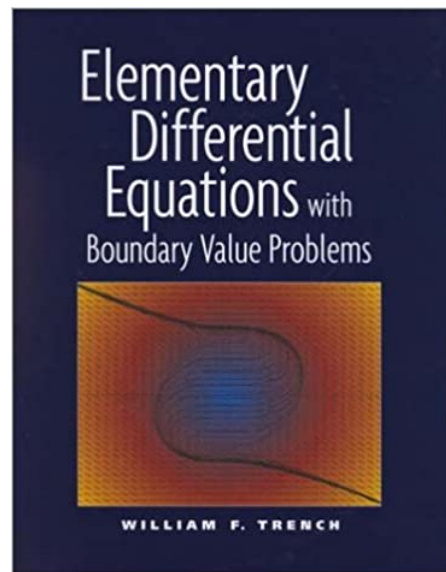


A Solution Manual For

**Elementary differential
equations with boundary value
problems. William F. Trench.
Brooks/Cole 2001**



Nasser M. Abbasi

March 3, 2024

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1.1 problem 2(a)

Internal problem ID [869]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 1, Introduction. Section 1.2 Page 14

Problem number: 2(a).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' - 2y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x) = 2*y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^{2x}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 16

```
DSolve[y'[x]== y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^x$$

$$y(x) \rightarrow 0$$

1.2 problem 2(b)

Internal problem ID [870]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 1, Introduction. Section 1.2 Page 14

Problem number: 2(b).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y + y'x = x^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x) +y(x)= x^2,y(x), singsol=all)
```

$$y(x) = \frac{\frac{x^3}{3} + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 19

```
DSolve[x*y'[x] +y[x]== x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x^2}{3} + \frac{c_1}{x}$$

1.3 problem 2(c)

Internal problem ID [871]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 1, Introduction. Section 1.2 Page 14

Problem number: 2(c).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' + 2yx = x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve(diff(y(x),x) +2*x*y(x)= x,y(x), singsol=all)
```

$$y(x) = \frac{1}{2} + e^{-x^2} c_1$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 26

```
DSolve[y'[x] +2*x*y[x]== x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2} + c_1 e^{-x^2}$$

$$y(x) \rightarrow \frac{1}{2}$$

1.4 problem 2(d)

Internal problem ID [872]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 1, Introduction. Section 1.2 Page 14

Problem number: 2(d).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$2y' + x(-1 + y^2) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(2*diff(y(x),x) +x*(y(x)^2-1)= 0,y(x), singsol=all)
```

$$y(x) = \tanh\left(\frac{x^2}{4} + \frac{c_1}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.242 (sec). Leaf size: 52

```
DSolve[2*y'[x] +x*(y[x]^2-1)== 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^{\frac{x^2}{2}} - e^{2c_1}}{e^{\frac{x^2}{2}} + e^{2c_1}}$$

$$y(x) \rightarrow -1$$

$$y(x) \rightarrow 1$$

1.5 problem 2(e)

Internal problem ID [873]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 1, Introduction. Section 1.2 Page 14

Problem number: 2(e).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' - (1 + y^2) x^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x) = x^2*(1+y(x)^2),y(x), singsol=all)
```

$$y(x) = \tan\left(\frac{x^3}{3} + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.171 (sec). Leaf size: 30

```
DSolve[y'[x] == x^2*(1+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \tan\left(\frac{x^3}{3} + c_1\right)$$

$$y(x) \rightarrow -i$$

$$y(x) \rightarrow i$$

1.6 problem 3(a)

Internal problem ID [874]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 1, Introduction. Section 1.2 Page 14

Problem number: 3(a).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [quadrature]

$$y' = -x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x) = -x,y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{2} + c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 15

```
DSolve[y'[x] == -x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{x^2}{2} + c_1$$

1.7 problem 3(b)

Internal problem ID [875]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 1, Introduction. Section 1.2 Page 14

Problem number: 3(b).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' = -\sin(x)x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x) = -x*sin(x),y(x), singsol=all)
```

$$y(x) = -\sin(x) + \cos(x)x + c_1$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 16

```
DSolve[y'[x] == -x*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\sin(x) + x \cos(x) + c_1$$

1.8 problem 3(c)

Internal problem ID [876]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 1, Introduction. Section 1.2 Page 14

Problem number: 3(c).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' = x \ln(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x) = x*ln(x),y(x), singsol=all)
```

$$y(x) = \frac{\ln(x) x^2}{2} - \frac{x^2}{4} + c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 24

```
DSolve[y'[x] == x*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{x^2}{4} + \frac{1}{2}x^2 \log(x) + c_1$$

1.9 problem 4(a)

Internal problem ID [877]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 1, Introduction. Section 1.2 Page 14

Problem number: 4(a).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' = -e^x x$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 11

```
dsolve([diff(y(x),x) = -x*exp(x),y(0) = 1],y(x), singsol=all)
```

$$y(x) = -(x - 1)e^x$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 13

```
DSolve[{y'[x] == -x*Exp[x],y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -e^x(x - 1)$$

1.10 problem 4(b)

Internal problem ID [878]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 1, Introduction. Section 1.2 Page 14

Problem number: 4(b).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' = x \sin(x^2)$$

With initial conditions

$$\left[y\left(\frac{\sqrt{2}\sqrt{\pi}}{2}\right) = 1 \right]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 12

```
dsolve([diff(y(x),x) = x*sin(x^2),y(1/2*2^(1/2)*Pi^(1/2)) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{\cos(x^2)}{2} + 1$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 15

```
DSolve[{y'[x] == x*Sin[x^2],y[Sqrt[Pi/2]]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 1 - \frac{\cos(x^2)}{2}$$

1.11 problem 4(c)

Internal problem ID [879]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 1, Introduction. Section 1.2 Page 14

Problem number: 4(c).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' = \tan(x)$$

With initial conditions

$$\left[y\left(\frac{\pi}{4}\right) = 3 \right]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve([diff(y(x),x) = tan(x),y(1/4*Pi) = 3],y(x), singsol=all)
```

$$y(x) = -\ln(\cos(x)) + 3 - \frac{\ln(2)}{2}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 18

```
DSolve[{y'[x] == Tan[x],y[Pi/4]==3},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\log(\cos(x)) + 3 - \frac{\log(2)}{2}$$

1.12 problem 5(a)

Internal problem ID [880]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 1, Introduction. Section 1.2 Page 14

Problem number: 5(a).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y' + \tan(x)y = \cos(x)$$

With initial conditions

$$\left[y\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}\pi}{8} \right]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve([diff(y(x),x) = cos(x)-y(x)*tan(x),y(1/4*Pi) = 1/8*2^(1/2)*Pi],y(x), singsol=all)
```

$$y(x) = \cos(x)x$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 9

```
DSolve[{y'[x] ==Cos[x]-y[x]*Tan[x],y[Pi/4]==Pi/(4*Sqrt[2])},y[x],x,IncludeSingularSolutions
```

$$y(x) \rightarrow x \cos(x)$$

1.13 problem 5(b)

Internal problem ID [881]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 1, Introduction. Section 1.2 Page 14

Problem number: 5(b).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y' - \frac{x^2 - 2x^2y + 2}{x^3} = 0$$

With initial conditions

$$\left[y(1) = \frac{3}{2} \right]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve([diff(y(x),x) = (x^2-2*x^2*y(x)+2)/x^3,y(1) = 3/2],y(x), singsol=all)
```

$$y(x) = \frac{\frac{x^2}{2} + 2 \ln(x) + 1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 19

```
DSolve[{y'[x] ==(x^2-2*x^2*y[x]+2)/x^3,y[1]==3/2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{x^2} + \frac{2 \log(x)}{x^2} + \frac{1}{2}$$

1.14 problem 5(c)

Internal problem ID [882]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 1, Introduction. Section 1.2 Page 14

Problem number: 5(c).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [`_separable`]

$$y' - x(1 + y^2) = 0$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 10

```
dsolve([diff(y(x),x) = x*(1+y(x)^2),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \tan\left(\frac{x^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.161 (sec). Leaf size: 13

```
DSolve[{y'[x] ==x*(1+y[x]^2),y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \tan\left(\frac{x^2}{2}\right)$$

1.15 problem 5(d)

Internal problem ID [883]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 1, Introduction. Section 1.2 Page 14

Problem number: 5(d).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' + \frac{y(1+y)}{x} = 0$$

With initial conditions

$$[y(1) = -2]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 11

```
dsolve([diff(y(x),x) = (- y(x)*(y(x)+1))/x,y(1) = -2],y(x), singsol=all)
```

$$y(x) = \frac{2}{-2+x}$$

✓ Solution by Mathematica

Time used: 0.224 (sec). Leaf size: 12

```
DSolve[{y'[x] ==(- y[x]*(y[x]+1))/x,y[1]==-2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{2}{x-2}$$

1.16 problem 8(a)

Internal problem ID [884]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 1, Introduction. Section 1.2 Page 14

Problem number: 8(a).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [quadrature]

$$y' - ay^{\frac{a-1}{a}} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 9

```
dsolve(diff(y(x),x) = a*y(x)^( (a-1)/a),y(x), singsol=all)
```

$$y(x) = (c_1 + x)^a$$

✓ Solution by Mathematica

Time used: 0.843 (sec). Leaf size: 28

```
DSolve[y'[x] ==a*y[x]^( (a-1)/a),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \left(x + \frac{c_1}{a}\right)^a$$

$$y(x) \rightarrow 0^{\frac{a}{a-1}}$$

1.17 problem 9

Internal problem ID [885]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 1, Introduction. Section 1.2 Page 14

Problem number: 9.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$y' - |y| = 1$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 19

```
dsolve([diff(y(x),x) = abs(y(x))+1,y(0) = 0],y(x), singsol=all)
```

$$y(x) = e^x - 1$$

$$y(x) = 1 - e^{-x}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'[x] ==Abs[y[x]]+1,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

```
{}
```

1.18 problem 10(a)

Internal problem ID [886]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 1, Introduction. Section 1.2 Page 14

Problem number: 10(a).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$y' - \frac{\sqrt{x^2 + 4x + 4y}}{2} = -1 - \frac{x}{2}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve(diff(y(x),x) = 1/2*(-(x+2)+sqrt(x^2+4*x+4*y(x))),y(x), singsol=all)
```

$$x - 2\sqrt{y(x) + \frac{x^2}{4}} + x - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.801 (sec). Leaf size: 47

```
DSolve[y' [x] ==1/2*(-(x+2)+Sqrt[x^2+4*x+4*y[x]]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{4}(-2x + 2e^{c_1}(x + 1) + 1 + e^{2c_1})$$

$$y(x) \rightarrow 1$$

$$y(x) \rightarrow \frac{1}{4}(1 - 2x)$$

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2.1 problem 1

Internal problem ID [887]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 1.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' + ay = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x) + a*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-ax}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 19

```
DSolve[y'[x] + a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{-ax}$$

$$y(x) \rightarrow 0$$

2.2 problem 2

Internal problem ID [888]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 2.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' + 3x^2y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x) + 3*x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x^3}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 20

```
DSolve[y'[x] + 3*x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{-x^3}$$

$$y(x) \rightarrow 0$$

2.3 problem 3

Internal problem ID [889]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 3.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y'x + \ln(x)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x*diff(y(x),x) + ln(x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{\ln(x)^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 23

```
DSolve[x*y'[x] + Log[x]*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{-\frac{1}{2} \log^2(x)}$$

$$y(x) \rightarrow 0$$

2.4 problem 4

Internal problem ID [890]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 4.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$3y + y'x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(x*diff(y(x),x) + 3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^3}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 16

```
DSolve[x*y'[x] + 3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_1}{x^3}$$

$$y(x) \rightarrow 0$$

2.5 problem 5

Internal problem ID [891]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 5.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y'x^2 + y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(x^2*diff(y(x),x) + y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{1}{x}}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 18

```
DSolve[x^2*y'[x] + y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{\frac{1}{x}}$$

$$y(x) \rightarrow 0$$

2.6 problem 6

Internal problem ID [892]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 6.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' + \frac{(x+1)y}{x} = 0$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve([diff(y(x),x) + ((1+x)/x)*y(x)=0,y(1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{e^{1-x}}{x}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 16

```
DSolve[{y'[x] + ((1+x)/x)*y[x]==0,y[1]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^{1-x}}{x}$$

2.7 problem 7

Internal problem ID [893]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 7.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y'x + \left(1 + \frac{1}{\ln(x)}\right) y = 0$$

With initial conditions

$$[y(e) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve([x*diff(y(x),x) + (1+1/ln(x))*y(x)=0,y(exp(1)) = 1],y(x), singsol=all)
```

$$y(x) = \frac{e}{x \ln(x)}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 18

```
DSolve[{y'[x] + (1+1/Log[x])*y[x]==0,y[Exp[1]]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-\text{LogIntegral}(x)+\text{LogIntegral}(e)-x+e}$$

2.8 problem 8

Internal problem ID [894]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 8.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [`_separable`]

$$y'x + (1 + x \cot(x))y = 0$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = 2 \right]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve([x*diff(y(x),x) + (1+x*cot(x))*y(x)=0,y(1/2*Pi) = 2],y(x), singsol=all)
```

$$y(x) = \frac{\csc(x)\pi}{x}$$

✓ Solution by Mathematica

Time used: 0.099 (sec). Leaf size: 66

```
DSolve[{y'[x] + (1+x*Cot[x])*y[x]==0,y[Pi/2]==2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 2^{1+\frac{\pi}{2}}(1 - e^{2ix})^{-x} \exp\left(-\frac{1}{12}i(-6 \text{PolyLog}(2, e^{2ix}) - 6x(x + 2i) + \pi^2 + 6i\pi)\right)$$

2.9 problem 9

Internal problem ID [895]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 9.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' - \frac{2xy}{x^2 + 1} = 0$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 11

```
dsolve([diff(y(x),x) - (2*x)/(1+x^2)*y(x)=0,y(0) = 2],y(x), singsol=all)
```

$$y(x) = 2x^2 + 2$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 12

```
DSolve[{y'[x] - (2*x)/(1+x^2)*y[x]==0,y[0]==2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 2(x^2 + 1)$$

2.10 problem 10

Internal problem ID [896]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 10.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' + \frac{ky}{x} = 0$$

With initial conditions

$$[y(1) = 3]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve([diff(y(x),x) +k/x*y(x)=0,y(1) = 3],y(x), singsol=all)
```

$$y(x) = 3x^{-k}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 12

```
DSolve[{y'[x] +k/x*y[x]==0,y[1]==3},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 3x^{-k}$$

2.11 problem 11

Internal problem ID [897]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 11.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' + \tan(kx)y = 0$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 18

```
dsolve([diff(y(x),x) +tan(k*x)*y(x)=0,y(0) = 2],y(x), singsol=all)
```

$$y(x) = 2(\sec(kx)^2)^{-\frac{1}{2k}}$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 15

```
DSolve[{y'[x] +Tan[k*x]*y[x]==0,y[0]==2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 2^k \sqrt[k]{\cos(kx)}$$

2.12 problem 12

Internal problem ID [898]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 12.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$3y + y' = 1$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x) +3*y(x)=1,y(x), singsol=all)
```

$$y(x) = \frac{1}{3} + e^{-3x}c_1$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 24

```
DSolve[y' [x] +3*y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{3} + c_1 e^{-3x}$$

$$y(x) \rightarrow \frac{1}{3}$$

2.13 problem 13

Internal problem ID [899]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 13.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y' + \left(\frac{1}{x} - 1\right)y = -\frac{2}{x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x) +(1/x-1)*y(x)=-2/x,y(x), singsol=all)
```

$$y(x) = \frac{2}{x} + \frac{e^x c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 17

```
DSolve[y'[x] +(1/x-1)*y[x]==-2/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{2 + c_1 e^x}{x}$$

2.14 problem 14

Internal problem ID [900]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 14.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y' + 2yx = e^{-x^2} x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x) +2*x*y(x)=x*exp(-x^2),y(x), singsol=all)
```

$$y(x) = \left(\frac{x^2}{2} + c_1 \right) e^{-x^2}$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 24

```
DSolve[y'[x] +2*x*y[x]==x*Exp[-x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2} e^{-x^2} (x^2 + 2c_1)$$

2.15 problem 15

Internal problem ID [901]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 15.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y' + \frac{2xy}{x^2 + 1} = \frac{e^{-x^2}}{x^2 + 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x) +(2*x)/(1+x^2)*y(x)=exp(-x^2)/(1+x^2),y(x), singsol=all)
```

$$y(x) = \frac{\frac{\sqrt{\pi} \operatorname{erf}(x)}{2} + c_1}{x^2 + 1}$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 28

```
DSolve[y'[x] +(2*x)/(1+x^2)*y[x]==Exp[-x^2]/(1+x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{\sqrt{\pi} \operatorname{erf}(x) + 2c_1}{2x^2 + 2}$$

2.16 problem 16

Internal problem ID [902]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 16.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y' + \frac{y}{x} = \frac{7}{x^2} + 3$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x) +1/x*y(x)=7/x^2+3,y(x), singsol=all)
```

$$y(x) = \frac{\frac{3x^2}{2} + 7 \ln(x) + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 24

```
DSolve[y'[x] +1/x*y[x]==7/x^2+3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{3x}{2} + \frac{7 \log(x)}{x} + \frac{c_1}{x}$$

2.17 problem 17

Internal problem ID [903]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 17.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y' + \frac{4y}{x-1} = \frac{1}{(x-1)^5} + \frac{\sin(x)}{(x-1)^4}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x) +4/(x-1)*y(x)=1/(x-1)^5+sin(x)/(x-1)^4,y(x), singsol=all)
```

$$y(x) = \frac{-\cos(x) + \ln(x-1) + c_1}{(x-1)^4}$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 22

```
DSolve[y'[x] +4/(x-1)*y[x]==1/(x-1)^5+Sin[x]/(x-1)^4,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \rightarrow \frac{\log(x-1) - \cos(x) + c_1}{(x-1)^4}$$

2.18 problem 18

Internal problem ID [904]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 18.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y'x + y(2x^2 + 1) = x^3e^{-x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x*diff(y(x),x) +(1+2*x^2)*y(x)=x^3*exp(-x^2),y(x), singsol=all)
```

$$y(x) = \frac{\left(\frac{x^4}{4} + c_1\right) e^{-x^2}}{x}$$

✓ Solution by Mathematica

Time used: 0.072 (sec). Leaf size: 27

```
DSolve[x*y'[x] +(1+2*x^2)*y[x]==x^3*Exp[-x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^{-x^2}(x^4 + 4c_1)}{4x}$$

2.19 problem 19

Internal problem ID [905]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 19.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$2y + y'x = \frac{2}{x^2} + 1$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x) +2*y(x)=2/x^2+1,y(x), singsol=all)
```

$$y(x) = \frac{\frac{x^2}{2} + 2 \ln(x) + c_1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 22

```
DSolve[x*y'[x] +2*y[x]==2/x^2+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{2 \log(x)}{x^2} + \frac{c_1}{x^2} + \frac{1}{2}$$

2.20 problem 20

Internal problem ID [906]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 20.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y' + \tan(x)y = \cos(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x) +tan(x)*y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = (c_1 + x) \cos(x)$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 12

```
DSolve[y'[x] +Tan[x]*y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow (x + c_1) \cos(x)$$

2.21 problem 21

Internal problem ID [907]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 21.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$2y + (x + 1)y' = \frac{\sin(x)}{x + 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve((1+x)*diff(y(x),x) +2*y(x)=sin(x)/(1+x),y(x), singsol=all)
```

$$y(x) = \frac{-\cos(x) + c_1}{(x + 1)^2}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 18

```
DSolve[(1+x)*y'[x] +2*y[x]==Sin[x]/(1+x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{-\cos(x) + c_1}{(x + 1)^2}$$

2.22 problem 22

Internal problem ID [908]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 22.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$(-2 + x)(x - 1)y' - (4x - 3)y = (-2 + x)^3$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 55

```
dsolve((x-2)*(x-1)*diff(y(x),x) -(4*x-3)*y(x)=(x-2)^3,y(x), singsol=all)
```

$$y(x) = \frac{(x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32)c_1}{x - 1} - \frac{x^3 - 6x^2 + 12x - 8}{2(x - 1)}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 30

```
DSolve[(x-2)*(x-1)*y'[x] -(4*x-3)*y[x]==(x-2)^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{(x - 2)^3 (-1 + 2c_1(x - 2)^2)}{2(x - 1)}$$

2.23 problem 23

Internal problem ID [909]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 23.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y' + 2 \cos(x) \sin(x) y = e^{-\sin(x)^2}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(diff(y(x),x) +(2*sin(x)*cos(x))*y(x)=exp(-sin(x)^2),y(x), singsol=all)
```

$$y(x) = (c_1 + x) e^{-\frac{1}{2} + \frac{\cos(2x)}{2}}$$

✓ Solution by Mathematica

Time used: 0.081 (sec). Leaf size: 24

```
DSolve[y'[x] +(2*Sin[x]*Cos[x])*y[x]==Exp[-Sin[x]^2],y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \rightarrow (x + \sqrt{e}c_1) e^{-\sin^2(x)}$$

2.24 problem 24

Internal problem ID [910]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 24.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y'x^2 + 3yx = e^x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x^2*diff(y(x),x) +3*x*y(x)=exp(x),y(x), singsol=all)
```

$$y(x) = \frac{(x - 1)e^x + c_1}{x^3}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 19

```
DSolve[x^2*y'[x] +3*x*y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^x(x - 1) + c_1}{x^3}$$

2.25 problem 25

Internal problem ID [911]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 25.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' + 7y = e^{3x}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve([diff(y(x),x) +7*y(x)=exp(3*x),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{(e^{10x} - 1)e^{-7x}}{10}$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 21

```
DSolve[{y'[x] +7*y[x]==Exp[3*x],y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{10}e^{-7x}(e^{10x} - 1)$$

2.26 problem 26

Internal problem ID [912]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 26.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$(x^2 + 1) y' + 4yx = \frac{2}{x^2 + 1}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([(1+x^2)*diff(y(x),x)+4*x*y(x)=2/(1+x^2),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{1 + 2x}{(x^2 + 1)^2}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 18

```
DSolve[{(1+x^2)*y'[x]+4*x*y[x]==2/(1+x^2),y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{2x + 1}{(x^2 + 1)^2}$$

2.27 problem 27

Internal problem ID [913]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 27.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$3y + y'x = \frac{2}{(x^2 + 1)x}$$

With initial conditions

$$[y(-1) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve([x*diff(y(x),x)+3*y(x)=2/(x*(1+x^2)),y(-1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{\ln(x^2 + 1) - \ln(2)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 19

```
DSolve[{x*y'[x]+3*y[x]==2/(x*(1+x^2)),y[-1]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{\log\left(\frac{1}{2}(x^2 + 1)\right)}{x^3}$$

2.28 problem 28

Internal problem ID [914]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 28.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\cot(x)y + y' = \cos(x)$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = 1 \right]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)+cot(x)*y(x)=cos(x),y(1/2*Pi) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{\cos(x)\cot(x)}{2} + \csc(x)$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 16

```
DSolve[{y'[x]+Cot[x]*y[x]==Cos[x],y[Pi/2]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \csc(x) - \frac{1}{2} \cos(x) \cot(x)$$

2.29 problem 29

Internal problem ID [915]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 29.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y' + \frac{y}{x} = \frac{2}{x^2} + 1$$

With initial conditions

$$[y(-1) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve([diff(y(x),x)+y(x)/x=2/x^2+1,y(-1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{-4i\pi + x^2 + 4 \ln(x) - 1}{2x}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 26

```
DSolve[{y'[x]+y[x]/x==2/x^2+1,y[-1]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x^2 + 4 \log(x) - 4i\pi - 1}{2x}$$

2.30 problem 30

Internal problem ID [916]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 30.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$(x-1)y' + 3y = \frac{1}{(x-1)^3} + \frac{\sin(x)}{(x-1)^2}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve([(x-1)*diff(y(x),x)+3*y(x)=1/(x-1)^3+sin(x)/(x-1)^2,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{-\cos(x) + \ln(x-1) - i\pi}{(x-1)^3}$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 25

```
DSolve[{(x-1)*y'[x]+3*y[x]==1/(x-1)^3+Sin[x]/(x-1)^2,y[0]==1},y[x],x,IncludeSingularSolution
```

$$y(x) \rightarrow \frac{\log(x-1) - \cos(x) - i\pi}{(x-1)^3}$$

2.31 problem 31

Internal problem ID [917]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 31.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$2y + y'x = 8x^2$$

With initial conditions

$$[y(1) = 3]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve([x*diff(y(x),x)+2*y(x)=8*x^2,y(1) = 3],y(x), singsol=all)
```

$$y(x) = \frac{2x^4 + 1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 14

```
DSolve[{x*y'[x]+2*y[x]==8*x^2,y[1]==3},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 2x^2 + \frac{1}{x^2}$$

2.32 problem 32

Internal problem ID [918]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 32.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y'x - 2y = -x^2$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve([x*diff(y(x),x)-2*y(x)=-x^2,y(1) = 1],y(x), singsol=all)
```

$$y(x) = (-\ln(x) + 1)x^2$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 14

```
DSolve[{x*y'[x]-2*y[x]==-x^2,y[1]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -x^2(\log(x) - 1)$$

2.33 problem 33

Internal problem ID [919]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 33.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' + 2yx = x$$

With initial conditions

$$[y(0) = 3]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)+2*x*y(x)=x,y(0) = 3],y(x), singsol=all)
```

$$y(x) = \frac{1}{2} + \frac{5e^{-x^2}}{2}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 20

```
DSolve[{y'[x]+2*x*y[x]==x,y[0]==3},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{5e^{-x^2}}{2} + \frac{1}{2}$$

2.34 problem 34

Internal problem ID [920]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 34.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$(x - 1)y' + 3y = \frac{1 + (x - 1)\sec(x)^2}{(x - 1)^3}$$

With initial conditions

$$[y(0) = -1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve([(x-1)*diff(y(x),x)+3*y(x)= (1+(x-1)*sec(x)^2)/(x-1)^3,y(0) = -1],y(x), singsol=all)
```

$$y(x) = \frac{\ln(x - 1) + \tan(x) + 1 - i\pi}{(x - 1)^3}$$

✓ Solution by Mathematica

Time used: 0.171 (sec). Leaf size: 21

```
DSolve[{(x-1)*y'[x]+3*y[x]==(1+(x-1)*Sec[x]^2)/(x-1)^3,y[0]==-1},y[x],x,IncludeSingularSolut
```

$$y(x) \rightarrow \frac{\log(1 - x) + \tan(x) + 1}{(x - 1)^3}$$

2.35 problem 35

Internal problem ID [921]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 35.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$(2+x)y' + 4y = \frac{2x^2 + 1}{x(2+x)^3}$$

With initial conditions

$$[y(-1) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve([(x+2)*diff(y(x),x)+4*y(x) = (1+2*x^2)/(x*(x+2)^3),y(-1) = 2],y(x), singsol=all)
```

$$y(x) = \frac{x^2 + \ln(x) + 1 - i\pi}{(2+x)^4}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 23

```
DSolve[{(x+2)*y'[x]+4*y[x]== (1+2*x^2)/(x*(x+2)^3),y[-1]==2},y[x],x,IncludeSingularSolutions
```

$$y(x) \rightarrow \frac{x^2 + \log(x) - i\pi + 1}{(x+2)^4}$$

2.36 problem 36

Internal problem ID [922]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 36.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$(x^2 - 1) y' - 2yx = x(x^2 - 1)$$

With initial conditions

$$[y(0) = 4]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve([(x^2-1)*diff(y(x),x)-2*x*y(x)= x*(x^2-1),y(0) = 4],y(x), singsol=all)
```

$$y(x) = -\frac{(i\pi - \ln(x+1) - \ln(x-1) + 8)(x^2 - 1)}{2}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 27

```
DSolve[{(x^2-1)*y'[x]-2*x*y[x]== x*(x^2-1),y[0]==4},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2}(x^2 - 1)(\log(x^2 - 1) - i\pi - 8)$$

2.37 problem 44

Internal problem ID [923]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 44.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y'x - 2y = -1$$

With initial conditions

$$\left[y(0) = \frac{1}{2} \right]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 11

```
dsolve([x*diff(y(x),x)-2*y(x)= -1,y(0) = 1/2],y(x), singsol=all)
```

$$y(x) = \frac{1}{2} + c_1x^2$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 8

```
DSolve[{x*y'[x]-2*y[x]== -1,y[0]==1/2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2}$$

2.38 problem 48(a)

Internal problem ID [924]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 48(a).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$\sec(y)^2 y' - 3 \tan(y) = -1$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve(sec(y(x))^2*diff(y(x),x)-3*tan(y(x))= -1,y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{c_1 e^{3x}}{3} + \frac{1}{3}\right)$$

✓ Solution by Mathematica

Time used: 60.217 (sec). Leaf size: 177

```
DSolve[Sec[y[x]]^2*y'[x]-3*Tan[y[x]]== -1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\arccos\left(\frac{3e^{6c_1}}{\sqrt{e^{6x} - 2e^{3x+6c_1} + 10e^{12c_1}}}\right)$$

$$y(x) \rightarrow \arccos\left(\frac{3e^{6c_1}}{\sqrt{e^{6x} - 2e^{3x+6c_1} + 10e^{12c_1}}}\right)$$

$$y(x) \rightarrow -\arccos\left(\frac{3e^{6c_1}}{\sqrt{e^{6x} - 2e^{3x+6c_1} + 10e^{12c_1}}}\right)$$

$$y(x) \rightarrow \arccos\left(\frac{3e^{6c_1}}{\sqrt{e^{6x} - 2e^{3x+6c_1} + 10e^{12c_1}}}\right)$$

2.39 problem 48(b)

Internal problem ID [925]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 48(b).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$e^{y^2} \left(2yy' + \frac{2}{x} \right) = \frac{1}{x^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(exp(y(x)^2)*(2*y(x)*diff(y(x),x)+2/x)= 1/x^2,y(x), singsol=all)
```

$$y(x) = \sqrt{\ln\left(-\frac{c_1 - x}{x^2}\right)}$$

$$y(x) = -\sqrt{\ln\left(-\frac{c_1 - x}{x^2}\right)}$$

✓ Solution by Mathematica

Time used: 7.286 (sec). Leaf size: 37

```
DSolve[Exp[y[x]^2]*(2*y[x]*y'[x]+2/x)== 1/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\sqrt{\log\left(\frac{x + c_1}{x^2}\right)}$$

$$y(x) \rightarrow \sqrt{\log\left(\frac{x + c_1}{x^2}\right)}$$

2.40 problem 48(c)

Internal problem ID [926]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 48(c).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$2 \ln(y) = -\frac{xy'}{y} + 4x^2$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x)/y(x)+2*ln(y(x))= 4*x^2,y(x), singsol=all)
```

$$y(x) = e^{x^2} e^{-\frac{c_1}{x^2}}$$

✓ Solution by Mathematica

Time used: 0.248 (sec). Leaf size: 17

```
DSolve[x*y'[x]/y[x]+2*Log[y[x]]== 4*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{x^2 + \frac{c_1}{x^2}}$$

2.41 problem 48(d)

Internal problem ID [927]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

Problem number: 48(d).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class C', _rational, _Riccati]`

$$\frac{y'}{(1+y)^2} - \frac{1}{x(1+y)} = -\frac{3}{x^2}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)/(1+y(x))^2-1/(x*(1+y(x))))=-3/x^2,y(x), singsol=all)
```

$$y(x) = -1 + \frac{x}{3 \ln(x) + 3c_1}$$

✓ Solution by Mathematica

Time used: 0.252 (sec). Leaf size: 31

```
DSolve[y'[x]/(1+y[x])^2-1/(x*(1+y[x]))== -3/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x - 3 \log(x) - 3c_1}{3(\log(x) + c_1)}$$

$$y(x) \rightarrow -1$$

3 Chapter 2, First order equations. separable equations. Section 2.2 Page 52

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3.1 problem 1

Internal problem ID [928]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 1.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' - \frac{3x^2 + 2x + 1}{y - 2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
dsolve(diff(y(x),x)= (3*x^2+2*x+1)/(y(x)-2),y(x), singsol=all)
```

$$y(x) = 2 - \sqrt{2x^3 + 2x^2 + 2c_1 + 2x + 4}$$

$$y(x) = 2 + \sqrt{2x^3 + 2x^2 + 2c_1 + 2x + 4}$$

✓ Solution by Mathematica

Time used: 0.158 (sec). Leaf size: 56

```
DSolve[y'[x]== (3*x^2+2*x+1)/(y[x]-2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 2 - \sqrt{2}\sqrt{x^3 + x^2 + x + 2 + c_1}$$

$$y(x) \rightarrow 2 + \sqrt{2}\sqrt{x^3 + x^2 + x + 2 + c_1}$$

3.2 problem 2

Internal problem ID [929]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 2.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$\sin(y) \sin(x) + \cos(y) y' = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 12

```
dsolve(sin(x)*sin(y(x))+cos(y(x))*diff(y(x),x)= 0,y(x), singsol=all)
```

$$y(x) = \arcsin\left(\frac{e^{\cos(x)}}{c_1}\right)$$

✓ Solution by Mathematica

Time used: 29.953 (sec). Leaf size: 22

```
DSolve[Sin[x]*Sin[y[x]]+Cos[y[x]]*y'[x]== 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \arcsin\left(e^{\cos(x)+\frac{c_1}{2}}\right)$$

$$y(x) \rightarrow 0$$

3.3 problem 3

Internal problem ID [930]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 3.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y'x + y^2 + y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x*diff(y(x),x)+y(x)^2+y(x)= 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{c_1x - 1}$$

✓ Solution by Mathematica

Time used: 0.251 (sec). Leaf size: 31

```
DSolve[x*y'[x]+y[x]^2+y[x]== 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{e^{c_1}}{-x + e^{c_1}}$$

$$y(x) \rightarrow -1$$

$$y(x) \rightarrow 0$$

3.4 problem 5

Internal problem ID [931]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 5.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$(3y^3 + 3y \cos(y) + 1) y' + \frac{(1 + 2x)y}{x^2 + 1} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve((3*y(x)^3+3*y(x)*cos(y(x))+1)*diff(y(x),x)+((2*x+1)*y(x))/(1+x^2)= 0,y(x), singsol=all
```

$$\ln(x^2 + 1) + \arctan(x) + y(x)^3 + 3 \sin(y(x)) + \ln(y(x)) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.379 (sec). Leaf size: 40

```
DSolve[(3*y[x]^3+3*y[x]*Cos[y[x]]+1)*y'[x]+((2*x+1)*y[x])/(1+x^2)== 0,y[x],x,IncludeSingular
```

$$y(x) \rightarrow \text{InverseFunction}\left[\#1^3 + \log(\#1) + 3 \sin(\#1)\&\right] \left[-\arctan(x) - \log(x^2 + 1) + c_1\right]$$

$$y(x) \rightarrow 0$$

3.5 problem 6

Internal problem ID [932]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 6.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y'x^2y - (-1 + y^2)^{\frac{3}{2}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x^2*y(x)*diff(y(x),x) = (y(x)^2-1)^(3/2),y(x), singsol=all)
```

$$-\frac{1}{x} + \frac{(y(x) - 1)(y(x) + 1)}{(y(x)^2 - 1)^{\frac{3}{2}}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.707 (sec). Leaf size: 111

```
DSolve[x^2*y[x]*y'[x]== (y[x]^2-1)^(3/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{\sqrt{(1+c_1^2)x^2-2c_1x+1}}{1-c_1x}$$

$$y(x) \rightarrow \frac{\sqrt{(1+c_1^2)x^2-2c_1x+1}}{-1+c_1x}$$

$$y(x) \rightarrow -1$$

$$y(x) \rightarrow 1$$

$$y(x) \rightarrow -\frac{\sqrt{x^2}}{x}$$

$$y(x) \rightarrow \frac{\sqrt{x^2}}{x}$$

3.6 problem 7

Internal problem ID [933]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 7.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' - (1 + y^2) x^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)= x^2*(1+y(x)^2),y(x), singsol=all)
```

$$y(x) = \tan\left(\frac{x^3}{3} + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.169 (sec). Leaf size: 30

```
DSolve[y'[x]== x^2*(1+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \tan\left(\frac{x^3}{3} + c_1\right)$$

$$y(x) \rightarrow -i$$

$$y(x) \rightarrow i$$

3.7 problem 8

Internal problem ID [934]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 8.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$(x^2 + 1) y' + yx = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)*(1+x^2)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{x^2 + 1}}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 22

```
DSolve[y'[x]*(1+x^2)+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_1}{\sqrt{x^2 + 1}}$$

$$y(x) \rightarrow 0$$

3.8 problem 9

Internal problem ID [935]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 9.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' - (x - 1)(y - 1)(y - 2) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
dsolve(diff(y(x),x)=(x-1)*(y(x)-1)*(y(x)-2),y(x), singsol=all)
```

$$y(x) = \frac{-2 + c_1 e^{\frac{1}{2}x^2 - x}}{c_1 e^{\frac{1}{2}x^2 - x} - 1}$$

✓ Solution by Mathematica

Time used: 0.308 (sec). Leaf size: 56

```
DSolve[y'[x]==(x-1)*(y[x]-1)*(y[x]-2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{2e^x - e^{\frac{x^2}{2} + c_1}}{e^x - e^{\frac{x^2}{2} + c_1}}$$

$$y(x) \rightarrow 1$$

$$y(x) \rightarrow 2$$

3.9 problem 10

Internal problem ID [936]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 10.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$(y - 1)^2 y' = 2x + 3$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 102

```
dsolve((y(x)-1)^2*diff(y(x),x)=2*x+3,y(x), singsol=all)
```

$$y(x) = (3x^2 + 3c_1 + 9x)^{\frac{1}{3}} + 1$$

$$y(x) = -\frac{(3x^2 + 3c_1 + 9x)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}(3x^2 + 3c_1 + 9x)^{\frac{1}{3}}}{2} + 1$$

$$y(x) = -\frac{(3x^2 + 3c_1 + 9x)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}(3x^2 + 3c_1 + 9x)^{\frac{1}{3}}}{2} + 1$$

✓ Solution by Mathematica

Time used: 0.482 (sec). Leaf size: 103

```
DSolve[(y[x]-1)^2*y'[x]==2*x+3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 1 + \sqrt[3]{3x^2 + 9x - 1 + 3c_1}$$

$$y(x) \rightarrow 1 + \frac{1}{2}i(\sqrt{3} + i) \sqrt[3]{3x^2 + 9x - 1 + 3c_1}$$

$$y(x) \rightarrow 1 - \frac{1}{2}(1 + i\sqrt{3}) \sqrt[3]{3x^2 + 9x - 1 + 3c_1}$$

3.10 problem 11

Internal problem ID [937]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 11.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' - \frac{x^2 + 3x + 2}{y - 2} = 0$$

With initial conditions

$$[y(1) = 4]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 25

```
dsolve([diff(y(x),x)=(x^2+3*x+2)/(y(x)-2),y(1) = 4],y(x), singsol=all)
```

$$y(x) = 2 + \frac{\sqrt{6x^3 + 27x^2 + 36x - 33}}{3}$$

✓ Solution by Mathematica

Time used: 0.167 (sec). Leaf size: 30

```
DSolve[{y'[x]==(x^2+3*x+2)/(y[x]-2),y[1]==4},y[x],x,IncludeSingularSolutions->True]
```

$$y(x) \rightarrow \sqrt{\frac{2x^3}{3} + 3x^2 + 4x - \frac{11}{3}} + 2$$

3.11 problem 12

Internal problem ID [938]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 12.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' + x(y^2 + y) = 0$$

With initial conditions

$$[y(2) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve([diff(y(x),x)+x*(y(x)^2+y(x))=0,y(2) = 1],y(x), singsol=all)
```

$$y(x) = \frac{1}{-1 + 2e^{\frac{(2+x)(-2+x)}{2}}}$$

✓ Solution by Mathematica

Time used: 0.241 (sec). Leaf size: 27

```
DSolve[{y'[x]+x*(y[x]^2+y[x])==0,y[2]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{e^2}{e^2 - 2e^{\frac{x^2}{2}}}$$

3.12 problem 13

Internal problem ID [939]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 13.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$(3y^2 + 4y) y' = -2x - \cos(x)$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.5 (sec). Leaf size: 102

```
dsolve([(3*y(x)^2+4*y(x))*diff(y(x),x)+2*x+cos(x)=0,y(0) = 1],y(x), singsol=all)
```

$y(x)$

$$= \frac{\left(260 - 108x^2 - 108 \sin(x) + 12\sqrt{441 - 390x^2 - 390 \sin(x) + 81x^4 + 162 \sin(x)x^2 + 81 \sin(x)^2}\right)^{\frac{1}{3}}}{6} + \frac{3 \left(260 - 108x^2 - 108 \sin(x) + 12\sqrt{441 - 390x^2 - 390 \sin(x) + 81x^4 + 162 \sin(x)x^2 + 81 \sin(x)^2}\right)}{8} - \frac{2}{3}$$

✓ Solution by Mathematica

Time used: 2.549 (sec). Leaf size: 127

```
DSolve[{(3*y[x]^2+4*y[x])*y'[x]+2*x+Cos[x]==0,y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$y(x)$

$$\rightarrow \frac{2^{2/3} \left(-27x^2 + \sqrt{(27x^2 + 27 \sin(x) - 65)^2 - 256 - 27 \sin(x) + 65} \right)^{2/3} - 4 \sqrt[3]{-27x^2 + \sqrt{(27x^2 + 27 \sin(x) - 65)^2 - 256 - 27 \sin(x) + 65}}}{6 \sqrt[3]{-27x^2 + \sqrt{(27x^2 + 27 \sin(x) - 65)^2 - 256 - 27 \sin(x) + 65}}}$$

3.13 problem 14

Internal problem ID [940]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 14.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' + \frac{(1+y)(y-1)(y-2)}{x+1} = 0$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple

Time used: 2.625 (sec). Leaf size: 111

```
dsolve([diff(y(x),x)+((y(x)+1)*(y(x)-1)*(y(x)-2))/(x+1)=0,y(1) = 0],y(x), singsol=all)
```

$y(x)$

$$= \text{RootOf}(-2048 + (x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 257) _Z^{18} + (-6x^6 - 36x^5 - 90x^4 - 120x^3 - 1$$

✓ Solution by Mathematica

Time used: 60.912 (sec). Leaf size: 1618

```
DSolve[{y'[x]+((y[x]+1)*(y[x]-1)*(y[x]-2))/(x+1)==0,y[1]==0},y[x],x,IncludeSingularSolutions
```

$y(x)$

$$(-1 - i\sqrt{3})x^{12} - 12i(-i + \sqrt{3})x^{11} - 66i(-i + \sqrt{3})x^{10} - 220i(-i + \sqrt{3})x^9 - 495i(-i + \sqrt{3})x^8 - 7$$

→

3.14 problem 15

Internal problem ID [941]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 15.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' + 2x(1 + y) = 0$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)+2*x*(y(x)+1)=0,y(0) = 2],y(x), singsol=all)
```

$$y(x) = -1 + 3e^{-x^2}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 16

```
DSolve[{y'[x]+2*x*(y[x]+1)==0,y[0]==2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 3e^{-x^2} - 1$$

3.15 problem 16

Internal problem ID [942]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 16.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' - 2xy(1 + y^2) = 0$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 16

```
dsolve([diff(y(x),x)=2*x*y(x)*(1+y(x)^2),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{2e^{-2x^2} - 1}}$$

✓ Solution by Mathematica

Time used: 60.086 (sec). Leaf size: 27

```
DSolve[{y'[x]==2*x*y[x]*(1+y[x]^2),y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{ie^{x^2}}{\sqrt{e^{2x^2} - 2}}$$

3.16 problem 17

Internal problem ID [943]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 17.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$(x^2 + 2)y' - 4x(y^2 + 2y + 1) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
dsolve(diff(y(x),x)*(x^2+2)=4*x*(y(x)^2+2*y(x)+1),y(x), singsol=all)
```

$$y(x) = -\frac{2 \ln(x^2 + 2) + 4c_1 + 1}{2(\ln(x^2 + 2) + 2c_1)}$$

✓ Solution by Mathematica

Time used: 0.195 (sec). Leaf size: 37

```
DSolve[y'[x]*(x^2+2)==4*x*(y[x]^2+2*y[x]+1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{2 \log(x^2 + 2) + 1 + c_1}{2 \log(x^2 + 2) + c_1}$$

$$y(x) \rightarrow -1$$

3.17 problem 18

Internal problem ID [944]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 18.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' + 2x(y^3 - 3y + 2) = 0$$

With initial conditions

$$[y(0) = 3]$$

✓ Solution by Maple

Time used: 0.421 (sec). Leaf size: 70

```
dsolve([diff(y(x),x)=-2*x*(y(x)^3-3*y(x)+2),y(0) = 3],y(x), singsol=all)
```

$y(x)$

$$= e^{\text{RootOf}(18x^2e^{-Z}-2\ln(e^{-Z}-3)e^{-Z}+2e^{-Z}\ln(2)-2e^{-Z}\ln(5)+2_Ze^{-Z}-54x^2+6\ln(e^{-Z}-3)+3e^{-Z}-6\ln(2)+6\ln(5)-6_Z-15)} - 2$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'[x]==-2*x*(y[x]^3-3*y[x]+2),y[1]==-1},y[x],x,IncludeSingularSolutions->True]
```

{}

3.18 problem 19

Internal problem ID [945]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 19.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' - \frac{2x}{1+2y} = 0$$

With initial conditions

$$[y(2) = 0]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 17

```
dsolve([diff(y(x),x)=2*x/(1+2*y(x)),y(2) = 0],y(x), singsol=all)
```

$$y(x) = -\frac{1}{2} + \frac{\sqrt{4x^2 - 15}}{2}$$

✓ Solution by Mathematica

Time used: 0.111 (sec). Leaf size: 22

```
DSolve[{y'[x]==2*x/(1+2*y[x]),y[2]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2} \left(\sqrt{4x^2 - 15} - 1 \right)$$

3.19 problem 20

Internal problem ID [946]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 20.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [quadrature]

$$y' - 2y + y^2 = 0$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)=2*y(x)-y(x)^2,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{2}{1 + e^{-2x}}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 21

```
DSolve[{y'[x]==2*y[x]-y[x]^2,y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{2e^{2x}}{e^{2x} + 1}$$

3.20 problem 21

Internal problem ID [947]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 21.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$yy' = -x$$

With initial conditions

$$[y(3) = -4]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 15

```
dsolve([x+y(x)*diff(y(x),x)=0,y(3) = -4],y(x), singsol=all)
```

$$y(x) = -\sqrt{-x^2 + 25}$$

✓ Solution by Mathematica

Time used: 0.083 (sec). Leaf size: 18

```
DSolve[{x+y[x]*y'[x]==0,y[3]==-4},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\sqrt{25 - x^2}$$

3.21 problem 22

Internal problem ID [948]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 22.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' + x^2(1 + y)(y - 2)^2 = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 35

```
dsolve(diff(y(x),x)+x^2*(y(x)+1)*(y(x)-2)^2=0,y(x), singsol=all)
```

$$y(x) = e^{\text{RootOf}(3x^3e^{-Z} + \ln(e^{-Z} + 3)e^{-Z} + 9c_1e^{-Z} - Ze^{-Z} - 3)} + 2$$

✓ Solution by Mathematica

Time used: 0.482 (sec). Leaf size: 52

```
DSolve[y'[x]+x^2*(y[x]+1)*(y[x]-2)^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \text{InverseFunction} \left[\frac{1}{9} \left(-\frac{3}{\#1 - 2} - \log(\#1 - 2) + \log(\#1 + 1) \right) \& \right] \left[-\frac{x^3}{3} + c_1 \right]$$

$$y(x) \rightarrow -1$$

$$y(x) \rightarrow 2$$

3.22 problem 23

Internal problem ID [949]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 23.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$(x + 1)(-2 + x)y' + y = 0$$

With initial conditions

$$[y(1) = -3]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 22

```
dsolve([(x+1)*(x-2)*diff(y(x),x)+y(x)=0,y(1) = -3],y(x), singsol=all)
```

$$y(x) = -\frac{3(1 + i\sqrt{3})2^{\frac{2}{3}}(x + 1)^{\frac{1}{3}}}{4(-2 + x)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 23

```
DSolve[{(x+1)*(x-2)*y'[x]+y[x]==0,y[1]==-3},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{3\sqrt[3]{x+1}}{\sqrt[3]{4-2x}}$$

3.23 problem 24

Internal problem ID [950]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 24.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' - \frac{1 + y^2}{x^2 + 1} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)=(1+y(x)^2)/(1+x^2),y(x), singsol=all)
```

$$y(x) = \tan(\arctan(x) + c_1)$$

✓ Solution by Mathematica

Time used: 0.227 (sec). Leaf size: 25

```
DSolve[y'[x]==(1+y[x]^2)/(1+x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \tan(\arctan(x) + c_1)$$

$$y(x) \rightarrow -i$$

$$y(x) \rightarrow i$$

3.24 problem 25

Internal problem ID [951]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 25.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' \sqrt{-x^2 + 1} + \sqrt{1 - y^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)*sqrt(1-x^2)+sqrt(1-y(x)^2)=0,y(x), singsol=all)
```

$$y(x) = -\sin(\arcsin(x) + c_1)$$

✓ Solution by Mathematica

Time used: 0.317 (sec). Leaf size: 47

```
DSolve[y'[x]*Sqrt[1-x^2]+Sqrt[1-y[x]^2]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \cos\left(2 \arctan\left(\frac{\sqrt{1-x^2}}{x+1}\right) + c_1\right)$$

$$y(x) \rightarrow -1$$

$$y(x) \rightarrow 1$$

$$y(x) \rightarrow \text{Interval}\{-1, 1\}$$

3.25 problem 26

Internal problem ID [952]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 26.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' - \frac{\cos(x)}{\sin(y)} = 0$$

With initial conditions

$$\left[y(\pi) = \frac{\pi}{2} \right]$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 11

```
dsolve([diff(y(x),x)=cos(x)/sin(y(x)),y(Pi) = 1/2*Pi],y(x), singsol=all)
```

$$y(x) = \frac{\pi}{2} + \arcsin(\sin(x))$$

✓ Solution by Mathematica

Time used: 0.439 (sec). Leaf size: 10

```
DSolve[{y'[x]==Cos[x]/Sin[y[x]],y[Pi]==Pi/2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \arccos(-\sin(x))$$

3.26 problem 27

Internal problem ID [953]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 27.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [quadrature]

$$y' - ay + by^2 = 0$$

With initial conditions

$$[y(0) = y_0]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 25

```
dsolve([diff(y(x),x)=a*y(x)-b*y(x)^2,y(0) = y0],y(x), singsol=all)
```

$$y(x) = \frac{a y_0}{(-y_0 b + a) e^{-ax} + y_0 b}$$

✓ Solution by Mathematica

Time used: 0.854 (sec). Leaf size: 27

```
DSolve[{y'[x]==a*y[x]-b*y[x]^2,y[0]==y0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{a y_0 e^{ax}}{b y_0 (e^{ax} - 1) + a}$$

3.27 problem 35

Internal problem ID [954]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 35.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$y' + y - \frac{2x e^{-x}}{1 + e^x y} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(diff(y(x),x)+y(x)=(2*x*exp(-x))/(1+y(x)*exp(x)),y(x), singsol=all)
```

$$y(x) = \left(-1 - \sqrt{2x^2 - 2c_1 + 1}\right) e^{-x}$$

$$y(x) = \left(-1 + \sqrt{2x^2 - 2c_1 + 1}\right) e^{-x}$$

✓ Solution by Mathematica

Time used: 32.427 (sec). Leaf size: 70

```
DSolve[y'[x]+y[x]==(2*x*Exp[-x])/(1+y[x]*Exp[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -e^{-2x} \left(e^x + \sqrt{e^{2x} (2x^2 + 1 + c_1)} \right)$$

$$y(x) \rightarrow e^{-2x} \left(-e^x + \sqrt{e^{2x} (2x^2 + 1 + c_1)} \right)$$

3.28 problem 36

Internal problem ID [955]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 36.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'x - 2y - \frac{x^6}{x^2 + y} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
dsolve(x*diff(y(x),x)-2*y(x)=x^6/(y(x)+x^2),y(x), singsol=all)
```

$$y(x) = \left(-1 - \sqrt{x^2 - 2c_1 + 1}\right) x^2$$

$$y(x) = \left(-1 + \sqrt{x^2 - 2c_1 + 1}\right) x^2$$

✓ Solution by Mathematica

Time used: 0.635 (sec). Leaf size: 70

```
DSolve[x*y'[x]-2*y[x]==x^6/(y[x]+x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -x^2 \left(1 + \sqrt{\frac{1}{x^5} x^2 \sqrt{x(x^2 + 1 + c_1)}}\right)$$

$$y(x) \rightarrow -x^2 + \sqrt{\frac{1}{x^5} x^4 \sqrt{x(x^2 + 1 + c_1)}}$$

3.29 problem 37

Internal problem ID [956]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 37.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]]'`

$$y' - y - \frac{(x+1)e^{4x}}{(y+e^x)^2} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 112

```
dsolve(diff(y(x),x)-y(x)=((x+1)*exp(4*x))/(y(x)+exp(x))^2,y(x), singsol=all)
```

$$y(x) = (3x e^x - 3c_1 + 1)^{\frac{1}{3}} e^x - e^x$$

$$y(x) = \left(-\frac{(3x e^x - 3c_1 + 1)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}(3x e^x - 3c_1 + 1)^{\frac{1}{3}}}{2} \right) e^x - e^x$$

$$y(x) = \left(-\frac{(3x e^x - 3c_1 + 1)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}(3x e^x - 3c_1 + 1)^{\frac{1}{3}}}{2} \right) e^x - e^x$$

✓ Solution by Mathematica

Time used: 19.018 (sec). Leaf size: 143

```
DSolve[y'[x]-y[x]==((x+1)*Exp[4*x])/(y[x]+Exp[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -e^x + e^{3x} \sqrt[3]{e^{-6x} (3e^x x + 1 + 3c_1)}$$

$$y(x) \rightarrow -e^x + \frac{1}{2}i(\sqrt{3} + i) e^{3x} \sqrt[3]{e^{-6x} (3e^x x + 1 + 3c_1)}$$

$$y(x) \rightarrow -e^x - \frac{1}{2}(1 + i\sqrt{3}) e^{3x} \sqrt[3]{e^{-6x} (3e^x x + 1 + 3c_1)}$$

3.30 problem 38

Internal problem ID [957]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52

Problem number: 38.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$y' - 2y - \frac{x e^{2x}}{1 - e^{-2x}y} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 84

```
dsolve(diff(y(x),x)-2*y(x)=x*exp(2*x)/(1-y(x)*exp(-2*x)),y(x), singsol=all)
```

$$y(x) = \left(e^{-2x} + \sqrt{-e^{-4x}x^2 + e^{-4x} - 2c_1e^{-4x}} \right) e^{4x}$$

$$y(x) = -\left(-e^{-2x} + \sqrt{-e^{-4x}x^2 + e^{-4x} - 2c_1e^{-4x}} \right) e^{4x}$$

✓ Solution by Mathematica

Time used: 0.777 (sec). Leaf size: 72

```
DSolve[y'[x]-2*y[x]==x*Exp[2*x]/(1-y[x]*Exp[-2*x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{2x} - \frac{\sqrt{x^2 - 1 - c_1}}{\sqrt{-e^{-4x}}}$$

$$y(x) \rightarrow e^{2x} + \frac{\sqrt{x^2 - 1 - c_1}}{\sqrt{-e^{-4x}}}$$

4 Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations.

Section 2.3 Page 60

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4.1 problem 1

Internal problem ID [958]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60

Problem number: 1.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [Riccati]

$$y' - \frac{x^2 + y^2}{\sin(x)} = 0$$

X Solution by Maple

```
dsolve(diff(y(x),x)=(x^2+y(x)^2)/sin(x),y(x), singsol=all)
```

No solution found

X Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==(x^2+y[x]^2)/Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.2 problem 2

Internal problem ID [959]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60

Problem number: 2.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type ['y=_G(x,y)']

$$y' - \frac{y + e^x}{x^2 + y^2} = 0$$

X Solution by Maple

```
dsolve(diff(y(x),x)=(exp(x)+y(x))/(x^2+y(x)^2),y(x), singsol=all)
```

No solution found

X Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==(Exp[x]+y[x])/(x^2+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.3 problem 3

Internal problem ID [960]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60

Problem number: 3.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [$y = G(x, y')$]

$$y' - \tan(xy) = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 44

```
dsolve(diff(y(x),x)=tan(x*y(x)),y(x), singsol=all)
```

$$y(x) = -i \operatorname{RootOf} \left(\sqrt{2} c_1 - \operatorname{erf} \left(\frac{(-x + _Z) \sqrt{2}}{2} \right) \sqrt{\pi} - \operatorname{erf} \left(\frac{\sqrt{2} (x + _Z)}{2} \right) \sqrt{\pi} \right)$$

✓ Solution by Mathematica

Time used: 0.311 (sec). Leaf size: 69

```
DSolve[y'[x]==Tan[x*y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\operatorname{Solve} \left[\frac{1}{2} \sqrt{\frac{\pi}{2}} e^{\frac{x^2}{2}} \left(\operatorname{erfi} \left(\frac{y(x) - ix}{\sqrt{2}} \right) + \operatorname{erfi} \left(\frac{y(x) + ix}{\sqrt{2}} \right) \right) = c_1 e^{\frac{x^2}{2}}, y(x) \right]$$

4.4 problem 4

Internal problem ID [961]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60

Problem number: 4.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [$y = G(x, y')$]

$$y' - \frac{x^2 + y^2}{\ln(yx)} = 0$$

X Solution by Maple

```
dsolve(diff(y(x),x)=(x^2+y(x)^2)/ln(x*y(x)),y(x), singsol=all)
```

No solution found

X Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==(x^2+y[x]^2)/Log[x*y[x]],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.5 problem 5

Internal problem ID [962]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60

Problem number: 5.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [$y = G(x, y')$]

$$y' - (x^2 + y^2) y^{\frac{1}{3}} = 0$$

X Solution by Maple

```
dsolve(diff(y(x),x)=(x^2+y(x)^2)*y(x)^(1/3),y(x), singsol=all)
```

No solution found

X Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==(x^2+y[x]^2)*y[x]^(1/3),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.6 problem 6

Internal problem ID [963]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60

Problem number: 6.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - 2yx = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=2*x*y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^{x^2}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 18

```
DSolve[y'[x]==2*x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{x^2}$$

$$y(x) \rightarrow 0$$

4.7 problem 7

Internal problem ID [964]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60

Problem number: 7.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [$y = G(x, y')$]

$$y' - \ln(1 + x^2 + y^2) = 0$$

X Solution by Maple

```
dsolve(diff(y(x),x)=ln(1+x^2+y(x)^2),y(x), singsol=all)
```

No solution found

X Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==Log[1+x^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.8 problem 8

Internal problem ID [965]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60

Problem number: 8.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, [_Abel, '2nd type', 'cl`

$$y' - \frac{2x + 3y}{x - 4y} = 0$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 46

```
dsolve(diff(y(x),x)=(2*x+3*y(x))/(x-4*y(x)),y(x), singsol=all)
```

$$y(x) = -\frac{x}{4} + \frac{\sqrt{7}x \tan\left(\text{RootOf}\left(\sqrt{7} \ln\left(\frac{7x^2}{8} + \frac{7x^2 \tan^2(-Z)}{8}\right) + 2\sqrt{7}c_1 - 4Z\right)\right)}{4}$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 53

```
DSolve[y'[x]==(2*x+3*y[x])/(x-4*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\log\left(\frac{2y(x)^2}{x^2} + \frac{y(x)}{x} + 1\right) - \frac{4 \arctan\left(\frac{4y(x)+1}{\sqrt{7}}\right)}{\sqrt{7}} = -2 \log(x) + c_1, y(x)\right]$$

4.9 problem 9

Internal problem ID [966]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60

Problem number: 9.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [$y = G(x, y')$]

$$y' - \sqrt{x^2 + y^2} = 0$$

X Solution by Maple

```
dsolve(diff(y(x),x)=(x^2+y(x)^2)^(1/2),y(x), singsol=all)
```

No solution found

X Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==(x^2+y[x]^2)^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.10 problem 10

Internal problem ID [967]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60

Problem number: 10.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - x(-1 + y^2)^{\frac{2}{3}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(diff(y(x),x)=x*(y(x)^2-1)^(2/3),y(x), singsol=all)
```

$$\frac{x^2}{2} - \frac{(-\text{signum}(y(x)^2 - 1))^{\frac{2}{3}} y(x) \text{hypergeom}\left(\left[\frac{1}{2}, \frac{2}{3}\right], \left[\frac{3}{2}\right], y(x)^2\right)}{\text{signum}(y(x)^2 - 1)^{\frac{2}{3}}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 6.768 (sec). Leaf size: 66

```
DSolve[y'[x]==x*(y[x]^2-1)^(2/3),y[x],x,IncludeSingularSolutions -> True]
```

$y(x)$

$$\rightarrow \text{InverseFunction}\left[\frac{\#1(1 - \#1^2)^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \#1^2\right)}{(\#1^2 - 1)^{2/3}} \&\right]\left[\frac{x^2}{2} + c_1\right]$$

$y(x) \rightarrow -1$

$y(x) \rightarrow 1$

4.11 problem 11

Internal problem ID [968]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60

Problem number: 11.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [$y = G(x, y')$]

$$y' - (x^2 + y^2)^2 = 0$$

X Solution by Maple

```
dsolve(diff(y(x),x)=(x^2+y(x)^2)^2,y(x), singsol=all)
```

No solution found

X Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==(x^2+y[x]^2)^2,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.12 problem 12

Internal problem ID [969]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60

Problem number: 12.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$y' - \sqrt{x+y} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve(diff(y(x),x)=(x+y(x))^(1/2),y(x), singsol=all)
```

$$\begin{aligned} x - 2\sqrt{x+y(x)} - \ln\left(-1 + \sqrt{x+y(x)}\right) \\ + \ln\left(1 + \sqrt{x+y(x)}\right) + \ln(x+y(x)-1) - c_1 = 0 \end{aligned}$$

✓ Solution by Mathematica

Time used: 9.242 (sec). Leaf size: 59

```
DSolve[y'[x]==(x+y[x])^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\rightarrow W\left(-e^{-\frac{x}{2}-1-\frac{c_1}{2}}\right)^2 + 2W\left(-e^{-\frac{x}{2}-1-\frac{c_1}{2}}\right) - x + 1 \\ y(x) &\rightarrow 1 - x \end{aligned}$$

4.13 problem 13

Internal problem ID [970]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60

Problem number: 13.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - \frac{\tan(y)}{x-1} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=tan(y(x))/(x-1),y(x), singsol=all)
```

$$y(x) = \arcsin(c_1(x-1))$$

✓ Solution by Mathematica

Time used: 7.817 (sec). Leaf size: 19

```
DSolve[y'[x]==Tan[y[x]]/(x-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \arcsin(e^{c_1}(x-1))$$

$$y(x) \rightarrow 0$$

4.14 problem 16

Internal problem ID [971]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60

Problem number: 16.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' - y^{\frac{2}{5}} = 0$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 11

```
dsolve([diff(y(x),x)=y(x)^(2/5),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{(3x + 5) \left(\frac{3x}{5} + 1\right)^{\frac{2}{3}}}{5}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 23

```
DSolve[{y'[x]==y[x]^(2/5),y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{(3x + 5)^{5/3}}{5 \cdot 5^{2/3}}$$

4.15 problem 18

Internal problem ID [972]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60

Problem number: 18.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' - 3x(y - 1)^{\frac{1}{3}} = 0$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=3*x*(y(x)-1)^(1/3),y(0) = 1],y(x), singsol=all)
```

$$y(x) = 1$$

✓ Solution by Mathematica

Time used: 0.296 (sec). Leaf size: 19

```
DSolve[{y'[x]==3*x*(y[x]-1)^(1/3),y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 1$$

$$y(x) \rightarrow (x^2)^{3/2} + 1$$

4.16 problem 19

Internal problem ID [973]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60

Problem number: 19.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - 3x(y - 1)^{\frac{1}{3}} = 0$$

With initial conditions

$$[y(0) = 9]$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)=3*x*(y(x)-1)^(1/3),y(0) = 9],y(x), singsol=all)
```

$$y(x) = x^2\sqrt{x^2 + 4} + 4\sqrt{x^2 + 4} + 1$$

✓ Solution by Mathematica

Time used: 0.125 (sec). Leaf size: 16

```
DSolve[{y'[x]==3*x*(y[x]-1)^(1/3),y[0]==9},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow (x^2 + 4)^{3/2} + 1$$

4.17 problem 20(a)

Internal problem ID [974]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60

Problem number: 20(a).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' - 3x(y - 1)^{\frac{1}{3}} = 0$$

With initial conditions

$$[y(3) = -7]$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 19

```
dsolve([diff(y(x),x)=3*x*(y(x)-1)^(1/3),y(3) = -7],y(x), singsol=all)
```

$$y(x) = 1 + \left(-11 + 2i\sqrt{3} + x^2\right)^{\frac{3}{2}}$$

✓ Solution by Mathematica

Time used: 0.134 (sec). Leaf size: 49

```
DSolve[{y'[x]==3*x*(y[x]-1)^(1/3),y[3]==-7},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 1 + \left(x^2 - 2i\sqrt{3} - 11\right)^{3/2}$$

$$y(x) \rightarrow 1 + \left(x^2 + 2i\sqrt{3} - 11\right)^{3/2}$$

5 Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

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5.1 problem Example 1

Internal problem ID [975]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: Example 1.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [Bernoulli]

$$y' - y - xy^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)-y(x)=x*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{1}{1 + e^{-x}c_1 - x}$$

✓ Solution by Mathematica

Time used: 0.11 (sec). Leaf size: 27

```
DSolve[y'[x]-y[x]==x*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^x}{-e^x(x-1) + c_1}$$

$$y(x) \rightarrow 0$$

5.2 problem Example 2

Internal problem ID [976]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: Example 2.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$y' - \frac{y + x e^{-\frac{y}{x}}}{x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)=(y(x)+x*exp(-y(x)/x))/x,y(x), singsol=all)
```

$$y(x) = \ln(\ln(x) + c_1) x$$

✓ Solution by Mathematica

Time used: 0.368 (sec). Leaf size: 13

```
DSolve[y'[x]==(y[x]+x*Exp[-y[x]/x])/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x \log(\log(x) + c_1)$$

5.3 problem Example 3(a) (As Riccati)

Internal problem ID [977]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: Example 3(a) (As Riccati).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Riccati]`

$$y'x^2 - y^2 - yx = -x^2$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve(x^2*diff(y(x),x)=y(x)^2+x*y(x)-x^2,y(x), singsol=all)
```

$$y(x) = -\tanh(\ln(x) + c_1) x$$

✓ Solution by Mathematica

Time used: 0.298 (sec). Leaf size: 298

```
DSolve[y'[x]==y[x]^2+x*y[x]-x^2,y[x],x,IncludeSingularSolutions -> True]
```

$y(x)$

$$\rightarrow \frac{5(\sqrt{5}-1)x \left(c_1 \operatorname{HermiteH}\left(\frac{1}{10}(-5+\sqrt{5}), \frac{\sqrt[4]{5}x}{\sqrt{2}}\right) - \operatorname{Hypergeometric1F1}\left(\frac{5}{4} - \frac{1}{4\sqrt{5}}, \frac{3}{2}, \frac{\sqrt{5}x^2}{2}\right) + \operatorname{Hypergeometric1F1}\left(\frac{1}{20}(5-\sqrt{5}), \frac{1}{2}, \frac{\sqrt{5}x^2}{2}\right) \right)}{10 \left(\operatorname{Hypergeometric1F1}\left(\frac{1}{20}(5-\sqrt{5}), \frac{1}{2}, \frac{\sqrt{5}x^2}{2}\right) \right)}$$

$$y(x) \rightarrow \frac{1}{2}(\sqrt{5}-1)x - \frac{(\sqrt{5}-5) \operatorname{HermiteH}\left(\frac{1}{10}(-15+\sqrt{5}), \frac{\sqrt[4]{5}x}{\sqrt{2}}\right)}{\sqrt{2}5^{3/4} \operatorname{HermiteH}\left(\frac{1}{10}(-5+\sqrt{5}), \frac{\sqrt[4]{5}x}{\sqrt{2}}\right)}$$

5.4 problem Example 3(b)

Internal problem ID [978]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: Example 3(b).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Riccati]`

$$y'x^2 - y^2 - yx = -x^2$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 19

```
dsolve([x^2*diff(y(x),x)=y(x)^2+x*y(x)-x^2,y(1) = 2],y(x), singsol=all)
```

$$y(x) = -\frac{x(x^2 + 3)}{x^2 - 3}$$

✓ Solution by Mathematica

Time used: 0.576 (sec). Leaf size: 20

```
DSolve[{x^2*y'[x]==y[x]^2+x*y[x]-x^2,y[1]==2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{x(x^2 + 3)}{x^2 - 3}$$

5.5 problem 1

Internal problem ID [979]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 1.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' + y - y^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)+y(x)=y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{1}{1 + c_1 e^x}$$

✓ Solution by Mathematica

Time used: 0.777 (sec). Leaf size: 54

```
DSolve[y'[x]+y[x]==y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{1}{\sqrt{1 + e^{2(x+c_1)}}}$$

$$y(x) \rightarrow \frac{1}{\sqrt{1 + e^{2(x+c_1)}}}$$

$$y(x) \rightarrow -1$$

$$y(x) \rightarrow 0$$

$$y(x) \rightarrow 1$$

5.6 problem 2

Internal problem ID [980]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 2.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class G', _rational, _Bernoulli]`

$$7y'x - 2y + \frac{x^2}{y^6} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 213

```
dsolve(7*x*diff(y(x),x)-2*y(x)=-x^2/y(x)^6,y(x), singsol=all)
```

$$\begin{aligned}y(x) &= (-\ln(x)x^2 + c_1x^2)^{\frac{1}{7}} \\y(x) &= \left(-\cos\left(\frac{\pi}{7}\right) - i\cos\left(\frac{5\pi}{14}\right)\right) (-\ln(x)x^2 + c_1x^2)^{\frac{1}{7}} \\y(x) &= \left(-\cos\left(\frac{\pi}{7}\right) + i\cos\left(\frac{5\pi}{14}\right)\right) (-\ln(x)x^2 + c_1x^2)^{\frac{1}{7}} \\y(x) &= \left(\cos\left(\frac{2\pi}{7}\right) - i\cos\left(\frac{3\pi}{14}\right)\right) (-\ln(x)x^2 + c_1x^2)^{\frac{1}{7}} \\y(x) &= \left(\cos\left(\frac{2\pi}{7}\right) + i\cos\left(\frac{3\pi}{14}\right)\right) (-\ln(x)x^2 + c_1x^2)^{\frac{1}{7}} \\y(x) &= \left(-\cos\left(\frac{3\pi}{7}\right) - i\cos\left(\frac{\pi}{14}\right)\right) (-\ln(x)x^2 + c_1x^2)^{\frac{1}{7}} \\y(x) &= \left(-\cos\left(\frac{3\pi}{7}\right) + i\cos\left(\frac{\pi}{14}\right)\right) (-\ln(x)x^2 + c_1x^2)^{\frac{1}{7}}\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.238 (sec). Leaf size: 181

```
DSolve[7*x*y'[x]-2*y[x]==-x^2/y[x]^6,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^{2/7} \sqrt[7]{-\log(x) + c_1}$$

$$y(x) \rightarrow -\sqrt[7]{-1} x^{2/7} \sqrt[7]{-\log(x) + c_1}$$

$$y(x) \rightarrow (-1)^{2/7} x^{2/7} \sqrt[7]{-\log(x) + c_1}$$

$$y(x) \rightarrow -(-1)^{3/7} x^{2/7} \sqrt[7]{-\log(x) + c_1}$$

$$y(x) \rightarrow (-1)^{4/7} x^{2/7} \sqrt[7]{-\log(x) + c_1}$$

$$y(x) \rightarrow -(-1)^{5/7} x^{2/7} \sqrt[7]{-\log(x) + c_1}$$

$$y(x) \rightarrow (-1)^{6/7} x^{2/7} \sqrt[7]{-\log(x) + c_1}$$

5.7 problem 3

Internal problem ID [981]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 3.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [Bernoulli]

$$y'x^2 + 2y - 2e^{\frac{1}{x}}\sqrt{y} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(x^2*diff(y(x),x)+2*y(x)=2*exp(1/x)*y(x)^(1/2),y(x), singsol=all)
```

$$\sqrt{y(x)} - \left(-\frac{1}{x} + c_1\right) e^{\frac{1}{x}} = 0$$

✓ Solution by Mathematica

Time used: 0.269 (sec). Leaf size: 39

```
DSolve[y'[x]+2*y[x]==2*Exp[1/x]*y[x]^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{4}e^{-2x} \left(\int_1^x 2e^{K[1]+\frac{1}{K[1]}} dK[1] + 2c_1 \right)^2$$

5.8 problem 4

Internal problem ID [982]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 4.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_rational, _Bernoulli]

$$(x^2 + 1)y' + 2yx - \frac{1}{(x^2 + 1)y} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 38

```
dsolve((1+x^2)*diff(y(x),x)+2*x*y(x)=1/((1+x^2)*y(x)),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{2x + c_1}}{x^2 + 1}$$

$$y(x) = -\frac{\sqrt{2x + c_1}}{x^2 + 1}$$

✓ Solution by Mathematica

Time used: 0.264 (sec). Leaf size: 46

```
DSolve[(1+x^2)*y'[x]+2*x*y[x]==1/((1+x^2)*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{\sqrt{2x + c_1}}{x^2 + 1}$$

$$y(x) \rightarrow \frac{\sqrt{2x + c_1}}{x^2 + 1}$$

5.9 problem 5

Internal problem ID [983]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 5.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [Bernoulli]

$$y' - yx - y^3x^3 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x)-x*y(x)=x^3*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{e^{-x^2}c_1 - x^2 + 1}}$$
$$y(x) = -\frac{1}{\sqrt{e^{-x^2}c_1 - x^2 + 1}}$$

✓ Solution by Mathematica

Time used: 1.866 (sec). Leaf size: 80

```
DSolve[y'[x]-x*y[x]==x^3*y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{ie^{\frac{x^2}{2}}}{\sqrt{e^{x^2}(x^2 - 1) - c_1}}$$
$$y(x) \rightarrow \frac{ie^{\frac{x^2}{2}}}{\sqrt{e^{x^2}(x^2 - 1) - c_1}}$$
$$y(x) \rightarrow 0$$

5.10 problem 6

Internal problem ID [984]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 6.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [rational, Bernoulli]

$$y' - \frac{(x+1)y}{3x} - y^4 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 179

```
dsolve(diff(y(x),x)-(1+x)/(3*x)*y(x)=y(x)^4,y(x), singsol=all)
```

$$y(x) = \frac{\left(x(e^{-x}c_1 - 3x + 3)^2\right)^{\frac{1}{3}}}{e^{-x}c_1 - 3x + 3}$$
$$y(x) = -\frac{\left(x(e^{-x}c_1 - 3x + 3)^2\right)^{\frac{1}{3}}}{2(e^{-x}c_1 - 3x + 3)} - \frac{i\sqrt{3}\left(x(e^{-x}c_1 - 3x + 3)^2\right)^{\frac{1}{3}}}{2(e^{-x}c_1 - 3x + 3)}$$
$$y(x) = -\frac{\left(x(e^{-x}c_1 - 3x + 3)^2\right)^{\frac{1}{3}}}{2(e^{-x}c_1 - 3x + 3)} + \frac{i\sqrt{3}\left(x(e^{-x}c_1 - 3x + 3)^2\right)^{\frac{1}{3}}}{2e^{-x}c_1 - 6x + 6}$$

✓ Solution by Mathematica

Time used: 1.88 (sec). Leaf size: 120

```
DSolve[y'[x]-(1+x)/(3*x)*y[x]==y[x]^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{e^{x/3}\sqrt[3]{x}}{\sqrt[3]{3e^x(x-1)-c_1}}$$

$$y(x) \rightarrow \frac{\sqrt[3]{-1}e^{x/3}\sqrt[3]{x}}{\sqrt[3]{3e^x(x-1)-c_1}}$$

$$y(x) \rightarrow -\frac{(-1)^{2/3}e^{x/3}\sqrt[3]{x}}{\sqrt[3]{3e^x(x-1)-c_1}}$$

$$y(x) \rightarrow 0$$

5.11 problem 7

Internal problem ID [985]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 7.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [Bernoulli]

$$y' - 2y - xy^3 = 0$$

With initial conditions

$$[y(0) = 2\sqrt{2}]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)-2*y(x)=x*y(x)^3,y(0) = 2*2^(1/2)],y(x), singsol=all)
```

$$y(x) = \frac{4}{\sqrt{-8x + 2}}$$

✓ Solution by Mathematica

Time used: 1.869 (sec). Leaf size: 34

```
DSolve[{y'[x]-2*y[x]==x*y[x]^3,y[0]==2*Sqrt[2]},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{2i\sqrt{2}e^{2x}}{\sqrt{e^{4x}(4x - 1)}}$$

5.12 problem 8

Internal problem ID [986]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 8.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' - yx - xy^{\frac{3}{2}} = 0$$

With initial conditions

$$[y(1) = 4]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 21

```
dsolve([diff(y(x),x)-x*y(x)=x*y(x)^(3/2),y(1) = 4],y(x), singsol=all)
```

$$y(x) = \frac{4}{\left(-2 + 3e^{-\frac{(x-1)(x+1)}{4}}\right)^2}$$

✓ Solution by Mathematica

Time used: 0.405 (sec). Leaf size: 55

```
DSolve[{y'[x]-x*y[x]==x*y[x]^(3/2),y[1]==4},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{4} \left(\tanh \left(\frac{1}{8} (-8 \operatorname{arctanh}(3) - x^2 + 1) \right) - 1 \right)^2$$

$$y(x) \rightarrow \frac{1}{4} \left(\tanh \left(\operatorname{arctanh}(5) - \frac{x^2}{8} + \frac{1}{8} \right) - 1 \right)^2$$

5.13 problem 9

Internal problem ID [987]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 9.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class G', _rational, _Bernoulli]`

$$y + y'x - x^4y^4 = 0$$

With initial conditions

$$\left[y(1) = \frac{1}{2} \right]$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 35

```
dsolve([x*diff(y(x),x)+y(x)=x^4*y(x)^4,y(1) = 1/2],y(x), singsol=all)
```

$$y(x) = \frac{(-(3x - 11)^2)^{\frac{1}{3}} (i\sqrt{3} - 1)}{6x^2 - 22x}$$

✓ Solution by Mathematica

Time used: 0.415 (sec). Leaf size: 19

```
DSolve[{x*y'[x]+y[x]==x^4*y[x]^4,y[1]==1/2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{\sqrt[3]{-x^3(3x - 11)}}$$

5.14 problem 10

Internal problem ID [988]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 10.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' - 2y - 2\sqrt{y} = 0$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 16

```
dsolve([diff(y(x),x)-2*y(x)=2*y(x)^(1/2),y(0) = 1],y(x), singsol=all)
```

$$y(x) = 4e^{2x} - 4e^x + 1$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 14

```
DSolve[{y'[x]-2*y[x]==2*y[x]^(1/2),y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow (1 - 2e^x)^2$$

5.15 problem 11

Internal problem ID [989]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 11.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$-4y + y' - \frac{48x}{y^2} = 0$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 17

```
dsolve([diff(y(x),x)-4*y(x)=48*x/y(x)^2,y(0) = 1],y(x), singsol=all)
```

$$y(x) = (2e^{12x} - 12x - 1)^{\frac{1}{3}}$$

✓ Solution by Mathematica

Time used: 4.195 (sec). Leaf size: 21

```
DSolve[{y'[x]-4*y[x]==48*x/y[x]^2,y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sqrt[3]{-12x + 2e^{12x} - 1}$$

5.16 problem 12

Internal problem ID [990]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 12.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class G', _rational, _Bernoulli]`

$$2yx + y'x^2 - y^3 = 0$$

With initial conditions

$$\left[y(1) = \frac{\sqrt{2}}{2} \right]$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 26

```
dsolve([x^2*diff(y(x),x)+2*x*y(x)=y(x)^3,y(1) = 1/2*2^(1/2)],y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{10} \sqrt{4x^6 + x}}{8x^5 + 2}$$

✓ Solution by Mathematica

Time used: 0.462 (sec). Leaf size: 29

```
DSolve[{x^2*y'[x]+2*x*y[x]==y[x]^3,y[1]==1/Sqrt[2]},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{\sqrt{\frac{5}{2}} \sqrt{x}}{\sqrt{4x^5 + 1}}$$

5.17 problem 13

Internal problem ID [991]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 13.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [Bernoulli]

$$y' - y - \sqrt{y}x = 0$$

With initial conditions

$$[y(0) = 4]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 26

```
dsolve([diff(y(x),x)-y(x)=x*y(x)^(1/2),y(0) = 4],y(x), singsol=all)
```

$$y(x) = (-8x - 16)e^{\frac{x}{2}} + x^2 + 4x + 16e^x + 4$$

✓ Solution by Mathematica

Time used: 0.146 (sec). Leaf size: 28

```
DSolve[{y'[x]-y[x]==x*y[x]^(1/2),y[0]==4},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow (x - 4e^{x/2} + 2)^2$$

$$y(x) \rightarrow (x + 2)^2$$

5.18 problem 15

Internal problem ID [992]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 15.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y' - \frac{x+y}{x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=(y(x)+x)/x,y(x), singsol=all)
```

$$y(x) = (\ln(x) + c_1) x$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 12

```
DSolve[y'[x]==(y[x]+x)/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x(\log(x) + c_1)$$

5.19 problem 16

Internal problem ID [993]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 16.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Bernoulli]`

$$y' - \frac{y^2 + 2yx}{x^2} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=(y(x)^2+2*x*y(x))/x^2,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{c_1 - x}$$

✓ Solution by Mathematica

Time used: 0.126 (sec). Leaf size: 23

```
DSolve[y'[x]==(y[x]^2+2*x*y[x])/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{x^2}{x - c_1}$$

$$y(x) \rightarrow 0$$

5.20 problem 17

Internal problem ID [994]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 17.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Bernoulli]`

$$xy^3y' - y^4 = x^4$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 58

```
dsolve(x*y(x)^3*diff(y(x),x)=y(x)^4+x^4,y(x), singsol=all)
```

$$y(x) = (4 \ln(x) + c_1)^{\frac{1}{4}} x$$

$$y(x) = -(4 \ln(x) + c_1)^{\frac{1}{4}} x$$

$$y(x) = -i(4 \ln(x) + c_1)^{\frac{1}{4}} x$$

$$y(x) = i(4 \ln(x) + c_1)^{\frac{1}{4}} x$$

✓ Solution by Mathematica

Time used: 0.2 (sec). Leaf size: 76

```
DSolve[x*y[x]^3*y'[x]==y[x]^4+x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -x\sqrt[4]{4\log(x) + c_1}$$

$$y(x) \rightarrow -ix\sqrt[4]{4\log(x) + c_1}$$

$$y(x) \rightarrow ix\sqrt[4]{4\log(x) + c_1}$$

$$y(x) \rightarrow x\sqrt[4]{4\log(x) + c_1}$$

5.21 problem 18

Internal problem ID [995]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 18.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$y' - \frac{y}{x} - \sec\left(\frac{y}{x}\right) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)=y(x)/x+sec(y(x)/x),y(x), singsol=all)
```

$$y(x) = \arcsin(\ln(x) + c_1) x$$

✓ Solution by Mathematica

Time used: 0.409 (sec). Leaf size: 13

```
DSolve[y'[x]==y[x]/x+Sec[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x \arcsin(\log(x) + c_1)$$

5.22 problem 19

Internal problem ID [996]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 19.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Riccati]`

$$y'x^2 - yx - y^2 = x^2$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 11

```
dsolve(x^2*diff(y(x),x)=x*y(x)+x^2+y(x)^2,y(x), singsol=all)
```

$$y(x) = \tan(\ln(x) + c_1) x$$

✓ Solution by Mathematica

Time used: 0.174 (sec). Leaf size: 13

```
DSolve[x^2*y'[x]==x*y[x]+x^2+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x \tan(\log(x) + c_1)$$

5.23 problem 20

Internal problem ID [997]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 20.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Bernoulli]`

$$xyy' - 2y^2 = x^2$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(x*y(x)*diff(y(x),x)=x^2+2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \sqrt{c_1 x^2 - 1} x$$

$$y(x) = -\sqrt{c_1 x^2 - 1} x$$

✓ Solution by Mathematica

Time used: 0.441 (sec). Leaf size: 38

```
DSolve[x*y[x]*y'[x]==x^2+2*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -x\sqrt{-1 + c_1 x^2}$$

$$y(x) \rightarrow x\sqrt{-1 + c_1 x^2}$$

5.24 problem 21

Internal problem ID [998]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 21.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A']]`

$$y' - \frac{2y^2 + x^2 e^{-\frac{y^2}{x^2}}}{2yx} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve(diff(y(x),x)=(2*y(x)^2+x^2*exp(-(y(x)/x)^2))/(2*x*y(x)),y(x), singsol=all)
```

$$y(x) = \sqrt{\ln(\ln(x) + c_1)} x$$

$$y(x) = -\sqrt{\ln(\ln(x) + c_1)} x$$

✓ Solution by Mathematica

Time used: 2.166 (sec). Leaf size: 38

```
DSolve[y'[x]==(2*y[x]^2+x^2*Exp[-(y[x]/x)^2])/(2*x*y[x]),y[x],x,IncludeSingularSolutions -
```

$$y(x) \rightarrow -x \sqrt{\log(\log(x) + 2c_1)}$$

$$y(x) \rightarrow x \sqrt{\log(\log(x) + 2c_1)}$$

5.25 problem 22

Internal problem ID [999]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 22.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Bernoulli]`

$$y' - \frac{yx + y^2}{x^2} = 0$$

With initial conditions

$$[y(-1) = 2]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 21

```
dsolve([diff(y(x),x)=(x*y(x)+y(x)^2)/x^2,y(-1) = 2],y(x), singsol=all)
```

$$y(x) = -\frac{2ix}{2i \ln(x) + i + 2\pi}$$

✓ Solution by Mathematica

Time used: 0.126 (sec). Leaf size: 25

```
DSolve[{y'[x]==(x*y[x]+y[x]^2)/x^2,y[-1]==2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{2ix}{2i \log(x) + 2\pi + i}$$

5.26 problem 23

Internal problem ID [1000]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 23.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Bernoulli]`

$$y' - \frac{x^3 + y^3}{y^2 x} = 0$$

With initial conditions

$$[y(1) = 3]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)=(x^3+y(x)^3)/(x*y(x)^2),y(1) = 3],y(x), singsol=all)
```

$$y(x) = (3 \ln(x) + 27)^{\frac{1}{3}} x$$

✓ Solution by Mathematica

Time used: 0.21 (sec). Leaf size: 20

```
DSolve[{y'[x]==(x^3+y[x]^3)/(x*y[x]^2),y[1]==3},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sqrt[3]{3x} \sqrt[3]{\log(x) + 9}$$

5.27 problem 24

Internal problem ID [1001]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 24.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Bernoulli]`

$$xyy' + y^2 = -x^2$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 18

```
dsolve([x*y(x)*diff(y(x),x)+x^2+y(x)^2=0,y(1) = 2],y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-2x^4 + 18}}{2x}$$

✓ Solution by Mathematica

Time used: 0.216 (sec). Leaf size: 25

```
DSolve[{x*y[x]*y'[x]+x^2+y[x]^2==0,y[1]==2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{\sqrt{9 - x^4}}{\sqrt{2}x}$$

5.28 problem 25

Internal problem ID [1002]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 25.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Riccati]`

$$y' - \frac{y^2 - 3yx - 5x^2}{x^2} = 0$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.39 (sec). Leaf size: 23

```
dsolve([diff(y(x),x)=(y(x)^2-3*x*y(x)-5*x^2)/x^2,y(1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{-2x^7 + 5x}{2x^6 + 1}$$

✓ Solution by Mathematica

Time used: 1.947 (sec). Leaf size: 24

```
DSolve[{y'[x]==(y[x]^2-3*x*y[x]-5*x^2)/x^2,y[1]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{5x - 2x^7}{2x^6 + 1}$$

5.29 problem 26

Internal problem ID [1003]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 26.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Riccati]`

$$y'x^2 - y^2 - 4yx = 2x^2$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 21

```
dsolve([x^2*diff(y(x),x)=2*x^2+y(x)^2+4*x*y(x),y(1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{-4x^2 + 3x}{2x - 3}$$

✓ Solution by Mathematica

Time used: 0.492 (sec). Leaf size: 20

```
DSolve[{x^2*y'[x]==2*x^2+y[x]^2+4*x*y[x],y[1]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{x(4x - 3)}{2x - 3}$$

5.30 problem 27

Internal problem ID [1004]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 27.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$xyy' - 4y^2 = 3x^2$$

With initial conditions

$$[y(1) = \sqrt{3}]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 15

```
dsolve([x*y(x)*diff(y(x),x)=3*x^2+4*y(x)^2,y(1) = 3^(1/2)],y(x), singsol=all)
```

$$y(x) = \sqrt{4x^6 - 1} x$$

✓ Solution by Mathematica

Time used: 0.603 (sec). Leaf size: 18

```
DSolve[{x*y[x]*y'[x]==3*x^2+4*y[x]^2,y[1]==Sqrt[3]},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x\sqrt{4x^6 - 1}$$

5.31 problem 28

Internal problem ID [1005]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 28.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, [_Abel, '2nd type', 'cl`

$$y' - \frac{x+y}{x-y} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)=(x+y(x))/(x-y(x)),y(x), singsol=all)
```

$$y(x) = \tan \left(\text{RootOf} \left(-2_Z + \ln \left(\frac{1}{\cos(_Z)^2} \right) + 2 \ln(x) + 2c_1 \right) \right) x$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 36

```
DSolve[y'[x]==(x+y[x])/(x-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve} \left[\frac{1}{2} \log \left(\frac{y(x)^2}{x^2} + 1 \right) - \arctan \left(\frac{y(x)}{x} \right) = -\log(x) + c_1, y(x) \right]$$

5.32 problem 29

Internal problem ID [1006]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 29.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A']]`

$$(y'x - y)(\ln(y) - \ln(x)) = x$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 19

```
dsolve((diff(y(x),x)*x-y(x))*(ln(y(x))-ln(x))=x,y(x), singsol=all)
```

$$y(x) = e^{\text{LambertW}(\ln(c_1x)e^{-1})+1}x$$

✓ Solution by Mathematica

Time used: 60.147 (sec). Leaf size: 24

```
DSolve[(y'[x]*x-y[x])*(Log[y[x]]-Log[x])==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x(\log(x) + c_1)}{W\left(\frac{\log(x)+c_1}{e}\right)}$$

5.33 problem 30

Internal problem ID [1007]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 30.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _dAlembert]`

$$y' - \frac{y^3 + 2xy^2 + x^2y + x^3}{x(x+y)^2} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 96

```
dsolve(diff(y(x),x)=(y(x)^3+2*x*y(x)^2+x^2*y(x)+x^3)/(x*(y(x)+x)^2),y(x), singsol=all)
```

$$y(x) = (3 \ln(x) + 3c_1)^{\frac{1}{3}} x - x$$

$$y(x) = \left(-\frac{(3 \ln(x) + 3c_1)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}(3 \ln(x) + 3c_1)^{\frac{1}{3}}}{2} \right) x - x$$

$$y(x) = \left(-\frac{(3 \ln(x) + 3c_1)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}(3 \ln(x) + 3c_1)^{\frac{1}{3}}}{2} \right) x - x$$

✓ Solution by Mathematica

Time used: 1.409 (sec). Leaf size: 109

```
DSolve[y'[x]==(y[x]^3+2*x*y[x]^2+x^2*y[x]+x^3)/(x*(y[x]+x)^2),y[x],x,IncludeSingularSolution
```

$$y(x) \rightarrow -x + \sqrt[3]{x^3(3\log(x) + 1 + 3c_1)}$$

$$y(x) \rightarrow -x + \frac{1}{2}i(\sqrt{3} + i) \sqrt[3]{x^3(3\log(x) + 1 + 3c_1)}$$

$$y(x) \rightarrow -x - \frac{1}{2}(1 + i\sqrt{3}) \sqrt[3]{x^3(3\log(x) + 1 + 3c_1)}$$

5.34 problem 31

Internal problem ID [1008]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 31.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$y' - \frac{x + 2y}{y + 2x} = 0$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 397

```
dsolve(diff(y(x),x)=(x+2*y(x))/(2*x+y(x)),y(x), singsol=all)
```

$$y(x) = -\frac{x \left(-c_1^2 - c_1^2 \left(\frac{\left(3\sqrt{3} \sqrt{27c_1^2 x^2 - 1 + 27c_1 x} \right)^{\frac{1}{3}}}{3xc_1} + \frac{1}{xc_1 \left(3\sqrt{3} \sqrt{27c_1^2 x^2 - 1 + 27c_1 x} \right)^{\frac{1}{3}}} \right) \right)}{c_1^2}$$

$$y(x) = \frac{x \left(-c_1^2 - c_1^2 \left(-\frac{\left(3\sqrt{3} \sqrt{27c_1^2 x^2 - 1 + 27c_1 x} \right)^{\frac{1}{3}}}{6xc_1} - \frac{1}{2xc_1 \left(3\sqrt{3} \sqrt{27c_1^2 x^2 - 1 + 27c_1 x} \right)^{\frac{1}{3}}} - \frac{i\sqrt{3} \left(\frac{\left(3\sqrt{3} \sqrt{27c_1^2 x^2 - 1 + 27c_1 x} \right)^{\frac{1}{3}}}{3xc_1} - \frac{1}{xc_1 \left(3\sqrt{3} \sqrt{27c_1^2 x^2 - 1 + 27c_1 x} \right)^{\frac{1}{3}}} \right)}{2} \right) \right)}{c_1^2}$$

$$y(x) = \frac{x \left(-c_1^2 - c_1^2 \left(-\frac{\left(3\sqrt{3} \sqrt{27c_1^2 x^2 - 1 + 27c_1 x} \right)^{\frac{1}{3}}}{6xc_1} - \frac{1}{2xc_1 \left(3\sqrt{3} \sqrt{27c_1^2 x^2 - 1 + 27c_1 x} \right)^{\frac{1}{3}}} + \frac{i\sqrt{3} \left(\frac{\left(3\sqrt{3} \sqrt{27c_1^2 x^2 - 1 + 27c_1 x} \right)^{\frac{1}{3}}}{3xc_1} - \frac{1}{xc_1 \left(3\sqrt{3} \sqrt{27c_1^2 x^2 - 1 + 27c_1 x} \right)^{\frac{1}{3}}} \right)}{2} \right) \right)}{c_1^2}$$

✓ Solution by Mathematica

Time used: 30.082 (sec). Leaf size: 382

`DSolve[y'[x]==(x+2*y[x])/(2*x+y[x]),y[x],x,IncludeSingularSolutions -> True]`

$$y(x) \rightarrow \frac{\sqrt[3]{\sqrt{3}\sqrt{27e^{4c_1}x^2 + e^{6c_1}} - 9e^{2c_1}x}}{3^{2/3}} - \frac{e^{2c_1}}{\sqrt[3]{3}\sqrt[3]{\sqrt{3}\sqrt{27e^{4c_1}x^2 + e^{6c_1}} - 9e^{2c_1}x}} + x$$

$$y(x) \rightarrow \frac{i(\sqrt{3} + i) \sqrt[3]{\sqrt{3}\sqrt{27e^{4c_1}x^2 + e^{6c_1}} - 9e^{2c_1}x}}{2 \cdot 3^{2/3}} + \frac{(1 + i\sqrt{3}) e^{2c_1}}{2\sqrt[3]{3}\sqrt[3]{\sqrt{3}\sqrt{27e^{4c_1}x^2 + e^{6c_1}} - 9e^{2c_1}x}} + x$$

$$y(x) \rightarrow -\frac{(1 + i\sqrt{3}) \sqrt[3]{\sqrt{3}\sqrt{27e^{4c_1}x^2 + e^{6c_1}} - 9e^{2c_1}x}}{2 \cdot 3^{2/3}} + \frac{(1 - i\sqrt{3}) e^{2c_1}}{2\sqrt[3]{3}\sqrt[3]{\sqrt{3}\sqrt{27e^{4c_1}x^2 + e^{6c_1}} - 9e^{2c_1}x}} + x$$

5.35 problem 32

Internal problem ID [1009]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 32.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$y' - \frac{y}{y - 2x} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 335

`dsolve(diff(y(x),x)=y(x)/(y(x)-2*x),y(x), singsol=all)`

$$y(x) = \frac{\left(-12c_1 + 8x^3 + 4\sqrt{-12c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{2} + \frac{\left(-12c_1 + 8x^3 + 4\sqrt{-12c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{2x^2} + x$$

$$y(x) = -\frac{\left(-12c_1 + 8x^3 + 4\sqrt{-12c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{4} - \frac{x^2}{\left(-12c_1 + 8x^3 + 4\sqrt{-12c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}} + x - \frac{i\sqrt{3} \left(\frac{\left(-12c_1 + 8x^3 + 4\sqrt{-12c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{2} - \frac{2x^2}{\left(-12c_1 + 8x^3 + 4\sqrt{-12c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}} \right)}{2}$$

$$y(x) = -\frac{\left(-12c_1 + 8x^3 + 4\sqrt{-12c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{4} - \frac{x^2}{\left(-12c_1 + 8x^3 + 4\sqrt{-12c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}} + x + \frac{i\sqrt{3} \left(\frac{\left(-12c_1 + 8x^3 + 4\sqrt{-12c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{2} - \frac{2x^2}{\left(-12c_1 + 8x^3 + 4\sqrt{-12c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}} \right)}{2}$$

✓ Solution by Mathematica

Time used: 37.127 (sec). Leaf size: 479

`DSolve[y'[x]==y[x]/(y[x]-2*x),y[x],x,IncludeSingularSolutions -> True]`

$$y(x) \rightarrow \frac{\sqrt[3]{2x^3 + \sqrt{e^{6c_1} - 4e^{3c_1}x^3} - e^{3c_1}}}{\sqrt[3]{2}} + \frac{\sqrt[3]{2}x^2}{\sqrt[3]{2x^3 + \sqrt{e^{6c_1} - 4e^{3c_1}x^3} - e^{3c_1}}} + x$$

$$y(x) \rightarrow \frac{i(\sqrt{3} + i) \sqrt[3]{2x^3 + \sqrt{e^{6c_1} - 4e^{3c_1}x^3} - e^{3c_1}}}{2\sqrt[3]{2}} - \frac{(1 + i\sqrt{3})x^2}{2^{2/3}\sqrt[3]{2x^3 + \sqrt{e^{6c_1} - 4e^{3c_1}x^3} - e^{3c_1}}} + x$$

$$y(x) \rightarrow -\frac{(1 + i\sqrt{3}) \sqrt[3]{2x^3 + \sqrt{e^{6c_1} - 4e^{3c_1}x^3} - e^{3c_1}}}{2\sqrt[3]{2}} + \frac{i(\sqrt{3} + i)x^2}{2^{2/3}\sqrt[3]{2x^3 + \sqrt{e^{6c_1} - 4e^{3c_1}x^3} - e^{3c_1}}} + x$$

$$y(x) \rightarrow 0$$

$$y(x) \rightarrow -\frac{i(\sqrt[3]{x^3} - x) \left((\sqrt{3} - i) \sqrt[3]{x^3} - 2ix \right)}{2x}$$

$$y(x) \rightarrow \frac{i(\sqrt[3]{x^3} - x) \left((\sqrt{3} + i) \sqrt[3]{x^3} + 2ix \right)}{2x}$$

$$y(x) \rightarrow \sqrt[3]{x^3} + \frac{(x^3)^{2/3}}{x} + x$$

5.36 problem 33

Internal problem ID [1010]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 33.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _dAlembert]`

$$y' - \frac{xy^2 + 2y^3}{x^3 + x^2y + xy^2} = 0$$

✓ Solution by Maple

Time used: 1.109 (sec). Leaf size: 129

```
dsolve(diff(y(x),x)=(x*y(x)^2+2*y(x)^3)/(x^3+x^2*y(x)+x*y(x)^2),y(x), singsol=all)
```

$$\begin{aligned} y(x) = & c_1 \text{RootOf} \left(_Z^8 c_1 x^2 + 2_Z^6 c_1 x^2 + _Z^4 c_1 x^2 - 2_Z^2 - 1 \right)^6 x^3 \\ & + 2c_1 \text{RootOf} \left(_Z^8 c_1 x^2 + 2_Z^6 c_1 x^2 + _Z^4 c_1 x^2 - 2_Z^2 - 1 \right)^4 x^3 \\ & + c_1 \text{RootOf} \left(_Z^8 c_1 x^2 + 2_Z^6 c_1 x^2 + _Z^4 c_1 x^2 - 2_Z^2 - 1 \right)^2 x^3 - x \end{aligned}$$

✓ Solution by Mathematica

Time used: 60.157 (sec). Leaf size: 1989

`DSolve[y'[x]==(x*y[x]^2+2*y[x]^3)/(x^3+x^2*y[x]+x*y[x]^2),y[x],x,IncludeSingularSolutions ->`

$$y(x) \rightarrow \frac{1}{6} \left(-\sqrt{3} \sqrt{-2e^{2c_1}x^4 + 3x^2 + \frac{e^{4c_1}x^8}{\sqrt[3]{e^{6c_1}x^{12} + 54e^{2c_1}x^8 + 6\sqrt{3}\sqrt{e^{4c_1}x^{16}(27 + e^{4c_1}x^4)}}}} + \sqrt[3]{e^{6c_1}x^{12} + 54e^{2c_1}x^8 + 6\sqrt{3}\sqrt{e^{4c_1}x^{16}(27 + e^{4c_1}x^4)}}} - \sqrt{3} \sqrt{-4e^{2c_1}x^4 + 6x^2 - \frac{e^{4c_1}x^8}{\sqrt[3]{e^{6c_1}x^{12} + 54e^{2c_1}x^8 + 6\sqrt{3}\sqrt{e^{4c_1}x^{16}(27 + e^{4c_1}x^4)}}}} - \sqrt[3]{e^{6c_1}x^{12} + 54e^{2c_1}x^8 + 6\sqrt{3}\sqrt{e^{4c_1}x^{16}(27 + e^{4c_1}x^4)}}} + 3x \right)$$

$$y(x) \rightarrow \frac{1}{6} \left(-\sqrt{3} \sqrt{-2e^{2c_1}x^4 + 3x^2 + \frac{e^{4c_1}x^8}{\sqrt[3]{e^{6c_1}x^{12} + 54e^{2c_1}x^8 + 6\sqrt{3}\sqrt{e^{4c_1}x^{16}(27 + e^{4c_1}x^4)}}}} + \sqrt[3]{e^{6c_1}x^{12} + 54e^{2c_1}x^8 + 6\sqrt{3}\sqrt{e^{4c_1}x^{16}(27 + e^{4c_1}x^4)}}} + \sqrt{3} \sqrt{-4e^{2c_1}x^4 + 6x^2 - \frac{e^{4c_1}x^8}{\sqrt[3]{e^{6c_1}x^{12} + 54e^{2c_1}x^8 + 6\sqrt{3}\sqrt{e^{4c_1}x^{16}(27 + e^{4c_1}x^4)}}}} - \sqrt[3]{e^{6c_1}x^{12} + 54e^{2c_1}x^8 + 6\sqrt{3}\sqrt{e^{4c_1}x^{16}(27 + e^{4c_1}x^4)}}} + 3x \right)$$

$$y(x) \left($$

5.37 problem 34

Internal problem ID [1011]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 34.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _dAlembert]`

$$y' - \frac{x^3 + x^2y + 3y^3}{x^3 + 3xy^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 369

`dsolve(diff(y(x),x)=(x^3+x^2*y(x)+3*y(x)^3)/(x^3+3*x*y(x)^2),y(x), singsol=all)`

$$y(x) = \left(\frac{\left(108 \ln(x) + 108c_1 + 12\sqrt{12 + 81 \ln(x)^2 + 162 \ln(x) c_1 + 81c_1^2}\right)^{\frac{1}{3}}}{6} - \frac{2}{\left(108 \ln(x) + 108c_1 + 12\sqrt{12 + 81 \ln(x)^2 + 162 \ln(x) c_1 + 81c_1^2}\right)^{\frac{1}{3}}} \right) x$$

$$y(x) = \left(-\frac{\left(108 \ln(x) + 108c_1 + 12\sqrt{12 + 81 \ln(x)^2 + 162 \ln(x) c_1 + 81c_1^2}\right)^{\frac{1}{3}}}{12} + \frac{1}{\left(108 \ln(x) + 108c_1 + 12\sqrt{12 + 81 \ln(x)^2 + 162 \ln(x) c_1 + 81c_1^2}\right)^{\frac{1}{3}}} \right) - \frac{i\sqrt{3} \left(\frac{\left(108 \ln(x) + 108c_1 + 12\sqrt{12 + 81 \ln(x)^2 + 162 \ln(x) c_1 + 81c_1^2}\right)^{\frac{1}{3}}}{6} + \frac{2}{\left(108 \ln(x) + 108c_1 + 12\sqrt{12 + 81 \ln(x)^2 + 162 \ln(x) c_1 + 81c_1^2}\right)^{\frac{1}{3}}} \right)}{2}$$

$$y(x) = \left(-\frac{\left(108 \ln(x) + 108c_1 + 12\sqrt{12 + 81 \ln(x)^2 + 162 \ln(x) c_1 + 81c_1^2}\right)^{\frac{1}{3}}}{12} + \frac{1}{\left(108 \ln(x) + 108c_1 + 12\sqrt{12 + 81 \ln(x)^2 + 162 \ln(x) c_1 + 81c_1^2}\right)^{\frac{1}{3}}} \right) + \frac{i\sqrt{3} \left(\frac{\left(108 \ln(x) + 108c_1 + 12\sqrt{12 + 81 \ln(x)^2 + 162 \ln(x) c_1 + 81c_1^2}\right)^{\frac{1}{3}}}{6 \cdot 165} + \frac{2}{\left(108 \ln(x) + 108c_1 + 12\sqrt{12 + 81 \ln(x)^2 + 162 \ln(x) c_1 + 81c_1^2}\right)^{\frac{1}{3}}} \right)}{2}$$

✓ Solution by Mathematica

Time used: 27.532 (sec). Leaf size: 398

`DSolve[y'[x]==(x^3+x^2*y[x]+3*y[x]^3)/(x^3+3*x*y[x]^2),y[x],x,IncludeSingularSolutions -> Tr`

$$y(x) \rightarrow \frac{-2\sqrt[3]{3}x^2 + \sqrt[3]{2}\left(\sqrt{3}\sqrt{x^6(4+27(\log(x)+c_1)^2)} + 9x^3\log(x) + 9c_1x^3\right)^{2/3}}{6^{2/3}\sqrt[3]{\sqrt{3}\sqrt{x^6(4+27(\log(x)+c_1)^2)} + 9x^3\log(x) + 9c_1x^3}}$$

$$y(x) \rightarrow \frac{2\sqrt[3]{3}(1+i\sqrt{3})x^2 + i\sqrt[3]{2}(\sqrt{3}+i)\left(\sqrt{3}\sqrt{x^6(4+27(\log(x)+c_1)^2)} + 9x^3\log(x) + 9c_1x^3\right)^{2/3}}{2\cdot 6^{2/3}\sqrt[3]{\sqrt{3}\sqrt{x^6(4+27(\log(x)+c_1)^2)} + 9x^3\log(x) + 9c_1x^3}}$$

$$y(x) \rightarrow \frac{2\sqrt[3]{3}(1-i\sqrt{3})x^2 - \sqrt[3]{2}(1+i\sqrt{3})\left(\sqrt{3}\sqrt{x^6(4+27(\log(x)+c_1)^2)} + 9x^3\log(x) + 9c_1x^3\right)^{2/3}}{2\cdot 6^{2/3}\sqrt[3]{\sqrt{3}\sqrt{x^6(4+27(\log(x)+c_1)^2)} + 9x^3\log(x) + 9c_1x^3}}$$

5.38 problem 35(a)

Internal problem ID [1012]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 35(a).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Riccati]`

$$y'x^2 - y^2 - yx = -4x^2$$

With initial conditions

$$[y(-1) = 0]$$

✓ Solution by Maple

Time used: 0.406 (sec). Leaf size: 19

```
dsolve([x^2*diff(y(x),x)=y(x)^2+x*y(x)-4*x^2,y(-1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{-2x^5 + 2x}{x^4 + 1}$$

✓ Solution by Mathematica

Time used: 2.186 (sec). Leaf size: 20

```
DSolve[{x^2*y'[x]==y[x]^2+x*y[x]-4*x^2,y[-1]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{2x(x^4 - 1)}{x^4 + 1}$$

5.39 problem 36(a)

Internal problem ID [1013]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 36(a).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, [_Abel, '2nd type', 'cl`

$$xyy' - y^2 + yx = x^2$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 25

```
dsolve(x*y(x)*diff(y(x),x)=x^2-x*y(x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = e^{-\text{LambertW}\left(\frac{e^{-1}e^{-c_1}}{x}\right) - c_1 - 1} + x$$

✓ Solution by Mathematica

Time used: 3.649 (sec). Leaf size: 25

```
DSolve[x*y[x]*y'[x]==x^2-x*y[x]+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x \left(1 + W \left(\frac{e^{-1+c_1}}{x} \right) \right)$$

$$y(x) \rightarrow x$$

5.40 problem 37(a)

Internal problem ID [1014]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 37(a).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, [_Abel, '2nd type', 'cl`

$$y' - \frac{2y^2 - yx + 2x^2}{yx + 2x^2} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 43

```
dsolve(diff(y(x),x)=(2*y(x)^2-x*y(x)+2*x^2)/(x*y(x)+2*x^2),y(x), singsol=all)
```

$$y(x) = \text{RootOf} \left(_Z^4 + c_1x + 16 + (-3c_1x - 32)_Z + (3c_1x + 24)_Z^2 + (-c_1x - 8)_Z^3 \right) x$$

✓ Solution by Mathematica

Time used: 60.166 (sec). Leaf size: 1913

`DSolve[y'[x]==(2*y[x]^2-x*y[x]+2*x^2)/(x*y[x]+2*x^2),y[x],x,IncludeSingularSolutions -> True`

$$y(x) \rightarrow \frac{1}{12} \left(- \sqrt{9e^{2c_1}x^4 + 6 \cdot 2^{2/3} \sqrt[3]{3\sqrt{3}\sqrt{e^{3c_1}x^{15}}(-256 + 27e^{c_1}x)} + 27e^{2c_1}x^8 + 24e^{c_1}x^3} \left(-3 + \frac{1}{\sqrt[3]{\sqrt{3}\sqrt{e^{3c_1}x^{15}}}} \right) \right. \\ \left. -6 \sqrt{\frac{1}{2}x^2(-8 + e^{c_1}x)^2 + 4x^2(-8 + e^{c_1}x) - \frac{\sqrt[3]{27e^{2c_1}x^8 + \sqrt{729e^{4c_1}x^{16} - 6912e^{3c_1}x^{15}}}}{3\sqrt[3]{2}} - \frac{1}{\sqrt[3]{\sqrt{3}\sqrt{e^{3c_1}x^{15}}}}} \right) - 3x(-8 + e^{c_1}x)} \right)$$

$$y(x) \rightarrow \frac{1}{12} \left(- \sqrt{9e^{2c_1}x^4 + 6 \cdot 2^{2/3} \sqrt[3]{3\sqrt{3}\sqrt{e^{3c_1}x^{15}}(-256 + 27e^{c_1}x)} + 27e^{2c_1}x^8 + 24e^{c_1}x^3} \left(-3 + \frac{1}{\sqrt[3]{\sqrt{3}\sqrt{e^{3c_1}x^{15}}}} \right) \right. \\ \left. +6 \sqrt{\frac{1}{2}x^2(-8 + e^{c_1}x)^2 + 4x^2(-8 + e^{c_1}x) - \frac{170 \sqrt[3]{27e^{2c_1}x^8 + \sqrt{729e^{4c_1}x^{16} - 6912e^{3c_1}x^{15}}}}{3\sqrt[3]{2}} - \frac{1}{\sqrt[3]{\sqrt{3}\sqrt{e^{3c_1}x^{15}}}}} \right)} \right)$$

5.41 problem 38

Internal problem ID [1015]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 38.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, [_Abel, '2nd type', 'cl`

$$y' - \frac{x^2 + yx + y^2}{yx} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve(diff(y(x),x)=(x*y(x)+x^2+y(x)^2)/(x*y(x)),y(x), singsol=all)
```

$$y(x) = e^{-\text{LambertW}\left(-\frac{e^{-1}e^{-c_1}}{x}\right)-c_1-1} - x$$

✓ Solution by Mathematica

Time used: 4.422 (sec). Leaf size: 31

```
DSolve[y'[x]==(x*y[x]+x^2+y[x]^2)/(x*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -x \left(1 + W \left(-\frac{e^{-1-c_1}}{x} \right) \right)$$

$$y(x) \rightarrow -x$$

5.42 problem 41

Internal problem ID [1016]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 41.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class C', _rational, [_Abel, '2nd type', 'cl`

$$y' - \frac{-6x + y - 3}{2x - y - 1} = 0$$

✓ Solution by Maple

Time used: 0.421 (sec). Leaf size: 51

```
dsolve(diff(y(x),x)=(-6*x+y(x)-3)/(2*x-y(x)-1),y(x), singsol=all)
```

$$y(x) = -3 - \frac{\text{RootOf}\left(_Z^{25} - 5(x+1)^5 c_1 _Z^5 - (x+1)^5 c_1\right)^{20}}{c_1} - 3(x+1)^5$$

$(x+1)^4$

✓ Solution by Mathematica

Time used: 60.095 (sec). Leaf size: 3011

```
DSolve[y'[x]==(-6*x+y[x]-3)/(2*x-y[x]-1),y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

5.43 problem 42

Internal problem ID [1017]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 42.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class C', _rational, [_Abel, '2nd type', 'cl`

$$y' - \frac{2x + y + 1}{x + 2y - 4} = 0$$

✓ Solution by Maple

Time used: 1.594 (sec). Leaf size: 64

```
dsolve(diff(y(x),x)=(2*x+y(x)+1)/(x+2*y(x)-4),y(x), singsol=all)
```

$$y(x) = 3 + \frac{(2+x) \left(\text{RootOf} \left(_Z^{16} + 2(2+x)^4 c_1 _Z^4 - (2+x)^4 c_1 \right)^4 - 1 \right)}{\text{RootOf} \left(_Z^{16} + 2(2+x)^4 c_1 _Z^4 - (2+x)^4 c_1 \right)^4}$$

✓ Solution by Mathematica

Time used: 60.312 (sec). Leaf size: 8077

```
DSolve[y'[x]==(2*x+y[x]+1)/(x+2*y[x]-4),y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

5.44 problem 43

Internal problem ID [1018]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 43.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class C', _rational, [_Abel, '2nd type', 'cl`

$$y' - \frac{-x + 3y - 14}{x + y - 2} = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 28

```
dsolve(diff(y(x),x)=(-x+3*y(x)-14)/(x+y(x)-2),y(x), singsol=all)
```

$$y(x) = 4 + \frac{(2+x)(\text{LambertW}(-2c_1(2+x)) + 2)}{\text{LambertW}(-2c_1(2+x))}$$

✓ Solution by Mathematica

Time used: 1.05 (sec). Leaf size: 144

```
DSolve[y'[x]==(-x+3*y[x]-14)/(x+y[x]-2),y[x],x,IncludeSingularSolutions->True]
```

$$\text{Solve} \left[\frac{2^{2/3} \left(x \log \left(\frac{y(x)-x-6}{y(x)+x-2} \right) - (x+6) \log \left(\frac{x+2}{y(x)+x-2} \right) + 6 \log \left(\frac{y(x)-x-6}{y(x)+x-2} \right) + y(x) \left(\log \left(\frac{x+2}{y(x)+x-2} \right) - \log \left(\frac{y(x)-x-6}{y(x)+x-2} \right) \right) \right)}{9(-y(x)+x+6)} \right]$$

5.45 problem 44

Internal problem ID [1019]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 44.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class G', _rational, _Bernoulli]`

$$3xy^2y' - y^3 = x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 78

```
dsolve(3*x*y(x)^2*diff(y(x),x)=y(x)^3+x,y(x), singsol=all)
```

$$y(x) = (x \ln(x) + c_1 x)^{\frac{1}{3}}$$

$$y(x) = -\frac{(x \ln(x) + c_1 x)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}(x \ln(x) + c_1 x)^{\frac{1}{3}}}{2}$$

$$y(x) = -\frac{(x \ln(x) + c_1 x)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}(x \ln(x) + c_1 x)^{\frac{1}{3}}}{2}$$

✓ Solution by Mathematica

Time used: 0.196 (sec). Leaf size: 69

```
DSolve[3*x*y[x]^2*y'[x]==y[x]^3+x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sqrt[3]{x} \sqrt[3]{\log(x) + c_1}$$

$$y(x) \rightarrow -\sqrt[3]{-1} \sqrt[3]{x} \sqrt[3]{\log(x) + c_1}$$

$$y(x) \rightarrow (-1)^{2/3} \sqrt[3]{x} \sqrt[3]{\log(x) + c_1}$$

5.46 problem 45

Internal problem ID [1020]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 45.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class G', _rational, _Bernoulli]`

$$xyy' - 6y^2 = 3x^6$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(x*y(x)*diff(y(x),x)=3*x^6+6*y(x)^2,y(x), singsol=all)
```

$$y(x) = \sqrt{c_1 x^6 - 1} x^3$$

$$y(x) = -\sqrt{c_1 x^6 - 1} x^3$$

✓ Solution by Mathematica

Time used: 0.672 (sec). Leaf size: 42

```
DSolve[x*y[x]*y'[x]==3*x^6+6*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -x^3 \sqrt{-1 + c_1 x^6}$$

$$y(x) \rightarrow x^3 \sqrt{-1 + c_1 x^6}$$

5.47 problem 46

Internal problem ID [1021]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 46.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class G', _rational, _Riccati]`

$$x^3y' - 2y^2 - 2x^2y = -2x^4$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(x^3*diff(y(x),x)=2*(y(x)^2+x^2*y(x)-x^4),y(x), singsol=all)
```

$$y(x) = \tanh(-2 \ln(x) + 2c_1) x^2$$

✓ Solution by Mathematica

Time used: 1.137 (sec). Leaf size: 62

```
DSolve[x^3*y'[x]==2*(y[x]^2+x^2*y[x]-x^4),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow ix^2 \tan(2i \log(x) + c_1)$$

$$y(x) \rightarrow \frac{x^2(-x^4 + e^{2i \text{Interval}\{0,\pi\}})}{x^4 + e^{2i \text{Interval}\{0,\pi\}}}$$

5.48 problem 47

Internal problem ID [1022]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 47.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Riccati]`

$$y' - y^2 e^{-x} - 4y = 2e^x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(diff(y(x),x)=y(x)^2*exp(-x)+4*y(x)+2*exp(x),y(x), singsol=all)
```

$$y(x) = -\frac{2e^x(c_1 e^x - 1)}{-2 + c_1 e^x}$$

✓ Solution by Mathematica

Time used: 0.265 (sec). Leaf size: 30

```
DSolve[y'[x]==y[x]^2*Exp[-x]+4*y[x]+2*Exp[x],y[x],x,IncludeSingularSolutions->True]
```

$$y(x) \rightarrow -2e^x + \frac{1}{e^{-x} + c_1}$$

$$y(x) \rightarrow -2e^x$$

5.49 problem 48

Internal problem ID [1023]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 48.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [Riccati]

$$y' - \frac{y^2 + \tan(x)y + \tan(x)^2}{\sin(x)^2} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 115

```
dsolve(diff(y(x),x)=(y(x)^2+y(x)*tan(x)+tan(x)^2)/sin(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\cos(x) \left(-c_1 \sin \left(-\ln(\sin(x)) + \frac{\ln(\sin(x)+1)}{2} + \frac{\ln(\sin(x)-1)}{2} \right) + \cos \left(-\ln(\sin(x)) + \frac{\ln(\sin(x)+1)}{2} + \frac{\ln(\sin(x)-1)}{2} \right) \right)}{(\sin(x)+1)(\sin(x)-1) \left(c_1 \cos \left(-\ln(\sin(x)) + \frac{\ln(\sin(x)+1)}{2} + \frac{\ln(\sin(x)-1)}{2} \right) + \sin \left(-\ln(\sin(x)) + \frac{\ln(\sin(x)+1)}{2} + \frac{\ln(\sin(x)-1)}{2} \right) \right)}$$

✓ Solution by Mathematica

Time used: 0.678 (sec). Leaf size: 20

```
DSolve[y'[x]==(y[x]^2+y[x]*Tan[x]+Tan[x]^2)/Sin[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \tan(x) \tan(\log(\sin(x)) - \log(\cos(x)) + c_1)$$

5.50 problem 49

Internal problem ID [1024]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 49.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, ‘_with_symmetry_[F(x),G(y)]’, _Riccati]`

$$x \ln(x)^2 y' - \ln(x) y - y^2 = -4 \ln(x)^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x*(ln(x))^2*diff(y(x),x)=-4*(ln(x))^2+y(x)*ln(x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = 2i \tan(2i \ln(\ln(x)) + c_1) \ln(x)$$

✓ Solution by Mathematica

Time used: 1.259 (sec). Leaf size: 64

```
DSolve[x*(Log[x])^2*y'[x]==-4*(Log[x])^2+y[x]*Log[x]+y[x]^2,y[x],x,IncludeSingularSolutions
```

$$y(x) \rightarrow 2i \log(x) \tan(2i \log(\log(x)) + c_1)$$
$$y(x) \rightarrow \frac{2 \log(x) (-\log^4(x) + e^{2i \text{Interval}\{0,\pi\}})}{\log^4(x) + e^{2i \text{Interval}\{0,\pi\}}}$$

5.51 problem 50

Internal problem ID [1025]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 50.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class G', _rational, [_Abel, '2nd type', 'cl`

$$2x(y + 2\sqrt{x})y' - (y + \sqrt{x})^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 64

```
dsolve(2*x*(y(x)+2*sqrt(x))*diff(y(x),x)=(y(x)+sqrt(x))^2,y(x), singsol=all)
```

$$y(x) = \frac{-2x + \sqrt{\ln(x)x^2 - c_1x^2 + 4x^2}}{\sqrt{x}}$$

$$y(x) = -\frac{2x + \sqrt{\ln(x)x^2 - c_1x^2 + 4x^2}}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.61 (sec). Leaf size: 68

```
DSolve[2*x*(y[x]+2*Sqrt[x])*y'[x]==(y[x]+Sqrt[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -2\sqrt{x} - \sqrt{\frac{1}{x^2}x\sqrt{x(\log(x) + 4 + c_1)}}$$

$$y(x) \rightarrow -2\sqrt{x} + \sqrt{\frac{1}{x^2}x\sqrt{x(\log(x) + 4 + c_1)}}$$

5.52 problem 51

Internal problem ID [1026]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 51.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]'], [_Abel, '2nd type`

$$(y + e^{x^2})y' - 2x(y^2 + ye^{x^2} + e^{2x^2}) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve((y(x)+exp(x^2))*diff(y(x),x)=2*x*(y(x)^2+y(x)*exp(x^2)+exp(2*x^2)),y(x), singsol=all)
```

$$y(x) = \left(-1 - \sqrt{2x^2 - 2c_1 + 1}\right) e^{x^2}$$

$$y(x) = \left(-1 + \sqrt{2x^2 - 2c_1 + 1}\right) e^{x^2}$$

✓ Solution by Mathematica

Time used: 0.744 (sec). Leaf size: 76

```
DSolve[(y[x]+Exp[x^2])*y'[x]==2*x*(y[x]^2+y[x]*Exp[x^2]+Exp[2*x^2]),y[x],x,IncludeSingularSo
```

$$y(x) \rightarrow -e^{x^2} - \frac{\sqrt{2x^2 + 1 + c_1}}{\sqrt{e^{-2x^2}}}$$

$$y(x) \rightarrow -e^{x^2} + \frac{\sqrt{2x^2 + 1 + c_1}}{\sqrt{e^{-2x^2}}}$$

5.53 problem 52

Internal problem ID [1027]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 52.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class G', _rational, [_Abel, '2nd type', 'cl`

$$y' + \frac{2y}{x} - \frac{3x^2y^2 + 6yx + 2}{x^2(2yx + 3)} = 0$$

With initial conditions

$$[y(2) = 2]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 18

```
dsolve([diff(y(x),x)+2/x*y(x)=(3*x^2*y(x)^2+6*x*y(x)+2)/(x^2*(2*x*y(x)+3)),y(2) = 2],y(x), s
```

$$y(x) = \frac{-3 + \sqrt{60x + 1}}{2x}$$

✓ Solution by Mathematica

Time used: 0.725 (sec). Leaf size: 35

```
DSolve[{y'[x]+2/x*y[x]==(3*x^2*y[x]^2+6*x*y[x]+2)/(x^2*(2*x*y[x]+3)),y[2]==2},y[x],x,Include
```

$$y(x) \rightarrow \frac{\sqrt{\frac{1}{x^2} \sqrt{x^2(60x + 1)}} - 3}{2x}$$

5.54 problem 53

Internal problem ID [1028]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 53.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class G', _rational, [_Abel, '2nd type', 'cl`

$$y' + \frac{3y}{x} - \frac{3y^2x^4 + 10x^2y + 6}{x^3(2x^2y + 5)} = 0$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 18

```
dsolve([diff(y(x),x)+3/x*y(x)=(3*x^4*y(x)^2+10*x^2*y(x)+6)/(x^3*(2*x^2*y(x)+5)),y(1) = 1],y(x))
```

$$y(x) = \frac{-5 + \sqrt{48x + 1}}{2x^2}$$

✓ Solution by Mathematica

Time used: 0.815 (sec). Leaf size: 37

```
DSolve[{y'[x]+3/x*y[x]==(3*x^4*y[x]^2+10*x^2*y[x]+6)/(x^3*(2*x^2*y[x]+5)),y[1]==1},y[x],x,Integrate->False]
```

$$y(x) \rightarrow \frac{\sqrt{\frac{1}{x^2} \sqrt{x^4(48x + 1)} - 5x}}{2x^3}$$

5.55 problem 56

Internal problem ID [1029]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 56.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [Riccati]

$$y' + (1 + 2x)y - xy^2 = x + 1$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)=1+x-(1+2*x)*y(x)+x*y(x)^2,y(x), singsol=all)
```

$$y(x) = 1 - \frac{2e^{-x}}{c_1 - 2(x+1)e^{-x}}$$

✓ Solution by Mathematica

Time used: 0.149 (sec). Leaf size: 31

```
DSolve[y'[x]==1+x-(1+2*x)*y[x]+x*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x + c_1 e^x + 2}{x + c_1 e^x + 1}$$

$$y(x) \rightarrow 1$$

6 Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

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6.1 problem 1

Internal problem ID [1030]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 1.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$6x^2y^2 + 4yy'x^3 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
dsolve(6*x^2*y(x)^2+4*x^3*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = -\frac{\sqrt{-2c_1x}}{2x^2}$$

$$y(x) = \frac{\sqrt{-2c_1x}}{2x^2}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 23

```
DSolve[6*x^2*y[x]^2+4*x^3*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 0$$

$$y(x) \rightarrow \frac{c_1}{x^{3/2}}$$

$$y(x) \rightarrow 0$$

6.2 problem 2

Internal problem ID [1031]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 2.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$3 \cos(x) y + 2x^3 y + (3 \sin(x) + 3) y' = -4e^x x$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 488

```
dsolve((3*y(x)*cos(x)+4*x*exp(x)+2*x^3*y(x))+(3*sin(x)+3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{4x(1-ie^{ix})^{4x^2} \int -\frac{4ix^3 e^{ix} + 24ix \operatorname{polylog}(2, ie^{ix}) e^{ix} + 3x - 3ix e^{ix} - 24i \operatorname{polylog}(3, ie^{ix}) - 3ix - 24 \operatorname{polylog}(3, ie^{ix}) e^{ix} - 3x e^{ix} - 24 \operatorname{polylog}(3, ie^{ix}) e^{ix} - 3x e^{ix} - 24 \operatorname{polylog}(3, ie^{ix}) e^{ix}}{3(e^{ix} + i)} dx}{(1 - ie^{ix})^{-4x^2}}$$

✓ Solution by Mathematica

Time used: 27.47 (sec). Leaf size: 193

```
DSolve[(3*y[x]*Cos[x]+4*x*Exp[x]+2*x^3*y[x])+(3*Sine[x]+3)*y'[x]==0,y[x],x,IncludeSingularSol
```

$$y(x) = \frac{(1 + ie^{-ix})^{-4x^2} \exp(-8ix \operatorname{PolyLog}(2, -ie^{-ix}) - 8 \operatorname{PolyLog}(3, -ie^{-ix}) + \frac{2}{3}x^3(\cot(\frac{1}{4}(2x + \pi)) - i))}{\int_1^x \dots dx}$$

6.3 problem 3

Internal problem ID [1032]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 3.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [quadrature]

$$14y^3x^2 + 21x^2y^2y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve((14*x^2*y(x)^3)+(21*x^2*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = c_1 e^{-\frac{2x}{3}}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 25

```
DSolve[(14*x^2*y[x]^3)+(21*x^2*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 0$$

$$y(x) \rightarrow c_1 e^{-2x/3}$$

$$y(x) \rightarrow 0$$

6.4 problem 4

Internal problem ID [1033]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 4.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [`_exact`, `_rational`]

$$-2y^2 + (12y^2 - 4yx) y' = -2x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 531

`dsolve((2*x-2*y(x)^2)+(12*y(x)^2-4*x*y(x))*diff(y(x),x)=0,y(x), singsol=all)`

$$\begin{aligned}
 y(x) &= \frac{\left(-27x^2 - 27c_1 + x^3 + 3\sqrt{-6x^5 - 6c_1x^3 + 81x^4 + 162c_1x^2 + 81c_1^2}\right)^{\frac{1}{3}}}{6x^2} \\
 &+ \frac{\left(-27x^2 - 27c_1 + x^3 + 3\sqrt{-6x^5 - 6c_1x^3 + 81x^4 + 162c_1x^2 + 81c_1^2}\right)^{\frac{1}{3}}}{6} + \frac{x}{6} \\
 y(x) &= -\frac{\left(-27x^2 - 27c_1 + x^3 + 3\sqrt{-6x^5 - 6c_1x^3 + 81x^4 + 162c_1x^2 + 81c_1^2}\right)^{\frac{1}{3}}}{12x^2} \\
 &- \frac{\left(-27x^2 - 27c_1 + x^3 + 3\sqrt{-6x^5 - 6c_1x^3 + 81x^4 + 162c_1x^2 + 81c_1^2}\right)^{\frac{1}{3}}}{12} + \frac{x}{6} \\
 &- \frac{i\sqrt{3}\left(\frac{\left(-27x^2 - 27c_1 + x^3 + 3\sqrt{-6x^5 - 6c_1x^3 + 81x^4 + 162c_1x^2 + 81c_1^2}\right)^{\frac{1}{3}}}{6} - \frac{x^2}{6\left(-27x^2 - 27c_1 + x^3 + 3\sqrt{-6x^5 - 6c_1x^3 + 81x^4 + 162c_1x^2 + 81c_1^2}\right)^{\frac{1}{3}}}\right)}{2} \\
 y(x) &= -\frac{\left(-27x^2 - 27c_1 + x^3 + 3\sqrt{-6x^5 - 6c_1x^3 + 81x^4 + 162c_1x^2 + 81c_1^2}\right)^{\frac{1}{3}}}{12x^2} \\
 &- \frac{\left(-27x^2 - 27c_1 + x^3 + 3\sqrt{-6x^5 - 6c_1x^3 + 81x^4 + 162c_1x^2 + 81c_1^2}\right)^{\frac{1}{3}}}{12} + \frac{x}{6} \\
 &+ \frac{i\sqrt{3}\left(\frac{\left(-27x^2 - 27c_1 + x^3 + 3\sqrt{-6x^5 - 6c_1x^3 + 81x^4 + 162c_1x^2 + 81c_1^2}\right)^{\frac{1}{3}}}{6} - \frac{x^2}{6\left(-27x^2 - 27c_1 + x^3 + 3\sqrt{-6x^5 - 6c_1x^3 + 81x^4 + 162c_1x^2 + 81c_1^2}\right)^{\frac{1}{3}}}\right)}{2}
 \end{aligned}$$

✓ Solution by Mathematica

Time used: 4.8 (sec). Leaf size: 414

`DSolve[(2*x-2*y[x]^2)+(12*y[x]^2-4*x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]`

$$y(x) \rightarrow -\frac{x^2}{3 \cdot 2^{2/3} \sqrt[3]{-2x^3 + 54x^2 + \sqrt{-4x^6 + 4(x^3 - 27x^2 - 54c_1)^2 + 108c_1}}} - \frac{\sqrt[3]{-2x^3 + 54x^2 + \sqrt{-4x^6 + 4(x^3 - 27x^2 - 54c_1)^2 + 108c_1}}}{6\sqrt[3]{2}} + \frac{x}{6}$$

$$y(x) \rightarrow \frac{(1 + i\sqrt{3})x^2}{6 \cdot 2^{2/3} \sqrt[3]{-2x^3 + 54x^2 + \sqrt{-4x^6 + 4(x^3 - 27x^2 - 54c_1)^2 + 108c_1}}} + \frac{(1 - i\sqrt{3}) \sqrt[3]{-2x^3 + 54x^2 + \sqrt{-4x^6 + 4(x^3 - 27x^2 - 54c_1)^2 + 108c_1}}}{12\sqrt[3]{2}} + \frac{x}{6}$$

$$y(x) \rightarrow \frac{(1 - i\sqrt{3})x^2}{6 \cdot 2^{2/3} \sqrt[3]{-2x^3 + 54x^2 + \sqrt{-4x^6 + 4(x^3 - 27x^2 - 54c_1)^2 + 108c_1}}} + \frac{(1 + i\sqrt{3}) \sqrt[3]{-2x^3 + 54x^2 + \sqrt{-4x^6 + 4(x^3 - 27x^2 - 54c_1)^2 + 108c_1}}}{12\sqrt[3]{2}} + \frac{x}{6}$$

6.5 problem 5

Internal problem ID [1034]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 5.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$(x + y)^2 + (x + y)^2 y' = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

```
dsolve((x+y(x))^2+(x+y(x))^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -x$$

$$y(x) = c_1 - x$$

$$y(x) = -\frac{c_1}{2} - \frac{i\sqrt{3}c_1}{2} - x$$

$$y(x) = -\frac{c_1}{2} + \frac{i\sqrt{3}c_1}{2} - x$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
DSolve[(x+y[x])^2+(x+y[x])^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -x$$

$$y(x) \rightarrow -x + c_1$$

6.6 problem 6

Internal problem ID [1035]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 6.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$7y + (3x + 4y)y' = -4x$$

✓ Solution by Maple

Time used: 0.281 (sec). Leaf size: 55

```
dsolve((4*x+7*y(x))+(3*x+4*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{x \left(2 \operatorname{RootOf} \left(_Z^{36} + 3 _Z^6 c_1 x^6 - 2 c_1 x^6 \right)^6 - 1 \right)}{\operatorname{RootOf} \left(_Z^{36} + 3 _Z^6 c_1 x^6 - 2 c_1 x^6 \right)^6}$$

✓ Solution by Mathematica

Time used: 3.222 (sec). Leaf size: 409

```
DSolve[(4*x+7*y[x])+(3*x+4*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \text{Root}\left[2\#1^6 + 21\#1^5x + 90\#1^4x^2 + 200\#1^3x^3 + 240\#1^2x^4 + 144\#1x^5 + 32x^6 - e^{3c_1}\&, 1\right]$$

$$y(x) \rightarrow \text{Root}\left[2\#1^6 + 21\#1^5x + 90\#1^4x^2 + 200\#1^3x^3 + 240\#1^2x^4 + 144\#1x^5 + 32x^6 - e^{3c_1}\&, 2\right]$$

$$y(x) \rightarrow \text{Root}\left[2\#1^6 + 21\#1^5x + 90\#1^4x^2 + 200\#1^3x^3 + 240\#1^2x^4 + 144\#1x^5 + 32x^6 - e^{3c_1}\&, 3\right]$$

$$y(x) \rightarrow \text{Root}\left[2\#1^6 + 21\#1^5x + 90\#1^4x^2 + 200\#1^3x^3 + 240\#1^2x^4 + 144\#1x^5 + 32x^6 - e^{3c_1}\&, 4\right]$$

$$y(x) \rightarrow \text{Root}\left[2\#1^6 + 21\#1^5x + 90\#1^4x^2 + 200\#1^3x^3 + 240\#1^2x^4 + 144\#1x^5 + 32x^6 - e^{3c_1}\&, 5\right]$$

$$y(x) \rightarrow \text{Root}\left[2\#1^6 + 21\#1^5x + 90\#1^4x^2 + 200\#1^3x^3 + 240\#1^2x^4 + 144\#1x^5 + 32x^6 - e^{3c_1}\&, 6\right]$$

6.7 problem 7

Internal problem ID [1036]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 7.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [`_exact`]

$$-2 \sin(x) y^2 + 3y^3 + (4 \cos(x) y + 9xy^2) y' = 2x$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 758

`dsolve((-2*y(x)^2*sin(x)+3*y(x)^3-2*x)+(4*y(x)*cos(x)+9*x*y(x)^2)*diff(y(x),x)=0,y(x), sings`

$$y(x) = \frac{\left(972x^4 + 36\sqrt{3}\sqrt{243x^6 - 32\cos(x)^3x^2 - 486x^4c_1 + 32\cos(x)^3c_1 + 243c_1^2x^2x - 64\cos(x)^3 - 972c_1x^2}\right)}{18x} + \frac{18x}{8\cos(x)^2} - \frac{9x\left(972x^4 + 36\sqrt{3}\sqrt{243x^6 - 32\cos(x)^3x^2 - 486x^4c_1 + 32\cos(x)^3c_1 + 243c_1^2x^2x - 64\cos(x)^3 - 972c_1x^2}\right) - \frac{2\cos(x)}{9x}}$$

$$y(x) = \frac{\left(972x^4 + 36\sqrt{3}\sqrt{243x^6 - 32\cos(x)^3x^2 - 486x^4c_1 + 32\cos(x)^3c_1 + 243c_1^2x^2x - 64\cos(x)^3 - 972c_1x^2}\right)}{36x} - \frac{36x}{4\cos(x)^2} - \frac{9x\left(972x^4 + 36\sqrt{3}\sqrt{243x^6 - 32\cos(x)^3x^2 - 486x^4c_1 + 32\cos(x)^3c_1 + 243c_1^2x^2x - 64\cos(x)^3 - 972c_1x^2}\right) - \frac{2\cos(x)}{9x}}{i\sqrt{3}\left(\frac{\left(972x^4 + 36\sqrt{3}\sqrt{243x^6 - 32\cos(x)^3x^2 - 486x^4c_1 + 32\cos(x)^3c_1 + 243c_1^2x^2x - 64\cos(x)^3 - 972c_1x^2}\right)^{\frac{1}{3}}}{18x} - \frac{9x\left(972x^4 + 36\sqrt{3}\sqrt{243x^6 - 32\cos(x)^3x^2 - 486x^4c_1 + 32\cos(x)^3c_1 + 243c_1^2x^2x - 64\cos(x)^3 - 972c_1x^2}\right)}{2}\right)}$$

$$y(x) = \frac{\left(972x^4 + 36\sqrt{3}\sqrt{243x^6 - 32\cos(x)^3x^2 - 486x^4c_1 + 32\cos(x)^3c_1 + 243c_1^2x^2x - 64\cos(x)^3 - 972c_1x^2}\right)}{36x} - \frac{36x}{4\cos(x)^2} - \frac{9x\left(972x^4 + 36\sqrt{3}\sqrt{243x^6 - 32\cos(x)^3x^2 - 486x^4c_1 + 32\cos(x)^3c_1 + 243c_1^2x^2x - 64\cos(x)^3 - 972c_1x^2}\right) - \frac{2\cos(x)}{9x}}{i\sqrt{3}\left(\frac{\left(972x^4 + 36\sqrt{3}\sqrt{243x^6 - 32\cos(x)^3x^2 - 486x^4c_1 + 32\cos(x)^3c_1 + 243c_1^2x^2x - 64\cos(x)^3 - 972c_1x^2}\right)^{\frac{1}{3}}}{18x} - \frac{9x\left(972x^4 + 36\sqrt{3}\sqrt{243x^6 - 32\cos(x)^3x^2 - 486x^4c_1 + 32\cos(x)^3c_1 + 243c_1^2x^2x - 64\cos(x)^3 - 972c_1x^2}\right)}{2}\right)}$$

✓ Solution by Mathematica

Time used: 34.362 (sec). Leaf size: 520

`DSolve[(-2*y[x]^2*Sin[x]+3*y[x]^3-2*x)+(4*y[x]*Cos[x]+9*x*y[x]^2)*y'[x]==0,y[x],x,IncludeSin`

$$y(x) \rightarrow \frac{2^{2/3} \sqrt[3]{-16 \cos^3(x) + 9 \left(27x^4 + 27c_1x^2 + \sqrt{3} \sqrt{x^2(x^2 + c_1)} (-32 \cos^3(x) + 243x^2(x^2 + c_1)) \right)} + \sqrt[3]{-8c_1}}{18x}$$

$$y(x) \rightarrow \frac{i 2^{2/3} (\sqrt{3} + i) \sqrt[3]{-16 \cos^3(x) + 9 \left(27x^4 + 27c_1x^2 + \sqrt{3} \sqrt{x^2(x^2 + c_1)} (-32 \cos^3(x) + 243x^2(x^2 + c_1)) \right)}}{36}$$

$$y(x) \rightarrow \frac{2^{2/3} (1 + i\sqrt{3}) \sqrt[3]{-16 \cos^3(x) + 9 \left(27x^4 + 27c_1x^2 + \sqrt{3} \sqrt{x^2(x^2 + c_1)} (-32 \cos^3(x) + 243x^2(x^2 + c_1)) \right)}}{36}$$

6.8 problem 8

Internal problem ID [1037]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 8.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, [_Abel, '2nd type', 'cl`

$$y + (2x + 2y)y' = -2x$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 46

```
dsolve((2*x+y(x))+(2*y(x)+2*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{3x}{4} + \frac{\sqrt{7}x \tan\left(\text{RootOf}\left(\sqrt{7} \ln\left(\frac{7x^2}{8} + \frac{7x^2 \tan\left(\frac{-Z}{8}\right)^2}{8}\right) + 2\sqrt{7}c_1 + 2_Z\right)\right)}{4}$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 62

```
DSolve[(2*x+y[x])+(2*y[x]+2*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\arctan\left(\frac{4y(x)+3}{x\sqrt{7}}\right)}{2\sqrt{7}} + \frac{1}{4} \log\left(\frac{2y(x)^2}{x^2} + \frac{3y(x)}{x} + 2\right) = -\frac{\log(x)}{2} + c_1, y(x)\right]$$

6.9 problem 9

Internal problem ID [1038]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 9.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_Abel, '2nd type', 'class B']]`

$$2yx + 4y^2 + (x^2 + 8yx + 18y) y' = -3x^2$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 75

```
dsolve((3*x^2+2*x*y(x)+4*y(x)^2)+(x^2+8*x*y(x)+18*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-x^2 + \sqrt{-15x^4 - 36x^3 - 16c_1x - 36c_1}}{8x + 18}$$

$$y(x) = -\frac{x^2 + \sqrt{-15x^4 - 36x^3 - 16c_1x - 36c_1}}{2(4x + 9)}$$

✓ Solution by Mathematica

Time used: 0.616 (sec). Leaf size: 84

```
DSolve[(3*x^2+2*x*y[x]+4*y[x]^2)+(x^2+8*x*y[x]+18*y[x])*y'[x]==0,y[x],x,IncludeSingularSolut
```

$$y(x) \rightarrow -\frac{x^2 + \sqrt{-15x^4 - 36x^3 + 16c_1x + 36c_1}}{8x + 18}$$

$$y(x) \rightarrow \frac{-x^2 + \sqrt{-15x^4 - 36x^3 + 16c_1x + 36c_1}}{8x + 18}$$

6.10 problem 10

Internal problem ID [1039]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 10.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [rational]

$$8yx + y^2 + \left(2x^2 + \frac{xy^3}{3}\right)y' = -2x^2$$

X Solution by Maple

```
dsolve((2*x^2+8*x*y(x)+y(x)^2)+(2*x^2+x*y(x)^3/3)*diff(y(x),x)=0,y(x), singsol=all)
```

No solution found

X Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(2*x^2+8*x*y[x]+y[x]^2)+(2*x^2+x*y[x]^3/3)*y'[x]==0,y[x],x,IncludeSingularSolutions -
```

Not solved

6.11 problem 11

Internal problem ID [1040]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 11.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$\left(\frac{1}{y} + 2y\right) y' = -\frac{1}{x} - 2x$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 36

```
dsolve((1/x+2*x)+(1/y(x)+2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{e^{-\frac{\text{LambertW}\left(\frac{2e^{-2x^2}-2c_1}{x^2}\right)-x^2-c_1}}{x}}$$

✓ Solution by Mathematica

Time used: 9.517 (sec). Leaf size: 71

```
DSolve[(1/x+2*x)+(1/y[x]+2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{\sqrt{W\left(\frac{2e^{-2x^2+2c_1}}{x^2}\right)}}{\sqrt{2}}$$

$$y(x) \rightarrow \frac{\sqrt{W\left(\frac{2e^{-2x^2+2c_1}}{x^2}\right)}}{\sqrt{2}}$$

$$y(x) \rightarrow 0$$

6.12 problem 12

Internal problem ID [1041]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 12.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [NONE]

$$y^2 \sin(x) + xy^3 \cos(x) + (x \sin(x)y + xy^3 \cos(x)) y' = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 5

```
dsolve((y(x)*sin(x)*y(x)+x*y(x)^2*cos(x)*y(x))+(x*sin(x)*y(x)+x*y(x)^2*cos(x))*diff(y(x)
```

$$y(x) = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y[x]*Sin[x]*y[x]+x*y[x]^2*Cos[x]*y[x])+(x*SIn[x]*y[x]+x*y[x]^2*Cos[x])*y'[x]==0
```

Not solved

6.13 problem 13

Internal problem ID [1042]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 13.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$\frac{x}{(x^2 + y^2)^{\frac{3}{2}}} + \frac{yy'}{(x^2 + y^2)^{\frac{3}{2}}} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve((x/(x^2+y(x)^2)^(3/2))+y(x)/(x^2+y(x)^2)^(3/2))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{-x^2 + c_1}$$

$$y(x) = -\sqrt{-x^2 + c_1}$$

✓ Solution by Mathematica

Time used: 0.094 (sec). Leaf size: 39

```
DSolve[(x/(x^2+y[x]^2)^(3/2))+y[x]/(x^2+y[x]^2)^(3/2))*y'[x]==0,y[x],x,IncludeSingularSolut
```

$$y(x) \rightarrow -\sqrt{-x^2 + 2c_1}$$

$$y(x) \rightarrow \sqrt{-x^2 + 2c_1}$$

6.14 problem 14

Internal problem ID [1043]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 14.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_exact, [_Abel, '2nd type', 'class B']]`

$$e^x(x^2y^2 + 2xy^2) + (2x^2ye^x + 2)y' = -6x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 66

```
dsolve((exp(x)*(x^2*y(x)^2+2*x*y(x)^2)+6*x)+(2*x^2*y(x)*exp(x)+2)*diff(y(x),x)=0,y(x), sings
```

$$y(x) = \frac{(-1 + \sqrt{-3e^xx^4 - e^xc_1x^2 + 1})e^{-x}}{x^2}$$

$$y(x) = -\frac{(1 + \sqrt{-3e^xx^4 - e^xc_1x^2 + 1})e^{-x}}{x^2}$$

✓ Solution by Mathematica

Time used: 36.472 (sec). Leaf size: 76

```
DSolve[(Exp[x]*(x^2*y[x]^2+2*x*y[x]^2)+6*x)+(2*x^2*y[x]*Exp[x]+2)*y'[x]==0,y[x],x,IncludeSin
```

$$y(x) \rightarrow -\frac{e^{-x}\left(1 + \sqrt{1 + e^x(-3x^4 + c_1x^2)}\right)}{x^2}$$

$$y(x) \rightarrow \frac{e^{-x}\left(-1 + \sqrt{1 + e^x(-3x^4 + c_1x^2)}\right)}{x^2}$$

6.15 problem 15

Internal problem ID [1044]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 15.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [exact]

$$x^2 e^{x^2+y} (2x^2 + 3) + (x^3 e^{x^2+y} - 12y^2) y' = -4x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve((x^2*exp(x^2+y(x)))*(2*x^2+3)+4*x)+(x^3*exp(x^2+y(x))-12*y(x)^2)*diff(y(x),x)=0,y(x),
```

$$x^3 e^{x^2+y(x)} - 4y(x)^3 + 2x^2 + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.428 (sec). Leaf size: 30

```
DSolve[(x^2*Exp[x^2+y[x]]*(2*x^2+3)+4*x)+(x^3*Exp[x^2+y[x]]-12*y[x]^2)*y'[x]==0,y[x],x,Inclu
```

$$\text{Solve}\left[2x^2 + x^3 e^{x^2+y(x)} - 4y(x)^3 = c_1, y(x)\right]$$

6.16 problem 16

Internal problem ID [1045]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 16.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_exact, [_1st_order, ‘_with_symmetry_[F(x),G(x)*y+H(x)]’]]

$$e^{yx}(yx^4 + 4x^3) + 3y + (x^5e^{yx} + 3x)y' = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve((exp(x*y(x))*(x^4*y(x)+4*x^3)+3*y(x))+( x^5*exp(x*y(x))+3*x)*diff(y(x),x)=0,y(x), sin
```

$$y(x) = -\frac{3 \operatorname{LambertW}\left(\frac{x^4 e^{-\frac{c_1}{3}}}{3}\right) + c_1}{3x}$$

✓ Solution by Mathematica

Time used: 4.486 (sec). Leaf size: 33

```
DSolve[(Exp[x*y[x]]*(x^4*y[x]+4*x^3)+3*y[x])+(x^5*Exp[x*y[x]]+3*x)*y'[x]==0,y[x],x,IncludeSi
```

$$y(x) \rightarrow \frac{c_1 - 3W\left(\frac{1}{3}e^{\frac{c_1}{3}}x^4\right)}{3x}$$

6.17 problem 17

Internal problem ID [1046]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 17.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$3 \cos(x) y x^2 - x^3 y^2 \sin(x) + (8y - y \sin(x) x^4) y' = -4x$$

X Solution by Maple

```
dsolve((3*x^2*cos(x)*y(x)-x^3*y(x)*sin(x)*y(x)+4*x)+(8*y(x)-x^4*sin(x)*y(x))*diff(y(x),x)=0,
```

No solution found

X Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(3*x^2*Cos[x]*y[x]-x^3*y[x]*Sin[x]*y[x]+4*x)+(8*y[x]-x^4*SIn[x]*y[x])*y'[x]==0,y[x],x
```

Not solved

6.18 problem 18

Internal problem ID [1047]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 18.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_Abel, '2nd type', 'class B']]`

$$4x^3y^2 - 6x^2y + (2yx^4 - 2x^3)y' = 2x + 3$$

With initial conditions

$$[y(1) = 3]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 22

```
dsolve([(4*x^3*y(x)^2-6*x^2*y(x)-2*x-3)+(2*x^4*y(x)-2*x^3)*diff(y(x),x)=0,y(1) = 3],y(x), si
```

$$y(x) = \frac{x + \sqrt{2x^2 + 3x - 1}}{x^2}$$

✓ Solution by Mathematica

Time used: 0.743 (sec). Leaf size: 31

```
DSolve[{(4*x^3*y[x]^2-6*x^2*y[x]-2*x-3)+(2*x^4*y[x]-2*x^3)*y'[x]==0,y[1]==3},y[x],x,IncludeS
```

$$y(x) \rightarrow \frac{x^3 + \sqrt{x^4(2x^2 + 3x - 1)}}{x^4}$$

6.19 problem 19

Internal problem ID [1048]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 19.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, ' _with_symmetry_[F(x),G(x)] '], [_Abel, ']`

$$-4 \cos(x) y + (4y - 4 \sin(x)) y' = -4 \cos(x) \sin(x) - \sec(x)^2$$

With initial conditions

$$\left[y\left(\frac{\pi}{4}\right) = 0 \right]$$

✓ Solution by Maple

Time used: 0.797 (sec). Leaf size: 32

```
dsolve([(-4*y(x)*cos(x)+4*sin(x)*cos(x)+sec(x)^2)+(4*y(x)-4*sin(x))*diff(y(x),x)=0,y(1/4*Pi)
```

$$y(x) = \sin(x) - \frac{\sec(x)^2 \sqrt{2} \sqrt{\cos(x)^3 (2 \cos(x) - \sin(x))}}{2}$$

✓ Solution by Mathematica

Time used: 11.693 (sec). Leaf size: 38

```
DSolve[{-4*y[x]*Cos[x]+4*Sin[x]*Cos[x]+Sec[x]^2+(4*y[x]-4*Sin[x])*y'[x]==0,y[Pi/4]==0},y[x]
```

$$y(x) \rightarrow \sin(x) + \frac{1}{2} \sqrt{-\sec^2(x)} \sqrt{\sin(2x) - 2 \cos(2x) - 2}$$

6.20 problem 20

Internal problem ID [1049]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 20.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$(y^3 - 1)e^x + 3y^2(1 + e^x)y' = 0$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 91

```
dsolve([(y(x)^3-1)*exp(x)+(3*y(x)^2*(exp(x)+1))*diff(y(x),x)=0,y(0) = 0],y(x), singsol=all
```

$$y(x) = \frac{((e^x - 1)(1 + e^x)^2)^{\frac{1}{3}}}{1 + e^x}$$

$$y(x) = \frac{(i\sqrt{3} - 1)((e^x - 1)(1 + e^x)^2)^{\frac{1}{3}}}{2 + 2e^x}$$

$$y(x) = -\frac{(1 + i\sqrt{3})((e^x - 1)(1 + e^x)^2)^{\frac{1}{3}}}{2 + 2e^x}$$

✓ Solution by Mathematica

Time used: 0.984 (sec). Leaf size: 77

```
DSolve[{((y[x]^3-1)*Exp[x])+(3*y[x]^2*(Exp[x]+1))*y'[x]==0,y[0]==0},y[x],x,IncludeSingularSo
```

$$y(x) \rightarrow \sqrt[3]{\frac{e^x - 1}{e^x + 1}}$$

$$y(x) \rightarrow -\sqrt[3]{-1} \sqrt[3]{\frac{e^x - 1}{e^x + 1}}$$

$$y(x) \rightarrow (-1)^{2/3} \sqrt[3]{\frac{e^x - 1}{e^x + 1}}$$

6.21 problem 21

Internal problem ID [1050]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 21.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$-\sin(x)y + y' \cos(x) = -\sin(x) + 2 \cos(x)$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 10

```
dsolve([(sin(x)-y(x)*sin(x)-2*cos(x))+(cos(x))*diff(y(x),x)=0,y(0) = 1],y(x), singsol=all)
```

$$y(x) = 2 \tan(x) + 1$$

✓ Solution by Mathematica

Time used: 0.093 (sec). Leaf size: 11

```
DSolve[{(Sin[x]-y[x]*Sin[x]-2*Cos[x])+(Cos[x])*y'[x]==0,y[0]==1},y[x],x,IncludeSingularSolut
```

$$y(x) \rightarrow 2 \tan(x) + 1$$

6.22 problem 22

Internal problem ID [1051]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 22.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$(2x - 1)(y - 1) + (2 + x)(x - 3)y' = 0$$

With initial conditions

$$[y(1) = -1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve([(2*x-1)*(y(x)-1))+((x+2)*(x-3))*diff(y(x),x)=0,y(1) = -1],y(x), singsol=all)
```

$$y(x) = \frac{x^2 - x + 6}{(2 + x)(x - 3)}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 24

```
DSolve[{{(2*x-1)*(y[x]-1))+((x+2)*(x-3))*y'[x]==0,y[1]==-1},y[x],x,IncludeSingularSolutions
```

$$y(x) \rightarrow \frac{x^2 - x + 6}{x^2 - x - 6}$$

6.23 problem 23

Internal problem ID [1052]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 23.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, [_Abel, '2nd ty`

$$4y + (4x + 3y)y' = -7x$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 53

```
dsolve((7*x+4*y(x))+(4*x+3*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-\frac{4c_1x}{3} - \frac{\sqrt{-5c_1^2x^2+3}}{3}}{c_1}$$

$$y(x) = \frac{-\frac{4c_1x}{3} + \frac{\sqrt{-5c_1^2x^2+3}}{3}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.483 (sec). Leaf size: 118

```
DSolve[(7*x+4*y[x])+(4*x+3*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{3} \left(-4x - \sqrt{-5x^2 + 3e^{2c_1}} \right)$$

$$y(x) \rightarrow \frac{1}{3} \left(-4x + \sqrt{-5x^2 + 3e^{2c_1}} \right)$$

$$y(x) \rightarrow \frac{1}{3} \left(-\sqrt{5}\sqrt{-x^2} - 4x \right)$$

$$y(x) \rightarrow \frac{1}{3} \left(\sqrt{5}\sqrt{-x^2} - 4x \right)$$

6.24 problem 24

Internal problem ID [1053]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 24.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [exact, Bernoulli]

$$e^x(y^2x^4 + 4x^3y^2 + 1) + (2x^4ye^x + 2y)y' = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 66

```
dsolve((exp(x)*(x^4*y(x)^2+4*x^3*y(x)^2+1))+(2*x^4*y(x)*exp(x)+2*y(x))*diff(y(x),x)=0,y(x),
```

$$y(x) = \frac{\sqrt{-(e^xx^4 + 1)(-c_1 + e^x)}}{e^xx^4 + 1}$$

$$y(x) = -\frac{\sqrt{-(e^xx^4 + 1)(-c_1 + e^x)}}{e^xx^4 + 1}$$

✓ Solution by Mathematica

Time used: 1.05 (sec). Leaf size: 64

```
DSolve[(Exp[x]*(x^4*y[x]^2+4*x^3*y[x]^2+1))+(2*x^4*y[x]*Exp[x]+2*y[x])*y'[x]==0,y[x],x,Inclu
```

$$y(x) \rightarrow -\frac{\sqrt{-2e^x + c_1}}{\sqrt{2e^xx^4 + 2}}$$

$$y(x) \rightarrow \frac{\sqrt{-2e^x + c_1}}{\sqrt{2e^xx^4 + 2}}$$

6.25 problem 25

Internal problem ID [1054]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 25.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [`_exact`, `_rational`]

$$x^3y^4 + (x^4y^3 + y)y' = -x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 111

```
dsolve((x^3*y(x)^4+x)+(x^4*y(x)^3+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-1 - \sqrt{-2x^6 - 4x^4c_1 + 1}}}{x^2}$$

$$y(x) = \frac{\sqrt{-1 + \sqrt{-2x^6 - 4x^4c_1 + 1}}}{x^2}$$

$$y(x) = -\frac{\sqrt{-1 - \sqrt{-2x^6 - 4x^4c_1 + 1}}}{x^2}$$

$$y(x) = -\frac{\sqrt{-1 + \sqrt{-2x^6 - 4x^4c_1 + 1}}}{x^2}$$

✓ Solution by Mathematica

Time used: 11.648 (sec). Leaf size: 135

```
DSolve[(x^3*y[x]^4+x)+(x^4*y[x]^3+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\sqrt{-\frac{1 + \sqrt{-2x^6 + 4c_1x^4 + 1}}{x^4}}$$

$$y(x) \rightarrow \sqrt{-\frac{1 + \sqrt{-2x^6 + 4c_1x^4 + 1}}{x^4}}$$

$$y(x) \rightarrow -\sqrt{\frac{-1 + \sqrt{-2x^6 + 4c_1x^4 + 1}}{x^4}}$$

$$y(x) \rightarrow \sqrt{\frac{-1 + \sqrt{-2x^6 + 4c_1x^4 + 1}}{x^4}}$$

6.26 problem 26

Internal problem ID [1055]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 26.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x),G(x)]]'`

$$2y + (2x + 2y)y' = -3x^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve((3*x^2+2*y(x))+(2*y(x)+2*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -x - \sqrt{-x^3 + x^2 - c_1}$$

$$y(x) = -x + \sqrt{-x^3 + x^2 - c_1}$$

✓ Solution by Mathematica

Time used: 0.131 (sec). Leaf size: 49

```
DSolve[(3*x^2+2*y[x])+(2*y[x]+2*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -x - \sqrt{-x^3 + x^2 + c_1}$$

$$y(x) \rightarrow -x + \sqrt{-x^3 + x^2 + c_1}$$

6.27 problem 27(a)

Internal problem ID [1056]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 27(a).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [`_exact`, `_rational`]

$$x^3y^4 + (x^4y^3 + 3y)y' = -2x$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 111

```
dsolve((x^3*y(x)^4+2*x)+(x^4*y(x)^3+3*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-3 - \sqrt{-4x^6 - 4x^4c_1 + 9}}}{x^2}$$

$$y(x) = \frac{\sqrt{-3 + \sqrt{-4x^6 - 4x^4c_1 + 9}}}{x^2}$$

$$y(x) = -\frac{\sqrt{-3 - \sqrt{-4x^6 - 4x^4c_1 + 9}}}{x^2}$$

$$y(x) = -\frac{\sqrt{-3 + \sqrt{-4x^6 - 4x^4c_1 + 9}}}{x^2}$$

✓ Solution by Mathematica

Time used: 11.546 (sec). Leaf size: 135

```
DSolve[(x^3*y[x]^4+2*x)+(x^4*y[x]^3+3*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \rightarrow -\sqrt{-\frac{3 + \sqrt{-4x^6 + 4c_1x^4 + 9}}{x^4}}$$

$$y(x) \rightarrow \sqrt{-\frac{3 + \sqrt{-4x^6 + 4c_1x^4 + 9}}{x^4}}$$

$$y(x) \rightarrow -\sqrt{\frac{-3 + \sqrt{-4x^6 + 4c_1x^4 + 9}}{x^4}}$$

$$y(x) \rightarrow \sqrt{\frac{-3 + \sqrt{-4x^6 + 4c_1x^4 + 9}}{x^4}}$$

6.28 problem 28(a)

Internal problem ID [1057]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 28(a).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, _Bernoulli]`

$$y^2 + 2xyy' = -x^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve((x^2+y(x)^2)+(2*x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{3} \sqrt{x(-x^3 + 3c_1)}}{3x}$$

$$y(x) = \frac{\sqrt{3} \sqrt{x(-x^3 + 3c_1)}}{3x}$$

✓ Solution by Mathematica

Time used: 0.22 (sec). Leaf size: 60

```
DSolve[(x^2+y[x]^2)+(2*x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{\sqrt{-x^3 + 3c_1}}{\sqrt{3}\sqrt{x}}$$

$$y(x) \rightarrow \frac{\sqrt{-x^3 + 3c_1}}{\sqrt{3}\sqrt{x}}$$

6.29 problem 38

Internal problem ID [1058]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 38.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_rational, [_Abel, '2nd type', 'class B']]

$$y' + \frac{2y}{x} + \frac{2xy}{x^2 + 2x^2y + 1} = 0$$

With initial conditions

$$[y(1) = -2]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)+2/x*y(x)= -(2*x*y(x))/(x^2+2*x^2*y(x)+1),y(1) = -2],y(x), singsol=all)
```

$$y(x) = \frac{-x^2 - 1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.694 (sec). Leaf size: 38

```
DSolve[{y'[x]+2/x*y[x]== -(2*x*y[x])/(x^2+2*x^2*y[x]+1),y[1]==-2},y[x],x,IncludeSingularSolu
```

$$y(x) \rightarrow \frac{1}{2} \left(-\frac{1}{x^2} - \frac{\sqrt{x^3(x^2+1)^2}}{x^{7/2}} - 1 \right)$$

6.30 problem 39

Internal problem ID [1059]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 39.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y' - \frac{3y}{x} - \frac{2x^4(4x^3 - 3y)}{3x^5 + 3x^3 + 2y} = 0$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 30

```
dsolve([diff(y(x),x)-3/x*y(x)= (2*x^4*(4*x^3-3*y(x)))/(3*x^5+3*x^3+2*y(x)),y(1) = 1],y(x), s
```

$$y(x) = \frac{(-3x^2 + \sqrt{9x^4 + 34x^2 + 21} - 3)x^3}{2}$$

✓ Solution by Mathematica

Time used: 0.734 (sec). Leaf size: 49

```
DSolve[{y'[x]-3/x*y[x]== (2*x^4*(4*x^3-3*y[x]))/(3*x^5+3*x^3+2*y[x]),y[1]==1},y[x],x,Include
```

$$y(x) \rightarrow \frac{1}{2} \left(-3x^5 - 3x^3 + \sqrt{\frac{1}{x^7}} \sqrt{x(9x^4 + 34x^2 + 21)} x^6 \right)$$

6.31 problem 40

Internal problem ID [1060]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

Problem number: 40.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$y' + 2yx + \frac{e^{-x^2}(3x + 2ye^{x^2})}{2x + 3ye^{x^2}} = 0$$

With initial conditions

$$[y(0) = -1]$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 36

```
dsolve([diff(y(x),x)+2*x*y(x)= -exp(-x^2)*(3*x+2*y(x)*exp(x^2))/(2*x+3*y(x)*exp(x^2)),y(0) =
```

$$y(x) = -\frac{(2x e^{x^2} + \sqrt{e^{2x^2}(-5x^2 + 9)}) e^{-2x^2}}{3}$$

✓ Solution by Mathematica

Time used: 33.105 (sec). Leaf size: 44

```
DSolve[{y'[x]+2*x*y[x]== -Exp[-x^2]*(3*x+2*y[x]*Exp[x^2])/(2*x+3*y[x]*Exp[x^2]),y[0]==-1},y[
```

$$y(x) \rightarrow -\frac{1}{3}e^{-2x^2} \left(2e^{x^2}x + \sqrt{e^{2x^2}(9 - 5x^2)} \right)$$

7 Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6 Page 91

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7.1 problem 1(a)

Internal problem ID [1061]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6

Page 91

Problem number: 1(a).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class G', _rational, [_Abel, '2nd type', 'cl`

$$y + \left(2x + \frac{1}{y}\right) y' = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

```
dsolve(y(x)+(2*x+1/y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-1 + \sqrt{4c_1x + 1}}{2x}$$

$$y(x) = -\frac{1 + \sqrt{4c_1x + 1}}{2x}$$

✓ Solution by Mathematica

Time used: 0.321 (sec). Leaf size: 54

```
DSolve[y[x]+(2*x+1/y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{1 + \sqrt{1 + 4c_1x}}{2x}$$

$$y(x) \rightarrow \frac{-1 + \sqrt{1 + 4c_1x}}{2x}$$

$$y(x) \rightarrow 0$$

7.2 problem 2(a)

Internal problem ID [1062]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6
Page 91

Problem number: 2(a).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$-y^2 + y'x^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(-y(x)^2+x^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x}{c_1x + 1}$$

✓ Solution by Mathematica

Time used: 0.104 (sec). Leaf size: 21

```
DSolve[-y[x]^2+x^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x}{1 - c_1x}$$

$$y(x) \rightarrow 0$$

7.3 problem 3

Internal problem ID [1063]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6
Page 91

Problem number: 3.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y - y'x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 7

```
dsolve(y(x)-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1x$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 14

```
DSolve[y[x]-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1x$$

$$y(x) \rightarrow 0$$

7.4 problem 4

Internal problem ID [1064]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6
Page 91

Problem number: 4.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$3x^2y + 2x^3y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(3*x^2*y(x)+2*x^3*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 18

```
DSolve[3*x^2*y[x]+2*x^3*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_1}{x^{3/2}}$$

$$y(x) \rightarrow 0$$

7.5 problem 5

Internal problem ID [1065]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6

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Problem number: 5.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$2y^3 + 3y^2y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(2*y(x)^3+3*y(x)^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = c_1 e^{-\frac{2x}{3}}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 25

```
DSolve[2*y[x]^3+3*y[x]^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 0$$

$$y(x) \rightarrow c_1 e^{-2x/3}$$

$$y(x) \rightarrow 0$$

7.6 problem 6

Internal problem ID [1066]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6

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Problem number: 6.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$5yx + 2y + 2y'x = -5$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve((5*x*y(x)+2*y(x)+5)+(2*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{x} + \frac{e^{-\frac{5x}{2}} c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 21

```
DSolve[(5*x*y[x]+2*y[x]+5)+(2*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{-1 + c_1 e^{-5x/2}}{x}$$

7.7 problem 7

Internal problem ID [1067]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6

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Problem number: 7.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$yx + 2y + (x + 1)y' = -x - 1$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve((x*y(x)+x+2*y(x)+1)+(x+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{(-x e^x + c_1) e^{-x}}{x + 1}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 23

```
DSolve[(x*y[x]+x+2*y[x]+1)+(x+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{x - c_1 e^{-x}}{x + 1}$$

7.8 problem 8

Internal problem ID [1068]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6

Page 91

Problem number: 8.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, [_Abel, '2nd type', 'cl`

$$27xy^2 + 8y^3 + (18x^2y + 12xy^2) y' = 0$$

✓ Solution by Maple

Time used: 0.313 (sec). Leaf size: 33

```
dsolve((27*x*y(x)^2+8*y(x)^3)+(18*x^2*y(x)+12*x*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \text{RootOf}(4_Z^{15}c_1x^5 + 9_Z^{10}c_1x^5 - 1)^5 x$$

✓ Solution by Mathematica

Time used: 59.109 (sec). Leaf size: 534

DSolve[(27*x*y[x]^2+8*y[x]^3)+(18*x^2*y[x]+12*x*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolut

$$y(x) \rightarrow 0$$

$$y(x) \rightarrow \frac{1}{4} \left(\frac{9x^2}{\sqrt[3]{\frac{-27x^5 + 4\sqrt{e^{6c_1}(-27x^5 + 4e^{6c_1})} + 8e^{6c_1}}{x^2}}} + \sqrt[3]{\frac{-27x^5 + 4\sqrt{e^{6c_1}(-27x^5 + 4e^{6c_1})} + 8e^{6c_1}}{x^2}} - 3x \right)$$

$$y(x) \rightarrow \frac{1}{8} \left(-\frac{9(1+i\sqrt{3})x^2}{\sqrt[3]{\frac{-27x^5 + 4\sqrt{e^{6c_1}(-27x^5 + 4e^{6c_1})} + 8e^{6c_1}}{x^2}}} + i(\sqrt{3}+i) \sqrt[3]{\frac{-27x^5 + 4\sqrt{e^{6c_1}(-27x^5 + 4e^{6c_1})} + 8e^{6c_1}}{x^2}} - 6x \right)$$

$$y(x) \rightarrow \frac{1}{8} \left(\frac{9i(\sqrt{3}+i)x^2}{\sqrt[3]{\frac{-27x^5 + 4\sqrt{e^{6c_1}(-27x^5 + 4e^{6c_1})} + 8e^{6c_1}}{x^2}}} - (1+i\sqrt{3}) \sqrt[3]{\frac{-27x^5 + 4\sqrt{e^{6c_1}(-27x^5 + 4e^{6c_1})} + 8e^{6c_1}}{x^2}} - 6x \right)$$

$$y(x) \rightarrow 0$$

$$y(x) \rightarrow \frac{3\left(\sqrt[3]{-x^3} + x\right) \left(-2x + (1-i\sqrt{3})\sqrt[3]{-x^3}\right)}{8x \quad 236}$$

$$y(x) \rightarrow \frac{3\left(\sqrt[3]{-x^3} + x\right) \left(-2x + (1+i\sqrt{3})\sqrt[3]{-x^3}\right)}{8x}$$

7.9 problem 9

Internal problem ID [1069]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6

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Problem number: 9.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$6y^2x + 2y + (12x^2y + 12y^2x) y' = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 5

```
dsolve((6*x*y(x)^2+2*y(x))+(12*x^2*y(x)+12*x*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(6*x*y[x]^2+2*y[x])+(12*x^2*y[x]+12*x*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolution
```

Not solved

7.10 problem 10

Internal problem ID [1070]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6

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Problem number: 10.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, ‘_with_symmetry_[F(x)*G(y),0]’]]`

$$y^2 + \left(xy^2 + 6yx + \frac{1}{y} \right) y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve((y(x)^2)+(x*y(x)^2+3*x*y(x)+3*x*y(x)+1/y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$x - \frac{3}{y(x)^4} + \frac{1}{y(x)^3} + \frac{6}{y(x)^5} - \frac{6}{y(x)^6} - \frac{e^{-y(x)}c_1}{y(x)^6} = 0$$

✓ Solution by Mathematica

Time used: 0.2 (sec). Leaf size: 41

```
DSolve[(y[x]^2)+(x*y[x]^2+3*x*y[x]+3*x*y[x]+1/y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions
```

$$\text{Solve} \left[x = -\frac{y(x)^3 - 3y(x)^2 + 6y(x) - 6}{y(x)^6} + \frac{c_1 e^{-y(x)}}{y(x)^6}, y(x) \right]$$

7.11 problem 11

Internal problem ID [1071]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6

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Problem number: 11.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$12x^3y + 24x^2y^2 + (9x^4 + 32x^3y + 4y)y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve((12*x^3*y(x)+24*x^2*y(x)^2)+(9*x^4+32*x^3*y(x)+4*y(x))*diff(y(x),x)=0,y(x), singsol=a
```

$$3x^4y(x)^3 + 8x^3y(x)^4 + y(x)^4 + c_1 = 0$$

✓ Solution by Mathematica

Time used: 61.708 (sec). Leaf size: 1733

`DSolve[(12*x^3*y[x]+24*x^2*y[x]^2)+(9*x^4+32*x^3*y[x]+4*y[x])*y'[x]==0,y[x],x,IncludeSingular`

$$y(x) \rightarrow -\frac{3x^4}{32x^3 + 4}$$

$$+ \frac{1}{2} \sqrt{\frac{9x^8}{4(8x^3 + 1)^2} + \frac{\sqrt[3]{\sqrt{59049c_1^2x^{16} + 6912c_1^3(8x^3 + 1)^3} - 243c_1x^8}}{3\sqrt[3]{2}(8x^3 + 1)}} - \frac{4\sqrt[3]{2}c_1}{\sqrt[3]{\sqrt{59049c_1^2x^{16} + 6912c_1^3(8x^3 + 1)^3}}}$$

$$- \frac{1}{2} \sqrt{\frac{9x^8}{2(8x^3 + 1)^2} - \frac{\sqrt[3]{\sqrt{59049c_1^2x^{16} + 6912c_1^3(8x^3 + 1)^3} - 243c_1x^8}}{3\sqrt[3]{2}(8x^3 + 1)}} + \frac{4\sqrt[3]{2}c_1}{\sqrt[3]{\sqrt{59049c_1^2x^{16} + 6912c_1^3(8x^3 + 1)^3}}}$$

$$y(x) \rightarrow -\frac{3x^4}{32x^3 + 4}$$

$$+ \frac{1}{2} \sqrt{\frac{9x^8}{4(8x^3 + 1)^2} + \frac{\sqrt[3]{\sqrt{59049c_1^2x^{16} + 6912c_1^3(8x^3 + 1)^3} - 243c_1x^8}}{3\sqrt[3]{2}(8x^3 + 1)}} - \frac{4\sqrt[3]{2}c_1}{\sqrt[3]{\sqrt{59049c_1^2x^{16} + 6912c_1^3(8x^3 + 1)^3}}}$$

$$+ \frac{1}{2} \sqrt{\frac{9x^8}{2(8x^3 + 1)^2} - \frac{\sqrt[3]{\sqrt{59049c_1^2x^{16} + 6912c_1^3(8x^3 + 1)^3} - 243c_1x^8}}{3\sqrt[3]{2}(8x^3 + 1)}} + \frac{4\sqrt[3]{2}c_1}{\sqrt[3]{\sqrt{59049c_1^2x^{16} + 6912c_1^3(8x^3 + 1)^3}}}$$

$$y(x) \rightarrow -\frac{3x^4}{32x^3 + 4}$$

$$- \frac{1}{2} \sqrt{\frac{9x^8}{4(8x^3 + 1)^2} + \frac{\sqrt[3]{\sqrt{59049c_1^2x^{16} + 6912c_1^3(8x^3 + 1)^3} - 243c_1x^8}}{3\sqrt[3]{2}(8x^3 + 1)}} - \frac{4\sqrt[3]{2}c_1}{\sqrt[3]{\sqrt{59049c_1^2x^{16} + 6912c_1^3(8x^3 + 1)^3}}}$$

$$- \frac{1}{2} \sqrt{\frac{9x^8}{2(8x^3 + 1)^2} - \frac{\sqrt[3]{\sqrt{59049c_1^2x^{16} + 6912c_1^3(8x^3 + 1)^3} - 243c_1x^8}}{3\sqrt[3]{2}(8x^3 + 1)}} + \frac{4\sqrt[3]{2}c_1}{\sqrt[3]{\sqrt{59049c_1^2x^{16} + 6912c_1^3(8x^3 + 1)^3}}}$$

7.12 problem 12

Internal problem ID [1072]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6

Page 91

Problem number: 12.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$x^2y + 4yx + 2y + (x^2 + x)y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((x^2*y(x)+4*x*y(x)+2*y(x))+(x^2+x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{-x}}{x^2(x+1)}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 26

```
DSolve[(x^2*y[x]+4*x*y[x]+2*y[x])+(x^2+x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_1 e^{-x}}{x^2(x+1)}$$

$$y(x) \rightarrow 0$$

7.13 problem 13

Internal problem ID [1073]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6

Page 91

Problem number: 13.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$-y + (x^4 - x) y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(-y(x)+(x^4-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2 + x + 1)^{\frac{1}{3}}(x - 1)^{\frac{1}{3}}}{x}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 27

```
DSolve[-y[x]+(x^4-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_1 \sqrt[3]{1-x^3}}{x}$$

$$y(x) \rightarrow 0$$

7.14 problem 14

Internal problem ID [1074]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6

Page 91

Problem number: 14.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\cos(x) \cos(y) + (\cos(y) \sin(x) - \sin(y) \sin(x) + y) y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve((cos(x)*cos(y(x)))+(sin(x)*cos(y(x))-sin(x)*sin(y(x))+y(x))*diff(y(x),x)=0,y(x),sing
```

$$\cos(y(x)) e^{y(x)} \sin(x) + (y(x) - 1) e^{y(x)} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.218 (sec). Leaf size: 28

```
DSolve[(Cos[x]*Cos[y[x]])+(Sin[x]*Cos[y[x]]-Sin[x]*Sin[y[x]]+y[x])*y'[x]==0,y[x],x,IncludeSi
```

$$\text{Solve}[-2e^{y(x)}(y(x) - 1) - 2e^{y(x)} \sin(x) \cos(y(x)) = c_1, y(x)]$$

7.15 problem 15

Internal problem ID [1075]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6

Page 91

Problem number: 15.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [rational]

$$2yx + y^2 + (2yx + x^2 - 2xy^2 - 2xy^3) y' = 0$$

X Solution by Maple

```
dsolve((2*x*y(x)+y(x)^2)+(2*x*y(x)+x^2-2*x*y(x)^2-2*x*y(x)^3)*diff(y(x),x)=0,y(x), singsol=a
```

No solution found

X Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(2*x*y[x]+y[x]^2)+(2*x*y[x]+x^2-2*x*y[x]^2-2*x*y[x]^3)*y'[x]==0,y[x],x,IncludeSingula
```

Not solved

7.16 problem 16

Internal problem ID [1076]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6

Page 91

Problem number: 16.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y \sin(y) + x(\sin(y) - y \cos(y)) y' = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 16

```
dsolve((y(x)*sin(y(x)))+(x*(sin(y(x))-y(x)*cos(y(x))))*diff(y(x),x)=0,y(x), singsol=all)
```

$$\ln(x) + \ln(y(x)) - \ln(\sin(y(x))) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.569 (sec). Leaf size: 27

```
DSolve[(y[x]*Sin[y[x]])+(x*(Sin[y[x]]-y[x]*Cos[y[x]]))*y'[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x) \rightarrow \text{InverseFunction}[\log(\sin(\#1)) - \log(\#1)\&][\log(x) + c_1]$$

$$y(x) \rightarrow 0$$

7.17 problem 18

Internal problem ID [1077]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6

Page 91

Problem number: 18.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$ay + ybx + (cx + dxy)y' = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 50

```
dsolve((a*y(x)+b*x*y(x))+(c*x+d*x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = e^{-\frac{a \ln(x) + bx + c \operatorname{LambertW}\left(\frac{dx - \frac{a}{c} e^{-\frac{bx+c_1}{c}}}{c}\right) + c_1}{c}}$$

✓ Solution by Mathematica

Time used: 1.133 (sec). Leaf size: 42

```
DSolve[(a*y[x]+b*x*y[x])+(c*x+d*x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{cW\left(\frac{dx - \frac{a}{c} e^{-\frac{bx+c_1}{c}}}{c}\right)}{d}$$
$$y(x) \rightarrow 0$$

7.18 problem 19

Internal problem ID [1078]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6

Page 91

Problem number: 19.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class C']]`

$$3x^2y^3 - y^2 + y + (-xy + 2x)y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve((3*x^2*y(x)^3-y(x)^2+y(x))+(-x*y(x)+2*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{4}{\sqrt{x} \sqrt{\frac{c_1 x + 48x^2 + 4}{x}} + 2}$$
$$y(x) = -\frac{4}{\sqrt{x} \sqrt{\frac{c_1 x + 48x^2 + 4}{x}} - 2}$$

✓ Solution by Mathematica

Time used: 0.776 (sec). Leaf size: 80

```
DSolve[(3*x^2*y[x]^3-y[x]^2+y[x])+(-x*y[x]+2*x)*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow \frac{2}{1 + \sqrt{-\frac{1}{x^2}x}\sqrt{-12x^2 - 4c_1x - 1}}$$

$$y(x) \rightarrow \frac{2x}{x + \frac{\sqrt{-12x^2 - 4c_1x - 1}}{\sqrt{-\frac{1}{x^2}}}}$$

$$y(x) \rightarrow 0$$

7.19 problem 20

Internal problem ID [1079]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6

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Problem number: 20.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$2y + 3(x^2 + y^3x^2)y' = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve((2*y(x))+3*(x^2+x^2*y(x)^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = e^{-\frac{x \operatorname{LambertW}\left(\frac{2}{e^x} - 2c_1\right) + 2c_1x - 2}{3x}}$$

✓ Solution by Mathematica

Time used: 4.79 (sec). Leaf size: 82

```
DSolve[(2*y[x])+3*(x^2+x^2*y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sqrt[3]{W\left(e^{\frac{2}{x}+3c_1}\right)}$$

$$y(x) \rightarrow -\sqrt[3]{-1} \sqrt[3]{W\left(e^{\frac{2}{x}+3c_1}\right)}$$

$$y(x) \rightarrow (-1)^{2/3} \sqrt[3]{W\left(e^{\frac{2}{x}+3c_1}\right)}$$

$$y(x) \rightarrow 0$$

7.20 problem 21

Internal problem ID [1080]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6

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Problem number: 21.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$a \cos(x) y - \sin(x) y^2 + (b \cos(x) y - y x \sin(x)) y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 67

```
dsolve((a*cos(x)*y(x)-y(x)*sin(x)*y(x))+(b*cos(x)*y(x)-x*sin(x)*y(x))*diff(y(x),x)=0,y(x), s
```

$$y(x) = 0$$

$$y(x) = \left(\int -\frac{a \cos(x) e^{\int -\frac{\sin(x)}{-\sin(x)x + \cos(x)b} dx}}{-\sin(x)x + \cos(x)b} dx + c_1 \right) e^{\int \frac{\sin(x)}{-\sin(x)x + \cos(x)b} dx}$$

✓ Solution by Mathematica

Time used: 3.564 (sec). Leaf size: 106

`DSolve[(a*Cos[x]*y[x]-y[x]*Sin[x]*y[x])+(b*Cos[x]*y[x]-x*Sin[x]*y[x])*y'[x]==0,y[x],x,Include`

$$y(x) \rightarrow 0$$

$$y(x) \rightarrow \exp\left(\int_1^x \frac{\sin(K[1])}{b \cos(K[1]) - K[1] \sin(K[1])} dK[1]\right) \left(\int_1^x \frac{a \exp\left(-\int_1^{K[2]} \frac{\sin(K[1])}{b \cos(K[1]) - K[1] \sin(K[1])} dK[1]\right) \cos(K[2])}{b \cos(K[2]) - K[2] \sin(K[2])} dK[2] + c_1\right)$$

$$y(x) \rightarrow 0$$

7.21 problem 22

Internal problem ID [1081]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6

Page 91

Problem number: 22.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$x^4 y^4 + x^5 y^3 y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve((x^4*y(x)^4)+(x^5*y(x)^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \frac{c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 21

```
DSolve[(x^4*y[x]^4)+(x^5*y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 0$$

$$y(x) \rightarrow \frac{c_1}{x}$$

$$y(x) \rightarrow 0$$

7.22 problem 23

Internal problem ID [1082]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6

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Problem number: 23.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y(x \cos(x) + 2 \sin(x)) + x(1 + y) y' = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 33

```
dsolve((y(x)*(x*cos(x)+2*sin(x)))+(x*(y(x)+1))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = e^{-\text{LambertW}(e^{-\sin(x)-2 \text{Si}(x)-c_1})-\sin(x)-2 \text{Si}(x)-c_1)}$$

✓ Solution by Mathematica

Time used: 2.869 (sec). Leaf size: 24

```
DSolve[(y[x]*(x*Cos[x]+2*Sin[x]))+(x*(y[x]+1))*y'[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \rightarrow W(e^{-2\text{Si}(x)-\sin(x)+c_1})$$

$$y(x) \rightarrow 0$$

7.23 problem 24

Internal problem ID [1083]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6

Page 91

Problem number: 24.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$x^4 y^3 + y + (y^2 x^5 - x) y' = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 304

```
dsolve((x^4*y(x)^3+y(x))+(x^5*y(x)^2-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\frac{(4x^3+4\sqrt{x^6-4c_1^3})^{\frac{1}{3}}}{2x^2} + \frac{2c_1}{x^2(4x^3+4\sqrt{x^6-4c_1^3})^{\frac{1}{3}}}}{\sqrt{c_1}}$$

$$y(x) = \frac{-\frac{(4x^3+4\sqrt{x^6-4c_1^3})^{\frac{1}{3}}}{4x^2} - \frac{c_1}{x^2(4x^3+4\sqrt{x^6-4c_1^3})^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{(4x^3+4\sqrt{x^6-4c_1^3})^{\frac{1}{3}}}{2x^2} - \frac{2c_1}{x^2(4x^3+4\sqrt{x^6-4c_1^3})^{\frac{1}{3}}}\right)}{2}}{\sqrt{c_1}}$$

$$y(x) = \frac{-\frac{(4x^3+4\sqrt{x^6-4c_1^3})^{\frac{1}{3}}}{4x^2} - \frac{c_1}{x^2(4x^3+4\sqrt{x^6-4c_1^3})^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{(4x^3+4\sqrt{x^6-4c_1^3})^{\frac{1}{3}}}{2x^2} - \frac{2c_1}{x^2(4x^3+4\sqrt{x^6-4c_1^3})^{\frac{1}{3}}}\right)}{2}}{\sqrt{c_1}}$$

✓ Solution by Mathematica

Time used: 39.244 (sec). Leaf size: 300

`DSolve[(x^4*y[x]^3+y[x])+(x^5*y[x]^2-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]`

$$y(x) \rightarrow \frac{2 + \frac{\sqrt[3]{2}(3c_1x^9 + \sqrt{x^{12}(-4 + 9c_1^2x^6)})^{2/3}}{x^4}}{2^{2/3}\sqrt[3]{3c_1x^9 + \sqrt{x^{12}(-4 + 9c_1^2x^6)}}$$

$$y(x) \rightarrow \frac{i\left((\sqrt{3} + i)\left(6c_1x^9 + 2\sqrt{x^{12}(-4 + 9c_1^2x^6)}\right)^{2/3} - 2\sqrt[3]{2}(\sqrt{3} - i)x^4\right)}{4x^4\sqrt[3]{3c_1x^9 + \sqrt{x^{12}(-4 + 9c_1^2x^6)}}$$

$$y(x) \rightarrow \frac{2i\sqrt[3]{2}(\sqrt{3} + i)x^4 + (-1 - i\sqrt{3})\left(6c_1x^9 + 2\sqrt{x^{12}(-4 + 9c_1^2x^6)}\right)^{2/3}}{4x^4\sqrt[3]{3c_1x^9 + \sqrt{x^{12}(-4 + 9c_1^2x^6)}}$$

7.24 problem 25

Internal problem ID [1084]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6

Page 91

Problem number: 25.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class D', _rational, [_Abel, '2nd type', 'cl`

$$3yx + 2y^2 + y + (x^2 + 2yx + x + 2y)y' = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 103

```
dsolve((3*x*y(x)+2*y(x)^2+y(x))+(x^2+2*x*y(x)+x+2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-\frac{c_1 x^2}{2} - \frac{c_1 x}{2} - \frac{\sqrt{c_1^2 x^4 + 2c_1^2 x^3 + c_1^2 x^2 + 4}}{2}}{c_1 (x + 1)}$$

$$y(x) = \frac{-\frac{c_1 x^2}{2} - \frac{c_1 x}{2} + \frac{\sqrt{c_1^2 x^4 + 2c_1^2 x^3 + c_1^2 x^2 + 4}}{2}}{c_1 (x + 1)}$$

✓ Solution by Mathematica

Time used: 14.424 (sec). Leaf size: 105

```
DSolve[(3*x*y[x]+2*y[x]^2+y[x])+(x^2+2*x*y[x]+x+2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolut
```

$$y(x) \rightarrow \frac{1}{2} \left(-x - \sqrt{x^2 + \frac{4e^{c_1}}{(x+1)^2}} \right)$$

$$y(x) \rightarrow \frac{1}{2} \left(-x + \sqrt{x^2 + \frac{4e^{c_1}}{(x+1)^2}} \right)$$

$$y(x) \rightarrow \frac{1}{2} \left(-\sqrt{x^2} - x \right)$$

$$y(x) \rightarrow \frac{1}{2} \left(\sqrt{x^2} - x \right)$$

7.25 problem 26

Internal problem ID [1085]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6

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Problem number: 26.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$12yx + 6y^3 + (9x^2 + 10xy^2) y' = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 35

```
dsolve((12*x*y(x)+6*y(x)^3)+(9*x^2+10*x*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$\ln(x) - c_1 + \frac{6 \ln\left(\frac{y(x)}{\sqrt{x}}\right)}{11} + \frac{2 \ln\left(\frac{2y(x)^2+3x}{x}\right)}{11} = 0$$

✓ Solution by Mathematica

Time used: 8.107 (sec). Leaf size: 151

```
DSolve[(12*x*y[x]+6*y[x]^3)+(9*x^2+10*x*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow \text{Root}[2\#1^5x^3 + 3\#1^3x^4 - c_1\&, 1]$$

$$y(x) \rightarrow \text{Root}[2\#1^5x^3 + 3\#1^3x^4 - c_1\&, 2]$$

$$y(x) \rightarrow \text{Root}[2\#1^5x^3 + 3\#1^3x^4 - c_1\&, 3]$$

$$y(x) \rightarrow \text{Root}[2\#1^5x^3 + 3\#1^3x^4 - c_1\&, 4]$$

$$y(x) \rightarrow \text{Root}[2\#1^5x^3 + 3\#1^3x^4 - c_1\&, 5]$$

7.26 problem 27

Internal problem ID [1086]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6

Page 91

Problem number: 27.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class G', _rational, _Bernoulli]`

$$3x^2y^2 + 2y + 2y'x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((3*x^2*y(x)^2+2*y(x))+(2*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{2}{(3x + 2c_1)x}$$

✓ Solution by Mathematica

Time used: 0.146 (sec). Leaf size: 25

```
DSolve[(3*x^2*y[x]^2+2*y[x])+(2*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{2}{3x^2 + 2c_1x}$$

$$y(x) \rightarrow 0$$

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8.1 problem 1

Internal problem ID [1087]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203

Problem number: 1.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 7y' + 10y = 0$$

With initial conditions

$$[y(0) = -1, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve([diff(y(x),x$2)-7*diff(y(x),x)+10*y(x)=0,y(0) = -1, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = e^{5x} - 2e^{2x}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 18

```
DSolve[{y'[x]-7*y'[x]+10*y[x]==0,{y[0]==-1,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{2x}(e^{3x} - 2)$$

8.2 problem 2c

Internal problem ID [1088]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203

Problem number: 2c.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 2y' + 2y = 0$$

With initial conditions

$$[y(0) = 3, y'(0) = -2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+2*y(x)=0,y(0) = 3, D(y)(0) = -2],y(x), singsol=all)
```

$$y(x) = e^x(-5 \sin(x) + 3 \cos(x))$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 18

```
DSolve[{y'[x]-2*y'[x]+2*y[x]==0,{y[0]==3,y'[0]==-2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x(3 \cos(x) - 5 \sin(x))$$

8.3 problem 2d

Internal problem ID [1089]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203

Problem number: 2d.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 2y' + 2y = 0$$

With initial conditions

$$[y(0) = k_0, y'(0) = k_1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+2*y(x)=0,y(0) = k__0, D(y)(0) = k__1],y(x), singsol=all)
```

$$y(x) = e^x((k_1 - k_0) \sin(x) + k_0 \cos(x))$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 22

```
DSolve[{y'[x]-2*y'[x]+2*y[x]==0,{y[0]==k0,y'[0]==k1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x((k1 - k0) \sin(x) + k0 \cos(x))$$

8.4 problem 3c

Internal problem ID [1090]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203

Problem number: 3c.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 2y' + y = 0$$

With initial conditions

$$[y(0) = 7, y'(0) = 4]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+y(x)=0,y(0) = 7, D(y)(0) = 4],y(x), singsol=all)
```

$$y(x) = e^x(7 - 3x)$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 14

```
DSolve[{y'[x]-2*y'[x]+y[x]==0,{y[0]==7,y'[0]==4}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x(7 - 3x)$$

8.5 problem 3d

Internal problem ID [1091]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203

Problem number: 3d.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 2y' + y = 0$$

With initial conditions

$$[y(0) = k_0, y'(0) = k_1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+y(x)=0,y(0) = k__0, D(y)(0) = k__1],y(x), singsol=all)
```

$$y(x) = -((x - 1)k_0 - xk_1)e^x$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 18

```
DSolve[{y'[x]-2*y'[x]+y[x]==0,{y[0]==k0,y'[0]==k1}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \rightarrow e^x(k_0(-x) + k_0 + k_1x)$$

8.6 problem 4

Internal problem ID [1092]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203

Problem number: 4.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(x^2 - 1)y'' + 4y'x + 2y = 0$$

With initial conditions

$$[y(0) = -5, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve([(x^2-1)*diff(y(x),x$2)+4*x*diff(y(x),x)+2*y(x)=0,y(0) = -5, D(y)(0) = 1],y(x), sings
```

$$y(x) = \frac{-x + 5}{x^2 - 1}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 18

```
DSolve[{(x^2-1)*y''[x]+4*x*y'[x]+2*y[x]==0,{y[0]==-5,y'[0]==1}},y[x],x,IncludeSingularSoluti
```

$$y(x) \rightarrow \frac{5 - x}{x^2 - 1}$$

8.7 problem 10

Internal problem ID [1093]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203

Problem number: 10.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 2y' - 3y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-x}c_1 + c_2e^{3x}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 22

```
DSolve[y''[x]-2*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x}(c_2e^{4x} + c_1)$$

8.8 problem 11

Internal problem ID [1094]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203

Problem number: 11.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 6y' + 9y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{3x} + c_2 e^{3x} x$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 18

```
DSolve[y''[x]-6*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{3x}(c_2 x + c_1)$$

8.9 problem 12

Internal problem ID [1095]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203

Problem number: 12.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 2ay' + a^2y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-2*a*diff(y(x),x)+a^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1e^{ax} + c_2e^{ax}x$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 18

```
DSolve[y''[x]-2*a*y'[x]+a^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{ax}(c_2x + c_1)$$

8.10 problem 13

Internal problem ID [1096]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203

Problem number: 13.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x^2y'' + y'x - y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + c_2x$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 16

```
DSolve[x^2*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_1}{x} + c_2x$$

8.11 problem 14

Internal problem ID [1097]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203

Problem number: 14.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^2 y'' - y' x + y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x + c_2 x \ln(x)$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 15

```
DSolve[x^2*y'[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x(c_2 \log(x) + c_1)$$

8.12 problem 15

Internal problem ID [1098]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203

Problem number: 15.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^2 y'' - (2a - 1) x y' + a^2 y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)-(2*a-1)*x*diff(y(x),x)+a^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^a + c_2 x^a \ln(x)$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 18

```
DSolve[x^2*y'[x]-(2*a-1)*x*y'[x]+a^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^a (a c_2 \log(x) + c_1)$$

8.13 problem 16

Internal problem ID [1099]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203

Problem number: 16.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' - 4y'x + (-16x^2 + 3)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(3-16*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x} \sinh(2x) + c_2\sqrt{x} \cosh(2x)$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 32

```
DSolve[4*x^2*y''[x]-4*x*y'[x]+(3-16*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{4}e^{-2x}\sqrt{x}(c_2e^{4x} + 4c_1)$$

8.14 problem 17

Internal problem ID [1100]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203

Problem number: 17.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x - 1)y'' - y'x + y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x + e^x c_2$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 17

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1e^x - c_2x$$

8.15 problem 18

Internal problem ID [1101]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203

Problem number: 18.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' - 2y'x + (x^2 + 2)y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(x) x + c_2 \cos(x) x$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 33

```
DSolve[x^2*y'[x]-2*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{-ix} x - \frac{1}{2} i c_2 e^{ix} x$$

8.16 problem 19

Internal problem ID [1102]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203

Problem number: 19.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2 \sin(x) y'' - 4x(x \cos(x) + \sin(x)) y' + (2x \cos(x) + 3 \sin(x)) y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 17

```
dsolve(4*x^2*sin(x)*diff(y(x),x$2)-4*x*(x*cos(x)+sin(x))*diff(y(x),x)+(2*x*cos(x)+3*sin(x))*
```

$$y(x) = c_1 \sqrt{x} + c_2 \sqrt{x} \cos(x)$$

✓ Solution by Mathematica

Time used: 0.356 (sec). Leaf size: 21

```
DSolve[4*x^2*Sin[x]*y''[x]-4*x*(x*Cos[x]+Sin[x])*y'[x]+(2*x*Cos[x]+3*Sin[x])*y[x]==0,y[x],x,
```

$$y(x) \rightarrow \sqrt{\arccos(\cos(x))}(c_2 \cos(x) + c_1)$$

8.17 problem 20

Internal problem ID [1103]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203

Problem number: 20.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(3x - 1)y'' - (3x + 2)y' + (6x - 8)y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 89

```
dsolve((3*x-1)*diff(y(x),x$2)-(3*x+2)*diff(y(x),x)+(6*x-8)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{x(i\sqrt{7}-1)}{2}} (3x-1)^2 \text{KummerM}\left(\frac{3}{2} - \frac{5i\sqrt{7}}{14}, 3, \frac{i\sqrt{7}(3x-1)}{3}\right) \\ + c_2 e^{-\frac{x(i\sqrt{7}-1)}{2}} (3x-1)^2 \text{KummerU}\left(\frac{3}{2} - \frac{5i\sqrt{7}}{14}, 3, \frac{i\sqrt{7}(3x-1)}{3}\right)$$

✓ Solution by Mathematica

Time used: 0.13 (sec). Leaf size: 109

```
DSolve[(3*x-1)*y'[x]-(3*x+2)*y'[x]+(6*x-8)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 4e^{\frac{1}{6}(1-i\sqrt{7})(3x-1)}(1-3x)^2 \left(c_1 \text{HypergeometricU}\left(\frac{3}{2} - \frac{5i}{2\sqrt{7}}, 3, \frac{1}{3}i\sqrt{7}(3x-1)\right) \right. \\ \left. + c_2 L^2_{-\frac{3}{2} + \frac{5i}{2\sqrt{7}}}\left(\frac{1}{3}i\sqrt{7}(3x-1)\right) \right)$$

8.18 problem 21

Internal problem ID [1104]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203

Problem number: 21.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(x^2 - 4)y'' + 4y'x + 2y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve((x^2-4)*diff(y(x),x$2)+4*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1x + c_2}{x^2 - 4}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 20

```
DSolve[(x^2-4)*y'[x]+4*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_2x + c_1}{x^2 - 4}$$

8.19 problem 22

Internal problem ID [1105]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203

Problem number: 22.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(2x + 1)y'' - 2(2x^2 - 1)y' - 4(1 + x)y = 0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 32

```
dsolve((2*x+1)*diff(y(x),x$2)-2*(2*x^2-1)*diff(y(x),x)-4*(x+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{HeunB}\left(-\frac{1}{2}, -2, -\frac{1}{2}, 3, \frac{1}{2} + x\right) + c_2 \operatorname{HeunB}\left(\frac{1}{2}, -2, -\frac{1}{2}, 3, \frac{1}{2} + x\right) \sqrt{2 + 4x}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(2*x+1)*y'[x]-2*(2*x^2-1)*y'[x]-4*(x+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

Not solved

8.20 problem 23

Internal problem ID [1106]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203

Problem number: 23.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 - 2x)y'' + (-x^2 + 2)y' + (2x - 2)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve((x^2-2*x)*diff(y(x),x$2)+(2-x^2)*diff(y(x),x)+(2*x-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^2 + e^xc_2$$

✓ Solution by Mathematica

Time used: 0.091 (sec). Leaf size: 18

```
DSolve[(x^2-2*x)*y''[x]+(2-x^2)*y'[x]+(2*x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_2x^2 + c_1e^x$$

9 Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

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9.1 problem 1

Internal problem ID [1107]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 1.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(1 + 2x)y'' - 2y' - (2x + 3)y = (1 + 2x)^2$$

Given that one solution of the ode is

$$y_1 = e^{-x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve([(2*x+1)*diff(y(x),x$2)-2*diff(y(x),x)-(2*x+3)*y(x)=(2*x+1)^2,exp(-x)],y(x), singsol=
```

$$y(x) = e^{-x}c_2 + x e^x c_1 + 1 - 2x$$

✓ Solution by Mathematica

Time used: 0.126 (sec). Leaf size: 33

```
DSolve[(2*x+1)*y'[x]-2*y'[x]-(2*x+3)*y[x]==(2*x+1)^2,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \rightarrow c_1 e^{-x-\frac{1}{2}} + x \left(-2 + c_2 e^{x+\frac{1}{2}} \right) + 1$$

9.2 problem 2

Internal problem ID [1108]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 2.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$x^2 y'' + y'x - y = \frac{4}{x^2}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve([x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=4/x^2,x],y(x), singsol=all)
```

$$y(x) = c_2 x + \frac{c_1}{x} + \frac{4}{3x^2}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 23

```
DSolve[x^2*y''[x]+x*y'[x]-y[x]==4/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{4}{3x^2} + \frac{c_1}{x} + c_2 x$$

9.3 problem 3

Internal problem ID [1109]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 3.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' - y'x + y = x$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve([x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=x,x],y(x), singsol=all)
```

$$y(x) = c_2 x + \ln(x) c_1 x + \frac{\ln(x)^2 x}{2}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 25

```
DSolve[x^2*y''[x]-x*y'[x]+y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2}x(\log^2(x) + 2c_2 \log(x) + 2c_1)$$

9.4 problem 4

Internal problem ID [1110]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 4.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 3y' + 2y = \frac{1}{1 + e^{-x}}$$

Given that one solution of the ode is

$$y_1 = e^{2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve([diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=1/(1+exp(-x)),exp(2*x)],y(x), singsol=all)
```

$$y(x) = (c_1 e^x - \ln(e^x) - \ln(e^x) e^x + \ln(1 + e^x)(1 + e^x) - 1 + c_2) e^x$$

✓ Solution by Mathematica

Time used: 0.078 (sec). Leaf size: 34

```
DSolve[y''[x]-3*y'[x]+2*y[x]==1/(1+Exp[-x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x(2(e^x + 1) \operatorname{arctanh}(2e^x + 1) + c_2 e^x - 1 + c_1)$$

9.5 problem 5

Internal problem ID [1111]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 5.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2y' + y = 7e^x x^{\frac{3}{2}}$$

Given that one solution of the ode is

$$y_1 = e^x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+y(x)=7*x^(3/2)*exp(x),exp(x)],y(x), singsol=all)
```

$$y(x) = e^x c_2 + x e^x c_1 + \frac{4x^{\frac{7}{2}} e^x}{5}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 29

```
DSolve[y''[x]-2*y'[x]+y[x]==7*x^(3/2)*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{5}e^x(4x^{7/2} + 5c_2x + 5c_1)$$

9.6 problem 6

Internal problem ID [1112]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 6.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$4x^2y'' + (-8x^2 + 4x)y' + (4x^2 - 4x - 1)y = 4\sqrt{x}e^x(4x + 1)$$

Given that one solution of the ode is

$$y_1 = e^x \sqrt{x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 32

```
dsolve([4*x^2*diff(y(x),x$2)+(4*x-8*x^2)*diff(y(x),x)+(4*x^2-4*x-1)*y(x)=4*x^(1/2)*exp(x)*(1+4*x)],y(x),x,implicit)
```

$$y(x) = \frac{e^x c_2}{\sqrt{x}} + \sqrt{x} e^x c_1 + e^x \sqrt{x} (\ln(x) + 2x - 1)$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 32

```
DSolve[4*x^2*y''[x]+(4*x-8*x^2)*y'[x]+(4*x^2-4*x-1)*y[x]==4*x^(1/2)*Exp[x]*(1+4*x),y[x],x,Implicit]
```

$$y(x) \rightarrow \frac{e^x(2x^2 + x \log(x) + (-1 + c_2)x + c_1)}{\sqrt{x}}$$

9.7 problem 7

Internal problem ID [1113]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 7.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2y' + 2y = e^x \sec(x)$$

Given that one solution of the ode is

$$y_1 = e^x \cos(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+2*y(x)=exp(x)*sec(x),exp(x)*cos(x)],y(x), singsol=all)
```

$$y(x) = c_2 \sin(x) e^x + c_1 \cos(x) e^x + e^x(\sin(x) x - \cos(x) \ln(\sec(x)))$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 26

```
DSolve[y''[x]-2*y'[x]+2*y[x]==Exp[x]*Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x((x + c_1) \sin(x) + \cos(x)(\log(\cos(x)) + c_2))$$

9.8 problem 8

Internal problem ID [1114]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 8.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y'x + (4x^2 + 2)y = 8e^{-x(2+x)}$$

Given that one solution of the ode is

$$y_1 = e^{-x^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve([diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2+2)*y(x)=8*exp(-x*(x+2)),exp(-x^2)],y(x),sing
```

$$y(x) = e^{-x^2}c_2 + e^{-x^2}xc_1 + 2e^{-x(2+x)}$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 29

```
DSolve[y''[x]+4*x*y'[x]+(4*x^2+2)*y[x]==8*Exp[-x*(x+2)],y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \rightarrow e^{-x(x+2)}(2 + e^{2x}(c_2x + c_1))$$

9.9 problem 9

Internal problem ID [1115]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 9.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + y'x - 4y = -6x - 4$$

Given that one solution of the ode is

$$y_1 = x^2$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve([x^2*diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=-6*x-4,x^2],y(x), singsol=all)
```

$$y(x) = x^2c_2 + \frac{c_1}{x^2} + 1 + 2x$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]+x*y'[x]-4*y[x]==-6*x-4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_2x^2 + \frac{c_1}{x^2} + 2x + 1$$

9.10 problem 10

Internal problem ID [1116]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 10.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x^2 y'' + 2x(x-1)y' + (x^2 - 2x + 2)y = e^{2x}x^3$$

Given that one solution of the ode is

$$y_1 = x e^{-x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve([x^2*diff(y(x),x$2)+2*x*(x-1)*diff(y(x),x)+(x^2-2*x+2)*y(x)=x^3*exp(2*x),x*exp(-x)],y
```

$$y(x) = c_2 e^{-x} x + x^2 e^{-x} c_1 + \frac{e^{2x} x}{9}$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 30

```
DSolve[x^2*y'[x]+2*x*(x-1)*y'[x]+(x^2-2*x+2)*y[x]==x^3*Exp[2*x],y[x],x,IncludeSingularSolut
```

$$y(x) \rightarrow \frac{1}{9} e^{-x} x (e^{3x} + 9(c_2 x + c_1))$$

9.11 problem 11

Internal problem ID [1117]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 11.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x^2 y'' - x(2x - 1) y' + (x^2 - x - 1) y = x^2 e^x$$

Given that one solution of the ode is

$$y_1 = e^x x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve([x^2*diff(y(x),x$2)-x*(2*x-1)*diff(y(x),x)+(x^2-x-1)*y(x)=x^2*exp(x),x*exp(x)],y(x),
```

$$y(x) = \frac{e^x c_2}{x} + x e^x c_1 + \frac{x^2 e^x}{3}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 32

```
DSolve[x^2*y'[x]-x*(2*x-1)*y'[x]+(x^2-x-1)*y[x]==x^2*Exp[x],y[x],x,IncludeSingularSolutions
```

$$y(x) \rightarrow \frac{e^x(2x^3 + 3c_2x^2 + 6c_1)}{6x}$$

9.12 problem 12

Internal problem ID [1118]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 12.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$(1 - 2x)y'' + 2y' + (-3 + 2x)y = (4x^2 - 4x + 1)e^x$$

Given that one solution of the ode is

$$y_1 = e^x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve([(1-2*x)*diff(y(x),x$2)+2*diff(y(x),x)+(2*x-3)*y(x)=(1-4*x+4*x^2)*exp(x),exp(x)],y(x))
```

$$y(x) = e^x c_2 + c_1 x e^{-x} - \frac{x(x-1)e^x}{2}$$

✓ Solution by Mathematica

Time used: 0.254 (sec). Leaf size: 77

```
DSolve[(1-2*x)*y'[x]+2*y'[x]+(2*x-3)*y[x]==(1-4*x+4*x^2)*Exp[x],y[x],x,IncludeSingularSolut
```

$$y(x) \rightarrow -\frac{1}{2}e^x(x-1)x - \frac{c_2 e^{\frac{1}{2}-x}\sqrt{1-2xx}}{\sqrt{2x-1}} + \frac{c_1 e^{x-\frac{1}{2}}\sqrt{1-2x}}{\sqrt{2x-1}}$$

9.13 problem 13

Internal problem ID [1119]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 13.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' - 3y'x + 4y = 4x^4$$

Given that one solution of the ode is

$$y_1 = x^2$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve([x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*y(x)=4*x^4,x^2],y(x), singsol=all)
```

$$y(x) = x^2 c_2 + \ln(x) c_1 x^2 + x^4$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 21

```
DSolve[x^2*y''[x]-3*x*y'[x]+4*y[x]==4*x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^2(x^2 + 2c_2 \log(x) + c_1)$$

9.14 problem 14

Internal problem ID [1120]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 14.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$2xy'' + (4x + 1)y' + (1 + 2x)y = 3\sqrt{x}e^{-x}$$

Given that one solution of the ode is

$$y_1 = e^{-x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve([2*x*diff(y(x),x$2)+(4*x+1)*diff(y(x),x)+(2*x+1)*y(x)=3*x^(1/2)*exp(-x),exp(-x)],y(x))
```

$$y(x) = e^{-x}c_2 + \sqrt{x}e^{-x}c_1 + x^{\frac{3}{2}}e^{-x}$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 28

```
DSolve[2*x*y'[x]+(4*x+1)*y'[x]+(2*x+1)*y[x]==3*x^(1/2)*Exp[-x],y[x],x,IncludeSingularSoluti
```

$$y(x) \rightarrow e^{-x}(x^{3/2} + 2c_2\sqrt{x} + c_1)$$

9.15 problem 15

Internal problem ID [1121]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 15.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$xy'' - (1 + 2x)y' + (x + 1)y = -e^{-x}$$

Given that one solution of the ode is

$$y_1 = e^x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve([x*diff(y(x),x$2)-(2*x+1)*diff(y(x),x)+(x+1)*y(x)=-exp(-x),exp(x)],y(x), singsol=all)
```

$$y(x) = e^x c_2 + e^x c_1 x^2 - \text{Ei}_1(2x) x^2 e^x + \frac{e^{-x}(2x - 1)}{4}$$

✓ Solution by Mathematica

Time used: 0.073 (sec). Leaf size: 52

```
DSolve[x*y''[x]-(2*x+1)*y'[x]+(x+1)*y[x]==-Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x x^2 \text{ExpIntegralEi}(-2x) + \frac{1}{4} e^{-x} (2c_2 e^{2x} x^2 + 2x + 4c_1 e^{2x} - 1)$$

9.16 problem 16

Internal problem ID [1122]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 16.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$4x^2y'' - 4x(x+1)y' + (2x+3)y = 4e^{2x}x^{\frac{5}{2}}$$

Given that one solution of the ode is

$$y_1 = \sqrt{x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve([4*x^2*diff(y(x),x$2)-4*x*(x+1)*diff(y(x),x)+(2*x+3)*y(x)=4*x^(5/2)*exp(2*x),x^(1/2)]
```

$$y(x) = \sqrt{x}c_2 + \sqrt{x}e^xc_1 + \frac{\sqrt{x}e^{2x}}{2}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 31

```
DSolve[4*x^2*y''[x]-4*x*(x+1)*y'[x]+(2*x+3)*y[x]==4*x^(5/2)*Exp[2*x],y[x],x,IncludeSingularS
```

$$y(x) \rightarrow \frac{1}{2}\sqrt{x}(e^{2x} + 2c_2e^x + 2c_1)$$

9.17 problem 17

Internal problem ID [1123]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 17.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' - 5y'x + 8y = 4x^2$$

Given that one solution of the ode is

$$y_1 = x^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve([x^2*diff(y(x),x$2)-5*x*diff(y(x),x)+8*y(x)=4*x^2,x^2],y(x), singsol=all)
```

$$y(x) = c_2 x^4 + c_1 x^2 + x^2(-1 - 2 \ln(x))$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 23

```
DSolve[x^2*y''[x]-5*x*y'[x]+8*y[x]==4*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^2(c_2 x^2 - 2 \log(x) - 1 + c_1)$$

9.18 problem 18

Internal problem ID [1124]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 18.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' + (-2x + 2)y' + (-2 + x)y = 0$$

Given that one solution of the ode is

$$y_1 = e^x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve([x*diff(y(x),x$2)+(2-2*x)*diff(y(x),x)+(x-2)*y(x)=0,exp(x)],y(x), singsol=all)
```

$$y(x) = c_1 e^x + \frac{e^x c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 19

```
DSolve[x*y''[x]+(2-2*x)*y'[x]+(x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^x(c_2 x + c_1)}{x}$$

9.19 problem 19

Internal problem ID [1125]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 19.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$x^2 y'' - 4y'x + 6y = 0$$

Given that one solution of the ode is

$$y_1 = x^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=0,x^2],y(x), singsol=all)
```

$$y(x) = c_1 x^3 + x^2 c_2$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 16

```
DSolve[x^2*y''[x]-4*x*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^2(c_2 x + c_1)$$

9.20 problem 20

Internal problem ID [1126]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 20.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 \ln(x)^2 y'' - 2x \ln(x) y' + (\ln(x) + 2) y = 0$$

Given that one solution of the ode is

$$y_1 = \ln(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve([x^2*(ln(x))^2*diff(y(x),x$2)-2*x*ln(x)*diff(y(x),x)+(2+ln(x))*y(x)=0,ln(x)],y(x), si
```

$$y(x) = \ln(x) c_1 + c_2 x \ln(x)$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 15

```
DSolve[x^2*Log[x]^2*y''[x]-2*x*Log[x]*y'[x]+(2+Log[x])*y[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x) \rightarrow (c_2 x + c_1) \log(x)$$

9.21 problem 21

Internal problem ID [1127]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction of order. Page 253

Problem number: 21.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$4xy'' + 2y' + y = 0$$

Given that one solution of the ode is

$$y_1 = \sin(\sqrt{x})$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve([4*x*diff(y(x),x$2)+2*diff(y(x),x)+y(x)=0,sin(sqrt(x))],y(x), singsol=all)
```

$$y(x) = c_1 \sin(\sqrt{x}) + c_2 \cos(\sqrt{x})$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 24

```
DSolve[4*x*y''[x]+2*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \cos(\sqrt{x}) + c_2 \sin(\sqrt{x})$$

9.22 problem 22

Internal problem ID [1128]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 22.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' - (2 + 2x)y' + (2 + x)y = 0$$

Given that one solution of the ode is

$$y_1 = e^x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve([x*diff(y(x),x$2)-(2*x+2)*diff(y(x),x)+(x+2)*y(x)=0,exp(x)],y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2 e^x x^3$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 23

```
DSolve[x*y''[x]-(2*x+2)*y'[x]+(x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{3} e^x (c_2 x^3 + 3c_1)$$

9.23 problem 23

Internal problem ID [1129]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 23.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^2 y'' - (2a - 1) x y' + a^2 y = 0$$

Given that one solution of the ode is

$$y_1 = x^a$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve([x^2*diff(y(x),x$2)-(2*a-1)*x*diff(y(x),x)+a^2*y(x)=0,x^a],y(x), singsol=all)
```

$$y(x) = c_1 x^a + c_2 x^a \ln(x)$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 18

```
DSolve[x^2*y'[x]-(2*a-1)*x*y'[x]+a^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^a (a c_2 \log(x) + c_1)$$

9.24 problem 24

Internal problem ID [1130]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 24.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' - 2y'x + (x^2 + 2)y = 0$$

Given that one solution of the ode is

$$y_1 = \sin(x)x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=0,x*sin(x)],y(x), singsol=all)
```

$$y(x) = c_1 \sin(x)x + c_2 \cos(x)x$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{-ix}x - \frac{1}{2} i c_2 e^{ix}x$$

9.25 problem 25

Internal problem ID [1131]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 25.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' - (4x + 1)y' + (2 + 4x)y = 0$$

Given that one solution of the ode is

$$y_1 = e^{2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve([x*diff(y(x),x$2)-(4*x+1)*diff(y(x),x)+(4*x+2)*y(x)=0,exp(2*x)],y(x), singsol=all)
```

$$y(x) = c_1 e^{2x} + c_2 e^{2x} x^2$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 25

```
DSolve[x*y''[x]-(4*x+1)*y'[x]+(4*x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2} e^{2x} (c_2 x^2 + 2c_1)$$

9.26 problem 26

Internal problem ID [1132]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 26.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2 \sin(x) y'' - 4x(x \cos(x) + \sin(x)) y' + (2x \cos(x) + 3 \sin(x)) y = 0$$

Given that one solution of the ode is

$$y_1 = \sqrt{x}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 17

```
dsolve([4*x^2*sin(x)*diff(y(x),x$2)-4*x*(x*cos(x)+sin(x))*diff(y(x),x)+(2*x*cos(x)+3*sin(x))*y(x)]=0,y(x),x)
```

$$y(x) = c_1 \sqrt{x} + c_2 \sqrt{x} \cos(x)$$

✓ Solution by Mathematica

Time used: 0.322 (sec). Leaf size: 21

```
DSolve[4*x^2*Sin[x]*y''[x]-4*x*(x*Cos[x]+Sin[x])*y'[x]+(2*x*Cos[x]+3*Sin[x])*y[x]==0,y[x],x,
```

$$y(x) \rightarrow \sqrt{\arccos(\cos(x))} (c_2 \cos(x) + c_1)$$

9.27 problem 27

Internal problem ID [1133]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 27.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' - 4y'x + (-16x^2 + 3)y = 0$$

Given that one solution of the ode is

$$y_1 = \sqrt{x} e^{2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve([4*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(3-16*x^2)*y(x)=0,sqrt(x)*exp(2*x)],y(x),sing
```

$$y(x) = c_1\sqrt{x} \sinh(2x) + c_2\sqrt{x} \cosh(2x)$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 32

```
DSolve[4*x^2*y'[x]-4*x*y'[x]+(3-16*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{4}e^{-2x}\sqrt{x}(c_2e^{4x} + 4c_1)$$

9.28 problem 28

Internal problem ID [1134]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 28.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(1 + 2x)xy'' - 2(2x^2 - 1)y' - 4(x + 1)y = 0$$

Given that one solution of the ode is

$$y_1 = \frac{1}{x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve([(2*x+1)*x*diff(y(x),x$2)-2*(2*x^2-1)*diff(y(x),x)-4*(x+1)*y(x)=0,1/x],y(x), singsol=
```

$$y(x) = \frac{c_1}{x} + c_2e^{2x}$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 28

```
DSolve[(2*x+1)*x*y'[x]-2*(2*x^2-1)*y'[x]-4*(x+1)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow \frac{c_2e^{2x+1}x + c_1}{\sqrt{ex}}$$

9.29 problem 29

Internal problem ID [1135]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 29.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 - 2x)y'' + (-x^2 + 2)y' + (2x - 2)y = 0$$

Given that one solution of the ode is

$$y_1 = e^x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve([(x^2-2*x)*diff(y(x),x$2)+(2-x^2)*diff(y(x),x)+(2*x-2)*y(x)=0,exp(x)],y(x), singsol=a
```

$$y(x) = c_1x^2 + e^xc_2$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 18

```
DSolve[(x^2-2*x)*y'[x]+(2-x^2)*y'[x]+(2*x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \rightarrow c_2x^2 + c_1e^x$$

9.30 problem 30

Internal problem ID [1136]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 30.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' - (4x + 1)y' + (2 + 4x)y = 0$$

Given that one solution of the ode is

$$y_1 = e^{2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve([x*diff(y(x),x$2)-(4*x+1)*diff(y(x),x)+(4*x+2)*y(x)=0,exp(2*x)],y(x), singsol=all)
```

$$y(x) = c_1 e^{2x} + c_2 e^{2x} x^2$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 25

```
DSolve[x*y''[x]-(4*x+1)*y'[x]+(4*x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2} e^{2x} (c_2 x^2 + 2c_1)$$

9.31 problem 31

Internal problem ID [1137]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 31.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' - 3y'x + 4y = 4x^4$$

Given that one solution of the ode is

$$y_1 = x^2$$

With initial conditions

$$[y(-1) = 7, y'(-1) = -8]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve([x^2*dif(dif(y(x),x),x)-3*x*dif(y(x),x)+4*y(x) = 4*x^4, x^2, y(-1) = 7, D(y)(-1) = -8])
```

$$y(x) = x^2(8i\pi + x^2 - 8 \ln(x) + 6)$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 37

```
DSolve[x^2*y''[x]-3*x*y'[x]+4*y[x]==4*x^2,{y[-1]==7,y'[-1]==8},y[x],x,IncludeSingularSolutions->True]
```

$$y(x) \rightarrow x^2(2 \log^2(x) + (-22 - 4i\pi) \log(x) - 2\pi^2 + 22i\pi + 7)$$

9.32 problem 32

Internal problem ID [1138]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 32.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(3x - 1)y'' - (3x + 2)y' - (6x - 8)y = 0$$

Given that one solution of the ode is

$$y_1 = e^{2x}$$

With initial conditions

$$[y(0) = 2, y'(0) = 3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve([(3*x-1)*diff(diff(y(x),x),x)-(3*x+2)*diff(y(x),x)-(6*x-8)*y(x) = 0, exp(2*x), y(0) =
```

$$y(x) = 2e^{2x} - xe^{-x}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(3*x-1)*y'[x]-(3*x+2)*x*y'[x]-(6*x-8)*y[x]==0,{y[0]==2,y'[0]==3},y[x],x,IncludeSingul
```

Not solved

9.33 problem 33

Internal problem ID [1139]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 33.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$(x + 1)^2 y'' - 2(x + 1) y' - (x^2 + 2x - 1) y = (x + 1)^3 e^x$$

Given that one solution of the ode is

$$y_1 = e^x(x + 1)$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve([(1+x)^2*diff(diff(y(x),x),x)-2*(1+x)*diff(y(x),x)-(x^2+2*x-1)*y(x) = (1+x)^3*exp(x),
```

$$y(x) = \frac{(x + 1)(x e^x - 5 \sinh(x) + 2 \cosh(x))}{2}$$

✓ Solution by Mathematica

Time used: 21.262 (sec). Leaf size: 5749

```
DSolve[(x+1)^2*y''[x]-2*(x+1)*x*y'[x]-(x^2+2*x-1)*y[x]==(x+1)^3*Exp[x],{y[0]==1,y'[0]==-1},y
```

Too large to display

9.34 problem 34

Internal problem ID [1140]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 34.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + 2y'x - 2y = x^2$$

Given that one solution of the ode is

$$y_1 = x$$

With initial conditions

$$\left[y(1) = \frac{5}{4}, y'(1) = \frac{3}{2} \right]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve([x^2*diff(diff(y(x),x),x)+2*x*diff(y(x),x)-2*y(x) = x^2, x, y(1) = 5/4, D(y)(1) = 3/2
```

$$y(x) = x + \frac{1}{4}x^2$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 13

```
DSolve[x^2*y'[x]+2*x*y'[x]-2*y[x]==x^2,{y[1]==5/4,y'[1]==3/2},y[x],x,IncludeSingularSolutio
```

$$y(x) \rightarrow \frac{1}{4}x(x + 4)$$

9.35 problem 35

Internal problem ID [1141]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction of order. Page 253

Problem number: 35.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$(x^2 - 4)y'' + 4y'x + 2y = 2 + x$$

Given that one solution of the ode is

$$y_1 = \frac{1}{-2 + x}$$

With initial conditions

$$\left[y(0) = -\frac{1}{3}, y'(0) = -1 \right]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve([(x^2-4)*diff(diff(y(x),x),x)+4*x*diff(y(x),x)+2*y(x) = 2+x, 1/(-2+x), y(0) = -1/3, D
```

$$y(x) = \frac{x^3 + 6x^2 + 24x + 8}{6x^2 - 24}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 30

```
DSolve[(x^2-4)*y'[x]+4*x*y'[x]+2*y[x]==x+2,{y[1]==5/4,y'[1]==3/2},y[x],x,IncludeSingularSol
```

$$y(x) \rightarrow \frac{-2x^3 - 12x^2 + 54x + 5}{48 - 12x^2}$$

9.36 problem 38 part (a)

Internal problem ID [1142]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 38 part (a).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' + y^2 = -k^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)+y(x)^2+k^2=0,y(x), singsol=all)
```

$$y(x) = -\tan(k(c_1 + x))k$$

✓ Solution by Mathematica

Time used: 4.112 (sec). Leaf size: 35

```
DSolve[y'[x]+y[x]^2+k^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -k \tan(k(x - c_1))$$

$$y(x) \rightarrow -ik$$

$$y(x) \rightarrow ik$$

9.37 problem 38 part (b)

Internal problem ID [1143]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 38 part (b).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [quadrature]

$$y' + y^2 - 3y = -2$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)+y(x)^2-3*y(x)+2=0,y(x), singsol=all)
```

$$y(x) = \frac{2c_1e^x - 1}{c_1e^x - 1}$$

✓ Solution by Mathematica

Time used: 0.93 (sec). Leaf size: 40

```
DSolve[y'[x]+y[x]^2-3*y[x]+2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{2e^x - e^{c_1}}{e^x - e^{c_1}}$$

$$y(x) \rightarrow 1$$

$$y(x) \rightarrow 2$$

9.38 problem 38 part (c)

Internal problem ID [1144]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 38 part (c).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' + y^2 + 5y = 6$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)+y(x)^2+5*y(x)-6=0,y(x), singsol=all)
```

$$y(x) = \frac{6 + e^{7x}c_1}{e^{7x}c_1 - 1}$$

✓ Solution by Mathematica

Time used: 0.71 (sec). Leaf size: 46

```
DSolve[y'[x]+y[x]^2+5*y[x]-6==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^{7x} + 6e^{7c_1}}{e^{7x} - e^{7c_1}}$$

$$y(x) \rightarrow -6$$

$$y(x) \rightarrow 1$$

9.39 problem 38 part (d)

Internal problem ID [1145]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 38 part (d).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [quadrature]

$$y' + y^2 + 8y = -7$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)+y(x)^2+8*y(x)+7=0,y(x), singsol=all)
```

$$y(x) = -\frac{-7 + e^{6x}c_1}{e^{6x}c_1 - 1}$$

✓ Solution by Mathematica

Time used: 0.655 (sec). Leaf size: 47

```
DSolve[y'[x]+y[x]^2+8*y[x]+7==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{e^{6x} - 7e^{6c_1}}{e^{6x} - e^{6c_1}}$$

$$y(x) \rightarrow -7$$

$$y(x) \rightarrow -1$$

9.40 problem 38 part (e)

Internal problem ID [1146]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 38 part (e).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' + y^2 + 14y = -50$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)+y(x)^2+14*y(x)+50=0,y(x), singsol=all)
```

$$y(x) = -7 - \tan(c_1 + x)$$

✓ Solution by Mathematica

Time used: 0.553 (sec). Leaf size: 30

```
DSolve[y'[x]+y[x]^2+14*y[x]+50==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -7 - \tan(x - c_1)$$

$$y(x) \rightarrow -7 - i$$

$$y(x) \rightarrow -7 + i$$

9.41 problem 38 part (f)

Internal problem ID [1147]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 38 part (f).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [quadrature]

$$6y' + 6y^2 - y = 1$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve(6*diff(y(x),x)+6*y(x)^2-y(x)-1=0,y(x), singsol=all)
```

$$y(x) = \frac{1 + e^{\frac{5x}{6}} c_1}{2 e^{\frac{5x}{6}} c_1 - 3}$$

✓ Solution by Mathematica

Time used: 0.234 (sec). Leaf size: 56

```
DSolve[6*y'[x]+6*y[x]^2-y[x]-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^{5x/6} - e^{5c_1}}{2e^{5x/6} + 3e^{5c_1}}$$

$$y(x) \rightarrow -\frac{1}{3}$$

$$y(x) \rightarrow \frac{1}{2}$$

9.42 problem 38 part (g)

Internal problem ID [1148]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 38 part (g).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [quadrature]

$$36y' + 36y^2 - 12y = -1$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(36*diff(y(x),x)+36*y(x)^2-12*y(x)+1=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 + x + 6}{6x + 6c_1}$$

✓ Solution by Mathematica

Time used: 0.119 (sec). Leaf size: 30

```
DSolve[36*y'[x]+36*y[x]^2-12*y[x]+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x + 6 - 36c_1}{6x - 216c_1}$$

$$y(x) \rightarrow \frac{1}{6}$$

9.43 problem 39 part(a)

Internal problem ID [1149]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 39 part(a).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, ‘_with_symmetry_[F(x),G(x)]’], _Riccati]`

$$x^2(y' + y^2) - x(2 + x)y = -x - 2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^2*(diff(y(x),x)+y(x)^2)-x*(x+2)*y(x)+x+2=0,y(x), singsol=all)
```

$$y(x) = -\frac{e^x}{-e^x + c_1} + \frac{1}{x}$$

✓ Solution by Mathematica

Time used: 0.179 (sec). Leaf size: 49

```
DSolve[x^2*(y'[x]+y[x])^2-x*(x+2)*y[x]+x+2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{\frac{1}{x}-x} \left(\int_1^x \frac{e^{K[1]-\frac{1}{K[1]}} (K[1]^2 + K[1] - 2)}{K[1]^2} dK[1] + c_1 \right)$$

9.44 problem 39 part(b)

Internal problem ID [1150]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 39 part(b).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _Riccati]`

$$y' + y^2 + 4yx = -4x^2 - 2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)+y(x)^2+4*x*y(x)+4*x^2+2=0,y(x), singsol=all)
```

$$y(x) = -\frac{2c_1x - 2x^2 + 1}{c_1 - x}$$

✓ Solution by Mathematica

Time used: 0.129 (sec). Leaf size: 22

```
DSolve[y'[x]+y[x]^2+4*x*y[x]+4*x^2+2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -2x + \frac{1}{x + c_1}$$

$$y(x) \rightarrow -2x$$

9.45 problem 39 part(c)

Internal problem ID [1151]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 39 part(c).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$(1 + 2x)(y' + y^2) - 2y = 2x + 3$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve((2*x+1)*(diff(y(x),x)+y(x)^2)-2*y(x)-(2*x+3)=0,y(x), singsol=all)
```

$$y(x) = -1 - \frac{e^{2x} + 2e^{2x}x}{-\frac{x(e^{2x}+2e^{2x}x)}{1+2x} - c_1}$$

✓ Solution by Mathematica

Time used: 0.403 (sec). Leaf size: 41

```
DSolve[(2*x+1)*(y'[x]+y[x]^2)-2*y[x]-(2*x+3)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^{2x+1}(x+1) - c_1}{e^{2x+1}x + c_1}$$

$$y(x) \rightarrow -1$$

9.46 problem 39 part(d)

Internal problem ID [1152]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 39 part(d).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$(3x - 1)(y' + y^2) - y(3x + 2) = 6x - 8$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 31

```
dsolve((3*x-1)*(diff(y(x),x)+y(x)^2)-(3*x+2)*y(x)-6*x+8=0,y(x), singsol=all)
```

$$y(x) = \frac{-c_1x + 2e^{3x-1} + c_1}{c_1x + e^{3x-1}}$$

✓ Solution by Mathematica

Time used: 0.557 (sec). Leaf size: 41

```
DSolve[(3*x-1)*(y'[x]+y[x]^2)-(3*x+2)*y[x]-6*x+8==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{2(-ex + c_1e^{3x} + e)}{2ex + c_1e^{3x}}$$

$$y(x) \rightarrow 2$$

9.47 problem 39 part(e)

Internal problem ID [1153]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 39 part(e).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, ‘_with_symmetry_[F(x),G(x)]’], _Riccati]`

$$x^2(y' + y^2) + yx = -x^2 + \frac{1}{4}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(x^2*(diff(y(x),x)+y(x)^2)+x*y(x)+x^2-1/4=0,y(x), singsol=all)
```

$$y(x) = \frac{2ix - 1}{2x} - \frac{e^{-2ix}}{c_1 - \frac{ie^{-2ix}}{2}}$$

✓ Solution by Mathematica

Time used: 0.377 (sec). Leaf size: 22

```
DSolve[x^2*(y'[x]+y[x]^2)+x*y[x]+x^2-1/4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{1}{2x} - \tan(x - c_1)$$

9.48 problem 39 part(f)

Internal problem ID [1154]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

Problem number: 39 part(f).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Riccati]`

$$x^2(y' + y^2) - 7yx = -7$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 24

```
dsolve(x^2*(diff(y(x),x)+y(x)^2)-7*x*y(x)+7=0,y(x), singsol=all)
```

$$y(x) = \frac{-7x^6 + c_1}{x(-x^6 + c_1)}$$

✓ Solution by Mathematica

Time used: 0.186 (sec). Leaf size: 34

```
DSolve[x^2*(y'[x]+y[x]^2)-7*x*y[x]+7==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{7x^6 - 6c_1}{x^7 - 6c_1x}$$

$$y(x) \rightarrow \frac{1}{x}$$

10 Chapter 5 linear second order equations.

Section 5.7 Variation of Parameters. Page 262

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10.1 problem 1

Internal problem ID [1155]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 1.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 9y = \tan(3x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)+9*y(x)=tan(3*x),y(x), singsol=all)
```

$$y(x) = \sin(3x)c_2 + \cos(3x)c_1 - \frac{\cos(3x)\ln(\sec(3x) + \tan(3x))}{9}$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 33

```
DSolve[y''[x]+9*y[x]==Tan[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{1}{9}\cos(3x)\operatorname{arctanh}(\sin(3x)) + c_1\cos(3x) + c_2\sin(3x)$$

10.2 problem 2

Internal problem ID [1156]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 2.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y = \sin(2x) \sec(2x)^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
dsolve(diff(y(x),x$2)+4*y(x)=sin(2*x)*sec(2*x)^2,y(x), singsol=all)
```

$$y(x) = \sin(2x) c_2 + \cos(2x) c_1 + \frac{\ln(\sec(2x)) \sin(2x)}{4} - \frac{\sin(2x)}{4} + \frac{x \cos(2x)}{2}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 33

```
DSolve[y''[x]+4*y[x]==Sin[2*x]*Sec[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow (-x + c_1) \cos(2x) + \sin(x) \cos(x) (2 \log(\cos(x)) - 1 + 2c_2)$$

10.3 problem 3

Internal problem ID [1157]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 3.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 3y' + 2y = \frac{4}{1 + e^{-x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=4/(1+exp(-x)),y(x), singsol=all)
```

$$y(x) = (c_1 e^x - 4 \ln(e^x) - 4 \ln(e^x) e^x + 4 \ln(1 + e^x) (1 + e^x) - 4 + c_2) e^x$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 34

```
DSolve[y''[x]-3*y'[x]+2*y[x]==4/(1+Exp[-x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x(8(e^x + 1) \operatorname{arctanh}(2e^x + 1) + c_2 e^x - 4 + c_1)$$

10.4 problem 4

Internal problem ID [1158]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 4.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2y' + 2y = 3e^x \sec(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+2*y(x)=3*exp(x)*sec(x),y(x), singsol=all)
```

$$y(x) = c_2 \sin(x) e^x + c_1 \cos(x) e^x - 3e^x (\cos(x) \ln(\sec(x)) - \sin(x) x)$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 30

```
DSolve[y''[x]-2*y'[x]+2*y[x]==3*Exp[x]*Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x((3x + c_1) \sin(x) + \cos(x)(3 \log(\cos(x)) + c_2))$$

10.5 problem 5

Internal problem ID [1159]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 5.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2y' + y = 14e^x x^{\frac{3}{2}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=14*x^(3/2)*exp(x),y(x), singsol=all)
```

$$y(x) = e^x c_2 + x e^x c_1 + \frac{8x^{\frac{7}{2}} e^x}{5}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 29

```
DSolve[y''[x]-2*y'[x]+y[x]==14*x^(3/2)*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{5}e^x(8x^{7/2} + 5c_2x + 5c_1)$$

10.6 problem 6

Internal problem ID [1160]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 6.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - y = \frac{4e^{-x}}{1 - e^{-2x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(diff(y(x),x$2)-y(x)=4*exp(-x)/(1-exp(-2*x)),y(x), singsol=all)
```

$$y(x) = e^{-x}c_2 + c_1e^x + \ln(1 - e^{-2x})e^x + e^{-x}(\ln(e^{-2x}) - \ln(-1 + e^{-2x}))$$

✓ Solution by Mathematica

Time used: 0.121 (sec). Leaf size: 67

```
DSolve[y''[x]-y[x]==4*Exp[-x]/(1-Exp[-2*x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x}(-2e^{2x} \log(e^x) + (e^{2x} - 1) \log(e^x - 1) + e^{2x} \log(e^x + 1) - \log(e^x + 1) + c_1e^{2x} + c_2)$$

10.7 problem 7

Internal problem ID [1161]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 7.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$x^2 y'' + y' x - y = 2x^2 + 2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=2*x^2+2,y(x), singsol=all)
```

$$y(x) = c_2 x + \frac{2x^2}{3} + \frac{c_1}{x} - 2$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 24

```
DSolve[x^2*y'[x]+x*y'[x]-y[x]==2*x^2+2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{2x^2}{3} + c_2 x + \frac{c_1}{x} - 2$$

10.8 problem 8

Internal problem ID [1162]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 8.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x^2 y'' + (-2x + 2) y' + (-2 + x) y = e^{2x}$$

X Solution by Maple

```
dsolve(x^2*diff(y(x),x$2)+(2-2*x)*diff(y(x),x)+(x-2)*y(x)=exp(2*x),y(x), singsol=all)
```

No solution found

X Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y''[x]+(2-2*x)*y'[x]+(x-2)*y[x]==exp(2*x),y[x],x,IncludeSingularSolutions -> True
```

Not solved

10.9 problem 9

Internal problem ID [1163]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 9.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$4x^2y'' + (-8x^2 + 4x)y' + (4x^2 - 4x - 1)y = 4e^x\sqrt{x}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 29

```
dsolve(4*x^2*diff(y(x),x$2)+(4*x-8*x^2)*diff(y(x),x)+(4*x^2-4*x-1)*y(x)=4*x^(1/2)*exp(x),y(x)
```

$$y(x) = \frac{e^x c_2}{\sqrt{x}} + \sqrt{x} e^x c_1 + \sqrt{x} e^x (-1 + \ln(x))$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 27

```
DSolve[4*x^2*y''[x]+(4*x-8*x^2)*y'[x]+(4*x^2-4*x-1)*y[x]==4*x^(1/2)*Exp[x],y[x],x,IncludeSin
```

$$y(x) \rightarrow \frac{e^x(x \log(x) + (-1 + c_2)x + c_1)}{\sqrt{x}}$$

10.10 problem 10

Internal problem ID [1164]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 10.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y'x + (4x^2 + 2)y = 4e^{-x(2+x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2+2)*y(x)=4*exp(-x*(x+2)),y(x), singsol=all)
```

$$y(x) = e^{-x^2}c_2 + e^{-x^2}xc_1 + e^{-x(2+x)}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 27

```
DSolve[4*x^2*y''[x]+(4*x-8*x^2)*y'[x]+(4*x^2-4*x-1)*y[x]==4*x^(1/2)*Exp[x],y[x],x,IncludeSin
```

$$y(x) \rightarrow \frac{e^x(x \log(x) + (-1 + c_2)x + c_1)}{\sqrt{x}}$$

10.11 problem 11

Internal problem ID [1165]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 11.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' - 4y'x + 6y = x^{\frac{5}{2}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=x^(5/2),y(x), singsol=all)
```

$$y(x) = x^3 c_2 + c_1 x^2 - 4x^{\frac{5}{2}}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 23

```
DSolve[x^2*y''[x]-4*x*y'[x]+6*y[x]==x^(5/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^2(-4\sqrt{x} + c_2 x + c_1)$$

10.12 problem 12

Internal problem ID [1166]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 12.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' - 3y'x + 3y = 2 \sin(x) x^4$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+3*y(x)=2*x^4*sin(x),y(x), singsol=all)
```

$$y(x) = \left(\frac{c_1 x^2}{2} - 2 \cos(x) - 2 \sin(x) x + c_2 \right) x$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 25

```
DSolve[x^2*y''[x]-3*x*y'[x]+3*y[x]==2*x^4*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x(c_2 x^2 - 2x \sin(x) - 2 \cos(x) + c_1)$$

10.13 problem 13

Internal problem ID [1167]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 13.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$(1 + 2x)y'' - 2y' - (2x + 3)y = e^{-x}(1 + 2x)^2$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 26

```
dsolve((2*x+1)*diff(y(x),x$2)-2*diff(y(x),x)-(2*x+3)*y(x)=(2*x+1)^2*exp(-x),y(x), singsol=all)
```

$$y(x) = e^{-x}c_2 + x e^x c_1 - \frac{(x+1)x e^{-x}}{2}$$

✓ Solution by Mathematica

Time used: 0.084 (sec). Leaf size: 42

```
DSolve[(2*x+1)*y'[x]-2*y'[x]-(2*x+3)*y[x]==(2*x+1)^2*Exp[-x],y[x],x,IncludeSingularSolution->True]
```

$$y(x) \rightarrow -\frac{1}{2}e^{-x}x(x+1) + c_1e^{-x-\frac{1}{2}} + c_2e^{x+\frac{1}{2}x}$$

10.14 problem 14

Internal problem ID [1168]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 14.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$2xy'' + 2y' + 2y = \sin(\sqrt{x})$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 72

```
dsolve(2*x*diff(y(x),x$2)+2*diff(y(x),x)+2*y(x)=sin(sqrt(x)),y(x), singsol=all)
```

$$y(x) = \text{BesselJ}(0, 2\sqrt{x}) c_2 + \text{BesselY}(0, 2\sqrt{x}) c_1 + \frac{\pi \left(\int \text{BesselJ}(0, 2\sqrt{x}) \sin(\sqrt{x}) dx \right) \text{BesselY}(0, 2\sqrt{x}) - \left(\int \text{BesselY}(0, 2\sqrt{x}) \sin(\sqrt{x}) dx \right) \text{BesselJ}(0, 2\sqrt{x})}{2}$$

✓ Solution by Mathematica

Time used: 11.396 (sec). Leaf size: 110

```
DSolve[2*x*y'[x]+2*y'[x]+2*y[x]==Sin[Sqrt[x]],y[x],x,IncludeSingularSolutions->True]
```

$$y(x) \rightarrow \text{BesselJ}(0, 2\sqrt{x}) \int_1^x -\frac{1}{2} \pi \text{BesselY}(0, 2\sqrt{K[1]}) \sin(\sqrt{K[1]}) dK[1] + 2 \text{BesselY}(0, 2\sqrt{x}) \int_1^x \frac{1}{4} \pi \text{BesselJ}(0, 2\sqrt{K[2]}) \sin(\sqrt{K[2]}) dK[2] + c_1 \text{BesselJ}(0, 2\sqrt{x}) + 2c_2 \text{BesselY}(0, 2\sqrt{x})$$

10.15 problem 15

Internal problem ID [1169]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 15.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$xy'' - (2 + 2x)y' + (2 + x)y = 6e^x x^3$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x$2)-(2*x+2)*diff(y(x),x)+(x+2)*y(x)=6*x^3*exp(x),y(x), singsol=all)
```

$$y(x) = e^x c_2 + e^x x^3 c_1 + \frac{3e^x x^4}{2}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 29

```
DSolve[x*y''[x]-(2*x+2)*y'[x]+(x+2)*y[x]==6*x^3*Exp[x],y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \rightarrow \frac{1}{6}e^x(9x^4 + 2c_2x^3 + 6c_1)$$

10.16 problem 16

Internal problem ID [1170]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 16.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' - (2a - 1) x y' + a^2 y = x^{1+a}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x$2)-(2*a-1)*x*diff(y(x),x)+a^2*y(x)=x^(a+1),y(x), singsol=all)
```

$$y(x) = x^a c_2 + x^a \ln(x) c_1 + x^{a+1}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 19

```
DSolve[x^2*y''[x]-(2*a-1)*x*y'[x]+a^2*y[x]==x^(a+1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^a (a c_2 \log(x) + x + c_1)$$

10.17 problem 17

Internal problem ID [1171]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 17.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x^2 y'' - 2y'x + (x^2 + 2)y = x^3 \cos(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=x^3*cos(x),y(x), singsol=all)
```

$$y(x) = \sin(x) x c_2 + \cos(x) x c_1 + \frac{\sin(x) x^2}{2}$$

✓ Solution by Mathematica

Time used: 0.066 (sec). Leaf size: 49

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2)*y[x]==x^3*Cos[x],y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \rightarrow \frac{1}{8} e^{-ix} x (2ix + e^{2ix} (-2ix + 1 - 4ic_2) + 1 + 8c_1)$$

10.18 problem 18

Internal problem ID [1172]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 18.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$xy'' - y' - 4x^3y = 8x^5$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x*diff(y(x),x$2)-diff(y(x),x)-4*x^3*y(x)=8*x^5,y(x), singsol=all)
```

$$y(x) = \sinh(x^2) c_2 + \cosh(x^2) c_1 - 2x^2$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 28

```
DSolve[x*y''[x]-y'[x]-4*x^3*y[x]==8*x^5,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -2x^2 + c_1 \cosh(x^2) + ic_2 \sinh(x^2)$$

10.19 problem 19

Internal problem ID [1173]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 19.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\sin(x) y'' + (2 \sin(x) - \cos(x)) y' + (\sin(x) - \cos(x)) y = e^{-x}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 31

```
dsolve(sin(x)*diff(y(x),x$2)+(2*sin(x)-cos(x))*diff(y(x),x)+(sin(x)-cos(x))*y(x)=exp(-x),y(x)
```

$$y(x) = e^{\frac{\pi}{2}-x} c_2 + \cos(x) e^{-x} c_1 - \sin(x) e^{-x}$$

✓ Solution by Mathematica

Time used: 0.809 (sec). Leaf size: 121

```
DSolve[Sin[x]*y''[x]+(2*SIn[x]-Cos[x])*y'[x]+(Sin[x]-Cos[x])*y[x]==Exp[-x],y[x],x,IncludeSin
```

$$\begin{aligned} y(x) \rightarrow & -\sqrt{\sin^2(x)} \exp\left(-\arccos(\cos(x)) - 4 \arctan\left(\frac{\sqrt{\sin^2(x)}}{\cos(x) + 1}\right)\right. \\ & \left. + 4 \cot^{-1}\left(\frac{\cos(x) + 1}{\sqrt{\sin^2(x)}}\right)\right) \\ & + c_2 \cos(x) \exp\left(2\left(\cot^{-1}\left(\frac{\cos(x) + 1}{\sqrt{\sin^2(x)}}\right) - 2 \arctan\left(\frac{\sqrt{\sin^2(x)}}{\cos(x) + 1}\right)\right)\right) \\ & + c_1 e^{-2 \cot^{-1}\left(\frac{\cos(x)+1}{\sqrt{\sin^2(x)}}\right)} \end{aligned}$$

10.20 problem 20

Internal problem ID [1174]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 20.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$4x^2y'' - 4y'x + (-16x^2 + 3)y = 8x^{\frac{5}{2}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(3-16*x^2)*y(x)=8*x^(5/2),y(x), singsol=all)
```

$$y(x) = \sqrt{x} \sinh(2x) c_2 + \sqrt{x} \cosh(2x) c_1 - \frac{\sqrt{x}}{2}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 39

```
DSolve[4*x^2*y''[x]-4*x*y'[x]+(3-16*x^2)*y[x]==8*x^(5/2),y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow \frac{1}{4}e^{-2x}\sqrt{x}(-2e^{2x} + c_2e^{4x} + 4c_1)$$

10.21 problem 21

Internal problem ID [1175]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 21.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$4x^2y'' - 4y'x + (4x^2 + 3)y = x^{\frac{7}{2}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2+3)*y(x)=x^(7/2),y(x), singsol=all)
```

$$y(x) = \sqrt{x} \sin(x) c_2 + \sqrt{x} \cos(x) c_1 + \frac{x^{\frac{3}{2}}}{4}$$

✓ Solution by Mathematica

Time used: 0.075 (sec). Leaf size: 48

```
DSolve[4*x^2*y''[x]-4*x*y'[x]+(4*x^2+3)*y[x]==x^(7/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{4}e^{-ix}\sqrt{x}(e^{ix}x - 2ic_2e^{2ix} + 4c_1)$$

10.22 problem 22

Internal problem ID [1176]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 22.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x^2 y'' - 2y'x - (x^2 - 2)y = 3x^4$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)-(x^2-2)*y(x)=3*x^4,y(x), singsol=all)
```

$$y(x) = \sinh(x) x c_2 + \cosh(x) x c_1 - 3x^2$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 29

```
DSolve[x^2*y'[x]-2*x*y'[x]-(x^2-2)*y[x]==3*x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2}x(-6x + 2c_1 e^{-x} + c_2 e^x)$$

10.23 problem 23

Internal problem ID [1177]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 23.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x^2 y'' - 2x(x+1)y' + (x^2 + 2x + 2)y = e^x x^3$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve(x^2*diff(y(x),x$2)-2*x*(x+1)*diff(y(x),x)+(x^2+2*x+2)*y(x)=x^3*exp(x),y(x), singsol=a
```

$$y(x) = c_2 e^x x + e^x c_1 x^2 + \frac{e^x x^3}{2}$$

✓ Solution by Mathematica

Time used: 0.867 (sec). Leaf size: 227

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2*x+2)*y[x]==x^3*Exp[x],y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow e^{ix} x \left(\text{HypergeometricU}(-i, 0, -2ix) \int_1^x \frac{ie^{(1-i)K[1]} L_i^{-1}(-2iK[1])}{4 \text{Hypergeometric1F1}(1-i, 2, -2iK[1]) \text{HypergeometricU}(1-i, 1, -2iK[1]) K[1] - 2 \text{Hypergeomet}} \right. \\ \left. + L_i^{-1}(-2ix) \int_1^x \frac{ie^{(1-i)K[2]} \text{HypergeometricU}(-i, 1, -2iK[2])}{4 \text{Hypergeometric1F1}(1-i, 2, -2iK[2]) \text{HypergeometricU}(1-i, 1, -2iK[2]) K[2] - 2 \text{Hypergeomet}} \right. \\ \left. + c_1 \text{HypergeometricU}(-i, 0, -2ix) + c_2 L_i^{-1}(-2ix) \right)$$

10.24 problem 24

Internal problem ID [1178]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 24.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$x^2y'' - y'x - 3y = x^{\frac{3}{2}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)-3*y(x)=x^(3/2),y(x), singsol=all)
```

$$y(x) = x^3c_2 - \frac{4x^{\frac{3}{2}}}{15} + \frac{c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 27

```
DSolve[x^2*y''[x]-x*y'[x]-3*y[x]==x^(3/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{4x^{3/2}}{15} + c_2x^3 + \frac{c_1}{x}$$

10.25 problem 25

Internal problem ID [1179]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 25.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x^2 y'' - x(x+4)y' + 2y(x+3) = x^4 e^x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)-x*(x+4)*diff(y(x),x)+2*(x+3)*y(x)=x^4*exp(x),y(x), singsol=all)
```

$$y(x) = c_2 x^2 e^x + c_1 x^2 + e^x x^3$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 21

```
DSolve[x^2*y''[x]-x*(x+4)*y'[x]+2*(x+3)*y[x]==x^4*Exp[x],y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow x^2(e^x(x-1+c_2) + c_1)$$

10.26 problem 26

Internal problem ID [1180]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 26.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x^2 y'' - 2x(2+x)y' + (x^2 + 4x + 6)y = 2e^x x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)-2*x*(x+2)*diff(y(x),x)+(x^2+4*x+6)*y(x)=2*x*exp(x),y(x), singsol=a
```

$$y(x) = c_2 x^2 e^x + e^x x^3 c_1 + x e^x$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]-2*x*(x+2)*y'[x]+(x^2+4*x+6)*y[x]==2*x*Exp[x],y[x],x,IncludeSingularSolutio
```

$$y(x) \rightarrow e^x x (c_2 x^2 + c_1 x + 1)$$

10.27 problem 27

Internal problem ID [1181]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 27.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x^2 y'' - 4y'x + (x^2 + 6)y = x^4$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(x^2+6)*y(x)=x^4,y(x), singsol=all)
```

$$y(x) = \sin(x) x^2 c_2 + \cos(x) x^2 c_1 + x^2$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 38

```
DSolve[x^2*y''[x]-4*x*y'[x]+(x^2+6)*y[x]==x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2}x^2(2c_1e^{-ix} - ic_2e^{ix} + 2)$$

10.28 problem 28

Internal problem ID [1182]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 28.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x - 1)y'' - y'x + y = 2(x - 1)^2 e^x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve((x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=2*(x-1)^2*exp(x),y(x), singsol=all)
```

$$y(x) = c_2x + c_1e^x + x(-2 + x)e^x$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 24

```
DSolve[(x-1)*y'[x]-x*y'[x]+y[x]==2*(x-1)^2*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x(x^2 - 2x + c_1) - c_2x$$

10.29 problem 29

Internal problem ID [1183]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 29.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$4x^2y'' - 4x(x+1)y' + (2x+3)y = x^{\frac{5}{2}}e^x$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*(x+1)*diff(y(x),x)+(2*x+3)*y(x)=x^(5/2)*exp(x),y(x),singsol
```

$$y(x) = \sqrt{x}c_2 + \sqrt{x}e^xc_1 + \frac{x^{\frac{3}{2}}e^x}{4}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 30

```
DSolve[4*x^2*y''[x]-4*x*(x+1)*y'[x]+(2*x+3)*y[x]==x^(5/2)*Exp[x],y[x],x,IncludeSingularSolut
```

$$y(x) \rightarrow \frac{1}{4}\sqrt{x}(e^x(x-1+4c_2)+4c_1)$$

10.30 problem 30

Internal problem ID [1184]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 30.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$(3x - 1)y'' - (3x + 2)y' - (6x - 8)y = (3x - 1)^2 e^{2x}$$

With initial conditions

$$[y(0) = 1, y'(0) = 2]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 28

```
dsolve([(3*x-1)*diff(y(x),x$2)-(3*x+2)*diff(y(x),x)-(6*x-8)*y(x)=(3*x-1)^2*exp(2*x),y(0) = 1,y'(0) = 2])
```

$$y(x) = \frac{(3x^2 - 2x + 6)e^{2x}}{6} + \frac{x e^{-x}}{3}$$

✓ Solution by Mathematica

Time used: 0.13 (sec). Leaf size: 34

```
DSolve[{(3*x-1)*y''[x]-(3*x+2)*y'[x]-(6*x-8)*y[x]==(3*x-1)^2*Exp[2*x],{y[0]==1,y'[0]==2}},y[x]]
```

$$y(x) \rightarrow \frac{1}{6}e^{-x}(e^{3x}(3x^2 - 2x + 6) + 2x)$$

10.31 problem 31

Internal problem ID [1185]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 31.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x - 1)^2 y'' - 2(x - 1) y' + 2y = (x - 1)^2$$

With initial conditions

$$[y(0) = 3, y'(0) = -6]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve([(x-1)^2*diff(y(x),x$2)-2*(x-1)*diff(y(x),x)+2*y(x)=(x-1)^2,y(0) = 3, D(y)(0) = -6],y
```

$$y(x) = -(x - 1)(i\pi x - i\pi - \ln(x - 1)x + \ln(x - 1) - 2x + 3)$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 30

```
DSolve[{(x-1)^2*y'[x]-2*(x-1)*y'[x]+2*y[x]==(x-1)^2,{y[0]==3,y'[0]==-6}},y[x],x,IncludeSing
```

$$y(x) \rightarrow (x - 1)(-i\pi(x - 1) + 2x + (x - 1)\log(x - 1) - 3)$$

10.32 problem 32

Internal problem ID [1186]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 32.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$(x-1)^2 y'' - (x^2-1) y' + (x-1)^3 y = (x-1)^3 e^x$$

With initial conditions

$$[y(0) = 4, y'(0) = -6]$$

X Solution by Maple

```
dsolve([(x-1)^2*diff(y(x),x$2)-(x^2-1)*diff(y(x),x)+(x-1)^3*y(x)=(x-1)^3*exp(x),y(0) = 4, D
```

No solution found

X Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{(x-1)^2*y''[x]-(x^2-1)*y'[x]+(x-1)^3*y[x]==(x-1)^3*Exp[x],{y[0]==4,y'[0]==-6}},y[x],
```

Not solved

10.33 problem 33

Internal problem ID [1187]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 33.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$(x - 1)^2 y'' + 4y'x + 2y = 2x$$

With initial conditions

$$[y(0) = 0, y'(0) = -2]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 67

```
dsolve([(x-1)^2*diff(y(x),x$2)+4*x*diff(y(x),x)+2*y(x)=2*x,y(0) = 0, D(y)(0) = -2],y(x), sin
```

$$y(x) = \frac{x^3 + 28 e^{\frac{4}{x-1}} \operatorname{Ei}_1\left(\frac{4}{x-1}\right) - 28 e^{\frac{4}{x-1}} \operatorname{Ei}_1(-4) - 7 e^{\frac{4x}{x-1}} - 2x^2 - 6x + 7}{3(x-1)^2}$$

✓ Solution by Mathematica

Time used: 0.242 (sec). Leaf size: 71

```
DSolve[{(x-1)^2*y''[x]+4*x*y'[x]+2*y[x]==2*x,{y[0]==0,y'[0]==-2}},y[x],x,IncludeSingularSolu
```

$$y(x) \rightarrow \frac{-28 e^{\frac{4}{x-1}} \operatorname{ExpIntegralEi}\left(-\frac{4}{x-1}\right) + 28 \operatorname{ExpIntegralEi}(4) e^{\frac{4}{x-1}} + x^3 - 2x^2 - 6x - 7 e^{\frac{4x}{x-1}} + 7}{3(x-1)^2}$$

10.34 problem 34

Internal problem ID [1188]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 34.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + 2y'x - 2y = -2x^2$$

With initial conditions

$$[y(1) = 1, y'(1) = -1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve([x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=-2*x^2,y(1) = 1, D(y)(1) = -1],y(x), sing
```

$$y(x) = \frac{1}{2x^2} + x - \frac{x^2}{2}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 21

```
DSolve[{x^2*y''[x]+2*x*y'[x]-2*y[x]==-2*x^2,{y[1]==1,y'[1]==-1}},y[x],x,IncludeSingularSolut
```

$$y(x) \rightarrow -\frac{x^2}{2} + \frac{1}{2x^2} + x$$

10.35 problem 35

Internal problem ID [1189]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

Problem number: 35.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$(x + 1)(2x + 3)y'' + 2(2 + x)y' - 2y = (2x + 3)^2$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve([(x+1)*(2*x+3)*diff(y(x),x$2)+2*(x+2)*diff(y(x),x)-2*y(x)=(2*x+3)^2,y(0) = 0, D(y)(0)
```

$$y(x) = \frac{x^2(4x + 9)}{6x + 6}$$

✓ Solution by Mathematica

Time used: 0.694 (sec). Leaf size: 22

```
DSolve[{(x+1)*(2*x+3)*y''[x]+2*(x+2)*y'[x]-2*y[x]==(2*x+3)^2,{y[0]==0,y'[0]==0}},y[x],x,Incl
```

$$y(x) \rightarrow \frac{x^2(4x + 9)}{6(x + 1)}$$

11 Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318

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11.1 problem 11

Internal problem ID [1190]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318

Problem number: 11.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(2 + x)y'' + y'x + 3y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;  
dsolve((2+x)*diff(y(x),x$2)+x*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{3}{4}x^2 + \frac{1}{8}x^3 + \frac{1}{8}x^4 - \frac{9}{160}x^5\right) y(0) + \left(x - \frac{1}{3}x^3 + \frac{1}{12}x^4 + \frac{1}{40}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[(2+x)*y''[x]+x*y'[x]+3*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{x^5}{40} + \frac{x^4}{12} - \frac{x^3}{3} + x \right) + c_1 \left(-\frac{9x^5}{160} + \frac{x^4}{8} + \frac{x^3}{8} - \frac{3x^2}{4} + 1 \right)$$

11.2 problem 12

Internal problem ID [1191]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318

Problem number: 12.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(3x^2 + 1)y'' + 3y'x^2 - 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
Order:=6;  
dsolve((1+3*x^2)*diff(y(x),x$2)+3*x^2*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + x^2 - \frac{1}{3}x^4 - \frac{3}{10}x^5\right)y(0) + \left(x + \frac{1}{3}x^3 - \frac{1}{4}x^4 - \frac{4}{15}x^5\right)D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 52

```
AsymptoticDSolveValue[(1+3*x^2)*y'[x]+3*x^2*y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(-\frac{4x^5}{15} - \frac{x^4}{4} + \frac{x^3}{3} + x \right) + c_1 \left(-\frac{3x^5}{10} - \frac{x^4}{3} + x^2 + 1 \right)$$

11.3 problem 13

Internal problem ID [1192]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318

Problem number: 13.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(2x^2 + 1)y'' + (-3x + 2)y' + 4y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
```

```
dsolve((1+2*x^2)*diff(y(x),x$2)+(2-3*x)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - 2x^2 + \frac{4}{3}x^3 - \frac{1}{3}x^4 - \frac{1}{3}x^5\right)y(0) + \left(x - x^2 + \frac{1}{2}x^3 - \frac{1}{12}x^4 - \frac{17}{120}x^5\right)D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 66

```
AsymptoticDSolveValue[(1+2*x^2)*y'[x]+(2-3*x)*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(-\frac{x^5}{3} - \frac{x^4}{3} + \frac{4x^3}{3} - 2x^2 + 1 \right) + c_2 \left(-\frac{17x^5}{120} - \frac{x^4}{12} + \frac{x^3}{2} - x^2 + x \right)$$

11.4 problem 14

Internal problem ID [1193]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318

Problem number: 14.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 + 1)y'' + (-x + 2)y' + 3y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 52

```
Order:=6;  
dsolve((1+x^2)*diff(y(x),x$2)+(2-x)*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{3}{2}x^2 + x^3 - \frac{1}{8}x^4 - \frac{1}{4}x^5\right) y(0) + \left(x - x^2 + \frac{1}{3}x^3 + \frac{1}{12}x^4 - \frac{2}{15}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 64

```
AsymptoticDSolveValue[(1+x^2)*y'[x]+(2-x)*y'[x]+3*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(-\frac{x^5}{4} - \frac{x^4}{8} + x^3 - \frac{3x^2}{2} + 1 \right) + c_2 \left(-\frac{2x^5}{15} + \frac{x^4}{12} + \frac{x^3}{3} - x^2 + x \right)$$

11.5 problem 15

Internal problem ID [1194]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318

Problem number: 15.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(3x^2 + 1)y'' - 2y'x + 4y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
Order:=6;  
dsolve((1+3*x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (x^4 - 2x^2 + 1)y(0) + \left(x - \frac{1}{3}x^3 + \frac{4}{15}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[(1+3*x^2)*y'[x]-2*x*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{4x^5}{15} - \frac{x^3}{3} + x \right) + c_1 (x^4 - 2x^2 + 1)$$

11.6 problem 16

Internal problem ID [1195]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318

Problem number: 16.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' + (2x + 4)y' + (2 + x)y = 0$$

With the expansion point for the power series method at $x = -1$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 52

```
Order:=6;  
dsolve(x*difff(y(x),x$2)+(4+2*x)*difff(y(x),x)+(2+x)*y(x)=0,y(x),type='series',x=-1);
```

$$y(x) = \left(1 + \frac{(x+1)^2}{2} + \frac{2(x+1)^3}{3} + \frac{7(x+1)^4}{8} + \frac{17(x+1)^5}{15} \right) y(-1) \\ + \left(x+1 + (x+1)^2 + \frac{3(x+1)^3}{2} + 2(x+1)^4 + \frac{103(x+1)^5}{40} \right) D(y)(-1) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 81

```
AsymptoticDSolveValue[(x)*y''[x]+(4+2*x)*y'[x]+(2+x)*y[x]==0,y[x],{x,-1,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{17}{15}(x+1)^5 + \frac{7}{8}(x+1)^4 + \frac{2}{3}(x+1)^3 + \frac{1}{2}(x+1)^2 + 1 \right) \\ + c_2 \left(\frac{103}{40}(x+1)^5 + 2(x+1)^4 + \frac{3}{2}(x+1)^3 + (x+1)^2 + x+1 \right)$$

11.7 problem 17

Internal problem ID [1196]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318

Problem number: 17.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^2 y'' + 2y'x - 3yx = 0$$

With the expansion point for the power series method at $x = 2$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;  
dsolve(x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-3*x*y(x)=0,y(x),type='series',x=2);
```

$$y(x) = \left(1 + \frac{3(-2+x)^2}{4} - \frac{3(-2+x)^3}{8} + \frac{9(-2+x)^4}{32} - \frac{27(-2+x)^5}{160} \right) y(2) \\ + \left(-2+x - \frac{(-2+x)^2}{2} + \frac{(-2+x)^3}{2} - \frac{5(-2+x)^4}{16} + \frac{31(-2+x)^5}{160} \right) D(y)(2) \\ + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 87

```
AsymptoticDSolveValue[x^2*y''[x]+2*x*y'[x]-3*x*y[x]==0,y[x],{x,2,5}]
```

$$y(x) \rightarrow c_1 \left(-\frac{27}{160}(x-2)^5 + \frac{9}{32}(x-2)^4 - \frac{3}{8}(x-2)^3 + \frac{3}{4}(x-2)^2 + 1 \right) \\ + c_2 \left(\frac{31}{160}(x-2)^5 - \frac{5}{16}(x-2)^4 + \frac{1}{2}(x-2)^3 - \frac{1}{2}(x-2)^2 + x - 2 \right)$$

11.8 problem 18

Internal problem ID [1197]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318

Problem number: 18.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$(-x + 2)y'' + 2y = 0$$

With initial conditions

$$[y(0) = a_0, y'(0) = a_1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
Order:=6;
dsolve([(2-x)*diff(y(x),x$2)+2*y(x)=0,y(0) = a__0, D(y)(0) = a__1],y(x),type='series',x=0);
```

$$y(x) = a_0 + a_1x - \frac{1}{2}a_0x^2 + \left(-\frac{a_1}{6} - \frac{a_0}{12}\right)x^3 + \left(\frac{a_0}{48} - \frac{a_1}{24}\right)x^4 + \left(-\frac{a_1}{240} + \frac{a_0}{96}\right)x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 79

```
AsymptoticDSolveValue[{(2-x)*y''[x]+2*y[x]==0,{y[0]==a0,y'[0]==a1}},y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{1}{20}x^5 \left(\frac{1}{6} \left(\frac{a_0}{2} + a_1 \right) + \frac{a_0}{8} - \frac{a_1}{4} \right) + \frac{1}{12}x^4 \left(\frac{a_0}{4} - \frac{a_1}{2} \right) + \frac{1}{6}x^3 \left(-\frac{a_0}{2} - a_1 \right) - \frac{a_0x^2}{2} + a_0 + a_1x$$

11.9 problem 19

Internal problem ID [1198]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318

Problem number: 19.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x + 1)y'' + 2(x - 1)^2 y' + 3y = 0$$

With initial conditions

$$[y(1) = a_0, y'(1) = a_1]$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

Order:=6;

`dsolve([(1+x)*diff(y(x),x$2)+2*(x-1)^2*diff(y(x),x)+3*y(x)=0,y(1) = a__0, D(y)(1) = a__1],y(x))`

$$y(x) = a_0 + a_1(x - 1) - \frac{3}{4}a_0(x - 1)^2 + \left(\frac{a_0}{8} - \frac{a_1}{4}\right)(x - 1)^3 \\ + \left(\frac{a_0}{16} - \frac{a_1}{48}\right)(x - 1)^4 + \left(\frac{3a_0}{64} + \frac{a_1}{40}\right)(x - 1)^5 + O((x - 1)^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 95

```
AsymptoticDSolveValue[{(1+x)*y'[x]+2*(x-1)^2*y'[x]+3*y[x]==0,{y[1]==a0,y'[1]==a1}},y[x],{x,
```

$$y(x) \rightarrow \frac{1}{20}(x-1)^5 \left(\frac{1}{4} \left(\frac{3a1}{2} - \frac{3a0}{4} \right) + \frac{9a0}{8} + \frac{a1}{8} \right) + \frac{1}{12}(x-1)^4 \left(\frac{3a0}{4} - \frac{a1}{4} \right) \\ + \frac{1}{6}(x-1)^3 \left(\frac{3a0}{4} - \frac{3a1}{2} \right) - \frac{3}{4}a0(x-1)^2 + a0 + a1(x-1)$$

11.10 problem 21

Internal problem ID [1199]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318

Problem number: 21.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-x)y'' + x(x+4)y' + (-x+2)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 55

```
Order:=6;
```

```
dsolve(x^2*(1-x)*diff(y(x),x$2)+x*(4+x)*diff(y(x),x)+(2-x)*y(x)=0,y(x),type='series',x=0);
```

$y(x)$

$$= \frac{\ln(x)(9x + 18x^2 + 3x^3 + O(x^6))c_2 + c_1(1 + 2x + \frac{1}{3}x^2 + O(x^6))x + (1 - 5x - 55x^2 - \frac{53}{3}x^3 + O(x^6))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 56

```
AsymptoticDSolveValue[x^2*(1-x)*y'[x]+x*(4+x)*y'[x]+(2-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{3(x^2 + 6x + 3) \log(x)}{x} - \frac{21x^3 + 75x^2 + 15x - 1}{x^2} \right) + c_2 \left(\frac{x}{3} + \frac{1}{x} + 2 \right)$$

11.11 problem 22

Internal problem ID [1200]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318

Problem number: 22.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x+1)y'' + (1+2x)xy' - (6x+4)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 51

```
Order:=6;
```

```
dsolve(x^2*(1+x)*diff(y(x),x$2)+x*(1+2*x)*diff(y(x),x)-(4+6*x)*y(x)=0,y(x),type='series',x=0
```

$$y(x) = c_1 x^2 (1 + O(x^6)) + \frac{c_2 (\ln(x) (576x^4 + O(x^6)) + (-144 + 192x - 288x^2 + 576x^3 - 576x^4 - 576x^5 + O(x^6)))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 48

```
AsymptoticDSolveValue[x^2*(1+x)*y''[x]+x*(1+2*x)*y'[x]-(4+6*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 x^2 + c_1 \left(\frac{3x^4 - 12x^3 + 6x^2 - 4x + 3}{3x^2} - 4x^2 \log(x) \right)$$

11.12 problem 23

Internal problem ID [1201]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318

Problem number: 23.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x+1)y'' - x(-x^2 - 6x + 1)y' + (x^2 + 6x + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 63

```
Order:=6;
```

```
dsolve(x^2*(1+x)*diff(y(x),x$2)-x*(1-6*x-x^2)*diff(y(x),x)+(1+6*x+x^2)*y(x)=0,y(x),type='ser
```

$$y(x) = x \left((c_2 \ln(x) + c_1) \left(1 - 12x + \frac{119}{2}x^2 - \frac{583}{3}x^3 + \frac{1981}{4}x^4 - \frac{80287}{75}x^5 + O(x^6) \right) \right. \\ \left. + \left(17x - \frac{471}{4}x^2 + 445x^3 - \frac{118285}{96}x^4 + \frac{702451}{250}x^5 + O(x^6) \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 114

```
AsymptoticDSolveValue[x^2*(1+x)*y'[x]-x*(1-6*x-x^2)*y'[x]+(1+6*x+x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 x \left(-\frac{80287x^5}{75} + \frac{1981x^4}{4} - \frac{583x^3}{3} + \frac{119x^2}{2} - 12x + 1 \right) \\ + c_2 \left(x \left(\frac{702451x^5}{250} - \frac{118285x^4}{96} + 445x^3 - \frac{471x^2}{4} + 17x \right) \right. \\ \left. + x \left(-\frac{80287x^5}{75} + \frac{1981x^4}{4} - \frac{583x^3}{3} + \frac{119x^2}{2} - 12x + 1 \right) \log(x) \right)$$

11.13 problem 24

Internal problem ID [1202]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318

Problem number: 24.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$x^2(3x + 1)y'' + x(x^2 + 12x + 2)y' + 2x(x + 3)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 44

```
Order:=6;
```

```
dsolve(x^2*(1+3*x)*diff(y(x),x$2)+x*(2+12*x+x^2)*diff(y(x),x)+2*x*(3+x)*y(x)=0,y(x),type='se
```

$$y(x) = c_1 \left(1 - 3x + \frac{26}{3}x^2 - \frac{101}{4}x^3 + \frac{4441}{60}x^4 - \frac{26141}{120}x^5 + O(x^6) \right) \\ + \frac{c_2 \left(1 - 6x + \frac{35}{2}x^2 - \frac{101}{2}x^3 + \frac{1177}{8}x^4 - \frac{17251}{40}x^5 + O(x^6) \right)}{x}$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 60

```
AsymptoticDSolveValue[x^2*(1+3*x)*y'[x]+x*(2+12*x+x^2)*y'[x]+2*x*(3+x)*y[x]==0,y[x],{x,0,5}
```

$$y(x) \rightarrow c_1 \left(\frac{571x^3}{8} - \frac{49x^2}{2} + \frac{17x}{2} + \frac{1}{x} - 3 \right) + c_2 \left(\frac{4441x^4}{60} - \frac{101x^3}{4} + \frac{26x^2}{3} - 3x + 1 \right)$$

11.14 problem 25

Internal problem ID [1203]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318

Problem number: 25.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(2x^2 + 1)y'' + x(2x^2 + 4)y' + 2(-x^2 + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
Order:=6;
```

```
dsolve(x^2*(1+2*x^2)*diff(y(x),x$2)+x*(4+2*x^2)*diff(y(x),x)+2*(1-x^2)*y(x)=0,y(x),type='ser
```

$$y(x) = \frac{c_1(1 + O(x^6))x + c_2(1 - 3x^2 - \frac{1}{2}x^4 + O(x^6))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 25

```
AsymptoticDSolveValue[x^2*(1+2*x^2)*y'[x]+x*(4+2*x^2)*y'[x]+2*(1-x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(-\frac{x^2}{2} + \frac{1}{x^2} - 3 \right) + \frac{c_2}{x}$$

11.15 problem 26

Internal problem ID [1204]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318

Problem number: 26.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2 + 2)y'' + 2x(x^2 + 5)y' + 2(-x^2 + 3)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
```

```
dsolve(x^2*(2+x^2)*diff(y(x),x$2)+2*x*(x^2+5)*diff(y(x),x)+2*(3-x^2)*y(x)=0,y(x),type='series')
```

$$y(x) = \frac{c_1(1 + \frac{1}{8}x^2 + O(x^6))}{x} + \frac{c_2(\ln(x)(2x^2 + \frac{1}{4}x^4 + O(x^6)) + (-2 - \frac{3}{2}x^2 - \frac{1}{4}x^4 + O(x^6)))}{x^3}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 51

```
AsymptoticDSolveValue[x^2*(2+x^2)*y'[x]+2*x*(x^2+5)*y'[x]+2*(3-x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{x^4 + 7x^2 + 4}{4x^3} - \frac{(x^2 + 8) \log(x)}{8x} \right) + c_2 \left(\frac{x}{8} + \frac{1}{x} \right)$$

12 Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

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12.1 problem 1

Internal problem ID [1205]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 1.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 + 1)y'' + 6y'x + 6y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;  
dsolve((1+x^2)*diff(y(x),x$2)+6*x*diff(y(x),x)+6*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (5x^4 - 3x^2 + 1)y(0) + (3x^5 - 2x^3 + x)D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[(1+x^2)*y'[x]+6*x*y'[x]+6*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2(3x^5 - 2x^3 + x) + c_1(5x^4 - 3x^2 + 1)$$

12.2 problem 2

Internal problem ID [1206]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 2.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 + 1)y'' + 2y'x - 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
Order:=6;  
dsolve((1+x^2)*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + x^2 - \frac{1}{3}x^4\right)y(0) + D(y)(0)x + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 23

```
AsymptoticDSolveValue[(1+x^2)*y''[x]+2*x*y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(-\frac{x^4}{3} + x^2 + 1\right) + c_2 x$$

12.3 problem 3

Internal problem ID [1207]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 3.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 + 1)y'' - 8y'x + 20y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;  
dsolve((1+x^2)*diff(y(x),x$2)-8*x*diff(y(x),x)+20*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (5x^4 - 10x^2 + 1)y(0) + \left(x - 2x^3 + \frac{1}{5}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[(1+x^2)*y''[x]-8*x*y'[x]+20*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{x^5}{5} - 2x^3 + x \right) + c_1 (5x^4 - 10x^2 + 1)$$

12.4 problem 4

Internal problem ID [1208]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 4.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [Gegenbauer]

$$(-x^2 + 1)y'' - 8y'x - 12y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((1-x^2)*diff(y(x),x$2)-8*x*diff(y(x),x)-12*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (15x^4 + 6x^2 + 1)y(0) + \left(x + \frac{10}{3}x^3 + 7x^5\right)D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-8*x*y'[x]-12*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(7x^5 + \frac{10x^3}{3} + x\right) + c_1(15x^4 + 6x^2 + 1)$$

12.5 problem 5

Internal problem ID [1209]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 5.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(2x^2 + 1)y'' + 7y'x + 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;  
dsolve((1+2*x^2)*diff(y(x),x$2)+7*x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x^2 + \frac{5}{3}x^4\right)y(0) + \left(x - \frac{3}{2}x^3 + \frac{21}{8}x^5\right)D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[(1+2*x^2)*y'[x]+7*x*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{21x^5}{8} - \frac{3x^3}{2} + x \right) + c_1 \left(\frac{5x^4}{3} - x^2 + 1 \right)$$

12.6 problem 6

Internal problem ID [1210]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 6.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 + 1)y'' + 2y'x + \frac{y}{4} = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;  
dsolve((1+x^2)*diff(y(x),x$2)+2*x*diff(y(x),x)+1/4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{8}x^2 + \frac{25}{384}x^4\right) y(0) + \left(x - \frac{3}{8}x^3 + \frac{147}{640}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[(1+x^2)*y''[x]+2*x*y'[x]+1/4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{147x^5}{640} - \frac{3x^3}{8} + x \right) + c_1 \left(\frac{25x^4}{384} - \frac{x^2}{8} + 1 \right)$$

12.7 problem 7

Internal problem ID [1211]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 7.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [Gegenbauer]

$$(-x^2 + 1)y'' - 5y'x - 4y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;  
dsolve((1-x^2)*diff(y(x),x$2)-5*x*diff(y(x),x)-4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + 2x^2 + \frac{8}{3}x^4\right)y(0) + \left(x + \frac{3}{2}x^3 + \frac{15}{8}x^5\right)D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-5*x*y'[x]-4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{15x^5}{8} + \frac{3x^3}{2} + x \right) + c_1 \left(\frac{8x^4}{3} + 2x^2 + 1 \right)$$

12.8 problem 8

Internal problem ID [1212]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 8.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 + 1)y'' - 10y'x + 28y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
Order:=6;  
dsolve((1+x^2)*diff(y(x),x$2)-10*x*diff(y(x),x)+28*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{35}{3}x^4 - 14x^2\right)y(0) + \left(x - 3x^3 + \frac{3}{5}x^5\right)D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 38

```
AsymptoticDSolveValue[(1+x^2)*y''[x]-10*x*y'[x]+28*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{3x^5}{5} - 3x^3 + x \right) + c_1 \left(\frac{35x^4}{3} - 14x^2 + 1 \right)$$

12.9 problem 9(a)

Internal problem ID [1213]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 9(a).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + y'x + 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;  
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x^2 + \frac{1}{3}x^4\right) y(0) + \left(x - \frac{1}{2}x^3 + \frac{1}{8}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[y'[x]+x*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{x^5}{8} - \frac{x^3}{2} + x \right) + c_1 \left(\frac{x^4}{3} - x^2 + 1 \right)$$

12.10 problem 10

Internal problem ID [1214]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 10.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 2y'x + 3y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;  
dsolve(diff(y(x),x$2)+2*x*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{3}{2}x^2 + \frac{7}{8}x^4\right) y(0) + \left(x - \frac{5}{6}x^3 + \frac{3}{8}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]+2*x*y'[x]+3*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{3x^5}{8} - \frac{5x^3}{6} + x \right) + c_1 \left(\frac{7x^4}{8} - \frac{3x^2}{2} + 1 \right)$$

12.11 problem 11

Internal problem ID [1215]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 11.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`, `[_2nd_order, _linear,`

$$(x^2 + 1)y'' + y'x + y = 0$$

With initial conditions

$$[y(0) = 2, y'(0) = -1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
```

```
dsolve([(1+x^2)*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(0) = 2, D(y)(0) = -1],y(x),type='series')
```

$$y(x) = 2 - x - x^2 + \frac{1}{3}x^3 + \frac{5}{12}x^4 - \frac{1}{6}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[{{(1+x^2)*y''[x]+x*y'[x]+y[x]==0,{y[0]==2,y'[0]==-1}},y[x],{x,0,5}]
```

$$y(x) \rightarrow -\frac{x^5}{6} + \frac{5x^4}{12} + \frac{x^3}{3} - x^2 - x + 2$$

12.12 problem 12

Internal problem ID [1216]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 12.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(2x^2 + 1)y'' - 9y'x - 6y = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
```

```
dsolve([(1+2*x^2)*diff(y(x),x$2)-9*x*diff(y(x),x)-6*y(x)=0,y(0) = 1, D(y)(0) = -1],y(x),type
```

$$y(x) = 1 - x + 3x^2 - \frac{5}{2}x^3 + 5x^4 - \frac{21}{8}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 32

```
AsymptoticDSolveValue[{{(1+2*x^2)*y'[x]-9*x*y'[x]-6*y[x]==0,{y[0]==1,y'[0]==-1}},y[x],{x,0,5
```

$$y(x) \rightarrow -\frac{21x^5}{8} + 5x^4 - \frac{5x^3}{2} + 3x^2 - x + 1$$

12.13 problem 13

Internal problem ID [1217]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 13.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$(8x^2 + 1)y'' + 2y = 0$$

With initial conditions

$$[y(0) = 2, y'(0) = -1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(1+8*x^2)*diff(y(x),x$2)+2*y(x)=0,y(0) = 2, D(y)(0) = -1],y(x),type='series',x=0);
```

$$y(x) = 2 - x - 2x^2 + \frac{1}{3}x^3 + 3x^4 - \frac{5}{6}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 32

```
AsymptoticDSolveValue[{(1+8*x^2)*y'[x]+2*y[x]==0,{y[0]==2,y'[0]==-1}},y[x],{x,0,5}]
```

$$y(x) \rightarrow -\frac{5x^5}{6} + 3x^4 + \frac{x^3}{3} - 2x^2 - x + 2$$

12.14 problem 16

Internal problem ID [1218]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 16.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - y = 0$$

With the expansion point for the power series method at $x = 3$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
Order:=6;  
dsolve(diff(y(x),x$2)-y(x)=0,y(x),type='series',x=3);
```

$$y(x) = \left(1 + \frac{(x-3)^2}{2} + \frac{(x-3)^4}{24}\right) y(3) + \left(x-3 + \frac{(x-3)^3}{6} + \frac{(x-3)^5}{120}\right) D(y)(3) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 51

```
AsymptoticDSolveValue[y'[x]-y[x]==0,y[x],{x,3,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{1}{24}(x-3)^4 + \frac{1}{2}(x-3)^2 + 1 \right) + c_2 \left(\frac{1}{120}(x-3)^5 + \frac{1}{6}(x-3)^3 + x-3 \right)$$

12.15 problem 17

Internal problem ID [1219]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 17.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$y'' - (x - 3)y' - y = 0$$

With the expansion point for the power series method at $x = 3$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;  
dsolve(diff(y(x),x$2)-(x-3)*diff(y(x),x)-y(x)=0,y(x),type='series',x=3);
```

$$y(x) = \left(1 + \frac{(x-3)^2}{2} + \frac{(x-3)^4}{8}\right) y(3) + \left(x-3 + \frac{(x-3)^3}{3} + \frac{(x-3)^5}{15}\right) D(y)(3) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 51

```
AsymptoticDSolveValue[y''[x]-(x-3)*y'[x]-y[x]==0,y[x],{x,3,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{1}{8}(x-3)^4 + \frac{1}{2}(x-3)^2 + 1 \right) + c_2 \left(\frac{1}{15}(x-3)^5 + \frac{1}{3}(x-3)^3 + x - 3 \right)$$

12.16 problem 18

Internal problem ID [1220]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 18.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(2x^2 - 4x + 1)y'' + 10(x - 1)y' + 6y = 0$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

Order:=6;

```
dsolve((1-4*x+2*x^2)*diff(y(x),x$2)+10*(x-1)*diff(y(x),x)+6*y(x)=0,y(x),type='series',x=1);
```

$$y(x) = \left(1 + 3(x-1)^2 + \frac{15(x-1)^4}{2}\right) y(1) + \left(x-1 + \frac{8(x-1)^3}{3} + \frac{32(x-1)^5}{5}\right) D(y)(1) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 49

```
AsymptoticDSolveValue[(1-4*x+2*x^2)*y''[x]+10*(x-1)*y'[x]+6*y[x]==0,y[x],{x,1,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{15}{2}(x-1)^4 + 3(x-1)^2 + 1 \right) + c_2 \left(\frac{32}{5}(x-1)^5 + \frac{8}{3}(x-1)^3 + x-1 \right)$$

12.17 problem 19

Internal problem ID [1221]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 19.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(2x^2 - 8x + 11)y'' - 16(-2 + x)y' + 36y = 0$$

With the expansion point for the power series method at $x = 2$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

Order:=6;

```
dsolve((11-8*x+2*x^2)*diff(y(x),x$2)-16*(x-2)*diff(y(x),x)+36*y(x)=0,y(x),type='series',x=2)
```

$$y(x) = \left(1 - 6(-2+x)^2 + \frac{4(-2+x)^4}{3}\right) y(2) + \left(-2+x - \frac{10(-2+x)^3}{9}\right) D(y)(2) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[(11-8*x+2*x^2)*y'[x]-16*(x-2)*y'[x]+36*y[x]==0,y[x],{x,2,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{4}{3}(x-2)^4 - 6(x-2)^2 + 1 \right) + c_2 \left(-\frac{10}{9}(x-2)^3 + x - 2 \right)$$

12.18 problem 20

Internal problem ID [1222]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 20.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(3x^2 + 6x + 5)y'' + 9(x + 1)y' + 3y = 0$$

With the expansion point for the power series method at $x = -1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
```

```
dsolve((5+6*x+3*x^2)*diff(y(x),x$2)+9*(x+1)*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=-1);
```

$$y(x) = \left(1 - \frac{3(x+1)^2}{4} + \frac{27(x+1)^4}{32}\right) y(-1) + \left(x+1 - (x+1)^3 + \frac{6(x+1)^5}{5}\right) D(y)(-1) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 70

```
AsymptoticDSolveValue[(5+6*x+2*x^2)*y'[x]+9*(x+1)*y'[x]+3*y[x]==0,y[x],{x,-1,5}]
```

$$y(x) \rightarrow c_1 \left(-\frac{93}{20}(x+1)^5 + \frac{17}{8}(x+1)^4 + (x+1)^3 - \frac{3}{2}(x+1)^2 + 1 \right) + c_2 \left(\frac{9}{5}(x+1)^5 + 2(x+1)^4 - 2(x+1)^3 + x + 1 \right)$$

12.19 problem 21

Internal problem ID [1223]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 21.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(x^2 - 4)y'' - y'x - 3y = 0$$

With initial conditions

$$[y(0) = -1, y'(0) = 2]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
Order:=6;
dsolve([(x^2-4)*diff(y(x),x$2)-x*diff(y(x),x)-3*y(x)=0,y(0) = -1, D(y)(0) = 2],y(x),type='series')
```

$$y(x) = -1 + 2x + \frac{3}{8}x^2 - \frac{1}{3}x^3 - \frac{3}{128}x^4 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 29

```
AsymptoticDSolveValue[{{(x^2-4)*y''[x]-x*y'[x]-3*y[x]==0,{y[0]==-1,y'[0]==2}},y[x],{x,0,5}]
```

$$y(x) \rightarrow -\frac{3x^4}{128} - \frac{x^3}{3} + \frac{3x^2}{8} + 2x - 1$$

12.20 problem 22

Internal problem ID [1224]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 22.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + (x - 3)y' + 3y = 0$$

With initial conditions

$$[y(3) = -2, y'(3) = 3]$$

With the expansion point for the power series method at $x = 3$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;  
dsolve([diff(y(x),x$2)+(x-3)*diff(y(x),x)+3*y(x)=0,y(3) = -2, D(y)(3) = 3],y(x),type='series
```

$$y(x) = -2 + 3(x - 3) + 3(x - 3)^2 - 2(x - 3)^3 - \frac{5}{4}(x - 3)^4 + \frac{3}{5}(x - 3)^5 + O((x - 3)^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[{y'[x]+(x-3)*y'[x]+3*y[x]==0,{y[3]==-2,y'[3]==3}},y[x],{x,3,5}]
```

$$y(x) \rightarrow \frac{3}{5}(x - 3)^5 - \frac{5}{4}(x - 3)^4 - 2(x - 3)^3 + 3(x - 3)^2 + 3(x - 3) - 2$$

12.21 problem 23

Internal problem ID [1225]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 23.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(3x^2 - 6x + 5)y'' + (x - 1)y' + 12y = 0$$

With initial conditions

$$[y(1) = -1, y'(1) = 1]$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
Order:=6;  
dsolve([(5-6*x+3*x^2)*diff(y(x),x$2)+(x-1)*diff(y(x),x)+12*y(x)=0,y(1) = -1, D(y)(1) = 1],y(x))
```

$$y(x) = -1 + (x - 1) + 3(x - 1)^2 - \frac{13}{12}(x - 1)^3 - \frac{5}{2}(x - 1)^4 + \frac{143}{160}(x - 1)^5 + O((x - 1)^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[{(5-6*x+3*x^2)*y'[x]+(x-1)*y'[x]+12*y[x]==0,{y[1]==-1,y'[1]==1}},y[x]]
```

$$y(x) \rightarrow \frac{143}{160}(x - 1)^5 - \frac{5}{2}(x - 1)^4 - \frac{13}{12}(x - 1)^3 + 3(x - 1)^2 + x - 2$$

12.22 problem 24

Internal problem ID [1226]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 24.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$(4x^2 - 24x + 37)y'' + y = 0$$

With initial conditions

$$[y(3) = 4, y'(3) = -6]$$

With the expansion point for the power series method at $x = 3$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(4*x^2-24*x+37)*diff(y(x),x$2)+y(x)=0,y(3) = 4, D(y)(3) = -6],y(x),type='series',x=3)
```

$$y(x) = 4 - 6(x - 3) - 2(x - 3)^2 + (x - 3)^3 + \frac{3}{2}(x - 3)^4 - \frac{5}{4}(x - 3)^5 + O((x - 3)^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[{(4*x^2-24*x+37)*y'[x]+y[x]==0,{y[3]==4,y'[3]==-6}},y[x],{x,3,5}]
```

$$y(x) \rightarrow -\frac{5}{4}(x - 3)^5 + \frac{3}{2}(x - 3)^4 + (x - 3)^3 - 2(x - 3)^2 - 6(x - 3) + 4$$

12.23 problem 25

Internal problem ID [1227]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 25.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 - 8x + 14)y'' - 8y'(x - 4) + 20y = 0$$

With initial conditions

$$[y(4) = 3, y'(4) = -4]$$

With the expansion point for the power series method at $x = 4$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
```

```
dsolve([(x^2-8*x+14)*diff(y(x),x$2)-8*(x-4)*diff(y(x),x)+20*y(x)=0,y(4) = 3, D(y)(4) = -4],y(x))
```

$$y(x) = 3 - 4(x - 4) + 15(x - 4)^2 - 4(x - 4)^3 + \frac{15}{4}(x - 4)^4 - \frac{1}{5}(x - 4)^5 + O((x - 4)^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 44

```
AsymptoticDSolveValue[{(x^2-8*x+14)*y'[x]+8*(x-4)*y'[x]+20*y[x]==0,{y[4]==3,y'[4]==-4}},y[x],{x,4,4}]
```

$$y(x) \rightarrow -\frac{35}{3}(x - 4)^5 + \frac{95}{4}(x - 4)^4 - \frac{28}{3}(x - 4)^3 + 15(x - 4)^2 - 4(x - 4) + 3$$

12.24 problem 26

Internal problem ID [1228]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 26.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(2x^2 + 4x + 5)y'' - 20(x + 1)y' + 60y = 0$$

With initial conditions

$$[y(-1) = 3, y'(-1) = -3]$$

With the expansion point for the power series method at $x = -1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
```

```
dsolve([(2*x^2+4*x+5)*diff(y(x),x$2)-20*(x+1)*diff(y(x),x)+60*y(x)=0,y(-1) = 3, D(y)(-1) = -3])
```

$$y(x) = 3 - 3(x + 1) - 30(x + 1)^2 + \frac{20}{3}(x + 1)^3 + 20(x + 1)^4 - \frac{4}{3}(x + 1)^5 + O((x + 1)^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[{(2*x^2+4*x+5)*y''[x]-20*(x+1)*y'[x]+60*y[x]==0,{y[-1]==3,y'[-1]==-3}}
```

$$y(x) \rightarrow -\frac{4}{3}(x + 1)^5 + 20(x + 1)^4 + \frac{20}{3}(x + 1)^3 - 30(x + 1)^2 - 3(x + 1) + 3$$

12.25 problem 27

Internal problem ID [1229]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 27.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(x^2 + 1)y'' + 4y'x + 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
Order:=6;  
dsolve((1+x^2)*diff(y(x),x$2)+4*x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (x^4 - x^2 + 1)y(0) + (x^5 - x^3 + x)D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 30

```
AsymptoticDSolveValue[(1+x^2)*y'[x]+4*x*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2(x^5 - x^3 + x) + c_1(x^4 - x^2 + 1)$$

12.26 problem 31

Internal problem ID [1230]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 31.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 2y'x + 2\alpha y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 63

```
Order:=6;  
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+2*alpha*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \alpha x^2 + \frac{\alpha(\alpha - 2)x^4}{6}\right) y(0) + \left(x - \frac{(\alpha - 1)x^3}{3} + \frac{(\alpha^2 - 4\alpha + 3)x^5}{30}\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 78

```
AsymptoticDSolveValue[y''[x]-2*x*y'[x]+2*\[Alpha]*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{\alpha^2 x^5}{30} - \frac{2\alpha x^5}{15} + \frac{x^5}{10} - \frac{\alpha x^3}{3} + \frac{x^3}{3} + x \right) + c_1 \left(\frac{\alpha^2 x^4}{6} - \frac{\alpha x^4}{3} - \alpha x^2 + 1 \right)$$

12.27 problem 33

Internal problem ID [1231]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 33.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$y'' - yx = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;  
dsolve(diff(y(x),x$2)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{x^3}{6}\right) y(0) + \left(x + \frac{1}{12}x^4\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y'[x]-x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{x^4}{12} + x\right) + c_1 \left(\frac{x^3}{6} + 1\right)$$

12.28 problem 34

Internal problem ID [1232]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 34.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(-2x^3 + 1)y'' - 10y'x^2 - 8yx = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve((1-2*x^3)*diff(y(x),x$2)-10*x^2*diff(y(x),x)-8*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{4x^3}{3}\right) y(0) + \left(x + \frac{3}{2}x^4\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[(1-2*x^3)*y'[x]-10*x^2*y'[x]-8*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{3x^4}{2} + x\right) + c_1 \left(\frac{4x^3}{3} + 1\right)$$

12.29 problem 35

Internal problem ID [1233]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 35.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^3 + 1)y'' + 7y'x^2 + 9yx = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;  
dsolve((1+x^3)*diff(y(x),x^2)+7*x^2*diff(y(x),x)+9*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{3x^3}{2}\right) y(0) + \left(x - \frac{4}{3}x^4\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[(1+x^3)*y''[x]+7*x^2*y'[x]+9*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(x - \frac{4x^4}{3}\right) + c_1 \left(1 - \frac{3x^3}{2}\right)$$

12.30 problem 36

Internal problem ID [1234]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 36.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(-2x^3 + 1)y'' + 6y'x^2 + 24yx = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve((1-2*x^3)*diff(y(x),x$2)+6*x^2*diff(y(x),x)+24*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (-4x^3 + 1)y(0) + \left(x - \frac{5}{2}x^4\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 26

```
AsymptoticDSolveValue[(1-2*x^3)*y'[x]+6*x^2*y'[x]+24*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(x - \frac{5x^4}{2}\right) + c_1(1 - 4x^3)$$

12.31 problem 37

Internal problem ID [1235]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 37.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(-x^3 + 1)y'' + 15y'x^2 - 36yx = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
Order:=6;  
dsolve((1-x^3)*diff(y(x),x$2)+15*x^2*diff(y(x),x)-36*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (6x^3 + 1)y(0) + \left(x + \frac{7}{4}x^4\right)D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[(1-2*x^3)*y'[x]-10*x^2*y'[x]-8*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{3x^4}{2} + x \right) + c_1 \left(\frac{4x^3}{3} + 1 \right)$$

12.32 problem 39

Internal problem ID [1236]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 39.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(2x^5 + 1)y'' + 14x^4y' + 10x^3y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
Order:=6;
```

```
dsolve((1+2*x^5)*diff(y(x),x$2)+14*x^4*diff(y(x),x)+10*x^3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^5}{2}\right)y(0) + D(y)(0)x + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 20

```
AsymptoticDSolveValue[(1+2*x^5)*y'[x]+14*x^4*y'[x]+10*x^3*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(1 - \frac{x^5}{2}\right) + c_2x$$

12.33 problem 40

Internal problem ID [1237]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 40.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$y'' + x^2y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;  
dsolve(diff(y(x),x$2)+x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^4}{12}\right) y(0) + \left(x - \frac{1}{20}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y'[x]+x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(x - \frac{x^5}{20}\right) + c_1 \left(1 - \frac{x^4}{12}\right)$$

12.34 problem 41

Internal problem ID [1238]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 41.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + y'x^6 + 7yx^5 = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
Order:=6;  
dsolve(diff(y(x),x$2)+x^6*diff(y(x),x)+7*x^5*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = y(0) + D(y)(0)x$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 10

```
AsymptoticDSolveValue[y''[x]+x^6*y'[x]+7*x^5*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2x + c_1$$

12.35 problem 42

Internal problem ID [1239]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 42.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^8 + 1)y'' - 16y'x^7 + 72yx^6 = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
Order:=6;  
dsolve((1+x^8)*diff(y(x),x$2)-16*x^7*diff(y(x),x)+72*x^6*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = y(0) + D(y)(0)x$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 10

```
AsymptoticDSolveValue[(1+x^8)*y'[x]-16*x^7*y'[x]+72*x^6*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2x + c_1$$

12.36 problem 43

Internal problem ID [1240]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 43.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(-x^6 + 1)y'' - 12y'x^5 - 30yx^4 = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve((1-x^6)*diff(y(x),x$2)-12*x^5*diff(y(x),x)-30*x^4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = y(0) + D(y)(0)x + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 10

```
AsymptoticDSolveValue[(1-x^6)*y''[x]-12*x^5*y'[x]-30*x^4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2x + c_1$$

12.37 problem 44

Internal problem ID [1241]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

Problem number: 44.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + y'x^5 + 6yx^4 = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;  
dsolve(diff(y(x),x$2)+x^5*diff(y(x),x)+6*x^4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = y(0) + D(y)(0)x + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 10

```
AsymptoticDSolveValue[y''[x]+x^5*y'[x]+6*x^4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2x + c_1$$

13 Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3.

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13.1 problem 1

Internal problem ID [1242]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 1.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(3x + 1)y'' + y'x + 2y = 0$$

With initial conditions

$$[y(0) = 2, y'(0) = -3]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(1+3*x)*diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(0) = 2, D(y)(0) = -3],y(x),type='series')
```

$$y(x) = 2 - 3x - 2x^2 + \frac{7}{2}x^3 - \frac{55}{12}x^4 + \frac{59}{8}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[{{(1+3*x)*y''[x]+x*y'[x]+2*y[x]==0,{y[0]==2,y'[0]==-3}},y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{59x^5}{8} - \frac{55x^4}{12} + \frac{7x^3}{2} - 2x^2 - 3x + 2$$

13.2 problem 2

Internal problem ID [1243]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 2.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(2x^2 + x + 1)y'' + (8x + 2)y' + 4y = 0$$

With initial conditions

$$[y(0) = -1, y'(0) = 2]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
Order:=6;  
dsolve([(1+x+2*x^2)*diff(y(x),x$2)+(2+8*x)*diff(y(x),x)+4*y(x)=0,y(0) = -1, D(y)(0) = 2],y(x))
```

$$y(x) = -1 + 2x - 4x^3 + 4x^4 + 4x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 23

```
AsymptoticDSolveValue[{(1+x+2*x^2)*y''[x]+(2+8*x)*y'[x]+4*y[x]==0,{y[0]==-1,y'[0]==2}},y[x],
```

$$y(x) \rightarrow 4x^5 + 4x^4 - 4x^3 + 2x - 1$$

13.3 problem 3

Internal problem ID [1244]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 3.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(-2x^2 + 1)y'' + (-6x + 2)y' - 2y = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
Order:=6;  
dsolve([(1-2*x^2)*diff(y(x),x$2)+(2-6*x)*diff(y(x),x)-2*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x),t
```

$$y(x) = 1 + x^2 - \frac{2}{3}x^3 + \frac{11}{6}x^4 - \frac{9}{5}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 29

```
AsymptoticDSolveValue[{(1-2*x^2)*y''[x]+(2-6*x)*y'[x]-2*y[x]==0,{y[0]==1,y'[0]==0}},y[x],{x,
```

$$y(x) \rightarrow -\frac{9x^5}{5} + \frac{11x^4}{6} - \frac{2x^3}{3} + x^2 + 1$$

13.4 problem 4

Internal problem ID [1245]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 4.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(3x^2 + x + 1)y'' + (2 + 15x)y' + 12y = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

Order:=6;

```
dsolve([(1+x+3*x^2)*diff(y(x),x$2)+(2+15*x)*diff(y(x),x)+12*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x))
```

$$y(x) = x - x^2 - \frac{7}{2}x^3 + \frac{15}{2}x^4 + \frac{45}{8}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 31

```
AsymptoticDSolveValue[{(1+x+3*x^2)*y''[x]+(2+15*x)*y'[x]+12*y[x]==0,{y[0]==0,y'[0]==1}},y[x]]
```

$$y(x) \rightarrow \frac{45x^5}{8} + \frac{15x^4}{2} - \frac{7x^3}{2} - x^2 + x$$

13.5 problem 5

Internal problem ID [1246]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 5.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(2+x)y'' + (x+1)y' + 3y = 0$$

With initial conditions

$$[y(0) = 4, y'(0) = 3]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(2+x)*diff(y(x),x$2)+(1+x)*diff(y(x),x)+3*y(x)=0,y(0) = 4, D(y)(0) = 3],y(x),type='s
```

$$y(x) = 4 + 3x - \frac{15}{4}x^2 + \frac{1}{4}x^3 + \frac{11}{16}x^4 - \frac{5}{16}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[{{(2+x)*y''[x]+(1+x)*y'[x]+3*y[x]==0,{y[0]==4,y'[0]==3}},y[x],{x,0,5}]
```

$$y(x) \rightarrow -\frac{5x^5}{16} + \frac{11x^4}{16} + \frac{x^3}{4} - \frac{15x^2}{4} + 3x + 4$$

13.6 problem 6

Internal problem ID [1247]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 6.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(x^2 + 3x + 3)y'' + (4x + 6)y' + 2y = 0$$

With initial conditions

$$[y(0) = 7, y'(0) = 3]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

Order:=6;

```
dsolve([(3+3*x+x^2)*diff(y(x),x$2)+(6+4*x)*diff(y(x),x)+2*y(x)=0,y(0) = 7, D(y)(0) = 3],y(x),f
```

$$y(x) = 7 + 3x - \frac{16}{3}x^2 + \frac{13}{3}x^3 - \frac{23}{9}x^4 + \frac{10}{9}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[{{(3+3*x+x^2)*y''[x]+(6+4*x)*y'[x]+2*y[x]==0,{y[0]==7,y'[0]==3}},y[x],f
```

$$y(x) \rightarrow \frac{10x^5}{9} - \frac{23x^4}{9} + \frac{13x^3}{3} - \frac{16x^2}{3} + 3x + 7$$

13.7 problem 7

Internal problem ID [1248]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 7.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x + 4)y'' + (2 + x)y' + 2y = 0$$

With initial conditions

$$[y(0) = 2, y'(0) = 5]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
Order:=6;
```

```
dsolve([(4+x)*diff(y(x),x$2)+(2+x)*diff(y(x),x)+2*y(x)=0,y(0) = 2, D(y)(0) = 5],y(x),type='s
```

$$y(x) = 2 + 5x - \frac{7}{4}x^2 - \frac{3}{16}x^3 + \frac{37}{192}x^4 - \frac{7}{192}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[{{(4+x)*y''[x]+(2+x)*y'[x]+2*y[x]==0,{y[0]==4,y'[0]==3}},y[x],{x,0,5}]
```

$$y(x) \rightarrow -\frac{7x^5}{192} + \frac{25x^4}{192} + \frac{x^3}{16} - \frac{7x^2}{4} + 3x + 4$$

13.8 problem 8

Internal problem ID [1249]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 8.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(2x^2 - 3x + 2)y'' - (-6x + 4)y' + 2y = 0$$

With initial conditions

$$[y(1) = 1, y'(1) = -1]$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
Order:=6;
dsolve([(2-3*x+2*x^2)*diff(y(x),x$2)-(4-6*x)*diff(y(x),x)+2*y(x)=0,y(1) = 1, D(y)(1) = -1],y
```

$$y(x) = 1 - (x - 1) + \frac{4}{3}(x - 1)^3 - \frac{4}{3}(x - 1)^4 - \frac{4}{5}(x - 1)^5 + O((x - 1)^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 35

```
AsymptoticDSolveValue[{{(2-3*x+2*x^2)*y'[x]-(4-6*x)*y'[x]+2*y[x]==0,{y[1]==1,y'[1]==-1}},y[x]
```

$$y(x) \rightarrow -\frac{4}{5}(x - 1)^5 - \frac{4}{3}(x - 1)^4 + \frac{4}{3}(x - 1)^3 - x + 2$$

13.9 problem 9

Internal problem ID [1250]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 9.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(2x^2 + 3x)y'' + 10(x + 1)y' + 8y = 0$$

With initial conditions

$$[y(-1) = 1, y'(-1) = -1]$$

With the expansion point for the power series method at $x = -1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
```

```
dsolve([(3*x+2*x^2)*diff(y(x),x$2)+10*(1+x)*diff(y(x),x)+8*y(x)=0,y(-1) = 1, D(y)(-1) = -1],y[
```

$$y(x) = 1 - (x + 1) + 4(x + 1)^2 - \frac{13}{3}(x + 1)^3 + \frac{77}{6}(x + 1)^4 - \frac{278}{15}(x + 1)^5 + O((x + 1)^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 41

```
AsymptoticDSolveValue[{{(3*x+2*x^2)*y''[x]+10*(1+x)*y'[x]+8*y[x]==0,{y[-1]==1,y'[-1]==-1}},y[
```

$$y(x) \rightarrow -\frac{278}{15}(x + 1)^5 + \frac{77}{6}(x + 1)^4 - \frac{13}{3}(x + 1)^3 + 4(x + 1)^2 - x$$

13.10 problem 10

Internal problem ID [1251]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 10.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(x^2 - x + 1)y'' - (-4x + 1)y' + 2y = 0$$

With initial conditions

$$[y(1) = 2, y'(1) = -1]$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
```

```
dsolve([(1-x+x^2)*diff(y(x),x$2)-(1-4*x)*diff(y(x),x)+2*y(x)=0,y(1) = 2, D(y)(1) = -1],y(x),
```

$$y(x) = 2 - (x - 1) - \frac{1}{2}(x - 1)^2 + \frac{5}{3}(x - 1)^3 - \frac{19}{12}(x - 1)^4 + \frac{7}{30}(x - 1)^5 + O((x - 1)^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 44

```
AsymptoticDSolveValue[{(1-x+x^2)*y''[x]-(1-4*x)*y'[x]+2*y[x]==0,{y[1]==2,y'[1]==-1}},y[x],{x
```

$$y(x) \rightarrow \frac{7}{30}(x - 1)^5 - \frac{19}{12}(x - 1)^4 + \frac{5}{3}(x - 1)^3 - \frac{1}{2}(x - 1)^2 - x + 3$$

13.11 problem 11

Internal problem ID [1252]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 11.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(2+x)y'' + (2+x)y' + y = 0$$

With initial conditions

$$[y(-1) = -2, y'(-1) = 3]$$

With the expansion point for the power series method at $x = -1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
```

```
dsolve([(2+x)*diff(y(x),x$2)+(2+x)*diff(y(x),x)+y(x)=0,y(-1) = -2, D(y)(-1) = 3],y(x),type='
```

$$y(x) = -2 + 3(x+1) - \frac{1}{2}(x+1)^2 - \frac{2}{3}(x+1)^3 + \frac{5}{8}(x+1)^4 - \frac{11}{30}(x+1)^5 + O((x+1)^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 46

```
AsymptoticDSolveValue[{(2+x)*y''[x]+(2+x)*y'[x]+y[x]==0,{y[-1]==-2,y'[-1]==3}},y[x],{x,-1,5}
```

$$y(x) \rightarrow -\frac{11}{30}(x+1)^5 + \frac{5}{8}(x+1)^4 - \frac{2}{3}(x+1)^3 - \frac{1}{2}(x+1)^2 + 3(x+1) - 2$$

13.12 problem 12

Internal problem ID [1253]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 12.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' - (6 - 7x)y' + 8y = 0$$

With initial conditions

$$[y(1) = 1, y'(1) = -2]$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
```

```
dsolve([x^2*diff(y(x),x$2)-(6-7*x)*diff(y(x),x)+8*y(x)=0,y(1) = 1, D(y)(1) = -2],y(x),type='
```

$$y(x) = 1 - 2(x - 1) - 3(x - 1)^2 + 8(x - 1)^3 - 4(x - 1)^4 - \frac{42}{5}(x - 1)^5 + O((x - 1)^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[{x^2*y'[x]-(6-7*x)*y'[x]+8*y[x]==0,{y[1]==1,y'[1]==-2}},y[x],{x,1,5}]
```

$$y(x) \rightarrow -\frac{42}{5}(x - 1)^5 - 4(x - 1)^4 + 8(x - 1)^3 - 3(x - 1)^2 - 2(x - 1) + 1$$

13.13 problem 13

Internal problem ID [1254]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 13.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(2x^2 + x + 1)y'' + (7x + 1)y' + 2y = 0$$

With initial conditions

$$[y(1) = 1, y'(1) = 0]$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

Order:=6;

```
dsolve([(1+x+2*x^2)*diff(y(x),x$2)+(1+7*x)*diff(y(x),x)+2*y(x)=0,y(1) = 1, D(y)(1) = 0],y(x),{
```

$$y(x) = 1 - \frac{1}{4}(x-1)^2 + \frac{13}{48}(x-1)^3 - \frac{77}{384}(x-1)^4 + \frac{287}{2560}(x-1)^5 + O((x-1)^6)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 41

```
AsymptoticDSolveValue[{{(1+x+2*x^2)*y''[x]+(1+7*x)*y'[x]+2*y[x]==0,{y[1]==1,y'[1]==0}},y[x],{
```

$$y(x) \rightarrow \frac{287(x-1)^5}{2560} - \frac{77}{384}(x-1)^4 + \frac{13}{48}(x-1)^3 - \frac{1}{4}(x-1)^2 + 1$$

13.14 problem 14

Internal problem ID [1255]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 14.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x + 3)y'' + (1 + 2x)y' - (-x + 2)y = 0$$

With initial conditions

$$[y(-1) = 1, y'(-1) = 0]$$

With the expansion point for the power series method at $x = -1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
Order:=6;
```

```
dsolve([(3+x)*diff(y(x),x$2)+(1+2*x)*diff(y(x),x)-(2-x)*y(x)=0,y(-1) = 1, D(y)(-1) = 0],y(x),{
```

$$y(x) = 1 + \frac{3}{4}(x + 1)^2 - \frac{1}{12}(x + 1)^3 - \frac{1}{48}(x + 1)^4 - \frac{1}{120}(x + 1)^5 + O((x + 1)^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 41

```
AsymptoticDSolveValue[{{(3+x)*y''[x]+(1+2*x)*y'[x]-(2-x)*y[x]==0,{y[-1]==1,y'[-1]==0}},y[x],{
```

$$y(x) \rightarrow -\frac{1}{120}(x + 1)^5 - \frac{1}{48}(x + 1)^4 - \frac{1}{12}(x + 1)^3 + \frac{3}{4}(x + 1)^2 + 1$$

13.15 problem 15

Internal problem ID [1256]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 15.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 3y'x + (2x^2 + 4)y = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;  
dsolve([diff(y(x),x$2)+3*x*diff(y(x),x)+(4+2*x^2)*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x),type='s'
```

$$y(x) = 1 - 2x^2 + \frac{3}{2}x^4 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 17

```
AsymptoticDSolveValue[{y'[x]+3*x*y'[x]+(4+2*x^2)*y[x]==0,{y[0]==1,y'[0]==0}},y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{3x^4}{2} - 2x^2 + 1$$

13.16 problem 19

Internal problem ID [1257]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 19.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(2 + 4x)y'' - 4y' - (4x + 6)y = 0$$

With initial conditions

$$[y(0) = 2, y'(0) = -7]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
```

```
dsolve([(2+4*x)*diff(y(x),x$2)-4*diff(y(x),x)-(6+4*x)*y(x)=0,y(0) = 2, D(y)(0) = -7],y(x),ty
```

$$y(x) = 2 - 7x - 4x^2 - \frac{17}{6}x^3 - \frac{3}{4}x^4 - \frac{9}{40}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[{(2+4*x)*y'[x]-4*y'[x]-(6+4*x)*y[x]==0,{y[0]==2,y'[0]==-7}},y[x],{x,0
```

$$y(x) \rightarrow -\frac{9x^5}{40} - \frac{3x^4}{4} - \frac{17x^3}{6} - 4x^2 - 7x + 2$$

13.17 problem 20

Internal problem ID [1258]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 20.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(1 + 2x)y'' - (1 - 2x)y' - (-2x + 3)y = 0$$

With initial conditions

$$[y(1) = 1, y'(1) = -2]$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
Order:=6;
dsolve([(1+2*x)*diff(y(x),x$2)-(1-2*x)*diff(y(x),x)-(3-2*x)*y(x)=0,y(1) = 1, D(y)(1) = -2],y(x))
```

$$y(x) = 1 - 2(x - 1) + \frac{1}{2}(x - 1)^2 - \frac{1}{6}(x - 1)^3 + \frac{5}{36}(x - 1)^4 - \frac{73}{1080}(x - 1)^5 + O((x - 1)^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 46

```
AsymptoticDSolveValue[{{(1+2*x)*y'[x]-(1-2*x)*y'[x]-(3-2*x)*y[x]==0,{y[1]==1,y'[1]==-2}},y[x]]
```

$$y(x) \rightarrow -\frac{73(x-1)^5}{1080} + \frac{5}{36}(x-1)^4 - \frac{1}{6}(x-1)^3 + \frac{1}{2}(x-1)^2 - 2(x-1) + 1$$

13.18 problem 21

Internal problem ID [1259]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 21.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(2x + 5)y'' - y' + (x + 5)y = 0$$

With initial conditions

$$[y(-2) = 2, y'(-2) = -1]$$

With the expansion point for the power series method at $x = -2$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(5+2*x)*diff(y(x),x$2)-diff(y(x),x)+(5+x)*y(x)=0,y(-2) = 2, D(y)(-2) = -1],y(x),type
```

$$y(x) = 2 - (2 + x) - \frac{7}{2}(2 + x)^2 + \frac{4}{3}(2 + x)^3 - \frac{1}{24}(2 + x)^4 + \frac{1}{60}(2 + x)^5 + O((2 + x)^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 43

```
AsymptoticDSolveValue[{{(5+2*x)*y'[x]-y'[x]+(5+x)*y[x]==0,{y[-2]==2,y'[-2]==-1}},y[x],{x,-2,
```

$$y(x) \rightarrow \frac{1}{60}(x + 2)^5 - \frac{1}{24}(x + 2)^4 + \frac{4}{3}(x + 2)^3 - \frac{7}{2}(x + 2)^2 - x$$

13.19 problem 22

Internal problem ID [1260]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 22.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x + 4)y'' - (2x + 4)y' + y(x + 6) = 0$$

With initial conditions

$$[y(-3) = 2, y'(-3) = -2]$$

With the expansion point for the power series method at $x = -3$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(4+x)*diff(y(x),x$2)-(4+2*x)*diff(y(x),x)+(6+x)*y(x)=0,y(-3) = 2, D(y)(-3) = -2],y(x)
```

$$y(x) = 2 - 2(x + 3) - (x + 3)^2 + (x + 3)^3 - \frac{11}{12}(x + 3)^4 + \frac{67}{60}(x + 3)^5 + O((x + 3)^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[{{(4+x)*y''[x]-(4+2*x)*y'[x]+(6+x)*y[x]==0,{y[-3]==2,y'[-3]==-2}},y[x],
```

$$y(x) \rightarrow \frac{67}{60}(x + 3)^5 - \frac{11}{12}(x + 3)^4 + (x + 3)^3 - (x + 3)^2 - 2(x + 3) + 2$$

13.20 problem 23

Internal problem ID [1261]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 23.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(3x + 2)y'' - y'x + 2yx = 0$$

With initial conditions

$$[y(0) = -1, y'(0) = 2]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
Order:=6;
```

```
dsolve([(2+3*x)*diff(y(x),x$2)-x*diff(y(x),x)+2*x*y(x)=0,y(0) = -1, D(y)(0) = 2],y(x),type='
```

$$y(x) = -1 + 2x + \frac{1}{3}x^3 - \frac{5}{12}x^4 + \frac{2}{5}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 29

```
AsymptoticDSolveValue[{{(2+3*x)*y'[x]-x*y'[x]+2*x*y[x]==0,{y[0]==-1,y'[0]==2}},y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{2x^5}{5} - \frac{5x^4}{12} + \frac{x^3}{3} + 2x - 1$$

13.21 problem 24

Internal problem ID [1262]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 24.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(2x + 3)y'' + 3y' - yx = 0$$

With initial conditions

$$[y(-1) = 2, y'(-1) = -3]$$

With the expansion point for the power series method at $x = -1$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
Order:=6;
```

```
dsolve([(3+2*x)*diff(y(x),x$2)+3*diff(y(x),x)-x*y(x)=0,y(-1) = 2, D(y)(-1) = -3],y(x),type='')
```

$$y(x) = 2 - 3(x + 1) + \frac{7}{2}(x + 1)^2 - 5(x + 1)^3 + \frac{197}{24}(x + 1)^4 - \frac{287}{20}(x + 1)^5 + O((x + 1)^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 44

```
AsymptoticDSolveValue[{{(3+2*x)*y'[x]+3*y'[x]-x*y[x]==0,{y[-1]==2,y'[-1]==-3}},y[x],{x,-1,5}]
```

$$y(x) \rightarrow -\frac{287}{20}(x + 1)^5 + \frac{197}{24}(x + 1)^4 - 5(x + 1)^3 + \frac{7}{2}(x + 1)^2 - 3(x + 1) + 2$$

13.22 problem 25

Internal problem ID [1263]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 25.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(2x + 3)y'' - 3y' - (2 + x)y = 0$$

With initial conditions

$$[y(-2) = -2, y'(-2) = 3]$$

With the expansion point for the power series method at $x = -2$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(3+2*x)*diff(y(x),x$2)-3*diff(y(x),x)-(2+x)*y(x)=0,y(-2) = -2, D(y)(-2) = 3],y(x),ty
```

$$y(x) = -2 + 3(2 + x) - \frac{9}{2}(2 + x)^2 + \frac{11}{6}(2 + x)^3 + \frac{5}{24}(2 + x)^4 + \frac{7}{20}(2 + x)^5 + O((2 + x)^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 46

```
AsymptoticDSolveValue[{(3+2*x)*y'[x]-3*y'[x]-(2+x)*y[x]==0,{y[-2]==-2,y'[-2]==3}},y[x],{x,-
```

$$y(x) \rightarrow \frac{7}{20}(x + 2)^5 + \frac{5}{24}(x + 2)^4 + \frac{11}{6}(x + 2)^3 - \frac{9}{2}(x + 2)^2 + 3(x + 2) - 2$$

13.23 problem 26

Internal problem ID [1264]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 26.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(-2x + 10)y'' + (x + 1)y = 0$$

With initial conditions

$$[y(2) = 2, y'(2) = -4]$$

With the expansion point for the power series method at $x = 2$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

Order:=6;

```
dsolve([(10-2*x)*diff(y(x),x$2)+(1+x)*y(x)=0,y(2) = 2, D(y)(2) = -4],y(x),type='series',x=2)
```

$$y(x) = 2 - 4(-2 + x) - \frac{1}{2}(-2 + x)^2 + \frac{2}{9}(-2 + x)^3 + \frac{49}{432}(-2 + x)^4 + \frac{23}{1080}(-2 + x)^5 + O((-2 + x)^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 46

```
AsymptoticDSolveValue[{(10-2*x)*y'[x]+(1+x)*y[x]==0,{y[2]==2,y'[2]==-4}},y[x],{x,2,5}]
```

$$y(x) \rightarrow \frac{23(x-2)^5}{1080} + \frac{49}{432}(x-2)^4 + \frac{2}{9}(x-2)^3 - \frac{1}{2}(x-2)^2 - 4(x-2) + 2$$

13.24 problem 27

Internal problem ID [1265]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 27.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x + 7)y'' + (2x + 8)y' + (x + 5)y = 0$$

With initial conditions

$$[y(-4) = 1, y'(-4) = 2]$$

With the expansion point for the power series method at $x = -4$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

Order:=6;

`dsolve([(7+x)*diff(y(x),x$2)+(8+2*x)*diff(y(x),x)+(5+x)*y(x)=0,y(-4) = 1, D(y)(-4) = 2],y(x))`

$$y(x) = 1 + 2(x + 4) - \frac{1}{6}(x + 4)^2 - \frac{10}{27}(x + 4)^3 + \frac{19}{648}(x + 4)^4 + \frac{13}{324}(x + 4)^5 + O((x + 4)^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 46

`AsymptoticDSolveValue[{(7+x)*y''[x]+(8+2*x)*y'[x]+(5+x)*y[x]==0,{y[-4]==1,y'[-4]==2}},y[x],{`

$$y(x) \rightarrow \frac{13}{324}(x + 4)^5 + \frac{19}{648}(x + 4)^4 - \frac{10}{27}(x + 4)^3 - \frac{1}{6}(x + 4)^2 + 2(x + 4) + 1$$

13.25 problem 28

Internal problem ID [1266]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 28.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(4x + 6)y'' + (1 + 2x)y = 0$$

With initial conditions

$$[y(-1) = -1, y'(-1) = 2]$$

With the expansion point for the power series method at $x = -1$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
Order:=6;
dsolve([(6+4*x)*diff(y(x),x$2)+(1+2*x)*y(x)=0,y(-1) = -1, D(y)(-1) = 2],y(x),type='series',x
```

$$y(x) = -1 + 2(x + 1) - \frac{1}{4}(x + 1)^2 + \frac{1}{2}(x + 1)^3 - \frac{65}{96}(x + 1)^4 + \frac{67}{80}(x + 1)^5 + O((x + 1)^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 46

```
AsymptoticDSolveValue[{{(6+4*x)*y'[x]+(1+2*x)*y[x]==0,{y[-1]==-1,y'[-1]==2}},y[x],{x,-1,5}]
```

$$y(x) \rightarrow \frac{67}{80}(x + 1)^5 - \frac{65}{96}(x + 1)^4 + \frac{1}{2}(x + 1)^3 - \frac{1}{4}(x + 1)^2 + 2(x + 1) - 1$$

13.26 problem 29

Internal problem ID [1267]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 29.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(\beta x^2 + \alpha x + 1) y'' + (\delta x + \gamma) y' + \epsilon y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 366

Order:=6;

dsolve((1+alpha*x+beta*x^2)*diff(y(x),x\$2)+(gamma+delta*x)*diff(y(x),x)+epsilon*y(x)=0,y(x),

$$\begin{aligned}
 y(x) = & \left(1 - \frac{\epsilon x^2}{2} + \frac{\epsilon(\alpha + \gamma) x^3}{6} + \frac{\epsilon(-\alpha^2 - \frac{3}{2}\alpha\gamma - \frac{1}{2}\gamma^2 + \beta + \delta + \frac{1}{2}\epsilon) x^4}{12} \right. \\
 & \left. - \frac{\left(\frac{(\alpha + \frac{\gamma}{2})\epsilon}{3} - \frac{\gamma^3}{12} - \frac{\alpha\gamma^2}{2} + \frac{(-\frac{11\alpha^2}{4} + 2\beta + \frac{5\delta}{4})\gamma}{3} + \alpha\left(-\frac{\alpha^2}{2} + \beta + \frac{3\delta}{4}\right) \right) \epsilon x^5}{10} \right) y(0) \\
 & + \left(x - \frac{\gamma x^2}{2} + \frac{(\alpha\gamma + \gamma^2 - \delta - \epsilon) x^3}{6} \right. \\
 & \quad \left. + \frac{((2\alpha + 2\gamma)\epsilon - \gamma^3 - 3\alpha\gamma^2 + (-2\alpha^2 + 2\beta + 3\delta)\gamma + 2\delta\alpha) x^4}{24} \right. \\
 & \quad \left. + \frac{(\epsilon^2 + (-6\alpha^2 - 9\alpha\gamma - 3\gamma^2 + 6\beta + 4\delta)\epsilon + \gamma^4 + 6\alpha\gamma^3 + (11\alpha^2 - 8\beta - 6\delta)\gamma^2 - 12\left(-\frac{\alpha^2}{2} + \beta + \frac{7\delta}{6}\right)\alpha\gamma}{120} \right) \\
 & + O(x^6)
 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 561

AsymptoticDSolveValue[(1+[Alpha]*x+[Beta]*x^2)*y''[x]+([Gamma]+[Delta]*x)*y'[x]+[Epsilon]

$$\begin{aligned}
 y(x) \rightarrow & c_1 \left(-\frac{1}{20}x^5\epsilon(2\alpha\beta - \alpha^3) - \frac{1}{20}x^5\epsilon(\alpha\delta - \gamma(\alpha^2 - \beta)) + \frac{1}{60}\gamma x^5\epsilon(\alpha^2 - \beta) \right. \\
 & + \frac{1}{120}\alpha\gamma^2x^5\epsilon + \frac{1}{40}\alpha x^5\epsilon(\alpha\gamma - \delta) + \frac{1}{24}\gamma x^5\epsilon(\alpha\gamma - \delta) - \frac{1}{30}\alpha x^5\epsilon^2 + \frac{1}{120}\gamma^3x^5\epsilon \\
 & - \frac{1}{60}\gamma x^5\epsilon^2 - \frac{1}{12}x^4\epsilon(\alpha^2 - \beta) - \frac{1}{12}x^4\epsilon(\alpha\gamma - \delta) - \frac{1}{24}\alpha\gamma x^4\epsilon - \frac{1}{24}\gamma^2x^4\epsilon + \frac{x^4\epsilon^2}{24} \\
 & \left. + \frac{1}{6}\alpha x^3\epsilon + \frac{1}{6}\gamma x^3\epsilon - \frac{x^2\epsilon}{2} + 1 \right) \\
 & + c_2 \left(\frac{1}{60}\gamma x^5(\gamma(\alpha^2 - \beta) - \alpha\delta) - \frac{1}{20}\gamma x^5(\alpha\delta - \gamma(\alpha^2 - \beta)) - \frac{1}{20}x^5\epsilon(\alpha^2 - \beta) \right. \\
 & - \frac{1}{20}x^5(\alpha^3(-\gamma) + \alpha^2\delta + 2\alpha\beta\gamma - \beta\delta) + \frac{1}{24}\gamma^2x^5(\alpha\gamma - \delta) - \frac{1}{120}\gamma^2x^5(\delta - \alpha\gamma) \\
 & - \frac{1}{40}x^5\epsilon(\alpha\gamma - \delta) + \frac{1}{120}x^5\epsilon(\delta - \alpha\gamma) - \frac{1}{40}x^5(\alpha\gamma - \delta)(\delta - \alpha\gamma) - \frac{1}{24}\alpha\gamma x^5\epsilon + \frac{\gamma^4x^5}{120} \\
 & - \frac{1}{40}\gamma^2x^5\epsilon + \frac{x^5\epsilon^2}{120} - \frac{1}{12}x^4(\gamma(\alpha^2 - \beta) - \alpha\delta) - \frac{1}{12}\gamma x^4(\alpha\gamma - \delta) + \frac{1}{24}\gamma x^4(\delta - \alpha\gamma) \\
 & \left. + \frac{1}{12}\alpha x^4\epsilon - \frac{\gamma^3x^4}{24} + \frac{1}{12}\gamma x^4\epsilon - \frac{1}{6}x^3(\delta - \alpha\gamma) + \frac{\gamma^2x^3}{6} - \frac{x^3\epsilon}{6} - \frac{\gamma x^2}{2} + x \right)
 \end{aligned}$$

13.27 problem 31(a)

Internal problem ID [1268]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 31(a).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(2x^2 + 3x + 1)y'' + (8x + 6)y' + 4y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
Order:=6;
dsolve((1+3*x+2*x^2)*diff(y(x),x$2)+(6+8*x)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (30x^5 - 14x^4 + 6x^3 - 2x^2 + 1)y(0) + (31x^5 - 15x^4 + 7x^3 - 3x^2 + x)D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 54

```
AsymptoticDSolveValue[(1+3*x+2*x^2)*y'[x]+(6+8*x)*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1(30x^5 - 14x^4 + 6x^3 - 2x^2 + 1) + c_2(31x^5 - 15x^4 + 7x^3 - 3x^2 + x)$$

13.28 problem 31(b)

Internal problem ID [1269]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 31(b).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(6x^2 - 5x + 1)y'' - (10 - 24x)y' + 12y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
```

```
dsolve((1-5*x+6*x^2)*diff(y(x),x$2)-(10-24*x)*diff(y(x),x)+12*y(x)=0,y(x),type='series',x=0)
```

$$y(x) = (-390x^5 - 114x^4 - 30x^3 - 6x^2 + 1)y(0) + (211x^5 + 65x^4 + 19x^3 + 5x^2 + x)D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 54

```
AsymptoticDSolveValue[(1-5*x+6*x^2)*y'[x]-(10-24*x)*y'[x]+12*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1(-390x^5 - 114x^4 - 30x^3 - 6x^2 + 1) + c_2(211x^5 + 65x^4 + 19x^3 + 5x^2 + x)$$

13.29 problem 31(c)

Internal problem ID [1270]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 31(c).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(4x^2 - 4x + 1)y'' - (8 - 16x)y' + 8y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 54

```
Order:=6;
dsolve((1-4*x+4*x^2)*diff(y(x),x$2)-(8-16*x)*diff(y(x),x)+8*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (-128x^5 - 48x^4 - 16x^3 - 4x^2 + 1)y(0) + (80x^5 + 32x^4 + 12x^3 + 4x^2 + x)D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 54

```
AsymptoticDSolveValue[(1-4*x+4*x^2)*y'[x]-(8-16*x)*y'[x]+8*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1(-128x^5 - 48x^4 - 16x^3 - 4x^2 + 1) + c_2(80x^5 + 32x^4 + 12x^3 + 4x^2 + x)$$

13.30 problem 31(d)

Internal problem ID [1271]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 31(d).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(x^2 + 4x + 4)y'' + (4x + 8)y' + 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

Order:=6;

```
dsolve((4+4*x+x^2)*diff(y(x),x$2)+(8+4*x)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{4}x^2 + \frac{1}{4}x^3 - \frac{3}{16}x^4 + \frac{1}{8}x^5\right) y(0) + \left(x - x^2 + \frac{3}{4}x^3 - \frac{1}{2}x^4 + \frac{5}{16}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 68

```
AsymptoticDSolveValue[(4+4*x+x^2)*y'[x]+(8+4*x)*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{x^5}{8} - \frac{3x^4}{16} + \frac{x^3}{4} - \frac{x^2}{4} + 1 \right) + c_2 \left(\frac{5x^5}{16} - \frac{x^4}{2} + \frac{3x^3}{4} - x^2 + x \right)$$

13.31 problem 31(e)

Internal problem ID [1272]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 31(e).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(3x^2 + 8x + 4)y'' + (16 + 12x)y' + 6y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
```

```
dsolve((4+8*x+3*x^2)*diff(y(x),x$2)+(16+12*x)*diff(y(x),x)+6*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{3}{4}x^2 + \frac{3}{2}x^3 - \frac{39}{16}x^4 + \frac{15}{4}x^5\right)y(0) + \left(x - 2x^2 + \frac{13}{4}x^3 - 5x^4 + \frac{121}{16}x^5\right)D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 66

```
AsymptoticDSolveValue[(4+8*x+3*x^2)*y'[x]+(16+12*x)*y'[x]+6*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{15x^5}{4} - \frac{39x^4}{16} + \frac{3x^3}{2} - \frac{3x^2}{4} + 1 \right) + c_2 \left(\frac{121x^5}{16} - 5x^4 + \frac{13x^3}{4} - 2x^2 + x \right)$$

13.32 problem 32

Internal problem ID [1273]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 32.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 2y'x + (2x^2 + 3)y = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = -2]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
```

```
dsolve([diff(y(x),x$2)+2*x*diff(y(x),x)+(3+2*x^2)*y(x)=0,y(0) = 1, D(y)(0) = -2],y(x),type='
```

$$y(x) = 1 - 2x - \frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{17}{24}x^4 - \frac{11}{20}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[{y'[x]+2*x*y'[x]+(3+2*x^2)*y[x]==0,{y[0]==1,y'[0]==-2}},y[x],{x,0,5}]
```

$$y(x) \rightarrow -\frac{11x^5}{20} + \frac{17x^4}{24} + \frac{5x^3}{3} - \frac{3x^2}{2} - 2x + 1$$

13.33 problem 33

Internal problem ID [1274]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 33.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 3y'x + (2x^2 + 5)y = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = -2]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
```

```
dsolve([diff(y(x),x$2)-3*x*diff(y(x),x)+(5+2*x^2)*y(x)=0,y(0) = 1, D(y)(0) = -2],y(x),type='
```

$$y(x) = 1 - 2x - \frac{5}{2}x^2 + \frac{2}{3}x^3 - \frac{3}{8}x^4 + \frac{1}{3}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[{y'[x]-3*x*y'[x]+(5+2*x^2)*y[x]==0,{y[0]==1,y'[0]==-2}},y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{x^5}{3} - \frac{3x^4}{8} + \frac{2x^3}{3} - \frac{5x^2}{2} - 2x + 1$$

13.34 problem 34

Internal problem ID [1275]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 34.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 5y'x - (-x^2 + 3)y = 0$$

With initial conditions

$$[y(0) = 6, y'(0) = -2]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;  
dsolve([diff(y(x),x$2)+5*x*diff(y(x),x)-(3-x^2)*y(x)=0,y(0) = 6, D(y)(0) = -2],y(x),type='series')
```

$$y(x) = 6 - 2x + 9x^2 + \frac{2}{3}x^3 - \frac{23}{4}x^4 - \frac{3}{10}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[{y'[x]+5*x*y'[x]-(3-x^2)*y[x]==0,{y[0]==6,y'[0]==-2}},y[x],{x,0,5}]
```

$$y(x) \rightarrow -\frac{3x^5}{10} - \frac{23x^4}{4} + \frac{2x^3}{3} + 9x^2 - 2x + 6$$

13.35 problem 35

Internal problem ID [1276]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 35.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 2y'x - (3x^2 + 2)y = 0$$

With initial conditions

$$[y(0) = 2, y'(0) = -5]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
Order:=6;
```

```
dsolve([diff(y(x),x$2)-2*x*diff(y(x),x)-(2+3*x^2)*y(x)=0,y(0) = 2, D(y)(0) = -5],y(x),type='
```

$$y(x) = 2 - 5x + 2x^2 - \frac{10}{3}x^3 + \frac{3}{2}x^4 - \frac{25}{12}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[{y'[x]-2*x*y'[x]-(2+3*x^2)*y[x]==0,{y[0]==2,y'[0]==-5}},y[x],{x,0,5}]
```

$$y(x) \rightarrow -\frac{25x^5}{12} + \frac{3x^4}{2} - \frac{10x^3}{3} + 2x^2 - 5x + 2$$

13.36 problem 36

Internal problem ID [1277]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 36.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 3y'x + (4x^2 + 2)y = 0$$

With initial conditions

$$[y(0) = 3, y'(0) = 6]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([diff(y(x),x$2)+3*x*diff(y(x),x)+(2+4*x^2)*y(x)=0,y(0) = 3, D(y)(0) = 6],y(x),type='s
```

$$y(x) = 3 + 6x - 3x^2 - 5x^3 + x^4 + \frac{31}{20}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[{y'[x]+3*x*y'[x]+(2+4*x^2)*y[x]==0,{y[0]==3,y'[0]==6}},y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{31x^5}{20} + x^4 - 5x^3 - 3x^2 + 6x + 3$$

13.37 problem 37

Internal problem ID [1278]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 37.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2y'' + 5y'x + (2x^2 + 4)y = 0$$

With initial conditions

$$[y(0) = 3, y'(0) = -2]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
Order:=6;  
dsolve([2*diff(y(x),x$2)+5*x*diff(y(x),x)+(4+2*x^2)*y(x)=0,y(0) = 3, D(y)(0) = -2],y(x),type
```

$$y(x) = 3 - 2x - 3x^2 + \frac{3}{2}x^3 + \frac{3}{2}x^4 - \frac{49}{80}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[{2*y''[x]+5*x*y'[x]+(4+2*x^2)*y[x]==0,{y[0]==3,y'[0]==-2}},y[x],{x,0,5
```

$$y(x) \rightarrow -\frac{49x^5}{80} + \frac{3x^4}{2} + \frac{3x^3}{2} - 3x^2 - 2x + 3$$

13.38 problem 38

Internal problem ID [1279]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 38.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3y'' + 2y'x + (-x^2 + 4)y = 0$$

With initial conditions

$$[y(0) = -2, y'(0) = 3]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
```

```
dsolve([3*diff(y(x),x$2)+2*x*diff(y(x),x)+(4-x^2)*y(x)=0,y(0) = -2, D(y)(0) = 3],y(x),type='
```

$$y(x) = -2 + 3x + \frac{4}{3}x^2 - x^3 - \frac{19}{54}x^4 + \frac{13}{60}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[{3*y''[x]+2*x*y'[x]+(4-x^2)*y[x]==0,{y[0]==-2,y'[0]==3}},y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{13x^5}{60} - \frac{19x^4}{54} - x^3 + \frac{4x^2}{3} + 3x - 2$$

13.39 problem 39 (a)

Internal problem ID [1280]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 39 (a).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 4y'x + (4x^2 + 2)y = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;  
dsolve([diff(y(x),x$2)+4*x*diff(y(x),x)+(2+4*x^2)*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x),type='s'
```

$$y(x) = 1 - x^2 + \frac{1}{2}x^4 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 17

```
AsymptoticDSolveValue[{y'[x]+4*x*y'[x]+(2+4*x^2)*y[x]==0,{y[0]==1,y'[0]==0}},y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{x^4}{2} - x^2 + 1$$

13.40 problem 39 (b)

Internal problem ID [1281]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 39 (b).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 4y'x + (4x^2 + 2)y = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
```

```
dsolve([diff(y(x),x$2)+4*x*diff(y(x),x)+(2+4*x^2)*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x),type='s
```

$$y(x) = x - x^3 + \frac{1}{2}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 17

```
AsymptoticDSolveValue[{y'[x]+4*x*y'[x]+(2+4*x^2)*y[x]==0,{y[0]==0,y'[0]==1}},y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{x^5}{2} - x^3 + x$$

13.41 problem 40

Internal problem ID [1282]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 40.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x + 1)y'' + y'x^2 + (1 + 2x)y = 0$$

With initial conditions

$$[y(0) = -2, y'(0) = 3]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(1+x)*diff(y(x),x$2)+x^2*diff(y(x),x)+(1+2*x)*y(x)=0,y(0) = -2, D(y)(0) = 3],y(x),t
```

$$y(x) = -2 + 3x + x^2 - \frac{1}{6}x^3 - \frac{3}{4}x^4 + \frac{31}{120}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 32

```
AsymptoticDSolveValue[{{(1+x)*y''[x]+x^2*y'[x]+(1+2*x)*y[x]==0,{y[0]==-2,y'[0]==3}},y[x],{x,0
```

$$y(x) \rightarrow \frac{31x^5}{120} - \frac{3x^4}{4} - \frac{x^3}{6} + x^2 + 3x - 2$$

13.42 problem 41

Internal problem ID [1283]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 41.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + (x^2 + 2x + 1)y' + 2y = 0$$

With initial conditions

$$[y(0) = 2, y'(0) = 3]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
```

```
dsolve([diff(y(x),x$2)+(1+2*x+x^2)*diff(y(x),x)+2*y(x)=0,y(0) = 2, D(y)(0) = 3],y(x),type='s
```

$$y(x) = 2 + 3x - \frac{7}{2}x^2 - \frac{5}{6}x^3 + \frac{41}{24}x^4 + \frac{41}{120}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[{y'[x]+(1+2*x+x^2)*y'[x]+2*y[x]==0,{y[0]==2,y'[0]==3}},y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{41x^5}{120} + \frac{41x^4}{24} - \frac{5x^3}{6} - \frac{7x^2}{2} + 3x + 2$$

13.43 problem 42

Internal problem ID [1284]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 42.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 + 1)y'' + (x^2 + 2)y' + yx = 0$$

With initial conditions

$$[y(0) = -3, y'(0) = 5]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
```

```
dsolve([(1+x^2)*diff(y(x),x$2)+(2+x^2)*diff(y(x),x)+x*y(x)=0,y(0) = -3, D(y)(0) = 5],y(x),ty
```

$$y(x) = -3 + 5x - 5x^2 + \frac{23}{6}x^3 - \frac{23}{12}x^4 + \frac{11}{30}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[{{(1+x^2)*y''[x]+(2+x^2)*y'[x]+x*y[x]==0,{y[0]==-3,y'[0]==5}},y[x],{x,0
```

$$y(x) \rightarrow \frac{11x^5}{30} - \frac{23x^4}{12} + \frac{23x^3}{6} - 5x^2 + 5x - 3$$

13.44 problem 43

Internal problem ID [1285]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 43.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x + 1)y'' + (2x^2 - 3x + 1)y' - y(x - 4) = 0$$

With initial conditions

$$[y(1) = -2, y'(1) = 3]$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

Order:=6;

```
dsolve([(1+x)*diff(y(x),x$2)+(1-3*x+2*x^2)*diff(y(x),x)-(x-4)*y(x)=0,y(1) = -2, D(y)(1) = 3]
```

$$y(x) = -2 + 3(x - 1) + \frac{3}{2}(x - 1)^2 - \frac{17}{12}(x - 1)^3 - \frac{1}{12}(x - 1)^4 + \frac{1}{8}(x - 1)^5 + O((x - 1)^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 46

```
AsymptoticDSolveValue[{{(1+x)*y''[x]+(1-3*x+x^2)*y'[x]-(x-4)*y[x]==0,{y[1]==-2,y'[1]==3}},y[x]
```

$$y(x) \rightarrow -\frac{13}{240}(x - 1)^5 - \frac{1}{96}(x - 1)^4 - \frac{2}{3}(x - 1)^3 + \frac{9}{4}(x - 1)^2 + 3(x - 1) - 2$$

13.45 problem 44

Internal problem ID [1286]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 44.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + (3x^2 + 12x + 13)y' + (2x + 5)y = 0$$

With initial conditions

$$[y(-2) = 2, y'(-2) = -3]$$

With the expansion point for the power series method at $x = -2$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

Order:=6;

`dsolve([diff(y(x),x$2)+(13+12*x+3*x^2)*diff(y(x),x)+(5+2*x)*y(x)=0,y(-2) = 2, D(y)(-2) = -3]`

$$y(x) = 2 - 3(2 + x) + \frac{1}{2}(2 + x)^2 - \frac{1}{3}(2 + x)^3 + \frac{31}{24}(2 + x)^4 - \frac{53}{120}(2 + x)^5 + O((2 + x)^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 46

`AsymptoticDSolveValue[{y'[x]+(13+12*x+3*x^2)*y'[x]+(5+2*x)*y[x]==0,{y[-2]==2,y'[-2]==-3}],y`

$$y(x) \rightarrow -\frac{53}{120}(x+2)^5 + \frac{31}{24}(x+2)^4 - \frac{1}{3}(x+2)^3 + \frac{1}{2}(x+2)^2 - 3(x+2) + 2$$

13.46 problem 45

Internal problem ID [1287]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 45.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(3x^2 + 2x + 1)y'' + (-x^2 + 2)y' + (x + 1)y = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = -2]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
```

```
dsolve([(1+2*x+3*x^2)*diff(y(x),x$2)+(2-x^2)*diff(y(x),x)+(1+x)*y(x)=0,y(0) = 1, D(y)(0) = -2])
```

$$y(x) = 1 - 2x + \frac{3}{2}x^2 - \frac{11}{6}x^3 + \frac{15}{8}x^4 - \frac{71}{60}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[{(1+2*x+3*x^2)*y'[x]+(2-x^2)*y'[x]+(1+x)*y[x]==0,{y[0]==1,y'[0]==-2}],y[x]
```

$$y(x) \rightarrow -\frac{71x^5}{60} + \frac{15x^4}{8} - \frac{11x^3}{6} + \frac{3x^2}{2} - 2x + 1$$

13.47 problem 46

Internal problem ID [1288]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 46.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 + 4x + 3)y'' - (-x^2 + 4x + 5)y' - (2 + x)y = 0$$

With initial conditions

$$[y(-2) = 2, y'(-2) = -1]$$

With the expansion point for the power series method at $x = -2$.

✓ [Solution by Maple](#)

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
```

```
dsolve([(3+4*x+x^2)*diff(y(x),x$2)-(5+4*x-x^2)*diff(y(x),x)-(2+x)*y(x)=0,y(-2) = 2, D(y)(-2)=
```

$$y(x) = 2 - (2 + x) - \frac{7}{2}(2 + x)^2 - \frac{43}{6}(2 + x)^3 - \frac{203}{24}(2 + x)^4 - \frac{167}{30}(2 + x)^5 + O((2 + x)^6)$$

✓ [Solution by Mathematica](#)

Time used: 0.001 (sec). Leaf size: 43

```
AsymptoticDSolveValue[{(3+4*x+x^2)*y'[x]-(5+4*x-x^2)*y'[x]-(2+x)*y[x]==0,{y[-2]==2,y'[-2]==
```

$$y(x) \rightarrow -\frac{167}{30}(x+2)^5 - \frac{203}{24}(x+2)^4 - \frac{43}{6}(x+2)^3 - \frac{7}{2}(x+2)^2 - x$$

13.48 problem 47

Internal problem ID [1289]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 47.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 + 2x + 1)y'' + (1 - x)y = 0$$

With initial conditions

$$[y(0) = 2, y'(0) = -1]$$

With the expansion point for the power series method at $x = 0$.

✓ [Solution by Maple](#)

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;  
dsolve([(1+2*x+x^2)*diff(y(x),x$2)+(1-x)*y(x)=0,y(0) = 2, D(y)(0) = -1],y(x),type='series',x
```

$$y(x) = 2 - x - x^2 + \frac{7}{6}x^3 - x^4 + \frac{89}{120}x^5 + O(x^6)$$

✓ [Solution by Mathematica](#)

Time used: 0.001 (sec). Leaf size: 32

```
AsymptoticDSolveValue[{{(1+2*x+x^2)*y'[x]+(1-x)*y[x]==0,{y[0]==2,y'[0]==-1}},y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{89x^5}{120} - x^4 + \frac{7x^3}{6} - x^2 - x + 2$$

13.49 problem 48

Internal problem ID [1290]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 48.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(-2x^2 + x)y'' + (-x^2 + 3x + 1)y' + (2 + x)y = 0$$

With initial conditions

$$[y(1) = 1, y'(1) = 0]$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

Order:=6;

`dsolve([(x-2*x^2)*diff(y(x),x$2)+(1+3*x-x^2)*diff(y(x),x)+(2+x)*y(x)=0,y(1) = 1, D(y)(1) = 0`

$$y(x) = 1 + \frac{3}{2}(x-1)^2 + \frac{1}{6}(x-1)^3 - \frac{1}{8}(x-1)^5 + O((x-1)^6)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 32

`AsymptoticDSolveValue[{(x-2*x^2)*y''[x]+(1+3*x-x^2)*y'[x]+(2+x)*y[x]==0,{y[1]==1,y'[1]==0}},`

$$y(x) \rightarrow -\frac{1}{8}(x-1)^5 + \frac{1}{6}(x-1)^3 + \frac{3}{2}(x-1)^2 + 1$$

13.50 problem 49

Internal problem ID [1291]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

Problem number: 49.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(2x^2 - 11x + 16)y'' + (x^2 - 6x + 10)y' - (-x + 2)y = 0$$

With initial conditions

$$[y(3) = 1, y'(3) = -2]$$

With the expansion point for the power series method at $x = 3$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
```

```
dsolve([(16-11*x+2*x^2)*diff(y(x),x$2)+(10-6*x+x^2)*diff(y(x),x)-(2-x)*y(x)=0,y(3) = 1, D(y
```

$$y(x) = 1 - 2(x - 3) + \frac{1}{2}(x - 3)^2 - \frac{1}{6}(x - 3)^3 + \frac{1}{4}(x - 3)^4 - \frac{1}{6}(x - 3)^5 + O((x - 3)^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 46

```
AsymptoticDSolveValue[{(16-11*x+2*x^2)*y'[x]+(10-6*x+x^2)*y'[x]-(2-x)*y[x]==0,{y[3]==1,y'[3
```

$$y(x) \rightarrow -\frac{1}{6}(x - 3)^5 + \frac{1}{4}(x - 3)^4 - \frac{1}{6}(x - 3)^3 + \frac{1}{2}(x - 3)^2 - 2(x - 3) + 1$$

14 Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

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Internal problem ID [1292]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: Example 7.5.1 page 353.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x^2 + x + 1)y'' + x(11x^2 + 11x + 9)y' + (7x^2 + 10x + 6)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 45

Order:=6;

`dsolve(2*x^2*(1+x+x^2)*diff(y(x),x$2)+x*(9+11*x+11*x^2)*diff(y(x),x)+(6+10*x+7*x^2)*y(x)=0,y`

$y(x)$

$$= \frac{\left(1 + \frac{1}{2}x^2 - \frac{1}{3}x^3 + \frac{1}{8}x^4 + \frac{1}{30}x^5 + O(x^6)\right) c_1 \sqrt{x} + \left(1 - \frac{1}{3}x + \frac{2}{5}x^2 - \frac{5}{21}x^3 + \frac{7}{135}x^4 + \frac{76}{1155}x^5 + O(x^6)\right) c_2 x}{x^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 83

`AsymptoticDSolveValue[2*x^2*(1+x+x^2)*y'[x]+x*(9+11*x+11*x^2)*y'[x]+(6+10*x+7*x^2)*y[x]==0,`

$$y(x) \rightarrow \frac{c_2 \left(\frac{x^5}{30} + \frac{x^4}{8} - \frac{x^3}{3} + \frac{x^2}{2} + 1 \right)}{x^2} + \frac{c_1 \left(\frac{76x^5}{1155} + \frac{7x^4}{135} - \frac{5x^3}{21} + \frac{2x^2}{5} - \frac{x}{3} + 1 \right)}{x^{3/2}}$$

14.2 problem Example 7.5.2 page 354

Internal problem ID [1293]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: Example 7.5.2 page 354.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x^2(x+3)y'' + 5x(x+1)y' - (-4x+1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

Order:=6;

```
dsolve(x^2*(3+x)*diff(y(x),x$2)+5*x*(1+x)*diff(y(x),x)-(1-4*x)*y(x)=0,y(x),type='series',x=0
```

$y(x)$

$$= \frac{c_2 x^{\frac{4}{3}} \left(1 - \frac{7}{9}x + \frac{35}{81}x^2 - \frac{455}{2187}x^3 + \frac{1820}{19683}x^4 - \frac{6916}{177147}x^5 + O(x^6) \right) + c_1 \left(1 + x - x^2 + \frac{3}{5}x^3 - \frac{3}{10}x^4 + \frac{3}{22}x^5 + O(x^6) \right)}{x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 82

```
AsymptoticDSolveValue[x^2*(3+x)*y'[x]+5*x*(1+x)*y'[x]-(1-4*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt[3]{x} \left(-\frac{6916x^5}{177147} + \frac{1820x^4}{19683} - \frac{455x^3}{2187} + \frac{35x^2}{81} - \frac{7x}{9} + 1 \right) + \frac{c_2 \left(\frac{3x^5}{22} - \frac{3x^4}{10} + \frac{3x^3}{5} - x^2 + x + 1 \right)}{x}$$

14.3 problem Example 7.5.3 page 356

Internal problem ID [1294]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: Example 7.5.3 page 356.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(-x^2 + 2)y'' - x(4x^2 + 3)y' + (2 + 2x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

`Order:=6;`

`dsolve(x^2*(2-x^2)*diff(y(x),x$2)-x*(3+4*x^2)*diff(y(x),x)+(2+2*x)*y(x)=0,y(x),type='series'`

$$y(x) = c_1\sqrt{x} \left(1 + 2x - \frac{9}{8}x^2 + \frac{7}{4}x^3 - \frac{607}{640}x^4 + \frac{13347}{11200}x^5 + O(x^6) \right) \\ + c_2x^2 \left(1 - \frac{2}{5}x + \frac{27}{35}x^2 - \frac{34}{105}x^3 + \frac{584}{1155}x^4 - \frac{768}{3575}x^5 + O(x^6) \right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 86

`AsymptoticDSolveValue[x^2*(2-x^2)*y'[x]-x*(3+4*x^2)*y'[x]+(2+2*x)*y[x]==0,y[x],{x,0,5}]`

$$y(x) \rightarrow c_1 \left(-\frac{768x^5}{3575} + \frac{584x^4}{1155} - \frac{34x^3}{105} + \frac{27x^2}{35} - \frac{2x}{5} + 1 \right) x^2 \\ + c_2 \left(\frac{13347x^5}{11200} - \frac{607x^4}{640} + \frac{7x^3}{4} - \frac{9x^2}{8} + 2x + 1 \right) \sqrt{x}$$

14.4 problem 1

Internal problem ID [1295]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 1.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x^2 + x + 1)y'' + x(5x^2 + 3x + 3)y' + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 1243

```
Order:=6;
```

```
dsolve(2*x^2*(1+x+x^2)*diff(y(x),x$2)+x*(3+3*x+5*x^2)*diff(y(x),x)+y(x)=0,y(x),type='series')
```

$y(x)$

$$= c_1 x^{-\frac{i\sqrt{7}}{4}} \left(1 - \frac{1}{-2+i\sqrt{7}}x + \frac{1}{4} \frac{11+i\sqrt{7}}{(-2+i\sqrt{7})(i\sqrt{7}-4)}x^2 - \frac{1}{12} \frac{49i\sqrt{7}-89}{(-2+i\sqrt{7})(i\sqrt{7}-4)(i\sqrt{7}-6)}x^3 - \frac{1}{48} \frac{395i\sqrt{7}+1553}{(-2+i\sqrt{7})(i\sqrt{7}-4)(i\sqrt{7}-6)}x^4 \right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 4838

```
AsymptoticDSolveValue[2*x^2*(1+x+x^2)*y''[x]+x*(3+3*x+5*x^2)*y'[x]+y[x]==0,y[x],{x,0,5}]
```

Too large to display

14.5 problem 2

Internal problem ID [1296]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 2.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$3x^2y'' + 2x(-2x^2 + x + 1)y' + (-8x^2 + 2x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

Order:=6;

```
dsolve(3*x^2*diff(y(x),x$2)+2*x*(1+x-2*x^2)*diff(y(x),x)+(2*x-8*x^2)*y(x)=0,y(x),type='series')
```

$$y(x) = c_1 x^{\frac{1}{3}} \left(1 - \frac{2}{3}x + \frac{8}{9}x^2 - \frac{40}{81}x^3 + \frac{92}{243}x^4 - \frac{664}{3645}x^5 + O(x^6) \right) \\ + c_2 \left(1 - x + \frac{6}{5}x^2 - \frac{4}{5}x^3 + \frac{32}{55}x^4 - \frac{24}{77}x^5 + O(x^6) \right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 83

```
AsymptoticDSolveValue[3*x^2*y'[x]+2*x*(1+x-2*x^2)*y'[x]+(2*x-8*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt[3]{x} \left(-\frac{664x^5}{3645} + \frac{92x^4}{243} - \frac{40x^3}{81} + \frac{8x^2}{9} - \frac{2x}{3} + 1 \right) \\ + c_2 \left(-\frac{24x^5}{77} + \frac{32x^4}{55} - \frac{4x^3}{5} + \frac{6x^2}{5} - x + 1 \right)$$

14.6 problem 3

Internal problem ID [1297]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 3.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2 + 3x + 3)y'' + x(7x^2 + 8x + 5)y' - (-9x^2 - 2x + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 43

Order:=6;

```
dsolve(x^2*(3+3*x+x^2)*diff(y(x),x$2)+x*(5+8*x+7*x^2)*diff(y(x),x)-(1-2*x-9*x^2)*y(x)=0,y(x))
```

$$y(x) = \frac{c_2 x^{\frac{4}{3}} \left(1 - \frac{4}{7}x - \frac{7}{45}x^2 + \frac{970}{2457}x^3 - \frac{5707}{22680}x^4 + \frac{13568}{300105}x^5 + O(x^6)\right) + c_1 \left(1 - x^2 + \frac{2}{3}x^3 - \frac{10}{33}x^5 + O(x^6)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 74

```
AsymptoticDSolveValue[x^2*(3+3*x+x^2)*y''[x]+x*(5+8*x+7*x^2)*y'[x]-(1-2*x-9*x^2)*y[x]==0,y[x],{x,0,1}]
```

$$y(x) \rightarrow \frac{c_2 \left(-\frac{10x^5}{33} + \frac{2x^3}{3} - x^2 + 1\right)}{x} + c_1 \sqrt[3]{x} \left(\frac{13568x^5}{300105} - \frac{5707x^4}{22680} + \frac{970x^3}{2457} - \frac{7x^2}{45} - \frac{4x}{7} + 1\right)$$

14.7 problem 4

Internal problem ID [1298]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 4.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + x(4x^2 + 2x + 7)y' - (-7x^2 - 4x + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

Order:=6;

```
dsolve(4*x^2*dif(y(x),x$2)+x*(7+2*x+4*x^2)*dif(y(x),x)-(1-4*x-7*x^2)*y(x)=0,y(x),type='ser
```

$$y(x) = \frac{c_2 x^{\frac{5}{4}} \left(1 - \frac{1}{2}x - \frac{19}{104}x^2 + \frac{1571}{10608}x^3 + \frac{3225}{198016}x^4 - \frac{752183}{29702400}x^5 + O(x^6)\right) + c_1 \left(1 + 2x - \frac{11}{6}x^2 - \frac{1}{7}x^3 + \frac{895}{1848}x^4 - \dots\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 86

```
AsymptoticDSolveValue[4*x^2*y'[x]+x*(7+2*x+4*x^2)*y'[x]-(1-4*x-7*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt[4]{x} \left(-\frac{752183x^5}{29702400} + \frac{3225x^4}{198016} + \frac{1571x^3}{10608} - \frac{19x^2}{104} - \frac{x}{2} + 1 \right) + \frac{c_2 \left(-\frac{499x^5}{13860} + \frac{895x^4}{1848} - \frac{x^3}{7} - \frac{11x^2}{6} + 2x + 1 \right)}{x}$$

14.8 problem 5

Internal problem ID [1299]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 5.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$12x^2(x+1)y'' + x(3x^2 + 35x + 11)y' - (-5x^2 - 10x + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 47

Order:=6;

`dsolve(12*x^2*(1+x)*diff(y(x),x$2)+x*(11+35*x+3*x^2)*diff(y(x),x)-(1-10*x-5*x^2)*y(x)=0,y(x))`

$y(x)$

$$= \frac{c_2 x^{\frac{7}{12}} \left(1 - x + \frac{28}{31} x^2 - \frac{1111}{1333} x^3 + \frac{57493}{73315} x^4 - \frac{3668716}{4912105} x^5 + O(x^6) \right) + c_1 \left(1 - x + \frac{7}{8} x^2 - \frac{19}{24} x^3 + \frac{283}{384} x^4 - \frac{1339}{1920} x^5 \right)}{x^{\frac{1}{4}}}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 86

`AsymptoticDSolveValue[12*x^2*(1+x)*y'[x]+x*(11+35*x+3*x^2)*y'[x]-(1-10*x-5*x^2)*y[x]==0,y[x]`

$$y(x) \rightarrow c_1 \sqrt[3]{x} \left(-\frac{3668716x^5}{4912105} + \frac{57493x^4}{73315} - \frac{1111x^3}{1333} + \frac{28x^2}{31} - x + 1 \right) + \frac{c_2 \left(-\frac{1339x^5}{1920} + \frac{283x^4}{384} - \frac{19x^3}{24} + \frac{7x^2}{8} - x + 1 \right)}{\sqrt[4]{x}}$$

14.9 problem 6

Internal problem ID [1300]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 6.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$x^2(10x^2 + x + 5)y'' + x(48x^2 + 3x + 4)y' + (36x^2 + x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 44

Order:=6;

```
dsolve(x^2*(5+x+10*x^2)*diff(y(x),x$2)+x*(4+3*x+48*x^2)*diff(y(x),x)+(x+36*x^2)*y(x)=0,y(x),
```

$$y(x) = c_1 x^{\frac{1}{5}} \left(1 - \frac{6}{25}x - \frac{1217}{625}x^2 + \frac{41972}{46875}x^3 + \frac{1447799}{390625}x^4 - \frac{375253322}{146484375}x^5 + O(x^6) \right) \\ + c_2 \left(1 - \frac{1}{4}x - \frac{35}{18}x^2 + \frac{11}{12}x^3 + \frac{632}{171}x^4 - \frac{2671}{1026}x^5 + O(x^6) \right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 85

```
AsymptoticDSolveValue[x^2*(5+x+10*x^2)*y'[x]+x*(4+3*x+48*x^2)*y'[x]+(x+36*x^2)*y[x]==0,y[x]
```

$$y(x) \rightarrow c_1 \sqrt[5]{x} \left(-\frac{375253322x^5}{146484375} + \frac{1447799x^4}{390625} + \frac{41972x^3}{46875} - \frac{1217x^2}{625} - \frac{6x}{25} + 1 \right) \\ + c_2 \left(-\frac{2671x^5}{1026} + \frac{632x^4}{171} + \frac{11x^3}{12} - \frac{35x^2}{18} - \frac{x}{4} + 1 \right)$$

14.10 problem 7

Internal problem ID [1301]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 7.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$8x^2y'' - 2x(-x^2 - 4x + 3)y' + (x^2 + 6x + 3)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

Order:=6;

```
dsolve(8*x^2*diff(y(x),x$2)-2*x*(3-4*x-x^2)*diff(y(x),x)+(3+6*x+x^2)*y(x)=0,y(x),type='series')
```

$$y(x) = c_1 x^{\frac{1}{4}} \left(1 + 4x - \frac{131}{24}x^2 + \frac{39}{14}x^3 - \frac{19865}{29568}x^4 + \frac{4421}{110880}x^5 + O(x^6) \right) \\ + c_2 x^{\frac{3}{2}} \left(1 - x + \frac{11}{26}x^2 - \frac{109}{1326}x^3 + \frac{5}{12376}x^4 + \frac{229}{71400}x^5 + O(x^6) \right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 86

```
AsymptoticDSolveValue[8*x^2*y'[x]-2*x*(3-4*x-x^2)*y'[x]+(3+6*x+x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{4421x^5}{110880} - \frac{19865x^4}{29568} + \frac{39x^3}{14} - \frac{131x^2}{24} + 4x + 1 \right) \sqrt[4]{x} \\ + c_1 \left(\frac{229x^5}{71400} + \frac{5x^4}{12376} - \frac{109x^3}{1326} + \frac{11x^2}{26} - x + 1 \right) x^{3/2}$$

14.11 problem 8

Internal problem ID [1302]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 8.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$18x^2(x+1)y'' + 3x(x^2 + 11x + 5)y' - (-5x^2 - 2x + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 45

Order:=6;

```
dsolve(18*x^2*(1+x)*diff(y(x),x$2)+3*x*(5+11*x+x^2)*diff(y(x),x)-(1-2*x-5*x^2)*y(x)=0,y(x),t
```

$$y(x) = \frac{c_2 \sqrt{x} \left(1 - \frac{1}{3}x + \frac{2}{15}x^2 - \frac{5}{63}x^3 + \frac{23}{405}x^4 - \frac{458}{10395}x^5 + O(x^6)\right) + c_1 \left(1 - \frac{1}{12}x^2 + \frac{1}{18}x^3 - \frac{11}{288}x^4 + \frac{31}{1080}x^5 + O(x^6)\right)}{x^{\frac{1}{6}}}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 85

```
AsymptoticDSolveValue[18*x^2*(1+x)*y''[x]+3*x*(5+11*x+x^2)*y'[x]-(1-2*x-5*x^2)*y[x]==0,y[x],t
```

$$y(x) \rightarrow c_1 \sqrt[3]{x} \left(-\frac{458x^5}{10395} + \frac{23x^4}{405} - \frac{5x^3}{63} + \frac{2x^2}{15} - \frac{x}{3} + 1 \right) + \frac{c_2 \left(\frac{31x^5}{1080} - \frac{11x^4}{288} + \frac{x^3}{18} - \frac{x^2}{12} + 1 \right)}{\sqrt[6]{x}}$$

14.12 problem 9

Internal problem ID [1303]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 9.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(x^2 + x + 3)y'' + (-x^2 + x + 4)y' + xy = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 42

```
Order:=6;
```

```
dsolve(x*(3+x+x^2)*diff(y(x),x$2)+(4+x-x^2)*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$y(x)$

$$= \frac{c_2 \left(1 - \frac{1}{14}x^2 + \frac{1}{105}x^3 - \frac{1}{3640}x^4 - \frac{23}{54600}x^5 + O(x^6)\right) x^{\frac{1}{3}} + c_1 \left(1 - \frac{1}{18}x - \frac{71}{405}x^2 + \frac{719}{34992}x^3 - \frac{1678}{1082565}x^4 - \frac{513}{99202}x^5\right) x^{\frac{1}{3}}}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 80

```
AsymptoticDSolveValue[x*(3+x+x^2)*y''[x]+(4+x-x^2)*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(-\frac{23x^5}{54600} - \frac{x^4}{3640} + \frac{x^3}{105} - \frac{x^2}{14} + 1 \right) + \frac{c_2 \left(-\frac{513547x^5}{992023200} - \frac{1678x^4}{1082565} + \frac{719x^3}{34992} - \frac{71x^2}{405} - \frac{x}{18} + 1 \right)}{\sqrt[3]{x}}$$

14.13 problem 10

Internal problem ID [1304]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 10.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$10x^2(2x^2 + x + 1)y'' + x(66x^2 + 13x + 13)y' - (10x^2 + 4x + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 47

```
Order:=6;
```

```
dsolve(10*x^2*(1+x+2*x^2)*diff(y(x),x$2)+x*(13+13*x+66*x^2)*diff(y(x),x)-(1+4*x+10*x^2)*y(x)
```

$$y(x) = \frac{c_2 x^{\frac{7}{10}} \left(1 + \frac{3}{17}x - \frac{7}{153}x^2 - \frac{547}{5661}x^3 + \frac{26942}{266067}x^4 + \frac{200432}{3991005}x^5 + O(x^6) \right) + c_1 \left(1 + x + \frac{14}{13}x^2 - \frac{556}{897}x^3 - \frac{5314}{9867}x^4 + \dots \right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 86

```
AsymptoticDSolveValue[10*x^2*(1+x+2*x^2)*y''[x]+x*(13+13*x+66*x^2)*y'[x]-(1+4*x+10*x^2)*y[x]
```

$$y(x) \rightarrow c_1 \sqrt{x} \left(\frac{200432x^5}{3991005} + \frac{26942x^4}{266067} - \frac{547x^3}{5661} - \frac{7x^2}{153} + \frac{3x}{17} + 1 \right) + \frac{c_2 \left(\frac{2092186x^5}{2121405} - \frac{5314x^4}{9867} - \frac{556x^3}{897} + \frac{14x^2}{13} + x + 1 \right)}{\sqrt{x}}$$

14.14 problem 14

Internal problem ID [1305]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 14.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2y'' + x(2x + 3)y' - (1 - x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
Order:=6;
```

```
dsolve(2*x^2*diff(y(x),x$2)+x*(3+2*x)*diff(y(x),x)-(1-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{3}{2}} \left(1 - \frac{2}{5}x + \frac{4}{35}x^2 - \frac{8}{315}x^3 + \frac{16}{3465}x^4 - \frac{32}{45045}x^5 + O(x^6)\right) + c_1 \left(1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + O(x^6)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 86

```
AsymptoticDSolveValue[2*x^2*y'[x]+x*(3+2*x)*y'[x]-(1-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt{x} \left(-\frac{32x^5}{45045} + \frac{16x^4}{3465} - \frac{8x^3}{315} + \frac{4x^2}{35} - \frac{2x}{5} + 1 \right) + \frac{c_2 \left(-\frac{x^5}{120} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} - x + 1 \right)}{x}$$

14.15 problem 15

Internal problem ID [1306]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 15.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x^2(x+3)y'' + x(4x+5)y' - (1-2x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
Order:=6;
```

```
dsolve(x^2*(3+x)*diff(y(x),x$2)+x*(5+4*x)*diff(y(x),x)-(1-2*x)*y(x)=0,y(x),type='series',x=0)
```

$$y(x) = \frac{c_2 x^{\frac{4}{3}} \left(1 - \frac{4}{9}x + \frac{14}{81}x^2 - \frac{140}{2187}x^3 + \frac{455}{19683}x^4 - \frac{1456}{177147}x^5 + O(x^6)\right) + c_1(1 + O(x^6))}{x}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 53

```
AsymptoticDSolveValue[x^2*(3+x)*y''[x]+x*(5+4*x)*y'[x]-(1-2*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt[3]{x} \left(-\frac{1456x^5}{177147} + \frac{455x^4}{19683} - \frac{140x^3}{2187} + \frac{14x^2}{81} - \frac{4x}{9} + 1 \right) + \frac{c_2}{x}$$

14.16 problem 16

Internal problem ID [1307]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 16.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2y'' + x(x+5)y' - (-3x+2)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
```

```
dsolve(2*x^2*diff(y(x),x$2)+x*(5+x)*diff(y(x),x)-(2-3*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{5}{2}} \left(1 - \frac{1}{2}x + \frac{1}{8}x^2 - \frac{1}{48}x^3 + \frac{1}{384}x^4 - \frac{1}{3840}x^5 + O(x^6)\right) + c_1 \left(1 + \frac{1}{3}x + \frac{1}{3}x^2 - \frac{1}{3}x^3 + \frac{1}{9}x^4 - \frac{1}{45}x^5 + O(x^6)\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 88

```
AsymptoticDSolveValue[2*x^2*y'[x]+x*(5+x)*y'[x]-(2-3*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt{x} \left(-\frac{x^5}{3840} + \frac{x^4}{384} - \frac{x^3}{48} + \frac{x^2}{8} - \frac{x}{2} + 1 \right) + \frac{c_2 \left(-\frac{x^5}{45} + \frac{x^4}{9} - \frac{x^3}{3} + \frac{x^2}{3} + \frac{x}{3} + 1 \right)}{x^2}$$

14.17 problem 17

Internal problem ID [1308]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 17.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3x^2y'' + x(x+1)y' - y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

```
Order:=6;
```

```
dsolve(3*x^2*diff(y(x),x$2)+x*(1+x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{4}{3}} \left(1 - \frac{1}{7}x + \frac{1}{70}x^2 - \frac{1}{910}x^3 + \frac{1}{14560}x^4 - \frac{1}{276640}x^5 + O(x^6)\right) + c_1 \left(1 - \frac{1}{3}x + \frac{1}{18}x^2 - \frac{1}{162}x^3 + \frac{1}{1944}x^4 - \frac{1}{2916}x^5 + O(x^6)\right)}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 86

```
AsymptoticDSolveValue[3*x^2*y''[x]+x*(1+x)*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 x \left(-\frac{x^5}{276640} + \frac{x^4}{14560} - \frac{x^3}{910} + \frac{x^2}{70} - \frac{x}{7} + 1 \right) + \frac{c_2 \left(-\frac{x^5}{29160} + \frac{x^4}{1944} - \frac{x^3}{162} + \frac{x^2}{18} - \frac{x}{3} + 1 \right)}{\sqrt[3]{x}}$$

14.18 problem 18

Internal problem ID [1309]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 18.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2y'' - y'x + (1 - 2x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;  
dsolve(2*x^2*diff(y(x),x$2)-x*diff(y(x),x)+(1-2*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x} \left(1 + 2x + \frac{2}{3}x^2 + \frac{4}{45}x^3 + \frac{2}{315}x^4 + \frac{4}{14175}x^5 + O(x^6) \right) \\ + c_2x \left(1 + \frac{2}{3}x + \frac{2}{15}x^2 + \frac{4}{315}x^3 + \frac{2}{2835}x^4 + \frac{4}{155925}x^5 + O(x^6) \right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 84

```
AsymptoticDSolveValue[2*x^2*y''[x]-x*y'[x]+(1-2*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1x \left(\frac{4x^5}{155925} + \frac{2x^4}{2835} + \frac{4x^3}{315} + \frac{2x^2}{15} + \frac{2x}{3} + 1 \right) \\ + c_2\sqrt{x} \left(\frac{4x^5}{14175} + \frac{2x^4}{315} + \frac{4x^3}{45} + \frac{2x^2}{3} + 2x + 1 \right)$$

14.19 problem 19

Internal problem ID [1310]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 19.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2y'' + 9y'x - (3x + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
dsolve(9*x^2*diff(y(x),x$2)+9*x*diff(y(x),x)-(1+3*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{2}{3}} \left(1 + \frac{1}{5}x + \frac{1}{80}x^2 + \frac{1}{2640}x^3 + \frac{1}{147840}x^4 + \frac{1}{12566400}x^5 + O(x^6) \right) + c_1 \left(1 + x + \frac{1}{8}x^2 + \frac{1}{168}x^3 + \frac{1}{6720}x^4 + \frac{1}{436800}x^5 \right)}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 86

```
AsymptoticDSolveValue[9*x^2*y''[x]+9*x*y'[x]-(1+3*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt[3]{x} \left(\frac{x^5}{12566400} + \frac{x^4}{147840} + \frac{x^3}{2640} + \frac{x^2}{80} + \frac{x}{5} + 1 \right) + \frac{c_2 \left(\frac{x^5}{436800} + \frac{x^4}{6720} + \frac{x^3}{168} + \frac{x^2}{8} + x + 1 \right)}{\sqrt[3]{x}}$$

14.20 problem 20

Internal problem ID [1311]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 20.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3x^2y'' + x(x+1)y' - (3x+1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
Order:=6;
```

```
dsolve(3*x^2*diff(y(x),x$2)+x*(1+x)*diff(y(x),x)-(1+3*x)*y(x)=0,y(x),type='series',x=0);
```

$y(x)$

$$= \frac{c_2 x^{\frac{4}{3}} \left(1 + \frac{2}{7}x + \frac{1}{70}x^2 + O(x^6)\right) + c_1 \left(1 - \frac{10}{3}x - \frac{35}{18}x^2 - \frac{14}{81}x^3 - \frac{7}{3888}x^4 + \frac{7}{320760}x^5 + O(x^6)\right)}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 65

```
AsymptoticDSolveValue[3*x^2*y''[x]+x*(1+x)*y'[x]-(1+3*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 x \left(\frac{x^2}{70} + \frac{2x}{7} + 1 \right) + \frac{c_2 \left(\frac{7x^5}{320760} - \frac{7x^4}{3888} - \frac{14x^3}{81} - \frac{35x^2}{18} - \frac{10x}{3} + 1 \right)}{\sqrt[3]{x}}$$

14.21 problem 21

Internal problem ID [1312]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 21.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x+3)y'' + x(5x+1)y' + (x+1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

Order:=6;

```
dsolve(2*x^2*(3+x)*diff(y(x),x$2)+x*(1+5*x)*diff(y(x),x)+(1+x)*y(x)=0,y(x),type='series',x=0
```

$$y(x) = c_1 x^{\frac{1}{3}} \left(1 - \frac{4}{9}x + \frac{14}{81}x^2 - \frac{140}{2187}x^3 + \frac{455}{19683}x^4 - \frac{1456}{177147}x^5 + O(x^6) \right) \\ + c_2 \sqrt{x} \left(1 - \frac{3}{7}x + \frac{15}{91}x^2 - \frac{15}{247}x^3 + \frac{27}{1235}x^4 - \frac{297}{38285}x^5 + O(x^6) \right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 90

```
AsymptoticDSolveValue[2*x^2*(3+x)*y'[x]+x*(1+5*x)*y'[x]+(1+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt{x} \left(-\frac{297x^5}{38285} + \frac{27x^4}{1235} - \frac{15x^3}{247} + \frac{15x^2}{91} - \frac{3x}{7} + 1 \right) \\ + c_2 \sqrt[3]{x} \left(-\frac{1456x^5}{177147} + \frac{455x^4}{19683} - \frac{140x^3}{2187} + \frac{14x^2}{81} - \frac{4x}{9} + 1 \right)$$

14.22 problem 22

Internal problem ID [1313]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 22.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x+4)y'' - x(1-3x)y' + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

Order:=6;

```
dsolve(x^2*(4+x)*diff(y(x),x$2)-x*(1-3*x)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{1}{4}} \left(1 - \frac{9}{16}x + \frac{117}{512}x^2 - \frac{663}{8192}x^3 + \frac{13923}{524288}x^4 - \frac{69615}{8388608}x^5 + O(x^6) \right) \\ + c_2 x \left(1 - \frac{3}{7}x + \frac{12}{77}x^2 - \frac{4}{77}x^3 + \frac{24}{1463}x^4 - \frac{24}{4807}x^5 + O(x^6) \right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 86

```
AsymptoticDSolveValue[x^2*(4+x)*y'[x]-x*(1-3*x)*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 x \left(-\frac{24x^5}{4807} + \frac{24x^4}{1463} - \frac{4x^3}{77} + \frac{12x^2}{77} - \frac{3x}{7} + 1 \right) \\ + c_2 \sqrt[4]{x} \left(-\frac{69615x^5}{8388608} + \frac{13923x^4}{524288} - \frac{663x^3}{8192} + \frac{117x^2}{512} - \frac{9x}{16} + 1 \right)$$

14.23 problem 23

Internal problem ID [1314]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 23.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2y'' + 5y'x + (x + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 \left(1 - x + \frac{1}{6}x^2 - \frac{1}{90}x^3 + \frac{1}{2520}x^4 - \frac{1}{113400}x^5 + O(x^6)\right) \sqrt{x} + c_2 \left(1 - \frac{1}{3}x + \frac{1}{30}x^2 - \frac{1}{630}x^3 + \frac{1}{22680}x^4 - \frac{1}{124740}x^5 + O(x^6)\right)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 86

```
AsymptoticDSolveValue[2*x^2*y''[x]+5*x*y'[x]+(1+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{c_1 \left(-\frac{x^5}{1247400} + \frac{x^4}{22680} - \frac{x^3}{630} + \frac{x^2}{30} - \frac{x}{3} + 1\right)}{\sqrt{x}} + \frac{c_2 \left(-\frac{x^5}{113400} + \frac{x^4}{2520} - \frac{x^3}{90} + \frac{x^2}{6} - x + 1\right)}{x}$$

14.24 problem 24

Internal problem ID [1315]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 24.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x^2(4x + 3)y'' + x(5 + 18x)y' - (1 - 12x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

Order:=6;

```
dsolve(x^2*(3+4*x)*diff(y(x),x$2)+x*(5+18*x)*diff(y(x),x)-(1-12*x)*y(x)=0,y(x),type='series'
```

$y(x)$

$$= \frac{c_2 x^{\frac{4}{3}} \left(1 - \frac{22}{9}x + \frac{374}{81}x^2 - \frac{17204}{2187}x^3 + \frac{249458}{19683}x^4 - \frac{3492412}{177147}x^5 + O(x^6) \right) + c_1 (1 + 2x - 6x^2 + 12x^3 - 21x^4 + \frac{37}{11}x^5)}{x}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 80

```
AsymptoticDSolveValue[x^2*(3+4*x)*y'[x]+x*(5+18*x)*y'[x]-(1-12*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt[3]{x} \left(-\frac{3492412x^5}{177147} + \frac{249458x^4}{19683} - \frac{17204x^3}{2187} + \frac{374x^2}{81} - \frac{22x}{9} + 1 \right) + \frac{c_2 \left(\frac{378x^5}{11} - 21x^4 + 12x^3 - 6x^2 + 2x + 1 \right)}{x}$$

14.25 problem 25

Internal problem ID [1316]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 25.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$6x^2y'' + x(10 - x)y' - (2 + x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
Order:=6;
```

```
dsolve(6*x^2*diff(y(x),x$2)+x*(10-x)*diff(y(x),x)-(2+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{4}{3}} \left(1 + \frac{2}{21}x + \frac{1}{180}x^2 + \frac{1}{4212}x^3 + \frac{1}{124416}x^4 + \frac{1}{4432320}x^5 + O(x^6) \right) + c_1(1 + O(x^6))}{x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 53

```
AsymptoticDSolveValue[6*x^2*y''[x]+x*(10-x)*y'[x]-(2+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt[3]{x} \left(\frac{x^5}{4432320} + \frac{x^4}{124416} + \frac{x^3}{4212} + \frac{x^2}{180} + \frac{2x}{21} + 1 \right) + \frac{c_2}{x}$$

14.26 problem 28

Internal problem ID [1317]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 28.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x+8)y'' + x(3x+2)y' + (x+1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

`Order:=6;`

`dsolve(x^2*(8+x)*diff(y(x),x$2)+x*(2+3*x)*diff(y(x),x)+(1+x)*y(x)=0,y(x),type='series',x=0);`

$$y(x) = c_1 x^{\frac{1}{4}} \left(1 - \frac{25}{96}x + \frac{675}{14336}x^2 - \frac{38025}{5046272}x^3 + \frac{732615}{645922816}x^4 - \frac{9230949}{56103010304}x^5 + O(x^6) \right) \\ + c_2 \sqrt{x} \left(1 - \frac{9}{40}x + \frac{5}{128}x^2 - \frac{245}{39936}x^3 + \frac{6615}{7241728}x^4 - \frac{7623}{57933824}x^5 + O(x^6) \right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 90

```
AsymptoticDSolveValue[x^2*(8+x)*y'[x]+x*(2+3*x)*y'[x]+(1+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt{x} \left(-\frac{7623x^5}{57933824} + \frac{6615x^4}{7241728} - \frac{245x^3}{39936} + \frac{5x^2}{128} - \frac{9x}{40} + 1 \right) \\ + c_2 \sqrt[4]{x} \left(-\frac{9230949x^5}{56103010304} + \frac{732615x^4}{645922816} - \frac{38025x^3}{5046272} + \frac{675x^2}{14336} - \frac{25x}{96} + 1 \right)$$

14.27 problem 29

Internal problem ID [1318]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 29.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(4x + 3)y'' + x(4x + 11)y' - (4x + 3)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 41

```
Order:=6;
```

```
dsolve(x^2*(3+4*x)*diff(y(x),x$2)+x*(11+4*x)*diff(y(x),x)-(3+4*x)*y(x)=0,y(x),type='series',
```

$$y(x) = \frac{c_2 x^{\frac{10}{3}} \left(1 + \frac{32}{117}x - \frac{28}{1053}x^2 + \frac{4480}{540189}x^3 - \frac{15680}{4113747}x^4 + \frac{401408}{185118615}x^5 + O(x^6)\right) + c_1 \left(1 + \frac{32}{7}x + \frac{48}{7}x^2 + O(x^6)\right)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 67

```
AsymptoticDSolveValue[x^2*(3+4*x)*y'[x]+x*(11+4*x)*y'[x]-(3+4*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{c_2 \left(\frac{48x^2}{7} + \frac{32x}{7} + 1\right)}{x^3} + c_1 \sqrt[3]{x} \left(\frac{401408x^5}{185118615} - \frac{15680x^4}{4113747} + \frac{4480x^3}{540189} - \frac{28x^2}{1053} + \frac{32x}{117} + 1\right)$$

14.28 problem 30

Internal problem ID [1319]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 30.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(3x + 2)y'' + x(11x + 4)y' - (1 - x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
```

```
dsolve(2*x^2*(2+3*x)*diff(y(x),x$2)+x*(4+11*x)*diff(y(x),x)-(1-x)*y(x)=0,y(x),type='series',
```

$y(x)$

$$= \frac{c_1 x \left(1 - \frac{5}{8}x + \frac{55}{96}x^2 - \frac{935}{1536}x^3 + \frac{4301}{6144}x^4 - \frac{124729}{147456}x^5 + O(x^6)\right) + c_2 \left(1 - \frac{5}{4}x + \frac{25}{32}x^2 - \frac{275}{384}x^3 + \frac{4675}{6144}x^4 - \frac{21505}{24576}x^5 + O(x^6)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 94

```
AsymptoticDSolveValue[2*x^2*(2+3*x)*y'[x]+x*(4+11*x)*y'[x]-(1-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(-\frac{935x^{7/2}}{6144} + \frac{55x^{5/2}}{384} - \frac{5x^{3/2}}{32} + \frac{\sqrt{x}}{4} + \frac{1}{\sqrt{x}} \right) + c_2 \left(\frac{4301x^{9/2}}{6144} - \frac{935x^{7/2}}{1536} + \frac{55x^{5/2}}{96} - \frac{5x^{3/2}}{8} + \sqrt{x} \right)$$

14.29 problem 31

Internal problem ID [1320]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 31.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(2+x)y'' + 5x(1-x)y' - (-8x+2)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
```

```
dsolve(x^2*(2+x)*diff(y(x),x$2)+5*x*(1-x)*diff(y(x),x)-(2-8*x)*y(x)=0,y(x),type='series',x=0)
```

$y(x)$

$$= \frac{c_2 x^{\frac{5}{2}} \left(1 - \frac{3}{4}x + \frac{5}{96}x^2 + \frac{5}{4224}x^3 + \frac{5}{292864}x^4 - \frac{1}{3514368}x^5 + O(x^6)\right) + c_1(1 + 8x + 60x^2 - 160x^3 + 40x^4 + O(x^5))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 73

```
AsymptoticDSolveValue[x^2*(2+x)*y'[x]+5*x*(1-x)*y'[x]-(2-8*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{c_2(40x^4 - 160x^3 + 60x^2 + 8x + 1)}{x^2} + c_1\sqrt{x} \left(-\frac{x^5}{3514368} + \frac{5x^4}{292864} + \frac{5x^3}{4224} + \frac{5x^2}{96} - \frac{3x}{4} + 1 \right)$$

14.30 problem 32

Internal problem ID [1321]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 32.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x+6)y'' + x(4x+11)y' + (1+2x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
```

```
dsolve(x^2*(6+x)*diff(y(x),x$2)+x*(11+4*x)*diff(y(x),x)+(1+2*x)*y(x)=0,y(x),type='series',x=
```

$$y(x) = \frac{\left(1 - \frac{10}{63}x + \frac{200}{7371}x^2 - \frac{17600}{3781323}x^3 + \frac{3872}{4861701}x^4 - \frac{921536}{6782072895}x^5 + O(x^6)\right) c_2 x^{\frac{1}{6}} + \left(1 - \frac{3}{20}x + \frac{9}{352}x^2 - \frac{105}{23936}x^3 + \dots\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 90

```
AsymptoticDSolveValue[x^2*(6+x)*y'[x]+x*(11+4*x)*y'[x]+(1+2*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{c_1 \left(-\frac{921536x^5}{6782072895} + \frac{3872x^4}{4861701} - \frac{17600x^3}{3781323} + \frac{200x^2}{7371} - \frac{10x}{63} + 1 \right)}{\sqrt[3]{x}} + \frac{c_2 \left(-\frac{11907x^5}{92889088} + \frac{6615x^4}{8808448} - \frac{105x^3}{23936} + \frac{9x^2}{352} - \frac{3x}{20} + 1 \right)}{\sqrt{x}}$$

14.31 problem 33

Internal problem ID [1322]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 33.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$8x^2y'' + x(x^2 + 2)y' + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
Order:=6;  
dsolve(8*x^2*dif(y(x),x$2)+x*(2+x^2)*dif(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{1}{4}} \left(1 - \frac{1}{112} x^2 + \frac{3}{17920} x^4 + O(x^6) \right) + c_2 \sqrt{x} \left(1 - \frac{1}{72} x^2 + \frac{5}{19584} x^4 + O(x^6) \right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 52

```
AsymptoticDSolveValue[8*x^2*y'[x]+x*(2+x^2)*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt{x} \left(\frac{5x^4}{19584} - \frac{x^2}{72} + 1 \right) + c_2 \sqrt[4]{x} \left(\frac{3x^4}{17920} - \frac{x^2}{112} + 1 \right)$$

14.32 problem 34

Internal problem ID [1323]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 34.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$8x^2(-x^2 + 1)y'' + 2x(-13x^2 + 1)y' + (-9x^2 + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

Order:=6;

```
dsolve(8*x^2*(1-x^2)*diff(y(x),x$2)+2*x*(1-13*x^2)*diff(y(x),x)+(1-9*x^2)*y(x)=0,y(x),type='
```

$$y(x) = c_1 x^{\frac{1}{4}} \left(1 + \frac{1}{2}x^2 + \frac{3}{8}x^4 + O(x^6) \right) + c_2 \sqrt{x} \left(1 + \frac{5}{9}x^2 + \frac{65}{153}x^4 + O(x^6) \right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 52

```
AsymptoticDSolveValue[8*x^2*(1-x^2)*y''[x]+2*x*(1-13*x^2)*y'[x]+(1-9*x^2)*y[x]==0,y[x],{x,0,
```

$$y(x) \rightarrow c_1 \sqrt{x} \left(\frac{65x^4}{153} + \frac{5x^2}{9} + 1 \right) + c_2 \sqrt[4]{x} \left(\frac{3x^4}{8} + \frac{x^2}{2} + 1 \right)$$

14.33 problem 35

Internal problem ID [1324]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 35.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2 + 1)y'' - 2x(-x^2 + 2)y' + 4y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
Order:=6;
```

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-2*x*(2-x^2)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^4 (1 - 2x^2 + 3x^4 + O(x^6)) + c_2 x (12 + 12x^2 - 36x^4 + O(x^6))$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 34

```
AsymptoticDSolveValue[x^2*(1+x^2)*y''[x]-2*x*(2-x^2)*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 (-3x^5 + x^3 + x) + c_2 (3x^8 - 2x^6 + x^4)$$

14.34 problem 36

Internal problem ID [1325]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 36.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x(x^2 + 3)y'' + (-x^2 + 2)y' - 8yx = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 32

```
Order:=6;
```

```
dsolve(x*(3+x^2)*diff(y(x),x$2)+(2-x^2)*diff(y(x),x)-8*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{1}{3}} \left(1 + \frac{11}{18}x^2 + \frac{55}{648}x^4 + O(x^6) \right) + c_2 \left(1 + \frac{4}{5}x^2 + \frac{8}{55}x^4 + O(x^6) \right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 47

```
AsymptoticDSolveValue[x*(3+x^2)*y'[x]+(2-x^2)*y'[x]-8*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt[3]{x} \left(\frac{55x^4}{648} + \frac{11x^2}{18} + 1 \right) + c_2 \left(\frac{8x^4}{55} + \frac{4x^2}{5} + 1 \right)$$

14.35 problem 37

Internal problem ID [1326]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 37.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(-x^2 + 1)y'' + x(-19x^2 + 7)y' - (14x^2 + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
Order:=6;
```

```
dsolve(4*x^2*(1-x^2)*diff(y(x),x$2)+x*(7-19*x^2)*diff(y(x),x)-(1+14*x^2)*y(x)=0,y(x),type='s
```

$$y(x) = \frac{c_2 x^{\frac{5}{4}} \left(1 + \frac{9}{13}x^2 + \frac{51}{91}x^4 + O(x^6)\right) + c_1 \left(1 + \frac{1}{2}x^2 + \frac{3}{8}x^4 + O(x^6)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 50

```
AsymptoticDSolveValue[4*x^2*(1-x^2)*y'[x]+x*(7-19*x^2)*y'[x]-(1+14*x^2)*y[x]==0,y[x],{x,0,5
```

$$y(x) \rightarrow c_1 \sqrt[4]{x} \left(\frac{51x^4}{91} + \frac{9x^2}{13} + 1 \right) + \frac{c_2 \left(\frac{3x^4}{8} + \frac{x^2}{2} + 1 \right)}{x}$$

14.36 problem 38

Internal problem ID [1327]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 38.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3x^2(-x^2 + 2)y'' + x(-11x^2 + 1)y' + (-5x^2 + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
Order:=6;
```

```
dsolve(3*x^2*(2-x^2)*diff(y(x),x$2)+x*(1-11*x^2)*diff(y(x),x)+(1-5*x^2)*y(x)=0,y(x),type='series')
```

$$y(x) = c_1 x^{\frac{1}{3}} \left(1 + \frac{4}{11} x^2 + \frac{40}{253} x^4 + O(x^6) \right) + c_2 \sqrt{x} \left(1 + \frac{3}{8} x^2 + \frac{21}{128} x^4 + O(x^6) \right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 52

```
AsymptoticDSolveValue[3*x^2*(2-x^2)*y''[x]+x*(1-11*x^2)*y'[x]+(1-5*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt{x} \left(\frac{21x^4}{128} + \frac{3x^2}{8} + 1 \right) + c_2 \sqrt[3]{x} \left(\frac{40x^4}{253} + \frac{4x^2}{11} + 1 \right)$$

14.37 problem 39

Internal problem ID [1328]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 39.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x^2 + 2)y'' - x(-7x^2 + 12)y' + (3x^2 + 7)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

Order:=6;

```
dsolve(2*x^2*(2+x^2)*diff(y(x),x$2)-x*(12-7*x^2)*diff(y(x),x)+(7+3*x^2)*y(x)=0,y(x),type='series')
```

$$y(x) = \sqrt{x} \left(x^3 \left(1 - \frac{9}{8}x^2 + \frac{117}{128}x^4 + O(x^6) \right) c_1 + \left(12 + 9x^2 - \frac{63}{4}x^4 + O(x^6) \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 58

```
AsymptoticDSolveValue[2*x^2*(2+x^2)*y''[x]-x*(12-7*x^2)*y'[x]+(7+3*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(-\frac{21x^{9/2}}{16} + \frac{3x^{5/2}}{4} + \sqrt{x} \right) + c_2 \left(\frac{117x^{15/2}}{128} - \frac{9x^{11/2}}{8} + x^{7/2} \right)$$

14.38 problem 40

Internal problem ID [1329]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 40.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x^2 + 2)y'' + x(7x^2 + 4)y' - (-3x^2 + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

Order:=6;

```
dsolve(2*x^2*(2+x^2)*diff(y(x),x$2)+x*(4+7*x^2)*diff(y(x),x)-(1-3*x^2)*y(x)=0,y(x),type='series')
```

$$y(x) = \frac{c_1 x \left(1 - \frac{1}{4}x^2 + \frac{7}{80}x^4 + O(x^6)\right) + c_2 \left(1 - \frac{1}{8}x^2 + \frac{5}{128}x^4 + O(x^6)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 58

```
AsymptoticDSolveValue[2*x^2*(2+x^2)*y'[x]+x*(4+7*x^2)*y'[x]-(1-3*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{5x^{7/2}}{128} - \frac{x^{3/2}}{8} + \frac{1}{\sqrt{x}} \right) + c_2 \left(\frac{7x^{9/2}}{80} - \frac{x^{5/2}}{4} + \sqrt{x} \right)$$

14.39 problem 41

Internal problem ID [1330]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 41.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(2x^2 + 1)y'' + 5x(6x^2 + 1)y' - (-40x^2 + 2)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

Order:=6;

```
dsolve(2*x^2*(1+2*x^2)*diff(y(x),x$2)+5*x*(1+6*x^2)*diff(y(x),x)-(2-40*x^2)*y(x)=0,y(x),type
```

$$y(x) = \frac{c_2 x^{\frac{5}{2}} \left(1 - 3x^2 + \frac{15}{2}x^4 + O(x^6)\right) + c_1 \left(1 + 2x^2 - \frac{20}{3}x^4 + O(x^6)\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 46

```
AsymptoticDSolveValue[2*x^2*(1+2*x^2)*y'[x]+5*x*(1+6*x^2)*y'[x]-(2-40*x^2)*y[x]==0,y[x],{x,
```

$$y(x) \rightarrow c_1 \sqrt{x} \left(\frac{15x^4}{2} - 3x^2 + 1 \right) + \frac{c_2 \left(-\frac{20x^4}{3} + 2x^2 + 1 \right)}{x^2}$$

14.40 problem 42

Internal problem ID [1331]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 42.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3x^2(x^2 + 1)y'' + 5xy'(x^2 + 1) - (-5x^2 + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
Order:=6;
```

```
dsolve(3*x^2*(1+x^2)*diff(y(x),x$2)+5*x*(1+x^2)*diff(y(x),x)-(1-5*x^2)*y(x)=0,y(x),type='ser
```

$$y(x) = \frac{c_2 x^{\frac{4}{3}} \left(1 - \frac{3}{10}x^2 + \frac{39}{320}x^4 + O(x^6)\right) + c_1 \left(1 - \frac{3}{2}x^2 + \frac{15}{32}x^4 + O(x^6)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 50

```
AsymptoticDSolveValue[3*x^2*(1+x^2)*y'[x]+5*x*(1+x^2)*y'[x]-(1-5*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt[3]{x} \left(\frac{39x^4}{320} - \frac{3x^2}{10} + 1 \right) + \frac{c_2 \left(\frac{15x^4}{32} - \frac{3x^2}{2} + 1 \right)}{x}$$

14.41 problem 43

Internal problem ID [1332]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 43.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x(x^2 + 1)y'' + (7x^2 + 4)y' + 8yx = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
Order:=6;
```

```
dsolve(x*(1+x^2)*diff(y(x),x$2)+(4+7*x^2)*diff(y(x),x)+8*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 \left(1 - \frac{4}{5}x^2 + \frac{24}{35}x^4 + O(x^6) \right) + \frac{c_2(12 - 6x^2 + \frac{9}{2}x^4 + O(x^6))}{x^3}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 42

```
AsymptoticDSolveValue[x*(1+x^2)*y''[x]+(4+7*x^2)*y'[x]+8*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{1}{x^3} + \frac{3x}{8} - \frac{1}{2x} \right) + c_2 \left(\frac{24x^4}{35} - \frac{4x^2}{5} + 1 \right)$$

14.42 problem 44

Internal problem ID [1333]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 44.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2 + 2)y'' + x(x^2 + 3)y' - y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
Order:=6;
```

```
dsolve(x^2*(2+x^2)*diff(y(x),x$2)+x*(3+x^2)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{3}{2}} \left(1 - \frac{1}{56}x^2 + \frac{25}{9856}x^4 + O(x^6)\right) + c_1 \left(1 - \frac{1}{2}x^2 + \frac{1}{40}x^4 + O(x^6)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 50

```
AsymptoticDSolveValue[x^2*(2+x^2)*y'[x]+x*(3+x^2)*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt{x} \left(\frac{25x^4}{9856} - \frac{x^2}{56} + 1 \right) + \frac{c_2 \left(\frac{x^4}{40} - \frac{x^2}{2} + 1 \right)}{x}$$

14.43 problem 45

Internal problem ID [1334]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 45.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x^2 + 1)y'' + x(8x^2 + 3)y' - (-4x^2 + 3)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

Order:=6;

```
dsolve(2*x^2*(1+x^2)*diff(y(x),x$2)+x*(3+8*x^2)*diff(y(x),x)-(3-4*x^2)*y(x)=0,y(x),type='ser
```

$$y(x) = \frac{c_2 x^{\frac{5}{2}} \left(1 - \frac{2}{3}x^2 + \frac{20}{39}x^4 + O(x^6)\right) + c_1 \left(1 - \frac{1}{4}x^2 + \frac{5}{32}x^4 + O(x^6)\right)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 48

```
AsymptoticDSolveValue[2*x^2*(1+x^2)*y'[x]+x*(3+8*x^2)*y'[x]-(3-4*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 x \left(\frac{20x^4}{39} - \frac{2x^2}{3} + 1 \right) + \frac{c_2 \left(\frac{5x^4}{32} - \frac{x^2}{4} + 1 \right)}{x^{3/2}}$$

14.44 problem 46

Internal problem ID [1335]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 46.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2y'' + 3x(x^2 + 3)y' - (-5x^2 + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

Order:=6;

```
dsolve(9*x^2*dif(y(x),x$2)+3*x*(3+x^2)*dif(y(x),x)-(1-5*x^2)*y(x)=0,y(x),type='series',x=0
```

$$y(x) = \frac{c_2 x^{\frac{2}{3}} \left(1 - \frac{1}{8}x^2 + \frac{1}{112}x^4 + O(x^6)\right) + c_1 \left(1 - \frac{1}{6}x^2 + \frac{1}{72}x^4 + O(x^6)\right)}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 52

```
AsymptoticDSolveValue[9*x^2*y'[x]+3*x*(3+x^2)*y'[x]-(1-5*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt[3]{x} \left(\frac{x^4}{112} - \frac{x^2}{8} + 1 \right) + \frac{c_2 \left(\frac{x^4}{72} - \frac{x^2}{6} + 1 \right)}{\sqrt[3]{x}}$$

14.45 problem 47

Internal problem ID [1336]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 47.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$6x^2y'' + x(6x^2 + 1)y' + (9x^2 + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
Order:=6;
```

```
dsolve(6*x^2*dif(y(x),x$2)+x*(1+6*x^2)*dif(y(x),x)+(1+9*x^2)*y(x)=0,y(x),type='series',x=0)
```

$$y(x) = c_1 x^{\frac{1}{3}} \left(1 - \frac{1}{2}x^2 + \frac{1}{8}x^4 + O(x^6) \right) + c_2 \sqrt{x} \left(1 - \frac{6}{13}x^2 + \frac{36}{325}x^4 + O(x^6) \right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 52

```
AsymptoticDSolveValue[6*x^2*y''[x]+x*(1+6*x^2)*y'[x]+(1+9*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt{x} \left(\frac{36x^4}{325} - \frac{6x^2}{13} + 1 \right) + c_2 \sqrt[3]{x} \left(\frac{x^4}{8} - \frac{x^2}{2} + 1 \right)$$

14.46 problem 48

Internal problem ID [1337]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 48.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x^2(x^2 + 8)y'' + 7x(x^2 + 2)y' - (-9x^2 + 2)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

Order:=6;

```
dsolve(x^2*(8+x^2)*diff(y(x),x$2)+7*x*(2+x^2)*diff(y(x),x)-(2-9*x^2)*y(x)=0,y(x),type='series')
```

$$y(x) = \frac{c_2 x^{\frac{5}{4}} \left(1 - \frac{13}{64}x^2 + \frac{273}{8192}x^4 + O(x^6)\right) + c_1 \left(1 - \frac{1}{3}x^2 + \frac{2}{33}x^4 + O(x^6)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 50

```
AsymptoticDSolveValue[x^2*(8+x^2)*y'[x]+7*x*(2+x^2)*y'[x]-(2-9*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt[4]{x} \left(\frac{273x^4}{8192} - \frac{13x^2}{64} + 1 \right) + \frac{c_2 \left(\frac{2x^4}{33} - \frac{x^2}{3} + 1 \right)}{x}$$

14.47 problem 49

Internal problem ID [1338]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 49.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2(x^2 + 1)y'' + 3x(13x^2 + 3)y' - (-25x^2 + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

Order:=6;

```
dsolve(9*x^2*(1+x^2)*diff(y(x),x$2)+3*x*(3+13*x^2)*diff(y(x),x)-(1-25*x^2)*y(x)=0,y(x),type=
```

$$y(x) = \frac{c_2 x^{\frac{2}{3}} \left(1 - \frac{3}{4}x^2 + \frac{9}{14}x^4 + O(x^6)\right) + c_1 \left(1 - \frac{2}{3}x^2 + \frac{5}{9}x^4 + O(x^6)\right)}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 52

```
AsymptoticDSolveValue[9*x^2*(1+x^2)*y'[x]+3*x*(3+13*x^2)*y'[x]-(1-25*x^2)*y[x]==0,y[x],{x,0
```

$$y(x) \rightarrow c_1 \sqrt[3]{x} \left(\frac{9x^4}{14} - \frac{3x^2}{4} + 1 \right) + \frac{c_2 \left(\frac{5x^4}{9} - \frac{2x^2}{3} + 1 \right)}{\sqrt[3]{x}}$$

14.48 problem 50

Internal problem ID [1339]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 50.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x^2 + 1)y'' + 4x(6x^2 + 1)y' - (-25x^2 + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

Order:=6;

```
dsolve(4*x^2*(1+x^2)*diff(y(x),x$2)+4*x*(1+6*x^2)*diff(y(x),x)-(1-25*x^2)*y(x)=0,y(x),type='
```

$$y(x) = \frac{c_1x\left(1 - \frac{3}{2}x^2 + \frac{15}{8}x^4 + O(x^6)\right) + c_2\left(1 - 2x^2 + \frac{8}{3}x^4 + O(x^6)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 56

```
AsymptoticDSolveValue[4*x^2*(1+x^2)*y'[x]+4*x*(1+6*x^2)*y'[x]-(1-25*x^2)*y[x]==0,y[x],{x,0,
```

$$y(x) \rightarrow c_1\left(\frac{8x^{7/2}}{3} - 2x^{3/2} + \frac{1}{\sqrt{x}}\right) + c_2\left(\frac{15x^{9/2}}{8} - \frac{3x^{5/2}}{2} + \sqrt{x}\right)$$

14.49 problem 51

Internal problem ID [1340]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 51.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$8x^2(2x^2 + 1)y'' + 2x(34x^2 + 5)y' - (-30x^2 + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

Order:=6;

```
dsolve(8*x^2*(1+2*x^2)*diff(y(x),x$2)+2*x*(5+34*x^2)*diff(y(x),x)-(1-30*x^2)*y(x)=0,y(x),typ
```

$$y(x) = \frac{c_2 x^{\frac{3}{4}} \left(1 - x^2 + \frac{3}{2}x^4 + O(x^6)\right) + c_1 \left(1 - \frac{2}{5}x^2 + \frac{36}{65}x^4 + O(x^6)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 50

```
AsymptoticDSolveValue[8*x^2*(1+2*x^2)*y'[x]+2*x*(5+34*x^2)*y'[x]-(1-30*x^2)*y[x]==0,y[x],{x
```

$$y(x) \rightarrow c_1 \sqrt[4]{x} \left(\frac{3x^4}{2} - x^2 + 1 \right) + \frac{c_2 \left(\frac{36x^4}{65} - \frac{2x^2}{5} + 1 \right)}{\sqrt{x}}$$

14.50 problem 61

Internal problem ID [1341]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 61.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x+1)y'' - x(1-3x)y' + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
```

```
dsolve(2*x^2*(1+x)*diff(y(x),x$2)-x*(1-3*x)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (-x^5 + x^4 - x^3 + x^2 - x + 1)(c_1\sqrt{x} + c_2x) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 58

```
AsymptoticDSolveValue[2*x^2*(1+x)*y''[x]-x*(1-3*x)*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1x(-x^5 + x^4 - x^3 + x^2 - x + 1) + c_2\sqrt{x}(-x^5 + x^4 - x^3 + x^2 - x + 1)$$

14.51 problem 62

Internal problem ID [1342]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 62.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$6x^2(2x^2 + 1)y'' + x(50x^2 + 1)y' + (30x^2 + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
Order:=6;
```

```
dsolve(6*x^2*(1+2*x^2)*diff(y(x),x$2)+x*(1+50*x^2)*diff(y(x),x)+(1+30*x^2)*y(x)=0,y(x),type=
```

$$y(x) = (4x^4 - 2x^2 + 1)x^{\frac{1}{3}}(c_2x^{\frac{1}{6}} + c_1) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 44

```
AsymptoticDSolveValue[6*x^2*(1+2*x^2)*y'[x]+x*(1+50*x^2)*y'[x]+(1+30*x^2)*y[x]==0,y[x],{x,0
```

$$y(x) \rightarrow c_1\sqrt{x}(4x^4 - 2x^2 + 1) + c_2\sqrt[3]{x}(4x^4 - 2x^2 + 1)$$

14.52 problem 63

Internal problem ID [1343]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 63.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$28x^2(1 - 3x)y'' - 7x(5 + 9x)y' + 7(2 + 9x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 47

```
Order:=6;
```

```
dsolve(28*x^2*(1-3*x)*diff(y(x),x$2)-7*x*(5+9*x)*diff(y(x),x)+7*(2+9*x)*y(x)=0,y(x),type='se
```

$$y(x) = (243x^5 + 81x^4 + 27x^3 + 9x^2 + 3x + 1) \left(c_1 x^{\frac{1}{4}} + x^2 c_2 \right) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 68

```
AsymptoticDSolveValue[28*x^2*(1-3*x)*y'[x]-7*x*(5+9*x)*y'[x]+7*(2+9*x)*y[x]==0,y[x],{x,0,5}
```

$$y(x) \rightarrow c_1 (243x^5 + 81x^4 + 27x^3 + 9x^2 + 3x + 1) x^2 + c_2 (243x^5 + 81x^4 + 27x^3 + 9x^2 + 3x + 1) \sqrt[4]{x}$$

14.53 problem 64

Internal problem ID [1344]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 64.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2(5+x)y'' + 9x(5+9x)y' - (5-8x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
```

```
dsolve(9*x^2*(5+x)*diff(y(x),x$2)+9*x*(5+9*x)*diff(y(x),x)-(5-8*x)*y(x)=0,y(x),type='series')
```

$y(x)$

$$= \frac{c_2 x^{\frac{2}{3}} \left(1 - \frac{11}{25}x + \frac{11}{50}x^2 - \frac{1}{10}x^3 + \frac{29}{700}x^4 - \frac{4727}{297500}x^5 + O(x^6) \right) + c_1 \left(1 + x - \frac{1}{2}x^2 + \frac{17}{70}x^3 - \frac{187}{1750}x^4 + \frac{24497}{568750}x^5 \right)}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 86

```
AsymptoticDSolveValue[9*x^2*(5+x)*y'[x]+9*x*(5+9*x)*y'[x]-(5-8*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt[3]{x} \left(-\frac{4727x^5}{297500} + \frac{29x^4}{700} - \frac{x^3}{10} + \frac{11x^2}{50} - \frac{11x}{25} + 1 \right) + \frac{c_2 \left(\frac{24497x^5}{568750} - \frac{187x^4}{1750} + \frac{17x^3}{70} - \frac{x^2}{2} + x + 1 \right)}{\sqrt[3]{x}}$$

14.54 problem 65

Internal problem ID [1345]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 65.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$8x^2(-x^2 + 2)y'' + 2x(-21x^2 + 10)y' - (35x^2 + 2)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

Order:=6;

```
dsolve(8*x^2*(2-x^2)*diff(y(x),x$2)+2*x*(10-21*x^2)*diff(y(x),x)-(2+35*x^2)*y(x)=0,y(x),type
```

$$y(x) = \frac{\left(1 + \frac{1}{2}x^2 + \frac{1}{4}x^4\right) \left(x^{\frac{3}{4}}c_2 + c_1\right)}{\sqrt{x}} + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 52

```
AsymptoticDSolveValue[8*x^2*(2-x^2)*y'[x]+2*x*(10-21*x^2)*y'[x]-(2+35*x^2)*y[x]==0,y[x],{x,
```

$$y(x) \rightarrow c_1 \sqrt[4]{x} \left(\frac{x^4}{4} + \frac{x^2}{2} + 1 \right) + \frac{c_2 \left(\frac{x^4}{4} + \frac{x^2}{2} + 1 \right)}{\sqrt{x}}$$

14.55 problem 66

Internal problem ID [1346]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 66.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x^2 + 3x + 1)y'' - 4x(-3x^2 - 3x + 1)y' + 3(x^2 - x + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

Order:=6;

```
dsolve(4*x^2*(1+3*x+x^2)*diff(y(x),x$2)-4*x*(1-3*x-3*x^2)*diff(y(x),x)+3*(1-x+x^2)*y(x)=0,y(x))
```

$$y(x) = \sqrt{x} \left(x(1 - 3x + 8x^2 - 21x^3 + 55x^4 - 144x^5 + O(x^6)) c_1 \right. \\ \left. + (1 - 6x + 17x^2 - 45x^3 + 118x^4 - 309x^5 + O(x^6)) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 78

```
AsymptoticDSolveValue[4*x^2*(1+3*x+x^2)*y''[x]-4*x*(1-3*x-3*x^2)*y'[x]+3*(1-x+x^2)*y[x]==0,y[x],{x,0},O[x]^6]
```

$$y(x) \rightarrow c_1(55x^{9/2} - 21x^{7/2} + 8x^{5/2} - 3x^{3/2} \\ + \sqrt{x}) + c_2(55x^{11/2} - 21x^{9/2} + 8x^{7/2} - 3x^{5/2} + x^{3/2})$$

14.56 problem 67

Internal problem ID [1347]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 67.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3x^2(x+1)^2 y'' - x(-11x^2 - 10x + 1) y' + (5x^2 + 1) y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

Order:=6;

```
dsolve(3*x^2*(1+x)^2*diff(y(x),x$2)-x*(1-10*x-11*x^2)*diff(y(x),x)+(1+5*x^2)*y(x)=0,y(x),typ
```

$$y(x) = (-6x^5 + 5x^4 - 4x^3 + 3x^2 - 2x + 1) \left(c_1 x^{\frac{1}{3}} + c_2 x \right) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 66

```
AsymptoticDSolveValue[3*x^2*(1+x)^2*y'[x]-x*(1-10*x-11*x^2)*y'[x]+(1+5*x^2)*y[x]==0,y[x],{x
```

$$y(x) \rightarrow c_1 x (-6x^5 + 5x^4 - 4x^3 + 3x^2 - 2x + 1) + c_2 \sqrt[3]{x} (-6x^5 + 5x^4 - 4x^3 + 3x^2 - 2x + 1)$$

14.57 problem 68

Internal problem ID [1348]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

Problem number: 68.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x^2 + 2x + 3)y'' - x(-15x^2 - 14x + 3)y' + (7x^2 + 3)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

Order:=6;

```
dsolve(4*x^2*(3+2*x+x^2)*diff(y(x),x$2)-x*(3-14*x-15*x^2)*diff(y(x),x)+(3+7*x^2)*y(x)=0,y(x))
```

$$y(x) = \left(1 - \frac{2}{3}x + \frac{1}{9}x^2 + \frac{4}{27}x^3 - \frac{11}{81}x^4 + \frac{10}{243}x^5\right) \left(c_1x^{\frac{1}{4}} + c_2x\right) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 86

```
AsymptoticDSolveValue[4*x^2*(3+2*x+x^2)*y''[x]-x*(3-14*x-15*x^2)*y'[x]+(3+7*x^2)*y[x]==0,y[x]]
```

$$y(x) \rightarrow c_1x \left(\frac{10x^5}{243} - \frac{11x^4}{81} + \frac{4x^3}{27} + \frac{x^2}{9} - \frac{2x}{3} + 1 \right) + c_2\sqrt[4]{x} \left(\frac{10x^5}{243} - \frac{11x^4}{81} + \frac{4x^3}{27} + \frac{x^2}{9} - \frac{2x}{3} + 1 \right)$$

15 Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

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15.1 problem Example 7.6.1 page 367

Internal problem ID [1349]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: Example 7.6.1 page 367.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2 - 2x + 1)y'' - x(3 + x)y' + (4 + x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

Order:=6;

```
dsolve(x^2*(1-2*x+x^2)*diff(y(x),x$2)-x*(3+x)*diff(y(x),x)+(4+x)*y(x)=0,y(x),type='series',x
```

$$y(x) = x^2 \left((c_2 \ln(x) + c_1) \left(1 + 5x + 17x^2 + \frac{143}{3}x^3 + \frac{355}{3}x^4 + \frac{4043}{15}x^5 + O(x^6) \right) \right. \\ \left. + \left((-3)x - \frac{29}{2}x^2 - \frac{859}{18}x^3 - \frac{4693}{36}x^4 - \frac{285181}{900}x^5 + O(x^6) \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 118

```
AsymptoticDSolveValue[x^2*(1-2*x+x^2)*y'[x]-x*(3+x)*y'[x]+(4+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{4043x^5}{15} + \frac{355x^4}{3} + \frac{143x^3}{3} + 17x^2 + 5x + 1 \right) x^2$$
$$+ c_2 \left(\left(-\frac{285181x^5}{900} - \frac{4693x^4}{36} - \frac{859x^3}{18} - \frac{29x^2}{2} - 3x \right) x^2 \right.$$
$$\left. + \left(\frac{4043x^5}{15} + \frac{355x^4}{3} + \frac{143x^3}{3} + 17x^2 + 5x + 1 \right) x^2 \log(x) \right)$$

15.2 problem Example 7.6.2 page 369

Internal problem ID [1350]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: Example 7.6.2 page 369.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x+2)y'' + 5y'x^2 + (x+1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

```
Order:=6;
```

```
dsolve(2*x^2*(2+x)*diff(y(x),x$2)+5*x^2*diff(y(x),x)+(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x} \left((c_2 \ln(x) + c_1) \left(1 - \frac{3}{4}x + \frac{15}{32}x^2 - \frac{35}{128}x^3 + \frac{315}{2048}x^4 - \frac{693}{8192}x^5 + O(x^6) \right) \right. \\ \left. + \left(\frac{1}{4}x - \frac{13}{64}x^2 + \frac{101}{768}x^3 - \frac{641}{8192}x^4 + \frac{7303}{163840}x^5 + O(x^6) \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 134

```
AsymptoticDSolveValue[2*x^2*(2+x)*y'[x]+5*x^2*y'[x]+(1+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt{x} \left(-\frac{693x^5}{8192} + \frac{315x^4}{2048} - \frac{35x^3}{128} + \frac{15x^2}{32} - \frac{3x}{4} + 1 \right) \\ + c_2 \left(\sqrt{x} \left(\frac{7303x^5}{163840} - \frac{641x^4}{8192} + \frac{101x^3}{768} - \frac{13x^2}{64} + \frac{x}{4} \right) \right. \\ \left. + \sqrt{x} \left(-\frac{693x^5}{8192} + \frac{315x^4}{2048} - \frac{35x^3}{128} + \frac{15x^2}{32} - \frac{3x}{4} + 1 \right) \log(x) \right)$$

15.3 problem Example 7.6.3 page 370

Internal problem ID [1351]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: Example 7.6.3 page 370.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(-x^2 + 2)y'' - 2x(2x^2 + 1)y' + (-2x^2 + 2)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

`Order:=6;`

`dsolve(x^2*(2-x^2)*diff(y(x),x$2)-2*x*(1+2*x^2)*diff(y(x),x)+(2-2*x^2)*y(x)=0,y(x),type='ser`

$$y(x) = x \left((c_2 \ln(x) + c_1) \left(1 + \frac{3}{4}x^2 + \frac{15}{32}x^4 + O(x^6) \right) + \left(-\frac{1}{8}x^2 - \frac{13}{128}x^4 + O(x^6) \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 65

`AsymptoticDSolveValue[x^2*(2-x^2)*y'[x]-2*x*(1+2*x^2)*y'[x]+(2-2*x^2)*y[x]==0,y[x],{x,0,5}]`

$$y(x) \rightarrow c_1 x \left(\frac{15x^4}{32} + \frac{3x^2}{4} + 1 \right) + c_2 \left(x \left(-\frac{13x^4}{128} - \frac{x^2}{8} \right) + x \left(\frac{15x^4}{32} + \frac{3x^2}{4} + 1 \right) \log(x) \right)$$

15.4 problem Example 7.6.4 page 372

Internal problem ID [1352]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: Example 7.6.4 page 372.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' - x(5 - x) y' + (9 - 4x) y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

Order:=6;

```
dsolve(x^2*diff(y(x),x$2)-x*(5-x)*diff(y(x),x)+(9-4*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x^3 \left((c_2 \ln(x) + c_1) (1 + x + O(x^6)) \right. \\ \left. + \left((-3)x - \frac{1}{4}x^2 + \frac{1}{36}x^3 - \frac{1}{288}x^4 + \frac{1}{2400}x^5 + O(x^6) \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 62

```
AsymptoticDSolveValue[x^2*y''[x]-x*(5-x)*y'[x]+(9-4*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1(x+1)x^3 + c_2 \left((x+1)x^3 \log(x) + \left(\frac{x^5}{2400} - \frac{x^4}{288} + \frac{x^3}{36} - \frac{x^2}{4} - 3x \right) x^3 \right)$$

15.5 problem 1

Internal problem ID [1353]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 1.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' - x(1-x)y' + (-x^2 + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 75

```
Order:=8;
```

```
dsolve(x^2*diff(y(x),x$2)-x*(1-x)*diff(y(x),x)+(1-x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x \left((c_2 \ln(x) + c_1) \left(1 - x + \frac{3}{4}x^2 - \frac{13}{36}x^3 + \frac{79}{576}x^4 - \frac{67}{1600}x^5 + \frac{5593}{518400}x^6 - \frac{60859}{25401600}x^7 + O(x^8) \right) + \left(x - x^2 + \frac{65}{108}x^3 - \frac{895}{3456}x^4 + \frac{12547}{144000}x^5 - \frac{41729}{1728000}x^6 + \frac{10121677}{1778112000}x^7 + O(x^8) \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 154

```
AsymptoticDSolveValue[x^2*y''[x]-x*(1-x)*y'[x]+(1-x^2)*y[x]==0,y[x],{x,0,7}]
```

$$\begin{aligned} y(x) \rightarrow & c_1 x \left(-\frac{60859x^7}{25401600} + \frac{5593x^6}{518400} - \frac{67x^5}{1600} + \frac{79x^4}{576} - \frac{13x^3}{36} + \frac{3x^2}{4} - x + 1 \right) \\ & + c_2 \left(x \left(\frac{10121677x^7}{1778112000} - \frac{41729x^6}{1728000} + \frac{12547x^5}{144000} - \frac{895x^4}{3456} + \frac{65x^3}{108} - x^2 + x \right) \right. \\ & \left. + x \left(-\frac{60859x^7}{25401600} + \frac{5593x^6}{518400} - \frac{67x^5}{1600} + \frac{79x^4}{576} - \frac{13x^3}{36} + \frac{3x^2}{4} - x + 1 \right) \log(x) \right) \end{aligned}$$

15.6 problem 2

Internal problem ID [1354]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 2.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x^2(2x^2 + x + 1)y'' + x(7x^2 + 6x + 3)y' + (-3x^2 + 6x + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 81

Order:=8;

```
dsolve(x^2*(1+x+2*x^2)*diff(y(x),x$2)+x*(3+6*x+7*x^2)*diff(y(x),x)+(1+6*x-3*x^2)*y(x)=0,y(x))
```

$$y(x) = \frac{(c_2 \ln(x) + c_1) \left(1 - 2x + \frac{9}{2}x^2 - \frac{20}{3}x^3 + \frac{173}{24}x^4 - \frac{93}{20}x^5 - \frac{419}{720}x^6 + \frac{6697}{1260}x^7 + O(x^8)\right) + \left(x - \frac{15}{4}x^2 + \frac{133}{18}x^3 - \dots\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 162

AsymptoticDSolveValue[x^2*(1+x+2*x^2)*y'[x]+x*(3+6*x+7*x^2)*y'[x]+(1+6*x-3*x^2)*y[x]==0,y[x]

$$\begin{aligned}
 y(x) \rightarrow & \frac{c_1 \left(\frac{6697x^7}{1260} - \frac{419x^6}{720} - \frac{93x^5}{20} + \frac{173x^4}{24} - \frac{20x^3}{3} + \frac{9x^2}{2} - 2x + 1 \right)}{x} \\
 & + c_2 \left(\frac{-\frac{125221x^7}{29400} - \frac{70949x^6}{14400} + \frac{4217x^5}{400} - \frac{3077x^4}{288} + \frac{133x^3}{18} - \frac{15x^2}{4} + x}{x} \right. \\
 & \left. + \frac{\left(\frac{6697x^7}{1260} - \frac{419x^6}{720} - \frac{93x^5}{20} + \frac{173x^4}{24} - \frac{20x^3}{3} + \frac{9x^2}{2} - 2x + 1 \right) \log(x)}{x} \right)
 \end{aligned}$$

15.7 problem 3

Internal problem ID [1355]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 3.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$x^2(x^2 + 2x + 1)y'' + x(4x^2 + 3x + 1)y' - x(1 - 2x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 71

```
Order:=8;
```

```
dsolve(x^2*(1+2*x+x^2)*diff(y(x),x$2)+x*(1+3*x+4*x^2)*diff(y(x),x)-x*(1-2*x)*y(x)=0,y(x),typ
```

$$y(x) = (c_2 \ln(x) + c_1) \left(1 + x - x^2 + \frac{1}{3}x^3 + \frac{1}{3}x^4 - \frac{11}{15}x^5 + \frac{37}{45}x^6 - \frac{209}{315}x^7 + O(x^8) \right) \\ + \left((-3)x + \frac{1}{2}x^2 + \frac{31}{18}x^3 - \frac{91}{36}x^4 + \frac{1897}{900}x^5 - \frac{301}{300}x^6 - \frac{3901}{14700}x^7 + O(x^8) \right) c_2$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 145

```
AsymptoticDSolveValue[x^2*(1+2*x+x^2)*y'[x]+x*(1+3*x+4*x^2)*y'[x]-x*(1-2*x)*y[x]==0,y[x],{x
```

$$y(x) \rightarrow c_1 \left(-\frac{209x^7}{315} + \frac{37x^6}{45} - \frac{11x^5}{15} + \frac{x^4}{3} + \frac{x^3}{3} - x^2 + x + 1 \right) \\ + c_2 \left(-\frac{3901x^7}{14700} - \frac{301x^6}{300} + \frac{1897x^5}{900} - \frac{91x^4}{36} + \frac{31x^3}{18} + \frac{x^2}{2} \right. \\ \left. + \left(-\frac{209x^7}{315} + \frac{37x^6}{45} - \frac{11x^5}{15} + \frac{x^4}{3} + \frac{x^3}{3} - x^2 + x + 1 \right) \log(x) - 3x \right)$$

15.8 problem 4

Internal problem ID [1356]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 4.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x^2 + x + 1)y'' + 12x^2(x + 1)y' + (3x^2 + 3x + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 81

Order:=8;

```
dsolve(4*x^2*(1+x+x^2)*diff(y(x),x$2)+12*x^2*(1+x)*diff(y(x),x)+(1+3*x+3*x^2)*y(x)=0,y(x),ty
```

$$y(x) = \sqrt{x} \left((c_2 \ln(x) + c_1) \left(1 - 2x + \frac{5}{2}x^2 - 2x^3 + \frac{5}{8}x^4 + \frac{17}{20}x^5 - \frac{121}{80}x^6 + x^7 + O(x^8) \right) \right. \\ \left. + \left(x - \frac{9}{4}x^2 + \frac{17}{6}x^3 - \frac{205}{96}x^4 + \frac{481}{1200}x^5 + \frac{2109}{1600}x^6 - \frac{1063}{560}x^7 + O(x^8) \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 156

```
AsymptoticDSolveValue[4*x^2*(1+x+x^2)*y'[x]+12*x^2*(1+x)*y'[x]+(1+3*x+3*x^2)*y[x]==0,y[x],{
```

$$\begin{aligned} y(x) \rightarrow & c_1 \sqrt{x} \left(x^7 - \frac{121x^6}{80} + \frac{17x^5}{20} + \frac{5x^4}{8} - 2x^3 + \frac{5x^2}{2} - 2x + 1 \right) \\ & + c_2 \left(\sqrt{x} \left(-\frac{1063x^7}{560} + \frac{2109x^6}{1600} + \frac{481x^5}{1200} - \frac{205x^4}{96} + \frac{17x^3}{6} - \frac{9x^2}{4} + x \right) \right. \\ & \left. + \sqrt{x} \left(x^7 - \frac{121x^6}{80} + \frac{17x^5}{20} + \frac{5x^4}{8} - 2x^3 + \frac{5x^2}{2} - 2x + 1 \right) \log(x) \right) \end{aligned}$$

15.9 problem 5

Internal problem ID [1357]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 5.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2 + x + 1)y'' - x(-2x^2 - 4x + 1)y' + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 75

Order:=8;

```
dsolve(x^2*(1+x+x^2)*diff(y(x),x$2)-x*(1-4*x-2*x^2)*diff(y(x),x)+y(x)=0,y(x),type='series',x
```

$$y(x) = x \left((c_2 \ln(x) + c_1) \left(1 - 4x + \frac{19}{2}x^2 - \frac{49}{3}x^3 + \frac{515}{24}x^4 - \frac{319}{15}x^5 + \frac{10093}{720}x^6 - \frac{647}{360}x^7 + O(x^8) \right) + \left(3x - \frac{43}{4}x^2 + \frac{208}{9}x^3 - \frac{10379}{288}x^4 + \frac{76321}{1800}x^5 - \frac{172499}{4800}x^6 + \frac{39091}{2400}x^7 + O(x^8) \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 158

```
AsymptoticDSolveValue[x^2*(1+x+x^2)*y'[x]-x*(1-4*x-2*x^2)*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$\begin{aligned} y(x) \rightarrow & c_1 x \left(-\frac{647x^7}{360} + \frac{10093x^6}{720} - \frac{319x^5}{15} + \frac{515x^4}{24} - \frac{49x^3}{3} + \frac{19x^2}{2} - 4x + 1 \right) \\ & + c_2 \left(x \left(\frac{39091x^7}{2400} - \frac{172499x^6}{4800} + \frac{76321x^5}{1800} - \frac{10379x^4}{288} + \frac{208x^3}{9} - \frac{43x^2}{4} + 3x \right) \right. \\ & \left. + x \left(-\frac{647x^7}{360} + \frac{10093x^6}{720} - \frac{319x^5}{15} + \frac{515x^4}{24} - \frac{49x^3}{3} + \frac{19x^2}{2} - 4x + 1 \right) \log(x) \right) \end{aligned}$$

15.10 problem 6

Internal problem ID [1358]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 6.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2y'' + 3x(-2x^2 + 3x + 5)y' + (-14x^2 + 12x + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 81

Order:=8;

```
dsolve(9*x^2*diff(y(x),x$2)+3*x*(5+3*x-2*x^2)*diff(y(x),x)+(1+12*x-14*x^2)*y(x)=0,y(x),type=
```

$$y(x) = \frac{(c_2 \ln(x) + c_1) \left(1 - x + \frac{5}{6}x^2 - \frac{1}{2}x^3 + \frac{19}{72}x^4 - \frac{43}{360}x^5 + \frac{319}{6480}x^6 - \frac{167}{9072}x^7 + O(x^8)\right) + \left(x - \frac{11}{12}x^2 + \frac{25}{36}x^3 - \frac{1}{28}x^4 + \frac{1}{120}x^5 - \frac{1}{1008}x^6 + \frac{1}{15120}x^7 - \frac{1}{120960}x^8 + O(x^9)\right)}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 168

AsymptoticDSolveValue[9*x^2*y''[x]+3*x*(5+3*x-2*x^2)*y'[x]+(1+12*x-14*x^2)*y[x]==0,y[x],{x,0

$$\begin{aligned}
 y(x) \rightarrow & \frac{c_1 \left(-\frac{167x^7}{9072} + \frac{319x^6}{6480} - \frac{43x^5}{360} + \frac{19x^4}{72} - \frac{x^3}{2} + \frac{5x^2}{6} - x + 1 \right)}{\sqrt[3]{x}} \\
 & + c_2 \left(\frac{\frac{126647x^7}{3810240} - \frac{32773x^6}{388800} + \frac{4211x^5}{21600} - \frac{113x^4}{288} + \frac{25x^3}{36} - \frac{11x^2}{12} + x}{\sqrt[3]{x}} \right. \\
 & \left. + \frac{\left(-\frac{167x^7}{9072} + \frac{319x^6}{6480} - \frac{43x^5}{360} + \frac{19x^4}{72} - \frac{x^3}{2} + \frac{5x^2}{6} - x + 1 \right) \log(x)}{\sqrt[3]{x}} \right)
 \end{aligned}$$

15.11 problem 7

Internal problem ID [1359]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 7.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + x(x^2 + x + 1) y' + x(2 - x) y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 71

```
Order:=8;
```

```
dsolve(x^2*diff(y(x),x$2)+x*(1+x+x^2)*diff(y(x),x)+x*(2-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1) \left(1 - 2x + \frac{7}{4}x^2 - \frac{7}{9}x^3 + \frac{77}{576}x^4 + \frac{217}{7200}x^5 - \frac{8813}{518400}x^6 + \frac{143}{453600}x^7 + O(x^8) \right) + \left(3x - \frac{15}{4}x^2 + \frac{239}{108}x^3 - \frac{2021}{3456}x^4 - \frac{1241}{54000}x^5 + \frac{93859}{1728000}x^6 - \frac{311177}{42336000}x^7 + O(x^8) \right) c_2$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 153

```
AsymptoticDSolveValue[x^2*y''[x]+x*(1+x+x^2)*y'[x]+x*(2-x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left(\frac{143x^7}{453600} - \frac{8813x^6}{518400} + \frac{217x^5}{7200} + \frac{77x^4}{576} - \frac{7x^3}{9} + \frac{7x^2}{4} - 2x + 1 \right) \\ + c_2 \left(-\frac{311177x^7}{42336000} + \frac{93859x^6}{1728000} - \frac{1241x^5}{54000} - \frac{2021x^4}{3456} + \frac{239x^3}{108} - \frac{15x^2}{4} \right. \\ \left. + \left(\frac{143x^7}{453600} - \frac{8813x^6}{518400} + \frac{217x^5}{7200} + \frac{77x^4}{576} - \frac{7x^3}{9} + \frac{7x^2}{4} - 2x + 1 \right) \log(x) + 3x \right)$$

15.12 problem 8

Internal problem ID [1360]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 8.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(2x + 1)y'' + x(3x^2 + 14x + 5)y' + (12x^2 + 18x + 4)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 79

Order:=8;

```
dsolve(x^2*(1+2*x)*diff(y(x),x$2)+x*(5+14*x+3*x^2)*diff(y(x),x)+(4+18*x+12*x^2)*y(x)=0,y(x),
```

$$y(x) = \frac{(c_2 \ln(x) + c_1) \left(1 - 2x + \frac{5}{2}x^2 - 3x^3 + \frac{33}{8}x^4 - \frac{129}{20}x^5 + \frac{867}{80}x^6 - \frac{1059}{56}x^7 + O(x^8)\right) + \left(\frac{3}{4}x^2 - \frac{13}{6}x^3 + \frac{407}{96}x^4 - \frac{13}{24}x^5 + \frac{13}{16}x^6 - \frac{1}{2}x^7 + \frac{1}{24}x^8\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 157

```
AsymptoticDSolveValue[x^2*(1+2*x)*y'[x]+x*(5+14*x+3*x^2)*y'[x]+(4+18*x+12*x^2)*y[x]==0,y[x]
```

$$y(x) \rightarrow \frac{c_1 \left(-\frac{1059x^7}{56} + \frac{867x^6}{80} - \frac{129x^5}{20} + \frac{33x^4}{8} - 3x^3 + \frac{5x^2}{2} - 2x + 1 \right)}{x^2} + c_2 \left(\frac{-\frac{559033x^7}{23520} + \frac{63851x^6}{4800} - \frac{9047x^5}{1200} + \frac{407x^4}{96} - \frac{13x^3}{6} + \frac{3x^2}{4}}{x^2} + \frac{\left(-\frac{1059x^7}{56} + \frac{867x^6}{80} - \frac{129x^5}{20} + \frac{33x^4}{8} - 3x^3 + \frac{5x^2}{2} - 2x + 1 \right) \log(x)}{x^2} \right)$$

15.13 problem 9

Internal problem ID [1361]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 9.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + 2x(x^2 + x + 4)y' + (3x^2 + 5x + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 81

Order:=8;

```
dsolve(4*x^2*diff(y(x),x$2)+2*x*(4+x*x^2)*diff(y(x),x)+(1+5*x+3*x^2)*y(x)=0,y(x),type='series')
```

$$y(x) = \frac{(c_2 \ln(x) + c_1) \left(1 - x + \frac{1}{4}x^2 + \frac{1}{18}x^3 - \frac{37}{1152}x^4 - \frac{17}{28800}x^5 + \frac{593}{259200}x^6 - \frac{1913}{12700800}x^7 + O(x^8)\right) + \left(\frac{3}{2}x - \frac{13}{16}x^2 - \frac{1}{\sqrt{x}}\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 172

AsymptoticDSolveValue[4*x^2*y'[x]+2*x*(4+x+x^2)*y'[x]+(1+5*x+3*x^2)*y[x]==0,y[x],{x,0,7}]

$$\begin{aligned}
 y(x) \rightarrow & \frac{c_1 \left(-\frac{1913x^7}{12700800} + \frac{593x^6}{259200} - \frac{17x^5}{28800} - \frac{37x^4}{1152} + \frac{x^3}{18} + \frac{x^2}{4} - x + 1 \right)}{\sqrt{x}} \\
 & + c_2 \left(\frac{\frac{982189x^7}{889056000} - \frac{98531x^6}{20736000} - \frac{19507x^5}{1728000} + \frac{1103x^4}{13824} + \frac{x^3}{54} - \frac{13x^2}{16} + \frac{3x}{2}}{\sqrt{x}} \right. \\
 & \left. + \frac{\left(-\frac{1913x^7}{12700800} + \frac{593x^6}{259200} - \frac{17x^5}{28800} - \frac{37x^4}{1152} + \frac{x^3}{18} + \frac{x^2}{4} - x + 1 \right) \log(x)}{\sqrt{x}} \right)
 \end{aligned}$$

15.14 problem 10

Internal problem ID [1362]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 10.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$16x^2y'' + 4x(2x^2 + x + 6)y' + (18x^2 + 5x + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 81

```
Order:=8;
```

```
dsolve(16*x^2*diff(y(x),x$2)+4*x*(6+x+2*x^2)*diff(y(x),x)+(1+5*x+18*x^2)*y(x)=0,y(x),type='s
```

$$y(x) = \frac{(c_2 \ln(x) + c_1) \left(1 - \frac{1}{4}x - \frac{7}{32}x^2 + \frac{23}{384}x^3 + \frac{145}{6144}x^4 - \frac{881}{122880}x^5 - \frac{4919}{2949120}x^6 + \frac{47207}{82575360}x^7 + O(x^8)\right) + \left(\frac{1}{4}x + \frac{1}{6}\right)}{x^{\frac{1}{4}}}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 176

AsymptoticDSolveValue[16*x^2*y''[x]+4*x*(6+x+2*x^2)*y'[x]+(1+5*x+18*x^2)*y[x]==0,y[x],{x,0,7}

$$\begin{aligned}
 y(x) \rightarrow & \frac{c_1 \left(\frac{47207x^7}{82575360} - \frac{4919x^6}{2949120} - \frac{881x^5}{122880} + \frac{145x^4}{6144} + \frac{23x^3}{384} - \frac{7x^2}{32} - \frac{x}{4} + 1 \right)}{\sqrt[4]{x}} \\
 & + c_2 \left(\frac{-\frac{953509x^7}{1284505600} + \frac{50791x^6}{58982400} + \frac{65017x^5}{7372800} - \frac{841x^4}{73728} - \frac{157x^3}{2304} + \frac{5x^2}{64} + \frac{x}{4}}{\sqrt[4]{x}} \right. \\
 & \left. + \frac{\left(\frac{47207x^7}{82575360} - \frac{4919x^6}{2949120} - \frac{881x^5}{122880} + \frac{145x^4}{6144} + \frac{23x^3}{384} - \frac{7x^2}{32} - \frac{x}{4} + 1 \right) \log(x)}{\sqrt[4]{x}} \right)
 \end{aligned}$$

15.15 problem 11

Internal problem ID [1363]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 11.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2(x+1)y'' + 3x(-x^2 + 11x + 5)y' + (-7x^2 + 16x + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 79

Order:=8;

```
dsolve(9*x^2*(1+x)*diff(y(x),x$2)+3*x*(5+11*x-x^2)*diff(y(x),x)+(1+16*x-7*x^2)*y(x)=0,y(x),t
```

$$y(x) = \frac{(c_2 \ln(x) + c_1) \left(1 - x + \frac{7}{6}x^2 - \frac{23}{18}x^3 + \frac{11}{8}x^4 - \frac{1577}{1080}x^5 + \frac{3319}{2160}x^6 - \frac{72853}{45360}x^7 + O(x^8)\right) + \left(-\frac{1}{12}x^2 + \frac{13}{108}x^3 - \frac{1}{x^{\frac{1}{3}}}\right)}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 167

AsymptoticDSolveValue[9*x^2*(1+x)*y'[x]+3*x*(5+11*x-x^2)*y'[x]+(1+16*x-7*x^2)*y[x]==0,y[x],

$$\begin{aligned}
 y(x) \rightarrow & \frac{c_1 \left(-\frac{72853x^7}{45360} + \frac{3319x^6}{2160} - \frac{1577x^5}{1080} + \frac{11x^4}{8} - \frac{23x^3}{18} + \frac{7x^2}{6} - x + 1 \right)}{\sqrt[3]{x}} \\
 & + c_2 \left(\frac{\frac{4118557x^7}{19051200} - \frac{76919x^6}{388800} + \frac{11449x^5}{64800} - \frac{131x^4}{864} + \frac{13x^3}{108} - \frac{x^2}{12}}{\sqrt[3]{x}} \right. \\
 & \left. + \frac{\left(-\frac{72853x^7}{45360} + \frac{3319x^6}{2160} - \frac{1577x^5}{1080} + \frac{11x^4}{8} - \frac{23x^3}{18} + \frac{7x^2}{6} - x + 1 \right) \log(x)}{\sqrt[3]{x}} \right)
 \end{aligned}$$

15.16 problem 12

Internal problem ID [1364]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 12.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + (4x + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 69

```
Order:=6;  
dsolve(4*x^2*diff(y(x),x$2)+(1+4*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x} \left((c_2 \ln(x) + c_1) \left(1 - x + \frac{1}{4}x^2 - \frac{1}{36}x^3 + \frac{1}{576}x^4 - \frac{1}{14400}x^5 + O(x^6) \right) \right. \\ \left. + \left(2x - \frac{3}{4}x^2 + \frac{11}{108}x^3 - \frac{25}{3456}x^4 + \frac{137}{432000}x^5 + O(x^6) \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 128

```
AsymptoticDSolveValue[4*x^2*y''[x]+(1+4*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt{x} \left(-\frac{x^5}{14400} + \frac{x^4}{576} - \frac{x^3}{36} + \frac{x^2}{4} - x + 1 \right) \\ + c_2 \left(\sqrt{x} \left(\frac{137x^5}{432000} - \frac{25x^4}{3456} + \frac{11x^3}{108} - \frac{3x^2}{4} + 2x \right) \right. \\ \left. + \sqrt{x} \left(-\frac{x^5}{14400} + \frac{x^4}{576} - \frac{x^3}{36} + \frac{x^2}{4} - x + 1 \right) \log(x) \right)$$

15.17 problem 13

Internal problem ID [1365]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 13.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$36x^2(1 - 2x)y'' + 24x(1 - 9x)y' + (1 - 70x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 69

```
Order:=6;
```

```
dsolve(36*x^2*(1-2*x)*diff(y(x),x$2)+24*x*(1-9*x)*diff(y(x),x)+(1-70*x)*y(x)=0,y(x),type='se
```

$$y(x) = x^{\frac{1}{6}} \left((c_2 \ln(x) + c_1) \left(1 + \frac{8}{3}x + \frac{56}{9}x^2 + \frac{1120}{81}x^3 + \frac{7280}{243}x^4 + \frac{46592}{729}x^5 + O(x^6) \right) \right. \\ \left. + \left(-\frac{2}{3}x - 2x^2 - \frac{1192}{243}x^3 - \frac{8168}{729}x^4 - \frac{270112}{10935}x^5 + O(x^6) \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 132

```
AsymptoticDSolveValue[36*x^2*(1-2*x)*y'[x]+24*x*(1-9*x)*y'[x]+(1-70*x)*y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned} y(x) \rightarrow & c_1 \sqrt[6]{x} \left(\frac{46592x^5}{729} + \frac{7280x^4}{243} + \frac{1120x^3}{81} + \frac{56x^2}{9} + \frac{8x}{3} + 1 \right) \\ & + c_2 \left(\sqrt[6]{x} \left(-\frac{270112x^5}{10935} - \frac{8168x^4}{729} - \frac{1192x^3}{243} - 2x^2 - \frac{2x}{3} \right) \right. \\ & \left. + \sqrt[6]{x} \left(\frac{46592x^5}{729} + \frac{7280x^4}{243} + \frac{1120x^3}{81} + \frac{56x^2}{9} + \frac{8x}{3} + 1 \right) \log(x) \right) \end{aligned}$$

15.18 problem 14

Internal problem ID [1366]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 14.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x+1)y'' - x(3-x)y' + 4y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 69

```
Order:=6;
```

```
dsolve(x^2*(1+x)*diff(y(x),x$2)-x*(3-x)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x^2((c_2 \ln(x) + c_1)(1 - 4x + 9x^2 - 16x^3 + 25x^4 - 36x^5 + O(x^6)) + (4x - 12x^2 + 24x^3 - 40x^4 + 60x^5 + O(x^6))c_2)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 98

```
AsymptoticDSolveValue[x^2*(1+x)*y''[x]-x*(3-x)*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1(-36x^5 + 25x^4 - 16x^3 + 9x^2 - 4x + 1)x^2 + c_2((60x^5 - 40x^4 + 24x^3 - 12x^2 + 4x)x^2 + (-36x^5 + 25x^4 - 16x^3 + 9x^2 - 4x + 1)x^2 \log(x))$$

15.19 problem 15

Internal problem ID [1367]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 15.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1 - 2x)y'' - x(5 - 4x)y' + (9 - 4x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 69

```
Order:=6;
```

```
dsolve(x^2*(1-2*x)*diff(y(x),x$2)-x*(5-4*x)*diff(y(x),x)+(9-4*x)*y(x)=0,y(x),type='series',x
```

$$y(x) = x^3((c_2 \ln(x) + c_1)(1 + 4x + 12x^2 + 32x^3 + 80x^4 + 192x^5 + O(x^6)) + ((-2)x - 8x^2 - 24x^3 - 64x^4 - 160x^5 + O(x^6))c_2)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 98

```
AsymptoticDSolveValue[x^2*(1-2*x)*y'[x]-x*(5-4*x)*y'[x]+(9-4*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1(192x^5 + 80x^4 + 32x^3 + 12x^2 + 4x + 1)x^3 + c_2((-160x^5 - 64x^4 - 24x^3 - 8x^2 - 2x)x^3 + (192x^5 + 80x^4 + 32x^3 + 12x^2 + 4x + 1)x^3 \log(x))$$

15.20 problem 16

Internal problem ID [1368]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 16.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$25x^2y'' + x(15 + x)y' + (x + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

```
Order:=6;
```

```
dsolve(25*x^2*diff(y(x),x$2)+x*(15+x)*diff(y(x),x)+(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x^{\frac{1}{5}} \left((c_2 \ln(x) + c_1) \left(1 - \frac{6}{125}x + \frac{33}{31250}x^2 - \frac{88}{5859375}x^3 + \frac{77}{488281250}x^4 - \frac{1001}{762939453125}x^5 + O(x^6) \right) + \left(\frac{7}{125}x - \frac{113}{62500}x^2 + \frac{1091}{35156250}x^3 - \frac{1721}{468750000}x^4 + \frac{609221}{183105468750000}x^5 + O(x^6) \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 134

```
AsymptoticDSolveValue[25*x^2*y''[x]+x*(15+x)*y'[x]+(1+x)*y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned} y(x) \rightarrow & c_1 \sqrt[5]{x} \left(-\frac{1001x^5}{762939453125} + \frac{77x^4}{488281250} - \frac{88x^3}{5859375} + \frac{33x^2}{31250} - \frac{6x}{125} + 1 \right) \\ & + c_2 \left(\sqrt[5]{x} \left(\frac{609221x^5}{183105468750000} - \frac{1721x^4}{4687500000} + \frac{1091x^3}{35156250} - \frac{113x^2}{62500} + \frac{7x}{125} \right) \right. \\ & \left. + \sqrt[5]{x} \left(-\frac{1001x^5}{762939453125} + \frac{77x^4}{488281250} - \frac{88x^3}{5859375} + \frac{33x^2}{31250} - \frac{6x}{125} + 1 \right) \log(x) \right) \end{aligned}$$

15.21 problem 17

Internal problem ID [1369]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 17.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x+2)y'' + y'x^2 + (1-x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

```
Order:=6;
```

```
dsolve(2*x^2*(2+x)*diff(y(x),x$2)+x^2*diff(y(x),x)+(1-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x} \left((c_2 \ln(x) + c_1) \left(1 + \frac{1}{4}x - \frac{1}{32}x^2 + \frac{1}{128}x^3 - \frac{5}{2048}x^4 + \frac{7}{8192}x^5 + O(x^6) \right) \right. \\ \left. + \left(-\frac{3}{4}x + \frac{3}{64}x^2 - \frac{7}{768}x^3 + \frac{61}{24576}x^4 - \frac{391}{491520}x^5 + O(x^6) \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 134

```
AsymptoticDSolveValue[2*x^2*(2+x)*y'[x]+x^2*y'[x]+(1-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt{x} \left(\frac{7x^5}{8192} - \frac{5x^4}{2048} + \frac{x^3}{128} - \frac{x^2}{32} + \frac{x}{4} + 1 \right) \\ + c_2 \left(\sqrt{x} \left(-\frac{391x^5}{491520} + \frac{61x^4}{24576} - \frac{7x^3}{768} + \frac{3x^2}{64} - \frac{3x}{4} \right) \right. \\ \left. + \sqrt{x} \left(\frac{7x^5}{8192} - \frac{5x^4}{2048} + \frac{x^3}{128} - \frac{x^2}{32} + \frac{x}{4} + 1 \right) \log(x) \right)$$

15.22 problem 18

Internal problem ID [1370]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 18.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(9 + 4x)y'' + 3xy' + (x + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

```
Order:=6;
```

```
dsolve(x^2*(9+4*x)*diff(y(x),x$2)+3*x*diff(y(x),x)+(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x^{\frac{1}{3}} \left((c_2 \ln(x) + c_1) \left(1 - \frac{1}{81}x + \frac{25}{26244}x^2 - \frac{3025}{19131876}x^3 + \frac{874225}{24794911296}x^4 - \frac{18498601}{2008387814976}x^5 + O(x^6) \right) + \left(\frac{14}{81}x - \frac{35}{2916}x^2 + \frac{110495}{57395628}x^3 - \frac{62786185}{148769467776}x^4 + \frac{1315043653}{12050326889856}x^5 + O(x^6) \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 134

```
AsymptoticDSolveValue[x^2*(9+4*x)*y'[x]+3*x*y'[x]+(1+x)*y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned} y(x) \rightarrow & c_1 \sqrt[3]{x} \left(-\frac{18498601x^5}{2008387814976} + \frac{874225x^4}{24794911296} - \frac{3025x^3}{19131876} + \frac{25x^2}{26244} - \frac{x}{81} + 1 \right) \\ & + c_2 \left(\sqrt[3]{x} \left(\frac{1315043653x^5}{12050326889856} - \frac{62786185x^4}{148769467776} + \frac{110495x^3}{57395628} - \frac{35x^2}{2916} + \frac{14x}{81} \right) \right. \\ & \left. + \sqrt[3]{x} \left(-\frac{18498601x^5}{2008387814976} + \frac{874225x^4}{24794911296} - \frac{3025x^3}{19131876} + \frac{25x^2}{26244} - \frac{x}{81} + 1 \right) \log(x) \right) \end{aligned}$$

15.23 problem 19

Internal problem ID [1371]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 19.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' - x(3 - 2x)y' + (4 + 3x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

```
Order:=6;
```

```
dsolve(x^2*diff(y(x),x$2)-x*(3-2*x)*diff(y(x),x)+(4+3*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((c_2 \ln(x) + c_1) \left(1 - 7x + \frac{63}{4}x^2 - \frac{77}{4}x^3 + \frac{1001}{64}x^4 - \frac{3003}{320}x^5 + O(x^6) \right) \right. \\ \left. + \left(12x - \frac{157}{4}x^2 + \frac{2063}{36}x^3 - \frac{59875}{1152}x^4 + \frac{323399}{9600}x^5 + O(x^6) \right) c_2 \right) x^2$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 122

```
AsymptoticDSolveValue[x^2*y''[x]-x*(3-2*x)*y'[x]+(4+3*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(-\frac{3003x^5}{320} + \frac{1001x^4}{64} - \frac{77x^3}{4} + \frac{63x^2}{4} - 7x + 1 \right) x^2 \\ + c_2 \left(\left(\frac{323399x^5}{9600} - \frac{59875x^4}{1152} + \frac{2063x^3}{36} - \frac{157x^2}{4} + 12x \right) x^2 \right. \\ \left. + \left(-\frac{3003x^5}{320} + \frac{1001x^4}{64} - \frac{77x^3}{4} + \frac{63x^2}{4} - 7x + 1 \right) x^2 \log(x) \right)$$

15.24 problem 20

Internal problem ID [1372]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 20.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x^2(1 - 4x)y'' + 3x(1 - 6x)y' + (1 - 12x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

Order:=6;

`dsolve(x^2*(1-4*x)*diff(y(x),x$2)+3*x*(1-6*x)*diff(y(x),x)+(1-12*x)*y(x)=0,y(x),type='series`

$y(x)$

$$= \frac{(c_2 \ln(x) + c_1)(1 + 2x + 6x^2 + 20x^3 + 70x^4 + 252x^5 + O(x^6)) + (2x + 7x^2 + \frac{74}{3}x^3 + \frac{533}{6}x^4 + \frac{1627}{5}x^5 + \dots}{x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 104

`AsymptoticDSolveValue[x^2*(1-4*x)*y'[x]+3*x*(1-6*x)*y'[x]+(1-12*x)*y[x]==0,y[x],{x,0,5}]`

$$y(x) \rightarrow \frac{c_1(252x^5 + 70x^4 + 20x^3 + 6x^2 + 2x + 1)}{x} + c_2 \left(\frac{\frac{1627x^5}{5} + \frac{533x^4}{6} + \frac{74x^3}{3} + 7x^2 + 2x}{x} + \frac{(252x^5 + 70x^4 + 20x^3 + 6x^2 + 2x + 1) \log(x)}{x} \right)$$

15.25 problem 21

Internal problem ID [1373]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 21.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x^2(2x + 1)y'' + x(3 + 5x)y' + (1 - 2x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 69

Order:=6;

`dsolve(x^2*(1+2*x)*diff(y(x),x$2)+x*(3+5*x)*diff(y(x),x)+(1-2*x)*y(x)=0,y(x),type='series',x`

$$y(x) = \frac{(c_2 \ln(x) + c_1) \left(1 + 3x + \frac{3}{2}x^2 - \frac{1}{2}x^3 + \frac{3}{8}x^4 - \frac{3}{8}x^5 + O(x^6)\right) + ((-5)x - \frac{25}{4}x^2 + \frac{5}{4}x^3 - \frac{25}{32}x^4 + \frac{113}{160}x^5 + O(x^6))}{x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 122

`AsymptoticDSolveValue[x^2*(1+2*x)*y'[x]+x*(3+5*x)*y'[x]+(1-2*x)*y[x]==0,y[x],{x,0,5}]`

$$y(x) \rightarrow \frac{c_1 \left(-\frac{3x^5}{8} + \frac{3x^4}{8} - \frac{x^3}{2} + \frac{3x^2}{2} + 3x + 1\right)}{x} + c_2 \left(\frac{\frac{113x^5}{160} - \frac{25x^4}{32} + \frac{5x^3}{4} - \frac{25x^2}{4} - 5x}{x} + \frac{\left(-\frac{3x^5}{8} + \frac{3x^4}{8} - \frac{x^3}{2} + \frac{3x^2}{2} + 3x + 1\right) \log(x)}{x} \right)$$

15.26 problem 22

Internal problem ID [1374]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 22.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x+1)y'' - x(6-x)y' + (8-x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 69

```
Order:=6;
```

```
dsolve(2*x^2*(1+x)*diff(y(x),x$2)-x*(6-x)*diff(y(x),x)+(8-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((c_2 \ln(x) + c_1) \left(1 - \frac{5}{2}x + \frac{35}{8}x^2 - \frac{105}{16}x^3 + \frac{1155}{128}x^4 - \frac{3003}{256}x^5 + O(x^6) \right) \right. \\ \left. + \left(\frac{3}{2}x - \frac{57}{16}x^2 + \frac{583}{96}x^3 - \frac{13771}{1536}x^4 + \frac{187339}{15360}x^5 + O(x^6) \right) c_2 \right) x^2$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 128

```
AsymptoticDSolveValue[2*x^2*(1+x)*y'[x]-x*(6-x)*y'[x]+(8-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(-\frac{3003x^5}{256} + \frac{1155x^4}{128} - \frac{105x^3}{16} + \frac{35x^2}{8} - \frac{5x}{2} + 1 \right) x^2 \\ + c_2 \left(\left(\frac{187339x^5}{15360} - \frac{13771x^4}{1536} + \frac{583x^3}{96} - \frac{57x^2}{16} + \frac{3x}{2} \right) x^2 \right. \\ \left. + \left(-\frac{3003x^5}{256} + \frac{1155x^4}{128} - \frac{105x^3}{16} + \frac{35x^2}{8} - \frac{5x}{2} + 1 \right) x^2 \log(x) \right)$$

15.27 problem 23

Internal problem ID [1375]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 23.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(2x + 1)y'' + x(5 + 9x)y' + (4 + 3x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 81

```
Order:=8;
```

```
dsolve(x^2*(1+2*x)*diff(y(x),x$2)+x*(5+9*x)*diff(y(x),x)+(4+3*x)*y(x)=0,y(x),type='series',x
```

$$y(x) = \frac{(c_2 \ln(x) + c_1) \left(1 + 3x + \frac{3}{2}x^2 - \frac{1}{2}x^3 + \frac{3}{8}x^4 - \frac{3}{8}x^5 + \frac{7}{16}x^6 - \frac{9}{16}x^7 + O(x^8)\right) + \left((-5)x - \frac{25}{4}x^2 + \frac{5}{4}x^3 - \frac{25}{32}x^4 + O(x^5)\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 164

```
AsymptoticDSolveValue[x^2*(1+2*x)*y'[x]+x*(5+9*x)*y'[x]+(4+3*x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow \frac{c_1 \left(-\frac{9x^7}{16} + \frac{7x^6}{16} - \frac{3x^5}{8} + \frac{3x^4}{8} - \frac{x^3}{2} + \frac{3x^2}{2} + 3x + 1 \right)}{x^2} + c_2 \left(\frac{\frac{2123x^7}{2240} - \frac{247x^6}{320} + \frac{113x^5}{160} - \frac{25x^4}{32} + \frac{5x^3}{4} - \frac{25x^2}{4} - 5x}{x^2} + \frac{\left(-\frac{9x^7}{16} + \frac{7x^6}{16} - \frac{3x^5}{8} + \frac{3x^4}{8} - \frac{x^3}{2} + \frac{3x^2}{2} + 3x + 1 \right) \log(x)}{x^2} \right)$$

15.28 problem 24

Internal problem ID [1376]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 24.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1 - 2x)y'' - x(5 + 4x)y' + (9 + 4x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 81

```
Order:=8;
```

```
dsolve(x^2*(1-2*x)*diff(y(x),x$2)-x*(5+4*x)*diff(y(x),x)+(9+4*x)*y(x)=0,y(x),type='series',x
```

$$y(x) = \left((c_2 \ln(x) + c_1) (1 + 20x + 180x^2 + 1120x^3 + 5600x^4 + 24192x^5 + 94080x^6 + 337920x^7 + O(x^8)) + \left((-26)x - 324x^2 - \frac{6968}{3}x^3 - \frac{37780}{3}x^4 - 57360x^5 - \frac{694736}{3}x^6 - \frac{2566144}{3}x^7 + O(x^8) \right) c_2 \right) x^3$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 136

```
AsymptoticDSolveValue[x^2*(1-2*x)*y'[x]-x*(5+4*x)*y'[x]+(9+4*x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1(337920x^7 + 94080x^6 + 24192x^5 + 5600x^4 + 1120x^3 + 180x^2 + 20x + 1) x^3 + c_2 \left(\left(-\frac{2566144x^7}{3} - \frac{694736x^6}{3} - 57360x^5 - \frac{37780x^4}{3} - \frac{6968x^3}{3} - 324x^2 - 26x \right) x^3 + (337920x^7 + 94080x^6 + 24192x^5 + 5600x^4 + 1120x^3 + 180x^2 + 20x + 1) x^3 \log(x) \right)$$

15.29 problem 25

Internal problem ID [1377]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 25.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(4x + 1)y'' - x(1 - 4x)y' + (x + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 75

Order:=8;

```
dsolve(x^2*(1+4*x)*diff(y(x),x$2)-x*(1-4*x)*diff(y(x),x)+(1+x)*y(x)=0,y(x),type='series',x=0
```

$$y(x) = \left((c_2 \ln(x) + c_1) \left(1 - 5x + \frac{85}{4}x^2 - \frac{3145}{36}x^3 + \frac{204425}{576}x^4 - \frac{825877}{576}x^5 + \frac{119752165}{20736}x^6 - \frac{23591176505}{1016064}x^7 + O(x^8) \right) + \left(2x - \frac{39}{4}x^2 + \frac{4499}{108}x^3 - \frac{594305}{3456}x^4 + \frac{2420617}{3456}x^5 - \frac{117547073}{41472}x^6 + \frac{162576422327}{14224896}x^7 + O(x^8) \right) c_2 \right) x$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 158

```
AsymptoticDSolveValue[x^2*(1+4*x)*y'[x]-x*(1-4*x)*y'[x]+(1+x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 x \left(-\frac{23591176505x^7}{1016064} + \frac{119752165x^6}{20736} - \frac{825877x^5}{576} + \frac{204425x^4}{576} - \frac{3145x^3}{36} + \frac{85x^2}{4} - 5x + 1 \right) + c_2 \left(x \left(\frac{162576422327x^7}{14224896} - \frac{117547073x^6}{41472} + \frac{2420617x^5}{3456} - \frac{594305x^4}{3456} + \frac{4499x^3}{108} - \frac{39x^2}{4} + 2x \right) + x \left(-\frac{23591176505x^7}{1016064} + \frac{119752165x^6}{20736} - \frac{825877x^5}{576} + \frac{204425x^4}{576} - \frac{3145x^3}{36} + \frac{85x^2}{4} - 5x + 1 \right) \log(x) \right)$$

15.30 problem 26

Internal problem ID [1378]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 26.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x+1)y'' + x(2x+1)y' + yx = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 71

```
Order:=8;
```

```
dsolve(x^2*(1+x)*diff(y(x),x$2)+x*(1+2*x)*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1) \left(1 - x + \frac{3}{4}x^2 - \frac{7}{12}x^3 + \frac{91}{192}x^4 - \frac{637}{1600}x^5 + \frac{19747}{57600}x^6 - \frac{17329}{57600}x^7 + O(x^8) \right) + \left(x - \frac{3}{4}x^2 + \frac{5}{9}x^3 - \frac{499}{1152}x^4 + \frac{16919}{48000}x^5 - \frac{56861}{192000}x^6 + \frac{1027717}{4032000}x^7 + O(x^8) \right) c_2$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 151

```
AsymptoticDSolveValue[x^2*(1+x)*y'[x]+x*(1+2*x)*y'[x]+x*y[x]==0,y[x],{x,0,7}]
```

$$\begin{aligned} y(x) \rightarrow & c_1 \left(-\frac{17329x^7}{57600} + \frac{19747x^6}{57600} - \frac{637x^5}{1600} + \frac{91x^4}{192} - \frac{7x^3}{12} + \frac{3x^2}{4} - x + 1 \right) \\ & + c_2 \left(\frac{1027717x^7}{4032000} - \frac{56861x^6}{192000} + \frac{16919x^5}{48000} - \frac{499x^4}{1152} + \frac{5x^3}{9} - \frac{3x^2}{4} \right. \\ & \left. + \left(-\frac{17329x^7}{57600} + \frac{19747x^6}{57600} - \frac{637x^5}{1600} + \frac{91x^4}{192} - \frac{7x^3}{12} + \frac{3x^2}{4} - x + 1 \right) \log(x) + x \right) \end{aligned}$$

15.31 problem 27

Internal problem ID [1379]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 27.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-x)y'' + x(7+x)y' + (9-x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 63

```
Order:=8;
```

```
dsolve(x^2*(1-x)*diff(y(x),x$2)+x*(7+x)*diff(y(x),x)+(9-x)*y(x)=0,y(x),type='series',x=0);
```

$y(x)$

$$= \frac{(c_2 \ln(x) + c_1)(1 + 16x + 36x^2 + 16x^3 + x^4 + O(x^8)) + ((-40)x - 150x^2 - \frac{280}{3}x^3 - \frac{25}{3}x^4 + O(x^8))c_2}{x^3}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 83

```
AsymptoticDSolveValue[x^2*(1-x)*y'[x]+x*(7+x)*y'[x]+(9-x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow \frac{c_1(x^4 + 16x^3 + 36x^2 + 16x + 1)}{x^3} + c_2 \left(\frac{-\frac{25x^4}{3} - \frac{280x^3}{3} - 150x^2 - 40x}{x^3} + \frac{(x^4 + 16x^3 + 36x^2 + 16x + 1) \log(x)}{x^3} \right)$$

15.32 problem 28

Internal problem ID [1380]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 28.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' - x(-x^2 + 1) y' + (x^2 + 1) y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

```
Order:=6;
```

```
dsolve(x^2*dif(y(x),x$2)-x*(1-x^2)*dif(y(x),x)+(1+x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x \left((c_2 \ln(x) + c_1) \left(1 - \frac{1}{2}x^2 + \frac{1}{8}x^4 + O(x^6) \right) + \left(\frac{1}{4}x^2 - \frac{3}{32}x^4 + O(x^6) \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 65

```
AsymptoticDSolveValue[x^2*y''[x]-x*(1-x^2)*y'[x]+(1+x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 x \left(\frac{x^4}{8} - \frac{x^2}{2} + 1 \right) + c_2 \left(x \left(\frac{x^2}{4} - \frac{3x^4}{32} \right) + x \left(\frac{x^4}{8} - \frac{x^2}{2} + 1 \right) \log(x) \right)$$

15.33 problem 29

Internal problem ID [1381]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 29.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2 + 1)y'' - 3x(-x^2 + 1)y' + 4y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 51

```
Order:=6;
```

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-3*x*(1-x^2)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((c_2 \ln(x) + c_1) (1 - 2x^2 + 3x^4 + O(x^6)) + \left(\frac{1}{2}x^2 - x^4 + O(x^6) \right) c_2 \right) x^2$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 61

```
AsymptoticDSolveValue[x^2*(1+x^2)*y''[x]-3*x*(1-x^2)*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1(3x^4 - 2x^2 + 1)x^2 + c_2 \left(\left(\frac{x^2}{2} - x^4 \right) x^2 + (3x^4 - 2x^2 + 1)x^2 \log(x) \right)$$

15.34 problem 30

Internal problem ID [1382]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 30.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + 2y'x^3 + (3x^2 + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 51

```
Order:=6;
```

```
dsolve(4*x^2*diff(y(x),x$2)+2*x^3*diff(y(x),x)+(1+3*x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x} \left((c_2 \ln(x) + c_1) \left(1 - \frac{1}{4}x^2 + \frac{1}{32}x^4 + O(x^6) \right) + \left(\frac{1}{8}x^2 - \frac{3}{128}x^4 + O(x^6) \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 77

```
AsymptoticDSolveValue[4*x^2*y''[x]+2*x^3*y'[x]+(1+3*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt{x} \left(\frac{x^4}{32} - \frac{x^2}{4} + 1 \right) + c_2 \left(\sqrt{x} \left(\frac{x^2}{8} - \frac{3x^4}{128} \right) + \sqrt{x} \left(\frac{x^4}{32} - \frac{x^2}{4} + 1 \right) \log(x) \right)$$

15.35 problem 31

Internal problem ID [1383]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 31.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2 + 1)y'' - x(-2x^2 + 1)y' + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
Order:=6;
```

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(1-2*x^2)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x \left((c_2 \ln(x) + c_1) \left(1 - \frac{1}{2}x^2 + \frac{3}{8}x^4 + O(x^6) \right) + \left(-\frac{1}{4}x^2 + \frac{7}{32}x^4 + O(x^6) \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 65

```
AsymptoticDSolveValue[x^2*(1+x^2)*y''[x]-x*(1-2*x^2)*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 x \left(\frac{3x^4}{8} - \frac{x^2}{2} + 1 \right) + c_2 \left(x \left(\frac{7x^4}{32} - \frac{x^2}{4} \right) + x \left(\frac{3x^4}{8} - \frac{x^2}{2} + 1 \right) \log(x) \right)$$

15.36 problem 32

Internal problem ID [1384]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 32.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x^2 + 2)y'' + 7y'x^3 + (3x^2 + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
Order:=6;
```

```
dsolve(2*x^2*(2+x^2)*diff(y(x),x$2)+7*x^3*diff(y(x),x)+(1+3*x^2)*y(x)=0,y(x),type='series',x
```

$$y(x) = \sqrt{x} \left((c_2 \ln(x) + c_1) \left(1 - \frac{3}{8}x^2 + \frac{21}{128}x^4 + O(x^6) \right) + \left(-\frac{1}{16}x^2 + \frac{17}{512}x^4 + O(x^6) \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 77

```
AsymptoticDSolveValue[2*x^2*(2+x^2)*y'[x]+7*x^3*y'[x]+(1+3*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt{x} \left(\frac{21x^4}{128} - \frac{3x^2}{8} + 1 \right) + c_2 \left(\sqrt{x} \left(\frac{17x^4}{512} - \frac{x^2}{16} \right) + \sqrt{x} \left(\frac{21x^4}{128} - \frac{3x^2}{8} + 1 \right) \log(x) \right)$$

15.37 problem 33

Internal problem ID [1385]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 33.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2 + 1)y'' - x(-4x^2 + 1)y' + (2x^2 + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

Order:=6;

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(1-4*x^2)*diff(y(x),x)+(1+2*x^2)*y(x)=0,y(x),type='series')
```

$$y(x) = x \left((c_2 \ln(x) + c_1) \left(1 - \frac{3}{2}x^2 + \frac{15}{8}x^4 + O(x^6) \right) + \left(\frac{1}{4}x^2 - \frac{13}{32}x^4 + O(x^6) \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 65

```
AsymptoticDSolveValue[x^2*(1+x^2)*y'[x]-x*(1-4*x^2)*y'[x]+(1+2*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 x \left(\frac{15x^4}{8} - \frac{3x^2}{2} + 1 \right) + c_2 \left(x \left(\frac{x^2}{4} - \frac{13x^4}{32} \right) + x \left(\frac{15x^4}{8} - \frac{3x^2}{2} + 1 \right) \log(x) \right)$$

15.38 problem 34

Internal problem ID [1386]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 34.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x^2 + 4)y'' + 3x(3x^2 + 8)y' + (-9x^2 + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

Order:=6;

```
dsolve(4*x^2*(4+x^2)*diff(y(x),x$2)+3*x*(8+3*x^2)*diff(y(x),x)+(1-9*x^2)*y(x)=0,y(x),type='s
```

$$y(x) = \frac{(c_2 \ln(x) + c_1) \left(1 + \frac{5}{32}x^2 - \frac{15}{2048}x^4 + O(x^6)\right) + \left(-\frac{13}{64}x^2 + \frac{13}{8192}x^4 + O(x^6)\right) c_2}{x^{\frac{1}{4}}}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 77

```
AsymptoticDSolveValue[4*x^2*(4+x^2)*y'[x]+3*x*(8+3*x^2)*y'[x]+(1-9*x^2)*y[x]==0,y[x],{x,0,5
```

$$y(x) \rightarrow \frac{c_1 \left(-\frac{15x^4}{2048} + \frac{5x^2}{32} + 1\right)}{\sqrt[4]{x}} + c_2 \left(\frac{\frac{13x^4}{8192} - \frac{13x^2}{64}}{\sqrt[4]{x}} + \frac{\left(-\frac{15x^4}{2048} + \frac{5x^2}{32} + 1\right) \log(x)}{\sqrt[4]{x}} \right)$$

15.39 problem 35

Internal problem ID [1387]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 35.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3x^2(x^2 + 3)y'' + x(11x^2 + 3)y' + (5x^2 + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
Order:=6;
```

```
dsolve(3*x^2*(3+x^2)*diff(y(x),x$2)+x*(3+11*x^2)*diff(y(x),x)+(1+5*x^2)*y(x)=0,y(x),type='series')
```

$$y(x) = x^{\frac{1}{3}} \left((c_2 \ln(x) + c_1) \left(1 - \frac{2}{9}x^2 + \frac{5}{81}x^4 + O(x^6) \right) + \left(-\frac{1}{18}x^2 + \frac{1}{54}x^4 + O(x^6) \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 42

```
AsymptoticDSolveValue[3*x^2*(3+x^2)*y'[x]+x*(3+11*x^2)*y'[x]+(1+5*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{c_1 \left(-\frac{1139x^5}{405} + \frac{53x^4}{81} + \frac{7x^3}{9} - \frac{11x^2}{9} + x + 1 \right)}{\sqrt[3]{x}}$$

15.40 problem 36

Internal problem ID [1388]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 36.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(4x^2 + 1)y'' + 32y'x^3 + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
Order:=6;
```

```
dsolve(4*x^2*(1+4*x^2)*diff(y(x),x$2)+32*x^3*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x} \left((c_2 \ln(x) + c_1) \left(1 - \frac{3}{4}x^2 + \frac{105}{64}x^4 + O(x^6) \right) + \left(-\frac{5}{4}x^2 + \frac{389}{128}x^4 + O(x^6) \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 77

```
AsymptoticDSolveValue[4*x^2*(1+4*x^2)*y'[x]+32*x^3*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt{x} \left(\frac{105x^4}{64} - \frac{3x^2}{4} + 1 \right) + c_2 \left(\sqrt{x} \left(\frac{389x^4}{128} - \frac{5x^2}{4} \right) + \sqrt{x} \left(\frac{105x^4}{64} - \frac{3x^2}{4} + 1 \right) \log(x) \right)$$

15.41 problem 37

Internal problem ID [1389]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 37.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2y'' - 3x(-2x^2 + 7)y' + (2x^2 + 25)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

Order:=6;

```
dsolve(9*x^2*diff(y(x),x$2)-3*x*(7-2*x^2)*diff(y(x),x)+(25+2*x^2)*y(x)=0,y(x),type='series',
```

$$y(x) = x^{\frac{5}{3}} \left((c_2 \ln(x) + c_1) \left(1 - \frac{1}{3}x^2 + \frac{1}{18}x^4 + O(x^6) \right) + \left(\frac{1}{6}x^2 - \frac{1}{24}x^4 + O(x^6) \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 77

```
AsymptoticDSolveValue[9*x^2*y''[x]-3*x*(7-2*x^2)*y'[x]+(25+2*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{x^4}{18} - \frac{x^2}{3} + 1 \right) x^{5/3} + c_2 \left(\left(\frac{x^2}{6} - \frac{x^4}{24} \right) x^{5/3} + \left(\frac{x^4}{18} - \frac{x^2}{3} + 1 \right) x^{5/3} \log(x) \right)$$

15.42 problem 38

Internal problem ID [1390]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 38.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x^2(2x^2 + 1)y'' + x(7x^2 + 3)y' + (-3x^2 + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

Order:=6;

```
dsolve(x^2*(1+2*x^2)*diff(y(x),x$2)+x*(3+7*x^2)*diff(y(x),x)+(1-3*x^2)*y(x)=0,y(x),type='ser
```

$$y(x) = \frac{(c_2 \ln(x) + c_1) \left(1 + \frac{3}{2}x^2 - \frac{3}{8}x^4 + O(x^6)\right) + \left(-\frac{7}{4}x^2 - \frac{7}{32}x^4 + O(x^6)\right) c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 71

```
AsymptoticDSolveValue[x^2*(1+2*x^2)*y'[x]+x*(3+7*x^2)*y'[x]+(1-3*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{c_1 \left(-\frac{3x^4}{8} + \frac{3x^2}{2} + 1\right)}{x} + c_2 \left(\frac{-\frac{7x^4}{32} - \frac{7x^2}{4}}{x} + \frac{\left(-\frac{3x^4}{8} + \frac{3x^2}{2} + 1\right) \log(x)}{x} \right)$$

15.43 problem 39

Internal problem ID [1391]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 39.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x^2(x^2 + 1)y'' + x(8x^2 + 3)y' + (12x^2 + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
Order:=6;
```

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)+x*(3+8*x^2)*diff(y(x),x)+(1+12*x^2)*y(x)=0,y(x),type='series')
```

$$y(x) = \frac{(c_2 \ln(x) + c_1) \left(1 - \frac{3}{2}x^2 + \frac{15}{8}x^4 + O(x^6)\right) + \left(\frac{1}{4}x^2 - \frac{13}{32}x^4 + O(x^6)\right) c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 71

```
AsymptoticDSolveValue[x^2*(1+x^2)*y'[x]+x*(3+8*x^2)*y'[x]+(1+12*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{c_1 \left(\frac{15x^4}{8} - \frac{3x^2}{2} + 1\right)}{x} + c_2 \left(\frac{\frac{x^2}{4} - \frac{13x^4}{32}}{x} + \frac{\left(\frac{15x^4}{8} - \frac{3x^2}{2} + 1\right) \log(x)}{x}\right)$$

15.44 problem 40

Internal problem ID [1392]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 40.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' - x(-x^2 + 1) y' + (x^2 + 1) y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
```

```
dsolve(x^2*dif(y(x),x$2)-x*(1-x^2)*dif(y(x),x)+(1+x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x \left((c_2 \ln(x) + c_1) \left(1 - \frac{1}{2}x^2 + \frac{1}{8}x^4 + O(x^6) \right) + \left(\frac{1}{4}x^2 - \frac{3}{32}x^4 + O(x^6) \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 65

```
AsymptoticDSolveValue[x^2*y''[x]-x*(1-x^2)*y'[x]+(1+x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 x \left(\frac{x^4}{8} - \frac{x^2}{2} + 1 \right) + c_2 \left(x \left(\frac{x^2}{4} - \frac{3x^4}{32} \right) + x \left(\frac{x^4}{8} - \frac{x^2}{2} + 1 \right) \log(x) \right)$$

15.45 problem 41

Internal problem ID [1393]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 41.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(-2x^2 + 1)y'' + x(-9x^2 + 5)y' + (-3x^2 + 4)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 51

Order:=6;

```
dsolve(x^2*(1-2*x^2)*diff(y(x),x$2)+x*(5-9*x^2)*diff(y(x),x)+(4-3*x^2)*y(x)=0,y(x),type='series')
```

$$y(x) = \frac{(c_2 \ln(x) + c_1) \left(1 - \frac{3}{4}x^2 - \frac{9}{64}x^4 + O(x^6)\right) + \left(\frac{1}{2}x^2 - \frac{21}{128}x^4 + O(x^6)\right) c_2}{x^2}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 71

```
AsymptoticDSolveValue[x^2*(1-2*x^2)*y'[x]+x*(5-9*x^2)*y'[x]+(4-3*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{c_1 \left(-\frac{9x^4}{64} - \frac{3x^2}{4} + 1\right)}{x^2} + c_2 \left(\frac{\frac{x^2}{2} - \frac{21x^4}{128}}{x^2} + \frac{\left(-\frac{9x^4}{64} - \frac{3x^2}{4} + 1\right) \log(x)}{x^2}\right)$$

15.46 problem 42

Internal problem ID [1394]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 42.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2 + 2)y'' + x(-x^2 + 14)y' + 2(x^2 + 9)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

Order:=6;

```
dsolve(x^2*(2+x^2)*diff(y(x),x$2)+x*(14-x^2)*diff(y(x),x)+2*(9+x^2)*y(x)=0,y(x),type='series
```

$$y(x) = \frac{(c_2 \ln(x) + c_1) \left(1 - \frac{17}{8}x^2 + \frac{85}{256}x^4 + O(x^6)\right) + \left(\frac{25}{8}x^2 - \frac{471}{512}x^4 + O(x^6)\right) c_2}{x^3}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 71

```
AsymptoticDSolveValue[x^2*(2+x^2)*y'[x]+x*(14-x^2)*y'[x]+2*(9+x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{c_1 \left(\frac{85x^4}{256} - \frac{17x^2}{8} + 1\right)}{x^3} + c_2 \left(\frac{\frac{25x^2}{8} - \frac{471x^4}{512}}{x^3} + \frac{\left(\frac{85x^4}{256} - \frac{17x^2}{8} + 1\right) \log(x)}{x^3}\right)$$

15.47 problem 43

Internal problem ID [1395]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 43.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2 + 1)y'' + x(7x^2 + 3)y' + (8x^2 + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 51

```
Order:=6;
```

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)+x*(3+7*x^2)*diff(y(x),x)+(1+8*x^2)*y(x)=0,y(x),type='series')
```

$$y(x) = \frac{(c_2 \ln(x) + c_1) \left(1 - \frac{3}{4}x^2 + \frac{45}{64}x^4 + O(x^6)\right) + \left(-\frac{1}{4}x^2 + \frac{33}{128}x^4 + O(x^6)\right) c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 71

```
AsymptoticDSolveValue[x^2*(1+x^2)*y'[x]+x*(3+7*x^2)*y'[x]+(1+8*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{c_1 \left(\frac{45x^4}{64} - \frac{3x^2}{4} + 1\right)}{x} + c_2 \left(\frac{\frac{33x^4}{128} - \frac{x^2}{4}}{x} + \frac{\left(\frac{45x^4}{64} - \frac{3x^2}{4} + 1\right) \log(x)}{x}\right)$$

15.48 problem 44

Internal problem ID [1396]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 44.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1 - 2x)y'' + 3xy' + (4x + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
```

```
dsolve(x^2*(1-2*x)*diff(y(x),x$2)+3*x*diff(y(x),x)+(1+4*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{(c_2 \ln(x) + c_1)(1 + O(x^6)) + ((-6)x + 6x^2 - \frac{8}{3}x^3 + O(x^6))c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 40

```
AsymptoticDSolveValue[x^2*(1-2*x)*y'[x]+3*x*y'[x]+(1+4*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{-\frac{8x^3}{3} + 6x^2 - 6x}{x} + \frac{\log(x)}{x} \right) + \frac{c_1}{x}$$

15.49 problem 45

Internal problem ID [1397]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 45.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(x+1)y'' + (1-x)y' + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
Order:=6;  
dsolve(x*(1+x)*diff(y(x),x$2)+(1-x)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1) (1 - x + O(x^6)) + (4x + O(x^6)) c_2$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 27

```
AsymptoticDSolveValue[x*(1+x)*y''[x]+(1-x)*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1(1-x) + c_2(4x + (1-x)\log(x))$$

15.50 problem 46

Internal problem ID [1398]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 46.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x^2(1-x)y'' + x(3-2x)y' + (2x+1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 57

```
Order:=6;
```

```
dsolve(x^2*(1-x)*diff(y(x),x$2)+x*(3-2*x)*diff(y(x),x)+(1+2*x)*y(x)=0,y(x),type='series',x=0)
```

$$y(x) = \frac{(c_2 \ln(x) + c_1)(1 - 2x + x^2 + O(x^6)) + (3x - 3x^2 + \frac{1}{3}x^3 + \frac{1}{12}x^4 + \frac{1}{30}x^5 + O(x^6))c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 70

```
AsymptoticDSolveValue[x^2*(1-x)*y'[x]+x*(3-2*x)*y'[x]+(1+2*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{c_1(x^2 - 2x + 1)}{x} + c_2 \left(\frac{(x^2 - 2x + 1) \log(x)}{x} + \frac{\frac{x^5}{30} + \frac{x^4}{12} + \frac{x^3}{3} - 3x^2 + 3x}{x} \right)$$

15.51 problem 47

Internal problem ID [1399]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 47.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x+1)y'' + 4y'x^2 + (1-5x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 69

```
Order:=6;
```

```
dsolve(4*x^2*(1+x)*diff(y(x),x$2)+4*x^2*diff(y(x),x)+(1-5*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x} \left((c_2 \ln(x) + c_1) \left(1 + x - \frac{1}{4}x^2 + \frac{5}{36}x^3 - \frac{55}{576}x^4 + \frac{209}{2880}x^5 + O(x^6) \right) \right. \\ \left. + \left((-3)x + \frac{1}{4}x^2 - \frac{5}{54}x^3 + \frac{175}{3456}x^4 - \frac{2863}{86400}x^5 + O(x^6) \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 124

```
AsymptoticDSolveValue[4*x^2*(1+x)*y'[x]+4*x^2*y'[x]+(1-5*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt{x} \left(\frac{209x^5}{2880} - \frac{55x^4}{576} + \frac{5x^3}{36} - \frac{x^2}{4} + x + 1 \right) \\ + c_2 \left(\sqrt{x} \left(-\frac{2863x^5}{86400} + \frac{175x^4}{3456} - \frac{5x^3}{54} + \frac{x^2}{4} - 3x \right) \right. \\ \left. + \sqrt{x} \left(\frac{209x^5}{2880} - \frac{55x^4}{576} + \frac{5x^3}{36} - \frac{x^2}{4} + x + 1 \right) \log(x) \right)$$

15.52 problem 48

Internal problem ID [1400]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 48.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-x)y'' - x(3-5x)y' + (4-5x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 61

```
Order:=6;
```

```
dsolve(x^2*(1-x)*diff(y(x),x$2)-x*(3-5*x)*diff(y(x),x)+(4-5*x)*y(x)=0,y(x),type='series',x=0)
```

$$y(x) = x^2 \left((c_2 \ln(x) + c_1) (1 - 3x + 3x^2 - x^3 + O(x^6)) \right. \\ \left. + \left(4x - 7x^2 + \frac{11}{3}x^3 - \frac{1}{4}x^4 - \frac{1}{20}x^5 + O(x^6) \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 84

```
AsymptoticDSolveValue[x^2*(1-x)*y'[x]-x*(3-5*x)*y'[x]+(4-5*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1(-x^3 + 3x^2 - 3x + 1)x^2 \\ + c_2 \left((-x^3 + 3x^2 - 3x + 1)x^2 \log(x) + \left(-\frac{x^5}{20} - \frac{x^4}{4} + \frac{11x^3}{3} - 7x^2 + 4x \right) x^2 \right)$$

15.53 problem 49

Internal problem ID [1401]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 49.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2 + 1)y'' - x(9x^2 + 1)y' + (25x^2 + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

Order:=6;

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(1+9*x^2)*diff(y(x),x)+(1+25*x^2)*y(x)=0,y(x),type='series')
```

$$y(x) = x((c_2 \ln(x) + c_1)(1 - 4x^2 + x^4 + O(x^6)) + (6x^2 - 3x^4 + O(x^6))c_2)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 49

```
AsymptoticDSolveValue[x^2*(1+x^2)*y'[x]-x*(1+9*x^2)*y'[x]+(1+25*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 x(x^4 - 4x^2 + 1) + c_2(x(6x^2 - 3x^4) + x(x^4 - 4x^2 + 1)\log(x))$$

15.54 problem 50

Internal problem ID [1402]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 50.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2y'' + 3x(-x^2 + 1)y' + (7x^2 + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

Order:=6;

```
dsolve(9*x^2*dif(y(x),x^2)+3*x*(1-x^2)*dif(y(x),x)+(1+7*x^2)*y(x)=0,y(x),type='series',x=0)
```

$$y(x) = x^{\frac{1}{3}} \left((c_2 \ln(x) + c_1) \left(1 - \frac{1}{6}x^2 + O(x^6) \right) + \left(\frac{1}{4}x^2 - \frac{1}{288}x^4 + O(x^6) \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 63

```
AsymptoticDSolveValue[9*x^2*y''[x]+3*x*(1-x^2)*y'[x]+(1+7*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt[3]{x} \left(1 - \frac{x^2}{6} \right) + c_2 \left(\sqrt[3]{x} \left(1 - \frac{x^2}{6} \right) \log(x) + \sqrt[3]{x} \left(\frac{x^2}{4} - \frac{x^4}{288} \right) \right)$$

15.55 problem 51

Internal problem ID [1403]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 51.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x(x^2 + 1)y'' + (-x^2 + 1)y' - 8yx = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 41

```
Order:=6;
```

```
dsolve(x*(1+x^2)*diff(y(x),x$2)+(1-x^2)*diff(y(x),x)-8*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1) (1 + 2x^2 + x^4 + O(x^6)) + \left(-\frac{3}{2}x^2 - \frac{3}{2}x^4 + O(x^6) \right) c_2$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 48

```
AsymptoticDSolveValue[x*(1+x^2)*y'[x]+(1-x^2)*y'[x]-8*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1(x^4 + 2x^2 + 1) + c_2 \left(-\frac{3x^4}{2} - \frac{3x^2}{2} + (x^4 + 2x^2 + 1) \log(x) \right)$$

15.56 problem 52

Internal problem ID [1404]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 52.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + 2x(-x^2 + 4)y' + (7x^2 + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

Order:=6;

```
dsolve(4*x^2*dif(y(x),x$2)+2*x*(4-x^2)*dif(y(x),x)+(1+7*x^2)*y(x)=0,y(x),type='series',x=0)
```

$$y(x) = \frac{(c_2 \ln(x) + c_1) \left(1 - \frac{1}{2}x^2 + \frac{1}{32}x^4 + O(x^6)\right) + \left(\frac{5}{8}x^2 - \frac{9}{128}x^4 + O(x^6)\right) c_2}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 77

```
AsymptoticDSolveValue[4*x^2*y'[x]+2*x*(4-x^2)*y'[x]+(1+7*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{c_1 \left(\frac{x^4}{32} - \frac{x^2}{2} + 1\right)}{\sqrt{x}} + c_2 \left(\frac{\frac{5x^2}{8} - \frac{9x^4}{128}}{\sqrt{x}} + \frac{\left(\frac{x^4}{32} - \frac{x^2}{2} + 1\right) \log(x)}{\sqrt{x}}\right)$$

15.57 problem 58

Internal problem ID [1405]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 58.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x+1)y'' + 8y'x^2 + (x+1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 49

```
Order:=6;
```

```
dsolve(4*x^2*(1+x)*diff(y(x),x$2)+8*x^2*diff(y(x),x)+(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x}(-x^5 + x^4 - x^3 + x^2 - x + 1)(c_2 \ln(x) + c_1) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 64

```
AsymptoticDSolveValue[4*x^2*(1+x)*y'[x]+8*x^2*y'[x]+(1+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt{x}(-x^5 + x^4 - x^3 + x^2 - x + 1) + c_2 \sqrt{x}(-x^5 + x^4 - x^3 + x^2 - x + 1) \log(x)$$

15.58 problem 59

Internal problem ID [1406]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 59.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2(3+x)y'' + 3x(3+7x)y' + (3+4x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

Order:=6;

```
dsolve(9*x^2*(3+x)*diff(y(x),x$2)+3*x*(3+7*x)*diff(y(x),x)+(3+4*x)*y(x)=0,y(x),type='series')
```

$$y(x) = x^{\frac{1}{3}} \left(1 - \frac{1}{3}x + \frac{1}{9}x^2 - \frac{1}{27}x^3 + \frac{1}{81}x^4 - \frac{1}{243}x^5 \right) (c_2 \ln(x) + c_1) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 92

```
AsymptoticDSolveValue[9*x^2*(3+x)*y'[x]+3*x*(3+7*x)*y'[x]+(3+4*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt[3]{x} \left(-\frac{x^5}{243} + \frac{x^4}{81} - \frac{x^3}{27} + \frac{x^2}{9} - \frac{x}{3} + 1 \right) + c_2 \sqrt[3]{x} \left(-\frac{x^5}{243} + \frac{x^4}{81} - \frac{x^3}{27} + \frac{x^2}{9} - \frac{x}{3} + 1 \right) \log(x)$$

15.59 problem 60

Internal problem ID [1407]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 60.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(-x^2 + 2)y'' - x(3x^2 + 2)y' + (-x^2 + 2)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
Order:=6;
```

```
dsolve(x^2*(2-x^2)*diff(y(x),x$2)-x*(2+3*x^2)*diff(y(x),x)+(2-x^2)*y(x)=0,y(x),type='series')
```

$$y(x) = \left(1 + \frac{1}{2}x^2 + \frac{1}{4}x^4\right) x(c_2 \ln(x) + c_1) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 46

```
AsymptoticDSolveValue[x^2*(2-x^2)*y'[x]-x*(2+3*x^2)*y'[x]+(2-x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 x \left(\frac{x^4}{4} + \frac{x^2}{2} + 1 \right) + c_2 x \left(\frac{x^4}{4} + \frac{x^2}{2} + 1 \right) \log(x)$$

15.60 problem 61

Internal problem ID [1408]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 61.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$16x^2(x^2 + 1)y'' + 8x(9x^2 + 1)y' + (49x^2 + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

Order:=6;

```
dsolve(16*x^2*(1+x^2)*diff(y(x),x$2)+8*x*(1+9*x^2)*diff(y(x),x)+(1+49*x^2)*y(x)=0,y(x),type=
```

$$y(x) = x^{\frac{1}{4}}(x^4 - x^2 + 1)(c_2 \ln(x) + c_1) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 42

```
AsymptoticDSolveValue[16*x^2*(1+x^2)*y'[x]+8*x*(1+9*x^2)*y'[x]+(1+49*x^2)*y[x]==0,y[x],{x,0
```

$$y(x) \rightarrow c_1 \sqrt[4]{x}(x^4 - x^2 + 1) + c_2 \sqrt[4]{x}(x^4 - x^2 + 1) \log(x)$$

15.61 problem 62

Internal problem ID [1409]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 62.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(4 + 3x)y'' - x(4 - 3x)y' + 4y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

```
Order:=6;
```

```
dsolve(x^2*(4+3*x)*diff(y(x),x$2)-x*(4-3*x)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{3}{4}x + \frac{9}{16}x^2 - \frac{27}{64}x^3 + \frac{81}{256}x^4 - \frac{243}{1024}x^5\right) x(c_2 \ln(x) + c_1) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 84

```
AsymptoticDSolveValue[x^2*(4+3*x)*y'[x]-x*(4-3*x)*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 x \left(-\frac{243x^5}{1024} + \frac{81x^4}{256} - \frac{27x^3}{64} + \frac{9x^2}{16} - \frac{3x}{4} + 1 \right) + c_2 x \left(-\frac{243x^5}{1024} + \frac{81x^4}{256} - \frac{27x^3}{64} + \frac{9x^2}{16} - \frac{3x}{4} + 1 \right) \log(x)$$

15.62 problem 63

Internal problem ID [1410]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 63.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x^2 + 3x + 1)y'' + 8x^2(2x + 3)y' + (9x^2 + 3x + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

Order:=6;

```
dsolve(4*x^2*(1+3*x+x^2)*diff(y(x),x$2)+8*x^2*(3+2*x)*diff(y(x),x)+(1+3*x+9*x^2)*y(x)=0,y(x))
```

$$y(x) = \sqrt{x}(-144x^5 + 55x^4 - 21x^3 + 8x^2 - 3x + 1)(c_2 \ln(x) + c_1) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 72

```
AsymptoticDSolveValue[4*x^2*(1+3*x+x^2)*y''[x]+8*x^2*(3+2*x)*y'[x]+(1+3*x+9*x^2)*y[x]==0,y[x],{x,0},6]
```

$$y(x) \rightarrow c_1 \sqrt{x}(-144x^5 + 55x^4 - 21x^3 + 8x^2 - 3x + 1) + c_2 \sqrt{x}(-144x^5 + 55x^4 - 21x^3 + 8x^2 - 3x + 1) \log(x)$$

15.63 problem 64

Internal problem ID [1411]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 64.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-x)^2 y'' - x(-3x^2 + 2x + 1) y' + (x^2 + 1) y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
```

```
dsolve(x^2*(1-x)^2*diff(y(x),x$2)-x*(1+2*x-3*x^2)*diff(y(x),x)+(1+x^2)*y(x)=0,y(x),type='ser
```

$$y(x) = (6x^5 + 5x^4 + 4x^3 + 3x^2 + 2x + 1) x(c_2 \ln(x) + c_1) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 64

```
AsymptoticDSolveValue[x^2*(1-x)^2*y'[x]-x*(1+2*x-3*x^2)*y'[x]+(1+x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 x(6x^5 + 5x^4 + 4x^3 + 3x^2 + 2x + 1) + c_2 x(6x^5 + 5x^4 + 4x^3 + 3x^2 + 2x + 1) \log(x)$$

15.64 problem 65

Internal problem ID [1412]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

Problem number: 65.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2(x^2 + x + 1)y'' + 3x(13x^2 + 7x + 1)y' + (25x^2 + 4x + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

Order:=6;

```
dsolve(9*x^2*(1+x+x^2)*diff(y(x),x$2)+3*x*(1+7*x+13*x^2)*diff(y(x),x)+(1+4*x+25*x^2)*y(x)=0,
```

$$y(x) = x^{\frac{1}{3}}(-x^4 + x^3 - x + 1)(c_2 \ln(x) + c_1) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 48

```
AsymptoticDSolveValue[9*x^2*(1+x+x^2)*y'[x]+3*x*(1+7*x+13*x^2)*y'[x]+(1+4*x+25*x^2)*y[x]==0,
```

$$y(x) \rightarrow c_1 \sqrt[3]{x}(-x^4 + x^3 - x + 1) + c_2 \sqrt[3]{x}(-x^4 + x^3 - x + 1) \log(x)$$

16 Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

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16.1 problem Example 7.7.1 page 381

Internal problem ID [1413]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: Example 7.7.1 page 381.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x+2)y'' - x(4-7x)y' - (5-3x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 65

Order:=6;

`dsolve(2*x^2*(2+x)*diff(y(x),x$2)-x*(4-7*x)*diff(y(x),x)-(5-3*x)*y(x)=0,y(x),type='series',x`

$$y(x) = \frac{c_1 x^3 \left(1 - \frac{7}{4}x + \frac{63}{32}x^2 - \frac{231}{128}x^3 + \frac{3003}{2048}x^4 - \frac{9009}{8192}x^5 + O(x^6)\right) + c_2 (\ln(x) \left(-\frac{45}{32}x^3 + \frac{315}{128}x^4 - \frac{2835}{1024}x^5 + O(x^6)\right))}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 98

`AsymptoticDSolveValue[2*x^2*(2+x)*y'[x]-x*(4-7*x)*y'[x]-(5-3*x)*y[x]==0,y[x],{x,0,5}]`

$$y(x) \rightarrow c_2 \left(\frac{3003x^{13/2}}{2048} - \frac{231x^{11/2}}{128} + \frac{63x^{9/2}}{32} - \frac{7x^{7/2}}{4} + x^{5/2} \right) + c_1 \left(\frac{15}{512} (7x-4)x^{5/2} \log(x) + \frac{809x^4 - 548x^3 + 96x^2 + 128x + 1024}{1024\sqrt{x}} \right)$$

16.2 problem Example 7.7.2 page 383

Internal problem ID [1414]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: Example 7.7.2 page 383.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1 - 2x)y'' + x(8 - 9x)y' + (6 - 3x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

Order:=6;

```
dsolve(x^2*(1-2*x)*diff(y(x),x$2)+x*(8-9*x)*diff(y(x),x)+(6-3*x)*y(x)=0,y(x),type='series',x
```

$$y(x) = \frac{c_1 \left(1 - \frac{1}{3}x - \frac{1}{14}x^2 - \frac{1}{28}x^3 - \frac{25}{1008}x^4 - \frac{1}{48}x^5 + O(x^6)\right)}{x} + \frac{c_2(2880 - 23760x + 71280x^2 - 83160x^3 + 62370x^5 + O(x^6))}{x^6}$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 61

```
AsymptoticDSolveValue[x^2*(1-2*x)*y'[x]+x*(8-9*x)*y'[x]+(6-3*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(-\frac{25x^3}{1008} - \frac{x^2}{28} - \frac{x}{14} + \frac{1}{x} - \frac{1}{3} \right) + c_1 \left(\frac{1}{x^6} - \frac{33}{4x^5} + \frac{99}{4x^4} - \frac{231}{8x^3} \right)$$

16.3 problem Example 7.7.3 page 385

Internal problem ID [1415]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: Example 7.7.3 page 385.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2 + 1)y'' + x(10x^2 + 3)y' - (-14x^2 + 15)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

Order:=6;

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)+x*(3+10*x^2)*diff(y(x),x)-(15-14*x^2)*y(x)=0,y(x),type='se
```

$$y(x) = c_1 x^3 \left(1 - \frac{5}{2}x^2 + \frac{35}{8}x^4 + O(x^6) \right) + \frac{c_2(-203212800 + 101606400x^2 - 25401600x^4 + O(x^6))}{x^5}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 46

```
AsymptoticDSolveValue[x^2*(1+x^2)*y'[x]+x*(3+10*x^2)*y'[x]-(15-14*x^2)*y[x]==0,y[x],{x,0,5}
```

$$y(x) \rightarrow c_1 \left(\frac{1}{x^5} - \frac{1}{2x^3} + \frac{1}{8x} \right) + c_2 \left(\frac{35x^7}{8} - \frac{5x^5}{2} + x^3 \right)$$

16.4 problem Example 7.7.4 page 387

Internal problem ID [1416]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: Example 7.7.4 page 387.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(-2x^2 + 1)y'' + x(-13x^2 + 7)y' - 14yx^2 = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
Order:=6;
```

```
dsolve(x^2*(1-2*x^2)*diff(y(x),x$2)+x*(7-13*x^2)*diff(y(x),x)-14*x^2*y(x)=0,y(x),type='series')
```

$$y(x) = c_1 \left(1 + \frac{7}{8}x^2 + \frac{77}{80}x^4 + O(x^6) \right) + \frac{c_2(-86400 + 216000x^2 - 54000x^4 + O(x^6))}{x^6}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 44

```
AsymptoticDSolveValue[x^2*(1-2*x^2)*y'[x]+x*(7-13*x^2)*y'[x]-14*x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{77x^4}{80} + \frac{7x^2}{8} + 1 \right) + c_1 \left(\frac{1}{x^6} - \frac{5}{2x^4} + \frac{5}{8x^2} \right)$$

16.5 problem 1

Internal problem ID [1417]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 1.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - 3xy' + (3 + 4x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 61

```
Order:=6;  
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+(3+4*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x \left(c_1 x^2 \left(1 - \frac{4}{3}x + \frac{2}{3}x^2 - \frac{8}{45}x^3 + \frac{4}{135}x^4 - \frac{16}{4725}x^5 + O(x^6) \right) \right. \\ \left. + c_2 \left(\ln(x) \left(16x^2 - \frac{64}{3}x^3 + \frac{32}{3}x^4 - \frac{128}{45}x^5 + O(x^6) \right) \right. \right. \\ \left. \left. + \left(-2 - 8x + \frac{256}{9}x^3 - \frac{200}{9}x^4 + \frac{5024}{675}x^5 + O(x^6) \right) \right) \right)$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 87

```
AsymptoticDSolveValue[x^2*y''[x]-3*x*y'[x]+(3+4*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{1}{9} x (124x^4 - 176x^3 + 36x^2 + 36x + 9) - \frac{8}{3} x^3 (2x^2 - 4x + 3) \log(x) \right) + c_2 \left(\frac{4x^7}{135} - \frac{8x^6}{45} + \frac{2x^5}{3} - \frac{4x^4}{3} + x^3 \right)$$

16.6 problem 2

Internal problem ID [1418]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 2.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$xy'' + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 58

```
Order:=6;  
dsolve(x*diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = & c_1 x \left(1 - \frac{1}{2}x + \frac{1}{12}x^2 - \frac{1}{144}x^3 + \frac{1}{2880}x^4 - \frac{1}{86400}x^5 + O(x^6) \right) \\ & + c_2 \left(\ln(x) \left(-x + \frac{1}{2}x^2 - \frac{1}{12}x^3 + \frac{1}{144}x^4 - \frac{1}{2880}x^5 + O(x^6) \right) \right. \\ & \left. + \left(1 - \frac{3}{4}x^2 + \frac{7}{36}x^3 - \frac{35}{1728}x^4 + \frac{101}{86400}x^5 + O(x^6) \right) \right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 85

```
AsymptoticDSolveValue[x*y''[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{1}{144} x(x^3 - 12x^2 + 72x - 144) \log(x) + \frac{-47x^4 + 480x^3 - 2160x^2 + 1728x + 1728}{1728} \right) + c_2 \left(\frac{x^5}{2880} - \frac{x^4}{144} + \frac{x^3}{12} - \frac{x^2}{2} + x \right)$$

16.7 problem 3

Internal problem ID [1419]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 3.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x+1)y'' + 4x(2x+1)y' - (3x+1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

`Order:=6;`

`dsolve(4*x^2*(1+x)*diff(y(x),x$2)+4*x*(1+2*x)*diff(y(x),x)-(1+3*x)*y(x)=0,y(x),type='series'`

$y(x)$

$$= \frac{c_1(1 + O(x^6))x + (x + O(x^6))\ln(x)c_2 + \left(1 - x - x^2 + \frac{1}{2}x^3 - \frac{1}{3}x^4 + \frac{1}{4}x^5 + O(x^6)\right)c_2}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 53

`AsymptoticDSolveValue[4*x^2*(1+x)*y'[x]+4*x*(1+2*x)*y'[x]-(1+3*x)*y[x]==0,y[x],{x,0,5}]`

$$y(x) \rightarrow c_1 \left(\sqrt{x} \log(x) - \frac{2x^4 - 3x^3 + 6x^2 + 6x - 6}{6\sqrt{x}} \right) + c_2 \sqrt{x}$$

16.8 problem 4

Internal problem ID [1420]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 4.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(x+1)y'' + xy' + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 60

```
Order:=6;
```

```
dsolve(x*(1+x)*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = & c_1 x \left(1 - x + \frac{5}{6}x^2 - \frac{25}{36}x^3 + \frac{85}{144}x^4 - \frac{221}{432}x^5 + O(x^6) \right) \\ & + \left(-x + x^2 - \frac{5}{6}x^3 + \frac{25}{36}x^4 - \frac{85}{144}x^5 + O(x^6) \right) \ln(x) c_2 \\ & + \left(1 - x + \frac{1}{2}x^2 - \frac{7}{18}x^3 + \frac{145}{432}x^4 - \frac{257}{864}x^5 + O(x^6) \right) c_2 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 85

```
AsymptoticDSolveValue[x*(1+x)*y''[x]+x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{1}{36} x (25x^3 - 30x^2 + 36x - 36) \log(x) + \frac{1}{432} (-455x^4 + 552x^3 - 648x^2 + 432x + 432) \right) + c_2 \left(\frac{85x^5}{144} - \frac{25x^4}{36} + \frac{5x^3}{6} - x^2 + x \right)$$

16.9 problem 5

Internal problem ID [1421]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 5.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(3x + 2)y'' + x(4 + 21x)y' - (1 - 9x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 69

Order:=6;

```
dsolve(2*x^2*(2+3*x)*diff(y(x),x$2)+x*(4+21*x)*diff(y(x),x)-(1-9*x)*y(x)=0,y(x),type='series
```

$$y(x) = \frac{c_1 x \left(1 - \frac{9}{4}x + \frac{135}{32}x^2 - \frac{945}{128}x^3 + \frac{25515}{2048}x^4 - \frac{168399}{8192}x^5 + O(x^6)\right) + c_2 (\ln(x) \left(-\frac{3}{4}x + \frac{27}{16}x^2 - \frac{405}{128}x^3 + \frac{2835}{512}x^4 - \dots\right))}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 108

```
AsymptoticDSolveValue[2*x^2*(2+3*x)*y'[x]+x*(4+21*x)*y'[x]-(1-9*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{25515x^{9/2}}{2048} - \frac{945x^{7/2}}{128} + \frac{135x^{5/2}}{32} - \frac{9x^{3/2}}{4} + \sqrt{x} \right) + c_1 \left(\frac{3}{512} \sqrt{x} (945x^3 - 540x^2 + 288x - 128) \log(x) + \frac{8613x^4 - 5076x^3 + 2880x^2 - 1536x + 1024}{1024\sqrt{x}} \right)$$

16.10 problem 6

Internal problem ID [1422]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 6.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + x(x+2)y' - (2-3x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 63

```
Order:=6;
```

```
dsolve(x^2*diff(y(x),x$2)+x*(2+x)*diff(y(x),x)-(2-3*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^3 \left(1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + O(x^6)\right) + c_2 (\ln(x) ((-6)x^3 + 6x^4 - 3x^5 + O(x^6)) + (12 + 6x^2))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 74

```
AsymptoticDSolveValue[x^2*y''[x]+x*(2+x)*y'[x]-(2-3*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{x^4 - 3x^3 + 2x^2 + 2x + 4}{4x^2} + \frac{1}{2}(x-1)x \log(x) \right) + c_2 \left(\frac{x^5}{24} - \frac{x^4}{6} + \frac{x^3}{2} - x^2 + x \right)$$

16.11 problem 7

Internal problem ID [1423]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 7.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + 4xy' - (9 - x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 63

```
Order:=6;
```

```
dsolve(4*x^2*diff(y(x),x$2)+4*x*diff(y(x),x)-(9-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^3 \left(1 - \frac{1}{16}x + \frac{1}{640}x^2 - \frac{1}{46080}x^3 + \frac{1}{5160960}x^4 - \frac{1}{825753600}x^5 + O(x^6)\right) + c_2 (\ln(x) \left(-\frac{1}{64}x^3 + \frac{1}{1024}x^4 - \frac{1}{40960}x^5 + \frac{1}{163840}x^6 + O(x^7)\right))}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 96

```
AsymptoticDSolveValue[4*x^2*y'[x]+4*x*y'[x]-(9-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{x^{11/2}}{5160960} - \frac{x^{9/2}}{46080} + \frac{x^{7/2}}{640} - \frac{x^{5/2}}{16} + x^{3/2} \right) + c_1 \left(\frac{(x-16)x^{3/2} \log(x)}{12288} - \frac{19x^4 - 64x^3 - 2304x^2 - 18432x - 147456}{147456x^{3/2}} \right)$$

16.12 problem 8

Internal problem ID [1424]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 8.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + 10xy' + (14 + x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 59

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+10*x*diff(y(x),x)+(14+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 \left(1 - \frac{1}{6}x + \frac{1}{84}x^2 - \frac{1}{2016}x^3 + \frac{1}{72576}x^4 - \frac{1}{3628800}x^5 + O(x^6)\right) x^5 + c_2 (\ln(x) (-x^5 + O(x^6))) + (2880 + 720 \ln(x))}{x^7}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 68

```
AsymptoticDSolveValue[x^2*y''[x]+10*x*y'[x]+(14+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{x^2}{72576} + \frac{1}{x^2} - \frac{x}{2016} - \frac{1}{6x} + \frac{1}{84} \right) + c_1 \left(\frac{1}{x^7} + \frac{1}{4x^6} + \frac{1}{24x^5} + \frac{1}{144x^4} + \frac{1}{576x^3} \right)$$

16.13 problem 9

Internal problem ID [1425]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 9.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x+1)y'' + 4x(3+8x)y' - (5-49x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 65

```
Order:=6;
```

```
dsolve(4*x^2*(1+x)*diff(y(x),x$2)+4*x*(3+8*x)*diff(y(x),x)-(5-49*x)*y(x)=0,y(x),type='series
```

$$y(x) = \frac{c_1 x^3 (1 - 4x + 10x^2 - 20x^3 + 35x^4 - 56x^5 + O(x^6)) + c_2 (\ln(x) ((-36)x^3 + 144x^4 - 360x^5 + O(x^6)) + x^{\frac{5}{2}})}{x^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 86

```
AsymptoticDSolveValue[4*x^2*(1+x)*y'[x]+4*x*(3+8*x)*y'[x]-(5-49*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 (35x^{9/2} - 20x^{7/2} + 10x^{5/2} - 4x^{3/2} + \sqrt{x}) + c_1 \left(\frac{62x^4 - 20x^3 + 2x^2 + x + 2}{2x^{5/2}} + 3\sqrt{x}(4x-1)\log(x) \right)$$

16.14 problem 10

Internal problem ID [1426]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 10.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x+1)y'' - x(3+10x)y' + 30yx = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 48

Order:=6;

```
dsolve(x^2*(1+x)*diff(y(x),x$2)-x*(3+10*x)*diff(y(x),x)+30*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^4 \left(1 - \frac{2}{5}x + O(x^6) \right) + (43200x^4 - 17280x^5 + O(x^6)) \ln(x) c_2 \\ + (-144 - 1440x - 7200x^2 - 28800x^3 - 90720x^4 + 82944x^5 + O(x^6)) c_2$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 48

```
AsymptoticDSolveValue[x^2*(1+x)*y'[x]-x*(3+10*x)*y'[x]+30*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(x^4 - \frac{2x^5}{5} \right) + c_1 (745x^4 - 300x^4 \log(x) + 200x^3 + 50x^2 + 10x + 1)$$

16.15 problem 11

Internal problem ID [1427]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 11.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + x(x+1)y' - 3(3+x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

```
Order:=6;
```

```
dsolve(x^2*diff(y(x),x$2)+x*(1+x)*diff(y(x),x)-3*(3+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^3 (1 + O(x^6)) + \frac{c_2 (-86400 + 103680x - 64800x^2 + 28800x^3 - 10800x^4 + 4320x^5 + O(x^6))}{x^3}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 39

```
AsymptoticDSolveValue[x^2*y''[x]+x*(1+x)*y'[x]-3*(3+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 x^3 + c_1 \left(\frac{1}{x^3} - \frac{6}{5x^2} + \frac{x}{8} + \frac{3}{4x} - \frac{1}{3} \right)$$

16.16 problem 12

Internal problem ID [1428]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 12.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + x(1 - 2x)y' - (4 + x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 61

```
Order:=6;
```

```
dsolve(x^2*diff(y(x),x$2)+x*(1-2*x)*diff(y(x),x)-(4+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^4 \left(1 + x + \frac{7}{12}x^2 + \frac{1}{4}x^3 + \frac{11}{128}x^4 + \frac{143}{5760}x^5 + O(x^6)\right) + c_2 (\ln(x) (9x^4 + 9x^5 + O(x^6))) + (-144 - 144x)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 75

```
AsymptoticDSolveValue[x^2*y''[x]+x*(1-2*x)*y'[x]-(4+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{3x^4 - 16x^3 + 48x^2 + 192x + 192}{192x^2} - \frac{1}{16}x^2 \log(x) \right) + c_2 \left(\frac{11x^6}{128} + \frac{x^5}{4} + \frac{7x^4}{12} + x^3 + x^2 \right)$$

16.17 problem 13

Internal problem ID [1429]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 13.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x(x+1)y'' - 4y' - 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 38.375 (sec). Leaf size: 40

```
Order:=6;  
dsolve(x*(1+x)*diff(y(x),x$2)-4*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^5 (1 - 3x + 6x^2 - 10x^3 + 15x^4 - 21x^5 + O(x^6)) \\ + c_2 (2880 - 1440x + 480x^2 - 480x^5 + O(x^6))$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 48

```
AsymptoticDSolveValue[x*(1+x)*y''[x]-4*y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{x^2}{6} - \frac{x}{2} + 1 \right) + c_2 (15x^9 - 10x^8 + 6x^7 - 3x^6 + x^5)$$

16.18 problem 14

Internal problem ID [1430]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 14.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(2x + 1)y'' + x(9 + 13x)y' + (7 + 5x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 41

```
Order:=6;
```

```
dsolve(x^2*(1+2*x)*diff(y(x),x$2)+x*(9+13*x)*diff(y(x),x)+(7+5*x)*y(x)=0,y(x),type='series',
```

$$y(x) = \frac{c_1 \left(1 + \frac{4}{7}x - \frac{5}{28}x^2 + \frac{5}{42}x^3 - \frac{5}{48}x^4 + \frac{7}{66}x^5 + O(x^6)\right)}{x} + \frac{c_2(-86400 - 449280x - 617760x^2 + O(x^6))}{x^7}$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 54

```
AsymptoticDSolveValue[x^2*(1+2*x)*y'[x]+x*(9+13*x)*y'[x]+(7+5*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(-\frac{5x^3}{48} + \frac{5x^2}{42} - \frac{5x}{28} + \frac{1}{x} + \frac{4}{7} \right) + c_1 \left(\frac{1}{x^7} + \frac{26}{5x^6} + \frac{143}{20x^5} \right)$$

16.19 problem 15

Internal problem ID [1431]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 15.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(2x + 1)y'' - 2x(4 - x)y' - (7 + 5x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

`Order:=6;`

`dsolve(4*x^2*(1+2*x)*diff(y(x),x$2)-2*x*(4-x)*diff(y(x),x)-(7+5*x)*y(x)=0,y(x),type='series'`

$$y(x) = \frac{c_1 x^4 \left(1 - \frac{18}{5}x + \frac{39}{4}x^2 - \frac{663}{28}x^3 + \frac{13923}{256}x^4 - \frac{7735}{64}x^5 + O(x^6)\right) + c_2 \left(-144 - \frac{405}{8}x^4 + \frac{729}{4}x^5 + O(x^6)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 67

`AsymptoticDSolveValue[4*x^2*(1+2*x)*y'[x]-2*x*(4-x)*y'[x]-(7+5*x)*y[x]==0,y[x],{x,0,5}]`

$$y(x) \rightarrow c_1 \left(\frac{1}{\sqrt{x}} - \frac{35x^{7/2}}{128} \right) + c_2 \left(\frac{13923x^{15/2}}{256} - \frac{663x^{13/2}}{28} + \frac{39x^{11/2}}{4} - \frac{18x^{9/2}}{5} + x^{7/2} \right)$$

16.20 problem 16

Internal problem ID [1432]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 16.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3x^2(3+x)y'' - x(15+x)y' - 20y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
```

```
dsolve(3*x^2*(3+x)*diff(y(x),x$2)-x*(15+x)*diff(y(x),x)-20*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^4 \left(1 - \frac{4}{9}x + \frac{13}{81}x^2 - \frac{832}{15309}x^3 + \frac{2470}{137781}x^4 - \frac{21736}{3720087}x^5 + O(x^6)\right) + c_2 \left(-144 - \frac{64}{3}x + \frac{16}{27}x^2 - \frac{112}{6561}x^4 + \frac{448}{5904}x^5\right)}{x^{\frac{2}{3}}}$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 85

```
AsymptoticDSolveValue[3*x^2*(3+x)*y'[x]-x*(15+x)*y'[x]-20*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{7x^{10/3}}{59049} - \frac{x^{4/3}}{243} + \frac{1}{x^{2/3}} + \frac{4\sqrt[3]{x}}{27} \right) + c_2 \left(\frac{2470x^{22/3}}{137781} - \frac{832x^{19/3}}{15309} + \frac{13x^{16/3}}{81} - \frac{4x^{13/3}}{9} + x^{10/3} \right)$$

16.21 problem 17

Internal problem ID [1433]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 17.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x+1)y'' + x(1-10x)y' - (9-10x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 45

Order:=6;

```
dsolve(x^2*(1+x)*diff(y(x),x$2)+x*(1-10*x)*diff(y(x),x)-(9-10*x)*y(x)=0,y(x),type='series',x
```

$$y(x) = c_1 x^3 \left(1 + 2x + \frac{9}{4}x^2 + \frac{5}{3}x^3 + \frac{5}{6}x^4 + \frac{3}{11}x^5 + O(x^6) \right) + \frac{c_2(-86400 - 898560x - 4043520x^2 - 9884160x^3 - 12355200x^4 + O(x^6))}{x^3}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 64

```
AsymptoticDSolveValue[x^2*(1+x)*y'[x]+x*(1-10*x)*y'[x]-(9-10*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{1}{x^3} + \frac{52}{5x^2} + 143x + \frac{234}{5x} + \frac{572}{5} \right) + c_2 \left(\frac{5x^7}{6} + \frac{5x^6}{3} + \frac{9x^5}{4} + 2x^4 + x^3 \right)$$

16.22 problem 18

Internal problem ID [1434]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 18.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x+1)y'' + 3y'x^2 - (6-x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 39

```
Order:=6;
```

```
dsolve(x^2*(1+x)*diff(y(x),x$2)+3*x^2*diff(y(x),x)-(6-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^3 \left(1 - \frac{8}{3}x + \frac{100}{21}x^2 - \frac{50}{7}x^3 + \frac{175}{18}x^4 - \frac{112}{9}x^5 + O(x^6) \right) + \frac{c_2(2880 + 720x + O(x^6))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 53

```
AsymptoticDSolveValue[x^2*(1+x)*y'[x]+3*x^2*y'[x]-(6-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{1}{x^2} + \frac{1}{4x} \right) + c_2 \left(\frac{175x^7}{18} - \frac{50x^6}{7} + \frac{100x^5}{21} - \frac{8x^4}{3} + x^3 \right)$$

16.23 problem 19

Internal problem ID [1435]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 19.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(2x + 1)y'' - 2x(3 + 14x)y' + (6 + 100x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

`Order:=6;`

`dsolve(x^2*(1+2*x)*diff(y(x),x$2)-2*x*(3+14*x)*diff(y(x),x)+(6+100*x)*y(x)=0,y(x),type='series')`

$$y(x) = c_1 x^6 \left(1 + \frac{4}{3}x + \frac{8}{7}x^2 + \frac{4}{7}x^3 + \frac{8}{63}x^4 + O(x^6) \right) + c_2 x (2880 + 51840x + 414720x^2 + 1935360x^3 + 5806080x^4 + 11612160x^5 + O(x^6))$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 64

`AsymptoticDSolveValue[x^2*(1+2*x)*y'[x]-2*x*(3+14*x)*y'[x]+(6+100*x)*y[x]==0,y[x],{x,0,5}]`

$$y(x) \rightarrow c_1 (2016x^5 + 672x^4 + 144x^3 + 18x^2 + x) + c_2 \left(\frac{8x^{10}}{63} + \frac{4x^9}{7} + \frac{8x^8}{7} + \frac{4x^7}{3} + x^6 \right)$$

16.24 problem 20

Internal problem ID [1436]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 20.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x+1)y'' - x(6+11x)y' + (6+32x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

`Order:=6;`

`dsolve(x^2*(1+x)*diff(y(x),x$2)-x*(6+11*x)*diff(y(x),x)+(6+32*x)*y(x)=0,y(x),type='series',x`

$$y(x) = c_1 x^6 \left(1 + \frac{2}{3}x + \frac{1}{7}x^2 + O(x^6) \right) + c_2 x (2880 + 15120x + 30240x^2 + 25200x^3 - 15120x^5 + O(x^6))$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 51

`AsymptoticDSolveValue[x^2*(1+x)*y'[x]-x*(6+11*x)*y'[x]+(6+32*x)*y[x]==0,y[x],{x,0,5}]`

$$y(x) \rightarrow c_2 \left(\frac{x^8}{7} + \frac{2x^7}{3} + x^6 \right) + c_1 \left(\frac{35x^4}{4} + \frac{21x^3}{2} + \frac{21x^2}{4} + x \right)$$

16.25 problem 21

Internal problem ID [1437]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 21.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x+1)y'' + 4x(4x+1)y' - (49+27x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

`Order:=6;`

`dsolve(4*x^2*(1+x)*diff(y(x),x$2)+4*x*(1+4*x)*diff(y(x),x)-(49+27*x)*y(x)=0,y(x),type='series')`

$$y(x) = \frac{c_1 x^7 (1 - 2x + 3x^2 - 4x^3 + 5x^4 - 6x^5 + O(x^6)) + c_2 (3628800 - 3024000x + 2419200x^2 - 1814400x^3 + \dots)}{x^{\frac{7}{2}}}$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 86

`AsymptoticDSolveValue[4*x^2*(1+x)*y'[x]+4*x*(1+4*x)*y'[x]-(49+27*x)*y[x]==0,y[x],{x,0,5}]`

$$y(x) \rightarrow c_1 \left(\frac{2}{3x^{3/2}} - \frac{5}{6x^{5/2}} + \frac{1}{x^{7/2}} + \frac{\sqrt{x}}{3} - \frac{1}{2\sqrt{x}} \right) + c_2 (5x^{15/2} - 4x^{13/2} + 3x^{11/2} - 2x^{9/2} + x^{7/2})$$

16.26 problem 22

Internal problem ID [1438]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 22.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`, `[_2nd_order, _linear,`

$$x^2(2x + 1)y'' - x(9 + 8x)y' - 12yx = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
Order:=6;
```

```
dsolve(x^2*(1+2*x)*diff(y(x),x$2)-x*(9+8*x)*diff(y(x),x)-12*x*y(x)=0,y(x),type='series',x=0)
```

$$y(x) = c_1 x^{10} (1 - 8x + 40x^2 - 160x^3 + 560x^4 - 1792x^5 + O(x^6)) \\ + c_2 (-1316818944000 + 1755758592000x - 2194698240000x^2 \\ + 2508226560000x^3 - 2508226560000x^4 + 2006581248000x^5 + O(x^6))$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 62

```
AsymptoticDSolveValue[x^2*(1+2*x)*y'[x]-x*(9+8*x)*y'[x]-12*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{40x^4}{21} - \frac{40x^3}{21} + \frac{5x^2}{3} - \frac{4x}{3} + 1 \right) + c_2 (560x^{14} - 160x^{13} + 40x^{12} - 8x^{11} + x^{10})$$

16.27 problem 23

Internal problem ID [1439]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 23.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2 + 1)y'' - x(-2x^2 + 7)y' + 12y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

Order:=6;

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(7-2*x^2)*diff(y(x),x)+12*y(x)=0,y(x),type='series',x=0)
```

$$y(x) = \left(c_1 x^4 \left(1 - \frac{7}{2}x^2 + \frac{63}{8}x^4 + O(x^6) \right) + c_2 \left(\ln(x) (1080x^4 + O(x^6)) + (-144 - 216x^2 + 2106x^4 + O(x^6)) \right) \right) x^2$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 57

```
AsymptoticDSolveValue[x^2*(1+x^2)*y'[x]-x*(7-2*x^2)*y'[x]+12*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{63x^{10}}{8} - \frac{7x^8}{2} + x^6 \right) + c_1 \left(-\frac{15}{2}x^6 \log(x) - \frac{1}{4}(31x^4 - 6x^2 - 4)x^2 \right)$$

16.28 problem 24

Internal problem ID [1440]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 24.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' - x(-x^2 + 7) y' + 12y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*(7-x^2)*diff(y(x),x)+12*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(c_1 x^4 \left(1 - \frac{1}{2} x^2 + \frac{1}{8} x^4 + O(x^6) \right) + c_2 (\ln(x) (72x^4 + O(x^6)) + (-144 - 72x^2 + 54x^4 + O(x^6))) \right) x^2$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 55

```
AsymptoticDSolveValue[x^2*y'[x]-x*(7-x^2)*y'[x]+12*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{x^{10}}{8} - \frac{x^8}{2} + x^6 \right) + c_1 \left(-\frac{1}{2} x^6 \log(x) - \frac{1}{4} (x^4 - 2x^2 - 4) x^2 \right)$$

16.29 problem 25

Internal problem ID [1441]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 25.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [Lienard]

$$xy'' - 5y' + yx = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
Order:=6;  
dsolve(x*diff(y(x),x$2)-5*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^6 \left(1 - \frac{1}{16} x^2 + \frac{1}{640} x^4 + O(x^6) \right) + c_2 (-86400 - 10800x^2 - 1350x^4 + O(x^6))$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 44

```
AsymptoticDSolveValue[x*y''[x]-5*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{x^4}{64} + \frac{x^2}{8} + 1 \right) + c_2 \left(\frac{x^{10}}{640} - \frac{x^8}{16} + x^6 \right)$$

16.30 problem 26

Internal problem ID [1442]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 26.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + x(2x^2 + 1) y' - (-10x^2 + 1) y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

Order:=6;

```
dsolve(x^2*diff(y(x),x$2)+x*(1+2*x^2)*diff(y(x),x)-(1-10*x^2)*y(x)=0,y(x),type='series',x=0)
```

$$y(x) = \frac{c_1 x^2 \left(1 - \frac{3}{2} x^2 + x^4 + O(x^6)\right) + c_2 (\ln(x) (8x^2 - 12x^4 + O(x^6)) + (-2 + 2x^2 + 4x^4 + O(x^6)))}{x}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 52

```
AsymptoticDSolveValue[x^2*y''[x]+x*(1+2*x^2)*y'[x]-(1-10*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(x^5 - \frac{3x^3}{2} + x \right) + c_1 \left(2x(3x^2 - 2) \log(x) - \frac{5x^4 - x^2 - 1}{x} \right)$$

16.31 problem 27

Internal problem ID [1443]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 27.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' - xy' - (-x^2 + 3)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)-(3-x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^4 \left(1 - \frac{1}{12} x^2 + \frac{1}{384} x^4 + O(x^6)\right) + c_2 (\ln(x) (9x^4 + O(x^6)) + (-144 - 36x^2 + O(x^6)))}{x}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 52

```
AsymptoticDSolveValue[x^2*y'[x]-x*y'[x]-(3-x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{(x^2 + 8)^2}{64x} - \frac{1}{16} x^3 \log(x) \right) + c_2 \left(\frac{x^7}{384} - \frac{x^5}{12} + x^3 \right)$$

16.32 problem 28

Internal problem ID [1444]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 28.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + 2x(x^2 + 8)y' + (3x^2 + 5)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
Order:=6;
```

```
dsolve(4*x^2*dif(y(x),x$2)+2*x*(8+x^2)*dif(y(x),x)+(5+3*x^2)*y(x)=0,y(x),type='series',x=0)
```

$y(x)$

$$= \frac{c_1 \left(1 - \frac{1}{16}x^2 + \frac{1}{256}x^4 + O(x^6)\right) x^2 + c_2 \left(\ln(x) \left(-\frac{1}{2}x^2 + \frac{1}{32}x^4 + O(x^6)\right) + \left(-2 + \frac{1}{2}x^2 - \frac{3}{128}x^4 + O(x^6)\right)\right)}{x^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 72

```
AsymptoticDSolveValue[4*x^2*y''[x]+2*x*(8+x^2)*y'[x]+(5+3*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{x^{7/2}}{256} - \frac{x^{3/2}}{16} + \frac{1}{\sqrt{x}} \right) + c_1 \left(\frac{5x^4 - 96x^2 + 256}{256x^{5/2}} - \frac{(x^2 - 16) \log(x)}{64\sqrt{x}} \right)$$

16.33 problem 29

Internal problem ID [1445]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 29.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x^2 y'' + x(x^2 + 1) y' - (-3x^2 + 1) y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
Order:=6;
```

```
dsolve(x^2*diff(y(x),x$2)+x*(1+x^2)*diff(y(x),x)-(1-3*x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^2 \left(1 - \frac{1}{2} x^2 + \frac{1}{8} x^4 + O(x^6)\right) + c_2 (\ln(x) (2x^2 - x^4 + O(x^6)) + (-2 + x^2 + O(x^6)))}{x}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 51

```
AsymptoticDSolveValue[x^2*y'[x]+x*(1+x^2)*y'[x]-(1-3*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{x^5}{8} - \frac{x^3}{2} + x \right) + c_1 \left(\frac{1}{2} x (x^2 - 2) \log(x) - \frac{x^4 - 4}{4x} \right)$$

16.34 problem 30

Internal problem ID [1446]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 30.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + x(-2x^2 + 1)y' - 4(2x^2 + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

```
Order:=6;
```

```
dsolve(x^2*diff(y(x),x$2)+x*(1-2*x^2)*diff(y(x),x)-4*(1+2*x^2)*y(x)=0,y(x),type='series',x=0)
```

$y(x)$

$$= \frac{c_1 x^4 (1 + x^2 + \frac{1}{2} x^4 + O(x^6)) + c_2 (\ln(x) (288 x^4 + O(x^6)) + (-144 + 144 x^2 + 216 x^4 + O(x^6)))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 45

```
AsymptoticDSolveValue[x^2*y''[x]+x*(1-2*x^2)*y'[x]-4*(1+2*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(-2x^2 \log(x) - \frac{x^4 + x^2 - 1}{x^2} \right) + c_2 \left(\frac{x^6}{2} + x^4 + x^2 \right)$$

16.35 problem 31

Internal problem ID [1447]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 31.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + 8xy' - (-x^2 + 35)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

Order:=6;

```
dsolve(4*x^2*diff(y(x),x$2)+8*x*diff(y(x),x)-(35-x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^6\left(1 - \frac{1}{64}x^2 + \frac{1}{10240}x^4 + O(x^6)\right) + c_2\left(-86400 - 2700x^2 - \frac{675}{8}x^4 + O(x^6)\right)}{x^{\frac{7}{2}}}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 58

```
AsymptoticDSolveValue[4*x^2*y''[x]+8*x*y'[x]-(35-x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1\left(\frac{1}{32x^{3/2}} + \frac{1}{x^{7/2}} + \frac{\sqrt{x}}{1024}\right) + c_2\left(\frac{x^{13/2}}{10240} - \frac{x^{9/2}}{64} + x^{5/2}\right)$$

16.36 problem 32

Internal problem ID [1448]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 32.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2y'' - 3x(2x^2 + 11)y' + (10x^2 + 13)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

Order:=6;

```
dsolve(9*x^2*diff(y(x),x$2)-3*x*(11+2*x^2)*diff(y(x),x)+(13+10*x^2)*y(x)=0,y(x),type='series
```

$$y(x) = \left(x^4 \left(1 + \frac{4}{27}x^2 + \frac{7}{486}x^4 + O(x^6) \right) c_1 + c_2 \left(\ln(x) \left(-\frac{32}{9}x^4 + O(x^6) \right) + \left(-144 - 32x^2 - \frac{8}{3}x^4 + O(x^6) \right) \right) \right) x^{\frac{1}{3}}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 62

```
AsymptoticDSolveValue[9*x^2*y'[x]-3*x*(11+2*x^2)*y'[x]+(13+10*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{7x^{25/3}}{486} + \frac{4x^{19/3}}{27} + x^{13/3} \right) + c_1 \left(\frac{2}{81}x^{13/3} \log(x) + \frac{1}{81}(x^2 + 9)^2 \sqrt[3]{x} \right)$$

16.37 problem 33

Internal problem ID [1449]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 33.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`, `[_2nd_order, _linear,`

$$x^2 y'' + x(-2x^2 + 1)y' - 4(1 - x^2)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
```

```
dsolve(x^2*diff(y(x),x$2)+x*(1-2*x^2)*diff(y(x),x)-4*(1-x^2)*y(x)=0,y(x),type='series',x=0);
```

$y(x)$

$$= c_1 x^2 (1 + O(x^6)) + \frac{c_2 (\ln(x) (288x^4 + O(x^6)) + (-144 - 288x^2 - 216x^4 + O(x^6)))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 37

```
AsymptoticDSolveValue[x^2*y''[x]+x*(1-2*x^2)*y'[x]-4*(1-x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 x^2 + c_1 \left(\frac{2x^4 + 2x^2 + 1}{x^2} - 2x^2 \log(x) \right)$$

16.38 problem 34

Internal problem ID [1450]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 34.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + x(-3x^2 + 1) y' - 4(-3x^2 + 1) y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

Order:=6;

```
dsolve(x^2*diff(y(x),x$2)+x*(1-3*x^2)*diff(y(x),x)-4*(1-3*x^2)*y(x)=0,y(x),type='series',x=0)
```

$$y(x) = c_1 x^2 \left(1 - \frac{1}{2} x^2 + O(x^6) \right) + \frac{c_2 (\ln(x) (1944x^4 + O(x^6)) + (-144 - 648x^2 - 810x^4 + O(x^6)))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 50

```
AsymptoticDSolveValue[x^2*y''[x]+x*(1-3*x^2)*y'[x]-4*(1-3*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(x^2 - \frac{x^4}{2} \right) + c_1 \left(\frac{18x^4 + 9x^2 + 2}{2x^2} - \frac{27}{2} x^2 \log(x) \right)$$

16.39 problem 35

Internal problem ID [1451]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 35.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2 + 1)y'' + x(11x^2 + 5)y' + 24yx^2 = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
Order:=6;
```

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)+x*(5+11*x^2)*diff(y(x),x)+24*x^2*y(x)=0,y(x),type='series')
```

$$y(x) = c_1(1 - 2x^2 + 3x^4 + O(x^6)) + \frac{c_2(-144 + 432x^4 + O(x^6))}{x^4}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 27

```
AsymptoticDSolveValue[x^2*(1+x^2)*y'[x]+x*(5+11*x^2)*y'[x]+24*x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1\left(\frac{1}{x^4} - 1\right) + c_2(3x^4 - 2x^2 + 1)$$

16.40 problem 36

Internal problem ID [1452]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 36.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x^2 + 1)y'' + 8xy' - (-x^2 + 35)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
Order:=6;
```

```
dsolve(4*x^2*(1+x^2)*diff(y(x),x$2)+8*x*diff(y(x),x)-(35-x^2)*y(x)=0,y(x),type='series',x=0)
```

$$y(x) = \frac{c_1 x^6 \left(1 - \frac{1}{4}x^2 + \frac{1}{10}x^4 + O(x^6)\right) + c_2 (-86400 - 172800x^2 - 86400x^4 + O(x^6))}{x^{\frac{7}{2}}}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 52

```
AsymptoticDSolveValue[4*x^2*(1+x^2)*y'[x]+8*x*y'[x]-(35-x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{2}{x^{3/2}} + \frac{1}{x^{7/2}} + \sqrt{x} \right) + c_2 \left(\frac{x^{13/2}}{10} - \frac{x^{9/2}}{4} + x^{5/2} \right)$$

16.41 problem 37

Internal problem ID [1453]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 37.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2 + 1)y'' - x(-x^2 + 5)y' - (25x^2 + 7)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

Order:=6;

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(5-x^2)*diff(y(x),x)-(7+25*x^2)*y(x)=0,y(x),type='series
```

$$y(x) = c_1 x^7 \left(1 - \frac{6}{5}x^2 + \frac{7}{5}x^4 + O(x^6) \right) + \frac{c_2(-203212800 + 406425600x^2 - 609638400x^4 + O(x^6))}{x}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 40

```
AsymptoticDSolveValue[x^2*(1+x^2)*y'[x]-x*(5-x^2)*y'[x]-(7+25*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(3x^3 - 2x + \frac{1}{x} \right) + c_2 \left(\frac{7x^{11}}{5} - \frac{6x^9}{5} + x^7 \right)$$

16.42 problem 38

Internal problem ID [1454]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 38.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2 + 1)y'' + x(2x^2 + 5)y' - 21y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
Order:=6;
```

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)+x*(5+2*x^2)*diff(y(x),x)-21*y(x)=0,y(x),type='series',x=0)
```

$$y(x) = c_1 x^3 \left(1 - \frac{1}{2}x^2 + \frac{15}{56}x^4 + O(x^6) \right) + \frac{c_2 (-1316818944000 - 3456649728000x^2 - 2880541440000x^4 + O(x^6))}{x^7}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 46

```
AsymptoticDSolveValue[x^2*(1+x^2)*y''[x]+x*(5+2*x^2)*y'[x]-21*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{1}{x^7} + \frac{21}{8x^5} + \frac{35}{16x^3} \right) + c_2 \left(\frac{15x^7}{56} - \frac{x^5}{2} + x^3 \right)$$

16.43 problem 39

Internal problem ID [1455]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 39.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1 + 2x^2)y'' - x(x^2 + 3)y' - 2yx = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 54

Order:=6;

```
dsolve(x^2*(1+2*x^2)*diff(y(x),x$2)-x*(3+x^2)*diff(y(x),x)-2*x*y(x)=0,y(x),type='series',x=0
```

$$y(x) = c_1 x^4 \left(1 + \frac{2}{5}x - \frac{8}{5}x^2 - \frac{86}{105}x^3 + \frac{445}{168}x^4 + \frac{9571}{6300}x^5 + O(x^6) \right) \\ + c_2 \left(\ln(x) \left(24x^4 + \frac{48}{5}x^5 + O(x^6) \right) \right. \\ \left. + \left(-144 + 96x - 48x^2 + 210x^4 + \frac{1812}{25}x^5 + O(x^6) \right) \right)$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 71

```
AsymptoticDSolveValue[x^2*(1+2*x^2)*y'[x]-x*(3+x^2)*y'[x]-2*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{1}{12}(3x^4 + 4x^2 - 8x + 12) - \frac{1}{6}x^4 \log(x) \right) + c_2 \left(\frac{445x^8}{168} - \frac{86x^7}{105} - \frac{8x^6}{5} + \frac{2x^5}{5} + x^4 \right)$$

16.44 problem 40

Internal problem ID [1456]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

Problem number: 40.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x^2 + 1)y'' + 4x(x^2 + 2)y' - (x^2 + 15)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
Order:=6;
```

```
dsolve(4*x^2*(1+x^2)*diff(y(x),x$2)+4*x*(2+x^2)*diff(y(x),x)-(15+x^2)*y(x)=0,y(x),type='series')
```

$$y(x) = \frac{c_1 x^4 \left(1 - \frac{1}{6}x^2 + \frac{1}{16}x^4 + O(x^6)\right) + c_2 (-144 - 216x^2 - 54x^4 + O(x^6))}{x^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 58

```
AsymptoticDSolveValue[4*x^2*(1+x^2)*y'[x]+4*x*(2+x^2)*y'[x]-(15+x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(\frac{3x^{3/2}}{8} + \frac{1}{x^{5/2}} + \frac{3}{2\sqrt{x}} \right) + c_2 \left(\frac{x^{11/2}}{16} - \frac{x^{7/2}}{6} + x^{3/2} \right)$$

17 Chapter 9 Introduction to Linear Higher Order Equations. Section 9.1. Page 471

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17.1 problem section 9.1, problem 2

Internal problem ID [1457]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.1. Page 471

Problem number: section 9.1, problem 2.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _homogeneous]]`

$$x^3 y''' - x^2 y'' - 2y'x + 6y = 0$$

With initial conditions

$$[y(-1) = -4, y'(-1) = -14, y''(-1) = -20]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 20

```
dsolve([x^3*diff(y(x),x$3)-x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+6*y(x)=0,y(-1) = -4, D(y)(-1)
```

$$y(x) = \frac{22x^2}{3} + \frac{25}{3x} + 3x^3$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 24

```
DSolve[{x^3*y'''[x]-x^2*y''[x]-2*x*y'[x]+6*y[x]==0,{y[-1]==-4,y'[-1]==-14,y''[-1]==-20}},y[x]
```

$$y(x) \rightarrow \frac{9x^4 + 22x^3 + 25}{3x}$$

17.2 problem section 9.1, problem 3

Internal problem ID [1458]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.1. Page 471

Problem number: section 9.1, problem 3.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' + y''' - 7y'' - y' + 6y = 0$$

With initial conditions

$$[y(0) = 5, y'(0) = -6, y''(0) = 10, y'''(0) = -36]$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 25

```
dsolve([diff(y(x),x$4)+diff(y(x),x$3)-7*diff(y(x),x$2)-diff(y(x),x)+6*y(x)=0,y(0) = 5, D(y)
```

$$y(x) = (-e^{5x} + 2e^{4x} + 3e^{2x} + 1)e^{-3x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 30

```
DSolve[{y''''[x]+y'''[x]-7*y''[x]-y'[x]+6*y[x]==0,{y[0]==5,y'[0]==-6,y''[0]==10,y'''[0]==-36
```

$$y(x) \rightarrow e^{-3x} + 3e^{-x} + 2e^x - e^{2x}$$

17.3 problem section 9.1, problem 5(b) 1

Internal problem ID [1459]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.1. Page 471

Problem number: section 9.1, problem 5(b) 1.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _homogeneous]]`

$$x^3 y''' - x^2 y'' - 2y'x + 6y = 0$$

With initial conditions

$$[y(1) = 1, y'(1) = 0, y''(1) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve([x^3*diff(y(x),x$3)-x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+6*y(x)=0,y(1) = 1, D(y)(1) =
```

$$y(x) = x^2 + \frac{1}{2x} - \frac{x^3}{2}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 23

```
DSolve[{x^3*y'''[x]-x^2*y''[x]-2*x*y'[x]+6*y[x]==0,{y[1]==1,y'[1]==0,y''[1]==0}},y[x],x,Incl
```

$$y(x) \rightarrow -\frac{x^3}{2} + x^2 + \frac{1}{2x}$$

17.4 problem section 9.1, problem 5(b) 2

Internal problem ID [1460]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.1. Page 471

Problem number: section 9.1, problem 5(b) 2.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _homogeneous]]`

$$x^3 y''' - x^2 y'' - 2y'x + 6y = 0$$

With initial conditions

$$[y(1) = 0, y'(1) = 1, y''(1) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve([x^3*diff(y(x),x$3)-x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+6*y(x)=0,y(1) = 0, D(y)(1) =
```

$$y(x) = \frac{x^3 - 1}{3x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 17

```
DSolve[{x^3*y'''[x]-x^2*y''[x]-2*x*y'[x]+6*y[x]==0,{y[1]==0,y'[1]==1,y''[1]==0}},y[x],x,Incl
```

$$y(x) \rightarrow \frac{x^3 - 1}{3x}$$

17.5 problem section 9.1, problem 5(b) 3

Internal problem ID [1461]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.1. Page 471

Problem number: section 9.1, problem 5(b) 3.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _homogeneous]]`

$$x^3y''' - x^2y'' - 2y'x + 6y = 0$$

With initial conditions

$$[y(1) = 0, y'(1) = 0, y''(1) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve([x^3*diff(y(x),x$3)-x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+6*y(x)=0,y(1) = 0, D(y)(1) =
```

$$y(x) = \frac{3x^4 - 4x^3 + 1}{12x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 24

```
DSolve[{x^3*y'''[x]-x^2*y''[x]-2*x*y'[x]+6*y[x]==0,{y[1]==0,y'[1]==0,y''[1]==1}},y[x],x,Incl
```

$$y(x) \rightarrow \frac{3x^4 - 4x^3 + 1}{12x}$$

17.6 problem section 9.1, problem 5(b)

Internal problem ID [1462]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.1. Page 471

Problem number: section 9.1, problem 5(b).

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _homogeneous]]`

$$x^3 y''' - x^2 y'' - 2y'x + 6y = 0$$

With initial conditions

$$[y(1) = k_0, y'(1) = k_1, y''(1) = k_2]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 40

```
dsolve([x^3*diff(y(x),x$3)-x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+6*y(x)=0,y(1) = k__0, D(y)(1)
```

$$y(x) = \frac{3(k_2 - 2k_0)x^4 + 4(3k_0 + k_1 - k_2)x^3 + 6k_0 - 4k_1 + k_2}{12x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 47

```
DSolve[{x^3*y'''[x]-x^2*y''[x]-2*x*y'[x]+6*y[x]==0,{y[1]==k0,y'[1]==k1,y''[1]==k2}},y[x],x,I
```

$$y(x) \rightarrow \frac{-6k_0(x^4 - 2x^3 - 1) + 4k_1(x^3 - 1) + 3k_2x^4 - 4k_2x^3 + k_2}{12x}$$

17.7 problem section 9.1, problem 6(a)

Internal problem ID [1463]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.1. Page 471

Problem number: section 9.1, problem 6(a).

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' + y'' - y' - y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$3)+diff(y(x),x$2)-diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + e^{-x} c_2 + c_3 e^{-x} x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 26

```
DSolve[y'''[x]+y''[x]-y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x} (c_2 x + c_3 e^{2x} + c_1)$$

17.8 problem section 9.1, problem 6(b)

Internal problem ID [1464]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.1. Page 471

Problem number: section 9.1, problem 6(b).

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' - 3y'' + 7y' - 5y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)+7*diff(y(x),x)-5*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2 e^x \sin(2x) + c_3 e^x \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 26

```
DSolve[y'''[x]-3*y''[x]+7*y'[x]-5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x(c_2 \cos(2x) + c_1 \sin(2x) + c_3)$$

18 Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient.

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18.1 problem section 9.2, problem 1

Internal problem ID [1465]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 1.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' - 3y'' + 3y' - y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)+3*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2 e^x x + c_3 e^x x^2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 21

```
DSolve[y'''[x]-3*y''[x]+3*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x(x(c_3 x + c_2) + c_1)$$

18.2 problem section 9.2, problem 2

Internal problem ID [1466]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 2.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' + 8y'' - 9y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$4)+8*diff(y(x),x$2)-9*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + e^{-x} c_2 + c_3 \sin(3x) + c_4 \cos(3x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 34

```
DSolve[y''''[x]+8*y''[x]-9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_3 e^{-x} + c_4 e^x + c_1 \cos(3x) + c_2 \sin(3x)$$

18.3 problem section 9.2, problem 3

Internal problem ID [1467]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 3.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' - y'' + 16y' - 16y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)+16*diff(y(x),x)-16*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2 \sin(4x) + c_3 \cos(4x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 26

```
DSolve[y'''[x]-y''[x]+16*y'[x]-16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_3 e^x + c_1 \cos(4x) + c_2 \sin(4x)$$

18.4 problem section 9.2, problem 4

Internal problem ID [1468]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 4.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$2y''' + 3y'' - 2y' - 3y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(2*diff(y(x),x$3)+3*diff(y(x),x$2)-2*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + e^{-x} c_2 + c_3 e^{-\frac{3x}{2}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[2*y'''[x]+3*y''[x]-2*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{-3x/2} + c_2 e^{-x} + c_3 e^x$$

18.5 problem section 9.2, problem 5

Internal problem ID [1469]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 5.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' + 5y'' + 9y' + 5y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$3)+5*diff(y(x),x$2)+9*diff(y(x),x)+5*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-x}c_1 + c_2e^{-2x} \sin(x) + c_3e^{-2x} \cos(x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 28

```
DSolve[y'''[x]+5*y''[x]+9*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-2x}(c_3e^x + c_2 \cos(x) + c_1 \sin(x))$$

18.6 problem section 9.2, problem 6

Internal problem ID [1470]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 6.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$4y''' - 8y'' + 5y' - y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(4*diff(y(x),x$3)-8*diff(y(x),x$2)+5*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2 e^{\frac{x}{2}} + c_3 e^{\frac{x}{2}} x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[4*y'''[x]-8*y''[x]+5*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{x/2}(c_2 x + c_3 e^{x/2} + c_1)$$

18.7 problem section 9.2, problem 7

Internal problem ID [1471]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 7.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$27y''' + 27y'' + 9y' + y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(27*diff(y(x),x$3)+27*diff(y(x),x$2)+9*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{x}{3}} + c_2 e^{-\frac{x}{3}} x + c_3 e^{-\frac{x}{3}} x^2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 25

```
DSolve[27*y'''[x]+27*y''[x]+9*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x/3}(x(c_3 x + c_2) + c_1)$$

18.8 problem section 9.2, problem 8

Internal problem ID [1472]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 8.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' + y'' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$4)+diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2x + c_3 \sin(x) + c_4 \cos(x)$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 24

```
DSolve[y''''[x]+y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_4x - c_1 \cos(x) - c_2 \sin(x) + c_3$$

18.9 problem section 9.2, problem 9

Internal problem ID [1473]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 9.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' - 16y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$4)-16*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{2x} + c_2 e^{-2x} + c_3 \sin(2x) + c_4 \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 36

```
DSolve[y''''[x]-16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{2x} + c_3 e^{-2x} + c_2 \cos(2x) + c_4 \sin(2x)$$

18.10 problem section 9.2, problem 10

Internal problem ID [1474]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 10.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' + 12y'' + 36y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$4)+12*diff(y(x),x$2)+36*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(\sqrt{6}x) + c_2 \cos(\sqrt{6}x) + c_3 \sin(\sqrt{6}x)x + c_4 \cos(\sqrt{6}x)x$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 38

```
DSolve[y''''[x]+12*y''[x]+36*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow (c_2x + c_1) \cos(\sqrt{6}x) + (c_4x + c_3) \sin(\sqrt{6}x)$$

18.11 problem section 9.2, problem 11

Internal problem ID [1475]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 11.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$16y'''' - 72y'' + 81y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
dsolve(16*diff(y(x),x$4)-72*diff(y(x),x$2)+81*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{3x}{2}} + c_2 e^{\frac{3x}{2}} x + c_3 e^{-\frac{3x}{2}} + c_4 e^{-\frac{3x}{2}} x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 37

```
DSolve[16*y''''[x]-72*y''[x]+81*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-3x/2} (c_3 e^{3x} + x(c_4 e^{3x} + c_2) + c_1)$$

18.12 problem section 9.2, problem 12

Internal problem ID [1476]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 12.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$6y'''' + 5y''' + 7y'' + 5y' + y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(6*diff(y(x),x$4)+5*diff(y(x),x$3)+7*diff(y(x),x$2)+5*diff(y(x),x)+y(x))=0,y(x), singular
```

$$y(x) = c_1 e^{-\frac{x}{2}} + c_2 e^{-\frac{x}{3}} + c_3 \sin(x) + c_4 \cos(x)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 37

```
DSolve[6*y''''[x]+5*y'''[x]+7*y''[x]+5*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \rightarrow e^{-x/2} (c_3 e^{x/6} + c_4) + c_1 \cos(x) + c_2 \sin(x)$$

18.13 problem section 9.2, problem 13

Internal problem ID [1477]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 13.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$4y'''' + 12y'''' + 3y'' - 13y' - 6y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(4*diff(y(x),x$4)+12*diff(y(x),x$3)+3*diff(y(x),x$2)-13*diff(y(x),x)-6*y(x)=0,y(x), si
```

$$y(x) = c_1 e^{-\frac{x}{2}} + e^x c_2 + c_3 e^{-2x} + c_4 e^{-\frac{3x}{2}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 42

```
DSolve[4*y''''[x]+12*y''''[x]+3*y''[x]-13*y'[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow e^{-2x} (c_1 e^{x/2} + c_2 e^{3x/2} + c_4 e^{3x} + c_3)$$

18.14 problem section 9.2, problem 14

Internal problem ID [1478]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 14.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' - 4y''' + 7y'' - 6y' + 2y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$4)-4*diff(y(x),x$3)+7*diff(y(x),x$2)-6*diff(y(x),x)+2*y(x))=0,y(x), singularities)
```

$$y(x) = c_1 e^x + c_2 e^x x + c_3 \sin(x) e^x + c_4 \cos(x) e^x$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 26

```
DSolve[y''''[x]-4*y'''[x]+7*y''[x]-6*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x (c_4 x + c_2 \cos(x) + c_1 \sin(x) + c_3)$$

18.15 problem section 9.2, problem 15

Internal problem ID [1479]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 15.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' - 2y'' + 4y' - 8y = 0$$

With initial conditions

$$[y(0) = 2, y'(0) = -2, y''(0) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$3)-2*diff(y(x),x$2)+4*diff(y(x),x)-8*y(x)=0,y(0) = 2, D(y)(0) = -2, (D@@
```

$$y(x) = \frac{5e^{2x}}{4} - \frac{9\sin(2x)}{4} + \frac{3\cos(2x)}{4}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 21

```
DSolve[{y'''[x]-2*y''[x]+4*y'[x]-8*y[x]==0,{y[0]==2,y'[0]==-2,y''[0]==0}},y[x],x,IncludeSing
```

$$y(x) \rightarrow e^{2x} - 2\sin(2x) + \cos(2x)$$

18.16 problem section 9.2, problem 16

Internal problem ID [1480]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 16.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' + 3y'' - y' - 3y = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 14, y''(0) = -40]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 21

```
dsolve([diff(y(x),x$3)+3*diff(y(x),x$2)-diff(y(x),x)-3*y(x)=0,y(0) = 0, D(y)(0) = 14, (D@@2)
```

$$y(x) = (2e^{4x} + 3e^{2x} - 5)e^{-3x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 25

```
DSolve[{y'''[x]+3*y''[x]-y'[x]-3*y[x]==0,{y[0]==0,y'[0]==14,y''[0]==-40}},y[x],x,IncludeSing
```

$$y(x) \rightarrow -5e^{-3x} + 3e^{-x} + 2e^x$$

18.17 problem section 9.2, problem 17

Internal problem ID [1481]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 17.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' - y'' - y' + y = 0$$

With initial conditions

$$[y(0) = -2, y'(0) = 9, y''(0) = 4]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 20

```
dsolve([diff(y(x),x$3)-diff(y(x),x$2)-diff(y(x),x)+y(x)=0,y(0) = -2, D(y)(0) = 9, (D@@2)(y)
```

$$y(x) = -4e^{-x} + (3x + 2)e^x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 22

```
DSolve[{y'''[x]-y''[x]-y'[x]+y[x]==0,{y[0]==-2,y'[0]==9,y''[0]==4}},y[x],x,IncludeSingularSo
```

$$y(x) \rightarrow e^x(3x + 2) - 4e^{-x}$$

18.18 problem section 9.2, problem 18

Internal problem ID [1482]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 18.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' - 2y' - 4y = 0$$

With initial conditions

$$[y(0) = 6, y'(0) = 3, y''(0) = 22]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 27

```
dsolve([diff(y(x),x$3)-2*diff(y(x),x)-4*y(x)=0,y(0) = 6, D(y)(0) = 3, (D@@2)(y)(0) = 22],y(x)
```

$$y(x) = (2 \cos(x) - 3 \sin(x)) e^{-x} + 4 e^{2x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 27

```
DSolve[{y'''[x]-2*y'[x]-4*y[x]==0,{y[0]==6,y'[0]==3,y''[0]==22}},y[x],x,IncludeSingularSolut
```

$$y(x) \rightarrow e^{-x} (4e^{3x} - 3 \sin(x) + 2 \cos(x))$$

18.19 problem section 9.2, problem 19

Internal problem ID [1483]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 19.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$3y''' - y'' - 7y' + 5y = 0$$

With initial conditions

$$\left[y(0) = \frac{14}{5}, y'(0) = 0, y''(0) = 10 \right]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 18

```
dsolve([3*diff(y(x),x$3)-diff(y(x),x$2)-7*diff(y(x),x)+5*y(x)=0,y(0) = 14/5, D(y)(0) = 0, D
```

$$y(x) = e^{-\frac{5x}{3}} \left(\frac{9}{5} + (1 + 2x) e^{\frac{8x}{3}} \right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 26

```
DSolve[{3*y'''[x]-y''[x]-7*y'[x]+5*y[x]==0,{y[0]==14/5,y'[0]==0,y''[0]==10}},y[x],x,IncludeS
```

$$y(x) \rightarrow e^x(2x + 1) + \frac{9}{5}e^{-5x/3}$$

18.20 problem section 9.2, problem 20

Internal problem ID [1484]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 20.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' - 6y'' + 12y' - 8y = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = -1, y''(0) = -4]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$3)-6*diff(y(x),x$2)+12*diff(y(x),x)-8*y(x)=0,y(0) = 1, D(y)(0) = -1, (D
```

$$y(x) = e^{2x}(2x^2 - 3x + 1)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 21

```
DSolve[{y'''[x]-6*y''[x]+12*y'[x]-8*y[x]==0,{y[0]==1,y'[0]==-1,y''[0]==-4}},y[x],x,IncludeSi
```

$$y(x) \rightarrow e^{2x}(2x^2 - 3x + 1)$$

18.21 problem section 9.2, problem 21

Internal problem ID [1485]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 21.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$2y''' - 11y'' + 12y' + 9y = 0$$

With initial conditions

$$[y(0) = 6, y'(0) = 3, y''(0) = 13]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 21

```
dsolve([2*diff(y(x),x$3)-11*diff(y(x),x$2)+12*diff(y(x),x)+9*y(x)=0,y(0) = 6, D(y)(0) = 3, (
```

$$y(x) = 4e^{-\frac{x}{2}} + (2 - x)e^{3x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 25

```
DSolve[{2*y'''[x]-11*y''[x]+12*y'[x]+9*y[x]==0,{y[0]==6,y'[0]==3,y''[0]==13}},y[x],x,Include
```

$$y(x) \rightarrow 4e^{-x/2} - e^{3x}(x - 2)$$

18.22 problem section 9.2, problem 22

Internal problem ID [1486]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 22.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$8y''' - 4y'' - 2y' + y = 0$$

With initial conditions

$$[y(0) = 4, y'(0) = -3, y''(0) = -1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 22

```
dsolve([8*dif(y(x),x$3)-4*dif(y(x),x$2)-2*dif(y(x),x)+y(x)=0,y(0) = 4, D(y)(0) = -3, (D@@
```

$$y(x) = 3e^{-\frac{x}{2}} + e^{\frac{x}{2}} - 2e^{\frac{x}{2}}x$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 57

```
DSolve[{8*y'''[x]-4*y''[x]-2*y'[x]-2*y[x]==0,{y[0]==4,y'[0]==-3,y''[0]==-1}},y[x],x,IncludeS
```

$$y(x) \rightarrow -\frac{2}{21}e^{-x/4} \left(9e^{5x/4} + 13\sqrt{3} \sin\left(\frac{\sqrt{3}x}{4}\right) - 51 \cos\left(\frac{\sqrt{3}x}{4}\right) \right)$$

18.23 problem section 9.2, problem 23

Internal problem ID [1487]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 23.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' - 16y = 0$$

With initial conditions

$$[y(0) = 2, y'(0) = 2, y''(0) = -2, y'''(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 29

```
dsolve([diff(y(x),x$4)-16*y(x)=0,y(0) = 2, D(y)(0) = 2, (D@@2)(y)(0) = -2, (D@@3)(y)(0) = 0])
```

$$y(x) = \frac{5e^{2x}}{8} + \frac{e^{-2x}}{8} + \frac{\sin(2x)}{2} + \frac{5\cos(2x)}{4}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 34

```
DSolve[{y''''[x]-16*y[x]==0,{y[0]==2,y'[0]==2,y''[0]==-2,y'''[0]==0}},y[x],x,IncludeSingular
```

$$y(x) \rightarrow \frac{1}{8}(e^{-2x} + 5e^{2x} + 4\sin(2x) + 10\cos(2x))$$

18.24 problem section 9.2, problem 24

Internal problem ID [1488]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 24.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' - 6y'''' + 7y'' + 6y' - 8y = 0$$

With initial conditions

$$[y(0) = -2, y'(0) = -8, y''(0) = -14, y'''(0) = -62]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 25

```
dsolve([diff(y(x),x$4)-6*diff(y(x),x$3)+7*diff(y(x),x$2)+6*diff(y(x),x)-8*y(x)=0,y(0) = -2,
```

$$y(x) = -e^{4x} - 4e^x + e^{2x} + 2e^{-x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[{y''''[x]-6*y''''[x]+7*y''[x]+6*y'[x]-8*y[x]==0,{y[0]==-2,y'[0]==-8,y''[0]==-14,y'''[0]
```

$$y(x) \rightarrow 2e^{-x} - 4e^x + e^{2x} - e^{4x}$$

18.25 problem section 9.2, problem 25

Internal problem ID [1489]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 25.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$4y'''' - 13y'' + 9y = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = 3, y''(0) = 1, y'''(0) = 3]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 15

```
dsolve([4*diff(y(x),x$4)-13*diff(y(x),x$2)+9*y(x)=0,y(0) = 1, D(y)(0) = 3, (D@@2)(y)(0) = 1,
```

$$y(x) = 2e^x - e^{-x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 18

```
DSolve[{4*y''''[x]-13*y''[x]+9*y[x]==0,{y[0]==1,y'[0]==3,y''[0]==1,y'''[0]==3}},y[x],x,Inclu
```

$$y(x) \rightarrow 2e^x - e^{-x}$$

18.26 problem section 9.2, problem 26

Internal problem ID [1490]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 26.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' + 2y''' - 2y'' - 8y' - 8y = 0$$

With initial conditions

$$[y(0) = 5, y'(0) = -2, y''(0) = 6, y'''(0) = 8]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 28

```
dsolve([diff(y(x),x$4)+2*diff(y(x),x$3)-2*diff(y(x),x$2)-8*diff(y(x),x)-8*y(x))=0,y(0) = 5, D
```

$$y(x) = (e^{4x} + 1 + (3 \cos(x) + \sin(x)) e^x) e^{-2x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 31

```
DSolve[{y''''[x]+2*y'''[x]-2*y''[x]-8*y'[x]-8*y[x]==0,{y[0]==5,y'[0]==-2,y''[0]==6,y'''[0]==
```

$$y(x) \rightarrow e^{-2x} (e^{4x} + e^x \sin(x) + 3e^x \cos(x) + 1)$$

18.27 problem section 9.2, problem 27

Internal problem ID [1491]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 27.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$4y'''' + 8y''' + 19y'' + 32y' + 12y = 0$$

With initial conditions

$$\left[y(0) = 3, y'(0) = -3, y''(0) = -\frac{7}{2}, y'''(0) = \frac{31}{4} \right]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 21

```
dsolve([4*diff(y(x),x$4)+8*diff(y(x),x$3)+19*diff(y(x),x$2)+32*diff(y(x),x)+12*y(x)=0,y(0) =
```

$$y(x) = 2e^{-\frac{x}{2}} - \sin(2x) + \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 31

```
DSolve[{y''''[x]+2*y'''[x]-2*y''[x]-8*y'[x]-8*y[x]==0,{y[0]==5,y'[0]==-2,y''[0]==6,y'''[0]==
```

$$y(x) \rightarrow e^{-2x}(e^{4x} + e^x \sin(x) + 3e^x \cos(x) + 1)$$

18.28 problem section 9.2, problem 43(a)

Internal problem ID [1492]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 43(a).

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' - y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$4)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + e^{-x} c_2 + c_3 \sin(x) + c_4 \cos(x)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 30

```
DSolve[y''''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^x + c_3 e^{-x} + c_2 \cos(x) + c_4 \sin(x)$$

18.29 problem section 9.2, problem 43(b)

Internal problem ID [1493]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 43(b).

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' + y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 71

```
dsolve(diff(y(x),x$4)+y(x)=0,y(x), singsol=all)
```

$$y(x) = -c_1 e^{-\frac{\sqrt{2}x}{2}} \sin\left(\frac{\sqrt{2}x}{2}\right) - c_2 e^{\frac{\sqrt{2}x}{2}} \sin\left(\frac{\sqrt{2}x}{2}\right) \\ + c_3 e^{-\frac{\sqrt{2}x}{2}} \cos\left(\frac{\sqrt{2}x}{2}\right) + c_4 e^{\frac{\sqrt{2}x}{2}} \cos\left(\frac{\sqrt{2}x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 65

```
DSolve[y''''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-\frac{x}{\sqrt{2}}} \left((c_1 e^{\sqrt{2}x} + c_2) \cos\left(\frac{x}{\sqrt{2}}\right) + (c_4 e^{\sqrt{2}x} + c_3) \sin\left(\frac{x}{\sqrt{2}}\right) \right)$$

18.30 problem section 9.2, problem 43(c)

Internal problem ID [1494]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 43(c).

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' + 64y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x$4)+64*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-2x} \sin(2x) + c_2 e^{-2x} \cos(2x) + c_3 e^{2x} \sin(2x) + c_4 e^{2x} \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 44

```
DSolve[y''''[x]+64*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-2x} ((c_4 e^{4x} + c_1) \cos(2x) + (c_3 e^{4x} + c_2) \sin(2x))$$

18.31 problem section 9.2, problem 43(d)

Internal problem ID [1495]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 43(d).

ODE order: 6.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y^{(6)} - y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 67

```
dsolve(diff(y(x),x$6)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + e^{-x} c_2 + c_3 e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}x}{2}\right) + c_4 e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}x}{2}\right) \\ + c_5 e^{\frac{x}{2}} \sin\left(\frac{\sqrt{3}x}{2}\right) + c_6 e^{\frac{x}{2}} \cos\left(\frac{\sqrt{3}x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 78

```
DSolve[y''''''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x} \left(c_1 e^{2x} + e^{x/2} (c_2 e^x + c_3) \cos\left(\frac{\sqrt{3}x}{2}\right) + e^{x/2} (c_6 e^x + c_5) \sin\left(\frac{\sqrt{3}x}{2}\right) + c_4 \right)$$

18.32 problem section 9.2, problem 43(e)

Internal problem ID [1496]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 43(e).

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' + 64y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x$4)+64*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-2x} \sin(2x) + c_2 e^{-2x} \cos(2x) + c_3 e^{2x} \sin(2x) + c_4 e^{2x} \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 44

```
DSolve[y''''[x]+64*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-2x} ((c_4 e^{4x} + c_1) \cos(2x) + (c_3 e^{4x} + c_2) \sin(2x))$$

18.33 problem section 9.2, problem 43(g)

Internal problem ID [1497]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

Problem number: section 9.2, problem 43(g).

ODE order: 5.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y^{(5)} + y'''' + y''' + y'' + y' + y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 63

```
dsolve(diff(y(x),x$5)+diff(y(x),x$4)+diff(y(x),x$3)+diff(y(x),x$2)+diff(y(x),x)+y(x)=0,y(x),x)
```

$$y(x) = e^{-x}c_1 + c_2e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}x}{2}\right) + c_3e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}x}{2}\right) \\ + c_4e^{\frac{x}{2}} \sin\left(\frac{\sqrt{3}x}{2}\right) + c_5e^{\frac{x}{2}} \cos\left(\frac{\sqrt{3}x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 70

```
DSolve[y'''''[x]+y''''[x]+y'''[x]+y''[x]+y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x} \left(e^{x/2}(c_3e^x + c_2) \cos\left(\frac{\sqrt{3}x}{2}\right) + e^{x/2}(c_4e^x + c_1) \sin\left(\frac{\sqrt{3}x}{2}\right) + c_5 \right)$$

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19.1 problem section 9.3, problem 1

Internal problem ID [1498]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 1.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - 6y'' + 11y' - 6y = -e^x(-24x^2 + 76x + 4)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 63

```
dsolve(diff(y(x),x$3)-6*diff(y(x),x$2)+11*diff(y(x),x)-6*y(x)=-exp(x)*(4+76*x-24*x^2),y(x),
```

$$y(x) = \frac{x(4x^2 - x - 17)(24x^2e^x - 76xe^x - 4e^x)}{24x^2 - 76x - 4} + c_1e^x + c_2e^{2x} + c_3e^{3x}$$

✓ Solution by Mathematica

Time used: 0.103 (sec). Leaf size: 47

```
DSolve[y'''[x]-6*y''[x]+11*y'[x]-6*y[x]==-Exp[x]*(4+76*x-24*x^2),y[x],x,IncludeSingularSolut
```

$$y(x) \rightarrow \frac{1}{2}e^x(8x^3 - 2x^2 - 34x + 2c_2e^x + 2c_3e^{2x} - 49 + 2c_1)$$

19.2 problem section 9.3, problem 2

Internal problem ID [1499]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 2.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - 2y'' - 5y' + 6y = e^{-3x}(6x^2 - 23x + 32)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$3)-2*diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=exp(-3*x)*(32-23*x+6*x^2),y(x),
```

$$y(x) = -\frac{(x^2 - x + 3)e^{-3x}}{4} + c_1e^x + c_2e^{-2x} + c_3e^{3x}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 45

```
DSolve[y'''[x]-2*y''[x]-5*y'[x]+6*y[x]==Exp[-3*x]*(32-23*x+6*x^2),y[x],x,IncludeSingularSolu
```

$$y(x) \rightarrow -\frac{1}{4}e^{-3x}(x^2 - x + 3) + c_1e^{-2x} + c_2e^x + c_3e^{3x}$$

19.3 problem section 9.3, problem 3

Internal problem ID [1500]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 3.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$4y''' + 8y'' - y' - 2y = -e^x(6x^2 + 45x + 4)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 64

```
dsolve(4*diff(y(x),x$3)+8*diff(y(x),x$2)-diff(y(x),x)-2*y(x)=-exp(x)*(4+45*x+6*x^2),y(x), si
```

$$y(x) = \frac{(18x^2 + 27x - 149)(-6x^2e^x - 45xe^x - 4e^x)}{162x^2 + 1215x + 108} + e^{-2x}c_1 + c_2e^{-\frac{x}{2}} + c_3e^{\frac{x}{2}}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 52

```
DSolve[4*y'''[x]+8*y''[x]-y'[x]-2*y[x]==-Exp[x]*(4+45*x+6*x^2),y[x],x,IncludeSingularSolutio
```

$$y(x) \rightarrow e^x \left(-\frac{2x^2}{3} - x + \frac{149}{27} \right) + c_1e^{-x/2} + c_2e^{x/2} + c_3e^{-2x}$$

19.4 problem section 9.3, problem 4

Internal problem ID [1501]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 4.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' + 3y'' - y' - 3y = e^{-2x}(3x^2 - 17x + 2)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$3)+3*diff(y(x),x$2)-diff(y(x),x)-3*y(x)=exp(-2*x)*(2-17*x+3*x^2),y(x), si
```

$$y(x) = (x^2 - 5x + 1)e^{-2x} + c_1e^x + c_2e^{-3x} + c_3e^{-x}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 42

```
DSolve[y'''[x]+3*y''[x]-y'[x]-3*y[x]==Exp[-2*x]*(2-17*x+3*x^2),y[x],x,IncludeSingularSolutio
```

$$y(x) \rightarrow e^{-3x}(e^x(x^2 - 5x + 1) + c_2e^{2x} + c_3e^{4x} + c_1)$$

19.5 problem section 9.3, problem 5

Internal problem ID [1502]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 5.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' + 3y'' - y' - 3y = e^x(16x^3 + 24x^2 + 2x - 1)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 76

```
dsolve(diff(y(x),x$3)+3*diff(y(x),x$2)-diff(y(x),x)-3*y(x)=exp(x)*(-1+2*x+24*x^2+16*x^3),y(x)
```

$$y(x) = \frac{x(x^3 - x^2 + x - 1)(16e^x x^3 + 24x^2 e^x + 2x e^x - e^x)}{32x^3 + 48x^2 + 4x - 2} + c_1 e^x + c_2 e^{-3x} + c_3 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 53

```
DSolve[y'''[x]+3*y''[x]-y'[x]-3*y[x]==Exp[x]*(-1+2*x+24*x^2+16*x^3),y[x],x,IncludeSingularSo
```

$$y(x) \rightarrow \frac{1}{4} e^x (2x^4 - 2x^3 + 2x^2 - 2x + 1 + 4c_3) + c_1 e^{-3x} + c_2 e^{-x}$$

19.6 problem section 9.3, problem 6

Internal problem ID [1503]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 6.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' + y'' - 2y = e^x(15x^2 + 34x + 14)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 61

```
dsolve(diff(y(x),x$3)+diff(y(x),x$2)-2*y(x)=exp(x)*(14+34*x+15*x^2),y(x), singsol=all)
```

$$y(x) = \frac{x^2(x+1)(15x^2e^x + 34xe^x + 14e^x)}{15x^2 + 34x + 14} + c_1e^x + c_2 \cos(x)e^{-x} + c_3e^{-x} \sin(x)$$

✓ Solution by Mathematica

Time used: 0.159 (sec). Leaf size: 49

```
DSolve[y'''[x]+y''[x]-2*y[x]==Exp[x]*(14+34*x+15*x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{5}e^{-x}(e^{2x}(5x^3 + 5x^2 - 2 + 5c_3) + 5c_2 \cos(x) + 5c_1 \sin(x))$$

19.7 problem section 9.3, problem 7

Internal problem ID [1504]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 7.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$4y''' + 8y'' - y' - 2y = -e^{-2x}(1 - 15x)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
dsolve(4*diff(y(x),x$3)+8*diff(y(x),x$2)-diff(y(x),x)-2*y(x)=-exp(-2*x)*(1-15*x),y(x),sings
```

$$y(x) = \frac{x(2+x)e^{-2x}}{2} + e^{-2x}c_1 + c_2e^{-\frac{x}{2}} + c_3e^{\frac{x}{2}}$$

✓ Solution by Mathematica

Time used: 0.075 (sec). Leaf size: 53

```
DSolve[4*y'''[x]+8*y''[x]-y'[x]-2*y[x]==-Exp[-2*x]*(1-15*x),y[x],x,IncludeSingularSolutions
```

$$y(x) \rightarrow \frac{1}{10}e^{-2x}(5x^2 + 10x + 2(5c_1e^{3x/2} + 5c_2e^{5x/2} + 4 + 5c_3))$$

19.8 problem section 9.3, problem 8

Internal problem ID [1505]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 8.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - y'' - y' + y = -e^x(7 + 6x)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)-diff(y(x),x)+y(x)=-exp(x)*(7+6*x),y(x), singsol=all)
```

$$y(x) = \frac{(2+x)x^2(-6xe^x - 7e^x)}{12x + 14} + c_1e^x + e^{-x}c_2 + c_3e^xx$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 41

```
DSolve[y'''[x]-y''[x]-y'[x]+y[x]==-Exp[x]*(7+6*x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x \left(-\frac{x^3}{2} - x^2 + x + c_3x - \frac{1}{2} + c_2 \right) + c_1e^{-x}$$

19.9 problem section 9.3, problem 9

Internal problem ID [1506]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 9.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$2y''' - 7y'' + 4y' + 4y = e^{2x}(17 + 30x)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 55

```
dsolve(2*diff(y(x),x$3)-7*diff(y(x),x$2)+4*diff(y(x),x)+4*y(x)=exp(2*x)*(17+30*x),y(x),sing
```

$$y(x) = \frac{x^2(1+2x)(30e^{2x}x + 17e^{2x})}{34 + 60x} + c_1e^{2x} + c_2e^{-\frac{x}{2}} + c_3e^{2x}x$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 46

```
DSolve[2*y'''[x]-7*y''[x]+4*y'[x]+4*y[x]==Exp[2*x]*(17+30*x),y[x],x,IncludeSingularSolutions
```

$$y(x) \rightarrow e^{2x} \left(x^3 + \frac{x^2}{2} + \left(-\frac{2}{5} + c_3 \right) x + \frac{4}{25} + c_2 \right) + c_1 e^{-x/2}$$

19.10 problem section 9.3, problem 10

Internal problem ID [1507]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 10.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - 5y'' + 3y' + 9y = 2e^{3x}(11 - 24x)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 55

```
dsolve(diff(y(x),x$3)-5*diff(y(x),x$2)+3*diff(y(x),x)+9*y(x)=2*exp(3*x)*(11-24*x),y(x),sing
```

$$y(x) = \frac{x^2(8x - 17)(-48xe^{3x} + 22e^{3x})}{192x - 88} + e^{-x}c_1 + c_2e^{3x} + c_3e^{3x}x$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 46

```
DSolve[y'''[x]-5*y''[x]+3*y'[x]+9*y[x]==2*Exp[3*x]*(11-24*x),y[x],x,IncludeSingularSolutions
```

$$y(x) \rightarrow e^{3x} \left(-2x^3 + \frac{17x^2}{4} + \left(-\frac{17}{8} + c_3 \right) x + \frac{17}{32} + c_2 \right) + c_1 e^{-x}$$

19.11 problem section 9.3, problem 11

Internal problem ID [1508]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 11.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - 7y'' + 8y' + 16y = 2e^{4x}(13 + 15x)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
dsolve(diff(y(x),x$3)-7*diff(y(x),x$2)+8*diff(y(x),x)+16*y(x)=2*exp(4*x)*(13+15*x),y(x), sin
```

$$y(x) = \frac{(2+x)x^2(30e^{4x}x + 26e^{4x})}{30x + 26} + e^{-x}c_1 + c_2e^{4x} + c_3e^{4x}x$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 42

```
DSolve[y'''[x]-7*y''[x]+8*y'[x]+16*y[x]==2*Exp[4*x]*(13+15*x),y[x],x,IncludeSingularSolution
```

$$y(x) \rightarrow e^{4x} \left(x^3 + 2x^2 + \left(-\frac{4}{5} + c_3 \right) x + \frac{4}{25} + c_2 \right) + c_1 e^{-x}$$

19.12 problem section 9.3, problem 12

Internal problem ID [1509]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 12.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$8y''' - 12y'' + 6y' - y = e^{\frac{x}{2}}(1 + 4x)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 51

```
dsolve(8*diff(y(x),x$3)-12*diff(y(x),x$2)+6*diff(y(x),x)-y(x)=exp(x/2)*(1+4*x),y(x), singsol
```

$$y(x) = \left(\frac{1}{192}x^3 + \frac{1}{256}x^2 \right) (4e^{\frac{x}{2}}x + e^{\frac{x}{2}}) + c_1e^{\frac{x}{2}} + c_2x^2e^{\frac{x}{2}} + c_3e^{\frac{x}{2}}x$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 39

```
DSolve[8*y'''[x]-12*y''[x]+6*y'[x]-y[x]==Exp[x/2]*(1+4*x),y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow \frac{1}{48}e^{x/2}(x^4 + x^3 + 48c_3x^2 + 48c_2x + 48c_1)$$

19.13 problem section 9.3, problem 13

Internal problem ID [1510]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 13.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' + 3y''' - 3y'' - 7y' + 6y = -3e^{-x}(-8x^2 + 8x + 12)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 62

```
dsolve(diff(y(x),x$4)+3*diff(y(x),x$3)-3*diff(y(x),x$2)-7*diff(y(x),x)+6*y(x))=-3*exp(-x)*(12+8*x-8*x^2),y(x),x,IncludeS
```

$$y(x) = \frac{(x^2 - 2x + 1)(24x^2 - 24x - 36)e^{-x}}{8x^2 - 8x - 12} + c_1e^x + c_2e^{-3x} + c_3e^{-2x} + c_4xe^x$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 45

```
DSolve[y''''[x]+3*y'''[x]-3*y''[x]-7*y'[x]+6*y[x]==-3*Exp[-x]*(12+8*x-8*x^2),y[x],x,IncludeS
```

$$y(x) \rightarrow e^{-3x}(3e^{2x}(x-1)^2 + c_2e^x + e^{4x}(c_4x + c_3) + c_1)$$

19.14 problem section 9.3, problem 14

Internal problem ID [1511]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 14.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' + 3y''' + y'' - 3y' - 2y = -3e^{2x}(11 + 12x)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$4)+3*diff(y(x),x$3)+diff(y(x),x$2)-3*diff(y(x),x)-2*y(x)=-3*exp(2*x)*(11+
```

$$y(x) = \frac{(x-1)(-36e^{2x}x - 33e^{2x})}{36x + 33} + c_1e^x + c_2e^{-2x} + c_3e^{-x} + c_4xe^{-x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 43

```
DSolve[y''''[x]+3*y'''[x]+y''[x]-3*y'[x]-2*y[x]==-3*Exp[2*x]*(11+12*x),y[x],x,IncludeSingular
```

$$y(x) \rightarrow e^{-2x}(-e^{4x}(x-1) + e^x(c_3x + c_2) + c_4e^{3x} + c_1)$$

19.15 problem section 9.3, problem 15

Internal problem ID [1512]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 15.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$y'''' + 8y''' + 24y'' + 32y' = -16e^{-2x}(-x^3 + x^2 + x + 1)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 94

```
dsolve(diff(y(x),x$4)+8*diff(y(x),x$3)+24*diff(y(x),x$2)+32*diff(y(x),x)=-16*exp(-2*x)*(1+x
```

$$y(x) = \frac{c_3 e^{-2x} (-2 \sin(2x) - 2 \cos(2x))}{8} + 2c_2 \left(\frac{(-2 \cos(x) + 2 \sin(x)) e^{-2x} \cos(x)}{8} - \frac{e^{-2x}}{8} \right) + \frac{c_2 e^{-2x}}{2} - \frac{c_1 e^{-4x}}{4} - e^{-2x} x^3 + e^{-2x} x^2 + e^{-2x} x + e^{-2x} + c_4$$

✓ Solution by Mathematica

Time used: 0.712 (sec). Leaf size: 64

```
DSolve[y''''[x]+8*y'''[x]+24*y''[x]+32*y'[x]==-16*Exp[-2*x]*(1+x+x^2-x^3),y[x],x,IncludeSing
```

$$y(x) \rightarrow \frac{1}{4} e^{-2x} (-4x^3 + 4x^2 + 4x - c_3 e^{-2x} - (c_1 + c_2) \cos(2x) + (c_2 - c_1) \sin(2x) + 4) + c_4$$

19.16 problem section 9.3, problem 16

Internal problem ID [1513]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 16.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$4y'''' - 11y'' - 9y' - 2y = -e^x(1 - 6x)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 52

```
dsolve(4*diff(y(x),x$4)-11*diff(y(x),x$2)-9*diff(y(x),x)-2*y(x)=-exp(x)*(1-6*x),y(x), singular
```

$$y(x) = -\frac{(x-1)(6xe^x - e^x)}{3(-1+6x)} + e^{-x}c_1 + c_2e^{2x} + c_3e^{-\frac{x}{2}} + c_4e^{-\frac{x}{2}}x$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 47

```
DSolve[4*y''''[x]-11*y''[x]-9*y'[x]-2*y[x]==-Exp[x]*(1-6*x),y[x],x,IncludeSingularSolutions
```

$$y(x) \rightarrow -\frac{1}{3}e^x(x-1) + e^{-x/2}(c_2x + c_1) + c_3e^{-x} + c_4e^{2x}$$

19.17 problem section 9.3, problem 17

Internal problem ID [1514]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 17.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 2y''' + 3y' - y = e^x(x^2 + 4x + 3)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 118

```
dsolve(diff(y(x),x$4)-2*diff(y(x),x$3)+0*diff(y(x),x$2)+3*diff(y(x),x)-y(x)=exp(x)*(3+4*x+x^2),y(x),x)
```

$$y(x) = \frac{(x+1)(x^2e^x + 4xe^x + 3e^x)}{x+3} + c_1 e^{\text{RootOf}(-Z^4-2Z^3+3Z-1, \text{index}=1)x} + c_2 e^{\text{RootOf}(-Z^4-2Z^3+3Z-1, \text{index}=2)x} + c_3 e^{\text{RootOf}(-Z^4-2Z^3+3Z-1, \text{index}=3)x} + c_4 e^{\text{RootOf}(-Z^4-2Z^3+3Z-1, \text{index}=4)x}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 123

```
DSolve[y''''[x]-2*y'''[x]+0*y''[x]+3*y'[x]-y[x]==Exp[x]*(3+4*x+x^2),y[x],x,IncludeSingularSolutions->True]
```

$$y(x) \rightarrow c_2 \exp(x \text{Root}[\#1^4 - 2\#1^3 + 3\#1 - 1\&, 2]) + c_3 \exp(x \text{Root}[\#1^4 - 2\#1^3 + 3\#1 - 1\&, 3]) + c_4 \exp(x \text{Root}[\#1^4 - 2\#1^3 + 3\#1 - 1\&, 4]) + c_1 \exp(x \text{Root}[\#1^4 - 2\#1^3 + 3\#1 - 1\&, 1]) + e^x(x+1)^2$$

19.18 problem section 9.3, problem 18

Internal problem ID [1515]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 18.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 4y''' + 6y'' - 4y' + 2y = e^{2x}(x^4 + x + 24)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 126

```
dsolve(diff(y(x),x$4)-4*diff(y(x),x$3)+6*diff(y(x),x$2)-4*diff(y(x),x)+2*y(x))=exp(2*x)*(24+x
```

$$y(x) = \frac{(x^4 - 8x^3 + 12x^2 + 49x - 62)(e^{2x}x^4 + e^{2x}x + 24e^{2x})}{2x^4 + 2x + 48} + c_1 e^{\frac{(2+\sqrt{2})x}{2}} \cos\left(\frac{\sqrt{2}x}{2}\right) + c_2 e^{\frac{(2+\sqrt{2})x}{2}} \sin\left(\frac{\sqrt{2}x}{2}\right) + c_3 e^{-\frac{(\sqrt{2}-2)x}{2}} \cos\left(\frac{\sqrt{2}x}{2}\right) + c_4 e^{-\frac{(\sqrt{2}-2)x}{2}} \sin\left(\frac{\sqrt{2}x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 102

```
DSolve[y''''[x]-4*y'''[x]+6*y''[x]-4*y'[x]+2*y[x]==Exp[2*x]*(24+x+x^4),y[x],x,IncludeSingular
```

$$y(x) \rightarrow \frac{1}{2} e^{x-\frac{x}{\sqrt{2}}} \left(e^{\frac{x}{\sqrt{2}}+x} (x^4 - 8x^3 + 12x^2 + 49x - 62) + 2 \left(c_4 e^{\sqrt{2}x} + c_2 \right) \cos\left(\frac{x}{\sqrt{2}}\right) + 2 \left(c_1 e^{\sqrt{2}x} + c_3 \right) \sin\left(\frac{x}{\sqrt{2}}\right) \right)$$

19.19 problem section 9.3, problem 19

Internal problem ID [1516]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 19.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$2y'''' + 5y''' - 5y' - 2y = 18e^x(5 + 2x)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 50

```
dsolve(2*diff(y(x),x$4)+5*diff(y(x),x$3)+0*diff(y(x),x$2)-5*diff(y(x),x)-2*y(x)=18*exp(x)*(5
```

$$y(x) = \frac{x(2+x)(36xe^x + 90e^x)}{36x + 90} + c_1e^x + c_2e^{-2x} + c_3e^{-x} + c_4e^{-\frac{x}{2}}$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 48

```
DSolve[2*y''''[x]+5*y'''[x]+0*y''[x]-5*y'[x]-2*y[x]==18*Exp[x]*(5+2*x),y[x],x,IncludeSingular
```

$$y(x) \rightarrow e^{-2x} \left(e^{3x} \left(x^2 + 2x - \frac{40}{9} + c_4 \right) + c_1 e^{3x/2} + c_3 e^x + c_2 \right)$$

19.20 problem section 9.3, problem 20

Internal problem ID [1517]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 20.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' + y''' - 2y'' - 6y' - 4y = -e^{2x}(15x^2 + 28x + 4)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 76

```
dsolve(1*diff(y(x),x$4)+1*diff(y(x),x$3)-2*diff(y(x),x$2)-6*diff(y(x),x)-4*y(x)=-exp(2*x)*(4
```

$$y(x) = \frac{x(x^2 - 1)(-15e^{2x}x^2 - 28e^{2x}x - 4e^{2x})}{90x^2 + 168x + 24} + e^{-x}c_1 + c_2e^{2x} + c_3e^{-x}\cos(x) + c_4e^{-x}\sin(x)$$

✓ Solution by Mathematica

Time used: 0.193 (sec). Leaf size: 65

```
DSolve[1*y''''[x]+1*y'''[x]-2*y''[x]-6*y'[x]-4*y[x]==-Exp[2*x]*(4+28*x+15*x^2),y[x],x,Includ
```

$$y(x) \rightarrow \frac{1}{90}e^{-x}(-15e^{3x}x^3 + 15e^{3x}x - 11e^{3x} + 90c_4e^{3x} + 90c_2\cos(x) + 90c_1\sin(x) + 90c_3)$$

19.21 problem section 9.3, problem 21

Internal problem ID [1518]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 21.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$2y'''' + y''' - 2y' - y = 3e^{-\frac{x}{2}}(1 - 6x)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 68

```
dsolve(2*diff(y(x),x$4)+1*diff(y(x),x$3)-0*diff(y(x),x$2)-2*diff(y(x),x)-1*y(x))=3*exp(-x/2)*
```

$$y(x) = -\frac{4(x+1)x(-18e^{-\frac{x}{2}}x + 3e^{-\frac{x}{2}})}{3(-1+6x)} + c_1e^x + c_2e^{-\frac{x}{2}} \\ + c_3e^{-\frac{x}{2}}\cos\left(\frac{\sqrt{3}x}{2}\right) + c_4e^{-\frac{x}{2}}\sin\left(\frac{\sqrt{3}x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.635 (sec). Leaf size: 63

```
DSolve[2*y''''[x]+1*y'''[x]-0*y''[x]-2*y'[x]-1*y[x]==3*Exp[-x/2]*(1-6*x),y[x],x,IncludeSingul
```

$$y(x) \rightarrow e^{-x/2}\left(4x^2 + 4x + c_4e^{3x/2} + c_2\cos\left(\frac{\sqrt{3}x}{2}\right) + c_1\sin\left(\frac{\sqrt{3}x}{2}\right) - 8 + c_3\right)$$

19.22 problem section 9.3, problem 22

Internal problem ID [1519]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 22.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 5y'' + 4y = e^x(-3x^2 + x + 3)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 63

```
dsolve(1*diff(y(x),x$4)+0*diff(y(x),x$3)-5*diff(y(x),x$2)-0*diff(y(x),x)+4*y(x)=exp(x)*(3+x-
```

$$y(x) = -\frac{x(x^2 + 1)(-3x^2e^x + xe^x + 3e^x)}{6(3x^2 - x - 3)} + c_1e^x + c_2e^{-2x} + c_3e^{-x} + c_4e^{2x}$$

✓ Solution by Mathematica

Time used: 0.111 (sec). Leaf size: 51

```
DSolve[1*y''''[x]+0*y'''[x]-5*y''[x]-0*y'[x]+4*y[x]==Exp[x]*(3+x-3*x^2),y[x],x,IncludeSingular
```

$$y(x) \rightarrow \frac{1}{36}e^x(6x^3 + 6x + 7 + 36c_3) + c_1e^{-2x} + c_2e^{-x} + c_4e^{2x}$$

19.23 problem section 9.3, problem 23

Internal problem ID [1520]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 23.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 2y''' - 3y'' + 4y' + 4y = e^{2x}(18x^2 + 33x + 13)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 77

```
dsolve(1*diff(y(x),x$4)-2*diff(y(x),x$3)-3*diff(y(x),x$2)+4*diff(y(x),x)+4*y(x)=exp(2*x)*(13
```

$$y(x) = \frac{x^2(x^2 + x + 1)(18e^{2x}x^2 + 33e^{2x}x + 13e^{2x})}{108x^2 + 198x + 78} + e^{-x}c_1 + c_2e^{2x} + c_3e^{-x}x + c_4e^{2x}x$$

✓ Solution by Mathematica

Time used: 0.104 (sec). Leaf size: 58

```
DSolve[1*y''''[x]-2*y'''[x]-3*y''[x]+4*y'[x]+4*y[x]==Exp[2*x]*(13+33*x+18*x^2),y[x],x,Includ
```

$$y(x) \rightarrow \frac{1}{54}e^{2x}(9x^4 + 9x^3 + 9x^2 + 18(-1 + 3c_4)x + 10 + 54c_3) + e^{-x}(c_2x + c_1)$$

19.24 problem section 9.3, problem 24

Internal problem ID [1521]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 24.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$y'''' - 3y''' + 4y' = e^{2x}(12x^2 + 26x + 15)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 76

```
dsolve(1*diff(y(x),x$4)-3*diff(y(x),x$3)-0*diff(y(x),x$2)+4*diff(y(x),x)+0*y(x)=exp(2*x)*(15
```

$$y(x) = \frac{c_2 e^{2x}}{2} - e^{-x} c_1 + \frac{e^{2x} x^3}{6} + \frac{e^{2x} x^4}{6} + \frac{e^{2x} x^2}{2} - \frac{e^{2x} x}{2} + \frac{e^{2x}}{4} + c_3 \left(\frac{e^{2x} x}{2} - \frac{e^{2x}}{4} \right) + c_4$$

✓ Solution by Mathematica

Time used: 0.189 (sec). Leaf size: 58

```
DSolve[1*y''''[x]-3*y'''[x]-0*y''[x]+4*y'[x]+0*y[x]==Exp[2*x]*(15+26*x+12*x^2),y[x],x,Includ
```

$$y(x) \rightarrow \frac{1}{12} e^{2x} (2x^4 + 2x^3 + 6x^2 + 6(-2 + c_3)x + 8 + 6c_2 - 3c_3) + c_1 (-e^{-x}) + c_4$$

19.25 problem section 9.3, problem 25

Internal problem ID [1522]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 25.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 2y''' + 2y' - y = e^x(1 + x)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
dsolve(1*diff(y(x),x$4)-2*diff(y(x),x$3)-0*diff(y(x),x$2)+2*diff(y(x),x)-1*y(x)=exp(x)*(1+x)
```

$$y(x) = \frac{x^3(2+x)(e^x + x e^x)}{48x + 48} + c_1 e^x + e^{-x} c_2 + c_3 e^x x + c_4 e^x x^2$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 55

```
DSolve[1*y''''[x]-2*y'''[x]-0*y''[x]+2*y'[x]-1*y[x]==Exp[x]*(1+x),y[x],x,IncludeSingularSolu
```

$$y(x) \rightarrow \frac{1}{96} e^x (2x^4 + 4x^3 + (-6 + 96c_4)x^2 + (6 + 96c_3)x - 3 + 96c_2) + c_1 e^{-x}$$

19.26 problem section 9.3, problem 26

Internal problem ID [1523]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 26.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$2y'''' - 5y''' + 3y'' + y' - y = e^x(11 + 12x)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 52

```
dsolve(2*diff(y(x),x$4)-5*diff(y(x),x$3)+3*diff(y(x),x$2)+1*diff(y(x),x)-1*y(x)=exp(x)*(11+12*x),y(x))
```

$$y(x) = \frac{x^3(x+1)(12xe^x + 11e^x)}{66 + 72x} + c_1e^x + c_2e^xx + c_3e^xx^2 + c_4e^{-\frac{x}{2}}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 58

```
DSolve[2*y''''[x]-5*y'''[x]+3*y''[x]+1*y'[x]-1*y[x]==Exp[x]*(11+12*x),y[x],x,IncludeSingular
```

$$y(x) \rightarrow e^x \left(\frac{x^4}{6} + \frac{x^3}{6} + \left(-\frac{1}{3} + c_4 \right) x^2 + \left(\frac{4}{9} + c_3 \right) x - \frac{8}{27} + c_2 \right) + c_1 e^{-x/2}$$

19.27 problem section 9.3, problem 27

Internal problem ID [1524]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 27.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$y'''' + 3y''' + 3y'' + y' = e^{-x}(10x^2 - 24x + 5)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 56

```
dsolve(1*diff(y(x),x$4)+3*diff(y(x),x$3)+3*diff(y(x),x$2)+1*diff(y(x),x)-0*y(x)=exp(-x)*(5-2
```

$$y(x) = -\frac{(x^5 - x^4 + 6c_3x^2 + x^3 + 6c_2x + 12c_3x + 3x^2 + 6c_1 + 6c_2 + 12c_3 + 6x + 6)e^{-x}}{6} + c_4$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 65

```
DSolve[1*y''''[x]+3*y'''[x]+3*y''[x]+1*y'[x]-0*y[x]==Exp[-x]*(5-24*x+10*x^2),y[x],x,IncludeS
```

$$y(x) \rightarrow \frac{1}{6}e^{-x}(-x^5 + x^4 - x^3 - 3(1 + 2c_3)x^2 - 6(1 + c_2 + 2c_3)x - 6(1 + c_1 + c_2 + 2c_3)) + c_4$$

19.28 problem section 9.3, problem 28

Internal problem ID [1525]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 28.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 7y''' + 18y'' - 20y' + 8y = e^{2x}(-5x^2 - 8x + 3)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 79

```
dsolve(1*diff(y(x),x$4)-7*diff(y(x),x$3)+18*diff(y(x),x$2)-20*diff(y(x),x)+8*y(x)=exp(2*x)*
```

$$y(x) = \frac{x^3(x^2 - x - 2)(-5e^{2x}x^2 - 8e^{2x}x + 3e^{2x})}{60x^2 + 96x - 36} + c_1e^x + c_2e^{2x} + c_3e^{2x}x + c_4e^{2x}x^2$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 59

```
DSolve[1*y''''[x]-7*y'''[x]+18*y''[x]-20*y'[x]+8*y[x]==Exp[2*x]*(3-8*x-5*x^2),y[x],x,Include
```

$$y(x) \rightarrow \frac{1}{12}e^{2x}(-x^5 + x^4 + 2x^3 + 6(-1 + 2c_4)x^2 + 12(1 + c_3)x + 12(-1 + c_2)) + c_1e^x$$

19.29 problem section 9.3, problem 29

Internal problem ID [1526]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 29.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - y'' - 4y' + 4y = e^{-x}((16 + 10x) \cos(x) + (30 - 10x) \sin(x))$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
dsolve(0*diff(y(x),x$4)+1*diff(y(x),x$3)-1*diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=exp(-x)*((16+10*x)*cos(x)+(30-10*x)*sin(x))
```

$$y(x) = \cos(x) e^{-x} x + e^{-x} \cos(x) - \sin(x) e^{-x} x + 2 \sin(x) e^{-x} + c_1 e^x + c_2 e^{-2x} + c_3 e^{2x}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 49

```
DSolve[0*y''''[x]+1*y'''[x]-1*y''[x]-4*y'[x]+4*y[x]==Exp[-x]*((16+10*x)*Cos[x]+(30-10*x)*Sin[x])
```

$$y(x) \rightarrow e^{-2x}(-e^x(x-2)\sin(x) + e^x(x+1)\cos(x) + c_2 e^{3x} + c_3 e^{4x} + c_1)$$

19.30 problem section 9.3, problem 30

Internal problem ID [1527]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 30.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' + y'' - 4y' - 4y = e^{-x}((1 - 22x) \cos(2x) - (1 + 6x) \sin(2x))$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve(1*diff(y(x),x$3)+1*diff(y(x),x$2)-4*diff(y(x),x)-4*y(x)=exp(-x)*((1-22*x)*cos(2*x)-(1
```

$$y(x) = -(x - 1)e^{-x} \cos(2x) + (x + 1)e^{-x} \sin(2x) - \frac{5e^{-x}}{3} + e^{-2x}c_1 + e^{-x}c_2 + c_3e^{2x}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 51

```
DSolve[1*y'''[x]+1*y''[x]-4*y'[x]-4*y[x]==Exp[-x]*((1-22*x)*Cos[2*x]-(1+6*x)*Sin[2*x]),y[x],
```

$$y(x) \rightarrow e^{-2x}(e^x(x + 1) \sin(2x) - e^x(x - 1) \cos(2x) + c_2e^x + c_3e^{4x} + c_1)$$

19.31 problem section 9.3, problem 31

Internal problem ID [1528]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 31.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - y'' + 2y' - 2y = e^{2x}((-x^2 + 5x + 27) \cos(x) + (9x^2 + 13x + 2) \sin(x))$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 217

```
dsolve(1*diff(y(x),x$3)-1*diff(y(x),x$2)+2*diff(y(x),x)-2*y(x)=exp(2*x)*((27+5*x-x^2)*cos(1*
```

$$\begin{aligned}
 y(x) = & -\frac{5 e^{2x} \cos(x) x^2}{3} + 5 e^{2x} \cos(x) + \frac{10 e^{2x} \sin(x) x}{3} \\
 & + \frac{4 e^{2x} \sin(x) x^2}{3} + \frac{7 e^{2x} \sin(x)}{3} + \frac{5 e^{2x} \cos(x) x}{3} \\
 & + \left(\int \frac{\sqrt{2} (\cos(x) x^2 - 9 \sin(x) x^2 - 5 \cos(x) x - 13 \sin(x) x - 27 \cos(x) - 2 \sin(x)) (\sqrt{2} \cos(\sqrt{2} x) - 1)}{6} dx \right) \\
 & + \left(\int \frac{\sqrt{2} (\cos(x) x^2 - 9 \sin(x) x^2 - 5 \cos(x) x - 13 \sin(x) x - 27 \cos(x) - 2 \sin(x)) (\sqrt{2} \sin(\sqrt{2} x) + 1)}{6} dx \right) \\
 & + c_1 e^x + c_2 \cos(\sqrt{2} x) + c_3 \sin(\sqrt{2} x)
 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 60

```
DSolve[1*y'''[x]-1*y''[x]+2*y'[x]-2*y[x]==Exp[2*x]*((27+5*x-x^2)*Cos[1*x]+(2+13*x+9*x^2)*Sin[1*x]),x]
```

$$y(x) \rightarrow e^{2x}((-x^2 + x + 1) \cos(x) + (2x + 1) \sin(x)) + c_3 e^x + c_1 \cos(\sqrt{2}x) + c_2 \sin(\sqrt{2}x)$$

19.32 problem section 9.3, problem 32

Internal problem ID [1529]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 32.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - 2y'' + y' - 2y = -e^x((4x^2 + 5x + 9) \cos(2x) - (-3x^2 - 5x + 6) \sin(2x))$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 50

```
dsolve(1*diff(y(x),x$3)-2*diff(y(x),x$2)+1*diff(y(x),x)-2*y(x)=-exp(x)*((9+5*x+4*x^2)*cos(2*x)-(-3*x^2-5*x+6)*sin(2*x)),y(x))
```

$$y(x) = \frac{(55x + 61)e^x \cos(2x)}{50} + \frac{(25x^2 + 15x - 27)e^x \sin(2x)}{50} + c_1 \cos(x) + c_2 \sin(x) + c_3 e^{2x}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 65

```
DSolve[1*y'''[x]-2*y''[x]+1*y'[x]-2*y[x]==Exp[2*x]*((9+5*x+4*x^2)*Cos[2*x]-(-3*x^2-5*x+6)*Sin[2*x]),y[x]]
```

$$y(x) \rightarrow -\frac{e^{2x}((6760x^2 - 17680x - 29907) \sin(2x) + 2(5915x^2 + 7345x + 3928) \cos(2x))}{43940} + c_3 e^{2x} + c_1 \cos(x) + c_2 \sin(x)$$

19.33 problem section 9.3, problem 33

Internal problem ID [1530]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 33.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' + 3y'' + 4y' + 12y = 8 \cos(2x) - 16 \sin(2x)$$

✓ Solution by Maple

Time used: 0.36 (sec). Leaf size: 43

```
dsolve(1*diff(y(x),x$3)+3*diff(y(x),x$2)+4*diff(y(x),x)+12*y(x)=8*cos(2*x)-16*sin(2*x),y(x),
```

$$y(x) = \left(\frac{56}{169} + \frac{8x}{13} \right) \cos(2x) + \left(-\frac{136}{169} + \frac{14x}{13} \right) \sin(2x) + \cos(2x) c_1 + c_2 e^{-3x} + c_3 \sin(2x)$$

✓ Solution by Mathematica

Time used: 0.199 (sec). Leaf size: 47

```
DSolve[1*y'''[x]+3*y''[x]+4*y'[x]+12*y[x]==8*Cos[2*x]-16*Sin[2*x],y[x],x,IncludeSingularSolu
```

$$y(x) \rightarrow \frac{1}{169} (169c_3 e^{-3x} + (104x + 43 + 169c_1) \cos(2x) + (182x - 32 + 169c_2) \sin(2x))$$

19.34 problem section 9.3, problem 34

Internal problem ID [1531]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 34.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - y'' + 2y = e^x((20 + 4x) \cos(x) - (12 + 12x) \sin(x))$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 55

```
dsolve(1*diff(y(x),x$3)-1*diff(y(x),x$2)+0*diff(y(x),x)+2*y(x)=exp(x)*((20+4*x)*cos(x)-(12+12*x)*sin(x)),y(x),x)
```

$$y(x) = \frac{e^x(5x^2 + 5x + 22) \cos(x)}{5} + \frac{e^x(5x^2 + 15x + 1) \sin(x)}{5} + e^{-x}c_1 + c_2 \cos(x) e^x + c_3 \sin(x) e^x$$

✓ Solution by Mathematica

Time used: 0.308 (sec). Leaf size: 60

```
DSolve[1*y'''[x]-1*y''[x]+0*y'[x]+2*y[x]==Exp[x]*((20+4*x)*Cos[x]-(12+12*x)*Sin[x]),y[x],x]
```

$$y(x) \rightarrow \frac{1}{10} e^x (10x^2 + 10x + 23 + 10c_2) \cos(x) + \frac{1}{10} e^x (10x^2 + 30x - 21 + 10c_1) \sin(x) + c_3 e^{-x}$$

19.35 problem section 9.3, problem 35

Internal problem ID [1532]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 35.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - 7y'' + 20y' - 24y = -e^{2x}((13 - 8x) \cos(2x) - (8 - 4x) \sin(2x))$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 66

```
dsolve(1*diff(y(x),x$3)-7*diff(y(x),x$2)+20*diff(y(x),x)-24*y(x)=-exp(2*x)*((13-8*x)*cos(2*x)
```

$$y(x) = -\frac{(20x^2 - 60x + 83) e^{2x} \cos(2x)}{40} + \frac{(10x - 47) e^{2x} \sin(2x)}{20} + c_1 e^{3x} + c_2 \cos(2x) e^{2x} + c_3 e^{2x} \sin(2x)$$

✓ Solution by Mathematica

Time used: 0.955 (sec). Leaf size: 55

```
DSolve[1*y'''[x]-7*y''[x]+20*y'[x]-24*y[x]==-Exp[2*x]*((13-8*x)*Cos[2*x]-(8-4*x)*Sin[2*x]),y
```

$$y(x) \rightarrow \frac{1}{40} e^{2x} ((-20x^2 + 60x + 21 + 40c_2) \cos(2x) + 40c_3 e^x + (20x - 37 + 40c_1) \sin(2x))$$

19.36 problem section 9.3, problem 36

Internal problem ID [1533]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 36.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$y''' - 6y'' + 18y' = -e^{3x}((2 - 3x) \cos(3x) - (3 + 3x) \sin(3x))$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 94

```
dsolve(1*diff(y(x),x$3)-6*diff(y(x),x$2)+18*diff(y(x),x)-0*y(x)=-exp(3*x)*((2-3*x)*cos(3*x)-
```

$$y(x) = -\frac{e^{3x} \cos(3x) x^2}{12} + \frac{11 \cos(3x) e^{3x}}{36} - \frac{e^{3x} \sin(3x) x}{12} - \frac{37 e^{3x} \sin(3x)}{24} \\ + \frac{\cos(3x) e^{3x} c_1}{6} + \frac{c_1 e^{3x} \sin(3x)}{6} - \frac{c_2 \cos(3x) e^{3x}}{6} + \frac{e^{3x} \sin(3x) c_2}{6} + c_3$$

✓ Solution by Mathematica

Time used: 3.765 (sec). Leaf size: 57

```
DSolve[1*y'''[x]-6*y''[x]+18*y'[x]-0*y[x]==-Exp[3*x]*((2-3*x)*Cos[3*x]-(3+3*x)*Sin[3*x]),y[x]
```

$$y(x) \rightarrow c_3 - \frac{1}{216} e^{3x} (6(3x^2 + 1 + 6c_1 - 6c_2) \cos(3x) + (18x + 1 - 36c_1 - 36c_2) \sin(3x))$$

19.37 problem section 9.3, problem 37

Internal problem ID [1534]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 37.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' + 2y''' - 2y'' - 8y' - 8y = e^x(8 \cos(x) + 16 \sin(x))$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(1*diff(y(x),x$4)+2*diff(y(x),x$3)-2*diff(y(x),x$2)-8*diff(y(x),x)-8*y(x)=exp(x)*(8*cos(x)+16*sin(x)),y(x),x,Incl
```

$$y(x) = -\frac{\cos(x) e^x}{10} - \frac{7 \sin(x) e^x}{10} + e^{-2x} c_1 + c_2 e^{2x} + c_3 e^{-x} \cos(x) + c_4 e^{-x} \sin(x)$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 56

```
DSolve[1*y''''[x]+2*y'''[x]-2*y''[x]-8*y'[x]-8*y[x]==Exp[x]*(8*Cos[x]+16*Sin[x]),y[x],x,Incl
```

$$y(x) \rightarrow c_3 e^{-2x} + c_4 e^{2x} - \frac{1}{10} e^x (7 \sin(x) + \cos(x)) + c_2 e^{-x} \cos(x) + c_1 e^{-x} \sin(x)$$

19.38 problem section 9.3, problem 38

Internal problem ID [1535]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 38.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 3y''' + 2y'' + 2y' - 4y = e^x(2 \cos(2x) - \sin(2x))$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(1*diff(y(x),x$4)-3*diff(y(x),x$3)+2*diff(y(x),x$2)+2*diff(y(x),x)-4*y(x)=exp(x)*(2*cos(2*x)-sin(2*x)),y(x),x,Inco
```

$$y(x) = \frac{e^x \cos(2x)}{12} - \frac{e^x \sin(2x)}{12} + e^{-x}c_1 + c_2e^{2x} + c_3 \cos(x) e^x + c_4 \sin(x) e^x$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 56

```
DSolve[1*y''''[x]-3*y'''[x]+2*y''[x]+2*y'[x]-4*y[x]==Exp[x]*(2*Cos[2*x]-Sin[2*x]),y[x],x,Inc
```

$$y(x) \rightarrow c_3e^{-x} + c_4e^{2x} + \frac{1}{12}e^x(\cos(2x) - \sin(2x)) + c_2e^x \cos(x) + c_1e^x \sin(x)$$

19.39 problem section 9.3, problem 39

Internal problem ID [1536]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 39.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 8y''' + 24y'' - 32y' + 15y = e^{2x}(15 \cos(2x)x + 32 \sin(2x))$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 41

```
dsolve(1*diff(y(x),x$4)-8*diff(y(x),x$3)+24*diff(y(x),x$2)-32*diff(y(x),x)+15*y(x)=exp(2*x)*
```

$$y(x) = \cos(2x)e^{2x}x + c_1e^x + c_2e^{3x} + c_3e^{2x}\cos(x) + c_4e^{2x}\sin(x)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 45

```
DSolve[1*y''''[x]-8*y'''[x]+24*y''[x]-32*y'[x]+15*y[x]==Exp[2*x]*(15*x*Cos[2*x]+32*Sin[2*x])
```

$$y(x) \rightarrow e^x(e^x x \cos(2x) + c_4 e^{2x} + c_2 e^x \cos(x) + c_1 e^x \sin(x) + c_3)$$

19.40 problem section 9.3, problem 40

Internal problem ID [1537]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 40.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' + 6y''' + 13y'' + 12y' + 4y = e^{-x}((4-x)\cos(x) - (x+5)\sin(x))$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 65

```
dsolve(1*diff(y(x),x$4)+6*diff(y(x),x$3)+13*diff(y(x),x$2)+12*diff(y(x),x)+4*y(x)=exp(-1*x)*
```

$$y(x) = -\frac{e^{-x}\cos(x)}{2} + \frac{\sin(x)e^{-x}x}{2} - \sin(x)e^{-x} - \frac{\cos(x)e^{-x}x}{2} + e^{-2x}c_1 + e^{-x}c_2 + c_3e^{-x}x + c_4e^{-2x}x$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 56

```
DSolve[1*y''''[x]+6*y'''[x]+13*y''[x]+12*y'[x]+4*y[x]==Exp[-1*x]*((4-x)*Cos[x]-(5+x)*Sin[x])
```

$$y(x) \rightarrow \frac{1}{2}e^{-2x}(e^x(x-2)\sin(x) - e^x(x+1)\cos(x) + 2(c_2x + c_3e^x + c_4e^xx + c_1))$$

19.41 problem section 9.3, problem 41

Internal problem ID [1538]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 41.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' + 3y''' + 2y'' - 2y' - 4y = -e^{-x}(\cos(x) - \sin(x))$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 57

```
dsolve(1*diff(y(x),x$4)+3*diff(y(x),x$3)+2*diff(y(x),x$2)-2*diff(y(x),x)-4*y(x)=-exp(-1*x)*
```

$$y(x) = \frac{e^{-x}(5x + 14) \cos(x)}{50} + \frac{e^{-x}(5x - 1) \sin(x)}{25} + c_1 e^x + c_2 e^{-2x} + c_3 e^{-x} \cos(x) + c_4 e^{-x} \sin(x)$$

✓ Solution by Mathematica

Time used: 0.136 (sec). Leaf size: 58

```
DSolve[1*y''''[x]+3*y'''[x]+2*y''[x]-2*y'[x]-4*y[x]==-Exp[-1*x]*(Cos[x]-Sin[x]),y[x],x,Inclu
```

$$y(x) \rightarrow \frac{1}{50} e^{-2x} (50(c_4 e^{3x} + c_3) + e^x (5x + 14 + 50c_2) \cos(x) + e^x (10x - 7 + 50c_1) \sin(x))$$

19.42 problem section 9.3, problem 42

Internal problem ID [1539]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 42.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 5y''' + 13y'' - 19y' + 10y = e^x(\cos(2x) + \sin(2x))$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 61

```
dsolve(1*diff(y(x),x$4)-5*diff(y(x),x$3)+13*diff(y(x),x$2)-19*diff(y(x),x)+10*y(x)=exp(x)*(cos(2*x)+sin(2*x)),y(x),x,In
```

$$y(x) = \frac{(15x + 1)e^x \cos(2x)}{200} - \frac{(10x + 39)e^x \sin(2x)}{400} + \frac{e^x}{8} + c_1 e^x + c_2 e^{2x} + c_3 e^x \cos(2x) + c_4 e^x \sin(2x)$$

✓ Solution by Mathematica

Time used: 0.186 (sec). Leaf size: 53

```
DSolve[1*y''''[x]-5*y'''[x]+13*y''[x]-19*y'[x]+10*y[x]==Exp[x]*(Cos[2*x]+Sin[2*x]),y[x],x,In
```

$$y(x) \rightarrow \frac{1}{400} e^x (400(c_4 e^x + c_3) + (30x - 13 + 400c_2) \cos(2x) - 2(5x + 17 - 200c_1) \sin(2x))$$

19.43 problem section 9.3, problem 43

Internal problem ID [1540]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 43.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' + 8y''' + 32y'' + 64y' + 39y = e^{-2x}((4 - 15x) \cos(3x) - (4 + 15x) \sin(3x))$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 77

```
dsolve(1*diff(y(x),x$4)+8*diff(y(x),x$3)+32*diff(y(x),x$2)+64*diff(y(x),x)+39*y(x)=exp(-2*x)
```

$$y(x) = -\frac{e^{-2x}(30x^2 - 30x - 11) \cos(3x)}{240} + \frac{e^{-2x}(30x^2 + 30x - 11) \sin(3x)}{240} + c_1 e^{-3x} + e^{-x} c_2 + c_3 e^{-2x} \cos(3x) + c_4 e^{-2x} \sin(3x)$$

✓ Solution by Mathematica

Time used: 0.841 (sec). Leaf size: 73

```
DSolve[1*y''''[x]+8*y'''[x]+32*y''[x]+64*y'[x]+39*y[x]==Exp[-2*x]*((4-15*x)*Cos[3*x]-(4+15*x)
```

$$y(x) \rightarrow \frac{1}{720} e^{-3x} (-5e^x (18x^2 - 18x - 5 - 144c_2) \cos(3x) + e^x (90x^2 + 90x - 41 + 720c_1) \sin(3x) + 720(c_4 e^{2x} + c_3))$$

19.44 problem section 9.3, problem 44

Internal problem ID [1541]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 44.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 5y''' + 13y'' - 19y' + 10y = e^x((7 + 8x) \cos(2x) + (8 - 4x) \sin(2x))$$

✓ Solution by Maple

Time used: 0.765 (sec). Leaf size: 61

```
dsolve(1*diff(y(x),x$4)-5*diff(y(x),x$3)+13*diff(y(x),x$2)-19*diff(y(x),x)+10*y(x)=exp(x)*((7+8*x)*cos(2*x)+(8-4*x)*sin(2*x)))
```

$$y(x) = \frac{3e^x \cos(2x)}{4} - \frac{(4x^2 + 4x + 23)e^x \sin(2x)}{16} + \frac{7e^x}{2} + c_1 e^x + c_2 e^{2x} + c_3 e^x \cos(2x) + c_4 e^x \sin(2x)$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 172

```
DSolve[1*y''''[x]-5*y'''[x]+13*y''[x]-19*y'[x]-10*y[x]==Exp[x]*((7+8*x)*Cos[2*x]+(8-4*x)*Sin[2*x]),y[x]]
```

$$y(x) \rightarrow c_1 \exp(x \text{Root}[\#1^4 - 5\#1^3 + 13\#1^2 - 19\#1 - 10\&, 1]) + c_3 \exp(x \text{Root}[\#1^4 - 5\#1^3 + 13\#1^2 - 19\#1 - 10\&, 3]) + c_4 \exp(x \text{Root}[\#1^4 - 5\#1^3 + 13\#1^2 - 19\#1 - 10\&, 4]) + c_2 \exp(x \text{Root}[\#1^4 - 5\#1^3 + 13\#1^2 - 19\#1 - 10\&, 2]) - \frac{1}{100} e^x (-20x \sin(2x) + 64 \sin(2x) + 40x \cos(2x) + 67 \cos(2x))$$

19.45 problem section 9.3, problem 45

Internal problem ID [1542]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 45.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' + 4y''' + 8y'' + 8y' + 4y = -2e^x(\cos(x) - \sin(x))$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 51

```
dsolve(1*diff(y(x),x$4)+4*diff(y(x),x$3)+8*diff(y(x),x$2)+8*diff(y(x),x)+4*y(x)=-2*exp(x)*(cos(x)-sin(x)),y(x),x)
```

$$y(x) = -\frac{\cos(x)e^x}{16} - \frac{\sin(x)e^x}{16} + c_1e^{-x}\cos(x) + c_2\sin(x)e^{-x} + c_3\cos(x)e^{-x}x + c_4\sin(x)e^{-x}x$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 54

```
DSolve[1*y''''[x]+4*y'''[x]+8*y''[x]+8*y'[x]+4*y[x]==-2*Exp[x]*(Cos[1*x]-Sin[1*x]),y[x],x,Integrate->False]
```

$$y(x) \rightarrow \frac{1}{16}e^{-x}((-e^{2x} + 16(c_4x + c_3))\cos(x) - (e^{2x} - 16(c_2x + c_1))\sin(x))$$

19.46 problem section 9.3, problem 46

Internal problem ID [1543]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 46.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 8y''' + 32y'' - 64y' + 64y = e^{2x}(-\sin(2x) + \cos(2x))$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 87

```
dsolve(1*diff(y(x),x$4)-8*diff(y(x),x$3)+32*diff(y(x),x$2)-64*diff(y(x),x)+64*y(x)=exp(2*x)*
```

$$y(x) = -\frac{e^{2x}(4x^2 - 6x - 41) \cos(2x)}{128} + \frac{e^{2x}(18x^2 + 9x - 517) \sin(2x)}{576} \\ + c_1 \cos(2x) e^{2x} + c_2 \sin(2x) e^{2x} + c_3 \cos(2x) e^{2x} x + c_4 \sin(2x) x e^{2x}$$

✓ Solution by Mathematica

Time used: 0.199 (sec). Leaf size: 65

```
DSolve[1*y''''[x]-8*y'''[x]+32*y''[x]-64*y'[x]+64*y[x]==Exp[2*x]*(Cos[2*x]-Sin[2*x]),y[x],x,
```

$$y(x) \rightarrow \frac{1}{256} e^{2x} ((-8x^2 + 4(1 + 64c_4)x + 5 + 256c_3) \cos(2x) \\ + (8x^2 + 8(1 + 32c_2)x - 1 + 256c_1) \sin(2x))$$

19.47 problem section 9.3, problem 47

Internal problem ID [1544]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 47.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 8y''' + 26y'' - 40y' + 25y = e^{2x}(3 \cos(x) - (1 + 3x) \sin(x))$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 71

```
dsolve(1*diff(y(x),x$4)-8*diff(y(x),x$3)+26*diff(y(x),x$2)-40*diff(y(x),x)+25*y(x)=exp(2*x)*
```

$$y(x) = \frac{e^{2x}(4x + 9) \cos(x)}{16} + \frac{e^{2x}(x^3 + x^2 + 3x - 2) \sin(x)}{8} + c_1 e^{2x} \cos(x) + c_2 e^{2x} \sin(x) + c_3 e^{2x} \cos(x) x + c_4 e^{2x} \sin(x) x$$

✓ Solution by Mathematica

Time used: 0.239 (sec). Leaf size: 60

```
DSolve[1*y''''[x]-8*y'''[x]+26*y''[x]-40*y'[x]+25*y[x]==Exp[2*x]*(3*Cos[1*x]-(1+3*x)*Sin[1*x]
```

$$y(x) \rightarrow \frac{1}{16} e^{2x} ((2x^3 + 2x^2 + (9 + 16c_2)x - 1 + 16c_1) \sin(x) + (2(1 + 8c_4)x + 3 + 16c_3) \cos(x))$$

19.48 problem section 9.3, problem 48

Internal problem ID [1545]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 48.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - 4y'' + 5y' - 2y = e^{2x} - 4e^x - 2\cos(x) + 4\sin(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 67

```
dsolve(1*diff(y(x),x$3)-4*diff(y(x),x$2)+5*diff(y(x),x)-2*y(x)=exp(2*x)-4*exp(x)-2*cos(x)+4*
```

$$y(x) = -(\cos(x)e^x - 2e^{2x}x^2 + 2e^{3x} - 4e^{2x}x - xe^{3x} - 4e^{2x})e^{-x} + c_1e^x + c_2e^{2x} + c_3e^xx$$

✓ Solution by Mathematica

Time used: 0.373 (sec). Leaf size: 38

```
DSolve[1*y'''[x]-4*y''[x]+5*y'[x]-2*y[x]==Exp[2*x]-4*Exp[x]-2*Cos[x]+4*Sin[x],y[x],x,Include
```

$$y(x) \rightarrow -\cos(x) + e^x(2x^2 + (4 + c_2)x + e^x(x - 2 + c_3) + 4 + c_1)$$

19.49 problem section 9.3, problem 49

Internal problem ID [1546]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 49.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - y'' + y' - y = 5e^{2x} + 2e^x - 4\cos(x) + 4\sin(x)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 41

```
dsolve(1*diff(y(x),x$3)-1*diff(y(x),x$2)+1*diff(y(x),x)-1*y(x)=5*exp(2*x)+2*exp(x)-4*cos(x)+4*sin(x),y(x),x,Includ
```

$$y(x) = (2x + 2)\cos(x) + xe^x - 2\sin(x) - e^x + e^{2x} + c_1\cos(x) + e^xc_2 + c_3\sin(x)$$

✓ Solution by Mathematica

Time used: 0.282 (sec). Leaf size: 35

```
DSolve[1*y'''[x]-1*y''[x]+1*y'[x]-1*y[x]==5*Exp[2*x]+2*Exp[x]-4*Cos[x]+4*Sin[x],y[x],x,Includ
```

$$y(x) \rightarrow e^x(x + e^x - 1 + c_3) + (2x + 1 + c_1)\cos(x) + (-2 + c_2)\sin(x)$$

19.50 problem section 9.3, problem 50

Internal problem ID [1547]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 50.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$y''' - y' = -2 - 2x + 4e^x - 6e^{-x} + 96e^{3x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 51

```
dsolve(1*diff(y(x),x$3)-0*diff(y(x),x$2)-1*diff(y(x),x)-0*y(x)=-2*(1+x)+4*exp(x)-6*exp(-x)+96*exp(3*x),y(x),x,Inco
```

$$y(x) = x^2 + 2xe^x - 3e^x - 3xe^{-x} - \frac{9e^{-x}}{2} + e^x c_2 - e^{-x} c_1 + 4e^{3x} + 2x + c_3$$

✓ Solution by Mathematica

Time used: 1.399 (sec). Leaf size: 49

```
DSolve[1*y'''[x]-0*y''[x]-1*y'[x]-0*y[x]==-2*(1+x)+4*Exp[x]-6*Exp[-x]+96*Exp[3*x],y[x],x,Inc
```

$$y(x) \rightarrow x(x+2) + 4e^{3x} + e^x(2x-3+c_1) - \frac{1}{2}e^{-x}(6x+9+2c_2) + c_3$$

19.51 problem section 9.3, problem 51

Internal problem ID [1548]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 51.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - 4y'' + 9y' - 10y = 10e^{2x} + 20\sin(2x)e^x - 10$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 91

```
dsolve(1*diff(y(x),x$3)-4*diff(y(x),x$2)+9*diff(y(x),x)-10*y(x)=10*exp(2*x)+20*exp(x)*sin(2*x)-10,y(x),x)
```

$$y(x) = \frac{e^{-2x}e^{3x}(5x-8)\cos(2x)}{5} - \frac{e^{-2x}e^{3x}(20x+13)\sin(2x)}{10} + \frac{e^{-2x}(10e^{4x}x+5e^{2x}-4e^{4x})}{5} + c_1e^{2x} + c_2e^x\cos(2x) + c_3e^x\sin(2x)$$

✓ Solution by Mathematica

Time used: 0.992 (sec). Leaf size: 72

```
DSolve[1*y'''[x]-4*y''[x]+9*y'[x]-10*y[x]==10*Exp[2*x]+20*Exp[x]*Sin[2*x]-10,y[x],x,IncludeS
```

$$y(x) \rightarrow 2e^{2x}x - \frac{4e^{2x}}{5} + c_3e^{2x} + \frac{1}{10}e^x(10x-11+10c_2)\cos(2x) - \frac{1}{20}e^x(40x+21-20c_1)\sin(2x) + 1$$

19.52 problem section 9.3, problem 52

Internal problem ID [1549]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 52.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' + 3y'' + 3y' + y = 12e^{-x} + 9\cos(2x) - 13\sin(2x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 46

```
dsolve(1*diff(y(x),x$3)+3*diff(y(x),x$2)+3*diff(y(x),x)+y(x)=12*exp(-x)+9*cos(2*x)-13*sin(2*x))
```

$$y(x) = 2e^{-x}x^3 - \cos(2x) + \sin(2x) + e^{-x}c_1 + c_2xe^{-x} + c_3x^2e^{-x}$$

✓ Solution by Mathematica

Time used: 0.295 (sec). Leaf size: 46

```
DSolve[1*y'''[x]+3*y''[x]+3*y'[x]+1*y[x]==12*Exp[-x]+9*Cos[2*x]-13*Sin[2*x],y[x],x,IncludeSi
```

$$y(x) \rightarrow e^{-x}(2x^3 + c_3x^2 + e^x \sin(2x) - e^x \cos(2x) + c_2x + c_1)$$

19.53 problem section 9.3, problem 53

Internal problem ID [1550]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 53.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' + y'' - y' - y = 4e^{-x}(1 - 6x) - 2\cos(x)x + 2\sin(x)(1 + x)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve(1*diff(y(x),x$3)+1*diff(y(x),x$2)-1*diff(y(x),x)-1*y(x)=4*exp(-x)*(1-6*x)-2*x*cos(x)+
```

$$y(x) = e^{-x}(e^x \cos(x)x + 2x^3 - 2\sin(x)e^x + 2x^2 + 2x + 1) + c_1e^x + e^{-x}c_2 + c_3e^{-x}x$$

✓ Solution by Mathematica

Time used: 0.627 (sec). Leaf size: 54

```
DSolve[1*y'''[x]+1*y''[x]-1*y'[x]-1*y[x]==4*Exp[-x]*(1-6*x)-2*x*Cos[x]+2*(1+x)*Sin[x],y[x],x
```

$$y(x) \rightarrow e^{-x}(2x^3 + 2x^2 + 2x - 2e^x \sin(x) + e^x x \cos(x) + c_2x + c_3e^{2x} + 1 + c_1)$$

19.54 problem section 9.3, problem 54

Internal problem ID [1551]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 54.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 5y'' + 4y = -12e^x + 6e^{-x} + 10\cos(x)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 68

```
dsolve(diff(y(x),x$4)-0*diff(y(x),x$3)-5*diff(y(x),x$2)-0*diff(y(x),x)+4*y(x))=-12*exp(x)+6*exp(-x)+10*cos(x),y(x),x,Inco
```

$$y(x) = \frac{(2e^{4x} + 6e^{3x}\cos(x) - e^{2x} + 12e^{4x}x + 6e^{2x}x)e^{-3x}}{6} + c_1e^x + c_2e^{-2x} + c_3e^{-x} + c_4e^{2x}$$

✓ Solution by Mathematica

Time used: 0.19 (sec). Leaf size: 58

```
DSolve[y''''[x]-0*y'''[x]-5*y''[x]-0*y'[x]+4*y[x]==-12*Exp[x]+6*Exp[-x]+10*Cos[x],y[x],x,Inc
```

$$y(x) \rightarrow \cos(x) + c_1e^{-2x} + \frac{1}{6}e^{-x}(6x - 1 + 6c_2) + \frac{1}{3}e^x(6x + 1 + 3c_3) + c_4e^{2x}$$

19.55 problem section 9.3, problem 55

Internal problem ID [1552]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 55.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 4y''' + 11y'' - 14y' + 10y = -e^x(\sin(x) + 2\cos(2x))$$

✓ Solution by Maple

Time used: 0.609 (sec). Leaf size: 63

```
dsolve(diff(y(x),x$4)-4*diff(y(x),x$3)+11*diff(y(x),x$2)-14*diff(y(x),x)+10*y(x))=-exp(x)*(sin(x)+2*cos(2*x)),y(x),x,inc
```

$$y(x) = \frac{7e^x \cos(2x)}{18} + \frac{e^x \sin(2x)x}{6} + \frac{e^x \cos(x)x}{6} + \frac{\sin(x)e^x}{9} + c_1 \cos(x)e^x + c_2 \sin(x)e^x + c_3 e^x \cos(2x) + c_4 e^x \sin(2x)$$

✓ Solution by Mathematica

Time used: 0.151 (sec). Leaf size: 53

```
DSolve[y''''[x]-4*y'''[x]+11*y''[x]-14*y'[x]+10*y[x]==-Exp[x]*(Sin[x]+2*Cos[2*x]),y[x],x,Inc
```

$$y(x) \rightarrow \frac{1}{36}e^x((11+36c_2)\cos(2x)+(1+36c_3)\sin(x)+6\cos(x)(x+2(x+6c_1)\sin(x)+6c_4))$$

19.56 problem section 9.3, problem 56

Internal problem ID [1553]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 56.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' + 2y''' - 3y'' - 4y' + 4y = 2e^x(1+x) + e^{-2x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 74

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$3)-3*diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=2*exp(x)*(1+x)
```

$$y(x) = \frac{e^{-2x}(18x^3e^{3x} + 18x^2e^{3x} - 36xe^{3x} + 27x^2 + 20e^{3x} + 36x + 18)}{486} + c_1e^x + c_2e^{-2x} + c_3e^xx + c_4e^{-2x}x$$

✓ Solution by Mathematica

Time used: 0.239 (sec). Leaf size: 66

```
DSolve[y''''[x]+2*y'''[x]-3*y''[x]-4*y'[x]+4*y[x]==2*Exp[x]*(1+x)+Exp[-2*x],y[x],x,IncludeSi
```

$$y(x) \rightarrow \frac{1}{54}e^{-2x}(3x^2 + (4 + 54c_2)x + 2 + 54c_1) + \frac{1}{243}e^x(9x^3 + 9x^2 + 9(-2 + 27c_4)x + 10 + 243c_3)$$

19.57 problem section 9.3, problem 57

Internal problem ID [1554]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 57.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' + 4y = \sinh(x) \cos(x) - \cosh(x) \sin(x)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 73

```
dsolve(diff(y(x),x$4)+0*diff(y(x),x$3)-0*diff(y(x),x$2)-0*diff(y(x),x)+4*y(x))=sinh(x)*cos(x)
```

$$y(x) = \frac{3e^{-x}(e^{2x} - 1) \cos(x)}{64} + \frac{e^{-x}(4e^{2x}x - 3e^{2x} - 4x - 3) \sin(x)}{64} \\ + c_1 \cos(x) e^x + c_2 \sin(x) e^x + c_3 e^{-x} \cos(x) + c_4 e^{-x} \sin(x)$$

✓ Solution by Mathematica

Time used: 0.676 (sec). Leaf size: 63

```
DSolve[y''''[x]+0*y'''[x]-0*y''[x]-0*y'[x]+4*y[x]==Sinh[x]*Cos[x]-Cosh[x]*Sin[x],y[x],x,Incl
```

$$y(x) \rightarrow \frac{1}{64} e^{-x} \left((3 + 64c_4) e^{2x} - 3 + 64c_1 \right) \cos(x) \\ + \left(-4x + e^{2x}(4x - 3 + 64c_3) - 3 + 64c_2 \right) \sin(x)$$

19.58 problem section 9.3, problem 58

Internal problem ID [1555]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 58.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' + 5y''' + 9y'' + 7y' + 2y = e^{-x}(30 + 24x) - e^{-2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 72

```
dsolve(diff(y(x),x$4)+5*diff(y(x),x$3)+9*diff(y(x),x$2)+7*diff(y(x),x)+2*y(x))=exp(-x)*(30+24*x)-exp(-2*x),y(x))
```

$$y(x) = -(-x^4 - x^3 - 6x + 3x^2 + 6 - x e^{-x} - 3 e^{-x}) e^{-x} + e^{-2x} c_1 + e^{-x} c_2 + c_3 e^{-x} x + c_4 x^2 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.274 (sec). Leaf size: 44

```
DSolve[y''''[x]+5*y'''[x]+9*y''[x]+7*y'[x]+2*y[x]==Exp[-x]*(30+24*x)-Exp[-2*x],y[x],x,IncludeSolutions->True]
```

$$y(x) \rightarrow e^{-2x} (e^x (x^4 + x^3 + (-3 + c_4)x^2 + (6 + c_3)x - 6 + c_2) + x + 3 + c_1)$$

19.59 problem section 9.3, problem 59

Internal problem ID [1556]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 59.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 4y''' + 7y'' - 6y' + 2y = e^x(12x - 2\cos(x) + 2\sin(x))$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 52

```
dsolve(diff(y(x),x$4)-4*diff(y(x),x$3)+7*diff(y(x),x$2)-6*diff(y(x),x)+2*y(x))=exp(x)*(12*x-2*cos(x)+2*sin(x)),y(x),x,In
```

$$y(x) = e^x(x + 3)\cos(x) + e^x(-2 + x)\sin(x) + 2e^xx(x^2 - 6) + c_1e^x + c_2xe^x + c_3\cos(x)e^x + c_4\sin(x)e^x$$

✓ Solution by Mathematica

Time used: 0.142 (sec). Leaf size: 40

```
DSolve[y''''[x]-4*y'''[x]+7*y''[x]-6*y'[x]+2*y[x]==Exp[x]*(12*x-2*Cos[x]+2*Sin[x]),y[x],x,In
```

$$y(x) \rightarrow e^x(2x^3 - 12x + c_4x + (x + 3 + c_2)\cos(x) + (x - 2 + c_1)\sin(x) + c_3)$$

19.60 problem section 9.3, problem 60

Internal problem ID [1557]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 60.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - y'' - y' + y = e^{2x}(10 + 3x)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
dsolve(diff(y(x),x$3)-1*diff(y(x),x$2)-1*diff(y(x),x)+1*y(x)=exp(2*x)*(10+3*x),y(x), singular
```

$$y(x) = \frac{(x+1)(3e^{2x}x + 10e^{2x})}{10+3x} + c_1e^x + e^{-x}c_2 + c_3e^xx$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 34

```
DSolve[y''''[x]-1*y''[x]-1*y'[x]+1*y[x]==Exp[2*x]*(10+3*x),y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow e^{2x}(x+1) + c_1e^{-x} + e^x(c_3x + c_2)$$

19.61 problem section 9.3, problem 61

Internal problem ID [1558]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 61.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' + y'' - 2y = -e^{3x}(17x^2 + 67x + 9)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 70

```
dsolve(diff(y(x),x$3)+1*diff(y(x),x$2)-0*diff(y(x),x)-2*y(x)=-exp(3*x)*(9+67*x+17*x^2),y(x),
```

$$y(x) = \frac{(x^2 + 2x - 2)(-17x^2 e^{3x} - 67x e^{3x} - 9 e^{3x})}{34x^2 + 134x + 18} + c_1 e^x + c_2 \cos(x) e^{-x} + c_3 e^{-x} \sin(x)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 49

```
DSolve[y'''[x]+1*y''[x]-0*y'[x]-2*y[x]==-Exp[3*x]*(9+67*x+17*x^2),y[x],x,IncludeSingularSolu
```

$$y(x) \rightarrow -\frac{1}{2}e^{3x}(x^2 + 2x - 2) + c_3 e^x + c_2 e^{-x} \cos(x) + c_1 e^{-x} \sin(x)$$

19.62 problem section 9.3, problem 62

Internal problem ID [1559]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 62.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - 6y'' + 11y' - 6y = e^{2x}(-3x^2 - 4x + 5)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 67

```
dsolve(diff(y(x),x$3)-6*diff(y(x),x$2)+11*diff(y(x),x)-6*y(x)=exp(2*x)*(5-4*x-3*x^2),y(x), s
```

$$y(x) = -\frac{x(x^2 + 2x + 1)(-3e^{2x}x^2 - 4e^{2x}x + 5e^{2x})}{3x^2 + 4x - 5} + c_1e^x + c_2e^{2x} + c_3e^{3x}$$

✓ Solution by Mathematica

Time used: 0.072 (sec). Leaf size: 37

```
DSolve[y'''[x]-6*y''[x]+11*y'[x]-6*y[x]==Exp[2*x]*(5-4*x-3*x^2),y[x],x,IncludeSingularSoluti
```

$$y(x) \rightarrow e^x(e^x(x^3 + 2x^2 + x + 4 + c_2) + c_3e^{2x} + c_1)$$

19.63 problem section 9.3, problem 63

Internal problem ID [1560]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 63.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$y''' + 2y'' + y' = -2e^{-x}(6x^2 - 18x + 7)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$3)+2*diff(y(x),x$2)+1*diff(y(x),x)-0*y(x)=-2*exp(-x)*(7-18*x+6*x^2),y(x),
```

$$y(x) = e^{-x}(x^4 - 2x^3 - c_1x + x^2 - c_1 - c_2 + 2x + 2) + c_3$$

✓ Solution by Mathematica

Time used: 0.101 (sec). Leaf size: 42

```
DSolve[y'''[x]+2*y''[x]+1*y'[x]-0*y[x]==-2*Exp[-x]*(7-18*x+6*x^2),y[x],x,IncludeSingularSolu
```

$$y(x) \rightarrow e^{-x}(x^4 - 2x^3 + x^2 - (-2 + c_2)x + 2 - c_1 - c_2) + c_3$$

19.64 problem section 9.3, problem 64

Internal problem ID [1561]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 64.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - 3y'' + 3y' - y = e^x(1 + x)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)+3*diff(y(x),x)-y(x)=exp(x)*(1+x),y(x), singsol=all)
```

$$y(x) = \left(\frac{1}{24}x^3 + \frac{1}{8}x^2\right)(e^x + x e^x) + c_1 e^x + c_2 x e^x + c_3 e^x x^2$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 37

```
DSolve[y'''[x]-3*y''[x]+3*y'[x]-1*y[x]==Exp[x]*(1+x),y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \rightarrow \frac{1}{24}e^x(x^4 + 4x^3 + 24c_3x^2 + 24c_2x + 24c_1)$$

19.65 problem section 9.3, problem 65

Internal problem ID [1562]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 65.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 2y'' + y = -e^{-x}(3x^2 - 9x + 4)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 66

```
dsolve(diff(y(x),x$4)-0*diff(y(x),x$3)-2*diff(y(x),x$2)+0*diff(y(x),x)+y(x)=-exp(-x)*(4-9*x
```

$$y(x) = \frac{x^2(x^2 - 2x - 1)(-3x^2 + 9x - 4)e^{-x}}{48x^2 - 144x + 64} + c_1e^x + e^{-x}c_2 + c_3e^x x + c_4x e^{-x}$$

✓ Solution by Mathematica

Time used: 0.113 (sec). Leaf size: 61

```
DSolve[y''''[x]-0*y'''[x]-2*y''[x]+0*y'[x]+1*y[x]==-Exp[-x]*(4-9*x+3*x^2),y[x],x,IncludeSing
```

$$y(x) \rightarrow \frac{1}{32}e^{-x}(-2x^4 + 4x^3 + 2x^2 + x(32c_4e^{2x} - 2 + 32c_2) + 32c_3e^{2x} - 3 + 32c_1)$$

19.66 problem section 9.3, problem 66

Internal problem ID [1563]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 66.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' + 2y'' - y' - 2y = e^{-2x}((23 - 2x) \cos(x) + (8 - 9x) \sin(x))$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 54

```
dsolve(diff(y(x), x$3)+2*diff(y(x), x$2)-diff(y(x), x)-2*y(x)=exp(-2*x)*((23-2*x)*cos(x)+(8-9*x)*sin(x)), y(x), x, Includ
```

$$y(x) = \frac{\cos(x) e^{-2x} x}{2} + e^{-2x} \cos(x) - 2 \sin(x) e^{-2x} x + \frac{3 e^{-2x} \sin(x)}{2} + c_1 e^x + c_2 e^{-2x} + c_3 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 48

```
DSolve[y'''[x]+2*y''[x]-y'[x]-2*y[x]==Exp[-2*x]*((23-2*x)*Cos[x]+(8-9*x)*Sin[x]), y[x], x, Includ
```

$$y(x) \rightarrow \frac{1}{2} e^{-2x} ((3 - 4x) \sin(x) + (x + 2) \cos(x) + 2(c_2 e^x + c_3 e^{3x} + c_1))$$

19.67 problem section 9.3, problem 67

Internal problem ID [1564]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 67.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$y'''' - 3y''' + 4y'' - 2y' = e^x((28 + 6x) \cos(2x) + (-12x + 11) \sin(2x))$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 51

```
dsolve(diff(y(x),x$4)-3*diff(y(x),x$3)+4*diff(y(x),x$2)-2*diff(y(x),x)-0*y(x))=exp(x)*((28+6*
```

$$y(x) = -e^x \sin(2x) x + \frac{3e^x}{2} - \frac{c_3 \cos(x) e^x}{2} + \frac{c_3 \sin(x) e^x}{2} + \frac{c_2 e^x \cos(x)}{2} + \frac{c_2 \sin(x) e^x}{2} + c_1 e^x + c_4$$

✓ Solution by Mathematica

Time used: 0.278 (sec). Leaf size: 88

```
DSolve[y''''[x]-3*y'''[x]+4*y''[x]-2*y'[x]-0*y[x]==Exp(x)*((28+6*x)*Cos[2*x]+(11-12*x)*Sin[2
```

$$y(x) \rightarrow \frac{1}{500} \text{Exp}(300x^2 + 715x + 676) \cos(2x) + \frac{1}{500} \cos(x) (\text{Exp}(300x^2 - 160x - 819) \sin(x) - 250(c_1 - c_2)e^x) + c_3 e^x + \frac{1}{2} c_1 e^x \sin(x) + \frac{1}{2} c_2 e^x \sin(x) + c_4$$

19.68 problem section 9.3, problem 68

Internal problem ID [1565]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 68.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 4y''' + 14y'' - 20y' + 25y = e^x((2 + 6x) \cos(2x) + 3 \sin(2x))$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 75

```
dsolve(diff(y(x),x$4)-4*diff(y(x),x$3)+14*diff(y(x),x$2)-20*diff(y(x),x)+25*y(x)=exp(x)*((2+
```

$$y(x) = -\frac{e^x(4x^3 + 4x^2 + 126x - 41) \cos(2x)}{64} + \frac{e^x(6x + 1111) \sin(2x)}{192} + c_1 e^x \cos(2x) + c_2 e^x \sin(2x) + c_3 e^x \cos(2x)x + c_4 e^x \sin(2x)x$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 74

```
DSolve[y''''[x]-4*y'''[x]+14*y''[x]-20*y'[x]+25*y[x]==Exp(x)*((2+6*x)*Cos[2*x]+3*Sin[2*x]),y
```

$$y(x) \rightarrow -\frac{\text{Exp}(13872x^2 + 51901x + 68676) \sin(2x)}{83521} - \frac{2\text{Exp}(13005x^2 + 62475x + 100292) \cos(2x)}{83521} + e^x(c_4x + c_3) \cos(2x) + e^x(c_2x + c_1) \sin(2x)$$

19.69 problem section 9.3, problem 69

Internal problem ID [1566]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 69.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - 2y'' - 5y' + 6y = 2e^x(1 - 6x)$$

With initial conditions

$$[y(0) = 2, y'(0) = 7, y''(0) = 9]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 29

```
dsolve([diff(y(x),x$3)-2*diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=2*exp(x)*(1-6*x),y(0) = 2, D(y
```

$$y(x) = (e^{5x} + x^2 e^{3x} + 2e^{3x} - 1)e^{-2x}$$

✓ Solution by Mathematica

Time used: 0.061 (sec). Leaf size: 27

```
DSolve[{y'''[x]-2*y''[x]-5*y'[x]+6*y[x]==2*Exp[x]*(1-6*x),{y[0]==2,y'[0]==7,y''[0]==9}},y[x]
```

$$y(x) \rightarrow e^x(x^2 + 2) - e^{-2x} + e^{3x}$$

19.70 problem section 9.3, problem 70

Internal problem ID [1567]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 70.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - y'' - y' + y = -e^{-x}(4 - 8x)$$

With initial conditions

$$[y(0) = 2, y'(0) = 0, y''(0) = 0]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$3)-1*diff(y(x),x$2)-1*diff(y(x),x)+1*y(x)=-exp(-x)*(4-8*x),y(0) = 2, D(y
```

$$y(x) = (x^2 + x + 1)e^{-x} - (x - 1)e^x$$

✓ Solution by Mathematica

Time used: 0.093 (sec). Leaf size: 27

```
DSolve[{y'''[x]-1*y''[x]-1*y'[x]+1*y[x]==-Exp[-x]*(4-8*x),{y[0]==2,y'[0]==0,y''[0]==0}},y[x]
```

$$y(x) \rightarrow e^{-x}(x^2 + x - e^{2x}(x - 1) + 1)$$

19.71 problem section 9.3, problem 71

Internal problem ID [1568]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 71.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$4y''' - 3y' - y = e^{-\frac{x}{2}}(2 - 3x)$$

With initial conditions

$$[y(0) = -1, y'(0) = 15, y''(0) = -17]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 22

```
dsolve([4*diff(y(x),x$3)-0*diff(y(x),x$2)-3*diff(y(x),x)-1*y(x)=exp(-x/2)*(2-3*x),y(0) = -1,
```

$$y(x) = \frac{(x^3 + 192x)e^{-\frac{x}{2}}}{12} - e^x$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 35

```
DSolve[{4*y'''[x]-0*y''[x]-3*y'[x]-1*y[x]==Exp[-x/2]*(2-3*x),{y[0]==-1,y'[0]==15,y''[0]==-17}},y
```

$$y(x) \rightarrow \frac{1}{36}e^{-x/2}(3x^3 + 24x + 8e^{3x/2} + 64)$$

19.72 problem section 9.3, problem 72

Internal problem ID [1569]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 72.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' + 2y''' + 2y'' + 2y' + y = e^{-x}(20 - 12x)$$

With initial conditions

$$[y(0) = 3, y'(0) = -4, y''(0) = 7, y'''(0) = -22]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 31

```
dsolve([diff(y(x),x$4)+2*diff(y(x),x$3)+2*diff(y(x),x$2)+2*diff(y(x),x)+1*y(x)=exp(-x)*(20-12*x),{y(0)=3,y'(0)=-4,y''(0)=7,y'''(0)=-22})
```

$$y(x) = (-x^3 + 2x^2 - x + 2)e^{-x} + \cos(x) - \sin(x)$$

✓ Solution by Mathematica

Time used: 0.085 (sec). Leaf size: 39

```
DSolve[{y''''[x]+2*y'''[x]+2*y''[x]+2*y'[x]+1*y[x]==Exp[-x]*(20-12*x),{y[0]==3,y'[0]==-4,y''[0]==7,y'''[0]==-22}]
```

$$y(x) \rightarrow e^{-x}(-x^3 + 2x^2 - x - e^x \sin(x) + e^x \cos(x) + 2)$$

19.73 problem section 9.3, problem 73

Internal problem ID [1570]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 73.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' + 2y'' + y' + 2y = 30 \cos(x) - 10 \sin(x)$$

With initial conditions

$$[y(0) = 3, y'(0) = -4, y''(0) = 16]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 25

```
dsolve([0*diff(y(x),x$4)+1*diff(y(x),x$3)+2*diff(y(x),x$2)+1*diff(y(x),x)+2*y(x)=30*cos(x)-10*sin(x),{y(0)=3,y'(0)=-4,y''(0)=16})
```

$$y(x) = e^{-2x} + (2 - x) \cos(x) + (7x - 1) \sin(x)$$

✓ Solution by Mathematica

Time used: 0.093 (sec). Leaf size: 26

```
DSolve[{0*y''''[x]+1*y'''[x]+2*y''[x]+1*y'[x]+2*y[x]==30*Cos[x]-10*Sin[x],{y[0]==3,y'[0]==-4,y''[0]==16}}
```

$$y(x) \rightarrow e^{-2x} + (7x - 1) \sin(x) - ((x - 2) \cos(x))$$

19.74 problem section 9.3, problem 74

Internal problem ID [1571]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

Problem number: section 9.3, problem 74.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$y'''' - 3y''' + 5y'' - 2y' = -2e^x(\cos(x) - \sin(x))$$

With initial conditions

$$[y(0) = 2, y'(0) = 0, y''(0) = -1, y'''(0) = -5]$$

✓ Solution by Maple

Time used: 42.938 (sec). Leaf size: 1299

```
dsolve([1*diff(y(x),x$4)-3*diff(y(x),x$3)+5*diff(y(x),x$2)-2*diff(y(x),x)+0*y(x)=-2*exp(x)*
```

Expression too large to display

✓ Solution by Mathematica

Time used: 0.103 (sec). Leaf size: 3484

```
DSolve[{1*y''''[x]-3*y'''[x]+5*y''[x]-2*y'[x]+0*y[x]==-2*Exp[x]*(Cos[x]-Sin[x]),{y[0]==2,y'
```

Too large to display

20 Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503

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20.1 problem section 9.4, problem 3

Internal problem ID [1572]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503

Problem number: section 9.4, problem 3.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^3 y''' - 3x^2 y'' + 6y'x - 6y = 2x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(x^3*diff(y(x),x$3)-3*x^2*diff(y(x),x$2)+6*x*diff(y(x),x)-6*y(x)=2*x,y(x), singsol=all
```

$$y(x) = x \ln(x) + \frac{3x}{2} + c_3 x^3 + x^2 c_2 + c_1 x$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 25

```
DSolve[x^3*y'''[x]-3*x^2*y''[x]+6*x*y'[x]-6*y[x]==2*x,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \rightarrow x \left(c_3 x^2 + \log(x) + c_2 x + \frac{3}{2} + c_1 \right)$$

20.2 problem section 9.4, problem 8

Internal problem ID [1573]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503

Problem number: section 9.4, problem 8.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$4x^3y''' + 4x^2y'' - 5y'x + 2y = 30x^2$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
dsolve(4*x^3*diff(y(x),x$3)+4*x^2*diff(y(x),x$2)-5*x*diff(y(x),x)+2*y(x)=30*x^2,y(x), singularSolutions = false)
```

$$y(x) = 2 \ln(x) x^2 - \frac{32x^2}{15} + c_1 x^2 + \frac{c_2}{\sqrt{x}} + c_3 \sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 38

```
DSolve[4*x^3*y'''[x]+4*x^2*y''[x]-5*x*y'[x]+2*y[x]==30*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 2x^2 \log(x) + \frac{\left(-\frac{32}{15} + c_3\right) x^{5/2} + c_2 x + c_1}{\sqrt{x}}$$

20.3 problem section 9.4, problem 11

Internal problem ID [1574]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503

Problem number: section 9.4, problem 11.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _nonhomogeneous]]`

$$x^3 y''' + x^2 y'' - 2y'x + 2y = x^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x^3*diff(y(x),x$3)+x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=x^2,y(x), singsol=all)
```

$$y(x) = c_2 x + c_3 x^2 + \frac{2x^3 \ln(x) + c_1}{6x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 35

```
DSolve[x^3*y'''[x]+x^2*y''[x]-2*x*y'[x]+2*y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{3}x^2 \log(x) + \left(-\frac{4}{9} + c_3\right)x^2 + c_2 x + \frac{c_1}{x}$$

20.4 problem section 9.4, problem 14

Internal problem ID [1575]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503

Problem number: section 9.4, problem 14.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$16x^4y'''' + 96x^3y'''' + 72x^2y'' - 24y'x + 9y = 96x^{\frac{5}{2}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(16*x^4*diff(y(x),x$4)+96*x^3*diff(y(x),x$3)+72*x^2*diff(y(x),x$2)-24*x*diff(y(x),x)+9*y(x),x)=96*x^(5/2),y(x),x,Inco
```

$$y(x) = \frac{x^{\frac{5}{2}}}{4} + \frac{c_1}{x^{\frac{3}{2}}} + \frac{c_2}{\sqrt{x}} + c_3\sqrt{x} + c_4x^{\frac{3}{2}}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 41

```
DSolve[16*x^4*y''''[x]+96*x^3*y''''[x]+72*x^2*y''[x]-24*x*y'[x]+9*y[x]==96*x^(5/2),y[x],x,Inc
```

$$y(x) \rightarrow \frac{x^4 + 4c_4x^3 + 4c_3x^2 + 4c_2x + 4c_1}{4x^{3/2}}$$

20.5 problem section 9.4, problem 16

Internal problem ID [1576]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503

Problem number: section 9.4, problem 16.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$x^4 y'''' - 4x^3 y''' + 12x^2 y'' - 24y'x + 24y = x^4$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 35

```
dsolve(x^4*diff(y(x),x$4)-4*x^3*diff(y(x),x$3)+12*x^2*diff(y(x),x$2)-24*x*diff(y(x),x)+24*y(x)=x^4)
```

$$y(x) = -\frac{11x^4}{36} + \frac{\ln(x)x^4}{6} + c_4x^4 + c_3x^3 + x^2c_2 + c_1x$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 40

```
DSolve[x^4*y''''[x]-4*x^3*y'''[x]+12*x^2*y''[x]-24*x*y'[x]+24*y[x]==x^4,y[x],x,IncludeSingularSolutions->True]
```

$$y(x) \rightarrow \frac{1}{6}x^4 \log(x) + x \left(\left(-\frac{11}{36} + c_4 \right) x^3 + c_3x^2 + c_2x + c_1 \right)$$

20.6 problem section 9.4, problem 18

Internal problem ID [1577]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503

Problem number: section 9.4, problem 18.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[_high_order, _exact, _linear, _nonhomogeneous]`

$$x^4 y'''' + 6x^3 y''' + 2x^2 y'' - 4y'x + 4y = 12x^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(x^4*diff(y(x),x$4)+6*x^3*diff(y(x),x$3)+2*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+4*y(x)=
```

$$y(x) = x^2 c_2 + c_3 x + \frac{c_4}{x^2} + \frac{12x^3 \ln(x) - 15x^3 + 2c_1}{12x}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 38

```
DSolve[x^4*y''''[x]+6*x^3*y'''[x]+2*x^2*y''[x]-4*x*y'[x]+4*y[x]==12*x^2,y[x],x,IncludeSingul
```

$$y(x) \rightarrow \frac{x^4 \log(x) + \left(-\frac{19}{12} + c_4\right) x^4 + c_3 x^3 + c_2 x + c_1}{x^2}$$

20.7 problem section 9.4, problem 22

Internal problem ID [1578]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503

Problem number: section 9.4, problem 22.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^3 y''' - 2x^2 y'' + 3y'x - 3y = 4x$$

With initial conditions

$$[y(1) = 4, y'(1) = 4, y''(1) = 2]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 23

```
dsolve([x^3*diff(y(x),x$3)-2*x^2*diff(y(x),x$2)+3*x*diff(y(x),x)-3*y(x)=4*x,y(1) = 4, D(y)(1) = 4, D(y)(1) = 2],y(x),x)
```

$$y(x) = x(x^2 - \ln(x)^2 - 2\ln(x) + 3)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 22

```
DSolve[{x^3*y'''[x]-2*x^2*y''[x]+3*x*y'[x]-3*y[x]==4*x,{y[1]==4,y'[1]==4,y''[1]==2}},y[x],x
```

$$y(x) \rightarrow x(x^2 - \log^2(x) - 2\log(x) + 3)$$

20.8 problem section 9.4, problem 23

Internal problem ID [1579]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503

Problem number: section 9.4, problem 23.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^3 y''' - 5x^2 y'' + 14y'x - 18y = x^3$$

With initial conditions

$$[y(1) = 0, y'(1) = 1, y''(1) = 7]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 25

```
dsolve([x^3*diff(y(x),x$3)-5*x^2*diff(y(x),x$2)+14*x*diff(y(x),x)-18*y(x)=x^3,y(1) = 0, D(y
```

$$y(x) = \frac{x^2(\ln(x)^2 x + 4x \ln(x) - 2x + 2)}{2}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 28

```
DSolve[{x^3*y'''[x]-5*x^2*y''[x]+14*x*y'[x]-18*y[x]==x^3,{y[1]==0,y'[1]==1,y''[1]==7}},y[x],
```

$$y(x) \rightarrow \frac{1}{2}x^2(-2x + x \log^2(x) + 4x \log(x) + 2)$$

20.9 problem section 9.4, problem 25

Internal problem ID [1580]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503

Problem number: section 9.4, problem 25.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^3 y''' - 6x^2 y'' + 16y'x - 16y = 9x^4$$

With initial conditions

$$[y(1) = 2, y'(1) = 1, y''(1) = 5]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 29

```
dsolve([x^3*diff(y(x),x$3)-6*x^2*diff(y(x),x$2)+16*x*diff(y(x),x)-16*y(x)=9*x^4,y(1) = 2, D
```

$$y(x) = -x^4 + \frac{3 \ln(x)^2 x^4}{2} + 2 \ln(x) x^4 + 3x$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 32

```
DSolve[{x^3*y'''[x]-6*x^2*y''[x]+16*x*y'[x]-16*y[x]==9*x^4,{y[1]==2,y'[1]==1,y''[1]==5}},y[x]
```

$$y(x) \rightarrow -x^4 + \frac{3}{2}x^4 \log^2(x) + 2x^4 \log(x) + 3x$$

20.10 problem section 9.4, problem 27

Internal problem ID [1581]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503

Problem number: section 9.4, problem 27.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _nonhomogeneous]]`

$$x^3 y''' + x^2 y'' - 2y'x + 2y = x(1 + x)$$

With initial conditions

$$\left[y(-1) = -6, y'(-1) = \frac{43}{6}, y''(-1) = -\frac{5}{2} \right]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 30

```
dsolve([x^3*diff(y(x),x$3)+x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=x*(x+1),y(-1) = -6, D
```

$$y(x) = -\frac{x(2i\pi x - 3i\pi - 2x \ln(x) + 3 \ln(x) + 12x - 24)}{6}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 34

```
DSolve[{x^3*y'''[x]+x^2*y''[x]-2*x*y'[x]+2*y[x]==x*(x+1),{y[-1]==-6,y'[-1]==43/6,y''[-1]==-5
```

$$y(x) \rightarrow \frac{1}{6}x(-2i\pi x - 12x + (2x - 3) \log(x) + 3i\pi + 24)$$

20.11 problem section 9.4, problem 30

Internal problem ID [1582]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503

Problem number: section 9.4, problem 30.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[_high_order, _exact, _linear, _nonhomogeneous]`

$$x^4 y'''' + 3x^3 y''' - x^2 y'' + 2y'x - 2y = 9x^2$$

With initial conditions

$$[y(1) = -7, y'(1) = -11, y''(1) = -5, y'''(1) = 6]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 14

```
dsolve([x^4*diff(y(x),x$4)+3*x^3*diff(y(x),x$3)-x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=9
```

$$y(x) = x^2(-7 + 3 \ln(x))$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 15

```
DSolve[{x^4*y''''[x]+3*x^3*y'''[x]-x^2*y''[x]+2*x*y'[x]-2*y[x]==9*x^2,{y[1]==-7,y'[1]==-11,y
```

$$y(x) \rightarrow x^2(3 \log(x) - 7)$$

20.12 problem section 9.4, problem 32

Internal problem ID [1583]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503

Problem number: section 9.4, problem 32.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _exact, _linear, _nonhomogeneous]]`

$$4x^4y'''' + 24x^3y''' + 23x^2y'' - y'x + y = 6x$$

With initial conditions

$$\left[y(1) = 2, y'(1) = 0, y''(1) = 4, y'''(1) = -\frac{37}{4} \right]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 27

```
dsolve([4*x^4*diff(y(x),x$4)+24*x^3*diff(y(x),x$3)+23*x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)]=6*x,
```

$$y(x) = \frac{\ln(x)x^{\frac{5}{2}} - x^2 + x^{\frac{5}{2}} + \sqrt{x} + x}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 26

```
DSolve[{4*x^4*y''''[x]+24*x^3*y'''[x]+23*x^2*y''[x]-x*y'[x]+y[x]==6*x,{y[1]==2,y'[1]==0,y''[1]==4,y'''[1]==-37/4}],y[x],x]
```

$$y(x) \rightarrow x - \sqrt{x} + \frac{1}{\sqrt{x}} + \frac{1}{x} + x \log(x)$$

20.13 problem section 9.4, problem 33

Internal problem ID [1584]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503

Problem number: section 9.4, problem 33.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[_high_order, _exact, _linear, _nonhomogeneous]`

$$x^4 y'''' + 5x^3 y''' - 3x^2 y'' - 6y'x + 6y = 40x^3$$

With initial conditions

$$[y(-1) = -1, y'(-1) = -7, y''(-1) = -1, y'''(-1) = -31]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 30

```
dsolve([x^4*diff(y(x),x$4)+5*x^3*diff(y(x),x$3)-3*x^2*diff(y(x),x$2)-6*x*diff(y(x),x)+6*y(x)
```

$$y(x) = \frac{\ln(x) x^5 - 1 + (-i\pi - 2) x^5 + x^3 + x}{x^2}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 32

```
DSolve[{x^4*y''''[x]+5*x^3*y'''[x]-3*x^2*y''[x]-6*x*y'[x]+6*y[x]==40*x^3,{y[-1]==-1,y'[-1]==
```

$$y(x) \rightarrow \frac{(-2 - i\pi)x^5 + x^5 \log(x) + x^3 + x - 1}{x^2}$$

20.14 problem section 9.4, problem 35

Internal problem ID [1585]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503

Problem number: section 9.4, problem 35.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' + 2y'' - y' - 2y = F(x)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 63

```
dsolve(diff(y(x),x$3)+2*diff(y(x),x$2)-diff(y(x),x)-2*y(x)=F(x),y(x), singsol=all)
```

$$y(x) = \left(\int \frac{e^{-x}F(x)}{6} dx \right) e^x + \left(\int \frac{F(x)e^{2x}}{3} dx \right) e^{-2x} - \left(\int \frac{e^x F(x)}{2} dx \right) e^{-x} + c_1 e^x + c_2 e^{-2x} + c_3 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 96

```
DSolve[y'''[x]+2*y''[x]-y'[x]-2*y[x]==f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-2x} \left(\int_1^x \frac{1}{3} e^{2K[1]} f(K[1]) dK[1] + e^x \int_1^x -\frac{1}{2} e^{K[2]} f(K[2]) dK[2] + e^{3x} \int_1^x \frac{1}{6} e^{-K[3]} f(K[3]) dK[3] + c_2 e^x + c_3 e^{3x} + c_1 \right)$$

20.15 problem section 9.4, problem 36

Internal problem ID [1586]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503

Problem number: section 9.4, problem 36.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _nonhomogeneous]]`

$$x^3 y''' + x^2 y'' - 2y'x + 2y = F(x)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve(x^3*diff(y(x),x$3)+x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=F(x),y(x), singsol=all)
```

$$y(x) = \left(c_3 + \int \frac{c_2 - \left(\int -\frac{c_1 - \left(\int -F(x) dx \right)}{x^3} dx \right)}{x^2} dx \right) x^2$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 82

```
DSolve[x^3*y'''[x]+x^2*y''[x]-2*x*y'[x]+2*y[x]==f[x],y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \rightarrow \frac{x^3 \int_1^x \frac{f(K[3])}{3K[3]^3} dK[3] + x^2 \int_1^x -\frac{f(K[2])}{2K[2]^2} dK[2] + \int_1^x \frac{1}{6} f(K[1]) dK[1] + c_3 x^3 + c_2 x^2 + c_1}{x}$$

20.16 problem section 9.4, problem 39

Internal problem ID [1587]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503

Problem number: section 9.4, problem 39.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 5y'' + 4y = F(x)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 85

```
dsolve(diff(y(x),x$4)-5*diff(y(x),x$2)+4*y(x)=F(x),y(x), singsol=all)
```

$$y(x) = -\left(\int \frac{e^{-x}F(x)}{6}dx\right)e^x - \left(\int \frac{F(x)e^{2x}}{12}dx\right)e^{-2x} + \left(\int \frac{e^x F(x)}{6}dx\right)e^{-x} \\ + \left(\int \frac{F(x)e^{-2x}}{12}dx\right)e^{2x} + c_1e^x + c_2e^{-2x} + c_3e^{-x} + c_4e^{2x}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 130

```
DSolve[y''''[x]-5*y''[x]+4*y[x]==f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-2x} \left(\int_1^x -\frac{1}{12} e^{2K[1]} f(K[1]) dK[1] + e^x \int_1^x \frac{1}{6} e^{K[2]} f(K[2]) dK[2] + e^{3x} \int_1^x \right. \\ \left. -\frac{1}{6} e^{-K[3]} f(K[3]) dK[3] + e^{4x} \int_1^x \frac{1}{12} e^{-2K[4]} f(K[4]) dK[4] + c_2 e^x + c_3 e^{3x} + c_4 e^{4x} \right. \\ \left. + c_1 \right)$$

20.17 problem section 9.4, problem 41

Internal problem ID [1588]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503

Problem number: section 9.4, problem 41.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[_high_order, _exact, _linear, _nonhomogeneous]`

$$x^4 y'''' + 6x^3 y''' + 2x^2 y'' - 4y'x + 4y = F(x)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 43

```
dsolve(x^4*diff(y(x),x$4)+6*x^3*diff(y(x),x$3)+2*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+4*y(x)=
```

$$y(x) = \frac{c_4 + \int \left(2c_2 x + c_3 - \left(\int \left(\int -\frac{c_1 - \left(\int \frac{-F(x) dx}{x^4} dx \right) dx \right) \right) x^2 dx}{x^2}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 104

```
DSolve[x^4*y''''[x]+6*x^3*y'''[x]+2*x^2*y''[x]-4*x*y'[x]+4*y[x]==f[x],y[x],x,IncludeSingular
```

$$y(x) \rightarrow \frac{x^4 \int_1^x \frac{f(K[4])}{12K[4]^3} dK[4] + x^3 \int_1^x -\frac{f(K[3])}{6K[3]^2} dK[3] + x \int_1^x \frac{1}{6} f(K[2]) dK[2] + \int_1^x -\frac{1}{12} f(K[1]) K[1] dK[1] + c_4 x^4 + c_5}{x^2}$$

21 Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540

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21.1 problem section 10.4, problem 1

Internal problem ID [1589]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540

Problem number: section 10.4, problem 1.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = y_1(t) + 2y_2(t)$$

$$y_2'(t) = 2y_1(t) + y_2(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 35

```
dsolve([diff(y__1(t),t)=y__1(t)+2*y__2(t),diff(y__2(t),t)=2*y__1(t)+1*y__2(t)], [y__1(t), y__2(t)])
```

$$y_1(t) = -e^{-t}c_1 + c_2e^{3t}$$

$$y_2(t) = e^{-t}c_1 + c_2e^{3t}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 68

```
DSolve[{y1'[t]==y1[t]+2*y2[t],y2'[t]==2*y1[t]+y2[t]},{y1[t],y2[t]},t,IncludeSingularSolution->False]
```

$$y_1(t) \rightarrow \frac{1}{2}e^{-t}(c_1(e^{4t} + 1) + c_2(e^{4t} - 1))$$

$$y_2(t) \rightarrow \frac{1}{2}e^{-t}(c_1(e^{4t} - 1) + c_2(e^{4t} + 1))$$

21.2 problem section 10.4, problem 2

Internal problem ID [1590]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540

Problem number: section 10.4, problem 2.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}y_1'(t) &= -\frac{5y_1(t)}{4} + \frac{3y_2(t)}{4} \\y_2'(t) &= \frac{3y_1(t)}{4} - \frac{5y_2(t)}{4}\end{aligned}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 35

```
dsolve([diff(y__1(t),t)=-5/4*y__1(t)+3/4*y__2(t),diff(y__2(t),t)=3/4*y__1(t)-5/4*y__2(t)], [y
```

$$y_1(t) = c_1 e^{-\frac{t}{2}} - e^{-2t} c_2$$

$$y_2(t) = c_1 e^{-\frac{t}{2}} + e^{-2t} c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 76

```
DSolve[{y1'[t]==-5/4*y1[t]+3/4*y2[t],y2'[t]==3/4*y1[t]-5/4*y2[t]},{y1[t],y2[t]},t,IncludeSin
```

$$y_1(t) \rightarrow \frac{1}{2} e^{-2t} (c_1 (e^{3t/2} + 1) + c_2 (e^{3t/2} - 1))$$

$$y_2(t) \rightarrow \frac{1}{2} e^{-2t} (c_1 (e^{3t/2} - 1) + c_2 (e^{3t/2} + 1))$$

21.3 problem section 10.4, problem 3

Internal problem ID [1591]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540

Problem number: section 10.4, problem 3.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}y_1'(t) &= -\frac{4y_1(t)}{5} + \frac{3y_2(t)}{5} \\y_2'(t) &= -\frac{2y_1(t)}{5} - \frac{11y_2(t)}{5}\end{aligned}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 36

```
dsolve([diff(y__1(t),t)=-4/5*y__1(t)+3/5*y__2(t),diff(y__2(t),t)=-2/5*y__1(t)-11/5*y__2(t)],
```

$$y_1(t) = -3e^{-t}c_1 - \frac{e^{-2t}c_2}{2}$$

$$y_2(t) = e^{-t}c_1 + e^{-2t}c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 65

```
DSolve[{y1'[t]==-4/5*y1[t]+3/5*y2[t],y2'[t]==-2/5*y1[t]-11/5*y2[t]},{y1[t],y2[t]},t,IncludeS
```

$$\begin{aligned}y_1(t) &\rightarrow \frac{1}{5}e^{-2t}(c_1(6e^t - 1) + 3c_2(e^t - 1)) \\y_2(t) &\rightarrow \frac{1}{5}e^{-2t}(-2c_1(e^t - 1) - c_2(e^t - 6))\end{aligned}$$

21.4 problem section 10.4, problem 4

Internal problem ID [1592]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540

Problem number: section 10.4, problem 4.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = -y_1(t) - 4y_2(t)$$

$$y_2'(t) = -y_1(t) - y_2(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 32

```
dsolve([diff(y__1(t),t)=-1*y__1(t)-4*y__2(t),diff(y__2(t),t)=-1*y__1(t)-1*y__2(t)], [y__1(t),
```

$$y_1(t) = 2c_1e^{-3t} - 2c_2e^t$$

$$y_2(t) = c_1e^{-3t} + c_2e^t$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 71

```
DSolve[{y1'[t]==-1*y1[t]-4*y2[t],y2'[t]==-1*y1[t]-1*y2[t]},{y1[t],y2[t]},t,IncludeSingularSo
```

$$y_1(t) \rightarrow \frac{1}{2}e^{-3t}(c_1(e^{4t} + 1) - 2c_2(e^{4t} - 1))$$

$$y_2(t) \rightarrow \frac{1}{4}e^{-3t}(2c_2(e^{4t} + 1) - c_1(e^{4t} - 1))$$

21.5 problem section 10.4, problem 5

Internal problem ID [1593]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540

Problem number: section 10.4, problem 5.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = 2y_1(t) - 4y_2(t)$$

$$y_2'(t) = -y_1(t) - y_2(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 35

```
dsolve([diff(y__1(t),t)=2*y__1(t)-4*y__2(t),diff(y__2(t),t)=-1*y__1(t)-1*y__2(t)],[y__1(t),
```

$$y_1(t) = -4c_1e^{3t} + e^{-2t}c_2$$

$$y_2(t) = c_1e^{3t} + e^{-2t}c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 72

```
DSolve[{y1'[t]==2*y1[t]-4*y2[t],y2'[t]==-1*y1[t]-1*y2[t]},{y1[t],y2[t]},t,IncludeSingularSol
```

$$y_1(t) \rightarrow \frac{1}{5}e^{-2t}(c_1(4e^{5t} + 1) - 4c_2(e^{5t} - 1))$$

$$y_2(t) \rightarrow \frac{1}{5}e^{-2t}(c_2(e^{5t} + 4) - c_1(e^{5t} - 1))$$

21.6 problem section 10.4, problem 6

Internal problem ID [1594]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540

Problem number: section 10.4, problem 6.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}y_1'(t) &= 4y_1(t) - 3y_2(t) \\y_2'(t) &= 2y_1(t) - y_2(t)\end{aligned}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 31

```
dsolve([diff(y__1(t),t)=4*y__1(t)-3*y__2(t),diff(y__2(t),t)=2*y__1(t)-1*y__2(t)],[y__1(t), y
```

$$y_1(t) = \frac{3c_1 e^{2t}}{2} + c_2 e^t$$

$$y_2(t) = c_1 e^{2t} + c_2 e^t$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 56

```
DSolve[{y1'[t]==4*y1[t]-3*y2[t],y2'[t]==2*y1[t]-1*y2[t]},{y1[t],y2[t]},t,IncludeSingularSolu
```

$$y_1(t) \rightarrow e^t(c_1(3e^t - 2) - 3c_2(e^t - 1))$$

$$y_2(t) \rightarrow e^t(2c_1(e^t - 1) + c_2(3 - 2e^t))$$

21.7 problem section 10.4, problem 7

Internal problem ID [1595]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540

Problem number: section 10.4, problem 7.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = -6y_1(t) - 3y_2(t)$$

$$y_2'(t) = y_1(t) - 2y_2(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 36

```
dsolve([diff(y__1(t),t)=-6*y__1(t)-3*y__2(t),diff(y__2(t),t)=1*y__1(t)-2*y__2(t)],[y__1(t),
```

$$y_1(t) = -c_1 e^{-3t} - 3c_2 e^{-5t}$$

$$y_2(t) = c_1 e^{-3t} + c_2 e^{-5t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 72

```
DSolve[{y1'[t]==-6*y1[t]-3*y2[t],y2'[t]==1*y1[t]-2*y2[t]},{y1[t],y2[t]},t,IncludeSingularSol
```

$$y_1(t) \rightarrow \frac{1}{2} e^{-5t} (-(c_1 (e^{2t} - 3)) - 3c_2 (e^{2t} - 1))$$

$$y_2(t) \rightarrow \frac{1}{2} e^{-5t} (c_1 (e^{2t} - 1) + c_2 (3e^{2t} - 1))$$

21.8 problem section 10.4, problem 8

Internal problem ID [1596]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540

Problem number: section 10.4, problem 8.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = y_1(t) - y_2(t) - 2y_3(t)$$

$$y_2'(t) = y_1(t) - 2y_2(t) - 3y_3(t)$$

$$y_3'(t) = -4y_1(t) + y_2(t) - y_3(t)$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 73

```
dsolve([diff(y__1(t),t)=1*y__1(t)-1*y__2(t)-2*y__3(t),diff(y__2(t),t)=1*y__1(t)-2*y__2(t)-3*
```

$$y_1(t) = c_1 e^{-3t} - c_2 e^{-t} - c_3 e^{2t}$$

$$y_2(t) = 2c_1 e^{-3t} - 4c_2 e^{-t} - c_3 e^{2t}$$

$$y_3(t) = c_1 e^{-3t} + c_2 e^{-t} + c_3 e^{2t}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 102

```
DSolve[{y1'[t]==1*y1[t]-1*y2[t]-2*y3[t],y2'[t]==1*y1[t]-2*y2[t]-3*y3[t],y1'[t]==-4*y1[t]+1*y
```

$$y1(t) \rightarrow \frac{1}{576}e^{-3t}(c_1(63 - 128e^t) + c_2(64e^t - 27))$$

$$y2(t) \rightarrow \frac{1}{864}e^{-3t}(c_2(224e^t - 81) - 7c_1(64e^t - 27))$$

$$y3(t) \rightarrow \frac{e^{-3t}(c_1(189 - 128e^t) + c_2(64e^t - 81))}{1728}$$

21.9 problem section 10.4, problem 9

Internal problem ID [1597]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540

Problem number: section 10.4, problem 9.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = -6y_1(t) - 4y_2(t) - 8y_3(t)$$

$$y_2'(t) = -4y_1(t) - 4y_3(t)$$

$$y_3'(t) = -8y_1(t) - 4y_2(t) - 6y_3(t)$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 66

```
dsolve([diff(y__1(t),t)=-6*y__1(t)-4*y__2(t)-8*y__3(t),diff(y__2(t),t)=-4*y__1(t)-0*y__2(t)-
```

$$y_1(t) = c_2 e^{-16t} - \frac{5c_3 e^{2t}}{4} - \frac{c_1 e^{2t}}{2}$$

$$y_2(t) = \frac{c_2 e^{-16t}}{2} + \frac{c_3 e^{2t}}{2} + c_1 e^{2t}$$

$$y_3(t) = c_2 e^{-16t} + c_3 e^{2t}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 116

```
DSolve[{y1'[t]==-6*y1[t]-4*y2[t]-8*y3[t],y2'[t]==-4*y1[t]-0*y2[t]-4*y3[t],y1'[t]==-8*y1[t]-4
```

$$y1(t) \rightarrow \frac{e^{-16t}(c_1(8 - 4913e^{18t}) + 2c_2(4913e^{18t} + 1))}{44217}$$

$$y2(t) \rightarrow \frac{e^{-16t}(4c_1(4913e^{18t} + 1) + c_2(1 - 39304e^{18t}))}{44217}$$

$$y3(t) \rightarrow \frac{e^{-16t}(c_1(8 - 4913e^{18t}) + 2c_2(4913e^{18t} + 1))}{44217}$$

21.10 problem section 10.4, problem 10

Internal problem ID [1598]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540

Problem number: section 10.4, problem 10.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = 3y_1(t) + 5y_2(t) + 8y_3(t)$$

$$y_2'(t) = y_1(t) - y_2(t) - 2y_3(t)$$

$$y_3'(t) = -y_1(t) - y_2(t) - y_3(t)$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 61

```
dsolve([diff(y__1(t),t)=3*y__1(t)+5*y__2(t)+8*y__3(t),diff(y__2(t),t)=1*y__1(t)-1*y__2(t)-2*y__3(t),diff(y__3(t),t)=-1*y__1(t)-1*y__2(t)-1*y__3(t)),y__1(0)=1,y__2(0)=1,y__3(0)=1)
```

$$y_1(t) = -\frac{7c_2e^{2t}}{4} - \frac{2c_3e^t}{3} - c_1e^{-2t}$$

$$y_2(t) = -\frac{4c_3e^t}{3} - \frac{5c_2e^{2t}}{4} + c_1e^{-2t}$$

$$y_3(t) = c_2e^{2t} + c_3e^t$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 193

```
DSolve[{y1'[t]==3*y1[t]+5*y2[t]+8*y3[t],y2'[t]==1*y1[t]-1*y2[t]-2*y3[t],y1'[t]==-1*y1[t]-1*y
```

$$y1(t) \rightarrow \frac{e^{-t/9} \left(\sqrt{35}(2c_2 - 121c_1) \sin\left(\frac{\sqrt{35}t}{9}\right) - 7(74c_1 + 53c_2) \cos\left(\frac{\sqrt{35}t}{9}\right) \right)}{1575}$$

$$y2(t) \rightarrow \frac{e^{-t/9} \left(7(901c_1 + 202c_2) \cos\left(\frac{\sqrt{35}t}{9}\right) - \sqrt{35}(34c_1 + 379c_2) \sin\left(\frac{\sqrt{35}t}{9}\right) \right)}{4725}$$

$$y3(t) \rightarrow \frac{e^{-t/9} \left(2\sqrt{35}(92c_1 + 125c_2) \sin\left(\frac{\sqrt{35}t}{9}\right) - 14(251c_1 + 32c_2) \cos\left(\frac{\sqrt{35}t}{9}\right) \right)}{4725}$$

21.11 problem section 10.4, problem 11

Internal problem ID [1599]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540

Problem number: section 10.4, problem 11.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = y_1(t) - y_2(t) + 2y_3(t)$$

$$y_2'(t) = 12y_1(t) - 4y_2(t) + 10y_3(t)$$

$$y_3'(t) = -6y_1(t) + y_2(t) - 7y_3(t)$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 74

```
dsolve([diff(y__1(t),t)=1*y__1(t)-1*y__2(t)+2*y__3(t),diff(y__2(t),t)=12*y__1(t)-4*y__2(t)+10*y__3(t),diff(y__3(t),t)=-6*y__1(t)+y__2(t)-7*y__3(t))
```

$$y_1(t) = -c_1 e^{-3t} - \frac{2c_2 e^{-5t}}{3} - c_3 e^{-2t}$$

$$y_2(t) = -2c_1 e^{-3t} - 2c_2 e^{-5t} - c_3 e^{-2t}$$

$$y_3(t) = c_1 e^{-3t} + c_2 e^{-5t} + c_3 e^{-2t}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 193

```
DSolve[{y1'[t]==1*y1[t]-1*y2[t]+2*y3[t],y2'[t]==12*y1[t]-4*y2[t]+10*y3[t],y1'[t]==-6*y1[t]+1
```

$$y_1(t) \rightarrow \frac{e^{-7t/6} \left(71(77c_1 - 109c_2) \cos\left(\frac{\sqrt{71}t}{6}\right) + \sqrt{71}(143c_2 - 2479c_1) \sin\left(\frac{\sqrt{71}t}{6}\right) \right)}{340800}$$

$$y_2(t) \rightarrow \frac{e^{-7t/6} \left(71(2071c_1 - 407c_2) \cos\left(\frac{\sqrt{71}t}{6}\right) - \sqrt{71}(2717c_1 + 5411c_2) \sin\left(\frac{\sqrt{71}t}{6}\right) \right)}{852000}$$

$$y_3(t) \rightarrow \frac{e^{-7t/6} \left(639(23c_1 + 9c_2) \cos\left(\frac{\sqrt{71}t}{6}\right) + 3\sqrt{71}(937c_1 - 329c_2) \sin\left(\frac{\sqrt{71}t}{6}\right) \right)}{568000}$$

21.12 problem section 10.4, problem 12

Internal problem ID [1600]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540

Problem number: section 10.4, problem 12.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = 4y_1(t) - y_2(t) - 4y_3(t)$$

$$y_2'(t) = 4y_1(t) - 3y_2(t) - 2y_3(t)$$

$$y_3'(t) = y_1(t) - y_2(t) - y_3(t)$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 71

```
dsolve([diff(y__1(t),t)=4*y__1(t)-1*y__2(t)-4*y__3(t),diff(y__2(t),t)=4*y__1(t)-3*y__2(t)-2*y__3(t),diff(y__3(t),t)=y__1(t)-y__2(t)-y__3(t))
```

$$y_1(t) = e^{-t}c_1 + 11c_2e^{3t} + c_3e^{-2t}$$

$$y_2(t) = e^{-t}c_1 + 7c_2e^{3t} + 2c_3e^{-2t}$$

$$y_3(t) = e^{-t}c_1 + c_2e^{3t} + c_3e^{-2t}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 102

```
DSolve[{y1'[t]==4*y1[t]-1*y2[t]-4*y3[t],y2'[t]==4*y1[t]-3*y2[t]-2*y3[t],y1'[t]==1*y1[t]-1*y2[t]}
```

$$y1(t) \rightarrow \frac{1}{216}e^{-2t}(c_1(54e^t - 8) + c_2(8 - 27e^t))$$

$$y2(t) \rightarrow \frac{1}{216}e^{-2t}(2c_1(27e^t - 8) + c_2(16 - 27e^t))$$

$$y3(t) \rightarrow \frac{1}{216}e^{-2t}(c_1(54e^t - 8) + c_2(8 - 27e^t))$$

21.13 problem section 10.4, problem 13

Internal problem ID [1601]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540

Problem number: section 10.4, problem 13.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = -2y_1(t) + 2y_2(t) - 6y_3(t)$$

$$y_2'(t) = 2y_1(t) + 6y_2(t) + 2y_3(t)$$

$$y_3'(t) = -2y_1(t) - 2y_2(t) + 2y_3(t)$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 67

```
dsolve([diff(y__1(t),t)=-2*y__1(t)+2*y__2(t)-6*y__3(t),diff(y__2(t),t)=2*y__1(t)+6*y__2(t)+2
```

$$y_1(t) = 4e^{-4t}c_1 - c_2e^{4t} - c_3e^{6t}$$

$$y_2(t) = -e^{-4t}c_1 - c_3e^{6t}$$

$$y_3(t) = e^{-4t}c_1 + c_2e^{4t} + c_3e^{6t}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 257

`DSolve[{y1'[t]==-2*y1[t]+2*y2[t]-6*y3[t],y2'[t]==2*y1[t]+6*y2[t]+2*y3[t],y1'[t]==-2*y1[t]-2*`

$$y1(t) \rightarrow \frac{e^{-\frac{1}{2}(\sqrt{73}-5)t} \left(2c_1 \left((841\sqrt{73} - 7227) e^{\sqrt{73}t} - 7227 - 841\sqrt{73} \right) + c_2 \left((171\sqrt{73} - 1825) e^{\sqrt{73}t} - 1825 - \right. \right.}{598016}$$

$$y2(t) \rightarrow \frac{e^{-\frac{1}{2}(\sqrt{73}-5)t} \left(c_1 \left((342\sqrt{73} - 3650) e^{\sqrt{73}t} - 3650 - 342\sqrt{73} \right) - c_2 \left((1971 + 143\sqrt{73}) e^{\sqrt{73}t} + 1971 - 143 \right. \right.}{598016}$$

$$y3(t) \rightarrow \frac{e^{-\frac{1}{2}(\sqrt{73}-5)t} \left(c_1 \left((342\sqrt{73} - 3650) e^{\sqrt{73}t} - 3650 - 342\sqrt{73} \right) - c_2 \left((1971 + 143\sqrt{73}) e^{\sqrt{73}t} + 1971 - 143 \right. \right.}{1196032}$$

21.14 problem section 10.4, problem 14

Internal problem ID [1602]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540

Problem number: section 10.4, problem 14.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}y_1'(t) &= 3y_1(t) + 2y_2(t) - 2y_3(t) \\y_2'(t) &= -2y_1(t) + 7y_2(t) - 2y_3(t) \\y_3'(t) &= -10y_1(t) + 10y_2(t) - 5y_3(t)\end{aligned}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 66

```
dsolve([diff(y__1(t),t)=3*y__1(t)+2*y__2(t)-2*y__3(t),diff(y__2(t),t)=-2*y__1(t)+7*y__2(t)-2
```

$$y_1(t) = -\frac{4c_2e^{5t}}{5} + \frac{c_3e^{-5t}}{5} + c_1e^{5t}$$

$$y_2(t) = \frac{c_2e^{5t}}{5} + \frac{c_3e^{-5t}}{5} + c_1e^{5t}$$

$$y_3(t) = c_2e^{5t} + c_3e^{-5t}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 126

```
DSolve[{y1'[t]==3*y1[t]+2*y2[t]-2*y3[t],y2'[t]==-2*y1[t]+7*y2[t]-2*y3[t],y1'[t]==-10*y1[t]+1
```

$$y_1(t) \rightarrow -\frac{e^{5t}(c_1(432e^{10t/3} - 1331) + c_2(1331 - 216e^{10t/3}))}{85184}$$

$$y_2(t) \rightarrow -\frac{e^{5t}(c_1(216e^{10t/3} - 1331) + c_2(1331 - 108e^{10t/3}))}{42592}$$

$$y_3(t) \rightarrow \frac{e^{5t}(c_1(720e^{10t/3} + 1331) - c_2(360e^{10t/3} + 1331))}{85184}$$

21.15 problem section 10.4, problem 15

Internal problem ID [1603]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540

Problem number: section 10.4, problem 15.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = 3y_1(t) + y_2(t) - y_3(t)$$

$$y_2'(t) = 3y_1(t) + 5y_2(t) + y_3(t)$$

$$y_3'(t) = -6y_1(t) + 2y_2(t) + 4y_3(t)$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 54

```
dsolve([diff(y__1(t),t)=3*y__1(t)+1*y__2(t)-1*y__3(t),diff(y__2(t),t)=3*y__1(t)+5*y__2(t)+1*
```

$$y_1(t) = -\frac{c_3 e^{6t}}{2} + \frac{c_2}{2} + \frac{e^{6t} c_1}{3}$$

$$y_2(t) = -\frac{c_2}{2} - \frac{c_3 e^{6t}}{2} + e^{6t} c_1$$

$$y_3(t) = c_2 + c_3 e^{6t}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 93

```
DSolve[{y1'[t]==3*y1[t]+1*y2[t]-1*y3[t],y2'[t]==3*y1[t]+5*y2[t]+1*y3[t],y1'[t]==-6*y1[t]+2*y
```

$$y1(t) \rightarrow \frac{1}{625}(-c_1(e^{6t} - 500)) - c_2(e^{6t} + 125))$$

$$y2(t) \rightarrow \frac{1}{625}(c_2(125 - 4e^{6t}) - 4c_1(e^{6t} + 125))$$

$$y3(t) \rightarrow \frac{1}{625}(-c_1(e^{6t} - 1000)) - c_2(e^{6t} + 250))$$

22 Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

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22.1 problem section 10.5, problem 1

Internal problem ID [1604]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 1.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}y_1'(t) &= 3y_1(t) + 4y_2(t) \\y_2'(t) &= -y_1(t) + 7y_2(t)\end{aligned}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 34

```
dsolve([diff(y__1(t),t)=3*y__1(t)+4*y__2(t),diff(y__2(t),t)=-1*y__1(t)+7*y__2(t)],[y__1(t),
```

$$y_1(t) = e^{5t}(2c_2t + 2c_1 - c_2)$$

$$y_2(t) = e^{5t}(c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 46

```
DSolve[{y1'[t]==3*y1[t]+4*y2[t],y2'[t]==-1*y1[t]+7*y2[t]},{y1[t],y2[t]},t,IncludeSingularSol
```

$$y_1(t) \rightarrow e^{5t}(-2c_1t + 4c_2t + c_1)$$

$$y_2(t) \rightarrow e^{5t}(c_1(-t) + 2c_2t + c_2)$$

22.2 problem section 10.5, problem 2

Internal problem ID [1605]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 2.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}y_1'(t) &= -y_2(t) \\ y_2'(t) &= y_1(t) - 2y_2(t)\end{aligned}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 29

```
dsolve([diff(y__1(t),t)=0*y__1(t)-1*y__2(t),diff(y__2(t),t)=1*y__1(t)-2*y__2(t)],[y__1(t), y
```

$$y_1(t) = e^{-t}(c_2t + c_1 + c_2)$$

$$y_2(t) = e^{-t}(c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 44

```
DSolve[{y1'[t]==0*y1[t]-1*y2[t],y2'[t]==1*y1[t]-2*y2[t]},{y1[t],y2[t]},t,IncludeSingularSolu
```

$$y_1(t) \rightarrow e^{-t}(c_1(t+1) - c_2t)$$

$$y_2(t) \rightarrow e^{-t}((c_1 - c_2)t + c_2)$$

22.3 problem section 10.5, problem 3

Internal problem ID [1606]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 3.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = -7y_1(t) + 4y_2(t)$$

$$y_2'(t) = -y_1(t) - 11y_2(t)$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 33

```
dsolve([diff(y__1(t),t)=-7*y__1(t)+4*y__2(t),diff(y__2(t),t)=-1*y__1(t)-11*y__2(t)], [y__1(t)
```

$$y_1(t) = -e^{-9t}(2c_2t + 2c_1 + c_2)$$

$$y_2(t) = e^{-9t}(c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 46

```
DSolve[{y1'[t]==-7*y1[t]+4*y2[t],y2'[t]==-1*y1[t]-11*y2[t]},{y1[t],y2[t]},t,IncludeSingularS
```

$$y1(t) \rightarrow e^{-9t}(2c_1t + 4c_2t + c_1)$$

$$y2(t) \rightarrow e^{-9t}(c_2 - (c_1 + 2c_2)t)$$

22.4 problem section 10.5, problem 4

Internal problem ID [1607]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 4.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = 3y_1(t) + y_2(t)$$

$$y_2'(t) = -y_1(t) + y_2(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 30

```
dsolve([diff(y__1(t),t)=3*y__1(t)+1*y__2(t),diff(y__2(t),t)=-1*y__1(t)+1*y__2(t)], [y__1(t),
```

$$y_1(t) = -e^{2t}(c_2t + c_1 + c_2)$$

$$y_2(t) = e^{2t}(c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 42

```
DSolve[{y1'[t]==3*y1[t]+1*y2[t],y2'[t]==-1*y1[t]+1*y2[t]},{y1[t],y2[t]},t,IncludeSingularSol
```

$$y_1(t) \rightarrow e^{2t}(c_1(t+1) + c_2t)$$

$$y_2(t) \rightarrow e^{2t}(c_2 - (c_1 + c_2)t)$$

22.5 problem section 10.5, problem 5

Internal problem ID [1608]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 5.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}y_1'(t) &= 4y_1(t) + 12y_2(t) \\y_2'(t) &= -3y_1(t) - 8y_2(t)\end{aligned}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 33

```
dsolve([diff(y__1(t),t)=4*y__1(t)+12*y__2(t),diff(y__2(t),t)=-3*y__1(t)-8*y__2(t)], [y__1(t),
```

$$y_1(t) = -\frac{e^{-2t}(6c_2t + 6c_1 + c_2)}{3}$$

$$y_2(t) = e^{-2t}(c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 46

```
DSolve[{y1'[t]==4*y1[t]+12*y2[t],y2'[t]==-3*y1[t]-8*y2[t]},{y1[t],y2[t]},t,IncludeSingularSo
```

$$y_1(t) \rightarrow e^{-2t}(6c_1t + 12c_2t + c_1)$$

$$y_2(t) \rightarrow e^{-2t}(c_2 - 3(c_1 + 2c_2)t)$$

22.6 problem section 10.5, problem 6

Internal problem ID [1609]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 6.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = -10y_1(t) + 9y_2(t)$$

$$y_2'(t) = -4y_1(t) + 2y_2(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 35

```
dsolve([diff(y__1(t),t)=-10*y__1(t)+9*y__2(t),diff(y__2(t),t)=-4*y__1(t)+2*y__2(t)], [y__1(t)
```

$$y_1(t) = \frac{e^{-4t}(6c_2t + 6c_1 - c_2)}{4}$$

$$y_2(t) = e^{-4t}(c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 46

```
DSolve[{y1'[t]==-10*y1[t]+9*y2[t],y2'[t]==-4*y1[t]+2*y2[t]},{y1[t],y2[t]},t,IncludeSingularS
```

$$y_1(t) \rightarrow e^{-4t}(-6c_1t + 9c_2t + c_1)$$

$$y_2(t) \rightarrow e^{-4t}(-4c_1t + 6c_2t + c_2)$$

22.7 problem section 10.5, problem 7

Internal problem ID [1610]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 7.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = -13y_1(t) + 16y_2(t)$$

$$y_2'(t) = -9y_1(t) + 11y_2(t)$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 35

```
dsolve([diff(y__1(t),t)=-13*y__1(t)+16*y__2(t),diff(y__2(t),t)=-9*y__1(t)+11*y__2(t)], [y__1(t), y__2(t)])
```

$$y_1(t) = \frac{e^{-t}(12c_2t + 12c_1 - c_2)}{9}$$

$$y_2(t) = e^{-t}(c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 46

```
DSolve[{y1'[t]==-13*y1[t]+16*y2[t],y2'[t]==-9*y1[t]+11*y2[t]},{y1[t],y2[t]},t,IncludeSingularSolutions->True]
```

$$y_1(t) \rightarrow e^{-t}(-12c_1t + 16c_2t + c_1)$$

$$y_2(t) \rightarrow e^{-t}(-9c_1t + 12c_2t + c_2)$$

22.8 problem section 10.5, problem 8

Internal problem ID [1611]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 8.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = 2y_2(t) + y_3(t)$$

$$y_2'(t) = -4y_1(t) + 6y_2(t) + y_3(t)$$

$$y_3'(t) = 4y_2(t) + 2y_3(t)$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 71

```
dsolve([diff(y__1(t),t)=0*y__1(t)+2*y__2(t)+1*y__3(t),diff(y__2(t),t)=-4*y__1(t)+6*y__2(t)+1
```

$$y_1(t) = \frac{c_2 e^{4t}}{2} + \frac{c_3 e^{4t} t}{2} - \frac{c_1}{2}$$

$$y_2(t) = \frac{c_2 e^{4t}}{2} + \frac{c_3 e^{4t} t}{2} + \frac{c_3 e^{4t}}{4} - \frac{c_1}{2}$$

$$y_3(t) = c_1 + c_2 e^{4t} + c_3 e^{4t} t$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 131

```
DSolve[{y1'[t]==0*y1[t]+2*y2[t]+1*y3[t],y2'[t]==-4*y1[t]+6*y2[t]+1*y3[t],y3'[t]==0*y1[t]+4*y
```

$$y_1(t) \rightarrow \frac{1}{4}(c_1(e^{4t}(2-8t)+2) + e^{4t}(8c_2t+c_3) - c_3)$$

$$y_2(t) \rightarrow \frac{1}{4}(-2c_1(e^{4t}(4t+1)-1) + e^{4t}(c_2(8t+4)+c_3) - c_3)$$

$$y_3(t) \rightarrow c_1(e^{4t}(1-4t)-1) + \frac{1}{2}(e^{4t}(8c_2t+c_3) + c_3)$$

22.9 problem section 10.5, problem 9

Internal problem ID [1612]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 9.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}y_1'(t) &= \frac{y_1(t)}{3} + \frac{y_2(t)}{3} - y_3(t) \\y_2'(t) &= -\frac{4y_1(t)}{3} - \frac{4y_2(t)}{3} + y_3(t) \\y_3'(t) &= -\frac{2y_1(t)}{3} + \frac{y_2(t)}{3}\end{aligned}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 75

```
dsolve([diff(y__1(t),t)=1/3*y__1(t)+1/3*y__2(t)-1*y__3(t),diff(y__2(t),t)=-4/3*y__1(t)-4/3*y
```

$$y_1(t) = -c_1e^t + c_2e^{-t} + c_3e^{-t}$$

$$y_2(t) = c_1e^t - c_2e^{-t} - c_3e^{-t} + 3c_3e^{-t}$$

$$y_3(t) = c_1e^t + c_2e^{-t} + c_3e^{-t}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 136

```
DSolve[{y1'[t]==1/3*y1[t]+1/3*y2[t]-1*y3[t],y2'[t]==-4/3*y1[t]-4/3*y2[t]+1*y3[t],y3'[t]==-2/
```

$$y_1(t) \rightarrow \frac{1}{6}e^{-t}(c_1(2t + 3e^{2t} + 3) + 2c_2t - 3c_3(e^{2t} - 1))$$

$$y_2(t) \rightarrow \frac{1}{6}e^{-t}(c_1(-2t - 3e^{2t} + 3) - 2c_2(t - 3) + 3c_3(e^{2t} - 1))$$

$$y_3(t) \rightarrow \frac{1}{6}e^{-t}(c_1(2t - 3e^{2t} + 3) + 2c_2t + 3c_3(e^{2t} + 1))$$

22.10 problem section 10.5, problem 10

Internal problem ID [1613]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 10.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = -y_1(t) + y_2(t) - y_3(t)$$

$$y_2'(t) = -2y_1(t) + 2y_3(t)$$

$$y_3'(t) = -y_1(t) + 3y_2(t) - y_3(t)$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 60

```
dsolve([diff(y__1(t),t)=-1*y__1(t)+1*y__2(t)-1*y__3(t),diff(y__2(t),t)=-2*y__1(t)+0*y__2(t)+
```

$$y_1(t) = \frac{e^{-2t}(2c_3t + 2c_2 + c_3)}{2}$$

$$y_2(t) = c_1e^{2t} + \frac{c_3e^{-2t}}{2}$$

$$y_3(t) = c_1e^{2t} + e^{-2t}c_2 + c_3e^{-2t}t$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 125

```
DSolve[{y1'[t]==-1*y1[t]+1*y2[t]-1*y3[t],y2'[t]==-2*y1[t]+0*y2[t]+2*y3[t],y3'[t]==-1*y1[t]+3
```

$$y1(t) \rightarrow e^{-2t}(c_1(t+1) + (c_2 - c_3)t)$$

$$y2(t) \rightarrow \frac{1}{2}e^{-2t}(-c_1(e^{4t} - 1)) + c_2(e^{4t} + 1) + c_3(e^{4t} - 1)$$

$$y3(t) \rightarrow \frac{1}{2}e^{-2t}(c_1(2t - e^{4t} + 1) + c_2(2t + e^{4t} - 1) + c_3(-2t + e^{4t} + 1))$$

22.11 problem section 10.5, problem 11

Internal problem ID [1614]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 11.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = 4y_1(t) - 2y_2(t) - 2y_3(t)$$

$$y_2'(t) = -2y_1(t) + 3y_2(t) - y_3(t)$$

$$y_3'(t) = 2y_1(t) - y_2(t) + 3y_3(t)$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 69

```
dsolve([diff(y__1(t),t)=4*y__1(t)-2*y__2(t)-2*y__3(t),diff(y__2(t),t)=-2*y__1(t)+3*y__2(t)-1
```

$$y_1(t) = -2c_1e^{2t} + \frac{c_3e^{4t}}{2}$$

$$y_2(t) = -3c_1e^{2t} - c_2e^{4t} - c_3e^{4t}t$$

$$y_3(t) = c_1e^{2t} + c_2e^{4t} + c_3e^{4t}t$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 126

```
DSolve[{y1'[t]==4*y1[t]-2*y2[t]-2*y3[t],y2'[t]==-2*y1[t]+3*y2[t]-1*y3[t],y3'[t]==2*y1[t]-1*y
```

$$y_1(t) \rightarrow e^{2t}((c_1 - c_2 - c_3)e^{2t} + c_2 + c_3)$$

$$y_2(t) \rightarrow \frac{1}{2}(3(c_2 + c_3)e^{2t} - e^{4t}(4(c_1 - c_2 - c_3)t + c_2 + 3c_3))$$

$$y_3(t) \rightarrow -\frac{1}{2}e^{2t}(-e^{2t}(4(c_1 - c_2 - c_3)t + c_2 + 3c_3) + c_2 + c_3)$$

22.12 problem section 10.5, problem 12

Internal problem ID [1615]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 12.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = 6y_1(t) - 5y_2(t) + 3y_3(t)$$

$$y_2'(t) = 2y_1(t) - y_2(t) + 3y_3(t)$$

$$y_3'(t) = 2y_1(t) + y_2(t) + y_3(t)$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 80

```
dsolve([diff(y__1(t),t)=6*y__1(t)-5*y__2(t)+3*y__3(t),diff(y__2(t),t)=2*y__1(t)-1*y__2(t)+3*
```

$$y_1(t) = -c_1 e^{-2t} + c_2 e^{4t} + c_3 e^{4t} t + \frac{c_3 e^{4t}}{2}$$

$$y_2(t) = -c_1 e^{-2t} + c_2 e^{4t} + c_3 e^{4t} t$$

$$y_3(t) = c_1 e^{-2t} + c_2 e^{4t} + c_3 e^{4t} t$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 127

```
DSolve[{y1'[t]==6*y1[t]-5*y2[t]+3*y3[t],y2'[t]==2*y1[t]-1*y2[t]+3*y3[t],y3'[t]==2*y1[t]+1*y2[t]}
```

$$y1(t) \rightarrow \frac{1}{2}e^{-2t}(e^{6t}(c_1(4t+2) - c_2(4t+1) + c_3) + c_2 - c_3)$$

$$y2(t) \rightarrow \frac{1}{2}e^{-2t}(e^{6t}(4(c_1 - c_2)t + c_2 + c_3) + c_2 - c_3)$$

$$y3(t) \rightarrow \frac{1}{2}e^{-2t}(e^{6t}(4(c_1 - c_2)t + c_2 + c_3) - c_2 + c_3)$$

22.13 problem section 10.5, problem 13

Internal problem ID [1616]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 13.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}y_1'(t) &= -11y_1(t) + 8y_2(t) \\y_2'(t) &= -2y_1(t) - 3y_2(t)\end{aligned}$$

With initial conditions

$$[y_1(0) = 6, y_2(0) = 2]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 29

```
dsolve([diff(y__1(t),t) = -11*y__1(t)+8*y__2(t), diff(y__2(t),t) = -2*y__1(t)-3*y__2(t), y__
```

$$y_1(t) = \frac{e^{-7t}(-16t + 12)}{2}$$

$$y_2(t) = e^{-7t}(-4t + 2)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[{y1'[t]==-11*y1[t]+8*y2[t],y2'[t]==-2*y1[t]-3*y2[t]},{y1[0]==6,y2[0]==2},{y1[t],y2[t]
```

$$y1(t) \rightarrow e^{-7t}(6 - 8t)$$

$$y2(t) \rightarrow e^{-7t}(2 - 4t)$$

22.14 problem section 10.5, problem 14

Internal problem ID [1617]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 14.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = 15y_1(t) - 9y_2(t)$$

$$y_2'(t) = 16y_1(t) - 9y_2(t)$$

With initial conditions

$$[y_1(0) = 5, y_2(0) = 8]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 29

```
dsolve([diff(y__1(t),t) = 15*y__1(t)-9*y__2(t), diff(y__2(t),t) = 16*y__1(t)-9*y__2(t), y__1
```

$$y_1(t) = \frac{e^{3t}(-192t + 80)}{16}$$

$$y_2(t) = e^{3t}(-16t + 8)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 31

```
DSolve[{y1'[t]==15*y1[t]-9*y2[t],y2'[t]==16*y1[t]-9*y2[t]},{y1[0]==5,y2[0]==8},{y1[t],y2[t]}
```

$$y_1(t) \rightarrow e^{3t}(5 - 12t)$$

$$y_2(t) \rightarrow -8e^{3t}(2t - 1)$$

22.15 problem section 10.5, problem 15

Internal problem ID [1618]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 15.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = -3y_1(t) - 4y_2(t)$$

$$y_2'(t) = y_1(t) - 7y_2(t)$$

With initial conditions

$$[y_1(0) = 2, y_2(0) = 3]$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 28

```
dsolve([diff(y__1(t),t) = -3*y__1(t)-4*y__2(t), diff(y__2(t),t) = y__1(t)-7*y__2(t), y__1(0)
```

$$y_1(t) = e^{-5t}(-8t + 2)$$

$$y_2(t) = e^{-5t}(-4t + 3)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[{y1'[t]==-3*y1[t]-4*y2[t],y2'[t]==1*y1[t]-7*y2[t]},{y1[0]==2,y2[0]==3},{y1[t],y2[t]},
```

$$y1(t) \rightarrow e^{-5t}(2 - 8t)$$

$$y2(t) \rightarrow e^{-5t}(3 - 4t)$$

22.16 problem section 10.5, problem 16

Internal problem ID [1619]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 16.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = -7y_1(t) + 24y_2(t)$$

$$y_2'(t) = -6y_1(t) + 17y_2(t)$$

With initial conditions

$$[y_1(0) = 3, y_2(0) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 29

```
dsolve([diff(y__1(t),t) = -7*y__1(t)+24*y__2(t), diff(y__2(t),t) = -6*y__1(t)+17*y__2(t), y_
```

$$y_1(t) = \frac{e^{5t}(-72t + 18)}{6}$$

$$y_2(t) = e^{5t}(-6t + 1)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 31

```
DSolve[{y1'[t]==-7*y1[t]+24*y2[t],y2'[t]==-6*y1[t]+17*y2[t]},{y1[0]==3,y2[0]==1},{y1[t],y2[t]
```

$$y_1(t) \rightarrow -3e^{5t}(4t - 1)$$

$$y_2(t) \rightarrow e^{5t}(1 - 6t)$$

22.17 problem section 10.5, problem 17

Internal problem ID [1620]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 17.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = -7y_1(t) + 3y_2(t)$$

$$y_2'(t) = -3y_1(t) - y_2(t)$$

With initial conditions

$$[y_1(0) = 0, y_2(0) = 2]$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 25

```
dsolve([diff(y__1(t),t) = -7*y__1(t)+3*y__2(t), diff(y__2(t),t) = -3*y__1(t)-y__2(t), y__1(0)=0, y__2(0)=2])
```

$$y_1(t) = 6e^{-4t}$$

$$y_2(t) = e^{-4t}(6t + 2)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 27

```
DSolve[{y1'[t]==-7*y1[t]+3*y2[t], y2'[t]==-3*y1[t]-1*y2[t]}, {y1[0]==0, y2[0]==2}, {y1[t], y2[t]}
```

$$y1(t) \rightarrow 6e^{-4t}$$

$$y2(t) \rightarrow e^{-4t}(6t + 2)$$

22.18 problem section 10.5, problem 18

Internal problem ID [1621]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 18.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = -y_1(t) + y_2(t)$$

$$y_2'(t) = y_1(t) - y_2(t) - 2y_3(t)$$

$$y_3'(t) = -y_1(t) - y_2(t) - y_3(t)$$

With initial conditions

$$[y_1(0) = 6, y_2(0) = 5, y_3(0) = -7]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 57

```
dsolve([diff(y__1(t),t) = -y__1(t)+y__2(t), diff(y__2(t),t) = y__1(t)-y__2(t)-2*y__3(t), dif
```

$$y_1(t) = 4e^t + 2e^{-2t} - te^{-2t}$$

$$y_2(t) = te^{-2t} + 8e^t - 3e^{-2t}$$

$$y_3(t) = -6e^t - e^{-2t}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 58

```
DSolve[{y1'[t]==-1*y1[t]+1*y2[t]+0*y3[t],y2'[t]==1*y1[t]-1*y2[t]-2*y3[t],y3'[t]==-1*y1[t]-1*
```

$$y1(t) \rightarrow e^{-2t}(-t + 4e^{3t} + 2)$$

$$y2(t) \rightarrow e^{-2t}(t + 8e^{3t} - 3)$$

$$y3(t) \rightarrow -e^{-2t} - 6e^t$$

22.19 problem section 10.5, problem 19

Internal problem ID [1622]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 19.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}y_1'(t) &= -2y_1(t) + 2y_2(t) + y_3(t) \\y_2'(t) &= -2y_1(t) + 2y_2(t) + y_3(t) \\y_3'(t) &= -3y_1(t) + 3y_2(t) + 2y_3(t)\end{aligned}$$

With initial conditions

$$[y_1(0) = -6, y_2(0) = -2, y_3(0) = 0]$$

✓ Solution by Maple

Time used: 38.844 (sec). Leaf size: 41

```
dsolve([diff(y__1(t),t) = -2*y__1(t)+2*y__2(t)+y__3(t), diff(y__2(t),t) = -2*y__1(t)+2*y__2(t)
```

$$y_1(t) = 2t + 3e^{2t} - 9$$

$$y_2(t) = 2t + 3e^{2t} - 5$$

$$y_3(t) = -6 + 6e^{2t}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 44

```
DSolve[{y1'[t]==-2*y1[t]+2*y2[t]+1*y3[t],y2'[t]==-2*y1[t]+2*y2[t]+1*y3[t],y3'[t]==-3*y1[t]+3
```

$$y1(t) \rightarrow 2t + 3e^{2t} - 9$$

$$y2(t) \rightarrow 2t + 3e^{2t} - 5$$

$$y3(t) \rightarrow 6(e^{2t} - 1)$$

22.20 problem section 10.5, problem 20

Internal problem ID [1623]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 20.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = -7y_1(t) - 4y_2(t) + 4y_3(t)$$

$$y_2'(t) = y_1(t) + y_3(t)$$

$$y_3'(t) = -9y_1(t) - 5y_2(t) + 6y_3(t)$$

With initial conditions

$$[y_1(0) = -6, y_2(0) = 9, y_3(0) = -1]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 80

```
dsolve([diff(y__1(t),t) = -7*y__1(t)-4*y__2(t)+4*y__3(t), diff(y__2(t),t) = y__1(t)+y__3(t),
```

$$y_1(t) = \frac{4e^{-t} \left(-\frac{17 \sin(2t)}{2} - \frac{51 \cos(2t)}{2} \right)}{17}$$

$$y_2(t) = \frac{9e^t}{2} - \frac{7e^{-t} \sin(2t)}{2} + \frac{9e^{-t} \cos(2t)}{2}$$

$$y_3(t) = \frac{9e^t}{2} - \frac{7e^{-t} \sin(2t)}{2} - \frac{11e^{-t} \cos(2t)}{2}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 50

```
DSolve[{y1'[t]==-7*y1[t]-4*y2[t]+4*y3[t],y2'[t]==-1*y1[t]-0*y2[t]+1*y3[t],y3'[t]==-9*y1[t]-5
```

$$y1(t) \rightarrow -2e^{-3t} - 4e^t$$

$$y2(t) \rightarrow e^t(9 - 4t)$$

$$y3(t) \rightarrow e^t(1 - 4t) - 2e^{-3t}$$

22.21 problem section 10.5, problem 21

Internal problem ID [1624]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 21.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}y_1'(t) &= -y_1(t) - 4y_2(t) - y_3(t) \\y_2'(t) &= 3y_1(t) + 6y_2(t) + y_3(t) \\y_3'(t) &= -3y_1(t) - 2y_2(t) + 3y_3(t)\end{aligned}$$

With initial conditions

$$[y_1(0) = -2, y_2(0) = 1, y_3(0) = 3]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 63

```
dsolve([diff(y__1(t),t) = -y__1(t)-4*y__2(t)-y__3(t), diff(y__2(t),t) = 3*y__1(t)+6*y__2(t)+
```

$$y_1(t) = -2e^{4t} + 3e^{2t}t$$

$$y_2(t) = 2e^{4t} - e^{2t} - 3e^{2t}t$$

$$y_3(t) = 2e^{4t} + e^{2t} + 3e^{2t}t$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 63

```
DSolve[{y1'[t]==-1*y1[t]-4*y2[t]-1*y3[t],y2'[t]==3*y1[t]+6*y2[t]+1*y3[t],y3'[t]==-3*y1[t]-2*
```

$$y_1(t) \rightarrow 3e^{2t}t - 2e^{4t}$$

$$y_2(t) \rightarrow e^{2t}(-3t + 2e^{2t} - 1)$$

$$y_3(t) \rightarrow e^{2t}(3t + 2e^{2t} + 1)$$

22.22 problem section 10.5, problem 22

Internal problem ID [1625]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 22.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = 4y_1(t) - 8y_2(t) - 4y_3(t)$$

$$y_2'(t) = -3y_1(t) - y_2(t) - 4y_3(t)$$

$$y_3'(t) = y_1(t) - y_2(t) + 9y_3(t)$$

With initial conditions

$$[y_1(0) = -4, y_2(0) = 1, y_3(0) = -3]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 62

```
dsolve([diff(y__1(t),t) = 4*y__1(t)-8*y__2(t)-4*y__3(t), diff(y__2(t),t) = -3*y__1(t)-y__2(t)
```

$$y_1(t) = -\frac{50 e^{7t}}{11} + \frac{22 e^{9t}}{13} - \frac{164 e^{-4t}}{143}$$

$$y_2(t) = \frac{5 e^{7t}}{11} + \frac{22 e^{9t}}{13} - \frac{164 e^{-4t}}{143}$$

$$y_3(t) = \frac{5 e^{7t}}{2} - \frac{11 e^{9t}}{2}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 57

```
DSolve[{y1'[t]==4*y1[t]-8*y2[t]-4*y3[t],y2'[t]==-3*y1[t]-1*y2[t]-3*y3[t],y3'[t]==1*y1[t]-1*y
```

$$y_1(t) \rightarrow e^{8t}(8t - 3) - e^{-4t}$$

$$y_2(t) \rightarrow 2e^{8t} - e^{-4t}$$

$$y_3(t) \rightarrow -e^{8t}(8t + 3)$$

22.23 problem section 10.5, problem 23

Internal problem ID [1626]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 23.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = -5y_1(t) - y_2(t) + 11y_3(t)$$

$$y_2'(t) = -7y_1(t) + y_2(t) + 13y_3(t)$$

$$y_3'(t) = -4y_1(t) + 8y_3(t)$$

With initial conditions

$$[y_1(0) = 0, y_2(0) = 2, y_3(0) = 2]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 44

```
dsolve([diff(y__1(t),t) = -5*y__1(t)-y__2(t)+11*y__3(t), diff(y__2(t),t) = -7*y__1(t)+y__2(t)
```

$$y_1(t) = -3 + 3e^{4t} + 8t$$

$$y_2(t) = 6e^{4t} - 4 + 4t$$

$$y_3(t) = -1 + 4t + 3e^{4t}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 47

```
DSolve[{y1'[t]==-5*y1[t]-1*y2[t]+11*y3[t],y2'[t]==-7*y1[t]+1*y2[t]+13*y3[t],y3'[t]==-4*y1[t]
```

$$y1(t) \rightarrow 8t + 3e^{4t} - 3$$

$$y2(t) \rightarrow 4t + 6e^{4t} - 4$$

$$y3(t) \rightarrow 4t + 3e^{4t} - 1$$

22.24 problem section 10.5, problem 24

Internal problem ID [1627]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 24.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = 5y_1(t) - y_2(t) + y_3(t)$$

$$y_2'(t) = -y_1(t) + 9y_2(t) - 3y_3(t)$$

$$y_3'(t) = -2y_1(t) + 2y_2(t) + 4y_3(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 67

```
dsolve([diff(y__1(t),t)=5*y__1(t)-1*y__2(t)+1*y__3(t),diff(y__2(t),t)=-1*y__1(t)+9*y__2(t)-3
```

$$y_1(t) = -\frac{e^{6t}(2c_3t + c_2 - c_3)}{4}$$

$$y_2(t) = \frac{e^{6t}(4c_3t^2 + 4c_2t + 2c_3t + 4c_1 + c_2 + c_3)}{4}$$

$$y_3(t) = e^{6t}(c_3t^2 + c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 106

```
DSolve[{y1'[t]==5*y1[t]-1*y2[t]+1*y3[t],y2'[t]==-1*y1[t]+9*y2[t]-3*y3[t],y3'[t]==-2*y1[t]+2*
```

$$y_1(t) \rightarrow -e^{6t}(c_1(t-1) + (c_2 - c_3)t)$$

$$y_2(t) \rightarrow e^{6t}(2(c_1 + c_2 - c_3)t^2 - (c_1 - 3c_2 + 3c_3)t + c_2)$$

$$y_3(t) \rightarrow e^{6t}(2(c_1 + c_2 - c_3)t^2 - 2(c_1 - c_2 + c_3)t + c_3)$$

22.25 problem section 10.5, problem 25

Internal problem ID [1628]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 25.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = y_1(t) + 10y_2(t) - 12y_3(t)$$

$$y_2'(t) = 2y_1(t) + 2y_2(t) + 3y_3(t)$$

$$y_3'(t) = 2y_1(t) - y_2(t) + 6y_3(t)$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 77

```
dsolve([diff(y__1(t),t)=1*y__1(t)+10*y__2(t)-12*y__3(t),diff(y__2(t),t)=2*y__1(t)+2*y__2(t)+
```

$$y_1(t) = -\frac{e^{3t}(18c_3t^2 + 18c_2t - 18c_3t + 18c_1 - 9c_2 - c_3)}{18}$$

$$y_2(t) = \frac{e^{3t}(9c_3t^2 + 9c_2t + 9c_1 + c_3)}{9}$$

$$y_3(t) = e^{3t}(c_3t^2 + c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 118

```
DSolve[{y1'[t]==1*y1[t]+10*y2[t]-12*y3[t],y2'[t]==2*y1[t]+2*y2[t]+3*y3[t],y3'[t]==2*y1[t]-1*
```

$$y_1(t) \rightarrow -e^{3t}(c_1(2t - 1) + c_2t(9t - 10) + 3c_3(4 - 3t)t)$$

$$y_2(t) \rightarrow e^{3t}(9(c_2 - c_3)t^2 + (2c_1 - c_2 + 3c_3)t + c_2)$$

$$y_3(t) \rightarrow e^{3t}(9(c_2 - c_3)t^2 + (2c_1 - c_2 + 3c_3)t + c_3)$$

22.26 problem section 10.5, problem 26

Internal problem ID [1629]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 26.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = -6y_1(t) - 4y_2(t) - 4y_3(t)$$

$$y_2'(t) = 2y_1(t) - y_2(t) + y_3(t)$$

$$y_3'(t) = 2y_1(t) + 3y_2(t) + y_3(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 67

```
dsolve([diff(y__1(t),t)=-6*y__1(t)-4*y__2(t)-4*y__3(t),diff(y__2(t),t)=2*y__1(t)-1*y__2(t)+1
```

$$y_1(t) = -\frac{e^{-2t}(4c_3t + 2c_2 - 3c_3)}{2}$$

$$y_2(t) = -e^{-2t}(c_3t^2 + c_2t - 2c_3t + c_1 - c_2 + c_3)$$

$$y_3(t) = e^{-2t}(c_3t^2 + c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 100

```
DSolve[{y1'[t]==-6*y1[t]-4*y2[t]-4*y3[t],y2'[t]==2*y1[t]-1*y2[t]+1*y3[t],y3'[t]==2*y1[t]+3*y
```

$$y1(t) \rightarrow e^{-2t}(c_1(1 - 4t) - 4(c_2 + c_3)t)$$

$$y2(t) \rightarrow e^{-2t}(-2(c_1 + c_2 + c_3)t^2 + (2c_1 + c_2 + c_3)t + c_2)$$

$$y3(t) \rightarrow e^{-2t}(2(c_1 + c_2 + c_3)t^2 + 2c_1t + 3(c_2 + c_3)t + c_3)$$

22.27 problem section 10.5, problem 27

Internal problem ID [1630]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 27.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = 2y_2(t) - 2y_3(t)$$

$$y_2'(t) = -y_1(t) + 5y_2(t) - 3y_3(t)$$

$$y_3'(t) = y_1(t) + y_2(t) + y_3(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 71

```
dsolve([diff(y__1(t),t)=0*y__1(t)+2*y__2(t)-2*y__3(t),diff(y__2(t),t)=-1*y__1(t)+5*y__2(t)-3
```

$$y_1(t) = \frac{e^{2t}(4c_3t + 2c_2 - c_3)}{4}$$

$$y_2(t) = \frac{e^{2t}(4c_3t^2 + 4c_2t + 4c_3t + 4c_1 + 2c_2 + c_3)}{4}$$

$$y_3(t) = e^{2t}(c_3t^2 + c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 108

```
DSolve[{y1'[t]==0*y1[t]+2*y2[t]-2*y3[t],y2'[t]==-1*y1[t]+5*y2[t]-3*y3[t],y3'[t]==1*y1[t]+1*y
```

$$y1(t) \rightarrow -e^{2t}(c_1(2t - 1) + 2(c_3 - c_2)t)$$

$$y2(t) \rightarrow e^{2t}(-2(c_1 - c_2 + c_3)t^2 - (c_1 - 3c_2 + 3c_3)t + c_2)$$

$$y3(t) \rightarrow e^{2t}(-2(c_1 - c_2 + c_3)t^2 + (c_1 + c_2 - c_3)t + c_3)$$

22.28 problem section 10.5, problem 28

Internal problem ID [1631]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 28.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = -2y_1(t) - 12y_2(t) + 10y_3(t)$$

$$y_2'(t) = 2y_1(t) - 24y_2(t) + 11y_3(t)$$

$$y_3'(t) = 2y_1(t) - 24y_2(t) + 8y_3(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 86

```
dsolve([diff(y__1(t),t)=-2*y__1(t)-12*y__2(t)+10*y__3(t),diff(y__2(t),t)=2*y__1(t)-24*y__2(t)
```

$$y_1(t) = -\frac{e^{-6t}(6c_3t^2 + 6c_2t + 6c_3t + 6c_1 + 3c_2 + 2c_3)}{6}$$

$$y_2(t) = \frac{e^{-6t}(18c_3t^2 + 18c_2t - 6c_3t + 18c_1 - 3c_2 - c_3)}{36}$$

$$y_3(t) = e^{-6t}(c_3t^2 + c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 131

```
DSolve[{y1'[t]==-2*y1[t]-12*y2[t]+10*y3[t],y2'[t]==2*y1[t]-24*y2[t]+11*y3[t],y3'[t]==2*y1[t]
```

$$y1(t) \rightarrow e^{-6t}(c_1(6t^2 + 4t + 1) + 2t(c_3(12t + 5) - 6c_2(3t + 1)))$$

$$y2(t) \rightarrow e^{-6t}(-3(c_1 - 6c_2 + 4c_3)t^2 + (2c_1 - 18c_2 + 11c_3)t + c_2)$$

$$y3(t) \rightarrow e^{-6t}(-6(c_1 - 6c_2 + 4c_3)t^2 + 2(c_1 - 12c_2 + 7c_3)t + c_3)$$

22.29 problem section 10.5, problem 29

Internal problem ID [1632]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 29.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = -y_1(t) - 12y_2(t) + 8y_3(t)$$

$$y_2'(t) = y_1(t) - 9y_2(t) + 4y_3(t)$$

$$y_3'(t) = y_1(t) - 6y_2(t) + y_3(t)$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 49

```
dsolve([diff(y__1(t),t)=-1*y__1(t)-12*y__2(t)+8*y__3(t),diff(y__2(t),t)=1*y__1(t)-9*y__2(t)+
```

$$y_1(t) = e^{-3t}(2c_3t + 6c_1 + 2c_2 + c_3)$$

$$y_2(t) = e^{-3t}(c_3t + c_1 + c_2)$$

$$y_3(t) = e^{-3t}(c_3t + c_2)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 79

```
DSolve[{y1'[t]==-1*y1[t]-12*y2[t]+8*y3[t],y2'[t]==1*y1[t]-9*y2[t]+4*y3[t],y3'[t]==1*y1[t]-6*
```

$$y_1(t) \rightarrow e^{-3t}(2c_1t - 12c_2t + 8c_3t + c_1)$$

$$y_2(t) \rightarrow e^{-3t}((c_1 - 6c_2 + 4c_3)t + c_2)$$

$$y_3(t) \rightarrow e^{-3t}((c_1 - 6c_2 + 4c_3)t + c_3)$$

22.30 problem section 10.5, problem 30

Internal problem ID [1633]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 30.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = -4y_1(t) - y_3(t)$$

$$y_2'(t) = -y_1(t) - 3y_2(t) - y_3(t)$$

$$y_3'(t) = y_1(t) - 2y_3(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 49

```
dsolve([diff(y__1(t),t)=-4*y__1(t)-0*y__2(t)-1*y__3(t),diff(y__2(t),t)=-1*y__1(t)-3*y__2(t)-
```

$$y_1(t) = -e^{-3t}(c_3t + c_2 - c_3)$$

$$y_2(t) = e^{-3t}(-c_3t + c_1 - c_2)$$

$$y_3(t) = e^{-3t}(c_3t + c_2)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 63

```
DSolve[{y1'[t]==-4*y1[t]-0*y2[t]-1*y3[t],y2'[t]==-1*y1[t]-3*y2[t]-1*y3[t],y3'[t]==1*y1[t]-0*y2[t]-1*y3[t]}
```

$$y1(t) \rightarrow e^{-3t}(c_1(-t) - c_3t + c_1)$$

$$y2(t) \rightarrow e^{-3t}(c_2 - (c_1 + c_3)t)$$

$$y3(t) \rightarrow e^{-3t}((c_1 + c_3)t + c_3)$$

22.31 problem section 10.5, problem 31

Internal problem ID [1634]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 31.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = -3y_1(t) - 3y_2(t) + 4y_3(t)$$

$$y_2'(t) = 4y_1(t) + 5y_2(t) - 8y_3(t)$$

$$y_3'(t) = 2y_1(t) + 3y_2(t) - 5y_3(t)$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 55

```
dsolve([diff(y__1(t),t)=-3*y__1(t)-3*y__2(t)+4*y__3(t),diff(y__2(t),t)=4*y__1(t)+5*y__2(t)-8
```

$$y_1(t) = -\frac{e^{-t}(2c_3t + 3c_1 + 2c_2 - c_3)}{2}$$

$$y_2(t) = e^{-t}(2c_3t + c_1 + 2c_2)$$

$$y_3(t) = e^{-t}(c_3t + c_2)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 83

```
DSolve[{y1'[t]==-3*y1[t]-3*y2[t]+4*y3[t],y2'[t]==4*y1[t]+5*y2[t]-8*y3[t],y3'[t]==2*y1[t]+3*y
```

$$y_1(t) \rightarrow e^{-t}(-2c_1t - 3c_2t + 4c_3t + c_1)$$

$$y_2(t) \rightarrow e^{-t}((4c_1 + 6c_2 - 8c_3)t + c_2)$$

$$y_3(t) \rightarrow e^{-t}((2c_1 + 3c_2 - 4c_3)t + c_3)$$

22.32 problem section 10.5, problem 32

Internal problem ID [1635]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

Problem number: section 10.5, problem 32.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = -3y_1(t) - y_2(t)$$

$$y_2'(t) = y_1(t) - y_2(t)$$

$$y_3'(t) = -y_1(t) - y_2(t) - 2y_3(t)$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 51

```
dsolve([diff(y__1(t),t)=-3*y__1(t)-1*y__2(t)+0*y__3(t),diff(y__2(t),t)=1*y__1(t)-1*y__2(t)+0
```

$$y_1(t) = -e^{-2t}(-c_3t + c_1 - c_2 + c_3)$$

$$y_2(t) = e^{-2t}(-c_3t + c_1 - c_2)$$

$$y_3(t) = e^{-2t}(c_3t + c_2)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 63

```
DSolve[{y1'[t]==-3*y1[t]-1*y2[t]+0*y3[t],y2'[t]==1*y1[t]-1*y2[t]+0*y3[t],y3'[t]==-1*y1[t]-1*y2[t]+0*y3[t]}
```

$$y1(t) \rightarrow e^{-2t}(c_1(-t) - c_2t + c_1)$$

$$y2(t) \rightarrow e^{-2t}((c_1 + c_2)t + c_2)$$

$$y3(t) \rightarrow e^{-2t}(c_3 - (c_1 + c_2)t)$$

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23.1 problem section 10.6, problem 1

Internal problem ID [1636]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.6, constant coefficient homogeneous system III. Page 566

Problem number: section 10.6, problem 1.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}y_1'(t) &= -y_1(t) + 2y_2(t) \\y_2'(t) &= -5y_1(t) + 5y_2(t)\end{aligned}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 48

```
dsolve([diff(y__1(t),t)=-1*y__1(t)+2*y__2(t),diff(y__2(t),t)=-5*y__1(t)+5*y__2(t)], [y__1(t),
```

$$y_1(t) = \frac{e^{2t}(3c_1 \sin(t) + c_2 \sin(t) - \cos(t) c_1 + 3c_2 \cos(t))}{5}$$

$$y_2(t) = e^{2t}(c_1 \sin(t) + c_2 \cos(t))$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 55

```
DSolve[{y1'[t]==-1*y1[t]+2*y2[t],y2'[t]==-5*y1[t]+5*y2[t]},{y1[t],y2[t]},t,IncludeSingularSo
```

$$y_1(t) \rightarrow e^{2t}(c_1 \cos(t) + (2c_2 - 3c_1) \sin(t))$$

$$y_2(t) \rightarrow e^{2t}(c_2(3 \sin(t) + \cos(t)) - 5c_1 \sin(t))$$

23.2 problem section 10.6, problem 2

Internal problem ID [1637]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.6, constant coefficient homogeneous system III. Page 566

Problem number: section 10.6, problem 2.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = -11y_1(t) + 4y_2(t)$$

$$y_2'(t) = -26y_1(t) + 9y_2(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 60

```
dsolve([diff(y__1(t),t)=-11*y__1(t)+4*y__2(t),diff(y__2(t),t)=-26*y__1(t)+9*y__2(t)], [y__1(t)
```

$$y_1(t) = \frac{e^{-t}(5c_1 \sin(2t) + c_2 \sin(2t) - c_1 \cos(2t) + 5c_2 \cos(2t))}{13}$$

$$y_2(t) = e^{-t}(c_1 \sin(2t) + c_2 \cos(2t))$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 64

```
DSolve[{y1'[t]==-11*y1[t]+4*y2[t],y2'[t]==-26*y1[t]+9*y2[t]},{y1[t],y2[t]},t,IncludeSingular
```

$$y1(t) \rightarrow e^{-t}(c_1 \cos(2t) + (2c_2 - 5c_1) \sin(2t))$$

$$y2(t) \rightarrow e^{-t}(c_2 \cos(2t) + (5c_2 - 13c_1) \sin(2t))$$

23.3 problem section 10.6, problem 3

Internal problem ID [1638]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.6, constant coefficient homogeneous system III. Page 566

Problem number: section 10.6, problem 3.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}y_1'(t) &= y_1(t) + 2y_2(t) \\y_2'(t) &= -4y_1(t) + 5y_2(t)\end{aligned}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 58

```
dsolve([diff(y__1(t),t)=1*y__1(t)+2*y__2(t),diff(y__2(t),t)=-4*y__1(t)+5*y__2(t)], [y__1(t),
```

$$y_1(t) = \frac{e^{3t}(c_1 \sin(2t) + c_2 \sin(2t) - c_1 \cos(2t) + c_2 \cos(2t))}{2}$$

$$y_2(t) = e^{3t}(c_1 \sin(2t) + c_2 \cos(2t))$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 60

```
DSolve[{y1'[t]==1*y1[t]+2*y2[t],y2'[t]==-4*y1[t]+5*y2[t]},{y1[t],y2[t]},t,IncludeSingularSol
```

$$y_1(t) \rightarrow e^{3t}(c_1 \cos(2t) + (c_2 - c_1) \sin(2t))$$

$$y_2(t) \rightarrow e^{3t}(c_2 \cos(2t) + (c_2 - 2c_1) \sin(2t))$$

23.4 problem section 10.6, problem 4

Internal problem ID [1639]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.6, constant coefficient homogeneous system III. Page 566

Problem number: section 10.6, problem 4.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = 5y_1(t) - 6y_2(t)$$

$$y_2'(t) = 3y_1(t) - y_2(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 57

```
dsolve([diff(y__1(t),t)=5*y__1(t)-6*y__2(t),diff(y__2(t),t)=3*y__1(t)-1*y__2(t)],[y__1(t), y
```

$$y_1(t) = e^{2t}(c_1 \cos(3t) + c_2 \cos(3t) + c_1 \sin(3t) - c_2 \sin(3t))$$

$$y_2(t) = e^{2t}(c_1 \sin(3t) + c_2 \cos(3t))$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 60

```
DSolve[{y1'[t]==5*y1[t]-6*y2[t],y2'[t]==3*y1[t]-1*y2[t]},{y1[t],y2[t]},t,IncludeSingularSolu
```

$$y_1(t) \rightarrow e^{2t}(c_1 \cos(3t) + (c_1 - 2c_2) \sin(3t))$$

$$y_2(t) \rightarrow e^{2t}(c_2 \cos(3t) + (c_1 - c_2) \sin(3t))$$

23.5 problem section 10.6, problem 5

Internal problem ID [1640]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.6, constant coefficient homogeneous system III. Page 566

Problem number: section 10.6, problem 5.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = -3y_1(t) - 3y_2(t) + y_3(t)$$

$$y_2'(t) = 2y_2(t) + 2y_3(t)$$

$$y_3'(t) = 5y_1(t) + y_2(t) + y_3(t)$$

✓ Solution by Maple

Time used: 0.563 (sec). Leaf size: 2820

```
dsolve([diff(y__1(t),t)=-3*y__1(t)-3*y__2(t)+1*y__3(t),diff(y__2(t),t)=0*y__1(t)+2*y__2(t)+2
```

Expression too large to display

Expression too large to display

$$\begin{aligned}
 y_3(t) = & c_2 e^{\frac{\left(\frac{(540+6\sqrt{6042})^{\frac{2}{3}}}{6}+7\right)t}{(540+6\sqrt{6042})^{\frac{1}{3}}}} \sin\left(\frac{\left(\left(540+6\sqrt{6042}\right)^{\frac{2}{3}}-42\right)t\sqrt{3}36^{\frac{1}{3}}}{36(90+\sqrt{6042})^{\frac{1}{3}}}\right) \\
 & + c_3 e^{\frac{\left(\frac{(540+6\sqrt{6042})^{\frac{2}{3}}}{6}+7\right)t}{(540+6\sqrt{6042})^{\frac{1}{3}}}} \cos\left(\frac{\left(\left(540+6\sqrt{6042}\right)^{\frac{2}{3}}-42\right)t\sqrt{3}36^{\frac{1}{3}}}{36(90+\sqrt{6042})^{\frac{1}{3}}}\right) \\
 & + c_1 e^{-\frac{\left(\frac{(540+6\sqrt{6042})^{\frac{2}{3}}}{6}+42\right)t}{3(540+6\sqrt{6042})^{\frac{1}{3}}}}
 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 187

```
DSolve[{y1'[t]==3*y1[t]-3*y2[t]+1*y3[t],y2'[t]==0*y1[t]+2*y2[t]+2*y3[t],y3'[t]==5*y1[t]+1*y2
```

$$y_1(t) \rightarrow \frac{1}{4}e^{-2t}((3c_1 - c_2 + c_3)e^{6t} \cos(2t) + (c_1 - 3c_2 - c_3)e^{6t} \sin(2t) + c_1 + c_2 - c_3)$$

$$y_2(t) \rightarrow \frac{1}{4}e^{-2t}(-(c_1 - 3c_2 - c_3)e^{6t} \cos(2t) + (3c_1 - c_2 + c_3)e^{6t} \sin(2t) + c_1 + c_2 - c_3)$$

$$y_3(t) \rightarrow \frac{1}{2}e^{-2t}((c_1 + c_2 + c_3)e^{6t} \cos(2t) + 2(c_1 - c_2)e^{6t} \sin(2t) - c_1 - c_2 + c_3)$$

23.6 problem section 10.6, problem 6

Internal problem ID [1641]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.6, constant coefficient homogeneous system III. Page 566

Problem number: section 10.6, problem 6.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = -3y_1(t) + 3y_2(t) + y_3(t)$$

$$y_2'(t) = y_1(t) - 5y_2(t) - 3y_3(t)$$

$$y_3'(t) = -3y_1(t) + 7y_2(t) + 3y_3(t)$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 142

```
dsolve([diff(y__1(t),t)=-3*y__1(t)+3*y__2(t)+1*y__3(t),diff(y__2(t),t)=1*y__1(t)-5*y__2(t)-3
```

$$y_1(t) = -e^{-t}c_1 + \frac{c_2e^{-2t}\sin(2t)}{2} + \frac{c_2e^{-2t}\cos(2t)}{2} + \frac{c_3e^{-2t}\cos(2t)}{2} - \frac{c_3e^{-2t}\sin(2t)}{2}$$

$$y_2(t) = -e^{-t}c_1 + \frac{c_2e^{-2t}\cos(2t)}{2} - \frac{c_3e^{-2t}\sin(2t)}{2} - \frac{c_2e^{-2t}\sin(2t)}{2} - \frac{c_3e^{-2t}\cos(2t)}{2}$$

$$y_3(t) = e^{-t}c_1 + c_2e^{-2t}\sin(2t) + c_3e^{-2t}\cos(2t)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 158

```
DSolve[{y1'[t]==-3*y1[t]+3*y2[t]+1*y3[t],y2'[t]==1*y1[t]-5*y2[t]-3*y3[t],y3'[t]==-3*y1[t]+7*y2[t]+1*y3[t]},y1,y2,y3,t]
```

$$y1(t) \rightarrow e^{-2t}((c_1 - c_2 - c_3)e^t + (c_2 + c_3) \cos(2t) + (-c_1 + 2c_2 + c_3) \sin(2t))$$

$$y2(t) \rightarrow e^{-2t}((c_1 - c_2 - c_3)e^t + (-c_1 + 2c_2 + c_3) \cos(2t) - (c_2 + c_3) \sin(2t))$$

$$y3(t) \rightarrow e^{-2t}((-c_1 + c_2 + c_3)e^t + (c_1 - c_2) \cos(2t) + (-c_1 + 3c_2 + 2c_3) \sin(2t))$$

23.7 problem section 10.6, problem 7

Internal problem ID [1642]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.6, constant coefficient homogeneous system III. Page 566

Problem number: section 10.6, problem 7.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = 2y_1(t) + y_2(t) - y_3(t)$$

$$y_2'(t) = y_2(t) + y_3(t)$$

$$y_3'(t) = y_1(t) + y_3(t)$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 70

```
dsolve([diff(y__1(t),t)=2*y__1(t)+1*y__2(t)-1*y__3(t),diff(y__2(t),t)=0*y__1(t)+1*y__2(t)+1*
```

$$y_1(t) = c_1 e^{2t} + c_2 e^t \cos(t) - c_3 e^t \sin(t)$$

$$y_2(t) = c_1 e^{2t} - c_2 e^t \cos(t) + c_3 e^t \sin(t)$$

$$y_3(t) = c_1 e^{2t} + c_2 e^t \sin(t) + c_3 e^t \cos(t)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 129

```
DSolve[{y1'[t]==2*y1[t]+1*y2[t]-1*y3[t],y2'[t]==0*y1[t]+1*y2[t]+1*y3[t],y3'[t]==1*y1[t]+0*y2
```

$$y1(t) \rightarrow \frac{1}{2}e^t(-2c_3 \sin(t) + c_2(e^t + \sin(t) - \cos(t)) + c_1(e^t + \sin(t) + \cos(t)))$$

$$y2(t) \rightarrow \frac{1}{2}e^t((c_1 + c_2)e^t + (c_2 - c_1) \cos(t) - (c_1 + c_2 - 2c_3) \sin(t))$$

$$y3(t) \rightarrow \frac{1}{2}e^t((c_1 + c_2)e^t - (c_1 + c_2 - 2c_3) \cos(t) + (c_1 - c_2) \sin(t))$$

23.8 problem section 10.6, problem 8

Internal problem ID [1643]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 10 Linear system of Differential equations. Section 10.6, constant coefficient homogeneous system III. Page 566

Problem number: section 10.6, problem 8.

ODE order: 1.

ODE degree: 1.

Solve

$$y_1'(t) = -3y_1(t) + y_2(t) - 3y_3(t)$$

$$y_2'(t) = 4y_1(t) - y_2(t) + 2y_3(t)$$

$$y_3'(t) = 4y_1(t) - 2y_2(t) + 3y_3(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 112

```
dsolve([diff(y__1(t),t)=-3*y__1(t)+1*y__2(t)-3*y__3(t),diff(y__2(t),t)=4*y__1(t)-1*y__2(t)+2
```

$$y_1(t) = -c_1 e^t - \frac{c_2 e^{-t} \sin(2t)}{2} - \frac{c_3 e^{-t} \cos(2t)}{2} + \frac{c_2 e^{-t} \cos(2t)}{2} - \frac{c_3 e^{-t} \sin(2t)}{2}$$

$$y_2(t) = -c_1 e^t + c_2 e^{-t} \sin(2t) + c_3 e^{-t} \cos(2t)$$

$$y_3(t) = c_1 e^t + c_2 e^{-t} \sin(2t) + c_3 e^{-t} \cos(2t)$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 163

```
DSolve[{y1'[t]==-3*y1[t]+1*y2[t]-3*y3[t],y2'[t]==4*y1[t]-1*y2[t]+2*y3[t],y3'[t]==4*y1[t]-2*y
```

$$y_1(t) \rightarrow \frac{1}{2}e^{-t}((c_2 - c_3)e^{2t} + (2c_1 - c_2 + c_3) \cos(2t) - 2(c_1 + c_3) \sin(2t))$$

$$y_2(t) \rightarrow \frac{1}{2}e^{-t}((c_2 - c_3)e^{2t} + (c_2 + c_3) \cos(2t) + (4c_1 - c_2 + 3c_3) \sin(2t))$$

$$y_3(t) \rightarrow \frac{1}{2}e^{-t}((c_3 - c_2)e^{2t} + (c_2 + c_3) \cos(2t) + (4c_1 - c_2 + 3c_3) \sin(2t))$$