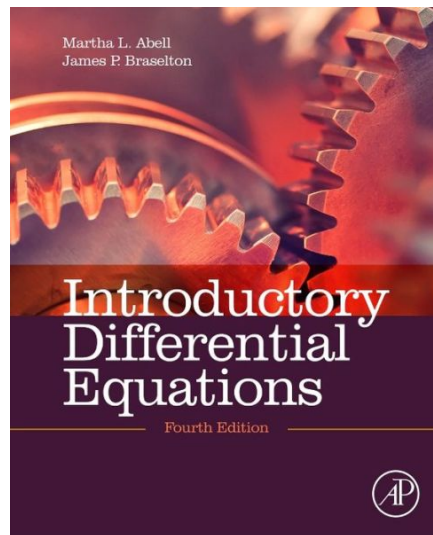


A Solution Manual For

**INTRODUCTORY  
DIFFERENTIAL EQUATIONS.**

**Martha L. Abell, James P.  
Braselton. Fourth edition 2014.  
ElScAe. 2014**



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## 1.1 problem 1

Internal problem ID [13725]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 1.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y' - 2y = x^3$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-2*y(x)=x^3,y(x), singsol=all)
```

$$y(x) = e^x c_2 + e^{-2x} c_1 - \frac{x^3}{2} - \frac{3x^2}{4} - \frac{9x}{4} - \frac{15}{8}$$

### ✓ Solution by Mathematica

Time used: 0.077 (sec). Leaf size: 39

```
DSolve[y''[x]+y'[x]-2*y[x]==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{8}(-4x^3 - 6x^2 - 18x - 15) + c_1 e^{-2x} + c_2 e^x$$

## 1.2 problem 2

Internal problem ID [13726]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 2.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [ $y = G(x, y')$ ]

$$yy' + y^4 = \sin(x)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 164

```
dsolve(y(x)*diff(y(x),x)+y(x)^4=sin(x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\left(c_1 \operatorname{MathieuC}\left(0, 8, -\frac{\pi}{4} + \frac{x}{2}\right) + \operatorname{MathieuS}\left(0, 8, -\frac{\pi}{4} + \frac{x}{2}\right)\right) \left(c_1 \operatorname{MathieuCPrime}\left(0, 8, -\frac{\pi}{4} + \frac{x}{2}\right) + \operatorname{MathieuSPrime}\left(0, 8, -\frac{\pi}{4} + \frac{x}{2}\right)\right)}{2c_1 \operatorname{MathieuC}\left(0, 8, -\frac{\pi}{4} + \frac{x}{2}\right) + 2 \operatorname{MathieuS}\left(0, 8, -\frac{\pi}{4} + \frac{x}{2}\right)}$$

$$y(x) = \frac{\sqrt{\left(c_1 \operatorname{MathieuC}\left(0, 8, -\frac{\pi}{4} + \frac{x}{2}\right) + \operatorname{MathieuS}\left(0, 8, -\frac{\pi}{4} + \frac{x}{2}\right)\right) \left(c_1 \operatorname{MathieuCPrime}\left(0, 8, -\frac{\pi}{4} + \frac{x}{2}\right) + \operatorname{MathieuSPrime}\left(0, 8, -\frac{\pi}{4} + \frac{x}{2}\right)\right)}{2c_1 \operatorname{MathieuC}\left(0, 8, -\frac{\pi}{4} + \frac{x}{2}\right) + 2 \operatorname{MathieuS}\left(0, 8, -\frac{\pi}{4} + \frac{x}{2}\right)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+y[x]^4==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

### 1.3 problem 4

Internal problem ID [13727]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 4.

**ODE order:** 3.

**ODE degree:** 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$y''' - 2y'' + 5y' + y = e^x$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 227

```
dsolve(diff(y(x),x$3)-2*diff(y(x),x$2)+5*diff(y(x),x)+y(x)=exp(x),y(x), singsol=all)
```

$$y(x) = \frac{e^x}{5} + c_1 e^{\frac{(15\sqrt{69}(404+60\sqrt{69})^{\frac{2}{3}} - 101(404+60\sqrt{69})^{\frac{2}{3}} - 484(404+60\sqrt{69})^{\frac{1}{3}} + 1936)x}{2904}}$$
$$+ c_2 e^{-\frac{(15\sqrt{69}(404+60\sqrt{69})^{\frac{2}{3}} - 101(404+60\sqrt{69})^{\frac{2}{3}} - 484(404+60\sqrt{69})^{\frac{1}{3}} - 3872)x}{5808}} \cos\left(\frac{(404+60\sqrt{69})^{\frac{1}{3}}\sqrt{3}(15(404+60\sqrt{69})^{\frac{1}{3}})}{5808}\right)$$
$$+ c_3 e^{-\frac{(15\sqrt{69}(404+60\sqrt{69})^{\frac{2}{3}} - 101(404+60\sqrt{69})^{\frac{2}{3}} - 484(404+60\sqrt{69})^{\frac{1}{3}} - 3872)x}{5808}} \sin\left(\frac{(404+60\sqrt{69})^{\frac{1}{3}}\sqrt{3}(15(404+60\sqrt{69})^{\frac{1}{3}})}{5808}\right)$$

#### ✓ Solution by Mathematica

Time used: 0.389 (sec). Leaf size: 2831

```
DSolve[y'''[x]-2*y''[x]+5*y'[x]+y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

## 1.4 problem 5

Internal problem ID [13728]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 5.

**ODE order:** 1.

**ODE degree:** 2.

CAS Maple gives this as type [quadrature]

$$y'^2 + y = 0$$

### ✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)^2+y(x)=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = -\frac{1}{4}x^2 + \frac{1}{2}c_1x - \frac{1}{4}c_1^2$$

### ✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 42

```
DSolve[y'[x]^2+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{1}{4}(x + ic_1)^2$$

$$y(x) \rightarrow -\frac{1}{4}(x - ic_1)^2$$

$$y(x) \rightarrow 0$$



## 1.5 problem 6

Internal problem ID [13729]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 6.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$t^2 y'' + t y' + 2y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(t^2*diff(y(t),t$2)+t*diff(y(t),t)+2*y(t)=0,y(t), singsol=all)
```

$$y = c_1 \sin(\sqrt{2} \ln(t)) + c_2 \cos(\sqrt{2} \ln(t))$$

### ✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 30

```
DSolve[t^2*y'[t]+t*y'[t]+2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow c_1 \cos(\sqrt{2} \log(t)) + c_2 \sin(\sqrt{2} \log(t))$$

## 1.6 problem 9

Internal problem ID [13730]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 9.

**ODE order:** 2.

**ODE degree:** 2.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy''^2 + 2y = 2x$$

**X** Solution by Maple

```
dsolve(x*diff(y(x),x$2)^2+2*y(x)=2*x,y(x), singsol=all)
```

No solution found

**X** Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]^2+2*y[x]==2*x,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

## 1.7 problem 10

Internal problem ID [13731]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 10.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type [NONE]

$$x'' + 2 \sin(x) = \sin(2t)$$

**X** Solution by Maple

```
dsolve(diff(x(t),t$2)+2*sin(x(t))=sin(2*t),x(t), singsol=all)
```

No solution found

**X** Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x''[t]+2*Sin[x[t]]==Sin[2*t],x[t],t,IncludeSingularSolutions -> True]
```

Not solved

## 1.8 problem 12

Internal problem ID [13732]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 12.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$-y' = -2x + 1$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((2*x-1)-diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = x^2 + c_1 - x$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 7

```
DSolve[(2*x-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1$$

## 1.9 problem 15

Internal problem ID [13733]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 15.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$-y - yy' = -2x$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 2300

`dsolve((2*x-y(x))-y(x)*diff(y(x),x)=0,y(x), singsol=all)`

$$y(x) = -\frac{8x \left( \left( 4c_1x^3 + 4\sqrt{4c_1^3x^9 + c_1^2x^6} \right)^{\frac{1}{3}} - \frac{4x^3c_1}{\left( 4c_1x^3 + 4\sqrt{4c_1^3x^9 + c_1^2x^6} \right)^{\frac{1}{3}}} - 1 \right)}{4 \left( 4c_1x^3 + 4\sqrt{4c_1^3x^9 + c_1^2x^6} \right)^{\frac{1}{3}} - \frac{16x^3c_1}{\left( 4c_1x^3 + 4\sqrt{4c_1^3x^9 + c_1^2x^6} \right)^{\frac{1}{3}}}}$$

$$y(x) = -\frac{8x \left( \frac{\left( -\frac{1}{2} - \frac{i\sqrt{3}}{2} \right)^3 \left( 4 \left( 4c_1x^3 + 4\sqrt{4c_1^3x^9 + c_1^2x^6} \right)^{\frac{1}{3}} - \frac{16x^3c_1}{\left( 4c_1x^3 + 4\sqrt{4c_1^3x^9 + c_1^2x^6} \right)^{\frac{1}{3}}} \right)}{4} - 1 \right)}{\left( -\frac{1}{2} - \frac{i\sqrt{3}}{2} \right)^3 \left( 4 \left( 4c_1x^3 + 4\sqrt{4c_1^3x^9 + c_1^2x^6} \right)^{\frac{1}{3}} - \frac{16x^3c_1}{\left( 4c_1x^3 + 4\sqrt{4c_1^3x^9 + c_1^2x^6} \right)^{\frac{1}{3}}} \right)}$$

$$y(x) = -\frac{8x \left( \frac{\left( -\frac{1}{2} + \frac{i\sqrt{3}}{2} \right)^3 \left( 4 \left( 4c_1x^3 + 4\sqrt{4c_1^3x^9 + c_1^2x^6} \right)^{\frac{1}{3}} - \frac{16x^3c_1}{\left( 4c_1x^3 + 4\sqrt{4c_1^3x^9 + c_1^2x^6} \right)^{\frac{1}{3}}} \right)}{4} - 1 \right)}{\left( -\frac{1}{2} + \frac{i\sqrt{3}}{2} \right)^3 \left( 4 \left( 4c_1x^3 + 4\sqrt{4c_1^3x^9 + c_1^2x^6} \right)^{\frac{1}{3}} - \frac{16x^3c_1}{\left( 4c_1x^3 + 4\sqrt{4c_1^3x^9 + c_1^2x^6} \right)^{\frac{1}{3}}} \right)}$$

$$y(x) = \frac{8x \left( -\frac{\left( 4c_1x^3 + 4\sqrt{4c_1^3x^9 + c_1^2x^6} \right)^{\frac{1}{3}}}{2} + \frac{2x^3c_1}{\left( 4c_1x^3 + 4\sqrt{4c_1^3x^9 + c_1^2x^6} \right)^{\frac{1}{3}}} - i\sqrt{3} \left( \frac{\left( 4c_1x^3 + 4\sqrt{4c_1^3x^9 + c_1^2x^6} \right)^{\frac{1}{3}}}{2} + \frac{2x^3c_1}{\left( 4c_1x^3 + 4\sqrt{4c_1^3x^9 + c_1^2x^6} \right)^{\frac{1}{3}}} \right) \right)}{-2 \left( 4c_1x^3 + 4\sqrt{4c_1^3x^9 + c_1^2x^6} \right)^{\frac{1}{3}} + \frac{8x^3c_1}{\left( 4c_1x^3 + 4\sqrt{4c_1^3x^9 + c_1^2x^6} \right)^{\frac{1}{3}}} - 4i\sqrt{3} \left( \frac{\left( 4c_1x^3 + 4\sqrt{4c_1^3x^9 + c_1^2x^6} \right)^{\frac{1}{3}}}{2} + \frac{2x^3c_1}{\left( 4c_1x^3 + 4\sqrt{4c_1^3x^9 + c_1^2x^6} \right)^{\frac{1}{3}}} \right)}$$

$$y(x) = \frac{8x \left( -\frac{\left( 4c_1x^3 + 4\sqrt{4c_1^3x^9 + c_1^2x^6} \right)^{\frac{1}{3}}}{2} + \frac{2x^3c_1}{\left( 4c_1x^3 + 4\sqrt{4c_1^3x^9 + c_1^2x^6} \right)^{\frac{1}{3}}} + i\sqrt{3} \left( \frac{\left( 4c_1x^3 + 4\sqrt{4c_1^3x^9 + c_1^2x^6} \right)^{\frac{1}{3}}}{2} + \frac{2x^3c_1}{\left( 4c_1x^3 + 4\sqrt{4c_1^3x^9 + c_1^2x^6} \right)^{\frac{1}{3}}} \right) \right)}{-2 \left( 4c_1x^3 + 4\sqrt{4c_1^3x^9 + c_1^2x^6} \right)^{\frac{1}{3}} + \frac{13}{13} \frac{8x^3c_1}{\left( 4c_1x^3 + 4\sqrt{4c_1^3x^9 + c_1^2x^6} \right)^{\frac{1}{3}}} + 4i\sqrt{3} \left( \frac{\left( 4c_1x^3 + 4\sqrt{4c_1^3x^9 + c_1^2x^6} \right)^{\frac{1}{3}}}{2} + \frac{2x^3c_1}{\left( 4c_1x^3 + 4\sqrt{4c_1^3x^9 + c_1^2x^6} \right)^{\frac{1}{3}}} \right)}$$

✓ Solution by Mathematica

Time used: 54.579 (sec). Leaf size: 496

`DSolve[(2*x-y[x])-y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]`

$$y(x) \rightarrow \frac{\sqrt[3]{2x^3 + \sqrt{e^{6c_1} - 4e^{3c_1}x^3} - e^{3c_1}}}{\sqrt[3]{2}} + \frac{\sqrt[3]{2}x^2}{\sqrt[3]{2x^3 + \sqrt{e^{6c_1} - 4e^{3c_1}x^3} - e^{3c_1}}} - x$$

$$y(x) \rightarrow \frac{i(\sqrt{3} + i) \sqrt[3]{2x^3 + \sqrt{e^{6c_1} - 4e^{3c_1}x^3} - e^{3c_1}}}{2\sqrt[3]{2}} - \frac{(1 + i\sqrt{3})x^2}{2^{2/3} \sqrt[3]{2x^3 + \sqrt{e^{6c_1} - 4e^{3c_1}x^3} - e^{3c_1}}} - x$$

$$y(x) \rightarrow -\frac{(1 + i\sqrt{3}) \sqrt[3]{2x^3 + \sqrt{e^{6c_1} - 4e^{3c_1}x^3} - e^{3c_1}}}{2\sqrt[3]{2}} + \frac{i(\sqrt{3} + i)x^2}{2^{2/3} \sqrt[3]{2x^3 + \sqrt{e^{6c_1} - 4e^{3c_1}x^3} - e^{3c_1}}} - x$$

$$y(x) \rightarrow \sqrt[3]{x^3} + \frac{(x^3)^{2/3}}{x} - x$$

$$y(x) \rightarrow \frac{1}{2} \left( i(\sqrt{3} + i) \sqrt[3]{x^3} + \frac{(-1 - i\sqrt{3})(x^3)^{2/3}}{x} - 2x \right)$$

$$y(x) \rightarrow \frac{1}{2} \left( (-1 - i\sqrt{3}) \sqrt[3]{x^3} + \frac{i(\sqrt{3} + i)(x^3)^{2/3}}{x} - 2x \right)$$

## 1.10 problem 17

Internal problem ID [13734]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 17.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' + 2y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-2x}c_1$$

### ✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 18

```
DSolve[y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{-2x}$$

$$y(x) \rightarrow 0$$



## 1.11 problem 18

Internal problem ID [13735]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 18.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$y' + yx = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{x^2}{2}}$$

### ✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 22

```
DSolve[y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{-\frac{x^2}{2}}$$

$$y(x) \rightarrow 0$$

## 1.12 problem 19

Internal problem ID [13736]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 19.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' + y = \sin(x)$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)+y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = \frac{\sin(x)}{2} - \frac{\cos(x)}{2} + c_1 e^{-x}$$

### ✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 25

```
DSolve[y'[x]+y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2}(\sin(x) - \cos(x) + 2c_1 e^{-x})$$

## 1.13 problem 20

Internal problem ID [13737]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 20.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - y' - 12y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(t),t$2)-diff(y(t),t)-12*y(t)=0,y(t), singsol=all)
```

$$y = c_1 e^{4t} + c_2 e^{-3t}$$

### ✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 22

```
DSolve[y''[t]-y'[t]-12*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow e^{-3t}(c_2 e^{7t} + c_1)$$

## 1.14 problem 21

Internal problem ID [13738]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 21.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 9y' = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(t),t$2)+9*diff(y(t),t)=0,y(t), singsol=all)
```

$$y = c_1 + c_2 e^{-9t}$$

### ✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 19

```
DSolve[y''[t]+9*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow c_2 - \frac{1}{9}c_1 e^{-9t}$$

## 1.15 problem 22

Internal problem ID [13739]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 22.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$x'' + 2x' - 10x = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(x(t),t$2)+2*diff(x(t),t)-10*x(t)=0,x(t), singsol=all)
```

$$x(t) = c_1 e^{(-1+\sqrt{11})t} + c_2 e^{-(1+\sqrt{11})t}$$

### ✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 34

```
DSolve[x''[t]+2*x'[t]-10*x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow e^{-((1+\sqrt{11})t)} (c_2 e^{2\sqrt{11}t} + c_1)$$

## 1.16 problem 23

Internal problem ID [13740]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 23.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x'' + x = t \cos(t) - \cos(t)$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
dsolve(diff(x(t),t$2)+x(t)=t*cos(t)-cos(t),x(t), singsol=all)
```

$$x(t) = c_2 \sin(t) + c_1 \cos(t) + \frac{(t^2 - 2t - 1) \sin(t)}{4} + \frac{\cos(t)(t - 2)}{4}$$

### ✓ Solution by Mathematica

Time used: 0.089 (sec). Leaf size: 38

```
DSolve[x''[t]+x[t]==t*Cos[t]-Cos[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow \frac{1}{8}((2t^2 - 4t - 1 + 8c_2) \sin(t) + 2(t - 2 + 4c_1) \cos(t))$$

## 1.17 problem 24

Internal problem ID [13741]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 24.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 12y' + 40y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)-12*diff(y(x),x)+40*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{6x} \sin(2x) + c_2 e^{6x} \cos(2x)$$

### ✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 26

```
DSolve[y''[x]-12*y'[x]+40*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{6x}(c_2 \cos(2x) + c_1 \sin(2x))$$

## 1.18 problem 25

Internal problem ID [13742]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 25.

**ODE order:** 3.

**ODE degree:** 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' - 4y'' = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$3)-4*diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = c_2x + c_1 + e^{4x}c_3$$

### ✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 23

```
DSolve[y'''[x]-4*y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{16}c_1e^{4x} + c_3x + c_2$$



## 1.19 problem 26

Internal problem ID [13743]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 26.

**ODE order:** 3.

**ODE degree:** 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' - 2y'' = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$3)-2*diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = c_2x + c_1 + e^{2x}c_3$$

### ✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 23

```
DSolve[y'''[x]-2*y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{4}c_1e^{2x} + c_3x + c_2$$

## 1.20 problem 27

Internal problem ID [13744]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 27.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^2y'' - 12y'x + 42y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-12*x*diff(y(x),x)+42*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^7 + c_2x^6$$

### ✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 16

```
DSolve[x^2*y''[x]-12*x*y'[x]+42*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^6(c_2x + c_1)$$

## 1.21 problem 28

Internal problem ID [13745]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 28.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$t^2 y'' + 3ty' + 5y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(t^2*diff(y(t),t$2)+3*t*diff(y(t),t)+5*y(t)=0,y(t), singsol=all)
```

$$y = \frac{c_1 \sin(2 \ln(t))}{t} + \frac{c_2 \cos(2 \ln(t))}{t}$$

### ✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 26

```
DSolve[t^2*y''[t]+3*t*y'[t]+5*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{c_2 \cos(2 \log(t)) + c_1 \sin(2 \log(t))}{t}$$

## 1.22 problem 29

Internal problem ID [13746]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 29.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$y' + \frac{x}{y} = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x)=-x/y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{-x^2 + c_1}$$

$$y(x) = -\sqrt{-x^2 + c_1}$$

### ✓ Solution by Mathematica

Time used: 0.201 (sec). Leaf size: 39

```
DSolve[y'[x]==-x/y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\sqrt{-x^2 + 2c_1}$$

$$y(x) \rightarrow \sqrt{-x^2 + 2c_1}$$

## 1.23 problem 30

Internal problem ID [13747]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 30.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _exact, _rational, [_Abel, '2nd ty`

$$3y(t^2 + y) + t(t^2 + 6y)y' = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
dsolve(3*y(t)*(t^2+y(t))+t*(t^2+6*y(t))*diff(y(t),t)=0,y(t), singsol=all)
```

$$y = \frac{-t^3 + \sqrt{t^6 + 12c_1t}}{6t}$$

$$y = -\frac{t^3 + \sqrt{t^6 + 12c_1t}}{6t}$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 114

```
DSolve[3*y[t]*(t^2+y[t])+t*(t^2+6*y[t])*y'[x]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{6} \left( -\sqrt{3} \sqrt{t^2 (3t^2 + 8ty'(x) + 12y'(x)^2)} - 3t^2 - 6ty'(x) \right)$$

$$y(t) \rightarrow \frac{1}{6} \left( \sqrt{3} \sqrt{t^2 (3t^2 + 8ty'(x) + 12y'(x)^2)} - 3t^2 - 6ty'(x) \right)$$

## 1.24 problem 31

Internal problem ID [13748]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 31.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$y' + \frac{2y}{x} = -3$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)=-2*y(x)/x-3,y(x), singsol=all)
```

$$y(x) = -x + \frac{c_1}{x^2}$$

### ✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 15

```
DSolve[y'[x]==-2*y[x]/x-3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -x + \frac{c_1}{x^2}$$

## 1.25 problem 32

Internal problem ID [13749]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 32.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_exact, [\_1st\_order, ‘\_with\_symmetry\_[F(x),G(y)]’], [\_Abel, ‘

$$y \cos(t) + (2y + \sin(t))y' = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
dsolve(y(t)*cos(t)+(2*y(t)+sin(t))*diff(y(t),t)=0,y(t), singsol=all)
```

$$y = -\frac{\sin(t)}{2} - \frac{\sqrt{\sin^2(t) - 4c_1}}{2}$$

$$y = -\frac{\sin(t)}{2} + \frac{\sqrt{\sin^2(t) - 4c_1}}{2}$$

### ✓ Solution by Mathematica

Time used: 0.151 (sec). Leaf size: 60

```
DSolve[y[t]*Cos[t]+(2*y[t]+Sin[t])*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{2} \left( -\sin(t) - \sqrt{\sin^2(t) + 4c_1} \right)$$

$$y(t) \rightarrow \frac{1}{2} \left( -\sin(t) + \sqrt{\sin^2(t) + 4c_1} \right)$$

$$y(t) \rightarrow 0$$

## 1.26 problem 33

Internal problem ID [13750]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 33.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [exact]

$$\frac{y}{x} + \cos(y) + (\ln(x) - x \sin(y)) y' = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((y(x)/x+cos(y(x)))+(ln(x)-x*sin(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$\cos(y(x)) x + \ln(x) y(x) + c_1 = 0$$

### ✓ Solution by Mathematica

Time used: 0.221 (sec). Leaf size: 19

```
DSolve[(y[x]/x+Cos[y[x]])+(Log[x]-x*Sin[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$\text{Solve}[-y(x) \log(x) - x \cos(y(x)) = c_1, y(x)]$$



## 1.27 problem 34

Internal problem ID [13751]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 34.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [quadrature]

$$y' = (x^2 - 1)(x^3 - 3x)^3$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x)=(x^2-1)*(x^3-3*x)^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{12}x^{12} - x^{10} + \frac{9}{2}x^8 - 9x^6 + \frac{27}{4}x^4 + c_1$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 39

```
DSolve[y'[x]==(x^2-1)*(x^3-3*x)^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x^{12}}{12} - x^{10} + \frac{9x^8}{2} - 9x^6 + \frac{27x^4}{4} + c_1$$

## 1.28 problem 35

Internal problem ID [13752]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 35.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [quadrature]

$$y' = x \sin(x^2)$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=x*sin(x^2),y(x), singsol=all)
```

$$y(x) = -\frac{\cos(x^2)}{2} + c_1$$

### ✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 16

```
DSolve[y'[x]==x*Sin[x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{\cos(x^2)}{2} + c_1$$

## 1.29 problem 36

Internal problem ID [13753]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 36.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' = \frac{x}{\sqrt{x^2 - 16}}$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)=x/sqrt(x^2-16),y(x), singsol=all)
```

$$y(x) = \frac{(x-4)(x+4)}{\sqrt{x^2-16}} + c_1$$

### ✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 17

```
DSolve[y'[x]==x/Sqrt[x^2-16],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sqrt{x^2 - 16} + c_1$$

### 1.30 problem 37

Internal problem ID [13754]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 37.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' = \frac{1}{x \ln(x)}$$

#### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)=1/(x*ln(x)),y(x), singsol=all)
```

$$y(x) = \ln(\ln(x)) + c_1$$

#### ✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 11

```
DSolve[y'[x]==1/(x*Log[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \log(\log(x)) + c_1$$

## 1.31 problem 38

Internal problem ID [13755]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 38.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' = x \ln(x)$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)=x*ln(x),y(x), singsol=all)
```

$$y(x) = \frac{\ln(x) x^2}{2} - \frac{x^2}{4} + c_1$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 24

```
DSolve[y'[x]==x*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{x^2}{4} + \frac{1}{2}x^2 \log(x) + c_1$$

## 1.32 problem 39

Internal problem ID [13756]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 39.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' = x e^{-x}$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=x*exp(-x),y(x), singsol=all)
```

$$y(x) = -(x + 1) e^{-x} + c_1$$

### ✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 18

```
DSolve[y'[x]==x*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -e^{-x}(x + 1) + c_1$$

### 1.33 problem 40

Internal problem ID [13757]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 40.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' = \frac{-2x - 10}{(x + 2)(x - 4)}$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=(-2*(x+5))/((x+2)*(x-4)),y(x), singsol=all)
```

$$y(x) = \ln(x + 2) - 3 \ln(x - 4) + c_1$$

#### ✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 20

```
DSolve[y'[x]==(-2*(x+5))/((x+2)*(x-4)),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -3 \log(4 - x) + \log(x + 2) + c_1$$

## 1.34 problem 41

Internal problem ID [13758]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 41.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' = \frac{-x^2 + x}{(x + 1)(x^2 + 1)}$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)=(x-x^2)/((x+1)*(x^2+1)),y(x), singsol=all)
```

$$y(x) = -\ln(x + 1) + \arctan(x) + c_1$$

### ✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 16

```
DSolve[y'[x]==(x-x^2)/((x+1)*(x^2+1)),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \arctan(x) - \log(x + 1) + c_1$$



## 1.35 problem 42

Internal problem ID [13759]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 42.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [quadrature]

$$y' = \frac{\sqrt{x^2 - 16}}{x}$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)=sqrt(x^2-16)/x,y(x), singsol=all)
```

$$y(x) = \sqrt{x^2 - 16} + 4 \arctan\left(\frac{4}{\sqrt{x^2 - 16}}\right) + c_1$$

### ✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 33

```
DSolve[y'[x]==Sqrt[x^2-16]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -4 \arctan\left(\frac{\sqrt{x^2 - 16}}{4}\right) + \sqrt{x^2 - 16} + c_1$$

## 1.36 problem 43

Internal problem ID [13760]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 43.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [quadrature]

$$y' = (-x^2 + 4)^{\frac{3}{2}}$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x)=(4-x^2)^(3/2),y(x), singsol=all)
```

$$y(x) = \frac{x(-x^2 + 4)^{\frac{3}{2}}}{4} + \frac{3\sqrt{-x^2 + 4}x}{2} + 6 \arcsin\left(\frac{x}{2}\right) + c_1$$

### ✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 49

```
DSolve[y'[x]==(4-x^2)^(3/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -12 \arctan\left(\frac{\sqrt{4-x^2}}{x+2}\right) - \frac{1}{4}x\sqrt{4-x^2}(x^2-10) + c_1$$

## 1.37 problem 44

Internal problem ID [13761]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 44.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [quadrature]

$$y' = \frac{1}{x^2 - 16}$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)=1/(x^2-16),y(x), singsol=all)
```

$$y(x) = -\frac{\ln(x+4)}{8} + \frac{\ln(x-4)}{8} + c_1$$

### ✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 26

```
DSolve[y'[x]==1/(x^2-16),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{8}(\log(4-x) - \log(x+4) + 8c_1)$$

## 1.38 problem 45

Internal problem ID [13762]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 45.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [quadrature]

$$y' = \cos(x) \cot(x)$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=cos(x)*cot(x),y(x), singsol=all)
```

$$y(x) = \cos(x) + \ln(\csc(x) - \cot(x)) + c_1$$

### ✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 26

```
DSolve[y'[x]==Cos[x]*Cot[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \cos(x) + \log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right) + c_1$$

## 1.39 problem 46

Internal problem ID [13763]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 46.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [quadrature]

$$y' = \sin(x)^3 \tan(x)$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x)=sin(x)^3*tan(x),y(x), singsol=all)
```

$$y(x) = -\frac{\sin(x)^3}{3} - \sin(x) + \ln(\sec(x) + \tan(x)) + c_1$$

### ✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 23

```
DSolve[y'[x]==Sin[x]^3*Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \operatorname{arctanh}(\sin(x)) - \frac{1}{3} \sin^3(x) - \sin(x) + c_1$$

## 1.40 problem 47

Internal problem ID [13764]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 47.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' + 2y = 0$$

With initial conditions

$$[y(0) = 2]$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 10

```
dsolve([diff(y(x),x)+2*y(x)=0,y(0) = 2],y(x), singsol=all)
```

$$y(x) = 2e^{-2x}$$

### ✓ Solution by Mathematica

Time used: 0.072 (sec). Leaf size: 12

```
DSolve[{y'[x]+2*y[x]==0,{y[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 2e^{-2x}$$

## 1.41 problem 48

Internal problem ID [13765]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 48.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' + y = \sin(t)$$

With initial conditions

$$[y(0) = -1]$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve([diff(y(t),t)+y(t)=sin(t),y(0) = -1],y(t), singsol=all)
```

$$y = -\frac{\cos(t)}{2} + \frac{\sin(t)}{2} - \frac{e^{-t}}{2}$$

### ✓ Solution by Mathematica

Time used: 0.057 (sec). Leaf size: 23

```
DSolve[{y'[t]+y[t]==Sin[t],{y[0]==-1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{2}(-e^{-t} + \sin(t) - \cos(t))$$

## 1.42 problem 49

Internal problem ID [13766]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 49.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - y' - 12y = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$2)-diff(y(x),x)-12*y(x)=0,y(0) = 1, D(y)(0) = -1],y(x), singsol=all)
```

$$y(x) = \frac{(2e^{7x} + 5)e^{-3x}}{7}$$

### ✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 23

```
DSolve[{y''[x]-y'[x]-12*y[x]==0,{y[0]==1,y'[0]==-1}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \rightarrow \frac{1}{7}e^{-3x}(2e^{7x} + 5)$$



## 1.43 problem 50

Internal problem ID [13767]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 50.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 9y' = 0$$

With initial conditions

$$[y(0) = 2, y'(0) = -1]$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 12

```
dsolve([diff(y(x),x$2)+9*diff(y(x),x)=0,y(0) = 2, D(y)(0) = -1],y(x), singsol=all)
```

$$y(x) = \frac{17}{9} + \frac{e^{-9x}}{9}$$

### ✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 16

```
DSolve[{y'[x]+9*y'[x]==0,{y[0]==2,y'[0]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{9}(e^{-9x} + 17)$$

## 1.44 problem 51

Internal problem ID [13768]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 51.

**ODE order:** 3.

**ODE degree:** 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' - 2y'' = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 1, y''(0) = 3]$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(x),x$3)-2*diff(y(x),x$2)=0,y(0) = 0, D(y)(0) = 1, (D@@2)(y)(0) = 3],y(x), sin
```

$$y(x) = -\frac{3}{4} - \frac{x}{2} + \frac{3e^{2x}}{4}$$

### ✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 21

```
DSolve[{y'''[x]-2*y''[x]==0,{y[0]==0,y'[0]==1,y''[0]==3}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow \frac{1}{4}(-2x + 3e^{2x} - 3)$$

## 1.45 problem 52

Internal problem ID [13769]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 52.

**ODE order:** 3.

**ODE degree:** 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' - 4y' = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = -1, y''(0) = 0]$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$3)-4*diff(y(x),x)=0,y(0) = 1, D(y)(0) = -1, (D@@2)(y)(0) = 0],y(x), sing
```

$$y(x) = 1 + \frac{e^{-2x}}{4} - \frac{e^{2x}}{4}$$

### ✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 23

```
DSolve[{y'''[x]-4*y'[x]==0,{y[0]==1,y'[0]==-1,y''[0]==0}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow \frac{1}{4}(e^{-2x} - e^{2x} + 4)$$

## 1.46 problem 53

Internal problem ID [13770]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 53.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$t^2 y'' - 12ty' + 42y = 0$$

With initial conditions

$$[y(1) = 0, y'(1) = -1]$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve([t^2*diff(y(t),t$2)-12*t*diff(y(t),t)+42*y(t)=0,y(1) = 0, D(y)(1) = -1],y(t), singsol
```

$$y = -t^7 + t^6$$

### ✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 13

```
DSolve[{t^2*y'[t]-12*t*y'[t]+42*y[t]==0,{y[1]==0,y'[1]==-1}},y[t],t,IncludeSingularSolution
```

$$y(t) \rightarrow -((t - 1)t^6)$$

## 1.47 problem 54

Internal problem ID [13771]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 54.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^2 y'' + 3y'x + 5y = 0$$

With initial conditions

$$[y(1) = 0, y'(1) = 1]$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve([x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+5*y(x)=0,y(1) = 0, D(y)(1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\sin(2 \ln(x))}{2x}$$

### ✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 15

```
DSolve[{x^2*y''[x]+3*x*y'[x]+5*y[x]==0,{y[1]==0,y'[1]==1}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \rightarrow \frac{\sin(\log(x)) \cos(\log(x))}{x}$$

## 1.48 problem 55

Internal problem ID [13772]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 55.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [quadrature]

$$y' = 4x^3 - x + 2$$

With initial conditions

$$[y(0) = 1]$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve([diff(y(x),x)=4*x^3-x+2,y(0) = 1],y(x), singsol=all)
```

$$y(x) = x^4 - \frac{1}{2}x^2 + 1 + 2x$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
DSolve[{y'[x]==4*x^3-x+2,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^4 - \frac{x^2}{2} + 2x + 1$$

## 1.49 problem 56

Internal problem ID [13773]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 56.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [quadrature]

$$y' = \sin(2t) - \cos(2t)$$

With initial conditions

$$[y(0) = 0]$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve([diff(y(t),t)=sin(2*t)-cos(2*t),y(0) = 0],y(t), singsol=all)
```

$$y = -\frac{\sin(2t)}{2} - \frac{\cos(2t)}{2} + \frac{1}{2}$$

### ✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 23

```
DSolve[{y'[t]==Sin[2*t]-Cos[2*t],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{2}(-\sin(2t) - \cos(2t) + 1)$$

## 1.50 problem 57

Internal problem ID [13774]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 57.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' = \frac{\cos\left(\frac{1}{x}\right)}{x^2}$$

With initial conditions

$$\left[ y\left(\frac{2}{\pi}\right) = 1 \right]$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)=cos(1/x)/x^2,y(2/Pi) = 1],y(x), singsol=all)
```

$$y(x) = -\sin\left(\frac{1}{x}\right) + 2$$

### ✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 13

```
DSolve[{y'[x]==Cos[1/x]/x^2,{y[2/Pi]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 2 - \sin\left(\frac{1}{x}\right)$$



## 1.51 problem 58

Internal problem ID [13775]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 58.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [quadrature]

$$y' = \frac{\ln(x)}{x}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve([diff(y(x),x)=ln(x)/x,y(1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{\ln(x)^2}{2}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 13

```
DSolve[{y'[x]==Log[x]/x,{y[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{\log^2(x)}{2}$$

## 1.52 problem 72

Internal problem ID [13776]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 72.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$y' - \frac{(x-4)y^3}{x^3(y-2)} = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 75

```
dsolve(diff(y(x),x)=(x-4)*y(x)^3/(x^3*(y(x)-2)),y(x), singsol=all)
```

$$y(x) = -\frac{(x + \sqrt{4c_1x^2 + x^2 - 4x + 8})x}{2(c_1x^2 - x + 2)}$$

$$y(x) = \frac{(-x + \sqrt{4c_1x^2 + x^2 - 4x + 8})x}{2c_1x^2 - 2x + 4}$$

### ✓ Solution by Mathematica

Time used: 0.66 (sec). Leaf size: 91

```
DSolve[y'[x]==(x-4)*y[x]^3/(x^3*(y[x]-2)),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x(-x + \sqrt{x^2 + 4c_1x^2 - 4x + 8})}{2c_1x^2 - 2x + 4}$$

$$y(x) \rightarrow -\frac{x(x + \sqrt{x^2 + 4c_1x^2 - 4x + 8})}{2c_1x^2 - 2x + 4}$$

$$y(x) \rightarrow 0$$

## 1.53 problem 73

Internal problem ID [13777]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 73.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Bernoulli]`

$$y' - \frac{y^2 + 2xy}{x^2} = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=(y(x)^2+2*x*y(x))/x^2,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{-x + c_1}$$

### ✓ Solution by Mathematica

Time used: 0.124 (sec). Leaf size: 23

```
DSolve[y'[x]==(y[x]^2+2*x*y[x])/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{x^2}{x - c_1}$$

$$y(x) \rightarrow 0$$

## 1.54 problem 75

Internal problem ID [13778]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 75.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$y'x + y = \cos(x)$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x)+y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = \frac{\sin(x) + c_1}{x}$$

### ✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 14

```
DSolve[x*y'[x]+y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{\sin(x) + c_1}{x}$$

## 1.55 problem 76

Internal problem ID [13779]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 76.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$16y'' + 24y' + 153y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(16*diff(y(x),x$2)+24*diff(y(x),x)+153*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{3x}{4}} \sin(3x) + c_2 e^{-\frac{3x}{4}} \cos(3x)$$

### ✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 28

```
DSolve[16*y''[x]+24*y'[x]+153*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-3x/4}(c_2 \cos(3x) + c_1 \sin(3x))$$

## 1.56 problem 77

Internal problem ID [13780]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 77.

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$\begin{aligned}x'(t) &= 4y(t) \\ y'(t) &= -x(t) - 2y(t)\end{aligned}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 77

```
dsolve([diff(x(t),t)=4*y(t),diff(y(t),t)=-x(t)-2*y(t)],[x(t), y(t)], singsol=all)
```

$$\begin{aligned}x(t) &= e^{-t} \left( \sqrt{3} \sin(\sqrt{3}t) c_2 - \sqrt{3} \cos(\sqrt{3}t) c_1 - \sin(\sqrt{3}t) c_1 - \cos(\sqrt{3}t) c_2 \right) \\ y &= e^{-t} \left( \sin(\sqrt{3}t) c_1 + \cos(\sqrt{3}t) c_2 \right)\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 93

```
DSolve[{x'[t]==4*y[t],y'[t]==-x[t]-2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions->True]
```

$$\begin{aligned}x(t) &\rightarrow \frac{1}{3}e^{-t} \left( 3c_1 \cos(\sqrt{3}t) + \sqrt{3}(c_1 + 4c_2) \sin(\sqrt{3}t) \right) \\ y(t) &\rightarrow \frac{1}{3}e^{-t} \left( 3c_2 \cos(\sqrt{3}t) - \sqrt{3}(c_1 + c_2) \sin(\sqrt{3}t) \right)\end{aligned}$$

## 1.57 problem 79

Internal problem ID [13781]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 79.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_rational]`

$$4x(y^2 + x^2) - 5y + 4y(x^2 + y^2 - 5x)y' = 0$$

**X** Solution by Maple

```
dsolve((4*x*(x^2+y(x)^2)-5*y(x))+(4*y(x)*(x^2+y(x)^2-5*x))*diff(y(x),x)=0,y(x), singsol=all)
```

No solution found

**X** Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(4*x*(x^2+y[x]^2)-5*y[x])+(4*y[x]*(x^2+y[x]^2-5*x))*y'[x]==0,y[x],x,IncludeSingularSo
```

Not solved

## 1.58 problem 80

Internal problem ID [13782]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 80.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [quadrature]

$$y' = \sin(x)^4$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve([diff(y(x),x)=sin(x)^4,y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{3x}{8} - \frac{\sin(2x)}{4} + \frac{\sin(4x)}{32}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 23

```
DSolve[{y'[x]==Sin[x]^4,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{32}(12x - 8\sin(2x) + \sin(4x))$$



## 1.59 problem 81

Internal problem ID [13783]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 81.

**ODE order:** 4.

**ODE degree:** 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' + \frac{25y''}{2} - 5y' + \frac{629y}{16} = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 1, y''(0) = -1, y'''(0) = 1]$$

### ✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 41

```
dsolve([diff(y(x),x$4)+25/2*diff(y(x),x$2)-5*diff(y(x),x)+629/16*y(x)=0,y(0) = 0, D(y)(0) =
```

$$y(x) = \frac{(74 \cos(3x) + 20 \sin(3x)) e^{-\frac{x}{2}}}{208} - \frac{37 e^{\frac{x}{2}} \left( \cos(2x) - \frac{3 \sin(2x)}{2} \right)}{104}$$

### ✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 47

```
DSolve[{y''''[x]+25/2*y''[x]-5*y'[x]+629/16*y[x]==0,{y[0]==0,y'[0]==1,y''[0]==-1,y'''[0]==1}
```

$$y(x) \rightarrow \frac{1}{208} e^{-x/2} (111 e^x \sin(2x) + 20 \sin(3x) - 74 e^x \cos(2x) + 74 \cos(3x))$$

## 1.60 problem 82

Internal problem ID [13784]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 82.

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$\begin{aligned}x'(t) &= 4y(t) \\ y'(t) &= -4x(t)\end{aligned}$$

With initial conditions

$$[x(0) = 4, y(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve([diff(x(t),t) = 4*y(t), diff(y(t),t) = -4*x(t), x(0) = 4, y(0) = 0],[x(t), y(t)], sin
```

$$\begin{aligned}x(t) &= 4 \cos(4t) \\ y &= -4 \sin(4t)\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[{x'[t]==4*y[t],y'[t]==-4*x[t]},{x[0]==4,y[0]==0},{x[t],y[t]},t,IncludeSingularSolutio
```

$$\begin{aligned}x(t) &\rightarrow 4 \cos(4t) \\ y(t) &\rightarrow -4 \sin(4t)\end{aligned}$$

## 1.61 problem 83

Internal problem ID [13785]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

**Problem number:** 83.

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$\begin{aligned}x'(t) &= -5x(t) + 4y(t) \\y'(t) &= 2x(t) + 2y(t)\end{aligned}$$

With initial conditions

$$[x(0) = 4, y(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
dsolve([diff(x(t),t) = -5*x(t)+4*y(t), diff(y(t),t) = 2*x(t)+2*y(t), x(0) = 4, y(0) = 0], [x(t), y(t)])
```

$$\begin{aligned}x(t) &= \frac{4e^{3t}}{9} + \frac{32e^{-6t}}{9} \\y &= \frac{8e^{3t}}{9} - \frac{8e^{-6t}}{9}\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 40

```
DSolve[{x'[t]==-5*x[t]+4*y[t], y'[t]==2*x[t]+2*y[t]}, {x[0]==4, y[0]==0}, {x[t], y[t]}, t, IncludeS
```

$$\begin{aligned}x(t) &\rightarrow \frac{4}{9}e^{-6t}(e^{9t} + 8) \\y(t) &\rightarrow \frac{8}{9}e^{-6t}(e^{9t} - 1)\end{aligned}$$

## 2 Chapter 1. Introduction to Differential Equations. Review exercises, page 23

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## 2.1 problem 6

Internal problem ID [13786]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Review exercises, page 23

**Problem number:** 6.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$y' + y \cos(x) = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)+y(x)*cos(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\sin(x)}$$

### ✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 19

```
DSolve[y'[x]+y[x]*Cos[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{-\sin(x)}$$

$$y(x) \rightarrow 0$$

## 2.2 problem 7

Internal problem ID [13787]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Review exercises, page 23

**Problem number:** 7.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' - y = \sin(x)$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)-y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = -\frac{\cos(x)}{2} - \frac{\sin(x)}{2} + e^x c_1$$

### ✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 24

```
DSolve[y'[x]-y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{\sin(x)}{2} - \frac{\cos(x)}{2} + c_1 e^x$$

## 2.3 problem 8

Internal problem ID [13788]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Review exercises, page 23

**Problem number:** 8.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 4y' - 5y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)-5*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^x c_1 + c_2 e^{-5x}$$

### ✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 20

```
DSolve[y''[x]+4*y'[x]-5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{-5x} + c_2 e^x$$

## 2.4 problem 9

Internal problem ID [13789]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Review exercises, page 23

**Problem number:** 9.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 6y' + 45y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+45*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{3x} \sin(6x) + c_2 e^{3x} \cos(6x)$$

### ✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 26

```
DSolve[y''[x]-6*y'[x]+45*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{3x}(c_2 \cos(6x) + c_1 \sin(6x))$$



## 2.5 problem 10

Internal problem ID [13790]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Review exercises, page 23

**Problem number:** 10.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^2y'' - y'x - 16y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)-16*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^{1+\sqrt{17}} + c_2x^{1-\sqrt{17}}$$

### ✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 32

```
DSolve[x^2*y'[x]-x*y'[x]-16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^{1-\sqrt{17}}(c_2x^{2\sqrt{17}} + c_1)$$

## 2.6 problem 11

Internal problem ID [13791]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Review exercises, page 23

**Problem number:** 11.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^2 y'' + 3y'x + 2y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(\ln(x))}{x} + \frac{c_2 \cos(\ln(x))}{x}$$

### ✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]+3*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_2 \cos(\log(x)) + c_1 \sin(\log(x))}{x}$$

## 2.7 problem 12

Internal problem ID [13792]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Review exercises, page 23

**Problem number:** 12.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 2y' + 2y = x$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+2*y(x)=x,y(x), singsol=all)
```

$$y(x) = \sin(x) e^{-x} c_2 + \cos(x) e^{-x} c_1 + \frac{x}{2} - \frac{1}{2}$$

### ✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 33

```
DSolve[y''[x]+2*y'[x]+2*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x-1}{2} + c_2 e^{-x} \cos(x) + c_1 e^{-x} \sin(x)$$

## 2.8 problem 13

Internal problem ID [13793]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Review exercises, page 23

**Problem number:** 13.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 7y' + 12y = 2$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-7*diff(y(x),x)+12*y(x)=2,y(x), singsol=all)
```

$$y(x) = e^{3x}c_2 + e^{4x}c_1 + \frac{1}{6}$$

### ✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 25

```
DSolve[y''[x]-7*y'[x]+12*y[x]==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{3x} + c_2 e^{4x} + \frac{1}{6}$$

## 2.9 problem 14

Internal problem ID [13794]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Review exercises, page 23

**Problem number:** 14.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, [_Abel, '2nd ty`

$$-3y + (2y - 3x)y' = -2x$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 53

```
dsolve((2*x-3*y(x))+(2*y(x)-3*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\frac{3c_1x}{2} - \frac{\sqrt{5c_1^2x^2+4}}{2}}{c_1}$$

$$y(x) = \frac{\frac{3c_1x}{2} + \frac{\sqrt{5c_1^2x^2+4}}{2}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.48 (sec). Leaf size: 110

```
DSolve[(2*x-3*y[x])+(2*y[x]-3*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2} \left( 3x - \sqrt{5x^2 + 4e^{c_1}} \right)$$

$$y(x) \rightarrow \frac{1}{2} \left( 3x + \sqrt{5x^2 + 4e^{c_1}} \right)$$

$$y(x) \rightarrow \frac{1}{2} \left( 3x - \sqrt{5}\sqrt{x^2} \right)$$

$$y(x) \rightarrow \frac{1}{2} \left( \sqrt{5}\sqrt{x^2} + 3x \right)$$

## 2.10 problem 15

Internal problem ID [13795]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Review exercises, page 23

**Problem number:** 15.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_exact, [\_1st\_order, ‘\_with\_symmetry\_[F(x),G(x)\*y+H(x)]’]]

$$y \cos(xy) + x \cos(xy) y' = -\sin(x)$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve((y(x)*cos(x*y(x))+sin(x))+(x*cos(x*y(x))))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\arcsin(-\cos(x) + c_1)}{x}$$

### ✓ Solution by Mathematica

Time used: 1.144 (sec). Leaf size: 15

```
DSolve[(y[x]*Cos[x*y[x]]+Sin[x])+(x*Cos[x*y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow \frac{\arcsin(\cos(x) + c_1)}{x}$$

## 2.11 problem 16

Internal problem ID [13796]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Review exercises, page 23

**Problem number:** 16.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [quadrature]

$$y' = x e^{-x^2}$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)=x*exp(-x^2),y(x), singsol=all)
```

$$y(x) = -\frac{e^{-x^2}}{2} + c_1$$

### ✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 19

```
DSolve[y'[x]==x*Exp[-x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{e^{-x^2}}{2} + c_1$$



## 2.12 problem 17

Internal problem ID [13797]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Review exercises, page 23

**Problem number:** 17.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' = x^2 \sin(x)$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x)=x^2*sin(x),y(x), singsol=all)
```

$$y(x) = -\cos(x)x^2 + 2\cos(x) + 2x\sin(x) + c_1$$

### ✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 22

```
DSolve[y'[x]==x^2*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -(x^2 - 2)\cos(x) + 2x\sin(x) + c_1$$

## 2.13 problem 18

Internal problem ID [13798]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Review exercises, page 23

**Problem number:** 18.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [quadrature]

$$y' = \frac{2x^2 - x + 1}{(x - 1)(x^2 + 1)}$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)=(2*x^2-x+1)/((x-1)*(x^2+1)),y(x), singsol=all)
```

$$y(x) = \frac{\ln(x^2 + 1)}{2} + \ln(x - 1) + c_1$$

### ✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 24

```
DSolve[y'[x]==(2*x^2-x+1)/((x-1)*(x^2+1)),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2} \log(x^2 + 1) + \log(1 - x) + c_1$$

## 2.14 problem 19

Internal problem ID [13799]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Review exercises, page 23

**Problem number:** 19.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [quadrature]

$$y' = \frac{x^2}{\sqrt{x^2 - 1}}$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x)=x^2/sqrt(x^2-1),y(x), singsol=all)
```

$$y(x) = \frac{x\sqrt{x^2 - 1}}{2} + \frac{\ln(x + \sqrt{x^2 - 1})}{2} + c_1$$

### ✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 40

```
DSolve[y'[x]==x^2/Sqrt[x^2-1],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2}\sqrt{x^2 - 1}x - \frac{1}{2}\log(\sqrt{x^2 - 1} - x) + c_1$$

## 2.15 problem 20

Internal problem ID [13800]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Review exercises, page 23

**Problem number:** 20.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' + 2y = x^2$$

With initial conditions

$$[y(0) = 1]$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve([diff(y(x),x)+2*y(x)=x^2,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} - \frac{x}{2} + \frac{1}{4} + \frac{3e^{-2x}}{4}$$

### ✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 26

```
DSolve[{y'[x]+2*y[x]==x^2,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{4}(2x^2 - 2x + 3e^{-2x} + 1)$$

## 2.16 problem 21

Internal problem ID [13801]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Review exercises, page 23

**Problem number:** 21.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 4y = t$$

With initial conditions

$$\left[ y(0) = 1, y'\left(\frac{\pi}{4}\right) = \frac{\pi}{16} \right]$$

**X** Solution by Maple

```
dsolve([diff(y(t),t$2)+4*y(t)=t,y(0) = 1, D(y)(1/4*Pi) = 1/16*Pi],y(t), singsol=all)
```

No solution found

**X** Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y''[t]+4*y[t]==t,{y[0]==1,y'[Pi/4]==Pi/16}},y[t],t,IncludeSingularSolutions -> True]
```

{}

## 2.17 problem 22

Internal problem ID [13802]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Review exercises, page 23

**Problem number:** 22.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^2 y'' + 5y'x + 4y = 0$$

With initial conditions

$$[y(1) = 1, y'(1) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve([x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+4*y(x)=0,y(1) = 1, D(y)(1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{1 + 2 \ln(x)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 15

```
DSolve[{x^2*y''[x]+5*x*y'[x]+4*y[x]==0,{y[1]==1,y'[1]==0}},y[x],x,IncludeSingularSolutions->True]
```

$$y(x) \rightarrow \frac{2 \log(x) + 1}{x^2}$$

## 2.18 problem 23

Internal problem ID [13803]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Review exercises, page 23

**Problem number:** 23.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [`_quadrature`]

$$y' = \cos(x)^2 \sin(x)$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)=cos(x)^2*sin(x),y(0) = 0],y(x), singsol=all)
```

$$y(x) = -\frac{\cos(x)^3}{3} + \frac{1}{3}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 17

```
DSolve[{y'[x]==Cos[x]^2*Sin[x],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{3}(1 - \cos^3(x))$$

## 2.19 problem 24

Internal problem ID [13804]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 1. Introduction to Differential Equations. Review exercises, page 23

**Problem number:** 24.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [`_quadrature`]

$$y' = \frac{4x - 9}{3(x - 3)^{\frac{2}{3}}}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 11

```
dsolve([diff(y(x),x)=1/3*(4*x-9)*(x-3)^(-2/3),y(0) = 0],y(x), singsol=all)
```

$$y(x) = (x - 3)^{\frac{1}{3}} x$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 14

```
DSolve[{y'[x]==1/3*(4*x-9)*(x-3)^(-2/3),{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sqrt[3]{x - 3} x$$



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### 3.1 problem 1 (a)

Internal problem ID [13805]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 1 (a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [Riccati]

$$y' - y^2 = -t^2$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.204 (sec). Leaf size: 59

```
dsolve([diff(y(t),t)+t^2=y(t)^2,y(0) = 0],y(t), singsol=all)
```

$$y = - \left( \begin{cases} 0 & t = 0 \\ \frac{\left(\text{BesselI}\left(-\frac{3}{4}, \frac{t^2}{2}\right)\pi\sqrt{2} - 2\text{BesselK}\left(\frac{3}{4}, \frac{t^2}{2}\right)\right)t}{\pi\sqrt{2}\text{BesselI}\left(\frac{1}{4}, \frac{t^2}{2}\right) + 2\text{BesselK}\left(\frac{1}{4}, \frac{t^2}{2}\right)} & \text{otherwise} \end{cases} \right)$$

✓ Solution by Mathematica

Time used: 0.663 (sec). Leaf size: 81

```
DSolve[{y'[t]+t^2==y[t]^2,{y[0]==0}],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow - \frac{it^2 \text{BesselJ}\left(-\frac{5}{4}, \frac{it^2}{2}\right) - it^2 \text{BesselJ}\left(\frac{3}{4}, \frac{it^2}{2}\right) + \text{BesselJ}\left(-\frac{1}{4}, \frac{it^2}{2}\right)}{2t \text{BesselJ}\left(-\frac{1}{4}, \frac{it^2}{2}\right)}$$

### 3.2 problem 1 (b)

Internal problem ID [13806]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 1 (b).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{1}{y^2} = -t^2$$

With initial conditions

$$[y(0) = 0]$$

**X** Solution by Maple

```
dsolve([diff(y(t),t)+t^2=1/y(t)^2,y(0) = 0],y(t), singsol=all)
```

No solution found

**X** Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'[t]+t^2==1/y[t]^2,{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

Not solved

### 3.3 problem 1 (c)

Internal problem ID [13807]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 1 (c).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$y' - y = \frac{1}{1-t}$$

With initial conditions

$$[y(1) = 0]$$

**X** Solution by Maple

```
dsolve([diff(y(t),t)=y(t)+1/(1-t),y(1) = 0],y(t), singsol=all)
```

No solution found

**X** Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'[t]==y[t]+1/(1-t)},{y[1]==0}],y[t],t,IncludeSingularSolutions -> True]
```

```
{}
```

### 3.4 problem 3

Internal problem ID [13808]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 3.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [quadrature]

$$y' - y^{\frac{1}{5}} = 0$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(t),t)=y(t)^(1/5),y(0) = 0],y(t), singsol=all)
```

$$y = 0$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 24

```
DSolve[{y'[t]==y[t]^(1/5)},{y[0]==0}],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{4\sqrt{2}t^{5/4}}{5\sqrt[4]{5}}$$

### 3.5 problem 4

Internal problem ID [13809]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 4.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$\frac{y'}{t} - \sqrt{y} = 0$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 5

```
dsolve([1/t*diff(y(t),t)=sqrt(y(t)),y(0) = 0],y(t), singsol=all)
```

$$y = 0$$

✓ Solution by Mathematica

Time used: 0.111 (sec). Leaf size: 17

```
DSolve[{1/t*y'[t]==Sqrt[y[t]},{y[0]==0}],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow 0$$

$$y(t) \rightarrow \frac{t^4}{16}$$

### 3.6 problem 6

Internal problem ID [13810]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 6.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [Riccati]

$$y' + ty^2 = 4t^2$$

With initial conditions

$$[y(2) = 1]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 127

```
dsolve([diff(y(t),t)=4*t^2-t*y(t)^2,y(2) = 1],y(t), singsol=all)
```

$$y = \frac{2 \left( \left( 2\sqrt{2} \operatorname{BesselK} \left( \frac{3}{5}, \frac{16\sqrt{2}}{5} \right) + \operatorname{BesselK} \left( \frac{2}{5}, \frac{16\sqrt{2}}{5} \right) \right) \operatorname{BesselI} \left( -\frac{3}{5}, \frac{4t^{\frac{5}{2}}}{5} \right) + \operatorname{BesselK} \left( \frac{3}{5}, \frac{4t^{\frac{5}{2}}}{5} \right) \left( -2\sqrt{2} \operatorname{BesselI} \left( \frac{2}{5}, \frac{4t^{\frac{5}{2}}}{5} \right) + \operatorname{BesselK} \left( \frac{2}{5}, \frac{16\sqrt{2}}{5} \right) \right) \right)}{\left( -2\sqrt{2} \operatorname{BesselK} \left( \frac{3}{5}, \frac{16\sqrt{2}}{5} \right) - \operatorname{BesselK} \left( \frac{2}{5}, \frac{16\sqrt{2}}{5} \right) \right) \operatorname{BesselI} \left( \frac{2}{5}, \frac{4t^{\frac{5}{2}}}{5} \right) + \operatorname{BesselK} \left( \frac{2}{5}, \frac{4t^{\frac{5}{2}}}{5} \right) \left( -2\sqrt{2} \operatorname{BesselI} \left( \frac{3}{5}, \frac{4t^{\frac{5}{2}}}{5} \right) + \operatorname{BesselK} \left( \frac{3}{5}, \frac{16\sqrt{2}}{5} \right) \right)}$$

✓ Solution by Mathematica

Time used: 0.268 (sec). Leaf size: 709

```
DSolve[{y'[t]==4*t^2-t*y[t]^2,{y[2]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{-3t^{5/2} \operatorname{BesselI} \left( \frac{2}{5}, \frac{16\sqrt{2}}{5} \right) \operatorname{BesselI} \left( \frac{3}{5}, \frac{4t^{5/2}}{5} \right) + 4\sqrt{2}t^{5/2} \operatorname{BesselI} \left( -\frac{3}{5}, \frac{16\sqrt{2}}{5} \right) \operatorname{BesselI} \left( \frac{3}{5}, \frac{4t^{5/2}}{5} \right) - 4\sqrt{2}t^{5/2} \operatorname{BesselI} \left( \frac{2}{5}, \frac{4t^{5/2}}{5} \right) \operatorname{BesselI} \left( \frac{3}{5}, \frac{4t^{5/2}}{5} \right) + \left( -2\sqrt{2} \operatorname{BesselK} \left( \frac{3}{5}, \frac{16\sqrt{2}}{5} \right) - \operatorname{BesselK} \left( \frac{2}{5}, \frac{16\sqrt{2}}{5} \right) \right) \operatorname{BesselI} \left( \frac{2}{5}, \frac{4t^{5/2}}{5} \right) + \operatorname{BesselK} \left( \frac{2}{5}, \frac{4t^{5/2}}{5} \right) \left( -2\sqrt{2} \operatorname{BesselI} \left( \frac{3}{5}, \frac{4t^{5/2}}{5} \right) + \operatorname{BesselK} \left( \frac{3}{5}, \frac{16\sqrt{2}}{5} \right) \right)}{\left( -2\sqrt{2} \operatorname{BesselK} \left( \frac{3}{5}, \frac{16\sqrt{2}}{5} \right) - \operatorname{BesselK} \left( \frac{2}{5}, \frac{16\sqrt{2}}{5} \right) \right) \operatorname{BesselI} \left( \frac{2}{5}, \frac{4t^{5/2}}{5} \right) + \operatorname{BesselK} \left( \frac{2}{5}, \frac{4t^{5/2}}{5} \right) \left( -2\sqrt{2} \operatorname{BesselI} \left( \frac{3}{5}, \frac{4t^{5/2}}{5} \right) + \operatorname{BesselK} \left( \frac{3}{5}, \frac{16\sqrt{2}}{5} \right) \right)}$$

### 3.7 problem 7

Internal problem ID [13811]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 7.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$y' - y\sqrt{t} = 0$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve([diff(y(t),t)=y(t)*sqrt(t),y(1) = 1],y(t), singsol=all)
```

$$y = e^{\frac{2(\sqrt{t}-1)(t+\sqrt{t}+1)}{3}}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 18

```
DSolve[{y'[t]==y[t]*Sqrt[t],{y[1]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow e^{\frac{2}{3}(t^{3/2}-1)}$$



### 3.8 problem 8

Internal problem ID [13812]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 8.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' - 6y^{\frac{2}{3}} = 0$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(t),t)=6*y(t)^(2/3),y(1) = 0],y(t), singsol=all)
```

$$y = 0$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y'[t]==6*y[t]^(2/3)},{y[1]==0}],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow 0$$

### 3.9 problem 9

Internal problem ID [13813]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 9.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [ $y' = G(x, y')$ ]

$$y' - \sin(y) = -\cos(t)$$

With initial conditions

$$[y(\pi) = 0]$$

**X** Solution by Maple

```
dsolve([diff(y(t),t)=sin(y(t))-cos(t),y(Pi) = 0],y(t), singsol=all)
```

No solution found

**X** Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'[t]==Sin[y[t]]-Cos[t],{y[Pi]==0}},y[t],t,IncludeSingularSolutions -> True]
```

Not solved

### 3.10 problem 10

Internal problem ID [13814]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 10.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$y't - y = 0$$

With initial conditions

$$[y(0) = 1]$$

**X** Solution by Maple

```
dsolve([t*diff(y(t),t)=y(t),y(0) = 1],y(t), singsol=all)
```

No solution found

**X** Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{t*y'[t]==y[t],{y[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

{}

### 3.11 problem 11

Internal problem ID [13815]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 11.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$y' - y \tan(t) = 0$$

With initial conditions

$$[y(0) = 1]$$

#### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 6

```
dsolve([diff(y(t),t)=y(t)*tan(t),y(0) = 1],y(t), singsol=all)
```

$$y = \sec(t)$$

#### ✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 7

```
DSolve[{y'[t]==y[t]*Tan[t],{y[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \sec(t)$$

### 3.12 problem 12

Internal problem ID [13816]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 12.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' = \frac{1}{t^2 + 1}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 6

```
dsolve([diff(y(t),t)=1/(1+t^2),y(0) = 0],y(t), singsol=all)
```

$$y = \arctan(t)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 7

```
DSolve[{y'[t]==1/(1+t^2)},{y[0]==0}],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \arctan(t)$$

### 3.13 problem 13 (a)

Internal problem ID [13817]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 13 (a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [`_quadrature`]

$$y' - \sqrt{y^2 - 1} = 0$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 27

```
dsolve([diff(y(t),t)=sqrt(y(t)^2-1),y(0) = 2],y(t), singsol=all)
```

$$y = \frac{e^t \sqrt{3}}{2} + e^t - \frac{\sqrt{3} e^{-t}}{2} + e^{-t}$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 49

```
DSolve[{y'[t]==Sqrt[y[t]^2-1},{y[0]==2}],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{2} e^{-t} \sqrt{2e^{2t} + (7 + 4\sqrt{3}) e^{4t} + 7 - 4\sqrt{3}}$$

### 3.14 problem 13 (b)

Internal problem ID [13818]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 13 (b).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' - \sqrt{y^2 - 1} = 0$$

With initial conditions

$$[y(4) = -1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(t),t)=sqrt(y(t)^2-1),y(4) = -1],y(t), singsol=all)
```

$$y = -1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 6

```
DSolve[{y'[t]==Sqrt[y[t]^2-1},{y[4]==-1}],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow -1$$

### 3.15 problem 13 (c)

Internal problem ID [13819]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 13 (c).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' - \sqrt{y^2 - 1} = 0$$

With initial conditions

$$\left[ y(0) = \frac{1}{2} \right]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 26

```
dsolve([diff(y(t),t)=sqrt(y(t)^2-1),y(0) = 1/2],y(t), singsol=all)
```

$$y = \frac{i\sqrt{3}e^t}{4} - \frac{i\sqrt{3}e^{-t}}{4} + \frac{e^t}{4} + \frac{e^{-t}}{4}$$

✓ Solution by Mathematica

Time used: 0.076 (sec). Leaf size: 59

```
DSolve[{y'[t]==Sqrt[y[t]^2-1},{y[0]==1/2}],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{e^{-t} \sqrt{4e^{2t} + i(\sqrt{3} + i)e^{4t} - 1 - i\sqrt{3}}}{2\sqrt{2}}$$



### 3.16 problem 13 (d)

Internal problem ID [13820]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 13 (d).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' - \sqrt{y^2 - 1} = 0$$

With initial conditions

$$[y(2) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(t),t)=sqrt(y(t)^2-1),y(2) = 1],y(t), singsol=all)
```

$$y = 1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 6

```
DSolve[{y'[t]==Sqrt[y[t]^2-1},{y[2]==1}],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow 1$$

### 3.17 problem 14 (a)

Internal problem ID [13821]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 14 (a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [quadrature]

$$y' - \sqrt{25 - y^2} = 0$$

With initial conditions

$$[y(-4) = 3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(t),t)=sqrt(25-y(t)^2),y(-4) = 3],y(t), singsol=all)
```

$$y = 5 \sin \left( t + 4 + \arcsin \left( \frac{3}{5} \right) \right)$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 17

```
DSolve[{y'[t]==Sqrt[25-y[t]^2],{y[-4]==3}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow 5 \cos \left( -2 \arctan \left( \frac{1}{2} \right) + t + 4 \right)$$

### 3.18 problem 14 (b)

Internal problem ID [13822]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 14 (b).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' - \sqrt{25 - y^2} = 0$$

With initial conditions

$$[y(0) = 5]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(t),t)=sqrt(25-y(t)^2),y(0) = 5],y(t), singsol=all)
```

$$y = 5$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y'[t]==Sqrt[25-y[t]^2],{y[0]==5}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow 5$$

### 3.19 problem 14 (c)

Internal problem ID [13823]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 14 (c).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' - \sqrt{25 - y^2} = 0$$

With initial conditions

$$[y(3) = -6]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 14

```
dsolve([diff(y(t),t)=sqrt(25-y(t)^2),y(3) = -6],y(t), singsol=all)
```

$$y = 5 \sin \left( t - 3 - \arcsin \left( \frac{6}{5} \right) \right)$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 23

```
DSolve[{y'[t]==Sqrt[25-y[t]^2},{y[3]==-6}],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow 5 \cos \left( -2i \operatorname{arctanh}(\sqrt{11}) - t + 3 \right)$$

### 3.20 problem 14 (c)

Internal problem ID [13824]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 14 (c).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_quadrature]`

$$y' - \sqrt{25 - y^2} = 0$$

With initial conditions

$$[y(4) = -5]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(t),t)=sqrt(25-y(t)^2),y(4) = -5],y(t), singsol=all)
```

$$y = -5$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y'[t]==Sqrt[25-y[t]^2},{y[4]==-5}],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow -5$$

### 3.21 problem 15

Internal problem ID [13825]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 15.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$y't + y = t^3$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve([t*diff(y(t),t)+y(t)=t^3,y(1) = 0],y(t), singsol=all)
```

$$y = \frac{t^4 - 1}{4t}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 17

```
DSolve[{t*y'[t]+y[t]==t^3,{y[1]==0}],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{t^4 - 1}{4t}$$

## 3.22 problem 16

Internal problem ID [13826]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 16.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$y't^3 + yt^4 = 2t^3$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve([t^3*diff(y(t),t)+t^4*y(t)=2*t^3,y(0) = 0],y(t), singsol=all)
```

$$y = -ie^{-\frac{t^2}{2}} \sqrt{\pi} \sqrt{2} \operatorname{erf}\left(\frac{i\sqrt{2}t}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 30

```
DSolve[{t^3*y'[t]+t^4*y[t]==2*t^3,{y[0]==0}],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \sqrt{2\pi} e^{-\frac{t^2}{2}} \operatorname{erfi}\left(\frac{t}{\sqrt{2}}\right)$$

### 3.23 problem 17

Internal problem ID [13827]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 17.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$2y' + ty = \ln(t)$$

With initial conditions

$$[y(e) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 28

```
dsolve([2*diff(y(t),t)+t*y(t)=ln(t),y(exp(1)) = 0],y(t), singsol=all)
```

$$y = \frac{\left(\int_e^t \ln(-z1) e^{-\frac{z1^2}{4}} d_{-z1}\right) e^{-\frac{t^2}{4}}}{2}$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 95

```
DSolve[{2*y'[t]+t*y[t]==Log[t],{y[Exp[1]]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{2} e^{-\frac{t^2}{4}} \left( -t {}_2F_2\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; \frac{t^2}{4}\right) + e {}_2F_2\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; \frac{e^2}{4}\right) + \sqrt{\pi} \operatorname{erfi}\left(\frac{t}{2}\right) \log(t) - \sqrt{\pi} \operatorname{erfi}\left(\frac{e}{2}\right) \right)$$



### 3.24 problem 18

Internal problem ID [13828]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 18.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$y' + y \sec(t) = t$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 59

```
dsolve([diff(y(t),t)+y(t)*sec(t)=t,y(0) = 0],y(t), singsol=all)
```

$$y = \frac{i\pi^2 + 12it^2 - 48t \ln(1 + ie^{it}) + 48i \operatorname{polylog}(2, -ie^{it}) - 48 \operatorname{Catalan}}{24 \sec(t) + 24 \tan(t)}$$

✓ Solution by Mathematica

Time used: 0.209 (sec). Leaf size: 146

```
DSolve[{y'[t]+y[t]*Sec[t]==t,{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{24} e^{-2 \operatorname{arctanh}(\tan(\frac{t}{2}))} \left( -48C + 48i \operatorname{PolyLog}(2, -ie^{it}) + 12it^2 - 36i\pi t \right. \\ \left. - 48t \log(1 + ie^{it}) - 48\pi \log(1 + e^{-it}) + 24\pi \log(1 + ie^{it}) + 48\pi \log\left(\cos\left(\frac{t}{2}\right)\right) \right. \\ \left. - 24\pi \log\left(-\cos\left(\frac{1}{4}(2t + \pi)\right)\right) \right) + 25i\pi^2 + 36\pi \log(2) - 24\pi \log(1 + i)$$

### 3.25 problem 19

Internal problem ID [13829]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 19.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$y' + \frac{y}{-3+t} = \frac{1}{t-1}$$

With initial conditions

$$[y(-1) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 27

```
dsolve([diff(y(t),t)+1/(t-3)*y(t)=1/(t-1),y(-1) = 0],y(t), singsol=all)
```

$$y = \frac{t - 2 \ln(t - 1) + 1 + 2 \ln(2) + 2i\pi}{t - 3}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 27

```
DSolve[{y'[t]+1/(t-3)*y[t]==1/(t-1),{y[-1]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{t - 2 \log(t - 1) + 2i\pi + 1 + \log(4)}{t - 3}$$

### 3.26 problem 20

Internal problem ID [13830]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 20.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$(t-2)y' + (t^2-4)y = \frac{1}{t+2}$$

With initial conditions

$$[y(0) = 3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve([(t-2)*diff(y(t),t)+(t^2-4)*y(t)=1/(t+2),y(0) = 3],y(t), singsol=all)
```

$$y = \left( \int_0^t \frac{e^{\frac{z1(-z1+4)}{2}}}{-z1^2 - 4} d_{-}z1 + 3 \right) e^{-\frac{t(t+4)}{2}}$$

✓ Solution by Mathematica

Time used: 0.435 (sec). Leaf size: 46

```
DSolve[{(t-2)*y'[t]+(t^2-4)*y[t]==1/(t+2),{y[0]==3}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \rightarrow e^{-\frac{1}{2}t(t+4)} \left( \int_0^t \frac{e^{\frac{1}{2}K[1](K[1]+4)}}{K[1]^2 - 4} dK[1] + 3 \right)$$

### 3.27 problem 21

Internal problem ID [13831]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 21.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$y' + \frac{y}{\sqrt{-t^2 + 4}} = t$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve([diff(y(t),t)+y(t)/sqrt(4-t^2)=t,y(0) = 0],y(t), singsol=all)
```

$$y = \left( \int_0^t z1 e^{\arcsin\left(\frac{-z1}{2}\right)} d_z1 \right) e^{-\arcsin\left(\frac{t}{2}\right)}$$

✓ Solution by Mathematica

Time used: 0.159 (sec). Leaf size: 62

```
DSolve[{y'[t]+y[t]/Sqrt[4-t^2]==t,{y[0]==0}],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow e^{2 \arctan\left(\frac{\sqrt{4-t^2}}{t+2}\right)} \int_0^t e^{-2 \arctan\left(\frac{\sqrt{4-K[1]^2}}{K[1]+2}\right)} K[1] dK[1]$$

### 3.28 problem 22

Internal problem ID [13832]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 22.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$y' + \frac{y}{\sqrt{-t^2 + 4}} = t$$

With initial conditions

$$[y(3) = -1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
dsolve([diff(y(t),t)+y(t)/sqrt(4-t^2)=t,y(3) = -1],y(t), singsol=all)
```

$$y = \left( \int_3^t -z1 e^{\arcsin\left(\frac{-z1}{2}\right)} d_z1 - e^{\arcsin\left(\frac{3}{2}\right)} \right) e^{-\arcsin\left(\frac{t}{2}\right)}$$

✓ Solution by Mathematica

Time used: 0.085 (sec). Leaf size: 88

```
DSolve[{y'[t]+y[t]/Sqrt[4-t^2]==t,{y[3]==-1}],y[t],t,IncludeSingularSolutions -> True]
```

$y(t)$

$$\rightarrow e^{2 \arctan\left(\frac{\sqrt{4-t^2}}{t+2}\right) - 2i \operatorname{arctanh}\left(\frac{1}{\sqrt{5}}\right)} \left( -1 + e^{2i \operatorname{arctanh}\left(\frac{1}{\sqrt{5}}\right)} \int_3^t e^{-2 \arctan\left(\frac{\sqrt{4-K[1]^2}}{K[1]+2}\right)} K[1] dK[1] \right)$$

### 3.29 problem 23

Internal problem ID [13833]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 23.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$y't + y = t \sin(t)$$

With initial conditions

$$[y(\pi) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve([t*diff(y(t),t)+y(t)=t*sin(t),y(Pi) = 1],y(t), singsol=all)
```

$$y = \frac{\sin(t) - \cos(t)t}{t}$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 16

```
DSolve[{t*y'[t]+y[t]==t*Sin[t],{y[Pi]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{\sin(t)}{t} - \cos(t)$$

### 3.30 problem 24

Internal problem ID [13834]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 24.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$y' + y \tan(t) = \sin(t)$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 11

```
dsolve([diff(y(t),t)+y(t)*tan(t)=sin(t),y(0) = 0],y(t), singsol=all)
```

$$y = -\cos(t) \ln(\cos(t))$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 12

```
DSolve[{y'[t]+y[t]*Tan[t]==Sin[t],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow -\cos(t) \log(\cos(t))$$

### 3.31 problem 25

Internal problem ID [13835]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 25.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' - y^2 = 0$$

With initial conditions

$$\left[ y(0) = \frac{1}{2} \right]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve([diff(y(t),t)=y(t)^2,y(0) = 1/2],y(t), singsol=all)
```

$$y = -\frac{1}{t-2}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 12

```
DSolve[{y'[t]==y[t]^2,{y[0]==1/2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{2-t}$$



### 3.32 problem 26

Internal problem ID [13836]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 26.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$y' - ty^2 = 0$$

With initial conditions

$$\left[ y(0) = \frac{1}{2} \right]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 13

```
dsolve([diff(y(t),t)=t*y(t)^2,y(0) = 1/2],y(t), singsol=all)
```

$$y = -\frac{2}{t^2 - 4}$$

✓ Solution by Mathematica

Time used: 0.101 (sec). Leaf size: 14

```
DSolve[{y'[t]==t*y[t]^2,{y[0]==1/2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow -\frac{2}{t^2 - 4}$$

### 3.33 problem 27

Internal problem ID [13837]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 27.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$y' + \frac{t}{y} = 0$$

With initial conditions

$$\left[ y(0) = \frac{1}{2} \right]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(t),t)=-t/y(t),y(0) = 1/2],y(t), singsol=all)
```

$$y = \frac{\sqrt{-4t^2 + 1}}{2}$$

✓ Solution by Mathematica

Time used: 0.084 (sec). Leaf size: 20

```
DSolve[{y'[t]==-t/y[t],{y[0]==1/2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{2}\sqrt{1 - 4t^2}$$

### 3.34 problem 28

Internal problem ID [13838]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.1, page 32

**Problem number:** 28.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [quadrature]

$$y' + y^3 = 0$$

With initial conditions

$$\left[ y(0) = \frac{1}{2} \right]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve([diff(y(t),t)=-y(t)^3,y(0) = 1/2],y(t), singsol=all)
```

$$y = \frac{1}{\sqrt{2t + 4}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[{y'[t]==-y[t]^3,{y[0]==1/2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{i}{\sqrt{-2t - 4}}$$

## 4 Chapter 2. First Order Equations. Exercises 2.2, page 39

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## 4.1 problem 1

Internal problem ID [13839]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 1.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$y' - \frac{x}{y^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)=x/y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{(12x^2 + 8c_1)^{\frac{1}{3}}}{2}$$

$$y(x) = -\frac{(12x^2 + 8c_1)^{\frac{1}{3}}}{4} - \frac{i\sqrt{3}(12x^2 + 8c_1)^{\frac{1}{3}}}{4}$$

$$y(x) = -\frac{(12x^2 + 8c_1)^{\frac{1}{3}}}{4} + \frac{i\sqrt{3}(12x^2 + 8c_1)^{\frac{1}{3}}}{4}$$

✓ Solution by Mathematica

Time used: 0.179 (sec). Leaf size: 79

```
DSolve[y'[x]==x/y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\sqrt[3]{-\frac{3}{2}\sqrt[3]{x^2 + 2c_1}}$$

$$y(x) \rightarrow \sqrt[3]{\frac{3}{2}\sqrt[3]{x^2 + 2c_1}}$$

$$y(x) \rightarrow (-1)^{2/3}\sqrt[3]{\frac{3}{2}\sqrt[3]{x^2 + 2c_1}}$$

## 4.2 problem 2

Internal problem ID [13840]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 2.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$y^2 y' = -\frac{1}{2\sqrt{t}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 73

```
dsolve(1/2/t^(1/2)+y(t)^2*diff(y(t),t)=0,y(t), singsol=all)
```

$$y = \left(-3\sqrt{t} + c_1\right)^{\frac{1}{3}}$$

$$y = -\frac{\left(-3\sqrt{t} + c_1\right)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}\left(-3\sqrt{t} + c_1\right)^{\frac{1}{3}}}{2}$$

$$y = -\frac{\left(-3\sqrt{t} + c_1\right)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}\left(-3\sqrt{t} + c_1\right)^{\frac{1}{3}}}{2}$$

✓ Solution by Mathematica

Time used: 1.899 (sec). Leaf size: 79

```
DSolve[1/2/t^(1/2)+y[t]^2*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow -\sqrt[3]{-3} \sqrt[3]{-\sqrt{t} + c_1}$$

$$y(t) \rightarrow \sqrt[3]{3} \sqrt[3]{-\sqrt{t} + c_1}$$

$$y(t) \rightarrow (-1)^{2/3} \sqrt[3]{3} \sqrt[3]{-\sqrt{t} + c_1}$$



### 4.3 problem 3

Internal problem ID [13841]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 3.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$y' - \frac{\sqrt{y}}{x^2} = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=sqrt(y(x))/x^2,y(x), singsol=all)
```

$$\sqrt{y(x)} + \frac{1}{2x} - c_1 = 0$$

#### ✓ Solution by Mathematica

Time used: 0.125 (sec). Leaf size: 25

```
DSolve[y'[x]==Sqrt[y[x]]/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{(-1 + c_1 x)^2}{4x^2}$$

$$y(x) \rightarrow 0$$

## 4.4 problem 4

Internal problem ID [13842]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 4.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' - \frac{1 + y^2}{y} = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(t),t)=(1+y(t)^2)/y(t),y(t), singsol=all)
```

$$y = \sqrt{e^{2t}c_1 - 1}$$
$$y = -\sqrt{e^{2t}c_1 - 1}$$

### ✓ Solution by Mathematica

Time used: 2.441 (sec). Leaf size: 53

```
DSolve[y'[t]==(1+y[t]^2)/y[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow -\sqrt{-1 + e^{2(t+c_1)}}$$
$$y(t) \rightarrow \sqrt{-1 + e^{2(t+c_1)}}$$
$$y(t) \rightarrow -i$$
$$y(t) \rightarrow i$$

## 4.5 problem 5

Internal problem ID [13843]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 5.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$\left(5 + \frac{9}{y^8}\right) y' = -4t^3 - 6$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve((6+4*t^3)+(5+9/y(t)^8)*diff(y(t),t)=0,y(t), singsol=all)
```

$$\frac{t^4}{2} + 3t + \frac{5y}{2} - \frac{9}{14y^7} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 3.593 (sec). Leaf size: 265

```
DSolve[(6+4*t^3)+(5+9/y[t]^8)*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \text{Root}[35\#1^8 + \#1^7(7t^4 + 42t - 7c_1) - 9\&, 1]$$

$$y(t) \rightarrow \text{Root}[35\#1^8 + \#1^7(7t^4 + 42t - 7c_1) - 9\&, 2]$$

$$y(t) \rightarrow \text{Root}[35\#1^8 + \#1^7(7t^4 + 42t - 7c_1) - 9\&, 3]$$

$$y(t) \rightarrow \text{Root}[35\#1^8 + \#1^7(7t^4 + 42t - 7c_1) - 9\&, 4]$$

$$y(t) \rightarrow \text{Root}[35\#1^8 + \#1^7(7t^4 + 42t - 7c_1) - 9\&, 5]$$

$$y(t) \rightarrow \text{Root}[35\#1^8 + \#1^7(7t^4 + 42t - 7c_1) - 9\&, 6]$$

$$y(t) \rightarrow \text{Root}[35\#1^8 + \#1^7(7t^4 + 42t - 7c_1) - 9\&, 7]$$

$$y(t) \rightarrow \text{Root}[35\#1^8 + \#1^7(7t^4 + 42t - 7c_1) - 9\&, 8]$$

## 4.6 problem 6

Internal problem ID [13844]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 6.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$\left(9 + \frac{1}{s^2} - 4s^8\right) s' = -\frac{6}{t^9} + \frac{6}{t^3} - t^7$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve((6/t^9-6/t^3+t^7)+(9+1/s(t)^2-4*s(t)^8)*diff(s(t),t)=0,s(t), singsol=all)
```

$$\frac{t^8}{8} - \frac{3}{4t^8} + \frac{3}{t^2} - \frac{4s(t)^9}{9} + 9s(t) - \frac{1}{s(t)} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 17.496 (sec). Leaf size: 531

```
DSolve[(6/t^9-6/t^3+t^7)+(9+1/s[t]^2-4*s[t]^8)*s'[t]==0,s[t],t,IncludeSingularSolutions -> T
```

$$\begin{aligned} s(t) &\rightarrow \text{Root}[32\#1^{10}t^8 - 648\#1^2t^8 + \#1(-9t^{16} - 72c_1t^8 - 216t^6 + 54) + 72t^8\&, 1] \\ s(t) &\rightarrow \text{Root}[32\#1^{10}t^8 - 648\#1^2t^8 + \#1(-9t^{16} - 72c_1t^8 - 216t^6 + 54) + 72t^8\&, 2] \\ s(t) &\rightarrow \text{Root}[32\#1^{10}t^8 - 648\#1^2t^8 + \#1(-9t^{16} - 72c_1t^8 - 216t^6 + 54) + 72t^8\&, 3] \\ s(t) &\rightarrow \text{Root}[32\#1^{10}t^8 - 648\#1^2t^8 + \#1(-9t^{16} - 72c_1t^8 - 216t^6 + 54) + 72t^8\&, 4] \\ s(t) &\rightarrow \text{Root}[32\#1^{10}t^8 - 648\#1^2t^8 + \#1(-9t^{16} - 72c_1t^8 - 216t^6 + 54) + 72t^8\&, 5] \\ s(t) &\rightarrow \text{Root}[32\#1^{10}t^8 - 648\#1^2t^8 + \#1(-9t^{16} - 72c_1t^8 - 216t^6 + 54) + 72t^8\&, 6] \\ s(t) &\rightarrow \text{Root}[32\#1^{10}t^8 - 648\#1^2t^8 + \#1(-9t^{16} - 72c_1t^8 - 216t^6 + 54) + 72t^8\&, 7] \\ s(t) &\rightarrow \text{Root}[32\#1^{10}t^8 - 648\#1^2t^8 + \#1(-9t^{16} - 72c_1t^8 - 216t^6 + 54) + 72t^8\&, 8] \\ s(t) &\rightarrow \text{Root}[32\#1^{10}t^8 - 648\#1^2t^8 + \#1(-9t^{16} - 72c_1t^8 - 216t^6 + 54) + 72t^8\&, 9] \\ s(t) &\rightarrow \text{Root}[32\#1^{10}t^8 - 648\#1^2t^8 + \#1(-9t^{16} - 72c_1t^8 - 216t^6 + 54) + 72t^8\&, 10] \end{aligned}$$

## 4.7 problem 7

Internal problem ID [13845]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 7.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$4 \sinh(4y) y' = 6 \cosh(3x)$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(4*sinh(4*y(x))*diff(y(x),x)=6*cosh(3*x),y(x), singsol=all)
```

$$y(x) = \frac{\operatorname{arccosh}(2 \sinh(3x) + 6c_1)}{4}$$

### ✓ Solution by Mathematica

Time used: 2.75 (sec). Leaf size: 41

```
DSolve[4*Sinh[4*y[x]]*y'[x]==6*Cosh[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{1}{4} \operatorname{arccosh}(2(\sinh(3x) + 2c_1))$$

$$y(x) \rightarrow \frac{1}{4} \operatorname{arccosh}(2(\sinh(3x) + 2c_1))$$

## 4.8 problem 8

Internal problem ID [13846]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 8.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$y' - \frac{y+1}{t+1} = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(t),t)=(y(t)+1)/(t+1),y(t), singsol=all)
```

$$y = -1 + (t + 1) c_1$$

### ✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 18

```
DSolve[y'[t]==(y[t]+1)/(t+1),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow -1 + c_1(t + 1)$$

$$y(t) \rightarrow -1$$



## 4.9 problem 9

Internal problem ID [13847]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 9.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$y' - \frac{y+2}{2t+1} = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(t),t)=(y(t)+2)/(2*t+1),y(t), singsol=all)
```

$$y = \sqrt{2t+1} c_1 - 2$$

### ✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 24

```
DSolve[y'[t]==(y[t]+2)/(2*t+1),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow -2 + c_1 \sqrt{2t+1}$$

$$y(t) \rightarrow -2$$

## 4.10 problem 10

Internal problem ID [13848]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 10.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$-\left(\frac{1}{\sqrt{y}} + \sqrt{y}\right) y' = -\frac{3}{t^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(3/t^2=(1/y(t)^(1/2)+y(t)^(1/2))*diff(y(t),t),y(t), singsol=all)
```

$$-\frac{1}{t} - \frac{2\sqrt{y}(y+3)}{9} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 6.594 (sec). Leaf size: 445

`DSolve[3/t^2==(1/y[t]^(1/2)+y[t]^(1/2))*y'[t],y[t],t,IncludeSingularSolutions -> True]`

$$y(t) \rightarrow \frac{\left(-2 + \sqrt[3]{\frac{81}{t^2}} + 3\sqrt{\frac{(-3 + c_1 t)^2 ((16 + 9c_1^2) t^2 - 54c_1 t + 81)}{t^4}} - \frac{54c_1}{t} + 8 + 9c_1^2\right)^2}{2\sqrt[3]{\frac{81}{t^2}} + 3\sqrt{\frac{(-3 + c_1 t)^2 ((16 + 9c_1^2) t^2 - 54c_1 t + 81)}{t^4}} - \frac{54c_1}{t} + 8 + 9c_1^2}$$

$$y(t) \rightarrow \frac{1}{4}i\left(\sqrt{3} + i\right) \frac{\sqrt[3]{\frac{81}{t^2}} + 3\sqrt{\frac{(-3 + c_1 t)^2 ((16 + 9c_1^2) t^2 - 54c_1 t + 81)}{t^4}} - \frac{54c_1}{t} + 8 + 9c_1^2}{-1 - i\sqrt{3}} - 2$$

$$y(t) \rightarrow -\frac{1}{4}i\left(\sqrt{3} - i\right) \frac{\sqrt[3]{\frac{81}{t^2}} + 3\sqrt{\frac{(-3 + c_1 t)^2 ((16 + 9c_1^2) t^2 - 54c_1 t + 81)}{t^4}} - \frac{54c_1}{t} + 8 + 9c_1^2}{-1 + i\sqrt{3}} - 2$$

## 4.11 problem 11

Internal problem ID [13849]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 11.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$-4 \cos(y) y' = -3 \sin(x)$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(3*sin(x)-4*cos(y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \arcsin\left(-\frac{3 \cos(x)}{4} + \frac{3c_1}{4}\right)$$

### ✓ Solution by Mathematica

Time used: 0.563 (sec). Leaf size: 29

```
DSolve[3*Sine[x]-4*Cos[y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \arcsin\left(-\frac{3 \cos(x)}{4} + c_1\right)$$

$$y(x) \rightarrow \arcsin\left(-\frac{3 \cos(x)}{4} + c_1\right)$$

## 4.12 problem 12

Internal problem ID [13850]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 12.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$\cos(y) y' = 8 \sin(8t)$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(cos(y(t))*diff(y(t),t)=8*sin(8*t),y(t), singsol=all)
```

$$y = \arcsin(-\cos(8t) + 64c_1)$$

### ✓ Solution by Mathematica

Time used: 0.509 (sec). Leaf size: 15

```
DSolve[Cos[y[t]]*y'[t]==8*Sin[8*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \arcsin(-\cos(8t) + c_1)$$

## 4.13 problem 13

Internal problem ID [13851]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 13.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' + ky = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)+k*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-kx}$$

### ✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 19

```
DSolve[y'[x]+k*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{-kx}$$

$$y(x) \rightarrow 0$$

## 4.14 problem 14

Internal problem ID [13852]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 14.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$(5x^5 - 4 \cos(x)) x' = -2 \cos(9t) - 2 \sin(7t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 28

```
dsolve((5*x(t)^5-4*cos(x(t)))*diff(x(t),t)+(2*cos(9*t)+2*sin(7*t))=0,x(t), singsol=all)
```

$$\frac{\sin(9t)}{9} - \frac{\cos(7t)}{7} + \frac{5x(t)^6}{12} - 2 \sin(x(t)) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.667 (sec). Leaf size: 40

```
DSolve[(5*x[t]^5-4*Cos[x[t]])*x'[t]+(2*Cos[9*t]+2*Sin[7*t])=0,x[t],t,IncludeSingularSolutio
```

$$x(t) \rightarrow \text{InverseFunction} \left[ \frac{5\#1^6}{6} - 4 \sin(\#1) \& \right] \left[ -\frac{2}{9} \sin(9t) + \frac{2}{7} \cos(7t) + c_1 \right]$$

## 4.15 problem 15

Internal problem ID [13853]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 15.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$20 \sinh(y) y' = -\cosh(6t) - 5 \sinh(4t)$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve((cosh(6*t)+5*sinh(4*t))+(20*sinh(y(t)))*diff(y(t),t)=0,y(t), singsol=all)
```

$$y = \operatorname{arccosh} \left( -\frac{\cosh(4t)}{16} - \frac{\sinh(6t)}{120} - \frac{c_1}{20} \right)$$

### ✓ Solution by Mathematica

Time used: 3.811 (sec). Leaf size: 51

```
DSolve[(Cosh[6*t]+5*Sinh[4*t])+(20*Sinh[y[t]])*y'[t]==0,y[t],t,IncludeSingularSolutions -> T
```

$$y(t) \rightarrow -\operatorname{arccosh} \left( -\frac{1}{120} \sinh(6t) - \frac{1}{16} \cosh(4t) + c_1 \right)$$

$$y(t) \rightarrow \operatorname{arccosh} \left( -\frac{1}{120} \sinh(6t) - \frac{1}{16} \cosh(4t) + c_1 \right)$$



## 4.16 problem 16

Internal problem ID [13854]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 16.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$y' - e^{2y+10t} = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(t),t)=exp(2*y(t)+10*t),y(t), singsol=all)
```

$$y = -\frac{\ln\left(-\frac{e^{10t}}{5} - 2c_1\right)}{2}$$

### ✓ Solution by Mathematica

Time used: 0.928 (sec). Leaf size: 24

```
DSolve[y'[t]==Exp[2*y[t]+10*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow -\frac{1}{2} \log\left(-\frac{e^{10t}}{5} - 2c_1\right)$$

## 4.17 problem 17

Internal problem ID [13855]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 17.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$y' - e^{3y+2t} = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(y(t),t)=exp(3*y(t)+2*t),y(t), singsol=all)
```

$$y = -\frac{\ln\left(-\frac{3e^{2t}}{2} - 3c_1\right)}{3}$$

### ✓ Solution by Mathematica

Time used: 0.927 (sec). Leaf size: 24

```
DSolve[y'[t]==Exp[3*y[t]+2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow -\frac{1}{3} \log\left(-\frac{3}{2}(e^{2t} + 2c_1)\right)$$

## 4.18 problem 18

Internal problem ID [13856]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 18.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$-\cos(y)^2 y' = -\sin(t)^2$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(sin(t)^2=cos(y(t))^2*diff(y(t),t),y(t), singsol=all)
```

$$y = \frac{\text{RootOf}(-\_Z + 2t + 4c_1 - \sin(2t) - \sin(\_Z))}{2}$$

### ✓ Solution by Mathematica

Time used: 0.492 (sec). Leaf size: 35

```
DSolve[Sin[t]^2==Cos[y[t]]^2*y'[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \text{InverseFunction} \left[ 2 \left( \frac{\#1}{2} + \frac{1}{4} \sin(2\#1) \right) \&\right] [t - \sin(t) \cos(t) + c_1]$$

## 4.19 problem 19

Internal problem ID [13857]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 19.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$-(\cos(4y) - 4\cos(y))y' = -3\sin(t) + \sin(3t)$$

### ✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 238

```
dsolve(3*sin(t)-sin(3*t)=(cos(4*y(t))-4*cos(y(t)))*diff(y(t),t),y(t), singsol=all)
```

$$y = \arctan\left(\frac{6 \operatorname{RootOf}(72_Z^8 - 144_Z^6 - 288_Z^5 + 90_Z^4 + 432_Z^3 + 432 \cos(t) c_1 - 48 c_1 \cos(3t) + 2 - 144_Z^6 - 288_Z^5 + 90_Z^4 + 432_Z^3 + 432 \cos(t) c_1 - 48 c_1 \cos(3t) + 288 c_1^2 + 270_Z^2 + \cos(6t) - 18 \cos(4t) + 63 \cos(2t) - 144_Z - 206)}{\dots}\right)$$

### ✓ Solution by Mathematica

Time used: 60.315 (sec). Leaf size: 2033

```
DSolve[3*Sin[t]-Sin[3*t]==(Cos[4*y[t]]-4*Cos[y[t]])*y'[t],y[t],t,IncludeSingularSolutions ->
```

Too large to display

## 4.20 problem 20

Internal problem ID [13858]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 20.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$x' - \frac{\sec(t)^2}{\sec(x)\tan(x)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(x(t),t)=sec(t)^2/(sec(x(t))*tan(x(t))),x(t), singsol=all)
```

$$x(t) = \arccos\left(\frac{1}{\tan(t) + c_1}\right)$$

✓ Solution by Mathematica

Time used: 0.848 (sec). Leaf size: 45

```
DSolve[x'[t]==Sec[t]^2/(Sec[x[t]]*Tan[x[t]]),x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow -\sec^{-1}(\tan(t) + 2c_1)$$

$$x(t) \rightarrow \sec^{-1}(\tan(t) + 2c_1)$$

$$x(t) \rightarrow -\frac{\pi}{2}$$

$$x(t) \rightarrow \frac{\pi}{2}$$

## 4.21 problem 21

Internal problem ID [13859]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 21.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$\left(2 - \frac{5}{y^2}\right) y' = -4 \cos(x)^2$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 107

```
dsolve((2-5/y(x)^2)*diff(y(x),x)+4*cos(x)^2=0,y(x), singsol=all)
```

$$y(x) = -c_1 - \frac{x}{2} - \frac{\sin(2x)}{4} - \frac{\sqrt{4c_1^2 + 4c_1x + x^2 - \frac{79}{8} - \frac{\cos(4x)}{8}} + 2c_1 \sin(2x) + x \sin(2x)}{2}$$

$$y(x) = -c_1 - \frac{x}{2} - \frac{\sin(2x)}{4} + \frac{\sqrt{4c_1^2 + 4c_1x + x^2 - \frac{79}{8} - \frac{\cos(4x)}{8}} + 2c_1 \sin(2x) + x \sin(2x)}{2}$$

### ✓ Solution by Mathematica

Time used: 0.672 (sec). Leaf size: 88

```
DSolve[(2-5/y[x]^2)*y'[x]+4*Cos[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{4} \left( -2x - \sin(2x) - \sqrt{-40 + (2x + \sin(2x) - c_1)^2} + c_1 \right)$$

$$y(x) \rightarrow \frac{1}{4} \left( -2x - \sin(2x) + \sqrt{-40 + (2x + \sin(2x) - c_1)^2} + c_1 \right)$$

$$y(x) \rightarrow 0$$

## 4.22 problem 22

Internal problem ID [13860]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 22.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' - \frac{t^3}{y\sqrt{(1-y^2)(t^4+9)}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 62

```
dsolve(diff(y(t),t)=t^3/(y(t)*sqrt((1-y(t)^2)*(t^4+9))),y(t), singsol=all)
```

$$\frac{(y-1)^{\frac{3}{2}}(y+1)^{\frac{3}{2}}}{3} + \int^t -\frac{-a^3\sqrt{y+1}\sqrt{y-1}}{\sqrt{-a^4y^2 + a^4 - 9y^2 + 9}} da + c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.286 (sec). Leaf size: 519

`DSolve[y'[t]==t^3/(y[t]*Sqrt[(1-y[t]^2)*(t^4+9)]),y[t],t,IncludeSingularSolutions -> True]`

$$y(t) \rightarrow -\sqrt{1 + \left(\frac{3}{2}\right)^{2/3} \sqrt[3]{-t^4 - 4ic_1\sqrt{t^4 + 9} - 9 + 4c_1^2}}$$

$$y(t) \rightarrow \sqrt{1 + \left(\frac{3}{2}\right)^{2/3} \sqrt[3]{-t^4 - 4ic_1\sqrt{t^4 + 9} - 9 + 4c_1^2}}$$

$$y(t) \rightarrow -\frac{1}{2} \sqrt{-\sqrt[3]{23^{2/3}} \sqrt[3]{-t^4 - 4ic_1\sqrt{t^4 + 9} - 9 + 4c_1^2} - 3i\sqrt[3]{2}\sqrt[3]{3} \sqrt[3]{-t^4 - 4ic_1\sqrt{t^4 + 9} - 9 + 4c_1^2} + 4}$$

$$y(t) \rightarrow \frac{1}{2} \sqrt{-\sqrt[3]{23^{2/3}} \sqrt[3]{-t^4 - 4ic_1\sqrt{t^4 + 9} - 9 + 4c_1^2} - 3i\sqrt[3]{2}\sqrt[3]{3} \sqrt[3]{-t^4 - 4ic_1\sqrt{t^4 + 9} - 9 + 4c_1^2} + 4}$$

$$y(t) \rightarrow -\frac{1}{2} \sqrt{-\sqrt[3]{23^{2/3}} \sqrt[3]{-t^4 - 4ic_1\sqrt{t^4 + 9} - 9 + 4c_1^2} + 3i\sqrt[3]{2}\sqrt[3]{3} \sqrt[3]{-t^4 - 4ic_1\sqrt{t^4 + 9} - 9 + 4c_1^2} + 4}$$

$$y(t) \rightarrow \frac{1}{2} \sqrt{-\sqrt[3]{23^{2/3}} \sqrt[3]{-t^4 - 4ic_1\sqrt{t^4 + 9} - 9 + 4c_1^2} + 3i\sqrt[3]{2}\sqrt[3]{3} \sqrt[3]{-t^4 - 4ic_1\sqrt{t^4 + 9} - 9 + 4c_1^2} + 4}$$



## 4.23 problem 23

Internal problem ID [13861]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 23.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$\tan(y) \sec(y)^2 y' = -\cos(2x)^3 \sin(2x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
dsolve(tan(y(x))*sec(y(x))^2*diff(y(x),x)+cos(2*x)^3*sin(2*x)=0,y(x), singsol=all)
```

$$y(x) = \operatorname{arccot} \left( \frac{8}{\sqrt{-256c_1 + 2 \cos(8x) + 8 \cos(4x)}} \right)$$
$$y(x) = \pi - \operatorname{arccot} \left( \frac{8}{\sqrt{-256c_1 + 2 \cos(8x) + 8 \cos(4x)}} \right)$$

✓ Solution by Mathematica

Time used: 2.982 (sec). Leaf size: 139

```
DSolve[Tan[y[x]]*Sec[y[x]]^2*y'[x]+Cos[2*x]^3*Sin[2*x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow -\sec^{-1}\left(-\frac{\sqrt{8\cos^4(2x)+c_1}}{4\sqrt{2}}\right)$$

$$y(x) \rightarrow \sec^{-1}\left(-\frac{\sqrt{8\cos^4(2x)+c_1}}{4\sqrt{2}}\right)$$

$$y(x) \rightarrow -\sec^{-1}\left(\frac{\sqrt{8\cos^4(2x)+c_1}}{4\sqrt{2}}\right)$$

$$y(x) \rightarrow \sec^{-1}\left(\frac{\sqrt{8\cos^4(2x)+c_1}}{4\sqrt{2}}\right)$$

$$y(x) \rightarrow -\frac{\pi}{2}$$

$$y(x) \rightarrow \frac{\pi}{2}$$

## 4.24 problem 24

Internal problem ID [13862]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 24.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$y' - \frac{(1 + 2e^y)e^{-y}}{t \ln(t)} = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 19

```
dsolve(diff(y(t),t)=(1+2*exp(y(t)))/(exp(y(t))*t*ln(t)),y(t), singsol=all)
```

$$y = -\ln\left(\frac{2}{\ln(t)^2 c_1 - 1}\right)$$

✓ Solution by Mathematica

Time used: 0.685 (sec). Leaf size: 51

```
DSolve[y'[t]==(1+2*Exp[y[t]])/(Exp[y[t]]*t*Log[t]),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \log\left(\frac{1}{2}(-1 + e^{2c_1} \log^2(t))\right)$$

$$y(t) \rightarrow -\log(2) - i\pi$$

$$y(t) \rightarrow -\log(2) + i\pi$$

## 4.25 problem 25

Internal problem ID [13863]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 25.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$-\frac{\cos(\sqrt{y}) y'}{\sqrt{y}} = -x \sin(x^2)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x*sin(x^2)=cos(sqrt(y(x)))/sqrt(y(x))*diff(y(x),x),y(x), singsol=all)
```

$$-\frac{\cos(x^2)}{2} - 2 \sin(\sqrt{y(x)}) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.996 (sec). Leaf size: 43

```
DSolve[x*Sin[x^2]==Cos[Sqrt[y[x]]]/Sqrt[y[x]]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \arcsin\left(\frac{1}{4}(-\cos(x^2) + 2c_1)\right)^2$$

$$y(x) \rightarrow \arcsin\left(\frac{1}{4}(\cos(x^2) - 2c_1)\right)^2$$

## 4.26 problem 26

Internal problem ID [13864]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 26.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$-\frac{\left(1 - \frac{1}{y}\right)^2 y'}{y^2} = -\frac{x - 2}{x^2 - 4x + 3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 1790

```
dsolve((x-2)/(x^2-4*x+3)=(1-1/y(x))^2*1/y(x)^2*diff(y(x),x),y(x), singsol=all)
```

$y(x)$

$$= \frac{\left(-9 \ln((x-1)(x-3))^2 + 3\sqrt{9 \ln((x-1)(x-3))^2 + 36 \ln((x-1)(x-3))} c_1 + 36c_1^2 + 12 \ln((x-1)(x-3))\right)}{2}$$

$$+ \frac{(\ln((x-1)(x-3)) + 2c_1) \left(-9 \ln((x-1)(x-3))^2 + 3\sqrt{9 \ln((x-1)(x-3))^2 + 36 \ln((x-1)(x-3))} c_1 + 36c_1^2 + 12 \ln((x-1)(x-3))\right)}{3(\ln((x-1)(x-3)) + 2c_1)}$$

Expression too large to display

Expression too large to display

✓ Solution by Mathematica

Time used: 1.679 (sec). Leaf size: 1134

```
DSolve[(x-2)/(x^2-4*x+3)==(1-1/y[x])^2*1/y[x]^2*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$y(x)$

$$\rightarrow \frac{-6 \log(x-3) - 6 \log(x-1) - \left(9 \log^2(x-3) + 9 \log^2(x-1) + 18 \log(x-3) + 18 \log(x-3) \log(x-1)\right)}{\dots}$$

$y(x)$

$$\rightarrow \frac{\sqrt[3]{9 \log^2(x-3) + 9 \log^2(x-1) + 18 \log(x-3) + 18 \log(x-3) \log(x-1) + 18 \log(x-1) + 3 \sqrt{(3 \log^2(x-3) + 3 \log^2(x-1) + 6 \log(x-3) \log(x-1))}}{\dots}$$

$y(x)$

$$\rightarrow \frac{\sqrt[3]{9 \log^2(x-3) + 9 \log^2(x-1) + 18 \log(x-3) + 18 \log(x-3) \log(x-1) + 18 \log(x-1) + 3 \sqrt{(3 \log^2(x-3) + 3 \log^2(x-1) + 6 \log(x-3) \log(x-1))}}{\dots}$$

$y(x) \rightarrow 0$

## 4.27 problem 27

Internal problem ID [13865]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 27.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$\frac{\cos(y) y'}{(1 - \sin(y))^2} = \sin(x)^3 \cos(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(cos(y(x))/(1-sin(y(x)))^2*diff(y(x),x)=sin(x)^3*cos(x),y(x), singsol=all)
```

$$y(x) = \arcsin\left(\frac{-32c_1 + 29 - \cos(4x) + 4 \cos(2x)}{-32c_1 - 3 - \cos(4x) + 4 \cos(2x)}\right)$$

✓ Solution by Mathematica

Time used: 28.843 (sec). Leaf size: 705

DSolve[Cos[y[x]]/(1-Sin[y[x]])^2\*y'[x]==Sin[x]^3\*Cos[x],y[x],x,IncludeSingularSolutions -> T

$$y(x) \rightarrow -2 \arccos \left( \frac{-4 + \sqrt{-4 \cos(2x) + \cos(4x) - 13 + c_1}}{\sqrt{2} \sqrt{8 \sin^4(x) + c_1}} \right)$$

$$y(x) \rightarrow 2 \arccos \left( \frac{-4 + \sqrt{-4 \cos(2x) + \cos(4x) - 13 + c_1}}{\sqrt{2} \sqrt{8 \sin^4(x) + c_1}} \right)$$

$$y(x) \rightarrow -2 \arccos \left( \frac{-4 + \sqrt{-4 \cos(2x) + \cos(4x) - 13 + c_1}}{\sqrt{2} \sqrt{8 \sin^4(x) + c_1}} \right)$$

$$y(x) \rightarrow 2 \arccos \left( \frac{-4 + \sqrt{-4 \cos(2x) + \cos(4x) - 13 + c_1}}{\sqrt{2} \sqrt{8 \sin^4(x) + c_1}} \right)$$

$$y(x) \rightarrow -2 \arccos \left( \frac{-4 + \sqrt{-4 \cos(2x) + \cos(4x) - 13 + c_1}}{\sqrt{2} \sqrt{8 \sin^4(x) + c_1}} \right)$$

$$y(x) \rightarrow 2 \arccos \left( \frac{-4 + \sqrt{-4 \cos(2x) + \cos(4x) - 13 + c_1}}{\sqrt{2} \sqrt{8 \sin^4(x) + c_1}} \right)$$

$$y(x) \rightarrow -2 \arccos \left( \frac{4 + \sqrt{-4 \cos(2x) + \cos(4x) - 13 + c_1}}{\sqrt{2} \sqrt{8 \sin^4(x) + c_1}} \right)$$

$$y(x) \rightarrow 2 \arccos \left( \frac{4 + \sqrt{-4 \cos(2x) + \cos(4x) - 13 + c_1}}{\sqrt{2} \sqrt{8 \sin^4(x) + c_1}} \right)$$

$$y(x) \rightarrow -\frac{3\pi}{2}$$

$$y(x) \rightarrow -\frac{\pi}{2}$$

$$y(x) \rightarrow \frac{\pi}{2}$$

$$y(x) \rightarrow \frac{3\pi}{2}$$

$$y(x) \rightarrow -2 \arccos \left( \frac{\sqrt{-4 \cos(2x) + \cos(4x) - 13 - 4}}{4 \sqrt{\sin^4(x)}} \right)$$

$$y(x) \rightarrow 2 \arccos \left( \frac{\sqrt{-4 \cos(2x) + \cos(4x) - 13 - 4}}{4 \sqrt{\sin^4(x)}} \right)$$

$$y(x) \rightarrow -2 \arccos \left( \frac{\sqrt{-4 \cos(2x) + \cos(4x) - 13 - 4}}{4 \sqrt{\sin^4(x)}} \right)$$



## 4.28 problem 28

Internal problem ID [13866]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 28.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$y' - \frac{(5 - 2 \cos(x))^3 \sin(x) \cos(y)^4}{\sin(y)} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 230

```
dsolve(diff(y(x),x)=((5-2*cos(x))^3*sin(x)*cos(y(x))^4)/sin(y(x)),y(x), singsol=all)
```

$$y(x) = \pi$$

$$-\arccos\left(\frac{2 \cdot 9^{\frac{1}{3}} \left((-8c_1 + 931 + 2 \cos(4x) + 308 \cos(2x) - 40 \cos(3x) - 1120 \cos(x))^2\right)^{\frac{1}{3}}}{24c_1 - 2793 - 6 \cos(4x) - 924 \cos(2x) + 120 \cos(3x) + 3360 \cos(x)}\right)$$

$$y(x) = \pi$$

$$-\arccos\left(\frac{(-1 + i\sqrt{3}) \cdot 9^{\frac{1}{3}} \left((-8c_1 + 931 + 2 \cos(4x) + 308 \cos(2x) - 40 \cos(3x) - 1120 \cos(x))^2\right)^{\frac{1}{3}}}{24c_1 - 2793 - 6 \cos(4x) - 924 \cos(2x) + 120 \cos(3x) + 3360 \cos(x)}\right)$$

$$y(x)$$

$$= \arccos\left(\frac{(1 + i\sqrt{3}) \cdot 9^{\frac{1}{3}} \left((-8c_1 + 931 + 2 \cos(4x) + 308 \cos(2x) - 40 \cos(3x) - 1120 \cos(x))^2\right)^{\frac{1}{3}}}{24c_1 - 2793 - 6 \cos(4x) - 924 \cos(2x) + 120 \cos(3x) + 3360 \cos(x)}\right)$$

✓ Solution by Mathematica

Time used: 9.673 (sec). Leaf size: 311

```
DSolve[y'[x]==((5-2*Cos[x])^3*Sin[x]*Cos[y[x]]^4)/Sin[y[x]],y[x],x,IncludeSingularSolutions
```

$$y(x) \rightarrow -\sec^{-1}\left(-\frac{1}{2}\sqrt[3]{-\frac{3}{2}}\sqrt[3]{-2240\cos(x)+616\cos(2x)-80\cos(3x)+4\cos(4x)+1862+c_1}\right)$$

$$y(x) \rightarrow \sec^{-1}\left(-\frac{1}{2}\sqrt[3]{-\frac{3}{2}}\sqrt[3]{-2240\cos(x)+616\cos(2x)-80\cos(3x)+4\cos(4x)+1862+c_1}\right)$$

$$y(x) \rightarrow -\sec^{-1}\left(\frac{1}{2}\sqrt[3]{\frac{3}{2}}\sqrt[3]{-2240\cos(x)+616\cos(2x)-80\cos(3x)+4\cos(4x)+1862+c_1}\right)$$

$$y(x) \rightarrow \sec^{-1}\left(\frac{1}{2}\sqrt[3]{\frac{3}{2}}\sqrt[3]{-2240\cos(x)+616\cos(2x)-80\cos(3x)+4\cos(4x)+1862+c_1}\right)$$

$$y(x) \rightarrow -\sec^{-1}\left(\frac{1}{2}(-1)^{2/3}\sqrt[3]{\frac{3}{2}}\sqrt[3]{-2240\cos(x)+616\cos(2x)-80\cos(3x)+4\cos(4x)+1862+c_1}\right)$$

$$y(x) \rightarrow \sec^{-1}\left(\frac{1}{2}(-1)^{2/3}\sqrt[3]{\frac{3}{2}}\sqrt[3]{-2240\cos(x)+616\cos(2x)-80\cos(3x)+4\cos(4x)+1862+c_1}\right)$$

$$y(x) \rightarrow -\frac{\pi}{2}$$

$$y(x) \rightarrow \frac{\pi}{2}$$

## 4.29 problem 29

Internal problem ID [13867]

**Book:** INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014

**Section:** Chapter 2. First Order Equations. Exercises 2.2, page 39

**Problem number:** 29.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$\frac{e^{\frac{3}{y}} y'}{y} = -\frac{\sqrt{\ln(x)}}{x}$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(sqrt(ln(x))/x==exp(3/y(x))/y(x)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = -\frac{3}{\text{RootOf}\left(2 \ln(x)^{\frac{3}{2}} - 3 \exp\text{Integral}_1(\_Z) + 3c_1\right)}$$

### ✓ Solution by Mathematica

Time used: 0.425 (sec). Leaf size: 29

```
DSolve[Sqrt[Log[x]]/x==Exp[3/y[x]]/y[x]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \text{InverseFunction}\left[-\text{ExpIntegralEi}\left(\frac{3}{\#1}\right) \& \right] \left[ \frac{2}{3} \log^{\frac{3}{2}}(x) + c_1 \right]$$