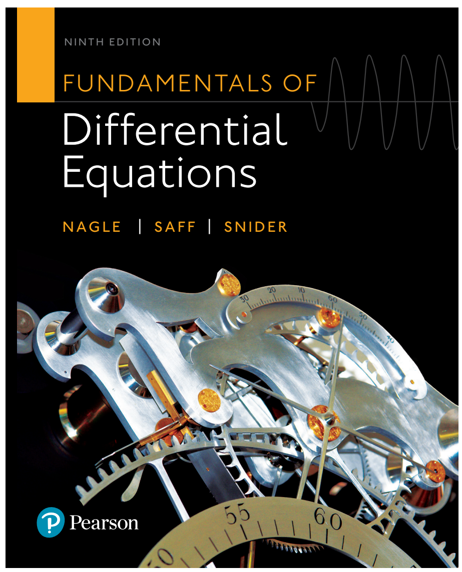
A Solution Manual For

Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.



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1.1 problem 1

Internal problem ID [4912]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - \sin(x + y) = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)-sin(x+y(x))=0,y(x), singsol=all)
```

$$y(x) = -x - 2\arctan\left(\frac{c_1 - x - 2}{c_1 - x}\right)$$

✓ Solution by Mathematica

Time used: 36.293 (sec). Leaf size: 541

```
DSolve[y'[x]-Sin[x+y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2\arccos\left(\frac{(x+c_1)\sin\left(\frac{x}{2}\right)-(x-2+c_1)\cos\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{x^2+2(-1+c_1)x+2+c_1{}^2-2c_1}}\right)$$

$$y(x) \to 2\arccos\left(\frac{(x+c_1)\sin\left(\frac{x}{2}\right)-(x-2+c_1)\cos\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{x^2+2(-1+c_1)x+2+c_1{}^2-2c_1}}\right)$$

$$y(x) \to -2\arccos\left(\frac{(x-2+c_1)\cos\left(\frac{x}{2}\right)-(x+c_1)\sin\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{x^2+2(-1+c_1)x+2+c_1{}^2-2c_1}}\right)$$

$$y(x) \to 2\arccos\left(\frac{(x-2+c_1)\cos\left(\frac{x}{2}\right)-(x+c_1)\sin\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{x^2+2(-1+c_1)x+2+c_1{}^2-2c_1}}\right)$$

$$y(x) \to -2\arccos\left(\frac{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}{\sqrt{2}}\right)$$

$$y(x) \to 2\arccos\left(\frac{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}{\sqrt{2}}\right)$$

$$y(x) \to -2\arccos\left(\frac{\sin\left(\frac{x}{2}\right)-\cos\left(\frac{x}{2}\right)}{\sqrt{2}}\right)$$

$$y(x) \to 2\arccos\left(\frac{\sin\left(\frac{x}{2}\right)-\cos\left(\frac{x}{2}\right)}{\sqrt{2}}\right)$$

$$y(x) \to -2\arccos\left(\frac{(x-2)\cos\left(\frac{x}{2}\right)-x\sin\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{x^2-2x+2}}\right)$$

$$y(x) \to 2\arccos\left(\frac{(x-2)\cos\left(\frac{x}{2}\right)-x\sin\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{x^2-2x+2}}\right)$$

$$y(x) \to -2\arccos\left(\frac{x\sin\left(\frac{x}{2}\right)-(x-2)\cos\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{x^2-2x+2}}\right)$$

$$y(x) \to 2\arccos\left(\frac{x\sin\left(\frac{x}{2}\right)-(x-2)\cos\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{x^2-2x+2}}\right)$$

1.2 problem 2

Internal problem ID [4913]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 4y^2 + 3y = 1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve(diff(y(x),x)=4*y(x)^2-3*y(x)+1,y(x), singsol=all)
```

$$y(x) = \frac{\left(3\sqrt{7} + 7\tan\left(\frac{(x+c_1)\sqrt{7}}{2}\right)\right)\sqrt{7}}{56}$$

✓ Solution by Mathematica

Time used: 1.272 (sec). Leaf size: 69

```
DSolve[y'[x]==4*y[x]^2-3*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}\left(3 + \sqrt{7}\tan\left(\frac{1}{2}\sqrt{7}(x + c_1)\right)\right)$$
$$y(x) \to \frac{1}{8}\left(3 - i\sqrt{7}\right)$$
$$y(x) \to \frac{1}{8}\left(3 + i\sqrt{7}\right)$$

1.3 problem 3

Internal problem ID [4914]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{s' - t\ln\left(s^{2t}\right) = 8t^2}$$

✗ Solution by Maple

```
dsolve(diff(s(t),t)=t*ln(s(t)^(2*t))+8*t^2,s(t), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 0.28 (sec). Leaf size: 34

```
DSolve[s'[t]==t*Log[s[t]^(2*t)]+8*t^2,s[t],t,IncludeSingularSolutions -> True]
```

$$s(t) \to \text{InverseFunction}\left[\frac{\text{ExpIntegralEi}(\log(\#1) + 4)}{e^4}\&\right]\left[\frac{2t^3}{3} + c_1\right]$$
$$s(t) \to \frac{1}{e^4}$$

1.4 problem 4

Internal problem ID [4915]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{y\,\mathrm{e}^{x+y}}{x^2+2} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 59

```
dsolve(diff(y(x),x)=y(x)*exp(x+y(x))/(x^2+2),y(x), singsol=all)
```

$$\frac{i\sqrt{2}\,\mathrm{e}^{i\sqrt{2}}\,\mathrm{Ei}_1\left(-x+i\sqrt{2}\right)}{4} - \frac{i\sqrt{2}\,\mathrm{e}^{-i\sqrt{2}}\,\mathrm{Ei}_1\left(-x-i\sqrt{2}\right)}{4} + \mathrm{Ei}_1\left(y(x)\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.932 (sec). Leaf size: 81

```
DSolve[y'[x]==y[x]*Exp[x+y[x]]/(x^2+2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}[\text{ExpIntegralEi}(-\#1)\&]\left[c_1 - \frac{ie^{-i\sqrt{2}}\left(e^{2i\sqrt{2}}\,\text{ExpIntegralEi}\left(x-i\sqrt{2}\right) - \text{ExpIntegralEi}\left(x+i\sqrt{2}\right)\right)}{2\sqrt{2}}\right]$$

$$y(x) \to 0$$

1.5 problem 5

Internal problem ID [4916]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(xy^2 + 3y^2)\, y' = 2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 93

```
dsolve((x*y(x)^2+3*y(x)^2)*diff(y(x),x)-2*x=0,y(x), singsol=all)
```

$$y(x) = (-18\ln(x+3) + c_1 + 6x)^{\frac{1}{3}}$$
$$y(x) = -\frac{(-18\ln(x+3) + c_1 + 6x)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}\,(-18\ln(x+3) + c_1 + 6x)^{\frac{1}{3}}}{2}$$
$$y(x) = -\frac{(-18\ln(x+3) + c_1 + 6x)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}\,(-18\ln(x+3) + c_1 + 6x)^{\frac{1}{3}}}{2}$$

✓ Solution by Mathematica

Time used: 0.238 (sec). Leaf size: 85

```
DSolve[(x*y[x]^2+3*y[x]^2)*y'[x]-2*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt[3]{-3}\,\sqrt[3]{2x - 6\log(x+3) + c_1}$$
$$y(x) \to \sqrt[3]{3}\,\sqrt[3]{2x - 6\log(x+3) + c_1}$$
$$y(x) \to (-1)^{2/3}\sqrt[3]{3}\,\sqrt[3]{2x - 6\log(x+3) + c_1}$$

1.6 problem 6

Internal problem ID [4917]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class C']]`

$$s^2 + s' - \frac{s+1}{ts} = 0$$

✗ Solution by Maple

```
dsolve(s(t)^2+diff(s(t),t)=(s(t)+1)/(s(t)*t),s(t), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[s[t]^2+s'[t]==(s[t]+1)/(s[t]*t),s[t],t,IncludeSingularSolutions -> True]
```

Not solved

1.7 problem 7

Internal problem ID [4918]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x - \frac{1}{y^3} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
dsolve(x*diff(y(x),x)=1/y(x)^3,y(x), singsol=all)
```

$$y(x) = (4\ln(x) + c_1)^{\frac{1}{4}}$$
$$y(x) = -(4\ln(x) + c_1)^{\frac{1}{4}}$$
$$y(x) = -i(4\ln(x) + c_1)^{\frac{1}{4}}$$
$$y(x) = i(4\ln(x) + c_1)^{\frac{1}{4}}$$

✓ Solution by Mathematica

Time used: 0.15 (sec). Leaf size: 84

```
DSolve[x*y'[x]==1/y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}\sqrt[4]{\log(x) + c_1}$$
$$y(x) \to -i\sqrt{2}\sqrt[4]{\log(x) + c_1}$$
$$y(x) \to i\sqrt{2}\sqrt[4]{\log(x) + c_1}$$
$$y(x) \to \sqrt{2}\sqrt[4]{\log(x) + c_1}$$

1.8 problem 8

Internal problem ID [4919]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x' - 3xt^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(x(t),t)=3*x(t)*t^2,x(t), singsol=all)
```

$$x(t) = c_1 e^{t^3}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 18

```
DSolve[x'[t]==3*x[t]*t^2,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 e^{t^3}$$
$$x(t) \to 0$$

1.9 problem 9

Internal problem ID [4920]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x' - \frac{t\,\mathrm{e}^{-t-2x}}{x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve(diff(x(t),t)=t/(x(t)*exp(t+2*x(t))),x(t), singsol=all)
```

$$x(t) = \frac{\operatorname{LambertW}\left(\left(4c_1\mathrm{e}^t - 4t - 4\right)\mathrm{e}^{-t-1}\right)}{2} + \frac{1}{2}$$

✓ Solution by Mathematica

Time used: 60.16 (sec). Leaf size: 31

```
DSolve[x'[t]==t/(x[t]*Exp[t+2*x[t]]),x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{2}\left(1 + W\left(-4e^{-t-1}\left(t - c_1e^t + 1\right)\right)\right)$$

1.10 problem 10

Internal problem ID [4921]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{x}{y^2\sqrt{1+x}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 123

```
dsolve(diff(y(x),x)=x/(y(x)^2*sqrt(1+x)),y(x), singsol=all)
```

$$y(x) = \left(2\sqrt{x+1}\,x - 4\sqrt{x+1} + c_1\right)^{\frac{1}{3}}$$
$$y(x) = -\frac{\left(2\sqrt{x+1}\,x - 4\sqrt{x+1} + c_1\right)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}\left(2\sqrt{x+1}\,x - 4\sqrt{x+1} + c_1\right)^{\frac{1}{3}}}{2}$$
$$y(x) = -\frac{\left(2\sqrt{x+1}\,x - 4\sqrt{x+1} + c_1\right)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}\left(2\sqrt{x+1}\,x - 4\sqrt{x+1} + c_1\right)^{\frac{1}{3}}}{2}$$

✓ Solution by Mathematica

Time used: 2.119 (sec). Leaf size: 110

```
DSolve[y'[x]==x/(y[x]^2*Sqrt[1+x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[3]{2\sqrt{x+1}x - 4\sqrt{x+1} + 3c_1}$$

$$y(x) \to -\sqrt[3]{-1}\sqrt[3]{2\sqrt{x+1}x - 4\sqrt{x+1} + 3c_1}$$

$$y(x) \to (-1)^{2/3}\sqrt[3]{2\sqrt{x+1}x - 4\sqrt{x+1} + 3c_1}$$

1.11 problem 11

Internal problem ID [4922]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xv' - \frac{1-4v^2}{3v} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
dsolve(x*diff(v(x),x)=(1-4*v(x)^2)/(3*v(x)),v(x), singsol=all)
```

$$v(x) = -\frac{\sqrt{x^{\frac{8}{3}}\left(x^{\frac{8}{3}}+4c_1\right)}}{2x^{\frac{8}{3}}}$$
$$v(x) = \frac{\sqrt{x^{\frac{8}{3}}\left(x^{\frac{8}{3}}+4c_1\right)}}{2x^{\frac{8}{3}}}$$

✓ Solution by Mathematica

Time used: 1.93 (sec). Leaf size: 67

```
DSolve[x*v'[x]==(1-4*v[x]^2)/(3*v[x]),v[x],x,IncludeSingularSolutions -> True]
```

$$v(x) \to -\frac{1}{2}\sqrt{1+\frac{e^{8c_1}}{x^{8/3}}}$$
$$v(x) \to \frac{1}{2}\sqrt{1+\frac{e^{8c_1}}{x^{8/3}}}$$
$$v(x) \to -\frac{1}{2}$$
$$v(x) \to \frac{1}{2}$$

1.12 problem 12

Internal problem ID [4923]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{\sec(y)^2}{x^2+1} = 0}$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 81

```
dsolve(diff(y(x),x)=sec(y(x))^2/(1+x^2),y(x), singsol=all)
```

$$y(x) = \frac{\arcsin\left(\mathrm{RootOf}\left(x^4\_Z + \_Z + 2x^2\_Z - x^4\sin(4c_1 - \_Z) + 4x^3\cos(4c_1 - \_Z) + 6x^2\sin(4c_1 - \_Z) - \right.\right.}{2}$$

✓ Solution by Mathematica

Time used: 0.517 (sec). Leaf size: 32

```
DSolve[y'[x]==Sec[y[x]]^2/(1+x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[2\left(\frac{\#1}{2} + \frac{1}{4}\sin(2\#1)\right)\&\right][2\arctan(x) + c_1]$$

1.13 problem 13

Internal problem ID [4924]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 3x^2(y^2+1)^{\frac{3}{2}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)=3*x^2*(1+y(x)^2)^(3/2),y(x), singsol=all)
```

$$c_1 + x^3 - \frac{y(x)}{\sqrt{1+y(x)^2}} = 0$$

✓ Solution by Mathematica

Time used: 0.242 (sec). Leaf size: 83

```
DSolve[y'[x]==3*x^2*(1+y[x]^2)^(3/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{i(x^3+c_1)}{\sqrt{x^6+2c_1x^3-1+c_1{}^2}}$$
$$y(x) \to \frac{i(x^3+c_1)}{\sqrt{x^6+2c_1x^3-1+c_1{}^2}}$$
$$y(x) \to -i$$
$$y(x) \to i$$

1.14 problem 14

Internal problem ID [4925]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - x^3 - x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(x(t),t)-x(t)^3=x(t),x(t), singsol=all)
```

$$x(t) = \frac{1}{\sqrt{\mathrm{e}^{-2t} c_1 - 1}}$$
$$x(t) = -\frac{1}{\sqrt{\mathrm{e}^{-2t} c_1 - 1}}$$

✓ Solution by Mathematica

Time used: 60.064 (sec). Leaf size: 57

```
DSolve[x'[t]-x[t]^3==x[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{i e^{t+c_1}}{\sqrt{-1 + e^{2(t+c_1)}}}$$
$$x(t) \to \frac{i e^{t+c_1}}{\sqrt{-1 + e^{2(t+c_1)}}}$$

1.15 problem 15

Internal problem ID [4926]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy^2 + y\,\mathrm{e}^{x^2} y' = -x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve((x+x*y(x)^2)+exp(x^2)*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{\mathrm{e}^{\mathrm{e}^{-x^2}} c_1 - 1}$$
$$y(x) = -\sqrt{\mathrm{e}^{\mathrm{e}^{-x^2}} c_1 - 1}$$

✓ Solution by Mathematica

Time used: 4.151 (sec). Leaf size: 65

```
DSolve[(x+x*y[x]^2)+Exp[x^2]*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-1 + e^{e^{-x^2} + 2c_1}}$$
$$y(x) \to \sqrt{-1 + e^{e^{-x^2} + 2c_1}}$$
$$y(x) \to -i$$
$$y(x) \to i$$

1.16 problem 16

Internal problem ID [4927]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y\,\mathrm{e}^{\cos(x)}\sin(x) = -\frac{y'}{y}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(1/y(x)*diff(y(x),x)+y(x)*exp(cos(x))*sin(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{\mathrm{e}^{\cos(x)} - c_1}$$

✓ Solution by Mathematica

Time used: 0.293 (sec). Leaf size: 21

```
DSolve[1/y[x]*y'[x]+y[x]*Exp[Cos[x]]*Sin[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{e^{\cos(x)} + c_1}$$
$$y(x) \to 0$$

1.17 problem 17

Internal problem ID [4928]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - (y^2 + 1)\tan(x) = 0}$$

With initial conditions

$$\left[y(0) = \sqrt{3}\right]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)=(1+y(x)^2)*tan(x),y(0) = 3^(1/2)],y(x), singsol=all)
```

$$y(x) = \cot\left(\frac{\pi}{6} + \ln(\cos(x))\right)$$

✓ Solution by Mathematica

Time used: 0.255 (sec). Leaf size: 15

```
DSolve[{y'[x]==(1+y[x]^2)*Tan[x],{y[0]==Sqrt[3]}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cot\left(\log(\cos(x)) + \frac{\pi}{6}\right)$$

1.18 problem 18

Internal problem ID [4929]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - x^3(1 - y) = 0}$$

With initial conditions

$$[y(0) = 3]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)=x^3*(1-y(x)),y(0) = 3],y(x), singsol=all)
```

$$y(x) = 1 + 2\,\mathrm{e}^{-\frac{x^4}{4}}$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 18

```
DSolve[{y'[x]==x^3*(1-y[x]),{y[0]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2e^{-\frac{x^4}{4}} + 1$$

1.19 problem 19

Internal problem ID [4930]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\frac{y'}{2} - \sqrt{1+y}\,\cos(x) = 0}$$

With initial conditions

$$[y(\pi) = 0]$$

✓ Solution by Maple

Time used: 0.391 (sec). Leaf size: 11

```
dsolve([1/2*diff(y(x),x)=sqrt(1+y(x))*cos(x),y(Pi) = 0],y(x), singsol=all)
```

$$y(x) = \sin(x)(\sin(x) + 2)$$

✓ Solution by Mathematica

Time used: 0.152 (sec). Leaf size: 23

```
DSolve[{1/2*y'[x]==Sqrt[1+y[x]]*Cos[x],{y[Pi]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (\sin(x) - 2)\sin(x)$$
$$y(x) \to \sin(x)(\sin(x) + 2)$$

1.20 problem 20

Internal problem ID [4931]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$x^2y' - \frac{4x^2 - x - 2}{(1+y)(1+x)} = 0$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 38

```
dsolve([x^2*diff(y(x),x)=(4*x^2-x-2)/((x+1)*(y(x)+1)),y(1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{-x + \sqrt{2}\,\sqrt{x(3\ln(x+1)x + x\ln(x) - 3\ln(2)x + 2)}}{x}$$

✓ Solution by Mathematica

Time used: 0.291 (sec). Leaf size: 36

```
DSolve[{x^2*y'[x]==(4*x^2-x-2)/((x+1)*(y[x]+1)),{y[1]==1}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{\sqrt{2x\log(x) + 6x\log(x+1) - 6x\log(2) + 4}}{\sqrt{x}} - 1$$

1.21 problem 21

Internal problem ID [4932]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\frac{y'}{\theta} - \frac{y\sin(\theta)}{y^2+1} = 0}$$

With initial conditions

$$[y(\pi) = 1]$$

✓ Solution by Maple

Time used: 0.531 (sec). Leaf size: 35

```
dsolve([1/theta*diff(y(theta),theta)= y(theta)*sin(theta)/(y(theta)^2+1),y(Pi) = 1],y(theta)
```

$$y(\theta) = \frac{e^{-\theta\cos(\theta)+\sin(\theta)+\frac{1}{2}}}{\sqrt{\frac{e^{-2\theta\cos(\theta)+2\sin(\theta)+1}}{\operatorname{LambertW}\left(e^{-2\theta\cos(\theta)-2\pi+2\sin(\theta)+1}\right)}}}$$

✓ Solution by Mathematica

Time used: 3.744 (sec). Leaf size: 26

```
DSolve[{1/\[Theta]*y'[\[Theta]]== y[\[Theta]]*Sin[\[Theta]]/(y[\[Theta]]^2+1),{y[Pi]==1}},y[
```

$$y(\theta) \to \sqrt{W\left(e^{2\sin(\theta)-2\theta\cos(\theta)-2\pi+1}\right)}$$

1.22 problem 22

Internal problem ID [4933]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2yy' = -x^2}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 15

```
dsolve([x^2+2*y(x)*diff(y(x),x)=0,y(0) = 2],y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-3x^3 + 36}}{3}$$

✓ Solution by Mathematica

Time used: 0.091 (sec). Leaf size: 18

```
DSolve[{x^2+2*y[x]*y'[x]==0,{y[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{4 - \frac{x^3}{3}}$$

1.23 problem 23

Internal problem ID [4934]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 2t\cos(y)^2 = 0}$$

With initial conditions

$$\left[y(0) = \frac{\pi}{4}\right]$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 10

```
dsolve([diff(y(t),t)=2*t*cos(y(t))^2,y(0) = 1/4*Pi],y(t), singsol=all)
```

$$y(t) = \arctan(t^2 + 1)$$

✓ Solution by Mathematica

Time used: 0.428 (sec). Leaf size: 11

```
DSolve[{y'[t]==2*t*Cos[y[t]]^2,{y[0]==Pi/4}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \arctan(t^2 + 1)$$

1.24 problem 24

Internal problem ID [4935]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 8x^3 \mathrm{e}^{-2y} = 0}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)=8*x^3*exp(-2*y(x)),y(1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{\ln(4x^4 - 3)}{2}$$

✓ Solution by Mathematica

Time used: 0.346 (sec). Leaf size: 17

```
DSolve[{y'[x]==8*x^3*Exp[-2*y[x]],{y[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} \log(4x^4 - 3)$$

1.25 problem 25

Internal problem ID [4936]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - x^2(1+y) = 0}$$

With initial conditions

$$[y(0) = 3]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)=x^2*(1+y(x)),y(0) = 3],y(x), singsol=all)
```

$$y(x) = -1 + 4\,e^{\frac{x^3}{3}}$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 18

```
DSolve[{y'[x]==x^2*(1+y[x]),{y[0]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 4e^{\frac{x^3}{3}} - 1$$

1.26 problem 26

Internal problem ID [4937]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sqrt{y} + y'(1+x) = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 14

```
dsolve([sqrt(y(x))+(1+x)*diff(y(x),x)=0,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{(\ln(x+1) - 2)^2}{4}$$

✓ Solution by Mathematica

Time used: 0.155 (sec). Leaf size: 33

```
DSolve[{Sqrt[y[x]]+(1+x)*y'[x]==0,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}(\log(x+1) - 2)^2$$
$$y(x) \to \frac{1}{4}(\log(x+1) + 2)^2$$

1.27 problem 27 part(a)

Internal problem ID [4938]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 27 part(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \mathrm{e}^{x^2}}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve([diff(y(x),x)=exp(x^2),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\pi}\,\mathrm{erfi}\,(x)}{2}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 16

```
DSolve[{y'[x]==Exp[x^2],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\sqrt{\pi}\mathrm{erfi}(x)$$

1.28 problem 27 part(b)

Internal problem ID [4939]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 27 part(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{e^{x^2}}{y^2} = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 17

```
dsolve([diff(y(x),x)=exp(x^2)/y(x)^2,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\left(8 + 12\sqrt{\pi}\,\operatorname{erfi}(x)\right)^{\frac{1}{3}}}{2}$$

✓ Solution by Mathematica

Time used: 0.317 (sec). Leaf size: 22

```
DSolve[{y'[x]==Exp[x^2]/y[x]^2,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[3]{\frac{3}{2}\sqrt{\pi}\operatorname{erfi}(x) + 1}$$

1.29 problem 27 part(c)

Internal problem ID [4940]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 27 part(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \sqrt{1 + \sin(x)}\,(y^2 + 1) = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 21

```
dsolve([diff(y(x),x)=sqrt(1+sin(x))*(1+y(x)^2),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \tan\left(\int_0^x \sqrt{1 + \sin(\_z1)}d\_z1 + \frac{\pi}{4}\right)$$

✓ Solution by Mathematica

Time used: 0.305 (sec). Leaf size: 29

```
DSolve[{y'[x]==Sqrt[1+Sin[x]]*(1+y[x]^2),{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan\left(\frac{1}{4}\left(8\sin\left(\frac{x}{2}\right) - 8\cos\left(\frac{x}{2}\right) + \pi + 8\right)\right)$$

1.30 problem 28

Internal problem ID [4941]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 2y + 2ty = 0}$$

With initial conditions

$$[y(0) = 3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve([diff(y(t),t)=2*y(t)-2*t*y(t),y(0) = 3],y(t), singsol=all)
```

$$y(t) = 3\,\mathrm{e}^{-t(t-2)}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 15

```
DSolve[{y'[t]==2*y[t]-2*t*y[t],{y[0]==3}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 3e^{-((t-2)t)}$$

1.31 problem 29 part(a)

Internal problem ID [4942]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 29 part(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^{\frac{1}{3}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)=y(x)^(1/3),y(x), singsol=all)
```

$$y(x)^{\frac{2}{3}} - \frac{2x}{3} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.165 (sec). Leaf size: 29

```
DSolve[y'[x]==y[x]^(1/3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{3}\sqrt{\frac{2}{3}}(x + c_1)^{3/2}$$
$$y(x) \to 0$$

1.32 problem 29 part(b)

Internal problem ID [4943]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 29 part(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^{\frac{1}{3}} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=y(x)^(1/3),y(0) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 21

```
DSolve[{y'[x]==y[x]^(1/3),{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{3}\sqrt{\frac{2}{3}}x^{3/2}$$

1.33 problem 30

Internal problem ID [4944]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - (x-3)(1+y)^{\frac{2}{3}} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)=(x-3)*(y(x)+1)^(2/3),y(x), singsol=all)
```

$$\frac{x^2}{2} - 3x - 3(y(x)+1)^{\frac{1}{3}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.197 (sec). Leaf size: 67

```
DSolve[y'[x]==(x-3)*(y[x]+1)^(2/3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{216}\left(x^6 - 18x^5 + 6(18+c_1)x^4 - 72(3+c_1)x^3 + 12c_1(18+c_1)x^2 - 72{c_1}^2x + 8\left(-27+{c_1}^3\right)\right)$$

$$y(x) \to -1$$

1.34 problem 31 part(a)

Internal problem ID [4945]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 31 part(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - x y^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x)=x*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{-x^2 + c_1}}$$
$$y(x) = -\frac{1}{\sqrt{-x^2 + c_1}}$$

✓ Solution by Mathematica

Time used: 0.146 (sec). Leaf size: 44

```
DSolve[y'[x]==x*y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{-x^2 - 2c_1}}$$
$$y(x) \to \frac{1}{\sqrt{-x^2 - 2c_1}}$$
$$y(x) \to 0$$

1.35 problem 31 part(b.1)

Internal problem ID [4946]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 31 part(b.1).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{y' - xy^3 = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)=x*y(x)^3,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{-x^2 + 1}}$$

✓ Solution by Mathematica

Time used: 0.096 (sec). Leaf size: 16

```
DSolve[{y'[x]==x*y[x]^3,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{\sqrt{1 - x^2}}$$

1.36 problem 31 part(b.2)

Internal problem ID [4947]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 31 part(b.2).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - x y^3 = 0}$$

With initial conditions

$$\left[y(0) = \frac{1}{2}\right]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)=x*y(x)^3,y(0) = 1/2],y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{-x^2 + 4}}$$

✓ Solution by Mathematica

Time used: 0.095 (sec). Leaf size: 16

```
DSolve[{y'[x]==x*y[x]^3,{y[0]==1/2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{\sqrt{4 - x^2}}$$

1.37 problem 31 part(b.3)

Internal problem ID [4948]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 31 part(b.3).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - xy^3 = 0}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 15

```
dsolve([diff(y(x),x)=x*y(x)^3,y(0) = 2],y(x), singsol=all)
```

$$y(x) = \frac{2}{\sqrt{-4x^2 + 1}}$$

✓ Solution by Mathematica

Time used: 0.097 (sec). Leaf size: 18

```
DSolve[{y'[x]==x*y[x]^3,{y[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{\sqrt{1 - 4x^2}}$$

1.38 problem 32

Internal problem ID [4949]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.2, Separable Equations. Exercises. page 46
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{-y^2 + y' + 3y = 2}$$

With initial conditions

$$\left[y(0) = \frac{3}{2}\right]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 15

```
dsolve([diff(y(x),x)=y(x)^2-3*y(x)+2,y(0) = 3/2],y(x), singsol=all)
```

$$y(x) = \frac{e^x + 2}{e^x + 1}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 18

```
DSolve[{y'[x]==y[x]^2-3*y[x]+2,{y[0]==3/2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x + 2}{e^x + 1}$$

2 Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54

2.1 problem 1

Internal problem ID [4950]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x^2y' - y = -\sin(x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 26

```
dsolve(x^2*diff(y(x),x)+sin(x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(\int -\frac{\sin(x)\,\mathrm{e}^{\frac{1}{x}}}{x^2}dx + c_1\right)\mathrm{e}^{-\frac{1}{x}}$$

✓ Solution by Mathematica

Time used: 1.622 (sec). Leaf size: 38

```
DSolve[x^2*y'[x]+Sin[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-1/x}\left(\int_1^x -\frac{e^{\frac{1}{K[1]}}\sin(K[1])}{K[1]^2}dK[1] + c_1\right)$$

2.2 problem 2

Internal problem ID [4951]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{x' + xt - e^{x} = 0}$$

✗ Solution by Maple

```
dsolve(diff(x(t),t)+x(t)*t=exp(x(t)),x(t), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x'[t]+x[t]*t==Exp[x[t]],x[t],t,IncludeSingularSolutions -> True]
```

Not solved

2.3 problem 3

Internal problem ID [4952]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(t^2+1)\,y'-ty+y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((t^2+1)*diff(y(t),t)=y(t)*t-y(t),y(t), singsol=all)
```

$$y(t)=c_1\sqrt{t^2+1}\,\mathrm{e}^{-\arctan(t)}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 28

```
DSolve[(t^2+1)*y'[t]==y[t]*t-y[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to c_1\sqrt{t^2+1}e^{-\arctan(t)}$$
$$y(t)\to 0$$

2.4 problem 4

Internal problem ID [4953]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{-\mathrm{e}^{t}y' - y\ln(t) = -3t}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
dsolve(3*t=exp(t)*diff(y(t),t)+y(t)*ln(t),y(t), singsol=all)
```

$$y(t) = \left(\int 3t^{1-\mathrm{e}^{-t}}\mathrm{e}^{-t-\mathrm{Ei}_1(t)}dt + c_1\right)t^{\mathrm{e}^{-t}}\mathrm{e}^{\mathrm{Ei}_1(t)}$$

✓ Solution by Mathematica

Time used: 0.237 (sec). Leaf size: 58

```
DSolve[3*t==Exp[t]*y'[t]+y[t]*Log[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t^{e^{-t}}e^{-\text{ExpIntegralEi}(-t)}\left(\int_1^t 3e^{\text{ExpIntegralEi}(-K[1])-K[1]}K[1]^{1-e^{-K[1]}}dK[1] + c_1\right)$$

2.5 problem 5

Internal problem ID [4954]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{xx' + xt^2 = \sin(t)}$$

✗ Solution by Maple

```
dsolve(x(t)*diff(x(t),t)+t^2*x(t)=sin(t),x(t), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x[t]*x'[t]+t^2*x[t]==Sin[t],x[t],t,IncludeSingularSolutions -> True]
```

Not solved

2.6 problem 6

Internal problem ID [4955]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{3r - r' = -\theta^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(3*r(theta)=diff(r(theta),theta)-theta^3,r(theta), singsol=all)
```

$$r(\theta) = -\frac{\theta^2}{3} - \frac{\theta^3}{3} - \frac{2\theta}{9} - \frac{2}{27} + e^{3\theta}c_1$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 33

```
DSolve[3*r[\[Theta]]==r'[\[Theta]]-\[Theta]^3,r[\[Theta]],\[Theta],IncludeSingularSolutions
```

$$r(\theta) \to \frac{1}{27}\left(-9\theta^3 - 9\theta^2 - 6\theta - 2\right) + c_1 e^{3\theta}$$

2.7 problem 7

Internal problem ID [4956]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - y = e^{3x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)-y(x)-exp(3*x)=0,y(x), singsol=all)
```

$$y(x) = \left(\frac{e^{2x}}{2} + c_1\right) e^x$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 21

```
DSolve[y'[x]-y[x]-Exp[3*x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{3x}}{2} + c_1 e^x$$

2.8 problem 8

Internal problem ID [4957]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{y}{x} = 1 + 2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)=y(x)/x+2*x+1,y(x), singsol=all)
```

$$y(x) = (2x + \ln(x) + c_1)\, x$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 15

```
DSolve[y'[x]==y[x]/x+2*x+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(2x + \log(x) + c_1)$$

2.9 problem 9

Internal problem ID [4958]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{r' + r\tan(\theta) = \sec(\theta)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(r(theta),theta)+r(theta)*tan(theta)=sec(theta),r(theta), singsol=all)
```

$$r(\theta) = (\tan(\theta) + c_1)\cos(\theta)$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 13

```
DSolve[r'[\[Theta]]+r[\[Theta]]*Tan[\[Theta]]==Sec[\[Theta]],r[\[Theta]],\[Theta],IncludeSin
```

$$r(\theta) \to \sin(\theta) + c_1 \cos(\theta)$$

2.10 problem 10

Internal problem ID [4959]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + 2y = \frac{1}{x^3}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x)+2*y(x)=1/x^3,y(x), singsol=all)
```

$$y(x) = \frac{-\frac{1}{x} + c_1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 15

```
DSolve[x*y'[x]+2*y[x]==1/x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-1 + c_1 x}{x^3}$$

2.11 problem 11

Internal problem ID [4960]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y - y' = -t - 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve((t+y(t)+1)-diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = -t - 2 + c_1 \mathrm{e}^t$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 16

```
DSolve[(t+y[t]+1)-y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -t + c_1 e^t - 2$$

2.12 problem 12

Internal problem ID [4961]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + 4y = x^2 e^{-4x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=x^2*exp(-4*x)-4*y(x),y(x), singsol=all)
```

$$y(x) = \left(\frac{x^3}{3} + c_1\right) e^{-4x}$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 22

```
DSolve[y'[x]==x^2*Exp[-4*x]-4*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} e^{-4x} \left(x^3 + 3c_1\right)$$

2.13 problem 13

Internal problem ID [4962]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class C']]`

$$\boxed{yy' - 5y^3 = -2x}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+2*x=5*y(x)^3,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+2*x==5*y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.14 problem 14

Internal problem ID [4963]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + 3y = -3x^2 + \frac{\sin(x)}{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x*diff(y(x),x)+3*(y(x)+x^2)=sin(x)/x,y(x), singsol=all)
```

$$y(x) = \frac{-x\cos(x) + \sin(x) - \frac{3x^5}{5} + c_1}{x^3}$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 31

```
DSolve[x*y'[x]+3*(y[x]+x^2)==Sin[x]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-3x^5 + 5\sin(x) - 5x\cos(x) + 5c_1}{5x^3}$$

2.15 problem 15

Internal problem ID [4964]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2+1)\,y'+yx=x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2+1)*diff(y(x),x)+x*y(x)-x=0,y(x), singsol=all)
```

$$y(x)=1+\frac{c_1}{\sqrt{x^2+1}}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 24

```
DSolve[(x^2+1)*y'[x]+x*y[x]-x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to 1+\frac{c_1}{\sqrt{x^2+1}}$$
$$y(x)\to 1$$

2.16 problem 16

Internal problem ID [4965]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(-x^2+1)\,y' - yx^2 = (1+x)\sqrt{-x^2+1}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
dsolve((1-x^2)*diff(y(x),x)-x^2*y(x)=(1+x)*sqrt(1-x^2),y(x), singsol=all)
```

$$y(x) = \frac{x+1}{\sqrt{-x^2+1}} + \frac{\mathrm{e}^{-x}\sqrt{x+1}\,c_1}{\sqrt{x-1}}$$

✓ Solution by Mathematica

Time used: 0.085 (sec). Leaf size: 33

```
DSolve[(1-x^2)*y'[x]-x^2*y[x]==(1+x)*Sqrt[1-x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}\sqrt{x+1}(e^x+c_1)}{\sqrt{1-x}}$$

2.17 problem 17

Internal problem ID [4966]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{y}{x} = x\,\mathrm{e}^{x}}$$

With initial conditions

$$[y(1) = \mathrm{e} - 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve([diff(y(x),x)-y(x)/x=x*exp(x),y(1) = -1+exp(1)],y(x), singsol=all)
```

$$y(x) = (\mathrm{e}^{x} - 1)\,x$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 12

```
DSolve[{y'[x]-y[x]/x==x*Exp[x],{y[1]==Exp[1]-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (e^{x} - 1)\,x$$

2.18 problem 18

Internal problem ID [4967]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + 4y = \mathrm{e}^{-x}}$$

With initial conditions

$$\left[y(0) = \frac{4}{3}\right]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve([diff(y(x),x)+4*y(x)-exp(-x)=0,y(0) = 4/3],y(x), singsol=all)
```

$$y(x) = \frac{(\mathrm{e}^{3x} + 3)\,\mathrm{e}^{-4x}}{3}$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 21

```
DSolve[{y'[x]+4*y[x]-Exp[-x]==0,{y[0]==4/3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}e^{-4x}\left(e^{3x} + 3\right)$$

2.19 problem 19

Internal problem ID [4968]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{t^2x' + 3xt = t^4 \ln(t) + 1}$$

With initial conditions

$$[x(1) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve([t^2*diff(x(t),t)+3*t*x(t)=t^4*ln(t)+1,x(1) = 0],x(t), singsol=all)
```

$$x(t) = \frac{6t^6 \ln(t) - t^6 + 18t^2 - 17}{36t^3}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 29

```
DSolve[{t^2*x'[t]+3*t*x[t]==t^4*Log[t]+1,{x[1]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{t^6 - 6t^6 \log(t) - 18t^2 + 17}{36t^3}$$

2.20 problem 20

Internal problem ID [4969]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{3y}{x} = 3x - 2}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve([diff(y(x),x)+3*y(x)/x+2=3*x,y(1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{3x^2}{5} - \frac{x}{2} + \frac{9}{10x^3}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 24

```
DSolve[{y'[x]+3*y[x]/x+2==3*x,{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{6x^5 - 5x^4 + 9}{10x^3}$$

2.21 problem 21

Internal problem ID [4970]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\sin(x)\,y + y'\cos(x) = 2\cos(x)^2 x}$$

With initial conditions

$$\left[y\left(\frac{\pi}{4}\right) = -\frac{15\sqrt{2}\,\pi^2}{32}\right]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve([cos(x)*diff(y(x),x)+y(x)*sin(x)=2*x*cos(x)^2,y(1/4*Pi) = -15/32*2^(1/2)*Pi^2],y(x),
```

$$y(x) = \left(-\pi^2 + x^2\right)\cos(x)$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 17

```
DSolve[{Cos[x]*y'[x]+y[x]*Sin[x]==2*x*Cos[x]^2,{y[Pi/4]==-15*Sqrt[2]*Pi^2/32}},y[x],x,Includ
```

$$y(x) \to \left(x^2 - \pi^2\right)\cos(x)$$

2.22 problem 22

Internal problem ID [4971]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' \sin(x) + \cos(x) y = \sin(x) x}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = 2\right]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve([sin(x)*diff(y(x),x)+y(x)*cos(x)=x*sin(x),y(1/2*Pi) = 2],y(x), singsol=all)
```

$$y(x) = -\cot(x) x + 1 + \csc(x)$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 14

```
DSolve[{Sin[x]*y'[x]+y[x]*Cos[x]==x*Sin[x],{y[Pi/2]==2}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -x\cot(x) + \csc(x) + 1$$

2.23 problem 27

Internal problem ID [4972]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\sqrt{1+\sin(x)^2} = x}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 0.469 (sec). Leaf size: 48

```
dsolve([diff(y(x),x)+y(x)*sqrt(1+sin(x)^2)=x,y(0) = 2],y(x), singsol=all)
```

$$y(x) = \left(\int_0^x \_z1\, \mathrm{e}^{-\mathrm{EllipticE}\left(\cos(\_z1), \frac{\sqrt{2}}{2}\right) \mathrm{csgn}(\sin(\_z1))\sqrt{2}} d\_z1 + 2\right) \mathrm{e}^{\mathrm{csgn}(\sin(x))\, \mathrm{EllipticE}\left(\cos(x), \frac{\sqrt{2}}{2}\right)\sqrt{2}}$$

✓ Solution by Mathematica

Time used: 0.209 (sec). Leaf size: 31

```
DSolve[{y'[x]+y[x]*Sqrt[1+Sin[x]^2]==x,{y[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-E(x|-1)} \left(\int_0^x e^{E(K[1]|-1)} K[1] dK[1] + 2\right)$$

2.24 problem 29

Internal problem ID [4973]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_exponential_symmetries]]`

$$\boxed{\left(\mathrm{e}^{4y}+2x\right)y'=1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
dsolve((exp(4*y(x)) + 2*x)*diff(y(x),x)-1=0,y(x), singsol=all)
```

$$y(x)=\frac{\ln\left(-c_1-\sqrt{c_1^2+2x}\right)}{2}$$
$$y(x)=\frac{\ln\left(-c_1+\sqrt{c_1^2+2x}\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.159 (sec). Leaf size: 113

```
DSolve[(Exp[4*y[x]]+2*x)*y'[x]-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log \left(-\sqrt{-\sqrt{2x + c_1^2} - c_1}\right)$$

$$y(x) \to \frac{1}{2} \log \left(-\sqrt{2x + c_1^2} - c_1\right)$$

$$y(x) \to \log \left(-\sqrt{\sqrt{2x + c_1^2} - c_1}\right)$$

$$y(x) \to \frac{1}{2} \log \left(\sqrt{2x + c_1^2} - c_1\right)$$

2.25 problem 30

Internal problem ID [4974]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{y' + 2y - \frac{x}{y^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 100

```
dsolve(diff(y(x),x)+2*y(x)=x*y(x)^(-2),y(x), singsol=all)
```

$$y(x) = \frac{\left(-18 + 216\,\mathrm{e}^{-6x} c_1 + 108x\right)^{\frac{1}{3}}}{6}$$
$$y(x) = -\frac{\left(-18 + 216\,\mathrm{e}^{-6x} c_1 + 108x\right)^{\frac{1}{3}}}{12} - \frac{i\sqrt{3}\left(-18 + 216\,\mathrm{e}^{-6x} c_1 + 108x\right)^{\frac{1}{3}}}{12}$$
$$y(x) = -\frac{\left(-18 + 216\,\mathrm{e}^{-6x} c_1 + 108x\right)^{\frac{1}{3}}}{12} + \frac{i\sqrt{3}\left(-18 + 216\,\mathrm{e}^{-6x} c_1 + 108x\right)^{\frac{1}{3}}}{12}$$

✓ Solution by Mathematica

Time used: 5.146 (sec). Leaf size: 99

```
DSolve[y'[x]+2*y[x]==x*y[x]^(-2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[3]{-\frac{1}{3}}\sqrt[3]{6x+12c_1e^{-6x}-1}}{2^{2/3}}$$

$$y(x) \to \frac{\sqrt[3]{2x+4c_1e^{-6x}-\frac{1}{3}}}{2^{2/3}}$$

$$y(x) \to \left(-\frac{1}{2}\right)^{2/3}\sqrt[3]{2x+4c_1e^{-6x}-\frac{1}{3}}$$

2.26 problem 36 part(b)

Internal problem ID [4975]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54
Problem number: 36 part(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{3y}{x} = x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+3/x*y(x)=x^2,y(x), singsol=all)
```

$$y(x) = \frac{\frac{x^6}{6} + c_1}{x^3}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 19

```
DSolve[y'[x]+3/x*y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{6} + \frac{c_1}{x^3}$$

2.27 problem 37

Internal problem ID [4976]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$x' + kx = \alpha - \beta \cos\left(\frac{\pi t}{12}\right)$$

With initial conditions

$$[x(0) = x_0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 86

```
dsolve([diff(x(t),t)=alpha-beta*cos(Pi*t/12)-k*x(t),x(0) = x__0],x(t), singsol=all)
```

$$x(t) = \frac{-144\cos\left(\frac{\pi t}{12}\right)\beta k^2 - 12\pi \sin\left(\frac{\pi t}{12}\right)\beta k + \left(144k^3 x_0 + 144(\beta-\alpha)k^2 + \pi^2 k x_0 - \pi^2\alpha\right)e^{-kt} + 144\alpha k^2 + \pi^2\alpha}{\pi^2 k + 144k^3}$$

✓ Solution by Mathematica

Time used: 0.291 (sec). Leaf size: 64

```
DSolve[{x'[t]==\[Alpha]-\[Beta]*Cos[Pi*t/12]-k*x[t],{}},x[t],t,IncludeSingularSolutions -> T
```

$$x(t) \to -\frac{12\pi\beta \sin\left(\frac{\pi t}{12}\right)}{144k^2+\pi^2} - \frac{144\beta k \cos\left(\frac{\pi t}{12}\right)}{144k^2+\pi^2} + \frac{\alpha}{k} + c_1 e^{-kt}$$

2.28 problem 40

Internal problem ID [4977]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.3, Linear equations. Exercises. page 54
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{u' - \alpha(1-u) + \beta u = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(u(t),t)=alpha*(1-u(t))-beta*u(t),u(t), singsol=all)
```

$$u(t) = \frac{\alpha}{\alpha+\beta} + \mathrm{e}^{-(\alpha+\beta)t} c_1$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 35

```
DSolve[u'[t]==\[Alpha]*(1-u[t])-\[Beta]*u[t],u[t],t,IncludeSingularSolutions -> True]
```

$$u(t) \to \frac{\alpha}{\alpha+\beta} + c_1 e^{-t(\alpha+\beta)}$$
$$u(t) \to \frac{\alpha}{\alpha+\beta}$$

3 Chapter 2, First order differential equations. Section 2.4, Exact equations. Exercises. page 64

3.1 problem 1

Internal problem ID [4978]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.4, Exact equations. Exercises. page 64
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{yx^2 - y'x^3 = -x^4 \cos(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve((x^2*y(x)+x^4*cos(x))-x^3*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = (\sin(x) + c_1)\, x$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 12

```
DSolve[(x^2*y[x]+x^4*Cos[x])-x^3*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(\sin(x) + c_1)$$

3.2 problem 2

Internal problem ID [4979]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.4, Exact equations. Exercises. page 64
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - 2y = -x^{\frac{10}{3}}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^(10/3)-2*y(x))+x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \left(-\frac{3x^{\frac{4}{3}}}{4} + c_1\right)x^2$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 21

```
DSolve[(x^(10/3)-2*y[x])+x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{3x^{10/3}}{4} + c_1x^2$$

3.3 problem 3

Internal problem ID [4980]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.4, Exact equations. Exercises. page 64
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sqrt{-2y-y^2}+\left(-x^2+2x+3\right)y'=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(sqrt(-2*y(x)-y(x)^2)+(3+2*x-x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=-1+\sin\left(\frac{\ln\left(x-3\right)}{4}-\frac{\ln\left(x+1\right)}{4}+c_1\right)$$

✓ Solution by Mathematica

Time used: 60.208 (sec). Leaf size: 385

```
DSolve[Sqrt[-2*y[x]-y[x]^2]+(3+2*x-x^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -1 - \frac{1}{4}\sqrt{8 - e^{-4ic_1}\left(-x^2+2x+3\right)^{-i}\sqrt{e^{4ic_1}\left(-x^2+2x+3\right)^{i}\left((x+1)^i+16e^{4ic_1}(3-x)^i\right)^2}}$$

$$y(x) \to \frac{1}{4}\left(-4 + \sqrt{8 - e^{-4ic_1}\left(-x^2+2x+3\right)^{-i}\sqrt{e^{4ic_1}\left(-x^2+2x+3\right)^{i}\left((x+1)^i+16e^{4ic_1}(3-x)^i\right)^2}}\right)$$

$$y(x) \to \frac{1}{4}\left(-4 - \sqrt{8 + e^{-4ic_1}\left(-x^2+2x+3\right)^{-i}\sqrt{e^{4ic_1}\left(-x^2+2x+3\right)^{i}\left((x+1)^i+16e^{4ic_1}(3-x)^i\right)^2}}\right)$$

$$y(x) \to \frac{1}{4}\left(-4 + \sqrt{8 + e^{-4ic_1}\left(-x^2+2x+3\right)^{-i}\sqrt{e^{4ic_1}\left(-x^2+2x+3\right)^{i}\left((x+1)^i+16e^{4ic_1}(3-x)^i\right)^2}}\right)$$

3.4 problem 4

Internal problem ID [4981]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.4, Exact equations. Exercises. page 64
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{y\,\mathrm{e}^{yx} + (x\,\mathrm{e}^{yx} - 2y)\,y' = -2x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve((y(x)*exp(x*y(x))+2*x)+(x*exp(x*y(x))-2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$\mathrm{e}^{y(x)x} + x^2 - y(x)^2 + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.268 (sec). Leaf size: 22

```
DSolve[(y[x]*Exp[x*y[x]]+2*x)+(x*Exp[x*y[x]]-2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolution
```

$$\text{Solve}\left[x^2 + e^{xy(x)} - y(x)^2 = c_1, y(x)\right]$$

3.5 problem 5

Internal problem ID [4982]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.4, Exact equations. Exercises. page 64
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yx + y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x*y(x)+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 22

```
DSolve[x*y[x]+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\frac{x^2}{2}}$$
$$y(x) \to 0$$

3.6 problem 6

Internal problem ID [4983]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.4, Exact equations. Exercises. page 64
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{y^2 + (2yx + \cos(y))\, y' = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 18

```
dsolve(y(x)^2+(2*x*y(x)+cos(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$x - \frac{-\sin(y(x)) + c_1}{y(x)^2} = 0$$

✓ Solution by Mathematica

Time used: 0.151 (sec). Leaf size: 22

```
DSolve[y[x]^2+(2*x*y[x]+Cos[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x = -\frac{\sin(y(x))}{y(x)^2} + \frac{c_1}{y(x)^2}, y(x)\right]$$

3.7 problem 7

Internal problem ID [4984]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.4, Exact equations. Exercises. page 64
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_exact]**

$$\boxed{y\cos(yx) + (x\cos(yx) - 2y)\, y' = -2x}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 29

```
dsolve((2*x+y(x)*cos(x*y(x)))+(x*cos(x*y(x))-2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{RootOf}\left(x^4 + x^2\sin(\_Z) + c_1x^2 - \_Z^2\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.186 (sec). Leaf size: 21

```
DSolve[(2*x+y[x]*Cos[x*y[x]])+(x*Cos[x*y[x]]-2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolution
```

$$\mathrm{Solve}\left[x^2 - y(x)^2 + \sin(xy(x)) = c_1, y(x)\right]$$

3.8 problem 8

Internal problem ID [4985]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.4, Exact equations. Exercises. page 64
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\theta r' + 3r = \theta + 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(theta*diff(r(theta),theta)+(3*r(theta)-theta-1)=0,r(theta), singsol=all)
```

$$r(\theta) = \frac{\theta}{4} + \frac{1}{3} + \frac{c_1}{\theta^3}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 20

```
DSolve[\[Theta]*r'[\[Theta]]+(3*r[\[Theta]]-\[Theta]-1)==0,r[\[Theta]],\[Theta],IncludeSingu
```

$$r(\theta) \to \frac{c_1}{\theta^3} + \frac{\theta}{4} + \frac{1}{3}$$

3.9 problem 9

Internal problem ID [4986]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.4, Exact equations. Exercises. page 64
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2yx + (x^2 - 1)\, y' = -3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve((2*x*y(x)+3)+(x^2-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 - 3x}{(x-1)(x+1)}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 19

```
DSolve[(2*x*y[x]+3)+(x^2-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-3x + c_1}{x^2 - 1}$$

3.10 problem 10

Internal problem ID [4987]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.4, Exact equations. Exercises. page 64
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{y + (x - 2y)\, y' = -2x}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 53

```
dsolve((2*x+y(x))+(x-2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\frac{c_1 x}{2} - \frac{\sqrt{5c_1^2 x^2 + 4}}{2}}{c_1}$$
$$y(x) = \frac{\frac{c_1 x}{2} + \frac{\sqrt{5c_1^2 x^2 + 4}}{2}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.452 (sec). Leaf size: 102

```
DSolve[(2*x+y[x])+(x-2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(x - \sqrt{5x^2 - 4e^{c_1}}\right)$$
$$y(x) \to \frac{1}{2}\left(x + \sqrt{5x^2 - 4e^{c_1}}\right)$$
$$y(x) \to \frac{1}{2}\left(x - \sqrt{5}\sqrt{x^2}\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{5}\sqrt{x^2} + x\right)$$

3.11 problem 11

Internal problem ID [4988]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.4, Exact equations. Exercises. page 64
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$e^x \sin(y) + \left(e^x \cos(y) + \frac{1}{3y^{\frac{2}{3}}}\right) y' = 3x^2$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve((exp(x)*sin(y(x))-3*x^2)+(exp(x)*cos(y(x))+y(x)^(-2/3)/3)*diff(y(x),x)=0,y(x), singsol
```

$$e^x \sin(y(x)) - x^3 + y(x)^{\frac{1}{3}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.416 (sec). Leaf size: 28

```
DSolve[(Exp[x]*Sin[y[x]]-3*x^2)+(Exp[x]*Cos[y[x]]+y[x]^(-2/3)/3)*y'[x]==0,y[x],x,IncludeSing
```

$$\text{Solve}\left[-3x^3 + 3\sqrt[3]{y(x)} + 3e^x \sin(y(x)) = c_1, y(x)\right]$$

3.12 problem 12

Internal problem ID [4989]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.4, Exact equations. Exercises. page 64
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{\cos(x)\cos(y) - (\sin(x)\sin(y) + 2y)\, y' = -2x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve((cos(x)*cos(y(x))+2*x)-(sin(x)*sin(y(x))+2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$\sin(x)\cos(y(x)) + x^2 - y(x)^2 + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.289 (sec). Leaf size: 25

```
DSolve[(Cos[x]*Cos[y[x]]+2*x)-(Sin[x]*Sin[y[x]]+2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolut
```

$$\text{Solve}\left[-2x^2 + 2y(x)^2 - 2\sin(x)\cos(y(x)) = c_1, y(x)\right]$$

3.13 problem 13

Internal problem ID [4990]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.4, Exact equations. Exercises. page 64
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{e^t(y-t)+\left(1+e^t\right)y'=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(exp(t)*(y(t)-t)+(1+exp(t))*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t)=\frac{(t-1)\,e^t+c_1}{1+e^t}$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 23

```
DSolve[Exp[t]*(y[t]-t)+(1+Exp[t])*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to\frac{e^t(t-1)+c_1}{e^t+1}$$

3.14 problem 14

Internal problem ID [4991]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.4, Exact equations. Exercises. page 64
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\ln(y) = -\frac{ty'}{y} - 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((t/y(t))*diff(y(t),t)+(1+ln(y(t)))=0,y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-\frac{c_1 t-1}{t c_1}}$$

✓ Solution by Mathematica

Time used: 0.256 (sec). Leaf size: 24

```
DSolve[(t/y[t])*y'[t]+(1+Log[y[t]])==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-1+\frac{e^{c_1}}{t}}$$
$$y(t) \to \frac{1}{e}$$

3.15 problem 15

Internal problem ID [4992]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.4, Exact equations. Exercises. page 64
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\cos(\theta) r' - \sin(\theta) r = -e^{\theta}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(cos(theta)*diff(r(theta),theta)-(r(theta)*sin(theta)-exp(theta))=0,r(theta), singsol=
```

$$r(\theta) = \frac{-e^{\theta} + c_1}{\cos(\theta)}$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 16

```
DSolve[Cos[\[Theta]]*r'[\[Theta]]-(r[\[Theta]]*Sin[\[Theta]]-Exp[\[Theta]])==0,r[\[Theta]],\
```

$$r(\theta) \to \left(-e^{\theta} + c_1\right) \sec(\theta)$$

3.16 problem 16

Internal problem ID [4993]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.4, Exact equations. Exercises. page 64
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{y\,e^{yx}-\frac{1}{y}+\left(x\,e^{yx}+\frac{x}{y^2}\right)y'=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve((y(x)*exp(x*y(x))-1/y(x))+(x*exp(x*y(x))+x/y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$e^{xy(x)}-\frac{x}{y(x)}+c_1=0$$

✓ Solution by Mathematica

Time used: 0.2 (sec). Leaf size: 20

```
DSolve[(y[x]*Exp[x*y[x]]-1/y[x])+(x*Exp[x*y[x]]+x/y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSol
```

$$\text{Solve}\left[e^{xy(x)}-\frac{x}{y(x)}=c_1,y(x)\right]$$

3.17 problem 17

Internal problem ID [4994]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.4, Exact equations. Exercises. page 64
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{\frac{1}{y} - \left(3y - \frac{x}{y^2}\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve(1/y(x)-(3*y(x)-x/y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$-\frac{c_1}{y(x)} + x - \frac{3y(x)^3}{4} = 0$$

✓ Solution by Mathematica

Time used: 32.895 (sec). Leaf size: 870

```
DSolve[1/y[x]-(3*y[x]-x/y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{4c_1}{\sqrt[3]{3x^2-\sqrt{9x^4-64{c_1}^3}}}+\sqrt[3]{3x^2-\sqrt{9x^4-64{c_1}^3}}+\sqrt{-\frac{2\sqrt{6}x}{\sqrt{\frac{4c_1}{\sqrt[3]{3x^2-\sqrt{9x^4-64{c_1}^3}}}+\sqrt[3]{3x^2-\sqrt{9x^4 \cdots}}}}}}{\sqrt{6}}$$

$$y(x) \to \frac{\sqrt{-\frac{2\sqrt{6}x}{\sqrt{\frac{4c_1}{\sqrt[3]{3x^2-\sqrt{9x^4-64{c_1}^3}}}+\sqrt[3]{3x^2-\sqrt{9x^4-64{c_1}^3}}}}-\sqrt[3]{3x^2-\sqrt{9x^4-64{c_1}^3}}-\frac{4c_1}{\sqrt[3]{3x^2-\sqrt{9x^4-6 \cdots}}}}}{\sqrt{6}}$$

$$y(x) \to \frac{\sqrt{\frac{4c_1}{\sqrt[3]{3x^2-\sqrt{9x^4-64{c_1}^3}}}+\sqrt[3]{3x^2-\sqrt{9x^4-64{c_1}^3}}-\sqrt{\frac{2\sqrt{6}x}{\sqrt{\frac{4c_1}{\sqrt[3]{3x^2-\sqrt{9x^4-64{c_1}^3}}}+\sqrt[3]{3x^2-\sqrt{9x^4-\cdots}}}}}}}{\sqrt{6}}$$

$$y(x) \to \frac{\sqrt{\frac{4c_1}{\sqrt[3]{3x^2-\sqrt{9x^4-64{c_1}^3}}}+\sqrt[3]{3x^2-\sqrt{9x^4-64{c_1}^3}}+\sqrt{\frac{2\sqrt{6}x}{\sqrt{\frac{4c_1}{\sqrt[3]{3x^2-\sqrt{9x^4-64{c_1}^3}}}+\sqrt[3]{3x^2-\sqrt{9x^4-\cdots}}}}}}}{\sqrt{6}}$$

3.18 problem 18

Internal problem ID [4995]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Section 2.4, Exact equations. Exercises. page 64
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y^2 - \cos(x+y) - (2yx - \cos(x+y) - e^y)\, y' = -2x}$$

✗ Solution by Maple

```
dsolve((2*x+y(x)^2-cos(x+y(x)))-(2*x*y(x)-cos(x+y(x))-exp(y(x)))*diff(y(x),x)=0,y(x), singso
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(2*x+y[x]^2-Cos[x+y[x]])-(2*x*y[x]-Cos[x+y[x]]-Exp[y[x]])*y'[x]==0,y[x],x,IncludeSing
```

Not solved

4 Chapter 2, First order differential equations. Review problems. page 79

4.1 problem 1

Internal problem ID [4996]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Review problems. page 79
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' - \frac{e^{x+y}}{-1+y} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)=exp(x+y(x))/(y(x)-1),y(x), singsol=all)
```

$$y(x) = -\operatorname{LambertW}\left(c_1 + e^x\right)$$

✓ Solution by Mathematica

Time used: 60.144 (sec). Leaf size: 14

```
DSolve[y'[x]==Exp[x+y[x]]/(y[x]-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -W(e^x + c_1)$$

4.2 problem 2

Internal problem ID [4997]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Review problems. page 79
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{-4y + y' = 32x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)-4*y(x)=32*x^2,y(x), singsol=all)
```

$$y(x) = -8x^2 - 4x - 1 + e^{4x}c_1$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 23

```
DSolve[y'[x]-4*y[x]==32*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -8x^2 - 4x + c_1e^{4x} - 1$$

4.3 problem 3

Internal problem ID [4998]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Review problems. page 79
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{\left(x^2 - \frac{2}{y^3}\right) y' + 2yx = 3x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 878

```
dsolve((x^2-2*y(x)^(-3))*diff(y(x),x)+(2*x*y(x)-3*x^2)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(8x^9 - 24c_1x^6 + 24c_1^2x^3 + 12\sqrt{3}\,\sqrt{-4x^9 + 12c_1x^6 - 12c_1^2x^3 + 27x^4 + 4c_1^3}\,x^2 - 108x^4 - 8c_1^3\right)^{\frac{1}{3}}}{6x^2} + \frac{2(-x^3 + c_1)^2}{3x^2\left(8x^9 - 24c_1x^6 + 24c_1^2x^3 + 12\sqrt{3}\,\sqrt{-4x^9 + 12c_1x^6 - 12c_1^2x^3 + 27x^4 + 4c_1^3}\,x^2 - 108x^4 - 8c_1^3\right)^{\frac{1}{3}}} - \frac{-x^3 + c_1}{3x^2}$$

$$y(x) = -\frac{\left(8x^9 - 24c_1x^6 + 24c_1^2x^3 + 12\sqrt{3}\,\sqrt{-4x^9 + 12c_1x^6 - 12c_1^2x^3 + 27x^4 + 4c_1^3}\,x^2 - 108x^4 - 8c_1^3\right)^{\frac{1}{3}}}{12x^2} - \frac{(-x^3 + c_1)^2}{3x^2\left(8x^9 - 24c_1x^6 + 24c_1^2x^3 + 12\sqrt{3}\,\sqrt{-4x^9 + 12c_1x^6 - 12c_1^2x^3 + 27x^4 + 4c_1^3}\,x^2 - 108x^4 - 8c_1^3\right)^{\frac{1}{3}}} - \frac{-x^3 + c_1}{3x^2} - \frac{i\sqrt{3}\left(\frac{\left(8x^9-24c_1x^6+24c_1^2x^3+12\sqrt{3}\,\sqrt{-4x^9+12c_1x^6-12c_1^2x^3+27x^4+4c_1^3}\,x^2-108x^4-8c_1^3\right)^{\frac{1}{3}}}{6x^2} - \frac{}{3x^2\left(8x^9-24c_1x^6+24c_1^2x^3+12\sqrt{3}\,\sqrt{-\right.}\right.}{2}$$

$$y(x) = -\frac{\left(8x^9 - 24c_1x^6 + 24c_1^2x^3 + 12\sqrt{3}\,\sqrt{-4x^9 + 12c_1x^6 - 12c_1^2x^3 + 27x^4 + 4c_1^3}\,x^2 - 108x^4 - 8c_1^3\right)^{\frac{1}{3}}}{12x^2} - \frac{(-x^3 + c_1)^2}{3x^2\left(8x^9 - 24c_1x^6 + 24c_1^2x^3 + 12\sqrt{3}\,\sqrt{-4x^9 + 12c_1x^6 - 12c_1^2x^3 + 27x^4 + 4c_1^3}\,x^2 - 108x^4 - 8c_1^3\right)^{\frac{1}{3}}} - \frac{-x^3 + c_1}{3x^2} + \frac{i\sqrt{3}\left(\frac{\left(8x^9-24c_1x^6+24c_1^2x^3+12\sqrt{3}\,\sqrt{-4x^9+12c_1x^6-12c_1^2x^3+27x^4+4c_1^3}\,x^2-108x^4-8c_1^3\right)^{\frac{1}{3}}}{6x^2} - \frac{}{3x^2\left(8x^9-24c_1x^6+24c_1^2x^3+12\sqrt{3}\,\sqrt{-\right.}\right.}{2}$$

✓ Solution by Mathematica

Time used: 13.843 (sec). Leaf size: 676

```
DSolve[(x^2-2*y[x]^(-3))*y'[x]+(2*x*y[x]-3*x^2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$y(x)$

$$\rightarrow \frac{2(x^3+c_1)+\frac{2(x^3+c_1)^2}{\sqrt[3]{x^9+3c_1x^6-\frac{27x^4}{2}+3c_1{}^2x^3+\frac{3}{2}\sqrt{3}\sqrt{-x^4\left(4x^9+12c_1x^6-27x^4+12c_1{}^2x^3+4c_1{}^3\right)}+c}}}{6x}$$

$y(x)$

$$\rightarrow \frac{4(x^3+c_1)-\frac{2i\left(\sqrt{3}-i\right)(x^3+c_1)^2}{\sqrt[3]{x^9+3c_1x^6-\frac{27x^4}{2}+3c_1{}^2x^3+\frac{3}{2}\sqrt{3}\sqrt{-x^4\left(4x^9+12c_1x^6-27x^4+12c_1{}^2x^3+4c_1{}^3\right)}+c}}}{}$$

$y(x)$

$$\rightarrow \frac{4(x^3+c_1)+\frac{2i\left(\sqrt{3}+i\right)(x^3+c_1)^2}{\sqrt[3]{x^9+3c_1x^6-\frac{27x^4}{2}+3c_1{}^2x^3+\frac{3}{2}\sqrt{3}\sqrt{-x^4\left(4x^9+12c_1x^6-27x^4+12c_1{}^2x^3+4c_1{}^3\right)}+c}}}{}$$

$y(x) \rightarrow 0$

4.4 problem 4

Internal problem ID [4999]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Review problems. page 79
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' + \frac{3y}{x} = x^2 - 4x + 3$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)+3*y(x)/x=x^2-4*x+3,y(x), singsol=all)
```

$$y(x) = \frac{x^3}{6} - \frac{4x^2}{5} + \frac{3x}{4} + \frac{c_1}{x^3}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 31

```
DSolve[y'[x]+3*y[x]/x==x^2-4*x+3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{6} + \frac{c_1}{x^3} - \frac{4x^2}{5} + \frac{3x}{4}$$

4.5 problem 6

Internal problem ID [5000]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Review problems. page 79
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2xy^3 - \left(-x^2+1\right)y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
dsolve(2*x*y(x)^3-(1-x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{c_1 + 2\ln(x-1) + 2\ln(x+1)}}$$
$$y(x) = -\frac{1}{\sqrt{c_1 + 2\ln(x-1) + 2\ln(x+1)}}$$

✓ Solution by Mathematica

Time used: 0.202 (sec). Leaf size: 57

```
DSolve[2*x*y[x]^3-(1-x^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{2}\sqrt{\log(x^2-1) - c_1}}$$
$$y(x) \to \frac{1}{\sqrt{2}\sqrt{\log(x^2-1) - c_1}}$$
$$y(x) \to 0$$

4.6 problem 7

Internal problem ID [5001]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 2, First order differential equations. Review problems. page 79
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{t^3y^2 + \frac{t^4y'}{y^6} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 164

```
dsolve(t^3*y(t)^2+t^4/(y(t)^6)*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \frac{1}{(c_1 + 7\ln(t))^{\frac{1}{7}}}$$

$$y(t) = \frac{-\cos\left(\frac{\pi}{7}\right) - i\cos\left(\frac{5\pi}{14}\right)}{(c_1 + 7\ln(t))^{\frac{1}{7}}}$$

$$y(t) = \frac{-\cos\left(\frac{\pi}{7}\right) + i\cos\left(\frac{5\pi}{14}\right)}{(c_1 + 7\ln(t))^{\frac{1}{7}}}$$

$$y(t) = \frac{\cos\left(\frac{2\pi}{7}\right) - i\cos\left(\frac{3\pi}{14}\right)}{(c_1 + 7\ln(t))^{\frac{1}{7}}}$$

$$y(t) = \frac{\cos\left(\frac{2\pi}{7}\right) + i\cos\left(\frac{3\pi}{14}\right)}{(c_1 + 7\ln(t))^{\frac{1}{7}}}$$

$$y(t) = \frac{-\cos\left(\frac{3\pi}{7}\right) - i\cos\left(\frac{\pi}{14}\right)}{(c_1 + 7\ln(t))^{\frac{1}{7}}}$$

$$y(t) = \frac{-\cos\left(\frac{3\pi}{7}\right) + i\cos\left(\frac{\pi}{14}\right)}{(c_1 + 7\ln(t))^{\frac{1}{7}}}$$

✓ Solution by Mathematica

Time used: 0.182 (sec). Leaf size: 183

```
DSolve[t^3*y[t]^2+t^4/(y[t]^6)*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{\sqrt[7]{-\frac{1}{7}}}{\sqrt[7]{\log(t) - c_1}}$$

$$y(t) \to \frac{1}{\sqrt[7]{7}\sqrt[7]{\log(t) - c_1}}$$

$$y(t) \to \frac{(-1)^{2/7}}{\sqrt[7]{7}\sqrt[7]{\log(t) - c_1}}$$

$$y(t) \to -\frac{(-1)^{3/7}}{\sqrt[7]{7}\sqrt[7]{\log(t) - c_1}}$$

$$y(t) \to \frac{(-1)^{4/7}}{\sqrt[7]{7}\sqrt[7]{\log(t) - c_1}}$$

$$y(t) \to -\frac{(-1)^{5/7}}{\sqrt[7]{7}\sqrt[7]{\log(t) - c_1}}$$

$$y(t) \to \frac{(-1)^{6/7}}{\sqrt[7]{7}\sqrt[7]{\log(t) - c_1}}$$

$$y(t) \to 0$$

5 Chapter 8, Series solutions of differential equations. Section 8.3. page 443

5.1 problem 1

Internal problem ID [5002]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.3. page 443
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1+x)\,y''-x^2y'+3y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

```
Order:=6;
dsolve((x+1)*diff(y(x),x$2)-x^2*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{3}{2}x^2+\frac{1}{2}x^3+\frac{1}{8}x^4-\frac{3}{10}x^5\right)y(0)+\left(x-\frac{1}{2}x^3+\frac{1}{3}x^4-\frac{1}{8}x^5\right)D(y)\,(0)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[(x+1)*y''[x]-x^2*y'[x]+3*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\rightarrow c_2\left(-\frac{x^5}{8}+\frac{x^4}{3}-\frac{x^3}{2}+x\right)+c_1\left(-\frac{3x^5}{10}+\frac{x^4}{8}+\frac{x^3}{2}-\frac{3x^2}{2}+1\right)$$

5.2 problem 2

Internal problem ID [5003]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.3. page 443
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 3y' - yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+3*diff(y(x),x)-x*y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 85

```
AsymptoticDSolveValue[x^2*y''[x]+3*y'[x]-x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 e^{3/x}\left(\frac{3001x^5}{1620} + \frac{613x^4}{648} + \frac{16x^3}{27} + \frac{x^2}{2} + \frac{2x}{3} + 1\right)x^2 + c_1\left(-\frac{23x^5}{810} + \frac{7x^4}{216} - \frac{x^3}{27} + \frac{x^2}{6} + 1\right)$$

5.3 problem 3

Internal problem ID [5004]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.3. page 443
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-2)\,y''+2y'+\sin(x)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve((x^2-2)*diff(y(x),x$2)+2*diff(y(x),x)+sin(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+\frac{1}{12}x^3+\frac{1}{48}x^4+\frac{1}{80}x^5\right)y(0)+\left(x+\frac{1}{2}x^2+\frac{1}{6}x^3+\frac{1}{8}x^4+\frac{1}{16}x^5\right)D(y)(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[(x^2-2)*y''[x]+2*y'[x]+Sin[x]*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{x^5}{80}+\frac{x^4}{48}+\frac{x^3}{12}+1\right)+c_2\left(\frac{x^5}{16}+\frac{x^4}{8}+\frac{x^3}{6}+\frac{x^2}{2}+x\right)$$

5.4 problem 4

Internal problem ID [5005]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.3. page 443
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{(x^2+x)\,y''+3y'-6yx=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 60

```
Order:=6;
dsolve((x^2+x)*diff(y(x),x$2)+3*diff(y(x),x)-6*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1+\frac{3}{4}x^2-\frac{1}{10}x^3+\frac{17}{80}x^4-\frac{9}{100}x^5+\mathrm{O}\left(x^6\right)\right)x^2+c_2\left(\ln\left(x\right)\left(6x^2+\frac{9}{2}x^4-\frac{3}{5}x^5+\mathrm{O}\left(x^6\right)\right)+\left(-2-12x-\right.\right.}{x^2}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 73

```
AsymptoticDSolveValue[(x^2+x)*y''[x]+2*y'[x]-6*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{7x^4}{20}-\frac{x^3}{6}+x^2+1\right)+c_1\left(\frac{1}{3}\left(x^3-6x^2-6\right)\log(x)+\frac{7x^4+240x^3+72x^2+180x+36}{36x}\right)$$

5.5 problem 5

Internal problem ID [5006]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.3. page 443
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(t^2 - t - 2)\, x'' + (t + 1)\, x' - (-2 + t)\, x = 0}$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve((t^2-t-2)*diff(x(t),t$2)+(t+1)*diff(x(t),t)-(t-2)*x(t)=0,x(t),type='series',t=0);
```

$$x(t) = \left(1 + \frac{1}{2}t^2 - \frac{1}{12}t^3 + \frac{13}{96}t^4 - \frac{1}{16}t^5\right) x(0) + \left(t + \frac{1}{4}t^2 + \frac{1}{4}t^3 - \frac{1}{96}t^4 + \frac{31}{480}t^5\right) D(x)\,(0) + O(t^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 70

```
AsymptoticDSolveValue[(t^2-t-2)*x''[t]+(t+1)*x'[t]-(t-2)*x[t]==0,x[t],{t,0,5}]
```

$$x(t) \to c_1\left(-\frac{t^5}{16} + \frac{13t^4}{96} - \frac{t^3}{12} + \frac{t^2}{2} + 1\right) + c_2\left(\frac{31t^5}{480} - \frac{t^4}{96} + \frac{t^3}{4} + \frac{t^2}{4} + t\right)$$

5.6 problem 6

Internal problem ID [5007]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.3. page 443
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2-1)\,y''+(1-x)\,y'+(x^2-2x+1)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
Order:=6;
dsolve((x^2-1)*diff(y(x),x$2)+(1-x)*diff(y(x),x)+(x^2-2*x+1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+\frac{1}{2}x^2-\frac{1}{6}x^3+\frac{1}{12}x^4-\frac{1}{15}x^5\right)y(0)+\left(x+\frac{1}{2}x^2+\frac{1}{6}x^3-\frac{1}{12}x^4+\frac{1}{60}x^5\right)D(y)\,(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 70

```
AsymptoticDSolveValue[(x^2-1)*y''[x]+(1-x)*y'[x]+(x^2-2*x+1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(-\frac{x^5}{15}+\frac{x^4}{12}-\frac{x^3}{6}+\frac{x^2}{2}+1\right)+c_2\left(\frac{x^5}{60}-\frac{x^4}{12}+\frac{x^3}{6}+\frac{x^2}{2}+x\right)$$

5.7 problem 7

Internal problem ID [5008]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.3. page 443
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\sin(x)\, y'' + \cos(x)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 58

```
Order:=6;
dsolve(sin(x)*diff(y(x),x$2)+cos(x)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = {} & c_1 x\left(1 - \frac{1}{2}x + \frac{1}{12}x^2 + \frac{1}{48}x^3 - \frac{3}{320}x^4 + \frac{19}{9600}x^5 + \mathrm{O}\left(x^6\right)\right) \\ & + c_2\left(\ln(x)\left(-x + \frac{1}{2}x^2 - \frac{1}{12}x^3 - \frac{1}{48}x^4 + \frac{3}{320}x^5 + \mathrm{O}\left(x^6\right)\right)\right. \\ & \left. + \left(1 - \frac{3}{4}x^2 + \frac{1}{4}x^3 - \frac{5}{576}x^4 - \frac{437}{28800}x^5 + \mathrm{O}\left(x^6\right)\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 85

```
AsymptoticDSolveValue[Sin[x]*y''[x]+Cos[x]*y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned} y(x) \to {} & c_1\left(\frac{1}{576}\left(7x^4 + 192x^3 - 720x^2 + 576x + 576\right) - \frac{1}{48}x\left(x^3 + 4x^2 - 24x + 48\right)\log(x)\right) \\ & + c_2\left(-\frac{3x^5}{320} + \frac{x^4}{48} + \frac{x^3}{12} - \frac{x^2}{2} + x\right) \end{aligned}$$

5.8 problem 8

Internal problem ID [5009]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.3. page 443
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{e^{x}y'' - \left(x^{2}-1\right)y' + 2yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve(exp(x)*diff(y(x),x$2)-(x^2-1)*diff(y(x),x)+2*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1-\frac{1}{3}x^{3}+\frac{1}{4}x^{4}-\frac{3}{20}x^{5}\right)y(0)+\left(x-\frac{1}{2}x^{2}+\frac{1}{3}x^{3}-\frac{7}{24}x^{4}+\frac{23}{120}x^{5}\right)D(y)(0)+O\left(x^{6}\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 63

```
AsymptoticDSolveValue[Exp[x]*y''[x]-(x^2-1)*y'[x]+2*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{3x^{5}}{20}+\frac{x^{4}}{4}-\frac{x^{3}}{3}+1\right)+c_2\left(\frac{23x^{5}}{120}-\frac{7x^{4}}{24}+\frac{x^{3}}{3}-\frac{x^{2}}{2}+x\right)$$

5.9 problem 9

Internal problem ID [5010]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.3. page 443
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\sin(x)\, y'' - \ln(x)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(sin(x)*diff(y(x),x$2)-ln(x)*y(x)=0,y(x),type='series',x=0);
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
AsymptoticDSolveValue[Sin[x]*y''[x]-Log[x]*y[x]==0,y[x],{x,0,5}]
```

Not solved

5.10 problem 11

Internal problem ID [5011]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.3. page 443
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + (x+2)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 36

```
Order:=6;
dsolve(diff(y(x),x)+(x+2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - 2x + \frac{3}{2}x^2 - \frac{1}{3}x^3 - \frac{5}{24}x^4 + \frac{3}{20}x^5\right) y(0) + O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 39

```
AsymptoticDSolveValue[y'[x]+(x+2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{3x^5}{20} - \frac{5x^4}{24} - \frac{x^3}{3} + \frac{3x^2}{2} - 2x + 1\right)$$

5.11 problem 12

Internal problem ID [5012]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.3. page 443
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5\right) y(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 37

```
AsymptoticDSolveValue[y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1\right)$$

5.12 problem 13

Internal problem ID [5013]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.3. page 443
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{z' - x^2 z = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
Order:=6;
dsolve(diff(z(x),x)-x^2*z(x)=0,z(x),type='series',x=0);
```

$$z(x) = \left(1 + \frac{x^3}{3}\right) z(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 15

```
AsymptoticDSolveValue[z'[x]-x^2*z[x]==0,z[x],{x,0,5}]
```

$$z(x) \to c_1 \left(\frac{x^3}{3} + 1\right)$$

5.13 problem 14

Internal problem ID [5014]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.3. page 443
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{(x^2+1)\, y''+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((x^2+1)*diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{1}{2}x^2+\frac{1}{8}x^4\right)y(0)+\left(x-\frac{1}{6}x^3+\frac{7}{120}x^5\right)D(y)(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[(x^2+1)*y''[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{7x^5}{120}-\frac{x^3}{6}+x\right)+c_1\left(\frac{x^4}{8}-\frac{x^2}{2}+1\right)$$

5.14 problem 15

Internal problem ID [5015]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.3. page 443
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + (x-1)\, y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve(diff(y(x),x$2)+(x-1)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{12}x^4 + \frac{1}{20}x^5\right) y(0) + \left(x + \frac{1}{2}x^2 - \frac{1}{6}x^3 - \frac{1}{6}x^4\right) D(y)\,(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[y''[x]+(x-1)*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(-\frac{x^4}{6} - \frac{x^3}{6} + \frac{x^2}{2} + x\right) + c_1\left(\frac{x^5}{20} + \frac{x^4}{12} - \frac{x^3}{6} - \frac{x^2}{2} + 1\right)$$

5.15 problem 16

Internal problem ID [5016]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.3. page 443
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 52

```
Order:=6;
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{8}x^4 - \frac{1}{30}x^5\right) y(0) + \left(x + x^2 + \frac{1}{2}x^3 + \frac{1}{6}x^4 + \frac{1}{24}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 66

```
AsymptoticDSolveValue[y''[x]-2*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{x^5}{30} - \frac{x^4}{8} - \frac{x^3}{3} - \frac{x^2}{2} + 1\right) + c_2\left(\frac{x^5}{24} + \frac{x^4}{6} + \frac{x^3}{2} + x^2 + x\right)$$

5.16 problem 17

Internal problem ID [5017]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.3. page 443
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{w'' - x^2 w' + w = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=6;
dsolve(diff(w(x),x$2)-x^2*diff(w(x),x)+w(x)=0,w(x),type='series',x=0);
```

$$w(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{20}x^5\right) w(0) + \left(x - \frac{1}{6}x^3 + \frac{1}{12}x^4 + \frac{1}{120}x^5\right) D(w)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[w''[x]-x^2*w'[x]+w[x]==0,w[x],{x,0,5}]
```

$$w(x) \to c_2\left(\frac{x^5}{120} + \frac{x^4}{12} - \frac{x^3}{6} + x\right) + c_1\left(-\frac{x^5}{20} + \frac{x^4}{24} - \frac{x^2}{2} + 1\right)$$

5.17 problem 18

Internal problem ID [5018]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.3. page 443
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x-3)\,y''-y'x+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((2*x-3)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+\frac{1}{6}x^2+\frac{1}{27}x^3+\frac{5}{648}x^4+\frac{1}{540}x^5\right)y(0)+D(y)\,(0)\,x+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 41

```
AsymptoticDSolveValue[(2*x-3)*y''[x]-x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{x^5}{540}+\frac{5x^4}{648}+\frac{x^3}{27}+\frac{x^2}{6}+1\right)+c_2x$$

6 Chapter 8, Series solutions of differential equations. Section 8.4. page 449

6.1 problem 1

Internal problem ID [5019]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.4. page 449
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(1+x)\,y''-3y'x+2y=0$$

With the expansion point for the power series method at $x=1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve((x+1)*diff(y(x),x$2)-3*x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=1);
```

$$y(x)=\left(1-\frac{(x-1)^2}{2}-\frac{(x-1)^3}{6}-\frac{5(x-1)^4}{48}-\frac{7(x-1)^5}{240}\right)y(1)+\left(x-1+\frac{3(x-1)^2}{4}+\frac{(x-1)^3}{3}+\frac{(x-1)^4}{6}+\frac{7(x-1)^5}{120}\right)D(y)\,(1)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 87

```
AsymptoticDSolveValue[(x+1)*y''[x]-3*x*y'[x]+2*y[x]==0,y[x],{x,1,5}]
```

$$y(x)\to c_1\left(-\frac{7}{240}(x-1)^5-\frac{5}{48}(x-1)^4-\frac{1}{6}(x-1)^3-\frac{1}{2}(x-1)^2+1\right)+c_2\left(\frac{7}{120}(x-1)^5+\frac{1}{6}(x-1)^4+\frac{1}{3}(x-1)^3+\frac{3}{4}(x-1)^2+x-1\right)$$

6.2 problem 2

Internal problem ID [5020]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.4. page 449
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Hermite]`

$$\boxed{y'' - y'x - 3y = 0}$$

With the expansion point for the power series method at $x = 2$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
Order:=6;
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-3*y(x)=0,y(x),type='series',x=2);
```

$$y(x) = \left(1 + \frac{3(x-2)^2}{2} + (x-2)^3 + \frac{9(x-2)^4}{8} + \frac{3(x-2)^5}{4}\right) y(2) + \left(x - 2 + (x-2)^2 + \frac{4(x-2)^3}{3} + \frac{13(x-2)^4}{12} + \frac{5(x-2)^5}{6}\right) D(y)(2) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 79

```
AsymptoticDSolveValue[y''[x]-x*y'[x]-3*y[x]==0,y[x],{x,2,5}]
```

$$y(x) \to c_1\left(\frac{3}{4}(x-2)^5 + \frac{9}{8}(x-2)^4 + (x-2)^3 + \frac{3}{2}(x-2)^2 + 1\right) + c_2\left(\frac{5}{6}(x-2)^5 + \frac{13}{12}(x-2)^4 + \frac{4}{3}(x-2)^3 + (x-2)^2 + x - 2\right)$$

6.3 problem 3

Internal problem ID [5021]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.4. page 449
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{(x^2 + x + 1)\, y'' - 3y = 0}$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve((1+x+x^2)*diff(y(x),x$2)-3*y(x)=0,y(x),type='series',x=1);
```

$$y(x) = \left(1 + \frac{(x-1)^2}{2} - \frac{(x-1)^3}{6} + \frac{7(x-1)^4}{72} - \frac{(x-1)^5}{20}\right) y(1) + \left(x - 1 + \frac{(x-1)^3}{6} - \frac{(x-1)^4}{12} + \frac{(x-1)^5}{24}\right) D(y)(1) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 78

```
AsymptoticDSolveValue[(1+x+x^2)*y''[x]-3*y[x]==0,y[x],{x,1,5}]
```

$$y(x) \to c_1\left(-\frac{1}{20}(x-1)^5 + \frac{7}{72}(x-1)^4 - \frac{1}{6}(x-1)^3 + \frac{1}{2}(x-1)^2 + 1\right) + c_2\left(\frac{1}{24}(x-1)^5 - \frac{1}{12}(x-1)^4 + \frac{1}{6}(x-1)^3 + x - 1\right)$$

6.4 problem 4

Internal problem ID [5022]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.4. page 449
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-5x+6)\,y''-3y'x-y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve((x^2-5*x+6)*diff(y(x),x$2)-3*x*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+\frac{1}{12}x^2+\frac{5}{216}x^3+\frac{5}{324}x^4+\frac{11}{1296}x^5\right)y(0)+\left(x+\frac{1}{9}x^3+\frac{5}{108}x^4+\frac{29}{1080}x^5\right)D(y)(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[(x^2-5*x+6)*y''[x]-3*x*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{29x^5}{1080}+\frac{5x^4}{108}+\frac{x^3}{9}+x\right)+c_1\left(\frac{11x^5}{1296}+\frac{5x^4}{324}+\frac{5x^3}{216}+\frac{x^2}{12}+1\right)$$

6.5 problem 5

Internal problem ID [5023]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.4. page 449
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{y'' - \tan(x)\, y' + y = 0}$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 106

```
Order:=6;
dsolve(diff(y(x),x$2)-tan(x)*diff(y(x),x)+y(x)=0,y(x),type='series',x=1);
```

$$y(x) = \left(1 - \frac{(x-1)^2}{2} - \frac{\tan(1)\,(x-1)^3}{6} + \left(\frac{1}{12} - \frac{\sec(1)^2}{8}\right)(x-1)^4 + \frac{\tan(1)\left(1 - 4\sec(1)^2\right)(x-1)^5}{40}\right) y(1) + \left(x - 1 + \frac{\tan(1)\,(x-1)^2}{2} + \frac{\tan(1)^2\,(x-1)^3}{3} + \frac{\tan(1)\left(2\sec(1)^2 - 1\right)(x-1)^4}{8} + \frac{\left(5 - 27\sec(1)^2 + 24\sec(1)^4\right)(x-1)^5}{120}\right) D(y)(1) + O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 442

```
AsymptoticDSolveValue[y''[x]-Tan[x]*y'[x]+y[x]==0,y[x],{x,1,5}]
```

$$
\begin{aligned}
y(x) \to\ & c_1\left(\frac{1}{24}(x-1)^4-\frac{1}{2}(x-1)^2+\frac{1}{20}(x-1)^5\left(-\tan^3(1)-\tan(1)\right)-\frac{1}{120}(x-1)^5\tan^3(1)\right.\\
&-\frac{1}{40}(x-1)^5\tan(1)\left(1+\tan^2(1)\right)+\frac{1}{60}(x-1)^5\tan(1)\left(-1-\tan^2(1)\right)\\
&+\frac{1}{12}(x-1)^4\left(-1-\tan^2(1)\right)-\frac{1}{24}(x-1)^4\tan^2(1)+\frac{1}{60}(x-1)^5\tan(1)\\
&\left.-\frac{1}{6}(x-1)^3\tan(1)+1\right)+c_2\left(\frac{1}{120}(x-1)^5-\frac{1}{6}(x-1)^3+x+\frac{1}{120}(x-1)^5\tan^4(1)\right.\\
&-\frac{1}{15}(x-1)^5\tan(1)\left(-\tan^3(1)-\tan(1)\right)-\frac{1}{12}(x-1)^4\left(-\tan^3(1)-\tan(1)\right)\\
&+\frac{1}{24}(x-1)^4\tan^3(1)-\frac{1}{40}(x-1)^5\left(-1-\tan^2(1)\right)\left(1+\tan^2(1)\right)\\
&+\frac{1}{40}(x-1)^5\tan^2(1)\left(1+\tan^2(1)\right)-\frac{1}{40}(x-1)^5\left(1+\tan^2(1)\right)\\
&-\frac{1}{40}(x-1)^5\tan^2(1)\left(-1-\tan^2(1)\right)+\frac{1}{120}(x-1)^5\left(-1-\tan^2(1)\right)\\
&-\frac{1}{40}(x-1)^5\tan^2(1)-\frac{1}{8}(x-1)^4\tan(1)\left(-1-\tan^2(1)\right)-\frac{1}{6}(x-1)^3\left(-1-\tan^2(1)\right)\\
&+\frac{1}{6}(x-1)^3\tan^2(1)-\frac{1}{60}(x-1)^5\left(-1-3\tan^4(1)-4\tan^2(1)\right)\\
&\left.-\frac{1}{12}(x-1)^4\tan(1)+\frac{1}{2}(x-1)^2\tan(1)-1\right)
\end{aligned}
$$

6.6 problem 6

Internal problem ID [5024]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.4. page 449
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^3+1)\,y''-y'x+2yx^2=0}$$

With the expansion point for the power series method at $x=1$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 54

```
Order:=6;
dsolve((1+x^3)*diff(y(x),x$2)-x*diff(y(x),x)+2*x^2*y(x)=0,y(x),type='series',x=1);
```

$$y(x)=\left(1-\frac{(x-1)^2}{2}-\frac{(x-1)^3}{6}+\frac{7(x-1)^4}{48}+\frac{7(x-1)^5}{240}\right)y(1)+\left(x-1+\frac{(x-1)^2}{4}-\frac{(x-1)^3}{6}-\frac{(x-1)^4}{8}+\frac{(x-1)^5}{12}\right)D(y)(1)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 78

```
AsymptoticDSolveValue[(1+x^3)*y''[x]-x*y'[x]+2*x*y[x]==0,y[x],{x,1,5}]
```

$$y(x)\to c_1\left(-\frac{1}{20}(x-1)^5+\frac{1}{8}(x-1)^4-\frac{1}{2}(x-1)^2+1\right)+c_2\left(\frac{19}{240}(x-1)^5-\frac{1}{24}(x-1)^4-\frac{1}{6}(x-1)^3+\frac{1}{4}(x-1)^2+x-1\right)$$

6.7 problem 7

Internal problem ID [5025]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.4. page 449
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + 2(x-1)\,y = 0}$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
Order:=6;
dsolve(diff(y(x),x)+2*(x-1)*y(x)=0,y(x),type='series',x=1);
```

$$y(x) = \left(1 - (x-1)^2 + \frac{(x-1)^4}{2}\right) y(1) + O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 24

```
AsymptoticDSolveValue[y'[x]+2*(x-1)*y[x]==0,y[x],{x,1,5}]
```

$$y(x) \to c_1 \left(\frac{1}{2}(x-1)^4 - (x-1)^2 + 1\right)$$

6.8 problem 8

Internal problem ID [5026]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.4. page 449
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{-2yx + y' = 0}$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
Order:=6;
dsolve(diff(y(x),x)-2*x*y(x)=0,y(x),type='series',x=1);
```

$$y(x) = \left(-1 + 2x + 3(x-1)^2 + \frac{10(x-1)^3}{3} + \frac{19(x-1)^4}{6} + \frac{13(x-1)^5}{5}\right) y(1) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 47

```
AsymptoticDSolveValue[y'[x]-2*x*y[x]==0,y[x],{x,1,5}]
```

$$y(x) \to c_1 \left(\frac{13}{5}(x-1)^5 + \frac{19}{6}(x-1)^4 + \frac{10}{3}(x-1)^3 + 3(x-1)^2 + 2(x-1) + 1\right)$$

6.9 problem 9

Internal problem ID [5027]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.4. page 449
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-2x)\,y''+2y=0}$$

With the expansion point for the power series method at $x=1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
Order:=6;
dsolve((x^2-2*x)*diff(y(x),x$2)+2*y(x)=0,y(x),type='series',x=1);
```

$$y(x)=\left(1+(x-1)^2+\frac{(x-1)^4}{3}\right)y(1)+\left(x-1+\frac{(x-1)^3}{3}+\frac{2(x-1)^5}{15}\right)D(y)\,(1)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 47

```
AsymptoticDSolveValue[(x^2-2*x)*y''[x]+2*y[x]==0,y[x],{x,1,5}]
```

$$y(x)\to c_1\left(\frac{1}{3}(x-1)^4+(x-1)^2+1\right)+c_2\left(\frac{2}{15}(x-1)^5+\frac{1}{3}(x-1)^3+x-1\right)$$

6.10 problem 10

Internal problem ID [5028]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.4. page 449
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - y'x + 2y = 0}$$

With the expansion point for the power series method at $x = 2$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=2);
```

$$y(x) = \left(1 - \frac{(x-2)^2}{4} + \frac{(x-2)^3}{24} - \frac{(x-2)^4}{192}\right) y(2) + \left(x - 2 + \frac{(x-2)^2}{4} - \frac{(x-2)^3}{12} + \frac{(x-2)^4}{48} - \frac{(x-2)^5}{192}\right) D(y)(2) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 78

```
AsymptoticDSolveValue[x^2*y''[x]-x*y'[x]+2*y[x]==0,y[x],{x,2,5}]
```

$$y(x) \to c_1\left(-\frac{1}{192}(x-2)^4 + \frac{1}{24}(x-2)^3 - \frac{1}{4}(x-2)^2 + 1\right) + c_2\left(-\frac{1}{192}(x-2)^5 + \frac{1}{48}(x-2)^4 - \frac{1}{12}(x-2)^3 + \frac{1}{4}(x-2)^2 + x - 2\right)$$

6.11 problem 11

Internal problem ID [5029]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.4. page 449
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - y' + y = 0}$$

With the expansion point for the power series method at $x = 2$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-diff(y(x),x)+y(x)=0,y(x),type='series',x=2);
```

$$y(x) = \left(1 - \frac{(x-2)^2}{8} + \frac{(x-2)^3}{32} - \frac{3(x-2)^4}{512} + \frac{(x-2)^5}{2048}\right) y(2) + \left(x - 2 + \frac{(x-2)^2}{8} - \frac{7(x-2)^3}{96} + \frac{37(x-2)^4}{1536} - \frac{211(x-2)^5}{30720}\right) D(y)(2) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 87

```
AsymptoticDSolveValue[x^2*y''[x]-y'[x]+y[x]==0,y[x],{x,2,5}]
```

$$y(x) \to c_1\left(\frac{(x-2)^5}{2048} - \frac{3}{512}(x-2)^4 + \frac{1}{32}(x-2)^3 - \frac{1}{8}(x-2)^2 + 1\right) + c_2\left(-\frac{211(x-2)^5}{30720} + \frac{37(x-2)^4}{1536} - \frac{7}{96}(x-2)^3 + \frac{1}{8}(x-2)^2 + x - 2\right)$$

6.12 problem 12

Internal problem ID [5030]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.4. page 449
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (3x - 1)\, y' - y = 0}$$

With the expansion point for the power series method at $x = -1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve(diff(y(x),x$2)+(3*x-1)*diff(y(x),x)-y(x)=0,y(x),type='series',x=-1);
```

$$y(x) = \left(1 + \frac{(x+1)^2}{2} + \frac{2(x+1)^3}{3} + \frac{11(x+1)^4}{24} + \frac{(x+1)^5}{10}\right) y(-1) + \left(x + 1 + 2(x+1)^2 + \frac{7(x+1)^3}{3} + \frac{3(x+1)^4}{2} + \frac{4(x+1)^5}{15}\right) D(y)\,(-1) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 85

```
AsymptoticDSolveValue[y''[x]+(3*x-1)*y'[x]-y[x]==0,y[x],{x,-1,5}]
```

$$y(x) \to c_1\left(\frac{1}{10}(x+1)^5 + \frac{11}{24}(x+1)^4 + \frac{2}{3}(x+1)^3 + \frac{1}{2}(x+1)^2 + 1\right) + c_2\left(\frac{4}{15}(x+1)^5 + \frac{3}{2}(x+1)^4 + \frac{7}{3}(x+1)^3 + 2(x+1)^2 + x + 1\right)$$

6.13 problem 13

Internal problem ID [5031]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.4. page 449
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x' + \sin(t)\,x = 0}$$

With initial conditions

$$[x(0) = 1]$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([diff(x(t),t)+sin(t)*x(t)=0,x(0) = 1],x(t),type='series',t=0);
```

$$x(t) = 1 - \frac{1}{2}t^2 + \frac{1}{6}t^4 + \mathrm{O}\left(t^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{x'[t]+Sin[t]*x[t]==0,{x[0]==1}},x[t],{t,0,5}]
```

$$x(t) \to \frac{t^4}{6} - \frac{t^2}{2} + 1$$

6.14 problem 14

Internal problem ID [5032]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.4. page 449
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - e^{x} y = 0}$$

With initial conditions

$$[y(0) = 1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([diff(y(x),x)-exp(x)*y(x)=0,y(0) = 1],y(x),type='series',x=0);
```

$$y(x) = 1 + x + x^{2} + \frac{5}{6}x^{3} + \frac{5}{8}x^{4} + \frac{13}{30}x^{5} + \mathrm{O}\left(x^{6}\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 30

```
AsymptoticDSolveValue[{y'[x]-Exp[x]*y[x]==0,{y[0]==1}},y[x],{x,0,5}]
```

$$y(x) \to \frac{13x^{5}}{30} + \frac{5x^{4}}{8} + \frac{5x^{3}}{6} + x^{2} + x + 1$$

6.15 problem 15

Internal problem ID [5033]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.4. page 449
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+1\right)y''-y'\mathrm{e}^x+y=0}$$

With initial conditions

$$[y(0)=1, y'(0)=1]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
Order:=6;
dsolve([(x^2+1)*diff(y(x),x$2)-exp(x)*diff(y(x),x)+y(x)=0,y(0) = 1, D(y)(0) = 1],y(x),type='
```

$$y(x)=1+x+\frac{1}{24}x^4+\frac{1}{60}x^5+\mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 20

```
AsymptoticDSolveValue[{(x^2+1)*y''[x]-Exp[x]*y'[x]+y[x]==0,{y[0]==1,y'[0]==1}},y[x],{x,0,5}]
```

$$y(x)\to\frac{x^5}{60}+\frac{x^4}{24}+x+1$$

6.16 problem 16

Internal problem ID [5034]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.4. page 449
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + ty' + e^t y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([diff(y(t),t$2)+t*diff(y(t),t)+exp(t)*y(t)=0,y(0) = 1, D(y)(0) = -1],y(t),type='serie
```

$$y(t) = 1 - t - \frac{1}{2}t^2 + \frac{1}{6}t^3 + \frac{1}{6}t^4 + \frac{1}{120}t^5 + O\left(t^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[{y''[t]+t*y'[t]+Exp[t]*y[t]==0,{y[0]==1,y'[0]==-1}},y[t],{t,0,5}]
```

$$y(t) \to \frac{t^5}{120} + \frac{t^4}{6} + \frac{t^3}{6} - \frac{t^2}{2} - t + 1$$

6.17 problem 19

Internal problem ID [5035]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.4. page 449
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y' \mathrm{e}^{2x} + \cos(x)\, y = 0}$$

With initial conditions

$$[y(0) = -1, y'(0) = 1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([diff(y(x),x$2)-exp(2*x)*diff(y(x),x)+cos(x)*y(x)=0,y(0) = -1, D(y)(0) = 1],y(x),type
```

$$y(x) = -1 + x + x^2 + \frac{1}{2}x^3 + \frac{1}{2}x^4 + \frac{31}{60}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 30

```
AsymptoticDSolveValue[{y''[x]-Exp[2*x]*y'[x]+Cos[x]*y[x]==0,{y[0]==-1,y'[0]==1}},y[x],{x,0,5
```

$$y(x) \to \frac{31x^5}{60} + \frac{x^4}{2} + \frac{x^3}{2} + x^2 + x - 1$$

6.18 problem 21

Internal problem ID [5036]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.4. page 449
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - yx = \sin(x)}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
Order:=6;
dsolve(diff(y(x),x)-x*y(x)=sin(x),y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 + \frac{1}{8}x^4\right) y(0) + \frac{x^2}{2} + \frac{x^4}{12} + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 37

```
AsymptoticDSolveValue[y'[x]-x*y[x]==Sin[x],y[x],{x,0,5}]
```

$$y(x) \to \frac{x^4}{12} + \frac{x^2}{2} + c_1\left(\frac{x^4}{8} + \frac{x^2}{2} + 1\right)$$

6.19 problem 22

Internal problem ID [5037]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.4. page 449
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{w' + wx = e^x}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
Order:=6;
dsolve(diff(w(x),x)+x*w(x)=exp(x),w(x),type='series',x=0);
```

$$w(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{8}x^4\right) w(0) + x + \frac{x^2}{2} - \frac{x^3}{6} - \frac{x^4}{12} + \frac{x^5}{24} + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 52

```
AsymptoticDSolveValue[w'[x]-x*w[x]==Exp[x],w[x],{x,0,5}]
```

$$w(x) \to \frac{13x^5}{120} + \frac{x^4}{6} + \frac{x^3}{2} + \frac{x^2}{2} + c_1\left(\frac{x^4}{8} + \frac{x^2}{2} + 1\right) + x$$

6.20 problem 23

Internal problem ID [5038]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.4. page 449
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$z'' + xz' + z = x^2 + 2x + 1$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
Order:=6;
dsolve(diff(z(x),x$2)+x*diff(z(x),x)+z(x)=x^2+2*x+1,z(x),type='series',x=0);
```

$$z(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{8}x^4\right) z(0) + \left(x - \frac{1}{3}x^3 + \frac{1}{15}x^5\right) D(z)(0) + \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{24} - \frac{x^5}{15} + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 70

```
AsymptoticDSolveValue[z''[x]+x*z'[x]+z[x]==x^2+2*x+1,z[x],{x,0,5}]
```

$$z(x) \to -\frac{x^5}{15} - \frac{x^4}{24} + \frac{x^3}{3} + \frac{x^2}{2} + c_2\left(\frac{x^5}{15} - \frac{x^3}{3} + x\right) + c_1\left(\frac{x^4}{8} - \frac{x^2}{2} + 1\right)$$

6.21 problem 24

Internal problem ID [5039]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.4. page 449
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y'x + 3y = x^2}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
Order:=6;
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+3*y(x)=x^2,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{3}{2}x^2 - \frac{1}{8}x^4\right) y(0) + \left(x - \frac{1}{6}x^3 - \frac{1}{40}x^5\right) D(y)(0) + \frac{x^4}{12} + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 49

```
AsymptoticDSolveValue[y''[x]-2*x*y'[x]+3*y[x]==x^2,y[x],{x,0,5}]
```

$$y(x) \to \frac{x^4}{12} + c_2\left(-\frac{x^5}{40} - \frac{x^3}{6} + x\right) + c_1\left(-\frac{x^4}{8} - \frac{3x^2}{2} + 1\right)$$

6.22 problem 25

Internal problem ID [5040]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.4. page 449
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''-y'x+y=\cos(x)}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
Order:=6;
dsolve((1+x^2)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=cos(x),y(x),type='series',x=0);
```

$$y(x)=\left(\frac{1}{24}x^4-\frac{1}{2}x^2+1\right)y(0)+D(y)(0)\,x+\frac{x^2}{2}-\frac{x^4}{12}+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 41

```
AsymptoticDSolveValue[(1+x^2)*y''[x]-x*y'[x]+y[x]==Cos[x],y[x],{x,0,5}]
```

$$y(x)\to-\frac{x^4}{12}+\frac{x^2}{2}+c_1\left(\frac{x^4}{24}-\frac{x^2}{2}+1\right)+c_2x$$

6.23 problem 26

Internal problem ID [5041]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.4. page 449
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y'x + 2y = \cos(x)}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
Order:=6;
dsolve(diff(y(x),x$2)-x*diff(y(x),x)+2*y(x)=cos(x),y(x),type='series',x=0);
```

$$y(x) = (-x^2 + 1)\, y(0) + \left(x - \frac{1}{6}x^3 - \frac{1}{120}x^5\right) D(y)(0) + \frac{x^2}{2} - \frac{x^4}{24} + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 47

```
AsymptoticDSolveValue[y''[x]-x*y'[x]+2*y[x]==Cos[x],y[x],{x,0,5}]
```

$$y(x) \to -\frac{x^4}{24} + \frac{x^2}{2} + c_1(1 - x^2) + c_2\left(-\frac{x^5}{120} - \frac{x^3}{6} + x\right)$$

6.24 problem 27

Internal problem ID [5042]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.4. page 449
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{(-x^2+1)\,y''-y'+y=\tan(x)}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
Order:=6;
dsolve((1-x^2)*diff(y(x),x$2)-diff(y(x),x)+y(x)=tan(x),y(x),type='series',x=0);
```

$$y(x) = \left(1-\frac{1}{2}x^2-\frac{1}{6}x^3-\frac{1}{12}x^4-\frac{7}{120}x^5\right)y(0) + \left(x+\frac{1}{2}x^2+\frac{1}{24}x^4+\frac{1}{120}x^5\right)D(y)(0)+\frac{x^3}{6}+\frac{x^4}{24}+\frac{x^5}{15}+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 197

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-y'[x]+y[x]==Tan[x],y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^6}{60}+\frac{x^5}{120}+\frac{x^4}{24}+\frac{x^2}{2}+x\right)+c_1\left(-\frac{7x^5}{120}-\frac{x^4}{12}-\frac{x^3}{6}-\frac{x^2}{2}+1\right)+\left(-\frac{7x^5}{120}-\frac{x^4}{12}-\frac{x^3}{6}-\frac{x^2}{2}+1\right)\left(\frac{7x^6}{48}-\frac{4x^5}{15}+\frac{x^4}{8}-\frac{x^3}{3}\right)+\left(\frac{x^6}{60}+\frac{x^5}{120}+\frac{x^4}{24}+\frac{x^2}{2}+x\right)\left(\frac{67x^6}{240}-\frac{3x^5}{10}+\frac{x^4}{3}-\frac{x^3}{3}+\frac{x^2}{2}\right)$$

6.25 problem 28

Internal problem ID [5043]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.4. page 449
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - \sin(x)\, y = \cos(x)}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
Order:=6;
dsolve(diff(y(x),x$2)-sin(x)*y(x)=cos(x),y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{6}x^3 - \frac{1}{120}x^5\right) y(0) + \left(x + \frac{1}{12}x^4\right) D(y)(0) + \frac{x^2}{2} - \frac{x^4}{24} + \frac{x^5}{40} + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 56

```
AsymptoticDSolveValue[y''[x]-Sin[x]*y[x]==Cos[x],y[x],{x,0,5}]
```

$$y(x) \to \frac{x^5}{40} - \frac{x^4}{24} + c_2\left(\frac{x^4}{12} + x\right) + \frac{x^2}{2} + c_1\left(-\frac{x^5}{120} + \frac{x^3}{6} + 1\right)$$

6.26 problem 29

Internal problem ID [5044]

Book: Fundamentals of Differential Equations. By Nagle, Saff and Snider. 9th edition. Boston. Pearson 2018.
Section: Chapter 8, Series solutions of differential equations. Section 8.4. page 449
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$(-x^2+1)\,y''-2y'x+n(n+1)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 101

```
Order:=6;
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+n*(n+1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{n(n+1)\,x^2}{2}+\frac{n(n^3+2n^2-5n-6)\,x^4}{24}\right)y(0)+\left(x-\frac{(n^2+n-2)\,x^3}{6}+\frac{(n^4+2n^3-13n^2-14n+24)\,x^5}{120}\right)D(y)\,(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 120

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-2*x*y'[x]+n*(n+1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{1}{120}(n^2+n)^2x^5+\frac{7}{60}(-n^2-n)\,x^5+\frac{1}{6}(-n^2-n)\,x^3+\frac{x^5}{5}+\frac{x^3}{3}+x\right)+c_1\left(\frac{1}{24}(n^2+n)^2x^4+\frac{1}{4}(-n^2-n)\,x^4+\frac{1}{2}(-n^2-n)\,x^2+1\right)$$