\,x^4}{24}-\frac{p^2(p^4-20p^2+64)\,x^6}{720}\right)y(0)+\left(x-\frac{(p^2-1)\,x^3}{6}+\frac{(p^4-10p^2+9)\,x^5}{120}-\frac{(p^6-35p^4+259p^2-225)\,x^7}{5040}\right)D(y)\,(0)+O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 155

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-x*y'[x]+p^2*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_2\left(-\frac{p^6x^7}{5040}+\frac{p^4x^7}{144}+\frac{p^4x^5}{120}-\frac{37p^2x^7}{720}-\frac{p^2x^5}{12}-\frac{p^2x^3}{6}+\frac{5x^7}{112}+\frac{3x^5}{40}+\frac{x^3}{6}+x\right)+c_1\left(-\frac{1}{720}p^6x^6+\frac{p^4x^6}{36}+\frac{p^4x^4}{24}-\frac{4p^2x^6}{45}-\frac{p^2x^4}{6}-\frac{p^2x^2}{2}+1\right)$$

## 18.14 problem 8

Internal problem ID [6440]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169
**Problem number**: 8.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'x + 2yp = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 109

```
Order:=8;
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+2*p*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - p\,x^2 + \frac{p(p-2)\,x^4}{6} - \frac{p(p-2)\,(p-4)\,x^6}{90}\right) y(0) + \left(x - \frac{(p-1)\,x^3}{3} + \frac{(p^2-4p+3)\,x^5}{30} - \frac{(p^3-9p^2+23p-15)\,x^7}{630}\right) D(y)\,(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 141

```
AsymptoticDSolveValue[y''[x]-2*x*y'[x]+2*p*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(-\frac{1}{630}p^3x^7 + \frac{p^2x^7}{70} + \frac{p^2x^5}{30} - \frac{23px^7}{630} - \frac{2px^5}{15} - \frac{px^3}{3} + \frac{x^7}{42} + \frac{x^5}{10} + \frac{x^3}{3} + x\right) + c_1\left(-\frac{1}{90}p^3x^6 + \frac{p^2x^6}{15} + \frac{p^2x^4}{6} - \frac{4px^6}{45} - \frac{px^4}{3} - px^2 + 1\right)$$

# 19 Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175

## 19.1 problem 1(a)

Internal problem ID [6441]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175
**Problem number**: 1(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^3(x-1)\,y''-2(x-1)\,y'+3yx=0}$$

With the expansion point for the power series method at $x=0$.

✗ Solution by Maple

```
Order:=8;
dsolve(x^3*(x-1)*diff(y(x),x$2)-2*(x-1)*diff(y(x),x)+3*x*y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 108

```
AsymptoticDSolveValue[x^3*(x-1)*y''[x]-2*(x-1)*y'[x]+3*x*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_2e^{-\frac{1}{x^2}}\left(\frac{1731x^7}{320}-\frac{795x^6}{128}-\frac{51x^5}{40}+\frac{63x^4}{32}+\frac{x^3}{2}-\frac{3x^2}{4}+1\right)x^3+c_1\left(-\frac{51x^7}{320}-\frac{19x^6}{128}-\frac{9x^5}{40}-\frac{9x^4}{32}-\frac{x^3}{2}-\frac{3x^2}{4}+1\right)$$

## 19.2 problem 1(b)

Internal problem ID [6442]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175
**Problem number**: 1(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x^2-1)\,y''-x(1-x)\,y'+2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 654

```
Order:=8;
dsolve(x^2*(x^2-1)*diff(y(x),x$2)-x*(1-x)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1x^{-\sqrt{2}}\left(1+\frac{\sqrt{2}}{-1+2\sqrt{2}}x+\frac{\sqrt{2}}{-5+3\sqrt{2}}x^2+\frac{6\sqrt{2}-8}{57\sqrt{2}-81}x^3+\frac{-49\sqrt{2}+69}{1104-780\sqrt{2}}x^4+\frac{293\sqrt{2}-414}{6108\sqrt{2}-8640}x^5+\frac{-2757\sqrt{2}+3898}{114408-80892\sqrt{2}}x^6+\frac{1}{126}\frac{77567\sqrt{2}-109686}{(-1+2\sqrt{2})(\sqrt{2}-1)(-3+2\sqrt{2})(-2+\sqrt{2})(-5+2\sqrt{2})(-3+\sqrt{2})(-7+2\sqrt{2})}x^7+\mathrm{O}(x^8)\right)+c_2x^{\sqrt{2}}\left(1+\frac{\sqrt{2}}{1+2\sqrt{2}}x+\frac{\sqrt{2}}{5+3\sqrt{2}}x^2+\frac{6\sqrt{2}+8}{57\sqrt{2}+81}x^3+\frac{49\sqrt{2}+69}{1104+780\sqrt{2}}x^4+\frac{293\sqrt{2}+414}{6108\sqrt{2}+8640}x^5+\frac{2757\sqrt{2}+3898}{114408+80892\sqrt{2}}x^6+\frac{1}{126}\frac{77567\sqrt{2}+109686}{(1+2\sqrt{2})(1+\sqrt{2})(3+2\sqrt{2})(2+\sqrt{2})(5+2\sqrt{2})(3+\sqrt{2})(7+2\sqrt{2})}x^7+\mathrm{O}(x^8)\right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 10352

```
AsymptoticDSolveValue[x^2*(x^2-1)*y''[x]-x*(1-x)*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

Too large to display

## 19.3 problem 1(c)

Internal problem ID [6443]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175
**Problem number**: 1(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x^2y'' + (-x+2)\,y' = 0}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+(2-x)*diff(y(x),x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 64

```
AsymptoticDSolveValue[x^2*y''[x]+(2-x)*y'[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2 e^{2/x}\left(\frac{2835x^7}{2} + 315x^6 + \frac{315x^5}{4} + \frac{45x^4}{2} + \frac{15x^3}{2} + 3x^2 + \frac{3x}{2} + 1\right)x^3 + c_1$$

## 19.4 problem 1(d)

Internal problem ID [6444]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175
**Problem number**: 1(d).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(3x+1)\,xy'' - y'(1+x) + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 72

```
Order:=8;
dsolve((3*x+1)*x*diff(y(x),x$2)-(x+1)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^2\left(1 - 2x + \frac{17}{4}x^2 - \frac{289}{30}x^3 + \frac{5491}{240}x^4 - \frac{236113}{4200}x^5 + \frac{28569673}{201600}x^6 - \frac{28569673}{78400}x^7 + \mathrm{O}\left(x^8\right)\right) + c_2\left(\ln\left(x\right)\left(2x^2 - 4x^3 + \frac{17}{2}x^4 - \frac{289}{15}x^5 + \frac{5491}{120}x^6 - \frac{236113}{2100}x^7 + \mathrm{O}\left(x^8\right)\right) + \left(-2 - 4x + 6x^2 - 12x^3 + \frac{209}{8}x^4 - \frac{54247}{900}x^5 + \frac{521849}{3600}x^6 - \frac{158526173}{441000}x^7 + \mathrm{O}\left(x^8\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.421 (sec). Leaf size: 118

```
AsymptoticDSolveValue[(3*x+1)*x*y''[x]-(x+1)*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 \left( \frac{27353x^6 - 12886x^5 + 6525x^4 - 3600x^3 + 1800x^2 + 7200x + 3600}{3600} - \frac{1}{240} x^2 \left(5491x^4 - 2312x^3 + 1020x^2 - 480x + 240\right) \log(x) \right) + c_2 \left( \frac{28569673x^8}{201600} - \frac{236113x^7}{4200} + \frac{5491x^6}{240} - \frac{289x^5}{30} + \frac{17x^4}{4} - 2x^3 + x^2 \right)$$

## 19.5 problem 2(a)

Internal problem ID [6445]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175
**Problem number**: 2(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \sin(x)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 49

```
Order:=8;
dsolve(diff(y(x),x$2)+sin(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{6}x^3 + \frac{1}{120}x^5 + \frac{1}{180}x^6 - \frac{1}{5040}x^7\right) y(0) + \left(x - \frac{1}{12}x^4 + \frac{1}{180}x^6 + \frac{1}{504}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 63

```
AsymptoticDSolveValue[y''[x]+Sin[x]*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(\frac{x^7}{504} + \frac{x^6}{180} - \frac{x^4}{12} + x\right) + c_1\left(-\frac{x^7}{5040} + \frac{x^6}{180} + \frac{x^5}{120} - \frac{x^3}{6} + 1\right)$$

## 19.6 problem 2(b)

Internal problem ID [6446]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175
**Problem number**: 2(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + \sin(x)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;
dsolve(x*diff(y(x),x$2)+sin(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{18}x^4 - \frac{53}{10800}x^6\right) y(0) + \left(x - \frac{1}{6}x^3 + \frac{1}{60}x^5 - \frac{19}{15120}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[x*y''[x]+Sin[x]*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(-\frac{19x^7}{15120} + \frac{x^5}{60} - \frac{x^3}{6} + x\right) + c_1\left(-\frac{53x^6}{10800} + \frac{x^4}{18} - \frac{x^2}{2} + 1\right)$$

## 19.7 problem 2(c)

Internal problem ID [6447]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175
**Problem number**: 2(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \sin(x)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 70

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+sin(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x\left(1 - \frac{1}{2}x + \frac{1}{12}x^2 + \frac{1}{144}x^3 - \frac{13}{2880}x^4 + \frac{29}{86400}x^5 + \frac{431}{3628800}x^6 - \frac{4961}{203212800}x^7 + \mathrm{O}\left(x^8\right)\right) + c_2\left(\ln(x)\left(-x + \frac{1}{2}x^2 - \frac{1}{12}x^3 - \frac{1}{144}x^4 + \frac{13}{2880}x^5 - \frac{29}{86400}x^6 - \frac{431}{3628800}x^7 + \mathrm{O}\left(x^8\right)\right) + \left(1 - \frac{3}{4}x^2 + \frac{2}{9}x^3 - \frac{25}{1728}x^4 - \frac{689}{86400}x^5 + \frac{263}{162000}x^6 + \frac{71809}{762048000}x^7 + \mathrm{O}\left(x^8\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 121

```
AsymptoticDSolveValue[x^2*y''[x]+Sin[x]*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{2539x^6 - 16185x^5 - 9750x^4 + 396000x^3 - 1620000x^2 + 1296000x + 1296000}{1296000} - \frac{x(29x^5 - 390x^4 + 600x^3 + 7200x^2 - 43200x + 86400)\log(x)}{86400}\right) + c_2\left(\frac{431x^7}{3628800} + \frac{29x^6}{86400} - \frac{13x^5}{2880} + \frac{x^4}{144} + \frac{x^3}{12} - \frac{x^2}{2} + x\right)$$

## 19.8 problem 2(d)

Internal problem ID [6448]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175
**Problem number**: 2(d).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^3y'' + \sin(x)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 427

```
Order:=8;
dsolve(x^3*diff(y(x),x$2)+sin(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x}\left(c_2 x^{\frac{i\sqrt{3}}{2}}\left(1 + \frac{1}{12i\sqrt{3}+24}x^2 - \frac{1}{1440}\frac{3i\sqrt{3}+1}{(i\sqrt{3}+4)\,(i\sqrt{3}+2)}x^4 + \frac{1}{362880}\frac{9i\sqrt{3}-115}{(i\sqrt{3}+6)\,(i\sqrt{3}+4)\,(i\sqrt{3}+2)}x^6 + \mathrm{O}\left(x^8\right)\right) + c_1 x^{-\frac{i\sqrt{3}}{2}}\left(1 - \frac{1}{12i\sqrt{3}-24}x^2 + \frac{1}{1440}\frac{3i\sqrt{3}-1}{(i\sqrt{3}-4)\,(-2+i\sqrt{3})}x^4 + \frac{1}{362880}\frac{9i\sqrt{3}+115}{(i\sqrt{3}-6)\,(i\sqrt{3}-4)\,(-2+i\sqrt{3})}x^6 + \mathrm{O}\left(x^8\right)\right)\right)$$

### ✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 410

```
AsymptoticDSolveValue[x^3*y''[x]+Sin[x]*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 \left( \frac{\left( \frac{1}{5040} - \frac{1}{720(1+(1-(-1)^{2/3})(2-(-1)^{2/3}))} + \frac{\frac{1}{36(1+(1-(-1)^{2/3})(2-(-1)^{2/3}))} - \frac{1}{120}}{6(1+(3-(-1)^{2/3})(4-(-1)^{2/3}))} \right) x^6}{1+(5-(-1)^{2/3})(6-(-1)^{2/3})} + \frac{\left( \frac{1}{36(1+(1-(-1)^{2/3})(2-(-1)^{2/3}))} - \frac{1}{120} \right) x^4}{1+(3-(-1)^{2/3})(4-(-1)^{2/3})} + \frac{x^2}{6(1+(1-(-1)^{2/3})(2-(-1)^{2/3}))} + 1 \right) x^{-(-1)^{2/3}} + c_2 \left( \frac{\left( \frac{1}{5040} - \frac{1}{720\left(1+\left(1+\sqrt[3]{-1}\right)\left(2+\sqrt[3]{-1}\right)\right)} + \frac{\frac{1}{36\left(1+\left(1+\sqrt[3]{-1}\right)\left(2+\sqrt[3]{-1}\right)\right)} - \frac{1}{120}}{6\left(1+\left(3+\sqrt[3]{-1}\right)\left(4+\sqrt[3]{-1}\right)\right)} \right) x^6}{1+\left(5+\sqrt[3]{-1}\right)\left(6+\sqrt[3]{-1}\right)} + \right.$$

## 19.9 problem 2(e)

Internal problem ID [6449]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175
**Problem number**: 2(e).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^4y'' + \sin(x)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=8;
dsolve(x^4*diff(y(x),x$2)+sin(x)*y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.11 (sec). Leaf size: 294

```
AsymptoticDSolveValue[x^4*y''[x]+Sin[x]*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 e^{-\frac{2i}{\sqrt{x}}} x^{3/4} \left( \frac{164874841524774786597462 23ix^{13/2}}{277358326363269177016 3200} - \frac{459493414836473518369 3ix^{11/2}}{632001394707970129920 0} + \frac{12579783586699513ix^{9/2}}{96185277197844480} - \frac{21896783401ix^{7/2}}{579820584960} + \frac{856783ix^{5/2}}{41943040} - \frac{3151ix^{3/2}}{73728} - \frac{39862632689408275722559635 29x^7}{20709421701790765217218560 0} + \frac{217307128883566287417723 37x^6}{109209841005537238450176 00} - \frac{1500040357444099007x^5}{5129881450551705600} + \frac{4885269094757x^4}{74217034874880} - \frac{2835642457x^3}{108716359680} + \frac{11659x^2}{524288} + \frac{15x}{512} - \frac{3i\sqrt{x}}{16} + 1 \right) + c_2 e^{\frac{2i}{\sqrt{x}}} x^{3/4} \left( -\frac{164874841524774786597462 23ix^{13/2}}{277358326363269177016 3200} + \frac{459493414836473518369 3ix^{11/2}}{632001394707970129920 0} - \frac{12579783\ldots}{961852\ldots} \right.$$

## 19.10 problem 3(a)

Internal problem ID [6450]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175
**Problem number**: 3(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^3y'' + (-1 + \cos(2x))\, y' + 2yx = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 37

```
Order:=8;
dsolve(x^3*diff(y(x),x$2)+(cos(2*x)-1)*diff(y(x),x)+2*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^2\left(1 - \frac{2}{9}x^2 + \frac{26}{675}x^4 - \frac{1742}{297675}x^6 + \mathrm{O}\left(x^8\right)\right) + c_2x\left(1 - \frac{1}{3}x^2 + \frac{17}{270}x^4 - \frac{173}{17010}x^6 + \mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 74

```
AsymptoticDSolveValue[x^3*y''[x]+(Cos[2*x]-1)*y'[x]+2*x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(\frac{32351x^8}{40186125} - \frac{1742x^6}{297675} + \frac{26x^4}{675} - \frac{2x^2}{9} + 1\right)x^2 + c_1\left(\frac{10471x^8}{7144200} - \frac{173x^6}{17010} + \frac{17x^4}{270} - \frac{x^2}{3} + 1\right)x$$

## 19.11 problem 3(b)

Internal problem ID [6451]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175
**Problem number**: 3(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + (2x^4 - 5x)\, y' + (3x^2 + 2)\, y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
Order:=8;
dsolve(4*x^2*diff(y(x),x$2)+(2*x^4-5*x)*diff(y(x),x)+(3*x^2+2)*y(x)=0,y(x),type='series',x=0
```

$$y(x) = c_1 x^{\frac{1}{4}} \left(1 - \frac{3}{2}x^2 - \frac{1}{30}x^3 + \frac{1}{8}x^4 + \frac{137}{1300}x^5 - \frac{19}{12240}x^6 - \frac{7169}{764400}x^7 + \mathrm{O}\left(x^8\right)\right) + c_2 x^2 \left(1 - \frac{1}{10}x^2 - \frac{4}{57}x^3 + \frac{3}{920}x^4 + \frac{32}{4275}x^5 + \frac{36287}{9753840}x^6 - \frac{4037}{16059750}x^7 + \mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 106

```
AsymptoticDSolveValue[4*x^2*y''[x]+(2*x^4-5*x)*y'[x]+(3*x^2+2)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 \left(-\frac{4037x^7}{16059750} + \frac{36287x^6}{9753840} + \frac{32x^5}{4275} + \frac{3x^4}{920} - \frac{4x^3}{57} - \frac{x^2}{10} + 1\right) x^2 + c_2 \left(-\frac{7169x^7}{764400} - \frac{19x^6}{12240} + \frac{137x^5}{1300} + \frac{x^4}{8} - \frac{x^3}{30} - \frac{3x^2}{2} + 1\right) \sqrt[4]{x}$$

## 19.12 problem 3(c)

Internal problem ID [6452]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175
**Problem number**: 3(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^2y'' + 3y'x + 4yx = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 74

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+4*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 - \frac{4}{3}x + \frac{2}{3}x^2 - \frac{8}{45}x^3 + \frac{4}{135}x^4 - \frac{16}{4725}x^5 + \frac{4}{14175}x^6 - \frac{16}{893025}x^7 + \mathrm{O}\left(x^8\right)\right)x^2 + c_2\left(\ln\left(x\right)\left(16x^2 - \frac{64}{3}x^3 + \right.\right.}{}$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 116

```
AsymptoticDSolveValue[x^2*y''[x]+3*x*y'[x]+4*x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(\frac{4x^6}{14175} - \frac{16x^5}{4725} + \frac{4x^4}{135} - \frac{8x^3}{45} + \frac{2x^2}{3} - \frac{4x}{3} + 1\right) + c_1\left(\frac{1696x^6 - 8976x^5 + 27900x^4 - 39600x^3 + 8100x^2 + 8100x + 2025}{2025x^2} - \frac{8}{135}\left(4x^4 - 24x^3 + 90x^2 - 180x + 135\right)\log(x)\right)$$

## 19.13 problem 3(d)

Internal problem ID [6453]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175
**Problem number**: 3(d).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^3 y'' - 4x^2 y' + 3yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
Order:=8;
dsolve(x^3*diff(y(x),x$2)-4*x^2*diff(y(x),x)+3*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x^{\frac{5}{2}} \left( x^{-\frac{\sqrt{13}}{2}} c_1 + x^{\frac{\sqrt{13}}{2}} c_2 \right) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 38

```
AsymptoticDSolveValue[x^3*y''[x]-4*x^2*y'[x]+3*x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 x^{\frac{1}{2}\left(5+\sqrt{13}\right)} + c_2 x^{\frac{1}{2}\left(5-\sqrt{13}\right)}$$

## 19.14 problem 4(a)

Internal problem ID [6454]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175
**Problem number**: 4(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{4y''x + 3y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 52

```
Order:=8;
dsolve(4*x*diff(y(x),x$2)+3*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{1}{4}} \left(1 - \frac{1}{5}x + \frac{1}{90}x^2 - \frac{1}{3510}x^3 + \frac{1}{238680}x^4 - \frac{1}{25061400}x^5 + \frac{1}{3759210000}x^6 - \frac{1}{763119630000}x^7 + \mathrm{O}\left(x^8\right)\right) + c_2 \left(1 - \frac{1}{3}x + \frac{1}{42}x^2 - \frac{1}{1386}x^3 + \frac{1}{83160}x^4 - \frac{1}{7900200}x^5 + \frac{1}{1090227600}x^6 - \frac{1}{206053016400}x^7 + \mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 113

```
AsymptoticDSolveValue[4*x*y''[x]+3*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\sqrt[4]{x}\left(-\frac{x^7}{763119630000}+\frac{x^6}{3759210000}-\frac{x^5}{25061400}+\frac{x^4}{238680}-\frac{x^3}{3510}+\frac{x^2}{90}-\frac{x}{5}+1\right)+c_2\left(-\frac{x^7}{206053016400}+\frac{x^6}{1090227600}-\frac{x^5}{7900200}+\frac{x^4}{83160}-\frac{x^3}{1386}+\frac{x^2}{42}-\frac{x}{3}+1\right)$$

## 19.15 problem 4(b)

Internal problem ID [6455]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175
**Problem number**: 4(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{2y''x + (-x+3)\,y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 52

```
Order:=8;
dsolve(2*x*diff(y(x),x$2)+(3-x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2\left(1+\frac{1}{3}x+\frac{1}{15}x^2+\frac{1}{105}x^3+\frac{1}{945}x^4+\frac{1}{10395}x^5+\frac{1}{135135}x^6+\frac{1}{2027025}x^7+\mathrm{O}\left(x^8\right)\right)\sqrt{x}+c_1\left(1+\frac{1}{2}x+\frac{1}{8}x^2+\frac{1}{4}\right.}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 113

```
AsymptoticDSolveValue[2*x*y''[x]+(3-x)*y'[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{x^7}{2027025}+\frac{x^6}{135135}+\frac{x^5}{10395}+\frac{x^4}{945}+\frac{x^3}{105}+\frac{x^2}{15}+\frac{x}{3}+1\right) + \frac{c_2\left(\frac{x^7}{645120}+\frac{x^6}{46080}+\frac{x^5}{3840}+\frac{x^4}{384}+\frac{x^3}{48}+\frac{x^2}{8}+\frac{x}{2}+1\right)}{\sqrt{x}}$$

## 19.16 problem 4(c)

Internal problem ID [6456]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175
**Problem number**: 4(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2y''x + y'(1+x) + 3y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 52

```
Order:=8;
dsolve(2*x*diff(y(x),x$2)+(x+1)*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1 - \frac{7}{6}x + \frac{21}{40}x^2 - \frac{11}{80}x^3 + \frac{143}{5760}x^4 - \frac{13}{3840}x^5 + \frac{17}{46080}x^6 - \frac{323}{9676800}x^7 + \mathrm{O}\left(x^8\right)\right) + c_2\left(1 - 3x + 2x^2 - \frac{2}{3}x^3 + \frac{1}{7}x^4 - \frac{1}{45}x^5 + \frac{4}{1485}x^6 - \frac{4}{15015}x^7 + \mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 106

```
AsymptoticDSolveValue[2*x*x*y''[x]+(x+1)*y'[x]+3*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{1386072x^7}{35} + \frac{20088x^6}{5} - \frac{2511x^5}{5} + 81x^4 - 18x^3 + 6x^2 - 3x + 1\right) + c_2 e^{\frac{1}{2}/x}\left(\frac{257243688x^7}{35} + \frac{2381886x^6}{5} + \frac{176436x^5}{5} + 3042x^4 + 312x^3 + 39x^2 + 6x + 1\right)x^{3/2}$$

## 19.17 problem 4(d)

Internal problem ID [6457]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175
**Problem number**: 4(d).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' + y'x - (1 + x)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
Order:=8;
dsolve(2*x^2*diff(y(x),x$2)+x*diff(y(x),x)-(x+1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{3}{2}} \left(1 + \frac{1}{5}x + \frac{1}{70}x^2 + \frac{1}{1890}x^3 + \frac{1}{83160}x^4 + \frac{1}{5405400}x^5 + \frac{1}{486486000}x^6 + \frac{1}{57891834000}x^7 + \mathrm{O}\left(x^8\right)\right) + c_1 \left(1 - x - \right.}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 112

```
AsymptoticDSolveValue[2*x^2*y''[x]+x*y'[x]-(x+1)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 x \left(\frac{x^7}{57891834000} + \frac{x^6}{486486000} + \frac{x^5}{5405400} + \frac{x^4}{83160} + \frac{x^3}{1890} + \frac{x^2}{70} + \frac{x}{5} + 1\right) + \frac{c_2 \left(-\frac{x^7}{52390800} - \frac{x^6}{680400} - \frac{x^5}{12600} - \frac{x^4}{360} - \frac{x^3}{18} - \frac{x^2}{2} - x + 1\right)}{\sqrt{x}}$$

## 19.18 problem 5

Internal problem ID [6458]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175
**Problem number**: 5.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{x^2y'' + y'x + yx^2 = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x)\, c_2 + c_1)\left(1 - \frac{1}{4}x^2 + \frac{1}{64}x^4 - \frac{1}{2304}x^6 + \mathrm{O}\left(x^8\right)\right) + \left(\frac{1}{4}x^2 - \frac{3}{128}x^4 + \frac{11}{13824}x^6 + \mathrm{O}\left(x^8\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 81

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+x^2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{x^6}{2304} + \frac{x^4}{64} - \frac{x^2}{4} + 1\right) + c_2\left(\frac{11x^6}{13824} - \frac{3x^4}{128} + \frac{x^2}{4} + \left(-\frac{x^6}{2304} + \frac{x^4}{64} - \frac{x^2}{4} + 1\right)\log(x)\right)$$

## 19.19 problem 6

Internal problem ID [6459]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175
**Problem number**: 6.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{y'' + \frac{y'}{x^2} - \frac{y}{x^3} = 0}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=8;
dsolve(diff(y(x),x$2)+1/x^2*diff(y(x),x)-1/x^3*y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 17

```
AsymptoticDSolveValue[y''[x]+1/x^2*y'[x]-1/x^3*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 x + c_2 e^{\frac{1}{x}} x$$

## 19.20 problem 8

Internal problem ID [6460]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175
**Problem number**: 8.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2y'' + (3x-1)\,y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+(3*x-1)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 53

```
AsymptoticDSolveValue[x^2*y''[x]+(3*x-1)*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(5040x^7 + 720x^6 + 120x^5 + 24x^4 + 6x^3 + 2x^2 + x + 1\right) + \frac{c_2e^{-1/x}}{x}$$

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## 20.1 problem 1

Internal problem ID [6461]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.5. More on Regular Singular Points. Page 183
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - 3y'x + (4x+4)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 81

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+(4*x+4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(\left(\ln(x)\,c_2 + c_1\right)\left(1 - 4x + 4x^2 - \frac{16}{9}x^3 + \frac{4}{9}x^4 - \frac{16}{225}x^5 + \frac{16}{2025}x^6 - \frac{64}{99225}x^7 + \mathrm{O}\left(x^8\right)\right) + \left(8x - 12x^2 + \frac{176}{27}x^3 - \frac{50}{27}x^4 + \frac{1096}{3375}x^5 - \frac{392}{10125}x^6 + \frac{3872}{1157625}x^7 + \mathrm{O}\left(x^8\right)\right)c_2\right)x^2$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 158

```
AsymptoticDSolveValue[x^2*y''[x]-3*x*y'[x]+(4*x+4)*y[x]==0,y[x],{x,0,7}]
```

$$
\begin{aligned}
y(x) \rightarrow\ & c_1\left(-\frac{64x^7}{99225}+\frac{16x^6}{2025}-\frac{16x^5}{225}+\frac{4x^4}{9}-\frac{16x^3}{9}+4x^2-4x+1\right)x^2 \\
& + c_2\left(\left(\frac{3872x^7}{1157625}-\frac{392x^6}{10125}+\frac{1096x^5}{3375}-\frac{50x^4}{27}+\frac{176x^3}{27}-12x^2+8x\right)x^2\right. \\
& \left. + \left(-\frac{64x^7}{99225}+\frac{16x^6}{2025}-\frac{16x^5}{225}+\frac{4x^4}{9}-\frac{16x^3}{9}+4x^2-4x+1\right)x^2\log(x)\right)
\end{aligned}
$$

## 20.2 problem 2

Internal problem ID [6462]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.5. More on Regular Singular Points. Page 183
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' - 8x^2y' + (4x^2 + 1)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 57

```
Order:=8;
dsolve(4*x^2*diff(y(x),x$2)-8*x^2*diff(y(x),x)+(4*x^2+1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x}\left(1+x+\frac{1}{2}x^2+\frac{1}{6}x^3+\frac{1}{24}x^4+\frac{1}{120}x^5+\frac{1}{720}x^6+\frac{1}{5040}x^7\right)(\ln(x)\,c_2+c_1)+O(x^8)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 112

```
AsymptoticDSolveValue[4*x^2*y''[x]-8*x^2*y'[x]+(4*x^2+1)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{x^7}{5040}+\frac{x^6}{720}+\frac{x^5}{120}+\frac{x^4}{24}+\frac{x^3}{6}+\frac{x^2}{2}+x+1\right) + c_2\sqrt{x}\left(\frac{x^7}{5040}+\frac{x^6}{720}+\frac{x^5}{120}+\frac{x^4}{24}+\frac{x^3}{6}+\frac{x^2}{2}+x+1\right)\log(x)$$

## 20.3 problem 3(a)

Internal problem ID [6463]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.5. More on Regular Singular Points. Page 183
**Problem number**: 3(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{y''x + 2y' + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 36

```
Order:=8;
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\left(1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 - \frac{1}{5040}x^6 + \mathrm{O}\left(x^8\right)\right) + \frac{c_2\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \mathrm{O}\left(x^8\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 56

```
AsymptoticDSolveValue[x*y''[x]+2*y'[x]+x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{x^5}{720} + \frac{x^3}{24} - \frac{x}{2} + \frac{1}{x}\right) + c_2\left(-\frac{x^6}{5040} + \frac{x^4}{120} - \frac{x^2}{6} + 1\right)$$

## 20.4 problem 3(b)

Internal problem ID [6464]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.5. More on Regular Singular Points. Page 183
**Problem number**: 3(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x^2y' + (x^2-2)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 55

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)-x^2*diff(y(x),x)+(x^2-2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^2\left(1+\frac{1}{2}x+\frac{1}{20}x^2-\frac{1}{60}x^3-\frac{1}{210}x^4-\frac{1}{3360}x^5+\frac{1}{20160}x^6+\frac{1}{100800}x^7+\mathrm{O}\left(x^8\right)\right) + \frac{c_2\left(12+6x+6x^2+5x^3+x^4-\frac{1}{5}x^5-\frac{1}{10}x^6-\frac{3}{280}x^7+\mathrm{O}\left(x^8\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 96

```
AsymptoticDSolveValue[x^2*y''[x]-x^2*y'[x]+(x^2-2)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{x^5}{120}-\frac{x^4}{60}+\frac{x^3}{12}+\frac{5x^2}{12}+\frac{x}{2}+\frac{1}{x}+\frac{1}{2}\right) + c_2\left(\frac{x^8}{20160}-\frac{x^7}{3360}-\frac{x^6}{210}-\frac{x^5}{60}+\frac{x^4}{20}+\frac{x^3}{2}+x^2\right)$$

## 20.5 problem 3(c)

Internal problem ID [6465]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.5. More on Regular Singular Points. Page 183
**Problem number**: 3(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{y''x - y' + 4yx^3 = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 28

```
Order:=8;
dsolve(x*diff(y(x),x$2)-diff(y(x),x)+4*x^3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^2 \left(1 - \frac{1}{6}x^4 + \mathrm{O}\left(x^8\right)\right) + c_2\left(-2 + x^4 + \mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 30

```
AsymptoticDSolveValue[x*y''[x]-y'[x]+4*x^3*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(1 - \frac{x^4}{2}\right) + c_2\left(x^2 - \frac{x^6}{6}\right)$$

## 20.6 problem 4

Internal problem ID [6466]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.5. More on Regular Singular Points. Page 183
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-1)^2 y'' - 3(x-1) y' + 2y = 0}$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
Order:=8;
dsolve((x-1)^2*diff(y(x),x$2)-3*(x-1)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=1);
```

$$y(x) = (x-1)^2 \left(c_1 (x-1)^{-\sqrt{2}} + c_2 (x-1)^{\sqrt{2}}\right) + O\left(x^8\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 34

```
AsymptoticDSolveValue[(x-1)^2*y''[x]-3*(x-1)*y'[x]+2*y[x]==0,y[x],{x,1,7}]
```

$$y(x) \to c_1 (x-1)^{2+\sqrt{2}} + c_2 (x-1)^{2-\sqrt{2}}$$

## 20.7 problem 5

Internal problem ID [6467]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.5. More on Regular Singular Points. Page 183
**Problem number**: 5.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3(1+x)^2 y'' - y'(1+x) - y = 0}$$

With the expansion point for the power series method at $x = -1$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 43

```
Order:=8;
dsolve(3*(x+1)^2*diff(y(x),x$2)-(x+1)*diff(y(x),x)-y(x)=0,y(x),type='series',x=-1);
```

$$y(x) = (x+1)^{\frac{2}{3}} \left((x+1)^{-\frac{\sqrt{7}}{3}} c_1 + (x+1)^{\frac{\sqrt{7}}{3}} c_2\right) + O\left(x^8\right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 42

```
AsymptoticDSolveValue[3*(x+1)^2*y''[x]-(x+1)*y'[x]-y[x]==0,y[x],{x,-1,7}]
```

$$y(x) \to c_1(x+1)^{\frac{1}{3}\left(2+\sqrt{7}\right)} + c_2(x+1)^{\frac{1}{3}\left(2-\sqrt{7}\right)}$$

## 20.8 problem 6

Internal problem ID [6468]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.5. More on Regular Singular Points. Page 183
**Problem number**: 6.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Bessel]`

$$x^2 y'' + y' x + y(x^2 - 1) = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^2 \left(1 - \frac{1}{8}x^2 + \frac{1}{192}x^4 - \frac{1}{9216}x^6 + \mathrm{O}(x^8)\right) + c_2\left(\ln(x)\left(x^2 - \frac{1}{8}x^4 + \frac{1}{192}x^6 + \mathrm{O}(x^8)\right) + \left(-2 + \frac{3}{32}x^4 - \frac{7}{1152}\right.\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 75

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-1)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(-\frac{x^7}{9216} + \frac{x^5}{192} - \frac{x^3}{8} + x\right) + c_1\left(\frac{5x^6 - 90x^4 + 288x^2 + 1152}{1152x} - \frac{1}{384}x(x^4 - 24x^2 + 192)\log(x)\right)$$

## 20.9 problem 7

Internal problem ID [6469]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.5. More on Regular Singular Points. Page 183
**Problem number**: 7.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + y'x + \left(-\frac{1}{4} + x^2\right) y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 39

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 - \frac{1}{5040}x^6 + \mathrm{O}\left(x^8\right)\right)x + c_2\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \mathrm{O}\left(x^8\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 76

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{x^{11/2}}{720} + \frac{x^{7/2}}{24} - \frac{x^{3/2}}{2} + \frac{1}{\sqrt{x}}\right) + c_2\left(-\frac{x^{13/2}}{5040} + \frac{x^{9/2}}{120} - \frac{x^{5/2}}{6} + \sqrt{x}\right)$$

# 21 Chapter 4. Power Series Solutions and Special Functions. Section 4.6. Gauss's Hypergeometric Equation. Page 187

## 21.1 problem 2(a)

Internal problem ID [6470]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.6. Gauss's Hypergeometric Equation. Page 187
**Problem number**: 2(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Jacobi]`

$$x(1-x)\,y'' + \left(\frac{3}{2} - 2x\right) y' + 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 40

```
Order:=8;
dsolve(x*(1-x)*diff(y(x),x$2)+(3/2-2*x)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2\left(1 - \frac{4}{3}x + \mathrm{O}\left(x^8\right)\right)\sqrt{x} + c_1\left(1 - \frac{9}{2}x + \frac{15}{8}x^2 + \frac{7}{16}x^3 + \frac{27}{128}x^4 + \frac{33}{256}x^5 + \frac{91}{1024}x^6 + \frac{135}{2048}x^7 + \mathrm{O}\left(x^8\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 71

```
AsymptoticDSolveValue[x*(1-x)*y''[x]+(3/2-2*x)*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to \frac{c_2\left(\frac{135x^7}{2048} + \frac{91x^6}{1024} + \frac{33x^5}{256} + \frac{27x^4}{128} + \frac{7x^3}{16} + \frac{15x^2}{8} - \frac{9x}{2} + 1\right)}{\sqrt{x}} + c_1\left(1 - \frac{4x}{3}\right)$$

## 21.2 problem 2(b)

Internal problem ID [6471]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.6. Gauss's Hypergeometric Equation. Page 187
**Problem number**: 2(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(2x^2+2x)\,y''+(1+5x)\,y'+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 52

```
Order:=8;
dsolve((2*x^2+2*x)*diff(y(x),x$2)+(1+5*x)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(-x^7+x^6-x^5+x^4-x^3+x^2-x+1\right)\left(c_1\sqrt{x}+c_2\right)+O\left(x^8\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 73

```
AsymptoticDSolveValue[(2*x^2+2*x)*y''[x]+(1+5*x)*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_1\sqrt{x}\left(-x^7+x^6-x^5+x^4-x^3+x^2-x+1\right)+c_2\left(-x^7+x^6-x^5+x^4-x^3+x^2-x+1\right)$$

## 21.3 problem 2(x)

Internal problem ID [6472]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.6. Gauss's Hypergeometric Equation. Page 187
**Problem number**: 2(x).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-1)\,y''+(5x+4)\,y'+4y=0}$$

With the expansion point for the power series method at $x=-1$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 54

```
Order:=8;
dsolve((x^2-1)*diff(y(x),x$2)+(5*x+4)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=-1);
```

$$y(x)=c_1\sqrt{x+1}\left(1+\frac{25}{12}(x+1)+\frac{245}{96}(x+1)^2+\frac{315}{128}(x+1)^3+\frac{4235}{2048}(x+1)^4+\frac{13013}{8192}(x+1)^5+\frac{75075}{65536}(x+1)^6+\frac{206635}{262144}(x+1)^7+\mathrm{O}\left((x+1)^8\right)\right)+c_2\left(1+4(x+1)+6(x+1)^2+\frac{32}{5}(x+1)^3+\frac{40}{7}(x+1)^4+\frac{32}{7}(x+1)^5+\frac{112}{33}(x+1)^6+\frac{1024}{429}(x+1)^7+\mathrm{O}\left((x+1)^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 139

```
AsymptoticDSolveValue[(x^2-1)*y''[x]+(5*x+4)*y'[x]+4*y[x]==0,y[x],{x,-1,7}]
```

$$y(x) \to c_1\sqrt{x+1}\left(\frac{206635(x+1)^7}{262144}+\frac{75075(x+1)^6}{65536}+\frac{13013(x+1)^5}{8192}+\frac{4235(x+1)^4}{2048}+\frac{315}{128}(x+1)^3+\frac{245}{96}(x+1)^2+\frac{25(x+1)}{12}+1\right)+c_2\left(\frac{1024}{429}(x+1)^7+\frac{112}{33}(x+1)^6+\frac{32}{7}(x+1)^5+\frac{40}{7}(x+1)^4+\frac{32}{5}(x+1)^3+6(x+1)^2+4(x+1)+1\right)$$

## 21.4 problem 2(d)

Internal problem ID [6473]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.6. Gauss's Hypergeometric Equation. Page 187
**Problem number**: 2(d).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\left(x^{2}-x-6\right) y''+\left(5+3x\right) y'+y=0$$

With the expansion point for the power series method at $x = 3$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 54

```
Order:=8;
dsolve((x^2-x-6)*diff(y(x),x$2)+(5+3*x)*diff(y(x),x)+y(x)=0,y(x),type='series',x=3);
```

$$y(x) = \frac{c_2\left(1-\frac{1}{14}(x-3)+\frac{1}{133}(x-3)^2-\frac{1}{1064}(x-3)^3+\frac{1}{7714}(x-3)^4-\frac{5}{262276}(x-3)^5+\frac{5}{1704794}(x-3)^6-\frac{5}{107158}\cdots\right)}{}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 145

```
AsymptoticDSolveValue[(x^2-x-6)*y''[x]+(5+3*x)*y'[x]+y[x]==0,y[x],{x,3,7}]
```

$$y(x) \to c_1\left(-\frac{5(x-3)^7}{10715848}+\frac{5(x-3)^6}{1704794}-\frac{5(x-3)^5}{262276}+\frac{(x-3)^4}{7714}-\frac{(x-3)^3}{1064}+\frac{1}{133}(x-3)^2+\frac{3-x}{14}+1\right)+\frac{c_2\left(\frac{2288(x-3)^7}{30517578125}-\frac{616(x-3)^6}{1220703125}+\frac{176(x-3)^5}{48828125}-\frac{11(x-3)^4}{390625}+\frac{4(x-3)^3}{15625}-\frac{2}{625}(x-3)^2+\frac{4(x-3)}{25}+1\right)}{(x-3)^{9/5}}$$

## 21.5 problem 3

Internal problem ID [6474]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.6. Gauss's Hypergeometric Equation. Page 187
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Gegenbauer, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']`

$$\boxed{(-x^2+1)\, y'' - y'x + p^2 y = 0}$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 338

```
Order:=8;
dsolve((1-x^2)*diff(y(x),x$2)-x*diff(y(x),x)+p^2*y(x)=0,y(x),type='series',x=1);
```

$$
\begin{aligned}
y(x) = c_1\sqrt{x-1}\,\Bigg(1 &+ \left(\frac{p^2}{3}-\frac{1}{12}\right)(x-1) + \left(\frac{1}{30}p^4-\frac{1}{12}p^2+\frac{3}{160}\right)(x-1)^2 \\
&+ \left(\frac{1}{630}p^6-\frac{1}{72}p^4+\frac{37}{1440}p^2-\frac{5}{896}\right)(x-1)^3 \\
&+ \left(\frac{1}{22680}p^8-\frac{1}{1080}p^6+\frac{47}{8640}p^4-\frac{3229}{362880}p^2+\frac{35}{18432}\right)(x-1)^4 \\
&+ \left(\frac{1}{1247400}p^{10}-\frac{1}{30240}p^8+\frac{19}{43200}p^6-\frac{1571}{725760}p^4+\frac{10679}{3225600}p^2-\frac{63}{90112}\right)(x-1)^5 \\
&+ \left(\frac{1}{97297200}p^{12}-\frac{1}{1360800}p^{10}+\frac{67}{3628800}p^8-\frac{2159}{10886400}p^6+\frac{153617}{174182400}p^4 \right.\\
&\left. -\frac{550499}{425779200}p^2+\frac{231}{851968}\right)(x-1)^6 \\
&+ \left(\frac{1}{10216206000}p^{14}-\frac{1}{89812800}p^{12}+\frac{11}{23328000}p^{10}-\frac{8521}{914457600}p^8 \right.\\
&\left. +\frac{230443}{2612736000}p^6-\frac{1206053}{3284582400}p^4+\frac{2430898831}{4649508864000}p^2-\frac{143}{1310720}\right)(x-1)^7 \\
&+ \mathrm{O}\left((x-1)^8\right)\Bigg) + c_2\Bigg(1+p^2(x-1)+\left(\frac{1}{6}p^4-\frac{1}{6}p^2\right)(x-1)^2 \\
&+ \left(\frac{1}{90}p^6-\frac{1}{18}p^4+\frac{2}{45}p^2\right)(x-1)^3 + \left(\frac{1}{2520}p^8-\frac{1}{180}p^6+\frac{7}{360}p^4-\frac{1}{70}p^2\right)(x-1)^4 \\
&+ \left(\frac{1}{113400}p^{10}-\frac{1}{3780}p^8+\frac{13}{5400}p^6-\frac{41}{5670}p^4+\frac{8}{1575}p^2\right)(x-1)^5 \\
&+ \left(\frac{1}{7484400}p^{12}-\frac{1}{136080}p^{10}+\frac{31}{226800}p^8-\frac{139}{136080}p^6+\frac{479}{170100}p^4 \right.\\
&\left. -\frac{4}{2079}p^2\right)(x-1)^6 + \left(\frac{1}{681080400}p^{14}-\frac{1}{7484400}p^{12}+\frac{1}{226800}p^{10}-\frac{311}{4762800}p^8 \right.\\
&\left. +\frac{37}{85050}p^6-\frac{59}{51975}p^4+\frac{16}{21021}p^2\right)(x-1)^7 + \mathrm{O}\left((x-1)^8\right)\Bigg)
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 5699

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-x*y'[x]+p^2*y[x]==0,y[x],{x,1,7}]
```

Too large to display

## 21.6 problem 5

Internal problem ID [6475]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Section 4.6. Gauss's Hypergeometric Equation. Page 187
**Problem number**: 5.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(1-\mathrm{e}^{x})\,y''+\frac{y'}{2}+\mathrm{e}^{x}y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.266 (sec). Leaf size: 52

```
Order:=8;
dsolve((1-exp(x))*diff(y(x),x$2)+1/2*diff(y(x),x)+exp(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1x^{\frac{3}{2}}\left(1+\frac{1}{4}x+\frac{3}{32}x^2+\frac{7}{384}x^3+\frac{109}{30720}x^4+\frac{13}{24576}x^5+\frac{4439}{61931520}x^6+\frac{2069}{247726080}x^7+\mathrm{O}\left(x^8\right)\right)+c_2\left(1-2x-x^2-\frac{1}{3}x^3-\frac{1}{12}x^4-\frac{1}{60}x^5-\frac{1}{360}x^6-\frac{1}{2520}x^7+\mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 109

```
AsymptoticDSolveValue[(1-Exp[x])*y''[x]+1/2*y'[x]+Exp[x]*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_2\left(-\frac{x^7}{2520}-\frac{x^6}{360}-\frac{x^5}{60}-\frac{x^4}{12}-\frac{x^3}{3}-x^2-2x+1\right)+c_1\left(\frac{2069x^7}{247726080}+\frac{4439x^6}{61931520}+\frac{13x^5}{24576}+\frac{109x^4}{30720}+\frac{7x^3}{384}+\frac{3x^2}{32}+\frac{x}{4}+1\right)x^{3/2}$$

# 22 Chapter 4. Power Series Solutions and Special Functions. Problems for review and discovert. (A) Drill Exercises . Page 194

## 22.1 problem 1(a)

Internal problem ID [6476]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Problems for review and discovert. (A) Drill Exercises . Page 194
**Problem number**: 1(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2yx = x^2}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
Order:=8;
dsolve(diff(y(x),x$2)+2*x*y(x)=x^2,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{3}x^3 + \frac{1}{45}x^6\right) y(0) + \left(x - \frac{1}{6}x^4 + \frac{1}{126}x^7\right) D(y)(0) + \frac{x^4}{12} - \frac{x^7}{252} + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 56

```
AsymptoticDSolveValue[y''[x]+2*x*y[x]==x^2,y[x],{x,0,7}]
```

$$y(x) \to -\frac{x^7}{252} + \frac{x^4}{12} + c_2\left(\frac{x^7}{126} - \frac{x^4}{6} + x\right) + c_1\left(\frac{x^6}{45} - \frac{x^3}{3} + 1\right)$$

## 22.2 problem 1(b)

Internal problem ID [6477]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Problems for review and discovert. (A) Drill Exercises . Page 194
**Problem number**: 1(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x + y = x}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
Order:=8;
dsolve(diff(y(x),x$2)-x*diff(y(x),x)+y(x)=x,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 - \frac{1}{24}x^4 - \frac{1}{240}x^6\right) y(0) + D(y)(0)\, x + \frac{x^3}{6} + \frac{x^5}{60} + \frac{x^7}{630} + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 55

```
AsymptoticDSolveValue[y''[x]-x*y'[x]+y[x]==x,y[x],{x,0,7}]
```

$$y(x) \to \frac{x^7}{630} + \frac{x^5}{60} + \frac{x^3}{6} + c_1\left(-\frac{x^6}{240} - \frac{x^4}{24} - \frac{x^2}{2} + 1\right) + c_2 x$$

## 22.3 problem 1(c)

Internal problem ID [6478]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Problems for review and discovert. (A) Drill Exercises . Page 194
**Problem number**: 1(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' + y = x^3 - x}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 63

```
Order:=8;
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=x^3-x,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = &\left(1 - \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{120}x^5 + \frac{1}{720}x^6\right) y(0) \\ &+ \left(x - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + \frac{1}{5040}x^7\right) D(y)(0) \\ &- \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{20} - \frac{7x^6}{720} + \frac{x^7}{5040} + O(x^8) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 105

```
AsymptoticDSolveValue[y''[x]+y'[x]+y[x]==x^3-x,y[x],{x,0,7}]
```

$$y(x) \to \frac{x^7}{5040} - \frac{7x^6}{720} + \frac{x^5}{20} + \frac{x^4}{24} - \frac{x^3}{6} + c_2\left(\frac{x^7}{5040} - \frac{x^5}{120} + \frac{x^4}{24} - \frac{x^2}{2} + x\right) + c_1\left(\frac{x^6}{720} - \frac{x^5}{120} + \frac{x^3}{6} - \frac{x^2}{2} + 1\right)$$

## 22.4 problem 1(d)

Internal problem ID [6479]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Problems for review and discovert. (A) Drill Exercises . Page 194
**Problem number**: 1(d).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{2y'' + y'x + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;
dsolve(2*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{4}x^2 + \frac{1}{32}x^4 - \frac{1}{384}x^6\right) y(0) + \left(x - \frac{1}{6}x^3 + \frac{1}{60}x^5 - \frac{1}{840}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[2*y''[x]+x*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2\left(-\frac{x^7}{840} + \frac{x^5}{60} - \frac{x^3}{6} + x\right) + c_1\left(-\frac{x^6}{384} + \frac{x^4}{32} - \frac{x^2}{4} + 1\right)$$

## 22.5 problem 1(e)

Internal problem ID [6480]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Problems for review and discovert. (A) Drill Exercises . Page 194
**Problem number**: 1(e).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+4)\,y''-y'+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 74

```
Order:=8;
dsolve((4+x^2)*diff(y(x),x$2)-diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{1}{8}x^2-\frac{1}{96}x^3+\frac{11}{1536}x^4+\frac{13}{10240}x^5-\frac{533}{737280}x^6-\frac{3809}{20643840}x^7\right)y(0)$$
$$+\left(x+\frac{1}{8}x^2-\frac{1}{32}x^3-\frac{5}{512}x^4+\frac{23}{10240}x^5+\frac{283}{245760}x^6-\frac{1649}{6881280}x^7\right)D(y)(0)$$
$$+O\left(x^8\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 98

```
AsymptoticDSolveValue[(4+x^2)*y''[x]-y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{3809x^7}{20643840} - \frac{533x^6}{737280} + \frac{13x^5}{10240} + \frac{11x^4}{1536} - \frac{x^3}{96} - \frac{x^2}{8} + 1\right) + c_2\left(-\frac{1649x^7}{6881280} + \frac{283x^6}{245760} + \frac{23x^5}{10240} - \frac{5x^4}{512} - \frac{x^3}{32} + \frac{x^2}{8} + x\right)$$

## 22.6 problem 1(f)

Internal problem ID [6481]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Problems for review and discovert. (A) Drill Exercises . Page 194
**Problem number**: 1(f).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\, y'' - y'x + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
Order:=8;
dsolve((x^2+1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{80}x^6\right) y(0) + D(y)(0)\, x + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[(x^2+1)*y''[x]-x*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{x^6}{80} + \frac{x^4}{24} - \frac{x^2}{2} + 1\right) + c_2 x$$

## 22.7 problem 1(g)

Internal problem ID [6482]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Problems for review and discovert. (A) Drill Exercises . Page 194
**Problem number**: 1(g).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'(1+x) - yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 69

```
Order:=8;
dsolve(diff(y(x),x$2)-(x+1)*diff(y(x),x)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{30}x^5 + \frac{1}{60}x^6 + \frac{37}{5040}x^7\right) y(0) + \left(x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \frac{1}{8}x^5 + \frac{47}{720}x^6 + \frac{19}{630}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 91

```
AsymptoticDSolveValue[y''[x]-(x+1)*y'[x]-x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{37x^7}{5040} + \frac{x^6}{60} + \frac{x^5}{30} + \frac{x^4}{24} + \frac{x^3}{6} + 1\right) + c_2\left(\frac{19x^7}{630} + \frac{47x^6}{720} + \frac{x^5}{8} + \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x\right)$$

## 22.8 problem 1(h)

Internal problem ID [6483]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Problems for review and discovert. (A) Drill Exercises . Page 194
**Problem number**: 1(h).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x-1)\,y''+y'(1+x)+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 74

```
Order:=8;
dsolve((x-1)*diff(y(x),x$2)+(x+1)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+\frac{1}{2}x^2+\frac{1}{3}x^3+\frac{3}{8}x^4+\frac{11}{30}x^5+\frac{53}{144}x^6+\frac{103}{280}x^7\right)y(0)+\left(x+\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{5}{8}x^4+\frac{19}{30}x^5+\frac{91}{144}x^6+\frac{177}{280}x^7\right)D(y)(0)+O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 98

```
AsymptoticDSolveValue[(x-1)*y''[x]+(x+1)*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_1\left(\frac{103x^7}{280}+\frac{53x^6}{144}+\frac{11x^5}{30}+\frac{3x^4}{8}+\frac{x^3}{3}+\frac{x^2}{2}+1\right)+c_2\left(\frac{177x^7}{280}+\frac{91x^6}{144}+\frac{19x^5}{30}+\frac{5x^4}{8}+\frac{2x^3}{3}+\frac{x^2}{2}+x\right)$$

## 22.9 problem 2(a)

Internal problem ID [6484]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Problems for review and discovert. (A) Drill Exercises . Page 194
**Problem number**: 2(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\, x^2 y'' - y'x + (x+2)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 87

```
Order:=8;
dsolve((x^2+1)*x^2*diff(y(x),x$2)-x*diff(y(x),x)+(2+x)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = {} & c_1 x^{1-i} \left(1 + \left(-\frac{1}{5} - \frac{2i}{5}\right) x + \left(-\frac{1}{40} + \frac{13i}{40}\right) x^2 + \left(\frac{71}{520} + \frac{17i}{520}\right) x^3 \right. \\ & + \left(-\frac{31}{832} - \frac{541i}{4160}\right) x^4 + \left(-\frac{1423}{20800} + \frac{7i}{4160}\right) x^5 + \left(\frac{12849}{416000} + \frac{10853i}{156000}\right) x^6 \\ & \left. + \left(\frac{209609}{5088000} - \frac{106907i}{17808000}\right) x^7 + \mathrm{O}\left(x^8\right)\right) \\ & + c_2 x^{1+i} \left(1 + \left(-\frac{1}{5} + \frac{2i}{5}\right) x + \left(-\frac{1}{40} - \frac{13i}{40}\right) x^2 + \left(\frac{71}{520} - \frac{17i}{520}\right) x^3 \right. \\ & + \left(-\frac{31}{832} + \frac{541i}{4160}\right) x^4 + \left(-\frac{1423}{20800} - \frac{7i}{4160}\right) x^5 + \left(\frac{12849}{416000} - \frac{10853i}{156000}\right) x^6 \\ & \left. + \left(\frac{209609}{5088000} + \frac{106907i}{17808000}\right) x^7 + \mathrm{O}\left(x^8\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 122

```
AsymptoticDSolveValue[(x^2+1)*x^2*y''[x]-x*y'[x]+(2+x)*y[x]==0,y[x],{x,0,7}]
```

$$
\begin{aligned}
y(x) \to & \left(\frac{1}{156000}+\frac{i}{1248000}\right) c_2 x^{1-i}\big((6080+10093i)x^6-(10476-1572i)x^5 \\
& -(8220+19260i)x^4+(21600+2400i)x^3+(2400+50400i)x^2-(38400+57600i)x \\
& +(153600-19200i)\big)-\left(\frac{1}{1248000}+\frac{i}{156000}\right) c_1 x^{1+i}\big((10093+6080i)x^6 \\
& +(1572-10476i)x^5-(19260+8220i)x^4+(2400+21600i)x^3 \\
& +(50400+2400i)x^2-(57600+38400i)x-(19200-153600i)\big)
\end{aligned}
$$

## 22.10 problem 2(b)

Internal problem ID [6485]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Problems for review and discovert. (A) Drill Exercises . Page 194
**Problem number**: 2(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + y'x + (1 + x)\, y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 85

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned}y(x) = c_1x^{-i}\bigg(1 &+ \left(-\frac{1}{5} - \frac{2i}{5}\right)x + \left(-\frac{1}{40} + \frac{3i}{40}\right)x^2 + \left(\frac{3}{520} - \frac{7i}{1560}\right)x^3 \\ &+ \left(-\frac{1}{2496} + \frac{i}{12480}\right)x^4 + \left(\frac{9}{603200} + \frac{i}{361920}\right)x^5 \\ &+ \left(-\frac{19}{54288000} - \frac{7i}{36192000}\right)x^6 + \left(\frac{1}{179829000} + \frac{223i}{40281696000}\right)x^7 + \mathrm{O}\left(x^8\right)\bigg) \\ &+ c_2x^{i}\bigg(1 + \left(-\frac{1}{5} + \frac{2i}{5}\right)x + \left(-\frac{1}{40} - \frac{3i}{40}\right)x^2 + \left(\frac{3}{520} + \frac{7i}{1560}\right)x^3 \\ &+ \left(-\frac{1}{2496} - \frac{i}{12480}\right)x^4 + \left(\frac{9}{603200} - \frac{i}{361920}\right)x^5 \\ &+ \left(-\frac{19}{54288000} + \frac{7i}{36192000}\right)x^6 + \left(\frac{1}{179829000} - \frac{223i}{40281696000}\right)x^7 + \mathrm{O}\left(x^8\right)\bigg)\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 118

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(1+x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to \left(\frac{7}{36192000} + \frac{19i}{54288000}\right) c_1 x^i \left(ix^6 + (12-36i)x^5 - (660-780i)x^4 + (16800-7200i)x^3 - (194400+36000i)x^2 + (633600+921600i)x + (1209600-2188800i)\right) - \left(\frac{19}{54288000} + \frac{7i}{36192000}\right) c_2 x^{-i} \left(x^6 - (36-12i)x^5 + (780-660i)x^4 - (7200-16800i)x^3 - (36000+194400i)x^2 + (921600+633600i)x - (2188800-1209600i)\right)$$

## 22.11 problem 2(c)

Internal problem ID [6486]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Problems for review and discovert. (A) Drill Exercises . Page 194
**Problem number**: 2(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' - 4y' + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
Order:=8;
dsolve(x*diff(y(x),x$2)-4*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^5 \left(1 - \frac{1}{14}x^2 + \frac{1}{504}x^4 - \frac{1}{33264}x^6 + O\left(x^8\right)\right) + c_2\left(2880 + 480x^2 + 120x^4 - 20x^6 + O\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 58

```
AsymptoticDSolveValue[x*y''[x]-4*y'[x]+x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{x^6}{144} + \frac{x^4}{24} + \frac{x^2}{6} + 1\right) + c_2\left(-\frac{x^{11}}{33264} + \frac{x^9}{504} - \frac{x^7}{14} + x^5\right)$$

## 22.12 problem 2(d)

Internal problem ID [6487]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Problems for review and discovert. (A) Drill Exercises . Page 194
**Problem number**: 2(d).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 4x^2y' + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 83

```
Order:=8;
dsolve(4*x^2*diff(y(x),x$2)+4*x^2*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned}
y(x) = {}& c_1 x^{\frac{1}{2}-\frac{i}{2}} \left(1 - \frac{1}{2}x + \left(\frac{7}{40} + \frac{i}{40}\right)x^2 + \left(-\frac{11}{240} - \frac{i}{80}\right)x^3 + \left(\frac{31}{3264} + \frac{i}{272}\right)x^4 \right.\\
& + \left(-\frac{53}{32640} - \frac{13i}{16320}\right)x^5 + \left(\frac{3421}{14492160} + \frac{223i}{1610240}\right)x^6 \\
& \left. + \left(-\frac{30269}{1014451200} - \frac{977i}{48307200}\right)x^7 + \mathrm{O}\left(x^8\right)\right) + c_2 x^{\frac{1}{2}+\frac{i}{2}} \left(1 - \frac{1}{2}x + \left(\frac{7}{40} - \frac{i}{40}\right)x^2 \right.\\
& + \left(-\frac{11}{240} + \frac{i}{80}\right)x^3 + \left(\frac{31}{3264} - \frac{i}{272}\right)x^4 + \left(-\frac{53}{32640} + \frac{13i}{16320}\right)x^5 \\
& \left. + \left(\frac{3421}{14492160} - \frac{223i}{1610240}\right)x^6 + \left(-\frac{30269}{1014451200} + \frac{977i}{48307200}\right)x^7 + \mathrm{O}\left(x^8\right)\right)
\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 226

```
AsymptoticDSolveValue[4*x^2*y''[x]+4*x^2*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$
\begin{aligned}
y(x) \rightarrow\; & c_1\left(\left(\frac{3421}{14492160}-\frac{223i}{1610240}\right)x^{\frac{13}{2}+\frac{i}{2}}-\left(\frac{53}{32640}-\frac{13i}{16320}\right)x^{\frac{11}{2}+\frac{i}{2}}\right.\\
&+\left(\frac{31}{3264}-\frac{i}{272}\right)x^{\frac{9}{2}+\frac{i}{2}}-\left(\frac{11}{240}-\frac{i}{80}\right)x^{\frac{7}{2}+\frac{i}{2}}+\left(\frac{7}{40}-\frac{i}{40}\right)x^{\frac{5}{2}+\frac{i}{2}}-\frac{1}{2}x^{\frac{3}{2}+\frac{i}{2}}+\left.x^{\frac{1}{2}+\frac{i}{2}}\right)\\
&+c_2\left(\left(\frac{3421}{14492160}+\frac{223i}{1610240}\right)x^{\frac{13}{2}-\frac{i}{2}}-\left(\frac{53}{32640}+\frac{13i}{16320}\right)x^{\frac{11}{2}-\frac{i}{2}}\right.\\
&+\left(\frac{31}{3264}+\frac{i}{272}\right)x^{\frac{9}{2}-\frac{i}{2}}-\left(\frac{11}{240}+\frac{i}{80}\right)x^{\frac{7}{2}-\frac{i}{2}}+\left(\frac{7}{40}+\frac{i}{40}\right)x^{\frac{5}{2}-\frac{i}{2}}-\frac{1}{2}x^{\frac{3}{2}-\frac{i}{2}}+\left.x^{\frac{1}{2}-\frac{i}{2}}\right)
\end{aligned}
$$

## 22.13 problem 2(e)

Internal problem ID [6488]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Problems for review and discovert. (A) Drill Exercises . Page 194
**Problem number**: 2(e).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + (1-x)\,y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 40

```
Order:=8;
dsolve(2*x*diff(y(x),x$2)+(1-x)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1 - \frac{1}{6}x - \frac{1}{120}x^2 - \frac{1}{1680}x^3 - \frac{1}{24192}x^4 - \frac{1}{380160}x^5 - \frac{1}{6589440}x^6 - \frac{1}{125798400}x^7 + \mathrm{O}\left(x^8\right)\right) + c_2\left(1 - x + \mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 111

```
AsymptoticDSolveValue[2*x*y''[x]+(1-x)*y'[x]+(2+x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{17333x^7}{48432384000} - \frac{34817x^6}{691891200} - \frac{1171x^5}{4435200} + \frac{121x^4}{40320} + \frac{37x^3}{1680} - \frac{3x^2}{40} - \frac{x}{2} + 1\right) + c_2\left(\frac{4x^7}{143325} - \frac{x^6}{8400} - \frac{19x^5}{6300} - \frac{x^4}{840} + \frac{2x^3}{15} + \frac{x^2}{6} - 2x + 1\right)$$

## 22.14 problem 2(f)

Internal problem ID [6489]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Problems for review and discovert. (A) Drill Exercises . Page 194
**Problem number**: 2(f).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Laguerre]`

$$xy'' - (x-1)\,y' + 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
Order:=8;
dsolve(x*diff(y(x),x$2)-(x-1)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x)\,c_2 + c_1)\left(1 - 2x + \frac{1}{2}x^2 + \mathrm{O}\left(x^8\right)\right) + \left(5x - \frac{9}{4}x^2 + \frac{1}{18}x^3 + \frac{1}{288}x^4 + \frac{1}{3600}x^5 + \frac{1}{43200}x^6 + \frac{1}{529200}x^7 + \mathrm{O}\left(x^8\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 83

```
AsymptoticDSolveValue[x*y''[x]-(x-1)*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{x^2}{2} - 2x + 1\right) + c_2\left(\frac{x^7}{529200} + \frac{x^6}{43200} + \frac{x^5}{3600} + \frac{x^4}{288} + \frac{x^3}{18} - \frac{9x^2}{4} + \left(\frac{x^2}{2} - 2x + 1\right)\log(x) + 5x\right)$$

## 22.15 problem 2(g)

Internal problem ID [6490]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Problems for review and discovert. (A) Drill Exercises . Page 194
**Problem number**: 2(g).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(1-x)\,y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 85

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*(1-x)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$
\begin{aligned}
y(x) = {} & c_1x^{-i}\left(1+\left(\frac{2}{5}-\frac{i}{5}\right)x+\left(\frac{1}{10}-\frac{i}{20}\right)x^2+\left(\frac{17}{780}-\frac{i}{130}\right)x^3+\left(\frac{5}{1248}-\frac{i}{1248}\right)x^4\right.\\
&+\left(\frac{113}{180960}-\frac{7i}{180960}\right)x^5+\left(\frac{911}{10857600}+\frac{19i}{3619200}\right)x^6\\
&\left.+\left(\frac{39799}{4028169600}+\frac{1009i}{575452800}\right)x^7+\mathrm{O}\left(x^8\right)\right)\\
&+c_2x^{i}\left(1+\left(\frac{2}{5}+\frac{i}{5}\right)x+\left(\frac{1}{10}+\frac{i}{20}\right)x^2+\left(\frac{17}{780}+\frac{i}{130}\right)x^3+\left(\frac{5}{1248}+\frac{i}{1248}\right)x^4\right.\\
&+\left(\frac{113}{180960}+\frac{7i}{180960}\right)x^5+\left(\frac{911}{10857600}-\frac{19i}{3619200}\right)x^6\\
&\left.+\left(\frac{39799}{4028169600}-\frac{1009i}{575452800}\right)x^7+\mathrm{O}\left(x^8\right)\right)
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 122

```
AsymptoticDSolveValue[x^2*y''[x]+x*(1-x)*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$
\begin{aligned}
y(x) \to & \left(\frac{59}{10857600}-\frac{17i}{10857600}\right)c_2x^{-i}\left((14+5i)x^6+(108+24i)x^5+(720+60i)x^4\right.\\
& \left.+(4080-240i)x^3+(19440-3600i)x^2+(77760-14400i)x+(169920+48960i)\right)\\
& +\left(\frac{59}{10857600}+\frac{17i}{10857600}\right)c_1x^{i}\left((14-5i)x^6+(108-24i)x^5+(720-60i)x^4\right.\\
& \left.+(4080+240i)x^3+(19440+3600i)x^2+(77760+14400i)x+(169920-48960i)\right)
\end{aligned}
$$

## 22.16 problem 2(h)

Internal problem ID [6491]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Problems for review and discovert. (A) Drill Exercises . Page 194
**Problem number**: 2(h).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{xy'' + y'(1+x) + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 71

```
Order:=8;
dsolve(x*diff(y(x),x$2)+(x+1)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x)\, c_2 + c_1)\left(1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + \frac{1}{720}x^6 - \frac{1}{5040}x^7 + \mathrm{O}\left(x^8\right)\right) + \left(x - \frac{3}{4}x^2 + \frac{11}{36}x^3 - \frac{25}{288}x^4 + \frac{137}{7200}x^5 - \frac{49}{14400}x^6 + \frac{121}{235200}x^7 + \mathrm{O}\left(x^8\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 151

```
AsymptoticDSolveValue[x*y''[x]+(x+1)*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{x^7}{5040} + \frac{x^6}{720} - \frac{x^5}{120} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} - x + 1\right) + c_2\left(\frac{121x^7}{235200} - \frac{49x^6}{14400} + \frac{137x^5}{7200} - \frac{25x^4}{288} + \frac{11x^3}{36} - \frac{3x^2}{4} + \left(-\frac{x^7}{5040} + \frac{x^6}{720} - \frac{x^5}{120} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} - x + 1\right)\log(x) + x\right)$$

## 22.17 problem 3(a)

Internal problem ID [6492]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Problems for review and discovert. (A) Drill Exercises . Page 194
**Problem number**: 3(a).
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

Solve

$$\boxed{x^3y''' + 2x^2y'' + \left(x^2 + x\right)y' + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

## ✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 1219

```
Order:=8;
dsolve(x^3*diff(y(x),x$3)+2*x^2*diff(y(x),x$2)+(x+x^2)*diff(y(x),x)+x*y(x)=0,y(x),type='seri
```

$$y(x) = c_1 x^{\frac{1}{2}-\frac{i\sqrt{3}}{2}} \left(1 + \frac{1}{-1+i\sqrt{3}}x + \frac{1}{2}\frac{1}{(-2+i\sqrt{3})(-1+i\sqrt{3})}x^2 + \frac{1}{6}\frac{1}{(-3+i\sqrt{3})(-2+i\sqrt{3})(-1+i\sqrt{3})}x^3 + \frac{1}{24}\frac{1}{(i\sqrt{3}-4)(-3+i\sqrt{3})(-2+i\sqrt{3})(-1+i\sqrt{3})}x^4 + \frac{1}{120}\frac{1}{(i\sqrt{3}-5)(i\sqrt{3}-4)(-3+i\sqrt{3})(-2+i\sqrt{3})(-1+i\sqrt{3})}x^5 + \frac{1}{720}\frac{1}{(i\sqrt{3}-6)(i\sqrt{3}-5)(i\sqrt{3}-4)(-3+i\sqrt{3})(-2+i\sqrt{3})(-1+i\sqrt{3})}x^6 + \frac{1}{5040}\frac{1}{(i\sqrt{3}-7)(i\sqrt{3}-6)(i\sqrt{3}-5)(i\sqrt{3}-4)(-3+i\sqrt{3})(-2+i\sqrt{3})(-1+i\sqrt{3})}x^7 + O(x^8)\right) + c_2 x^{\frac{1}{2}+\frac{i\sqrt{3}}{2}} \left(1 - \frac{1}{1+i\sqrt{3}}x + \frac{1}{2}\frac{1}{(i\sqrt{3}+2)(1+i\sqrt{3})}x^2 - \frac{1}{6}\frac{1}{(i\sqrt{3}+3)(i\sqrt{3}+2)(1+i\sqrt{3})}x^3 + \frac{1}{24}\frac{1}{(i\sqrt{3}+4)(i\sqrt{3}+3)(i\sqrt{3}+2)(1+i\sqrt{3})}x^4 - \frac{1}{120}\frac{1}{(i\sqrt{3}+5)(i\sqrt{3}+4)(i\sqrt{3}+3)(i\sqrt{3}+2)(1+i\sqrt{3})}x^5 + \frac{1}{720}\frac{1}{(i\sqrt{3}+6)(i\sqrt{3}+5)(i\sqrt{3}+4)(i\sqrt{3}+3)(i\sqrt{3}+2)(1+i\sqrt{3})}x^6 - \frac{1}{5040}\frac{1}{(i\sqrt{3}+7)(i\sqrt{3}+6)(i\sqrt{3}+5)(i\sqrt{3}+4)(i\sqrt{3}+3)(i\sqrt{3}+2)(1+i\sqrt{3})}x^7 + O(x^8)\right) + c_3\left(1 - x + \frac{1}{3}x^2 - \frac{1}{21}x^3 + \frac{1}{273}x^4 - \frac{1}{5733}x^5 + \frac{1}{177723}x^6 - \frac{1}{7642089}x^7 + O(x^8)\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 3447

```
AsymptoticDSolveValue[x^3*y'''[x]+2*x^2*y''[x]+(x+x^2)*y'[x]+x*y[x]==0,y[x],{x,0,7}]
```

Too large to display

## 22.18 problem 3(b)

Internal problem ID [6493]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Problems for review and discovert. (A) Drill Exercises . Page 194
**Problem number**: 3(b).
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

Solve

$$x^3y''' + x^2y'' - 3xy' + y(x-1) = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 12916

```
Order:=8;
dsolve(x^3*diff(y(x),x$3)+x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+(x-1)*y(x)=0,y(x),type='series
```

Expression too large to display

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 11815

```
AsymptoticDSolveValue[x^3*y'''[x]+x^2*y''[x]-3*x*y'[x]+(x-1)*y[x]==0,y[x],{x,0,7}]
```

Too large to display

## 22.19 problem 3(c)

Internal problem ID [6494]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Problems for review and discovert. (A) Drill Exercises . Page 194
**Problem number**: 3(c).
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

Solve

$$\boxed{x^3y''' - 2x^2y'' + (x^2 + 2x)\, y' - yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 120

```
Order:=8;
dsolve(x^3*diff(y(x),x$3)-2*x^2*diff(y(x),x$2)+(x^2+2*x)*diff(y(x),x)-x*y(x)=0,y(x),type='se
```

$$\begin{aligned} y(x) = {} & c_1 x^3 \left(1 - \frac{1}{4}x + \frac{1}{40}x^2 - \frac{1}{720}x^3 + \frac{1}{20160}x^4 - \frac{1}{806400}x^5 + \frac{1}{43545600}x^6 \right. \\ & \left. - \frac{1}{3048192000}x^7 + \mathrm{O}\left(x^8\right)\right) + c_2 x^2 \left(\ln(x)\left((-240)\, x + 60x^2 - 6x^3 + \frac{1}{3}x^4 \right.\right. \\ & \left. - \frac{1}{84}x^5 + \frac{1}{3360}x^6 - \frac{1}{181440}x^7 + \mathrm{O}\left(x^8\right)\right) + \left(720 - 908x + 152x^2 - 11x^3 + \frac{4}{9}x^4 \right. \\ & \left.\left. - \frac{79}{7056}x^5 + \frac{517}{2822400}x^6 - \frac{851}{457228800}x^7 + \mathrm{O}\left(x^8\right)\right)\right) \\ & + c_3 \left(2\ln(x)\left(x^3 - \frac{1}{4}x^4 + \frac{1}{40}x^5 - \frac{1}{720}x^6 + \frac{1}{20160}x^7 + \mathrm{O}\left(x^8\right)\right) \right. \\ & \left. + \left(-24 - 12x - 6x^2 + \frac{5}{8}x^4 - \frac{39}{400}x^5 + \frac{49}{7200}x^6 - \frac{199}{705600}x^7 + \mathrm{O}\left(x^8\right)\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.664 (sec). Leaf size: 186

```
AsymptoticDSolveValue[x^3*y'''[x]-2*x^2*y''[x]+(x^2+2*x)*y'[x]-x*y[x]==0,y[x],{x,0,7}]
```

$$
\begin{aligned}
y(x) \rightarrow\; & c_1\left(\frac{(x^3-18x^2+180x-720)\,x^3\log(x)}{4320}\right. \\
&\left.+\frac{-167x^6+2466x^5-17100x^4+14400x^3+129600x^2+259200x+518400}{259200}\right) \\
&+c_2\left(\frac{x^3(x^5-40x^4+1120x^3-20160x^2+201600x-806400)\log(x)}{2419200}\right. \\
&\left.-\frac{x^2(2941x^6-106720x^5+2618560x^4-38666880x^3+268128000x^2-225792000x-2032128000)}{2032128000}\right) \\
&+c_3\left(\frac{x^9}{43545600}-\frac{x^8}{806400}+\frac{x^7}{20160}-\frac{x^6}{720}+\frac{x^5}{40}-\frac{x^4}{4}+x^3\right)
\end{aligned}
$$

## 22.20 problem 3(d)

Internal problem ID [6495]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Problems for review and discovert. (A) Drill Exercises . Page 194
**Problem number**: 3(d).
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

Solve

$$\boxed{x^3y''' + (2x^3 - x^2)\, y'' - y'x + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 1506

```
Order:=8;
dsolve(x^3*diff(y(x),x$3)+(2*x^3-x^2)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x),type='series
```

$$y(x) = c_3x\left(1+\mathrm{O}\left(x^8\right)\right) + c_2x^{\frac{3}{2}-\frac{\sqrt{13}}{2}}\left(1 - x + \frac{-3+\sqrt{13}}{2\sqrt{13}-4}x^2 + \frac{5-\sqrt{13}}{6\sqrt{13}-12}x^3 + \frac{1}{24}\frac{\left(-5+\sqrt{13}\right)\left(-7+\sqrt{13}\right)}{\left(-2+\sqrt{13}\right)\left(-4+\sqrt{13}\right)}x^4 + \frac{1}{30}\frac{-19+4\sqrt{13}}{\left(-2+\sqrt{13}\right)\left(-4+\sqrt{13}\right)}x^5 + \frac{1}{20}\frac{-29+7\sqrt{13}}{\left(-2+\sqrt{13}\right)\left(-4+\sqrt{13}\right)\left(-6+\sqrt{13}\right)}x^6 + \frac{-\frac{117}{35}+\frac{6\sqrt{13}}{7}}{\left(-2+\sqrt{13}\right)\left(-4+\sqrt{13}\right)\left(-6+\sqrt{13}\right)\left(-7+\sqrt{13}\right)}x^7 + \mathrm{O}\left(x^8\right)\right) + c_1x^{\frac{3}{2}+\frac{\sqrt{13}}{2}}\left(1 - x + \frac{3+\sqrt{13}}{4+2\sqrt{13}}x^2 + \frac{-5-\sqrt{13}}{6\sqrt{13}+12}x^3 + \frac{1}{24}\frac{\left(5+\sqrt{13}\right)\left(7+\sqrt{13}\right)}{\left(2+\sqrt{13}\right)\left(4+\sqrt{13}\right)}x^4 - \frac{1}{30}\frac{19+4\sqrt{13}}{\left(2+\sqrt{13}\right)\left(4+\sqrt{13}\right)}x^5 + \frac{1}{20}\frac{29+7\sqrt{13}}{\left(2+\sqrt{13}\right)\left(4+\sqrt{13}\right)\left(6+\sqrt{13}\right)}x^6 + \frac{-\frac{117}{35}-\frac{6\sqrt{13}}{7}}{\left(2+\sqrt{13}\right)\left(4+\sqrt{13}\right)\left(6+\sqrt{13}\right)\left(7+\sqrt{13}\right)}x^7 + \mathrm{O}\left(x^8\right)\right)$$

## ✓ Solution by Mathematica

Time used: 0.24 (sec). Leaf size: 310

```
AsymptoticDSolveValue[x^3*y'''[x]+(2*x^3-x^2)*y''[x]-y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$
\begin{aligned}
y(x) \to\ & c_1\left(\frac{99473x^7}{1008}+\frac{1043x^6}{144}+\frac{19x^5}{24}+\frac{11x^4}{24}-\frac{x^3}{6}+\frac{x^2}{2}+x+1\right) \\
& + c_2e^{-\frac{2}{\sqrt{x}}}\left(-\frac{279112936065458899252220570230691x^{13/2}}{160251477454333302276096}\right. \\
& -\frac{24300579025340445956934704 83x^{11/2}}{10031768169967779840 0}-\frac{1545013796231079344731x^{9/2}}{3562417673994240} \\
& -\frac{2005991558758787x^{7/2}}{193273528320}-\frac{43999069453x^{5/2}}{125829120}-\frac{438565x^{3/2}}{24576} \\
& +\frac{1443631997259645004783532051693861578 3x^7}{8974082737442664927461376 00} \\
& +\frac{384086400743305395636666 53617 51x^6}{192609948863381372928 00}+\frac{1786308115320202497636167x^5}{569986827839078400} \\
& +\frac{319234145332261451x^4}{4947802324992}+\frac{21959100963217x^3}{12079595520}+\frac{117706529x^2}{1572864}+\frac{2353x}{512}-\frac{29\sqrt{x}}{16} \\
& \left.+1\right)x^{11/4}+c_3e^{\frac{2}{\sqrt{x}}}\left(\frac{279112936065458899252220570230691x^{13/2}}{160251477454333302276096}+\frac{243005790253404459569347048 3x^{11/}}{100317681699677798400}\right.
\end{aligned}
$$

# 23 Chapter 4. Power Series Solutions and Special Functions. Problems for review and discovert. (B) Challenge Problems . Page 194

## 23.1 problem 1(a)

Internal problem ID [6496]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Problems for review and discovert. (B) Challenge Problems . Page 194
**Problem number**: 1(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^3y'' + x^2y' + y = 0}$$

With the expansion point for the power series method at $x = \infty$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 207

```
Order:=8;
dsolve(x^3*diff(y(x),x$2)+x^2*diff(y(x),x)+y(x)=0,y(x),type='series',x=Infinity);
```

$$y(x) = \left(1 - \frac{(x-\text{Infinity})^2}{2\,\text{Infinity}^3} + \frac{2(x-\text{Infinity})^3}{3\,\text{Infinity}^4} + \frac{(-18\,\text{Infinity}+1)\,(x-\text{Infinity})^4}{24\,\text{Infinity}^6} + \frac{(96\,\text{Infinity}-14)\,(x-\text{Infinity})^5}{120\,\text{Infinity}^7} + \frac{(-600\,\text{Infinity}^2+156\,\text{Infinity}-1)\,(x-\text{Infinity})^6}{720\,\text{Infinity}^9} + \frac{(4320\,\text{Infinity}^2-1692\,\text{Infinity}+30)\,(x-\text{Infinity})^7}{5040\,\text{Infinity}^{10}}\right) y(\text{Infinity}) + \left(x-\text{Infinity} - \frac{(x-\text{Infinity})^2}{2\,\text{Infinity}} + \frac{(2\,\text{Infinity}^2-\text{Infinity})\,(x-\text{Infinity})^3}{6\,\text{Infinity}^4} - \frac{\left(\text{Infinity}-\frac{4}{3}\right)(x-\text{Infinity})^4}{4\,\text{Infinity}^4} + \frac{(24\,\text{Infinity}^3-58\,\text{Infinity}^2+\text{Infinity})\,(x-\text{Infinity})^5}{120\,\text{Infinity}^7} + \frac{(-120\,\text{Infinity}^4+444\,\text{Infinity}^3-21\,\text{Infinity}^2)\,(x-\text{Infinity})^6}{720\,\text{Infinity}^9} + \frac{(720\,\text{Infinity}^4-3708\,\text{Infinity}^3+324\,\text{Infinity}^2-\text{Infinity})\,(x-\text{Infinity})^7}{5040\,\text{Infinity}^{10}}\right) D(y)(\text{Infinity}) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 171

```
AsymptoticDSolveValue[x^3*y''[x]+x^2*y'[x]+y[x]==0,y[x],{x,Infinity,7}]
```

$$y(x) \to c_1\left(-\frac{1}{25401600x^7}+\frac{1}{518400x^6}-\frac{1}{14400x^5}+\frac{1}{576x^4}-\frac{1}{36x^3}+\frac{1}{4x^2}-\frac{1}{x}+1\right) + c_2\left(\frac{121}{592704000x^7}+\frac{\log(x)}{25401600x^7}-\frac{49}{5184000x^6}-\frac{\log(x)}{518400x^6}+\frac{137}{432000x^5} + \frac{\log(x)}{14400x^5}-\frac{25}{3456x^4}-\frac{\log(x)}{576x^4}+\frac{11}{108x^3}+\frac{\log(x)}{36x^3}-\frac{3}{4x^2}-\frac{\log(x)}{4x^2}+\frac{2}{x}+\frac{\log(x)}{x} - \log(x)\right)$$

## 23.2 problem 1(b)

Internal problem ID [6497]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Problems for review and discovert. (B) Challenge Problems . Page 194
**Problem number**: 1(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9(x-2)^2 (x-3) y'' + 6x(x-2) y' + 16y = 0$$

With the expansion point for the power series method at $x = \infty$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 414

```
Order:=8;
dsolve(9*(x-2)^2*(x-3)*diff(y(x),x$2)+6*x*(x-2)*diff(y(x),x)+16*y(x)=0,y(x),type='series',x=
```

Expression too large to display

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 130

```
AsymptoticDSolveValue[9*(x-2)^2*(x-3)*y''[x]+6*x*(x-2)*y'[x]+16*y[x]==0,y[x],{x,Infinity,7}]
```

$$y(x) \to c_2 \left( -\frac{13}{3x^{2/3}} - \frac{251}{45x^{5/3}} - \frac{7781}{810x^{8/3}} - \frac{22151}{1215x^{11/3}} - \frac{669229}{18225x^{14/3}} - \frac{216463313}{2788425x^{17/3}} - \frac{7179886604}{41826375x^{20/3}} + \sqrt[3]{x} \right) + c_1 \left( -\frac{401483448544}{1336967775x^7} - \frac{4666732192}{40514175x^6} - \frac{822592}{18225x^5} - \frac{285704}{15795x^4} - \frac{3004}{405x^3} - \frac{28}{9x^2} - \frac{4}{3x} + 1 \right)$$

## 23.3 problem 1(c)

Internal problem ID [6498]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 4. Power Series Solutions and Special Functions. Problems for review and discovert. (B) Challenge Problems . Page 194
**Problem number**: 1(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$(-x^2+1)\,y''-2y'x+p(p+1)\,y=0$$

With the expansion point for the power series method at $x=\infty$.

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 1124

```
Order:=8;
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+p*(p+1)*y(x)=0,y(x),type='series',x=Infinity)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 2707

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-2*x*y'[x]+p*(p+1)*y[x]==0,y[x],{x,Infinity,7}]
```

$$
\begin{aligned}
y(x) \to \Bigg( & \frac{p^2x^{-p-7}}{-p^2-p+(p+6)(p+7)} + \frac{3px^{-p-7}}{-p^2-p+(p+6)(p+7)} \\
& + \frac{p^4x^{-p-7}}{(-p^2-p+(p+2)(p+3))\,(-p^2-p+(p+6)(p+7))} \\
& + \frac{6p^3x^{-p-7}}{(-p^2-p+(p+2)(p+3))\,(-p^2-p+(p+6)(p+7))} \\
& + \frac{17p^2x^{-p-7}}{(-p^2-p+(p+2)(p+3))\,(-p^2-p+(p+6)(p+7))} \\
& + \frac{24px^{-p-7}}{(-p^2-p+(p+2)(p+3))\,(-p^2-p+(p+6)(p+7))} \\
& + \frac{12x^{-p-7}}{(-p^2-p+(p+2)(p+3))\,(-p^2-p+(p+6)(p+7))} \\
& + \frac{p^4x^{-p-7}}{(-p^2-p+(p+4)(p+5))\,(-p^2-p+(p+6)(p+7))} \\
& + \frac{6p^3x^{-p-7}}{(-p^2-p+(p+4)(p+5))\,(-p^2-p+(p+6)(p+7))} \\
& + \frac{21p^2x^{-p-7}}{(-p^2-p+(p+4)(p+5))\,(-p^2-p+(p+6)(p+7))} \\
& + \frac{36px^{-p-7}}{(-p^2-p+(p+4)(p+5))\,(-p^2-p+(p+6)(p+7))} \\
& + \frac{p^6x^{-p-7}}{(-p^2-p+(p+2)(p+3))\,(-p^2-p+(p+4)(p+5))\,(-p^2-p+(p+6)(p+7))} \\
& + \frac{9p^5x^{-p-7}}{(-p^2-p+(p+2)(p+3))\,(-p^2-p+(p+4)(p+5))\,(-p^2-p+(p+6)(p+7))} \\
& + \frac{45p^4x^{-p-7}}{(-p^2-p+(p+2)(p+3))\,(-p^2-p+(p+4)(p+5))\,(-p^2-p+(p+6)(p+7))} \\
& + \frac{135p^3x^{-p-7}}{(-p^2-p+(p+2)(p+3))\,(-p^2-p+(p+4)(p+5))\,(-p^2-p+(p+6)(p+7))} \\
& + \frac{254p^2x^{-p-7}}{(-p^2-p+(p+2)(p+3))\,(-p^2-p+(p+4)(p+5))\,(-p^2-p+(p+6)(p+7))} \\
& + \frac{276px^{-p-7}}{(-p^2-p+(p+2)(p+3))\,(-p^2-p+(p+4)(p+5))\,(-p^2-p+(p+6)(p+7))} \\
& + \frac{120x^{-p-7}}{(-p^2-p+(p+2)(p+3))\,(-p^2-p+(p+4)(p+5))\,(-p^2-p+(p+6)(p+7))} \\
& + \frac{20x^{-p-7}}{(-p^2-p+(p+4)(p+5))\,(-p^2-p+(p+6)(p+7))} \\
& + \frac{2x^{-p-7}}{} + \frac{p^2x^{-p-5}}{} + \frac{3px^{-p-5}}{}
\end{aligned}
$$

# 24 Chapter 7. Laplace Transforms. Section 7.5 The Unit Step and Impulse Functions. Page 303

## 24.1 problem 1(a)

Internal problem ID [6499]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 7. Laplace Transforms. Section 7.5 The Unit Step and Impulse Functions. Page 303
**Problem number**: 1(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 5y' + 6y = 5\,\mathrm{e}^{3t}}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve([diff(y(t),t$2)+5*diff(y(t),t)+6*y(t)=5*exp(3*t),y(0) = 0, D(y)(0) = 0],y(t), singsol
```

$$y(t) = \frac{(\mathrm{e}^{6t} - 6\,\mathrm{e}^{t} + 5)\,\mathrm{e}^{-3t}}{6}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 26

```
DSolve[{y''[t]+5*y'[t]+6*y[t]==5*Exp[3*t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolution
```

$$y(t) \to \frac{1}{6}e^{-3t}\left(-6e^{t} + e^{6t} + 5\right)$$

## 24.2 problem 1(b)

Internal problem ID [6500]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 7. Laplace Transforms. Section 7.5 The Unit Step and Impulse Functions. Page 303
**Problem number**: 1(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' - 6y = t}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve([diff(y(t),t$2)+diff(y(t),t)-6*y(t)=t,y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{(9\,e^{5t} - 30t\,e^{3t} - 5\,e^{3t} - 4)\,e^{-3t}}{180}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 28

```
DSolve[{y''[t]+y'[t]-6*y[t]==t,{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{180}\left(-30t - 4e^{-3t} + 9e^{2t} - 5\right)$$

## 24.3 problem 1(c)

Internal problem ID [6501]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 7. Laplace Transforms. Section 7.5 The Unit Step and Impulse Functions. Page 303
**Problem number**: 1(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y = t^2}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)-y(t)=t^2,y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = e^{-t} + e^t - t^2 - 2$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 20

```
DSolve[{y''[t]-y[t]==t^2,{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -t^2 + e^{-t} + e^t - 2$$

## 24.4 problem 7(a)

Internal problem ID [6502]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 7. Laplace Transforms. Section 7.5 The Unit Step and Impulse Functions. Page 303
**Problem number**: 7(a).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{Li' + Ri = E_0 \operatorname{Heaviside}(t)}$$

With initial conditions

$$[i(0) = 0]$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 22

```
dsolve([L*diff(i(t),t)+R*i(t)=E__0*Heaviside(t),i(0) = 0],i(t), singsol=all)
```

$$i(t) = -\frac{E_0 \operatorname{Heaviside}(t)\left(e^{-\frac{Rt}{L}} - 1\right)}{R}$$

✓ Solution by Mathematica

Time used: 0.077 (sec). Leaf size: 25

```
DSolve[{L*i'[t]+R*i[t]==E0*UnitStep[t],{i[0]==0}},i[t],t,IncludeSingularSolutions -> True]
```

$$i(t) \to \frac{\mathrm{E0}\theta(t)\left(1 - e^{-\frac{Rt}{L}}\right)}{R}$$

## 24.5 problem 7(b)

Internal problem ID [6503]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 7. Laplace Transforms. Section 7.5 The Unit Step and Impulse Functions. Page 303
**Problem number**: 7(b).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{Li' + Ri = E_0(\delta(t))}$$

With initial conditions

$$[i(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 21

```
dsolve([L*diff(i(t),t)+R*i(t)=E__0*Dirac(t),i(0) = 0],i(t), singsol=all)
```

$$i(t) = \frac{E_0 \mathrm{e}^{-\frac{Rt}{L}} (2 \operatorname{Heaviside}(t) - 1)}{2L}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 26

```
DSolve[{L*i'[t]+R*i[t]==E0*DiracDelta[t],{i[0]==0}},i[t],t,IncludeSingularSolutions -> True]
```

$$i(t) \to \frac{\mathrm{E0}(\theta(t) - \theta(0))e^{-\frac{Rt}{L}}}{L}$$

## 24.6 problem 7(c)

Internal problem ID [6504]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 7. Laplace Transforms. Section 7.5 The Unit Step and Impulse Functions. Page 303
**Problem number**: 7(c).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{Li' + Ri = E_0 \sin(\omega t)}$$

With initial conditions

$$[i(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 45

```
dsolve([L*diff(i(t),t)+R*i(t)=E__0*sin(omega*t),i(0) = 0],i(t), singsol=all)
```

$$i(t) = \frac{E_0 \left(\mathrm{e}^{-\frac{Rt}{L}} L\omega - L\cos(\omega t)\,\omega + \sin(\omega t)\,R\right)}{\omega^2 L^2 + R^2}$$

✓ Solution by Mathematica

Time used: 0.114 (sec). Leaf size: 47

```
DSolve[{L*i'[t]+R*i[t]==E0*Sin[\[Omega]*t],{i[0]==0}},i[t],t,IncludeSingularSolutions -> Tru
```

$$i(t) \to \frac{\mathrm{E0}\left(L\omega e^{-\frac{Rt}{L}} - L\omega\cos(t\omega) + R\sin(t\omega)\right)}{L^2\omega^2 + R^2}$$

# 25 Chapter 7. Laplace Transforms. Section 7.5 Problesm for review and discovery. Section A, Drill exercises. Page 309

## 25.1 problem 3(a)

Internal problem ID [6505]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 7. Laplace Transforms. Section 7.5 Problesm for review and discovery. Section A, Drill exercises. Page 309
**Problem number**: 3(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 3y' - 5y = 1}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 40

```
dsolve([diff(y(t),t$2)+3*diff(y(t),t)-5*y(t)=1,y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = \frac{\left(29 + 13\sqrt{29}\right) e^{\frac{\left(-3+\sqrt{29}\right)t}{2}}}{290} - \frac{1}{5} + \frac{\left(29 - 13\sqrt{29}\right) e^{-\frac{\left(3+\sqrt{29}\right)t}{2}}}{290}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 67

```
DSolve[{y''[t]+3*y'[t]-5*y[t]==1,{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to \frac{1}{290} e^{-\frac{1}{2}\left(3+\sqrt{29}\right)t}\left(\left(29 + 13\sqrt{29}\right) e^{\sqrt{29}t} - 58e^{\frac{1}{2}\left(3+\sqrt{29}\right)t} + 29 - 13\sqrt{29}\right)$$

## 25.2 problem 3(b)

Internal problem ID [6506]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 7. Laplace Transforms. Section 7.5 Problesm for review and discovery. Section A, Drill exercises. Page 309
**Problem number**: 3(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 3y' - 2y = -6\,e^{\pi - t}}$$

With initial conditions

$$[y(\pi) = 1, y'(\pi) = 4]$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 57

```
dsolve([diff(y(t),t$2)+3*diff(y(t),t)-2*y(t)=-6*exp(Pi-t),y(Pi) = 1, D(y)(Pi) = 4],y(t), sin
```

$$y(t) = \frac{\left(19\sqrt{17} - 17\right) e^{-\frac{\left(-3+\sqrt{17}\right)(\pi - t)}{2}}}{68} + \frac{\left(-19\sqrt{17} - 17\right) e^{\frac{\left(3+\sqrt{17}\right)(\pi - t)}{2}}}{68} + \frac{3\,e^{\pi - t}}{2}$$

✓ Solution by Mathematica

Time used: 0.354 (sec). Leaf size: 103

```
DSolve[{y''[t]+3*y'[t]-2*y[t]==-6*Exp[Pi-t],{y[Pi]==1,y'[Pi]==4}},y[t],t,IncludeSingularSolu
```

$$y(t) \to \frac{1}{68} e^{-\frac{1}{2}\left(3+\sqrt{17}\right)t - \frac{1}{2}\left(\sqrt{17}-3\right)\pi} \left( \left(19\sqrt{17} - 17\right) e^{\sqrt{17}t} + 102 e^{\frac{1}{2}\left(\left(1+\sqrt{17}\right)t + \left(\sqrt{17}-1\right)\pi\right)} - \left(\left(17 + 19\sqrt{17}\right) e^{\sqrt{17}\pi}\right) \right)$$

## 25.3 problem 3(c)

Internal problem ID [6507]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 7. Laplace Transforms. Section 7.5 Problesm for review and discovery. Section A, Drill exercises. Page 309
**Problem number**: 3(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' - y = t\,\mathrm{e}^{-t}}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 39

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)-y(t)=t*exp(-t),y(0) = 0, D(y)(0) = 1],y(t), singsol=al
```

$$y(t) = \frac{3\,\mathrm{e}^{\left(\sqrt{2}-1\right)t}\sqrt{2}}{8} - \frac{3\,\mathrm{e}^{-\left(1+\sqrt{2}\right)t}\sqrt{2}}{8} - \frac{\mathrm{e}^{-t}t}{2}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 51

```
DSolve[{y''[t]+2*y'[t]-y[t]==t*Exp[-t],{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions -
```

$$y(t) \rightarrow \frac{1}{8}e^{-t}\left(-4t - 3\sqrt{2}e^{-\sqrt{2}t} + 3\sqrt{2}e^{\sqrt{2}t}\right)$$

## 25.4 problem 3(d)

Internal problem ID [6508]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 7. Laplace Transforms. Section 7.5 Problesm for review and discovery. Section A, Drill exercises. Page 309
**Problem number**: 3(d).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y' + y = 3\,\mathrm{e}^{-t}}$$

With initial conditions

$$[y(0) = 3, y'(0) = 2]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 38

```
dsolve([diff(y(t),t$2)-diff(y(t),t)+y(t)=3*exp(-t),y(0) = 3, D(y)(0) = 2],y(t), singsol=all)
```

$$y(t) = \frac{\left(4\,\mathrm{e}^{\frac{3t}{2}} \sin\left(\frac{\sqrt{3}\,t}{2}\right)\sqrt{3} + 6\,\mathrm{e}^{\frac{3t}{2}} \cos\left(\frac{\sqrt{3}\,t}{2}\right) + 3\right)\mathrm{e}^{-t}}{3}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 56

```
DSolve[{y''[t]-y'[t]+y[t]==3*Exp[-t],{y[0]==3,y'[0]==2}},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \to e^{-t} + \frac{4e^{t/2}\sin\left(\frac{\sqrt{3}t}{2}\right)}{\sqrt{3}} + 2e^{t/2}\cos\left(\frac{\sqrt{3}t}{2}\right)$$

## 25.5 problem 4(a)

Internal problem ID [6509]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 7. Laplace Transforms. Section 7.5 Problesm for review and discovery. Section A, Drill exercises. Page 309
**Problem number**: 4(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 5y' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(diff(y(t),t$2)-5*diff(y(t),t)+4*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 e^t + c_2 e^{4t}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 20

```
DSolve[y''[t]-5*y'[t]+4*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^t \left(c_2 e^{3t} + c_1\right)$$

## 25.6 problem 4(b)

Internal problem ID [6510]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 7. Laplace Transforms. Section 7.5 Problesm for review and discovery. Section A, Drill exercises. Page 309
**Problem number**: 4(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 3y' + 3y = 2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(t),t$2)+3*diff(y(t),t)+3*y(t)=2,y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-\frac{3t}{2}} \sin\left(\frac{\sqrt{3}\,t}{2}\right) c_2 + \mathrm{e}^{-\frac{3t}{2}} \cos\left(\frac{\sqrt{3}\,t}{2}\right) c_1 + \frac{2}{3}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 51

```
DSolve[y''[t]+3*y'[t]+3*y[t]==2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_2 e^{-3t/2} \cos\left(\frac{\sqrt{3}t}{2}\right) + c_1 e^{-3t/2} \sin\left(\frac{\sqrt{3}t}{2}\right) + \frac{2}{3}$$

## 25.7 problem 4(c)

Internal problem ID [6511]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 7. Laplace Transforms. Section 7.5 Problesm for review and discovery. Section A, Drill exercises. Page 309
**Problem number**: 4(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' + 2y = t}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(t),t$2)+diff(y(t),t)+2*y(t)=t,y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-\frac{t}{2}} \sin\left(\frac{\sqrt{7}\,t}{2}\right) c_2 + \mathrm{e}^{-\frac{t}{2}} \cos\left(\frac{\sqrt{7}\,t}{2}\right) c_1 - \frac{1}{4} + \frac{t}{2}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 56

```
DSolve[y''[t]+y'[t]+2*y[t]==t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{t}{2} + c_2 e^{-t/2} \cos\left(\frac{\sqrt{7}t}{2}\right) + c_1 e^{-t/2} \sin\left(\frac{\sqrt{7}t}{2}\right) - \frac{1}{4}$$

## 25.8 problem 4(d)

Internal problem ID [6512]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 7. Laplace Transforms. Section 7.5 Problesm for review and discovery. Section A, Drill exercises. Page 309
**Problem number**: 4(d).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 7y' + 12y = t\,e^{2t}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(t),t$2)-7*diff(y(t),t)+12*y(t)=t*exp(2*t),y(t), singsol=all)
```

$$y(t) = c_2 e^{3t} + e^{4t} c_1 + \frac{(2t+3)\,e^{2t}}{4}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 35

```
DSolve[y''[t]-7*y'[t]+12*y[t]==t*Exp[2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{4}e^{2t}\left(2t + 4c_1 e^t + 4c_2 e^{2t} + 3\right)$$

# 26 Chapter 7. Laplace Transforms. Section 7.5 Problesm for review and discovery. Section B, Challenge Problems. Page 310

## 26.1 problem 3

Internal problem ID [6513]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 7. Laplace Transforms. Section 7.5 Problesm for review and discovery. Section B, Challenge Problems. Page 310
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$i'' + 2i' + 3i = \begin{cases} 30 & 0 < t < 2\pi \\ 0 & 2\pi \leq t \leq 5\pi \\ 10 & 5\pi < t < \infty \end{cases}$$

With initial conditions

$$[i(0) = 8, i'(0) = 0]$$

✗ Solution by Maple

```
dsolve([diff(i(t),t$2)+2*diff(i(t),t)+3*i(t)=piecewise(0<t and t<2*Pi,30,2*Pi<= t and t<= 5*
```

No solution found

✓ Solution by Mathematica

Time used: 0.234 (sec). Leaf size: 297

```
DSolve[{i''[t]+2*i'[t]+3*i[t]==Piecewise[{{30,0<t<2*Pi},{0,2*Pi<= t <= 5*Pi},{10,5*Pi<t<Infi
```

$i(t)$

$$\to \{ \begin{array}{c} e^{-t}\left(-2\cos\left(\sqrt{2}t\right)+10e^t-\sqrt{2}\sin\left(\sqrt{2}t\right)\right) \\ 4e^{-t}\left(2\cos\left(\sqrt{2}t\right)+\sqrt{2}\sin\left(\sqrt{2}t\right)\right) \\ -e^{-t}\left(2\cos\left(\sqrt{2}t\right)-10e^{2\pi}\cos\left(\sqrt{2}(t-2\pi)\right)+\sqrt{2}\left(\sin\left(\sqrt{2}t\right)-5e\right.\right. \\ \frac{1}{3}e^{-t}\left(-6\cos\left(\sqrt{2}t\right)+10e^t-10e^{5\pi}\cos\left(\sqrt{2}(t-5\pi)\right)+30e^{2\pi}\cos\left(\sqrt{2}(t-2\pi)\right)-3\sqrt{2}\sin\left(\sqrt{2}t\right)-\right. \end{array}$$

# 27 Chapter 10. Systems of First-Order Equations. Section 10.2 Linear Systems. Page 380

## 27.1 problem 2(a)

Internal problem ID [6514]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section 10.2 Linear Systems. Page 380
**Problem number**: 2(a).
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = x(t) + 3y(t)$$
$$y'(t) = 3x(t) + y(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 35

```
dsolve([diff(x(t),t)=x(t)+3*y(t),diff(y(t),t)=3*x(t)+y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -c_1 e^{-2t} + c_2 e^{4t}$$

$$y(t) = c_1 e^{-2t} + c_2 e^{4t}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 68

```
DSolve[{x'[t]==x[t]+3*y[t],y'[t]==3*x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tru
```

$$x(t) \to \frac{1}{2}e^{-2t}\left(c_1\left(e^{6t}+1\right)+c_2\left(e^{6t}-1\right)\right)$$
$$y(t) \to \frac{1}{2}e^{-2t}\left(c_1\left(e^{6t}-1\right)+c_2\left(e^{6t}+1\right)\right)$$

## 27.2 problem 2(c)

Internal problem ID [6515]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section 10.2 Linear Systems. Page 380
**Problem number**: 2(c).
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = x(t) + 3y(t)$$
$$y'(t) = 3x(t) + y(t)$$

With initial conditions

$$[x(0) = 5, y(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve([diff(x(t),t) = x(t)+3*y(t), diff(y(t),t) = 3*x(t)+y(t), x(0) = 5, y(0) = 1],[x(t), y
```

$$x(t) = 2\,\mathrm{e}^{-2t} + 3\,\mathrm{e}^{4t}$$

$$y(t) = -2\,\mathrm{e}^{-2t} + 3\,\mathrm{e}^{4t}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 38

```
DSolve[{x'[t]==x[t]+3*y[t],y'[t]==3*x[t]+y[t]},{x[0]==5,y[0]==1},{x[t],y[t]},t,IncludeSingul
```

$$x(t) \to e^{-2t}\left(3e^{6t} + 2\right)$$
$$y(t) \to e^{-2t}\left(3e^{6t} - 2\right)$$

## 27.3 problem 3(a)

Internal problem ID [6516]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section 10.2 Linear Systems. Page 380
**Problem number**: 3(a).
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = x(t) + 2y(t)$$
$$y'(t) = 3x(t) + 2y(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=x(t)+2*y(t),diff(y(t),t)=3*x(t)+2*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -c_1 e^{-t} + \frac{2c_2 e^{4t}}{3}$$

$$y(t) = c_1 e^{-t} + c_2 e^{4t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 74

```
DSolve[{x'[t]==x[t]+2*y[t],y'[t]==3*x[t]+2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> T
```

$$x(t) \to \frac{1}{5}e^{-t}\left(c_1\left(2e^{5t}+3\right)+2c_2\left(e^{5t}-1\right)\right)$$
$$y(t) \to \frac{1}{5}e^{-t}\left(3c_1\left(e^{5t}-1\right)+c_2\left(3e^{5t}+2\right)\right)$$

## 27.4 problem 3(c)

Internal problem ID [6517]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section 10.2 Linear Systems. Page 380
**Problem number**: 3(c).
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = x(t) + 2y(t) + t - 1$$
$$y'(t) = 3x(t) + 2y(t) - 5t - 2$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 44

```
dsolve([diff(x(t),t)=x(t)+2*y(t)+t-1,diff(y(t),t)=3*x(t)+2*y(t)-5*t-2],[x(t), y(t)], singsol
```

$$x(t) = -e^{-t}c_2 + \frac{2\,e^{4t}c_1}{3} - 2 + 3t$$

$$y(t) = e^{-t}c_2 + e^{4t}c_1 - 2t + 3$$

✓ Solution by Mathematica

Time used: 0.188 (sec). Leaf size: 88

```
DSolve[{x'[t]==x[t]+2*y[t]+t-1,y'[t]==3*x[t]+2*y[t]-5*t-2},{x[t],y[t]},t,IncludeSingularSolu
```

$$x(t) \to \frac{1}{5}e^{-t}\left(5e^t(3t-2) + 2(c_1+c_2)e^{5t} + 3c_1 - 2c_2\right)$$
$$y(t) \to \frac{1}{5}e^{-t}\left(-5e^t(2t-3) + 3(c_1+c_2)e^{5t} - 3c_1 + 2c_2\right)$$

## 27.5 problem 5

Internal problem ID [6518]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section 10.2 Linear Systems. Page 380
**Problem number**: 5.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = x(t) + y(t)$$
$$y'(t) = y(t)$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 20

```
dsolve([diff(x(t),t)=x(t)+y(t),diff(y(t),t)=y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = (c_2t + c_1)\,\mathrm{e}^t$$

$$y(t) = c_2\mathrm{e}^t$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 25

```
DSolve[{x'[t]==x[t]+y[t],y'[t]==y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^t(c_2t + c_1)$$
$$y(t) \to c_2e^t$$

## 27.6 problem 6(a)

Internal problem ID [6519]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section 10.2 Linear Systems. Page 380
**Problem number**: 6(a).
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = x(t)$$
$$y'(t) = y(t)$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 16

```
dsolve([diff(x(t),t)=x(t),diff(y(t),t)=y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = c_1 e^t$$

$$y(t) = c_2 e^t$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 57

```
DSolve[{x'[t]==x[t],y'[t]==y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 e^t$$
$$y(t) \to c_2 e^t$$
$$x(t) \to c_1 e^t$$
$$y(t) \to 0$$
$$x(t) \to 0$$
$$y(t) \to c_2 e^t$$
$$x(t) \to 0$$
$$y(t) \to 0$$

# 28 Chapter 10. Systems of First-Order Equations. Section 10.3 Homogeneous Linear Systems with Constant Coefficients. Page 387

## 28.1 problem 1(a)

Internal problem ID [6520]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section 10.3 Homogeneous Linear Systems with Constant Coefficients. Page 387
**Problem number**: 1(a).
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = -3x(t) + 4y(t)$$
$$y'(t) = -2x(t) + 3y(t)$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 31

```
dsolve([diff(x(t),t)=-3*x(t)+4*y(t),diff(y(t),t)=-2*x(t)+3*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = 2c_1 e^{-t} + c_2 e^{t}$$

$$y(t) = c_1 e^{-t} + c_2 e^{t}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 67

```
DSolve[{x'[t]==-3*x[t]+4*y[t],y'[t]==-2*x[t]+3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions
```

$$x(t) \to e^{-t}\left(2c_2\left(e^{2t} - 1\right) - c_1\left(e^{2t} - 2\right)\right)$$
$$y(t) \to e^{-t}\left(c_2\left(2e^{2t} - 1\right) - c_1\left(e^{2t} - 1\right)\right)$$

## 28.2 problem 1(b)

Internal problem ID [6521]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section 10.3 Homogeneous Linear Systems with Constant Coefficients. Page 387
**Problem number**: 1(b).
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = 4x(t) - 2y(t)$$
$$y'(t) = 5x(t) + 2y(t)$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 59

```
dsolve([diff(x(t),t)=4*x(t)-2*y(t),diff(y(t),t)=5*x(t)+2*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = \frac{e^{3t}(\sin(3t)\,c_1 - 3\sin(3t)\,c_2 + 3\cos(3t)\,c_1 + \cos(3t)\,c_2)}{5}$$

$$y(t) = e^{3t}(\sin(3t)\,c_1 + \cos(3t)\,c_2)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 70

```
DSolve[{x'[t]==4*x[t]-2*y[t],y'[t]==5*x[t]+2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{1}{3}e^{3t}(3c_1\cos(3t) + (c_1 - 2c_2)\sin(3t))$$
$$y(t) \to \frac{1}{3}e^{3t}(3c_2\cos(3t) + (5c_1 - c_2)\sin(3t))$$

## 28.3 problem 1(c)

Internal problem ID [6522]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section 10.3 Homogeneous Linear Systems with Constant Coefficients. Page 387
**Problem number**: 1(c).
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = 5x(t) + 4y(t)$$
$$y'(t) = -x(t) + y(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 33

```
dsolve([diff(x(t),t)=5*x(t)+4*y(t),diff(y(t),t)=-x(t)+y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -e^{3t}(2c_2t + 2c_1 + c_2)$$

$$y(t) = e^{3t}(c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 46

```
DSolve[{x'[t]==5*x[t]+4*y[t],y'[t]==-x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tr
```

$$x(t) \to e^{3t}(2c_1t + 4c_2t + c_1)$$
$$y(t) \to e^{3t}(c_2 - (c_1 + 2c_2)t)$$

## 28.4 problem 1(d)

Internal problem ID [6523]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section 10.3 Homogeneous Linear Systems with Constant Coefficients. Page 387
**Problem number**: 1(d).
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = 4x(t) - 3y(t)$$
$$y'(t) = 8x(t) - 6y(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 27

```
dsolve([diff(x(t),t)=4*x(t)-3*y(t),diff(y(t),t)=8*x(t)-6*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = \frac{e^{-2t}c_2}{2} + \frac{3c_1}{4}$$

$$y(t) = c_1 + e^{-2t}c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 59

```
DSolve[{x'[t]==4*x[t]-3*y[t],y'[t]==8*x[t]-6*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to c_1(3 - 2e^{-2t}) + \frac{3}{2}c_2(e^{-2t} - 1)$$
$$y(t) \to c_1(4 - 4e^{-2t}) + c_2(3e^{-2t} - 2)$$

## 28.5 problem 1(e)

Internal problem ID [6524]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section 10.3 Homogeneous Linear Systems with Constant Coefficients. Page 387
**Problem number**: 1(e).
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = 2x(t)$$
$$y'(t) = 3y(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 20

```
dsolve([diff(x(t),t)=2*x(t),diff(y(t),t)=3*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = c_1 \mathrm{e}^{2t}$$

$$y(t) = c_2 \mathrm{e}^{3t}$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 65

```
DSolve[{x'[t]==2*x[t],y'[t]==3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t)\to c_1e^{2t}$$
$$y(t)\to c_2e^{3t}$$
$$x(t)\to c_1e^{2t}$$
$$y(t)\to 0$$
$$x(t)\to 0$$
$$y(t)\to c_2e^{3t}$$
$$x(t)\to 0$$
$$y(t)\to 0$$

## 28.6 problem 1(f)

Internal problem ID [6525]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section 10.3 Homogeneous Linear Systems with Constant Coefficients. Page 387
**Problem number**: 1(f).
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = -4x(t) - y(t)$$
$$y'(t) = x(t) - 2y(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 32

```
dsolve([diff(x(t),t)=-4*x(t)-y(t),diff(y(t),t)=x(t)-2*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -e^{-3t}(c_2 t + c_1 - c_2)$$

$$y(t) = e^{-3t}(c_2 t + c_1)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 43

```
DSolve[{x'[t]==-4*x[t]-y[t],y'[t]==x[t]-2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tr
```

$$x(t) \to e^{-3t}(c_1(-t) - c_2 t + c_1)$$
$$y(t) \to e^{-3t}((c_1 + c_2)t + c_2)$$

## 28.7 problem 1(g)

Internal problem ID [6526]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section 10.3 Homogeneous Linear Systems with Constant Coefficients. Page 387
**Problem number**: 1(g).
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = 7x(t) + 6y(t)$$
$$y'(t) = 2x(t) + 6y(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=7*x(t)+6*y(t),diff(y(t),t)=2*x(t)+6*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{3c_1 e^{3t}}{2} + 2c_2 e^{10t}$$

$$y(t) = c_1 e^{3t} + c_2 e^{10t}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 74

```
DSolve[{x'[t]==7*x[t]+6*y[t],y'[t]==2*x[t]+6*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{1}{7}e^{3t}\left(c_1\left(4e^{7t}+3\right)+6c_2\left(e^{7t}-1\right)\right)$$
$$y(t) \to \frac{1}{7}e^{3t}\left(2c_1\left(e^{7t}-1\right)+c_2\left(3e^{7t}+4\right)\right)$$

## 28.8 problem 1(h)

Internal problem ID [6527]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section 10.3 Homogeneous Linear Systems with Constant Coefficients. Page 387
**Problem number**: 1(h).
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = x(t) - 2y(t)$$
$$y'(t) = 4x(t) + 5y(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 58

```
dsolve([diff(x(t),t)=x(t)-2*y(t),diff(y(t),t)=4*x(t)+5*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{e^{3t}(\sin(2t)c_1 + \sin(2t)c_2 - \cos(2t)c_1 + \cos(2t)c_2)}{2}$$

$$y(t) = e^{3t}(\sin(2t)c_1 + \cos(2t)c_2)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 59

```
DSolve[{x'[t]==x[t]-2*y[t],y'[t]==4*x[t]+5*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> T
```

$$x(t) \to e^{3t}(c_1 \cos(2t) - (c_1 + c_2)\sin(2t))$$
$$y(t) \to e^{3t}(c_2 \cos(2t) + (2c_1 + c_2)\sin(2t))$$

## 28.9 problem 5(b)

Internal problem ID [6528]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section 10.3 Homogeneous Linear Systems with Constant Coefficients. Page 387
**Problem number**: 5(b).
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = x(t) + y(t) - 5t + 2$$
$$y'(t) = 4x(t) - 2y(t) - 8t - 8$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 43

```
dsolve([diff(x(t),t)=x(t)+y(t)-5*t+2,diff(y(t),t)=4*x(t)-2*y(t)-8*t-8],[x(t), y(t)], singsol
```

$$x(t) = -\frac{c_2 \mathrm{e}^{-3t}}{4} + c_1 \mathrm{e}^{2t} + 2 + 3t$$

$$y(t) = c_2 \mathrm{e}^{-3t} + c_1 \mathrm{e}^{2t} + 2t - 1$$

✓ Solution by Mathematica

Time used: 0.15 (sec). Leaf size: 92

```
DSolve[{x'[t]==x[t]+y[t]-5*t+2,y'[t]==4*x[t]-2*y[t]-8*t-8},{x[t],y[t]},t,IncludeSingularSolu
```

$$x(t) \to \frac{1}{5} e^{-3t} \left(5e^{3t}(3t+2) + (4c_1 + c_2)e^{5t} + c_1 - c_2\right)$$
$$y(t) \to \frac{1}{5} e^{-3t} \left(5e^{3t}(2t-1) + (4c_1 + c_2)e^{5t} - 4c_1 + 4c_2\right)$$

# 29 Chapter 10. Systems of First-Order Equations. Section A. Drill exercises. Page 400

## 29.1 problem 2(a)

Internal problem ID [6529]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section A. Drill exercises. Page 400
**Problem number**: 2(a).
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = 3x(t) - 4y(t)$$
$$y'(t) = 4x(t) - 7y(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 32

```
dsolve([diff(x(t),t)=3*x(t)-4*y(t),diff(y(t),t)=4*x(t)-7*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = 2c_1 e^t + \frac{c_2 e^{-5t}}{2}$$

$$y(t) = c_1 e^t + c_2 e^{-5t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 73

```
DSolve[{x'[t]==3*x[t]-4*y[t],y'[t]==4*x[t]-7*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{1}{3} e^{-5t} \left(c_1 \left(4e^{6t} - 1\right) - 2c_2 \left(e^{6t} - 1\right)\right)$$
$$y(t) \to \frac{1}{3} e^{-5t} \left(2c_1 \left(e^{6t} - 1\right) - c_2 \left(e^{6t} - 4\right)\right)$$

## 29.2 problem 2(b)

Internal problem ID [6530]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section A. Drill exercises. Page 400
**Problem number**: 2(b).
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = x(t) + y(t)$$
$$y'(t) = 4x(t) + y(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=x(t)+y(t),diff(y(t),t)=4*x(t)+y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = \frac{c_1 \mathrm{e}^{3t}}{2} - \frac{\mathrm{e}^{-t} c_2}{2}$$

$$y(t) = c_1 \mathrm{e}^{3t} + \mathrm{e}^{-t} c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 70

```
DSolve[{x'[t]==x[t]+y[t],y'[t]==4*x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{4} e^{-t} \left(2c_1 \left(e^{4t} + 1\right) + c_2 \left(e^{4t} - 1\right)\right)$$
$$y(t) \to \frac{1}{2} e^{-t} \left(2c_1 \left(e^{4t} - 1\right) + c_2 \left(e^{4t} + 1\right)\right)$$

## 29.3 problem 2(c)

Internal problem ID [6531]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section A. Drill exercises. Page 400
**Problem number**: 2(c).
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = -3x(t) + \sqrt{2}\, y(t)$$
$$y'(t) = \sqrt{2}\, x(t) - 2y(t)$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 41

```
dsolve([diff(x(t),t)=-3*x(t)+sqrt(2)*y(t),diff(y(t),t)=sqrt(2)*x(t)-2*y(t)],[x(t), y(t)], si
```

$$x(t) = -\frac{\left(2c_1 e^{-4t} - e^{-t} c_2\right)\sqrt{2}}{2}$$

$$y(t) = c_1 e^{-4t} + e^{-t} c_2$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 80

```
DSolve[{x'[t]==-3*x[t]+Sqrt[2]*y[t],y'[t]==Sqrt[2]*x[t]-2*y[t]},{x[t],y[t]},t,IncludeSingula
```

$$x(t) \to \frac{1}{3} e^{-4t} \left(c_1 \left(e^{3t} + 2\right) + \sqrt{2} c_2 \left(e^{3t} - 1\right)\right)$$
$$y(t) \to \frac{1}{3} e^{-4t} \left(\sqrt{2} c_1 \left(e^{3t} - 1\right) + c_2 \left(2e^{3t} + 1\right)\right)$$

## 29.4 problem 2(d)

Internal problem ID [6532]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section A. Drill exercises. Page 400
**Problem number**: 2(d).
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = 5x(t) + 3y(t)$$
$$y'(t) = -6x(t) - 4y(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=5*x(t)+3*y(t),diff(y(t),t)=-6*x(t)-4*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{c_1 \mathrm{e}^{-t}}{2} - c_2 \mathrm{e}^{2t}$$

$$y(t) = c_1 \mathrm{e}^{-t} + c_2 \mathrm{e}^{2t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 66

```
DSolve[{x'[t]==5*x[t]+3*y[t],y'[t]==-6*x[t]-4*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -
```

$$x(t) \to e^{-t}\left(c_1\left(2e^{3t} - 1\right) + c_2\left(e^{3t} - 1\right)\right)$$
$$y(t) \to e^{-t}\left(-2c_1\left(e^{3t} - 1\right) - c_2\left(e^{3t} - 2\right)\right)$$

## 29.5 problem 3(a)

Internal problem ID [6533]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section A. Drill exercises. Page 400
**Problem number**: 3(a).
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = 3x(t) + 2y(t)$$
$$y'(t) = -2x(t) - y(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 29

```
dsolve([diff(x(t),t)=3*x(t)+2*y(t),diff(y(t),t)=-2*x(t)-y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{e^t(2c_2t + 2c_1 + c_2)}{2}$$

$$y(t) = (c_2t + c_1)\,e^t$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 40

```
DSolve[{x'[t]==3*x[t]+2*y[t],y'[t]==-2*x[t]-y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to e^t(2c_1t + 2c_2t + c_1)$$
$$y(t) \to e^t(c_2 - 2(c_1 + c_2)t)$$

## 29.6 problem 3(b)

Internal problem ID [6534]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section A. Drill exercises. Page 400
**Problem number**: 3(b).
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = x(t) + y(t)$$
$$y'(t) = -x(t) + y(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 34

```
dsolve([diff(x(t),t)=x(t)+y(t),diff(y(t),t)=-x(t)+y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -e^{t}(\cos(t) c_1 - \sin(t) c_2)$$

$$y(t) = e^{t}(c_2 \cos(t) + c_1 \sin(t))$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 39

```
DSolve[{x'[t]==x[t]+y[t],y'[t]==-x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^t(c_1 \cos(t) + c_2 \sin(t))$$
$$y(t) \to e^t(c_2 \cos(t) - c_1 \sin(t))$$

## 29.7 problem 3(c)

Internal problem ID [6535]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section A. Drill exercises. Page 400
**Problem number**: 3(c).
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = 3x(t) - 5y(t)$$
$$y'(t) = -x(t) + 2y(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 86

```
dsolve([diff(x(t),t)=3*x(t)-5*y(t),diff(y(t),t)=-x(t)+2*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{c_1 \mathrm{e}^{\frac{(5+\sqrt{21})t}{2}}\sqrt{21}}{2} + \frac{c_2 \mathrm{e}^{-\frac{(-5+\sqrt{21})t}{2}}\sqrt{21}}{2} - \frac{c_1 \mathrm{e}^{\frac{(5+\sqrt{21})t}{2}}}{2} - \frac{c_2 \mathrm{e}^{-\frac{(-5+\sqrt{21})t}{2}}}{2}$$

$$y(t) = c_1 \mathrm{e}^{\frac{(5+\sqrt{21})t}{2}} + c_2 \mathrm{e}^{-\frac{(-5+\sqrt{21})t}{2}}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 144

```
DSolve[{x'[t]==3*x[t]-5*y[t],y'[t]==-x[t]+2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{1}{42}e^{-\frac{1}{2}\left(\sqrt{21}-5\right)t}\left(c_1\left(\left(21+\sqrt{21}\right)e^{\sqrt{21}t} + 21 - \sqrt{21}\right) - 10\sqrt{21}c_2\left(e^{\sqrt{21}t} - 1\right)\right)$$
$$y(t) \to -\frac{1}{42}e^{-\frac{1}{2}\left(\sqrt{21}-5\right)t}\left(2\sqrt{21}c_1\left(e^{\sqrt{21}t} - 1\right) + c_2\left(\left(\sqrt{21} - 21\right)e^{\sqrt{21}t} - 21 - \sqrt{21}\right)\right)$$

## 29.8 problem 3(d)

Internal problem ID [6536]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section A. Drill exercises. Page 400
**Problem number**: 3(d).
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = x(t) + 2y(t)$$
$$y'(t) = -4x(t) + y(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 57

```
dsolve([diff(x(t),t)=x(t)+2*y(t),diff(y(t),t)=-4*x(t)+y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{\sqrt{2}\,e^{t}\left(\cos\left(2\sqrt{2}\,t\right)c_1 - \sin\left(2\sqrt{2}\,t\right)c_2\right)}{2}$$

$$y(t) = e^{t}\left(c_2\cos\left(2\sqrt{2}\,t\right) + c_1\sin\left(2\sqrt{2}\,t\right)\right)$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 79

```
DSolve[{x'[t]==x[t]+2*y[t],y'[t]==-4*x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tr
```

$$x(t) \to c_1e^{t}\cos\left(2\sqrt{2}t\right) + \frac{c_2e^{t}\sin\left(2\sqrt{2}t\right)}{\sqrt{2}}$$
$$y(t) \to e^{t}\left(c_2\cos\left(2\sqrt{2}t\right) - \sqrt{2}c_1\sin\left(2\sqrt{2}t\right)\right)$$

## 29.9 problem 3(e)

Internal problem ID [6537]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section A. Drill exercises. Page 400
**Problem number**: 3(e).
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = 3x(t) + 2y(t) + z(t)$$
$$y'(t) = -2x(t) - y(t) + 3z(t)$$
$$z'(t) = x(t) + y(t) + z(t)$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 63

```
dsolve([diff(x(t),t)=3*x(t)+2*y(t)+z(t),diff(y(t),t)=-2*x(t)-y(t)+3*z(t),diff(z(t),t)=x(t)+y
```

$$x(t) = \frac{5c_2 e^{3t}}{2} - c_1 e^t + \frac{3c_3 e^{-t}}{2}$$

$$y(t) = -\frac{c_2 e^{3t}}{2} + c_1 e^t - \frac{7c_3 e^{-t}}{2}$$

$$z(t) = c_2 e^{3t} + c_3 e^{-t}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 180

```
DSolve[{x'[t]==3*x[t]+2*y[t]+z[t],y'[t]==-2*x[t]-y[t]+3*z[t],z'[t]==x[t]+y[t]+z[t]},{x[t],y[
```

$$x(t) \to \frac{1}{8}e^{-t}\left(c_1\left(6e^{2t}+5e^{4t}-3\right)+\left(e^{2t}-1\right)\left(c_2\left(5e^{2t}+3\right)+2c_3\left(5e^{2t}-3\right)\right)\right)$$

$$y(t) \to \frac{1}{8}e^{-t}\left(-\left(c_1\left(6e^{2t}+e^{4t}-7\right)\right)+c_2\left(2e^{2t}-e^{4t}+7\right)-2c_3\left(-8e^{2t}+e^{4t}+7\right)\right)$$

$$z(t) \to \frac{1}{4}e^{-t}\left(c_1\left(e^{4t}-1\right)+c_2\left(e^{4t}-1\right)+2c_3\left(e^{4t}+1\right)\right)$$

## 29.10 problem 3(f)

Internal problem ID [6538]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section A. Drill exercises. Page 400
**Problem number**: 3(f).
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$
\begin{aligned}
x'(t) &= -x(t) + y(t) - z(t) \\
y'(t) &= 2x(t) - y(t) - 4z(t) \\
z'(t) &= 3x(t) - y(t) + z(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.265 (sec). Leaf size: 2535

```
dsolve([diff(x(t),t)=-x(t)+y(t)-z(t),diff(y(t),t)=2*x(t)-y(t)-4*z(t),diff(z(t),t)=3*x(t)-y(t
```

Expression too large to display

Expression too large to display

$$
\begin{aligned}
z(t) = {} & c_2 \mathrm{e}^{\frac{\left(13+\left(154+3\sqrt{2391}\right)^{\frac{2}{3}}-2\left(154+3\sqrt{2391}\right)^{\frac{1}{3}}\right)t}{6\left(154+3\sqrt{2391}\right)^{\frac{1}{3}}}} \sin\left(\frac{\sqrt{3}\left(\left(154+3\sqrt{2391}\right)^{\frac{2}{3}}-13\right)t}{6\left(154+3\sqrt{2391}\right)^{\frac{1}{3}}}\right) \\
& + c_3 \mathrm{e}^{\frac{\left(13+\left(154+3\sqrt{2391}\right)^{\frac{2}{3}}-2\left(154+3\sqrt{2391}\right)^{\frac{1}{3}}\right)t}{6\left(154+3\sqrt{2391}\right)^{\frac{1}{3}}}} \cos\left(\frac{\sqrt{3}\left(\left(154+3\sqrt{2391}\right)^{\frac{2}{3}}-13\right)t}{6\left(154+3\sqrt{2391}\right)^{\frac{1}{3}}}\right) \\
& + c_1 \mathrm{e}^{-\frac{\left(\left(154+3\sqrt{2391}\right)^{\frac{2}{3}}+\left(154+3\sqrt{2391}\right)^{\frac{1}{3}}+13\right)t}{3\left(154+3\sqrt{2391}\right)^{\frac{1}{3}}}}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 501

```
DSolve[{x'[t]==-x[t]+y[t]-z[t],y'[t]==2*x[t]-y[t]-4*z[t],z'[t]==3*x[t]-y[t]+z[t]},{x[t],y[t]
```

$$
\begin{aligned}
x(t) \to\ & c_2\text{RootSum}\left[\#1^3+\#1^2-4\#1+10\&,\frac{\#1e^{\#1t}}{3\#1^2+2\#1-4}\&\right]\\
&- c_3\text{RootSum}\left[\#1^3+\#1^2-4\#1+10\&,\frac{\#1e^{\#1t}+5e^{\#1t}}{3\#1^2+2\#1-4}\&\right]\\
&+ c_1\text{RootSum}\left[\#1^3+\#1^2-4\#1+10\&,\frac{\#1^2e^{\#1t}-5e^{\#1t}}{3\#1^2+2\#1-4}\&\right]\\
y(t) \to\ & 2c_1\text{RootSum}\left[\#1^3+\#1^2-4\#1+10\&,\frac{\#1e^{\#1t}-7e^{\#1t}}{3\#1^2+2\#1-4}\&\right]\\
&- 2c_3\text{RootSum}\left[\#1^3+\#1^2-4\#1+10\&,\frac{2\#1e^{\#1t}+3e^{\#1t}}{3\#1^2+2\#1-4}\&\right]\\
&+ c_2\text{RootSum}\left[\#1^3+\#1^2-4\#1+10\&,\frac{\#1^2e^{\#1t}+2e^{\#1t}}{3\#1^2+2\#1-4}\&\right]\\
z(t) \to\ & -c_2\text{RootSum}\left[\#1^3+\#1^2-4\#1+10\&,\frac{\#1e^{\#1t}-2e^{\#1t}}{3\#1^2+2\#1-4}\&\right]\\
&+ c_1\text{RootSum}\left[\#1^3+\#1^2-4\#1+10\&,\frac{3\#1e^{\#1t}+e^{\#1t}}{3\#1^2+2\#1-4}\&\right]\\
&+ c_3\text{RootSum}\left[\#1^3+\#1^2-4\#1+10\&,\frac{\#1^2e^{\#1t}+2\#1e^{\#1t}-e^{\#1t}}{3\#1^2+2\#1-4}\&\right]
\end{aligned}
$$

## 29.11 problem 4(a)

Internal problem ID [6539]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section A. Drill exercises. Page 400
**Problem number**: 4(a).
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = x(t) + 2y(t) - 4t + 1$$
$$y'(t) = -x(t) + 2y(t) + 3t + 4$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 106

```
dsolve([diff(x(t),t)=x(t)+2*y(t)-4*t+1,diff(y(t),t)=-x(t)+2*y(t)+3*t+4],[x(t), y(t)], singsc
```

$$x(t) = \frac{e^{\frac{3t}{2}} \sin\left(\frac{\sqrt{7}\,t}{2}\right) c_2}{2} - \frac{e^{\frac{3t}{2}} \sqrt{7}\cos\left(\frac{\sqrt{7}\,t}{2}\right) c_2}{2} + \frac{e^{\frac{3t}{2}} \cos\left(\frac{\sqrt{7}\,t}{2}\right) c_1}{2} + \frac{e^{\frac{3t}{2}} \sqrt{7}\sin\left(\frac{\sqrt{7}\,t}{2}\right) c_1}{2} + \frac{25}{8} + \frac{7t}{2}$$

$$y(t) = e^{\frac{3t}{2}} \sin\left(\frac{\sqrt{7}\,t}{2}\right) c_2 + e^{\frac{3t}{2}} \cos\left(\frac{\sqrt{7}\,t}{2}\right) c_1 + \frac{t}{4} - \frac{5}{16}$$

✓ Solution by Mathematica

Time used: 2.624 (sec). Leaf size: 128

```
DSolve[{x'[t]==x[t]+2*y[t]-4+t+1,y'[t]==-x[t]+2*y[t]+3*t+4},{x[t],y[t]},t,IncludeSingularSol
```

$$x(t) \to t + c_1 e^{3t/2} \cos\left(\frac{\sqrt{7}t}{2}\right) - \frac{(c_1 - 4c_2)e^{3t/2} \sin\left(\frac{\sqrt{7}t}{2}\right)}{\sqrt{7}} + \frac{9}{2}$$

$$y(t) \to -t + c_2 e^{3t/2} \cos\left(\frac{\sqrt{7}t}{2}\right) - \frac{(2c_1 - c_2)e^{3t/2} \sin\left(\frac{\sqrt{7}t}{2}\right)}{\sqrt{7}} - \frac{1}{4}$$

## 29.12 problem 4(b)

Internal problem ID [6540]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section A. Drill exercises. Page 400
**Problem number**: 4(b).
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = -2x(t) + y(t) - t + 3$$
$$y'(t) = x(t) + 4y(t) + t - 2$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 90

```
dsolve([diff(x(t),t)=-2*x(t)+y(t)-t+3,diff(y(t),t)=x(t)+4*y(t)+t-2],[x(t), y(t)], singsol=al
```

$$x(t) = e^{\left(1+\sqrt{10}\right)t} c_2 \sqrt{10} - e^{-\left(-1+\sqrt{10}\right)t} c_1 \sqrt{10} - 3\, e^{\left(1+\sqrt{10}\right)t} c_2 - 3\, e^{-\left(-1+\sqrt{10}\right)t} c_1 - \frac{5t}{9} + \frac{145}{81}$$

$$y(t) = e^{\left(1+\sqrt{10}\right)t} c_2 + e^{-\left(-1+\sqrt{10}\right)t} c_1 - \frac{t}{9} + \frac{2}{81}$$

✓ Solution by Mathematica

Time used: 10.617 (sec). Leaf size: 190

```
DSolve[{x'[t]==-2*x[t]+y[t]-t+3,y'[t]==x[t]+4*y[t]+t-2},{x[t],y[t]},t,IncludeSingularSolutio
```

$$x(t) \to -\frac{e^{t-\sqrt{10}t}\left(100e^{\left(\sqrt{10}-1\right)t}(9t-29)+81\left(\left(3\sqrt{10}-10\right)c_1-\sqrt{10}c_2\right)e^{2\sqrt{10}t}-81\left(10+3\sqrt{10}\right)c_1+81\sqrt{10}c_2}{1620}$$

$$y(t) \to \frac{e^{t-\sqrt{10}t}\left(-20e^{\left(\sqrt{10}-1\right)t}(9t-2)+81\left(\sqrt{10}c_1+\left(10+3\sqrt{10}\right)c_2\right)e^{2\sqrt{10}t}-81\left(\sqrt{10}c_1+\left(3\sqrt{10}-10\right)c_2\right)\right)}{1620}$$

## 29.13 problem 4(c)

Internal problem ID [6541]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section A. Drill exercises. Page 400
**Problem number**: 4(c).
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = -4x(t) + y(t) - t + 3$$
$$y'(t) = -x(t) - 5y(t) + t + 1$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 106

```
dsolve([diff(x(t),t)=-4*x(t)+y(t)-t+3,diff(y(t),t)=-x(t)-5*y(t)+t+1],[x(t), y(t)], singsol=a
```

$$x(t) = -\frac{e^{-\frac{9t}{2}} \sin\left(\frac{\sqrt{3}\,t}{2}\right) c_2}{2} - \frac{e^{-\frac{9t}{2}} \sqrt{3}\,\cos\left(\frac{\sqrt{3}\,t}{2}\right) c_2}{2} - \frac{e^{-\frac{9t}{2}} \cos\left(\frac{\sqrt{3}\,t}{2}\right) c_1}{2} + \frac{e^{-\frac{9t}{2}} \sqrt{3}\,\sin\left(\frac{\sqrt{3}\,t}{2}\right) c_1}{2} + \frac{39}{49} - \frac{4t}{21}$$

$$y(t) = e^{-\frac{9t}{2}} \sin\left(\frac{\sqrt{3}\,t}{2}\right) c_2 + e^{-\frac{9t}{2}} \cos\left(\frac{\sqrt{3}\,t}{2}\right) c_1 + \frac{5t}{21} - \frac{1}{147}$$

✓ Solution by Mathematica

Time used: 2.267 (sec). Leaf size: 131

```
DSolve[{x'[t]==-4*x[t]+y[t]-t+3,y'[t]==-x[t]-5*y[t]+t+1},{x[t],y[t]},t,IncludeSingularSoluti
```

$$x(t) \to -\frac{4t}{21} + c_1 e^{-9t/2} \cos\left(\frac{\sqrt{3}t}{2}\right) + \frac{(c_1 + 2c_2)e^{-9t/2} \sin\left(\frac{\sqrt{3}t}{2}\right)}{\sqrt{3}} + \frac{39}{49}$$

$$y(t) \to \frac{5t}{21} + c_2 e^{-9t/2} \cos\left(\frac{\sqrt{3}t}{2}\right) - \frac{(2c_1 + c_2)e^{-9t/2} \sin\left(\frac{\sqrt{3}t}{2}\right)}{\sqrt{3}} - \frac{1}{147}$$

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## 30.1 problem 1

Internal problem ID [6542]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section B. Challenge Problems. Page 401
**Problem number**: 1.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = x(t)\, y(t) + 1$$
$$y'(t) = -x(t) + y(t)$$

With initial conditions

$$[x(0) = 2, y(0) = -1]$$

✗ Solution by Maple

```
dsolve([diff(x(t),t) = x(t)*y(t)+1, diff(y(t),t) = -x(t)+y(t), x(0) = 2, y(0) = -1],[x(t), y
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x'[t]==x[t]*y[t]+1,y'[t]==-x[t]+y[t]},{x[0]==2,y[0]==-1},{x[t],y[t]},t,IncludeSingul
```

Not solved

## 30.2 problem 2

Internal problem ID [6543]

**Book**: Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.
**Section**: Chapter 10. Systems of First-Order Equations. Section B. Challenge Problems. Page 401
**Problem number**: 2.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = ty(t) + 1$$
$$y'(t) = -x(t)\,t + y(t)$$

With initial conditions

$$[x(0) = 0, y(0) = -1]$$

✗ Solution by Maple

```
dsolve([diff(x(t),t) = t*y(t)+1, diff(y(t),t) = -t*x(t)+y(t), x(0) = 0, y(0) = -1],[x(t), y(
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x'[t]==x[t]*y[t]+1,y'[t]==-x[t]+y[t]},{x[0]==2,y[0]==-1},{x[t],y[t]},t,IncludeSingul
```

Not solved