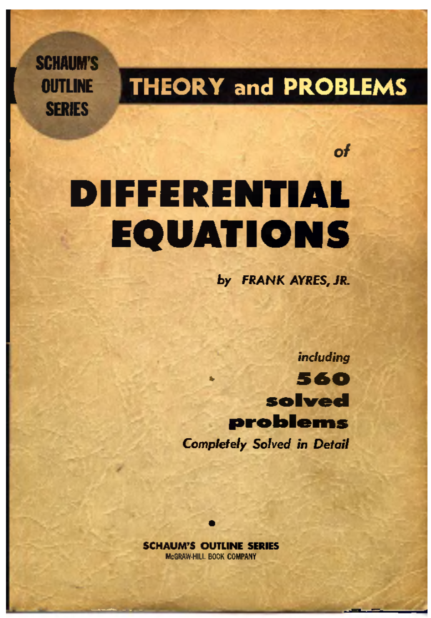
A Solution Manual For

Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952



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Contents

1 Chapter 2. Solutions of differential equations. Supplemetary problems. Page 11

1.1 problem 13

Internal problem ID [5226]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 2. Solutions of differential equations. Supplemetary problems. Page 11
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(x*diff(y(x),x)=2*y(x),y(x), singsol=all)
```

$$y = c_1 x^2$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 16

```
DSolve[x*y'[x]==2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x^2$$
$$y(x) \to 0$$

1.2 problem 14

Internal problem ID [5227]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 2. Solutions of differential equations. Supplemetary problems. Page 11
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yy' = -x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(y(x)*diff(y(x),x)+x=0,y(x), singsol=all)
```

$$y = \sqrt{-x^2 + c_1}$$
$$y = -\sqrt{-x^2 + c_1}$$

✓ Solution by Mathematica

Time used: 0.078 (sec). Leaf size: 39

```
DSolve[y[x]*y'[x]+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^2 + 2c_1}$$
$$y(x) \to \sqrt{-x^2 + 2c_1}$$

1.3 problem 15

Internal problem ID [5228]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 2. Solutions of differential equations. Supplemetary problems. Page 11
Problem number: 15.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y - y'x - y'^4 = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 19

```
dsolve(y(x)=x*diff(y(x),x)+diff(y(x),x)^4,y(x), singsol=all)
```

$$y = c_1^4 + c_1 x$$
$$y = c_1 x^{\frac{4}{3}}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 75

```
DSolve[y[x]==x*y'[x]+(y'[x])^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \left(x + c_1{}^3\right)$$
$$y(x) \to -\frac{3}{4}\left(-\frac{1}{2}\right)^{2/3} x^{4/3}$$
$$y(x) \to -\frac{3x^{4/3}}{4\ 2^{2/3}}$$
$$y(x) \to \frac{3\sqrt[3]{-1}x^{4/3}}{4\ 2^{2/3}}$$

1.4 problem 16

Internal problem ID [5229]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 2. Solutions of differential equations. Supplemetary problems. Page 11
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{2y'x^3 - y(y^2 + 3x^2) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(2*x^3*diff(y(x),x)=y(x)*(y(x)^2+3*x^2),y(x), singsol=all)
```

$$y = \frac{\sqrt{(-x + c_1)\, x}\, x}{-x + c_1}$$
$$y = -\frac{\sqrt{(-x + c_1)\, x}\, x}{-x + c_1}$$

✓ Solution by Mathematica

Time used: 0.179 (sec). Leaf size: 47

```
DSolve[2*x^3*y'[x]==y[x]*(y[x]^2+3*x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^{3/2}}{\sqrt{-x + c_1}}$$
$$y(x) \to \frac{x^{3/2}}{\sqrt{-x + c_1}}$$
$$y(x) \to 0$$

1.5 problem 17

Internal problem ID [5230]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 2. Solutions of differential equations. Supplemetary problems. Page 11
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^x + x e^x c_2$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 16

```
DSolve[y''[x]-2*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x (c_2 x + c_1)$$

1.6 problem 18

Internal problem ID [5231]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 2. Solutions of differential equations. Supplemetary problems. Page 11
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-1)\, y'' - y'x + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y = c_1 x + c_2 \mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 17

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x - c_2 x$$

1.7 problem 19

Internal problem ID [5232]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 2. Solutions of differential equations. Supplemetary problems. Page 11
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)-y(x)=0,y(x), singsol=all)
```

$$y = c_1 \mathrm{e}^{-x} + c_2 \mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 20

```
DSolve[y''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x} + c_2 e^{-x}$$

1.8 problem 20

Internal problem ID [5233]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 2. Solutions of differential equations. Supplemetary problems. Page 11
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y = 4 - x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-y(x)=4-x,y(x), singsol=all)
```

$$y = c_2 e^{-x} + c_1 e^{x} - 4 + x$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 22

```
DSolve[y''[x]-y[x]==4-x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + c_1 e^{x} + c_2 e^{-x} - 4$$

1.9 problem 21

Internal problem ID [5234]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 2. Solutions of differential equations. Supplemetary problems. Page 11
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 3y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^{2x} + c_2 e^{x}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 18

```
DSolve[y''[x]-3*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x (c_2 e^x + c_1)$$

1.10 problem 22

Internal problem ID [5235]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 2. Solutions of differential equations. Supplemetary problems. Page 11
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 3y' + 2y = 2\,\mathrm{e}^{x}(1 - x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=2*exp(x)*(1-x),y(x), singsol=all)
```

$$y = \left(c_1\mathrm{e}^{x} + x^2 + c_2\right)\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 21

```
DSolve[y''[x]-3*y'[x]+2*y[x]==2*Exp[x]*(1-x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}\left(x^2 + c_2e^{x} + c_1\right)$$

2 Chapter 4. Equations of first order and first degree (Variable separable). Supplemetary problems. Page 22

2.1 problem 24

Internal problem ID [5236]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 4. Equations of first order and first degree (Variable separable). Suplemetary problems. Page 22
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(4*y(x)+x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{c_1}{x^4}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 16

```
DSolve[4*y[x]+x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x^4}$$
$$y(x) \to 0$$

2.2 problem 25

Internal problem ID [5237]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 4. Equations of first order and first degree (Variable separable). Supplemetary problems. Page 22
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2y + \left(-x^2 + 4\right) y' = -1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((1+2*y(x))+(4-x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = -\frac{1}{2} + \frac{\sqrt{x-2}\, c_1}{\sqrt{x+2}}$$

✓ Solution by Mathematica

Time used: 0.076 (sec). Leaf size: 35

```
DSolve[(1+2*y[x])+(4-x^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2} + \frac{c_1\sqrt{2-x}}{\sqrt{x+2}}$$
$$y(x) \to -\frac{1}{2}$$

2.3 problem 26

Internal problem ID [5238]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 4. Equations of first order and first degree (Variable separable). Supplemetary problems. Page 22
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^2 - x^2 y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(y(x)^2-x^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{x}{c_1 x + 1}$$

✓ Solution by Mathematica

Time used: 0.115 (sec). Leaf size: 21

```
DSolve[y[x]^2-x^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{1 - c_1 x}$$
$$y(x) \to 0$$

2.4 problem 27

Internal problem ID [5239]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 4. Equations of first order and first degree (Variable separable). Suplemetary problems. Page 22
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y - y'(1 + x) = -1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve((1+y(x))-(1+x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = -1 + (x + 1) c_1$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 18

```
DSolve[(1+y[x])-(1+x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -1 + c_1(x + 1)$$
$$y(x) \to -1$$

2.5 problem 28

Internal problem ID [5240]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 4. Equations of first order and first degree (Variable separable). Suplemetary problems. Page 22
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{xy^2 + y + (yx^2 - x)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 23

```
dsolve((x*y(x)^2+y(x))+(x^2*y(x)-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = x\,\mathrm{e}^{-\operatorname{LambertW}\left(-x^2\mathrm{e}^{-2c_1}\right)-2c_1}$$

✓ Solution by Mathematica

Time used: 13.386 (sec). Leaf size: 33

```
DSolve[(x*y[x]^2+y[x])+(x^2*y[x]-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{W\left(e^{-1+\frac{9c_1}{2^{2/3}}}x^2\right)}{x}$$
$$y(x) \to 0$$

2.6 problem 29

Internal problem ID [5241]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 4. Equations of first order and first degree (Variable separable). Supplemetary problems. Page 22
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{x \sin\left(\frac{y}{x}\right) - y \cos\left(\frac{y}{x}\right) + x \cos\left(\frac{y}{x}\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve((x*sin(y(x)/x)-y(x)*cos(y(x)/x))+(x*cos(y(x)/x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = x \arcsin\left(\frac{1}{x c_1}\right)$$

✓ Solution by Mathematica

Time used: 12.962 (sec). Leaf size: 21

```
DSolve[(x*Sin[y[x]/x]-y[x]*Cos[y[x]/x])+(x*Cos[y[x]/x])*y'[x]==0,y[x],x,IncludeSingularSolut
```

$$y(x) \to x \arcsin\left(\frac{e^{c_1}}{x}\right)$$
$$y(x) \to 0$$

2.7 problem 30

Internal problem ID [5242]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 4. Equations of first order and first degree (Variable separable). Supplemetary problems. Page 22
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational]`

$$\boxed{y^2(x^2+2)+(x^3+y^3)(-y'x+y)=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 1062

```
dsolve(y(x)^2*(x^2+2)+(x^3+y(x)^3)*(y(x)-x*diff(y(x),x))=0,y(x), singsol=all)
```

$$y = \frac{\left(27x^3 + 3\sqrt{-24c_1^3x^6 - 72c_1^2x^6\ln(x) + 72c_1^2x^4 - 72c_1x^6\ln(x)^2 + 144c_1x^4\ln(x) - 72c_1x^2 - 24x^6\ln(x)^3}\right.}{3} - \frac{3\left(-\frac{2c_1x^2}{3} - \frac{2x^2\ln(x)}{3} + \frac{2}{3}\right)}{\left(27x^3 + 3\sqrt{-24c_1^3x^6 - 72c_1^2x^6\ln(x) + 72c_1^2x^4 - 72c_1x^6\ln(x)^2 + 144c_1x^4\ln(x) - 72c_1x^2 - 24x^6\ln(x)}\right.}$$

$$y = -\frac{\left(27x^3 + 3\sqrt{-24c_1^3x^6 - 72c_1^2x^6\ln(x) + 72c_1^2x^4 - 72c_1x^6\ln(x)^2 + 144c_1x^4\ln(x) - 72c_1x^2 - 24x^6\ln(x)}\right.}{6} + \frac{-c_1x^2 - x^2\ln(x) + 1}{\left(27x^3 + 3\sqrt{-24c_1^3x^6 - 72c_1^2x^6\ln(x) + 72c_1^2x^4 - 72c_1x^6\ln(x)^2 + 144c_1x^4\ln(x) - 72c_1x^2 - 24x^6\ln(x)}\right.} - \frac{i\sqrt{3}\left(\frac{\left(27x^3+3\sqrt{-24c_1^3x^6-72c_1^2x^6\ln(x)+72c_1^2x^4-72c_1x^6\ln(x)^2+144c_1x^4\ln(x)-72c_1x^2-24x^6\ln(x)^3+72x^4\ln(x)^2-72x^2\ln(x)+24+8}\right.}{3}\right.}{}$$

$$y = -\frac{\left(27x^3 + 3\sqrt{-24c_1^3x^6 - 72c_1^2x^6\ln(x) + 72c_1^2x^4 - 72c_1x^6\ln(x)^2 + 144c_1x^4\ln(x) - 72c_1x^2 - 24x^6\ln(x)}\right.}{6} + \frac{-c_1x^2 - x^2\ln(x) + 1}{\left(27x^3 + 3\sqrt{-24c_1^3x^6 - 72c_1^2x^6\ln(x) + 72c_1^2x^4 - 72c_1x^6\ln(x)^2 + 144c_1x^4\ln(x) - 72c_1x^2 - 24x^6\ln(x)}\right.} + \frac{i\sqrt{3}\left(\frac{\left(27x^3+3\sqrt{-24c_1^3x^6-72c_1^2x^6\ln(x)+72c_1^2x^4-72c_1x^6\ln(x)^2+144c_1x^4\ln(x)-72c_1x^2-24x^6\ln(x)^3+72x^4\ln(x)^2-72x^2\ln(x)+24+8}\right.}{3}\right.}{}$$

✓ Solution by Mathematica

Time used: 54.35 (sec). Leaf size: 396

```
DSolve[y[x]^2*(x^2+2)+(x^3+y[x]^3)*(y[x]-x*y'[x])==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{6x^2\log(x)+6c_1x^2+3^{2/3}\left(9x^3+\frac{1}{3}\sqrt{729x^6+(-6x^2\log(x)-6c_1x^2+6)^3}\right)^{2/3}-6}{3\sqrt[3]{3}\sqrt[3]{9x^3+\frac{1}{3}\sqrt{729x^6+(-6x^2\log(x)-6c_1x^2+6)^3}}}$$

$$y(x) \to \frac{i(\sqrt{3}+i)\sqrt[3]{9x^3+\frac{1}{3}\sqrt{729x^6+(-6x^2\log(x)-6c_1x^2+6)^3}}}{2\ 3^{2/3}} - \frac{i\sqrt[3]{2}(\sqrt{3}-i)(x^2\log(x)+c_1x^2-1)}{\sqrt[3]{54x^3+2\sqrt{729x^6+(-6x^2\log(x)-6c_1x^2+6)^3}}}$$

$$y(x) \to \frac{i\sqrt[3]{2}(\sqrt{3}+i)(x^2\log(x)+c_1x^2-1)}{\sqrt[3]{54x^3+2\sqrt{729x^6+(-6x^2\log(x)-6c_1x^2+6)^3}}} - \frac{(1+i\sqrt{3})\sqrt[3]{54x^3+2\sqrt{729x^6+(-6x^2\log(x)-6c_1x^2+6)^3}}}{6\sqrt[3]{2}}$$

2.8 problem 31

Internal problem ID [5243]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 4. Equations of first order and first degree (Variable separable). Supplemetary problems. Page 22
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _dAlembert]`

$$\boxed{y\sqrt{x^2+y^2}-x\left(x+\sqrt{x^2+y^2}\right)y'=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 51

```
dsolve(y(x)*sqrt(x^2+y(x)^2)-x*(x+sqrt(x^2+y(x)^2))*diff(y(x),x)=0,y(x), singsol=all)
```

$$\ln\left(\frac{2x\left(x+\sqrt{x^2+y^2}\right)}{y}\right)-\ln(y)-\frac{\sqrt{x^2+y^2}}{x}-\ln(x)-c_1=0$$

✓ Solution by Mathematica

Time used: 0.319 (sec). Leaf size: 43

```
DSolve[y[x]*Sqrt[x^2+y[x]^2]-x*(x+Sqrt[x^2+y[x]^2])*y'[x]==0,y[x],x,IncludeSingularSolutions
```

$$\text{Solve}\left[\sqrt{\frac{y(x)^2}{x^2}+1}+\log\left(\sqrt{\frac{y(x)^2}{x^2}+1}-1\right)=-\log(x)+c_1,y(x)\right]$$

2.9 problem 32

Internal problem ID [5244]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 4. Equations of first order and first degree (Variable separable). Suplemetary problems. Page 22
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y + (2x + 2y + 1)\, y' = -x - 1}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 25

```
dsolve((x+y(x)+1)+(2*x+2*y(x)+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \mathrm{e}^{-\operatorname{LambertW}\left(2\,\mathrm{e}^{-c_1}\mathrm{e}^{x}\right)+x-c_1} - x$$

✓ Solution by Mathematica

Time used: 4.251 (sec). Leaf size: 30

```
DSolve[(x+y[x]+1)+(2*x+2*y[x]+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-2x + W\left(-e^{x-1+c_1}\right)\right)$$
$$y(x) \to -x$$

2.10 problem 34

Internal problem ID [5245]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 4. Equations of first order and first degree (Variable separable). Supplemetary problems. Page 22
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2y - y'(4 - x) = -1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve((1+2*y(x))-(4-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{4x - \frac{1}{2}x^2 + c_1}{(x - 4)^2}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 34

```
DSolve[(1+2*y[x])-(4-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-x^2 + 8x + 2c_1}{2(x - 4)^2}$$

$$y(x) \to -\frac{1}{2}$$

2.11 problem 35

Internal problem ID [5246]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 4. Equations of first order and first degree (Variable separable). Supplemetary problems. Page 22
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2+1)\,y'+yx=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve((x*y(x))+(1+x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y=\frac{c_1}{\sqrt{x^2+1}}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 22

```
DSolve[(x*y[x])+(1+x^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_1}{\sqrt{x^2+1}}$$
$$y(x)\to 0$$

2.12 problem 37

Internal problem ID [5247]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 4. Equations of first order and first degree (Variable separable). Supplemetary problems. Page 22
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class A‘]]`

$$\boxed{2yx + (2x + 3y)\, y' = 0}$$

✗ Solution by Maple

```
dsolve((x*2*y(x))+(2*x+3*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(x*2*y[x])+(2*x+3*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.13 problem 38

Internal problem ID [5248]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 4. Equations of first order and first degree (Variable separable). Supplemetary problems. Page 22
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{2y'x - 2y - \sqrt{x^2 + 4y^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(2*x*diff(y(x),x)-2*y(x)= sqrt(x^2+4*y(x)^2),y(x), singsol=all)
```

$$\frac{2y}{x^2} + \frac{\sqrt{x^2 + 4y^2}}{x^2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.394 (sec). Leaf size: 27

```
DSolve[2*x*y'[x]-2*y[x]== Sqrt[x^2+4*y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{-2c_1}\left(-1 + e^{4c_1}x^2\right)$$

2.14 problem 39

Internal problem ID [5249]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 4. Equations of first order and first degree (Variable separable). Supplemetary problems. Page 22
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{3y + (7y - 3x + 3)\, y' = 7x - 7}$$

✓ Solution by Maple

Time used: 0.453 (sec). Leaf size: 705

```
dsolve((3*y(x)-7*x+7)+(7*y(x)-3*x+3)*diff(y(x),x)= 0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 60.746 (sec). Leaf size: 7785

```
DSolve[(3*y[x]-7*x+7)+(7*y[x]-3*x+3)*y'[x]== 0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

2.15 problem 40

Internal problem ID [5250]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 4. Equations of first order and first degree (Variable separable). Suplemetary problems. Page 22
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xyy' - (1+y)(1-x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(x*y(x)*diff(y(x),x)= (y(x)+1)*(1-x),y(x), singsol=all)
```

$$y = -\operatorname{LambertW}\left(-\frac{c_1 e^{x-1}}{x}\right) - 1$$

✓ Solution by Mathematica

Time used: 6.202 (sec). Leaf size: 29

```
DSolve[x*y[x]*y'[x]== (y[x]+1)*(1-x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -1 - W\left(-\frac{e^{x-1-c_1}}{x}\right)$$
$$y(x) \rightarrow -1$$

2.16 problem 41

Internal problem ID [5251]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 4. Equations of first order and first degree (Variable separable). Supplemetary problems. Page 22
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{y^2 + xyy' = x^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve((y(x)^2-x^2)+x*y(x)*diff(y(x),x)= 0,y(x), singsol=all)
```

$$y = -\frac{\sqrt{2x^4 + 4c_1}}{2x}$$
$$y = \frac{\sqrt{2x^4 + 4c_1}}{2x}$$

✓ Solution by Mathematica

Time used: 0.199 (sec). Leaf size: 46

```
DSolve[(y[x]^2-x^2)+x*y[x]*y'[x]== 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{x^4}{2} + c_1}}{x}$$
$$y(x) \to \frac{\sqrt{\frac{x^4}{2} + c_1}}{x}$$

2.17 problem 42

Internal problem ID [5252]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 4. Equations of first order and first degree (Variable separable). Suplemetary problems. Page 22
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y(1+2yx)+x(1-yx)\,y'=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve(y(x)*(1+2*x*y(x))+x*(1-x*y(x))*diff(y(x),x)= 0,y(x), singsol=all)
```

$$y=-\frac{1}{\operatorname{LambertW}\left(-\frac{c_1}{x^3}\right)x}$$

✓ Solution by Mathematica

Time used: 6.645 (sec). Leaf size: 35

```
DSolve[y[x]*(1+2*x*y[x])+x*(1-x*y[x])*y'[x]== 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{1}{xW\left(\frac{e^{-1+\frac{9c_1}{2^{2/3}}}}{x^3}\right)}$$
$$y(x)\to 0$$

2.18 problem 43

Internal problem ID [5253]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 4. Equations of first order and first degree (Variable separable). Supplemetary problems. Page 22
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\left(-x^{2}+1\right)\cot\left(y\right)y'=-1}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(1+(1-x^2)*cot(y(x))*diff(y(x),x)= 0,y(x), singsol=all)
```

$$y=\arcsin\left(\frac{\sqrt{-x^{2}+1}\,c_{1}}{x+1}\right)$$

✓ Solution by Mathematica

Time used: 0.125 (sec). Leaf size: 27

```
DSolve[1+(1-x^2)*Cot[y[x]]*y'[x]== 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\arcsin\left(\frac{e^{c_{1}}\sqrt{1-x}}{\sqrt{x+1}}\right)$$

2.19 problem 44

Internal problem ID [5254]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 4. Equations of first order and first degree (Variable separable). Supplemetary problems. Page 22
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _Bernoulli]`

$$\boxed{y^3 + 3xy^2y' = -x^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 120

```
dsolve((x^3+y(x)^3)+3*x*y(x)^2*diff(y(x),x)= 0,y(x), singsol=all)
```

$$y = \frac{\left(\left(-2x^4 + 8c_1\right)x^2\right)^{\frac{1}{3}}}{2x}$$
$$y = -\frac{\left(\left(-2x^4 + 8c_1\right)x^2\right)^{\frac{1}{3}}}{4x} - \frac{i\sqrt{3}\left(\left(-2x^4 + 8c_1\right)x^2\right)^{\frac{1}{3}}}{4x}$$
$$y = -\frac{\left(\left(-2x^4 + 8c_1\right)x^2\right)^{\frac{1}{3}}}{4x} + \frac{i\sqrt{3}\left(\left(-2x^4 + 8c_1\right)x^2\right)^{\frac{1}{3}}}{4x}$$

✓ Solution by Mathematica

Time used: 0.212 (sec). Leaf size: 99

```
DSolve[(x^3+y[x]^3)+3*x*y[x]^2*y'[x]== 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{-x^4+4c_1}}{2^{2/3}\sqrt[3]{x}}$$

$$y(x) \to -\frac{\sqrt[3]{-1}\sqrt[3]{-x^4+4c_1}}{2^{2/3}\sqrt[3]{x}}$$

$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{-x^4+4c_1}}{2^{2/3}\sqrt[3]{x}}$$

2.20 problem 45

Internal problem ID [5255]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 4. Equations of first order and first degree (Variable separable). Supplemetary problems. Page 22
Problem number: 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{2y-(3x+2y-1)\,y'=-3x-1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve((3*x+2*y(x)+1)-(3*x+2*y(x)-1)*diff(y(x),x)= 0,y(x), singsol=all)
```

$$y=-\frac{3x}{2}-\frac{2\,\mathrm{LambertW}\left(-\frac{e^{\frac{1}{4}}e^{-\frac{25x}{4}}c_1}{4}\right)}{5}+\frac{1}{10}$$

✓ Solution by Mathematica

Time used: 4.841 (sec). Leaf size: 43

```
DSolve[(3*x+2*y[x]+1)-(3*x+2*y[x]-1)*y'[x]== 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{10}\left(-4W\left(-e^{-\frac{25x}{4}-1+c_1}\right)-15x+1\right)$$
$$y(x)\to\frac{1}{10}-\frac{3x}{2}$$

2.21 problem 46

Internal problem ID [5256]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 4. Equations of first order and first degree (Variable separable). Supplemetary problems. Page 22
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x + 2y = 0}$$

With initial conditions

$$[y(2) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve([x*diff(y(x),x)+2*y(x)= 0,y(2) = 1],y(x), singsol=all)
```

$$y = \frac{4}{x^2}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 10

```
DSolve[{x*y'[x]+2*y[x]==0,{y[2]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4}{x^2}$$

2.22 problem 47

Internal problem ID [5257]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 4. Equations of first order and first degree (Variable separable). Suplemetary problems. Page 22
Problem number: 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y^2 + xyy' = -x^2}$$

With initial conditions

$$[y(1) = -1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 18

```
dsolve([(x^2+y(x)^2)+x*y(x)*diff(y(x),x)= 0,y(1) = -1],y(x), singsol=all)
```

$$y = -\frac{\sqrt{-2x^4 + 6}}{2x}$$

✓ Solution by Mathematica

Time used: 0.202 (sec). Leaf size: 26

```
DSolve[{(x^2+y[x]^2)+x*y[x]*y'[x]==0,{y[1]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{3 - x^4}}{\sqrt{2}x}$$

2.23 problem 48

Internal problem ID [5258]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 4. Equations of first order and first degree (Variable separable). Supplemetary problems. Page 22
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\cos(y) + (1 + e^{-x}) \sin(y) y' = 0}$$

With initial conditions

$$\left[y(0) = \frac{\pi}{4}\right]$$

✓ Solution by Maple

Time used: 0.391 (sec). Leaf size: 14

```
dsolve([cos(y(x))+(1+exp(-x))*sin(y(x))*diff(y(x),x)= 0,y(0) = 1/4*Pi],y(x), singsol=all)
```

$$y = \arccos\left(\frac{\sqrt{2}\,(e^x + 1)}{4}\right)$$

✓ Solution by Mathematica

Time used: 46.229 (sec). Leaf size: 20

```
DSolve[{Cos[y[x]]+(1+Exp[-x])*Sin[y[x]]*y'[x]== 0,{y[0]==Pi/4}},y[x],x,IncludeSingularSoluti
```

$$y(x) \to \arccos\left(\frac{e^x + 1}{2\sqrt{2}}\right)$$

2.24 problem 49

Internal problem ID [5259]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 4. Equations of first order and first degree (Variable separable). Supplemetary problems. Page 22
Problem number: 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{y^2 + yx - y'x = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 22

```
dsolve([(y(x)^2+x*y(x))-x*diff(y(x),x)= 0,y(1) = 1],y(x), singsol=all)
```

$$y = \frac{e^x}{\mathrm{Ei}_1(-x) + e - \mathrm{Ei}_1(-1)}$$

✓ Solution by Mathematica

Time used: 0.161 (sec). Leaf size: 19

```
DSolve[{(y[x]^2+x*y[x])-x*y'[x]== 0,{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x}{-\mathrm{ExpIntegralEi}(x) + \mathrm{ExpIntegralEi}(1) + e}$$

2.25 problem 51

Internal problem ID [5260]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 4. Equations of first order and first degree (Variable separable). Supplemetary problems. Page 22
Problem number: 51.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Riccati]`

$$\boxed{y' + 2(2x + 3y)^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x)= -2*(2*x+3*y(x))^2,y(x), singsol=all)
```

$$y = -\frac{\left(2\sqrt{3}\,x + \tanh\left(2(-x + c_1)\sqrt{3}\right)\right)\sqrt{3}}{9}$$

✓ Solution by Mathematica

Time used: 0.142 (sec). Leaf size: 59

```
DSolve[y'[x]==-2*(2*x+3*y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{9}\left(-6x - \frac{6}{\sqrt{3} + 12c_1 e^{4\sqrt{3}x}} + \sqrt{3}\right)$$
$$y(x) \to \frac{1}{9}\left(\sqrt{3} - 6x\right)$$

2.26 problem 52

Internal problem ID [5261]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 4. Equations of first order and first degree (Variable separable). Supplemetary problems. Page 22
Problem number: 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, ‘_with_symmetry_[F(x),G(y)]‘]]`

$$\boxed{-2\sin(y) + (2x - 4\sin(y) - 3)\cos(y)\, y' = -x - 3}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 22

```
dsolve((x-2*sin(y(x))+3)+(2*x-4*sin(y(x))-3)*cos(y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \arcsin\left(\frac{9\,\mathrm{LambertW}\left(\frac{\mathrm{e}^{-\frac{8x}{9}}\mathrm{e}^{-\frac{1}{3}c_1}}{9}\right)}{8} + \frac{x}{2} + \frac{3}{8}\right)$$

✓ Solution by Mathematica

Time used: 60.95 (sec). Leaf size: 73

```
DSolve[(x-2*Sin[y[x]]+3)+(2*x-4*Sin[y[x]]-3)*Cos[y[x]]*y'[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to \arcsin\left(\frac{1}{8}\left(9W\left(-\frac{1}{9}e^{-\frac{2}{9}(4x+3-8c_1)}\right) + 4x + 3\right)\right)$$
$$y(x) \to \arcsin\left(\frac{1}{8}\left(9W\left(-\frac{1}{9}e^{-\frac{2}{9}(4x+3-8c_1)}\right) + 4x + 3\right)\right)$$

3 Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33

3.1 problem 23 (a)

Internal problem ID [5262]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 23 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{-y - y'x = -x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2-y(x))-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{\frac{x^3}{3} + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 19

```
DSolve[(x^2-y[x])-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{3} + \frac{c_1}{x}$$

3.2 problem 23 (d)

Internal problem ID [5263]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 23 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _Bernoulli]`

$$\boxed{y^2 + 2xyy' = -x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve((x^2+y(x)^2)+2*x*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = -\frac{\sqrt{3}\,\sqrt{x\,(-x^3 + 3c_1)}}{3x}$$
$$y = \frac{\sqrt{3}\,\sqrt{x\,(-x^3 + 3c_1)}}{3x}$$

✓ Solution by Mathematica

Time used: 0.233 (sec). Leaf size: 60

```
DSolve[(x^2+y[x]^2)+2*x*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-x^3 + 3c_1}}{\sqrt{3}\sqrt{x}}$$
$$y(x) \to \frac{\sqrt{-x^3 + 3c_1}}{\sqrt{3}\sqrt{x}}$$

3.3 problem 23 (e)

Internal problem ID [5264]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 23 (e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' \sin(x) + \cos(x)\, y = -x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve((x+y(x)*cos(x))+sin(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{-\frac{x^2}{2} + c_1}{\sin(x)}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 19

```
DSolve[(x+y[x]*Cos[x])+Sin[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}\left(x^2 - 2c_1\right)\csc(x)$$

3.4 problem 23 (h)

Internal problem ID [5265]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 23 (h).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{3y+(3x+4y+5)\,y'=-2x-4}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 33

```
dsolve((2*x+3*y(x)+4)+(3*x+4*y(x)+5)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y=-2-\frac{\frac{3(x-1)c_1}{4}+\frac{\sqrt{(x-1)^2c_1^2+8}}{4}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.134 (sec). Leaf size: 61

```
DSolve[(2*x+3*y[x]+4)+(3*x+4*y[x]+5)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{4}\left(-\sqrt{x^2-2x+25+16c_1}-3x-5\right)$$
$$y(x)\to\frac{1}{4}\left(\sqrt{x^2-2x+25+16c_1}-3x-5\right)$$

3.5 problem 23 (i)

Internal problem ID [5266]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 23 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _exact, _rational]`

$$\boxed{4y^3x^3 + \left(3y^2x^4 - \frac{1}{y}\right)y' = -\frac{1}{x}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve((4*x^3*y(x)^3+1/x)+(3*x^4*y(x)^2-1/y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{1}{\left(-\frac{3x^4}{\operatorname{LambertW}(-3c_1x^7)}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 4.154 (sec). Leaf size: 108

```
DSolve[(4*x^3*y[x]^3+1/x)+(3*x^4*y[x]^2-1/y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{\sqrt[3]{-\frac{1}{3}}\sqrt[3]{W\left(-3e^{-3c_1}x^7\right)}}{x^{4/3}}$$

$$y(x) \to -\frac{\sqrt[3]{W\left(-3e^{-3c_1}x^7\right)}}{\sqrt[3]{3}x^{4/3}}$$

$$y(x) \to -\frac{(-1)^{2/3}\sqrt[3]{W\left(-3e^{-3c_1}x^7\right)}}{\sqrt[3]{3}x^{4/3}}$$

$$y(x) \to 0$$

3.6 problem 23 (j)

Internal problem ID [5267]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 23 (j).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$\boxed{2uv + \left(u^2 + v^2\right) v' = -2u^2}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 417

```
dsolve(2*(u^2+u*v(u))+(u^2+v(u)^2)*diff(v(u),u)=0,v(u), singsol=all)
```

$$v(u) = \frac{\frac{\left(4-8u^3c_1^{\frac{3}{2}}+4\sqrt{8u^6c_1^3-4u^3c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}{2} - \frac{2u^2c_1}{\left(4-8u^3c_1^{\frac{3}{2}}+4\sqrt{8u^6c_1^3-4u^3c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}}{\sqrt{c_1}}$$

$$v(u) = \frac{-\frac{\left(4-8u^3c_1^{\frac{3}{2}}+4\sqrt{8u^6c_1^3-4u^3c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}{4} + \frac{u^2c_1}{\left(4-8u^3c_1^{\frac{3}{2}}+4\sqrt{8u^6c_1^3-4u^3c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(4-8u^3c_1^{\frac{3}{2}}+4\sqrt{8u^6c_1^3-4u^3c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}{2} + \frac{\cdots}{\left(4-8u^3c_1^{\frac{3}{2}}+\cdots\right.}\right.}{2}}{\sqrt{c_1}}$$

$$v(u) = \frac{-\frac{\left(4-8u^3c_1^{\frac{3}{2}}+4\sqrt{8u^6c_1^3-4u^3c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}{4} + \frac{u^2c_1}{\left(4-8u^3c_1^{\frac{3}{2}}+4\sqrt{8u^6c_1^3-4u^3c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(4-8u^3c_1^{\frac{3}{2}}+4\sqrt{8u^6c_1^3-4u^3c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}{2} + \frac{\cdots}{\left(4-8u^3c_1^{\frac{3}{2}}+\cdots\right.}\right.}{2}}{\sqrt{c_1}}$$

✓ Solution by Mathematica

Time used: 15.565 (sec). Leaf size: 593

```
DSolve[2*(u^2+u*v[u])+(u^2+v[u]^2)*v'[u]==0,v[u],u,IncludeSingularSolutions -> True]
```

$$v(u) \to \frac{\sqrt[3]{-2u^3+\sqrt{8u^6-4e^{3c_1}u^3+e^{6c_1}}+e^{3c_1}}}{\sqrt[3]{2}} - \frac{\sqrt[3]{2}u^2}{\sqrt[3]{-2u^3+\sqrt{8u^6-4e^{3c_1}u^3+e^{6c_1}}+e^{3c_1}}}$$

$$v(u) \to \frac{\sqrt[3]{2}\left(2+2i\sqrt{3}\right)u^2+i2^{2/3}\left(\sqrt{3}+i\right)\left(-2u^3+\sqrt{8u^6-4e^{3c_1}u^3+e^{6c_1}}+e^{3c_1}\right)^{2/3}}{4\sqrt[3]{-2u^3+\sqrt{8u^6-4e^{3c_1}u^3+e^{6c_1}}+e^{3c_1}}}$$

$$v(u) \to \frac{\left(1-i\sqrt{3}\right)u^2}{2^{2/3}\sqrt[3]{-2u^3+\sqrt{8u^6-4e^{3c_1}u^3+e^{6c_1}}+e^{3c_1}}} - \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{-2u^3+\sqrt{8u^6-4e^{3c_1}u^3+e^{6c_1}}+e^{3c_1}}}{2\sqrt[3]{2}}$$

$$v(u) \to \sqrt[3]{\sqrt{2}\sqrt{u^6}-u^3} - \frac{u^2}{\sqrt[3]{\sqrt{2}\sqrt{u^6}-u^3}}$$

$$v(u) \to \frac{\left(1-i\sqrt{3}\right)u^2+\left(-1-i\sqrt{3}\right)\left(\sqrt{2}\sqrt{u^6}-u^3\right)^{2/3}}{2\sqrt[3]{\sqrt{2}\sqrt{u^6}-u^3}}$$

$$v(u) \to \frac{\left(1+i\sqrt{3}\right)u^2+i\left(\sqrt{3}+i\right)\left(\sqrt{2}\sqrt{u^6}-u^3\right)^{2/3}}{2\sqrt[3]{\sqrt{2}\sqrt{u^6}-u^3}}$$

3.7 problem 23 (k)

Internal problem ID [5268]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 23 (k).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{x\sqrt{x^2+y^2}-y+\left(y\sqrt{x^2+y^2}-x\right)y'=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve((x*sqrt(x^2+y(x)^2)-y(x))+(y(x)*sqrt(x^2+y(x)^2)-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$\frac{(x^2+y^2)^{\frac{3}{2}}}{3}-yx+c_1=0$$

✓ Solution by Mathematica

Time used: 30.753 (sec). Leaf size: 319

```
DSolve[(x*Sqrt[x^2+y[x]^2]-y[x])+(y[x]*Sqrt[x^2+y[x]^2]-x)*y'[x]==0,y[x],x,IncludeSingularSo
```

$$y(x) \to \text{Root}\left[\#1^6 + 3\#1^4x^2 + \#1^2\left(3x^4 - 9x^2\right) - 18\#1c_1x + x^6 - 9c_1{}^2\&, 1\right]$$
$$y(x) \to \text{Root}\left[\#1^6 + 3\#1^4x^2 + \#1^2\left(3x^4 - 9x^2\right) - 18\#1c_1x + x^6 - 9c_1{}^2\&, 2\right]$$
$$y(x) \to \text{Root}\left[\#1^6 + 3\#1^4x^2 + \#1^2\left(3x^4 - 9x^2\right) - 18\#1c_1x + x^6 - 9c_1{}^2\&, 3\right]$$
$$y(x) \to \text{Root}\left[\#1^6 + 3\#1^4x^2 + \#1^2\left(3x^4 - 9x^2\right) - 18\#1c_1x + x^6 - 9c_1{}^2\&, 4\right]$$
$$y(x) \to \text{Root}\left[\#1^6 + 3\#1^4x^2 + \#1^2\left(3x^4 - 9x^2\right) - 18\#1c_1x + x^6 - 9c_1{}^2\&, 5\right]$$
$$y(x) \to \text{Root}\left[\#1^6 + 3\#1^4x^2 + \#1^2\left(3x^4 - 9x^2\right) - 18\#1c_1x + x^6 - 9c_1{}^2\&, 6\right]$$

3.8 problem 23 (m)

Internal problem ID [5269]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 23 (m).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{y - (y - x + 3)\, y' = -x - 1}$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 32

```
dsolve((x+y(x)+1)-(y(x)-x+3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = -2 - \frac{-(x-1)\, c_1 + \sqrt{2\,(x-1)^2\, c_1^2 + 1}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.125 (sec). Leaf size: 59

```
DSolve[(x+y[x]+1)-(y[x]-x+3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -i\sqrt{-2x^2 + 4x - 9 - c_1} + x - 3$$
$$y(x) \to i\sqrt{-2x^2 + 4x - 9 - c_1} + x - 3$$

3.9 problem 23 (o)

Internal problem ID [5270]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 23 (o).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{y^2 - \frac{y}{x(x+y)} + \left(\frac{1}{x+y} + 2(1+x)\,y\right)y' = -2}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 130

```
dsolve((y(x)^2- y(x)/(x*(x+y(x)))+2)+( 1/(x+y(x)) + 2*y(x)*(1+x))*diff(y(x),x)=0,y(x), sings
```

$$y = \left(-x\,\mathrm{e}^{\mathrm{RootOf}\left(x^3\mathrm{e}^{2\_Z}+x^2\mathrm{e}^{2\_Z}-2x^3\mathrm{e}^{\_Z}+c_1\mathrm{e}^{2\_Z}-\_Z\mathrm{e}^{2\_Z}+2x\,\mathrm{e}^{2\_Z}-2x^2\mathrm{e}^{\_Z}+x^3+x^2\right)} + x\right)\mathrm{e}^{-\mathrm{RootOf}\left(x^3\mathrm{e}^{2\_Z}+x^2\mathrm{e}^{2\_Z}-2x^3\mathrm{e}^{\_Z}+c_1\mathrm{e}^{2\_Z}-\_Z\mathrm{e}^{2\_Z}+2x\,\mathrm{e}^{2\_Z}-2x^2\mathrm{e}^{\_Z}+x^3+x^2\right)}$$

✓ Solution by Mathematica

Time used: 0.43 (sec). Leaf size: 29

```
DSolve[(y[x]^2- y[x]/(x*(x+y[x]))+2)+( 1/(x+y[x]) + 2*y[x]*(1+x))*y'[x]==0,y[x],x,IncludeSin
```

$$\mathrm{Solve}\left[xy(x)^2 + y(x)^2 + \log(y(x)+x) + 2x - \log(x) = c_1, y(x)\right]$$

3.10 problem 23 (p)

Internal problem ID [5271]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 23 (p).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$2xy\, e^{y x^2} + y^2 e^{x y^2} + \left(x^2 e^{y x^2} + 2xy\, e^{x y^2} - 2y\right) y' = -1$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
dsolve((2*x*y(x)*exp(x^2*y(x))+ y(x)^2*exp(x*y(x)^2)+1)+(x^2*exp(x^2*y(x))+ 2*x*y(x)*exp(x*y
```

$$y = \frac{\operatorname{RootOf}\left(e^{\_Z} x^4 + e^{\frac{\_Z^2}{x^3}} x^4 + c_1 x^4 + x^5 - \_Z^2\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.385 (sec). Leaf size: 30

```
DSolve[(2*x*y[x]*Exp[x^2*y[x]]+ y[x]^2*Exp[x*y[x]^2]+1)+(x^2*Exp[x^2*y[x]]+ 2*x*y[x]*Exp[x*y
```

$$\text{Solve}\left[e^{x^2 y(x)} - y(x)^2 + e^{x y(x)^2} + x = c_1, y(x)\right]$$

3.11 problem 24 (p)

Internal problem ID [5272]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 24 (p).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y(x-2y)-x^2y'=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(y(x)*(x-2*y(x))-x^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y=\frac{x}{2\ln(x)+c_1}$$

✓ Solution by Mathematica

Time used: 0.145 (sec). Leaf size: 21

```
DSolve[y[x]*(x-2*y[x])-x^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x}{2\log(x)+c_1}$$
$$y(x)\to 0$$

3.12 problem 24 (c)

Internal problem ID [5273]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 24 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{y^2 + xyy' = -x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve((x^2+y(x)^2)+x*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = -\frac{\sqrt{-2x^4 + 4c_1}}{2x}$$
$$y = \frac{\sqrt{-2x^4 + 4c_1}}{2x}$$

✓ Solution by Mathematica

Time used: 0.198 (sec). Leaf size: 46

```
DSolve[(x^2+y[x]^2)+x*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-\frac{x^4}{2} + c_1}}{x}$$
$$y(x) \to \frac{\sqrt{-\frac{x^4}{2} + c_1}}{x}$$

3.13 problem 24 (d)

Internal problem ID [5274]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 24 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, _Bernoulli]`

$$\boxed{y^2 + 2xyy' = -x^2}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 49

```
dsolve((x^2+y(x)^2)+2*x*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = -\frac{\sqrt{3}\,\sqrt{x\left(-x^3+3c_1\right)}}{3x}$$
$$y = \frac{\sqrt{3}\,\sqrt{x\left(-x^3+3c_1\right)}}{3x}$$

✓ Solution by Mathematica

Time used: 0.207 (sec). Leaf size: 60

```
DSolve[(x^2+y[x]^2)+2*x*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-x^3+3c_1}}{\sqrt{3}\sqrt{x}}$$
$$y(x) \to \frac{\sqrt{-x^3+3c_1}}{\sqrt{3}\sqrt{x}}$$

3.14 problem 24 (g)

Internal problem ID [5275]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 24 (g).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{-\sqrt{a^2 - x^2}\, y' = -1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(1-(sqrt(a^2-x^2))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \arctan\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + c_1$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 24

```
DSolve[1-(Sqrt[a^2-x^2])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arctan\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + c_1$$

3.15 problem 24 (L)

Internal problem ID [5276]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 24 (L).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y-(x-y-3)\,y'=-x-1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve((x+y(x)+1)-(x-y(x)-3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y=-2-\tan\left(\operatorname{RootOf}\left(2\_Z+\ln\left(\frac{1}{\cos(\_Z)^2}\right)+2\ln(x-1)+2c_1\right)\right)(x-1)$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 58

```
DSolve[(x+y[x]+1)-(x-y[x]-3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2\arctan\left(\frac{y(x)+x+1}{y(x)-x+3}\right)+\log\left(\frac{x^2+y(x)^2+4y(x)-2x+5}{2(x-1)^2}\right)+2\log(x-1)+c_1=0,y(x)\right]$$

3.16 problem 25 (a)

Internal problem ID [5277]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 25 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _rational, _Bernoulli]`

$$\boxed{-y^2 + yy' = x^2 - x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve((x-x^2-y(x)^2)+y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \sqrt{c_1 e^{2x} - x^2}$$
$$y = -\sqrt{c_1 e^{2x} - x^2}$$

✓ Solution by Mathematica

Time used: 4.613 (sec). Leaf size: 47

```
DSolve[(x-x^2-y[x]^2)+y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^2 + c_1 e^{2x}}$$
$$y(x) \to \sqrt{-x^2 + c_1 e^{2x}}$$

3.17 problem 25 (b)

Internal problem ID [5278]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 25 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + 2y = 3x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve((2*y(x)-3*x)+x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = x + \frac{c_1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 13

```
DSolve[(2*y[x]-3*x)+x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + \frac{c_1}{x^2}$$

3.18 problem 25 (c)

Internal problem ID [5279]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 25 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$\boxed{-y^2 + 2xyy' = -x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve((x-y(x)^2)+2*x*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \sqrt{-\ln(x)\, x + c_1 x}$$
$$y = -\sqrt{-\ln(x)\, x + c_1 x}$$

✓ Solution by Mathematica

Time used: 0.212 (sec). Leaf size: 44

```
DSolve[(x-y[x]^2)+2*x*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x}\sqrt{-\log(x) + c_1}$$
$$y(x) \to \sqrt{x}\sqrt{-\log(x) + c_1}$$

3.19 problem 25 (d)

Internal problem ID [5280]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 25 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Riccati]`

$$-y - 3x^2(x^2 + y^2) + y'x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve((-y(x)-3*x^2*(x^2+y(x)^2))+x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \tan(x^3 + 3c_1)\, x$$

✓ Solution by Mathematica

Time used: 0.184 (sec). Leaf size: 14

```
DSolve[(-y[x]-3*x^2*(x^2+y[x]^2))+x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \tan(x^3 + c_1)$$

3.20 problem 25 (e)

Internal problem ID [5281]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 25 (e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{-y'x + y = \ln(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve((y(x)-ln(x))-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = c_1 x + \ln(x) + 1$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 13

```
DSolve[(y[x]-Log[x])-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(x) + c_1 x + 1$$

3.21 problem 25 (f)

Internal problem ID [5282]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 25 (f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y^2 - 2xyy' = -3x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve((3*x^2+y(x)^2)-2*x*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \sqrt{c_1x + 3x^2}$$
$$y = -\sqrt{c_1x + 3x^2}$$

✓ Solution by Mathematica

Time used: 0.182 (sec). Leaf size: 42

```
DSolve[(3*x^2+y[x]^2)-2*x*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x}\sqrt{3x + c_1}$$
$$y(x) \to \sqrt{x}\sqrt{3x + c_1}$$

3.22 problem 25 (g)

Internal problem ID [5283]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 25 (g).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{yx - 2y^2 - (x^2 - 3yx)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve((x*y(x)-2*y(x)^2)-(x^2-3*x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \mathrm{e}^{\mathrm{LambertW}\left(-\frac{\mathrm{e}^{\frac{c_1}{3}} x^{\frac{1}{3}}}{3}\right) - \frac{c_1}{3} - \frac{\ln(x)}{3}} x$$

✓ Solution by Mathematica

Time used: 4.722 (sec). Leaf size: 35

```
DSolve[(x*y[x]-2*y[x]^2)-(x^2-3*x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{3W\left(-\frac{1}{3}e^{-\frac{c_1}{3}}\sqrt[3]{x}\right)}$$

$$y(x) \to 0$$

3.23 problem 25 (h)

Internal problem ID [5284]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 25 (h).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$y-(x-y)\,y'=-x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve((x+y(x))-(x-y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y=\tan\left(\operatorname{RootOf}\left(-2\_Z+\ln\left(\frac{1}{\cos(\_Z)^2}\right)+2\ln(x)+2c_1\right)\right)x$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 36

```
DSolve[(x+y[x])-(x-y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right)-\arctan\left(\frac{y(x)}{x}\right)=-\log(x)+c_1,y(x)\right]$$

3.24 problem 25 (L)

Internal problem ID [5285]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 25 (L).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$\boxed{2y - 3xy^2 - y'x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((2*y(x)-3*x*y(x)^2)-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{x^2}{x^3 + c_1}$$

✓ Solution by Mathematica

Time used: 0.145 (sec). Leaf size: 22

```
DSolve[(2*y[x]-3*x*y[x]^2)-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{x^3 + c_1}$$
$$y(x) \to 0$$

3.25 problem 25 (j)

Internal problem ID [5286]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 25 (j).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y + x\left(yx^2 - 1\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 789

```
dsolve(y(x)+x*(x^2*y(x)-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{\left(\frac{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} - \frac{c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}\right)^2 + 3}{2x^2}$$

$$y = \frac{\left(-\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)^6 \left(\frac{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} - \frac{c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}\right)^2 + 3}{2x^2}$$

$$y = \frac{\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^6 \left(\frac{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} - \frac{c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}\right)^2 + 3}{2x^2}$$

$$y = \frac{\frac{\left(-\frac{4\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} + \frac{4c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}} - 4i\sqrt{3}\left(\frac{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} + \frac{c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}\right)\right)^2}{64} + 3}{2x^2}$$

$$y = \frac{\frac{\left(-\frac{4\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} + \frac{4c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}} + 4i\sqrt{3}\left(\frac{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} + \frac{c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}\right)\right)^2}{64} + 3}{2x^2}$$

$$y = \frac{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)^6 \left(-\frac{4\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} + \frac{4c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}} - 4i\sqrt{3}\left(\frac{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} + \frac{c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}\right)\right)^2}{64} + 3}{2x^2}$$

$$y = \frac{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)^6 \left(-\frac{4\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} + \frac{4c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}} + 4i\sqrt{3}\left(\frac{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} + \frac{c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}\right)\right)^2}{64} + 3}{2x^2}$$

$$y = \frac{\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^6 \left(-\frac{4\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} + \frac{4c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}} - 4i\sqrt{3}\left(\frac{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} + \frac{c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}\right)\right)^2}{64} + 3}{2x^2}$$

$$\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^6 \left(-\frac{4\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} + \frac{4c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}} + 4i\sqrt{3}\left(\frac{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} + \frac{c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}\right)\right)^2 \quad + 3$$

✓ Solution by Mathematica

Time used: 56.665 (sec). Leaf size: 452

```
DSolve[y[x]+x*(x^2*y[x]-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-6c_1}\sqrt[3]{-2e^{12c_1}x^6+2\sqrt{-e^{24c_1}x^6\left(-x^6+e^{6c_1}\right)}+e^{18c_1}}+\frac{e^{6c_1}}{\sqrt[3]{-2e^{12c_1}x^6+2\sqrt{-e^{24c_1}x^6\left(-x^6+e^{6c_1}\right)}+e^{18c}}}}{2x^2}$$

$$y(x) \to \frac{i\left(\sqrt{3}+i\right)e^{-6c_1}\sqrt[3]{-2e^{12c_1}x^6+2\sqrt{-e^{24c_1}x^6\left(-x^6+e^{6c_1}\right)}+e^{18c_1}}-\frac{\left(1+i\sqrt{3}\right)e^{6c_1}}{\sqrt[3]{-2e^{12c_1}x^6+2\sqrt{-e^{24c_1}x^6\left(-x^6+e\right.}}}}{4x^2}$$

$$y(x) \to \frac{-\left(\left(1+i\sqrt{3}\right)e^{-6c_1}\sqrt[3]{-2e^{12c_1}x^6+2\sqrt{-e^{24c_1}x^6\left(-x^6+e^{6c_1}\right)}+e^{18c_1}}\right)+\frac{i\left(\sqrt{3}+i\right)e^{6c_1}}{\sqrt[3]{-2e^{12c_1}x^6+2\sqrt{-e^{24c_1}x^6\left(-\right.}}}}{4x^2}$$

$$y(x) \to 0$$

$$y(x) \to \frac{3}{2x^2}$$

3.26 problem 25 (k)

Internal problem ID [5287]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 25 (k).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{yx^3 + y + (x + 4y^4x + 8y^3)\, y' = -2x^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve((y(x)+x^3*y(x)+2*x^2)+(x+4*x*y(x)^4+8*y(x)^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$-\frac{x^3}{3} - \ln (yx + 2) - y^4 + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.188 (sec). Leaf size: 25

```
DSolve[(y[x]+x^3*y[x]+2*x^2)+(x+4*x*y[x]^4+8*y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolution
```

$$\text{Solve}\left[\frac{x^3}{3} + y(x)^4 + \log(xy(x) + 2) = c_1, y(x)\right]$$

3.27 problem 26 (a)

Internal problem ID [5288]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 26 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - y = x^2 e^x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve((-y(x)-x^2*exp(x))+x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = (e^x + c_1)\,x$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 13

```
DSolve[(-y[x]-x^2*Exp[x])+x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(e^x + c_1)$$

3.28 problem 26 (b)

Internal problem ID [5289]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 26 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^2 - (x^2 + x)\, y' = -1}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(1+y(x)^2=(x+x^2)*diff(y(x),x),y(x), singsol=all)
```

$$y = \tan(-\ln(x+1) + \ln(x) + c_1)$$

✓ Solution by Mathematica

Time used: 0.228 (sec). Leaf size: 31

```
DSolve[1+y[x]^2==(x+x^2)*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan(\log(x) - \log(x+1) + c_1)$$
$$y(x) \to -i$$
$$y(x) \to i$$

3.29 problem 26 (c)

Internal problem ID [5290]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 26 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + 2y = x^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((2*y(x)-x^3)+x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{\frac{x^5}{5} + c_1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 19

```
DSolve[(2*y[x]-x^3)+x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{5} + \frac{c_1}{x^2}$$

3.30 problem 26 (d)

Internal problem ID [5291]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 26 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$\boxed{y + (-x + y^2)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(y(x)+(-x+y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{c_1}{2} - \frac{\sqrt{c_1^2 - 4x}}{2}$$
$$y = \frac{c_1}{2} + \frac{\sqrt{c_1^2 - 4x}}{2}$$

✓ Solution by Mathematica

Time used: 0.276 (sec). Leaf size: 54

```
DSolve[y[x]+(-x+y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(c_1 - \sqrt{-4x + c_1{}^2}\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{-4x + c_1{}^2} + c_1\right)$$
$$y(x) \to 0$$

3.31 problem 26 (e)

Internal problem ID [5292]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 26 (e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$3y^3 - yx - (x^2 + 6xy^2)\, y' = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve((3*y(x)^3-x*y(x))-(x^2+6*x*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{e^{-\frac{\mathrm{LambertW}\left(\frac{6\,e^{3c_1}}{x^3}\right)}{2} + \frac{3c_1}{2}}}{x}$$

✓ Solution by Mathematica

Time used: 3.943 (sec). Leaf size: 69

```
DSolve[(3*y[x]^3-x*y[x])-(x^2+6*x*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{x}\sqrt{W\left(\frac{6e^{3c_1}}{x^3}\right)}}{\sqrt{6}}$$

$$y(x) \to \frac{\sqrt{x}\sqrt{W\left(\frac{6e^{3c_1}}{x^3}\right)}}{\sqrt{6}}$$

$$y(x) \to 0$$

3.32 problem 26 (f)

Internal problem ID [5293]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 26 (f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{3x^2y^2 + 4(yx^3 - 3)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.703 (sec). Leaf size: 30

```
dsolve((3*x^2*y(x)^2)+4*(x^3*y(x)-3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{\operatorname{RootOf}\left(\_Z^{12}c_1 + 4\_Z^3c_1 - x^3\right)^9 + 4}{x^3}$$

✓ Solution by Mathematica

Time used: 60.296 (sec). Leaf size: 1175

```
DSolve[(3*x^2*y[x]^2)+4*(x^3*y[x]-3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{x^3} - \frac{\sqrt{-\frac{3^{2/3}c_1}{\sqrt[3]{\sqrt{3}\sqrt{c_1{}^2\left(27+c_1x^9\right)}-9c_1}} + \frac{6}{x^6} + \frac{\sqrt[3]{3\sqrt{3}\sqrt{c_1{}^2\left(27+c_1x^9\right)}-27c_1}}{x^3}}}{\sqrt{6}} - \frac{1}{2}\sqrt{\frac{2c_1}{\sqrt[3]{3\sqrt{3}\sqrt{c_1{}^2\left(27+c_1x^9\right)}-27c_1}} + \frac{8}{x^6} - \frac{2\sqrt[3]{\sqrt{3}\sqrt{c_1{}^2\left(27+c_1x^9\right)}-9c_1}}{3^{2/3}x^3} - \frac{\cdots}{x^9\sqrt{-\frac{3^{2/3}c\cdots}{\sqrt[3]{\sqrt{3}\sqrt{c_1{}^2\left(27\cdots\right.}}}}}}$$

$$y(x) \to \frac{1}{x^3} - \frac{\sqrt{-\frac{3^{2/3}c_1}{\sqrt[3]{\sqrt{3}\sqrt{c_1{}^2\left(27+c_1x^9\right)}-9c_1}} + \frac{6}{x^6} + \frac{\sqrt[3]{3\sqrt{3}\sqrt{c_1{}^2\left(27+c_1x^9\right)}-27c_1}}{x^3}}}{\sqrt{6}} + \frac{1}{2}\sqrt{\frac{2c_1}{\sqrt[3]{3\sqrt{3}\sqrt{c_1{}^2\left(27+c_1x^9\right)}-27c_1}} + \frac{8}{x^6} - \frac{2\sqrt[3]{\sqrt{3}\sqrt{c_1{}^2\left(27+c_1x^9\right)}-9c_1}}{3^{2/3}x^3} - \frac{\cdots}{x^9\sqrt{-\frac{3^{2/3}c\cdots}{\sqrt[3]{\sqrt{3}\sqrt{c_1{}^2\left(27\cdots\right.}}}}}}$$

$$y(x) \to \frac{1}{x^3} + \frac{\sqrt{-\frac{3^{2/3}c_1}{\sqrt[3]{\sqrt{3}\sqrt{c_1{}^2\left(27+c_1x^9\right)}-9c_1}} + \frac{6}{x^6} + \frac{\sqrt[3]{3\sqrt{3}\sqrt{c_1{}^2\left(27+c_1x^9\right)}-27c_1}}{x^3}}}{\sqrt{6}} - \frac{1}{2}\sqrt{\frac{2c_1}{\sqrt[3]{3\sqrt{3}\sqrt{c_1{}^2\left(27+c_1x^9\right)}-27c_1}} + \frac{8}{x^6} - \frac{2\sqrt[3]{\sqrt{3}\sqrt{c_1{}^2\left(27+c_1x^9\right)}-9c_1}}{3^{2/3}x^3} + \frac{\cdots}{x^9\sqrt{-\frac{3^{2/3}c\cdots}{\sqrt[3]{\sqrt{3}\sqrt{c_1{}^2\left(27\cdots\right.}}}}}}$$

$$y(x) \to \frac{1}{x^3} + \frac{\sqrt{-\frac{3^{2/3}c_1}{\sqrt[3]{\sqrt{3}\sqrt{c_1{}^2\left(27+c_1x^9\right)}-9c_1}} + \frac{6}{x^6} + \frac{\sqrt[3]{3\sqrt{3}\sqrt{c_1{}^2\left(27+c_1x^9\right)}-27c_1}}{x^3}}}{\sqrt{6}} + \frac{1}{2}\sqrt{\frac{2c_1}{\sqrt[3]{3\sqrt{3}\sqrt{c_1{}^2\left(27+c_1x^9\right)}-27c_1}} + \frac{8}{x^6} - \frac{2\sqrt[3]{\sqrt{3}\sqrt{c_1{}^2\left(27+c_1x^9\right)}-9c_1}}{3^{2/3}x^3} + \cdots}$$

3.33 problem 26 (g)

Internal problem ID [5294]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 26 (g).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y(x+y) - x^2 y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(y(x)*(x+y(x))-x^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = -\frac{x}{\ln(x) - c_1}$$

✓ Solution by Mathematica

Time used: 0.14 (sec). Leaf size: 21

```
DSolve[y[x]*(x+y[x])-x^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{-\log(x) + c_1}$$
$$y(x) \to 0$$

3.34 problem 26 (h)

Internal problem ID [5295]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 26 (h).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{2y + 3xy^2 + (x + 2yx^2)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 43

```
dsolve((2*y(x)+3*x*y(x)^2)+(x+2*x^2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{-x + \sqrt{4c_1x + x^2}}{2x^2}$$
$$y = -\frac{x + \sqrt{4c_1x + x^2}}{2x^2}$$

✓ Solution by Mathematica

Time used: 0.526 (sec). Leaf size: 69

```
DSolve[(2*y[x]+3*x*y[x]^2)+(x+2*x^2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^{3/2} + \sqrt{x^2(x + 4c_1)}}{2x^{5/2}}$$
$$y(x) \to \frac{-x^{3/2} + \sqrt{x^2(x + 4c_1)}}{2x^{5/2}}$$

3.35 problem 26 (i)

Internal problem ID [5296]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 26 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{y(y^2-2x^2)+x(2y^2-x^2)\,y'=0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 69

```
dsolve(y(x)*(y(x)^2-2*x^2)+x*(2*y(x)^2-x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y=\sqrt{\frac{\frac{c_1x^3}{2}-\frac{\sqrt{c_1^2x^6+4}}{2}}{c_1x^3}}\,x$$

$$y=\sqrt{\frac{\frac{c_1x^3}{2}+\frac{\sqrt{c_1^2x^6+4}}{2}}{c_1x^3}}\,x$$

✓ Solution by Mathematica

Time used: 11.861 (sec). Leaf size: 277

```
DSolve[y[x]*(y[x]^2-2*x^2)+x*(2*y[x]^2-x^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -\frac{\sqrt{x^2 - \frac{\sqrt{x^6 - 4e^{2c_1}}}{x}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{x^2 - \frac{\sqrt{x^6 - 4e^{2c_1}}}{x}}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{\frac{x^3 + \sqrt{x^6 - 4e^{2c_1}}}{x}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{\frac{x^3 + \sqrt{x^6 - 4e^{2c_1}}}{x}}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{x^2 - \frac{\sqrt{x^6}}{x}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{x^2 - \frac{\sqrt{x^6}}{x}}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{\frac{\sqrt{x^6} + x^3}{x}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{\frac{\sqrt{x^6} + x^3}{x}}}{\sqrt{2}}$$

3.36 problem 27

Internal problem ID [5297]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 5. Equations of first order and first degree (Exact equations). Supplemetary problems. Page 33
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 7

```
dsolve(-y(x)+x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = c_1 x$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 14

```
DSolve[-y[x]+x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x$$
$$y(x) \to 0$$

4 Chapter 6. Equations of first order and first degree (Linear equations). Supplemetary problems. Page 39

4.1 problem 19 (a)

Internal problem ID [5298]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 6. Equations of first order and first degree (Linear equations). Supplemetary problems. Page 39
Problem number: 19 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + y = 2x + 2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+y(x)=2+2*x,y(x), singsol=all)
```

$$y = 2x + c_1 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 17

```
DSolve[y'[x]+y[x]==2+2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x + c_1 e^{-x}$$

4.2 problem 19 (c)

Internal problem ID [5299]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 6. Equations of first order and first degree (Linear equations). Supplemetary problems. Page 39
Problem number: 19 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y - yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)-y(x)=x*y(x),y(x), singsol=all)
```

$$y = c_1 \mathrm{e}^{\frac{x(x+2)}{2}}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 23

```
DSolve[y'[x]-y[x]==x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{\frac{1}{2}x(x+2)}$$
$$y(x) \to 0$$

4.3 problem 19 (d)

Internal problem ID [5300]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 6. Equations of first order and first degree (Linear equations). Supplemetary problems. Page 39
Problem number: 19 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{-3y + y'x = (x-2)\,\mathrm{e}^x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve((-2*y(x)-(x-2)*exp(x))+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y = \left(-\frac{\mathrm{e}^x}{6x^2} - \frac{\mathrm{e}^x}{6x} - \frac{\mathrm{Ei}_1(-x)}{6} + \frac{2\,\mathrm{e}^x}{3x^3} + c_1\right)x^3$$

✓ Solution by Mathematica

Time used: 0.109 (sec). Leaf size: 33

```
DSolve[(-2*y[x]-(x-2)*Exp[x])+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}x^3\left(\mathrm{ExpIntegralEi}(x) - \frac{e^x(x^2+x-4)}{x^3} + 6c_1\right)$$

4.4 problem 19 (e)

Internal problem ID [5301]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 6. Equations of first order and first degree (Linear equations). Supplemetary problems. Page 39
Problem number: 19 (e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{i' - 6i = 10\sin(2t)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(i(t),t)-6*i(t)=10*sin(2*t),i(t), singsol=all)
```

$$i(t) = -\frac{\cos(2t)}{2} - \frac{3\sin(2t)}{2} + e^{6t}c_1$$

✓ Solution by Mathematica

Time used: 0.094 (sec). Leaf size: 28

```
DSolve[i'[t]-6*i[t]==10*Sin[2*t],i[t],t,IncludeSingularSolutions -> True]
```

$$i(t) \to -\frac{1}{2}\cos(2t) + c_1 e^{6t} - 3\sin(t)\cos(t)$$

4.5 problem 19 (f)

Internal problem ID [5302]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 6. Equations of first order and first degree (Linear equations). Supplemetary problems. Page 39
Problem number: 19 (f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Bernoulli]`

$$\boxed{y' + y - y^2 e^x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)+y(x)=y(x)^2*exp(x),y(x), singsol=all)
```

$$y = \frac{e^{-x}}{-x + c_1}$$

✓ Solution by Mathematica

Time used: 0.204 (sec). Leaf size: 25

```
DSolve[y'[x]+y[x]==y[x]^2*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{-x}}{x - c_1}$$
$$y(x) \to 0$$

4.6 problem 19 (g)

Internal problem ID [5303]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 6. Equations of first order and first degree (Linear equations). Supplemetary problems. Page 39
Problem number: 19 (g).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]'], [_Ab`

$$\boxed{y + (yx + x - 3y)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 30

```
dsolve(y(x)+(x*y(x)+x-3*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \operatorname{LambertW}\left(\frac{\mathrm{e}^{\frac{3}{x-3}}}{c_1 (x-3)}\right) - \frac{3}{x-3}$$

✓ Solution by Mathematica

Time used: 60.04 (sec). Leaf size: 31

```
DSolve[y[x]+(x*y[x]+x-3*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{3}{x-3} + W\left(\frac{c_1 e^{\frac{3}{x-3}}}{x-3}\right)$$

4.7 problem 19 (h)

Internal problem ID [5304]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 6. Equations of first order and first degree (Linear equations). Supplemetary problems. Page 39
Problem number: 19 (h).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x),G(x)]'], [_Abel, '`

$$\left(2s-\mathrm{e}^{2t}\right)s'-2s\,\mathrm{e}^{2t}=-2\cos\left(2t\right)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 57

```
dsolve((2*s(t)-exp(2*t))*diff(s(t),t)=2*(s(t)*exp(2*t)-cos(2*t)),s(t), singsol=all)
```

$$s(t)=\frac{\mathrm{e}^{2t}}{2}-\frac{\sqrt{\mathrm{e}^{4t}-4\sin\left(2t\right)-4c_1}}{2}$$
$$s(t)=\frac{\mathrm{e}^{2t}}{2}+\frac{\sqrt{\mathrm{e}^{4t}-4\sin\left(2t\right)-4c_1}}{2}$$

✓ Solution by Mathematica

Time used: 15.59 (sec). Leaf size: 81

```
DSolve[(2*s[t]-Exp[2*t])*s'[t]==2*(s[t]*Exp[2*t]-Cos[2*t]),s[t],t,IncludeSingularSolutions -
```

$$s(t)\to\frac{1}{2}\left(e^{2t}-i\sqrt{-e^{4t}+4\sin(2t)-4c_1}\right)$$
$$s(t)\to\frac{1}{2}\left(e^{2t}+i\sqrt{-e^{4t}+4\sin(2t)-4c_1}\right)$$

4.8 problem 19 (i)

Internal problem ID [5305]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 6. Equations of first order and first degree (Linear equations). Supplemetary problems. Page 39
Problem number: 19 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y'x + y - x^3y^6 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 265

```
dsolve(x*diff(y(x),x)+(y(x)-x^3*y(x)^6)=0,y(x), singsol=all)
```

$$y = \frac{2^{\frac{1}{5}} \left(x^2 (2c_1x^2 + 5)^4\right)^{\frac{1}{5}}}{x (2c_1x^2 + 5)}$$

$$y = \frac{\left(-\frac{\sqrt{5}}{4} - \frac{1}{4} - \frac{i\sqrt{2}\,\sqrt{5-\sqrt{5}}}{4}\right) 2^{\frac{1}{5}} \left(x^2 (2c_1x^2 + 5)^4\right)^{\frac{1}{5}}}{x (2c_1x^2 + 5)}$$

$$y = \frac{\left(-\frac{\sqrt{5}}{4} - \frac{1}{4} + \frac{i\sqrt{2}\,\sqrt{5-\sqrt{5}}}{4}\right) 2^{\frac{1}{5}} \left(x^2 (2c_1x^2 + 5)^4\right)^{\frac{1}{5}}}{x (2c_1x^2 + 5)}$$

$$y = \frac{\left(\frac{\sqrt{5}}{4} - \frac{1}{4} - \frac{i\sqrt{2}\,\sqrt{5+\sqrt{5}}}{4}\right) 2^{\frac{1}{5}} \left(x^2 (2c_1x^2 + 5)^4\right)^{\frac{1}{5}}}{x (2c_1x^2 + 5)}$$

$$y = \frac{\left(\frac{\sqrt{5}}{4} - \frac{1}{4} + \frac{i\sqrt{2}\,\sqrt{5+\sqrt{5}}}{4}\right) 2^{\frac{1}{5}} \left(x^2 (2c_1x^2 + 5)^4\right)^{\frac{1}{5}}}{x (2c_1x^2 + 5)}$$

✓ Solution by Mathematica

Time used: 0.472 (sec). Leaf size: 141

```
DSolve[x*y'[x]+(y[x]-x^3*y[x]^6)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[5]{-2}}{\sqrt[5]{x^3 (5 + 2c_1x^2)}}$$

$$y(x) \to \frac{1}{\sqrt[5]{\frac{5x^3}{2} + c_1x^5}}$$

$$y(x) \to \frac{(-1)^{2/5}}{\sqrt[5]{\frac{5x^3}{2} + c_1x^5}}$$

$$y(x) \to -\frac{(-1)^{3/5}}{\sqrt[5]{\frac{5x^3}{2} + c_1x^5}}$$

$$y(x) \to \frac{(-1)^{4/5}}{\sqrt[5]{\frac{5x^3}{2} + c_1x^5}}$$

$$y(x) \to 0$$

4.9 problem 19 (j)

Internal problem ID [5306]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 6. Equations of first order and first degree (Linear equations). Supplemetary problems. Page 39
Problem number: 19 (j).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{r' + 2r\cos(\theta) = -\sin(2\theta)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(r(theta),theta)+(2*r(theta)*cos(theta)+sin(2*theta))=0,r(theta), singsol=all)
```

$$r(\theta) = -\sin(\theta) + \frac{1}{2} + \mathrm{e}^{-2\sin(\theta)} c_1$$

✓ Solution by Mathematica

Time used: 0.095 (sec). Leaf size: 22

```
DSolve[r'[t]+(2*r[t]*Cos[t]+Sin[2*t])==0,r[t],t,IncludeSingularSolutions -> True]
```

$$r(t) \to -\sin(t) + c_1 e^{-2\sin(t)} + \frac{1}{2}$$

4.10 problem 19 (k)

Internal problem ID [5307]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 6. Equations of first order and first degree (Linear equations). Supplemetary problems. Page 39
Problem number: 19 (k).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, ‘_with_symmetry_[F(x)*G(y),0]‘]]`

$$\boxed{y(y^2+1)-2(1-2xy^2)\,y'=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(y(x)*(1+y(x)^2)=2*(1-2*x*y(x)^2)*diff(y(x),x),y(x), singsol=all)
```

$$y=\mathrm{e}^{\mathrm{RootOf}\left(-x\,\mathrm{e}^{4\_Z}-2x\,\mathrm{e}^{2\_Z}+\mathrm{e}^{2\_Z}+c_1+2\_Z-x\right)}$$

✓ Solution by Mathematica

Time used: 0.176 (sec). Leaf size: 36

```
DSolve[y[x]*(1+y[x]^2)==2*(1-2*x*y[x]^2)*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x=\frac{y(x)^2+2\log(y(x))}{(y(x)^2+1)^2}+\frac{c_1}{(y(x)^2+1)^2},y(x)\right]$$

4.11 problem 19 (L)

Internal problem ID [5308]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 6. Equations of first order and first degree (Linear equations). Supplemetary problems. Page 39
Problem number: 19 (L).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yy' - xy^2 = -x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(y(x)*diff(y(x),x)-x*y(x)^2+x=0,y(x), singsol=all)
```

$$y = \sqrt{e^{x^2} c_1 + 1}$$
$$y = -\sqrt{e^{x^2} c_1 + 1}$$

✓ Solution by Mathematica

Time used: 1.859 (sec). Leaf size: 53

```
DSolve[y[x]*y'[x]-x*y[x]^2+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{1 + e^{x^2 + 2c_1}}$$
$$y(x) \to \sqrt{1 + e^{x^2 + 2c_1}}$$
$$y(x) \to -1$$
$$y(x) \to 1$$

4.12 problem 19 (m)

Internal problem ID [5309]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 6. Equations of first order and first degree (Linear equations). Supplemetary problems. Page 39
Problem number: 19 (m).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{\left(x - x\sqrt{x^2 - y^2}\right) y' - y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve((x-x*sqrt(x^2-y(x)^2))*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y - \arctan\left(\frac{y}{\sqrt{-y^2 + x^2}}\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.518 (sec). Leaf size: 29

```
DSolve[(x-x*Sqrt[x^2-y[x]^2])*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\arctan\left(\frac{\sqrt{x^2 - y(x)^2}}{y(x)}\right) + y(x) = c_1, y(x)\right]$$

4.13 problem 19 (o)

Internal problem ID [5310]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 6. Equations of first order and first degree (Linear equations). Supplemetary problems. Page 39
Problem number: 19 (o).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{2x' - \frac{x}{y} + x^3 \cos(y) = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 54

```
dsolve(2*diff(x(y),y)-x(y)/y+x(y)^3*cos(y)=0,x(y), singsol=all)
```

$$x(y) = \frac{\sqrt{(\cos(y) + y\sin(y) + c_1)\, y}}{\cos(y) + y\sin(y) + c_1}$$

$$x(y) = -\frac{\sqrt{(\cos(y) + y\sin(y) + c_1)\, y}}{\cos(y) + y\sin(y) + c_1}$$

✓ Solution by Mathematica

Time used: 0.261 (sec). Leaf size: 53

```
DSolve[2*x'[y]-x[y]/y+x[y]^3*Cos[y]==0,x[y],y,IncludeSingularSolutions -> True]
```

$$x(y) \to -\frac{\sqrt{y}}{\sqrt{y\sin(y) + \cos(y) + c_1}}$$

$$x(y) \to \frac{\sqrt{y}}{\sqrt{y\sin(y) + \cos(y) + c_1}}$$

$$x(y) \to 0$$

4.14 problem 19 (p)

Internal problem ID [5311]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 6. Equations of first order and first degree (Linear equations). Supplemetary problems. Page 39
Problem number: 19 (p).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y'x - y(1 - x\tan(x)) = \cos(x)\,x^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x*diff(y(x),x)=y(x)*(1-x*tan(x))+x^2*cos(x),y(x), singsol=all)
```

$$y = (x + c_1)\,x\cos(x)$$

✓ Solution by Mathematica

Time used: 0.081 (sec). Leaf size: 13

```
DSolve[x*y'[x]==y[x]*(1-x*Tan[x])+x^2*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(x + c_1)\cos(x)$$

4.15 problem 19 (q)

Internal problem ID [5312]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 6. Equations of first order and first degree (Linear equations). Supplemetary problems. Page 39
Problem number: 19 (q).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{y^2 - \left(yx + 2y + y^3\right) y' = -2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve((2+y(x)^2)-(x*y(x)+2*y(x)+y(x)^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$x - y^2 - 2 - \sqrt{y^2 + 2}\, c_1 = 0$$

✓ Solution by Mathematica

Time used: 5.808 (sec). Leaf size: 189

```
DSolve[(2+y[x]^2)-(x*y[x]+2*y[x]+y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{2x - \sqrt{4c_1^2x + c_1^4} - 4 + c_1^2}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{2x - \sqrt{4c_1^2x + c_1^4} - 4 + c_1^2}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{2x + \sqrt{4c_1^2x + c_1^4} - 4 + c_1^2}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{2x + \sqrt{4c_1^2x + c_1^4} - 4 + c_1^2}}{\sqrt{2}}$$

$$y(x) \to -i\sqrt{2}$$

$$y(x) \to i\sqrt{2}$$

4.16 problem 19 (r)

Internal problem ID [5313]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 6. Equations of first order and first degree (Linear equations). Supplemetary problems. Page 39
Problem number: 19 (r).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{y^2 - (\arctan(y) - x)\, y' = -1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((1+y(x)^2)=(arctan(y(x))-x)*diff(y(x),x),y(x), singsol=all)
```

$$y = \tan\left(\operatorname{LambertW}\left(-c_1 \mathrm{e}^{-x-1}\right) + x + 1\right)$$

✓ Solution by Mathematica

Time used: 60.157 (sec). Leaf size: 21

```
DSolve[(1+y[x]^2)==(ArcTan[y[x]]-x)*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan\left(W\left(c_1\left(-e^{-x-1}\right)\right) + x + 1\right)$$

4.17 problem 19 (s)

Internal problem ID [5314]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 6. Equations of first order and first degree (Linear equations). Supplemetary problems. Page 39
Problem number: 19 (s).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$\boxed{2xy^5 - y + 2y'x = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 131

```
dsolve((2*x*y(x)^5-y(x))+2*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{\sqrt{-3\sqrt{12x^3+9c_1}\,x}}{\sqrt{12x^3+9c_1}}$$
$$y = \frac{\sqrt{3}\,\sqrt{\sqrt{12x^3+9c_1}\,x}}{\sqrt{12x^3+9c_1}}$$
$$y = -\frac{\sqrt{-3\sqrt{12x^3+9c_1}\,x}}{\sqrt{12x^3+9c_1}}$$
$$y = -\frac{\sqrt{3}\,\sqrt{\sqrt{12x^3+9c_1}\,x}}{\sqrt{12x^3+9c_1}}$$

✓ Solution by Mathematica

Time used: 0.214 (sec). Leaf size: 109

```
DSolve[(2*x*y[x]^5-y[x])+2*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{x}}{\sqrt[4]{\frac{4x^3}{3}+c_1}}$$

$$y(x) \to -\frac{i\sqrt{x}}{\sqrt[4]{\frac{4x^3}{3}+c_1}}$$

$$y(x) \to \frac{i\sqrt{x}}{\sqrt[4]{\frac{4x^3}{3}+c_1}}$$

$$y(x) \to \frac{\sqrt{x}}{\sqrt[4]{\frac{4x^3}{3}+c_1}}$$

$$y(x) \to 0$$

4.18 problem 19 (t)

Internal problem ID [5315]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 6. Equations of first order and first degree (Linear equations). Supplemetary problems. Page 39
Problem number: 19 (t).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, ‘_with_symmetry_[F(x)*G(y),0]‘]]`

$$\sin(y) - (2y\cos(y) - x(\sec(y) + \tan(y)))\, y' = -1$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 22

```
dsolve((1+sin(y(x)))=(2*y(x)*cos(y(x))-x*(sec(y(x))+tan(y(x))) )*diff(y(x),x),y(x), singsol=
```

$$x - \frac{y^2 + c_1}{\sec(y) + \tan(y)} = 0$$

✓ Solution by Mathematica

Time used: 1.489 (sec). Leaf size: 66

```
DSolve[(1+Sin[y[x]])==(2*y[x]*Cos[y[x]]-x*(Sec[y[x]]+Tan[y[x]]) )*y'[x],y[x],x,IncludeSingul
```

$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{3\pi}{2}$$
$$\text{Solve}\left[x = y(x)^2 e^{-2\text{arctanh}\left(\tan\left(\frac{y(x)}{2}\right)\right)} + c_1 e^{-2\text{arctanh}\left(\tan\left(\frac{y(x)}{2}\right)\right)}, y(x)\right]$$
$$y(x) \to -\frac{\pi}{2}$$

4.19 problem 22 (a)

Internal problem ID [5316]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 6. Equations of first order and first degree (Linear equations). Supplemetary problems. Page 39
Problem number: 22 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - 2y = x^3 \mathrm{e}^x}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([x*diff(y(x),x)=2*y(x)+x^3*exp(x),y(1) = 0],y(x), singsol=all)
```

$$y = (\mathrm{e}^x - \mathrm{e})\, x^2$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 16

```
DSolve[{x*y'[x]==2*y[x]+x^3*Exp[x],{y[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (e^x - e)\, x^2$$

4.20 problem 22 (b)

Internal problem ID [5317]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 6. Equations of first order and first degree (Linear equations). Supplemetary problems. Page 39
Problem number: 22 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{Li' + Ri = E\sin(2t)}$$

With initial conditions

$$[i(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
dsolve([L*diff(i(t),t)+R*i(t)=E*sin(2*t),i(0) = 0],i(t), singsol=all)
```

$$i(t) = -\frac{2E\left(L\cos(2t) - L\,\mathrm{e}^{-\frac{Rt}{L}} - \frac{\sin(2t)R}{2}\right)}{4L^2 + R^2}$$

✓ Solution by Mathematica

Time used: 0.152 (sec). Leaf size: 49

```
DSolve[{L*i'[t]+R*i[t]==e*Sin[2*t],{i[0]==0}},i[t],t,IncludeSingularSolutions -> True]
```

$$i(t) \to \frac{2e\left(L\left(e^{-\frac{Rt}{L}} + \sin^2(t)\right) - L\cos^2(t) + R\sin(t)\cos(t)\right)}{4L^2 + R^2}$$

4.21 problem 23 (a)

Internal problem ID [5318]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 6. Equations of first order and first degree (Linear equations). Supplemetary problems. Page 39
Problem number: 23 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' \cos(y)\, x^2 - 2\sin(y)\, x = -1}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve(x^2*cos(y(x))*diff(y(x),x)=2*x*sin(y(x))-1,y(x), singsol=all)
```

$$y = -\arcsin\left(\frac{3c_1x^3 - 1}{3x}\right)$$

✓ Solution by Mathematica

Time used: 10.185 (sec). Leaf size: 21

```
DSolve[x^2*Cos[y[x]]*y'[x]==2*x*Sin[y[x]]-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin\left(\frac{1}{3x} + 2c_1x^2\right)$$

4.22 problem 23 (b)

Internal problem ID [5319]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 6. Equations of first order and first degree (Linear equations). Supplemetary problems. Page 39
Problem number: 23 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$4x^2yy' - 3x(3y^2+2) - 2(3y^2+2)^3 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 177

```
dsolve(4*x^2*y(x)*diff(y(x),x)=3*x*(3*y(x)^2+2)+2*(3*y(x)^2+2)^3,y(x), singsol=all)
```

$$y = -\frac{\sqrt{-\frac{6\left(3c_1x^8+\sqrt{-3c_1^2x^{17}+c_1x^9}-1\right)}{3c_1x^8-1}}}{3}$$

$$y = \frac{\sqrt{-\frac{6\left(3c_1x^8+\sqrt{-3c_1^2x^{17}+c_1x^9}-1\right)}{3c_1x^8-1}}}{3}$$

$$y = -\frac{\sqrt{6}\,\sqrt{\frac{-3c_1x^8+\sqrt{-3c_1^2x^{17}+c_1x^9}+1}{3c_1x^8-1}}}{3}$$

$$y = \frac{\sqrt{6}\,\sqrt{\frac{-3c_1x^8+\sqrt{-3c_1^2x^{17}+c_1x^9}+1}{3c_1x^8-1}}}{3}$$

✓ Solution by Mathematica

Time used: 19.518 (sec). Leaf size: 277

```
DSolve[4*x^2*y[x]*y'[x]==3*x*(3*y[x]^2+2)+2*(3*y[x]^2+2)^3,y[x],x,IncludeSingularSolutions -
```

$$y(x)\to -\frac{1}{3}\sqrt{2}\sqrt{-\frac{3x^8+\sqrt{3}\sqrt{-x^9\left(x^8+72c_1\right)}+216c_1}{x^8+72c_1}}$$

$$y(x)\to \frac{1}{3}\sqrt{2}\sqrt{-\frac{3x^8+\sqrt{3}\sqrt{-x^9\left(x^8+72c_1\right)}+216c_1}{x^8+72c_1}}$$

$$y(x)\to -\frac{1}{3}\sqrt{2}\sqrt{\frac{-3x^8+\sqrt{3}\sqrt{-x^9\left(x^8+72c_1\right)}-216c_1}{x^8+72c_1}}$$

$$y(x)\to \frac{1}{3}\sqrt{2}\sqrt{\frac{-3x^8+\sqrt{3}\sqrt{-x^9\left(x^8+72c_1\right)}-216c_1}{x^8+72c_1}}$$

$$y(x)\to -i\sqrt{\frac{2}{3}}$$

$$y(x)\to i\sqrt{\frac{2}{3}}$$

4.23 problem 23 (c)

Internal problem ID [5320]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 6. Equations of first order and first degree (Linear equations). Supplemetary problems. Page 39
Problem number: 23 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{xy^3 - y^3 + 3xy^2y' = x^2 \mathrm{e}^x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 140

```
dsolve((x*y(x)^3-y(x)^3-x^2*exp(x))+(3*x*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{\mathrm{e}^{-x}\left(\left(4\,\mathrm{e}^{2x} + 8c_1\right) x\,\mathrm{e}^{2x}\right)^{\frac{1}{3}}}{2}$$

$$y = -\frac{\mathrm{e}^{-x}\left(\left(4\,\mathrm{e}^{2x} + 8c_1\right) x\,\mathrm{e}^{2x}\right)^{\frac{1}{3}}}{4} - \frac{i\sqrt{3}\,\mathrm{e}^{-x}\left(\left(4\,\mathrm{e}^{2x} + 8c_1\right) x\,\mathrm{e}^{2x}\right)^{\frac{1}{3}}}{4}$$

$$y = -\frac{\mathrm{e}^{-x}\left(\left(4\,\mathrm{e}^{2x} + 8c_1\right) x\,\mathrm{e}^{2x}\right)^{\frac{1}{3}}}{4} + \frac{i\sqrt{3}\,\mathrm{e}^{-x}\left(\left(4\,\mathrm{e}^{2x} + 8c_1\right) x\,\mathrm{e}^{2x}\right)^{\frac{1}{3}}}{4}$$

✓ Solution by Mathematica

Time used: 0.854 (sec). Leaf size: 117

```
DSolve[(x*y[x]^3-y[x]^3-x^2*Exp[x])+(3*x*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\sqrt[3]{-\frac{1}{2}}e^{-x/3}\sqrt[3]{x}\sqrt[3]{e^{2x}+2c_1}$$

$$y(x) \to \frac{e^{-x/3}\sqrt[3]{x}\sqrt[3]{e^{2x}+2c_1}}{\sqrt[3]{2}}$$

$$y(x) \to \frac{(-1)^{2/3}e^{-x/3}\sqrt[3]{x}\sqrt[3]{e^{2x}+2c_1}}{\sqrt[3]{2}}$$

4.24 problem 23 (d)

Internal problem ID [5321]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 6. Equations of first order and first degree (Linear equations). Supplemetary problems. Page 39
Problem number: 23 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$\boxed{y' + x(x+y) - x^3(x+y)^3 = -1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 66

```
dsolve(diff(y(x),x)+x*(x+y(x))=x^3*(x+y(x))^3-1,y(x), singsol=all)
```

$$y = -\frac{e^{-\frac{x^2}{2}}}{\sqrt{c_1 + (x^2+1)\,e^{-x^2}}} - x$$

$$y = \frac{e^{-\frac{x^2}{2}}}{\sqrt{c_1 + (x^2+1)\,e^{-x^2}}} - x$$

✓ Solution by Mathematica

Time used: 10.062 (sec). Leaf size: 85

```
DSolve[y'[x]+x*(x+y[x])==x^3*(x+y[x])^3-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x - \frac{e^{-\frac{x^2}{2}}}{\sqrt{e^{-x^2}\left(x^2+1\right)+c_1}}$$

$$y(x) \to -x + \frac{e^{-\frac{x^2}{2}}}{\sqrt{e^{-x^2}\left(x^2+1\right)+c_1}}$$

$$y(x) \to -x$$

4.25 problem 23 (e)

Internal problem ID [5322]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 6. Equations of first order and first degree (Linear equations). Supplemetary problems. Page 39
Problem number: 23 (e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y + e^{y} + (1 + e^{y})\, y' = e^{-x}}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 36

```
dsolve((y(x)+exp(y(x))-exp(-x))+(1+exp(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = -\left(\text{LambertW}\left(e^{-c_1 e^{-x}} e^{x\, e^{-x}}\right) e^{x} + c_1 - x\right) e^{-x}$$

✓ Solution by Mathematica

Time used: 6.265 (sec). Leaf size: 33

```
DSolve[(y[x]+Exp[y[x]]-Exp[-x])+(1+Exp[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to e^{-x}\left(-e^{x} W\left(e^{e^{-x}(x+c_1)}\right) + x + c_1\right)$$

5 Chapter 9. Equations of first order and higher degree. Supplemetary problems. Page 65

5.1 problem 17

Internal problem ID [5323]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 9. Equations of first order and higher degree. Supplemetary problems. Page 65
Problem number: 17.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2 {y'}^2 + xyy' - 6y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x)^2+x*y(x)*diff(y(x),x)-6*y(x)^2=0,y(x), singsol=all)
```

$$y = c_1 x^2$$
$$y = \frac{c_1}{x^3}$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 26

```
DSolve[x^2*(y'[x])^2+x*y[x]*y'[x]-6*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x^3}$$
$$y(x) \to c_1 x^2$$
$$y(x) \to 0$$

5.2 problem 18

Internal problem ID [5324]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 9. Equations of first order and higher degree. Supplemetary problems. Page 65
Problem number: 18.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xy'^2 + (y - 1 - x^2)\,y' - x(-1 + y) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x*diff(y(x),x)^2+(y(x)-1-x^2)*diff(y(x),x)-x*(y(x)-1)=0,y(x), singsol=all)
```

$$y = \frac{x^2}{2} + c_1$$
$$y = \frac{x + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 32

```
DSolve[x*(y'[x])^2+(y[x]-1-x^2)*y'[x]-x*(y[x]-1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2} + c_1$$
$$y(x) \to \frac{x + c_1}{x}$$
$$y(x) \to 1$$

5.3 problem 19

Internal problem ID [5325]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 9. Equations of first order and higher degree. Supplemetary problems. Page 65
Problem number: 19.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{xy'^2 - 2yy' = -4x}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 29

```
dsolve(x*diff(y(x),x)^2-2*y(x)*diff(y(x),x)+4*x=0,y(x), singsol=all)
```

$$y = -2x$$
$$y = 2x$$
$$y = -\frac{\left(-\frac{x^2}{c_1^2} - 4\right) c_1}{2}$$

✓ Solution by Mathematica

Time used: 0.274 (sec). Leaf size: 43

```
DSolve[x*(y'[x])^2-2*y[x]*y'[x]+4*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -2x \cosh(-\log(x) + c_1)$$
$$y(x) \rightarrow -2x \cosh(\log(x) + c_1)$$
$$y(x) \rightarrow -2x$$
$$y(x) \rightarrow 2x$$

5.4 problem 20

Internal problem ID [5326]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 9. Equations of first order and higher degree. Supplemetary problems. Page 65
Problem number: 20.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{3x^4y'^2 - y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 147

```
dsolve(3*x^4*diff(y(x),x)^2-x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y = -\frac{1}{12x^2}$$

$$y = \frac{-c_1^2 - c_1\left(-c_1 + 2ix\sqrt{3}\right) - 6x^2}{6c_1^2x^2}$$

$$y = \frac{-c_1^2 - c_1\left(-c_1 - 2ix\sqrt{3}\right) - 6x^2}{6c_1^2x^2}$$

$$y = \frac{c_1\left(c_1 + 2ix\sqrt{3}\right) - 6x^2 - c_1^2}{6c_1^2x^2}$$

$$y = \frac{c_1\left(c_1 - 2ix\sqrt{3}\right) - 6x^2 - c_1^2}{6c_1^2x^2}$$

✓ Solution by Mathematica

Time used: 0.512 (sec). Leaf size: 123

```
DSolve[3*x^4*y'[x]^2-x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{x\sqrt{12x^2y(x)+1}\text{arctanh}\left(\sqrt{12x^2y(x)+1}\right)}{\sqrt{12x^4y(x)+x^2}}-\frac{1}{2}\log(y(x))=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{x\sqrt{12x^2y(x)+1}\text{arctanh}\left(\sqrt{12x^2y(x)+1}\right)}{\sqrt{12x^4y(x)+x^2}}-\frac{1}{2}\log(y(x))=c_1,y(x)\right]$$

$$y(x)\to 0$$

5.5 problem 21

Internal problem ID [5327]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 9. Equations of first order and higher degree. Supplemetary problems. Page 65
Problem number: 21.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$8yy'^2 - 2y'x + y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 185

```
dsolve(8*y(x)*diff(y(x),x)^2-2*x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y = -\frac{\sqrt{2}\,x}{4}$$

$$y = \frac{\sqrt{2}\,x}{4}$$

$$y = 0$$

$$\ln(x) - \sqrt{\frac{x^2-8y^2}{x^2}} + \operatorname{arctanh}\left(\frac{1}{\sqrt{\frac{x^2-8y^2}{x^2}}}\right) + \frac{\sqrt{2}\sqrt{\frac{(\sqrt{2}\,x+4y)(\sqrt{2}\,x-4y)}{x^2}}}{2} + \ln\left(\frac{y}{x}\right) - c_1 = 0$$

$$\ln(x) + \sqrt{\frac{x^2-8y^2}{x^2}} - \operatorname{arctanh}\left(\frac{1}{\sqrt{\frac{x^2-8y^2}{x^2}}}\right) - \frac{\sqrt{2}\sqrt{\frac{(\sqrt{2}\,x+4y)(\sqrt{2}\,x-4y)}{x^2}}}{2} + \ln\left(\frac{y}{x}\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.347 (sec). Leaf size: 174

```
DSolve[8*y[x]*y'[x]^2-2*x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{4c_1}\sqrt{e^{8c_1}-2ix}}{2\sqrt{2}}$$
$$y(x) \to \frac{e^{4c_1}\sqrt{e^{8c_1}-2ix}}{2\sqrt{2}}$$
$$y(x) \to -\frac{e^{4c_1}\sqrt{2ix+e^{8c_1}}}{2\sqrt{2}}$$
$$y(x) \to \frac{e^{4c_1}\sqrt{2ix+e^{8c_1}}}{2\sqrt{2}}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{x}{2\sqrt{2}}$$
$$y(x) \to \frac{x}{2\sqrt{2}}$$

5.6 problem 22

Internal problem ID [5328]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 9. Equations of first order and higher degree. Supplemetary problems. Page 65
Problem number: 22.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$\boxed{y^2 y'^2 + 3y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 119

```
dsolve(y(x)^2*diff(y(x),x)^2+3*x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y = \frac{(-18x^2)^{\frac{1}{3}}}{2}$$
$$y = -\frac{(-18x^2)^{\frac{1}{3}}}{4} - \frac{i\sqrt{3}\,(-18x^2)^{\frac{1}{3}}}{4}$$
$$y = -\frac{(-18x^2)^{\frac{1}{3}}}{4} + \frac{i\sqrt{3}\,(-18x^2)^{\frac{1}{3}}}{4}$$
$$y = 0$$
$$y = \text{RootOf}\left(-\ln(x) + \int^{\_Z} -\frac{3\left(4\_a^3 + 3\sqrt{4\_a^3+9} + 9\right)}{2\_a\,(4\_a^3+9)} d\_a + c_1\right) x^{\frac{2}{3}}$$

✓ Solution by Mathematica

Time used: 0.574 (sec). Leaf size: 239

```
DSolve[y[x]^2*y'[x]^2+3*x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{c_1}{3}} \sqrt[3]{-3x + e^{c_1}}$$
$$y(x) \to -\sqrt[3]{-1} e^{\frac{c_1}{3}} \sqrt[3]{-3x + e^{c_1}}$$
$$y(x) \to (-1)^{2/3} e^{\frac{c_1}{3}} \sqrt[3]{-3x + e^{c_1}}$$
$$y(x) \to e^{\frac{c_1}{3}} \sqrt[3]{3x + e^{c_1}}$$
$$y(x) \to -\sqrt[3]{-1} e^{\frac{c_1}{3}} \sqrt[3]{3x + e^{c_1}}$$
$$y(x) \to (-1)^{2/3} e^{\frac{c_1}{3}} \sqrt[3]{3x + e^{c_1}}$$
$$y(x) \to 0$$
$$y(x) \to -\left(-\frac{3}{2}\right)^{2/3} x^{2/3}$$
$$y(x) \to -\left(\frac{3}{2}\right)^{2/3} x^{2/3}$$
$$y(x) \to \frac{\sqrt[3]{-1} 3^{2/3} x^{2/3}}{2^{2/3}}$$

5.7 problem 23

Internal problem ID [5329]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 9. Equations of first order and higher degree. Supplemetary problems. Page 65
Problem number: 23.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y'^2 - y'x + y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)^2-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y = \frac{x^2}{4}$$
$$y = -c_1^2 + c_1 x$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 25

```
DSolve[y'[x]^2-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x - c_1)$$
$$y(x) \to \frac{x^2}{4}$$

5.8 problem 24

Internal problem ID [5330]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 9. Equations of first order and higher degree. Supplemetary problems. Page 65
Problem number: 24.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$\boxed{16y^3y'^2 - 4y'x + y = 0}$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 97

```
dsolve(16*y(x)^3*diff(y(x),x)^2-4*x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y = -\frac{\sqrt{-2x}}{2}$$
$$y = \frac{\sqrt{-2x}}{2}$$
$$y = -\frac{\sqrt{2}\,\sqrt{x}}{2}$$
$$y = \frac{\sqrt{2}\,\sqrt{x}}{2}$$
$$y = 0$$
$$y = \operatorname{RootOf}\left(-\ln(x) + \int^{\_Z} -\frac{2\left(4\_a^4 - \sqrt{-4\_a^4+1} - 1\right)}{\_a\left(4\_a^4-1\right)} d\_a + c_1\right)\sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.563 (sec). Leaf size: 303

```
DSolve[16*y[x]^3*y'[x]^2-4*x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{\frac{c_1}{4}}\sqrt[4]{e^{c_1} - ix}$$
$$y(x) \to -ie^{\frac{c_1}{4}}\sqrt[4]{e^{c_1} - ix}$$
$$y(x) \to ie^{\frac{c_1}{4}}\sqrt[4]{e^{c_1} - ix}$$
$$y(x) \to e^{\frac{c_1}{4}}\sqrt[4]{e^{c_1} - ix}$$
$$y(x) \to -e^{\frac{c_1}{4}}\sqrt[4]{ix + e^{c_1}}$$
$$y(x) \to -ie^{\frac{c_1}{4}}\sqrt[4]{ix + e^{c_1}}$$
$$y(x) \to ie^{\frac{c_1}{4}}\sqrt[4]{ix + e^{c_1}}$$
$$y(x) \to e^{\frac{c_1}{4}}\sqrt[4]{ix + e^{c_1}}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{\sqrt{x}}{\sqrt{2}}$$
$$y(x) \to -\frac{i\sqrt{x}}{\sqrt{2}}$$
$$y(x) \to \frac{i\sqrt{x}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{x}}{\sqrt{2}}$$

5.9 problem 25

Internal problem ID [5331]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 9. Equations of first order and higher degree. Supplemetary problems. Page 65
Problem number: 25.
ODE order: 1.
ODE degree: 5.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Clairaut]`

$$\boxed{xy'^5 - yy'^4 + (x^2+1)\, y'^3 - 2xyy'^2 + (x+y^2)\, y' - y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 37

```
dsolve(x*diff(y(x),x)^5-y(x)*diff(y(x),x)^4+(1+x^2)*diff(y(x),x)^3-2*x*y(x)*diff(y(x),x)^2+(
```

$$y = c_1^3 + c_1 x$$
$$y = c_1 x^{\frac{3}{2}}$$
$$y = c_1 x + \frac{1}{c_1}$$
$$y = c_1 \sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 142

```
DSolve[x*y'[x]^5-y[x]*y'[x]^4+(1+x^2)*y'[x]^3-2*x*y[x]*y'[x]^2+(x+y[x]^2)*y'[x]-y[x]==0,y[x]
```

$$y(x) \to c_1 x + \frac{1}{c_1}$$

$$y(x) \to c_1 (x + c_1{}^2)$$

$$y(x) \to \text{Indeterminate}$$

$$y(x) \to -x - 1$$

$$y(x) \to -2\sqrt{x}$$

$$y(x) \to 2\sqrt{x}$$

$$y(x) \to -\frac{2ix^{3/2}}{3\sqrt{3}}$$

$$y(x) \to \frac{2ix^{3/2}}{3\sqrt{3}}$$

$$y(x) \to x + 1$$

$$y(x) \to -\sqrt{-(x-1)^2}$$

$$y(x) \to \sqrt{-(x-1)^2}$$

5.10 problem 26

Internal problem ID [5332]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 9. Equations of first order and higher degree. Supplemetary problems. Page 65
Problem number: 26.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{xy'^2 - yy' - y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 32

```
dsolve(x*diff(y(x),x)^2-y(x)*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y = 0$$
$$y = \frac{\left(\operatorname{LambertW}\left(\frac{x\,\mathrm{e}}{c_1}\right) - 1\right)^2 x}{\operatorname{LambertW}\left(\frac{x\,\mathrm{e}}{c_1}\right)}$$

✓ Solution by Mathematica

Time used: 2.255 (sec). Leaf size: 158

```
DSolve[x*y'[x]^2-y[x]*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{y(x)}{4x}+\frac{1}{4}\sqrt{\frac{y(x)}{x}}\sqrt{\frac{y(x)}{x}+4}-\log\left(\sqrt{\frac{y(x)}{x}+4}-\sqrt{\frac{y(x)}{x}}\right)=-\frac{\log(x)}{2}+c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{4}\left(\frac{y(x)}{x}+\sqrt{\frac{y(x)}{x}}\sqrt{\frac{y(x)}{x}+4}-4\log\left(\sqrt{\frac{y(x)}{x}+4}-\sqrt{\frac{y(x)}{x}}\right)\right)=\frac{\log(x)}{2}+c_1,y(x)\right]$$

$$y(x)\to 0$$

5.11 problem 27

Internal problem ID [5333]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 9. Equations of first order and higher degree. Supplemetary problems. Page 65
Problem number: 27.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y - 2y'x - y^2 {y'}^3 = 0}$$

✓ Solution by Maple

Time used: 0.485 (sec). Leaf size: 107

```
dsolve(y(x)=2*x*diff(y(x),x)+y(x)^2*diff(y(x),x)^3,y(x), singsol=all)
```

$$y = -\frac{2\, 2^{\frac{1}{4}} 3^{\frac{1}{4}} (-x^3)^{\frac{1}{4}}}{3}$$
$$y = \frac{2\, 2^{\frac{1}{4}} 3^{\frac{1}{4}} (-x^3)^{\frac{1}{4}}}{3}$$
$$y = -\frac{2i 2^{\frac{1}{4}} 3^{\frac{1}{4}} (-x^3)^{\frac{1}{4}}}{3}$$
$$y = \frac{2i 2^{\frac{1}{4}} 3^{\frac{1}{4}} (-x^3)^{\frac{1}{4}}}{3}$$
$$y = 0$$
$$y = \sqrt{c_1^3 + 2c_1 x}$$
$$y = -\sqrt{c_1^3 + 2c_1 x}$$

✓ Solution by Mathematica

Time used: 0.111 (sec). Leaf size: 119

```
DSolve[y[x]==2*x*y'[x]+y[x]^2*y'[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2c_1x + c_1{}^3}$$
$$y(x) \to \sqrt{2c_1x + c_1{}^3}$$
$$y(x) \to (-1-i)\left(\frac{2}{3}\right)^{3/4} x^{3/4}$$
$$y(x) \to (1-i)\left(\frac{2}{3}\right)^{3/4} x^{3/4}$$
$$y(x) \to (-1+i)\left(\frac{2}{3}\right)^{3/4} x^{3/4}$$
$$y(x) \to (1+i)\left(\frac{2}{3}\right)^{3/4} x^{3/4}$$

5.12 problem 28

Internal problem ID [5334]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 9. Equations of first order and higher degree. Supplemetary problems. Page 65
Problem number: 28.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y'^2 - y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 77

```
dsolve(diff(y(x),x)^2-x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$\frac{c_1}{\sqrt{2x - 2\sqrt{x^2 + 4y}}} + \frac{2x}{3} + \frac{\sqrt{x^2 + 4y}}{3} = 0$$
$$\frac{c_1}{\sqrt{2x + 2\sqrt{x^2 + 4y}}} + \frac{2x}{3} - \frac{\sqrt{x^2 + 4y}}{3} = 0$$

✓ Solution by Mathematica

Time used: 60.129 (sec). Leaf size: 1003

```
DSolve[y'[x]^2-x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\left(x^2+\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)^2+8e^{3c_1}x}{4\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}$$

$$y(x) \to \frac{1}{8}\left(4x^2-\frac{i\left(\sqrt{3}-i\right)x\left(x^3+8e^{3c_1}\right)}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}\right.$$
$$\left.+i\left(\sqrt{3}+i\right)\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{8}\left(4x^2+\frac{i\left(\sqrt{3}+i\right)x\left(x^3+8e^{3c_1}\right)}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}\right.$$
$$\left.-\left(1+i\sqrt{3}\right)\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x)$$
$$\to \frac{2\sqrt[3]{2}x^4+2^{2/3}\left(-2x^6-10e^{3c_1}x^3+\sqrt{e^{3c_1}\left(4x^3+e^{3c_1}\right)^3}+e^{6c_1}\right)^{2/3}+4x^2\sqrt[3]{-2x^6-10e^{3c_1}x^3+\sqrt{e^{3c_1}(4x}}}{8\sqrt[3]{-2x^6-10e^{3c_1}x^3+\sqrt{e^{3c_1}\left(4x^3+e^{3c_1}\right)^3}+e^{6c_1}}}$$

$$y(x) \to \frac{1}{16}\left(8x^2+\frac{2\sqrt[3]{2}\left(1+i\sqrt{3}\right)x\left(-x^3+2e^{3c_1}\right)}{\sqrt[3]{-2x^6-10e^{3c_1}x^3+\sqrt{e^{3c_1}\left(4x^3+e^{3c_1}\right)^3}+e^{6c_1}}}\right.$$
$$\left.+i2^{2/3}\left(\sqrt{3}+i\right)\sqrt[3]{-2x^6-10e^{3c_1}x^3+\sqrt{e^{3c_1}\left(4x^3+e^{3c_1}\right)^3}+e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{16}\left(8x^2+\frac{2i\sqrt[3]{2}\left(\sqrt{3}+i\right)x\left(x^3-2e^{3c_1}\right)}{\sqrt[3]{-2x^6-10e^{3c_1}x^3+\sqrt{e^{3c_1}\left(4x^3+e^{3c_1}\right)^3}+e^{6c_1}}}\right.$$
$$\left.-2^{2/3}\left(1+i\sqrt{3}\right)\sqrt[3]{-2x^6-10e^{3c_1}x^3+\sqrt{e^{3c_1}\left(4x^3+e^{3c_1}\right)^3}+e^{6c_1}}\right)$$

5.13 problem 29

Internal problem ID [5335]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 9. Equations of first order and higher degree. Supplemetary problems. Page 65
Problem number: 29.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$y - (1 + y')\, x - {y'}^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve(y(x)=(1+diff(y(x),x))*x+diff(y(x),x)^2,y(x), singsol=all)
```

$$y = \left(\mathrm{LambertW}\left(\frac{c_1 \mathrm{e}^{\frac{x}{2}-1}}{2}\right) - \frac{x}{2} + 2\right) x + \left(\mathrm{LambertW}\left(\frac{c_1 \mathrm{e}^{\frac{x}{2}-1}}{2}\right) - \frac{x}{2} + 1\right)^2$$

✓ Solution by Mathematica

Time used: 1.048 (sec). Leaf size: 177

```
DSolve[y[x]==(1+y'[x])*x+y'[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\Big[-\sqrt{x^2 + 4y(x) - 4x} + 2\log\left(\sqrt{x^2 + 4y(x) - 4x} - x + 2\right) - 2\log\left(-x\sqrt{x^2 + 4y(x) - 4x} + x^2 + 4y(x) - 2x - 4\right) + x = c_1, y(x)\Big]$$

$$\mathrm{Solve}\left[-4\mathrm{arctanh}\left(\frac{(x-5)\sqrt{x^2 + 4y(x) - 4x} - x^2 - 4y(x) + 7x - 6}{(x-3)\sqrt{x^2 + 4y(x) - 4x} - x^2 - 4y(x) + 5x - 2}\right) + \sqrt{x^2 + 4y(x) - 4x} + x = c_1, y(x)\right]$$

5.14 problem 30

Internal problem ID [5336]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 9. Equations of first order and higher degree. Supplemetary problems. Page 65
Problem number: 30.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y - 2y' - \sqrt{y'^2 + 1} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 221

```
dsolve(y(x)=2*diff(y(x),x)+sqrt(1+diff(y(x),x)^2),y(x), singsol=all)
```

$$x+\frac{\sqrt{(y+1)^2-2y+2}}{2}-\operatorname{arcsinh}\left(\frac{\sqrt{3}\,y}{3}\right)-\operatorname{arctanh}\left(\frac{6-2y}{4\sqrt{(y+1)^2-2y+2}}\right)-\frac{\sqrt{(y-1)^2+2y+2}}{2}+\operatorname{arctanh}\left(\frac{6+2y}{4\sqrt{(y-1)^2+2y+2}}\right)-\ln(y-1)-\ln(y+1)-c_1=0$$

$$x-\frac{\sqrt{(y+1)^2-2y+2}}{2}+\operatorname{arcsinh}\left(\frac{\sqrt{3}\,y}{3}\right)+\operatorname{arctanh}\left(\frac{6-2y}{4\sqrt{(y+1)^2-2y+2}}\right)+\frac{\sqrt{(y-1)^2+2y+2}}{2}-\operatorname{arctanh}\left(\frac{6+2y}{4\sqrt{(y-1)^2+2y+2}}\right)-\ln(y-1)-\ln(y+1)-c_1=0$$

✓ Solution by Mathematica

Time used: 60.301 (sec). Leaf size: 4821

```
DSolve[y[x]==2*y'[x]+Sqrt[1+y'[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

5.15 problem 31

Internal problem ID [5337]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 9. Equations of first order and higher degree. Supplemetary problems. Page 65
Problem number: 31.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$yy'^2 - y'x + 3y = 0$$

✓ Solution by Maple

Time used: 2.609 (sec). Leaf size: 153

```
dsolve(y(x)*diff(y(x),x)^2-x*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y = 0$$

$$\ln(x) - \frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{\frac{x^2-12y^2}{x^2}}}\right)}{4} + \frac{5\operatorname{arctanh}\left(\frac{\sqrt{\frac{x^2-12y^2}{x^2}}}{5}\right)}{4} + \frac{5\ln\left(\frac{2x^2+y^2}{x^2}\right)}{8} - \frac{\ln\left(\frac{y}{x}\right)}{4} - c_1 = 0$$

$$\ln(x) + \frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{\frac{x^2-12y^2}{x^2}}}\right)}{4} - \frac{5\operatorname{arctanh}\left(\frac{\sqrt{\frac{x^2-12y^2}{x^2}}}{5}\right)}{4} + \frac{5\ln\left(\frac{2x^2+y^2}{x^2}\right)}{8} - \frac{\ln\left(\frac{y}{x}\right)}{4} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.281 (sec). Leaf size: 1131

```
DSolve[y[x]*y'[x]^2-x*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\text{Root}\left[62208\#1^5 + 622080\#1^4x^2 + \#1^3\left(2488320x^4 - 864e^{8c_1}\right) + \#1^2\left(4976640x^6 + 16416e^{8c_1}x^2\right) +\right.}$$

$$y(x) \to \sqrt{\text{Root}\left[62208\#1^5 + 622080\#1^4x^2 + \#1^3\left(2488320x^4 - 864e^{8c_1}\right) + \#1^2\left(4976640x^6 + 16416e^{8c_1}x^2\right) +\right.}$$

$$y(x) \to -\sqrt{\text{Root}\left[62208\#1^5 + 622080\#1^4x^2 + \#1^3\left(2488320x^4 - 864e^{8c_1}\right) + \#1^2\left(4976640x^6 + 16416e^{8c_1}x^2\right) +\right.}$$

$$y(x) \to \sqrt{\text{Root}\left[62208\#1^5 + 622080\#1^4x^2 + \#1^3\left(2488320x^4 - 864e^{8c_1}\right) + \#1^2\left(4976640x^6 + 16416e^{8c_1}x^2\right) +\right.}$$

$$y(x) \to -\sqrt{\text{Root}\left[62208\#1^5 + 622080\#1^4x^2 + \#1^3\left(2488320x^4 - 864e^{8c_1}\right) + \#1^2\left(4976640x^6 + 16416e^{8c_1}x^2\right) +\right.}$$

$$y(x) \to \sqrt{\text{Root}\left[62208\#1^5 + 622080\#1^4x^2 + \#1^3\left(2488320x^4 - 864e^{8c_1}\right) + \#1^2\left(4976640x^6 + 16416e^{8c_1}x^2\right) +\right.}$$

$$y(x) \to -\sqrt{\text{Root}\left[62208\#1^5 + 622080\#1^4x^2 + \#1^3\left(2488320x^4 - 864e^{8c_1}\right) + \#1^2\left(4976640x^6 + 16416e^{8c_1}x^2\right) +\right.}$$

$$y(x) \to \sqrt{\text{Root}\left[62208\#1^5 + 622080\#1^4x^2 + \#1^3\left(2488320x^4 - 864e^{8c_1}\right) + \#1^2\left(4976640x^6 + 16416e^{8c_1}x^2\right) +\right.}$$

$$y(x) \to -\sqrt{\text{Root}\left[62208\#1^5 + 622080\#1^4x^2 + \#1^3\left(2488320x^4 - 864e^{8c_1}\right) + \#1^2\left(4976640x^6 + 16416e^{8c_1}x^2\right) +\right.}$$

$$y(x) \to \sqrt{\text{Root}\left[62208\#1^5 + 622080\#1^4x^2 + \#1^3\left(2488320x^4 - 864e^{8c_1}\right) + \#1^2\left(4976640x^6 + 16416e^{8c_1}x^2\right) +\right.}$$

6 Chapter 10. Singular solutions, Extraneous loci. Supplemetary problems. Page 74

6.1 problem 10

Internal problem ID [5338]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 10. Singular solutions, Extraneous loci. Supplemetary problems. Page 74
Problem number: 10.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y - y'x + 2y'^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(y(x)=diff(y(x),x)*x-2*diff(y(x),x)^2,y(x), singsol=all)
```

$$y = \frac{x^2}{8}$$
$$y = -2c_1^2 + c_1 x$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 25

```
DSolve[y[x]==y'[x]*x-2*y'[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x - 2c_1)$$
$$y(x) \to \frac{x^2}{8}$$

6.2 problem 11

Internal problem ID [5339]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 10. Singular solutions, Extraneous loci. Supplemetary problems. Page 74
Problem number: 11.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$\boxed{y^2 {y'}^2 + 3y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 119

```
dsolve(y(x)^2*diff(y(x),x)^2+3*x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y = \frac{(-18x^2)^{\frac{1}{3}}}{2}$$
$$y = -\frac{(-18x^2)^{\frac{1}{3}}}{4} - \frac{i\sqrt{3}\,(-18x^2)^{\frac{1}{3}}}{4}$$
$$y = -\frac{(-18x^2)^{\frac{1}{3}}}{4} + \frac{i\sqrt{3}\,(-18x^2)^{\frac{1}{3}}}{4}$$
$$y = 0$$
$$y = \operatorname{RootOf}\left(-\ln(x) + \int^{\_Z} -\frac{3\left(4\_a^3 - 3\sqrt{4\_a^3 + 9} + 9\right)}{2\_a\,(4\_a^3 + 9)} d\_a + c_1\right) x^{\frac{2}{3}}$$

✓ Solution by Mathematica

Time used: 0.597 (sec). Leaf size: 239

```
DSolve[y[x]^2*y'[x]^2+3*x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{c_1}{3}} \sqrt[3]{-3x + e^{c_1}}$$
$$y(x) \to -\sqrt[3]{-1} e^{\frac{c_1}{3}} \sqrt[3]{-3x + e^{c_1}}$$
$$y(x) \to (-1)^{2/3} e^{\frac{c_1}{3}} \sqrt[3]{-3x + e^{c_1}}$$
$$y(x) \to e^{\frac{c_1}{3}} \sqrt[3]{3x + e^{c_1}}$$
$$y(x) \to -\sqrt[3]{-1} e^{\frac{c_1}{3}} \sqrt[3]{3x + e^{c_1}}$$
$$y(x) \to (-1)^{2/3} e^{\frac{c_1}{3}} \sqrt[3]{3x + e^{c_1}}$$
$$y(x) \to 0$$
$$y(x) \to -\left(-\frac{3}{2}\right)^{2/3} x^{2/3}$$
$$y(x) \to -\left(\frac{3}{2}\right)^{2/3} x^{2/3}$$
$$y(x) \to \frac{\sqrt[3]{-1} 3^{2/3} x^{2/3}}{2^{2/3}}$$

6.3 problem 12

Internal problem ID [5340]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 10. Singular solutions, Extraneous loci. Supplemetary problems. Page 74
Problem number: 12.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{xy'^2 - 2yy' = -4x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(x*diff(y(x),x)^2-2*y(x)*diff(y(x),x)+4*x=0,y(x), singsol=all)
```

$$y = -2x$$
$$y = 2x$$
$$y = -\frac{\left(-\frac{x^2}{c_1^2} - 4\right) c_1}{2}$$

✓ Solution by Mathematica

Time used: 0.271 (sec). Leaf size: 43

```
DSolve[x*y'[x]^2-2*y[x]*y'[x]+4*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2x \cosh(-\log(x) + c_1)$$
$$y(x) \to -2x \cosh(\log(x) + c_1)$$
$$y(x) \to -2x$$
$$y(x) \to 2x$$

6.4 problem 13

Internal problem ID [5341]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 10. Singular solutions, Extraneous loci. Supplemetary problems. Page 74
Problem number: 13.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{xy'^2 - 2yy' + 2y = -x}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 36

```
dsolve(x*diff(y(x),x)^2-2*y(x)*diff(y(x),x)+x+2*y(x)=0,y(x), singsol=all)
```

$$y = -\frac{\left(\frac{(x+c_1)^2}{c_1^2} + 1\right)x}{-\frac{2(x+c_1)}{c_1} + 2}$$
$$y = c_1 x$$

✓ Solution by Mathematica

Time used: 0.153 (sec). Leaf size: 78

```
DSolve[x*y'[x]^2-2*y[x]*y'[x]+x+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}e^{-c_1}x^2 + x - e^{c_1}$$
$$y(x) \to -e^{c_1}x^2 + x - \frac{e^{-c_1}}{2}$$
$$y(x) \to x - \sqrt{2}x$$
$$y(x) \to \left(1+\sqrt{2}\right)x$$

6.5 problem 14

Internal problem ID [5342]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 10. Singular solutions, Extraneous loci. Supplemetary problems. Page 74
Problem number: 14.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(3y-1)^2 y'^2 - 4y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 689

```
dsolve((3*y(x)-1)^2*diff(y(x),x)^2=4*y(x),y(x), singsol=all)
```

$$y = 0$$

$$y = \left(\frac{\left(108c_1 - 108x + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 - 12}\right)^{\frac{1}{3}}}{6} + \frac{2}{\left(108c_1 - 108x + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 - \cdots}\right.}\right.$$

$$y = \left(-\frac{\left(108c_1 - 108x + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 - 12}\right)^{\frac{1}{3}}}{12} - \frac{1}{\left(108c_1 - 108x + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 \cdots}\right.}\right.$$

$$y = \left(-\frac{\left(108c_1 - 108x + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 - 12}\right)^{\frac{1}{3}}}{12} - \frac{1}{\left(108c_1 - 108x + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 \cdots}\right.}\right.$$

$$y = \left(\frac{\left(108x - 108c_1 + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 - 12}\right)^{\frac{1}{3}}}{6} + \frac{2}{\left(108x - 108c_1 + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 - \cdots}\right.}\right.$$

$$y = \left(-\frac{\left(108x - 108c_1 + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 - 12}\right)^{\frac{1}{3}}}{12} - \frac{1}{\left(108x - 108c_1 + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 \cdots}\right.}\right.$$

$$y = \left(-\frac{\left(108x - 108c_1 + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 - 12}\right)^{\frac{1}{3}}}{12} - \frac{1}{\left(108x - 108c_1 + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 \cdots}\right.}\right.$$

✓ Solution by Mathematica

Time used: 4.472 (sec). Leaf size: 892

```
DSolve[(3*y[x]-1)^2*y'[x]^2==4*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\left(2+\sqrt[3]{108x^2+3\sqrt{3}\sqrt{(-2x+c_1)^2\,(108x^2-108c_1x-16+27c_1{}^2)}-108c_1x-8+27c_1{}^2}\right)^2}{6\sqrt[3]{108x^2+3\sqrt{3}\sqrt{(-2x+c_1)^2\,(108x^2-108c_1x-16+27c_1{}^2)}-108c_1x-8+27c_1{}^2}}$$

$$y(x) \to \frac{1}{24}\left(2i\left(\sqrt{3}+i\right)\sqrt[3]{108x^2+3\sqrt{3}\sqrt{(-2x+c_1)^2\,(108x^2-108c_1x-16+27c_1{}^2)}-108c_1x-8+27c_1{}^2}+\frac{-8-8i\sqrt{3}}{\sqrt[3]{108x^2+3\sqrt{3}\sqrt{(-2x+c_1)^2\,(108x^2-108c_1x-16+27c_1{}^2)}-108c_1x-8+27c_1{}^2}}+16\right)$$

$$y(x) \to \frac{1}{24}\left(-2\left(1+i\sqrt{3}\right)\sqrt[3]{108x^2+3\sqrt{3}\sqrt{(-2x+c_1)^2\,(108x^2-108c_1x-16+27c_1{}^2)}-108c_1x-8+27c_1{}^2}+\frac{-8+8i\sqrt{3}}{\sqrt[3]{108x^2+3\sqrt{3}\sqrt{(-2x+c_1)^2\,(108x^2-108c_1x-16+27c_1{}^2)}-108c_1x-8+27c_1{}^2}}+16\right)$$

$$y(x) \to \frac{\left(2+\sqrt[3]{108x^2+3\sqrt{3}\sqrt{(2x+c_1)^2\,(108x^2+108c_1x-16+27c_1{}^2)}+108c_1x-8+27c_1{}^2}\right)^2}{6\sqrt[3]{108x^2+3\sqrt{3}\sqrt{(2x+c_1)^2\,(108x^2+108c_1x-16+27c_1{}^2)}+108c_1x-8+27c_1{}^2}}$$

$$y(x) \to \frac{1}{24}\left(2i\left(\sqrt{3}+i\right)\sqrt[3]{108x^2+3\sqrt{3}\sqrt{(2x+c_1)^2\,(108x^2+108c_1x-16+27c_1{}^2)}+108c_1x-8+27c_1{}^2}-\frac{8\left(1+i\sqrt{3}\right)}{\cdots}\right.$$

6.6 problem 15

Internal problem ID [5343]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 10. Singular solutions, Extraneous loci. Supplemetary problems. Page 74
Problem number: 15.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{y + y'x - x^4 {y'}^2 = 0}$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 135

```
dsolve(y(x)=-x*diff(y(x),x)+x^4*diff(y(x),x)^2,y(x), singsol=all)
```

$$y = -\frac{1}{4x^2}$$
$$y = \frac{-c_1(2ix - c_1) - c_1^2 - 2x^2}{2c_1^2x^2}$$
$$y = \frac{-c_1(-2ix - c_1) - c_1^2 - 2x^2}{2c_1^2x^2}$$
$$y = \frac{c_1(2ix + c_1) - 2x^2 - c_1^2}{2c_1^2x^2}$$
$$y = \frac{c_1(-2ix + c_1) - 2x^2 - c_1^2}{2c_1^2x^2}$$

✓ Solution by Mathematica

Time used: 0.498 (sec). Leaf size: 123

```
DSolve[y[x]==-x*y'[x]+x^4*y'[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{x\sqrt{4x^2y(x)+1}\text{arctanh}\left(\sqrt{4x^2y(x)+1}\right)}{\sqrt{4x^4y(x)+x^2}}-\frac{1}{2}\log(y(x))=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{x\sqrt{4x^2y(x)+1}\text{arctanh}\left(\sqrt{4x^2y(x)+1}\right)}{\sqrt{4x^4y(x)+x^2}}-\frac{1}{2}\log(y(x))=c_1,y(x)\right]$$

$$y(x)\to 0$$

6.7 problem 16

Internal problem ID [5344]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 10. Singular solutions, Extraneous loci. Supplemetary problems. Page 74
Problem number: 16.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{2y - {y'}^2 - 4y'x = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 696

```
dsolve(2*y(x)=diff(y(x),x)^2+4*x*diff(y(x),x),y(x), singsol=all)
```

$$y = \frac{\left(\frac{\left(12c_1-8x^3+4\sqrt{-12c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}{2}+\frac{2x^2}{\left(12c_1-8x^3+4\sqrt{-12c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}-x\right)^2}{2} + 2\left(\frac{\left(12c_1-8x^3+4\sqrt{-12c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}{2}+\frac{2x^2}{\left(12c_1-8x^3+4\sqrt{-12c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}-x\right)x$$

$$y = \frac{\left(-\frac{\left(12c_1-8x^3+4\sqrt{-12c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}{4}-\frac{x^2}{\left(12c_1-8x^3+4\sqrt{-12c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}-x-\frac{i\sqrt{3}\left(\frac{\left(12c_1-8x^3+4\sqrt{-12c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}{2}-\frac{\cdots}{\left(12c_1-8x^3+\cdots\right.}\right.}{2}\cdots\right.}{2} + 2\left(-\frac{\left(12c_1-8x^3+4\sqrt{-12c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}{4}-\frac{x^2}{\left(12c_1-8x^3+4\sqrt{-12c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}-x-\frac{i\sqrt{3}\left(\frac{\left(12c_1-8x^3+4\sqrt{-12c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}{2}-\frac{2x^2}{\left(12c_1-8x^3+4\sqrt{-12c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}\right)}{2}\right)x$$

$$y = \frac{\left(-\frac{\left(12c_1-8x^3+4\sqrt{-12c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}{4}-\frac{x^2}{\left(12c_1-8x^3+4\sqrt{-12c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}-x+\frac{i\sqrt{3}\left(\frac{\left(12c_1-8x^3+4\sqrt{-12c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}{2}-\frac{\cdots}{\left(12c_1-8x^3+\cdots\right.}\right.}{2}\cdots\right.}{2}\cdots$$

✓ Solution by Mathematica

Time used: 60.241 (sec). Leaf size: 1344

```
DSolve[2*y[x]==y'[x]^2+4*x*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-x^2 + \frac{x\left(x^3+2\sqrt{2}e^{3c_1}\right)}{\sqrt[3]{-x^6+5\sqrt{2}e^{3c_1}x^3+\sqrt{e^{3c_1}\left(-16\sqrt{2}x^9+24e^{3c_1}x^6-6\sqrt{2}e^{6c_1}x^3+e^{9c_1}\right)}+e^{6c_1}}} + \sqrt[3]{-x^6+5\sqrt{2}e^{3c_1}x^3+\sqrt{e^{3c_1}\left(-16\sqrt{2}x^9+24e^{3c_1}x^6-6\sqrt{2}e^{6c_1}x^3+e^{9c_1}\right)}+e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{4}\left(-2x^2 - \frac{\left(1+i\sqrt{3}\right)x\left(x^3+2\sqrt{2}e^{3c_1}\right)}{\sqrt[3]{-x^6+5\sqrt{2}e^{3c_1}x^3+\sqrt{e^{3c_1}\left(-16\sqrt{2}x^9+24e^{3c_1}x^6-6\sqrt{2}e^{6c_1}x^3+e^{9c_1}\right)}+e^{6c_1}}} + i\left(\sqrt{3}+i\right)\sqrt[3]{-x^6+5\sqrt{2}e^{3c_1}x^3+\sqrt{e^{3c_1}\left(-16\sqrt{2}x^9+24e^{3c_1}x^6-6\sqrt{2}e^{6c_1}x^3+e^{9c_1}\right)}+e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{4}\left(-2x^2 + \frac{i\left(\sqrt{3}+i\right)x\left(x^3+2\sqrt{2}e^{3c_1}\right)}{\sqrt[3]{-x^6+5\sqrt{2}e^{3c_1}x^3+\sqrt{e^{3c_1}\left(-16\sqrt{2}x^9+24e^{3c_1}x^6-6\sqrt{2}e^{6c_1}x^3+e^{9c_1}\right)}+e^{6c_1}}}\right.$$

6.8 problem 17

Internal problem ID [5345]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 10. Singular solutions, Extraneous loci. Supplemetary problems. Page 74
Problem number: 17.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y(3-4y)^2 {y'}^2 + 4y = 4}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 58

```
dsolve(y(x)*(3-4*y(x))^2*diff(y(x),x)^2=4*(1-y(x)),y(x), singsol=all)
```

$$y = 1$$
$$x + \frac{y^2(y-1)}{\sqrt{-y\,(y-1)}} - c_1 = 0$$
$$x - \frac{y^2(y-1)}{\sqrt{-y\,(y-1)}} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.264 (sec). Leaf size: 3751

```
DSolve[y[x]*(3-4*y[x])^2*y'[x]^2==4*(1-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

6.9 problem 18

Internal problem ID [5346]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 10. Singular solutions, Extraneous loci. Supplemetary problems. Page 74
Problem number: 18.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y'^3 - 4y'x^4 + 8yx^3 = 0}$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 26

```
dsolve(diff(y(x),x)^3-4*x^4*diff(y(x),x)+8*x^3*y(x)=0,y(x), singsol=all)
```

$$y = \frac{x^2}{2c_1} - \frac{1}{8c_1^3}$$
$$y = c_1x^3$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]^3-4*x^4*y'[x]+8*x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

6.10 problem 19

Internal problem ID [5347]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 10. Singular solutions, Extraneous loci. Supplemetary problems. Page 74
Problem number: 19.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\left(y'^2+1\right)(x-y)^2-(x+yy')^2=0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 106

```
dsolve((diff(y(x),x)^2+1)*(x-y(x))^2=(x+y(x)*diff(y(x),x))^2,y(x), singsol=all)
```

$$y=0$$

$$y=\operatorname{RootOf}\left(-2\ln(x)-\left(\int^{\_Z}\frac{2\_a^2+\sqrt{2\_a^3-4\_a^2+2\_a}}{\_a\,(\_a^2+1)}d\_a\right)+2c_1\right)x$$

$$y=\operatorname{RootOf}\left(-2\ln(x)+\int^{\_Z}\frac{\sqrt{2}\sqrt{\_a\,(\_a-1)^2}-2\_a^2}{\_a\,(\_a^2+1)}d\_a+2c_1\right)x$$

✓ Solution by Mathematica

Time used: 4.36 (sec). Leaf size: 167

```
DSolve[(y'[x]^2+1)*(x-y[x])^2==(x+y[x]*y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x\left(x+2e^{\frac{c_1}{2}}\right)} - e^{\frac{c_1}{2}}$$
$$y(x) \to \sqrt{-x\left(x+2e^{\frac{c_1}{2}}\right)} - e^{\frac{c_1}{2}}$$
$$y(x) \to e^{\frac{c_1}{2}} - \sqrt{x\left(-x+2e^{\frac{c_1}{2}}\right)}$$
$$y(x) \to \sqrt{x\left(-x+2e^{\frac{c_1}{2}}\right)} + e^{\frac{c_1}{2}}$$
$$y(x) \to -\sqrt{-x^2}$$
$$y(x) \to \sqrt{-x^2}$$

7 Chapter 12. Linear equations of order n. Supplemetary problems. Page 81

7.1 problem 10

Internal problem ID [5348]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 12. Linear equations of order n. Supplemetary problems. Page 81
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - 6y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-6*y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^{2x} + c_2 e^{-3x}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 22

```
DSolve[y''[x]+y'[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(c_2 e^{5x} + c_1\right)$$

7.2 problem 11

Internal problem ID [5349]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 12. Linear equations of order n. Supplemetary problems. Page 81
Problem number: 11.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 6y'' + 12y' - 8y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$3)-6*diff(y(x),x$2)+12*diff(y(x),x)-8*y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^{2x} + x\,e^{2x} c_2 + x^2 e^{2x} c_3$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 23

```
DSolve[y'''[x]-6*y''[x]+12*y'[x]-8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}(x(c_3 x + c_2) + c_1)$$

7.3 problem 12

Internal problem ID [5350]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 12. Linear equations of order n. Supplemetary problems. Page 81
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 3y' + 2y = e^{5x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=exp(5*x),y(x), singsol=all)
```

$$y = \left(\frac{e^{4x}}{12} + c_1 e^x + c_2\right) e^x$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 29

```
DSolve[y''[x]-3*y'[x]+2*y[x]==Exp[5*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{5x}}{12} + c_1 e^x + c_2 e^{2x}$$

7.4 problem 13

Internal problem ID [5351]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 12. Linear equations of order n. Supplemetary problems. Page 81
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = \cos(x)\, x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+9*y(x)=x*cos(x),y(x), singsol=all)
```

$$y = c_2 \sin(3x) + c_1 \cos(3x) + \frac{\sin(x)}{32} + \frac{x\cos(x)}{8}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 32

```
DSolve[y''[x]+9*y[x]==x*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{32}(\sin(x) + 4x\cos(x)) + c_1 \cos(3x) + c_2 \sin(3x)$$

7.5 problem 14

Internal problem ID [5352]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 12. Linear equations of order n. Supplemetary problems. Page 81
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' - 3y'x + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y = c_1x^2 + \ln(x)\,x^2c_2$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]-3*x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(2c_2\log(x) + c_1)$$

7.6 problem 15

Internal problem ID [5353]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 12. Linear equations of order n. Supplemetary problems. Page 81
Problem number: 15.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''' + y'x - y = 3x^4}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(x^3*diff(y(x),x$3)+x*diff(y(x),x)-y(x)=3*x^4,y(x), singsol=all)
```

$$y = \frac{x^4}{9} + c_1 x + c_2 \ln(x)\, x + c_3 \ln(x)^2\, x$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 31

```
DSolve[x^3*y'''[x]+x*y'[x]-y[x]==3*x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^4}{9} + c_1 x + c_3 x \log^2(x) + c_2 x \log(x)$$

7.7 problem 16

Internal problem ID [5354]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 12. Linear equations of order n. Supplemetary problems. Page 81
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{xy'' - y' + 4yx^3 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x$2)-diff(y(x),x)+4*x^3*y(x)=0,y(x), singsol=all)
```

$$y = c_1 \sin\left(x^2\right) + c_2 \cos\left(x^2\right)$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 20

```
DSolve[x*y''[x]-y'[x]+4*x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos\left(x^2\right) + c_2 \sin\left(x^2\right)$$

7.8 problem 17

Internal problem ID [5355]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 12. Linear equations of order n. Supplemetary problems. Page 81
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{y'' + y'^2 = -1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)+diff(y(x),x)^2+1=0,y(x), singsol=all)
```

$$y = \ln\left(-\frac{c_1 \tan(x) - c_2}{\sec(x)}\right)$$

✓ Solution by Mathematica

Time used: 1.861 (sec). Leaf size: 16

```
DSolve[y''[x]+y'[x]^2+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(\cos(x - c_1)) + c_2$$

7.9 problem 18

Internal problem ID [5356]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 12. Linear equations of order n. Supplemetary problems. Page 81
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], [`

$$\boxed{yy'' + y'^2 = 2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve(y(x)*diff(y(x),x$2)+diff(y(x),x)^2=2,y(x), singsol=all)
```

$$y = \sqrt{-2c_1x + 2x^2 + 2c_2}$$
$$y = -\sqrt{-2c_1x + 2x^2 + 2c_2}$$

✓ Solution by Mathematica

Time used: 6.295 (sec). Leaf size: 101

```
DSolve[y[x]*y''[x]+y'[x]^2==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{4(x + c_2)^2 - e^{2c_1}}}{\sqrt{2}}$$
$$y(x) \to \sqrt{2(x + c_2)^2 - \frac{e^{2c_1}}{2}}$$
$$y(x) \to -\sqrt{2}\sqrt{(x + c_2)^2}$$
$$y(x) \to \sqrt{2}\sqrt{(x + c_2)^2}$$

7.10 problem 19

Internal problem ID [5357]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 12. Linear equations of order n. Supplemetary problems. Page 81
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1],`

$$\boxed{yy'' + y'^3 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(y(x)*diff(y(x),x$2)+diff(y(x),x)^3=0,y(x), singsol=all)
```

$$y = 0$$
$$y = c_1$$
$$y = e^{\text{LambertW}\left((x+c_2)e^{c_1}e^{-1}\right)-c_1+1}$$

✓ Solution by Mathematica

Time used: 60.091 (sec). Leaf size: 26

```
DSolve[y[x]*y''[x]+y'[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x + c_2}{W\left(e^{-1-c_1}(x + c_2)\right)}$$

8 Chapter 13. Homogeneous Linear equations with constant coefficients. Supplemetary problems. Page 86

8.1 problem 16

Internal problem ID [5358]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 13. Homogeneous Linear equations with constant coefficients. Supplemetary problems. Page 86
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' - 15y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)-15*y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^{3x} + c_2 e^{-5x}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 22

```
DSolve[y''[x]+2*y'[x]-15*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-5x}\left(c_2 e^{8x} + c_1\right)$$

8.2 problem 17

Internal problem ID [5359]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 13. Homogeneous Linear equations with constant coefficients. Supplemetary problems. Page 86
Problem number: 17.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + y'' - 2y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$3)+diff(y(x),x$2)-2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = c_1 + \mathrm{e}^{-2x} c_2 + \mathrm{e}^{x} c_3$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 25

```
DSolve[y'''[x]+y''[x]-2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2} c_1 e^{-2x} + c_2 e^{x} + c_3$$

8.3 problem 18

Internal problem ID [5360]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 13. Homogeneous Linear equations with constant coefficients. Supplemetary problems. Page 86
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 6y' + 9y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)+6*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^{-3x} + c_2 x\, e^{-3x}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 18

```
DSolve[y''[x]+6*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}(c_2 x + c_1)$$

8.4 problem 19

Internal problem ID [5361]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 13. Homogeneous Linear equations with constant coefficients. Supplemetary problems. Page 86
Problem number: 19.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 6y''' + 12y'' - 8y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$4)-6*diff(y(x),x$3)+12*diff(y(x),x$2)-8*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = c_1 + c_2 e^{2x} + x\, e^{2x} c_3 + x^2 e^{2x} c_4$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 43

```
DSolve[y''''[x]-6*y'''[x]+12*y''[x]-8*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} e^{2x} \left(c_3 \left(2x^2 - 2x + 1\right) + c_2(2x - 1) + 2c_1\right) + c_4$$

8.5 problem 20

Internal problem ID [5362]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 13. Homogeneous Linear equations with constant coefficients. Supplemetary problems. Page 86
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y' + 13y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+13*y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^{2x} \sin(3x) + c_2 e^{2x} \cos(3x)$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 26

```
DSolve[y''[x]-4*y'[x]+13*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}(c_2 \cos(3x) + c_1 \sin(3x))$$

8.6 problem 21

Internal problem ID [5363]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 13. Homogeneous Linear equations with constant coefficients. Supplemetary problems. Page 86
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 25y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+25*y(x)=0,y(x), singsol=all)
```

$$y = c_1 \sin(5x) + c_2 \cos(5x)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 20

```
DSolve[y''[x]+25*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(5x) + c_2 \sin(5x)$$

8.7 problem 22

Internal problem ID [5364]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 13. Homogeneous Linear equations with constant coefficients. Supplemetary problems. Page 86
Problem number: 22.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - y'' + 9y' - 9y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)+9*diff(y(x),x)-9*y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^x + c_2 \sin(3x) + c_3 \cos(3x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 26

```
DSolve[y'''[x]-y''[x]+9*y'[x]-9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3 e^x + c_1 \cos(3x) + c_2 \sin(3x)$$

8.8 problem 23

Internal problem ID [5365]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 13. Homogeneous Linear equations with constant coefficients. Supplemetary problems. Page 86
Problem number: 23.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 4y'' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$4)+4*diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y = c_1 + c_2 x + c_3 \sin(2x) + c_4 \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 32

```
DSolve[y''''[x]+4*y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_4 x - \frac{1}{4} c_1 \cos(2x) - \frac{1}{4} c_2 \sin(2x) + c_3$$

8.9 problem 24

Internal problem ID [5366]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 13. Homogeneous Linear equations with constant coefficients. Supplemetary problems. Page 86
Problem number: 24.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 6y''' + 13y'' - 12y' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$4)-6*diff(y(x),x$3)+13*diff(y(x),x$2)-12*diff(y(x),x)+4*y(x)=0,y(x), sing
```

$$y = c_1 e^{2x} + x\,e^{2x} c_2 + e^{x} c_3 + c_4 e^{x} x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 29

```
DSolve[y''''[x]-6*y'''[x]+13*y''[x]-12*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to e^{x}(c_3 e^{x} + x(c_4 e^{x} + c_2) + c_1)$$

8.10 problem 25

Internal problem ID [5367]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 13. Homogeneous Linear equations with constant coefficients. Supplemetary problems. Page 86
Problem number: 25.
ODE order: 6.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(6)} + 9y'''' + 24y'' + 16y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$6)+9*diff(y(x),x$4)+24*diff(y(x),x$2)+16*y(x)=0,y(x), singsol=all)
```

$$y = c_1 \sin(x) + c_2 \cos(x) + c_3 \sin(2x) + c_4 \cos(2x) + c_5 \sin(2x)\, x + c_6 \cos(2x)\, x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 40

```
DSolve[y''''''[x]+9*y''''[x]+24*y''[x]+16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (c_2 x + c_1)\cos(2x) + c_6 \sin(x) + \cos(x)(2(c_4 x + c_3)\sin(x) + c_5)$$

9 Chapter 14. Linear equations with constant coefficients. Supplemetary problems. Page 92

9.1 problem 11

Internal problem ID [5368]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 14. Linear equations with constant coefficients. Supplemetary problems. Page 92
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y' + 3y = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+3*y(x)=1,y(x), singsol=all)
```

$$y = c_2 \mathrm{e}^{3x} + c_1 \mathrm{e}^{x} + \frac{1}{3}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 23

```
DSolve[y''[x]-4*y'[x]+3*y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x} + c_2 e^{3x} + \frac{1}{3}$$

9.2 problem 12

Internal problem ID [5369]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 14. Linear equations with constant coefficients. Supplemetary problems. Page 92
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y' = 5}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)=5,y(x), singsol=all)
```

$$y = \frac{c_1 e^{4x}}{4} - \frac{5x}{4} + c_2$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 24

```
DSolve[y''[x]-4*y'[x]==5,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{5x}{4} + \frac{1}{4} c_1 e^{4x} + c_2$$

9.3 problem 13

Internal problem ID [5370]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 14. Linear equations with constant coefficients. Supplemetary problems. Page 92
Problem number: 13.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 4y'' = 5}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)-4*diff(y(x),x$2)=5,y(x), singsol=all)
```

$$y = \frac{c_1 e^{4x}}{16} - \frac{5x^2}{8} + c_2 x + c_3$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 30

```
DSolve[y'''[x]-4*y''[x]==5,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{5x^2}{8} + c_3 x + \frac{1}{16} c_1 e^{4x} + c_2$$

9.4 problem 14

Internal problem ID [5371]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 14. Linear equations with constant coefficients. Supplemetary problems. Page 92
Problem number: 14.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(5)} - 4y''' = 5}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$5)-4*diff(y(x),x$3)=5,y(x), singsol=all)
```

$$y = -\frac{5x^3}{24} + \frac{c_1 e^{2x}}{8} + \frac{c_3 x^2}{2} - \frac{e^{-2x} c_2}{8} + c_4 x + c_5$$

✓ Solution by Mathematica

Time used: 0.079 (sec). Leaf size: 47

```
DSolve[y'''''[x]-4*y'''[x]==5,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{5x^3}{24} + c_5 x^2 + c_4 x + \frac{1}{8} c_1 e^{2x} - \frac{1}{8} c_2 e^{-2x} + c_3$$

9.5 problem 15

Internal problem ID [5372]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 14. Linear equations with constant coefficients. Supplemetary problems. Page 92
Problem number: 15.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - 4y' = x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$3)-4*diff(y(x),x)=x,y(x), singsol=all)
```

$$y = -\frac{x^2}{8} + \frac{c_1 e^{2x}}{2} - \frac{e^{-2x} c_2}{2} + c_3$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 37

```
DSolve[y'''[x]-4*y'[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2}{8} + \frac{1}{2} c_1 e^{2x} - \frac{1}{2} c_2 e^{-2x} + c_3$$

9.6 problem 16

Internal problem ID [5373]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 14. Linear equations with constant coefficients. Supplemetary problems. Page 92
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 6y' + 9y = e^{2x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+9*y(x)=exp(2*x),y(x), singsol=all)
```

$$y = c_1 x\, e^{3x} + c_2 e^{3x} + e^{2x}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 24

```
DSolve[y''[x]-6*y'[x]+9*y[x]==Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}(1 + e^x(c_2 x + c_1))$$

9.7 problem 17

Internal problem ID [5374]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 14. Linear equations with constant coefficients. Supplemetary problems. Page 92
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' - 2y = -2x^2 + 2x + 2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-2*y(x)=2*(1+x-x^2),y(x), singsol=all)
```

$$y = c_1 e^x + e^{-2x} c_2 + x^2$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 23

```
DSolve[y''[x]+y'[x]-2*y[x]==2*(1+x-x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + c_1 e^{-2x} + c_2 e^x$$

9.8 problem 18

Internal problem ID [5375]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 14. Linear equations with constant coefficients. Supplemetary problems. Page 92
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y = 4x\,\mathrm{e}^{x}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)-y(x)=4*x*exp(x),y(x), singsol=all)
```

$$y = c_2\mathrm{e}^{-x} + c_1\mathrm{e}^{x} + (x-1)\,x\,\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 30

```
DSolve[y''[x]-y[x]==4*x*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}\left(x^2 - x + \frac{1}{2} + c_1\right) + c_2e^{-x}$$

9.9 problem 19

Internal problem ID [5376]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 14. Linear equations with constant coefficients. Supplemetary problems. Page 92
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y = \sin(x)^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)-y(x)=sin(x)^2,y(x), singsol=all)
```

$$y = c_2 \mathrm{e}^{-x} + c_1 \mathrm{e}^{x} + \frac{\cos(x)^2}{5} - \frac{3}{5}$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 30

```
DSolve[y''[x]-y[x]==Sin[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{10}(\cos(2x) - 5) + c_1 e^x + c_2 e^{-x}$$

9.10 problem 20

Internal problem ID [5377]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 14. Linear equations with constant coefficients. Supplemetary problems. Page 92
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y = \frac{1}{(1 + \mathrm{e}^{-x})^2}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)-y(x)=(1+exp(-x))^(-2),y(x), singsol=all)
```

$$y = c_2\mathrm{e}^{-x} + c_1\mathrm{e}^{x} + \frac{\mathrm{e}^{x}}{2} - 1 + \ln\left(\mathrm{e}^{x} + 1\right)\mathrm{e}^{-x} + \frac{\mathrm{e}^{-x}}{2}$$

✓ Solution by Mathematica

Time used: 0.085 (sec). Leaf size: 42

```
DSolve[y''[x]-y[x]==(1+Exp[-x])^(-2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(-2e^{x} + 2\log\left(e^{x} + 1\right) + 2c_1e^{2x} + 1 + 2c_2\right)$$

9.11 problem 21

Internal problem ID [5378]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 14. Linear equations with constant coefficients. Supplemetary problems. Page 92
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \csc(x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)+y(x)=csc(x),y(x), singsol=all)
```

$$y = \sin(x) c_2 + \cos(x) c_1 - \ln(\csc(x)) \sin(x) - x \cos(x)$$

✓ Solution by Mathematica

Time used: 0.082 (sec). Leaf size: 86

```
DSolve[y''[x]-y[x]==Csc[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\frac{1}{2} + \frac{i}{2}\right) e^{ix} \left(\text{Hypergeometric2F1}\left(\frac{1}{2} - \frac{i}{2}, 1, \frac{3}{2} - \frac{i}{2}, e^{2ix}\right) + i\,\text{Hypergeometric2F1}\left(\frac{1}{2} + \frac{i}{2}, 1, \frac{3}{2} + \frac{i}{2}, e^{2ix}\right)\right) + c_1 e^x + c_2 e^{-x}$$

9.12 problem 22

Internal problem ID [5379]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 14. Linear equations with constant coefficients. Supplemetary problems. Page 92
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 3y' + 2y = \sin\left(\mathrm{e}^{-x}\right)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=sin(exp(-x)),y(x), singsol=all)
```

$$y = \left(c_1\mathrm{e}^x - \mathrm{e}^x \sin\left(\mathrm{e}^{-x}\right) + c_2\right)\mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.085 (sec). Leaf size: 29

```
DSolve[y''[x]-3*y'[x]+2*y[x]==Sin[Exp[-x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x\left(-e^x \sin\left(e^{-x}\right) + c_2e^x + c_1\right)$$

10 Chapter 15. Linear equations with constant coefficients (Variation of parameters). Supplemetary problems. Page 98

10.1 problem 10

Internal problem ID [5380]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 15. Linear equations with constant coefficients (Variation of parameters). Supplemetary problems. Page 98
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \csc(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)+y(x)=csc(x),y(x), singsol=all)
```

$$y = \sin(x) c_2 + \cos(x) c_1 - \ln(\csc(x)) \sin(x) - x\cos(x)$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 24

```
DSolve[y''[x]+y[x]==Csc[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (-x + c_1)\cos(x) + \sin(x)(\log(\sin(x)) + c_2)$$

10.2 problem 11

Internal problem ID [5381]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 15. Linear equations with constant coefficients (Variation of parameters). Supplemetary problems. Page 98
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = 4\sec(x)^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$2)+4*y(x)=4*sec(x)^2,y(x), singsol=all)
```

$$y = c_2 \sin(2x) + c_1 \cos(2x) + \left(-8\cos(x)^2 + 4\right)\ln(\sec(x)) + 8\cos(x)\sin(x)\,x - 4\sin(x)^2$$

✓ Solution by Mathematica

Time used: 0.075 (sec). Leaf size: 44

```
DSolve[y''[x]+4*y[x]==4*Sec[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\sin(2x)\arctan(\tan(x)) + 2x\sin(2x) + c_2\sin(2x) + \cos(2x)(4\log(\cos(x)) + 2 + c_1) - 2$$

10.3 problem 12

Internal problem ID [5382]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 15. Linear equations with constant coefficients (Variation of parameters). Supplemetary problems. Page 98
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 3y = \frac{1}{1 + e^{-x}}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 56

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+3*y(x)=1/(1+exp(-x)),y(x), singsol=all)
```

$$y = c_2 e^{3x} + c_1 e^{x} - \frac{e^{3x} \ln(e^x + 1)}{2} + \frac{e^{3x} \ln(e^x)}{2} + \frac{\ln(1 + e^{-x}) e^x}{2} + \frac{e^{2x}}{2} - \frac{e^x}{4}$$

✓ Solution by Mathematica

Time used: 0.077 (sec). Leaf size: 49

```
DSolve[y''[x]-4*y'[x]+3*y[x]==1/(1+Exp[-x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} e^x \left(-4\left(e^{2x} - 1\right) \operatorname{arctanh}(2e^x + 1) + 2e^x + 4c_2 e^{2x} - 1 + 4c_1\right)$$

10.4 problem 13

Internal problem ID [5383]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 15. Linear equations with constant coefficients (Variation of parameters). Supplemetary problems. Page 98
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y = \sin\left(\mathrm{e}^{-x}\right)\mathrm{e}^{-x} + \cos\left(\mathrm{e}^{-x}\right)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)-y(x)=exp(-x)*sin(exp(-x))+cos(exp(-x)),y(x), singsol=all)
```

$$y = c_2\mathrm{e}^{-x} + c_1\mathrm{e}^{x} - 2\cos\left(\frac{\mathrm{e}^{-x}}{2}\right)\mathrm{e}^{x}\sin\left(\frac{\mathrm{e}^{-x}}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.166 (sec). Leaf size: 31

```
DSolve[y''[x]-y[x]==Exp[-x]*Sin[Exp[-x]]+Cos[Exp[-x]],y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to -e^{x}\sin\left(e^{-x}\right) + c_1e^{x} + c_2e^{-x}$$

10.5 problem 14

Internal problem ID [5384]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 15. Linear equations with constant coefficients (Variation of parameters). Supplemetary problems. Page 98
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y = \frac{1}{(1+\mathrm{e}^{-x})^2}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)-y(x)=1/(1+exp(-x))^2,y(x), singsol=all)
```

$$y = c_2\mathrm{e}^{-x} + c_1\mathrm{e}^{x} + \frac{\mathrm{e}^{x}}{2} - 1 + \ln(\mathrm{e}^{x}+1)\,\mathrm{e}^{-x} + \frac{\mathrm{e}^{-x}}{2}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 42

```
DSolve[y''[x]-y[x]==1/(1+Exp[-x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(-2e^{x} + 2\log(e^{x}+1) + 2c_1e^{2x} + 1 + 2c_2\right)$$

10.6 problem 15

Internal problem ID [5385]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 15. Linear equations with constant coefficients (Variation of parameters). Supplemetary problems. Page 98
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y = 2 + e^{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+2*y(x)=2+exp(x),y(x), singsol=all)
```

$$y = c_1 \cos\left(\sqrt{2}\, x\right) + c_2 \sin\left(\sqrt{2}\, x\right) + 1 + \frac{e^{x}}{3}$$

✓ Solution by Mathematica

Time used: 0.216 (sec). Leaf size: 36

```
DSolve[y''[x]+2*y[x]==2+Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x}{3} + c_1 \cos\left(\sqrt{2}x\right) + c_2 \sin\left(\sqrt{2}x\right) + 1$$

10.7 problem 16

Internal problem ID [5386]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 15. Linear equations with constant coefficients (Variation of parameters). Supplemetary problems. Page 98
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y = \mathrm{e}^{x} \sin (2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)-y(x)=exp(x)*sin(2*x),y(x), singsol=all)
```

$$y = c_2 \mathrm{e}^{-x} + c_1 \mathrm{e}^{x} - \frac{\mathrm{e}^{x} (\cos (2x) + \sin (2x))}{8}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 37

```
DSolve[y''[x]-y[x]==Exp[x]*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x} + c_2 e^{-x} - \frac{1}{8} e^{x} (\sin(2x) + \cos(2x) + 2)$$

10.8 problem 17

Internal problem ID [5387]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 15. Linear equations with constant coefficients (Variation of parameters). Supplemetary problems. Page 98
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + 2y = x^2 + \sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+2*y(x)=x^2+sin(x),y(x), singsol=all)
```

$$y = \cos(x)\, \mathrm{e}^{-x} c_1 + \sin(x)\, \mathrm{e}^{-x} c_2 + \frac{x^2}{2} - \frac{2\cos(x)}{5} + \frac{\sin(x)}{5} - x + \frac{1}{2}$$

✓ Solution by Mathematica

Time used: 0.308 (sec). Leaf size: 50

```
DSolve[y''[x]+2*y'[x]+2*y[x]==x^2+Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{10} e^{-x} \left(5e^x(x-1)^2 + (-4e^x + 10c_2)\cos(x) + 2(e^x + 5c_1)\sin(x)\right)$$

10.9 problem 18

Internal problem ID [5388]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 15. Linear equations with constant coefficients (Variation of parameters). Supplemetary problems. Page 98
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 9y = x + e^{2x} - \sin(2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)-9*y(x)=x+exp(2*x)-sin(2*x),y(x), singsol=all)
```

$$y = c_2 e^{3x} + c_1 e^{-3x} - \frac{e^{2x}}{5} + \frac{\sin(2x)}{13} - \frac{x}{9}$$

✓ Solution by Mathematica

Time used: 0.846 (sec). Leaf size: 44

```
DSolve[y''[x]-9*y[x]==x+Exp[2*x]-Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{9} - \frac{e^{2x}}{5} + \frac{1}{13}\sin(2x) + c_1 e^{3x} + c_2 e^{-3x}$$

10.10 problem 19

Internal problem ID [5389]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 15. Linear equations with constant coefficients (Variation of parameters). Supplemetary problems. Page 98
Problem number: 19.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + 3y'' + 2y' = x^2 + 4x + 8}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$3)+3*diff(y(x),x$2)+2*diff(y(x),x)=x^2+4*x+8,y(x), singsol=all)
```

$$y = \frac{x^2}{4} + \frac{x^3}{6} + \frac{c_1 e^{-2x}}{2} - c_2 e^{-x} + \frac{11x}{4} + c_3$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 47

```
DSolve[y'''[x]+3*y''[x]+2*y'[x]==x^2+4*x+8,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{6} + \frac{x^2}{4} + \frac{11x}{4} - \frac{1}{2}c_1 e^{-2x} - c_2 e^{-x} + c_3$$

10.11 problem 20

Internal problem ID [5390]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 15. Linear equations with constant coefficients (Variation of parameters). Supplemetary problems. Page 98
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = -2\sin(x) + 4\cos(x)\, x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)+y(x)=-2*sin(x)+4*x*cos(x),y(x), singsol=all)
```

$$y = \sin(x)\, c_2 + \cos(x)\, c_1 + \sin(x)\, x^2 + 2x\cos(x) - \sin(x)$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 32

```
DSolve[y''[x]+y[x]==-2*Sin[x]+4*x*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(2x^2 - 1 + 2c_2\right)\sin(x) + (2x + c_1)\cos(x)$$

10.12 problem 21

Internal problem ID [5391]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 15. Linear equations with constant coefficients (Variation of parameters). Supplemetary problems. Page 98
Problem number: 21.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - y'' - 4y' + 4y = 2x^2 - 4x - 1 + 2x^2 e^{2x} + 5x\, e^{2x} + e^{2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 46

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=2*x^2-4*x-1+2*x^2*exp(2*x)+5*x*ex
```

$$y = \frac{(12x^2 e^{2x} + 4x^3 e^{4x})\, e^{-2x}}{24} + c_1 e^x + e^{-2x} c_2 + e^{2x} c_3$$

✓ Solution by Mathematica

Time used: 0.519 (sec). Leaf size: 44

```
DSolve[y'''[x]-y''[x]-4*y'[x]+4*y[x]==2*x^2-4*x-1+2*x^2*Exp[2*x]+5*x*Exp[2*x]+Exp[2*x],y[x],
```

$$y(x) \to \frac{1}{6}\left(e^{2x} x + 3\right) x^2 + c_1 e^{-2x} + c_2 e^x + c_3 e^{2x}$$

11 Chapter 16. Linear equations with constant coefficients (Short methods). Supplemetary problems. Page 107

11.1 problem 26

Internal problem ID [5392]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 16. Linear equations with constant coefficients (Short methods). Suplemetary problems. Page 107
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y' + y = e^{3x} + 6\,e^{x} - 3\,e^{-2x} + 5$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=exp(3*x)+6*exp(x)-3*exp(-2*x)+5,y(x), singsol=all)
```

$$y = e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right) c_2 + e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right) c_1 + \frac{e^{-2x}(26\,e^{3x} + e^{5x} + 65\,e^{2x} - 13)}{13}$$

✓ Solution by Mathematica

Time used: 6.996 (sec). Leaf size: 70

```
DSolve[y''[x]+y'[x]+y[x]==Exp[3*x]+6*Exp[x]-3*Exp[-2*x]+5,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -e^{-2x} + 2e^{x} + \frac{e^{3x}}{13} + c_2 e^{-x/2} \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 e^{-x/2} \sin\left(\frac{\sqrt{3}x}{2}\right) + 5$$

11.2 problem 27

Internal problem ID [5393]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 16. Linear equations with constant coefficients (Short methods). Supplemetary problems. Page 107
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y = e^x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)-y(x)=exp(x),y(x), singsol=all)
```

$$y = c_1 e^x + c_2 e^{-x} + \frac{x e^x}{2}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 29

```
DSolve[y''[x]-y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x \left(\frac{x}{2} - \frac{1}{4} + c_1\right) + c_2 e^{-x}$$

11.3 problem 28

Internal problem ID [5394]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 16. Linear equations with constant coefficients (Short methods). Suplemetary problems. Page 107
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 4y = e^{x} + e^{2x}x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=exp(x)+x*exp(2*x),y(x), singsol=all)
```

$$y = c_2 e^{2x} + c_1 x\, e^{2x} + \frac{(x^3 e^{x} + 6)\, e^{x}}{6}$$

✓ Solution by Mathematica

Time used: 0.161 (sec). Leaf size: 31

```
DSolve[y''[x]-4*y'[x]+4*y[x]==Exp[x]+x*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}e^{x}\left(6 + e^{x}\left(x^3 + 6c_2 x + 6c_1\right)\right)$$

11.4 problem 29

Internal problem ID [5395]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 16. Linear equations with constant coefficients (Short methods). Suplemetary problems. Page 107
Problem number: 29.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' - y = \sin(2x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$4)-y(x)=sin(2*x),y(x), singsol=all)
```

$$y = \frac{\sin(2x)}{15} + \cos(x) c_1 + c_2 e^{x} + c_3 \sin(x) + c_4 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 37

```
DSolve[y''''[x]-y[x]==Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x} + c_3 e^{-x} + c_4 \sin(x) + \cos(x) \left(\frac{2\sin(x)}{15} + c_2 \right)$$

11.5 problem 30

Internal problem ID [5396]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 16. Linear equations with constant coefficients (Short methods). Supplemetary problems. Page 107
Problem number: 30.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' + y = \cos(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 73

```
dsolve(diff(y(x),x$3)+y(x)=cos(x),y(x), singsol=all)
```

$$y = -\frac{\cos(x)}{2\left(2+\sqrt{3}\right)\left(\sqrt{3}-2\right)} + \frac{\sin(x)}{2\left(2+\sqrt{3}\right)\left(\sqrt{3}-2\right)} + c_1 \mathrm{e}^{-x} + c_2 \cos\left(\frac{\sqrt{3}\,x}{2}\right)\mathrm{e}^{\frac{x}{2}} + c_3 \sin\left(\frac{\sqrt{3}\,x}{2}\right)\mathrm{e}^{\frac{x}{2}}$$

✓ Solution by Mathematica

Time used: 0.515 (sec). Leaf size: 68

```
DSolve[y'''[x]+y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sin(x)}{2} + \frac{\cos(x)}{2} + c_1 e^{-x} + c_3 e^{x/2}\cos\left(\frac{\sqrt{3}x}{2}\right) + c_2 e^{x/2}\sin\left(\frac{\sqrt{3}x}{2}\right)$$

11.6 problem 31

Internal problem ID [5397]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 16. Linear equations with constant coefficients (Short methods). Supplemetary problems. Page 107
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \sin(2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+4*y(x)=sin(2*x),y(x), singsol=all)
```

$$y = c_2 \sin(2x) + c_1 \cos(2x) - \frac{x \cos(2x)}{4}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 33

```
DSolve[y''[x]+4*y[x]==Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(-\frac{x}{4} + c_1\right) \cos(2x) + \frac{1}{8}(1 + 16c_2) \sin(x) \cos(x)$$

11.7 problem 32

Internal problem ID [5398]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 16. Linear equations with constant coefficients (Short methods). Suplemetary problems. Page 107
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 5y = \cos\left(\sqrt{5}\, x\right)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
dsolve(diff(y(x),x$2)+5*y(x)=cos(sqrt(5)*x),y(x), singsol=all)
```

$$y = \sin\left(x\sqrt{5}\right) c_2 + c_1 \cos\left(x\sqrt{5}\right) + \frac{\cos\left(x\sqrt{5}\right)}{10} + \frac{\sqrt{5}\, x \sin\left(x\sqrt{5}\right)}{10}$$

✓ Solution by Mathematica

Time used: 0.16 (sec). Leaf size: 45

```
DSolve[y''[x]+5*y[x]==Cos[Sqrt[5]*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\frac{1}{20} + c_1\right) \cos\left(\sqrt{5}x\right) + \frac{1}{10}\left(\sqrt{5}x + 10c_2\right) \sin\left(\sqrt{5}x\right)$$

11.8 problem 33

Internal problem ID [5399]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 16. Linear equations with constant coefficients (Short methods). Supplemetary problems. Page 107
Problem number: 33.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' + y'' + y' + y = \mathrm{e}^{x} + \mathrm{e}^{-x} + \sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve(diff(y(x),x$3)+diff(y(x),x$2)+diff(y(x),x)+y(x)=exp(x)+exp(-x)+sin(x),y(x), singsol=a
```

$$y = -\frac{x\cos(x)}{4} + \left(-\frac{x}{4} + \frac{1}{4}\right)\sin(x) + \frac{x\,\mathrm{e}^{-x}}{2} + \frac{\mathrm{e}^{x}}{4} + \frac{\mathrm{e}^{-x}}{2} + \cos(x)\,c_1 + \sin(x)\,c_2 + c_3\mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.265 (sec). Leaf size: 55

```
DSolve[y'''[x]+y''[x]+y'[x]+y[x]==Exp[x]+Exp[-x]+Sin[x],y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{1}{8}\left(2e^{-x}\left(2x + e^{2x} + 2 + 4c_3\right) + (-2x - 1 + 8c_1)\cos(x) + (-2x + 3 + 8c_2)\sin(x)\right)$$

11.9 problem 34

Internal problem ID [5400]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 16. Linear equations with constant coefficients (Short methods). Supplemetary problems. Page 107
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y = x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-y(x)=x^2,y(x), singsol=all)
```

$$y = c_1 e^x + c_2 e^{-x} - x^2 - 2$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 26

```
DSolve[y''[x]-y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^2 + c_1 e^x + c_2 e^{-x} - 2$$

11.10 problem 36

Internal problem ID [5401]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 16. Linear equations with constant coefficients (Short methods). Suplemetary problems. Page 107
Problem number: 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y = x^3 + x^2 + e^{-2x} + \cos(3x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(diff(y(x),x$2)+2*y(x)=x^3+x^2+exp(-2*x)+cos(3*x),y(x), singsol=all)
```

$$y = c_1 \cos\left(\sqrt{2}\, x\right) + c_2 \sin\left(\sqrt{2}\, x\right) + \frac{e^{-2x}}{6} - \frac{\cos(3x)}{7} - \frac{1}{2} - \frac{3x}{2} + \frac{x^2}{2} + \frac{x^3}{2}$$

✓ Solution by Mathematica

Time used: 4.775 (sec). Leaf size: 69

```
DSolve[y''[x]+2*y[x]==x^3+x^2+Exp[-2*x]+Cos[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{42}\left(21x^3 + 21x^2 - 63x + 7e^{-2x} + 9\sin(x)\sin(2x) - 6\cos^3(x) + 42c_1 \cos\left(\sqrt{2}x\right) + 42c_2 \sin\left(\sqrt{2}x\right) - 21\right)$$

11.11 problem 37

Internal problem ID [5402]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 16. Linear equations with constant coefficients (Short methods). Suplemetary problems. Page 107
Problem number: 37.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' - y = \cos(x)\,\mathrm{e}^{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-y(x)=exp(x)*cos(x),y(x), singsol=all)
```

$$y = \mathrm{e}^{x\left(1+\sqrt{2}\right)}c_2 + c_1\mathrm{e}^{-x\left(\sqrt{2}-1\right)} - \frac{\cos(x)\,\mathrm{e}^{x}}{3}$$

✓ Solution by Mathematica

Time used: 0.159 (sec). Leaf size: 56

```
DSolve[y''[x]-2*y'[x]-y[x]==Exp[x]*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}e^{-\sqrt{2}x}\left(-e^{\left(1+\sqrt{2}\right)x}\cos(x) + 3e^{x}\left(c_2e^{2\sqrt{2}x} + c_1\right)\right)$$

11.12 problem 38

Internal problem ID [5403]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 16. Linear equations with constant coefficients (Short methods). Supplemetary problems. Page 107
Problem number: 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 4y = \frac{e^{2x}}{x^2}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=exp(2*x)/x^2,y(x), singsol=all)
```

$$y = c_2 e^{2x} + c_1 x\, e^{2x} + e^{2x}(-1 - \ln(x))$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 23

```
DSolve[y''[x]-4*y'[x]+4*y[x]==Exp[2*x]/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}(-\log(x) + c_2 x - 1 + c_1)$$

11.13 problem 39

Internal problem ID [5404]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 16. Linear equations with constant coefficients (Short methods). Suplemetary problems. Page 107
Problem number: 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y = x\,e^{3x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)-y(x)=x*exp(3*x),y(x), singsol=all)
```

$$y = c_2 e^{-x} + c_1 e^{x} + \frac{(4x-3)\,e^{3x}}{32}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 34

```
DSolve[y''[x]-y[x]==x*Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{32} e^{3x}(4x-3) + c_1 e^{x} + c_2 e^{-x}$$

11.14 problem 40

Internal problem ID [5405]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 16. Linear equations with constant coefficients (Short methods). Supplemetary problems. Page 107
Problem number: 40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 5y' + 6y = e^{-2x} \sec(x)^2 (1 + 2\tan(x))$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 69

```
dsolve(diff(y(x),x$2)+5*diff(y(x),x)+6*y(x)=exp(-2*x)*sec(x)^2*(1+2*tan(x)),y(x), singsol=al
```

$$y = e^{-2x}c_2 + c_1 e^{-3x} + \frac{2\,e^{-2x}\left(\left(\tan(x)^2 + \tan(x) - 2\right)e^{2ix} + \frac{\tan(x)\left(e^{4ix}+1\right)\left(\tan(x)+1\right)}{2}\right)}{e^{4ix} + 2\,e^{2ix} + 1}$$

✓ Solution by Mathematica

Time used: 0.147 (sec). Leaf size: 26

```
DSolve[y''[x]+5*y'[x]+6*y[x]==Exp[-2*x]*Sec[x]^2*(1+2*Tan[x]),y[x],x,IncludeSingularSolution
```

$$y(x) \to e^{-3x}(e^x \tan(x) + c_2 e^x + c_1)$$

12 Chapter 17. Linear equations with variable coefficients (Cauchy and Legndre). Supplemetary problems. Page 110

12.1 problem 6

Internal problem ID [5406]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 17. Linear equations with variable coefficients (Cauchy and Legndre). Supplemetary problems. Page 110
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' - 3y'x + 4y = x + \ln(x)\, x^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*y(x)=x+x^2*ln(x),y(x), singsol=all)
```

$$y = c_2x^2 + x^2\ln(x)\, c_1 + \frac{x\left(\ln(x)^3 x + 6\right)}{6}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 30

```
DSolve[x^2*y''[x]-3*x*y'[x]+4*y[x]==x+x^2*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}x\left(x\log^3(x) + 6c_1x + 12c_2x\log(x) + 6\right)$$

12.2 problem 7

Internal problem ID [5407]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 17. Linear equations with variable coefficients (Cauchy and Legndre). Supplemetary problems. Page 110
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2y'x + 2y = \ln(x)^2 - \ln(x^2)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=(ln(x))^2-ln(x^2),y(x), singsol=all)
```

$$y = c_2x^2 + c_1x + \frac{\ln(x)^2}{2} + \frac{3\ln(x)}{2} - \frac{\ln(x^2)}{2} + \frac{1}{4}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 38

```
DSolve[x^2*y''[x]-2*x*y'[x]+2*y[x]==(Log[x])^2-Log[x^2],y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{1}{4}\left(-2\log\left(x^2\right) + 2\log^2(x) + 6\log(x) + 1\right) + c_2x^2 + c_1x$$

12.3 problem 8

Internal problem ID [5408]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 17. Linear equations with variable coefficients (Cauchy and Legndre). Supplemetary problems. Page 110
Problem number: 8.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{x^3 y''' + 2x^2 y'' = x + \sin(\ln(x))}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(x^3*diff(y(x),x$3)+2*x^2*diff(y(x),x$2)=x+sin(ln(x)),y(x), singsol=all)
```

$$y = -c_1 \ln(x) - \frac{-\tan\left(\frac{\ln(x)}{2}\right) - 1}{1 + \tan\left(\frac{\ln(x)}{2}\right)^2} + \ln(x)\, x - x + c_2 x + c_3$$

✓ Solution by Mathematica

Time used: 0.173 (sec). Leaf size: 36

```
DSolve[x^3*y'''[x]+2*x^2*y''[x]==x+Sin[Log[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(\sin(\log(x)) + \cos(\log(x)) + 2((-1 + c_3)x + (x - c_1)\log(x) + c_2))$$

12.4 problem 9

Internal problem ID [5409]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 17. Linear equations with variable coefficients (Cauchy and Legndre). Supplemetary problems. Page 110
Problem number: 9.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''' + y'x - y = 3x^4}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(x^3*diff(y(x),x$3)+x*diff(y(x),x)-y(x)=3*x^4,y(x), singsol=all)
```

$$y = \frac{x^4}{9} + c_1x + c_2\ln(x)\,x + c_3\ln(x)^2\,x$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 31

```
DSolve[x^3*y'''[x]+x*y'[x]-y[x]==3*x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^4}{9} + c_1x + c_3x\log^2(x) + c_2x\log(x)$$

12.5 problem 10

Internal problem ID [5410]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 17. Linear equations with variable coefficients (Cauchy and Legndre). Supplemetary problems. Page 110
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{(1+x)^2 y'' + y'(1+x) - y = \ln(1+x)^2 + x - 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 65

```
dsolve((x+1)^2*diff(y(x),x$2)+(x+1)*diff(y(x),x)-y(x)=(ln(x+1))^2+x-1,y(x), singsol=all)
```

$$y = \frac{c_1}{x+1} + (x+1)\,c_2 - \frac{4\ln(x+1)^2 x - 2\ln(x+1)\,x^2 + 4\ln(x+1)^2 - 4\ln(x+1)\,x - 2\ln(x+1) + 3}{4(x+1)}$$

✓ Solution by Mathematica

Time used: 0.262 (sec). Leaf size: 72

```
DSolve[(x+1)^2*y''[x]+(x+1)*y'[x]-y[x]==(Log[x+1])^2+x-1,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{(-1+2c_1+2ic_2)x^2 - 4(x+1)\log^2(x+1) + 2(x+1)^2\log(x+1) + (-2+4c_1+4ic_2)x - 1 + 4c_1}{4(x+1)}$$

12.6 problem 11

Internal problem ID [5411]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 17. Linear equations with variable coefficients (Cauchy and Legndre). Supplemetary problems. Page 110
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{(1+2x)^2 y'' - 2(1+2x) y' - 12y = 6x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve((2*x+1)^2*diff(y(x),x$2)-2*(2*x+1)*diff(y(x),x)-12*y(x)=6*x,y(x), singsol=all)
```

$$y = \frac{c_1}{2x+1} + (2x+1)^3 c_2 - \frac{24x^2+8x+1}{32(2x+1)}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 41

```
DSolve[(2*x+1)^2*y''[x]-2*(2*x+1)*y'[x]-12*y[x]==6*x,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{-24x^2 - 8x + 32c_1(2x+1)^4 - 1 + 32c_2}{32(2x+1)}$$

13 Chapter 18. Linear equations with variable coefficients (Equations of second order). Supplemetary problems. Page 120

13.1 problem 21

Internal problem ID [5412]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 18. Linear equations with variable coefficients (Equations of second order). Supplemetary problems. Page 120
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{xy'' - (x+2)\,y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)-(x+2)*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^x + c_2(x^2 + 2x + 2)$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 24

```
DSolve[x*y''[x]-(x+2)*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x - c_2(x^2 + 2x + 2)$$

13.2 problem 22

Internal problem ID [5413]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 18. Linear equations with variable coefficients (Equations of second order). Supplemetary problems. Page 120
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''-2y'x+2y=2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve((1+x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=2,y(x), singsol=all)
```

$$y=c_2x+(x^2-1)\,c_1+1$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 22

```
DSolve[(1+x^2)*y''[x]-2*x*y'[x]+2*y[x]==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -c_1(x-i)^2+c_2x+1$$

13.3 problem 23

Internal problem ID [5414]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 18. Linear equations with variable coefficients (Equations of second order). Supplemetary problems. Page 120
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+4)\,y''-2y'x+2y=8}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve((x^2+4)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=8,y(x), singsol=all)
```

$$y=c_2x+\left(x^2-4\right)c_1+4$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 22

```
DSolve[(x^2+4)*y''[x]-2*x*y'[x]+2*y[x]==8,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1(x-2i)^2-c_2x+4$$

13.4 problem 24

Internal problem ID [5415]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 18. Linear equations with variable coefficients (Equations of second order). Supplemetary problems. Page 120
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$(1+x)\,y''-(2x+3)\,y'+(x+2)\,y=\left(x^2+2x+1\right)\mathrm{e}^{2x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve((x+1)*diff(y(x),x$2)-(2*x+3)*diff(y(x),x)+(x+2)*y(x)=(x^2+2*x+1)*exp(2*x),y(x), sings
```

$$y=c_2\mathrm{e}^x+\mathrm{e}^x x(x+2)\,c_1+x\,\mathrm{e}^{2x}$$

✓ Solution by Mathematica

Time used: 0.117 (sec). Leaf size: 32

```
DSolve[(x+1)*y''[x]-(2*x+3)*y'[x]+(x+2)*y[x]==(x^2+2*x+1)*Exp[2*x],y[x],x,IncludeSingularSol
```

$$y(x)\to\frac{1}{2}e^x(2e^x x+ec_2(x+2)x+2ec_1)$$

13.5 problem 25

Internal problem ID [5416]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 18. Linear equations with variable coefficients (Equations of second order). Supplemetary problems. Page 120
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2\tan(x)\, y' - 10y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-2*tan(x)*diff(y(x),x)-10*y(x)=0,y(x), singsol=all)
```

$$y = c_1 \sec(x) \sinh(3x) + c_2 \sec(x) \cosh(3x)$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 29

```
DSolve[y''[x]-2*Tan[x]*y'[x]-10*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6} e^{-3x} \left(c_2 e^{6x} + 6c_1\right) \sec(x)$$

13.6 problem 26

Internal problem ID [5417]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 18. Linear equations with variable coefficients (Equations of second order). Supplemetary problems. Page 120
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2 y'' - x(2x+3)\, y' + \left(x^2+3x+3\right) y = \left(-x^2+6\right) e^x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 25

```
dsolve(x^2*diff(y(x),x$2)-x*(2*x+3)*diff(y(x),x)+(x^2+3*x+3)*y(x)=(6-x^2)*exp(x),y(x), sings
```

$$y = x\, e^x c_2 + x^3 e^x c_1 + e^x \left(x^2+2\right)$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 30

```
DSolve[x^2*y''[x]-x*(2*x+3)*y'[x]+(x^2+3*x+3)*y[x]==(6-x^2)*Exp[x],y[x],x,IncludeSingularSol
```

$$y(x) \to \frac{1}{2} e^x \left(c_2 x^3 + 2x^2 + 2c_1 x + 4\right)$$

13.7 problem 27

Internal problem ID [5418]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 18. Linear equations with variable coefficients (Equations of second order). Supplemetary problems. Page 120
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 4y'x^3 + \left(x^2+1\right)^2 y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(4*x^2*diff(y(x),x$2)+4*x^3*diff(y(x),x)+(x^2+1)^2*y(x)=0,y(x), singsol=all)
```

$$y = c_1\sqrt{x}\,\mathrm{e}^{-\frac{x^2}{4}} + c_2\sqrt{x}\,\mathrm{e}^{-\frac{x^2}{4}}\ln(x)$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 28

```
DSolve[4*x^2*y''[x]+4*x^3*y'[x]+(x^2+1)^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{x^2}{4}}\sqrt{x}(c_2\log(x) + c_1)$$

13.8 problem 28

Internal problem ID [5419]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 18. Linear equations with variable coefficients (Equations of second order). Supplemetary problems. Page 120
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x^2y'' + (-4x^2 + x)\, y' + (4x^2 - 2x + 1)\, y = (x^2 - x + 1)\, e^x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(x^2*diff(y(x),x$2)+(x-4*x^2)*diff(y(x),x)+(1-2*x+4*x^2)*y(x)=(x^2-x+1)*exp(x),y(x), s
```

$$y = x^i e^{2x} c_2 + c_1 x^{-i} e^{2x} + e^x$$

✓ Solution by Mathematica

Time used: 0.184 (sec). Leaf size: 104

```
DSolve[x^2*y''[x]+(x-4*x^2)*y'[x]+(1-2*x+4*x^2)*y[x]==(x^2-x+1)*Exp[x],y[x],x,IncludeSingula
```

$$y(x) \to \frac{1}{2} e^{2x} x^{-i} \big(i x^{2i} \Gamma(-i, x) - i x^{2i} \Gamma(1-i, x) + i x^{2i} \Gamma(2-i, x) - i c_2 x^{2i} - i \Gamma(i, x) + i \Gamma(1+i, x) - i \Gamma(2+i, x) + 2c_1 \big)$$

13.9 problem 29

Internal problem ID [5420]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 18. Linear equations with variable coefficients (Equations of second order). Supplemetary problems. Page 120
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{xy'' - y' + 4yx^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x$2)-diff(y(x),x)+4*x^3*y(x)=0,y(x), singsol=all)
```

$$y = c_1 \sin(x^2) + c_2 \cos(x^2)$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 20

```
DSolve[x*y''[x]-y'[x]+4*x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(x^2) + c_2 \sin(x^2)$$

13.10 problem 30

Internal problem ID [5421]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 18. Linear equations with variable coefficients (Equations of second order). Supplemetary problems. Page 120
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^4y'' + 2y'x^3 + y = \frac{1+x}{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x^4*diff(y(x),x$2)+2*x^3*diff(y(x),x)+y(x)=(1+x)/x,y(x), singsol=all)
```

$$y = c_2 \sin\left(\frac{1}{x}\right) + c_1 \cos\left(\frac{1}{x}\right) + \frac{x+1}{x}$$

✓ Solution by Mathematica

Time used: 0.072 (sec). Leaf size: 25

```
DSolve[x^4*y''[x]+2*x^3*y'[x]+y[x]==(1+x)/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{x} + c_1 \cos\left(\frac{1}{x}\right) - c_2 \sin\left(\frac{1}{x}\right) + 1$$

13.11 problem 31

Internal problem ID [5422]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 18. Linear equations with variable coefficients (Equations of second order). Supplemetary problems. Page 120
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^8 y'' + 4x^7 y' + y = \frac{1}{x^3}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x^8*diff(y(x),x$2)+4*x^7*diff(y(x),x)+y(x)=1/x^3,y(x), singsol=all)
```

$$y = c_2 \sin\left(\frac{1}{3x^3}\right) + c_1 \cos\left(\frac{1}{3x^3}\right) + \frac{1}{x^3}$$

✓ Solution by Mathematica

Time used: 0.057 (sec). Leaf size: 32

```
DSolve[x^8*y''[x]+4*x^7*y'[x]+y[x]==1/x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{x^3} + c_1 \cos\left(\frac{1}{3x^3}\right) - c_2 \sin\left(\frac{1}{3x^3}\right)$$

13.12 problem 32

Internal problem ID [5423]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 18. Linear equations with variable coefficients (Equations of second order). Supplemetary problems. Page 120
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(\sin(x)\,x + \cos(x))\,y'' - y'\cos(x)\,x + \cos(x)\,y = x$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 23

```
dsolve((x*sin(x)+cos(x))*diff(y(x),x$2)-x*cos(x)*diff(y(x),x)+y(x)*cos(x)=x,y(x), singsol=al
```

$$y = \left(-\frac{c_1\cos(x)}{x} - \frac{\sin(x)}{x} + c_2\right)x$$

✓ Solution by Mathematica

Time used: 0.764 (sec). Leaf size: 20

```
DSolve[(x*Sin[x]+Cos[x])*y''[x]-x*Cos[x]*y'[x]+y[x]*Cos[x]==x,y[x],x,IncludeSingularSolution
```

$$y(x) \to -\sin(x) + c_1 x - c_2\cos(x)$$

13.13 problem 33

Internal problem ID [5424]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 18. Linear equations with variable coefficients (Equations of second order). Supplemetary problems. Page 120
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - 3y' + \frac{3y}{x} = x + 2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x*diff(y(x),x$2)-3*diff(y(x),x)+3*y(x)/x=x+2,y(x), singsol=all)
```

$$y = \left(-x - \ln(x) + \frac{c_1 x^2}{2} + c_2\right) x$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 30

```
DSolve[x*y''[x]-3*y'[x]+3*y[x]/x==x+2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} x \left(2c_2 x^2 - 2x - 2\log(x) - 1 + 2c_1\right)$$

13.14 problem 35

Internal problem ID [5425]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 18. Linear equations with variable coefficients (Equations of second order). Supplemetary problems. Page 120
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(1+x)\,y'' - (4+3x)\,y' + 3y = (3x+2)\,\mathrm{e}^{3x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve((x+1)*diff(y(x),x$2)-(3*x+4)*diff(y(x),x)+3*y(x)=(3*x+2)*exp(3*x),y(x), singsol=all)
```

$$y = \left(x+\frac{4}{3}\right)c_2 + c_1\mathrm{e}^{3x} + x\,\mathrm{e}^{3x}$$

✓ Solution by Mathematica

Time used: 0.364 (sec). Leaf size: 48

```
DSolve[(x+1)*y''[x]-(3*x+4)*y'[x]+3*y[x]==(3*x+2)*Exp[3*x],y[x],x,IncludeSingularSolutions -
```

$$y(x) \to e^{3x}\left(x+\frac{2}{3}\right) + \frac{c_1 e^{3x+3}}{\sqrt{2}} - \frac{1}{9}\sqrt{2}c_2(3x+4)$$

13.15 problem 36

Internal problem ID [5426]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 18. Linear equations with variable coefficients (Equations of second order). Supplemetary problems. Page 120
Problem number: 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 4y'x + (9x^2 + 6)\, y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(6+9*x^2)*y(x)=0,y(x), singsol=all)
```

$$y = c_1x^2 \sin(3x) + c_2x^2 \cos(3x)$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 37

```
DSolve[x^2*y''[x]-4*x*y'[x]+(6+9*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}e^{-3ix}x^2\left(6c_1 - ic_2e^{6ix}\right)$$

13.16 problem 37

Internal problem ID [5427]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 18. Linear equations with variable coefficients (Equations of second order). Supplemetary problems. Page 120
Problem number: 37.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{xy'' + 2y' + 4yx = 4}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+4*x*y(x)=4,y(x), singsol=all)
```

$$y = \frac{c_2 \sin(2x)}{x} + \frac{c_1 \cos(2x)}{x} + \frac{1}{x}$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 38

```
DSolve[x*y''[x]+2*y'[x]+4*x*y[x]==4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4c_1 e^{-2ix} - ic_2 e^{2ix} + 4}{4x}$$

13.17 problem 38

Internal problem ID [5428]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 18. Linear equations with variable coefficients (Equations of second order). Supplemetary problems. Page 120
Problem number: 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''-2y'x+2y=\frac{-x^2+1}{x}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve((1+x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=(1-x^2)/x,y(x), singsol=all)
```

$$y=c_2x+(x^2-1)\,c_1+(1+\ln(x))\,x$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 27

```
DSolve[(1+x^2)*y''[x]-2*x*y'[x]+2*y[x]==(1-x^2)/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x(\log(x)+1)-c_1(x-i)^2+c_2x$$

14 Chapter 19. Linear equations with variable coefficients (Misc. types). Supplemetary problems. Page 132

14.1 problem 22

Internal problem ID [5429]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 19. Linear equations with variable coefficients (Misc. types). Supplemetary problems. Page 132
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{y'' + y'^2 = -1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)+diff(y(x),x)^2+1=0,y(x), singsol=all)
```

$$y = \ln\left(-\frac{c_1 \tan(x) - c_2}{\sec(x)}\right)$$

✓ Solution by Mathematica

Time used: 1.79 (sec). Leaf size: 16

```
DSolve[y''[x]+y'[x]^2+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(\cos(x - c_1)) + c_2$$

14.2 problem 23

Internal problem ID [5430]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 19. Linear equations with variable coefficients (Misc. types). Supplemetary problems. Page 132
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{(x^2+1)\,y''+2y'x=\frac{2}{x^3}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((1+x^2)*diff(y(x),x$2)+2*x*diff(y(x),x)=2*x^(-3),y(x), singsol=all)
```

$$y=(c_1+1)\arctan(x)+\frac{1}{x}+c_2$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 18

```
DSolve[(1+x^2)*y''[x]+2*x*y'[x]==2*x^(-3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to(1+c_1)\arctan(x)+\frac{1}{x}+c_2$$

14.3 problem 24

Internal problem ID [5431]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 19. Linear equations with variable coefficients (Misc. types). Supplemetary problems. Page 132
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' - y' = -\frac{2}{x} - \ln(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x*diff(y(x),x$2)-diff(y(x),x)=-2/x-ln(x),y(x), singsol=all)
```

$$y = \frac{c_1 x^2}{2} + \ln(x)\,x + \ln(x) + c_2$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 23

```
DSolve[x*y''[x]-y'[x]==-2/x-Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 x^2}{2} + (x+1)\log(x) + c_2$$

14.4 problem 25

Internal problem ID [5432]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 19. Linear equations with variable coefficients (Misc. types). Supplemetary problems. Page 132
Problem number: 25.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + y'' = x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$3)+diff(y(x),x$2)=x^2,y(x), singsol=all)
```

$$y = \frac{x^4}{12} + x^2 - \frac{x^3}{3} + \mathrm{e}^{-x} c_1 + c_2 x + c_3$$

✓ Solution by Mathematica

Time used: 0.09 (sec). Leaf size: 37

```
DSolve[y'''[x]+y''[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^4}{12} - \frac{x^3}{3} + x^2 + c_3 x + c_1 e^{-x} + c_2$$

14.5 problem 26

Internal problem ID [5433]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 19. Linear equations with variable coefficients (Misc. types). Supplemetary problems. Page 132
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1],`

$$\boxed{yy'' + y'^3 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(y(x)*diff(y(x),x$2)+diff(y(x),x)^3=0,y(x), singsol=all)
```

$$y = 0$$
$$y = c_1$$
$$y = e^{\mathrm{LambertW}\left((x+c_2)e^{c_1}e^{-1}\right)-c_1+1}$$

✓ Solution by Mathematica

Time used: 60.092 (sec). Leaf size: 26

```
DSolve[y[x]*y''[x]+y'[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x + c_2}{W\left(e^{-1-c_1}(x + c_2)\right)}$$

14.6 problem 27

Internal problem ID [5434]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 19. Linear equations with variable coefficients (Misc. types). Supplemetary problems. Page 132
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _`

$$\boxed{yy'' + {y'}^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(y(x)*diff(y(x),x$2)+diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y = 0$$
$$y = \sqrt{2c_1x + 2c_2}$$
$$y = -\sqrt{2c_1x + 2c_2}$$

✓ Solution by Mathematica

Time used: 0.172 (sec). Leaf size: 20

```
DSolve[y[x]*y''[x]+y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2\sqrt{2x - c_1}$$

14.7 problem 28

Internal problem ID [5435]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 19. Linear equations with variable coefficients (Misc. types). Supplemetary problems. Page 132
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_y_y1]]`

$$yy'' - y'^2(1 - y'\cos(y) + yy'\sin(y)) = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 22

```
dsolve(y(x)*diff(y(x),x$2)=diff(y(x),x)^2*(1-diff(y(x),x)*cos(y(x))+y(x)*diff(y(x),x)*sin(y(
```

$$y = c_1$$
$$\sin(y) + c_1 \ln(y) - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 0.425 (sec). Leaf size: 63

```
DSolve[y[x]*y''[x]==y'[x]^2*(1-y'[x]*Cos[y[x]]+y[x]*y'[x]*Sin[y[x]] ),y[x],x,IncludeSingular
```

$$y(x) \to \text{InverseFunction}[\sin(\#1) + c_1 \log(\#1)\&][x + c_2]$$
$$y(x) \to \text{InverseFunction}[\sin(\#1) - c_1 \log(\#1)\&][x + c_2]$$
$$y(x) \to \text{InverseFunction}[\sin(\#1) + c_1 \log(\#1)\&][x + c_2]$$

14.8 problem 29

Internal problem ID [5436]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 19. Linear equations with variable coefficients (Misc. types). Supplemetary problems. Page 132
Problem number: 29.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{(2x-3)\,y'''-(6x-7)\,y''+4y'x-4y=8}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve((2*x-3)*diff(y(x),x$3)-(6*x-7)*diff(y(x),x$2)+4*x*diff(y(x),x)-4*y(x)=8,y(x), singsol
```

$$y=-2+c_1x+c_2\mathrm{e}^{x}+c_3\mathrm{e}^{2x}$$

✓ Solution by Mathematica

Time used: 0.159 (sec). Leaf size: 26

```
DSolve[(2*x-3)*y'''[x]-(6*x-7)*y''[x]+4*x*y'[x]-4*y[x]==8,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to c_1x+c_2e^{x}-c_3e^{2x}-2$$

14.9 problem 30

Internal problem ID [5437]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 19. Linear equations with variable coefficients (Misc. types). Supplemetary problems. Page 132
Problem number: 30.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{(2x^3-1)\,y'''-6x^2y''+6y'x=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve((2*x^3-1)*diff(y(x),x$3)-6*x^2*diff(y(x),x$2)+6*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y=c_1+c_2x^2+c_3x\left(x^3+4\right)$$

✓ Solution by Mathematica

Time used: 1.357 (sec). Leaf size: 31

```
DSolve[(2*x^3-1)*y'''[x]-6*x^2*y''[x]+6*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{c_2x^4}{4}+\frac{c_1x^2}{2}-c_2x+c_3$$

14.10 problem 31

Internal problem ID [5438]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 19. Linear equations with variable coefficients (Misc. types). Supplemetary problems. Page 132
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{yy'' - y'^2 - y^2 \ln(y) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(y(x)*diff(y(x),x$2)-diff(y(x),x)^2=y(x)^2*ln(y(x)),y(x), singsol=all)
```

$$y = \mathrm{e}^{\frac{\mathrm{e}^{-2x} c_1 \mathrm{e}^{x}}{2}} \mathrm{e}^{-\frac{c_2 \mathrm{e}^{x}}{2}}$$

✓ Solution by Mathematica

Time used: 2.66 (sec). Leaf size: 73

```
DSolve[y[x]*y''[x]-y'[x]^2==y[x]^2*Log[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \exp\left(-\frac{1}{2}\sqrt{c_1} e^{-x-c_2}\left(-1+e^{2(x+c_2)}\right)\right)$$
$$y(x) \to \exp\left(\frac{1}{2}\sqrt{c_1} e^{-x-c_2}\left(-1+e^{2(x+c_2)}\right)\right)$$

14.11 problem 32

Internal problem ID [5439]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 19. Linear equations with variable coefficients (Misc. types). Supplemetary problems. Page 132
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _nonlinear], [_2nd_order, _with_linear_s`

$$(x+2y)\,y''+2y'^2+2y'=2$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

```
dsolve((x+2*y(x))*diff(y(x),x$2)+2*diff(y(x),x)^2+2*diff(y(x),x)=2,y(x), singsol=all)
```

$$y=-\frac{x}{2}-\frac{\sqrt{-4c_1x+5x^2+4c_2}}{2}$$
$$y=-\frac{x}{2}+\frac{\sqrt{-4c_1x+5x^2+4c_2}}{2}$$

✓ Solution by Mathematica

Time used: 0.645 (sec). Leaf size: 77

```
DSolve[(x+2*y[x])*y''[x]+2*y'[x]^2+2*y'[x]==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{1}{2}x\left(1+\sqrt{\frac{1}{x^2}}\sqrt{5x^2+4c_2x+4c_1}\right)$$
$$y(x)\to \frac{1}{2}x\left(-1+\sqrt{\frac{1}{x^2}}\sqrt{5x^2+4c_2x+4c_1}\right)$$

14.12 problem 33

Internal problem ID [5440]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 19. Linear equations with variable coefficients (Misc. types). Supplemetary problems. Page 132
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _nonlinear]]`

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 1500

```
dsolve((1+2*y(x)+3*y(x)^2)*diff(y(x),x$3)+6*diff(y(x),x)*( diff(y(x),x$2)+diff(y(x),x)^2+3*y
```

y

$= \frac{(224 + 36x^4 - 432c_1x^2 - 432c_1^2 - 864c_2x + 864c_3 + 12\sqrt{9x^8 - 216c_1x^6 + 1080c_1^2x^4 - 432c_2x^5 + 2592c_1^3}}{}$

$- \frac{}{3(224 + 36x^4 - 432c_1x^2 - 432c_1^2 - 864c_2x + 864c_3 + 12\sqrt{9x^8 - 216c_1x^6 + 1080c_1^2x^4 - 432c_2x^5 + 25}}$

$- \frac{1}{3}$

$y =$

$- \frac{(224 + 36x^4 - 432c_1x^2 - 432c_1^2 - 864c_2x + 864c_3 + 12\sqrt{9x^8 - 216c_1x^6 + 1080c_1^2x^4 - 432c_2x^5 + 259}}{}$

$+ \frac{}{3(224 + 36x^4 - 432c_1x^2 - 432c_1^2 - 864c_2x + 864c_3 + 12\sqrt{9x^8 - 216c_1x^6 + 1080c_1^2x^4 - 432c_2x^5 + 25}}$

$- \frac{1}{3}$

$- \frac{i\sqrt{3}\,(\frac{(224+36x^4-432c_1x^2-432c_1^2-864c_2x+864c_3+12\sqrt{9x^8-216c_1x^6+1080c_1^2x^4-432c_2x^5+2592c_1^3x^2+5184c_1c_2x^3+432c_3x^4+1296c_1^4}}{6}}{}$

$y =$

$- \frac{(224 + 36x^4 - 432c_1x^2 - 432c_1^2 - 864c_2x + 864c_3 + 12\sqrt{9x^8 - 216c_1x^6 + 1080c_1^2x^4 - 432c_2x^5 + 259}}{}$

$+ \frac{}{3(224 + 36x^4 - 432c_1x^2 - 432c_1^2 - 864c_2x + 864c_3 + 12\sqrt{9x^8 - 216c_1x^6 + 1080c_1^2x^4 - 432c_2x^5 + 25}}$

$- \frac{1}{3}$

$+ \frac{i\sqrt{3}\,(\frac{(224+36x^4-432c_1x^2-432c_1^2-864c_2x+864c_3+12\sqrt{9x^8-216c_1x^6+1080c_1^2x^4-432c_2x^5+2592c_1^3x^2+5184c_1c_2x^3+432c_3x^4+1296c_1^4}}{6}}{}$

✓ Solution by Mathematica

Time used: 0.369 (sec). Leaf size: 523

```
DSolve[(1+2*y[x]+3*y[x]^2)*y'''[x]+6*y'[x]*( y''[x]+y'[x]^2+3*y[x]*y''[x] )==x,y[x],x,Includ
```

$$y(x) \to \frac{2^{2/3}\left(9x^4+108c_1x^2+\sqrt{2048+(9x^4+108c_1x^2+27c_3x+56+216c_2)^2}+27c_3x+56+216c_2\right)^{2/3}-4}{12\sqrt[3]{9x^4+108c_1x^2+\sqrt{2048+(9x^4+108c_1x^2-}}}$$

$$y(x) \to \frac{1}{24}\left(i2^{2/3}\left(\sqrt{3}+i\right)\sqrt[3]{9x^4+108c_1x^2+\sqrt{2048+(9x^4+108c_1x^2+27c_3x+56+216c_2)^2}+27c_3x+56} \right. + \frac{16\sqrt[3]{2}(1+i\sqrt{3})}{\sqrt[3]{9x^4+108c_1x^2+\sqrt{2048+(9x^4+108c_1x^2+27c_3x+56+216c_2)^2}+27c_3x+56+216c_2}} \left. -8\right)$$

$$y(x) \to \frac{1}{24}\left(-2^{2/3}\left(1+i\sqrt{3}\right)\sqrt[3]{9x^4+108c_1x^2+\sqrt{2048+(9x^4+108c_1x^2+27c_3x+56+216c_2)^2}+27c_3x+5} \right. + \frac{16\sqrt[3]{2}(1-i\sqrt{3})}{\sqrt[3]{9x^4+108c_1x^2+\sqrt{2048+(9x^4+108c_1x^2+27c_3x+56+216c_2)^2}+27c_3x+56+216c_2}} \left. -8\right)$$

14.13 problem 34

Internal problem ID [5441]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 19. Linear equations with variable coefficients (Misc. types). Supplemetary problems. Page 132
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _nonlinear], [_3rd_order, _with_linear_s`

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 150

```
dsolve(3*x*( y(x)^2* diff(y(x),x$3)+6*y(x)*diff(y(x),x)*diff(y(x),x$2)+2*diff(y(x),x)^3   )-
```

$$y = \frac{(-8c_3x^3 + 8\ln(x)\,x + 12c_1x + 8c_2 - 4x)^{\frac{1}{3}}}{2}$$

$$y = -\frac{(-8c_3x^3 + 8\ln(x)\,x + 12c_1x + 8c_2 - 4x)^{\frac{1}{3}}}{4} - \frac{i\sqrt{3}\,(-8c_3x^3 + 8\ln(x)\,x + 12c_1x + 8c_2 - 4x)^{\frac{1}{3}}}{4}$$

$$y = -\frac{(-8c_3x^3 + 8\ln(x)\,x + 12c_1x + 8c_2 - 4x)^{\frac{1}{3}}}{4} + \frac{i\sqrt{3}\,(-8c_3x^3 + 8\ln(x)\,x + 12c_1x + 8c_2 - 4x)^{\frac{1}{3}}}{4}$$

✓ Solution by Mathematica

Time used: 0.297 (sec). Leaf size: 121

```
DSolve[3*x*( y[x]^2* y'''[x]+6*y[x]*y'[x]*y''[x]+2*y'[x]^3   )-3*y[x]* (y[x]*y''[x]+2* y'[x]
```

$$y(x) \to -\sqrt[3]{-\frac{1}{6}}\sqrt[3]{6c_3x^3 + 6x\log(x) + (3+9c_2)x + 2c_1}$$

$$y(x) \to \sqrt[3]{c_3x^3 + x\log(x) + \frac{1}{2}(1+3c_2)x + \frac{c_1}{3}}$$

$$y(x) \to (-1)^{2/3}\sqrt[3]{c_3x^3 + x\log(x) + \frac{1}{2}(1+3c_2)x + \frac{c_1}{3}}$$

14.14 problem 35

Internal problem ID [5442]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952

Section: Chapter 19. Linear equations with variable coefficients (Misc. types). Supplemetary problems. Page 132

Problem number: 35.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _nonlinear], [_3rd_order, _with_linear_s`

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 51

```
dsolve(y(x)*diff(y(x),x$3)+3*diff(y(x),x)*diff(y(x),x$2)-2*y(x)*diff(y(x),x$2)-2*diff(y(x),x
```

$$y = \sqrt{-2c_3 e^x x + e^{2x} + 2c_2 e^x - 2c_1}$$
$$y = -\sqrt{-2c_3 e^x x + e^{2x} + 2c_2 e^x - 2c_1}$$

✓ Solution by Mathematica

Time used: 0.387 (sec). Leaf size: 65

```
DSolve[y[x]*y'''[x]+3*y'[x]*y''[x]-2*y[x]*y''[x]-2*y'[x]^2+y[x]*y'[x]==Exp[2*x],y[x],x,Inclu
```

$$y(x) \to -\sqrt{e^{2x} + e^x(c_3 x + 2c_2) + 2c_1}$$
$$y(x) \to \sqrt{e^{2x} + e^x(c_3 x + 2c_2) + 2c_1}$$

14.15 problem 36

Internal problem ID [5443]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 19. Linear equations with variable coefficients (Misc. types). Supplemetary problems. Page 132
Problem number: 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{2(1+y)\, y''+2{y'}^{2}+y^{2}+2y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
dsolve(2*(y(x)+1)*diff(y(x),x$2)+2*diff(y(x),x)^2+y(x)^2+2*y(x)=0,y(x), singsol=all)
```

$$y = -1-\sqrt{1+2c_{2}\cos\left(x\right)-2c_{1}\sin\left(x\right)}$$
$$y = -1+\sqrt{1+2c_{2}\cos\left(x\right)-2c_{1}\sin\left(x\right)}$$

✓ Solution by Mathematica

Time used: 24.469 (sec). Leaf size: 5629

```
DSolve[2*(y[x]+1)*y''[x]+2*y'[x]^2+y[x]^2+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

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15 Chapter 21. System of simultaneous linear equations. Supplemetary problems. Page 163

15.1 problem 10

Internal problem ID [5444]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 21. System of simultaneous linear equations. Supplemetary problems. Page 163
Problem number: 10.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -x(t) + e^{2t} - e^{t}$$
$$y'(t) = -x(t) + y(t) + e^{2t}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 52

```
dsolve([diff(x(t),t)-diff(y(t),t)+y(t)=-exp(t),x(t)+diff(y(t),t)-y(t)=exp(2*t)],[x(t), y(t)]
```

$$x(t) = \frac{e^{2t}}{3} + 2\,e^{-t}c_2 - \frac{e^{t}}{2}$$
$$y(t) = e^{-t}c_2 + c_1 e^{t} + \frac{2\,e^{2t}}{3} + \frac{e^{t}t}{2} - \frac{e^{t}}{4}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 72

```
DSolve[{x'[t]-y'[t]+y[t]==-Exp[t],x[t]+y'[t]-y[t]==Exp[2*t]},{x[t],y[t]},t,IncludeSingularSo
```

$$x(t) \to \frac{1}{6}e^{t}\left(2e^{t} - 3\right) + c_1 e^{-t}$$
$$y(t) \to \frac{2e^{2t}}{3} + \frac{c_1 e^{-t}}{2} + \frac{1}{4}e^{t}(2t - 1 - 2c_1 + 4c_2)$$

15.2 problem 11

Internal problem ID [5445]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 21. System of simultaneous linear equations. Supplemetary problems. Page 163
Problem number: 11.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 3x(t) - t^2 + 2y(t) + t$$
$$y'(t) = t^2 - 3y(t) - 5x(t)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve([diff(x(t),t)+2*x(t)+diff(y(t),t)+y(t)=t,5*x(t)+diff(y(t),t)+3*y(t)=t^2],[x(t), y(t)]
```

$$x(t) = -t^2 - \frac{\cos(t) c_2}{5} + \frac{\sin(t) c_1}{5} + t + 3 - \frac{3\sin(t) c_2}{5} - \frac{3\cos(t) c_1}{5}$$
$$y(t) = \sin(t) c_2 + \cos(t) c_1 + 2t^2 - 3t - 4$$

✓ Solution by Mathematica

Time used: 0.122 (sec). Leaf size: 61

```
DSolve[{x'[t]+2*x[t]+y'[t]+y[t]==t,5*x[t]+y'[t]+3*y[t]==t^2},{x[t],y[t]},t,IncludeSingularSo
```

$$x(t) \to -t^2 + t + c_1 \cos(t) + (3c_1 + 2c_2)\sin(t) + 3$$
$$y(t) \to 2t^2 - 3t + c_2 \cos(t) - (5c_1 + 3c_2)\sin(t) - 4$$

15.3 problem 12

Internal problem ID [5446]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 21. System of simultaneous linear equations. Supplemetary problems. Page 163
Problem number: 12.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 4 - 5x(t) - y(t) - e^t$$
$$y'(t) = 2x(t) - 3y(t) + e^t - 1$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 72

```
dsolve([diff(x(t),t)+x(t)+2*diff(y(t),t)+7*y(t)=exp(t)+2,-2*x(t)+diff(y(t),t)+3*y(t)=exp(t)-
```

$$x(t) = -\frac{e^{-4t}\sin(t)\,c_2}{2} + \frac{e^{-4t}\cos(t)\,c_2}{2} - \frac{e^{-4t}\cos(t)\,c_1}{2} - \frac{e^{-4t}\sin(t)\,c_1}{2} - \frac{5\,e^t}{26} + \frac{13}{17}$$
$$y(t) = e^{-4t}\sin(t)\,c_2 + e^{-4t}\cos(t)\,c_1 + \frac{3}{17} + \frac{2\,e^t}{13}$$

✓ Solution by Mathematica

Time used: 0.239 (sec). Leaf size: 79

```
DSolve[{x'[t]+x[t]+2*y'[t]+7*y[t]==Exp[t]+2,-2*x[t]+y'[t]+3*y[t]==Exp[t]-1},{x[t],y[t]},t,In
```

$$x(t) \to -\frac{5e^t}{26} + c_1e^{-4t}\cos(t) - (c_1 + c_2)e^{-4t}\sin(t) + \frac{13}{17}$$
$$y(t) \to \frac{2e^t}{13} + c_2e^{-4t}\cos(t) + (2c_1 + c_2)e^{-4t}\sin(t) + \frac{3}{17}$$

15.4 problem 13

Internal problem ID [5447]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 21. System of simultaneous linear equations. Supplemetary problems. Page 163
Problem number: 13.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) + y'(t) = x(t) - 3y(t) - 1 + e^{-t}$$
$$x'(t) + y'(t) = -2x(t) - 3y(t) + e^{2t} + 1$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve([diff(x(t),t)-x(t)+diff(y(t),t)+3*y(t)=exp(-t)-1,diff(x(t),t)+2*x(t)+diff(y(t),t)+3*y
```

$$x(t) = \frac{e^{2t}}{3} + \frac{2}{3} - \frac{e^{-t}}{3}$$
$$y(t) = -\frac{1}{9} - \frac{e^{2t}}{15} + \frac{e^{-t}}{6} + e^{-3t}c_1$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 62

```
DSolve[{x'[t]-x[t]+y'[t]+3*y[t]==Exp[-t]-1,x'[t]+2*x[t]+y'[t]+3*y[t]==Exp[2*t]+1},{x[t],y[t]
```

$$x(t) \to \frac{1}{3}e^{-t}\left(2e^{t} + e^{3t} - 1\right)$$
$$y(t) \to \frac{e^{-t}}{6} - \frac{e^{2t}}{15} + \frac{1}{16}c_1e^{-3t} - \frac{1}{9}$$

15.5 problem 17

Internal problem ID [5448]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 21. System of simultaneous linear equations. Supplemetary problems. Page 163
Problem number: 17.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 1 + x(t) + \frac{e^t}{2}$$
$$y'(t) = -2y(t) + \frac{e^t}{2}$$
$$z'(t) = 2 - z(t) + \frac{e^t}{2}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 45

```
dsolve([diff(x(t),t)-x(t)+diff(y(t),t)+2*y(t)=1+exp(t),diff(y(t),t)+2*y(t)+diff(z(t),t)+z(t)
```

$$x(t) = -1 + \frac{e^t(2c_1 + t)}{2}$$
$$y(t) = \frac{e^t}{6} + e^{-2t}c_2$$
$$z(t) = 2 + \frac{e^t}{4} + e^{-t}c_3$$

✓ Solution by Mathematica

Time used: 0.116 (sec). Leaf size: 60

```
DSolve[{x'[t]-x[t]+y'[t]+2*y[t]==1+Exp[t],y'[t]+2*y[t]+z'[t]+z[t]==2+Exp[t],x'[t]-x[t]+z'[t]
```

$$x(t) \to -1 + e^t\left(\frac{t}{2} + c_1\right)$$

$$y(t) \to \frac{e^t}{6} + c_2 e^{-2t}$$

$$z(t) \to \frac{e^t}{4} + (4 + c_3)e^{-t} + 2$$

16 Chapter 25. Integration in series. Supplemetary problems. Page 205

16.1 problem 9

Internal problem ID [5449]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 25. Integration in series. Supplemetary problems. Page 205
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(1-x)\,y' + y = x^2}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 22

```
Order:=6;
dsolve((1-x)*diff(y(x),x)=x^2-y(x),y(x),type='series',x=0);
```

$$y = (1-x)\,y(0) + \frac{x^3}{3} + \frac{x^4}{6} + \frac{x^5}{10} + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 33

```
AsymptoticDSolveValue[(1-x)*y'[x]==x^2-y[x],y[x],{x,0,5}]
```

$$y(x) \to \frac{x^5}{10} + \frac{x^4}{6} + \frac{x^3}{3} + c_1(1-x)$$

16.2 problem 10A

Internal problem ID [5450]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 25. Integration in series. Supplemetary problems. Page 205
Problem number: 10A.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - 2y = 1 - x}$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(x*diff(y(x),x)=1-x+2*y(x),y(x),type='series',x=1);
```

$$y = y(1)\,x^2 - \frac{(-1+x)^2}{2}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 28

```
AsymptoticDSolveValue[x*y'[x]==1-x+2*y[x],y[x],{x,1,5}]
```

$$y(x) \to -\frac{1}{2}(x-1)^2 + c_1\left((x-1)^2 + 2(x-1) + 1\right)$$

16.3 problem 10B

Internal problem ID [5451]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 25. Integration in series. Supplemetary problems. Page 205
Problem number: 10B.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - 2y = 1 - x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x)=1-x+2*y(x),y(x), singsol=all)
```

$$y = -\frac{1}{2} + x + c_1x^2$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 16

```
DSolve[x*y'[x]==1-x+2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1x^2 + x - \frac{1}{2}$$

16.4 problem 11

Internal problem ID [5452]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 25. Integration in series. Supplemetary problems. Page 205
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - 3y = 2x^2}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
Order:=6;
dsolve(diff(y(x),x)=2*x^2+3*y(x),y(x),type='series',x=0);
```

$$y = \left(1 + 3x + \frac{9}{2}x^2 + \frac{9}{2}x^3 + \frac{27}{8}x^4 + \frac{81}{40}x^5\right) y(0) + \frac{2x^3}{3} + \frac{x^4}{2} + \frac{3x^5}{10} + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 61

```
AsymptoticDSolveValue[y'[x]==2*x^2+3*y[x],y[x],{x,0,5}]
```

$$y(x) \to \frac{3x^5}{10} + \frac{x^4}{2} + \frac{2x^3}{3} + c_1\left(\frac{81x^5}{40} + \frac{27x^4}{8} + \frac{9x^3}{2} + \frac{9x^2}{2} + 3x + 1\right)$$

16.5 problem 12

Internal problem ID [5453]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 25. Integration in series. Supplemetary problems. Page 205
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x+1)\,y'-y=x^2-2x}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
Order:=6;
dsolve((x+1)*diff(y(x),x)=x^2-2*x+y(x),y(x),type='series',x=0);
```

$$y=(1+x)\,y(0)-x^2+\frac{2x^3}{3}-\frac{x^4}{3}+\frac{x^5}{5}+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 36

```
AsymptoticDSolveValue[(x+1)*y'[x]==x^2-2*x+y[x],y[x],{x,0,5}]
```

$$y(x)\to\frac{x^5}{5}-\frac{x^4}{3}+\frac{2x^3}{3}-x^2+c_1(x+1)$$

16.6 problem 13

Internal problem ID [5454]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 25. Integration in series. Supplemetary problems. Page 205
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)+x*y(x)=0,y(x),type='series',x=0);
```

$$y = \left(1 - \frac{x^3}{6}\right) y(0) + \left(x - \frac{1}{12}x^4\right) y'(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x - \frac{x^4}{12}\right) + c_1\left(1 - \frac{x^3}{6}\right)$$

16.7 problem 14

Internal problem ID [5455]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 25. Integration in series. Supplemetary problems. Page 205
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + 2yx^2 = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)+2*x^2*y(x)=0,y(x),type='series',x=0);
```

$$y = \left(1 - \frac{x^4}{6}\right) y(0) + \left(x - \frac{1}{10}x^5\right) y'(0) + O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]+2*x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x - \frac{x^5}{10}\right) + c_1\left(1 - \frac{x^4}{6}\right)$$

16.8 problem 15

Internal problem ID [5456]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 25. Integration in series. Supplemetary problems. Page 205
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x + yx^2 = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
Order:=6;
dsolve(diff(y(x),x$2)-x*diff(y(x),x)+x^2*y(x)=0,y(x),type='series',x=0);
```

$$y = \left(1 - \frac{x^4}{12}\right) y(0) + \left(x + \frac{1}{6}x^3 - \frac{1}{40}x^5\right) y'(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 35

```
AsymptoticDSolveValue[y''[x]-x*y'[x]+x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(1 - \frac{x^4}{12}\right) + c_2 \left(-\frac{x^5}{40} + \frac{x^3}{6} + x\right)$$

16.9 problem 16

Internal problem ID [5457]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 25. Integration in series. Supplemetary problems. Page 205
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$(-x^2+1)\,y''-2y'x+p(p+1)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 101

```
Order:=6;
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+p*(p+1)*y(x)=0,y(x),type='series',x=0);
```

$$y=\left(1-\frac{p(p+1)\,x^2}{2}+\frac{p(p^3+2p^2-5p-6)\,x^4}{24}\right)y(0)+\left(x-\frac{(p^2+p-2)\,x^3}{6}+\frac{(p^4+2p^3-13p^2-14p+24)\,x^5}{120}\right)y'(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 120

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-2*x*y'[x]+p*(p+1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{1}{120}(p^2+p)^2x^5+\frac{7}{60}(-p^2-p)\,x^5+\frac{1}{6}(-p^2-p)\,x^3+\frac{x^5}{5}+\frac{x^3}{3}+x\right)+c_1\left(\frac{1}{24}(p^2+p)^2x^4+\frac{1}{4}(-p^2-p)\,x^4+\frac{1}{2}(-p^2-p)\,x^2+1\right)$$

16.10 problem 17

Internal problem ID [5458]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 25. Integration in series. Supplemetary problems. Page 205
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + yx^2 = x^2 + x + 1}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
Order:=6;
dsolve(diff(y(x),x$2)+x^2*y(x)=1+x+x^2,y(x),type='series',x=0);
```

$$y = \left(1 - \frac{x^4}{12}\right) y(0) + \left(x - \frac{1}{20}x^5\right) y'(0) + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{12} + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 49

```
AsymptoticDSolveValue[y''[x]+x^2*y[x]==1+x+x^2,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x - \frac{x^5}{20}\right) + \frac{x^4}{12} + c_1\left(1 - \frac{x^4}{12}\right) + \frac{x^3}{6} + \frac{x^2}{2}$$

17 Chapter 26. Integration in series (singular points). Supplemetary problems. Page 218

17.1 problem 11

Internal problem ID [5459]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 26. Integration in series (singular points). Supplemetary problems. Page 218
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2\left(x^3+x^2\right)y''-\left(-3x^2+x\right)y'+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
Order:=6;
dsolve(2*(x^2+x^3)*diff(y(x),x$2)-(x-3*x^2)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y=\left(-x^5+x^4-x^3+x^2-x+1\right)\left(c_1\sqrt{x}+c_2x\right)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 58

```
AsymptoticDSolveValue[2*(x^2+x^3)*y''[x]-(x-3*x^2)*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1x\left(-x^5+x^4-x^3+x^2-x+1\right)+c_2\sqrt{x}\left(-x^5+x^4-x^3+x^2-x+1\right)$$

17.2 problem 12

Internal problem ID [5460]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 26. Integration in series (singular points). Supplemetary problems. Page 218
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4xy'' + 2(1-x)\,y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 44

```
Order:=6;
dsolve(4*x*diff(y(x),x$2)+2*(1-x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y = c_1\sqrt{x}\left(1+\frac{1}{3}x+\frac{1}{15}x^2+\frac{1}{105}x^3+\frac{1}{945}x^4+\frac{1}{10395}x^5+\mathrm{O}\left(x^6\right)\right) + c_2\left(1+\frac{1}{2}x+\frac{1}{8}x^2+\frac{1}{48}x^3+\frac{1}{384}x^4+\frac{1}{3840}x^5+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 85

```
AsymptoticDSolveValue[4*x*y''[x]+2*(1-x)*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{x^5}{10395}+\frac{x^4}{945}+\frac{x^3}{105}+\frac{x^2}{15}+\frac{x}{3}+1\right)+c_2\left(\frac{x^5}{3840}+\frac{x^4}{384}+\frac{x^3}{48}+\frac{x^2}{8}+\frac{x}{2}+1\right)$$

17.3 problem 13

Internal problem ID [5461]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 26. Integration in series (singular points). Supplemetary problems. Page 218
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' - y'x + (-x^2 + 1)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)-x*diff(y(x),x)+(1-x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y = c_1\sqrt{x}\left(1 + \frac{1}{6}x^2 + \frac{1}{168}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2 x\left(1 + \frac{1}{10}x^2 + \frac{1}{360}x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 48

```
AsymptoticDSolveValue[2*x^2*y''[x]-x*y'[x]+(1-x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 x\left(\frac{x^4}{360} + \frac{x^2}{10} + 1\right) + c_2\sqrt{x}\left(\frac{x^4}{168} + \frac{x^2}{6} + 1\right)$$

17.4 problem 14

Internal problem ID [5462]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 26. Integration in series (singular points). Supplemetary problems. Page 218
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + y' + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
Order:=6;
dsolve(x*diff(y(x),x$2)+diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y = (\ln(x) c_2 + c_1)\left(1 - \frac{1}{4}x^2 + \frac{1}{64}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(\frac{1}{4}x^2 - \frac{3}{128}x^4 + \mathrm{O}\left(x^6\right)\right) c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 60

```
AsymptoticDSolveValue[x*y''[x]+y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^4}{64} - \frac{x^2}{4} + 1\right) + c_2\left(-\frac{3x^4}{128} + \frac{x^2}{4} + \left(\frac{x^4}{64} - \frac{x^2}{4} + 1\right)\log(x)\right)$$

17.5 problem 15

Internal problem ID [5463]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 26. Integration in series (singular points). Supplemetary problems. Page 218
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - y'x + (x^2 + 1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)+(x^2+1)*y(x)=0,y(x),type='series',x=0);
```

$$y = x\left((\ln(x)\,c_2 + c_1)\left(1 - \frac{1}{4}x^2 + \frac{1}{64}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(\frac{1}{4}x^2 - \frac{3}{128}x^4 + \mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 65

```
AsymptoticDSolveValue[x^2*y''[x]-x*y'[x]+(x^2+1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1x\left(\frac{x^4}{64} - \frac{x^2}{4} + 1\right) + c_2\left(x\left(\frac{x^2}{4} - \frac{3x^4}{128}\right) + x\left(\frac{x^4}{64} - \frac{x^2}{4} + 1\right)\log(x)\right)$$

17.6 problem 16

Internal problem ID [5464]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 26. Integration in series (singular points). Supplemetary problems. Page 218
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' - 2y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 56

```
Order:=6;
dsolve(x*diff(y(x),x$2)-2*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y = {} & c_1 x^3 \left(1 - \frac{1}{4}x + \frac{1}{40}x^2 - \frac{1}{720}x^3 + \frac{1}{20160}x^4 - \frac{1}{806400}x^5 + \mathrm{O}\left(x^6\right)\right) \\ & + c_2 \left(\ln\left(x\right)\left(-x^3 + \frac{1}{4}x^4 - \frac{1}{40}x^5 + \mathrm{O}\left(x^6\right)\right) \right. \\ & \left. + \left(12 + 6x + 3x^2 - \frac{5}{16}x^4 + \frac{39}{800}x^5 + \mathrm{O}\left(x^6\right)\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 79

```
AsymptoticDSolveValue[x*y''[x]-2*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{1}{48}(x-4)x^3\log(x)+\frac{1}{576}\left(-19x^4+16x^3+144x^2+288x+576\right)\right)+c_2\left(\frac{x^7}{20160}-\frac{x^6}{720}+\frac{x^5}{40}-\frac{x^4}{4}+x^3\right)$$

17.7 problem 17

Internal problem ID [5465]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 26. Integration in series (singular points). Supplemetary problems. Page 218
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + 2y' + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
Order:=6;
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y = c_1\left(1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + \mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 42

```
AsymptoticDSolveValue[x*y''[x]+2*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1\left(\frac{x^3}{24} - \frac{x}{2} + \frac{1}{x}\right) + c_2\left(\frac{x^4}{120} - \frac{x^2}{6} + 1\right)$$

17.8 problem 18

Internal problem ID [5466]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 26. Integration in series (singular points). Supplemetary problems. Page 218
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x+1)\,y''+x(x+1)\,y'-y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
Order:=6;
dsolve(x^2*(x+1)*diff(y(x),x$2)+x*(x+1)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y=c_1x\left(1-\frac{1}{3}x+\frac{1}{6}x^2-\frac{1}{10}x^3+\frac{1}{15}x^4-\frac{1}{21}x^5+\mathrm{O}\left(x^6\right)\right)+\frac{c_2(-2-2x+\mathrm{O}\left(x^6\right))}{x}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 45

```
AsymptoticDSolveValue[x^2*(x+1)*y''[x]+x*(x+1)*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{x^5}{15}-\frac{x^4}{10}+\frac{x^3}{6}-\frac{x^2}{3}+x\right)+c_1\left(\frac{1}{x}+1\right)$$

17.9 problem 19

Internal problem ID [5467]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 26. Integration in series (singular points). Supplemetary problems. Page 218
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + y' - y = 1 + x}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 60

```
Order:=6;
dsolve(2*x*diff(y(x),x$2)+diff(y(x),x)-y(x)=x+1,y(x),type='series',x=0);
```

$$\begin{aligned} y = {} & c_1\sqrt{x}\left(1+\frac{1}{3}x+\frac{1}{30}x^2+\frac{1}{630}x^3+\frac{1}{22680}x^4+\frac{1}{1247400}x^5+\mathrm{O}\left(x^6\right)\right) \\ & + c_2\left(1+x+\frac{1}{6}x^2+\frac{1}{90}x^3+\frac{1}{2520}x^4+\frac{1}{113400}x^5+\mathrm{O}\left(x^6\right)\right) \\ & + x\left(1+\frac{1}{3}x+\frac{1}{45}x^2+\frac{1}{1260}x^3+\frac{1}{56700}x^4+\mathrm{O}\left(x^5\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 246

```
AsymptoticDSolveValue[2*x*y''[x]+y'[x]-y[x]==x+1,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^5}{113400}+\frac{x^4}{2520}+\frac{x^3}{90}+\frac{x^2}{6}+x+1\right)+c_2\sqrt{x}\left(\frac{x^5}{1247400}+\frac{x^4}{22680}+\frac{x^3}{630}+\frac{x^2}{30}+\frac{x}{3}+1\right)+\sqrt{x}\left(\frac{x^5}{1247400}+\frac{x^4}{22680}+\frac{x^3}{630}+\frac{x^2}{30}+\frac{x}{3}+1\right)\left(\frac{23x^{11/2}}{311850}+\frac{29x^{9/2}}{11340}+\frac{16x^{7/2}}{315}+\frac{7x^{5/2}}{15}+\frac{4x^{3/2}}{3}+2\sqrt{x}\right)+\left(\frac{x^5}{113400}+\frac{x^4}{2520}+\frac{x^3}{90}+\frac{x^2}{6}+x+1\right)\left(-\frac{x^6}{133650}-\frac{37x^5}{113400}-\frac{11x^4}{1260}-\frac{11x^3}{90}-\frac{2x^2}{3}-x\right)$$

17.10 problem 20

Internal problem ID [5468]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 26. Integration in series (singular points). Supplemetary problems. Page 218
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2x^3y'' + y'x^2 + y = 0}$$

With the expansion point for the power series method at $x = \infty$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 117

```
Order:=6;
dsolve(2*x^3*diff(y(x),x$2)+x^2*diff(y(x),x)+y(x)=0,y(x),type='series',x=Infinity);
```

$$y = \left(1 - \frac{(x - \text{Infinity})^2}{4\,\text{Infinity}^3} + \frac{7(x - \text{Infinity})^3}{24\,\text{Infinity}^4} + \frac{(-59\,\text{Infinity} + 2)\,(x - \text{Infinity})^4}{192\,\text{Infinity}^6} + \frac{(605\,\text{Infinity} - 52)\,(x - \text{Infinity})^5}{1920\,\text{Infinity}^7}\right) y(\text{Infinity}) + \left(x - \text{Infinity} - \frac{(x - \text{Infinity})^2}{4\,\text{Infinity}} + \frac{(3\,\text{Infinity}^2 - 2\,\text{Infinity})\,(x - \text{Infinity})^3}{24\,\text{Infinity}^4} - \frac{5\left(\text{Infinity} - \frac{28}{15}\right)(x - \text{Infinity})^4}{64\,\text{Infinity}^4} + \frac{(105\,\text{Infinity}^3 - 370\,\text{Infinity}^2 + 4\,\text{Infinity})\,(x - \text{Infinity})^5}{1920\,\text{Infinity}^7}\right) y'(\text{Infinity}) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 96

```
AsymptoticDSolveValue[2*x^3*y''[x]+x^2*y'[x]+y[x]==0,y[x],{x,Infinity,5}]
```

$$y(x) \to c_2\left(\frac{1}{6x^{3/2}} - \frac{1}{90x^{5/2}} + \frac{1}{2520x^{7/2}} - \frac{1}{113400x^{9/2}} + \sqrt{x} - \frac{1}{\sqrt{x}}\right) + c_1\left(-\frac{1}{1247400x^5} + \frac{1}{22680x^4} - \frac{1}{630x^3} + \frac{1}{30x^2} - \frac{1}{3x} + 1\right)$$

17.11 problem 21

Internal problem ID [5469]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 26. Integration in series (singular points). Supplemetary problems. Page 218
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^3y'' + \left(x^2 + x\right) y' - y = 0}$$

With the expansion point for the power series method at $x = \infty$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 179

```
Order:=6;
dsolve(x^3*diff(y(x),x$2)+(x^2+x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=Infinity);
```

$$y = \left(1 + \frac{(x - \text{Infinity})^2}{2\,\text{Infinity}^3} + \frac{(-4\,\text{Infinity} - 1)(x - \text{Infinity})^3}{6\,\text{Infinity}^5} + \frac{(18\,\text{Infinity}^2 + 10\,\text{Infinity} + 1)(x - \text{Infinity})^4}{24\,\text{Infinity}^7} + \frac{(-96\,\text{Infinity}^3 - 86\,\text{Infinity}^2 - 18\,\text{Infinity} - 1)(x - \text{Infinity})^5}{120\,\text{Infinity}^9}\right) y(\text{Infinity}) + \left(x - \text{Infinity} + \frac{(-\text{Infinity}^2 - \text{Infinity})(x - \text{Infinity})^2}{2\,\text{Infinity}^3} + \frac{(2\,\text{Infinity}^3 + 5\,\text{Infinity}^2 + \text{Infinity})(x - \text{Infinity})^3}{6\,\text{Infinity}^5} + \frac{(-6\,\text{Infinity}^4 - 26\,\text{Infinity}^3 - 11\,\text{Infinity}^2 - \text{Infinity})(x - \text{Infinity})^4}{24\,\text{Infinity}^7} + \frac{(24\,\text{Infinity}^5 + 154\,\text{Infinity}^4 + 102\,\text{Infinity}^3 + 19\,\text{Infinity}^2 + \text{Infinity})(x - \text{Infinity})^5}{120\,\text{Infinity}^9}\right) y'(\text{Infinity}) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 124

```
AsymptoticDSolveValue[x^3*y''[x]+(x^2+x)*y'[x]-y[x]==0,y[x],{x,Infinity,5}]
```

$$y(x) \to c_1\left(\frac{1}{120x^5} + \frac{1}{24x^4} + \frac{1}{6x^3} + \frac{1}{2x^2} + \frac{1}{x} + 1\right) + c_2\left(-\frac{137}{7200x^5} - \frac{\log(x)}{120x^5} - \frac{25}{288x^4} - \frac{\log(x)}{24x^4} - \frac{11}{36x^3} - \frac{\log(x)}{6x^3} - \frac{3}{4x^2} - \frac{\log(x)}{2x^2} - \frac{1}{x} - \frac{\log(x)}{x} - \log(x)\right)$$

18 Chapter 27. The Legendre, Bessel and Gauss Equations. Supplemetary problems. Page 230

18.1 problem 20

Internal problem ID [5470]

Book: Schaums Outline. Theory and problems of Differential Equations, 1st edition. Frank Ayres. McGraw Hill 1952
Section: Chapter 27. The Legendre, Bessel and Gauss Equations. Supplemetary problems. Page 230
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$z'' + tz' + \left(t^2 - \frac{1}{9}\right) z = 0$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(z(t),t$2)+t*diff(z(t),t)+(t^2-1/9)*z(t)=0,z(t),type='series',t=0);
```

$$z(t) = \left(1 + \frac{1}{18}t^2 - \frac{179}{1944}t^4\right) z(0) + \left(t - \frac{4}{27}t^3 - \frac{139}{4860}t^5\right) D(z)(0) + O(t^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[z''[t]+t*z'[t]+(t^2-1/9)*z[t]==0,z[t],{t,0,5}]
```

$$z(t) \to c_2\left(-\frac{139t^5}{4860} - \frac{4t^3}{27} + t\right) + c_1\left(-\frac{179t^4}{1944} + \frac{t^2}{18} + 1\right)$$