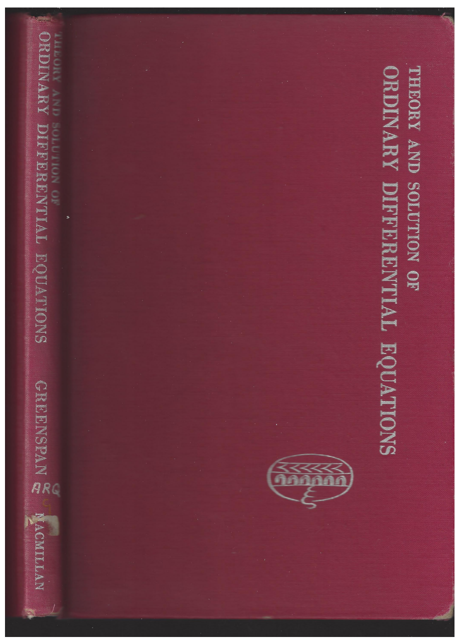
A Solution Manual For

Theory and solutions of Ordinary Differential equations, Donald Greenspan, 1960



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1 Exercises, page 14

1.1 problem 1(a)

Internal problem ID [3002]

Book: Theory and solutions of Ordinary Differential equations, Donald Greenspan, 1960
Section: Exercises, page 14
Problem number: 1(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \mathrm{e}^{-x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=exp(-x),y(x), singsol=all)
```

$$y(x) = -\mathrm{e}^{-x} + c_1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 15

```
DSolve[y'[x]==Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{-x} + c_1$$

1.2 problem 1(b)

Internal problem ID [3003]

Book: Theory and solutions of Ordinary Differential equations, Donald Greenspan, 1960
Section: Exercises, page 14
Problem number: 1(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = 1 - x^5 + \sqrt{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=1-x^5+sqrt(x),y(x), singsol=all)
```

$$y(x) = \frac{2x^{\frac{3}{2}}}{3} - \frac{x^6}{6} + x + c_1$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 25

```
DSolve[y'[x]==1-x^5+Sqrt[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x^{3/2}}{3} - \frac{x^6}{6} + x + c_1$$

1.3 problem 1(c)

Internal problem ID [3004]

Book: Theory and solutions of Ordinary Differential equations, Donald Greenspan, 1960
Section: Exercises, page 14
Problem number: 1(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{3y + (3x - 2)\, y' = 2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve((3*y(x)-2*x)+(3*x-2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x^2 + c_1}{-2 + 3x}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 21

```
DSolve[(3*y[x]-2*x)+(3*x-2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2 - c_1}{3x - 2}$$

1.4 problem 1(d)

Internal problem ID [3005]

Book: Theory and solutions of Ordinary Differential equations, Donald Greenspan, 1960
Section: Exercises, page 14
Problem number: 1(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$(2yx + y)\, y' = -x^2 - x + 1$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 55

```
dsolve((x^2+x-1)+(2*x*y(x)+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2x^2 + 5\ln(1+2x) + 4c_1 - 2x}}{2}$$
$$y(x) = \frac{\sqrt{-2x^2 + 5\ln(1+2x) + 4c_1 - 2x}}{2}$$

✓ Solution by Mathematica

Time used: 0.119 (sec). Leaf size: 73

```
DSolve[(x^2+x-1)+(2*x*y[x]+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}\sqrt{-2x^2 - 2x + 5\log(2x+1) - \frac{1}{2} + 8c_1}$$
$$y(x) \to \frac{1}{2}\sqrt{-2x^2 - 2x + 5\log(2x+1) - \frac{1}{2} + 8c_1}$$

1.5 problem 1(e)

Internal problem ID [3006]

Book: Theory and solutions of Ordinary Differential equations, Donald Greenspan, 1960
Section: Exercises, page 14
Problem number: 1(e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{e^{2y} + (x+1)\,y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(exp(2*y(x))+(1+x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\ln(2\ln(x+1) + 2c_1)}{2}$$

✓ Solution by Mathematica

Time used: 0.376 (sec). Leaf size: 21

```
DSolve[Exp[2*y[x]]+(1+x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}\log(2(\log(x+1) - c_1))$$

1.6 problem 1(f)

Internal problem ID [3007]

Book: Theory and solutions of Ordinary Differential equations, Donald Greenspan, 1960
Section: Exercises, page 14
Problem number: 1(f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x+1)\,y' - y^2x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve((x+1)*diff(y(x),x)-x^2*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = -\frac{2}{x^2 + 2\ln(x+1) - 2c_1 - 2x}$$

✓ Solution by Mathematica

Time used: 0.162 (sec). Leaf size: 32

```
DSolve[(x+1)*y'[x]-x^2*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{x^2 - 2x + 2\log(x+1) - 3 + 2c_1}$$
$$y(x) \to 0$$

1.7 problem 1(g)

Internal problem ID [3008]

Book: Theory and solutions of Ordinary Differential equations, Donald Greenspan, 1960
Section: Exercises, page 14
Problem number: 1(g).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{-2x + y}{x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=(y(x)-2*x)/x,y(x), singsol=all)
```

$$y(x) = (-2\ln(x) + c_1)\, x$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 14

```
DSolve[y'[x]==(y[x]-2*x)/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(-2\log(x) + c_1)$$

1.8 problem 1(h)

Internal problem ID [3009]

Book: Theory and solutions of Ordinary Differential equations, Donald Greenspan, 1960
Section: Exercises, page 14
Problem number: 1(h).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y^3 - y'y^2x = -x^3}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 74

```
dsolve((x^3+y(x)^3)-x*y(x)^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = (3\ln(x) + c_1)^{\frac{1}{3}}\, x$$
$$y(x) = \left(-\frac{(3\ln(x) + c_1)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}\,(3\ln(x) + c_1)^{\frac{1}{3}}}{2}\right) x$$
$$y(x) = \left(-\frac{(3\ln(x) + c_1)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}\,(3\ln(x) + c_1)^{\frac{1}{3}}}{2}\right) x$$

✓ Solution by Mathematica

Time used: 0.193 (sec). Leaf size: 63

```
DSolve[(x^3+y[x]^3)-x*y[x]^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\sqrt[3]{3\log(x) + c_1}$$
$$y(x) \to -\sqrt[3]{-1}x\sqrt[3]{3\log(x) + c_1}$$
$$y(x) \to (-1)^{2/3}x\sqrt[3]{3\log(x) + c_1}$$

1.9 problem 1(i)

Internal problem ID [3010]

Book: Theory and solutions of Ordinary Differential equations, Donald Greenspan, 1960
Section: Exercises, page 14
Problem number: 1(i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 18

```
DSolve[y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x}$$
$$y(x) \to 0$$

1.10 problem 1(j)

Internal problem ID [3011]

Book: Theory and solutions of Ordinary Differential equations, Donald Greenspan, 1960
Section: Exercises, page 14
Problem number: 1(j).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = x^2 + 2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)+y(x)=x^2+2,y(x), singsol=all)
```

$$y(x) = x^2 - 2x + 4 + c_1 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.068 (sec). Leaf size: 21

```
DSolve[y'[x]+y[x]==x^2+2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 - 2x + c_1 e^{-x} + 4$$

1.11 problem 2(a)

Internal problem ID [3012]

Book: Theory and solutions of Ordinary Differential equations, Donald Greenspan, 1960
Section: Exercises, page 14
Problem number: 2(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - y\tan(x) = x}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)-y(x)*tan(x)=x,y(0) = 0],y(x), singsol=all)
```

$$y(x) = 1 + \tan(x)\, x - \sec(x)$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 15

```
DSolve[{y'[x]-y[x]*Tan[x]==x,y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\tan(x) - \sec(x) + 1$$

1.12 problem 2(b)

Internal problem ID [3013]

Book: Theory and solutions of Ordinary Differential equations, Donald Greenspan, 1960
Section: Exercises, page 14
Problem number: 2(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - e^{x-2y} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)=exp(x-2*y(x)),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{\ln(2\,e^{x} - 1)}{2}$$

✓ Solution by Mathematica

Time used: 0.824 (sec). Leaf size: 17

```
DSolve[{y'[x]==Exp[x-2*y[x]],y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\log(2e^{x} - 1)$$

1.13 problem 2(c)

Internal problem ID [3014]

Book: Theory and solutions of Ordinary Differential equations, Donald Greenspan, 1960
Section: Exercises, page 14
Problem number: 2(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Riccati]`

$$\boxed{y' - \frac{y^2 + x^2}{2x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=(x^2+y(x)^2)/(2*x^2),y(x), singsol=all)
```

$$y(x) = \frac{x(\ln(x) + c_1 - 2)}{\ln(x) + c_1}$$

✓ Solution by Mathematica

Time used: 0.149 (sec). Leaf size: 29

```
DSolve[y'[x]==(x^2+y[x]^2)/(2*x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x(\log(x) - 2 + 2c_1)}{\log(x) + 2c_1}$$
$$y(x) \to x$$

1.14 problem 2(d)

Internal problem ID [3015]

Book: Theory and solutions of Ordinary Differential equations, Donald Greenspan, 1960
Section: Exercises, page 14
Problem number: 2(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' - y = x}$$

With initial conditions

$$[y(-1) = -1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve([x*diff(y(x),x)=x+y(x),y(-1) = -1],y(x), singsol=all)
```

$$y(x) = -(i\pi - \ln(x) - 1)\,x$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 16

```
DSolve[{x*y'[x]==x+y[x],y[-1]==-1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(\log(x) - i\pi + 1)$$

1.15 problem 2(e)

Internal problem ID [3016]

Book: Theory and solutions of Ordinary Differential equations, Donald Greenspan, 1960
Section: Exercises, page 14
Problem number: 2(e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{e^{-y} + (x^2 + 1)\, y' = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 11

```
dsolve([exp(-y(x))+(1+x^2)*diff(y(x),x)=0,y(0) = 0],y(x), singsol=all)
```

$$y(x) = \ln(-\arctan(x) + 1)$$

✓ Solution by Mathematica

Time used: 0.391 (sec). Leaf size: 12

```
DSolve[{Exp[-y[x]]+(1+x^2)*y'[x]==0,y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(1 - \arctan(x))$$

1.16 problem 2(f)

Internal problem ID [3017]

Book: Theory and solutions of Ordinary Differential equations, Donald Greenspan, 1960
Section: Exercises, page 14
Problem number: 2(f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \sin(x)\,e^{x}}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve([diff(y(x),x)=exp(x)*sin(x),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{1}{2} + \frac{e^{x}(-\cos(x) + \sin(x))}{2}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 24

```
DSolve[{y'[x]==Exp[x]*Sin[x],y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(e^{x}\sin(x) - e^{x}\cos(x) + 1)$$

1.17 problem 2(g)

Internal problem ID [3018]

Book: Theory and solutions of Ordinary Differential equations, Donald Greenspan, 1960
Section: Exercises, page 14
Problem number: 2(g).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - 3y = e^{3x} + e^{-3x}}$$

With initial conditions

$$[y(5) = 5]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 31

```
dsolve([diff(y(x),x)-3*y(x)=exp(3*x)+exp(-3*x),y(5) = 5],y(x), singsol=all)
```

$$y(x) = \frac{e^{3x-30}}{6} + 5\,e^{3x-15} + (-5+x)\,e^{3x} - \frac{e^{-3x}}{6}$$

✓ Solution by Mathematica

Time used: 0.077 (sec). Leaf size: 48

```
DSolve[{y'[x]-3*y[x]==Exp[3*x]+Exp[-3*x],y[5]==5},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}e^{-3(x+10)}\left(6e^{6(x+5)}(x-5) + e^{6x} + 30e^{6x+15} - e^{30}\right)$$

1.18 problem 2(h)

Internal problem ID [3019]

Book: Theory and solutions of Ordinary Differential equations, Donald Greenspan, 1960
Section: Exercises, page 14
Problem number: 2(h).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = x + \frac{1}{x}}$$

With initial conditions

$$[y(-2) = 5]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 21

```
dsolve([diff(y(x),x)=x+1/x,y(-2) = 5],y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} + \ln(x) + 3 - \ln(2) - i\pi$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 25

```
DSolve[{y'[x]==x+1/x,y[-2]==5},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2} + \log\left(\frac{x}{2}\right) - i\pi + 3$$

1.19 problem 2(i)

Internal problem ID [3020]

Book: Theory and solutions of Ordinary Differential equations, Donald Greenspan, 1960
Section: Exercises, page 14
Problem number: 2(i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' + 2y = (3x+2)\,\mathrm{e}^{3x}}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve([x*diff(y(x),x)+2*y(x)=(3*x+2)*exp(3*x),y(1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{3x}x^2 - \mathrm{e}^3 + 1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.086 (sec). Leaf size: 22

```
DSolve[{x*y'[x]+2*y[x]==(3*x+2)*Exp[3*x],y[1]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^3}{x^2} + \frac{1}{x^2} + e^{3x}$$

1.20 problem 2(j)

Internal problem ID [3021]

Book: Theory and solutions of Ordinary Differential equations, Donald Greenspan, 1960
Section: Exercises, page 14
Problem number: 2(j).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2\sin(3x)\sin(2y)\,y'-3\cos(3x)\cos(2y)=0}$$

With initial conditions

$$\left[y\left(\frac{\pi}{12}\right)=\frac{\pi}{8}\right]$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 19

```
dsolve([2*sin(3*x)*sin(2*y(x))*diff(y(x),x)-3*cos(3*x)*cos(2*y(x))=0,y(1/12*Pi) = 1/8*Pi],y(
```

$$y(x)=\frac{\operatorname{arccot}\left(\frac{1}{\sqrt{-4\cos(3x)^2+3}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 6.727 (sec). Leaf size: 18

```
DSolve[{2*Sin[3*x]*Sin[2*y[x]]*y'[x]-3*Cos[3*x]*Cos[2*y[x]]==0,y[Pi/12]==Pi/8},y[x],x,Includ
```

$$y(x)\to\frac{1}{2}\arccos\left(\frac{1}{2}\csc(3x)\right)$$

1.21 problem 2(k)

Internal problem ID [3022]

Book: Theory and solutions of Ordinary Differential equations, Donald Greenspan, 1960
Section: Exercises, page 14
Problem number: 2(k).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'yx - (x+1)(y+1) = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 21

```
dsolve([x*y(x)*diff(y(x),x)=(x+1)*(y(x)+1),y(1) = 1],y(x), singsol=all)
```

$$y(x) = -\operatorname{LambertW}\left(-1, -\frac{2\,\mathrm{e}^{-x-1}}{x}\right) - 1$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x*y[x]*y'[x]==(x+1)*(y[x]+1),y[1]==1},y[x],x,IncludeSingularSolutions -> True]
```

{}

1.22 problem 2(L)

Internal problem ID [3023]

Book: Theory and solutions of Ordinary Differential equations, Donald Greenspan, 1960
Section: Exercises, page 14
Problem number: 2(L).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$y' - \frac{2x-y}{y+2x} = 0$$

With initial conditions

$$[y(2) = 2]$$

✓ Solution by Maple

Time used: 0.735 (sec). Leaf size: 66

```
dsolve([diff(y(x),x)=(2*x-y(x))/(2*x+y(x)),y(2) = 2],y(x), singsol=all)
```

$$y(x) = \mathrm{RootOf}\left(2\sqrt{17}\,\mathrm{arctanh}\left(\frac{(3x+2\_Z)\sqrt{17}}{17x}\right) - 2\sqrt{17}\,\mathrm{arctanh}\left(\frac{5\sqrt{17}}{17}\right) - 17\ln\left(\frac{\_Z^2+3\_Zx-2x^2}{x^2}\right) + 51\ln(2) - 34\ln(x)\right)$$

✓ Solution by Mathematica

Time used: 0.126 (sec). Leaf size: 137

```
DSolve[{y'[x]==(2*x-y[x])/(2*x+y[x]),y[2]==2},y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{34}\left(\left(17+\sqrt{17}\right)\log\left(-\frac{2y(x)}{x}+\sqrt{17}-3\right)-\left(\sqrt{17}-17\right)\log\left(\frac{2y(x)}{x}+\sqrt{17}+3\right)\right)=-\log(x)+\frac{1}{34}i\left(17+\sqrt{17}\right)\pi+\frac{1}{34}\left(34\log(2)+17\log\left(5-\sqrt{17}\right)+\sqrt{17}\log\left(5-\sqrt{17}\right)+17\log\left(5+\sqrt{17}\right)-\sqrt{17}\log\left(5+\sqrt{17}\right)\right),y(x)\right]$$

1.23 problem 2(m)

Internal problem ID [3024]

Book: Theory and solutions of Ordinary Differential equations, Donald Greenspan, 1960
Section: Exercises, page 14
Problem number: 2(m).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$y' - \frac{3x - y + 1}{3y - x + 5} = 0$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 1.75 (sec). Leaf size: 84

```
dsolve([diff(y(x),x)=(3*x-y(x)+1)/(3*y(x)-x+5),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{\left(-324 + 12\sqrt{96x^3 + 288x^2 + 288x + 825}\right)^{\frac{4}{3}} - 12\left(-324 + 12\sqrt{96x^3 + 288x^2 + 288x + 825}\right)^{\frac{2}{3}} x - 84(-3}{36\left(-324 + 12\sqrt{96x^3 + 288x^2 + 288x + 82}\right.}$$

✓ Solution by Mathematica

Time used: 60.775 (sec). Leaf size: 341

```
DSolve[{y'[x]==(3*x-y[x]+1)/(3*y[x]-x+5),y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x\text{Root}\left[\#1^6(1024x^6 + 6144x^5 + 15360x^4 + 20480x^3 + 15360x^2 + 6144x - 58025) + \#1^4(-384x^4 - 153\right.}{}$$

1.24 problem 2(n)

Internal problem ID [3025]

Book: Theory and solutions of Ordinary Differential equations, Donald Greenspan, 1960
Section: Exercises, page 14
Problem number: 2(n).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$3y + (7y - 3x + 3) y' = 7x - 7$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.844 (sec). Leaf size: 5735

```
dsolve([(3*y(x)-7*x+7)+(7*y(x)-3*x+3)*diff(y(x),x)=0,y(0) = 0],y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 88.015 (sec). Leaf size: 1602

```
DSolve[{(3*y[x]-7*x+7)+(7*y[x]-3*x+3)*y'[x]==0,y[0]==0},y[x],x,IncludeSingularSolutions -> T
```

Too large to display

1.25 problem 2(o)

Internal problem ID [3026]

Book: Theory and solutions of Ordinary Differential equations, Donald Greenspan, 1960
Section: Exercises, page 14
Problem number: 2(o).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(2-x+2y)\,y'-xy(y'-1)=-x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x+(2-x+2*y(x))*diff(y(x),x)=x*y(x)*(diff(y(x),x)-1),y(x), singsol=all)
```

$$y(x)=-1$$
$$y(x)=x+2\ln(x-2)+c_1$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 20

```
DSolve[x+(2-x+2*y[x])*y'[x]==x*y[x]*(y'[x]-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -1$$
$$y(x)\to x+2\log(x-2)+c_1$$

1.26 problem 2(p)

Internal problem ID [3027]

Book: Theory and solutions of Ordinary Differential equations, Donald Greenspan, 1960
Section: Exercises, page 14
Problem number: 2(p).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\cos(x) y' + y \sin(x) = 1}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 6

```
dsolve([diff(y(x),x)*cos(x)+y(x)*sin(x)=1,y(0) = 0],y(x), singsol=all)
```

$$y(x) = \sin(x)$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 7

```
DSolve[{y'[x]*Cos[x]+y[x]*Sin[x]==1,y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x)$$

1.27 problem 2(q)

Internal problem ID [3028]

Book: Theory and solutions of Ordinary Differential equations, Donald Greenspan, 1960
Section: Exercises, page 14
Problem number: 2(q).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$(x + y^2)\, y' + y = x^2$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 56

```
dsolve([(x+y(x)^2)*diff(y(x),x)+(y(x)-x^2)=0,y(1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\left(12 + 4x^3 + 4\sqrt{x^6 + 10x^3 + 9}\right)^{\frac{2}{3}} - 4x}{2\left(12 + 4x^3 + 4\sqrt{x^6 + 10x^3 + 9}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 3.931 (sec). Leaf size: 66

```
DSolve[{(x+y[x]^2)*y'[x]+(y[x]-x^2)==0,y[1]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{x^3 + \sqrt{x^6 + 10x^3 + 9} + 3}}{\sqrt[3]{2}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{x^3 + \sqrt{x^6 + 10x^3 + 9} + 3}}$$