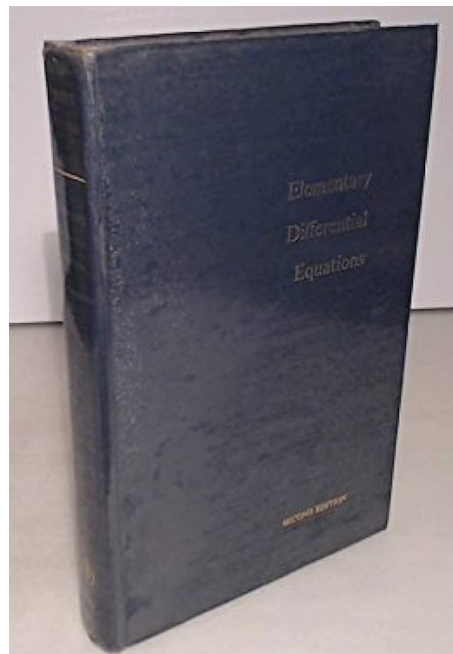


A Solution Manual For

# Elementary Differential equations, Chaundy, 1969



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# Contents

1 Exercises 3, page 60

2

## 1 Exercises 3, page 60

1.1	problem 1(a)	3
1.2	problem 1(b)	4
1.3	problem 1(c)	5
1.4	problem 1(d)	6
1.5	problem 1(e)	7
1.6	problem 1(f)	8
1.7	problem 2(a)	9
1.8	problem 2(b)	10
1.9	problem 2(c)	11
1.10	problem 2(d)	12
1.11	problem 2(e)	13
1.12	problem 3(a)	14
1.13	problem 3(b)	15
1.14	problem 3(c)	16
1.15	problem 3(a)	17
1.16	problem 4(a)	18
1.17	problem 4(b)	19
1.18	problem 4(c)	20
1.19	problem 4(d)	21
1.20	problem 5(a)	22
1.21	problem 5(b)	23
1.22	problem 5(c)	24
1.23	problem 5(d)	25

## 1.1 problem 1(a)

Internal problem ID [3029]

**Book:** Elementary Differential equations, Chaundy, 1969

**Section:** Exercises 3, page 60

**Problem number:** 1(a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$y'y = x$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)*y(x)=x,y(x), singsol=all)
```

$$y(x) = \sqrt{x^2 + c_1}$$

$$y(x) = -\sqrt{x^2 + c_1}$$

### ✓ Solution by Mathematica

Time used: 0.091 (sec). Leaf size: 35

```
DSolve[y'[x]*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\sqrt{x^2 + 2c_1}$$

$$y(x) \rightarrow \sqrt{x^2 + 2c_1}$$

## 1.2 problem 1(b)

Internal problem ID [3030]

**Book:** Elementary Differential equations, Chaundy, 1969

**Section:** Exercises 3, page 60

**Problem number:** 1(b).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear, 'class A']`

$$y' - y = x^3$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)-y(x)=x^3,y(x), singsol=all)
```

$$y(x) = -x^3 - 3x^2 - 6x - 6 + c_1e^x$$

### ✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 26

```
DSolve[y'[x]-y[x]==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -x^3 - 3x^2 - 6x + c_1e^x - 6$$

### 1.3 problem 1(c)

Internal problem ID [3031]

**Book:** Elementary Differential equations, Chaundy, 1969

**Section:** Exercises 3, page 60

**Problem number:** 1(c).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear]`

$$y' + y \cot(x) = x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)+y(x)*cot(x)=x,y(x), singsol=all)
```

$$y(x) = \frac{\sin(x) - x \cos(x) + c_1}{\sin(x)}$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 17

```
DSolve[y'[x]+y[x]*Cot[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -x \cot(x) + c_1 \csc(x) + 1$$

## 1.4 problem 1(d)

Internal problem ID [3032]

**Book:** Elementary Differential equations, Chaundy, 1969

**Section:** Exercises 3, page 60

**Problem number:** 1(d).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear]`

$$y' + y \cot(x) = \tan(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)+y(x)*cot(x)=tan(x),y(x), singsol=all)
```

$$y(x) = \frac{-\sin(x) + \ln(\sec(x) + \tan(x)) + c_1}{\sin(x)}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 18

```
DSolve[y'[x]+y[x]*Cot[x]==Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \csc(x)\operatorname{arctanh}(\sin(x)) + c_1 \csc(x) - 1$$

## 1.5 problem 1(e)

Internal problem ID [3033]

**Book:** Elementary Differential equations, Chaundy, 1969

**Section:** Exercises 3, page 60

**Problem number:** 1(e).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear]`

$$y' + y \tan(x) = \cot(x)$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+y(x)*tan(x)=cot(x),y(x), singsol=all)
```

$$y(x) = (-\ln(\csc(x) + \cot(x)) + c_1) \cos(x)$$

### ✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 16

```
DSolve[y'[x]+y[x]*Tan[x]==Cot[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \cos(x)(-\operatorname{arctanh}(\cos(x)) + c_1)$$



## 1.6 problem 1(f)

Internal problem ID [3034]

**Book:** Elementary Differential equations, Chaundy, 1969

**Section:** Exercises 3, page 60

**Problem number:** 1(f).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear]`

$$y' + y \ln(x) = x^{-x}$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)+y(x)*ln(x)=x^(-x),y(x), singsol=all)
```

$$y(x) = -x^{-x} + x^{-x}e^x c_1$$

### ✓ Solution by Mathematica

Time used: 0.08 (sec). Leaf size: 19

```
DSolve[y'[x]+y[x]*Log[x]==x^(-x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^{-x}(-1 + c_1 e^x)$$

## 1.7 problem 2(a)

Internal problem ID [3035]

**Book:** Elementary Differential equations, Chaundy, 1969

**Section:** Exercises 3, page 60

**Problem number:** 2(a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear]`

$$xy' + y = x$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x*diff(y(x),x)+y(x)=x,y(x), singsol=all)
```

$$y(x) = \frac{x}{2} + \frac{c_1}{x}$$

### ✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 17

```
DSolve[x*y'[x]+y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x}{2} + \frac{c_1}{x}$$

## 1.8 problem 2(b)

Internal problem ID [3036]

**Book:** Elementary Differential equations, Chaundy, 1969

**Section:** Exercises 3, page 60

**Problem number:** 2(b).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$xy' - y = x^3$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve(x*diff(y(x),x)-y(x)=x^3,y(x), singsol=all)
```

$$y(x) = \left( \frac{x^2}{2} + c_1 \right) x$$

### ✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 17

```
DSolve[x*y'[x]-y[x]==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x^3}{2} + c_1 x$$

## 1.9 problem 2(c)

Internal problem ID [3037]

**Book:** Elementary Differential equations, Chaundy, 1969

**Section:** Exercises 3, page 60

**Problem number:** 2(c).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear]`

$$xy' + ny = x^n$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x*diff(y(x),x)+n*y(x)=x^n,y(x), singsol=all)
```

$$y(x) = \frac{x^n}{2n} + x^{-n}c_1$$

### ✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 24

```
DSolve[x*y'[x]+n*y[x]==x^n,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x^n}{2n} + c_1x^{-n}$$

## 1.10 problem 2(d)

Internal problem ID [3038]

**Book:** Elementary Differential equations, Chaundy, 1969

**Section:** Exercises 3, page 60

**Problem number:** 2(d).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear]`

$$xy' - ny = x^n$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x)-n*y(x)=x^n,y(x), singsol=all)
```

$$y(x) = (\ln(x) + c_1) x^n$$

### ✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 14

```
DSolve[x*y'[x]-n*y[x]==x^n,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^n(\log(x) + c_1)$$

## 1.11 problem 2(e)

Internal problem ID [3039]

**Book:** Elementary Differential equations, Chaundy, 1969

**Section:** Exercises 3, page 60

**Problem number:** 2(e).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear]`

$$(x^3 + x)y' + y = x$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve((x^3+x)*diff(y(x),x)+y(x)=x,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{x^2 + 1} c_1}{x} - \frac{1}{x}$$

### ✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 23

```
DSolve[(x^3+x)*y'[x]+y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{-1 + c_1 \sqrt{x^2 + 1}}{x}$$

## 1.12 problem 3(a)

Internal problem ID [3040]

**Book:** Elementary Differential equations, Chaundy, 1969

**Section:** Exercises 3, page 60

**Problem number:** 3(a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$\cot(x) y' + y = x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(cot(x)*diff(y(x),x)+y(x)=x,y(x), singsol=all)
```

$$y(x) = \left( \frac{x}{\cos(x)} - \ln(\sec(x) + \tan(x)) + c_1 \right) \cos(x)$$

✓ Solution by Mathematica

Time used: 0.081 (sec). Leaf size: 45

```
DSolve[Cot[x]*y'[x]+y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x + \cos(x) \left( \log \left( \cos \left( \frac{x}{2} \right) - \sin \left( \frac{x}{2} \right) \right) - \log \left( \sin \left( \frac{x}{2} \right) + \cos \left( \frac{x}{2} \right) \right) + c_1 \right)$$

### 1.13 problem 3(b)

Internal problem ID [3041]

**Book:** Elementary Differential equations, Chaundy, 1969

**Section:** Exercises 3, page 60

**Problem number:** 3(b).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear]`

$$\cot(x)y' + y = \tan(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(cot(x)*diff(y(x),x)+y(x)=tan(x),y(x), singsol=all)
```

$$y(x) = \left( \frac{\sin(x)^3}{2\cos(x)^2} + \frac{\sin(x)}{2} - \frac{\ln(\sec(x) + \tan(x))}{2} + c_1 \right) \cos(x)$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 25

```
DSolve[Cot[x]*y'[x]+y[x]==Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2}(\cos(x)(-\operatorname{arctanh}(\sin(x))) + \tan(x) + 2c_1 \cos(x))$$



## 1.14 problem 3(c)

Internal problem ID [3042]

**Book:** Elementary Differential equations, Chaundy, 1969

**Section:** Exercises 3, page 60

**Problem number:** 3(c).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear]`

$$y' \tan(x) + y = \cot(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(tan(x)*diff(y(x),x)+y(x)=cot(x),y(x), singsol=all)
```

$$y(x) = \frac{\cos(x) + \ln(\csc(x) - \cot(x)) + c_1}{\sin(x)}$$

✓ Solution by Mathematica

Time used: 0.085 (sec). Leaf size: 29

```
DSolve[Tan[x]*y'[x]+y[x]==Cot[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \csc(x) \left( \cos(x) + \log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right) + c_1 \right)$$

## 1.15 problem 3(a)

Internal problem ID [3043]

**Book:** Elementary Differential equations, Chaundy, 1969

**Section:** Exercises 3, page 60

**Problem number:** 3(a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear]`

$$y' \tan(x) - y = -\cos(x)$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(tan(x)*diff(y(x),x)=y(x)-cos(x),y(x), singsol=all)
```

$$y(x) = \left( \cot(x) - \frac{\pi}{2} + x + c_1 \right) \sin(x)$$

### ✓ Solution by Mathematica

Time used: 0.061 (sec). Leaf size: 28

```
DSolve[Tan[x]*y'[x]==y[x]-Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \cos(x) \operatorname{Hypergeometric2F1} \left( -\frac{1}{2}, 1, \frac{1}{2}, -\tan^2(x) \right) + c_1 \sin(x)$$

## 1.16 problem 4(a)

Internal problem ID [3044]

**Book:** Elementary Differential equations, Chaundy, 1969

**Section:** Exercises 3, page 60

**Problem number:** 4(a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear]`

$$y' + y \cos(x) = \sin(2x)$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+y(x)*cos(x)=sin(2*x),y(x), singsol=all)
```

$$y(x) = 2 \sin(x) - 2 + e^{-\sin(x)} c_1$$

### ✓ Solution by Mathematica

Time used: 0.083 (sec). Leaf size: 20

```
DSolve[y'[x]+y[x]*Cos[x]==Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 2 \sin(x) + c_1 e^{-\sin(x)} - 2$$

## 1.17 problem 4(b)

Internal problem ID [3045]

**Book:** Elementary Differential equations, Chaundy, 1969

**Section:** Exercises 3, page 60

**Problem number:** 4(b).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear]`

$$\cos(x) y' + y = \sin(2x)$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(cos(x)*diff(y(x),x)+y(x)=sin(2*x),y(x), singsol=all)
```

$$y(x) = \frac{-2 \sin(x) - 2 \ln(\sin(x) - 1) + c_1}{\sec(x) + \tan(x)}$$

### ✓ Solution by Mathematica

Time used: 0.091 (sec). Leaf size: 42

```
DSolve[Cos[x]*y'[x]+y[x]==Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-2\arctanh(\tan(\frac{x}{2}))} \left( -2 \sin(x) - 4 \log \left( \cos \left( \frac{x}{2} \right) - \sin \left( \frac{x}{2} \right) \right) + c_1 \right)$$

## 1.18 problem 4(c)

Internal problem ID [3046]

**Book:** Elementary Differential equations, Chaundy, 1969

**Section:** Exercises 3, page 60

**Problem number:** 4(c).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear]`

$$y' + y \sin(x) = \sin(2x)$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+y(x)*sin(x)=sin(2*x),y(x), singsol=all)
```

$$y(x) = 2 \cos(x) + 2 + e^{\cos(x)} c_1$$

### ✓ Solution by Mathematica

Time used: 0.075 (sec). Leaf size: 18

```
DSolve[y'[x]+y[x]*Sin[x]==Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 2 \cos(x) + c_1 e^{\cos(x)} + 2$$

## 1.19 problem 4(d)

Internal problem ID [3047]

**Book:** Elementary Differential equations, Chaundy, 1969

**Section:** Exercises 3, page 60

**Problem number:** 4(d).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear]`

$$\sin(x) y' + y = \sin(2x)$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(sin(x)*diff(y(x),x)+y(x)=sin(2*x),y(x), singsol=all)
```

$$y(x) = (-2 \cos(x) + 2 \ln(\cos(x) + 1) + c_1) (\csc(x) + \cot(x))$$

### ✓ Solution by Mathematica

Time used: 0.288 (sec). Leaf size: 38

```
DSolve[Sin[x]*y'[x]+y[x]==Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{\operatorname{arctanh}(\cos(x))} \left( -2 \sqrt{\sin^2(x)} \csc(x) \left( \cos(x) + \log \left( \sec^2 \left( \frac{x}{2} \right) \right) \right) + c_1 \right)$$

## 1.20 problem 5(a)

Internal problem ID [3048]

**Book:** Elementary Differential equations, Chaundy, 1969

**Section:** Exercises 3, page 60

**Problem number:** 5(a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear]`

$$\sqrt{x^2 + 1} y' + y = 2x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
dsolve(sqrt(1+x^2)*diff(y(x),x)+y(x)=2*x,y(x), singsol=all)
```

$$y(x) = \frac{x^2 + x\sqrt{x^2 + 1} - \operatorname{arcsinh}(x) + c_1}{x + \sqrt{x^2 + 1}}$$

✓ Solution by Mathematica

Time used: 0.136 (sec). Leaf size: 50

```
DSolve[Sqrt[1+x^2]*y'[x]+y[x]==2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow (\sqrt{x^2 + 1} - x) \left( x^2 + \sqrt{x^2 + 1}x + \log(\sqrt{x^2 + 1} - x) + c_1 \right)$$

## 1.21 problem 5(b)

Internal problem ID [3049]

**Book:** Elementary Differential equations, Chaundy, 1969

**Section:** Exercises 3, page 60

**Problem number:** 5(b).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$\sqrt{x^2 + 1}y' - y = 2\sqrt{x^2 + 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(sqrt(1+x^2)*diff(y(x),x)-y(x)=2*sqrt(1+x^2),y(x), singsol=all)
```

$$y(x) = \left( x\sqrt{x^2 + 1} + \operatorname{arcsinh}(x) - x^2 + c_1 \right) \left( x + \sqrt{x^2 + 1} \right)$$

✓ Solution by Mathematica

Time used: 0.082 (sec). Leaf size: 55

```
DSolve[Sqrt[1+x^2]*y'[x]-y[x]==2*Sqrt[1+x^2],y[x],x,IncludeSingularSolutions->True]
```

$$y(x) \rightarrow \frac{x^2 - \sqrt{x^2 + 1}x + \log(\sqrt{x^2 + 1} - x) - c_1}{x - \sqrt{x^2 + 1}}$$



## 1.22 problem 5(c)

Internal problem ID [3050]

**Book:** Elementary Differential equations, Chaundy, 1969

**Section:** Exercises 3, page 60

**Problem number:** 5(c).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear]`

$$\sqrt{(x+a)(x+b)}(2y' - 3) + y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 63

```
dsolve(sqrt((x+a)*(x+b))*(2*diff(y(x),x)-3)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{2 \left( \int \frac{3\sqrt{2a+2b+4x+4\sqrt{(x+a)(x+b)}}}{4} dx \right) + 2c_1}{\sqrt{2a+2b+4x+4\sqrt{x^2+(a+b)x+ab}}}$$

### ✓ Solution by Mathematica

Time used: 0.433 (sec). Leaf size: 115

```
DSolve[Sqrt[(x+a)*(x+b)]*(2*y'[x]-3)+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \exp \left( -\frac{\sqrt{a+x}\sqrt{b+x} \operatorname{arctanh} \left( \frac{\sqrt{b+x}}{\sqrt{a+x}} \right)}{\sqrt{(a+x)(b+x)}} \right) \left( \int_1^x \frac{3}{2} \exp \left( \frac{\operatorname{arctanh} \left( \frac{\sqrt{b+K[1]}}{\sqrt{a+K[1]}} \right) \sqrt{a+K[1]}\sqrt{b+K[1]}}{\sqrt{(a+K[1])(b+K[1])}} \right) dK + c_1 \right)$$

## 1.23 problem 5(d)

Internal problem ID [3051]

**Book:** Elementary Differential equations, Chaundy, 1969

**Section:** Exercises 3, page 60

**Problem number:** 5(d).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$\sqrt{(x+a)(x+b)}y' + y = \sqrt{x+a} - \sqrt{x+b}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 103

```
dsolve(sqrt((x+a)*(x+b))*diff(y(x),x)+y(x)=sqrt(x+a)-sqrt(x+b),y(x), singsol=all)
```

$$y(x) = \frac{\frac{2(x+a)^{\frac{3}{2}}}{3} - \frac{2(x+b)^{\frac{3}{2}}}{3} + \frac{\sqrt{x+a}(x+b)(2x-b+3a)}{3\sqrt{(x+a)(x+b)}} - \frac{\sqrt{x+b}(x+a)(2x-a+3b)}{3\sqrt{(x+a)(x+b)}} + C_1}{\frac{a}{2} + \frac{b}{2} + x + \sqrt{x^2 + (a+b)x + ab}}$$

✓ Solution by Mathematica

Time used: 2.411 (sec). Leaf size: 145

`DSolve[Sqrt[(x+a)*(x+b)]*y'[x]+y[x]==Sqrt[x+a]-Sqrt[x+b],y[x],x,IncludeSingularSolutions ->`

$y(x)$

$$\rightarrow \exp\left(-\frac{2\sqrt{a+x}\sqrt{b+x}\operatorname{arctanh}\left(\frac{\sqrt{b+x}}{\sqrt{a+x}}\right)}{\sqrt{(a+x)(b+x)}}\right) \left( \int_1^x \frac{\exp\left(\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b+K[1]}}{\sqrt{a+K[1]}}\right)\sqrt{a+K[1]}\sqrt{b+K[1]}}{\sqrt{(a+K[1])(b+K[1])}}\right)}{\sqrt{(a+K[1])(b+K[1])}} \left(\sqrt{a+K[1]}\right) dx + c_1 \right)$$