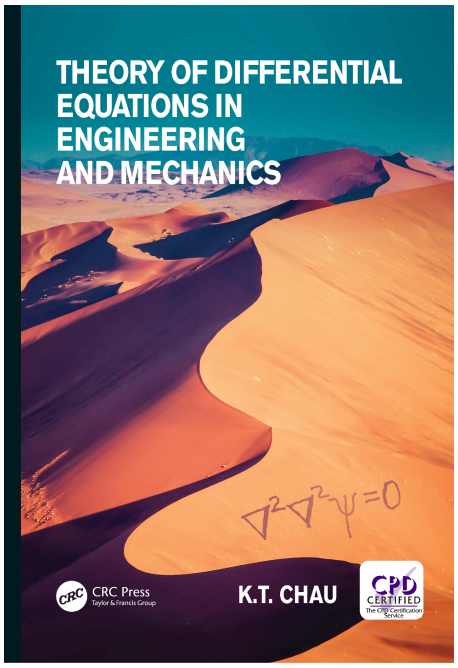
A Solution Manual For

THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018



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March 3, 2024

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1 Chapter 3. Ordinary Differential Equations. Section 3.2 FIRST ORDER ODE. Page 114

1.1 problem Example 3.1

Internal problem ID [5834]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.2 FIRST ORDER ODE. Page 114
Problem number: Example 3.1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \left(y^2 + 1\right) x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=x^2*(y(x)^2+1),y(x), singsol=all)
```

$$y(x) = \tan\left(\frac{x^3}{3} + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.191 (sec). Leaf size: 30

```
DSolve[y'[x]==x^2*(y[x]^2+1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan\left(\frac{x^3}{3} + c_1\right)$$
$$y(x) \to -i$$
$$y(x) \to i$$

1.2 problem Example 3.2

Internal problem ID [5835]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.2 FIRST ORDER ODE. Page 114
Problem number: Example 3.2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{x^2}{1-y^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 357

```
dsolve(diff(y(x),x)=x^2/(1-y(x)^2),y(x), singsol=all)
```

$$y(x) = \frac{\left(-4x^3 - 12c_1 + 4\sqrt{x^6 + 6c_1x^3 + 9c_1^2 - 4}\right)^{\frac{1}{3}}}{2} + \frac{2}{\left(-4x^3 - 12c_1 + 4\sqrt{x^6 + 6c_1x^3 + 9c_1^2 - 4}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(-4x^3 - 12c_1 + 4\sqrt{x^6 + 6c_1x^3 + 9c_1^2 - 4}\right)^{\frac{1}{3}}}{4} - \frac{1}{\left(-4x^3 - 12c_1 + 4\sqrt{x^6 + 6c_1x^3 + 9c_1^2 - 4}\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(-4x^3-12c_1+4\sqrt{x^6+6c_1x^3+9c_1^2-4}\right)^{\frac{1}{3}}}{2} - \frac{2}{\left(-4x^3-12c_1+4\sqrt{x^6+6c_1x^3+9c_1^2-4}\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(-4x^3 - 12c_1 + 4\sqrt{x^6 + 6c_1x^3 + 9c_1^2 - 4}\right)^{\frac{1}{3}}}{4} - \frac{1}{\left(-4x^3 - 12c_1 + 4\sqrt{x^6 + 6c_1x^3 + 9c_1^2 - 4}\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(-4x^3-12c_1+4\sqrt{x^6+6c_1x^3+9c_1^2-4}\right)^{\frac{1}{3}}}{2} - \frac{2}{\left(-4x^3-12c_1+4\sqrt{x^6+6c_1x^3+9c_1^2-4}\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 2.485 (sec). Leaf size: 320

```
DSolve[y'[x]==x^2/(1-y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{-x^3+\sqrt{x^6-6c_1x^3-4+9{c_1}^2}+3c_1}}{\sqrt[3]{2}}+\frac{\sqrt[3]{2}}{\sqrt[3]{-x^3+\sqrt{x^6-6c_1x^3-4+9{c_1}^2}+3c_1}}$$

$$y(x) \to \frac{i\left(\sqrt{3}+i\right)\sqrt[3]{-x^3+\sqrt{x^6-6c_1x^3-4+9{c_1}^2}+3c_1}}{2\sqrt[3]{2}}-\frac{1+i\sqrt{3}}{2^{2/3}\sqrt[3]{-x^3+\sqrt{x^6-6c_1x^3-4+9{c_1}^2}+3c_1}}$$

$$y(x) \to \frac{i\left(\sqrt{3}+i\right)}{2^{2/3}\sqrt[3]{-x^3+\sqrt{x^6-6c_1x^3-4+9{c_1}^2}+3c_1}}-\frac{\left(1+i\sqrt{3}\right)\sqrt[3]{-x^3+\sqrt{x^6-6c_1x^3-4+9{c_1}^2}+3c_1}}{2\sqrt[3]{2}}$$

1.3 problem Example 3.3

Internal problem ID [5836]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.2 FIRST ORDER ODE. Page 114
Problem number: Example 3.3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{3x^2 + 4x + 2}{-2 + 2y} = 0}$$

With initial conditions

$$[y(0) = -1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve([diff(y(x),x)=(3*x^2+4*x+2)/(2*(y(x)-1)),y(0) = -1],y(x), singsol=all)
```

$$y(x) = 1 - \sqrt{(x+2)\left(x^2+2\right)}$$

✓ Solution by Mathematica

Time used: 0.134 (sec). Leaf size: 26

```
DSolve[{y'[x]==(3*x^2+4*x+2)/(2*(y[x]-1)),{y[0]==-1}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to 1 - \sqrt{x^3 + 2x^2 + 2x + 4}$$

1.4 problem Example 3.4

Internal problem ID [5837]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.2 FIRST ORDER ODE. Page 114
Problem number: Example 3.4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{y'x - 2\sqrt{yx} - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x)-2*sqrt(x*y(x))=y(x),y(x), singsol=all)
```

$$-\frac{y(x)}{\sqrt{y(x)\,x}} + \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.182 (sec). Leaf size: 19

```
DSolve[x*y'[x]-2*Sqrt[x*y[x]]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}x(2\log(x) + c_1)^2$$

1.5 problem Example 3.5

Internal problem ID [5838]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.2 FIRST ORDER ODE. Page 114
Problem number: Example 3.5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{y' - \frac{y + x - 1}{x - y + 3} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 31

```
dsolve(diff(y(x),x)=(x+y(x)-1)/(x-y(x)+3),y(x), singsol=all)
```

$$y(x) = 2 - \tan\left(\text{RootOf}\left(2\_Z + \ln\left(\frac{1}{\cos(\_Z)^2}\right) + 2\ln(x+1) + 2c_1\right)\right)(x+1)$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 59

```
DSolve[y'[x]==(x+y[x]-1)/(x-y[x]+3),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2\arctan\left(1 - \frac{2(x+1)}{-y(x) + x + 3}\right) + \log\left(\frac{x^2 + y(x)^2 - 4y(x) + 2x + 5}{2(x+1)^2}\right) + 2\log(x+1) + c_1 = 0, y(x)\right]$$

1.6 problem Example 3.6

Internal problem ID [5839]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.2 FIRST ORDER ODE. Page 114
Problem number: Example 3.6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{y + (x - 2\sin(y))\, y' = -\mathrm{e}^{x}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve((exp(x)+y(x))+(x-2*sin(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)\, x + \mathrm{e}^{x} + 2\cos(y(x)) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.233 (sec). Leaf size: 19

```
DSolve[(Exp[x]+y[x])+(x-2*Sin[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}[xy(x) + 2\cos(y(x)) + e^{x} = c_1, y(x)]$$

1.7 problem Example 3.7

Internal problem ID [5840]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.2 FIRST ORDER ODE. Page 114
Problem number: Example 3.7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{\frac{6}{y}+\left(\frac{x^2}{y}+\frac{3y}{x}\right)y'=-3x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 430

```
dsolve((3*x+6/y(x))+(x^2/y(x)+3*y(x)/x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(-324x^2 - 108c_1 + 12\sqrt{12x^9 + 729x^4 + 486c_1x^2 + 81c_1^2}\right)^{\frac{1}{3}}}{6} - \frac{2x^3}{\left(-324x^2 - 108c_1 + 12\sqrt{12x^9 + 729x^4 + 486c_1x^2 + 81c_1^2}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(-324x^2 - 108c_1 + 12\sqrt{12x^9 + 729x^4 + 486c_1x^2 + 81c_1^2}\right)^{\frac{1}{3}}}{12} + \frac{x^3}{\left(-324x^2 - 108c_1 + 12\sqrt{12x^9 + 729x^4 + 486c_1x^2 + 81c_1^2}\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(-324x^2-108c_1+12\sqrt{12x^9+729x^4+486c_1x^2+81c_1^2}\right)^{\frac{1}{3}}}{6} + \frac{2x^3}{\left(-324x^2-108c_1+12\sqrt{12x^9+729x^4+486c_1x^2+81c_1^2}\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(-324x^2 - 108c_1 + 12\sqrt{12x^9 + 729x^4 + 486c_1x^2 + 81c_1^2}\right)^{\frac{1}{3}}}{12} + \frac{x^3}{\left(-324x^2 - 108c_1 + 12\sqrt{12x^9 + 729x^4 + 486c_1x^2 + 81c_1^2}\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(-324x^2-108c_1+12\sqrt{12x^9+729x^4+486c_1x^2+81c_1^2}\right)^{\frac{1}{3}}}{6} + \frac{2x^3}{\left(-324x^2-108c_1+12\sqrt{12x^9+729x^4+486c_1x^2+81c_1^2}\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 4.542 (sec). Leaf size: 331

```
DSolve[(3*x+6/y[x])+(x^2/y[x]+3*y[x]/x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{-81x^2+\sqrt{108x^9+729\left(-3x^2+c_1\right)^2}+27c_1}}{3\sqrt[3]{2}} - \frac{\sqrt[3]{2}x^3}{\sqrt[3]{-81x^2+\sqrt{108x^9+729\left(-3x^2+c_1\right)^2}+27c_1}}$$

$$y(x) \to \frac{\left(-1+i\sqrt{3}\right)\sqrt[3]{-81x^2+\sqrt{108x^9+729\left(-3x^2+c_1\right)^2}+27c_1}}{6\sqrt[3]{2}} + \frac{\left(1+i\sqrt{3}\right)x^3}{2^{2/3}\sqrt[3]{-81x^2+\sqrt{108x^9+729\left(-3x^2+c_1\right)^2}+27c_1}}$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)x^3}{2^{2/3}\sqrt[3]{-81x^2+\sqrt{108x^9+729\left(-3x^2+c_1\right)^2}+27c_1}} - \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{-81x^2+\sqrt{108x^9+729\left(-3x^2+c_1\right)^2}+27c_1}}{6\sqrt[3]{2}}$$

1.8 problem Example 3.8

Internal problem ID [5841]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.2 FIRST ORDER ODE. Page 114
Problem number: Example 3.8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{y^2 - yx + x^2 y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((y(x)^2-x*y(x))+x^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x}{\ln(x) + c_1}$$

✓ Solution by Mathematica

Time used: 0.143 (sec). Leaf size: 19

```
DSolve[(y[x]^2-x*y[x])+x^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{\log(x) + c_1}$$
$$y(x) \to 0$$

1.9 problem Example 3.9

Internal problem ID [5842]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.2 FIRST ORDER ODE. Page 114
Problem number: Example 3.9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{y-(x-y)\,y'=-x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve((x+y(x))-(x-y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\tan\left(\text{RootOf}\left(-2\_Z+\ln\left(\frac{1}{\cos\left(\_Z\right)^2}\right)+2\ln\left(x\right)+2c_1\right)\right)x$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 36

```
DSolve[(x+y[x])-(x-y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right)-\arctan\left(\frac{y(x)}{x}\right)=-\log(x)+c_1,y(x)\right]$$

1.10 problem Example 3.10

Internal problem ID [5843]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.2 FIRST ORDER ODE. Page 114
Problem number: Example 3.10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$\boxed{y' - \frac{y}{2x} - \frac{x^2}{2y} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
dsolve(diff(y(x),x)=y(x)/(2*x)+x^2/(2*y(x)),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{2x^3 + 4c_1 x}}{2}$$
$$y(x) = \frac{\sqrt{2x^3 + 4c_1 x}}{2}$$

✓ Solution by Mathematica

Time used: 0.194 (sec). Leaf size: 56

```
DSolve[y'[x]==y[x]/(2*x)+x^2/(2*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{x}\sqrt{x^2 + 2c_1}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{x}\sqrt{x^2 + 2c_1}}{\sqrt{2}}$$

1.11 problem Example 3.11

Internal problem ID [5844]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.2 FIRST ORDER ODE. Page 114
Problem number: Example 3.11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{y}{t} - \frac{y^2}{t} = -\frac{2}{t}}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 23

```
dsolve(diff(y(t),t)=-2/t+1/t*y(t)+1/t*y(t)^2,y(t), singsol=all)
```

$$y(t) = -\frac{2c_1 t^3 + 1}{c_1 t^3 - 1}$$

✓ Solution by Mathematica

Time used: 1.263 (sec). Leaf size: 43

```
DSolve[y'[t]==-2/t+1/t*y[t]+1/t*y[t]^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1 - 2e^{3c_1} t^3}{1 + e^{3c_1} t^3}$$
$$y(t) \to -2$$
$$y(t) \to 1$$

1.12 problem Example 3.12

Internal problem ID [5845]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.2 FIRST ORDER ODE. Page 114
Problem number: Example 3.12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y' + \frac{y}{t} + y^2 = -1}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 36

```
dsolve(diff(y(t),t)=-y(t)/t-1-y(t)^2,y(t), singsol=all)
```

$$y(t) = \frac{2\,\mathrm{BesselK}\left(1, it\right) c_1 - \mathrm{BesselJ}\left(1, t\right)}{2i\,\mathrm{BesselK}\left(0, it\right) c_1 + \mathrm{BesselJ}\left(0, t\right)}$$

✓ Solution by Mathematica

Time used: 0.189 (sec). Leaf size: 43

```
DSolve[y'[t]==-y[t]/t-1-y[t]^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{\mathrm{BesselY}(1,t) + c_1\,\mathrm{BesselJ}(1,t)}{\mathrm{BesselY}(0,t) + c_1\,\mathrm{BesselJ}(0,t)}$$
$$y(t) \to -\frac{\mathrm{BesselJ}(1,t)}{\mathrm{BesselJ}(0,t)}$$

1.13 problem Example 3.14

Internal problem ID [5846]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.2 FIRST ORDER ODE. Page 114
Problem number: Example 3.14.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_dAlembert]`

$$\boxed{yy' - ay'^2 = -x}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 264

```
dsolve(x+y(x)*diff(y(x),x)=a*(diff(y(x),x))^2,y(x), singsol=all)
```

$$\frac{c_1\left(y(x)+\sqrt{4ax+y(x)^2}\right)}{\sqrt{\frac{y(x)\sqrt{4ax+y(x)^2}+2a^2+2ax+y(x)^2}{a^2}}}+x$$
$$-\frac{\sqrt{2}\left(y(x)+\sqrt{4ax+y(x)^2}\right)\operatorname{arcsinh}\left(\frac{y(x)+\sqrt{4ax+y(x)^2}}{2a}\right)}{2\sqrt{\frac{y(x)\sqrt{4ax+y(x)^2}+2a^2+2ax+y(x)^2}{a^2}}}=0$$

$$\frac{c_1\left(-y(x)+\sqrt{4ax+y(x)^2}\right)}{\sqrt{-\frac{2\left(y(x)\sqrt{4ax+y(x)^2}-2a^2-2ax-y(x)^2\right)}{a^2}}}+x$$
$$-\frac{\left(-y(x)+\sqrt{4ax+y(x)^2}\right)\operatorname{arcsinh}\left(\frac{-y(x)+\sqrt{4ax+y(x)^2}}{2a}\right)}{\sqrt{-\frac{2\left(y(x)\sqrt{4ax+y(x)^2}-2a^2-2ax-y(x)^2\right)}{a^2}}}=0$$

✓ Solution by Mathematica

Time used: 1.371 (sec). Leaf size: 71

```
DSolve[x+y[x]*y'[x]==a*(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x=-\frac{aK[1]\log\left(\sqrt{K[1]^2+1}-K[1]\right)}{\sqrt{K[1]^2+1}}+\frac{c_1K[1]}{\sqrt{K[1]^2+1}},y(x)=aK[1]-\frac{x}{K[1]}\right\},\{y(x),K[1]\}\right]$$

1.14 problem Example 3.15

Internal problem ID [5847]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.2 FIRST ORDER ODE. Page 114
Problem number: Example 3.15.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - y^2 a^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)^2-a^2*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{ax}$$
$$y(x) = c_1 \mathrm{e}^{-ax}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 31

```
DSolve[(y'[x])^2-a^2*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-ax}$$
$$y(x) \to c_1 e^{ax}$$
$$y(x) \to 0$$

1.15 problem Example 3.16

Internal problem ID [5848]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.2 FIRST ORDER ODE. Page 114
Problem number: Example 3.16.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 = 4x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)^2=4*x^2,y(x), singsol=all)
```

$$y(x) = x^2 + c_1$$
$$y(x) = -x^2 + c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 23

```
DSolve[(y'[x])^2==4*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^2 + c_1$$
$$y(x) \to x^2 + c_1$$

2 Chapter 3. Ordinary Differential Equations. Section 3.3 SECOND ORDER ODE. Page 147

2.1 problem Example 3.17

Internal problem ID [5849]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.3 SECOND ORDER ODE. Page 147
Problem number: Example 3.17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' - 3y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{3x} + c_2 \mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 22

```
DSolve[y''[x]-2*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_2 e^{4x} + c_1\right)$$

2.2 problem Example 3.18

Internal problem ID [5850]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.3 SECOND ORDER ODE. Page 147
Problem number: Example 3.18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{s'' + 2s' + s = 0}$$

With initial conditions

$$[s(0) = 4, s'(0) = -2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve([diff(s(t),t$2)+2*diff(s(t),t)+s(t)=0,s(0) = 4, D(s)(0) = -2],s(t), singsol=all)
```

$$s(t) = 2\,\mathrm{e}^{-t}(t+2)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 15

```
DSolve[{s''[t]+2*s'[t]+s[t]==0,{s[0]==4,s'[0]==-2}},s[t],t,IncludeSingularSolutions -> True]
```

$$s(t) \to 2e^{-t}(t+2)$$

2.3 problem Example 3.19

Internal problem ID [5851]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.3 SECOND ORDER ODE. Page 147
Problem number: Example 3.19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + 5y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+5*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x} \sin(2x) + c_2 \mathrm{e}^{x} \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 24

```
DSolve[y''[x]-2*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(c_2 \cos(2x) + c_1 \sin(2x))$$

2.4 problem Example 3.21

Internal problem ID [5852]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.3 SECOND ORDER ODE. Page 147
Problem number: Example 3.21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y' - 3y = 3x + 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-3*y(x)=3*x+1,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{3x} c_2 + \mathrm{e}^{-x} c_1 - x + \frac{1}{3}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 28

```
DSolve[y''[x]-2*y'[x]-3*y[x]==3*x+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + c_1 e^{-x} + c_2 e^{3x} + \frac{1}{3}$$

2.5 problem Example 3.22

Internal problem ID [5853]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.3 SECOND ORDER ODE. Page 147
Problem number: Example 3.22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 3y' + 2y = \mathrm{e}^{2x} x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=x*exp(2*x),y(x), singsol=all)
```

$$y(x) = \left(\frac{\mathrm{e}^{x} x^{2}}{2} - \mathrm{e}^{x} x + \mathrm{e}^{x} + \mathrm{e}^{x} c_1 + c_2 \right) \mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 33

```
DSolve[y''[x]-3*y'[x]+2*y[x]==x*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{x} \left(e^{x} \left(x^{2} - 2x + 2 + 2c_2 \right) + 2c_1 \right)$$

2.6 problem Example 3.23

Internal problem ID [5854]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.3 SECOND ORDER ODE. Page 147
Problem number: Example 3.23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = 4\sin(x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+y(x)=4*sin(x),y(x), singsol=all)
```

$$y(x) = \sin(x)\, c_2 + \cos(x)\, c_1 + 2\sin(x) - 2\cos(x)\, x$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 20

```
DSolve[y''[x]+y[x]==4*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (-2x + c_1)\cos(x) + c_2 \sin(x)$$

2.7 problem Example 3.24

Internal problem ID [5855]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.3 SECOND ORDER ODE. Page 147
Problem number: Example 3.24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2x^2 y' + \left(x^4 + 2x - 1\right) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+2*x^2*diff(y(x),x)+(x^4+2*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x\left(x^2-3\right)}{3}} + c_2 \mathrm{e}^{-\frac{x\left(x^2+3\right)}{3}}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 34

```
DSolve[y''[x]+2*x^2*y'[x]+(x^4+2*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-\frac{1}{3}x\left(x^2+3\right)} \left(c_2 e^{2x} + 2c_1\right)$$

2.8 problem Example 3.26

Internal problem ID [5856]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.3 SECOND ORDER ODE. Page 147
Problem number: Example 3.26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{p\,x^2u'' + qxu' + ru = f(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 222

```
dsolve(p*x^2*diff(u(x),x$2)+q*x*diff(u(x),x)+r*u(x)=f(x),u(x), singsol=all)
```

$$u(x) = x^{\frac{-q+p+\sqrt{p^2-2qp-4rp+q^2}}{2p}}c_2 + x^{-\frac{q-p+\sqrt{p^2-2qp-4rp+q^2}}{2p}}c_1$$
$$-x^{-\frac{q-p+\sqrt{p^2+(-2q-4r)p+q^2}}{2p}}\left(\int x^{\frac{\sqrt{p^2+(-2q-4r)p+q^2}-3p+q}{2p}}f(x)\,dx\right) + x^{\frac{-q+p+\sqrt{p^2+(-2q-4r)p+q^2}}{2p}}\left(\int x^{-\frac{\sqrt{p^2+(-2q-4r)p+q^2}}{2p}}$$
$$+\frac{}{\sqrt{p^2+(-2q-4r)\,p+q^2}}$$

✓ Solution by Mathematica

Time used: 1.17 (sec). Leaf size: 342

```
DSolve[p*x^2*u''[x]+q*x*u'[x]+r*u[x]==f[x],u[x],x,IncludeSingularSolutions -> True]
```

$$u(x)$$

$$\to x^{-\frac{\sqrt{p}\sqrt{r}\sqrt{\frac{p^2-2p(q+2r)+q^2}{pr}}-p+q}{2p}}\left(x^{\frac{\sqrt{r}\sqrt{\frac{p^2-2p(q+2r)+q^2}{pr}}}{\sqrt{p}}}\int_1^x \frac{f(K[2])K[2]^{\frac{-3p-\sqrt{r}\sqrt{\frac{p^2-2(q+2r)p+q^2}{pr}}\sqrt{p}+q}{2p}}}{\sqrt{p}\sqrt{r}\sqrt{\frac{p^2-2(q+2r)p+q^2}{pr}}}dK[2]\right.$$

$$\left.+\int_1^x -\frac{f(K[1])K[1]^{\frac{-3p+\sqrt{r}\sqrt{\frac{p^2-2(q+2r)p+q^2}{pr}}\sqrt{p}+q}{2p}}}{\sqrt{p}\sqrt{r}\sqrt{\frac{p^2-2(q+2r)p+q^2}{pr}}}dK[1]+c_2 x^{\frac{\sqrt{r}\sqrt{\frac{p^2-2p(q+2r)+q^2}{pr}}}{\sqrt{p}}}+c_1\right)$$

3 Chapter 3. Ordinary Differential Equations. Section 3.5 HIGHER ORDER ODE. Page 181

3.1 problem Example 3.29

Internal problem ID [5857]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.5 HIGHER ORDER ODE. Page 181
Problem number: Example 3.29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{\sin(x)\,u'' + 2\cos(x)\,u' + \sin(x)\,u = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(sin(x)*diff(u(x),x$2)+2*cos(x)*diff(u(x),x)+sin(x)*u(x)=0,u(x), singsol=all)
```

$$u(x) = c_1 \csc(x) \sin\left(\sqrt{2}\,x\right) + c_2 \csc(x) \cos\left(\sqrt{2}\,x\right)$$

✓ Solution by Mathematica

Time used: 0.09 (sec). Leaf size: 51

```
DSolve[Sin[x]*u''[x]+2*Cos[x]*u'[x]+Sin[x]*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to \frac{1}{4} e^{-i\sqrt{2}x} \left(4c_1 - i\sqrt{2}c_2 e^{2i\sqrt{2}x}\right) \csc(x)$$

3.2 problem Example 3.30

Internal problem ID [5858]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.5 HIGHER ORDER ODE. Page 181
Problem number: Example 3.30.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x], [_3rd_order, _missing_y], [_3rd_ord`

Solve

$$\boxed{3(y'')^2 - y'y''' - y''(y')^2 = 0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 42

```
dsolve(3*diff(y(x),x$2)^2-diff(y(x),x)*diff(y(x),x$3)-diff(y(x),x$2)*diff(y(x),x)^2=0,y(x),
```

$$y(x) = c_1$$

$$y(x) = \frac{\text{LambertW}\left(-\frac{e^{\frac{c_3}{c_1}} e^{\frac{x}{c_1}}}{c_2 c_1}\right) c_1 - c_3 - x}{c_1}$$

✓ Solution by Mathematica

Time used: 3.622 (sec). Leaf size: 79

```
DSolve[3*(y''[x])^2-y'[x]*y'''[x]-y''[x]*(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \log\left(\text{InverseFunction}\left[-\frac{1}{\#1} - c_1 \log(\#1) + c_1 \log(1 + \#1 c_1)\&\right][x + c_2]\right) - \log\left(1 + c_1 \text{InverseFunction}\left[-\frac{1}{\#1} - c_1 \log(\#1) + c_1 \log(1 + \#1 c_1)\&\right][x + c_2]\right) + c_3$$

3.3 problem Example 3.32

Internal problem ID [5859]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.5 HIGHER ORDER ODE. Page 181
Problem number: Example 3.32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']]`

$$\boxed{y'' - \frac{xy'}{-x^2+1} + \frac{y}{-x^2+1} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)-x/(1-x^2)*diff(y(x),x)+y(x)/(1-x^2)=0,y(x), singsol=all)
```

$$y(x) = c_1 x + c_2 \sqrt{x-1}\,\sqrt{x+1}$$

✓ Solution by Mathematica

Time used: 0.177 (sec). Leaf size: 97

```
DSolve[y''[x]-x/(1-x^2)*y'[x]+y[x]/(1-x^2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cosh\left(\frac{2\sqrt{1-x^2}\arctan\left(\frac{\sqrt{1-x^2}}{x+1}\right)}{\sqrt{x^2-1}}\right) - ic_2 \sinh\left(\frac{2\sqrt{1-x^2}\arctan\left(\frac{\sqrt{1-x^2}}{x+1}\right)}{\sqrt{x^2-1}}\right)$$

3.4 problem Example 3.33

Internal problem ID [5860]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.5 HIGHER ORDER ODE. Page 181
Problem number: Example 3.33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _reducibl`

$$\boxed{x^2 y y'' - x^2 y'^2 + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 19

```
dsolve(x^2*y(x)*diff(y(x),x$2)=x^2*(diff(y(x),x))^2-y(x)^2,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \mathrm{e}^{-c_1 x} c_2 \mathrm{e} x$$

✓ Solution by Mathematica

Time used: 0.212 (sec). Leaf size: 15

```
DSolve[x^2*y[x]*y''[x]==x^2*(y'[x])^2-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 x e^{c_1 x}$$

3.5 problem Example 3.34

Internal problem ID [5861]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.5 HIGHER ORDER ODE. Page 181
Problem number: Example 3.34.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - 3y'' + 3y' - y = 4\,\mathrm{e}^{t}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve(diff(y(t),t$3)-3*diff(y(t),t$2)+3*diff(y(t),t)-y(t)=4*exp(t),y(t), singsol=all)
```

$$y(t) = \frac{2t^3\mathrm{e}^{t}}{3} + c_1\mathrm{e}^{t} + c_2\mathrm{e}^{t}t + c_3\mathrm{e}^{t}t^2$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 34

```
DSolve[y'''[t]-3*y''[t]+3*y'[t]-y[t]==4*Exp[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{3}e^{t}\left(2t^3 + 3c_3t^2 + 3c_2t + 3c_1\right)$$

3.6 problem Example 3.35

Internal problem ID [5862]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.5 HIGHER ORDER ODE. Page 181
Problem number: Example 3.35.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' + 2y'' + y = 3\sin(t) - 5\cos(t)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve(diff(y(t),t$4)+2*diff(y(t),t$2)+y(t)=3*sin(t)-5*cos(t),y(t), singsol=all)
```

$$y(t) = \left(-\frac{5}{4} - \frac{3}{4}t + \frac{5}{8}t^2\right)\cos(t) + \left(\frac{3}{4} - \frac{5}{4}t - \frac{3}{8}t^2\right)\sin(t) + \cos(t)\,c_1 + c_2\sin(t) + c_3 t\cos(t) + c_4 t\sin(t)$$

✓ Solution by Mathematica

Time used: 0.128 (sec). Leaf size: 56

```
DSolve[y''''[t]+2*y''[t]+y[t]==3*Sin[t]-5*Cos[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{16}\left(\left(10t^2 + 2(-3 + 8c_2)t - 25 + 16c_1\right)\cos(t) + \left(-6t^2 + 2(-15 + 8c_4)t + 3 + 16c_3\right)\sin(t)\right)$$

3.7 problem Example 3.36

Internal problem ID [5863]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.5 HIGHER ORDER ODE. Page 181
Problem number: Example 3.36.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - y'' - y' + y = g(t)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 66

```
dsolve(diff(y(t),t$3)-diff(y(t),t$2)-diff(y(t),t)+y(t)=g(t),y(t), singsol=all)
```

$$\begin{aligned} y(t) = &-\left(\int \frac{(2t+1)\,g(t)\,\mathrm{e}^{-t}}{4}dt\right)\mathrm{e}^{t} + \left(\int \frac{\mathrm{e}^{t}g(t)}{4}dt\right)\mathrm{e}^{-t} \\ &+ \left(\int \frac{\mathrm{e}^{-t}g(t)}{2}dt\right)\mathrm{e}^{t}t + c_1\mathrm{e}^{t} + c_2\mathrm{e}^{-t} + c_3\mathrm{e}^{t}t \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 106

```
DSolve[y'''[t]-y''[t]-y'[t]+y[t]==g[t],y[t],t,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(t) \to\ & e^{-t}\int_1^t \frac{1}{4}e^{K[1]}g(K[1])dK[1] + e^{t}t\int_1^t \frac{1}{2}e^{-K[3]}g(K[3])dK[3] \\ &+ e^{t}\int_1^t -\frac{1}{4}e^{-K[2]}g(K[2])(2K[2]+1)dK[2] + c_1e^{-t} + c_2e^{t} + c_3e^{t}t \end{aligned}$$

3.8 problem Example 3.37

Internal problem ID [5864]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.5 HIGHER ORDER ODE. Page 181
Problem number: Example 3.37.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y^{(5)} - \frac{y''''}{t} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(t),t$5)-1/t*diff(y(t),t$4)=0,y(t), singsol=all)
```

$$y(t) = c_4 t^5 + c_3 t^3 + c_2 t^2 + c_5 t + c_1$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 33

```
DSolve[y'''''[t]-1/t*y''''[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_1 t^5}{120} + c_5 t^3 + c_4 t^2 + c_3 t + c_2$$

3.9 problem Example 3.38

Internal problem ID [5865]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.5 HIGHER ORDER ODE. Page 181
Problem number: Example 3.38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{xx'' - x'^2 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 14

```
dsolve(x(t)*diff(x(t),t$2)-diff(x(t),t)^2=0,x(t), singsol=all)
```

$$x(t) = 0$$
$$x(t) = \mathrm{e}^{c_1 t} c_2$$

✓ Solution by Mathematica

Time used: 0.108 (sec). Leaf size: 14

```
DSolve[x[t]*x''[t]-(x'[t])^2==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_2 e^{c_1 t}$$

3.10 problem Example 3.39

Internal problem ID [5866]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.5 HIGHER ORDER ODE. Page 181
Problem number: Example 3.39.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' + 4y''' + 3y'' - 4y' - 4y = f(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 91

```
dsolve(diff(y(x),x$4)+4*diff(y(x),x$3)+3*diff(y(x),x$2)-4*diff(y(x),x)-4*y(x)=f(x),y(x), sin
```

$$\begin{aligned} y(x) = &\left(\int \frac{f(x)\,\mathrm{e}^{-x}}{18}dx\right)\mathrm{e}^{x} + \left(\int -\frac{f(x)\,(3x-4)\,\mathrm{e}^{2x}}{9}dx\right)\mathrm{e}^{-2x} - \left(\int \frac{f(x)\,\mathrm{e}^{x}}{2}dx\right)\mathrm{e}^{-x} \\ &+ \left(\int \frac{f(x)\,\mathrm{e}^{2x}}{3}dx\right)\mathrm{e}^{-2x}x + \mathrm{e}^{x}c_1 + c_2\mathrm{e}^{-2x} + c_3\mathrm{e}^{-x} + c_4\mathrm{e}^{-2x}x \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 128

```
DSolve[y''''[x]+4*y'''[x]+3*y''[x]-4*y'[x]-4*y[x]==f[x],y[x],x,IncludeSingularSolutions -> T
```

$$\begin{aligned} y(x) \to e^{-2x}\Bigg(&x\int_1^x \frac{1}{3}e^{2K[2]}f(K[2])dK[2] + e^{x}\int_1^x -\frac{1}{2}e^{K[3]}f(K[3])dK[3] \\ &+ e^{3x}\int_1^x \frac{1}{18}e^{-K[4]}f(K[4])dK[4] + \int_1^x -\frac{1}{9}e^{2K[1]}f(K[1])(3K[1]-4)dK[1] + c_2x \\ &+ c_3e^{x} + c_4e^{3x} + c_1\Bigg) \end{aligned}$$

3.11 problem Example 3.40

Internal problem ID [5867]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.5 HIGHER ORDER ODE. Page 181
Problem number: Example 3.40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{u'' - (1+2x)\,u' + \left(x^2+x-1\right)u = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(u(x),x$2)-(2*x+1)*diff(u(x),x)+(x^2+x-1)*u(x)=0,u(x), singsol=all)
```

$$u(x) = \mathrm{e}^{\frac{x^2}{2}} c_1 + c_2 \mathrm{e}^{\frac{x(x+2)}{2}}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 24

```
DSolve[u''[x]-(2*x+1)*u'[x]+(x^2+x-1)*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to e^{\frac{x^2}{2}} \left(c_2 e^x + c_1\right)$$

3.12 problem Example 3.41

Internal problem ID [5868]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.5 HIGHER ORDER ODE. Page 181
Problem number: Example 3.41.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 6y' + 9y = 50\,\mathrm{e}^{2x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+6*diff(y(x),x)+9*y(x)=50*exp(2*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-3x} c_2 + \mathrm{e}^{-3x} x c_1 + 2\,\mathrm{e}^{2x}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 25

```
DSolve[y''[x]+6*y'[x]+9*y[x]==50*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(2e^{5x} + c_2 x + c_1\right)$$

3.13 problem Example 3.42

Internal problem ID [5869]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.5 HIGHER ORDER ODE. Page 181
Problem number: Example 3.42.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y' + 4y = 50\,\mathrm{e}^{2x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=50*exp(2*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2x} c_2 + \mathrm{e}^{2x} x c_1 + 25\,\mathrm{e}^{2x} x^2$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 23

```
DSolve[y''[x]-4*y'[x]+4*y[x]==50*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}\left(25x^2 + c_2 x + c_1\right)$$

3.14 problem Example 3.43

Internal problem ID [5870]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.5 HIGHER ORDER ODE. Page 181
Problem number: Example 3.43.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 3y' + 2y = \cos(2x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=cos(2*x),y(x), singsol=all)
```

$$y(x) = -\mathrm{e}^{-2x} c_1 + c_2 \mathrm{e}^{-x} - \frac{\cos(2x)}{20} + \frac{3\sin(2x)}{20}$$

✓ Solution by Mathematica

Time used: 0.119 (sec). Leaf size: 37

```
DSolve[y''[x]+3*y'[x]+2*y[x]==Cos[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3}{20}\sin(2x) - \frac{1}{20}\cos(2x) + e^{-2x}(c_2 e^{x} + c_1)$$

3.15 problem Example 3.44

Internal problem ID [5871]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.5 HIGHER ORDER ODE. Page 181
Problem number: Example 3.44.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' + 6y'' + 11y' + 6y = 2\sin(3x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$3)+6*diff(y(x),x$2)+11*diff(y(x),x)+6*y(x)=2*sin(3*x),y(x), singsol=all)
```

$$y(x) = -\frac{\cos(3x)}{195} - \frac{8\sin(3x)}{195} + c_1 e^{-3x} + c_2 e^{-2x} + c_3 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 44

```
DSolve[y'''[x]+6*y''[x]+11*y'[x]+6*y[x]==2*Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{8}{195}\sin(3x) - \frac{1}{195}\cos(3x) + e^{-3x}(e^x(c_3 e^x + c_2) + c_1)$$

3.16 problem Example 3.45

Internal problem ID [5872]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.5 HIGHER ORDER ODE. Page 181
Problem number: Example 3.45.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y = x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+4*y(x)=x^2,y(x), singsol=all)
```

$$y(x) = \sin(2x)\, c_2 + \cos(2x)\, c_1 + \frac{x^2}{4} - \frac{1}{8}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 30

```
DSolve[y''[x]+4*y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{4} + c_1 \cos(2x) + c_2 \sin(2x) - \frac{1}{8}$$

3.17 problem Example 3.46

Internal problem ID [5873]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.5 HIGHER ORDER ODE. Page 181
Problem number: Example 3.46.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 3y = x^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+3*y(x)=x^3,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{3x} c_2 + \mathrm{e}^{x} c_1 + \frac{x^3}{3} + \frac{4x^2}{3} + \frac{26x}{9} + \frac{80}{27}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 39

```
DSolve[y''[x]-4*y'[x]+3*y[x]==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{27}\left(9x^3 + 36x^2 + 78x + 80\right) + c_1 e^{x} + c_2 e^{3x}$$

3.18 problem Example 3.47

Internal problem ID [5874]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.5 HIGHER ORDER ODE. Page 181
Problem number: Example 3.47.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y' + \left(1 + \frac{2}{(3x+1)^2}\right) y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+(1+2/(1+3*x)^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 (3x+1)^{\frac{1}{3}} \mathrm{e}^{-x} + c_2 (3x+1)^{\frac{2}{3}} \mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 35

```
DSolve[y''[x]+2*y'[x]+(1+2/(1+3*x)^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x} \sqrt[3]{3x+1} \left(c_2 \sqrt[3]{3x+1} + c_1\right)$$

4 Chapter 3. Ordinary Differential Equations. Section 3.6 Summary and Problems. Page 218

4.1 problem Problem 3.1

Internal problem ID [5875]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.6 Summary and Problems. Page 218
Problem number: Problem 3.1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{y + \sqrt{x^2 + y^2} - y'x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(y(x)+sqrt(x^2+y(x)^2)-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$\frac{y(x)}{x^2} + \frac{\sqrt{x^2 + y(x)^2}}{x^2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.347 (sec). Leaf size: 27

```
DSolve[y[x]+Sqrt[x^2+y[x]^2]-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-c_1} \left(-1 + e^{2c_1} x^2\right)$$

4.2 problem Problem 3.2

Internal problem ID [5876]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.6 Summary and Problems. Page 218
Problem number: Problem 3.2.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 + y^2 = a^2}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 68

```
dsolve(diff(y(x),x)^2=a^2-y(x)^2,y(x), singsol=all)
```

$$y(x) = -a$$
$$y(x) = a$$
$$y(x) = -\tan\left(-x + c_1\right)\sqrt{\frac{a^2}{\tan\left(-x + c_1\right)^2 + 1}}$$
$$y(x) = \tan\left(-x + c_1\right)\sqrt{\frac{a^2}{\tan\left(-x + c_1\right)^2 + 1}}$$

✓ Solution by Mathematica

Time used: 3.336 (sec). Leaf size: 111

```
DSolve[(y'[x])^2==a^2-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a \tan(x - c_1)}{\sqrt{\sec^2(x - c_1)}}$$

$$y(x) \to \frac{a \tan(x - c_1)}{\sqrt{\sec^2(x - c_1)}}$$

$$y(x) \to -\frac{a \tan(x + c_1)}{\sqrt{\sec^2(x + c_1)}}$$

$$y(x) \to \frac{a \tan(x + c_1)}{\sqrt{\sec^2(x + c_1)}}$$

$$y(x) \to -a$$

$$y(x) \to a$$

4.3 problem Problem 3.3

Internal problem ID [5877]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.6 Summary and Problems. Page 218
Problem number: Problem 3.3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2y'x + y\left(x^2+2\right) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(x)\, x + c_2 \cos(x)\, x$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-ix} x - \frac{1}{2} i c_2 e^{ix} x$$

4.4 problem Problem 3.4

Internal problem ID [5878]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.6 Summary and Problems. Page 218
Problem number: Problem 3.4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{2y'}{x} - \frac{2y}{(1+x)^2} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+2/x*diff(y(x),x)-2/(1+x)^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x\,(x+1)} + \frac{c_2(x^2+3x+3)}{x+1}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 34

```
DSolve[y''[x]+2/x*y'[x]-2/(1+x)^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 x(x^2+3x+3) + 3c_1}{3x(x+1)}$$

4.5 problem Problem 3.6

Internal problem ID [5879]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.6 Summary and Problems. Page 218
Problem number: Problem 3.6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$\boxed{\left(x^2y^2+1\right)y+\left(x^2y^2-1\right)xy'=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 23

```
dsolve((x^2*y(x)^2+1)*y(x)+(x^2*y(x)^2-1)*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-\frac{\mathrm{LambertW}\left(-x^4\mathrm{e}^{-4c_1}\right)}{2}-2c_1}x$$

✓ Solution by Mathematica

Time used: 6.032 (sec). Leaf size: 60

```
DSolve[(x^2*y[x]^2+1)*y[x]+(x^2*y[x]^2-1)*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x)\to -\frac{i\sqrt{W\left(-e^{-2c_1}x^4\right)}}{x}$$
$$y(x)\to \frac{i\sqrt{W\left(-e^{-2c_1}x^4\right)}}{x}$$
$$y(x)\to 0$$

4.6 problem Problem 3.7

Internal problem ID [5880]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.6 Summary and Problems. Page 218
Problem number: Problem 3.7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{2y^2x^3 - y + \left(2y^3x^2 - x\right)y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 522

```
dsolve((2*x^3*y(x)^2-y(x))+(2*x^2*y(x)^3-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\frac{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}}{6x}-\frac{6\left(\frac{x^2}{3}-\frac{c_1}{3}\right)x}{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}}$$

$$y(x)=-\frac{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}}{12x}+\frac{3\left(\frac{x^2}{3}-\frac{c_1}{3}\right)x}{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}}-\frac{i\sqrt{3}\left(\frac{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}}{6x}+\frac{6\left(\frac{x^2}{3}-\frac{c_1}{3}\right)x}{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x)=-\frac{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}}{12x}+\frac{3\left(\frac{x^2}{3}-\frac{c_1}{3}\right)x}{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}}+\frac{i\sqrt{3}\left(\frac{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}}{6x}+\frac{6\left(\frac{x^2}{3}-\frac{c_1}{3}\right)x}{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 44.412 (sec). Leaf size: 358

```
DSolve[(2*x^3*y[x]^2-y[x])+(2*x^2*y[x]^3-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{\sqrt[3]{2}(-x^3+c_1 x)}{\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3\left(x^3-c_1 x\right)^3}}} + \frac{\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3\left(x^3-c_1 x\right)^3}}}{3\sqrt[3]{2}x}$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)\left(x^3-c_1 x\right)}{2^{2/3}\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3\left(x^3-c_1 x\right)^3}}} - \frac{\left(1-i\sqrt{3}\right)\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3\left(x^3-c_1 x\right)^3}}}{6\sqrt[3]{2}x}$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)\left(x^3-c_1 x\right)}{2^{2/3}\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3\left(x^3-c_1 x\right)^3}}} - \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3\left(x^3-c_1 x\right)^3}}}{6\sqrt[3]{2}x}$$

4.7 problem Problem 3.8

Internal problem ID [5881]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.6 Summary and Problems. Page 218
Problem number: Problem 3.8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D']]`

$$\boxed{\frac{1}{y}+\sec\left(\frac{y}{x}\right)-\frac{xy'}{y^2}=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve((1/y(x)+sec(y(x)/x))-x/y(x)^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\text{RootOf}\left(\_Z\,\text{Si}\left(\_Z\right)+\_Zc_1+\_Zx+\cos\left(\_Z\right)\right)x$$

✓ Solution by Mathematica

Time used: 0.145 (sec). Leaf size: 32

```
DSolve[(1/y[x]+Sec[y[x]/x])-x/y[x]^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\text{Si}\left(\frac{y(x)}{x}\right)-\frac{x\cos\left(\frac{y(x)}{x}\right)}{y(x)}=x+c_1,y(x)\right]$$

4.8 problem Problem 3.11

Internal problem ID [5882]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.6 Summary and Problems. Page 218
Problem number: Problem 3.11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{\phi' - \frac{\phi^2}{2} - \phi \cot(\theta) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(phi(theta),theta)-1/2*phi(theta)^2-phi(theta)*cot(theta)=0,phi(theta), singsol=a
```

$$\phi(\theta) = \frac{2\sin(\theta)}{\cos(\theta) + 2c_1}$$

✓ Solution by Mathematica

Time used: 0.3 (sec). Leaf size: 23

```
DSolve[\[Phi]'[\[Theta]]-1/2*\[Phi][\[Theta]]^2-\[Phi][\[Theta]]*Cot[\[Theta]]==0,\[Phi][\[T
```

$$\phi(\theta) \to \frac{2\sin(\theta)}{\cos(\theta) + 2c_1}$$
$$\phi(\theta) \to 0$$

4.9 problem Problem 3.12

Internal problem ID [5883]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.6 Summary and Problems. Page 218
Problem number: Problem 3.12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{u'' - \cot(\theta)\, u' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(u(theta),theta$2)-cot(theta)*diff(u(theta),theta)=0,u(theta), singsol=all)
```

$$u(\theta) = c_1 + \cos(\theta)\, c_2$$

✓ Solution by Mathematica

Time used: 0.085 (sec). Leaf size: 13

```
DSolve[u''[\[Theta]]-Cot[\[Theta]]*u'[\[Theta]]==0,u[\[Theta]],\[Theta],IncludeSingularSolut
```

$$u(\theta) \to c_2 \cos(\theta) + c_1$$

4.10 problem Problem 3.14

Internal problem ID [5884]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.6 Summary and Problems. Page 218
Problem number: Problem 3.14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$\boxed{\left(\phi' - \frac{\phi^2}{2}\right)\sin(\theta)^2 - \phi\cos(\theta)\sin(\theta) = \frac{\cos(2\theta)}{2} + 1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve((diff(phi(theta),theta)-1/2*phi(theta)^2)*sin(theta)^2-phi(theta)*sin(theta)*cos(thet
```

$$\phi(\theta) = -\frac{\sinh\left(\frac{\theta}{2}\right)c_1 + \cosh\left(\frac{\theta}{2}\right)}{\cosh\left(\frac{\theta}{2}\right)c_1 + \sinh\left(\frac{\theta}{2}\right)} - \frac{\cos(\theta)}{\sin(\theta)}$$

✓ Solution by Mathematica

Time used: 0.64 (sec). Leaf size: 36

```
DSolve[(\[Phi]'[\[Theta]]-1/2\[Phi][\[Theta]]^2)*Sin[\[Theta]]^2-\[Phi][\[Theta]]*Sin[\[Thet
```

$$\phi(\theta) \to -\cot(\theta) - \frac{2e^{\theta}}{e^{\theta} - 2c_1} + 1$$
$$\phi(\theta) \to 1 - \cot(\theta)$$

4.11 problem Problem 3.18

Internal problem ID [5885]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.6 Summary and Problems. Page 218
Problem number: Problem 3.18.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x], [_3rd_order, _missing_y], [_3rd_ord`

Solve

$$\boxed{ay''y''' - \sqrt{1 + (y'')^2} = 0}$$

✓ Solution by Maple

Time used: 0.218 (sec). Leaf size: 237

```
dsolve(a*diff(y(x),x$2)*diff(y(x),x$3)=sqrt(1+ diff(y(x),x$2)^2),y(x), singsol=all)
```

$$y(x) = -\frac{1}{2}ix^2 + c_1 x + c_2$$

$$y(x) = \frac{1}{2}ix^2 + c_1 x + c_2$$

$$y(x) = \frac{(-a^2 + c_1^2 + 2c_1 x + x^2)^{\frac{3}{2}}}{6a} - \frac{a \ln\left(c_1 + x + \sqrt{-a^2 + c_1^2 + 2c_1 x + x^2}\right) x}{2} - \frac{a \ln\left(c_1 + x + \sqrt{-a^2 + c_1^2 + 2c_1 x + x^2}\right) c_1}{2} + \frac{a\sqrt{-a^2 + c_1^2 + 2c_1 x + x^2}}{2} + xc_2 + c_3$$

$$y(x) = -\frac{(-a^2 + c_1^2 + 2c_1 x + x^2)^{\frac{3}{2}}}{6a} + \frac{a \ln\left(c_1 + x + \sqrt{-a^2 + c_1^2 + 2c_1 x + x^2}\right) x}{2} + \frac{a \ln\left(c_1 + x + \sqrt{-a^2 + c_1^2 + 2c_1 x + x^2}\right) c_1}{2} - \frac{a\sqrt{-a^2 + c_1^2 + 2c_1 x + x^2}}{2} + xc_2 + c_3$$

✓ Solution by Mathematica

Time used: 11.484 (sec). Leaf size: 209

```
DSolve[a*y''[x]*y'''[x]==Sqrt[1+ y''[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a^2\left(-1 + {c_1}^2\right) + 2ac_1 x + x^2}\left(a^2\left(2 + {c_1}^2\right) + 2ac_1 x + x^2\right)}{6a} - \frac{1}{2}a(x + ac_1)\log\left(\sqrt{a^2\left(-1 + {c_1}^2\right) + 2ac_1 x + x^2} + ac_1 + x\right) + c_3 x + c_2$$

$$y(x) \to -\frac{\sqrt{a^2\left(-1 + {c_1}^2\right) + 2ac_1 x + x^2}\left(a^2\left(2 + {c_1}^2\right) + 2ac_1 x + x^2\right)}{6a} + \frac{1}{2}a(x + ac_1)\log\left(\sqrt{a^2\left(-1 + {c_1}^2\right) + 2ac_1 x + x^2} + ac_1 + x\right) + c_3 x + c_2$$

4.12 problem Problem 3.19

Internal problem ID [5886]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.6 Summary and Problems. Page 218
Problem number: Problem 3.19.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{a^2 y'''' - y'' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(a^2*diff(y(x),x$4)=diff(y(x),x$2),y(x), singsol=all)
```

$$y(x) = c_1 + x c_2 + c_3 \mathrm{e}^{\frac{x}{a}} + c_4 \mathrm{e}^{-\frac{x}{a}}$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 38

```
DSolve[a^2*y''''[x]==y''[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to a^2 e^{-\frac{x}{a}} \left(c_1 e^{\frac{2x}{a}} + c_2 \right) + c_4 x + c_3$$

4.13 problem Problem 3.20

Internal problem ID [5887]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.6 Summary and Problems. Page 218
Problem number: Problem 3.20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y\,\mathrm{e}^{yx} + x\,\mathrm{e}^{yx} y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 9

```
dsolve(y(x)*exp(x*y(x))+x*exp(x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 16

```
DSolve[y[x]*Exp[x*y[x]]+x*Exp[x*y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x}$$
$$y(x) \to 0$$

4.14 problem Problem 3.21

Internal problem ID [5888]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.6 Summary and Problems. Page 218
Problem number: Problem 3.21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{-2yx + \mathrm{e}^{y} + \left(y - x^{2} + \mathrm{e}^{y} x\right) y' = -x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve((x-2*x*y(x)+exp(y(x)))+(y(x)-x^2+x*exp(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$-x^{2} y(x) + x\,\mathrm{e}^{y(x)} + \frac{x^{2}}{2} + \frac{y(x)^{2}}{2} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.341 (sec). Leaf size: 35

```
DSolve[(x-2*x*y[x]+Exp[y[x]])+(y[x]-x^2+x*Exp[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolution
```

$$\text{Solve}\left[x^{2}(-y(x)) + \frac{x^{2}}{2} + x e^{y(x)} + \frac{y(x)^{2}}{2} = c_1, y(x)\right]$$

4.15 problem Problem 3.22

Internal problem ID [5889]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.6 Summary and Problems. Page 218
Problem number: Problem 3.22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \frac{y'}{\sqrt{x}} + \frac{\left(x + \sqrt{x} - 8\right) y}{4x^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)-1/x^(1/2)*diff(y(x),x)+1/(4*x^2)*(x+x^(1/2)-8)*y(x)=0,y(x), singsol=al
```

$$y(x) = \frac{c_1 \mathrm{e}^{\sqrt{x}}}{x} + c_2 \mathrm{e}^{\sqrt{x}} x^2$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 30

```
DSolve[y''[x]-1/x^(1/2)*y'[x]+1/(4*x^2)*(x+x^(1/2)-8)*y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{e^{\sqrt{x}}(c_2 x^3 + 3c_1)}{3x}$$

4.16 problem Problem 3.23

Internal problem ID [5890]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.6 Summary and Problems. Page 218
Problem number: Problem 3.23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(-x^2+1\right)z''+(1-3x)z'+kz=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 99

```
dsolve((1-x^2)*diff(z(x),x$2)+(1-3*x)*diff(z(x),x)+k*z(x)=0,z(x), singsol=all)
```

$$z(x)=c_1(x+1)^{-1-\sqrt{k+1}}\,\text{hypergeom}\left(\left[\sqrt{k+1},1+\sqrt{k+1}\right],\left[1+2\sqrt{k+1}\right],\frac{2}{x+1}\right)$$
$$+c_2(x+1)^{-1+\sqrt{k+1}}\,\text{hypergeom}\left(\left[-\sqrt{k+1},1-\sqrt{k+1}\right],\left[1-2\sqrt{k+1}\right],\frac{2}{x+1}\right)$$

✓ Solution by Mathematica

Time used: 0.407 (sec). Leaf size: 77

```
DSolve[(1-x^2)*z''[x]+(1-3*x)*z'[x]+k*z[x]==0,z[x],x,IncludeSingularSolutions -> True]
```

$$z(x)\to c_2 G^{2,0}_{2,2}\left(\frac{1-x}{2}\middle|\begin{matrix}-\sqrt{k+1},\sqrt{k+1}\\0,0\end{matrix}\right)$$
$$+c_1\,\text{Hypergeometric2F1}\left(1-\sqrt{k+1},\sqrt{k+1}+1,1,\frac{1-x}{2}\right)$$

4.17 problem Problem 3.24

Internal problem ID [5891]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.6 Summary and Problems. Page 218
Problem number: Problem 3.24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(-x^2+1\right)\eta''-(1+x)\eta'+(k+1)\eta=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 95

```
dsolve((1-x^2)*diff(eta(x),x$2)-(1+x)*diff(eta(x),x)+(k+1)*eta(x)=0,eta(x), singsol=all)
```

$$\eta(x)=c_1(x+1)^{\sqrt{k+1}}\,\text{hypergeom}\left(\left[-\sqrt{k+1},1-\sqrt{k+1}\right],\left[1-2\sqrt{k+1}\right],\frac{2}{x+1}\right)+c_2(x+1)^{-\sqrt{k+1}}\,\text{hypergeom}\left(\left[\sqrt{k+1},1+\sqrt{k+1}\right],\left[1+2\sqrt{k+1}\right],\frac{2}{x+1}\right)$$

✓ Solution by Mathematica

Time used: 0.283 (sec). Leaf size: 77

```
DSolve[(1-x^2)*z''[x]-(1+x)*z'[x]+(k+1)*z[x]==0,z[x],x,IncludeSingularSolutions -> True]
```

$$z(x)\to c_2 G^{2,0}_{2,2}\left(\frac{1-x}{2}\middle|\begin{matrix}1-\sqrt{k+1},\sqrt{k+1}+1\\0,0\end{matrix}\right)+c_1\,\text{Hypergeometric2F1}\left(-\sqrt{k+1},\sqrt{k+1},1,\frac{1-x}{2}\right)$$

4.18 problem Problem 3.31

Internal problem ID [5892]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.6 Summary and Problems. Page 218
Problem number: Problem 3.31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{y^2 - 2xyy' = -x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve((x^2+y(x)^2)-2*x*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{c_1 x + x^2}$$
$$y(x) = -\sqrt{c_1 x + x^2}$$

✓ Solution by Mathematica

Time used: 0.2 (sec). Leaf size: 38

```
DSolve[(x^2+y[x]^2)-2*x*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x}\sqrt{x + c_1}$$
$$y(x) \to \sqrt{x}\sqrt{x + c_1}$$

4.19 problem Problem 3.32

Internal problem ID [5893]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.6 Summary and Problems. Page 218
Problem number: Problem 3.32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{-y^2+2xyy'=-x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve((x^2-y(x)^2)+2*x*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\sqrt{c_1x-x^2}$$
$$y(x)=-\sqrt{c_1x-x^2}$$

✓ Solution by Mathematica

Time used: 0.355 (sec). Leaf size: 37

```
DSolve[(x^2-y[x]^2)+2*x*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\sqrt{-x(x-c_1)}$$
$$y(x)\to \sqrt{-x(x-c_1)}$$

4.20 problem Problem 3.33

Internal problem ID [5894]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.6 Summary and Problems. Page 218
Problem number: Problem 3.33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _rational, _Riccati]`

$$\boxed{y'x - y - y^2 = x^2}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 10

```
dsolve(x*diff(y(x),x)-y(x)=(x^2+y(x)^2),y(x), singsol=all)
```

$$y(x) = \tan\left(x + c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.18 (sec). Leaf size: 12

```
DSolve[x*y'[x]-y[x]==(x^2+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \tan(x + c_1)$$

4.21 problem Problem 3.34

Internal problem ID [5895]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.6 Summary and Problems. Page 218
Problem number: Problem 3.34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y'x - y - x\sqrt{x^2 - y^2}\, y' = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x)-y(x)=x*sqrt(x^2-y(x)^2)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) - \arctan\left(\frac{y(x)}{\sqrt{x^2 - y\left(x\right)^2}}\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.51 (sec). Leaf size: 29

```
DSolve[x*y'[x]-y[x]==x*Sqrt[x^2-y[x]^2]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\arctan\left(\frac{\sqrt{x^2 - y(x)^2}}{y(x)}\right) + y(x) = c_1, y(x)\right]$$

4.22 problem Problem 3.35

Internal problem ID [5896]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.6 Summary and Problems. Page 218
Problem number: Problem 3.35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{yy' - y'x + y = -x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x+y(x)*diff(y(x),x)+y(x)-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \tan\left(\text{RootOf}\left(-2\_Z + \ln\left(\frac{1}{\cos(\_Z)^2}\right) + 2\ln(x) + 2c_1\right)\right) x$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 36

```
DSolve[x+y[x]*y'[x]+y[x]-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\log\left(\frac{y(x)^2}{x^2} + 1\right) - \arctan\left(\frac{y(x)}{x}\right) = -\log(x) + c_1, y(x)\right]$$

4.23 problem Problem 3.38

Internal problem ID [5897]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 3. Ordinary Differential Equations. Section 3.6 Summary and Problems. Page 218
Problem number: Problem 3.38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _with_potential_symmet`

$$\boxed{yy'' - y'^2 - y^2 y' = 0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 32

```
dsolve(y(x)*diff(y(x),x$2)-(diff(y(x),x))^2-y(x)^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = -\frac{c_1 \mathrm{e}^{c_1 c_2} \mathrm{e}^{c_1 x}}{-1 + \mathrm{e}^{c_1 c_2} \mathrm{e}^{c_1 x}}$$

✓ Solution by Mathematica

Time used: 1.53 (sec). Leaf size: 43

```
DSolve[y[x]*y''[x]-(y'[x])^2-y[x]^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{c_1 e^{c_1 (x + c_2)}}{-1 + e^{c_1 (x + c_2)}}$$
$$y(x) \to -\frac{1}{x + c_2}$$

5 Chapter 5. Systems of First Order Differential Equations. Section 5.11 Problems. Page 360

5.1 problem Problem 5.1

Internal problem ID [5898]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 5. Systems of First Order Differential Equations. Section 5.11 Problems. Page 360
Problem number: Problem 5.1.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 3x_1(t) - 18x_2(t)$$
$$x_2'(t) = 2x_1(t) - 9x_2(t)$$

With initial conditions

$$[x_1(0) = 2, x_2(0) = 1]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 29

```
dsolve([diff(x__1(t),t) = 3*x__1(t)-18*x__2(t), diff(x__2(t),t) = 2*x__1(t)-9*x__2(t), x__1(
```

$$x_1(t) = \frac{\mathrm{e}^{-3t}(-12t+4)}{2}$$

$$x_2(t) = \mathrm{e}^{-3t}(-2t+1)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 30

```
DSolve[{x1'[t]==3*x1[t]-18*x2[t],x2'[t]==2*x1[t]-9*x2[t]},{x1[0]==2,x2[0]==1},{x1[t],x2[t]},
```

$$\text{x1}(t) \to e^{-3t}(2-6t)$$
$$\text{x2}(t) \to e^{-3t}(1-2t)$$

5.2 problem Problem 5.2

Internal problem ID [5899]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 5. Systems of First Order Differential Equations. Section 5.11 Problems. Page 360
Problem number: Problem 5.2.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) + 3x_2(t)$$
$$x_2'(t) = 5x_1(t) + 3x_2(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 36

```
dsolve([diff(x__1(t),t)=x__1(t)+3*x__2(t),diff(x__2(t),t)=5*x__1(t)+3*x__2(t)],[x__1(t), x__
```

$$x_1(t) = \frac{3c_1 \mathrm{e}^{6t}}{5} - \mathrm{e}^{-2t} c_2$$

$$x_2(t) = c_1 \mathrm{e}^{6t} + \mathrm{e}^{-2t} c_2$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 74

```
DSolve[{x1'[t]==x1[t]+3*x2[t],x2'[t]==5*x1[t]+3*x2[t]},{x1[t],x2[t]},t,IncludeSingularSoluti
```

$$\mathrm{x1}(t) \to \frac{1}{8} e^{-2t} \left(c_1 \left(3e^{8t} + 5 \right) + 3c_2 \left(e^{8t} - 1 \right) \right)$$
$$\mathrm{x2}(t) \to \frac{1}{8} e^{-2t} \left(5c_1 \left(e^{8t} - 1 \right) + c_2 \left(5e^{8t} + 3 \right) \right)$$

5.3 problem Problem 5.3

Internal problem ID [5900]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 5. Systems of First Order Differential Equations. Section 5.11 Problems. Page 360
Problem number: Problem 5.3.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -x_1(t) + 3x_2(t)$$
$$x_2'(t) = -3x_1(t) + 5x_2(t)$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 2]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 29

```
dsolve([diff(x__1(t),t) = -x__1(t)+3*x__2(t), diff(x__2(t),t) = -3*x__1(t)+5*x__2(t), x__1(0
```

$$x_1(t) = \frac{e^{2t}(9t+3)}{3}$$

$$x_2(t) = e^{2t}(3t+2)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[{x1'[t]==-x1[t]+3*x2[t],x2'[t]==-3*x1[t]+5*x2[t]},{x1[0]==1,x2[0]==2},{x1[t],x2[t]},t
```

$$\text{x1}(t) \to e^{2t}(3t+1)$$
$$\text{x2}(t) \to e^{2t}(3t+2)$$

5.4 problem Problem 5.4

Internal problem ID [5901]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 5. Systems of First Order Differential Equations. Section 5.11 Problems. Page 360
Problem number: Problem 5.4.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 4x_1(t) - x_2(t)$$
$$x_2'(t) = 5x_1(t) + 2x_2(t)$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 59

```
dsolve([diff(x__1(t),t)=4*x__1(t)-x__2(t),diff(x__2(t),t)=5*x__1(t)+2*x__2(t)],[x__1(t), x__
```

$$x_1(t) = \frac{e^{3t}(\sin(2t)\,c_1 - 2\sin(2t)\,c_2 + 2\cos(2t)\,c_1 + \cos(2t)\,c_2)}{5}$$

$$x_2(t) = e^{3t}(\sin(2t)\,c_1 + \cos(2t)\,c_2)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 70

```
DSolve[{x1'[t]==4*x1[t]-x2[t],x2'[t]==5*x1[t]+2*x2[t]},{x1[t],x2[t]},t,IncludeSingularSoluti
```

$$\text{x1}(t) \to \frac{1}{2}e^{3t}(2c_1\cos(2t) + (c_1 - c_2)\sin(2t))$$
$$\text{x2}(t) \to \frac{1}{2}e^{3t}(2c_2\cos(2t) + (5c_1 - c_2)\sin(2t))$$

5.5 problem Problem 5.6

Internal problem ID [5902]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 5. Systems of First Order Differential Equations. Section 5.11 Problems. Page 360
Problem number: Problem 5.6.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -2x_1(t) + x_2(t)$$
$$x_2'(t) = x_1(t) - 2x_2(t)$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 35

```
dsolve([diff(x__1(t),t)=-2*x__1(t)+x__2(t),diff(x__2(t),t)=x__1(t)-2*x__2(t)],[x__1(t), x__2
```

$$x_1(t) = c_1 \mathrm{e}^{-t} - c_2 \mathrm{e}^{-3t}$$

$$x_2(t) = c_1 \mathrm{e}^{-t} + c_2 \mathrm{e}^{-3t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 68

```
DSolve[{x1'[t]==-2*x1[t]+x2[t],x2'[t]==x1[t]-2*x2[t]},{x1[t],x2[t]},t,IncludeSingularSolutio
```

$$\mathrm{x1}(t) \to \frac{1}{2} e^{-3t} \left(c_1 \left(e^{2t} + 1 \right) + c_2 \left(e^{2t} - 1 \right) \right)$$
$$\mathrm{x2}(t) \to \frac{1}{2} e^{-3t} \left(c_1 \left(e^{2t} - 1 \right) + c_2 \left(e^{2t} + 1 \right) \right)$$

5.6 problem Problem 5.7

Internal problem ID [5903]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 5. Systems of First Order Differential Equations. Section 5.11 Problems. Page 360
Problem number: Problem 5.7.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -2x_1(t) + x_2(t) + 2\,\mathrm{e}^{-t}$$
$$x_2'(t) = x_1(t) - 2x_2(t) + 3t$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 65

```
dsolve([diff(x__1(t),t)=-2*x__1(t)+x__2(t)+2*exp(-t),diff(x__2(t),t)=x__1(t)-2*x__2(t)+3*t],
```

$$x_1(t) = c_2\mathrm{e}^{-t} - c_1\mathrm{e}^{-3t} + \mathrm{e}^{-t}t + \frac{\mathrm{e}^{-t}}{2} - \frac{4}{3} + t$$

$$x_2(t) = c_2\mathrm{e}^{-t} + c_1\mathrm{e}^{-3t} + \mathrm{e}^{-t}t + 2t - \frac{5}{3} - \frac{\mathrm{e}^{-t}}{2}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 93

```
DSolve[{x1'[t]==-2*x1[t]+x2[t]+2*Exp[-t],x2'[t]==x1[t]-2*x2[t]+3*t},{x1[t],x2[t]},t,IncludeS
```

$$\mathrm{x1}(t) \to \frac{1}{6}\left(6t + 3(c_1 - c_2)e^{-3t} + 3e^{-t}(2t + 1 + c_1 + c_2) - 8\right)$$
$$\mathrm{x2}(t) \to \frac{1}{6}e^{-3t}\left(2e^{3t}(6t - 5) + 3e^{2t}(2t - 1 + c_1 + c_2) - 3c_1 + 3c_2\right)$$

5.7 problem Problem 5.8

Internal problem ID [5904]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 5. Systems of First Order Differential Equations. Section 5.11 Problems. Page 360
Problem number: Problem 5.8.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 3x_1(t) - x_2(t)$$
$$x_2'(t) = 16x_1(t) - 5x_2(t)$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 1]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 29

```
dsolve([diff(x__1(t),t) = 3*x__1(t)-x__2(t), diff(x__2(t),t) = 16*x__1(t)-5*x__2(t), x__1(0)
```

$$x_1(t) = \frac{\mathrm{e}^{-t}(48t + 16)}{16}$$

$$x_2(t) = \mathrm{e}^{-t}(12t + 1)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[{x1'[t]==3*x1[t]-x2[t],x2'[t]==16*x1[t]-5*x2[t]},{x1[0]==1,x2[0]==1},{x1[t],x2[t]},t,
```

$$\text{x1}(t) \to e^{-t}(3t + 1)$$
$$\text{x2}(t) \to e^{-t}(12t + 1)$$

5.8 problem Problem 5.9

Internal problem ID [5905]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 5. Systems of First Order Differential Equations. Section 5.11 Problems. Page 360
Problem number: Problem 5.9.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) - 2x_2(t)$$
$$x_2'(t) = 3x_1(t) - 4x_2(t)$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 34

```
dsolve([diff(x__1(t),t) = x__1(t)-2*x__2(t), diff(x__2(t),t) = 3*x__1(t)-4*x__2(t), x__1(0)
```

$$x_1(t) = -2\,\mathrm{e}^{-2t} + 3\,\mathrm{e}^{-t}$$

$$x_2(t) = -3\,\mathrm{e}^{-2t} + 3\,\mathrm{e}^{-t}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 33

```
DSolve[{x1'[t]==x1[t]-2*x2[t],x2'[t]==3*x1[t]-4*x2[t]},{x1[0]==1,x2[0]==0},{x1[t],x2[t]},t,I
```

$$\text{x1}(t) \to e^{-2t}\left(3e^{t} - 2\right)$$
$$\text{x2}(t) \to 3e^{-2t}\left(e^{t} - 1\right)$$

5.9 problem Problem 5.10

Internal problem ID [5906]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 5. Systems of First Order Differential Equations. Section 5.11 Problems. Page 360
Problem number: Problem 5.10.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 3x_1(t) - 18x_2(t)$$
$$x_2'(t) = 2x_1(t) - 9x_2(t)$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve([diff(x__1(t),t) = 3*x__1(t)-18*x__2(t), diff(x__2(t),t) = 2*x__1(t)-9*x__2(t), x__1(
```

$$x_1(t) = \frac{\mathrm{e}^{-3t}(-60t + 2)}{2}$$

$$x_2(t) = \mathrm{e}^{-3t}(-10t + 2)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[{x1'[t]==3*x1[t]-18*x2[t],x2'[t]==2*x1[t]-9*x2[t]},{x1[0]==1,x2[0]==2},{x1[t],x2[t]},
```

$$\text{x1}(t) \to e^{-3t}(1 - 30t)$$
$$\text{x2}(t) \to e^{-3t}(2 - 10t)$$

5.10 problem Problem 5.11

Internal problem ID [5907]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 5. Systems of First Order Differential Equations. Section 5.11 Problems. Page 360
Problem number: Problem 5.11.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -x_1(t) + 3x_2(t)$$
$$x_2'(t) = -3x_1(t) + 5x_2(t)$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 2]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve([diff(x__1(t),t) = -x__1(t)+3*x__2(t), diff(x__2(t),t) = -3*x__1(t)+5*x__2(t), x__1(0
```

$$x_1(t) = \frac{e^{2t}(9t+3)}{3}$$

$$x_2(t) = e^{2t}(3t+2)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[{x1'[t]==-x1[t]+3*x2[t],x2'[t]==-3*x1[t]+5*x2[t]},{x1[0]==1,x2[0]==2},{x1[t],x2[t]},t
```

$$\text{x1}(t) \to e^{2t}(3t+1)$$
$$\text{x2}(t) \to e^{2t}(3t+2)$$

5.11 problem Problem 5.12

Internal problem ID [5908]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 5. Systems of First Order Differential Equations. Section 5.11 Problems. Page 360
Problem number: Problem 5.12.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 3x_1(t) - 18x_2(t)$$
$$x_2'(t) = 2x_1(t) - 9x_2(t)$$

With initial conditions

$$[x_1(0) = 2, x_2(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve([diff(x__1(t),t) = 3*x__1(t)-18*x__2(t), diff(x__2(t),t) = 2*x__1(t)-9*x__2(t), x__1(
```

$$x_1(t) = \frac{\mathrm{e}^{-3t}(-12t+4)}{2}$$

$$x_2(t) = \mathrm{e}^{-3t}(-2t+1)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[{x1'[t]==3*x1[t]-18*x2[t],x2'[t]==2*x1[t]-9*x2[t]},{x1[0]==2,x2[0]==1},{x1[t],x2[t]},
```

$$\text{x1}(t) \to e^{-3t}(2-6t)$$
$$\text{x2}(t) \to e^{-3t}(1-2t)$$

5.12 problem Problem 5.13

Internal problem ID [5909]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 5. Systems of First Order Differential Equations. Section 5.11 Problems. Page 360
Problem number: Problem 5.13.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 3x_1(t) - x_2(t)$$
$$x_2'(t) = 4x_1(t) - 2x_2(t)$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 1]$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 16

```
dsolve([diff(x__1(t),t) = 3*x__1(t)-x__2(t), diff(x__2(t),t) = 4*x__1(t)-2*x__2(t), x__1(0)
```

$$x_1(t) = \mathrm{e}^{2t}$$

$$x_2(t) = \mathrm{e}^{2t}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 18

```
DSolve[{x1'[t]==3*x1[t]-x2[t],x2'[t]==4*x1[t]-2*x2[t]},{x1[0]==1,x2[0]==1},{x1[t],x2[t]},t,I
```

$$\mathrm{x1}(t) \to e^{2t}$$
$$\mathrm{x2}(t) \to e^{2t}$$

5.13 problem Problem 5.15 part 1

Internal problem ID [5910]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 5. Systems of First Order Differential Equations. Section 5.11 Problems. Page 360
Problem number: Problem 5.15 part 1.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) + x_2(t) - 8$$
$$x_2'(t) = x_1(t) + x_2(t) + 3$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 35

```
dsolve([diff(x__1(t),t)=x__1(t)+x__2(t)-8,diff(x__2(t),t)=x__1(t)+x__2(t)+3],[x__1(t), x__2(
```

$$x_1(t) = \frac{c_1 \mathrm{e}^{2t}}{2} + \frac{5}{2} - \frac{11t}{2} - c_2$$

$$x_2(t) = \frac{c_1 \mathrm{e}^{2t}}{2} + \frac{11t}{2} + c_2$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 74

```
DSolve[{x1'[t]==x1[t]+x2[t]-8,x2'[t]==x1[t]+x2[t]+3},{x1[t],x2[t]},t,IncludeSingularSolution
```

$$\mathrm{x1}(t) \to \frac{1}{4}\left(-22t + 2c_1\left(e^{2t}+1\right) + 2c_2 e^{2t} + 5 - 2c_2\right)$$
$$\mathrm{x2}(t) \to \frac{1}{4}\left(22t + 2c_1\left(e^{2t}-1\right) + 2c_2 e^{2t} + 5 + 2c_2\right)$$

5.14 problem Problem 5.15 part 3

Internal problem ID [5911]

Book: THEORY OF DIFFERENTIAL EQUATIONS IN ENGINEERING AND MECHANICS. K.T. CHAU, CRC Press. Boca Raton, FL. 2018
Section: Chapter 5. Systems of First Order Differential Equations. Section 5.11 Problems. Page 360
Problem number: Problem 5.15 part 3.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) + x_2(t) - 8$$
$$x_2'(t) = x_1(t) + x_2(t) + 3$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 2]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 30

```
dsolve([diff(x__1(t),t) = x__1(t)+x__2(t)-8, diff(x__2(t),t) = x__1(t)+x__2(t)+3, x__1(0) =
```

$$x_1(t) = \frac{e^{2t}}{4} + \frac{3}{4} - \frac{11t}{2}$$

$$x_2(t) = \frac{e^{2t}}{4} + \frac{11t}{2} + \frac{7}{4}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 36

```
DSolve[{x1'[t]==x1[t]+x2[t]-8,x2'[t]==x1[t]+x2[t]+3},{x1[0]==1,x2[0]==2},{x1[t],x2[t]},t,Inc
```

$$\text{x1}(t) \to \frac{1}{4}\left(-22t + e^{2t} + 3\right)$$
$$\text{x2}(t) \to \frac{1}{4}\left(22t + e^{2t} + 7\right)$$