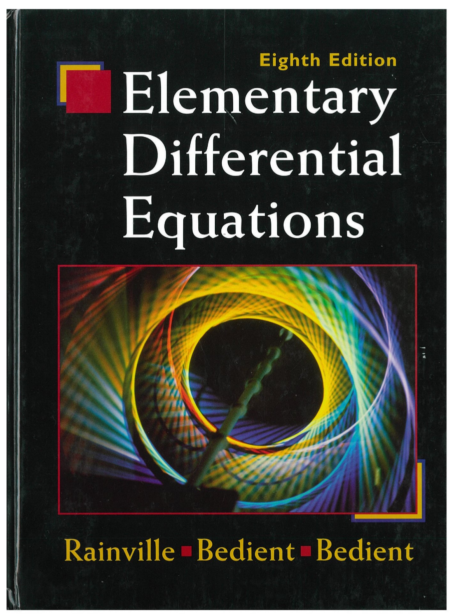
A Solution Manual For

# Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.

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# 1 CHAPTER 8. Nonhomogeneous Equations: Undetermined Coefficients. Exercises Page 142


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## 1.1 problem 1

Internal problem ID [6861]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 8. Nonhomogeneous Equations: Undetermined Coefficients. Exercises Page 142
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = -\cos(x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)+y(x)=-cos(x),y(x), singsol=all)
```

$$y(x) = \sin(x)\, c_2 + \cos(x)\, c_1 - \frac{\sin(x)\, x}{2}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 27

```
DSolve[y''[x]+y[x]==-Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(-\frac{1}{2} + c_1\right)\cos(x) - \frac{1}{2}(x - 2c_2)\sin(x)$$

## 1.2 problem 2

Internal problem ID [6862]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 8. Nonhomogeneous Equations: Undetermined Coefficients. Exercises Page 142
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 6y' + 9y = e^{x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+9*y(x)=exp(x),y(x), singsol=all)
```

$$y(x) = e^{3x}c_2 + e^{3x}xc_1 + \frac{e^{x}}{4}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 26

```
DSolve[y''[x]-6*y'[x]+9*y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{x}}{4} + e^{3x}(c_2x + c_1)$$

## 1.3 problem 3

Internal problem ID [6863]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 8. Nonhomogeneous Equations: Undetermined Coefficients. Exercises Page 142
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 3y' + 2y = 12x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=12*x^2,y(x), singsol=all)
```

$$y(x) = -\mathrm{e}^{-2x}c_1 + c_2\mathrm{e}^{-x} + 6x^2 - 18x + 21$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 31

```
DSolve[y''[x]+3*y'[x]+2*y[x]==12*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 6x^2 - 18x + c_1e^{-2x} + c_2e^{-x} + 21$$

## 1.4 problem 4

Internal problem ID [6864]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 8. Nonhomogeneous Equations: Undetermined Coefficients. Exercises Page 142
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 3y' + 2y = x^2 + 2x + 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=1+2*x+x^2,y(x), singsol=all)
```

$$y(x) = \frac{3}{4} - \frac{x}{2} + \frac{x^2}{2} - e^{-2x}c_1 + c_2e^{-x}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 36

```
DSolve[y''[x]+3*y'[x]+2*y[x]==1+2*x+x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}(2x^2 - 2x + 3) + c_1e^{-2x} + c_2e^{-x}$$

# 2 CHAPTER 16. Nonlinear equations. Miscellaneous Exercises. Page 340


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## 2.1 problem 1

Internal problem ID [6865]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 16. Nonlinear equations. Miscellaneous Exercises. Page 340
**Problem number**: 1.
**ODE order**: 1.
**ODE degree**: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{x^3 {y'}^2 + x^2 y y' = -4}$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 53

```
dsolve(x^3*diff(y(x),x)^2+x^2*y(x)*diff(y(x),x)+4=0,y(x), singsol=all)
```

$$y(x) = -\frac{4}{\sqrt{x}}$$

$$y(x) = \frac{4}{\sqrt{x}}$$

$$y(x) = \frac{x\,c_1^2 + 16}{2xc_1}$$

$$y(x) = \frac{c_1^2 + 16x}{2xc_1}$$

✓ Solution by Mathematica

Time used: 0.558 (sec). Leaf size: 77

```
DSolve[x^3*(y'[x])^2+x^2*y[x]*y'[x]+4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{e^{-\frac{c_1}{2}}(x + 64e^{c_1})}{4x}$$

$$y(x) \rightarrow \frac{e^{-\frac{c_1}{2}}(x + 64e^{c_1})}{4x}$$

$$y(x) \rightarrow -\frac{4}{\sqrt{x}}$$

$$y(x) \rightarrow \frac{4}{\sqrt{x}}$$

## 2.2 problem 2

Internal problem ID [6866]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 16. Nonlinear equations. Miscellaneous Exercises. Page 340
**Problem number**: 2.
**ODE order**: 1.
**ODE degree**: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{6xy'^2 - (3x + 2y)\, y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(6*x*diff(y(x),x)^2-(3*x+2*y(x))*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{\frac{1}{3}}$$
$$y(x) = \frac{x}{2} + c_1$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 30

```
DSolve[6*x*(y'[x])^2-(3*x+2*y[x])*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \sqrt[3]{x}$$
$$y(x) \to \frac{x}{2} + c_1$$
$$y(x) \to 0$$

## 2.3 problem 3

Internal problem ID [6867]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 16. Nonlinear equations. Miscellaneous Exercises. Page 340
**Problem number**: 3.
**ODE order**: 1.
**ODE degree**: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{9y'^2 + 3y^4y'x + y^5 = 0}$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 109

```
dsolve(9*diff(y(x),x)^2+3*x*y(x)^4*diff(y(x),x)+y(x)^5=0,y(x), singsol=all)
```

$$y(x) = \frac{4^{\frac{1}{3}}}{x^{\frac{2}{3}}}$$

$$y(x) = -\frac{4^{\frac{1}{3}}}{2x^{\frac{2}{3}}} - \frac{i\sqrt{3}\,4^{\frac{1}{3}}}{2x^{\frac{2}{3}}}$$

$$y(x) = -\frac{4^{\frac{1}{3}}}{2x^{\frac{2}{3}}} + \frac{i\sqrt{3}\,4^{\frac{1}{3}}}{2x^{\frac{2}{3}}}$$

$$y(x) = 0$$

$$y(x) = \frac{\operatorname{RootOf}\left(-\ln(x) + \int^{\_Z} \frac{\frac{3\_a^3}{2} + \frac{3\sqrt{\_a^3(\_a^3-4)}}{2} - 6}{\_a(\_a^3-4)} d\_a + c_1\right)}{x^{\frac{2}{3}}}$$

✓ Solution by Mathematica

Time used: 1.025 (sec). Leaf size: 212

```
DSolve[9*(y'[x])^2+3*x*y[x]^4*y'[x]+y[x]^5==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{\sqrt{x^2y(x)^3-4}\,y(x)^{5/2}\text{arctanh}\left(\frac{xy(x)^{3/2}}{\sqrt{x^2y(x)^3-4}}\right)}{\sqrt{y(x)^5\left(x^2y(x)^3-4\right)}}-\frac{3}{2}\log(y(x))=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{y(x)^{5/2}\sqrt{x^2y(x)^3-4}\,\text{arctanh}\left(\frac{xy(x)^{3/2}}{\sqrt{x^2y(x)^3-4}}\right)}{\sqrt{y(x)^5\left(x^2y(x)^3-4\right)}}-\frac{3}{2}\log(y(x))=c_1,y(x)\right]$$

$$y(x)\to 0$$

$$y(x)\to \frac{(-2)^{2/3}}{x^{2/3}}$$

$$y(x)\to \frac{2^{2/3}}{x^{2/3}}$$

$$y(x)\to -\frac{\sqrt[3]{-1}2^{2/3}}{x^{2/3}}$$

## 2.4 problem 4

Internal problem ID [6868]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 16. Nonlinear equations. Miscellaneous Exercises. Page 340
**Problem number**: 4.
**ODE order**: 1.
**ODE degree**: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$\boxed{4y^3y'^2 - 4y'x + y = 0}$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 81

```
dsolve(4*y(x)^3*diff(y(x),x)^2-4*x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{-x}$$
$$y(x) = -\sqrt{-x}$$
$$y(x) = \sqrt{x}$$
$$y(x) = -\sqrt{x}$$
$$y(x) = 0$$
$$y(x) = \text{RootOf}\left(-\ln(x) + \int^{\_Z} -\frac{2(\_a^4 + \sqrt{-\_a^4 + 1} - 1)}{\_a(\_a^4 - 1)} d\_a + c_1\right)\sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.581 (sec). Leaf size: 282

```
DSolve[4*y[x]^3*(y'[x])^2-4*x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{\frac{c_1}{4}} \sqrt[4]{e^{c_1} - 2ix}$$
$$y(x) \to -ie^{\frac{c_1}{4}} \sqrt[4]{e^{c_1} - 2ix}$$
$$y(x) \to ie^{\frac{c_1}{4}} \sqrt[4]{e^{c_1} - 2ix}$$
$$y(x) \to e^{\frac{c_1}{4}} \sqrt[4]{e^{c_1} - 2ix}$$
$$y(x) \to -e^{\frac{c_1}{4}} \sqrt[4]{2ix + e^{c_1}}$$
$$y(x) \to -ie^{\frac{c_1}{4}} \sqrt[4]{2ix + e^{c_1}}$$
$$y(x) \to ie^{\frac{c_1}{4}} \sqrt[4]{2ix + e^{c_1}}$$
$$y(x) \to e^{\frac{c_1}{4}} \sqrt[4]{2ix + e^{c_1}}$$
$$y(x) \to 0$$
$$y(x) \to -\sqrt{x}$$
$$y(x) \to -i\sqrt{x}$$
$$y(x) \to i\sqrt{x}$$
$$y(x) \to \sqrt{x}$$

## 2.5 problem 5

Internal problem ID [6869]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 16. Nonlinear equations. Miscellaneous Exercises. Page 340
**Problem number**: 5.
**ODE order**: 1.
**ODE degree**: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{x^6 {y'}^2 - 2y'x - 4y = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 143

```
dsolve(x^6*diff(y(x),x)^2-2*x*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{4x^4}$$

$$y(x) = \frac{-2x^4 - c_1^2 - c_1(2ix^2 - c_1)}{2c_1^2x^4}$$

$$y(x) = \frac{-2x^4 - c_1^2 - c_1(-2ix^2 - c_1)}{2c_1^2x^4}$$

$$y(x) = \frac{-2x^4 + c_1(2ix^2 + c_1) - c_1^2}{2c_1^2x^4}$$

$$y(x) = \frac{-2x^4 + c_1(-2ix^2 + c_1) - c_1^2}{2c_1^2x^4}$$

✓ Solution by Mathematica

Time used: 0.532 (sec). Leaf size: 128

```
DSolve[x^6*(y'[x])^2-2*x*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{x\sqrt{4x^4y(x)+1}\text{arctanh}\left(\sqrt{4x^4y(x)+1}\right)}{2\sqrt{4x^6y(x)+x^2}}-\frac{1}{4}\log(y(x))=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{x\sqrt{4x^4y(x)+1}\text{arctanh}\left(\sqrt{4x^4y(x)+1}\right)}{2\sqrt{4x^6y(x)+x^2}}-\frac{1}{4}\log(y(x))=c_1,y(x)\right]$$

$$y(x)\to 0$$

## 2.6 problem 6

Internal problem ID [6870]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 16. Nonlinear equations. Miscellaneous Exercises. Page 340
**Problem number**: 6.
**ODE order**: 1.
**ODE degree**: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{5{y'}^2 + 6y'x - 2y = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 85

```
dsolve(5*diff(y(x),x)^2+6*x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$\frac{c_1}{\left(-15x - 5\sqrt{9x^2 + 10y(x)}\right)^{\frac{3}{2}}} + \frac{2x}{5} - \frac{\sqrt{9x^2 + 10y(x)}}{5} = 0$$

$$\frac{c_1}{\left(-15x + 5\sqrt{9x^2 + 10y(x)}\right)^{\frac{3}{2}}} + \frac{2x}{5} + \frac{\sqrt{9x^2 + 10y(x)}}{5} = 0$$

✓ Solution by Mathematica

Time used: 14.054 (sec). Leaf size: 771

```
DSolve[5*(y'[x])^2+6*x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{Root}\left[4\#1^5 + 4\#1^4x^2 + \#1^3x^4 + 1000\#1^2e^{5c_1}x + 900\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 25000e^{10c_1}\&, 1\right]$$

$$y(x) \to \text{Root}\left[4\#1^5 + 4\#1^4x^2 + \#1^3x^4 + 1000\#1^2e^{5c_1}x + 900\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 25000e^{10c_1}\&, 2\right]$$

$$y(x) \to \text{Root}\left[4\#1^5 + 4\#1^4x^2 + \#1^3x^4 + 1000\#1^2e^{5c_1}x + 900\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 25000e^{10c_1}\&, 3\right]$$

$$y(x) \to \text{Root}\left[4\#1^5 + 4\#1^4x^2 + \#1^3x^4 + 1000\#1^2e^{5c_1}x + 900\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 25000e^{10c_1}\&, 4\right]$$

$$y(x) \to \text{Root}\left[4\#1^5 + 4\#1^4x^2 + \#1^3x^4 + 1000\#1^2e^{5c_1}x + 900\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 25000e^{10c_1}\&, 5\right]$$

$$y(x) \to \text{Root}\left[100000\#1^5 + 100000\#1^4x^2 + 25000\#1^3x^4 - 1000\#1^2e^{5c_1}x - 900\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 1\right]$$

$$y(x) \to \text{Root}\left[100000\#1^5 + 100000\#1^4x^2 + 25000\#1^3x^4 - 1000\#1^2e^{5c_1}x - 900\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 2\right]$$

$$y(x) \to \text{Root}\left[100000\#1^5 + 100000\#1^4x^2 + 25000\#1^3x^4 - 1000\#1^2e^{5c_1}x - 900\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 3\right]$$

$$y(x) \to \text{Root}\left[100000\#1^5 + 100000\#1^4x^2 + 25000\#1^3x^4 - 1000\#1^2e^{5c_1}x - 900\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 4\right]$$

$$y(x) \to \text{Root}\left[100000\#1^5 + 100000\#1^4x^2 + 25000\#1^3x^4 - 1000\#1^2e^{5c_1}x - 900\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 5\right]$$

$$y(x) \to 0$$

## 2.7 problem 8

Internal problem ID [6871]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 16. Nonlinear equations. Miscellaneous Exercises. Page 340
**Problem number**: 8.
**ODE order**: 1.
**ODE degree**: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y^2 y'^2 - y(x+1)\, y' = -x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(y(x)^2*diff(y(x),x)^2-y(x)*(x+1)*diff(y(x),x)+x=0,y(x), singsol=all)
```

$$y(x) = \sqrt{2x + c_1}$$
$$y(x) = -\sqrt{2x + c_1}$$
$$y(x) = \sqrt{x^2 + c_1}$$
$$y(x) = -\sqrt{x^2 + c_1}$$

✓ Solution by Mathematica

Time used: 0.141 (sec). Leaf size: 72

```
DSolve[y[x]^2*(y'[x])^2-y[x]*(x+1)*y'[x]+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}\sqrt{x + c_1}$$
$$y(x) \to \sqrt{2}\sqrt{x + c_1}$$
$$y(x) \to -\sqrt{x^2 + 2c_1}$$
$$y(x) \to \sqrt{x^2 + 2c_1}$$

## 2.8 problem 9

Internal problem ID [6872]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 16. Nonlinear equations. Miscellaneous Exercises. Page 340
**Problem number**: 9.
**ODE order**: 1.
**ODE degree**: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{4x^5y'^2 + 12yy'x^4 = -9}$$

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 53

```
dsolve(4*x^5*diff(y(x),x)^2+12*x^4*y(x)*diff(y(x),x)+9=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{x^{\frac{3}{2}}}$$

$$y(x) = -\frac{1}{x^{\frac{3}{2}}}$$

$$y(x) = \frac{c_1^2x^3 + 1}{2c_1x^3}$$

$$y(x) = \frac{x^3 + c_1^2}{2c_1x^3}$$

✓ Solution by Mathematica

Time used: 6.994 (sec). Leaf size: 75

```
DSolve[4*x^5*(y'[x])^2+12*x^4*y[x]*y'[x]+9==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{x^3 \operatorname{sech}^2\left(\frac{3}{2}(-\log(x)+c_1)\right)}}$$

$$y(x) \to \frac{1}{\sqrt{x^3 \operatorname{sech}^2\left(\frac{3}{2}(-\log(x)+c_1)\right)}}$$

$$y(x) \to -\frac{1}{x^{3/2}}$$

$$y(x) \to \frac{1}{x^{3/2}}$$

## 2.9 problem 10

Internal problem ID [6873]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 16. Nonlinear equations. Miscellaneous Exercises. Page 340
**Problem number**: 10.
**ODE order**: 1.
**ODE degree**: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{4{y'}^3 y^2 - 2y'x + y = 0}$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 95

```
dsolve(4*y(x)^2*diff(y(x),x)^3-2*x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{2^{\frac{3}{4}} 3^{\frac{1}{4}} x^{\frac{3}{4}}}{3}$$
$$y(x) = \frac{2^{\frac{3}{4}} 3^{\frac{1}{4}} x^{\frac{3}{4}}}{3}$$
$$y(x) = -\frac{i 2^{\frac{3}{4}} 3^{\frac{1}{4}} x^{\frac{3}{4}}}{3}$$
$$y(x) = \frac{i 2^{\frac{3}{4}} 3^{\frac{1}{4}} x^{\frac{3}{4}}}{3}$$
$$y(x) = 0$$
$$y(x) = \sqrt{-4c_1^3 + 2c_1 x}$$
$$y(x) = -\sqrt{-4c_1^3 + 2c_1 x}$$

✓ Solution by Mathematica

Time used: 83.967 (sec). Leaf size: 11250

```
DSolve[4*y[x]^2*(y'[x])^3-2*x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

## 2.10 problem 11

Internal problem ID [6874]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 16. Nonlinear equations. Miscellaneous Exercises. Page 340
**Problem number**: 11.
**ODE order**: 1.
**ODE degree**: 4.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y'^4 + y'x - 3y = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 37

```
dsolve(diff(y(x),x)^4+x*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$\left[x(\_T) = \sqrt{\_T}\left(\frac{4\_T^{\frac{5}{2}}}{5} + c_1\right), y(\_T) = \frac{\_T^4}{3} + \frac{\_T^{\frac{3}{2}}\left(\frac{4\_T^{\frac{5}{2}}}{5} + c_1\right)}{3}\right]$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y'[x])^4+x*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

## 2.11 problem 13

Internal problem ID [6875]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 16. Nonlinear equations. Miscellaneous Exercises. Page 340
**Problem number**: 13.
**ODE order**: 1.
**ODE degree**: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{x^2y'^3 - 2xyy'^2 + y'y^2 = -1}$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 81

```
dsolve(x^2*diff(y(x),x)^3-2*x*y(x)*diff(y(x),x)^2+y(x)^2*diff(y(x),x)+1=0,y(x), singsol=all)
```

$$y(x) = \frac{3(-2x)^{\frac{1}{3}}}{2}$$
$$y(x) = -\frac{3(-2x)^{\frac{1}{3}}}{4} - \frac{3i\sqrt{3}\,(-2x)^{\frac{1}{3}}}{4}$$
$$y(x) = -\frac{3(-2x)^{\frac{1}{3}}}{4} + \frac{3i\sqrt{3}\,(-2x)^{\frac{1}{3}}}{4}$$
$$y(x) = c_1x - \frac{1}{\sqrt{-c_1}}$$
$$y(x) = c_1x + \frac{1}{\sqrt{-c_1}}$$

✓ Solution by Mathematica

Time used: 66.431 (sec). Leaf size: 33909

```
DSolve[x^2*(y'[x])^3-2*x*y[x]*(y'[x])^2+y[x]^2*y'[x]+1==0,y[x],x,IncludeSingularSolutions ->
```

Too large to display

## 2.12 problem 14

Internal problem ID [6876]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 16. Nonlinear equations. Miscellaneous Exercises. Page 340
**Problem number**: 14.
**ODE order**: 1.
**ODE degree**: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{16xy'^2 + 8y'y + y^6 = 0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 103

```
dsolve(16*x*diff(y(x),x)^2+8*y(x)*diff(y(x),x)+y(x)^6=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{x^{\frac{1}{4}}}$$
$$y(x) = -\frac{1}{x^{\frac{1}{4}}}$$
$$y(x) = -\frac{i}{x^{\frac{1}{4}}}$$
$$y(x) = \frac{i}{x^{\frac{1}{4}}}$$
$$y(x) = 0$$
$$y(x) = \frac{\operatorname{RootOf}\left(-\ln(x) + c_1 + 4\left(\int^{\_Z} \frac{1}{\_a\sqrt{-\_a^4+1}}d\_a\right)\right)}{x^{\frac{1}{4}}}$$
$$y(x) = \frac{\operatorname{RootOf}\left(-\ln(x) + c_1 - 4\left(\int^{\_Z} \frac{1}{\_a\sqrt{-\_a^4+1}}d\_a\right)\right)}{x^{\frac{1}{4}}}$$

✓ Solution by Mathematica

Time used: 0.684 (sec). Leaf size: 171

```
DSolve[16*x*(y'[x])^2+8*y[x]*y'[x]+y[x]^6==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{2}e^{\frac{c_1}{4}}}{\sqrt{x+e^{c_1}}}$$
$$y(x) \to -\frac{i\sqrt{2}e^{\frac{c_1}{4}}}{\sqrt{x+e^{c_1}}}$$
$$y(x) \to \frac{i\sqrt{2}e^{\frac{c_1}{4}}}{\sqrt{x+e^{c_1}}}$$
$$y(x) \to \frac{\sqrt{2}e^{\frac{c_1}{4}}}{\sqrt{x+e^{c_1}}}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{1}{\sqrt[4]{x}}$$
$$y(x) \to -\frac{i}{\sqrt[4]{x}}$$
$$y(x) \to \frac{i}{\sqrt[4]{x}}$$
$$y(x) \to \frac{1}{\sqrt[4]{x}}$$

## 2.13 problem 15

Internal problem ID [6877]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 16. Nonlinear equations. Miscellaneous Exercises. Page 340
**Problem number**: 15.
**ODE order**: 1.
**ODE degree**: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x{y'}^2-\left(x^2+1\right)y'=-x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x*diff(y(x),x)^2-(x^2+1)*diff(y(x),x)+x=0,y(x), singsol=all)
```

$$y(x)=\ln\left(x\right)+c_1$$
$$y(x)=\frac{x^2}{2}+c_1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 24

```
DSolve[x*(y'[x])^2-(x^2+1)*y'[x]+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\rightarrow\frac{x^2}{2}+c_1$$
$$y(x)\rightarrow\log(x)+c_1$$

## 2.14 problem 16

Internal problem ID [6878]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 16. Nonlinear equations. Miscellaneous Exercises. Page 340
**Problem number**: 16.
**ODE order**: 1.
**ODE degree**: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y'^3 - 2y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 496

```
dsolve(diff(y(x),x)^3-2*x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$-\frac{c_1}{\left(\frac{\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}+24x}{\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{1}{3}}}\right)^{\frac{2}{3}}}+x-\frac{\left(\left(108y(x)+12\sqrt{-96x^3+81y\left(x\right)^2}\right)^{\frac{2}{3}}+24x\right)^2}{96\left(108y\left(x\right)+12\sqrt{-96x^3+81y\left(x\right)^2}\right)^{\frac{2}{3}}}=0$$

$$-\frac{c_1}{\left(\frac{i\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}\sqrt{3}-24ix\sqrt{3}-\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}-24x}{\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{1}{3}}}\right)^{\frac{2}{3}}}+x-\frac{\left(i\left(108y(x)+12\sqrt{-96x^3+81y\left(x\right)^2}\right)^{\frac{2}{3}}\sqrt{3}-24ix\sqrt{3}-\left(108y(x)+12\sqrt{-96x^3+81y\left(x\right)^2}\right)^{\frac{2}{3}}-24x\right)}{384\left(108y\left(x\right)+12\sqrt{-96x^3+81y\left(x\right)^2}\right)^{\frac{2}{3}}}=0$$

$$-\frac{12^{\frac{2}{3}}c_1}{\left(\frac{-i\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}\sqrt{3}+24ix\sqrt{3}-\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}-24x}{\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{1}{3}}}\right)^{\frac{2}{3}}}+x-\frac{\left(i\left(108y(x)+12\sqrt{-96x^3+81y\left(x\right)^2}\right)^{\frac{2}{3}}\sqrt{3}-24ix\sqrt{3}+\left(108y(x)+12\sqrt{-96x^3+81y\left(x\right)^2}\right)^{\frac{2}{3}}+24x\right)}{384\left(108y\left(x\right)+12\sqrt{-96x^3+81y\left(x\right)^2}\right)^{\frac{2}{3}}}=0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y'[x])^3-2*x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

## 2.15 problem 18

Internal problem ID [6879]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 16. Nonlinear equations. Miscellaneous Exercises. Page 340
**Problem number**: 18.
**ODE order**: 1.
**ODE degree**: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{9xy^4y'^2 - 3y^5y' = 1}$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 283

```
dsolve(9*x*y(x)^4*diff(y(x),x)^2-3*y(x)^5*diff(y(x),x)-1=0,y(x), singsol=all)
```

$$y(x) = 2^{\frac{1}{3}}(-x)^{\frac{1}{6}}$$
$$y(x) = -2^{\frac{1}{3}}(-x)^{\frac{1}{6}}$$
$$y(x) = \left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)2^{\frac{1}{3}}(-x)^{\frac{1}{6}}$$
$$y(x) = \left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)2^{\frac{1}{3}}(-x)^{\frac{1}{6}}$$
$$y(x) = \left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)2^{\frac{1}{3}}(-x)^{\frac{1}{6}}$$
$$y(x) = \left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)2^{\frac{1}{3}}(-x)^{\frac{1}{6}}$$
$$y(x) = \frac{\left(\left(c_1^2-2c_1x+x^2\right)c_1^5\right)^{\frac{1}{6}}}{c_1}$$
$$y(x) = -\frac{\left(\left(c_1^2-2c_1x+x^2\right)c_1^5\right)^{\frac{1}{6}}}{c_1}$$
$$y(x) = \frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(\left(c_1^2-2c_1x+x^2\right)c_1^5\right)^{\frac{1}{6}}}{c_1}$$
$$y(x) = \frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\left(c_1^2-2c_1x+x^2\right)c_1^5\right)^{\frac{1}{6}}}{c_1}$$
$$y(x) = \frac{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(\left(c_1^2-2c_1x+x^2\right)c_1^5\right)^{\frac{1}{6}}}{c_1}$$
$$y(x) = \frac{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\left(c_1^2-2c_1x+x^2\right)c_1^5\right)^{\frac{1}{6}}}{c_1}$$

✓ Solution by Mathematica

Time used: 3.005 (sec). Leaf size: 322

```
DSolve[9*x*y[x]^4*(y'[x])^2-3*y[x]^5*y'[x]-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt[3]{-\frac{1}{2}}e^{-\frac{c_1}{6}}\sqrt[3]{-4x+e^{c_1}}$$

$$y(x) \to \frac{e^{-\frac{c_1}{6}}\sqrt[3]{-4x+e^{c_1}}}{\sqrt[3]{2}}$$

$$y(x) \to \frac{(-1)^{2/3}e^{-\frac{c_1}{6}}\sqrt[3]{-4x+e^{c_1}}}{\sqrt[3]{2}}$$

$$y(x) \to -\sqrt[3]{-\frac{1}{2}}\sqrt[3]{-e^{-\frac{c_1}{2}}\left(-4x+e^{c_1}\right)}$$

$$y(x) \to \frac{\sqrt[3]{e^{-\frac{c_1}{2}}\left(4x-e^{c_1}\right)}}{\sqrt[3]{2}}$$

$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{-e^{-\frac{c_1}{2}}\left(-4x+e^{c_1}\right)}}{\sqrt[3]{2}}$$

$$y(x) \to -i\sqrt[3]{2}\sqrt[6]{x}$$

$$y(x) \to i\sqrt[3]{2}\sqrt[6]{x}$$

$$y(x) \to -\sqrt[6]{-1}\sqrt[3]{2}\sqrt[6]{x}$$

$$y(x) \to \sqrt[6]{-1}\sqrt[3]{2}\sqrt[6]{x}$$

$$y(x) \to -(-1)^{5/6}\sqrt[3]{2}\sqrt[6]{x}$$

$$y(x) \to (-1)^{5/6}\sqrt[3]{2}\sqrt[6]{x}$$

## 2.16 problem 19

Internal problem ID [6880]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 16. Nonlinear equations. Miscellaneous Exercises. Page 340
**Problem number**: 19.
**ODE order**: 1.
**ODE degree**: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Clairaut]`

$$\boxed{x^2y'^2-(2yx+1)\,y'+y^2=-1}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 42

```
dsolve(x^2*diff(y(x),x)^2-(2*x*y(x)+1)*diff(y(x),x)+y(x)^2+1=0,y(x), singsol=all)
```

$$y(x)=\frac{4x^2-1}{4x}$$
$$y(x)=c_1x-\sqrt{c_1-1}$$
$$y(x)=c_1x+\sqrt{c_1-1}$$

✓ Solution by Mathematica

Time used: 1.527 (sec). Leaf size: 66

```
DSolve[x^2*y'[x]^2-(2*x*y[x]+1)*y'[x]+y[x]^2+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x+e^{-2c_1}x+e^{-c_1}$$
$$y(x)\to x+\frac{1}{4}e^{-2c_1}x+\frac{e^{-c_1}}{2}$$
$$y(x)\to x$$
$$y(x)\to x-\frac{1}{4x}$$

## 2.17 problem 20

Internal problem ID [6881]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 16. Nonlinear equations. Miscellaneous Exercises. Page 340
**Problem number**: 20.
**ODE order**: 1.
**ODE degree**: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{x^6 {y'}^2 - 16y - 8y'x = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 141

```
dsolve(x^6*diff(y(x),x)^2=8*(2*y(x)+x*diff(y(x),x)),y(x), singsol=all)
```

$$y(x) = -\frac{1}{x^4}$$
$$y(x) = \frac{-x^4 + 2c_1(i x^2 + c_1) - 2c_1^2}{c_1^2 x^4}$$
$$y(x) = \frac{-x^4 + 2c_1(-i x^2 + c_1) - 2c_1^2}{c_1^2 x^4}$$
$$y(x) = \frac{-x^4 - 2c_1(i x^2 - c_1) - 2c_1^2}{c_1^2 x^4}$$
$$y(x) = \frac{-x^4 - 2c_1(-i x^2 - c_1) - 2c_1^2}{c_1^2 x^4}$$

✓ Solution by Mathematica

Time used: 0.535 (sec). Leaf size: 122

```
DSolve[x^6*y'[x]^2==8*(2*y[x]+x*y'[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{x\sqrt{x^4y(x)+1}\text{arctanh}\left(\sqrt{x^4y(x)+1}\right)}{2\sqrt{x^6y(x)+x^2}}-\frac{1}{4}\log(y(x))=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{x\sqrt{x^4y(x)+1}\text{arctanh}\left(\sqrt{x^4y(x)+1}\right)}{2\sqrt{x^6y(x)+x^2}}-\frac{1}{4}\log(y(x))=c_1,y(x)\right]$$

$$y(x)\to 0$$

## 2.18 problem 21

Internal problem ID [6882]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 16. Nonlinear equations. Miscellaneous Exercises. Page 340
**Problem number**: 21.
**ODE order**: 1.
**ODE degree**: 2.

CAS Maple gives this as type `[_linear]`

$$x^2 {y'}^2 - (x-y)^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x^2*diff(y(x),x)^2=(x-y(x))^2,y(x), singsol=all)
```

$$y(x) = (-\ln(x) + c_1) x$$
$$y(x) = \frac{x}{2} + \frac{c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.073 (sec). Leaf size: 30

```
DSolve[x^2*y'[x]^2==(x-y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{2} + \frac{c_1}{x}$$
$$y(x) \to x(-\log(x) + c_1)$$

## 2.19 problem 22

Internal problem ID [6883]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 16. Nonlinear equations. Miscellaneous Exercises. Page 340
**Problem number**: 22.
**ODE order**: 1.
**ODE degree**: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$(y'+1)^2\,(y-y'x)=1$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 106

```
dsolve((diff(y(x),x)+1)^2*(y(x)-diff(y(x),x)*x)=1,y(x), singsol=all)
```

$$y(x)=\frac{3\,2^{\frac{1}{3}}\left(x^2\right)^{\frac{1}{3}}}{2}-x$$

$$y(x)=-\frac{3\,2^{\frac{1}{3}}\left(x^2\right)^{\frac{1}{3}}}{4}-\frac{3i\sqrt{3}\,2^{\frac{1}{3}}\left(x^2\right)^{\frac{1}{3}}}{4}-x$$

$$y(x)=-\frac{3\,2^{\frac{1}{3}}\left(x^2\right)^{\frac{1}{3}}}{4}+\frac{3i\sqrt{3}\,2^{\frac{1}{3}}\left(x^2\right)^{\frac{1}{3}}}{4}-x$$

$$y(x)=\frac{\left(c_1^3+2c_1^2+c_1\right)x}{\left(1+c_1\right)^2}+\frac{1}{\left(1+c_1\right)^2}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 102

```
DSolve[(y'[x]+1)^2*(y[x]-y'[x]*x)==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x + \frac{1}{(1 + c_1)^2}$$

$$y(x) \to \frac{3x^{2/3}}{2^{2/3}} - x$$

$$y(x) \to -x + \frac{3i\left(\sqrt{3} + i\right) x^{2/3}}{2\ 2^{2/3}}$$

$$y(x) \to -x - \frac{3\left(1 + i\sqrt{3}\right) x^{2/3}}{2\ 2^{2/3}}$$

## 2.20 problem 23

Internal problem ID [6884]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 16. Nonlinear equations. Miscellaneous Exercises. Page 340
**Problem number**: 23.
**ODE order**: 1.
**ODE degree**: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y'^3 - y'^2 + y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 70

```
dsolve(diff(y(x),x)^3-diff(y(x),x)^2+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x}{3} - \frac{2}{27} - \frac{2\sqrt{-27x^3 + 27x^2 - 9x + 1}}{27}$$
$$y(x) = \frac{x}{3} - \frac{2}{27} + \frac{2\sqrt{-27x^3 + 27x^2 - 9x + 1}}{27}$$
$$y(x) = c_1^3 - c_1^2 + c_1 x$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 74

```
DSolve[y'[x]^3-y'[x]^2+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x + (-1 + c_1)c_1)$$
$$y(x) \to \frac{1}{27}\left(9x - 2\left(\sqrt{-(3x-1)^3} + 1\right)\right)$$
$$y(x) \to \frac{1}{27}\left(9x + 2\sqrt{-(3x-1)^3} - 2\right)$$

## 2.21 problem 24

Internal problem ID [6885]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 16. Nonlinear equations. Miscellaneous Exercises. Page 340
**Problem number**: 24.
**ODE order**: 1.
**ODE degree**: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xy'^2 + y(1-x)\,y' - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x)^2+y(x)*(1-x)*diff(y(x),x)-y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x}$$
$$y(x) = \mathrm{e}^x c_1$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 26

```
DSolve[x*y'[x]^2+y[x]*(1-x)*y'[x]-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x$$
$$y(x) \to \frac{c_1}{x}$$
$$y(x) \to 0$$

## 2.22 problem 25

Internal problem ID [6886]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 16. Nonlinear equations. Miscellaneous Exercises. Page 340
**Problem number**: 25.
**ODE order**: 1.
**ODE degree**: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$yy'^2 - (x + y)\, y' + y = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 271

```
dsolve(y(x)*diff(y(x),x)^2-(x+y(x))*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = x$$

$$y(x) = 0$$

$$\ln(x) - \frac{x\left(\frac{x^2+2y(x)x-3y(x)^2}{x^2}\right)^{\frac{3}{2}}}{2y(x)} - \operatorname{arctanh}\left(\frac{y(x)+x}{x\sqrt{\frac{x^2+2y(x)x-3y(x)^2}{x^2}}}\right) + \ln\left(\frac{y(x)}{x}\right) + \sqrt{\frac{x^2+2y(x)x-3y(x)^2}{x^2}} - \frac{3\sqrt{\frac{x^2+2y(x)x-3y(x)^2}{x^2}}\, y(x)}{2x} - \frac{x}{2y(x)} - c_1 = 0$$

$$\ln(x) + \frac{x\left(\frac{x^2+2y(x)x-3y(x)^2}{x^2}\right)^{\frac{3}{2}}}{2y(x)} + \operatorname{arctanh}\left(\frac{y(x)+x}{x\sqrt{\frac{x^2+2y(x)x-3y(x)^2}{x^2}}}\right) + \ln\left(\frac{y(x)}{x}\right) - \sqrt{\frac{x^2+2y(x)x-3y(x)^2}{x^2}} + \frac{3\sqrt{\frac{x^2+2y(x)x-3y(x)^2}{x^2}}\, y(x)}{2x} - \frac{x}{2y(x)} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 4.268 (sec). Leaf size: 320

```
DSolve[y[x]*y'[x]^2-(x+y[x])*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{x\left(-i\sqrt{\frac{y(x)}{x}-1}\sqrt{\frac{3y(x)}{x}+1}+\frac{4y(x)\log\left(\sqrt{\frac{3y(x)}{x}-3}-\sqrt{\frac{3y(x)}{x}+1}\right)}{x}-\frac{4y(x)\log\left(-i\left(\frac{3y(x)}{x}+1\right)+i\sqrt{\frac{3y(x)}{x}-3}\sqrt{\frac{3y(x)}{x}+1+}\right.}{x}\right.}{4y(x)}\right.$$

$$\left.-\frac{\log(x)}{2}+c_1,y(x)\right]$$

$$\text{Solve}\left[-\frac{x\left(i\sqrt{\frac{y(x)}{x}-1}\sqrt{\frac{3y(x)}{x}+1}+\frac{4y(x)\log\left(\sqrt{\frac{3y(x)}{x}-3}-\sqrt{\frac{3y(x)}{x}+1}\right)}{x}-\frac{4y(x)\log\left(i\left(\frac{3y(x)}{x}+1\right)-i\sqrt{\frac{3y(x)}{x}-3}\sqrt{\frac{3y(x)}{x}+1}+\sqrt{2-}\right.}{x}\right.}{4y(x)}\right.$$

$$\left.-\frac{\log(x)}{2}+c_1,y(x)\right]$$

$$y(x)\to 0$$

## 2.23 problem 26

Internal problem ID [6887]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 16. Nonlinear equations. Miscellaneous Exercises. Page 340
**Problem number**: 26.
**ODE order**: 1.
**ODE degree**: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _dAlembert]`

$$\boxed{xy'^2 + (k - x - y)\, y' + y = 0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 59

```
dsolve(x*diff(y(x),x)^2+(k-x-y(x))*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = k + x - 2\sqrt{kx}$$
$$y(x) = k + x + 2\sqrt{kx}$$
$$y(x) = -\frac{(c_1^2 - c_1)\, x}{1 - c_1} - \frac{kc_1}{1 - c_1}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 54

```
DSolve[x*y'[x]^2+(k-x-y[x])*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \left(x + \frac{k}{-1 + c_1}\right)$$
$$y(x) \to -2\sqrt{k}\sqrt{x} + k + x$$
$$y(x) \to \left(\sqrt{k} + \sqrt{x}\right)^2$$

## 2.24 problem 27

Internal problem ID [6888]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 16. Nonlinear equations. Miscellaneous Exercises. Page 340
**Problem number**: 27.
**ODE order**: 1.
**ODE degree**: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{xy'^3 - 2yy'^2 = -4x^2}$$

## ✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 831

```
dsolve(x*diff(y(x),x)^3-2*y(x)*diff(y(x),x)^2+4*x^2=0,y(x), singsol=all)
```

$$y(x) = \frac{3x^{\frac{4}{3}}}{2}$$

$$y(x) = \frac{3\left(-\frac{x^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}\,x^{\frac{1}{3}}}{2}\right)x}{2}$$

$$y(x) = \frac{3\left(-\frac{x^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}\,x^{\frac{1}{3}}}{2}\right)x}{2}$$

$$y(x) = -\frac{4x^2}{c_1} + \frac{c_1^2}{32}$$

$$y(x) = \frac{4x^2}{c_1} + \frac{c_1^2}{32}$$

$$y(x) = \frac{c_1\left(-1728x^2 + c_1^3 + 24\sqrt{-6c_1^3x^2 + 5184x^4}\right)^{\frac{1}{3}}}{96} + \frac{c_1^3}{96\left(-1728x^2 + c_1^3 + 24\sqrt{-6c_1^3x^2 + 5184x^4}\right)^{\frac{1}{3}}} + \frac{c_1^2}{96}$$

$$y(x) = \frac{c_1\left(1728x^2 + c_1^3 + 24\sqrt{6c_1^3x^2 + 5184x^4}\right)^{\frac{1}{3}}}{96} + \frac{c_1^3}{96\left(1728x^2 + c_1^3 + 24\sqrt{6c_1^3x^2 + 5184x^4}\right)^{\frac{1}{3}}} + \frac{c_1^2}{96}$$

$$y(x) = -\frac{c_1\left(-1728x^2 + c_1^3 + 24\sqrt{-6c_1^3x^2 + 5184x^4}\right)^{\frac{1}{3}}}{192} - \frac{c_1^3}{192\left(-1728x^2 + c_1^3 + 24\sqrt{-6c_1^3x^2 + 5184x^4}\right)^{\frac{1}{3}}} + \frac{c_1^2}{96} - \frac{ic_1\sqrt{3}\left(-1728x^2 + c_1^3 + 24\sqrt{-6c_1^3x^2 + 5184x^4}\right)^{\frac{1}{3}}}{192} + \frac{i\sqrt{3}\,c_1^3}{192\left(-1728x^2 + c_1^3 + 24\sqrt{-6c_1^3x^2 + 5184x^4}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{c_1\left(-1728x^2 + c_1^3 + 24\sqrt{-6c_1^3x^2 + 5184x^4}\right)^{\frac{1}{3}}}{192}$$

✓ Solution by Mathematica

Time used: 171.698 (sec). Leaf size: 15120

```
DSolve[x*y'[x]^3-2*y[x]*y'[x]^2+4*x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

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# 3 CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355


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## 3.1 problem 1

Internal problem ID [6889]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;
dsolve(diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6\right) y(0) + \left(x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[y''[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(-\frac{x^7}{5040} + \frac{x^5}{120} - \frac{x^3}{6} + x\right) + c_1\left(-\frac{x^6}{720} + \frac{x^4}{24} - \frac{x^2}{2} + 1\right)$$

## 3.2 problem 2

Internal problem ID [6890]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 9y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 44

```
Order:=8;
dsolve(diff(y(x),x$2)-9*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{9}{2}x^2 + \frac{27}{8}x^4 + \frac{81}{80}x^6\right) y(0) + \left(x + \frac{3}{2}x^3 + \frac{27}{40}x^5 + \frac{81}{560}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[y''[x]-9*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(\frac{81x^7}{560} + \frac{27x^5}{40} + \frac{3x^3}{2} + x\right) + c_1\left(\frac{81x^6}{80} + \frac{27x^4}{8} + \frac{9x^2}{2} + 1\right)$$

## 3.3 problem 3

Internal problem ID [6891]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + 3y'x + 3y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;
dsolve(diff(y(x),x$2)+3*x*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{3}{2}x^2 + \frac{9}{8}x^4 - \frac{9}{16}x^6\right) y(0) + \left(x - x^3 + \frac{3}{5}x^5 - \frac{9}{35}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 54

```
AsymptoticDSolveValue[y''[x]+3*x*y'[x]+3*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(-\frac{9x^7}{35} + \frac{3x^5}{5} - x^3 + x\right) + c_1\left(-\frac{9x^6}{16} + \frac{9x^4}{8} - \frac{3x^2}{2} + 1\right)$$

## 3.4 problem 4

Internal problem ID [6892]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(4x^2+1)\,y''-8y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=8;
dsolve((1+4*x^2)*diff(y(x),x$2)-8*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(4x^2+1)\,y(0)+\left(x+\frac{4}{3}x^3-\frac{16}{15}x^5+\frac{64}{35}x^7\right)D(y)\,(0)+O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[(1+4*x^2)*y''[x]-8*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_1(4x^2+1)+c_2\left(\frac{64x^7}{35}-\frac{16x^5}{15}+\frac{4x^3}{3}+x\right)$$

### 3.5 problem 5

Internal problem ID [6893]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355
**Problem number**: 5.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(-4x^2+1)\,y''+8y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=8;
dsolve((1-4*x^2)*diff(y(x),x$2)+8*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(-4x^2+1)\,y(0)+\left(x-\frac{4}{3}x^3-\frac{16}{15}x^5-\frac{64}{35}x^7\right)D(y)(0)+O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[(1-4*x^2)*y''[x]+8*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_1(1-4x^2)+c_2\left(-\frac{64x^7}{35}-\frac{16x^5}{15}-\frac{4x^3}{3}+x\right)$$

## 3.6 problem 6

Internal problem ID [6894]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355
**Problem number**: 6.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''-4y'x+6y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
Order:=8;
dsolve((1+x^2)*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=y(0)+D(y)\,(0)\,x-3x^2y(0)-\frac{D(y)\,(0)\,x^3}{3}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 26

```
AsymptoticDSolveValue[(1+x^2)*y''[x]-4*x*y'[x]+6*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_2\left(x-\frac{x^3}{3}\right)+c_1(1-3x^2)$$

## 3.7 problem 7

Internal problem ID [6895]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355
**Problem number**: 7.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''+10y'x+20y=0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;
dsolve((1+x^2)*diff(y(x),x$2)+10*x*diff(y(x),x)+20*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (-84x^6+35x^4-10x^2+1)\,y(0)+(-30x^7+14x^5-5x^3+x)\,D(y)(0)+O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 44

```
AsymptoticDSolveValue[(1+x^2)*y''[x]+10*x*y'[x]+20*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2(-30x^7+14x^5-5x^3+x)+c_1(-84x^6+35x^4-10x^2+1)$$

## 3.8 problem 8

Internal problem ID [6896]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355
**Problem number**: 8.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+4)\,y''+2y'x-12y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=8;
dsolve((x^2+4)*diff(y(x),x$2)+2*x*diff(y(x),x)-12*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+\frac{3}{2}x^2+\frac{3}{16}x^4-\frac{1}{80}x^6\right)y(0)+\left(x+\frac{5}{12}x^3\right)D(y)(0)+O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[(x^2+4)*y''[x]+2*x*y'[x]-12*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_2\left(\frac{5x^3}{12}+x\right)+c_1\left(-\frac{x^6}{80}+\frac{3x^4}{16}+\frac{3x^2}{2}+1\right)$$

## 3.9 problem 9

Internal problem ID [6897]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355
**Problem number**: 9.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-9)\,y''+3y'x-3y=0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
Order:=8;
dsolve((x^2-9)*diff(y(x),x$2)+3*x*diff(y(x),x)-3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1-\frac{1}{6}x^2-\frac{5}{648}x^4-\frac{7}{11664}x^6\right)y(0)+D(y)\,(0)\,x+O\left(x^8\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[(x^2-9)*y''[x]+3*x*y'[x]-3*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{7x^6}{11664}-\frac{5x^4}{648}-\frac{x^2}{6}+1\right)+c_2x$$

## 3.10 problem 10

Internal problem ID [6898]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355
**Problem number**: 10.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y'x + 5y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;
dsolve(diff(y(x),x$2)+2*x*diff(y(x),x)+5*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{5}{2}x^2 + \frac{15}{8}x^4 - \frac{13}{16}x^6\right) y(0) + \left(x - \frac{7}{6}x^3 + \frac{77}{120}x^5 - \frac{11}{48}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[y''[x]+2*x*y'[x]+5*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(-\frac{11x^7}{48} + \frac{77x^5}{120} - \frac{7x^3}{6} + x\right) + c_1\left(-\frac{13x^6}{16} + \frac{15x^4}{8} - \frac{5x^2}{2} + 1\right)$$

## 3.11 problem 11

Internal problem ID [6899]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355
**Problem number**: 11.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x^2+4)\,y''+6y'x+4y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 44

```
Order:=8;
dsolve((x^2+4)*diff(y(x),x$2)+6*x*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{1}{2}x^2+\frac{3}{16}x^4-\frac{1}{16}x^6\right)y(0)+\left(x-\frac{5}{12}x^3+\frac{7}{48}x^5-\frac{3}{64}x^7\right)D(y)(0)+O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[(x^2+4)*y''[x]+6*x*y'[x]+4*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_2\left(-\frac{3x^7}{64}+\frac{7x^5}{48}-\frac{5x^3}{12}+x\right)+c_1\left(-\frac{x^6}{16}+\frac{3x^4}{16}-\frac{x^2}{2}+1\right)$$

## 3.12 problem 12

Internal problem ID [6900]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355
**Problem number**: 12.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x^2+1)\,y''-5y'x+3y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=8;
dsolve((1+2*x^2)*diff(y(x),x$2)-5*x*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{3}{2}x^2-\frac{3}{8}x^4+\frac{7}{80}x^6\right)y(0)+\left(x+\frac{1}{3}x^3\right)D(y)(0)+O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[(1+2*x^2)*y''[x]-5*x*y'[x]+3*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_2\left(\frac{x^3}{3}+x\right)+c_1\left(\frac{7x^6}{80}-\frac{3x^4}{8}-\frac{3x^2}{2}+1\right)$$

## 3.13 problem 13

Internal problem ID [6901]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355
**Problem number**: 13.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + x^2 y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=8;
dsolve(diff(y(x),x$2)+x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^4}{12}\right) y(0) + \left(x - \frac{1}{20}x^5\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]+x^2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(x - \frac{x^5}{20}\right) + c_1\left(1 - \frac{x^4}{12}\right)$$

## 3.14 problem 14

Internal problem ID [6902]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355
**Problem number**: 14.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-4x^2+1)\,y''+6y'x-4y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=8;
dsolve((1-4*x^2)*diff(y(x),x$2)+6*x*diff(y(x),x)-4*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(2x^2+1)\,y(0)+\left(x-\frac{1}{3}x^3-\frac{1}{6}x^5-\frac{3}{14}x^7\right)D(y)\,(0)+O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 40

```
AsymptoticDSolveValue[(1-4*x^2)*y''[x]+6*x*y'[x]-4*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_1(2x^2+1)+c_2\left(-\frac{3x^7}{14}-\frac{x^5}{6}-\frac{x^3}{3}+x\right)$$

## 3.15 problem 15

Internal problem ID [6903]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355
**Problem number**: 15.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x^2+1)\,y''+3y'x-3y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
Order:=8;
dsolve((1+2*x^2)*diff(y(x),x$2)+3*x*diff(y(x),x)-3*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+\frac{3}{2}x^2-\frac{7}{8}x^4+\frac{77}{80}x^6\right)y(0)+D(y)\,(0)\,x+O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 34

```
AsymptoticDSolveValue[(1+2*x^2)*y''[x]+3*x*y'[x]-3*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_1\left(\frac{77x^6}{80}-\frac{7x^4}{8}+\frac{3x^2}{2}+1\right)+c_2x$$

## 3.16 problem 16

Internal problem ID [6904]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355
**Problem number**: 16.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y''' + x^2y'' + 5y'x + 3y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 101

```
dsolve(diff(y(x),x$3)+x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{x^3}{3}} x + \frac{c_2 x^2 \left(3\Gamma\left(\frac{1}{3}, -\frac{x^3}{3}\right)\Gamma\left(\frac{2}{3}\right) - 2\sqrt{3}\,\pi\right) e^{-\frac{x^3}{3}}}{\left(-x^3\right)^{\frac{1}{3}}} + \frac{c_3 \left(\left(-x^3\right)^{\frac{2}{3}} 3^{\frac{1}{3}} - \Gamma\left(\frac{2}{3}\right) x^3 e^{-\frac{x^3}{3}} + \Gamma\left(\frac{2}{3}, -\frac{x^3}{3}\right) x^3 e^{-\frac{x^3}{3}}\right)}{\left(-x^3\right)^{\frac{2}{3}}}$$

✓ Solution by Mathematica

Time used: 0.093 (sec). Leaf size: 88

```
DSolve[y'''[x]+x^2*y''[x]+5*x*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{x^3}{3}} \left(-2\ 3^{2/3} c_3 \sqrt[3]{-x^3}\, x \Gamma\left(-\frac{1}{3}, -\frac{x^3}{3}\right) + 3\sqrt[3]{3} c_1 \left(-x^3\right)^{2/3} \Gamma\left(\frac{1}{3}, -\frac{x^3}{3}\right) + 18 c_2 x^2\right)}{18x}$$

### 3.17 problem 17

Internal problem ID [6905]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355
**Problem number**: 17.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y'x + 3y = x^2}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
Order:=8;
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+3*y(x)=x^2,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{3}{2}x^2 + \frac{5}{8}x^4 - \frac{7}{48}x^6\right) y(0) + \left(x - \frac{2}{3}x^3 + \frac{1}{5}x^5 - \frac{4}{105}x^7\right) D(y)(0) + \frac{x^4}{12} - \frac{7x^6}{360} + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 70

```
AsymptoticDSolveValue[y''[x]+x*y'[x]+3*y[x]==x^2,y[x],{x,0,7}]
```

$$y(x) \to -\frac{7x^6}{360} + \frac{x^4}{12} + c_2\left(-\frac{4x^7}{105} + \frac{x^5}{5} - \frac{2x^3}{3} + x\right) + c_1\left(-\frac{7x^6}{48} + \frac{5x^4}{8} - \frac{3x^2}{2} + 1\right)$$

## 3.18 problem 18

Internal problem ID [6906]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355
**Problem number**: 18.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + 2y'x + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;
dsolve(diff(y(x),x$2)+2*x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x^2 + \frac{1}{2}x^4 - \frac{1}{6}x^6\right) y(0) + \left(x - \frac{2}{3}x^3 + \frac{4}{15}x^5 - \frac{8}{105}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 54

```
AsymptoticDSolveValue[y''[x]+2*x*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(-\frac{8x^7}{105} + \frac{4x^5}{15} - \frac{2x^3}{3} + x\right) + c_1\left(-\frac{x^6}{6} + \frac{x^4}{2} - x^2 + 1\right)$$

## 3.19 problem 19

Internal problem ID [6907]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355
**Problem number**: 19.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 3y'x + 7y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;
dsolve(diff(y(x),x$2)+3*x*diff(y(x),x)+7*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{7}{2}x^2 + \frac{91}{24}x^4 - \frac{1729}{720}x^6\right) y(0) + \left(x - \frac{5}{3}x^3 + \frac{4}{3}x^5 - \frac{44}{63}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[y''[x]+3*x*y'[x]+7*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(-\frac{44x^7}{63} + \frac{4x^5}{3} - \frac{5x^3}{3} + x\right) + c_1\left(-\frac{1729x^6}{720} + \frac{91x^4}{24} - \frac{7x^2}{2} + 1\right)$$

## 3.20 problem 20

Internal problem ID [6908]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355
**Problem number**: 20.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2y'' + 9y'x - 36y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
Order:=8;
dsolve(2*diff(y(x),x$2)+9*x*diff(y(x),x)-36*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(\frac{27}{4}x^4 + 9x^2 + 1\right) y(0) + \left(x + \frac{9}{4}x^3 + \frac{81}{160}x^5 - \frac{243}{4480}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 47

```
AsymptoticDSolveValue[2*y''[x]+9*x*y'[x]-36*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{27x^4}{4} + 9x^2 + 1\right) + c_2\left(-\frac{243x^7}{4480} + \frac{81x^5}{160} + \frac{9x^3}{4} + x\right)$$

## 3.21 problem 21

Internal problem ID [6909]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355
**Problem number**: 21.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{(x^2+4)\,y''+y'x-9y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
Order:=8;
dsolve((x^2+4)*diff(y(x),x$2)+x*diff(y(x),x)-9*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+\frac{9}{8}x^2+\frac{15}{128}x^4-\frac{7}{1024}x^6\right)y(0)+\left(x+\frac{1}{3}x^3\right)D(y)(0)+O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[(x^2+4)*y''[x]+x*y'[x]-9*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_2\left(\frac{x^3}{3}+x\right)+c_1\left(-\frac{7x^6}{1024}+\frac{15x^4}{128}+\frac{9x^2}{8}+1\right)$$

## 3.22 problem 22

Internal problem ID [6910]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355
**Problem number**: 22.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+4)\,y''+3y'x-8y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
Order:=8;
dsolve((x^2+4)*diff(y(x),x$2)+3*x*diff(y(x),x)-8*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(x^2+1)\,y(0)+\left(x+\frac{5}{24}x^3-\frac{7}{384}x^5+\frac{3}{1024}x^7\right)D(y)\,(0)+O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 38

```
AsymptoticDSolveValue[(x^2+4)*y''[x]+3*x*y'[x]-8*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_1(x^2+1)+c_2\left(\frac{3x^7}{1024}-\frac{7x^5}{384}+\frac{5x^3}{24}+x\right)$$

## 3.23 problem 23

Internal problem ID [6911]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355
**Problem number**: 23.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(9x^2+1)\,y''-18y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=8;
dsolve((1+9*x^2)*diff(y(x),x$2)-18*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(9x^2+1)\,y(0)+\left(x+3x^3-\frac{27}{5}x^5+\frac{729}{35}x^7\right)D(y)(0)+O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 38

```
AsymptoticDSolveValue[(1+9*x^2)*y''[x]-18*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_1(9x^2+1)+c_2\left(\frac{729x^7}{35}-\frac{27x^5}{5}+3x^3+x\right)$$

## 3.24 problem 24

Internal problem ID [6912]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355
**Problem number**: 24.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(3x^2+1)\,y''+13y'x+7y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;
dsolve((1+3*x^2)*diff(y(x),x$2)+13*x*diff(y(x),x)+7*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{7}{2}x^2+\frac{91}{8}x^4-\frac{1729}{48}x^6\right)y(0)+\left(x-\frac{10}{3}x^3+\frac{32}{3}x^5-\frac{704}{21}x^7\right)D(y)(0)+O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[(1+3*x^2)*y''[x]+13*x*y'[x]+7*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_2\left(-\frac{704x^7}{21}+\frac{32x^5}{3}-\frac{10x^3}{3}+x\right)+c_1\left(-\frac{1729x^6}{48}+\frac{91x^4}{8}-\frac{7x^2}{2}+1\right)$$

## 3.25 problem 25

Internal problem ID [6913]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355
**Problem number**: 25.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x^2+1)\,y''+11y'x+9y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;
dsolve((1+2*x^2)*diff(y(x),x$2)+11*x*diff(y(x),x)+9*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{9}{2}x^2+\frac{105}{8}x^4-\frac{539}{16}x^6\right)y(0)+\left(x-\frac{10}{3}x^3+9x^5-\frac{156}{7}x^7\right)D(y)(0)+O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 54

```
AsymptoticDSolveValue[(1+2*x^2)*y''[x]+11*x*y'[x]+9*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_2\left(-\frac{156x^7}{7}+9x^5-\frac{10x^3}{3}+x\right)+c_1\left(-\frac{539x^6}{16}+\frac{105x^4}{8}-\frac{9x^2}{2}+1\right)$$

## 3.26 problem 26

Internal problem ID [6914]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355
**Problem number**: 26.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2(x+3)\, y' - 3y = 0}$$

With the expansion point for the power series method at $x = -3$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;
dsolve(diff(y(x),x$2)-2*(x+3)*diff(y(x),x)-3*y(x)=0,y(x),type='series',x=-3);
```

$$y(x) = \left(1 + \frac{3(x+3)^2}{2} + \frac{7(x+3)^4}{8} + \frac{77(x+3)^6}{240}\right) y(-3) + \left(x + 3 + \frac{5(x+3)^3}{6} + \frac{3(x+3)^5}{8} + \frac{13(x+3)^7}{112}\right) D(y)\,(-3) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 69

```
AsymptoticDSolveValue[y''[x]-2*(x+3)*y'[x]-3*y[x]==0,y[x],{x,-3,7}]
```

$$y(x) \to c_1 \left(\frac{77}{240}(x+3)^6 + \frac{7}{8}(x+3)^4 + \frac{3}{2}(x+3)^2 + 1\right) + c_2 \left(\frac{13}{112}(x+3)^7 + \frac{3}{8}(x+3)^5 + \frac{5}{6}(x+3)^3 + x + 3\right)$$

## 3.27 problem 27

Internal problem ID [6915]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355
**Problem number**: 27.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (x-2)\,y = 0}$$

With the expansion point for the power series method at $x = 2$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=8;
dsolve(diff(y(x),x$2)+(x-2)*y(x)=0,y(x),type='series',x=2);
```

$$y(x) = \left(1 - \frac{(x-2)^3}{6} + \frac{(x-2)^6}{180}\right) y(2) + \left(x - 2 - \frac{(x-2)^4}{12} + \frac{(x-2)^7}{504}\right) D(y)(2) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 51

```
AsymptoticDSolveValue[y''[x]+(x-2)*y[x]==0,y[x],{x,2,7}]
```

$$y(x) \to c_1\left(\frac{1}{180}(x-2)^6 - \frac{1}{6}(x-2)^3 + 1\right) + c_2\left(\frac{1}{504}(x-2)^7 - \frac{1}{12}(x-2)^4 + x - 2\right)$$

## 3.28 problem 28

Internal problem ID [6916]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 17. Power series solutions. 17.5. Solutions Near an Ordinary Point. Exercises page 355
**Problem number**: 28.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-2x+2)\,y''-4(x-1)\,y'+6y=0}$$

With the expansion point for the power series method at $x=1$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
Order:=8;
dsolve((x^2-2*x+2)*diff(y(x),x$2)-4*(x-1)*diff(y(x),x)+6*y(x)=0,y(x),type='series',x=1);
```

$$y(x)=\frac{(-x^3+3x^2-2)\,D(y)\,(1)}{3}-3y(1)\left(x^2-2x+\frac{2}{3}\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 31

```
AsymptoticDSolveValue[(x^2-2*x+2)*y''[x]-4*(x-1)*y'[x]+6*y[x]==0,y[x],{x,1,7}]
```

$$y(x)\to c_1(1-3(x-1)^2)+c_2\left(-\frac{1}{3}(x-1)^3+x-1\right)$$

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## 4.1 problem 1

Internal problem ID [6917]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{2x(x+1)\,y''+3(x+1)\,y'-y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 40

```
Order:=8;
dsolve(2*x*(x+1)*diff(y(x),x$2)+3*(x+1)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2\left(1+\frac{1}{3}x-\frac{1}{15}x^2+\frac{1}{35}x^3-\frac{1}{63}x^4+\frac{1}{99}x^5-\frac{1}{143}x^6+\frac{1}{195}x^7+\mathrm{O}\left(x^8\right)\right)\sqrt{x}+c_1\left(1+x+\mathrm{O}\left(x^8\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 67

```
AsymptoticDSolveValue[2*x*(x+1)*y''[x]+3*(x+1)*y'[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_1\left(\frac{x^7}{195}-\frac{x^6}{143}+\frac{x^5}{99}-\frac{x^4}{63}+\frac{x^3}{35}-\frac{x^2}{15}+\frac{x}{3}+1\right)+\frac{c_2(x+1)}{\sqrt{x}}$$

## 4.2 problem 2

Internal problem ID [6918]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 4y'x + (4x^2 - 1)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
Order:=8;
dsolve(4*x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 - \frac{1}{5040}x^6 + \mathrm{O}\left(x^8\right)\right)x + c_2\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \mathrm{O}\left(x^8\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 76

```
AsymptoticDSolveValue[4*x^2*y''[x]+4*x*y'[x]+(4*x^2-1)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{x^{11/2}}{720} + \frac{x^{7/2}}{24} - \frac{x^{3/2}}{2} + \frac{1}{\sqrt{x}}\right) + c_2\left(-\frac{x^{13/2}}{5040} + \frac{x^{9/2}}{120} - \frac{x^{5/2}}{6} + \sqrt{x}\right)$$

## 4.3 problem 3

Internal problem ID [6919]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 4y'x - (4x^2+1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
Order:=8;
dsolve(4*x^2*diff(y(x),x$2)+4*x*diff(y(x),x)-(4*x^2+1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x\left(1+\frac{1}{6}x^2+\frac{1}{120}x^4+\frac{1}{5040}x^6+\mathrm{O}\left(x^8\right)\right)+c_2\left(1+\frac{1}{2}x^2+\frac{1}{24}x^4+\frac{1}{720}x^6+\mathrm{O}\left(x^8\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 76

```
AsymptoticDSolveValue[4*x^2*y''[x]+4*x*y'[x]-(4*x^2+1)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{x^{11/2}}{720}+\frac{x^{7/2}}{24}+\frac{x^{3/2}}{2}+\frac{1}{\sqrt{x}}\right)+c_2\left(\frac{x^{13/2}}{5040}+\frac{x^{9/2}}{120}+\frac{x^{5/2}}{6}+\sqrt{x}\right)$$

## 4.4 problem 4

Internal problem ID [6920]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{4xy'' + 3y' + 3y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 52

```
Order:=8;
dsolve(4*x*diff(y(x),x$2)+3*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{1}{4}} \left(1 - \frac{3}{5}x + \frac{1}{10}x^2 - \frac{1}{130}x^3 + \frac{3}{8840}x^4 - \frac{3}{309400}x^5 + \frac{3}{15470000}x^6 - \frac{9}{3140410000}x^7 + \mathrm{O}\left(x^8\right)\right) + c_2\left(1 - x + \frac{3}{14}x^2 - \frac{3}{154}x^3 + \frac{3}{3080}x^4 - \frac{9}{292600}x^5 + \frac{9}{13459600}x^6 - \frac{1}{94217200}x^7 + \mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 111

```
AsymptoticDSolveValue[4*x*y''[x]+3*y'[x]+3*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 \sqrt[4]{x} \left(-\frac{9x^7}{3140410000} + \frac{3x^6}{15470000} - \frac{3x^5}{309400} + \frac{3x^4}{8840} - \frac{x^3}{130} + \frac{x^2}{10} - \frac{3x}{5} + 1\right) + c_2 \left(-\frac{x^7}{94217200} + \frac{9x^6}{13459600} - \frac{9x^5}{292600} + \frac{3x^4}{3080} - \frac{3x^3}{154} + \frac{3x^2}{14} - x + 1\right)$$

## 4.5 problem 5

Internal problem ID [6921]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 5.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(1-x)\,y''-x(7x+1)\,y'+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
Order:=8;
dsolve(2*x^2*(1-x)*diff(y(x),x$2)-x*(1+7*x)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1\sqrt{x}\left(1+3x+6x^2+10x^3+15x^4+21x^5+28x^6+36x^7+\mathrm{O}\left(x^8\right)\right)\\
+c_2x\left(1+\frac{7}{3}x+\frac{21}{5}x^2+\frac{33}{5}x^3+\frac{143}{15}x^4+13x^5+17x^6+\frac{323}{15}x^7+\mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 96

```
AsymptoticDSolveValue[2*x^2*(1-x)*y''[x]-x*(1+7*x)*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_1x\left(\frac{323x^7}{15}+17x^6+13x^5+\frac{143x^4}{15}+\frac{33x^3}{5}+\frac{21x^2}{5}+\frac{7x}{3}+1\right)\\
+c_2\sqrt{x}\left(36x^7+28x^6+21x^5+15x^4+10x^3+6x^2+3x+1\right)$$

## 4.6 problem 6

Internal problem ID [6922]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 6.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + 5(-2x+1)\,y' - 5y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 40

```
Order:=8;
dsolve(2*x*diff(y(x),x$2)+5*(1-2*x)*diff(y(x),x)-5*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2\left(1 + x + \frac{15}{14}x^2 + \frac{125}{126}x^3 + \frac{625}{792}x^4 + \frac{625}{1144}x^5 + \frac{625}{1872}x^6 + \frac{3125}{17136}x^7 + \mathrm{O}\left(x^8\right)\right)x^{\frac{3}{2}} + c_1\left(1 + 10x + \mathrm{O}\left(x^8\right)\right)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 65

```
AsymptoticDSolveValue[2*x*y''[x]+5*(1-2*x)*y'[x]-5*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to \frac{c_2(10x+1)}{x^{3/2}} + c_1\left(\frac{3125x^7}{17136} + \frac{625x^6}{1872} + \frac{625x^5}{1144} + \frac{625x^4}{792} + \frac{125x^3}{126} + \frac{15x^2}{14} + x + 1\right)$$

## 4.7 problem 7

Internal problem ID [6923]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 7.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{8x^2y'' + 10y'x - y(1+x) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 55

```
Order:=8;
dsolve(8*x^2*diff(y(x),x$2)+10*x*diff(y(x),x)-(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{3}{4}} \left(1 + \frac{1}{14}x + \frac{1}{616}x^2 + \frac{1}{55440}x^3 + \frac{1}{8426880}x^4 + \frac{1}{1938182400}x^5 + \frac{1}{627971097600}x^6 + \frac{1}{272539456358400}x^7 + \mathrm{O}\left(x^8\right)\right) -}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 118

```
AsymptoticDSolveValue[8*x^2*y''[x]+10*x*y'[x]-(1+x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 \sqrt[4]{x}\left(\frac{x^7}{272539456358400} + \frac{x^6}{627971097600} + \frac{x^5}{1938182400} + \frac{x^4}{8426880} + \frac{x^3}{55440} + \frac{x^2}{616} + \frac{x}{14} + 1\right) + \frac{c_2\left(\frac{x^7}{3368252160000} + \frac{x^6}{9623577600} + \frac{x^5}{38188800} + \frac{x^4}{224640} + \frac{x^3}{2160} + \frac{x^2}{40} + \frac{x}{2} + 1\right)}{\sqrt{x}}$$

## 4.8 problem 8

Internal problem ID [6924]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 8.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + (-x+2)\,y' - 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 71

```
Order:=8;
dsolve(2*x*diff(y(x),x$2)+(2-x)*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x)\,c_2 + c_1)\left(1 + x + \frac{3}{8}x^2 + \frac{1}{12}x^3 + \frac{5}{384}x^4 + \frac{1}{640}x^5 + \frac{7}{46080}x^6 + \frac{1}{80640}x^7 + \mathrm{O}\left(x^8\right)\right) + \left(-\frac{3}{2}x - \frac{13}{16}x^2 - \frac{31}{144}x^3 - \frac{173}{4608}x^4 - \frac{187}{38400}x^5 - \frac{463}{921600}x^6 - \frac{971}{22579200}x^7 + \mathrm{O}\left(x^8\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 151

```
AsymptoticDSolveValue[2*x*y''[x]+(2-x)*y'[x]-2*y[x]==0,y[x],{x,0,7}]
```

$$
\begin{aligned}
y(x) \to\ & c_1\left(\frac{x^7}{80640}+\frac{7x^6}{46080}+\frac{x^5}{640}+\frac{5x^4}{384}+\frac{x^3}{12}+\frac{3x^2}{8}+x+1\right) \\
& + c_2\left(-\frac{971x^7}{22579200}-\frac{463x^6}{921600}-\frac{187x^5}{38400}-\frac{173x^4}{4608}-\frac{31x^3}{144}-\frac{13x^2}{16}\right. \\
& \left.+\left(\frac{x^7}{80640}+\frac{7x^6}{46080}+\frac{x^5}{640}+\frac{5x^4}{384}+\frac{x^3}{12}+\frac{3x^2}{8}+x+1\right)\log(x)-\frac{3x}{2}\right)
\end{aligned}
$$

## 4.9 problem 9

Internal problem ID [6925]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 9.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x(x+3)\,y'' - 3(1+x)\,y' + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 42

```
Order:=8;
dsolve(2*x*(x+3)*diff(y(x),x$2)-3*(x+1)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{3}{2}}\left(1+\frac{1}{15}x-\frac{1}{315}x^2+\frac{1}{2835}x^3-\frac{1}{18711}x^4+\frac{1}{104247}x^5-\frac{1}{521235}x^6 + \frac{1}{2416635}x^7+\mathrm{O}\left(x^8\right)\right)+c_2\left(1+\frac{2}{3}x+\frac{1}{9}x^2+\mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 78

```
AsymptoticDSolveValue[2*x*(x+3)*y''[x]-3*(x+1)*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(\frac{x^2}{9}+\frac{2x}{3}+1\right) + c_1\left(\frac{x^7}{2416635}-\frac{x^6}{521235}+\frac{x^5}{104247}-\frac{x^4}{18711}+\frac{x^3}{2835}-\frac{x^2}{315}+\frac{x}{15}+1\right)x^{3/2}$$

## 4.10 problem 10

Internal problem ID [6926]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 10.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{2xy'' + \left(-2x^2 + 1\right) y' - 4yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
Order:=8;
dsolve(2*x*diff(y(x),x$2)+(1-2*x^2)*diff(y(x),x)-4*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1+\frac{1}{2}x^2+\frac{1}{8}x^4+\frac{1}{48}x^6+\mathrm{O}\left(x^8\right)\right)+c_2\left(1+\frac{2}{3}x^2+\frac{4}{21}x^4+\frac{8}{231}x^6+\mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 61

```
AsymptoticDSolveValue[2*x*y''[x]+(1-2*x^2)*y'[x]-4*x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{x^6}{48}+\frac{x^4}{8}+\frac{x^2}{2}+1\right)+c_2\left(\frac{8x^6}{231}+\frac{4x^4}{21}+\frac{2x^2}{3}+1\right)$$

## 4.11 problem 11

Internal problem ID [6927]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 11.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{x(-x+4)\,y''+(-x+2)\,y'+4y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 42

```
Order:=8;
dsolve(x*(4-x)*diff(y(x),x$2)+(2-x)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1\sqrt{x}\left(1-\frac{5}{8}x+\frac{7}{128}x^2+\frac{3}{1024}x^3+\frac{11}{32768}x^4+\frac{13}{262144}x^5+\frac{35}{4194304}x^6+\frac{51}{33554432}x^7+\mathrm{O}\left(x^8\right)\right)+c_2\left(1-2x+\frac{1}{2}x^2+\mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 76

```
AsymptoticDSolveValue[x*(4-x)*y''[x]+(2-x)*y'[x]+4*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_2\left(\frac{x^2}{2}-2x+1\right)+c_1\sqrt{x}\left(\frac{51x^7}{33554432}+\frac{35x^6}{4194304}+\frac{13x^5}{262144}+\frac{11x^4}{32768}+\frac{3x^3}{1024}+\frac{7x^2}{128}-\frac{5x}{8}+1\right)$$

## 4.12 problem 12

Internal problem ID [6928]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 12.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3x^2y'' + y'x - y(1+x) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 53

```
Order:=8;
dsolve(3*x^2*diff(y(x),x$2)+x*diff(y(x),x)-(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{4}{3}}\left(1 + \frac{1}{7}x + \frac{1}{140}x^2 + \frac{1}{5460}x^3 + \frac{1}{349440}x^4 + \frac{1}{33196800}x^5 + \frac{1}{4381977600}x^6 + \frac{1}{766846080000}x^7 + \mathrm{O}\left(x^8\right)\right) + c_1(1 -}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 112

```
AsymptoticDSolveValue[3*x^2*y''[x]+x*y'[x]-(1+x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 x\left(\frac{x^7}{766846080000} + \frac{x^6}{4381977600} + \frac{x^5}{33196800} + \frac{x^4}{349440} + \frac{x^3}{5460} + \frac{x^2}{140} + \frac{x}{7} + 1\right) + \frac{c_2\left(-\frac{x^7}{1055577600} - \frac{x^6}{8870400} - \frac{x^5}{105600} - \frac{x^4}{1920} - \frac{x^3}{60} - \frac{x^2}{4} - x + 1\right)}{\sqrt[3]{x}}$$

## 4.13 problem 13

Internal problem ID [6929]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 13.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + (2x+1)\,y' + 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 52

```
Order:=8;
dsolve(2*x*diff(y(x),x$2)+(1+2*x)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1 - \frac{5}{3}x + \frac{7}{6}x^2 - \frac{1}{2}x^3 + \frac{11}{72}x^4 - \frac{13}{360}x^5 + \frac{1}{144}x^6 - \frac{17}{15120}x^7 + \mathrm{O}\left(x^8\right)\right) + c_2\left(1 - 4x + 4x^2 - \frac{32}{15}x^3 + \frac{16}{21}x^4 - \frac{64}{315}x^5 + \frac{64}{1485}x^6 - \frac{1024}{135135}x^7 + \mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 109

```
AsymptoticDSolveValue[2*x*y''[x]+(1+2*x)*y'[x]+4*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\sqrt{x}\left(-\frac{17x^7}{15120} + \frac{x^6}{144} - \frac{13x^5}{360} + \frac{11x^4}{72} - \frac{x^3}{2} + \frac{7x^2}{6} - \frac{5x}{3} + 1\right) + c_2\left(-\frac{1024x^7}{135135} + \frac{64x^6}{1485} - \frac{64x^5}{315} + \frac{16x^4}{21} - \frac{32x^3}{15} + 4x^2 - 4x + 1\right)$$

## 4.14 problem 14

Internal problem ID [6930]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 14.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + (2x+1)\,y' - 5y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 42

```
Order:=8;
dsolve(2*x*diff(y(x),x$2)+(1+2*x)*diff(y(x),x)-5*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1 + \frac{4}{3}x + \frac{4}{15}x^2 + \mathrm{O}\left(x^8\right)\right) + c_2\left(1 + 5x + \frac{5}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{168}x^4 + \frac{1}{2520}x^5 - \frac{1}{33264}x^6 + \frac{1}{432432}x^7 + \mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 76

```
AsymptoticDSolveValue[2*x*y''[x]+(1+2*x)*y'[x]-5*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{4x^2}{15} + \frac{4x}{3} + 1\right) + c_2\left(\frac{x^7}{432432} - \frac{x^6}{33264} + \frac{x^5}{2520} - \frac{x^4}{168} + \frac{x^3}{6} + \frac{5x^2}{2} + 5x + 1\right)$$

## 4.15 problem 15

Internal problem ID [6931]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 15.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' - 3x(1-x)\,y' + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 55

```
Order:=8;
dsolve(2*x^2*diff(y(x),x$2)-3*x*(1-x)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1+\frac{3}{2}x-\frac{27}{8}x^2+\frac{45}{16}x^3-\frac{189}{128}x^4+\frac{729}{1280}x^5-\frac{891}{5120}x^6+\frac{3159}{71680}x^7+\mathrm{O}\left(x^8\right)\right) + c_2x^2\left(1-\frac{6}{5}x+\frac{27}{35}x^2-\frac{12}{35}x^3+\frac{9}{77}x^4-\frac{162}{5005}x^5+\frac{27}{3575}x^6-\frac{648}{425425}x^7+\mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 116

```
AsymptoticDSolveValue[2*x^2*y''[x]-3*x*(1-x)*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{648x^7}{425425}+\frac{27x^6}{3575}-\frac{162x^5}{5005}+\frac{9x^4}{77}-\frac{12x^3}{35}+\frac{27x^2}{35}-\frac{6x}{5}+1\right)x^2 + c_2\left(\frac{3159x^7}{71680}-\frac{891x^6}{5120}+\frac{729x^5}{1280}-\frac{189x^4}{128}+\frac{45x^3}{16}-\frac{27x^2}{8}+\frac{3x}{2}+1\right)\sqrt{x}$$

## 4.16 problem 16

Internal problem ID [6932]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 16.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' + x(4x-1)\,y' + 2(3x-1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 55

```
Order:=8;
dsolve(2*x^2*diff(y(x),x$2)+x*(4*x-1)*diff(y(x),x)+2*(3*x-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{5}{2}}\left(1-2x+2x^2-\frac{4}{3}x^3+\frac{2}{3}x^4-\frac{4}{15}x^5+\frac{4}{45}x^6-\frac{8}{315}x^7+\mathrm{O}\left(x^8\right)\right)+c_1\left(1+\frac{4}{3}x+\frac{16}{3}x^2-\frac{64}{3}x^3+\frac{256}{9}x^4-\right.}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 112

```
AsymptoticDSolveValue[2*x^2*y''[x]+x*(4*x-1)*y'[x]+2*(3*x-1)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{8x^7}{315}+\frac{4x^6}{45}-\frac{4x^5}{15}+\frac{2x^4}{3}-\frac{4x^3}{3}+2x^2-2x+1\right)x^2 + \frac{c_2\left(-\frac{16384x^7}{2835}+\frac{4096x^6}{315}-\frac{1024x^5}{45}+\frac{256x^4}{9}-\frac{64x^3}{3}+\frac{16x^2}{3}+\frac{4x}{3}+1\right)}{\sqrt{x}}$$

## 4.17 problem 17

Internal problem ID [6933]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 17.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' - (2x^2+1)\,y' - yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 36

```
Order:=8;
dsolve(2*x*diff(y(x),x$2)-(1+2*x^2)*diff(y(x),x)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{3}{2}}\left(1+\frac{2}{7}x^2+\frac{4}{77}x^4+\frac{8}{1155}x^6+\mathrm{O}\left(x^8\right)\right)+c_2\left(1+\frac{1}{2}x^2+\frac{1}{8}x^4+\frac{1}{48}x^6+\mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 61

```
AsymptoticDSolveValue[2*x*y''[x]-(1+2*x^2)*y'[x]-x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(\frac{x^6}{48}+\frac{x^4}{8}+\frac{x^2}{2}+1\right)+c_1\left(\frac{8x^6}{1155}+\frac{4x^4}{77}+\frac{2x^2}{7}+1\right)x^{3/2}$$

## 4.18 problem 19

Internal problem ID [6934]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 19.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2x^2y'' + y'x - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
Order:=8;
dsolve(2*x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{x^{\frac{3}{2}}c_2 + c_1}{\sqrt{x}} + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 16

```
AsymptoticDSolveValue[2*x^2*y''[x]+x*y'[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 x + \frac{c_2}{\sqrt{x}}$$

## 4.19 problem 20

Internal problem ID [6935]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 20.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$2x^2y'' - 3y'x + 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
Order:=8;
dsolve(2*x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x} + c_2x^2 + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
AsymptoticDSolveValue[2*x^2*y''[x]-3*x*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1x^2 + c_2\sqrt{x}$$

## 4.20 problem 21

Internal problem ID [6936]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 21.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{9x^2y'' + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
Order:=8;
dsolve(9*x^2*diff(y(x),x$2)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x^{\frac{1}{3}}\left(c_2 x^{\frac{1}{3}} + c_1\right) + O\left(x^8\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
AsymptoticDSolveValue[9*x^2*y''[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 x^{2/3} + c_2 \sqrt[3]{x}$$

## 4.21 problem 22

Internal problem ID [6937]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 22.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{2x^2y'' + 5y'x - 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
Order:=8;
dsolve(2*x^2*diff(y(x),x$2)+5*x*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{x^{\frac{5}{2}}c_2 + c_1}{x^2} + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
AsymptoticDSolveValue[2*x^2*y''[x]+5*x*y'[x]-2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to \frac{c_2}{x^2} + c_1\sqrt{x}$$

## 4.22 problem 25

Internal problem ID [6938]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 25.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2x^2y'' + y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve(2*x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{x}} + xc_2$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 18

```
DSolve[2*x^2*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{\sqrt{x}} + c_2x$$

## 4.23 problem 26

Internal problem ID [6939]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 26.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{2x^2y'' - 3y'x + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(2*x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^2 + \sqrt{x}\,c_2$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 20

```
DSolve[2*x^2*y''[x]-3*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x^2 + c_1\sqrt{x}$$

## 4.24 problem 27

Internal problem ID [6940]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 27.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{9x^2y'' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(9*x^2*diff(y(x),x$2)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{\frac{2}{3}} + c_2 x^{\frac{1}{3}}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 22

```
DSolve[9*x^2*y''[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[3]{x}\left(c_2\sqrt[3]{x} + c_1\right)$$

## 4.25 problem 28

Internal problem ID [6941]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 28.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{2x^2y'' + 5y'x - 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(2*x^2*diff(y(x),x$2)+5*x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^2} + \sqrt{x}\, c_2$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 20

```
DSolve[2*x^2*y''[x]+5*x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 x^{5/2} + c_1}{x^2}$$

## 4.26 problem 29

Internal problem ID [6942]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 29.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 2y'x - 12y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-12*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^4} + c_2x^3$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+2*x*y'[x]-12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^7 + c_1}{x^4}$$

## 4.27 problem 30

Internal problem ID [6943]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 30.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' + y'x - 9y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-9*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^3} + c_2x^3$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+x*y'[x]-9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^6 + c_1}{x^3}$$

## 4.28 problem 31

Internal problem ID [6944]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 31.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' - 3y'x + 4y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^2 + c_2x^2 \ln(x)$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]-3*x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(2c_2 \log(x) + c_1)$$

## 4.29 problem 32

Internal problem ID [6945]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 32.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 5y'x + 9y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)-5*x*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^3 + c_2x^3 \ln(x)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]-5*x*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^3(3c_2 \log(x) + c_1)$$

## 4.30 problem 33

Internal problem ID [6946]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 33.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 5y'x + 5y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+5*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(\ln(x))}{x^2} + \frac{c_2 \cos(\ln(x))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]+5*x*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 \cos(\log(x)) + c_1 \sin(\log(x))}{x^2}$$

## 4.31 problem 34

Internal problem ID [6947]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.4 Indicial Equation with Difference of Roots Nonintegral. Exercises page 365
**Problem number**: 34.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''' + 4x^2y'' - 8y'x + 8y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve(x^3*diff(y(x),x$3)+4*x^2*diff(y(x),x$2)-8*x*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^4} + c_2x^2 + c_3x$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 22

```
DSolve[x^3*y'''[x]+4*x^2*y''[x]-8*x*y'[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_1}{x^4} + c_3x^2 + c_2x$$

# 5 CHAPTER 18. Power series solutions. 18.6. Indicial Equation with Equal Roots. Exercises page 373


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## 5.1 problem 1

Internal problem ID [6948]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.6. Indicial Equation with Equal Roots. Exercises page 373
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(1+x)\,y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 75

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)-x*(1+x)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((\ln(x)\, c_2 + c_1)\left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \frac{1}{5040}x^7 + \mathrm{O}\left(x^8\right)\right) + \left(-x - \frac{3}{4}x^2 - \frac{11}{36}x^3 - \frac{25}{288}x^4 - \frac{137}{7200}x^5 - \frac{49}{14400}x^6 - \frac{121}{235200}x^7 + \mathrm{O}\left(x^8\right)\right) c_2\right) x$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 154

```
AsymptoticDSolveValue[x^2*y''[x]-x*(1+x)*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 x \left(\frac{x^7}{5040} + \frac{x^6}{720} + \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1\right) + c_2 \left(x \left(-\frac{121x^7}{235200} - \frac{49x^6}{14400} - \frac{137x^5}{7200} - \frac{25x^4}{288} - \frac{11x^3}{36} - \frac{3x^2}{4} - x\right) + x \left(\frac{x^7}{5040} + \frac{x^6}{720} + \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1\right) \log(x)\right)$$

## 5.2 problem 2

Internal problem ID [6949]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.6. Indicial Equation with Equal Roots. Exercises page 373
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + (-2x+1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 81

```
Order:=8;
dsolve(4*x^2*diff(y(x),x$2)+(1-2*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x}\left((\ln(x)\,c_2 + c_1)\left(1 + \frac{1}{2}x + \frac{1}{16}x^2 + \frac{1}{288}x^3 + \frac{1}{9216}x^4 + \frac{1}{460800}x^5 + \frac{1}{33177600}x^6 + \frac{1}{3251404800}x^7 + \mathrm{O}\left(x^8\right)\right) + \left(-x - \frac{3}{16}x^2 - \frac{11}{864}x^3 - \frac{25}{55296}x^4 - \frac{137}{13824000}x^5 - \frac{49}{331776000}x^6 - \frac{121}{75866112000}x^7 + \mathrm{O}\left(x^8\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 174

```
AsymptoticDSolveValue[4*x^2*y''[x]+(1-2*x)*y[x]==0,y[x],{x,0,7}]
```

$$
\begin{aligned}
y(x) \to\ & c_1\sqrt{x}\left(\frac{x^7}{3251404800}+\frac{x^6}{33177600}+\frac{x^5}{460800}+\frac{x^4}{9216}+\frac{x^3}{288}+\frac{x^2}{16}+\frac{x}{2}+1\right) \\
& +c_2\left(\sqrt{x}\left(-\frac{121x^7}{75866112000}-\frac{49x^6}{331776000}-\frac{137x^5}{13824000}-\frac{25x^4}{55296}-\frac{11x^3}{864}-\frac{3x^2}{16}-x\right)\right. \\
& \left.+\sqrt{x}\left(\frac{x^7}{3251404800}+\frac{x^6}{33177600}+\frac{x^5}{460800}+\frac{x^4}{9216}+\frac{x^3}{288}+\frac{x^2}{16}+\frac{x}{2}+1\right)\log(x)\right)
\end{aligned}
$$

## 5.3 problem 3

Internal problem ID [6950]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.6. Indicial Equation with Equal Roots. Exercises page 373
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(x-3)\,y' + 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 81

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*(x-3)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((\ln(x)\,c_2 + c_1)\left(1 - 2x + \frac{3}{2}x^2 - \frac{2}{3}x^3 + \frac{5}{24}x^4 - \frac{1}{20}x^5 + \frac{7}{720}x^6 - \frac{1}{630}x^7 + \mathrm{O}\left(x^8\right)\right) + \left(3x - \frac{13}{4}x^2 + \frac{31}{18}x^3 - \frac{173}{288}x^4 + \frac{187}{1200}x^5 - \frac{463}{14400}x^6 + \frac{971}{176400}x^7 + \mathrm{O}\left(x^8\right)\right)c_2\right)x^2$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 164

```
AsymptoticDSolveValue[x^2*y''[x]+x*(x-3)*y'[x]+4*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 \left( -\frac{x^7}{630} + \frac{7x^6}{720} - \frac{x^5}{20} + \frac{5x^4}{24} - \frac{2x^3}{3} + \frac{3x^2}{2} - 2x + 1 \right) x^2 + c_2 \left( \left( \frac{971x^7}{176400} - \frac{463x^6}{14400} + \frac{187x^5}{1200} - \frac{173x^4}{288} + \frac{31x^3}{18} - \frac{13x^2}{4} + 3x \right) x^2 + \left( -\frac{x^7}{630} + \frac{7x^6}{720} - \frac{x^5}{20} + \frac{5x^4}{24} - \frac{2x^3}{3} + \frac{3x^2}{2} - 2x + 1 \right) x^2 \log(x) \right)$$

## 5.4 problem 4

Internal problem ID [6951]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.6. Indicial Equation with Equal Roots. Exercises page 373
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 3y'x + (4x^2 + 1)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 57

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+(1+4*x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{(\ln(x)\, c_2 + c_1)\left(1 - x^2 + \frac{1}{4}x^4 - \frac{1}{36}x^6 + \mathrm{O}(x^8)\right) + \left(x^2 - \frac{3}{8}x^4 + \frac{11}{216}x^6 + \mathrm{O}(x^8)\right) c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 84

```
AsymptoticDSolveValue[x^2*y''[x]+3*x*y'[x]+(1+4*x^2)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to \frac{c_1\left(-\frac{x^6}{36} + \frac{x^4}{4} - x^2 + 1\right)}{x} + c_2\left(\frac{\frac{11x^6}{216} - \frac{3x^4}{8} + x^2}{x} + \frac{\left(-\frac{x^6}{36} + \frac{x^4}{4} - x^2 + 1\right)\log(x)}{x}\right)$$

## 5.5 problem 5

Internal problem ID [6952]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.6. Indicial Equation with Equal Roots. Exercises page 373
**Problem number**: 5.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x(1+x)\,y''+(5x+1)\,y'+3y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 71

```
Order:=8;
dsolve(x*(1+x)*diff(y(x),x$2)+(1+5*x)*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(\ln(x)\,c_2+c_1)\left(1-3x+6x^2-10x^3+15x^4-21x^5+28x^6-36x^7+\mathrm{O}\left(x^8\right)\right)+\left(2x-\frac{11}{2}x^2+\frac{21}{2}x^3-17x^4+25x^5-\frac{69}{2}x^6+\frac{91}{2}x^7+\mathrm{O}\left(x^8\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 125

```
AsymptoticDSolveValue[x*(1+x)*y''[x]+(1+5*x)*y'[x]+3*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_1\left(-36x^7+28x^6-21x^5+15x^4-10x^3+6x^2-3x+1\right)+c_2\left(\frac{91x^7}{2}-\frac{69x^6}{2}+25x^5-17x^4+\frac{21x^3}{2}-\frac{11x^2}{2}+\left(-36x^7+28x^6-21x^5+15x^4-10x^3+6x^2-3x+1\right)\log(x)+2x\right)$$

## 5.6 problem 6

Internal problem ID [6953]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.6. Indicial Equation with Equal Roots. Exercises page 373
**Problem number**: 6.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(3x+1)\,y' + (-6x+1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 75

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)-x*(1+3*x)*diff(y(x),x)+(1-6*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((\ln(x)\,c_2 + c_1)\left(1 + 9x + 27x^2 + 45x^3 + \frac{405}{8}x^4 + \frac{1701}{40}x^5 + \frac{567}{20}x^6 + \frac{2187}{140}x^7 + \mathrm{O}\left(x^8\right)\right) + \left((-15)\,x - \frac{261}{4}x^2 - \frac{519}{4}x^3 - \frac{5211}{32}x^4 - \frac{118179}{800}x^5 - \frac{83511}{800}x^6 - \frac{2361717}{39200}x^7 + \mathrm{O}\left(x^8\right)\right)c_2\right)x$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 150

```
AsymptoticDSolveValue[x^2*y''[x]-x*(1+3*x)*y'[x]+(1-6*x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 x \left(\frac{2187x^7}{140} + \frac{567x^6}{20} + \frac{1701x^5}{40} + \frac{405x^4}{8} + 45x^3 + 27x^2 + 9x + 1\right) + c_2 \left(x\left(-\frac{2361717x^7}{39200} - \frac{83511x^6}{800} - \frac{118179x^5}{800} - \frac{5211x^4}{32} - \frac{519x^3}{4} - \frac{261x^2}{4} - 15x\right) + x\left(\frac{2187x^7}{140} + \frac{567x^6}{20} + \frac{1701x^5}{40} + \frac{405x^4}{8} + 45x^3 + 27x^2 + 9x + 1\right)\log(x)\right)$$

## 5.7 problem 7

Internal problem ID [6954]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.6. Indicial Equation with Equal Roots. Exercises page 373
**Problem number**: 7.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(x-1)\,y' + y(1-x) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*(x-1)*diff(y(x),x)+(1-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((\ln(x)\,c_2 + c_1)\left(1 + \mathrm{O}\left(x^8\right)\right) + \left(-x + \frac{1}{4}x^2 - \frac{1}{18}x^3 + \frac{1}{96}x^4 - \frac{1}{600}x^5 + \frac{1}{4320}x^6 - \frac{1}{35280}x^7 + \mathrm{O}\left(x^8\right)\right)c_2\right)x$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 64

```
AsymptoticDSolveValue[x^2*y''[x]+x*(x-1)*y'[x]+(1-x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(x\left(-\frac{x^7}{35280} + \frac{x^6}{4320} - \frac{x^5}{600} + \frac{x^4}{96} - \frac{x^3}{18} + \frac{x^2}{4} - x\right) + x\log(x)\right) + c_1x$$

## 5.8 problem 8

Internal problem ID [6955]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.6. Indicial Equation with Equal Roots. Exercises page 373
**Problem number**: 8.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(x-2)\,y''+2(x-1)\,y'-2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 47

```
Order:=8;
dsolve(x*(x-2)*diff(y(x),x$2)+2*(x-1)*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(\ln(x)\,c_2+c_1)\left(1-x+\mathrm{O}\left(x^8\right)\right)+\left(\frac{5}{2}x-\frac{3}{8}x^2-\frac{1}{12}x^3-\frac{5}{192}x^4-\frac{3}{320}x^5-\frac{7}{1920}x^6-\frac{1}{672}x^7+\mathrm{O}\left(x^8\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 71

```
AsymptoticDSolveValue[x*(x-2)*y''[x]+2*(x-1)*y'[x]-2*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_2\left(-\frac{x^7}{672}-\frac{7x^6}{1920}-\frac{3x^5}{320}-\frac{5x^4}{192}-\frac{x^3}{12}-\frac{3x^2}{8}+\frac{5x}{2}+(1-x)\log(x)\right)+c_1(1-x)$$

## 5.9 problem 9

Internal problem ID [6956]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.6. Indicial Equation with Equal Roots. Exercises page 373
**Problem number**: 9.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(x-2)\,y''+2(x-1)\,y'-2y=0$$

With the expansion point for the power series method at $x=2$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 49

```
Order:=8;
dsolve(x*(x-2)*diff(y(x),x$2)+2*(x-1)*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=2);
```

$$y(x)=(c_2\ln(x-2)+c_1)\left(1+(x-2)+\mathrm{O}\left((x-2)^8\right)\right)+\left(-\frac{5}{2}(x-2)-\frac{3}{8}(x-2)^2+\frac{1}{12}(x-2)^3-\frac{5}{192}(x-2)^4+\frac{3}{320}(x-2)^5-\frac{7}{1920}(x-2)^6+\frac{1}{672}(x-2)^7+\mathrm{O}\left((x-2)^8\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 90

```
AsymptoticDSolveValue[x*(x-2)*y''[x]+2*(x-1)*y'[x]-2*y[x]==0,y[x],{x,2,7}]
```

$$y(x)\to c_1(x-1)+c_2\left(\frac{1}{672}(x-2)^7-\frac{7(x-2)^6}{1920}+\frac{3}{320}(x-2)^5-\frac{5}{192}(x-2)^4+\frac{1}{12}(x-2)^3-\frac{3}{8}(x-2)^2-2(x-2)+\frac{2-x}{2}+(x-1)\log(x-2)\right)$$

## 5.10 problem 10

Internal problem ID [6957]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.6. Indicial Equation with Equal Roots. Exercises page 373
**Problem number**: 10.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4(x-4)^2 y'' + (x-4)(x-8) y' + yx = 0$$

With the expansion point for the power series method at $x = 4$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 83

```
Order:=8;
dsolve(4*(x-4)^2*diff(y(x),x$2)+(x-4)*(x-8)*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=4);
```

$$\begin{aligned} y(x) = (x-4)\Bigg( & (c_2\ln(x-4)+c_1)\left(1-\frac{1}{2}(x-4)+\frac{3}{32}(x-4)^2-\frac{1}{96}(x-4)^3+\frac{5}{6144}(x-4)^4\right. \\ & \left.-\frac{1}{20480}(x-4)^5+\frac{7}{2949120}(x-4)^6-\frac{1}{10321920}(x-4)^7+\mathrm{O}\left((x-4)^8\right)\right) \\ & +\left(\frac{3}{4}(x-4)-\frac{13}{64}(x-4)^2+\frac{31}{1152}(x-4)^3-\frac{173}{73728}(x-4)^4+\frac{187}{1228800}(x-4)^5\right. \\ & \left.-\frac{463}{58982400}(x-4)^6+\frac{971}{2890137600}(x-4)^7+\mathrm{O}\left((x-4)^8\right)\right)c_2\Bigg) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 222

```
AsymptoticDSolveValue[4*(x-4)^2*y''[x]+(x-4)*(x-8)*y'[x]+x*y[x]==0,y[x],{x,4,7}]
```

$$y(x) \to c_1\left(-\frac{(x-4)^7}{10321920}+\frac{7(x-4)^6}{2949120}-\frac{(x-4)^5}{20480}+\frac{5(x-4)^4}{6144}-\frac{1}{96}(x-4)^3+\frac{3}{32}(x-4)^2+\frac{4-x}{2}+1\right)(x-4)+c_2\left((x-4)\left(\frac{971(x-4)^7}{2890137600}-\frac{463(x-4)^6}{58982400}+\frac{187(x-4)^5}{1228800}-\frac{173(x-4)^4}{73728}+\frac{31(x-4)^3}{1152}-\frac{13}{64}(x-4)^2+\frac{4-x}{4}+x-4\right)+\left(-\frac{(x-4)^7}{10321920}+\frac{7(x-4)^6}{2949120}-\frac{(x-4)^5}{20480}+\frac{5(x-4)^4}{6144}-\frac{1}{96}(x-4)^3+\frac{3}{32}(x-4)^2+\frac{4-x}{2}+1\right)(x-4)\log(x-4)\right)$$

## 5.11 problem 11 (solved as direct Bessel)

Internal problem ID [6958]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.6. Indicial Equation with Equal Roots. Exercises page 373
**Problem number**: 11 (solved as direct Bessel).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + y' - yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x$2)+diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselI}(0, x) + c_2 \operatorname{BesselK}(0, x)$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 26

```
DSolve[x*y''[x]+y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}(0, ix) + c_2 \operatorname{BesselY}(0, -ix)$$

## 5.12 problem 11 (solved as series)

Internal problem ID [6959]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.6. Indicial Equation with Equal Roots. Exercises page 373
**Problem number**: 11 (solved as series).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + y' - yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
Order:=8;
dsolve(x*diff(y(x),x$2)+diff(y(x),x)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x) c_2 + c_1)\left(1 + \frac{1}{4}x^2 + \frac{1}{64}x^4 + \frac{1}{2304}x^6 + \mathrm{O}\left(x^8\right)\right) + \left(-\frac{1}{4}x^2 - \frac{3}{128}x^4 - \frac{11}{13824}x^6 + \mathrm{O}\left(x^8\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 81

```
AsymptoticDSolveValue[x*y''[x]+y'[x]-x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{x^6}{2304} + \frac{x^4}{64} + \frac{x^2}{4} + 1\right) + c_2\left(-\frac{11x^6}{13824} - \frac{3x^4}{128} - \frac{x^2}{4} + \left(\frac{x^6}{2304} + \frac{x^4}{64} + \frac{x^2}{4} + 1\right)\log(x)\right)$$

## 5.13 problem 12

Internal problem ID [6960]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.6. Indicial Equation with Equal Roots. Exercises page 373
**Problem number**: 12.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + \left(-x^2+1\right) y' - yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=8;
dsolve(x*diff(y(x),x$2)+(1-x^2)*diff(y(x),x)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x)\, c_2 + c_1)\left(1 + \frac{1}{4}x^2 + \frac{3}{64}x^4 + \frac{5}{768}x^6 + \mathrm{O}\left(x^8\right)\right) + \left(-\frac{1}{128}x^4 - \frac{1}{512}x^6 + \mathrm{O}\left(x^8\right)\right) c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 74

```
AsymptoticDSolveValue[x*y''[x]+(1-x^2)*y'[x]-x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{5x^6}{768} + \frac{3x^4}{64} + \frac{x^2}{4} + 1\right) + c_2\left(-\frac{x^6}{512} - \frac{x^4}{128} + \left(\frac{5x^6}{768} + \frac{3x^4}{64} + \frac{x^2}{4} + 1\right)\log(x)\right)$$

## 5.14 problem 14

Internal problem ID [6961]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.6. Indicial Equation with Equal Roots. Exercises page 373
**Problem number**: 14.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(2x+3)\,y' + (3x+1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 79

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*(3+2*x)*diff(y(x),x)+(1+3*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{(\ln(x)\,c_2 + c_1)\left(1 - x + \frac{3}{4}x^2 - \frac{5}{12}x^3 + \frac{35}{192}x^4 - \frac{21}{320}x^5 + \frac{77}{3840}x^6 - \frac{143}{26880}x^7 + \mathrm{O}(x^8)\right) + \left(-\frac{1}{4}x^2 + \frac{1}{4}x^3 - \frac{19}{128}\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 161

```
AsymptoticDSolveValue[x^2*y''[x]+x*(3+2*x)*y'[x]+(1+3*x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to \frac{c_1\left(-\frac{143x^7}{26880}+\frac{77x^6}{3840}-\frac{21x^5}{320}+\frac{35x^4}{192}-\frac{5x^3}{12}+\frac{3x^2}{4}-x+1\right)}{x} + c_2\left(\frac{\frac{469x^7}{69120}-\frac{317x^6}{13824}+\frac{25x^5}{384}-\frac{19x^4}{128}+\frac{x^3}{4}-\frac{x^2}{4}}{x} + \frac{\left(-\frac{143x^7}{26880}+\frac{77x^6}{3840}-\frac{21x^5}{320}+\frac{35x^4}{192}-\frac{5x^3}{12}+\frac{3x^2}{4}-x+1\right)\log(x)}{x}\right)$$

## 5.15 problem 15

Internal problem ID [6962]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.6. Indicial Equation with Equal Roots. Exercises page 373
**Problem number**: 15.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 8x(1+x)\,y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 81

```
Order:=8;
dsolve(4*x^2*diff(y(x),x$2)+8*x*(x+1)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{(\ln(x)\,c_2 + c_1)\left(1 + x - \frac{1}{4}x^2 + \frac{1}{12}x^3 - \frac{5}{192}x^4 + \frac{7}{960}x^5 - \frac{7}{3840}x^6 + \frac{11}{26880}x^7 + \mathrm{O}\left(x^8\right)\right) + \left((-4)\,x + \frac{3}{4}x^2 - \frac{1}{4}\right.}{\sqrt{x}}$$

## ✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 166

```
AsymptoticDSolveValue[4*x^2*y''[x]+8*x*(x+1)*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$
\begin{aligned}
y(x) \to{}& \frac{c_1\left(\frac{11x^7}{26880}-\frac{7x^6}{3840}+\frac{7x^5}{960}-\frac{5x^4}{192}+\frac{x^3}{12}-\frac{x^2}{4}+x+1\right)}{\sqrt{x}} \\
&+ c_2\left(\frac{-\frac{97x^7}{69120}+\frac{419x^6}{69120}-\frac{3x^5}{128}+\frac{31x^4}{384}-\frac{x^3}{4}+\frac{3x^2}{4}-4x}{\sqrt{x}}\right. \\
&\left.+\frac{\left(\frac{11x^7}{26880}-\frac{7x^6}{3840}+\frac{7x^5}{960}-\frac{5x^4}{192}+\frac{x^3}{12}-\frac{x^2}{4}+x+1\right)\log(x)}{\sqrt{x}}\right)
\end{aligned}
$$

## 5.16 problem 16

Internal problem ID [6963]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.6. Indicial Equation with Equal Roots. Exercises page 373
**Problem number**: 16.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 3x(1+x)\,y' + (-3x+1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 61

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+3*x*(1+x)*diff(y(x),x)+(1-3*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{(\ln(x)\,c_2 + c_1)\left(1 + 6x + \frac{9}{2}x^2 + \mathrm{O}(x^8)\right) + \left((-15)\,x - \frac{81}{4}x^2 - \frac{3}{2}x^3 + \frac{9}{32}x^4 - \frac{27}{400}x^5 + \frac{27}{1600}x^6 - \frac{81}{19600}x^7 + \mathrm{(}\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 94

```
AsymptoticDSolveValue[x^2*y''[x]+3*x*(1+x)*y'[x]+(1-3*x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to \frac{c_1\left(\frac{9x^2}{2} + 6x + 1\right)}{x} + c_2\left(\frac{\left(\frac{9x^2}{2} + 6x + 1\right)\log(x)}{x} + \frac{-\frac{81x^7}{19600} + \frac{27x^6}{1600} - \frac{27x^5}{400} + \frac{9x^4}{32} - \frac{3x^3}{2} - \frac{81x^2}{4} - 15x}{x}\right)$$

## 5.17 problem 17

Internal problem ID [6964]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.6. Indicial Equation with Equal Roots. Exercises page 373
**Problem number**: 17.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{xy'' + (1 - x)\, y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 71

```
Order:=8;
dsolve(x*diff(y(x),x$2)+(1-x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x)\, c_2 + c_1) \left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \frac{1}{5040}x^7 + \mathrm{O}\left(x^8\right)\right) + \left(-x - \frac{3}{4}x^2 - \frac{11}{36}x^3 - \frac{25}{288}x^4 - \frac{137}{7200}x^5 - \frac{49}{14400}x^6 - \frac{121}{235200}x^7 + \mathrm{O}\left(x^8\right)\right) c_2$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 149

```
AsymptoticDSolveValue[x*y''[x]+(1-x)*y'[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 \left(\frac{x^7}{5040} + \frac{x^6}{720} + \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1\right) + c_2 \left(-\frac{121x^7}{235200} - \frac{49x^6}{14400} - \frac{137x^5}{7200} - \frac{25x^4}{288} - \frac{11x^3}{36} - \frac{3x^2}{4} + \left(\frac{x^7}{5040} + \frac{x^6}{720} + \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1\right) \log(x) - x\right)$$

# 6 CHAPTER 18. Power series solutions. 18.8 Indicial Equation with Difference of Roots a Positive Integer: Nonlogarithmic Case. Exercises page 380


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## 6.1 problem 1

Internal problem ID [6965]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.8 Indicial Equation with Difference of Roots a Positive Integer: Nonlogarithmic Case. Exercises page 380
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y''+2x(x-2)\,y'+2(-3x+2)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 53

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+2*x*(x-2)*diff(y(x),x)+2*(2-3*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1x^4\left(1-\frac{1}{2}x+\frac{1}{5}x^2-\frac{1}{15}x^3+\frac{2}{105}x^4-\frac{1}{210}x^5+\frac{1}{945}x^6-\frac{1}{4725}x^7+\mathrm{O}\left(x^8\right)\right)$$
$$+c_2x\left(12-24x+24x^2-16x^3+8x^4-\frac{16}{5}x^5+\frac{16}{15}x^6-\frac{32}{105}x^7+\mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.101 (sec). Leaf size: 96

```
AsymptoticDSolveValue[x^2*y''[x]+2*x*(x-2)*y'[x]+2*(2-3*x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_1\left(\frac{4x^7}{45}-\frac{4x^6}{15}+\frac{2x^5}{3}-\frac{4x^4}{3}+2x^3-2x^2+x\right)$$
$$+c_2\left(\frac{x^{10}}{945}-\frac{x^9}{210}+\frac{2x^8}{105}-\frac{x^7}{15}+\frac{x^6}{5}-\frac{x^5}{2}+x^4\right)$$

## 6.2 problem 2

Internal problem ID [6966]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.8 Indicial Equation with Difference of Roots a Positive Integer: Nonlogarithmic Case. Exercises page 380
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(2x+1)\,y''+2x(6x+1)\,y'-2y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
Order:=8;
dsolve(x^2*(1+2*x)*diff(y(x),x$2)+2*x*(1+6*x)*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x\left(1-3x+\frac{42}{5}x^2-\frac{112}{5}x^3+\frac{288}{5}x^4-144x^5+352x^6-\frac{4224}{5}x^7+\mathrm{O}\left(x^8\right)\right) + \frac{c_2\left(12-72x+288x^2-960x^3+2880x^4-8064x^5+21504x^6-55296x^7+\mathrm{O}\left(x^8\right)\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.138 (sec). Leaf size: 76

```
AsymptoticDSolveValue[x^2*(1+2*x)*y''[x]+2*x*(1+6*x)*y'[x]-2*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_1\left(1792x^4-672x^3+240x^2+\frac{1}{x^2}-80x-\frac{6}{x}+24\right) + c_2\left(352x^7-144x^6+\frac{288x^5}{5}-\frac{112x^4}{5}+\frac{42x^3}{5}-3x^2+x\right)$$

## 6.3 problem 3

Internal problem ID [6967]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.8 Indicial Equation with Difference of Roots a Positive Integer: Nonlogarithmic Case. Exercises page 380
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(3x+2)\,y' - 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*(2+3*x)*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x\left(1 - \frac{3}{4}x + \frac{9}{20}x^2 - \frac{9}{40}x^3 + \frac{27}{280}x^4 - \frac{81}{2240}x^5 + \frac{27}{2240}x^6 - \frac{81}{22400}x^7 + \mathrm{O}\left(x^8\right)\right) + \frac{c_2\left(12 - 36x + 54x^2 - 54x^3 + \frac{81}{2}x^4 - \frac{243}{10}x^5 + \frac{243}{20}x^6 - \frac{729}{140}x^7 + \mathrm{O}\left(x^8\right)\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 92

```
AsymptoticDSolveValue[x^2*y''[x]+x*(2+3*x)*y'[x]-2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{81x^4}{80} - \frac{81x^3}{40} + \frac{27x^2}{8} + \frac{1}{x^2} - \frac{9x}{2} - \frac{3}{x} + \frac{9}{2}\right) + c_2\left(\frac{27x^7}{2240} - \frac{81x^6}{2240} + \frac{27x^5}{280} - \frac{9x^4}{40} + \frac{9x^3}{20} - \frac{3x^2}{4} + x\right)$$

## 6.4 problem 4

Internal problem ID [6968]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.8 Indicial Equation with Difference of Roots a Positive Integer: Nonlogarithmic Case. Exercises page 380
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{xy'' - (x+3)\,y' + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 50

```
Order:=8;
dsolve(x*diff(y(x),x$2)-(3+x)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^4 \left(1 + \frac{2}{5}x + \frac{1}{10}x^2 + \frac{2}{105}x^3 + \frac{1}{336}x^4 + \frac{1}{2520}x^5 + \frac{1}{21600}x^6 + \frac{1}{207900}x^7 + \mathrm{O}\left(x^8\right)\right) + c_2\left(-144 - 96x - 24x^2 + 2x^4 + \frac{4}{5}x^5 + \frac{1}{5}x^6 + \frac{4}{105}x^7 + \mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.101 (sec). Leaf size: 91

```
AsymptoticDSolveValue[x*y''[x]-(3+x)*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{x^6}{720} - \frac{x^5}{180} - \frac{x^4}{72} + \frac{x^2}{6} + \frac{2x}{3} + 1\right) + c_2\left(\frac{x^{10}}{21600} + \frac{x^9}{2520} + \frac{x^8}{336} + \frac{2x^7}{105} + \frac{x^6}{10} + \frac{2x^5}{5} + x^4\right)$$

## 6.5 problem 5

Internal problem ID [6969]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.8 Indicial Equation with Difference of Roots a Positive Integer: Nonlogarithmic Case. Exercises page 380
**Problem number**: 5.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(1+x)\,y''+(x+5)\,y'-4y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 38

```
Order:=8;
dsolve(x*(1+x)*diff(y(x),x$2)+(x+5)*diff(y(x),x)-4*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1\left(1+\frac{4}{5}x+\frac{1}{5}x^2+\mathrm{O}\left(x^8\right)\right)+\frac{c_2(-144-576x-720x^2+720x^4+576x^5+144x^6+\mathrm{O}\left(x^8\right))}{x^4}$$

✓ Solution by Mathematica

Time used: 0.13 (sec). Leaf size: 47

```
AsymptoticDSolveValue[x*(1+x)*y''[x]+(x+5)*y'[x]-4*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_2\left(\frac{x^2}{5}+\frac{4x}{5}+1\right)+c_1\left(\frac{1}{x^4}+\frac{4}{x^3}-x^2+\frac{5}{x^2}-4x-5\right)$$

## 6.6 problem 6

Internal problem ID [6970]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.8 Indicial Equation with Difference of Roots a Positive Integer: Nonlogarithmic Case. Exercises page 380
**Problem number**: 6.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(1+x)\,y''+(x+5)\,y'-4y=0}$$

With the expansion point for the power series method at $x=-1$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 50

```
Order:=8;
dsolve(x*(1+x)*diff(y(x),x$2)+(x+5)*diff(y(x),x)-4*y(x)=0,y(x),type='series',x=-1);
```

$$y(x)=c_1(x+1)^5\left(1+\frac{7}{2}(x+1)+8(x+1)^2+15(x+1)^3+25(x+1)^4+\frac{77}{2}(x+1)^5+56(x+1)^6+78(x+1)^7+\mathrm{O}\left((x+1)^8\right)\right)+c_2\left(2880+2880(x+1)+1440(x+1)^2+2880(x+1)^5+10080(x+1)^6+23040(x+1)^7+\mathrm{O}\left((x+1)^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.12 (sec). Leaf size: 88

```
AsymptoticDSolveValue[x*(1+x)*y''[x]+(x+5)*y'[x]-4*y[x]==0,y[x],{x,-1,7}]
```

$$y(x) \to c_1\left(\frac{7}{2}(x+1)^6+(x+1)^5+\frac{1}{2}(x+1)^2+x+2\right)+c_2\left(56(x+1)^{11}+\frac{77}{2}(x+1)^{10}+25(x+1)^9+15(x+1)^8+8(x+1)^7+\frac{7}{2}(x+1)^6+(x+1)^5\right)$$

## 6.7 problem 7

Internal problem ID [6971]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.8 Indicial Equation with Difference of Roots a Positive Integer: Nonlogarithmic Case. Exercises page 380
**Problem number**: 7.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x^2y' - 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x^2*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^2 \left(1 - \frac{1}{2}x + \frac{3}{20}x^2 - \frac{1}{30}x^3 + \frac{1}{168}x^4 - \frac{1}{1120}x^5 + \frac{1}{8640}x^6 - \frac{1}{75600}x^7 + \mathrm{O}\left(x^8\right)\right) + \frac{c_2\left(12 - 6x + x^3 - \frac{1}{2}x^4 + \frac{3}{20}x^5 - \frac{1}{30}x^6 + \frac{1}{168}x^7 + \mathrm{O}\left(x^8\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 91

```
AsymptoticDSolveValue[x^2*y''[x]+x^2*y'[x]-2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{x^5}{360} + \frac{x^4}{80} - \frac{x^3}{24} + \frac{x^2}{12} + \frac{1}{x} - \frac{1}{2}\right) + c_2\left(\frac{x^8}{8640} - \frac{x^7}{1120} + \frac{x^6}{168} - \frac{x^5}{30} + \frac{3x^4}{20} - \frac{x^3}{2} + x^2\right)$$

## 6.8 problem 8

Internal problem ID [6972]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.8 Indicial Equation with Difference of Roots a Positive Integer: Nonlogarithmic Case. Exercises page 380
**Problem number**: 8.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x(1-x)\,y''-3y'+2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 50

```
Order:=8;
dsolve(x*(1-x)*diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned}y(x) &= c_1x^4\left(1+2x+3x^2+4x^3+5x^4+6x^5+7x^6+8x^7+\mathrm{O}\left(x^8\right)\right)\\ &\quad+c_2\left(-144-96x-48x^2+48x^4+96x^5+144x^6+192x^7+\mathrm{O}\left(x^8\right)\right)\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.438 (sec). Leaf size: 77

```
AsymptoticDSolveValue[x*(1-x)*y''[x]-3*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_1\left(-x^6-\frac{2x^5}{3}-\frac{x^4}{3}+\frac{x^2}{3}+\frac{2x}{3}+1\right)+c_2\left(7x^{10}+6x^9+5x^8+4x^7+3x^6+2x^5+x^4\right)$$

## 6.9 problem 9

Internal problem ID [6973]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.8 Indicial Equation with Difference of Roots a Positive Integer: Nonlogarithmic Case. Exercises page 380
**Problem number**: 9.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x(1-x)\,y''-3y'+2y=0}$$

With the expansion point for the power series method at $x=1$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 38

```
Order:=8;
dsolve(x*(1-x)*diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=1);
```

$$y(x)=c_1\left(1+\frac{2}{3}(x-1)+\frac{1}{6}(x-1)^2+\mathrm{O}\left((x-1)^8\right)\right)+\frac{c_2\left(-2-8(x-1)-12(x-1)^2-8(x-1)^3-2(x-1)^4+\mathrm{O}\left((x-1)^8\right)\right)}{(x-1)^2}$$

✓ Solution by Mathematica

Time used: 0.454 (sec). Leaf size: 52

```
AsymptoticDSolveValue[x*(1-x)*y''[x]-3*y'[x]+2*y[x]==0,y[x],{x,1,7}]
```

$$y(x)\to c_1\left((x-1)^2+4(x-1)+\frac{4}{x-1}+\frac{1}{(x-1)^2}+6\right)+c_2\left(\frac{1}{6}(x-1)^2+\frac{2(x-1)}{3}+1\right)$$

## 6.10 problem 10

Internal problem ID [6974]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.8 Indicial Equation with Difference of Roots a Positive Integer: Nonlogarithmic Case. Exercises page 380
**Problem number**: 10.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{xy'' + (3x+4)\,y' + 3y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 52

```
Order:=8;
dsolve(x*diff(y(x),x$2)+(4+3*x)*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\left(1-\frac{3}{4}x+\frac{9}{20}x^2-\frac{9}{40}x^3+\frac{27}{280}x^4-\frac{81}{2240}x^5+\frac{27}{2240}x^6-\frac{81}{22400}x^7+\mathrm{O}\left(x^8\right)\right) + \frac{c_2\left(12-36x+54x^2-54x^3+\frac{81}{2}x^4-\frac{243}{10}x^5+\frac{243}{20}x^6-\frac{729}{140}x^7+\mathrm{O}\left(x^8\right)\right)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.11 (sec). Leaf size: 90

```
AsymptoticDSolveValue[x*y''[x]+(4+3*x)*y'[x]+3*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{81x^3}{80}+\frac{1}{x^3}-\frac{81x^2}{40}-\frac{3}{x^2}+\frac{27x}{8}+\frac{9}{2x}-\frac{9}{2}\right) + c_2\left(\frac{27x^6}{2240}-\frac{81x^5}{2240}+\frac{27x^4}{280}-\frac{9x^3}{40}+\frac{9x^2}{20}-\frac{3x}{4}+1\right)$$

## 6.11 problem 11

Internal problem ID [6975]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.8 Indicial Equation with Difference of Roots a Positive Integer: Nonlogarithmic Case. Exercises page 380
**Problem number**: 11.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - 2(x+2)\,y' + 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 48

```
Order:=8;
dsolve(x*diff(y(x),x$2)-2*(x+2)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^5 \left(1 + x + \frac{4}{7}x^2 + \frac{5}{21}x^3 + \frac{5}{63}x^4 + \frac{1}{45}x^5 + \frac{8}{1485}x^6 + \frac{4}{3465}x^7 + \mathrm{O}\left(x^8\right)\right) + c_2\left(2880 + 2880x + 960x^2 + 128x^5 + 128x^6 + \frac{512}{7}x^7 + \mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.11 (sec). Leaf size: 76

```
AsymptoticDSolveValue[x*y''[x]-2*(x+2)*y'[x]+4*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{2x^6}{45} + \frac{2x^5}{45} + \frac{x^2}{3} + x + 1\right) + c_2\left(\frac{8x^{11}}{1485} + \frac{x^{10}}{45} + \frac{5x^9}{63} + \frac{5x^8}{21} + \frac{4x^7}{7} + x^6 + x^5\right)$$

## 6.12 problem 12

Internal problem ID [6976]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.8 Indicial Equation with Difference of Roots a Positive Integer: Nonlogarithmic Case. Exercises page 380
**Problem number**: 12.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (2x+3)\,y' + 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 50

```
Order:=8;
dsolve(x*diff(y(x),x$2)+(3+2*x)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\left(1 - \frac{4}{3}x + x^2 - \frac{8}{15}x^3 + \frac{2}{9}x^4 - \frac{8}{105}x^5 + \frac{1}{45}x^6 - \frac{16}{2835}x^7 + \mathrm{O}\left(x^8\right)\right) + \frac{c_2\left(-2 + 4x^2 - \frac{16}{3}x^3 + 4x^4 - \frac{32}{15}x^5 + \frac{8}{9}x^6 - \frac{32}{105}x^7 + \mathrm{O}\left(x^8\right)\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.106 (sec). Leaf size: 77

```
AsymptoticDSolveValue[x*y''[x]+(3+2*x)*y'[x]+4*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{4x^4}{9} + \frac{16x^3}{15} - 2x^2 + \frac{1}{x^2} + \frac{8x}{3} - 2\right) + c_2\left(\frac{x^6}{45} - \frac{8x^5}{105} + \frac{2x^4}{9} - \frac{8x^3}{15} + x^2 - \frac{4x}{3} + 1\right)$$

## 6.13 problem 13

Internal problem ID [6977]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.8 Indicial Equation with Difference of Roots a Positive Integer: Nonlogarithmic Case. Exercises page 380
**Problem number**: 13.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(x+3)\,y''-9y'-6y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 52

```
Order:=8;
dsolve(x*(x+3)*diff(y(x),x$2)-9*diff(y(x),x)-6*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1x^4\left(1-\frac{2}{5}x+\frac{7}{45}x^2-\frac{8}{135}x^3+\frac{1}{45}x^4-\frac{2}{243}x^5+\frac{11}{3645}x^6-\frac{4}{3645}x^7+\mathrm{O}\left(x^8\right)\right)\\+c_2\left(-144+96x-48x^2+\frac{64}{3}x^3-\frac{80}{9}x^4+\frac{32}{9}x^5-\frac{112}{81}x^6+\frac{128}{243}x^7+\mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.419 (sec). Leaf size: 98

```
AsymptoticDSolveValue[x*(x+3)*y''[x]-9*y'[x]-6*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_1\left(\frac{7x^6}{729}-\frac{2x^5}{81}+\frac{5x^4}{81}-\frac{4x^3}{27}+\frac{x^2}{3}-\frac{2x}{3}+1\right)\\+c_2\left(\frac{11x^{10}}{3645}-\frac{2x^9}{243}+\frac{x^8}{45}-\frac{8x^7}{135}+\frac{7x^6}{45}-\frac{2x^5}{5}+x^4\right)$$

## 6.14 problem 14

Internal problem ID [6978]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.8 Indicial Equation with Difference of Roots a Positive Integer: Nonlogarithmic Case. Exercises page 380
**Problem number**: 14.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(-2x+1)\,y''-2(x+2)\,y'+8y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 48

```
Order:=8;
dsolve(x*(1-2*x)*diff(y(x),x$2)-2*(2+x)*diff(y(x),x)+8*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = {} & c_1x^5\left(1+7x+32x^2+120x^3+400x^4+1232x^5+3584x^6+9984x^7+\mathrm{O}\left(x^8\right)\right) \\ & + c_2\left(2880+5760x+5760x^2+92160x^5+645120x^6+2949120x^7+\mathrm{O}\left(x^8\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.123 (sec). Leaf size: 64

```
AsymptoticDSolveValue[x*(1-2*x)*y''[x]-2*(2+x)*y'[x]+8*y[x]==0,y[x],{x,0,7}]
```

$$\begin{aligned} y(x) \to {} & c_1\left(224x^6+32x^5+2x^2+2x+1\right) \\ & + c_2\left(3584x^{11}+1232x^{10}+400x^9+120x^8+32x^7+7x^6+x^5\right) \end{aligned}$$

## 6.15 problem 15

Internal problem ID [6979]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.8 Indicial Equation with Difference of Roots a Positive Integer: Nonlogarithmic Case. Exercises page 380
**Problem number**: 15.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (x^3 - 1)\, y' + x^2 y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 32

```
Order:=8;
dsolve(x*diff(y(x),x$2)+(x^3-1)*diff(y(x),x)+x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^2 \left(1 - \frac{1}{5}x^3 + \frac{1}{40}x^6 + \mathrm{O}\left(x^8\right)\right) + c_2 \left(-2 + \frac{2}{3}x^3 - \frac{1}{9}x^6 + \mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 44

```
AsymptoticDSolveValue[x*y''[x]+(x^3-1)*y'[x]+x^2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 \left(\frac{x^6}{18} - \frac{x^3}{3} + 1\right) + c_2 \left(\frac{x^8}{40} - \frac{x^5}{5} + x^2\right)$$

## 6.16 problem 16

Internal problem ID [6980]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.8 Indicial Equation with Difference of Roots a Positive Integer: Nonlogarithmic Case. Exercises page 380
**Problem number**: 16.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(4x-1)\,y''+x(5x+1)\,y'+3y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
Order:=8;
dsolve(x^2*(4*x-1)*diff(y(x),x$2)+x*(5*x+1)*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1x^3\left(1+\frac{39}{5}x+\frac{221}{5}x^2+221x^3+\frac{16575}{16}x^4+\frac{224315}{48}x^5+\frac{493493}{24}x^6+\frac{711399}{8}x^7+\mathrm{O}\left(x^8\right)\right)+\frac{c_2\left(-144+144x+270x^4+2106x^5+11934x^6+59670x^7+\mathrm{O}\left(x^8\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.171 (sec). Leaf size: 80

```
AsymptoticDSolveValue[x^2*(4*x-1)*y''[x]+x*(5*x+1)*y'[x]+3*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_1\left(-\frac{663x^5}{8}-\frac{117x^4}{8}-\frac{15x^3}{8}+\frac{1}{x}-1\right)+c_2\left(\frac{493493x^9}{24}+\frac{224315x^8}{48}+\frac{16575x^7}{16}+221x^6+\frac{221x^5}{5}+\frac{39x^4}{5}+x^3\right)$$

# 7 CHAPTER 18. Power series solutions. 18.9 Indicial Equation with Difference of Roots a Positive Integer: Logarithmic Case. Exercises page 384


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## 7.1 problem 1

Internal problem ID [6981]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.9 Indicial Equation with Difference of Roots a Positive Integer: Logarithmic Case. Exercises page 384
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 70

```
Order:=8;
dsolve(x*diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x \left(1 - \frac{1}{2}x + \frac{1}{12}x^2 - \frac{1}{144}x^3 + \frac{1}{2880}x^4 - \frac{1}{86400}x^5 + \frac{1}{3628800}x^6 - \frac{1}{203212800}x^7 + \mathrm{O}\left(x^8\right)\right) + c_2 \left(\ln\left(x\right)\left(-x + \frac{1}{2}x^2 - \frac{1}{12}x^3 + \frac{1}{144}x^4 - \frac{1}{2880}x^5 + \frac{1}{86400}x^6 - \frac{1}{3628800}x^7 + \mathrm{O}\left(x^8\right)\right) + \left(1 - \frac{3}{4}x^2 + \frac{7}{36}x^3 - \frac{35}{1728}x^4 + \frac{101}{86400}x^5 - \frac{7}{162000}x^6 + \frac{283}{254016000}x^7 + \mathrm{O}\left(x^8\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 119

```
AsymptoticDSolveValue[x*y''[x]+y[x]==0,y[x],{x,0,7}]
```

$$
\begin{aligned}
y(x) \\
\rightarrow c_1\bigg(&\frac{x(x^5-30x^4+600x^3-7200x^2+43200x-86400)\log(x)}{86400} \\
&+\frac{-71x^6+1965x^5-35250x^4+360000x^3-1620000x^2+1296000x+1296000}{1296000}\bigg) \\
&+c_2\left(\frac{x^7}{3628800}-\frac{x^6}{86400}+\frac{x^5}{2880}-\frac{x^4}{144}+\frac{x^3}{12}-\frac{x^2}{2}+x\right)
\end{aligned}
$$

## 7.2 problem 2

Internal problem ID [6982]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.9 Indicial Equation with Difference of Roots a Positive Integer: Logarithmic Case. Exercises page 384
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 3y'x + (4x+3)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 73

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+(3+4*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(c_1x^2\left(1-\frac{4}{3}x+\frac{2}{3}x^2-\frac{8}{45}x^3+\frac{4}{135}x^4-\frac{16}{4725}x^5+\frac{4}{14175}x^6-\frac{16}{893025}x^7+\mathrm{O}\left(x^8\right)\right)\right.$$
$$+c_2\left(\ln(x)\left(16x^2-\frac{64}{3}x^3+\frac{32}{3}x^4-\frac{128}{45}x^5+\frac{64}{135}x^6-\frac{256}{4725}x^7+\mathrm{O}\left(x^8\right)\right)\right.$$
$$\left.\left.+\left(-2-8x+\frac{256}{9}x^3-\frac{200}{9}x^4+\frac{5024}{675}x^5-\frac{2912}{2025}x^6+\frac{90752}{496125}x^7+\mathrm{O}\left(x^8\right)\right)\right)\right)x$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 121

```
AsymptoticDSolveValue[x^2*y''[x]-3*x*y'[x]+(3+4*x)*y[x]==0,y[x],{x,0,7}]
```

$$
y(x) \to c_1\left(\frac{x(1696x^6-8976x^5+27900x^4-39600x^3+8100x^2+8100x+2025)}{2025} - \frac{8}{135}x^3(4x^4-24x^3+90x^2-180x+135)\log(x)\right) + c_2\left(\frac{4x^9}{14175}-\frac{16x^8}{4725}+\frac{4x^7}{135}-\frac{8x^6}{45}+\frac{2x^5}{3}-\frac{4x^4}{3}+x^3\right)
$$

## 7.3 problem 3

Internal problem ID [6983]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.9 Indicial Equation with Difference of Roots a Positive Integer: Logarithmic Case. Exercises page 384
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2xy'' + 6y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 74

```
Order:=8;
dsolve(2*x*diff(y(x),x$2)+6*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 - \frac{1}{6}x + \frac{1}{96}x^2 - \frac{1}{2880}x^3 + \frac{1}{138240}x^4 - \frac{1}{9676800}x^5 + \frac{1}{928972800}x^6 - \frac{1}{117050572800}x^7 + \mathrm{O}\left(x^8\right)\right)x^2 + c_2\left(\ln\left(x\right)\right.}{}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 114

```
AsymptoticDSolveValue[2*x*y''[x]+6*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(\frac{x^6}{928972800}-\frac{x^5}{9676800}+\frac{x^4}{138240}-\frac{x^3}{2880}+\frac{x^2}{96}-\frac{x}{6}+1\right) + c_1\left(\frac{53x^6-2244x^5+55800x^4-633600x^3+1036800x^2+8294400x+16588800}{16588800x^2} - \frac{(x^4-48x^3+1440x^2-23040x+138240)\log(x)}{1105920}\right)$$

## 7.4 problem 4

Internal problem ID [6984]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.9 Indicial Equation with Difference of Roots a Positive Integer: Logarithmic Case. Exercises page 384
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + 2x(-x+2)\,y' - (3x+1)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 79

```
Order:=8;
dsolve(4*x^2*diff(y(x),x$2)+2*x*(2-x)*diff(y(x),x)-(1+3*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x\left(1+\frac{1}{2}x+\frac{1}{8}x^2+\frac{1}{48}x^3+\frac{1}{384}x^4+\frac{1}{3840}x^5+\frac{1}{46080}x^6+\frac{1}{645120}x^7+\mathrm{O}\left(x^8\right)\right)+c_2\left(\ln\left(x\right)\left(\frac{1}{2}x+\frac{1}{4}x^2+\frac{1}{16}x^3\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 0.094 (sec). Leaf size: 141

```
AsymptoticDSolveValue[4*x^2*y''[x]+2*x*(2-x)*y'[x]-(1+3*x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(\frac{x^{13/2}}{46080}+\frac{x^{11/2}}{3840}+\frac{x^{9/2}}{384}+\frac{x^{7/2}}{48}+\frac{x^{5/2}}{8}+\frac{x^{3/2}}{2}+\sqrt{x}\right)+c_1\left(\frac{\sqrt{x}(x^5+10x^4+80x^3+480x^2+1920x+3840)\log(x)}{7680}-\frac{137x^6+1250x^5+8800x^4+432}{460800\sqrt{}}\right.$$

## 7.5 problem 5

Internal problem ID [6985]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.9 Indicial Equation with Difference of Roots a Positive Integer: Logarithmic Case. Exercises page 384
**Problem number**: 5.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(x+6)\,y' + 10y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 77

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)-x*(6+x)*diff(y(x),x)+10*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(c_1x^3\left(1+\frac{5}{4}x+\frac{3}{4}x^2+\frac{7}{24}x^3+\frac{1}{12}x^4+\frac{3}{160}x^5+\frac{1}{288}x^6+\frac{11}{20160}x^7+\mathrm{O}\left(x^8\right)\right)\right.$$
$$+ c_2\left(\ln(x)\left(24x^3+30x^4+18x^5+7x^6+2x^7+\mathrm{O}\left(x^8\right)\right)\right.$$
$$\left.\left.+\left(12-12x+18x^2+26x^3+x^4-9x^5-6x^6-\frac{9}{4}x^7+\mathrm{O}\left(x^8\right)\right)\right)\right)x^2$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 118

```
AsymptoticDSolveValue[x^2*y''[x]-x*(6+x)*y'[x]+10*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 \left( \frac{1}{12} x^5 (7x^3 + 18x^2 + 30x + 24) \log(x) - \frac{1}{36} x^2 (25x^6 + 45x^5 + 27x^4 - 54x^3 - 54x^2 + 36x - 36) \right) + c_2 \left( \frac{x^{11}}{288} + \frac{3x^{10}}{160} + \frac{x^9}{12} + \frac{7x^8}{24} + \frac{3x^7}{4} + \frac{5x^6}{4} + x^5 \right)$$

## 7.6 problem 6

Internal problem ID [6986]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.9 Indicial Equation with Difference of Roots a Positive Integer: Logarithmic Case. Exercises page 384
**Problem number**: 6.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' + (2x + 3)\,y' + 8y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 76

```
Order:=8;
dsolve(x*diff(y(x),x$2)+(3+2*x)*diff(y(x),x)+8*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 - \frac{8}{3}x + \frac{10}{3}x^2 - \frac{8}{3}x^3 + \frac{14}{9}x^4 - \frac{32}{45}x^5 + \frac{4}{15}x^6 - \frac{16}{189}x^7 + \mathrm{O}\left(x^8\right)\right)x^2 + c_2\left(\ln\left(x\right)\left(24x^2 - 64x^3 + 80x^4 - \right.\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.107 (sec). Leaf size: 116

```
AsymptoticDSolveValue[x*y''[x]+(3+2*x)*y'[x]+8*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(\frac{4x^6}{15} - \frac{32x^5}{45} + \frac{14x^4}{9} - \frac{8x^3}{3} + \frac{10x^2}{3} - \frac{8x}{3} + 1\right) + c_1\left(\frac{326x^6 - 480x^5 + 468x^4 - 216x^3 - 36x^2 + 36x + 9}{9x^2} - \frac{4}{3}\left(14x^4 - 24x^3 + 30x^2 - 24x + 9\right)\log(x)\right)$$

## 7.7 problem 7

Internal problem ID [6987]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.9 Indicial Equation with Difference of Roots a Positive Integer: Logarithmic Case. Exercises page 384
**Problem number**: 7.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Jacobi]`

$$\boxed{x(1-x)\, y'' + 2(1-x)\, y' + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 54

```
Order:=8;
dsolve(x*(1-x)*diff(y(x),x$2)+2*(1-x)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{((-2)\, x + 2x^2 + \mathrm{O}(x^8)) \ln(x)\, c_2 + c_1(1 - x + \mathrm{O}(x^8))\, x + \left(1 - 4x^2 + x^3 + \frac{1}{3}x^4 + \frac{1}{6}x^5 + \frac{1}{10}x^6 + \frac{1}{15}x^7 + \cdots\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.408 (sec). Leaf size: 60

```
AsymptoticDSolveValue[x*(1-x)*y''[x]+2*(1-x)*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{3x^6 + 5x^5 + 10x^4 + 30x^3 - 150x^2 + 30x + 30}{30x} + 2(x-1)\log(x)\right) + c_2(1-x)$$

## 7.8 problem 9

Internal problem ID [6988]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.9 Indicial Equation with Difference of Roots a Positive Integer: Logarithmic Case. Exercises page 384
**Problem number**: 9.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Jacobi]`

$$\boxed{x(1-x)\,y''+2(1-x)\,y'+2y=0}$$

With the expansion point for the power series method at $x=1$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 50

```
Order:=8;
dsolve(x*(1-x)*diff(y(x),x$2)+2*(1-x)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=1);
```

$$y(x)=\left(1-2(x-1)-3(x-1)^2+2(x-1)^3-\frac{5}{3}(x-1)^4+\frac{3}{2}(x-1)^5-\frac{7}{5}(x-1)^6+\frac{4}{3}(x-1)^7+\mathrm{O}\left((x-1)^8\right)\right)c_2+c_1(x-1)\left(1+\mathrm{O}\left((x-1)^8\right)\right)+\left(2(x-1)+\mathrm{O}\left((x-1)^8\right)\right)\ln(x-1)\,c_2$$

✓ Solution by Mathematica

Time used: 0.422 (sec). Leaf size: 69

```
AsymptoticDSolveValue[x*(1-x)*y''[x]+2*(1-x)*y'[x]+2*y[x]==0,y[x],{x,1,7}]
```

$$y(x) \to c_2(x-1) + c_1\left(\frac{1}{30}\left(-42(x-1)^6 + 45(x-1)^5 - 50(x-1)^4 + 60(x-1)^3 - 90(x-1)^2 - 90(x-1) + 30\right) + 2(x-1)\log(x-1)\right)$$

## 7.9 problem 10 (as direct Bessel)

Internal problem ID [6989]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.9 Indicial Equation with Difference of Roots a Positive Integer: Logarithmic Case. Exercises page 384
**Problem number**: 10 (as direct Bessel).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Bessel]`

$$\boxed{x^2y'' + y'x + (x^2 - 1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}(1, x) + c_2 \operatorname{BesselY}(1, x)$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}(1, x) + c_2 \operatorname{BesselY}(1, x)$$

## 7.10 problem 10 (as series)

Internal problem ID [6990]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.9 Indicial Equation with Difference of Roots a Positive Integer: Logarithmic Case. Exercises page 384
**Problem number**: 10 (as series).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Bessel]`

$$x^2y'' + y'x + (x^2 - 1)\, y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^2\left(1 - \frac{1}{8}x^2 + \frac{1}{192}x^4 - \frac{1}{9216}x^6 + \mathrm{O}\left(x^8\right)\right) + c_2\left(\ln(x)\left(x^2 - \frac{1}{8}x^4 + \frac{1}{192}x^6 + \mathrm{O}\left(x^8\right)\right) + \left(-2 + \frac{3}{32}x^4 - \frac{7}{1152}x\right.\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 75

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-1)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(-\frac{x^7}{9216} + \frac{x^5}{192} - \frac{x^3}{8} + x\right) + c_1\left(\frac{5x^6 - 90x^4 + 288x^2 + 1152}{1152x} - \frac{1}{384}x\left(x^4 - 24x^2 + 192\right)\log(x)\right)$$

## 7.11 problem 11

Internal problem ID [6991]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.9 Indicial Equation with Difference of Roots a Positive Integer: Logarithmic Case. Exercises page 384
**Problem number**: 11.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 5y'x + (8+5x)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 77

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)-5*x*diff(y(x),x)+(8+5*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(c_1x^2\left(1-\frac{5}{3}x+\frac{25}{24}x^2-\frac{25}{72}x^3+\frac{125}{1728}x^4-\frac{125}{12096}x^5+\frac{625}{580608}x^6-\frac{3125}{36578304}x^7+\mathrm{O}\left(x^8\right)\right)+c_2\left(\ln(x)\left(25x^2-\frac{125}{3}x^3+\frac{625}{24}x^4-\frac{625}{72}x^5+\frac{3125}{1728}x^6-\frac{3125}{12096}x^7+\mathrm{O}\left(x^8\right)\right)+\left(-2-10x+\frac{500}{9}x^3-\frac{15625}{288}x^4+\frac{19625}{864}x^5-\frac{56875}{10368}x^6+\frac{443125}{508032}x^7+\mathrm{O}\left(x^8\right)\right)\right)\right)x^2$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 123

```
AsymptoticDSolveValue[x^2*y''[x]-5*x*y'[x]+(8+5*x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{x^2(33125x^6 - 140250x^5 + 348750x^4 - 396000x^3 + 64800x^2 + 51840x + 10368)}{10368} - \frac{25x^4(125x^4 - 600x^3 + 1800x^2 - 2880x + 1728)\log(x)}{3456}\right) + c_2\left(\frac{625x^{10}}{580608} - \frac{125x^9}{12096} + \frac{125x^8}{1728} - \frac{25x^7}{72} + \frac{25x^6}{24} - \frac{5x^5}{3} + x^4\right)$$

## 7.12 problem 12

Internal problem ID [6992]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.9 Indicial Equation with Difference of Roots a Positive Integer: Logarithmic Case. Exercises page 384
**Problem number**: 12.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{xy'' + (-x+3)\,y' - 5y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 76

```
Order:=8;
dsolve(x*diff(y(x),x$2)+(3-x)*diff(y(x),x)-5*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1+\frac{5}{3}x+\frac{5}{4}x^2+\frac{7}{12}x^3+\frac{7}{36}x^4+\frac{1}{20}x^5+\frac{1}{96}x^6+\frac{11}{6048}x^7+\mathrm{O}\left(x^8\right)\right)x^2+c_2\left(\ln\left(x\right)\left(12x^2+20x^3+15x^4+\right.\right.}{x^2}$$

✓ Solution by Mathematica

Time used: 0.103 (sec). Leaf size: 116

```
AsymptoticDSolveValue[x*y''[x]+(3-x)*y'[x]-5*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(\frac{x^6}{96}+\frac{x^5}{20}+\frac{7x^4}{36}+\frac{7x^3}{12}+\frac{5x^2}{4}+\frac{5x}{3}+1\right) + c_1\left(\frac{389x^6+1020x^5+1764x^4+1512x^3-72x^2-432x+144}{144x^2} - \frac{1}{6}\left(7x^4+21x^3+45x^2+60x+36\right)\log(x)\right)$$

## 7.13 problem 13

Internal problem ID [6993]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.9 Indicial Equation with Difference of Roots a Positive Integer: Logarithmic Case. Exercises page 384
**Problem number**: 13.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9x^2y'' - 15y'x + 7y(1+x) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 77

```
Order:=8;
dsolve(9*x^2*diff(y(x),x$2)-15*x*diff(y(x),x)+7*(x+1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(\left(1 - \frac{7}{27}x + \frac{49}{1944}x^2 - \frac{343}{262440}x^3 + \frac{2401}{56687040}x^4 - \frac{2401}{2550916800}x^5 + \frac{16807}{1101996057600}x^6 - \frac{16807}{89261680665600}x^7 + \mathrm{O}\left(x^8\right)\right)x^2c_1 + c_2\left(\ln(x)\left(\frac{49}{81}x^2 - \frac{343}{2187}x^3 + \frac{2401}{157464}x^4 - \frac{16807}{21257640}x^5 + \frac{117649}{4591650240}x^6 - \frac{117649}{206624260800}x^7 + \mathrm{O}\left(x^8\right)\right) + \left(-2 - \frac{14}{9}x + \frac{1372}{6561}x^3 - \frac{60025}{1889568}x^4 + \frac{2638699}{1275458400}x^5 - \frac{10706059}{137749507200}x^6 + \frac{11916163}{6198727824000}x^7 + \mathrm{O}\left(x^8\right)\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.064 (sec). Leaf size: 141

```
AsymptoticDSolveValue[9*x^2*y''[x]-15*x*y'[x]+7*(x+1)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(\frac{16807x^{25/3}}{1101996057600} - \frac{2401x^{22/3}}{2550916800} + \frac{2401x^{19/3}}{56687040} - \frac{343x^{16/3}}{262440} + \frac{49x^{13/3}}{1944} - \frac{7x^{10/3}}{27} + x^{7/3}\right) + c_1\left(\frac{\sqrt[3]{x}(6235397x^6 - 169717086x^5 + 2713009950x^4 - 19803722400x^3 + 20832487200x^2 + 10}{137749507200}\right.$$

## 7.14 problem 14

Internal problem ID [6994]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.9 Indicial Equation with Difference of Roots a Positive Integer: Logarithmic Case. Exercises page 384
**Problem number**: 14.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(-2x+1)\,y' - y(1+x) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 75

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*(1-2*x)*diff(y(x),x)-(x+1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^2\left(1+x+\frac{5}{8}x^2+\frac{7}{24}x^3+\frac{7}{64}x^4+\frac{11}{320}x^5+\frac{143}{15360}x^6+\frac{143}{64512}x^7+\mathrm{O}\left(x^8\right)\right)+c_2\left(\ln\left(x\right)\left(-x^2-x^3-\frac{5}{8}x^4-\right.\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.109 (sec). Leaf size: 115

```
AsymptoticDSolveValue[x^2*y''[x]+x*(1-2*x)*y'[x]-(x+1)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{1}{384}x\left(21x^4+56x^3+120x^2+192x+192\right)\log(x) - \frac{617x^6+1482x^5+2730x^4+3360x^3+1440x^2-5760x-5760}{5760x}\right) + c_2\left(\frac{143x^7}{15360}+\frac{11x^6}{320}+\frac{7x^5}{64}+\frac{7x^4}{24}+\frac{5x^3}{8}+x^2+x\right)$$

# 8 CHAPTER 18. Power series solutions. 18.11 Many-Term Recurrence Relations. Exercises page 391


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## 8.1 problem 1

Internal problem ID [6995]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.11 Many-Term Recurrence Relations. Exercises page 391
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' + 3y'x + (x^3 + x + 1)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 81

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+(1+x+x^3)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{(\ln(x)\, c_2 + c_1)\left(1 - x + \frac{1}{4}x^2 - \frac{5}{36}x^3 + \frac{41}{576}x^4 - \frac{37}{2880}x^5 + \frac{437}{103680}x^6 - \frac{7817}{5080320}x^7 + \mathrm{O}(x^8)\right) + \left(2x - \frac{3}{4}x^2 + \frac{1}{10}\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 164

```
AsymptoticDSolveValue[x^2*y''[x]+3*x*y'[x]+(1+x+x^3)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to \frac{c_1\left(-\frac{7817x^7}{5080320}+\frac{437x^6}{103680}-\frac{37x^5}{2880}+\frac{41x^4}{576}-\frac{5x^3}{36}+\frac{x^2}{4}-x+1\right)}{x} + c_2\left(\frac{\frac{485257x^7}{118540800}-\frac{7733x^6}{1036800}+\frac{3629x^5}{86400}-\frac{593x^4}{3456}+\frac{19x^3}{108}-\frac{3x^2}{4}+2x}{x} + \frac{\left(-\frac{7817x^7}{5080320}+\frac{437x^6}{103680}-\frac{37x^5}{2880}+\frac{41x^4}{576}-\frac{5x^3}{36}+\frac{x^2}{4}-x+1\right)\log(x)}{x}\right)$$

## 8.2 problem 2

Internal problem ID [6996]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.11 Many-Term Recurrence Relations. Exercises page 391
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Jacobi]`

$$\boxed{2x(1-x)\,y''+(-2x+1)\,y'+(x+2)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 52

```
Order:=8;
dsolve(2*x*(1-x)*diff(y(x),x$2)+(1-2*x)*diff(y(x),x)+(2+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1-\frac{1}{2}x-\frac{9}{40}x^2-\frac{149}{1680}x^3-\frac{661}{13440}x^4-\frac{16171}{492800}x^5-\frac{5530601}{230630400}x^6-\frac{299137703}{16144128000}x^7+\mathrm{O}\left(x^8\right)\right)+c_2\left(1-2x-\frac{1}{6}x^2+\frac{1}{15}x^3+\frac{37}{840}x^4+\frac{527}{18900}x^5+\frac{16309}{831600}x^6+\frac{14339}{970200}x^7+\mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 111

```
AsymptoticDSolveValue[2*x*(1-x)*y''[x]+(1-2*x)*y'[x]+(2+x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\sqrt{x}\left(-\frac{299137703x^7}{16144128000}-\frac{5530601x^6}{230630400}-\frac{16171x^5}{492800}-\frac{661x^4}{13440}-\frac{149x^3}{1680}-\frac{9x^2}{40}-\frac{x}{2}+1\right)+c_2\left(\frac{14339x^7}{970200}+\frac{16309x^6}{831600}+\frac{527x^5}{18900}+\frac{37x^4}{840}+\frac{x^3}{15}-\frac{x^2}{6}-2x+1\right)$$

## 8.3 problem 3

Internal problem ID [6997]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.11 Many-Term Recurrence Relations. Exercises page 391
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + y' + x(1 + x)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 65

```
Order:=8;
dsolve(x*diff(y(x),x$2)+diff(y(x),x)+x*(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x)\, c_2 + c_1)\left(1 - \frac{1}{4}x^2 - \frac{1}{9}x^3 + \frac{1}{64}x^4 + \frac{13}{900}x^5 + \frac{55}{20736}x^6 - \frac{433}{705600}x^7 + \mathrm{O}\left(x^8\right)\right) + \left(\frac{1}{4}x^2 + \frac{2}{27}x^3 - \frac{3}{128}x^4 - \frac{253}{13500}x^5 - \frac{95}{41472}x^6 + \frac{153527}{148176000}x^7 + \mathrm{O}\left(x^8\right)\right) c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 144

```
AsymptoticDSolveValue[x*y''[x]+y'[x]+x*(1+x)*y[x]==0,y[x],{x,0,7}]
```

$$
\begin{aligned}
y(x) \to\; & c_1\left(-\frac{433x^7}{705600}+\frac{55x^6}{20736}+\frac{13x^5}{900}+\frac{x^4}{64}-\frac{x^3}{9}-\frac{x^2}{4}+1\right) \\
& + c_2\left(\frac{153527x^7}{148176000}-\frac{95x^6}{41472}-\frac{253x^5}{13500}-\frac{3x^4}{128}+\frac{2x^3}{27}+\frac{x^2}{4}\right. \\
& \left. + \left(-\frac{433x^7}{705600}+\frac{55x^6}{20736}+\frac{13x^5}{900}+\frac{x^4}{64}-\frac{x^3}{9}-\frac{x^2}{4}+1\right)\log(x)\right)
\end{aligned}
$$

## 8.4 problem 4

Internal problem ID [6998]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.11 Many-Term Recurrence Relations. Exercises page 391
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x(1+x)\,y' - \left(6x^2 - 3x + 1\right) y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*(1+x)*diff(y(x),x)-(1-3*x+6*x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x \left(1 - \frac{4}{3}x + \frac{19}{12}x^2 - \frac{7}{6}x^3 + \frac{53}{72}x^4 - \frac{116}{315}x^5 + \frac{3247}{20160}x^6 - \frac{5501}{90720}x^7 + \mathrm{O}\left(x^8\right)\right) + \frac{c_2\left(-2 - 4x + 5x^2 - \frac{44}{3}x^3 + \frac{155}{12}x^4 - \frac{331}{30}x^5 + \frac{2321}{360}x^6 - \frac{212}{63}x^7 + \mathrm{O}\left(x^8\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.149 (sec). Leaf size: 92

```
AsymptoticDSolveValue[x^2*y''[x]+x*(1+x)*y'[x]-(1-3*x+6*x^2)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{2321x^5}{720} + \frac{331x^4}{60} - \frac{155x^3}{24} + \frac{22x^2}{3} - \frac{5x}{2} + \frac{1}{x} + 2\right) + c_2\left(\frac{3247x^7}{20160} - \frac{116x^6}{315} + \frac{53x^5}{72} - \frac{7x^4}{6} + \frac{19x^3}{12} - \frac{4x^2}{3} + x\right)$$

## 8.5 problem 6

Internal problem ID [6999]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.11 Many-Term Recurrence Relations. Exercises page 391
**Problem number**: 6.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + y'x + (x^4 + 1)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 70

```
Order:=8;
dsolve(x*diff(y(x),x$2)+x*diff(y(x),x)+(1+x^4)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = {} & c_1 x\left(1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{24}x^5 + \frac{31}{1008}x^6 - \frac{47}{3528}x^7 + \mathrm{O}\left(x^8\right)\right) \\ & + c_2\left(\ln(x)\left(-x + x^2 - \frac{1}{2}x^3 + \frac{1}{6}x^4 - \frac{1}{24}x^5 + \frac{1}{24}x^6 - \frac{31}{1008}x^7 + \mathrm{O}\left(x^8\right)\right) \right. \\ & \left. + \left(1 - x + \frac{1}{4}x^3 - \frac{5}{36}x^4 - \frac{7}{1440}x^5 + \frac{49}{2400}x^6 + \frac{10847}{2116800}x^7 + \mathrm{O}\left(x^8\right)\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.087 (sec). Leaf size: 114

```
AsymptoticDSolveValue[x*y''[x]+x*y'[x]+(1+x^4)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{1}{24}x\left(x^5 - x^4 + 4x^3 - 12x^2 + 24x - 24\right)\log(x) + \frac{-153x^6 + 265x^5 - 2200x^4 + 5400x^3 - 7200x^2 + 7200}{7200}\right) + c_2\left(\frac{31x^7}{1008} - \frac{x^6}{24} + \frac{x^5}{24} - \frac{x^4}{6} + \frac{x^3}{2} - x^2 + x\right)$$

## 8.6 problem 8

Internal problem ID [7000]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.11 Many-Term Recurrence Relations. Exercises page 391
**Problem number**: 8.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(x-2)^2 y'' - 2(x-2) y' + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=8;
dsolve(x*(x-2)^2*diff(y(x),x$2)-2*(x-2)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x) c_2 + c_1)\left(1 - \frac{1}{2}x + \mathrm{O}\left(x^8\right)\right) + \left(\frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{48}x^3 - \frac{1}{192}x^4 - \frac{1}{640}x^5 - \frac{1}{1920}x^6 - \frac{1}{5376}x^7 + \mathrm{O}\left(x^8\right)\right) c_2$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 75

```
AsymptoticDSolveValue[x*(x-2)^2*y''[x]-2*(x-2)*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(-\frac{x^7}{5376} - \frac{x^6}{1920} - \frac{x^5}{640} - \frac{x^4}{192} - \frac{x^3}{48} - \frac{x^2}{8} + \frac{x}{2} + \left(1 - \frac{x}{2}\right)\log(x)\right) + c_1\left(1 - \frac{x}{2}\right)$$

## 8.7 problem 9

Internal problem ID [7001]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.11 Many-Term Recurrence Relations. Exercises page 391
**Problem number**: 9.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(x-2)^2 y'' - 2(x-2) y' + 2y = 0}$$

With the expansion point for the power series method at $x = 2$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 55

```
Order:=8;
dsolve(x*(x-2)^2*diff(y(x),x$2)-2*(x-2)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=2);
```

$$y(x) = (x-2)\left((c_2 \ln(x-2)+c_1)\left(1+\mathrm{O}\left((x-2)^8\right)\right)+\left(-\frac{1}{2}(x-2)+\frac{1}{8}(x-2)^2-\frac{1}{24}(x-2)^3 + \frac{1}{64}(x-2)^4 - \frac{1}{160}(x-2)^5 + \frac{1}{384}(x-2)^6 - \frac{1}{896}(x-2)^7 + \mathrm{O}\left((x-2)^8\right)\right) c_2\right)$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 90

```
AsymptoticDSolveValue[x*(x-2)^2*y''[x]-2*(x-2)*y'[x]+2*y[x]==0,y[x],{x,2,7}]
```

$$y(x) \to c_1(x-2) + c_2\left(\left(-\frac{1}{896}(x-2)^7 + \frac{1}{384}(x-2)^6 - \frac{1}{160}(x-2)^5 + \frac{1}{64}(x-2)^4 - \frac{1}{24}(x-2)^3 + \frac{1}{8}(x-2)^2 + \frac{2-x}{2}\right)(x-2) + (x-2)\log(x-2)\right)$$

## 8.8 problem 10

Internal problem ID [7002]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. 18.11 Many-Term Recurrence Relations. Exercises page 391
**Problem number**: 10.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + (1-x)\,y' - y(1+x) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 52

```
Order:=8;
dsolve(2*x*diff(y(x),x$2)+(1-x)*diff(y(x),x)-(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1+\frac{1}{2}x+\frac{9}{40}x^2+\frac{103}{1680}x^3+\frac{187}{13440}x^4+\frac{247}{98560}x^5+\frac{17861}{46126080}x^6 + \frac{23767}{461260800}x^7+\mathrm{O}\left(x^8\right)\right) + c_2\left(1+x+\frac{1}{2}x^2+\frac{1}{6}x^3+\frac{1}{24}x^4+\frac{1}{120}x^5+\frac{1}{720}x^6+\frac{1}{5040}x^7+\mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 75

```
AsymptoticDSolveValue[x*(x-2)^2*y''[x]-2*(x-2)*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(-\frac{x^7}{5376}-\frac{x^6}{1920}-\frac{x^5}{640}-\frac{x^4}{192}-\frac{x^3}{48}-\frac{x^2}{8}+\frac{x}{2}+\left(1-\frac{x}{2}\right)\log(x)\right)+c_1\left(1-\frac{x}{2}\right)$$

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## 9.1 problem 1

Internal problem ID [7003]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. Miscellaneous Exercises. page 394
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{xy'' - (x+2)\,y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 68

```
Order:=8;
dsolve(x*diff(y(x),x$2)-(2+x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^3\left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \frac{1}{5040}x^7 + \mathrm{O}\left(x^8\right)\right) + c_2\left(\ln\left(x\right)\left(6x^3 + 6x^4 + 3x^5 + x^6 + \frac{1}{4}x^7 + \mathrm{O}\left(x^8\right)\right) + \left(12 - 6x + 6x^2 + 11x^3 + 5x^4 + x^5 - \frac{1}{16}x^7 + \mathrm{O}\left(x^8\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.095 (sec). Leaf size: 104

```
AsymptoticDSolveValue[x*y''[x]-(2+x)*y'[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{1}{12}\left(x^3+3x^2+6x+6\right)x^3\log(x) + \frac{1}{36}\left(-x^6+9x^4+27x^3+18x^2-18x+36\right)\right) + c_2\left(\frac{x^9}{720} + \frac{x^8}{120} + \frac{x^7}{24} + \frac{x^6}{6} + \frac{x^5}{2} + x^4 + x^3\right)$$

## 9.2 problem 2

Internal problem ID [7004]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. Miscellaneous Exercises. page 394
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{xy'' - (x+2)\,y' - 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 70

```
Order:=8;
dsolve(x*diff(y(x),x$2)-(2+x)*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^3 \left(1 + \frac{5}{4}x + \frac{3}{4}x^2 + \frac{7}{24}x^3 + \frac{1}{12}x^4 + \frac{3}{160}x^5 + \frac{1}{288}x^6 + \frac{11}{20160}x^7 + \mathrm{O}\left(x^8\right)\right) + c_2 \left(\ln(x)\left(24x^3 + 30x^4 + 18x^5 + 7x^6 + 2x^7 + \mathrm{O}\left(x^8\right)\right) + \left(12 - 12x + 18x^2 + 26x^3 + x^4 - 9x^5 - 6x^6 - \frac{9}{4}x^7 + \mathrm{O}\left(x^8\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.103 (sec). Leaf size: 115

```
AsymptoticDSolveValue[x*y''[x]-(2+x)*y'[x]-2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{1}{12}\left(7x^3+18x^2+30x+24\right)x^3\log(x) + \frac{1}{36}\left(-25x^6-45x^5-27x^4+54x^3+54x^2-36x+36\right)\right) + c_2\left(\frac{x^9}{288}+\frac{3x^8}{160}+\frac{x^7}{12}+\frac{7x^6}{24}+\frac{3x^5}{4}+\frac{5x^4}{4}+x^3\right)$$

## 9.3 problem 3

Internal problem ID [7005]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. Miscellaneous Exercises. page 394
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 2x^2y' - 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+2*x^2*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^2\left(1 - x + \frac{3}{5}x^2 - \frac{4}{15}x^3 + \frac{2}{21}x^4 - \frac{1}{35}x^5 + \frac{1}{135}x^6 - \frac{8}{4725}x^7 + \mathrm{O}\left(x^8\right)\right) + \frac{c_2\left(12 - 12x + 8x^3 - 8x^4 + \frac{24}{5}x^5 - \frac{32}{15}x^6 + \frac{16}{21}x^7 + \mathrm{O}\left(x^8\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 87

```
AsymptoticDSolveValue[x^2*y''[x]+2*x^2*y'[x]-2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{8x^5}{45} + \frac{2x^4}{5} - \frac{2x^3}{3} + \frac{2x^2}{3} + \frac{1}{x} - 1\right) + c_2\left(\frac{x^8}{135} - \frac{x^7}{35} + \frac{2x^6}{21} - \frac{4x^5}{15} + \frac{3x^4}{5} - x^3 + x^2\right)$$

## 9.4 problem 4

Internal problem ID [7006]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. Miscellaneous Exercises. page 394
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' - x(2x+7)\,y' + 2(x+5)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 55

```
Order:=8;
dsolve(2*x^2*diff(y(x),x$2)-x*(2*x+7)*diff(y(x),x)+2*(x+5)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^2\left(1 + 2x + \frac{4}{3}x^2 + \frac{8}{15}x^3 + \frac{16}{105}x^4 + \frac{32}{945}x^5 + \frac{64}{10395}x^6 + \frac{128}{135135}x^7 + \mathrm{O}\left(x^8\right)\right) + c_2x^{\frac{5}{2}}\left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \frac{1}{5040}x^7 + \mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 110

```
AsymptoticDSolveValue[2*x^2*y''[x]-x*(2*x+7)*y'[x]+2*(x+5)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(\frac{128x^7}{135135} + \frac{64x^6}{10395} + \frac{32x^5}{945} + \frac{16x^4}{105} + \frac{8x^3}{15} + \frac{4x^2}{3} + 2x + 1\right)x^2 + c_1\left(\frac{x^7}{5040} + \frac{x^6}{720} + \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1\right)x^{5/2}$$

## 9.5 problem 5

Internal problem ID [7007]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. Miscellaneous Exercises. page 394
**Problem number**: 5.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+1)\,y''+2x(x^2+3)\,y'+6y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
Order:=8;
dsolve(x^2*(1+x^2)*diff(y(x),x$2)+2*x*(3+x^2)*diff(y(x),x)+6*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\frac{c_1\left(1-\frac{1}{3}x^2+\mathrm{O}\left(x^8\right)\right)x+c_2\left(1-3x^2+\mathrm{O}\left(x^8\right)\right)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 26

```
AsymptoticDSolveValue[x^2*(1+x^2)*y''[x]+2*x*(3+x^2)*y'[x]+6*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_1\left(\frac{1}{x^3}-\frac{3}{x}\right)+c_2\left(\frac{1}{x^2}-\frac{1}{3}\right)$$

## 9.6 problem 6

Internal problem ID [7008]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. Miscellaneous Exercises. page 394
**Problem number**: 6.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-x^2+1)\,y''-10y'x-18y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
Order:=8;
dsolve((1-x^2)*diff(y(x),x$2)-10*x*diff(y(x),x)-18*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(70x^6+30x^4+9x^2+1)\,y(0)+\left(x+\frac{14}{3}x^3+\frac{63}{5}x^5+\frac{132}{5}x^7\right)D(y)(0)+O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 50

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-10*x*y'[x]-18*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_2\left(\frac{132x^7}{5}+\frac{63x^5}{5}+\frac{14x^3}{3}+x\right)+c_1(70x^6+30x^4+9x^2+1)$$

## 9.7 problem 7

Internal problem ID [7009]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. Miscellaneous Exercises. page 394
**Problem number**: 7.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + (2x+1)\,y' - 3y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 40

```
Order:=8;
dsolve(2*x*diff(y(x),x$2)+(1+2*x)*diff(y(x),x)-3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1+\frac{2}{3}x+\mathrm{O}\left(x^8\right)\right) + c_2\left(1+3x+\frac{1}{2}x^2-\frac{1}{30}x^3+\frac{1}{280}x^4-\frac{1}{2520}x^5+\frac{1}{23760}x^6-\frac{1}{240240}x^7+\mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 69

```
AsymptoticDSolveValue[2*x*y''[x]+(1+2*x)*y'[x]-3*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(-\frac{x^7}{240240}+\frac{x^6}{23760}-\frac{x^5}{2520}+\frac{x^4}{280}-\frac{x^3}{30}+\frac{x^2}{2}+3x+1\right)+c_1\left(\frac{2x}{3}+1\right)\sqrt{x}$$

## 9.8 problem 8

Internal problem ID [7010]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. Miscellaneous Exercises. page 394
**Problem number**: 8.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_erf]`

$$\boxed{y'' + 2y'x - 8y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
Order:=8;
dsolve(diff(y(x),x$2)+2*x*diff(y(x),x)-8*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(\frac{4}{3}x^4 + 4x^2 + 1\right) y(0) + \left(x + x^3 + \frac{1}{10}x^5 - \frac{1}{210}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 43

```
AsymptoticDSolveValue[y''[x]+2*x*y'[x]-8*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{4x^4}{3} + 4x^2 + 1\right) + c_2\left(-\frac{x^7}{210} + \frac{x^5}{10} + x^3 + x\right)$$

## 9.9 problem 9

Internal problem ID [7011]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. Miscellaneous Exercises. page 394
**Problem number**: 9.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x(-x^2+1)\,y''-(x^2+7)\,y'+4yx=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 32

```
Order:=8;
dsolve(x*(1-x^2)*diff(y(x),x$2)-(7+x^2)*diff(y(x),x)+4*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1x^8\left(1+3x^2+6x^4+10x^6+\mathrm{O}\left(x^8\right)\right)+c_2\left(-203212800-67737600x^2+\mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 38

```
AsymptoticDSolveValue[x*(1-x^2)*y''[x]-(7+x^2)*y'[x]+4*x*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_1\left(\frac{x^2}{3}+1\right)+c_2\left(10x^{14}+6x^{12}+3x^{10}+x^8\right)$$

## 9.10 problem 10

Internal problem ID [7012]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. Miscellaneous Exercises. page 394
**Problem number**: 10.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' - x(2x+1)\,y' + (4x+1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 41

```
Order:=8;
dsolve(2*x^2*diff(y(x),x$2)-x*(1+2*x)*diff(y(x),x)+(1+4*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1-3x+\frac{1}{2}x^2+\frac{1}{30}x^3+\frac{1}{280}x^4+\frac{1}{2520}x^5+\frac{1}{23760}x^6+\frac{1}{240240}x^7+\mathrm{O}\left(x^8\right)\right) + c_2x\left(1-\frac{2}{3}x+\mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 70

```
AsymptoticDSolveValue[2*x^2*y''[x]-x*(1+2*x)*y'[x]+(1+4*x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\sqrt{x}\left(\frac{x^7}{240240}+\frac{x^6}{23760}+\frac{x^5}{2520}+\frac{x^4}{280}+\frac{x^3}{30}+\frac{x^2}{2}-3x+1\right)+c_1\left(1-\frac{2x}{3}\right)x$$

## 9.11 problem 11

Internal problem ID [7013]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. Miscellaneous Exercises. page 394
**Problem number**: 11.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' - 2x(x+2)\,y' + (x+3)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 55

```
Order:=8;
dsolve(4*x^2*diff(y(x),x$2)-2*x*(2+x)*diff(y(x),x)+(3+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x}\left(x\left(1+\frac{1}{4}x+\frac{1}{24}x^2+\frac{1}{192}x^3+\frac{1}{1920}x^4+\frac{1}{23040}x^5+\frac{1}{322560}x^6+\frac{1}{5160960}x^7 + \mathrm{O}\left(x^8\right)\right)c_1 + \left(1+\frac{1}{2}x+\frac{1}{8}x^2+\frac{1}{48}x^3+\frac{1}{384}x^4+\frac{1}{3840}x^5+\frac{1}{46080}x^6+\frac{1}{645120}x^7+\mathrm{O}\left(x^8\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.114 (sec). Leaf size: 130

```
AsymptoticDSolveValue[4*x^2*y''[x]-2*x*(2+x)*y'[x]+(3+x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 \left( \frac{x^{13/2}}{46080} + \frac{x^{11/2}}{3840} + \frac{x^{9/2}}{384} + \frac{x^{7/2}}{48} + \frac{x^{5/2}}{8} + \frac{x^{3/2}}{2} + \sqrt{x} \right) + c_2 \left( \frac{x^{15/2}}{322560} + \frac{x^{13/2}}{23040} + \frac{x^{11/2}}{1920} + \frac{x^{9/2}}{192} + \frac{x^{7/2}}{24} + \frac{x^{5/2}}{4} + x^{3/2} \right)$$

## 9.12 problem 12

Internal problem ID [7014]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. Miscellaneous Exercises. page 394
**Problem number**: 12.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(x^2+1)\,y' + (-x^2+1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)-x*(1+x^2)*diff(y(x),x)+(1-x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((\ln(x)\,c_2 + c_1)\left(1 + \frac{1}{2}x^2 + \frac{1}{8}x^4 + \frac{1}{48}x^6 + \mathrm{O}\left(x^8\right)\right) + \left(-\frac{1}{4}x^2 - \frac{3}{32}x^4 - \frac{11}{576}x^6 + \mathrm{O}\left(x^8\right)\right)c_2\right)x$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 86

```
AsymptoticDSolveValue[x^2*y''[x]-x*(1+x^2)*y'[x]+(1-x^2)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1x\left(\frac{x^6}{48} + \frac{x^4}{8} + \frac{x^2}{2} + 1\right) + c_2\left(x\left(-\frac{11x^6}{576} - \frac{3x^4}{32} - \frac{x^2}{4}\right) + x\left(\frac{x^6}{48} + \frac{x^4}{8} + \frac{x^2}{2} + 1\right)\log(x)\right)$$

## 9.13 problem 13

Internal problem ID [7015]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. Miscellaneous Exercises. page 394
**Problem number**: 13.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{2xy'' + y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 52

```
Order:=8;
dsolve(2*x*diff(y(x),x$2)+diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1 - \frac{1}{3}x + \frac{1}{30}x^2 - \frac{1}{630}x^3 + \frac{1}{22680}x^4 - \frac{1}{1247400}x^5 + \frac{1}{97297200}x^6 - \frac{1}{10216206000}x^7 + \mathrm{O}\left(x^8\right)\right) + c_2\left(1 - x + \frac{1}{6}x^2 - \frac{1}{90}x^3 + \frac{1}{2520}x^4 - \frac{1}{113400}x^5 + \frac{1}{7484400}x^6 - \frac{1}{681080400}x^7 + \mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 111

```
AsymptoticDSolveValue[2*x*y''[x]+y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\sqrt{x}\left(-\frac{x^7}{10216206000} + \frac{x^6}{97297200} - \frac{x^5}{1247400} + \frac{x^4}{22680} - \frac{x^3}{630} + \frac{x^2}{30} - \frac{x}{3} + 1\right) + c_2\left(-\frac{x^7}{681080400} + \frac{x^6}{7484400} - \frac{x^5}{113400} + \frac{x^4}{2520} - \frac{x^3}{90} + \frac{x^2}{6} - x + 1\right)$$

## 9.14 problem 14

Internal problem ID [7016]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. Miscellaneous Exercises. page 394
**Problem number**: 14.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(x^2-3)\,y' + 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 57

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*(x^2-3)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((\ln(x)\,c_2 + c_1)\left(1 - \frac{1}{2}x^2 + \frac{1}{8}x^4 - \frac{1}{48}x^6 + \mathrm{O}\left(x^8\right)\right) + \left(\frac{1}{4}x^2 - \frac{3}{32}x^4 + \frac{11}{576}x^6 + \mathrm{O}\left(x^8\right)\right)c_2\right)x^2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 92

```
AsymptoticDSolveValue[x^2*y''[x]+x*(x^2-3)*y'[x]+4*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{x^6}{48} + \frac{x^4}{8} - \frac{x^2}{2} + 1\right)x^2 + c_2\left(\left(\frac{11x^6}{576} - \frac{3x^4}{32} + \frac{x^2}{4}\right)x^2 + \left(-\frac{x^6}{48} + \frac{x^4}{8} - \frac{x^2}{2} + 1\right)x^2\log(x)\right)$$

## 9.15 problem 15

Internal problem ID [7017]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. Miscellaneous Exercises. page 394
**Problem number**: 15.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' - x^2y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 79

```
Order:=8;
dsolve(4*x^2*diff(y(x),x$2)-x^2*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(\left(\ln(x)\, c_2 + c_1\right)\left(1 + \frac{1}{8}x + \frac{3}{256}x^2 + \frac{5}{6144}x^3 + \frac{35}{786432}x^4 + \frac{21}{10485760}x^5 + \frac{77}{1006632960}x^6 + \frac{143}{56371445760}x^7 + \mathrm{O}\left(x^8\right)\right) + \left(-\frac{1}{256}x^2 - \frac{1}{2048}x^3 - \frac{19}{524288}x^4 - \frac{25}{12582912}x^5 - \frac{317}{3623878656}x^6 - \frac{469}{144955146240}x^7 + \mathrm{O}\left(x^8\right)\right) c_2\right)\sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 171

```
AsymptoticDSolveValue[4*x^2*y''[x]-x^2*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{143x^7}{56371445760}+\frac{77x^6}{1006632960}+\frac{21x^5}{10485760}+\frac{35x^4}{786432}+\frac{5x^3}{6144}+\frac{3x^2}{256}+\frac{x}{8}+1\right) + c_2\left(\sqrt{x}\left(-\frac{469x^7}{144955146240}-\frac{317x^6}{3623878656}-\frac{25x^5}{12582912}-\frac{19x^4}{524288}-\frac{x^3}{2048}-\frac{x^2}{256}\right) + \sqrt{x}\left(\frac{143x^7}{56371445760}+\frac{77x^6}{1006632960}+\frac{21x^5}{10485760}+\frac{35x^4}{786432}+\frac{5x^3}{6144}+\frac{3x^2}{256}+\frac{x}{8} + 1\right)\log(x)\right)$$

## 9.16 problem 16

Internal problem ID [7018]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. Miscellaneous Exercises. page 394
**Problem number**: 16.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x^2+1)\,y''-2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
Order:=8;
dsolve((1+x^2)*diff(y(x),x$2)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(x^2+1)\,y(0)+\left(x+\frac{1}{3}x^3-\frac{1}{15}x^5+\frac{1}{35}x^7\right)D(y)\,(0)+O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 38

```
AsymptoticDSolveValue[(1+x^2)*y''[x]-2*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_1(x^2+1)+c_2\left(\frac{x^7}{35}-\frac{x^5}{15}+\frac{x^3}{3}+x\right)$$

## 9.17 problem 17

Internal problem ID [7019]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. Miscellaneous Exercises. page 394
**Problem number**: 17.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' - x(2x+1)\,y' + (3x+1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
Order:=8;
dsolve(2*x^2*diff(y(x),x$2)-x*(1+2*x)*diff(y(x),x)+(1+3*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1 - 2x + \mathrm{O}\left(x^8\right)\right) + c_2 x\left(1 - \frac{1}{3}x - \frac{1}{30}x^2 - \frac{1}{210}x^3 - \frac{1}{1512}x^4 - \frac{1}{11880}x^5 - \frac{1}{102960}x^6 - \frac{1}{982800}x^7 + \mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 70

```
AsymptoticDSolveValue[2*x^2*y''[x]-x*(1+2*x)*y'[x]+(1+3*x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 x\left(-\frac{x^7}{982800} - \frac{x^6}{102960} - \frac{x^5}{11880} - \frac{x^4}{1512} - \frac{x^3}{210} - \frac{x^2}{30} - \frac{x}{3} + 1\right) + c_2(1 - 2x)\sqrt{x}$$

## 9.18 problem 19

Internal problem ID [7020]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. Miscellaneous Exercises. page 394
**Problem number**: 19.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 3x^2y' + (3x+1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 81

```
Order:=8;
dsolve(4*x^2*diff(y(x),x$2)+3*x^2*diff(y(x),x)+(1+3*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((\ln(x)\,c_2 + c_1)\left(1 - \frac{9}{8}x + \frac{135}{256}x^2 - \frac{315}{2048}x^3 + \frac{8505}{262144}x^4 - \frac{56133}{10485760}x^5 + \frac{243243}{335544320}x^6 - \frac{312741}{3758096384}x^7 + \mathrm{O}\left(x^8\right)\right) + \left(\frac{3}{2}x - \frac{261}{256}x^2 + \frac{729}{2048}x^3 - \frac{44091}{524288}x^4 + \frac{63099}{4194304}x^5 - \frac{1454463}{671088640}x^6 + \frac{1403811}{5368709120}x^7 + \mathrm{O}\left(x^8\right)\right)c_2\right)\sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 176

```
AsymptoticDSolveValue[4*x^2*y''[x]+3*x^2*y'[x]+(1+3*x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\sqrt{x}\left(-\frac{312741x^7}{3758096384} + \frac{243243x^6}{335544320} - \frac{56133x^5}{10485760} + \frac{8505x^4}{262144} - \frac{315x^3}{2048} + \frac{135x^2}{256} - \frac{9x}{8} + 1\right) + c_2\left(\sqrt{x}\left(\frac{1403811x^7}{5368709120} - \frac{1454463x^6}{671088640} + \frac{63099x^5}{4194304} - \frac{44091x^4}{524288} + \frac{729x^3}{2048} - \frac{261x^2}{256} + \frac{3x}{2}\right) + \sqrt{x}\left(-\frac{312741x^7}{3758096384} + \frac{243243x^6}{335544320} - \frac{56133x^5}{10485760} + \frac{8505x^4}{262144} - \frac{315x^3}{2048} + \frac{135x^2}{256} - \frac{9x}{8} + 1\right)\log(x)\right)$$

## 9.19 problem 20

Internal problem ID [7021]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. Miscellaneous Exercises. page 394
**Problem number**: 20.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (-x^2+1)\,y' + 2yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
Order:=8;
dsolve(x*diff(y(x),x$2)+(1-x^2)*diff(y(x),x)+2*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x)\,c_2 + c_1)\left(1 - \frac{1}{2}x^2 + \mathrm{O}\left(x^8\right)\right) + \left(\frac{3}{4}x^2 - \frac{1}{32}x^4 - \frac{1}{576}x^6 + \mathrm{O}\left(x^8\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 53

```
AsymptoticDSolveValue[x*y''[x]+(1-x^2)*y'[x]+2*x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(1 - \frac{x^2}{2}\right) + c_2\left(-\frac{x^6}{576} - \frac{x^4}{32} + \frac{3x^2}{4} + \left(1 - \frac{x^2}{2}\right)\log(x)\right)$$

## 9.20 problem 21

Internal problem ID [7022]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. Miscellaneous Exercises. page 394
**Problem number**: 21.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 2x^2y' - (x+3)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 55

```
Order:=8;
dsolve(4*x^2*diff(y(x),x$2)+2*x^2*diff(y(x),x)-(x+3)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^2\left(1 - \frac{1}{6}x + \frac{1}{48}x^2 - \frac{1}{480}x^3 + \frac{1}{5760}x^4 - \frac{1}{80640}x^5 + \frac{1}{1290240}x^6 - \frac{1}{23224320}x^7 + \mathrm{O}\left(x^8\right)\right) + c_2\left(-2 + x - \frac{1}{4}x^2 + \right.}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.106 (sec). Leaf size: 130

```
AsymptoticDSolveValue[4*x^2*y''[x]+2*x^2*y'[x]-(x+3)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{x^{11/2}}{46080} - \frac{x^{9/2}}{3840} + \frac{x^{7/2}}{384} - \frac{x^{5/2}}{48} + \frac{x^{3/2}}{8} - \frac{\sqrt{x}}{2} + \frac{1}{\sqrt{x}}\right) + c_2\left(\frac{x^{15/2}}{1290240} - \frac{x^{13/2}}{80640} + \frac{x^{11/2}}{5760} - \frac{x^{9/2}}{480} + \frac{x^{7/2}}{48} - \frac{x^{5/2}}{6} + x^{3/2}\right)$$

## 9.21 problem 22

Internal problem ID [7023]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. Miscellaneous Exercises. page 394
**Problem number**: 22.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x(-x^2+1)\,y''+5(-x^2+1)\,y'-4yx=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
Order:=8;
dsolve(x*(1-x^2)*diff(y(x),x$2)+5*(1-x^2)*diff(y(x),x)-4*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1\left(1+\frac{1}{3}x^2+\frac{1}{6}x^4+\frac{1}{10}x^6+\mathrm{O}\left(x^8\right)\right)+\frac{c_2\left(-144+144x^2+\mathrm{O}\left(x^8\right)\right)}{x^4}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 42

```
AsymptoticDSolveValue[x*(1-x^2)*y''[x]+5*(1-x^2)*y'[x]-4*x*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_1\left(\frac{1}{x^4}-\frac{1}{x^2}\right)+c_2\left(\frac{x^6}{10}+\frac{x^4}{6}+\frac{x^2}{3}+1\right)$$

## 9.22 problem 23

Internal problem ID [7024]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. Miscellaneous Exercises. page 394
**Problem number**: 23.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x^2y'' + x(x+3)\,y' + (2x+1)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 81

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*(3+x)*diff(y(x),x)+(1+2*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{(\ln(x)\,c_2 + c_1)\left(1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + \frac{1}{720}x^6 - \frac{1}{5040}x^7 + \mathrm{O}(x^8)\right) + \left(x - \frac{3}{4}x^2 + \frac{11}{36}x^3 - \frac{25}{288}\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 162

```
AsymptoticDSolveValue[x^2*y''[x]+x*(3+x)*y'[x]+(1+2*x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to \frac{c_1\left(-\frac{x^7}{5040}+\frac{x^6}{720}-\frac{x^5}{120}+\frac{x^4}{24}-\frac{x^3}{6}+\frac{x^2}{2}-x+1\right)}{x} + c_2\left(\frac{\frac{121x^7}{235200}-\frac{49x^6}{14400}+\frac{137x^5}{7200}-\frac{25x^4}{288}+\frac{11x^3}{36}-\frac{3x^2}{4}+x}{x} + \frac{\left(-\frac{x^7}{5040}+\frac{x^6}{720}-\frac{x^5}{120}+\frac{x^4}{24}-\frac{x^3}{6}+\frac{x^2}{2}-x+1\right)\log(x)}{x}\right)$$

## 9.23 problem 24

Internal problem ID [7025]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. Miscellaneous Exercises. page 394
**Problem number**: 24.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Bessel, _modified]]`

$$\boxed{x^2y'' + y'x - (x^2+4)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-(x^2+4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^4\left(1+\frac{1}{12}x^2+\frac{1}{384}x^4+\frac{1}{23040}x^6+O(x^8)\right)+c_2\left(\ln(x)\left(9x^4+\frac{3}{4}x^6+O(x^8)\right)+\left(-144+36x^2-\frac{1}{2}x^6+O\right.\right.}{x^2}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 74

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]-(x^2+4)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{11x^6+36x^4-576x^2+2304}{2304x^2}-\frac{1}{192}x^2\left(x^2+12\right)\log(x)\right) + c_2\left(\frac{x^8}{23040}+\frac{x^6}{384}+\frac{x^4}{12}+x^2\right)$$

## 9.24 problem 25

Internal problem ID [7026]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. Miscellaneous Exercises. page 394
**Problem number**: 25.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(1-2x)\,y''-2(x+2)\,y'+18y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 50

```
Order:=8;
dsolve(x*(1-2*x)*diff(y(x),x$2)-2*(2+x)*diff(y(x),x)+18*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1x^5\left(1+\frac{16}{3}x+\frac{144}{7}x^2+\frac{480}{7}x^3+\frac{4400}{21}x^4+\frac{4224}{7}x^5+1664x^6+\frac{13312}{3}x^7+\mathrm{O}\left(x^8\right)\right)+c_2\left(2880+12960x+34560x^2+57600x^3-483840x^5-2580480x^6-9953280x^7+\mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.128 (sec). Leaf size: 81

```
AsymptoticDSolveValue[x*(1-2*x)*y''[x]-2*(2+x)*y'[x]+18*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_1\left(-896x^6-168x^5+20x^3+12x^2+\frac{9x}{2}+1\right)+c_2\left(1664x^{11}+\frac{4224x^{10}}{7}+\frac{4400x^9}{21}+\frac{480x^8}{7}+\frac{144x^7}{7}+\frac{16x^6}{3}+x^5\right)$$

## 9.25 problem 26

Internal problem ID [7027]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. Miscellaneous Exercises. page 394
**Problem number**: 26.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{xy'' + (-x+2)\,y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 52

```
Order:=8;
dsolve(x*diff(y(x),x$2)+(2-x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\left(1+\frac{1}{2}x+\frac{1}{6}x^2+\frac{1}{24}x^3+\frac{1}{120}x^4+\frac{1}{720}x^5+\frac{1}{5040}x^6+\frac{1}{40320}x^7+\mathrm{O}\left(x^8\right)\right) + \frac{c_2\left(1+x+\frac{1}{2}x^2+\frac{1}{6}x^3+\frac{1}{24}x^4+\frac{1}{120}x^5+\frac{1}{720}x^6+\frac{1}{5040}x^7+\mathrm{O}\left(x^8\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.078 (sec). Leaf size: 90

```
AsymptoticDSolveValue[x*y''[x]+(2-x)*y'[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{x^5}{720}+\frac{x^4}{120}+\frac{x^3}{24}+\frac{x^2}{6}+\frac{x}{2}+\frac{1}{x}+1\right)+c_2\left(\frac{x^6}{5040}+\frac{x^5}{720}+\frac{x^4}{120}+\frac{x^3}{24}+\frac{x^2}{6}+\frac{x}{2}+1\right)$$

## 9.26 problem 27

Internal problem ID [7028]

**Book**: Elementary differential equations. Rainville, Bedient, Bedient. Prentice Hall. NJ. 8th edition. 1997.
**Section**: CHAPTER 18. Power series solutions. Miscellaneous Exercises. page 394
**Problem number**: 27.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 3y'x + 4y(1 + x) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 81

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((\ln(x)\, c_2 + c_1)\left(1 - 4x + 4x^2 - \frac{16}{9}x^3 + \frac{4}{9}x^4 - \frac{16}{225}x^5 + \frac{16}{2025}x^6 - \frac{64}{99225}x^7 + \mathrm{O}\left(x^8\right)\right) + \left(8x - 12x^2 + \frac{176}{27}x^3 - \frac{50}{27}x^4 + \frac{1096}{3375}x^5 - \frac{392}{10125}x^6 + \frac{3872}{1157625}x^7 + \mathrm{O}\left(x^8\right)\right) c_2\right) x^2$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 158

```
AsymptoticDSolveValue[x^2*y''[x]-3*x*y'[x]+4*(1+x)*y[x]==0,y[x],{x,0,7}]
```

$$
\begin{aligned}
y(x) \to\ & c_1\left(-\frac{64x^7}{99225}+\frac{16x^6}{2025}-\frac{16x^5}{225}+\frac{4x^4}{9}-\frac{16x^3}{9}+4x^2-4x+1\right)x^2 \\
&+c_2\left(\left(\frac{3872x^7}{1157625}-\frac{392x^6}{10125}+\frac{1096x^5}{3375}-\frac{50x^4}{27}+\frac{176x^3}{27}-12x^2+8x\right)x^2\right. \\
&\left.+\left(-\frac{64x^7}{99225}+\frac{16x^6}{2025}-\frac{16x^5}{225}+\frac{4x^4}{9}-\frac{16x^3}{9}+4x^2-4x+1\right)x^2\log(x)\right)
\end{aligned}
$$