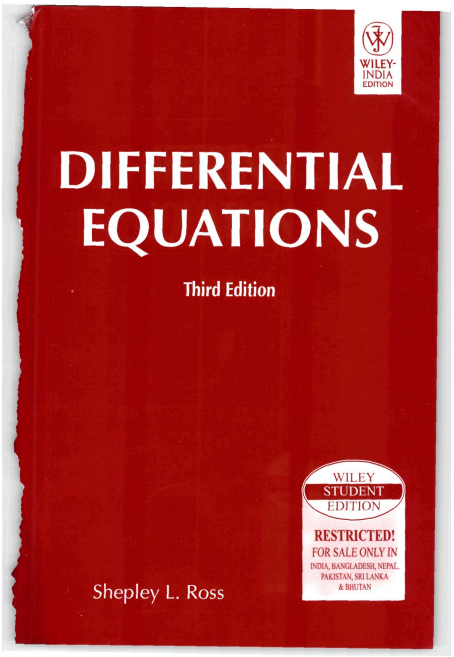
A Solution Manual For

Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.



Nasser M. Abbasi

March 3, 2024

Contents

1 Chapter 1, Differential equations and their solutions. Exercises page 13

1.1 problem 1(a)

Internal problem ID [11580]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 1, Differential equations and their solutions. Exercises page 13
Problem number: 1(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = x + 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)+y(x)=1+x,y(x), singsol=all)
```

$$y(x) = x + \mathrm{e}^{-x} c_1$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 15

```
DSolve[y'[x]+y[x]==1+x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + c_1 e^{-x}$$

1.2 problem 1(b)

Internal problem ID [11581]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 1, Differential equations and their solutions. Exercises page 13
Problem number: 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 7y' + 12y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-7*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{3x} c_1 + c_2 \mathrm{e}^{4x}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 20

```
DSolve[y''[x]-7*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{3x}(c_2 e^{x} + c_1)$$

1.3 problem 1(c)

Internal problem ID [11582]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 1, Differential equations and their solutions. Exercises page 13
Problem number: 1(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 3y' + 2y = 4x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=4*x^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2x} c_1 + c_2 \mathrm{e}^{x} + 2x^2 + 6x + 7$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 29

```
DSolve[y''[x]-3*y'[x]+2*y[x]==4*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x^2 + 6x + c_1 e^{x} + c_2 e^{2x} + 7$$

1.4 problem 1(d)

Internal problem ID [11583]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 1, Differential equations and their solutions. Exercises page 13
Problem number: 1(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{\left(x^2+1\right)y''+4xy'+2y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve((1+x^2)*diff(y(x),x$2)+4*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{xc_1+c_2}{x^2+1}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 20

```
DSolve[(1+x^2)*y''[x]+4*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_2x+c_1}{x^2+1}$$

1.5 problem 2(a)

Internal problem ID [11584]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 1, Differential equations and their solutions. Exercises page 13
Problem number: 2(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, _Bernoulli]`

$$\boxed{2xyy' + y^2 = -x^2}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 49

```
dsolve(2*x*y(x)*diff(y(x),x)+x^2+y(x)^2=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{3}\sqrt{x\left(-x^3+3c_1\right)}}{3x}$$
$$y(x) = \frac{\sqrt{3}\sqrt{x\left(-x^3+3c_1\right)}}{3x}$$

✓ Solution by Mathematica

Time used: 0.377 (sec). Leaf size: 60

```
DSolve[2*x*y[x]*y'[x]+x^2+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-x^3+3c_1}}{\sqrt{3}\sqrt{x}}$$
$$y(x) \to \frac{\sqrt{-x^3+3c_1}}{\sqrt{3}\sqrt{x}}$$

1.6 problem 2(b)

Internal problem ID [11585]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 1, Differential equations and their solutions. Exercises page 13
Problem number: 2(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$\boxed{xy' + y - y^3x^3 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 30

```
dsolve(x*diff(y(x),x)+y(x)=x^3*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{-2x + c_1}\, x}$$
$$y(x) = -\frac{1}{\sqrt{-2x + c_1}\, x}$$

✓ Solution by Mathematica

Time used: 0.6 (sec). Leaf size: 44

```
DSolve[x*y'[x]+y[x]==x^3*y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{x^2(-2x + c_1)}}$$
$$y(x) \to \frac{1}{\sqrt{x^2(-2x + c_1)}}$$
$$y(x) \to 0$$

1.7 problem 3(a)

Internal problem ID [11586]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 1, Differential equations and their solutions. Exercises page 13
Problem number: 3(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + 3y = 3x^2 \mathrm{e}^{-3x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+3*y(x)=3*x^2*exp(-3*x),y(x), singsol=all)
```

$$y(x) = \left(x^3 + c_1\right) \mathrm{e}^{-3x}$$

✓ Solution by Mathematica

Time used: 0.098 (sec). Leaf size: 17

```
DSolve[y'[x]+3*y[x]==3*x^2*Exp[-3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(x^3 + c_1\right)$$

1.8 problem 3(b)

Internal problem ID [11587]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 1, Differential equations and their solutions. Exercises page 13
Problem number: 3(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + 4yx = 8x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+4*x*y(x)=8*x,y(x), singsol=all)
```

$$y(x) = 2 + \mathrm{e}^{-2x^2} c_1$$

✓ Solution by Mathematica

Time used: 0.074 (sec). Leaf size: 22

```
DSolve[y'[x]+4*x*y[x]==8*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2 + c_1 e^{-2x^2}$$
$$y(x) \to 2$$

1.9 problem 4(a)

Internal problem ID [11588]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 1, Differential equations and their solutions. Exercises page 13
Problem number: 4(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' - 8y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-8*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{4x} + c_2 \mathrm{e}^{-2x}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 22

```
DSolve[y''[x]-2*y'[x]-8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(c_2 e^{6x} + c_1\right)$$

1.10 problem 4(b)

Internal problem ID [11589]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 1, Differential equations and their solutions. Exercises page 13
Problem number: 4(b).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 2y'' - 4y' + 8y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$3)-2*diff(y(x),x$2)-4*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-2x} c_1 + c_2 \mathrm{e}^{2x} + c_3 \mathrm{e}^{2x} x$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 27

```
DSolve[y'''[x]-2*y''[x]-4*y'[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x} \left(e^{4x} (c_3 x + c_2) + c_1 \right)$$

1.11 problem 5(a)

Internal problem ID [11590]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 1, Differential equations and their solutions. Exercises page 13
Problem number: 5(a).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 3y'' - 4y' + 12y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)-4*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{3x} c_1 + c_2 \mathrm{e}^{-2x} + c_3 \mathrm{e}^{2x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 29

```
DSolve[y'''[x]-3*y''[x]-4*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(e^{4x}(c_3 e^{x} + c_2) + c_1\right)$$

1.12 problem 5(b)

Internal problem ID [11591]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 1, Differential equations and their solutions. Exercises page 13
Problem number: 5(b).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _fully, _exact, _linear]]`

$$x^3y''' + 2x^2y'' - 10y'x - 8y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(x^3*diff(y(x),x$3)+2*x^2*diff(y(x),x$2)-10*x*diff(y(x),x)-8*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + c_2x^4 + \frac{c_3}{x^2}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 22

```
DSolve[x^3*y'''[x]+2*x^2*y''[x]-10*x*y'[x]-8*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{c_3x^6 + c_2x + c_1}{x^2}$$

1.13 problem 6(a)

Internal problem ID [11592]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 1, Differential equations and their solutions. Exercises page 13
Problem number: 6(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + 2y = 6\,\mathrm{e}^{x} + 4x\,\mathrm{e}^{-2x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x)+2*y(x)=6*exp(x)+4*x*exp(-2*x),y(x), singsol=all)
```

$$y(x) = \left(2x^2 + 2\,\mathrm{e}^{3x} + c_1\right)\mathrm{e}^{-2x}$$

✓ Solution by Mathematica

Time used: 0.129 (sec). Leaf size: 26

```
DSolve[y'[x]+2*y[x]==6*Exp[x]+4*x*Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(2x^2 + 2e^{3x} + c_1\right)$$

1.14 problem 6(b)

Internal problem ID [11593]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 1, Differential equations and their solutions. Exercises page 13
Problem number: 6(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 4y = -8\sin(2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=-8*sin(2*x),y(x), singsol=all)
```

$$y(x) = c_2 \mathrm{e}^{2x} + x\,\mathrm{e}^{2x} c_1 - \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 25

```
DSolve[y''[x]-4*y'[x]+4*y[x]==-8*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\cos(2x) + e^{2x}(c_2 x + c_1)$$

1.15 problem 7(a)

Internal problem ID [11594]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 1, Differential equations and their solutions. Exercises page 13
Problem number: 7(a).
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - 4y = 0}$$

✓ Solution by Maple

Time used: 0.171 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)^2-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = c_1^2 - 2xc_1 + x^2$$

✓ Solution by Mathematica

Time used: 0.079 (sec). Leaf size: 38

```
DSolve[(y'[x])^2-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}(-2x + c_1)^2$$
$$y(x) \to \frac{1}{4}(2x + c_1)^2$$
$$y(x) \to 0$$

2 Chapter 1, section 1.3. Exercises page 22

2.1 problem 1

Internal problem ID [11595]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 1, section 1.3. Exercises page 22
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - 6y = 0}$$

With initial conditions

$$[y(0) = 6, y'(0) = 2]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)+diff(y(x),x)-6*y(x)=0,y(0) = 6, D(y)(0) = 2],y(x), singsol=all)
```

$$y(x) = \left(4\,\mathrm{e}^{5x} + 2\right)\mathrm{e}^{-3x}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 20

```
DSolve[{y''[x]+y'[x]-6*y[x]==0,{y[0]==6,y'[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(4e^{5x} + 2\right)$$

2.2 problem 2(a)

Internal problem ID [11596]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 1, section 1.3. Exercises page 22
Problem number: 2(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = 2x\,\mathrm{e}^{-x}}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)+y(x)=2*x*exp(-x),y(0) = 2],y(x), singsol=all)
```

$$y(x) = \left(x^2 + 2\right)\mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.087 (sec). Leaf size: 16

```
DSolve[{y'[x]+y[x]==2*x*Exp[-x],{y[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(x^2 + 2\right)$$

2.3 problem 2(b)

Internal problem ID [11597]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 1, section 1.3. Exercises page 22
Problem number: 2(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = 2x\,\mathrm{e}^{-x}}$$

With initial conditions

$$[y(-1) = \mathrm{e} + 3]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 17

```
dsolve([diff(y(x),x)+y(x)=2*x*exp(-x),y(-1) = exp(1)+3],y(x), singsol=all)
```

$$y(x) = \left(x^2 + 3\,\mathrm{e}^{-1}\right)\mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.086 (sec). Leaf size: 20

```
DSolve[{y'[x]+y[x]==2*x*Exp[-x],{y[-1]==Exp[1]+3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x-1}\left(ex^2 + 3\right)$$

2.4 problem 3(a)

Internal problem ID [11598]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 1, section 1.3. Exercises page 22
Problem number: 3(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' - 12y = 0}$$

With initial conditions

$$[y(0) = 5, y'(0) = 6]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)-diff(y(x),x)-12*y(x)=0,y(0) = 5, D(y)(0) = 6],y(x), singsol=all)
```

$$y(x) = \left(3\,\mathrm{e}^{7x} + 2\right)\mathrm{e}^{-3x}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 20

```
DSolve[{y''[x]-y'[x]-12*y[x]==0,{y[0]==5,y'[0]==6}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(3e^{7x} + 2\right)$$

2.5 problem 4(a)

Internal problem ID [11599]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 1, section 1.3. Exercises page 22
Problem number: 4(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

With initial conditions

$$\left[y(0) = 0, y'\left(\frac{\pi}{2}\right) = 1\right]$$

✗ Solution by Maple

```
dsolve([diff(y(x),x$2)+y(x)=0,y(0) = 0, D(y)(1/2*Pi) = 1],y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y''[x]+y[x]==0,{y[0]==0,y'[Pi/2]==1}},y[x],x,IncludeSingularSolutions -> True]
```

{}

2.6 problem 4(b)

Internal problem ID [11600]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 1, section 1.3. Exercises page 22
Problem number: 4(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

With initial conditions

$$\left[y(0) = 0, y'\left(\frac{\pi}{2}\right) = -1\right]$$

✗ Solution by Maple

```
dsolve([diff(y(x),x$2)+y(x)=0,y(0) = 0, D(y)(1/2*Pi) = -1],y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y''[x]+y[x]==0,{y[0]==0,y'[Pi/2]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

{}

2.7 problem 4(c)

Internal problem ID [11601]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 1, section 1.3. Exercises page 22
Problem number: 4(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

With initial conditions

$$[y(0) = 0, y'(\pi) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 8

```
dsolve([diff(y(x),x$2)+y(x)=0,y(0) = 0, D(y)(Pi) = 1],y(x), singsol=all)
```

$$y(x) = -\sin(x)$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 9

```
DSolve[{y''[x]+y[x]==0,{y[0]==0,y'[Pi]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sin(x)$$

2.8 problem 5

Internal problem ID [11602]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 1, section 1.3. Exercises page 22
Problem number: 5.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''' - 3x^2y'' + 6y'x - 6y = 0}$$

With initial conditions

$$[y(2) = 0, y'(2) = 2, y''(2) = 6]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve([x^3*diff(y(x),x$3)-3*x^2*diff(y(x),x$2)+6*x*diff(y(x),x)-6*y(x)=0,y(2) = 0, D(y)(2)
```

$$y(x) = x^3 - 3x^2 + 2x$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 15

```
DSolve[{x^3*y'''[x]-3*x^2*y''[x]+6*x*y'[x]-6*y[x]==0,{y[2]==0,y'[2]==2,y''[2]==6}},y[x],x,In
```

$$y(x) \to x\left(x^2 - 3x + 2\right)$$

2.9 problem 6(a)

Internal problem ID [11603]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 1, section 1.3. Exercises page 22
Problem number: 6(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - x^2 \sin(y) = 0}$$

With initial conditions

$$[y(1) = -2]$$

✓ Solution by Maple

Time used: 2.719 (sec). Leaf size: 97

```
dsolve([diff(y(x),x)=x^2*sin(y(x)),y(1) = -2],y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{2\sin(2)\,\mathrm{e}^{\frac{(x-1)\left(x^2+x+1\right)}{3}}}{(-1+\cos(2))\,\mathrm{e}^{\frac{2(x-1)\left(x^2+x+1\right)}{3}} - \cos(2) - 1}, \frac{(-\cos(2)+1)\,\mathrm{e}^{\frac{2(x-1)\left(x^2+x+1\right)}{3}} - \cos(2) - 1}{(-1+\cos(2))\,\mathrm{e}^{\frac{2(x-1)\left(x^2+x+1\right)}{3}} - \cos(2) - 1}\right)$$

✓ Solution by Mathematica

Time used: 0.68 (sec). Leaf size: 23

```
DSolve[{y'[x]==x^2*Sin[y[x]],{y[1]==-2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arccos\left(\tanh\left(\operatorname{arctanh}(\cos(2)) - \frac{x^3}{3} + \frac{1}{3}\right)\right)$$

2.10 problem 6(b)

Internal problem ID [11604]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 1, section 1.3. Exercises page 22
Problem number: 6(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{y^2}{x-2} = 0}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=y(x)^2/(x-2),y(1) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 6

```
DSolve[{y'[x]==y[x]^2/(x-2),{y[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

2.11 problem 8

Internal problem ID [11605]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 1, section 1.3. Exercises page 22
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^{\frac{1}{3}} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=y(x)^(1/3),y(0) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 21

```
DSolve[{y'[x]==y[x]^(1/3),{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{3}\sqrt{\frac{2}{3}}x^{3/2}$$

3 Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37

3.1 problem 1

Internal problem ID [11606]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, [_Abel, '2nd ty`

$$\boxed{2y + (2x + y)\,y' = -3x}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 49

```
dsolve((3*x+2*y(x))+(2*x+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-2xc_1 - \sqrt{c_1^2x^2 + 1}}{c_1}$$
$$y(x) = \frac{-2xc_1 + \sqrt{c_1^2x^2 + 1}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.781 (sec). Leaf size: 79

```
DSolve[(3*x+2*y[x])+(2*x+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2x - \sqrt{x^2 + e^{2c_1}}$$
$$y(x) \to -2x + \sqrt{x^2 + e^{2c_1}}$$
$$y(x) \to -\sqrt{x^2} - 2x$$
$$y(x) \to \sqrt{x^2} - 2x$$

3.2 problem 2

Internal problem ID [11607]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]`

$$\boxed{y^2 + (2yx - 4)\, y' = -3}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve((y(x)^2+3)+(2*x*y(x)-4)*diff(y(x),x)=0,y(x), singsol=all)
```

$$c_1 + \frac{1}{\left(i\sqrt{3} - y\left(x\right)\right)\left(i\left(xy\left(x\right) - 4\right)\sqrt{3} - 3x\right)} = 0$$

✓ Solution by Mathematica

Time used: 0.615 (sec). Leaf size: 79

```
DSolve[(y[x]^2+3)+(2*x*y[x]-4)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2 - \sqrt{-3x^2 + c_1 x + 4}}{x}$$
$$y(x) \to \frac{2 + \sqrt{-3x^2 + c_1 x + 4}}{x}$$
$$y(x) \to -i\sqrt{3}$$
$$y(x) \to i\sqrt{3}$$

3.3 problem 3

Internal problem ID [11608]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x),G(x)]']`

$$\boxed{2yx + \left(x^2 + 4y\right) y' = -1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve((2*x*y(x)+1)+(x^2+4*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{4} - \frac{\sqrt{x^4 - 8c_1 - 8x}}{4}$$
$$y(x) = -\frac{x^2}{4} + \frac{\sqrt{x^4 - 8c_1 - 8x}}{4}$$

✓ Solution by Mathematica

Time used: 0.223 (sec). Leaf size: 61

```
DSolve[(2*x*y[x]+1)+(x^2+4*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-x^2 - \sqrt{x^4 - 8x + 16c_1}\right)$$
$$y(x) \to \frac{1}{4}\left(-x^2 + \sqrt{x^4 - 8x + 16c_1}\right)$$

3.4 problem 4

Internal problem ID [11609]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{3yx^2 - \left(x^3 + y\right) y' = -2}$$

✗ Solution by Maple

```
dsolve((3*x^2*y(x)+2)-(x^3+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(3*x^2+2)-(x^3+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

3.5 problem 5

Internal problem ID [11610]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{6yx + 2y^2 + \left(3x^2 + 4yx - 6\right) y' = 5}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 67

```
dsolve((6*x*y(x)+2*y(x)^2-5)+(3*x^2+4*x*y(x)-6)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-3x^2 + 6 + \sqrt{9x^4 - 8xc_1 + 4x^2 + 36}}{4x}$$
$$y(x) = -\frac{3x^2 + \sqrt{9x^4 - 8xc_1 + 4x^2 + 36} - 6}{4x}$$

✓ Solution by Mathematica

Time used: 0.709 (sec). Leaf size: 79

```
DSolve[(6*x*y[x]+2*y[x]^2-5)+(3*x^2+4*x*y[x]-6)*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{3x^2 + \sqrt{9x^4 + 4x^2 + 16c_1x + 36} - 6}{4x}$$
$$y(x) \to \frac{-3x^2 + \sqrt{9x^4 + 4x^2 + 16c_1x + 36} + 6}{4x}$$

3.6 problem 7

Internal problem ID [11611]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_Abel, '2nd type', 'class A']]`

$$\boxed{y \sec(x)^2 + (\tan(x) + 2y)\, y' = -\sec(x) \tan(x)}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 63

```
dsolve((y(x)*sec(x)^2+sec(x)*tan(x))+(tan(x)+2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sin(x) - \sqrt{-4\cos(x)^2 c_1 + \sin(x)^2 - 4\cos(x)}}{2\cos(x)}$$
$$y(x) = -\frac{\sin(x) + \sqrt{-4\cos(x)^2 c_1 + \sin(x)^2 - 4\cos(x)}}{2\cos(x)}$$

✓ Solution by Mathematica

Time used: 1.831 (sec). Leaf size: 101

```
DSolve[(y[x]*Sec[x]^2+Sec[x]*Tan[x])+(Tan[x]+2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{1}{4}\left(-2\tan(x) - \sqrt{2}\sqrt{\sec^2(x)}\sqrt{-8\cos(x) + (-1 + 4c_1)\cos(2x) + 1 + 4c_1}\right)$$
$$y(x) \to \frac{1}{4}\left(-2\tan(x) + \sqrt{\sec^2(x)}\sqrt{-16\cos(x) + (-2 + 8c_1)\cos(2x) + 2 + 8c_1}\right)$$

3.7 problem 8

Internal problem ID [11612]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{\frac{x}{y^2}+\left(\frac{x^2}{y^3}+y\right)y'=-x}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 47

```
dsolve((x/y(x)^2+x)+(x^2/y(x)^3+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$\frac{y(x)^2x^2}{2y(x)^2+2}+\frac{y(x)^2}{2}-\frac{1}{2\left(y(x)^2+1\right)}-\ln\left(y(x)^2+1\right)+c_1=0$$

✓ Solution by Mathematica

Time used: 0.4 (sec). Leaf size: 55

```
DSolve[(x/y[x]^2+x)+(x^2/y[x]^3+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{x^2y(x)^2}{2\left(y(x)^2+1\right)}+\frac{y(x)^2}{2}-\frac{1}{2\left(y(x)^2+1\right)}-\log\left(y(x)^2+1\right)=c_1,y(x)\right]$$

3.8 problem 9

Internal problem ID [11613]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\frac{(2s-1)\,s'}{t}+\frac{s-s^2}{t^2}=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 31

```
dsolve((2*s(t)-1)/t*diff(s(t),t)+(s(t)-s(t)^2)/t^2=0,s(t), singsol=all)
```

$$s(t)=\frac{1}{2}-\frac{\sqrt{4c_1t+1}}{2}$$
$$s(t)=\frac{1}{2}+\frac{\sqrt{4c_1t+1}}{2}$$

✓ Solution by Mathematica

Time used: 0.682 (sec). Leaf size: 59

```
DSolve[(2*s[t]-1)/t*s'[t]+(s[t]-s[t]^2)/t^2==0,s[t],t,IncludeSingularSolutions -> True]
```

$$s(t) \to \frac{1}{2}\left(1 - \sqrt{1 - 4e^{c_1} t}\right)$$

$$s(t) \to \frac{1}{2}\left(1 + \sqrt{1 - 4e^{c_1} t}\right)$$

$$s(t) \to 0$$

$$s(t) \to 1$$

3.9 problem 10

Internal problem ID [11614]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\frac{2y^{\frac{3}{2}}+1}{x^{\frac{1}{3}}}+\left(3\sqrt{x}\,\sqrt{y}-1\right)y'=0$$

✗ Solution by Maple

```
dsolve((2*y(x)^(3/2)+1)/x^(1/3)+(3*x^(1/2)*y(x)^(1/2)-1)*diff(y(x),x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(2*y[x]^(3/2)+1)/x^(1/3)+(3*x^(1/2)*y[x]^(1/2)-1)*y'[x]==0,y[x],x,IncludeSingularSolu
```

Timed out

3.10 problem 11

Internal problem ID [11615]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x),G(x)]']`

$$\boxed{2yx + \left(x^2 + 4y\right) y' = 3}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 22

```
dsolve([(2*x*y(x)-3)+(x^2+4*y(x))*diff(y(x),x)=0,y(1) = 2],y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{4} + \frac{\sqrt{x^4 + 24x + 56}}{4}$$

✓ Solution by Mathematica

Time used: 0.218 (sec). Leaf size: 27

```
DSolve[{(2*x*y[x]-3)+(x^2+4*y[x])*y'[x]==0,{y[1]==2}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{4}\left(\sqrt{x^4 + 24x + 56} - x^2\right)$$

3.11 problem 12

Internal problem ID [11616]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{3y^2x^2 - y^3 + \left(2yx^3 - 3y^2x + 1\right)y' = -2x}$$

With initial conditions

$$[y(-2) = 1]$$

✗ Solution by Maple

```
dsolve([(3*x^2*y(x)^2-y(x)^3+2*x)+(2*x^3*y(x)-3*x*y(x)^2+1)*diff(y(x),x)=0,y(-2) = 1],y(x),
```

No solution found

✓ Solution by Mathematica

Time used: 60.368 (sec). Leaf size: 250

```
DSolve[{(3*x^2*y[x]^2-y[x]^3+2*x)+(2*x^3*y[x]-3*x*y[x]^2+1)*y'[x]==0,{y[-2]==1}},y[x],x,Incl
```

$$y(x)$$
$$\to \frac{2\sqrt[3]{2}\left(1-i\sqrt{3}\right)x^6 + 4\sqrt[3]{-2x^9-36x^4-27x^2+3\sqrt{3}\sqrt{x^3\left(4x^{10}+4x^8+44x^5+72x^3+27x-4\right)}}x^3 + (1}{12x\sqrt[3]{-2x^9-36x^4-27x^2+3\sqrt{3}\sqrt{x}}$$

3.12 problem 13

Internal problem ID [11617]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_Abel, '2nd type', 'class B']]`

$$\boxed{2y\sin(x)\cos(x)+y^2\sin(x)+\left(\sin(x)^2-2y\cos(x)\right)y'=0}$$

With initial conditions

$$[y(0)=3]$$

✓ Solution by Maple

Time used: 0.359 (sec). Leaf size: 24

```
dsolve([(2*y(x)*sin(x)*cos(x)+y(x)^2*sin(x))+(sin(x)^2-2*y(x)*cos(x))*diff(y(x),x)=0,y(0) =
```

$$y(x)=\frac{\left(\sin(x)^2+\sqrt{\sin(x)^4+36\cos(x)}\right)\sec(x)}{2}$$

✓ Solution by Mathematica

Time used: 2.029 (sec). Leaf size: 34

```
DSolve[{(2*y[x]*Sin[x]*Cos[x]+y[x]^2*Sin[x])+(Sin[x]^2-2*y[x]*Cos[x])*y'[x]==0,{y[0]==3}},y[
```

$$y(x)\to\frac{1}{4}\sec(x)\left(-\cos(2x)+2\sqrt{\sin^4(x)+36\cos(x)}+1\right)$$

3.13 problem 14

Internal problem ID [11618]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_Abel, '2nd type', 'class B']]`

$$\boxed{y\,\mathrm{e}^{x}+y^{2}+\left(\mathrm{e}^{x}+2yx\right)y'=-2\,\mathrm{e}^{x}}$$

With initial conditions

$$[y(0)=6]$$

✓ Solution by Maple

Time used: 0.437 (sec). Leaf size: 29

```
dsolve([(y(x)*exp(x)+2*exp(x)+y(x)^2)+(exp(x)+2*x*y(x))*diff(y(x),x)=0,y(0) = 6],y(x), sings
```

$$y(x)=\frac{-\mathrm{e}^{x}+\sqrt{\mathrm{e}^{2x}-8x\,\mathrm{e}^{x}+32x}}{2x}$$

✓ Solution by Mathematica

Time used: 32.264 (sec). Leaf size: 37

```
DSolve[{(y[x]*Exp[x]+2*Exp[x]+y[x]^2)+(Exp[x]+2*x*y[x])*y'[x]==0,{y[0]==6}},y[x],x,IncludeSi
```

$$y(x)\to\frac{\sqrt{-8e^{x}x+32x+e^{2x}}-e^{x}}{2x}$$

3.14 problem 15

Internal problem ID [11619]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]`

$$\boxed{\frac{3-y}{x^2}+\frac{\left(y^2-2x\right)y'}{y^2x}=0}$$

With initial conditions

$$[y(-1)=2]$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 21

```
dsolve([(3-y(x))/x^2+((y(x)^2-2*x)/(x*y(x)^2))*diff(y(x),x)=0,y(-1) = 2],y(x), singsol=all)
```

$$y(x)=x+\frac{3}{2}+\frac{\sqrt{4x^2+4x+9}}{2}$$

✓ Solution by Mathematica

Time used: 1.961 (sec). Leaf size: 28

```
DSolve[{(3-y[x])/x^2+( (y[x]^2-2*x)/(x*y[x]^2) )*y'[x]==0,{y[-1]==2}},y[x],x,IncludeSingular
```

$$y(x)\to\frac{1}{2}\left(\sqrt{4x^2+4x+9}+2x+3\right)$$

3.15 problem 16

Internal problem ID [11620]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _exact, _rational]`

$$\boxed{\frac{1+8xy^{\frac{2}{3}}}{x^{\frac{2}{3}}y^{\frac{1}{3}}}+\frac{\left(2x^{\frac{4}{3}}y^{\frac{2}{3}}-x^{\frac{1}{3}}\right)y'}{y^{\frac{4}{3}}}=0}$$

With initial conditions

$$[y(1)=8]$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 55

```
dsolve([(1+8*x*y(x)^(2/3))/(x^(2/3)*y(x)^(1/3))+((2*x^(4/3)*y(x)^(2/3)-x^(1/3))/(y(x)^(4/3))
```

$$y(x)=\text{RootOf}\left(64\_Z^{\frac{7}{3}}x^4+96\_Z^{\frac{5}{3}}x^3-729\_Z^{\frac{4}{3}}+48x^2\_Z+8x\_Z^{\frac{1}{3}}\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{(1+8*x*y[x]^(2/3))/(x^(2/3)*y[x]^(1/3))+((2*x^(4/3)*y[x]^(2/3)-x^(1/3))/(y[x]^(4/3))
```

{}

3.16 problem 21

Internal problem ID [11621]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$\boxed{3y^2 + 2xyy' = -4x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 38

```
dsolve((4*x+3*y(x)^2)+(2*x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{x\left(-x^4 + c_1\right)}}{x^2}$$
$$y(x) = -\frac{\sqrt{x\left(-x^4 + c_1\right)}}{x^2}$$

✓ Solution by Mathematica

Time used: 0.349 (sec). Leaf size: 46

```
DSolve[(4*x+3*y[x]^2)+(2*x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-x^4 + c_1}}{x^{3/2}}$$
$$y(x) \to \frac{\sqrt{-x^4 + c_1}}{x^{3/2}}$$

3.17 problem 22

Internal problem ID [11622]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{y^2 + 2yx - x^2 y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((y(x)^2+2*x*y(x))-x^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{-x + c_1}$$

✓ Solution by Mathematica

Time used: 0.221 (sec). Leaf size: 23

```
DSolve[(y[x]^2+2*x*y[x])-x^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2}{x - c_1}$$
$$y(x) \to 0$$

3.18 problem 24

Internal problem ID [11623]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$\boxed{y + x\left(x^2+y^2\right)^2 + \left(y\left(x^2+y^2\right)^2 - x\right)y' = 0}$$

✓ Solution by Maple

Time used: 0.218 (sec). Leaf size: 39

```
dsolve((y(x)+x*(x^2+y(x)^2)^2)+(y(x)*(x^2+y(x)^2)^2-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\tan\left(\text{RootOf}\left(-\tan\left(\_Z\right)^4 x^4 - 2\tan\left(\_Z\right)^2 x^4 - x^4 + 4c_1 - 4\_Z\right)\right)x$$

✓ Solution by Mathematica

Time used: 0.144 (sec). Leaf size: 40

```
DSolve[(y[x]+x*(x^2+y[x]^2)^2)+(y[x]*(x^2+y[x]^2)^2-x)*y'[x]==0,y[x],x,IncludeSingularSoluti
```

$$\text{Solve}\left[\arctan\left(\frac{x}{y(x)}\right) + \frac{x^4}{4} + \frac{1}{2}x^2 y(x)^2 + \frac{y(x)^4}{4} = c_1, y(x)\right]$$

4 Chapter 2, section 2.2 (Separable equations). Exercises page 47

4.1 problem 1

Internal problem ID [11624]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.2 (Separable equations). Exercises page 47
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{4yx + \left(x^2+1\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve((4*x*y(x))+(x^2+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\left(x^2+1\right)^2}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 20

```
DSolve[(4*x*y[x])+(x^2+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{\left(x^2+1\right)^2}$$
$$y(x) \to 0$$

4.2 problem 2

Internal problem ID [11625]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.2 (Separable equations). Exercises page 47
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yx + y + \left(x^2 + 2x\right) y' = -2x - 2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve((x*y(x)+2*x+y(x)+2)+(x^2+2*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -2 + \frac{c_1}{\sqrt{x\left(x+2\right)}}$$

✓ Solution by Mathematica

Time used: 0.149 (sec). Leaf size: 27

```
DSolve[(x*y[x]+2*x+y[x]+2)+(x^2+2*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2 + \frac{c_1}{\sqrt{x}\sqrt{x+2}}$$
$$y(x) \to -2$$

4.3 problem 3

Internal problem ID [11626]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.2 (Separable equations). Exercises page 47
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2r\left(s^2+1\right)+\left(r^4+1\right)s'=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(2*r*(s(r)^2+1)+(r^4+1)*diff(s(r),r)=0,s(r), singsol=all)
```

$$s(r)=-\tan\left(\arctan\left(r^2\right)+2c_1\right)$$

✓ Solution by Mathematica

Time used: 0.478 (sec). Leaf size: 31

```
DSolve[2*r*(s[r]^2+1)+(r^4+1)*s'[r]==0,s[r],r,IncludeSingularSolutions -> True]
```

$$s(r)\to-\tan\left(\arctan\left(r^2\right)-c_1\right)$$
$$s(r)\to-i$$
$$s(r)\to i$$

4.4 problem 4

Internal problem ID [11627]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.2 (Separable equations). Exercises page 47
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\csc(y) + y' \sec(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(csc(y(x))+sec(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \arccos(\sin(x) + c_1)$$

✓ Solution by Mathematica

Time used: 0.696 (sec). Leaf size: 27

```
DSolve[Csc[y[x]]+Sec[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arccos(\sin(x) - c_1)$$
$$y(x) \to \arccos(\sin(x) - c_1)$$

4.5 problem 5

Internal problem ID [11628]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.2 (Separable equations). Exercises page 47
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\tan(\theta) + 2r\theta' = 0}$$

✓ Solution by Maple

Time used: 0.469 (sec). Leaf size: 21

```
dsolve(tan(theta(r))+2*r*diff(theta(r),r)=0,theta(r), singsol=all)
```

$$\theta(r) = \arcsin\left(\frac{1}{\sqrt{c_1 r}}\right)$$
$$\theta(r) = -\arcsin\left(\frac{1}{\sqrt{c_1 r}}\right)$$

✓ Solution by Mathematica

Time used: 15.319 (sec). Leaf size: 21

```
DSolve[Tan[theta[r]]+2*r*theta'[r]==0,theta[r],r,IncludeSingularSolutions -> True]
```

$$\theta(r) \to \arcsin\left(\frac{e^{c_1}}{\sqrt{r}}\right)$$
$$\theta(r) \to 0$$

4.6 problem 6

Internal problem ID [11629]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.2 (Separable equations). Exercises page 47
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(\mathrm{e}^{v}+1)\cos(u)+\mathrm{e}^{v}(1+\sin(u))\,v'=0}$$

✓ Solution by Maple

Time used: 0.922 (sec). Leaf size: 28

```
dsolve((exp(v(u))+1)*cos(u) + exp(v(u))*(1+sin(u))*diff(v(u),u)=0,v(u), singsol=all)
```

$$v(u)=-\ln\left(-\frac{1+\sin(u)}{-1+\sin(u)\,\mathrm{e}^{c_1}+\mathrm{e}^{c_1}}\right)-c_1$$

✓ Solution by Mathematica

Time used: 5.457 (sec). Leaf size: 37

```
DSolve[(Exp[v[u]]+1)*Cos[u] + Exp[v[u]]*(1+Sin[u])*v'[u]==0,v[u],u,IncludeSingularSolutions
```

$$v(u)\to\log\left(-1+\frac{e^{c_1}}{\left(\sin\left(\frac{u}{2}\right)+\cos\left(\frac{u}{2}\right)\right)^2}\right)$$
$$v(u)\to i\pi$$

4.7 problem 7

Internal problem ID [11630]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.2 (Separable equations). Exercises page 47
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(4+x)\left(y^2+1\right)+y\left(x^2+3x+2\right)y'=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 122

```
dsolve((x+4)*(y(x)^2+1) + y(x)*(x^2+3*x+2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-x^6+c_1x^4-6x^5+8c_1x^3+24x^2c_1+100x^3+32xc_1+345x^2+16c_1+474x+239}}{(x+1)^3}$$

$$y(x) = -\frac{\sqrt{-x^6+c_1x^4-6x^5+8c_1x^3+24x^2c_1+100x^3+32xc_1+345x^2+16c_1+474x+239}}{(x+1)^3}$$

✓ Solution by Mathematica

Time used: 5.501 (sec). Leaf size: 126

```
DSolve[(x+4)*(y[x]^2+1) + y[x]*(x^2+3*x+2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-(x+1)^6 + e^{2c_1}(x+2)^4}}{(x+1)^3}$$

$$y(x) \to \frac{\sqrt{-(x+1)^6 + e^{2c_1}(x+2)^4}}{(x+1)^3}$$

$$y(x) \to -i$$

$$y(x) \to i$$

$$y(x) \to \frac{(x+1)^3}{\sqrt{-(x+1)^6}}$$

$$y(x) \to \frac{\sqrt{-(x+1)^6}}{(x+1)^3}$$

4.8 problem 8

Internal problem ID [11631]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.2 (Separable equations). Exercises page 47
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y - xy' = -x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve((x+y(x))- x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = (\ln(x) + c_1)\, x$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 12

```
DSolve[(x+y[x])- x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(\log(x) + c_1)$$

4.9 problem 9

Internal problem ID [11632]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.2 (Separable equations). Exercises page 47
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{2yx + 3y^2 - \left(2yx + x^2\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 35

```
dsolve((2*x*y(x)+3*y(x)^2)- (2*x*y(x)+x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \left(-\frac{1}{2} - \frac{\sqrt{4xc_1 + 1}}{2}\right) x$$
$$y(x) = \left(-\frac{1}{2} + \frac{\sqrt{4xc_1 + 1}}{2}\right) x$$

✓ Solution by Mathematica

Time used: 0.618 (sec). Leaf size: 61

```
DSolve[(2*x*y[x]+3*y[x]^2)- (2*x*y[x]+x^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}x\left(1 + \sqrt{1 + 4e^{c_1}x}\right)$$
$$y(x) \to \frac{1}{2}x\left(-1 + \sqrt{1 + 4e^{c_1}x}\right)$$
$$y(x) \to 0$$
$$y(x) \to -x$$

4.10 problem 10

Internal problem ID [11633]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.2 (Separable equations). Exercises page 47
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{v^3 + \left(u^3 - uv^2\right) v' = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 21

```
dsolve(v(u)^3+ (u^3-u*v(u)^2)*diff(v(u),u)=0,v(u), singsol=all)
```

$$v(u) = \mathrm{e}^{-\frac{\mathrm{LambertW}\left(-\frac{\mathrm{e}^{-2c_1}}{u^2}\right)}{2} - c_1}$$

✓ Solution by Mathematica

Time used: 9.023 (sec). Leaf size: 56

```
DSolve[v[u]^3+ (u^3-u*v[u]^2)*v'[u]==0,v[u],u,IncludeSingularSolutions -> True]
```

$$v(u) \to -iu\sqrt{W\left(-\frac{e^{-2c_1}}{u^2}\right)}$$
$$v(u) \to iu\sqrt{W\left(-\frac{e^{-2c_1}}{u^2}\right)}$$
$$v(u) \to 0$$

4.11 problem 11

Internal problem ID [11634]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.2 (Separable equations). Exercises page 47
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{x \tan\left(\frac{y}{x}\right) + y - xy' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 10

```
dsolve((x*tan(y(x)/x)+y(x))- x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \arcsin\left(xc_1\right) x$$

✓ Solution by Mathematica

Time used: 8.002 (sec). Leaf size: 19

```
DSolve[(x*Tan[y[x]/x]+y[x])- x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \arcsin\left(e^{c_1} x\right)$$
$$y(x) \to 0$$

4.12 problem 12

Internal problem ID [11635]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.2 (Separable equations). Exercises page 47
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, _dAlembert]`

$$\boxed{\left(2s^2+2st+t^2\right)s'+s^2+2st=t^2}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 421

```
dsolve((2*s(t)^2+2*s(t)*t+t^2)*diff(s(t),t)+(s(t)^2+2*s(t)*t-t^2)=0,s(t), singsol=all)
```

$$s(t)=\frac{\frac{\left(4t^3c_1^3+2+\sqrt{17c_1^6t^6+16t^3c_1^3+4}\right)^{\frac{1}{3}}}{2}-\frac{t^2c_1^2}{2\left(4t^3c_1^3+2+\sqrt{17c_1^6t^6+16t^3c_1^3+4}\right)^{\frac{1}{3}}}-\frac{c_1t}{2}}{c_1}$$

$$s(t)$$

$$=\frac{-\frac{\left(4t^3c_1^3+2+\sqrt{17c_1^6t^6+16t^3c_1^3+4}\right)^{\frac{1}{3}}}{4}+\frac{t^2c_1^2}{4\left(4t^3c_1^3+2+\sqrt{17c_1^6t^6+16t^3c_1^3+4}\right)^{\frac{1}{3}}}-\frac{c_1t}{2}-\frac{i\sqrt{3}\left(\frac{\left(4t^3c_1^3+2+\sqrt{17c_1^6t^6+16t^3c_1^3+4}\right)^{\frac{1}{3}}}{2}+\frac{}{2\left(4t^3c_1^3+\right.}\right.}{2}}{c_1}$$

$$s(t)$$

$$=\frac{-\frac{\left(4t^3c_1^3+2+\sqrt{17c_1^6t^6+16t^3c_1^3+4}\right)^{\frac{1}{3}}}{4}+\frac{t^2c_1^2}{4\left(4t^3c_1^3+2+\sqrt{17c_1^6t^6+16t^3c_1^3+4}\right)^{\frac{1}{3}}}-\frac{c_1t}{2}+\frac{i\sqrt{3}\left(\frac{\left(4t^3c_1^3+2+\sqrt{17c_1^6t^6+16t^3c_1^3+4}\right)^{\frac{1}{3}}}{2}+\frac{}{2\left(4t^3c_1^3+\right.}\right.}{2}}{c_1}$$

✓ Solution by Mathematica

Time used: 48.03 (sec). Leaf size: 616

```
DSolve[(2*s[t]^2+2*s[t]*t+t^2)*s'[t]+(s[t]^2+2*s[t]*t-t^2)==0,s[t],t,IncludeSingularSolution
```

$$s(t) \to \frac{1}{2}\left(\sqrt[3]{4t^3+\sqrt{17t^6+16e^{3c_1}t^3+4e^{6c_1}}+2e^{3c_1}} - \frac{t^2}{\sqrt[3]{4t^3+\sqrt{17t^6+16e^{3c_1}t^3+4e^{6c_1}}+2e^{3c_1}}} - t\right)$$

$$s(t) \to \frac{1}{8}\left(2i\left(\sqrt{3}+i\right)\sqrt[3]{4t^3+\sqrt{17t^6+16e^{3c_1}t^3+4e^{6c_1}}+2e^{3c_1}} + \frac{2\left(1+i\sqrt{3}\right)t^2}{\sqrt[3]{4t^3+\sqrt{17t^6+16e^{3c_1}t^3+4e^{6c_1}}+2e^{3c_1}}} - 4t\right)$$

$$s(t) \to \frac{1}{8}\left(-2\left(1+i\sqrt{3}\right)\sqrt[3]{4t^3+\sqrt{17t^6+16e^{3c_1}t^3+4e^{6c_1}}+2e^{3c_1}} + \frac{2\left(1-i\sqrt{3}\right)t^2}{\sqrt[3]{4t^3+\sqrt{17t^6+16e^{3c_1}t^3+4e^{6c_1}}+2e^{3c_1}}} - 4t\right)$$

$$s(t) \to \frac{1}{2}\left(\sqrt[3]{\sqrt{17}\sqrt{t^6}+4t^3} - \frac{t^2}{\sqrt[3]{\sqrt{17}\sqrt{t^6}+4t^3}} - t\right)$$

$$s(t) \to \frac{1}{4}\left(\left(-1-i\sqrt{3}\right)\sqrt[3]{\sqrt{17}\sqrt{t^6}+4t^3} + \frac{\left(1-i\sqrt{3}\right)t^2}{\sqrt[3]{\sqrt{17}\sqrt{t^6}+4t^3}} - 2t\right)$$

$$s(t) \to \frac{1}{4}\left(i\left(\sqrt{3}+i\right)\sqrt[3]{\sqrt{17}\sqrt{t^6}+4t^3} + \frac{\left(1+i\sqrt{3}\right)t^2}{\sqrt[3]{\sqrt{17}\sqrt{t^6}+4t^3}} - 2t\right)$$

4.13 problem 13

Internal problem ID [11636]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.2 (Separable equations). Exercises page 47
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{y^2\sqrt{x^2+y^2}-xy\sqrt{x^2+y^2}\,y'=-x^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve((x^3+y(x)^2*sqrt(x^2+y(x)^2))-x*y(x)*sqrt(x^2+y(x)^2)*diff(y(x),x)=0,y(x), singsol=al
```

$$-\frac{\left(y(x)^2+x^2\right)^{\frac{3}{2}}}{x^3}+3\ln(x)-c_1=0$$

✓ Solution by Mathematica

Time used: 28.664 (sec). Leaf size: 265

```
DSolve[(x^3+y[x]^2*Sqrt[x^2+y[x]^2])-x*y[x]*Sqrt[x^2+y[x]^2]*y'[x]==0,y[x],x,IncludeSingular
```

$$y(x) \to -\sqrt{-x^2 - \frac{1}{2}\sqrt[6]{3}\left(\sqrt{3}+3i\right)\sqrt[3]{x^6(\log(x)+c_1)^2}}$$

$$y(x) \to \sqrt{-x^2 - \frac{1}{2}\sqrt[6]{3}\left(\sqrt{3}+3i\right)\sqrt[3]{x^6(\log(x)+c_1)^2}}$$

$$y(x) \to -\sqrt{-x^2 - \frac{1}{2}\sqrt[6]{3}\left(\sqrt{3}-3i\right)\sqrt[3]{x^6(\log(x)+c_1)^2}}$$

$$y(x) \to \sqrt{-x^2 - \frac{1}{2}\sqrt[6]{3}\left(\sqrt{3}-3i\right)\sqrt[3]{x^6(\log(x)+c_1)^2}}$$

$$y(x) \to -\sqrt{-x^2 + 3^{2/3}\sqrt[3]{x^6(\log(x)+c_1)^2}}$$

$$y(x) \to \sqrt{-x^2 + 3^{2/3}\sqrt[3]{x^6(\log(x)+c_1)^2}}$$

4.14 problem 14

Internal problem ID [11637]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.2 (Separable equations). Exercises page 47
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$\boxed{\sqrt{x+y}+\sqrt{-y+x}+\left(\sqrt{-y+x}-\sqrt{x+y}\right)y'=0}$$

✓ Solution by Maple

Time used: 3.781 (sec). Leaf size: 32

```
dsolve((sqrt(x+y(x))+sqrt(x-y(x)))+(sqrt(x-y(x))-sqrt(x+y(x)))*diff(y(x),x)=0,y(x), singsol=
```

$$\ln(x)+\ln\left(\frac{y(x)}{x}\right)-\operatorname{arctanh}\left(\frac{1}{\sqrt{-\frac{y(x)^2}{x^2}+1}}\right)-c_1=0$$

✓ Solution by Mathematica

Time used: 2.828 (sec). Leaf size: 84

```
DSolve[(Sqrt[x+y[x]]+Sqrt[x-y[x]])+(Sqrt[x-y[x]]-Sqrt[x+y[x]])*y'[x]==0,y[x],x,IncludeSingul
```

$$y(x)\to -\frac{1}{4}\left(\cosh\left(\frac{c_1}{2}\right)+\sinh\left(\frac{c_1}{2}\right)\right)\sqrt{-8ix+\cosh(c_1)+\sinh(c_1)}$$
$$y(x)\to \frac{1}{4}\left(\cosh\left(\frac{c_1}{2}\right)+\sinh\left(\frac{c_1}{2}\right)\right)\sqrt{-8ix+\cosh(c_1)+\sinh(c_1)}$$
$$y(x)\to 0$$

4.15 problem 15

Internal problem ID [11638]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.2 (Separable equations). Exercises page 47
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y + y(4+x)\,y' = -2}$$

With initial conditions

$$[y(-3) = -1]$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 18

```
dsolve([(y(x)+2)+(y(x)*(x+4))*diff(y(x),x)=0,y(-3) = -1],y(x), singsol=all)
```

$$y(x) = -2\,\mathrm{LambertW}\left(-\frac{\sqrt{x+4}\,\mathrm{e}^{-\frac{1}{2}}}{2}\right) - 2$$

✓ Solution by Mathematica

Time used: 12.779 (sec). Leaf size: 26

```
DSolve[{(y[x]+2)+(y[x]*(x+4))*y'[x]==0,{y[-3]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2\left(W\left(-\frac{\sqrt{x+4}}{2\sqrt{e}}\right) + 1\right)$$

4.16 problem 16

Internal problem ID [11639]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.2 (Separable equations). Exercises page 47
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{8\cos(y)^2+\csc(x)^2y'=0}$$

With initial conditions

$$\left[y\left(\frac{\pi}{12}\right)=\frac{\pi}{4}\right]$$

✓ Solution by Maple

Time used: 0.234 (sec). Leaf size: 20

```
dsolve([(8*cos(y(x))^2)+csc(x)^2*diff(y(x),x)=0,y(1/12*Pi) = 1/4*Pi],y(x), singsol=all)
```

$$y(x)=-\arctan\left(-\frac{\pi}{3}+4x-2\sin(2x)\right)$$

✓ Solution by Mathematica

Time used: 1.156 (sec). Leaf size: 21

```
DSolve[{(8*Cos[y[x]]^2)+Csc[x]^2*y'[x]==0,{y[Pi/12]==Pi/4}},y[x],x,IncludeSingularSolutions
```

$$y(x)\to\arctan\left(-4x+2\sin(2x)+\frac{\pi}{3}\right)$$

4.17 problem 17

Internal problem ID [11640]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.2 (Separable equations). Exercises page 47
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(3x+8)\left(y^2+4\right)-4y\left(x^2+5x+6\right)y'=0}$$

With initial conditions

$$[y(1)=2]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 21

```
dsolve([(3*x+8)*(y(x)^2+4)-4*y(x)*(x^2+5*x+6)*diff(y(x),x)=0,y(1) = 2],y(x), singsol=all)
```

$$y(x)=\frac{2\sqrt{-9+(3x+6)\sqrt{3+x}}}{3}$$

✓ Solution by Mathematica

Time used: 4.88 (sec). Leaf size: 36

```
DSolve[{(3*x+8)*(y[x]^2+4)-4*y[x]*(x^2+5*x+6)*y'[x]==0,{y[1]==2}},y[x],x,IncludeSingularSolu
```

$$y(x)\to\frac{2\sqrt{\sqrt{x+3}x+2\sqrt{x+3}-3}}{\sqrt{3}}$$

4.18 problem 18

Internal problem ID [11641]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.2 (Separable equations). Exercises page 47
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{3y^2 - 2xyy' = -x^2}$$

With initial conditions

$$[y(2) = 6]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 13

```
dsolve([(x^2+3*y(x)^2)-2*x*y(x)*diff(y(x),x)=0,y(2) = 6],y(x), singsol=all)
```

$$y(x) = \sqrt{5x - 1}\, x$$

✓ Solution by Mathematica

Time used: 0.455 (sec). Leaf size: 16

```
DSolve[{(x^2+3*y[x]^2)-2*x*y[x]*y'[x]==0,{y[2]==6}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\sqrt{5x - 1}$$

4.19 problem 19

Internal problem ID [11642]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.2 (Separable equations). Exercises page 47
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{-5y + (4x - y)\,y' = -2x}$$

With initial conditions

$$[y(1) = 4]$$

✓ Solution by Maple

Time used: 0.297 (sec). Leaf size: 35

```
dsolve([(2*x-5*y(x))+(4*x-y(x))*diff(y(x),x)=0,y(1) = 4],y(x), singsol=all)
```

$$y(x) = 6 - 2x - 6\sqrt{1 - x}$$
$$y(x) = 6 - 2x + 6\sqrt{1 - x}$$

✓ Solution by Mathematica

Time used: 2.199 (sec). Leaf size: 41

```
DSolve[{(2*x-5*y[x])+(4*x-y[x])*y'[x]==0,{y[1]==4}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2x - 6i\sqrt{x - 1} + 6$$
$$y(x) \to -2x + 6i\sqrt{x - 1} + 6$$

4.20 problem 20

Internal problem ID [11643]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.2 (Separable equations). Exercises page 47
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{9yx + 5y^2 - \left(6x^2 + 4yx\right) y' = -3x^2}$$

With initial conditions

$$[y(2) = -6]$$

✓ Solution by Maple

Time used: 0.329 (sec). Leaf size: 21

```
dsolve([(3*x^2+9*x*y(x)+5*y(x)^2)-(6*x^2+4*x*y(x))*diff(y(x),x)=0,y(2) = -6],y(x), singsol=a
```

$$y(x) = -\frac{\left(3 + \sqrt{-3 + 6\sqrt{x}\,\sqrt{2}}\right) x}{2}$$

✓ Solution by Mathematica

Time used: 37.251 (sec). Leaf size: 30

```
DSolve[{(3*x^2+9*x*y[x]+5*y[x]^2)-(6*x^2+4*x*y[x])*y'[x]==0,{y[2]==-6}},y[x],x,IncludeSingul
```

$$y(x) \to -\frac{1}{2}\left(\sqrt{6\sqrt{2}\sqrt{x} - 3} + 3\right) x$$

4.21 problem 22(a)

Internal problem ID [11644]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.2 (Separable equations). Exercises page 47
Problem number: 22(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, [_Abel, '2nd ty`

$$\boxed{2y + (2x - y)\,y' = -x}$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 51

```
dsolve((x+2*y(x))+(2*x-y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{2xc_1 - \sqrt{5c_1^2x^2 + 1}}{c_1}$$
$$y(x) = \frac{2xc_1 + \sqrt{5c_1^2x^2 + 1}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.777 (sec). Leaf size: 94

```
DSolve[(x+2*y[x])+(2*x-y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x - \sqrt{5x^2 + e^{2c_1}}$$
$$y(x) \to 2x + \sqrt{5x^2 + e^{2c_1}}$$
$$y(x) \to 2x - \sqrt{5}\sqrt{x^2}$$
$$y(x) \to \sqrt{5}\sqrt{x^2} + 2x$$

4.22 problem 22(b)

Internal problem ID [11645]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.2 (Separable equations). Exercises page 47
Problem number: 22(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, [_Abel, '2nd ty`

$$\boxed{-y-(x+y)\,y'=-3x}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 51

```
dsolve((3*x-y(x))-(x+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\frac{-xc_1-\sqrt{4c_1^2x^2+1}}{c_1}$$
$$y(x)=\frac{-xc_1+\sqrt{4c_1^2x^2+1}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.752 (sec). Leaf size: 85

```
DSolve[(3*x-y[x])-(x+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -x-\sqrt{4x^2+e^{2c_1}}$$
$$y(x)\to -x+\sqrt{4x^2+e^{2c_1}}$$
$$y(x)\to -2\sqrt{x^2}-x$$
$$y(x)\to 2\sqrt{x^2}-x$$

4.23 problem 23(a)

Internal problem ID [11646]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.2 (Separable equations). Exercises page 47
Problem number: 23(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, _dAlembert]`

$$\boxed{2y^2 + (4yx - y^2)\, y' = -x^2}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 441

```
dsolve((x^2+2*y(x)^2)+(4*x*y(x)-y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\frac{\left(4+68x^3c_1^3+4\sqrt{33c_1^6x^6+34x^3c_1^3+1}\right)^{\frac{1}{3}}}{2} + \frac{8c_1^2x^2}{\left(4+68x^3c_1^3+4\sqrt{33c_1^6x^6+34x^3c_1^3+1}\right)^{\frac{1}{3}}} + 2xc_1}{c_1}$$

$$y(x) = -\frac{\left(4+68x^3c_1^3+4\sqrt{33c_1^6x^6+34x^3c_1^3+1}\right)^{\frac{1}{3}}}{4} - \frac{4c_1^2x^2}{\left(4+68x^3c_1^3+4\sqrt{33c_1^6x^6+34x^3c_1^3+1}\right)^{\frac{1}{3}}} + 2xc_1 - \frac{i\sqrt{3}\left(\frac{\left(4+68x^3c_1^3+4\sqrt{33c_1^6x^6+34x^3c_1^3+1}\right)^{\frac{1}{3}}}{2}\right.}{2}$$

$$= \frac{}{c_1}$$

$$y(x) = -\frac{\left(4+68x^3c_1^3+4\sqrt{33c_1^6x^6+34x^3c_1^3+1}\right)^{\frac{1}{3}}}{4} - \frac{4c_1^2x^2}{\left(4+68x^3c_1^3+4\sqrt{33c_1^6x^6+34x^3c_1^3+1}\right)^{\frac{1}{3}}} + 2xc_1 + \frac{i\sqrt{3}\left(\frac{\left(4+68x^3c_1^3+4\sqrt{33c_1^6x^6+34x^3c_1^3+1}\right)^{\frac{1}{3}}}{2}\right.}{2}$$

$$= \frac{}{c_1}$$

✓ Solution by Mathematica

Time used: 33.481 (sec). Leaf size: 731

```
DSolve[(x^2+2*y[x]^2)+(4*x*y[x]-y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{17x^3+\sqrt{33x^6+34e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}{\sqrt[3]{2}} + \frac{4\sqrt[3]{2}x^2}{\sqrt[3]{17x^3+\sqrt{33x^6+34e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}} + 2x$$

$$y(x) \to -\frac{\left(1-i\sqrt{3}\right)\sqrt[3]{17x^3+\sqrt{33x^6+34e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}{2\sqrt[3]{2}} - \frac{2\sqrt[3]{2}\left(1+i\sqrt{3}\right)x^2}{\sqrt[3]{17x^3+\sqrt{33x^6+34e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}} + 2x$$

$$y(x) \to -\frac{\left(1+i\sqrt{3}\right)\sqrt[3]{17x^3+\sqrt{33x^6+34e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}{2\sqrt[3]{2}} - \frac{2\sqrt[3]{2}\left(1-i\sqrt{3}\right)x^2}{\sqrt[3]{17x^3+\sqrt{33x^6+34e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}} + 2x$$

$$y(x) \to \frac{8\sqrt[3]{2}x^2+4\sqrt[3]{\sqrt{33}\sqrt{x^6}+17x^3}x+2^{2/3}\left(\sqrt{33}\sqrt{x^6}+17x^3\right)^{2/3}}{2\sqrt[3]{\sqrt{33}\sqrt{x^6}+17x^3}}$$

$$y(x) \to \frac{8i\sqrt[3]{2}\sqrt{3}x^2-8\sqrt[3]{2}x^2+8\sqrt[3]{\sqrt{33}\sqrt{x^6}+17x^3}x-i2^{2/3}\sqrt{3}\left(\sqrt{33}\sqrt{x^6}+17x^3\right)^{2/3}-2^{2/3}\left(\sqrt{33}\sqrt{x^6}+17x^3\right)^{2/3}}{4\sqrt[3]{\sqrt{33}\sqrt{x^6}+17x^3}}$$

$$y(x) \to \frac{\left(\sqrt{33}\sqrt{x^6}+17x^3\right)^{2/3}\text{Root}\left[2\#1^3-1\&,3\right]-4\sqrt[3]{-2}x^2+2\sqrt[3]{\sqrt{33}\sqrt{x^6}+17x^3}x}{\sqrt[3]{\sqrt{33}\sqrt{x^6}+17x^3}}$$

4.24 problem 23(b)

Internal problem ID [11647]

Book: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
Section: Chapter 2, section 2.2 (Separable equations). Exercises page 47
Problem number: 23(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, [_Abel, '2nd ty`

$$\boxed{2yx + y^2 + \left(2yx + x^2\right) y' = -2x^2}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 73

```
dsolve((2*x^2+2*x*y(x)+y(x)^2)+(x^2+2*x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{3c_1^2x^2 - \sqrt{-15x^4c_1^4 + 12xc_1}}{6x\,c_1^2}$$

$$y(x) = -\frac{3c_1^2x^2 + \sqrt{-15x^4c_1^4 + 12xc_1}}{6x\,c_1^2}$$

✓ Solution by Mathematica

Time used: 1.277 (sec). Leaf size: 150

```
DSolve[(2*x^2+2*x*y[x]+y[x]^2)+(x^2+2*x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{1}{6}\left(-3x - \frac{\sqrt{3}\sqrt{-5x^3+4e^{3c_1}}}{\sqrt{x}}\right)$$

$$y(x) \to \frac{1}{6}\left(-3x + \frac{\sqrt{3}\sqrt{-5x^3+4e^{3c_1}}}{\sqrt{x}}\right)$$

$$y(x) \to \frac{1}{6}x\left(\frac{\sqrt{15}x^{3/2}}{\sqrt{-x^3}} - 3\right)$$

$$y(x) \to \frac{\sqrt{\frac{5}{3}}\sqrt{-x^3}}{2\sqrt{x}} - \frac{x}{2}$$