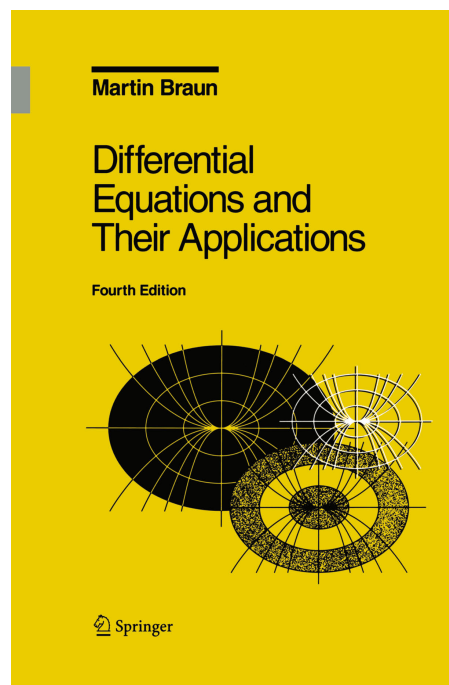


A Solution Manual For

Differential equations and their applications, 4th ed., M. Braun



Nasser M. Abbasi

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**1 Section 3.8, Systems of differential equations.
The eigenvalue-eigenvector method. Page 339**

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1.1 problem 1

Internal problem ID [1824]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.8, Systems of differential equations. The eigenvalue-eigenvector method.

Page 339

Problem number: 1.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = 6x_1(t) - 3x_2(t)$$

$$x_2'(t) = 2x_1(t) + x_2(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 35

```
dsolve([diff(x__1(t),t)=6*x__1(t)-3*x__2(t),diff(x__2(t),t)=2*x__1(t)+1*x__2(t)],[x__1(t), x
```

$$x_1(t) = \frac{3c_1 e^{4t}}{2} + c_2 e^{3t}$$

$$x_2(t) = c_1 e^{4t} + c_2 e^{3t}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 60

```
DSolve[{x1'[t]==6*x1[t]-3*x2[t],x2'[t]==2*x1[t]+1*x2[t]},{x1[t],x2[t]},t,IncludeSingularSolu
```

$$x_1(t) \rightarrow e^{3t}(c_1(3e^t - 2) - 3c_2(e^t - 1))$$

$$x_2(t) \rightarrow e^{3t}(2c_1(e^t - 1) + c_2(3 - 2e^t))$$

1.2 problem 2

Internal problem ID [1825]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.8, Systems of differential equations. The eigenvalue-eigenvector method.

Page 339

Problem number: 2.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x_1'(t) &= -2x_1(t) + x_2(t) \\x_2'(t) &= -4x_1(t) + 3x_2(t)\end{aligned}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 35

```
dsolve([diff(x__1(t),t)=-2*x__1(t)+1*x__2(t),diff(x__2(t),t)=-4*x__1(t)+3*x__2(t)],[x__1(t),
```

$$x_1(t) = e^{-t}c_1 + \frac{c_2e^{2t}}{4}$$

$$x_2(t) = e^{-t}c_1 + c_2e^{2t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 72

```
DSolve[{x1'[t]==-2*x1[t]+1*x2[t],x2'[t]==-4*x1[t]+3*x2[t]},{x1[t],x2[t]},t,IncludeSingularSo
```

$$x_1(t) \rightarrow \frac{1}{3}e^{-t}(c_2(e^{3t} - 1) - c_1(e^{3t} - 4))$$

$$x_2(t) \rightarrow \frac{1}{3}e^{-t}(c_2(4e^{3t} - 1) - 4c_1(e^{3t} - 1))$$

1.3 problem 3

Internal problem ID [1826]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.8, Systems of differential equations. The eigenvalue-eigenvector method.

Page 339

Problem number: 3.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = 3x_1(t) + 2x_2(t) + 4x_3(t)$$

$$x_2'(t) = 2x_1(t) + 2x_3(t)$$

$$x_3'(t) = 4x_1(t) + 2x_2(t) + 3x_3(t)$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 66

```
dsolve([diff(x__1(t),t)=3*x__1(t)+2*x__2(t)+4*x__3(t),diff(x__2(t),t)=2*x__1(t)+0*x__2(t)+2*
```

$$x_1(t) = c_2 e^{8t} - \frac{5c_3 e^{-t}}{4} - \frac{e^{-t} c_1}{2}$$

$$x_2(t) = \frac{c_2 e^{8t}}{2} + \frac{c_3 e^{-t}}{2} + e^{-t} c_1$$

$$x_3(t) = c_2 e^{8t} + c_3 e^{-t}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 135

```
DSolve[{x1'[t]==3*x1[t]+2*x2[t]+4*x3[t],x2'[t]==2*x1[t]+0*x2[t]+2*x3[t],x3'[t]==4*x1[t]+2*x2[t]}
```

$$x1(t) \rightarrow \frac{1}{9}e^{-t}(c_1(4e^{9t} + 5) + 2(c_2 + 2c_3)(e^{9t} - 1))$$

$$x2(t) \rightarrow \frac{1}{9}e^{-t}(2c_1(e^{9t} - 1) + c_2(e^{9t} + 8) + 2c_3(e^{9t} - 1))$$

$$x3(t) \rightarrow \frac{1}{9}e^{-t}(4c_1(e^{9t} - 1) + 2c_2(e^{9t} - 1) + c_3(4e^{9t} + 5))$$

1.4 problem 4

Internal problem ID [1827]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.8, Systems of differential equations. The eigenvalue-eigenvector method.

Page 339

Problem number: 4.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = 7x_1(t) - x_2(t) + 6x_3(t)$$

$$x_2'(t) = -10x_1(t) + 4x_2(t) - 12x_3(t)$$

$$x_3'(t) = -2x_1(t) + x_2(t) - x_3(t)$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 73

```
dsolve([diff(x__1(t),t)=7*x__1(t)-1*x__2(t)+6*x__3(t),diff(x__2(t),t)=-10*x__1(t)+4*x__2(t)-
```

$$x_1(t) = -c_1 e^{2t} - \frac{3c_2 e^{5t}}{2} - c_3 e^{3t}$$

$$x_2(t) = c_1 e^{2t} + 3c_2 e^{5t} + 2c_3 e^{3t}$$

$$x_3(t) = c_1 e^{2t} + c_2 e^{5t} + c_3 e^{3t}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 153

```
DSolve[{x1'[t]==7*x1[t]-1*x2[t]+6*x3[t],x2'[t]==-10*x1[t]+4*x2[t]-12*x3[t],x3'[t]==-2*x1[t]+
```

$$x1(t) \rightarrow e^{2t}(c_1(-4e^t + 3e^{3t} + 2) - c_2(e^t - 1) + 3c_3e^t(e^{2t} - 1))$$

$$x2(t) \rightarrow -e^{2t}(c_1(-8e^t + 6e^{3t} + 2) + c_2(1 - 2e^t) + 6c_3e^t(e^{2t} - 1))$$

$$x3(t) \rightarrow e^{2t}(-2c_1(-2e^t + e^{3t} + 1) + c_2(e^t - 1) + c_3e^t(3 - 2e^{2t}))$$

1.5 problem 5

Internal problem ID [1828]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.8, Systems of differential equations. The eigenvalue-eigenvector method.

Page 339

Problem number: 5.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = -7x_1(t) + 6x_3(t)$$

$$x_2'(t) = 5x_2(t)$$

$$x_3'(t) = 6x_1(t) + 2x_3(t)$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 45

```
dsolve([diff(x__1(t),t)=-7*x__1(t)+0*x__2(t)+6*x__3(t),diff(x__2(t),t)=0*x__1(t)+5*x__2(t)+0
```

$$x_1(t) = -2c_2e^{-10t} + \frac{c_3e^{5t}}{2}$$

$$x_2(t) = c_1e^{5t}$$

$$x_3(t) = c_2e^{-10t} + c_3e^{5t}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 158

```
DSolve[{x1'[t]==-7*x1[t]+0*x2[t]+6*x3[t],x2'[t]==0*x1[t]+5*x2[t]+0*x3[t],x3'[t]==6*x1[t]+0*x
```

$$x1(t) \rightarrow \frac{1}{5}e^{-10t}(c_1(e^{15t} + 4) + 2c_2(e^{15t} - 1))$$

$$x3(t) \rightarrow \frac{1}{5}e^{-10t}(2c_1(e^{15t} - 1) + c_2(4e^{15t} + 1))$$

$$x2(t) \rightarrow c_3e^{5t}$$

$$x1(t) \rightarrow \frac{1}{5}e^{-10t}(c_1(e^{15t} + 4) + 2c_2(e^{15t} - 1))$$

$$x3(t) \rightarrow \frac{1}{5}e^{-10t}(2c_1(e^{15t} - 1) + c_2(4e^{15t} + 1))$$

$$x2(t) \rightarrow 0$$

1.6 problem 6

Internal problem ID [1829]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.8, Systems of differential equations. The eigenvalue-eigenvector method.

Page 339

Problem number: 6.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) + 2x_2(t) + 3x_3(t) + 6x_4(t)$$

$$x_2'(t) = 3x_1(t) + 6x_2(t) + 9x_3(t) + 18x_4(t)$$

$$x_3'(t) = 5x_1(t) + 10x_2(t) + 15x_3(t) + 30x_4(t)$$

$$x_4'(t) = 7x_1(t) + 14x_2(t) + 21x_3(t) + 42x_4(t)$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 63

```
dsolve([diff(x__1(t),t)=1*x__1(t)+2*x__2(t)+3*x__3(t)+6*x__4(t),diff(x__2(t),t)=3*x__1(t)+6*
```

$$x_1(t) = \frac{c_4 e^{64t}}{7} - 9c_3 - 2c_1 - 3c_2$$

$$x_2(t) = \frac{3c_3}{7} + \frac{3c_4 e^{64t}}{7} + c_1$$

$$x_3(t) = \frac{5c_3}{7} + \frac{5c_4 e^{64t}}{7} + c_2$$

$$x_4(t) = c_3 + c_4 e^{64t}$$

✓ Solution by Mathematica

Time used: 0.089 (sec). Leaf size: 554

`DSolve[{x1'[t]==1*x1[t]+2*x2[t]+3*x3[t]+6*x4[t],x2'[t]==3*x1[t]+6*x2[t]+9*x3[t]+19*x4[t],x3'`

$$x1(t) \rightarrow \frac{e^{-\sqrt{1038}t} \left(2076(7c_1 - c_4)e^{\sqrt{1038}t} - (7\sqrt{1038}c_1 + 14\sqrt{1038}c_2 + 21\sqrt{1038}c_3 + 10\sqrt{1038}c_4 - 1038c_4) e^{32t} \right)}{14532}$$

$$x2(t) \rightarrow \frac{(7(519 + 13\sqrt{1038})c_1 + 14(519 + 13\sqrt{1038})c_2 + 273\sqrt{1038}c_3 + 10899c_3 - 389\sqrt{1038}c_4 - 8304c_4) e^{-\sqrt{1038}t}}{14532}$$

$$x3(t) \rightarrow \frac{e^{-\sqrt{1038}t} \left(2076(7c_3 - 5c_4)e^{\sqrt{1038}t} - 5(7\sqrt{1038}c_1 + 14\sqrt{1038}c_2 + 21\sqrt{1038}c_3 + 10\sqrt{1038}c_4 - 1038c_4) e^{32t} \right)}{14532}$$

$$x4(t) \rightarrow \frac{e^{-((\sqrt{1038}-32)t)} \left(7c_1 \left(e^{2\sqrt{1038}t} - 1 \right) + 14c_2 \left(e^{2\sqrt{1038}t} - 1 \right) + 21c_3 e^{2\sqrt{1038}t} + \sqrt{1038}c_4 e^{2\sqrt{1038}t} + 10c_4 e^{2\sqrt{1038}t} \right)}{2\sqrt{1038}}$$

1.7 problem 7

Internal problem ID [1830]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.8, Systems of differential equations. The eigenvalue-eigenvector method.

Page 339

Problem number: 7.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) + x_2(t)$$

$$x_2'(t) = 4x_1(t) + x_2(t)$$

With initial conditions

$$[x_1(0) = 2, x_2(0) = 3]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 34

```
dsolve([diff(x__1(t),t) = x__1(t)+x__2(t), diff(x__2(t),t) = 4*x__1(t)+x__2(t)], x__1(0) = 2,
```

$$x_1(t) = \frac{e^{-t}}{4} + \frac{7e^{3t}}{4}$$

$$x_2(t) = -\frac{e^{-t}}{2} + \frac{7e^{3t}}{2}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 44

```
DSolve[{x1'[t]==1*x1[t]+1*x2[t],x2'[t]==4*x1[t]+1*x2[t]},{x1[0]==2,x2[0]==3},{x1[t],x2[t]},t
```

$$x1(t) \rightarrow \frac{1}{4}e^{-t}(7e^{4t} + 1)$$

$$x2(t) \rightarrow \frac{1}{2}e^{-t}(7e^{4t} - 1)$$

1.8 problem 8

Internal problem ID [1831]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.8, Systems of differential equations. The eigenvalue-eigenvector method.

Page 339

Problem number: 8.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x_1'(t) &= x_1(t) - 3x_2(t) \\x_2'(t) &= -2x_1(t) + 2x_2(t)\end{aligned}$$

With initial conditions

$$[x_1(0) = 0, x_2(0) = 5]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 34

```
dsolve([diff(x__1(t),t) = x__1(t)-3*x__2(t), diff(x__2(t),t) = -2*x__1(t)+2*x__2(t), x__1(0)
```

$$x_1(t) = 3e^{-t} - 3e^{4t}$$

$$x_2(t) = 2e^{-t} + 3e^{4t}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 37

```
DSolve[{x1'[t]==1*x1[t]-3*x2[t],x2'[t]==-2*x1[t]+2*x2[t]},{x1[0]==0,x2[0]==5},{x1[t],x2[t]},
```

$$x1(t) \rightarrow -3e^{-t}(e^{5t} - 1)$$

$$x2(t) \rightarrow e^{-t}(3e^{5t} + 2)$$

1.9 problem 9

Internal problem ID [1832]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.8, Systems of differential equations. The eigenvalue-eigenvector method.

Page 339

Problem number: 9.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x_1'(t) &= 3x_1(t) + x_2(t) - x_3(t) \\x_2'(t) &= x_1(t) + 3x_2(t) - x_3(t) \\x_3'(t) &= 3x_1(t) + 3x_2(t) - x_3(t)\end{aligned}$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = -2, x_3(0) = -1]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 27

```
dsolve([diff(x__1(t),t) = 3*x__1(t)+x__2(t)-x__3(t), diff(x__2(t),t) = x__1(t)+3*x__2(t)-x__3(t), diff(x__3(t),t) = 3*x__1(t)+3*x__2(t)-x__3(t)], t)
```

$$x_1(t) = e^{2t}$$

$$x_2(t) = -2e^{2t}$$

$$x_3(t) = -e^{2t}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 30

```
DSolve[{x1'[t]==3*x1[t]+1*x2[t]-1*x3[t],x2'[t]==1*x1[t]+3*x2[t]-1*x3[t],x3'[t]==3*x1[t]+3*x2[t]-1*x3[t]}
```

$$x1(t) \rightarrow e^{2t}$$

$$x2(t) \rightarrow -2e^{2t}$$

$$x3(t) \rightarrow -e^{2t}$$

1.10 problem 10

Internal problem ID [1833]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.8, Systems of differential equations. The eigenvalue-eigenvector method.

Page 339

Problem number: 10.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) - x_2(t)$$

$$x_2'(t) = x_1(t) + 2x_2(t) + x_3(t)$$

$$x_3'(t) = x_1(t) + 10x_2(t) + 2x_3(t)$$

With initial conditions

$$[x_1(0) = -1, x_2(0) = -4, x_3(0) = 13]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 37

```
dsolve([diff(x__1(t),t) = x__1(t)-x__2(t), diff(x__2(t),t) = x__1(t)+2*x__2(t)+x__3(t), diff
```

$$x_1(t) = -2e^{-t} + e^t$$

$$x_2(t) = -4e^{-t}$$

$$x_3(t) = 14e^{-t} - e^t$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 42

```
DSolve[{x1'[t]==1*x1[t]-1*x2[t]-0*x3[t],x2'[t]==1*x1[t]+2*x2[t]+1*x3[t],x3'[t]==1*x1[t]+10*x
```

$$x1(t) \rightarrow e^t - 2e^{-t}$$

$$x2(t) \rightarrow -4e^{-t}$$

$$x3(t) \rightarrow 14e^{-t} - e^t$$

1.11 problem 11

Internal problem ID [1834]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.8, Systems of differential equations. The eigenvalue-eigenvector method.

Page 339

Problem number: 11.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) - 3x_2(t) + 2x_3(t)$$

$$x_2'(t) = -x_2(t)$$

$$x_3'(t) = -x_2(t) - 2x_3(t)$$

With initial conditions

$$[x_1(0) = -2, x_2(0) = 0, x_3(0) = 3]$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 24

```
dsolve([diff(x__1(t),t) = x__1(t)-3*x__2(t)+2*x__3(t), diff(x__2(t),t) = -x__2(t), diff(x__3
```

$$x_1(t) = -2e^{-2t}$$

$$x_2(t) = 0$$

$$x_3(t) = 3e^{-2t}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 26

```
DSolve[{x1'[t]==1*x1[t]-3*x2[t]+2*x3[t],x2'[t]==0*x1[t]-1*x2[t]+0*x3[t],x3'[t]==0*x1[t]-1*x2[t]}
```

$$x1(t) \rightarrow -2e^{-2t}$$

$$x2(t) \rightarrow 0$$

$$x3(t) \rightarrow 3e^{-2t}$$

1.12 problem 12

Internal problem ID [1835]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.8, Systems of differential equations. The eigenvalue-eigenvector method.

Page 339

Problem number: 12.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = 3x_1(t) + x_2(t) - 2x_3(t)$$

$$x_2'(t) = -x_1(t) + 2x_2(t) + x_3(t)$$

$$x_3'(t) = 4x_1(t) + x_2(t) - 3x_3(t)$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 4, x_3(0) = -7]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 58

```
dsolve([diff(x__1(t),t) = 3*x__1(t)+x__2(t)-2*x__3(t), diff(x__2(t),t) = -x__1(t)+2*x__2(t)+
```

$$x_1(t) = \frac{4e^{2t}}{3} - \frac{28e^{-t}}{3} + 9e^t$$

$$x_2(t) = \frac{4e^{2t}}{3} + \frac{8e^{-t}}{3}$$

$$x_3(t) = \frac{4e^{2t}}{3} - \frac{52e^{-t}}{3} + 9e^t$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 75

```
DSolve[{x1'[t]==3*x1[t]+1*x2[t]-2*x3[t],x2'[t]==-1*x1[t]+2*x2[t]+1*x3[t],x3'[t]==4*x1[t]+1*x
```

$$x1(t) \rightarrow -\frac{28e^{-t}}{3} + 9e^t + \frac{4e^{2t}}{3}$$

$$x2(t) \rightarrow \frac{4}{3}e^{-t}(e^{3t} + 2)$$

$$x3(t) \rightarrow -\frac{52e^{-t}}{3} + 9e^t + \frac{4e^{2t}}{3}$$

**2 Section 3.9, Systems of differential equations.
Complex roots. Page 344**

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2.1 problem 1

Internal problem ID [1836]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.9, Systems of differential equations. Complex roots. Page 344

Problem number: 1.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = -3x_1(t) + 2x_2(t)$$

$$x_2'(t) = -x_1(t) - x_2(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 45

```
dsolve([diff(x__1(t),t)=-3*x__1(t)+2*x__2(t),diff(x__2(t),t)=-1*x__1(t)-1*x__2(t)],[x__1(t),
```

$$x_1(t) = e^{-2t}(c_1 \sin(t) + c_2 \sin(t) - \cos(t) c_1 + c_2 \cos(t))$$

$$x_2(t) = e^{-2t}(c_1 \sin(t) + c_2 \cos(t))$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 53

```
DSolve[{x1'[t]==-3*x1[t]+2*x2[t],x2'[t]==-1*x1[t]-1*x2[t]},{x1[t],x2[t]},t,IncludeSingularSo
```

$$x1(t) \rightarrow e^{-2t}(c_1 \cos(t) - (c_1 - 2c_2) \sin(t))$$

$$x2(t) \rightarrow e^{-2t}(c_2 \cos(t) + (c_2 - c_1) \sin(t))$$

2.2 problem 2

Internal problem ID [1837]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.9, Systems of differential equations. Complex roots. Page 344

Problem number: 2.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) - 5x_2(t)$$

$$x_2'(t) = x_1(t) - 3x_2(t)$$

$$x_3'(t) = x_3(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 54

```
dsolve([diff(x__1(t),t)=1*x__1(t)-5*x__2(t)+0*x__3(t),diff(x__2(t),t)=1*x__1(t)-3*x__2(t)+0*x__3(t),diff(x__3(t),t)=x__3(t)),x__1(t),x__2(t),x__3(t))
```

$$x_1(t) = e^{-t}(\cos(t) c_1 - c_2 \sin(t) + 2c_1 \sin(t) + 2c_2 \cos(t))$$

$$x_2(t) = e^{-t}(c_1 \sin(t) + c_2 \cos(t))$$

$$x_3(t) = c_3 e^t$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 120

```
DSolve[{x1'[t]==1*x1[t]-5*x2[t]+0*x3[t],x2'[t]==1*x1[t]-3*x2[t]+0*x3[t],x3'[t]==0*x1[t]-0*x2[t]}
```

$$x1(t) \rightarrow e^{-t}(c_1 \cos(t) + (2c_1 - 5c_2) \sin(t))$$

$$x2(t) \rightarrow e^{-t}(c_2 \cos(t) + (c_1 - 2c_2) \sin(t))$$

$$x3(t) \rightarrow c_3 e^t$$

$$x1(t) \rightarrow e^{-t}(c_1 \cos(t) + (2c_1 - 5c_2) \sin(t))$$

$$x2(t) \rightarrow e^{-t}(c_2 \cos(t) + (c_1 - 2c_2) \sin(t))$$

$$x3(t) \rightarrow 0$$

2.3 problem 3

Internal problem ID [1838]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.9, Systems of differential equations. Complex roots. Page 344

Problem number: 3.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = x_1(t)$$

$$x_2'(t) = 3x_1(t) + x_2(t) - 2x_3(t)$$

$$x_3'(t) = 2x_1(t) + 2x_2(t) + x_3(t)$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 55

```
dsolve([diff(x__1(t),t)=1*x__1(t)-0*x__2(t)+0*x__3(t),diff(x__2(t),t)=3*x__1(t)+1*x__2(t)-2*
```

$$x_1(t) = \frac{2c_1 e^t}{3}$$

$$x_2(t) = \frac{e^t(3c_2 \cos(2t) - 3c_3 \sin(2t) - 2c_1)}{3}$$

$$x_3(t) = e^t(c_2 \sin(2t) + c_3 \cos(2t) + c_1)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 91

```
DSolve[{x1'[t]==1*x1[t]-0*x2[t]+0*x3[t],x2'[t]==3*x1[t]+1*x2[t]-2*x3[t],x3'[t]==2*x1[t]+2*x2[t]}
```

$$x1(t) \rightarrow c_1 e^t$$

$$x2(t) \rightarrow \frac{1}{2} e^t (2(c_1 + c_2) \cos(2t) + (3c_1 - 2c_3) \sin(2t) - 2c_1)$$

$$x3(t) \rightarrow \frac{1}{2} e^t ((2c_3 - 3c_1) \cos(2t) + 2(c_1 + c_2) \sin(2t) + 3c_1)$$

2.4 problem 4

Internal problem ID [1839]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.9, Systems of differential equations. Complex roots. Page 344

Problem number: 4.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) + x_3(t)$$

$$x_2'(t) = x_2(t) - x_3(t)$$

$$x_3'(t) = -2x_1(t) - x_3(t)$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 66

```
dsolve([diff(x__1(t),t)=1*x__1(t)-0*x__2(t)+1*x__3(t),diff(x__2(t),t)=0*x__1(t)+1*x__2(t)-1*
```

$$x_1(t) = \frac{c_3 \sin(t)}{2} - \frac{c_2 \cos(t)}{2} - \frac{c_3 \cos(t)}{2} - \frac{c_2 \sin(t)}{2}$$

$$x_2(t) = \frac{c_2 \sin(t)}{2} - \frac{c_3 \sin(t)}{2} + \frac{c_2 \cos(t)}{2} + \frac{c_3 \cos(t)}{2} + c_1 e^t$$

$$x_3(t) = c_3 \cos(t) + c_2 \sin(t)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 67

```
DSolve[{x1'[t]==1*x1[t]-0*x2[t]+1*x3[t],x2'[t]==0*x1[t]+1*x2[t]-1*x3[t],x3'[t]==-2*x1[t]-0*x
```

$$x1(t) \rightarrow c_1 \cos(t) + (c_1 + c_3) \sin(t)$$

$$x2(t) \rightarrow (c_1 + c_2)e^t - c_1 \cos(t) - (c_1 + c_3) \sin(t)$$

$$x3(t) \rightarrow c_3 \cos(t) - (2c_1 + c_3) \sin(t)$$

2.5 problem 5

Internal problem ID [1840]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.9, Systems of differential equations. Complex roots. Page 344

Problem number: 5.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x_1'(t) &= x_1(t) - x_2(t) \\x_2'(t) &= 5x_1(t) - 3x_2(t)\end{aligned}$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 2]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 27

```
dsolve([diff(x__1(t),t) = x__1(t)-x__2(t), diff(x__2(t),t) = 5*x__1(t)-3*x__2(t), x__1(0) =
```

$$x_1(t) = e^{-t} \cos(t)$$

$$x_2(t) = e^{-t}(\sin(t) + 2 \cos(t))$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 29

```
DSolve[{x1'[t]==1*x1[t]-1*x2[t],x2'[t]==5*x1[t]-3*x2[t]},{x1[0]==1,x2[0]==2},{x1[t],x2[t]},t
```

$$x1(t) \rightarrow e^{-t} \cos(t)$$

$$x2(t) \rightarrow e^{-t}(\sin(t) + 2 \cos(t))$$

2.6 problem 6

Internal problem ID [1841]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.9, Systems of differential equations. Complex roots. Page 344

Problem number: 6.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = 3x_1(t) - 2x_2(t)$$

$$x_2'(t) = 4x_1(t) - x_2(t)$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 5]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 41

```
dsolve([diff(x__1(t),t) = 3*x__1(t)-2*x__2(t), diff(x__2(t),t) = 4*x__1(t)-x__2(t), x__1(0)
```

$$x_1(t) = \frac{e^t(-8 \sin(2t) + 2 \cos(2t))}{2}$$

$$x_2(t) = e^t(-3 \sin(2t) + 5 \cos(2t))$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 40

```
DSolve[{x1'[t]==3*x1[t]-2*x2[t],x2'[t]==4*x1[t]-1*x2[t]},{x1[0]==1,x2[0]==5},{x1[t],x2[t]},t
```

$$x1(t) \rightarrow e^t(\cos(2t) - 4 \sin(2t))$$

$$x2(t) \rightarrow e^t(5 \cos(2t) - 3 \sin(2t))$$

2.7 problem 7

Internal problem ID [1842]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.9, Systems of differential equations. Complex roots. Page 344

Problem number: 7.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = -3x_1(t) + 2x_3(t)$$

$$x_2'(t) = x_1(t) - x_2(t)$$

$$x_3'(t) = -2x_1(t) - x_2(t)$$

With initial conditions

$$[x_1(0) = 0, x_2(0) = -1, x_3(0) = -2]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 95

```
dsolve([diff(x__1(t),t) = -3*x__1(t)+2*x__3(t), diff(x__2(t),t) = x__1(t)-x__2(t), diff(x__3
```

$$x_1(t) = 2e^{-2t} - 2e^{-t} \cos(t\sqrt{2}) - e^{-t}\sqrt{2} \sin(t\sqrt{2})$$

$$x_2(t) = -2e^{-2t} + e^{-t} \cos(t\sqrt{2}) - e^{-t}\sqrt{2} \sin(t\sqrt{2})$$

$$x_3(t) = e^{-2t} - 3e^{-t} \cos(t\sqrt{2})$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 109

```
DSolve[{x1'[t]==-3*x1[t]-0*x2[t]+2*x3[t],x2'[t]==1*x1[t]-1*x2[t]-0*x3[t],x3'[t]==-2*x1[t]-1*
```

$$x1(t) \rightarrow -e^{-2t} \left(\sqrt{2}e^t \sin(\sqrt{2}t) + 2e^t \cos(\sqrt{2}t) - 2 \right)$$

$$x2(t) \rightarrow e^{-2t} \left(-\sqrt{2}e^t \sin(\sqrt{2}t) + e^t \cos(\sqrt{2}t) - 2 \right)$$

$$x3(t) \rightarrow e^{-2t} \left(1 - 3e^t \cos(\sqrt{2}t) \right)$$

2.8 problem 8

Internal problem ID [1843]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.9, Systems of differential equations. Complex roots. Page 344

Problem number: 8.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = 2x_2(t)$$

$$x_2'(t) = -2x_1(t)$$

$$x_3'(t) = -3x_4(t)$$

$$x_4'(t) = 3x_3(t)$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 1, x_3(0) = 1, x_4(0) = 0]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 42

```
dsolve([diff(x__1(t),t) = 2*x__2(t), diff(x__2(t),t) = -2*x__1(t), diff(x__3(t),t) = -3*x__4(t), diff(x__4(t),t) = 3*x__3(t)],t)
```

$$x_1(t) = \cos(2t) + \sin(2t)$$

$$x_2(t) = -\sin(2t) + \cos(2t)$$

$$x_3(t) = \cos(3t)$$

$$x_4(t) = \sin(3t)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 42

```
DSolve[{x1'[t]==-0*x1[t]+2*x2[t]+0*x3[t]+0*x4[t],x2'[t]==-2*x1[t]-0*x2[t]-0*x3[t]+0*x4[t],x3
```

$$x1(t) \rightarrow \sin(2t) + \cos(2t)$$

$$x2(t) \rightarrow \cos(2t) - \sin(2t)$$

$$x3(t) \rightarrow \cos(3t)$$

$$x4(t) \rightarrow \sin(3t)$$

3 Section 3.10, Systems of differential equations. Equal roots. Page 352

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3.1 problem Example 1, page 348

Internal problem ID [1844]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.10, Systems of differential equations. Equal roots. Page 352

Problem number: Example 1, page 348.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) + x_2(t)$$

$$x_2'(t) = x_2(t)$$

$$x_3'(t) = 2x_3(t)$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 29

```
dsolve([diff(x__1(t),t)=1*x__1(t)+1*x__2(t)+0*x__3(t),diff(x__2(t),t)=0*x__1(t)+1*x__2(t)-0*
```

$$x_1(t) = (c_2 t + c_1) e^t$$

$$x_2(t) = c_2 e^t$$

$$x_3(t) = c_3 e^{2t}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 64

```
DSolve[{x1'[t]==1*x1[t]+1*x2[t]+0*x3[t],x2'[t]==0*x1[t]+1*x2[t]-0*x3[t],x3'[t]==0*x1[t]-0*x2[t]}
```

$$x1(t) \rightarrow e^t(c_2t + c_1)$$

$$x2(t) \rightarrow c_2e^t$$

$$x3(t) \rightarrow c_3e^{2t}$$

$$x1(t) \rightarrow e^t(c_2t + c_1)$$

$$x2(t) \rightarrow c_2e^t$$

$$x3(t) \rightarrow 0$$

3.2 problem Example 2, page 349

Internal problem ID [1845]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.10, Systems of differential equations. Equal roots. Page 352

Problem number: Example 2, page 349.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = 2x_1(t) + x_2(t) + 3x_3(t)$$

$$x_2'(t) = 2x_2(t) - x_3(t)$$

$$x_3'(t) = 2x_3(t)$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 2, x_3(0) = 1]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 41

```
dsolve([diff(x__1(t),t) = 2*x__1(t)+x__2(t)+3*x__3(t), diff(x__2(t),t) = 2*x__2(t)-x__3(t),
```

$$x_1(t) = \frac{(-t^2 + 10t + 2) e^{2t}}{2}$$

$$x_2(t) = (2 - t) e^{2t}$$

$$x_3(t) = e^{2t}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 43

```
DSolve[{x1'[t]==2*x1[t]+1*x2[t]+3*x3[t],x2'[t]==0*x1[t]+2*x2[t]-1*x3[t],x3'[t]==0*x1[t]-0*x2
```

$$x1(t) \rightarrow -\frac{1}{2}e^{2t}(t^2 - 10t - 2)$$

$$x2(t) \rightarrow -e^{2t}(t - 2)$$

$$x3(t) \rightarrow e^{2t}$$

3.3 problem 1

Internal problem ID [1846]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.10, Systems of differential equations. Equal roots. Page 352

Problem number: 1.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x_1'(t) &= -x_2(t) + x_3(t) \\x_2'(t) &= 2x_1(t) - 3x_2(t) + x_3(t) \\x_3'(t) &= x_1(t) - x_2(t) - x_3(t)\end{aligned}$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 51

```
dsolve([diff(x__1(t),t)=0*x__1(t)-1*x__2(t)+1*x__3(t),diff(x__2(t),t)=2*x__1(t)-3*x__2(t)+1*x__3(t),diff(x__3(t),t)=x__1(t)-x__2(t)-x__3(t)),t)
```

$$x_1(t) = e^{-t}(c_2 t + c_1)$$

$$x_2(t) = e^{-t}(c_2 t + c_1) + c_3 e^{-2t}$$

$$x_3(t) = c_2 e^{-t} + c_3 e^{-2t}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 99

```
DSolve[{x1'[t]==0*x1[t]-1*x2[t]+1*x3[t],x2'[t]==2*x1[t]-3*x2[t]+1*x3[t],x3'[t]==1*x1[t]-1*x2[t]-x3[t]},x1[t],x2[t],x3[t],t]
```

$$x1(t) \rightarrow e^{-t}(c_1(t+1) + (c_3 - c_2)t)$$

$$x2(t) \rightarrow e^{-2t}(c_1(e^t(t+1) - 1) - c_2 e^t t + c_3 e^t t + c_2)$$

$$x3(t) \rightarrow e^{-2t}(c_1(e^t - 1) - c_2 e^t + c_3 e^t + c_2)$$

3.4 problem 2

Internal problem ID [1847]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.10, Systems of differential equations. Equal roots. Page 352

Problem number: 2.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x_1'(t) &= x_1(t) + x_2(t) + x_3(t) \\x_2'(t) &= 2x_1(t) + x_2(t) - x_3(t) \\x_3'(t) &= -3x_1(t) + 2x_2(t) + 4x_3(t)\end{aligned}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 65

```
dsolve([diff(x__1(t),t)=1*x__1(t)+1*x__2(t)+1*x__3(t),diff(x__2(t),t)=2*x__1(t)+1*x__2(t)-1*
```

$$x_1(t) = -e^{2t}(2c_3t + c_2 + 4c_3)$$

$$x_2(t) = -e^{2t}(c_3t^2 + c_2t + 2c_3t + c_1 + c_2 + 6c_3)$$

$$x_3(t) = e^{2t}(c_3t^2 + c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 110

```
DSolve[{x1'[t]==1*x1[t]+1*x2[t]+1*x3[t],x2'[t]==2*x1[t]+1*x2[t]-1*x3[t],x3'[t]==-3*x1[t]+2*x
```

$$x1(t) \rightarrow e^{2t}((c_2 + c_3)t - c_1(t - 1))$$

$$x2(t) \rightarrow \frac{1}{2}e^{2t}(c_2(t^2 - 2t + 2) - (c_1(t - 4)t) + c_3(t - 2)t)$$

$$x3(t) \rightarrow \frac{1}{2}e^{2t}((c_1 - c_2 - c_3)t^2 - 6c_1t + 4(c_2 + c_3)t + 2c_3)$$

3.5 problem 3

Internal problem ID [1848]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.10, Systems of differential equations. Equal roots. Page 352

Problem number: 3.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = -x_1(t) - x_2(t)$$

$$x_2'(t) = -x_2(t)$$

$$x_3'(t) = -2x_3(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 34

```
dsolve([diff(x__1(t),t)=-1*x__1(t)-1*x__2(t)+0*x__3(t),diff(x__2(t),t)=0*x__1(t)-1*x__2(t)+0
```

$$x_1(t) = (-c_2 t + c_1) e^{-t}$$

$$x_2(t) = c_2 e^{-t}$$

$$x_3(t) = c_3 e^{-2t}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 74

```
DSolve[{x1'[t]==-1*x1[t]-1*x2[t]+0*x3[t],x2'[t]==0*x1[t]-1*x2[t]+0*x3[t],x3'[t]==0*x1[t]-0*x2[t]+0*x3[t]}
```

$$x1(t) \rightarrow e^{-t}(c_1 - c_2 t)$$

$$x2(t) \rightarrow c_2 e^{-t}$$

$$x3(t) \rightarrow c_3 e^{-2t}$$

$$x1(t) \rightarrow e^{-t}(c_1 - c_2 t)$$

$$x2(t) \rightarrow c_2 e^{-t}$$

$$x3(t) \rightarrow 0$$

3.6 problem 4

Internal problem ID [1849]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.10, Systems of differential equations. Equal roots. Page 352

Problem number: 4.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = 2x_1(t) - x_3(t)$$

$$x_2'(t) = 2x_2(t) + x_3(t)$$

$$x_3'(t) = 2x_3(t)$$

$$x_4'(t) = -x_3(t) + 2x_4(t)$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 56

```
dsolve([diff(x__1(t),t)=2*x__1(t)+0*x__2(t)-1*x__3(t)+0*x__4(t),diff(x__2(t),t)=0*x__1(t)+2*
```

$$x_1(t) = e^{2t}(c_4t + c_1 + c_3)$$

$$x_2(t) = e^{2t}(-c_4t + c_2 - c_3)$$

$$x_3(t) = -c_4e^{2t}$$

$$x_4(t) = (c_4t + c_3)e^{2t}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 63

```
DSolve[{x1'[t]==2*x1[t]+0*x2[t]-1*x3[t]+0*x4[t],x2'[t]==0*x1[t]+2*x2[t]+1*x3[t]+0*x4[t],x3'[t]==0*x1[t]+0*x2[t]+0*x3[t]+0*x4[t],x4'[t]==0*x1[t]+0*x2[t]+0*x3[t]+0*x4[t]}
```

$$x1(t) \rightarrow e^{2t}(c_1 - c_3 t)$$

$$x2(t) \rightarrow e^{2t}(c_3 t + c_2)$$

$$x3(t) \rightarrow c_3 e^{2t}$$

$$x4(t) \rightarrow e^{2t}(c_4 - c_3 t)$$

3.7 problem 5

Internal problem ID [1850]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.10, Systems of differential equations. Equal roots. Page 352

Problem number: 5.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = -x_1(t) + x_2(t) + 2x_3(t)$$

$$x_2'(t) = -x_1(t) + x_2(t) + x_3(t)$$

$$x_3'(t) = -2x_1(t) + x_2(t) + 3x_3(t)$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 0, x_3(0) = 1]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 16

```
dsolve([diff(x__1(t),t) = -x__1(t)+x__2(t)+2*x__3(t), diff(x__2(t),t) = -x__1(t)+x__2(t)+x__3(t), diff(x__3(t),t) = -2*x__1(t)+x__2(t)+3*x__3(t)], [x__1(t), x__2(t), x__3(t)], [1, 0, 1])
```

$$x_1(t) = e^t$$

$$x_2(t) = 0$$

$$x_3(t) = e^t$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 18

```
DSolve[{x1'[t]==-1*x1[t]+1*x2[t]+2*x3[t],x2'[t]==-1*x1[t]+1*x2[t]+1*x3[t],x3'[t]==-2*x1[t]+1
```

$$x1(t) \rightarrow e^t$$

$$x2(t) \rightarrow 0$$

$$x3(t) \rightarrow e^t$$

3.8 problem 6

Internal problem ID [1851]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.10, Systems of differential equations. Equal roots. Page 352

Problem number: 6.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = -4x_1(t) - 4x_2(t)$$

$$x_2'(t) = 10x_1(t) + 9x_2(t) + x_3(t)$$

$$x_3'(t) = -4x_1(t) - 3x_2(t) + x_3(t)$$

With initial conditions

$$[x_1(0) = 2, x_2(0) = 1, x_3(0) = -1]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 57

```
dsolve([diff(x__1(t),t) = -4*x__1(t)-4*x__2(t), diff(x__2(t),t) = 10*x__1(t)+9*x__2(t)+x__3(t), diff(x__3(t),t) = -4*x__1(t)-3*x__2(t)+x__3(t)], [x__1(t), x__2(t), x__3(t)], [2, 1, -1])
```

$$x_1(t) = e^{2t}(-4t^2 - 16t + 2)$$

$$x_2(t) = -e^{2t}(-6t^2 - 26t - 1)$$

$$x_3(t) = e^{2t}(-2t^2 - 10t - 1)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 61

```
DSolve[{x1'[t]==-4*x1[t]-4*x2[t]+0*x3[t],x2'[t]==10*x1[t]+9*x2[t]+1*x3[t],x3'[t]==-4*x1[t]-3
```

$$x_1(t) \rightarrow -2e^{2t}(2t^2 + 8t - 1)$$

$$x_2(t) \rightarrow e^{2t}(6t^2 + 26t + 1)$$

$$x_3(t) \rightarrow -e^{2t}(2t^2 + 10t + 1)$$

3.9 problem 7

Internal problem ID [1852]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.10, Systems of differential equations. Equal roots. Page 352

Problem number: 7.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) + 2x_2(t) - 3x_3(t)$$

$$x_2'(t) = x_1(t) + x_2(t) + 2x_3(t)$$

$$x_3'(t) = x_1(t) - x_2(t) + 4x_3(t)$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 0, x_3(0) = 0]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 32

```
dsolve([diff(x__1(t),t) = x__1(t)+2*x__2(t)-3*x__3(t), diff(x__2(t),t) = x__1(t)+x__2(t)+2*x__3(t), diff(x__3(t),t) = x__1(t)-x__2(t)+4*x__3(t)], [x__1(t), x__2(t), x__3(t)])
```

$$x_1(t) = -(t - 1)e^{2t}$$

$$x_2(t) = e^{2t}t$$

$$x_3(t) = e^{2t}t$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 35

```
DSolve[{x1'[t]==1*x1[t]+2*x2[t]-3*x3[t],x2'[t]==1*x1[t]+1*x2[t]+2*x3[t],x3'[t]==1*x1[t]-1*x2
```

$$x1(t) \rightarrow -e^{2t}(t - 1)$$

$$x2(t) \rightarrow e^{2t}t$$

$$x3(t) \rightarrow e^{2t}t$$

3.10 problem 8

Internal problem ID [1853]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.10, Systems of differential equations. Equal roots. Page 352

Problem number: 8.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x_1'(t) &= 3x_1(t) \\x_2'(t) &= x_1(t) + 3x_2(t) \\x_3'(t) &= 3x_3(t) \\x_4'(t) &= 2x_3(t) + 3x_4(t)\end{aligned}$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 1, x_3(0) = 1, x_4(0) = 1]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 40

```
dsolve([diff(x__1(t),t) = 3*x__1(t), diff(x__2(t),t) = x__1(t)+3*x__2(t), diff(x__3(t),t) =
```

$$x_1(t) = e^{3t}$$

$$x_2(t) = e^{3t}(t + 1)$$

$$x_3(t) = e^{3t}$$

$$x_4(t) = e^{3t}(1 + 2t)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 44

```
DSolve[{x1'[t]==3*x1[t]+0*x2[t]+0*x3[t]+0*x4[t],x2'[t]==1*x1[t]+3*x2[t]-0*x3[t]+0*x4[t],x3'[t]==0*x1[t]+0*x2[t]+3*x3[t]+0*x4[t],x4'[t]==0*x1[t]+0*x2[t]+0*x3[t]+3*x4[t]}
```

$$x1(t) \rightarrow e^{3t}$$

$$x2(t) \rightarrow e^{3t}(t + 1)$$

$$x3(t) \rightarrow e^{3t}$$

$$x4(t) \rightarrow e^{3t}(2t + 1)$$

**4 Section 3.12, Systems of differential equations.
The nonhomogeneous equation. variation of
parameters. Page 366**

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4.1 problem Example 1, page 361

Internal problem ID [1854]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366

Problem number: Example 1, page 361.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = x_1(t)$$

$$x_2'(t) = 2x_1(t) + x_2(t) - 2x_3(t)$$

$$x_3'(t) = 3x_1(t) + 2x_2(t) + x_3(t) + 2e^t \cos(t)^2 - e^t$$

✓ Solution by Maple

Time used: 0.641 (sec). Leaf size: 85

```
dsolve([diff(x__1(t),t)=1*x__1(t)+0*x__2(t)+0*x__3(t),diff(x__2(t),t)=2*x__1(t)+1*x__2(t)-2*
```

$$x_1(t) = c_1 e^t$$

$$x_2(t) = \frac{e^t(2c_3 \cos(2t) - 2c_2 \sin(2t) - \sin(2t)t - \cos(2t) - 3c_1)}{2}$$

$$x_3(t) = \frac{e^t(4c_2 \cos(2t) + 2 \cos(2t)t + 4c_3 \sin(2t) - \sin(2t) + 4c_1)}{4}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 103

```
DSolve[{x1'[t]==1*x1[t]+0*x2[t]+0*x3[t],x2'[t]==2*x1[t]+1*x2[t]-2*x3[t],x3'[t]==3*x1[t]+2*x2[t]}
```

$$x1(t) \rightarrow c_1 e^t$$

$$x2(t) \rightarrow -\frac{1}{8} e^t ((1 - 12c_1 - 8c_2) \cos(2t) + 4(t - 2c_1 + 2c_3) \sin(2t) + 12c_1)$$

$$x3(t) \rightarrow \frac{1}{8} e^t (4(t - 2c_1 + 2c_3) \cos(2t) + (1 + 12c_1 + 8c_2) \sin(2t) + 8c_1)$$

4.2 problem Example 2, page 364

Internal problem ID [1855]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366

Problem number: Example 2, page 364.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x_1'(t) &= x_1(t) + e^{ct} \\x_2'(t) &= 2x_1(t) + x_2(t) - 2x_3(t) \\x_3'(t) &= 3x_1(t) + 2x_2(t) + x_3(t)\end{aligned}$$

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 225

```
dsolve([diff(x__1(t),t)=1*x__1(t)+0*x__2(t)+0*x__3(t)+exp(c*t),diff(x__2(t),t)=2*x__1(t)+1*x__2(t)-2*x__3(t),diff(x__3(t),t)=3*x__1(t)+2*x__2(t)+x__3(t)),t)
```

$$x_1(t) = \frac{e^t c_1 c - c_1 e^t + e^{ct}}{c - 1}$$

$$x_2(t) = \frac{2 \cos(2t) e^t c_3 c^3 - 2 \sin(2t) e^t c_2 c^3 - 6 \cos(2t) e^t c_3 c^2 + 6 \sin(2t) e^t c_2 c^2 - 3 e^t c_1 c^3 + 14 \cos(2t) e^t c_3 c - 14 \sin(2t) e^t c_2 c}{2(c-1)(c^2-2c)}$$

$$x_3(t) = \frac{3c e^{ct} + e^{ct}}{c^3 - 3c^2 + 7c - 5} + c_1 e^t + c_2 e^t \cos(2t) + c_3 e^t \sin(2t)$$

✓ Solution by Mathematica

Time used: 0.496 (sec). Leaf size: 256

```
DSolve[{x1'[t]==1*x1[t]+0*x2[t]+0*x3[t]+Exp[c*t],x2'[t]==2*x1[t]+1*x2[t]-2*x3[t],x3'[t]==3*x
```

$$x1(t) \rightarrow e^t \left(\frac{e^{(c-1)t}}{c-1} + c_1 \right)$$

$$x2(t) \rightarrow \frac{e^t (-3c^3 c_1 + 9c^2 c_1 + (c^3 - 3c^2 + 7c - 5)(3c_1 + 2c_2) \cos(2t) + 2(c^3 - 3c^2 + 7c - 5)(c_1 - c_3) \sin(2t) + 4c_2 \cos(2t) - 4c_3 \sin(2t))}{2(c-1)(c^2 - 2c + 5)}$$

$$x3(t) \rightarrow \frac{e^t (-2(c^3 - 3c^2 + 7c - 5)(c_1 - c_3) \cos(2t) + (c^3 - 3c^2 + 7c - 5)(3c_1 + 2c_2) \sin(2t) + 2(c^3 - 3c^2 + 7c - 5)c_2 \cos(2t) - 2(c^3 - 3c^2 + 7c - 5)c_3 \sin(2t))}{2(c-1)(c^2 - 2c + 5)}$$

4.3 problem 1

Internal problem ID [1856]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366

Problem number: 1.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x_1'(t) &= 4x_1(t) + 5x_2(t) + 4e^t \cos(t) \\x_2'(t) &= -2x_1(t) - 2x_2(t)\end{aligned}$$

With initial conditions

$$[x_1(0) = 0, x_2(0) = 0]$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 34

```
dsolve([diff(x__1(t),t) = 4*x__1(t)+5*x__2(t)+4*exp(t)*cos(t), diff(x__2(t),t) = -2*x__1(t)-
```

$$x_1(t) = \frac{e^t(12 \sin(t)t + 4 \cos(t)t + 4 \sin(t))}{2}$$

$$x_2(t) = -4 \sin(t) e^t t$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 33

```
DSolve[{x1'[t]==4*x1[t]+5*x2[t]+4*Exp[t]*Cos[t],x2'[t]==-2*x1[t]-2*x2[t]},{x1[0]==0,x2[0]==0
```

$$x1(t) \rightarrow 2e^t(3t \sin(t) + \sin(t) + t \cos(t))$$

$$x2(t) \rightarrow -4e^t t \sin(t)$$

4.4 problem 2

Internal problem ID [1857]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366

Problem number: 2.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x_1'(t) &= 3x_1(t) - 4x_2(t) + e^t \\x_2'(t) &= x_1(t) - x_2(t) + e^t\end{aligned}$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 1]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 32

```
dsolve([diff(x__1(t),t) = 3*x__1(t)-4*x__2(t)+exp(t), diff(x__2(t),t) = x__1(t)-x__2(t)+exp(t)
```

$$x_1(t) = e^t(-t^2 - t + 1)$$

$$x_2(t) = \frac{e^t(-t^2 + 2)}{2}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 31

```
DSolve[{x1'[t]==3*x1[t]-4*x2[t]+Exp[t],x2'[t]==1*x1[t]-1*x2[t]+Exp[t]},{x1[0]==1,x2[0]==1},t
```

$$x1(t) \rightarrow -e^t(t^2 + t - 1)$$

$$x2(t) \rightarrow -\frac{1}{2}e^t(t^2 - 2)$$

4.5 problem 3

Internal problem ID [1858]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366

Problem number: 3.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x_1'(t) &= 2x_1(t) - 5x_2(t) + \sin(t) \\x_2'(t) &= x_1(t) - 2x_2(t) + \tan(t)\end{aligned}$$

With initial conditions

$$[x_1(0) = 0, x_2(0) = 0]$$

✓ Solution by Maple

Time used: 0.469 (sec). Leaf size: 172

```
dsolve([diff(x__1(t),t) = 2*x__1(t)-5*x__2(t)+sin(t), diff(x__2(t),t) = x__1(t)-2*x__2(t)+tan(t)], [x__1(t), x__2(t)])
```

$$x_1(t) = \frac{-2 \sec(t) \tan(t) + 2 \sin(t) + 4 \cos(t) - 4 \sec(t) - 4 \tan(t) - 2 \tan(t)^2 - 8 \sin(t) \sec(t) + 10 \ln(\sec(t) + \tan(t))}{2}$$

$$\begin{aligned}x_2(t) &= -\frac{3 \sin(t)}{2} + \cos(t) - 1 + \ln(\sec(t) + \tan(t)) \sin(t) \\&\quad + 2 \ln(\sec(t) + \tan(t)) \cos(t) - \frac{\cos(t)t}{2}\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 58

```
DSolve[{x1'[t]==2*x1[t]-5*x2[t]+Sin[t],x2'[t]==1*x1[t]-2*x2[t]+Tan[t]},{x1[0]==0,x2[0]==0},t]
```

$$x1(t) \rightarrow 5 \cos(t) \operatorname{arctanh}(\sin(t)) + \frac{1}{2}(t - 8) \sin(t) - t \cos(t)$$

$$x2(t) \rightarrow \operatorname{arctanh}(\sin(t))(\sin(t) + 2 \cos(t)) - \frac{3 \sin(t)}{2} - \frac{1}{2}t \cos(t) + \cos(t) - 1$$

4.6 problem 4

Internal problem ID [1859]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366

Problem number: 4.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x_1'(t) &= x_2(t) + f_1(t) \\x_2'(t) &= -x_1(t) + f_2(t)\end{aligned}$$

With initial conditions

$$[x_1(0) = 0, x_2(0) = 0]$$

✓ Solution by Maple

Time used: 0.516 (sec). Leaf size: 110

```
dsolve([diff(x__1(t),t) = x__2(t)+f__1(t), diff(x__2(t),t) = -x__1(t)+f__2(t), x__1(0) = 0,
```

$$\begin{aligned}x_1(t) &= -f_2(0) \cos(t) - \left(\int_0^t \cos(_z1) \left(\frac{d}{d_z1} f_2(_z1) - f_1(_z1) \right) d_z1 \right) \cos(t) \\&\quad - \left(\int_0^t \sin(_z1) \left(\frac{d}{d_z1} f_2(_z1) - f_1(_z1) \right) d_z1 \right) \sin(t) + f_2(t)\end{aligned}$$

$$\begin{aligned}x_2(t) &= f_2(0) \sin(t) + \left(\int_0^t \cos(_z1) \left(\frac{d}{d_z1} f_2(_z1) - f_1(_z1) \right) d_z1 \right) \sin(t) \\&\quad - \left(\int_0^t \sin(_z1) \left(\frac{d}{d_z1} f_2(_z1) - f_1(_z1) \right) d_z1 \right) \cos(t)\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 216

`DSolve[{x1'[t]==0*x1[t]+1*x2[t]+f1[t],x2'[t]==-1*x1[t]-0*x2[t]+f2[t]},{x1[0]==0,x2[0]==0},{t}]`

$$\begin{aligned}
 x1(t) \rightarrow & -\cos(t) \int_1^0 (\cos(K[1])f1(K[1]) - f2(K[1]) \sin(K[1]))dK[1] \\
 & + \cos(t) \int_1^t (\cos(K[1])f1(K[1]) - f2(K[1]) \sin(K[1]))dK[1] \\
 & + \sin(t) \left(\int_1^t (\cos(K[2])f2(K[2]) + f1(K[2]) \sin(K[2]))dK[2] \right. \\
 & \quad \left. - \int_1^0 (\cos(K[2])f2(K[2]) + f1(K[2]) \sin(K[2]))dK[2] \right)
 \end{aligned}$$

$$\begin{aligned}
 x2(t) \rightarrow & \sin(t) \int_1^0 (\cos(K[1])f1(K[1]) - f2(K[1]) \sin(K[1]))dK[1] \\
 & - \sin(t) \int_1^t (\cos(K[1])f1(K[1]) - f2(K[1]) \sin(K[1]))dK[1] \\
 & + \cos(t) \left(\int_1^t (\cos(K[2])f2(K[2]) + f1(K[2]) \sin(K[2]))dK[2] \right. \\
 & \quad \left. - \int_1^0 (\cos(K[2])f2(K[2]) + f1(K[2]) \sin(K[2]))dK[2] \right)
 \end{aligned}$$

4.7 problem 5

Internal problem ID [1860]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366

Problem number: 5.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = 2x_1(t) + x_3(t) + e^{2t}$$

$$x_2'(t) = 2x_2(t)$$

$$x_3'(t) = x_2(t) + 3x_3(t) + e^{2t}$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 1, x_3(0) = 1]$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 45

```
dsolve([diff(x__1(t),t) = 2*x__1(t)+x__3(t)+exp(2*t), diff(x__2(t),t) = 2*x__2(t), diff(x__3
```

$$x_1(t) = (-t - 2)e^{2t} + 3e^{3t}$$

$$x_2(t) = e^{2t}$$

$$x_3(t) = -2e^{2t} + 3e^{3t}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 45

```
DSolve[{x1'[t]==2*x1[t]+0*x2[t]+1*x3[t]+Exp[2*t],x2'[t]==0*x1[t]+2*x2[t]+0*x3[t],x3'[t]==0*x
```

$$x1(t) \rightarrow e^{2t}(-t + 3e^t - 2)$$

$$x2(t) \rightarrow e^{2t}$$

$$x3(t) \rightarrow e^{2t}(3e^t - 2)$$

4.8 problem 6

Internal problem ID [1861]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366

Problem number: 6.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = -x_1(t) - x_2(t) - 2x_3(t) + e^t$$

$$x_2'(t) = x_1(t) + x_2(t) + x_3(t)$$

$$x_3'(t) = 2x_1(t) + x_2(t) + 3x_3(t)$$

With initial conditions

$$[x_1(0) = 0, x_2(0) = 0, x_3(0) = 0]$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 47

```
dsolve([diff(x__1(t),t) = -x__1(t)-x__2(t)-2*x__3(t)+exp(t), diff(x__2(t),t) = x__1(t)+x__2(t)
```

$$x_1(t) = -\frac{e^t(t^3 + 6t^2 - 6t)}{6}$$

$$x_2(t) = \frac{e^t t^2}{2}$$

$$x_3(t) = \frac{e^t(t^3 + 6t^2)}{6}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 50

```
DSolve[{x1'[t]==-1*x1[t]-1*x2[t]-2*x3[t]+Exp[t],x2'[t]==1*x1[t]+1*x2[t]+1*x3[t],x3'[t]==2*x1
```

$$x1(t) \rightarrow -\frac{1}{6}e^t t(t^2 + 6t - 6)$$

$$x2(t) \rightarrow \frac{e^t t^2}{2}$$

$$x3(t) \rightarrow \frac{1}{6}e^t t^2(t + 6)$$

4.9 problem 10

Internal problem ID [1862]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366

Problem number: 10.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x_1'(t) &= 2x_1(t) + x_2(t) + e^{3t} \\x_2'(t) &= 3x_1(t) - 2x_2(t) + e^{3t}\end{aligned}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 83

```
dsolve([diff(x__1(t),t)=2*x__1(t)+1*x__2(t)+1*exp(3*t),diff(x__2(t),t)=3*x__1(t)-2*x__2(t)+e
```

$$x_1(t) = \frac{\sqrt{7} e^{\sqrt{7}t} c_2}{3} - \frac{\sqrt{7} e^{-\sqrt{7}t} c_1}{3} + 3e^{3t} + \frac{2e^{\sqrt{7}t} c_2}{3} + \frac{2e^{-\sqrt{7}t} c_1}{3}$$

$$x_2(t) = e^{\sqrt{7}t} c_2 + e^{-\sqrt{7}t} c_1 + 2e^{3t}$$

✓ Solution by Mathematica

Time used: 0.411 (sec). Leaf size: 171

```
DSolve[{x1'[t]==2*x1[t]+1*x2[t]+Exp[3*t],x2'[t]==3*x1[t]-2*x2[t]+Exp[3*t]},{x1[t],x2[t]},t,I
```

$$x_1(t) \rightarrow \frac{1}{14} e^{-\sqrt{7}t} \left(42e^{(3+\sqrt{7})t} + \left((7+2\sqrt{7})c_1 + \sqrt{7}c_2 \right) e^{2\sqrt{7}t} + (7-2\sqrt{7})c_1 - \sqrt{7}c_2 \right)$$

$$x_2(t) \rightarrow \frac{1}{14} e^{-\sqrt{7}t} \left(28e^{(3+\sqrt{7})t} + \left(3\sqrt{7}c_1 + (7-2\sqrt{7})c_2 \right) e^{2\sqrt{7}t} - 3\sqrt{7}c_1 + (7+2\sqrt{7})c_2 \right)$$

4.10 problem 11

Internal problem ID [1863]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366

Problem number: 11.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x_1'(t) &= x_1(t) - x_2(t) - t^2 \\x_2'(t) &= x_1(t) + 3x_2(t) + 2t\end{aligned}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 62

```
dsolve([diff(x__1(t),t)=1*x__1(t)-1*x__2(t)-t^2,diff(x__2(t),t)=1*x__1(t)+3*x__2(t)+2*t],[x__1(t),x__2(t)],t)
```

$$x_1(t) = -c_2 e^{2t} - e^{2t} t c_1 + c_1 e^{2t} + \frac{t}{2} + \frac{1}{8} + \frac{3t^2}{4}$$

$$x_2(t) = c_2 e^{2t} + e^{2t} t c_1 - \frac{t^2}{4} - t - \frac{3}{8}$$

✓ Solution by Mathematica

Time used: 0.27 (sec). Leaf size: 94

```
DSolve[{x1'[t]==1*x1[t]+3*x2[t]-t^2,x2'[t]==1*x1[t]+3*x2[t]+2*t},{x1[t],x2[t]},t,IncludeSingularSolutions->True]
```

$$x_1(t) \rightarrow \frac{1}{128} (-32t^3 - 88t^2 - 44t + 32c_1(e^{4t} + 3) + 96c_2 e^{4t} - 11 - 96c_2)$$

$$x_2(t) \rightarrow \frac{1}{384} (32t^3 + 120t^2 - 132t + 96c_1(e^{4t} - 1) + 288c_2 e^{4t} - 33 + 96c_2)$$

4.11 problem 12

Internal problem ID [1864]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366

Problem number: 12.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) + 3x_2(t) + 2x_3(t) + \sin(t)$$

$$x_2'(t) = -x_1(t) + 2x_2(t) + x_3(t)$$

$$x_3'(t) = 4x_1(t) - x_2(t) - x_3(t)$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 84

```
dsolve([diff(x__1(t),t)=1*x__1(t)+3*x__2(t)+2*x__3(t)+sin(t),diff(x__2(t),t)=-1*x__1(t)+2*x__3(t),diff(x__3(t),t)=4*x__1(t)-x__2(t)-x__3(t)),x__1(t),x__2(t),x__3(t))
```

$$x_1(t) = -\frac{\sin(t)}{10} - \frac{\cos(t)}{5} + \frac{c_1 e^t}{3} - \frac{e^{-2t} c_2}{3} + c_3 e^{3t}$$

$$x_2(t) = \frac{\cos(t)}{10} + \frac{3 \sin(t)}{10} - \frac{2c_1 e^t}{3} - \frac{e^{-2t} c_2}{3}$$

$$x_3(t) = -\frac{\cos(t)}{10} - \frac{4 \sin(t)}{5} + c_1 e^t + e^{-2t} c_2 + c_3 e^{3t}$$

✓ Solution by Mathematica

Time used: 0.318 (sec). Leaf size: 211

```
DSolve[{x1'[t]==1*x1[t]+3*x2[t]+2*x3[t]+Sin[t],x2'[t]==-1*x1[t]+2*x2[t]+1*x3[t],x3'[t]==4*x1[t]}
```

$$x1(t) \rightarrow \frac{1}{30}(-3 \sin(t) - 6 \cos(t) + 5e^{-2t}(c_1(e^{3t} + 3e^{5t} + 2) + c_2(-4e^{3t} + 6e^{5t} - 2) + c_3(-e^{3t} + 3e^{5t} - 2)))$$

$$x2(t) \rightarrow \frac{1}{30}(9 \sin(t) + 3 \cos(t) - 10e^{-2t}(c_1(e^{3t} - 1) - 4c_2e^{3t} - c_3e^{3t} + c_2 + c_3))$$

$$x3(t) \rightarrow \frac{1}{10}(-8 \sin(t) - \cos(t) + 5e^{-2t}(c_1(e^{3t} + e^{5t} - 2) + 2c_2(-2e^{3t} + e^{5t} + 1) + c_3(-e^{3t} + e^{5t} + 2)))$$

4.12 problem 13

Internal problem ID [1865]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366

Problem number: 13.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) + 2x_2(t) - 3x_3(t) + e^t$$

$$x_2'(t) = x_1(t) + x_2(t) + 2x_3(t)$$

$$x_3'(t) = x_1(t) - x_2(t) + 4x_3(t) - e^t$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 123

```
dsolve([diff(x__1(t),t)=1*x__1(t)+2*x__2(t)-3*x__3(t)+exp(t),diff(x__2(t),t)=1*x__1(t)+1*x__
```

$$x_1(t) = 2e^t - c_1e^{2t} - c_2e^{2t}t + c_2e^{2t} - e^{2t}c_3t^2 + 2e^{2t}c_3t + 2c_3e^{2t}$$

$$x_2(t) = -2e^t + c_1e^{2t} + c_2e^{2t}t + e^{2t}c_3t^2 + 2c_3e^{2t}$$

$$x_3(t) = -e^t + c_1e^{2t} + c_2e^{2t}t + e^{2t}c_3t^2$$

✓ Solution by Mathematica

Time used: 0.098 (sec). Leaf size: 133

```
DSolve[{x1'[t]==1*x1[t]+2*x2[t]-3*x3[t]+Exp[t],x2'[t]==1*x1[t]+1*x2[t]+2*x3[t],x3'[t]==1*x1[t]
```

$$x1(t) \rightarrow \frac{1}{2}e^t(4 + e^t(-2c_1(t-1) - c_2(t-4)t + c_3(t-6)t))$$

$$x2(t) \rightarrow \frac{1}{2}e^t(-4 + e^t((c_2 - c_3)t^2 + 2(c_1 - c_2 + 2c_3)t + 2c_2))$$

$$x3(t) \rightarrow \frac{1}{2}e^t(-2 + e^t((c_2 - c_3)t^2 + 2(c_1 - c_2 + 2c_3)t + 2c_3))$$

4.13 problem 14

Internal problem ID [1866]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366

Problem number: 14.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x_1'(t) &= -x_1(t) - x_2(t) + 1 \\x_2'(t) &= -4x_2(t) - x_3(t) + t \\x_3'(t) &= 5x_2(t) + e^t\end{aligned}$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 124

```
dsolve([diff(x__1(t),t)=-1*x__1(t)-1*x__2(t)+0*x__3(t)+1,diff(x__2(t),t)=0*x__1(t)-4*x__2(t)
```

$$x_1(t) = \frac{c_2 e^{-2t} \sin(t)}{10} - \frac{3 e^{-2t} \sin(t) c_3}{10} - \frac{3 c_2 e^{-2t} \cos(t)}{10} - \frac{e^{-2t} \cos(t) c_3}{10} + \frac{e^t}{20} + \frac{4}{5} + e^{-t} c_1$$

$$x_2(t) = -\frac{2 e^{-2t} \sin(t) c_3}{5} + \frac{e^{-2t} \cos(t) c_3}{5} - \frac{2 c_2 e^{-2t} \cos(t)}{5} - \frac{c_2 e^{-2t} \sin(t)}{5} + \frac{1}{5} - \frac{e^t}{10}$$

$$x_3(t) = e^{-2t} \sin(t) c_3 + c_2 e^{-2t} \cos(t) + t - \frac{4}{5} + \frac{e^t}{2}$$

✓ Solution by Mathematica

Time used: 1.816 (sec). Leaf size: 144

```
DSolve[{x1'[t]==-1*x1[t]-1*x2[t]+0*x3[t]+1,x2'[t]==0*x1[t]-4*x2[t]-1*x3[t]+t,x3'[t]==0*x1[t]
```

$$x1(t) \rightarrow \frac{1}{20}e^{-2t}(e^t(16e^t + e^{2t} + 10(2c_1 + c_2 + c_3)) - 10(c_2 + c_3)\cos(t) - 10(3c_2 + c_3)\sin(t))$$

$$x2(t) \rightarrow \frac{1}{10}(2 - e^t) + c_2e^{-2t}\cos(t) - (2c_2 + c_3)e^{-2t}\sin(t)$$

$$x3(t) \rightarrow t + \frac{e^t}{2} + c_3e^{-2t}\cos(t) + (5c_2 + 2c_3)e^{-2t}\sin(t) - \frac{4}{5}$$

4.14 problem 16

Internal problem ID [1867]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366

Problem number: 16.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x_1'(t) &= x_1(t) + x_2(t) - x_3(t) + e^{2t} \\x_2'(t) &= 2x_1(t) + 3x_2(t) - 4x_3(t) + 2e^{2t} \\x_3'(t) &= 4x_1(t) + x_2(t) - 4x_3(t) + e^{2t}\end{aligned}$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 84

```
dsolve([diff(x__1(t),t)=1*x__1(t)+1*x__2(t)-1*x__3(t)+exp(2*t),diff(x__2(t),t)=2*x__1(t)+3*x__2(t)-4*x__3(t)+2*exp(2*t),diff(x__3(t),t)=4*x__1(t)+x__2(t)-4*x__3(t)+exp(2*t)),t)
```

$$x_1(t) = e^{2t}t + c_1e^t + \frac{c_2e^{-3t}}{11} + c_3e^{2t}$$

$$x_2(t) = 2e^{2t}t + c_1e^t + \frac{7c_2e^{-3t}}{11} + 2c_3e^{2t}$$

$$x_3(t) = e^{2t}t + c_1e^t + c_2e^{-3t} + c_3e^{2t}$$

✓ Solution by Mathematica

Time used: 0.128 (sec). Leaf size: 2491

```
DSolve[{x1'[t]==1*x1[t]+1*x2[t]-1*x3[t]+Exp[2*t],x2'[t]==2*x1[t]+3*x2[t]-4*x3[t]+2*Exp[2*t],x3'[t]==4*x1[t]+x2[t]-4*x3[t]+Exp[2*t]},t]
```

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4.15 problem 17

Internal problem ID [1868]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366

Problem number: 17.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x_1'(t) &= x_1(t) - x_2(t) - x_3(t) + e^{3t} \\x_2'(t) &= x_1(t) + 3x_2(t) + x_3(t) - e^{3t} \\x_3'(t) &= -3x_1(t) + x_2(t) - x_3(t) - e^{3t}\end{aligned}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 86

```
dsolve([diff(x__1(t),t)=1*x__1(t)-1*x__2(t)-1*x__3(t)+exp(3*t),diff(x__2(t),t)=1*x__1(t)+3*x__2(t)+1*x__3(t)-exp(3*t),diff(x__3(t),t)=-3*x__1(t)+x__2(t)-x__3(t)-exp(3*t)],t)
```

$$x_1(t) = e^{3t}t + \frac{c_1 e^{-2t}}{4} - c_2 e^{2t} - c_3 e^{3t}$$

$$x_2(t) = -e^{3t}t - \frac{c_1 e^{-2t}}{4} + c_3 e^{3t}$$

$$x_3(t) = -e^{3t}t + c_1 e^{-2t} + c_2 e^{2t} + c_3 e^{3t}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 142

```
DSolve[{x1'[t]==1*x1[t]-1*x2[t]-1*x3[t]+Exp[3*t],x2'[t]==1*x1[t]+3*x2[t]+1*x3[t]-Exp[3*t],x3
```

$$x1(t) \rightarrow \frac{1}{5}e^{-2t}(5(c_1 + c_2)e^{4t} + e^{5t}(5t - c_1 - 5c_2 - c_3) + c_1 + c_3)$$

$$x2(t) \rightarrow \frac{1}{5}e^{-2t}(e^{5t}(-5t + c_1 + 5c_2 + c_3) - c_1 - c_3)$$

$$x3(t) \rightarrow \frac{1}{5}e^{-2t}(-5(c_1 + c_2)e^{4t} + e^{5t}(-5t + c_1 + 5c_2 + c_3) + 4(c_1 + c_3))$$

4.16 problem 18

Internal problem ID [1869]

Book: Differential equations and their applications, 4th ed., M. Braun

Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366

Problem number: 18.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = 3x_1(t) + 2x_2(t) + 4x_3(t) + 2e^{8t}$$

$$x_2'(t) = 2x_1(t) + 2x_3(t) + e^{8t}$$

$$x_3'(t) = 4x_1(t) + 2x_2(t) + 3x_3(t) + 2e^{8t}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 86

```
dsolve([diff(x__1(t),t)=3*x__1(t)+2*x__2(t)+4*x__3(t)+2*exp(8*t),diff(x__2(t),t)=2*x__1(t)+0
```

$$x_1(t) = c_3 e^{8t} - \frac{5c_2 e^{-t}}{4} + 2t e^{8t} - \frac{e^{-t} c_1}{2}$$

$$x_2(t) = \frac{c_3 e^{8t}}{2} + \frac{c_2 e^{-t}}{2} + t e^{8t} + e^{-t} c_1$$

$$x_3(t) = c_3 e^{8t} + c_2 e^{-t} + 2t e^{8t}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 139

```
DSolve[{x1'[t]==3*x1[t]+2*x2[t]+4*x3[t]+2*Exp[8*t],x2'[t]==2*x1[t]+0*x2[t]+2*x3[t]+Exp[8*t],
```

$$x1(t) \rightarrow \frac{1}{9}e^{-t}(2e^{9t}(9t + 2c_1 + c_2 + 2c_3) + 5c_1 - 2(c_2 + 2c_3))$$

$$x2(t) \rightarrow \frac{1}{9}e^{-t}(e^{9t}(9t + 2c_1 + c_2 + 2c_3) - 2(c_1 - 4c_2 + c_3))$$

$$x3(t) \rightarrow \frac{1}{9}e^{-t}(2e^{9t}(9t + 2c_1 + c_2 + 2c_3) - 4c_1 - 2c_2 + 5c_3)$$