A Solution Manual For

Selected problems from homeworks from different courses

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Contents

1 Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota

1.1 problem HW 1 problem 6(a)

Internal problem ID [7029]

Book: Selected problems from homeworks from different courses
Section: Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota
Problem number: HW 1 problem 6(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{y}{\ln(x)\, x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x)=y(x)/(x*ln(x)),y(x), singsol=all)
```

$$y(x) = c_1 \ln(x)$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 15

```
DSolve[y'[x]==y[x]/(x*Log[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \log(x)$$
$$y(x) \to 0$$

1.2 problem HW 1 problem 6(b)

Internal problem ID [7030]

Book: Selected problems from homeworks from different courses
Section: Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota
Problem number: HW 1 problem 6(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\left(x^2+1\right)y' + y^2 = -1}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 11

```
dsolve([(x^2+1)*diff(y(x),x)+y(x)^2=-1,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \cot\left(\arctan\left(x\right) + \frac{\pi}{4}\right)$$

✓ Solution by Mathematica

Time used: 0.264 (sec). Leaf size: 14

```
DSolve[{(x^2+1)*y'[x]+y[x]^2==-1,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cot\left(\arctan(x) + \frac{\pi}{4}\right)$$

1.3 problem HW 1 problem 7(a)

Internal problem ID [7031]

Book: Selected problems from homeworks from different courses
Section: Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota
Problem number: HW 1 problem 7(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{2y}{x} = 5x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)+2/x*y(x)=5*x^2,y(x), singsol=all)
```

$$y(x) = \frac{x^5 + c_1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 15

```
DSolve[y'[x]+2/x*y[x]==5*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^5 + c_1}{x^2}$$

1.4 problem HW 1 problem 7(b)

Internal problem ID [7032]

Book: Selected problems from homeworks from different courses
Section: Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota
Problem number: HW 1 problem 7(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{tx' + 2x = 4\,\mathrm{e}^t}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(t*diff(x(t),t)+2*x(t)=4*exp(t),x(t), singsol=all)
```

$$x(t) = \frac{4(t-1)\,\mathrm{e}^t + c_1}{t^2}$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 20

```
DSolve[t*x'[t]+2*x[t]==4*Exp[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{4e^t(t-1) + c_1}{t^2}$$

1.5 problem HW 1 problem 10

Internal problem ID [7033]

Book: Selected problems from homeworks from different courses
Section: Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota
Problem number: HW 1 problem 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{y' - \frac{2x - y}{x + 4y} = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 19

```
dsolve([diff(y(x),x)=(2*x-y(x))/(x+4*y(x)),y(1) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{x}{4} + \frac{\sqrt{9x^2 + 16}}{4}$$

✓ Solution by Mathematica

Time used: 0.482 (sec). Leaf size: 24

```
DSolve[{y'[x]==(2*x-y[x])/(x+4*y[x]),{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(\sqrt{9x^2 + 16} - x\right)$$

1.6 problem HW 1 problem 11

Internal problem ID [7034]

Book: Selected problems from homeworks from different courses
Section: Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota
Problem number: HW 1 problem 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$\boxed{y' + \frac{2y}{x} - 6x^4y^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+2*y(x)/x=6*y(x)^2*x^4,y(x), singsol=all)
```

$$y(x) = \frac{1}{(-2x^3 + c_1)\, x^2}$$

✓ Solution by Mathematica

Time used: 0.153 (sec). Leaf size: 24

```
DSolve[y'[x]+2*y[x]/x==6*y[x]^2*x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{-2x^5 + c_1 x^2}$$
$$y(x) \to 0$$

1.7 problem HW 1 problem 13

Internal problem ID [7035]

Book: Selected problems from homeworks from different courses
Section: Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota
Problem number: HW 1 problem 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{y^2 + (2yx + \sin(y))\, y' = -\cos(x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve((y(x)^2+cos(x))+(2*x*y(x)+sin(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$xy(x)^2 + \sin(x) - \cos(y(x)) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.229 (sec). Leaf size: 20

```
DSolve[(y[x]^2+Cos[x])+(2*x*y[x]+Sin[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$\text{Solve}\left[xy(x)^2 - \cos(y(x)) + \sin(x) = c_1, y(x)\right]$$

1.8 problem HW 1 problem 14

Internal problem ID [7036]

Book: Selected problems from homeworks from different courses
Section: Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota
Problem number: HW 1 problem 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{yx + x^2 y' = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x*y(x)-1)+x^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\ln(x) + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 14

```
DSolve[(x*y[x]-1)+x^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\log(x) + c_1}{x}$$

1.9 problem HW 5 problem 1(a)

Internal problem ID [7037]

Book: Selected problems from homeworks from different courses
Section: Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota
Problem number: HW 5 problem 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y' - 2y = 5\,\mathrm{e}^{2x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-2*y(x)=5*exp(2*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2x} c_2 + \mathrm{e}^{-x} c_1 + \frac{5\,\mathrm{e}^{2x} x}{3}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 31

```
DSolve[y''[x]-y'[x]-2*y[x]==5*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x} + e^{2x}\left(\frac{5x}{3} - \frac{5}{9} + c_2\right)$$

1.10 problem HW 5 problem 1(b)

Internal problem ID [7038]

Book: Selected problems from homeworks from different courses
Section: Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota
Problem number: HW 5 problem 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 16y = 4\cos(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+16*y(x)=4*cos(x),y(x), singsol=all)
```

$$y(x) = \sin(4x)\, c_2 + \cos(4x)\, c_1 + \frac{4\cos(x)}{15}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 26

```
DSolve[y''[x]+16*y[x]==4*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4\cos(x)}{15} + c_1 \cos(4x) + c_2 \sin(4x)$$

1.11 problem HW 5 problem 1(c)

Internal problem ID [7039]

Book: Selected problems from homeworks from different courses
Section: Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota
Problem number: HW 5 problem 1(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y' + 3y = 9x^2 + 4}$$

With initial conditions

$$[y(0) = 6, y'(0) = 8]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve([diff(y(x),x$2)-4*diff(y(x),x)+3*y(x)=9*x^2+4,y(0) = 6, D(y)(0) = 8],y(x), singsol=al
```

$$y(x) = -6\,\mathrm{e}^{x} + 2\,\mathrm{e}^{3x} + 3x^2 + 8x + 10$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 27

```
DSolve[{y''[x]-4*y'[x]+3*y[x]==9*x^2+4,{y[0]==6,y'[0]==8}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to 3x^2 + 8x - 6e^{x} + 2e^{3x} + 10$$

1.12 problem HW 5 problem 2

Internal problem ID [7040]

Book: Selected problems from homeworks from different courses
Section: Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota
Problem number: HW 5 problem 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \tan(x)^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+y(x)=tan(x)^2,y(x), singsol=all)
```

$$y(x) = \sin(x)\, c_2 + \cos(x)\, c_1 - 2 + \sin(x) \ln(\sec(x) + \tan(x))$$

✓ Solution by Mathematica

Time used: 0.129 (sec). Leaf size: 23

```
DSolve[y''[x]+y[x]==Tan[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x)\text{arctanh}(\sin(x)) + c_1 \cos(x) + c_2 \sin(x) - 2$$

1.13 problem HW 5 problem 5

Internal problem ID [7041]

Book: Selected problems from homeworks from different courses
Section: Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota
Problem number: HW 5 problem 5.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -2x(t) + 3y(t)$$
$$y'(t) = -2x(t) + 5y(t)$$

With initial conditions

$$[x(0) = -2, y(0) = 1]$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 32

```
dsolve([diff(x(t),t) = -2*x(t)+3*y(t), diff(y(t),t) = -2*x(t)+5*y(t), x(0) = -2, y(0) = 1],[
```

$$x(t) = -3\,\mathrm{e}^{-t} + \mathrm{e}^{4t}$$

$$y(t) = -\mathrm{e}^{-t} + 2\,\mathrm{e}^{4t}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 36

```
DSolve[{x'[t]==-2*x[t]+3*y[t],y'[t]==-2*x[t]+5*y[t]},{x[0]==-2,y[0]==1},{x[t],y[t]},t,Includ
```

$$x(t) \to e^{-t}\left(e^{5t} - 3\right)$$
$$y(t) \to e^{-t}\left(2e^{5t} - 1\right)$$

1.14 problem HW 5 problem 6

Internal problem ID [7042]

Book: Selected problems from homeworks from different courses
Section: Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota
Problem number: HW 5 problem 6.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -x(t) + 4y(t)$$
$$y'(t) = 2x(t) - 3y(t)$$

With initial conditions

$$[x(0) = 3, y(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 26

```
dsolve([diff(x(t),t) = -x(t)+4*y(t), diff(y(t),t) = 2*x(t)-3*y(t), x(0) = 3, y(0) = 0],[x(t)
```

$$x(t) = 2\,\mathrm{e}^{t} + \mathrm{e}^{-5t}$$

$$y(t) = \mathrm{e}^{t} - \mathrm{e}^{-5t}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 30

```
DSolve[{x'[t]==-x[t]+4*y[t],y'[t]==2*x[t]-3*y[t]},{x[0]==3,y[0]==0},{x[t],y[t]},t,IncludeSin
```

$$x(t) \to e^{-5t} + 2e^{t}$$
$$y(t) \to e^{t} - e^{-5t}$$

1.15 problem HW 5 problem 7

Internal problem ID [7043]

Book: Selected problems from homeworks from different courses
Section: Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota
Problem number: HW 5 problem 7.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 2x(t) - y(t)$$
$$y'(t) = -x(t) + 2y(t) + 4\,\mathrm{e}^{t}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 45

```
dsolve([diff(x(t),t)=2*x(t)-y(t),diff(y(t),t)=-x(t)+2*y(t)+4*exp(t)],[x(t), y(t)], singsol=a
```

$$x(t) = c_2\mathrm{e}^{t} - c_1\mathrm{e}^{3t} + 2\,\mathrm{e}^{t}t + 2\,\mathrm{e}^{t}$$

$$y(t) = c_2\mathrm{e}^{t} + c_1\mathrm{e}^{3t} + 2\,\mathrm{e}^{t}t$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 74

```
DSolve[{x'[t]==2*x[t]-y[t],y'[t]==-x[t]+2*y[t]+4*Exp[t]},{x[t],y[t]},t,IncludeSingularSoluti
```

$$x(t) \to \frac{1}{2}e^{t}\left(4t + c_1\left(e^{2t}+1\right) - c_2e^{2t} + 2 + c_2\right)$$
$$y(t) \to \frac{1}{2}e^{t}\left(4t - c_1e^{2t} + c_2e^{2t} - 2 + c_1 + c_2\right)$$

1.16 problem Example 8.3.4 from Handout chapter 8.2

Internal problem ID [7044]

Book: Selected problems from homeworks from different courses
Section: Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota
Problem number: Example 8.3.4 from Handout chapter 8.2.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 6x(t) - 7y(t) + 10$$
$$y'(t) = x(t) - 2y(t) - 2\,\mathrm{e}^{t}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 45

```
dsolve([diff(x(t),t)=6*x(t)-7*y(t)+10,diff(y(t),t)=x(t)-2*y(t)-2*exp(t)],[x(t), y(t)], sings
```

$$x(t) = \mathrm{e}^{-t}c_2 + 7c_1\mathrm{e}^{5t} - \frac{7\,\mathrm{e}^{t}}{4} - 4$$

$$y(t) = \mathrm{e}^{-t}c_2 + c_1\mathrm{e}^{5t} - \frac{5\,\mathrm{e}^{t}}{4} - 2$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 90

```
DSolve[{x'[t]==6*x[t]-7*y[t]+10,y'[t]==x[t]-2*y[t]-2*Exp[t]},{x[t],y[t]},t,IncludeSingularSo
```

$$x(t) \to -\frac{7e^{t}}{4} - \frac{1}{6}(c_1 - 7c_2)e^{-t} + \frac{7}{6}(c_1 - c_2)e^{5t} - 4$$
$$y(t) \to -\frac{5e^{t}}{4} - \frac{1}{6}(c_1 - 7c_2)e^{-t} + \frac{1}{6}(c_1 - c_2)e^{5t} - 2$$