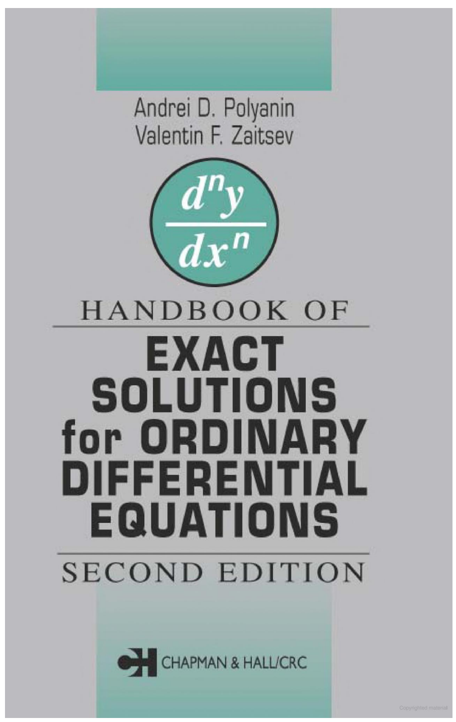
A Solution Manual For

Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition



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Contents

1 Chapter 1, First-Order differential equations

1.1 problem 1.1.1

Internal problem ID [10335]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, First-Order differential equations
Problem number: 1.1.1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = f(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=f(x),y(x), singsol=all)
```

$$y(x) = \int f(x)\, dx + c_1$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 18

```
DSolve[y'[x]==f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \int_1^x f(K[1])dK[1] + c_1$$

1.2 problem 1.1.2

Internal problem ID [10336]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, First-Order differential equations
Problem number: 1.1.2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - f(y) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=f(y(x)),y(x), singsol=all)
```

$$x - \left(\int^{y(x)} \frac{1}{f(\_a)} d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.442 (sec). Leaf size: 33

```
DSolve[y'[x]==f[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{f(K[1])} dK[1]\&\right][x + c_1]$$

$$y(x) \to f^{(-1)}(0)$$

1.3 problem 1.1.3

Internal problem ID [10337]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, First-Order differential equations
Problem number: 1.1.3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - f(x)\,g(y) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)=f(x)*g(y(x)),y(x), singsol=all)
```

$$\int f(x)\,dx - \left(\int^{y(x)} \frac{1}{g\left(\_a\right)} d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.452 (sec). Leaf size: 42

```
DSolve[y'[x]==f[x]*g[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{g(K[1])} dK[1]\&\right]\left[\int_1^x f(K[2])dK[2] + c_1\right]$$
$$y(x) \to g^{(-1)}(0)$$

1.4 problem 1.1.4

Internal problem ID [10338]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, First-Order differential equations
Problem number: 1.1.4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{g(x)\,y' - f_1(x)\,y = f_0(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(g(x)*diff(y(x),x)=f__1(x)*y(x)+f__0(x),y(x), singsol=all)
```

$$y(x) = \left(\int \frac{f_0(x)\,\mathrm{e}^{-\left(\int \frac{f_1(x)}{g(x)}dx\right)}}{g\left(x\right)}dx + c_1\right)\mathrm{e}^{\int \frac{f_1(x)}{g(x)}dx}$$

✓ Solution by Mathematica

Time used: 0.135 (sec). Leaf size: 64

```
DSolve[g[x]*y'[x]==f1[x]*y[x]+f0[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \exp\left(\int_1^x \frac{\mathrm{f1}(K[1])}{g(K[1])}dK[1]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[2]} \frac{\mathrm{f1}(K[1])}{g(K[1])}dK[1]\right)\mathrm{f0}(K[2])}{g(K[2])}dK[2] + c_1\right)$$

1.5 problem 1.1.5

Internal problem ID [10339]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, First-Order differential equations
Problem number: 1.1.5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{g(x)\, y' - f_1(x)\, y - f_n(x)\, y^n = 0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 119

```
dsolve(g(x)*diff(y(x),x)=f__1(x)*y(x)+f__n(x)*y(x)^n,y(x), singsol=all)
```

$$y(x) = \left(\int \left(-\frac{n\,\mathrm{e}^{\int \left(\frac{f_1(x) n}{g(x)}-\frac{f_1(x)}{g(x)}\right)dx} f_n(x)}{g\left(x\right)} + \frac{\mathrm{e}^{\int \left(\frac{f_1(x) n}{g(x)}-\frac{f_1(x)}{g(x)}\right)dx} f_n(x)}{g\left(x\right)}\right)dx + c_1\right)^{-\frac{1}{n-1}} \mathrm{e}^{\frac{\left(\int \frac{f_1(x)}{g(x)}dx\right)n}{n-1}} \mathrm{e}^{\int -\frac{f_1(x)}{(n-1)g(x)}dx}$$

✓ Solution by Mathematica

Time used: 14.019 (sec). Leaf size: 84

```
DSolve[g[x]*y'[x]==f1[x]*y[x]+fn[x]*y[x]^n,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\exp\left(-\left((n-1)\int_1^x \frac{\mathrm{f1}(K[1])}{g(K[1])}dK[1]\right)\right)\left(-(n-1)\int_1^x \frac{\exp\left((n-1)\int_1^{K[2]} \frac{\mathrm{f1}(K[1])}{g(K[1])}dK[1]\right)\mathrm{fn}(K[2])}{g(K[2])}dK[2 + c_1\right)\right)^{\frac{1}{1-n}}$$

1.6 problem 1.1.6

Internal problem ID [10340]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, First-Order differential equations
Problem number: 1.1.6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y' - f\left(\frac{y}{x}\right) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)=f(y(x)/x),y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(\int^{\_Z} \frac{1}{-f(\_a) + \_a} d\_a + \ln(x) + c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.106 (sec). Leaf size: 33

```
DSolve[y'[x]==f[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{\frac{y(x)}{x}} \frac{1}{K[1] - f(K[1])} dK[1] = -\log(x) + c_1, y(x)\right]$$

2 Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions

2.1 problem 1

Internal problem ID [10341]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - ay^2 = bx + c}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)=a*y(x)^2+b*x+c,y(x), singsol=all)
```

$$y(x) = \frac{\left(\frac{b}{\sqrt{a}}\right)^{\frac{1}{3}} \left(\mathrm{AiryAi}\left(1, -\frac{xb+c}{\left(\frac{b}{\sqrt{a}}\right)^{\frac{2}{3}}}\right) c_1 + \mathrm{AiryBi}\left(1, -\frac{xb+c}{\left(\frac{b}{\sqrt{a}}\right)^{\frac{2}{3}}}\right)\right)}{\sqrt{a} \left(c_1 \,\mathrm{AiryAi}\left(-\frac{xb+c}{\left(\frac{b}{\sqrt{a}}\right)^{\frac{2}{3}}}\right) + \mathrm{AiryBi}\left(-\frac{xb+c}{\left(\frac{b}{\sqrt{a}}\right)^{\frac{2}{3}}}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.325 (sec). Leaf size: 143

```
DSolve[y'[x]==a*y[x]^2+b*x+c,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{b\left(\text{AiryBiPrime}\left(-\frac{a(c+bx)}{(-ab)^{2/3}}\right) + c_1 \text{AiryAiPrime}\left(-\frac{a(c+bx)}{(-ab)^{2/3}}\right)\right)}{(-ab)^{2/3}\left(\text{AiryBi}\left(-\frac{a(c+bx)}{(-ab)^{2/3}}\right) + c_1 \text{AiryAi}\left(-\frac{a(c+bx)}{(-ab)^{2/3}}\right)\right)}$$

$$y(x) \to \frac{b\,\text{AiryAiPrime}\left(-\frac{a(c+bx)}{(-ab)^{2/3}}\right)}{(-ab)^{2/3}\,\text{AiryAi}\left(-\frac{a(c+bx)}{(-ab)^{2/3}}\right)}$$

2.2 problem 2

Internal problem ID [10342]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = -a^2x^2 + 3a}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 108

```
dsolve(diff(y(x),x)=y(x)^2-a^2*x^2+3*a,y(x), singsol=all)
```

$$y(x) = \frac{e^{\frac{a x^2}{2}} c_1 a x + \left(\left(-c_1 x^2 \sqrt{\pi}\,(-a)^{\frac{3}{2}} - \sqrt{\pi}\,\sqrt{-a}\, c_1\right) \operatorname{erf}\left(x\sqrt{-a}\right) + a\,x^2 - 1\right) e^{-\frac{a\,x^2}{2}}}{e^{\frac{a x^2}{2}} c_1 + \left(\operatorname{erf}\left(x\sqrt{-a}\right)\sqrt{\pi}\,\sqrt{-a}\, c_1 x + x\right) e^{-\frac{a\,x^2}{2}}}$$

✓ Solution by Mathematica

Time used: 0.79 (sec). Leaf size: 192

```
DSolve[y'[x]==y[x]^2-a^2*x^2+3*a,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{ax\,\text{ParabolicCylinderD}\left(-2, i\sqrt{2}\sqrt{a}x\right) + i\sqrt{2}\sqrt{a}\,\text{ParabolicCylinderD}\left(-1, i\sqrt{2}\sqrt{a}x\right) - ac_1x\,\text{Parabolic}}{\text{ParabolicCylinderD}\left(-2, i\sqrt{2}\sqrt{a}x\right) + c_1\,\text{ParabolicCyli}}$$

$$y(x) \to \frac{\sqrt{2}\sqrt{a}\,\text{ParabolicCylinderD}\left(2, \sqrt{2}\sqrt{a}x\right)}{\text{ParabolicCylinderD}\left(1, \sqrt{2}\sqrt{a}x\right)} - ax$$

2.3 problem 3

Internal problem ID [10343]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = a^2x^2 + bx + c}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 688

```
dsolve(diff(y(x),x)=y(x)^2+a^2*x^2+b*x+c,y(x), singsol=all)
```

$$y(x) = -\frac{(48a^7c_1x^2i - 16a^6cc_1x^2 + 48a^5bc_1xi + 4a^4b^2c_1x^2 - 16a^4bcc_1x + 12a^3b^2c_1i + 4a^2b^3c_1x - 4a^2b^2cc_1 + b^4}{24a^4\left((2a^2c_1x + c_1b)\,\mathrm{hypergeom}\left(\left[\frac{4ia^2c+12a^3-ib^2}{16a^3}\right], \left[\frac{3}{2}\right], \frac{i(2a^2x+b)^2}{4a^3}\right) + \mathrm{hypergeom}\left(\left[\right.\right.\right.}$$
$$-\frac{(24ia^5x - 24a^4cx + 12ia^3b + 6a^2b^2x - 12a^2bc + 3b^3)\,\mathrm{hypergeom}\left(\left[\frac{4ia^2c+20a^3-ib^2}{16a^3}\right], \left[\frac{3}{2}\right], \frac{i(2a^2x+}{4a^3}\right.}{24a^4\left((2a^2c_1x + c_1b)\,\mathrm{hypergeom}\left(\left[\frac{4ia^2c+12a^3-ib^2}{16a^3}\right], \left[\frac{3}{2}\right], \frac{i(2a^2x+b)^2}{4a^3}\right) + \mathrm{hypergeom}\left(\left[\frac{4ia^2c+4a^3-ib^2}{16a^3}\right], \left[\frac{1}{2}\right],\right.\right.}$$
$$-\frac{(-48a^7c_1x^2i - 48a^5bc_1xi + 48a^6c_1 - 12a^3b^2c_1i)\,\mathrm{hypergeom}\left(\left[\frac{4ia^2c+12a^3-ib^2}{16a^3}\right], \left[\frac{3}{2}\right], \frac{i(2a^2x+b)^2}{4a^3}\right) + (-24a}{24a^4\left((2a^2c_1x + c_1b)\,\mathrm{hypergeom}\left(\left[\frac{4ia^2c+12a^3-ib^2}{16a^3}\right], \left[\frac{3}{2}\right], \frac{i(2a^2x+b)^2}{4a^3}\right) + \mathrm{hypergeo}\right.}$$

✓ Solution by Mathematica

Time used: 1.582 (sec). Leaf size: 664

```
DSolve[y'[x]==y[x]^2+a^2*x^2+b*x+c,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2i\sqrt{2}a^2x\,\text{ParabolicCylinderD}\left(\frac{1}{8}\left(-\frac{ib^2}{a^3}+\frac{4ic}{a}-4\right),-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)(2xa^2+b)}{a^{3/2}}\right)+4(-1)^{3/4}a^{3/2}\,\text{ParabolicCylinderI}}{}$$

$$y(x) \to \frac{(1+i)\sqrt{a}\,\text{ParabolicCylinderD}\left(\frac{1}{8}\left(\frac{ib^2}{a^3}-\frac{4ic}{a}+4\right),\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(2xa^2+b)}{a^{3/2}}\right)}{\text{ParabolicCylinderD}\left(\frac{1}{8}\left(\frac{ib^2}{a^3}-\frac{4ic}{a}-4\right),\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(2xa^2+b)}{a^{3/2}}\right)}-\frac{i(2a^2x+b)}{2a}$$

$$y(x) \to \frac{(1+i)\sqrt{a}\,\text{ParabolicCylinderD}\left(\frac{1}{8}\left(\frac{ib^2}{a^3}-\frac{4ic}{a}+4\right),\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(2xa^2+b)}{a^{3/2}}\right)}{\text{ParabolicCylinderD}\left(\frac{1}{8}\left(\frac{ib^2}{a^3}-\frac{4ic}{a}-4\right),\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(2xa^2+b)}{a^{3/2}}\right)}-\frac{i(2a^2x+b)}{2a}$$

2.4 problem 4

Internal problem ID [10344]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{y' - ay^2 = b\,x^n}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 207

```
dsolve(diff(y(x),x)=a*y(x)^2+b*x^n,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{ba}\,x^{\frac{n}{2}+1}\,\mathrm{BesselJ}\left(\frac{n+3}{n+2}, \frac{2\sqrt{ba}\,x^{\frac{n}{2}+1}}{n+2}\right) c_1 + \mathrm{BesselY}\left(\frac{n+3}{n+2}, \frac{2\sqrt{ba}\,x^{\frac{n}{2}+1}}{n+2}\right)\sqrt{ba}\,x^{\frac{n}{2}+1} - c_1\,\mathrm{BesselJ}\left(\frac{1}{n+2}, \frac{2\sqrt{ba}\,x^{\frac{n}{2}+1}}{n+2}\right.}{xa\left(c_1\,\mathrm{BesselJ}\left(\frac{1}{n+2}, \frac{2\sqrt{ba}\,x^{\frac{n}{2}+1}}{n+2}\right) + \mathrm{BesselY}\left(\frac{1}{n+2}, \frac{2\sqrt{ba}\,x^{\frac{n}{2}+1}}{n+2}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.696 (sec). Leaf size: 605

```
DSolve[y'[x]==a*y[x]^2+b*x^n,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}\text{Gamma}\left(1+\frac{1}{n+2}\right)\text{BesselJ}\left(\frac{1}{n+2}-1,\frac{2\sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}}{n+2}\right)-\sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}\text{Gamma}\left(1+\frac{1}{n+2}\right)\text{BesselJ}}{}$$

$$y(x) \to \frac{\frac{\sqrt{a}\sqrt{b}x^{n/2}\left(\text{BesselJ}\left(\frac{n+1}{n+2},\frac{2\sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}}{n+2}\right)-\text{BesselJ}\left(-\frac{n+3}{n+2},\frac{2\sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}}{n+2}\right)\right)}{\text{BesselJ}\left(-\frac{1}{n+2},\frac{2\sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}}{n+2}\right)}-\frac{1}{x}}{2a}$$

2.5 problem 5

Internal problem ID [10345]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = an\,x^{-1+n} - a^2x^{2n}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 555

```
dsolve(diff(y(x),x)=y(x)^2+a*n*x^(n-1)-a^2*x^(2*n),y(x), singsol=all)
```

$$y(x) = \frac{2a\,x^{1+n}\mathrm{e}^{-\frac{a\,x^{1+n}}{1+n}} + \left(2x^{-\frac{3n}{2}-1}c_1n^3 + 11x^{-\frac{3n}{2}-1}c_1n^2 + 20x^{-\frac{3n}{2}-1}c_1n + 12x^{-\frac{3n}{2}-1}c_1\right)\mathrm{WhittakerM}\left(\frac{3n+4}{2+2n}, \frac{3+}{2+}\right.}{}$$

✓ Solution by Mathematica

Time used: 1.61 (sec). Leaf size: 227

```
DSolve[y'[x]==y[x]^2+a*n*x^(n-1)-a^2*x^(2*n),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2^{\frac{1}{n+1}}(n+1)\left(-\frac{ax^{n+1}}{n+1}\right)^{\frac{1}{n+1}}\left(ax^n - c_1e^{\frac{2ax^{n+1}}{n+1}}\right) - ac_1x^{n+1}\Gamma\left(\frac{1}{n+1}, -\frac{2ax^{n+1}}{n+1}\right)}{2^{\frac{1}{n+1}}(n+1)\left(-\frac{ax^{n+1}}{n+1}\right)^{\frac{1}{n+1}} - c_1x\Gamma\left(\frac{1}{n+1}, -\frac{2ax^{n+1}}{n+1}\right)}$$

$$y(x) \to \frac{2^{\frac{1}{n+1}}(n+1)e^{\frac{2ax^{n+1}}{n+1}}\left(-\frac{ax^{n+1}}{n+1}\right)^{\frac{1}{n+1}}}{x\Gamma\left(\frac{1}{n+1}, -\frac{2ax^{n+1}}{n+1}\right)} + ax^n$$

2.6 problem 6

Internal problem ID [10346]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - ay^2 = b\,x^{2n} + c\,x^{-1+n}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 499

```
dsolve(diff(y(x),x)=a*y(x)^2+b*x^(2*n)+c*x^(n-1),y(x), singsol=all)
```

$$y(x) = -\frac{\left(-2b^{\frac{3}{2}}c_1 n - 2b^{\frac{3}{2}}c_1\right)\mathrm{WhittakerW}\left(-\frac{i\sqrt{a}\,c-2\sqrt{b}\,n-2\sqrt{b}}{2\sqrt{b}\,(1+n)}, \frac{1}{2+2n}, \frac{2i\sqrt{b}\,\sqrt{a}\,x^{1+n}}{1+n}\right)}{2b^{\frac{3}{2}}\left(\mathrm{WhittakerW}\left(-\frac{i\sqrt{a}\,c}{2\sqrt{b}\,(1+n)}, \frac{1}{2+2n}, \frac{2i\sqrt{b}\,\sqrt{a}\,x^{1+n}}{1+n}\right)c_1 + \mathrm{WhittakerM}\left(-\frac{i\sqrt{a}\,c}{2\sqrt{b}\,(1+n)}, \frac{1}{2+2n}, \frac{2i\sqrt{b}\,\sqrt{a}\,x^{1+n}}{1+n}\right)\right) a}$$
$$-\frac{\left(2i\sqrt{a}\,x^{1+n}c_1 b^2 + i\sqrt{a}\,c_1 bc - b^{\frac{3}{2}}c_1 n\right)\mathrm{WhittakerW}\left(-\frac{i\sqrt{a}\,c}{2\sqrt{b}\,(1+n)}, \frac{1}{2+2n}, \frac{2i\sqrt{b}\,\sqrt{a}\,x^{1+n}}{1+n}\right) + \left(-i\sqrt{a}\,bc + b^{\frac{3}{2}}n\right.}{2b^{\frac{3}{2}}\left(\mathrm{WhittakerW}\left(-\frac{i\sqrt{a}\,c}{2\sqrt{b}\,(1+n)}, \frac{1}{2+\cdots}\right.\right.}$$

✓ Solution by Mathematica

Time used: 1.818 (sec). Leaf size: 982

```
DSolve[y'[x]==a*y[x]^2+b*x^(2*n)+c*x^(n-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^n\left(\sqrt{b}c_1(n+1)\sqrt{-(n+1)^2}\,\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{a}c}{\sqrt{b}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right),\frac{n}{n+1},\frac{2\sqrt{a}\sqrt{b}x^{n+1}}{\sqrt{-(n+1)^2}}\right)+c_1\left(\sqrt{a}c(n\right.\right.}{\sqrt{a}(n+1)^2}$$

$$y(x) \to \frac{x^n\left(-\frac{\left(\sqrt{a}c(n+1)+\sqrt{b}\sqrt{-(n+1)^2}n\right)\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{a}c}{\sqrt{b}\sqrt{-(n+1)^2}}+\frac{n}{n+1}+2\right),\frac{n}{n+1}+1,\frac{2\sqrt{a}\sqrt{b}x^{n+1}}{\sqrt{-(n+1)^2}}\right)}{\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{a}c}{\sqrt{b}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right),\frac{n}{n+1},\frac{2\sqrt{a}\sqrt{b}x^{n+1}}{\sqrt{-(n+1)^2}}\right)}-\sqrt{b}\sqrt{-(n+1)^2}(n\right.}{\sqrt{a}(n+1)^2}$$

$$y(x) \to \frac{x^n\left(-\frac{\left(\sqrt{a}c(n+1)+\sqrt{b}\sqrt{-(n+1)^2}n\right)\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{a}c}{\sqrt{b}\sqrt{-(n+1)^2}}+\frac{n}{n+1}+2\right),\frac{n}{n+1}+1,\frac{2\sqrt{a}\sqrt{b}x^{n+1}}{\sqrt{-(n+1)^2}}\right)}{\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{a}c}{\sqrt{b}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right),\frac{n}{n+1},\frac{2\sqrt{a}\sqrt{b}x^{n+1}}{\sqrt{-(n+1)^2}}\right)}-\sqrt{b}\sqrt{-(n+1)^2}(n\right.}{\sqrt{a}(n+1)^2}$$

2.7 problem 7

Internal problem ID [10347]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _Riccati]`

$$\boxed{y' - a\,x^n y^2 = b\,x^{-2-n}}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 61

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2+b*x^(-n-2),y(x), singsol=all)
```

$$y(x) = -\frac{x^{-1-n}\left(n+1+\tan\left(\frac{\sqrt{4ba-n^2-2n-1}\,(-\ln(x)+c_1)}{2}\right)\sqrt{4ba-n^2-2n-1}\right)}{2a}$$

✓ Solution by Mathematica

Time used: 0.778 (sec). Leaf size: 135

```
DSolve[y'[x]==a*x^n*y[x]^2+b*x^(-n-2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{-n-1}\left(-\left(\sqrt{(n+1)^2-4ab}+n+1\right)x^{\sqrt{(n+1)^2-4ab}}+c_1\left(\sqrt{(n+1)^2-4ab}-n-1\right)\right)}{2a\left(x^{\sqrt{(n+1)^2-4ab}}+c_1\right)}$$

$$y(x) \to \frac{x^{-n-1}\left(\sqrt{(n+1)^2-4ab}-n-1\right)}{2a}$$

2.8 problem 8

Internal problem ID [10348]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a\,x^n y^2 = b\,x^m}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 177

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2+b*x^m,y(x), singsol=all)
```

$$y(x) = \frac{\left(\text{BesselY}\left(\frac{m+1}{m+n+2}, \frac{2\sqrt{ba}\,x^{\frac{m}{2}+\frac{n}{2}+1}}{m+n+2}\right) c_1 + \text{BesselJ}\left(\frac{m+1}{m+n+2}, \frac{2\sqrt{ba}\,x^{\frac{m}{2}+\frac{n}{2}+1}}{m+n+2}\right)\right) x^{\frac{m}{2}+\frac{n}{2}+1}\sqrt{ba}\,x^{-n}}{\left(\text{BesselY}\left(-\frac{1+n}{m+n+2}, \frac{2\sqrt{ba}\,x^{\frac{m}{2}+\frac{n}{2}+1}}{m+n+2}\right) c_1 + \text{BesselJ}\left(-\frac{1+n}{m+n+2}, \frac{2\sqrt{ba}\,x^{\frac{m}{2}+\frac{n}{2}+1}}{m+n+2}\right)\right) ax}$$

✓ Solution by Mathematica

Time used: 2.978 (sec). Leaf size: 1805

```
DSolve[y'[x]==a*x^n*y[x]^2+b*x^m,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a^{-\frac{2m+3n+5}{2(m+n+2)}} b^{-\frac{n+1}{2(m+n+2)}} (m+n+1)^{\frac{n+1}{m+n+2}} \left((m+n+1)^2\right)^{\frac{n+1}{m+n+2}-\frac{1}{2}} x^{-n-1} \left(x^{m+n+1}\right)^{-\frac{n+1}{2(m+n+1)}} \left(a^{\frac{n+1}{2(m+n+2)}} b^{\frac{}{2(}}\right.}{}$$

$$y(x) \to \frac{x^{-n-1} \left(\sqrt{a}\sqrt{b}(m+n+1) \left(x^{m+n+1}\right)^{\frac{1}{2}\left(\frac{1}{m+n+1}+1\right)} \text{BesselJ}\left(\frac{m+1}{m+n+2}, \frac{2\sqrt{a}\sqrt{b}(m+n+1)\left(x^{m+n+1}\right)^{\frac{1}{2}\left(1+\frac{1}{m+n+1}\right)}}{\sqrt{(m+n+1)^2}(m+n+2)}\right) - \sqrt{}\right.}{2}$$

$$y(x) \to \frac{x^{-n-1} \left(\sqrt{a}\sqrt{b}(m+n+1) \left(x^{m+n+1}\right)^{\frac{1}{2}\left(\frac{1}{m+n+1}+1\right)} \text{BesselJ}\left(\frac{m+1}{m+n+2}, \frac{2\sqrt{a}\sqrt{b}(m+n+1)\left(x^{m+n+1}\right)^{\frac{1}{2}\left(1+\frac{1}{m+n+1}\right)}}{\sqrt{(m+n+1)^2}(m+n+2)}\right) - \sqrt{}\right.}{2}$$

2.9 problem 9

Internal problem ID [10349]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = k(ax + b)^n (cx + d)^{-n-4}}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+k*(a*x+b)^n*(c*x+d)^(-n-4),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+k*(a*x+b)^n*(c*x+d)^(-n-4),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.10 problem 10

Internal problem ID [10350]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,x^n y^2 = bm\,x^{m-1} - a\,b^2 x^{n+2m}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 1166

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2+b*m*x^(m-1)-a*b^2*x^(n+2*m),y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 2.322 (sec). Leaf size: 306

```
DSolve[y'[x]==a*x^n*y[x]^2+b*m*x^(m-1)-a*b^2*x^(n+2*m),y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{2^{\frac{n+1}{m+n+1}}(m+n+1)\left(-\frac{abx^{m+n+1}}{m+n+1}\right)^{\frac{n+1}{m+n+1}}\left(abx^m - c_1 e^{\frac{2abx^{m+n+1}}{m+n+1}}\right) - abc_1 x^{m+n+1}\Gamma\left(\frac{n+1}{m+n+1}, -\frac{2abx^{m+n+1}}{m+n+1}\right)}{a\left(2^{\frac{n+1}{m+n+1}}(m+n+1)\left(-\frac{abx^{m+n+1}}{m+n+1}\right)^{\frac{n+1}{m+n+1}} - c_1 x^{n+1}\Gamma\left(\frac{n+1}{m+n+1}, -\frac{2abx^{m+n+1}}{m+n+1}\right)\right)}$$

$$y(x) \to bx^m - \frac{b2^{\frac{n+1}{m+n+1}}x^m e^{\frac{2abx^{m+n+1}}{m+n+1}}\left(-\frac{abx^{m+n+1}}{m+n+1}\right)^{-\frac{m}{m+n+1}}}{\Gamma\left(\frac{n+1}{m+n+1}, -\frac{2abx^{m+n+1}}{m+n+1}\right)}$$

2.11 problem 11

Internal problem ID [10351]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \left(a\,x^{2n} + b\,x^{-1+n}\right) y^2 = c$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8014

```
dsolve(diff(y(x),x)=(a*x^(2*n)+b*x^(n-1))*y(x)^2+c,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 2.128 (sec). Leaf size: 1384

```
DSolve[y'[x]==(a*x^(2*n)+b*x^(n-1))*y[x]^2+c,y[x],x,IncludeSingularSolutions -> True]
```

$y(x)$

$$\rightarrow \frac{\sqrt{c}(n+1)^2x^{-n}\Bigg(\cdots}{\sqrt{a}c_1(n+1)\sqrt{-(n+1)^2}\,\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{c}b}{\sqrt{a}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right),\frac{n}{n+1},\frac{2\sqrt{a}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)+c_1\left(\sqrt{a}\sqrt{-(n\cdots\right.}$$

$y(x)$

$$\rightarrow \frac{\sqrt{c}(n+1)^2x^{-n}\,\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{c}b}{\sqrt{a}\sqrt{-(n\cdots}\right.\right.}{\sqrt{a}(n+1)\sqrt{-(n+1)^2}\,\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{c}b}{\sqrt{a}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right),\frac{n}{n+1},\frac{2\sqrt{a}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)+\left(\sqrt{a}\sqrt{-(n+1)^2}\cdots\right.}$$

$y(x)$

$$\rightarrow \frac{\sqrt{c}(n+1)^2x^{-n}\,\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{c}b}{\sqrt{a}\sqrt{-(n\cdots}\right.\right.}{\sqrt{a}(n+1)\sqrt{-(n+1)^2}\,\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{c}b}{\sqrt{a}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right),\frac{n}{n+1},\frac{2\sqrt{a}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)+\left(\sqrt{a}\sqrt{-(n+1)^2}\cdots\right.}$$

$y(x)$

$$\rightarrow \frac{\sqrt{c}(n+1)x^{-n}L^{-\frac{1}{n+1}}_{-\frac{\sqrt{c}b}{2\sqrt{a}\sqrt{-(n+1)^2}}-\frac{n}{2(n+1)}}\left(\frac{2\sqrt{a}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)}{\sqrt{a}\sqrt{-(n+1)^2}\left(2L^{\frac{n}{n+1}}_{-\frac{\sqrt{c}b}{2\sqrt{a}\sqrt{-(n+1)^2}}-\frac{n}{2(n+1)}-1}\left(\frac{2\sqrt{a}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)+L^{-\frac{1}{n+1}}_{-\frac{\sqrt{c}b}{2\sqrt{a}\sqrt{-(n+1)^2}}-\frac{n}{2(n+1)}}\left(\frac{2\sqrt{a}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)\right)}$$

2.12 problem 12

Internal problem ID [10352]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$(a_2x + b_2)\left(y' + \lambda y^2\right) = -a_0x - b_0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 1030

```
dsolve((a__2*x+b__2)*(diff(y(x),x)+lambda*y(x)^2)+a__0*x+b__0=0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 1.895 (sec). Leaf size: 690

```
DSolve[(a2*x+b2)*(y'[x]+\[Lambda]*y[x]^2)+a0*x+b0==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{c_1\sqrt{\lambda}(\mathrm{a2b0}-\mathrm{a0b2})\,\mathrm{HypergeometricU}\left(\frac{i\sqrt{\lambda}(\mathrm{a2b0}-\mathrm{a0b2})}{2\sqrt{\mathrm{a0}}\mathrm{a2}^{3/2}}+1,1,\frac{2i\sqrt{\mathrm{a0}}(\mathrm{b2}+\mathrm{a2}x)\sqrt{\lambda}}{\mathrm{a2}^{3/2}}\right)-i\sqrt{\mathrm{a0}}\mathrm{a2}^{3/2}\left(c_1\,\mathrm{Hy}\right.}{\mathrm{a2}^2\sqrt{\lambda}\left(c_1\,\mathrm{HypergeometricU}\left(\right.\right.}$$

$$y(x) \to \frac{(\mathrm{a2b0}-\mathrm{a0b2})\,\mathrm{HypergeometricU}\left(\frac{i\sqrt{\lambda}(\mathrm{a2b0}-\mathrm{a0b2})}{2\sqrt{\mathrm{a0}}\mathrm{a2}^{3/2}}+1,1,\frac{2i\sqrt{\mathrm{a0}}(\mathrm{b2}+\mathrm{a2}x)\sqrt{\lambda}}{\mathrm{a2}^{3/2}}\right)}{\mathrm{a2}^2\,\mathrm{HypergeometricU}\left(\frac{i(\mathrm{a2b0}-\mathrm{a0b2})\sqrt{\lambda}}{2\sqrt{\mathrm{a0}}\mathrm{a2}^{3/2}},0,\frac{2i\sqrt{\mathrm{a0}}(\mathrm{b2}+\mathrm{a2}x)\sqrt{\lambda}}{\mathrm{a2}^{3/2}}\right)} - \frac{i\sqrt{\mathrm{a0}}}{\sqrt{\mathrm{a2}}\sqrt{\lambda}}$$

$$y(x) \to \frac{(\mathrm{a2b0}-\mathrm{a0b2})\,\mathrm{HypergeometricU}\left(\frac{i\sqrt{\lambda}(\mathrm{a2b0}-\mathrm{a0b2})}{2\sqrt{\mathrm{a0}}\mathrm{a2}^{3/2}}+1,1,\frac{2i\sqrt{\mathrm{a0}}(\mathrm{b2}+\mathrm{a2}x)\sqrt{\lambda}}{\mathrm{a2}^{3/2}}\right)}{\mathrm{a2}^2\,\mathrm{HypergeometricU}\left(\frac{i(\mathrm{a2b0}-\mathrm{a0b2})\sqrt{\lambda}}{2\sqrt{\mathrm{a0}}\mathrm{a2}^{3/2}},0,\frac{2i\sqrt{\mathrm{a0}}(\mathrm{b2}+\mathrm{a2}x)\sqrt{\lambda}}{\mathrm{a2}^{3/2}}\right)} - \frac{i\sqrt{\mathrm{a0}}}{\sqrt{\mathrm{a2}}\sqrt{\lambda}}$$

2.13 problem 13

Internal problem ID [10353]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Riccati, _special]]`

$$\boxed{y'x^2 - a\,x^2y^2 = b}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 40

```
dsolve(x^2*diff(y(x),x)=a*x^2*y(x)^2+b,y(x), singsol=all)
```

$$y(x) = -\frac{1 + \tan\left(\frac{\sqrt{4ba-1}\,(-\ln(x)+c_1)}{2}\right)\sqrt{4ba-1}}{2ax}$$

✓ Solution by Mathematica

Time used: 0.292 (sec). Leaf size: 77

```
DSolve[x^2*y'[x]==a*x^2*y[x]^2+b,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-1 + \sqrt{1-4ab}\left(-1 + \frac{2c_1}{x^{\sqrt{1-4ab}}+c_1}\right)}{2ax}$$

$$y(x) \to \frac{\sqrt{1-4ab}-1}{2ax}$$

2.14 problem 14

Internal problem ID [10354]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x^2 - y^2x^2 = -a^2x^4 + a(1-2b)\,x^2 - b(b+1)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 121

```
dsolve(x^2*diff(y(x),x)=x^2*y(x)^2-a^2*x^4+a*(1-2*b)*x^2-b*(b+1),y(x), singsol=all)
```

$$y(x) = \frac{2(-a\,x^2)^{b-\frac{1}{2}}\,c_1 a x\,\mathrm{e}^{a\,x^2}}{c_1\Gamma\left(b+\frac{1}{2}\right) - c_1\Gamma\left(b+\frac{1}{2}, -a\,x^2\right) + 1} + \frac{(-ac_1x^2 - c_1 b)\,\Gamma\left(b+\frac{1}{2}, -a\,x^2\right) + (ac_1x^2 + c_1 b)\,\Gamma\left(b+\frac{1}{2}\right) + a\,x^2 + b}{x\left(c_1\Gamma\left(b+\frac{1}{2}\right) - c_1\Gamma\left(b+\frac{1}{2}, -a\,x^2\right) + 1\right)}$$

✓ Solution by Mathematica

Time used: 1.225 (sec). Leaf size: 128

```
DSolve[x^2*y'[x]==x^2*y[x]^2-a^2*x^4+a*(1-2*b)*x^2-b*(b+1),y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{x^{2b+1}(ax^2+b)\,\Gamma\left(b+\frac{1}{2}, -ax^2\right) - 2(-ax^2)^{b+\frac{1}{2}}\left(-e^{ax^2}x^{2b+1} + c_1(ax^2+b)\right)}{x^{2b+2}\Gamma\left(b+\frac{1}{2}, -ax^2\right) - 2c_1 x\left(-ax^2\right)^{b+\frac{1}{2}}}$$

$$y(x) \to ax + \frac{b}{x}$$

2.15 problem 15

Internal problem ID [10355]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$y'x^2 - a\,x^2y^2 = b\,x^n + c$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 239

```
dsolve(x^2*diff(y(x),x)=a*x^2*y(x)^2+b*x^n+c,y(x), singsol=all)
```

$$y(x) = -\frac{\left(\sqrt{-4ac+1}\,c_1 + c_1\right)\text{BesselY}\left(\frac{\sqrt{-4ac+1}}{n}, \frac{2\sqrt{ba}\,x^{\frac{n}{2}}}{n}\right) - 2\,\text{BesselY}\left(\frac{\sqrt{-4ac+1}+n}{n}, \frac{2\sqrt{ba}\,x^{\frac{n}{2}}}{n}\right)\sqrt{ba}\,x^{\frac{n}{2}}c_1 + \left(\sqrt{-\right.}{2xa\left(\text{BesselY}\left(\frac{\sqrt{-4ac+1}}{n}, \frac{2\sqrt{ba}\,x^{\frac{n}{2}}}{n}\right)c_1 + \text{Bess}\right.}$$

✓ Solution by Mathematica

Time used: 1.898 (sec). Leaf size: 1779

```
DSolve[x^2*y'[x]==a*x^2*y[x]^2+b*x^n+c,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-a^{\frac{i\sqrt{4ac-1}}{n}+\frac{1}{2}} n^{\frac{2\sqrt{(1-4ac)n^2}}{n^2}+1} (x^n)^{\frac{i\sqrt{4ac-1}}{n}+1} \text{BesselJ}\left(\frac{\sqrt{(1-4ac)n^2}}{n^2}-1, \frac{2\sqrt{a}\sqrt{b}\sqrt{x^n}}{n}\right) \text{Gamma}\left(\frac{n+\sqrt{1-4ac}}{n}\right) b^{\frac{i\sqrt{4ac-1}}{n}}}{}$$

$$y(x) \to \frac{\frac{\sqrt{a}\sqrt{b}\sqrt{x^n}\left(\text{BesselJ}\left(1-\frac{\sqrt{(1-4ac)n^2}}{n^2}, \frac{2\sqrt{a}\sqrt{b}\sqrt{x^n}}{n}\right)-\text{BesselJ}\left(-\frac{\sqrt{(1-4ac)n^2}}{n^2}-1, \frac{2\sqrt{a}\sqrt{b}\sqrt{x^n}}{n}\right)\right)}{\text{BesselJ}\left(-\frac{\sqrt{(1-4ac)n^2}}{n^2}, \frac{2\sqrt{a}\sqrt{b}\sqrt{x^n}}{n}\right)} - \frac{\sqrt{n^2(1-4ac)}}{n} + i\sqrt{4ac-1} - 1}{2ax}$$

2.16 problem 16

Internal problem ID [10356]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$
\boxed{y'x^2 - y^2x^2 = a\,x^{2m}(b\,x^m + c)^n - \frac{n^2}{4} + \frac{1}{4}}
$$

✗ Solution by Maple

```
dsolve(x^2*diff(y(x),x)=x^2*y(x)^2+a*x^(2*m)*(b*x^m+c)^n+1/4*(1-n^2),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y'[x]==x^2*y[x]^2+a*x^(2*m)*(b*x^m+c)^n+1/4*(1-n^2),y[x],x,IncludeSingularSolutio
```

Not solved

2.17 problem 17

Internal problem ID [10357]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$(c_2x^2 + b_2x + a_2)(y' + \lambda y^2) = -a_0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 2906

```
dsolve((c__2*x^2+b__2*x+a__2)*(diff(y(x),x)+lambda*y(x)^2)+a__0=0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 5.964 (sec). Leaf size: 1046

```
DSolve[(c2*x^2+b2*x+a2)*(y'[x]+\[Lambda]*y[x]^2)+a0==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{(\mathrm{b2}+2\mathrm{c2}x)\left(8\mathrm{c2}\left(\mathrm{b2}^2-4\mathrm{a2c2}\right)G_{2,2}^{2,0}\left(-\frac{4\mathrm{c2}(\mathrm{a2}+x(\mathrm{b2}+\mathrm{c2}x))}{\mathrm{b2}^2-4\mathrm{a2c2}}\middle|\begin{array}{c}\frac{1}{4}-\frac{\sqrt{\mathrm{c2}-4\mathrm{a0}\lambda}}{4\sqrt{\mathrm{c2}}},\frac{1}{4}\left(\frac{\sqrt{\mathrm{c2}-4\mathrm{a0}\lambda}}{\sqrt{\mathrm{c2}}}+1\right)\\0,0\end{array}\right)+c_1\left(\right.}{2\lambda\left(\mathrm{b2}^2\right.}$$

$$y(x) \to \frac{(\mathrm{b2}+2\mathrm{c2}x)\left(2\left(\mathrm{b2}^2-4\mathrm{a2c2}\right)\mathrm{Hypergeometric2F1}\left(\frac{3\mathrm{c2}+\sqrt{\mathrm{c2}(\mathrm{c2}-4\mathrm{a0}\lambda)}}{4\mathrm{c2}},\frac{1}{4}\left(3-\frac{\sqrt{\mathrm{c2}(\mathrm{c2}-4\mathrm{a0}\lambda)}}{\mathrm{c2}}\right),2,-\frac{4\mathrm{c2}}{}\right.\right.}{2\lambda\left(\mathrm{b2}^2-4\mathrm{a2c2}\right)(\mathrm{a2}+x(\mathrm{b2}+\mathrm{c2}x))\,\mathrm{Hyperg}}$$

$$y(x) \to \frac{(\mathrm{b2}+2\mathrm{c2}x)\left(2\left(\mathrm{b2}^2-4\mathrm{a2c2}\right)\mathrm{Hypergeometric2F1}\left(\frac{3\mathrm{c2}+\sqrt{\mathrm{c2}(\mathrm{c2}-4\mathrm{a0}\lambda)}}{4\mathrm{c2}},\frac{1}{4}\left(3-\frac{\sqrt{\mathrm{c2}(\mathrm{c2}-4\mathrm{a0}\lambda)}}{\mathrm{c2}}\right),2,-\frac{4\mathrm{c2}}{}\right.\right.}{2\lambda\left(\mathrm{b2}^2-4\mathrm{a2c2}\right)(\mathrm{a2}+x(\mathrm{b2}+\mathrm{c2}x))\,\mathrm{Hyperg}}$$

2.18 problem 18

Internal problem ID [10358]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Riccati, _special]]`

$$\boxed{x^4y' + x^4y^2 = -a^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(x^4*diff(y(x),x)=-x^4*y(x)^2-a^2,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{a^2}\tan\left(\frac{\sqrt{a^2}\,(xc_1-1)}{x}\right) - x}{x^2}$$

✓ Solution by Mathematica

Time used: 1.107 (sec). Leaf size: 87

```
DSolve[x^4*y'[x]==-x^4*y[x]^2-a^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-2ia^2c_1e^{\frac{2ia}{x}} + 2ac_1xe^{\frac{2ia}{x}} + a - ix}{x^2\left(2ac_1e^{\frac{2ia}{x}} - i\right)}$$

$$y(x) \to \frac{x - ia}{x^2}$$

2.19 problem 19

Internal problem ID [10359]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$a\,x^2(x-1)^2\left(y'+\lambda y^2\right) = -b\,x^2 - cx - s$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 2636

```
dsolve(a*x^2*(x-1)^2*(diff(y(x),x)+lambda*y(x)^2)+b*x^2+c*x+s=0,y(x), singsol=all)
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[a*x^2*(x-1)^2*(y'[x]+\[Lambda]*y[x]^2)+b*x^2+c*x+s==0,y[x],x,IncludeSingularSolutions
```

Timed out

2.20 problem 20

Internal problem ID [10360]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{\left(a\,x^{2}+bx+c\right)^{2}\left(y'+y^{2}\right)=-A}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 846

```
dsolve((a*x^2+b*x+c)^2*(diff(y(x),x)+y(x)^2)+A=0,y(x), singsol=all)
```

$$y(x) = -\frac{2\left(-i\sqrt{-\frac{4ac-b^2+4A}{a^2}}\,\sqrt{4ac-b^2}\left(\frac{i\sqrt{4ac-b^2}-2ax-b}{2ax+b+i\sqrt{4ac-b^2}}\right)^{-\frac{a\sqrt{-\frac{4ac-b^2+4A}{a^2}}}{2\sqrt{-4ac+b^2}}} c_1 a + i\sqrt{-\frac{4ac-b^2+4A}{a^2}}\,\sqrt{4ac-b^2}\left(\frac{i\sqrt{4ac-b}}{2ax+b+i}\right.\cdots}{\sqrt{-4c\cdots}}$$

✓ Solution by Mathematica

Time used: 5.579 (sec). Leaf size: 743

```
DSolve[(a*x^2+b*x+c)^2*(y'[x]+y[x]^2)+A==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{b^2 c_1 \left(-\exp\left(\frac{2\sqrt{4ac-b^2}\sqrt{1-\frac{4A}{b^2-4ac}}\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{b^2-4ac}}\right)\right) + bc_1\sqrt{b^2-4ac}\sqrt{1-\frac{4A}{b^2-4ac}}\exp\left(\frac{2\sqrt{4ac-b^2}\sqrt{1-\frac{4A}{b^2-4ac}}\,\mathrm{a}}{\sqrt{b^2-4a}}\right.}{}$$

$$y(x) \rightarrow \frac{2ax\sqrt{b^2-4ac}\sqrt{1-\frac{4A}{b^2-4ac}} + b\sqrt{b^2-4ac}\sqrt{1-\frac{4A}{b^2-4ac}} + 4ac + 4A - b^2}{2\sqrt{b^2-4ac}\sqrt{1-\frac{4A}{b^2-4ac}}(x(ax+b)+c)}$$

2.21 problem 21

Internal problem ID [10361]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{x^{n+1}y' - x^{2n}y^2 a = c\,x^m + d}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 270

```
dsolve(x^(n+1)*diff(y(x),x)=a*x^(2*n)*y(x)^2+c*x^m+d,y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(-\sqrt{-4ad+n^2}\,c_1 - c_1 n\right) \operatorname{BesselY}\left(\frac{\sqrt{-4ad+n^2}}{m}, \frac{2\sqrt{ac}\,x^{\frac{m}{2}}}{m}\right) + 2x^{\frac{m}{2}}\sqrt{ac}\, \operatorname{BesselY}\left(\frac{\sqrt{-4ad+n^2}+m}{m}, \frac{2\sqrt{ac}\,x^{\frac{m}{2}}}{m}\right) c_1}{2xa\left(\operatorname{BesselY}\left(\frac{\sqrt{-4ad+n^2}}{m}, \frac{2\sqrt{ac}\,x^{\frac{m}{2}}}{m}\right)\right.}$$

✓ Solution by Mathematica

Time used: 2.124 (sec). Leaf size: 1890

```
DSolve[x^(n+1)*y'[x]==a*x^(2*n)*y[x]^2+c*x^m+d,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{-n}\left(a^{\frac{\sqrt{n^2-4ad}}{m}} m^{\frac{2\sqrt{m^2(n^2-4ad)}}{m^2}}\left(\sqrt{m^2(n^2-4ad)}-m\left(n+\sqrt{n^2-4ad}\right)\right)(x^m)^{\frac{\sqrt{n^2-4ad}}{m}+\frac{1}{2}}\operatorname{BesselJ}\left(\frac{\sqrt{m^2(n^2-}{m^2}\right.\right.}{}$$

$$y(x) \to \frac{x^{-n}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{x^m}\left(\operatorname{BesselJ}\left(1-\frac{\sqrt{m^2(n^2-4ad)}}{m^2},\frac{2\sqrt{a}\sqrt{c}\sqrt{x^m}}{m}\right)-\operatorname{BesselJ}\left(-\frac{\sqrt{m^2(n^2-4ad)}}{m^2}-1,\frac{2\sqrt{a}\sqrt{c}\sqrt{x^m}}{m}\right)\right)}{\operatorname{BesselJ}\left(-\frac{\sqrt{m^2(n^2-4ad)}}{m^2},\frac{2\sqrt{a}\sqrt{c}\sqrt{x^m}}{m}\right)}-\frac{\sqrt{m^2(n^2-4ad)}}{m}+\sqrt{n^2}\right.}{2a}$$

2.22 problem 22

Internal problem ID [10362]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$(x^n a + b)\, y' - b y^2 = a\, x^{n-2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 333

```
dsolve((a*x^n+b)*diff(y(x),x)=b*y(x)^2+a*x^(n-2),y(x), singsol=all)
```

$$y(x) = -\frac{\left(-x^{2n} c_1 a^2 n - x^n c_1 a b n\right) \mathrm{hypergeom}\left(\left[2, \frac{1+n}{n}\right], \left[\frac{2n-1}{n}\right], -\frac{a\, x^n}{b}\right)}{\left(\mathrm{hypergeom}\left(\left[\frac{2}{n}\right], [\,], -\frac{a\, x^n}{b}\right) x + \mathrm{hypergeom}\left(\left[1, \frac{1}{n}\right], \left[\frac{n-1}{n}\right], -\frac{a\, x^n}{b}\right) c_1\right)(n-1)\, b^2 x} - \frac{\left(x^n c_1 a b n - x^n c_1 a b\right) \mathrm{hypergeom}\left(\left[1, \frac{1}{n}\right], \left[\frac{n-1}{n}\right], -\frac{a\, x^n}{b}\right) + \left(2abn\, x^{1+n} - 2ab\, x^{1+n} + b^2 n x - b^2 x\right) \mathrm{hyperg}}{\left(\mathrm{hypergeom}\left(\left[\frac{2}{n}\right], [\,], -\frac{a\, x^n}{b}\right) x + \mathrm{hyperg}}$$

✓ Solution by Mathematica

Time used: 1.899 (sec). Leaf size: 289

```
DSolve[(a*x^n+b)*y'[x]==b*y[x]^2+a*x^(n-2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-b^2(-1)^{\frac{1}{n}}(n-1)\left(-\frac{ax^n}{b}\right)^{\frac{1}{n}} - abc_1(n-1)x^n\left(\frac{ax^n}{b}+1\right)^{2/n}\text{Hypergeometric2F1}\left(1,\frac{1}{n},\frac{n-1}{n},-\frac{ax^n}{b}\right) + ac_1nx}{b^2(n-1)x\left((-1)^{\frac{1}{n}}\left(-\frac{ax^n}{b}\right)^{\frac{1}{n}} + c_1\left(\frac{ax^n}{b}+1\right)^{2/n}\text{Hypergeomet}\right.}$$

$$y(x) \to \frac{ax^{n-1}\left(\frac{n(ax^n+b)\,\text{Hypergeometric2F1}\left(2,1+\frac{1}{n},2-\frac{1}{n},-\frac{ax^n}{b}\right)}{\text{Hypergeometric2F1}\left(1,\frac{1}{n},\frac{n-1}{n},-\frac{ax^n}{b}\right)} + b(-n) + b\right)}{b^2(n-1)}$$

2.23 problem 23

Internal problem ID [10363]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$
\boxed{(x^n a + b\,x^m + c)\left(y' - y^2\right) = -an(-1+n)\,x^{n-2} - bm(m-1)\,x^{m-2}}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 173

```
dsolve((a*x^n+b*x^m+c)*(diff(y(x),x)-y(x)^2)+a*n*(n-1)*x^(n-2)+b*m*(m-1)*x^(m-2)=0,y(x), sin
```

$$
y(x) = -\frac{\left(a^2x^{2n}n + abm\,x^{m+n} + abn\,x^{m+n} + x^{2m}b^2m + can\,x^n + x^m bcm\right)\left(\int \frac{1}{(a\,x^n+b\,x^m+c)^2}dx\right) + x^{2n}c_1a^2n + ab}{(a\,x^n + b\,x^m + c)^2\,x\left(c_1 + \int \frac{1}{(a\,x^n+b\,x^m+c)^2}\right.}
$$

✓ Solution by Mathematica

Time used: 35.099 (sec). Leaf size: 201

```
DSolve[(a*x^n+b*x^m+c)*(y'[x]-y[x]^2)+a*n*(n-1)*x^(n-2)+b*m*(m-1)*x^(m-2)==0,y[x],x,IncludeS
```

$$
y(x) \to -\frac{c_1\left(\frac{(anx^n+bmx^m)\int_1^x \frac{1}{(bK[1]^m+aK[1]^n+c)^2}dK[1]}{x} + \frac{1}{ax^n+bx^m+c}\right) + anx^{n-1} + bmx^{m-1}}{(ax^n+bx^m+c)\left(1 + c_1\int_1^x \frac{1}{(bK[1]^m+aK[1]^n+c)^2}dK[1]\right)}
$$

$$
y(x) \to -\frac{\frac{1}{\int_1^x \frac{1}{(bK[1]^m+aK[1]^n+c)^2}dK[1]} + \frac{(anx^n+bmx^m)(ax^n+bx^m+c)}{x}}{(ax^n+bx^m+c)^2}
$$

2.24 problem 24

Internal problem ID [10364]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - ay^2 - by = cx + k}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 194

```
dsolve(diff(y(x),x)=a*y(x)^2+b*y(x)+c*x+k,y(x), singsol=all)
```

$$y(x) = \frac{2\sqrt{a}\left(\frac{c}{\sqrt{a}}\right)^{\frac{1}{3}}\left(\operatorname{AiryAi}\left(1, -\frac{a(cx+k)-\frac{b^2}{4}}{\left(\frac{c}{\sqrt{a}}\right)^{\frac{2}{3}}a}\right)c_1 + \operatorname{AiryBi}\left(1, -\frac{a(cx+k)-\frac{b^2}{4}}{\left(\frac{c}{\sqrt{a}}\right)^{\frac{2}{3}}a}\right)\right) - \left(c_1\operatorname{AiryAi}\left(-\frac{a(cx+k)-\frac{b^2}{4}}{\left(\frac{c}{\sqrt{a}}\right)^{\frac{2}{3}}a}\right) \cdots}{2a\left(c_1\operatorname{AiryAi}\left(-\frac{a(cx+k)-\frac{b^2}{4}}{\left(\frac{c}{\sqrt{a}}\right)^{\frac{2}{3}}a}\right) + \operatorname{AiryBi}\left(-\frac{a(cx+k)-\frac{b^2}{4}}{\left(\frac{c}{\sqrt{a}}\right)^{\frac{2}{3}}a}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.51 (sec). Leaf size: 359

```
DSolve[y'[x]==a*y[x]^2+b*y[x]+c*x+k,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to$$
$$-\frac{c\left(-b(-ac)^{2/3}\operatorname{AiryBi}\left(\frac{b^2-4a(k+cx)}{4(-ac)^{2/3}}\right)+2ac\operatorname{AiryBiPrime}\left(\frac{b^2-4a(k+cx)}{4(-ac)^{2/3}}\right)+c_1\left(2ac\operatorname{AiryAiPrime}\left(\frac{b^2-4a(k+}{4(-ac)^2}\right.\right.\right.}{2(-ac)^{5/3}\left(\operatorname{AiryBi}\left(\frac{b^2-4a(k+cx)}{4(-ac)^{2/3}}\right)+c_1\operatorname{AiryAi}\left(\frac{b^2-4a(k+cx)}{4(-ac)^{2/3}}\right)\right)}$$

$$y(x) \to -\frac{\frac{2\sqrt[3]{-ac}\operatorname{AiryAiPrime}\left(\frac{b^2-4a(k+cx)}{4(-ac)^{2/3}}\right)}{\operatorname{AiryAi}\left(\frac{b^2-4a(k+cx)}{4(-ac)^{2/3}}\right)}+b}{2a}$$

$$y(x) \to -\frac{\frac{2\sqrt[3]{-ac}\operatorname{AiryAiPrime}\left(\frac{b^2-4a(k+cx)}{4(-ac)^{2/3}}\right)}{\operatorname{AiryAi}\left(\frac{b^2-4a(k+cx)}{4(-ac)^{2/3}}\right)}+b}{2a}$$

2.25 problem 25

Internal problem ID [10365]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a\,x^n y = a\,x^{-1+n}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 385

```
dsolve(diff(y(x),x)=y(x)^2+a*x^n*y(x)+a*x^(n-1),y(x), singsol=all)
```

$$y(x) = \frac{\quad}{x^2 \left(c_1 - \frac{\left(\frac{a}{-1-n}\right)^{\frac{1}{1+n}} \left(\frac{(-1-n)^2 x^{-1-\frac{1}{1+n}-\frac{n}{1+n}-n} \left(\frac{a}{-1-n}\right)^{-\frac{1}{1+n}} \left(\frac{x^{1+n} a n^2}{-1-n} + \frac{2x^{1+n} a n}{-1-n} + n^2 + \frac{x^{1+n} a}{-1-n} + n\right) \left(\frac{x^{1+n} a}{-1-n}\right)^{-\frac{n}{2(1+n)}} \mathrm{e}^{-\frac{x^{1+n} a}{2(-1-n)}} \mathrm{W}}{n(2n+1)a} \right.}{} \right)} - \frac{1}{x}$$

✓ Solution by Mathematica

Time used: 2.82 (sec). Leaf size: 136

```
DSolve[y'[x]==y[x]^2+a*x^n*y[x]+a*x^(n-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{\left(-\frac{ax^{n+1}}{n+1}\right)^{\frac{1}{n+1}}\Gamma\left(-\frac{1}{n+1},-\frac{ax^{n+1}}{n+1}\right)-(n+1)\left(e^{\frac{ax^{n+1}}{n+1}}+c_1x\right)}{x\left(-\left(-\frac{ax^{n+1}}{n+1}\right)^{\frac{1}{n+1}}\Gamma\left(-\frac{1}{n+1},-\frac{ax^{n+1}}{n+1}\right)+c_1(n+1)x\right)}$$

$$y(x)\to -\frac{1}{x}$$

2.26 problem 26

Internal problem ID [10366]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a\,x^n y = b\,x^{-1+n}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 376

```
dsolve(diff(y(x),x)=y(x)^2+a*x^n*y(x)+b*x^(n-1),y(x), singsol=all)
```

$$y(x) = \frac{(c_1 a n + c_1 a)\operatorname{KummerU}\left(-\frac{an+b}{a(1+n)}, \frac{n+2}{1+n}, \frac{a\,x^{1+n}}{1+n}\right)}{\left(\operatorname{KummerU}\left(\frac{a-b}{a(1+n)}, \frac{n+2}{1+n}, \frac{a\,x^{1+n}}{1+n}\right) c_1 + \operatorname{KummerM}\left(\frac{a-b}{a(1+n)}, \frac{n+2}{1+n}, \frac{a\,x^{1+n}}{1+n}\right)\right) a x}$$
$$+ \frac{(-x^{1+n} c_1 a^2 + c_1 a n + c_1 b)\operatorname{KummerU}\left(\frac{a-b}{a(1+n)}, \frac{n+2}{1+n}, \frac{a\,x^{1+n}}{1+n}\right) + (-an - a - b)\operatorname{KummerM}\left(-\frac{an+b}{a(1+n)}, \frac{n+2}{1+n}\right.}{\left(\operatorname{KummerU}\left(\frac{a-b}{a(1+n)}, \frac{n+2}{1+n}, \frac{a\,x^{1+n}}{1+n}\right) c_1 + \operatorname{KummerM}\left(\frac{a-b}{a(1+n}\right.\right.}$$

✓ Solution by Mathematica

Time used: 0.956 (sec). Leaf size: 453

```
DSolve[y'[x]==y[x]^2+a*x^n*y[x]+b*x^(n-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(x^n)^{\frac{1}{n}} \left(-(-1)^{\frac{1}{n+1}} n(n+2) a^{\frac{1}{n+1}} \text{Hypergeometric1F1}\left(\frac{a-b}{na+a}, \frac{n+2}{n+1}, \frac{a(x^n)^{1+\frac{1}{n}}}{n+1}\right) + x^n \left(-(-1)^{\frac{1}{n+1}} n(a-b) a^{\cdots}\right.\right.}{n(n+2)x \left((-1)^{\frac{1}{n+1}} a^{\frac{1}{n+1}} (x^n)^{\frac{1}{n}} \text{Hypergeometric1F1}\left(\cdots\right.\right.}$$

$$y(x) \to \frac{b x^{n-1} (x^n)^{\frac{1}{n}} \text{Hypergeometric1F1}\left(\frac{na+a-b}{na+a}, \frac{2n+1}{n+1}, \frac{a(x^n)^{1+\frac{1}{n}}}{n+1}\right)}{n \,\text{Hypergeometric1F1}\left(-\frac{b}{na+a}, \frac{n}{n+1}, \frac{a(x^n)^{1+\frac{1}{n}}}{n+1}\right)}$$

2.27 problem 27

Internal problem ID [10367]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - (\alpha x + \beta)\, y = a\, x^2 + bx + c$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 4562

```
dsolve(diff(y(x),x)=y(x)^2+(alpha*x+beta)*y(x)+a*x^2+b*x+c,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 4.264 (sec). Leaf size: 1291

```
DSolve[y'[x]==y[x]^2+(\[Alpha]*x+\[Beta])*y[x]+a*x^2+b*x+c,y[x],x,IncludeSingularSolutions -
```

$y(x) \to$

$$-\frac{2\left(2b+4ax+\left(\sqrt{\alpha^2-4a}-\alpha\right)(x\alpha+\beta)\right)\text{Hypergeometric1F1}\left(-\frac{2b^2-2\alpha\beta b+\alpha^2\left(2c+\alpha-\sqrt{\alpha^2-4a}\right)+2a\left(\beta^2-4c-\right.}{4(\alpha^2-4a)^{3/2}}\right.}{}$$

$y(x)$

$$\to \frac{(4a-\alpha^2)\left(\left(\sqrt{\alpha^2-4a}-\alpha\right)(\beta+\alpha x)+4ax+2b\right)-\frac{\sqrt{2}\sqrt[4]{\alpha^2-4a}\left(2a\left(2\sqrt{\alpha^2-4a}-2\alpha+\beta^2-4c\right)+\alpha^2\left(-\sqrt{\alpha^2-4a}+\alpha+\right.\right.}{\text{HermiteH}\left(-\frac{-2b^2+2\alpha\beta b+\alpha}{}\right.}}{2\left(\alpha^2-4a\right)^{3/2}}$$

2.28 problem 28

Internal problem ID [10368]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a\,x^n y = -ab\,x^n - b^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 89

```
dsolve(diff(y(x),x)=y(x)^2+a*x^n*y(x)-a*b*x^n-b^2,y(x), singsol=all)
```

$$c_1 + \int^x -\frac{(-\_ab + \_ay(x))\,\mathrm{e}^{\frac{2\_abn + a\_a^{1+n} + 2\_ab}{1+n}}}{(b - y(x))\_a}d\_a - \frac{\mathrm{e}^{\frac{2bnx + a\,x^{1+n} + 2xb}{1+n}}}{b - y(x)} = 0$$

✓ Solution by Mathematica

Time used: 1.948 (sec). Leaf size: 195

```
DSolve[y'[x]==y[x]^2+a*x^n*y[x]-a*b*x^n-b^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{e^{\frac{ax^{n+1}}{n+1}+2bx}}{an(K[2]-b)^2}\right.\right.$$
$$\left.-\int_1^x\left(\frac{e^{\frac{aK[1]^{n+1}}{n+1}+2bK[1]}(aK[1]^n+b+K[2])}{an(b-K[2])^2}+\frac{e^{\frac{aK[1]^{n+1}}{n+1}+2bK[1]}}{an(b-K[2])}\right)dK[1]\right)dK[2]$$
$$\left.+\int_1^x\frac{e^{\frac{aK[1]^{n+1}}{n+1}+2bK[1]}(aK[1]^n+b+y(x))}{an(b-y(x))}dK[1]=c_1,y(x)\right]$$

2.29 problem 29

Internal problem ID [10369]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + (n+1)\, x^n y^2 = a\, x^{m+1+n} - a\, x^m}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=-(n+1)*x^n*y(x)^2+a*x^(n+m+1)-a*x^m,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-(n+1)*x^n*y[x]^2+a*x^(n+m+1)-a*x^m,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.30 problem 30

Internal problem ID [10370]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a\,x^n y^2 - b\,x^m y = bc\,x^m - a\,c^2 x^n}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 149

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2+b*x^m*y(x)+b*c*x^m-a*c^2*x^n,y(x), singsol=all)
```

$$c_1 + \int^x -\frac{\left(-ac\_a^{1+n} - ay(x)\_a^{1+n}\right) \mathrm{e}^{-\frac{2ac\_a^{1+n}m - \_a^{m+1}bn + 2ac\_a^{1+n} - \_a^{m+1}b}{(1+n)(m+1)}}}{\left(c + y\left(x\right)\right)\_a} d\_a + \frac{\mathrm{e}^{-\frac{2ac\,x^{1+n}m - x^{m+1}bn + 2ac\,x^{1+n} - x^{m+1}b}{(1+n)(m+1)}}}{c + y\left(x\right)} = 0$$

✓ Solution by Mathematica

Time used: 3.436 (sec). Leaf size: 286

```
DSolve[y'[x]==a*x^n*y[x]^2+b*x^m*y[x]+b*c*x^m-a*c^2*x^n,y[x],x,IncludeSingularSolutions -> T
```

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{e^{\frac{bx^{m+1}}{m+1}-\frac{2acx^{n+1}}{n+1}}}{ab(m-n)(c+K[2])^2}\right.\right.$$

$$-\int_1^x\left(-\frac{\exp\left(\frac{bK[1]^{m+1}}{m+1}-\frac{2acK[1]^{n+1}}{n+1}\right)K[1]^n}{b(m-n)(c+K[2])}-\frac{\exp\left(\frac{bK[1]^{m+1}}{m+1}-\frac{2acK[1]^{n+1}}{n+1}\right)(-bK[1]^m+acK[1]^n-aK[2]K[}{ab(m-n)(c+K[2])^2}\right.$$

$$\left.+\int_1^x\frac{\exp\left(\frac{bK[1]^{m+1}}{m+1}-\frac{2acK[1]^{n+1}}{n+1}\right)(-bK[1]^m+acK[1]^n-ay(x)K[1]^n)}{ab(m-n)(c+y(x))}dK[1]=c_1,y(x)\right]$$

2.31 problem 31

Internal problem ID [10371]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,x^n y^2 + a\,x^n (b\,x^m + c)\,y = bm\,x^{m-1}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2-a*x^n*(b*x^m+c)*y(x)+b*m*x^(m-1),y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 59.342 (sec). Leaf size: 353

```
DSolve[y'[x]==a*x^n*y[x]^2-a*x^n*(b*x^m+c)*y[x]+b*m*x^(m-1),y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{bm(bx^m+c)^2\left(1+c_1\int_1^x \frac{\exp\left(aK[1]^{n+1}\left(\frac{bK[1]^m}{m+n+1}+\frac{c}{n+1}\right)\right)K[1]^{m-1}}{(bK[1]^m+c)^2}dK[1]\right)}{bc_1m\left(bx^m+c\right)\int_1^x \frac{\exp\left(aK[1]^{n+1}\left(\frac{bK[1]^m}{m+n+1}+\frac{c}{n+1}\right)\right)K[1]^{m-1}}{(bK[1]^m+c)^2}dK[1]+c_1e^{ax^{n+1}\left(\frac{bx^m}{m+n+1}+\frac{c}{n+1}\right)}+b^2mx^m+bcm}$$

$$y(x) \to \frac{bm(bx^m+c)^2\int_1^x \frac{\exp\left(aK[1]^{n+1}\left(\frac{bK[1]^m}{m+n+1}+\frac{c}{n+1}\right)\right)K[1]^{m-1}}{(bK[1]^m+c)^2}dK[1]}{bm\left(bx^m+c\right)\int_1^x \frac{\exp\left(aK[1]^{n+1}\left(\frac{bK[1]^m}{m+n+1}+\frac{c}{n+1}\right)\right)K[1]^{m-1}}{(bK[1]^m+c)^2}dK[1]+e^{ax^{n+1}\left(\frac{bx^m}{m+n+1}+\frac{c}{n+1}\right)}}$$

2.32 problem 32

Internal problem ID [10372]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + an\,x^{-1+n}y^2 - c\,x^m(x^n a + b)\,y = -c\,x^m}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 400

```
dsolve(diff(y(x),x)=-a*n*x^(n-1)*y(x)^2+c*x^m*(a*x^n+b)*y(x)-c*x^m,y(x), singsol=all)
```

$$y(x) = - \frac{-\left(\int -\frac{e^{\frac{cx\,x^m(x^n am + a\,x^n + mb + bn + b)}{(m+1)(m+n+1)}} an\,x^n}{(a\,x^n+b)^2 x}dx\right)x^n a - c_1 a\,x^n - \left(\int -\frac{e^{\frac{cx\,x^m(x^n am + a\,x^n + mb + bn + b)}{(m+1)(m+n+1)}}}{(a\,x^n+b)^2 x}\right.}{\left(\int -\frac{e^{\frac{cx\,x^m(x^n am + a\,x^n + mb + bn + b)}{(m+1)(m+n+1)}} an\,x^n}{(a\,x^n+b)^2 x}dx\right)x^{2n}a^2 + x^{2n}c_1 a^2 + 2\left(\int -\frac{e^{\frac{cx\,x^m(x^n am + a\,x^n + mb + bn + b)}{(m+1)(m+n+1)}} an\,x^n}{(a\,x^n+b)^2 x}dx\right)x^n ab -}$$

✓ Solution by Mathematica

Time used: 8.659 (sec). Leaf size: 304

```
DSolve[y'[x]==-a*n*x^(n-1)*y[x]^2+c*x^m*(a*x^n+b)*y[x]-c*x^m,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{ac_1n(ax^n+b)\int_1^x \frac{\exp\left(cK[1]^{m+1}\left(\frac{aK[1]^n}{m+n+1}+\frac{b}{m+1}\right)\right)K[1]^{n-1}}{(aK[1]^n+b)^2}dK[1]+a^2nx^n+c_1e^{cx^{m+1}\left(\frac{ax^n}{m+n+1}+\frac{b}{m+1}\right)}+abn}{an\left(ax^n+b\right)^2\left(1+c_1\int_1^x \frac{\exp\left(cK[1]^{m+1}\left(\frac{aK[1]^n}{m+n+1}+\frac{b}{m+1}\right)\right)K[1]^{n-1}}{(aK[1]^n+b)^2}dK[1]\right)}$$

$$y(x) \to \frac{\frac{e^{cx^{m+1}\left(\frac{ax^n}{m+n+1}+\frac{b}{m+1}\right)}}{an\int_1^x \frac{\exp\left(cK[1]^{m+1}\left(\frac{aK[1]^n}{m+n+1}+\frac{b}{m+1}\right)\right)K[1]^{n-1}}{(aK[1]^n+b)^2}dK[1]}+ax^n+b}{\left(ax^n+b\right)^2}$$

2.33 problem 33

Internal problem ID [10373]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,x^n y^2 - b\,x^m y = ck\,x^{k-1} - bc\,x^{m+k} - a\,c^2 x^{n+2k}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2+b*x^m*y(x)+c*k*x^(k-1)-b*c*x^(m+k)-a*c^2*x^(n+2*k),y(x), si
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*x^n*y[x]^2+b*x^m*y[x]+c*k*x^(k-1)-b*c*x^(m+k)-a*c^2*x^(n+2*k),y[x],x,Include
```

Not solved

2.34 problem 34

Internal problem ID [10374]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - ay^2 - by = c\,x^{2b}}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 38

```
dsolve(x*diff(y(x),x)=a*y(x)^2+b*y(x)+c*x^(2*b),y(x), singsol=all)
```

$$y(x) = \frac{\tan\left(\frac{\sqrt{a}\,\sqrt{c}\,x^b - c_1 b}{b}\right)\sqrt{c}\,x^b}{\sqrt{a}}$$

✓ Solution by Mathematica

Time used: 0.544 (sec). Leaf size: 139

```
DSolve[x*y'[x]==a*y[x]^2+b*y[x]+c*x^(2*b),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{c}x^b\left(-\cos\left(\frac{\sqrt{a}\sqrt{c}x^b}{b}\right) + c_1\sin\left(\frac{\sqrt{a}\sqrt{c}x^b}{b}\right)\right)}{\sqrt{a}\left(\sin\left(\frac{\sqrt{a}\sqrt{c}x^b}{b}\right) + c_1\cos\left(\frac{\sqrt{a}\sqrt{c}x^b}{b}\right)\right)}$$

$$y(x) \to \frac{\sqrt{c}x^b\tan\left(\frac{\sqrt{a}\sqrt{c}x^b}{b}\right)}{\sqrt{a}}$$

2.35 problem 35

Internal problem ID [10375]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$y'x - ay^2 - by = c\,x^n$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 225

```
dsolve(x*diff(y(x),x)=a*y(x)^2+b*y(x)+c*x^n,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{ac}\,x^{\frac{n}{2}} c_1 \operatorname{BesselY}\left(\frac{b+n}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)}{a\left(\operatorname{BesselY}\left(\frac{b}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right) c_1 + \operatorname{BesselJ}\left(\frac{b}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)\right)} + \frac{\operatorname{BesselJ}\left(\frac{b+n}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)\sqrt{ac}\,x^{\frac{n}{2}} - \operatorname{BesselY}\left(\frac{b}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right) c_1 b - b\operatorname{BesselJ}\left(\frac{b}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)}{a\left(\operatorname{BesselY}\left(\frac{b}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right) c_1 + \operatorname{BesselJ}\left(\frac{b}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.513 (sec). Leaf size: 402

```
DSolve[x*y'[x]==a*y[x]^2+b*y[x]+c*x^n,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a}\sqrt{c}x^{n/2}\left(-2\,\text{BesselJ}\left(\frac{b}{n}-1,\frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right)+c_1\left(\text{BesselJ}\left(1-\frac{b}{n},\frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right)-\text{BesselJ}\left(-\frac{b+n}{n},\frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right.\right.\right.}{2a\left(\text{BesselJ}\left(\frac{b}{n},\frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right)+c_1\,\text{BesselJ}\left(-\frac{b}{n},\frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right)\right)}$$

$$y(x) \to -\frac{-\sqrt{a}\sqrt{c}x^{n/2}\,\text{BesselJ}\left(1-\frac{b}{n},\frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right)+\sqrt{a}\sqrt{c}x^{n/2}\,\text{BesselJ}\left(-\frac{b+n}{n},\frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right)+b\,\text{BesselJ}\left(-\frac{b}{n},\frac{2\sqrt{a}\right.}{2a\,\text{BesselJ}\left(-\frac{b}{n},\frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right)}$$

2.36 problem 36

Internal problem ID [10376]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - ay^2 - (n + b\,x^n)\,y = c\,x^{2n}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 81

```
dsolve(x*diff(y(x),x)=a*y(x)^2+(n+b*x^n)*y(x)+c*x^(2*n),y(x), singsol=all)
```

$$y(x) = \frac{x^{2n-1}\left(\sqrt{4b^2ac - b^4}\,\tan\left(\frac{\sqrt{4b^2ac-b^4}\,(b\,x^n+c_1n)}{2b^2n}\right) - b^2\right)x^{-n+1}}{2ab}$$

✓ Solution by Mathematica

Time used: 1.07 (sec). Leaf size: 114

```
DSolve[x*y'[x]==a*y[x]^2+(n+b*x^n)*y[x]+c*x^(2*n),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^n\left(-b + \frac{\sqrt{b^2-4ac}\left(-e^{\frac{x^n\sqrt{b^2-4ac}}{n}}+c_1\right)}{e^{\frac{x^n\sqrt{b^2-4ac}}{n}}+c_1}\right)}{2a}$$

$$y(x) \to \frac{x^n\left(\sqrt{b^2-4ac} - b\right)}{2a}$$

2.37 problem 37

Internal problem ID [10377]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - y^2x - ay = b\,x^n}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 216

```
dsolve(x*diff(y(x),x)=x*y(x)^2+a*y(x)+b*x^n,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^{\frac{n}{2}+\frac{1}{2}}\sqrt{b}\,\text{BesselY}\left(-\frac{a-n}{1+n}, \frac{2\sqrt{b}\,x^{\frac{n}{2}+\frac{1}{2}}}{1+n}\right)}{\left(\text{BesselY}\left(-\frac{a+1}{1+n}, \frac{2\sqrt{b}\,x^{\frac{n}{2}+\frac{1}{2}}}{1+n}\right)c_1 + \text{BesselJ}\left(-\frac{a+1}{1+n}, \frac{2\sqrt{b}\,x^{\frac{n}{2}+\frac{1}{2}}}{1+n}\right)\right)x} + \frac{\text{BesselJ}\left(-\frac{a-n}{1+n}, \frac{2\sqrt{b}\,x^{\frac{n}{2}+\frac{1}{2}}}{1+n}\right)\sqrt{b}\,x^{\frac{n}{2}+\frac{1}{2}}}{\left(\text{BesselY}\left(-\frac{a+1}{1+n}, \frac{2\sqrt{b}\,x^{\frac{n}{2}+\frac{1}{2}}}{1+n}\right)c_1 + \text{BesselJ}\left(-\frac{a+1}{1+n}, \frac{2\sqrt{b}\,x^{\frac{n}{2}+\frac{1}{2}}}{1+n}\right)\right)x}$$

✓ Solution by Mathematica

Time used: 1.381 (sec). Leaf size: 855

```
DSolve[x*y'[x]==x*y[x]^2+a*y[x]+b*x^n,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to$$

$$-\frac{\sqrt{b}(x^n)^{\frac{n+1}{2n}}\,\text{Gamma}\left(\frac{a+n+2}{n+1}\right)\text{BesselJ}\left(\frac{a-n}{n+1},\frac{2\sqrt{b}(x^n)^{\frac{n+1}{2n}}}{n+1}\right)-\sqrt{b}(x^n)^{\frac{n+1}{2n}}\,\text{Gamma}\left(\frac{a+n+2}{n+1}\right)\text{BesselJ}\left(\frac{a+n+2}{n+1}\right.}{}$$

$$y(x) \to -\frac{\frac{\sqrt{b}(x^n)^{\frac{n+1}{2n}}\left(\text{BesselJ}\left(-\frac{a+n+2}{n+1},\frac{2\sqrt{b}(x^n)^{\frac{n+1}{2n}}}{n+1}\right)-\text{BesselJ}\left(\frac{n-a}{n+1},\frac{2\sqrt{b}(x^n)^{\frac{n+1}{2n}}}{n+1}\right)\right)}{\text{BesselJ}\left(-\frac{a+1}{n+1},\frac{2\sqrt{b}(x^n)^{\frac{n+1}{2n}}}{n+1}\right)}+a+1}{2x}$$

$$y(x) \to -\frac{\frac{\sqrt{b}(x^n)^{\frac{n+1}{2n}}\left(\text{BesselJ}\left(-\frac{a+n+2}{n+1},\frac{2\sqrt{b}(x^n)^{\frac{n+1}{2n}}}{n+1}\right)-\text{BesselJ}\left(\frac{n-a}{n+1},\frac{2\sqrt{b}(x^n)^{\frac{n+1}{2n}}}{n+1}\right)\right)}{\text{BesselJ}\left(-\frac{a+1}{n+1},\frac{2\sqrt{b}(x^n)^{\frac{n+1}{2n}}}{n+1}\right)}+a+1}{2x}$$

2.38 problem 38

Internal problem ID [10378]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x + a_3xy^2 + a_2y = -a_1x - a_0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 848

```
dsolve(x*diff(y(x),x)+a__3*x*y(x)^2+a__2*y(x)+a__1*x+a__0=0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 0.748 (sec). Leaf size: 541

```
DSolve[x*y'[x]+a3*x*y[x]^2+a2*y[x]+a1*x+a0==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{i\left(\sqrt{\text{a1}}c_1 \text{HypergeometricU}\left(\frac{1}{2}\left(\frac{i\sqrt{\text{a3}}\text{a0}}{\sqrt{\text{a1}}}+\text{a2}\right),\text{a2},2i\sqrt{\text{a1}}\sqrt{\text{a3}}x\right)+c_1\left(\sqrt{\text{a1}}\text{a2}+i\text{a0}\sqrt{\text{a3}}\right)\text{Hypergeom}}{\sqrt{\text{a3}}\left(c_1 \text{HypergeometricU}\left(\frac{1}{2}\left(\frac{i\sqrt{\text{a3}}}{\sqrt{\text{a}}}\right.\right.\right.}$$

$$y(x) \to \frac{\frac{\left(\text{a0}\sqrt{\text{a3}}-i\sqrt{\text{a1}}\text{a2}\right)\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{i\sqrt{\text{a3}}\text{a0}}{\sqrt{\text{a1}}}+\text{a2}+2\right),\text{a2}+1,2i\sqrt{\text{a1}}\sqrt{\text{a3}}x\right)}{\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{i\sqrt{\text{a3}}\text{a0}}{\sqrt{\text{a1}}}+\text{a2}\right),\text{a2},2i\sqrt{\text{a1}}\sqrt{\text{a3}}x\right)}-i\sqrt{\text{a1}}}{\sqrt{\text{a3}}}$$

$$y(x) \to \frac{\frac{\left(\text{a0}\sqrt{\text{a3}}-i\sqrt{\text{a1}}\text{a2}\right)\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{i\sqrt{\text{a3}}\text{a0}}{\sqrt{\text{a1}}}+\text{a2}+2\right),\text{a2}+1,2i\sqrt{\text{a1}}\sqrt{\text{a3}}x\right)}{\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{i\sqrt{\text{a3}}\text{a0}}{\sqrt{\text{a1}}}+\text{a2}\right),\text{a2},2i\sqrt{\text{a1}}\sqrt{\text{a3}}x\right)}-i\sqrt{\text{a1}}}{\sqrt{\text{a3}}}$$

2.39 problem 39

Internal problem ID [10379]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Riccati]`

$$y'x - a\,x^n y^2 - by = c\,x^{-n}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 69

```
dsolve(x*diff(y(x),x)=a*x^n*y(x)^2+b*y(x)+c*x^(-n),y(x), singsol=all)
```

$$y(x) = -\frac{x^{-n}\left(b+n+\tan\left(\frac{\sqrt{4ac-b^2-2bn-n^2}\,(-\ln(x)+c_1)}{2}\right)\sqrt{4ac-b^2-2bn-n^2}\right)}{2a}$$

✓ Solution by Mathematica

Time used: 0.978 (sec). Leaf size: 138

```
DSolve[x*y'[x]==a*x^n*y[x]^2+b*y[x]+c*x^(-n),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{-n}\left(\frac{\sqrt{-4ac+b^2+2bn+n^2}\left(-x^{\sqrt{-4ac+b^2+2bn+n^2}}+c_1\right)}{x^{\sqrt{-4ac+b^2+2bn+n^2}}+c_1}-b-n\right)}{2a}$$

$$y(x) \to \frac{x^{-n}\left(\sqrt{-4ac+b^2+2bn+n^2}-b-n\right)}{2a}$$

2.40 problem 40

Internal problem ID [10380]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - a\,x^n y^2 - ym = -a\,b^2 x^{n+2m}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(x*diff(y(x),x)=a*x^n*y(x)^2+m*y(x)-a*b^2*x^(n+2*m),y(x), singsol=all)
```

$$y(x) = i\tan\left(\frac{iab\,x^{m+n} + c_1 m + c_1 n}{m+n}\right) b\,x^m$$

✓ Solution by Mathematica

Time used: 1.736 (sec). Leaf size: 43

```
DSolve[x*y'[x]==a*x^n*y[x]^2+m*y[x]-a*b^2*x^(n+2*m),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{-b^2}x^m \tan\left(\frac{a\sqrt{-b^2}x^{m+n}}{m+n} + c_1\right)$$

2.41 problem 41

Internal problem ID [10381]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - x^{2n}y^2 - (m-n)\,y = x^{2m}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(x*diff(y(x),x)=x^(2*n)*y(x)^2+(m-n)*y(x)+x^(2*m),y(x), singsol=all)
```

$$y(x) = \tan\left(\frac{-c_1 m - c_1 n + x^{m+n}}{m+n}\right) x^{m-n}$$

✓ Solution by Mathematica

Time used: 0.727 (sec). Leaf size: 28

```
DSolve[x*y'[x]==x^(2*n)*y[x]^2+(m-n)*y[x]+x^(2*m),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^{m-n} \tan\left(\frac{x^{m+n}}{m+n} + c_1\right)$$

2.42 problem 42

Internal problem ID [10382]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - a\,x^n y^2 - by = c\,x^m}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 172

```
dsolve(x*diff(y(x),x)=a*x^(n)*y(x)^2+b*y(x)+c*x^(m),y(x), singsol=all)
```

$$y(x) = \frac{\left(\operatorname{BesselY}\left(-\frac{b-m}{m+n}, \frac{2\sqrt{ac}\,x^{\frac{m}{2}+\frac{n}{2}}}{m+n}\right) c_1 + \operatorname{BesselJ}\left(-\frac{b-m}{m+n}, \frac{2\sqrt{ac}\,x^{\frac{m}{2}+\frac{n}{2}}}{m+n}\right)\right) x^{\frac{m}{2}+\frac{n}{2}}\sqrt{ac}\,x^{-n+1}}{\left(\operatorname{BesselY}\left(-\frac{b+n}{m+n}, \frac{2\sqrt{ac}\,x^{\frac{m}{2}+\frac{n}{2}}}{m+n}\right) c_1 + \operatorname{BesselJ}\left(-\frac{b+n}{m+n}, \frac{2\sqrt{ac}\,x^{\frac{m}{2}+\frac{n}{2}}}{m+n}\right)\right) ax}$$

✓ Solution by Mathematica

Time used: 1.49 (sec). Leaf size: 1321

```
DSolve[x*y'[x]==a*x^(n)*y[x]^2+b*y[x]+c*x^(m),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x^{-n}\left(\sqrt{a}\sqrt{c}(m+n)x^{m+n}\,\text{BesselJ}\left(\frac{m-b}{m+n},\frac{2\sqrt{a}\sqrt{c}\sqrt{x^{m+n}}}{\sqrt{(m+n)^2}}\right)c_1\,\text{Gamma}\left(\frac{m-b}{m+n}\right)\left((m+n)^2\right)^{\frac{b+n}{m+n}}-\sqrt{a}\sqrt{c}mx^{m+n}\right.}{}$$

$$y(x) \rightarrow \frac{x^{-n}\left(\sqrt{a}\sqrt{c}(m+n)\sqrt{x^{m+n}}\,\text{BesselJ}\left(\frac{m-b}{m+n},\frac{2\sqrt{a}\sqrt{c}\sqrt{x^{m+n}}}{\sqrt{(m+n)^2}}\right)-(b+n)\sqrt{(m+n)^2}\,\text{BesselJ}\left(-\frac{b+n}{m+n},\frac{2\sqrt{a}\sqrt{c}\sqrt{x^{m}}}{\sqrt{(m+n)}}\right.\right.}{2a\sqrt{(m+n)^2}\,\text{BesselJ}\left(-\frac{b+n}{m+n},\frac{2\sqrt{a}\sqrt{c}\sqrt{x^{m+}}}{\sqrt{(m+n)^2}}\right.}$$

2.43 problem 43

Internal problem ID [10383]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - x^{2n}y^2a - (b\,x^n - n)\,y = c}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 72

```
dsolve(x*diff(y(x),x)=a*x^(2*n)*y(x)^2+(b*x^n-n)*y(x)+c,y(x), singsol=all)
```

$$y(x) = \frac{\left(\sqrt{4b^2ac - b^4}\,\tan\left(\frac{\sqrt{4b^2ac-b^4}\,(b\,x^n+c_1n)}{2b^2n}\right) - b^2\right)x^{-n}}{2ab}$$

✓ Solution by Mathematica

Time used: 1.071 (sec). Leaf size: 118

```
DSolve[x*y'[x]==a*x^(2*n)*y[x]^2+(b*x^n-n)*y[x]+c,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{-n}\left(-b + \frac{\sqrt{b^2-4ac}\left(-e^{\frac{x^n\sqrt{b^2-4ac}}{n}}+c_1\right)}{e^{\frac{x^n\sqrt{b^2-4ac}}{n}}+c_1}\right)}{2a}$$

$$y(x) \to \frac{x^{-n}\left(\sqrt{b^2-4ac} - b\right)}{2a}$$

2.44 problem 44

Internal problem ID [10384]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$y'x - a\,x^{m+2n}y^2 - \left(b\,x^{m+n} - n\right)y = c\,x^m$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 87

```
dsolve(x*diff(y(x),x)=a*x^(2*n+m)*y(x)^2+(b*x^(n+m)-n)*y(x)+c*x^m,y(x), singsol=all)
```

$$y(x) = \frac{x^{m-1}\left(\sqrt{4b^2ac-b^4}\,\tan\left(\frac{\sqrt{4b^2ac-b^4}\,(x^{m+n}b+c_1m+c_1n)}{2b^2(m+n)}\right)-b^2\right)x^{-m-n+1}}{2ab}$$

✓ Solution by Mathematica

Time used: 1.566 (sec). Leaf size: 126

```
DSolve[x*y'[x]==a*x^(2*n+m)*y[x]^2+(b*x^(n+m)-n)*y[x]+c*x^m,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{x^{-n}\left(-b+\frac{\sqrt{b^2-4ac}\left(-e^{\frac{\sqrt{b^2-4ac}x^{m+n}}{m+n}}+c_1\right)}{e^{\frac{\sqrt{b^2-4ac}x^{m+n}}{m+n}}+c_1}\right)}{2a}$$

$$y(x) \to \frac{x^{-n}\left(\sqrt{b^2-4ac}-b\right)}{2a}$$

2.45 problem 45

Internal problem ID [10385]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{(a_2x+b_2)\left(y'+\lambda y^2\right)+(a_1x+b_1)\,y=-a_0x-b_0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 964

```
dsolve((a__2*x+b__2)*(diff(y(x),x)+lambda*y(x)^2)+(a__1*x+b__1)*y(x)+a__0*x+b__0=0,y(x), sin
```

Expression too large to display

✓ Solution by Mathematica

Time used: 3.165 (sec). Leaf size: 1418

```
DSolve[(a2*x+b2)*(y'[x]+\[Lambda]*y[x]^2)+(a1*x+b1)*y[x]+a0*x+b0==0,y[x],x,IncludeSingularSo
```

Too large to display

2.46 problem 46

Internal problem ID [10386]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Riccati]`

$$(ax+c)\,y' - \alpha(ay+bx)^2 - \beta(ay+bx) = -bx+\gamma$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 94

```
dsolve((a*x+c)*diff(y(x),x)=alpha*(a*y(x)+b*x)^2+beta*(a*y(x)+b*x)-b*x+gamma,y(x), singsol=a
```

$$y(x) = \frac{-2a^2\alpha bx - a^2\beta + \tan\left(\frac{-2c_1a^2+\ln(ax+c)\sqrt{a^3(4\alpha\gamma a-a\,\beta^2+4\alpha bc)}}{2a^2}\right)\sqrt{a^3\left(4\alpha\gamma a-a\,\beta^2+4\alpha bc\right)}}{2a^3\alpha}$$

✓ Solution by Mathematica

Time used: 60.527 (sec). Leaf size: 98

```
DSolve[(a*x+c)*y'[x]==\[Alpha]*(a*y[x]+b*x)^2+\[Beta]*(a*y[x]+b*x)-b*x+\[Gamma],y[x],x,Inclu
```

$$y(x) \to -\frac{-a\alpha\sqrt{\frac{4a\alpha\gamma-a\beta^2+4\alpha bc}{a^3\alpha^2}}\tan\left(\frac{1}{2}a\alpha\log(ax+c)\sqrt{\frac{4a\alpha\gamma-a\beta^2+4\alpha bc}{a^3\alpha^2}}+c_1\right)+2\alpha bx+\beta}{2a\alpha}$$

2.47 problem 47

Internal problem ID [10387]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{2y'x^2 - 2y^2 - yx = -2a^2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(2*x^2*diff(y(x),x)=2*y(x)^2+x*y(x)-2*a^2*x,y(x), singsol=all)
```

$$y(x) = -i\tan\left(\frac{2ia - c_1\sqrt{x}}{\sqrt{x}}\right)a\sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.619 (sec). Leaf size: 43

```
DSolve[2*x^2*y'[x]==2*y[x]^2+x*y[x]-2*a^2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-a^2}\sqrt{x}\tan\left(\frac{2\sqrt{-a^2}}{\sqrt{x}} - c_1\right)$$

2.48 problem 48

Internal problem ID [10388]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{2y'x^2 - 2y^2 - 3yx = -2a^2x}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 102

```
dsolve(2*x^2*diff(y(x),x)=2*y(x)^2+3*x*y(x)-2*a^2*x,y(x), singsol=all)
```

$$y(x) = \frac{\left(-2c_1x\sqrt{-\frac{a^2}{x}} - x\right)\sin\left(2\sqrt{-\frac{a^2}{x}}\right) - x\left(c_1 - 2\sqrt{-\frac{a^2}{x}}\right)\cos\left(2\sqrt{-\frac{a^2}{x}}\right)}{2\cos\left(2\sqrt{-\frac{a^2}{x}}\right)c_1 + 2\sin\left(2\sqrt{-\frac{a^2}{x}}\right)}$$

✓ Solution by Mathematica

Time used: 0.457 (sec). Leaf size: 94

```
DSolve[2*x^2*y'[x]==2*y[x]^2+3*x*y[x]-2*a^2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4a^2c_1\sqrt{x} + 2a\sqrt{x}e^{\frac{4a}{\sqrt{x}}} - xe^{\frac{4a}{\sqrt{x}}} + 2ac_1x}{2e^{\frac{4a}{\sqrt{x}}} - 4ac_1}$$
$$y(x) \to a(-\sqrt{x}) - \frac{x}{2}$$

2.49 problem 49

Internal problem ID [10389]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Riccati]`

$$\boxed{y'x^2 - a\,x^2y^2 - ybx = c}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 57

```
dsolve(x^2*diff(y(x),x)=a*x^2*y(x)^2+b*x*y(x)+c,y(x), singsol=all)
```

$$y(x) = -\frac{b+1+\tan\left(\frac{\sqrt{4ac-b^2-2b-1}\,(-\ln(x)+c_1)}{2}\right)\sqrt{4ac-b^2-2b-1}}{2ax}$$

✓ Solution by Mathematica

Time used: 0.43 (sec). Leaf size: 99

```
DSolve[x^2*y'[x]==a*x^2*y[x]^2+b*x*y[x]+c,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-4ac+b^2+2b+1}\left(1-\frac{2c_1}{x^{\sqrt{-4ac+b^2+2b+1}}+c_1}\right)+b+1}{2ax}$$

$$y(x) \to -\frac{-\sqrt{-4ac+b^2+2b+1}+b+1}{2ax}$$

2.50 problem 50

Internal problem ID [10390]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x^2 - y^2 c\, x^2 - \left(a\, x^2 + bx\right) y = \alpha\, x^2 + \beta x + \gamma}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 724

```
dsolve(x^2*diff(y(x),x)=c*x^2*y(x)^2+(a*x^2+b*x)*y(x)+alpha*x^2+beta*x+gamma,y(x), singsol=a
```

$$y(x) = -\frac{\left(\sqrt{a^2-4\alpha c}\, c_1 a^3 x - 4\sqrt{a^2-4\alpha c}\, c_1 a\alpha c x + c_1 a^4 x - 8c_1 a^2 \alpha c x + 16 c_1 \alpha^2 c^2 x + \sqrt{a^2-4\alpha c}\, c_1 a^2 b - 4\sqrt{a^2}\right.}{}$$

✓ Solution by Mathematica

Time used: 1.712 (sec). Leaf size: 1584

```
DSolve[x^2*y'[x]==c*x^2*y[x]^2+(a*x^2+b*x)*y[x]+\[Alpha]*x^2+\[Beta]*x+\[Gamma],y[x],x,Inclu
```

$$y(x) \to -\frac{\left(b + ax - x\sqrt{a^2 - 4c\alpha} + \sqrt{b^2 + 2b - 4c\gamma + 1} + 1\right) c_1 \operatorname{HypergeometricU}\left(\frac{ab - 2c\beta + \sqrt{a^2 - 4c\alpha}\left(\sqrt{b^2 + 2b - 4c\gamma + 1}\right.}{2\sqrt{a^2 - 4c\alpha}}\right.}{}$$

$$y(x) \to \frac{\frac{\left(\sqrt{a^2-4\alpha c}\left(\sqrt{b^2+2b-4c\gamma+1}+1\right)+ab-2\beta c\right) \operatorname{HypergeometricU}\left(\frac{ab-2c\beta+\sqrt{a^2-4c\alpha}\left(\sqrt{b^2+2b-4c\gamma+1}+3\right)}{2\sqrt{a^2-4c\alpha}}, \sqrt{b^2+2b-4c\gamma+1}+2, x\sqrt{a^2-4c\alpha}\right)}{\operatorname{HypergeometricU}\left(\frac{ab-2c\beta+\sqrt{a^2-4c\alpha}\left(\sqrt{b^2+2b-4c\gamma+1}+1\right)}{2\sqrt{a^2-4c\alpha}}, \sqrt{b^2+2b-4c\gamma+1}+1, x\sqrt{a^2-4c\alpha}\right)} - }{2c}$$

$$y(x) \to \frac{\frac{\left(\sqrt{a^2-4\alpha c}\left(\sqrt{b^2+2b-4c\gamma+1}+1\right)+ab-2\beta c\right) \operatorname{HypergeometricU}\left(\frac{ab-2c\beta+\sqrt{a^2-4c\alpha}\left(\sqrt{b^2+2b-4c\gamma+1}+3\right)}{2\sqrt{a^2-4c\alpha}}, \sqrt{b^2+2b-4c\gamma+1}+2, x\sqrt{a^2-4c\alpha}\right)}{\operatorname{HypergeometricU}\left(\frac{ab-2c\beta+\sqrt{a^2-4c\alpha}\left(\sqrt{b^2+2b-4c\gamma+1}+1\right)}{2\sqrt{a^2-4c\alpha}}, \sqrt{b^2+2b-4c\gamma+1}+1, x\sqrt{a^2-4c\alpha}\right)} - }{2c}$$

2.51 problem 51

Internal problem ID [10391]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 51.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y' x^2 - a\, x^2 y^2 - ybx = c\, x^n + s}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 299

```
dsolve(x^2*diff(y(x),x)=a*x^2*y(x)^2+b*x*y(x)+c*x^n+s,y(x), singsol=all)
```

$$y(x) = \frac{\left(-\sqrt{-4as+b^2+2b+1}\, c_1 - c_1 b - c_1\right) \text{BesselY}\left(\frac{\sqrt{-4as+b^2+2b+1}}{n}, \frac{2\sqrt{ac}\, x^{\frac{n}{2}}}{n}\right) + 2\sqrt{ac}\, \text{BesselY}\left(\frac{\sqrt{-4as+b^2+2b-}}{n}\right.}{2xa\left(\text{BesselY}\left(\frac{\sqrt{-4as+b}}{n}\right.\right.}$$

✓ Solution by Mathematica

Time used: 2.637 (sec). Leaf size: 2281

```
DSolve[x^2*y'[x]==a*x^2*y[x]^2+b*x*y[x]+c*x^n+s,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

2.52 problem 52

Internal problem ID [10392]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x^2 - a\,x^2y^2 - ybx = c\,x^{2n} + s\,x^n}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 442

```
dsolve(x^2*diff(y(x),x)=a*x^2*y(x)^2+b*x*y(x)+c*x^(2*n)+s*x^n,y(x), singsol=all)
```

$$y(x) = -\frac{\left(2ix^n\sqrt{a}\,c_1c + i\sqrt{a}\,c_1s + \sqrt{c}\,c_1b - \sqrt{c}\,c_1n + \sqrt{c}\,c_1\right)\operatorname{KummerU}\left(\frac{i\sqrt{a}\,s+\sqrt{c}\,b+\sqrt{c}\,n+\sqrt{c}}{2\sqrt{c}\,n}, \frac{b+n+1}{n}, \frac{2i\sqrt{a}\,\sqrt{c}\,x^n}{n}\right) -}{}$$

✓ Solution by Mathematica

Time used: 1.839 (sec). Leaf size: 819

```
DSolve[x^2*y'[x]==a*x^2*y[x]^2+b*x*y[x]+c*x^(2*n)+s*x^n,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to$$

$$-\frac{i\sqrt{a}c_1x^n\left(\sqrt{c}(b+n+1)-i\sqrt{a}s\right)\text{HypergeometricU}\left(\frac{b+3n-\frac{i\sqrt{a}s}{\sqrt{c}}+1}{2n},\frac{b+2n+1}{n},-\frac{2i\sqrt{a}\sqrt{c}x^n}{n}\right)+c_1n(i\sqrt{a}\sqrt{c}}{anx\left(c_1\,\text{Hyperge}\right.}$$

$$y(x) \to -\frac{\frac{\sqrt{a}x^n\left(\sqrt{a}s+i\sqrt{c}(b+n+1)\right)\text{HypergeometricU}\left(\frac{b+3n-\frac{i\sqrt{a}s}{\sqrt{c}}+1}{2n},\frac{b+2n+1}{n},-\frac{2i\sqrt{a}\sqrt{c}x^n}{n}\right)}{n\,\text{HypergeometricU}\left(\frac{b+n-\frac{i\sqrt{a}s}{\sqrt{c}}+1}{2n},\frac{b+n+1}{n},-\frac{2i\sqrt{a}\sqrt{c}x^n}{n}\right)}+i\sqrt{a}\sqrt{c}x^n+b+1}{ax}$$

$$y(x) \to -\frac{\frac{\sqrt{a}x^n\left(\sqrt{a}s+i\sqrt{c}(b+n+1)\right)\text{HypergeometricU}\left(\frac{b+3n-\frac{i\sqrt{a}s}{\sqrt{c}}+1}{2n},\frac{b+2n+1}{n},-\frac{2i\sqrt{a}\sqrt{c}x^n}{n}\right)}{n\,\text{HypergeometricU}\left(\frac{b+n-\frac{i\sqrt{a}s}{\sqrt{c}}+1}{2n},\frac{b+n+1}{n},-\frac{2i\sqrt{a}\sqrt{c}x^n}{n}\right)}+i\sqrt{a}\sqrt{c}x^n+b+1}{ax}$$

2.53 problem 53

Internal problem ID [10393]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 53.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x^2 - y^2 c\, x^2 - (x^n a + b)\, xy = \alpha\, x^{2n} + \beta\, x^n + \gamma}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 1036

```
dsolve(x^2*diff(y(x),x)=c*x^2*y(x)^2+(a*x^n+b)*x*y(x)+alpha*x^(2*n)+beta*x^n+gamma,y(x), sin
```

Expression too large to display

✓ Solution by Mathematica

Time used: 3.672 (sec). Leaf size: 1837

```
DSolve[x^2*y'[x]==c*x^2*y[x]^2+(a*x^n+b)*x*y[x]+\[Alpha]*x^(2*n)+\[Beta]*x^n+\[Gamma],y[x],x
```

$y(x)$

$$\to \frac{-\left(\left(-\left(\left(n^2+\sqrt{n^2\left(b^2+2b-4c\gamma+1\right)}\right)a^2\right)+n(-b+n-1)\sqrt{a^2-4c\alpha}a+2c\left(2\alpha n^2+\sqrt{a^2-4c\alpha}\beta n\right.\right.}{}$$

$y(x)$

$$\to \frac{-\frac{x^n\left(2c\left(\beta n\sqrt{a^2-4\alpha c}+2\alpha\sqrt{n^2(b^2+2b-4c\gamma+1)}+2\alpha n^2\right)-\left(a^2\left(\sqrt{n^2(b^2+2b-4c\gamma+1)}+n^2\right)\right)+an(-b+n-1)\sqrt{a^2-4\alpha c}\right)\text{HypergeometricU}\left(\left(\right.\right.}{\text{HypergeometricU}\left(\frac{\left(n^2+\sqrt{n^2\left(b^2+2b-4c\gamma+1\right)}\right)a^2+(b-n+1)n\sqrt{a^2-4c\alpha}a-2c\left(2\alpha n^2\right.}{2n^2\left(a^2-4c\alpha\right)}\right.}}{}$$

2.54 problem 54

Internal problem ID [10394]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 54.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$y'x^2 - (\alpha\, x^{2n} + \beta\, x^n + \gamma)\, y^2 - (x^n a + b)\, xy = c\, x^2$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 215224

```
dsolve(x^2*diff(y(x),x)=(alpha*x^(2*n)+beta*x^n+gamma)*y(x)^2+(a*x^n+b)*x*y(x)+c*x^2,y(x), s
```

Expression too large to display

✓ Solution by Mathematica

Time used: 4.676 (sec). Leaf size: 2649

```
DSolve[x^2*y'[x]==(\[Alpha]*x^(2*n)+\[Beta]*x^n+\[Gamma])*y[x]^2+(a*x^n+b)*x*y[x]+c*x^2,y[x]
```

Too large to display

2.55 problem 55

Internal problem ID [10395]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 55.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$(x^2-1)\,y'+\lambda(y^2-2yx+1)=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 231

```
dsolve((x^2-1)*diff(y(x),x)+lambda*(y(x)^2-2*x*y(x)+1)=0,y(x), singsol=all)
```

$$y(x) = \frac{8c_1\left(\left(\lambda-\frac{1}{2}\right)x-\frac{\lambda}{2}+\frac{1}{2}\right)(x+1)\operatorname{HeunC}\left(0,-2\lambda+1,0,0,\lambda^2-\lambda+\frac{1}{2},\frac{2}{x+1}\right)-\lambda\left(-\frac{x}{2}-\frac{1}{2}\right)^{-2\lambda+1}(x+1)\operatorname{He}}{4\lambda\left(\operatorname{HeunC}\left(0,-2\lambda+1,0,\right.\right.}$$

✓ Solution by Mathematica

Time used: 0.643 (sec). Leaf size: 47

```
DSolve[(x^2-1)*y'[x]+\[Lambda]*(y[x]^2-2*x*y[x]+1)==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to\frac{\operatorname{LegendreQ}(\lambda,x)+c_1\operatorname{LegendreP}(\lambda,x)}{\operatorname{LegendreQ}(\lambda-1,x)+c_1\operatorname{LegendreP}(\lambda-1,x)}$$

$$y(x)\to\frac{\operatorname{LegendreP}(\lambda,x)}{\operatorname{LegendreP}(\lambda-1,x)}$$

2.56 problem 56

Internal problem ID [10396]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 56.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\left(a\,x^{2}+b\right)y'+\alpha y^{2}+\beta x y=-\frac{b(a+\beta)}{\alpha}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 563

```
dsolve((a*x^2+b)*diff(y(x),x)+alpha*y(x)^2+beta*x*y(x)+b/alpha*(a+beta)=0,y(x), singsol=all)
```

$$y(x) = -\frac{b\left(-\frac{\left(-\frac{-ax+\sqrt{-ba}}{2\sqrt{-ba}}\right)^{\frac{a+\beta}{a}}\left(ax-\sqrt{-ba}\right)\operatorname{HeunC}\left(0,\frac{-a-\beta}{a},\frac{2a+\beta}{2a},0,\frac{2a^{2}+2a\beta+\beta^{2}}{4a^{2}},\frac{2\sqrt{-ba}}{-ax+\sqrt{-ba}}\right)}{2}+\left(-\frac{-ax+\sqrt{-ba}}{2\sqrt{-ba}}\right)^{\frac{a+\beta}{a}}\left(ax+\sqrt{-ba}\right.\cdots}{\alpha\left(\cdots\right.}$$

✓ Solution by Mathematica

Time used: 1.111 (sec). Leaf size: 27

```
DSolve[(a*x^2+b)*y'[x]+\[Alpha]*y[x]^2+\[Beta]*x*y[x]+b/\[Alpha]*(a+\[Beta])==0,y[x],x,Inclu
```

$$y(x) \to -\frac{x(a+\beta)}{\alpha}$$

$$y(x) \to -\frac{x(a+\beta)}{\alpha}$$

2.57 problem 57

Internal problem ID [10397]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 57.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{\left(a\,x^2+b\right)y'+\alpha y^2+\beta x y=-\gamma}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 682

```
dsolve((a*x^2+b)*diff(y(x),x)+alpha*y(x)^2+beta*x*y(x)+gamma=0,y(x), singsol=all)
```

$y(x) =$

$$-\frac{}{2c_1\left(\frac{\left(-\sqrt{-ba}\,x-b\right)\sqrt{4\gamma\alpha ba+b^2\beta^2}}{2}+b\left(\frac{\sqrt{-ba}\,\beta x}{2}+\left(a-\frac{\beta}{2}\right)b+a\,x^2\left(a-\beta\right)\right)\right)\operatorname{HeunC}\left(0,\frac{a-\beta}{a},-\frac{\sqrt{4\gamma\alpha ba+b^2\beta^2}}{2ab},\right.}$$

✓ Solution by Mathematica

Time used: 1.098 (sec). Leaf size: 598

```
DSolve[(a*x^2+b)*y'[x]+\[Alpha]*y[x]^2+\[Beta]*x*y[x]+\[Gamma]==0,y[x],x,IncludeSingularSolu
```

$$y(x) \to \frac{i\left(c_1\left(\sqrt{4a\alpha\gamma+b\beta^2}-2a\sqrt{b}-\sqrt{b}\beta\right)P_{\frac{\beta}{2a}+1}^{\frac{\sqrt{b\beta^2+4a\alpha\gamma}}{2a\sqrt{b}}}\left(\frac{i\sqrt{a}x}{\sqrt{b}}\right)+2i\sqrt{a}x(a+\beta)Q_{\frac{\beta}{2a}}^{\frac{\sqrt{b\beta^2+4a\alpha\gamma}}{2a\sqrt{b}}}\left(\frac{i\sqrt{a}x}{\sqrt{b}}\right)+\left(\sqrt{4a\alpha\gamma\right.\right.}{2\sqrt{a}\alpha\left(c_1P_{\frac{\beta}{2a}}^{\frac{\sqrt{b\beta^2+4a\alpha\gamma}}{2a\sqrt{b}}}\left(\frac{i\sqrt{a}x}{\sqrt{b}}\right)+Q_{\frac{\beta}{2a}}\right.}$$

$$y(x) \to \frac{-2x(a+\beta)+\frac{i\left(\sqrt{4a\alpha\gamma+b\beta^2}-2a\sqrt{b}-\sqrt{b}\beta\right)P_{\frac{\beta}{2a}+1}^{\frac{\sqrt{b\beta^2+4a\alpha\gamma}}{2a\sqrt{b}}}\left(\frac{i\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}P_{\frac{\beta}{2a}}^{\frac{\sqrt{b\beta^2+4a\alpha\gamma}}{2a\sqrt{b}}}\left(\frac{i\sqrt{a}x}{\sqrt{b}}\right)}}{2\alpha}$$

2.58 problem 58

Internal problem ID [10398]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 58.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Ricca`

$$\boxed{(a\,x^2+b)\,y'+y^2-2yx=-(-a+1)\,x^2+b}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve((a*x^2+b)*diff(y(x),x)+y(x)^2-2*x*y(x)+(1-a)*x^2-b=0,y(x), singsol=all)
```

$$y(x)=x+\frac{1}{c_1+\frac{\arctan\left(\frac{ax}{\sqrt{ba}}\right)}{\sqrt{ba}}}$$

✓ Solution by Mathematica

Time used: 0.562 (sec). Leaf size: 41

```
DSolve[(a*x^2+b)*y'[x]+y[x]^2-2*x*y[x]+(1-a)*x^2-b==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to x+\frac{1}{\frac{\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}+c_1}$$

$$y(x)\to x$$

2.59 problem 59

Internal problem ID [10399]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 59.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Ricca`

$$(a\,x^2+bx+c)\,y'-y^2-(2\lambda x+b)\,y=\lambda(\lambda-a)\,x^2+\mu$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5761

```
dsolve((a*x^2+b*x+c)*diff(y(x),x)=y(x)^2+(2*lambda*x+b)*y(x)+lambda*(lambda-a)*x^2+mu,y(x),
```

Expression too large to display

✓ Solution by Mathematica

Time used: 17.168 (sec). Leaf size: 93

```
DSolve[(a*x^2+b*x+c)*y'[x]==y[x]^2+(2*\[Lambda]*x+b)*y[x]+\[Lambda]*(\[Lambda]-a)*x^2+\[Mu],
```

$$y(x)\to\frac{1}{2}\left(\sqrt{4(c\lambda+\mu)-b^2}\tan\left(\frac{\sqrt{-b^2+4c\lambda+4\mu}\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+c_1\right)-b-2\lambda x\right)$$

2.60 problem 60

Internal problem ID [10400]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 60.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$(a\,x^2+bx+c)\,y'-y^2-(ax+\mu)\,y=-\lambda^2x^2+\lambda(b-\mu)\,x+c\lambda$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 476004

```
dsolve((a*x^2+b*x+c)*diff(y(x),x)=y(x)^2+(a*x+mu)*y(x)-lambda^2*x^2+lambda*(b-mu)*x+lambda*c
```

Expression too large to display

✓ Solution by Mathematica

Time used: 23.352 (sec). Leaf size: 433

```
DSolve[(a*x^2+b*x+c)*y'[x]==y[x]^2+(a*x+\[Mu])*y[x]-\[Lambda]^2*x^2+\[Lambda]*(b-\[Mu])*x+\[
```

$$y(x) \to \frac{(x(ax+b)+c)^{\frac{\lambda}{a}-\frac{1}{2}}\exp\left(-\frac{(a(b-2\mu)+2b\lambda)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{a\sqrt{4ac-b^2}}\right)\left(\lambda x(x(ax+b)+c)^{\frac{1}{2}-\frac{\lambda}{a}}\exp\left(\frac{(a(b-2\mu)+2b\lambda)\arctan}{a\sqrt{4ac-b}}\right.\right.}{\int_1^x \exp}$$

$$y(x) \to \lambda x$$

2.61 problem 61

Internal problem ID [10401]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$(a_2x^2 + b_2x + c_2)\, y' - y^2 - (a_1x + b_1)\, y = -\lambda(\lambda + a_1 - a_2)\, x^2 + \lambda(b_2 - b_1)\, x + \lambda c_2$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 545907

```
dsolve((a__2*x^2+b__2*x+c__2)*diff(y(x),x)=y(x)^2+(a__1*x+b__1)*y(x)-lambda*(lambda+a__1-a__
```

Expression too large to display

✓ Solution by Mathematica

Time used: 34.34 (sec). Leaf size: 284

```
DSolve[(a2*x^2+b2*x+c2)*y'[x]==y[x]^2+(a1*x+b1)*y[x]-\[Lambda]*(\[Lambda]+a1-a2)*x^2+\[Lambd
```

$$y(x) \rightarrow \frac{\lambda x \int_1^x \exp\left(\frac{(\text{a1}-2\text{a2}+2\lambda)\log(\text{c2}+K[1](\text{b2}+\text{a2}K[1]))-\frac{2(\text{b2}(\text{a1}+2\lambda)-2\text{a2}\text{b1})\arctan\left(\frac{\text{b2}+2\text{a2}K[1]}{\sqrt{4\text{a2}\text{c2}-\text{b2}^2}}\right)}{\sqrt{4\text{a2}\text{c2}-\text{b2}^2}}}{2\text{a2}}\right)dK[1]+(x(\text{a2}x - \cdots}{\int_1^x \exp\left(\frac{(\text{a1}-2\text{a2}+2\lambda)\log(\text{c2}+K[1](\text{b2}+\text{a2}K[1]))-\frac{2(\text{b2}(\text{a1}+2\lambda)-2\text{a2}\text{b1} \cdots}{\sqrt{4\text{a2} \cdots}}}{2\text{a2}}\right. \cdots}$$

$$y(x) \rightarrow \lambda x$$

2.62 problem 62

Internal problem ID [10402]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 62.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$(a_2x^2 + b_2x + c_2)\, y' - y^2 - (a_1x + b_1)\, y = a_0x^2 + b_0x + c_0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 404913

```
dsolve((a__2*x^2+b__2*x+c__2)*diff(y(x),x)=y(x)^2+(a__1*x+b__1)*y(x)+a__0*x^2+b__0*x+c__0,y(
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a2*x^2+b2*x+c2)*y'[x]==y[x]^2+(a1*x+b1)*y[x]+a0*x^2+b0*x+c0,y[x],x,IncludeSingularSo
```

Timed out

2.63 problem 63

Internal problem ID [10403]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 63.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Ricca`

$$(x-a)(x-b)\,y'+y^2+k(y+x-a)(y+x-b)=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 128

```
dsolve((x-a)*(x-b)*diff(y(x),x)+y(x)^2+k*(y(x)+x-a)*(y(x)+x-b)=0,y(x), singsol=all)
```

$$y(x)=\frac{k\left(\frac{bc_1(-x+b)^k}{c_1(-x+b)^k+(-x+a)^k}-\frac{xc_1(-x+b)^k}{c_1(-x+b)^k+(-x+a)^k}+\frac{a(-x+a)^k}{c_1(-x+b)^k+(-x+a)^k}-\frac{x(-x+a)^k}{c_1(-x+b)^k+(-x+a)^k}\right)}{1+k}$$

✓ Solution by Mathematica

Time used: 60.572 (sec). Leaf size: 99

```
DSolve[(x-a)*(x-b)*y'[x]+y[x]^2+k*(y[x]+x-a)*(y[x]+x-b)==0,y[x],x,IncludeSingularSolutions -
```

$$y(x)\to\frac{1}{2}\left(\frac{k(a+b-2x)}{k+1}+\sqrt{-\frac{k^2(a-b)^2}{(k+1)^2}}\tan\left(\frac{(k+1)\sqrt{-\frac{k^2(a-b)^2}{(k+1)^2}}(\log(x-b)-\log(x-a))}{2(a-b)}+c_1\right)\right)$$

2.64 problem 64

Internal problem ID [10404]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 64.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$(c_2x^2 + b_2x + a_2)\,(y' + \lambda y^2) + (b_1x + a_1)\,y = -a_0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18039

```
dsolve((c__2*x^2+b__2*x+a__2)*(diff(y(x),x)+lambda*y(x)^2)+(b__1*x+a__1)*y(x)+a__0=0,y(x), s
```

Expression too large to display

✓ Solution by Mathematica

Time used: 14.836 (sec). Leaf size: 1986

```
DSolve[(c2*x^2+b2*x+a2)*(y'[x]+\[Lambda]*y[x]^2)+(b1*x+a1)*y[x]+a0==0,y[x],x,IncludeSingular
```

Too large to display

2.65 problem 65

Internal problem ID [10405]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 65.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$x^3y' - a\,x^3y^2 - \left(b\,x^2+c\right)y = sx$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 327

```
dsolve(x^3*diff(y(x),x)=a*x^3*y(x)^2+(b*x^2+c)*y(x)+s*x,y(x), singsol=all)
```

$$y(x) = -\frac{2c_1 \operatorname{KummerU}\left(\frac{\sqrt{-4as+b^2+2b+1}}{4}+\frac{b}{4}+\frac{1}{4}, 1+\frac{\sqrt{-4as+b^2+2b+1}}{2}, \frac{c}{2x^2}\right)}{xa\left(\operatorname{KummerU}\left(\frac{5}{4}+\frac{\sqrt{-4as+b^2+2b+1}}{4}+\frac{b}{4}, 1+\frac{\sqrt{-4as+b^2+2b+1}}{2}, \frac{c}{2x^2}\right)c_1+\operatorname{KummerM}\left(\frac{5}{4}+\frac{\sqrt{-4as+b^2+2b+1}}{4}\right.\right.}$$
$$+\frac{\left(-1+\sqrt{-4as+b^2+2b+1}-b\right)\operatorname{KummerM}\left(\frac{\sqrt{-4as+b^2+2b+1}}{4}+\frac{b}{4}+\frac{1}{4}, 1+\frac{\sqrt{-4as+b^2}}{2}\right.}{2xa\left(\operatorname{KummerU}\left(\frac{5}{4}+\frac{\sqrt{-4as+b^2+2b+1}}{4}+\frac{b}{4}, 1+\frac{\sqrt{-4as+b^2+2b+1}}{2}, \frac{c}{2x^2}\right)c_1+\operatorname{KummerM}\left(\frac{5}{4}+\frac{\sqrt{-4as+b^2+2b+1}}{4}\right.\right.}$$

✓ Solution by Mathematica

Time used: 3.199 (sec). Leaf size: 907

```
DSolve[x^3*y'[x]==a*x^3*y[x]^2+(b*x^2+c)*y[x]+s*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to$$

$$-\frac{-\left(\left(\sqrt{-4as+b^2+2b+1}-b-1\right)c^{\frac{1}{2}\sqrt{-4as+b^2+2b+1}}\left(\frac{1}{x}\right)^{\sqrt{-4as+b^2+2b+1}}\text{Hypergeometric1F1}\left(\frac{1}{4}\left(-b+\sqrt{b\right.\right.\right.}{}$$

$$y(x) \to \frac{\frac{c\left(b\left(\sqrt{-4as+b^2+2b+1}+4\right)+3\sqrt{-4as+b^2+2b+1}-4as+b^2+3\right)\text{Hypergeometric1F1}\left(\frac{1}{4}\left(-b-\sqrt{b^2+2b-4as+1}+3\right),2-\frac{1}{2}\sqrt{b^2+2b-4as+1},-\frac{c}{2x^2}\right.}{\text{Hypergeometric1F1}\left(\frac{1}{4}\left(-b-\sqrt{b^2+2b-4as+1}-1\right),1-\frac{1}{2}\sqrt{b^2+2b-4as+1},-\frac{c}{2x^2}\right)}}{2ax^3\left(4as-b^2-2b+3\right)}$$

2.66 problem 66

Internal problem ID [10406]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 66.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{x^3y' - a\,x^3y^2 - x(bx + c)\,y = \alpha x + \beta}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 615

```
dsolve(x^3*diff(y(x),x)=a*x^3*y(x)^2+x*(b*x+c)*y(x)+alpha*x+beta,y(x), singsol=all)
```

$$y(x) = \frac{(2a^2\beta^2c_1x + 2a\alpha\,c^2c_1x - 2ab\beta cc_1x - 6a\beta cc_1x + 2b\,c^2c_1x + 4c^2c_1x)\,\mathrm{KummerU}\left(\frac{\sqrt{-4a\alpha+b^2+2b+1}\,c\cdots}{2c}\right.}{2x^2c^2a\left(\mathrm{KummerU}\left(\frac{\sqrt{-4a\alpha+b^2+2b+1}\,c-2a\beta+bc+3c}{2c}, 1+\sqrt{-4a\alpha+b^2+2b+1}, \frac{c}{x}\right)c_1 + \mathrm{KummerM}\left(\frac{\sqrt{-4a\alpha+b\cdots}}{}\right.\right.}$$
$$+ \frac{(2a\beta cc_1x - 2b\,c^2c_1x - 2c^3c_1 - 4c^2c_1x)\,\mathrm{KummerU}\left(\frac{\sqrt{-4a\alpha+b^2+2b+1}\,c-2a\beta+bc+3c}{2c}, 1+\sqrt{-4a\alpha+b^2+2b+\cdots}\right.}{2x^2c^2a\left(\mathrm{K}\cdots\right.}$$

✓ Solution by Mathematica

Time used: 2.395 (sec). Leaf size: 908

```
DSolve[x^3*y'[x]==a*x^3*y[x]^2+x*(b*x+c)*y[x]+\[Alpha]*x+\[Beta],y[x],x,IncludeSingularSolut
```

$$y(x) \to$$

$$-\frac{\frac{c^{\sqrt{-4a\alpha+b^2+2b+1}}\left(\frac{1}{x}\right)^{\sqrt{-4a\alpha+b^2+2b+1}+1}\left(c\left(\sqrt{-4a\alpha+b^2+2b+1}-b-1\right)+2a\beta\right)\text{Hypergeometric1F1}\left(\frac{1}{2}\left(-b+\frac{2a\beta}{c}+\sqrt{b^2+2b-4a\alpha+1}+1\right),\sqrt{b^2}\right.}{\sqrt{-4a\alpha+b^2+2b+1}+1}}{}$$

$$y(x)$$

$$\to \frac{\frac{\left(c\left(b\left(\sqrt{-4a\alpha+b^2+2b+1}+3\right)+2\left(-2a\alpha+\sqrt{-4a\alpha+b^2+2b+1}+1\right)+b^2\right)-2a\beta\left(\sqrt{-4a\alpha+b^2+2b+1}+1\right)\right)\text{Hypergeometric1F1}\left(\frac{2a\beta-c\left(b+\sqrt{b^2+2b}\right.}{2c}\right.}{\text{Hypergeometric1F1}\left(\frac{2a\beta-c\left(b+\sqrt{b^2+2b-4a\alpha+1}+1\right)}{2c},1-\sqrt{b^2+2b-4a\alpha+1},-\frac{c}{x}\right)}}{2ax^2\left(4a\alpha-b^2-2b\right)}$$

2.67 problem 67

Internal problem ID [10407]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 67.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$x(x^2+a)\left(y'+\lambda y^2\right)+\left(b\,x^2+c\right)y=-sx$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 1348

```
dsolve(x*(x^2+a)*(diff(y(x),x)+lambda*y(x)^2)+(b*x^2+c)*y(x)+s*x=0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 2.874 (sec). Leaf size: 862

```
DSolve[x*(x^2+a)*(y'[x]+\[Lambda]*y[x]^2)+(b*x^2+c)*y[x]+s*x==0,y[x],x,IncludeSingularSoluti
```

$y(x)$

$$\rightarrow \frac{a^{\frac{1}{2}\left(\frac{c}{a}-3\right)}(a-c)x^{-\frac{c}{a}}\text{Hypergeometric2F1}\left(\frac{ba-\sqrt{b^2-2b-4s\lambda+1}a+a-2c}{4a},\frac{a\left(b+\sqrt{b^2-2b-4s\lambda+1}+1\right)-2c}{4a},\frac{3}{2}-\frac{c}{2a},-\frac{x^2}{a}\right)+}{\lambda a^{\frac{1}{2}\left(\frac{c}{a}-1\right)}x^{1-\frac{c}{a}}\text{Hypergeometric2F1}\left(\frac{ba-\sqrt{b^2-2b-4s\lambda-}}{4a}\right.}$$

$y(x) \rightarrow$

$$-\frac{sx\,\text{Hypergeometric2F1}\left(\frac{1}{4}\left(b-\sqrt{b^2-2b-4s\lambda+1}+3\right),\frac{1}{4}\left(b+\sqrt{b^2-2b-4s\lambda+1}+3\right),\frac{1}{2}\left(\frac{c}{a}+3\right),\right.}{(a+c)\,\text{Hypergeometric2F1}\left(\frac{1}{4}\left(b-\sqrt{b^2-2b-4s\lambda+1}-1\right),\frac{1}{4}\left(b+\sqrt{b^2-2b-4s\lambda+1}-1\right),\frac{a+c}{2a},\right.}$$

$y(x) \rightarrow$

$$-\frac{sx\,\text{Hypergeometric2F1}\left(\frac{1}{4}\left(b-\sqrt{b^2-2b-4s\lambda+1}+3\right),\frac{1}{4}\left(b+\sqrt{b^2-2b-4s\lambda+1}+3\right),\frac{1}{2}\left(\frac{c}{a}+3\right),\right.}{(a+c)\,\text{Hypergeometric2F1}\left(\frac{1}{4}\left(b-\sqrt{b^2-2b-4s\lambda+1}-1\right),\frac{1}{4}\left(b+\sqrt{b^2-2b-4s\lambda+1}-1\right),\frac{a+c}{2a},\right.}$$

2.68 problem 68

Internal problem ID [10408]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 68.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$x^2(x+a)\left(y'+\lambda y^2\right)+x(bx+c)\,y=-\alpha x-\beta$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5852

```
dsolve(x^2*(x+a)*(diff(y(x),x)+lambda*y(x)^2)+x*(b*x+c)*y(x)+alpha*x+beta=0,y(x), singsol=al
```

Expression too large to display

✓ Solution by Mathematica

Time used: 5.239 (sec). Leaf size: 1770

```
DSolve[x^2*(x+a)*(y'[x]+\[Lambda]*y[x]^2)+x*(b*x+c)*y[x]+\[Alpha]*x+\[Beta]==0,y[x],x,Includ
```

$y(x)$

$$\rightarrow \frac{2a\left(a-c+\sqrt{a^2-2(c+2\beta\lambda)a+c^2}\right)\text{Hypergeometric2F1}\left(\frac{-c+a\left(b-\sqrt{b^2-2b-4\alpha\lambda+1}\right)+\sqrt{a^2-2(c+2\beta\lambda)a+c^2}}{2a},\frac{-c\cdots}{}\right.}{\cdots}$$

$y(x)$

$$\rightarrow \frac{\frac{a\left(c^2-2a(2\beta\lambda+c)\right)\left(\sqrt{a^2-2a(2\beta\lambda+c)+c^2}-a+c\right)}{x}-\frac{\left(2\alpha a^3\lambda+a^2\left(2\alpha\lambda\sqrt{a^2-2a(2\beta\lambda+c)+c^2}+4b\beta\lambda+bc-2\beta\lambda\right)-a\left(bc\sqrt{a^2-2a(2\beta\lambda+c)+c^2}+2\cdots\right.\right.}{\text{Hy}\cdots}}{\cdots}$$

2.69 problem 69

Internal problem ID [10409]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Riccati]`

$$\boxed{(a\,x^2+bx+e)\,(y'x-y)-y^2=-x^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 58

```
dsolve((a*x^2+b*x+e)*(x*diff(y(x),x)-y(x))-y(x)^2+x^2=0,y(x), singsol=all)
```

$$y(x)=-\tanh\left(\frac{c_1\sqrt{4ea-b^2}+2\arctan\left(\frac{2ax+b}{\sqrt{4ea-b^2}}\right)}{\sqrt{4ea-b^2}}\right)x$$

✓ Solution by Mathematica

Time used: 1.973 (sec). Leaf size: 116

```
DSolve[(a*x^2+b*x+e)*(x*y'[x]-y[x])-y[x]^2+x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\frac{x\left(-1+\exp\left(\frac{4\arctan\left(\frac{2ax+b}{\sqrt{4ae-b^2}}\right)}{\sqrt{4ae-b^2}}+2c_1\right)\right)}{1+\exp\left(\frac{4\arctan\left(\frac{2ax+b}{\sqrt{4ae-b^2}}\right)}{\sqrt{4ae-b^2}}+2c_1\right)}$$

$$y(x)\to-x$$

$$y(x)\to x$$

2.70 problem 70

Internal problem ID [10410]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 70.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$x^2(x^2+a)(y'+\lambda y^2)+x(b\,x^2+c)\,y=-s$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5730

```
dsolve(x^2*(x^2+a)*(diff(y(x),x)+lambda*y(x)^2)+x*(b*x^2+c)*y(x)+s=0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 7.158 (sec). Leaf size: 1470

```
DSolve[x^2*(x^2+a)*(y'[x]+\[Lambda]*y[x]^2)+x*(b*x^2+c)*y[x]+s==0,y[x],x,IncludeSingularSolu
```

$$y(x) \to \frac{\frac{\left(a-c-\sqrt{a^2-2(c+2s\lambda)a+c^2}\right)c_1\left(\left(-2ba+a+c+\sqrt{a^2-2(c+2s\lambda)a+c^2}\right)\text{Hypergeometric2F1}\left(-\frac{-5a+c+\sqrt{a^2-2(c+2s\lambda)a+c^2}}{4a},-\frac{-a(2b+3)+c+\sqrt{a}}{4}\right.\right.}{}}{}$$

$$y(x) \to \frac{x\left(a^3(-b)+a^2\left(b\sqrt{a^2-2a(c+2\lambda s)+c^2}-4(b-1)\lambda s+c\right)+a\left(bc\left(\sqrt{a^2-2a(c+2\lambda s)+c^2}+c\right)-c\right.\right.}{2a^2\lambda\left(3a^2+2a\right.}$$

$$-\frac{\sqrt{a^2-2a(c+2\lambda s)+c^2}-a+c}{2a\lambda x}$$

2.71 problem 71

Internal problem ID [10411]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 71.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$a(x^2-1)(y'+\lambda y^2)+bx(x^2-1)y=-c\,x^2-dx-s$$

✗ Solution by Maple

```
dsolve(a*(x^2-1)*(diff(y(x),x)+lambda*y(x)^2)+b*x*(x^2-1)*y(x)+c*x^2+d*x+s=0,y(x), singsol=a
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[a*(x^2-1)*(y'[x]+\[Lambda]*y[x]^2)+b*x*(x^2-1)*y[x]+c*x^2+d*x+s==0,y[x],x,IncludeSing
```

Not solved

2.72 problem 72

Internal problem ID [10412]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 72.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{x^{n+1}y' - x^{2n}y^2a - yx^nb = c\,x^m + d}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 333

```
dsolve(x^(n+1)*diff(y(x),x)=a*x^(2*n)*y(x)^2+b*x^n*y(x)+c*x^m+d,y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(-\sqrt{-4ad+b^2+2bn+n^2}\,c_1 - c_1 b - c_1 n\right)\operatorname{BesselY}\left(\frac{\sqrt{-4ad+b^2+2bn+n^2}}{m}, \frac{2\sqrt{ac}\,x^{\frac{m}{2}}}{m}\right) + 2\sqrt{ac}\operatorname{BesselY}\left(\frac{\sqrt{-4}}{}\right.\right.}{2xa\left(\operatorname{Besse}\right.}$$

✓ Solution by Mathematica

Time used: 3.153 (sec). Leaf size: 2576

```
DSolve[x^(n+1)*y'[x]==a*x^(2*n)*y[x]^2+b*x^n*y[x]+c*x^m+d,y[x],x,IncludeSingularSolutions ->
```

Too large to display

2.73 problem 73

Internal problem ID [10413]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 73.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$x(a\,x^k+b)\,y'-\alpha\,x^n y^2-(\beta-an\,x^k)\,y=\gamma\,x^{-n}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 257

```
dsolve(x*(a*x^k+b)*diff(y(x),x)=alpha*x^n*y(x)^2+(beta-a*n*x^k)*y(x)+gamma*x^(-n),y(x), sing
```

$$y(x) = -\frac{\left(x^n x^{-n} b^2 n^2 + 2x^n x^{-n} b\beta n + x^n x^{-n}\beta^2 + \tanh\left(\frac{\sqrt{b^4n^4+4b^3\beta n^3-4\alpha b^2\gamma n^2+6b^2\beta^2n^2-8\alpha b\beta\gamma n+4b\beta^3n-4\alpha\beta^2\gamma+\beta^4}\,(\ln(}{2bk(b^2n^2+2b\beta n+\beta^2}\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 4.641 (sec). Leaf size: 663

```
DSolve[x*(a*x^k+b)*y'[x]==\[Alpha]*x^n*y[x]^2+(\[Beta]-a*n*x^k)*y[x]+\[Gamma]*x^(-n),y[x],x,
```

$$y(x) \to \frac{x^{-n}\left(b\left(n\left(-\exp\left(-\frac{\left(\log\left(ax^k+b\right)+\log(b)-k\log(x)+\log(k)\right)\left(\sqrt{\alpha}\sqrt{\gamma}\sqrt{\frac{-4\alpha\gamma+b^2n^2+\beta^2+2b\beta n}{\alpha\gamma}}+bn+\beta\right)}{2bk}\right)\right)\right)-c_1n\exp\left(-\frac{(1}{}\right.\right.}{}$$

$$y(x) \to \frac{x^{-n}\left(\sqrt{\alpha}\sqrt{\gamma}\sqrt{\frac{(bn+\beta)^2}{\alpha\gamma}-4}-bn-\beta\right)}{2\alpha}$$

2.74 problem 74

Internal problem ID [10414]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 74.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$x^2(x^n a - 1)\left(y' + \lambda y^2\right) + (p\,x^n + q)\,xy = -r\,x^n - s$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 3716

```
dsolve(x^2*(a*x^n-1)*(diff(y(x),x)+lambda*y(x)^2)+(p*x^n+q)*x*y(x)+r*x^n+s=0,y(x), singsol=a
```

Expression too large to display

✓ Solution by Mathematica

Time used: 7.968 (sec). Leaf size: 2419

```
DSolve[x^2*(a*x^n-1)*(y'[x]+\[Lambda]*y[x]^2)+(p*x^n+q)*x*y[x]+r*x^n+s==0,y[x],x,IncludeSing
```

Too large to display

2.75 problem 75

Internal problem ID [10415]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 75.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{(x^n a + b\,x^m + c)\,y' - cy^2 + b\,x^{m-1}y = a\,x^{n-2}}$$

✗ Solution by Maple

```
dsolve((a*x^n+b*x^m+c)*diff(y(x),x)=c*y(x)^2-b*x^(m-1)*y(x)+a*x^(n-2),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a*x^n+b*x^m+c)*y'[x]==c*y[x]^2-b*x^(m-1)*y[x]+a*x^(n-2),y[x],x,IncludeSingularSoluti
```

Not solved

2.76 problem 76

Internal problem ID [10416]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 76.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$(x^n a + b\,x^m + c)\,y' - a\,x^{n-2}y^2 - b\,x^{m-1}y = c$$

✗ Solution by Maple

```
dsolve((a*x^n+b*x^m+c)*diff(y(x),x)=a*x^(n-2)*y(x)^2+b*x^(m-1)*y(x)+c,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a*x^n+b*x^m+c)*y'[x]==a*x^(n-2)*y[x]^2+b*x^(m-1)*y[x]+c,y[x],x,IncludeSingularSoluti
```

Not solved

2.77 problem 77

Internal problem ID [10417]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 77.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$(a\,x^n + b\,x^m + c)\,y' - \alpha\,x^k y^2 - \beta\,x^s y = -\alpha\,\lambda^2 x^k + \beta\lambda\,x^s$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 225

```
dsolve((a*x^n+b*x^m+c)*diff(y(x),x)=alpha*x^k*y(x)^2+beta*x^s*y(x)-alpha*lambda^2*x^k+beta*l
```

$$y(x) = -\frac{\left(\left(\int \frac{\alpha\,x^k \mathrm{e}^{\int -\frac{2x^k\alpha\lambda-\beta\,x^s}{a\,x^n+b\,x^m+c}dx}}{a\,x^n+b\,x^m+c}dx\right)\mathrm{e}^{\int \frac{2x^k\alpha\lambda-\beta\,x^s}{a\,x^n+b\,x^m+c}dx}\lambda + c_1 \mathrm{e}^{\int \frac{2x^k\alpha\lambda-\beta\,x^s}{a\,x^n+b\,x^m+c}dx}\lambda + 1\right)\mathrm{e}^{\int -\frac{2x^k\alpha\lambda-\beta\,x^s}{a\,x^n+b\,x^m+c}dx}}{c_1 + \int \frac{\alpha\,x^k \mathrm{e}^{\int -\frac{2x^k\alpha\lambda-\beta\,x^s}{a\,x^n+b\,x^m+c}dx}}{a\,x^n+b\,x^m+c}dx}$$

✓ Solution by Mathematica

Time used: 13.649 (sec). Leaf size: 389

```
DSolve[(a*x^n+b*x^m+c)*y'[x]==\[Alpha]*x^k*y[x]^2+\[Beta]*x^s*y[x]-\[Alpha]*\[Lambda]^2*x^k+
```

$$\text{Solve}\left[\int_1^x \frac{\exp\left(-\int_1^{K[2]} -\frac{\beta K[1]^s-2\alpha\lambda K[1]^k}{bK[1]^m+aK[1]^n+c}dK[1]\right)\left(-\alpha\lambda K[2]^k+\alpha y(x)K[2]^k+\beta K[2]^s\right)}{(k-s)\alpha\beta\left(bK[2]^m+aK[2]^n+c\right)(\lambda+y(x))}dK[2]\right.$$
$$+\int_1^{y(x)}\left(-\int_1^x\left(\frac{\exp\left(-\int_1^{K[2]} -\frac{\beta K[1]^s-2\alpha\lambda K[1]^k}{bK[1]^m+aK[1]^n+c}dK[1]\right)K[2]^k}{(k-s)\beta\left(bK[2]^m+aK[2]^n+c\right)(\lambda+K[3])}-\frac{\exp\left(-\int_1^{K[2]} -\frac{\beta K[1]^s-2\alpha\lambda K[1]^k}{bK[1]^m+aK[1]^n+c}dK[1]\right)(-\alpha}{(k-s)\alpha\beta\left(bK[2]^m+aK[2\right.}\right.\right.$$
$$\left.\left.-\frac{\exp\left(-\int_1^x -\frac{\beta K[1]^s-2\alpha\lambda K[1]^k}{bK[1]^m+aK[1]^n+c}dK[1]\right)}{(k-s)\alpha\beta(\lambda+K[3])^2}\right)dK[3]=c_1,y(x)\right]$$

2.78 problem 78

Internal problem ID [10418]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 78.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Riccati]`

$$(x^n a + b\,x^m + c)\,(y'x - y) + s\,x^k\left(y^2 - \lambda\,x^2\right) = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 42

```
dsolve((a*x^n+b*x^m+c)*(x*diff(y(x),x)-y(x))+s*x^k*(y(x)^2-lambda*x^2)=0,y(x), singsol=all)
```

$$y(x) = \tanh\left(\left(\int \frac{x^k}{a\,x^n + b\,x^m + c}dx\right) s\sqrt{\lambda} + c_1 s\sqrt{\lambda}\right) x\sqrt{\lambda}$$

✓ Solution by Mathematica

Time used: 22.652 (sec). Leaf size: 53

```
DSolve[(a*x^n+b*x^m+c)*(x*y'[x]-y[x])+s*x^k*(y[x]^2-\[Lambda]*x^2)==0,y[x],x,IncludeSingular
```

$$y(x) \to \sqrt{\lambda}(-x)\tanh\left(\sqrt{\lambda}\left(\int_1^x -\frac{sK[1]^k}{bK[1]^m + aK[1]^n + c}dK[1] + c_1\right)\right)$$

3 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions

3.1 problem 1

Internal problem ID [10419]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - ay^2 = b\,\mathrm{e}^{\lambda x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 144

```
dsolve(diff(y(x),x)=a*y(x)^2+b*exp(lambda*x),y(x), singsol=all)
```

$$y(x) = \left(\frac{\sqrt{b}\,c_1 \operatorname{BesselY}\left(1, \frac{2\sqrt{b}\,\sqrt{a}\,\mathrm{e}^{\frac{\lambda x}{2}}}{\lambda}\right)}{\sqrt{a}\left(c_1 \operatorname{BesselY}\left(0, \frac{2\sqrt{b}\,\sqrt{a}\,\mathrm{e}^{\frac{\lambda x}{2}}}{\lambda}\right) + \operatorname{BesselJ}\left(0, \frac{2\sqrt{b}\,\sqrt{a}\,\mathrm{e}^{\frac{\lambda x}{2}}}{\lambda}\right)\right)} + \frac{\sqrt{b}\operatorname{BesselJ}\left(1, \frac{2\sqrt{b}\,\sqrt{a}\,\mathrm{e}^{\frac{\lambda x}{2}}}{\lambda}\right)}{\sqrt{a}\left(c_1 \operatorname{BesselY}\left(0, \frac{2\sqrt{b}\,\sqrt{a}\,\mathrm{e}^{\frac{\lambda x}{2}}}{\lambda}\right) + \operatorname{BesselJ}\left(0, \frac{2\sqrt{b}\,\sqrt{a}\,\mathrm{e}^{\frac{\lambda x}{2}}}{\lambda}\right)\right)}\right)\mathrm{e}^{\frac{\lambda x}{2}}$$

✓ Solution by Mathematica

Time used: 0.551 (sec). Leaf size: 266

```
DSolve[y'[x]==a*y[x]^2+b*Exp[\[Lambda]*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{be^{\lambda x}}\left(2\,\text{BesselY}\left(1,\frac{2\sqrt{a}\sqrt{be^{x\lambda}}}{\lambda}\right)+c_1\,\text{BesselJ}\left(1,\frac{2\sqrt{a}\sqrt{be^{x\lambda}}}{\lambda}\right)\right)}{\sqrt{a}\left(2\,\text{BesselY}\left(0,\frac{2\sqrt{a}\sqrt{be^{x\lambda}}}{\lambda}\right)+c_1\,\text{BesselJ}\left(0,\frac{2\sqrt{a}\sqrt{be^{x\lambda}}}{\lambda}\right)\right)}$$

$$y(x) \to \frac{\sqrt{be^{\lambda x}}\,\text{BesselJ}\left(1,\frac{2\sqrt{a}\sqrt{be^{x\lambda}}}{\lambda}\right)}{\sqrt{a}\,\text{BesselJ}\left(0,\frac{2\sqrt{a}\sqrt{be^{x\lambda}}}{\lambda}\right)}$$

$$y(x) \to \frac{\sqrt{be^{\lambda x}}\,\text{BesselJ}\left(1,\frac{2\sqrt{a}\sqrt{be^{x\lambda}}}{\lambda}\right)}{\sqrt{a}\,\text{BesselJ}\left(0,\frac{2\sqrt{a}\sqrt{be^{x\lambda}}}{\lambda}\right)}$$

3.2 problem 2

Internal problem ID [10420]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = a\lambda\, \mathrm{e}^{\lambda x} - a^2 \mathrm{e}^{2\lambda x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 63

```
dsolve(diff(y(x),x)=y(x)^2+a*lambda*exp(lambda*x)-a^2*exp(2*lambda*x),y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{Ei}_1\left(-\frac{2\,\mathrm{e}^{\lambda x} a}{\lambda}\right) \mathrm{e}^{\lambda x} c_1 a + \mathrm{e}^{\frac{2\,\mathrm{e}^{\lambda x} a}{\lambda}} c_1 \lambda + \mathrm{e}^{\lambda x} a}{\mathrm{Ei}_1\left(-\frac{2\,\mathrm{e}^{\lambda x} a}{\lambda}\right) c_1 + 1}$$

✓ Solution by Mathematica

Time used: 2.507 (sec). Leaf size: 79

```
DSolve[y'[x]==y[x]^2+a*\[Lambda]*Exp[\[Lambda]*x]-a^2*Exp[2*\[Lambda]*x],y[x],x,IncludeSingu
```

$$y(x) \to \frac{ae^{\lambda x}\,\mathrm{ExpIntegralEi}\left(\frac{2ae^{x\lambda}}{\lambda}\right) + \lambda\left(-e^{\frac{2ae^{\lambda x}}{\lambda}}\right) + ac_1 e^{\lambda x}}{\mathrm{ExpIntegralEi}\left(\frac{2ae^{x\lambda}}{\lambda}\right) + c_1}$$

$$y(x) \to ae^{\lambda x}$$

3.3 problem 3

Internal problem ID [10421]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \sigma y^2 = a + b\,e^{\lambda x} + c\,e^{2\lambda x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 650

```
dsolve(diff(y(x),x)=sigma*y(x)^2+a+b*exp(lambda*x)+c*exp(2*lambda*x),y(x), singsol=all)
```

$$y(x) = -\frac{\left(2i\,\mathrm{WhittakerW}\left(-\frac{i\sqrt{\sigma}\,b}{2\lambda\sqrt{c}}, \frac{i\sqrt{a}\,\sqrt{\sigma}}{\lambda}, \frac{2i\sqrt{c}\,\sqrt{\sigma}\,e^{\lambda x}}{\lambda}\right)\sqrt{\sigma}\,c_1c^2 + 2i\,\mathrm{WhittakerM}\left(-\frac{i\sqrt{\sigma}\,b}{2\lambda\sqrt{c}}, \frac{i\sqrt{a}\,\sqrt{\sigma}}{\lambda}, \frac{2i\sqrt{c}\,\sqrt{\sigma}\,e^{\lambda x}}{\lambda}\right)\sqrt{\sigma}\right.}{2c^{\frac{3}{2}}\sigma\left(\mathrm{WhittakerW}\left(-\frac{i\sqrt{\sigma}\,b}{2\lambda\sqrt{c}}, \frac{i\sqrt{a}\,\sqrt{\sigma}}{\lambda}, \frac{2i\sqrt{c}\,\sqrt{\sigma}\,e^{\lambda x}}{\lambda}\right)c_1 + \mathrm{WhittakerM}\left(-\frac{i\sqrt{\sigma}\,b}{2\lambda\sqrt{c}}, \frac{i\sqrt{a}\,\sqrt{\sigma}}{\lambda}, \frac{2i\sqrt{c}\,\sqrt{\sigma}\,e^{\lambda x}}{\lambda}\right)\right.}$$
$$+ \frac{c_1\lambda\,\mathrm{WhittakerW}\left(-\frac{i\sqrt{\sigma}\,b-2\lambda\sqrt{c}}{2\lambda\sqrt{c}}, \frac{i\sqrt{a}\,\sqrt{\sigma}}{\lambda}, \frac{2i\sqrt{c}\,\sqrt{\sigma}\,e^{\lambda x}}{\lambda}\right)}{\sigma\left(\mathrm{WhittakerW}\left(-\frac{i\sqrt{\sigma}\,b}{2\lambda\sqrt{c}}, \frac{i\sqrt{a}\,\sqrt{\sigma}}{\lambda}, \frac{2i\sqrt{c}\,\sqrt{\sigma}\,e^{\lambda x}}{\lambda}\right)c_1 + \mathrm{WhittakerM}\left(-\frac{i\sqrt{\sigma}\,b}{2\lambda\sqrt{c}}, \frac{i\sqrt{a}\,\sqrt{\sigma}}{\lambda}, \frac{2i\sqrt{c}\,\sqrt{\sigma}\,e^{\lambda x}}{\lambda}\right)\right)}$$
$$- \frac{\left(i\sqrt{\sigma}\,c_1bc - c^{\frac{3}{2}}c_1\lambda\right)\mathrm{WhittakerW}\left(-\frac{i\sqrt{\sigma}\,b}{2\lambda\sqrt{c}}, \frac{i\sqrt{a}\,\sqrt{\sigma}}{\lambda}, \frac{2i\sqrt{c}\,\sqrt{\sigma}\,e^{\lambda x}}{\lambda}\right) + \left(2i\sqrt{\sigma}\,\sqrt{a}\,c^{\frac{3}{2}} - i\sqrt{\sigma}\,bc + c^{\frac{3}{2}}\lambda\right)\mathrm{Whitt}}{2c^{\frac{3}{2}}\sigma\left(\mathrm{WhittakerW}\left(-\frac{i\sqrt{\sigma}\,b}{2\lambda\sqrt{c}}, \frac{i\sqrt{a}\,\sqrt{\sigma}}{\lambda}, \frac{2i\sqrt{c}\,\sqrt{\sigma}\,e^{\lambda x}}{\lambda}\right)\right.}$$

✓ Solution by Mathematica

Time used: 3.251 (sec). Leaf size: 1081

```
DSolve[y'[x]==sigma*y[x]^2+a+b*Exp[\[Lambda]*x]+c*Exp[2*\[Lambda]*x],y[x],x,IncludeSingularS
```

$$y(x) \to -\frac{i\left(c_1\lambda\left(\sqrt{a}-\sqrt{c}e^{\lambda x}\right)\text{HypergeometricU}\left(\frac{\frac{i\sqrt{\sigma}b}{\sqrt{c}}+\lambda+2i\sqrt{a}\sqrt{\sigma}}{2\lambda},\frac{2i\sqrt{a}\sqrt{\sigma}}{\lambda}+1,\frac{2i\sqrt{c}e^{x\lambda}\sqrt{\sigma}}{\lambda}\right)-ic_1e^{\lambda x}\left(b\sqrt{\sigma}+\sqrt{c}(\right.\right.}{\lambda\sqrt{\sigma}\left(c_1\,\text{Hyperge}\right.}$$

$$y(x) \to \frac{-\frac{e^{\lambda x}\left(b\sqrt{\sigma}+\sqrt{c}\left(2\sqrt{a}\sqrt{\sigma}-i\lambda\right)\right)\text{HypergeometricU}\left(\frac{\frac{i\sqrt{\sigma}b}{\sqrt{c}}+3\lambda+2i\sqrt{a}\sqrt{\sigma}}{2\lambda},\frac{2i\sqrt{a}\sqrt{\sigma}}{\lambda}+2,\frac{2i\sqrt{c}e^{x\lambda}\sqrt{\sigma}}{\lambda}\right)}{\lambda\,\text{HypergeometricU}\left(\frac{\frac{i\sqrt{\sigma}b}{\sqrt{c}}+\lambda+2i\sqrt{a}\sqrt{\sigma}}{2\lambda},\frac{2i\sqrt{a}\sqrt{\sigma}}{\lambda}+1,\frac{2i\sqrt{c}e^{x\lambda}\sqrt{\sigma}}{\lambda}\right)}-i\left(\sqrt{a}-\sqrt{c}e^{\lambda x}\right)}{\sqrt{\sigma}}$$

$$y(x) \to \frac{-\frac{e^{\lambda x}\left(b\sqrt{\sigma}+\sqrt{c}\left(2\sqrt{a}\sqrt{\sigma}-i\lambda\right)\right)\text{HypergeometricU}\left(\frac{\frac{i\sqrt{\sigma}b}{\sqrt{c}}+3\lambda+2i\sqrt{a}\sqrt{\sigma}}{2\lambda},\frac{2i\sqrt{a}\sqrt{\sigma}}{\lambda}+2,\frac{2i\sqrt{c}e^{x\lambda}\sqrt{\sigma}}{\lambda}\right)}{\lambda\,\text{HypergeometricU}\left(\frac{\frac{i\sqrt{\sigma}b}{\sqrt{c}}+\lambda+2i\sqrt{a}\sqrt{\sigma}}{2\lambda},\frac{2i\sqrt{a}\sqrt{\sigma}}{\lambda}+1,\frac{2i\sqrt{c}e^{x\lambda}\sqrt{\sigma}}{\lambda}\right)}-i\left(\sqrt{a}-\sqrt{c}e^{\lambda x}\right)}{\sqrt{\sigma}}$$

3.4 problem 4

Internal problem ID [10422]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \sigma y^2 - ay = b\,\mathrm{e}^x + c}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 317

```
dsolve(diff(y(x),x)=sigma*y(x)^2+a*y(x)+b*exp(x)+c,y(x), singsol=all)
```

$$
\begin{aligned}
y(x) &= \left(\frac{\sqrt{b}\, c_1 \operatorname{BesselY}\left(\sqrt{a^2-4c\sigma}+1, 2\sqrt{\sigma}\,\sqrt{b}\,\mathrm{e}^{\frac{x}{2}}\right)}{\sqrt{\sigma}\,\left(\operatorname{BesselY}\left(\sqrt{a^2-4c\sigma}, 2\sqrt{\sigma}\,\sqrt{b}\,\mathrm{e}^{\frac{x}{2}}\right) c_1 + \operatorname{BesselJ}\left(\sqrt{a^2-4c\sigma}, 2\sqrt{\sigma}\,\sqrt{b}\,\mathrm{e}^{\frac{x}{2}}\right)\right)} \right. \\
&\quad \left. + \frac{\operatorname{BesselJ}\left(\sqrt{a^2-4c\sigma}+1, 2\sqrt{\sigma}\,\sqrt{b}\,\mathrm{e}^{\frac{x}{2}}\right)\sqrt{b}}{\sqrt{\sigma}\,\left(\operatorname{BesselY}\left(\sqrt{a^2-4c\sigma}, 2\sqrt{\sigma}\,\sqrt{b}\,\mathrm{e}^{\frac{x}{2}}\right) c_1 + \operatorname{BesselJ}\left(\sqrt{a^2-4c\sigma}, 2\sqrt{\sigma}\,\sqrt{b}\,\mathrm{e}^{\frac{x}{2}}\right)\right)} \right) \mathrm{e}^{\frac{x}{2}} \\
&\quad + \frac{\left(-\sqrt{a^2-4c\sigma}\, c_1 - c_1 a\right) \operatorname{BesselY}\left(\sqrt{a^2-4c\sigma}, 2\sqrt{\sigma}\,\sqrt{b}\,\mathrm{e}^{\frac{x}{2}}\right) + \left(-\sqrt{a^2-4c\sigma} - a\right) \operatorname{BesselJ}\left(\sqrt{a^2-4c\sigma},\right.}{2\sigma\left(\operatorname{BesselY}\left(\sqrt{a^2-4c\sigma}, 2\sqrt{\sigma}\,\sqrt{b}\,\mathrm{e}^{\frac{x}{2}}\right) c_1 + \operatorname{BesselJ}\left(\sqrt{a^2-4c\sigma}, 2\sqrt{\sigma}\,\sqrt{b}\,\mathrm{e}^{\frac{x}{2}}\right)\right)}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.971 (sec). Leaf size: 546

```
DSolve[y'[x]==sigma*y[x]^2+a*y[x]+b*Exp[x]+c,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to$$

$$-\frac{a\sqrt{b\sigma e^x}\,\text{Gamma}\left(\sqrt{a^2-4c\sigma}+1\right)\text{BesselJ}\left(\sqrt{a^2-4c\sigma},2\sqrt{be^x\sigma}\right)+b\sigma e^x\,\text{Gamma}\left(\sqrt{a^2-4c\sigma}+1\right)\text{Be}}{}$$

$$y(x) \to \frac{\frac{\sqrt{b\sigma e^x}\left(\text{BesselJ}\left(1-\sqrt{a^2-4c\sigma},2\sqrt{be^x\sigma}\right)-\text{BesselJ}\left(-\sqrt{a^2-4c\sigma}-1,2\sqrt{be^x\sigma}\right)\right)}{\text{BesselJ}\left(-\sqrt{a^2-4c\sigma},2\sqrt{be^x\sigma}\right)}-a}{2\sigma}$$

3.5 problem 5

Internal problem ID [10423]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - by = a(\lambda - b)\,\mathrm{e}^{\lambda x} - a^2\mathrm{e}^{2\lambda x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 117

```
dsolve(diff(y(x),x)=y(x)^2+b*y(x)+a*(lambda-b)*exp(lambda*x)-a^2*exp(2*lambda*x),y(x), sings
```

$$y(x) = -\frac{\left(-\left(\int \mathrm{e}^{xb+\frac{2\,\mathrm{e}^{\lambda x}a}{\lambda}}dx\right)a - c_1 a\right)\mathrm{e}^{\frac{2\,\mathrm{e}^{\lambda x}a}{\lambda}}\,\mathrm{e}^{\lambda x-\frac{2\,\mathrm{e}^{\lambda x}a}{\lambda}}}{\int \mathrm{e}^{xb+\frac{2\,\mathrm{e}^{\lambda x}a}{\lambda}}dx + c_1} - \frac{\mathrm{e}^{xb}\mathrm{e}^{\frac{2\,\mathrm{e}^{\lambda x}a}{\lambda}}}{\int \mathrm{e}^{xb+\frac{2\,\mathrm{e}^{\lambda x}a}{\lambda}}dx + c_1}$$

✓ Solution by Mathematica

Time used: 3.226 (sec). Leaf size: 191

```
DSolve[y'[x]==y[x]^2+b*y[x]+a*(\[Lambda]-b)*Exp[\[Lambda]*x]-a^2*Exp[2*\[Lambda]*x],y[x],x,I
```

$$y(x) \to \frac{-2^{b/\lambda}\left(b - ae^{\lambda x}\right)\left(\frac{ae^{\lambda x}}{\lambda}\right)^{b/\lambda} L_{-\frac{b}{\lambda}}^{\frac{b}{\lambda}}\left(\frac{2ae^{x\lambda}}{\lambda}\right) + ae^{\lambda x}\left(2^{\frac{b+\lambda}{\lambda}}\left(\frac{ae^{\lambda x}}{\lambda}\right)^{b/\lambda} L_{-\frac{b+\lambda}{\lambda}}^{\frac{b+\lambda}{\lambda}}\left(\frac{2ae^{x\lambda}}{\lambda}\right) + c_1\right)}{2^{b/\lambda}\left(\frac{ae^{\lambda x}}{\lambda}\right)^{b/\lambda} L_{-\frac{b}{\lambda}}^{\frac{b}{\lambda}}\left(\frac{2ae^{x\lambda}}{\lambda}\right) + c_1}$$

$$y(x) \to ae^{\lambda x}$$

3.6 problem 6

Internal problem ID [10424]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a\,\mathrm{e}^{\lambda x} y = -ab\,\mathrm{e}^{\lambda x} - b^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 46

```
dsolve(diff(y(x),x)=y(x)^2+a*exp(lambda*x)*y(x)-a*b*exp(lambda*x)-b^2,y(x), singsol=all)
```

$$y(x) = b - \frac{\mathrm{e}^{\frac{\mathrm{e}^{\lambda x} a}{\lambda}+2xb}}{\int \mathrm{e}^{\frac{\mathrm{e}^{\lambda x} a}{\lambda}+2xb} dx - c_1}$$

✓ Solution by Mathematica

Time used: 0.944 (sec). Leaf size: 115

```
DSolve[y'[x]==y[x]^2+a*Exp[\[Lambda]*x]*y[x]-a*b*Exp[\[Lambda]*x]-b^2,y[x],x,IncludeSingular
```

$$y(x) \to \frac{b\left(-2\lambda e^{\frac{ae^{\lambda x}}{\lambda}}\left(-\frac{ae^{\lambda x}}{\lambda}\right)^{\frac{2b}{\lambda}} + 2b\Gamma\left(\frac{2b}{\lambda}, 0, -\frac{ae^{x\lambda}}{\lambda}\right) + c_1\lambda(-1)^{b/\lambda}\right)}{2b\Gamma\left(\frac{2b}{\lambda}, 0, -\frac{ae^{x\lambda}}{\lambda}\right) + c_1\lambda(-1)^{b/\lambda}}$$

$$y(x) \to b$$

3.7 problem 7

Internal problem ID [10425]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = a\,\mathrm{e}^{2\lambda x}\left(\mathrm{e}^{\lambda x} + b\right)^n - \frac{\lambda^2}{4}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 739

```
dsolve(diff(y(x),x)=y(x)^2+a*exp(2*lambda*x)*(exp(lambda*x)+b)^n-1/4*lambda^2,y(x), singsol=
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+a*Exp[2*\[Lambda]*x]*(Exp[\[Lambda]*x]+b)^n-1/4*\[Lambda]^2,y[x],x,Incl
```

Not solved

3.8 problem 8

Internal problem ID [10426]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = a\,e^{8\lambda x} + b\,e^{6\lambda x} + c\,e^{4\lambda x} - \lambda^2$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 2495

```
dsolve(diff(y(x),x)=y(x)^2+a*exp(8*lambda*x)+b*exp(6*lambda*x)+c*exp(4*lambda*x)-lambda^2,y(
```

Expression too large to display

✓ Solution by Mathematica

Time used: 4.991 (sec). Leaf size: 1282

```
DSolve[y'[x]==y[x]^2+a*Exp[8*\[Lambda]*x]+b*Exp[6*\[Lambda]*x]+c*Exp[4*\[Lambda]*x]-\[Lambda
```

$$y(x) \rightarrow \frac{-e^{2x\lambda}\,\text{Hypergeometric1F1}\left(\frac{-ib^2+4iac+40a^{3/2}\lambda}{32a^{3/2}\lambda}, \frac{3}{2}, \frac{i\left(2e^{2x\lambda}a+b\right)^2}{8a^{3/2}\lambda}\right) b^3 - 2ae^{4x\lambda}\,\text{Hypergeometric1F1}\left(\frac{-ib^2+4iac+}{32a^{3/2}}\right.}{}$$

$$y(x) \rightarrow \frac{\left(\frac{1}{8}+\frac{i}{8}\right) e^{2\lambda x}\left(8a^{3/2}\lambda + 4iac - ib^2\right) \text{HermiteH}\left(\frac{i\left(b^2-4ac+24ia^{3/2}\lambda\right)}{16a^{3/2}\lambda}, \frac{\left(\frac{1}{4}+\frac{i}{4}\right)\left(2e^{2x\lambda}a+b\right)}{a^{3/4}\sqrt{\lambda}}\right)}{a^{5/4}\sqrt{\lambda}\,\text{HermiteH}\left(\frac{i\left(b^2-4ac+8ia^{3/2}\lambda\right)}{16a^{3/2}\lambda}, \frac{\left(\frac{1}{4}+\frac{i}{4}\right)\left(2e^{2x\lambda}a+b\right)}{a^{3/4}\sqrt{\lambda}}\right)} + \frac{ibe^{2\lambda x}}{2\sqrt{a}} + i\sqrt{a}e^{4\lambda x} + \lambda$$

3.9 problem 9

Internal problem ID [10427]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a\,e^{kx}y^2 = b\,e^{sx}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 145

```
dsolve(diff(y(x),x)=a*exp(k*x)*y(x)^2+b*exp(s*x),y(x), singsol=all)
```

$$y(x) = \frac{\left(\operatorname{BesselY}\left(\frac{s}{s+k}, \frac{2\sqrt{b}\,\sqrt{a}\,e^{\frac{x(s+k)}{2}}}{s+k}\right)c_1 + \operatorname{BesselJ}\left(\frac{s}{s+k}, \frac{2\sqrt{b}\,\sqrt{a}\,e^{\frac{x(s+k)}{2}}}{s+k}\right)\right)\sqrt{b}\,e^{-xk+\frac{x(s+k)}{2}}}{\sqrt{a}\,\left(\operatorname{BesselY}\left(-\frac{k}{s+k}, \frac{2\sqrt{b}\,\sqrt{a}\,e^{\frac{x(s+k)}{2}}}{s+k}\right)c_1 + \operatorname{BesselJ}\left(-\frac{k}{s+k}, \frac{2\sqrt{b}\,\sqrt{a}\,e^{\frac{x(s+k)}{2}}}{s+k}\right)\right)}$$

✓ Solution by Mathematica

Time used: 6.491 (sec). Leaf size: 1097

```
DSolve[y'[x]==a*Exp[k*x]*y[x]^2+b*Exp[s*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-kx}\left(-kK_{\frac{k\log\left(e^{k+s}\right)}{(k+s)^2}}\left(2\sqrt{-\frac{ab\left(\left(e^{k+s}\right)^x\right)^{\frac{k+s}{\log\left(e^{k+s}\right)}}\log^2\left(e^{k+s}\right)}{(k+s)^4}}\right)-c_1k(-1)^{\frac{k\log\left(e^{k+s}\right)}{(k+s)^2}}\text{BesselI}\left(\frac{k\log\left(e^{k+s}\right)}{(k+s)^2},2\sqrt{-\frac{ab\cdots}{\cdots}}\right.\cdots}{\cdots}$$

$$y(x) \to \frac{e^{-kx}\left(-\frac{(k+s)\sqrt{-\frac{ab\log^2\left(e^{k+s}\right)\left(\left(e^{k+s}\right)^x\right)^{\frac{k+s}{\log\left(e^{k+s}\right)}}}{(k+s)^4}}\left(\text{BesselI}\left(\frac{k\log\left(e^{k+s}\right)}{(k+s)^2}-1,2\sqrt{-\frac{ab\left(\left(e^{k+s}\right)^x\right)^{\frac{k+s}{\log\left(e^{k+s}\right)}}\log^2\left(e^{k+s}\right)}{(k+s)^4}}\right)+\text{BesselI}\left(\frac{k\log\cdots}{(k\cdots}\right.\cdots\right.}{\text{BesselI}\left(\frac{k\log\left(e^{k+s}\right)}{(k+s)^2},2\sqrt{-\frac{ab\left(\left(e^{k+s}\right)^x\right)^{\frac{k+s}{\log\left(e^{k+s}\right)}}\log^2\left(e^{k+s}\right)}{(k+s)^4}}\right)}\cdots\right.}{2a}$$

$$y(x) \to \frac{e^{-kx}\left(-\frac{(k+s)\sqrt{-\frac{ab\log^2\left(e^{k+s}\right)\left(\left(e^{k+s}\right)^x\right)^{\frac{k+s}{\log\left(e^{k+s}\right)}}}{(k+s)^4}}\left(\text{BesselI}\left(\frac{k\log\left(e^{k+s}\right)}{(k+s)^2}-1,2\sqrt{-\frac{ab\left(\left(e^{k+s}\right)^x\right)^{\frac{k+s}{\log\left(e^{k+s}\right)}}\log^2\left(e^{k+s}\right)}{(k+s)^4}}\right)+\text{BesselI}\left(\frac{k\log\cdots}{(k\cdots}\right.\cdots\right.}{\text{BesselI}\left(\frac{k\log\left(e^{k+s}\right)}{(k+s)^2},2\sqrt{-\frac{ab\left(\left(e^{k+s}\right)^x\right)^{\frac{k+s}{\log\left(e^{k+s}\right)}}\log^2\left(e^{k+s}\right)}{(k+s)^4}}\right)}\cdots\right.}{2a}$$

3.10 problem 10

Internal problem ID [10428]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - b\,\mathrm{e}^{\mu x} y^2 = a\lambda\,\mathrm{e}^{\lambda x} - a^2 b\,\mathrm{e}^{(\mu+2\lambda)x}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=b*exp(mu*x)*y(x)^2+a*lambda*exp(lambda*x)-a^2*b*exp((mu+2*lambda)*x),y(x
```

No solution found

✓ Solution by Mathematica

Time used: 8.808 (sec). Leaf size: 844

```
DSolve[y'[x]==b*Exp[\[Mu]*x]*y[x]^2+a*\[Lambda]*Exp[\[Lambda]*x]-a^2*b*Exp[(\[Mu]+2*\[Lambda]
```

$$y(x) \to \frac{e^{\mu(-x)}\left(-2ab\log\left(e^{\lambda+\mu}\right)\left(\left(e^{\lambda+\mu}\right)^{x}\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}\left(2(\lambda+\mu)L_{-\frac{\log\left(e^{\lambda+\mu}\right)}{2(\lambda+\mu)}-\frac{3}{2}}^{\frac{\mu\log\left(e^{\lambda+\mu}\right)}{(\lambda+\mu)^2}+1}\left(-\frac{2ab\left(\left(e^{\lambda+\mu}\right)^{x}\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}\log\left(e^{\lambda+\mu}\right)}{(\lambda+\mu)^2}\right)+c\right.\right.}{}$$

$$y(x) \to -\frac{ae^{\mu(-x)}\log\left(e^{\lambda+\mu}\right)\left(\log\left(e^{\lambda+\mu}\right)+\lambda+\mu\right)\left(\left(e^{\lambda+\mu}\right)^{x}\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\log\left(e^{\lambda+\mu}\right)}{\lambda+\mu}+3\right),\frac{2(\lambda+\mu\right.}{}}{(\lambda+\mu)^2\,\text{HypergeometricU}\left(\frac{\lambda+\mu+\log\left(e^{\lambda+\mu}\right)}{2(\lambda+\mu)},\frac{\mu\log\left(e^{\lambda+\mu}\right)}{(\lambda+\mu)^2}+1,-\frac{2ab\left(\left(e^{\lambda+\mu}\right)^{x}\right)^{\frac{}{\log(}}}{(\lambda+}\right.}$$

$$-\frac{e^{\mu(-x)}\left(\log\left(e^{\lambda+\mu}\right)\left(2ab\left(\left(e^{\lambda+\mu}\right)^{x}\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}+\mu\right)+\mu(\lambda+\mu)\right)}{2b(\lambda+\mu)}$$

3.11 problem 11

Internal problem ID [10429]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Riccati]`

$$y' - a\,\mathrm{e}^{\lambda x} y^2 - by = c\,\mathrm{e}^{-\lambda x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 165

```
dsolve(diff(y(x),x)=a*exp(lambda*x)*y(x)^2+b*y(x)+c*exp(-lambda*x),y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{-\lambda x}\left(\sqrt{4b^2ac+8abc\lambda+4\lambda^2ac-b^4-4b^3\lambda-6b^2\lambda^2-4b\,\lambda^3-\lambda^4}\,\tan\left(\frac{\sqrt{4b^2ac+8abc\lambda+4\lambda^2ac-b^4-4b^3\lambda-6b^2\lambda^2-4b}}{2b^2+4b\lambda+2\lambda^2}\right.\right.}{2a\,(b+\lambda)}$$

✓ Solution by Mathematica

Time used: 0.927 (sec). Leaf size: 188

```
DSolve[y'[x]==a*Exp[\[Lambda]*x]*y[x]^2+b*y[x]+c*Exp[-\[Lambda]*x],y[x],x,IncludeSingularSol
```

$$y(x) \to \frac{e^{\lambda(-x)}\left(-\sqrt{-4ac+b^2+2b\lambda+\lambda^2}+\frac{2}{\frac{1}{\sqrt{-4ac+b^2+2b\lambda+\lambda^2}}+c_1e^{x\sqrt{-4ac+b^2+2b\lambda+\lambda^2}}}-b-\lambda\right)}{2a}$$

$$y(x) \to -\frac{e^{\lambda(-x)}\left(b\left(\sqrt{-4ac+b^2+2b\lambda+\lambda^2}+2\lambda\right)+\lambda\left(\sqrt{-4ac+b^2+2b\lambda+\lambda^2}+\lambda\right)-4ac+b^2\right)}{2a\sqrt{-4ac+b^2+2b\lambda+\lambda^2}}$$

3.12 problem 12

Internal problem ID [10430]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,\mathrm{e}^{\mu x} y^2 - y\lambda = -a\,b^2 \mathrm{e}^{(\mu+2\lambda)x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 86

```
dsolve(diff(y(x),x)=a*exp(mu*x)*y(x)^2+lambda*y(x)-a*b^2*exp((mu+2*lambda)*x),y(x), singsol=
```

$$y(x) = -\frac{b\left(c_1 \sinh\left(\frac{ba\,\mathrm{e}^{x(\lambda+\mu)}}{\lambda+\mu}\right) + \cosh\left(\frac{ba\,\mathrm{e}^{x(\lambda+\mu)}}{\lambda+\mu}\right)\right) \mathrm{e}^{x(\lambda+\mu)-\mu x}}{c_1 \cosh\left(\frac{ba\,\mathrm{e}^{x(\lambda+\mu)}}{\lambda+\mu}\right) + \sinh\left(\frac{ba\,\mathrm{e}^{x(\lambda+\mu)}}{\lambda+\mu}\right)}$$

✓ Solution by Mathematica

Time used: 2.706 (sec). Leaf size: 286

```
DSolve[y'[x]==a*Exp[\[Mu]*x]*y[x]^2+\[Lambda]*y[x]-a*b^2*Exp[(\[Mu]+2*\[Lambda])*x],y[x],x,I
```

$$y(x) \to \boxed{-\frac{\tan\left(\frac{ab^2 e^{x(2\lambda+\mu)}\sqrt{-\frac{e^{-2x\lambda}}{b^2}}}{\lambda+\mu} - c_1\right)}{\sqrt{-\frac{e^{-2x\lambda}}{b^2}}} \text{ if condition}}$$

3.13 problem 13

Internal problem ID [10431]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 e^{\lambda x} - a\, e^{\mu x} y = a\lambda\, e^{(\mu-\lambda)x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 140

```
dsolve(diff(y(x),x)=exp(lambda*x)*y(x)^2+a*exp(mu*x)*y(x)+a*lambda*exp((mu-lambda)*x),y(x),
```

$$y(x) = \left(-\frac{\lambda \operatorname{hypergeom}\left(\left[-\frac{\lambda-\mu}{\mu}\right], \left[-\frac{\lambda-2\mu}{\mu}\right], \frac{a\, e^{\mu x}}{\mu}\right) c_1 a\, e^{\mu x}}{(\lambda-\mu)\left(c_1 \operatorname{hypergeom}\left(\left[-\frac{\lambda}{\mu}\right], \left[-\frac{\lambda-\mu}{\mu}\right], \frac{a\, e^{\mu x}}{\mu}\right) + e^{\lambda x}\right)} - \frac{\lambda\, e^{\lambda x}}{c_1 \operatorname{hypergeom}\left(\left[-\frac{\lambda}{\mu}\right], \left[-\frac{\lambda-\mu}{\mu}\right], \frac{a\, e^{\mu x}}{\mu}\right) + e^{\lambda x}}\right) e^{-\lambda x}$$

✓ Solution by Mathematica

Time used: 4.392 (sec). Leaf size: 148

```
DSolve[y'[x]==Exp[\[Lambda]*x]*y[x]^2+a*Exp[\[Mu]*x]*y[x]+a*\[Lambda]*Exp[(\[Mu]-\[Lambda])*
```

$$y(x) \to -\frac{e^{\lambda(-x)}\left(-\lambda\left(-\frac{ae^{\mu x}}{\mu}\right)^{\lambda/\mu}\Gamma\left(-\frac{\lambda}{\mu},-\frac{ae^{x\mu}}{\mu}\right)+\mu e^{\frac{ae^{\mu x}}{\mu}}+c_1\lambda\left(e^{\mu x}\right)^{\lambda/\mu}\right)}{-\left(-\frac{ae^{\mu x}}{\mu}\right)^{\lambda/\mu}\Gamma\left(-\frac{\lambda}{\mu},-\frac{ae^{x\mu}}{\mu}\right)+c_1\left(e^{\mu x}\right)^{\lambda/\mu}}$$

$$y(x) \to \lambda\left(-e^{\lambda(-x)}\right)$$

3.14 problem 14

Internal problem ID [10432]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + y^2 e^{\lambda x}\lambda - a\,e^{\mu x} y = -a\,e^{(\mu-\lambda)x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 136

```
dsolve(diff(y(x),x)=-lambda*exp(lambda*x)*y(x)^2+a*exp(mu*x)*y(x)-a*exp((mu-lambda)*x),y(x),
```

$$y(x) = \left(\frac{\text{hypergeom}\left(\left[-\frac{\lambda-\mu}{\mu}\right], \left[-\frac{\lambda-2\mu}{\mu}\right], \frac{a\,e^{\mu x}}{\mu}\right) c_1 a\,e^{\mu x}}{(\lambda-\mu)\left(c_1 \,\text{hypergeom}\left(\left[-\frac{\lambda}{\mu}\right], \left[-\frac{\lambda-\mu}{\mu}\right], \frac{a\,e^{\mu x}}{\mu}\right) + e^{\lambda x}\right)} + \frac{e^{\lambda x}}{c_1 \,\text{hypergeom}\left(\left[-\frac{\lambda}{\mu}\right], \left[-\frac{\lambda-\mu}{\mu}\right], \frac{a\,e^{\mu x}}{\mu}\right) + e^{\lambda x}}\right) e^{-\lambda x}$$

✓ Solution by Mathematica

Time used: 4.358 (sec). Leaf size: 147

```
DSolve[y'[x]==-\[Lambda]*Exp[\[Lambda]*x]*y[x]^2+a*Exp[\[Mu]*x]*y[x]-a*Exp[(\[Mu]-\[Lambda])
```

$$y(x) \to \frac{e^{\lambda(-x)}\left(-\lambda\left(-\frac{ae^{\mu x}}{\mu}\right)^{\lambda/\mu}\Gamma\left(-\frac{\lambda}{\mu},-\frac{ae^{x\mu}}{\mu}\right)+\mu e^{\frac{ae^{\mu x}}{\mu}}+c_1\lambda(e^{\mu x})^{\lambda/\mu}\right)}{\lambda\left(-\left(-\frac{ae^{\mu x}}{\mu}\right)^{\lambda/\mu}\Gamma\left(-\frac{\lambda}{\mu},-\frac{ae^{x\mu}}{\mu}\right)+c_1\left(e^{\mu x}\right)^{\lambda/\mu}\right)}$$

$$y(x) \to e^{\lambda(-x)}$$

3.15 problem 15

Internal problem ID [10433]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,e^{\mu x} y^2 - ab\,e^{x(\lambda+\mu)} y = -b\lambda\,e^{\lambda x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 1648

```
dsolve(diff(y(x),x)=a*exp(mu*x)*y(x)^2+a*b*exp((lambda+mu)*x)*y(x)-b*lambda*exp(lambda*x),y(
```

Expression too large to display

✓ Solution by Mathematica

Time used: 12.587 (sec). Leaf size: 902

```
DSolve[y'[x]==a*Exp[\[Mu]*x]*y[x]^2+a*b*Exp[(\[Lambda]+\[Mu])*x]*y[x]-b*\[Lambda]*Exp[\[Lamb
```

$$y(x) \to \frac{e^{\mu(-x)}\left(ab\log\left(e^{\lambda+\mu}\right)\left(\left(e^{\lambda+\mu}\right)^x\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}\left(2(\lambda+\mu)L_{-\frac{\log\left(e^{\lambda+\mu}\right)}{2(\lambda+\mu)}-\frac{3}{2}}^{\frac{\mu\log\left(e^{\lambda+\mu}\right)}{(\lambda+\mu)^2}+1}\left(\frac{ab\left(\left(e^{\lambda+\mu}\right)^x\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}\log\left(e^{\lambda+\mu}\right)}{(\lambda+\mu)^2}\right)+c_1\left(\log\right.\right.\right.}{}$$

$$y(x) \to \frac{be^{\mu(-x)}\log\left(e^{\lambda+\mu}\right)\left(\log\left(e^{\lambda+\mu}\right)+\lambda+\mu\right)\left(\left(e^{\lambda+\mu}\right)^x\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\log\left(e^{\lambda+\mu}\right)}{\lambda+\mu}+3\right),\frac{\mu\log\left(e^{\lambda+\right.}}{(\lambda+\mu)^2}\right.}{2(\lambda+\mu)^2\text{HypergeometricU}\left(\frac{\lambda+\mu+\log\left(e^{\lambda+\mu}\right)}{2(\lambda+\mu)},\frac{\mu\log\left(e^{\lambda+\mu}\right)}{(\lambda+\mu)^2}+1,\frac{ab\left(\left(e^{\lambda+\mu}\right)^x\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}}{(\lambda+\mu)^2}\right.}$$

$$-\frac{e^{\mu(-x)}\left((\lambda+\mu)\left(ab\left(\left(e^{\lambda+\mu}\right)^x\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}+\mu\right)+\log\left(e^{\lambda+\mu}\right)\left(\mu-ab\left(\left(e^{\lambda+\mu}\right)^x\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}\right)\right)}{2a(\lambda+\mu)}$$

3.16 problem 16

Internal problem ID [10434]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,e^{kx} y^2 - by = c\,e^{sx} + d\,e^{-kx}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 563

```
dsolve(diff(y(x),x)=a*exp(k*x)*y(x)^2+b*y(x)+c*exp(s*x)+d*exp(-k*x),y(x), singsol=all)
```

$$y(x) = \left(\frac{\sqrt{c}\,c_1 \operatorname{BesselY}\left(\frac{\sqrt{-4ad+b^2+2bk+k^2}+s+k}{s+k}, \frac{2\sqrt{a}\sqrt{c}\,e^{\frac{x(s+k)}{2}}}{s+k}\right)}{\sqrt{a}\left(\operatorname{BesselY}\left(\frac{\sqrt{-4ad+b^2+2bk+k^2}}{s+k}, \frac{2\sqrt{a}\sqrt{c}\,e^{\frac{x(s+k)}{2}}}{s+k}\right) c_1 + \operatorname{BesselJ}\left(\frac{\sqrt{-4ad+b^2+2bk+k^2}}{s+k}, \frac{2\sqrt{a}\sqrt{c}\,e^{\frac{x(s+k)}{2}}}{s+k}\right)\right)} + \frac{\operatorname{BesselJ}\left(\frac{\sqrt{-4ad+b^2+2bk+k^2}+s+k}{s+k}, \frac{2\sqrt{a}\sqrt{c}\,e^{\frac{x(s+k)}{2}}}{s+k}\right)\sqrt{c}}{\sqrt{a}\left(\operatorname{BesselY}\left(\frac{\sqrt{-4ad+b^2+2bk+k^2}}{s+k}, \frac{2\sqrt{a}\sqrt{c}\,e^{\frac{x(s+k)}{2}}}{s+k}\right) c_1 + \operatorname{BesselJ}\left(\frac{\sqrt{-4ad+b^2+2bk+k^2}}{s+k}, \frac{2\sqrt{a}\sqrt{c}\,e^{\frac{x(s+k)}{2}}}{s+k}\right)\right)} \right) e^{-x} + \frac{\left(\left(-\sqrt{-4ad+b^2+2bk+k^2}\,c_1 - c_1 b - c_1 k\right)\operatorname{BesselY}\left(\frac{\sqrt{-4ad+b^2+2bk+k^2}}{s+k}, \frac{2\sqrt{a}\sqrt{c}\,e^{\frac{x(s+k)}{2}}}{s+k}\right) + \left(-\sqrt{-4ad+\right.\right.}{2\left(\operatorname{BesselY}\left(\frac{\sqrt{-4ad+b^2+2bk+k^2}}{s+k}, \frac{2\sqrt{a}\sqrt{c}\,e^{\frac{x(s+k)}{2}}}{s+k}\right) c_1 + \operatorname{BesselJ}\left(\sqrt{-\right.\right.}$$

✓ Solution by Mathematica

Time used: 18.386 (sec). Leaf size: 1636

```
DSolve[y'[x]==a*Exp[k*x]*y[x]^2+b*y[x]+c*Exp[s*x]+d*Exp[-k*x],y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{e^{-kx}\left(-\left((b+k)K_{\frac{\sqrt{(b^2+2kb+k^2-4ad)(k+s)^4\log^2(e^{k+s})}}{(k+s)^4}}\left(2\sqrt{-\frac{ac\left((e^{k+s})^x\right)^{\frac{k+s}{\log(e^{k+s})}}\log^2(e^{k+s})}{(k+s)^4}}\right)\right)+(-1)^{\frac{k^4+4sk^3+6s^2k^{\cdots}}{\cdots}}\cdots}{\cdots}$$

$$y(x) \to \frac{e^{-kx}\left(-(b+k)(k+s)^3\sqrt{-\frac{ac\log^2(e^{k+s})\left((e^{k+s})^x\right)^{\frac{k+s}{\log(e^{k+s})}}}{(k+s)^4}}\,\text{BesselI}\left(\frac{\sqrt{(b^2+2kb+k^2-4ad)(k+s)^4\log^2(e^{k+s})}}{(k+s)^4},2\sqrt{-\frac{ac(\cdots}{\cdots}}\cdots\right)\cdots\right)}{\cdots}$$

3.17 problem 17

Internal problem ID [10435]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a\,\mathrm{e}^{(2\lambda+\mu)x} y^2 - \left(b\,\mathrm{e}^{(\lambda+\mu)x} - \lambda\right) y = c\,\mathrm{e}^{\mu x}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 87

```
dsolve(diff(y(x),x)=a*exp((2*lambda+mu)*x)*y(x)^2+(b*exp((lambda+mu)*x)-lambda)*y(x)+c*exp(m
```

$$y(x) = \frac{\mathrm{e}^{\mu x}\left(\sqrt{4b^2ac-b^4}\,\tan\left(\frac{\sqrt{4b^2ac-b^4}\,(\mathrm{e}^{x(\lambda+\mu)}b+c_1\lambda+c_1\mu)}{2b^2(\lambda+\mu)}\right)-b^2\right)\mathrm{e}^{-x(\lambda+\mu)}}{2ab}$$

✓ Solution by Mathematica

Time used: 6.375 (sec). Leaf size: 349

```
DSolve[y'[x]==a*Exp[(2*\[Lambda]+\[Mu])*x]*y[x]^2+(b*Exp[(\[Lambda]+\[Mu])*x]-\[Lambda])*y[x
```

$$y(x) \to \frac{e^{\lambda(-x)}\left(b^2 e^{x(\lambda+\mu)}\left(\pi + ic_1\left(e^{\sqrt{\frac{\left(b^2-4ac\right)e^{2x(\lambda+\mu)}}{(\lambda+\mu)^2}}} - 1\right)\right)\right) - b(\lambda+\mu)\sqrt{\frac{(b^2-4ac)e^{2x(\lambda+\mu)}}{(\lambda+\mu)^2}}\left(\pi - ic_1\left(e^{\sqrt{\frac{\left(b^2-4ac\right)e^{2}}{(\lambda+\mu)}}}\right.\right.}{2a(\lambda+\mu)\sqrt{\frac{(b^2-4ac)e^{2x(\lambda+\mu)}}{(\lambda+\mu)^2}}\left(\pi - ic_1\left(e^{\sqrt{\frac{\left(b^2-4ac\right)e^{2x(}}{(\lambda+\mu)^2}}}\right.\right.}$$

$$y(x) \to \frac{e^{\lambda(-x)}\left(-(\lambda+\mu)e^{-x(\lambda+\mu)}\sqrt{\frac{(b^2-4ac)e^{2x(\lambda+\mu)}}{(\lambda+\mu)^2}}\tanh\left(\frac{1}{2}\sqrt{\frac{(b^2-4ac)e^{2x(\lambda+\mu)}}{(\lambda+\mu)^2}}\right) - b\right)}{2a}$$

3.18 problem 18

Internal problem ID [10436]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$
y' - a\,\mathrm{e}^{kx}y^2 - by = c\,\mathrm{e}^{knx} + d\,\mathrm{e}^{k(2n+1)x}
$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*exp(k*x)*y(x)^2+b*y(x)+c*exp(k*n*x)+d*exp(k*(2*n+1)*x),y(x), singsol=a
```

No solution found

✓ Solution by Mathematica

Time used: 27.598 (sec). Leaf size: 2503

```
DSolve[y'[x]==a*Exp[k*x]*y[x]^2+b*y[x]+c*Exp[k*n*x]+d*Exp[k*(2*n+1)*x],y[x],x,IncludeSingula
```

Too large to display

3.19 problem 19

Internal problem ID [10437]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - e^{\mu x}\left(y - b\, e^{\lambda x}\right)^2 = b\lambda\, e^{\lambda x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 74

```
dsolve(diff(y(x),x)=exp(mu*x)*(y(x)-b*exp(lambda*x))^2+b*lambda*exp(lambda*x),y(x), singsol=
```

$$y(x) = \left(e^{x(\lambda+\mu)}b - \mu + \frac{e^{-\mu x - \frac{2b\, e^{\lambda x+\mu x}}{\lambda+\mu} + \frac{2\, e^{x(\lambda+\mu)}b}{\lambda+\mu}}}{c_1 + \frac{e^{-\mu x}}{\mu}}\right) e^{-\mu x}$$

✓ Solution by Mathematica

Time used: 1.524 (sec). Leaf size: 40

```
DSolve[y'[x]==Exp[\[Mu]*x]*(y[x]-b*Exp[\[Lambda]*x])^2+b*\[Lambda]*Exp[\[Lambda]*x],y[x],x,I
```

$$y(x) \to be^{\lambda x} + \frac{\mu}{-e^{\mu x} + c_1\mu}$$

$$y(x) \to be^{\lambda x}$$

3.20 problem 20

Internal problem ID [10438]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$(e^{\lambda x}a + b\,e^{\mu x} + c)\,y' - y^2 - k\,e^{\nu x}y = -m^2 + km\,e^{\nu x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 129

```
dsolve((a*exp(lambda*x)+b*exp(mu*x)+c)*diff(y(x),x)=y(x)^2+k*exp(nu*x)*y(x)-m^2+k*m*exp(nu*x
```

$$y(x) = -m - \frac{e^{\int \frac{k\,e^{\nu x}}{e^{\lambda x}a+b\,e^{\mu x}+c}dx - 2m\left(\int \frac{1}{e^{\lambda x}a+b\,e^{\mu x}+c}dx\right)}}{\int \frac{e^{\int \frac{k\,e^{\nu x}}{e^{\lambda x}a+b\,e^{\mu x}+c}dx - 2m\left(\int \frac{1}{e^{\lambda x}a+b\,e^{\mu x}+c}dx\right)}}{e^{\lambda x}a+b\,e^{\mu x}+c}dx - c_1}$$

✓ Solution by Mathematica

Time used: 16.545 (sec). Leaf size: 358

```
DSolve[(a*Exp[\[Lambda]*x]+b*Exp[\[Mu]*x]+c)*y'[x]==y[x]^2+k*Exp[\[Nu]*x]*y[x]-m^2+k*m*Exp[\
```

$$\text{Solve}\left[\int_1^x -\frac{\exp\left(-\int_1^{K[2]} -\frac{e^{\nu K[1]}k-2m}{e^{\lambda K[1]}a+be^{\mu K[1]}+c}dK[1]\right)\left(e^{\nu K[2]}k-m+y(x)\right)}{\left(e^{\lambda K[2]}a+be^{\mu K[2]}+c\right)k\nu(m+y(x))}dK[2]\right.$$

$$+\int_1^{y(x)}\left(\frac{\exp\left(-\int_1^{x} -\frac{e^{\nu K[1]}k-2m}{e^{\lambda K[1]}a+be^{\mu K[1]}+c}dK[1]\right)}{k\nu(m+K[3])^2}\right.$$

$$-\int_1^x\left(\frac{\exp\left(-\int_1^{K[2]} -\frac{e^{\nu K[1]}k-2m}{e^{\lambda K[1]}a+be^{\mu K[1]}+c}dK[1]\right)\left(e^{\nu K[2]}k-m+K[3]\right)}{\left(e^{\lambda K[2]}a+be^{\mu K[2]}+c\right)k\nu(m+K[3])^2}-\frac{\exp\left(-\int_1^{K[2]} -\frac{e^{\nu K[1]}k-2m}{e^{\lambda K[1]}a+be^{\mu K[1]}+c}dK[1]\right.}{\left(e^{\lambda K[2]}a+be^{\mu K[2]}+c\right)k\nu(m+K[3]\right.}\right.$$

3.21 problem 21

Internal problem ID [10439]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$
\left(e^{\lambda x} a+b\,e^{\mu x}+c\right)\left(y'-y^{2}\right)=-a\,\lambda^{2} e^{\lambda x}-b\,\mu^{2} e^{\mu x}
$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 350

```
dsolve((a*exp(lambda*x)+b*exp(mu*x)+c)*(diff(y(x),x)-y(x)^2)+a*lambda^2*exp(lambda*x)+b*mu^2
```

$$
\begin{aligned}
y(x)= & -\frac{\left(\left(ab\lambda+\mu ba\right)\left(\int \frac{1}{\left(e^{\lambda x}a+b\,e^{\mu x}+c\right)^{2}}dx\right)+c_1 ab\lambda+c_1 ab\mu\right)e^{\lambda x+\mu x}}{\left(e^{\lambda x}a+b\,e^{\mu x}+c\right)^{2}\left(c_1+\int \frac{1}{\left(e^{\lambda x}a+b\,e^{\mu x}+c\right)^{2}}dx\right)} \\
& -\frac{\left(\left(\int \frac{1}{\left(e^{\lambda x}a+b\,e^{\mu x}+c\right)^{2}}dx\right)b^{2}\mu+c_1 b^{2}\mu\right)e^{2\mu x}}{\left(e^{\lambda x}a+b\,e^{\mu x}+c\right)^{2}\left(c_1+\int \frac{1}{\left(e^{\lambda x}a+b\,e^{\mu x}+c\right)^{2}}dx\right)} \\
& -\frac{\left(\left(\int \frac{1}{\left(e^{\lambda x}a+b\,e^{\mu x}+c\right)^{2}}dx\right)a^{2}\lambda+c_1 a^{2}\lambda\right)e^{2\lambda x}}{\left(e^{\lambda x}a+b\,e^{\mu x}+c\right)^{2}\left(c_1+\int \frac{1}{\left(e^{\lambda x}a+b\,e^{\mu x}+c\right)^{2}}dx\right)} \\
& -\frac{\left(\left(\int \frac{1}{\left(e^{\lambda x}a+b\,e^{\mu x}+c\right)^{2}}dx\right)bc\mu+c_1 bc\mu\right)e^{\mu x}+1+\left(\left(\int \frac{1}{\left(e^{\lambda x}a+b\,e^{\mu x}+c\right)^{2}}dx\right)ac\lambda+c_1 ac\lambda\right)e^{\lambda x}}{\left(e^{\lambda x}a+b\,e^{\mu x}+c\right)^{2}\left(c_1+\int \frac{1}{\left(e^{\lambda x}a+b\,e^{\mu x}+c\right)^{2}}dx\right)}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 24.922 (sec). Leaf size: 393

```
DSolve[(a*Exp[\[Lambda]*x]+b*Exp[\[Mu]*x]+c)*(y'[x]-y[x]^2)+a*\[Lambda]^2*Exp[\[Lambda]*x]+b
```

$$\text{Solve}\Bigg[\int_1^x -\frac{-ae^{\lambda K[1]}\lambda^2 - be^{\mu K[1]}\mu^2 + ae^{\lambda K[1]}y(x)^2 + be^{\mu K[1]}y(x)^2 + cy(x)^2}{\left(e^{\lambda K[1]}a + be^{\mu K[1]} + c\right)\left(ae^{\lambda K[1]}\lambda + be^{\mu K[1]}\mu + ae^{\lambda K[1]}y(x) + be^{\mu K[1]}y(x) + cy(x)\right)^2} dK[1]$$

$$+ \int_1^{y(x)} \left(\frac{1}{\left(ae^{x\lambda}\lambda + be^{x\mu}\mu + ae^{x\lambda}K[2] + be^{x\mu}K[2] + cK[2]\right)^2}\right.$$

$$- \int_1^x \left(\frac{2\left(-ae^{\lambda K[1]}\lambda^2 - be^{\mu K[1]}\mu^2 + ae^{\lambda K[1]}K[2]^2 + be^{\mu K[1]}K[2]^2 + cK[2]^2\right)}{\left(ae^{\lambda K[1]}\lambda + be^{\mu K[1]}\mu + ae^{\lambda K[1]}K[2] + be^{\mu K[1]}K[2] + cK[2]\right)^3} - \frac{2ae^{\lambda K[1]}\cdots}{\left(e^{\lambda K[1]}a + be^{\mu K[1]} + c\right)\left(ae^{\lambda K[1]}\cdots\right.}\right.$$

4 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions

4.1 problem 22

Internal problem ID [10440]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - ax\,\mathrm{e}^{\lambda x} y = \mathrm{e}^{\lambda x} a}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 76

```
dsolve(diff(y(x),x)=y(x)^2+a*x*exp(lambda*x)*y(x)+a*exp(lambda*x),y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{\frac{ax\,\mathrm{e}^{\lambda x}}{\lambda} - \frac{a\,\mathrm{e}^{\lambda x}}{\lambda^2}}}{x^2\lambda^2\left(c_1 - \left(\int \frac{\mathrm{e}^{\frac{ax\,\mathrm{e}^{\lambda x}}{\lambda} - \frac{a\,\mathrm{e}^{\lambda x}}{\lambda^2}}}{x^2\lambda^2}dx\right)\right)} - \frac{1}{x}$$

✓ Solution by Mathematica

Time used: 2.132 (sec). Leaf size: 110

```
DSolve[y'[x]==y[x]^2+a*x*Exp[\[Lambda]*x]*y[x]+a*Exp[\[Lambda]*x],y[x],x,IncludeSingularSolu
```

$$y(x) \to -\frac{x\int_1^x \frac{e^{\frac{ae^{\lambda K[1]}(\lambda K[1]-1)}{\lambda^2}}}{K[1]^2}dK[1] + e^{\frac{ae^{\lambda x}(\lambda x-1)}{\lambda^2}} + c_1 x}{x^2\left(\int_1^x \frac{e^{\frac{ae^{\lambda K[1]}(\lambda K[1]-1)}{\lambda^2}}}{K[1]^2}dK[1] + c_1\right)}$$

$$y(x) \to -\frac{1}{x}$$

4.2 problem 23

Internal problem ID [10441]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Riccati]`

$$y' - a\,e^{\lambda x} y^2 = e^{-\lambda x} b$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 75

```
dsolve(diff(y(x),x)=a*exp(lambda*x)*y(x)^2+b*exp(-lambda*x),y(x), singsol=all)
```

$$y(x) = \frac{\left(-e^{\lambda x} e^{-\lambda x} \lambda^2 + \tan\left(\frac{\sqrt{4ab\,\lambda^2-\lambda^4}\,(\lambda x+c_1)}{2\lambda^2}\right)\sqrt{4ab\,\lambda^2-\lambda^4}\right) e^{-\lambda x}}{2a\lambda}$$

✓ Solution by Mathematica

Time used: 0.624 (sec). Leaf size: 123

```
DSolve[y'[x]==a*Exp[\[Lambda]*x]*y[x]^2+b*Exp[-\[Lambda]*x],y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{e^{\lambda(-x)}\left(-\sqrt{\lambda^2-4ab} + \frac{2}{\frac{1}{\sqrt{\lambda^2-4ab}} + c_1 e^{x\sqrt{\lambda^2-4ab}}} - \lambda\right)}{2a}$$

$$y(x) \to \frac{e^{\lambda(-x)}\left(4ab - \lambda\left(\sqrt{\lambda^2-4ab} + \lambda\right)\right)}{2a\sqrt{\lambda^2-4ab}}$$

4.3 problem 24

Internal problem ID [10442]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a\,\mathrm{e}^{\lambda x} y^2 = bn\,x^{-1+n} - a\,b^2 \mathrm{e}^{\lambda x} x^{2n}}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*exp(lambda*x)*y(x)^2+b*n*x^(n-1)-a*b^2*exp(lambda*x)*x^(2*n),y(x), sin
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*Exp[\[Lambda]*x]*y[x]^2+b*n*x^(n-1)-a*b^2*Exp[\[Lambda]*x]*x^(2*n),y[x],x,In
```

Not solved

4.4 problem 25

Internal problem ID [10443]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 \mathrm{e}^{\lambda x} - a\, x^n y = a\lambda\, x^n \mathrm{e}^{-\lambda x}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 89

```
dsolve(diff(y(x),x)=exp(lambda*x)*y(x)^2+a*x^(n)*y(x)+a*lambda*x^n*exp(-lambda*x),y(x), sing
```

$$y(x) = -\frac{\left(\left(\int \mathrm{e}^{\frac{x(a\,x^n-\lambda n-\lambda)}{1+n}}dx\right)\lambda + c_1\lambda + \mathrm{e}^{\frac{x(a\,x^n-\lambda n-\lambda)}{1+n}}\right)\mathrm{e}^{-\lambda x}}{c_1 + \int \mathrm{e}^{\frac{x(a\,x^n-\lambda n-\lambda)}{1+n}}dx}$$

✓ Solution by Mathematica

Time used: 1.93 (sec). Leaf size: 254

```
DSolve[y'[x]==Exp[\[Lambda]*x]*y[x]^2+a*x^(n)*y[x]+a*\[Lambda]*x^n*Exp[-\[Lambda]*x],y[x],x,
```

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{e^{\frac{ax^{n+1}}{n+1}}}{(\lambda+e^{x\lambda}K[2])^2}\right.\right.$$

$$-\int_1^x\left(\frac{2e^{\frac{aK[1]^{n+1}}{n+1}}\left(a\lambda K[1]^n+ae^{\lambda K[1]}K[2]K[1]^n+e^{2\lambda K[1]}K[2]^2\right)}{(\lambda+e^{\lambda K[1]}K[2])^3}-\frac{e^{\frac{aK[1]^{n+1}}{n+1}-\lambda K[1]}\left(ae^{\lambda K[1]}K[1]^n+2e^{2\lambda K[1]}K\right.}{(\lambda+e^{\lambda K[1]}K[2])^2}\right.$$

$$\left.+\int_1^x-\frac{e^{\frac{aK[1]^{n+1}}{n+1}-\lambda K[1]}\left(a\lambda K[1]^n+ae^{\lambda K[1]}y(x)K[1]^n+e^{2\lambda K[1]}y(x)^2\right)}{(\lambda+e^{\lambda K[1]}y(x))^2}dK[1]=c_1,y(x)\right]$$

4.5 problem 26

Internal problem ID [10444]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + y^2 e^{\lambda x}\lambda - a\,x^n y\,e^{\lambda x} = -x^n a$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 135

```
dsolve(diff(y(x),x)=-lambda*exp(lambda*x)*y(x)^2+a*x^(n)*exp(lambda*x)*y(x)-a*x^n,y(x), sing
```

$$y(x) = \frac{c_1 e^{-\lambda x} e^{-\lambda x + a\left(\int e^{\lambda x} x^n dx\right)}}{\lambda^2 \left(\left(\int \frac{e^{-\lambda x + a\left(\int e^{\lambda x} x^n dx\right)}}{\lambda} dx\right) c_1 + 1\right)} + \frac{e^{-\lambda x}\left(\left(\int \frac{e^{-\lambda x + a\left(\int e^{\lambda x} x^n dx\right)}}{\lambda} dx\right) c_1 \lambda^2 + \lambda^2\right)}{\lambda^2 \left(\left(\int \frac{e^{-\lambda x + a\left(\int e^{\lambda x} x^n dx\right)}}{\lambda} dx\right) c_1 + 1\right)}$$

✓ Solution by Mathematica

Time used: 6.627 (sec). Leaf size: 185

```
DSolve[y'[x]==-\[Lambda]*Exp[\[Lambda]*x]*y[x]^2+a*x^(n)*Exp[\[Lambda]*x]*y[x]-a*x^n,y[x],x,
```

$$y(x) \to \frac{e^{-2\lambda x}\left(e^{\lambda x}\int_1^{e^{x\lambda}} \frac{\exp\left(\frac{a\Gamma(n+1,-\log(K[1]))(-\log(K[1]))^{-n}\left(\frac{\log(K[1])}{\lambda}\right)^n}{\lambda}\right)}{K[1]^2} dK[1] + \exp\left(\frac{a(-\log(e^{\lambda x}))^{-n}\left(\frac{\log(e^{\lambda x})}{\lambda}\right)^n \Gamma(n+1,-\log\right.}{\lambda}\right.}{\int_1^{e^{x\lambda}} \frac{\exp\left(\frac{a\Gamma(n+1,-\log(K[1]))(-\log(K[1]))^{-n}\left(\frac{\log(K[1])}{\lambda}\right)^n}{\lambda}\right)}{K[1]^2} dK[1] + c_1}$$

$$y(x) \to e^{\lambda(-x)}$$

4.6 problem 27

Internal problem ID [10445]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,e^{\lambda x}y^2 + ab\,x^n e^{\lambda x}y = bn\,x^{-1+n}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 143

```
dsolve(diff(y(x),x)=a*exp(lambda*x)*y(x)^2-a*b*x^(n)*exp(lambda*x)*y(x)+b*n*x^(n-1),y(x), si
```

$$y(x) = -\frac{c_1\lambda\,e^{-\lambda x}e^{\lambda x+ab\left(\int e^{\lambda x}x^n dx\right)}}{a\left(\left(\int \lambda\,e^{\lambda x+ab\left(\int e^{\lambda x}x^n dx\right)}dx\right)c_1+1\right)} - \frac{\left(-x^n\left(\int \lambda\,e^{\lambda x+ab\left(\int e^{\lambda x}x^n dx\right)}dx\right)c_1ab - x^n ab\right)e^{\lambda x}e^{-\lambda x}}{a\left(\left(\int \lambda\,e^{\lambda x+ab\left(\int e^{\lambda x}x^n dx\right)}dx\right)c_1+1\right)}$$

✓ Solution by Mathematica

Time used: 63.132 (sec). Leaf size: 188

```
DSolve[y'[x]==a*Exp[\[Lambda]*x]*y[x]^2-a*b*x^(n)*Exp[\[Lambda]*x]*y[x]+b*n*x^(n-1),y[x],x,I
```

$y(x)$

$$\rightarrow \frac{abc_1\left(\frac{\log(e^{\lambda x})}{\lambda}\right)^n \int_1^{e^{x\lambda}} \exp\left(\frac{ab\Gamma(n+1,-\log(K[1]))(-\log(K[1]))^{-n}\left(\frac{\log(K[1])}{\lambda}\right)^n}{\lambda}\right) dK[1] - c_1\lambda \exp\left(\frac{ab(-\log(e^{\lambda x}))^{-n}\left(\frac{\log(}{}\right.}{}\right.}{a + ac_1\int_1^{e^{x\lambda}} \exp\left(\frac{ab\Gamma(n+1,-\log(K[1]))(-\log(K[1]))^{-n}\left(\frac{\log(K[1])}{\lambda}\right)^n}{\lambda}\right) dK[1]}$$

4.7 problem 28

Internal problem ID [10446]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,x^n y^2 = b\lambda\,e^{\lambda x} - a\,b^2 x^n e^{2\lambda x}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2+b*lambda*exp(lambda*x)-a*b^2*x^n*exp(2*lambda*x),y(x), sing
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*x^n*y[x]^2+b*\[Lambda]*Exp[\[Lambda]*x]-a*b^2*x^n*Exp[2*\[Lambda]*x],y[x],x,
```

Not solved

4.8 problem 29

Internal problem ID [10447]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,x^n y^2 - y\lambda = -a\,b^2 x^n \mathrm{e}^{2\lambda x}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 79

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2+lambda*y(x)-a*b^2*x^n*exp(2*lambda*x),y(x), singsol=all)
```

$$y(x) = -i\tan\left(\frac{i\Gamma(n)\,an\,x^n(-\lambda x)^{-n}\,b - ian\,x^n\Gamma(n,-\lambda x)\,(-\lambda x)^{-n}\,b - iba\,\mathrm{e}^{\lambda x}x^n - c_1\lambda}{\lambda}\right)b\,\mathrm{e}^{\lambda x}$$

✓ Solution by Mathematica

Time used: 1.69 (sec). Leaf size: 57

```
DSolve[y'[x]==a*x^n*y[x]^2+\[Lambda]*y[x]-a*b^2*x^n*Exp[2*\[Lambda]*x],y[x],x,IncludeSingula
```

$$y(x) \to \sqrt{-b^2}e^{\lambda x}\tan\left(\frac{a\sqrt{-b^2}x^n(\lambda(-x))^{-n}\Gamma(n+1,-x\lambda)}{\lambda}+c_1\right)$$

4.9 problem 30

Internal problem ID [10448]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,x^n y^2 + ab\,x^n \mathrm{e}^{\lambda x} y = b\lambda\,\mathrm{e}^{\lambda x}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2-a*b*x^n*exp(lambda*x)*y(x)+b*lambda*exp(lambda*x),y(x), sin
```

No solution found

✓ Solution by Mathematica

Time used: 53.05 (sec). Leaf size: 190

```
DSolve[y'[x]==a*x^n*y[x]^2-a*b*x^n*Exp[\[Lambda]*x]*y[x]+b*\[Lambda]*Exp[\[Lambda]*x],y[x],x
```

$$y(x) \to \frac{be^{2\lambda x}\left(\int_1^{e^{x\lambda}} \frac{\exp\left(\frac{ab\Gamma(n+1,-\log(K[1]))(-\log(K[1]))^{-n}\left(\frac{\log(K[1])}{\lambda}\right)^n}{\lambda}\right)}{K[1]^2}dK[1] + c_1\right)}{e^{\lambda x}\int_1^{e^{x\lambda}} \frac{\exp\left(\frac{ab\Gamma(n+1,-\log(K[1]))(-\log(K[1]))^{-n}\left(\frac{\log(K[1])}{\lambda}\right)^n}{\lambda}\right)}{K[1]^2}dK[1] + \exp\left(\frac{ab(-\log(e^{\lambda x}))^{-n}\left(\frac{\log(e^{\lambda x})}{\lambda}\right)^n\Gamma(n+1,-\log(e^{x\lambda}))}{\lambda}\right.}$$

$$y(x) \to be^{\lambda x}$$

4.10 problem 31

Internal problem ID [10449]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + (k+1)\, x^k y^2 - a\, x^{k+1} \mathrm{e}^{\lambda x} y = -\mathrm{e}^{\lambda x} a}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 196

```
dsolve(diff(y(x),x)=-(k+1)*x^k*y(x)^2+a*x^(k+1)*exp(lambda*x)*y(x)-a*exp(lambda*x),y(x), sin
```

$$y(x) = \frac{\left(\mathrm{e}^{\int \frac{x^2 x^k \mathrm{e}^{\lambda x} a - 2k - 2}{x} dx} x\, x^k + \int \left(x^k k\, \mathrm{e}^{a\left(\int x^{1+k} \mathrm{e}^{\lambda x} dx\right) - 2k\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)} + x^k \mathrm{e}^{a\left(\int x^{1+k} \mathrm{e}^{\lambda x} dx\right) - 2k\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)}\right) dx}{x \left(\int \left(x^k k\, \mathrm{e}^{a\left(\int x^{1+k} \mathrm{e}^{\lambda x} dx\right) - 2k\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)} + x^k \mathrm{e}^{a\left(\int x^{1+k} \mathrm{e}^{\lambda x} dx\right) - 2k\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)}\right) dx + c_1\right)}$$

✓ Solution by Mathematica

Time used: 86.249 (sec). Leaf size: 280

```
DSolve[y'[x]==-(k+1)*x^k*y[x]^2+a*x^(k+1)*Exp[\[Lambda]*x]*y[x]-a*Exp[\[Lambda]*x],y[x],x,In
```

$y(x)$

$$\to \frac{a\lambda \exp\left(\frac{a(-\log(e^{\lambda x}))^{-k}\left(\frac{\log(e^{\lambda x})}{\lambda}\right)^{k}\Gamma(k+2,-\log(e^{x\lambda}))}{\lambda^2}\right)\left(1+c_1\int_1^{e^{x\lambda}} \mathrm{ex}\right.}{ac_1\lambda\left(\frac{\log(e^{\lambda x})}{\lambda}\right)^{k+1}\exp\left(\frac{a(-\log(e^{\lambda x}))^{-k}\left(\frac{\log(e^{\lambda x})}{\lambda}\right)^{k}\Gamma(k+2,-\log(e^{x\lambda}))}{\lambda^2}\right)\int_1^{e^{x\lambda}}\exp\left(-\frac{a\Gamma(k+2,-\log(K[1]))(-\log(K[1]))^{-}}{\lambda^2}\right.}$$

4.11 problem 32

Internal problem ID [10450]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,x^n y^2 + a\,x^n\left(b\,e^{\lambda x} + c\right) y = c\,x^n$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2-a*x^n*(b*exp(lambda*x)+c)*y(x)+c*x^n,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*x^n*y[x]^2-a*x^n*(b*Exp[\[Lambda]*x]+c)*y[x]+c*x^n,y[x],x,IncludeSingularSol
```

Not solved

4.12 problem 33

Internal problem ID [10451]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,x^n e^{2\lambda x} y^2 - \left(e^{\lambda x} x^n b - \lambda\right) y = c\,x^n$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 114

```
dsolve(diff(y(x),x)=a*x^n*exp(2*lambda*x)*y(x)^2+(b*x^n*exp(lambda*x)-lambda)*y(x)+c*x^n,y(x
```

$$y(x) = \frac{\left(\tan\left(\frac{\sqrt{4b^2ac-b^4}\left(x^n(-\lambda x)^{-n}\Gamma(n,-\lambda x)bn - x^n\Gamma(n)(-\lambda x)^{-n}bn + x^n e^{\lambda x}b + c_1\lambda\right)}{2b^2\lambda}\right)\sqrt{4b^2ac-b^4} - b^2\right)e^{-\lambda x}}{2ab}$$

✓ Solution by Mathematica

Time used: 3.112 (sec). Leaf size: 102

```
DSolve[y'[x]==a*x^n*Exp[2*\[Lambda]*x]*y[x]^2+(b*x^n*Exp[\[Lambda]*x]-\[Lambda])*y[x]+c*x^n,
```

$$\text{Solve}\left[\int_1^{\sqrt{\frac{ae^{2x\lambda}}{c}}y(x)} \frac{1}{K[1]^2 - \sqrt{\frac{b^2}{ac}}K[1]+1}dK[1] = \frac{cx^n e^{\lambda(-x)}(\lambda(-x))^{-n}\sqrt{\frac{ae^{2\lambda x}}{c}}\Gamma(n+1,-x\lambda)}{\lambda} + c_1, y(x)\right]$$

4.13 problem 34

Internal problem ID [10452]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - a\,\mathrm{e}^{\lambda x}(y - b\,x^n - c)^2 = bn\,x^{-1+n}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve(diff(y(x),x)=a*exp(lambda*x)*(y(x)-b*x^n-c)^2+b*n*x^(n-1),y(x), singsol=all)
```

$$y(x) = \frac{\left(ba\,\mathrm{e}^{\lambda x}x^n + \mathrm{e}^{\lambda x}ac - \lambda + \frac{\mathrm{e}^{-\lambda x}}{c_1 + \frac{\mathrm{e}^{-\lambda x}}{\lambda}}\right)\mathrm{e}^{-\lambda x}}{a}$$

✓ Solution by Mathematica

Time used: 1.563 (sec). Leaf size: 40

```
DSolve[y'[x]==a*Exp[\[Lambda]*x]*(y[x]-b*x^n-c)^2+b*n*x^(n-1),y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{\lambda}{-ae^{\lambda x} + c_1\lambda} + bx^n + c$$
$$y(x) \to bx^n + c$$

4.14 problem 35

Internal problem ID [10453]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - a\,\mathrm{e}^{\lambda x}y^2 - ky = a\,b^2x^{2k}\mathrm{e}^{\lambda x}}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 54

```
dsolve(x*diff(y(x),x)=a*exp(lambda*x)*y(x)^2+k*y(x)+a*b^2*x^(2*k)*exp(lambda*x),y(x), singso
```

$$y(x) = -\tan\left(-a\,x^k(-\lambda x)^{-k}\,\Gamma(k)\,b + a\,x^k(-\lambda x)^{-k}\,\Gamma(k,-\lambda x)\,b + c_1\right)b\,x^k$$

✓ Solution by Mathematica

Time used: 1.593 (sec). Leaf size: 47

```
DSolve[x*y'[x]==a*Exp[\[Lambda]*x]*y[x]^2+k*y[x]+a*b^2*x^(2*k)*Exp[\[Lambda]*x],y[x],x,Inclu
```

$$y(x) \to \sqrt{b^2}x^k\tan\left(-a\sqrt{b^2}x^k(\lambda(-x))^{-k}\Gamma(k,-x\lambda) + c_1\right)$$

4.15 problem 36

Internal problem ID [10454]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - a\,x^{2n}\mathrm{e}^{\lambda x}y^2 - \left(\mathrm{e}^{\lambda x}x^n b - n\right)y = \mathrm{e}^{\lambda x}c}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 97

```
dsolve(x*diff(y(x),x)=a*x^(2*n)*exp(lambda*x)*y(x)^2+(b*x^n*exp(lambda*x)-n)*y(x)+c*exp(lamb
```

$$y(x) = -\frac{\left(\tan\left(\frac{\sqrt{4b^2ac-b^4}\,\left(x^n(-\lambda x)^{-n}\Gamma(n,-\lambda x)b-x^n\Gamma(n)(-\lambda x)^{-n}b-c_1\right)}{2b^2}\right)\sqrt{4b^2ac-b^4}+b^2\right)x^{-n}}{2ab}$$

✓ Solution by Mathematica

Time used: 3.62 (sec). Leaf size: 87

```
DSolve[x*y'[x]==a*x^(2*n)*Exp[\[Lambda]*x]*y[x]^2+(b*x^n*Exp[\[Lambda]*x]-n)*y[x]+c*Exp[\[La
```

$$\text{Solve}\left[\int_1^{\sqrt{\frac{ax^{2n}}{c}}y(x)}\frac{1}{K[1]^2-\sqrt{\frac{b^2}{ac}}K[1]+1}dK[1] = -c(\lambda(-x))^{-n}\sqrt{\frac{ax^{2n}}{c}}\Gamma(n,-x\lambda)+c_1,y(x)\right]$$

4.16 problem 37

Internal problem ID [10455]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = 2a\lambda x\,\mathrm{e}^{\lambda\,x^2} - a^2\mathrm{e}^{2\lambda\,x^2}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+2*a*lambda*x*exp(lambda*x^2)-a^2*exp(2*lambda*x^2),y(x), singsol=
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+2*a*\[Lambda]*x*Exp[\[Lambda]*x^2]-a^2*Exp[2*\[Lambda]*x^2],y[x],x,Incl
```

Not solved

4.17 problem 38

Internal problem ID [10456]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a\,\mathrm{e}^{-\lambda\,x^2} y^2 - y\lambda x = a\,b^2}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 49

```
dsolve(diff(y(x),x)=a*exp(-lambda*x^2)*y(x)^2+lambda*x*y(x)+a*b^2,y(x), singsol=all)
```

$$y(x) = -\tan\left(\frac{-ba\sqrt{\pi}\,\sqrt{2}\,\operatorname{erf}\left(\frac{\sqrt{2}\,\sqrt{\lambda}\,x}{2}\right) + 2c_1\sqrt{\lambda}}{2\sqrt{\lambda}}\right) b\,\mathrm{e}^{\frac{x^2\lambda}{2}}$$

✓ Solution by Mathematica

Time used: 2.252 (sec). Leaf size: 63

```
DSolve[y'[x]==a*Exp[-\[Lambda]*x^2]*y[x]^2+\[Lambda]*x*y[x]+a*b^2,y[x],x,IncludeSingularSolu
```

$$y(x) \to \sqrt{b^2}e^{\frac{\lambda x^2}{2}} \tan\left(\frac{\sqrt{\frac{\pi}{2}}a\sqrt{b^2}\operatorname{erf}\left(\frac{\sqrt{\lambda}x}{\sqrt{2}}\right)}{\sqrt{\lambda}} + c_1\right)$$

4.18 problem 39

Internal problem ID [10457]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,x^n y^2 - y\lambda x = a\,b^2 x^n \mathrm{e}^{\lambda\,x^2}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 141

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2+lambda*x*y(x)+a*b^2*x^n*exp(lambda*x^2),y(x), singsol=all)
```

$$y(x) = -\tan\left(-\frac{ba2^{\frac{n}{2}+\frac{1}{2}}\lambda^{-\frac{n}{2}-\frac{1}{2}}(-1)^{-\frac{n}{2}}\,x^{1+n}\lambda^{\frac{n}{2}+\frac{1}{2}}(-1)^{\frac{n}{2}}\,(-x^2\lambda)^{-\frac{n}{2}-\frac{1}{2}}\,\Gamma\left(\frac{n}{2}+\frac{1}{2}\right)}{2} + \frac{ba2^{\frac{n}{2}+\frac{1}{2}}\lambda^{-\frac{n}{2}-\frac{1}{2}}(-1)^{-\frac{n}{2}}\,x^{1+n}\lambda^{\frac{n}{2}+\frac{1}{2}}(-1)^{\frac{n}{2}}\,(-x^2\lambda)^{-\frac{n}{2}-\frac{1}{2}}\,\Gamma\left(\frac{n}{2}+\frac{1}{2},-\frac{x^2\lambda}{2}\right)}{2} + c_1\right) b\,\mathrm{e}^{\frac{x^2\lambda}{2}}$$

✓ Solution by Mathematica

Time used: 2.366 (sec). Leaf size: 83

```
DSolve[y'[x]==a*x^n*y[x]^2+\[Lambda]*x*y[x]+a*b^2*x^n*Exp[\[Lambda]*x^2],y[x],x,IncludeSingu
```

$$y(x) \to \sqrt{b^2}e^{\frac{\lambda x^2}{2}}\tan\left(a\sqrt{b^2}\lambda 2^{\frac{n-1}{2}}x^{n+3}\left(\lambda(-x^2)\right)^{-\frac{n}{2}-\frac{3}{2}}\Gamma\left(\frac{n+1}{2},-\frac{x^2\lambda}{2}\right)+c_1\right)$$

4.19 problem 40

Internal problem ID [10458]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$x^4(y' - y^2) = a + b\,\mathrm{e}^{\frac{k}{x}} + c\,\mathrm{e}^{\frac{2k}{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 564

```
dsolve(x^4*(diff(y(x),x)-y(x)^2)=a+b*exp(k/x)+c*exp(2*k/x),y(x), singsol=all)
```

$$y(x) = \frac{\left(2i\,\mathrm{WhittakerW}\left(-\frac{ib}{2k\sqrt{c}}, \frac{i\sqrt{a}}{k}, \frac{2i\sqrt{c}\,\mathrm{e}^{\frac{k}{x}}}{k}\right) c_1 c^2 + 2i\,\mathrm{WhittakerM}\left(-\frac{ib}{2k\sqrt{c}}, \frac{i\sqrt{a}}{k}, \frac{2i\sqrt{c}\,\mathrm{e}^{\frac{k}{x}}}{k}\right) c^2\right) \mathrm{e}^{\frac{k}{x}}}{2c^{\frac{3}{2}} x^2 \left(\mathrm{WhittakerW}\left(-\frac{ib}{2k\sqrt{c}}, \frac{i\sqrt{a}}{k}, \frac{2i\sqrt{c}\,\mathrm{e}^{\frac{k}{x}}}{k}\right) c_1 + \mathrm{WhittakerM}\left(-\frac{ib}{2k\sqrt{c}}, \frac{i\sqrt{a}}{k}, \frac{2i\sqrt{c}\,\mathrm{e}^{\frac{k}{x}}}{k}\right)\right)}$$
$$- \frac{c_1 k\,\mathrm{WhittakerW}\left(-\frac{ib-2k\sqrt{c}}{2k\sqrt{c}}, \frac{i\sqrt{a}}{k}, \frac{2i\sqrt{c}\,\mathrm{e}^{\frac{k}{x}}}{k}\right)}{x^2 \left(\mathrm{WhittakerW}\left(-\frac{ib}{2k\sqrt{c}}, \frac{i\sqrt{a}}{k}, \frac{2i\sqrt{c}\,\mathrm{e}^{\frac{k}{x}}}{k}\right) c_1 + \mathrm{WhittakerM}\left(-\frac{ib}{2k\sqrt{c}}, \frac{i\sqrt{a}}{k}, \frac{2i\sqrt{c}\,\mathrm{e}^{\frac{k}{x}}}{k}\right)\right)}$$
$$+ \frac{\left(-c^{\frac{3}{2}} c_1 k - 2c^{\frac{3}{2}} c_1 x + i c_1 b c\right) \mathrm{WhittakerW}\left(-\frac{ib}{2k\sqrt{c}}, \frac{i\sqrt{a}}{k}, \frac{2i\sqrt{c}\,\mathrm{e}^{\frac{k}{x}}}{k}\right) + \left(2i\sqrt{a}\,c^{\frac{3}{2}} + c^{\frac{3}{2}} k - ibc\right) \mathrm{WhittakerM}}{2c^{\frac{3}{2}} x^2 \left(\mathrm{WhittakerW}\left(-\frac{ib}{2k\sqrt{c}}, \frac{i\sqrt{a}}{k}, \frac{2i\sqrt{c}\,\mathrm{e}^{\frac{k}{x}}}{k}\right) c_1 + \mathrm{W}\right.}$$

✓ Solution by Mathematica

Time used: 4.039 (sec). Leaf size: 940

```
DSolve[x^4*(y'[x]-y[x]^2)==a+b*Exp[k/x]+c*Exp[2*k/x],y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{e^{k/x}\log\left(e^{k/x}\right)\left(c_1\left(b+\sqrt{c}(2\sqrt{a}-ik)\right)\text{HypergeometricU}\left(\frac{\frac{ib}{\sqrt{c}}+3k+2i\sqrt{a}}{2k},\frac{2i\sqrt{a}}{k}+2,\frac{2i\sqrt{c}e^{k/x}}{k}\right)-2i\sqrt{c}kL\cdots\right.}{kx^2\log\left(e^{k/x}\right)\cdots}$$

$$y(x) \to \frac{\frac{e^{k/x}\left(b+\sqrt{c}(2\sqrt{a}-ik)\right)\text{HypergeometricU}\left(\frac{\frac{ib}{\sqrt{c}}+3k+2i\sqrt{a}}{2k},\frac{2i\sqrt{a}}{k}+2,\frac{2i\sqrt{c}e^{k/x}}{k}\right)}{k\,\text{HypergeometricU}\left(\frac{\frac{ib}{\sqrt{c}}+k+2i\sqrt{a}}{2k},\frac{2i\sqrt{a}}{k}+1,\frac{2i\sqrt{c}e^{k/x}}{k}\right)}+i\left(\sqrt{a}-\sqrt{c}e^{k/x}\right)-\frac{k}{\log\left(e^{k/x}\right)}}{x^2}$$

$$y(x) \to \frac{\frac{e^{k/x}\left(b+\sqrt{c}(2\sqrt{a}-ik)\right)\text{HypergeometricU}\left(\frac{\frac{ib}{\sqrt{c}}+3k+2i\sqrt{a}}{2k},\frac{2i\sqrt{a}}{k}+2,\frac{2i\sqrt{c}e^{k/x}}{k}\right)}{k\,\text{HypergeometricU}\left(\frac{\frac{ib}{\sqrt{c}}+k+2i\sqrt{a}}{2k},\frac{2i\sqrt{a}}{k}+1,\frac{2i\sqrt{c}e^{k/x}}{k}\right)}+i\left(\sqrt{a}-\sqrt{c}e^{k/x}\right)-\frac{k}{\log\left(e^{k/x}\right)}}{x^2}$$

5 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine

5.1 problem 1

Internal problem ID [10459]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = -a^2 + a\lambda \sinh(\lambda x) - a^2 \sinh(\lambda x)^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 508

```
dsolve(diff(y(x),x)=y(x)^2-a^2+a*lambda*sinh(lambda*x)-a^2*sinh(lambda*x)^2,y(x), singsol=al
```

$$y(x) = \frac{\cosh(\lambda x)\left(i\,\mathrm{HeunCPrime}\left(\frac{4ia}{\lambda}, \frac{1}{2}, -\frac{1}{2}, \frac{2ia}{\lambda}, -\frac{8ia-3\lambda}{8\lambda}, \frac{1}{2} - \frac{i\sinh(\lambda x)}{2}\right) c_1\lambda - 2\,\mathrm{HeunC}\left(\frac{4ia}{\lambda}, \frac{1}{2}, -\frac{1}{2}, \frac{2ia}{\lambda}, -\frac{8ia-3}{8\lambda}\right.\right.}{2\sqrt{\sinh(\lambda x) + i}\,\left(\mathrm{HeunC}\left(\frac{4ia}{\lambda}, \frac{1}{2}, -\frac{1}{2}, \frac{2ia}{\lambda}, -\frac{8ia-3\lambda}{8\lambda}, \frac{1}{2} - \frac{i\sinh(\lambda x)}{2}\right)\sqrt{\sinh(\lambda x) + i}\, c_1 + \mathrm{HeunC}\left(\frac{4ia}{\lambda}, -\frac{1}{2}, -\right.\right.} + \frac{\left((-2ac_1 i - c_1\lambda)\,\mathrm{HeunC}\left(\frac{4ia}{\lambda}, \frac{1}{2}, -\frac{1}{2}, \frac{2ia}{\lambda}, -\frac{8ia-3\lambda}{8\lambda}, \frac{1}{2} - \frac{i\sinh(\lambda x)}{2}\right) + i\lambda\,\mathrm{HeunCPrime}\left(\frac{4ia}{\lambda}, -\frac{1}{2}, -\frac{1}{2}, \frac{2ia}{\lambda}, -\right.\right.}{2\sqrt{\sinh(\lambda x) + i}\,\left(\mathrm{HeunC}\left(\frac{4ia}{\lambda}\right.\right.}$$

✓ Solution by Mathematica

Time used: 11.807 (sec). Leaf size: 162

```
DSolve[y'[x]==y[x]^2-a^2+a*\[Lambda]*Sinh[\[Lambda]*x]-a^2*Sinh[\[Lambda]*x]^2,y[x],x,Includ
```

$$y(x) \to \frac{e^{\lambda(-x)}\left(a\left(e^{2\lambda x}+1\right)\int_1^{e^{x\lambda}}\frac{e^{\frac{a\left(K[1]^2-1\right)}{\lambda K[1]}}}{K[1]}dK[1]-2\lambda e^{\frac{ae^{\lambda(-x)}\left(e^{2\lambda x}-1\right)}{\lambda}+\lambda x}+ac_1e^{2\lambda x}+ac_1\right)}{2\left(\int_1^{e^{x\lambda}}\frac{e^{\frac{a\left(K[1]^2-1\right)}{\lambda K[1]}}}{K[1]}dK[1]+c_1\right)}$$

$$y(x) \to \frac{1}{2}ae^{\lambda(-x)}\left(e^{2\lambda x}+1\right)$$

5.2 problem 2

Internal problem ID [10460]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a\sinh(\beta x)\, y = ab\sinh(\beta x) - b^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 48

```
dsolve(diff(y(x),x)=y(x)^2+a*sinh(beta*x)*y(x)+a*b*sinh(beta*x)-b^2,y(x), singsol=all)
```

$$y(x) = -b - \frac{e^{\frac{a\cosh(\beta x)}{\beta} - 2xb}}{\int e^{\frac{a\cosh(\beta x)}{\beta} - 2xb} dx - c_1}$$

✓ Solution by Mathematica

Time used: 9.168 (sec). Leaf size: 183

```
DSolve[y'[x]==y[x]^2+a*Sinh[\[Beta]*x]*y[x]+a*b*Sinh[\[Beta]*x]-b^2,y[x],x,IncludeSingularSo
```

$$\text{Solve}\left[\int_1^x -\frac{e^{\frac{a\cosh(\beta K[1])}{\beta} - 2bK[1]}(-b + a\sinh(\beta K[1]) + y(x))}{a\beta(b + y(x))} dK[1] + \int_1^{y(x)} \left(\frac{e^{\frac{a\cosh(x\beta)}{\beta} - 2bx}}{a\beta(b + K[2])^2} - \int_1^x \left(\frac{e^{\frac{a\cosh(\beta K[1])}{\beta} - 2bK[1]}(-b + K[2] + a\sinh(\beta K[1]))}{a\beta(b + K[2])^2} - \frac{e^{\frac{a\cosh(\beta K[1])}{\beta} - 2bK[1]}}{a\beta(b + K[2])}\right) dK[1]\right) dK[2] = c_1, y(x)\right]$$

5.3 problem 3

Internal problem ID [10461]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - ax \sinh(bx)^m y = a \sinh(bx)^m$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)=y(x)^2+a*x*sinh(b*x)^m*y(x)+a*sinh(b*x)^m,y(x), singsol=all)
```

$$y(x) = -\frac{e^{\int \frac{a \sinh(xb)^m x^2 - 2}{x} dx} x + \int e^{\int \frac{a \sinh(xb)^m x^2 - 2}{x} dx} dx - c_1}{\left(-c_1 + \int e^{\int \frac{a \sinh(xb)^m x^2 - 2}{x} dx} dx\right) x}$$

✓ Solution by Mathematica

Time used: 7.437 (sec). Leaf size: 379

```
DSolve[y'[x]==y[x]^2+a*x*Sinh[b*x]^m*y[x]+a*Sinh[b*x]^m,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to$$

$$-\frac{\int_1^x \frac{\exp\left(-\frac{a\left(-e^{-bK[1]}+e^{bK[1]}\right)^m\left(2-2e^{2bK[1]}\right)^{-m}\left({}_3F_2\left(-m,-\frac{m}{2},-\frac{m}{2};1-\frac{m}{2},1-\frac{m}{2};e^{2bK[1]}\right)+bm\,\text{Hypergeometric2F1}\left(-m,-\frac{m}{2},1-\frac{m}{2},e^{2bK[1]}\right)K[1\right.}{b^2m^2}\right.}{K[1]^2}}{x\left(\int_1^x \frac{\exp\left(-\frac{a\left(-e^{-bK[1]}+e^{bK[1]}\right)^m\left(2-2e^{2bK[1]}\right)^{-m}\left({}_3F_2\left(-m,-\frac{m}{2},-\frac{m}{2};1-\frac{m}{2}\right.\right.}{b^2}\right.}{K[1]}\right.}$$

$$y(x) \to -\frac{1}{x}$$

5.4 problem 4

Internal problem ID [10462]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \sinh(\lambda x) y^2 = -\lambda \sinh(\lambda x)^3$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 67

```
dsolve(diff(y(x),x)=lambda*sinh(lambda*x)*y(x)^2-lambda*sinh(lambda*x)^3,y(x), singsol=all)
```

$$y(x) = -\frac{2c_1 \mathrm{e}^{\cosh(\lambda x)^2}}{\sqrt{\pi}\,(\mathrm{erfi}(\cosh(\lambda x))\, c_1 + 1)} + \frac{\cosh(\lambda x)\sqrt{\pi}\,\mathrm{erfi}(\cosh(\lambda x))\, c_1 + \cosh(\lambda x)\sqrt{\pi}}{\sqrt{\pi}\,(\mathrm{erfi}(\cosh(\lambda x))\, c_1 + 1)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*Sinh[\[Lambda]*x]*y[x]^2-\[Lambda]*Sinh[\[Lambda]*x]^3,y[x],x,Includ
```

Not solved

5.5 problem 5

Internal problem ID [10463]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \left(a\sinh(\lambda x)^2 - \lambda\right) y^2 = -a\sinh(\lambda x)^2 + \lambda - a$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 470

```
dsolve(diff(y(x),x)=(a*sinh(lambda*x)^2-lambda)*y(x)^2-a*sinh(lambda*x)^2+lambda-a,y(x), sin
```

$$y(x) = \frac{\sinh(2\lambda x)\left(-4\cosh(2\lambda x)\sqrt{-1+\cosh(2\lambda x)}\, c_1 a\lambda + 4\sqrt{-1+\cosh(2\lambda x)}\, c_1 a\lambda + 8\sqrt{-1+\cosh(2\lambda x)}\, c\right.}{2\left(-1+\cosh(2\lambda x)\right)^2\sqrt{1+\cosh(2\lambda x)}\,\left(\sinh(\lambda x)^2 a - \lambda\right)\left(\left(\left(\int \frac{2(a\cosh(2\lambda x)-a-2\lambda)\mathrm{e}^{\frac{a\cosh(2\lambda x)}{2\lambda}}\lambda\sinh(2\lambda x)}{(-1+\cosh(2\lambda x))^{\frac{3}{2}}\sqrt{1+\cosh(2\lambda x)}}dx\right.\right.\right.}$$
$$+ \frac{\sinh(2\lambda x)\left(\left(\cosh(2\lambda x)^2\sqrt{1+\cosh(2\lambda x)}\, c_1 a + \left(-2\sqrt{1+\cosh(2\lambda x)}\, c_1 a - 2\sqrt{1+\cosh(2\lambda x)}\, c_1\lambda\right)\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 50.151 (sec). Leaf size: 211

```
DSolve[y'[x]==(a*Sinh[\[Lambda]*x]^2-\[Lambda])*y[x]^2-a*Sinh[\[Lambda]*x]^2+\[Lambda]-a,y[x
```

$$y(x) \to \frac{\operatorname{csch}^2(\lambda x)\left(c_1\sinh(2\lambda x)\int_1^x e^{\frac{a\sinh^2(\lambda K[1])}{\lambda}}\operatorname{csch}^2(\lambda K[1])\left(\lambda - a\sinh^2(\lambda K[1])\right)dK[1] + 2c_1 e^{\frac{a\sinh^2(\lambda x)}{\lambda}} + \sinh(\right.}{2 + 2c_1\int_1^x e^{\frac{a\sinh^2(\lambda K[1])}{\lambda}}\operatorname{csch}^2(\lambda K[1])\left(\lambda - a\sinh^2(\lambda K[1])\right)dK[1]}$$

$$y(x) \to \frac{1}{2}\operatorname{csch}^2(\lambda x)\left(\frac{2e^{\frac{a\sinh^2(\lambda x)}{\lambda}}}{\int_1^x e^{\frac{a\sinh^2(\lambda K[1])}{\lambda}}\operatorname{csch}^2(\lambda K[1])\left(\lambda - a\sinh^2(\lambda K[1])\right)dK[1]} + \sinh(2\lambda x)\right)$$

5.6 problem 6

Internal problem ID [10464]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$(a\sinh(x\lambda)+b)\,y'-y^2-c\sinh(\mu x)\,y=-d^2+cd\sinh(\mu x)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 147

```
dsolve((a*sinh(lambda*x)+b)*diff(y(x),x)=y(x)^2+c*sinh(mu*x)*y(x)-d^2+c*d*sinh(mu*x),y(x), s
```

$$y(x)=-d-\frac{e^{\int \frac{c\sinh(\mu x)}{\sinh(\lambda x)a+b}dx-\frac{4d\operatorname{arctanh}\left(\frac{2b\tanh\left(\frac{\lambda x}{2}\right)-2a}{2\sqrt{a^2+b^2}}\right)}{\lambda\sqrt{a^2+b^2}}}}{\int \frac{e^{\int \frac{c\sinh(\mu x)}{\sinh(\lambda x)a+b}dx-\frac{4d\operatorname{arctanh}\left(\frac{2b\tanh\left(\frac{\lambda x}{2}\right)-2a}{2\sqrt{a^2+b^2}}\right)}{\lambda\sqrt{a^2+b^2}}}}{\sinh(\lambda x)a+b}dx-c_1}$$

✓ Solution by Mathematica

Time used: 28.506 (sec). Leaf size: 289

```
DSolve[(a*Sinh[\[Lambda]*x]+b)*y'[x]==y[x]^2+c*Sinh[\[Mu]*x]*y[x]-d^2+c*d*Sinh[\[Mu]*x],y[x]
```

$$\text{Solve}\left[\int_1^x -\frac{\exp\left(-\int_1^{K[2]} \frac{2d-c\sinh(\mu K[1])}{b+a\sinh(\lambda K[1])}dK[1]\right)(-d+c\sinh(\mu K[2])+y(x))}{c\mu(b+a\sinh(\lambda K[2]))(d+y(x))}dK[2]\right.$$
$$+\int_1^{y(x)}\left(\frac{\exp\left(-\int_1^x \frac{2d-c\sinh(\mu K[1])}{b+a\sinh(\lambda K[1])}dK[1]\right)}{c\mu(d+K[3])^2}\right.$$
$$-\int_1^x\left(\frac{\exp\left(-\int_1^{K[2]} \frac{2d-c\sinh(\mu K[1])}{b+a\sinh(\lambda K[1])}dK[1]\right)(-d+K[3]+c\sinh(\mu K[2]))}{c\mu(d+K[3])^2(b+a\sinh(\lambda K[2]))}-\frac{\exp\left(-\int_1^{K[2]} \frac{2d-c\sinh(\mu K[1])}{b+a\sinh(\lambda K[1])}dK[1\right.}{c\mu(d+K[3])(b+a\sinh(\lambda K[2]\right.}\right.$$

5.7 problem 7

Internal problem ID [10465]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{(a\sinh(\lambda x)+b)\left(y'-y^2\right)=-a\,\lambda^2\sinh(\lambda x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 938

```
dsolve((a*sinh(lambda*x)+b)*(diff(y(x),x)-y(x)^2)+a*lambda^2*sinh(lambda*x)=0,y(x), singsol=
```

Expression too large to display

✓ Solution by Mathematica

Time used: 24.532 (sec). Leaf size: 202

```
DSolve[(a*Sinh[\[Lambda]*x]+b)*(y'[x]-y[x]^2)+a*\[Lambda]^2*Sinh[\[Lambda]*x]==0,y[x],x,Incl
```

$$y(x)\to -\frac{\lambda\left(\sqrt{-a^2-b^2}(b-a\sinh(\lambda x))+a\cosh(\lambda x)\left(2b\arctan\left(\frac{a-b\tanh\left(\frac{\lambda x}{2}\right)}{\sqrt{-a^2-b^2}}\right)-c_1\lambda(-a^2-b^2)^{3/2}\right)\right)}{-a\sqrt{-a^2-b^2}\cosh(\lambda x)+(a\sinh(\lambda x)+b)\left(2b\arctan\left(\frac{a-b\tanh\left(\frac{\lambda x}{2}\right)}{\sqrt{-a^2-b^2}}\right)-c_1\lambda\left(-a^2-b^2\right)^{3/2}\right)}$$

$$y(x)\to -\frac{a\lambda\cosh(\lambda x)}{a\sinh(\lambda x)+b}$$

5.8 problem 8

Internal problem ID [10466]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \alpha y^2 = \beta + \gamma \cosh(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 70

```
dsolve(diff(y(x),x)=alpha*y(x)^2+beta+gamma*cosh(x),y(x), singsol=all)
```

$$y(x) = -\frac{i\left(c_1 \operatorname{MathieuSPrime}\left(-4\alpha\beta, 2\gamma\alpha, \frac{ix}{2}\right) + \operatorname{MathieuCPrime}\left(-4\alpha\beta, 2\gamma\alpha, \frac{ix}{2}\right)\right)}{2\alpha\left(c_1 \operatorname{MathieuS}\left(-4\alpha\beta, 2\gamma\alpha, \frac{ix}{2}\right) + \operatorname{MathieuC}\left(-4\alpha\beta, 2\gamma\alpha, \frac{ix}{2}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.543 (sec). Leaf size: 140

```
DSolve[y'[x]==\[Alpha]*y[x]^2+\[Beta]+\[Gamma]*Cosh[x],y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -\frac{ic_1\text{MathieuCPrime}\left[-4\alpha\beta, 2\alpha\gamma, \frac{ix}{2}\right] - i\text{MathieuSPrime}\left[-4\alpha\beta, 2\alpha\gamma, \frac{ix}{2}\right]}{2\alpha c_1\text{MathieuC}\left[-4\alpha\beta, 2\alpha\gamma, \frac{ix}{2}\right] - 2\alpha\text{MathieuS}\left[-4\alpha\beta, 2\alpha\gamma, \frac{ix}{2}\right]}$$

$$y(x) \to -\frac{i\text{MathieuCPrime}\left[-4\alpha\beta, 2\alpha\gamma, \frac{ix}{2}\right]}{2\alpha\text{MathieuC}\left[-4\alpha\beta, 2\alpha\gamma, \frac{ix}{2}\right]}$$

5.9 problem 9

Internal problem ID [10467]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - a\cosh(\beta x)\, y = ab\cosh(\beta x) - b^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(diff(y(x),x)=y(x)^2+a*cosh(beta*x)*y(x)+a*b*cosh(beta*x)-b^2,y(x), singsol=all)
```

$$y(x) = -b - \frac{e^{\frac{a\sinh(\beta x)}{\beta}-2xb}}{\int e^{\frac{a\sinh(\beta x)}{\beta}-2xb}dx - c_1}$$

✓ Solution by Mathematica

Time used: 9.815 (sec). Leaf size: 242

```
DSolve[y'[x]==y[x]^2+a*Cosh[\[Beta]*x]*y[x]+a*b*Cosh[\[Beta]*x]-b^2,y[x],x,IncludeSingularSo
```

$$y(x) \to -\frac{b \int_1^{e^{x\beta}} e^{\frac{a\left(K[1]^2-1\right)}{2\beta K[1]}} K[1]^{-\frac{2b}{\beta}-1} dK[1] + \beta e^{\frac{a e^{\beta(-x)}\left(e^{2\beta x}-1\right)}{2\beta}} \left(e^{\beta x}\right)^{-\frac{2b}{\beta}} + b c_1}{\int_1^{e^{x\beta}} e^{\frac{a\left(K[1]^2-1\right)}{2\beta K[1]}} K[1]^{-\frac{2b}{\beta}-1} dK[1] + c_1}$$

$$y(x) \to -b$$

$$y(x) \to -\frac{\beta e^{\frac{a e^{\beta(-x)}\left(e^{2\beta x}-1\right)}{2\beta}} \left(e^{\beta x}\right)^{-\frac{2b}{\beta}}}{\int_1^{e^{x\beta}} e^{\frac{a\left(K[1]^2-1\right)}{2\beta K[1]}} K[1]^{-\frac{2b}{\beta}-1} dK[1]} - b$$

5.10 problem 10

Internal problem ID [10468]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - ax\cosh(bx)^m y = a\cosh(bx)^m$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)=y(x)^2+a*x*cosh(b*x)^m*y(x)+a*cosh(b*x)^m,y(x), singsol=all)
```

$$y(x) = -\frac{e^{\int \frac{a\cosh(xb)^m x^2-2}{x}dx} x + \int e^{\int \frac{a\cosh(xb)^m x^2-2}{x}dx} dx - c_1}{\left(-c_1 + \int e^{\int \frac{a\cosh(xb)^m x^2-2}{x}dx} dx\right) x}$$

✓ Solution by Mathematica

Time used: 7.557 (sec). Leaf size: 394

```
DSolve[y'[x]==y[x]^2+a*x*Cosh[b*x]^m*y[x]+a*Cosh[b*x]^m,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to$$

$$-\frac{\int_1^x \frac{\exp\left(-\frac{2^{-m}a\left(e^{-bK[1]}+e^{bK[1]}\right)^m\left(1+e^{2bK[1]}\right)^{-m}\left({}_3F_2\left(-m,-\frac{m}{2},-\frac{m}{2};1-\frac{m}{2},1-\frac{m}{2};-e^{2bK[1]}\right)+bm\,\text{Hypergeometric2F1}\left(-m,-\frac{m}{2},1-\frac{m}{2},-e^{2bK[1}\right.\right.}{b^2m^2}\right.}{K[1]^2}}{x\left(\int_1^x \frac{\exp\left(-\frac{2^{-m}a\left(e^{-bK[1]}+e^{bK[1]}\right)^m\left(1+e^{2bK[1]}\right)^{-m}\left({}_3F_2\left(-m,-\frac{m}{2},-\frac{m}{2};\right.\right.}{}\right.}{}\right.}$$

$$y(x) \to -\frac{1}{x}$$

5.11 problem 11

Internal problem ID [10469]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \left(a \cosh(\lambda x)^2 - \lambda\right) y^2 = -a \cosh(\lambda x)^2 + a + \lambda$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 464

```
dsolve(diff(y(x),x)=(a*cosh(lambda*x)^2-lambda)*y(x)^2+a+lambda-a*cosh(lambda*x)^2,y(x), sin
```

$$y(x) = \frac{\sinh(2\lambda x)\left(-4\cosh(2\lambda x)\sqrt{1+\cosh(2\lambda x)}\,c_1 a\lambda - 4\sqrt{1+\cosh(2\lambda x)}\,c_1 a\lambda + 8\sqrt{1+\cosh(2\lambda x)}\,c_1\lambda^2\right) e}{2\left(1+\cosh(2\lambda x)\right)^2\sqrt{-1+\cosh(2\lambda x)}\left(a\cosh(\lambda x)^2-\lambda\right)\left(\left(\int \frac{2(a\cosh(2\lambda x)+a-2\lambda)e^{\frac{a\cosh(2\lambda x)}{2\lambda}}\lambda\sinh(2\lambda x)}{\sqrt{-1+\cosh(2\lambda x)}\,(1+\cosh(2\lambda x))^{\frac{3}{2}}}dx\right)\right.}$$

$$+ \frac{\sinh(2\lambda x)\left(\left(\cosh(2\lambda x)^2\sqrt{-1+\cosh(2\lambda x)}\,c_1 a + \left(2\sqrt{-1+\cosh(2\lambda x)}\,c_1 a - 2\sqrt{-1+\cosh(2\lambda x)}\,c\right.\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 49.81 (sec). Leaf size: 211

```
DSolve[y'[x]==(a*Cosh[\[Lambda]*x]^2-\[Lambda])*y[x]^2+a+\[Lambda]-a*Cosh[\[Lambda]*x]^2,y[x
```

$$y(x) \to \frac{\operatorname{sech}^2(\lambda x)\left(c_1\sinh(2\lambda x)\int_1^x e^{\frac{a\cosh^2(\lambda K[1])}{\lambda}}\left(\lambda - a\cosh^2(\lambda K[1])\right)\operatorname{sech}^2(\lambda K[1])dK[1] + 2c_1 e^{\frac{a\cosh^2(\lambda x)}{\lambda}} + \sinh(\right.}{2 + 2c_1\int_1^x e^{\frac{a\cosh^2(\lambda K[1])}{\lambda}}\left(\lambda - a\cosh^2(\lambda K[1])\right)\operatorname{sech}^2(\lambda K[1])dK[1]}$$

$$y(x) \to \frac{1}{2}\operatorname{sech}^2(\lambda x)\left(\frac{2e^{\frac{a\cosh^2(\lambda x)}{\lambda}}}{\int_1^x e^{\frac{a\cosh^2(\lambda K[1])}{\lambda}}\left(\lambda - a\cosh^2(\lambda K[1])\right)\operatorname{sech}^2(\lambda K[1])dK[1]} + \sinh(2\lambda x)\right)$$

5.12 problem 12

Internal problem ID [10470]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{2y' - (a - \lambda + a\cosh(\lambda x))\, y^2 = a + \lambda - a\cosh(\lambda x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 255

```
dsolve(2*diff(y(x),x)=(a-lambda+a*cosh(lambda*x))*y(x)^2+a+lambda-a*cosh(lambda*x),y(x), sin
```

$$y(x) = -\frac{2c_1\lambda\sinh(\lambda x)\,\mathrm{e}^{\frac{a\cosh(\lambda x)}{\lambda}}}{(\cosh(\lambda x)+1)^{\frac{3}{2}}\left(\left(\int \frac{(a-\lambda+\cosh(\lambda x)a)\mathrm{e}^{\frac{a\cosh(\lambda x)}{\lambda}}\lambda\sinh(\lambda x)}{\sqrt{\cosh(\lambda x)-1}\,(\cosh(\lambda x)+1)^{\frac{3}{2}}}dx\right)c_1+1\right)\sqrt{\cosh(\lambda x)-1}}$$
$$+\frac{\left(\left(\cosh(\lambda x)\sqrt{\cosh(\lambda x)-1}\,c_1+\sqrt{\cosh(\lambda x)-1}\,c_1\right)\left(\int \frac{(a-\lambda+\cosh(\lambda x)a)\mathrm{e}^{\frac{a\cosh(\lambda x)}{\lambda}}\lambda\sinh(\lambda x)}{\sqrt{\cosh(\lambda x)-1}\,(\cosh(\lambda x)+1)^{\frac{3}{2}}}dx\right)+\cosh(\ldots}{\left(\left(\int \frac{(a-\lambda+\cosh(\lambda x)a)\mathrm{e}^{\frac{a\cosh(\lambda x)}{\lambda}}\lambda\sinh(\lambda x)}{\sqrt{\cosh(\lambda x)-1}\,(\cosh(\lambda x)+1)^{\frac{3}{2}}}dx\right)c_1+1\right)\sqrt{\cosh(\lambda x)-1}\,(\ldots}$$

✓ Solution by Mathematica

Time used: 59.899 (sec). Leaf size: 338

```
DSolve[2*y'[x]==(a-\[Lambda]+a*Cosh[\[Lambda]*x])*y[x]^2+a+\[Lambda]-a*Cosh[\[Lambda]*x],y[x
```

$$y(x) \to \frac{\operatorname{sech}^2\left(\frac{\lambda x}{2}\right)\left(c_1 \sinh(\lambda x)\int_1^x -e^{\frac{2a\cosh^2\left(\frac{1}{2}\lambda K[1]\right)}{\lambda}}(\cosh(\lambda K[1])a+a-\lambda)\operatorname{sech}^2\left(\frac{1}{2}\lambda K[1]\right)dK[1]+4c_1 e^{\frac{2a\cosh^2\left(\frac{\lambda x}{2}\right)}{\lambda}}\right.}{2+2c_1\int_1^x -e^{\frac{2a\cosh^2\left(\frac{1}{2}\lambda K[1]\right)}{\lambda}}(\cosh(\lambda K[1])a+a-\lambda)\operatorname{sech}^2\left(\frac{1}{2}\lambda K[1]\right)dK[1]}$$

$$y(x) \to \frac{1}{2}\operatorname{sech}^2\left(\frac{\lambda x}{2}\right)\left(\frac{4e^{\frac{2a\cosh^2\left(\frac{\lambda x}{2}\right)}{\lambda}}}{\int_1^x -e^{\frac{2a\cosh^2\left(\frac{1}{2}\lambda K[1]\right)}{\lambda}}(\cosh(\lambda K[1])a+a-\lambda)\operatorname{sech}^2\left(\frac{1}{2}\lambda K[1]\right)dK[1]}+\sinh(\lambda x)\right)$$

$$y(x) \to \frac{1}{2}\operatorname{sech}^2\left(\frac{\lambda x}{2}\right)\left(\frac{4e^{\frac{2a\cosh^2\left(\frac{\lambda x}{2}\right)}{\lambda}}}{\int_1^x -e^{\frac{2a\cosh^2\left(\frac{1}{2}\lambda K[1]\right)}{\lambda}}(\cosh(\lambda K[1])a+a-\lambda)\operatorname{sech}^2\left(\frac{1}{2}\lambda K[1]\right)dK[1]}+\sinh(\lambda x)\right)$$

5.13 problem 13

Internal problem ID [10471]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = -\lambda^2 + a\cosh(\lambda x)^n \sinh(\lambda x)^{-n-4}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2-lambda^2+a*cosh(lambda*x)^n*sinh(lambda*x)^(-n-4),y(x), singsol=a
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2-\[Lambda]^2+a*Cosh[\[Lambda]*x]^n*Sinh[\[Lambda]*x]^(-n-4),y[x],x,Inclu
```

Not solved

5.14 problem 14

Internal problem ID [10472]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \sinh(\lambda x)\, y^2 a = b \sinh(\lambda x) \cosh(\lambda x)^n$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 333

```
dsolve(diff(y(x),x)=a*sinh(lambda*x)*y(x)^2+b*sinh(lambda*x)*cosh(lambda*x)^n,y(x), singsol=
```

$$y(x) = \frac{\frac{\sqrt{b}\cosh(\lambda x)^{\frac{n}{2}+1} c_1 \operatorname{BesselY}\left(\frac{n+3}{n+2}, \frac{2\sqrt{a}\sqrt{b}\cosh(\lambda x)^{\frac{n}{2}+1}}{\lambda(n+2)}\right)}{\sqrt{a}\left(\operatorname{BesselY}\left(\frac{1}{n+2}, \frac{2\sqrt{a}\sqrt{b}\cosh(\lambda x)^{\frac{n}{2}+1}}{\lambda(n+2)}\right) c_1 + \operatorname{BesselJ}\left(\frac{1}{n+2}, \frac{2\sqrt{a}\sqrt{b}\cosh(\lambda x)^{\frac{n}{2}+1}}{\lambda(n+2)}\right)\right)} + \frac{\operatorname{BesselJ}\left(\frac{n+3}{n+2}, \frac{2\sqrt{a}\sqrt{b}\cosh(\lambda x)^{\frac{n}{2}+1}}{\lambda(n+2)}\right)\sqrt{a}\sqrt{b}\cosh(\lambda x}{\left(\operatorname{BesselY}\left(\frac{1}{n+2}, \frac{2\sqrt{a}\sqrt{b}}{}\right.\right.}}{\cosh(\lambda x)}$$

✓ Solution by Mathematica

Time used: 1.376 (sec). Leaf size: 667

```
DSolve[y'[x]==a*Sinh[\[Lambda]*x]*y[x]^2+b*Sinh[\[Lambda]*x]*Cosh[\[Lambda]*x]^n,y[x],x,Incl
```

$$y(x) \to \frac{\sqrt{a}\sqrt{b}c_1 \operatorname{Gamma}\left(\frac{n+1}{n+2}\right)\cosh^{\frac{n}{2}}(\lambda x)\operatorname{BesselJ}\left(\frac{n+1}{n+2},\frac{2\sqrt{a}\sqrt{b}\cosh^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right) - \operatorname{sech}(\lambda x)\left(\operatorname{Gamma}\left(1+\frac{1}{n+2}\right)\left(\sqrt{a}\right.\right.}{}$$

$$y(x) \to \frac{\frac{\sqrt{a}\sqrt{b}\cosh^{\frac{n}{2}}(\lambda x)\left(\operatorname{BesselJ}\left(\frac{n+1}{n+2},\frac{2\sqrt{a}\sqrt{b}\cosh^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right)-\operatorname{BesselJ}\left(-\frac{n+3}{n+2},\frac{2\sqrt{a}\sqrt{b}\cosh^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right)\right)}{\operatorname{BesselJ}\left(-\frac{1}{n+2},\frac{2\sqrt{a}\sqrt{b}\cosh^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right)} - \lambda\operatorname{sech}(\lambda x)}{2a}$$

5.15 problem 15

Internal problem ID [10473]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 \cosh(\lambda x)\, a = b \cosh(\lambda x) \sinh(\lambda x)^n}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 333

```
dsolve(diff(y(x),x)=a*cosh(lambda*x)*y(x)^2+b*cosh(lambda*x)*sinh(lambda*x)^n,y(x), singsol=
```

$$y(x) = \frac{\frac{\sqrt{b}\, \sinh(\lambda x)^{\frac{n}{2}+1} c_1 \operatorname{BesselY}\left(\frac{n+3}{n+2}, \frac{2\sqrt{a}\,\sqrt{b}\, \sinh(\lambda x)^{\frac{n}{2}+1}}{\lambda(n+2)}\right)}{\sqrt{a}\left(\operatorname{BesselY}\left(\frac{1}{n+2}, \frac{2\sqrt{a}\,\sqrt{b}\, \sinh(\lambda x)^{\frac{n}{2}+1}}{\lambda(n+2)}\right) c_1 + \operatorname{BesselJ}\left(\frac{1}{n+2}, \frac{2\sqrt{a}\,\sqrt{b}\, \sinh(\lambda x)^{\frac{n}{2}+1}}{\lambda(n+2)}\right)\right)} + \frac{\operatorname{BesselJ}\left(\frac{n+3}{n+2}, \frac{2\sqrt{a}\,\sqrt{b}\, \sinh(\lambda x)^{\frac{n}{2}+1}}{\lambda(n+2)}\right)\sqrt{a}\,\sqrt{b}\, \sinh(\lambda x)^{\cdots}}{\left(\operatorname{BesselY}\left(\frac{1}{n+2}, \frac{2\sqrt{a}\,\sqrt{b}\cdots}{\cdots}\right.\right.}}{\sinh(\lambda x)}$$

✓ Solution by Mathematica

Time used: 1.277 (sec). Leaf size: 633

```
DSolve[y'[x]==a*Cosh[\[Lambda]*x]*y[x]^2+b*Cosh[\[Lambda]*x]*Sinh[\[Lambda]*x]^n,y[x],x,Incl
```

$$y(x) \to \frac{\operatorname{csch}(\lambda x)\left(-\lambda\,\text{Gamma}\left(1+\frac{1}{n+2}\right)\text{BesselJ}\left(\frac{1}{n+2},\frac{2\sqrt{a}\sqrt{b}\sinh^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right)+\sqrt{a}\sqrt{b}\sinh^{\frac{n}{2}+1}(\lambda x)\left(\text{Gamma}\left(1+\right.\right.\right.}{}$$

$$y(x) \to \frac{\frac{\sqrt{a}\sqrt{b}\sinh^{\frac{n}{2}}(\lambda x)\left(\text{BesselJ}\left(\frac{n+1}{n+2},\frac{2\sqrt{a}\sqrt{b}\sinh^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right)-\text{BesselJ}\left(-\frac{n+3}{n+2},\frac{2\sqrt{a}\sqrt{b}\sinh^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right)\right)}{\text{BesselJ}\left(-\frac{1}{n+2},\frac{2\sqrt{a}\sqrt{b}\sinh^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right)}-\lambda\operatorname{csch}(\lambda x)}{2a}$$

5.16 problem 16

Internal problem ID [10474]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$(a\cosh(x\lambda)+b)\,y'-y^2-c\cosh(\mu x)\,y=-d^2+cd\cosh(\mu x)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 149

```
dsolve((a*cosh(lambda*x)+b)*diff(y(x),x)=y(x)^2+c*cosh(mu*x)*y(x)-d^2+c*d*cosh(mu*x),y(x), s
```

$$y(x)=-d-\frac{e^{\int \frac{c\cosh(\mu x)}{\cosh(\lambda x)a+b}dx-\frac{4d\arctan\left(\frac{(a-b)\tanh\left(\frac{\lambda x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\lambda\sqrt{(a-b)(a+b)}}}}{\int \frac{e^{\int \frac{c\cosh(\mu x)}{\cosh(\lambda x)a+b}dx-\frac{4d\arctan\left(\frac{(a-b)\tanh\left(\frac{\lambda x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\lambda\sqrt{(a-b)(a+b)}}}}{\cosh(\lambda x)a+b}dx-c_1}$$

✓ Solution by Mathematica

Time used: 24.309 (sec). Leaf size: 289

```
DSolve[(a*Cosh[\[Lambda]*x]+b)*y'[x]==y[x]^2+c*Cosh[\[Mu]*x]*y[x]-d^2+c*d*Cosh[\[Mu]*x],y[x]
```

$$\text{Solve}\left[\int_1^x -\frac{\exp\left(-\int_1^{K[2]} \frac{2d-c\cosh(\mu K[1])}{b+a\cosh(\lambda K[1])}dK[1]\right)(-d+c\cosh(\mu K[2])+y(x))}{c\mu(b+a\cosh(\lambda K[2]))(d+y(x))}dK[2]\right.$$
$$+\int_1^{y(x)}\left(\frac{\exp\left(-\int_1^x \frac{2d-c\cosh(\mu K[1])}{b+a\cosh(\lambda K[1])}dK[1]\right)}{c\mu(d+K[3])^2}\right.$$
$$-\int_1^x\left(\frac{\exp\left(-\int_1^{K[2]} \frac{2d-c\cosh(\mu K[1])}{b+a\cosh(\lambda K[1])}dK[1]\right)(-d+c\cosh(\mu K[2])+K[3])}{c\mu(b+a\cosh(\lambda K[2]))(d+K[3])^2}-\frac{\exp\left(-\int_1^{K[2]} \frac{2d-c\cosh(\mu K[1])}{b+a\cosh(\lambda K[1])}dK[1\right.}{c\mu(b+a\cosh(\lambda K[2]))(d+K[3}\right.$$

5.17 problem 17

Internal problem ID [10475]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{(a\cosh(\lambda x)+b)\left(y'-y^2\right)=-a\,\lambda^2\cosh(\lambda x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 758

```
dsolve((a*cosh(lambda*x)+b)*(diff(y(x),x)-y(x)^2)+a*lambda^2*cosh(lambda*x)=0,y(x), singsol=
```

$$y(x) = \frac{\lambda\left(\left(\left(\left(2\sqrt{a^2-b^2}\,a^3b-4\sqrt{a^2-b^2}\,a^2b^2+2\sqrt{a^2-b^2}\,a\,b^3\right)\arctan\left(\frac{(a-b)\tanh\left(\frac{\lambda x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)-2\sqrt{a^2-b^2}\,c_1a^3+4\right.\right.\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 7.749 (sec). Leaf size: 246

```
DSolve[(a*Cosh[\[Lambda]*x]+b)*(y'[x]-y[x]^2)+a*\[Lambda]^2*Cosh[\[Lambda]*x]==0,y[x],x,Incl
```

$$y(x) \to -\frac{\lambda\left(a\sinh(\lambda x)\left(2b\arctan\left(\frac{(b-a)\tanh\left(\frac{\lambda x}{2}\right)}{\sqrt{a^2-b^2}}\right)+c_1\lambda(a^2-b^2)^{3/2}\right)+a\sqrt{a^2-b^2}\cosh(\lambda x)+}{b\left(2b\arctan\left(\frac{(b-a)\tanh\left(\frac{\lambda x}{2}\right)}{\sqrt{a^2-b^2}}\right)+c_1\lambda\left(a^2-b^2\right)^{3/2}\right)+a\cosh(\lambda x)\left(2b\arctan\left(\frac{(b-a)\tanh\left(\frac{\lambda x}{2}\right)}{\sqrt{a^2-b^2}}\right)+c_1\lambda\left(a^2\right.\right.}$$

$$y(x) \to -\frac{a\lambda\sinh(\lambda x)}{a\cosh(\lambda x)+b}$$

6 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.

6.1 problem 18

Internal problem ID [10476]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = \lambda a - a(a+\lambda)\tanh(\lambda x)^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 198

```
dsolve(diff(y(x),x)=y(x)^2+a*lambda-a*(a+lambda)*tanh(lambda*x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{\left((c_1 a + c_1\lambda)\,\mathrm{LegendreQ}\left(\frac{a}{\lambda},\frac{a}{\lambda},\tanh(\lambda x)\right) + (a+\lambda)\,\mathrm{LegendreP}\left(\frac{a}{\lambda},\frac{a}{\lambda},\tanh(\lambda x)\right)\right)\tanh(\lambda x)}{c_1\,\mathrm{LegendreQ}\left(\frac{a}{\lambda},\frac{a}{\lambda},\tanh(\lambda x)\right) + \mathrm{LegendreP}\left(\frac{a}{\lambda},\frac{a}{\lambda},\tanh(\lambda x)\right)} + \frac{c_1\lambda\,\mathrm{LegendreQ}\left(\frac{a+\lambda}{\lambda},\frac{a}{\lambda},\tanh(\lambda x)\right)}{c_1\,\mathrm{LegendreQ}\left(\frac{a}{\lambda},\frac{a}{\lambda},\tanh(\lambda x)\right) + \mathrm{LegendreP}\left(\frac{a}{\lambda},\frac{a}{\lambda},\tanh(\lambda x)\right)} + \frac{\mathrm{LegendreP}\left(\frac{a+\lambda}{\lambda},\frac{a}{\lambda},\tanh(\lambda x)\right)\lambda}{c_1\,\mathrm{LegendreQ}\left(\frac{a}{\lambda},\frac{a}{\lambda},\tanh(\lambda x)\right) + \mathrm{LegendreP}\left(\frac{a}{\lambda},\frac{a}{\lambda},\tanh(\lambda x)\right)}$$

✓ Solution by Mathematica

Time used: 8.574 (sec). Leaf size: 177

```
DSolve[y'[x]==y[x]^2+a*\[Lambda]-a*(a+\[Lambda])*Tanh[\[Lambda]*x]^2,y[x],x,IncludeSingularS
```

$$y(x) \to \frac{a\left(-\lambda\left(e^{2\lambda x}-1\right)\text{Hypergeometric2F1}\left(-\frac{2a}{\lambda},-\frac{a}{\lambda},1-\frac{a}{\lambda},-e^{2x\lambda}\right)-2\lambda\left(e^{2\lambda x}+1\right)^{\frac{2a}{\lambda}+1}+ac_1\left(e^{2\lambda x}-1\right)\left(e^{2\lambda x}\right)\right.}{\left(e^{2\lambda x}+1\right)\left(-\lambda\,\text{Hypergeometric2F1}\left(-\frac{2a}{\lambda},-\frac{a}{\lambda},1-\frac{a}{\lambda},-e^{2x\lambda}\right)+ac_1\left(e^{2\lambda x}\right)^{a/\lambda}\right)}$$

$$y(x) \to \frac{a\left(e^{2\lambda x}-1\right)}{e^{2\lambda x}+1}$$

6.2 problem 19

Internal problem ID [10477]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = 3\lambda a - \lambda^2 - a(a+\lambda)\tanh(\lambda x)^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 245

```
dsolve(diff(y(x),x)=y(x)^2+3*a*lambda-lambda^2-a*(a+lambda)*tanh(lambda*x)^2,y(x), singsol=a
```

$$y(x) = \frac{\left(\left(-c_1 a - c_1 \lambda\right) \mathrm{LegendreQ}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \tanh(\lambda x)\right) + (-a-\lambda)\,\mathrm{LegendreP}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \tanh(\lambda x)\right)\right)\tanh(\lambda x)}{c_1 \mathrm{LegendreQ}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \tanh(\lambda x)\right) + \mathrm{LegendreP}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \tanh(\lambda x)\right)} + \frac{2c_1 \lambda\,\mathrm{LegendreQ}\left(\frac{a+\lambda}{\lambda}, \frac{a-\lambda}{\lambda}, \tanh(\lambda x)\right)}{c_1 \mathrm{LegendreQ}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \tanh(\lambda x)\right) + \mathrm{LegendreP}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \tanh(\lambda x)\right)} + \frac{2\,\mathrm{LegendreP}\left(\frac{a+\lambda}{\lambda}, \frac{a-\lambda}{\lambda}, \tanh(\lambda x)\right)\lambda}{c_1 \mathrm{LegendreQ}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \tanh(\lambda x)\right) + \mathrm{LegendreP}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \tanh(\lambda x)\right)}$$

✓ Solution by Mathematica

Time used: 12.804 (sec). Leaf size: 631

```
DSolve[y'[x]==y[x]^2+3*a*\[Lambda]-\[Lambda]^2-a*(a+\[Lambda])*Tanh[\[Lambda]*x]^2,y[x],x,In
```

$$y(x) \to \frac{-\lambda(a-2\lambda)\left(e^{2\lambda x}-1\right)\left(e^{2\lambda x}+1\right)^{\frac{2a}{\lambda}}\left(\frac{1}{e^{2\lambda x}-1}+1\right)^{a/\lambda}\left(a\left(4e^{2\lambda x}+e^{4\lambda x}-1\right)+\lambda-\lambda e^{4\lambda x}\right)\text{AppellF1}\left(1-\right.}{}$$

$$y(x) \to \frac{a\left(e^{2\lambda x}-1\right)^2-\lambda\left(e^{2\lambda x}+1\right)^2}{e^{4\lambda x}-1}$$

6.3 problem 20

Internal problem ID [10478]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - ax \tanh(bx)^m y = a \tanh(bx)^m$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)=y(x)^2+a*x*tanh(b*x)^m*y(x)+a*tanh(b*x)^m,y(x), singsol=all)
```

$$y(x) = -\frac{e^{\int \frac{a \tanh(xb)^m x^2 - 2}{x} dx} x + \int e^{\int \frac{a \tanh(xb)^m x^2 - 2}{x} dx} dx - c_1}{\left(-c_1 + \int e^{\int \frac{a \tanh(xb)^m x^2 - 2}{x} dx} dx\right) x}$$

✓ Solution by Mathematica

Time used: 12.331 (sec). Leaf size: 126

```
DSolve[y'[x]==y[x]^2+a*x*Tanh[b*x]^m*y[x]+a*Tanh[b*x]^m,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to -\frac{\exp\left(-\int_1^x -aK[1] \tanh^m(bK[1]) dK[1]\right) + x \int_1^x \frac{\exp\left(-\int_1^{K[2]} -aK[1] \tanh^m(bK[1]) dK[1]\right)}{K[2]^2} dK[2] + c_1 x}{x^2 \left(\int_1^x \frac{\exp\left(-\int_1^{K[2]} -aK[1] \tanh^m(bK[1]) dK[1]\right)}{K[2]^2} dK[2] + c_1\right)}$$

$$y(x) \to -\frac{1}{x}$$

6.4 problem 21

Internal problem ID [10479]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{(\tanh(\lambda x)\, a + b)\, y' - y^2 - c\tanh(\mu x)\, y = -d^2 + cd\tanh(\mu x)}$$

✗ Solution by Maple

```
dsolve((a*tanh(lambda*x)+b)*diff(y(x),x)=y(x)^2+c*tanh(mu*x)*y(x)-d^2+c*d*tanh(mu*x),y(x), s
```

No solution found

✓ Solution by Mathematica

Time used: 163.692 (sec). Leaf size: 800

```
DSolve[(a*Tanh[\[Lambda]*x]+b)*y'[x]==y[x]^2+c*Tanh[\[Mu]*x]*y[x]-d^2+c*d*Tanh[\[Mu]*x],y[x]
```

$$\text{Solve}\left[\int_1^x \frac{e^{-\int_1^{K[2]} \frac{\text{sech}(\mu K[1])(2d\cosh(\lambda K[1]-\mu K[1])+2d\cosh(\lambda K[1]+\mu K[1])+c\sinh(\lambda K[1]-\mu K[1])-c\sinh(\lambda K[1]+\mu K[1]))}{2(b\cosh(\lambda K[1])+a\sinh(\lambda K[1]))}dK[1]}(d\cosh(\lambda K[2]}{c\mu(b\cosh(\lambda K[2]-\mu K[2])+b\,\text{co}}\right.$$

$$+\int_1^{y(x)}\left(\frac{e^{-\int_1^x \frac{\text{sech}(\mu K[1])(2d\cosh(\lambda K[1]-\mu K[1])+2d\cosh(\lambda K[1]+\mu K[1])+c\sinh(\lambda K[1]-\mu K[1])-c\sinh(\lambda K[1]+\mu K[1]))}{2(b\cosh(\lambda K[1])+a\sinh(\lambda K[1]))}dK[1]}}{c\mu(d+K[3])^2}\right.$$

$$-\int_1^x\left(\frac{e^{-\int_1^{K[2]} \frac{\text{sech}(\mu K[1])(2d\cosh(\lambda K[1]-\mu K[1])+2d\cosh(\lambda K[1]+\mu K[1])+c\sinh(\lambda K[1]-\mu K[1])-c\sinh(\lambda K[1]+\mu K[1]))}{2(b\cosh(\lambda K[1])+a\sinh(\lambda K[1]))}dK[1]}(-\cosh(\lambda K[2]-}{c\mu(d+K[3])(b\cosh(\lambda K[2]-\mu K[2])+b\cosh(\lambda K[2]+\mu K[2])+a\sinh(\lambda K[2]-\mu K[2])+}\right.$$

6.5 problem 22

Internal problem ID [10480]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = \lambda a - a(a+\lambda)\coth(\lambda x)^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 198

```
dsolve(diff(y(x),x)=y(x)^2+a*lambda-a*(a+lambda)*coth(lambda*x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{\left((c_1 a + c_1\lambda)\,\mathrm{LegendreQ}\left(\frac{a}{\lambda}, \frac{a}{\lambda}, \coth(\lambda x)\right) + (a+\lambda)\,\mathrm{LegendreP}\left(\frac{a}{\lambda}, \frac{a}{\lambda}, \coth(\lambda x)\right)\right)\coth(\lambda x)}{c_1\,\mathrm{LegendreQ}\left(\frac{a}{\lambda}, \frac{a}{\lambda}, \coth(\lambda x)\right) + \mathrm{LegendreP}\left(\frac{a}{\lambda}, \frac{a}{\lambda}, \coth(\lambda x)\right)} + \frac{c_1\lambda\,\mathrm{LegendreQ}\left(\frac{a+\lambda}{\lambda}, \frac{a}{\lambda}, \coth(\lambda x)\right)}{c_1\,\mathrm{LegendreQ}\left(\frac{a}{\lambda}, \frac{a}{\lambda}, \coth(\lambda x)\right) + \mathrm{LegendreP}\left(\frac{a}{\lambda}, \frac{a}{\lambda}, \coth(\lambda x)\right)} + \frac{\mathrm{LegendreP}\left(\frac{a+\lambda}{\lambda}, \frac{a}{\lambda}, \coth(\lambda x)\right)\lambda}{c_1\,\mathrm{LegendreQ}\left(\frac{a}{\lambda}, \frac{a}{\lambda}, \coth(\lambda x)\right) + \mathrm{LegendreP}\left(\frac{a}{\lambda}, \frac{a}{\lambda}, \coth(\lambda x)\right)}$$

✓ Solution by Mathematica

Time used: 8.402 (sec). Leaf size: 175

```
DSolve[y'[x]==y[x]^2+a*\[Lambda]-a*(a+\[Lambda])*Coth[\[Lambda]*x]^2,y[x],x,IncludeSingularS
```

$$y(x) \to \frac{a\left(-\lambda\left(e^{2\lambda x}+1\right)\text{Hypergeometric2F1}\left(-\frac{2a}{\lambda},-\frac{a}{\lambda},1-\frac{a}{\lambda},e^{2x\lambda}\right)+2\lambda\left(1-e^{2\lambda x}\right)^{\frac{2a}{\lambda}+1}+ac_1\left(e^{2\lambda x}+1\right)\left(e^{2\lambda x}\right)}{\left(e^{2\lambda x}-1\right)\left(-\lambda\,\text{Hypergeometric2F1}\left(-\frac{2a}{\lambda},-\frac{a}{\lambda},1-\frac{a}{\lambda},e^{2x\lambda}\right)+ac_1\left(e^{2\lambda x}\right)^{a/\lambda}\right)}$$

$$y(x) \to \frac{a\left(e^{2\lambda x}+1\right)}{e^{2\lambda x}-1}$$

6.6 problem 23

Internal problem ID [10481]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = 3\lambda a - \lambda^2 - a(a+\lambda)\coth(\lambda x)^2$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 240

```
dsolve(diff(y(x),x)=y(x)^2-lambda^2+3*a*lambda-a*(a+lambda)*coth(lambda*x)^2,y(x), singsol=a
```

$$y(x) = -\frac{\left((c_1 a + c_1\lambda)\,\mathrm{LegendreQ}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \coth(\lambda x)\right) + (a+\lambda)\,\mathrm{LegendreP}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \coth(\lambda x)\right)\right)\coth(\lambda x)}{c_1\,\mathrm{LegendreQ}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \coth(\lambda x)\right) + \mathrm{LegendreP}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \coth(\lambda x)\right)} + \frac{2c_1\lambda\,\mathrm{LegendreQ}\left(\frac{a+\lambda}{\lambda}, \frac{a-\lambda}{\lambda}, \coth(\lambda x)\right)}{c_1\,\mathrm{LegendreQ}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \coth(\lambda x)\right) + \mathrm{LegendreP}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \coth(\lambda x)\right)} + \frac{2\,\mathrm{LegendreP}\left(\frac{a+\lambda}{\lambda}, \frac{a-\lambda}{\lambda}, \coth(\lambda x)\right)\lambda}{c_1\,\mathrm{LegendreQ}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \coth(\lambda x)\right) + \mathrm{LegendreP}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \coth(\lambda x)\right)}$$

✓ Solution by Mathematica

Time used: 14.312 (sec). Leaf size: 659

```
DSolve[y'[x]==y[x]^2-\[Lambda]^2+3*a*\[Lambda]-a*(a+\[Lambda])*Coth[\[Lambda]*x]^2,y[x],x,In
```

$$y(x) \to \frac{-\lambda(a-2\lambda)\left(e^{2\lambda x}+1\right)\left(1-e^{2\lambda x}\right)^{\frac{2a}{\lambda}}\left(\frac{e^{2\lambda x}}{e^{2\lambda x}+1}\right)^{a/\lambda}\left(a\left(-4e^{2\lambda x}+e^{4\lambda x}-1\right)+\lambda-\lambda e^{4\lambda x}\right)\text{AppellF1}\left(1-\frac{a}{\lambda},\right.}{}$$

$$y(x) \to \frac{a\left(e^{2\lambda x}+1\right)^2-\lambda\left(e^{2\lambda x}-1\right)^2}{e^{4\lambda x}-1}$$

6.7 problem 24

Internal problem ID [10482]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - ax \coth(bx)^m y = a \coth(bx)^m$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)=y(x)^2+a*x*coth(b*x)^m*y(x)+a*coth(b*x)^m,y(x), singsol=all)
```

$$y(x) = -\frac{e^{\int \frac{a \coth(xb)^m x^2 - 2}{x} dx} x + \int e^{\int \frac{a \coth(xb)^m x^2 - 2}{x} dx} dx - c_1}{\left(-c_1 + \int e^{\int \frac{a \coth(xb)^m x^2 - 2}{x} dx} dx\right) x}$$

✓ Solution by Mathematica

Time used: 11.817 (sec). Leaf size: 126

```
DSolve[y'[x]==y[x]^2+a*x*Coth[b*x]^m*y[x]+a*Coth[b*x]^m,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \rightarrow -\frac{\exp\left(-\int_1^x -a \coth^m(bK[1])K[1]dK[1]\right) + x \int_1^x \frac{\exp\left(-\int_1^{K[2]} -a \coth^m(bK[1])K[1]dK[1]\right)}{K[2]^2} dK[2] + c_1 x}{x^2 \left(\int_1^x \frac{\exp\left(-\int_1^{K[2]} -a \coth^m(bK[1])K[1]dK[1]\right)}{K[2]^2} dK[2] + c_1\right)}$$

$$y(x) \rightarrow -\frac{1}{x}$$

6.8 problem 25

Internal problem ID [10483]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$(a\coth(x\lambda)+b)\,y'-y^2-c\coth(\mu x)\,y=-d^2+cd\coth(\mu x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 217

```
dsolve((a*coth(lambda*x)+b)*diff(y(x),x)=y(x)^2+c*coth(mu*x)*y(x)-d^2+c*d*coth(mu*x),y(x), s
```

$$y(x)=-d-\frac{e^{\int\frac{c\coth(\mu x)}{a\coth(\lambda x)+b}dx}(a\coth(\lambda x)+b)^{-\frac{2ad}{\lambda(a-b)(a+b)}}(\coth(\lambda x)-1)^{\frac{d}{\lambda(a+b)}}(\coth(\lambda x)+1)^{\frac{d}{\lambda(a-b)}}}{\int\frac{e^{\int\frac{c\coth(\mu x)}{a\coth(\lambda x)+b}dx}(a\coth(\lambda x)+b)^{-\frac{2ad}{\lambda(a-b)(a+b)}}(\coth(\lambda x)-1)^{\frac{d}{\lambda(a+b)}}(\coth(\lambda x)+1)^{\frac{d}{\lambda(a-b)}}}{a\coth(\lambda x)+b}dx-c_1}$$

✓ Solution by Mathematica

Time used: 153.106 (sec). Leaf size: 808

```
DSolve[(a*Coth[\[Lambda]*x]+b)*y'[x]==y[x]^2+c*Coth[\[Mu]*x]*y[x]-d^2+c*d*Coth[\[Mu]*x],y[x]
```

$$\text{Solve}\Bigg[\int_1^x -\frac{e^{-\int_1^{K[2]}\frac{\operatorname{csch}(\mu K[1])(-2d\cosh(\lambda K[1]-\mu K[1])+2d\cosh(\lambda K[1]+\mu K[1])-c\sinh(\lambda K[1]-\mu K[1])-c\sinh(\lambda K[1]+\mu K[1]))}{2(a\cosh(\lambda K[1])+b\sinh(\lambda K[1]))}dK[1]}(d\cosh(\lambda K[2]-\mu K[2]}{c\mu(b\cosh(\lambda K[2]-\mu K[2])-b\cosh(\lambda K[2}$$
$$+\int_1^{y(x)}\left(-\int_1^x\left(\frac{e^{-\int_1^{K[2]}\frac{\operatorname{csch}(\mu K[1])(-2d\cosh(\lambda K[1]-\mu K[1])+2d\cosh(\lambda K[1]+\mu K[1])-c\sinh(\lambda K[1]-\mu K[1])-c\sinh(\lambda K[1]+\mu K[1]))}{2(a\cosh(\lambda K[1])+b\sinh(\lambda K[1]))}dK[1]}(d\cosh}{c\mu(d+K[3])^2(b\cosh(\lambda K}\right.\right.$$
$$\left.-\frac{e^{-\int_1^x\frac{\operatorname{csch}(\mu K[1])(-2d\cosh(\lambda K[1]-\mu K[1])+2d\cosh(\lambda K[1]+\mu K[1])-c\sinh(\lambda K[1]-\mu K[1])-c\sinh(\lambda K[1]+\mu K[1]))}{2(a\cosh(\lambda K[1])+b\sinh(\lambda K[1]))}dK[1]}}{c\mu(d+K[3])^2}\right)dK[3]=c_1,y(x)\Bigg]$$

6.9 problem 26

Internal problem ID [10484]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = -2\tanh(\lambda x)^2 \lambda^2 - 2\coth(\lambda x)^2 \lambda^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 330

```
dsolve(diff(y(x),x)=y(x)^2-2*lambda^2*tanh(lambda*x)^2-2*lambda^2*coth(lambda*x)^2,y(x), sin
```

$$y(x) = \frac{\lambda\left(\left(\left(-3\coth(\lambda x)^2 c_1 - c_1\right)\operatorname{csch}(\lambda x)^2 \sinh(\lambda x)^2 + 2\coth(\lambda x)\cosh(\lambda x)\sinh(\lambda x)\operatorname{csch}(\lambda x)^2 c_1\right)\ln(\coth}{}$$

✓ Solution by Mathematica

Time used: 7.989 (sec). Leaf size: 132

```
DSolve[y'[x]==y[x]^2-2*\[Lambda]^2*Tanh[\[Lambda]*x]^2-2*\[Lambda]^2*Coth[\[Lambda]*x]^2,y[x
```

$$y(x) \rightarrow -\frac{2\lambda\left(e^{12\lambda x} + 2e^{4\lambda x}\left(e^{4\lambda x}+1\right)\log\left(e^{4\lambda x}\right) + (-7+c_1)\left(-e^{4\lambda x}\right) - (7+c_1)e^{8\lambda x} - 1\right)}{\left(e^{4\lambda x}-1\right)\left(e^{8\lambda x} - 2e^{4\lambda x}\log\left(e^{4\lambda x}\right) + c_1 e^{4\lambda x} - 1\right)}$$

$$y(x) \rightarrow \frac{2\lambda\left(e^{4\lambda x}+1\right)}{e^{4\lambda x}-1}$$

6.10 problem 27

Internal problem ID [10485]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = -2ba + \lambda a + b\lambda - a(a+\lambda)\tanh(\lambda x)^2 - b(b+\lambda)\coth(\lambda x)^2$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 1111

```
dsolve(diff(y(x),x)=y(x)^2+a*lambda+b*lambda-2*a*b-a*(a+lambda)*tanh(lambda*x)^2-b*(b+lambda
```

Expression too large to display

✓ Solution by Mathematica

Time used: 40.238 (sec). Leaf size: 493

```
DSolve[y'[x]==y[x]^2+a*\[Lambda]+b*\[Lambda]-2*a*b-a*(a+\[Lambda])*Tanh[\[Lambda]*x]^2-b*(b+
```

$$y(x) \to -\frac{(a+b)\left(e^{2\lambda x}\right)^{\frac{a+b}{\lambda}}\left(\frac{2\lambda\left(a\left(e^{2\lambda x}-1\right)^2+b\left(e^{2\lambda x}+1\right)^2\right)\left(e^{2\lambda x}\right)^{-\frac{a+b}{\lambda}}\text{AppellF1}\left(-\frac{a+b}{\lambda},-\frac{2b}{\lambda},-\frac{2a}{\lambda},-\frac{a+b-\lambda}{\lambda},e^{2x\lambda},-e^{2x\lambda}\right)}{(a+b)\left(e^{2\lambda x}-1\right)\left(e^{2\lambda x}+1\right)} + 4\lambda\left(e^{2\lambda x}\right)^{-}\right.}{}$$

$$y(x) \to \frac{a\left(e^{2\lambda x}-1\right)^2 + b\left(e^{2\lambda x}+1\right)^2}{e^{4\lambda x}-1}$$

7 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions

7.1 problem 1

Internal problem ID [10486]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\ln(x)^n y^2 = bm\, x^{m-1} - a\, b^2 x^{2m} \ln(x)^n$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*(ln(x))^n*y(x)^2+b*m*x^(m-1)-a*b^2*x^(2*m)*(ln(x))^n,y(x), singsol=all
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*(Log[x])^n*y[x]^2+b*m*x^(m-1)-a*b^2*x^(2*m)*(Log[x])^n,y[x],x,IncludeSingula
```

Not solved

7.2 problem 2

Internal problem ID [10487]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **unknown**

$$\boxed{y'x - ay^2 = b\ln(x) + c}$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)=a*y(x)^2+b*ln(x)+c,y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 1.682 (sec). Leaf size: 149

```
DSolve[x*y'[x]==a*y[x]^2+b*Log[x]+c,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{b\left(\text{AiryBiPrime}\left(-\frac{a(c+b\log(x))}{(-ab)^{2/3}}\right) + c_1\,\text{AiryAiPrime}\left(-\frac{a(c+b\log(x))}{(-ab)^{2/3}}\right)\right)}{(-ab)^{2/3}\left(\text{AiryBi}\left(-\frac{a(c+b\log(x))}{(-ab)^{2/3}}\right) + c_1\,\text{AiryAi}\left(-\frac{a(c+b\log(x))}{(-ab)^{2/3}}\right)\right)}$$

$$y(x) \to \frac{b\,\text{AiryAiPrime}\left(-\frac{a(c+b\log(x))}{(-ab)^{2/3}}\right)}{(-ab)^{2/3}\,\text{AiryAi}\left(-\frac{a(c+b\log(x))}{(-ab)^{2/3}}\right)}$$

7.3 problem 3

Internal problem ID [10488]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - ay^2 = b\ln(x)^k + c\ln(x)^{2k+2}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 660

```
dsolve(x*diff(y(x),x)=a*y(x)^2+b*(ln(x))^k+c*(ln(x))^(2*k+2),y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(-i\sqrt{a}\,\ln(x)^{k+2}\sqrt{c}\,c_1k^2 - 4i\sqrt{a}\,\ln(x)^{k+2}\sqrt{c}\,c_1k - 3i\sqrt{a}\,\ln(x)^{k+2}\sqrt{c}\,c_1 + \ln(x)^{k+2}c_1abk + \ln(x)^{k+2}\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 3.775 (sec). Leaf size: 807

```
DSolve[x*y'[x]==a*y[x]^2+b*(Log[x])^k+c*(Log[x])^(2*k+2),y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{\log^{k+1}(x)\left(\sqrt{c}c_1(k+2)\sqrt{-(k+2)^2}\,\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{a}b}{\sqrt{c}\sqrt{-(k+2)^2}}+\frac{k+1}{k+2}\right),\frac{k+1}{k+2},\frac{2\sqrt{a}\sqrt{c}\log^{k+2}(x)}{\sqrt{-(k+2)^2}}\right)+\right.}{\sqrt{a}(}$$

$$y(x) \to \frac{\log^{k+1}(x)\left(-\frac{\left(\sqrt{a}b(k+2)+\sqrt{c}\sqrt{-(k+2)^2}(k+1)\right)\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{a}b}{\sqrt{c}\sqrt{-(k+2)^2}}+\frac{3k+5}{k+2}\right),\frac{2k+3}{k+2},\frac{2\sqrt{a}\sqrt{c}\log^{k+2}(x)}{\sqrt{-(k+2)^2}}\right)}{\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{a}b}{\sqrt{c}\sqrt{-(k+2)^2}}+\frac{k+1}{k+2}\right),\frac{k+1}{k+2},\frac{2\sqrt{a}\sqrt{c}\log^{k+2}(x)}{\sqrt{-(k+2)^2}}\right)}-\sqrt{c}\sqrt{-(\right.}{\sqrt{a}(k+2)^2}$$

7.4 problem 4

Internal problem ID [10489]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - y^2x = -a^2x\ln(\beta x)^2 + a$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)=x*y(x)^2-a^2*x*(ln(beta*x))^2+a,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]==x*y[x]^2-a^2*x*(Log[\[Beta]*x])^2+a,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.5 problem 5

Internal problem ID [10490]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - y^2x = -a^2x\ln(\beta x)^{2k} + ak\ln(\beta x)^{k-1}$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)=x*y(x)^2-a^2*x*(ln(beta*x))^(2*k)+a*k*(ln(beta*x))^(k-1),y(x), singsol
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]==x*y[x]^2-a^2*x*(Log[\[Beta]*x])^(2*k)+a*k*(Log[\[Beta]*x])^(k-1),y[x],x,Incl
```

Not solved

7.6 problem 6

Internal problem ID [10491]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - a\,x^n y^2 = b - a\,b^2 x^n \ln\left(x\right)^2$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)=a*x^n*y(x)^2+b-a*b^2*x^n*(ln(x))^2,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]==a*x^n*y[x]^2+b-a*b^2*x^n*(Log[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.7 problem 7

Internal problem ID [10492]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x^2 - y^2x^2 = a\ln(x)^2 + b\ln(x) + c}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 850

```
dsolve(x^2*diff(y(x),x)=x^2*y(x)^2+a*(ln(x))^2+b*ln(x)+c,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 1.151 (sec). Leaf size: 868

```
DSolve[x^2*y'[x]==x^2*y[x]^2+a*(Log[x])^2+b*Log[x]+c,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{ib\,\text{ParabolicCylinderD}\left(\frac{-ib^2-4a^{3/2}+ia(4c-1)}{8a^{3/2}}, -\frac{\left(\frac{1}{2}-\frac{i}{2}\right)(b+2a\log(x))}{a^{3/4}}\right) + 2ia\log(x)\,\text{ParabolicCylinderD}\left(\frac{-ib^2-4\right.}{}$$

$$y(x) \to -\frac{-\frac{2\sqrt[4]{-1}\sqrt{2}\sqrt[4]{a}\,\text{ParabolicCylinderD}\left(\frac{ib^2+4a^{3/2}-ia(4c-1)}{8a^{3/2}}, \frac{\left(\frac{1}{2}+\frac{i}{2}\right)(b+2a\log(x))}{a^{3/4}}\right)}{\text{ParabolicCylinderD}\left(\frac{ib^2-4a^{3/2}-ia(4c-1)}{8a^{3/2}}, \frac{\left(\frac{1}{2}+\frac{i}{2}\right)(b+2a\log(x))}{a^{3/4}}\right)} + \frac{ib}{\sqrt{a}} + 2i\sqrt{a}\log(x) + 1}{2x}$$

$$y(x) \to -\frac{-\frac{2\sqrt[4]{-1}\sqrt{2}\sqrt[4]{a}\,\text{ParabolicCylinderD}\left(\frac{ib^2+4a^{3/2}-ia(4c-1)}{8a^{3/2}}, \frac{\left(\frac{1}{2}+\frac{i}{2}\right)(b+2a\log(x))}{a^{3/4}}\right)}{\text{ParabolicCylinderD}\left(\frac{ib^2-4a^{3/2}-ia(4c-1)}{8a^{3/2}}, \frac{\left(\frac{1}{2}+\frac{i}{2}\right)(b+2a\log(x))}{a^{3/4}}\right)} + \frac{ib}{\sqrt{a}} + 2i\sqrt{a}\log(x) + 1}{2x}$$

7.8 problem 8

Internal problem ID [10493]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x^2 - y^2x^2 = a(b\ln(x) + c)^n + \frac{1}{4}}$$

✗ Solution by Maple

```
dsolve(x^2*diff(y(x),x)=x^2*y(x)^2+a*(b*ln(x)+c)^n+1/4,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y'[x]==x^2*y[x]^2+a*(b*Log[x]+c)^n+1/4,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.9 problem 9

Internal problem ID [10494]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{x^2 \ln(ax)\left(y' - y^2\right) = 1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
dsolve(x^2*ln(a*x)*(diff(y(x),x)-y(x)^2)=1,y(x), singsol=all)
```

$$y(x) = -\frac{c_1 \operatorname{Ei}_1(-\ln(ax)) - 1}{x\left((c_1 \operatorname{Ei}_1(-\ln(ax)) - 1)\ln(ax) + axc_1\right)}$$

✓ Solution by Mathematica

Time used: 0.616 (sec). Leaf size: 74

```
DSolve[x^2*Log[a*x]*(y'[x]-y[x]^2)==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{a + c_1 \operatorname{LogIntegral}(ax)}{-c_1 x \operatorname{LogIntegral}(ax)\log(ax) + ac_1 x^2 - ax\log(ax)}$$

$$y(x) \to \frac{\operatorname{LogIntegral}(ax)}{ax^2 - x\operatorname{LogIntegral}(ax)\log(ax)}$$

8 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2

8.1 problem 10

Internal problem ID [10495]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a\ln(\beta x)\, y = -ab\ln(\beta x) - b^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
dsolve(diff(y(x),x)=y(x)^2+a*ln(beta*x)*y(x)-a*b*ln(beta*x)-b^2,y(x), singsol=all)
```

$$y(x) = b - \frac{(\beta x)^{ax}\, \mathrm{e}^{-ax}\mathrm{e}^{2xb}}{\int (\beta x)^{ax}\, \mathrm{e}^{-ax}\mathrm{e}^{2xb} dx - c_1}$$

✓ Solution by Mathematica

Time used: 1.584 (sec). Leaf size: 187

```
DSolve[y'[x]==y[x]^2+a*Log[\[Beta]*x]*y[x]-a*b*Log[\[Beta]*x]-b^2,y[x],x,IncludeSingularSolu
```

$$\text{Solve}\left[\int_1^x \frac{e^{2bK[1]-aK[1]}(\beta K[1])^{aK[1]}(b + a\log(\beta K[1]) + y(x))}{a(b - y(x))} dK[1] + \int_1^{y(x)} \left(\frac{e^{2bx-ax}(x\beta)^{ax}}{a(K[2]-b)^2} - \int_1^x \left(\frac{e^{2bK[1]-aK[1]}(b + K[2] + a\log(\beta K[1]))(\beta K[1])^{aK[1]}}{a(b-K[2])^2} + \frac{e^{2bK[1]-aK[1]}(\beta K[1])^{aK[1]}}{a(b-K[2])}\right) dK[1]\right) dK[2] = c_1,\right.$$

8.2 problem 11

Internal problem ID [10496]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - ax\ln(bx)^m y = a\ln(bx)^m}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(diff(y(x),x)=y(x)^2+a*x*(ln(b*x))^m*y(x)+a*(ln(b*x))^m,y(x), singsol=all)
```

$$y(x) = \frac{e^{\int \frac{a\ln(xb)^m x^2 - 2}{x} dx}}{c_1 - \left(\int e^{\int \frac{a\ln(xb)^m x^2 - 2}{x} dx} dx\right)} - \frac{1}{x}$$

✓ Solution by Mathematica

Time used: 3.589 (sec). Leaf size: 181

```
DSolve[y'[x]==y[x]^2+a*x*(Log[b*x])^m*y[x]+a*(Log[b*x])^m,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{x\int_1^x \frac{\exp\left(\frac{2^{-m-1}a\Gamma(m+1,-2\log(bK[1]))(-\log(bK[1]))^{-m}\log^m(bK[1])}{b^2}\right)}{K[1]^2}dK[1] + \exp\left(\frac{a2^{-m-1}(-\log(bx))^{-m}\log^m(bx)\Gamma(m+1,-2\log(}{b^2}\right.}{x^2\left(\int_1^x \frac{\exp\left(\frac{2^{-m-1}a\Gamma(m+1,-2\log(bK[1]))(-\log(bK[1]))^{-m}\log^m(bK[1])}{b^2}\right)}{K[1]^2}dK[1] + c_1\right)}$$

$$y(x) \to -\frac{1}{x}$$

8.3 problem 12

Internal problem ID [10497]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,x^n y^2 + ab\,x^{n+1} \ln(x)\, y = b \ln(x) + b$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2-a*b*x^(n+1)*ln(x)*y(x)+b*ln(x)+b,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*x^n*y[x]^2-a*b*x^(n+1)*Log[x]*y[x]+b*Log[x]+b,y[x],x,IncludeSingularSolution
```

Not solved

8.4 problem 13

Internal problem ID [10498]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + (n+1)\,x^n y^2 - a\,x^{n+1}\ln(x)^m\,y = -a\ln(x)^m$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 201

```
dsolve(diff(y(x),x)=-(n+1)*x^n*y(x)^2+a*x^(n+1)*(ln(x))^m*y(x)-a*(ln(x))^m,y(x), singsol=all
```

$$y(x) = \frac{\left(-\mathrm{e}^{\int \frac{\ln(x)^m x^n a\, x^2 - 2n - 2}{x}dx} x^n x + \int \left(-x^n n\, \mathrm{e}^{a\left(\int x^{1+n}\ln(x)^m dx\right) - 2n\left(\int \frac{1}{x}dx\right) - 2\left(\int \frac{1}{x}dx\right)} - x^n \mathrm{e}^{a\left(\int x^{1+n}\ln(x)^m dx\right) - 2n\left(\int \frac{1}{x}dx\right) \cdots}\right.\right.}{x\left(\int \left(-x^n n\, \mathrm{e}^{a\left(\int x^{1+n}\ln(x)^m dx\right) - 2n\left(\int \frac{1}{x}dx\right) - 2\left(\int \frac{1}{x}dx\right)} - x^n \mathrm{e}^{a\left(\int x^{1+n}\ln(x)^m dx\right) - 2n\left(\int \frac{1}{x}dx\right) - 2\left(\int \frac{1}{x}dx\right)}\right) \cdots\right.}$$

✓ Solution by Mathematica

Time used: 5.364 (sec). Leaf size: 311

```
DSolve[y'[x]==-(n+1)*x^n*y[x]^2+a*x^(n+1)*(Log[x])^m*y[x]-a*(Log[x])^m,y[x],x,IncludeSingula
```

$$y(x) \to \frac{x^{-2(n+1)}\left(c_1(n+1)x^{n+1}\int_1^x \exp\left(\frac{a\Gamma(m+1,-((n+2)\log(K[1])))\log^m(K[1])(-((n+2)\log(K[1])))^{-m}}{n+2} - (n+2)\log(K[1])\right)\right.}{(n+1)\left(1+c_1\int_1^x \exp\left(\frac{a\Gamma(m+1,-((n+2)\log(K[1])))\log^m(K[1])(-((n+2)\log\right.\right.}$$

$$y(x) \to \frac{x^{-2(n+1)}\left(\frac{\exp\left(\frac{a\log^m(x)(-((n+2)\log(x)))^{-m}\Gamma(m+1,-((n+2)\log(x)))}{n+2}\right)}{\int_1^x \exp\left(\frac{a\Gamma(m+1,-((n+2)\log(K[1])))\log^m(K[1])(-((n+2)\log(K[1])))^{-m}}{n+2}-(n+2)\log(K[1])\right)dK[1]} + (n+1)x^{n+1}\right)}{n+1}$$

8.5 problem 14

Internal problem ID [10499]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - a\ln(x)^n y + abx\ln(x)^{n+1} y = b\ln(x) + b}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
dsolve(diff(y(x),x)=a*(ln(x))^n*y(x)-a*b*x*(ln(x))^(n+1)*y(x)+b*ln(x)+b,y(x), singsol=all)
```

$$y(x) = \left(\int \mathrm{e}^{a\left(\int \ln(x)^n(-1+\ln(x)bx)dx\right)} b(\ln(x)+1)\,dx + c_1\right) \mathrm{e}^{\int \left(-\ln(x)^{1+n}abx + a\ln(x)^n\right)dx}$$

✓ Solution by Mathematica

Time used: 0.86 (sec). Leaf size: 124

```
DSolve[y'[x]==a*(Log[x])^n*y[x]-a*b*x*(Log[x])^(n+1)*y[x]+b*Log[x]+b,y[x],x,IncludeSingularS
```

$$y(x) \to \exp\left(a2^{-n-2}(-\log(x))^{-n}\log^n(x)\left(b\Gamma(n+2,-2\log(x))\right.\right.$$
$$\left.\left. +2^{n+2}\Gamma(n+1,-\log(x))\right)\right)\left(\int_1^x b\exp\left(-2^{-n-2}a\left(2^{n+2}\Gamma(n+1,-\log(K[1]))+b\Gamma(n+2,-2\log(K[1]))\right)(-\right.\right.$$
$$\left. +1)dK[1] + c_1\right)$$

8.6 problem 15

Internal problem ID [10500]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$
\boxed{y' - a\ln(x)^k (y - b\,x^n - c)^2 = bn\,x^{-1+n}}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 50

```
dsolve(diff(y(x),x)=a*(ln(x))^k*(y(x)-b*x^n-c)^2+b*n*x^(n-1),y(x), singsol=all)
```

$$
y(x) = -\frac{\left(-2a\ln(x)^k x^n b - 2a\ln(x)^k c\right)\ln(x)^{-k}}{2a} + \frac{1}{c_1 - \left(\int a\ln(x)^k\,dx\right)}
$$

✓ Solution by Mathematica

Time used: 1.572 (sec). Leaf size: 51

```
DSolve[y'[x]==a*(Log[x])^k*(y[x]-b*x^n-c)^2+b*n*x^(n-1),y[x],x,IncludeSingularSolutions -> T
```

$$
\begin{aligned}
y(x) &\to \frac{1}{-a(-\log(x))^{-k}\log^k(x)\Gamma(k+1,-\log(x)) + c_1} + bx^n + c \\
y(x) &\to bx^n + c
\end{aligned}
$$

8.7 problem 16

Internal problem ID [10501]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\ln(x)^n y^2 - b\ln(x)^m y = bc\ln(x)^m - a c^2 \ln(x)^n$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 134

```
dsolve(diff(y(x),x)=a*(ln(x))^n*y(x)^2+b*(ln(x))^m*y(x)+b*c*(ln(x))^m-a*c^2*(ln(x))^n,y(x),
```

$$y(x) = -\frac{\left(\left(\int a\ln(x)^n e^{\int(-2\ln(x)^n ac+\ln(x)^m b)dx}dx\right)e^{\int(2\ln(x)^n ac-\ln(x)^m b)dx}c + c_1 e^{\int(2\ln(x)^n ac-\ln(x)^m b)dx}c + 1\right)e^{\int(-2\ln(x)^n}}{c_1 + \int a\ln(x)^n e^{\int(-2\ln(x)^n ac+\ln(x)^m b)dx}dx}$$

✓ Solution by Mathematica

Time used: 3.927 (sec). Leaf size: 385

```
DSolve[y'[x]==a*(Log[x])^n*y[x]^2+b*(Log[x])^m*y[x]+b*c*(Log[x])^m-a*c^2*(Log[x])^n,y[x],x,I
```

$$\text{Solve}\left[\int_1^x \frac{\exp(b\Gamma(m+1,-\log(K[1]))(-\log(K[1]))^{-m}\log^m(K[1]) - 2ac\Gamma(n+1,-\log(K[1]))(-\log(K[}{ab(m-n)(c+y(x))}\right.$$
$$+\int_1^{y(x)}\left(\frac{\exp(b\Gamma(m+1,-\log(x))(-\log(x))^{-m}\log^m(x) - 2ac\Gamma(n+1,-\log(x))(-\log(x))^{-n}\log^n(x))}{ab(m-n)(c+K[2])^2}\right.$$
$$-\int_1^x\left(-\frac{\exp(b\Gamma(m+1,-\log(K[1]))(-\log(K[1]))^{-m}\log^m(K[1]) - 2ac\Gamma(n+1,-\log(K[1]))(-\log(K[1}{b(m-n)(c+K[2])}\right.$$

8.8 problem 17

Internal problem ID [10502]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Riccati]`

$$\boxed{y'x - (ay + b\ln(x))^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(x*diff(y(x),x)=(a*y(x)+b*ln(x))^2,y(x), singsol=all)
```

$$y(x) = -\frac{\ln(x)\,ab - \tan\left(c_1 a\sqrt{ba} + \ln(x)\sqrt{ba}\right)\sqrt{ba}}{a^2}$$

✓ Solution by Mathematica

Time used: 6.524 (sec). Leaf size: 43

```
DSolve[x*y'[x]==(a*y[x]+b*Log[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{b\log(x)}{a} + \sqrt{\frac{b}{a^3}}\tan\left(a^2\sqrt{\frac{b}{a^3}}\log(x) + c_1\right)$$

8.9 problem 18

Internal problem ID [10503]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - a\ln(\lambda x)^m y^2 - ky = a\,b^2x^{2k}\ln(\lambda x)^m}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 36

```
dsolve(x*diff(y(x),x)=a*(ln(lambda*x))^m*y(x)^2+k*y(x)+a*b^2*x^(2*k)*(ln(lambda*x))^m,y(x),
```

$$y(x) = -\tan\left(-ba\left(\int \frac{x^k \ln(\lambda x)^m}{x}dx\right) + c_1\right) b\,x^k$$

✓ Solution by Mathematica

Time used: 2.161 (sec). Leaf size: 70

```
DSolve[x*y'[x]==a*(Log[\[Lambda]*x])^m*y[x]^2+k*y[x]+a*b^2*x^(2*k)*(Log[\[Lambda]*x])^m,y[x]
```

$$y(x) \to \sqrt{b^2}x^k \tan\left(\frac{a\sqrt{b^2}x^k(\lambda x)^{-k}\log^m(\lambda x)(-k\log(\lambda x))^{-m}\Gamma(m+1,-k\log(x\lambda))}{k} + c_1\right)$$

8.10 problem 19

Internal problem ID [10504]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$\boxed{y'x - a\,x^n(y + b\ln(x))^2 = -b}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x)=a*x^n*(y(x)+b*ln(x))^2-b,y(x), singsol=all)
```

$$y(x) = -\ln(x)\,b + \frac{1}{c_1 - \frac{a\,x^n}{n}}$$

✓ Solution by Mathematica

Time used: 0.649 (sec). Leaf size: 35

```
DSolve[x*y'[x]==a*x^n*(y[x]+b*Log[x])^2-b,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -b\log(x) + \frac{n}{-ax^n + c_1 n}$$
$$y(x) \to -b\log(x)$$

8.11 problem 20

Internal problem ID [10505]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - a\,x^{2n}\ln(x)\,y^2 - (\ln(x)\,x^n b - n)\,y = \ln(x)\,c}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 83

```
dsolve(x*diff(y(x),x)=a*x^(2*n)*ln(x)*y(x)^2+(b*x^n*ln(x)-n)*y(x)+c*ln(x),y(x), singsol=all)
```

$$y(x) = \frac{\left(\tan\left(\frac{\sqrt{4b^2ac-b^4}\,(\ln(x)x^n bn+c_1n^2-b\,x^n)}{2b^2n^2}\right)\sqrt{4b^2ac-b^4}-b^2\right)x^{-n}}{2ab}$$

✓ Solution by Mathematica

Time used: 1.872 (sec). Leaf size: 130

```
DSolve[x*y'[x]==a*x^(2*n)*Log[x]*y[x]^2+(b*x^n*Log[x]-n)*y[x]+c*Log[x],y[x],x,IncludeSingula
```

$$y(x) \to \frac{x^{-n}\left(-b+\frac{\sqrt{b^2-4ac}\left(-e^{\frac{x^n\sqrt{b^2-4ac}(n\log(x)-1)}{n^2}}+c_1\right)}{e^{\frac{x^n\sqrt{b^2-4ac}(n\log(x)-1)}{n^2}}+c_1}\right)}{2a}$$

$$y(x) \to \frac{x^{-n}\left(\sqrt{b^2-4ac}-b\right)}{2a}$$

8.12 problem 21

Internal problem ID [10506]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x^2 - a^2x^2y^2 + yx = b^2\ln(x)^n}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 324

```
dsolve(x^2*diff(y(x),x)=a^2*x^2*y(x)^2-x*y(x)+b^2*(ln(x))^n,y(x), singsol=all)
```

$$y(x) = \frac{\frac{\ln(x)^{\frac{n}{2}+1}\sqrt{b^2a^2}\,c_1\,\mathrm{BesselY}\left(\frac{n+3}{n+2}, \frac{2\sqrt{b^2a^2}\,\ln(x)^{\frac{n}{2}+1}}{n+2}\right)}{\left(\mathrm{BesselY}\left(\frac{1}{n+2}, \frac{2\sqrt{b^2a^2}\,\ln(x)^{\frac{n}{2}+1}}{n+2}\right)c_1+\mathrm{BesselJ}\left(\frac{1}{n+2}, \frac{2\sqrt{b^2a^2}\,\ln(x)^{\frac{n}{2}+1}}{n+2}\right)\right)a^2x} + \frac{\mathrm{BesselJ}\left(\frac{n+3}{n+2}, \frac{2\sqrt{b^2a^2}\,\ln(x)^{\frac{n}{2}+1}}{n+2}\right)\sqrt{b^2a^2}\,\ln(x)^{\frac{n}{2}+1}-\mathrm{Besse}}{\left(\mathrm{BesselY}\left(\frac{1}{n+2}, \frac{2\sqrt{b^2a^2}\,\ln(x)^{\frac{n}{2}+1}}{n+2}\right)\right.}}{\ln(x)}$$

✓ Solution by Mathematica

Time used: 45.846 (sec). Leaf size: 1769

```
DSolve[x^2*y'[x]==a^2*x^2*y[x]^2-x*y[x]+b^2*(Log[x])^n,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{}{x\left(2a(n+2)^{\frac{2(n+1)}{n+2}}\operatorname{BesselJ}\left(-\frac{1}{n+2},\frac{2a\sqrt{\left(b^2\log^{n+1}(x)\right)^{1+\frac{1}{n+1}}}}{\sqrt{b^{\frac{2}{n+1}}(n+2)^2}}\right)\operatorname{Gamma}\left(\frac{2n+3}{n+2}\right)\left(b^2\log^{n+1}(x)\right)^{1+\frac{1}{n+1}}b^{\frac{2}{n+2}}+an}$$

$$y(x) \to \frac{2b^2\sqrt{(n+2)^2b^{\frac{2}{n+1}}}\log^{n+1}(x)\sqrt{(b}}{x\left(-a(n+2)\left(b^2\log^{n+1}(x)\right)^{\frac{1}{n+1}+1}\operatorname{BesselJ}\left(\frac{1}{n+2},\frac{2a\sqrt{\left(b^2\log^{n+1}(x)\right)^{1+\frac{1}{n+1}}}}{\sqrt{b^{\frac{2}{n+1}}(n+2)^2}}\right)+a(n+2)\left(b^2\log^{n+1}(x)\right)^{\frac{1}{n+1}+1}\right.}$$

8.13 problem 22

Internal problem ID [10507]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{(a\ln(x)+b)\,y'-y^2-c\ln(x)^n\,y=-\lambda^2+\lambda c\ln(x)^n}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 160

```
dsolve((a*ln(x)+b)*diff(y(x),x)=y(x)^2+c*(ln(x))^n*y(x)-lambda^2+lambda*c*(ln(x))^n,y(x), si
```

$$y(x)=-\frac{\left(\left(\int \frac{e^{\int \frac{\ln(x)^n c-2\lambda}{a\ln(x)+b}dx}}{a\ln(x)+b}dx\right)e^{\int -\frac{\ln(x)^n c-2\lambda}{a\ln(x)+b}dx}\lambda+c_1e^{\int -\frac{\ln(x)^n c-2\lambda}{a\ln(x)+b}dx}\lambda+1\right)e^{\int \frac{\ln(x)^n c-2\lambda}{a\ln(x)+b}dx}}{c_1+\int \frac{e^{\int \frac{\ln(x)^n c-2\lambda}{a\ln(x)+b}dx}}{a\ln(x)+b}dx}$$

✓ Solution by Mathematica

Time used: 5.348 (sec). Leaf size: 275

```
DSolve[(a*Log[x]+b)*y'[x]==y[x]^2+c*(Log[x])^n*y[x]-\[Lambda]^2+\[Lambda]*c*(Log[x])^n,y[x],
```

$$\text{Solve}\left[\int_1^x -\frac{\exp\left(-\int_1^{K[2]}\frac{2\lambda-c\log^n(K[1])}{b+a\log(K[1])}dK[1]\right)(c\log^n(K[2])-\lambda+y(x))}{cn(b+a\log(K[2]))(\lambda+y(x))}dK[2]\right.$$
$$+\int_1^{y(x)}\left(\frac{\exp\left(-\int_1^x\frac{2\lambda-c\log^n(K[1])}{b+a\log(K[1])}dK[1]\right)}{cn(\lambda+K[3])^2}\right.$$
$$-\int_1^x\left(\frac{\exp\left(-\int_1^{K[2]}\frac{2\lambda-c\log^n(K[1])}{b+a\log(K[1])}dK[1]\right)(c\log^n(K[2])-\lambda+K[3])}{cn(\lambda+K[3])^2(b+a\log(K[2]))}-\frac{\exp\left(-\int_1^{K[2]}\frac{2\lambda-c\log^n(K[1])}{b+a\log(K[1])}dK[1]\right)}{cn(\lambda+K[3])(b+a\log(K[2]))}\right)$$

8.14 problem 23

Internal problem ID [10508]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$(a \ln(x) + b) y' - \ln(x)^n y^2 - cy = -\lambda^2 \ln(x)^n + c\lambda$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 168

```
dsolve((a*ln(x)+b)*diff(y(x),x)=(ln(x))^n*y(x)^2+c*y(x)-lambda^2*(ln(x))^n+c*lambda,y(x), si
```

$$y(x) = -\frac{\left(\left(\int \frac{\ln(x)^n e^{\int -\frac{2\ln(x)^n \lambda - c}{a\ln(x)+b}dx}}{a\ln(x)+b}dx\right) e^{\int \frac{2\ln(x)^n \lambda - c}{a\ln(x)+b}dx}\lambda + c_1 e^{\int \frac{2\ln(x)^n \lambda - c}{a\ln(x)+b}dx}\lambda + 1\right) e^{\int -\frac{2\ln(x)^n \lambda - c}{a\ln(x)+b}dx}}{c_1 + \int \frac{\ln(x)^n e^{\int -\frac{2\ln(x)^n \lambda - c}{a\ln(x)+b}dx}}{a\ln(x)+b}dx}$$

✓ Solution by Mathematica

Time used: 5.137 (sec). Leaf size: 286

```
DSolve[(a*Log[x]+b)*y'[x]==(Log[x])^n*y[x]^2+c*y[x]-\[Lambda]^2*(Log[x])^n+c*\[Lambda],y[x],
```

$$\text{Solve}\left[\int_1^x \frac{\exp\left(-\int_1^{K[2]} -\frac{c-2\lambda\log^n(K[1])}{b+a\log(K[1])}dK[1]\right)(-\lambda\log^n(K[2])+y(x)\log^n(K[2])+c)}{cn(b+a\log(K[2]))(\lambda+y(x))}dK[2]\right.$$
$$+\int_1^{y(x)}\left(-\int_1^x\left(\frac{\exp\left(-\int_1^{K[2]} -\frac{c-2\lambda\log^n(K[1])}{b+a\log(K[1])}dK[1]\right)\log^n(K[2])}{cn(\lambda+K[3])(b+a\log(K[2]))}-\frac{\exp\left(-\int_1^{K[2]} -\frac{c-2\lambda\log^n(K[1])}{b+a\log(K[1])}dK[1]\right)(-\lambda}{cn(\lambda+K[3])^2(b+}\right.\right.$$
$$\left.\left.-\frac{\exp\left(-\int_1^x -\frac{c-2\lambda\log^n(K[1])}{b+a\log(K[1])}dK[1]\right)}{cn(\lambda+K[3])^2}\right)dK[3]=c_1,y(x)\right]$$

9 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine

9.1 problem 1

Internal problem ID [10509]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \alpha y^2 = \beta + \gamma \sin(\lambda x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 110

```
dsolve(diff(y(x),x)=alpha*y(x)^2+beta+gamma*sin(lambda*x),y(x), singsol=all)
```

$$y(x) = -\frac{\lambda\left(c_1 \,\mathrm{MathieuSPrime}\left(\frac{4\alpha\beta}{\lambda^2}, -\frac{2\gamma\alpha}{\lambda^2}, -\frac{\pi}{4} + \frac{\lambda x}{2}\right) + \mathrm{MathieuCPrime}\left(\frac{4\alpha\beta}{\lambda^2}, -\frac{2\gamma\alpha}{\lambda^2}, -\frac{\pi}{4} + \frac{\lambda x}{2}\right)\right)}{2\alpha\left(c_1 \,\mathrm{MathieuS}\left(\frac{4\alpha\beta}{\lambda^2}, -\frac{2\gamma\alpha}{\lambda^2}, -\frac{\pi}{4} + \frac{\lambda x}{2}\right) + \mathrm{MathieuC}\left(\frac{4\alpha\beta}{\lambda^2}, -\frac{2\gamma\alpha}{\lambda^2}, -\frac{\pi}{4} + \frac{\lambda x}{2}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.612 (sec). Leaf size: 191

```
DSolve[y'[x]==\[Alpha]*y[x]^2+\[Beta]+\[Gamma]*Sin[\[Lambda]*x],y[x],x,IncludeSingularSoluti
```

$$y(x) \to -\frac{\lambda\left(\mathrm{MathieuSPrime}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{1}{4}(\pi - 2\lambda x)\right] + c_1\mathrm{MathieuCPrime}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{1}{4}(2\lambda x - \pi)\right]\right)}{2\alpha\left(\mathrm{MathieuS}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{1}{4}(2\lambda x - \pi)\right] + c_1\mathrm{MathieuC}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{1}{4}(\pi - 2\lambda x)\right]\right)}$$

$$y(x) \to \frac{\lambda\mathrm{MathieuCPrime}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{1}{4}(\pi - 2\lambda x)\right]}{2\alpha\mathrm{MathieuC}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{1}{4}(\pi - 2\lambda x)\right]}$$

9.2 problem 2

Internal problem ID [10510]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = -a^2 + a\lambda \sin(\lambda x) + a^2 \sin(\lambda x)^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 415

```
dsolve(diff(y(x),x)=y(x)^2-a^2+a*lambda*sin(lambda*x)+a^2*sin(lambda*x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{\left(\left(4\sqrt{-\cos(\lambda x)^2+1}\, c_1 a + 4c_1 a + 2c_1\lambda\right)\operatorname{HeunC}\left(\frac{4a}{\lambda}, \frac{1}{2}, -\frac{1}{2}, -\frac{2a}{\lambda}, \frac{8a+3\lambda}{8\lambda}, \frac{\sqrt{-\cos(\lambda x)^2+1}}{2} + \frac{1}{2}\right) + 2a\,\mathrm{He}}{2\sqrt{2}}$$

✓ Solution by Mathematica

Time used: 4.337 (sec). Leaf size: 132

```
DSolve[y'[x]==y[x]^2-a^2+a*\[Lambda]*Sin[\[Lambda]*x]+a^2*Sin[\[Lambda]*x]^2,y[x],x,IncludeS
```

$$y(x) \to -\frac{ac_1 \cos(\lambda x) \int_1^x e^{-\frac{2a\sin(\lambda K[1])}{\lambda}} dK[1] + a\cos(\lambda x) + c_1 e^{-\frac{2a\sin(\lambda x)}{\lambda}}}{1 + c_1 \int_1^x e^{-\frac{2a\sin(\lambda K[1])}{\lambda}} dK[1]}$$

$$y(x) \to -\frac{e^{-\frac{2a\sin(\lambda x)}{\lambda}}}{\int_1^x e^{-\frac{2a\sin(\lambda K[1])}{\lambda}} dK[1]} - a\cos(\lambda x)$$

9.3 problem 3

Internal problem ID [10511]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = \lambda^2 + c\sin(\lambda x + a)^n \sin(\lambda x + b)^{-n-4}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+lambda^2+c*sin(lambda*x+a)^n*sin(lambda*x+b)^(-n-4),y(x), singsol
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+\[Lambda]^2+c*Sin[\[Lambda]*x+a]^n*Sin[\[Lambda]*x+b]^(-n-4),y[x],x,Inc
```

Not solved

9.4 problem 4

Internal problem ID [10512]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a\sin(\beta x)\, y = ab\sin(\beta x) - b^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 50

```
dsolve(diff(y(x),x)=y(x)^2+a*sin(beta*x)*y(x)+a*b*sin(beta*x)-b^2,y(x), singsol=all)
```

$$y(x) = -b - \frac{\mathrm{e}^{-\frac{a\cos(\beta x)}{\beta} - 2xb}}{\int \mathrm{e}^{-\frac{a\cos(\beta x)}{\beta} - 2xb} dx - c_1}$$

✓ Solution by Mathematica

Time used: 9.066 (sec). Leaf size: 187

```
DSolve[y'[x]==y[x]^2+a*Sin[\[Beta]*x]*y[x]+a*b*Sin[\[Beta]*x]-b^2,y[x],x,IncludeSingularSolu
```

$$\text{Solve}\left[\int_1^x -\frac{e^{-\frac{a\cos(\beta K[1])}{\beta} - 2bK[1]}(-b + a\sin(\beta K[1]) + y(x))}{a\beta(b + y(x))} dK[1] + \int_1^{y(x)} \left(\frac{e^{-2bx - \frac{a\cos(x\beta)}{\beta}}}{a\beta(b + K[2])^2} - \int_1^x \left(\frac{e^{-\frac{a\cos(\beta K[1])}{\beta} - 2bK[1]}(-b + K[2] + a\sin(\beta K[1]))}{a\beta(b + K[2])^2} - \frac{e^{-\frac{a\cos(\beta K[1])}{\beta} - 2bK[1]}}{a\beta(b + K[2])}\right) dK[1]\right) dK[2] = c_1, y(x)\right]$$

9.5 problem 5

Internal problem ID [10513]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a\sin(bx)^m y = a\sin(bx)^m}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+a*sin(b*x)^m*y(x)+a*sin(b*x)^m,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+a*Sin[b*x]^m*y[x]+a*Sin[b*x]^m,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

9.6 problem 6

Internal problem ID [10514]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \lambda \sin(\lambda x)\, y^2 = \lambda \sin(\lambda x)^3}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 63

```
dsolve(diff(y(x),x)=lambda*sin(lambda*x)*y(x)^2+lambda*sin(lambda*x)^3,y(x), singsol=all)
```

$$y(x) = \frac{2c_1 e^{\cos(\lambda x)^2}}{\sqrt{\pi}\,(\mathrm{erfi}(\cos(\lambda x))\, c_1 + 1)} - \frac{\left(\sqrt{\pi}\,\mathrm{erfi}(\cos(\lambda x))\, c_1 + \sqrt{\pi}\right)\cos(\lambda x)}{\sqrt{\pi}\,(\mathrm{erfi}(\cos(\lambda x))\, c_1 + 1)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*Sin[\[Lambda]*x]*y[x]^2+\[Lambda]*Sin[\[Lambda]*x]^3,y[x],x,IncludeS
```

Not solved

9.7 problem 7

Internal problem ID [10515]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{2y' - (\lambda + a - \sin(\lambda x)\, a)\, y^2 = -a + \lambda - \sin(\lambda x)\, a}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 3136

```
dsolve(2*diff(y(x),x)=(lambda+a-a*sin(lambda*x))*y(x)^2+lambda-a-a*sin(lambda*x),y(x), sings
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[2*y'[x]==(\[Lambda]+a-a*Sin[\[Lambda]*x])*y[x]^2+\[Lambda]-a-a*Sin[\[Lambda]*x],y[x],
```

Not solved

9.8 problem 8

Internal problem ID [10516]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \left(\lambda + \sin(\lambda x)^2 a\right) y^2 = -a + \lambda + \sin(\lambda x)^2 a$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 467

```
dsolve(diff(y(x),x)=(lambda+a*sin(lambda*x)^2)*y(x)^2+lambda-a+a*sin(lambda*x)^2,y(x), sings
```

$$y(x) = -\frac{\left(\left(-4\cos(2\lambda x)\sqrt{-1+\cos(2\lambda x)}\,c_1 a\lambda + 4\sqrt{-1+\cos(2\lambda x)}\,c_1 a\lambda + 8\sqrt{-1+\cos(2\lambda x)}\,c_1\lambda^2\right) e^{\frac{a\cos(2\lambda x}{2\lambda}}}{}$$

✓ Solution by Mathematica

Time used: 41.676 (sec). Leaf size: 187

```
DSolve[y'[x]==(\[Lambda]+a*Sin[\[Lambda]*x]^2)*y[x]^2+\[Lambda]-a+a*Sin[\[Lambda]*x]^2,y[x],
```

$$y(x) \to -\frac{2\left(c_1 \cot(\lambda x) \int_1^x e^{-\frac{a \sin^2(\lambda K[1])}{\lambda}} (\lambda \csc^2(\lambda K[1]) + a)\, dK[1] + c_1 \csc^2(\lambda x) e^{-\frac{a \sin^2(\lambda x)}{\lambda}} + \cot(\lambda x)\right)}{2 + 2c_1 \int_1^x e^{-\frac{a \sin^2(\lambda K[1])}{\lambda}} (\lambda \csc^2(\lambda K[1]) + a)\, dK[1]}$$

$$y(x) \to -\frac{\csc^2(\lambda x) e^{-\frac{a \sin^2(\lambda x)}{\lambda}}}{\int_1^x e^{-\frac{a \sin^2(\lambda K[1])}{\lambda}} (\lambda \csc^2(\lambda K[1]) + a)\, dK[1]} - \cot(\lambda x)$$

9.9 problem 9

Internal problem ID [10517]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + (k+1)\, x^k y^2 - a\, x^{k+1} \sin(x)^m\, y = -a \sin(x)^m$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 172

```
dsolve(diff(y(x),x)=-(k+1)*x^k*y(x)^2+a*x^(k+1)*sin(x)^m*y(x)-a*sin(x)^m,y(x), singsol=all)
```

$$y(x) = \frac{\left(\mathrm{e}^{\int \frac{\sin(x)^m x^k a\, x^2 - 2k - 2}{x} dx} x\, x^k + \left(\int x^k \mathrm{e}^{\int \frac{a\, x^{k+2} \sin(x)^m - 2k - 2}{x} dx} dx\right) k + \int x^k \mathrm{e}^{\int \frac{a\, x^{k+2} \sin(x)^m - 2k - 2}{x} dx} dx - c_1\right) x^{-k}}{x \left(\left(\int x^k \mathrm{e}^{\int \frac{a\, x^{k+2} \sin(x)^m - 2k - 2}{x} dx} dx\right) k + \int x^k \mathrm{e}^{\int \frac{a\, x^{k+2} \sin(x)^m - 2k - 2}{x} dx} dx - c_1\right)}$$

✓ Solution by Mathematica

Time used: 16.483 (sec). Leaf size: 248

```
DSolve[y'[x]==-(k+1)*x^k*y[x]^2+a*x^(k+1)*Sin[x]^m*y[x]-a*Sin[x]^m,y[x],x,IncludeSingularSol
```

$$y(x) \rightarrow \frac{x^{-k-1}\left(c_1 x \exp\left(\int_1^x -\frac{-a\sin^m(K[1])K[1]^{k+2}+k+2}{K[1]}dK[1]\right) + c_1(k+1)\int_1^x \exp\left(\int_1^{K[2]} -\frac{-a\sin^m(K[1])K[1]^{k+2}+k+2}{K[1]}dK\right.\right.}{(k+1)\left(1+c_1\int_1^x \exp\left(\int_1^{K[2]} -\frac{-a\sin^m(K[1])K[1]^{k+2}+k+2}{K[1]}dK[1]\right)dK[2]\right)}$$

$$y(x) \rightarrow \frac{x^{-k}\left(\frac{\exp\left(\int_1^x -\frac{-a\sin^m(K[1])K[1]^{k+2}+k+2}{K[1]}dK[1]\right)}{\int_1^x \exp\left(\int_1^{K[2]} -\frac{-a\sin^m(K[1])K[1]^{k+2}+k+2}{K[1]}dK[1]\right)dK[2]} + \frac{k+1}{x}\right)}{k+1}$$

9.10 problem 10

Internal problem ID [10518]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - a\sin(\lambda x+\mu)^k (y - b\,x^n - c)^2 = bn\,x^{-1+n}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 102

```
dsolve(diff(y(x),x)=a*sin(lambda*x+mu)^k*(y(x)-b*x^n-c)^2+b*n*x^(n-1),y(x), singsol=all)
```

$$y(x) = -\frac{\left(-2a\,x^n(\sin(\lambda x)\cos(\mu)+\cos(\lambda x)\sin(\mu))^k b - 2ac(\sin(\lambda x)\cos(\mu)+\cos(\lambda x)\sin(\mu))^k\right)\sin(\lambda x + \mu}{2a}$$
$$+ \frac{1}{c_1 - \left(\int a(\sin(\lambda x)\cos(\mu)+\cos(\lambda x)\sin(\mu))^k dx\right)}$$

✓ Solution by Mathematica

Time used: 5.928 (sec). Leaf size: 93

```
DSolve[y'[x]==a*Sin[\[Lambda]*x+\[Mu]]^k*(y[x]-b*x^n-c)^2+b*n*x^(n-1),y[x],x,IncludeSingular
```

$$y(x) \to \frac{1}{-\frac{a\sqrt{\cos^2(\mu+\lambda x)}\sec(\mu+\lambda x)\sin^{k+1}(\mu+\lambda x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{k+1}{2},\frac{k+3}{2},\sin^2(x\lambda+\mu)\right)}{(k+1)\lambda} + c_1} + bx^n + c$$
$$y(x) \to bx^n + c$$

9.11 problem 11

Internal problem ID [10519]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - a\sin(\lambda x)^m y^2 - ky = a\,b^2x^{2k}\sin(\lambda x)^m}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 36

```
dsolve(x*diff(y(x),x)=a*sin(lambda*x)^m*y(x)^2+k*y(x)+a*b^2*x^(2*k)*sin(lambda*x)^m,y(x), si
```

$$y(x) = -\tan\left(-ba\left(\int \frac{x^k \sin(\lambda x)^m}{x}dx\right) + c_1\right) b\,x^k$$

✓ Solution by Mathematica

Time used: 1.774 (sec). Leaf size: 50

```
DSolve[x*y'[x]==a*Sin[\[Lambda]*x]^m*y[x]^2+k*y[x]+a*b^2*x^(2*k)*Sin[\[Lambda]*x]^m,y[x],x,I
```

$$y(x) \to \sqrt{b^2}x^k \tan\left(\sqrt{b^2}\int_1^x aK[1]^{k-1}\sin^m(\lambda K[1])dK[1] + c_1\right)$$

9.12 problem 12

Internal problem ID [10520]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{(a\sin(x\lambda)+b)\,y'-y^2-c\sin(\mu x)\,y=-d^2+cd\sin(\mu x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 155

```
dsolve((a*sin(lambda*x)+b)*diff(y(x),x)=y(x)^2+c*sin(mu*x)*y(x)-d^2+c*d*sin(mu*x),y(x), sing
```

$$y(x)=-d-\frac{e^{\int \frac{c\sin(\mu x)}{a\sin(\lambda x)+b}dx-\frac{4d\arctan\left(\frac{2b\tan\left(\frac{\lambda x}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{\lambda\sqrt{-a^2+b^2}}}}{\int \frac{e^{\int \frac{c\sin(\mu x)}{a\sin(\lambda x)+b}dx-\frac{4d\arctan\left(\frac{2b\tan\left(\frac{\lambda x}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{\lambda\sqrt{-a^2+b^2}}}}{a\sin(\lambda x)+b}dx-c_1}$$

✓ Solution by Mathematica

Time used: 15.846 (sec). Leaf size: 289

```
DSolve[(a*Sin[\[Lambda]*x]+b)*y'[x]==y[x]^2+c*Sin[\[Mu]*x]*y[x]-d^2+c*d*Sin[\[Mu]*x],y[x],x,
```

$$\text{Solve}\left[\int_1^x -\frac{\exp\left(-\int_1^{K[2]} \frac{2d-c\sin(\mu K[1])}{b+a\sin(\lambda K[1])}dK[1]\right)(-d+c\sin(\mu K[2])+y(x))}{c\mu(b+a\sin(\lambda K[2]))(d+y(x))}dK[2]\right.$$

$$+\int_1^{y(x)}\left(\frac{\exp\left(-\int_1^x \frac{2d-c\sin(\mu K[1])}{b+a\sin(\lambda K[1])}dK[1]\right)}{c\mu(d+K[3])^2}\right.$$

$$-\int_1^x\left(\frac{\exp\left(-\int_1^{K[2]} \frac{2d-c\sin(\mu K[1])}{b+a\sin(\lambda K[1])}dK[1]\right)(-d+K[3]+c\sin(\mu K[2]))}{c\mu(d+K[3])^2(b+a\sin(\lambda K[2]))}-\frac{\exp\left(-\int_1^{K[2]} \frac{2d-c\sin(\mu K[1])}{b+a\sin(\lambda K[1])}dK[1]\right)}{c\mu(d+K[3])(b+a\sin(\lambda K[2]))}\right)$$

9.13 problem 13

Internal problem ID [10521]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_Riccati]

$$\boxed{(\sin(\lambda x) a + b)(y' - y^2) = a \lambda^2 \sin(\lambda x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 650

```
dsolve((a*sin(lambda*x)+b)*(diff(y(x),x)-y(x)^2)-a*lambda^2*sin(lambda*x)=0,y(x), singsol=al
```

$y(x) =$

$$-\frac{\lambda\left(\left(\left(\left(-2\arctan\left(\frac{b\tan\left(\frac{\lambda x}{2}\right)+a}{\sqrt{-a^2+b^2}}\right)\sqrt{-a^2+b^2}\,a\,b^3+a^4b-b^3a^2\right)\sin\left(\frac{\lambda x}{2}\right)^2+\left(2\arctan\left(\frac{b\tan\left(\frac{\lambda x}{2}\right)+a}{\sqrt{-a^2+b^2}}\right)\sqrt{-\right.\right.\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 24.795 (sec). Leaf size: 189

```
DSolve[(a*Sin[\[Lambda]*x]+b)*(y'[x]-y[x]^2)-a*\[Lambda]^2*Sin[\[Lambda]*x]==0,y[x],x,Includ
```

$$y(x) \to \frac{\lambda\left(2ab\cos(\lambda x)\arctan\left(\frac{a+b\tan\left(\frac{\lambda x}{2}\right)}{\sqrt{b^2-a^2}}\right)+\sqrt{b^2-a^2}(-ac_1\lambda(a^2-b^2)\cos(\lambda x)-a\sin(\lambda x)+b)\right)}{-2b(a\sin(\lambda x)+b)\arctan\left(\frac{a+b\tan\left(\frac{\lambda x}{2}\right)}{\sqrt{b^2-a^2}}\right)+\sqrt{b^2-a^2}\left(-a\cos(\lambda x)+c_1\lambda\left(a^2-b^2\right)\left(a\sin(\lambda x)+b\right)\right)}$$

$$y(x) \to -\frac{a\lambda\cos(\lambda x)}{a\sin(\lambda x)+b}$$

10 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.

10.1 problem 14

Internal problem ID [10522]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \alpha y^2 = \beta + \gamma \cos(\lambda x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 94

```
dsolve(diff(y(x),x)=alpha*y(x)^2+beta+gamma*cos(lambda*x),y(x), singsol=all)
```

$$y(x) = -\frac{\lambda\left(c_1 \operatorname{MathieuSPrime}\left(\frac{4\alpha\beta}{\lambda^2}, -\frac{2\gamma\alpha}{\lambda^2}, \frac{\lambda x}{2}\right) + \operatorname{MathieuCPrime}\left(\frac{4\alpha\beta}{\lambda^2}, -\frac{2\gamma\alpha}{\lambda^2}, \frac{\lambda x}{2}\right)\right)}{2\alpha\left(c_1 \operatorname{MathieuS}\left(\frac{4\alpha\beta}{\lambda^2}, -\frac{2\gamma\alpha}{\lambda^2}, \frac{\lambda x}{2}\right) + \operatorname{MathieuC}\left(\frac{4\alpha\beta}{\lambda^2}, -\frac{2\gamma\alpha}{\lambda^2}, \frac{\lambda x}{2}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.577 (sec). Leaf size: 163

```
DSolve[y'[x]==\[Alpha]*y[x]^2+\[Beta]+\[Gamma]*Cos[\[Lambda]*x],y[x],x,IncludeSingularSoluti
```

$$y(x) \to -\frac{\lambda\left(\operatorname{MathieuSPrime}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{\lambda x}{2}\right] + c_1 \operatorname{MathieuCPrime}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{\lambda x}{2}\right]\right)}{2\alpha\left(\operatorname{MathieuS}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{\lambda x}{2}\right] + c_1 \operatorname{MathieuC}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{\lambda x}{2}\right]\right)}$$

$$y(x) \to -\frac{\lambda \operatorname{MathieuCPrime}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{\lambda x}{2}\right]}{2\alpha \operatorname{MathieuC}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{\lambda x}{2}\right]}$$

10.2 problem 15

Internal problem ID [10523]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = -a^2 + \lambda \cos(\lambda x) a + a^2 \cos(\lambda x)^2}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 467

```
dsolve(diff(y(x),x)=y(x)^2-a^2+a*lambda*cos(lambda*x)+a^2*cos(lambda*x)^2,y(x), singsol=all)
```

$$y(x) = \left(\frac{\left(4\,\mathrm{HeunC}\left(\frac{4a}{\lambda}, \frac{1}{2}, -\frac{1}{2}, -\frac{2a}{\lambda}, \frac{8a+3\lambda}{8\lambda}, \frac{\cos(\lambda x)}{2} + \frac{1}{2}\right) c_1 a + 2\,\mathrm{HeunCPrime}\left(\frac{4a}{\lambda}, \frac{1}{2}, -\frac{1}{2}, -\frac{2a}{\lambda}, \frac{8a+3\lambda}{8\lambda}, \frac{\cos(\lambda}{2}\right.\right.}{2\sqrt{2\cos(\lambda x)+2}\,\left(\sqrt{2\cos(\lambda x)+2}\,\mathrm{HeunC}\left(\frac{4a}{\lambda}, \frac{1}{2}, -\frac{1}{2}, -\frac{2a}{\lambda}, \frac{8a+3\lambda}{8\lambda}, \frac{\cos(\lambda x)}{2} + \frac{1}{2}\right) c_1 + \mathrm{HeunC}\left(\frac{4a}{\lambda}, -\frac{1}{2}, -\right.\right.} \right.$$
$$+ \frac{(4c_1 a + 2c_1\lambda)\,\mathrm{HeunC}\left(\frac{4a}{\lambda}, \frac{1}{2}, -\frac{1}{2}, -\frac{2a}{\lambda}, \frac{8a+3\lambda}{8\lambda}, \frac{\cos(\lambda x)}{2} + \frac{1}{2}\right) + 2a\,\mathrm{HeunC}\left(\frac{4a}{\lambda}, -\frac{1}{2}, -\frac{1}{2}, -\frac{2a}{\lambda}, \frac{8a+3\lambda}{8\lambda}, \frac{\cos(\lambda x)}{2}\right.}{2\sqrt{2\cos(\lambda x)+2}\,\left(\sqrt{2\cos(\lambda x)+2}\,\mathrm{Heu}\right.}$$

✓ Solution by Mathematica

Time used: 3.942 (sec). Leaf size: 131

```
DSolve[y'[x]==y[x]^2-a^2+a*\[Lambda]*Cos[\[Lambda]*x]+a^2*Cos[\[Lambda]*x]^2,y[x],x,IncludeS
```

$$y(x) \to \frac{ac_1 \sin(\lambda x) \int_1^x e^{-\frac{2a\cos(\lambda K[1])}{\lambda}} dK[1] + a\sin(\lambda x) + c_1 \left(-e^{-\frac{2a\cos(\lambda x)}{\lambda}}\right)}{1 + c_1 \int_1^x e^{-\frac{2a\cos(\lambda K[1])}{\lambda}} dK[1]}$$

$$y(x) \to a\sin(\lambda x) - \frac{e^{-\frac{2a\cos(\lambda x)}{\lambda}}}{\int_1^x e^{-\frac{2a\cos(\lambda K[1])}{\lambda}} dK[1]}$$

10.3 problem 16

Internal problem ID [10524]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = \lambda^2 + c\cos(\lambda x + a)^n \cos(\lambda x + b)^{-n-4}}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+lambda^2+c*cos(lambda*x+a)^n*cos(lambda*x+b)^(-n-4),y(x), singsol
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+\[Lambda]^2+c*Cos[\[Lambda]*x+a]^n*Cos[\[Lambda]*x+b]^(-n-4),y[x],x,Inc
```

Not solved

10.4 problem 17

Internal problem ID [10525]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a\cos(\beta x)\, y = ab\cos(\beta x) - b^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(diff(y(x),x)=y(x)^2+a*cos(beta*x)*y(x)+a*b*cos(beta*x)-b^2,y(x), singsol=all)
```

$$y(x) = -b - \frac{\mathrm{e}^{\frac{a\sin(\beta x)}{\beta}-2xb}}{\int \mathrm{e}^{\frac{a\sin(\beta x)}{\beta}-2xb}dx - c_1}$$

✓ Solution by Mathematica

Time used: 9.071 (sec). Leaf size: 183

```
DSolve[y'[x]==y[x]^2+a*Cos[\[Beta]*x]*y[x]+a*b*Cos[\[Beta]*x]-b^2,y[x],x,IncludeSingularSolu
```

$$\text{Solve}\left[\int_1^x \frac{e^{\frac{a\sin(\beta K[1])}{\beta}-2bK[1]}(-b+a\cos(\beta K[1])+y(x))}{a\beta(b+y(x))}dK[1] + \int_1^{y(x)}\left(-\int_1^x\left(\frac{e^{\frac{a\sin(\beta K[1])}{\beta}-2bK[1]}}{a\beta(b+K[2])} - \frac{e^{\frac{a\sin(\beta K[1])}{\beta}-2bK[1]}(-b+a\cos(\beta K[1])+K[2])}{a\beta(b+K[2])^2}\right)dK[1] - \frac{e^{\frac{a\sin(x\beta)}{\beta}-2bx}}{a\beta(b+K[2])^2}\right)dK[2] = c_1, y(x)\right]$$

10.5 problem 18

Internal problem ID [10526]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a\cos(bx)^m y = a\cos(bx)^m}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+a*cos(b*x)^m*y(x)+a*cos(b*x)^m,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+a*Cos[b*x]^m*y[x]+a*Cos[b*x]^m,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

10.6 problem 19

Internal problem ID [10527]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \cos(\lambda x)\, y^2 \lambda = \cos(\lambda x)^3 \lambda}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(diff(y(x),x)=lambda*cos(lambda*x)*y(x)^2+lambda*cos(lambda*x)^3,y(x), singsol=all)
```

$$y(x) = \sin(\lambda x) + \frac{2c_1 - 1}{\left(\mathrm{KummerU}\left(1, \frac{3}{2}, -\sin(\lambda x)^2\right) c_1 + \mathrm{KummerM}\left(1, \frac{3}{2}, -\sin(\lambda x)^2\right)\right) \sin(\lambda x)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*Cos[\[Lambda]*x]*y[x]^2+\[Lambda]*Cos[\[Lambda]*x]^3,y[x],x,IncludeS
```

Not solved

10.7 problem 20

Internal problem ID [10528]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$2y' - (\lambda + a - a\cos(\lambda x))\, y^2 = -a + \lambda - a\cos(\lambda x)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 307

```
dsolve(2*diff(y(x),x)=(lambda+a-a*cos(lambda*x))*y(x)^2+lambda-a-a*cos(lambda*x),y(x), sings
```

$$y(x) = \left(-\frac{2c_1\lambda\, \mathrm{e}^{\frac{\cos(\lambda x)a}{\lambda}}}{(\cos(\lambda x)-1)^{\frac{3}{2}} \left(\left(\int -\frac{(-\lambda-a+\cos(\lambda x)a)\mathrm{e}^{\frac{\cos(\lambda x)a}{\lambda}}\lambda\sin(\lambda x)}{(\cos(\lambda x)-1)^{\frac{3}{2}}\sqrt{\cos(\lambda x)+1}}dx \right) c_1 + 1 \right) \sqrt{\cos(\lambda x)+1}} + \frac{\left(\left(\int -\frac{(-\lambda-a+\cos(\lambda x)a)\mathrm{e}^{\frac{\cos(\lambda x)a}{\lambda}}\lambda\sin(\lambda x)}{(\cos(\lambda x)-1)^{\frac{3}{2}}\sqrt{\cos(\lambda x)+1}}dx \right)\sqrt{\cos(\lambda x)+1}\, c_1 + \sqrt{\cos(\lambda x)+1} \right)\cos(\lambda x) - \left(\int -\frac{(-\lambda-a+c}{(\cos(}\right.}{\left(\left(\int -\frac{(-\lambda-a+\cos(\lambda x)a)\mathrm{e}^{\frac{\cos(\lambda x)a}{\lambda}}\lambda\sin(\lambda x)}{(\cos(\lambda x)-1)^{\frac{3}{2}}\sqrt{\cos(\lambda x)+1}}dx \right) c_1 + 1 \right)\sqrt{\cos(\lambda x) -}$$

✓ Solution by Mathematica

Time used: 34.139 (sec). Leaf size: 234

```
DSolve[2*y'[x]==(\[Lambda]+a-a*Cos[\[Lambda]*x])*y[x]^2+\[Lambda]-a-a*Cos[\[Lambda]*x],y[x],
```

$$y(x) \to -\frac{2\left(c_1 \cot\left(\frac{\lambda x}{2}\right) \int_1^x e^{-\frac{2a\sin^2\left(\frac{1}{2}\lambda K[1]\right)}{\lambda}} \left(\lambda \csc^2\left(\frac{1}{2}\lambda K[1]\right) + 2a\right) dK[1] + 2c_1 \csc^2\left(\frac{\lambda x}{2}\right) e^{-\frac{2a\sin^2\left(\frac{\lambda x}{2}\right)}{\lambda}} + \cot\left(\frac{\lambda x}{2}\right)\right)}{2 + 2c_1 \int_1^x e^{-\frac{2a\sin^2\left(\frac{1}{2}\lambda K[1]\right)}{\lambda}} \left(\lambda \csc^2\left(\frac{1}{2}\lambda K[1]\right) + 2a\right) dK[1]}$$

$$y(x) \to \frac{1}{2}\csc^2\left(\frac{\lambda x}{2}\right)\left(-\frac{4e^{-\frac{2a\sin^2\left(\frac{\lambda x}{2}\right)}{\lambda}}}{\int_1^x e^{-\frac{2a\sin^2\left(\frac{1}{2}\lambda K[1]\right)}{\lambda}} \left(\lambda \csc^2\left(\frac{1}{2}\lambda K[1]\right) + 2a\right) dK[1]} - \sin(\lambda x)\right)$$

10.8 problem 21

Internal problem ID [10529]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_Riccati]

$$y' - \left(\lambda + \cos(\lambda x)^2 a\right) y^2 = -a + \lambda + \cos(\lambda x)^2 a$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 550

```
dsolve(diff(y(x),x)=(lambda+a*cos(lambda*x)^2)*y(x)^2+lambda-a+a*cos(lambda*x)^2,y(x), sings
```

$$y(x) = \left(\frac{\left(4\cos(2\lambda x)\sqrt{1+\cos(2\lambda x)}\,c_1 a\lambda + 4\sqrt{1+\cos(2\lambda x)}\,c_1 a\lambda + 8\sqrt{1+\cos(2\lambda x)}\,c_1\lambda^2\right) e^{-\frac{a\cos(2\lambda x)}{2\lambda}}}{2\left(1+\cos(2\lambda x)\right)^2\sqrt{-1+\cos(2\lambda x)}\,\left(\lambda + a\cos(\lambda x)^2\right)\left(\left(\left(\int -\frac{2(a\cos(2\lambda x)+a+2\lambda)e^{-\frac{a\cos(2\lambda x)}{2\lambda}}\sin(2\lambda x)\lambda}{\sqrt{-1+\cos(2\lambda x)}\,(1+\cos(2\lambda x))^{\frac{3}{2}}}dx\right) c_1\right.\right.} + \frac{\left(\left(\int -\frac{2(a\cos(2\lambda x)+a+2\lambda)e^{-\frac{a\cos(2\lambda x)}{2\lambda}}\sin(2\lambda x)\lambda}{\sqrt{-1+\cos(2\lambda x)}\,(1+\cos(2\lambda x))^{\frac{3}{2}}}dx\right)\sqrt{-1+\cos(2\lambda x)}\,c_1 a + a\sqrt{-1+\cos(2\lambda x)}\right)\cos(2\lambda x)^2}{}\right.$$

✓ Solution by Mathematica

Time used: 36.333 (sec). Leaf size: 263

```
DSolve[y'[x]==(\[Lambda]+a*Cos[\[Lambda]*x]^2)*y[x]^2+\[Lambda]-a+a*Cos[\[Lambda]*x]^2,y[x],
```

$$y(x) \to \frac{2\left(c_1\tan(\lambda x)\int_1^x e^{-\frac{a\cos^2(\lambda K[1])}{\lambda}}(\lambda\sec^2(\lambda K[1])+a)\,dK[1]+c_1\sec^2(\lambda x)\left(-e^{-\frac{a\cos^2(\lambda x)}{\lambda}}\right)+\tan(\lambda x)\right)}{2+2c_1\int_1^x e^{-\frac{a\cos^2(\lambda K[1])}{\lambda}}(\lambda\sec^2(\lambda K[1])+a)\,dK[1]}$$

$$y(x) \to \frac{1}{2}\sec^2(\lambda x)\left(\sin(2\lambda x)-\frac{2e^{-\frac{a\cos^2(\lambda x)}{\lambda}}}{\int_1^x e^{-\frac{a\cos^2(\lambda K[1])}{\lambda}}(\lambda\sec^2(\lambda K[1])+a)\,dK[1]}\right)$$

$$y(x) \to \frac{1}{2}\sec^2(\lambda x)\left(\sin(2\lambda x)-\frac{2e^{-\frac{a\cos^2(\lambda x)}{\lambda}}}{\int_1^x e^{-\frac{a\cos^2(\lambda K[1])}{\lambda}}(\lambda\sec^2(\lambda K[1])+a)\,dK[1]}\right)$$

10.9 problem 22

Internal problem ID [10530]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + (k+1)\, x^k y^2 - a\, x^{k+1} \cos(x)^m\, y = -a \cos(x)^m$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 168

```
dsolve(diff(y(x),x)=-(k+1)*x^k*y(x)^2+a*x^(k+1)*cos(x)^m*y(x)-a*cos(x)^m,y(x), singsol=all)
```

$$y(x) = \frac{\left(e^{\int \frac{x^k \cos(x)^m a\, x^2 - 2k - 2}{x} dx} x^k x + \left(\int x^k e^{\int \frac{a\, x^{k+2} \cos(x)^m - 2k - 2}{x} dx} dx\right) k + \int x^k e^{\int \frac{a\, x^{k+2} \cos(x)^m - 2k - 2}{x} dx} dx + c_1\right) x^{-k}}{x \left(\left(\int x^k e^{\int \frac{a\, x^{k+2} \cos(x)^m - 2k - 2}{x} dx} dx\right) k + \int x^k e^{\int \frac{a\, x^{k+2} \cos(x)^m - 2k - 2}{x} dx} dx + c_1\right)}$$

✓ Solution by Mathematica

Time used: 20.002 (sec). Leaf size: 248

```
DSolve[y'[x]==-(k+1)*x^k*y[x]^2+a*x^(k+1)*Cos[x]^m*y[x]-a*Cos[x]^m,y[x],x,IncludeSingularSol
```

$$y(x) \to \frac{x^{-k-1}\left(c_1 x \exp\left(\int_1^x -\frac{-a\cos^m(K[1])K[1]^{k+2}+k+2}{K[1]}dK[1]\right) + c_1(k+1)\int_1^x \exp\left(\int_1^{K[2]} -\frac{-a\cos^m(K[1])K[1]^{k+2}+k+2}{K[1]}dK\right.\right.}{(k+1)\left(1+c_1\int_1^x \exp\left(\int_1^{K[2]} -\frac{-a\cos^m(K[1])K[1]^{k+2}+k+2}{K[1]}dK[1]\right)dK[2]\right)}$$

$$y(x) \to \frac{x^{-k}\left(\frac{\exp\left(\int_1^x -\frac{-a\cos^m(K[1])K[1]^{k+2}+k+2}{K[1]}dK[1]\right)}{\int_1^x \exp\left(\int_1^{K[2]} -\frac{-a\cos^m(K[1])K[1]^{k+2}+k+2}{K[1]}dK[1]\right)dK[2]} + \frac{k+1}{x}\right)}{k+1}$$

10.10 problem 23

Internal problem ID [10531]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - a\cos(\lambda x + \mu)^k (y - b\,x^n - c)^2 = bn\,x^{-1+n}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 106

```
dsolve(diff(y(x),x)=a*cos(lambda*x+mu)^k*(y(x)-b*x^n-c)^2+b*n*x^(n-1),y(x), singsol=all)
```

$$y(x) = -\frac{\left(-2a\,x^n(\cos(\lambda x)\cos(\mu) - \sin(\lambda x)\sin(\mu))^k b - 2ac(\cos(\lambda x)\cos(\mu) - \sin(\lambda x)\sin(\mu))^k\right)\cos(\lambda x + \mu}{2a} + \frac{1}{c_1 - \left(\int a(\cos(\lambda x)\cos(\mu) - \sin(\lambda x)\sin(\mu))^k dx\right)}$$

✓ Solution by Mathematica

Time used: 6.016 (sec). Leaf size: 92

```
DSolve[y'[x]==a*Cos[\[Lambda]*x+\[Mu]]^k*(y[x]-b*x^n-c)^2+b*n*x^(n-1),y[x],x,IncludeSingular
```

$$y(x) \to \frac{1}{\frac{a\sqrt{\sin^2(\mu+\lambda x)}\csc(\mu+\lambda x)\cos^{k+1}(\mu+\lambda x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{k+1}{2},\frac{k+3}{2},\cos^2(x\lambda+\mu)\right)}{(k+1)\lambda} + c_1} + bx^n + c$$

$$y(x) \to bx^n + c$$

10.11 problem 24

Internal problem ID [10532]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - a\cos(\lambda x)^m y^2 - ky = a\,b^2 x^{2k}\cos(\lambda x)^m}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 36

```
dsolve(x*diff(y(x),x)=a*cos(lambda*x)^m*y(x)^2+k*y(x)+a*b^2*x^(2*k)*cos(lambda*x)^m,y(x), si
```

$$y(x) = -\tan\left(-ba\left(\int \frac{\cos(\lambda x)^m x^k}{x}dx\right) + c_1\right) b\,x^k$$

✓ Solution by Mathematica

Time used: 1.628 (sec). Leaf size: 50

```
DSolve[x*y'[x]==a*Cos[\[Lambda]*x]^m*y[x]^2+k*y[x]+a*b^2*x^(2*k)*Cos[\[Lambda]*x]^m,y[x],x,I
```

$$y(x) \to \sqrt{b^2}x^k \tan\left(\sqrt{b^2}\int_1^x a\cos^m(\lambda K[1])K[1]^{k-1}dK[1] + c_1\right)$$

10.12 problem 25

Internal problem ID [10533]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$(a\cos(x\lambda)+b)\,y'-y^2-c\cos(\mu x)\,y=-d^2+cd\cos(\mu x)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 149

```
dsolve((a*cos(lambda*x)+b)*diff(y(x),x)=y(x)^2+c*cos(mu*x)*y(x)-d^2+c*d*cos(mu*x),y(x), sing
```

$$y(x)=-d-\frac{e^{\int \frac{c\cos(\mu x)}{\cos(\lambda x)a+b}dx-\frac{4d\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{\lambda x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\lambda\sqrt{(a-b)(a+b)}}}}{\int \frac{e^{\int \frac{c\cos(\mu x)}{\cos(\lambda x)a+b}dx-\frac{4d\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{\lambda x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\lambda\sqrt{(a-b)(a+b)}}}}{\cos(\lambda x)a+b}dx-c_1}$$

✓ Solution by Mathematica

Time used: 12.31 (sec). Leaf size: 289

```
DSolve[(a*Cos[\[Lambda]*x]+b)*y'[x]==y[x]^2+c*Cos[\[Mu]*x]*y[x]-d^2+c*d*Cos[\[Mu]*x],y[x],x,
```

$$
\text{Solve}\left[\int_1^x \frac{\exp\left(-\int_1^{K[2]} \frac{2d-c\cos(\mu K[1])}{b+a\cos(\lambda K[1])}dK[1]\right)(-d+c\cos(\mu K[2])+y(x))}{c\mu(b+a\cos(\lambda K[2]))(d+y(x))}dK[2]\right.
$$

$$
+\int_1^{y(x)}\left(-\int_1^x\left(\frac{\exp\left(-\int_1^{K[2]} \frac{2d-c\cos(\mu K[1])}{b+a\cos(\lambda K[1])}dK[1]\right)}{c\mu(b+a\cos(\lambda K[2]))(d+K[3])}-\frac{\exp\left(-\int_1^{K[2]} \frac{2d-c\cos(\mu K[1])}{b+a\cos(\lambda K[1])}dK[1]\right)(-d+c\cos(\mu K[2]}{c\mu(b+a\cos(\lambda K[2]))(d+K[3])^2}\right.\right.
$$

$$
\left.\left.-\frac{\exp\left(-\int_1^x \frac{2d-c\cos(\mu K[1])}{b+a\cos(\lambda K[1])}dK[1]\right)}{c\mu(d+K[3])^2}\right)dK[3]=c_1,y(x)\right]
$$

10.13 problem 26

Internal problem ID [10534]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{(a\cos(\lambda x)+b)\left(y'-y^2\right)=a\,\lambda^2\cos(\lambda x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 539

```
dsolve((a*cos(lambda*x)+b)*(diff(y(x),x)-y(x)^2)-a*lambda^2*cos(lambda*x)=0,y(x), singsol=al
```

$$y(x) = \frac{\lambda\left(\left(\left(a^4-b\,a^3-b^2a^2+a\,b^3\right)\sin\left(\frac{\lambda x}{2}\right)^2+\left(-4\sqrt{a^2-b^2}\,a^2b+4\sqrt{a^2-b^2}\,a\,b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{\lambda x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)\cos\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 7.903 (sec). Leaf size: 202

```
DSolve[(a*Cos[\[Lambda]*x]+b)*(y'[x]-y[x]^2)-a*\[Lambda]^2*Cos[\[Lambda]*x]==0,y[x],x,Includ
```

$$y(x) \to -\frac{\lambda\left(-2ab\sin(\lambda x)\text{arctanh}\left(\frac{(b-a)\tan\left(\frac{\lambda x}{2}\right)}{\sqrt{a^2-b^2}}\right)+\sqrt{a^2-b^2}(-ac_1\lambda(a^2-b^2)\sin(\lambda x)+a\cos(\lambda x)-b)\right)}{2b(a\cos(\lambda x)+b)\text{arctanh}\left(\frac{(b-a)\tan\left(\frac{\lambda x}{2}\right)}{\sqrt{a^2-b^2}}\right)+\sqrt{a^2-b^2}\left(bc_1\lambda\left(a^2-b^2\right)+ac_1\lambda\left(a^2-b^2\right)\cos(\lambda x)+a\sin(\right.}$$

$$y(x) \to \frac{a\lambda\sin(\lambda x)}{a\cos(\lambda x)+b}$$

11 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.

11.1 problem 27

Internal problem ID [10535]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = \lambda a + a(\lambda - a)\tan(\lambda x)^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 380

```
dsolve(diff(y(x),x)=y(x)^2+a*lambda+a*(lambda-a)*tan(lambda*x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\left(\frac{c_1 \lambda \operatorname{LegendreQ}\left(\frac{2a+\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \sqrt{-\cos(\lambda x)^2+1}\right)}{\sqrt{-\cos(\lambda x)^2+1}\left(\operatorname{LegendreQ}\left(\frac{2a-\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \sqrt{-\cos(\lambda x)^2+1}\right)c_1 + \operatorname{LegendreP}\left(\frac{2a-\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \sqrt{-\cos(\lambda x)^2+1}\right)\right)} + \frac{-\operatorname{LegendreQ}\left(\frac{2a-\lambda}{2\lambda}, \frac{2a}{2}\right.}{}\right.}{}$$

✓ Solution by Mathematica

Time used: 2.982 (sec). Leaf size: 259

```
DSolve[y'[x]==y[x]^2+a*\[Lambda]+a*(\[Lambda]-a)*Tan[\[Lambda]*x]^2,y[x],x,IncludeSingularSo
```

$$y(x) \to \frac{2\left(ac_1 \sin^2(\lambda x)\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}-\frac{a}{\lambda}, \frac{3}{2}-\frac{a}{\lambda}, \cos^2(x\lambda)\right) + (2a-\lambda)\sqrt{\sin^2(\lambda x)}\left(a\sin(\lambda x)\cos^{\frac{2a}{\lambda}-1}\right.\right.}{2(2a-\lambda)\sqrt{\sin^2(\lambda x)}\cos^{\frac{2a}{\lambda}}(\lambda x) + c_1\sin(2\lambda x)\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}-\frac{a}{\lambda}, \frac{3}{2}-\frac{a}{\lambda}, \cos^2(x\lambda)\right)}$$

$$y(x) \to \frac{\tan(\lambda x)\left(a\sqrt{\sin^2(\lambda x)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}-\frac{a}{\lambda}, \frac{3}{2}-\frac{a}{\lambda}, \cos^2(x\lambda)\right) - 2a + \lambda\right)}{\sqrt{\sin^2(\lambda x)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}-\frac{a}{\lambda}, \frac{3}{2}-\frac{a}{\lambda}, \cos^2(x\lambda)\right)}$$

11.2 problem 28

Internal problem ID [10536]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = 3\lambda a + \lambda^2 + a(\lambda - a)\tan(\lambda x)^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 320

```
dsolve(diff(y(x),x)=y(x)^2+lambda^2+3*a*lambda+a*(lambda-a)*tan(lambda*x)^2,y(x), singsol=al
```

$$y(x) = \frac{\left(\left(-\sqrt{-\cos(\lambda x)^2+1}\, c_1 a - \sqrt{-\cos(\lambda x)^2+1}\, c_1 \lambda\right) \text{LegendreQ}\left(\frac{2a+\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \sqrt{-\cos(\lambda x)^2+1}\right) + 2\,\text{L}}{\sqrt{-\cos(\lambda x)}}$$

✓ Solution by Mathematica

Time used: 76.241 (sec). Leaf size: 319

```
DSolve[y'[x]==y[x]^2+\[Lambda]^2+3*a*\[Lambda]+a*(\[Lambda]-a)*Tan[\[Lambda]*x]^2,y[x],x,Inc
```

$$y(x) \to -\frac{\sin^{-\frac{a+\lambda}{\lambda}}(2\lambda x)e^{-\frac{a\operatorname{arctanh}(\cos(2\lambda x))}{\lambda}}\left(c_1 \sin^{\frac{a}{\lambda}}(2\lambda x)((a+\lambda)\cos(2\lambda x)-a+\lambda)e^{\frac{a\operatorname{arctanh}(\cos(2\lambda x))}{\lambda}}\int_1^x e^{-\frac{(a-\lambda)\operatorname{arctanh}}{\lambda}}\right.}{1+c_1\int_1^x e^{-\frac{(a-\lambda)\operatorname{arctanh}(\mathrm{co}}{\lambda}}}$$

$$y(x) \to \csc(2\lambda x)\left(-\frac{\sin^{-\frac{a}{\lambda}}(2\lambda x)e^{-\frac{(a-\lambda)\operatorname{arctanh}(\cos(2\lambda x))}{\lambda}}}{\int_1^x e^{-\frac{(a-\lambda)\operatorname{arctanh}(\cos(2\lambda K[1]))}{\lambda}}\sin^{-\frac{a+\lambda}{\lambda}}(2\lambda K[1])dK[1]}-(a+\lambda)\cos(2\lambda x)+a-\lambda\right)$$

11.3 problem 29

Internal problem ID [10537]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - ay^2 - b\tan(x)\, y = c}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 359

```
dsolve(diff(y(x),x)=a*y(x)^2+b*tan(x)*y(x)+c,y(x), singsol=all)
```

$$y(x) = \frac{\left((-c_1 b + c_1)\,\mathrm{LegendreQ}\left(\frac{\sqrt{4ac+b^2}}{2} - \frac{1}{2}, \frac{b}{2} - \frac{1}{2}, \sin(x)\right) + (1-b)\,\mathrm{LegendreP}\left(\frac{\sqrt{4ac+b^2}}{2} - \frac{1}{2}, \frac{b}{2} - \frac{1}{2}, \sin(x)\right)\right.}{}$$

✓ Solution by Mathematica

Time used: 2.211 (sec). Leaf size: 608

```
DSolve[y'[x]==a*y[x]^2+b*Tan[x]*y[x]+c,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sin(x)\left(\left(-b^3+3b^2+b-3\right)\text{Hypergeometric2F1}\left(\frac{1}{4}\left(-b-\sqrt{b^2+4ac}+2\right),\frac{1}{4}\left(-b+\sqrt{b^2+4ac}+2\right),\frac{3-b}{2}\right.\right.}{a(b-3)(b+1)\left(\cos(x)\,\text{Hyperg}\right.}$$

$$y(x) \to -\frac{c\sin(x)\cos(x)\,\text{Hypergeometric2F1}\left(\frac{1}{4}\left(b-\sqrt{b^2+4ac}+4\right),\frac{1}{4}\left(b+\sqrt{b^2+4ac}+4\right),\frac{b+3}{2},\cos^2(x)\right)}{(b+1)\,\text{Hypergeometric2F1}\left(\frac{1}{4}\left(b-\sqrt{b^2+4ac}\right),\frac{1}{4}\left(b+\sqrt{b^2+4ac}\right),\frac{b+1}{2},\cos^2(x)\right)}$$

$$y(x) \to -\frac{c\sin(x)\cos(x)\,\text{Hypergeometric2F1}\left(\frac{1}{4}\left(b-\sqrt{b^2+4ac}+4\right),\frac{1}{4}\left(b+\sqrt{b^2+4ac}+4\right),\frac{b+3}{2},\cos^2(x)\right)}{(b+1)\,\text{Hypergeometric2F1}\left(\frac{1}{4}\left(b-\sqrt{b^2+4ac}\right),\frac{1}{4}\left(b+\sqrt{b^2+4ac}\right),\frac{b+1}{2},\cos^2(x)\right)}$$

11.4 problem 30

Internal problem ID [10538]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - ay^2 - 2ab\tan(x)\,y = b(ba-1)\tan(x)^2$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 60

```
dsolve(diff(y(x),x)=a*y(x)^2+2*a*b*tan(x)*y(x)+b*(a*b-1)*tan(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{-b\tan(x)\,a - i\sqrt{b}\,\sqrt{a} + \frac{e^{-2i\sqrt{b}\,\sqrt{a}\,x}}{c_1 - \frac{i e^{-2i\sqrt{b}\,\sqrt{a}\,x}}{2\sqrt{b}\,\sqrt{a}}}}{a}$$

✓ Solution by Mathematica

Time used: 12.833 (sec). Leaf size: 37

```
DSolve[y'[x]==a*y[x]^2+2*a*b*Tan[x]*y[x]+b*(a*b-1)*Tan[x]^2,y[x],x,IncludeSingularSolutions
```

$$y(x) \to -b\tan(x) + \sqrt{\frac{b}{a}}\tan\left(ax\sqrt{\frac{b}{a}} + c_1\right)$$

11.5 problem 31

Internal problem ID [10539]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - a\tan(\beta x)\, y = ab\tan(\beta x) - b^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 61

```
dsolve(diff(y(x),x)=y(x)^2+a*tan(beta*x)*y(x)+a*b*tan(beta*x)-b^2,y(x), singsol=all)
```

$$y(x) = -b - \frac{\left(1+\tan(\beta x)^2\right)^{\frac{a}{2\beta}} \mathrm{e}^{-2xb}}{\int \left(1+\tan(\beta x)^2\right)^{\frac{a}{2\beta}} \mathrm{e}^{-2xb} dx - c_1}$$

✓ Solution by Mathematica

Time used: 25.611 (sec). Leaf size: 408

```
DSolve[y'[x]==y[x]^2+a*Tan[\[Beta]*x]*y[x]+a*b*Tan[\[Beta]*x]-b^2,y[x],x,IncludeSingularSolu
```

$$y(x) \to \frac{2^{-\frac{a}{\beta}} \cos^{-\frac{a}{\beta}}(\beta x) \left(ib(a+2ib+2\beta)\,\text{Hypergeometric2F1}\left(1, -\frac{a-2ib}{2\beta}, \frac{a+2ib+2\beta}{2\beta}, -e^{2ix\beta}\right) (\sin(2\beta x)\csc(\beta x))^{a}\right.}{(a+2ib)\left(a\beta c_1 e^{2bx}(a+2ib+2\beta)\cos^{\frac{a}{\beta}}(\beta x) - ie^{2i\beta x}\,\text{Hyper}\right.}$$

$$y(x) \to -b$$

11.6 problem 32

Internal problem ID [10540]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - ax\tan(bx)^m y = a\tan(bx)^m}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)=y(x)^2+a*x*tan(b*x)^m*y(x)+a*tan(b*x)^m,y(x), singsol=all)
```

$$y(x) = -\frac{e^{\int \frac{a\tan(xb)^m x^2-2}{x}dx} x + \int e^{\int \frac{a\tan(xb)^m x^2-2}{x}dx}dx - c_1}{x\left(-c_1 + \int e^{\int \frac{a\tan(xb)^m x^2-2}{x}dx}dx\right)}$$

✓ Solution by Mathematica

Time used: 8.199 (sec). Leaf size: 126

```
DSolve[y'[x]==y[x]^2+a*x*Tan[b*x]^m*y[x]+a*Tan[b*x]^m,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to$$
$$-\frac{\exp\left(-\int_1^x -aK[1]\tan^m(bK[1])dK[1]\right) + x\int_1^x \frac{\exp\left(-\int_1^{K[2]} -aK[1]\tan^m(bK[1])dK[1]\right)}{K[2]^2}dK[2] + c_1 x}{x^2\left(\int_1^x \frac{\exp\left(-\int_1^{K[2]} -aK[1]\tan^m(bK[1])dK[1]\right)}{K[2]^2}dK[2] + c_1\right)}$$

$$y(x) \to -\frac{1}{x}$$

11.7 problem 33

Internal problem ID [10541]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + (k+1)\,x^k y^2 - a\,x^{k+1}\tan(x)^m\,y = -a\tan(x)^m$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 172

```
dsolve(diff(y(x),x)=-(k+1)*x^k*y(x)^2+a*x^(k+1)*tan(x)^m*y(x)-a*tan(x)^m,y(x), singsol=all)
```

$$y(x) = \frac{\left(\mathrm{e}^{\int \frac{x^k \tan(x)^m a\,x^2-2k-2}{x}dx} x\,x^k + \left(\int x^k \mathrm{e}^{\int \frac{a\,x^{k+2}\tan(x)^m-2k-2}{x}dx}dx\right)k + \int x^k \mathrm{e}^{\int \frac{a\,x^{k+2}\tan(x)^m-2k-2}{x}dx}dx - c_1\right)x^{-k}}{x\left(\left(\int x^k \mathrm{e}^{\int \frac{a\,x^{k+2}\tan(x)^m-2k-2}{x}dx}dx\right)k + \int x^k \mathrm{e}^{\int \frac{a\,x^{k+2}\tan(x)^m-2k-2}{x}dx}dx - c_1\right)}$$

✓ Solution by Mathematica

Time used: 19.083 (sec). Leaf size: 248

```
DSolve[y'[x]==-(k+1)*x^k*y[x]^2+a*x^(k+1)*Tan[x]^m*y[x]-a*Tan[x]^m,y[x],x,IncludeSingularSol
```

$$y(x) \to \frac{x^{-k-1}\left(c_1 x \exp\left(\int_1^x -\frac{-a\tan^m(K[1])K[1]^{k+2}+k+2}{K[1]}dK[1]\right) + c_1(k+1)\int_1^x \exp\left(\int_1^{K[2]} -\frac{-a\tan^m(K[1])K[1]^{k+2}+k+2}{K[1]}d\right.\right.}{(k+1)\left(1 + c_1\int_1^x \exp\left(\int_1^{K[2]} -\frac{-a\tan^m(K[1])K[1]^{k+2}+k+2}{K[1]}dK[1]\right)dK[2]\right)}$$

$$y(x) \to \frac{x^{-k}\left(\frac{\exp\left(\int_1^x -\frac{-a\tan^m(K[1])K[1]^{k+2}+k+2}{K[1]}dK[1]\right)}{\int_1^x \exp\left(\int_1^{K[2]} -\frac{-a\tan^m(K[1])K[1]^{k+2}+k+2}{K[1]}dK[1]\right)dK[2]} + \frac{k+1}{x}\right)}{k+1}$$

11.8 problem 34

Internal problem ID [10542]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a \tan(\lambda x)^n y^2 = -a b^2 \tan(\lambda x)^{2+n} + b\lambda \tan(\lambda x)^2 + b\lambda$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*tan(lambda*x)^n*y(x)^2-a*b^2*tan(lambda*x)^(n+2)+b*lambda*tan(lambda*x
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*Tan[\[Lambda]*x]^n*y[x]^2-a*b^2*Tan[\[Lambda]*x]^(n+2)+b*\[Lambda]*Tan[\[Lam
```

Not solved

11.9 problem 35

Internal problem ID [10543]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - a\tan(x\lambda+\mu)^k (y - b\,x^n - c)^2 = bn\,x^{-1+n}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 134

```
dsolve(diff(y(x),x)=a*tan(lambda*x+mu)^k*(y(x)-b*x^n-c)^2+b*n*x^(n-1),y(x), singsol=all)
```

$$y(x) = -\frac{\left(-2a\,x^n\left(\frac{-\tan(\mu)-\tan(\lambda x)}{\tan(\mu)\tan(\lambda x)-1}\right)^k b - 2ac\left(\frac{-\tan(\mu)-\tan(\lambda x)}{\tan(\mu)\tan(\lambda x)-1}\right)^k\right)\left(-\frac{\tan(\mu)+\tan(\lambda x)}{\tan(\mu)\tan(\lambda x)-1}\right)^{-k}}{2a} + \frac{1}{c_1 - \left(\int a\left(\frac{-\tan(\mu)-\tan(\lambda x)}{\tan(\mu)\tan(\lambda x)-1}\right)^k dx\right)}$$

✓ Solution by Mathematica

Time used: 6.024 (sec). Leaf size: 75

```
DSolve[y'[x]==a*Tan[\[Lambda]*x+mu]^k*(y[x]-b*x^n-c)^2+b*n*x^(n-1),y[x],x,IncludeSingularSol
```

$$y(x) \to \frac{1}{-\frac{a\tan^{k+1}(\mu+\lambda x)\,\text{Hypergeometric2F1}\left(1,\frac{k+1}{2},\frac{k+3}{2},-\tan^2(\mu+x\lambda)\right)}{(k+1)\lambda} + c_1} + bx^n + c$$

$$y(x) \to bx^n + c$$

11.10 problem 36

Internal problem ID [10544]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - a\tan(\lambda x)^m y^2 - ky = a\,b^2x^{2k}\tan(\lambda x)^m$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 36

```
dsolve(x*diff(y(x),x)=a*tan(lambda*x)^m*y(x)^2+k*y(x)+a*b^2*x^(2*k)*tan(lambda*x)^m,y(x), si
```

$$y(x) = -\tan\left(-ba\left(\int \frac{x^k\tan(\lambda x)^m}{x}dx\right)+c_1\right)b\,x^k$$

✓ Solution by Mathematica

Time used: 1.817 (sec). Leaf size: 50

```
DSolve[x*y'[x]==a*Tan[\[Lambda]*x]^m*y[x]^2+k*y[x]+a*b^2*x^(2*k)*Tan[\[Lambda]*x]^m,y[x],x,I
```

$$y(x) \to \sqrt{b^2}x^k\tan\left(\sqrt{b^2}\int_1^x aK[1]^{k-1}\tan^m(\lambda K[1])dK[1]+c_1\right)$$

11.11 problem 37

Internal problem ID [10545]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$
(a\tan(x\lambda)+b)\,y'-y^2-k\tan(\mu x)\,y=-d^2+kd\tan(\mu x)
$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 213

```
dsolve((a*tan(lambda*x)+b)*diff(y(x),x)=y(x)^2+k*tan(mu*x)*y(x)-d^2+k*d*tan(mu*x),y(x), sing
```

$$
y(x) = -d-\frac{e^{\int \frac{\tan(\mu x)k}{a\tan(\lambda x)+b}dx}(a\tan(\lambda x)+b)^{-\frac{2ad}{\lambda(a^2+b^2)}}\left(1+\tan(\lambda x)^2\right)^{\frac{ad}{\lambda(a^2+b^2)}}e^{-\frac{2db\arctan(\tan(\lambda x))}{\lambda(a^2+b^2)}}}{\int \frac{e^{\int \frac{\tan(\mu x)k}{a\tan(\lambda x)+b}dx}(a\tan(\lambda x)+b)^{-\frac{2ad}{\lambda(a^2+b^2)}}\left(1+\tan(\lambda x)^2\right)^{\frac{ad}{\lambda(a^2+b^2)}}e^{-\frac{2db\arctan(\tan(\lambda x))}{\lambda(a^2+b^2)}}}{a\tan(\lambda x)+b}dx-c_1}
$$

✓ Solution by Mathematica

Time used: 130.719 (sec). Leaf size: 800

```
DSolve[(a*Tan[\[Lambda]*x]+b)*y'[x]==y[x]^2+k*Tan[\[Mu]*x]*y[x]-d^2+k*d*Tan[\[Mu]*x],y[x],x,
```

$$\text{Solve}\left[\int_1^x \frac{e^{-\int_1^{K[2]} \frac{\sec(\mu K[1])(2d\cos(\lambda K[1]-\mu K[1])+2d\cos(\lambda K[1]+\mu K[1])+k\sin(\lambda K[1]-\mu K[1])-k\sin(\lambda K[1]+\mu K[1]))}{2(b\cos(\lambda K[1])+a\sin(\lambda K[1]))}dK[1]}(d\cos(\lambda K[2]-\mu K}{k\mu(b\cos(\lambda K[2]-\mu K[2])+b\cos(\lambda\right.$$

$$+\int_1^{y(x)}\left(\frac{e^{-\int_1^{x} \frac{\sec(\mu K[1])(2d\cos(\lambda K[1]-\mu K[1])+2d\cos(\lambda K[1]+\mu K[1])+k\sin(\lambda K[1]-\mu K[1])-k\sin(\lambda K[1]+\mu K[1]))}{2(b\cos(\lambda K[1])+a\sin(\lambda K[1]))}dK[1]}}{k\mu(d+K[3])^2}\right.$$

$$-\int_1^x\left(\frac{e^{-\int_1^{K[2]} \frac{\sec(\mu K[1])(2d\cos(\lambda K[1]-\mu K[1])+2d\cos(\lambda K[1]+\mu K[1])+k\sin(\lambda K[1]-\mu K[1])-k\sin(\lambda K[1]+\mu K[1]))}{2(b\cos(\lambda K[1])+a\sin(\lambda K[1]))}dK[1]}(-\cos(\lambda K[2]-\mu K[2}{k\mu(d+K[3])(b\cos(\lambda K[2]-\mu K[2])+b\cos(\lambda K[2]+\mu K[2])+a\sin(\lambda K[2]-\mu K[2])+a\sin}\right.$$

12 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.

12.1 problem 38

Internal problem ID [10546]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = \lambda a + a(\lambda - a)\cot(\lambda x)^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 353

```
dsolve(diff(y(x),x)=y(x)^2+a*lambda+a*(lambda-a)*cot(lambda*x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\left(\mathrm{LegendreQ}\left(\frac{2a-\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \cos(\lambda x)\right) c_1 a + \mathrm{LegendreP}\left(\frac{2a-\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \cos(\lambda x)\right) a\right) \cos(3\lambda x) + \left(-2\,\mathrm{Legendre}\right.}{}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+a*\[Lambda]+a*(\[Lambda]-a)*Cot[\[Lambda]*x]^2,y[x],x,IncludeSingularSo
```

Not solved

12.2 problem 39

Internal problem ID [10547]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = 3\lambda a + \lambda^2 + a(\lambda - a)\cot(\lambda x)^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 436

```
dsolve(diff(y(x),x)=y(x)^2+lambda^2+3*a*lambda+a*(lambda-a)*cot(lambda*x)^2,y(x), singsol=al
```

$$y(x) = -\frac{\left(\left((2c_1a + 3c_1\lambda)\,\text{LegendreQ}\left(\frac{2a+\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \cos(\lambda x)\right) + (2a + 3\lambda)\,\text{LegendreP}\left(\frac{2a+\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \cos(\lambda x)\right)\right)\cos(\right.}{2\left(\cos(\lambda x)^2 - 1\right)\left(\text{LegendreQ}\left(\frac{2a+\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \cos(\lambda\right.\right.}$$
$$- \frac{\left(\text{LegendreQ}\left(\frac{2a+\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \cos(\lambda x)\right)c_1\lambda + \text{LegendreP}\left(\frac{2a+\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \cos(\lambda x)\right)\lambda\right)\cos(\lambda x)^3 + \left(-\,\text{Legendre}\right.}{2\left(\cos(\lambda x)^2 - 1\right)\left(\text{LegendreQ}\left(\frac{2a+\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \cos(\lambda x)\right)c_1 + \text{Legen}\right.}$$

✓ Solution by Mathematica

Time used: 67.099 (sec). Leaf size: 306

```
DSolve[y'[x]==y[x]^2+\[Lambda]^2+3*a*\[Lambda]+a*(\[Lambda]-a)*Cot[\[Lambda]*x]^2,y[x],x,Inc
```

$$y(x) \to -\frac{\sin^{-\frac{a+\lambda}{\lambda}}(2\lambda x)e^{-\operatorname{arctanh}(\cos(2\lambda x))}\left(c_1\sin^{\frac{a}{\lambda}}(2\lambda x)((a+\lambda)\cos(2\lambda x)+a-\lambda)e^{\operatorname{arctanh}(\cos(2\lambda x))}\int_1^x e^{\frac{(a-\lambda)\operatorname{arctanh}(\text{co}}{\lambda}}\right.}{1+c_1\int_1^x e^{\frac{(a-\lambda)\operatorname{arctanh}(\cos(}{\lambda}}}$$

$$y(x) \to \csc(2\lambda x)\left(-\frac{\sin^{-\frac{a}{\lambda}}(2\lambda x)e^{\frac{(a-\lambda)\operatorname{arctanh}(\cos(2\lambda x))}{\lambda}}}{\int_1^x e^{\frac{(a-\lambda)\operatorname{arctanh}(\cos(2\lambda K[1]))}{\lambda}}\sin^{-\frac{a+\lambda}{\lambda}}(2\lambda K[1])dK[1]}-(a+\lambda)\cos(2\lambda x)-a+\lambda\right)$$

12.3 problem 40

Internal problem ID [10548]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 + 2ab\cot(ax)\, y = -a^2 + b^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 565

```
dsolve(diff(y(x),x)=y(x)^2-2*a*b*cot(a*x)*y(x)+b^2-a^2,y(x), singsol=all)
```

$$y(x) = \frac{\left(\frac{\left(bac_1 - \sqrt{b^2a^2 - a^2 + b^2}\, c_1 - c_1 a\right) \operatorname{LegendreQ}\left(\frac{a + 2\sqrt{b^2a^2 - a^2 + b^2}}{2a}, b - \frac{1}{2}, \sqrt{-\sin(ax)^2 + 1}\right)}{\sqrt{-\sin(ax)^2 + 1}\left(\operatorname{LegendreQ}\left(\frac{-a + 2\sqrt{b^2a^2 - a^2 + b^2}}{2a}, b - \frac{1}{2}, \sqrt{-\sin(ax)^2 + 1}\right) c_1 + \operatorname{LegendreP}\left(\frac{-a + 2\sqrt{b^2a^2 - a^2 + b^2}}{2a}, b - \frac{1}{2}, \sqrt{-\sin(ax)^2 + 1}\right)\right)} + \right.}{}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2-2*a*b*Cot[a*x]*y[x]+b^2-a^2,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

12.4 problem 41

Internal problem ID [10549]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a\cot(\beta x)\, y = ab\cot(\beta x) - b^2}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 61

```
dsolve(diff(y(x),x)=y(x)^2+a*cot(beta*x)*y(x)+a*b*cot(beta*x)-b^2,y(x), singsol=all)
```

$$y(x) = -b - \frac{\left(\cot(\beta x)^2 + 1\right)^{-\frac{a}{2\beta}} e^{-2xb}}{\int \left(\cot(\beta x)^2 + 1\right)^{-\frac{a}{2\beta}} e^{-2xb} dx - c_1}$$

✓ Solution by Mathematica

Time used: 26.26 (sec). Leaf size: 462

```
DSolve[y'[x]==y[x]^2+a*Cot[\[Beta]*x]*y[x]+a*b*Cot[\[Beta]*x]-b^2,y[x],x,IncludeSingularSolu
```

$$y(x) \to \frac{b(ia + 2b - 2i\beta)\left(-ie^{-i\beta x}\left(-1 + e^{2i\beta x}\right)\right)^{a/\beta} \text{Hypergeometric2F1}\left(1, \frac{a+2ib}{2\beta}, -\frac{a-2ib-2\beta}{2\beta}, e^{2ix\beta}\right) + (a - 2ib)}{i(-a + 2ib + 2\beta)\left(-ie^{-i\beta x}\left(-1 + e^{2i\beta x}\right)\right)^{a/\beta} \text{Hypergeometric2F1}\left(1, \frac{a+2ib}{2\beta}, -\frac{a-2ib-2\beta}{2\beta}, e^{2}\right.}$$

$$y(x) \to -b$$

12.5 problem 42

Internal problem ID [10550]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - ax\cot(bx)^m y = a\cot(bx)^m$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)=y(x)^2+a*x*cot(b*x)^m*y(x)+a*cot(b*x)^m,y(x), singsol=all)
```

$$y(x) = -\frac{e^{\int \frac{a\cot(xb)^m x^2-2}{x}dx}x + \int e^{\int \frac{a\cot(xb)^m x^2-2}{x}dx}dx - c_1}{\left(-c_1 + \int e^{\int \frac{a\cot(xb)^m x^2-2}{x}dx}dx\right)x}$$

✓ Solution by Mathematica

Time used: 8.36 (sec). Leaf size: 126

```
DSolve[y'[x]==y[x]^2+a*x*Cot[b*x]^m*y[x]+a*Cot[b*x]^m,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to -\frac{\exp\left(-\int_1^x -a\cot^m(bK[1])K[1]dK[1]\right) + x\int_1^x \frac{\exp\left(-\int_1^{K[2]} -a\cot^m(bK[1])K[1]dK[1]\right)}{K[2]^2}dK[2] + c_1 x}{x^2\left(\int_1^x \frac{\exp\left(-\int_1^{K[2]} -a\cot^m(bK[1])K[1]dK[1]\right)}{K[2]^2}dK[2] + c_1\right)}$$

$$y(x) \to -\frac{1}{x}$$

12.6 problem 43

Internal problem ID [10551]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + (k+1)\, x^k y^2 - a\, x^{k+1} \cot(x)^m\, y = -a \cot(x)^m$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 168

```
dsolve(diff(y(x),x)=-(k+1)*x^k*y(x)^2+a*x^(k+1)*cot(x)^m*y(x)-a*cot(x)^m,y(x), singsol=all)
```

$$y(x) = \frac{\left(\mathrm{e}^{\int \frac{x^k \cot(x)^m a\, x^2 - 2k - 2}{x} dx} x^k x + \left(\int x^k \mathrm{e}^{\int \frac{a\, x^{k+2} \cot(x)^m - 2k - 2}{x} dx} dx\right) k + \int x^k \mathrm{e}^{\int \frac{a\, x^{k+2} \cot(x)^m - 2k - 2}{x} dx} dx + c_1\right) x^{-k}}{x \left(\left(\int x^k \mathrm{e}^{\int \frac{a\, x^{k+2} \cot(x)^m - 2k - 2}{x} dx} dx\right) k + \int x^k \mathrm{e}^{\int \frac{a\, x^{k+2} \cot(x)^m - 2k - 2}{x} dx} dx + c_1\right)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-(k+1)*x^k*y[x]^2+a*x^(k+1)*Cot[x]^m*y[x]-a*Cot[x]^m,y[x],x,IncludeSingularSol
```

Not solved

12.7 problem 44

Internal problem ID [10552]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - a\cot\left(x\lambda+\mu\right)^{k}\left(y-b\,x^{n}-c\right)^{2}=bn\,x^{-1+n}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 118

```
dsolve(diff(y(x),x)=a*cot(lambda*x+mu)^k*(y(x)-b*x^n-c)^2+b*n*x^(n-1),y(x), singsol=all)
```

$$y(x) = -\frac{\left(-2x^{n}ab\left(\frac{-1+\cot(\mu)\cot(\lambda x)}{\cot(\mu)+\cot(\lambda x)}\right)^{k}-2ac\left(\frac{-1+\cot(\mu)\cot(\lambda x)}{\cot(\mu)+\cot(\lambda x)}\right)^{k}\right)\left(\frac{-1+\cot(\mu)\cot(\lambda x)}{\cot(\mu)+\cot(\lambda x)}\right)^{-k}}{2a} + \frac{1}{c_{1}-\left(\int a\left(\frac{-1+\cot(\mu)\cot(\lambda x)}{\cot(\mu)+\cot(\lambda x)}\right)^{k}dx\right)}$$

✓ Solution by Mathematica

Time used: 5.758 (sec). Leaf size: 74

```
DSolve[y'[x]==a*Cot[\[Lambda]*x+mu]^k*(y[x]-b*x^n-c)^2+b*n*x^(n-1),y[x],x,IncludeSingularSol
```

$$y(x) \to \frac{1}{\frac{a\cot^{k+1}(\mu+\lambda x)\,\text{Hypergeometric2F1}\left(1,\frac{k+1}{2},\frac{k+3}{2},-\cot^{2}(\mu+x\lambda)\right)}{(k+1)\lambda}+c_{1}}+bx^{n}+c$$

$$y(x) \to bx^{n}+c$$

12.8 problem 45

Internal problem ID [10553]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.
Problem number: 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - a\cot(\lambda x)^m y^2 - ky = a\,b^2 x^{2k}\cot(\lambda x)^m}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 36

```
dsolve(x*diff(y(x),x)=a*cot(lambda*x)^m*y(x)^2+k*y(x)+a*b^2*x^(2*k)*cot(lambda*x)^m,y(x), si
```

$$y(x) = -\tan\left(-ba\left(\int \frac{x^k \cot(\lambda x)^m}{x}dx\right) + c_1\right) b\,x^k$$

✓ Solution by Mathematica

Time used: 1.805 (sec). Leaf size: 50

```
DSolve[x*y'[x]==a*Cot[\[Lambda]*x]^m*y[x]^2+k*y[x]+a*b^2*x^(2*k)*Cot[\[Lambda]*x]^m,y[x],x,I
```

$$y(x) \to \sqrt{b^2}x^k \tan\left(\sqrt{b^2}\int_1^x a\cot^m(\lambda K[1])K[1]^{k-1}dK[1] + c_1\right)$$

12.9 problem 46

Internal problem ID [10554]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$(a\cot(x\lambda)+b)\,y'-y^2-c\cot(\mu x)\,y=-d^2+cd\cot(\mu x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 251

```
dsolve((a*cot(lambda*x)+b)*diff(y(x),x)=y(x)^2+c*cot(mu*x)*y(x)-d^2+c*d*cot(mu*x),y(x), sing
```

$$y(x)=-d-\frac{e^{\int\frac{c\cot(\mu x)}{a\cot(\lambda x)+b}dx}(a\cot(\lambda x)+b)^{\frac{2ad}{\lambda(a^2+b^2)}}\left(\cot(\lambda x)^2+1\right)^{-\frac{ad}{\lambda(a^2+b^2)}}e^{\frac{\pi bd}{\lambda(a^2+b^2)}}e^{-\frac{2db\,\mathrm{arccot}(\cot(\lambda x))}{\lambda(a^2+b^2)}}}{\int\frac{e^{\int\frac{c\cot(\mu x)}{a\cot(\lambda x)+b}dx}(a\cot(\lambda x)+b)^{\frac{2ad}{\lambda(a^2+b^2)}}\left(\cot(\lambda x)^2+1\right)^{-\frac{ad}{\lambda(a^2+b^2)}}e^{\frac{\pi bd}{\lambda(a^2+b^2)}}e^{-\frac{2db\,\mathrm{arccot}(\cot(\lambda x))}{\lambda(a^2+b^2)}}}{a\cot(\lambda x)+b}dx-c_1}$$

✓ Solution by Mathematica

Time used: 87.594 (sec). Leaf size: 799

```
DSolve[(a*Cot[\[Lambda]*x]+b)*y'[x]==y[x]^2+c*Cot[\[Mu]*x]*y[x]-d^2+c*d*Cot[\[Mu]*x],y[x],x,
```

$$\text{Solve}\left[\int_1^x \frac{e^{-\int_1^{K[2]} -\frac{\csc(\mu K[1])(-2d\cos(\lambda K[1]-\mu K[1])+2d\cos(\lambda K[1]+\mu K[1])+c\sin(\lambda K[1]-\mu K[1])+c\sin(\lambda K[1]+\mu K[1]))}{2(a\cos(\lambda K[1])+b\sin(\lambda K[1]))}dK[1]}(-d\cos(\lambda K[2]-}{c\mu(b\cos(\lambda K[2]-\mu K[2])-b\cos(}\right.$$

$$+\int_1^{y(x)}\left(-\int_1^x\left(\frac{e^{-\int_1^{K[2]} -\frac{\csc(\mu K[1])(-2d\cos(\lambda K[1]-\mu K[1])+2d\cos(\lambda K[1]+\mu K[1])+c\sin(\lambda K[1]-\mu K[1])+c\sin(\lambda K[1]+\mu K[1]))}{2(a\cos(\lambda K[1])+b\sin(\lambda K[1]))}dK[1]}(\cos(\lambda K[2}{c\mu(d+K[3])(b\cos(\lambda K[2]-\mu K[2])-b\cos(\lambda K[2]+\mu K[2])-a\sin(\lambda K[2]-\mu K}\right.\right.$$

$$\left.\left.-\frac{e^{-\int_1^x -\frac{\csc(\mu K[1])(-2d\cos(\lambda K[1]-\mu K[1])+2d\cos(\lambda K[1]+\mu K[1])+c\sin(\lambda K[1]-\mu K[1])+c\sin(\lambda K[1]+\mu K[1]))}{2(a\cos(\lambda K[1])+b\sin(\lambda K[1]))}dK[1]}}{c\mu(d+K[3])^2}\right)dK[3]=c_1,y(x)\right]$$

13 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.

13.1 problem 47

Internal problem ID [10555]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = \lambda^2 + c\sin(\lambda x)^n \cos(\lambda x)^{-n-4}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+lambda^2+c*sin(lambda*x)^n*cos(lambda*x)^(-n-4),y(x), singsol=all
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+\[Lambda]^2+c*Sin[\[Lambda]*x]^n*Cos[\[Lambda]*x]^(-n-4),y[x],x,Include
```

Not solved

13.2 problem 48

Internal problem ID [10556]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 \sin(\lambda x)\, a = b \sin(\lambda x) \cos(\lambda x)^n}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 334

```
dsolve(diff(y(x),x)=a*sin(lambda*x)*y(x)^2+b*sin(lambda*x)*cos(lambda*x)^n,y(x), singsol=all
```

$$y(x) = \frac{-\frac{\sqrt{\frac{ab}{\lambda^2}}\cos(\lambda x)^{\frac{n}{2}+1} c_1 \lambda \,\mathrm{BesselY}\left(\frac{n+3}{n+2}, \frac{2\sqrt{\frac{ab}{\lambda^2}}\cos(\lambda x)^{\frac{n}{2}+1}}{n+2}\right)}{\left(\mathrm{BesselY}\left(\frac{1}{n+2}, \frac{2\sqrt{\frac{ab}{\lambda^2}}\cos(\lambda x)^{\frac{n}{2}+1}}{n+2}\right) c_1 + \mathrm{BesselJ}\left(\frac{1}{n+2}, \frac{2\sqrt{\frac{ab}{\lambda^2}}\cos(\lambda x)^{\frac{n}{2}+1}}{n+2}\right)\right) a} - \frac{\left(\mathrm{BesselJ}\left(\frac{n+3}{n+2}, \frac{2\sqrt{\frac{ab}{\lambda^2}}\cos(\lambda x)^{\frac{n}{2}+1}}{n+2}\right)\right) \sqrt{\frac{ab}{\lambda^2}}\cos(\lambda x)^{\frac{n}{2}+}}{\left(\mathrm{BesselY}\left(\frac{1}{n+2}, \frac{2\sqrt{\frac{ab}{\lambda^2}}\,\mathrm{co}}{n}\right.\right.}}{\cos(\lambda x)}$$

✓ Solution by Mathematica

Time used: 1.409 (sec). Leaf size: 695

```
DSolve[y'[x]==a*Sin[\[Lambda]*x]*y[x]^2+b*Sin[\[Lambda]*x]*Cos[\[Lambda]*x]^n,y[x],x,Include
```

$$y(x) \to \frac{\sqrt{a}\sqrt{b}\,\text{Gamma}\left(1+\frac{1}{n+2}\right)\cos^{\frac{n}{2}}(\lambda x)\,\text{BesselJ}\left(\frac{1}{n+2}-1,\frac{2\sqrt{a}\sqrt{b}\cos^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right)-\sqrt{a}\sqrt{b}\,\text{Gamma}\left(1+\frac{1}{n+2}\right)\cos^{\frac{n}{2}}}{}$$

$$y(x) \to \frac{\frac{\sqrt{a}\sqrt{b}\cos^{\frac{n}{2}}(\lambda x)\left(\text{BesselJ}\left(-\frac{n+3}{n+2},\frac{2\sqrt{a}\sqrt{b}\cos^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right)-\text{BesselJ}\left(\frac{n+1}{n+2},\frac{2\sqrt{a}\sqrt{b}\cos^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right)\right)}{\text{BesselJ}\left(-\frac{1}{n+2},\frac{2\sqrt{a}\sqrt{b}\cos^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right)}+\lambda\sec(\lambda x)}{2a}$$

13.3 problem 49

Internal problem ID [10557]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \sin(\lambda x)\, y^2 - a \cos(\lambda x)^n\, y = -a \cos(\lambda x)^{-1+n}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=lambda*sin(lambda*x)*y(x)^2+a*cos(lambda*x)^n*y(x)-a*cos(lambda*x)^(n-1)
```

No solution found

✓ Solution by Mathematica

Time used: 150.623 (sec). Leaf size: 467

```
DSolve[y'[x]==\[Lambda]*Sin[\[Lambda]*x]*y[x]^2+a*Cos[\[Lambda]*x]^n*y[x]-a*Cos[\[Lambda]*x]
```

$$\text{Solve}\Bigg[\int_1^x -\frac{\exp\left(-\frac{a\cos^{n+1}(\lambda K[1])\csc(\lambda K[1])\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(\lambda K[1])\right)\sqrt{\sin^2(\lambda K[1])}}{(n+1)\lambda}\right)\tan(\lambda K[1])\left(-a\csc(\lambda K[1])\,\text{co}\right.}{(\cos(\lambda K[1])y(x)-1)^2}$$

$$+\int_1^{y(x)}\left(\frac{\exp\left(-\frac{a\cos^{n+1}(x\lambda)\csc(x\lambda)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(x\lambda)\right)\sqrt{\sin^2(x\lambda)}}{(n+1)\lambda}\right)}{(\cos(x\lambda)K[2]-1)^2}\right.$$

$$-\int_1^x\left(\frac{2\exp\left(-\frac{a\cos^{n+1}(\lambda K[1])\csc(\lambda K[1])\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(\lambda K[1])\right)\sqrt{\sin^2(\lambda K[1])}}{(n+1)\lambda}\right)\left(-a\csc(\lambda K[1])\cos^n(\lambda\right.}{(\cos(\lambda K[1])K[2]-1)^3}\right.$$

13.4 problem 50

Internal problem ID [10558]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \cos(\lambda x)\, y^2 a = b \cos(\lambda x) \sin(\lambda x)^n}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 284

```
dsolve(diff(y(x),x)=a*cos(lambda*x)*y(x)^2+b*cos(lambda*x)*sin(lambda*x)^n,y(x), singsol=all
```

$$y(x) = \frac{\left(\left(\sin(\lambda x)^{n+2} c_1 abn + \sin(\lambda x)^{n+2} c_1 ab\right) \operatorname{hypergeom}\left([\,], \left[\frac{2n+5}{n+2}\right], -\frac{\sin(\lambda x)^{n+2} ab}{\lambda^2 (n+2)^2}\right) + \left(-c_1 \lambda^2 n^2 - 4 c_1 \lambda^2 n - 3\right.\right.}{(1+n)\, \lambda \sin(\lambda x)\, (n+3)\, a \left(c_1 \sin(\lambda x) \operatorname{hyperg}\right.}$$

✓ Solution by Mathematica

Time used: 1.376 (sec). Leaf size: 633

```
DSolve[y'[x]==a*Cos[\[Lambda]*x]*y[x]^2+b*Cos[\[Lambda]*x]*Sin[\[Lambda]*x]^n,y[x],x,Include
```

$$y(x) \to \frac{\csc(\lambda x)\left(-\lambda\,\text{Gamma}\left(1+\frac{1}{n+2}\right)\text{BesselJ}\left(\frac{1}{n+2},\frac{2\sqrt{a}\sqrt{b}\sin^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right)+\sqrt{a}\sqrt{b}\sin^{\frac{n}{2}+1}(\lambda x)\left(\text{Gamma}\left(1+\frac{1}{n+2}\right)\right.\right.}{}$$

$$y(x) \to \frac{\frac{\sqrt{a}\sqrt{b}\sin^{\frac{n}{2}}(\lambda x)\left(\text{BesselJ}\left(\frac{n+1}{n+2},\frac{2\sqrt{a}\sqrt{b}\sin^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right)-\text{BesselJ}\left(-\frac{n+3}{n+2},\frac{2\sqrt{a}\sqrt{b}\sin^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right)\right)}{\text{BesselJ}\left(-\frac{1}{n+2},\frac{2\sqrt{a}\sqrt{b}\sin^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right)}-\lambda\csc(\lambda x)}{2a}$$

13.5 problem 51

Internal problem ID [10559]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 51.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \lambda \sin(\lambda x) y^2 - a x^n \cos(\lambda x) y = -x^n a}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 122

```
dsolve(diff(y(x),x)=lambda*sin(lambda*x)*y(x)^2+a*x^n*cos(lambda*x)*y(x)-a*x^n,y(x), singsol
```

$$y(x) = \frac{c_1 e^{\int \frac{\sqrt{\frac{1}{2}-\frac{\cos(2\lambda x)}{2}}\left(\cos(\lambda x)\sin(\lambda x)x^n a+2\sqrt{\frac{1}{2}-\frac{\cos(2\lambda x)}{2}}\,\lambda\tan(\lambda x)\right)}{\sin(\lambda x)^2}dx}}{\left(\int -e^{\int \frac{\sqrt{\frac{1}{2}-\frac{\cos(2\lambda x)}{2}}\left(\cos(\lambda x)\sin(\lambda x)x^n a+2\sqrt{\frac{1}{2}-\frac{\cos(2\lambda x)}{2}}\,\lambda\tan(\lambda x)\right)}{\sin(\lambda x)^2}dx}\sin(\lambda x)\lambda dx\right)c_1+1} + \frac{1}{\cos(\lambda x)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*Sin[\[Lambda]*x]*y[x]^2+a*x^n*Cos[\[Lambda]*x]*y[x]-a*x^n,y[x],x,Inc
```

Not solved

13.6 problem 52

Internal problem ID [10560]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\sin(2x)^{n+1} y' - a y^2 \sin(x)^{2n} = b\cos(x)^{2n}$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 325

```
dsolve(sin(2*x)^(n+1)*diff(y(x),x)=a*y(x)^2*sin(x)^(2*n)+b*cos(x)^(2*n),y(x), singsol=all)
```

$$y(x) = -\frac{\sin(2x)^n \left(-\sin(x)^{-2n+1-\frac{\sqrt{n^2-ab4^{-n}}}{2}} \cos(x)^{\frac{\sqrt{n^2-ab4^{-n}}}{2}} \sqrt{n^2-ab4^{-n}}\, c_1 + \sin(x)^{-2n+1-\frac{\sqrt{n^2-ab4^{-n}}}{2}} \cos(x)\right.}{a\left(\sin(x)^{-\frac{\sqrt{n^2-ab4^{-n}}}{2}} \cos(x)^{\frac{\sqrt{n^2-ab4}}{2}}\right.}$$

✓ Solution by Mathematica

Time used: 33.745 (sec). Leaf size: 132

```
DSolve[Sin[2*x]^(n+1)*y'[x]==a*y[x]^2*Sin[x]^(2*n)+b*Cos[x]^(2*n),y[x],x,IncludeSingularSolu
```

$$\text{Solve}\left[\int_1^{\sqrt{\frac{a\cos^{-2n}(x)\sin^{2n}(x)}{b}}y(x)} \frac{1}{K[1]^2 - \sqrt{\frac{2^{2n+2}n^2}{ab}}K[1]+1} dK[1] = \frac{1}{2} b \sin^{-n}(2x)\cos^{2n}(x)\left(\log\left(\tan\left(\frac{x}{2}\right)\right)\right.\right.$$
$$\left.\left. - \log\left(\cos(x)\sec^2\left(\frac{x}{2}\right)\right)\right)\sqrt{\frac{a\sin^{2n}(x)\cos^{-2n}(x)}{b}} + c_1, y(x)\right]$$

13.7 problem 53

Internal problem ID [10561]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 53.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 + \tan(x)\,y = a(-a+1)\cot(x)^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 70

```
dsolve(diff(y(x),x)=y(x)^2-y(x)*tan(x)+a*(1-a)*cot(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\left(\frac{a c_1 \sin(x)}{c_1 \sin(x)+\sin(x)^{2a}} - \frac{c_1 \sin(x)}{c_1 \sin(x)+\sin(x)^{2a}} - \frac{\sin(x)^{2a} a}{c_1 \sin(x)+\sin(x)^{2a}}\right)\cos(x)}{\sin(x)}$$

✓ Solution by Mathematica

Time used: 7.444 (sec). Leaf size: 230

```
DSolve[y'[x]==y[x]^2-y[x]*Tan[x]+a*(1-a)*Cot[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{i\cot(x)\left(\left(\sqrt{a-1}\sqrt{a}\sqrt{-\frac{(2a-1)^2}{(a-1)a}} - i\right)\left(-\sin^2(x)\right)^{\frac{1}{2}i\sqrt{a-1}\sqrt{a}\sqrt{-\frac{(2a-1)^2}{(a-1)a}}} - \left(\sqrt{a-1}\sqrt{a}\sqrt{-\frac{(2a-1)^2}{(a-1)a}} + i\right)c_1\right.}{2\left(\left(-\sin^2(x)\right)^{\frac{1}{2}i\sqrt{a-1}\sqrt{a}\sqrt{\frac{1}{a-a^2}-4}} + c_1\right)}$$

$$y(x) \to \frac{1}{2}i\left(\sqrt{a-1}\sqrt{a}\sqrt{\frac{1}{a-a^2} - 4} + i\right)\cot(x)$$

13.8 problem 54

Internal problem ID [10562]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 54.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 + my\tan(x) = b^2\cos(x)^{2m}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 346

```
dsolve(diff(y(x),x)=y(x)^2-m*y(x)*tan(x)+b^2*cos(x)^(2*m),y(x), singsol=all)
```

$y(x) =$

$$-\frac{\left((m-1)\,\mathrm{hypergeom}\left(\left[\frac{3}{2}, -\frac{m}{2}+\frac{3}{2}\right], \left[\frac{5}{2}\right], \sin(x)^2\right)\sin(x)^2 - 3\,\mathrm{hypergeom}\left(\left[\frac{1}{2}, -\frac{m}{2}+\frac{1}{2}\right], \left[\frac{3}{2}\right], \sin(x)^2\right.}{3\left(c_1\cos\left(b\sqrt{\cos(x)^{-2+2m}}\cos(x)^{-m+1}\sin(x)\,\mathrm{hypergeom}\left(\left[\frac{1}{2}, -\frac{m}{2}+\frac{1}{2}\right],\right.\right.\right.}$$

$$+\frac{b\sqrt{\cos(x)^{2m}}\cos(x)^{-m}\cos(x)\left((m-1)\,\mathrm{hypergeom}\left(\left[\frac{3}{2}, -\frac{m}{2}+\frac{3}{2}\right], \left[\frac{5}{2}\right], \sin(x)^2\right)\sin(x)^2 - 3\,\mathrm{hyperge}\right.}{3c_1\cos\left(b\sqrt{\cos(x)^{-2+2m}}\cos(x)^{-m+1}\sin(x)\,\mathrm{hypergeom}\left(\left[\frac{1}{2}, -\frac{m}{2}+\frac{1}{2}\right], \left[\frac{3}{2}\right], \sin\right.\right.}$$

✓ Solution by Mathematica

Time used: 4.179 (sec). Leaf size: 73

```
DSolve[y'[x]==y[x]^2-m*y[x]*Tan[x]+b^2*Cos[x]^(2*m),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{b^2}\cos^m(x)\tan\left(-\frac{\sqrt{b^2}\sqrt{\sin^2(x)}\csc(x)\cos^{m+1}(x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(x)\right)}{m+1}+c_1\right)$$

13.9 problem 55

Internal problem ID [10563]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 55.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - my\cot(x) = b^2\sin(x)^{2m}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 301

```
dsolve(diff(y(x),x)=y(x)^2+m*y(x)*cot(x)+b^2*sin(x)^(2*m),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\left(\csc(x)^2\right)^{-m-2}}\left(\csc(x)^2\right)^{\frac{m}{2}+1} b\left((-m-2)\operatorname{hypergeom}\left(\left[\frac{3}{2}, 2+\frac{m}{2}\right], \left[\frac{5}{2}\right], -\cot(x)^2\right)\cot(x)^4 + ((-m -\right.}{}$$

✓ Solution by Mathematica

Time used: 5.352 (sec). Leaf size: 72

```
DSolve[y'[x]==y[x]^2+m*y[x]*Cot[x]+b^2*Sin[x]^(2*m),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{b^2}\sin^m(x)\tan\left(\frac{\sqrt{b^2}\sqrt{\cos^2(x)}\sec(x)\sin^{m+1}(x)\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \sin^2(x)\right)}{m+1} + c_1\right)$$

13.10 problem 56

Internal problem ID [10564]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 56.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = -2\lambda^2 \tan(x)^2 - 2\lambda^2 \cot(\lambda x)^2$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2-2*lambda^2*tan(x)^2-2*lambda^2*cot(lambda*x)^2,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2-2*\[Lambda]^2*Tan[x]^2-2*\[Lambda]^2*Cot[\[Lambda]*x]^2,y[x],x,IncludeS
```

Not solved

13.11 problem 57

Internal problem ID [10565]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 57.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = 2ba + \lambda a + b\lambda + a(\lambda - a)\tan(\lambda x)^2 + b(\lambda - b)\cot(\lambda x)^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 885

```
dsolve(diff(y(x),x)=y(x)^2+lambda*a+lambda*b+2*a*b+a*(lambda-a)*tan(lambda*x)^2+b*(lambda-b)
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+\[Lambda]*a+\[Lambda]*b+2*a*b+a*(\[Lambda]-a)*Tan[\[Lambda]*x]^2+b*(\[L
```

Not solved

13.12 problem 58

Internal problem ID [10566]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition

Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.

Problem number: 58.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = -\frac{\lambda^2}{2} - \frac{3\tan(\lambda x)^2 \lambda^2}{4} + a\cos(\lambda x)^2 \sin(\lambda x)^n$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 411

```
dsolve(diff(y(x),x)=y(x)^2-1/2*lambda^2-3/4*lambda^2*tan(lambda*x)^2+a*cos(lambda*x)^2*sin(l
```

$y(x) =$

$$-\frac{(c_1\lambda^2 n^2 + 4c_1\lambda^2 n + 3c_1\lambda^2)\,\mathrm{hypergeom}\left([\,],\left[\frac{n+3}{n+2}\right],-\frac{\sin(\lambda x)^{n+2}a}{\lambda^2(n+2)^2}\right)\sin(\lambda x)^3 + (\lambda^2 n^2 + 4\lambda^2 n + 3\lambda^2)\,\mathrm{hype}}{}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2-1/2*\[Lambda]^2-3/4*\[Lambda]^2*Tan[\[Lambda]*x]^2+a*Cos[\[Lambda]*x]^2
```

Not solved

13.13 problem 59

Internal problem ID [10567]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 59.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$
\boxed{y' - \lambda \sin(\lambda x) y^2 - a \sin(\lambda x) y = -a \tan(\lambda x)}
$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 63

```
dsolve(diff(y(x),x)=lambda*sin(lambda*x)*y(x)^2+a*sin(lambda*x)*y(x)-a*tan(lambda*x),y(x), s
```

$$
y(x) = \frac{\mathrm{Ei}_1\left(\frac{\cos(\lambda x)a}{\lambda}\right) c_1 a - 1}{\mathrm{Ei}_1\left(\frac{\cos(\lambda x)a}{\lambda}\right) \cos(\lambda x) c_1 a - \mathrm{e}^{-\frac{\cos(\lambda x)a}{\lambda}} c_1 \lambda - \cos(\lambda x)}
$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*Sin[\[Lambda]*x]*y[x]^2+a*Sin[\[Lambda]*x]*y[x]-a*Tan[\[Lambda]*x],y
```

Not solved

14 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.

14.1 problem 1

Internal problem ID [10568]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - \lambda \arcsin(x)^n y = -a^2 + a\lambda \arcsin(x)^n$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 97

```
dsolve(diff(y(x),x)=y(x)^2+lambda*arcsin(x)^n*y(x)-a^2+a*lambda*arcsin(x)^n,y(x), singsol=al
```

$$y(x) = -\frac{\left(\left(\int e^{\int (\arcsin(x)^n\lambda-2a)dx}dx\right) e^{\int (-\arcsin(x)^n\lambda+2a)dx} a + c_1 e^{\int (-\arcsin(x)^n\lambda+2a)dx} a + 1\right) e^{\int (\arcsin(x)^n\lambda-2a)dx}}{c_1 + \int e^{\int (\arcsin(x)^n\lambda-2a)dx}dx}$$

✓ Solution by Mathematica

Time used: 6.556 (sec). Leaf size: 398

```
DSolve[y'[x]==y[x]^2+\[Lambda]*ArcSin[x]^n*y[x]-a^2+a*\[Lambda]*ArcSin[x]^n,y[x],x,IncludeSi
```

$$\text{Solve}\Bigg[\int_1^x$$

$$-\frac{\exp\left(\frac{1}{2}i\lambda\arcsin(K[1])^n\left(\arcsin(K[1])^2\right)^{-n}\left((-i\arcsin(K[1]))^n\Gamma(n+1,i\arcsin(K[1]))-(i\arcsin(K[1]))^n\right.\right.}{n\lambda(a+y(x))}$$

$$+\int_1^{y(x)}\left(\frac{\exp\left(\frac{1}{2}i\lambda\arcsin(x)^n\left(\arcsin(x)^2\right)^{-n}\left((-i\arcsin(x))^n\Gamma(n+1,i\arcsin(x))-(i\arcsin(x))^n\Gamma(n+1,\right.\right.}{n\lambda(a+K[2])^2}\right.$$

$$-\int_1^x\left(\frac{\exp\left(\frac{1}{2}i\lambda\arcsin(K[1])^n\left(\arcsin(K[1])^2\right)^{-n}\left((-i\arcsin(K[1]))^n\Gamma(n+1,i\arcsin(K[1]))-(i\arcsin(\right.\right.}{n\lambda(a+K[2])^2}\right.$$

14.2 problem 2

Internal problem ID [10569]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - \lambda x \arcsin(x)^n y = \arcsin(x)^n \lambda}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 55

```
dsolve(diff(y(x),x)=y(x)^2+lambda*x*arcsin(x)^n*y(x)+lambda*arcsin(x)^n,y(x), singsol=all)
```

$$y(x) = \frac{e^{\int \frac{\arcsin(x)^n \lambda x^2 - 2}{x} dx}}{c_1 - \left(\int e^{\int \frac{\arcsin(x)^n \lambda x^2 - 2}{x} dx} dx\right)} - \frac{1}{x}$$

✓ Solution by Mathematica

Time used: 4.147 (sec). Leaf size: 256

```
DSolve[y'[x]==y[x]^2+\[Lambda]*x*ArcSin[x]^n*y[x]+\[Lambda]*ArcSin[x]^n,y[x],x,IncludeSingul
```

$$y(x) \to$$

$$-\frac{\int_1^x \frac{\exp\left(-2^{-n-3}\lambda \arcsin(K[1])^n \left(\arcsin(K[1])^2\right)^{-n} (\Gamma(n+1,2i\arcsin(K[1]))(-i\arcsin(K[1]))^n + (i\arcsin(K[1]))^n \Gamma(n+1,-2i\arcsin(K[1]))}{K[1]^2}}{x\left(\int_1^x \frac{\exp\left(-2^{-n-3}\lambda \arcsin(K[1])^n (\arcsin(K[1])^2)^{-n} (\Gamma(n+1,2i\arcsin(K[1]))}{K[1]^2}\right.}$$

$$y(x) \to -\frac{1}{x}$$

14.3 problem 3

Internal problem ID [10570]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + (k+1)\,x^k y^2 - \lambda \arcsin(x)^n \left(x^{k+1} y - 1\right) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 196

```
dsolve(diff(y(x),x)=-(k+1)*x^k*y(x)^2+lambda*arcsin(x)^n*(x^(k+1)*y(x)-1),y(x), singsol=all)
```

$$y(x) = \frac{\left(e^{\int \frac{x^k \arcsin(x)^n \lambda x^2 - 2k - 2}{x} dx} x^k x + \int \left(x^k k\, e^{\lambda\left(\int x^{1+k} \arcsin(x)^n dx\right) - 2k\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)} + x^k e^{\lambda\left(\int x^{1+k} \arcsin(x)^n dx\right) - 2k\left(\int \frac{1}{x}\right.}\right.\right.}{x\left(\int \left(x^k k\, e^{\lambda\left(\int x^{1+k} \arcsin(x)^n dx\right) - 2k\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)} + x^k e^{\lambda\left(\int x^{1+k} \arcsin(x)^n dx\right) - 2k\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)}\right.\right.}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-(k+1)*x^k*y[x]^2+\[Lambda]*ArcSin[x]^n*(x^(k+1)*y[x]-1),y[x],x,IncludeSingula
```

Not solved

14.4 problem 4

Internal problem ID [10571]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \lambda \arcsin(x)^n y^2 - ay = ba - b^2 \lambda \arcsin(x)^n}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 114

```
dsolve(diff(y(x),x)=lambda*arcsin(x)^n*y(x)^2+a*y(x)+a*b-b^2*lambda*arcsin(x)^n,y(x), singso
```

$$y(x) = -\frac{\left(\left(\int \arcsin(x)^n \lambda \, e^{\int (-2\arcsin(x)^n \lambda b + a)dx} dx\right) e^{\int (2\arcsin(x)^n \lambda b - a)dx} b + c_1 e^{\int (2\arcsin(x)^n \lambda b - a)dx} b + 1\right) e^{\int (-2\arcsin(x}}{c_1 + \int \arcsin(x)^n \lambda \, e^{\int (-2\arcsin(x)^n \lambda b + a)dx} dx}$$

✓ Solution by Mathematica

Time used: 7.093 (sec). Leaf size: 428

```
DSolve[y'[x]==\[Lambda]*ArcSin[x]^n*y[x]^2+a*y[x]+a*b-b^2*\[Lambda]*ArcSin[x]^n,y[x],x,Inclu
```

$$\text{Solve}\left[\int_1^x \frac{i\exp\left(aK[1]-ib\lambda\arcsin(K[1])^n\left(\arcsin(K[1])^2\right)^{-n}\left((-i\arcsin(K[1]))^n\Gamma(n+1,i\arcsin(K[1]))\right.\right.}{an\lambda(b\,\cdot}\right.$$

$$+\int_1^{y(x)}\left(-\int_1^x\left(\frac{i\exp\left(aK[1]-ib\lambda\arcsin(K[1])^n\left(\arcsin(K[1])^2\right)^{-n}\left((-i\arcsin(K[1]))^n\Gamma(n+1,i\arcsin(\right.\right.}{an(b+K[2])}\right.\right.$$

$$-\frac{i\exp\left(ax-ib\lambda\arcsin(x)^n\left(\arcsin(x)^2\right)^{-n}\left((-i\arcsin(x))^n\Gamma(n+1,i\arcsin(x))-(i\arcsin(x))^n\Gamma(n+1,\right.\right.}{an\lambda(b+K[2])^2}$$

14.5 problem 5

Internal problem ID [10572]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \arcsin(x)^n y^2 + b\lambda x^m \arcsin(x)^n y = bm x^{m-1}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=lambda*arcsin(x)^n*y(x)^2-b*lambda*x^m*arcsin(x)^n*y(x)+b*m*x^(m-1),y(x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*ArcSin[x]^n*y[x]^2-b*\[Lambda]*x^m*ArcSin[x]^n*y[x]+b*m*x^(m-1),y[x]
```

Not solved

14.6 problem 6

Internal problem ID [10573]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \arcsin(x)^n y^2 = \beta m\, x^{m-1} - \lambda \beta^2 x^{2m} \arcsin(x)^n$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=lambda*arcsin(x)^n*y(x)^2+beta*m*x^(m-1)-lambda*beta^2*x^(2*m)*arcsin(x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*ArcSin[x]^n*y[x]^2+\[Beta]*m*x^(m-1)-\[Lambda]*\[Beta]^2*x^(2*m)*Arc
```

Not solved

14.7 problem 7

Internal problem ID [10574]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$\boxed{y' - \lambda \arcsin(x)^n (y - a\,x^m - b)^2 = am\,x^{m-1}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve(diff(y(x),x)=lambda*arcsin(x)^n*(y(x)-a*x^m-b)^2+a*m*x^(m-1),y(x), singsol=all)
```

$$y(x) = -\frac{(-2a\,x^m \arcsin(x)^n \lambda - 2\arcsin(x)^n \lambda b) \arcsin(x)^{-n}}{2\lambda} + \frac{1}{c_1 - \left(\int \arcsin(x)^n \lambda dx\right)}$$

✓ Solution by Mathematica

Time used: 4.054 (sec). Leaf size: 87

```
DSolve[y'[x]==\[Lambda]*ArcSin[x]^n*(y[x]-a*x^m-b)^2+a*m*x^(m-1),y[x],x,IncludeSingularSolut
```

$$y(x) \to ax^m + \frac{1}{\frac{1}{2}i\lambda \arcsin(x)^n (\arcsin(x)^2)^{-n} ((i\arcsin(x))^n\Gamma(n+1,-i\arcsin(x)) - (-i\arcsin(x))^n\Gamma(n+1,i\arcsin(} + b$$

$$y(x) \to ax^m + b$$

14.8 problem 8

Internal problem ID [10575]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - \lambda \arcsin(x)^n y^2 - ky = \lambda b^2 x^{2k} \arcsin(x)^n}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 34

```
dsolve(x*diff(y(x),x)=lambda*arcsin(x)^n*y(x)^2+k*y(x)+lambda*b^2*x^(2*k)*arcsin(x)^n,y(x),
```

$$y(x) = -\tan\left(-b\lambda\left(\int \frac{\arcsin(x)^n x^k}{x} dx\right) + c_1\right) b\, x^k$$

✓ Solution by Mathematica

Time used: 1.716 (sec). Leaf size: 48

```
DSolve[x*y'[x]==\[Lambda]*ArcSin[x]^n*y[x]^2+k*y[x]+\[Lambda]*b^2*x^(2*k)*ArcSin[x]^n,y[x],x
```

$$y(x) \to \sqrt{b^2} x^k \tan\left(\sqrt{b^2} \int_1^x \lambda \arcsin(K[1])^n K[1]^{k-1} dK[1] + c_1\right)$$

14.9 problem 9

Internal problem ID [10576]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - \left(a\, x^{2m} y^2 + y x^n b + c\right) \arcsin\left(x\right)^m + ny = 0$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)=(a*x^(2*m)*y(x)^2+b*x^n*y(x)+c)*arcsin(x)^m-n*y(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]==(a*x^(2*m)*y[x]^2+b*x^n*y[x]+c)*ArcSin[x]^m-n*y[x],y[x],x,IncludeSingularSol
```

Not solved

15 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arccosine.

15.1 problem 10

Internal problem ID [10577]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arccosine.
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - \lambda \arccos(x)^n y = -a^2 + a\lambda \arccos(x)^n$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 595

```
dsolve(diff(y(x),x)=y(x)^2+lambda*arccos(x)^n*y(x)-a^2+a*lambda*arccos(x)^n,y(x), singsol=al
```

Expression too large to display

✓ Solution by Mathematica

Time used: 8.046 (sec). Leaf size: 404

```
DSolve[y'[x]==y[x]^2+\[Lambda]*ArcCos[x]^n*y[x]-a^2+a*\[Lambda]*ArcCos[x]^n,y[x],x,IncludeSi
```

$$\text{Solve}\left[\int_1^x \frac{i\exp\left(\frac{1}{2}\lambda\arccos(K[1])^n\Gamma(n+1,-i\arccos(K[1]))(-i\arccos(K[1]))^{-n}+\frac{1}{2}\lambda(i\arccos(K[1]))^{-n}\,\mathrm{a}}{n\lambda(a+y(x))}\right.$$
$$+\int_1^{y(x)}\left(-\int_1^x\left(\frac{i\exp\left(\frac{1}{2}\lambda\arccos(K[1])^n\Gamma(n+1,-i\arccos(K[1]))(-i\arccos(K[1]))^{-n}+\frac{1}{2}\lambda(i\arccos(K[1}{n\lambda(a+K[2])}\right.\right.$$
$$-\frac{i\exp\left(\frac{1}{2}\lambda\arccos(x)^n\Gamma(n+1,-i\arccos(x))(-i\arccos(x))^{-n}-2ax+\frac{1}{2}\lambda(i\arccos(x))^{-n}\arccos(x)^n\Gamma(n+}{n\lambda(a+K[2])^2}$$

15.2 problem 11

Internal problem ID [10578]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arccosine.
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - \lambda x \arccos(x)^n y = \arccos(x)^n \lambda$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
dsolve(diff(y(x),x)=y(x)^2+lambda*x*arccos(x)^n*y(x)+lambda*arccos(x)^n,y(x), singsol=all)
```

$$y(x) = \frac{e^{\int \frac{\arccos(x)^n \lambda x^2 - 2}{x} dx}}{c_1 - \left(\int e^{\int \frac{\arccos(x)^n \lambda x^2 - 2}{x} dx} dx\right)} - \frac{1}{x}$$

✓ Solution by Mathematica

Time used: 5.617 (sec). Leaf size: 253

```
DSolve[y'[x]==y[x]^2+\[Lambda]*x*ArcCos[x]^n*y[x]+\[Lambda]*ArcCos[x]^n,y[x],x,IncludeSingul
```

$$y(x) \to$$

$$-\frac{x\int_1^x \frac{\exp\left(2^{-n-3}\lambda \arccos(K[1])^n \left(\arccos(K[1])^2\right)^{-n} (\Gamma(n+1,2i\arccos(K[1]))(-i\arccos(K[1]))^n + (i\arccos(K[1]))^n\Gamma(n+1,-2i\arccos(K[1}{K[1]^2}}{x^2\left(\int_1^x \frac{\exp\left(2^{-n-3}\lambda \arccos(K[1])^n (\arccos(K[1])^2)^{-n}}{}\right.}$$

$$y(x) \to -\frac{1}{x}$$

15.3 problem 12

Internal problem ID [10579]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arccosine.
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + (k+1)\, x^k y^2 - \lambda \arccos(x)^n \left(x^{k+1} y - 1\right) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 205

```
dsolve(diff(y(x),x)=-(k+1)*x^k*y(x)^2+lambda*arccos(x)^n*(x^(k+1)*y(x)-1),y(x), singsol=all)
```

$$y(x) = -\frac{\left(\mathrm{e}^{\int \frac{x^k \arccos(x)^n \lambda\, x^2 - 2k - 2}{x} dx} x^k x - \left(\int \left(-x^k k\, \mathrm{e}^{\lambda\left(\int x^{1+k} \arccos(x)^n dx\right) - 2k\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)} - x^k \mathrm{e}^{\lambda\left(\int x^{1+k} \arccos(x)^n dx\right)}\right.\right.\right.}{x \left(\int \left(-x^k k\, \mathrm{e}^{\lambda\left(\int x^{1+k} \arccos(x)^n dx\right) - 2k\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)} - x^k \mathrm{e}^{\lambda\left(\int x^{1+k} \arccos(x)^n dx\right) - 2k\left(\int \frac{1}{x} dx\right) -}\right.\right.}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-(k+1)*x^k*y[x]^2+\[Lambda]*ArcCos[x]^n*(x^(k+1)*y[x]-1),y[x],x,IncludeSingula
```

Not solved

15.4 problem 13

Internal problem ID [10580]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arccosine.
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \arccos(x)^n y^2 - ay = ba - b^2\lambda \arccos(x)^n$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 587

```
dsolve(diff(y(x),x)=lambda*arccos(x)^n*y(x)^2+a*y(x)+a*b-b^2*lambda*arccos(x)^n,y(x), singso
```

Expression too large to display

✓ Solution by Mathematica

Time used: 9.288 (sec). Leaf size: 420

```
DSolve[y'[x]==\[Lambda]*ArcCos[x]^n*y[x]^2+a*y[x]+a*b-b^2*\[Lambda]*ArcCos[x]^n,y[x],x,Inclu
```

$$\text{Solve}\Bigg[\int_1^x$$
$$-\frac{i\exp\left(-b\lambda\arccos(K[1])^n\Gamma(n+1,-i\arccos(K[1]))(-i\arccos(K[1]))^{-n} - b\lambda(i\arccos(K[1]))^{-n}\arccos(K}{an\lambda(b+y(x))}$$
$$+\int_1^{y(x)}\left(\frac{i\exp\left(-b\lambda\arccos(x)^n\Gamma(n+1,-i\arccos(x))(-i\arccos(x))^{-n} + ax - b\lambda(i\arccos(x))^{-n}\arccos(x)}{an\lambda(b+K[2])^2}\right.$$
$$-\int_1^x\left(\frac{i\exp\left(-b\lambda\arccos(K[1])^n\Gamma(n+1,-i\arccos(K[1]))(-i\arccos(K[1]))^{-n} - b\lambda(i\arccos(K[1]))^{-n}\text{arc}}{an\lambda(b+K}\right.$$

15.5 problem 14

Internal problem ID [10581]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arccosine.
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \arccos(x)^n y^2 + b\lambda x^m \arccos(x)^n y = bm x^{m-1}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=lambda*arccos(x)^n*y(x)^2-b*lambda*x^m*arccos(x)^n*y(x)+b*m*x^(m-1),y(x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*ArcCos[x]^n*y[x]^2-b*\[Lambda]*x^m*ArcCos[x]^n*y[x]+b*m*x^(m-1),y[x]
```

Not solved

15.6 problem 15

Internal problem ID [10582]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arccosine.
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \arccos(x)^n y^2 = \beta m x^{m-1} - \lambda \beta^2 x^{2m} \arccos(x)^n$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=lambda*arccos(x)^n*y(x)^2+beta*m*x^(m-1)-lambda*beta^2*x^(2*m)*arccos(x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*ArcCos[x]^n*y[x]^2+\[Beta]*m*x^(m-1)-\[Lambda]*\[Beta]^2*x^(2*m)*Arc
```

Not solved

15.7 problem 16

Internal problem ID [10583]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arccosine.
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - \lambda \arccos(x)^n \left(y - a\,x^m - b\right)^2 = am\,x^{m-1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 162

```
dsolve(diff(y(x),x)=lambda*arccos(x)^n*(y(x)-a*x^m-b)^2+a*m*x^(m-1),y(x), singsol=all)
```

$$y(x) = -\frac{\left(-2a\,x^m \arccos(x)^n \lambda - 2\arccos(x)^n \lambda b\right)\arccos(x)^{-n}}{2\lambda} + \frac{1}{c_1 + \lambda\sqrt{\pi}\,2^n \left(\frac{\arccos(x)^{1+n} 2^{-n}\sqrt{-x^2+1}}{\sqrt{\pi}\,(n+2)} - \frac{2^{-n}\sqrt{\arccos(x)}\ \mathrm{LommelS1}\left(\frac{3}{2}+n,\frac{3}{2},\arccos(x)\right)\sqrt{-x^2+1}}{\sqrt{\pi}\,(n+2)} - \frac{3\,2^{-1-n}\left(\frac{2n}{3}+\frac{4}{3}\right)\left(\arccos(x\right.}{\sqrt{}}}$$

✓ Solution by Mathematica

Time used: 4.776 (sec). Leaf size: 86

```
DSolve[y'[x]==\[Lambda]*ArcCos[x]^n*(y[x]-a*x^m-b)^2+a*m*x^(m-1),y[x],x,IncludeSingularSolut
```

$$y(x) \to ax^m + \frac{1}{-\frac{1}{2}\lambda \arccos(x)^n(-i\arccos(x))^{-n}\Gamma(n+1,-i\arccos(x)) - \frac{1}{2}\lambda(i\arccos(x))^{-n}\arccos(x)^n\Gamma(n+1,i\arccc}} + b$$

$$y(x) \to ax^m + b$$

15.8 problem 17

Internal problem ID [10584]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arccosine.
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - \lambda \arccos(x)^n y^2 - ky = \lambda b^2 x^{2k} \arccos(x)^n$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 34

```
dsolve(x*diff(y(x),x)=lambda*arccos(x)^n*y(x)^2+k*y(x)+lambda*b^2*x^(2*k)*arccos(x)^n,y(x),
```

$$y(x) = -\tan\left(-b\lambda\left(\int \frac{x^k \arccos(x)^n}{x} dx\right) + c_1\right) b\, x^k$$

✓ Solution by Mathematica

Time used: 2.128 (sec). Leaf size: 48

```
DSolve[x*y'[x]==\[Lambda]*ArcCos[x]^n*y[x]^2+k*y[x]+\[Lambda]*b^2*x^(2*k)*ArcCos[x]^n,y[x],x
```

$$y(x) \to \sqrt{b^2} x^k \tan\left(\sqrt{b^2} \int_1^x \lambda \arccos(K[1])^n K[1]^{k-1} dK[1] + c_1\right)$$

15.9 problem 18

Internal problem ID [10585]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arccosine.
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - \left(a\,x^{2m}y^2 + yx^nb + c\right)\arccos\left(x\right)^m + ny = 0$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)=(a*x^(2*m)*y(x)^2+b*x^n*y(x)+c)*arccos(x)^m-n*y(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]==(a*x^(2*m)*y[x]^2+b*x^n*y[x]+c)*ArcCos[x]^m-n*y[x],y[x],x,IncludeSingularSol
```

Not solved

16 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.

16.1 problem 19

Internal problem ID [10586]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - \lambda \arctan(x)^n y = -a^2 + a\lambda \arctan(x)^n$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 97

```
dsolve(diff(y(x),x)=y(x)^2+lambda*arctan(x)^n*y(x)-a^2+a*lambda*arctan(x)^n,y(x), singsol=al
```

$$y(x) = -\frac{\left(\left(\int e^{\int (\arctan(x)^n\lambda-2a)dx}dx\right) e^{\int (-\arctan(x)^n\lambda+2a)dx} a + c_1 e^{\int (-\arctan(x)^n\lambda+2a)dx} a + 1\right) e^{\int (\arctan(x)^n\lambda-2a)dx}}{c_1 + \int e^{\int (\arctan(x)^n\lambda-2a)dx}dx}$$

✓ Solution by Mathematica

Time used: 7.862 (sec). Leaf size: 210

```
DSolve[y'[x]==y[x]^2+\[Lambda]*ArcTan[x]^n*y[x]-a^2+a*\[Lambda]*ArcTan[x]^n,y[x],x,IncludeSi
```

$$\text{Solve}\left[\int_1^x \frac{\exp\left(-\int_1^{K[2]} (2a-\lambda\arctan(K[1])^n)\,dK[1]\right)(-\lambda\arctan(K[2])^n+a-y(x))}{n\lambda(a+y(x))}dK[2]\right.$$

$$+\int_1^{y(x)}\left(\frac{\exp\left(-\int_1^x (2a-\lambda\arctan(K[1])^n)\,dK[1]\right)}{n\lambda(a+K[3])^2}\right.$$

$$-\int_1^x\left(-\frac{\exp\left(-\int_1^{K[2]} (2a-\lambda\arctan(K[1])^n)\,dK[1]\right)(-\lambda\arctan(K[2])^n+a-K[3])}{n\lambda(a+K[3])^2}-\frac{\exp\left(-\int_1^{K[2]} (2a\right.}{n\lambda}\right.$$

16.2 problem 20

Internal problem ID [10587]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - \lambda x \arctan(x)^n y = \arctan(x)^n \lambda$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
dsolve(diff(y(x),x)=y(x)^2+lambda*x*arctan(x)^n*y(x)+lambda*arctan(x)^n,y(x), singsol=all)
```

$$y(x) = \frac{e^{\int \frac{\arctan(x)^n \lambda x^2 - 2}{x} dx}}{c_1 - \left(\int e^{\int \frac{\arctan(x)^n \lambda x^2 - 2}{x} dx} dx\right)} - \frac{1}{x}$$

✓ Solution by Mathematica

Time used: 7.063 (sec). Leaf size: 120

```
DSolve[y'[x]==y[x]^2+\[Lambda]*x*ArcTan[x]^n*y[x]+\[Lambda]*ArcTan[x]^n,y[x],x,IncludeSingul
```

$$y(x) \to -\frac{\exp\left(-\int_1^x -\lambda \arctan(K[1])^n K[1] dK[1]\right) + x \int_1^x \frac{\exp\left(-\int_1^{K[2]} -\lambda \arctan(K[1])^n K[1] dK[1]\right)}{K[2]^2} dK[2] + c_1 x}{x^2 \left(\int_1^x \frac{\exp\left(-\int_1^{K[2]} -\lambda \arctan(K[1])^n K[1] dK[1]\right)}{K[2]^2} dK[2] + c_1\right)}$$

$$y(x) \to -\frac{1}{x}$$

16.3 problem 21

Internal problem ID [10588]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + (k+1)\,x^k y^2 - \lambda \arctan(x)^n\,(x^{k+1}y - 1) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 205

```
dsolve(diff(y(x),x)=-(k+1)*x^k*y(x)^2+lambda*arctan(x)^n*(x^(k+1)*y(x)-1),y(x), singsol=all)
```

$$y(x) = -\frac{\Big(e^{\int \frac{x^k \arctan(x)^n \lambda x^2 - 2k - 2}{x}dx} x^k x - \Big(\int \Big(-x^k k\, e^{\lambda(\int x^{1+k}\arctan(x)^n dx) - 2k(\int \frac{1}{x}dx) - 2(\int \frac{1}{x}dx)} - x^k e^{\lambda(\int x^{1+k}\arctan(x)^n dx}}{x\Big(\int \Big(-x^k k\, e^{\lambda(\int x^{1+k}\arctan(x)^n dx) - 2k(\int \frac{1}{x}dx) - 2(\int \frac{1}{x}dx)} - x^k e^{\lambda(\int x^{1+k}\arctan(x)^n dx) - 2k(\int \frac{1}{x}dx) -}}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-(k+1)*x^k*y[x]^2+\[Lambda]*ArcTan[x]^n*(x^(k+1)*y[x]-1),y[x],x,IncludeSingula
```

Not solved

16.4 problem 22

Internal problem ID [10589]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \arctan(x)^n y^2 - ay = ba - b^2\lambda \arctan(x)^n$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 114

```
dsolve(diff(y(x),x)=lambda*arctan(x)^n*y(x)^2+a*y(x)+a*b-b^2*lambda*arctan(x)^n,y(x), singso
```

$$y(x) = -\frac{\left(\left(\int \arctan(x)^n \lambda \, e^{\int (-2\arctan(x)^n \lambda b+a)dx} dx\right) e^{\int (2\arctan(x)^n \lambda b-a)dx} b + c_1 e^{\int (2\arctan(x)^n \lambda b-a)dx} b + 1\right) e^{\int (-2\arctan}}{c_1 + \int \arctan(x)^n \lambda \, e^{\int (-2\arctan(x)^n \lambda b+a)dx} dx}$$

✓ Solution by Mathematica

Time used: 10.998 (sec). Leaf size: 240

```
DSolve[y'[x]==\[Lambda]*ArcTan[x]^n*y[x]^2+a*y[x]+a*b-b^2*\[Lambda]*ArcTan[x]^n,y[x],x,Inclu
```

$$\text{Solve}\left[\int_1^x \frac{\exp\left(-\int_1^{K[2]} (2b\lambda \arctan(K[1])^n - a)\, dK[1]\right)(-b\lambda \arctan(K[2])^n + \lambda y(x) \arctan(K[2])^n + a)}{an\lambda(b+y(x))}\, d\right.$$

$$+\int_1^{y(x)} \left(-\int_1^x \left(\frac{\exp\left(-\int_1^{K[2]} (2b\lambda \arctan(K[1])^n - a)\, dK[1]\right) \arctan(K[2])^n}{an(b+K[3])} - \frac{\exp\left(-\int_1^{K[2]} (2b\lambda \arctan(\right.}{}\right.\right.$$

$$\left.\left. - \frac{\exp\left(-\int_1^x (2b\lambda \arctan(K[1])^n - a)\, dK[1]\right)}{an\lambda(b+K[3])^2}\right) dK[3] = c_1, y(x)\right]$$

16.5 problem 23

Internal problem ID [10590]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \arctan(x)^n y^2 + b\lambda x^m \arctan(x)^n y = bm x^{m-1}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=lambda*arctan(x)^n*y(x)^2-b*lambda*x^m*arctan(x)^n*y(x)+b*m*x^(m-1),y(x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*ArcTan[x]^n*y[x]^2-b*\[Lambda]*x^m*ArcTan[x]^n*y[x]+b*m*x^(m-1),y[x]
```

Not solved

16.6 problem 24

Internal problem ID [10591]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \arctan(x)^n y^2 = \beta m x^{m-1} - \lambda \beta^2 x^{2m} \arctan(x)^n$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=lambda*arctan(x)^n*y(x)^2+beta*m*x^(m-1)-lambda*beta^2*x^(2*m)*arctan(x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*ArcTan[x]^n*y[x]^2+\[Beta]*m*x^(m-1)-\[Lambda]*\[Beta]^2*x^(2*m)*Arc
```

Not solved

16.7 problem 25

Internal problem ID [10592]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - \lambda \arctan(x)^n (y - a x^m - b)^2 = am\, x^{m-1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve(diff(y(x),x)=lambda*arctan(x)^n*(y(x)-a*x^m-b)^2+a*m*x^(m-1),y(x), singsol=all)
```

$$y(x) = -\frac{(-2a\, x^m \arctan(x)^n \lambda - 2\arctan(x)^n \lambda b)\arctan(x)^{-n}}{2\lambda} + \frac{1}{c_1 - \left(\int \arctan(x)^n \lambda dx\right)}$$

✓ Solution by Mathematica

Time used: 2.089 (sec). Leaf size: 44

```
DSolve[y'[x]==\[Lambda]*ArcTan[x]^n*(y[x]-a*x^m-b)^2+a*m*x^(m-1),y[x],x,IncludeSingularSolut
```

$$y(x) \to \frac{1}{-\int_1^x \lambda \arctan(K[2])^n dK[2] + c_1} + ax^m + b$$
$$y(x) \to ax^m + b$$

16.8 problem 26

Internal problem ID [10593]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - \lambda \arctan(x)^n y^2 - ky = \lambda b^2 x^{2k} \arctan(x)^n}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 34

```
dsolve(x*diff(y(x),x)=lambda*arctan(x)^n*y(x)^2+k*y(x)+lambda*b^2*x^(2*k)*arctan(x)^n,y(x),
```

$$y(x) = -\tan\left(-b\lambda\left(\int \frac{\arctan(x)^n x^k}{x} dx\right) + c_1\right) b\, x^k$$

✓ Solution by Mathematica

Time used: 1.992 (sec). Leaf size: 48

```
DSolve[x*y'[x]==\[Lambda]*ArcTan[x]^n*y[x]^2+k*y[x]+\[Lambda]*b^2*x^(2*k)*ArcTan[x]^n,y[x],x
```

$$y(x) \to \sqrt{b^2} x^k \tan\left(\sqrt{b^2} \int_1^x \lambda \arctan(K[1])^n K[1]^{k-1} dK[1] + c_1\right)$$

16.9 problem 27

Internal problem ID [10594]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - \left(a\,x^{2m}y^2 + yx^n b + c\right)\arctan\left(x\right)^m + ny = 0}$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)=(a*x^(2*m)*y(x)^2+b*x^n*y(x)+c)*arctan(x)^m-n*y(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]==(a*x^(2*m)*y[x]^2+b*x^n*y[x]+c)*ArcTan[x]^m-n*y[x],y[x],x,IncludeSingularSol
```

Not solved

17 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-4. Equations containing arccotangent.

17.1 problem 28

Internal problem ID [10595]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-4. Equations containing arccotangent.
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - \lambda \operatorname{arccot}(x)^n y = -a^2 + a\lambda \operatorname{arccot}(x)^n$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 97

```
dsolve(diff(y(x),x)=y(x)^2+lambda*arccot(x)^n*y(x)-a^2+a*lambda*arccot(x)^n,y(x), singsol=al
```

$$y(x) = -\frac{\left(\left(\int e^{\int (\operatorname{arccot}(x)^n\lambda-2a)dx}dx\right) e^{\int (-\operatorname{arccot}(x)^n\lambda+2a)dx}a + c_1 e^{\int (-\operatorname{arccot}(x)^n\lambda+2a)dx}a + 1\right) e^{\int (\operatorname{arccot}(x)^n\lambda-2a)dx}}{c_1 + \int e^{\int (\operatorname{arccot}(x)^n\lambda-2a)dx}dx}$$

✓ Solution by Mathematica

Time used: 8.548 (sec). Leaf size: 210

```
DSolve[y'[x]==y[x]^2+\[Lambda]*ArcCot[x]^n*y[x]-a^2+a*\[Lambda]*ArcCot[x]^n,y[x],x,IncludeSi
```

$$\text{Solve}\left[\int_1^x -\frac{\exp\left(-\int_1^{K[2]}\left(2a-\lambda\cot^{-1}(K[1])^n\right)dK[1]\right)\left(-\lambda\cot^{-1}(K[2])^n+a-y(x)\right)}{n\lambda(a+y(x))}dK[2] + \int_1^{y(x)}\left(-\int_1^x\left(\frac{\exp\left(-\int_1^{K[2]}\left(2a-\lambda\cot^{-1}(K[1])^n\right)dK[1]\right)\left(-\lambda\cot^{-1}(K[2])^n+a-K[3]\right)}{n\lambda(a+K[3])^2}+\frac{\exp\left(-\int_1^{K[}\right.}{}\right.\right.\right.$$

$$\left.\left.-\frac{\exp\left(-\int_1^x\left(2a-\lambda\cot^{-1}(K[1])^n\right)dK[1]\right)}{n\lambda(a+K[3])^2}\right)dK[3]=c_1,y(x)\right]$$

18 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.

18.1 problem 29

Internal problem ID [10596]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - \lambda x \operatorname{arccot}(x)^n y = \operatorname{arccot}(x)^n \lambda$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 55

```
dsolve(diff(y(x),x)=y(x)^2+lambda*x*arccot(x)^n*y(x)+lambda*arccot(x)^n,y(x), singsol=all)
```

$$y(x) = \frac{e^{\int \frac{\operatorname{arccot}(x)^n \lambda x^2 - 2}{x} dx}}{c_1 - \left(\int e^{\int \frac{\operatorname{arccot}(x)^n \lambda x^2 - 2}{x} dx} dx\right)} - \frac{1}{x}$$

✓ Solution by Mathematica

Time used: 7.258 (sec). Leaf size: 120

```
DSolve[y'[x]==y[x]^2+\[Lambda]*x*ArcCot[x]^n*y[x]+\[Lambda]*ArcCot[x]^n,y[x],x,IncludeSingul
```

$$y(x) \to -\frac{\exp\left(-\int_1^x -\lambda \cot^{-1}(K[1])^n K[1] dK[1]\right) + x \int_1^x \frac{\exp\left(-\int_1^{K[2]} -\lambda \cot^{-1}(K[1])^n K[1] dK[1]\right)}{K[2]^2} dK[2] + c_1 x}{x^2 \left(\int_1^x \frac{\exp\left(-\int_1^{K[2]} -\lambda \cot^{-1}(K[1])^n K[1] dK[1]\right)}{K[2]^2} dK[2] + c_1\right)}$$

$$y(x) \to -\frac{1}{x}$$

18.2 problem 30

Internal problem ID [10597]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + (k+1)\, x^k y^2 - \lambda \operatorname{arccot}(x)^n \left(x^{k+1} y - 1\right) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 205

```
dsolve(diff(y(x),x)=-(k+1)*x^k*y(x)^2+lambda*arccot(x)^n*(x^(k+1)*y(x)-1),y(x), singsol=all)
```

$$y(x) = -\frac{\left(e^{\int \frac{x^k \operatorname{arccot}(x)^n \lambda x^2 - 2k - 2}{x} dx} x^k x - \left(\int \left(-x^k k\, e^{\lambda\left(\int x^{1+k} \operatorname{arccot}(x)^n dx\right) - 2k\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)} - x^k e^{\lambda\left(\int x^{1+k} \operatorname{arccot}(x)^n dx\right)} \cdots\right.\right.\right.}{x \left(\int \left(-x^k k\, e^{\lambda\left(\int x^{1+k} \operatorname{arccot}(x)^n dx\right) - 2k\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)} - x^k e^{\lambda\left(\int x^{1+k} \operatorname{arccot}(x)^n dx\right) - 2k\left(\int \frac{1}{x} dx\right) - \cdots}\right.\right.}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-(k+1)*x^k*y[x]^2+\[Lambda]*ArcCot[x]^n*(x^(k+1)*y[x]-1),y[x],x,IncludeSingula
```

Not solved

18.3 problem 31

Internal problem ID [10598]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \operatorname{arccot}(x)^n y^2 - ay = ba - b^2 \lambda \operatorname{arccot}(x)^n$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 114

```
dsolve(diff(y(x),x)=lambda*arccot(x)^n*y(x)^2+a*y(x)+a*b-b^2*lambda*arccot(x)^n,y(x), singso
```

$$y(x) = -\frac{\left(\left(\int \operatorname{arccot}(x)^n \lambda \, e^{\int (-2 \operatorname{arccot}(x)^n \lambda b + a) dx} dx\right) e^{\int (2 \operatorname{arccot}(x)^n \lambda b - a) dx} b + c_1 e^{\int (2 \operatorname{arccot}(x)^n \lambda b - a) dx} b + 1\right) e^{\int (-2 \operatorname{arccot}(}}{c_1 + \int \operatorname{arccot}(x)^n \lambda \, e^{\int (-2 \operatorname{arccot}(x)^n \lambda b + a) dx} dx}$$

✓ Solution by Mathematica

Time used: 11.807 (sec). Leaf size: 240

```
DSolve[y'[x]==\[Lambda]*ArcCot[x]^n*y[x]^2+a*y[x]+a*b-b^2*\[Lambda]*ArcCot[x]^n,y[x],x,Inclu
```

$$\text{Solve}\Bigg[\int_1^x -\frac{\exp\left(-\int_1^{K[2]}\left(2b\lambda\cot^{-1}(K[1])^n-a\right)dK[1]\right)\left(-b\lambda\cot^{-1}(K[2])^n+\lambda y(x)\cot^{-1}(K[2])^n+a\right)}{an\lambda(b+y(x))}dK[2]$$
$$+\int_1^{y(x)}\Bigg(\frac{\exp\left(-\int_1^x\left(2b\lambda\cot^{-1}(K[1])^n-a\right)dK[1]\right)}{an\lambda(b+K[3])^2}$$
$$-\int_1^x\Bigg(\frac{\exp\left(-\int_1^{K[2]}\left(2b\lambda\cot^{-1}(K[1])^n-a\right)dK[1]\right)\left(-b\lambda\cot^{-1}(K[2])^n+\lambda K[3]\cot^{-1}(K[2])^n+a\right)}{an\lambda(b+K[3])^2}-\frac{\exp(}{}$$

18.4 problem 32

Internal problem ID [10599]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \operatorname{arccot}(x)^n y^2 + b\lambda x^m \operatorname{arccot}(x)^n y = bm x^{m-1}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=lambda*arccot(x)^n*y(x)^2-b*lambda*x^m*arccot(x)^n*y(x)+b*m*x^(m-1),y(x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*ArcCot[x]^n*y[x]^2-b*\[Lambda]*x^m*ArcCot[x]^n*y[x]+b*m*x^(m-1),y[x]
```

Not solved

18.5 problem 33

Internal problem ID [10600]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \operatorname{arccot}(x)^n y^2 = \beta m\, x^{m-1} - \lambda\, \beta^2 x^{2m} \operatorname{arccot}(x)^n$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=lambda*arccot(x)^n*y(x)^2+beta*m*x^(m-1)-lambda*beta^2*x^(2*m)*arccot(x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*ArcCot[x]^n*y[x]^2+\[Beta]*m*x^(m-1)-\[Lambda]*\[Beta]^2*x^(2*m)*Arc
```

Not solved

18.6 problem 34

Internal problem ID [10601]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - \lambda \operatorname{arccot}(x)^n (y - a\,x^m - b)^2 = am\,x^{m-1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve(diff(y(x),x)=lambda*arccot(x)^n*(y(x)-a*x^m-b)^2+a*m*x^(m-1),y(x), singsol=all)
```

$$y(x) = -\frac{(-2a\,x^m \operatorname{arccot}(x)^n \lambda - 2\operatorname{arccot}(x)^n \lambda b) \operatorname{arccot}(x)^{-n}}{2\lambda} + \frac{1}{c_1 - \left(\int \operatorname{arccot}(x)^n \lambda dx\right)}$$

✓ Solution by Mathematica

Time used: 2.259 (sec). Leaf size: 44

```
DSolve[y'[x]==\[Lambda]*ArcCot[x]^n*(y[x]-a*x^m-b)^2+a*m*x^(m-1),y[x],x,IncludeSingularSolut
```

$$y(x) \to \frac{1}{-\int_1^x \lambda \cot^{-1}(K[2])^n dK[2] + c_1} + ax^m + b$$

$$y(x) \to ax^m + b$$

18.7 problem 35

Internal problem ID [10602]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x-\lambda \operatorname{arccot}(x)^{n} y^{2}-k y=\lambda b^{2} x^{2 k} \operatorname{arccot}(x)^{n}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 34

```
dsolve(x*diff(y(x),x)=lambda*arccot(x)^n*y(x)^2+k*y(x)+lambda*b^2*x^(2*k)*arccot(x)^n,y(x),
```

$$y(x)=-\tan\left(-b\lambda\left(\int \frac{x^{k} \operatorname{arccot}(x)^{n}}{x}dx\right)+c_{1}\right) b\, x^{k}$$

✓ Solution by Mathematica

Time used: 2.591 (sec). Leaf size: 48

```
DSolve[x*y'[x]==\[Lambda]*ArcCot[x]^n*y[x]^2+k*y[x]+\[Lambda]*b^2*x^(2*k)*ArcCot[x]^n,y[x],x
```

$$y(x)\to \sqrt{b^2}x^k \tan\left(\sqrt{b^2}\int_1^x \lambda \cot^{-1}(K[1])^n K[1]^{k-1}dK[1]+c_1\right)$$

18.8 problem 36

Internal problem ID [10603]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - \left(a\,x^{2m}y^2 + yx^n b + c\right)\operatorname{arccot}\left(x\right)^m + ny = 0}$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)=(a*x^(2*m)*y(x)^2+b*x^n*y(x)+c)*arccot(x)^m-n*y(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]==(a*x^(2*m)*y[x]^2+b*x^n*y[x]+c)*ArcCot[x]^m-n*y[x],y[x],x,IncludeSingularSol
```

Not solved

19 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).

19.1 problem 1

Internal problem ID [10604]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - f(x)\,y = -a^2 - af(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x)=y(x)^2+f(x)*y(x)-a^2-a*f(x),y(x), singsol=all)
```

$$y(x) = a - \frac{e^{\int f(x)dx+2ax}}{\int e^{\int f(x)dx+2ax}dx - c_1}$$

✓ Solution by Mathematica

Time used: 0.719 (sec). Leaf size: 166

```
DSolve[y'[x]==y[x]^2+f[x]*y[x]-a^2-a*f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^x \frac{\exp\left(-\int_1^{K[2]}(-2a-f(K[1]))dK[1]\right)(a+f(K[2])+y(x))}{a-y(x)}dK[2]\right.$$
$$+\int_1^{y(x)}\left(\frac{\exp\left(-\int_1^x(-2a-f(K[1]))dK[1]\right)}{(K[3]-a)^2}\right.$$
$$-\int_1^x\left(\frac{\exp\left(-\int_1^{K[2]}(-2a-f(K[1]))dK[1]\right)(a+f(K[2])+K[3])}{(a-K[3])^2}+\frac{\exp\left(-\int_1^{K[2]}(-2a-f(K[1]))dK[1]\right)}{a-K[3]}\right.$$

19.2 problem 2

Internal problem ID [10605]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 f(x) + ay = -ba - b^2 f(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 83

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*y(x)-a*b-b^2*f(x),y(x), singsol=all)
```

$$y(x) = -\frac{\left(\left(\int f(x)\,\mathrm{e}^{\int (-2bf(x)-a)dx}dx\right)\mathrm{e}^{\int (2bf(x)+a)dx}b + c_1\mathrm{e}^{\int (2bf(x)+a)dx}b + 1\right)\mathrm{e}^{\int (-2bf(x)-a)dx}}{c_1 + \int f(x)\,\mathrm{e}^{\int (-2bf(x)-a)dx}dx}$$

✓ Solution by Mathematica

Time used: 0.955 (sec). Leaf size: 185

```
DSolve[y'[x]==f[x]*y[x]^2-a*y[x]-a*b-b^2*f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^x \frac{\exp\left(-\int_1^{K[2]}(a+2bf(K[1]))dK[1]\right)(a+bf(K[2])-f(K[2])y(x))}{a(b+y(x))}dK[2]\right.$$
$$+\int_1^{y(x)}\left(\frac{\exp\left(-\int_1^x(a+2bf(K[1]))dK[1]\right)}{a(b+K[3])^2}\right.$$
$$-\int_1^x\left(-\frac{\exp\left(-\int_1^{K[2]}(a+2bf(K[1]))dK[1]\right)f(K[2])}{a(b+K[3])}-\frac{\exp\left(-\int_1^{K[2]}(a+2bf(K[1]))dK[1]\right)(a+bf(K[2}{a(b+K[3])^2}\right.$$

19.3 problem 3

Internal problem ID [10606]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - xf(x)\, y = f(x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

```
dsolve(diff(y(x),x)=y(x)^2+x*f(x)*y(x)+f(x),y(x), singsol=all)
```

$$y(x) = \frac{e^{\int \frac{f(x)x^2-2}{x}dx}}{c_1 - \left(\int e^{\int \frac{f(x)x^2-2}{x}dx}dx\right)} - \frac{1}{x}$$

✓ Solution by Mathematica

Time used: 1.074 (sec). Leaf size: 111

```
DSolve[y'[x]==y[x]^2+x*f[x]*y[x]+f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\exp\left(-\int_1^x -f(K[1])K[1]dK[1]\right) + x\int_1^x \frac{\exp\left(-\int_1^{K[2]} -f(K[1])K[1]dK[1]\right)}{K[2]^2}dK[2] + c_1 x}{x^2\left(\int_1^x \frac{\exp\left(-\int_1^{K[2]} -f(K[1])K[1]dK[1]\right)}{K[2]^2}dK[2] + c_1\right)}$$

$$y(x) \to -\frac{1}{x}$$

19.4 problem 4

Internal problem ID [10607]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) + a\,x^n f(x)\,y = an\,x^{-1+n}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*x^n*f(x)*y(x)+a*n*x^(n-1),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a*x^n*f[x]*y[x]+a*n*x^(n-1),y[x],x,IncludeSingularSolutions -> Tru
```

Not solved

19.5 problem 5

Internal problem ID [10608]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) = an\, x^{-1+n} - x^{2n} f(x)\, a^2$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+a*n*x^(n-1)-a^2*x^(2*n)*f(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2+a*n*x^(n-1)-a^2*x^(2*n)*f[x],y[x],x,IncludeSingularSolutions -> Tr
```

Not solved

19.6 problem 6

Internal problem ID [10609]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$
y' + (n+1)\, x^n y^2 - x^{n+1} f(x)\, y = -f(x)
$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 182

```
dsolve(diff(y(x),x)=-(n+1)*x^n*y(x)^2+x^(n+1)*f(x)*y(x)-f(x),y(x), singsol=all)
```

$$
y(x) = \frac{\left(-\mathrm{e}^{\int \frac{x^n f(x) x^2 - 2n - 2}{x} dx} x^n x + \int \left(-x^n n\, \mathrm{e}^{\int x^{1+n} f(x) dx - 2n\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)} - x^n \mathrm{e}^{\int x^{1+n} f(x) dx - 2n\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)}\right) dx}{x \left(\int \left(-x^n n\, \mathrm{e}^{\int x^{1+n} f(x) dx - 2n\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)} - x^n \mathrm{e}^{\int x^{1+n} f(x) dx - 2n\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)}\right) dx + c_1\right)}
$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-(n+1)*x^n*y[x]^2+x^(n+1)*f[x]*y[x]-f[x],y[x],x,IncludeSingularSolutions -> Tr
```

Not solved

19.7 problem 7

Internal problem ID [10610]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - y^2 f(x) - ny = x^{2n} f(x)\, a}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 35

```
dsolve(x*diff(y(x),x)=f(x)*y(x)^2+n*y(x)+a*x^(2*n)*f(x),y(x), singsol=all)
```

$$y(x) = -\tan\left(-\sqrt{a}\left(\int \frac{x^n f(x)}{x} dx\right) + c_1\right)\sqrt{a}\, x^n$$

✓ Solution by Mathematica

Time used: 0.577 (sec). Leaf size: 41

```
DSolve[x*y'[x]==f[x]*y[x]^2+n*y[x]+a*x^(2*n)*f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{a} x^n \tan\left(\sqrt{a}\int_1^x f(K[1])K[1]^{n-1} dK[1] + c_1\right)$$

19.8 problem 8

Internal problem ID [10611]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - x^{2n} f(x)\, y^2 - (x^n f(x)\, a - n)\, y = f(x)\, b}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 64

```
dsolve(x*diff(y(x),x)=x^(2*n)*f(x)*y(x)^2+(a*x^n*f(x)-n)*y(x)+b*f(x),y(x), singsol=all)
```

$$y(x) = -\frac{\left(\tanh\left(\frac{\sqrt{a^4-4a^2b}\left(a\left(\int \frac{x^n f(x)}{x}dx\right)+c_1\right)}{2a^2}\right)\sqrt{a^4-4a^2b}+a^2\right)x^{-n}}{2a}$$

✓ Solution by Mathematica

Time used: 2.272 (sec). Leaf size: 82

```
DSolve[x*y'[x]==x^(2*n)*f[x]*y[x]^2+(a*x^n*f[x]-n)*y[x]+b*f[x],y[x],x,IncludeSingularSolutio
```

$$\text{Solve}\left[\int_1^{\sqrt{\frac{x^{2n}}{b}}y(x)} \frac{1}{K[1]^2-\sqrt{\frac{a^2}{b}}K[1]+1}dK[1] = \int_1^x \frac{bf(K[2])\sqrt{\frac{K[2]^{2n}}{b}}}{K[2]}dK[2]+c_1, y(x)\right]$$

19.9 problem 9

Internal problem ID [10612]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 f(x) - g(x)\, y = -f(x)\, a^2 - a g(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+g(x)*y(x)-a^2*f(x)-a*g(x),y(x), singsol=all)
```

$$y(x) = a - \frac{e^{\int g(x)dx + 2a\left(\int f(x)dx\right)}}{\int e^{\int g(x)dx + 2a\left(\int f(x)dx\right)} f(x)\, dx - c_1}$$

✓ Solution by Mathematica

Time used: 1.122 (sec). Leaf size: 201

```
DSolve[y'[x]==f[x]*y[x]^2+g[x]*y[x]-a^2*f[x]-a*g[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^x -\frac{\exp\left(-\int_1^{K[2]}(-2af(K[1])-g(K[1]))dK[1]\right)(af(K[2])+y(x)f(K[2])+g(K[2]))}{a-y(x)}dK[2] + \int_1^{y(x)}\left(-\int_1^x\left(-\frac{\exp\left(-\int_1^{K[2]}(-2af(K[1])-g(K[1]))dK[1]\right)f(K[2])}{a-K[3]} - \frac{\exp\left(-\int_1^{K[2]}(-2af(K[1])-g\right.}{}\right.\right.\right.$$
$$\left.\left. - \frac{\exp\left(-\int_1^x(-2af(K[1])-g(K[1]))dK[1]\right)}{(K[3]-a)^2}\right)dK[3]=c_1,y(x)\right]$$

19.10 problem 10

Internal problem ID [10613]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) - g(x)\, y = an\, x^{-1+n} - a\, x^n g(x) - x^{2n} f(x)\, a^2$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+g(x)*y(x)+a*n*x^(n-1)-a*x^n*g(x)-a^2*f(x)*x^(2*n),y(x), sing
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2+g[x]*y[x]+a*n*x^(n-1)-a*x^n*g[x]-a^2*f[x]*x^(2*n),y[x],x,IncludeSi
```

Not solved

19.11 problem 11

Internal problem ID [10614]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) + a\,x^n g(x)\,y = an\,x^{-1+n} + a^2 x^{2n}(g(x) - f(x))$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*x^n*g(x)*y(x)+a*n*x^(n-1)+a^2*x^(2*n)*(g(x)-f(x)),y(x), si
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a*x^n*g[x]*y[x]+a*n*x^(n-1)+a^2*x^(2*n)*(g[x]-f[x]),y[x],x,Include
```

Not solved

19.12 problem 12

Internal problem ID [10615]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a\,\mathrm{e}^{\lambda x} y^2 - a\,\mathrm{e}^{\lambda x} f(x)\,y = \lambda f(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 139

```
dsolve(diff(y(x),x)=a*exp(lambda*x)*y(x)^2+a*exp(lambda*x)*f(x)*y(x)+lambda*f(x),y(x), sings
```

$$y(x) = -\frac{\mathrm{e}^{-\lambda x} c_1 \mathrm{e}^{-\lambda x + a\left(\int f(x)\mathrm{e}^{\lambda x}dx\right)}}{\lambda a \left(\left(\int \frac{\mathrm{e}^{-\lambda x + a\left(\int f(x)\mathrm{e}^{\lambda x}dx\right)}}{\lambda}dx\right) c_1 + 1\right)} - \frac{\mathrm{e}^{-\lambda x}\left(\left(\int \frac{\mathrm{e}^{-\lambda x + a\left(\int f(x)\mathrm{e}^{\lambda x}dx\right)}}{\lambda}dx\right) c_1 \lambda^2 + \lambda^2\right)}{\lambda a \left(\left(\int \frac{\mathrm{e}^{-\lambda x + a\left(\int f(x)\mathrm{e}^{\lambda x}dx\right)}}{\lambda}dx\right) c_1 + 1\right)}$$

✓ Solution by Mathematica

Time used: 4.45 (sec). Leaf size: 166

```
DSolve[y'[x]==a*Exp[\[Lambda]*x]*y[x]^2+a*Exp[\[Lambda]*x]*f[x]*y[x]+\[Lambda]*f[x],y[x],x,I
```

$$y(x) \to -\frac{\lambda e^{-2\lambda x}\left(\exp\left(-\int_1^{e^{x\lambda}} -\frac{af\left(\frac{\log(K[1])}{\lambda}\right)}{\lambda} dK[1]\right) + e^{\lambda x}\int_1^{e^{x\lambda}} \frac{\exp\left(-\int_1^{K[2]} -\frac{af\left(\frac{\log(K[1])}{\lambda}\right)}{\lambda} dK[1]\right)}{K[2]^2} dK[2] + c_1 e^{\lambda x}\right)}{a\left(\int_1^{e^{x\lambda}} \frac{\exp\left(-\int_1^{K[2]} -\frac{af\left(\frac{\log(K[1])}{\lambda}\right)}{\lambda} dK[1]\right)}{K[2]^2} dK[2] + c_1\right)}$$

$$y(x) \to -\frac{\lambda e^{\lambda(-x)}}{a}$$

19.13 problem 13

Internal problem ID [10616]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 f(x) + a\,\mathrm{e}^{\lambda x} f(x)\,y = a\lambda\,\mathrm{e}^{\lambda x}}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*exp(lambda*x)*f(x)*y(x)+a*lambda*exp(lambda*x),y(x), sings
```

No solution found

✓ Solution by Mathematica

Time used: 48.456 (sec). Leaf size: 207

```
DSolve[y'[x]==f[x]*y[x]^2-a*Exp[\[Lambda]*x]*f[x]*y[x]+a*\[Lambda]*Exp[\[Lambda]*x],y[x],x,I
```

$$y(x) \to \frac{a\exp\left(\int_1^{e^{x\lambda}} -\frac{af\left(\frac{\log(K[1])}{\lambda}\right)}{\lambda}dK[1] + 2\lambda x\right)\left(\int_1^{e^{x\lambda}} \frac{\exp\left(-\int_1^{K[2]} -\frac{af\left(\frac{\log(K[1])}{\lambda}\right)}{\lambda}dK[1]\right)}{K[2]^2}dK[2] + c_1 \cdots\right.}{\exp\left(\int_1^{e^{x\lambda}} -\frac{af\left(\frac{\log(K[1])}{\lambda}\right)}{\lambda}dK[1] + \lambda x\right)\int_1^{e^{x\lambda}} \frac{\exp\left(-\int_1^{K[2]} -\frac{af\left(\frac{\log(K[1])}{\lambda}\right)}{\lambda}dK[1]\right)}{K[2]^2}dK[2] + c_1\exp\left(\int_1^{e^{x\lambda}} -\frac{af\left(\frac{\log(K[1])}{\lambda}\right)}{\lambda} \cdots\right.}$$

$$y(x) \to ae^{\lambda x}$$

19.14 problem 14

Internal problem ID [10617]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) = a\lambda\, e^{\lambda x} - a^2 e^{2\lambda x} f(x)$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+a*lambda*exp(lambda*x)-a^2*exp(2*lambda*x)*f(x),y(x), singso
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2+a*\[Lambda]*Exp[\[Lambda]*x]-a^2*Exp[2*\[Lambda]*x]*f[x],y[x],x,In
```

Not solved

19.15 problem 15

Internal problem ID [10618]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 f(x) - y\lambda = a^2 \mathrm{e}^{2\lambda x} f(x)}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+lambda*y(x)+a^2*exp(2*lambda*x)*f(x),y(x), singsol=all)
```

$$y(x) = -\tan\left(-a\left(\int f(x)\,\mathrm{e}^{\lambda x} dx\right) + c_1\right) a\,\mathrm{e}^{\lambda x}$$

✓ Solution by Mathematica

Time used: 0.615 (sec). Leaf size: 47

```
DSolve[y'[x]==f[x]*y[x]^2+\[Lambda]*y[x]+a^2*Exp[2*\[Lambda]*x]*f[x],y[x],x,IncludeSingularS
```

$$y(x) \to \sqrt{a^2} e^{\lambda x} \tan\left(\sqrt{a^2} \int_1^x e^{\lambda K[1]} f(K[1]) dK[1] + c_1\right)$$

19.16 problem 16

Internal problem ID [10619]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) + f(x) \left(e^{\lambda x} a + b\right) y = a\lambda\, e^{\lambda x}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-f(x)*(a*exp(lambda*x)+b)*y(x)+a*lambda*exp(lambda*x),y(x), s
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-f[x]*(a*Exp[\[Lambda]*x]+b)*y[x]+a*\[Lambda]*Exp[\[Lambda]*x],y[x]
```

Not solved

19.17 problem 17

Internal problem ID [10620]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - e^{\lambda x} f(x)\, y^2 - (a f(x) - \lambda)\, y = b\, e^{-\lambda x} f(x)$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 68

```
dsolve(diff(y(x),x)=exp(lambda*x)*f(x)*y(x)^2+(a*f(x)-lambda)*y(x)+b*exp(-lambda*x)*f(x),y(x
```

$$y(x) = -\frac{\left(e^{\lambda x} e^{-\lambda x} a^2 + \tanh\left(\frac{\sqrt{a^4-4a^2b}\,\left(a\left(\int f(x)dx\right)+c_1\right)}{2a^2}\right)\sqrt{a^4-4a^2b}\right) e^{-\lambda x}}{2a}$$

✓ Solution by Mathematica

Time used: 1.392 (sec). Leaf size: 87

```
DSolve[y'[x]==Exp[\[Lambda]*x]*f[x]*y[x]^2+(a*f[x]-\[Lambda])*y[x]+b*Exp[-\[Lambda]*x]*f[x],
```

$$\text{Solve}\left[\int_1^{\sqrt{\frac{e^{2x\lambda}}{b}}y(x)} \frac{1}{K[1]^2 - \sqrt{\frac{a^2}{b}}K[1]+1} dK[1] = \int_1^x be^{-\lambda K[2]}\sqrt{\frac{e^{2\lambda K[2]}}{b}} f(K[2])dK[2] + c_1, y(x)\right]$$

19.18 problem 18

Internal problem ID [10621]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) - g(x)\, y = a\lambda\, e^{\lambda x} - a\, e^{\lambda x} g(x) - a^2 e^{2\lambda x} f(x)$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+g(x)*y(x)+a*lambda*exp(lambda*x)-a*exp(lambda*x)*g(x)-a^2*ex
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2+g[x]*y[x]+a*\[Lambda]*Exp[\[Lambda]*x]-a*Exp[\[Lambda]*x]*g[x]-a^2
```

Not solved

19.19 problem 19

Internal problem ID [10622]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) + a\,e^{\lambda x} g(x)\,y = a\lambda\,e^{\lambda x} + a^2 e^{2\lambda x}(g(x) - f(x))$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*exp(lambda*x)*g(x)*y(x)+a*lambda*exp(lambda*x)+a^2*exp(2*l
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a*Exp[\[Lambda]*x]*g[x]*y[x]+a*\[Lambda]*Exp[\[Lambda]*x]+a^2*Exp[
```

Not solved

19.20 problem 20

Internal problem ID [10623]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) = 2a\lambda x \,\mathrm{e}^{\lambda x^2} - a^2 f(x) \,\mathrm{e}^{2\lambda x^2}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+2*a*lambda*x*exp(lambda*x^2)-a^2*f(x)*exp(2*lambda*x^2),y(x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2+2*a*\[Lambda]*x*Exp[\[Lambda]*x^2]-a^2*f[x]*Exp[2*\[Lambda]*x^2],y
```

Not solved

19.21 problem 21

Internal problem ID [10624]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) - y\lambda x = a\,\mathrm{e}^{\lambda x} f(x)$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+lambda*x*y(x)+a*f(x)*exp(lambda*x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2+\[Lambda]*x*y[x]+a*f[x]*Exp[\[Lambda]*x],y[x],x,IncludeSingularSol
```

Not solved

19.22 problem 22

Internal problem ID [10625]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) = -a \tanh(\lambda x)^2 (a f(x) + \lambda) + \lambda a$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*tanh(lambda*x)^2*(a*f(x)+lambda)+a*lambda,y(x), singsol=al
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a*Tanh[\[Lambda]*x]^2*(a*f[x]+\[Lambda])+a*\[Lambda],y[x],x,Includ
```

Not solved

19.23 problem 23

Internal problem ID [10626]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) = -a \coth(\lambda x)^2 (a f(x) + \lambda) + \lambda a$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*coth(lambda*x)^2*(a*f(x)+lambda)+a*lambda,y(x), singsol=al
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a*Coth[\[Lambda]*x]^2*(a*f[x]+\[Lambda])+a*\[Lambda],y[x],x,Includ
```

Not solved

19.24 problem 24

Internal problem ID [10627]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) = -f(x)\, a^2 + a\lambda \sinh(\lambda x) - f(x) \sinh(\lambda x)^2 a^2$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a^2*f(x)+a*lambda*sinh(lambda*x)-a^2*f(x)*sinh(lambda*x)^2,y
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a^2*f[x]+a*\[Lambda]*Sinh[\[Lambda]*x]-a^2*f[x]*Sinh[\[Lambda]*x]^
```

Not solved

19.25 problem 25

Internal problem ID [10628]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - y^2 f(x) = a - a^2 f(x) \ln(x)^2}$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)=f(x)*y(x)^2+a-a^2*f(x)*(ln(x))^2,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]==f[x]*y[x]^2+a-a^2*f[x]*(Log[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

19.26 problem 26

Internal problem ID [10629]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, `_with_symmetry_[F(x),G(x)]`], _Riccati]`

$$\boxed{y'x - f(x)\left(y + a\ln(x)\right)^2 = -a}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x*diff(y(x),x)=f(x)*(y(x)+a*ln(x))^2-a,y(x), singsol=all)
```

$$y(x) = -a\ln(x) + \frac{1}{c_1 - \left(\int \frac{f(x)}{x}dx\right)}$$

✓ Solution by Mathematica

Time used: 0.48 (sec). Leaf size: 42

```
DSolve[x*y'[x]==f[x]*(y[x]+a*Log[x])^2-a,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -a\log(x) + \frac{1}{-\int_1^x \frac{f(K[2])}{K[2]}dK[2] + c_1}$$

$$y(x) \to -a\log(x)$$

19.27 problem 27

Internal problem ID [10630]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) + ax\ln(x) f(x) y = a\ln(x) + a$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*x*ln(x)*f(x)*y(x)+a*ln(x)+a,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a*x*Log[x]*f[x]*y[x]+a*Log[x]+a,y[x],x,IncludeSingularSolutions ->
```

Not solved

19.28 problem 28

Internal problem ID [10631]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + y^2 \ln(x)\, a - af(x)\left(\ln(x)\, x - x\right) y = -f(x)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 348

```
dsolve(diff(y(x),x)=-a*ln(x)*y(x)^2+a*f(x)*(x*ln(x)-x)*y(x)-f(x),y(x), singsol=all)
```

$$y(x) = \frac{x\,\mathrm{e}^{\int \frac{f(x)\ln(x)^2 a\,x^2-2f(x)\ln(x)a\,x^2+a\,x^2 f(x)-2\ln(x)}{x(-1+\ln(x))}dx}\ln(x) - x\,\mathrm{e}^{\int \frac{f(x)\ln(x)^2 a\,x^2-2f(x)\ln(x)a\,x^2+a\,x^2 f(x)-2\ln(x)}{x(-1+\ln(x))}dx} - c_1 a}{ax\left(-\ln(x)\,c_1 a + \ln(x)\left(\int \ln(x)\,\mathrm{e}^{a\left(\int \frac{xf(x)\ln(x)^2}{-1+\ln(x)}dx\right)-2a\left(\int \frac{xf(x)\ln(x)}{-1+\ln(x)}dx\right)+a\left(\int \frac{xf(x)}{-1+\ln(x)}dx\right)-2\left(\int \frac{\ln(x)}{x(-1+\ln(x))}dx\right)}dx\right) +}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-a*Log[x]*y[x]^2+a*f[x]*(x*Log[x]-x)*y[x]-f[x],y[x],x,IncludeSingularSolutions
```

Not solved

19.29 problem 29

Internal problem ID [10632]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \lambda \sin(\lambda x)\, y^2 - f(x)\cos(\lambda x)\, y = -f(x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 118

```
dsolve(diff(y(x),x)=lambda*sin(lambda*x)*y(x)^2+f(x)*cos(lambda*x)*y(x)-f(x),y(x), singsol=a
```

$$y(x) = \frac{c_1 \mathrm{e}^{\int \frac{\sqrt{\frac{1}{2}-\frac{\cos(2\lambda x)}{2}}\left(f(x)\sin(\lambda x)\cos(\lambda x)+2\sqrt{\frac{1}{2}-\frac{\cos(2\lambda x)}{2}}\,\lambda\tan(\lambda x)\right)}{\sin(\lambda x)^2}dx}}{\left(\int -\mathrm{e}^{\int \frac{\sqrt{\frac{1}{2}-\frac{\cos(2\lambda x)}{2}}\left(f(x)\sin(\lambda x)\cos(\lambda x)+2\sqrt{\frac{1}{2}-\frac{\cos(2\lambda x)}{2}}\,\lambda\tan(\lambda x)\right)}{\sin(\lambda x)^2}dx}\sin(\lambda x)\,\lambda dx\right)c_1+1} + \frac{1}{\cos(\lambda x)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*Sin[\[Lambda]*x]*y[x]^2+f[x]*Cos[\[Lambda]*x]*y[x]-f[x],y[x],x,Inclu
```

Not solved

19.30 problem 30

Internal problem ID [10633]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) = -f(x)\,a^2 + a\lambda \sin(\lambda x) + a^2 f(x) \sin(\lambda x)^2$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a^2*f(x)+a*lambda*sin(lambda*x)+a^2*f(x)*sin(lambda*x)^2,y(x
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a^2*f[x]+a*\[Lambda]*Sin[\[Lambda]*x]+a^2*f[x]*Sin[\[Lambda]*x]^2,
```

Not solved

19.31 problem 31

Internal problem ID [10634]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) = -f(x)\, a^2 + \lambda \cos(\lambda x)\, a + f(x)\, a^2 \cos(\lambda x)^2$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a^2*f(x)+a*lambda*cos(lambda*x)+a^2*f(x)*cos(lambda*x)^2,y(x
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a^2*f[x]+a*\[Lambda]*cos[\[Lambda]*x]+a^2*f[x]*Cos[\[Lambda]*x]^2,
```

Not solved

19.32 problem 32

Internal problem ID [10635]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) = -a \tan(\lambda x)^2 (a f(x) - \lambda) + \lambda a$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*tan(lambda*x)^2*(a*f(x)-lambda)+a*lambda,y(x), singsol=all
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a*Tan[\[Lambda]*x]^2*(a*f[x]-\[Lambda])+a*\[Lambda],y[x],x,Include
```

Not solved

19.33 problem 33

Internal problem ID [10636]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 f(x) = -a \cot(\lambda x)^2 (a f(x) - \lambda) + \lambda a}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*cot(lambda*x)^2*(a*f(x)-lambda)+a*lambda,y(x), singsol=all
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a*Cot[\[Lambda]*x]^2*(a*f[x]-\[Lambda])+a*\[Lambda],y[x],x,Include
```

Not solved

20 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.

20.1 problem 34

Internal problem ID [10637]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = -f(x)^2 + f'(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x)=y(x)^2-f(x)^2+diff(f(x),x),y(x), singsol=all)
```

$$y(x) = f(x) + \frac{e^{\int 2f(x)dx}}{c_1 - \left(\int e^{\int 2f(x)dx} dx\right)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2-f[x]^2+f'[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

20.2 problem 35

Internal problem ID [10638]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 f(x) + f(x)\, g(x)\, y = g'(x)}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-f(x)*g(x)*y(x)+diff(g(x),x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-f[x]*g[x]*y[x]+g'[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

20.3 problem 36

Internal problem ID [10639]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + f'(x)\, y^2 - f(x)\, g(x)\, y = -g(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 102

```
dsolve(diff(y(x),x)=-diff(f(x),x)*y(x)^2+f(x)*g(x)*y(x)-g(x),y(x), singsol=all)
```

$$y(x) = \frac{f(x)\,\mathrm{e}^{\int \frac{g(x)f(x)^2-2\frac{d}{dx}f(x)}{f(x)}dx} + \int \left(\frac{d}{dx}f(x)\right)\mathrm{e}^{\int g(x)f(x)dx-2\left(\int \frac{\frac{d}{dx}f(x)}{f(x)}dx\right)}dx - c_1}{f(x)\left(\int \left(\frac{d}{dx}f(x)\right)\mathrm{e}^{\int g(x)f(x)dx-2\left(\int \frac{\frac{d}{dx}f(x)}{f(x)}dx\right)}dx - c_1\right)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-f'[x]*y[x]^2+f[x]*g[x]*y[x]-g[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

20.4 problem 37

Internal problem ID [10640]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$\boxed{y' - g(x)\,(y - f(x))^2 = f'(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=g(x)*(y(x)-f(x))^2+diff(f(x),x),y(x), singsol=all)
```

$$y(x) = f(x) + \frac{1}{c_1 - \left(\int g(x)\,dx\right)}$$

✓ Solution by Mathematica

Time used: 0.35 (sec). Leaf size: 31

```
DSolve[y'[x]==g[x]*(y[x]-f[x])^2+f'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to f(x) + \frac{1}{-\int_1^x g(K[2])dK[2] + c_1}$$

$$y(x) \to f(x)$$

20.5 problem 38

Internal problem ID [10641]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \frac{f'(x)\, y^2}{g(x)} = -\frac{g'(x)}{f(x)}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 57

```
dsolve(diff(y(x),x)=diff(f(x),x)/g(x)*y(x)^2-diff(g(x),x)/f(x),y(x), singsol=all)
```

$$y(x) = -\frac{g(x)\left(\int \frac{\frac{d}{dx}f(x)}{g(x)f(x)^2}dx\right)f(x) + c_1 f(x)\, g(x) + 1}{f(x)^2\left(\int \frac{\frac{d}{dx}f(x)}{g(x)f(x)^2}dx + c_1\right)}$$

✓ Solution by Mathematica

Time used: 0.347 (sec). Leaf size: 160

```
DSolve[y'[x]==f'[x]/g[x]*y[x]^2-g'[x]/f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{1}{(g(x)+f(x)K[2])^2}\right.\right.$$
$$\left.-\int_1^x\left(\frac{2(f(K[1])K[2]^2f'(K[1])-g(K[1])g'(K[1]))}{g(K[1])(g(K[1])+f(K[1])K[2])^3}-\frac{2K[2]f'(K[1])}{g(K[1])(g(K[1])+f(K[1])K[2])^2}\right)dK[1]\right)dK[2]$$
$$\left.+\int_1^x-\frac{f(K[1])y(x)^2f'(K[1])-g(K[1])g'(K[1])}{f(K[1])g(K[1])(g(K[1])+f(K[1])y(x))^2}dK[1]=c_1,y(x)\right]$$

20.6 problem 39

Internal problem ID [10642]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' f(x)^2 - f'(x)\, y^2 + g(x)\,(y - f(x)) = 0}$$

✗ Solution by Maple

```
dsolve(f(x)^2*diff(y(x),x)-diff(f(x),x)*y(x)^2+g(x)*(y(x)-f(x))=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[f[x]^2*y'[x]-f'[x]*y[x]^2+g[x]*(y[x]-f[x])==0,y[x],x,IncludeSingularSolutions -> True
```

Not solved

20.7 problem 40

Internal problem ID [10643]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - f'(x)\,y^2 - a\,\mathrm{e}^{\lambda x} f(x)\,y = \mathrm{e}^{\lambda x} a}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 110

```
dsolve(diff(y(x),x)=diff(f(x),x)*y(x)^2+a*exp(lambda*x)*f(x)*y(x)+a*exp(lambda*x),y(x), sing
```

$$y(x) = -\frac{f(x)\,\mathrm{e}^{\int \frac{f(x)^2 \mathrm{e}^{\lambda x} a - 2\frac{d}{dx} f(x)}{f(x)} dx} + \int \left(\frac{d}{dx} f(x)\right) \mathrm{e}^{a\left(\int f(x)\mathrm{e}^{\lambda x} dx\right) - 2\left(\int \frac{\frac{d}{dx} f(x)}{f(x)} dx\right)} dx + c_1}{f(x)\left(\int \left(\frac{d}{dx} f(x)\right) \mathrm{e}^{a\left(\int f(x)\mathrm{e}^{\lambda x} dx\right) - 2\left(\int \frac{\frac{d}{dx} f(x)}{f(x)} dx\right)} dx + c_1\right)}$$

✓ Solution by Mathematica

Time used: 84.356 (sec). Leaf size: 167

```
DSolve[y'[x]==f'[x]*y[x]^2+a*Exp[\[Lambda]*x]*f[x]*y[x]+a*Exp[\[Lambda]*x],y[x],x,IncludeSin
```

$y(x) \to$

$$-\frac{a \exp\left(\int_1^{e^{x\lambda}} -\frac{af\left(\frac{\log(K[1])}{\lambda}\right)}{\lambda} dK[1]\right)\left(1 + c_1 \int_1^{e^{x\lambda}} \exp\left(-\int_1^{K[2]} -\frac{af\left(\frac{\log(K[1])}{\lambda}\right)}{\lambda} dK[1]\right) dK[2]\right)}{af\left(\frac{\log(e^{\lambda x})}{\lambda}\right) \exp\left(\int_1^{e^{x\lambda}} -\frac{af\left(\frac{\log(K[1])}{\lambda}\right)}{\lambda} dK[1]\right)\left(1 + c_1 \int_1^{e^{x\lambda}} \exp\left(-\int_1^{K[2]} -\frac{af\left(\frac{\log(K[1])}{\lambda}\right)}{\lambda} dK[1]\right) dK[2]\right) -}$$

20.8 problem 41

Internal problem ID [10644]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) - y g'(x) = a f(x) \, \mathrm{e}^{2g(x)}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 28

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+diff(g(x),x)*y(x)+a*f(x)*exp(2*g(x)),y(x), singsol=all)
```

$$y(x) = -\tan\left(-\sqrt{a}\left(\int f(x)\, \mathrm{e}^{g(x)} dx\right) + c_1\right)\sqrt{a}\, \mathrm{e}^{g(x)}$$

✓ Solution by Mathematica

Time used: 0.635 (sec). Leaf size: 41

```
DSolve[y'[x]==f[x]*y[x]^2+g'[x]*y[x]+a*f[x]*Exp[2*g[x]],y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \sqrt{a} e^{g(x)} \tan\left(\sqrt{a} \int_1^x e^{g(K[1])} f(K[1]) dK[1] + c_1\right)$$

20.9 problem 42

Internal problem ID [10645]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.
Problem number: 42.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = -\frac{f''(x)}{f(x)}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 55

```
dsolve(diff(y(x),x)=y(x)^2-diff(f(x),x$2)/f(x),y(x), singsol=all)
```

$$y(x) = -\frac{\left(\int \frac{1}{f(x)^2}dx\right)\left(\frac{d}{dx}f(x)\right) + c_1\left(\frac{d}{dx}f(x)\right)}{\left(\int \frac{1}{f(x)^2}dx + c_1\right)f(x)} - \frac{1}{\left(\int \frac{1}{f(x)^2}dx + c_1\right)f(x)^2}$$

✓ Solution by Mathematica

Time used: 0.365 (sec). Leaf size: 132

```
DSolve[y'[x]==y[x]^2-f''[x]/f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{1}{(f(x)K[2]+f'(x))^2} - \int_1^x\left(\frac{2(f(K[1])K[2]^2-f''(K[1]))}{(f(K[1])K[2]+f'(K[1]))^3}-\frac{2K[2]}{(f(K[1])K[2]+f'(K[1]))^2}\right)dK[1]\right)dK[2] + \int_1^x -\frac{f(K[1])y(x)^2-f''(K[1])}{f(K[1])\,(f(K[1])y(x)+f'(K[1]))^2}dK[1]=c_1,y(x)\right]$$

21 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations

21.1 problem 1

Internal problem ID [10646]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = a^2 f(ax + b)}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+a^2*f(a*x+b),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+a^2*f[a*x+b],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

21.2 problem 2

Internal problem ID [10647]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = \frac{f\left(\frac{1}{x}\right)}{x^4}}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+1/x^4*f(1/x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+1/x^4*f[1/x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

21.3 problem 3

Internal problem ID [10648]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = \frac{f\left(\frac{ax+b}{cx+d}\right)}{(cx+d)^4}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+1/(c*x+d)^4*f((a*x+b)/(c*x+d)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+1/(c*x+d)^4*f[(a*x+b)/(c*x+d)],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

21.4 problem 4

Internal problem ID [10649]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' x^2 - x^4 f(x)\, y^2 = 1}$$

✗ Solution by Maple

```
dsolve(x^2*diff(y(x),x)=x^4*f(x)*y(x)^2+1,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y'[x]==x^4*f[x]*y[x]^2+1,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

21.5 problem 5

Internal problem ID [10650]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x^2 - x^4y^2 = x^{2n}f(x^n a + b) - \frac{n^2}{4} + \frac{1}{4}$$

✗ Solution by Maple

```
dsolve(x^2*diff(y(x),x)=x^4*y(x)^2+x^(2*n)*f(a*x^n+b)+1/4*(1-n^2),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y'[x]==x^4*y[x]^2+x^(2*n)*f[a*x^n+b]+1/4*(1-n^2),y[x],x,IncludeSingularSolutions
```

Not solved

21.6 problem 6

Internal problem ID [10651]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 f(x) - g(x)\, y = h(x)}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+g(x)*y(x)+h(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2+g[x]*y[x]+h[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

21.7 problem 7

Internal problem ID [10652]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = e^{2\lambda x} f\left(e^{\lambda x}\right) - \frac{\lambda^2}{4}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+exp(2*lambda*x)*f(exp(lambda*x))-1/4*lambda^2,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+Exp[2*\[Lambda]*x]*f[Exp[\[Lambda]*x]]-1/4*\[Lambda]^2,y[x],x,IncludeSi
```

Not solved

21.8 problem 8

Internal problem ID [10653]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = -\frac{\lambda^2}{4} + \frac{e^{2\lambda x} f\left(\frac{e^{\lambda x} a+b}{e^{\lambda x} c+d}\right)}{\left(e^{\lambda x} c+d\right)^4}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2-lambda^2/4+exp(2*lambda*x)/(c*exp(lambda*x)+d)^4*f((a*exp(lambda*
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2-\[Lambda]^2/4+Exp[2*\[Lambda]*x]/(c*Exp[\[Lambda]*x]+d)^4*f[(a*Exp[\[La
```

Not solved

21.9 problem 9

Internal problem ID [10654]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = -\lambda^2 + \frac{f(\coth(\lambda x))}{\sinh(\lambda x)^4}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2-lambda^2+sinh(lambda*x)^(-4)*f(coth(lambda*x)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2-\[Lambda]^2+Sinh[\[Lambda]*x]^(-4)*f[Coth[\[Lambda]*x]],y[x],x,IncludeS
```

Not solved

21.10 problem 10

Internal problem ID [10655]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = -\lambda^2 + \frac{f(\tanh(\lambda x))}{\cosh(\lambda x)^4}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2-lambda^2+cosh(lambda*x)^(-4)*f(tanh(lambda*x)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2-\[Lambda]^2+Cosh[\[Lambda]*x]^(-4)*f[Tanh[\[Lambda]*x]],y[x],x,IncludeS
```

Not solved

21.11 problem 11

Internal problem ID [10656]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x^2 - y^2x^2 = f(a\ln(x) + b) + \frac{1}{4}$$

✗ Solution by Maple

```
dsolve(x^2*diff(y(x),x)=x^2*y(x)^2+f(a*ln(x)+b)+1/4,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y'[x]==x^2*y[x]^2+f[a*Log[x]+b]+1/4,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

21.12 problem 12

Internal problem ID [10657]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = \lambda^2 + \frac{f(\cot(\lambda x))}{\sin(\lambda x)^4}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+lambda^2+sin(lambda*x)^(-4)*f(cot(lambda*x)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+\[Lambda]^2+Sin[\[Lambda]*x]^(-4)*f[Cot[\[Lambda]*x]],y[x],x,IncludeSin
```

Not solved

21.13 problem 13

Internal problem ID [10658]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = \lambda^2 + \frac{f(\tan(\lambda x))}{\cos(\lambda x)^4}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+lambda^2+cos(lambda*x)^(-4)*f(tan(lambda*x)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+\[Lambda]^2+Cos[\[Lambda]*x]^(-4)*f[Tan[\[Lambda]*x]],y[x],x,IncludeSin
```

Not solved

21.14 problem 14

Internal problem ID [10659]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = \lambda^2 + \frac{f\left(\frac{\sin(\lambda x+a)}{\sin(\lambda x+b)}\right)}{\sin(\lambda x + b)^4}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+lambda^2+sin(lambda*x+b)^(-4)*f(sin(lambda*x+a)/sin(lambda*x+b)),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+\[Lambda]^2+Sin[\[Lambda]*x+b]^(-4)*f[Sin[\[Lambda]*x+a]/Sin[\[Lambda]*
```

Not solved

22 Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions

22.1 problem 1

Internal problem ID [10660]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'y - y = A}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(y(x)*diff(y(x),x)-y(x)=A,y(x), singsol=all)
```

$$y(x) = -A\left(\text{LambertW}\left(-\frac{e^{-1-\frac{c_1}{A}-\frac{x}{A}}}{A}\right)+1\right)$$

✓ Solution by Mathematica

Time used: 60.032 (sec). Leaf size: 28

```
DSolve[y[x]*y'[x]-y[x]==A,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -A\left(1+W\left(-\frac{e^{-\frac{A+x+c_1}{A}}}{A}\right)\right)$$

22.2 problem 2

Internal problem ID [10661]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y'y - y = xA + B}$$

✓ Solution by Maple

Time used: 1.0 (sec). Leaf size: 119

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*x+B,y(x), singsol=all)
```

$$y(x) = -\frac{\operatorname{RootOf}\left(\_Z^2 - A + e^{\operatorname{RootOf}\left((xA+B)^2\left(4\tanh\left(-\frac{\_Z\sqrt{4A+1}}{2}+\ln(xA+B)\sqrt{4A+1}+c_1\sqrt{4A+1}\right)^2 A+\tanh\left(-\frac{\_Z\sqrt{4A+1}}{2}+\ln(xA+B)\right.\right.\right.}\right.}{A}$$

✓ Solution by Mathematica

Time used: 0.184 (sec). Leaf size: 88

```
DSolve[y[x]*y'[x]-y[x]==A*x+B,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{\frac{2\arctan\left(\frac{\frac{2Ay(x)}{Ax+B}-1}{\sqrt{-4A-1}}\right)}{\sqrt{-4A-1}} + \log\left(-\frac{Ay(x)^2}{(Ax+B)^2} + \frac{y(x)}{Ax+B} + 1\right)}{2A} = \frac{\log(Ax+B)}{A} + c_1, y(x)\right]$$

22.3 problem 3

Internal problem ID [10662]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{2x}{9} + A + \frac{B}{\sqrt{x}}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 89

```
dsolve(y(x)*diff(y(x),x)-y(x)=-2/9*x+A+B*x^(-1/2),y(x), singsol=all)
```

$$y(x) = -\frac{A\left(9A\sqrt{x} - 2x^{\frac{3}{2}} + 9B\right)}{3\left(A\sqrt{x} + \operatorname{RootOf}\left(9A^3\left(\int^{-Z} \frac{1}{-2\_a^3B^2 + 9\_aA^3 - 9A^3}d\_a\right) + \int -\frac{9A}{2(9xA - 2x^2 + 9B\sqrt{x})}dx + c_1\right)B\right)}$$

✓ Solution by Mathematica

Time used: 8.154 (sec). Leaf size: 415

```
DSolve[y[x]*y'[x]-y[x]==-2/9*x+A+B*x^(-1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\Bigg[6\text{RootSum}\Bigg[8\#1^6 - 72\#1^4A - 36\#1^4y(x) - 72\#1^3B + 162\#1^2A^2$$
$$+ 162\#1^2Ay(x) + 54\#1^2y(x)^2 + 324\#1AB + 162\#1By(x) - 81Ay(x)^2 + 162B^2$$
$$-27y(x)^3\&, \frac{-2\#1^3\log\left(\sqrt{x}-\#1\right) + 9\#1A\log\left(\sqrt{x}-\#1\right) + 9B\log\left(\sqrt{x}-\#1\right) + 9\#1y(x)\log\left(\sqrt{x}-\right.}{8\#1^5 - 48\#1^3A - 24\#1^3y(x) - 36\#1^2B + 54\#1A^2 + 54\#1Ay(x) + 18\#1y(x)^2 + 54AB +}$$
$$+\int_1^{y(x)}\left(\frac{162K[1]}{8x^3 - 72Ax^2 - 36K[1]x^2 - 72Bx^{3/2} + 162A^2x + 54K[1]^2x + 162AK[1]x + 324AB\sqrt{x} + 162BK}\right.$$
$$+\frac{162K[1]}{-8x^3 + 72Ax^2 + 36K[1]x^2 + 72Bx^{3/2} - 162A^2x - 54K[1]^2x - 162AK[1]x - 324AB\sqrt{x} - 162BK[1]\sqrt{}}$$

22.4 problem 4

Internal problem ID [10663]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$yy' - y = 2A\left(\sqrt{x} + 4A + \frac{3A^2}{\sqrt{x}}\right)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 109

```
dsolve(y(x)*diff(y(x),x)-y(x)=2*A*(x^(1/2)+4*A+3*A^2*x^(-1/2)),y(x), singsol=all)
```

$$c_1 + \frac{4\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{A^2}{y(x)}}\,(3A+\sqrt{x})}{\sqrt{\frac{-3A^2-4A\sqrt{x}-x+y(x)}{y(x)}}\,A}\right)\sqrt{-\frac{A^2}{y(x)}} - \sqrt{2}\,\sqrt{\frac{-6A^2-8A\sqrt{x}-2x+2y(x)}{y(x)}}}{\sqrt{-\frac{A^2}{y(x)}}} = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==2*A*(x^(1/2)+4*A+3*A^2*x^(-1/2)),y[x],x,IncludeSingularSolutions ->
```

Not solved

22.5 problem 5

Internal problem ID [10664]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = Ax + \frac{B}{x} - \frac{B^2}{x^3}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 171

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*x+B/x-B^2*x^(-3),y(x), singsol=all)
```

$$c_1 + \frac{\left(-y(x)\, B\, x^2 - B^2 x\right)\left(\int^{-\frac{x^2}{2xy(x)+2B}} \frac{e^{\frac{2\,\operatorname{arctanh}\left(\frac{4A\_a-1}{\sqrt{4A+1}}\right)}{\sqrt{4A+1}}}\,(4A\_a^2-2\_a-1)}{\_a^2} d\_a\right) + 2y(x)\, e^{-\frac{2\,\operatorname{arctanh}\left(\frac{2A\,x^2+xy(x)+B}{\sqrt{4A+1}\,(xy(x)+B)}\right)}{\sqrt{4A+1}}}}{x\,(xy(x)+B)} = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==A*x+B/x-B^2*x^(-3),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.6 problem 6

Internal problem ID [10665]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y'y - y = A\,x^{k-1} - kB\,x^k + k\,B^2x^{-1+2k}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*x^(k-1)-k*B*x^k+k*B^2*x^(2*k-1),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==A*x^(k-1)-k*B*x^k+k*B^2*x^(2*k-1),y[x],x,IncludeSingularSolutions ->
```

Not solved

22.7 problem 7

Internal problem ID [10666]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$yy' - y = \frac{A}{x} - \frac{A^2}{x^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 117

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*x^(-1)-A^2*x^(-3),y(x), singsol=all)
```

$$y(x) = \frac{\left(x^2 c_1 - A\, e^{\operatorname{RootOf}\left(2\_ZA\, e^{2\_Z} - e^{2\_Z}x^2 + 2c_1 e^{\_Z}x^2 - c_1^2x^2 - 2A\, e^{2\_Z} + 2Ac_1 e^{\_Z}\right)}\right) e^{-\operatorname{RootOf}\left(2\_ZA\, e^{2\_Z} - e^{2\_Z}x^2 + 2c_1 e^{\_Z}x^2 - c_1^2x^2 - 2\ldots\right.}}{x}$$

✓ Solution by Mathematica

Time used: 0.569 (sec). Leaf size: 63

```
DSolve[y[x]*y'[x]-y[x]==A*x^(-1)-A^2*x^(-3),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x^2\left(-\frac{1}{A} + \frac{2x^2 \log\left(\frac{x^2}{A+xy(x)}\right) + 2A - c_1x^2 + 2xy(x)}{(A - x^2 + xy(x))^2}\right) = 0, y(x)\right]$$

22.8 problem 8

Internal problem ID [10667]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - y = A + B\,\mathrm{e}^{-\frac{2x}{A}}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 76

```
dsolve(y(x)*diff(y(x),x)-y(x)=A+B*exp(-2*x/A),y(x), singsol=all)
```

$$c_1 - 2A\arctan\left(\frac{y(x)+A}{y\left(x\right)\sqrt{\frac{-AB\,\mathrm{e}^{-\frac{2x}{A}}-(y(x)+A)^2}{y(x)^2}}}\right) - 2\sqrt{\frac{-AB\,\mathrm{e}^{-\frac{2x}{A}}-\left(y\left(x\right)+A\right)^2}{y\left(x\right)^2}}\,y(x) = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==A+B*Exp[-2*x/A],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.9 problem 9

Internal problem ID [10668]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - y = A\left(e^{\frac{2x}{A}} - 1\right)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 82

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*(exp(2*x/A)-1),y(x), singsol=all)
```

$$c_1 + 2\arctan\left(\frac{A - y(x)}{y(x)\sqrt{\frac{e^{\frac{2x}{A}}A^2 - (A - y(x))^2}{y(x)^2}}}\right)A + 2\sqrt{\frac{e^{\frac{2x}{A}}A^2 - (A - y(x))^2}{y(x)^2}}\,y(x) = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==A*(Exp[2*x/A]-1),y[x],x,IncludeSingularSolutions -> True]
```

{}

22.10 problem 10

Internal problem ID [10669]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class A‘]]`

$$y'y - y = -\frac{2(m+1)}{(m+3)^2} + A\,x^m$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-(2*(m+1))/(m+3)^2+A*x^m,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-(2*(m+1))/(m+3)^2+A*x^m,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.11 problem 11

Internal problem ID [10670]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$\boxed{yy' - y = -\frac{2x}{9} + 6A^2\left(\frac{2A}{\sqrt{x}} + 1\right)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 331

```
dsolve(y(x)*diff(y(x),x)-y(x)=-2/9*x+6*A^2*(1+2*A*x^(-1/2)),y(x), singsol=all)
```

$$y(x) = \frac{}{3\,\mathrm{e}^{\mathrm{RootOf}\left(18A^2\ln\left(\frac{4(3A-\sqrt{x})(6A-\sqrt{x})(36A^2-x)}{(9A^2-x)(6A+\sqrt{x})(3A+\sqrt{x})(\mathrm{e}^{\_Z}+9)^2}\right)\mathrm{e}^{\_Z}+108A^2c_1\mathrm{e}^{\_Z}+36A^2\mathrm{e}^{\_Z}\_Z+3A\sqrt{x}\,\ln\left(\frac{4(3A-\sqrt{x})(6A-\sqrt{x})(36A^2-x)}{(9A^2-x)(6A+\sqrt{x})(3A+\sqrt{x})(\mathrm{e}^{\_Z}+9)^2}\right.}}$$

✓ Solution by Mathematica

Time used: 12.331 (sec). Leaf size: 488

```
DSolve[y[x]*y'[x]-y[x]==-2/9*x+6*A^2*(1+2*A*x^(-1/2)),y[x],x,IncludeSingularSolutions -> Tru
```

$$\text{Solve}\left[\frac{2^{2/3}\left(\frac{-\frac{6(6A-\sqrt{x})(3A+\sqrt{x})^2}{y(x)}-9\sqrt{x}}{\sqrt[3]{A^3}}+54\right)\left(\frac{6(6A-\sqrt{x})(3A+\sqrt{x})^2+9\sqrt{x}y(x)}{\sqrt[3]{A^3}y(x)}+27\right)\left(-\frac{\left(3\left(3\sqrt[3]{A^3}+\sqrt{x}\right)y(x)+2(6A-\sqrt{x}\cdots}{\cdots}\right.}{6561\left(\frac{\left(2(6A-\cdots}{\cdots}\right.}\right.$$

$$\left.+c_1, y(x)\right]$$

22.12 problem 12

Internal problem ID [10671]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y = \frac{2m-2}{(-3+m)^2} + \frac{2A\left(m(m+3)\sqrt{x} + (4m^2+3m+9)A + \frac{3m(m+3)A^2}{\sqrt{x}}\right)}{(-3+m)^2}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=2*(m-1)/(m-3)^2+2*A/(m-3)^2*(m*(m+3)*x^(1/2)+(4*m^2+3*m+9)*A+3
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==2*(m-1)/(m-3)^2+2*A/(m-3)^2*(m*(m+3)*x^(1/2)+(4*m^2+3*m+9)*A+3*m*(m+
```

Not solved

22.13 problem 13

Internal problem ID [10672]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = \frac{(2m+1)\,x}{4m^2} + \frac{A}{x} - \frac{A^2}{x^3}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 165

```
dsolve(y(x)*diff(y(x),x)-y(x)=(2*m+1)/(4*m^2)*x+A*1/x-A^2*1/(x^3),y(x), singsol=all)
```

$$c_1 + \frac{y(x)\,2^{-\frac{m}{m+1}}\left(\frac{-2y(x)mx-2Am-x^2}{2xy(x)+2A}\right)^{\frac{1}{m+1}}(xy(x)+A)\left(\frac{(-2x^2+2xy(x)+2A)m-x^2}{xy(x)+A}\right)^{\frac{1+2m}{m+1}} - \left(\int^{-\frac{x^2}{2xy(x)+2A}} \frac{(-m+\_a)^{\frac{1}{m+1}}(}{}\right.}{x} = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==(2*m+1)/(4*m^2)*x+A*1/x-A^2*1/(x^3),y[x],x,IncludeSingularSolutions
```

Not solved

22.14 problem 14

Internal problem ID [10673]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - y = \frac{4}{9}x + 2A\,x^2 + 2A^2x^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 177

```
dsolve(y(x)*diff(y(x),x)-y(x)=4/9*x+2*A*x^2+2*A^2*x^3,y(x), singsol=all)
```

$$c_1 - \frac{9\left(-\frac{\sqrt{\frac{(3xA+1)^2}{1+(3x-9y(x))A}}\left(\frac{1}{3}+(-3y(x)+x)A\right)\left(\int^{\frac{(3xA+1)^2}{1+(3x-9y(x))A}}\frac{\left(\_a^2-1\right)^{\frac{1}{4}}}{\sqrt{\_a}}d\_a\right)}{3} + \sqrt{3}\,Ay(x)\,(3xA+1)\left(\frac{\left(A\,x^2+\frac{x}{3}+y(x)\right)A\left(\frac{2}{9}+\right.}{\left(\frac{1}{3}+(-3y(x)\right.}\right.\right.}{\sqrt{\frac{(3xA+1)^2}{1+(3x-9y(x))A}}\,(1+3\,(-3y\,(x)+x)\,A)}$$

$$= 0$$

✓ Solution by Mathematica

Time used: 3.439 (sec). Leaf size: 170

```
DSolve[y[x]*y'[x]-y[x]==4/9*x+2*A*x^2+2*A^2*x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\sqrt[4]{\frac{(-9Ay(x)+3Ax+1)^2}{(3Ax+1)^4}-1}\left(\frac{(-9Ay(x)+3Ax+1)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\frac{(3Ax}{}\right.}{2\sqrt[4]{3}(3Ax+1)\sqrt{(3Ax+1)^2}\sqrt[4]{\frac{A\left(6(3Ax+1)y(x)-27Ay(x)^2+x\right.}{(3Ax+1)^4}}}\right.\right.$$
$$\left.\left.+\sqrt{(3Ax+1)^2}\right)+c_1=0,y(x)\right]$$

22.15 problem 15

Internal problem ID [10674]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y = -\frac{3x}{16} + \frac{5A}{x^{\frac{1}{3}}} - \frac{12A^2}{x^{\frac{5}{3}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-3/16*x+5*A*x^(-1/3)-12*A^2*x^(-5/3),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-3/16*x+5*A*x^(-1/3)-12*A^2*x^(-5/3),y[x],x,IncludeSingularSolutions
```

Not solved

22.16 problem 16

Internal problem ID [10675]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$\boxed{yy' - y = \frac{A}{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 57

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*1/x,y(x), singsol=all)
```

$$c_1 + \frac{-\operatorname{erf}\left(\frac{(x-y(x))\sqrt{2}}{2\sqrt{-A}}\right)\sqrt{2}\,\sqrt{\pi}\,x - 2\,\mathrm{e}^{\frac{(x-y(x))^2}{2A}}\sqrt{-A}}{x} = 0$$

✓ Solution by Mathematica

Time used: 0.827 (sec). Leaf size: 64

```
DSolve[y[x]*y'[x]-y[x]==A*1/x,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{x}{\sqrt{A}} = \frac{2e^{\frac{(x-y(x))^2}{2A}}}{\sqrt{2\pi}\text{erfi}\left(\frac{y(x)-x}{\sqrt{2}\sqrt{A}}\right) + 2c_1}, y(x)\right]$$

22.17 problem 17

Internal problem ID [10676]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$yy' - y = -\frac{x}{4} + \frac{A\left(\sqrt{x} + 5A + \frac{3A^2}{\sqrt{x}}\right)}{4}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 252

```
dsolve(y(x)*diff(y(x),x)-y(x)=-1/4*x+1/4*A*(x^(1/2)+5*A+3*A^2*x^(-1/2)),y(x), singsol=all)
```

$$c_1 - \frac{-2A\left(\int^{\frac{6A\sqrt{x}-2x+3y(x)}{12A^2-4A\sqrt{x}+2y(x)}} \frac{e^{-\frac{2}{2\_a+1}}\sqrt{2\_a+1}}{\sqrt{2\_a-3}}d\_a\right)\left(6A^2-2A\sqrt{x}+y(x)\right)\sqrt{-\frac{\left(3A-\sqrt{x}\right)^2}{6A^2-2A\sqrt{x}+y(x)}} + y(x)\,e^{\frac{-6A^2+2A\sqrt{x}}{3A^2+2A\sqrt{x}-x}}}{\sqrt{-\frac{\left(3A-\sqrt{x}\right)^2}{6A^2-2A\sqrt{x}+y(x)}}\left(6A^2-2A\sqrt{x}+y\left(x\right)\right)} = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-1/4*x+1/4*A*(x^(1/2)+5*A+3*A^2*x^(-1/2)),y[x],x,IncludeSingularSolu
```

Not solved

22.18 problem 18

Internal problem ID [10677]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$yy' - y = \frac{2a^2}{\sqrt{8a^2 + x^2}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 720

```
dsolve(y(x)*diff(y(x),x)-y(x)=2*a^2/sqrt(x^2+8*a^2),y(x), singsol=all)
```

$$c_1 + \frac{128\sqrt{-\sqrt{8a^2+x^2}\,x + 4a^2 + x^2}\,\sqrt{2}\,\sqrt{\pi}\left(\frac{\left(-\frac{33a^4x}{16} + a^4y(x) - \frac{23a^2x^3}{32} + \frac{21a^2x^2y(x)}{32} - \frac{x^5}{32} + \frac{x^4y(x)}{32}\right)\sqrt{8a^2+x^2}}{4} + a^6 + \frac{75a^4x^2}{64} \cdots\right.}{\sqrt{-\cdots}}$$
$$= 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==2*a^2/Sqrt[x^2+8*a^2],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.19 problem 19

Internal problem ID [10678]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = 2x + \frac{A}{x^2}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 169

```
dsolve(y(x)*diff(y(x),x)-y(x)=2*x+A*x^(-2),y(x), singsol=all)
```

$$c_1 + \frac{-6\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{\frac{x\left(A^2\right)^{\frac{1}{3}}}{A}}\,(-y(x)+2x)}{\sqrt{\frac{\left(4x^3-4y(x)x^2+xy(x)^2+2A\right)\left(A^2\right)^{\frac{1}{3}}}{y(x)^2 A}}\,y(x)}\right) Ax\sqrt{\frac{x(A^2)^{\frac{1}{3}}}{A}} + \sqrt{\frac{\left(4x^3-4y(x)x^2+xy(x)^2+2A\right)(A^2)^{\frac{1}{3}}}{y(x)^2 A}}\,y(x)\,(-2x}{x\sqrt{\frac{x(A^2)^{\frac{1}{3}}}{A}}} = 0$$

✓ Solution by Mathematica

Time used: 2.08 (sec). Leaf size: 233

```
DSolve[y[x]*y'[x]-y[x]==2*x+A*x^(-2),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[c_1=\right.$$
$$-\frac{i\sqrt{-\frac{2A+4x^3-4x^2y(x)+xy(x)^2}{A}}\left(-6\sqrt{A}x^{3/2}\text{arcsinh}\left(\frac{\sqrt{x}(2x-y(x))}{\sqrt{2}\sqrt{A}}\right)+x^2(-y(x))\sqrt{\frac{2A+4x^3-4x^2y(x)+xy(x)^2}{A}}+xy(x)^2\right.}{4\sqrt{A}x^{3/2}\sqrt{\frac{2A+4x^3-4x^2y(x)+xy(x)^2}{A}}}$$

22.20 problem 20

Internal problem ID [10679]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y = -\frac{6X}{25} + \frac{2A\left(2\sqrt{x} + 19A + \frac{6A^2}{\sqrt{x}}\right)}{25}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-6/25*X+2/25*A*(2*x^(1/2)+19*A+6*A^2*x^(-1/2)),y(x), singsol=a
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-6/25*X+2/25*A*(2*x^(1/2)+19*A+6*A^2*x^(-1/2)),y[x],x,IncludeSingula
```

Not solved

22.21 problem 21

Internal problem ID [10680]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$y'y - y = \frac{3x}{8} + \frac{3\sqrt{a^2 + x^2}}{8} - \frac{a^2}{16\sqrt{a^2 + x^2}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=3/8*x+3/8*sqrt(x^2+a^2)-a^2/(16*sqrt(x^2+a^2)),y(x), singsol=a
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==3/8*x+3/8*Sqrt[x^2+a^2]-a^2/(16*Sqrt[x^2+a^2]),y[x],x,IncludeSingula
```

Not solved

22.22 problem 22

Internal problem ID [10681]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{4x}{25} + \frac{A}{\sqrt{x}}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 165

```
dsolve(y(x)*diff(y(x),x)-y(x)=-4/25*x+A*x^(-1/2),y(x), singsol=all)
```

$$c_1 + \frac{\frac{4\left(10xA-4\sqrt{A\,x^{\frac{3}{2}}}\,x+5\sqrt{A\,x^{\frac{3}{2}}}\,y(x)\right)\left(10xA+4\sqrt{A\,x^{\frac{3}{2}}}\,x-5\sqrt{A\,x^{\frac{3}{2}}}\,y(x)\right)(4x-5y(x))\left(150A\sqrt{x}-16x^2+40xy(x)-25y(x)^2\right)}{A^2x^{\frac{3}{2}}\left(100A\sqrt{x}-16x^2+40xy(x)-25y(x)^2\right)} - 5000A}{\left(\frac{100A\sqrt{x}-16x^2+40xy(x)-25y(x)^2}{A\sqrt{x}}\right)^{\frac{3}{2}}\sqrt{A\,x^{\frac{3}{2}}}} = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-4/25*x+A*x^(-1/2),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.23 problem 23

Internal problem ID [10682]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$yy' - y = -\frac{9x}{100} + \frac{A}{x^{\frac{5}{3}}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 551

```
dsolve(y(x)*diff(y(x),x)-y(x)=-9/100*x+A*x^(-5/3),y(x), singsol=all)
```

$$c_1 + \frac{10460353203000A\,x^{\frac{44}{3}} - 895371796800000A\,x^{\frac{35}{3}}y(x)^3 + 1205308188000000A\,x^{\frac{32}{3}}y(x)^4 - 8928208800000}{}$$

$= 0$

✓ Solution by Mathematica

Time used: 60.566 (sec). Leaf size: 7909

```
DSolve[y[x]*y'[x]-y[x]==-9/100*x+A*x^(-5/3),y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

22.24 problem 24

Internal problem ID [10683]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$yy' - y = -\frac{12x}{49} + \frac{2A\left(5\sqrt{x} + 34A + \frac{15A^2}{\sqrt{x}}\right)}{49}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 274

```
dsolve(y(x)*diff(y(x),x)-y(x)=-12/49*x+2/49*A*(5*x^(1/2)+34*A+15*A^2*x^(-1/2)),y(x), singsol
```

$$\frac{\left(3A-\sqrt{x}\right)\left(36A^4+120A^3\sqrt{x}-80A\,x^{\frac{3}{2}}+52A^2x+84A^2y(x)+140A\sqrt{x}\,y(x)+16x^2-56xy(x)+49y(x\right.}{8\left(\frac{15A^2+4A\sqrt{x}-3x+7y(x)}{6A^2-2A\sqrt{x}+y(x)}\right)^{\frac{3}{2}}\sqrt{-\frac{\left(3A-\sqrt{x}\right)^2}{6A^2-2A\sqrt{x}+y(x)}}\left(6A^2-2A\sqrt{x}+y\left(x\right)\right)^3A}$$
$$-\frac{\left(54A^2+6A\sqrt{x}-8x+21y(x)\right)\sqrt{-\frac{2\left(9A^2-6A\sqrt{x}+x\right)}{6A^2-2A\sqrt{x}+y(x)}}\sqrt{2}}{12\left(6A^2-2A\sqrt{x}+y\left(x\right)\right)\sqrt{\frac{15A^2+4A\sqrt{x}-3x+7y(x)}{6A^2-2A\sqrt{x}+y(x)}}}+c_1=0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-12/49*x+2/49*A*(5*x^(1/2)+34*A+15*A^2*x^(-1/2)),y[x],x,IncludeSingu
```

Not solved

22.25 problem 25

Internal problem ID [10684]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$yy' - y = -\frac{12x}{49} + \frac{A\left(25\sqrt{x} + 41A + \frac{10A^2}{\sqrt{x}}\right)}{98}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 1093

```
dsolve(y(x)*diff(y(x),x)-y(x)=-12/49*x+1/98*A*(25*x^(1/2)+41*A+10*A^2*x^(-1/2)),y(x), singso
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-12/49*x+1/98*A*(25*x^(1/2)+41*A+10*A^2*x^(-1/2)),y[x],x,IncludeSing
```

Not solved

22.26 problem 26

Internal problem ID [10685]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$\boxed{yy' - y = -\frac{2x}{9} + \frac{A}{\sqrt{x}}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 148

```
dsolve(y(x)*diff(y(x),x)-y(x)=-2/9*x+A*x^(-1/2),y(x), singsol=all)
```

$$y(x) = \frac{2\, 6^{\frac{1}{3}} \sqrt{3} \left(-2x^{\frac{3}{2}} + 9A\right)}{3\sqrt{x} \left(9 \tan \left(\mathrm{RootOf}\left(18\sqrt{3}\, 6^{\frac{1}{3}} \left(\int \frac{\left(\frac{A}{x^{\frac{3}{2}}}\right)^{\frac{2}{3}} \sqrt{x}}{-2x^{\frac{3}{2}}+9A} dx\right) + \ln\left(\frac{\tan(\_Z)^4 - 4\sqrt{3}\tan(\_Z)^3 + 18\tan(\_Z)^2 - 12\sqrt{3}\tan(\_}{\tan(\_Z)^4 + 2\tan(\_Z)^2 + 1}\right.\right.\right.\right.}$$

✓ Solution by Mathematica

Time used: 1.355 (sec). Leaf size: 282

```
DSolve[y[x]*y'[x]-y[x]==-2/9*x+A*x^(-1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\Bigg[\log\left(9A^{2/3}+3\sqrt[3]{6}\sqrt[3]{A}\sqrt{x}\right.$$

$$\left.+6^{2/3}x\right)+2\sqrt{3}\arctan\left(\frac{-\frac{6\sqrt[3]{6}(9A-2x^{3/2}+3\sqrt{x}y(x))}{\sqrt[3]{A}y(x)}-27}{27\sqrt{3}}\right)+2\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{6}\sqrt{x}}{\sqrt[3]{A}}+3}{3\sqrt{3}}\right)+2\log\left(\frac{1}{27}\left(27-\frac{3\sqrt[3]{}}{}\right.\right.$$

22.27 problem 27

Internal problem ID [10686]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y = -\frac{5x}{36} + \frac{A}{x^{\frac{7}{5}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-5/36*x+A*x^(-7/5),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-5/36*x+A*x^(-7/5),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.28 problem 28

Internal problem ID [10687]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y = -\frac{12x}{49} + \frac{6A\left(-3\sqrt{x} + 23A + \frac{12A^2}{\sqrt{x}}\right)}{49}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-12/49*x+6/49*A*(-3*x^(1/2)+23*A+12*A^2*x^(-1/2)),y(x), singso
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-12/49*x+6/49*A*(-3*x^(1/2)+23*A+12*A^2*x^(-1/2)),y[x],x,IncludeSing
```

Not solved

22.29 problem 29

Internal problem ID [10688]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y = -\frac{30x}{121} + \frac{3A\left(21\sqrt{x} + 35A + \frac{6A^2}{\sqrt{x}}\right)}{242}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-30/121*x+3/242*A*(21*x^(1/2)+35*A+6*A^2*x^(-1/2)),y(x), sings
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-30/121*x+3/242*A*(21*x^(1/2)+35*A+6*A^2*x^(-1/2)),y[x],x,IncludeSin
```

Not solved

22.30 problem 30

Internal problem ID [10689]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{3x}{16} + \frac{A}{x^{\frac{5}{3}}}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8477

```
dsolve(y(x)*diff(y(x),x)-y(x)=-3/16*x+A*x^(-5/3),y(x), singsol=all)
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-3/16*x+A*x^(-5/3),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.31 problem 31

Internal problem ID [10690]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y = -\frac{12x}{49} + \frac{4A\left(-10\sqrt{x} + 27A + \frac{10A^2}{\sqrt{x}}\right)}{49}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-12/49*x+4/49*A*(-10*x^(1/2)+27*A+10*A^2*x^(-1/2)),y(x), sings
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-12/49*x+4/49*A*(-10*x^(1/2)+27*A+10*A^2*x^(-1/2)),y[x],x,IncludeSin
```

Not solved

22.32 problem 32

Internal problem ID [10691]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$\boxed{yy' - y = \frac{A}{\sqrt{x}}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 159

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*x^(-1/2),y(x), singsol=all)
```

$$c_1 + \frac{-2^{\frac{2}{3}}\left(-A^2x^{\frac{3}{2}}\right)^{\frac{1}{3}} \operatorname{AiryAi}\left(\frac{2^{\frac{1}{3}}\left(-A^2x^{\frac{3}{2}}\right)^{\frac{2}{3}}(x-y(x))}{2A^2x}\right) - 2\operatorname{AiryAi}\left(1, \frac{2^{\frac{1}{3}}\left(-A^2x^{\frac{3}{2}}\right)^{\frac{2}{3}}(x-y(x))}{2A^2x}\right)A}{2^{\frac{2}{3}}\left(-A^2x^{\frac{3}{2}}\right)^{\frac{1}{3}} \operatorname{AiryBi}\left(\frac{2^{\frac{1}{3}}\left(-A^2x^{\frac{3}{2}}\right)^{\frac{2}{3}}(x-y(x))}{2A^2x}\right) + 2\operatorname{AiryBi}\left(1, \frac{2^{\frac{1}{3}}\left(-A^2x^{\frac{3}{2}}\right)^{\frac{2}{3}}(x-y(x))}{2A^2x}\right)A} = 0$$

✓ Solution by Mathematica

Time used: 0.566 (sec). Leaf size: 139

```
DSolve[y[x]*y'[x]-y[x]==A*x^(-1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\sqrt[3]{-1}2^{2/3}\sqrt{x}\,\text{AiryAi}\left(\frac{\left(-\frac{1}{2}\right)^{2/3}(x-y(x))}{A^{2/3}}\right)+2\sqrt[3]{A}\,\text{AiryAiPrime}\left(\frac{\left(-\frac{1}{2}\right)^{2/3}(x-y(x))}{A^{2/3}}\right)}{\sqrt[3]{-1}2^{2/3}\sqrt{x}\,\text{AiryBi}\left(\frac{\left(-\frac{1}{2}\right)^{2/3}(x-y(x))}{A^{2/3}}\right)+2\sqrt[3]{A}\,\text{AiryBiPrime}\left(\frac{\left(-\frac{1}{2}\right)^{2/3}(x-y(x))}{A^{2/3}}\right)}+c_1=0,y(x)\right]$$

22.33 problem 33

Internal problem ID [10692]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = \frac{A}{x^2}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 197

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*x^(-2),y(x), singsol=all)
```

$$c_1 + \frac{-2^{\frac{1}{3}} A(x-y(x)) \operatorname{AiryAi}\left(-\frac{\left(x^3-2y(x)x^2+xy(x)^2+2A\right)2^{\frac{2}{3}}}{4(-A^2)^{\frac{1}{3}}x}\right) + 2 \operatorname{AiryAi}\left(1, -\frac{\left(x^3-2y(x)x^2+xy(x)^2+2A\right)2^{\frac{2}{3}}}{4(-A^2)^{\frac{1}{3}}x}\right)(-A^2)^{\frac{2}{3}}}{2^{\frac{1}{3}} A\left(x-y\left(x\right)\right) \operatorname{AiryBi}\left(-\frac{\left(x^3-2y(x)x^2+xy(x)^2+2A\right)2^{\frac{2}{3}}}{4(-A^2)^{\frac{1}{3}}x}\right) - 2 \operatorname{AiryBi}\left(1, -\frac{\left(x^3-2y(x)x^2+xy(x)^2+2A\right)2^{\frac{2}{3}}}{4(-A^2)^{\frac{1}{3}}x}\right)(-A^2)^{\frac{2}{3}}} = 0$$

✓ Solution by Mathematica

Time used: 1.053 (sec). Leaf size: 201

```
DSolve[y[x]*y'[x]-y[x]==A*x^(-2),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\text{AiryAiPrime}\left(\frac{x^3-2y(x)x^2+y(x)^2x+2A}{2\sqrt[3]{2}A^{2/3}x}\right)-\frac{(x-y(x))\,\text{AiryAi}\left(\frac{x^3-2y(x)x^2+y(x)^2x+2A}{2\sqrt[3]{2}A^{2/3}x}\right)}{2^{2/3}\sqrt[3]{A}}}{\text{AiryBiPrime}\left(\frac{x^3-2y(x)x^2+y(x)^2x+2A}{2\sqrt[3]{2}A^{2/3}x}\right)-\frac{(x-y(x))\,\text{AiryBi}\left(\frac{x^3-2y(x)x^2+y(x)^2x+2A}{2\sqrt[3]{2}A^{2/3}x}\right)}{2^{2/3}\sqrt[3]{A}}}+c_1=0,y(x)\right]$$

22.34 problem 34

Internal problem ID [10693]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$yy' - y = A(2+n)\left(\sqrt{x} + 2(2+n)A + \frac{(1+n)(n+3)A^2}{\sqrt{x}}\right)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 309

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*(n+2)*(x^(1/2)+2*(n+2)*A+(n+1)*(n+3)*A^2*x^(-1/2)),y(x), sin
```

$$c_1 + \frac{A\sqrt{\frac{2(n+2)A\sqrt{x}+A^2(n^2+4n+3)+x-y(x)}{(n+2)^2A^2}}\,(n+2)\,\mathrm{BesselK}\left(\frac{n+3}{n+2}, -\sqrt{\frac{2(n+2)A\sqrt{x}+A^2(n^2+4n+3)+x-y(x)}{(n+2)^2A^2}}\right) - \mathrm{BesselK}\left(\frac{1}{n-}\right.}{A\sqrt{\frac{2(n+2)A\sqrt{x}+A^2(n^2+4n+3)+x-y(x)}{(n+2)^2A^2}}\,(n+2)\,\mathrm{BesselI}\left(\frac{n+3}{n+2}, -\sqrt{\frac{2(n+2)A\sqrt{x}+A^2(n^2+4n+3)+x-y(x)}{(n+2)^2A^2}}\right) + \mathrm{BesselI}\left(\frac{1}{n+}\right.}$$
$$= 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==A*(n+2)*(x^(1/2)+2*(n+2)*A+(n+1)*(n+3)*A^2*x^(-1/2)),y[x],x,IncludeS
```

Not solved

22.35 problem 35

Internal problem ID [10694]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$yy' - y = A(2+n)\left(\sqrt{x} + 2(2+n)\,A + \frac{(3+2n)\,A^2}{\sqrt{x}}\right)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 359

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*(n+2)*(x^(1/2)+2*(n+2)*A+(2*n+3)*A^2*x^(-1/2)),y(x), singsol
```

$$c_1 + \frac{\left(A\sqrt{\frac{(1+n)^2}{(n+2)^2}}\,(n+2) - \sqrt{x} + (-n-2)\,A\right)\mathrm{BesselK}\left(\sqrt{\frac{(1+n)^2}{(n+2)^2}}, -\sqrt{\frac{2(n+2)A\sqrt{x}+(3+2n)A^2+x-y(x)}{(n+2)^2A^2}}\right) + \mathrm{BesselK}\left(\right.}{\left(-A\sqrt{\frac{(1+n)^2}{(n+2)^2}}\,(n+2) + \sqrt{x} + (n+2)\,A\right)\mathrm{BesselI}\left(\sqrt{\frac{(1+n)^2}{(n+2)^2}}, -\sqrt{\frac{2(n+2)A\sqrt{x}+(3+2n)A^2+x-y(x)}{(n+2)^2A^2}}\right) + A\,\mathrm{BesselI}}$$
$$= 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==A*(n+2)*(x^(1/2)+2*(n+2)*A+(2*n+3)*A^2*x^(-1/2)),y[x],x,IncludeSingu
```

Not solved

22.36 problem 36

Internal problem ID [10695]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = A\sqrt{x} + 2A^2 + \frac{B}{\sqrt{x}}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 273

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*x^(1/2)+2*A^2+B*x^(-1/2),y(x), singsol=all)
```

$$c_1 + \frac{\left(\sqrt{\frac{A^3-B}{A^3}}\,A - A - \sqrt{x}\right)\mathrm{BesselK}\left(\sqrt{\frac{A^3-B}{A^3}}, -\sqrt{\frac{2A^2\sqrt{x}+(x-y(x))A+B}{A^3}}\right) + \mathrm{BesselK}\left(1+\sqrt{\frac{A^3-B}{A^3}}, -\sqrt{\frac{2A^2\sqrt{x}}{}}\right.}{\left(-\sqrt{\frac{A^3-B}{A^3}}\,A + A + \sqrt{x}\right)\mathrm{BesselI}\left(\sqrt{\frac{A^3-B}{A^3}}, -\sqrt{\frac{2A^2\sqrt{x}+(x-y(x))A+B}{A^3}}\right) + A\,\mathrm{BesselI}\left(1+\sqrt{\frac{A^3-B}{A^3}}, -\sqrt{\frac{2A^2}{}}\right.}$$
$$= 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==A*x^(1/2)+2*A^2+B*x^(-1/2),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.37 problem 37

Internal problem ID [10696]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$yy' - y = 2A^2 - A\sqrt{x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 156

```
dsolve(y(x)*diff(y(x),x)-y(x)=2*A^2-A*x^(1/2),y(x), singsol=all)
```

$$c_1 + \frac{\left(-2A+\sqrt{x}\right) \operatorname{BesselK}\left(1, -\sqrt{-\frac{2A\sqrt{x}-x+y(x)}{A^2}}\right) + \operatorname{BesselK}\left(0, -\sqrt{-\frac{2A\sqrt{x}-x+y(x)}{A^2}}\right) \sqrt{-\frac{2A\sqrt{x}-x+y(x)}{A^2}}\, A}{A \operatorname{BesselI}\left(0, \sqrt{-\frac{2A\sqrt{x}-x+y(x)}{A^2}}\right) \sqrt{-\frac{2A\sqrt{x}-x+y(x)}{A^2}} + \left(-2A+\sqrt{x}\right) \operatorname{BesselI}\left(1, \sqrt{-\frac{2A\sqrt{x}-x+y(x)}{A^2}}\right)} = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==2*A^2-A*x^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.38 problem 38

Internal problem ID [10697]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y = -\frac{x}{4} + \frac{6A\left(\sqrt{x} + 8A + \frac{5A^2}{\sqrt{x}}\right)}{49}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-12/48*x+6/49*A*(x^(1/2)+8*A+5*A^2*x^(-1/2)),y(x), singsol=all
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-12/48*x+6/49*A*(x^(1/2)+8*A+5*A^2*x^(-1/2)),y[x],x,IncludeSingularS
```

Not solved

22.39 problem 39

Internal problem ID [10698]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y = -\frac{6x}{25} + \frac{6A\left(2\sqrt{x} + 7A + \frac{4A^2}{\sqrt{x}}\right)}{25}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-6/25*x+6/25*A*(2*x^(1/2)+7*A+4*A^2*x^(-1/2)),y(x), singsol=al
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-6/25*x+6/25*A*(2*x^(1/2)+7*A+4*A^2*x^(-1/2)),y[x],x,IncludeSingular
```

Not solved

22.40 problem 40

Internal problem ID [10699]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$yy' - y = -\frac{3x}{16} + \frac{3A}{x^{\frac{1}{3}}} - \frac{12A^2}{x^{\frac{5}{3}}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9138

```
dsolve(y(x)*diff(y(x),x)-y(x)=-3/16*x+3*A*x^(-1/3)-12*A^2*x^(-5/3),y(x), singsol=all)
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-3/16*x+3*A*x^(-1/3)-12*A^2*x^(-5/3),y[x],x,IncludeSingularSolutions
```

Not solved

22.41 problem 41

Internal problem ID [10700]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, ‘2nd type‘, ‘class B‘]]`

$$y'y - y = \frac{3x}{8} + \frac{3\sqrt{b^2 + x^2}}{8} + \frac{3b^2}{16\sqrt{b^2 + x^2}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=3/8*x+3/8*sqrt(x^2+b^2)+3*b^2/(16*sqrt(x^2+b^2)),y(x), singsol
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==3/8*x+3/8*Sqrt[x^2+b^2]+3*b^2/(16*Sqrt[x^2+b^2]),y[x],x,IncludeSingu
```

Not solved

22.42 problem 42

Internal problem ID [10701]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$y'y - y = \frac{9x}{32} + \frac{15\sqrt{b^2 + x^2}}{32} + \frac{3b^2}{64\sqrt{b^2 + x^2}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=9/32*x+15/32*sqrt(x^2+b^2)+3*b^2/(64*sqrt(x^2+b^2)),y(x), sing
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==9/32*x+15/32*Sqrt[x^2+b^2]+3*b^2/(64*Sqrt[x^2+b^2]),y[x],x,IncludeSi
```

Not solved

22.43 problem 43

Internal problem ID [10702]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$y'y - y = -\frac{3x}{32} - \frac{3\sqrt{a^2 + x^2}}{32} + \frac{15a^2}{64\sqrt{a^2 + x^2}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-3/32*x-3/32*sqrt(x^2+a^2)+15*a^2/(64*sqrt(x^2+a^2)),y(x), sin
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-3/32*x-3/32*Sqrt[x^2+a^2]+15*a^2/(64*Sqrt[x^2+a^2]),y[x],x,IncludeS
```

Not solved

22.44 problem 44

Internal problem ID [10703]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_rational, [_Abel, '2nd type', 'class A']]**

$$\boxed{yy' - y = A\,x^2 - \frac{9}{625A}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 196

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*x^2-9/625*A^(-1),y(x), singsol=all)
```

$$c_1 + \frac{2\left(\frac{(25xA+3)^{\frac{3}{2}}}{50xA-125Ay(x)+6}\right)^{\frac{1}{3}} (6+(50x-125y(x))\,A)\left(\int^{\frac{2(25xA+3)^{\frac{3}{2}}}{6+(50x-125y(x))A}} \frac{(\_a^2-6)^{\frac{1}{6}}}{\_a^{\frac{1}{3}}} d\_a\right) - 125\, 2^{\frac{5}{6}}\left(\frac{-54+31250A^3x^3+}{}\right.}{\left(\frac{(25xA+3)^{\frac{3}{2}}}{6+(50x-125y(x))A}\right)^{\frac{1}{3}} (12+(100x-250y\,(x}$$
$$= 0$$

✓ Solution by Mathematica

Time used: 2.438 (sec). Leaf size: 198

```
DSolve[y[x]*y'[x]-y[x]==A*x^2-9/625*A^(-1),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\sqrt[6]{\frac{46875A^2y(x)^2-1500A(25Ax+3)y(x)-2(25Ax-3)(25Ax+3)^2}{(25Ax+3)^3}}\left(\frac{(-125Ay(x)+ \cdots}{\sqrt[3]{2}\sqrt{3}(25Ax+3)^{3/2}\sqrt[6]{\frac{-46875 \cdots}{\cdots}}}\cdots\right)}{\sqrt[6]{2}}\right.$$

$$\left.+c_1=0,y(x)\right]$$

22.45 problem 45

Internal problem ID [10704]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - y = -\frac{6}{25}x - A\,x^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 160

```
dsolve(y(x)*diff(y(x),x)-y(x)=-6/25*x-A*x^2,y(x), singsol=all)
```

$$c_1 + \frac{(2x-5y(x))\left(\int^{\frac{10\sqrt{-xA}\,x}{2x-5y(x)}} \frac{(\_a^2-6)^{\frac{1}{6}}}{\_a^{\frac{1}{3}}} d\_a\right) - \frac{5\,2^{\frac{1}{6}}\left(-\frac{50x^3A}{(2x-5y(x))^2} - \frac{12x^2}{(2x-5y(x))^2} + \frac{60y(x)x}{(2x-5y(x))^2} - \frac{75y(x)^2}{(2x-5y(x))^2}\right)^{\frac{1}{6}} 10^{\frac{2}{3}}\sqrt{-xA}\,y(x)}{2\left(\frac{\sqrt{-xA}\,x}{2x-5y(x)}\right)^{\frac{1}{3}}}}{2x-5y\left(x\right)} = 0$$

✓ Solution by Mathematica

Time used: 2.139 (sec). Leaf size: 162

```
DSolve[y[x]*y'[x]-y[x]==-6/25*x-A*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[c_1 = \frac{i\sqrt[6]{\frac{-2x^2(25Ax+6)+60xy(x)-75y(x)^2}{Ax^3}}\left(25Ax^2 - \frac{\sqrt[6]{2}\sqrt[3]{5}(2x-5y(x))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},-\frac{3(2\cdots}{\cdots}\right)}{\sqrt[6]{\frac{2x^2(25Ax+6)-60xy(x)+75y(\cdots}{Ax^3}}}\cdots\right.}{5\ 2^{2/3}\sqrt{3}\sqrt[3]{5}\sqrt{A}x^{3/2}}\cdots\right.$$

22.46 problem 46

Internal problem ID [10705]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - y = \frac{6}{25}x - A\,x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 196

```
dsolve(y(x)*diff(y(x),x)-y(x)=6/25*x-A*x^2,y(x), singsol=all)
```

$$c_1 + \frac{-125\,5^{\frac{1}{3}}2^{\frac{5}{6}}\left(\frac{-1250A^3x^3+\left(600x^2+1500xy(x)-1875y(x)^2\right)A^2+(-72x-360y(x))A}{(50xA-125Ay(x)-12)^2}\right)^{\frac{1}{6}} Ay(x)\sqrt{-25xA+6}+100\left(-\frac{6}{25}+\left(\right.\right.}{\left(-\frac{(-25xA+6)^{\frac{3}{2}}}{-12+(50x-125y(x))A}\right)^{\frac{1}{3}}(-24+(100x-25}$$
$$= 0$$

✓ Solution by Mathematica

Time used: 3.324 (sec). Leaf size: 189

```
DSolve[y[x]*y'[x]-y[x]==6/25*x-A*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\sqrt[3]{5}\sqrt[6]{-\frac{A\left(1875Ay(x)^2-60(25Ax-6)y(x)+2x(6-25Ax)^2\right)}{(25Ax-6)^3}}\left(\frac{(-125Ay(x)+50Ax-12)\,\text{Hyperge}}{\sqrt[3]{10}\sqrt{18-75Ax}(25Ax-6)\sqrt[6]{\frac{A\left(1875Ay\right.}{}}}\right.}{\sqrt[6]{2}} \quad % expression cut off at page edge
\right.$$

$$+c_1=0,y(x)\Bigg]$$

22.47 problem 47

Internal problem ID [10706]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = 12x + \frac{A}{x^{\frac{5}{2}}}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 119

```
dsolve(y(x)*diff(y(x),x)-y(x)=12*x+A*x^(-5/2),y(x), singsol=all)
```

$$c_1 + \frac{-168x^{\frac{5}{2}}\sqrt{3}\,(-y(x)+4x)\,\mathrm{hypergeom}\left(\left[-\frac{1}{6},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^{\frac{3}{2}}(-y(x)+4x)^2}{4A}\right) + 3\,2^{\frac{2}{3}}\left(\frac{48x^{\frac{7}{2}}-24y(x)x^{\frac{5}{2}}+3y(x)^2x^{\frac{3}{2}}+4A}{A}\right.}{\sqrt{-A\,x^{\frac{7}{2}}}} = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==12*x+A*x^(-5/2),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.48 problem 48

Internal problem ID [10707]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y = \frac{63x}{4} + \frac{A}{x^{\frac{5}{3}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=63/4*x+A*x^(-5/3),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==63/4*x+A*x^(-5/3),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.49 problem 49

Internal problem ID [10708]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$yy' - y = 2x + 2A\left(10\sqrt{x} + 31A + \frac{30A^2}{\sqrt{x}}\right)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 196

```
dsolve(y(x)*diff(y(x),x)-y(x)=2*x+2*A*(10*x^(1/2)+31*A+30*A^2*x^(-1/2)),y(x), singsol=all)
```

$$c_1 - \frac{\left(3A+\sqrt{x}\right) 2^{\frac{1}{3}} \left(\frac{12A^2+10A\sqrt{x}+2x-y(x)}{6A^2+2A\sqrt{x}+y(x)}\right)^{\frac{1}{3}} \left(\frac{15A^2+8A\sqrt{x}+x+y(x)}{6A^2+2A\sqrt{x}+y(x)}\right)^{\frac{1}{6}} y(x)}{4\sqrt{\frac{\left(3A+\sqrt{x}\right)^2}{6A^2+2A\sqrt{x}+y(x)}}\left(6A^2+2A\sqrt{x}+y\left(x\right)\right) A} - \left(\int^{\frac{6A\sqrt{x}+2x-3y(x)}{12A^2+4A\sqrt{x}+2y(x)}} \frac{(\_a+1)^{\frac{1}{3}} (2\_a+5)^{\frac{1}{6}}}{\sqrt{2\_a+3}} d\_a\right) = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==2*x+2*A*(10*x^(1/2)+31*A+30*A^2*x^(-1/2)),y[x],x,IncludeSingularSolu
```

Not solved

22.50 problem 50

Internal problem ID [10709]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y = 2x + 2A\left(-10\sqrt{x} + 19A + \frac{30A^2}{\sqrt{x}}\right)$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=2*x+2*A*(-10*x^(1/2)+19*A+30*A^2*x^(-1/2)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==2*x+2*A*(-10*x^(1/2)+19*A+30*A^2*x^(-1/2)),y[x],x,IncludeSingularSol
```

Not solved

22.51 problem 51

Internal problem ID [10710]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 51.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y = -\frac{28x}{121} + \frac{2A\left(5\sqrt{x} + 106A + \frac{65A^2}{\sqrt{x}}\right)}{121}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-28/121*x+2/121*A*(5*x^(1/2)+106*A+65*A^2*x^(-1/2)),y(x), sing
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-28/121*x+2/121*A*(5*x^(1/2)+106*A+65*A^2*x^(-1/2)),y[x],x,IncludeSi
```

Not solved

22.52 problem 52

Internal problem ID [10711]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$yy' - y = -\frac{12x}{49} + \frac{A\left(5\sqrt{x} + 262A + \frac{65A^2}{\sqrt{x}}\right)}{49}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 697

```
dsolve(y(x)*diff(y(x),x)-y(x)=-12/49*x+1/49*A*(5*x^(1/2)+262*A+65*A^2*x^(-1/2)),y(x), singso
```

$$c_1 + \frac{2i\sqrt{3}\,4^{\frac{2}{3}}\left(\left(\left(A\left(3+\frac{5i\sqrt{3}}{3}\right)\sqrt{x} + \frac{i(-25A^2-x)\sqrt{3}}{6} - x + \frac{7y(x)}{4} + 10A^2\right)\sqrt{-35A^2+7A\sqrt{x}} + \frac{7iAx\sqrt{3}}{6} + \left(-\frac{35iA}{3}\right.\right.\right.}{\left(\frac{i(5A-\sqrt{x})\sqrt{3}\sqrt{-35A^2+7A\sqrt{x}}}{10i\sqrt{3}\left(A-\frac{\sqrt{x}}{5}\right)\sqrt{-35A^2+7A\sqrt{x}}-120A^2-36A\sqrt{x}+12x-21y(x)}\right)^{\frac{1}{3}}\left(2\left(\left(2A\left(18-7i\sqrt{3}\right)\sqrt{x}+70iA^2\sqrt{3}+120A^2-\right.\right.\right.}$$
$$= 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-28/121*x+2/121*A*(5*x^(1/2)+262*A+65*A^2*x^(-1/2)),y[x],x,IncludeSi
```

Not solved

22.53 problem 53

Internal problem ID [10712]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 53.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$yy' - y = -\frac{12x}{49} + A\sqrt{x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 127

```
dsolve(y(x)*diff(y(x),x)-y(x)=-12/49*x+A*x^(1/2),y(x), singsol=all)
```

$$c_1 + \frac{-7\,14^{\frac{1}{3}} A\sqrt{3} + \frac{\sqrt{3}\,(4x-7y(x))\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{7}{6}\right],\left[\frac{3}{2}\right],\frac{3(4x-7y(x))^2}{196x^{\frac{3}{2}}A}\right)\left(\frac{196A\,x^{\frac{3}{2}}-48x^2+168xy(x)-147y(x)^2}{A\,x^{\frac{3}{2}}}\right)^{\frac{1}{6}}}{7\sqrt{x}}}{\left(\frac{196A\,x^{\frac{3}{2}}-48x^2+168xy(x)-147y(x)^2}{A\,x^{\frac{3}{2}}}\right)^{\frac{1}{6}}\sqrt{A\sqrt{x}}}$$
$$= 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-12/49*x+A*x^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.54 problem 54

Internal problem ID [10713]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 54.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = 6x + \frac{A}{x^4}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 308

```
dsolve(y(x)*diff(y(x),x)-y(x)=6*x+A*x^(-4),y(x), singsol=all)
```

$$c_1 + \frac{2\left(-\frac{x^{\frac{11}{2}} 625^{\frac{5}{6}} 2^{\frac{2}{3}} 16^{\frac{1}{6}} A \operatorname{hypergeom}\left(\left[\frac{1}{6}, \frac{2}{3}\right], \left[\frac{5}{3}\right], -\frac{2A}{3x^3(y(x)+2x)^2}\right) 243^{\frac{1}{6}} \left(-\frac{1}{x^{\frac{3}{2}}(y(x)+2x)}\right)^{\frac{5}{3}} \left(\frac{y(x)}{2}+x\right)^3 \left(\frac{12x^5+12x^4y(x)+3x^3y(x)^2+2A}{x^9(y(x)+2x)^6}\right)^{\frac{1}{6}} \left(-\cdots\right.}{10}}{\left(-\frac{2}{x^{\frac{3}{2}}(10x+5y(x))}\cdots\right.}$$
$$= 0$$

✓ Solution by Mathematica

Time used: 2.079 (sec). Leaf size: 213

```
DSolve[y[x]*y'[x]-y[x]==6*x+A*x^(-4),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[c_1 = \frac{i\left(-\frac{2A+12x^5+12x^4y(x)+3x^3y(x)^2}{A}\right)^{5/6}\left(-10\ 2^{5/6}x^5\ \text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},-\frac{3x^3(2x+y(x))^2}{2A}\right) - 5\right.}{2\sqrt[3]{2}\sqrt{3}\sqrt{A}x^{5/2}\left(\frac{2A+12x^5}{}\right.}\right.$$

22.55 problem 55

Internal problem ID [10714]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 55.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$\boxed{y'y - y = 20x + \frac{A}{\sqrt{x}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=20*x+A*x^(-1/2),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==20*x+A*x^(-1/2),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.56 problem 56

Internal problem ID [10715]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 56.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{y'y - y = \frac{15x}{4} + \frac{A}{x^7}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=15/4*x+A*x^(-7),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==15/4*x+A*x^(-7),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.57 problem 57

Internal problem ID [10716]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 57.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$yy' - y = -\frac{10x}{49} + \frac{2A\left(4\sqrt{x} + 61A + \frac{12A^2}{\sqrt{x}}\right)}{49}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 200

```
dsolve(y(x)*diff(y(x),x)-y(x)=-10/49*x+2/49*A*(4*x^(1/2)+61*A+12*A^2*x^(-1/2)),y(x), singsol
```

$$c_1 - \frac{\left(3A+\sqrt{x}\right) 2^{\frac{2}{3}} \left(\frac{3A^2+16A\sqrt{x}+5x-7y(x)}{6A^2+2A\sqrt{x}+y(x)}\right)^{\frac{5}{6}} y(x)}{2\sqrt{\frac{\left(3A+\sqrt{x}\right)^2}{6A^2+2A\sqrt{x}+y(x)}}\, \left(\frac{-24A^2-2A\sqrt{x}+2x-7y(x)}{6A^2+2A\sqrt{x}+y(x)}\right)^{\frac{1}{3}} \left(6A^2+2A\sqrt{x}+y(x)\right) A} - \left(\int^{\frac{6A\sqrt{x}+2x-3y(x)}{12A^2+4A\sqrt{x}+2y(x)}} \frac{(10\_a+1)^{\frac{5}{6}}}{\sqrt{2\_a+3}\, (\_a-2)^{\frac{1}{3}}} d\_a\right) = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-10/49*x+2/49*A*(4*x^(1/2)+61*A+12*A^2*x^(-1/2)),y[x],x,IncludeSingu
```

Not solved

22.58 problem 58

Internal problem ID [10717]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 58.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$yy' - y = -\frac{12x}{49} + \frac{2A\left(\sqrt{x} + 166A + \frac{55A^2}{\sqrt{x}}\right)}{49}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 683

```
dsolve(y(x)*diff(y(x),x)-y(x)=-12/49*x+2/49*A*(x^(1/2)+166*A+55*A^2*x^(-1/2)),y(x), singsol=
```

$$c_1 + \frac{3i\sqrt{6}\,4^{\frac{2}{3}}\left(\left(\left(A\left(3+\frac{5i\sqrt{6}}{3}\right)\sqrt{x}+\frac{i(25A^2+x)\sqrt{6}}{6} - \right.\right.\right.}{4\left(\frac{i(5A+\sqrt{x})\sqrt{6}\,\sqrt{-35A^2-7A\sqrt{x}}}{10i\sqrt{6}\left(A+\frac{\sqrt{x}}{5}\right)\sqrt{-35A^2-7A\sqrt{x}}-120A^2+36A\sqrt{x}+12x-21y(x)}\right)^{\frac{1}{3}}\left(\left(\left(2A\left(18-7i\sqrt{6}\right)\sqrt{x}-70iA^2\sqrt{6}-120A^2+12\right.\right.\right.}$$
$$= 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-12/49*x+2/49*A*(x^(1/2)+166*A+55*A^2*x^(-1/2)),y[x],x,IncludeSingul
```

Not solved

22.59 problem 59

Internal problem ID [10718]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 59.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y = -\frac{4x}{25} + \frac{A\left(7\sqrt{x} + 49A + \frac{6A^2}{\sqrt{x}}\right)}{50}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-4/25*x+1/50*A*(7*x^(1/2)+49*A+6*A^2*x^(-1/2)),y(x), singsol=a
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-4/25*x+1/50*A*(7*x^(1/2)+49*A+6*A^2*x^(-1/2)),y[x],x,IncludeSingula
```

Not solved

22.60 problem 60

Internal problem ID [10719]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 60.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y = \frac{15x}{4} + \frac{6A}{x^{\frac{1}{3}}} - \frac{3A^2}{x^{\frac{5}{3}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=15/4*x+6*A*x^(-1/3)-3*A^2*x^(-5/3),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==15/4*x+6*A*x^(-1/3)-3*A^2*x^(-5/3),y[x],x,IncludeSingularSolutions -
```

Not solved

22.61 problem 61

Internal problem ID [10720]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y = -\frac{3x}{16} + \frac{A}{x^{\frac{1}{3}}} + \frac{B}{x^{\frac{5}{3}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-3/16*x+A*x^(-1/3)+B*x^(-5/3),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-3/16*x+A*x^(-1/3)+B*x^(-5/3),y[x],x,IncludeSingularSolutions -> Tru
```

Not solved

22.62 problem 62

Internal problem ID [10721]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 62.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$y'y - y = -\frac{5x}{36} + \frac{A}{x^{\frac{3}{5}}} - \frac{B}{x^{\frac{7}{5}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-5/36*x+A*x^(-3/5)-B*x^(-7/5),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-5/36*x+A*x^(-3/5)-B*x^(-7/5),y[x],x,IncludeSingularSolutions -> Tru
```

Timed out

22.63 problem 63

Internal problem ID [10722]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 63.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$\boxed{y'y - y = \frac{k}{\sqrt{A\,x^2 + Bx + c}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=k*(A*x^2+B*x+c)^(-1/2),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==k*(A*x^2+B*x+c)^(-1/2),y[x],x,IncludeSingularSolutions -> True]
```

Timed out

22.64 problem 64

Internal problem ID [10723]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 64.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y = -\frac{12x}{49} + 3A\left(\frac{1}{49} + B\right)\sqrt{x} + 3A^2\left(\frac{4}{49} - \frac{5B}{2}\right) + \frac{15A^3\left(\frac{1}{49} - \frac{5B}{4}\right)}{4\sqrt{x}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-12/49*x+3*A*(1/49+B)*x^(1/2)+3*A^2*(4/49-5/2*B)+15/4*A^3*(1/4
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-12/49*x+3*A*(1/49+B)*x^(1/2)+3*A^2*(4/49-5/2*B)+15/4*A^3*(1/49-5/4*
```

Not solved

22.65 problem 65

Internal problem ID [10724]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 65.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$yy' - y = -\frac{6x}{25} + \frac{4B^2\left((2-A)\,x^{\frac{1}{3}} - \frac{3B(2A+1)}{2} + \frac{B^2(1-3A)}{x^{\frac{1}{3}}} - \frac{A\,B^3}{x^{\frac{2}{3}}}\right)}{75}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 2369

```
dsolve(y(x)*diff(y(x),x)-y(x)=-6/25*x+4/75*B^2*((2-A)*x^(1/3)-3/2*B*(2*A+1)+B^2*(1-3*A)*x^(-
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-6/25*x+4/75*B^2*((2-A)*x^(1/3)-3/2*B*(2*A+1)+B^2*(1-3*A)*x^(-1/3)-A
```

Not solved

22.66 problem 66

Internal problem ID [10725]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 66.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y = \frac{3x}{4} - \frac{3A\,x^{\frac{1}{3}}}{2} + \frac{3A^2}{4x^{\frac{1}{3}}} - \frac{27A^4}{625x^{\frac{5}{3}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=3/4*x-3/2*A*x^(1/3)+3/4*A^2*x^(-1/3)-27/625*A^4*x^(-5/3),y(x),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==3/4*x-3/2*A*x^(1/3)+3/4*A^2*x^(-1/3)-27/625*A^4*x^(-5/3),y[x],x,Incl
```

Not solved

22.67 problem 67

Internal problem ID [10726]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 67.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y = -\frac{6x}{25} + \frac{7A\, x^{\frac{1}{3}}}{5} + \frac{31A^2}{3x^{\frac{1}{3}}} - \frac{100A^4}{3x^{\frac{5}{3}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-6/25*x+7/5*A*x^(1/3)+31/3*A^2*x^(-1/3)-100/3*A^4*x^(-5/3),y(x
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-6/25*x+7/5*A*x^(1/3)+31/3*A^2*x^(-1/3)-100/3*A^4*x^(-5/3),y[x],x,In
```

Not solved

22.68 problem 68

Internal problem ID [10727]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 68.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y = -\frac{10x}{49} + \frac{13A^2}{5x^{\frac{1}{5}}} - \frac{7A^3}{20x^{\frac{4}{5}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-10/49*x+13/5*A^2*x^(-1/5)-7/20*A^3*x^(-4/5),y(x), singsol=all
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-10/49*x+13/5*A^2*x^(-1/5)-7/20*A^3*x^(-4/5),y[x],x,IncludeSingularS
```

Not solved

22.69 problem 69

Internal problem ID [10728]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y = -\frac{33x}{169} + \frac{286A^2}{3x^{\frac{5}{11}}} - \frac{770A^3}{9x^{\frac{13}{11}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-33/169*x+286/3*A^2*x^(-5/11)-770/9*A^3*x^(-13/11),y(x), sings
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-33/169*x+286/3*A^2*x^(-5/11)-770/9*A^3*x^(-13/11),y[x],x,IncludeSin
```

Timed out

22.70 problem 70

Internal problem ID [10729]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 70.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y = -\frac{21x}{100} + \frac{7A^2\left(\frac{123}{x^{\frac{1}{7}}} + \frac{280A}{x^{\frac{5}{7}}} - \frac{400A^2}{x^{\frac{9}{7}}}\right)}{9}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-21/100*x+7/9*A^2*(123*x^(-1/7)+280*A*x^(-5/7)-400*A^2*x^(-9/7
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-21/100*x+7/9*A^2*(123*x^(-1/7)+280*A*x^(-5/7)-400*A^2*x^(-9/7)),y[x
```

Not solved

22.71 problem 71

Internal problem ID [10730]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 71.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{y'y - y = ax + b\,x^m}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=a*x+b*x^m,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==a*x+b*x^m,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.72 problem 72

Internal problem ID [10731]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 72.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y'y - y = -\frac{(m+1)\,x}{(m+2)^2} + A\,x^{2m+1} + B\,x^{1+3m}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-(m+1)/(m+2)^2*x+A*x^(2*m+1)+B*x^(3*m+1),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-(m+1)/(m+2)^2*x+A*x^(2*m+1)+B*x^(3*m+1),y[x],x,IncludeSingularSolut
```

Not solved

22.73 problem 73

Internal problem ID [10732]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 73.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$y'y - y = a^2\lambda\,\mathrm{e}^{2\lambda x} - a(b\lambda + 1)\,\mathrm{e}^{\lambda x} + b$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=a^2*lambda*exp(2*lambda*x)-a*(b*lambda+1)*exp(lambda*x)+b,y(x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==a^2*\[Lambda]*Exp[2*\[Lambda]*x]-a*(b*\[Lambda]+1)*Exp[\[Lambda]*x]+
```

Not solved

22.74 problem 74

Internal problem ID [10733]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 74.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$y'y - y = a^2\lambda\,\mathrm{e}^{2\lambda x} + \mathrm{e}^{\lambda x}a\lambda x + b\,\mathrm{e}^{\lambda x}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=a^2*lambda*exp(2*lambda*x)+a*lambda*x*exp(lambda*x)+b*exp(lamb
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==a^2*\[Lambda]*Exp[2*\[Lambda]*x]+a*\[Lambda]*x*Exp[\[Lambda]*x]+b*Ex
```

Not solved

22.75 problem 75

Internal problem ID [10734]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 75.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$y'y - y = 2a^2\lambda \sin(2\lambda x) + 2\sin(\lambda x) a$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=2*a^2*lambda*sin(2*lambda*x)+2*a*sin(lambda*x),y(x), singsol=a
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==2*a^2*\[Lambda]*Sin[2*\[Lambda]*x]+2*a*Sin[\[Lambda]*x],y[x],x,Inclu
```

Not solved

22.76 problem 76

Internal problem ID [10735]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 76.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, ‘2nd type‘, ‘class B‘]]`

$$y'y - y = a^2 f'(x)\, f''(x) - \frac{(f(x) + b)^2 f''(x)}{f'(x)^3}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=a^2*diff(f(x),x)*diff(f(x),x$2)-(f(x)+b)^2/( diff(f(x),x)^3)*d
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==a^2*f'[x]*f''[x]-(f[x]+b)^2/( (f'[x])^3)*f''[x],y[x],x,IncludeSingul
```

Timed out

23 Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$

23.1 problem 1

Internal problem ID [10736]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - (ax + b)\, y = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 191

```
dsolve(y(x)*diff(y(x),x)=(a*x+b)*y(x)+1,y(x), singsol=all)
```

$$c_1 + \frac{-2^{\frac{1}{3}} (-a^2)^{\frac{1}{3}} (ax+b) \operatorname{AiryAi}\left(-\frac{(a^2x^2+(2xb-2y(x))a+b^2)2^{\frac{2}{3}}}{4(-a^2)^{\frac{1}{3}}}\right) - 2 \operatorname{AiryAi}\left(1, -\frac{(a^2x^2+(2xb-2y(x))a+b^2)2^{\frac{2}{3}}}{4(-a^2)^{\frac{1}{3}}}\right) a}{2^{\frac{1}{3}} (-a^2)^{\frac{1}{3}} (ax+b) \operatorname{AiryBi}\left(-\frac{(a^2x^2+(2xb-2y(x))a+b^2)2^{\frac{2}{3}}}{4(-a^2)^{\frac{1}{3}}}\right) + 2 \operatorname{AiryBi}\left(1, -\frac{(a^2x^2+(2xb-2y(x))a+b^2)2^{\frac{2}{3}}}{4(-a^2)^{\frac{1}{3}}}\right) a} = 0$$

✓ Solution by Mathematica

Time used: 0.901 (sec). Leaf size: 161

```
DSolve[y[x]*y'[x]==(a*x+b)*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\sqrt[3]{2}(ax+b)\,\text{AiryAi}\left(\frac{(b+ax)^2-2ay(x)}{2\sqrt[3]{2}a^{2/3}}\right)-2\sqrt[3]{a}\,\text{AiryAiPrime}\left(\frac{(b+ax)^2-2ay(x)}{2\sqrt[3]{2}a^{2/3}}\right)}{\sqrt[3]{2}(ax+b)\,\text{AiryBi}\left(\frac{(b+ax)^2-2ay(x)}{2\sqrt[3]{2}a^{2/3}}\right)-2\sqrt[3]{a}\,\text{AiryBiPrime}\left(\frac{(b+ax)^2-2ay(x)}{2\sqrt[3]{2}a^{2/3}}\right)}+c_1=0,y(x)\right]$$

23.2 problem 2

Internal problem ID [10737]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - \frac{y}{(ax+b)^2} = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 415

```
dsolve(y(x)*diff(y(x),x)=(a*x+b)^(-2)*y(x)+1,y(x), singsol=all)
```

$$c_1 + \frac{-a2^{\frac{1}{3}}\left(y(x)\,a^2x + y(x)\,ab + 1\right)\mathrm{AiryAi}\left(-\frac{2^{\frac{2}{3}}\left(-\frac{1}{2}+x^2\left(-\frac{y(x)^2}{2}+x\right)a^4+3xb\left(-\frac{y(x)^2}{3}+x\right)a^3+\left(\left(-\frac{y(x)^2}{2}+3x\right)b^2-xy(x)\right)a^2+b\right.}{2(a^2)^{\frac{1}{3}}(ax+b)^2}\right.}{a2^{\frac{1}{3}}\left(y\left(x\right)a^2x + y\left(x\right)ab + 1\right)\mathrm{AiryBi}\left(-\frac{2^{\frac{2}{3}}\left(-\frac{1}{2}+x^2\left(-\frac{y(x)^2}{2}+x\right)a^4+3xb\left(-\frac{y(x)^2}{3}+x\right)a^3+\left(\left(-\frac{y(x)^2}{2}+3x\right)b^2-xy(x)\right)a^2+b\right.}{2(a^2)^{\frac{1}{3}}(ax+b)^2}\right.}$$
$$= 0$$

✓ Solution by Mathematica

Time used: 2.233 (sec). Leaf size: 561

```
DSolve[y[x]*y'[x]==(a*x+b)^(-2)*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{ay(x)(ax+b)\,\text{AiryAi}\left(\frac{-2x^3a^4-6bx^2a^3+(b+ax)^2y(x)^2a^2-6b^2xa^2-2b^3a+2(b+ax)y(x)a+1}{2\sqrt[3]{2}(a(b+ax)^3)^{2/3}}\right)+\text{AiryAi}\left(\frac{-2x^3a^4-6bx^2}{}\right.}{ay(x)(ax+b)\,\text{AiryBi}\left(\frac{-2x^3a^4-6bx^2a^3+(b+ax)^2y(x)^2a^2-6b^2xa^2-2b^3a+2(b+ax)y(x)a+1}{2\sqrt[3]{2}(a(b+ax)^3)^{2/3}}\right)+\text{AiryBi}\left(\frac{-2x^3a^4-6bx^2}{}\right.}\right.$$

$$\left.+c_1=0,y(x)\right]$$

23.3 problem 3

Internal problem ID [10738]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - \left(a - \frac{1}{ax}\right) y = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(y(x)*diff(y(x),x)=(a-1/(a*x))*y(x)+1,y(x), singsol=all)
```

$$y(x) = \frac{a^2x - \operatorname{RootOf}\left(-e^{\_Z} - \operatorname{Ei}_1\left(-\_Z\right) a^2x + c_1a^2x\right)}{a}$$

✓ Solution by Mathematica

Time used: 0.268 (sec). Leaf size: 37

```
DSolve[y[x]*y'[x]==(a-1/(a*x))*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\text{ExpIntegralEi}(a(ax - y(x))) + c_1 = \frac{e^{a(ax-y(x))}}{a^2x}, y(x)\right]$$

23.4 problem 4

Internal problem ID [10739]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], [_Abel, '2nd type', 'c`

$$\boxed{y'y - \frac{y}{\sqrt{ax+b}} = 1}$$

✓ Solution by Maple

Time used: 0.344 (sec). Leaf size: 171

```
dsolve(y(x)*diff(y(x),x)=(a*x+b)^(-1/2)*y(x)+1,y(x), singsol=all)
```

$$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{ax+b}\,y(x)a-ax-b}{\sqrt{(2a+1)(ax+b)^2}}\right)ax}{\sqrt{(2a+1)(ax+b)^2}} + \ln\left(ay(x)^2\sqrt{ax+b} - 2\sqrt{ax+b}\,ax - 2axy(x) - 2\sqrt{ax+b}\,b - 2by(x)\right) - \frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{ax+b}\,y(x)a-ax-b}{\sqrt{(2a+1)(ax+b)^2}}\right)b}{\sqrt{(2a+1)(ax+b)^2}} - \frac{\ln(ax+b)}{2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.256 (sec). Leaf size: 90

```
DSolve[y[x]*y'[x]==(a*x+b)^(-1/2)*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{\frac{2\arctan\left(\frac{\frac{ay(x)}{\sqrt{ax+b}}-1}{\sqrt{-2a-1}}\right)}{\sqrt{-2a-1}}+\log\left(-\frac{ay(x)^2}{ax+b}+\frac{2y(x)}{\sqrt{ax+b}}+2\right)}{a}=\frac{\log(ax+b)}{a}+c_1,y(x)\right]$$

23.5 problem 5

Internal problem ID [10740]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - \frac{3y}{\sqrt{a\,x^{\frac{3}{2}} + 8x}} = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 292

```
dsolve(y(x)*diff(y(x),x)=3*(a*x^(3/2)+8*x)^(-1/2)*y(x)+1,y(x), singsol=all)
```

$$c_1 + \frac{\left(-\frac{a\sqrt{x}\left(-2a\,x^{\frac{3}{2}}+\sqrt{x}\,ay(x)^2-8\sqrt{x\left(8+a\sqrt{x}\right)}\,y(x)-16x\right)}{\left(\sqrt{x}\,ay(x)-4\sqrt{x\left(8+a\sqrt{x}\right)}\right)^2}\right)^{\frac{1}{4}}\sqrt{2a\sqrt{x}+16}\,a\sqrt{x}\,y(x)+4\left(\int^{-\frac{\sqrt{2a\sqrt{x}+16}\,\sqrt{x\left(8+a\sqrt{x}\right)}}{\sqrt{x}\,ay(x)-4\sqrt{x\left(8+a\sqrt{x}\right)}}}\frac{\left(\_a^2-1\right)}{\sqrt{\_a}}\right.}{\sqrt{-\frac{\sqrt{2a\sqrt{x}+16}\,\sqrt{x\left(8+a\sqrt{x}\right)}}{\sqrt{x}\,ay(x)-4\sqrt{x\left(8+a\sqrt{x}\right)}}}\left(\sqrt{x}\,ay\left(x\right)-4\sqrt{x\left(8+\right.}\right.}$$

$$= 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==3*(a*x^(3/2)+8*x)^(-1/2)*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

23.6 problem 6

Internal problem ID [10741]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$yy' - \left(\frac{a}{x^{\frac{2}{3}}} - \frac{2}{3a\,x^{\frac{1}{3}}}\right)y = 1$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 128

```
dsolve(y(x)*diff(y(x),x)=(a*x^(-2/3)-2/3*a^(-1)*x^(-1/3))*y(x)+1,y(x), singsol=all)
```

$$c_1 + \frac{\operatorname{BesselK}\left(1, -\frac{2\sqrt{\frac{x^{\frac{2}{3}}+ay(x)}{a^4}}}{3}\right)\sqrt{\frac{x^{\frac{2}{3}}+ay(x)}{a^4}}\,a^2 - x^{\frac{1}{3}}\operatorname{BesselK}\left(0, -\frac{2\sqrt{\frac{x^{\frac{2}{3}}+ay(x)}{a^4}}}{3}\right)}{-\operatorname{BesselI}\left(1, \frac{2\sqrt{\frac{x^{\frac{2}{3}}+ay(x)}{a^4}}}{3}\right)\sqrt{\frac{x^{\frac{2}{3}}+ay(x)}{a^4}}\,a^2 + x^{\frac{1}{3}}\operatorname{BesselI}\left(0, \frac{2\sqrt{\frac{x^{\frac{2}{3}}+ay(x)}{a^4}}}{3}\right)} = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*x^(-2/3)-2/3*a^(-1)*x^(-1/3))*y[x]+1,y[x],x,IncludeSingularSolutions -
```

Not solved

23.7 problem 7

Internal problem ID [10742]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - a\,\mathrm{e}^{x\lambda}y = 1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 84

```
dsolve(y(x)*diff(y(x),x)=a*exp(lambda*x)*y(x)+1,y(x), singsol=all)
```

$$c_1 - a\ \mathrm{erf}\left(\frac{(-\lambda y(x) + \mathrm{e}^{\lambda x}a)\sqrt{2}}{2\sqrt{-\lambda}}\right)\sqrt{2}\,\sqrt{\pi} - 2\sqrt{-\lambda}\,\mathrm{e}^{\frac{a^2\mathrm{e}^{2\lambda x} - 2\,\mathrm{e}^{\lambda x}a\lambda y(x) + \lambda^2 y(x)^2 - 2\lambda^2 x}{2\lambda}} = 0$$

✓ Solution by Mathematica

Time used: 1.687 (sec). Leaf size: 83

```
DSolve[y[x]*y'[x]==a*Exp[\[Lambda]*x]*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{ae^{\lambda x}}{\sqrt{\lambda}} = \frac{2e^{\frac{(ae^{\lambda x} - \lambda y(x))^2}{2\lambda}}}{\sqrt{2\pi}\text{erfi}\left(\frac{\lambda y(x) - ae^{\lambda x}}{\sqrt{2}\sqrt{\lambda}}\right) + 2c_1}, y(x)\right]$$

23.8 problem 8

Internal problem ID [10743]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{y'y - \left(e^{\lambda x}a + e^{-\lambda x}b\right) y = 1}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=(a*exp(lambda*x)+b*exp(-lambda*x))*y(x)+1,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*Exp[\[Lambda]*x]+b*Exp[-\[Lambda]*x])*y[x]+1,y[x],x,IncludeSingularSol
```

Not solved

23.9 problem 9

Internal problem ID [10744]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{y'y - ay\cosh(x) = 1}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=a*y(x)*cosh(x)+1,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==a*y[x]*Cosh[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

23.10 problem 10

Internal problem ID [10745]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{y'y - ay\sinh(x) = 1}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=a*y(x)*sinh(x)+1,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==a*y[x]*Sinh[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

23.11 problem 11

Internal problem ID [10746]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{y'y - a\cos(\lambda x)\, y = 1}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=a*cos(lambda*x)*y(x)+1,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==a*Cos[\[Lambda]*x]*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

23.12 problem 12

Internal problem ID [10747]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{y'y - a\sin(\lambda x)\, y = 1}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=a*sin(lambda*x)*y(x)+1,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==a*Sin[\[Lambda]*x]*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

24 Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$

24.1 problem 1

Internal problem ID [10748]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$yy' - (ax + 3b)\,y = -ab\,x^2 + x^3c - 2b^2x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 233

```
dsolve(y(x)*diff(y(x),x)=(a*x+3*b)*y(x)+c*x^3-a*b*x^2-2*b^2*x,y(x), singsol=all)
```

$$c_1 + \frac{x\left(\frac{-ab\,x^3+c\,x^4+a\,x^2y(x)-2b^2x^2+4bxy(x)-2y(x)^2}{(xb-y(x))^2}\right)^{\frac{1}{4}} \mathrm{e}^{-\frac{a\ \mathrm{arctanh}\left(\frac{-2c\,x^2+a(xb-y(x))}{(xb-y(x))\sqrt{a^2+8c}}\right)}{2\sqrt{a^2+8c}}} y(x) + \sqrt{-\frac{x^2}{xb-y(x)}}\left(\int^{-\frac{x^2}{xb-y(x)}} \frac{(\_a^2c+\_}{\cdots}\right.}{\sqrt{-\frac{x^2}{xb-y(x)}}\,(xb-y(x))}$$
$$= 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*x+3*b)*y[x]+c*x^3-a*b*x^2-2*b^2*x,y[x],x,IncludeSingularSolutions -> T
```

Not solved

24.2 problem 2

Internal problem ID [10749]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y'y - (3ax + b)\,y = -a^2x^3 - ba\,x^2 + cx$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 943

```
dsolve(y(x)*diff(y(x),x)=(3*a*x+b)*y(x)-a^2*x^3-a*b*x^2+c*x,y(x), singsol=all)
```

$y(x) =$

$$-\frac{}{-9a\,b^2x - 27acx + b\,\mathrm{e}^{\mathrm{RootOf}\left(2a\,b^8 \operatorname{arctanh}\left(\frac{b^2\left(9b^2+27c+2\,\mathrm{e}^{\_Z}\right)}{9\sqrt{b^8+10b^6c+33b^4c^2+36b^2c^3}}\right)+20a\,b^6c \operatorname{arctanh}\left(\frac{b^2\left(9b^2+27c+2\,\mathrm{e}^{\_Z}\right)}{9\sqrt{b^8+10b^6c+33b^4c^2+36b^2c^3}}\right)+66a \cdots\right.}}$$

✓ Solution by Mathematica

Time used: 6.592 (sec). Leaf size: 194

```
DSolve[y[x]*y'[x]==(3*a*x+b)*y[x]-a^2*x^3-a*b*x^2+c*x,y[x],x,IncludeSingularSolutions -> Tru
```

$$\mathrm{Solve}\left[\frac{2ab\left(\mathrm{RootSum}\left[\#1^4a^2 + \#1^3ab - 2\#1^2ay(x) - \#1^2c - \#1by(x) + y(x)^2\&, \frac{-2\#1^3a^2\log(x-\#1) - \#1^2 \cdots}{\cdots}\right.\right.}{c(3a + b \cdots}\right.$$

24.3 problem 3

Internal problem ID [10750]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$2yy' - (7ax + 5b)\, y = -3a^2x^3 - 3b^2x - 2c\, x^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 2909

```
dsolve(2*y(x)*diff(y(x),x)=(7*a*x+5*b)*y(x)-3*a^2*x^3-2*c*x^2-3*b^2*x,y(x), singsol=all)
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[2*y[x]*y'[x]==(7*a*x+5*b)*y[x]-3*a^2*x^3-2*c*x^2-3*b^2*x,y[x],x,IncludeSingularSoluti
```

Not solved

24.4 problem 4

Internal problem ID [10751]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{y'y - ((-m+3)\,x - 1)\,y = -(m-1)\,ax}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=((3-m)*x-1)*y(x)+(m-1)*(x^2-x^2-a*x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==((3-m)*x-1)*y[x]+(m-1)*(x^2-x^2-a*x),y[x],x,IncludeSingularSolutions -> T
```

Not solved

24.5 problem 5

Internal problem ID [10752]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class A‘]]`

$$\boxed{yy' + x(a\,x^2 + b)\,y = -x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 157

```
dsolve(y(x)*diff(y(x),x)+x*(a*x^2+b)*y(x)+x=0,y(x), singsol=all)
```

$$c_1 + \frac{-2\,\mathrm{AiryAi}\left(1, \frac{a^2x^4+2ab\,x^2+4ay(x)+b^2}{4a^{\frac{2}{3}}}\right)a^{\frac{1}{3}} + (-a\,x^2 - b)\,\mathrm{AiryAi}\left(\frac{a^2x^4+2ab\,x^2+4ay(x)+b^2}{4a^{\frac{2}{3}}}\right)}{(a\,x^2+b)\,\mathrm{AiryBi}\left(\frac{a^2x^4+2ab\,x^2+4ay(x)+b^2}{4a^{\frac{2}{3}}}\right) + 2\,\mathrm{AiryBi}\left(1, \frac{a^2x^4+2ab\,x^2+4ay(x)+b^2}{4a^{\frac{2}{3}}}\right)a^{\frac{1}{3}}} = 0$$

✓ Solution by Mathematica

Time used: 0.492 (sec). Leaf size: 143

```
DSolve[y[x]*y'[x]+x*(a*x^2+b)*y[x]+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\frac{(ax^2+b)\,\mathrm{AiryAi}\left(\frac{(ax^2+b)^2+4ay(x)}{4a^{2/3}}\right) + 2\sqrt[3]{a}\,\mathrm{AiryAiPrime}\left(\frac{(ax^2+b)^2+4ay(x)}{4a^{2/3}}\right)}{(ax^2+b)\,\mathrm{AiryBi}\left(\frac{(ax^2+b)^2+4ay(x)}{4a^{2/3}}\right) + 2\sqrt[3]{a}\,\mathrm{AiryBiPrime}\left(\frac{(ax^2+b)^2+4ay(x)}{4a^{2/3}}\right)} + c_1 = 0, y(x)\right]$$

24.6 problem 6

Internal problem ID [10753]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' + a\left(1 - \frac{1}{x}\right)y = a^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve(y(x)*diff(y(x),x)+a*(1-x^(-1))*y(x)=a^2,y(x), singsol=all)
```

$$y(x) = -ax + \text{RootOf}\left(-e^{-\_Z} - \text{Ei}_1\left(-\_Z\right)x + xc_1\right)a$$

✓ Solution by Mathematica

Time used: 0.208 (sec). Leaf size: 30

```
DSolve[y[x]*y'[x]+a*(1-x^(-1))*y[x]==a^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\text{ExpIntegralEi}\left(x + \frac{y(x)}{a}\right) + c_1 = \frac{e^{\frac{y(x)}{a}+x}}{x}, y(x)\right]$$

24.7 problem 7

Internal problem ID [10754]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - a\left(1 - \frac{b}{x}\right)y = b\,a^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(y(x)*diff(y(x),x)-a*(1-b*x^(-1))*y(x)=a^2*b,y(x), singsol=all)
```

$$y(x) = -\operatorname{RootOf}\left(-b\,\mathrm{e}^{\_Z} - \operatorname{Ei}_1\left(-\_Z\right)x + xc_1\right)ba + ax$$

✓ Solution by Mathematica

Time used: 0.293 (sec). Leaf size: 45

```
DSolve[y[x]*y'[x]-a*(1-b*x^(-1))*y[x]==a^2*b,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\text{ExpIntegralEi}\left(\frac{ax - y(x)}{ab}\right) + c_1 = \frac{be^{\frac{ax-y(x)}{ab}}}{x}, y(x)\right]$$

24.8 problem 8

Internal problem ID [10755]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$yy' - x^{-1+n}((2n+1)\,x + an)\,y = -n\,x^{2n}(x+a)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 153

```
dsolve(y(x)*diff(y(x),x)=x^(n-1)*((1+2*n)*x+a*n)*y(x)-n*x^(2*n)*(x+a),y(x), singsol=all)
```

$$y(x) = -\frac{2\left(-\frac{\sqrt{-n^2}\,\tan\left(\frac{\operatorname{RootOf}\left(-\sqrt{-n^2}\,\tan\left(\frac{\_a\sqrt{-n^2}}{2}\right)\_Z x - 2an\,e^{\_a+\_Z} - nx\,e^{\_a+\_Z} + 2xc_1 e^{\_a}\right)\sqrt{-n^2}}{2}\right)x}{2} + n\left(a+\frac{x}{2}\right)\right)x^n}{\tan\left(\frac{\operatorname{RootOf}\left(-\sqrt{-n^2}\,\tan\left(\frac{\_a\sqrt{-n^2}}{2}\right)\_Z x - 2an\,e^{\_a+\_Z} - nx\,e^{\_a+\_Z} + 2xc_1 e^{\_a}\right)\sqrt{-n^2}}{2}\right)\sqrt{-n^2} - n}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==x^(n-1)*((1+2*n)*x+a*n)*y[x]-n*x^(2*n)*(x+a),y[x],x,IncludeSingularSoluti
```

Not solved

24.9 problem 9

Internal problem ID [10756]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$yy' - a(-nb + x)\, x^{-1+n} y = c\left(x^2 - (2n+1)\, bx + n(1+n)\, b^2\right) x^{2n-1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10211

```
dsolve(y(x)*diff(y(x),x)=a*(x-n*b)*x^(n-1)*y(x)+c*(x^2-(2*n+1)*b*x+n*(n+1)*b^2)*x^(2*n-1),y(
```

Expression too large to display

✓ Solution by Mathematica

Time used: 0.744 (sec). Leaf size: 200

```
DSolve[y[x]*y'[x]==a*(x-n*b)*x^(n-1)*y[x]+c*(x^2-(2*n+1)*b*x+n*(n+1)*b^2)*x^(2*n-1),y[x],x,I
```

$$\text{Solve}\left[\frac{a^2\left(-\frac{2a\operatorname{arctanh}\left(\frac{a^2-\frac{2ac(n+1)y(x)}{-bcx^n-bcnx^n+cx^{n+1}}}{a\sqrt{a^2+4c(n+1)}}\right)}{\sqrt{a^2+4c(n+1)}}-\log\left(a^2\left(\frac{ay(x)}{-bcx^n-bcnx^n+cx^{n+1}}+1\right)-\frac{a^2c(n+1)y(x)^2}{(-bcx^n-bcnx^n+cx^{n+1})^2}\right)\right)}{2c(n+1)}\right.$$

$$\left.+c_1,y(x)\right]$$

24.10 problem 10

Internal problem ID [10757]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, `2nd type`, `class A`]]`

$$y'y - \left(a(2n+k)\,x^k + b\right)x^{-1+n}y = \left(-a^2n\,x^{2k} - ab\,x^k + c\right)x^{2n-1}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=(a*(2*n+k)*x^k+b)*x^(n-1)*y(x)+(-a^2*n*x^(2*k)-a*b*x^k+c)*x^(2*n-1)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*(2*n+k)*x^k+b)*x^(n-1)*y[x]+(-a^2*n*x^(2*k)-a*b*x^k+c)*x^(2*n-1),y[x],
```

Not solved

24.11 problem 11

Internal problem ID [10758]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - \left(a(2n+k)\,x^{2k} + b(2m-k)\right)x^{m-k-1}y = -\frac{a^2m\,x^{4k} + c\,x^{2k} + m\,b^2}{x}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=(a*(2*n+k)*x^(2*k)+b*(2*m-k))*x^(m-k-1)*y(x)-(a^2*m*x^(4*k)+c*x^(2*
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*(2*n+k)*x^(2*k)+b*(2*m-k))*x^(m-k-1)*y[x]-(a^2*m*x^(4*k)+c*x^(2*k)+b^2
```

Timed out

24.12 problem 12

Internal problem ID [10759]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - \frac{((m+2L-3)\,x+n-2L+3)\,y}{x} = ((m-L-1)\,x^2+(n-m-2L+3)\,x-n+L-2)\,x^{1-2L}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=((m+2*L-3)*x+n-2*L+3)*1/x*y(x)+((m-L-1)*x^2+(n-m-2*L+3)*x-n+L-2)*x^
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==((m+2*L-3)*x+n-2*L+3)*1/x*y[x]+((m-L-1)*x^2+(n-m-2*L+3)*x-n+L-2)*x^(1-2*L
```

Timed out

24.13 problem 13

Internal problem ID [10760]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y'y - \left(a(2n+1)\,x^2 + cx + b(2n-1)\right)x^{n-2}y = -\left(n\,a^2x^4 + ac\,x^3 + b^2n + bcx + d\,x^2\right)x^{-3+2n}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=(a*(2*n+1)*x^2+c*x+b*(2*n-1))*x^(n-2)*y(x)-(n*a^2*x^4+a*c*x^3+d*x^2
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*(2*n+1)*x^2+c*x+b*(2*n-1))*x^(n-2)*y[x]-(n*a^2*x^4+a*c*x^3+d*x^2+b*c*x
```

Timed out

24.14 problem 14

Internal problem ID [10761]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$y'y - (a(-1+n)x + b(2\lambda+n))\, x^{\lambda-1}(ax+b)^{-\lambda-2}\, y = -(axn + b(\lambda+n))\, x^{2\lambda-1}(ax+b)^{-2\lambda-3}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=(a*(n-1)*x+b*(2*lambda+n))*x^(lambda-1)*(a*x+b)^(-lambda-2)*y(x)-(a
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*(n-1)*x+b*(2*\[Lambda]+n))*x^(\[Lambda]-1)*(a*x+b)^(-\[Lambda]-2)*y[x]
```

Not solved

24.15 problem 15

Internal problem ID [10762]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$
\boxed{yy' - \frac{a((m-1)\,x+1)\,y}{x} = \frac{a^2(mx+1)\,(x-1)}{x}}
$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 521

```
dsolve(y(x)*diff(y(x),x)-a*((m-1)*x+1)*1/x*y(x)=a^2*1/x*(m*x+1)*(x-1),y(x), singsol=all)
```

$$
c_1 - \frac{9(3mx-m+1)\,m\left(\frac{(amx+a-y(x))m}{am-y(x)m+2a-2y(x)}\right)^{\frac{m}{m+1}}\left(\frac{a\,m^2x}{am-y(x)m-a+y(x)}\right)^{-\frac{m}{m+1}}\left(\frac{a\,m^2x}{am-y(x)m-a+y(x)}\right)^{-\frac{1}{m+1}}\left(\frac{(ax-a+y(x}{2am-2y(x)m}\right.}{2m^3+3m^2-3m-2}
$$

$$
-\left(\int^{-\frac{9m(3amx+y(x)m-am-y(x)+a)}{2y(x)m^3-2a\,m^3+3y(x)m^2-3a\,m^2-3y(x)m+3am-2y(x)+2a}} \frac{\_a(2\_a\,m^2-\_am-\_a+9m)^{\frac{2m^3}{(m+1)\left(2m^2-m-1\right)}}\left(2\_a\,m^2-\right.}{}\right.
$$

$$
= 0
$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a*((m-1)*x+1)*1/x*y[x]==a^2*1/x*(m*x+1)*(x-1),y[x],x,IncludeSingularSoluti
```

Not solved

24.16 problem 16

Internal problem ID [10763]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$yy' - a\left(1 - \frac{b}{\sqrt{x}}\right)y = \frac{a^2 b}{\sqrt{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 183

```
dsolve(y(x)*diff(y(x),x)-a*(1-b*x^(-1/2))*y(x)=a^2*b*x^(-1/2),y(x), singsol=all)
```

$$c_1 + \frac{-(b^2)^{\frac{1}{3}} 2^{\frac{2}{3}} \left(-\sqrt{x}+b\right) \operatorname{AiryAi}\left(\frac{2^{\frac{1}{3}}\left(-2\sqrt{x}\,ab+(b^2+x)a-y(x)\right)}{2(b^2)^{\frac{1}{3}}a}\right) - 2\operatorname{AiryAi}\left(1, \frac{2^{\frac{1}{3}}\left(-2\sqrt{x}\,ab+(b^2+x)a-y(x)\right)}{2(b^2)^{\frac{1}{3}}a}\right) b}{(b^2)^{\frac{1}{3}} 2^{\frac{2}{3}} \left(-\sqrt{x}+b\right) \operatorname{AiryBi}\left(\frac{2^{\frac{1}{3}}\left(-2\sqrt{x}\,ab+(b^2+x)a-y(x)\right)}{2(b^2)^{\frac{1}{3}}a}\right) + 2\operatorname{AiryBi}\left(1, \frac{2^{\frac{1}{3}}\left(-2\sqrt{x}\,ab+(b^2+x)a-y(x)\right)}{2(b^2)^{\frac{1}{3}}a}\right) b} = 0$$

✓ Solution by Mathematica

Time used: 1.905 (sec). Leaf size: 323

```
DSolve[y[x]*y'[x]-a*(1-b*x^(-1/2))*y[x]==a^2*b*x^(-1/2),y[x],x,IncludeSingularSolutions -> T
```

$$\text{Solve}\left[\frac{\sqrt[3]{-1}2^{2/3}\sqrt[3]{\left(b-\sqrt{x}\right)^3}\,\text{AiryAi}\left(\frac{\left(-\frac{1}{2}\right)^{2/3}\left(\left(b-\sqrt{x}\right)^3\right)^{2/3}\left(a\left(b-\sqrt{x}\right)^2-y(x)\right)}{ab^{2/3}\left(b-\sqrt{x}\right)^2}\right)-2\sqrt[3]{b}\,\text{AiryAiPrime}\left(\frac{\left(-\frac{1}{2}\right)^{2/3}\left(\left(b-\right.\right.}{}\right.}{\sqrt[3]{-1}2^{2/3}\sqrt[3]{\left(b-\sqrt{x}\right)^3}\,\text{AiryBi}\left(\frac{\left(-\frac{1}{2}\right)^{2/3}\left(\left(b-\sqrt{x}\right)^3\right)^{2/3}\left(a\left(b-\sqrt{x}\right)^2-y(x)\right)}{ab^{2/3}\left(b-\sqrt{x}\right)^2}\right)-2\sqrt[3]{b}\,\text{AiryBiPrime}\left(\frac{\left(-\frac{1}{2}\right)^{2/3}\left(\left(b-\right.\right.}{}\right.}\right.$$

$$\left.+c_1=0,y(x)\right]$$

24.17 problem 17

Internal problem ID [10764]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$yy' - \frac{3y}{(ax+b)^{\frac{1}{3}} x^{\frac{5}{3}}} = \frac{3}{(ax+b)^{\frac{2}{3}} x^{\frac{7}{3}}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 147

```
dsolve(y(x)*diff(y(x),x)=3*(a*x+b)^(-1/3)*x^(-5/3)*y(x)+3*(a*x+b)^(-2/3)*x^(-7/3),y(x), sing
```

$$y(x) = -\frac{6\sqrt{3}}{(ax+b)^{\frac{1}{3}} x^{\frac{2}{3}} \left(\left(\frac{a}{(ax+b)^2 x^4}\right)^{\frac{1}{3}} \sqrt{3}\,(ax+b)^{\frac{1}{3}} x^{\frac{5}{3}} - 3x^{\frac{5}{3}} \tan\left(\mathrm{RootOf}\left(\sqrt{3}\ln\left(\frac{\tan(\_Z)^2+1}{\left(\sqrt{3}-\tan(\_Z)\right)^2}\right) + 6\sqrt{3}\,c\right.\right.\right.}$$

✓ Solution by Mathematica

Time used: 1.769 (sec). Leaf size: 312

```
DSolve[y[x]*y'[x]==3*(a*x+b)^(-1/3)*x^(-5/3)*y[x]+3*(a*x+b)^(-2/3)*x^(-7/3),y[x],x,IncludeSi
```

$$\mathrm{Solve}\left[\frac{1}{6}\left(2\sqrt{3}\arctan\left(\frac{-\frac{2\left(x^{2/3}y(x)\sqrt[3]{ax+b}+3\right)}{\sqrt[3]{ax^3}y(x)}-1}{\sqrt{3}}\right)+2\log\left(\frac{-x^{2/3}y(x)\sqrt[3]{ax+b}-3}{\sqrt[3]{ax^3}y(x)}+1\right)-\log\left(\frac{\left(x^{2/3}\right.}{}\right.\right.\right.$$

24.18 problem 18

Internal problem ID [10765]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$3y'y - \frac{(-7\lambda s(3s+4\lambda)x + 6s - 2\lambda)y}{x^{\frac{1}{3}}} = \frac{6\lambda sx - 6}{x^{\frac{2}{3}}} + 2(\lambda s(3s+4\lambda)x + 5\lambda)(-\lambda s(3s+4\lambda)x + 3s + 4\lambda$$

✗ Solution by Maple

```
dsolve(3*y(x)*diff(y(x),x)=(-7*lambda*s*(3*s+4*lambda)*x+6*s-2*lambda)*x^(-1/3)*y(x)+6*(lamb
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[3*y[x]*y'[x]==(-7*\[Lambda]*s*(3*s+4*\[Lambda])*x+6*s-2*\[Lambda])*x^(-1/3)*y[x]+6*(\
```

Timed out

24.19 problem 19

Internal problem ID [10766]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' + \frac{a(6x-1)\,y}{2x} = -\frac{a^2(x-1)\,(4x-1)}{2x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 364

```
dsolve(y(x)*diff(y(x),x)+1/2*a*(6*x-1)*1/x*y(x)=-1/2*a^2*(x-1)*(4*x-1)*1/x,y(x), singsol=all
```

$$c_1 + \frac{\sqrt{2}\left(\frac{i\left(i\sqrt{-x}\,a+2ax+y(x)-a\right)\sqrt{-x}}{xa}\right)^{\frac{3}{2}}\left(-\frac{i\left(i\sqrt{-x}\,a+2ax+y(x)-a\right)\sqrt{-x}\ \mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{2}\right],\left[\frac{7}{2}\right],\frac{i\left(i\sqrt{-x}\,a+2ax+y(x)-a\right)\sqrt{-x}}{2xa}\right)}{8xa} + \frac{5\left(-4i\sqrt{}\right.}{}\right.}{2\left(\frac{3}{2}-\frac{-4i\sqrt{2}\,x+6i\sqrt{-x}+i\sqrt{2}-4x-2}{2\left(-4i\sqrt{2}\,x+2i\sqrt{-x}+4\sqrt{2}\,\sqrt{-x}-i\sqrt{2}-4x+2\right)}\right)\mathrm{hypergeom}\left(\left[-2,-1\right],\left[-\frac{1}{2}\right],\frac{i\left(i\sqrt{-x}\,a+2a\right.}{}\right.} = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/2*a*(6*x-1)*1/x*y[x]==-1/2*a^2*(x-1)*(4*x-1)*1/x,y[x],x,IncludeSingularS
```

Not solved

24.20 problem 20

Internal problem ID [10767]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - \frac{a\left(1 + \frac{2b}{x^2}\right) y}{2} = \frac{a^2\left(3x + \frac{4b}{x}\right)}{16}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-1/2*a*(1+2*b*x^(-2))*y(x)=1/16*a^2*(3*x+4*b/x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/2*a*(1+2*b*x^(-2))*y[x]==1/16*a^2*(3*x+4*b/x),y[x],x,IncludeSingularSolu
```

Not solved

24.21 problem 21

Internal problem ID [10768]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$
y'y + \frac{a(13x-20)\,y}{14x^{\frac{9}{7}}} = -\frac{3a^2(x-1)\,(x-8)}{14x^{\frac{11}{17}}}
$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+1/14*a*(13*x-20)*x^(-9/7)*y(x)=-3/14*a^2*(x-1)*(x-8)*x^(-11/17),y(x
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/14*a*(13*x-20)*x^(-9/7)*y[x]==-3/14*a^2*(x-1)*(x-8)*x^(-11/17),y[x],x,In
```

Timed out

24.22 problem 22

Internal problem ID [10769]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$y'y + \frac{5a(23x-16)\,y}{56x^{\frac{9}{7}}} = -\frac{3a^2(x-1)\,(25x-32)}{56x^{\frac{11}{17}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+5/56*a*(23*x-16)*x^(-9/7)*y(x)=-3/56*a^2*(x-1)*(25*x-32)*x^(-11/17)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+5/56*a*(23*x-16)*x^(-9/7)*y[x]==-3/56*a^2*(x-1)*(25*x-32)*x^(-11/17),y[x],
```

Timed out

24.23 problem 23

Internal problem ID [10770]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y + \frac{a(19x + 85)\,y}{26x^{\frac{18}{13}}} = -\frac{3a^2(x - 1)\,(x + 25)}{26x^{\frac{23}{13}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+1/26*a*(19*x+85)*x^(-18/13)*y(x)=-3/26*a^2*(x-1)*(x+25)*x^(-23/13),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/26*a*(19*x+85)*x^(-18/13)*y[x]==-3/26*a^2*(x-1)*(x+25)*x^(-23/13),y[x],x
```

Timed out

24.24 problem 24

Internal problem ID [10771]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$y'y + \frac{a(13x - 18)\,y}{15x^{\frac{7}{5}}} = -\frac{4a^2(x-1)\,(x-6)}{15x^{\frac{9}{5}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+1/15*a*(13*x-18)*x^(-7/5)*y(x)=-4/15*a^2*(x-1)*(x-6)*x^(-9/5),y(x),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/15*a*(13*x-18)*x^(-7/5)*y[x]==-4/15*a^2*(x-1)*(x-6)*x^(-9/5),y[x],x,Incl
```

Timed out

24.25 problem 25

Internal problem ID [10772]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$yy' + \frac{a(5x+1)\,y}{2\sqrt{x}} = a^2(-x^2+1)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13536

```
dsolve(y(x)*diff(y(x),x)+1/2*a*(5*x+1)*x^(-1/2)*y(x)=a^2*(1-x^2),y(x), singsol=all)
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/2*a*(5*x+1)*x^(-1/2)*y[x]==a^2*(1-x^2),y[x],x,IncludeSingularSolutions -
```

Not solved

24.26 problem 26

Internal problem ID [10773]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class A‘]]`

$$y'y + \frac{3a(19x-14)\,x^{\frac{7}{5}}y}{35} = -\frac{4a^2(x-1)\,(9x-14)\,x^{\frac{9}{5}}}{35}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+3/35*a*(19*x-14)*x^(7/5)*y(x)=-4/35*a^2*(x-1)*(9*x-14)*x^(9/5),y(x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+3/35*a*(19*x-14)*x^(7/5)*y[x]==-4/35*a^2*(x-1)*(9*x-14)*x^(9/5),y[x],x,Inc
```

Timed out

24.27 problem 27

Internal problem ID [10774]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y + \frac{3a(3x+7)\,y}{10x^{\frac{13}{10}}} = -\frac{a^2(x-1)\,(x+9)}{5x^{\frac{8}{5}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+3/10*a*(3*x+7)*x^(-13/10)*y(x)=-1/5*a^2*(x-1)*(x+9)*x^(-8/5),y(x),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+3/10*a*(3*x+7)*x^(-13/10)*y[x]==-1/5*a^2*(x-1)*(x+9)*x^(-8/5),y[x],x,Inclu
```

Timed out

24.28 problem 28

Internal problem ID [10775]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$yy' + \frac{a(7x-12)\,y}{10x^{\frac{7}{5}}} = -\frac{a^2(x-1)\,(x-16)}{10x^{\frac{9}{5}}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 290

```
dsolve(y(x)*diff(y(x),x)+1/10*a*(7*x-12)*x^(-7/5)*y(x)=-1/10*a^2*(x-1)*(x-16)*x^(-9/5),y(x),
```

$$c_1 + \frac{\left(-\frac{x^{\frac{8}{5}}a+4x^{\frac{3}{5}}a-5x^{\frac{11}{10}}a+xy(x)}{10x^{\frac{11}{10}}a}\right)^{\frac{3}{2}}\left(\frac{5\left(x^{\frac{8}{5}}a+4x^{\frac{3}{5}}a-5x^{\frac{11}{10}}a+xy(x)\right)\mathrm{hypergeom}\left(\left[-\frac{3}{2}\right],[],-\frac{x^{\frac{8}{5}}a+4x^{\frac{3}{5}}a-5x^{\frac{11}{10}}a+xy(x)}{10x^{\frac{11}{10}}a}\right)}{8x^{\frac{11}{10}}a}+\frac{25(-x-2+3\sqrt{\cdots}}{\cdots}\cdots\right.}{\frac{\left(\frac{3}{2}-\frac{5(-x-2+3\sqrt{x})}{2(-x+2+\sqrt{x})}\right)\mathrm{hypergeom}\left([-4,1],\left[-\frac{1}{2}\right],-\frac{x^{\frac{8}{5}}a+4x^{\frac{3}{5}}a-5x^{\frac{11}{10}}a+xy(x)}{10x^{\frac{11}{10}}a}\right)}{2}+\frac{2\left(x^{\frac{8}{5}}a+4x^{\frac{3}{5}}a-5x^{\frac{11}{10}}a+xy(x)\right)\mathrm{hypergeom}\left(\left[\cdots\right.\right.}{5x^{\frac{11}{10}}a\cdots}} = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/10*a*(7*x-12)*x^(-7/5)*y[x]==-1/10*a^2*(x-1)*(x-16)*x^(-9/5),y[x],x,Incl
```

Timed out

24.29 problem 29

Internal problem ID [10776]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y + \frac{3a(13x-8)\,y}{20x^{\frac{7}{5}}} = -\frac{a^2(x-1)\,(27x-32)}{20x^{\frac{9}{5}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+3/20*a*(13*x-8)*x^(-7/5)*y(x)=-1/20*a^2*(x-1)*(27*x-32)*x^(-9/5),y(
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+3/20*a*(13*x-8)*x^(-7/5)*y[x]==-1/20*a^2*(x-1)*(27*x-32)*x^(-9/5),y[x],x,I
```

Timed out

24.30 problem 30

Internal problem ID [10777]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y + \frac{3a(3x+11)\,y}{14x^{\frac{10}{7}}} = -\frac{a^2(x-1)\,(x-27)}{14x^{\frac{13}{7}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+3/14*a*(3*x+11)*x^(-10/7)*y(x)=-1/14*a^2*(x-1)*(x-27)*x^(-13/7),y(x
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+3/14*a*(3*x+11)*x^(-10/7)*y[x]==-1/14*a^2*(x-1)*(x-27)*x^(-13/7),y[x],x,In
```

Timed out

24.31 problem 31

Internal problem ID [10778]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - \frac{a(1+x)\,y}{2x^{\frac{7}{4}}} = \frac{a^2(x-1)\,(3x+5)}{4x^{\frac{5}{2}}}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 191

```
dsolve(y(x)*diff(y(x),x)-1/2*a*(x+1)*x^(-7/4)*y(x)=1/4*a^2*(x-1)*(3*x+5)*x^(-5/2),y(x), sing
```

$$c_1 - \frac{\left(\int^{-\frac{90\left(2x^{\frac{3}{4}}y(x)+2ax-15a\right)}{143\left(x^{\frac{3}{4}}y(x)+ax\right)}} \frac{\_a\sqrt{11\_a-90}\,(13\_a+90)^{\frac{5}{6}}}{(143\_a+180)^{\frac{4}{3}}\left(-\frac{20449}{1458000}\_a^3+\frac{49}{60}\_a+1\right)}d\_a\right)x\left(\frac{a}{x^{\frac{3}{4}}y(x)+ax}\right)^{\frac{4}{3}} - \frac{\sqrt{78}\,11^{\frac{1}{6}}\left(\frac{(3x+5)a+3x^{\frac{3}{4}}y(x)}{x^{\frac{3}{4}}y(x)+ax}\right)^{\frac{5}{6}}}{}}{\left(\frac{a}{x^{\frac{3}{4}}y(x)+ax}\right)^{\frac{4}{3}}x}$$
$$= 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/2*a*(x+1)*x^(-7/4)*y[x]==1/4*a^2*(x-1)*(3*x+5)*x^(-5/2),y[x],x,IncludeSi
```

Timed out

24.32 problem 32

Internal problem ID [10779]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{y'y - \frac{a(x+1)\,y}{2x^{\frac{7}{4}}} = \frac{a^2(x-1)\,(x+5)}{4x^{\frac{5}{2}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-1/2*a*(x+1)*x^(-7/4)*y(x)=1/4*a^2*(x-1)*(x+5)*x^(-5/2),y(x), singso
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/2*a*(x+1)*x^(-7/4)*y[x]==1/4*a^2*(x-1)*(x+5)*x^(-5/2),y[x],x,IncludeSing
```

Timed out

24.33 problem 33

Internal problem ID [10780]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - \frac{a(4x+3)\,y}{14x^{\frac{8}{7}}} = -\frac{a^2(x-1)\,(16x+5)}{14x^{\frac{9}{7}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-1/14*a*(4*x+3)*x^(-8/7)*y(x)=-1/14*a^2*(x-1)*(16*x+5)*x^(-9/7),y(x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/14*a*(4*x+3)*x^(-8/7)*y[x]==-1/14*a^2*(x-1)*(16*x+5)*x^(-9/7),y[x],x,Inc
```

Timed out

24.34 problem 34

Internal problem ID [10781]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, `2nd type`, `class B`]]`

$$y'y + \frac{a(13x-3)\,y}{6x^{\frac{2}{3}}} = -\frac{a^2(x-1)\,(5x-1)}{6x^{\frac{1}{3}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+1/6*a*(13*x-3)*x^(-2/3)*y(x)=-1/6*a^2*(x-1)*(5*x-1)*x^(-1/3),y(x),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/6*a*(13*x-3)*x^(-2/3)*y[x]==-1/6*a^2*(x-1)*(5*x-1)*x^(-1/3),y[x],x,Inclu
```

Not solved

24.35 problem 35

Internal problem ID [10782]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$y'y - \frac{a(8x-1)\,y}{28x^{\frac{8}{7}}} = \frac{a^2(x-1)\,(32x+3)}{28x^{\frac{9}{7}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-1/28*a*(8*x-1)*x^(-8/7)*y(x)=1/28*a^2*(x-1)*(32*x+3)*x^(-9/7),y(x),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/28*a*(8*x-1)*x^(-8/7)*y[x]==1/28*a^2*(x-1)*(32*x+3)*x^(-9/7),y[x],x,Incl
```

Timed out

24.36 problem 36

Internal problem ID [10783]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$yy' - \frac{a(5x-4)\,y}{x^4} = \frac{a^2(x-1)\,(3x-1)}{x^7}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 169

```
dsolve(y(x)*diff(y(x),x)-a*(5*x-4)*x^(-4)*y(x)=a^2*(x-1)*(3*x-1)*x^(-7),y(x), singsol=all)
```

$$c_1 - \frac{4^{\frac{1}{3}} 27^{\frac{2}{3}} 5^{\frac{1}{6}} \left(x-\frac{3}{4}\right) \sqrt{\frac{y(x)x^2+a-\frac{a}{x}}{y(x)x^2+a}}}{5x \left(\frac{3y(x)x^2+3a-\frac{a}{x}}{y(x)x^2+a}\right)^{\frac{1}{6}} \left(\frac{a}{x(-y(x)x^2-a)}\right)^{\frac{1}{3}}} - \left(\int^{\frac{\frac{9y(x)x^3}{5}+\frac{9ax}{5}-\frac{27a}{20}}{x\left(y(x)x^2+a\right)}} \frac{\_a\sqrt{20\_a-9}}{(9+4\_a)^{\frac{1}{6}}\,(5\_a-9)^{\frac{1}{3}}\left(\frac{400}{729}\_a^3-\frac{7}{3}\_a+1\right)}d\_a\right) = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a*(5*x-4)*x^(-4)*y[x]==a^2*(x-1)*(3*x-1)*x^(-7),y[x],x,IncludeSingularSolu
```

Not solved

24.37 problem 37

Internal problem ID [10784]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - \frac{2a(3x-10)\,y}{5x^4} = \frac{a^2(x-1)\,(8x-5)}{5x^7}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-2/5*a*(3*x-10)*x^(-4)*y(x)=1/5*a^2*(x-1)*(8*x-5)*x^(-7),y(x), sings
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-2/5*a*(3*x-10)*x^(-4)*y[x]==1/5*a^2*(x-1)*(8*x-5)*x^(-7),y[x],x,IncludeSin
```

Not solved

24.38 problem 38

Internal problem ID [10785]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y + \frac{a(39x-4)\,y}{42x^{\frac{9}{7}}} = -\frac{a^2(x-1)\,(9x-1)}{42x^{\frac{11}{7}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+1/42*a*(39*x-4)*x^(-9/7)*y(x)=-1/42*a^2*(x-1)*(9*x-1)*x^(-11/7),y(x
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/42*a*(39*x-4)*x^(-9/7)*y[x]==-1/42*a^2*(x-1)*(9*x-1)*x^(-11/7),y[x],x,In
```

Timed out

24.39 problem 39

Internal problem ID [10786]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' + \frac{a(x-2)\,y}{x} = \frac{2a^2(x-1)}{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 116

```
dsolve(y(x)*diff(y(x),x)+a*(x-2)*x^(-1)*y(x)=2*a^2*(x-1)*x^(-1),y(x), singsol=all)
```

$$c_1 + \frac{x\sqrt{\frac{a}{ax+y(x)}}\,(ax+y(x))\left(\int^{\frac{a}{ax+y(x)}} \frac{\sqrt{\_a-1}\,\mathrm{e}^{\frac{1}{2\_a}}}{\sqrt{\_a}}d\_a\right) + \sqrt{\frac{(1-x)a-y(x)}{ax+y(x)}}\,\mathrm{e}^{\frac{ax+y(x)}{2a}}y(x)}{\sqrt{\frac{a}{ax+y(x)}}\,(ax+y(x))\,x} = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+a*(x-2)*x^(-1)*y[x]==2*a^2*(x-1)*x^(-1),y[x],x,IncludeSingularSolutions ->
```

Not solved

24.40 problem 40

Internal problem ID [10787]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y + \frac{a(3x-2)\,y}{x} = -\frac{2a^2(x-1)^2}{x}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+a*(3*x-2)*x^(-1)*y(x)=-2*a^2*(x-1)^2*x^(-1),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+a*(3*x-2)*x^(-1)*y[x]==-2*a^2*(x-1)^2*x^(-1),y[x],x,IncludeSingularSolutio
```

Not solved

24.41 problem 41

Internal problem ID [10788]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y + \frac{a\left(1 - \frac{b}{x^2}\right)y}{x} = \frac{a^2b}{x}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+a*(1-b*x^(-2))*x^(-1)*y(x)=a^2*b*x^(-1),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+a*(1-b*x^(-2))*x^(-1)*y[x]==a^2*b*x^(-1),y[x],x,IncludeSingularSolutions -
```

Not solved

24.42 problem 42

Internal problem ID [10789]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - \frac{a(3x-4)\,y}{4x^{\frac{5}{2}}} = \frac{a^2(x-1)\,(x+2)}{4x^4}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-1/4*a*(3*x-4)*x^(-5/2)*y(x)=1/4*a^2*(x-1)*(x+2)*x^(-4),y(x), singso
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/4*a*(3*x-4)*x^(-5/2)*y[x]==1/4*a^2*(x-1)*(x+2)*x^(-4),y[x],x,IncludeSing
```

Not solved

24.43 problem 43

Internal problem ID [10790]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$yy' + \frac{a(33x+2)\,y}{30x^{\frac{6}{5}}} = -\frac{a^2(x-1)\,(9x-4)}{30x^{\frac{7}{5}}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 4330

```
dsolve(y(x)*diff(y(x),x)+1/30*a*(33*x+2)*x^(-6/5)*y(x)=-1/30*a^2*(x-1)*(9*x-4)*x^(-7/5),y(x)
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/30*a*(33*x+2)*x^(-6/5)*y[x]==-1/30*a^2*(x-1)*(9*x-4)*x^(-7/5),y[x],x,Inc
```

Timed out

24.44 problem 44

Internal problem ID [10791]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - \frac{a(x-8)\,y}{8x^{\frac{5}{2}}} = -\frac{a^2(x-1)\,(3x-4)}{8x^4}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-1/8*a*(x-8)*x^(-5/2)*y(x)=-1/8*a^2*(x-1)*(3*x-4)*x^(-4),y(x), sings
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/8*a*(x-8)*x^(-5/2)*y[x]==-1/8*a^2*(x-1)*(3*x-4)*x^(-4),y[x],x,IncludeSin
```

Not solved

24.45 problem 45

Internal problem ID [10792]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$y'y + \frac{a(17x+18)\,y}{30x^{\frac{22}{15}}} = -\frac{a^2(x-1)\,(x+4)}{30x^{\frac{29}{15}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+1/30*a*(17*x+18)*x^(-22/15)*y(x)=-1/30*a^2*(x-1)*(x+4)*x^(-29/15),y
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/30*a*(17*x+18)*x^(-22/15)*y[x]==-1/30*a^2*(x-1)*(x+4)*x^(-29/15),y[x],x,
```

Timed out

24.46 problem 46

Internal problem ID [10793]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$y'y - \frac{a(6x-13)\,y}{13x^{\frac{5}{2}}} = -\frac{a^2(x-1)\,(x-13)}{26x^4}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-1/13*a*(6*x-13)*x^(-5/2)*y(x)=-1/26*a^2*(x-1)*(x-13)*x^(-4),y(x), s
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/13*a*(6*x-13)*x^(-5/2)*y[x]==-1/26*a^2*(x-1)*(x-13)*x^(-4),y[x],x,Includ
```

Not solved

24.47 problem 47

Internal problem ID [10794]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y + \frac{a(24x+11)\,x^{\frac{27}{20}}y}{30} = -\frac{a^2(x-1)\,(9x+1)}{60x^{\frac{17}{10}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+1/30*a*(24*x+11)*x^(27/20)*y(x)=-1/60*a^2*(x-1)*(9*x+1)*x^(-17/10),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/30*a*(24*x+11)*x^(27/20)*y[x]==-1/60*a^2*(x-1)*(9*x+1)*x^(-17/10),y[x],x
```

Timed out

24.48 problem 48

Internal problem ID [10795]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$y'y - \frac{2a(3x+2)\,y}{5x^{\frac{8}{5}}} = \frac{a^2(x-1)\,(8x+1)}{5x^{\frac{11}{5}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-2/5*a*(3*x+2)*x^(-8/5)*y(x)=1/5*a^2*(x-1)*(8*x+1)*x^(-11/5),y(x), s
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-2/5*a*(3*x+2)*x^(-8/5)*y[x]==1/5*a^2*(x-1)*(8*x+1)*x^(-11/5),y[x],x,Includ
```

Timed out

24.49 problem 49

Internal problem ID [10796]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - \frac{6a(1+4x)\,y}{5x^{\frac{7}{5}}} = \frac{a^2(x-1)\,(27x+8)}{5x^{\frac{9}{5}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-6/5*a*(4*x+1)*x^(-7/5)*y(x)=1/5*a^2*(x-1)*(27*x+8)*x^(-9/5),y(x), s
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-6/5*a*(4*x+1)*x^(-7/5)*y[x]==1/5*a^2*(x-1)*(27*x+8)*x^(-9/5),y[x],x,Includ
```

Timed out

24.50 problem 50

Internal problem ID [10797]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$
y'y - \frac{a(x+4)\,y}{5x^{\frac{8}{5}}} = \frac{a^2(x-1)\,(3x+7)}{5x^{\frac{3}{5}}}
$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-1/5*a*(x+4)*x^(-8/5)*y(x)=1/5*a^2*(x-1)*(3*x+7)*x^(-3/5),y(x), sing
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/5*a*(x+4)*x^(-8/5)*y[x]==1/5*a^2*(x-1)*(3*x+7)*x^(-3/5),y[x],x,IncludeSi
```

Not solved

24.51 problem 51

Internal problem ID [10798]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 51.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$yy' - \frac{a(x+4)\, y}{5x^{\frac{8}{5}}} = \frac{a^2(x-1)\,(3x+7)}{5x^{\frac{11}{5}}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 194

```
dsolve(y(x)*diff(y(x),x)-1/5*a*(x+4)*x^(-8/5)*y(x)=1/5*a^2*(x-1)*(3*x+7)*x^(-11/5),y(x), sin
```

$$c_1 - \frac{\left(\int^{-\frac{315\left(4y(x)x^{\frac{3}{5}}+4ax-21a\right)}{884\left(y(x)x^{\frac{3}{5}}+ax\right)}} \frac{\_a\sqrt{52\_a-315}\,(68\_a+315)^{\frac{7}{6}}}{(221\_a+315)^{\frac{5}{3}}\left(-\frac{781456}{31255875}\_a^3+\frac{79}{105}\_a+1\right)}d\_a\right) x\left(\frac{a}{y(x)x^{\frac{3}{5}}+ax}\right)^{\frac{5}{3}} - \frac{8\,105^{\frac{1}{6}}\left(\frac{(3x+7)a+3y(x)x^{\frac{3}{5}}}{y(x)x^{\frac{3}{5}}+ax}\right)}{}}{\left(\frac{a}{y(x)x^{\frac{3}{5}}+ax}\right)^{\frac{5}{3}} x}$$

$= 0$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/5*a*(x+4)*x^(-8/5)*y[x]==1/5*a^2*(x-1)*(3*x+7)*x^(-11/5),y[x],x,IncludeS
```

Timed out

24.52 problem 52

Internal problem ID [10799]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$yy' - \frac{a(2x-1)\,y}{x^{\frac{5}{2}}} = \frac{a^2(x-1)\,(3x+1)}{2x^4}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 195

```
dsolve(y(x)*diff(y(x),x)-a*(2*x-1)*x^(-5/2)*y(x)=1/2*a^2*(x-1)*(3*x+1)*x^(-4),y(x), singsol=
```

$$c_1 - \frac{\left(\int^{\frac{-\frac{18x^{\frac{3}{2}}y(x)}{35}+\frac{9(-2x-3)a}{35}}{x(\sqrt{x}\,y(x)+a)}} \frac{\_\_a(5\_\_a-9)^{\frac{1}{6}}\sqrt{7\_\_a+9}}{(35\_\_a+18)^{\frac{2}{3}}\left(-\frac{1225}{1458}\_\_a^3+\frac{13}{6}\_\_a+1\right)}d\_\_a\right) x\left(\frac{a}{x(-\sqrt{x}\,y(x)-a)}\right)^{\frac{2}{3}} - \frac{6\,7^{\frac{2}{3}}\left(x+\frac{3}{2}\right)\sqrt{\frac{(x-1)a+x^{\frac{3}{2}}y(x)}{x(\sqrt{x}\,y(x)+a)}}\,21^{\frac{1}{6}}}{1225}}{\left(-\frac{a}{x(\sqrt{x}\,y(x)+a)}\right)^{\frac{2}{3}} x}$$
$$= 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a*(2*x-1)*x^(-5/2)*y[x]==1/2*a^2*(x-1)*(3*x+1)*x^(-4),y[x],x,IncludeSingul
```

Not solved

24.53 problem 53

Internal problem ID [10800]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 53.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$yy' + \frac{a(x-6)\,y}{5x^{\frac{7}{5}}} = \frac{2a^2(x-1)\,(x+4)}{5x^{\frac{9}{5}}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 157

```
dsolve(y(x)*diff(y(x),x)+1/5*a*(x-6)*x^(-7/5)*y(x)=2/5*a^2*(x-1)*(x+4)*x^(-9/5),y(x), singso
```

$$c_1 + \frac{\left(-12ay(x)\,x^{\frac{2}{5}} - \frac{3x^{\frac{4}{5}}y(x)^2}{2} + \left(-\frac{y(x)x^{\frac{7}{5}}}{2} + a(x+24)\,(x-1)\right)a\right)\left(8ay(x)\,x^{\frac{2}{5}} + x^{\frac{4}{5}}y(x)^2 + a\left(2y(x)\,x^{\frac{7}{5}} + a\right.\right.}{54\left(\frac{a}{y(x)x^{\frac{2}{5}}+ax}\right)^{\frac{5}{2}}\left(a\,(x+4) + y\,(x)\,x^{\frac{2}{5}}\right)^2 x\left(y\,(x)\,x^{\frac{2}{5}} + ax\right)^2}$$
$$= 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/5*a*(x-6)*x^(-7/5)*y[x]==2/5*a^2*(x-1)*(x+4)*x^(-9/5),y[x],x,IncludeSing
```

Timed out

24.54 problem 54

Internal problem ID [10801]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 54.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y + \frac{a(21x + 19)\,y}{5x^{\frac{7}{5}}} = -\frac{2a^2(x-1)\,(9x-4)}{5x^{\frac{9}{5}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+1/5*a*(21*x+19)*x^(-7/5)*y(x)=-2/5*a^2*(x-1)*(9*x-4)*x^(-9/5),y(x),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/5*a*(21*x+19)*x^(-7/5)*y[x]==-2/5*a^2*(x-1)*(9*x-4)*x^(-9/5),y[x],x,Incl
```

Timed out

24.55 problem 55

Internal problem ID [10802]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 55.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - \frac{3ay}{x^{\frac{7}{4}}} = \frac{a^2(x-1)(x-9)}{4x^{\frac{5}{2}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-3*a*x^(-7/4)*y(x)=1/4*a^2*(x-1)*(x-9)*x^(-5/2),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-3*a*x^(-7/4)*y[x]==1/4*a^2*(x-1)*(x-9)*x^(-5/2),y[x],x,IncludeSingularSolu
```

Not solved

24.56 problem 56

Internal problem ID [10803]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 56.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$yy' - \frac{a((k+1)x-1)y}{x^2} = \frac{a^2(k+1)(x-1)}{x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 141

```
dsolve(y(x)*diff(y(x),x)-a*((k+1)*x-1)*x^(-2)*y(x)=a^2*(k+1)*(x-1)*x^(-2),y(x), singsol=all)
```

$$c_1 + \frac{(xy(x)-a)\left(\int^{\frac{ax}{-xy(x)+a}} (\_a-1)^{\frac{1}{1+k}} e^{\frac{1}{(1+k)\_a}} \_a^{-\frac{1}{1+k}} d\_a\right) + \left(\frac{ax}{-xy(x)+a}\right)^{-\frac{1}{1+k}} x^2 \left(\frac{(x-1)a+xy(x)}{-xy(x)+a}\right)^{\frac{1}{1+k}} e^{\frac{-xy(x)+a}{a(1+k)x}} y}{-xy(x)+a}$$
$$= 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a*((k+1)*x-1)*x^(-2)*y[x]==a^2*(k+1)*(x-1)*x^(-2),y[x],x,IncludeSingularSo
```

Not solved

24.57 problem 57

Internal problem ID [10804]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 57.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y'y - a((-2+k)x + 2k - 3)x^{-k}y = a^2(-2+k)(x-1)^2 x^{1-2k}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-a*( (k-2)*x + 2*k - 3)*x^(-k)*y(x)=a^2*(k-2)*(x-1)^2*x^(1-2*k),y(x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a*( (k-2)*x + 2*k - 3)*x^(-k)*y[x]==a^2*(k-2)*(x-1)^2*x^(1-2*k),y[x],x,Inc
```

Not solved

24.58 problem 58

Internal problem ID [10805]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 58.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y'y - \frac{a((4k-7)x-4k+5)x^{-k}y}{2} = \frac{a^2(-3+2k)(x-1)^2x^{1-2k}}{2}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-1/2*a*( (4*k-7)*x - 4*k + 5)*x^(-k)*y(x)=1/2*a^2*(2*k-3)*(x-1)^2*x^
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/2*a*( (4*k-7)*x - 4*k + 5)*x^(-k)*y[x]==1/2*a^2*(2*k-3)*(x-1)^2*x^(1-2*k
```

Not solved

24.59 problem 59

Internal problem ID [10806]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 59.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$yy' - ((2n-1)\,x - an)\,x^{-1-n}y = n(x-a)\,x^{-2n}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 150

```
dsolve(y(x)*diff(y(x),x)-((2*n-1)*x-a*n)*x^(-n-1)*y(x)=n*(x-a)*x^(-2*n),y(x), singsol=all)
```

$$y(x) = -\frac{\left(\sqrt{-n^2}\,x\tan\left(\frac{\mathrm{RootOf}\left(-\sqrt{-n^2}\,\tan\left(\frac{\_Z a\sqrt{-n^2}}{2}\right)\_Z x-2\,\mathrm{e}^{\_Z n a}\,\mathrm{e}^{\_Z a}+n x\,\mathrm{e}^{\_Z}\mathrm{e}^{\_Z a}+2 x c_1\mathrm{e}^{\_Z a}\right)\sqrt{-n^2}}{2}\right)-2an+nx\right)x^{-n}}{\sqrt{-n^2}\,\tan\left(\frac{\mathrm{RootOf}\left(-\sqrt{-n^2}\,\tan\left(\frac{\_Z a\sqrt{-n^2}}{2}\right)\_Z x-2\,\mathrm{e}^{\_Z n a}\,\mathrm{e}^{\_Z a}+n x\,\mathrm{e}^{\_Z}\mathrm{e}^{\_Z a}+2 x c_1\mathrm{e}^{\_Z a}\right)\sqrt{-n^2}}{2}\right)+n}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-((2*n-1)*x-a*n)*x^(-n-1)*y[x]==n*(x-a)*x^(-2*n),y[x],x,IncludeSingularSolu
```

Not solved

24.60 problem 60

Internal problem ID [10807]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 60.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$y'y - ((n+1)\,x - an)\,x^{-1+n}(x-a)^{-2-n}\,y = n\,x^{2n}(x-a)^{-3-2n}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-((n+1)*x-a*n)*x^(n-1)*(x-a)^(-n-2)*y(x)=n*x^(2*n)*(x-a)^(-2*n-3),y(
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-((n+1)*x-a*n)*x^(n-1)*(x-a)^(-n-2)*y[x]==n*x^(2*n)*(x-a)^(-2*n-3),y[x],x,I
```

Not solved

24.61 problem 61

Internal problem ID [10808]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y'y - a((-3+2k)\,x+1)\,x^{-k}y = a^2(-2+k)\,((k-1)\,x+1)\,x^{2-2k}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-a*((2*k-3)*x+1)*x^(-k)*y(x)=a^2*(k-2)*((k-1)*x+1)*x^(2*(1-k)),y(x),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a*((2*k-3)*x+1)*x^(-k)*y[x]==a^2*(k-2)*((k-1)*x+1)*x^(2*(1-k)),y[x],x,Incl
```

Not solved

24.62 problem 62

Internal problem ID [10809]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 62.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y'y - a((n + 2k - 3)x + 3 - 2k)x^{-k}y = a^2\left((n + k - 1)x^2 - (n + 2k - 3)x + k - 2\right)x^{1-2k}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-a*((n+2*k-3)*x+3-2*k)*x^(-k)*y(x)=a^2*((n+k-1)*x^2-(n+2*k-3)*x+k-2)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a*((n+2*k-3)*x+3-2*k)*x^(-k)*y[x]==a^2*((n+k-1)*x^2-(n+2*k-3)*x+k-2)*x^(1-
```

Timed out

24.63 problem 63

Internal problem ID [10810]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 63.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y'y - \frac{a((2+n)x-2)x^{-\frac{2n+1}{n}}y}{n} = \frac{a^2((n+1)x^2-2x-n+1)x^{-\frac{2+3n}{n}}}{n}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-a/n*((n+2)*x-2)*x^(-(2*n+1)/n)*y(x)=a^2/n*((n+1)*x^2-2*x-n+1)*x^(-(
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a/n*((n+2)*x-2)*x^(-(2*n+1)/n)*y[x]==a^2/n*((n+1)*x^2-2*x-n+1)*x^(-(3*n+2)
```

Not solved

24.64 problem 64

Internal problem ID [10811]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 64.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y'y - \frac{a\left(\frac{(4+n)x}{2+n} - 2\right) x^{-\frac{2n+1}{n}} y}{n} = \frac{a^2(2x^2 + (n^2+n-4)\, x - (-1+n)\,(2+n))\, x^{-\frac{2+3n}{n}}}{n\,(2+n)}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-a/n*((n+4)/(n+2)*x-2)*x^(-(2*n+1)/n)*y(x)=a^2/(n*(n+2))*(2*x^2+(n^2
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a/n*((n+4)/(n+2)*x-2)*x^(-(2*n+1)/n)*y[x]==a^2/(n*(n+2))*(2*x^2+(n^2+n-4)*
```

Not solved

24.65 problem 65

Internal problem ID [10812]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 65.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y'y + \frac{a\left(\frac{(5+3n)x}{2} + \frac{-1+n}{n+1}\right) x^{-\frac{4+n}{n+3}} y}{n+3} = -\frac{a^2\left((n+1)\,x^2 - \frac{(n^2+2n+5)x}{n+1} + \frac{4}{n+1}\right) x^{-\frac{5+n}{n+3}}}{2n+6}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+a/(n+3)*((3*n+5)/(2)*x+(n-1)/(n+1))*x^(-(n+4)/(n+3))*y(x)=-a^2/(2*(
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+a/(n+3)*((3*n+5)/(2)*x+(n-1)/(n+1))*x^(-(n+4)/(n+3))*y[x]==-a^2/(2*(n+3))*
```

Timed out

24.66 problem 66

Internal problem ID [10813]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 66.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$yy' - a\left(\frac{2+n}{n} + b\,x^n\right)y = -\frac{a^2 x\left(\frac{1+n}{n} + b\,x^n\right)}{n}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 202

```
dsolve(y(x)*diff(y(x),x)-a*((n+2)/n+b*x^n)*y(x)=-a^2/n*x*((n+1)/n+b*x^n),y(x), singsol=all)
```

$$c_1 - n\sqrt{-\frac{(1+n)^2}{n^2}}\left(\int^{\frac{2\arctan\left(\frac{2abn\,x^{1+n}+anx-n^2y(x)+ax-y(x)n}{n\sqrt{-\frac{(1+n)^2}{n^2}}\,(ax-y(x)n)}\right)}{\sqrt{-\frac{(1+n)^2}{n^2}}}}\tan\left(\frac{\_a\sqrt{-\frac{(1+n)^2}{n^2}}}{2}\right)\mathrm{e}^{-\_a}d\_a\right) + (-2x^n nb - n - 1)\,\mathrm{e}^{-\frac{2\arctan\left(\frac{2abn\,x^{1+n}+anx-n^2y(x)+ax-y(x)n}{n\sqrt{-\frac{(1+n)^2}{n^2}}\,(ax-y(x)n)}\right)}{\sqrt{-\frac{(1+n)^2}{n^2}}}} = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a*((n+2)/n+b*x^n)*y[x]==-a^2/n*x*((n+1)/n+b*x^n),y[x],x,IncludeSingularSol
```

Not solved

24.67 problem 67

Internal problem ID [10814]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 67.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - (a\,e^{x} + b)\,y = c\,e^{2x} - ab\,e^{x} - b^{2}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 154

```
dsolve(y(x)*diff(y(x),x)=(a*exp(x)+b)*y(x)+c*exp(2*x)-a*b*exp(x)-b^2,y(x), singsol=all)
```

$$c_1 + \sqrt{\frac{c\,e^{2x} - (b - y(x))\,(e^{x}a + b - y(x))}{(b - y(x))^2}}\, y(x)\, e^{-\frac{a \operatorname{arctanh}\left(\frac{(b-y(x))a-2\,e^{x}c}{\sqrt{a^2+4c}\,(b-y(x))}\right)}{\sqrt{a^2+4c}}} - b\left(\int^{-\frac{e^{x}}{b-y(x)}} \frac{\sqrt{\_a^2c + \_aa - 1}\, e^{-\frac{a \operatorname{arctanh}\left(\frac{2\_ac+a}{\sqrt{a^2+4c}}\right)}{\sqrt{a^2+4c}}}}{\_a} d\_a\right) = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*Exp[x]+b)*y[x]+c*Exp[2*x]-a*b*Exp[x]-b^2,y[x],x,IncludeSingularSolutio
```

Not solved

24.68 problem 68

Internal problem ID [10815]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 68.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$y'y - \left(a(\lambda + 2\mu)\,\mathrm{e}^{\lambda x} + b\right)\mathrm{e}^{\mu x} y = \left(-a^2\mu\,\mathrm{e}^{2\lambda x} - ab\,\mathrm{e}^{\lambda x} + c\right)\mathrm{e}^{2\mu x}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=(a*(2*mu+lambda)*exp(lambda*x)+b)*exp(mu*x)*y(x)+(-a^2*mu*exp(2*lam
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*(2*\[Mu]+\[Lambda])*Exp[\[Lambda]*x]+b)*Exp[\[Mu]*x]*y[x]+(-a^2*\[Mu]*
```

Not solved

24.69 problem 69

Internal problem ID [10816]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[[_Abel, '2nd type', 'class A']]**

$$yy' - \left(a\,e^{x\lambda} + b\right) y = c\left(a^2 e^{2x\lambda} + ab(x\lambda + 1)\,e^{x\lambda} + b^2\lambda x\right)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 545

```
dsolve(y(x)*diff(y(x),x)=(a*exp(lambda*x)+b)*y(x)+c*(a^2*exp(2*lambda*x)+a*b*(lambda*x+1)*ex
```

$$-\frac{(3c\lambda+1)\left(6\,\operatorname{arctanh}\left(\frac{6b\,c^2\lambda^2x+6\,e^{\lambda x}a\,c^2\lambda+2bc\lambda x+2\,e^{\lambda x}ac+3c\lambda y(x)+y(x)}{y(x)\sqrt{36c^3\lambda^3+33\lambda^2c^2+10c\lambda+1}}\right)c\lambda+2\ln\left(\frac{9\left(3bc\,\lambda^2x+3\,e^{\lambda x}ac\lambda+b\lambda x+e^{\lambda x}a\right)c}{y(x)}\right)\sqrt{}\right.}{}$$

$$+\frac{(3c\lambda+1)\ln\left(b\lambda x+e^{\lambda x}a\right)}{3c\lambda}-c_1=0$$

✓ Solution by Mathematica

Time used: 0.494 (sec). Leaf size: 134

```
DSolve[y[x]*y'[x]==(a*Exp[\[Lambda]*x]+b)*y[x]+c*(a^2*Exp[2*\[Lambda]*x]+a*b*(\[Lambda]*x+1)
```

$$\text{Solve}\left[-\frac{\frac{2\arctan\left(\frac{\frac{2c\lambda y(x)}{ace^{\lambda x}+bc\lambda x}-1}{\sqrt{-4c\lambda-1}}\right)}{\sqrt{-4c\lambda-1}}+\log\left(-\frac{c\lambda y(x)^2}{(ace^{\lambda x}+bc\lambda x)^2}+\frac{y(x)}{ace^{\lambda x}+bc\lambda x}+1\right)}{2c\lambda}=\frac{\log\left(ace^{\lambda x}+bc\lambda x\right)}{c\lambda}\right.$$

$$\left.+c_1,y(x)\right]$$

24.70 problem 70

Internal problem ID [10817]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 70.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$yy' - e^{x\lambda}(2a\lambda x + a + b)\, y = -e^{2x\lambda}\left(a^2\lambda\, x^2 + abx + c\right)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 119

```
dsolve(y(x)*diff(y(x),x)=exp(lambda*x)*(2*a*lambda*x+a+b)*y(x)-exp(2*lambda*x)*(a^2*lambda*x
```

$$y(x) = \frac{\left(\tan\left(\frac{\mathrm{RootOf}\left(2\,e^{\_Z a\lambda x}\, e^{a} - \sqrt{-\frac{b^2-4c\lambda}{a^2}}\,\tan\left(\frac{\_a\sqrt{-\frac{b^2-4c\lambda}{a^2}}}{2}\right)\_Z a + e^{\_Z b}\, e^{\_a} + 2c_1 a\, e^{\_a}\right)\sqrt{-\frac{b^2-4c\lambda}{a^2}}}{2}\right) a\sqrt{-\frac{b^2-4c\lambda}{a^2}} + 2a}{2\lambda}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==Exp[\[Lambda]*x]*(2*a*\[Lambda]*x+a+b)*y[x]-Exp[2*\[Lambda]*x]*(a^2*\[Lam
```

Not solved

24.71 problem 71

Internal problem ID [10818]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 71.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[[_Abel, '2nd type', 'class A']]**

$$y'y - e^{ax}(2a\,x^2 + b + 2x)\,y = e^{2ax}(-a\,x^4 - b\,x^2 + c)$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=exp(a*x)*(2*a*x^2+2*x+b)*y(x)+exp(2*a*x)*(-a*x^4-b*x^2+c),y(x), sin
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==Exp[a*x]*(2*a*x^2+2*x+b)*y[x]+Exp[2*a*x]*(-a*x^4-b*x^2+c),y[x],x,IncludeS
```

Not solved

24.72 problem 72

Internal problem ID [10819]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 72.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$yy' + a(2bx+1)\,\mathrm{e}^{bx}y = -a^2b\,x^2\mathrm{e}^{2bx}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
dsolve(y(x)*diff(y(x),x)+a*(1+2*b*x)*exp(b*x)*y(x)=-a^2*b*x^2*exp(2*b*x),y(x), singsol=all)
```

$$y(x) = -\frac{\mathrm{e}^{xb}a\left(bx\,\mathrm{RootOf}\left(-\mathrm{e}^{\_Z}xb - \mathrm{Ei}_1\left(-\_Z\right) + c_1\right) - 1\right)}{\mathrm{RootOf}\left(-\mathrm{e}^{\_Z}xb - \mathrm{Ei}_1\left(-\_Z\right) + c_1\right)b}$$

✓ Solution by Mathematica

Time used: 0.736 (sec). Leaf size: 59

```
DSolve[y[x]*y'[x]+a*(1+2*b*x)*Exp[b*x]*y[x]==-a^2*b*x^2*Exp[2*b*x],y[x],x,IncludeSingularSol
```

$$\text{Solve}\left[bxe^{\frac{ae^{bx}}{abxe^{bx}+by(x)}} = \text{ExpIntegralEi}\left(\frac{ae^{bx}}{abe^{bx}x + by(x)}\right) + c_1, y(x)\right]$$

24.73 problem 73

Internal problem ID [10820]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 73.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$
yy' - a(1 + 2n + 2(1 + n)\, nx)\, e^{(1+n)x} y = -a^2 n(1 + n)\,(nx + 1)\, x\, e^{2(1+n)x}
$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 130

```
dsolve(y(x)*diff(y(x),x)-a*(1+2*n+2*n*(n+1)*x)*exp((n+1)*x)*y(x)=-a^2*n*(n+1)*(1+n*x)*x*exp(
```

$$
y(x) = \frac{\left(1 + 2n^2 x + \left(\sqrt{-\frac{(1+n)^2}{n^2}}\ \tan\left(\frac{\operatorname{RootOf}\left(2x\, n^2 e^{\_a+\_Z} - \tan\left(\frac{\_a\sqrt{-\frac{(1+n)^2}{n^2}}}{2}\right)\_Z\sqrt{-\frac{(1+n)^2}{n^2}}\, n + 2nx\, e^{\_a+\_Z} + n\, e^{\_a+\_Z} + 2\right.}{2}\right.\right.\right.}{2 + 2n}
$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a*(1+2*n+2*n*(n+1)*x)*Exp[(n+1)*x]*y[x]==-a^2*n*(n+1)*(1+n*x)*x*Exp[2*(n+1
```

Not solved

24.74 problem 74

Internal problem ID [10821]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 74.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$yy' + a\left(1 + 2b\sqrt{x}\right) e^{2b\sqrt{x}}y = -a^2 b\, x^{\frac{3}{2}} e^{4b\sqrt{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 209

```
dsolve(y(x)*diff(y(x),x)+a*(1+2*b*x^(1/2))*exp(2*b*x^(1/2))*y(x)=-a^2*b*x^(3/2)*exp(4*b*x^(1
```

$$c_1 + \frac{-\operatorname{BesselK}\left(1, -\sqrt{\frac{a\,e^{2b\sqrt{x}}}{b^2\left(e^{2b\sqrt{x}}ax+y(x)\right)}}\right)\sqrt{\frac{a\,e^{2b\sqrt{x}}}{b^2\left(e^{2b\sqrt{x}}ax+y(x)\right)}}\,b\sqrt{x} + \operatorname{BesselK}\left(0, -\sqrt{\frac{a\,e^{2b\sqrt{x}}}{b^2\left(e^{2b\sqrt{x}}ax+y(x)\right)}}\right)}{\operatorname{BesselI}\left(1, \sqrt{\frac{a\,e^{2b\sqrt{x}}}{b^2\left(e^{2b\sqrt{x}}ax+y(x)\right)}}\right)\sqrt{\frac{a\,e^{2b\sqrt{x}}}{b^2\left(e^{2b\sqrt{x}}ax+y(x)\right)}}\,b\sqrt{x} - \operatorname{BesselI}\left(0, \sqrt{\frac{a\,e^{2b\sqrt{x}}}{b^2\left(e^{2b\sqrt{x}}ax+y(x)\right)}}\right)} = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+a*(1+2*b*x^(1/2))*Exp[2*b*x^(1/2)]*y[x]==-a^2*b*x^(3/2)*exp(4*b*x^(1/2)),y
```

Not solved

24.75 problem 75

Internal problem ID [10822]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 75.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$y'y - (a \cosh(x) + b) y = -ab \sinh(x) + c$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=(a*cosh(x)+b)*y(x)-a*b*sinh(x)+c,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*Cosh[x]+b)*y[x]-a*b*Sinh[x]+c,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

24.76 problem 76

Internal problem ID [10823]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 76.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$y'y - (a\sinh(x) + b)\, y = -ab\cosh(x) + c$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=(a*sinh(x)+b)*y(x)-a*b*cosh(x)+c,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*Sinh[x]+b)*y[x]-a*b*Cosh[x]+c,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

24.77 problem 77

Internal problem ID [10824]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 77.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$yy' - (2\ln(x) + a + 1)\,y = x\left(-\ln(x)^2 - a\ln(x) + b\right)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 163

```
dsolve(y(x)*diff(y(x),x)=(2*ln(x)+a+1)*y(x)+x*( -(ln(x))^2-a*ln(x)+b),y(x), singsol=all)
```

$$y(x) = \frac{x\left(-\tanh\left(\frac{\mathrm{RootOf}\left(\mathrm{e}^{-\frac{2\,\mathrm{arctanh}\left(\frac{2\_a-a}{\sqrt{a^2+4b}}\right)}{\sqrt{a^2+4b}}}\tanh\left(\frac{\_Z\sqrt{a^2+4b}}{2}\right)\sqrt{a^2+4b}-\sqrt{a^2+4b}\tanh\left(\frac{\_Z\sqrt{a^2+4b}}{2}\right)\mathrm{e}^{\_Z}+2\,\mathrm{e}^{\_Z}\ln(x)-\mathrm{e}^{-\frac{2\,\mathrm{arctan}}{\sqrt{}}}\right.}{2}\right.\right.}{2}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(2*Log[x]+a+1)*y[x]+x*( -(Log[x])^2-a*Log[x]+b),y[x],x,IncludeSingularSol
```

Not solved

24.78 problem 78

Internal problem ID [10825]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 78.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$y'y - \left(2\ln(x)^2 + 2\ln(x) + a\right)y = x\left(-\ln(x)^4 - a\ln(x)^2 + b\right)$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=(2*(ln(x))^2+2*ln(x)+a)*y(x)+x*(- (ln(x))^4-a*(ln(x))^2+b),y(x), si
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(2*(Log[x])^2+2*Log[x]+a)*y[x]+x*(- (Log[x])^4-a*(Log[x])^2+b),y[x],x,Inc
```

Not solved

24.79 problem 79

Internal problem ID [10826]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 79.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{y'y - ax\cos\left(\lambda\, x^2\right) y = x}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=a*x*cos(lambda*x^2)*y(x)+x,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==a*x*Cos[\[Lambda]*x^2]*y[x]+x,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

24.80 problem 80

Internal problem ID [10827]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 80.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$y'y - ax\sin\left(\lambda\,x^2\right)y = x$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=a*x*sin(lambda*x^2)*y(x)+x,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==a*x*Sin[\[Lambda]*x^2]*y[x]+x,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

25 Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.4-2. Equations of the form $(g_1(x) + g_0(x))y' = f_2(x)y^2 + f_1(x)y + f_0(x)$

25.1 problem 1

Internal problem ID [10828]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.4-2. Equations of the form $(g_1(x) + g_0(x))y' = f_2(x)y^2 + f_1(x)y + f_0(x)$
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{(Ay + Bx + a)\, y' + By = -kx - b}$$

✓ Solution by Maple

Time used: 0.703 (sec). Leaf size: 113

```
dsolve((A*y(x)+B*x+a)*diff(y(x),x)+B*y(x)+k*x+b=0,y(x), singsol=all)
```

$$y(x) = \frac{-Bb + ak + \frac{B(x(Ak-B^2)+Ab-aB)c_1 + \sqrt{-A\,c_1^2 k(x(Ak-B^2)+Ab-aB)^2 + B^2(x(Ak-B^2)+Ab-aB)^2 c_1^2 + A}}{Ac_1}}{-Ak + B^2}$$

✓ Solution by Mathematica

Time used: 18.19 (sec). Leaf size: 106

```
DSolve[(A*y[x]+B*x+a)*y'[x]+B*y[x]+k*x+b==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\frac{\sqrt{\frac{(a+Bx)^2}{A} + Ac_1 - x(2b+kx)}}{\sqrt{\frac{1}{A}}} + a + Bx}{A}$$

$$y(x) \to -\frac{a+Bx}{A} + \sqrt{\frac{1}{A}}\sqrt{\frac{(a+Bx)^2}{A} + Ac_1 - x(2b+kx)}$$

25.2 problem 2

Internal problem ID [10829]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.4-2. Equations of the form $(g_1(x) + g_0(x))y' = f_2(x)y^2 + f_1(x)y + f_0(x)$
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$(y + ax + b)\, y' - \alpha y = \beta x + \gamma$$

✓ Solution by Maple

Time used: 0.672 (sec). Leaf size: 186

```
dsolve((y(x)+a*x+b)*diff(y(x),x)=alpha*y(x)+beta*x+gamma,y(x), singsol=all)
```

$$y(x) = \frac{a\gamma - b\beta - \frac{(x(a\alpha-\beta)+b\alpha-\gamma)\left(\tan\left(\operatorname{RootOf}\left(\sqrt{-a^2+2a\alpha-\alpha^2-4\beta}\ln\left(-\frac{(x(a\alpha-\beta)+b\alpha-\gamma)^2\left(\tan\left(\_Z\right)^2 a^2-2\tan\left(\_Z\right)^2 a\alpha+\alpha^2\tan\left(\_Z\right.\right.\right.\right.\right.\right.}{4}}{2}}{-a\alpha+\beta}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y[x]*a*x+b)*y'[x]==\[Alpha]*y[x]+\[Beta]*x+\[Gamma],y[x],x,IncludeSingularSolutions
```

Not solved

25.3 problem 3

Internal problem ID [10830]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.4-2. Equations of the form $(g_1(x) + g_0(x))y' = f_2(x)y^2 + f_1(x)y + f_0(x)$
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{(y + k\,x^2 a + bx + c)\,y' + ay^2 - 2yakx - ym = k(k + b - m)\,x + s}$$

✗ Solution by Maple

```
dsolve((y(x)+a*k*x^2+b*x+c)*diff(y(x),x)=-a*y(x)^2+2*a*k*x*y(x)+m*y(x)+k*(k+b-m)*x+s,y(x), s
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y[x]+a*k*x^2+b*x+c)*y'[x]==-a*y[x]^2+2*a*k*x*y[x]+m*y[x]+k*(k+b-m)*x+s,y[x],x,Includ
```

Timed out

25.4 problem 4

Internal problem ID [10831]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.4-2. Equations of the form $(g_1(x) + g_0(x))y' = f_2(x)y^2 + f_1(x)y + f_0(x)$
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x),G(x)]']`

$$(y + A\,x^n + a)\,y' + nA\,x^{-1+n}y = -k\,x^m - b$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 299

```
dsolve((y(x)+A*x^n+a)*diff(y(x),x)+n*A*x^(n-1)*y(x)+k*x^m+b=0,y(x), singsol=all)
```

$$y(x) = -\frac{Am\,x^n + A\,x^n + am - \sqrt{A^2x^{2n}m^2 + 2A^2x^{2n}m + 2A\,x^n a\,m^2 + x^{2n}A^2 + 4A\,x^n am + m^2a^2 - 2b\,m^2x + \cdots}}{m+1}$$

$$y(x) = -\frac{Am\,x^n + A\,x^n + am + \sqrt{A^2x^{2n}m^2 + 2A^2x^{2n}m + 2A\,x^n a\,m^2 + x^{2n}A^2 + 4A\,x^n am + m^2a^2 - 2b\,m^2x + \cdots}}{m+1}$$

✓ Solution by Mathematica

Time used: 21.171 (sec). Leaf size: 118

```
DSolve[(y[x]+A*x^n+a)*y'[x]+n*A*x^(n-1)*y[x]+k*x^m+b==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to -\sqrt{\frac{1}{x}}\sqrt{x\left((a+Ax^n)^2 - \frac{2x\,(bm+b+kx^m)}{m+1} + c_1\right)} - a - Ax^n$$

$$y(x) \to \sqrt{\frac{1}{x}}\sqrt{x\left((a+Ax^n)^2 - \frac{2x\,(bm+b+kx^m)}{m+1} + c_1\right)} - a - Ax^n$$

25.5 problem 5

Internal problem ID [10832]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.4-2. Equations of the form $(g_1(x)+g_0(x))y' = f_2(x)y^2 + f_1(x)y + f_0(x)$
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\left(y+a\,x^{n+1}+b\,x^{n}\right)y'-\left(an\,x^{n}+c\,x^{-1+n}\right)y=0$$

✗ Solution by Maple

```
dsolve((y(x)+a*x^(n+1)+b*x^n)*diff(y(x),x)=(a*n*x^n+c*x^(n-1))*y(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y[x]+a*x^(n+1)+b*x^n)*y'[x]==(a*n*x^n+c*x^(n-1))*y[x],y[x],x,IncludeSingularSolution
```

Not solved

25.6 problem 6

Internal problem ID [10833]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.4-2. Equations of the form $(g_1(x) + g_0(x))y' = f_2(x)y^2 + f_1(x)y + f_0(x)$
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{y'xy - ay^2 - by = c\,x^n + s}$$

✗ Solution by Maple

```
dsolve(x*y(x)*diff(y(x),x)=a*y(x)^2+b*y(x)+c*x^n+s,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y[x]*y'[x]==a*y[x]^2+b*y[x]+c*x^n+s,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

25.7 problem 7

Internal problem ID [10834]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.4-2. Equations of the form $(g_1(x) + g_0(x))y' = f_2(x)y^2 + f_1(x)y + f_0(x)$
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$xyy' + y^2n - a(2n+1)\,xy - by = -a^2n\,x^2 - abx + c$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 223

```
dsolve(x*y(x)*diff(y(x),x)=-n*y(x)^2+a*(2*n+1)*x*y(x)+b*y(x)-a^2*n*x^2-a*b*x+c,y(x), singsol
```

$$c_1 + \frac{\left(\frac{1}{ax-y(x)}\right)^{\frac{1}{n}} \left(\frac{-n\,a^2x^2-x(-2y(x)n+b)a-ny(x)^2+by(x)+c}{(ax-y(x))^2}\right)^{-\frac{1}{2n}} y(x)\,\mathrm{e}^{\frac{b\,\operatorname{arctanh}\left(\frac{b(ax-y(x))-2c}{\sqrt{b^2+4cn}\,(ax-y(x))}\right)}{\sqrt{b^2+4cn}\,n}} - \left(\int^{\frac{1}{ax-y(x)}} \_a^{\frac{1}{n}} (\_a^2c\right.}{(ax - y(x))\,x} = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y[x]*y'[x]==-n*y[x]^2+a*(2*n+1)*x*y[x]+b*y[x]-a^2*n*x^2-a*b*x+c,y[x],x,IncludeSingu
```

Not solved

26 Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213

26.1 problem 1

Internal problem ID [10835]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + ay = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(\sqrt{a}\, x\right) + c_2 \cos\left(\sqrt{a}\, x\right)$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 28

```
DSolve[y''[x]+a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos\left(\sqrt{a}x\right) + c_2 \sin\left(\sqrt{a}x\right)$$

26.2 problem 2

Internal problem ID [10836]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - (ax + b)\, y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)-(a*x+b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{AiryAi}\left(\frac{ax+b}{(-a)^{\frac{2}{3}}}\right) + c_2 \operatorname{AiryBi}\left(\frac{ax+b}{(-a)^{\frac{2}{3}}}\right)$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 36

```
DSolve[y''[x]-(a*x+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{AiryAi}\left(\frac{b+ax}{a^{2/3}}\right) + c_2 \operatorname{AiryBi}\left(\frac{b+ax}{a^{2/3}}\right)$$

26.3 problem 3

Internal problem ID [10837]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - (a^2x^2 + a)\, y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)-(a^2*x^2+a)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{a\,x^2}{2}} + c_2 \mathrm{e}^{\frac{a\,x^2}{2}} \operatorname{erf}\left(\sqrt{a}\,x\right)$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 43

```
DSolve[y''[x]-(a^2*x^2+a)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \,\text{ParabolicCylinderD}\left(-1, \sqrt{2}\sqrt{a}x\right) + c_2 \,\text{ParabolicCylinderD}\left(0, i\sqrt{2}\sqrt{a}x\right)$$

26.4 problem 4

Internal problem ID [10838]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - (a\,x^2 + b)\,y = 0}$$

✓ Solution by Maple

Time used: 0.266 (sec). Leaf size: 45

```
dsolve(diff(y(x),x$2)-(a*x^2+b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \operatorname{WhittakerM}\left(-\frac{b}{4\sqrt{a}}, \frac{1}{4}, x^2\sqrt{a}\right)}{\sqrt{x}} + \frac{c_2 \operatorname{WhittakerW}\left(-\frac{b}{4\sqrt{a}}, \frac{1}{4}, x^2\sqrt{a}\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 68

```
DSolve[y''[x]-(a*x^2+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \to\ & c_1 \operatorname{ParabolicCylinderD}\left(-\frac{b}{2\sqrt{a}} - \frac{1}{2}, \sqrt{2}\sqrt[4]{a}x\right) \\ & + c_2 \operatorname{ParabolicCylinderD}\left(\frac{1}{2}\left(\frac{b}{\sqrt{a}} - 1\right), i\sqrt{2}\sqrt[4]{a}x\right) \end{aligned}$$

26.5 problem 5

Internal problem ID [10839]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + a^3x(-ax+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 41

```
dsolve(diff(y(x),x$2)+a^3*x*(2-a*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{ax(ax-2)}{2}} + c_2 \mathrm{e}^{-\frac{1}{2}a^2x^2+ax} \operatorname{erf}(iax - i)$$

✓ Solution by Mathematica

Time used: 0.28 (sec). Leaf size: 50

```
DSolve[y''[x]+a^3*x*(2-a*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{1}{2}ax(ax-2)-1}\left(2eac_1 - \sqrt{\pi}c_2 \operatorname{erfi}(1-ax)\right)}{2a}$$

26.6 problem 6

Internal problem ID [10840]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - (a\,x^2 + bcx)\,y = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 119

```
dsolve(diff(y(x),x$2)-(a*x^2+b*x*c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \,\mathrm{hypergeom}\left(\left[-\frac{b^2c^2 - 4a^{\frac{3}{2}}}{16a^{\frac{3}{2}}}\right], \left[\frac{1}{2}\right], \frac{(2ax+bc)^2}{4a^{\frac{3}{2}}}\right) \mathrm{e}^{-\frac{x(ax+bc)}{2\sqrt{a}}} + c_2(2ax+bc)\,\mathrm{hypergeom}\left(\left[-\frac{b^2c^2 - 12a^{\frac{3}{2}}}{16a^{\frac{3}{2}}}\right], \left[\frac{3}{2}\right], \frac{(2ax+bc)^2}{4a^{\frac{3}{2}}}\right) \mathrm{e}^{-\frac{x(ax+bc)}{2\sqrt{a}}}$$

✓ Solution by Mathematica

Time used: 0.08 (sec). Leaf size: 92

```
DSolve[y''[x]-(a*x^2+b*x*c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \,\mathrm{ParabolicCylinderD}\left(-\frac{b^2c^2}{8a^{3/2}} - \frac{1}{2}, \frac{i(bc+2ax)}{\sqrt{2}a^{3/4}}\right) + c_1 \,\mathrm{ParabolicCylinderD}\left(\frac{1}{8}\left(\frac{b^2c^2}{a^{3/2}} - 4\right), \frac{bc+2ax}{\sqrt{2}a^{3/4}}\right)$$

26.7 problem 7

Internal problem ID [10841]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - a\,x^n y = 0}$$

✓ Solution by Maple

Time used: 0.875 (sec). Leaf size: 65

```
dsolve(diff(y(x),x$2)-a*x^n*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\ \mathrm{BesselJ}\left(\frac{1}{n+2}, \frac{2\sqrt{-a}\,x^{\frac{n}{2}+1}}{n+2}\right) + c_2\sqrt{x}\ \mathrm{BesselY}\left(\frac{1}{n+2}, \frac{2\sqrt{-a}\,x^{\frac{n}{2}+1}}{n+2}\right)$$

✓ Solution by Mathematica

Time used: 0.125 (sec). Leaf size: 119

```
DSolve[y''[x]-a*x^n*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (n+2)^{-\frac{1}{n+2}}\sqrt{x}a^{\frac{1}{2n+4}}\left(c_1\ \mathrm{Gamma}\left(\frac{n+1}{n+2}\right)\mathrm{BesselI}\left(-\frac{1}{n+2}, \frac{2\sqrt{a}x^{\frac{n}{2}+1}}{n+2}\right) + c_2(-1)^{\frac{1}{n+2}}\ \mathrm{Gamma}\left(1+\frac{1}{n+2}\right)\mathrm{BesselI}\left(\frac{1}{n+2}, \frac{2\sqrt{a}x^{\frac{n}{2}+1}}{n+2}\right)\right)$$

26.8 problem 8

Internal problem ID [10842]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - a\left(a\,x^{2n} + n\,x^{-1+n}\right) y = 0$$

✓ Solution by Maple

Time used: 0.407 (sec). Leaf size: 137

```
dsolve(diff(y(x),x$2)-a*(a*x^(2*n)+n*x^(n-1))*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{a\,x^{1+n}}{1+n}} + c_2 \left(\frac{x^{-\frac{3n}{2}-1}(n+2)^2 \operatorname{WhittakerM}\left(\frac{n+2}{2+2n}, \frac{3+2n}{2+2n}, \frac{2a\,x^{1+n}}{1+n}\right)}{2} + \operatorname{WhittakerM}\left(-\frac{n}{2+2n}, \frac{3+2n}{2+2n}, \frac{2a\,x^{1+n}}{1+n}\right) \left(\left(\frac{n}{2}+1\right) x^{-\frac{3n}{2}-1} + a\,x^{-\frac{n}{2}}\right)(1+n) \right)$$

✓ Solution by Mathematica

Time used: 0.596 (sec). Leaf size: 81

```
DSolve[y''[x]-a*(a*x^(2*n)+n*x^(n-1))*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{ax^{n+1}}{n+1}} \left(c_2 - \frac{c_1 2^{-\frac{1}{n+1}} x \left(\frac{ax^{n+1}}{n+1}\right)^{-\frac{1}{n+1}} \Gamma\left(\frac{1}{n+1}, \frac{2ax^{n+1}}{n+1}\right)}{n+1} \right)$$

26.9 problem 9

Internal problem ID [10843]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - a\,x^{n-2}(x^n a + n + 1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.562 (sec). Leaf size: 109

```
dsolve(diff(y(x),x$2)-a*x^(n-2)*(a*x^n+n+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x\,\mathrm{e}^{\frac{a\,x^n}{n}} + c_2\left(\left(\left(n^2-n\right)x^{-\frac{3n}{2}+\frac{1}{2}} + 2an\,x^{-\frac{n}{2}+\frac{1}{2}}\right)\mathrm{WhittakerM}\left(-\frac{1}{2}-\frac{1}{2n}, -\frac{1}{2n}+1, \frac{2a\,x^n}{n}\right) + x^{-\frac{3n}{2}+\frac{1}{2}}\,\mathrm{WhittakerM}\left(\frac{1}{2}-\frac{1}{2n}, -\frac{1}{2n}+1, \frac{2a\,x^n}{n}\right)(n-1)^2\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]-a*x^(n-2)*(a*x^n+n+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

26.10 problem 10

Internal problem ID [10844]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(a\,x^{2n} + b\,x^{-1+n}\right) y = 0}$$

✓ Solution by Maple

Time used: 0.86 (sec). Leaf size: 95

```
dsolve(diff(y(x),x$2)+(a*x^(2*n)+b*x^(n-1))*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{WhittakerM}\left(-\frac{ib}{\sqrt{a}\,(2+2n)}, \frac{1}{2+2n}, \frac{2i\sqrt{a}\,x^{1+n}}{1+n}\right) x^{-\frac{n}{2}} + c_2 \operatorname{WhittakerW}\left(-\frac{ib}{\sqrt{a}\,(2+2n)}, \frac{1}{2+2n}, \frac{2i\sqrt{a}\,x^{1+n}}{1+n}\right) x^{-\frac{n}{2}}$$

✓ Solution by Mathematica

Time used: 0.399 (sec). Leaf size: 225

```
DSolve[y''[x]+(a*x^(2*n)+b*x^(n-1))*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 2^{\frac{n}{2n+2}} x^{-n/2} \left(x^{n+1}\right)^{\frac{n}{2n+2}} e^{-\frac{\sqrt{a}x^{n+1}}{\sqrt{-(n+1)^2}}} \left(c_1 \operatorname{HypergeometricU}\left(-\frac{(n+1)\left(nb+b+\sqrt{a}n\sqrt{-(n+1)^2}\right)}{2\sqrt{a}\left(-(n+1)^2\right)^{3/2}}, \frac{n}{n+1} \right.\right.$$

27 Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$

27.1 problem 11

Internal problem ID [10845]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + ay' + by = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x$2)+a*diff(y(x),x)+b*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\left(-\frac{a}{2}+\frac{\sqrt{a^2-4b}}{2}\right)x} + c_2 e^{\left(-\frac{a}{2}-\frac{\sqrt{a^2-4b}}{2}\right)x}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 47

```
DSolve[y''[x]+a*y'[x]+b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{1}{2}x\left(\sqrt{a^2-4b}+a\right)}\left(c_2 e^{x\sqrt{a^2-4b}} + c_1\right)$$

27.2 problem 12

Internal problem ID [10846]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + ay' + (bx + c)\, y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 53

```
dsolve(diff(y(x),x$2)+a*diff(y(x),x)+(b*x+c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{ax}{2}} \operatorname{AiryAi}\left(\frac{a^2 - 4xb - 4c}{4b^{\frac{2}{3}}}\right) + c_2 \mathrm{e}^{-\frac{ax}{2}} \operatorname{AiryBi}\left(\frac{a^2 - 4xb - 4c}{4b^{\frac{2}{3}}}\right)$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 67

```
DSolve[y''[x]+a*y'[x]+(b*x+c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-\frac{ax}{2}} \left(c_1 \operatorname{AiryAi}\left(\frac{a^2 - 4(c + bx)}{4(-b)^{2/3}}\right) + c_2 \operatorname{AiryBi}\left(\frac{a^2 - 4(c + bx)}{4(-b)^{2/3}}\right)\right)$$

27.3 problem 13

Internal problem ID [10847]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + ay' - (b\,x^2 + c)\,y = 0}$$

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 85

```
dsolve(diff(y(x),x$2)+a*diff(y(x),x)-(b*x^2+c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \,\mathrm{KummerM}\left(\frac{a^2 + 12\sqrt{b} + 4c}{16\sqrt{b}}, \frac{3}{2}, \sqrt{b}\,x^2\right) \mathrm{e}^{-\frac{x\left(\sqrt{b}\,x + a\right)}{2}} + c_2 x \,\mathrm{KummerU}\left(\frac{a^2 + 12\sqrt{b} + 4c}{16\sqrt{b}}, \frac{3}{2}, \sqrt{b}\,x^2\right) \mathrm{e}^{-\frac{x\left(\sqrt{b}\,x + a\right)}{2}}$$

✓ Solution by Mathematica

Time used: 0.088 (sec). Leaf size: 96

```
DSolve[y''[x]+a*y'[x]-(b*x^2+c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{1}{2}x\left(a+\sqrt{b}x\right)}\left(c_1 \operatorname{HermiteH}\left(\frac{-a^2-4\left(c+\sqrt{b}\right)}{8\sqrt{b}}, \sqrt[4]{b}x\right) + c_2 \operatorname{Hypergeometric1F1}\left(\frac{a^2+4\left(c+\sqrt{b}\right)}{16\sqrt{b}}, \frac{1}{2}, \sqrt{b}x^2\right)\right)$$

27.4 problem 14

Internal problem ID [10848]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{y'' + ay' + b\left(-b\,x^2 + ax + 1\right) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$2)+a*diff(y(x),x)+b*(-b*x^2+a*x+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \operatorname{erf}\left(x\sqrt{-b} + \frac{a}{2\sqrt{-b}}\right) \mathrm{e}^{-\frac{b\,x^2}{2}} c_1 + c_2 \mathrm{e}^{-\frac{b\,x^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.28 (sec). Leaf size: 67

```
DSolve[y''[x]+a*y'[x]+b*(-b*x^2+a*x+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-\frac{bx^2}{2}} \left(\frac{\sqrt{\pi} c_2 e^{-\frac{a^2}{4b}} \operatorname{erfi}\left(\frac{2bx-a}{2\sqrt{b}}\right)}{\sqrt{b}} + 2c_1 \right)$$

27.5 problem 15

Internal problem ID [10849]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{y'' + ay' + bx(-b\,x^3 + ax + 2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)+a*diff(y(x),x)+b*x*(-b*x^3+a*x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(\left(\int \mathrm{e}^{-ax}\mathrm{e}^{\frac{2x^3b}{3}}dx\right)c_1 + c_2\right)\mathrm{e}^{-\frac{x^3b}{3}}$$

✓ Solution by Mathematica

Time used: 0.913 (sec). Leaf size: 46

```
DSolve[y''[x]+a*y'[x]+b*x*(-b*x^3+a*x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{bx^3}{3}}\left(c_2\int_1^x e^{\frac{2}{3}bK[1]^3 - aK[1]}dK[1] + c_1\right)$$

27.6 problem 16

Internal problem ID [10850]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$y'' + ay' + b\left(-b\,x^{2n} + a\,x^{n} + n\,x^{-1+n}\right) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(diff(y(x),x$2)+a*diff(y(x),x)+b*(-b*x^(2*n)+a*x^n+n*x^(n-1))*y(x)=0,y(x), singsol=all
```

$$y(x) = \left(\left(\int \mathrm{e}^{-ax}\mathrm{e}^{\frac{2b\,x^{1+n}}{1+n}}dx\right)c_1 + c_2\right)\mathrm{e}^{-\frac{b\,x^{1+n}}{1+n}}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+a*y'[x]+b*(-b*x^(2*n)+a*x^n+n*x^(n-1))*y[x]==0,y[x],x,IncludeSingularSolutions
```

Not solved

27.7 problem 17

Internal problem ID [10851]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + ay' + b\left(-b\,x^{2n} - a\,x^n + n\,x^{-1+n}\right) y = 0$$

✓ Solution by Maple

Time used: 0.406 (sec). Leaf size: 80

```
dsolve(diff(y(x),x$2)+a*diff(y(x),x)+b*(-b*x^(2*n)-a*x^n+n*x^(n-1))*y(x)=0,y(x), singsol=all
```

$$y(x) = c_1 \mathrm{e}^{-\frac{anx+b\,x^{1+n}+ax}{1+n}} + c_2 \left(\int \mathrm{e}^{\frac{anx+2b\,x^{1+n}+ax}{1+n}} dx\right) \mathrm{e}^{-\frac{anx+b\,x^{1+n}+ax}{1+n}}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+a*y'[x]+b*(-b*x^(2*n)-a*x^n+n*x^(n-1))*y[x]==0,y[x],x,IncludeSingularSolutions
```

Not solved

27.8 problem 18

Internal problem ID [10852]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + (-1+n)\, y = 0}$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 47

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+(n-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x^2}{2}} \operatorname{KummerM}\left(\frac{3}{2} - \frac{n}{2}, \frac{3}{2}, \frac{x^2}{2}\right) x + c_2 \mathrm{e}^{-\frac{x^2}{2}} \operatorname{KummerU}\left(\frac{3}{2} - \frac{n}{2}, \frac{3}{2}, \frac{x^2}{2}\right) x$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 51

```
DSolve[y''[x]+x*y'[x]+(n-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{x^2}{2}} \left(c_1 \operatorname{HermiteH}\left(n-2, \frac{x}{\sqrt{2}}\right) + c_2 \operatorname{Hypergeometric1F1}\left(1 - \frac{n}{2}, \frac{1}{2}, \frac{x^2}{2}\right)\right)$$

27.9 problem 19

Internal problem ID [10853]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'x + 2ny = 0}$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+2*n*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \operatorname{KummerM}\left(-\frac{n}{2} + \frac{1}{2}, \frac{3}{2}, x^2\right) + c_2 x \operatorname{KummerU}\left(-\frac{n}{2} + \frac{1}{2}, \frac{3}{2}, x^2\right)$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 27

```
DSolve[y''[x]-2*x*y'[x]+2*n*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{HermiteH}(n, x) + c_2 \operatorname{Hypergeometric1F1}\left(-\frac{n}{2}, \frac{1}{2}, x^2\right)$$

27.10 problem 20

Internal problem ID [10854]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + axy' + by = 0}$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 65

```
dsolve(diff(y(x),x$2)+a*x*diff(y(x),x)+b*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{a x^2}{2}} \operatorname{KummerM}\left(\frac{2a-b}{2a}, \frac{3}{2}, \frac{a x^2}{2}\right) x + c_2 e^{-\frac{a x^2}{2}} \operatorname{KummerU}\left(\frac{2a-b}{2a}, \frac{3}{2}, \frac{a x^2}{2}\right) x$$

✓ Solution by Mathematica

Time used: 0.061 (sec). Leaf size: 67

```
DSolve[y''[x]+a*x*y'[x]+b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{ax^2}{2}} \left(c_1 \operatorname{HermiteH}\left(\frac{b}{a} - 1, \frac{\sqrt{a}x}{\sqrt{2}}\right) + c_2 \operatorname{Hypergeometric1F1}\left(\frac{a-b}{2a}, \frac{1}{2}, \frac{ax^2}{2}\right)\right)$$

27.11 problem 21

Internal problem ID [10855]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + axy' + bxy = 0}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)+a*x*diff(y(x),x)+b*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{bx}{a}} \operatorname{KummerM}\left(\frac{b^2}{2a^3}, \frac{1}{2}, -\frac{(a^2x - 2b)^2}{2a^3}\right) + c_2 e^{-\frac{bx}{a}} \operatorname{KummerU}\left(\frac{b^2}{2a^3}, \frac{1}{2}, -\frac{(a^2x - 2b)^2}{2a^3}\right)$$

✓ Solution by Mathematica

Time used: 0.106 (sec). Leaf size: 96

```
DSolve[y''[x]+a*x*y'[x]+b*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{bx}{a} - \frac{ax^2}{2}} \left(c_2 \operatorname{Hypergeometric1F1}\left(\frac{1}{2} - \frac{b^2}{2a^3}, \frac{1}{2}, \frac{(a^2x - 2b)^2}{2a^3}\right) + c_1 \operatorname{HermiteH}\left(\frac{b^2}{a^3} - 1, \frac{a^2x - 2b}{\sqrt{2}a^{3/2}}\right)\right)$$

27.12 problem 22

Internal problem ID [10856]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + axy' + (bx + c)\, y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 89

```
dsolve(diff(y(x),x$2)+a*x*diff(y(x),x)+(b*x+c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{bx}{a}} \operatorname{KummerM}\left(\frac{c\, a^2 + b^2}{2a^3}, \frac{1}{2}, -\frac{(a^2 x - 2b)^2}{2a^3}\right) + c_2 \mathrm{e}^{-\frac{bx}{a}} \operatorname{KummerU}\left(\frac{c\, a^2 + b^2}{2a^3}, \frac{1}{2}, -\frac{(a^2 x - 2b)^2}{2a^3}\right)$$

✓ Solution by Mathematica

Time used: 0.085 (sec). Leaf size: 108

```
DSolve[y''[x]+a*x*y'[x]+(b*x+c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{bx}{a} - \frac{ax^2}{2}} \left(c_2 \operatorname{Hypergeometric1F1}\left(-\frac{-a^3 + ca^2 + b^2}{2a^3}, \frac{1}{2}, \frac{(a^2 x - 2b)^2}{2a^3}\right) + c_1 \operatorname{HermiteH}\left(\frac{b^2}{a^3} + \frac{c}{a} - 1, \frac{a^2 x - 2b}{\sqrt{2} a^{3/2}}\right)\right)$$

27.13 problem 23

Internal problem ID [10857]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 2axy' + (b\,x^4 + a^2x^2 + xc + a)\,y = 0$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 97

```
dsolve(diff(y(x),x$2)+2*a*x*diff(y(x),x)+(b*x^4+a^2*x^2+c*x+a)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x\,\mathrm{e}^{-\frac{x^2\left(i\sqrt{b}\,x+\frac{3a}{2}\right)}{3}}\,\mathrm{KummerM}\left(\frac{ic+4\sqrt{b}}{6\sqrt{b}}, \frac{4}{3}, \frac{2i\sqrt{b}\,x^3}{3}\right) + c_2 x\,\mathrm{e}^{-\frac{x^2\left(i\sqrt{b}\,x+\frac{3a}{2}\right)}{3}}\,\mathrm{KummerU}\left(\frac{ic+4\sqrt{b}}{6\sqrt{b}}, \frac{4}{3}, \frac{2i\sqrt{b}\,x^3}{3}\right)$$

✓ Solution by Mathematica

Time used: 0.322 (sec). Leaf size: 121

```
DSolve[y''[x]+2*a*x*y'[x]+(b*x^4+a^2*x^2+c*x+a)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{\sqrt[3]{2}\sqrt[3]{x^3}e^{\frac{1}{6}ix^2\left(2\sqrt{b}x+3ia\right)}\left(c_1\,\mathrm{HypergeometricU}\left(\frac{1}{3}-\frac{ic}{6\sqrt{b}}, \frac{2}{3}, -\frac{2}{3}i\sqrt{b}x^3\right) + c_2 L_{\frac{ic}{6\sqrt{b}}-\frac{1}{3}}^{-\frac{1}{3}}\left(-\frac{2}{3}i\sqrt{b}x^3\right)\right)}{x}$$

27.14 problem 24

Internal problem ID [10858]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'(ax+b) - ay = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 66

```
dsolve(diff(y(x),x$2)+(a*x+b)*diff(y(x),x)-a*y(x)=0,y(x), singsol=all)
```

$$y(x) = (ax+b)\,c_1 + c_2\left(\pi(ax+b)\,\mathrm{e}^{\frac{b^2}{2a}}\,\mathrm{erf}\left(\frac{\sqrt{2}\,(ax+b)}{2\sqrt{a}}\right) + \sqrt{2}\,\sqrt{\pi}\,\sqrt{a}\,\mathrm{e}^{-\frac{x(ax+2b)}{2}}\right)$$

✓ Solution by Mathematica

Time used: 0.959 (sec). Leaf size: 82

```
DSolve[y''[x]+(a*x+b)*y'[x]-a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(ax+b)\left(-\frac{\sqrt{\frac{\pi}{2}}c_2\mathrm{erf}\left(\frac{ax+b}{\sqrt{2}\sqrt{a}}\right)}{a^{3/2}} - \frac{c_2e^{-\frac{(ax+b)^2}{2a}}}{a(ax+b)} + c_1\right)}{b}$$

27.15 problem 25

Internal problem ID [10859]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + y'(ax + b) + ay = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 52

```
dsolve(diff(y(x),x$2)+(a*x+b)*diff(y(x),x)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = \operatorname{erf}\left(-\frac{\sqrt{-2a}\,x}{2} + \frac{b}{\sqrt{-2a}}\right) \mathrm{e}^{-\frac{1}{2}a\,x^2 - xb} c_1 + c_2 \mathrm{e}^{-\frac{1}{2}a\,x^2 - xb}$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 79

```
DSolve[y''[x]+(a*x+b)*y'[x]+a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{(ax+b)^2}{2a}}\left(2\sqrt{a}c_2 e^{\frac{b^2}{2a}} + \sqrt{2\pi}c_1 \operatorname{erfi}\left(\frac{ax+b}{\sqrt{2}\sqrt{a}}\right)\right)}{2\sqrt{a}}$$

27.16 problem 26

Internal problem ID [10860]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'(ax+b) + c(ax+b-c)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)+(a*x+b)*diff(y(x),x)+c*(a*x+b-c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-cx} + c_2 \mathrm{e}^{-cx} \operatorname{erf}\left(\frac{\sqrt{2}\,(ax+b-2c)}{2\sqrt{a}}\right)$$

✓ Solution by Mathematica

Time used: 0.091 (sec). Leaf size: 70

```
DSolve[y''[x]+(a*x+b)*y'[x]+c*(a*x+b-c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{1}{2}x(ax+2b-2c)}\left(c_1 \operatorname{HermiteH}\left(-1, \frac{b-2c+ax}{\sqrt{2}\sqrt{a}}\right) + c_2 e^{\frac{(ax+b-2c)^2}{2a}}\right)$$

27.17 problem 27

Internal problem ID [10861]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + (ax + 2b)\, y' + \left(bax + b^2 - a\right) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
dsolve(diff(y(x),x$2)+(a*x+2*b)*diff(y(x),x)+(a*b*x-a+b^2)*y(x)=0,y(x), singsol=all)
```

$$y = c_1 x\, \mathrm{e}^{-bx} + c_2 \left(\mathrm{e}^{-bx} \sqrt{2}\, \sqrt{\pi}\, \sqrt{a}\, \mathrm{erf}\left(\frac{\sqrt{2}\, \sqrt{a}\, x}{2}\right) x + 2\, \mathrm{e}^{-\frac{(ax+2b)x}{2}} \right)$$

✓ Solution by Mathematica

Time used: 0.405 (sec). Leaf size: 64

```
DSolve[y''[x]+(a*x+2*b)*y'[x]+(a*b*x-a+b^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x e^{-bx} \left(-\sqrt{\frac{\pi}{2}} \sqrt{a} c_2 \mathrm{erf}\left(\frac{\sqrt{a} x}{\sqrt{2}}\right) - \frac{c_2 e^{-\frac{ax^2}{2}}}{x} + c_1 \right)$$

27.18 problem 28

Internal problem ID [10862]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (ax + b)\, y' + (xc + d)\, y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 105

```
dsolve(diff(y(x),x$2)+(a*x+b)*diff(y(x),x)+(c*x+d)*y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^{-\frac{cx}{a}} \operatorname{KummerM}\left(\frac{a^2 d - abc + c^2}{2a^3}, \frac{1}{2}, -\frac{(a^2 x + ab - 2c)^2}{2a^3}\right) + c_2 e^{-\frac{cx}{a}} \operatorname{KummerU}\left(\frac{a^2 d - abc + c^2}{2a^3}, \frac{1}{2}, -\frac{(a^2 x + ab - 2c)^2}{2a^3}\right)$$

✓ Solution by Mathematica

Time used: 0.116 (sec). Leaf size: 132

```
DSolve[y''[x]+(a*x+b)*y'[x]+(c*x+d)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{cx}{a} - \frac{ax^2}{2} - bx}\left(c_2 \operatorname{Hypergeometric1F1}\left(\frac{a^3 - da^2 + bca - c^2}{2a^3}, \frac{1}{2}, \frac{(xa^2 + ba - 2c)^2}{2a^3}\right) + c_1 \operatorname{HermiteH}\left(\frac{-a^3 + da^2 - bca + c^2}{a^3}, \frac{xa^2 + ba - 2c}{\sqrt{2}a^{3/2}}\right)\right)$$

27.19 problem 29

Internal problem ID [10863]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (ax + b)\, y' + c\big((-c + a)\, x^2 + bx + 1\big)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
dsolve(diff(y(x),x$2)+(a*x+b)*diff(y(x),x)+c*((a-c)*x^2+b*x+1)*y(x)=0,y(x), singsol=all)
```

$$y = c_1 \mathrm{e}^{-\frac{x^2 c}{2}} + c_2 \mathrm{e}^{-\frac{x^2 c}{2}} \operatorname{erf}\left(\frac{(a - 2c)\, x + b}{\sqrt{2a - 4c}}\right)$$

✓ Solution by Mathematica

Time used: 0.135 (sec). Leaf size: 81

```
DSolve[y''[x]+(a*x+b)*y'[x]+c*((a-c)*x^2+b*x+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to e^{-\frac{1}{2}x(x(a-c)+2b)}\left(c_1 \,\text{HermiteH}\left(-1, \frac{b + (a - 2c)x}{\sqrt{2}\sqrt{a - 2c}}\right) + c_2 e^{\frac{(x(a-2c)+b)^2}{2(a-2c)}}\right)$$

27.20 problem 30

Internal problem ID [10864]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 2y'(ax + b) + \left(a^2x^2 + 2bax + c\right) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
dsolve(diff(y(x),x$2)+2*(a*x+b)*diff(y(x),x)+(a^2*x^2+2*a*b*x+c)*y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^{-\frac{x\left(ax - 2\sqrt{b^2+a-c}+2b\right)}{2}} + e^{-\frac{x\left(ax+2\sqrt{b^2+a-c}+2b\right)}{2}} c_2$$

✓ Solution by Mathematica

Time used: 0.228 (sec). Leaf size: 86

```
DSolve[y''[x]+2*(a*x+b)*y'[x]+(a^2*x^2+2*a*b*x+c)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{e^{-\frac{1}{2}x\left(2\sqrt{a+b^2-c}+ax+2b\right)}\left(c_2 e^{2x\sqrt{a+b^2-c}} + 2c_1\sqrt{a + b^2 - c}\right)}{2\sqrt{a + b^2 - c}}$$

27.21 problem 31

Internal problem ID [10865]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + (ax + b)\, y' + \left(\alpha\, x^2 + \beta x + \gamma\right) y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 317

```
dsolve(diff(y(x),x$2)+(a*x+b)*diff(y(x),x)+(alpha*x^2+beta*x+gamma)*y(x)=0,y(x), singsol=all
```

$$y$$

$$= c_1 \operatorname{hypergeom}\left(\left[\frac{(a^2-4\alpha)^{\frac{3}{2}} + a^3 - 2\gamma\, a^2 + (2b\beta - 4\alpha)\, a + (-2b^2+8\gamma)\,\alpha - 2\beta^2}{4\,(a^2-4\alpha)^{\frac{3}{2}}}\right], \left[\frac{1}{2}\right], \frac{(a^2x + ab - 4}{2\,(a^2 - }\right.$$
$$+ c_2\big(a^2x + ab - 4\alpha x$$
$$-2\beta\big) \operatorname{hypergeom}\left(\left[\frac{3(a^2-4\alpha)^{\frac{3}{2}} + a^3 - 2\gamma\, a^2 + (2b\beta - 4\alpha)\, a + (-2b^2+8\gamma)\,\alpha - 2\beta^2}{4\,(a^2-4\alpha)^{\frac{3}{2}}}\right], \left[\frac{3}{2}\right], \frac{(a^2x + a}{2\,(}\right.$$

✓ Solution by Mathematica

Time used: 0.467 (sec). Leaf size: 307

```
DSolve[y''[x]+(a*x+b)*y'[x]+(\[Alpha]*x^2+\[Beta]*x+\[Gamma])*y[x]==0,y[x],x,IncludeSingular
```

$$y(x) \to \exp\left(-\frac{x\left(2b\sqrt{a^2-4\alpha}+a\left(x\sqrt{a^2-4\alpha}+2b\right)+a^2x-4(\beta+\alpha x)\right)}{4\sqrt{a^2-4\alpha}}\right)\left(c_1 \operatorname{HermiteH}\left(\frac{-a^3-\left(\sqrt{a^2-4\alpha\right.}\right.\right.\right.$$

27.22 problem 32

Internal problem ID [10866]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + y'(ax + b) + c\left(-c\,x^{2n} + a\,x^{n+1} + b\,x^{n} + n\,x^{-1+n}\right) y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+(a*x+b)*diff(y(x),x)+c*(-c*x^(2*n)+a*x^(n+1)+b*x^n+n*x^(n-1) )*y(x)=0,
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+(a*x+b)*y'[x]+c*(-c*x^(2*n)+a*x^(n+1)+b*x^n+n*x^(n-1) )*y[x]==0,y[x],x,Include
```

Not solved

27.23 problem 33

Internal problem ID [10867]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + a(-b^2 + x^2)\, y' - a(x + b)\, y = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 181

```
dsolve(diff(y(x),x$2)+a*(x^2-b^2)*diff(y(x),x)-a*(x+b)*y(x)=0,y(x), singsol=all)
```

$$y = c_1 \operatorname{HeunT}\left(-\frac{3^{\frac{2}{3}} a^3 b}{(a^2)^{\frac{4}{3}}}, -\frac{6\sqrt{a^2}}{a}, -\frac{a^2 b^2 3^{\frac{1}{3}}}{(a^2)^{\frac{2}{3}}}, \frac{3^{\frac{2}{3}} (a^2)^{\frac{1}{6}}\, x}{3}\right) \mathrm{e}^{\frac{ax(3b^2 - x^2)\left((a^2)^{\frac{2}{3}} + (a^2)^{\frac{1}{6}} a\right)}{6(a^2)^{\frac{2}{3}}}}$$
$$+ c_2 \operatorname{HeunT}\left(-\frac{3^{\frac{2}{3}} a^3 b}{(a^2)^{\frac{4}{3}}}, \frac{6\sqrt{a^2}}{a}, -\frac{a^2 b^2 3^{\frac{1}{3}}}{(a^2)^{\frac{2}{3}}}, -\frac{3^{\frac{2}{3}} (a^2)^{\frac{1}{6}}\, x}{3}\right) \mathrm{e}^{-\frac{\left(b^2 - \frac{x^2}{3}\right) xa\left(-(a^2)^{\frac{2}{3}} + (a^2)^{\frac{1}{6}} a\right)}{2(a^2)^{\frac{2}{3}}}}$$

✓ Solution by Mathematica

Time used: 3.893 (sec). Leaf size: 55

```
DSolve[y''[x]+a*(x^2-b^2)*y'[x]-a*(x+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(b - x)\left(c_2 \int_1^x \frac{e^{ab^2 K[1] - \frac{1}{3} a K[1]^3}}{(b - K[1])^2} dK[1] + c_1\right)}{b}$$

27.24 problem 34

Internal problem ID [10868]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(a\,x^2 + b\right)y' + c\left(a\,x^2 + b - c\right)y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 174

```
dsolve(diff(y(x),x$2)+(a*x^2+b)*diff(y(x),x)+c*(a*x^2+b-c)*y(x)=0,y(x), singsol=all)
```

$$y = c_1 \operatorname{HeunT}\left(0, -\frac{3\sqrt{a^2}}{a}, \frac{a(b-2c)\,3^{\frac{1}{3}}}{(a^2)^{\frac{2}{3}}}, \frac{3^{\frac{2}{3}}(a^2)^{\frac{1}{6}}\,x}{3}\right) e^{-\frac{x\left(\left(a\,x^2+3b\right)\left(a^2\right)^{\frac{2}{3}}+a\left(a^2\right)^{\frac{1}{6}}\left(a\,x^2+3b-6c\right)\right)}{6\left(a^2\right)^{\frac{2}{3}}}} + c_2 \operatorname{HeunT}\left(0, \frac{3\sqrt{a^2}}{a}, \frac{a(b-2c)\,3^{\frac{1}{3}}}{(a^2)^{\frac{2}{3}}}, -\frac{3^{\frac{2}{3}}(a^2)^{\frac{1}{6}}\,x}{3}\right) e^{\frac{x\left(\left(-a\,x^2-3b\right)\left(a^2\right)^{\frac{2}{3}}+a\left(a^2\right)^{\frac{1}{6}}\left(a\,x^2+3b-6c\right)\right)}{6\left(a^2\right)^{\frac{2}{3}}}}$$

✓ Solution by Mathematica

Time used: 0.915 (sec). Leaf size: 46

```
DSolve[y''[x]+(a*x^2+b)*y'[x]+c*(a*x^2+b-c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-cx}\left(c_2 \int_1^x e^{-\frac{1}{3}K[1](aK[1]^2+3b-6c)}dK[1] + c_1\right)$$

27.25 problem 35

Internal problem ID [10869]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (a\,x^2 + 2b)\,y' + (ba\,x^2 - ax + b^2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 74

```
dsolve(diff(y(x),x$2)+(a*x^2+2*b)*diff(y(x),x)+(a*b*x^2-a*x+b^2)*y(x)=0,y(x), singsol=all)
```

$$y = c_1 x\,\mathrm{e}^{-bx} + \frac{c_2 \mathrm{e}^{-\frac{x(a\,x^2+6b)}{6}} \left(x^6 a^2 \,\mathrm{WhittakerM}\left(\frac{1}{3}, \frac{5}{6}, \frac{a\,x^3}{3}\right) + 5\,\mathrm{WhittakerM}\left(\frac{4}{3}, \frac{5}{6}, \frac{a\,x^3}{3}\right) a\,x^3 + 10\,\mathrm{WhittakerM}\left(\frac{4}{3}, \frac{5}{6},\right.\right.}{x^4}$$

✓ Solution by Mathematica

Time used: 0.407 (sec). Leaf size: 51

```
DSolve[y''[x]+(a*x^2+2*b)*y'[x]+(a*b*x^2-a*x+b^2)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{9} e^{-bx} \left(9c_1 x - 3^{2/3} c_2 \sqrt[3]{ax^3}\,\Gamma\left(-\frac{1}{3}, \frac{ax^3}{3}\right)\right)$$

27.26 problem 36

Internal problem ID [10870]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(2x^2 + a\right) y' + \left(x^4 + a\,x^2 + b + 2x\right) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 57

```
dsolve(diff(y(x),x$2)+(2*x^2+a)*diff(y(x),x)+(x^4+a*x^2+2*x+b)*y(x)=0,y(x), singsol=all)
```

$$y = c_1 \mathrm{e}^{-\frac{x\left(2x^2-3\sqrt{a^2-4b}+3a\right)}{6}} + c_2 \mathrm{e}^{-\frac{x\left(2x^2+3\sqrt{a^2-4b}+3a\right)}{6}}$$

✓ Solution by Mathematica

Time used: 0.217 (sec). Leaf size: 79

```
DSolve[y''[x]+(2*x^2+a)*y'[x]+(x^4+a*x^2+2*x+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{e^{-\frac{1}{6}x\left(3\sqrt{a^2-4b}+3a+2x^2\right)}\left(c_2 e^{x\sqrt{a^2-4b}} + c_1\sqrt{a^2-4b}\right)}{\sqrt{a^2-4b}}$$

27.27 problem 37

Internal problem ID [10871]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 37.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + (a\,x^2 + bx)\,y' + (\alpha\,x^2 + \beta x + \gamma)\,y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 298

```
dsolve(diff(y(x),x$2)+(a*x^2+b*x)*diff(y(x),x)+(alpha*x^2+beta*x+gamma)*y(x)=0,y(x), singsol
```

$$y = c_1 e^{-\frac{(\sqrt{a^2}+a)\sqrt{a^2}\,x^3}{6a} - \frac{b(\sqrt{a^2}+a)\sqrt{a^2}\,x^2}{4a^2} + \frac{\alpha\sqrt{a^2}\,x}{a^2}} \operatorname{HeunT}\left(\frac{3^{\frac{2}{3}}(2\gamma\,a^2 - ab\beta + b^2\alpha + 2\alpha^2)}{2\,(a^2)^{\frac{4}{3}}}, -\frac{3(a^2 - \beta a + b\alpha)\sqrt{a^2}}{a^3}, -\frac{3^{\frac{1}{3}}(b^2 + 8\alpha)}{4\,(a^2)^{\frac{2}{3}}}, \frac{3^{\frac{2}{3}}a(2ax + b)}{6\,(a^2)^{\frac{5}{6}}}\right) + c_2 e^{-\frac{(\sqrt{a^2}-a)\sqrt{a^2}\,x^3}{6a} - \frac{b(\sqrt{a^2}-a)\sqrt{a^2}\,x^2}{4a^2} - \frac{\alpha\sqrt{a^2}\,x}{a^2}} \operatorname{HeunT}\left(\frac{3^{\frac{2}{3}}(2\gamma\,a^2 - ab\beta + b^2\alpha + 2\alpha^2)}{2\,(a^2)^{\frac{4}{3}}}, \frac{3(a^2 - \beta a + b\alpha)\sqrt{a^2}}{a^3}, -\frac{3^{\frac{1}{3}}(b^2 + 8\alpha)}{4\,(a^2)^{\frac{2}{3}}}, -\frac{3^{\frac{2}{3}}a(2ax + b)}{6\,(a^2)^{\frac{5}{6}}}\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+(a*x^2+b*x)*y'[x]+(\[Alpha]*x^2+\[Beta]*x+\[Gamma])*y[x]==0,y[x],x,IncludeSing
```

Not solved

27.28 problem 38

Internal problem ID [10872]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(ba\,x^2 + bx + 2a\right) y' + a^2\left(b\,x^2 + 1\right) y = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 254

```
dsolve(diff(y(x),x$2)+(a*b*x^2+b*x+2*a)*diff(y(x),x)+a^2*(b*x^2+1)*y(x)=0,y(x), singsol=all)
```

$$y = c_1 \mathrm{e}^{-\frac{\left(\frac{\left(a^5b^5+\left(b^2a^2\right)^{\frac{5}{2}}\right)\left(ax+\frac{3}{2}\right)xb}{\left(b^2a^2\right)^{\frac{5}{2}}}+6a\right)x}{6}} \operatorname{HeunT}\left(\frac{b^3 3^{\frac{2}{3}} a^2}{2\left(b^2a^2\right)^{\frac{4}{3}}}, -\frac{6\sqrt{b^2}\,\sqrt{a^2}}{ba}, -\frac{b^2 3^{\frac{1}{3}}}{4\left(b^2a^2\right)^{\frac{2}{3}}}, \frac{3^{\frac{2}{3}} a\, b^2(2ax+1)}{6\left(b^2a^2\right)^{\frac{5}{6}}}\right) + c_2 \mathrm{e}^{\frac{\left(\frac{\left(ax+\frac{3}{2}\right)\left(a^5b^5-\left(b^2a^2\right)^{\frac{5}{2}}\right)xb}{\left(b^2a^2\right)^{\frac{5}{2}}}-6a\right)x}{6}} \operatorname{HeunT}\left(\frac{b^3 3^{\frac{2}{3}} a^2}{2\left(b^2a^2\right)^{\frac{4}{3}}}, \frac{6\sqrt{b^2}\,\sqrt{a^2}}{ba}, -\frac{b^2 3^{\frac{1}{3}}}{4\left(b^2a^2\right)^{\frac{2}{3}}}, -\frac{\left(ax+\frac{1}{2}\right) a 3^{\frac{2}{3}} b^2}{3\left(b^2a^2\right)^{\frac{5}{6}}}\right)$$

✓ Solution by Mathematica

Time used: 2.136 (sec). Leaf size: 57

```
DSolve[y''[x]+(a*b*x^2+b*x+2*a)*y'[x]+a^2*(b*x^2+1)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to e^{-ax}(ax+1)\left(c_2\int_1^x \frac{e^{-\frac{1}{6}bK[1]^2(2aK[1]+3)}}{(aK[1]+1)^2}dK[1]+c_1\right)$$

27.29 problem 39

Internal problem ID [10873]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(a\,x^2 + bx + c\right) y' + x\left(ba\,x^2 + bc + 2a\right) y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 240

```
dsolve(diff(y(x),x$2)+(a*x^2+b*x+c)*diff(y(x),x)+x*(a*b*x^2+b*c+2*a)*y(x)=0,y(x), singsol=al
```

$$y = c_1 \mathrm{e}^{-\frac{\left(\sqrt{a^2}+a\right)\sqrt{a^2}\,x^3}{6a} - \frac{b\left(\sqrt{a^2}-a\right)\sqrt{a^2}\,x^2}{4a^2} - \frac{c\left(\sqrt{a^2}+a\right)\sqrt{a^2}\,x}{2a^2}} \operatorname{HeunT}\left(0, \frac{3\sqrt{a^2}}{a}, \frac{3^{\frac{1}{3}}(4ac-b^2)}{4\left(a^2\right)^{\frac{2}{3}}}, \frac{3^{\frac{2}{3}}a(2ax-b)}{6\left(a^2\right)^{\frac{5}{6}}}\right) + c_2 \mathrm{e}^{-\frac{\left(\sqrt{a^2}-a\right)\sqrt{a^2}\,x^3}{6a} - \frac{b\left(\sqrt{a^2}+a\right)\sqrt{a^2}\,x^2}{4a^2} - \frac{c\left(\sqrt{a^2}-a\right)\sqrt{a^2}\,x}{2a^2}} \operatorname{HeunT}\left(0, -\frac{3\sqrt{a^2}}{a}, \frac{3^{\frac{1}{3}}(4ac-b^2)}{4\left(a^2\right)^{\frac{2}{3}}}, -\frac{\left(ax-\frac{b}{2}\right)a3^{\frac{2}{3}}}{3\left(a^2\right)^{\frac{5}{6}}}\right)$$

✓ Solution by Mathematica

Time used: 1.085 (sec). Leaf size: 59

```
DSolve[y''[x]+(a*x^2+b*x+c)*y'[x]+x*(a*b*x^2+b*c+2*a)*y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to e^{-\frac{1}{3}x(ax^2+3c)}\left(c_2 \int_1^x \exp\left(\frac{1}{6}K[1](6c+K[1](2aK[1]-3b))\right) dK[1] + c_1\right)$$

27.30 problem 40

Internal problem ID [10874]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(a\,x^{2} + bx + c\right) y' + \left(b\,x^{3}a + ac\,x^{2} + b\right) y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 236

```
dsolve(diff(y(x),x$2)+(a*x^2+b*x+c)*diff(y(x),x)+(a*b*x^3+a*c*x^2+b)*y(x)=0,y(x), singsol=al
```

$$y = c_1 \mathrm{e}^{-\frac{\left(\sqrt{a^2}+a\right)\sqrt{a^2}\,x^3}{6a} - \frac{b\left(\sqrt{a^2}-a\right)\sqrt{a^2}\,x^2}{4a^2} - \frac{c\left(\sqrt{a^2}-a\right)\sqrt{a^2}\,x}{2a^2}} \operatorname{HeunT}\left(0, -\frac{3\sqrt{a^2}}{a}, -\frac{3^{\frac{1}{3}}\left(4ac+b^2\right)}{4\left(a^2\right)^{\frac{2}{3}}}, \frac{3^{\frac{2}{3}} a(2ax-b)}{6\left(a^2\right)^{\frac{5}{6}}}\right) + c_2 \mathrm{e}^{-\frac{\left(\sqrt{a^2}-a\right)\sqrt{a^2}\,x^3}{6a} - \frac{b\left(\sqrt{a^2}+a\right)\sqrt{a^2}\,x^2}{4a^2} - \frac{c\left(\sqrt{a^2}+a\right)\sqrt{a^2}\,x}{2a^2}} \operatorname{HeunT}\left(0, \frac{3\sqrt{a^2}}{a}, -\frac{3^{\frac{1}{3}}\left(4ac+b^2\right)}{4\left(a^2\right)^{\frac{2}{3}}}, -\frac{\left(ax-\frac{b}{2}\right) a 3^{\frac{2}{3}}}{3\left(a^2\right)^{\frac{5}{6}}}\right)$$

✓ Solution by Mathematica

Time used: 1.096 (sec). Leaf size: 57

```
DSolve[y''[x]+(a*x^2+b*x+c)*y'[x]+(a*b*x^3+a*c*x^2+b)*y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x) \rightarrow e^{-\frac{1}{2}x(bx+2c)}\left(c_2\int_1^x \exp\left(\frac{1}{6}K[1](6c+K[1](3b-2aK[1]))\right)dK[1]+c_1\right)$$

27.31 problem 41

Internal problem ID [10875]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 41.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(a\,x^3 + 2b\right) y' + \left(b\,x^3 a - a\,x^2 + b^2\right) y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 74

```
dsolve(diff(y(x),x$2)+(a*x^3+2*b)*diff(y(x),x)+(a*b*x^3-a*x^2+b^2)*y(x)=0,y(x), singsol=all)
```

$$y = c_1 x\,\mathrm{e}^{-bx} + \frac{c_2 \mathrm{e}^{-\frac{x\left(a\,x^3+8b\right)}{8}} \left(x^8 a^2\,\mathrm{WhittakerM}\left(\frac{3}{8}, \frac{7}{8}, \frac{a\,x^4}{4}\right) + 7\,\mathrm{WhittakerM}\left(\frac{11}{8}, \frac{7}{8}, \frac{a\,x^4}{4}\right) a\,x^4 + 21\,\mathrm{WhittakerM}\left(\frac{11}{8}, \frac{7}{8}\right.\right.}{x^{\frac{11}{2}}}$$

✓ Solution by Mathematica

Time used: 0.431 (sec). Leaf size: 51

```
DSolve[y''[x]+(a*x^3+2*b)*y'[x]+(a*b*x^3-a*x^2+b^2)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{8} e^{-bx} \left(8c_1 x - \sqrt{2} c_2 \sqrt[4]{ax^4}\,\Gamma\left(-\frac{1}{4}, \frac{ax^4}{4}\right)\right)$$

27.32 problem 42

Internal problem ID [10876]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 42.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + (a\,x^3 + bx)\,y' + 2(2a\,x^2 + b)\,y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 84

```
dsolve(diff(y(x),x$2)+(a*x^3+b*x)*diff(y(x),x)+2*(2*a*x^2+b)*y(x)=0,y(x), singsol=all)
```

$$y = c_1 x\,\mathrm{HeunB}\left(\frac{1}{2}, \frac{b}{\sqrt{a}}, \frac{5}{2}, -\frac{3b}{2\sqrt{a}}, \frac{\sqrt{a}\,x^2}{2}\right) \mathrm{e}^{-\frac{x^2\left(a\,x^2+2b\right)}{4}} + c_2\,\mathrm{HeunB}\left(-\frac{1}{2}, \frac{b}{\sqrt{a}}, \frac{5}{2}, -\frac{3b}{2\sqrt{a}}, \frac{\sqrt{a}\,x^2}{2}\right) \mathrm{e}^{-\frac{x^2\left(a\,x^2+2b\right)}{4}}$$

✓ Solution by Mathematica

Time used: 2.589 (sec). Leaf size: 63

```
DSolve[y''[x]+(a*x^3+b*x)*y'[x]+2*(2*a*x^2+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to xe^{-\frac{1}{4}x^2(ax^2+2b)}\left(c_2\int_1^x \frac{e^{\frac{1}{4}(aK[1]^4+2bK[1]^2)}}{K[1]^2}dK[1] + c_1\right)$$

27.33 problem 43

Internal problem ID [10877]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 43.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(b\,x^3 a + b\,x^2 + 2a\right) y' + a^2\left(b\,x^3 + 1\right) y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 51

```
dsolve(diff(y(x),x$2)+(a*b*x^3+b*x^2+2*a)*diff(y(x),x)+a^2*(b*x^3+1)*y(x)=0,y(x), singsol=al
```

$$y = c_1 \mathrm{e}^{-ax}(ax+1) + c_2 \mathrm{e}^{-ax}(ax+1)\left(\int \frac{\mathrm{e}^{-\frac{\left(ax+\frac{4}{3}\right)x^3 b}{4}}}{\left(ax+1\right)^2} dx\right)$$

✓ Solution by Mathematica

Time used: 3.356 (sec). Leaf size: 57

```
DSolve[y''[x]+(a*b*x^3+b*x^2+2*a)*y'[x]+a^2*(b*x^3+1)*y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to e^{-ax}(ax+1)\left(c_2 \int_1^x \frac{e^{-\frac{1}{12}bK[1]^3(3aK[1]+4)}}{(aK[1]+1)^2} dK[1] + c_1\right)$$

27.34 problem 44

Internal problem ID [10878]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition

Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$

Problem number: 44.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + a\,x^n y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 788

```
dsolve(diff(y(x),x$2)+a*x^n*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = c_1 + \left(\frac{(n+1)\left(ax\,n^2\left(\frac{a}{n+1}\right)^{\frac{1}{n+1}} \mathrm{e}^{-\frac{a\,x^{n+1}}{2(n+1)}}\left(\frac{a\,x^{n+1}}{n+1}\right)^{-\frac{2+n}{2(n+1)}} \mathrm{WhittakerM}\left(-\frac{n}{2(n+1)}, \frac{3}{2(n+1)}+\frac{n}{n+1}, \frac{a\,x^{n+1}}{n+1}\right) + 2anx\right.}{} \right.$$

$$\left. + \frac{(n+1)^2 x^{-n}\left(\frac{a}{n+1}\right)^{\frac{1}{n+1}}(2+n)\left(\frac{a\,x^{n+1}}{n+1}\right)^{-\frac{2+n}{2(n+1)}} \mathrm{e}^{-\frac{a\,x^{n+1}}{2(n+1)}} \mathrm{WhittakerM}\left(\frac{2+n}{2+2n}, \frac{3+2n}{2+2n}, \frac{a\,x^{n+1}}{n+1}\right)}{(3+2n)\,a}\right) c_2$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 56

```
DSolve[y''[x]+a*x^n*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{c_1 x \left(\frac{a x^{n+1}}{n+1}\right)^{-\frac{1}{n+1}} \Gamma\left(\frac{1}{n+1}, \frac{a x^{n+1}}{n+1}\right)}{n+1}$$

27.35 problem 45

Internal problem ID [10879]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 45.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + a\,x^n y' + b\,x^{-1+n} y = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 119

```
dsolve(diff(y(x),x$2)+a*x^n*diff(y(x),x)+b*x^(n-1)*y(x)=0,y(x), singsol=all)
```

$$y = c_1 x\,\mathrm{e}^{-\frac{a\,x^{n+1}}{n+1}} \operatorname{KummerM}\left(\frac{a(n+1)-b}{a\,(n+1)}, \frac{2+n}{n+1}, \frac{a\,x^{n+1}}{n+1}\right) + c_2 x\,\mathrm{e}^{-\frac{a\,x^{n+1}}{n+1}} \operatorname{KummerU}\left(\frac{a(n+1)-b}{a\,(n+1)}, \frac{2+n}{n+1}, \frac{a\,x^{n+1}}{n+1}\right)$$

✓ Solution by Mathematica

Time used: 0.143 (sec). Leaf size: 120

```
DSolve[y''[x]+a*x^n*y'[x]+b*x^(n-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \left(\frac{1}{n}+1\right)^{-\frac{1}{n+1}} n^{-\frac{1}{n+1}} a^{\frac{1}{n+1}} (x^n)^{\frac{1}{n}} \operatorname{Hypergeometric1F1}\left(\frac{a+b}{na+a}, \frac{n+2}{n+1}, -\frac{a(x^n)^{1+\frac{1}{n}}}{n+1}\right) + c_1 \operatorname{Hypergeometric1F1}\left(\frac{b}{na+a}, \frac{n}{n+1}, -\frac{a(x^n)^{1+\frac{1}{n}}}{n+1}\right)$$

27.36 problem 46

Internal problem ID [10880]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 46.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2a\,x^n y' + a\left(a\,x^{2n} + n\,x^{-1+n}\right) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x$2)+2*a*x^n*diff(y(x),x)+a*(a*x^(2*n)+n*x^(n-1))*y(x)=0,y(x), singsol=all)
```

$$y = c_1 \mathrm{e}^{-\frac{a\,x^{n+1}}{n+1}} + c_2 \mathrm{e}^{-\frac{a\,x^{n+1}}{n+1}} x$$

✓ Solution by Mathematica

Time used: 0.112 (sec). Leaf size: 28

```
DSolve[y''[x]+2*a*x^n*y'[x]+a*(a*x^(2*n)+n*x^(n-1))*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to (c_2 x + c_1) e^{-\frac{ax^{n+1}}{n+1}}$$

27.37 problem 47

Internal problem ID [10881]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 47.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + a\,x^n y' + \left(b\,x^{2n} + c\,x^{-1+n}\right) y = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 197

```
dsolve(diff(y(x),x$2)+a*x^n*diff(y(x),x)+(b*x^(2*n)+c*x^(n-1))*y(x)=0,y(x), singsol=all)
```

$$y = c_1 x\,\mathrm{e}^{-\frac{x^{n+1}\left(\sqrt{a^2-4b}+a\right)}{2+2n}} \operatorname{KummerM}\left(\frac{(2+n)\sqrt{a^2-4b}+na-2c}{\sqrt{a^2-4b}\,(2+2n)}, \frac{2+n}{n+1}, \frac{\sqrt{a^2-4b}\,x^{n+1}}{n+1}\right) + c_2 x\,\mathrm{e}^{-\frac{x^{n+1}\left(\sqrt{a^2-4b}+a\right)}{2+2n}} \operatorname{KummerU}\left(\frac{(2+n)\sqrt{a^2-4b}+na-2c}{\sqrt{a^2-4b}\,(2+2n)}, \frac{2+n}{n+1}, \frac{\sqrt{a^2-4b}\,x^{n+1}}{n+1}\right)$$

✓ Solution by Mathematica

Time used: 0.492 (sec). Leaf size: 333

```
DSolve[y''[x]+a*x^n*y'[x]+(b*x^(2*n)+c*x^(n-1))*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to 2^{\frac{n}{2n+2}} x^{-n/2} \left(x^{n+1}\right)^{\frac{n}{2n+2}} \exp\left(-\frac{1}{2} x^{n+1} \left(\frac{\sqrt{a^2-4b}}{\sqrt{(n+1)^2}} + \frac{a}{n+1}\right)\right) \left(c_1 \operatorname{HypergeometricU}\left(\frac{n\left(\sqrt{(n+1)^2}a^2\right.}{}\right.\right.$$

$$+ c_2 L^{-\frac{1}{n+1}}_{\frac{2\sqrt{a^2-4b}c(n+1)-n\left(\sqrt{(n+1)^2}a^2+\sqrt{a^2-4b}(n+1)a-4b\sqrt{(n+1)^2}\right)}{2\left(a^2-4b\right)(n+1)\sqrt{(n+1)^2}}}\left(\frac{\sqrt{a^2-4b}x^{n+1}}{\sqrt{(n+1)^2}}\right)\Bigg)$$

27.38 problem 48

Internal problem ID [10882]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 48.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + a\,x^n y' - b\left(a\,x^{m+n} + b\,x^{2m} + m\,x^{m-1}\right)y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+a*x^n*diff(y(x),x)-b*(a*x^(n+m)+b*x^(2*m)+m*x^(m-1))*y(x)=0,y(x), sing
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+a*x^n*y'[x]-b*(a*x^(n+m)+b*x^(2*m)+m*x^(m-1))*y[x]==0,y[x],x,IncludeSingularSo
```

Not solved

27.39 problem 49

Internal problem ID [10883]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y''+f(x)y'+g(x)y=0$
Problem number: 49.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y''+2a\,x^n y'+\left(x^{2n}a^2+b\,x^{2m}+x^{-1+n}an+c\,x^{m-1}\right)y=0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 185

```
dsolve(diff(y(x),x$2)+2*a*x^n*diff(y(x),x)+(a^2*x^(2*n)+b*x^(2*m)+a*n*x^(n-1)+c*x^(m-1))*y(x
```

$$y=c_1x\,\mathrm{e}^{\frac{-i\sqrt{b}\,(n+1)x^{1+m}-a\,x^{n+1}(1+m)}{(n+1)(1+m)}}\,\mathrm{KummerM}\left(\frac{(m+2)\sqrt{b}+ic}{\sqrt{b}\,(2+2m)},\frac{m+2}{1+m},\frac{2i\sqrt{b}\,x^{1+m}}{1+m}\right)+c_2x\,\mathrm{e}^{\frac{-i\sqrt{b}\,(n+1)x^{1+m}-a\,x^{n+1}(1+m)}{(n+1)(1+m)}}\,\mathrm{KummerU}\left(\frac{(m+2)\sqrt{b}+ic}{\sqrt{b}\,(2+2m)},\frac{m+2}{1+m},\frac{2i\sqrt{b}\,x^{1+m}}{1+m}\right)$$

✓ Solution by Mathematica

Time used: 0.376 (sec). Leaf size: 236

```
DSolve[y''[x]+2*a*x^n*y'[x]+(a^2*x^(2*n)+b*x^(2*m)+a*n*x^(n-1)+c*x^(m-1))*y[x]==0,y[x],x,Inc
```

$$y(x)\to 2^{\frac{m}{2m+2}}x^{-m/2}\left(x^{m+1}\right)^{\frac{m}{2m+2}}\exp\left(-x\left(\frac{ax^n}{n+1}+\frac{\sqrt{b}x^m}{\sqrt{-(m+1)^2}}\right)\right)\left(c_1\,\mathrm{HypergeometricU}\left(-\frac{(m+1)\left(mc+\right.}{2\sqrt{b}\,(}\right.\right.$$

27.40 problem 50

Internal problem ID [10884]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 50.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + (x^n a + b)\, y' + c(x^n a + b - c)\, y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+(a*x^n+b)*diff(y(x),x)+c*(a*x^n+b-c)*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+(a*x^n+b)*y'[x]+c*(a*x^n+b-c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

27.41 problem 51

Internal problem ID [10885]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 51.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (a\,x^n + 2b)\,y' + \left(ab\,x^n - a\,x^{-1+n} + b^2\right)y = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 147

```
dsolve(diff(y(x),x$2)+(a*x^n+2*b)*diff(y(x),x)+(a*b*x^n-a*x^(n-1)+b^2)*y(x)=0,y(x), singsol=
```

$$y = c_1 x\,\mathrm{e}^{-bx} + c_2 \mathrm{e}^{\frac{-a\,x^{n+1}-2bx(n+1)}{2+2n}}\left((n+1)\left(x^{-\frac{n}{2}}a + x^{-\frac{3n}{2}-1}n\right)\mathrm{WhittakerM}\left(\frac{-2-n}{2+2n}, \frac{1+2n}{2+2n}, \frac{a\,x^{n+1}}{n+1}\right) + x^{-\frac{3n}{2}-1}\,\mathrm{WhittakerM}\left(\frac{n}{2+2n}, \frac{1+2n}{2+2n}, \frac{a\,x^{n+1}}{n+1}\right)n^2\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+(a*x^n+2*b)*y'[x]+(a*b*x^n-a*x^(n-1)+b^2)*y[x]==0,y[x],x,IncludeSingularSoluti
```

Not solved

27.42 problem 52

Internal problem ID [10886]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 52.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(ab\, x^{n} + b\, x^{-1+n} + 2a\right) y' + a^{2}\left(b\, x^{n} + 1\right) y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+(a*b*x^n+b*x^(n-1)+2*a)*diff(y(x),x)+a^2*(b*x^n+1)*y(x)=0,y(x), singso
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+(a*b*x^n+b*x^(n-1)+2*a)*y'[x]+a^2*(b*x^n+1)*y[x]==0,y[x],x,IncludeSingularSolu
```

Not solved

27.43 problem 53

Internal problem ID [10887]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 53.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(ab\,x^{n} + 2b\,x^{-1+n} - a^{2}x\right)y' + a\left(ab\,x^{n} + b\,x^{-1+n} - a^{2}x\right)y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+(a*b*x^n+2*b*x^(n-1)-a^2*x)*diff(y(x),x)+a*(a*b*x^n+b*x^(n-1)-a^2*x)*y
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+(a*b*x^n+2*b*x^(n-1)-a^2*x)*y'[x]+a*(a*b*x^n+b*x^(n-1)-a^2*x)*y[x]==0,y[x],x,I
```

Not solved

27.44 problem 54

Internal problem ID [10888]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 54.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + x^n\left(a\,x^2 + (ac+b)\,x + bc\right)y' - x^n(ax+b)\,y = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 131

```
dsolve(diff(y(x),x$2)+x^n*(a*x^2+(a*c+b)*x+b*c)*diff(y(x),x)-x^n*(a*x+b)*y(x)=0,y(x), singso
```

$$y = -c_1\left(\int e^{-\frac{a\,x^3x^n}{n+3}-\frac{(ac+b)x^2x^n}{2+n}-\frac{bcx\,x^n}{n+1}-2\ln(c+x)}dx\right)x - c_1\left(\int e^{-\frac{a\,x^3x^n}{n+3}-\frac{(ac+b)x^2x^n}{2+n}-\frac{bcx\,x^n}{n+1}-2\ln(c+x)}dx\right)c - c_2x - c_2c$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+x^n*(a*x^2+(a*c+b)*x+b*c)*y'[x]-x^n*(a*x+b)*y[x]==0,y[x],x,IncludeSingularSolu
```

Not solved

27.45 problem 55

Internal problem ID [10889]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 55.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type **[[_2nd_order, _with_linear_symmetries]]**

$$y'' + (x^n a + b\,x^m)\,y' - \left(a\,x^{-1+n} + x^{m-1} b\right) y = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)+(a*x^n+b*x^m)*diff(y(x),x)-(a*x^(n-1)+b*x^(m-1))*y(x)=0,y(x), singsol=
```

$$y = c_1 x + c_2 x \left(\int \frac{e^{-\frac{x(x^n a m + x^m b n + a\,x^n + x^m b)}{(n+1)(1+m)}}}{x^2} dx \right)$$

✓ Solution by Mathematica

Time used: 1.216 (sec). Leaf size: 55

```
DSolve[y''[x]+(a*x^n+b*x^m)*y'[x]-(a*x^(n-1)+b*x^(m-1))*y[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to x \left(c_2 \int_1^x \frac{\exp\left(K[1]\left(-\frac{bK[1]^m}{m+1} - \frac{aK[1]^n}{n+1}\right)\right)}{K[1]^2} dK[1] + c_1 \right)$$

27.46 problem 56

Internal problem ID [10890]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 56.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{y'' + (x^n a + b\,x^m)\,y' + \left(an\,x^{-1+n} + bm\,x^{m-1}\right) y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 65

```
dsolve(diff(y(x),x$2)+(a*x^n+b*x^m)*diff(y(x),x)+(a*n*x^(n-1)+b*m*x^(m-1))*y(x)=0,y(x), sing
```

$$y = \left(c_1 \left(\int \mathrm{e}^{\frac{a\,x^{n+1}}{n+1}+\frac{b\,x^{1+m}}{1+m}} dx\right) + c_2\right) \mathrm{e}^{-\frac{a\,x^{n+1}}{n+1}-\frac{b\,x^{1+m}}{1+m}}$$

✓ Solution by Mathematica

Time used: 0.139 (sec). Leaf size: 74

```
DSolve[y''[x]+(a*x^n+b*x^m)*y'[x]+(a*n*x^(n-1)+b*m*x^(m-1))*y[x]==0,y[x],x,IncludeSingularSo
```

$$y(x) \to e^{x\left(-\frac{ax^n}{n+1}-\frac{bx^m}{m+1}\right)} \left(\int_1^x \exp\left(K[1]\left(\frac{bK[1]^m}{m+1} + \frac{aK[1]^n}{n+1}\right)\right) c_1 dK[1] + c_2\right)$$

27.47 problem 57

Internal problem ID [10891]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 57.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + (x^n a + b\,x^m)\,y' + \left(a(n+1)\,x^{-1+n} + b(m+1)\,x^{m-1}\right)y = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 148

```
dsolve(diff(y(x),x$2)+(a*x^n+b*x^m)*diff(y(x),x)+(a*(n+1)*x^(n-1)+b*(m+1)*x^(m-1))*y(x)=0,y(
```

$$y = c_1 \mathrm{e}^{-\frac{am\,x^{n+1}+bn\,x^{1+m}+a\,x^{n+1}+b\,x^{1+m}}{(n+1)(1+m)}} x + c_2 \mathrm{e}^{-\frac{am\,x^{n+1}+bn\,x^{1+m}+a\,x^{n+1}+b\,x^{1+m}}{(n+1)(1+m)}} \left(\int \frac{\mathrm{e}^{\frac{am\,x^{n+1}+bn\,x^{1+m}+a\,x^{n+1}+b\,x^{1+m}}{(n+1)(1+m)}}}{x^2} dx\right) x$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+(a*x^n+b*x^m)*y'[x]+(a*(n+1)*x^(n-1)+b*(m+1)*x^(m-1))*y[x]==0,y[x],x,IncludeSi
```

Not solved

27.48 problem 58

Internal problem ID [10892]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 58.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + (x^n a + b\,x^m)\,y' + c(x^n a + b\,x^m - c)\,y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+(a*x^n+b*x^m)*diff(y(x),x)+c*(a*x^n+b*x^m-c)*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+(a*x^n+b*x^m)*y'[x]+c*(a*x^n+b*x^m-c)*y[x]==0,y[x],x,IncludeSingularSolutions
```

Not solved

27.49 problem 59

Internal problem ID [10893]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 59.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + (x^n a + b\,x^m)\,y' + \left(ab\,x^{m+n} + b(m+1)\,x^{m-1} - a\,x^{-1+n}\right)y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+(a*x^n+b*x^m)*diff(y(x),x)+(a*b*x^(n+m)+b*(m+1)*x^(m-1)-a*x^(n-1))*y(x
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+(a*x^n+b*x^m)*y'[x]+(a*b*x^(n+m)+b*(m+1)*x^(m-1)-a*x^(n-1))*y[x]==0,y[x],x,Inc
```

Not solved

27.50 problem 60

Internal problem ID [10894]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 60.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + (x^n a + b\,x^m + c)\,y' + \left(ab\,x^{m+n} + bc\,x^m + an\,x^{-1+n}\right) y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+(a*x^n+b*x^m+c)*diff(y(x),x)+(a*b*x^(n+m)+b*c*x^m+a*n*x^(n-1))*y(x)=0,
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+(a*x^n+b*x^m+c)*y'[x]+(a*b*x^(n+m)+b*c*x^m+a*n*x^(n-1))*y[x]==0,y[x],x,Include
```

Not solved

28 Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$

28.1 problem 61

Internal problem ID [10895]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 61.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{xy''+\frac{y'}{2}+ay=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x$2)+1/2*diff(y(x),x)+a*y(x)=0,y(x), singsol=all)
```

$$y=c_1\sin\left(2\sqrt{a}\,\sqrt{x}\right)+c_2\cos\left(2\sqrt{a}\,\sqrt{x}\right)$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 38

```
DSolve[x*y''[x]+1/2*y'[x]+a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\cos\left(2\sqrt{a}\sqrt{x}\right)+c_2\sin\left(2\sqrt{a}\sqrt{x}\right)$$

28.2 problem 62

Internal problem ID [10896]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 62.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy''+ay'+by=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(x*diff(y(x),x$2)+a*diff(y(x),x)+b*y(x)=0,y(x), singsol=all)
```

$$y=c_1x^{\frac{1}{2}-\frac{a}{2}}\operatorname{BesselJ}\left(a-1,2\sqrt{b}\,\sqrt{x}\right)+c_2x^{\frac{1}{2}-\frac{a}{2}}\operatorname{BesselY}\left(a-1,2\sqrt{b}\,\sqrt{x}\right)$$

✓ Solution by Mathematica

Time used: 0.111 (sec). Leaf size: 77

```
DSolve[x*y''[x]+a*y'[x]+b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\rightarrow b^{\frac{1}{2}-\frac{a}{2}}x^{\frac{1}{2}-\frac{a}{2}}\left(c_2\operatorname{Gamma}(2-a)\operatorname{BesselJ}\left(1-a,2\sqrt{b}\sqrt{x}\right)\right.$$
$$\left.+c_1\operatorname{Gamma}(a)\operatorname{BesselJ}\left(a-1,2\sqrt{b}\sqrt{x}\right)\right)$$

28.3 problem 63

Internal problem ID [10897]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 63.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+ay'+ybx=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(x*diff(y(x),x$2)+a*diff(y(x),x)+b*x*y(x)=0,y(x), singsol=all)
```

$$y=c_1x^{\frac{1}{2}-\frac{a}{2}}\operatorname{BesselJ}\left(\frac{a}{2}-\frac{1}{2},\sqrt{b}\,x\right)+c_2x^{\frac{1}{2}-\frac{a}{2}}\operatorname{BesselY}\left(\frac{a}{2}-\frac{1}{2},\sqrt{b}\,x\right)$$

✓ Solution by Mathematica

Time used: 0.057 (sec). Leaf size: 54

```
DSolve[x*y''[x]+a*y'[x]+b*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x^{\frac{1}{2}-\frac{a}{2}}\left(c_1\operatorname{BesselJ}\left(\frac{a-1}{2},\sqrt{b}x\right)+c_2\operatorname{BesselY}\left(\frac{a-1}{2},\sqrt{b}x\right)\right)$$

28.4 problem 64

Internal problem ID [10898]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 64.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+ay'+(bx+c)\,y=0}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 73

```
dsolve(x*diff(y(x),x$2)+a*diff(y(x),x)+(b*x+c)*y(x)=0,y(x), singsol=all)
```

$$y = c_1 \mathrm{e}^{-i\sqrt{b}\,x}\,\mathrm{KummerM}\left(\frac{ic+a\sqrt{b}}{2\sqrt{b}}, a, 2i\sqrt{b}\,x\right) + c_2 \mathrm{e}^{-i\sqrt{b}\,x}\,\mathrm{KummerU}\left(\frac{ic+a\sqrt{b}}{2\sqrt{b}}, a, 2i\sqrt{b}\,x\right)$$

✓ Solution by Mathematica

Time used: 0.099 (sec). Leaf size: 85

```
DSolve[x*y''[x]+a*y'[x]+(b*x+c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{-i\sqrt{b}x}\left(c_1\,\mathrm{HypergeometricU}\left(\frac{1}{2}\left(a+\frac{ic}{\sqrt{b}}\right), a, 2i\sqrt{b}x\right)+c_2 L_{-\frac{a}{2}-\frac{ic}{2\sqrt{b}}}^{a-1}\left(2i\sqrt{b}x\right)\right)$$

28.5 problem 65

Internal problem ID [10899]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 65.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{xy''+ny'+b\,x^{-2n+1}y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(x*diff(y(x),x$2)+n*diff(y(x),x)+b*x^(1-2*n)*y(x)=0,y(x), singsol=all)
```

$$y=c_1\sin\left(\frac{x^{1-n}\sqrt{b}}{n-1}\right)+c_2\cos\left(\frac{x^{1-n}\sqrt{b}}{n-1}\right)$$

✓ Solution by Mathematica

Time used: 0.072 (sec). Leaf size: 52

```
DSolve[x*y''[x]+n*y'[x]+b*x^(1-2*n)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\cos\left(\frac{\sqrt{b}x^{1-n}}{n-1}\right)+c_2\sin\left(\frac{\sqrt{b}x^{1-n}}{1-n}\right)$$

28.6 problem 66

Internal problem ID [10900]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 66.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$xy''+(1-3n)\,y'-a^2n^2x^{2n-1}y=0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 61

```
dsolve(x*diff(y(x),x$2)+(1-3*n)*diff(y(x),x)-a^2*n^2*x^(2*n-1)*y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^{a\,x^n}\left(-a\,x^n+x^{-n}\sqrt{x^{2n}}\right)+c_2 e^{-a\,x^n}\left(a\,x^n+x^{-n}\sqrt{x^{2n}}\right)$$

✓ Solution by Mathematica

Time used: 0.196 (sec). Leaf size: 77

```
DSolve[x*y''[x]+(1-3*n)*y'[x]-a^2*n^2*x^(2*n-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\to\left(c_1-\frac{3}{8}iac_2\sqrt{x^{2n}}\right)\cosh\left(a\sqrt{x^{2n}}\right)+\frac{1}{8}\left(3ic_2-8ac_1\sqrt{x^{2n}}\right)\sinh\left(a\sqrt{x^{2n}}\right)$$

28.7 problem 67

Internal problem ID [10901]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 67.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy''+ay'+yx^n b=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 77

```
dsolve(x*diff(y(x),x$2)+a*diff(y(x),x)+b*x^n*y(x)=0,y(x), singsol=all)
```

$$y=c_1 x^{\frac{1}{2}-\frac{a}{2}} \operatorname{BesselJ}\left(\frac{a-1}{n+1}, \frac{2\sqrt{b}\, x^{\frac{n}{2}+\frac{1}{2}}}{n+1}\right)+c_2 x^{\frac{1}{2}-\frac{a}{2}} \operatorname{BesselY}\left(\frac{a-1}{n+1}, \frac{2\sqrt{b}\, x^{\frac{n}{2}+\frac{1}{2}}}{n+1}\right)$$

✓ Solution by Mathematica

Time used: 0.245 (sec). Leaf size: 165

```
DSolve[x*y''[x]+a*y'[x]+b*x^n*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\left(\frac{1}{n}+1\right)^{\frac{a-1}{n+1}} n^{\frac{a-1}{n+1}} b^{\frac{1-a}{2n+2}} (x^n)^{-\frac{a-1}{2n}}\left(c_2 \operatorname{Gamma}\left(\frac{-a+n+2}{n+1}\right) \operatorname{BesselJ}\left(\frac{1-a}{n+1}, \frac{2\sqrt{b}(x^n)^{\frac{n+1}{2n}}}{n+1}\right)+c_1 \operatorname{Gamma}\left(\frac{a+n}{n+1}\right) \operatorname{BesselJ}\left(\frac{a-1}{n+1}, \frac{2\sqrt{b}(x^n)^{\frac{n+1}{2n}}}{n+1}\right)\right)$$

28.8 problem 68

Internal problem ID [10902]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 68.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy''+ay'+b\,x^n\left(-x^{n+1}b+a+n\right)y=0$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 167

```
dsolve(x*diff(y(x),x$2)+a*diff(y(x),x)+b*x^n*(-b*x^(n+1)+a+n)*y(x)=0,y(x), singsol=all)
```

$$y=c_1\mathrm{e}^{-\frac{b\,x^{n+1}}{n+1}}+c_2\left(-(n+1)\left((a-2-n)\,x^{-\frac{3n}{2}-\frac{a}{2}-1}+2b\,x^{-\frac{n}{2}-\frac{a}{2}}\right)\mathrm{WhittakerM}\left(\frac{-a-n}{2+2n},\frac{-a+3+2n}{2+2n},-\frac{2b\,x^{n+1}}{n+1}\right)+x^{-\frac{3n}{2}-\frac{a}{2}-1}\,\mathrm{WhittakerM}\left(\frac{-a+2+n}{2+2n},\frac{-a+3+2n}{2+2n},-\frac{2b\,x^{n+1}}{n+1}\right)(a-2-n)^2\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]+a*y'[x]+b*x^n*(-b*x^(n+1)+a+n)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

Not solved

28.9 problem 69

Internal problem ID [10903]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 69.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{xy''+axy'+ay=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(x*diff(y(x),x$2)+a*x*diff(y(x),x)+a*y(x)=0,y(x), singsol=all)
```

$$y=\left(\mathrm{Ei}_1\left(-ax\right)ax+\mathrm{e}^{ax}\right)\mathrm{e}^{-ax}c_1+\mathrm{e}^{-ax}c_2x$$

✓ Solution by Mathematica

Time used: 0.177 (sec). Leaf size: 35

```
DSolve[x*y''[x]+a*x*y'[x]+a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{-ax}(ac_2x\,\mathrm{ExpIntegralEi}(ax)-c_2e^{ax}+c_1x)$$

28.10 problem 70

Internal problem ID [10904]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 70.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{xy''+(b-x)\,y'-ya=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x$2)+(b-x)*diff(y(x),x)-a*y(x)=0,y(x), singsol=all)
```

$$y=c_1\,\mathrm{KummerM}\,(a,b,x)+c_2\,\mathrm{KummerU}\,(a,b,x)$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 24

```
DSolve[x*y''[x]+(b-x)*y'[x]-a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\,\mathrm{HypergeometricU}(a,b,x)+c_2L_{-a}^{b-1}(x)$$

28.11 problem 71

Internal problem ID [10905]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 71.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' + (ax+b)\,y' + c((-c+a)\,x+b)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(x*diff(y(x),x$2)+(a*x+b)*diff(y(x),x)+c*((a-c)*x+b)*y(x)=0,y(x), singsol=all)
```

$$y = c_1 \mathrm{e}^{-cx} + c_2 \,\mathrm{WhittakerM}\left(-\frac{b}{2}, -\frac{b}{2}+\frac{1}{2}, (a-2c)\,x\right) x^{-\frac{b}{2}} \mathrm{e}^{-\frac{ax}{2}}$$

✓ Solution by Mathematica

Time used: 0.333 (sec). Leaf size: 50

```
DSolve[x*y''[x]+(a*x+b)*y'[x]+c*((a-c)*x+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-cx}\left(c_1 - c_2 x^{1-b}(x(a-2c))^{b-1}\Gamma(1-b,(a-2c)x)\right)$$

28.12 problem 72

Internal problem ID [10906]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 72.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+(2ax+b)\,y'+a(ax+b)\,y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 26

```
dsolve(x*diff(y(x),x$2)+(2*a*x+b)*diff(y(x),x)+a*(a*x+b)*y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^{-ax} + x^{-b+1} c_2 e^{-ax}$$

✓ Solution by Mathematica

Time used: 0.233 (sec). Leaf size: 70

```
DSolve[x*y''[x]+(2*a*x+b)*y'[x]+a*(a*x+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ax} x^{\frac{1}{2}\left(-b-\sqrt{(b-1)^2}+1\right)} \left(c_2 x^{\sqrt{(b-1)^2}} + \sqrt{(b-1)^2} c_1\right)}{\sqrt{(b-1)^2}}$$

28.13 problem 73

Internal problem ID [10907]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 73.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy''+((a+b)x+m+n)y'+(abx+an+bm)y=0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 43

```
dsolve(x*diff(y(x),x$2)+((a+b)*x+n+m)*diff(y(x),x)+(a*b*x+a*n+b*m)*y(x)=0,y(x), singsol=all)
```

$$y=c_1\mathrm{e}^{-ax}\,\mathrm{KummerM}\left(m,n+m,(a-b)\,x\right)+c_2\mathrm{e}^{-ax}\,\mathrm{KummerU}\left(m,n+m,(a-b)\,x\right)$$

✓ Solution by Mathematica

Time used: 0.131 (sec). Leaf size: 46

```
DSolve[x*y''[x]+((a+b)*x+n+m)*y'[x]+(a*b*x+a*n+b*m)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to e^{-ax}(c_1\,\text{HypergeometricU}(m,m+n,(a-b)x)+c_2L_{-m}^{m+n-1}((a-b)x))$$

28.14 problem 74

Internal problem ID [10908]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 74.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy''+(ax+b)\,y'+(xc+d)\,y=0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 123

```
dsolve(x*diff(y(x),x$2)+(a*x+b)*diff(y(x),x)+(c*x+d)*y(x)=0,y(x), singsol=all)
```

$$y=c_1\mathrm{e}^{-\frac{x\left(a+\sqrt{a^2-4c}\right)}{2}}\operatorname{KummerM}\left(\frac{b\sqrt{a^2-4c}+ab-2d}{2\sqrt{a^2-4c}},b,\sqrt{a^2-4c}\,x\right)+c_2\mathrm{e}^{-\frac{x\left(a+\sqrt{a^2-4c}\right)}{2}}\operatorname{KummerU}\left(\frac{b\sqrt{a^2-4c}+ab-2d}{2\sqrt{a^2-4c}},b,\sqrt{a^2-4c}\,x\right)$$

✓ Solution by Mathematica

Time used: 0.135 (sec). Leaf size: 135

```
DSolve[x*y''[x]+(a*x+b)*y'[x]+(c*x+d)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{-\frac{1}{2}x\left(\sqrt{a^2-4c}+a\right)}\left(c_1\operatorname{HypergeometricU}\left(\frac{ab+\sqrt{a^2-4c}b-2d}{2\sqrt{a^2-4c}},b,\sqrt{a^2-4c}x\right)+c_2L^{b-1}_{-\frac{ab+\sqrt{a^2-4c}b-2d}{2\sqrt{a^2-4c}}}\left(\sqrt{a^2-4c}x\right)\right)$$

28.15 problem 75

Internal problem ID [10909]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 75.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' - (ax+1)\,y' - b\,x^2(bx+a)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 73

```
dsolve(x*diff(y(x),x$2)-(a*x+1)*diff(y(x),x)-b*x^2*(b*x+a)*y(x)=0,y(x), singsol=all)
```

$$y = c_1 \mathrm{e}^{-\frac{x^2 b}{2}} + c_2 \left(a\pi \,\mathrm{erf}\left(\frac{2bx+a}{2\sqrt{-b}}\right) \mathrm{e}^{-\frac{2b^2x^2+a^2}{4b}} + 2\sqrt{-b}\,\sqrt{\pi}\,\mathrm{e}^{\frac{1}{2}x^2b+ax}\right)$$

✓ Solution by Mathematica

Time used: 0.444 (sec). Leaf size: 88

```
DSolve[x*y''[x]-(a*x+1)*y'[x]-b*x^2*(b*x+a)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{bx^2}{2}}\left(2\sqrt{b}\left(c_2 e^{x(a+bx)} + 2bc_1\right) - \sqrt{\pi} a c_2 e^{-\frac{a^2}{4b}} \operatorname{erfi}\left(\frac{a+2bx}{2\sqrt{b}}\right)\right)}{4b^{3/2}}$$

28.16 problem 76

Internal problem ID [10910]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 76.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''-(2ax+1)\,y'+\left(b\,x^3+a^2x+a\right)y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(x*diff(y(x),x$2)-(2*a*x+1)*diff(y(x),x)+(b*x^3+a^2*x+a)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\mathrm{e}^{ax+\frac{x^2\sqrt{-b}}{2}}+c_2\mathrm{e}^{ax-\frac{x^2\sqrt{-b}}{2}}$$

✓ Solution by Mathematica

Time used: 0.276 (sec). Leaf size: 59

```
DSolve[x*y''[x]-(2*a*x+1)*y'[x]+(b*x^3+a^2*x+a)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\rightarrow\frac{1}{2}e^{ax-\frac{1}{2}i\sqrt{b}x^2}\left(2c_1-\frac{ic_2e^{i\sqrt{b}x^2}}{\sqrt{b}}\right)$$

28.17 problem 77

Internal problem ID [10911]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 77.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$xy''+y'(ax+b)=-cx\left(-c\,x^2+ax+b+1\right)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 180

```
dsolve(x*diff(y(x),x$2)+(a*x+b)*diff(y(x),x)+c*x*(-c*x^2+a*x+b+1)=0,y(x), singsol=all)
```

$$y(x)=\int -\frac{-c^2\mathrm{e}^{-ax}(-ax)^{-b}\,\Gamma(b,-ax)\,b^3-3c^2\mathrm{e}^{-ax}(-ax)^{-b}\,\Gamma(b,-ax)\,b^2-2c^2\mathrm{e}^{-ax}(-ax)^{-b}\,\Gamma(b,-ax)\,b-\mathrm{e}^{-ax}x^-}{a^3} + c_2$$

✓ Solution by Mathematica

Time used: 61.322 (sec). Leaf size: 92

```
DSolve[x*y''[x]+(a*x+b)*y'[x]+c*x*(-c*x^2+a*x+b+1)==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to\int_1^x e^{-aK[1]}K[1]^{-b}\left(\frac{c(-((b+1)\Gamma(b+1,-aK[1])a^2)+\Gamma(b+2,-aK[1])a^2+c\Gamma(b+3,-aK[1]))\,K[1]^b(}{a^3} + c_1\right)dK[1]+c_2$$

28.18 problem 78

Internal problem ID [10912]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 78.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''-(2ax+1)\,y'+yb\,x^3=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 154

```
dsolve(x*diff(y(x),x$2)-(2*a*x+1)*diff(y(x),x)+b*x^3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^2\,\mathrm{HeunB}\left(2,0,\frac{a^2}{\sqrt{-b}},-\frac{2ia}{(-b)^{\frac{1}{4}}},i(-b)^{\frac{1}{4}}\,x\right)\mathrm{e}^{ax+\frac{x^2\sqrt{-b}}{2}}+c_2x^2\,\mathrm{HeunB}\left(2,0,\frac{a^2}{\sqrt{-b}},-\frac{2ia}{(-b)^{\frac{1}{4}}},i(-b)^{\frac{1}{4}}\,x\right)\mathrm{e}^{ax+\frac{x^2\sqrt{-b}}{2}}\left(\int\frac{\mathrm{e}^{-x^2\sqrt{-b}}}{\mathrm{HeunB}\left(2,0,\frac{a^2}{\sqrt{-b}},-\frac{2ia}{(-b)^{\frac{1}{4}}},i\,(-b)^{\frac{1}{4}}\,x\right)^2x^3}dx\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]-(2*a*x+1)*y'[x]+b*x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

28.19 problem 79

Internal problem ID [10913]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 79.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+\left(ab\,x^2+b-5\right)y'+2a^2(-2+b)\,x^3y=0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 69

```
dsolve(x*diff(y(x),x$2)+(a*b*x^2+b-5)*diff(y(x),x)+2*a^2*(b-2)*x^3*y(x)=0,y(x), singsol=all)
```

$$\begin{aligned}y(x) = {} & c_1 \mathrm{e}^{-\frac{a(b-2)x^2}{2}} \operatorname{KummerM}\left(-1+\frac{b}{2}, -2+\frac{b}{2}, \frac{a(b-4)\,x^2}{2}\right) \\ & + c_2 \mathrm{e}^{-\frac{a(b-2)x^2}{2}} \operatorname{KummerU}\left(-1+\frac{b}{2}, -2+\frac{b}{2}, \frac{a(b-4)\,x^2}{2}\right)\end{aligned}$$

✓ Solution by Mathematica

Time used: 3.578 (sec). Leaf size: 67

```
DSolve[x*y''[x]+(a*b*x^2+b-5)*y'[x]+2*a^2*(b-2)*x^3*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to e^{-ax^2}\left(ax^2+1\right)\left(c_2 \int_1^x \frac{e^{-\frac{1}{2}a(b-4)K[1]^2}K[1]^{5-b}}{\left(aK[1]^2+1\right)^2}dK[1]+c_1\right)$$

28.20 problem 80

Internal problem ID [10914]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 80.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy''+\left(a\,x^2+bx\right)y'-\left(ac\,x^2+\left(bc+c^2+a\right)x+b+2c\right)y=0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 39

```
dsolve(x*diff(y(x),x$2)+(a*x^2+b*x)*diff(y(x),x)-(a*c*x^2+(a+b*c+c^2)*x+b+2*c)*y(x)=0,y(x),
```

$$y(x)=c_1\mathrm{e}^{cx}x+c_2\mathrm{e}^{cx}x\left(\int\frac{\mathrm{e}^{-\frac{x(ax+2b+4c)}{2}}}{x^2}dx\right)$$

✓ Solution by Mathematica

Time used: 3.129 (sec). Leaf size: 49

```
DSolve[x*y''[x]+(a*x^2+b*x)*y'[x]-(a*c*x^2+(a+b*c+c^2)*x+b+2*c)*y[x]==0,y[x],x,IncludeSingul
```

$$y(x)\to xe^{cx}\left(c_2\int_1^x\frac{e^{-\frac{1}{2}K[1](2b+4c+aK[1])}}{K[1]^2}dK[1]+c_1\right)$$

28.21 problem 81

Internal problem ID [10915]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 81.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+\left(a\,x^{2}+bx+2\right)y'+by=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 72

```
dsolve(x*diff(y(x),x$2)+(a*x^2+b*x+2)*diff(y(x),x)+b*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(ax+b)}{x}+\frac{c_2\left(\pi(ax+b)\,\mathrm{e}^{\frac{b^2}{2a}}\,\mathrm{erf}\left(\frac{\sqrt{2}\,(ax+b)}{2\sqrt{a}}\right)+\mathrm{e}^{-\frac{x(ax+2b)}{2}}\sqrt{2}\,\sqrt{\pi}\,\sqrt{a}\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.535 (sec). Leaf size: 85

```
DSolve[x*y''[x]+(a*x^2+b*x+2)*y'[x]+b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{(ax+b)\left(-\frac{\sqrt{\frac{\pi}{2}}c_2\mathrm{erf}\left(\frac{ax+b}{\sqrt{2}\sqrt{a}}\right)}{a^{3/2}}-\frac{c_2e^{-\frac{(ax+b)^2}{2a}}}{a(ax+b)}+c_1\right)}{bx}$$

28.22 problem 82

Internal problem ID [10916]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 82.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{xy''+\left(a\,x^2+bx+c\right)y'+(2ax+b)\,y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 50

```
dsolve(x*diff(y(x),x$2)+(a*x^2+b*x+c)*diff(y(x),x)+(2*a*x+b)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\left(c_1\left(\int\frac{x^c\,\mathrm{e}^{\frac{a\,x^2}{2}}\,\mathrm{e}^{bx}}{x^2}dx\right)+c_2\right)\mathrm{e}^{-\frac{a\,x^2}{2}}\,\mathrm{e}^{-bx}x^{-c}x$$

✓ Solution by Mathematica

Time used: 1.772 (sec). Leaf size: 63

```
DSolve[x*y''[x]+(a*x^2+b*x+c)*y'[x]+(2*a*x+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to x^{1-c}e^{-\frac{1}{2}x(ax+2b)}\left(c_2\int_1^x e^{\frac{1}{2}aK[1]^2+bK[1]}K[1]^{c-2}dK[1]+c_1\right)$$

28.23 problem 83

Internal problem ID [10917]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 83.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+\left(a\,x^{2}+bx+c\right)y'+(c-1)\,(ax+b)\,y=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 112

```
dsolve(x*diff(y(x),x$2)+(a*x^2+b*x+c)*diff(y(x),x)+(c-1)*(a*x+b)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\mathrm{e}^{-\frac{x(ax+2b)}{2}}\,\mathrm{HeunB}\left(c-1,\frac{b\sqrt{2}}{\sqrt{a}},c-3,-\frac{\sqrt{2}\,b(c-2)}{\sqrt{a}},\frac{\sqrt{2}\,\sqrt{a}\,x}{2}\right)$$
$$+c_2x^{-c+1}\mathrm{e}^{-\frac{x(ax+2b)}{2}}\,\mathrm{HeunB}\left(-c+1,\frac{b\sqrt{2}}{\sqrt{a}},c-3,-\frac{\sqrt{2}\,b(c-2)}{\sqrt{a}},\frac{\sqrt{2}\,\sqrt{a}\,x}{2}\right)$$

✓ Solution by Mathematica

Time used: 1.579 (sec). Leaf size: 49

```
DSolve[x*y''[x]+(a*x^2+b*x+c)*y'[x]+(c-1)*(a*x+b)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to x^{1-c}\left(c_2\int_1^x e^{-\frac{1}{2}K[1](2b+aK[1])}K[1]^{c-2}dK[1]+c_1\right)$$

28.24 problem 84

Internal problem ID [10918]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 84.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy''+\left(a\,x^2+bx+c\right)y'+\left(A\,x^2+Bx+\mathrm{C0}\right)y=0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 206

```
dsolve(x*diff(y(x),x$2)+(a*x^2+b*x+c)*diff(y(x),x)+(A*x^2+B*x+C0)*y(x)=0,y(x), singsol=all)
```

$$\begin{aligned}y(x) &= c_1\mathrm{e}^{\frac{x\left(-a^2x-2ab+2A\right)}{2a}}\,\mathrm{HeunB}\left(c-1,\right.\\&\left.-\frac{\sqrt{2}\,(-ab+2A)}{a^{\frac{3}{2}}},\frac{(-c-1)\,a^3+2B\,a^2-2Aab+2A^2}{a^3},\frac{(cb-2\,\mathrm{C0})\sqrt{2}}{\sqrt{a}},\frac{\sqrt{2}\,\sqrt{a}\,x}{2}\right)\\&+c_2x^{-c+1}\mathrm{e}^{\frac{x\left(-a^2x-2ab+2A\right)}{2a}}\,\mathrm{HeunB}\left(-c+1,\right.\\&\left.-\frac{\sqrt{2}\,(-ab+2A)}{a^{\frac{3}{2}}},\frac{(-c-1)\,a^3+2B\,a^2-2Aab+2A^2}{a^3},\frac{(cb-2\,\mathrm{C0})\sqrt{2}}{\sqrt{a}},\frac{\sqrt{2}\,\sqrt{a}\,x}{2}\right)\end{aligned}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]+(a*x^2+b*x+c)*y'[x]+(A*x^2+B*x+C0)*y[x]==0,y[x],x,IncludeSingularSolutions -
```

Not solved

28.25 problem 85

Internal problem ID [10919]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 85.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy''+\left(a\,x^2+bx+2\right)y'+\left(c\,x^2+dx+b\right)y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 165

```
dsolve(x*diff(y(x),x$2)+(a*x^2+b*x+2)*diff(y(x),x)+(c*x^2+d*x+b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1\,\mathrm{hypergeom}\left(\left[\frac{2a^3-d\,a^2+abc-c^2}{2a^3}\right],\left[\frac{1}{2}\right],\frac{(a^2x+ab-2c)^2}{2a^3}\right)\mathrm{e}^{-\frac{x\left(a^2x+2ab-2c\right)}{2a}}}{x} + \frac{c_2(a^2x+ab-2c)\,\mathrm{hypergeom}\left(\left[\frac{3a^3-d\,a^2+abc-c^2}{2a^3}\right],\left[\frac{3}{2}\right],\frac{(a^2x+ab-2c)^2}{2a^3}\right)\mathrm{e}^{-\frac{x\left(a^2x+2ab-2c\right)}{2a}}}{x}$$

✓ Solution by Mathematica

Time used: 0.155 (sec). Leaf size: 134

```
DSolve[x*y''[x]+(a*x^2+b*x+2)*y'[x]+(c*x^2+d*x+b)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{e^{-\frac{1}{2}x\left(-\frac{2c}{a}+ax+2b\right)}\left(c_2\,\text{Hypergeometric1F1}\left(-\frac{-2a^3+da^2-bca+c^2}{2a^3},\frac{1}{2},\frac{(xa^2+ba-2c)^2}{2a^3}\right)+c_1\,\text{HermiteH}\left(\frac{-2a^3+da^2-bc}{a^3}\right.\right.}{x}$$

28.26 problem 86

Internal problem ID [10920]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 86.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy''+\left(a\,x^3+b\right)y'+a(b-1)\,x^2y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 84

```
dsolve(x*diff(y(x),x$2)+(a*x^3+b)*diff(y(x),x)+a*(b-1)*x^2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x^{-b+1}+c_2\mathrm{e}^{-\frac{a\,x^3}{6}}\left(\left(b+5\right)\left(a\,x^3+b+2\right)\mathrm{WhittakerM}\left(\frac{4}{3}+\frac{b}{6},\frac{b}{6}+\frac{5}{6},\frac{a\,x^3}{3}\right)+a^2\,\mathrm{WhittakerM}\left(\frac{1}{3}+\frac{b}{6},\frac{b}{6}+\frac{5}{6},\frac{a\,x^3}{3}\right)x^6\right)x^{-\frac{b}{2}-4}$$

✓ Solution by Mathematica

Time used: 0.424 (sec). Leaf size: 60

```
DSolve[x*y''[x]+(a*x^3+b)*y'[x]+a*(b-1)*x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1x^{1-b}-3^{\frac{b-4}{3}}c_2\left(ax^3\right)^{\frac{1}{3}-\frac{b}{3}}\Gamma\left(\frac{b-1}{3},\frac{ax^3}{3}\right)$$

28.27 problem 87

Internal problem ID [10921]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 87.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{xy''+x\left(a\,x^{2}+b\right)y'+\left(3a\,x^{2}+b\right)y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve(x*diff(y(x),x$2)+x*(a*x^2+b)*diff(y(x),x)+(3*a*x^2+b)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\left(c_{1}\left(\int\frac{\mathrm{e}^{\frac{a\,x^{3}}{3}}\mathrm{e}^{bx}}{x^{2}}dx\right)+c_{2}\right)\mathrm{e}^{-\frac{a\,x^{3}}{3}}\mathrm{e}^{-bx}x$$

✓ Solution by Mathematica

Time used: 2.43 (sec). Leaf size: 56

```
DSolve[x*y''[x]+x*(a*x^2+b)*y'[x]+(3*a*x^2+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to xe^{-\frac{ax^{3}}{3}-bx}\left(c_{2}\int_{1}^{x}\frac{e^{\frac{1}{3}aK[1]^{3}+bK[1]}}{K[1]^{2}}dK[1]+c_{1}\right)$$

28.28 problem 88

Internal problem ID [10922]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 88.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy''+\left(a\,x^{3}+b\,x^{2}+2\right)y'+ybx=0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 185

```
dsolve(x*diff(y(x),x$2)+(a*x^3+b*x^2+2)*diff(y(x),x)+b*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1 \mathrm{e}^{-\frac{\left(\sqrt{a^2}+a\right)\sqrt{a^2}\,x^2(2ax+3b)}{12a^2}}\operatorname{HeunT}\left(\frac{3^{\frac{2}{3}}b\,a^2}{2(a^2)^{\frac{4}{3}}},-\frac{6\sqrt{a^2}}{a},-\frac{b^2 3^{\frac{1}{3}}}{4(a^2)^{\frac{2}{3}}},\frac{3^{\frac{2}{3}}a(2ax+b)}{6(a^2)^{\frac{5}{6}}}\right)}{x}+\frac{c_2 \mathrm{e}^{\frac{\left(-\sqrt{a^2}+a\right)\sqrt{a^2}\,x^2(2ax+3b)}{12a^2}}\operatorname{HeunT}\left(\frac{3^{\frac{2}{3}}b\,a^2}{2(a^2)^{\frac{4}{3}}},\frac{6\sqrt{a^2}}{a},-\frac{b^2 3^{\frac{1}{3}}}{4(a^2)^{\frac{2}{3}}},-\frac{3^{\frac{2}{3}}a(2ax+b)}{6(a^2)^{\frac{5}{6}}}\right)}{x}$$

✓ Solution by Mathematica

Time used: 1.962 (sec). Leaf size: 58

```
DSolve[x*y''[x]+(a*x^3+b*x^2+2)*y'[x]+b*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{(ax+b)\left(c_2\int_1^x\frac{e^{-\frac{1}{6}K[1]^2(3b+2aK[1])}}{(b+aK[1])^2}dK[1]+c_1\right)}{bx}$$

28.29 problem 89

Internal problem ID [10923]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 89.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy''+\left(b\,x^{3}a+b\,x^{2}+ax-1\right)y'+a^{2}b\,x^{3}y=0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 58

```
dsolve(x*diff(y(x),x$2)+(a*b*x^3+b*x^2+a*x-1)*diff(y(x),x)+a^2*b*x^3*y(x)=0,y(x), singsol=al
```

$$y(x)=c_1\mathrm{e}^{-ax}(ax+1)+c_2\mathrm{e}^{-ax}(ax+1)\left(\int \frac{x\,\mathrm{e}^{-\frac{x\left(ab\,x^{2}+\frac{3}{2}bx-3a\right)}{3}}}{\left(ax+1\right)^{2}}dx\right)$$

✓ Solution by Mathematica

Time used: 4.606 (sec). Leaf size: 72

```
DSolve[x*y''[x]+(a*b*x^3+b*x^2+a*x-1)*y'[x]+a^2*b*x^3*y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x)\to \frac{e^{-ax}(ax+1)\left(c_2\int_1^x \frac{a^2\exp\left(-\frac{1}{6}K[1](3bK[1]+2a(bK[1]^2-3))\right)K[1]}{(aK[1]+1)^2}dK[1]+c_1\right)}{a}$$

28.30 problem 90

Internal problem ID [10924]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 90.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy''+\left(a\,x^{3}+b\,x^{2}+cx+d\right)y'+(d-1)\left(a\,x^{2}+bx+c\right)y=0$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 48

```
dsolve(x*diff(y(x),x$2)+(a*x^3+b*x^2+c*x+d)*diff(y(x),x)+(d-1)*(a*x^2+b*x+c)*y(x)=0,y(x), si
```

$$y(x)=c_{1}x^{-d+1}+c_{2}x^{-d+1}\left(\int x^{d-2}\mathrm{e}^{-\frac{\left(a\,x^{2}+\frac{3}{2}bx+3c\right)x}{3}}dx\right)$$

✓ Solution by Mathematica

Time used: 1.839 (sec). Leaf size: 57

```
DSolve[x*y''[x]+(a*x^3+b*x^2+c*x+d)*y'[x]+(d-1)*(a*x^2+b*x+c)*y[x]==0,y[x],x,IncludeSingular
```

$$y(x)\to x^{1-d}\left(c_{2}\int_{1}^{x}\exp\left(-\frac{1}{6}K[1](6c+K[1](3b+2aK[1]))\right)K[1]^{d-2}dK[1]+c_{1}\right)$$

28.31 problem 91

Internal problem ID [10925]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 91.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+a\,x^n y'+\left(ab\,x^n-a\,x^{-1+n}-b^2x+2b\right)y=0}$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x$2)+a*x^n*diff(y(x),x)+(a*b*x^n-a*x^(n-1)-b^2*x+2*b)*y(x)=0,y(x), singso
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]+a*x^n*y'[x]+(a*b*x^n-a*x^(n-1)-b^2*x+2*b)*y[x]==0,y[x],x,IncludeSingularSolu
```

Not solved

28.32 problem 92

Internal problem ID [10926]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 92.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy''+(x^n a+2)\,y'+x^{-1+n}ya=0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 105

```
dsolve(x*diff(y(x),x$2)+(a*x^n+2)*diff(y(x),x)+a*x^(n-1)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1}{x}+c_2\left(\left((n+1)\,x^{-\frac{3n}{2}-\frac{1}{2}}+a\,x^{-\frac{n}{2}-\frac{1}{2}}\right)n\,\mathrm{WhittakerM}\left(-\frac{1}{2}+\frac{1}{2n},1+\frac{1}{2n},\frac{a\,x^n}{n}\right)+x^{-\frac{3n}{2}-\frac{1}{2}}\,\mathrm{WhittakerM}\left(\frac{1}{2}+\frac{1}{2n},1+\frac{1}{2n},\frac{a\,x^n}{n}\right)(n+1)^2\right)\mathrm{e}^{-\frac{a\,x^n}{2n}}$$

✓ Solution by Mathematica

Time used: 0.099 (sec). Leaf size: 62

```
DSolve[x*y''[x]+(a*x^n+2)*y'[x]+a*x^(n-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to(-1)^{-1/n}n^{\frac{1}{n}-1}a^{-1/n}(x^n)^{-1/n}\left(c_1(-1)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},0,\frac{ax^n}{n}\right)+c_2n\right)$$

28.33 problem 93

Internal problem ID [10927]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 93.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+(x^n+1-n)\,y'+b\,x^{2n-1}y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 61

```
dsolve(x*diff(y(x),x$2)+(x^n+1-n)*diff(y(x),x)+b*x^(2*n-1)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\mathrm{e}^{-\frac{x^n}{2n}}\sinh\left(\frac{x^n\sqrt{\frac{-4b+1}{n^2}}}{2}\right)+c_2\mathrm{e}^{-\frac{x^n}{2n}}\cosh\left(\frac{x^n\sqrt{\frac{-4b+1}{n^2}}}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.094 (sec). Leaf size: 53

```
DSolve[x*y''[x]+(x^n+1-n)*y'[x]+b*x^(2*n-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{-\frac{\left(\sqrt{1-4b}+1\right)x^n}{2n}}\left(c_2e^{\frac{\sqrt{1-4b}x^n}{n}}+c_1\right)$$

28.34 problem 94

Internal problem ID [10928]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 94.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$xy'' + (x^n a + b)\, y' + x^{-1+n} yan = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 62

```
dsolve(x*diff(y(x),x$2)+(a*x^n+b)*diff(y(x),x)+a*n*x^(n-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{a\,x^n}{n}} \operatorname{hypergeom}\left(\left[\frac{b-1}{n}\right], \left[\frac{n+b-1}{n}\right], \frac{a\,x^n}{n}\right) + c_2 x^{-b+1} \mathrm{e}^{-\frac{a\,x^n}{n}}$$

✓ Solution by Mathematica

Time used: 0.244 (sec). Leaf size: 121

```
DSolve[x*y''[x]+(a*x^n+b)*y'[x]+a*n*x^(n-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (-1)^{-\frac{b}{n}} n^{\frac{b-n-1}{n}} a^{\frac{1-b}{n}} e^{-\frac{ax^n}{n}} (x^n)^{\frac{1-b}{n}} \left(-(b-1)c_1(-1)^{\frac{1}{n}} \Gamma\left(\frac{b-1}{n}, -\frac{ax^n}{n}\right) + c_2 n(-1)^{b/n} + (b-1)c_1(-1)^{\frac{1}{n}} \operatorname{Gamma}\left(\frac{b-1}{n}\right)\right)$$

28.35 problem 95

Internal problem ID [10929]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 95.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+(x^n a+b)\,y'+a(b-1)\,x^{-1+n}y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 134

```
dsolve(x*diff(y(x),x$2)+(a*x^n+b)*diff(y(x),x)+a*(b-1)*x^(n-1)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x^{-b+1}+c_2\mathrm{e}^{-\frac{a\,x^n}{2n}}\left(n\left((n+b-1)\,x^{\frac{1}{2}-\frac{3n}{2}-\frac{b}{2}}+a\,x^{\frac{1}{2}-\frac{b}{2}-\frac{n}{2}}\right)\mathrm{WhittakerM}\left(\frac{b-n-1}{2n},\frac{2n+b-1}{2n},\frac{a\,x^n}{n}\right)+\mathrm{WhittakerM}\left(\frac{n+b-1}{2n},\frac{2n+b-1}{2n},\frac{a\,x^n}{n}\right)x^{\frac{1}{2}-\frac{3n}{2}-\frac{b}{2}}(n+b-1)^2\right)$$

✓ Solution by Mathematica

Time used: 0.182 (sec). Leaf size: 90

```
DSolve[x*y''[x]+(a*x^n+b)*y'[x]+a*(b-1)*x^(n-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\to(-1)^{-\frac{b}{n}}n^{\frac{b-n-1}{n}}a^{\frac{1-b}{n}}(x^n)^{\frac{1-b}{n}}\left((b-1)c_1(-1)^{b/n}\Gamma\left(\frac{b-1}{n},0,\frac{ax^n}{n}\right)+c_2(-1)^{\frac{1}{n}}n\right)$$

28.36 problem 96

Internal problem ID [10930]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 96.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+(x^n a+b)\,y'+a(-1+b+n)\,x^{-1+n}y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 66

```
dsolve(x*diff(y(x),x$2)+(a*x^n+b)*diff(y(x),x)+a*(b+n-1)*x^(n-1)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\mathrm{e}^{-\frac{a\,x^n}{n}}+c_2x^{-b+1}\mathrm{e}^{-\frac{a\,x^n}{n}}\operatorname{hypergeom}\left(\left[\frac{-b+1}{n}\right],\left[\frac{-b+n+1}{n}\right],\frac{a\,x^n}{n}\right)$$

✓ Solution by Mathematica

Time used: 0.178 (sec). Leaf size: 93

```
DSolve[x*y''[x]+(a*x^n+b)*y'[x]+a*(b+n-1)*x^(n-1)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{(-1)^{-1/n}e^{-\frac{ax^n}{n}}\left((b-1)c_2(-1)^{b/n}\Gamma\left(\frac{1-b}{n},-\frac{ax^n}{n}\right)-(b-1)c_2(-1)^{b/n}\operatorname{Gamma}\left(\frac{1-b}{n}\right)+c_1(-1)^{\frac{1}{n}}n\right)}{n}$$

28.37 problem 97

Internal problem ID [10931]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 97.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy''+(x^n a+b)\,y'+c(x^n a-cx+b)\,y=0$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x$2)+(a*x^n+b)*diff(y(x),x)+c*(a*x^n-c*x+b)*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]+(a*x^n+b)*y'[x]+c*(a*x^n-c*x+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

Not solved

28.38 problem 98

Internal problem ID [10932]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 98.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy''+(ab\,x^n+b-3n+1)\,y'+a^2n(b-n)\,x^{2n-1}y=0$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x$2)+(a*b*x^n+b-3*n+1)*diff(y(x),x)+a^2*n*(b-n)*x^(2*n-1)*y(x)=0,y(x), si
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]+(a*b*x^n+b-3*n+1)*y'[x]+a^2*n*(b-n)*x^(2*n-1)*y[x]==0,y[x],x,IncludeSingular
```

Not solved

28.39 problem 99

Internal problem ID [10933]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 99.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x y''+\left(a\,x^{n}+b\right) y'+\left(c\,x^{2n-1}+d\,x^{-1+n}\right) y=0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 175

```
dsolve(x*diff(y(x),x$2)+(a*x^n+b)*diff(y(x),x)+(c*x^(2*n-1)+d*x^(n-1))*y(x)=0,y(x), singsol=
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x^{n}\left(a+\sqrt{a^{2}-4c}\right)}{2n}} \operatorname{KummerM}\left(\frac{(n+b-1)\sqrt{a^{2}-4c}+(n+b-1)\,a-2d}{2\sqrt{a^{2}-4c}\,n}, \frac{n+b-1}{n}, \frac{\sqrt{a^{2}-4c}\,x^{n}}{n}\right) + c_2 \mathrm{e}^{-\frac{x^{n}\left(a+\sqrt{a^{2}-4c}\right)}{2n}} \operatorname{KummerU}\left(\frac{(n+b-1)\sqrt{a^{2}-4c}+(n+b-1)\,a-2d}{2\sqrt{a^{2}-4c}\,n}, \frac{n+b-1}{n}, \frac{\sqrt{a^{2}-4c}\,x^{n}}{n}\right)$$

✓ Solution by Mathematica

Time used: 0.38 (sec). Leaf size: 255

```
DSolve[x*y''[x]+(a*x^n+b)*y'[x]+(c*x^(2*n-1)+d*x^(n-1))*y[x]==0,y[x],x,IncludeSingularSoluti
```

$y(x)$

$$\to 2^{\frac{b+n-1}{2n}} x^{\frac{1}{2}-\frac{n}{2}} (x^n)^{\frac{n-1}{2n}} e^{-\frac{\left(\sqrt{a^2-4c}+a\right)x^n}{2n}} \left(c_1 \, \text{HypergeometricU}\left(\frac{(b+n-1)a^2+\sqrt{a^2-4c}(b+n-1)a-2\sqrt{}}{2\left(a^2-4c\right)n} \right. \right.$$

$$\left. + c_2 L^{\frac{b-1}{n}}_{-\frac{(b+n-1)a^2+\sqrt{a^2-4c}(b+n-1)a-2\sqrt{a^2-4c}d-4c(b+n-1)}{2\left(a^2-4c\right)n}} \left(\frac{\sqrt{a^2-4c}x^n}{n} \right) \right)$$

28.40 problem 100

Internal problem ID [10934]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 100.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy''+\left(a\,x^{n}+b\,x^{-1+n}+2\right)y'+b\,x^{-2+n}y=0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 61

```
dsolve(x*diff(y(x),x$2)+(a*x^n+b*x^(n-1)+2)*diff(y(x),x)+(b*x^(n-2))*y(x)=0,y(x), singsol=al
```

$$y(x)=\frac{c_1(ax+b)}{x}+\frac{c_2(ax+b)\left(\int\frac{\mathrm{e}^{-\frac{x^{n-1}(ax(n-1)+nb)}{n(n-1)}}}{(ax+b)^2}dx\right)}{x}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]+(a*x^n+b*x^(n-1)+2)*y'[x]+(b*x^(n-2))*y[x]==0,y[x],x,IncludeSingularSolution
```

Not solved

28.41 problem 101

Internal problem ID [10935]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 101.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy''+(x^n a+bx)\,y'+\left(ab\,x^n+an\,x^{-1+n}-b\right)y=0$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x$2)+(a*x^n+b*x)*diff(y(x),x)+(a*b*x^n+a*n*x^(n-1)-b)*y(x)=0,y(x), singso
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]+(a*x^n+b*x)*y'[x]+(a*b*x^n+a*n*x^(n-1)-b)*y[x]==0,y[x],x,IncludeSingularSolu
```

Not solved

28.42 problem 102

Internal problem ID [10936]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 102.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy''+\left(ab\,x^{n}+b\,x^{-1+n}+ax-1\right)y'+a^{2}b\,x^{n}y=0$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x$2)+(a*b*x^n+b*x^(n-1)+a*x-1)*diff(y(x),x)+(a^2*b*x^n)*y(x)=0,y(x), sing
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]+(a*b*x^n+b*x^(n-1)+a*x-1)*y'[x]+(a^2*b*x^n)*y[x]==0,y[x],x,IncludeSingularSo
```

Not solved

28.43 problem 103

Internal problem ID [10937]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 103.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' + (x^n a + b\,x^m + c)\,y' + (c-1)\left(a\,x^{-1+n} + x^{m-1}b\right)y = 0$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x$2)+(a*x^n+b*x^m+c)*diff(y(x),x)+(c-1)*(a*x^(n-1)+b*x^(m-1))*y(x)=0,y(x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]+(a*x^n+b*x^m+c)*y'[x]+(c-1)*(a*x^(n-1)+b*x^(m-1))*y[x]==0,y[x],x,IncludeSing
```

Not solved

28.44 problem 104

Internal problem ID [10938]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 104.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy''+\left(ab\,x^{m+n}+an\,x^{n}+b\,x^{m}+1-2n\right)y'+a^{2}bn\,x^{m-1+2n}y=0$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x$2)+(a*b*x^(n+m)+a*n*x^n+b*x^m+1-2*n)*diff(y(x),x)+a^2*b*n*x^(2*n+m-1)*y
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]+(a*b*x^(n+m)+a*n*x^n+b*x^m+1-2*n)*y'[x]+a^2*b*n*x^(2*n+m-1)*y[x]==0,y[x],x,I
```

Not solved

28.45 problem 105

Internal problem ID [10939]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 105.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x+a)\,y''+(bx+c)\,y'+by=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 52

```
dsolve((x+a)*diff(y(x),x$2)+(b*x+c)*diff(y(x),x)+b*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\mathrm{e}^{-bx}(x+a)^{ab-c+1}+c_2\mathrm{e}^{-bx}\,\mathrm{hypergeom}\left([-ab+c-1],[-ab+c],b(x+a)\right)$$

✓ Solution by Mathematica

Time used: 0.559 (sec). Leaf size: 90

```
DSolve[(x+a)*y''[x]+(b*x+c)*y'[x]+b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{-b(a+x)}(a+x)^{1-c}(-b(a+x))^{-c}\left(c_1e^{ab}(a+x)^{ab}(-b(a+x))^{c}\right.$$
$$\left.+bc_2(-b(a+x))^{ab}(a+x)^{c}\Gamma(-ab+c-1,-b(a+x))\right)$$

28.46 problem 106

Internal problem ID [10940]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 106.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(a_1x+a_0)\,y''+(b_1x+b_0)\,y'-mb_1y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 129

```
dsolve((a__1*x+a__0)*diff(y(x),x$2)+(b__1*x+b__0)*diff(y(x),x)-m*b__1*y(x)=0,y(x), singsol=a
```

$$y(x)=c_1\mathrm{e}^{-\frac{b_1x}{a_1}}\,\mathrm{KummerM}\left(m+1,\frac{a_0b_1+2a_1^2-a_1b_0}{a_1^2},\frac{b_1(a_1x+a_0)}{a_1^2}\right)(a_1x+a_0)^{\frac{a_0b_1+a_1^2-a_1b_0}{a_1^2}}+c_2\mathrm{e}^{-\frac{b_1x}{a_1}}\,\mathrm{KummerU}\left(m+1,\frac{a_0b_1+2a_1^2-a_1b_0}{a_1^2},\frac{b_1(a_1x+a_0)}{a_1^2}\right)(a_1x+a_0)^{\frac{a_0b_1+a_1^2-a_1b_0}{a_1^2}}$$

✓ Solution by Mathematica

Time used: 0.278 (sec). Leaf size: 102

```
DSolve[(a1*x+a0)*y''[x]+(b1*x+b0)*y'[x]-m*b1*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x)\to e^{-\frac{\mathrm{b1}x}{\mathrm{a1}}}(\mathrm{a0}+\mathrm{a1}x)^{\frac{\mathrm{a0b1}+\mathrm{a1}^2-\mathrm{a1b0}}{\mathrm{a1}^2}}\left(c_1\,\mathrm{HypergeometricU}\left(m+1,-\frac{\mathrm{b0}}{\mathrm{a1}}+\frac{\mathrm{a0b1}}{\mathrm{a1}^2}+2,\frac{\mathrm{b1}(\mathrm{a0}+\mathrm{a1}x)}{\mathrm{a1}^2}\right)+c_2L_{-m-1}^{\frac{\mathrm{a1}^2-\mathrm{b0a1}+\mathrm{a0b1}}{\mathrm{a1}^2}}\left(\frac{\mathrm{b1}(\mathrm{a0}+\mathrm{a1}x)}{\mathrm{a1}^2}\right)\right)$$

28.47 problem 107

Internal problem ID [10941]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 107.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(ax+b)\,y''+s(xc+d)\,y'-s^2((c+a)\,x+b+d)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 149

```
dsolve((a*x+b)*diff(y(x),x$2)+s*(c*x+d)*diff(y(x),x)-s^2*((a+c)*x+b+d)*y(x)=0,y(x), singsol=
```

$$y(x)=c_1\mathrm{e}^{-\frac{s(a+c)x}{a}}\operatorname{KummerM}\left(1,\frac{-dsa+csb+2a^2}{a^2},\frac{s(2a+c)\,(ax+b)}{a^2}\right)(ax+b)^{\frac{-dsa+csb+a^2}{a^2}}+c_2\mathrm{e}^{-\frac{s(a+c)x}{a}}\operatorname{KummerU}\left(1,\frac{-dsa+csb+2a^2}{a^2},\frac{s(2a+c)\,(ax+b)}{a^2}\right)(ax+b)^{\frac{-dsa+csb+a^2}{a^2}}$$

✓ Solution by Mathematica

Time used: 1.269 (sec). Leaf size: 122

```
DSolve[(a*x+b)*y''[x]+s*(c*x+d)*y'[x]-s^2*((a+c)*x+b+d)*y[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to c_1 e^{sx} - \frac{c_2 e^{s\left(\frac{b(2a+c)}{a^2}+x\right)}(ax+b)^{\frac{s(bc-ad)}{a^2}+1}\left(\frac{s(2a+c)(ax+b)}{a^2}\right)^{\frac{s(ad-bc)}{a^2}-1}\Gamma\left(\frac{a^2-dsa+bcs}{a^2},\frac{(2a+c)s(b+ax)}{a^2}\right)}{a}$$

28.48 problem 108

Internal problem ID [10942]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 108.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(a_2x+b_2)\,y''+(a_1x+b_1)\,y'+(a_0x+b_0)\,y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 287

```
dsolve((a__2*x+b__2)*diff(y(x),x$2)+(a__1*x+b__1)*diff(y(x),x)+(a__0*x+b__0)*y(x)=0,y(x), si
```

$$y(x) = c_1 \mathrm{e}^{-\frac{\left(\sqrt{-4a_0a_2+a_1^2}+a_1\right)x}{2a_2}} \operatorname{KummerM}\left(\frac{(a_1b_2+2a_2^2-a_2b_1)\sqrt{-4a_0a_2+a_1^2}-2a_2^2b_0+(2a_0b_2+b_1a_1)a_2-a_1^2b_2}{2\sqrt{-4a_0a_2+a_1^2}\,a_2^2} \right. \left. + b_2)^{\frac{a_1b_2+a_2^2-a_2b_1}{a_2^2}} \right.$$
$$+ c_2 \mathrm{e}^{-\frac{\left(\sqrt{-4a_0a_2+a_1^2}+a_1\right)x}{2a_2}} \operatorname{KummerU}\left(\frac{(a_1b_2+2a_2^2-a_2b_1)\sqrt{-4a_0a_2+a_1^2}-2a_2^2b_0+(2a_0b_2+b_1a_1)a_2-a_1^2}{2\sqrt{-4a_0a_2+a_1^2}\,a_2^2} \right. \left. + b_2)^{\frac{a_1b_2+a_2^2-a_2b_1}{a_2^2}} \right.$$

✓ Solution by Mathematica

Time used: 0.526 (sec). Leaf size: 301

```
DSolve[(a2*x+b2)*y''[x]+(a1*x+b1)*y'[x]+(a0*x+b0)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to e^{-\frac{x\left(\sqrt{\mathrm{a1}^2-4\mathrm{a0a2}}+\mathrm{a1}\right)}{2\mathrm{a2}}}(\mathrm{a2}x+\mathrm{b2})^{\frac{\mathrm{a1b2}+\mathrm{a2}^2-\mathrm{a2b1}}{\mathrm{a2}^2}}\left(c_1\,\mathrm{HypergeometricU}\left(\frac{2\left(\sqrt{\mathrm{a1}^2-4\mathrm{a0a2}}-\mathrm{b0}\right)\mathrm{a2}^2+\left(\mathrm{a1b1}-\sqrt{\mathrm{a1}^2-4\mathrm{a0a2}}\mathrm{b1}+2\right.}{2\mathrm{a2}^2\sqrt{\mathrm{a1}^2-4\mathrm{a0a2}}}\right.\right.$$

$$\left.-\frac{\mathrm{b1}}{\mathrm{a2}}+\frac{\mathrm{a1b2}}{\mathrm{a2}^2}+2,\frac{\sqrt{\mathrm{a1}^2-4\mathrm{a0a2}}(\mathrm{b2}+\mathrm{a2}x)}{\mathrm{a2}^2}\right)$$

$$\left.+c_2L^{\frac{\mathrm{a2}^2-\mathrm{b1a2}+\mathrm{a1b2}}{\mathrm{a2}^2}}_{\frac{-2\left(\sqrt{\mathrm{a1}^2-4\mathrm{a0a2}}-\mathrm{b0}\right)\mathrm{a2}^2+\left(-\mathrm{a1b1}+\sqrt{\mathrm{a1}^2-4\mathrm{a0a2}}\mathrm{b1}-2\mathrm{a0b2}\right)\mathrm{a2}+\mathrm{a1}\left(\mathrm{a1}-\sqrt{\mathrm{a1}^2-4\mathrm{a0a2}}\right)\mathrm{b2}}{2\mathrm{a2}^2\sqrt{\mathrm{a1}^2-4\mathrm{a0a2}}}}\left(\frac{\sqrt{\mathrm{a1}^2-4\mathrm{a0a2}}(\mathrm{b2}+\mathrm{a2}x)}{\mathrm{a2}^2}\right)\right)$$

28.49 problem 109

Internal problem ID [10943]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 109.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$(x+\gamma)\,y''+(x^n a+b\,x^m+c)\,y'+\left(an\,x^{-1+n}+bm\,x^{m-1}\right)y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 63

```
dsolve((x+gamma)*diff(y(x),x$2)+(a*x^n+b*x^m+c)*diff(y(x),x)+(a*n*x^(n-1)+b*m*x^(m-1))*y(x)=
```

$$y(x)=\left(c_1\left(\int\frac{\mathrm{e}^{\int\frac{a\,x^n+b\,x^m+c-1}{x+\gamma}dx}}{x+\gamma}dx\right)+c_2\right)\mathrm{e}^{\int-\frac{a\,x^n+b\,x^m+c-1}{x+\gamma}dx}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(x+\[Gamma])*y''[x]+(a*x^n+b*x^m+c)*y'[x]+(a*n*x^(n-1)+b*m*x^(m-1))*y[x]==0,y[x],x,In
```

Not solved

29 Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$

29.1 problem 110

Internal problem ID [10944]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 110.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + ay = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(x^2*diff(y(x),x$2)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{\frac{1}{2}+\frac{\sqrt{1-4a}}{2}} + c_2 x^{\frac{1}{2}-\frac{\sqrt{1-4a}}{2}}$$

✓ Solution by Mathematica

Time used: 0.057 (sec). Leaf size: 42

```
DSolve[x^2*y''[x]+a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^{\frac{1}{2}-\frac{1}{2}\sqrt{1-4a}}\left(c_2 x^{\sqrt{1-4a}} + c_1\right)$$

29.2 problem 111

Internal problem ID [10945]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 111.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + (ax + b)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(x^2*diff(y(x),x$2)+(a*x+b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\mathrm{BesselJ}\left(\sqrt{-4b+1}, 2\sqrt{a}\,\sqrt{x}\right) + c_2\sqrt{x}\,\mathrm{BesselY}\left(\sqrt{-4b+1}, 2\sqrt{a}\,\sqrt{x}\right)$$

✓ Solution by Mathematica

Time used: 0.147 (sec). Leaf size: 95

```
DSolve[x^2*y''[x]+(a*x+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{a}\sqrt{x}\Big(c_1\,\mathrm{Gamma}\left(1-\sqrt{1-4b}\right)\mathrm{BesselJ}\left(-\sqrt{1-4b}, 2\sqrt{a}\sqrt{x}\right) + c_2\,\mathrm{Gamma}\left(\sqrt{1-4b}+1\right)\mathrm{BesselJ}\left(\sqrt{1-4b}, 2\sqrt{a}\sqrt{x}\right)\Big)$$

29.3 problem 112

Internal problem ID [10946]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 112.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + (a^2x^2 - n(n+1))\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x^2*diff(y(x),x$2)+(a^2*x^2-n*(n+1))*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\text{BesselJ}\left(n+\frac{1}{2}, ax\right) + c_2\sqrt{x}\,\text{BesselY}\left(n+\frac{1}{2}, ax\right)$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 36

```
DSolve[x^2*y''[x]+(a^2*x^2-n*(n+1))*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}\left(c_1\,\text{BesselJ}\left(n+\frac{1}{2}, ax\right) + c_2\,\text{BesselY}\left(n+\frac{1}{2}, ax\right)\right)$$

29.4 problem 113

Internal problem ID [10947]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 113.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - (a^2x^2 + n(n+1))\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
dsolve(x^2*diff(y(x),x$2)-(a^2*x^2+n*(n+1))*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\, \text{BesselJ}\left(n+\frac{1}{2}, \sqrt{-a^2}\, x\right) + c_2\sqrt{x}\, \text{BesselY}\left(n+\frac{1}{2}, \sqrt{-a^2}\, x\right)$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 42

```
DSolve[x^2*y''[x]-(a^2*x^2+n*(n+1))*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}\left(c_1\, \text{BesselJ}\left(n+\frac{1}{2}, -iax\right) + c_2\, \text{BesselY}\left(n+\frac{1}{2}, -iax\right)\right)$$

29.5 problem 114

Internal problem ID [10948]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 114.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - \left(a^2x^2 + 2bax + b^2 - b\right) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x^2*diff(y(x),x$2)-(a^2*x^2+2*a*b*x+b^2-b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^b \mathrm{e}^{ax} + c_2 \operatorname{WhittakerM}\left(-b, -b + \frac{1}{2}, 2ax\right)$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 38

```
DSolve[x^2*y''[x]-(a^2*x^2+2*a*b*x+b^2-b)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 M_{-b,b-\frac{1}{2}}(2ax) + c_2 W_{-b,b-\frac{1}{2}}(2ax)$$

29.6 problem 115

Internal problem ID [10949]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 115.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + \left(a\,x^2 + bx + c\right)y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 57

```
dsolve(x^2*diff(y(x),x$2)+(a*x^2+b*x+c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \,\mathrm{WhittakerM}\left(-\frac{ib}{2\sqrt{a}}, \frac{\sqrt{1-4c}}{2}, 2i\sqrt{a}\,x\right) + c_2 \,\mathrm{WhittakerW}\left(-\frac{ib}{2\sqrt{a}}, \frac{\sqrt{1-4c}}{2}, 2i\sqrt{a}\,x\right)$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 88

```
DSolve[x^2*y''[x]+(a*x^2+b*x+c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 M_{-\frac{ib}{2\sqrt{a}},-\frac{1}{2}i\sqrt{4c-1}}\left(2i\sqrt{a}x\right) + c_2 W_{-\frac{ib}{2\sqrt{a}},-\frac{1}{2}i\sqrt{4c-1}}\left(2i\sqrt{a}x\right)$$

29.7 problem 116

Internal problem ID [10950]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 116.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - \left(a\,x^3 + \frac{5}{16}\right)y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
dsolve(x^2*diff(y(x),x$2)-(a*x^3+5/16)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sinh\left(\frac{2x^{\frac{3}{2}}\sqrt{a}}{3}\right)}{x^{\frac{1}{4}}} + \frac{c_2 \cosh\left(\frac{2x^{\frac{3}{2}}\sqrt{a}}{3}\right)}{x^{\frac{1}{4}}}$$

✓ Solution by Mathematica

Time used: 0.102 (sec). Leaf size: 60

```
DSolve[x^2*y''[x]-(a*x^3+5/16)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{2}{3}\sqrt{a}x^{3/2}}\left(2c_1e^{\frac{4}{3}\sqrt{a}x^{3/2}} - \frac{c_2}{\sqrt{a}}\right)}{2\sqrt[4]{x}}$$

29.8 problem 117

Internal problem ID [10951]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 117.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - \left(a^2x^4 + a(2b-1)\,x^2 + b(b+1)\right)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve(x^2*diff(y(x),x$2)-(a^2*x^4+a*(2*b-1)*x^2+b*(b+1))*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{-b} \mathrm{e}^{-\frac{a\,x^2}{2}} + c_2 x^{-b} \mathrm{e}^{-\frac{a\,x^2}{2}} \left(\Gamma\left(b+\frac{1}{2}\right) - \Gamma\left(b+\frac{1}{2}, -a\,x^2\right)\right)$$

✓ Solution by Mathematica

Time used: 0.45 (sec). Leaf size: 66

```
DSolve[x^2*y''[x]-(a^2*x^4+a*(2*b-1)*x^2+b*(b+1))*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{2} e^{-\frac{ax^2}{2}} x^{-b} \left(a c_2 x^{2b+3} \left(-ax^2\right)^{-b-\frac{3}{2}} \Gamma\left(b+\frac{1}{2}, -ax^2\right) + 2c_1\right)$$

29.9 problem 118

Internal problem ID [10952]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 118.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + (x^n a + b)\, y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 65

```
dsolve(x^2*diff(y(x),x$2)+(a*x^n+b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\ \mathrm{BesselJ}\left(\frac{\sqrt{-4b+1}}{n}, \frac{2\sqrt{a}\, x^{\frac{n}{2}}}{n}\right) + c_2\sqrt{x}\ \mathrm{BesselY}\left(\frac{\sqrt{-4b+1}}{n}, \frac{2\sqrt{a}\, x^{\frac{n}{2}}}{n}\right)$$

✓ Solution by Mathematica

Time used: 0.343 (sec). Leaf size: 351

```
DSolve[x^2*y''[x]+(a*x^n+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to n^{-\frac{\sqrt{(1-4b)n^2}+i\sqrt{4b-1}n+n}{n^2}} a^{\frac{-\sqrt{(1-4b)n^2}-i\sqrt{4b-1}n+n}{2n^2}} (x^n)^{\frac{-\sqrt{(1-4b)n^2}-i\sqrt{4b-1}n+n}{2n^2}} \left(c_2 n^{\frac{2\sqrt{(1-4b)n^2}}{n^2}} a^{\frac{i\sqrt{4b-1}}{n}} (x^n)^{\frac{i\sqrt{4b-1}}{n}} \mathrm{Gamma}\right.$$
$$\left. + c_1 n^{\frac{2i\sqrt{4b-1}}{n}} a^{\frac{\sqrt{(1-4b)n^2}}{n^2}} (x^n)^{\frac{\sqrt{(1-4b)n^2}}{n^2}} \mathrm{Gamma}\left(1 - \frac{\sqrt{1-4b}}{n}\right) \mathrm{BesselJ}\left(-\frac{\sqrt{(1-4b)n^2}}{n^2}, \frac{2\sqrt{a}\sqrt{x^n}}{n}\right)\right)$$

29.10 problem 119

Internal problem ID [10953]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 119.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - \left(a^2x^{2n} + a(2b+n-1)\,x^n + b(b-1)\right)y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 138

```
dsolve(x^2*diff(y(x),x$2)-(a^2*x^(2*n)+a*(2*b+n-1)*x^n+b*(b-1))*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^b\mathrm{e}^{\frac{a\,x^n}{n}} + c_2\left(2\left(\left(-b+\frac{1}{2}+\frac{n}{2}\right)x^{-\frac{3n}{2}+\frac{1}{2}} + a\,x^{-\frac{n}{2}+\frac{1}{2}}\right)n\,\mathrm{WhittakerM}\left(\frac{-2b-n+1}{2n}, -\frac{b}{n}+1+\frac{1}{2n}, \frac{2a\,x^n}{n}\right) + 4x^{-\frac{3n}{2}+\frac{1}{2}}\,\mathrm{WhittakerM}\left(\frac{n-2b+1}{2n}, -\frac{b}{n}+1+\frac{1}{2n}, \frac{2a\,x^n}{n}\right)\left(b-\frac{1}{2}-\frac{n}{2}\right)^2\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y''[x]-(a^2*x^(2*n)+a*(2*b+n-1)*x^n+b*(b-1))*y[x]==0,y[x],x,IncludeSingularSoluti
```

Not solved

29.11 problem 120

Internal problem ID [10954]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 120.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \left(a\,x^{2n} + b\,x^n + c\right)y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 95

```
dsolve(x^2*diff(y(x),x$2)+(a*x^(2*n)+b*x^n+c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{-\frac{n}{2}+\frac{1}{2}}\,\text{WhittakerM}\left(-\frac{ib}{2\sqrt{a}\,n}, \frac{i\sqrt{-1+4c}}{2n}, \frac{2i\sqrt{a}\,x^n}{n}\right) + c_2 x^{-\frac{n}{2}+\frac{1}{2}}\,\text{WhittakerW}\left(-\frac{ib}{2\sqrt{a}\,n}, \frac{i\sqrt{-1+4c}}{2n}, \frac{2i\sqrt{a}\,x^n}{n}\right)$$

✓ Solution by Mathematica

Time used: 0.313 (sec). Leaf size: 236

```
DSolve[x^2*y''[x]+(a*x^(2*n)+b*x^n+c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2^{\frac{\sqrt{(1-4c)n^2}+n^2}{2n^2}} x^{\frac{1}{2}-\frac{n}{2}} e^{\frac{i\sqrt{a}x^n}{n}} (x^n)^{\frac{\sqrt{(1-4c)n^2}+n^2}{2n^2}} \left(c_1 \operatorname{HypergeometricU}\left(\frac{1}{2}\left(-\frac{ib}{\sqrt{a}n}+\frac{\sqrt{(1-4c)n^2}}{n^2}+1\right), \frac{\sqrt{(1-}{n}\right.\right.$$

$$\left.\left. + 1, -\frac{2i\sqrt{a}x^n}{n}\right) + c_2 L_{\frac{1}{2}\left(\frac{ib}{\sqrt{a}n}-\frac{\sqrt{(1-4c)n^2}}{n^2}-1\right)}^{\frac{\sqrt{(1-4c)n^2}}{n^2}}\left(-\frac{2i\sqrt{a}x^n}{n}\right)\right)$$

29.12 problem 121

Internal problem ID [10955]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 121.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + \left(a\,x^{3n} + b\,x^{2n} + \frac{1}{4} - \frac{n^2}{4}\right) y = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 104

```
dsolve(x^2*diff(y(x),x$2)+(a*x^(3*n)+b*x^(2*n)+1/4-1/4*n^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{-\frac{n}{2}+\frac{1}{2}} \operatorname{hypergeom}\left([\,], \left[\frac{2}{3}\right], -\frac{(a\,x^n+b)^3}{9a^2n^2}\right) + c_2 \operatorname{hypergeom}\left([\,], \left[\frac{4}{3}\right], \frac{-x^{3n}a^3 - 3x^{2n}a^2b - 3x^n a\,b^2 - b^3}{9n^2a^2}\right)\left(a\,x^{\frac{n}{2}+\frac{1}{2}} + b\,x^{-\frac{n}{2}+\frac{1}{2}}\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y''[x]+(a*x^(3*n)+b*x^(2*n)+1/4-1/4*n^2)*y[x]==0,y[x],x,IncludeSingularSolutions
```

Not solved

29.13 problem 122

Internal problem ID [10956]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 122.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + \left(a\,x^{2n}(b\,x^n + c)^m + \frac{1}{4} - \frac{n^2}{4}\right) y = 0$$

✗ Solution by Maple

```
dsolve(x^2*diff(y(x),x$2)+(a*x^(2*n)*(b*x^n+c)^m+1/4-1/4*n^2)*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y''[x]+(a*x^(2*n)*(b*x^n+c)^m+1/4-1/4*n^2)*y[x]==0,y[x],x,IncludeSingularSolution
```

Not solved

29.14 problem 123

Internal problem ID [10957]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 123.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + axy' + by = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
dsolve(x^2*diff(y(x),x$2)+a*x*diff(y(x),x)+b*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{-\frac{a}{2}+\frac{1}{2}+\frac{\sqrt{a^2-2a-4b+1}}{2}} + c_2 x^{-\frac{a}{2}+\frac{1}{2}-\frac{\sqrt{a^2-2a-4b+1}}{2}}$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 57

```
DSolve[x^2*y''[x]+a*x*y'[x]+b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^{\frac{1}{2}\left(-\sqrt{a^2-2a-4b+1}-a+1\right)}\left(c_2 x^{\sqrt{a^2-2a-4b+1}} + c_1\right)$$

29.15 problem 124

Internal problem ID [10958]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 124.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + y'x + \left(x^2 - \left(n + \frac{1}{2}\right)^2\right) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-(n+1/2)^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \,\text{BesselJ}\left(n + \frac{1}{2}, x\right) + c_2 \,\text{BesselY}\left(n + \frac{1}{2}, x\right)$$

✓ Solution by Mathematica

Time used: 0.387 (sec). Leaf size: 26

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-(n+1/2)^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \,\text{BesselJ}\left(n + \frac{1}{2}, x\right) + c_2 \,\text{BesselY}\left(n + \frac{1}{2}, x\right)$$

29.16 problem 125

Internal problem ID [10959]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 125.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + y'x - \left(x^2 + \left(n + \frac{1}{2}\right)^2\right)y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-(x^2+(n+1/2)^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselI}\left(n + \frac{1}{2}, x\right) + c_2 \operatorname{BesselK}\left(n + \frac{1}{2}, x\right)$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 34

```
DSolve[x^2*y''[x]+x*y'[x]-(x^2+(n+1/2)^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}\left(n + \frac{1}{2}, -ix\right) + c_2 \operatorname{BesselY}\left(n + \frac{1}{2}, -ix\right)$$

29.17 problem 126

Internal problem ID [10960]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 126.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Bessel]`

$$\boxed{x^2y'' + y'x + (-\nu^2 + x^2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-nu^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}(\nu, x) + c_2 \operatorname{BesselY}(\nu, x)$$

✓ Solution by Mathematica

Time used: 0.102 (sec). Leaf size: 26

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-\[Nu])*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}\left(\sqrt{\nu}, x\right) + c_2 \operatorname{BesselY}\left(\sqrt{\nu}, x\right)$$

29.18 problem 127

Internal problem ID [10961]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 127.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Bessel, _modified]]`

$$\boxed{x^2y'' + y'x - (\nu^2 + x^2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-(x^2+nu^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselI}(\nu, x) + c_2 \operatorname{BesselK}(\nu, x)$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 34

```
DSolve[x^2*y''[x]+x*y'[x]-(x^2+\[Nu])*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}\left(\sqrt{\nu}, -ix\right) + c_2 \operatorname{BesselY}\left(\sqrt{\nu}, -ix\right)$$

29.19 problem 128

Internal problem ID [10962]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 128.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 2y'x - (a^2x^2 + 2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-(a^2*x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{ax}(ax - 1)}{x^2} + \frac{c_2 e^{-ax}(ax + 1)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 29

```
DSolve[x^2*y''[x]+2*x*y'[x]-(a^2*x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 j_{-2}(iax) - c_2 y_{-2}(iax)$$

29.20 problem 129

Internal problem ID [10963]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 129.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - 2axy' + \left(b^2x^2 + a(a+1)\right) y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)-2*a*x*diff(y(x),x)+(b^2*x^2+a*(a+1))*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^a \sin(bx) + c_2x^a \cos(bx)$$

✓ Solution by Mathematica

Time used: 0.066 (sec). Leaf size: 42

```
DSolve[x^2*y''[x]-2*a*x*y'[x]+(b^2*x^2+a*(a+1))*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to c_1x^a e^{-ibx} - \frac{ic_2x^a e^{ibx}}{2b}$$

29.21 problem 130

Internal problem ID [10964]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 130.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2axy' + \left(-b^2x^2 + a(a+1)\right)y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)-2*a*x*diff(y(x),x)+(-b^2*x^2+a*(a+1))*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^a \sinh(bx) + c_2x^a \cosh(bx)$$

✓ Solution by Mathematica

Time used: 0.08 (sec). Leaf size: 35

```
DSolve[x^2*y''[x]-2*a*x*y'[x]+(-b^2*x^2+a*(a+1))*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to c_1x^a e^{-bx} + \frac{c_2x^a e^{bx}}{2b}$$

29.22 problem 131

Internal problem ID [10965]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 131.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \lambda xy' + (a\,x^2 + bx + c)\,y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 79

```
dsolve(x^2*diff(y(x),x$2)+lambda*x*diff(y(x),x)+(a*x^2+b*x+c)*y(x)=0,y(x), singsol=all)
```

$$\begin{aligned}y(x) &= c_1x^{-\frac{\lambda}{2}}\,\mathrm{WhittakerM}\left(-\frac{ib}{2\sqrt{a}}, \frac{\sqrt{\lambda^2-4c-2\lambda+1}}{2}, 2i\sqrt{a}\,x\right)\\ &\quad + c_2x^{-\frac{\lambda}{2}}\,\mathrm{WhittakerW}\left(-\frac{ib}{2\sqrt{a}}, \frac{\sqrt{\lambda^2-4c-2\lambda+1}}{2}, 2i\sqrt{a}\,x\right)\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.224 (sec). Leaf size: 159

```
DSolve[x^2*y''[x]+\[Lambda]*x*y'[x]+(a*x^2+b*x+c)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$\begin{aligned}&y(x)\\ &\to e^{-i\sqrt{a}x}x^{\frac{1}{2}\left(\sqrt{(\lambda-1)^2-4c}-\lambda+1\right)}\left(c_1\,\mathrm{HypergeometricU}\left(\frac{1}{2}\left(\frac{ib}{\sqrt{a}}+\sqrt{(\lambda-1)^2-4c}+1\right), \sqrt{(\lambda-1)^2-4c}\right.\right.\\ &\qquad\left.\left.+1, 2i\sqrt{a}x\right) + c_2L_{\frac{1}{2}\left(-\frac{ib}{\sqrt{a}}-\sqrt{(\lambda-1)^2-4c}-1\right)}^{\sqrt{(\lambda-1)^2-4c}}\left(2i\sqrt{a}x\right)\right)\end{aligned}$$

29.23 problem 132

Internal problem ID [10966]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 132.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + axy' + (b\,x^n + c)\,y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 85

```
dsolve(x^2*diff(y(x),x$2)+a*x*diff(y(x),x)+(b*x^n+c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{-\frac{a}{2}+\frac{1}{2}}\,\mathrm{BesselJ}\left(\frac{\sqrt{a^2-2a-4c+1}}{n}, \frac{2\sqrt{b}\,x^{\frac{n}{2}}}{n}\right) + c_2 x^{-\frac{a}{2}+\frac{1}{2}}\,\mathrm{BesselY}\left(\frac{\sqrt{a^2-2a-4c+1}}{n}, \frac{2\sqrt{b}\,x^{\frac{n}{2}}}{n}\right)$$

✓ Solution by Mathematica

Time used: 0.287 (sec). Leaf size: 168

```
DSolve[x^2*y''[x]+a*x*y'[x]+(b*x^n+c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to n^{\frac{a-1}{n}} b^{-\frac{a-1}{2n}} (x^n)^{-\frac{a-1}{2n}} \left(c_1 \operatorname{Gamma}\left(1 - \frac{\sqrt{a^2-2a-4c+1}}{n}\right) \operatorname{BesselJ}\left(-\frac{\sqrt{a^2-2a-4c+1}}{n}, \frac{2\sqrt{b}\sqrt{x^n}}{n}\right) + c_2 \operatorname{Gamma}\left(\frac{n+\sqrt{a^2-2a-4c+1}}{n}\right) \operatorname{BesselJ}\left(\frac{\sqrt{a^2-2a-4c+1}}{n}, \frac{2\sqrt{b}\sqrt{x^n}}{n}\right)\right)$$

29.24 problem 133

Internal problem ID [10967]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 133.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + axy' + x^n(b\,x^n + c)\,y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 91

```
dsolve(x^2*diff(y(x),x$2)+a*x*diff(y(x),x)+x^n*(b*x^n+c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{-\frac{a}{2}-\frac{n}{2}+\frac{1}{2}} \operatorname{WhittakerM}\left(-\frac{ic}{2n\sqrt{b}}, \frac{a-1}{2n}, \frac{2i\sqrt{b}\,x^n}{n}\right) + c_2 x^{-\frac{a}{2}-\frac{n}{2}+\frac{1}{2}} \operatorname{WhittakerW}\left(-\frac{ic}{2n\sqrt{b}}, \frac{a-1}{2n}, \frac{2i\sqrt{b}\,x^n}{n}\right)$$

✓ Solution by Mathematica

Time used: 0.269 (sec). Leaf size: 165

```
DSolve[x^2*y''[x]+a*x*y'[x]+x^n*(b*x^n+c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2^{\frac{a+n-1}{2n}} x^{\frac{1}{2}(-a-n+1)} (x^n)^{\frac{a+n-1}{2n}} e^{\frac{i\sqrt{b}x^n}{n}} \left(c_1 \operatorname{HypergeometricU}\left(-\frac{-a+\frac{ic}{\sqrt{b}}-n+1}{2n}, \frac{a+n-1}{n}, -\frac{2i\sqrt{b}x^n}{n}\right) + c_2 L_{-\frac{a-\frac{ic}{\sqrt{b}}+n-1}{2n}}^{\frac{a-1}{n}}\left(-\frac{2i\sqrt{b}x^n}{n}\right)\right)$$

29.25 problem 134

Internal problem ID [10968]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 134.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + (ax + b)\,y' + yc = 0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 135

```
dsolve(x^2*diff(y(x),x$2)+(a*x+b)*diff(y(x),x)+c*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{-\frac{\sqrt{a^2-2a-4c+1}}{2}-\frac{a}{2}+\frac{1}{2}} \operatorname{KummerM}\left(-\frac{1}{2}+\frac{\sqrt{a^2-2a-4c+1}}{2}+\frac{a}{2}, 1 + \sqrt{a^2-2a-4c+1}, \frac{b}{x}\right) + c_2 x^{-\frac{\sqrt{a^2-2a-4c+1}}{2}-\frac{a}{2}+\frac{1}{2}} \operatorname{KummerU}\left(-\frac{1}{2} + \frac{\sqrt{a^2-2a-4c+1}}{2}+\frac{a}{2}, 1+\sqrt{a^2-2a-4c+1}, \frac{b}{x}\right)$$

✓ Solution by Mathematica

Time used: 0.574 (sec). Leaf size: 243

```
DSolve[x^2*y''[x]+(a*x+b)*y'[x]+c*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -i^{-\sqrt{a^2-2a-4c+1}+a+1} b^{\frac{1}{2}\left(-\sqrt{a^2-2a-4c+1}+a-1\right)} \left(\frac{1}{x}\right)^{\frac{1}{2}\left(-\sqrt{a^2-2a-4c+1}+a-1\right)} \left(c_2 i^{2\sqrt{a^2-2a-4c+1}} b^{\sqrt{a^2-2a-4c+1}} \left(\frac{1}{x}\right)^{\sqrt{\ldots}} \ldots + 1, \frac{b}{x}\right) + c_1 \text{Hypergeometric1F1}\left(\frac{1}{2}\left(a - \sqrt{a^2-2a-4c+1} - 1\right), 1 - \sqrt{a^2-2a-4c+1}, \frac{b}{x}\right)\right)$$

29.26 problem 135

Internal problem ID [10969]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 135.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + a\,x^2y' + \left(b\,x^2 + xc + d\right)y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 83

```
dsolve(x^2*diff(y(x),x$2)+a*x^2*diff(y(x),x)+(b*x^2+c*x+d)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{ax}{2}} \operatorname{WhittakerM}\left(\frac{c}{\sqrt{a^2-4b}}, \frac{\sqrt{1-4d}}{2}, \sqrt{a^2-4b}\,x\right) + c_2 \mathrm{e}^{-\frac{ax}{2}} \operatorname{WhittakerW}\left(\frac{c}{\sqrt{a^2-4b}}, \frac{\sqrt{1-4d}}{2}, \sqrt{a^2-4b}\,x\right)$$

✓ Solution by Mathematica

Time used: 0.228 (sec). Leaf size: 157

```
DSolve[x^2*y''[x]+a*x^2*y'[x]+(b*x^2+c*x+d)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^{\frac{1}{2}\left(\sqrt{1-4d}+1\right)} e^{-\frac{1}{2}x\left(\sqrt{a^2-4b}+a\right)} \left(c_1 \operatorname{HypergeometricU}\left(\frac{1}{2}\left(-\frac{2c}{\sqrt{a^2-4b}}+\sqrt{1-4d}+1\right), \sqrt{1-4d}+1, \sqrt{a^2-4b}x\right) + c_2 L^{\sqrt{1-4d}}_{\frac{c}{\sqrt{a^2-4b}}-\frac{1}{2}\sqrt{1-4d}-\frac{1}{2}}\left(\sqrt{a^2-4b}x\right)\right)$$

29.27 problem 136

Internal problem ID [10970]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 136.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + \left(a\,x^2 + b\right) y' + c\left((-c + a)\,x^2 + b\right) y = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 245

```
dsolve(x^2*diff(y(x),x$2)+(a*x^2+b)*diff(y(x),x)+c*((a-c)*x^2+b)*y(x)=0,y(x), singsol=all)
```

$$\begin{aligned}y(x) = {} & c_1\sqrt{x}\,\mathrm{e}^{\frac{-c\,x^2+b}{x}}\operatorname{HeunD}\Bigg(4\sqrt{-b\,(-2c+a)}, -4\sqrt{-b\,(-2c+a)} - 1 \\ & + (-4a+8c)\,b, 8\sqrt{-b\,(-2c+a)}, -4\sqrt{-b\,(-2c+a)} + 1 \\ & + (-8c+4a)\,b, \frac{\sqrt{-b\,(-2c+a)}\,x - b}{\sqrt{-b\,(-2c+a)}\,x + b}\Bigg) \\ & + c_2\sqrt{x}\,\mathrm{e}^{-x(a-c)}\operatorname{HeunD}\Bigg(-4\sqrt{-b\,(-2c+a)}, -4\sqrt{-b\,(-2c+a)} - 1 \\ & + (-4a+8c)\,b, 8\sqrt{-b\,(-2c+a)}, -4\sqrt{-b\,(-2c+a)} + 1 \\ & + (-8c+4a)\,b, \frac{\sqrt{-b\,(-2c+a)}\,x - b}{\sqrt{-b\,(-2c+a)}\,x + b}\Bigg)\end{aligned}$$

✓ Solution by Mathematica

Time used: 1.026 (sec). Leaf size: 44

```
DSolve[x^2*y''[x]+(a*x^2+b)*y'[x]+c*((a-c)*x^2+b)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to e^{-cx} \left(c_2 \int_1^x e^{\frac{b}{K[1]} - aK[1] + 2cK[1]} dK[1] + c_1 \right)$$

29.28 problem 137

Internal problem ID [10971]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 137.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \left(a\,x^2 + bx\right)y' - by = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 48

```
dsolve(x^2*diff(y(x),x$2)+(a*x^2+b*x)*diff(y(x),x)-b*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^{-b}\mathrm{e}^{-ax} + c_2\left(1 - b\,\mathrm{e}^{-ax}(\Gamma(b) - \Gamma(b, -ax))\,(-ax)^{-b}\right)$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 43

```
DSolve[x^2*y''[x]+(a*x^2+b*x)*y'[x]-b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-ax}\left(\frac{c_1(-ax)^{-b}\Gamma(b+1, -ax)}{a} + c_2x^{-b}\right)$$

29.29 problem 138

Internal problem ID [10972]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 138.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + \left(a\,x^2 + bx\right)y' + \left(k(a-k)\,x^2 + (an + bk - 2kn)\,x + n(-n + b - 1)\right)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(x^2*diff(y(x),x$2)+(a*x^2+b*x)*diff(y(x),x)+(k*(a-k)*x^2+(a*n+b*k-2*k*n)*x+n*(b-n-1))
```

$$y(x) = c_1 x^{-n} \mathrm{e}^{-kx} + c_2 \,\mathrm{WhittakerM}\left(-\frac{b}{2} + n, -\frac{b}{2} + n + \frac{1}{2}, (-2k + a)\,x\right) x^{-\frac{b}{2}} \mathrm{e}^{-\frac{ax}{2}}$$

✓ Solution by Mathematica

Time used: 0.504 (sec). Leaf size: 64

```
DSolve[x^2*y''[x]+(a*x^2+b*x)*y'[x]+(k*(a-k)*x^2+(a*n+b*k-2*k*n)*x+n*(b-n-1))*y[x]==0,y[x],x
```

$$y(x) \to e^{-kx} x^{-n} \left(c_1 - c_2 x^{-b+2n+1} (x(a-2k))^{b-2n-1} \Gamma(-b + 2n + 1, (a-2k)x)\right)$$

29.30 problem 139

Internal problem ID [10973]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 139.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{a_2x^2y'' + \left(a_1x^2 + b_1x\right)y' + \left(x^2a_0 + b_0x + c_0\right)y = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 165

```
dsolve(a__2*x^2*diff(y(x),x$2)+(a__1*x^2+b__1*x)*diff(y(x),x)+(a__0*x^2+b__0*x+c__0)*y(x)=0,
```

$$y(x) = c_1 \mathrm{e}^{-\frac{a_1x}{2a_2}} x^{-\frac{b_1}{2a_2}} \operatorname{WhittakerM}\left(-\frac{b_1a_1 - 2a_2b_0}{2a_2\sqrt{-4a_0a_2 + a_1^2}}, \frac{\sqrt{a_2^2 + (-2b_1 - 4c_0)a_2 + b_1^2}}{2a_2}, \frac{\sqrt{-4a_0a_2 + a_1^2}\,x}{a_2}\right) + c_2 \mathrm{e}^{-\frac{a_1x}{2a_2}} x^{-\frac{b_1}{2a_2}} \operatorname{WhittakerW}\left(-\frac{b_1a_1 - 2a_2b_0}{2a_2\sqrt{-4a_0a_2 + a_1^2}}, \frac{\sqrt{a_2^2 + (-2b_1 - 4c_0)a_2 + b_1^2}}{2a_2}, \frac{\sqrt{-4a_0a_2 + a_1^2}\,x}{a_2}\right)$$

✓ Solution by Mathematica

Time used: 0.538 (sec). Leaf size: 272

```
DSolve[a2*x^2*y''[x]+(a1*x^2+b1*x)*y'[x]+(a0*x^2+b0*x+c0)*y[x]==0,y[x],x,IncludeSingularSolu
```

$$y(x) \to e^{-\frac{x\left(\sqrt{a1^2-4a0a2}+a1\right)}{2a2}} x^{\frac{\sqrt{a2^2-2a2(b1+2c0)+b1^2}+a2-b1}{2a2}} \left(c_1 \text{HypergeometricU}\left(\frac{-\frac{2b0a2}{\sqrt{a1^2-4a0a2}}+a2+\frac{a1b1}{\sqrt{a1^2-4a0a2}}}{2a2}\right.\right.$$

$$\left.\left. + c_2 L_{-\frac{-\frac{2b0a2}{\sqrt{a1^2-4a0a2}}+a2+\frac{a1b1}{\sqrt{a1^2-4a0a2}}+\sqrt{a2^2-2(b1+2c0)a2+b1^2}}{2a2}}^{\frac{\sqrt{a2^2-2(b1+2c0)a2+b1^2}}{a2}}\left(\frac{\sqrt{a1^2-4a0a2}x}{a2}\right)\right)$$

29.31 problem 140

Internal problem ID [10974]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 140.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + \left(a\,x^2 + (ba-1)\,x + b\right)y' + a^2bxy = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 298

```
dsolve(x^2*diff(y(x),x$2)+(a*x^2+(a*b-1)*x+b)*diff(y(x),x)+a^2*b*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{1-\frac{ab}{2}} \mathrm{e}^{\frac{-a\,x^2+b}{x}} \operatorname{HeunD}\left(4\sqrt{ab}, \frac{(-2ab-2)\,(ab)^{\frac{3}{2}} - ba\left((-2ab+6)\sqrt{ab} + (ab-2)^2\right)}{ab}, -8\sqrt{ab}\,(ab-1)\,, \frac{(-2ab-2)\,(ab)^{\frac{3}{2}} + ba\left((2ab-6)\sqrt{ab} + (ab-2)^2\right)}{ab}, \frac{\sqrt{ab}\,x-b}{\sqrt{ab}\,x+b}\right) + c_2 x^{1-\frac{ab}{2}} \operatorname{HeunD}\left(-4\sqrt{ab}, \frac{(-2ab-2)\,(ab)^{\frac{3}{2}} - ba\left((-2ab+6)\sqrt{ab} + (ab-2)^2\right)}{ab}, -8\sqrt{ab}\,(ab-1)\,, \frac{(-2ab-2)\,(ab)^{\frac{3}{2}} + ba\left((2ab-6)\sqrt{ab} + (ab-2)^2\right)}{ab}, \frac{\sqrt{ab}\,x-b}{\sqrt{ab}\,x+b}\right)$$

✓ Solution by Mathematica

Time used: 4.002 (sec). Leaf size: 67

```
DSolve[x^2*y''[x]+(a*x^2+(a*b-1)*x+b)*y'[x]+a^2*b*x*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{e^{-ax}(ax+1)\left(c_2 \int_1^x \frac{a^2 e^{\frac{b}{K[1]}+aK[1]} K[1]^{1-ab}}{(aK[1]+1)^2} dK[1] + c_1\right)}{a}$$

29.32 problem 141

Internal problem ID [10975]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 141.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - 2x(x^2-a)\,y' + \left(2n\,x^2 + ((-1)^n - 1)\,a\right)y = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 93

```
dsolve(x^2*diff(y(x),x$2)-2*x*(x^2-a)*diff(y(x),x)+(2*n*x^2+( (-1)^n-1)*a )*y(x)=0,y(x), sin
```

$$y(x) = c_1 x^{-a-\frac{1}{2}} \mathrm{e}^{\frac{x^2}{2}} \operatorname{WhittakerM}\left(\frac{a}{2}+\frac{n}{2}+\frac{1}{4}, \frac{\sqrt{1-4a(-1)^n+4a^2}}{4}, x^2\right) + c_2 x^{-a-\frac{1}{2}} \mathrm{e}^{\frac{x^2}{2}} \operatorname{WhittakerW}\left(\frac{a}{2}+\frac{n}{2}+\frac{1}{4}, \frac{\sqrt{1-4a(-1)^n+4a^2}}{4}, x^2\right)$$

✓ Solution by Mathematica

Time used: 0.646 (sec). Leaf size: 231

```
DSolve[x^2*y''[x]-2*x*(x^2-a)*y'[x]+(2*n*x^2+( (-1)^n-1)*a )*y[x]==0,y[x],x,IncludeSingularS
```

$$y(x) \to i^{-a}(-1)^{\frac{1}{4}\left(1-\sqrt{4a^2-4a(-1)^n+1}\right)}x^{\frac{1}{2}\left(-\sqrt{4a^2-4a(-1)^n+1}-2a+1\right)}\left(c_1 \text{Hypergeometric1F1}\left(\frac{1}{4}\left(-2a-2n-\sqrt{4a^2-4\right.\right.\right.$$

$$\left.\left. -\frac{1}{2}\sqrt{4a^2-4(-1)^n a+1}, x^2\right)$$

$$+c_2 i^{\sqrt{4a^2-4a(-1)^n+1}}x^{\sqrt{4a^2-4a(-1)^n+1}} \text{Hypergeometric1F1}\left(\frac{1}{4}\left(-2a-2n+\sqrt{4a^2-4(-1)^n a+1}+1\right), \frac{1}{2}\left(\sqrt{\right.\right.$$

29.33 problem 142

Internal problem ID [10976]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 142.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x(a x^2 + bx + c) y' + (A x^3 + B x^2 + Cx + d) y = 0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 263

```
dsolve(x^2*diff(y(x),x$2)+x*(a*x^2+b*x+c)*diff(y(x),x)+(A*x^3+B*x^2+C*x+d)*y(x)=0,y(x), sing
```

$$y(x) = c_1 x^{-\frac{c}{2}+\frac{\sqrt{c^2-2c-4d+1}}{2}+\frac{1}{2}} e^{\frac{x(-a^2x-2ab+2A)}{2a}} \operatorname{HeunB}\left(\sqrt{c^2-2c-4d+1}, \frac{\sqrt{2}(-ab+2A)}{a^{\frac{3}{2}}}, \frac{(-c-1)a^3+2Ba^2-}{a^3} \right.$$
$$\left. -\frac{\sqrt{2}\sqrt{a}\,x}{2}\right)$$
$$+ c_2 x^{-\frac{c}{2}-\frac{\sqrt{c^2-2c-4d+1}}{2}+\frac{1}{2}} e^{\frac{x(-a^2x-2ab+2A)}{2a}} \operatorname{HeunB}\left(-\sqrt{c^2-2c-4d+1}, \frac{\sqrt{2}(-ab+2A)}{a^{\frac{3}{2}}}, \frac{(-c-1)a^3+2B a}{a^3} \right.$$
$$\left. -\frac{\sqrt{2}\sqrt{a}\,x}{2}\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y''[x]+x*(a*x^2+b*x+c)*y'[x]+(A*x^3+B*x^2+C0*x+d)*y[x]==0,y[x],x,IncludeSingularS
```

Not solved

29.34 problem 143

Internal problem ID [10977]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 143.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + a\,x^n y' - \left(ab\,x^n + ac\,x^{-1+n} + b^2x^2 + 2bcx + c^2 - c\right)y = 0$$

✗ Solution by Maple

```
dsolve(x^2*diff(y(x),x$2)+a*x^n*diff(y(x),x)-(a*b*x^n+a*c*x^(n-1)+b^2*x^2+2*b*c*x+c^2-c)*y(x
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y''[x]+a*x^n*y'[x]-(a*b*x^n+a*c*x^(n-1)+b^2*x^2+2*b*c*x+c^2-c)*y[x]==0,y[x],x,Inc
```

Not solved

29.35 problem 144

Internal problem ID [10978]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 144.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + a\,x^n y' + \left(ab\,x^{n+2m} - b^2x^{4m+2} + am\,x^{-1+n} - m^2 - m\right)y = 0$$

✗ Solution by Maple

```
dsolve(x^2*diff(y(x),x$2)+a*x^n*diff(y(x),x)+(a*b*x^(n+2*m)-b^2*x^(4*m+2)+a*m*x^(n-1)-m^2-m)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y''[x]+a*x^n*y'[x]+(a*b*x^(n+2*m)-b^2*x^(4*m+2)+a*m*x^(n-1)-m^2-m)*y[x]==0,y[x],x
```

Not solved

29.36 problem 145

Internal problem ID [10979]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 145.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x(x^n a + b)\,y' + b(x^n a - 1)\,y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 132

```
dsolve(x^2*diff(y(x),x$2)+x*(a*x^n+b)*diff(y(x),x)+b*(a*x^n-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^{-b} + c_2\mathrm{e}^{-\frac{a\,x^n}{2n}}\left(n\left((b+n+1)\,x^{\frac{1}{2}-\frac{3n}{2}-\frac{b}{2}} + a\,x^{\frac{1}{2}-\frac{b}{2}-\frac{n}{2}}\right)\mathrm{WhittakerM}\left(\frac{1+b-n}{2n}, \frac{b+2n+1}{2n}, \frac{a\,x^n}{n}\right) + x^{\frac{1}{2}-\frac{3n}{2}-\frac{b}{2}}\,\mathrm{WhittakerM}\left(\frac{b+n+1}{2n}, \frac{b+2n+1}{2n}, \frac{a\,x^n}{n}\right)(b+n+1)^2\right)$$

✓ Solution by Mathematica

Time used: 0.146 (sec). Leaf size: 76

```
DSolve[x^2*y''[x]+x*(a*x^n+b)*y'[x]+b*(a*x^n-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to (-1)^{-\frac{b}{n}}n^{\frac{b}{n}-1}a^{-\frac{b}{n}}(x^n)^{-\frac{b}{n}}\left((b+1)c_1(-1)^{b/n}\Gamma\left(\frac{b+1}{n}, 0, \frac{ax^n}{n}\right) + c_2n\right)$$

29.37 problem 146

Internal problem ID [10980]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 146.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + (a\,x^n + b)\,y'x + \left(\alpha\,x^{2n} + \beta\,x^n + \gamma\right)y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 167

```
dsolve(x^2*diff(y(x),x$2)+x*(a*x^n+b)*diff(y(x),x)+(alpha*x^(2*n)+beta*x^n+gamma)*y(x)=0,y(x
```

$$y(x) = c_1 x^{\frac{1}{2}-\frac{b}{2}-\frac{n}{2}} e^{-\frac{a\,x^n}{2n}} \operatorname{WhittakerM}\left(-\frac{(n+b-1)\,a-2\beta}{2\sqrt{a^2-4\alpha}\,n}, \frac{\sqrt{b^2-2b-4\gamma+1}}{2n}, \frac{\sqrt{a^2-4\alpha}\,x^n}{n}\right) + c_2 x^{\frac{1}{2}-\frac{b}{2}-\frac{n}{2}} e^{-\frac{a\,x^n}{2n}} \operatorname{WhittakerW}\left(-\frac{(n+b-1)\,a-2\beta}{2\sqrt{a^2-4\alpha}\,n}, \frac{\sqrt{b^2-2b-4\gamma+1}}{2n}, \frac{\sqrt{a^2-4\alpha}\,x^n}{n}\right)$$

✓ Solution by Mathematica

Time used: 0.485 (sec). Leaf size: 420

```
DSolve[x^2*y''[x]+x*(a*x^n+b)*y'[x]+(\[Alpha]*x^(2*n)+\[Beta]*x^n+\[Gamma])*y[x]==0,y[x],x,I
```

$$y(x) \to x^{\frac{1}{2}-\frac{n}{2}} 2^{\frac{1}{2}\left(\frac{\sqrt{n^2\left(b^2-2b-4\gamma+1\right)}}{n^2}+1\right)} e^{-\frac{\left(\sqrt{a^2-4\alpha}+a\right)x^n}{2n}} (x^n)^{\frac{\sqrt{n^2\left(b^2-2b-4\gamma+1\right)}-bn+n^2}{2n^2}} \left(c_1 \text{HypergeometricU}\left(\frac{\left(n^2+\sqrt{n^2\right.}}{}\right.\right.$$

$$\left.+c_2 L^{\frac{\sqrt{n^2\left(b^2-2b-4\gamma+1\right)}}{n^2}}_{\frac{-\left(\left(n^2+\sqrt{n^2\left(b^2-2b-4\gamma+1\right)}\right)a^2\right)-n(b+n-1)\sqrt{a^2-4\alpha}a+4n^2\alpha+2n\sqrt{a^2-4\alpha}\beta+4\alpha\sqrt{n^2\left(b^2-2b-4\gamma+1\right)}}{2n^2\left(a^2-4\alpha\right)}}\left(\frac{x^n\sqrt{a^2-4\alpha}}{n}\right)\right)$$

29.38 problem 147

Internal problem ID [10981]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 147.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x(2a\,x^n + b)\,y' + \left(x^{2n}a^2 + a(b+n-1)\,x^n + \alpha\,x^{2m} + \beta\,x^m + \gamma\right)y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 131

```
dsolve(x^2*diff(y(x),x$2)+x*(2*a*x^n+b)*diff(y(x),x)+(a^2*x^(2*n)+a*(b+n-1)*x^n+alpha*x^(2*m
```

$$y(x) = c_1 x^{-\frac{b}{2}+\frac{1}{2}-\frac{m}{2}} \mathrm{e}^{-\frac{a\,x^n}{n}} \operatorname{WhittakerM}\left(-\frac{i\beta}{2m\sqrt{\alpha}}, \frac{\sqrt{b^2-2b-4\gamma+1}}{2m}, \frac{2i\sqrt{\alpha}\,x^m}{m}\right) + c_2 x^{-\frac{b}{2}+\frac{1}{2}-\frac{m}{2}} \mathrm{e}^{-\frac{a\,x^n}{n}} \operatorname{WhittakerW}\left(-\frac{i\beta}{2m\sqrt{\alpha}}, \frac{\sqrt{b^2-2b-4\gamma+1}}{2m}, \frac{2i\sqrt{\alpha}\,x^m}{m}\right)$$

✓ Solution by Mathematica

Time used: 0.47 (sec). Leaf size: 291

```
DSolve[x^2*y''[x]+x*(2*a*x^n+b)*y'[x]+(a^2*x^(2*n)+a*(b+n-1)*x^n+\[Alpha]*x^(2*m)+\[Beta]*x^
```

$$y(x) \to x^{\frac{1}{2}-\frac{m}{2}} 2^{\frac{1}{2}\left(\frac{\sqrt{m^2(b^2-2b-4\gamma+1)}}{m^2}+1\right)} (x^n)^{-\frac{b}{2n}} (x^m)^{\frac{1}{2}\left(\frac{\sqrt{m^2(b^2-2b-4\gamma+1)}}{m^2}+1\right)} e^{-\frac{ax^n}{n}+\frac{i\sqrt{\alpha}x^m}{m}} \left(c_1 \operatorname{HypergeometricU}\left(\frac{m^2 \right. \right.$$

$$\left. -\frac{2ix^m\sqrt{\alpha}}{m}\right) + c_2 L^{\frac{\sqrt{m^2(b^2-2b-4\gamma+1)}}{m^2}}_{-\frac{m^2-\frac{i\beta m}{\sqrt{\alpha}}+\sqrt{m^2(b^2-2b-4\gamma+1)}}{2m^2}}\left(-\frac{2ix^m\sqrt{\alpha}}{m}\right)\right)$$

29.39 problem 148

Internal problem ID [10982]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 148.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + \left(x^{2+n}a + b\,x^2 + c\right)y' + \left(a\,x^{n+1}n + a\,x^n c + bc\right)y = 0$$

✗ Solution by Maple

```
dsolve(x^2*diff(y(x),x$2)+(a*x^(n+2)+b*x^2+c)*diff(y(x),x)+(a*n*x^(n+1)+a*c*x^n+b*c)*y(x)=0,
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y''[x]+(a*x^(n+2)+b*x^2+c)*y'[x]+(a*n*x^(n+1)+a*c*x^n+b*c)*y[x]==0,y[x],x,Include
```

Not solved

30 Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2 + bx + c)y'' + f(x)y' + g(x)y = 0$

30.1 problem 149

Internal problem ID [10983]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 149.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-x^2+1)\,y''+n(-1+n)\,y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 57

```
dsolve((1-x^2)*diff(y(x),x$2)+n*(n-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(-x^2+1)\,\mathrm{hypergeom}\left(\left[-\frac{n}{2}+1,\frac{n}{2}+\frac{1}{2}\right],\left[\frac{1}{2}\right],x^2\right) + c_2(-x^3+x)\,\mathrm{hypergeom}\left(\left[1+\frac{n}{2},\frac{3}{2}-\frac{n}{2}\right],\left[\frac{3}{2}\right],x^2\right)$$

✓ Solution by Mathematica

Time used: 0.149 (sec). Leaf size: 56

```
DSolve[(1-x^2)*y''[x]+n*(n-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to ic_2x\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2}-\frac{n}{2},\frac{n}{2},\frac{3}{2},x^2\right) + c_1\,\mathrm{Hypergeometric2F1}\left(\frac{n-1}{2},-\frac{n}{2},\frac{1}{2},x^2\right)$$

30.2 problem 150

Internal problem ID [10984]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 150.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(-a^2+x^2\right)y''+y'b-6y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 75

```
dsolve((x^2-a^2)*diff(y(x),x$2)+b*diff(y(x),x)-6*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(\frac{b^3}{24}+\frac{b^2x}{4}+\frac{(-5a^2+9x^2)\,b}{12}-a^2x+x^3\right)+c_2(a-x)\,(x+a)\,(b-4x)\left(\frac{x+a}{a-x}\right)^{\frac{b}{2a}}$$

✓ Solution by Mathematica

Time used: 13.059 (sec). Leaf size: 1171

```
DSolve[(x^2-a^2)*y''[x]+b*y'[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{b\,\text{arctanh}\left(\frac{x}{a}\right)}{2a}+\frac{\left(b^5-20a^2b^3+64a^4b+\sqrt{b^2\left(64a^4-20b^2a^2+b^4\right)^2}\right)\text{RootSum}\left[-b^3-6\#1b^2+10a^2b-18\#1^2b-24\#1^3+24a^2\#1\&,\log(x-\#1)\&\right]}{2\left(b^5-20a^2b^3+64a^4b\right)}}(a+x)^{\frac{1}{2}-\frac{\sqrt{b^2\left(64a^4-20b^2a^2+b^4\right)^2}}{4ab\left(32a^3-16ba^2-2b^2a+b^3\right)}}(4x-b)^{\frac{b^5}{2\left(b^5-20a^2b^3+64a^4b\right)}-\frac{10a^2b^3}{b^5-20a^2b^3+64a^4b}+\frac{32a^4b}{b^5-20a^2b^3+64a^4b}-\frac{\sqrt{b^2\left(64a^4-20b^2a^2+b^4\right)^2}}{2\left(b^5-20a^2b^3+64a^4b\right)}}c_2\int_1^x -\frac{e^{-\frac{\left(b^5-20a^2b^3+64a^4b+\sqrt{b^2\left(64a^4-20b^2a^2+b^4\right)^2}\right)\text{RootSum}\left[-b^3-6\#1b^2+10a^2b-18\#1^2b-24\#1^3+24a^2\#1\&,\log(K[1]-\#1)\&\right]}{b^5-20a^2b^3+64a^4b}}(K[1]-a)^{\frac{\sqrt{b^2\ldots}}{2a\ldots}}}{\ldots}$$

$$(x-a)^{\frac{1}{2}-\frac{\sqrt{b^2\left(64a^4-20b^2a^2+b^4\right)^2}}{4a(4a-b)b(2a+b)(4a+b)}}+e^{\frac{1}{2}\left(\frac{b\,\text{arctanh}\left(\frac{x}{a}\right)}{a}+\frac{\left(b^5-20a^2b^3+64a^4b+\sqrt{\left(b^5-20a^2b^3+64a^4b\right)^2}\right)\text{RootSum}\left[-b^3-6\#1b^2+10a^2b-18\#1^2b-24\#1^3+24a^2\#1\&,\log(x-\#1)\&\right]}{b^5-20a^2b^3+64a^4b}\right)}(a+x)^{\frac{1}{4}\left(2-\frac{\sqrt{\left(b^5-20a^2b^3+64a^4b\right)^2}}{ab\left(32a^3-16ba^2-2b^2a+b^3\right)}\right)}(4x-b)^{\frac{b^5-20a^2b^3+64a^4b-\sqrt{\left(b^5-20a^2b^3+64a^4b\right)^2}}{2\left(b^5-20a^2b^3+64a^4b\right)}}c_1(x-a)^{\frac{1}{2}-\frac{\sqrt{\left(b^5-20a^2b^3+64a^4b\right)^2}}{4a(4a-b)b(2a+b)(4a+b)}}$$

30.3 problem 151

Internal problem ID [10985]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2 + bx + c)y'' + f(x)y' + g(x)y = 0$
Problem number: 151.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']]`

$$\boxed{(x^2 - 1)\, y'' + y'x + ay = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve((x^2-1)*diff(y(x),x$2)+x*diff(y(x),x)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \left(x + \sqrt{x^2 - 1}\right)^{i\sqrt{a}} + c_2 \left(x + \sqrt{x^2 - 1}\right)^{-i\sqrt{a}}$$

✓ Solution by Mathematica

Time used: 0.144 (sec). Leaf size: 97

```
DSolve[(x^2-1)*y''[x]+x*y'[x]+a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \to\ & c_1 \cos\left(\frac{1}{2}\sqrt{a}\left(\log\left(1 - \frac{x}{\sqrt{x^2 - 1}}\right) - \log\left(\frac{x}{\sqrt{x^2 - 1}} + 1\right)\right)\right) \\ & - c_2 \sin\left(\frac{1}{2}\sqrt{a}\left(\log\left(1 - \frac{x}{\sqrt{x^2 - 1}}\right) - \log\left(\frac{x}{\sqrt{x^2 - 1}} + 1\right)\right)\right) \end{aligned}$$

30.4 problem 152

Internal problem ID [10986]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 152.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer, [_2nd_order, _linear, `_with_symmetry_[0,F(x)]`]`

$$\boxed{(-x^2+1)\, y''-y'x+y n^2=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve((1-x^2)*diff(y(x),x$2)-x*diff(y(x),x)+n^2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x+\sqrt{x^2-1}\right)^{-n}+c_2\left(x+\sqrt{x^2-1}\right)^{n}$$

✓ Solution by Mathematica

Time used: 0.139 (sec). Leaf size: 91

```
DSolve[(1-x^2)*y''[x]-x*y'[x]+n^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}y(x)\to c_1\cosh\left(\frac{1}{2}n\left(\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)-\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)\right)\right)\\ -ic_2\sinh\left(\frac{1}{2}n\left(\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)-\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)\right)\right)\end{aligned}$$

30.5 problem 153

Internal problem ID [10987]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2 + bx + c)y'' + f(x)y' + g(x)y = 0$
Problem number: 153.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-x^2 + 1)\, y'' - 2y'x + n(1 + n)\, y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 15

```
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+n*(n+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{LegendreP}(n, x) + c_2 \operatorname{LegendreQ}(n, x)$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 18

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+n*(n+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{LegendreP}(n, x) + c_2 \operatorname{LegendreQ}(n, x)$$

30.6 problem 154

Internal problem ID [10988]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 154.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-x^2+1)\,y''-2y'x+\nu(\nu+1)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+nu*(nu+1)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\mathrm{LegendreP}\,(\nu,x)+c_2\,\mathrm{LegendreQ}\,(\nu,x)$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 18

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+\[Nu]*(\[Nu]+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\to c_1\,\mathrm{LegendreP}(\nu,x)+c_2\,\mathrm{LegendreQ}(\nu,x)$$

30.7 problem 155

Internal problem ID [10989]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2 + bx + c)y'' + f(x)y' + g(x)y = 0$
Problem number: 155.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\left(-x^2 + 1\right) y'' - 3y'x + ny(2 + n) = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 62

```
dsolve((1-x^2)*diff(y(x),x$2)-3*x*diff(y(x),x)+n*(n+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1\left(x + \sqrt{x^2 - 1}\right)^{-1-n}}{\sqrt{x^2 - 1}} + \frac{c_2\left(x + \sqrt{x^2 - 1}\right)^{n}}{\left(\sqrt{x^2 - 1} - x\right)\sqrt{x^2 - 1}}$$

✓ Solution by Mathematica

Time used: 0.061 (sec). Leaf size: 42

```
DSolve[(1-x^2)*y''[x]-3*x*y'[x]+n*(n+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 P_{n+\frac{1}{2}}^{\frac{1}{2}}(x) + c_2 Q_{n+\frac{1}{2}}^{\frac{1}{2}}(x)}{\sqrt[4]{x^2 - 1}}$$

30.8 problem 156

Internal problem ID [10990]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 156.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(x^2-1)\,y''+2(1+n)\,xy'-(\nu+n+1)\,(\nu-n)\,y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 35

```
dsolve((x^2-1)*diff(y(x),x$2)+2*(n+1)*x*diff(y(x),x)-(nu+n+1)*(nu-n)*y(x)=0,y(x), singsol=al
```

$$y(x)=c_1\left(x^2-1\right)^{-\frac{n}{2}}\operatorname{LegendreP}\left(\nu,n,x\right)+c_2\left(x^2-1\right)^{-\frac{n}{2}}\operatorname{LegendreQ}\left(\nu,n,x\right)$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 32

```
DSolve[(x^2-1)*y''[x]+2*(n+1)*x*y'[x]-(\[Nu]+n+1)*(\[Nu]-n)*y[x]==0,y[x],x,IncludeSingularSo
```

$$y(x)\to\left(x^2-1\right)^{-n/2}\left(c_1P_\nu^n(x)+c_2Q_\nu^n(x)\right)$$

30.9 problem 157

Internal problem ID [10991]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2 + bx + c)y'' + f(x)y' + g(x)y = 0$
Problem number: 157.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$(x^2 - 1)\, y'' - 2(-1 + n)\, xy' - (\nu - n + 1)\, (\nu + n)\, y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve((x^2-1)*diff(y(x),x$2)-2*(n-1)*x*diff(y(x),x)-(nu-n+1)*(nu+n)*y(x)=0,y(x), singsol=al
```

$$y(x) = c_1 (x^2 - 1)^{\frac{n}{2}} \operatorname{LegendreP}(\nu, n, x) + c_2 (x^2 - 1)^{\frac{n}{2}} \operatorname{LegendreQ}(\nu, n, x)$$

✓ Solution by Mathematica

Time used: 0.064 (sec). Leaf size: 32

```
DSolve[(x^2-1)*y''[x]-2*(n-1)*x*y'[x]-(\[Nu]-n+1)*(\[Nu]+n)*y[x]==0,y[x],x,IncludeSingularSo
```

$$y(x) \to (x^2 - 1)^{n/2} (c_1 P_\nu^n(x) + c_2 Q_\nu^n(x))$$

30.10 problem 158

Internal problem ID [10992]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 158.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2-1)\,y''+(1+2a)\,y'-b(2a+b)\,y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 112

```
dsolve((x^2-1)*diff(y(x),x$2)+(2*a+1)*diff(y(x),x)-b*(2*a+b)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\mathrm{hypergeom}\left(\left[-\frac{1}{2}-\frac{\sqrt{8ab+4b^2+1}}{2},\frac{\sqrt{8ab+4b^2+1}}{2}-\frac{1}{2}\right],\left[-a-\frac{1}{2}\right],\frac{x}{2}+\frac{1}{2}\right)+c_2\left(\frac{x}{2}+\frac{1}{2}\right)^{\frac{3}{2}+a}\mathrm{hypergeom}\left(\left[1-\frac{\sqrt{8ab+4b^2+1}}{2}+a,\frac{\sqrt{8ab+4b^2+1}}{2}+1+a\right],\left[\frac{5}{2}+a\right],\frac{x}{2}+\frac{1}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.304 (sec). Leaf size: 152

```
DSolve[(x^2-1)*y''[x]+(2*a+1)*y'[x]-b*(2*a+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to 2^{a-\frac{1}{2}}c_2(x-1)^{\frac{1}{2}-a}\,\text{Hypergeometric2F1}\left(-a-\frac{1}{2}\sqrt{4b^2+8ab+1},\frac{1}{2}\sqrt{4b^2+8ab+1}-a,\frac{3}{2}-a,\frac{1}{2}-\frac{x}{2}\right)+c_1\,\text{Hypergeometric2F1}\left(\frac{1}{2}\left(-\sqrt{4b^2+8ab+1}-1\right),\frac{1}{2}\left(\sqrt{4b^2+8ab+1}-1\right),a+\frac{1}{2},\frac{1-x}{2}\right)$$

30.11 problem 159

Internal problem ID [10993]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2 + bx + c)y'' + f(x)y' + g(x)y = 0$
Problem number: 159.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-x^2+1)\, y'' + (2a-3)\, xy' + (1+n)\,(n+2a-1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

```
dsolve((1-x^2)*diff(y(x),x$2)+(2*a-3)*x*diff(y(x),x)+(n+1)*(n+2*a-1)*y(x)=0,y(x), singsol=al
```

$$y(x) = c_1 (x^2-1)^{-\frac{1}{4}+\frac{a}{2}} \operatorname{LegendreP}\left(n+a-\frac{1}{2}, -\frac{1}{2}+a, x\right) + c_2 (x^2-1)^{-\frac{1}{4}+\frac{a}{2}} \operatorname{LegendreQ}\left(n+a-\frac{1}{2}, -\frac{1}{2}+a, x\right)$$

✓ Solution by Mathematica

Time used: 0.325 (sec). Leaf size: 158

```
DSolve[(1-x^2)*y''[x]+(2*a-3)*y'[x]+(n+1)*(n+2*a-1)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to 2^{\frac{1}{2}-a} c_2 (x-1)^{a-\frac{1}{2}} \text{Hypergeometric2F1}\left(a - \frac{1}{2}\sqrt{4n^2+8a(n+1)-3} - 1, a + \frac{1}{2}\sqrt{4n^2+8a(n+1)-3} - 1, a + \frac{1}{2}, \frac{1-x}{2}\right) + c_1 \text{Hypergeometric2F1}\left(\frac{1}{2}\left(-\sqrt{4n^2+8a(n+1)-3} - 1\right), \frac{1}{2}\left(\sqrt{4n^2+8a(n+1)-3} - 1\right), \frac{3}{2} - a, \frac{1-x}{2}\right)$$

30.12 problem 160

Internal problem ID [10994]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 160.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(-x^2+1)\,y''+(\beta-\alpha-(\alpha+\beta+2)\,x)\,y'+n(n+\alpha+\beta+1)\,y=0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 61

```
dsolve((1-x^2)*diff(y(x),x$2)+(beta-alpha-(alpha+beta+2)*x)*diff(y(x),x)+n*(n+alpha+beta+1)*
```

$$y(x)=c_1\operatorname{hypergeom}\left([-n,n+\alpha+\beta+1]\,,[\beta+1]\,,\frac{x}{2}+\frac{1}{2}\right)+c_2\left(\frac{x}{2}+\frac{1}{2}\right)^{-\beta}\operatorname{hypergeom}\left([-n-\beta,n+\alpha+1]\,,[1-\beta]\,,\frac{x}{2}+\frac{1}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.28 (sec). Leaf size: 69

```
DSolve[(1-x^2)*y''[x]+(\[Beta]-\[Alpha]-(\[Alpha]+\[Beta]+2)*x)*y'[x]+n*(n+\[Alpha]+\[Beta]+
```

$$y(x)\to 2^{\alpha}c_2(x-1)^{-\alpha}\,\text{Hypergeometric2F1}\left(-n-\alpha,n+\beta+1,1-\alpha,\frac{1-x}{2}\right)+c_1\,\text{Hypergeometric2F1}\left(-n,n+\alpha+\beta+1,\alpha+1,\frac{1-x}{2}\right)$$

30.13 problem 161

Internal problem ID [10995]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2 + bx + c)y'' + f(x)y' + g(x)y = 0$
Problem number: 161.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(-x^2 + 1)\, y'' + (\alpha - \beta + (-2 + \beta + \alpha)\, x)\, y' + (1 + n)\,(n + \alpha + \beta)\, y = 0$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 64

```
dsolve((1-x^2)*diff(y(x),x$2)+(alpha-beta+(alpha+beta-2)*x)*diff(y(x),x)+(n+1)*(n+alpha+beta
```

$$y(x) = c_1 \operatorname{hypergeom}\left([n+1, -n-\alpha-\beta], [1-\beta], \frac{x}{2}+\frac{1}{2}\right) + c_2 \left(\frac{x}{2}+\frac{1}{2}\right)^{\beta} \operatorname{hypergeom}\left([-n-\alpha, n+\beta+1], [\beta+1], \frac{x}{2}+\frac{1}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.225 (sec). Leaf size: 74

```
DSolve[(1-x^2)*y''[x]+(\[Alpha]-\[Beta]+(\[Alpha]+\[Beta]-2)*x)*y'[x]+(n+1)*(n+\[Alpha]+\[Be
```

$$y(x) \to 2^{-\alpha} c_2 (x-1)^{\alpha} \operatorname{Hypergeometric2F1}\left(n+\alpha+1, -n-\beta, \alpha+1, \frac{1-x}{2}\right) + c_1 \operatorname{Hypergeometric2F1}\left(n+1, -n-\alpha-\beta, 1-\alpha, \frac{1-x}{2}\right)$$

30.14 problem 162

Internal problem ID [10996]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 162.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{(a\,x^2+b)\,y''+axy'+cy=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 61

```
dsolve((a*x^2+b)*diff(y(x),x$2)+a*x*diff(y(x),x)+c*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(\sqrt{a}\,x+\sqrt{a\,x^2+b}\right)^{\frac{i\sqrt{c}}{\sqrt{a}}}+c_2\left(\sqrt{a}\,x+\sqrt{a\,x^2+b}\right)^{-\frac{i\sqrt{c}}{\sqrt{a}}}$$

✓ Solution by Mathematica

Time used: 0.196 (sec). Leaf size: 74

```
DSolve[(a*x^2+b)*y''[x]+a*x*y'[x]+c*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\cos\left(\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+b}}\right)}{\sqrt{a}}\right)+c_2\sin\left(\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+b}}\right)}{\sqrt{a}}\right)$$

30.15 problem 163

Internal problem ID [10997]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2 + bx + c)y'' + f(x)y' + g(x)y = 0$
Problem number: 163.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x^2 + a)\, y'' + 2bxy' + 2(b-1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
dsolve((x^2+a)*diff(y(x),x$2)+2*b*x*diff(y(x),x)+2*(b-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \left(\frac{x^2 + a}{a}\right)^{-b+1} + c_2 x \,\mathrm{hypergeom}\left(\left[1, b - \frac{1}{2}\right], \left[\frac{3}{2}\right], -\frac{x^2}{a}\right)$$

✓ Solution by Mathematica

Time used: 0.426 (sec). Leaf size: 64

```
DSolve[(x^2+a)*y''[x]+2*b*x*y'[x]+2*(b-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (a + x^2) \left(\frac{c_2 x \left(\frac{a+x^2}{a}\right)^{-b} \text{Hypergeometric2F1}\left(\frac{1}{2}, 2 - b, \frac{3}{2}, -\frac{x^2}{a}\right)}{a^2} + c_1 (a + x^2)^{-b}\right)$$

30.16 problem 164

Internal problem ID [10998]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 164.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-a^2+x^2)\,y''+2bxy'+b(-1+b)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve((x^2-a^2)*diff(y(x),x$2)+2*b*x*diff(y(x),x)+b*(b-1)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(x+a)^{-b+1}+c_2(a-x)^{-b+1}$$

✓ Solution by Mathematica

Time used: 0.727 (sec). Leaf size: 127

```
DSolve[(x^2-a^2)*y''[x]+2*b*x*y'[x]+b*(b-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(x-a)^{\frac{1}{2}-\frac{1}{2}\sqrt{(b-1)^2}}(a+x)^{\frac{1}{2}-\frac{1}{2}\sqrt{(b-1)^2}}\left(x^2-a^2\right)^{-b/2}\left(2a\sqrt{(b-1)^2}c_1(x-a)^{\sqrt{(b-1)^2}}-c_2(a+x)^{\sqrt{(b-1)^2}}\right)}{2a\sqrt{(b-1)^2}}$$

30.17 problem 165

Internal problem ID [10999]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2 + bx + c)y'' + f(x)y' + g(x)y = 0$
Problem number: 165.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(a^2 + x^2)\, y'' + 2bxy' + b(b-1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 33

```
dsolve((x^2+a^2)*diff(y(x),x$2)+2*b*x*diff(y(x),x)+b*(b-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(-ix + a)^{-b+1} + c_2(ix + a)^{-b+1}$$

✓ Solution by Mathematica

Time used: 0.813 (sec). Leaf size: 101

```
DSolve[(x^2+a^2)*y''[x]+2*b*x*y'[x]+b*(b-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(a^2 + x^2)^{\frac{1}{2}-\frac{b}{2}}\, e^{-i\sqrt{(b-1)^2}\arctan\left(\frac{a}{x}\right)} \left(ic_2 e^{2i\sqrt{(b-1)^2}\arctan\left(\frac{a}{x}\right)} + 2a\sqrt{(b-1)^2}c_1\right)}{2a\sqrt{(b-1)^2}}$$

30.18 problem 166

Internal problem ID [11000]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 166.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(a\,x^2+b)\,y''+(2n+1)\,axy'+yc=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 105

```
dsolve((a*x^2+b)*diff(y(x),x$2)+(2*n+1)*a*x*diff(y(x),x)+c*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,(a\,x^2+b)^{\frac{1}{4}-\frac{n}{2}}\,\mathrm{LegendreP}\left(-\frac{-2\sqrt{n^2a-c}+\sqrt{a}}{2\sqrt{a}},n-\frac{1}{2},\frac{ax}{\sqrt{-ab}}\right)$$
$$+c_2\,(a\,x^2+b)^{\frac{1}{4}-\frac{n}{2}}\,\mathrm{LegendreQ}\left(-\frac{-2\sqrt{n^2a-c}+\sqrt{a}}{2\sqrt{a}},n-\frac{1}{2},\frac{ax}{\sqrt{-ab}}\right)$$

✓ Solution by Mathematica

Time used: 0.149 (sec). Leaf size: 118

```
DSolve[(a*x^2+b)*y''[x]+(2*n+1)*a*x*y'[x]+c*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to(ax^2+b)^{\frac{1}{4}-\frac{n}{2}}\left(c_1P^{n-\frac{1}{2}}_{\frac{\sqrt{an^2-c}}{\sqrt{a}}-\frac{1}{2}}\left(\frac{i\sqrt{a}x}{\sqrt{b}}\right)+c_2Q^{n-\frac{1}{2}}_{\frac{\sqrt{an^2-c}}{\sqrt{a}}-\frac{1}{2}}\left(\frac{i\sqrt{a}x}{\sqrt{b}}\right)\right)$$

30.19 problem 167

Internal problem ID [11001]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2 + bx + c)y'' + f(x)y' + g(x)y = 0$
Problem number: 167.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(-x^2 + 1)\, y'' - y'x + (2a\, x^2 + b)\, y = 0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 27

```
dsolve((1-x^2)*diff(y(x),x$2)-x*diff(y(x),x)+(2*a*x^2+b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{MathieuC}\left(a + b, -\frac{a}{2}, \arccos(x)\right) + c_2 \operatorname{MathieuS}\left(a + b, -\frac{a}{2}, \arccos(x)\right)$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 34

```
DSolve[(1-x^2)*y''[x]-x*y'[x]+(2*a*x^2+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \text{MathieuC}\left[a + b, -\frac{a}{2}, \arccos(x)\right] + c_2 \text{MathieuS}\left[a + b, -\frac{a}{2}, \arccos(x)\right]$$

30.20 problem 168

Internal problem ID [11002]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2 + bx + c)y'' + f(x)y' + g(x)y = 0$
Problem number: 168.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(-x^2+1\right)y''+(ax+b)\,y'+yc=0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 134

```
dsolve((1-x^2)*diff(y(x),x$2)+(a*x+b)*diff(y(x),x)+c*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\mathrm{hypergeom}\left(\left[-\frac{1}{2}-\frac{a}{2}-\frac{\sqrt{a^2+2a+4c+1}}{2},-\frac{1}{2}-\frac{a}{2}+\frac{\sqrt{a^2+2a+4c+1}}{2}\right],\left[-\frac{a}{2}+\frac{b}{2}\right],\frac{x}{2}+\frac{1}{2}\right)+c_2\left(\frac{x}{2}+\frac{1}{2}\right)^{1+\frac{a}{2}-\frac{b}{2}}\mathrm{hypergeom}\left(\left[\frac{1}{2}-\frac{\sqrt{a^2+2a+4c+1}}{2}-\frac{b}{2},\frac{1}{2}+\frac{\sqrt{a^2+2a+4c+1}}{2}-\frac{b}{2}\right],\left[2+\frac{a}{2}-\frac{b}{2}\right],\frac{x}{2}+\frac{1}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.317 (sec). Leaf size: 184

```
DSolve[(1-x^2)*y''[x]+(a*x+b)*y'[x]+c*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2^{\frac{1}{2}(-a-b-2)}\Bigg(c_2(x$$
$$-1)^{\frac{1}{2}(a+b+2)}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(b-\sqrt{a^2+2a+4c+1}+1\right),\frac{1}{2}\left(b+\sqrt{a^2+2a+4c+1}+1\right),\frac{1}{2}(a+$$
$$+c_1 2^{\frac{1}{2}(a+b+2)}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(-a-\sqrt{a^2+2a+4c+1}-1\right),\frac{1}{2}\left(-a+\sqrt{a^2+2a+4c+1}-1\right),\frac{1}{2}$$

30.21 problem 169

Internal problem ID [11003]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 169.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(a\,x^2+b\right)y''+\left(c\,x^2+d\right)y'+\lambda\left(\left(-\lambda a+c\right)x^2+d-b\lambda\right)y=0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 1074

```
dsolve((a*x^2+b)*diff(y(x),x$2)+(c*x^2+d)*diff(y(x),x)+lambda*((c-a*lambda)*x^2+d-b*lambda)*
```

Expression too large to display

✓ Solution by Mathematica

Time used: 2.859 (sec). Leaf size: 74

```
DSolve[(a*x^2+b)*y''[x]+(c*x^2+d)*y'[x]+\[Lambda]*((c-a*\[Lambda])*x^2+d-b*\[Lambda])*y[x]==
```

$$y(x)\to e^{\lambda(-x)}\left(c_2\int_1^x\exp\left(\frac{(bc-ad)\arctan\left(\frac{\sqrt{a}K[1]}{\sqrt{b}}\right)}{a^{3/2}\sqrt{b}}+\left(2\lambda-\frac{c}{a}\right)K[1]\right)dK[1]+c_1\right)$$

30.22 problem 170

Internal problem ID [11004]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 170.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(a\,x^2+b)\,y''+\left(\lambda(c+a)\,x^2+(c-a)\,x+2b\lambda\right)y'+\lambda^2\left(c\,x^2+b\right)y=0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 1571

```
dsolve((a*x^2+b)*diff(y(x),x$2)+(lambda*(c+a)*x^2+(c-a)*x+2*b*lambda)*diff(y(x),x)+lambda^2*
```

Expression too large to display

✓ Solution by Mathematica

Time used: 5.408 (sec). Leaf size: 104

```
DSolve[(a*x^2+b)*y''[x]+(\[Lambda]*(c+a)*x^2+(c-a)*x+2*b*\[Lambda])*y'[x]+\[Lambda]^2*(c*x^2
```

$$y(x)\to e^{\lambda(-x)}(\lambda x+1)\left(c_2\int_1^x\frac{\exp\left(\frac{(a-c)\lambda\left(\sqrt{a}K[1]-\sqrt{b}\arctan\left(\frac{\sqrt{a}K[1]}{\sqrt{b}}\right)\right)}{a^{3/2}}\right)\left(aK[1]^2+b\right)^{\frac{a-c}{2a}}}{(\lambda K[1]+1)^2}dK[1]+c_1\right)$$

30.23 problem 171

Internal problem ID [11005]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 171.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$x(x-1)\,y''+((\beta+\alpha+1)\,x-\gamma)\,y'+\alpha\beta y=0$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 44

```
dsolve(x*(x-1)*diff(y(x),x$2)+((alpha+beta+1)*x-gamma)*diff(y(x),x)+alpha*beta*y(x)=0,y(x),
```

$$y(x)=c_1\,\mathrm{hypergeom}\left([\alpha,\beta],[\gamma],x\right)+c_2x^{1-\gamma}\,\mathrm{hypergeom}\left([\beta+1-\gamma,\alpha+1-\gamma],[2-\gamma],x\right)$$

✓ Solution by Mathematica

Time used: 0.281 (sec). Leaf size: 49

```
DSolve[x*(x-1)*y''[x]+((\[Alpha]+\[Beta]+1)*x-\[Gamma])*y'[x]+\[Alpha]*\[Beta]*y[x]==0,y[x],
```

$$y(x)\to c_1\,\mathrm{Hypergeometric2F1}(\alpha,\beta,\gamma,x)\\ -(-1)^{-\gamma}c_2x^{1-\gamma}\,\mathrm{Hypergeometric2F1}(\alpha-\gamma+1,\beta-\gamma+1,2-\gamma,x)$$

30.24 problem 172

Internal problem ID [11006]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2 + bx + c)y'' + f(x)y' + g(x)y = 0$
Problem number: 172.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(x+a)\,y'' + (bx+c)\,y' + dy = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 249

```
dsolve(x*(x+a)*diff(y(x),x$2)+(b*x+c)*diff(y(x),x)+d*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{hypergeom}\left(\left[-\frac{1}{2}+\frac{b}{2}-\frac{\sqrt{b^2-2b-4d+1}}{2}, -\frac{1}{2}+\frac{b}{2} + \frac{\sqrt{b^2-2b-4d+1}}{2}\right], \left[\frac{b\sqrt{a^2}+ab-2c}{2\sqrt{a^2}}\right], \frac{\sqrt{a^2}+a+2x}{2\sqrt{a^2}}\right) + c_2\left(\sqrt{a^2}+a +2x\right)^{-\frac{\left((b-2)\sqrt{a^2}+ab-2c\right)\sqrt{a^2}}{2a^2}} \operatorname{hypergeom}\left(\left[-\frac{\sqrt{b^2-2b-4d+1}\,\sqrt{a^2}-\sqrt{a^2}+ab-2c}{2\sqrt{a^2}}, \frac{\sqrt{a^2}+\sqrt{b^2-2b-}\cdots}{2\cdots}\right.\right.$$

✓ Solution by Mathematica

Time used: 0.423 (sec). Leaf size: 165

```
DSolve[x*(x+a)*y''[x]+(b*x+c)*y'[x]+d*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 a^{\frac{c}{a}-1} x^{1-\frac{c}{a}} \text{Hypergeometric2F1}\left(\frac{1}{2}\left(b - \frac{2c}{a} + \sqrt{b^2-2b-4d+1} + 1\right), \frac{ba - \sqrt{b^2-2b-4d+1}a + a - 2c}{2a}, 2 - \frac{c}{a}, -\frac{x}{a}\right) + c_1 \text{Hypergeometric2F1}\left(\frac{1}{2}\left(b - \sqrt{b^2-2b-4d+1} - 1\right), \frac{1}{2}\left(b + \sqrt{b^2-2b-4d+1} - 1\right), \frac{c}{a}, -\frac{x}{a}\right)$$

30.25 problem 173

Internal problem ID [11007]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 173.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$\boxed{2x(x-1)\,y''+(2x-1)\,y'+(ax+b)\,y=0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 39

```
dsolve(2*x*(x-1)*diff(y(x),x$2)+(2*x-1)*diff(y(x),x)+(a*x+b)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\mathrm{MathieuC}\left(-2b-a,\frac{a}{2},\arccos\left(\sqrt{x}\right)\right)+c_2\,\mathrm{MathieuS}\left(-2b-a,\frac{a}{2},\arccos\left(\sqrt{x}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.258 (sec). Leaf size: 50

```
DSolve[2*x*(x-1)*y''[x]+(2*x-1)*y'[x]+(a*x+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to c_1\mathrm{MathieuC}\left[-a-2b,\frac{a}{2},\arccos\left(\sqrt{x}\right)\right]+c_2\mathrm{MathieuS}\left[-a-2b,\frac{a}{2},\arccos\left(\sqrt{x}\right)\right]$$

30.26 problem 174

Internal problem ID [11008]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2 + bx + c)y'' + f(x)y' + g(x)y = 0$
Problem number: 174.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{(2ax + x^2 + b)\, y'' + (x + a)\, y' - ym^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve((x^2+2*a*x+b)*diff(y(x),x$2)+(x+a)*diff(y(x),x)-m^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \left(x + a + \sqrt{2ax + x^2 + b}\right)^{-m} + c_2 \left(x + a + \sqrt{2ax + x^2 + b}\right)^{m}$$

✓ Solution by Mathematica

Time used: 0.301 (sec). Leaf size: 63

```
DSolve[(x^2+2*a*x+b)*y''[x]+(x+a)*y'[x]-m^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cosh\left(m \log\left(\sqrt{2ax + b + x^2} - a - x\right)\right) - ic_2 \sinh\left(m \log\left(\sqrt{2ax + b + x^2} - a - x\right)\right)$$

30.27 problem 175

Internal problem ID [11009]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 175.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(a\,x^2+bx+c)\,y''+(dx+k)\,y'+(d-2a)\,y=0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 1626

```
dsolve((a*x^2+b*x+c)*diff(y(x),x$2)+(d*x+k)*diff(y(x),x)+(d-2*a)*y(x)=0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 15.225 (sec). Leaf size: 164

```
DSolve[(a*x^2+b*x+c)*y''[x]+(d*x+k)*y'[x]+(d-2*a)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to (x(ax+b) +c)^{1-\frac{d}{2a}}\exp\left(\frac{(bd-2ak)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{a\sqrt{4ac-b^2}}\right)\left(c_2\int_1^x\exp\left(\frac{(d-4a)\log(c+K[1](b+aK[1]))-\frac{2(bd-2}{}}{2a}\right. + c_1\right)$$

30.28 problem 176

Internal problem ID [11010]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2 + bx + c)y'' + f(x)y' + g(x)y = 0$
Problem number: 176.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(a\,x^2 + bx + c)\,y'' + (kx + d)\,y' - ky = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 315

```
dsolve((a*x^2+b*x+c)*diff(y(x),x$2)+(k*x+d)*diff(y(x),x)-k*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(kx + d)$$
$$+ c_2\left(2\sqrt{\frac{-4ac + b^2}{a^2}}\,x\,a^2 + \sqrt{\frac{-4ac + b^2}{a^2}}\,ba - 4ac + b^2\right)^{\frac{a\left(a-\frac{k}{2}\right)\sqrt{\frac{-4ac+b^2}{a^2}}+ad-\frac{bk}{2}}{\sqrt{\frac{-4ac+b^2}{a^2}}\,a^2}} \text{hypergeom}\left(\left[-\frac{k\sqrt{\frac{-4ac+}{a^2}}}{2a^2}\right.\right.$$

✓ Solution by Mathematica

Time used: 4.256 (sec). Leaf size: 107

```
DSolve[(a*x^2+b*x+c)*y''[x]+(k*x+d)*y'[x]-k*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(d + kx)\left(c_2\int_1^x \frac{\exp\left(\frac{(bk-2ad)\arctan\left(\frac{b+2aK[1]}{\sqrt{4ac-b^2}}\right)}{a\sqrt{4ac-b^2}}\right)(c+K[1](b+aK[1]))^{-\frac{k}{2a}}}{(d+kK[1])^2}dK[1] + c_1\right)}{d}$$

30.29 problem 177

Internal problem ID [11011]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 177.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\left(a\,x^{2}+2bx+c\right)y''+y'(ax+b)+dy=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 77

```
dsolve((a*x^2+2*b*x+c)*diff(y(x),x$2)+(a*x+b)*diff(y(x),x)+d*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(\frac{ax+b}{\sqrt{a}}+\sqrt{a\,x^{2}+2bx+c}\right)^{\frac{i\sqrt{d}}{\sqrt{a}}}+c_2\left(\frac{ax+b}{\sqrt{a}}+\sqrt{a\,x^{2}+2bx+c}\right)^{-\frac{i\sqrt{d}}{\sqrt{a}}}$$

✓ Solution by Mathematica

Time used: 0.404 (sec). Leaf size: 93

```
DSolve[(a*x^2+2*b*x+c)*y''[x]+(a*x+b)*y'[x]+d*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$\begin{aligned}y(x)\to{}& c_1\cos\left(\frac{\sqrt{d}\log\left(-\sqrt{a}\sqrt{ax^2+2bx+c}+ax+b\right)}{\sqrt{a}}\right)\\&-c_2\sin\left(\frac{\sqrt{d}\log\left(-\sqrt{a}\sqrt{ax^2+2bx+c}+ax+b\right)}{\sqrt{a}}\right)\end{aligned}$$

30.30 problem 178

Internal problem ID [11012]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 178.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(a\,x^{2}+2bx+c\right)y''+3y'(ax+b)+dy=0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 100

```
dsolve((a*x^2+2*b*x+c)*diff(y(x),x$2)+3*(a*x+b)*diff(y(x),x)+d*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1\left(\sqrt{a\left(a\,x^2+2bx+c\right)}+ax+b\right)^{\frac{\sqrt{-d+a}}{\sqrt{a}}}}{\sqrt{a\,x^2+2bx+c}}+\frac{c_2\left(\sqrt{a\left(a\,x^2+2bx+c\right)}+ax+b\right)^{-\frac{\sqrt{-d+a}}{\sqrt{a}}}}{\sqrt{a\,x^2+2bx+c}}$$

✓ Solution by Mathematica

Time used: 0.171 (sec). Leaf size: 152

```
DSolve[(a*x^2+2*b*x+c)*y''[x]+3*(a*x+b)*y'[x]+d*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\to\frac{c_1P^{\frac{1}{2}}_{\frac{\sqrt{a-d}}{\sqrt{a}}-\frac{1}{2}}\left(\frac{\sqrt{-b^2-ac}(b+ax)}{a\sqrt{c^2-\frac{b^4}{a^2}}}\right)+c_2Q^{\frac{1}{2}}_{\frac{\sqrt{a-d}}{\sqrt{a}}-\frac{1}{2}}\left(\frac{\sqrt{-b^2-ac}(b+ax)}{a\sqrt{c^2-\frac{b^4}{a^2}}}\right)}{\sqrt[4]{x(ax+2b)+c}}$$

30.31 problem 179

Internal problem ID [11013]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 179.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(a_2x^2+b_2x+c_2\right)y''+\left(b_1x+c_1\right)y'+c_0y=0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 482

```
dsolve((a__2*x^2+b__2*x+c__2)*diff(y(x),x$2)+(b__1*x+c__1)*diff(y(x),x)+c__0*y(x)=0,y(x), si
```

$$y(x) = c_3\,\mathrm{hypergeom}\left(\left[\frac{-a_2+b_1+\sqrt{a_2^2+\left(-2b_1-4c_0\right)a_2+b_1^2}}{2a_2},\right.\right.$$

$$\left.-\frac{a_2-b_1+\sqrt{a_2^2+\left(-2b_1-4c_0\right)a_2+b_1^2}}{2a_2}\right],\left[\frac{b_1\sqrt{\frac{-4c_2a_2+b_2^2}{a_2^2}}\,a_2-2a_2c_1+b_1b_2}{2a_2^2\sqrt{\frac{-4c_2a_2+b_2^2}{a_2^2}}}\right],\frac{\left(-2a_2^2x-a_2b_2\right)\sqrt{\frac{-4c_2a}{a_2^2}}}{8c_2a_2-2}$$

$$+c_4\left(2\sqrt{\frac{-4c_2a_2+b_2^2}{a_2^2}}\,x\,a_2^2+\sqrt{\frac{-4c_2a_2+b_2^2}{a_2^2}}\,b_2a_2-4c_2a_2+b_2^2\right)^{\frac{a_2\left(a_2-\frac{b_1}{2}\right)\sqrt{\frac{-4c_2a_2+b_2^2}{a_2^2}}+a_2c_1-\frac{b_1b_2}{2}}{\sqrt{\frac{-4c_2a_2+b_2^2}{a_2^2}}\,a_2^2}}\,\mathrm{hypergeom}$$

✓ Solution by Mathematica

Time used: 6.771 (sec). Leaf size: 498

```
DSolve[(a2*x^2+b2*x+c2)*y''[x]+(b1*x+c1)*y'[x]+c0*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to c_1 \text{Hypergeometric2F1}\left(-\frac{\text{a2}-\text{b1}+\sqrt{(\text{a2}-\text{b1})^2-4\text{a2c0}}}{2\text{a2}}, \frac{-\text{a2}+\text{b1}+\sqrt{(\text{a2}-\text{b1})^2-4\text{a2c0}}}{2\text{a2}}, \frac{\text{b1}\Big(\text{b2}\right.$$

$$-c_2 2^{\frac{\frac{\text{b1b2}}{\sqrt{\text{b2}^2-4\text{a2c2}}}+\text{b1}}{2\text{a2}}-\frac{\text{c1}}{\sqrt{\text{b2}^2-4\text{a2c2}}}-1} \exp\left(-\frac{i\pi\left(\text{b1}\left(\sqrt{\text{b2}^2-4\text{a2c2}}+\text{b2}\right)-2\text{a2c1}\right)}{2\text{a2}\sqrt{\text{b2}^2-4\text{a2c2}}}\right)\left(\frac{\sqrt{\text{b2}^2-4\text{a2c2}}+2\text{a}}{\sqrt{\text{b2}^2-4\text{a2c}}}\right.$$

$$\left.-\frac{\frac{\text{b2b1}}{\sqrt{\text{b2}^2-4\text{a2c2}}}+\text{b1}+\text{a2}\left(-\frac{2\text{c1}}{\sqrt{\text{b2}^2-4\text{a2c2}}}-4\right)}{2\text{a2}}, \frac{\text{b2}+2\text{a2}x+\sqrt{\text{b2}^2-4\text{a2c2}}}{2\sqrt{\text{b2}^2-4\text{a2c2}}}\right)$$

30.32 problem 180

Internal problem ID [11014]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 180.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(a\,x^2+bx+c\right)y''-\left(-k^2+x^2\right)y'+(x+k)\,y=0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 1802

```
dsolve((a*x^2+b*x+c)*diff(y(x),x$2)-(x^2-k^2)*diff(y(x),x)+(x+k)*y(x)=0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 2.442 (sec). Leaf size: 119

```
DSolve[(a*x^2+b*x+c)*y''[x]-(x^2-k^2)*y'[x]+(x+k)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{(k-x)\left(c_2\int_1^x \frac{\exp\left(\frac{\frac{\left(b^2-2a\left(ak^2+c\right)\right)\arctan\left(\frac{b+2aK[1]}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+aK[1]}{a^2}\right)(c+K[1](b+aK[1]))^{-\frac{b}{2a^2}}}{(k-K[1])^2}dK[1]+c_1\right)}{k}$$

30.33 problem 181

Internal problem ID [11015]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 181.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(a\,x^2+bx+c)\,y''+(k^3+x^3)\,y'-(k^2-kx+x^2)\,y=0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 249

```
dsolve((a*x^2+b*x+c)*diff(y(x),x$2)+(x^3+k^3)*diff(y(x),x)-(x^2-k*x+k^2)*y(x)=0,y(x), singso
```

$$y(x)=c_1(x+k)+c_2(x+k)\left(\int\frac{\left(\frac{-2ax-b+\sqrt{-4ac+b^2}}{2ax+b+\sqrt{-4ac+b^2}}\right)^{-\frac{3cb}{2\sqrt{-4ac+b^2}\,a^2}}(a\,x^2+bx+c)^{\frac{ac-b^2}{2a^3}}\left(\frac{2ax+b+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}\right)^{\frac{k^3}{\sqrt{-4ac+b^2}}}(2ax+b-\sqrt{-4}}{(x+k)^2}\right.$$

✓ Solution by Mathematica

Time used: 3.224 (sec). Leaf size: 137

```
DSolve[(a*x^2+b*x+c)*y''[x]+(x^3+k^3)*y'[x]-(x^2-k*x+k^2)*y[x]==0,y[x],x,IncludeSingularSolu
```

$$y(x)\to\frac{(k+x)\left(c_2\int_1^x\frac{\exp\left(\frac{(b^3-3acb-2a^3k^3)\arctan\left(\frac{b+2aK[1]}{\sqrt{4ac-b^2}}\right)}{a^3\sqrt{4ac-b^2}}-\frac{K[1](aK[1]-2b)}{2a^2}\right)(c+K[1](b+aK[1]))^{-\frac{b^2-ac}{2a^3}}}{(k+K[1])^2}dK[1]+c_1\right)}{k}$$

31 Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$

31.1 problem 182

Internal problem ID [11016]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 182.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^3y'' + (ax + b)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 51

```
dsolve(x^3*diff(y(x),x$2)+(a*x+b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\text{BesselJ}\left(-\sqrt{1-4a}, \frac{2\sqrt{b}}{\sqrt{x}}\right) + c_2\sqrt{x}\,\text{BesselY}\left(-\sqrt{1-4a}, \frac{2\sqrt{b}}{\sqrt{x}}\right)$$

✓ Solution by Mathematica

Time used: 0.128 (sec). Leaf size: 101

```
DSolve[x^3*y''[x]+(a*x+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1\,\text{Gamma}\left(1-\sqrt{1-4a}\right)\text{BesselJ}\left(-\sqrt{1-4a}, 2\sqrt{b}\sqrt{\frac{1}{x}}\right) + c_2\,\text{Gamma}\left(\sqrt{1-4a}+1\right)\text{BesselJ}\left(\sqrt{1-4a}\right.}{\sqrt{b}\sqrt{\frac{1}{x}}}$$

31.2 problem 183

Internal problem ID [11017]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 183.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^3y'' + \left(a\,x^2 + b\right)y' + cxy = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 141

```
dsolve(x^3*diff(y(x),x$2)+(a*x^2+b)*diff(y(x),x)+c*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{-\frac{\sqrt{a^2-2a-4c+1}}{2}-\frac{a}{2}+\frac{1}{2}} \operatorname{KummerM}\left(-\frac{1}{4} + \frac{\sqrt{a^2-2a-4c+1}}{4} + \frac{a}{4}, 1 + \frac{\sqrt{a^2-2a-4c+1}}{2}, \frac{b}{2x^2}\right) + c_2 x^{-\frac{\sqrt{a^2-2a-4c+1}}{2}-\frac{a}{2}+\frac{1}{2}} \operatorname{KummerU}\left(-\frac{1}{4} + \frac{\sqrt{a^2-2a-4c+1}}{4} + \frac{a}{4}, 1 + \frac{\sqrt{a^2-2a-4c+1}}{2}, \frac{b}{2x^2}\right)$$

✓ Solution by Mathematica

Time used: 0.646 (sec). Leaf size: 308

```
DSolve[x^3*y''[x]+(a*x^2+b)*y'[x]+c*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -(-1)^{\frac{1}{4}\left(-\sqrt{a^2-2a-4c+1}+a+3\right)} 2^{\frac{1}{4}\left(-\sqrt{a^2-2a-4c+1}-a+1\right)} b^{\frac{1}{4}\left(-\sqrt{a^2-2a-4c+1}+a-1\right)} \left(\frac{1}{x}\right)^{\frac{1}{2}\left(-\sqrt{a^2-2a-4c+1}+a-1\right)} \left(c_2 i^{\sqrt{a}} \right.$$

$$\left. + c_1 2^{\frac{1}{2}\sqrt{a^2-2a-4c+1}} \text{Hypergeometric1F1}\left(\frac{1}{4}\left(a-\sqrt{a^2-2a-4c+1}-1\right), 1 - \frac{1}{2}\sqrt{a^2-2a-4c+1}, \frac{b}{2x^2}\right)\right)$$

31.3 problem 184

Internal problem ID [11018]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 184.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^3y'' + \left(a\,x^2 + bx\right)y' + by = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 118

```
dsolve(x^3*diff(y(x),x$2)+(a*x^2+b*x)*diff(y(x),x)+b*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1\left(-(a-2)\,x(-1)^{-a}\left(\Gamma\left(a,-\frac{b}{x}\right)-\Gamma(a)\right)b^{-\frac{a}{2}+1} - (-1)^{-a}\left(\Gamma\left(a,-\frac{b}{x}\right)-\Gamma(a)\right)b^{-\frac{a}{2}+2} - \mathrm{e}^{\frac{b}{x}}x\left(b^{\frac{a}{2}+1}x^{-a} - x\right.\right.}{x} + \frac{c_2(x(a-2)+b)}{x}$$

✓ Solution by Mathematica

Time used: 2.653 (sec). Leaf size: 62

```
DSolve[x^3*y''[x]+(a*x^2+b*x)*y'[x]+b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{((a-2)x+b)\left(c_2\int_1^x \frac{e^{\frac{b}{K[1]}}K[1]^{2-a}}{(b+(a-2)K[1])^2}dK[1] + c_1\right)}{x(a+b-2)}$$

31.4 problem 185

Internal problem ID [11019]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 185.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''+\left(a\,x^2+bx\right)y'+yc=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 63

```
dsolve(x^3*diff(y(x),x$2)+(a*x^2+b*x)*diff(y(x),x)+c*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x^{1-a}\,\mathrm{KummerM}\left(\frac{b(a-1)-c}{b},a,\frac{b}{x}\right)+c_2x^{1-a}\,\mathrm{KummerU}\left(\frac{b(a-1)-c}{b},a,\frac{b}{x}\right)$$

✓ Solution by Mathematica

Time used: 0.435 (sec). Leaf size: 62

```
DSolve[x^3*y''[x]+(a*x^2+b*x)*y'[x]+c*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\,\mathrm{Hypergeometric1F1}\left(-\frac{c}{b},2-a,\frac{b}{x}\right)-(-1)^ac_2b^{a-1}\left(\frac{1}{x}\right)^{a-1}\mathrm{Hypergeometric1F1}\left(a-\frac{b+c}{b},a,\frac{b}{x}\right)$$

31.5 problem 186

Internal problem ID [11020]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 186.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^3y'' + \left(a\,x^2 + bx\right)y' + (xc + d)\,y = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 153

```
dsolve(x^3*diff(y(x),x$2)+(a*x^2+b*x)*diff(y(x),x)+(c*x+d)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{-\frac{\sqrt{a^2-2a-4c+1}}{2}-\frac{a}{2}+\frac{1}{2}} \operatorname{KummerM}\left(\frac{\sqrt{a^2-2a-4c+1}\,b + b(a-1) - 2d}{2b}, 1 + \sqrt{a^2-2a-4c+1}, \frac{b}{x}\right) + c_2 x^{-\frac{\sqrt{a^2-2a-4c+1}}{2}-\frac{a}{2}+\frac{1}{2}} \operatorname{KummerU}\left(\frac{\sqrt{a^2-2a-4c+1}\,b + b(a-1) - 2d}{2b}, 1 + \sqrt{a^2-2a-4c+1}, \frac{b}{x}\right)$$

✓ Solution by Mathematica

Time used: 0.641 (sec). Leaf size: 255

```
DSolve[x^3*y''[x]+(a*x^2+b*x)*y'[x]+(c*x+d)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -i^{-\sqrt{a^2-2a-4c+1}+a+1} b^{\frac{1}{2}\left(-\sqrt{a^2-2a-4c+1}+a-1\right)} \left(\frac{1}{x}\right)^{\frac{1}{2}\left(-\sqrt{a^2-2a-4c+1}+a-1\right)} \left(c_2 i^{2\sqrt{a^2-2a-4c+1}} b^{\sqrt{a^2-2a-4c+1}} \left(\frac{1}{x}\right)^{\sqrt{}} \right.$$

$$\left. + 1, \frac{b}{x}\right) + c_1 \text{Hypergeometric1F1}\left(\frac{1}{2}\left(a - \frac{2d}{b} - \sqrt{a^2-2a-4c+1} - 1\right), 1 - \sqrt{a^2-2a-4c+1}, \frac{b}{x}\right)\right)$$

31.6 problem 187

Internal problem ID [11021]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 187.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^3y'' + (x^3a + abx - x^2 + b)\, y' + a^2bxy = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 59

```
dsolve(x^3*diff(y(x),x$2)+(a*x^3-x^2+a*b*x+b)*diff(y(x),x)+a^2*b*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-ax}(ax+1) + c_2 e^{-ax}(ax+1)\left(\int \frac{x\, e^{\frac{2a\,x^3+2abx+b}{2x^2}}}{(ax+1)^2}dx\right)$$

✓ Solution by Mathematica

Time used: 1.347 (sec). Leaf size: 70

```
DSolve[x^3*y''[x]+(a*x^3-x^2+a*b*x+b)*y'[x]+a^2*b*x*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{e^{-ax}(ax+1)\left(c_2 \int_1^x \frac{a^2 e^{aK[1]+\frac{2aK[1]b+b}{2K[1]^2}}K[1]}{(aK[1]+1)^2}dK[1] + c_1\right)}{a}$$

31.7 problem 188

Internal problem ID [11022]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 188.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^3y'' + x(x^n a + b)\, y' - \left(x^n a - ab\, x^{-1+n} + b\right) y = 0$$

✗ Solution by Maple

```
dsolve(x^3*diff(y(x),x$2)+x*(a*x^n+b)*diff(y(x),x)-(a*x^n-a*b*x^(n-1)+b)*y(x)=0,y(x), singso
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^3*y''[x]+x*(a*x^n+b)*y'[x]-(a*x^n-a*b*x^(n-1)+b)*y[x]==0,y[x],x,IncludeSingularSolu
```

Not solved

31.8 problem 189

Internal problem ID [11023]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 189.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x\left(a\,x^{2}+b\right) y''+2\left(a\,x^{2}+b\right) y'-2yax=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve(x*(a*x^2+b)*diff(y(x),x$2)+2*(a*x^2+b)*diff(y(x),x)-2*a*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_{1}\left(a\,x^{2}+b\right)}{x}+\frac{c_{2}\left(\left(a\,x^{2}+b\right)\arctan\left(\frac{\sqrt{ab}\,x}{b}\right)+\sqrt{ab}\,x\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.16 (sec). Leaf size: 78

```
DSolve[x*(a*x^2+b)*y''[x]+2*(a*x^2+b)*y'[x]-2*a*x*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{c_2(ax^2+b)\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)+\sqrt{a}\sqrt{b}(2abc_1x^2+2b^2c_1+c_2x)}{2\sqrt{a}b^{3/2}x}$$

31.9 problem 190

Internal problem ID [11024]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 190.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x\left(x^{2}+a\right)y''+\left(b\,x^{2}+c\right)y'+sxy=0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 203

```
dsolve(x*(x^2+a)*diff(y(x),x$2)+(b*x^2+c)*diff(y(x),x)+s*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{\frac{a-c}{a}} \left(x^2+a\right)^{\frac{(-b+2)a+c}{2a}} \operatorname{hypergeom}\left(\left[-\frac{b}{4}+\frac{5}{4}-\frac{\sqrt{b^2-2b-4s+1}}{4}, -\frac{b}{4}+\frac{5}{4} + \frac{\sqrt{b^2-2b-4s+1}}{4}\right], \left[\frac{3a-c}{2a}\right], -\frac{x^2}{a}\right) + c_2 \operatorname{hypergeom}\left(\left[-\frac{\sqrt{b^2-2b-4s+1}\,a+ab-3a-2c}{4a}, \frac{\sqrt{b^2-2b-4s+1}\,a+(-b+3)\,a+2c}{4a}\right], \left[\frac{a}{2}\right. \right. -\frac{x^2}{a}\Big) \left(x^2+a\right)^{\frac{(-b+2)a+c}{2a}}$$

✓ Solution by Mathematica

Time used: 0.967 (sec). Leaf size: 185

```
DSolve[x*(x^2+a)*y''[x]+(b*x^2+c)*y'[x]+s*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_2 a^{\frac{1}{2}\left(\frac{c}{a}-1\right)} x^{1-\frac{c}{a}} \text{Hypergeometric2F1}\left(\frac{a\left(b+\sqrt{b^2-2b-4s+1}+1\right)-2c}{4a}, \frac{ba-\sqrt{b^2-2b-4s+1}a+a}{4a} -\frac{c}{2a}, -\frac{x^2}{a}\right) + c_1 \text{Hypergeometric2F1}\left(\frac{1}{4}\left(b-\sqrt{b^2-2b-4s+1}-1\right), \frac{1}{4}\left(b+\sqrt{b^2-2b-4s+1}-1\right), \frac{a+c}{2a}, -\frac{x^2}{a}\right)$$

31.10 problem 191

Internal problem ID [11025]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 191.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(ax+b)\,y'' + \left(c\,x^2 + (\lambda a + 2b)\,x + b\lambda\right) y' + \lambda(-2a+c)\,y = 0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 203

```
dsolve(x^2*(a*x+b)*diff(y(x),x$2)+(c*x^2+(2*b+a*lambda)*x+b*lambda)*diff(y(x),x)+lambda*(c-2
```

$$y(x) = c_1(ax+b)^{\frac{3a-c}{a}} x^{\frac{-3a+c}{a}} \operatorname{HeunC}\left(\frac{\lambda a}{b}, \frac{a-c}{a}, \frac{3a-c}{a}, 0, \frac{-2a^3\lambda + (c\lambda+5b)\,a^2 - 4cba + c^2b}{2a^2b}, -\frac{b}{ax}\right) + \frac{c_2(ax+b)^{\frac{3a-c}{a}} \operatorname{HeunC}\left(\frac{\lambda a}{b}, \frac{-a+c}{a}, \frac{3a-c}{a}, 0, \frac{-2a^3\lambda+(c\lambda+5b)a^2-4cba+c^2b}{2a^2b}, -\frac{b}{ax}\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 1.48 (sec). Leaf size: 55

```
DSolve[x^2*(a*x+b)*y''[x]+(c*x^2+(2*b+a*\[Lambda])*x+b*\[Lambda])*y'[x]+\[Lambda]*(c-2*a)*y[
```

$$y(x) \to e^{\frac{\lambda}{x}}\left(c_2\int_1^x \frac{e^{-\frac{\lambda}{K[1]}}(b+aK[1])^{2-\frac{c}{a}}}{K[1]^2}dK[1] + c_1\right)$$

31.11 problem 192

Internal problem ID [11026]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 192.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(ax+b)\,y''-2x(ax+2b)\,y'+2(ax+3b)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x^2*(a*x+b)*diff(y(x),x$2)-2*x*(a*x+2*b)*diff(y(x),x)+2*(a*x+3*b)*y(x)=0,y(x), singso
```

$$y(x)=\frac{c_1x^2}{ax+b}+\frac{c_2x^3}{ax+b}$$

✓ Solution by Mathematica

Time used: 0.068 (sec). Leaf size: 23

```
DSolve[x^2*(a*x+b)*y''[x]-2*x*(a*x+2*b)*y'[x]+2*(a*x+3*b)*y[x]==0,y[x],x,IncludeSingularSolu
```

$$y(x)\to\frac{x^2(c_2x+c_1)}{ax+b}$$

31.12 problem 193

Internal problem ID [11027]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 193.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(ax+b)\,y'' + \left(a(2-m-n)\,x^2 - b(m+n)\,x\right)y' + \left(am(-1+n)\,x + bn(m+1)\right)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(x^2*(a*x+b)*diff(y(x),x$2)+(a*(2-n-m)*x^2-b*(n+m)*x)*diff(y(x),x)+(a*m*(n-1)*x+b*n*(m
```

$$y(x) = \frac{c_1 x^n}{ax+b} + \frac{c_2 x^{m+1}}{ax+b}$$

✓ Solution by Mathematica

Time used: 0.403 (sec). Leaf size: 82

```
DSolve[x^2*(a*x+b)*y''[x]+(a*(2-n-m)*x^2-b*(n+m)*x)*y'[x]+(a*m*(n-1)*x+b*n*(m+1))*y[x]==0,y[
```

$$y(x) \to \frac{x^{\frac{1}{2}\left(-\sqrt{(m-n+1)^2}+m+n+1\right)}\left(c_2 x^{\sqrt{(m-n+1)^2}} + c_1\sqrt{(m-n+1)^2}\right)}{\sqrt{(m-n+1)^2}(ax+b)}$$

31.13 problem 194

Internal problem ID [11028]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 194.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x + a_2)\, y'' + x(b_1x + a_1)\, y' + (b_0x + a_0)\, y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 319

```
dsolve(x^2*(x+a__2)*diff(y(x),x$2)+x*(b__1*x+a__1)*diff(y(x),x)+(b__0*x+a__0)*y(x)=0,y(x), s
```

$$y(x) = c_1 x^{\frac{a_2-a_1+\sqrt{a_2^2+(-4a_0-2a_1)a_2+a_1^2}}{2a_2}} \operatorname{hypergeom}\left(\left[\frac{a_2b_1 - a_1 + \sqrt{a_2^2 + (-4a_0 - 2a_1)\, a_2 + a_1^2} + \sqrt{b_1^2 - 4b_0 - 2b_1 +}{2a_2}\right.\right.$$
$$\left.- \frac{\sqrt{b_1^2 - 4b_0 - 2b_1 + 1}\, a_2 - a_2b_1 - \sqrt{a_2^2 + (-4a_0 - 2a_1)\, a_2 + a_1^2} + a_1}{2a_2}\right], \left[\frac{a_2 + \sqrt{a_2^2 + (-4a_0 - 2a_1)\, a_2}}{a_2}\right.$$
$$\left. -\frac{x}{a_2}\right)$$
$$+ c_2 x^{-\frac{-a_2+a_1+\sqrt{a_2^2+(-4a_0-2a_1)a_2+a_1^2}}{2a_2}} \operatorname{hypergeom}\left(\left[\frac{a_2b_1 - a_1 - \sqrt{a_2^2 + (-4a_0 - 2a_1)\, a_2 + a_1^2} + \sqrt{b_1^2 - 4b_0 -}{2a_2}\right.\right.$$
$$\left.- \frac{\sqrt{b_1^2 - 4b_0 - 2b_1 + 1}\, a_2 - a_2b_1 + \sqrt{a_2^2 + (-4a_0 - 2a_1)\, a_2 + a_1^2} + a_1}{2a_2}\right], \left[\frac{a_2 - \sqrt{a_2^2 + (-4a_0 - 2a_1)\, a_2}}{a_2}\right.$$
$$\left. -\frac{x}{a_2}\right)$$

✓ Solution by Mathematica

Time used: 1.236 (sec). Leaf size: 384

```
DSolve[x^2*(x+a2)*y''[x]+x*(b1*x+a1)*y'[x]+(b0*x+a0)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \mathrm{a2}^{-\frac{\sqrt{-4\mathrm{a0a2}+\mathrm{a1}^2-2\mathrm{a1a2}+\mathrm{a2}^2}-\mathrm{a1}+\mathrm{a2}}{2\mathrm{a2}}} x^{-\frac{\sqrt{-4\mathrm{a0a2}+\mathrm{a1}^2-2\mathrm{a1a2}+\mathrm{a2}^2}+\mathrm{a1}-\mathrm{a2}}{2\mathrm{a2}}} \left(c_2 x^{\frac{\sqrt{-4\mathrm{a0a2}+\mathrm{a1}^2-2\mathrm{a1a2}+\mathrm{a2}^2}}{\mathrm{a2}}} \text{Hypergeometric2F1} \ldots -\frac{x}{\mathrm{a2}}\right)$$

$$+c_1 \mathrm{a2}^{\frac{\sqrt{-4\mathrm{a0a2}+\mathrm{a1}^2-2\mathrm{a1a2}+\mathrm{a2}^2}}{\mathrm{a2}}} \text{Hypergeometric2F1}\left(-\frac{\mathrm{a1}-\mathrm{a2b1}+\sqrt{\mathrm{a1}^2-2\mathrm{a2a1}+\mathrm{a2}(\mathrm{a2}-4\mathrm{a0})}+\mathrm{a2}\sqrt{(\ldots}}{2\mathrm{a2}}\right.$$

$$-\frac{\mathrm{a1}-\mathrm{a2}\left(\mathrm{b1}+\sqrt{(\mathrm{b1}-1)^2-4\mathrm{b0}}\right)+\sqrt{\mathrm{a1}^2-2\mathrm{a2a1}+\mathrm{a2}(\mathrm{a2}-4\mathrm{a0})}}{2\mathrm{a2}}, 1$$

$$\left.\left.-\frac{\sqrt{\mathrm{a1}^2-2\mathrm{a2a1}+\mathrm{a2}^2-4\mathrm{a0a2}}}{\mathrm{a2}}, -\frac{x}{\mathrm{a2}}\right)\right)$$

31.14 problem 195

Internal problem ID [11029]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 195.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$
\boxed{(x^3a + b\,x^2 + xc)\,y'' + (\alpha\,x^2 + \beta x + 2c)\,y' + (\beta - 2b)\,y = 0}
$$

✓ Solution by Maple

Time used: 0.61 (sec). Leaf size: 1747

```
dsolve((a*x^3+b*x^2+c*x)*diff(y(x),x$2)+(alpha*x^2+beta*x+2*c)*diff(y(x),x)+(beta-2*b)*y(x)=
```

Expression too large to display

✓ Solution by Mathematica

Time used: 4.64 (sec). Leaf size: 139

```
DSolve[(a*x^3+b*x^2+c*x)*y''[x]+(\[Alpha]*x^2+\[Beta]*x+2*c)*y'[x]+(\[Beta]-2*b)*y[x]==0,y[x
```

$$
y(x) \to \frac{(2ax + 2b - \beta - \alpha x)\left(c_2 \int_1^x \frac{\exp\left(\frac{(b\alpha+2a(b-\beta))\arctan\left(\frac{b+2aK[1]}{\sqrt{4ac-b^2}}\right)}{a\sqrt{4ac-b^2}}\right)(c+K[1](b+aK[1]))^{1-\frac{\alpha}{2a}}}{(-2b+\beta+(\alpha-2a)K[1])^2}dK[1] + c_1\right)}{x(2b-\beta)}
$$

31.15 problem 196

Internal problem ID [11030]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 196.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^3a + b\,x^2 + xc)\,y'' + (\alpha\,x^2 + \beta x + 2c)\,y' - (\alpha x + 2b - \beta)\,y = 0$$

✓ Solution by Maple

Time used: 3.5 (sec). Leaf size: 1747

```
dsolve((a*x^3+b*x^2+c*x)*diff(y(x),x$2)+(alpha*x^2+beta*x+2*c)*diff(y(x),x)-(alpha*x+2*b-bet
```

Expression too large to display

✓ Solution by Mathematica

Time used: 6.092 (sec). Leaf size: 224

```
DSolve[(a*x^3+b*x^2+c*x)*y''[x]+(\[Alpha]*x^2+\[Beta]*x+2*c)*y'[x]-(\[Alpha]*x+2*b-\[Beta])*
```

$$y(x) \to \frac{(b(2ax - 3\beta - 2\alpha x) - a\alpha x^2 - 2a\beta x + 2b^2 + \beta^2 + \alpha c + \alpha^2x^2 + 2\alpha\beta x)\left(c_2 \int_1^x \frac{\exp\left(\frac{(b\alpha+2a(b-\beta))\arctan\left(\frac{b}{\sqrt{}}\right.}{a\sqrt{4ac-b^2}}\right.}{(2b^2-3\beta b+2(a-\alpha)K[1]b+\beta^2+\alpha^2}\right.}{x\,(2b^2 + \beta^2 - 3\beta b + \alpha c)}$$

31.16 problem 197

Internal problem ID [11031]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 197.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^3a + b\,x^2 + xc)\,y'' + \left(-2a\,x^2 - (1+b)\,x + k\right)y' + 2(ax+1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.297 (sec). Leaf size: 2565

```
dsolve((a*x^3+b*x^2+c*x)*diff(y(x),x$2)+(-2*a*x^2-(b+1)*x+k)*diff(y(x),x)+2*(a*x+1)*y(x)=0,y
```

Expression too large to display

✓ Solution by Mathematica

Time used: 10.126 (sec). Leaf size: 186

```
DSolve[(a*x^3+b*x^2+c*x)*y''[x]+(-2*a*x^2-(b+1)*x+k)*y'[x]+2*(a*x+1)*y[x]==0,y[x],x,IncludeS
```

$$y(x) \to$$

$$-\frac{(-kx(ax+2) - (b-1)x^2 + c(k-2x) + k^2)\left(c_2 \int_1^x \frac{\exp\left(\frac{(2c+bk)\arctan\left(\frac{b+2aK[1]}{\sqrt{4ac-b^2}}\right)}{c\sqrt{4ac-b^2}}\right)K[1]^{-\frac{k}{c}}(c+K[1](b+aK[1]))^{\frac{k}{2c}+1}}{(k^2-K[1](aK[1]+2)k-(b-1)K[1]^2+c(k-2K[1]))^2}\right.}{ak+b-c(k-2)-k^2+2k-1}$$

31.17 problem 198

Internal problem ID [11032]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 198.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(a\,x^3 + b\,x^2 + cx)\,y'' + (x^2n + mx + k)\,y' + (k-1)\,((-ak+n)\,x + m - bk)\,y = 0$$

✗ Solution by Maple

```
dsolve((a*x^3+b*x^2+c*x)*diff(y(x),x$2)+(n*x^2+m*x+k)*diff(y(x),x)+(k-1)*((n-a*k)*x+m-b*k)*y
```

No solution found

✓ Solution by Mathematica

Time used: 148.451 (sec). Leaf size: 570

```
DSolve[(a*x^3+b*x^2+c*x)*y''[x]+(n*x^2+m*x+k)*y'[x]+(k-1)*((n-a*k)*x+m-b*k)*y[x]==0,y[x],x,I
```

$y(x) \to$

$$-\frac{2^{-\frac{k}{c}}\left(-\frac{ax}{\sqrt{b^2-4ac}+b}\right)^{1-\frac{k}{c}}\left(ac_1x\left(-2^{\frac{k}{c}}\right)\left(-\frac{ax}{\sqrt{b^2-4ac}+b}\right)^{\frac{k}{c}-1}\text{HeunG}\left[\frac{b-\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}+b},-\frac{2(k-1)(bk-m)}{\sqrt{b^2-4ac}+b},\frac{1}{2}\left(-\sqrt{\frac{(-2ak+}{a}}\right.\right.\right.}{}$$

31.18 problem 199

Internal problem ID [11033]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 199.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(a\,x^3 + b\,x^2 + cx)\,y'' + \left((m-a)\,x^2 + (2cm-1)\,x - c\right)y' + (-2mx+1)\,y = 0$$

✗ Solution by Maple

```
dsolve((a*x^3+b*x^2+c*x)*diff(y(x),x$2)+((m-a)*x^2+(2*c*m-1)*x-c)*diff(y(x),x)+(-2*m*x+1)*y(
```

No solution found

✓ Solution by Mathematica

Time used: 17.694 (sec). Leaf size: 192

```
DSolve[(a*x^3+b*x^2+c*x)*y''[x]+((m-a)*x^2+(2*c*m-1)*x-c)*y'[x]+(-2*m*x+1)*y[x]==0,y[x],x,In
```

$$y(x) \to \frac{(x(ax+2b+mx-1)+c(2b+4mx-1)+4c^2m)\left(c_2\int_1^x \frac{\exp\left(\frac{(bm-2a(b+2cm-1))\arctan\left(\frac{b+2aK[1]}{\sqrt{4ac-b^2}}\right)}{a\sqrt{4ac-b^2}}\right)K[1](c+K[1](b\cdots}{(4mc^2+(2b+4mK[1]-1)c+K[1](2b+aK[1]+mK[1]\cdots}\right.}{a+2b(c+1)+4c^2m+4cm-c+m-1}$$

31.19 problem 200

Internal problem ID [11034]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 200.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(a\,x^3 + b\,x^2 + cx)\,y'' + (x^2n + mx + k)\,y' + (-2(a+n)\,x + 1)\,y = 0$$

✗ Solution by Maple

```
dsolve((a*x^3+b*x^2+c*x)*diff(y(x),x$2)+(n*x^2+m*x+k)*diff(y(x),x)+(-2*(a+n)*x+1)*y(x)=0,y(x
```

No solution found

✓ Solution by Mathematica

Time used: 121.038 (sec). Leaf size: 552

```
DSolve[(a*x^3+b*x^2+c*x)*y''[x]+(n*x^2+m*x+k)*y'[x]+(-2*(a+n)*x+1)*y[x]==0,y[x],x,IncludeSin
```

$y(x) \to$

$$-\frac{2^{-\frac{k}{c}}\left(-\frac{ax}{\sqrt{b^2-4ac}+b}\right)^{1-\frac{k}{c}}\left(ac_1x\left(-2^{\frac{k}{c}}\right)\left(-\frac{ax}{\sqrt{b^2-4ac}+b}\right)^{\frac{k}{c}-1}\text{HeunG}\left[\frac{b-\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}+b},\frac{2}{\sqrt{b^2-4ac}+b},\frac{1}{2}\left(-\sqrt{\frac{(3a+n)^2}{a^2}}+\right.\right.\right.}{}$$

31.20 problem 201

Internal problem ID [11035]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 201.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^3a + x^2 + b)\, y'' + a^2x(x^2 - b)\, y' - a^3bxy = 0$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 88

```
dsolve((a*x^3+x^2+b)*diff(y(x),x$2)+a^2*x*(x^2-b)*diff(y(x),x)-a^3*b*x*y(x)=0,y(x), singsol=
```

$$y(x) = c_1\mathrm{e}^{-ax}(ax + 2) + c_2\mathrm{e}^{-ax}(ax + 2)\left(\int \mathrm{e}^{\int \frac{(a^2x^4+2a\,x^3+(a^2b+2)x^2+4abx+2b)a}{(a\,x^3+x^2+b)(ax+2)}dx}dx\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a*x^3+x^2+b)*y''[x]+a^2*x*(x^2-b)*y'[x]-a^3*b*x*y[x]==0,y[x],x,IncludeSingularSoluti
```

Timed out

31.21 problem 202

Internal problem ID [11036]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 202.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$2y''x(a\,x^2 + bx + c) + (a\,x^2 - c)\,y' + \lambda\,x^2y = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 65

```
dsolve(2*x*(a*x^2+b*x+c)*diff(y(x),x$2)+(a*x^2-c)*diff(y(x),x)+lambda*x^2*y(x)=0,y(x), sings
```

$$y(x) = c_1 e^{\int \frac{i\sqrt{2}\,\sqrt{\lambda}\,\sqrt{x}}{2\sqrt{a\,x^2+bx+c}}dx} + c_2 e^{-\left(\int \frac{i\sqrt{2}\,\sqrt{\lambda}\,\sqrt{x}}{2\sqrt{a\,x^2+bx+c}}dx\right)}$$

✓ Solution by Mathematica

Time used: 144.69 (sec). Leaf size: 501

```
DSolve[2*x*(a*x^2+b*x+c)*y''[x]+(a*x^2-c)*y'[x]+\[Lambda]*x^2*y[x]==0,y[x],x,IncludeSingular
```

$$y(x) \to c_1 \cosh\left(\frac{\sqrt{\lambda}\left(\sqrt{b^2-4ac}-b\right)\sqrt{\sqrt{b^2-4ac}+2ax+b}\sqrt{\frac{2ax}{b-\sqrt{b^2-4ac}}+1}\left(E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|\frac{b+\sqrt{b^2-}}{b-\sqrt{b^2-}}\right.\right.}{2a^{3/2}\sqrt{x(ax+b)+c}}\right.$$
$$+ic_2 \sinh\left(\frac{\sqrt{\lambda}\left(\sqrt{b^2-4ac}-b\right)\sqrt{\sqrt{b^2-4ac}+2ax+b}\sqrt{\frac{2ax}{b-\sqrt{b^2-4ac}}+1}\left(E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|\frac{b+\sqrt{}}{b-\sqrt{}}\right.\right.}{2a^{3/2}\sqrt{x(ax+b)+c}}\right.$$

31.22 problem 203

Internal problem ID [11037]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 203.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(a\,x^2 + bx + 1)\,y'' + (\alpha\,x^2 + \beta x + \gamma)\,y' + (nx + m)\,y = 0$$

✗ Solution by Maple

```
dsolve(x*(a*x^2+b*x+1)*diff(y(x),x$2)+(alpha*x^2+beta*x+gamma)*diff(y(x),x)+(n*x+m)*y(x)=0,y
```

No solution found

✓ Solution by Mathematica

Time used: 108.623 (sec). Leaf size: 524

```
DSolve[x*(a*x^2+b*x+1)*y''[x]+(\[Alpha]*x^2+\[Beta]*x+\[Gamma])*y'[x]+(n*x+m)*y[x]==0,y[x],x
```

$$y(x) \to -\frac{2^{-\gamma}\left(-\frac{ax}{\sqrt{b^2-4a}+b}\right)^{1-\gamma}\left(a(-2^{\gamma})\,c_1x\left(-\frac{ax}{\sqrt{b^2-4a}+b}\right)^{\gamma-1}\text{HeunG}\left[\frac{b-\sqrt{b^2-4a}}{\sqrt{b^2-4a}+b}, \frac{2m}{\sqrt{b^2-4a}+b}, \frac{1}{2}\left(-\sqrt{\frac{a^2+\alpha^2-2a(\alpha+2n)}{a^2}}\right.\right.\right.}{}$$

31.23 problem 204

Internal problem ID [11038]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 204.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(x-1)(x-a)\,y'' + \left((\beta+\alpha+1)\,x^2 - (\alpha+\beta+1+a(\gamma+d)-a)\,x + a\gamma\right)y' + (\alpha\beta x - q)\,y = 0$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 82

```
dsolve(x*(x-1)*(x-a)*diff(y(x),x$2)+((alpha+beta+1)*x^2-(alpha+beta+1+a*(gamma+d)-a)*x+a*gam
```

$$y(x) = c_1 \operatorname{HeunG}\left(a, q, \alpha, \beta, \gamma, \frac{a(d-1)}{a-1}, x\right) + c_2 x^{1-\gamma} \operatorname{HeunG}\left(a, q - (-1+\gamma)\left(a(d-1)+\alpha+\beta-\gamma+1\right), \beta+1-\gamma, \alpha+1-\gamma, 2-\gamma, \frac{a(d-1)}{a-1}, x\right)$$

✓ Solution by Mathematica

Time used: 2.215 (sec). Leaf size: 85

```
DSolve[x*(x-1)*(x-a)*y''[x]+((\[Alpha]+\[Beta]+1)*x^2-(\[Alpha]+\[Beta]+1+a*(\[Gamma]+d)-a)*
```

$$y(x) \to c_2 x^{1-\gamma}\text{HeunG}\left[a, q-(\gamma-1)(a(d-1)+\alpha+\beta-\gamma+1), \alpha-\gamma+1, \beta-\gamma+1, 2-\gamma, \frac{a(d-1)}{a-1}, x\right] + c_1\text{HeunG}\left[a, q, \alpha, \beta, \gamma, \frac{a(d-1)}{a-1}, x\right]$$

31.24 problem 205

Internal problem ID [11039]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 205.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^3a + b\,x^2 + xc + d)\,y'' - (-\lambda^2 + x^2)\,y' + (\lambda + x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 87

```
dsolve((a*x^3+b*x^2+c*x+d)*diff(y(x),x$2)-(x^2-lambda^2)*diff(y(x),x)+(x+lambda)*y(x)=0,y(x)
```

$$y(x) = c_1(-\lambda + x) + c_2(\lambda - x)\left(\int e^{\int \frac{(-2a+1)x^3+(-2b-\lambda)x^2+\left(-\lambda^2-2c\right)x+\lambda^3-2d}{\left(a\,x^3+b\,x^2+cx+d\right)(-\lambda+x)}dx} dx\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a*x^3+b*x^2+c*x+d)*y''[x]-(x^2-\[Lambda]^2)*y'[x]+(x+\[Lambda])*y[x]==0,y[x],x,Inclu
```

Timed out

31.25 problem 206

Internal problem ID [11040]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 206.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$2\left(a\,x^3 + b\,x^2 + cx + d\right) y'' + \left(3a\,x^2 + 2bx + c\right) y' + y\lambda = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 69

```
dsolve(2*(a*x^3+b*x^2+c*x+d)*diff(y(x),x$2)+(3*a*x^2+2*b*x+c)*diff(y(x),x)+lambda*y(x)=0,y(x
```

$$y(x) = c_1 \mathrm{e}^{\int \frac{i\sqrt{2}\,\sqrt{\lambda}}{2\sqrt{a\,x^3+b\,x^2+cx+d}}dx} + c_2 \mathrm{e}^{-\left(\int \frac{i\sqrt{2}\,\sqrt{\lambda}}{2\sqrt{a\,x^3+b\,x^2+cx+d}}dx\right)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[2*(a*x^3+b*x^2+c*x+d)*y''[x]+(3*a*x^2+2*b*x+c)*y'[x]+lambda*y[x]==0,y[x],x,IncludeSin
```

Timed out

31.26 problem 207

Internal problem ID [11041]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 207.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2\left(x^3 a + b\,x^2 + xc + d\right) y'' + 3\left(3a\,x^2 + 2bx + c\right) y' + \left(6ax + 2b + \lambda\right) y = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 113

```
dsolve(2*(a*x^3+b*x^2+c*x+d)*diff(y(x),x$2)+3*(3*a*x^2+2*b*x+c)*diff(y(x),x)+(6*a*x+2*b+lamb
```

$$y(x) = \frac{c_1 \mathrm{e}^{\int \frac{\sqrt{-\frac{2\lambda}{a}}}{2\sqrt{\frac{a\,x^3+b\,x^2+cx+d}{a}}}dx}}{\sqrt{a\,x^3+b\,x^2+cx+d}} + \frac{c_2 \mathrm{e}^{-\left(\int \frac{\sqrt{-\frac{2\lambda}{a}}}{2\sqrt{\frac{a\,x^3+b\,x^2+cx+d}{a}}}dx\right)}}{\sqrt{a\,x^3+b\,x^2+cx+d}}$$

✓ Solution by Mathematica

Time used: 135.727 (sec). Leaf size: 3202

```
DSolve[2*(a*x^3+b*x^2+c*x+d)*y''[x]+3*(3*a*x^2+2*b*x+c)*y'[x]+(6*a*x+2*b+\[Lambda])*y[x]==0,
```

Too large to display

31.27 problem 208

Internal problem ID [11042]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 208.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(a\,x^3 + b\,x^2 + cx + d\right) y'' + \left(\alpha\,x^2 + (\gamma\alpha + \beta)\,x + \beta\lambda\right) y' - (\alpha x + \beta)\,y = 0$$

✗ Solution by Maple

```
dsolve((a*x^3+b*x^2+c*x+d)*diff(y(x),x$2)+(alpha*x^2+(alpha*gamma+beta)*x+beta*lambda)*diff(
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a*x^3+b*x^2+c*x+d)*y''[x]+(\[Alpha]*x^2+(\[Alpha]*\[Gamma]+\[Beta])*x+\[Beta]*\[Lamb
```

Timed out

31.28 problem 209

Internal problem ID [11043]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 209.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^3a + b\,x^2 + xc + d)\,y'' + (\lambda^3 + x^3)\,y' - (\lambda^2 - x\lambda + x^2)\,y = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 85

```
dsolve((a*x^3+b*x^2+c*x+d)*diff(y(x),x$2)+(x^3+lambda^3)*diff(y(x),x)-(x^2-lambda*x+lambda^2
```

$$y(x) = c_1(x+\lambda) + c_2(x+\lambda)\left(\int e^{\int \frac{-x^4+(-2a-\lambda)x^3-2b\,x^2+(-\lambda^3-2c)x-\lambda^4-2d}{(a\,x^3+b\,x^2+cx+d)(x+\lambda)}dx}dx\right)$$

✓ Solution by Mathematica

Time used: 1.343 (sec). Leaf size: 240

```
DSolve[(a*x^3+b*x^2+c*x+d)*y''[x]+(x^3+\[Lambda]^3)*y'[x]-(x^2-\[Lambda]*x+\[Lambda]^2)*y[x]
```

$$y(x) \to \frac{c_2(\lambda+x)\int_1^x \exp\left(-\frac{\lambda+K[1]+2a\log(\lambda+K[1])+\text{RootSum}\left[-a\lambda^3+b\lambda^2+3a\#1\lambda^2-3a\#1^2\lambda-c\lambda-2b\#1\lambda+a\#1^3+b\#1^2+d+c\right.}{}\right.}{} + \frac{c_1(\lambda+x)}{\lambda}$$

31.29 problem 210

Internal problem ID [11044]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 210.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x\left(a\,x^2 + bx + c\right)y'' + \left(a(2-k)\,x^2 + b(-k+1)\,x - ck\right)y' + \lambda\,x^{k+1}y = 0$$

✗ Solution by Maple

```
dsolve(2*x(a*x^2+b*x+c)*diff(y(x),x$2)+(a*(2-k)*x^2+b*(1-k)*x-c*k)*diff(y(x),x)+(lambda*x^(k
```

No solution found

✓ Solution by Mathematica

Time used: 157.344 (sec). Leaf size: 790

```
DSolve[2*x(a*x^2+b*x+c)*y''[x]+(a*(2-k)*x^2+b*(1-k)*x-c*k)*y'[x]+(\[Lambda]*x^(k+1))*y[x]==0
```

$$y(x) \to \frac{\sqrt{2}\sqrt{c_1}\tan\left(\frac{\sqrt{2}x\sqrt{\frac{-\sqrt{b^2-4ac}+2ax+b}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+2ax+b}{\sqrt{b^2-4ac}+b}}\,\text{AppellF1}\left(\frac{k+2}{2},\frac{1}{2},\frac{1}{2},\frac{k+4}{2},-\frac{2ax}{b+\sqrt{b^2-4ac}},\frac{2ax}{\sqrt{b^2-4ac}-b}\right)}{(k+2)\sqrt{\frac{x^{-k}(x(ax+b)+c)}{\lambda}}}-c_2\right)}{\sqrt{-1-\tan^2\left(\frac{\sqrt{2}x\sqrt{\frac{-\sqrt{b^2-4ac}+2ax+b}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+2ax+b}{\sqrt{b^2-4ac}+b}}\,\text{AppellF1}\left(\frac{k+2}{2},\frac{1}{2},\frac{1}{2},\frac{k+4}{2},-\frac{2ax}{b+\sqrt{b^2-4ac}},\frac{2ax}{\sqrt{b^2-4ac}-b}\right)}{(k+2)\sqrt{\frac{x^{-k}(x(ax+b)+c)}{\lambda}}}-c_2\right)}}$$

$$y(x) \to -\frac{\sqrt{2}\sqrt{c_1}\tan\left(\frac{\sqrt{2}x\sqrt{\frac{-\sqrt{b^2-4ac}+2ax+b}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+2ax+b}{\sqrt{b^2-4ac}+b}}\,\text{AppellF1}\left(\frac{k+2}{2},\frac{1}{2},\frac{1}{2},\frac{k+4}{2},-\frac{2ax}{b+\sqrt{b^2-4ac}},\frac{2ax}{\sqrt{b^2-4ac}-b}\right)}{(k+2)\sqrt{\frac{x^{-k}(x(ax+b)+c)}{\lambda}}}+c_2\right)}{\sqrt{-1-\tan^2\left(\frac{\sqrt{2}x\sqrt{\frac{-\sqrt{b^2-4ac}+2ax+b}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+2ax+b}{\sqrt{b^2-4ac}+b}}\,\text{AppellF1}\left(\frac{k+2}{2},\frac{1}{2},\frac{1}{2},\frac{k+4}{2},-\frac{2ax}{b+\sqrt{b^2-4ac}},\frac{2ax}{\sqrt{b^2-4ac}-b}\right)}{(k+2)\sqrt{\frac{x^{-k}(x(ax+b)+c)}{\lambda}}}+c_2\right)}}$$

32 Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4 + a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$

32.1 problem 211

Internal problem ID [11045]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4 + a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 211.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^4y'' + ay = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(x^4*diff(y(x),x$2)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sinh\left(\frac{\sqrt{-a}}{x}\right) + c_2 x \cosh\left(\frac{\sqrt{-a}}{x}\right)$$

✓ Solution by Mathematica

Time used: 0.199 (sec). Leaf size: 52

```
DSolve[x^4*y''[x]+a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x e^{\frac{i\sqrt{a}}{x}} - \frac{ic_2 x e^{-\frac{i\sqrt{a}}{x}}}{2\sqrt{a}}$$

32.2 problem 212

Internal problem ID [11046]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4 + a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 212.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^4y'' + \left(a\,x^2 + bx + c\right) y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 63

```
dsolve(x^4*diff(y(x),x$2)+(a*x^2+b*x+c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \,\mathrm{WhittakerM}\left(-\frac{ib}{2\sqrt{c}}, \frac{\sqrt{1-4a}}{2}, \frac{2i\sqrt{c}}{x}\right) + c_2 x \,\mathrm{WhittakerW}\left(-\frac{ib}{2\sqrt{c}}, \frac{\sqrt{1-4a}}{2}, \frac{2i\sqrt{c}}{x}\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^4*y''[x]+(a*x^2+b*x+c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

32.3 problem 213

Internal problem ID [11047]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4 + a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 213.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^4y'' - (a+b)\,y'x^2 + ((a+b)\,x + ba)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 25

```
dsolve(x^4*diff(y(x),x$2)-(a+b)*x^2*diff(y(x),x)+((a+b)*x+a*b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x\,\mathrm{e}^{-\frac{a}{x}} + c_2x\,\mathrm{e}^{-\frac{b}{x}}$$

✓ Solution by Mathematica

Time used: 0.214 (sec). Leaf size: 37

```
DSolve[x^4*y''[x]-(a+b)*x^2*y'[x]+((a+b)*x+a*b)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{c_2xe^{-\frac{a}{x}}}{a-b} + c_1xe^{-\frac{b}{x}}$$

32.4 problem 214

Internal problem ID [11048]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4 + a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 214.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^4y'' + 2(x + a)\, y'x^2 + by = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 43

```
dsolve(x^4*diff(y(x),x$2)+2*x^2*(x+a)*diff(y(x),x)+b*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{a-\sqrt{a^2-b}}{x}} + c_2 e^{\frac{a+\sqrt{a^2-b}}{x}}$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 51

```
DSolve[x^4*y''[x]+2*x^2*(x+a)*y'[x]+b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{a-\sqrt{a^2-b}}{x}} \left(c_1 e^{\frac{2\sqrt{a^2-b}}{x}} + c_2\right)$$

32.5 problem 215

Internal problem ID [11049]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4 + a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 215.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^4y'' + a\,x^n y' - \left(a\,x^{-1+n} + x^{n-2}ab + b^2\right) y = 0$$

✗ Solution by Maple

```
dsolve(x^4*diff(y(x),x$2)+a*x^n*diff(y(x),x)-(a*x^(n-1)+a*b*x^(n-2)+b^2)*y(x)=0,y(x), singso
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^4*y''[x]+a*x^n*y'[x]-(a*x^(n-1)+a*b*x^(n-2)+b^2)*y[x]==0,y[x],x,IncludeSingularSolu
```

Not solved

32.6 problem 216

Internal problem ID [11050]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4 + a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 216.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x-a)^2 y'' + by = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 75

```
dsolve(x^2*(x-a)^2*diff(y(x),x$2)+b*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x(-x+a)}\left(\frac{-x+a}{x}\right)^{\frac{\sqrt{a^2-4b}}{2a}} + c_2\sqrt{x(-x+a)}\left(\frac{x}{-x+a}\right)^{\frac{\sqrt{a^2-4b}}{2a}}$$

✓ Solution by Mathematica

Time used: 0.487 (sec). Leaf size: 121

```
DSolve[x^2*(x-a)^2*y''[x]+b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{\frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{4b}{a^2}}}(x-a)^{\frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{4b}{a^2}}}\left(ac_1\sqrt{1-\frac{4b}{a^2}}x^{\sqrt{1-\frac{4b}{a^2}}} + c_2(x-a)^{\sqrt{1-\frac{4b}{a^2}}}\right)}{a\sqrt{1-\frac{4b}{a^2}}}$$

32.7 problem 217

Internal problem ID [11051]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4 + a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 217.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x^2(x-a)^2 y'' + by = c\,x^2(x-a)^2$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 219

```
dsolve(x^2*(x-a)^2*diff(y(x),x$2)+b*y(x)=c*x^2*(x-a)^2,y(x), singsol=all)
```

$$y(x) = \sqrt{x\,(-x+a)}\left(\frac{-x+a}{x}\right)^{\frac{\sqrt{a^2-4b}}{2a}} c_2 + \sqrt{x\,(-x+a)}\left(\frac{x}{-x+a}\right)^{\frac{\sqrt{a^2-4b}}{2a}} c_1$$
$$+ \frac{\sqrt{x\,(-x+a)}\,c\left(\left(\frac{x}{-x+a}\right)^{\frac{\sqrt{a^2-4b}}{2a}}\left(\int \sqrt{x\,(-x+a)}\left(\frac{x}{-x+a}\right)^{-\frac{\sqrt{a^2-4b}}{2a}} dx\right) - \left(\frac{-x+a}{x}\right)^{\frac{\sqrt{a^2-4b}}{2a}}\left(\int \sqrt{x\,(-x+a)}\right.\right.}{\sqrt{a^2-4b}}$$

✓ Solution by Mathematica

Time used: 0.958 (sec). Leaf size: 371

```
DSolve[x^2*(x-a)^2*y''[x]+b*y[x]==c*x^2*(x-a)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{acx^2(a-x)\left(1-\frac{x}{a}\right)^{-\frac{1}{2}\sqrt{1-\frac{4b}{a^2}}-\frac{1}{2}}\left(\left(\sqrt{1-\frac{4b}{a^2}}-3\right)\left(1-\frac{x}{a}\right)^{\sqrt{1-\frac{4b}{a^2}}}\text{Hypergeometric2F1}\left(\frac{1}{2}\sqrt{1-\frac{4b}{a^2}}-\frac{1}{2},\frac{1}{2}\right.\right.}{}$$

$$+c_1x^{\frac{1}{2}\sqrt{1-\frac{4b}{a^2}}+\frac{1}{2}}(x-a)^{\frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{4b}{a^2}}}+\frac{c_2x^{\frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{4b}{a^2}}}(x-a)^{\frac{1}{2}\sqrt{1-\frac{4b}{a^2}}+\frac{1}{2}}}{a\sqrt{1-\frac{4b}{a^2}}}$$

32.8 problem 218

Internal problem ID [11052]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4 + a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 218.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$a\,x^2(x-1)^2\,y'' + \left(b\,x^2 + cx + d\right)y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 299

```
dsolve(a*x^2*(x-1)^2*diff(y(x),x$2)+(b*x^2+c*x+d)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{\frac{\sqrt{a}+\sqrt{a-4d}}{2\sqrt{a}}}\,\mathrm{hypergeom}\left(\left[\frac{-\sqrt{a-4b-4c-4d}+\sqrt{a}+\sqrt{a-4d}+\sqrt{a-4b}}{2\sqrt{a}}, -\frac{\sqrt{a-4b-4c-4d}-\sqrt{a}-\sqrt{a-4d}+\sqrt{a-4b}}{2\sqrt{a}}\right], \left[\frac{\sqrt{a}+\sqrt{a-4d}}{\sqrt{a}}\right], x\right)(x-1)^{\frac{-\sqrt{a-4b-4c-4d}+\sqrt{a}}{2\sqrt{a}}} + c_2(x-1)^{-\frac{\sqrt{a-4b-4c-4d}-\sqrt{a}}{2\sqrt{a}}}\,\mathrm{hypergeom}\left(\left[-\frac{\sqrt{a-4b-4c-4d}-\sqrt{a}+\sqrt{a-4d}+\sqrt{a-4b}}{2\sqrt{a}}, -\frac{\sqrt{a-4b-4c-4d}-\sqrt{a}+\sqrt{a-4d}-\sqrt{a-4b}}{2\sqrt{a}}\right], \left[\frac{\sqrt{a}-\sqrt{a-4d}}{\sqrt{a}}\right], x\right)x^{\frac{\sqrt{a}-\sqrt{a-4d}}{2\sqrt{a}}}$$

✓ Solution by Mathematica

Time used: 135.53 (sec). Leaf size: 413606

```
DSolve[a*x^2*(x-1)^2*y''[x]+(b*x^2+c*x+d)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

32.9 problem 219

Internal problem ID [11053]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4 + a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 219.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+a)\,y'' + (b\,x^2+c)\,xy' + dy = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 309

```
dsolve(x^2*(x^2+a)*diff(y(x),x$2)+(b*x^2+c)*x*diff(y(x),x)+d*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{\frac{a-c+\sqrt{a^2+(-2c-4d)a+c^2}}{2a}} (x^2 + a)^{\frac{(-b+2)a+c}{2a}} \operatorname{hypergeom}\left(\left[\frac{3a+c+\sqrt{a^2+(-2c-4d)\,a+c^2}}{4a}, \frac{\sqrt{a^2+(-2c-4d)\,a+c^2}+(-2b+5)\,a}{4a} \right. \right. -\frac{x^2}{a}\Big) + c_2 x^{-\frac{-a+c+\sqrt{a^2+(-2c-4d)a+c^2}}{2a}} (x^2 + a)^{\frac{(-b+2)a+c}{2a}} \operatorname{hypergeom}\left(\left[-\frac{-3a-c+\sqrt{a^2+(-2c-4d)\,a+c^2}}{4a}, \frac{-\sqrt{a^2+(-2c-4d)\,a+c^2}+(-2b}{4a} \right.\right. -\frac{x^2}{a}\Big)$$

✓ Solution by Mathematica

Time used: 2.385 (sec). Leaf size: 336

```
DSolve[x^2*(x^2+a)*y''[x]+(b*x^2+c)*x*y'[x]+d*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to a^{-\frac{\sqrt{a^2-2a(c+2d)+c^2}+a-c}{4a}} x^{-\frac{\sqrt{a^2-2a(c+2d)+c^2}-a+c}{2a}} \left(c_2 x^{\frac{\sqrt{a^2-2a(c+2d)+c^2}}{a}} \text{Hypergeometric2F1}\left(-\frac{-2ba+a+c-\sqrt{a}}{4a} \right.\right.$$

$$\left. +1, -\frac{x^2}{a}\right)$$

$$+ c_1 a^{\frac{\sqrt{a^2-2a(c+2d)+c^2}}{2a}} \text{Hypergeometric2F1}\left(-\frac{-a+c+\sqrt{a^2-2(c+2d)a+c^2}}{4a},\right.$$

$$\left.\left. -\frac{-2ba+a+c+\sqrt{a^2-2(c+2d)a+c^2}}{4a}, 1-\frac{\sqrt{a^2-2(c+2d)a+c^2}}{2a}, -\frac{x^2}{a}\right)\right)$$

32.10 problem 220

Internal problem ID [11054]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4 + a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 220.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Halm]`

$$\boxed{(x^2+1)^2 y'' + ay = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 65

```
dsolve((x^2+1)^2*diff(y(x),x$2)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x^2+1}\left(\frac{x+i}{-x+i}\right)^{\frac{\sqrt{a+1}}{2}} + c_2\sqrt{x^2+1}\left(\frac{x+i}{-x+i}\right)^{-\frac{\sqrt{a+1}}{2}}$$

✓ Solution by Mathematica

Time used: 0.215 (sec). Leaf size: 83

```
DSolve[(x^2+1)^2*y''[x]+a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\sqrt{x^2+1}e^{i\sqrt{a+1}\arctan(x)}\left(\frac{ic_2(1-ix)^{\sqrt{a+1}}(1+ix)^{-\sqrt{a+1}}}{\sqrt{a+1}} + 2c_1\right)$$

32.11 problem 221

Internal problem ID [11055]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4 + a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 221.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-1)^2 y'' + ay = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 61

```
dsolve((x^2-1)^2*diff(y(x),x$2)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x^2-1}\left(\frac{x-1}{x+1}\right)^{\frac{\sqrt{-a+1}}{2}} + c_2\sqrt{x^2-1}\left(\frac{x-1}{x+1}\right)^{-\frac{\sqrt{-a+1}}{2}}$$

✓ Solution by Mathematica

Time used: 0.249 (sec). Leaf size: 88

```
DSolve[(x^2-1)^2*y''[x]+a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(1-x^2)^{\frac{1}{2}-\frac{\sqrt{1-a}}{2}}\left(2\sqrt{1-a}c_1(1-x)^{\sqrt{1-a}} + c_2(x+1)^{\sqrt{1-a}}\right)}{2\sqrt{1-a}}$$

32.12 problem 222 A

Internal problem ID [11056]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4 + a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 222 A.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{(a^2 + x^2)^2 y'' + yb^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 91

```
dsolve((x^2+a^2)^2*diff(y(x),x$2)+b^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{a^2 + x^2}\left(\frac{ix - a}{ix + a}\right)^{\frac{\sqrt{a^2+b^2}}{2a}} + c_2\sqrt{a^2 + x^2}\left(\frac{ix - a}{ix + a}\right)^{-\frac{\sqrt{a^2+b^2}}{2a}}$$

✓ Solution by Mathematica

Time used: 0.489 (sec). Leaf size: 97

```
DSolve[(x^2+a^2)^2*y''[x]+b^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\sqrt{a^2 + x^2}e^{-i\sqrt{\frac{b^2}{a^2}+1}\arctan\left(\frac{a}{x}\right)}\left(\frac{ic_2e^{2i\sqrt{\frac{b^2}{a^2}+1}\arctan\left(\frac{a}{x}\right)}}{a\sqrt{\frac{b^2}{a^2}+1}} + 2c_1\right)$$

32.13 problem 222 B

Internal problem ID [11057]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4 + a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 222 B.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-a^2 + x^2)^2 y'' + y b^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 87

```
dsolve((x^2-a^2)^2*diff(y(x),x$2)+b^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sqrt{(-x+a)(x+a)} \left(\frac{-x+a}{x+a}\right)^{\frac{\sqrt{a^2-b^2}}{2a}} + c_2 \sqrt{(-x+a)(x+a)} \left(\frac{-x+a}{x+a}\right)^{-\frac{\sqrt{a^2-b^2}}{2a}}$$

✓ Solution by Mathematica

Time used: 0.529 (sec). Leaf size: 142

```
DSolve[(x^2-a^2)^2*y''[x]+b^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(x-a)^{\frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{b^2}{a^2}}}(a+x)^{\frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{b^2}{a^2}}}\left(2ac_1\sqrt{1-\frac{b^2}{a^2}}(x-a)^{\sqrt{1-\frac{b^2}{a^2}}} - c_2(a+x)^{\sqrt{1-\frac{b^2}{a^2}}}\right)}{2a\sqrt{1-\frac{b^2}{a^2}}}$$

32.14 problem 223

Internal problem ID [11058]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4 + a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 223.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Halm]`

$$4(x^2+1)^2 y'' + (a\,x^2+a-3)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 61

```
dsolve(4*(x^2+1)^2*diff(y(x),x$2)+(a*x^2+a-3)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x^2+1)^{\frac{1}{4}}\left(x+\sqrt{x^2+1}\right)^{\frac{\sqrt{-a+1}}{2}} + c_2(x^2+1)^{\frac{1}{4}}\left(x+\sqrt{x^2+1}\right)^{-\frac{\sqrt{-a+1}}{2}}$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 70

```
DSolve[4*(x^2+1)^2*y''[x]+(a*x^2+a-3)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x^2+1}\left(c_1 P^{\frac{1}{2}}_{\frac{1}{2}(\sqrt{1-a}-1)}(ix) + c_2 Q^{\frac{1}{2}}_{\frac{1}{2}(\sqrt{1-a}-1)}(ix)\right)$$

32.15 problem 224

Internal problem ID [11059]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4 + a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 224.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{(a\,x^2+b)^2\,y''+2ax\,(a\,x^2+b)\,y'+cy=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve((a*x^2+b)^2*diff(y(x),x$2)+2*a*x*(a*x^2+b)*diff(y(x),x)+c*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\sin\left(\frac{\sqrt{c}\,\arctan\left(\frac{xa}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)+c_2\cos\left(\frac{\sqrt{c}\,\arctan\left(\frac{xa}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)$$

✓ Solution by Mathematica

Time used: 2.133 (sec). Leaf size: 72

```
DSolve[(a*x^2+b)^2*y''[x]+2*a*x*(a*x^2+b)*y'[x]+c*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to c_1\cos\left(\frac{\sqrt{c}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}\right)+c_2\sin\left(\frac{\sqrt{c}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}\right)$$

32.16 problem 225

Internal problem ID [11060]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4 + a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 225.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2-1)^2 y'' + 2x(x^2-1)\, y' - (\nu(\nu+1)\,(x^2-1) + n^2)\, y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve((x^2-1)^2*diff(y(x),x$2)+2*x*(x^2-1)*diff(y(x),x)-(nu*(nu+1)*(x^2-1)+n^2)*y(x)=0,y(x)
```

$$y(x) = c_1 \operatorname{LegendreP}(\nu, n, x) + c_2 \operatorname{LegendreQ}(\nu, n, x)$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 20

```
DSolve[(x^2-1)^2*y''[x]+2*x*(x^2-1)*y'[x]-(\[Nu]*(\[Nu]+1)*(x^2-1)+n^2)*y[x]==0,y[x],x,Inclu
```

$$y(x) \to c_1 P_\nu^n(x) + c_2 Q_\nu^n(x)$$

32.17 problem 226

Internal problem ID [11061]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4 + a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 226.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(-x^2+1)^2 y'' - 2x(-x^2+1) y' + (\nu(\nu+1)(-x^2+1) - \mu^2) y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve((1-x^2)^2*diff(y(x),x$2)-2*x*(1-x^2)*diff(y(x),x)+(nu*(nu+1)*(1-x^2)-mu^2)*y(x)=0,y(x
```

$$y(x) = c_1 \operatorname{LegendreP}(\nu, \mu, x) + c_2 \operatorname{LegendreQ}(\nu, \mu, x)$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 20

```
DSolve[(1-x^2)^2*y''[x]-2*x*(1-x^2)*y'[x]+(\[Nu]*(\[Nu]+1)*(1-x^2)-\[Mu]^2)*y[x]==0,y[x],x,I
```

$$y(x) \to c_1 P_\nu^\mu(x) + c_2 Q_\nu^\mu(x)$$

32.18 problem 227

Internal problem ID [11062]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4 + a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 227.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$a(x^2-1)^2 y'' + bx(x^2-1)\,y' + (c\,x^2 + dx + e)\,y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 613

```
dsolve(a*(x^2-1)^2*diff(y(x),x$2)+b*x*(x^2-1)*diff(y(x),x)+(c*x^2+d*x+e)*y(x)=0,y(x), singso
```

$$y(x) = c_1\left(\frac{x}{2}-\frac{1}{2}\right)^{\frac{2a+\sqrt{4a^2+(-4b-4c-4d-4e)a+b^2}}{4a}} (x^2$$
$$-1)^{-\frac{b}{4a}} \text{hypergeom}\left(\left[-\frac{-\sqrt{4a^2+(-4b-4c-4d-4e)\,a+b^2}+2\sqrt{a^2+(-2b-4c)\,a+b^2}+\sqrt{4a^2+}}{4a}\right.\right.$$
$$-\frac{-\sqrt{4a^2+(-4b-4c-4d-4e)\,a+b^2}-2\sqrt{a^2+(-2b-4c)\,a+b^2}+\sqrt{4a^2+(-4b-4c+4d-4e)}}{4a}$$
$$\left.+\frac{1}{2}\right)\left(\frac{x}{2}+\frac{1}{2}\right)^{\frac{2a-\sqrt{4a^2+(-4b-4c+4d-4e)a+b^2}}{4a}} + c_2\left(\frac{x}{2}-\frac{1}{2}\right)^{\frac{2a+\sqrt{4a^2+(-4b-4c-4d-4e)a+b^2}}{4a}} (x^2$$
$$-1)^{-\frac{b}{4a}} \text{hypergeom}\left(\left[\frac{\sqrt{4a^2+(-4b-4c-4d-4e)\,a+b^2}-2\sqrt{a^2+(-2b-4c)\,a+b^2}+\sqrt{4a^2+(-}}{4a}\right.\right.$$
$$\left.+\frac{1}{2}\right)\left(\frac{x}{2}+\frac{1}{2}\right)^{\frac{2a+\sqrt{4a^2+(-4b-4c+4d-4e)a+b^2}}{4a}}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[a*(x^2-1)^2*y''[x]+b*x*(x^2-1)*y'[x]+(c*x^2+d*x+e)*y[x]==0,y[x],x,IncludeSingularSolu
```

Timed out

32.19 problem 228

Internal problem ID [11063]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4 + a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 228.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(a\,x^2+b)^2\,y''+(2ax+c)\left(a\,x^2+b\right)y'+ky=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 162

```
dsolve((a*x^2+b)^2*diff(y(x),x$2)+(2*a*x+c)*(a*x^2+b)*diff(y(x),x)+k*y(x)=0,y(x), singsol=al
```

$$y(x)=c_1\left(\frac{\sqrt{ab}+iax}{iax-\sqrt{ab}}\right)^{\frac{i\sqrt{ab}\,c\sqrt{-ab}+a^2\sqrt{\frac{c^2-4k}{a^2}}\,b}{4ab\sqrt{-ab}}}+c_2\left(\frac{\sqrt{ab}+iax}{iax-\sqrt{ab}}\right)^{\frac{i\sqrt{ab}\,c\sqrt{-ab}-a^2\sqrt{\frac{c^2-4k}{a^2}}\,b}{4ab\sqrt{-ab}}}$$

✓ Solution by Mathematica

Time used: 2.188 (sec). Leaf size: 91

```
DSolve[(a*x^2+b)^2*y''[x]+(2*a*x+c)*(a*x^2+b)*y'[x]+k*y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x)\to e^{-\frac{\left(\sqrt{c^2-4k}+c\right)\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}}\left(c_2e^{\frac{\sqrt{c^2-4k}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}}+c_1\right)$$

32.20 problem 229

Internal problem ID [11064]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 229.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(a\,x^2+b\right)^2 y''+\left(a\,x^2+b\right)\left(c\,x^2+d\right)y'+2(-da+bc)\,xy=0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 1000

```
dsolve((a*x^2+b)^2*diff(y(x),x$2)+(a*x^2+b)*(c*x^2+d)*diff(y(x),x)+2*(b*c-a*d)*x*y(x)=0,y(x)
```

$$y(x)=c_1\left(ax+\sqrt{-ab}\right)^{\frac{2a^2b+\sqrt{4a^2b(ad-cb)\sqrt{-ab}+4a^4b^2-b\,d^2a^3+2b^2cd\,a^2-b^3c^2a}}{4a^2b}}\left(-ax\right.$$
$$\left.+\sqrt{-ab}\right)^{\frac{2a^2b+\sqrt{-ab\left(4\sqrt{-ab}\,a^2d-4\sqrt{-ab}\,abc-4a^3b+d^2a^2-2abcd+b^2c^2\right)}}{4a^2b}}\mathrm{e}^{-\frac{-a^{\frac{3}{2}}\sqrt{-ab}\,\sqrt{b}\,c+\arctan\left(\frac{x\sqrt{a}}{\sqrt{b}}\right)d\,a^3-b\arctan\left(\frac{x\sqrt{a}}{\sqrt{b}}\right)c\,a^2}{2a^{\frac{7}{2}}\sqrt{b}}}\operatorname{HeunC}\Bigg($$
$$+c_2\operatorname{HeunC}\left(\frac{2c\sqrt{-\frac{b}{a}}}{a},\right.$$
$$-\frac{\sqrt{4a^2b\,(ad-cb)\sqrt{-ab}+4a^4b^2-b\,d^2a^3+2b^2cd\,a^2-b^3c^2a}}{2a^2b},\frac{\sqrt{-ab\,(4\sqrt{-ab}\,a^2d-4\sqrt{-ab}\,abc-4a^3b}}{2a^2b}$$
$$+\sqrt{-ab}\Big)^{\frac{2a^2b+\sqrt{-ab\left(4\sqrt{-ab}\,a^2d-4\sqrt{-ab}\,abc-4a^3b+d^2a^2-2abcd+b^2c^2\right)}}{4a^2b}}\mathrm{e}^{\frac{i\pi\,a^{\frac{3}{2}}\sqrt{4a^2b(ad-cb)\sqrt{-ab}+4a^4b^2-b\,d^2a^3+2b^2cd\,a^2-b^3c^2a}\,\sqrt{b}-i\pi\,a^{\frac{3}{2}}\sqrt{b}}{}}$$
$$+\sqrt{-ab}\Big)^{\frac{2a^2b-\sqrt{4a^2b(ad-cb)\sqrt{-ab}+4a^4b^2-b\,d^2a^3+2b^2cd\,a^2-b^3c^2a}}{4a^2b}}$$

✓ Solution by Mathematica

Time used: 0.165 (sec). Leaf size: 104

```
DSolve[(a*x^2+b)^2*y''[x]+(a*x^2+b)*(c*x^2+d)*y'[x]+2*(b*c-a*d)*x*y[x]==0,y[x],x,IncludeSing
```

$$y(x) \to \exp\left(\frac{\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)(bc-ad)}{a^{3/2}\sqrt{b}} - \frac{cx}{a}\right)\left(\int_1^x \exp\left(\frac{(ad-bc)\arctan\left(\frac{\sqrt{a}K[1]}{\sqrt{b}}\right)}{a^{3/2}\sqrt{b}} + \frac{cK[1]}{a}\right)c_1dK[1] + c_2\right)$$

32.21 problem 230

Internal problem ID [11065]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4 + a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 230.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(x^2 + a\right)^2 y'' + b\, x^n \left(x^2 + a\right) y' - \left(x^{n+1}b + a\right) y = 0$$

✗ Solution by Maple

```
dsolve((x^2+a)^2*diff(y(x),x$2)+b*x^n*(x^2+a)*diff(y(x),x)-(b*x^(n+1)+a)*y(x)=0,y(x), singso
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(x^2+a)^2*y''[x]+b*x^n*(x^2+a)*y'[x]-(b*x^(n+1)+a)*y[x]==0,y[x],x,IncludeSingularSolu
```

Not solved

32.22 problem 231

Internal problem ID [11066]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4 + a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(x^2+a\right)^2 y''+b\,x^n\left(x^2+a\right)y'-m\left(x^{n+1}b+(m-1)\,x^2+a\right)y=0$$

✗ Solution by Maple

```
dsolve((x^2+a)^2*diff(y(x),x$2)+b*x^n*(x^2+a)*diff(y(x),x)-m*(b*x^(n+1)+(m-1)*x^2+a)*y(x)=0,
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(x^2+a)^2*y''[x]+b*x^n*(x^2+a)*y'[x]-m*(b*x^(n+1)+(m-1)*x^2+a)*y[x]==0,y[x],x,Include
```

Not solved

32.23 problem 232

Internal problem ID [11067]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4 + a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 232.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-a)^2 (x-b)^2 y'' - cy = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 116

```
dsolve((x-a)^2*(x-b)^2*diff(y(x),x$2)-c*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{(-x+a)(-x+b)}\left(\frac{-x+a}{-x+b}\right)^{\frac{\sqrt{a^2-2ab+b^2+4c}}{2a-2b}} + c_2\sqrt{(-x+a)(-x+b)}\left(\frac{-x+a}{-x+b}\right)^{-\frac{\sqrt{a^2-2ab+b^2+4c}}{2a-2b}}$$

✓ Solution by Mathematica

Time used: 1.085 (sec). Leaf size: 141

```
DSolve[(x-a)^2*(x-b)^2*y''[x]-c*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x-a)^{\frac{1}{2}\left(1-\sqrt{\frac{4c}{(a-b)^2}+1}\right)}(x-b)^{\frac{1}{2}\left(1-\sqrt{\frac{4c}{(a-b)^2}+1}\right)}\left(c_1(x-a)^{\sqrt{\frac{4c}{(a-b)^2}+1}} - \frac{c_2(x-b)^{\sqrt{\frac{4c}{(a-b)^2}+1}}}{(a-b)\sqrt{\frac{4c}{(a-b)^2}+1}}\right)$$

32.24 problem 233

Internal problem ID [11068]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4 + a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 233.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x-a)^2 (x-b)^2 y'' + (x-a)(x-b)(2x+\lambda) y' + y\mu = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 170

```
dsolve((x-a)^2*(x-b)^2*diff(y(x),x$2)+(x-a)*(x-b)*(2*x+lambda)*diff(y(x),x)+mu*y(x)=0,y(x),
```

$$y(x) = c_1 \left(\frac{-x+b}{-x+a}\right)^{\frac{b+a+\lambda}{2a-2b}} \left(\frac{-x+a}{-x+b}\right)^{\frac{\sqrt{\lambda^2+(2a+2b)\lambda+a^2+2ab+b^2-4\mu}}{2a-2b}} + c_2 \left(\frac{-x+b}{-x+a}\right)^{\frac{b+a+\lambda}{2a-2b}} \left(\frac{-x+a}{-x+b}\right)^{-\frac{\sqrt{\lambda^2+(2a+2b)\lambda+a^2+2ab+b^2-4\mu}}{2a-2b}}$$

✓ Solution by Mathematica

Time used: 2.299 (sec). Leaf size: 152

```
DSolve[(x-a)^2*(x-b)^2*y''[x]+(x-a)*(x-b)*(2*x+\[Lambda])*y'[x]+mu*y[x]==0,y[x],x,IncludeSin
```

$$y(x) \to e^{-\frac{(a+b+\lambda)(\log(x-a)-\log(x-b))}{a-b}}\left(c_1\exp\left(\frac{\left(\sqrt{\mu}\sqrt{\frac{(a+b+\lambda)^2}{\mu}-4}+a+b+\lambda\right)(\log(x-a)-\log(x-b))}{2(a-b)}\right)+c_2\exp\left(\frac{\left(-\sqrt{\mu}\sqrt{\frac{(a+b+\lambda)^2}{\mu}-4}+a+b+\lambda\right)(\log(x-a)-\log(x-b))}{2(a-b)}\right)\right)$$

32.25 problem 234

Internal problem ID [11069]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4 + a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 234.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{\left(a\,x^2 + bx + c\right)^2 y'' + Ay = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 189

```
dsolve((a*x^2+b*x+c)^2*diff(y(x),x$2)+A*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{a\,x^2 + xb + c}\left(\frac{i\sqrt{4ac-b^2} - 2ax - b}{2ax + b + i\sqrt{4ac-b^2}}\right)^{\frac{a\sqrt{\frac{-4ac+b^2-4A}{a^2}}}{2\sqrt{-4ac+b^2}}} + c_2\sqrt{a\,x^2 + xb + c}\left(\frac{i\sqrt{4ac-b^2} - 2ax - b}{2ax + b + i\sqrt{4ac-b^2}}\right)^{-\frac{a\sqrt{\frac{-4ac+b^2-4A}{a^2}}}{2\sqrt{-4ac+b^2}}}$$

✓ Solution by Mathematica

Time used: 2.154 (sec). Leaf size: 199

```
DSolve[(a*x^2+b*x+c)^2*y''[x]+A*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$y(x)$

$$\to \sqrt{x(ax+b)+c}\exp\left(-\frac{\sqrt{4ac-b^2}\sqrt{1-\frac{4A}{b^2-4ac}}\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{b^2-4ac}}\right)\left(c_1\exp\left(\frac{2\sqrt{4ac-b^2}\sqrt{1-\frac{4A}{b^2-4ac}}\ a}{\sqrt{b^2-4a}}\right.\right.$$

$$\left.+\frac{c_2}{\sqrt{b^2-4ac}\sqrt{1-\frac{4A}{b^2-4ac}}}\right)$$

32.26 problem 235

Internal problem ID [11070]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4 + a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 235.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2-1)^2 y'' + 2x(x^2-1) y' + ((x^2-1)(a^2x^2-\lambda) - m^2) y = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 72

```
dsolve((x^2-1)^2*diff(y(x),x$2)+2*x*(x^2-1)*diff(y(x),x)+( (x^2-1)*(a^2*x^2-lambda)-m^2)*y(x
```

$$y(x) = c_1(x^2-1)^{\frac{m}{2}} \operatorname{HeunC}\left(0, -\frac{1}{2}, m, \frac{a^2}{4}, \frac{1}{4} + \frac{m^2}{4} - \frac{\lambda}{4}, x^2\right) + c_2(x^2-1)^{\frac{m}{2}} x \operatorname{HeunC}\left(0, \frac{1}{2}, m, \frac{a^2}{4}, \frac{1}{4} + \frac{m^2}{4} - \frac{\lambda}{4}, x^2\right)$$

✓ Solution by Mathematica

Time used: 0.602 (sec). Leaf size: 234

```
DSolve[(x^2-1)^2*y''[x]+2*x*(x^2-1)*y'[x]+( (x^2-1)*(a^2*x^2-\[Lambda])-m^2)*y[x]==0,y[x],x,
```

$$y(x) \to e^{i\sqrt{a^2}x}\left(\frac{x+1}{x-1}\right)^{\frac{\sqrt{m^2}}{2}}\left(c_2(x-1)^{\sqrt{m^2}}\text{HeunC}\left[-\left(\sqrt{m^2}+1\right)\left(\sqrt{m^2}+2i\sqrt{a^2}\right)-a^2\right.\right.$$
$$\left.+\lambda,-4i\sqrt{a^2}\left(\sqrt{m^2}+1\right),\sqrt{m^2}+1,\sqrt{m^2}+1,-4i\sqrt{a^2},\frac{1-x}{2}\right]$$
$$+c_1\text{HeunC}\left[2i\sqrt{a^2}\left(\sqrt{m^2}-1\right)-a^2+\lambda,-4i\sqrt{a^2},1-\sqrt{m^2},\sqrt{m^2}+1,\right.$$
$$\left.\left.-4i\sqrt{a^2},\frac{1-x}{2}\right]\right)$$

32.27 problem 236

Internal problem ID [11071]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4 + a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 236.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)^2 y'' + 2x(x^2+1)\,y' + \left((x^2+1)\left(a^2x^2-\lambda\right)+m^2\right)y = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 76

```
dsolve((x^2+1)^2*diff(y(x),x$2)+2*x*(x^2+1)*diff(y(x),x)+( (x^2+1)*(a^2*x^2-lambda)+m^2)*y(x
```

$$y(x) = c_1(x^2+1)^{\frac{m}{2}} \operatorname{HeunC}\left(0, -\frac{1}{2}, m, -\frac{a^2}{4}, \frac{1}{4} + \frac{m^2}{4} - \frac{\lambda}{4}, -x^2\right) + c_2(x^2+1)^{\frac{m}{2}}\, x \operatorname{HeunC}\left(0, \frac{1}{2}, m, -\frac{a^2}{4}, \frac{1}{4} + \frac{m^2}{4} - \frac{\lambda}{4}, -x^2\right)$$

✓ Solution by Mathematica

Time used: 0.605 (sec). Leaf size: 124

```
DSolve[(x^2+1)^2*y''[x]+2*x*(x^2+1)*y'[x]+( (x^2+1)*(a^2*x^2-\[Lambda])+m^2)*y[x]==0,y[x],x,
```

$$y(x) \to (x^2+1)^{\frac{\sqrt{m^2}}{2}} \left(c_2 x \operatorname{HeunC}\left[\frac{1}{4}\left(\lambda - m^2 - 3\sqrt{m^2} - 2\right), -\frac{a^2}{4}, \frac{3}{2}, \sqrt{m^2}+1, 0, -x^2\right] + c_1 \operatorname{HeunC}\left[\frac{1}{4}\left(\lambda - m^2 - \sqrt{m^2}\right), -\frac{a^2}{4}, \frac{1}{2}, \sqrt{m^2}+1, 0, -x^2\right]\right)$$

32.28 problem 237

Internal problem ID [11072]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4 + a_3x^3 + a_2x^2x + a_1x + a_0)y'' + f(x)y' + g(x)y = 0$
Problem number: 237.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(a\,x^2 + bx + c\right)^2 y'' + (2ax + k)\left(a\,x^2 + bx + c\right) y' + ym = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 375

```
dsolve((a*x^2+b*x+c)^2*diff(y(x),x$2)+(2*a*x+k)*(a*x^2+b*x+c)*diff(y(x),x)+m*y(x)=0,y(x), si
```

$$y(x)$$

$$= c_1 \left(\frac{2ax + \sqrt{-4ac + b^2} + b}{-2ax + \sqrt{-4ac + b^2} - b}\right)^{-\frac{b}{2\sqrt{-4ac+b^2}}} \left(\frac{-2ax + \sqrt{-4ac + b^2} - b}{2ax + \sqrt{-4ac + b^2} + b}\right)^{-\frac{k}{2\sqrt{-4ac+b^2}}} \left(\frac{i\sqrt{4ac - b^2} - 2ax - b}{2ax + b + i\sqrt{4ac - b^2}}\right.$$

$$+ c_2 \left(\frac{2ax + \sqrt{-4ac + b^2} + b}{-2ax + \sqrt{-4ac + b^2} - b}\right)^{-\frac{b}{2\sqrt{-4ac+b^2}}} \left(\frac{-2ax + \sqrt{-4ac + b^2} - b}{2ax + \sqrt{-4ac + b^2} + b}\right)^{-\frac{k}{2\sqrt{-4ac+b^2}}} \left(\frac{i\sqrt{4ac - b^2} - 2ax -}{2ax + b + i\sqrt{4ac -}}\right.$$

✓ Solution by Mathematica

Time used: 2.382 (sec). Leaf size: 157

```
DSolve[(a*x^2+b*x+c)^2*y''[x]+(2*a*x+k)*(a*x^2+b*x+c)*y'[x]+m*y[x]==0,y[x],x,IncludeSingular
```

$$y(x) \to c_1 \exp\left(\frac{\left(-\sqrt{m}\sqrt{\frac{b^2-2bk+k^2-4m}{m}}+b-k\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}\right) + c_2 \exp\left(\frac{\left(\sqrt{m}\sqrt{\frac{b^2-2bk+k^2-4m}{m}}+b-k\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}\right)$$

33 Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.

33.1 problem 238

Internal problem ID [11073]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 238.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^6y'' - y'x^5 + ay = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve(x^6*diff(y(x),x$2)-x^5*diff(y(x),x)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^2\sinh\left(\frac{\sqrt{-a}}{2x^2}\right) + c_2x^2\cosh\left(\frac{\sqrt{-a}}{2x^2}\right)$$

✓ Solution by Mathematica

Time used: 0.239 (sec). Leaf size: 58

```
DSolve[x^6*y''[x]-x^5*y'[x]+a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}x^2e^{-\frac{i\sqrt{a}}{2x^2}}\left(2c_1e^{\frac{i\sqrt{a}}{x^2}} - \frac{ic_2}{\sqrt{a}}\right)$$

33.2 problem 239

Internal problem ID [11074]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 239.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^6 y'' + (3x^2 + a)\, y' x^3 + by = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
dsolve(x^6*diff(y(x),x$2)+(3*x^2+a)*x^3*diff(y(x),x)+b*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{a-\sqrt{a^2-4b}}{4x^2}} + c_2 \mathrm{e}^{\frac{a+\sqrt{a^2-4b}}{4x^2}}$$

✓ Solution by Mathematica

Time used: 0.066 (sec). Leaf size: 56

```
DSolve[x^6*y''[x]+(3*x^2+a)*x^3*y'[x]+b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{a-\sqrt{a^2-4b}}{4x^2}} \left(c_1 e^{\frac{\sqrt{a^2-4b}}{2x^2}} + c_2\right)$$

33.3 problem 241

Internal problem ID [11075]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 241.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^n y'' + c(ax+b)^{-4+n} y = 0$$

✗ Solution by Maple

```
dsolve(x^n*diff(y(x),x$2)+c*(a*x+b)^(n-4)*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^n*y''[x]+c*(a*x+b)^(n-4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

33.4 problem 242

Internal problem ID [11076]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 242.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^n y'' + axy' - \left(x^n b^2 + 2b\, x^{-1+n} + bax + a\right) y = 0$$

✗ Solution by Maple

```
dsolve(x^n*diff(y(x),x$2)+a*x*diff(y(x),x)-(b^2*x^n+2*b*x^(n-1)+a*b*x+a)*y(x)=0,y(x), singso
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^n*y''[x]+a*x*y'[x]-(b^2*x^n+2*b*x^(n-1)+a*b*x+a)*y[x]==0,y[x],x,IncludeSingularSolu
```

Not solved

33.5 problem 243

Internal problem ID [11077]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 243.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^n y'' + y'(ax+b) - ay = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 95

```
dsolve(x^n*diff(y(x),x$2)+(a*x+b)*diff(y(x),x)-a*y(x)=0,y(x), singsol=all)
```

$$y(x) = -c_1 \left(\int \frac{e^{\left(\frac{a\,x^2}{n-2}+\frac{bx}{n-1}\right)x^{-n}}}{(ax+b)^2}dx\right) xa - c_1 \left(\int \frac{e^{\left(\frac{a\,x^2}{n-2}+\frac{bx}{n-1}\right)x^{-n}}}{(ax+b)^2}dx\right) b - c_2 xa - c_2 b$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^n*y''[x]+(a*x+b)*y'[x]-a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

33.6 problem 244

Internal problem ID [11078]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 244.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^n y'' + \left(a\,x^{-1+n} + bx\right) y' + (a-1)\,y = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 161

```
dsolve(x^n*diff(y(x),x$2)+(a*x^(n-1)+b*x)*diff(y(x),x)+(a-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{-\frac{1}{2}-\frac{a}{2}+\frac{n}{2}} \mathrm{e}^{\frac{b\,x^{-n+2}}{-4+2n}} \operatorname{WhittakerM}\left(\frac{(-a+n-1)\,b+2a-2}{2b\,(n-2)}, \frac{a-1}{-4+2n}, \frac{b\,x^{-n+2}}{n-2}\right) + c_2 x^{-\frac{1}{2}-\frac{a}{2}+\frac{n}{2}} \mathrm{e}^{\frac{b\,x^{-n+2}}{-4+2n}} \operatorname{WhittakerW}\left(\frac{(-a+n-1)\,b+2a-2}{2b\,(n-2)}, \frac{a-1}{-4+2n}, \frac{b\,x^{-n+2}}{n-2}\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^n*y''[x]+(a*x^(n-1)+b*x)*y'[x]+(a-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

Not solved

33.7 problem 245

Internal problem ID [11079]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 245.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^n y'' + \left(2x^{-1+n} + a\,x^2 + bx\right) y' + by = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 84

```
dsolve(x^n*diff(y(x),x$2)+(2*x^(n-1)+a*x^2+b*x)*diff(y(x),x)+b*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(ax+b)}{x} + \frac{c_2(ax+b)\left(\int \frac{e^{\frac{b(n-3)x^{-n+2}+(n-2)\left(a\,x^{-n+3}-2(n-3)\ln(x)\right)}{(n-3)(n-2)}}x^2}{(ax+b)^2}dx\right)}{x}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^n*y''[x]+(2*x^(n-1)+a*x^2+b*x)*y'[x]+b*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

Not solved

33.8 problem 246

Internal problem ID [11080]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 246.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^{n}y''+(x^{n}a+b)\,y'+c((a-c)\,x^{n}+b)\,y=0}$$

✗ Solution by Maple

```
dsolve(x^n*diff(y(x),x$2)+(a*x^n+b)*diff(y(x),x)+c*((a-c)*x^n+b)*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^n*y''[x]+(a*x^n+b)*y'[x]+c*((a-c)*x^n+b)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

Not solved

33.9 problem 247

Internal problem ID [11081]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition

Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.

Problem number: 247.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^n y'' + \left(x^n a - x^{-1+n} + bax + b\right) y' + a^2 bxy = 0$$

✗ Solution by Maple

```
dsolve(x^n*diff(y(x),x$2)+(a*x^n-x^(n-1)+a*b*x+b)*diff(y(x),x)+a^2*b*x*y(x)=0,y(x), singsol=
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^n*y''[x]+(a*x^n-x^(n-1)+a*b*x+b)*y'[x]+a^2*b*x*y[x]==0,y[x],x,IncludeSingularSoluti
```

Not solved

33.10 problem 248

Internal problem ID [11082]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 248.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^n y'' + \left(a\,x^{m+n} + 1\right) y' + a\,x^m \left(1 + m\,x^{-1+n}\right) y = 0$$

✗ Solution by Maple

```
dsolve(x^n*diff(y(x),x$2)+(a*x^(n+m)+1)*diff(y(x),x)+a*x^m*(1+m*x^(n-1))*y(x)=0,y(x), singso
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^n*y''[x]+(a*x^(n+m)+1)*y'[x]+a*x^m*(1+m*x^(n-1))*y[x]==0,y[x],x,IncludeSingularSolu
```

Not solved

33.11 problem 249

Internal problem ID [11083]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 249.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^n a + b)\, y'' + (c\, x^n + d)\, y' + \lambda((-\lambda a + c)\, x^n + d - b\lambda)\, y = 0}$$

✗ Solution by Maple

```
dsolve((a*x^n+b)*diff(y(x),x$2)+(c*x^n+d)*diff(y(x),x)+lambda*((c-a*lambda)*x^n+d-b*lambda)*
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a*x^n+b)*y''[x]+(c*x^n+d)*y'[x]+\[Lambda]*((c-a*\[Lambda])*x^n+d-b*\[Lambda])*y[x]==
```

Not solved

33.12 problem 250

Internal problem ID [11084]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 250.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{(x^n a + bx + c)\, y'' - an(-1+n)\, x^{n-2} y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve((a*x^n+b*x+c)*diff(y(x),x$2)=a*n*(n-1)*x^(n-2)*y(x),y(x), singsol=all)
```

$$y(x) = \left(\left(\int \frac{1}{(a\,x^n + xb + c)^2} dx\right) c_1 + c_2\right) (a\,x^n + xb + c)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a*x^n+b*x+c)*y''[x]==a*n*(n-1)*x^(n-2)*y[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

33.13 problem 251

Internal problem ID [11085]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 251.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(x^n+1)\,y''+((a-b)\,x^n+a-n)\,y'+b(1-a)\,x^{-1+n}y=0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 76

```
dsolve(x*(x^n+1)*diff(y(x),x$2)+((a-b)*x^n+a-n)*diff(y(x),x)+b*(1-a)*x^(n-1)*y(x)=0,y(x), si
```

$$y(x)=c_1(x^n+1)^{\frac{b}{n}} + c_2x^{-a+n+1}(x^n+1)^{\frac{b}{n}}\,\text{hypergeom}\left(\left[\frac{b+n}{n},\frac{-a+n+1}{n}\right],\left[\frac{2n-a+1}{n}\right],-x^n\right)$$

✓ Solution by Mathematica

Time used: 0.164 (sec). Leaf size: 69

```
DSolve[x*(x^n+1)*y''[x]+((a-b)*x^n+a-n)*y'[x]+b*(1-a)*x^(n-1)*y[x]==0,y[x],x,IncludeSingular
```

$$y(x)\to c_2(x^n)^{\frac{-a+n+1}{n}}\,\text{Hypergeometric2F1}\left(1,\frac{-a-b+n+1}{n},\frac{-a+2n+1}{n},-x^n\right) + c_1(x^n+1)^{b/n}$$

33.14 problem 252

Internal problem ID [11086]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 252.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$x(x^{2n}+a)\,y''+(x^{2n}+a-an)\,y'-b^2x^{2n-1}y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 61

```
dsolve(x*(x^(2*n)+a)*diff(y(x),x$2)+(x^(2*n)+a-a*n)*diff(y(x),x)-b^2*x^(2*n-1)*y(x)=0,y(x),
```

$$y(x)=c_1\mathrm{e}^{\int ib\,x^{n-1}\sqrt{-\frac{1}{x^{2n}+a}}dx}+c_2\mathrm{e}^{-\left(\int ib\,x^{n-1}\sqrt{-\frac{1}{x^{2n}+a}}dx\right)}$$

✓ Solution by Mathematica

Time used: 0.458 (sec). Leaf size: 47

```
DSolve[x*(x^(2*n)+a)*y''[x]+(x^(2*n)+a-a*n)*y'[x]-b^2*x^(2*n-1)*y[x]==0,y[x],x,IncludeSingul
```

$$y(x)\to c_1\cosh\left(\frac{b\operatorname{arcsinh}\left(\frac{x^n}{\sqrt{a}}\right)}{n}\right)+ic_2\sinh\left(\frac{b\operatorname{arcsinh}\left(\frac{x^n}{\sqrt{a}}\right)}{n}\right)$$

33.15 problem 253

Internal problem ID [11087]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 253.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2\left(x^{2n}a^2-1\right)y''+x\left(a^2(1+n)\,x^{2n}+n-1\right)y'-\nu(\nu+1)\,a^2n^2x^{2n}y=0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 23

```
dsolve(x^2*(a^2*x^(2*n)-1)*diff(y(x),x$2)+x*(a^2*(n+1)*x^(2*n)+n-1)*diff(y(x),x)-nu*(nu+1)*a
```

$$y(x)=c_1\,\mathrm{LegendreP}\left(\nu,a\,x^n\right)+c_2\,\mathrm{LegendreQ}\left(\nu,a\,x^n\right)$$

✓ Solution by Mathematica

Time used: 0.21 (sec). Leaf size: 79

```
DSolve[x^2*(a^2*x^(2*n)-1)*y''[x]+x*(a^2*(n+1)*x^(2*n)+n-1)*y'[x]-\[Nu]*(\[Nu]+1)*a^2*n^2*x^
```

$$y(x)\to iac_2\sqrt{x^{2n}}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2}-\frac{\nu}{2},\frac{\nu}{2}+1,\frac{3}{2},a^2x^{2n}\right)+c_1\,\mathrm{Hypergeometric2F1}\left(-\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},a^2x^{2n}\right)$$

33.16 problem 254

Internal problem ID [11088]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 254.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2\left(x^{2n}a^2-1\right)y''+x(ap\,x^n+q)\,y'+(ar\,x^n+s)\,y=0$$

✓ Solution by Maple

Time used: 0.328 (sec). Leaf size: 445

```
dsolve(x^2*(a^2*x^(2*n)-1)*diff(y(x),x$2)+x*(a*p*x^n+q)*diff(y(x),x)+(a*r*x^n+s)*y(x)=0,y(x)
```

$$y(x) = c_1 x^{\frac{\sqrt{q^2+2q+4s+1}}{2}+\frac{q}{2}+\frac{1}{2}}\,\mathrm{HeunG}\left(-1, \frac{-(q^2+2q+4s+1)^{\frac{3}{2}}+(-p^2+(2n-q-2)p+q^2+2q+4s+1)\sqrt{q}}{2}\right.$$

$$\left. -\frac{p-q}{2n}, -a\,x^n\right)$$

$$+c_2 x^{\frac{q}{2}+\frac{1}{2}-\frac{\sqrt{q^2+2q+4s+1}}{2}}\,\mathrm{HeunG}\left(-1, \frac{-(q^2+2q+4s+1)^{\frac{3}{2}}+(p^2+(-2n+q+2)p+q^2+2q+4s+1)\sqrt{}}{}\right.$$

$$-\frac{\sqrt{q^2+2q+4s+1}-q-1}{2n}, -\frac{\sqrt{q^2+2q+4s+1}-q+1}{2n}, \frac{n-\sqrt{q^2+2q+4s+1}}{n},$$

$$\left. -\frac{p-q}{2n}, -a\,x^n\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*(a^2*x^(2*n)-1)*y''[x]+x*(a*p*x^n+q)*y'[x]+(a*r*x^n+s)*y[x]==0,y[x],x,IncludeSing
```

Not solved

33.17 problem 255

Internal problem ID [11089]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 255.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^n+a)^2 y''-b\,x^{n-2}((b-1)\,x^n+a(-1+n))\,y=0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 89

```
dsolve((x^n+a)^2*diff(y(x),x$2)-b*x^(n-2)*( (b-1)*x^n+a*(n-1))*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(x^n+a)^{\frac{-b+n}{n}}\,\text{hypergeom}\left(\left[\frac{-2b+n}{n}\right],[],-\frac{x^n}{a}\right)+c_2x(x^n+a)^{\frac{-b+n}{n}}\,\text{hypergeom}\left(\left[1,\frac{-2b+n+1}{n}\right],\left[\frac{1+n}{n}\right],-\frac{x^n}{a}\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(x^n+a)^2*y''[x]-b*x^(n-2)*( (b-1)*x^n+a*(n-1))*y[x]==0,y[x],x,IncludeSingularSolutio
```

Not solved

33.18 problem 256

Internal problem ID [11090]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 256.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^n a+b)^2 y''+(x^n a+b)(c\,x^n+d)\,y'+n(-da+bc)\,x^{-1+n}y=0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 82

```
dsolve((a*x^n+b)^2*diff(y(x),x$2)+(a*x^n+b)*(c*x^n+d)*diff(y(x),x)+n*(b*c-a*d)*x^(n-1)*y(x)=
```

$$y(x)=c_1\mathrm{e}^{\int \frac{-x^n c-d}{a\,x^n+b}dx}+c_2\left(\int \mathrm{e}^{-\left(\int \frac{-x^n c-d}{a\,x^n+b}dx\right)}dx\right)\mathrm{e}^{\int \frac{-x^n c-d}{a\,x^n+b}dx}$$

✓ Solution by Mathematica

Time used: 0.923 (sec). Leaf size: 106

```
DSolve[(a*x^n+b)^2*y''[x]+(a*x^n+b)*(c*x^n+d)*y'[x]+n*(b*c-a*d)*x^(n-1)*y[x]==0,y[x],x,Inclu
```

$$y(x)\to \exp\left(-\frac{x\left((ad-bc)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ax^n}{b}\right)+bc\right)}{ab}\right)\left(\int_1^x \exp\left(\frac{\left(bc+(ad-bc)\,\text{Hyper}\right.}{}\right.\right.$$

$$\left.+c_2\right)$$

33.19 problem 257

Internal problem ID [11091]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 257.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^n + a)^2 y'' + b\, x^m (x^n + a)\, y' - x^{n-2}\left(x^{m+1} b + an - a\right) y = 0$$

✗ Solution by Maple

```
dsolve((x^n+a)^2*diff(y(x),x$2)+b*x^m*(x^n+a)*diff(y(x),x)-x^(n-2)*(b*x^(m+1)+a*n-a)*y(x)=0,
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(x^n+a)^2*y''[x]+b*x^m*(x^n+a)*y'[x]-x^(n-2)*(b*x^(m+1)+a*n-a)*y[x]==0,y[x],x,Include
```

Not solved

33.20 problem 258

Internal problem ID [11092]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 258.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^n a + b)^2 y'' + c\,x^m (x^n a + b)\, y' + \left(c\,x^m - an\,x^{-1+n} - 1\right) y = 0$$

✗ Solution by Maple

```
dsolve((a*x^n+b)^2*diff(y(x),x$2)+c*x^m*(a*x^n+b)*diff(y(x),x)+(c*x^m-a*n*x^(n-1)-1)*y(x)=0,
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a*x^n+b)^2*y''[x]+c*x^m*(a*x^n+b)*y'[x]+(c*x^m-a*n*x^(n-1)-1)*y[x]==0,y[x],x,Include
```

Not solved

33.21 problem 259

Internal problem ID [11093]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 259.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(a\,x^n+b)^2\,y''+(1+n)\,x\left(x^{2n}a^2-b^2\right)y'+yc=0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 159

```
dsolve(x^2*(a*x^n+b)^2*diff(y(x),x$2)+(n+1)*x*(a^2*x^(2*n)-b^2)*diff(y(x),x)+c*y(x)=0,y(x),
```

$$y(x)=c_1x(a\,x^n+b)^{\frac{-1-n}{n}}\sqrt{x^{2n}a+x^nb}\left(\frac{x^n}{a\,x^n+b}\right)^{\frac{\sqrt{\frac{(n+2)^2b^2-4c}{n^2a^2}}\,a}{2b}}+c_2x(a\,x^n+b)^{\frac{-1-n}{n}}\sqrt{x^{2n}a+x^nb}\left(\frac{x^n}{a\,x^n+b}\right)^{-\frac{\sqrt{\frac{(n+2)^2b^2-4c}{n^2a^2}}\,a}{2b}}$$

✓ Solution by Mathematica

Time used: 0.407 (sec). Leaf size: 149

```
DSolve[x^2*(a*x^n+b)^2*y''[x]+(n+1)*x*(a^2*x^(2*n)-b^2)*y'[x]+c*y[x]==0,y[x],x,IncludeSingul
```

$$y(x) \to c_1 \exp\left(\frac{\left(b(n+2)-\sqrt{c}\sqrt{\frac{b^2(n+2)^2-4c}{c}}\right)(-\log(ax^n+b)-\log(b)+n\log(x)-\log(n))}{2bn}\right) + c_2 \exp\left(\frac{\left(\sqrt{c}\sqrt{\frac{b^2(n+2)^2-4c}{c}}+b(n+2)\right)(-\log(ax^n+b)-\log(b)+n\log(x)-\log(n))}{2bn}\right)$$

33.22 problem 260

Internal problem ID [11094]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 260.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(a\,x^{n+1}+b\,x^n+c)^2\,y''+(\alpha\,x^n+\beta\,x^{-1+n}+\gamma)\,y'+(n(-an-a+\alpha)\,x^{-1+n}+(-1+n)\,(-nb+\beta)\,x^{n-2}$$

✗ Solution by Maple

```
dsolve((a*x^(n+1)+b*x^n+c)^2*diff(y(x),x$2)+(alpha*x^n+beta*x^(n-1)+gamma)*diff(y(x),x)+(n*(
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a*x^(n+1)+b*x^n+c)^2*y''[x]+(\[Alpha]*x^n+\[Beta]*x^(n-1)+\[Gamma])*y'[x]+(n*(\[Alph
```

Not solved

33.23 problem 261

Internal problem ID [11095]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 261.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^n a + b\,x^m + c)\,y'' + (-x+\lambda)\,y' + y = 0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 157

```
dsolve((a*x^n+b*x^m+c)*diff(y(x),x$2)+(lambda-x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\int e^{\int \frac{-2+\frac{x^2}{a\,x^n+b\,x^m+c}-\frac{2x\lambda}{a\,x^n+b\,x^m+c}+\frac{\lambda^2}{a\,x^n+b\,x^m+c}}{-\lambda+x}dx}dx\right)x - c_1\left(\int e^{\int \frac{-2+\frac{x^2}{a\,x^n+b\,x^m+c}-\frac{2x\lambda}{a\,x^n+b\,x^m+c}+\frac{\lambda^2}{a\,x^n+b\,x^m+c}}{-\lambda+x}dx}dx\right)\lambda + c_2x - c_2\lambda$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a*x^n+b*x^m+c)*y''[x]+(\[Lambda]-x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions ->
```

Not solved

33.24 problem 262

Internal problem ID [11096]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 262.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(a\,x^n + b\,x^m + c)\,y'' + (\lambda^2 - x^2)\,y' + (\lambda + x)\,y = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 175

```
dsolve((a*x^n+b*x^m+c)*diff(y(x),x$2)+(lambda^2-x^2)*diff(y(x),x)+(x+lambda)*y(x)=0,y(x), si
```

$$y(x) = c_1 \left(\int e^{\int \frac{-2+\frac{(x+\lambda)x^2}{a\,x^n+b\,x^m+c}-\frac{2(x+\lambda)x\lambda}{a\,x^n+b\,x^m+c}+\frac{(x+\lambda)\lambda^2}{a\,x^n+b\,x^m+c}}{-\lambda+x}dx}dx\right) x - c_1 \left(\int e^{\int \frac{-2+\frac{(x+\lambda)x^2}{a\,x^n+b\,x^m+c}-\frac{2(x+\lambda)x\lambda}{a\,x^n+b\,x^m+c}+\frac{(x+\lambda)\lambda^2}{a\,x^n+b\,x^m+c}}{-\lambda+x}dx}dx\right) \lambda + c_2 x - c_2 \lambda$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a*x^n+b*x^m+c)*y''[x]+(\[Lambda]^2-x^2)*y'[x]+(x+\[Lambda])*y[x]==0,y[x],x,IncludeSi
```

Not solved

33.25 problem 263

Internal problem ID [11097]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 263.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2(x^{n}a + b\,x^{m} + c)\,y'' + an\,x^{-1+n}bm\,x^{m-1}y' + dy = 0}$$

✗ Solution by Maple

```
dsolve(2*(a*x^n+b*x^m+c)*diff(y(x),x$2)+(a*n*x^(n-1)*b*m*x^(m-1))*diff(y(x),x)+d*y(x)=0,y(x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[2*(a*x^n+b*x^m+c)*y''[x]+(a*n*x^(n-1)*b*m*x^(m-1))*y'[x]+d*y[x]==0,y[x],x,IncludeSing
```

Not solved

33.26 problem 264

Internal problem ID [11098]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 264.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$(x^n a+b)^{m+1} y''+(x^n a+b) y'-anm\, x^{-1+n} y=0$$

✗ Solution by Maple

```
dsolve((a*x^n+b)^(m+1)*diff(y(x),x$2)+(a*x^n+b)*diff(y(x),x)-a*n*m*x^(n-1)*y(x)=0,y(x), sing
```

No solution found

✓ Solution by Mathematica

Time used: 0.249 (sec). Leaf size: 116

```
DSolve[(a*x^n+b)^(m+1)*y''[x]+(a*x^n+b)*y'[x]-a*n*m*x^(n-1)*y[x]==0,y[x],x,IncludeSingularSo
```

$$y(x)\to \exp\left(-x(ax^n+b)^{-m}\left(\frac{ax^n}{b}+1\right)^m \text{Hypergeometric2F1}\left(m,\frac{1}{n},1+\frac{1}{n},-\frac{ax^n}{b}\right)\right)\left(\int_1^x \exp\left(\text{Hypergeometric2F1}\left(m,\frac{1}{n},1+\frac{1}{n},-\frac{aK[1]^n}{b}\right)K[1]\left(aK[1]^n+b\right)^{-m}\left(\frac{aK[1]^n}{b}+1\right)\right. \right. + c_2\Bigg)$$

34 Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions

34.1 problem 1

Internal problem ID [11099]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + a\,\mathrm{e}^{\lambda x} y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$2)+a*exp(lambda*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}\left(0, \frac{2\sqrt{a}\,\mathrm{e}^{\frac{\lambda x}{2}}}{\lambda}\right) + c_2 \operatorname{BesselY}\left(0, \frac{2\sqrt{a}\,\mathrm{e}^{\frac{\lambda x}{2}}}{\lambda}\right)$$

✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 55

```
DSolve[y''[x]+a*Exp[\[Lambda]*x]*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}\left(0, \frac{2\sqrt{a}\sqrt{e^{x\lambda}}}{\lambda}\right) + 2c_2 \operatorname{BesselY}\left(0, \frac{2\sqrt{a}\sqrt{e^{x\lambda}}}{\lambda}\right)$$

34.2 problem 2

Internal problem ID [11100]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (a\,e^{x} - b)\,y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$2)+(a*exp(x)-b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}\left(2\sqrt{b}, 2\sqrt{a}\,e^{\frac{x}{2}}\right) + c_2 \operatorname{BesselY}\left(2\sqrt{b}, 2\sqrt{a}\,e^{\frac{x}{2}}\right)$$

✓ Solution by Mathematica

Time used: 0.081 (sec). Leaf size: 76

```
DSolve[y''[x]+(a*Exp[x]-b)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{Gamma}\left(1 - 2\sqrt{b}\right) \operatorname{BesselJ}\left(-2\sqrt{b}, 2\sqrt{a}\sqrt{e^x}\right) + c_2 \operatorname{Gamma}\left(2\sqrt{b} + 1\right) \operatorname{BesselJ}\left(2\sqrt{b}, 2\sqrt{a}\sqrt{e^x}\right)$$

34.3 problem 3

Internal problem ID [11101]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + a\left(\lambda\, e^{x\lambda} - a\, e^{2x\lambda}\right) y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 43

```
dsolve(diff(y(x),x$2)+a*(lambda*exp(lambda*x)-a*exp(2*lambda*x))*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{a\, e^{\lambda x}}{\lambda}} + c_2 e^{-\frac{a\, e^{\lambda x}}{\lambda}} \operatorname{Ei}_1\left(-\frac{2a\, e^{\lambda x}}{\lambda}\right)$$

✓ Solution by Mathematica

Time used: 1.32 (sec). Leaf size: 37

```
DSolve[y''[x]+a*(\[Lambda]*Exp[\[Lambda]*x]-a*Exp[2*\[Lambda]*x])*y[x]==0,y[x],x,IncludeSing
```

$$y(x) \to e^{-\frac{ae^{\lambda x}}{\lambda}} \left(c_2 \operatorname{ExpIntegralEi}\left(\frac{2ae^{x\lambda}}{\lambda}\right) + c_1\right)$$

34.4 problem 4

Internal problem ID [11102]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \left(a^2 e^{2x} + a(2b+1)\, e^{x} + b^2\right) y = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 71

```
dsolve(diff(y(x),x$2)-(a^2*exp(2*x)+a*(2*b+1)*exp(x)+b^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{xb+a\,e^{x}} + c_2\left(a^{-2b-1}\left(-\frac{1}{2}+b\right) e^{-x}\,\mathrm{WhittakerM}\left(-b+1, -b+\frac{1}{2}, 2a\,e^{x}\right) - \mathrm{WhittakerM}\left(-b, -b+\frac{1}{2}, 2a\,e^{x}\right) a^{-2b}\right)$$

✓ Solution by Mathematica

Time used: 1.8 (sec). Leaf size: 57

```
DSolve[y''[x]-(a^2*Exp[2*x]+a*(2*b+1)*Exp[x]+b^2)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to e^{ae^{x}}\left(e^{x}\right)^{-b}\left(c_1\left(e^{x}\right)^{2b} - 4^{b} c_2\left(ae^{x}\right)^{2b}\Gamma\left(-2b, 2ae^{x}\right)\right)$$

34.5 problem 5

Internal problem ID [11103]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \left(a\,e^{2x\lambda} + b\,e^{x\lambda} + c\right) y = 0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)-(a*exp(2*lambda*x)+b*exp(lambda*x)+c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{\lambda x}{2}} \operatorname{WhittakerM}\left(-\frac{b}{2\lambda\sqrt{a}}, \frac{\sqrt{c}}{\lambda}, \frac{2\sqrt{a}\,e^{\lambda x}}{\lambda}\right) + c_2 e^{-\frac{\lambda x}{2}} \operatorname{WhittakerW}\left(-\frac{b}{2\lambda\sqrt{a}}, \frac{\sqrt{c}}{\lambda}, \frac{2\sqrt{a}\,e^{\lambda x}}{\lambda}\right)$$

✓ Solution by Mathematica

Time used: 1.158 (sec). Leaf size: 145

```
DSolve[y''[x]-(a*Exp[2*\[Lambda]*x]+b*Exp[\[Lambda]*x]+c)*y[x]==0,y[x],x,IncludeSingularSolu
```

$$y(x) \to e^{-\frac{\sqrt{a}e^{\lambda x}}{\lambda}} \left(e^{\lambda x}\right)^{\frac{\sqrt{c}}{\lambda}} \left(c_1 \operatorname{HypergeometricU}\left(\frac{\frac{b}{\sqrt{a}} + \lambda + 2\sqrt{c}}{2\lambda}, \frac{2\sqrt{c}}{\lambda} + 1, \frac{2\sqrt{a}e^{x\lambda}}{\lambda}\right) + c_2 L_{-\frac{\frac{b}{\sqrt{a}}+\lambda+2\sqrt{c}}{2\lambda}}^{\frac{2\sqrt{c}}{\lambda}}\left(\frac{2\sqrt{a}e^{x\lambda}}{\lambda}\right)\right)$$

34.6 problem 6

Internal problem ID [11104]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(a\,\mathrm{e}^{4x\lambda} + b\,\mathrm{e}^{3x\lambda} + \mathrm{e}^{2x\lambda}c - \frac{\lambda^2}{4}\right)y = 0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 221

```
dsolve(diff(y(x),x$2)+(a*exp(4*lambda*x)+b*exp(3*lambda*x)+c*exp(2*lambda*x)-1/4*lambda^2)*y
```

$$y(x) = c_1 \,\mathrm{hypergeom}\left(\left[\frac{4\lambda\,a^{\frac{3}{2}} + 4iac - ib^2}{16\lambda\,a^{\frac{3}{2}}}\right], \left[\frac{1}{2}\right], \frac{i\left(2a\,\mathrm{e}^{\lambda x} + b\right)^2}{4\lambda\,a^{\frac{3}{2}}}\right)\mathrm{e}^{-\frac{i\mathrm{e}^{2\lambda x}a + \lambda^2 x\sqrt{a} + i\mathrm{e}^{\lambda x}b}{2\lambda\sqrt{a}}} + c_2 \,\mathrm{hypergeom}\left(\left[\frac{12\lambda\,a^{\frac{3}{2}} + 4iac - ib^2}{16\lambda\,a^{\frac{3}{2}}}\right], \left[\frac{3}{2}\right], \frac{i\left(2a\,\mathrm{e}^{\lambda x} + b\right)^2}{4\lambda\,a^{\frac{3}{2}}}\right)\left(2\,\mathrm{e}^{-\frac{i\mathrm{e}^{2\lambda x}a - \lambda^2 x\sqrt{a} + i\mathrm{e}^{\lambda x}b}{2\lambda\sqrt{a}}}a + \mathrm{e}^{-\frac{i\mathrm{e}^{2\lambda x}a + \lambda^2 x\sqrt{a} + i\mathrm{e}^{\lambda x}b}{2\lambda\sqrt{a}}}b\right)$$

✓ Solution by Mathematica

Time used: 1.895 (sec). Leaf size: 178

```
DSolve[y''[x]+(a*Exp[4*\[Lambda]*x]+b*Exp[3*\[Lambda]*x]+c*Exp[2*\[Lambda]*x]-1/4*\[Lambda]^
```

$y(x)$

$$\rightarrow \frac{e^{-\frac{ie^{\lambda x}\left(ae^{\lambda x}+b\right)}{2\sqrt{a}\lambda}}\left(c_1 \operatorname{HermiteH}\left(\frac{i(b^2-4ac+4ia^{3/2}\lambda)}{8a^{3/2}\lambda}, \frac{\sqrt[4]{-1}(2e^{x\lambda}a+b)}{2a^{3/4}\sqrt{\lambda}}\right) + c_2 \operatorname{Hypergeometric1F1}\left(\frac{-ib^2+4iac+4a^{3/2}\lambda}{16a^{3/2}\lambda}\right.\right.}{\sqrt{e^{\lambda x}}}$$

34.7 problem 7

Internal problem ID [11105]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$
y'' + \left(a\,\mathrm{e}^{2x\lambda}\left(b\,\mathrm{e}^{x\lambda}+c\right)^{n} - \frac{\lambda^2}{4}\right) y = 0
$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 105

```
dsolve(diff(y(x),x$2)+(a*exp(2*lambda*x)*(b*exp(lambda*x)+c)^n-1/4*lambda^2)*y(x)=0,y(x), si
```

$$
\begin{aligned}
y(x) = {} & c_1\,\mathrm{hypergeom}\left([\,],\left[\frac{1+n}{n+2}\right],-\frac{a\left(b\,\mathrm{e}^{\lambda x}+c\right)^{n+2}}{\lambda^2 b^2\left(n+2\right)^2}\right)\mathrm{e}^{-\frac{\lambda x}{2}} \\
& + c_2\,\mathrm{hypergeom}\left([\,],\left[\frac{n+3}{n+2}\right],-\frac{a\left(b\,\mathrm{e}^{\lambda x}+c\right)^{n+2}}{\lambda^2 b^2\left(n+2\right)^2}\right)\left(\mathrm{e}^{-\frac{\lambda x}{2}}c+\mathrm{e}^{\frac{\lambda x}{2}}b\right)
\end{aligned}
$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+(a*Exp[2*\[Lambda]*x]*(b*Exp[\[Lambda]*x]+c)^n-1/4*\[Lambda]^2)*y[x]==0,y[x],x
```

Not solved

34.8 problem 8

Internal problem ID [11106]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + ay' + b\,e^{2ax}y = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 43

```
dsolve(diff(y(x),x$2)+a*diff(y(x),x)+b*exp(2*a*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-ax} \sin\left(\frac{\sqrt{b}\,e^{ax}}{a}\right) + c_2 e^{-ax} \cos\left(\frac{\sqrt{b}\,e^{ax}}{a}\right)$$

✓ Solution by Mathematica

Time used: 0.133 (sec). Leaf size: 78

```
DSolve[y''[x]+a*y'[x]+b*Exp[2*a*x]*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a}e^{-\frac{ax}{2}}\left(2c_1\cos\left(\frac{\sqrt{be^{2ax}}}{a}\right) + c_2\sin\left(\frac{\sqrt{be^{2ax}}}{a}\right)\right)}{\sqrt{2}\sqrt[4]{be^{2ax}}}$$

34.9 problem 9

Internal problem ID [11107]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{y'' - ay' + b\,\mathrm{e}^{2ax}y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)-a*diff(y(x),x)+b*exp(2*a*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(\frac{\sqrt{b}\,\mathrm{e}^{ax}}{a}\right) + c_2 \cos\left(\frac{\sqrt{b}\,\mathrm{e}^{ax}}{a}\right)$$

✓ Solution by Mathematica

Time used: 0.061 (sec). Leaf size: 42

```
DSolve[y''[x]-a*y'[x]+b*Exp[2*a*x]*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos\left(\frac{\sqrt{b}e^{ax}}{a}\right) + c_2 \sin\left(\frac{\sqrt{b}e^{ax}}{a}\right)$$

34.10 problem 10

Internal problem ID [11108]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type **[[_2nd_order, _with_linear_symmetries]]**

$$\boxed{y'' + ay' + \left(b\,\mathrm{e}^{\lambda x} + c\right) y = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 73

```
dsolve(diff(y(x),x$2)+a*diff(y(x),x)+(b*exp(lambda*x)+c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{ax}{2}} \operatorname{BesselJ}\left(\frac{\sqrt{a^2-4c}}{\lambda}, \frac{2\sqrt{b}\,\mathrm{e}^{\frac{\lambda x}{2}}}{\lambda}\right) + c_2 \mathrm{e}^{-\frac{ax}{2}} \operatorname{BesselY}\left(\frac{\sqrt{a^2-4c}}{\lambda}, \frac{2\sqrt{b}\,\mathrm{e}^{\frac{\lambda x}{2}}}{\lambda}\right)$$

✓ Solution by Mathematica

Time used: 0.185 (sec). Leaf size: 123

```
DSolve[y''[x]+a*y'[x]+(b*Exp[\[Lambda]*x]+c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to e^{-\frac{ax}{2}} \left(c_1 \operatorname{Gamma}\left(1 - \frac{\sqrt{a^2-4c}}{\lambda}\right) \operatorname{BesselJ}\left(-\frac{\sqrt{a^2-4c}}{\lambda}, \frac{2\sqrt{be^{x\lambda}}}{\lambda}\right) + c_2 \operatorname{Gamma}\left(\frac{\lambda + \sqrt{a^2-4c}}{\lambda}\right) \operatorname{BesselJ}\left(\frac{\sqrt{a^2-4c}}{\lambda}, \frac{2\sqrt{be^{x\lambda}}}{\lambda}\right)\right)$$

34.11 problem 11

Internal problem ID [11109]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - y' + \left(a\,\mathrm{e}^{3x\lambda} + b\,\mathrm{e}^{2x\lambda} + \frac{1}{4} - \frac{\lambda^2}{4}\right) y = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 57

```
dsolve(diff(y(x),x$2)-diff(y(x),x)+(a*exp(3*lambda*x)+b*exp(2*lambda*x)+1/4-1/4*lambda^2  )*
```

$$y(x) = c_1 \mathrm{e}^{-\frac{(\lambda-1)x}{2}} \operatorname{AiryAi}\left(-\frac{a\,\mathrm{e}^{\lambda x} + b}{\lambda^{\frac{2}{3}} a^{\frac{2}{3}}}\right) + c_2 \mathrm{e}^{-\frac{(\lambda-1)x}{2}} \operatorname{AiryBi}\left(-\frac{a\,\mathrm{e}^{\lambda x} + b}{\lambda^{\frac{2}{3}} a^{\frac{2}{3}}}\right)$$

✓ Solution by Mathematica

Time used: 1.332 (sec). Leaf size: 77

```
DSolve[y''[x]-y'[x]+(a*Exp[3*\[Lambda]*x]+b*Exp[2*\[Lambda]*x]+1/4-1/4*\[Lambda]^2  )*y[x]==
```

$$y(x) \to \frac{e^{x/2}\left(c_1 \operatorname{AiryAi}\left(\frac{(e^{x\lambda}a+b)\sqrt[3]{-\frac{a}{\lambda^2}}}{a}\right) + c_2 \operatorname{AiryBi}\left(\frac{(e^{x\lambda}a+b)\sqrt[3]{-\frac{a}{\lambda^2}}}{a}\right)\right)}{\sqrt{e^{\lambda x}}}$$

34.12 problem 12

Internal problem ID [11110]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''-y'+\left(a\,\mathrm{e}^{2x\lambda}\left(b\,\mathrm{e}^{x\lambda}+c\right)^{n}+\frac{1}{4}-\frac{\lambda^{2}}{4}\right)y=0}$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 111

```
dsolve(diff(y(x),x$2)-diff(y(x),x)+(a*exp(2*lambda*x)*(b*exp(lambda*x)+c)^n+1/4-1/4*lambda^2
```

$$y(x)=c_1\mathrm{e}^{-\frac{(\lambda-1)x}{2}}\operatorname{hypergeom}\left([\,],\left[\frac{1+n}{n+2}\right],-\frac{a\left(b\,\mathrm{e}^{\lambda x}+c\right)^{n+2}}{\lambda^{2}b^{2}\left(n+2\right)^{2}}\right)+c_2\operatorname{hypergeom}\left([\,],\left[\frac{n+3}{n+2}\right],-\frac{a\left(b\,\mathrm{e}^{\lambda x}+c\right)^{n+2}}{\lambda^{2}b^{2}\left(n+2\right)^{2}}\right)\left(\mathrm{e}^{\frac{x(\lambda+1)}{2}}b+\mathrm{e}^{-\frac{(\lambda-1)x}{2}}c\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]-y'[x]+(a*Exp[2*\[Lambda]*x]*(b*Exp[\[Lambda]*x]+c)^n+1/4-1/4*\[Lambda]^2 )*y[
```

Not solved

34.13 problem 13

Internal problem ID [11111]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 2a\,e^{\lambda x} y' + a\,e^{\lambda x}\left(e^{\lambda x} a + \lambda\right) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)+2*a*exp(lambda*x)*diff(y(x),x)+a*exp(lambda*x)*(a*exp(lambda*x)+lambda
```

$$y(x) = c_1 e^{-\frac{a\,e^{\lambda x}}{\lambda}} + c_2 e^{-\frac{a\,e^{\lambda x}}{\lambda}} x$$

✓ Solution by Mathematica

Time used: 0.109 (sec). Leaf size: 26

```
DSolve[y''[x]+2*a*Exp[\[Lambda]*x]*y'[x]+a*Exp[\[Lambda]*x]*(a*Exp[\[Lambda]*x]+\[Lambda])*y
```

$$y(x) \to (c_2 x + c_1) e^{-\frac{a e^{\lambda x}}{\lambda}}$$

34.14 problem 14

Internal problem ID [11112]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + (a+b)\,\mathrm{e}^{x\lambda} y' + a\,\mathrm{e}^{x\lambda}\left(b\,\mathrm{e}^{x\lambda} + \lambda\right) y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 47

```
dsolve(diff(y(x),x$2)+(a+b)*exp(lambda*x)*diff(y(x),x)+a*exp(lambda*x)*(b*exp(lambda*x)+lamb
```

$$y(x) = c_1 \mathrm{e}^{-\frac{a\,\mathrm{e}^{\lambda x}}{\lambda}} + c_2 \mathrm{e}^{-\frac{a\,\mathrm{e}^{\lambda x}}{\lambda}} \operatorname{Ei}_1\left(-\frac{\mathrm{e}^{\lambda x}(a-b)}{\lambda}\right)$$

✓ Solution by Mathematica

Time used: 2.377 (sec). Leaf size: 40

```
DSolve[y''[x]+(a+b)*Exp[\[Lambda]*x]*y'[x]+a*Exp[\[Lambda]*x]*(b*Exp[\[Lambda]*x]+\[Lambda])
```

$$y(x) \to e^{-\frac{ae^{\lambda x}}{\lambda}}\left(c_2 \,\text{ExpIntegralEi}\left(\frac{(a-b)e^{x\lambda}}{\lambda}\right) + c_1\right)$$

34.15 problem 15

Internal problem ID [11113]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + e^{x\lambda} a y' - b\, e^{\mu x} \left(a\, e^{x\lambda} + b\, e^{\mu x} + \mu\right) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x$2)+a*exp(lambda*x)*diff(y(x),x)-b*exp(mu*x)*(a*exp(lambda*x)+b*exp(mu*x)+
```

$$y(x) = \left(\left(\int e^{-\frac{a\, e^{\lambda x}}{\lambda}} e^{-\frac{2b\, e^{\mu x}}{\mu}} dx\right) c_1 + c_2\right) e^{\frac{b\, e^{\mu x}}{\mu}}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+a*Exp[\[Lambda]*x]*y'[x]-b*Exp[\[Mu]*x]*(a*Exp[\[Lambda]*x]+b*Exp[\[Mu]*x]+\[M
```

Not solved

34.16 problem 16

Internal problem ID [11114]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 2k\,e^{\mu x}y' + \left(e^{2\lambda x}a + b\,e^{\lambda x} + k^2e^{2\mu x} + k\mu\,e^{\mu x} + c\right)y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+2*k*exp(mu*x)*diff(y(x),x)+(a*exp(2*lambda*x)+b*exp(lambda*x)+k^2*exp(
```

No solution found

✓ Solution by Mathematica

Time used: 2.531 (sec). Leaf size: 232

```
DSolve[y''[x]+2*k*Exp[\[Mu]*x]*y'[x]+(a*Exp[2*\[Lambda]*x]+b*Exp[\[Lambda]*x]+k^2*Exp[2*\[Mu
```

$$y(x) \to 2^{\frac{1}{2}-\frac{i\sqrt{c}}{\lambda}}(e^x)^{\frac{1}{2}-\frac{\lambda}{2}}\left((e^x)^\mu\right)^{-\frac{1}{2}/\mu}\left((e^x)^\lambda\right)^{\frac{1}{2}-\frac{i\sqrt{c}}{\lambda}}e^{-\frac{k(e^x)^\mu}{\mu}+\frac{i\sqrt{a}(e^x)^\lambda}{\lambda}}\left(c_1\,\text{HypergeometricU}\left(-\frac{\frac{ib}{\sqrt{a}}-\lambda+2i\sqrt{c}}{2\lambda},1-\frac{2i\sqrt{c}}{\lambda},-\frac{2i\sqrt{a}(e^x)^\lambda}{\lambda}\right)+c_2L_{\frac{\frac{ib}{\sqrt{a}}-\lambda+2i\sqrt{c}}{2\lambda}}^{-\frac{2i\sqrt{c}}{\lambda}}\left(-\frac{2i\sqrt{a}(e^x)^\lambda}{\lambda}\right)\right)$$

34.17 problem 17

Internal problem ID [11115]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - (a + 2\,\mathrm{e}^{ax} b)\, y' + b^2 \mathrm{e}^{2ax} y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 57

```
dsolve(diff(y(x),x$2)-(a+2*b*exp(a*x))*diff(y(x),x)+b^2*exp(2*a*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{a^2 x + 2b\,\mathrm{e}^{ax}}{2a}} \sinh\left(\frac{ax}{2}\right) + c_2 \mathrm{e}^{\frac{a^2 x + 2b\,\mathrm{e}^{ax}}{2a}} \cosh\left(\frac{ax}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.079 (sec). Leaf size: 35

```
DSolve[y''[x]-(a+2*b*Exp[a*x])*y'[x]+b^2*Exp[2*a*x]*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{e^{\frac{be^{ax}}{a}}\left(bc_2 e^{ax} + ac_1\right)}{a}$$

34.18 problem 18

Internal problem ID [11116]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(a\,e^{2x\lambda} + \lambda\right) y' - y\,e^{2x\lambda} a\lambda = 0$$

✓ Solution by Maple

Time used: 0.234 (sec). Leaf size: 80

```
dsolve(diff(y(x),x$2)+(a*exp(2*lambda*x)+lambda)*diff(y(x),x)-a*lambda*exp(2*lambda*x)*y(x)=
```

$$y(x) = c_1\left(a\,e^{\lambda x} + e^{-\lambda x}\lambda\right) + c_2\left(\sqrt{\pi}\left(a\,e^{\lambda x} + e^{-\lambda x}\lambda\right)\operatorname{erf}\left(\frac{\sqrt{2}\,e^{\lambda x}\sqrt{a}}{2\sqrt{\lambda}}\right) + \sqrt{\lambda}\,\sqrt{a}\,\sqrt{2}\,e^{-\frac{a\,e^{2\lambda x}}{2\lambda}}\right)$$

✓ Solution by Mathematica

Time used: 0.283 (sec). Leaf size: 129

```
DSolve[y''[x]+(a*Exp[2*\[Lambda]*x]+\[Lambda])*y'[x]-a*\[Lambda]*Exp[2*\[Lambda]*x]*y[x]==0,
```

$$y(x) \to \frac{\sqrt{2\pi}c_2\left(ae^{2\lambda x} + \lambda\right)\operatorname{erf}\left(\frac{\sqrt{a\lambda e^{2\lambda x}}}{\sqrt{2}\lambda}\right) - 4i\sqrt{2}ac_1e^{2\lambda x} + 2c_2e^{-\frac{ae^{2\lambda x}}{2\lambda}}\sqrt{a\lambda e^{2\lambda x}} - 4i\sqrt{2}c_1\lambda}{4\sqrt{a\lambda e^{2\lambda x}}}$$

34.19 problem 19

Internal problem ID [11117]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(e^{\lambda x} a - \lambda\right) y' + y b\, e^{2\lambda x} = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 53

```
dsolve(diff(y(x),x$2)+(a*exp(lambda*x)-lambda)*diff(y(x),x)+b*exp(2*lambda*x)*y(x)=0,y(x), s
```

$$y(x) = c_1 e^{-\frac{\left(a-\sqrt{a^2-4b}\right) e^{\lambda x}}{2\lambda}} + c_2 e^{-\frac{\left(a+\sqrt{a^2-4b}\right) e^{\lambda x}}{2\lambda}}$$

✓ Solution by Mathematica

Time used: 0.133 (sec). Leaf size: 61

```
DSolve[y''[x]+(a*Exp[\[Lambda]*x]-\[Lambda])*y'[x]+b*Exp[2*\[Lambda]*x]*y[x]==0,y[x],x,Inclu
```

$$y(x) \rightarrow e^{-\frac{\left(\sqrt{a^2-4b}+a\right) e^{\lambda x}}{2\lambda}} \left(c_2 e^{\frac{\sqrt{a^2-4b} e^{\lambda x}}{\lambda}} + c_1\right)$$

34.20 problem 20

Internal problem ID [11118]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(a\,\mathrm{e}^{x\lambda} + b\right) y' + c\left(a\,\mathrm{e}^{x\lambda} + b - c\right) y = 0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 177

```
dsolve(diff(y(x),x$2)+(a*exp(lambda*x)+b)*diff(y(x),x)+c*(a*exp(lambda*x)+b-c)*y(x)=0,y(x),
```

$$y(x) = c_1 \mathrm{e}^{-cx} + c_2 \Bigg((-\lambda - 2c + b)^2 \,\mathrm{WhittakerM}\left(-\frac{-\lambda - 2c + b}{2\lambda}, -\frac{-2\lambda - 2c + b}{2\lambda}, \frac{a\,\mathrm{e}^{\lambda x}}{\lambda}\right) \mathrm{e}^{\frac{-a\,\mathrm{e}^{\lambda x} - (b+3\lambda)x\lambda}{2\lambda}} + \lambda \left((\lambda + 2c - b)\,\mathrm{e}^{\frac{-a\,\mathrm{e}^{\lambda x} - (b+3\lambda)x\lambda}{2\lambda}} + a\,\mathrm{e}^{\frac{-a\,\mathrm{e}^{\lambda x} - \lambda x(b+\lambda)}{2\lambda}} \right) \mathrm{WhittakerM}\left(-\frac{b - 2c + \lambda}{2\lambda}, -\frac{-2\lambda - 2c + b}{2\lambda}, \frac{a\,\mathrm{e}^{\lambda x}}{\lambda}\right) \Bigg)$$

✓ Solution by Mathematica

Time used: 0.244 (sec). Leaf size: 96

```
DSolve[y''[x]+(a*Exp[\[Lambda]*x]+b)*y'[x]+c*(a*Exp[\[Lambda]*x]+b-c)*y[x]==0,y[x],x,Include
```

$$y(x) \to (-1)^{-\frac{c}{\lambda}} c^{c/\lambda} \lambda^{\frac{c}{\lambda}-1} a^{-\frac{c}{\lambda}} \left(ce^{\lambda x}\right)^{-\frac{c}{\lambda}} \left(c_2(2c - b)(-1)^{c/\lambda} \Gamma\left(-\frac{b - 2c}{\lambda}, 0, \frac{ae^{x\lambda}}{\lambda}\right) + c_1 \lambda \right)$$

34.21 problem 21

Internal problem ID [11119]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(a + b\,e^{2\lambda x}\right) y' + \lambda\left(a - \lambda - b\,e^{2\lambda x}\right) y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+(a+b*exp(2*lambda*x))*diff(y(x),x)+lambda*(a-lambda-b*exp(2*lambda*x))
```

No solution found

✓ Solution by Mathematica

Time used: 0.513 (sec). Leaf size: 248

```
DSolve[y''[x]+(a+b*Exp[2*\[Lambda]*x])*y'[x]+\[Lambda]*(a-\[Lambda]-b*Exp[2*\[Lambda]*x])*y[
```

$$y(x) \rightarrow \frac{-\frac{1}{2}c_2(a-2\lambda)e^{-\frac{be^{2\lambda x}}{2\lambda}}\left(b\lambda e^{2\lambda x}\right)^{-\frac{a}{2\lambda}}\left(b2^{\frac{a}{2\lambda}}\lambda^{a/\lambda}e^{2\lambda x} + \text{Gamma}\left(1-\frac{a}{2\lambda}\right)e^{\frac{be^{2\lambda x}}{2\lambda}}\left(a+be^{2\lambda x}\right)\left(b\lambda e^{2\lambda x}\right)^{\frac{a}{2\lambda}} - e^{\frac{be^{2\lambda x}}{2\lambda}}\right.}{\sqrt{2}\lambda\sqrt{b\lambda e^{2\lambda x}}}$$

34.22 problem 22

Internal problem ID [11120]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(a + b\,e^{x\lambda} + b - 3\lambda\right) y' + a^2\lambda(b-\lambda)\,e^{2x\lambda} y = 0$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 233

```
dsolve(diff(y(x),x$2)+(a+b*exp(lambda*x)+b-3*lambda)*diff(y(x),x)+a^2*lambda*(b-lambda)*exp(
```

$$y(x) = c_1 e^{-\frac{e^{\lambda x}\left(b+\sqrt{-4\lambda(b-\lambda)a^2+b^2}\right)}{2\lambda}} \operatorname{KummerM}\left(\frac{\left(b+\sqrt{-4\lambda\left(b-\lambda\right)a^2+b^2}\right)\left(a+b-2\lambda\right)}{2\sqrt{-4\lambda\left(b-\lambda\right)a^2+b^2}\,\lambda}, \frac{a+b-2\lambda}{\lambda}, \sqrt{-4\lambda\left(b-\right.}\right.$$
$$+ c_2 e^{-\frac{e^{\lambda x}\left(b+\sqrt{-4\lambda(b-\lambda)a^2+b^2}\right)}{2\lambda}} \operatorname{KummerU}\left(\frac{\left(b+\sqrt{-4\lambda\left(b-\lambda\right)a^2+b^2}\right)\left(a+b-2\lambda\right)}{2\sqrt{-4\lambda\left(b-\lambda\right)a^2+b^2}\,\lambda}, \frac{a+b-2\lambda}{\lambda}, \sqrt{-4\lambda\left(b\right.}\right.$$

✓ Solution by Mathematica

Time used: 3.799 (sec). Leaf size: 260

```
DSolve[y''[x]+(a+b*Exp[\[Lambda]*x]+b-3*\[Lambda])*y'[x]+a^2*\[Lambda]*(b-\[Lambda])*Exp[2*\
```

$$
\begin{aligned}
&y(x) \\
&\to \exp\left(-\frac{e^{\lambda x}\left(\sqrt{-4a^2b\lambda+4a^2\lambda^2+b^2}+b\right)}{2\lambda}\right)\left(c_1\,\text{HypergeometricU}\left(\frac{(a+b-2\lambda)\left(b+\sqrt{4\lambda^2a^2-4b\lambda a^2\right.}}{2\lambda\sqrt{4\lambda^2a^2-4b\lambda a^2+b^2}}\right.\right. \\
&\qquad + c_2 L^{\frac{a+b-3\lambda}{\lambda}}_{-\frac{(a+b-2\lambda)\left(b+\sqrt{4\lambda^2a^2-4b\lambda a^2+b^2}\right)}{2\lambda\sqrt{4\lambda^2a^2-4b\lambda a^2+b^2}}}\left(\frac{e^{x\lambda}\sqrt{4\lambda^2a^2-4b\lambda a^2+b^2}}{\lambda}\right)\Bigg)\Bigg)
\end{aligned}
$$

34.23 problem 23

Internal problem ID [11121]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(2\,\mathrm{e}^{\lambda x} a - \lambda\right) y' + \left(a^2 \mathrm{e}^{2\lambda x} + c\,\mathrm{e}^{\mu x}\right) y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+(2*a*exp(lambda*x)-lambda)*diff(y(x),x)+(a^2*exp(2*lambda*x)+c*exp(mu*
```

No solution found

✓ Solution by Mathematica

Time used: 1.858 (sec). Leaf size: 164

```
DSolve[y''[x]+(2*a*Exp[\[Lambda]*x]-\[Lambda])*y'[x]+(a^2*Exp[2*\[Lambda]*x]+c*Exp[\[Mu]*x])
```

$$y(x) \to (-1)^{-\frac{\lambda}{\mu}} 2^{\frac{\lambda+\mu}{2\mu}} \left((e^x)^\lambda\right)^{\frac{\lambda-1}{2\lambda}} (e^x)^{\frac{1}{2}-\frac{\mu}{2}} e^{-\frac{a(e^x)^\lambda}{\lambda}} \left((e^x)^\mu\right)^{\frac{\lambda+\mu}{2\mu}} \left(-\frac{c(e^x)^\mu}{\mu^2}\right)^{-\frac{\lambda}{2\mu}} \left(c_1(-1)^{\lambda/\mu} \operatorname{BesselI}\left(\frac{\lambda}{\mu}, 2\sqrt{-\frac{c\,(e^x)}{\mu^2}}\right.\right.$$
$$\left.\left. + c_2 K_{\frac{\lambda}{\mu}}\left(2\sqrt{-\frac{c\,(e^x)^\mu}{\mu^2}}\right)\right)\right)$$

34.24 problem 24

Internal problem ID [11122]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(2\,\mathrm{e}^{\lambda x} a + b\right) y' + \left(a^2 \mathrm{e}^{2\lambda x} + a(b+\lambda)\,\mathrm{e}^{\lambda x} + c\right) y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 71

```
dsolve(diff(y(x),x$2)+(2*a*exp(lambda*x)+b)*diff(y(x),x)+(a^2*exp(2*lambda*x)+a*(b+lambda)*e
```

$$y(x) = c_1 \mathrm{e}^{-\frac{\lambda x b + 2a\,\mathrm{e}^{\lambda x}}{2\lambda}} \sinh\left(\frac{\sqrt{b^2-4c}\,x}{2}\right) + c_2 \mathrm{e}^{-\frac{\lambda x b + 2a\,\mathrm{e}^{\lambda x}}{2\lambda}} \cosh\left(\frac{\sqrt{b^2-4c}\,x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.589 (sec). Leaf size: 82

```
DSolve[y''[x]+(2*a*Exp[\[Lambda]*x]+b)*y'[x]+(a^2*Exp[2*\[Lambda]*x]+a*(b+\[Lambda])*Exp[\[L
```

$$y(x) \to \frac{\left(c_2 e^{x\sqrt{b^2-4c}} + c_1\sqrt{b^2-4c}\right) e^{-\frac{a e^{\lambda x}}{\lambda} - \frac{1}{2}x\left(\sqrt{b^2-4c}+b\right)}}{\sqrt{b^2-4c}}$$

34.25 problem 25

Internal problem ID [11123]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(a\,e^{x\lambda} + 2b - \lambda\right) y' + \left(e^{2x\lambda}c + ab\,e^{x\lambda} + b^2 - b\lambda\right) y = 0}$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 74

```
dsolve(diff(y(x),x$2)+(a*exp(lambda*x)+2*b-lambda)*diff(y(x),x)+(c*exp(2*lambda*x)+a*b*exp(l
```

$$y(x) = c_1 e^{-\frac{2\lambda x b - e^{\lambda x}\sqrt{a^2-4c} + a\,e^{\lambda x}}{2\lambda}} + c_2 e^{-\frac{2\lambda x b + e^{\lambda x}\sqrt{a^2-4c} + a\,e^{\lambda x}}{2\lambda}}$$

✓ Solution by Mathematica

Time used: 2.119 (sec). Leaf size: 97

```
DSolve[y''[x]+(a*Exp[\[Lambda]*x]+2*b-\[Lambda])*y'[x]+(c*Exp[2*\[Lambda]*x]+a*b*Exp[\[Lambd
```

$$y(x) \to \frac{\left(e^{\lambda x}\right)^{-\frac{b}{\lambda}} e^{-\frac{\left(\sqrt{a^2-4c}+a\right)e^{\lambda x}}{2\lambda}} \left(c_2 \lambda e^{\frac{\sqrt{a^2-4c}e^{\lambda x}}{\lambda}} + c_1\sqrt{a^2-4c}\right)}{\sqrt{a^2-4c}}$$

34.26 problem 26

Internal problem ID [11124]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + (a\,e^{x} + b)\,y' + \left(c(-c + a)\,e^{2x} + (ak + bc - 2ck + c)\,e^{x} + k(b - k)\right) y = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 100

```
dsolve(diff(y(x),x$2)+(a*exp(x)+b)*diff(y(x),x)+( c*(a-c)*exp(2*x)+ (a*k+b*c+c-2*c*k)*exp(x)
```

$$y(x) = c_1 e^{-kx-e^{x}c} + c_2\left(-e^{-\frac{a\,e^{x}}{2}-\frac{(b+2)x}{2}}(-1+b-2k)\,\mathrm{WhittakerM}\left(-\frac{b}{2}+k+1, -\frac{b}{2}+k+\frac{1}{2}, (a-2c)\,e^{x}\right) + (a-2c)\,\mathrm{WhittakerM}\left(-\frac{b}{2}+k, -\frac{b}{2}+k+\frac{1}{2}, (a-2c)\,e^{x}\right) e^{-\frac{a\,e^{x}}{2}-\frac{xb}{2}}\right)$$

✓ Solution by Mathematica

Time used: 3.806 (sec). Leaf size: 71

```
DSolve[y''[x]+(a*Exp[x]+b)*y'[x]+( c*(a-c)*Exp[2*x]+ (a*k+b*c+c-2*c*k)*Exp[x] + k*(b-k) )*y[
```

$$y(x) \to e^{-ce^{x}}(e^{x})^{-k}\left(c_1 - c_2(e^{x})^{2k-b}\,(e^{x}(a-2c))^{b-2k}\,\Gamma(2k-b, (a-2c)e^{x})\right)$$

34.27 problem 27

Internal problem ID [11125]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(a\,\mathrm{e}^{x\lambda} + b\right) y' + \left(\alpha\,\mathrm{e}^{2x\lambda} + \beta\,\mathrm{e}^{x\lambda} + \gamma\right) y = 0$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 161

```
dsolve(diff(y(x),x$2)+(a*exp(lambda*x)+b)*diff(y(x),x)+( alpha*exp(2*lambda*x)+ beta*exp(lam
```

$$y(x) = c_1 \,\mathrm{WhittakerM}\left(-\frac{a(b+\lambda)-2\beta}{2\sqrt{a^2-4\alpha}\,\lambda}, \frac{\sqrt{b^2-4\gamma}}{2\lambda}, \frac{\sqrt{a^2-4\alpha}\,\mathrm{e}^{\lambda x}}{\lambda}\right)\mathrm{e}^{\frac{-a\,\mathrm{e}^{\lambda x}-\lambda x(b+\lambda)}{2\lambda}} + c_2 \,\mathrm{WhittakerW}\left(-\frac{a(b+\lambda)-2\beta}{2\sqrt{a^2-4\alpha}\,\lambda}, \frac{\sqrt{b^2-4\gamma}}{2\lambda}, \frac{\sqrt{a^2-4\alpha}\,\mathrm{e}^{\lambda x}}{\lambda}\right)\mathrm{e}^{\frac{-a\,\mathrm{e}^{\lambda x}-\lambda x(b+\lambda)}{2\lambda}}$$

✓ Solution by Mathematica

Time used: 2.375 (sec). Leaf size: 248

```
DSolve[y''[x]+(a*Exp[\[Lambda]*x]+b)*y'[x]+( alpha*Exp[2*\[Lambda]*x]+ \[Beta]*Exp[\[Lambda]
```

$$y(x) \to e^{-\frac{\left(\sqrt{a^2-4\alpha}+a\right)e^{\lambda x}}{2\lambda}}\left(e^{\lambda x}\right)^{\frac{\sqrt{b^2-4\gamma}-b}{2\lambda}}\left(c_1\,\text{HypergeometricU}\left(\frac{-2\beta+a(b+\lambda)+\sqrt{a^2-4\alpha}\left(\lambda+\sqrt{b^2-4\gamma}\right)}{2\sqrt{a^2-4\alpha}\lambda},\frac{\lambda+\right.\right.$$

$$\left.\left.+c_2 L^{\frac{\sqrt{b^2-4\gamma}}{\lambda}}_{\frac{2\beta-a(b+\lambda)-\sqrt{a^2-4\alpha}\left(\lambda+\sqrt{b^2-4\gamma}\right)}{2\sqrt{a^2-4\alpha}\lambda}}\left(\frac{\sqrt{a^2-4\alpha}e^{x\lambda}}{\lambda}\right)\right)\right)$$

34.28 problem 28

Internal problem ID [11126]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(2\,\mathrm{e}^{\lambda x} a - \lambda\right) y' + \left(a^2 \mathrm{e}^{2\lambda x} + \mathrm{e}^{2\mu x} b + c\,\mathrm{e}^{\mu x} + k\right) y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+(2*a*exp(lambda*x)-lambda)*diff(y(x),x)+( a^2*exp(2*lambda*x) + b*exp(
```

No solution found

✓ Solution by Mathematica

Time used: 2.29 (sec). Leaf size: 290

```
DSolve[y''[x]+(2*a*Exp[\[Lambda]*x]-\[Lambda])*y'[x]+( a^2*Exp[2*\[Lambda]*x] + b*Exp[2*\[Mu
```

$$y(x) \to \left((e^x)^\lambda\right)^{\frac{\lambda-1}{2\lambda}} (e^x)^{\frac{1}{2}-\frac{\mu}{2}} 2^{\frac{\sqrt{\mu^2\left(\lambda^2-4k\right)}+\mu^2}{2\mu^2}} \left((e^x)^\mu\right)^{\frac{\sqrt{\mu^2\left(\lambda^2-4k\right)}+\mu^2}{2\mu^2}} e^{-\frac{a(e^x)^\lambda}{\lambda}+\frac{i\sqrt{b}(e^x)^\mu}{\mu}} \left(c_1 \text{HypergeometricU}\left(\frac{\mu^2-\frac{ic\mu}{\sqrt{b}}\right. \right.$$

$$\left.\left. -\frac{2i\sqrt{b}(e^x)^\mu}{\mu}\right) + c_2 L_{\frac{ic}{2\sqrt{b}\mu}-\frac{\mu^2+\sqrt{\left(\lambda^2-4k\right)\mu^2}}{2\mu^2}}^{\frac{\sqrt{\left(\lambda^2-4k\right)\mu^2}}{\mu^2}}\left(-\frac{2i\sqrt{b}(e^x)^\mu}{\mu}\right)\right)$$

34.29 problem 29

Internal problem ID [11127]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(2\,\mathrm{e}^{\lambda x}a + b - \lambda\right) y' + \left(a^2\mathrm{e}^{2\lambda x} + ab\,\mathrm{e}^{\lambda x} + \mathrm{e}^{2\mu x}c + d\,\mathrm{e}^{\mu x} + k\right) y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+(2*a*exp(lambda*x)+b-lambda)*diff(y(x),x)+( a^2*exp(2*lambda*x) + a*b*
```

No solution found

✓ Solution by Mathematica

Time used: 2.625 (sec). Leaf size: 332

```
DSolve[y''[x]+(2*a*Exp[\[Lambda]*x]+b-\[Lambda])*y'[x]+( a^2*Exp[2*\[Lambda]*x] + a*b*Exp[\[
```

$$y(x) \rightarrow (e^x)^{\frac{1}{2}-\frac{\mu}{2}} \left((e^x)^\lambda\right)^{-\frac{b-\lambda+1}{2\lambda}} 2^{\frac{1}{2}\left(\sqrt{\frac{\mu^2\left(b^2-2b\lambda+\lambda^2-4k\right)}{\mu^2}}+1\right)} e^{-\frac{a(e^x)^\lambda}{\lambda}+\frac{i\sqrt{c}(e^x)^\mu}{\mu}} \left((e^x)^\mu\right)^{\frac{1}{2}\left(\sqrt{\frac{\mu^2\left(b^2-2b\lambda+\lambda^2-4k\right)}{\mu^2}}+1\right)} \Bigg(c_1 \text{Hyper}$$

$$-\frac{2i\sqrt{c}(e^x)^\mu}{\mu}\Bigg) + c_2 L^{\sqrt{\frac{\left(b^2-2\lambda b+\lambda^2-4k\right)\mu^2}{\mu^2}}}_{-\frac{\mu^2-\frac{id\mu}{\sqrt{c}}+\sqrt{\left(b^2-2\lambda b+\lambda^2-4k\right)\mu^2}}{2\mu^2}}\left(-\frac{2i\sqrt{c}(e^x)^\mu}{\mu}\right)\Bigg)$$

34.30 problem 30

Internal problem ID [11128]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(e^{\lambda x} a + b\, e^{\mu x}\right) y' + a\, e^{\lambda x} (b\, e^{\mu x} + \lambda)\, y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+(a*exp(lambda*x)+b*exp(mu*x))*diff(y(x),x)+a*exp(lambda*x)*(b*exp(mu*x
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+(a*Exp[\[Lambda]*x]+b*Exp[\[Mu]*x])*y'[x]+a*Exp[\[Lambda]*x]*(b*Exp[\[Mu]*x]+\
```

Not solved

34.31 problem 31

Internal problem ID [11129]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + e^{x\lambda}\left(a\,e^{2\mu x} + b\right) y' + \mu\left(e^{x\lambda}\left(b - a\,e^{2\mu x}\right) - \mu\right) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 93

```
dsolve(diff(y(x),x$2)+exp(lambda*x)*(a*exp(2*mu*x)+b)*diff(y(x),x)+mu*(exp(lambda*x)*(b-a*ex
```

$$y(x) = c_1\left(a\,e^{\mu x} + e^{-\mu x} b\right) + c_2\left(\int \frac{e^{\frac{-e^{x(\lambda+2\mu)} a\lambda - 2\,e^{\lambda x}\left(\mu+\frac{\lambda}{2}\right) b}{\lambda(\lambda+2\mu)}}}{\left(a\,e^{\mu x} + e^{-\mu x} b\right)^2} dx\right)\left(a\,e^{\mu x} + e^{-\mu x} b\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+Exp[\[Lambda]*x]*(a*Exp[2*\[Mu]*x]+b)*y'[x]+\[Mu]*(Exp[\[Lambda]*x]*(b-a*Exp[2
```

Not solved

34.32 problem 32

Internal problem ID [11130]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$y'' + \left(e^{\lambda x} a + b\, e^{\mu x} + c\right) y' + \left(a\lambda\, e^{\lambda x} + b\mu\, e^{\mu x}\right) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(diff(y(x),x$2)+(a*exp(lambda*x)+b*exp(mu*x)+c)*diff(y(x),x)+(a*lambda*exp(lambda*x)+b
```

$$y = \left(c_1 \left(\int e^{xc+\frac{a\,e^{\lambda x}}{\lambda}+\frac{b\,e^{\mu x}}{\mu}} dx\right) + c_2\right) e^{-xc-\frac{a\,e^{\lambda x}}{\lambda}-\frac{b\,e^{\mu x}}{\mu}}$$

✓ Solution by Mathematica

Time used: 0.146 (sec). Leaf size: 77

```
DSolve[y''[x]+(a*Exp[\[Lambda]*x]+b*Exp[\[Mu]*x]+c)*y'[x]+(a*\[Lambda]*Exp[\[Lambda]*x]+b*\[
```

$$y(x) \to e^{-\frac{ae^{\lambda x}}{\lambda}-\frac{be^{\mu x}}{\mu}-cx} \left(\int_1^x e^{\frac{e^{\lambda K[1]}a}{\lambda}+cK[1]+\frac{be^{\mu K[1]}}{\mu}} c_1 dK[1] + c_2\right)$$

34.33 problem 33

Internal problem ID [11131]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(\mathrm{e}^{\lambda x} a + b\,\mathrm{e}^{\mu x} + c\right) y' + \left(\mathrm{e}^{x(\lambda+\mu)} ab + \mathrm{e}^{\lambda x} ac + b\mu\,\mathrm{e}^{\mu x}\right) y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+(a*exp(lambda*x)+b*exp(mu*x)+c)*diff(y(x),x)+(a*b*exp((lambda+mu)*x)+a
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+(a*Exp[\[Lambda]*x]+b*Exp[\[Mu]*x]+c)*y'[x]+(a*b*Exp[(\[Lambda]+\[Mu])*x]+a*c*
```

Not solved