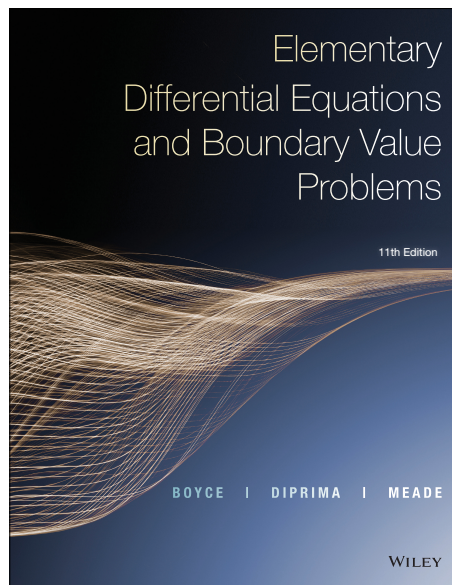


A Solution Manual For

**Elementary differential
equations and boundary value
problems, 11th ed., Boyce,
DiPrima, Meade**



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1 Chapter 4.1, Higher order linear differential equations. General theory. page 173

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1.1 problem 1

Internal problem ID [812]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 4.1, Higher order linear differential equations. General theory. page 173

Problem number: 1.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$y'''' + 4y''' + 3y = t$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 227

```
dsolve(diff(y(t),t$4)+4*diff(y(t),t$3)+3*y(t)=t,y(t), singsol=all)
```

$$y(t) = \frac{t}{3} + e^{-t}c_1 + c_2 e^{\frac{(\sqrt{2}(4+2\sqrt{2})^{\frac{2}{3}} - 2(4+2\sqrt{2})^{\frac{2}{3}} - 2(4+2\sqrt{2})^{\frac{1}{3}} - 2)t}{2}}$$
$$+ c_3 e^{-\frac{(\sqrt{2}(4+2\sqrt{2})^{\frac{2}{3}} - 2(4+2\sqrt{2})^{\frac{2}{3}} - 2(4+2\sqrt{2})^{\frac{1}{3}} + 4)t}{4}} \cos\left(\frac{\sqrt{3}(4+2\sqrt{2})^{\frac{1}{3}}\left((4+2\sqrt{2})^{\frac{1}{3}}\sqrt{2} - 2(4+2\sqrt{2})^{\frac{1}{3}} + 2\right)t}{4}\right)$$
$$+ c_4 e^{-\frac{(\sqrt{2}(4+2\sqrt{2})^{\frac{2}{3}} - 2(4+2\sqrt{2})^{\frac{2}{3}} - 2(4+2\sqrt{2})^{\frac{1}{3}} + 4)t}{4}} \sin\left(\frac{\sqrt{3}(4+2\sqrt{2})^{\frac{1}{3}}\left((4+2\sqrt{2})^{\frac{1}{3}}\sqrt{2} - 2(4+2\sqrt{2})^{\frac{1}{3}} + 2\right)t}{4}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 100

```
DSolve[y''''[t]+4*y'''[t]+3*y[t]==t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow c_2 \exp(t\text{Root}[\#1^3 + 3\#1^2 - 3\#1 + 3\&, 2]) \\ + c_3 \exp(t\text{Root}[\#1^3 + 3\#1^2 - 3\#1 + 3\&, 3]) \\ + c_1 \exp(t\text{Root}[\#1^3 + 3\#1^2 - 3\#1 + 3\&, 1]) + \frac{t}{3} + c_4 e^{-t}$$

1.2 problem 2

Internal problem ID [813]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 4.1, Higher order linear differential equations. General theory. page 173

Problem number: 2.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$t(t-1)y'''' + e^t y'' + 4t^2 y = 0$$

X Solution by Maple

```
dsolve(t*(t-1)*diff(y(t),t$4)+exp(t)*diff(y(t),t$2)+4*t^2*y(t)=0,y(t), singsol=all)
```

No solution found

X Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[t*(t-1)*y''''[t]+Exp[t]*y''[t]+4*t^2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

Not solved

1.3 problem 8

Internal problem ID [814]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 4.1, Higher order linear differential equations. General theory. page 173

Problem number: 8.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' + y'' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(t),t$4)+diff(y(t),t$2)=0,y(t), singsol=all)
```

$$y(t) = c_1 + c_2 t + c_3 \sin(t) + c_4 \cos(t)$$

✓ Solution by Mathematica

Time used: 0.098 (sec). Leaf size: 24

```
DSolve[y''''[t]+y''[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow c_4 t - c_1 \cos(t) - c_2 \sin(t) + c_3$$

1.4 problem 9

Internal problem ID [815]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 4.1, Higher order linear differential equations. General theory. page 173

Problem number: 9.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' + 2y'' - y' - 2y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(t),t$3)+2*diff(y(t),t$2)-diff(y(t),t)-2*y(t)=0,y(t), singsol=all)
```

$$y(t) = e^{-t}c_1 + e^{-2t}c_2 + c_3e^t$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 28

```
DSolve[y'''[t]+2*y''[t]-y'[t]-2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow e^{-2t}(c_2e^t + c_3e^{3t} + c_1)$$

1.5 problem 10

Internal problem ID [816]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 4.1, Higher order linear differential equations. General theory. page 173

Problem number: 10.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$xy''' - y'' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x*diff(y(x),x$3)-diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = c_2x^3 + c_3x + c_1$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 21

```
DSolve[x*y'''[x]-y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_1x^3}{6} + c_3x + c_2$$

1.6 problem 11

Internal problem ID [817]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 4.1, Higher order linear differential equations. General theory. page 173

Problem number: 11.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _homogeneous]]`

$$x^3y''' + x^2y'' - 2y'x + 2y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^3*diff(y(x),x$3)+x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + x^2c_2 + c_3x$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 22

```
DSolve[x^3*y'''[x]+x^2*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_3x^2 + c_2x + \frac{c_1}{x}$$

1.7 problem 16

Internal problem ID [818]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 4.1, Higher order linear differential equations. General theory. page 173

Problem number: 16.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' + 2y'' - y' - 3y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 181

```
dsolve(diff(y(x),x$3)+2*diff(y(x),x$2)-diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{\left(\left(188+12\sqrt{93}\right)^{\frac{2}{3}}-4\left(188+12\sqrt{93}\right)^{\frac{1}{3}}+28\right)x}{6\left(188+12\sqrt{93}\right)^{\frac{1}{3}}}} - c_2 e^{-\frac{\left(28+\left(188+12\sqrt{93}\right)^{\frac{2}{3}}+8\left(188+12\sqrt{93}\right)^{\frac{1}{3}}\right)x}{12\left(188+12\sqrt{93}\right)^{\frac{1}{3}}}} \sin\left(\frac{\left(\sqrt{3}\left(188+12\sqrt{93}\right)^{\frac{2}{3}}-28\sqrt{3}\right)x}{12\left(188+12\sqrt{93}\right)^{\frac{1}{3}}}\right) + c_3 e^{-\frac{\left(28+\left(188+12\sqrt{93}\right)^{\frac{2}{3}}+8\left(188+12\sqrt{93}\right)^{\frac{1}{3}}\right)x}{12\left(188+12\sqrt{93}\right)^{\frac{1}{3}}}} \cos\left(\frac{\left(\sqrt{3}\left(188+12\sqrt{93}\right)^{\frac{2}{3}}-28\sqrt{3}\right)x}{12\left(188+12\sqrt{93}\right)^{\frac{1}{3}}}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 87

```
DSolve[y'''[x]+2*y''[x]-y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}y(x) \rightarrow & c_2 \exp(x\text{Root}[\#1^3 + 2\#1^2 - \#1 - 3\&, 2]) \\ & + c_3 \exp(x\text{Root}[\#1^3 + 2\#1^2 - \#1 - 3\&, 3]) \\ & + c_1 \exp(x\text{Root}[\#1^3 + 2\#1^2 - \#1 - 3\&, 1])\end{aligned}$$

1.8 problem 17

Internal problem ID [819]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 4.1, Higher order linear differential equations. General theory. page 173

Problem number: 17.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$ty''' + 2y'' - y' + ty = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 183

```
dsolve(t*diff(y(t),t$3)+2*diff(y(t),t$2)-diff(y(t),t)+t*y(t)=0,y(t), singsol=all)
```

$$\begin{aligned} y(t) = & c_1 \text{KummerM} \left(\frac{1}{2} - \frac{i\sqrt{3}}{6}, 1, i\sqrt{3}t \right) e^{-\frac{t(i\sqrt{3}-1)}{2}} \\ & + c_2 \text{KummerU} \left(\frac{1}{2} - \frac{i\sqrt{3}}{6}, 1, i\sqrt{3}t \right) e^{-\frac{t(i\sqrt{3}-1)}{2}} \\ & + c_3 e^{-\frac{t(i\sqrt{3}-1)}{2}} \left(\left(\int \text{KummerU} \left(\frac{1}{2} - \frac{i\sqrt{3}}{6}, 1, i\sqrt{3}t \right) e^{-\frac{t(i\sqrt{3}+3)}{2}} dt \right) \text{KummerM} \left(\frac{1}{2} \right. \right. \\ & \qquad \qquad \qquad \left. \left. - \frac{i\sqrt{3}}{6}, 1, i\sqrt{3}t \right) - \text{KummerU} \left(\frac{1}{2} \right. \right. \\ & \qquad \qquad \qquad \left. \left. - \frac{i\sqrt{3}}{6}, 1, i\sqrt{3}t \right) \left(\int \text{KummerM} \left(\frac{1}{2} - \frac{i\sqrt{3}}{6}, 1, i\sqrt{3}t \right) e^{-\frac{t(i\sqrt{3}+3)}{2}} dt \right) \right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.639 (sec). Leaf size: 520

`DSolve[t*y''[t]+2*y'[t]-y[t]+t*y[t]==0,y[t],t,IncludeSingularSolutions -> True]`

$$\begin{aligned}
 & y(t) \\
 & \rightarrow e^{\frac{1}{2}(t-i\sqrt{3}t)} \left(c_3 \operatorname{HypergeometricU} \left(\frac{1}{6}(3-i\sqrt{3}), 1, i\sqrt{3}t \right) \int_1^t \frac{1}{(-1-i\sqrt{3}) K[1] \operatorname{Hypergeometric1F1} \left(\frac{1}{6}(3-i\sqrt{3}), 1, i\sqrt{3}t \right)} dt \right. \\
 & \quad \left. + c_3 \operatorname{LaguerreL} \left(\frac{1}{6}i(3i+\sqrt{3}), i\sqrt{3}t \right) \int_1^t \frac{1}{2ie^{\frac{1}{2}i(3i+\sqrt{3})K[2]} \operatorname{HypergeometricU} \left(\frac{1}{6}(3-i\sqrt{3}), 1, i\sqrt{3}t \right)} dt \right. \\
 & \quad \left. - \frac{1}{(-i+\sqrt{3}) K[2] \operatorname{Hypergeometric1F1} \left(\frac{1}{6}(9-i\sqrt{3}), 2, i\sqrt{3}K[2] \right) \operatorname{HypergeometricU} \left(\frac{1}{6}(3-i\sqrt{3}), 1, i\sqrt{3}t \right)} \right. \\
 & \quad \left. + c_1 \operatorname{HypergeometricU} \left(\frac{1}{6}(3-i\sqrt{3}), 1, i\sqrt{3}t \right) \right. \\
 & \quad \left. + c_2 \operatorname{LaguerreL} \left(\frac{1}{6}i(3i+\sqrt{3}), i\sqrt{3}t \right) \right)
 \end{aligned}$$

1.9 problem 20

Internal problem ID [820]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 4.1, Higher order linear differential equations. General theory. page 173

Problem number: 20.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$(2 - t)y''' + (2t - 3)y'' - ty' + y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve([(2-t)*diff(y(t),t$3)+(2*t-3)*diff(y(t),t$2)-t*diff(y(t),t)+y(t)=0,exp(t)],y(t),sing
```

$$y(t) = c_1t + c_2e^t + c_3e^t t$$

✓ Solution by Mathematica

Time used: 0.079 (sec). Leaf size: 28

```
DSolve[(2-t)*y'''[t]+(2*t-3)*y''[t]-t*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow t(c_2e^t + c_1) + (c_3 - 4c_2)e^t$$

1.10 problem 21

Internal problem ID [821]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 4.1, Higher order linear differential equations. General theory. page 173

Problem number: 21.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$t^2(t+3)y''' - 3t(t+2)y'' + 6(1+t)y' - 6y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve([t^2*(t+3)*diff(y(t),t$3)-3*t*(t+2)*diff(y(t),t$2)+6*(1+t)*diff(y(t),t)-6*y(t)=0, [t^2
```

$$y(t) = c_1 t^2 + t^3 c_2 + c_3(t+1)$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 58

```
DSolve[t^2*(t+3)*y'''[t]-3*t*(t+2)*y''[t]+6*(1+t)*y'[t]-6*y[t]==0,y[t],t,IncludeSingularSolu
```

$$y(t) \rightarrow \frac{1}{8}(2c_1(t^3 - 3t^2 + 3t + 3) - (t - 1)(4c_2(t^2 - 2t - 1) + c_3(-3t^2 + 2t + 1)))$$

2 Chapter 4.2, Higher order linear differential equations. Constant coefficients. page 180

| | | |
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| 2.3 | problem 10 | 19 |
| 2.4 | problem 11 | 20 |
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| 2.6 | problem 13 | 22 |
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2.1 problem 8

Internal problem ID [822]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 4.2, Higher order linear differential equations. Constant coefficients. page 180

Problem number: 8.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' - y'' - y' + y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)-diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-x}c_1 + e^x c_2 + c_3 e^x x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 25

```
DSolve[y'''[x]-y''[x]-y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{-x} + e^x (c_3 x + c_2)$$

2.2 problem 9

Internal problem ID [823]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 4.2, Higher order linear differential equations. Constant coefficients. page 180

Problem number: 9.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' - 3y'' + 3y' + y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 61

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)+3*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{(-2^{\frac{1}{3}}+1)x} + c_2 e^{\left(\frac{\frac{1}{2}^{\frac{1}{3}}+1}{2}\right)x} \sin\left(\frac{\sqrt{3} 2^{\frac{1}{3}} x}{2}\right) + c_3 e^{\left(\frac{\frac{1}{2}^{\frac{1}{3}}+1}{2}\right)x} \cos\left(\frac{\sqrt{3} 2^{\frac{1}{3}} x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 87

```
DSolve[y'''[x]-3*y''[x]+3*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \exp(x \text{Root}[\#1^3 - 3\#1^2 + 3\#1 + 1\&, 1]) \\ + c_2 \exp(x \text{Root}[\#1^3 - 3\#1^2 + 3\#1 + 1\&, 2]) \\ + c_3 \exp(x \text{Root}[\#1^3 - 3\#1^2 + 3\#1 + 1\&, 3])$$

2.3 problem 10

Internal problem ID [824]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 4.2, Higher order linear differential equations. Constant coefficients. page 180

Problem number: 10.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' - 4y''' + 4y'' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$4)-4*diff(y(x),x$3)+4*diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2x + c_3e^{2x} + c_4e^{2x}x$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 22

```
DSolve[y''''[x]-4*y'''[x]+4*y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x(x(c_4x + c_3) + c_2) + c_1$$

2.4 problem 11

Internal problem ID [825]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 4.2, Higher order linear differential equations. Constant coefficients. page 180

Problem number: 11.

ODE order: 6.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y^{(6)} + y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 66

```
dsolve(diff(y(x),x$6)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(x) + \cos(x) c_2 + c_3 e^{\frac{\sqrt{3}x}{2}} \sin\left(\frac{x}{2}\right) - c_4 e^{-\frac{\sqrt{3}x}{2}} \sin\left(\frac{x}{2}\right) + c_5 e^{\frac{\sqrt{3}x}{2}} \cos\left(\frac{x}{2}\right) + c_6 e^{-\frac{\sqrt{3}x}{2}} \cos\left(\frac{x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 92

```
DSolve[y''''''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-\frac{\sqrt{3}x}{2}} \left(c_1 e^{\sqrt{3}x} + c_3 \right) \cos\left(\frac{x}{2}\right) + c_2 \cos(x) + c_4 e^{-\frac{\sqrt{3}x}{2}} \sin\left(\frac{x}{2}\right) + c_6 e^{\frac{\sqrt{3}x}{2}} \sin\left(\frac{x}{2}\right) + c_5 \sin(x)$$

2.5 problem 12

Internal problem ID [826]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 4.2, Higher order linear differential equations. Constant coefficients. page 180

Problem number: 12.

ODE order: 6.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y^{(6)} - 3y'''' + 3y'' - y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x$6)-3*diff(y(x),x$4)+3*diff(y(x),x$2)-y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-x}c_1 + c_2e^{-x}x + c_3e^{-x}x^2 + c_4e^x + c_5e^xx + c_6e^xx^2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 50

```
DSolve[y''''''[x]-3*y''''[x]+3*y''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x}(x^2(c_6e^{2x} + c_3) + x(c_5e^{2x} + c_2) + c_4e^{2x} + c_1)$$

2.6 problem 13

Internal problem ID [827]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 4.2, Higher order linear differential equations. Constant coefficients. page 180

Problem number: 13.

ODE order: 6.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y^{(6)} - y'' = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$6)-diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2x + c_3e^{-x} + c_4e^x + c_5 \sin(x) + c_6 \cos(x)$$

✓ Solution by Mathematica

Time used: 0.112 (sec). Leaf size: 38

```
DSolve[y''''''[x]-y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1e^x + c_3e^{-x} + c_6x - c_2 \cos(x) - c_4 \sin(x) + c_5$$

2.7 problem 14

Internal problem ID [828]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 4.2, Higher order linear differential equations. Constant coefficients. page 180

Problem number: 14.

ODE order: 5.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y^{(5)} - 3y'''' + 3y''' - 3y'' + 2y' = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$5)-3*diff(y(x),x$4)+3*diff(y(x),x$3)-3*diff(y(x),x$2)+2*diff(y(x),x)=0,y(x)
```

$$y(x) = c_1 + c_2 e^{2x} + c_3 e^x + c_4 \sin(x) + c_5 \cos(x)$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 36

```
DSolve[y'''''[x]-3*y''''[x]+3*y'''[x]-3*y''[x]+2*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow c_3 e^x + \frac{1}{2} c_4 e^{2x} - c_2 \cos(x) + c_1 \sin(x) + c_5$$

2.8 problem 15

Internal problem ID [829]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 4.2, Higher order linear differential equations. Constant coefficients. page 180

Problem number: 15.

ODE order: 8.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y^{(8)} + 8y'''' + 16y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 65

```
dsolve(diff(y(x),x$8)+8*diff(y(x),x$4)+16*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x} \sin(x) + c_2 e^{-x} \cos(x) + c_3 e^{-x} \sin(x) x + c_4 e^{-x} \cos(x) x \\ + c_5 \sin(x) e^x + c_6 \cos(x) e^x + c_7 \sin(x) e^x x + c_8 \cos(x) e^x x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 238

```
DSolve[D[y[x] ,{x,8}]+8*y''''[x]+3*y'''[x]+16*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \rightarrow c_1 \exp(x\text{Root}[\#1^8 + 8\#1^4 + 3\#1^3 + 16\&, 1]) \\ + c_2 \exp(x\text{Root}[\#1^8 + 8\#1^4 + 3\#1^3 + 16\&, 2]) \\ + c_5 \exp(x\text{Root}[\#1^8 + 8\#1^4 + 3\#1^3 + 16\&, 5]) \\ + c_6 \exp(x\text{Root}[\#1^8 + 8\#1^4 + 3\#1^3 + 16\&, 6]) \\ + c_3 \exp(x\text{Root}[\#1^8 + 8\#1^4 + 3\#1^3 + 16\&, 3]) \\ + c_4 \exp(x\text{Root}[\#1^8 + 8\#1^4 + 3\#1^3 + 16\&, 4]) \\ + c_7 \exp(x\text{Root}[\#1^8 + 8\#1^4 + 3\#1^3 + 16\&, 7]) \\ + c_8 \exp(x\text{Root}[\#1^8 + 8\#1^4 + 3\#1^3 + 16\&, 8])$$

2.9 problem 16

Internal problem ID [830]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 4.2, Higher order linear differential equations. Constant coefficients. page 180

Problem number: 16.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' + 2y'' + y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$2)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(x) + \cos(x) c_2 + c_3 \sin(x) x + c_4 \cos(x) x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 26

```
DSolve[y''''[x]+2*y''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow (c_2 x + c_1) \cos(x) + (c_4 x + c_3) \sin(x)$$

2.10 problem 17

Internal problem ID [831]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 4.2, Higher order linear differential equations. Constant coefficients. page 180

Problem number: 17.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' + 5y'' + 6y' + 2y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$3)+5*diff(y(x),x$2)+6*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-x}c_1 + c_2e^{(\sqrt{2}-2)x} + c_3e^{-(2+\sqrt{2})x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 43

```
DSolve[y'''[x]+5*y''[x]+6*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x} \left(c_1 e^{-((1+\sqrt{2})x)} + c_2 e^{(\sqrt{2}-1)x} + c_3 \right)$$

2.11 problem 18

Internal problem ID [832]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 4.2, Higher order linear differential equations. Constant coefficients. page 180

Problem number: 18.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' - 7y''' + 6y'' + 30y' - 36y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x$4)-7*diff(y(x),x$3)+6*diff(y(x),x$2)+30*diff(y(x),x)-36*y(x)=0,y(x),sing
```

$$y(x) = c_1 e^{3x} + c_2 e^{-2x} + c_3 e^{(3+\sqrt{3})x} + c_4 e^{-(3+\sqrt{3})x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 51

```
DSolve[y''''[x]-7*y'''[x]+6*y''[x]+30*y'[x]-36*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \rightarrow c_1 e^{-((\sqrt{3}-3)x)} + c_2 e^{(3+\sqrt{3})x} + c_3 e^{-2x} + c_4 e^{3x}$$

3 Chapter 6.2, The Laplace Transform. Solution of Initial Value Problems. page 255

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3.1 problem 8

Internal problem ID [833]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.2, The Laplace Transform. Solution of Initial Value Problems. page 255

Problem number: 8.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - y' - 6y = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 16

```
dsolve([diff(y(t),t$2)-diff(y(t),t)-6*y(t)=0,y(0) = 1, D(y)(0) = -1],y(t), singsol=all)
```

$$y(t) = \frac{(e^{5t} + 4)e^{-2t}}{5}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 21

```
DSolve[{y''[t]-y'[t]-6*y[t]==0,{y[0]==1,y'[0]==-1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{5}e^{-2t}(e^{5t} + 4)$$

3.2 problem 9

Internal problem ID [834]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.2, The Laplace Transform. Solution of Initial Value Problems. page 255

Problem number: 9.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 3y' + 2y = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)+3*diff(y(t),t)+2*y(t)=0,y(0) = 1, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = 2e^{-t} - e^{-2t}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 18

```
DSolve[{y'[t]+3*y'[t]+2*y[t]==0,{y[0]==1,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \rightarrow e^{-2t}(2e^t - 1)$$

3.3 problem 10

Internal problem ID [835]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.2, The Laplace Transform. Solution of Initial Value Problems. page 255

Problem number: 10.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 2y' + 2y = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve([diff(y(t),t$2)-2*diff(y(t),t)+2*y(t)=0,y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = \sin(t) e^t$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 11

```
DSolve[{y'[t]-2*y'[t]+2*y[t]==0,{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \rightarrow e^t \sin(t)$$

3.4 problem 11

Internal problem ID [836]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.2, The Laplace Transform. Solution of Initial Value Problems. page 255

Problem number: 11.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 2y' + 4y = 0$$

With initial conditions

$$[y(0) = 2, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 27

```
dsolve([diff(y(t),t$2)-2*diff(y(t),t)+4*y(t)=0,y(0) = 2, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = -\frac{2e^t(\sqrt{3}\sin(\sqrt{3}t) - 3\cos(\sqrt{3}t))}{3}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 37

```
DSolve[{y'[t]-2*y'[t]+4*y[t]==0,{y[0]==2,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \rightarrow -\frac{2}{3}e^t(\sqrt{3}\sin(\sqrt{3}t) - 3\cos(\sqrt{3}t))$$

3.5 problem 12

Internal problem ID [837]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.2, The Laplace Transform. Solution of Initial Value Problems. page 255

Problem number: 12.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 2y' + 5y = 0$$

With initial conditions

$$[y(0) = 2, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+5*y(t)=0,y(0) = 2, D(y)(0) = -1],y(t), singsol=all)
```

$$y(t) = \frac{e^{-t}(\sin(2t) + 4 \cos(2t))}{2}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 25

```
DSolve[{y''[t]+2*y'[t]+5*y[t]==0,{y[0]==2,y'[0]==-1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{2}e^{-t}(\sin(2t) + 4 \cos(2t))$$

3.6 problem 13

Internal problem ID [838]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.2, The Laplace Transform. Solution of Initial Value Problems. page 255

Problem number: 13.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' - 4y''' + 6y'' - 4y' + y = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 1, y''(0) = 0, y'''(0) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve([diff(y(t),t$4)-4*diff(y(t),t$3)+6*diff(y(t),t$2)-4*diff(y(t),t)+y(t)=0,y(0) = 0, D(y
```

$$y(t) = \frac{e^t t(2t^2 - 3t + 3)}{3}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 23

```
DSolve[{y''''[t]-4*y'''[t]+6*y''[t]-4*y'[t]+y[t]==0,{y[0]==0,y'[0]==1,y''[0]==0,y'''[0]==1}]
```

$$y(t) \rightarrow \frac{1}{3} e^t t(2t^2 - 3t + 3)$$

3.7 problem 14

Internal problem ID [839]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.2, The Laplace Transform. Solution of Initial Value Problems. page 255

Problem number: 14.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' - 4y = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = 0, y''(0) = 1, y'''(0) = 0]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 30

```
dsolve([diff(y(t),t$4)-4*y(t)=0,y(0) = 1, D(y)(0) = 0, (D@@2)(y)(0) = 1, (D@@3)(y)(0) = 0],y
```

$$y(t) = \frac{3e^{t\sqrt{2}}}{8} + \frac{3e^{-t\sqrt{2}}}{8} + \frac{\cos(t\sqrt{2})}{4}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 43

```
DSolve[{y''''[t]-4*y[t]==0,{y[0]==1,y'[0]==0,y''[0]==1,y'''[0]==0}},y[t],t,IncludeSingularSo
```

$$y(t) \rightarrow \frac{1}{8} \left(3e^{-\sqrt{2}t} + 3e^{\sqrt{2}t} + 2 \cos(\sqrt{2}t) \right)$$

3.8 problem 15

Internal problem ID [840]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.2, The Laplace Transform. Solution of Initial Value Problems. page 255

Problem number: 15.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + \omega^2 y = \cos(2t)$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 31

```
dsolve([diff(y(t),t$2)+omega^2*y(t)=cos(2*t),y(0) = 1, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{\cos(\omega t) \omega^2 - 5 \cos(\omega t) + \cos(2t)}{\omega^2 - 4}$$

✓ Solution by Mathematica

Time used: 0.209 (sec). Leaf size: 28

```
DSolve[{y'[t]+w^2*y[t]==Cos[2*t],{y[0]==1,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{(w^2 - 5) \cos(tw) + \cos(2t)}{w^2 - 4}$$

3.9 problem 16

Internal problem ID [841]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.2, The Laplace Transform. Solution of Initial Value Problems. page 255

Problem number: 16.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 2y' + 2y = e^{-t}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$2)-2*diff(y(t),t)+2*y(t)=exp(-t),y(0) = 0, D(y)(0) = 1],y(t), singsol=all
```

$$y(t) = \frac{e^{-t}}{5} + \frac{(-\cos(t) + 7\sin(t))e^t}{5}$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 29

```
DSolve[{y''[t]-2*y'[t]+2*y[t]==Exp[-t],{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions -
```

$$y(t) \rightarrow \frac{1}{5}(e^{-t} + 7e^t \sin(t) - e^t \cos(t))$$

3.10 problem 17

Internal problem ID [842]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.2, The Laplace Transform. Solution of Initial Value Problems. page 255

Problem number: 17.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y = \begin{cases} 1 & 0 \leq t < \pi \\ 0 & \pi \leq t < \infty \end{cases}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 30

```
dsolve([diff(y(t),t$2)+4*y(t)=piecewise(0<=t and t<Pi,1,Pi<=t and t<infinity,0),y(0) = 1, D
```

$$y(t) = \begin{cases} \cos(2t) & t < 0 \\ \frac{1}{4} + \frac{3\cos(2t)}{4} & t < \pi \\ \cos(2t) & \pi \leq t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 31

```
DSolve[{y'[t]+4*y[t]==Piecewise[{{1,0<t<Pi},{0,Pi<=t<Infinity}}],{y[0]==1,y'[0]==0}],y[t],t
```

$$y(t) \rightarrow \begin{cases} \cos(2t) & t > \pi \vee t \leq 0 \\ \frac{1}{4}(3 \cos(2t) + 1) & \text{True} \end{cases}$$

3.11 problem 18

Internal problem ID [843]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.2, The Laplace Transform. Solution of Initial Value Problems. page 255

Problem number: 18.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y = \begin{cases} 1 & 0 \leq t < 1 \\ 0 & 1 \leq t < \infty \end{cases}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 38

```
dsolve([diff(y(t),t$2)+4*y(t)=piecewise(0<=t and t<1,1,1<=t and t<infinity,0),y(0) = 0, D(y)
```

$$y(t) = \frac{\begin{pmatrix} 0 & t < 0 \\ 1 - \cos(2t) & t < 1 \\ \cos(2t - 2) - \cos(2t) & 1 \leq t \end{pmatrix}}{4}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 39

```
DSolve[{y'[t]+4*y[t]==Piecewise[{{1,0<t<1},{0,1<=t<Infinity}}],{y[0]==0,y'[0]==0}],y[t],t,I
```

$$y(t) \rightarrow \begin{cases} 0 & t \leq 0 \\ \frac{\sin^2(t)}{2} & 0 < t \leq 1 \\ -\frac{1}{2} \sin(1) \sin(1 - 2t) & \text{True} \end{cases}$$

3.12 problem 19

Internal problem ID [844]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.2, The Laplace Transform. Solution of Initial Value Problems. page 255

Problem number: 19.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y = \begin{cases} t & 0 \leq t < 1 \\ -t + 2 & 1 \leq t < 2 \\ 0 & 2 \leq t < \infty \end{cases}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 59

```
dsolve([diff(y(t),t$2)+y(t)=piecewise(0<=t and t<1,t,1<=t and t<2,2-t,2<=t and t<infinity,0)
```

$$y(t) = \cos(t) + \begin{pmatrix} 0 & t < 0 \\ -\sin(t) + t & t < 1 \\ 2 \sin(t-1) - t - \sin(t) + 2 & t < 2 \\ 2 \sin(t-1) - \sin(t) - \sin(t-2) & 2 \leq t \end{pmatrix}$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 68

```
DSolve[{y'[t]+y[t]==Piecewise[{{t,0<t<1},{2-t,1<=t<2},{0,2<=t<Infinity}}],{y[0]==1,y'[0]==0}
```

$$y(t) \rightarrow \begin{cases} \cos(t) & t \leq 0 \\ \cos(t) - 4 \sin^2\left(\frac{1}{2}\right) \sin(1-t) & t > 2 \\ t + \cos(t) - \sin(t) & 0 < t \leq 1 \\ -t + \cos(t) - 2 \sin(1-t) - \sin(t) + 2 & \text{True} \end{cases}$$

4 Chapter 6.4, The Laplace Transform.
Differential equations with discontinuous forcing
functions. page 268

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4.1 problem 1

Internal problem ID [845]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.4, The Laplace Transform. Differential equations with discontinuous forcing functions. page 268

Problem number: 1.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y = \begin{cases} 1 & 0 \leq t < 3\pi \\ 0 & 3\pi \leq t < \infty \end{cases}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 32

```
dsolve([diff(y(t),t$2)+y(t)=piecewise(0<=t and t<3*Pi,1,3*Pi<=t and t<infinity,0),y(0) = 0,
```

$$y(t) = \sin(t) - \begin{pmatrix} \begin{cases} 0 & t < 0 \\ \cos(t) - 1 & 0 \leq t < 3\pi \\ 2 \cos(t) & 3\pi \leq t \end{cases} \end{pmatrix}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 34

```
DSolve[{y'[t]+y[t]==Piecewise[{{1,0<=t<3*Pi},{0,3*Pi<=t<Infinity}}],{y[0]==0,y'[0]==1}],y[t]
```

$$y(t) \rightarrow \begin{cases} \sin(t) & t \leq 0 \\ \sin(t) - 2\cos(t) & t > 3\pi \\ -\cos(t) + \sin(t) + 1 & \text{True} \end{cases}$$

4.2 problem 2

Internal problem ID [846]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.4, The Laplace Transform. Differential equations with discontinuous forcing functions. page 268

Problem number: 2.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' + 2y = \begin{cases} 1 & \pi \leq t < 2\pi \\ 0 & \text{otherwise} \end{cases}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.5 (sec). Leaf size: 76

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+2*y(t)=piecewise(Pi<=t and t<2*Pi,1,true,0),y(0) = 0,
```

$$y(t) = \begin{cases} e^{-t} \sin(t) & t < \pi \\ \frac{(\cos(t)+\sin(t))e^{\pi-t}}{2} + e^{-t} \sin(t) + \frac{1}{2} & \pi < t < 2\pi \\ \frac{e^{-t}((\cos(t)+\sin(t))e^{2\pi}+(\cos(t)+\sin(t))e^{\pi}+2\sin(t))}{2} & 2\pi \leq t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 89

```
DSolve[{y'[t]+2*y'[t]+2*y[t]==Piecewise[{{1,Pi<=t<2*Pi},{0,True}}],{y[0]==0,y'[0]==1}},y[t]
```

$$y(t) \rightarrow \begin{cases} e^{-t} \sin(t) & t \leq \pi \\ \frac{1}{2}e^{-t}(e^{\pi} \cos(t) + e^t + (2 + e^{\pi}) \sin(t)) & \pi < t \leq 2\pi \\ \frac{1}{2}e^{-t}(e^{\pi}(1 + e^{\pi}) \cos(t) + (2 + e^{\pi} + e^{2\pi}) \sin(t)) & \text{True} \end{cases}$$

4.3 problem 3

Internal problem ID [847]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.4, The Laplace Transform. Differential equations with discontinuous forcing functions. page 268

Problem number: 3.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y = \sin(t) - \text{Heaviside}(-2\pi + t) \sin(t)$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 20

```
dsolve([diff(y(t),t$2)+4*y(t)=sin(t)-Heaviside(t-2*Pi)*sin(t-2*Pi),y(0) = 0, D(y)(0) = 0],y(t))
```

$$y(t) = \frac{\sin(t) (-1 + \text{Heaviside}(-2\pi + t)) (\cos(t) - 1)}{3}$$

✓ Solution by Mathematica

Time used: 0.061 (sec). Leaf size: 27

```
DSolve[{y''[t]+4*y[t]==Sin[t]-UnitStep[t-2*Pi]*Sin[t-2*Pi],{y[0]==0,y'[0]==0}},y[t],t,IncludeS
```

$$y(t) \rightarrow \frac{2}{3} \theta(2\pi - t) \sin^2\left(\frac{t}{2}\right) \sin(t)$$

4.4 problem 4

Internal problem ID [848]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.4, The Laplace Transform. Differential equations with discontinuous forcing functions. page 268

Problem number: 4.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 3y' + 2y = \begin{cases} 1 & 0 \leq t < 10 \\ 0 & \text{otherwise} \end{cases}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.235 (sec). Leaf size: 56

```
dsolve([diff(y(t),t$2)+3*diff(y(t),t)+2*y(t)=piecewise(0<=t and t<10,1,true,0),y(0) = 0, D(y
```

$$y(t) = \frac{\begin{pmatrix} \begin{cases} 0 & t < 0 \\ 1 - 2e^{-t} + e^{-2t} & 0 \leq t < 10 \\ 2e^{10-t} - e^{-2t+20} - 2e^{-t} + e^{-2t} & 10 \leq t \end{cases} \end{pmatrix}}{2}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 61

```
DSolve[{y''[t]+3*y'[t]+2*y[t]==Piecewise[{{1,0<=t<10},{0,True}}],{y[0]==0,y'[0]==0}},y[t],t,
```

$$y(t) \rightarrow \begin{cases} 0 & t \leq 0 \\ \frac{1}{2}e^{-2t}(-1 + e^t)^2 & 0 < t \leq 10 \\ \frac{1}{2}e^{-2t}(-1 + e^{10})(-1 - e^{10} + 2e^t) & \text{True} \end{cases}$$

4.5 problem 5

Internal problem ID [849]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.4, The Laplace Transform. Differential equations with discontinuous forcing functions. page 268

Problem number: 5.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y' + \frac{5y}{4} = t - \text{Heaviside}\left(t - \frac{\pi}{2}\right) \left(t - \frac{\pi}{2}\right)$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 63

```
dsolve([diff(y(t),t$2)+diff(y(t),t)+5/4*y(t)=t-Heaviside(t-Pi/2)*(t-Pi/2),y(0) = 0, D(y)(0)
```

$$y(t) = -\frac{16}{25} - \frac{12 \text{Heaviside}\left(t - \frac{\pi}{2}\right) \left(\cos(t) + \frac{4\sin(t)}{3}\right) e^{-\frac{t}{2} + \frac{\pi}{4}}}{25} + \frac{2(8 - 10t + 5\pi) \text{Heaviside}\left(t - \frac{\pi}{2}\right)}{25} + \frac{4(4\cos(t) - 3\sin(t)) e^{-\frac{t}{2}}}{25} + \frac{4t}{5}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 96

```
DSolve[{y''[t]+y'[t]+5/4*y[t]==t-UnitStep[t-Pi/2]*(t-Pi/2),{y[0]==0,y'[0]==0}},y[t],t,Includ
```

$$y(t) \rightarrow \begin{cases} \frac{4}{25}e^{-t/2}(e^{t/2}(5t-4)+4\cos(t)-3\sin(t)) & 2t \leq \pi \\ -\frac{2}{25}e^{-t/2}((-8+6e^{\pi/4})\cos(t)+(6+8e^{\pi/4})\sin(t)-5e^{t/2}\pi) & \text{True} \end{cases}$$

4.6 problem 6

Internal problem ID [850]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.4, The Laplace Transform. Differential equations with discontinuous forcing functions. page 268

Problem number: 6.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y' + \frac{5y}{4} = \begin{cases} \sin(t) & 0 \leq t < \pi \\ 0 & \text{otherwise} \end{cases}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.453 (sec). Leaf size: 59

```
dsolve([diff(y(t),t$2)+diff(y(t),t)+5/4*y(t)=piecewise(0<=t and t<Pi,sin(t),true,0),y(0) = 0
```

$$y(t) = \frac{4 \left(\begin{cases} 0 & t < 0 \\ (4 \cos(t) + \sin(t)) e^{-\frac{t}{2}} - 4 \cos(t) + \sin(t) & t < \pi \\ (4 \cos(t) + \sin(t)) \left(e^{-\frac{t}{2}} - e^{-\frac{t}{2} + \frac{\pi}{2}} \right) & \pi \leq t \end{cases} \right)}{17}$$

✓ Solution by Mathematica

Time used: 0.129 (sec). Leaf size: 77

```
DSolve[{y''[t]+y'[t]+5/4*y[t]==Piecewise[{{Sin[t],0<=t<Pi},{0,True}}],{y[0]==0,y'[0]==0}],y[t]
```

$$y(t) \rightarrow \begin{cases} 0 & t \leq 0 \\ \frac{4}{17}((-4 + 4e^{-t/2}) \cos(t) + (1 + e^{-t/2}) \sin(t)) & 0 < t \leq \pi \\ -\frac{4}{17}e^{-t/2}(-1 + e^{\pi/2})(4 \cos(t) + \sin(t)) & \text{True} \end{cases}$$

4.7 problem 7

Internal problem ID [851]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.4, The Laplace Transform. Differential equations with discontinuous forcing functions. page 268

Problem number: 7.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y = \text{Heaviside}(-\pi + t) - \text{Heaviside}(-3\pi + t)$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve([diff(y(t),t$2)+4*y(t)=Heaviside(t-Pi)-Heaviside(t-3*Pi),y(0) = 0, D(y)(0) = 0],y(t),
```

$$y(t) = \frac{(-1 + \cos(2t))(-\text{Heaviside}(-\pi + t) + \text{Heaviside}(-3\pi + t))}{4}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 25

```
DSolve[{y'[t]+4*y[t]==UnitStep[t-Pi]-UnitStep[t-3*Pi],{y[0]==0,y'[0]==0}},y[t],t,IncludeSin
```

$$y(t) \rightarrow \begin{cases} \frac{\sin^2(t)}{2} & \pi < t \leq 3\pi \\ 0 & \text{True} \end{cases}$$

4.8 problem 8

Internal problem ID [852]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.4, The Laplace Transform. Differential equations with discontinuous forcing functions. page 268

Problem number: 8.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' + 5y'' + 4y = 1 - \text{Heaviside}(t - \pi)$$

With initial conditions

$$[y(0) = 0, y'(0) = 0, y''(0) = 0, y'''(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 27

```
dsolve([diff(y(t),t$4)+5*diff(y(t),t$2)+4*y(t)=1-Heaviside(t-Pi),y(0) = 0, D(y)(0) = 0, (D@@
```

$$y(t) = -\frac{(\cos(t) + 1)^2 \text{Heaviside}(-\pi + t)}{6} + \frac{(\cos(t) - 1)^2}{6}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 29

```
DSolve[{y''''[t]+5*y''[t]+4*y[t]==1-UnitStep[t-Pi]},{y[0]==0,y'[0]==0,y''[0]==0,y'''[0]==0}],
```

$$y(t) \rightarrow \begin{cases} \frac{2}{3} \sin^4\left(\frac{t}{2}\right) & t \leq \pi \\ -\frac{2 \cos(t)}{3} & \text{True} \end{cases}$$

4.9 problem 11(b)

Internal problem ID [853]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.4, The Laplace Transform. Differential equations with discontinuous forcing functions. page 268

Problem number: 11(b).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$u'' + \frac{u'}{4} + u = k \left(\text{Heaviside} \left(t - \frac{3}{2} \right) - \text{Heaviside} \left(t - \frac{5}{2} \right) \right)$$

With initial conditions

$$[u(0) = 0, u'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 117

```
dsolve([diff(u(t),t$2)+1/4*diff(u(t),t)+u(t)=k*(Heaviside(t-3/2)-Heaviside(t-5/2)),u(0) = 0,
```

$u(t) =$

$$k \left(\text{Heaviside} \left(t - \frac{3}{2} \right) \sqrt{7} e^{-\frac{t}{8} + \frac{3}{16}} \sin \left(\frac{3\sqrt{7}(2t-3)}{16} \right) - \text{Heaviside} \left(t - \frac{5}{2} \right) \sqrt{7} e^{-\frac{t}{8} + \frac{5}{16}} \sin \left(\frac{3\sqrt{7}(2t-5)}{16} \right) - 21 H \right)$$

✓ Solution by Mathematica

Time used: 0.163 (sec). Leaf size: 192

`DSolve[{u''[t]+1/4*u'[t]+u[t]==k*(UnitStep[t-3/2]-UnitStep[t-5/2]),{u[0]==0,u'[0]==0}},u[t],`

$u(t)$

$$\rightarrow \left\{ \begin{array}{l} -e^{\frac{3}{16}-\frac{t}{8}} \cos\left(\frac{3}{16}\sqrt{7}(3-2t)\right) k + \frac{e^{\frac{3}{16}-\frac{t}{8}} \sin\left(\frac{3}{16}\sqrt{7}(3-2t)\right) k}{3\sqrt{7}} + k \\ \frac{1}{21} e^{\frac{3}{16}-\frac{t}{8}} k \left(-21 \cos\left(\frac{3}{16}\sqrt{7}(3-2t)\right) + 21\sqrt[8]{e} \cos\left(\frac{3}{16}\sqrt{7}(5-2t)\right) + \sqrt{7} \left(\sin\left(\frac{3}{16}\sqrt{7}(3-2t)\right) - \sqrt[8]{e} \sin\right) \right) \end{array} \right.$$

4.10 problem 11(c) k=1/2

Internal problem ID [854]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.4, The Laplace Transform. Differential equations with discontinuous forcing functions. page 268

Problem number: 11(c) k=1/2.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$u'' + \frac{u'}{4} + u = \frac{\text{Heaviside}\left(t - \frac{3}{2}\right)}{2} - \frac{\text{Heaviside}\left(t - \frac{5}{2}\right)}{2}$$

With initial conditions

$$[u(0) = 0, u'(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 115

```
dsolve([diff(u(t),t$2)+1/4*diff(u(t),t)+u(t)=1/2*(Heaviside(t-3/2)-Heaviside(t-5/2)),u(0) =
```

$$u(t) = \frac{\text{Heaviside}\left(t - \frac{3}{2}\right)}{2} - \frac{\text{Heaviside}\left(t - \frac{5}{2}\right)}{2} + \frac{\text{Heaviside}\left(t - \frac{5}{2}\right) \sqrt{7} e^{-\frac{t}{8} + \frac{5}{16}} \sin\left(\frac{3\sqrt{7}(2t-5)}{16}\right)}{42} - \frac{\text{Heaviside}\left(t - \frac{3}{2}\right) \sqrt{7} e^{-\frac{t}{8} + \frac{3}{16}} \sin\left(\frac{3\sqrt{7}(2t-3)}{16}\right)}{42} + \frac{\text{Heaviside}\left(t - \frac{5}{2}\right) e^{-\frac{t}{8} + \frac{5}{16}} \cos\left(\frac{3\sqrt{7}(2t-5)}{16}\right)}{2} - \frac{\text{Heaviside}\left(t - \frac{3}{2}\right) e^{-\frac{t}{8} + \frac{3}{16}} \cos\left(\frac{3\sqrt{7}(2t-3)}{16}\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.115 (sec). Leaf size: 190

```
DSolve[{u'[t]+1/4*u'[t]+u[t]==1/2*(UnitStep[t-3/2]-UnitStep[t-5/2]),{u[0]==0,u'[0]==0}},u[t]
```

$u(t)$

$$\rightarrow \left\{ \begin{array}{l} \frac{1}{42} \left(-21 e^{\frac{3}{16} - \frac{t}{8}} \cos \left(\frac{3}{16} \sqrt{7} (3 - 2t) \right) + \sqrt{7} e^{\frac{3}{16} - \frac{t}{8}} \sin \left(\frac{3}{16} \sqrt{7} (3 - 2t) \right) + 21 \right) \\ \frac{1}{42} e^{\frac{3}{16} - \frac{t}{8}} \left(-21 \cos \left(\frac{3}{16} \sqrt{7} (3 - 2t) \right) + 21 \sqrt[8]{e} \cos \left(\frac{3}{16} \sqrt{7} (5 - 2t) \right) + \sqrt{7} \left(\sin \left(\frac{3}{16} \sqrt{7} (3 - 2t) \right) - \sqrt[8]{e} \sin \left(\frac{3}{16} \sqrt{7} (5 - 2t) \right) \right) \right) \end{array} \right.$$

4.11 problem 12

Internal problem ID [855]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.4, The Laplace Transform. Differential equations with discontinuous forcing functions. page 268

Problem number: 12.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$u'' + \frac{u'}{4} + u = \frac{\text{Heaviside}(t - 5)(t - 5) - \text{Heaviside}(t - 5 - k)(t - 5 - k)}{k}$$

With initial conditions

$$[u(0) = 0, u'(0) = 0]$$

✓ Solution by Maple

Time used: 0.453 (sec). Leaf size: 182

```
dsolve([diff(u(t), t$2)+1/4*diff(u(t), t)+u(t)=1/k*(Heaviside(t-5)*(t-5)-Heaviside(t-(5+k))*(t-(5+k))), u(0)=0, D[1]u(0)=0])
```

$u(t)$

$$= \frac{-31(\text{Heaviside}(5 + k) + \text{Heaviside}(t - 5 - k) - 1) \left(\left(\left(\sin\left(\frac{15\sqrt{7}}{8}\right) \sqrt{7} + \frac{21 \cos\left(\frac{15\sqrt{7}}{8}\right)}{31} \right) \cos\left(\frac{3\sqrt{7}k}{8}\right) + \sin\left(\frac{3\sqrt{7}k}{8}\right) \right) \right)}{k^2}$$

✓ Solution by Mathematica

Time used: 13.449 (sec). Leaf size: 486

`DSolve[{u''[t]+1/4*u'[t]+u[t]==1/k*(UnitStep[t-5]*(t-5)-UnitStep[t-(5+k)]*(t-(5+k))),{u[0]=`

$u(t)$

$$\rightarrow \frac{e^{-t/8} \left(21e^{\frac{k+5}{8}} \cos\left(\frac{3}{8}\sqrt{7}(k-t+5)\right) - 84k \cos\left(\frac{3\sqrt{7}t}{8}\right) - 441 \cos\left(\frac{3\sqrt{7}t}{8}\right) + 31\sqrt{7}e^{\frac{k+5}{8}} \sin\left(\frac{3}{8}\sqrt{7}(k-t+5)\right) - 4\sqrt{7}k \sin\left(\frac{3\sqrt{7}t}{8}\right) + 11\sqrt{7} \sin\left(\frac{3\sqrt{7}t}{8}\right) \right)}{\dots}$$

$u(t)$

$$\rightarrow \frac{e^{-t/8} \left(\left(3\sqrt{7}e^{t/8}(4t-21) + 3\sqrt{7}e^{5/8} \cos\left(\frac{3}{8}\sqrt{7}(t-5)\right) - 31e^{5/8} \sin\left(\frac{3}{8}\sqrt{7}(t-5)\right) \right) \theta(t-5) - \left(3\sqrt{7}e^{t/8}(-4k+4t-21) + 3\sqrt{7}e^{\frac{k+5}{8}} \cos\left(\frac{3}{8}\sqrt{7}(k-t+5)\right) \right) \theta(t-(5+k)) \right)}{12\sqrt{7}k}$$

5 Chapter 6.5, The Laplace Transform. Impulse functions. page 273

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5.1 problem 1

Internal problem ID [856]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.5, The Laplace Transform. Impulse functions. page 273

Problem number: 1.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' + 2y = \delta(-\pi + t)$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 31

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+2*y(t)=Dirac(t-Pi),y(0) = 1, D(y)(0) = 0],y(t), singso
```

$$y(t) = -\sin(t) \operatorname{Heaviside}(-\pi + t) e^{\pi-t} + e^{-t}(\cos(t) + \sin(t))$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 29

```
DSolve[{y'[t]+2*y'[t]+2*y[t]==DiracDelta[t-Pi],{y[0]==1,y'[0]==0}},y[t],t,IncludeSingularSo
```

$$y(t) \rightarrow e^{-t}(-e^{\pi}\theta(t - \pi) \sin(t) + \sin(t) + \cos(t))$$

5.2 problem 2

Internal problem ID [857]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.5, The Laplace Transform. Impulse functions. page 273

Problem number: 2.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y = \delta(-\pi + t) - (\delta(-2\pi + t))$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve([diff(y(t),t$2)+4*y(t)=Dirac(t-Pi)-Dirac(t-2*Pi),y(0) = 0, D(y)(0) = 0],y(t), singsol
```

$$y(t) = -\frac{(\text{Heaviside}(-2\pi + t) - \text{Heaviside}(-\pi + t)) \sin(2t)}{2}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 26

```
DSolve[{y''[t]+4*y[t]==DiracDelta[t-Pi]-DiracDelta[t-2*Pi],{y[0]==0,y'[0]==0}},y[t],t,Includ
```

$$y(t) \rightarrow (\theta(t - 2\pi) - \theta(t - \pi)) \sin(t)(-\cos(t))$$

5.3 problem 3

Internal problem ID [858]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.5, The Laplace Transform. Impulse functions. page 273

Problem number: 3.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 3y' + 2y = \delta(t - 5) + \text{Heaviside}(-10 + t)$$

With initial conditions

$$\left[y(0) = 0, y'(0) = \frac{1}{2} \right]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 70

```
dsolve([diff(y(t),t$2)+3*diff(y(t),t)+2*y(t)=Dirac(t-5)+Heaviside(t-10),y(0) = 0, D(y)(0) =
```

$$y(t) = -\frac{e^{-2t}}{2} - \text{Heaviside}(t - 5) e^{-2t+10} + \text{Heaviside}(t - 5) e^{-t+5} + \frac{\text{Heaviside}(-10 + t)}{2} \\ - \text{Heaviside}(-10 + t) e^{10-t} + \frac{\text{Heaviside}(-10 + t) e^{-2t+20}}{2} + \frac{e^{-t}}{2}$$

✓ Solution by Mathematica

Time used: 0.226 (sec). Leaf size: 71

```
DSolve[{y'[t]+3*y'[t]+2*y[t]==DiracDelta[t-5]+UnitStep[t-10],{y[0]==0,y'[0]==1/2}},y[t],t,I
```

$$y(t) \rightarrow \frac{1}{2} e^{-2t} \left(2e^5 (e^t - e^5) \theta(t - 5) + (e^{10} - e^t)^2 (-\theta(10 - t)) + e^t + e^{2t} - 2e^{t+10} + e^{20} - 1 \right)$$

5.4 problem 4

Internal problem ID [859]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.5, The Laplace Transform. Impulse functions. page 273

Problem number: 4.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' + 3y = \sin(t) + \delta(-3\pi + t)$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 54

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+3*y(t)=sin(t)+Dirac(t-3*Pi),y(0) = 0, D(y)(0) = 0],y(t)
```

$$y(t) = \frac{e^{-t} \cos(t\sqrt{2})}{4} + \frac{\sqrt{2} e^{3\pi-t} \text{Heaviside}(-3\pi + t) \sin(\sqrt{2}(-3\pi + t))}{2} + \frac{\sin(t)}{4} - \frac{\cos(t)}{4}$$

✓ Solution by Mathematica

Time used: 1.726 (sec). Leaf size: 82

```
DSolve[{y''[t]+2*y'[t]+3*y[t]==Sin[t]+DiracDelta[t-3*Pi],{y[0]==0,y'[0]==1/2}},y[t],t,Includ
```

$$y(t) \rightarrow \frac{1}{4}e^{-t} \left(-2\sqrt{2}e^{3\pi} \theta(t-3\pi) \sin(\sqrt{2}(3\pi-t)) + e^t \sin(t) + \sqrt{2} \sin(\sqrt{2}t) - e^t \cos(t) + \cos(\sqrt{2}t) \right)$$

5.5 problem 5

Internal problem ID [860]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.5, The Laplace Transform. Impulse functions. page 273

Problem number: 5.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y = \cos(t) (\delta(-2\pi + t))$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve([diff(y(t),t$2)+y(t)=Dirac(t-2*Pi)*cos(t),y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = \sin(t) (\text{Heaviside}(-2\pi + t) + 1)$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 16

```
DSolve[{y'[t]+y[t]==DiracDelta[t-2*Pi]*Cos[t],{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSol
```

$$y(t) \rightarrow (\theta(t - 2\pi) + 1) \sin(t)$$

5.6 problem 6

Internal problem ID [861]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.5, The Laplace Transform. Impulse functions. page 273

Problem number: 6.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y = 2\left(\delta\left(t - \frac{\pi}{4}\right)\right)$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve([diff(y(t),t$2)+4*y(t)=2*Dirac(t-Pi/4),y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = -\text{Heaviside}\left(t - \frac{\pi}{4}\right) \cos(2t)$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 28

```
DSolve[{y'[t]+4*y[t]==2*DiracDelta[t-Pi/4],{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSoluti
```

$$y(t) \rightarrow \frac{1}{2}(\sin(2t) - 2\theta(4t - \pi) \cos(2t))$$

5.7 problem 7

Internal problem ID [862]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.5, The Laplace Transform. Impulse functions. page 273

Problem number: 7.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' + 2y = \cos(t) + \delta\left(t - \frac{\pi}{2}\right)$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 47

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+2*y(t)=cos(t)+Dirac(t-Pi/2),y(0) = 0, D(y)(0) = 0],y(t)
```

$$y(t) = -\cos(t) \operatorname{Heaviside}\left(t - \frac{\pi}{2}\right) e^{-t+\frac{\pi}{2}} + \frac{(-\cos(t) - 3\sin(t))e^{-t}}{5} + \frac{\cos(t)}{5} + \frac{2\sin(t)}{5}$$

✓ Solution by Mathematica

Time used: 0.176 (sec). Leaf size: 52

```
DSolve[{y''[t]+2*y'[t]+2*y[t]==Cos[t]+DiracDelta[t-Pi/2],{y[0]==0,y'[0]==0}},y[t],t,IncludeS
```

$$y(t) \rightarrow \frac{1}{5}e^{-t}(-5e^{\pi/2}\theta(2t - \pi)\cos(t) + (2e^t - 3)\sin(t) + (e^t - 1)\cos(t))$$

5.8 problem 8

Internal problem ID [863]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.5, The Laplace Transform. Impulse functions. page 273

Problem number: 8.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - y = \delta(t - 1)$$

With initial conditions

$$[y(0) = 0, y'(0) = 0, y''(0) = 0, y'''(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 29

```
dsolve([diff(y(t),t$4)-y(t)=Dirac(t-1),y(0) = 0, D(y)(0) = 0, (D@@2)(y)(0) = 0, (D@@3)(y)(0) = 0],y(t),t,Includ
```

$$y(t) = \frac{\text{Heaviside}(t - 1) (e^{t-1} - e^{-t+1} - 2 \sin(t - 1))}{4}$$

✓ Solution by Mathematica

Time used: 0.11 (sec). Leaf size: 44

```
DSolve[{y''''[t]-y[t]==DiracDelta[t-1],{y[0]==0,y'[0]==0,y''[0]==0,y'''[0]==0}},y[t],t,Includ
```

$$y(t) \rightarrow \frac{1}{4} e^{-t-1} \theta(t - 1) (e^{2t} + 2e^{t+1} \sin(1 - t) - e^2)$$

5.9 problem 10(a)

Internal problem ID [864]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.5, The Laplace Transform. Impulse functions. page 273

Problem number: 10(a).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + \frac{y'}{2} + y = \delta(t - 1)$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 28

```
dsolve([diff(y(t),t$2)+1/2*diff(y(t),t)+y(t)=Dirac(t-1),y(0) = 0, D(y)(0) = 0],y(t), singsol
```

$$y(t) = \frac{4 \operatorname{Heaviside}(t - 1) e^{\frac{1}{4} - \frac{t}{4}} \sqrt{15} \sin\left(\frac{\sqrt{15}(t-1)}{4}\right)}{15}$$

✓ Solution by Mathematica

Time used: 0.097 (sec). Leaf size: 40

```
DSolve[{y''[t]+1/2*y'[t]+y[t]==DiracDelta[t-1],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSol
```

$$y(t) \rightarrow \frac{4e^{\frac{1}{4} - \frac{t}{4}} \theta(t - 1) \sin\left(\frac{1}{4}\sqrt{15}(t - 1)\right)}{\sqrt{15}}$$

5.10 problem 10(c)

Internal problem ID [865]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.5, The Laplace Transform. Impulse functions. page 273

Problem number: 10(c).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + \frac{y'}{4} + y = \delta(t - 1)$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve([diff(y(t),t$2)+1/4*diff(y(t),t)+y(t)=Dirac(t-1),y(0) = 0, D(y)(0) = 0],y(t), singsol
```

$$y(t) = \frac{8 \operatorname{Heaviside}(t - 1) e^{\frac{1}{8} - \frac{t}{8}} \sqrt{7} \sin\left(\frac{3\sqrt{7}(t-1)}{8}\right)}{21}$$

✓ Solution by Mathematica

Time used: 0.075 (sec). Leaf size: 42

```
DSolve[{y''[t]+1/4*y'[t]+y[t]==DiracDelta[t-1],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSol
```

$$y(t) \rightarrow \frac{8e^{\frac{1}{8} - \frac{t}{8}} \theta(t - 1) \sin\left(\frac{3}{8}\sqrt{7}(t - 1)\right)}{3\sqrt{7}}$$

5.11 problem 12

Internal problem ID [866]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.5, The Laplace Transform. Impulse functions. page 273

Problem number: 12.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y = \frac{\text{Heaviside}(t - 4 + k) - \text{Heaviside}(t - 4 - k)}{2k}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 83

```
dsolve([diff(y(t),t$2)+y(t)=1/(2*k)*(Heaviside(t-(4-k)) - Heaviside(t-(4+k)) ),y(0) = 0, D
```

$$y(t) = \frac{(\text{Heaviside}(4 + k) + \text{Heaviside}(t - 4 - k) - 1) \cos(-t + 4 + k) - \text{Heaviside}(t - 4 - k) + (-\cos(t - 4 - k))}{2k}$$

✓ Solution by Mathematica

Time used: 1.204 (sec). Leaf size: 181

```
DSolve[{y'[t]+y[t]==1/(2*k)*(UnitStep[t-(4-k)] - UnitStep[t-(4+k)] ),{y[0]==0,y'[0]==0}},y
```

$$y(t) \rightarrow \frac{(\cos(k-t+4)-1)\theta(-k+t-4)-(\cos(-k-t+4)-1)\theta(k+t-4)}{2k} \text{ if } -4 < k < 4$$

$$y(t) \rightarrow \frac{\cos(-k-t+4)-\cos(t)+(\cos(k-t+4)-1)\theta(-k+t-4)-(\cos(-k-t+4)-1)\theta(k+t-4)}{2k} \text{ if } k > 4$$

$$y(t) \rightarrow \frac{-\cos(k-t+4)+\cos(t)+(\cos(k-t+4)-1)\theta(-k+t-4)-(\cos(-k-t+4)-1)\theta(k+t-4)}{2k} \text{ if } k < -4$$

5.12 problem 19(a)

Internal problem ID [867]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.5, The Laplace Transform. Impulse functions. page 273

Problem number: 19(a).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' + 2y = f(t)$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 43

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+2*y(t)=f(t),y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \left(\sin(t) \left(\int_0^t f(_z1) \cos(_z1) e^{-z1} d_z1 \right) - \cos(t) \left(\int_0^t f(_z1) \sin(_z1) e^{-z1} d_z1 \right) \right) e^{-t}$$

✓ Solution by Mathematica

Time used: 0.104 (sec). Leaf size: 99

```
DSolve[{y'[t]+2*y'[t]+2*y[t]==f[t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -> T
```

$$y(t) \rightarrow e^{-t} \left(-\sin(t) \int_1^0 e^{K[1]} \cos(K[1]) f(K[1]) dK[1] \right. \\ \left. + \sin(t) \int_1^t e^{K[1]} \cos(K[1]) f(K[1]) dK[1] + \cos(t) \left(\int_1^t \right. \right. \\ \left. \left. -e^{K[2]} f(K[2]) \sin(K[2]) dK[2] - \int_1^0 -e^{K[2]} f(K[2]) \sin(K[2]) dK[2] \right) \right)$$

5.13 problem 19(b)

Internal problem ID [868]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade

Section: Chapter 6.5, The Laplace Transform. Impulse functions. page 273

Problem number: 19(b).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' + 2y = \delta(-\pi + t)$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+2*y(t)=Dirac(t-Pi),y(0) = 0, D(y)(0) = 0],y(t), singularso
```

$$y(t) = -\sin(t) \operatorname{Heaviside}(-\pi + t) e^{\pi - t}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 22

```
DSolve[{y'[t]+2*y'[t]+2*y[t]==DiracDelta[t-Pi],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSo
```

$$y(t) \rightarrow -e^{\pi - t} \theta(t - \pi) \sin(t)$$