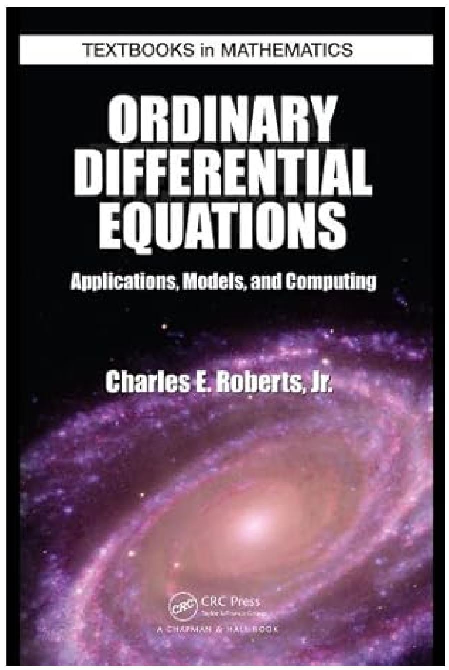
A Solution Manual For

Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010



Nasser M. Abbasi

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1.1 problem 15

Internal problem ID [12254]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises page 14
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2y'' + xy' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + c_2x$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 16

```
DSolve[x^2*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x} + c_2x$$

1.2 problem 16

Internal problem ID [12255]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises page 14
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 7

```
dsolve(x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 14

```
DSolve[x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x$$
$$y(x) \to 0$$

1.3 problem 17

Internal problem ID [12256]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises page 14
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{2x^2y'' + 3xy' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(2*x^2*diff(y(x),x$2)+3*x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + c_2\sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 20

```
DSolve[2*x^2*y''[x]+3*x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^{3/2} + c_1}{x}$$

1.4 problem 18

Internal problem ID [12257]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises page 14
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 3y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{2x} + c_2 e^{x}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 18

```
DSolve[y''[x]-3*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x (c_2 e^x + c_1)$$

1.5 problem 19

Internal problem ID [12258]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises page 14
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2y'' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^2 + \frac{c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_2x^3 + c_1}{x}$$

1.6 problem 20

Internal problem ID [12259]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises page 14
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + \frac{1}{2y} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)+1/(2*y(x))=0,y(x), singsol=all)
```

$$y(x) = \sqrt{-x + c_1}$$
$$y(x) = -\sqrt{-x + c_1}$$

✓ Solution by Mathematica

Time used: 0.113 (sec). Leaf size: 35

```
DSolve[y'[x]+1/(2*y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x + 2c_1}$$
$$y(x) \to \sqrt{-x + 2c_1}$$

1.7 problem 21

Internal problem ID [12260]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises page 14
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{y}{x} = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)-y(x)/x=1,y(x), singsol=all)
```

$$y(x) = x(\ln(x) + c_1)$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 12

```
DSolve[y'[x]-y[x]/x==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(\log(x) + c_1)$$

1.8 problem 22

Internal problem ID [12261]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises page 14
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 2\sqrt{|y|} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x)-2*sqrt(abs(y(x)))=0,y(x), singsol=all)
```

$$x - \frac{\left(\begin{cases} -2\sqrt{-y\left(x\right)} & y(x) \leq 0 \\ 2\sqrt{y\left(x\right)} & 0 < y(x) \end{cases}\right)}{2} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.291 (sec). Leaf size: 31

```
DSolve[y'[x]-Sqrt[Abs[y[x]]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{\sqrt{|K[1]|}} dK[1]\&\right][x + c_1]$$

$$y(x) \to 0$$

1.9 problem 23

Internal problem ID [12262]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises page 14
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2y' + 2yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(x^2*diff(y(x),x)+2*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.064 (sec). Leaf size: 16

```
DSolve[x^2*y'[x]+2*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x^2}$$
$$y(x) \to 0$$

1.10 problem 24

Internal problem ID [12263]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises page 14
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^2 = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x)-y(x)^2=1,y(x), singsol=all)
```

$$y(x) = \tan(x + c_1)$$

✓ Solution by Mathematica

Time used: 0.213 (sec). Leaf size: 24

```
DSolve[y'[x]-y[x]^2==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan(x + c_1)$$
$$y(x) \to -i$$
$$y(x) \to i$$

1.11 problem 25

Internal problem ID [12264]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises page 14
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2x^2y'' + xy' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(2*x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x + \frac{c_2}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 18

```
DSolve[2*x^2*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{\sqrt{x}} + c_2x$$

1.12 problem 26

Internal problem ID [12265]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises page 14
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xy' = \sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(x*diff(y(x),x)-sin(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{Si}(x) + c_1$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 10

```
DSolve[x*y'[x]-Sin[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \mathrm{Si}(x) + c_1$$

1.13 problem 27

Internal problem ID [12266]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises page 14
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-3x}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 18

```
DSolve[y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-3x}$$
$$y(x) \to 0$$

1.14 problem 28

Internal problem ID [12267]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises page 14
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 3y' - 10y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)-10*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{5x} + c_2 e^{-2x}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 22

```
DSolve[y''[x]-3*y'[x]-10*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x} \left(c_2 e^{7x} + c_1\right)$$

1.15 problem 29

Internal problem ID [12268]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises page 14
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x} + c_2 x \,\mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 18

```
DSolve[y''[x]+2*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_2 x + c_1)$$

1.16 problem 30

Internal problem ID [12269]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises page 14
Problem number: 30.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 7y'' + 12y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$3)-7*diff(y(x),x$2)+12*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + e^{3x}c_2 + e^{4x}c_3$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 30

```
DSolve[y'''[x]-7*y''[x]+12*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}c_1e^{3x} + \frac{1}{4}c_2e^{4x} + c_3$$

1.17 problem 31

Internal problem ID [12270]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises page 14
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2xy' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(2*x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 18

```
DSolve[2*x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\sqrt{x}$$
$$y(x) \to 0$$

1.18 problem 32

Internal problem ID [12271]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises page 14
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x^2y'' - xy' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_2x^2 + c_1$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 17

```
DSolve[x^2*y''[x]-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1x^2}{2} + c_2$$

1.19 problem 33

Internal problem ID [12272]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises page 14
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2y'' + 6xy' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+6*x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^4} + \frac{c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+6*x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^3 + c_1}{x^4}$$

1.20 problem 34

Internal problem ID [12273]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises page 14
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 5xy' + 9y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)-5*x*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^3 + c_2x^3 \ln(x)$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]-5*x*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^3(3c_2 \log(x) + c_1)$$

1.21 problem 35

Internal problem ID [12274]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises page 14
Problem number: 35.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - 4y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)^2-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = c_1^2 - 2c_1x + x^2$$

✓ Solution by Mathematica

Time used: 0.078 (sec). Leaf size: 38

```
DSolve[(y'[x])^2-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}(-2x + c_1)^2$$
$$y(x) \to \frac{1}{4}(2x + c_1)^2$$
$$y(x) \to 0$$

1.22 problem 36

Internal problem ID [12275]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises page 14
Problem number: 36.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{y'^2 - 9yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 101

```
dsolve(diff(y(x),x)^2-9*x*y(x)=0,y(x), singsol=all)
```

$$
\begin{aligned}
y(x) &= 0 \\
y(x) &= -x^3 - 2\left(-x^2 - \sqrt{c_1 x}\right) x + c_1 \\
y(x) &= -x^3 - 2\left(-x^2 + \sqrt{c_1 x}\right) x + c_1 \\
y(x) &= -x^3 + 2\left(x^2 - \sqrt{c_1 x}\right) x + c_1 \\
y(x) &= -x^3 + 2\left(x^2 + \sqrt{c_1 x}\right) x + c_1
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.087 (sec). Leaf size: 46

```
DSolve[(y'[x])^2-9*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) &\to \frac{1}{4}\left(-2x^{3/2} + c_1\right)^2 \\
y(x) &\to \frac{1}{4}\left(2x^{3/2} + c_1\right)^2 \\
y(x) &\to 0
\end{aligned}
$$

1.23 problem 37

Internal problem ID [12276]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises page 14
Problem number: 37.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 = x^6}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)^2=x^6,y(x), singsol=all)
```

$$y(x) = \frac{x^4}{4} + c_1$$
$$y(x) = -\frac{x^4}{4} + c_1$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 29

```
DSolve[(y'[x])^2==x^6,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^4}{4} + c_1$$
$$y(x) \to \frac{x^4}{4} + c_1$$

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2.1 problem 1

Internal problem ID [12277]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises 1.3, page 27
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 2yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)-2*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{x^2}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 18

```
DSolve[y'[x]-2*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x^2}$$
$$y(x) \to 0$$

2.2 problem 2

Internal problem ID [12278]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises 1.3, page 27
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + y = x^2 + 2x - 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+y(x)=x^2+2*x-1,y(x), singsol=all)
```

$$y(x) = x^2 - 1 + c_1 \mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.088 (sec). Leaf size: 18

```
DSolve[y'[x]+y[x]==x^2+2*x-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + c_1 e^{-x} - 1$$

2.3 problem 3

Internal problem ID [12279]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises 1.3, page 27
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' - 6y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-6*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{3x} + c_2 e^{-2x}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 22

```
DSolve[y''[x]-y'[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(c_2 e^{5x} + c_1\right)$$

2.4 problem 4

Internal problem ID [12280]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises 1.3, page 27
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - x\sqrt{y} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=x*y(x)^(1/2),y(x), singsol=all)
```

$$\sqrt{y(x)} - \frac{x^2}{4} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.2 (sec). Leaf size: 24

```
DSolve[y'[x]==x*y[x]^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{16}(x^2 + 2c_1)^2$$
$$y(x) \to 0$$

2.5 problem 5

Internal problem ID [12281]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises 1.3, page 27
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)-y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{x} c_1 + e^{-x} c_2$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 20

```
DSolve[y''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x} + c_2 e^{-x}$$

2.6 problem 6

Internal problem ID [12282]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises 1.3, page 27
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 3y^{\frac{2}{3}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)=3*y(x)^(2/3),y(x), singsol=all)
```

$$y(x)^{\frac{1}{3}} - c_1 - x = 0$$

✓ Solution by Mathematica

Time used: 0.166 (sec). Leaf size: 22

```
DSolve[y'[x]==3*y[x]^(2/3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{27}(3x + c_1)^3$$
$$y(x) \to 0$$

2.7 problem 7

Internal problem ID [12283]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises 1.3, page 27
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy' \ln (x) - (\ln (x) + 1) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve((x*ln(x))*diff(y(x),x)-(1+ln(x))*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \ln (x)$$

✓ Solution by Mathematica

Time used: 0.087 (sec). Leaf size: 16

```
DSolve[(x*Log[x])*y'[x]-(1+Log[x])*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x \log(x)$$
$$y(x) \to 0$$

2.8 problem 8 a(i)

Internal problem ID [12284]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises 1.3, page 27
Problem number: 8 a(i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' - 2y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = -5]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)-diff(y(x),x)-2*y(x)=0,y(0) = 2, D(y)(0) = -5],y(x), singsol=all)
```

$$y(x) = -\mathrm{e}^{2x} + 3\,\mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 19

```
DSolve[{y''[x]-y'[x]-2*y[x]==0,{y[0]==2,y'[0]==-5}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{-x}\left(e^{3x} - 3\right)$$

2.9 problem 8 a(ii)

Internal problem ID [12285]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises 1.3, page 27
Problem number: 8 a(ii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' - 2y = 0}$$

With initial conditions

$$[y(1) = 3, y'(1) = -1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve([diff(y(x),x$2)-diff(y(x),x)-2*y(x)=0,y(1) = 3, D(y)(1) = -1],y(x), singsol=all)
```

$$y(x) = \frac{2\,e^{2x-2}}{3} + \frac{7\,e^{1-x}}{3}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 28

```
DSolve[{y''[x]-y'[x]-2*y[x]==0,{y[1]==3,y'[1]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{7e^{1-x}}{3} + \frac{2}{3}e^{2x-2}$$

2.10 problem 8 b(i)

Internal problem ID [12286]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises 1.3, page 27
Problem number: 8 b(i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' - 2y = 0}$$

With initial conditions

$$[y(0) = 1, y(2) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve([diff(y(x),x$2)-diff(y(x),x)-2*y(x)=0,y(0) = 1, y(2) = 0],y(x), singsol=all)
```

$$y(x) = \frac{-\mathrm{e}^{2x} + \mathrm{e}^{-x+6}}{\mathrm{e}^{6} - 1}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 29

```
DSolve[{y''[x]-y'[x]-2*y[x]==0,{y[0]==1,y[2]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}(e^{6} - e^{3x})}{e^{6} - 1}$$

2.11 problem 8 b(ii)

Internal problem ID [12287]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises 1.3, page 27
Problem number: 8 b(ii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' - 2y = 0}$$

With initial conditions

$$[y(0) = 0, y'(2) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve([diff(y(x),x$2)-diff(y(x),x)-2*y(x)=0,y(0) = 0, D(y)(2) = 1],y(x), singsol=all)
```

$$y(x) = \frac{e^{2-x}(e^{3x} - 1)}{2\,e^{6} + 1}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 29

```
DSolve[{y''[x]-y'[x]-2*y[x]==0,{y[0]==0,y'[2]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{2-x}(e^{3x} - 1)}{1 + 2e^{6}}$$

2.12 problem 9

Internal problem ID [12288]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises 1.3, page 27
Problem number: 9.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''' - 3x^2y'' + 6y'x - 6y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^3*diff(y(x),x$3)-3*x^2*diff(y(x),x$2)+6*x*diff(y(x),x)-6*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^3 + c_2x^2 + c_3x$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 19

```
DSolve[x^3*y'''[x]-3*x^2*y''[x]+6*x*y'[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(x(c_3x + c_2) + c_1)$$

2.13 problem 10 (a)

Internal problem ID [12289]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises 1.3, page 27
Problem number: 10 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' - 4xy' + 6y = 0}$$

With initial conditions

$$[y(1) = 0, y(2) = -4]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve([x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=0,y(1) = 0, y(2) = -4],y(x), singsol=all)
```

$$y(x) = -x^3 + x^2$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 13

```
DSolve[{x^2*y''[x]-4*x*y'[x]+6*y[x]==0,{y[1]==0,y[2]==-4}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to -\left((x-1)x^2\right)$$

2.14 problem 10 (b)

Internal problem ID [12290]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises 1.3, page 27
Problem number: 10 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' - 4xy' + 6y = 0}$$

With initial conditions

$$[y(2) = 4, y'(1) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=0,y(2) = 4, D(y)(1) = 0],y(x), singsol=al
```

$$y(x) = 2x^3 - 3x^2$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 14

```
DSolve[{x^2*y''[x]-4*x*y'[x]+6*y[x]==0,{y'[1]==0,y[2]==4}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to x^2(2x - 3)$$

2.15 problem 10 (c)

Internal problem ID [12291]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises 1.3, page 27
Problem number: 10 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' - 4xy' + 6y = 0}$$

With initial conditions

$$[y(1) = 1, y'(2) = -12]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=0,y(1) = 1, D(y)(2) = -12],y(x), singsol=
```

$$y(x) = -2x^3 + 3x^2$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 14

```
DSolve[{x^2*y''[x]-4*x*y'[x]+6*y[x]==0,{y[1]==1,y'[2]==-12}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to (3 - 2x)x^2$$

2.16 problem 10 (d)

Internal problem ID [12292]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises 1.3, page 27
Problem number: 10 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' - 4xy' + 6y = 0}$$

With initial conditions

$$[y'(1) = 3, y'(2) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=0,D(y)(1) = 3, D(y)(2) = 0],y(x), singsol
```

$$y(x) = -x^3 + 3x^2$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 13

```
DSolve[{x^2*y''[x]-4*x*y'[x]+6*y[x]==0,{y'[1]==3,y'[2]==0}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to -\left((x - 3)x^2\right)$$

2.17 problem 10 (e)

Internal problem ID [12293]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises 1.3, page 27
Problem number: 10 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' - 4xy' + 6y = 0}$$

With initial conditions

$$[y(0) = 0, y(2) = 4]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=0,y(0) = 0, y(2) = 4],y(x), singsol=all)
```

$$y(x) = (1 + (-2 + x) c_1) x^2$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 23

```
DSolve[{x^2*y''[x]-4*x*y'[x]+6*y[x]==0,{y[0]==0,y[2]==4}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{2}x^2(x - c_1x + 2c_1)$$

2.18 problem 10 (f)

Internal problem ID [12294]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 1. Introduction. Exercises 1.3, page 27
Problem number: 10 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' - 4xy' + 6y = 0}$$

With initial conditions

$$[y(0) = 2, y'(2) = -1]$$

✗ Solution by Maple

```
dsolve([x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=0,y(0) = 2, D(y)(2) = -1],y(x), singsol=a
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x^2*y''[x]-4*x*y'[x]+6*y[x]==0,{y[0]==2,y'[2]==-1}},y[x],x,IncludeSingularSolutions
```

{}

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3.1 problem 1 (A)

Internal problem ID [12295]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.1, page 40
Problem number: 1 (A).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = 1 - x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=1-x,y(x), singsol=all)
```

$$y(x) = -\frac{1}{2}x^2 + x + c_1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 16

```
DSolve[y'[x]==1-x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2}{2} + x + c_1$$

3.2 problem 1 (B)

Internal problem ID [12296]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.1, page 40
Problem number: 1 (B).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = x - 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)=x-1,y(x), singsol=all)
```

$$y(x) = \frac{1}{2}x^2 - x + c_1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[y'[x]==x-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2} - x + c_1$$

3.3 problem 2 (C)

Internal problem ID [12297]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.1, page 40
Problem number: 2 (C).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + y = 1}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=1-y(x),y(x), singsol=all)
```

$$y(x) = 1 + c_1 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 20

```
DSolve[y'[x]==1-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 + c_1 e^{-x}$$
$$y(x) \to 1$$

3.4 problem 2 (D)

Internal problem ID [12298]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.1, page 40
Problem number: 2 (D).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=1+y(x),y(x), singsol=all)
```

$$y(x) = -1 + c_1 e^x$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 18

```
DSolve[y'[x]==1+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -1 + c_1 e^x$$
$$y(x) \to -1$$

3.5 problem 3 (E)

Internal problem ID [12299]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.1, page 40
Problem number: 3 (E).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^2 = -4}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)=y(x)^2-4,y(x), singsol=all)
```

$$y(x) = -\frac{2(c_1 e^{4x} + 1)}{-1 + c_1 e^{4x}}$$

✓ Solution by Mathematica

Time used: 1.066 (sec). Leaf size: 40

```
DSolve[y'[x]==y[x]^2-4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2 - 2e^{4(x+c_1)}}{1 + e^{4(x+c_1)}}$$
$$y(x) \to -2$$
$$y(x) \to 2$$

3.6 problem 3 (F)

Internal problem ID [12300]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.1, page 40
Problem number: 3 (F).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + y^2 = 4}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)=4-y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{2c_1 e^{4x} + 2}{-1 + c_1 e^{4x}}$$

✓ Solution by Mathematica

Time used: 0.278 (sec). Leaf size: 45

```
DSolve[y'[x]==4-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2(e^{4x} - e^{4c_1})}{e^{4x} + e^{4c_1}}$$
$$y(x) \to -2$$
$$y(x) \to 2$$

3.7 problem 4 (G)

Internal problem ID [12301]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.1, page 40
Problem number: 4 (G).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=x*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{x^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 22

```
DSolve[y'[x]==x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{\frac{x^2}{2}}$$
$$y(x) \to 0$$

3.8 problem 4 (H)

Internal problem ID [12302]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.1, page 40
Problem number: 4 (H).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=-x*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 22

```
DSolve[y'[x]==-x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\frac{x^2}{2}}$$
$$y(x) \to 0$$

3.9 problem 5 (I)

Internal problem ID [12303]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.1, page 40
Problem number: 5 (I).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + y^2 = x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(diff(y(x),x)=x^2-y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x\left(\operatorname{BesselI}\left(-\frac{3}{4}, \frac{x^2}{2}\right) c_1 - \operatorname{BesselK}\left(\frac{3}{4}, \frac{x^2}{2}\right)\right)}{c_1 \operatorname{BesselI}\left(\frac{1}{4}, \frac{x^2}{2}\right) + \operatorname{BesselK}\left(\frac{1}{4}, \frac{x^2}{2}\right)}$$

✓ Solution by Mathematica

Time used: 0.184 (sec). Leaf size: 197

```
DSolve[y'[x]==x^2-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{-ix^2\left(2\operatorname{BesselJ}\left(-\frac{3}{4}, \frac{ix^2}{2}\right) + c_1\left(\operatorname{BesselJ}\left(-\frac{5}{4}, \frac{ix^2}{2}\right) - \operatorname{BesselJ}\left(\frac{3}{4}, \frac{ix^2}{2}\right)\right)\right) - c_1 \operatorname{BesselJ}\left(-\frac{1}{4}, \frac{ix^2}{2}\right)}{2x\left(\operatorname{BesselJ}\left(\frac{1}{4}, \frac{ix^2}{2}\right) + c_1 \operatorname{BesselJ}\left(-\frac{1}{4}, \frac{ix^2}{2}\right)\right)}$$

$$y(x) \to \frac{ix^2 \operatorname{BesselJ}\left(-\frac{5}{4}, \frac{ix^2}{2}\right) - ix^2 \operatorname{BesselJ}\left(\frac{3}{4}, \frac{ix^2}{2}\right) + \operatorname{BesselJ}\left(-\frac{1}{4}, \frac{ix^2}{2}\right)}{2x \operatorname{BesselJ}\left(-\frac{1}{4}, \frac{ix^2}{2}\right)}$$

3.10 problem 5 (J)

Internal problem ID [12304]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.1, page 40
Problem number: 5 (J).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = -x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x)=y(x)^2-x^2,y(x), singsol=all)
```

$$y(x) = \frac{x\left(-\operatorname{BesselI}\left(-\frac{3}{4}, \frac{x^2}{2}\right) c_1 + \operatorname{BesselK}\left(\frac{3}{4}, \frac{x^2}{2}\right)\right)}{c_1 \operatorname{BesselI}\left(\frac{1}{4}, \frac{x^2}{2}\right) + \operatorname{BesselK}\left(\frac{1}{4}, \frac{x^2}{2}\right)}$$

✓ Solution by Mathematica

Time used: 0.178 (sec). Leaf size: 196

```
DSolve[y'[x]==y[x]^2-x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{ix^2\left(2\operatorname{BesselJ}\left(-\frac{3}{4}, \frac{ix^2}{2}\right) + c_1\left(\operatorname{BesselJ}\left(-\frac{5}{4}, \frac{ix^2}{2}\right) - \operatorname{BesselJ}\left(\frac{3}{4}, \frac{ix^2}{2}\right)\right)\right) + c_1 \operatorname{BesselJ}\left(-\frac{1}{4}, \frac{ix^2}{2}\right)}{2x\left(\operatorname{BesselJ}\left(\frac{1}{4}, \frac{ix^2}{2}\right) + c_1 \operatorname{BesselJ}\left(-\frac{1}{4}, \frac{ix^2}{2}\right)\right)}$$

$$y(x) \rightarrow -\frac{ix^2 \operatorname{BesselJ}\left(-\frac{5}{4}, \frac{ix^2}{2}\right) - ix^2 \operatorname{BesselJ}\left(\frac{3}{4}, \frac{ix^2}{2}\right) + \operatorname{BesselJ}\left(-\frac{1}{4}, \frac{ix^2}{2}\right)}{2x \operatorname{BesselJ}\left(-\frac{1}{4}, \frac{ix^2}{2}\right)}$$

3.11 problem 6

Internal problem ID [12305]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.1, page 40
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - y = x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)=x+y(x),y(x), singsol=all)
```

$$y(x) = -x - 1 + c_1 e^x$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 16

```
DSolve[y'[x]==x+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + c_1 e^x - 1$$

3.12 problem 7

Internal problem ID [12306]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.1, page 40
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=x*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{x^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 22

```
DSolve[y'[x]==x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{\frac{x^2}{2}}$$
$$y(x) \to 0$$

3.13 problem 8

Internal problem ID [12307]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.1, page 40
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{x}{y} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)=x/y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{x^2 + c_1}$$
$$y(x) = -\sqrt{x^2 + c_1}$$

✓ Solution by Mathematica

Time used: 0.127 (sec). Leaf size: 35

```
DSolve[y'[x]==x/y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x^2 + 2c_1}$$
$$y(x) \to \sqrt{x^2 + 2c_1}$$

3.14 problem 9

Internal problem ID [12308]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.1, page 40
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{y}{x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 7

```
dsolve(diff(y(x),x)=y(x)/x,y(x), singsol=all)
```

$$y(x) = c_1 x$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 14

```
DSolve[y'[x]==y[x]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x$$
$$y(x) \to 0$$

3.15 problem 10

Internal problem ID [12309]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.1, page 40
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^2 = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x)=1+y(x)^2,y(x), singsol=all)
```

$$y(x) = \tan(x + c_1)$$

✓ Solution by Mathematica

Time used: 0.145 (sec). Leaf size: 24

```
DSolve[y'[x]==1+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \tan(x + c_1)$$
$$y(x) \rightarrow -i$$
$$y(x) \rightarrow i$$

3.16 problem 11

Internal problem ID [12310]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.1, page 40
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^2 + 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=y(x)^2-3*y(x),y(x), singsol=all)
```

$$y(x) = \frac{3}{1 + 3c_1 e^{3x}}$$

✓ Solution by Mathematica

Time used: 0.352 (sec). Leaf size: 29

```
DSolve[y'[x]==y[x]^2-3*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3}{1 + e^{3(x+c_1)}}$$
$$y(x) \to 0$$
$$y(x) \to 3$$

3.17 problem 12

Internal problem ID [12311]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.1, page 40
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$\boxed{y' - y^3 = x^3}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=x^3+y(x)^3,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==x^3+y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

3.18 problem 13

Internal problem ID [12312]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.1, page 40
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - |y| = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)=abs(y(x)),y(x), singsol=all)
```

$$y(x) = \frac{e^{-x}}{c_1}$$
$$y(x) = c_1 e^{x}$$

✓ Solution by Mathematica

Time used: 0.229 (sec). Leaf size: 29

```
DSolve[y'[x]==Abs[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{|K[1]|} dK[1]\&\right][x + c_1]$$
$$y(x) \rightarrow 0$$

3.19 problem 14

Internal problem ID [12313]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.1, page 40
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \mathrm{e}^{-y+x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)=exp(x-y(x)),y(x), singsol=all)
```

$$y(x) = \ln\left(\mathrm{e}^{x} + c_1\right)$$

✓ Solution by Mathematica

Time used: 1.319 (sec). Leaf size: 12

```
DSolve[y'[x]==Exp[x-y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(e^{x} + c_1\right)$$

3.20 problem 15

Internal problem ID [12314]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.1, page 40
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$\boxed{y' - \ln(x + y) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)=ln(x+y(x)),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{RootOf}\left(c_1 \mathrm{e} - x\,\mathrm{e} - \mathrm{Ei}_1(-\_Z - 1)\right)} - x$$

✓ Solution by Mathematica

Time used: 0.207 (sec). Leaf size: 22

```
DSolve[y'[x]==Log[x+y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\text{ExpIntegralEi}(\log(x + y(x)) + 1)}{e} - x = c_1, y(x)\right]$$

3.21 problem 16

Internal problem ID [12315]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.1, page 40
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{2x - y}{x + 3y} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 53

```
dsolve(diff(y(x),x)=(2*x-y(x))/(x+3*y(x)),y(x), singsol=all)
```

$$y(x) = \frac{-\frac{c_1 x}{3} - \frac{\sqrt{7c_1^2x^2+3}}{3}}{c_1}$$

$$y(x) = \frac{-\frac{c_1 x}{3} + \frac{\sqrt{7c_1^2x^2+3}}{3}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.812 (sec). Leaf size: 114

```
DSolve[y'[x]==(2*x-y[x])/(x+3*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(-x - \sqrt{7x^2 + 3e^{2c_1}}\right)$$
$$y(x) \to \frac{1}{3}\left(-x + \sqrt{7x^2 + 3e^{2c_1}}\right)$$
$$y(x) \to \frac{1}{3}\left(-\sqrt{7}\sqrt{x^2} - x\right)$$
$$y(x) \to \frac{1}{3}\left(\sqrt{7}\sqrt{x^2} - x\right)$$

3.22 problem 17

Internal problem ID [12316]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.1, page 40
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - \frac{1}{\sqrt{15 - x^2 - y^2}} = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=1/sqrt(15-x^2-y(x)^2),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==1/Sqrt[15-x^2-y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4 Chapter 2. The Initial Value Problem. Exercises 2.2, page 53

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4.1 problem 1

Internal problem ID [12317]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.2, page 53
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{3y}{(x-5)(x+3)} = \mathrm{e}^{-x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x)=3*y(x)/( (x-5)*(x+3))+exp(-x),y(x), singsol=all)
```

$$y(x) = \frac{(x-5)^{\frac{3}{8}} \left(\int \frac{(x+3)^{\frac{3}{8}} \mathrm{e}^{-x}}{(x-5)^{\frac{3}{8}}} dx + c_1\right)}{(x+3)^{\frac{3}{8}}}$$

✓ Solution by Mathematica

Time used: 15.323 (sec). Leaf size: 57

```
DSolve[y'[x]==3*y[x]/( (x-5)*(x+3))+Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(5-x)^{3/8} \left(\int_1^x \frac{e^{-K[1]}(K[1]+3)^{3/8}}{(5-K[1])^{3/8}} dK[1] + c_1\right)}{(x+3)^{3/8}}$$

4.2 problem 2

Internal problem ID [12318]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.2, page 53
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y' - \frac{xy}{x^2 + y^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=x*y(x)/( x^2+y(x)^2),y(x), singsol=all)
```

$$y(x) = \sqrt{\frac{1}{\text{LambertW}\left(c_1 x^2\right)}}\, x$$

✓ Solution by Mathematica

Time used: 11.187 (sec). Leaf size: 49

```
DSolve[y'[x]==x*y[x]/( x^2+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{\sqrt{W\left(e^{-2c_1}x^2\right)}}$$
$$y(x) \to \frac{x}{\sqrt{W\left(e^{-2c_1}x^2\right)}}$$
$$y(x) \to 0$$

4.3 problem 3

Internal problem ID [12319]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.2, page 53
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{1}{yx} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)=1/( x*y(x)),y(x), singsol=all)
```

$$y(x) = \sqrt{2\ln(x) + c_1}$$
$$y(x) = -\sqrt{2\ln(x) + c_1}$$

✓ Solution by Mathematica

Time used: 0.1 (sec). Leaf size: 40

```
DSolve[y'[x]==1/(x*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}\sqrt{\log(x) + c_1}$$
$$y(x) \to \sqrt{2}\sqrt{\log(x) + c_1}$$

4.4 problem 4

Internal problem ID [12320]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.2, page 53
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \ln(y-1) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=ln(y(x)-1),y(x), singsol=all)
```

$$y(x) = e^{\mathrm{RootOf}(\mathrm{Ei}_1(-\_Z)+x+c_1)} + 1$$

✓ Solution by Mathematica

Time used: 0.29 (sec). Leaf size: 21

```
DSolve[y'[x]==Log[y[x]-1],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \mathrm{InverseFunction}[\mathrm{LogIntegral}(\#1-1)\&][x+c_1]$$
$$y(x) \to 2$$

4.5 problem 5

Internal problem ID [12321]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.2, page 53
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \sqrt{(y+2)(y-1)} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)=sqrt( (y(x)+2)*(y(x)-1)),y(x), singsol=all)
```

$$x - \ln\left(y(x) + \frac{1}{2} + \sqrt{-2 + y(x)^2 + y(x)}\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.497 (sec). Leaf size: 41

```
DSolve[y'[x]==Sqrt[ (y[x]+2)*(y[x]-1)],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-e^{-x-c_1} - 9e^{x+c_1} - 2\right)$$
$$y(x) \to -2$$
$$y(x) \to 1$$

4.6 problem 6

Internal problem ID [12322]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.2, page 53
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{y}{y - x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x)=y(x)/(y(x)-x),y(x), singsol=all)
```

$$y(x) = x - \sqrt{x^2 - 2c_1}$$
$$y(x) = x + \sqrt{x^2 - 2c_1}$$

✓ Solution by Mathematica

Time used: 0.836 (sec). Leaf size: 80

```
DSolve[y'[x]==y[x]/(y[x]-x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - \sqrt{x^2 - e^{2c_1}}$$
$$y(x) \to x + \sqrt{x^2 - e^{2c_1}}$$
$$y(x) \to 0$$
$$y(x) \to x - \sqrt{x^2}$$
$$y(x) \to \sqrt{x^2} + x$$

4.7 problem 7

Internal problem ID [12323]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.2, page 53
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{x}{y^2} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)=x/y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{(12x^2 + 8c_1)^{\frac{1}{3}}}{2}$$
$$y(x) = -\frac{(12x^2 + 8c_1)^{\frac{1}{3}}}{4} - \frac{i\sqrt{3}\,(12x^2 + 8c_1)^{\frac{1}{3}}}{4}$$
$$y(x) = -\frac{(12x^2 + 8c_1)^{\frac{1}{3}}}{4} + \frac{i\sqrt{3}\,(12x^2 + 8c_1)^{\frac{1}{3}}}{4}$$

✓ Solution by Mathematica

Time used: 0.283 (sec). Leaf size: 79

```
DSolve[y'[x]==x/y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt[3]{-\frac{3}{2}}\sqrt[3]{x^2+2c_1}$$
$$y(x) \to \sqrt[3]{\frac{3}{2}}\sqrt[3]{x^2+2c_1}$$
$$y(x) \to (-1)^{2/3}\sqrt[3]{\frac{3}{2}}\sqrt[3]{x^2+2c_1}$$

4.8 problem 8

Internal problem ID [12324]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.2, page 53
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{\sqrt{y}}{x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=sqrt(y(x))/x,y(x), singsol=all)
```

$$\sqrt{y\left(x\right)} - \frac{\ln\left(x\right)}{2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.202 (sec). Leaf size: 21

```
DSolve[y'[x]==Sqrt[y[x]]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}(\log(x) + c_1)^2$$
$$y(x) \to 0$$

4.9 problem 9

Internal problem ID [12325]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.2, page 53
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{xy}{1-y} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)=x*y(x)/(1-y(x)),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\operatorname{LambertW}\left(-\mathrm{e}^{\frac{x^2}{2}+c_1}\right)+\frac{x^2}{2}+c_1}$$

✓ Solution by Mathematica

Time used: 3.96 (sec). Leaf size: 29

```
DSolve[y'[x]==x*y[x]/(1-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -W\left(-e^{\frac{x^2}{2}-c_1}\right)$$
$$y(x) \to 0$$

4.10 problem 10

Internal problem ID [12326]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.2, page 53
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{y' - (yx)^{\frac{1}{3}} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 120

```
dsolve(diff(y(x),x)=(x*y(x))^(1/3),y(x), singsol=all)
```

$$\frac{4(xy(x))^{\frac{4}{3}}}{\left(-x^4+8y\left(x\right)^2\right)\left(2\left(xy\left(x\right)\right)^{\frac{2}{3}}-x^2\right)^2}+\frac{2x^2(xy(x))^{\frac{2}{3}}}{\left(-x^4+8y\left(x\right)^2\right)\left(2\left(xy\left(x\right)\right)^{\frac{2}{3}}-x^2\right)^2}+\frac{x^4}{\left(-x^4+8y\left(x\right)^2\right)\left(2\left(xy\left(x\right)\right)^{\frac{2}{3}}-x^2\right)^2}-c_1=0$$

✓ Solution by Mathematica

Time used: 4.979 (sec). Leaf size: 35

```
DSolve[y'[x]==(x*y[x])^(1/3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\left(3x^{4/3}+4c_1\right)^{3/2}}{6\sqrt{6}}$$
$$y(x) \to 0$$

4.11 problem 11

Internal problem ID [12327]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.2, page 53
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - \sqrt{\frac{y-4}{x}} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 41

```
dsolve(diff(y(x),x)=sqrt( (y(x)-4)/x ),y(x), singsol=all)
```

$$-\ln\left(\frac{x-y(x)+4}{x}\right)+2\,\mathrm{arctanh}\left(\sqrt{-\frac{-y\left(x\right)+4}{x}}\right)-\ln\left(x\right)-c_1=0$$

✓ Solution by Mathematica

Time used: 0.294 (sec). Leaf size: 29

```
DSolve[y'[x]==Sqrt[ (y[x]-4)/x ],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + c_1\sqrt{x} + 4 + \frac{c_1{}^2}{4}$$
$$y(x) \to 4$$

4.12 problem 12

Internal problem ID [12328]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.2, page 53
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y' + \frac{y}{x} - y^{\frac{1}{4}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=-y(x)/x+y(x)^(1/4),y(x), singsol=all)
```

$$y(x)^{\frac{3}{4}} - \frac{3x}{7} - \frac{c_1}{x^{\frac{3}{4}}} = 0$$

✓ Solution by Mathematica

Time used: 9.843 (sec). Leaf size: 31

```
DSolve[y'[x]==-y[x]/x+y[x]^(1/4),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\left(3x + \frac{7c_1}{x^{3/4}}\right)^{4/3}}{7\sqrt[3]{7}}$$

4.13 problem 13

Internal problem ID [12329]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.2, page 53
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 4y = -5}$$

With initial conditions

$$[y(1) = 4]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)=4*y(x)-5,y(1) = 4],y(x), singsol=all)
```

$$y(x) = \frac{11\,\mathrm{e}^{4x-4}}{4} + \frac{5}{4}$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 20

```
DSolve[{y'[x]==4*y[x]-5,{y[1]==4}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{11}{4}e^{4x-4} + \frac{5}{4}$$

4.14 problem 14

Internal problem ID [12330]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.2, page 53
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + 3y = 1}$$

With initial conditions

$$[y(-2) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)+3*y(x)=1,y(-2) = 1],y(x), singsol=all)
```

$$y(x) = \frac{2\,\mathrm{e}^{-3x-6}}{3} + \frac{1}{3}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 20

```
DSolve[{y'[x]+3*y[x]==1,{y[-2]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{3}e^{-3(x+2)} + \frac{1}{3}$$

4.15 problem 15

Internal problem ID [12331]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.2, page 53
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - ay = b}$$

With initial conditions

$$[y(c) = d]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve([diff(y(x),x)=a*y(x)+b,y(c) = d],y(x), singsol=all)
```

$$y(x) = \frac{(da + b)\,\mathrm{e}^{-a(c-x)} - b}{a}$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 39

```
DSolve[{y'[x]==a*y[x]+b,{y[c]==d}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ac}(b(e^{ax} - e^{ac}) + ade^{ax})}{a}$$

4.16 problem 16

Internal problem ID [12332]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.2, page 53
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = x^2 + e^x - \sin(x)}$$

With initial conditions

$$[y(2) = -1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve([diff(y(x),x)=x^2+exp(x)-sin(x),y(2) = -1],y(x), singsol=all)
```

$$y(x) = \frac{x^3}{3} + \cos(x) + e^x - \frac{11}{3} - \cos(2) - e^2$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 30

```
DSolve[{y'[x]==x^2+Exp[x]-Sin[x],{y[2]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{3} + e^x + \cos(x) - e^2 - \frac{11}{3} - \cos(2)$$

4.17 problem 17

Internal problem ID [12333]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.2, page 53
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - yx = \frac{1}{x^2 + 1}}$$

With initial conditions

$$[y(-5) = 0]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 31

```
dsolve([diff(y(x),x)=x*y(x)+1/(1+x^2),y(-5) = 0],y(x), singsol=all)
```

$$y(x) = \left(\int_{-5}^{x} \frac{e^{-\frac{\_z1^2}{2}}}{\_z1^2 + 1} d\_z1\right) e^{\frac{x^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.478 (sec). Leaf size: 41

```
DSolve[{y'[x]==x*y[x]+1/(1+x^2),{y[-5]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{x^2}{2}} \int_{-5}^{x} \frac{e^{-\frac{1}{2}K[1]^2}}{K[1]^2 + 1} dK[1]$$

4.18 problem 18

Internal problem ID [12334]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.2, page 53
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{y}{x} = \cos(x)}$$

With initial conditions

$$[y(-1) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(x),x)=y(x)/x+cos(x),y(-1) = 0],y(x), singsol=all)
```

$$y(x) = -(i\pi + \mathrm{Ci}(1) - \mathrm{Ci}(x))\, x$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 14

```
DSolve[{y'[x]==y[x]/x+Cos[x],{y[-1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(\mathrm{CosIntegral}(x) - \mathrm{CosIntegral}(-1))$$

4.19 problem 19

Internal problem ID [12335]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.2, page 53
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{y}{x} = \tan(x)}$$

With initial conditions

$$[y(\pi) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve([diff(y(x),x)=y(x)/x+tan(x),y(Pi) = 0],y(x), singsol=all)
```

$$y(x) = \left(\int_{\pi}^{x} \frac{\tan(\_z1)}{\_z1} d\_z1\right) x$$

✓ Solution by Mathematica

Time used: 1.98 (sec). Leaf size: 22

```
DSolve[{y'[x]==y[x]/x+Tan[x],{y[Pi]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \int_{\pi}^{x} \frac{\tan(K[1])}{K[1]} dK[1]$$

4.20 problem 20

Internal problem ID [12336]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.2, page 53
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{y}{-x^2+4} = \sqrt{x}}$$

With initial conditions

$$[y(3) = 4]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 41

```
dsolve([diff(y(x),x)=y(x)/(4-x^2)+sqrt(x),y(3) = 4],y(x), singsol=all)
```

$$y(x) = \frac{\left(4\,5^{\frac{3}{4}} + 5\left(\int_3^x \frac{(\_z1-2)^{\frac{1}{4}}\sqrt{\_z1}}{(\_z1+2)^{\frac{1}{4}}} d\_z1\right)\right)(x+2)^{\frac{1}{4}}}{5\,(x-2)^{\frac{1}{4}}}$$

✓ Solution by Mathematica

Time used: 2.843 (sec). Leaf size: 202

```
DSolve[{y'[x]==y[x]/(4-x^2)+Sqrt[x],{y[3]==4}},y[x],x,IncludeSingularSolutions -> True]
```

$y(x)$

$$\to \frac{\left(\frac{1}{45}+\frac{i}{45}\right)\sqrt[4]{x+2}\left((10-10i)x^{3/2}\,\text{AppellF1}\left(\frac{3}{2},\frac{3}{4},\frac{1}{4},\frac{5}{2},\frac{x}{2},-\frac{x}{2}\right)-(30-30i)\sqrt{x}\,\text{AppellF1}\left(\frac{1}{2},\frac{3}{4},\frac{1}{4},\frac{3}{2},\frac{x}{2},-\right.\right.}{}$$

4.21 problem 21

Internal problem ID [12337]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.2, page 53
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{y}{-x^2+4} = \sqrt{x}}$$

With initial conditions

$$[y(1) = -3]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 44

```
dsolve([diff(y(x),x)=y(x)/(4-x^2)+sqrt(x),y(1) = -3],y(x), singsol=all)
```

$$y(x) = -\frac{(x+2)^{\frac{1}{4}} \left(-2\left(\int_1^x \frac{(\_z1-2)^{\frac{1}{4}}\sqrt{\_z1}}{(\_z1+2)^{\frac{1}{4}}} d\_z1\right) + (1+i)\sqrt{2}\, 3^{\frac{3}{4}}\right)}{2(x-2)^{\frac{1}{4}}}$$

✓ Solution by Mathematica

Time used: 0.145 (sec). Leaf size: 158

```
DSolve[{y'[x]==y[x]/(4-x^2)+Sqrt[x],{y[1]==-3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[4]{x+2}\left(4x^{3/2}\,\text{AppellF1}\left(\frac{3}{2},\frac{3}{4},\frac{1}{4},\frac{5}{2},\frac{x}{2},-\frac{x}{2}\right) - 12\sqrt{x}\,\text{AppellF1}\left(\frac{1}{2},\frac{3}{4},\frac{1}{4},\frac{3}{2},\frac{x}{2},-\frac{x}{2}\right) - 4\,\text{AppellF1}\left(\frac{3}{2},\frac{3}{4},\frac{1}{4},\frac{5}{2},\right.\right.}{9\sqrt[4]{2-x}}$$

4.22 problem 22

Internal problem ID [12338]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.2, page 53
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - y\cot(x) = \csc(x)}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = 1\right]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve([diff(y(x),x)=cot(x)*y(x)+csc(x),y(1/2*Pi) = 1],y(x), singsol=all)
```

$$y(x) = -\cos(x) + \sin(x)$$

✓ Solution by Mathematica

Time used: 0.082 (sec). Leaf size: 12

```
DSolve[{y'[x]==Cot[x]*y[x]+Csc[x],{y[Pi/2]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x) - \cos(x)$$

4.23 problem 23

Internal problem ID [12339]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.2, page 53
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + x\sqrt{-y^2 + 1} = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=-x*sqrt(1-y(x)^2),y(0) = 1],y(x), singsol=all)
```

$$y(x) = 1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 6

```
DSolve[{y'[x]==-x*Sqrt[1-y[x]^2],{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1$$

4.24 problem 24

Internal problem ID [12340]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.2, page 53
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y' - \frac{\sqrt{x^2+4y}}{2} = -\frac{x}{2}}$$

With initial conditions

$$[y(6) = -9]$$

✓ Solution by Maple

Time used: 0.844 (sec). Leaf size: 17

```
dsolve([diff(y(x),x)=(-x+sqrt(x^2+4*y(x)))/2,y(6) = -9],y(x), singsol=all)
```

$$y(x) = 9 - 3x$$
$$y(x) = -\frac{x^2}{4}$$

✓ Solution by Mathematica

Time used: 0.987 (sec). Leaf size: 10

```
DSolve[{y'[x]==(-x+Sqrt[x^2+4*y[x]])/2,{y[6]==-9}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 9 - 3x$$

5 Chapter 2. The Initial Value Problem. Exercises 2.3.1, page 57

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5.1 problem 1

Internal problem ID [12341]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.1, page 57
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = 3x + 1}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)=3*x+1,y(1) = 2],y(x), singsol=all)
```

$$y(x) = \frac{3}{2}x^2 + x - \frac{1}{2}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 17

```
DSolve[{y'[x]==3*x+1,{y[1]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3x^2}{2} + x - \frac{1}{2}$$

5.2 problem 2

Internal problem ID [12342]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.1, page 57
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = x + \frac{1}{x}}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)=x+1/x,y(1) = 2],y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} + \ln(x) + \frac{3}{2}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 18

```
DSolve[{y'[x]==x+1/x,{y[1]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(x^2 + 2\log(x) + 3)$$

5.3 problem 3

Internal problem ID [12343]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.1, page 57
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = 2\sin(x)}$$

With initial conditions

$$[y(\pi) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve([diff(y(x),x)=2*sin(x),y(Pi) = 1],y(x), singsol=all)
```

$$y(x) = -2\cos(x) - 1$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 11

```
DSolve[{y'[x]==2*Sin[x],{y[Pi]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2\cos(x) - 1$$

5.4 problem 4

Internal problem ID [12344]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.1, page 57
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = x \sin(x)}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = 1\right]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)=x*sin(x),y(1/2*Pi) = 1],y(x), singsol=all)
```

$$y(x) = \sin(x) - x\cos(x)$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 13

```
DSolve[{y'[x]==x*Sin[x],{y[Pi/2]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x) - x\cos(x)$$

5.5 problem 5

Internal problem ID [12345]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.1, page 57
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{1}{x-1}}$$

With initial conditions

$$[y(2) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 10

```
dsolve([diff(y(x),x)=1/(x-1),y(2) = 1],y(x), singsol=all)
```

$$y(x) = \ln(x-1) + 1$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 11

```
DSolve[{y'[x]==1/(x-1),{y[2]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(x-1) + 1$$

5.6 problem 6

Internal problem ID [12346]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.1, page 57
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{1}{x-1}}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)=1/(x-1),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \ln(x-1) - i\pi + 1$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 16

```
DSolve[{y'[x]==1/(x-1),{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(x-1) - i\pi + 1$$

5.7 problem 7

Internal problem ID [12347]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.1, page 57
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{1}{x^2 - 1}}$$

With initial conditions

$$[y(2) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve([diff(y(x),x)=1/(x^2-1),y(2) = 1],y(x), singsol=all)
```

$$y(x) = -\operatorname{arctanh}(x) + \operatorname{arctanh}\left(\frac{1}{2}\right) - \frac{i\pi}{2} + 1$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 28

```
DSolve[{y'[x]==1/(x^2-1),{y[2]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(\log(3 - 3x) - \log(x + 1) - i\pi + 2)$$

5.8 problem 8

Internal problem ID [12348]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.1, page 57
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{1}{x^2 - 1}}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve([diff(y(x),x)=1/(x^2-1),y(0) = 1],y(x), singsol=all)
```

$$y(x) = -\operatorname{arctanh}(x) + 1$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 23

```
DSolve[{y'[x]==1/(x^2-1),{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(\log(1 - x) - \log(x + 1) + 2)$$

5.9 problem 9

Internal problem ID [12349]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.1, page 57
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \tan(x)}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 9

```
dsolve([diff(y(x),x)=tan(x),y(0) = 0],y(x), singsol=all)
```

$$y(x) = -\ln(\cos(x))$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 10

```
DSolve[{y'[x]==Tan[x],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\log(\cos(x))$$

5.10 problem 10

Internal problem ID [12350]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.1, page 57
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \tan(x)}$$

With initial conditions

$$[y(\pi) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)=tan(x),y(Pi) = 0],y(x), singsol=all)
```

$$y(x) = -\ln(\cos(x)) + i\pi$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 16

```
DSolve[{y'[x]==Tan[x],{y[Pi]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\log(\cos(x)) + i\pi$$

6 Chapter 2. The Initial Value Problem. Exercises 2.3.2, page 63

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6.1 problem 1

Internal problem ID [12351]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.2, page 63
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 3y = 0}$$

With initial conditions

$$[y(0) = -1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 10

```
dsolve([diff(y(x),x)=3*y(x),y(0) = -1],y(x), singsol=all)
```

$$y(x) = -\mathrm{e}^{3x}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 12

```
DSolve[{y'[x]==3*y[x],{y[0]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{3x}$$

6.2 problem 2

Internal problem ID [12352]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.2, page 63
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + y = 1}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=-y(x)+1,y(0) = 1],y(x), singsol=all)
```

$$y(x) = 1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 6

```
DSolve[{y'[x]==-y[x]+1,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1$$

6.3 problem 3

Internal problem ID [12353]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.2, page 63
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + y = 1}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve([diff(y(x),x)=-y(x)+1,y(0) = 2],y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} + 1$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 12

```
DSolve[{y'[x]==-y[x]+1,{y[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x} + 1$$

6.4 problem 4

Internal problem ID [12354]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.2, page 63
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - x\,\mathrm{e}^{-x^2+y} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 19

```
dsolve([diff(y(x),x)=x*exp(y(x)-x^2),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \ln(2) - \ln\left(\mathrm{e}^{x^2} + 1\right) + x^2$$

✓ Solution by Mathematica

Time used: 2.407 (sec). Leaf size: 21

```
DSolve[{y'[x]==x*Exp[y[x]-x^2],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\log\left(\frac{1}{2}\left(e^{-x^2} + 1\right)\right)$$

6.5 problem 5

Internal problem ID [12355]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.2, page 63
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{y}{x} = 0}$$

With initial conditions

$$[y(-1) = 2]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 7

```
dsolve([diff(y(x),x)=y(x)/x,y(-1) = 2],y(x), singsol=all)
```

$$y(x) = -2x$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 8

```
DSolve[{y'[x]==y[x]/x,{y[-1]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2x$$

6.6 problem 6

Internal problem ID [12356]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.2, page 63
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{2x}{y} = 0}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)=2*x/y(x),y(0) = 2],y(x), singsol=all)
```

$$y(x) = \sqrt{2x^2 + 4}$$

✓ Solution by Mathematica

Time used: 0.121 (sec). Leaf size: 20

```
DSolve[{y'[x]==2*x/y[x],{y[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{2}\sqrt{x^2 + 2}$$

6.7 problem 7

Internal problem ID [12357]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.2, page 63
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + 2y - y^2 = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)=-2*y(x)+y(x)^2,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{2}{\mathrm{e}^{2x} + 1}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 16

```
DSolve[{y'[x]==-2*y[x]+y[x]^2,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{e^{2x} + 1}$$

6.8 problem 8

Internal problem ID [12358]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.2, page 63
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - yx = x}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(x),x)=x*y(x)+x,y(1) = 2],y(x), singsol=all)
```

$$y(x) = -1 + 3\,\mathrm{e}^{\frac{(x-1)(x+1)}{2}}$$

✓ Solution by Mathematica

Time used: 0.099 (sec). Leaf size: 20

```
DSolve[{y'[x]==x*y[x]+x,{y[1]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3e^{\frac{1}{2}(x^2-1)} - 1$$

6.9 problem 9

Internal problem ID [12359]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.2, page 63
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x\,\mathrm{e}^{y} + y' = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 15

```
dsolve([x*exp(y(x))+diff(y(x),x)=0,y(0) = 0],y(x), singsol=all)
```

$$y(x) = \ln(2) - \ln(x^2 + 2)$$

✓ Solution by Mathematica

Time used: 0.476 (sec). Leaf size: 16

```
DSolve[{x*Exp[y[x]]+y'[x]==0,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(2) - \log(x^2 + 2)$$

6.10 problem 10

Internal problem ID [12360]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.2, page 63
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y - x^2 y' = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve([y(x)-x^2*diff(y(x),x)=0,y(1) = 1],y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{x-1}{x}}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 14

```
DSolve[{y[x]-x^2*y'[x]==0,{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{1-\frac{1}{x}}$$

6.11 problem 11

Internal problem ID [12361]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.2, page 63
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{2yy' = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(2*y(x)*diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x) = \sqrt{x + c_1}$$
$$y(x) = -\sqrt{x + c_1}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 31

```
DSolve[2*y[x]*y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x + 2c_1}$$
$$y(x) \to \sqrt{x + 2c_1}$$

6.12 problem 12

Internal problem ID [12362]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.2, page 63
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2xyy' + y^2 = -1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(2*x*y(x)*diff(y(x),x)+y(x)^2=-1,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{x(-x+c_1)}}{x}$$
$$y(x) = -\frac{\sqrt{x(-x+c_1)}}{x}$$

✓ Solution by Mathematica

Time used: 0.471 (sec). Leaf size: 98

```
DSolve[2*x*y[x]*y'[x]+y[x]^2==-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-x+e^{2c_1}}}{\sqrt{x}}$$
$$y(x) \to \frac{\sqrt{-x+e^{2c_1}}}{\sqrt{x}}$$
$$y(x) \to -i$$
$$y(x) \to i$$
$$y(x) \to \frac{\sqrt{-x}}{\sqrt{x}}$$
$$y(x) \to \frac{\sqrt{x}}{\sqrt{-x}}$$

6.13 problem 13

Internal problem ID [12363]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.2, page 63
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{1 - yx}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=(1-x*y(x))/x^2,y(x), singsol=all)
```

$$y(x) = \frac{\ln(x) + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 14

```
DSolve[y'[x]==(1-x*y[x])/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\log(x) + c_1}{x}$$

6.14 problem 14

Internal problem ID [12364]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.2, page 63
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' + \frac{y(2x + y)}{x\,(x + 2y)} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 69

```
dsolve(diff(y(x),x)=-y(x)*(2*x+y(x))/(x*(2*y(x)+x)),y(x), singsol=all)
```

$$y(x) = -\frac{x^2 c_1^2 - \sqrt{c_1^4 x^4 + 4c_1 x}}{2x\,c_1^2}$$
$$y(x) = -\frac{x^2 c_1^2 + \sqrt{c_1^4 x^4 + 4c_1 x}}{2x\,c_1^2}$$

✓ Solution by Mathematica

Time used: 1.084 (sec). Leaf size: 118

```
DSolve[y'[x]==-y[x]*(2*x+y[x])/(x*(2*y[x]+x)),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-x - \frac{\sqrt{x^3 + 4e^{c_1}}}{\sqrt{x}}\right)$$

$$y(x) \to \frac{1}{2}\left(-x + \frac{\sqrt{x^3 + 4e^{c_1}}}{\sqrt{x}}\right)$$

$$y(x) \to -\frac{x^{3/2} + \sqrt{x^3}}{2\sqrt{x}}$$

$$y(x) \to \frac{\sqrt{x^3}}{2\sqrt{x}} - \frac{x}{2}$$

6.15 problem 15

Internal problem ID [12365]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.2, page 63
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{y^2}{1 - yx} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)=y(x)^2/(1-x*y(x)),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\operatorname{LambertW}\left(-x\,\mathrm{e}^{-c_1}\right)-c_1}$$

✓ Solution by Mathematica

Time used: 3.256 (sec). Leaf size: 25

```
DSolve[y'[x]==y[x]^2/(1-x*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{W(-e^{-c_1}x)}{x}$$
$$y(x) \to 0$$

7 Chapter 2. The Initial Value Problem. Exercises 2.3.3, page 71

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7.1 problem 1

Internal problem ID [12366]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.3, page 71
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 4y = 1}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)=4*y(x)+1,y(0) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{1}{4} + \frac{5\,\mathrm{e}^{4x}}{4}$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 18

```
DSolve[{y'[x]==4*y[x]+1,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(5e^{4x} - 1\right)$$

7.2 problem 2

Internal problem ID [12367]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.3, page 71
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - yx = 2}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve([diff(y(x),x)=x*y(x)+2,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \left(\sqrt{\pi}\,\sqrt{2}\,\operatorname{erf}\left(\frac{\sqrt{2}\,x}{2}\right) + 1\right) \mathrm{e}^{\frac{x^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 33

```
DSolve[{y'[x]==x*y[x]+2,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{x^2}{2}}\left(\sqrt{2\pi}\operatorname{erf}\left(\frac{x}{\sqrt{2}}\right) + 1\right)$$

7.3 problem 3

Internal problem ID [12368]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.3, page 71
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{y}{x} = 0}$$

With initial conditions

$$[y(-1) = 2]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 7

```
dsolve([diff(y(x),x)=y(x)/x,y(-1) = 2],y(x), singsol=all)
```

$$y(x) = -2x$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 8

```
DSolve[{y'[x]==y[x]/x,{y[-1]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2x$$

7.4 problem 4

Internal problem ID [12369]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.3, page 71
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{y}{x-1} = x^2}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve([diff(y(x),x)=y(x)/(x-1)+x^2,y(0) = 1],y(x), singsol=all)
```

$$y(x) = -(x-1)\left(i\pi - \frac{x^2}{2} - x - \ln(x-1) + 1\right)$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 31

```
DSolve[{y'[x]==y[x]/(x-1)+x^2,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(x-1)\left(x^2 + 2x + 2\log(x-1) - 2i\pi - 2\right)$$

7.5 problem 5

Internal problem ID [12370]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.3, page 71
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{y}{x} = \sin\left(x^2\right)}$$

With initial conditions

$$[y(-1) = -1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(x),x)=y(x)/x+sin(x^2),y(-1) = -1],y(x), singsol=all)
```

$$y(x) = -\frac{x(-\operatorname{Si}\left(x^2\right) - 2 + \operatorname{Si}(1))}{2}$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 20

```
DSolve[{y'[x]==y[x]/x+Sin[x^2],{y[-1]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}x\left(\operatorname{Si}\left(x^2\right) - \operatorname{Si}(1) + 2\right)$$

7.6 problem 6

Internal problem ID [12371]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.3, page 71
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{2y}{x} = \mathrm{e}^{x}}$$

With initial conditions

$$\left[y(1) = \frac{1}{2}\right]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve([diff(y(x),x)=2*y(x)/x+exp(x),y(1) = 1/2],y(x), singsol=all)
```

$$y(x) = -\operatorname{Ei}_1(-x)\,x^2 + \operatorname{Ei}_1(-1)\,x^2 + \frac{x(2x\,\mathrm{e} + x - 2\,\mathrm{e}^{x})}{2}$$

✓ Solution by Mathematica

Time used: 0.116 (sec). Leaf size: 31

```
DSolve[{y'[x]==2*y[x]/x+Exp[x],{y[1]==1/2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}x(2x\,\text{ExpIntegralEi}(x) - 2\,\text{ExpIntegralEi}(1)x + 2ex + x - 2e^{x})$$

7.7 problem 7

Internal problem ID [12372]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.3, page 71
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - y\cot(x) = \sin(x)}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = 0\right]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)=cot(x)*y(x)+sin(x),y(1/2*Pi) = 0],y(x), singsol=all)
```

$$y(x) = \left(x - \frac{\pi}{2}\right)\sin(x)$$

✓ Solution by Mathematica

Time used: 0.088 (sec). Leaf size: 16

```
DSolve[{y'[x]==Cot[x]*y[x]+Sin[x],{y[Pi/2]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}(\pi - 2x)\sin(x)$$

7.8 problem 12

Internal problem ID [12373]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.3, page 71
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{-yy' = -x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x-y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x^2 + c_1}$$
$$y(x) = -\sqrt{x^2 + c_1}$$

✓ Solution by Mathematica

Time used: 0.132 (sec). Leaf size: 35

```
DSolve[x-y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x^2 + 2c_1}$$
$$y(x) \to \sqrt{x^2 + 2c_1}$$

7.9 problem 13

Internal problem ID [12374]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.3, page 71
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y - xy' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 7

```
dsolve(y(x)-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 14

```
DSolve[y[x]-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x$$
$$y(x) \to 0$$

7.10 problem 14

Internal problem ID [12375]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.3, page 71
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{-y + xy' = -x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve((x^2-y(x))+x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = x(-x + c_1)$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 13

```
DSolve[(x^2-y[x])+x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(-x + c_1)$$

7.11 problem 15

Internal problem ID [12376]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.3, page 71
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy(1-y)-2y'=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*y(x)*(1-y(x))-2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\frac{1}{1+\mathrm{e}^{-\frac{x^2}{4}}c_1}$$

✓ Solution by Mathematica

Time used: 0.392 (sec). Leaf size: 41

```
DSolve[x*y[x]*(1-y[x])-2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{e^{\frac{x^2}{4}}}{e^{\frac{x^2}{4}}+e^{c_1}}$$
$$y(x)\to 0$$
$$y(x)\to 1$$

7.12 problem 16

Internal problem ID [12377]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.3, page 71
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x(1-y^3)-3y^2y'=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 88

```
dsolve(x*(1-y(x)^3)-3*y(x)^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \left(1+c_1\mathrm{e}^{-\frac{x^2}{2}}\right)^{\frac{1}{3}}$$
$$y(x) = -\frac{\left(1+c_1\mathrm{e}^{-\frac{x^2}{2}}\right)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}\left(1+c_1\mathrm{e}^{-\frac{x^2}{2}}\right)^{\frac{1}{3}}}{2}$$
$$y(x) = -\frac{\left(1+c_1\mathrm{e}^{-\frac{x^2}{2}}\right)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}\left(1+c_1\mathrm{e}^{-\frac{x^2}{2}}\right)^{\frac{1}{3}}}{2}$$

✓ Solution by Mathematica

Time used: 2.121 (sec). Leaf size: 111

```
DSolve[x*(1-y[x]^3)-3*y[x]^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[3]{1 + e^{-\frac{x^2}{2}+3c_1}}$$
$$y(x) \to -\sqrt[3]{-1}\sqrt[3]{1 + e^{-\frac{x^2}{2}+3c_1}}$$
$$y(x) \to (-1)^{2/3}\sqrt[3]{1 + e^{-\frac{x^2}{2}+3c_1}}$$
$$y(x) \to 1$$
$$y(x) \to -\sqrt[3]{-1}$$
$$y(x) \to (-1)^{2/3}$$

7.13 problem 17

Internal problem ID [12378]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.3.3, page 71
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y(2x-1)+x(x+1)\,y'=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(y(x)*(2*x-1)+x*(x+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x}{(x+1)^3}$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 19

```
DSolve[y[x]*(2*x-1)+x*(x+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_1x}{(x+1)^3}$$
$$y(x)\to 0$$

8 Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115

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8.1 problem 1

Internal problem ID [12379]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{1}{x-1}}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)=1/(x-1),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \ln (x-1) - i\pi + 1$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 16

```
DSolve[{y'[x]==1/(x-1),{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(x-1) - i\pi + 1$$

8.2 problem 2

Internal problem ID [12380]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - y = x}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve([diff(y(x),x)=y(x)+x,y(0) = 0],y(x), singsol=all)
```

$$y(x) = -1 + e^x - x$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 13

```
DSolve[{y'[x]==y[x]+x,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + e^x - 1$$

8.3 problem 3 (a)

Internal problem ID [12381]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 3 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{y}{x} = 0}$$

With initial conditions

$$[y(-1) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 7

```
dsolve([diff(y(x),x)=y(x)/x,y(-1) = 1],y(x), singsol=all)
```

$$y(x) = -x$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 8

```
DSolve[{y'[x]==y[x]/x,{y[-1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x$$

8.4 problem 3 (b)

Internal problem ID [12382]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 3 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{y}{x} = 0}$$

With initial conditions

$$[y(-1) = -1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=y(x)/x,y(-1) = -1],y(x), singsol=all)
```

$$y(x) = x$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 6

```
DSolve[{y'[x]==y[x]/x,{y[-1]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x$$

8.5 problem 4 (a)

Internal problem ID [12383]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 4 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{y}{-x^2+1} = \sqrt{x}}$$

With initial conditions

$$\left[y\left(\frac{1}{2}\right) = 1\right]$$

✓ Solution by Maple

Time used: 0.344 (sec). Leaf size: 145

```
dsolve([diff(y(x),x)=y(x)/(1-x^2)+sqrt(x),y(1/2) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{\left(4i\sqrt{2}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) - 12i\sqrt{2}\,\mathrm{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) + \sqrt{3}\,\sqrt{2} - 2\sqrt{3}\right)(x+1)}{6\sqrt{-x^2+1}} + \frac{-2\sqrt{x+1}\,\sqrt{-2x+2}\,\sqrt{-x}\,\mathrm{EllipticF}\left(\sqrt{x+1}, \frac{\sqrt{2}}{2}\right) + 6\sqrt{x+1}\,\sqrt{-2x+2}\,\sqrt{-x}\,\mathrm{EllipticE}\left(\sqrt{x+1}, \frac{\sqrt{2}}{2}\right.}{\sqrt{x}\,(3x-3)}$$

✓ Solution by Mathematica

Time used: 1.562 (sec). Leaf size: 215

```
DSolve[{y'[x]==y[x]/(1-x^2)+Sqrt[x],{y[1/2]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4\sqrt{1-x^2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},x^2\right)-4\sqrt{1-x^2}x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},x^2\right)-\sqrt{2}\,\text{Hype}}{}$$

8.6 problem 4 (b)

Internal problem ID [12384]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 4 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{y}{-x^2+1} = \sqrt{x}}$$

With initial conditions

$$[y(1) = 1]$$

✗ Solution by Maple

```
dsolve([diff(y(x),x)=y(x)/(1-x^2)+sqrt(x),y(1) = 1],y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'[x]==y[x]/(1-x^2)+Sqrt[x],{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

Not solved

8.7 problem 4 (c)

Internal problem ID [12385]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 4 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{y}{-x^2+1} = \sqrt{x}}$$

With initial conditions

$$[y(2) = 1]$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 136

```
dsolve([diff(y(x),x)=y(x)/(1-x^2)+sqrt(x),y(2) = 1],y(x), singsol=all)
```

$$y(x) = \frac{2i\left(\sqrt{2}\,\sqrt{3}\ \mathrm{EllipticE}\left(\sqrt{3}, \frac{\sqrt{2}}{2}\right) - \frac{\sqrt{2}\,\sqrt{3}\ \mathrm{EllipticF}\left(\sqrt{3}, \frac{\sqrt{2}}{2}\right)}{3} - \sqrt{2} + \frac{1}{2}\right)(x+1)\sqrt{3}}{3\sqrt{-x^2+1}} + \frac{-2\sqrt{x+1}\,\sqrt{-2x+2}\,\sqrt{-x}\ \mathrm{EllipticF}\left(\sqrt{x+1}, \frac{\sqrt{2}}{2}\right) + 6\sqrt{x+1}\,\sqrt{-2x+2}\,\sqrt{-x}\ \mathrm{EllipticE}\left(\sqrt{x+1}, \frac{\sqrt{}}{2}\right.}{\sqrt{x}\,(3x-3)}$$

✓ Solution by Mathematica

Time used: 0.121 (sec). Leaf size: 215

```
DSolve[{y'[x]==y[x]/(1-x^2)+Sqrt[x],{y[2]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2\sqrt{1-x^2}x^2\ \mathrm{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, x^2\right) - 2\sqrt{1-x^2}x\ \mathrm{Hypergeometric2F1}\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, x^2\right) - 4\sqrt{2}\,\mathrm{Hyp}}{}$$

8.8 problem 5 (a)

Internal problem ID [12386]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 5 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^2 = 0}$$

With initial conditions

$$[y(-1) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve([diff(y(x),x)=y(x)^2,y(-1) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{1}{x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 10

```
DSolve[{y'[x]==y[x]^2,{y[-1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{x}$$

8.9 problem 5 (b)

Internal problem ID [12387]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 5 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^2 = 0}$$

With initial conditions

$$[y(-1) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=y(x)^2,y(-1) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y'[x]==y[x]^2,{y[-1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

8.10 problem 5 (c)

Internal problem ID [12388]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 5 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^2 = 0}$$

With initial conditions

$$\left[y(1) = \frac{1}{2}\right]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 11

```
dsolve([diff(y(x),x)=y(x)^2,y(1) = 1/2],y(x), singsol=all)
```

$$y(x) = -\frac{1}{x-3}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 12

```
DSolve[{y'[x]==y[x]^2,{y[1]==1/2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3-x}$$

8.11 problem 6 (a)

Internal problem ID [12389]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 6 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^3 = 0}$$

With initial conditions

$$[y(-1) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve([diff(y(x),x)=y(x)^3,y(-1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{-2x-1}}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 14

```
DSolve[{y'[x]==y[x]^3,{y[-1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{\sqrt{-2x-1}}$$

8.12 problem 6 (b)

Internal problem ID [12390]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 6 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^3 = 0}$$

With initial conditions

$$[y(-1) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=y(x)^3,y(-1) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y'[x]==y[x]^3,{y[-1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 0$$

8.13 problem 6 (c)

Internal problem ID [12391]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 6 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^3 = 0}$$

With initial conditions

$$[y(-1) = -1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)=y(x)^3,y(-1) = -1],y(x), singsol=all)
```

$$y(x) = -\frac{1}{\sqrt{-2x-1}}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 16

```
DSolve[{y'[x]==y[x]^3,{y[-1]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{-2x-1}}$$

8.14 problem 7 (a)

Internal problem ID [12392]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 7 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \frac{3x^2}{2y} = 0}$$

With initial conditions

$$[y(-1) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)=-3*x^2/(2*y(x)),y(-1) = 1],y(x), singsol=all)
```

$$y(x) = (-x)^{\frac{3}{2}}$$

✓ Solution by Mathematica

Time used: 0.144 (sec). Leaf size: 14

```
DSolve[{y'[x]==-3*x^2/(2*y[x]),{y[-1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{-x^3}$$

8.15 problem 7 (b)

Internal problem ID [12393]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 7 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \frac{3x^2}{2y} = 0}$$

With initial conditions

$$\left[y(-1) = \frac{1}{2}\right]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(x),x)=-3*x^2/(2*y(x)),y(-1) = 1/2],y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-4x^3 - 3}}{2}$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 20

```
DSolve[{y'[x]==-3*x^2/(2*y[x]),{y[-1]==1/2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\sqrt{-4x^3 - 3}$$

8.16 problem 7 (c)

Internal problem ID [12394]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 7 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \frac{3x^2}{2y} = 0}$$

With initial conditions

$$[y(-1) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve([diff(y(x),x)=-3*x^2/(2*y(x)),y(-1) = 0],y(x), singsol=all)
```

$$y(x) = \sqrt{-x^3 - 1}$$
$$y(x) = -\sqrt{-x^3 - 1}$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 33

```
DSolve[{y'[x]==-3*x^2/(2*y[x]),{y[-1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^3 - 1}$$
$$y(x) \to \sqrt{-x^3 - 1}$$

8.17 problem 7 (d)

Internal problem ID [12395]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 7 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \frac{3x^2}{2y} = 0}$$

With initial conditions

$$[y(-1) = -1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve([diff(y(x),x)=-3*x^2/(2*y(x)),y(-1) = -1],y(x), singsol=all)
```

$$y(x) = -(-x)^{\frac{3}{2}}$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 16

```
DSolve[{y'[x]==-3*x^2/(2*y[x]),{y[-1]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^3}$$

8.18 problem 8 (a)

Internal problem ID [12396]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 8 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{\sqrt{y}}{x} = 0}$$

With initial conditions

$$[y(-1) = 1]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 29

```
dsolve([diff(y(x),x)=sqrt(y(x))/x,y(-1) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{i \ln(x) \pi}{2} - i\pi - \frac{\pi^2}{4} + \frac{\ln(x)^2}{4} + \ln(x) + 1$$

✓ Solution by Mathematica

Time used: 0.235 (sec). Leaf size: 43

```
DSolve[{y'[x]==Sqrt[y[x]]/x,{y[-1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{4}(i \log(x) + \pi - 2i)^2$$
$$y(x) \to -\frac{1}{4}(i \log(x) + \pi + 2i)^2$$

8.19 problem 8 (b)

Internal problem ID [12397]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 8 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{\sqrt{y}}{x} = 0}$$

With initial conditions

$$[y(-1) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=sqrt(y(x))/x,y(-1) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.157 (sec). Leaf size: 24

```
DSolve[{y'[x]==Sqrt[y[x]]/x,{y[-1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$
$$y(x) \to -\frac{1}{4}(\pi + i\log(x))^2$$

8.20 problem 8 (c)

Internal problem ID [12398]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 8 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{\sqrt{y}}{x} = 0}$$

With initial conditions

$$[y(-1) = -1]$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 28

```
dsolve([diff(y(x),x)=sqrt(y(x))/x,y(-1) = -1],y(x), singsol=all)
```

$$y(x) = \frac{\ln(x)^2}{4} + \frac{i(2-\pi)\ln(x)}{2} - \frac{(\pi-2)^2}{4}$$

✓ Solution by Mathematica

Time used: 0.151 (sec). Leaf size: 39

```
DSolve[{y'[x]==Sqrt[y[x]]/x,{y[-1]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{4}(i\log(x) + \pi + 2)^2$$
$$y(x) \to -\frac{1}{4}(i\log(x) + \pi - 2)^2$$

8.21 problem 8 (d)

Internal problem ID [12399]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 8 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{\sqrt{y}}{x} = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)=sqrt(y(x))/x,y(1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{(\ln(x) + 2)^2}{4}$$

✓ Solution by Mathematica

Time used: 0.151 (sec). Leaf size: 29

```
DSolve[{y'[x]==Sqrt[y[x]]/x,{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}(\log(x) - 2)^2$$

$$y(x) \to \frac{1}{4}(\log(x) + 2)^2$$

8.22 problem 9 (a)

Internal problem ID [12400]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 9 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 3xy^{\frac{1}{3}} = 0}$$

With initial conditions

$$\left[y(-1) = \frac{3}{2}\right]$$

✓ Solution by Maple

Time used: 0.516 (sec). Leaf size: 23

```
dsolve([diff(y(x),x)=3*x*y(x)^(1/3),y(-1) = 3/2],y(x), singsol=all)
```

$$y(x) = \frac{\left(3^{\frac{2}{3}}2^{\frac{1}{3}} + 2x^2 - 2\right)\sqrt{2\,3^{\frac{2}{3}}2^{\frac{1}{3}} + 4x^2 - 4}}{4}$$

✓ Solution by Mathematica

Time used: 0.374 (sec). Leaf size: 36

```
DSolve[{y'[x]==3*x*y[x]^(1/3),{y[-1]==3/2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\left(2x^2 + \sqrt[3]{2}3^{2/3} - 2\right)^{3/2}}{2\sqrt{2}}$$

8.23 problem 9 (b)

Internal problem ID [12401]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 9 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 3xy^{\frac{1}{3}} = 0}$$

With initial conditions

$$[y(-1) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 9

```
dsolve([diff(y(x),x)=3*x*y(x)^(1/3),y(-1) = 1],y(x), singsol=all)
```

$$y(x) = -x^3$$

✓ Solution by Mathematica

Time used: 0.214 (sec). Leaf size: 12

```
DSolve[{y'[x]==3*x*y[x]^(1/3),{y[-1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \left(x^2\right)^{3/2}$$

8.24 problem 9 (c)

Internal problem ID [12402]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 9 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 3xy^{\frac{1}{3}} = 0}$$

With initial conditions

$$\left[y(-1) = \frac{1}{2}\right]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 20

```
dsolve([diff(y(x),x)=3*x*y(x)^(1/3),y(-1) = 1/2],y(x), singsol=all)
```

$$y(x) = \frac{\left(2x^2 + 2^{\frac{1}{3}} - 2\right)\sqrt{4x^2 + 2\,2^{\frac{1}{3}} - 4}}{4}$$

✓ Solution by Mathematica

Time used: 0.175 (sec). Leaf size: 30

```
DSolve[{y'[x]==3*x*y[x]^(1/3),{y[-1]==1/2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\left(2x^2 + \sqrt[3]{2} - 2\right)^{3/2}}{2\sqrt{2}}$$

8.25 problem 9 (d)

Internal problem ID [12403]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 9 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 3xy^{\frac{1}{3}} = 0}$$

With initial conditions

$$[y(-1) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=3*x*y(x)^(1/3),y(-1) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.159 (sec). Leaf size: 19

```
DSolve[{y'[x]==3*x*y[x]^(1/3),{y[-1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$
$$y(x) \to (x^2 - 1)^{3/2}$$

8.26 problem 9 (e)

Internal problem ID [12404]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 9 (e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 3xy^{\frac{1}{3}} = 0}$$

With initial conditions

$$[y(-1) = -1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 7

```
dsolve([diff(y(x),x)=3*x*y(x)^(1/3),y(-1) = -1],y(x), singsol=all)
```

$$y(x) = x^3$$

✓ Solution by Mathematica

Time used: 0.175 (sec). Leaf size: 67

```
DSolve[{y'[x]==3*x*y[x]^(1/3),{y[-1]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\left(2x^2 - i\sqrt{3} - 3\right)^{3/2}}{2\sqrt{2}}$$
$$y(x) \to \frac{\left(2x^2 + i\sqrt{3} - 3\right)^{3/2}}{2\sqrt{2}}$$

8.27 problem 10 (a)

Internal problem ID [12405]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 10 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \sqrt{(y+2)(y-1)} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 34

```
dsolve([diff(y(x),x)=sqrt( (y(x)+2)*( y(x)-1)),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{i e^{x} \sqrt{2}}{2} + \frac{e^{x}}{4} - \frac{i\sqrt{2}\, e^{-x}}{2} - \frac{1}{2} + \frac{e^{-x}}{4}$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 45

```
DSolve[{y'[x]==Sqrt[ (y[x]+2)*( y[x]-1)],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} e^{-x}(e^{x} - 1)\left(\left(1 + 2i\sqrt{2}\right) e^{x} - 1 + 2i\sqrt{2}\right)$$

8.28 problem 10 (b)

Internal problem ID [12406]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 10 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \sqrt{(y+2)(y-1)} = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=sqrt( (y(x)+2)*( y(x)-1)),y(0) = 1],y(x), singsol=all)
```

$$y(x) = 1$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 23

```
DSolve[{y'[x]==Sqrt[ (y[x]+2)*( y[x]-1)],{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(3e^{-x} + 3e^{x} - 2\right)$$

8.29 problem 10 (c)

Internal problem ID [12407]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 10 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \sqrt{(y+2)(y-1)} = 0}$$

With initial conditions

$$[y(0) = -3]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 16

```
dsolve([diff(y(x),x)=sqrt( (y(x)+2)*( y(x)-1)),y(0) = -3],y(x), singsol=all)
```

$$y(x) = -\frac{1}{2} - \frac{e^{x}}{4} - \frac{9\,e^{-x}}{4}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 23

```
DSolve[{y'[x]==Sqrt[ (y[x]+2)*( y[x]-1)],{y[0]==-3}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{1}{4}\left(-9e^{-x} - e^{x} - 2\right)$$

8.30 problem 11 (a)

Internal problem ID [12408]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 11 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{y}{y - x} = 0}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 7

```
dsolve([diff(y(x),x)=y(x)/(y(x)-x),y(1) = 2],y(x), singsol=all)
```

$$y(x) = 2x$$

✓ Solution by Mathematica

Time used: 0.838 (sec). Leaf size: 14

```
DSolve[{y'[x]==y[x]/(y[x]-x),{y[1]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x^2} + x$$

8.31 problem 11 (b)

Internal problem ID [12409]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 11 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{y' - \frac{y}{y - x} = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve([diff(y(x),x)=y(x)/(y(x)-x),y(1) = 1],y(x), singsol=all)
```

$$y(x) = x - \sqrt{x^2 - 1}$$
$$y(x) = x + \sqrt{x^2 - 1}$$

✓ Solution by Mathematica

Time used: 0.129 (sec). Leaf size: 33

```
DSolve[{y'[x]==y[x]/(y[x]-x),{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - \sqrt{x^2 - 1}$$
$$y(x) \to \sqrt{x^2 - 1} + x$$

8.32 problem 11 (c)

Internal problem ID [12410]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 11 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{y}{y - x} = 0}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=y(x)/(y(x)-x),y(1) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 6

```
DSolve[{y'[x]==y[x]/(y[x]-x),{y[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

8.33 problem 11 (d)

Internal problem ID [12411]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 11 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{y}{y - x} = 0}$$

With initial conditions

$$[y(1) = -1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(x),x)=y(x)/(y(x)-x),y(1) = -1],y(x), singsol=all)
```

$$y(x) = x - \sqrt{x^2 + 3}$$

✓ Solution by Mathematica

Time used: 0.127 (sec). Leaf size: 18

```
DSolve[{y'[x]==y[x]/(y[x]-x),{y[1]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - \sqrt{x^2 + 3}$$

8.34 problem 12 (a)

Internal problem ID [12412]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 12 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y' - \frac{xy}{x^2 + y^2} = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.578 (sec). Leaf size: 11

```
dsolve([diff(y(x),x)=x*y(x)/(x^2+y(x)^2),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \sqrt{\frac{x^2}{\text{LambertW}\,(x^2)}}$$

✓ Solution by Mathematica

Time used: 10.851 (sec). Leaf size: 15

```
DSolve[{y'[x]==x*y[x]/(x^2+y[x]^2),{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{\sqrt{W\,(x^2)}}$$

8.35 problem 12 (b)

Internal problem ID [12413]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 12 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y' - \frac{xy}{x^2 + y^2} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=x*y(x)/(x^2+y(x)^2),y(0) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 6

```
DSolve[{y'[x]==x*y[x]/(x^2+y[x]^2),{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

8.36 problem 12 (c)

Internal problem ID [12414]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 12 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y' - \frac{xy}{x^2 + y^2} = 0}$$

With initial conditions

$$[y(0) = -1]$$

✓ Solution by Maple

Time used: 0.313 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)=x*y(x)/(x^2+y(x)^2),y(0) = -1],y(x), singsol=all)
```

$$y(x) = -\sqrt{\frac{x^2}{\operatorname{LambertW}(x^2)}}$$

✓ Solution by Mathematica

Time used: 0.443 (sec). Leaf size: 16

```
DSolve[{y'[x]==x*y[x]/(x^2+y[x]^2),{y[0]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{\sqrt{W(x^2)}}$$

8.37 problem 13 (a)

Internal problem ID [12415]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 13 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - x\sqrt{-y^2+1} = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=x*sqrt(1-y(x)^2),y(0) = 1],y(x), singsol=all)
```

$$y(x) = 1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 6

```
DSolve[{y'[x]==x*Sqrt[1-y[x]^2],{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1$$

8.38 problem 13 (b)

Internal problem ID [12416]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 13 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - x\sqrt{-y^2+1} = 0}$$

With initial conditions

$$\left[y(0) = \frac{9}{10}\right]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)=x*sqrt(1-y(x)^2),y(0) = 9/10],y(x), singsol=all)
```

$$y(x) = \sin\left(\frac{x^2}{2} + \arcsin\left(\frac{9}{10}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.368 (sec). Leaf size: 43

```
DSolve[{y'[x]==x*Sqrt[1-y[x]^2],{y[0]==9/10}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos\left(\frac{1}{2}\left(4\arctan\left(\frac{1}{\sqrt{19}}\right) + x^2\right)\right)$$

$$y(x) \to \cos\left(\frac{1}{2}\left(x^2 - 4\arctan\left(\frac{1}{\sqrt{19}}\right)\right)\right)$$

8.39 problem 13 (c)

Internal problem ID [12417]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 13 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - x\sqrt{-y^2 + 1} = 0}$$

With initial conditions

$$\left[y(0) = \frac{1}{2}\right]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)=x*sqrt(1-y(x)^2),y(0) = 1/2],y(x), singsol=all)
```

$$y(x) = \sin\left(\frac{x^2}{2} + \frac{\pi}{6}\right)$$

✓ Solution by Mathematica

Time used: 0.215 (sec). Leaf size: 33

```
DSolve[{y'[x]==x*Sqrt[1-y[x]^2],{y[0]==1/2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin\left(\frac{1}{6}(\pi - 3x^2)\right)$$

$$y(x) \to \sin\left(\frac{1}{6}(3x^2 + \pi)\right)$$

8.40 problem 13 (d)

Internal problem ID [12418]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 13 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - x\sqrt{-y^2 + 1} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve([diff(y(x),x)=x*sqrt(1-y(x)^2),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \sin\left(\frac{x^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.21 (sec). Leaf size: 27

```
DSolve[{y'[x]==x*Sqrt[1-y[x]^2],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sin\left(\frac{x^2}{2}\right)$$
$$y(x) \to \sin\left(\frac{x^2}{2}\right)$$

8.41 problem 14 (a)

Internal problem ID [12419]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 14 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y' - \frac{\sqrt{x^2+4y}}{2} = -\frac{x}{2}}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 15

```
dsolve([diff(y(x),x)=(-x+sqrt(x^2+4*y(x)))/2,y(0) = 1],y(x), singsol=all)
```

$$y(x) = 1 - x$$
$$y(x) = x + 1$$

✓ Solution by Mathematica

Time used: 0.443 (sec). Leaf size: 17

```
DSolve[{y'[x]==(-x+Sqrt[x^2+4*y[x]])/2,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 - x$$
$$y(x) \to x + 1$$

8.42 problem 14 (b)

Internal problem ID [12420]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 14 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y' - \frac{\sqrt{x^2+4y}}{2} = -\frac{x}{2}}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)=(-x+sqrt(x^2+4*y(x)))/2,y(0) = 0],y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = -\frac{x^2}{4}$$

✓ Solution by Mathematica

Time used: 0.287 (sec). Leaf size: 6

```
DSolve[{y'[x]==(-x+Sqrt[x^2+4*y[x]])/2,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

8.43 problem 14 (c)

Internal problem ID [12421]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 14 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y' - \frac{\sqrt{x^2 + 4y}}{2} = -\frac{x}{2}}$$

With initial conditions

$$[y(0) = -1]$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 19

```
dsolve([diff(y(x),x)=(-x+sqrt(x^2+4*y(x)))/2,y(0) = -1],y(x), singsol=all)
```

$$y(x) = -ix - 1$$
$$y(x) = ix - 1$$

✓ Solution by Mathematica

Time used: 0.293 (sec). Leaf size: 23

```
DSolve[{y'[x]==(-x+Sqrt[x^2+4*y[x]])/2,{y[0]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -1 - ix$$
$$y(x) \to -1 + ix$$

8.44 problem 14 (d)

Internal problem ID [12422]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 14 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y' - \frac{\sqrt{x^2+4y}}{2} = -\frac{x}{2}}$$

With initial conditions

$$\left[y(1) = -\frac{1}{5}\right]$$

✓ Solution by Maple

Time used: 0.515 (sec). Leaf size: 69

```
dsolve([diff(y(x),x)=(-x+sqrt(x^2+4*y(x)))/2,y(1) = -1/5],y(x), singsol=all)
```

$$y(x) = \frac{(x-1)\sqrt{5}}{10} - \frac{x}{2} + \frac{3}{10}$$
$$y(x) = \frac{\left(-5+\sqrt{5}\right)\left(\sqrt{5}-5+10x\right)}{100}$$
$$y(x) = -\frac{2^{\frac{1}{3}}\left(2^{\frac{1}{3}}x - \frac{\left(50+20\sqrt{5}\right)^{\frac{1}{3}}}{5}\right)\left(50+20\sqrt{5}\right)^{\frac{1}{3}}}{10}$$

✓ Solution by Mathematica

Time used: 0.301 (sec). Leaf size: 51

```
DSolve[{y'[x]==(-x+Sqrt[x^2+4*y[x]])/2,{y[1]==-2/10}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{100}\left(5+\sqrt{5}\right)\left(-10x+\sqrt{5}+5\right)$$
$$y(x) \to \frac{1}{100}\left(\sqrt{5}-5\right)\left(10x+\sqrt{5}-5\right)$$

8.45 problem 14 (e)

Internal problem ID [12423]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 2. The Initial Value Problem. Exercises 2.4.4, page 115
Problem number: 14 (e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y' - \frac{\sqrt{x^2+4y}}{2} = -\frac{x}{2}}$$

With initial conditions

$$\left[y(1) = -\frac{1}{4}\right]$$

✓ Solution by Maple

Time used: 0.296 (sec). Leaf size: 17

```
dsolve([diff(y(x),x)=(-x+sqrt(x^2+4*y(x)))/2,y(1) = -1/4],y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{4}$$
$$y(x) = \frac{1}{4} - \frac{x}{2}$$

✓ Solution by Mathematica

Time used: 0.282 (sec). Leaf size: 14

```
DSolve[{y'[x]==(-x+Sqrt[x^2+4*y[x]])/2,{y[1]==-1/4}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{1}{4}(1-2x)$$

9 Chapter 4. N-th Order Linear Differential Equations. Exercises 4.1, page 186

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9.1 problem 1

Internal problem ID [12424]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.1, page 186
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3y'' - 2y' + 4y = x}$$

With initial conditions

$$[y(-1) = 2, y'(-1) = 3]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 85

```
dsolve([3*diff(y(x),x$2)-2*diff(y(x),x)+4*y(x)=x,y(-1) = 2, D(y)(-1) = 3],y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(49\sqrt{11}\,\sin\left(\frac{\sqrt{11}}{3}\right) + 187\cos\left(\frac{\sqrt{11}}{3}\right)\right)\cos\left(\frac{\sqrt{11}\,x}{3}\right) + 49\sin\left(\frac{\sqrt{11}\,x}{3}\right)\left(\sqrt{11}\,\cos\left(\frac{\sqrt{11}}{3}\right) - \frac{187\sin\left(\frac{\sqrt{11}}{3}\right)}{49}\right)\right) \mathrm{e}}{88} + \frac{x}{4} + \frac{1}{8}$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 67

```
DSolve[{3*y''[x]-2*y'[x]+4*y[x]==x,{y[-1]==2,y'[-1]==3}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{88}\left(22x + 49\sqrt{11}e^{\frac{x+1}{3}}\sin\left(\frac{1}{3}\sqrt{11}(x+1)\right) + 187e^{\frac{x+1}{3}}\cos\left(\frac{1}{3}\sqrt{11}(x+1)\right) + 11\right)$$

9.2 problem 2

Internal problem ID [12425]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.1, page 186
Problem number: 2.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{xy''' + y'x = 4}$$

With initial conditions

$$[y(1) = 0, y'(1) = 1, y''(1) = -1]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 49

```
dsolve([x*diff(y(x),x$3)+x*diff(y(x),x)=4,y(1) = 0, D(y)(1) = 1, (D@@2)(y)(1) = -1],y(x), si
```

$$\begin{aligned}y(x) = &(4\operatorname{Ci}(1) - 4\operatorname{Ci}(x) + \cos(1) - \sin(1))\cos(x)\\ &+ (4\operatorname{Si}(1) - 4\operatorname{Si}(x) + \cos(1) + \sin(1))\sin(x) + 4\ln(x) - 1\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.184 (sec). Leaf size: 85

```
DSolve[{x*y'''[x]+x*y'[x]==4,{y[1]==0,y'[1]==1,y''[1]==-1}},y[x],x,IncludeSingularSolutions
```

$$\begin{aligned}y(x) \to &-4\,\text{CosIntegral}(x)\cos(x) + 4\,\text{CosIntegral}(1)\cos(x) - 2\text{sinc}(1)\cos(2-x)\\ &- 6\text{sinc}(1)\cos(x) + 8\text{sinc}(1)\cos(1) - 4\text{Si}(x)\sin(x) + 4\text{Si}(1)\sin(x) + 4\log(x)\\ &+ \sin(1-x) + \sin(3-x) + 3\sin(x+1) + \cos(1-x) - 1 - 4\sin(2)\end{aligned}$$

9.3 problem 3

Internal problem ID [12426]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.1, page 186
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x(x-3)\,y''+3y'=x^2}$$

With initial conditions

$$[y(1)=0, y'(1)=1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 11

```
dsolve([x*(x-3)*diff(y(x),x$2)+3*diff(y(x),x)=x^2,y(1) = 0, D(y)(1) = 1],y(x), singsol=all)
```

$$y(x)=-\frac{1}{2}+\frac{x^2}{2}$$

✓ Solution by Mathematica

Time used: 0.08 (sec). Leaf size: 14

```
DSolve[{x*(x-3)*y''[x]+3*y'[x]==x^2,{y[1]==0,y'[1]==1}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\to\frac{1}{2}(x^2-1)$$

9.4 problem 4

Internal problem ID [12427]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.1, page 186
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x(x-3)\,y''+3y'=x^2}$$

With initial conditions

$$[y(5)=0, y'(5)=1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve([x*(x-3)*diff(y(x),x$2)+3*diff(y(x),x)=x^2,y(5) = 0, D(y)(5) = 1],y(x), singsol=all)
```

$$y(x)=\frac{x^2}{2}-\frac{8x}{5}-\frac{24\ln(x-3)}{5}-\frac{9}{2}+\frac{24\ln(2)}{5}$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 29

```
DSolve[{x*(x-3)*y''[x]+3*y'[x]==x^2,{y[5]==0,y'[5]==1}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\to\frac{1}{10}\left(5x^2-16x-48\log(x-3)-45+48\log(2)\right)$$

9.5 problem 5

Internal problem ID [12428]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.1, page 186
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{\sqrt{1-x}\,y''-4y=\sin(x)}$$

With initial conditions

$$[y(-2)=3, y'(-2)=-1]$$

✓ Solution by Maple

Time used: 0.562 (sec). Leaf size: 185

```
dsolve([sqrt(1-x)*diff(y(x),x$2)-4*y(x)=sin(x),y(-2) = 3, D(y)(-2) = -1],y(x), singsol=all)
```

$$y(x) = \frac{8\pi\left(\frac{\left(\left(\int_{-2}^{x}\frac{\sin(\_z1)\sqrt{1-\_z1}\,\mathrm{BesselI}\left(\frac{2}{3},\frac{8\sqrt{(1-\_z1)^{\frac{3}{2}}}}{3}\right)}{\left((1-\_z1)^{\frac{3}{2}}\right)^{\frac{1}{3}}}d\_z1\right)\sqrt{3}+6\,\mathrm{BesselI}\left(-\frac{1}{3},\frac{8\,3^{\frac{3}{4}}}{3}\right)3^{\frac{3}{4}}-3\,\mathrm{BesselI}\left(\frac{2}{3},\frac{8\,3^{\frac{3}{4}}}{3}\right)\right)\left(\left((1-x)^{\frac{3}{2}}\right)^{\frac{2}{3}}\mathrm{Be}\cdots}{6}\cdots}{\cdots}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{Sqrt[1-x]*y''[x]-4*y[x]==Sin[x],{y[-2]==3,y'[-2]==-1}},y[x],x,IncludeSingularSolutio
```

Not solved

9.6 problem 6

Internal problem ID [12429]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.1, page 186
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{(x^2-4)\,y''+y\ln(x)=x\,\mathrm{e}^{x}}$$

With initial conditions

$$[y(1)=1, y'(1)=2]$$

✗ Solution by Maple

```
dsolve([(x^2-4)*diff(y(x),x$2)+ln(x)*y(x)=x*exp(x),y(1) = 1, D(y)(1) = 2],y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{(x^2-4)*y''[x]+Log[x]*y[x]==x*Exp[x],{y[1]==1,y'[1]==2}},y[x],x,IncludeSingularSolut
```

Not solved

9.7 problem 7

Internal problem ID [12430]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.1, page 186
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x} + c_2 \mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 20

```
DSolve[y''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x} + c_2 e^{-x}$$

9.8 problem 8

Internal problem ID [12431]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.1, page 186
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x$2)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(x) + c_2 \cos(x)$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 16

```
DSolve[y''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(x) + c_2 \sin(x)$$

9.9 problem 9

Internal problem ID [12432]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.1, page 186
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y''x^2 + 2xy' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 13

```
dsolve(x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x + \frac{c_2}{x^2}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 16

```
DSolve[x^2*y''[x]+2*x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x^2} + c_2x$$

9.10 problem 10

Internal problem ID [12433]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.1, page 186
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{2yy'' - y'^2 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 27

```
dsolve(2*y(x)*diff(y(x),x$2)-diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{1}{4}x^2c_1^2 + \frac{1}{2}c_1xc_2 + \frac{1}{4}c_2^2$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 29

```
DSolve[2*y[x]*y''[x]-(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(c_1x + 2c_2)^2}{4c_2}$$
$$y(x) \to \text{Indeterminate}$$

9.11 problem 13

Internal problem ID [12434]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.1, page 186
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve([diff(y(x),x$2)-y(x)=0,y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{e^{x}}{2} - \frac{e^{-x}}{2}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 21

```
DSolve[{y''[x]-y[x]==0,{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}(e^{2x} - 1)$$

9.12 problem 14

Internal problem ID [12435]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.1, page 186
Problem number: 14.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + y' = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0, y''(0) = -1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 6

```
dsolve([diff(y(x),x$3)+diff(y(x),x)=0,y(0) = 1, D(y)(0) = 0, (D@@2)(y)(0) = -1],y(x), singso
```

$$y(x) = \cos(x)$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 7

```
DSolve[{y'''[x]+y'[x]==0,{y[0]==1,y'[0]==0,y''[0]==-1}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \cos(x)$$

9.13 problem 15

Internal problem ID [12436]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.1, page 186
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y''x^2 - xy' + y = 0}$$

With initial conditions

$$[y(1) = 2, y'(1) = -1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve([x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(1) = 2, D(y)(1) = -1],y(x), singsol=all)
```

$$y(x) = (-3\ln(x) + 2)\, x$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 13

```
DSolve[{x^2*y''[x]-x*y'[x]+y[x]==0,{y[1]==2,y'[1]==-1}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to x(2 - 3\log(x))$$

9.14 problem 16

Internal problem ID [12437]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.1, page 186
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y = 31}$$

With initial conditions

$$[y(0) = -9, y'(0) = 6]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$2)-4*y(x)=31,y(0) = -9, D(y)(0) = 6],y(x), singsol=all)
```

$$y(x) = -\frac{31}{4} + \frac{7\,\mathrm{e}^{2x}}{8} - \frac{17\,\mathrm{e}^{-2x}}{8}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 25

```
DSolve[{y''[x]-4*y[x]==31,{y[0]==-9,y'[0]==6}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}\left(-17e^{-2x} + 7e^{2x} - 62\right)$$

9.15 problem 17

Internal problem ID [12438]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.1, page 186
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 9y = 27x + 18}$$

With initial conditions

$$[y(0) = 23, y'(0) = 21]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve([diff(y(x),x$2)+9*y(x)=27*x+18,y(0) = 23, D(y)(0) = 21],y(x), singsol=all)
```

$$y(x) = 2 + 21\cos(3x) + 6\sin(3x) + 3x$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 22

```
DSolve[{y''[x]+9*y[x]==27*x+18,{y[0]==23,y'[0]==21}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to 3x + 6\sin(3x) + 21\cos(3x) + 2$$

9.16 problem 18

Internal problem ID [12439]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.1, page 186
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x^2 + xy' - 4y = -3x - \frac{3}{x}}$$

With initial conditions

$$[y(1) = 3, y'(1) = -6]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve([x^2*diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=-3*x-3/x,y(1) = 3, D(y)(1) = -6],y(x), sing
```

$$y(x) = \frac{-x^4 + x^3 + x + 2}{x^2}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 20

```
DSolve[{x^2*y''[x]+x*y'[x]-4*y[x]==-3*x-3/x,{y[1]==3,y'[1]==-6}},y[x],x,IncludeSingularSolut
```

$$y(x) \to \frac{-x^4 + x^3 + x + 2}{x^2}$$

10 Chapter 4. N-th Order Linear Differential Equations. Exercises 4.3, page 210

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10.1 problem 1

Internal problem ID [12440]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.3, page 210
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' + 4y' - 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(4*diff(y(x),x$2)+4*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{x}{2}} + c_2 e^{-\frac{3x}{2}}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 24

```
DSolve[4*y''[x]+4*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x/2}\left(c_2 e^{2x} + c_1\right)$$

10.2 problem 2

Internal problem ID [12441]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.3, page 210
Problem number: 2.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 4y'' + 6y' - 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$3)-4*diff(y(x),x$2)+6*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{2x} + c_2 e^{x} \sin(x) + c_3 e^{x} \cos(x)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 26

```
DSolve[y'''[x]-4*y''[x]+6*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(c_3 e^{x} + c_2 \cos(x) + c_1 \sin(x))$$

10.3 problem 3

Internal problem ID [12442]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.3, page 210
Problem number: 3.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 16y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$4)-16*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{2x} + c_2 e^{-2x} + c_3 \sin(2x) + c_4 \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 36

```
DSolve[y''''[x]-16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{2x} + c_3 e^{-2x} + c_2 \cos(2x) + c_4 \sin(2x)$$

10.4 problem 4

Internal problem ID [12443]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.3, page 210
Problem number: 4.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 16y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 65

```
dsolve(diff(y(x),x$4)+16*y(x)=0,y(x), singsol=all)
```

$$y(x) = -c_1 e^{-\sqrt{2}\,x} \sin\left(\sqrt{2}\,x\right) - c_2 e^{\sqrt{2}\,x} \sin\left(\sqrt{2}\,x\right) + c_3 e^{-\sqrt{2}\,x} \cos\left(\sqrt{2}\,x\right) + c_4 e^{\sqrt{2}\,x} \cos\left(\sqrt{2}\,x\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 67

```
DSolve[y''''[x]+16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\sqrt{2}x}\left(\left(c_1 e^{2\sqrt{2}x} + c_2\right)\cos\left(\sqrt{2}x\right) + \left(c_4 e^{2\sqrt{2}x} + c_3\right)\sin\left(\sqrt{2}x\right)\right)$$

10.5 problem 5

Internal problem ID [12444]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.3, page 210
Problem number: 5.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 4y''' + 8y'' - 8y' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$4)-4*diff(y(x),x$3)+8*diff(y(x),x$2)-8*diff(y(x),x)+4*y(x)=0,y(x), singso
```

$$y(x) = c_1 e^x \sin(x) + c_2 e^x \cos(x) + c_3 e^x \sin(x) x + c_4 e^x \cos(x) x$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 30

```
DSolve[y''''[x]-4*y'''[x]+8*y''[x]-8*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to e^x((c_4 x + c_3)\cos(x) + (c_2 x + c_1)\sin(x))$$

10.6 problem 6

Internal problem ID [12445]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.3, page 210
Problem number: 6.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 8y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$4)-8*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 e^{2x} + c_3 e^{-x} \sin\left(\sqrt{3}\, x\right) + c_4 e^{-x} \cos\left(\sqrt{3}\, x\right)$$

✓ Solution by Mathematica

Time used: 0.658 (sec). Leaf size: 70

```
DSolve[y''''[x]-8*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} e^{-x} \left(2c_1 e^{3x} - \left(c_2 + \sqrt{3} c_3\right) \cos\left(\sqrt{3} x\right) + \left(\sqrt{3} c_2 - c_3\right) \sin\left(\sqrt{3} x\right)\right) + c_4$$

10.7 problem 7

Internal problem ID [12446]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.3, page 210
Problem number: 7.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{36y'''' - 12y''' - 11y'' + 2y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(36*diff(y(x),x$4)-12*diff(y(x),x$3)-11*diff(y(x),x$2)+2*diff(y(x),x)+y(x)=0,y(x), sin
```

$$y(x) = c_1 e^{-\frac{x}{3}} + c_2 e^{-\frac{x}{3}} x + c_3 e^{\frac{x}{2}} + c_4 e^{\frac{x}{2}} x$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 41

```
DSolve[36*y''''[x]-12*y'''[x]-11*y''[x]+2*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to e^{-x/3}\left(c_3 e^{5x/6} + x\left(c_4 e^{5x/6} + c_2\right) + c_1\right)$$

10.8 problem 8

Internal problem ID [12447]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.3, page 210
Problem number: 8.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(5)} - 3y'''' + 3y''' - 3y'' + 2y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$5)-3*diff(y(x),x$4)+3*diff(y(x),x$3)-3*diff(y(x),x$2)+2*diff(y(x),x)=0,y(
```

$$y(x) = c_1 + c_2 e^{2x} + e^x c_3 + c_4 \sin(x) + c_5 \cos(x)$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 36

```
DSolve[y'''''[x]-3*y''''[x]+3*y'''[x]-3*y''[x]+2*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to c_3 e^x + \frac{1}{2} c_4 e^{2x} - c_2 \cos(x) + c_1 \sin(x) + c_5$$

10.9 problem 9

Internal problem ID [12448]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.3, page 210
Problem number: 9.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(5)} - y'''' + y''' + 35y'' + 16y' - 52y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$5)-diff(y(x),x$4)+diff(y(x),x$3)+35*diff(y(x),x$2)+16*diff(y(x),x)-52*y(x
```

$$y(x) = c_1\mathrm{e}^{x} + c_2\mathrm{e}^{-2x} + c_3\mathrm{e}^{-2x}x + c_4\mathrm{e}^{2x}\sin(3x) + c_5\mathrm{e}^{2x}\cos(3x)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 50

```
DSolve[y'''''[x]-y''''[x]+y'''[x]+35*y''[x]+16*y'[x]-52*y[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to e^{-2x}\left(c_4x + c_5e^{3x} + c_2e^{4x}\cos(3x) + c_1e^{4x}\sin(3x) + c_3\right)$$

10.10 problem 10

Internal problem ID [12449]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.3, page 210
Problem number: 10.
ODE order: 8.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(8)} + 8y'''' + 16y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 65

```
dsolve(diff(y(x),x$8)+8*diff(y(x),x$4)+16*y(x)=0,y(x), singsol=all)
```

$$\begin{aligned}y(x) = {} & c_1 \mathrm{e}^{-x} \sin(x) + c_2 \mathrm{e}^{-x} \cos(x) + c_3 \mathrm{e}^{-x} \sin(x)\, x + c_4 \mathrm{e}^{-x} \cos(x)\, x \\ & + c_5 \mathrm{e}^{x} \sin(x) + c_6 \mathrm{e}^{x} \cos(x) + c_7 \mathrm{e}^{x} \sin(x)\, x + c_8 \mathrm{e}^{x} \cos(x)\, x\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 66

```
DSolve[D[y[x],{x,8}]+8*y''''[x]+16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x} \left(\left(c_4 x + c_7 e^{2x} + c_8 e^{2x} x + c_3 \right) \cos(x) + \left(c_2 x + c_5 e^{2x} + c_6 e^{2x} x + c_1 \right) \sin(x) \right)$$

10.11 problem 11

Internal problem ID [12450]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.3, page 210
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + \alpha y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+alpha*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(\sqrt{\alpha}\, x\right) + c_2 \cos\left(\sqrt{\alpha}\, x\right)$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 28

```
DSolve[y''[x]+a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos\left(\sqrt{a}x\right) + c_2 \sin\left(\sqrt{a}x\right)$$

10.12 problem 17

Internal problem ID [12451]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.3, page 210
Problem number: 17.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + (-3-4i)\,y'' + (-4+12i)\,y' + 12y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$3)-(3+4*I)*diff(y(x),x$2)-(4-12*I)*diff(y(x),x)+12*y(x)=0,y(x), singsol=a
```

$$y(x) = c_1 \mathrm{e}^{3x} + \mathrm{e}^{2ix} c_2 + x\,\mathrm{e}^{2ix} c_3$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 29

```
DSolve[y'''[x]-(3+4*I)*y''[x]-(4-12*I)*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to e^{2ix}(c_2 x + c_1) + c_3 e^{3x}$$

10.13 problem 18

Internal problem ID [12452]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.3, page 210
Problem number: 18.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + (-3 - i)\, y''' + (4 + 3i)\, y'' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$4)-(3+I)*diff(y(x),x$3)+(4+3*I)*diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{(2-i)x} + c_2 e^{(1+2i)x} + c_3 + c_4 x$$

✓ Solution by Mathematica

Time used: 0.156 (sec). Leaf size: 46

```
DSolve[y''''[x]-(3+I)*y'''[x]+(4+3*I)*y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(-\frac{3}{25} - \frac{4i}{25}\right) c_1 e^{(1+2i)x} + \left(\frac{3}{25} + \frac{4i}{25}\right) c_2 e^{(2-i)x} + c_4 x + c_3$$

10.14 problem 19

Internal problem ID [12453]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.3, page 210
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - iy = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve([diff(y(x),x)-I*y(x)=0,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{ix}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 12

```
DSolve[{y'[x]-I*y[x]==0,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{ix}$$

11 Chapter 4. N-th Order Linear Differential Equations. Exercises 4.4, page 218

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11.1 problem 1

Internal problem ID [12454]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.4, page 218
Problem number: 1.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' - 6y''' + 13y'' - 12y' + 4y = 2\,e^{x} - 4\,e^{2x}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 78

```
dsolve(diff(y(x),x$4)-6*diff(y(x),x$3)+13*diff(y(x),x$2)-12*diff(y(x),x)+4*y(x)=2*exp(x)-4*e
```

$$y(x) = -\left(-x^2 e^{2x} + 2x^2 e^{3x} + 12\,e^{3x} - 4\,e^{2x} x - 8x\,e^{3x} - 6\,e^{2x}\right) e^{-x} + c_1 e^{x} + c_2 e^{2x} + x\,e^{x} c_3 + x\,e^{2x} c_4$$

✓ Solution by Mathematica

Time used: 0.187 (sec). Leaf size: 41

```
DSolve[y''''[x]-6*y'''[x]+13*y''[x]-12*y'[x]+4*y[x]==2*Exp[x]-4*Exp[2*x],y[x],x,IncludeSingu
```

$$y(x) \to e^x\left(x^2 + e^x\left(-2x^2 + (8 + c_4)x - 12 + c_3\right) + (4 + c_2)x + 6 + c_1\right)$$

11.2 problem 2

Internal problem ID [12455]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.4, page 218
Problem number: 2.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y'''' + 4y'' = 24x^2 - 6x + 14 + 32\cos(2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 51

```
dsolve(diff(y(x),x$4)+4*diff(y(x),x$2)=24*x^2-6*x+14+32*cos(2*x),y(x), singsol=all)
```

$$y(x) = \frac{x^4}{2} - \frac{x^3}{4} + \frac{x^2}{4} - \frac{\cos(2x)\, c_1}{4} - \frac{\sin(2x)\, c_2}{4} - \frac{5\cos(2x)}{2} - 2x\sin(2x) + c_3 x + c_4$$

✓ Solution by Mathematica

Time used: 1.052 (sec). Leaf size: 54

```
DSolve[y''''[x]+4*y''[x]==24*x^2-6*x+14+32*Cos[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(2x^4 - x^3 + x^2 + 4c_4 x - (12 + c_1)\cos(2x) - (8x + c_2)\sin(2x) + 4c_3\right)$$

11.3 problem 3

Internal problem ID [12456]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.4, page 218
Problem number: 3.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' + 2y'' + y = 3 + \cos(2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$2)+y(x)=3+cos(2*x),y(x), singsol=all)
```

$$y(x) = \frac{\cos(2x)}{9} + 3 + c_1 \cos(x) + c_2 \sin(x) + c_3 x \cos(x) + c_4 \sin(x) x$$

✓ Solution by Mathematica

Time used: 0.199 (sec). Leaf size: 36

```
DSolve[y''''[x]+2*y''[x]+y[x]==3+Cos[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{9}\cos(2x) + (c_2 x + c_1)\cos(x) + c_3 \sin(x) + c_4 x \sin(x) + 3$$

11.4 problem 4

Internal problem ID [12457]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.4, page 218
Problem number: 4.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y'''' - 3y''' + 3y'' - y' = 6x - 20 - 120x^2 \mathrm{e}^x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 84

```
dsolve(diff(y(x),x$4)-3*diff(y(x),x$3)+3*diff(y(x),x$2)-diff(y(x),x)=6*x-20-120*x^2*exp(x),y
```

$$\begin{aligned} y(x) = {} & c_1 \mathrm{e}^x + c_2(\mathrm{e}^x x - \mathrm{e}^x) + c_3\left(\mathrm{e}^x x^2 - 2\,\mathrm{e}^x x + 2\,\mathrm{e}^x\right) - 3x^2 - 2x^5 \mathrm{e}^x \\ & + 10x^4 \mathrm{e}^x - 40\,\mathrm{e}^x x^3 + 120\,\mathrm{e}^x x^2 - 240\,\mathrm{e}^x x + 240\,\mathrm{e}^x + 2x + c_4 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.569 (sec). Leaf size: 65

```
DSolve[y''''[x]-3*y'''[x]+3*y''[x]-y'[x]==6*x-20-120*x^2*Exp[x],y[x],x,IncludeSingularSoluti
```

$$\begin{aligned} y(x) \to {} & -3x^2 \\ & + e^x\left(-2x^5 + 10x^4 - 40x^3 + (120 + c_3)x^2 + (-240 + c_2 - 2c_3)x + 240 + c_1 - c_2 + 2c_3\right) \\ & + 2x + c_4 \end{aligned}$$

11.5 problem 5

Internal problem ID [12458]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.4, page 218
Problem number: 5.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 6y'' + 21y' - 26y = 36\,\mathrm{e}^{2x}\sin(3x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 52

```
dsolve(diff(y(x),x$3)-6*diff(y(x),x$2)+21*diff(y(x),x)-26*y(x)=36*exp(2*x)*sin(3*x),y(x), si
```

$$y(x) = -\frac{2\,\mathrm{e}^{2x}\cos(3x)}{3} - 2\,\mathrm{e}^{2x}\sin(3x)\,x + c_1\mathrm{e}^{2x} + c_2\mathrm{e}^{2x}\cos(3x) + c_3\mathrm{e}^{2x}\sin(3x)$$

✓ Solution by Mathematica

Time used: 0.103 (sec). Leaf size: 34

```
DSolve[y'''[x]-6*y''[x]+21*y'[x]-26*y[x]==36*Exp[2*x]*Sin[3*x],y[x],x,IncludeSingularSolutio
```

$$y(x) \to e^{2x}((-1 + c_2)\cos(3x) + (-2x + c_1)\sin(3x) + c_3)$$

11.6 problem 6

Internal problem ID [12459]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.4, page 218
Problem number: 6.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' + y'' - y' - y = (2x^2 + 4x + 8)\cos(x) + (6x^2 + 8x + 12)\sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 51

```
dsolve(diff(y(x),x$3)+diff(y(x),x$2)-diff(y(x),x)-y(x)=(2*x^2+4*x+8)*cos(x)+(6*x^2+8*x+12)*s
```

$$\begin{aligned} y(x) = &\cos(x)\, x^2 - 2\sin(x)\, x^2 - 6x\cos(x) - 4\sin(x)\, x \\ &- 2\cos(x) + \sin(x) + c_1 e^x + c_2 e^{-x} + c_3 e^{-x} x \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 55

```
DSolve[y'''[x]+y''[x]-y'[x]-y[x]==(2*x^2+4*x+8)*Cos[x]+(6*x^2+8*x+12)*Sin[x],y[x],x,IncludeS
```

$$y(x) \to (x^2 - 6x - 2)\cos(x) + e^{-x}\left(-e^x(2x^2 + 4x - 1)\sin(x) + c_2 x + c_3 e^{2x} + c_1\right)$$

11.7 problem 7

Internal problem ID [12460]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.4, page 218
Problem number: 7.
ODE order: 6.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y^{(6)} - 12y^{(5)} + 63y'''' - 18y''' + 315y'' - 300y' + 125y = \mathrm{e}^x(48\cos(x) + 96\sin(x))}$$

✓ Solution by Maple

Time used: 1.547 (sec). Leaf size: 2807

```
dsolve(diff(y(x),x$6)-12*diff(y(x),x$5)+63*diff(y(x),x$4)-18*diff(y(x),x$3)+315*diff(y(x),x$
```

Expression too large to display

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 292

```
DSolve[y''''''[x]-12*y'''''[x]+63*y''''[x]-18*y'''[x]+315*y''[x]-300*y'[x]+125*y[x]==Exp[x]*
```

$$\begin{aligned} y(x) \to\ & c_3 \exp\left(x\text{Root}\left[\#1^6 - 12\#1^5 + 63\#1^4 - 18\#1^3 + 315\#1^2 - 300\#1 + 125\&, 3\right]\right) \\ & + c_4 \exp\left(x\text{Root}\left[\#1^6 - 12\#1^5 + 63\#1^4 - 18\#1^3 + 315\#1^2 - 300\#1 + 125\&, 4\right]\right) \\ & + c_1 \exp\left(x\text{Root}\left[\#1^6 - 12\#1^5 + 63\#1^4 - 18\#1^3 + 315\#1^2 - 300\#1 + 125\&, 1\right]\right) \\ & + c_2 \exp\left(x\text{Root}\left[\#1^6 - 12\#1^5 + 63\#1^4 - 18\#1^3 + 315\#1^2 - 300\#1 + 125\&, 2\right]\right) \\ & + c_5 \exp\left(x\text{Root}\left[\#1^6 - 12\#1^5 + 63\#1^4 - 18\#1^3 + 315\#1^2 - 300\#1 + 125\&, 5\right]\right) \\ & + c_6 \exp\left(x\text{Root}\left[\#1^6 - 12\#1^5 + 63\#1^4 - 18\#1^3 + 315\#1^2 - 300\#1 + 125\&, 6\right]\right) \\ & - \frac{48e^x(352\sin(x) + 1011\cos(x))}{229205} \end{aligned}$$

12 Chapter 4. N-th Order Linear Differential Equations. Exercises 4.5, page 221

12.1 problem 1 231
12.2 problem 2 232
12.3 problem 3 233
12.4 problem 4 234

12.1 problem 1

Internal problem ID [12461]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.5, page 221
Problem number: 1.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 3y'' - 4y' + 12y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 5, y''(0) = -1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$3)-3*diff(y(x),x$2)-4*diff(y(x),x)+12*y(x)=0,y(0) = 1, D(y)(0) = 5, (D@@
```

$$y(x) = \left(-e^{5x} + 3\,e^{4x} - 1\right) e^{-2x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 26

```
DSolve[{y'''[x]-3*y''[x]-4*y'[x]+12*y[x]==0,{y[0]==1,y'[0]==5,y''[0]==-1}},y[x],x,IncludeSin
```

$$y(x) \to -e^{-2x}\left(-3e^{4x} + e^{5x} + 1\right)$$

12.2 problem 2

Internal problem ID [12462]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.5, page 221
Problem number: 2.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 2y''' + 2y' - y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -1, y''(0) = -3, y'''(0) = 3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve([diff(y(x),x$4)-2*diff(y(x),x$3)+2*diff(y(x),x)-y(x)=0,y(0) = 1, D(y)(0) = -1, (D@@2)
```

$$y(x) = -\mathrm{e}^{-x} + (2x^2 - 4x + 2)\,\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 25

```
DSolve[{y''''[x]-2*y'''[x]+2*y'[x]-y[x]==0,{y[0]==1,y'[0]==-1,y''[0]==-3,y'''[0]==3}},y[x],x
```

$$y(x) \to e^{-x}\left(2e^{2x}(x-1)^2 - 1\right)$$

12.3 problem 3

Internal problem ID [12463]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.5, page 221
Problem number: 3.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - y'' + y' - y = 2\,\mathrm{e}^{x}}$$

With initial conditions

$$[y(0) = 1, y'(0) = 3, y''(0) = -3]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$3)-diff(y(x),x$2)+diff(y(x),x)-y(x)=2*exp(x),y(0) = 1, D(y)(0) = 3, (D@@
```

$$y(x) = (x-2)\,\mathrm{e}^{x} + 3\cos(x) + 4\sin(x)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'''[x]-y'''[x]+y'[x]-y[x]==2*Exp[x],{y[0]==1,y'[0]==3,y''[0]==-3}},y[x],x,IncludeSi
```

{}

12.4 problem 4

Internal problem ID [12464]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 4. N-th Order Linear Differential Equations. Exercises 4.5, page 221
Problem number: 4.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$\boxed{y'''' + 2y'' + y = 3x + 4}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0, y''(0) = 1, y'''(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve([diff(y(x),x$4)+2*diff(y(x),x$2)+y(x)=3*x+4,y(0) = 0, D(y)(0) = 0, (D@@2)(y)(0) = 1,
```

$$y(x) = 4 + (-4 + x)\cos(x) + \frac{(-3x - 8)\sin(x)}{2} + 3x$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 27

```
DSolve[{y''''[x]+2*y''[x]+y[x]==3*x+4,{y[0]==0,y'[0]==0,y''[0]==1,y'''[0]==1}},y[x],x,Includ
```

$$y(x) \to 3x - \frac{1}{2}(3x + 8)\sin(x) + (x - 4)\cos(x) + 4$$

13 Chapter 5. The Laplace Transform Method. Exercises 5.2, page 248

13.1 problem 1 236
13.2 problem 2 237
13.3 problem 3 238
13.4 problem 4 239
13.5 problem 5 240
13.6 problem 6 241
13.7 problem 7 242
13.8 problem 8 243
13.9 problem 9 244
13.10 problem 10 245
13.11 problem 11 246
13.12 problem 12 247
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13.1 problem 1

Internal problem ID [12465]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.2, page 248
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y = 0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = y(0)\,\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 16

```
DSolve[y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x$$
$$y(x) \to 0$$

13.2 problem 2

Internal problem ID [12466]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.2, page 248
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + 5y = 0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+5*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{e^{x}(2y(0)\cos(2x) + (-y(0) + D(y)(0))\sin(2x))}{2}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 24

```
DSolve[y''[x]-2*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(c_2 \cos(2x) + c_1 \sin(2x))$$

13.3 problem 3

Internal problem ID [12467]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.2, page 248
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + 2y = 4}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+2*y(x)=4,y(x), singsol=all)
```

$$y(x) = (y(0) - 2)\,\mathrm{e}^{-2x} + 2$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 20

```
DSolve[y'[x]+2*y[x]==4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2 + c_1 e^{-2x}$$
$$y(x) \to 2$$

13.4 problem 4

Internal problem ID [12468]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.2, page 248
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 9y = 2\sin(3x)}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)-9*y(x)=2*sin(3*x),y(x), singsol=all)
```

$$y(x) = -\frac{\sin(3x)}{9} + y(0)\cosh(3x) + \frac{\sinh(3x)(1 + 3D(y)(0))}{9}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 30

```
DSolve[y''[x]-9*y[x]==2*Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{9}\sin(3x) + c_1 e^{3x} + c_2 e^{-3x}$$

13.5 problem 5

Internal problem ID [12469]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.2, page 248
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = 2\sin(3x)}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+9*y(x)=2*sin(3*x),y(x), singsol=all)
```

$$y(x) = -\frac{\cos(3x)(x - 3y(0))}{3} + \frac{\sin(3x)(1 + 3D(y)(0))}{9}$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 33

```
DSolve[y''[x]+9*y[x]==2*Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(-\frac{x}{3} + c_1\right)\cos(3x) + \frac{1}{18}(1 + 18c_2)\sin(3x)$$

13.6 problem 6

Internal problem ID [12470]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.2, page 248
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' - 2y = x\,\mathrm{e}^{x} - 3x^{2}}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 52

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-2*y(x)=x*exp(x)-3*x^2,y(x), singsol=all)
```

$$y(x) = \frac{9}{4} + \frac{3x^{2}}{2} + \frac{3x}{2} + \frac{\mathrm{e}^{x}(9x^{2} + 36y(0) + 18D(y)(0) - 6x - 106)}{54} + \frac{\mathrm{e}^{-2x}(36y(0) - 36D(y)(0) - 31)}{108}$$

✓ Solution by Mathematica

Time used: 0.313 (sec). Leaf size: 49

```
DSolve[y''[x]+y'[x]-2*y[x]==x*Exp[x]-3*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3}{4}(2x^{2} + 2x + 3) + \frac{1}{54}e^{x}(9x^{2} - 6x + 2 + 54c_{2}) + c_{1}e^{-2x}$$

13.7 problem 7

Internal problem ID [12471]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.2, page 248
Problem number: 7.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y'''' - 2y''' + y'' = x\,\mathrm{e}^{x} - 3x^{2}}$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 79

```
dsolve(diff(y(x),x$4)-2*diff(y(x),x$3)+diff(y(x),x$2)=x*exp(x)-3*x^2,y(x), singsol=all)
```

$$\begin{aligned} y(x) &= -26 - 9x^{2} - \frac{x^{4}}{4} - 2x^{3} + y(0) \\ &+ \frac{\mathrm{e}^{x}\left(x^{3} + 6xD^{(3)}(y)\,(0) - 6xD^{(2)}(y)\,(0) - 6x^{2} - 12D^{(3)}(y)\,(0) + 18D^{(2)}(y)\,(0) - 18x + 156\right)}{6} \\ &- D^{(2)}(y)\,(0)\,(3+2x) + D^{(3)}(y)\,(0)\,(x+2) + x(-23 + D(y)\,(0)) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.812 (sec). Leaf size: 59

```
DSolve[y''''[x]-2*y'''[x]+y''[x]==x*Exp[x]-3*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^{4}}{4} - 2x^{3} - 9x^{2} + e^{x}\left(\frac{x^{3}}{6} - x^{2} + (3+c_{2})x - 4 + c_{1} - 2c_{2}\right) + c_{4}x + c_{3}$$

13.8 problem 8

Internal problem ID [12472]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.2, page 248
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = e^x}$$

With initial conditions

$$[y(0) = -1]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 8

```
dsolve([diff(y(x),x)=exp(x),y(0) = -1],y(x), singsol=all)
```

$$y(x) = e^x - 2$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 10

```
DSolve[{y'[x]==Exp[x],{y[0]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x - 2$$

13.9 problem 9

Internal problem ID [12473]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.2, page 248
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, `class A`]]`

$$\boxed{y' - y = 2\,\mathrm{e}^{x}}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)-y(x)=2*exp(x),y(0) = 1],y(x), singsol=all)
```

$$y(x) = (2x + 1)\,\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 14

```
DSolve[{y'[x]-y[x]==2*Exp[x],{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(2x + 1)$$

13.10 problem 10

Internal problem ID [12474]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.2, page 248
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 9y = x + 2}$$

With initial conditions

$$[y(0) = -1, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 21

```
dsolve([diff(y(x),x$2)-9*y(x)=x+2,y(0) = -1, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{7\cosh(3x)}{9} + \frac{10\sinh(3x)}{27} - \frac{x}{9} - \frac{2}{9}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 33

```
DSolve[{y''[x]-9*y[x]==x+2,{y[0]==-1,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{54}e^{-3x}\left(-6e^{3x}(x+2) - 11e^{6x} - 31\right)$$

13.11 problem 11

Internal problem ID [12475]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.2, page 248
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 9y = x + 2}$$

With initial conditions

$$[y(0) = -1, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 21

```
dsolve([diff(y(x),x$2)+9*y(x)=x+2,y(0) = -1, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{11\cos(3x)}{9} + \frac{8\sin(3x)}{27} + \frac{x}{9} + \frac{2}{9}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 26

```
DSolve[{y''[x]+9*y[x]==x+2,{y[0]==-1,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{27}(3x + 8\sin(3x) - 33\cos(3x) + 6)$$

13.12 problem 12

Internal problem ID [12476]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.2, page 248
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y' + 6y = -2\sin(3x)}$$

With initial conditions

$$[y(0) = 0, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 45

```
dsolve([diff(y(x),x$2)-diff(y(x),x)+6*y(x)=-2*sin(3*x),y(0) = 0, D(y)(0) = -1],y(x), singsol
```

$$y(x) = -\frac{13\,\mathrm{e}^{\frac{x}{2}}\sqrt{23}\,\sin\left(\frac{\sqrt{23}\,x}{2}\right)}{69} + \frac{\mathrm{e}^{\frac{x}{2}}\cos\left(\frac{\sqrt{23}\,x}{2}\right)}{3} - \frac{\cos(3x)}{3} + \frac{\sin(3x)}{3}$$

✓ Solution by Mathematica

Time used: 0.057 (sec). Leaf size: 67

```
DSolve[{y''[x]-y'[x]+6*y[x]==-2*Sin[3*x],{y[0]==0,y'[0]==-1}},y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{1}{69}\left(23\sin(3x) - 13\sqrt{23}e^{x/2}\sin\left(\frac{\sqrt{23}x}{2}\right) - 23\cos(3x) + 23e^{x/2}\cos\left(\frac{\sqrt{23}x}{2}\right)\right)$$

13.13 problem 13

Internal problem ID [12477]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.2, page 248
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y' + 2y = -x^2 + 1}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+2*y(x)=1-x^2,y(0) = 1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{2} + e^x \cos(x) - x$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 20

```
DSolve[{y''[x]-2*y'[x]+2*y[x]==1-x^2,{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to e^x \cos(x) - \frac{1}{2}x(x+2)$$

13.14 problem 14

Internal problem ID [12478]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.2, page 248
Problem number: 14.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + 3y'' + 2y' = x + \cos(x)}$$

With initial conditions

$$[y(0) = 1, y'(0) = -1, y''(0) = 2]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 34

```
dsolve([diff(y(x),x$3)+3*diff(y(x),x$2)+2*diff(y(x),x)=x+cos(x),y(0) = 1, D(y)(0) = -1, (D@@
```

$$y(x) = \frac{17\,\mathrm{e}^{-2x}}{40} + \frac{x^2}{4} - \frac{3\cos(x)}{10} + \frac{\sin(x)}{10} - \frac{3x}{4} - \frac{\mathrm{e}^{-x}}{2} + \frac{11}{8}$$

✓ Solution by Mathematica

Time used: 0.565 (sec). Leaf size: 41

```
DSolve[{y'''[x]+3*y''[x]+2*y'[x]==x+Cos[x],{y[0]==1,y'[0]==-1,y''[0]==2}},y[x],x,IncludeSing
```

$$y(x) \to \frac{1}{40}\left(10x^2 - 30x + 17e^{-2x} - 20e^{-x} + 4\sin(x) - 12\cos(x) + 55\right)$$

14 Chapter 5. The Laplace Transform Method. Exercises 5.3, page 255

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14.1 problem 7

Internal problem ID [12479]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.3, page 255
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 2y = 6}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 15

```
dsolve([diff(y(x),x)-2*y(x)=6,y(0) = 2],y(x), singsol=all)
```

$$y(x) = 2\,\mathrm{e}^{x}(\cosh(x) + 4\sinh(x))$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 14

```
DSolve[{y'[x]-2*y[x]==6,{y[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 5e^{2x} - 3$$

14.2 problem 8

Internal problem ID [12480]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.3, page 255
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + y = \mathrm{e}^{x}}$$

With initial conditions

$$\left[y(0) = \frac{5}{2}\right]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)+y(x)=exp(x),y(0) = 5/2],y(x), singsol=all)
```

$$y(x) = \frac{5\cosh(x)}{2} - \frac{3\sinh(x)}{2}$$

✓ Solution by Mathematica

Time used: 0.066 (sec). Leaf size: 20

```
DSolve[{y'[x]+y[x]==Exp[x],{y[0]==5/2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2e^{-x} + \frac{e^{x}}{2}$$

14.3 problem 9

Internal problem ID [12481]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.3, page 255
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 9y = 1}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 12

```
dsolve([diff(y(x),x$2)+9*y(x)=1,y(0) = 0, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = -\frac{\cos(3x)}{9} + \frac{1}{9}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 17

```
DSolve[{y''[x]+9*y[x]==1,{y[0]==0,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{9}\sin^2\left(\frac{3x}{2}\right)$$

14.4 problem 10

Internal problem ID [12482]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.3, page 255
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 9y = 18\,\mathrm{e}^{3x}}$$

With initial conditions

$$[y(0) = -1, y'(0) = 6]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$2)+9*y(x)=18*exp(3*x),y(0) = -1, D(y)(0) = 6],y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{3x} - 2\cos(3x) + \sin(3x)$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 21

```
DSolve[{y''[x]+9*y[x]==18*Exp[3*x],{y[0]==-1,y'[0]==6}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to e^{3x} + \sin(3x) - 2\cos(3x)$$

14.5 problem 11

Internal problem ID [12483]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.3, page 255
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' - 2y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 3]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 15

```
dsolve([diff(y(x),x$2)-diff(y(x),x)-2*y(x)=0,y(0) = 0, D(y)(0) = 3],y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2x} - \mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 18

```
DSolve[{y''[x]-y'[x]-2*y[x]==0,{y[0]==0,y'[0]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(e^{3x} - 1)$$

14.6 problem 12

Internal problem ID [12484]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.3, page 255
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y' - 2y = x^2}$$

With initial conditions

$$\left[y(0) = \frac{11}{4}, y'(0) = \frac{1}{2}\right]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 26

```
dsolve([diff(y(x),x$2)-diff(y(x),x)-2*y(x)=x^2,y(0) = 11/4, D(y)(0) = 1/2],y(x), singsol=all
```

$$y(x) = -\frac{x^2}{2} + \frac{x}{2} + \frac{7\,\mathrm{e}^{2x}}{6} + \frac{7\,\mathrm{e}^{-x}}{3} - \frac{3}{4}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 33

```
DSolve[{y''[x]-y'[x]-2*y[x]==x^2,{y[0]==11/4,y'[0]==1/2}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{12}\left(-6x^2 + 6x + 28e^{-x} + 14e^{2x} - 9\right)$$

14.7 problem 13

Internal problem ID [12485]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.3, page 255
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + y = 2\sin(x)}$$

With initial conditions

$$[y(0) = -2, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 14

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+y(x)=2*sin(x),y(0) = -2, D(y)(0) = 0],y(x), singsol=al
```

$$y(x) = (3x - 3)\,\mathrm{e}^{x} + \cos(x)$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 16

```
DSolve[{y''[x]-2*y'[x]+y[x]==2*Sin[x],{y[0]==-2,y'[0]==0}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to 3e^{x}(x-1) + \cos(x)$$

14.8 problem 14

Internal problem ID [12486]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.3, page 255
Problem number: 14.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - y'' + 4y' - 4y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 5, y''(0) = 5]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 20

```
dsolve([diff(y(x),x$3)-diff(y(x),x$2)+4*diff(y(x),x)-4*y(x)=0,y(0) = 0, D(y)(0) = 5, (D@@2)(
```

$$y(x) = -2\cos(x)^2 + 4\cos(x)\sin(x) + e^x + 1$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 21

```
DSolve[{y'''[x]-y''[x]+4*y'[x]-4*y[x]==0,{y[0]==0,y'[0]==5,y''[0]==5}},y[x],x,IncludeSingula
```

$$y(x) \to e^x + 2\sin(2x) - \cos(2x)$$

15 Chapter 5. The Laplace Transform Method. Exercises 5.4, page 265

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15.1 problem 4 (a)

Internal problem ID [12487]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.4, page 265
Problem number: 4 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + 2y = \begin{cases} 2 & 0 \leq x < 1 \\ 1 & 1 \leq x \end{cases}}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 22

```
dsolve([diff(y(x),x)+2*y(x)=piecewise(0<=x and x<1,2,1<=x,1),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \begin{cases} 1 & x < 1 \\ \frac{1}{2} + \frac{e^{-2x+2}}{2} & 1 \leq x \end{cases}$$

✓ Solution by Mathematica

Time used: 0.113 (sec). Leaf size: 37

```
DSolve[{y'[x]+2*y[x]==Piecewise[{ {2,0<=x<1},{1,1<=x}}],{y[0]==1}},y[x],x,IncludeSingularSol
```

$$y(x) \to \begin{cases} e^{-2x} & x \leq 0 \\ 1 & 0 < x \leq 1 \\ \frac{1}{2}(1 + e^{2-2x}) & \text{True} \end{cases}$$

15.2 problem 4 (b)

Internal problem ID [12488]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.4, page 265
Problem number: 4 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y' - 2y = \begin{cases} 1 & 2 \leq x < 4 \\ 0 & \text{otherwise} \end{cases}}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 136

```
dsolve([diff(y(x),x$2)-diff(y(x),x)-2*y(x)=piecewise(2<=x and x<4,1,true,0),y(0) = 0, D(y)(0
```

$$y(x) = \frac{\left(\begin{cases} e^{2x} - e^{-x} & x < 2 \\ -\frac{1}{2} - e^{-2} + e^{4} & x = 2 \\ -e^{-x} + e^{2x} - \frac{3}{2} + e^{2-x} + \frac{e^{2x-4}}{2} & x < 4 \\ \frac{(2\,e^{12} + e^{8} - 2\,e^{4} + 2\,e^{2} - 2)\,e^{-4}}{2} & x = 4 \\ -e^{-x} + e^{2x} - e^{4-x} + e^{2-x} - \frac{e^{2x-8}}{2} + \frac{e^{2x-4}}{2} & 4 < x \end{cases}\right)}{3}$$

✓ Solution by Mathematica

Time used: 0.068 (sec). Leaf size: 127

```
DSolve[{y''[x]-y'[x]-2*y[x]==Piecewise[{ {1,2<=x<4},{0,True}}],{y[0]==0,y'[0]==1}},y[x],x,In
```

$$y(x) \to \begin{cases} \frac{1}{3}e^{-x}(-1+e^{3x}) & x \leq 2 \\ \frac{1}{6}e^{-x-4}(-2e^4+2e^6+e^{3x}-3e^{x+4}+2e^{3x+4}) & 2 < x \leq 4 \\ \frac{1}{6}e^{-x-8}(-2e^8+2e^{10}-2e^{12}-e^{3x}+e^{3x+4}+2e^{3x+8}) & \text{True} \end{cases}$$

15.3 problem 4 (c)

Internal problem ID [12489]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.4, page 265
Problem number: 4 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - 2y' = \begin{cases} 0 & 0 \le x < 1 \\ (x-1)^2 & 1 \le x \end{cases}}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 39

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)=piecewise(0<=x and x<1,0,1<=x,(x-1)^2),y(0) = 1, D(y)(
```

$$y(x) = \begin{cases} 1 & x < 1 \\ \frac{7}{8} & x = 1 \\ \frac{25}{24} + \frac{e^{2x-2}}{8} + \frac{x^2}{4} - \frac{x^3}{6} - \frac{x}{4} & 1 < x \end{cases}$$

✓ Solution by Mathematica

Time used: 0.269 (sec). Leaf size: 40

```
DSolve[{y''[x]-2*y'[x]==Piecewise[{ {0,0<=x<1},{(x-1)^2,x>=1}}],{y[0]==1,y'[0]==0}},y[x],x,I
```

$$y(x) \to \begin{cases} 1 & x \le 1 \\ \frac{1}{24}(-4x^3 + 6x^2 - 6x + 3e^{2x-2} + 25) & \text{True} \end{cases}$$

15.4 problem 4 (d)

Internal problem ID [12490]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.4, page 265
Problem number: 4 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + y = \begin{cases} 0 & 0 \leq x < 1 \\ x^2 - 2x + 3 & 1 \leq x \end{cases}}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 43

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+y(x)=piecewise(0<=x and x<1,0,1<=x,x^2-2*x+3),y(0) = 0
```

$$y(x) = \begin{cases} x\,\mathrm{e}^{x} & x < 1 \\ \mathrm{e} + 8 & x = 1 \\ x\,\mathrm{e}^{x} + 5 + 4(x-3)\,\mathrm{e}^{x-1} + x^2 + 2x & 1 < x \end{cases}$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 39

```
DSolve[{y''[x]-2*y'[x]+y[x]==Piecewise[{ {0,0<=x<1},{x^2-2*x+3,x>=1}}],{y[0]==0,y'[0]==1}},y
```

$$y(x) \to \begin{cases} e^x x & x \leq 1 \\ x^2 + e^x x + 2x + 4e^{x-1}(x-3) + 5 & \text{True} \end{cases}$$

15.5 problem 4 (e)

Internal problem ID [12491]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.4, page 265
Problem number: 4 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \begin{cases} 0 & 0 \le x < \pi \\ -\sin(3x) & \pi \le x \end{cases}}$$

With initial conditions

$$[y(0) = 1, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 39

```
dsolve([diff(y(x),x$2)+4*y(x)=piecewise(0<=x and x<Pi,0,Pi<=x,sin(3*(x-Pi))),y(0) = 1, D(y)(
```

$$y(x) = \cos(2x) + \left(\begin{cases} \frac{\sin(2x)}{2} & x < \pi \\ \frac{4\sin(2x)}{5} + \frac{\sin(3x)}{5} & \pi \le x \end{cases} \right)$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 42

```
DSolve[{y''[x]+4*y[x]==Piecewise[{ {0,0<=x<Pi},{Sin[3*(x-Pi)],x>=Pi}}],{y[0]==1,y'[0]==1}},y
```

$$y(x) \to \begin{cases} \cos(2x) + \cos(x)\sin(x) & x \le \pi \\ \frac{1}{5}(5\cos(2x) + 4\sin(2x) + \sin(3x)) & \text{True} \end{cases}$$

15.6 problem 4 (g)

Internal problem ID [12492]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.4, page 265
Problem number: 4 (g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y = \begin{cases} x & 0 \leq x < 1 \\ 1 & 1 \leq x \end{cases}}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 46

```
dsolve([diff(y(x),x$2)-4*y(x)=piecewise(0<=x and x<1,x,1<=x,1),y(0) = 0, D(y)(0) = 0],y(x),
```

$$y(x) = \frac{\left(\begin{cases} \sinh(2x) - 2x & x < 1 \\ \sinh(2) - 4 & x = 1 \\ \sinh(2x) - \sinh(2x - 2) - 2 & 1 < x \end{cases}\right)}{8}$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 36

```
DSolve[{y''[x]-4*y[x]==Piecewise[{ {x,0<=x<1},{x,x>=1}}],{y[0]==0,y'[0]==0}},y[x],x,IncludeS
```

$$y(x) \to \left\{ \begin{array}{cc} 0 & x \leq 0 \\ \frac{1}{16}e^{-2x}(-4e^{2x}x + e^{4x} - 1) & \text{True} \end{array} \right.$$

15.7 problem 4 (h)

Internal problem ID [12493]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.4, page 265
Problem number: 4 (h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 5y = \begin{cases} x & 0 \leq x < 1 \\ 1 & 1 \leq x \end{cases}}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 87

```
dsolve([diff(y(x),x$2)-4*diff(y(x),x)+5*y(x)=piecewise(0<=x and x<1,x,1<=x,1),y(0) = 1, D(y)
```

$$y(x) = \frac{\left(\begin{cases} (21\cos(x) - 47\sin(x))\,\mathrm{e}^{2x} + 5x + 4 & x < 1 \\ 10 + (21\cos(1) - 47\sin(1))\,\mathrm{e}^{2} & x = 1 \\ (4\cos(x-1) - 3\sin(x-1))\,\mathrm{e}^{2x-2} + 5 + (21\cos(x) - 47\sin(x))\,\mathrm{e}^{2x} & 1 < x \end{cases}\right)}{25}$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 51

```
DSolve[{y''[x]-4*y'[x]+5*y[x]==Piecewise[{ {x,0<=x<1},{x,x>=1}}],{y[0]==1,y'[0]==0}},y[x],x,
```

$$y(x) \to \begin{cases} e^{2x}(\cos(x) - 2\sin(x)) & x \leq 0 \\ \frac{1}{25}(5x + 21e^{2x}\cos(x) - 47e^{2x}\sin(x) + 4) & \text{True} \end{cases}$$

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16.1 problem 1

Internal problem ID [12494]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.5, page 273
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + 3y = \delta(x - 2)}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 20

```
dsolve([diff(y(x),x)+3*y(x)=Dirac(x-2),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \text{Heaviside}\,(x - 2)\,\mathrm{e}^{6-3x} + \mathrm{e}^{-3x}$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 21

```
DSolve[{y'[x]+3*y[x]==DiracDelta[x-2],{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(e^6\theta(x - 2) + 1\right)$$

16.2 problem 2

Internal problem ID [12495]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.5, page 273
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - 3y = \delta(x-1) + 2\,\mathrm{Heaviside}\,(x-2)}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 46

```
dsolve([diff(y(x),x)-3*y(x)=Dirac(x-1)+2*Heaviside(x-2),y(0) = 0],y(x), singsol=all)
```

$$y(x) = -\frac{2\,\mathrm{Heaviside}\,(x-2)}{3} + \frac{2\,\mathrm{Heaviside}\,(x-2)\,\mathrm{e}^{-6+3x}}{3} + \mathrm{Heaviside}\,(x-1)\,\mathrm{e}^{3x-3}$$

✓ Solution by Mathematica

Time used: 0.212 (sec). Leaf size: 44

```
DSolve[{y'[x]-3*y[x]==DiracDelta[x-1]+2*UnitStep[x-2],{y[0]==0}},y[x],x,IncludeSingularSolut
```

$$y(x) \to e^{3x-3}\theta(x-1) + \frac{2(e^6 - e^{3x})\,(\theta(2-x) - 1)}{3e^6}$$

16.3 problem 3

Internal problem ID [12496]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.5, page 273
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = \delta(x-\pi) + \delta(x-3\pi)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$2)+9*y(x)=Dirac(x-Pi)+Dirac(x-3*Pi),y(0) = 0, D(y)(0) = 0],y(x), singsol
```

$$y(x) = -\frac{(\text{Heaviside}(x-3\pi) + \text{Heaviside}(x-\pi))\sin(3x)}{3}$$

✓ Solution by Mathematica

Time used: 0.085 (sec). Leaf size: 26

```
DSolve[{y''[x]+9*y[x]==DiracDelta[x-Pi]+DiracDelta[x-3*Pi],{y[0]==0,y'[0]==0}},y[x],x,Includ
```

$$y(x) \to -\frac{1}{3}(\theta(x-3\pi) + \theta(x-\pi))\sin(3x)$$

16.4 problem 4

Internal problem ID [12497]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.5, page 273
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + y = 2(\delta(x-1))}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 28

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+y(x)=2*Dirac(x-1),y(0) = 0, D(y)(0) = 1],y(x), singsol
```

$$y(x) = 2\,\mathrm{Heaviside}\,(x-1)\,\mathrm{e}^{x-1}(x-1) + x\,\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 24

```
DSolve[{y''[x]-2*y'[x]+y[x]==2*DiracDelta[x-1],{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSol
```

$$y(x) \to e^{x-1}(2(x-1)\theta(x-1) + ex)$$

16.5 problem 5

Internal problem ID [12498]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.5, page 273
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + 5y = \cos(x) + 2(\delta(x - \pi))}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 50

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+5*y(x)=cos(x)+2*Dirac(x-Pi),y(0) = 1, D(y)(0) = 0],y(x
```

$$y(x) = \sin(2x)\,\mathrm{Heaviside}(x - \pi)\,\mathrm{e}^{x-\pi} - \frac{7\,\mathrm{e}^{x}\sin(2x)}{20} + \frac{4\,\mathrm{e}^{x}\cos(2x)}{5} + \frac{\cos(x)}{5} - \frac{\sin(x)}{10}$$

✓ Solution by Mathematica

Time used: 0.506 (sec). Leaf size: 54

```
DSolve[{y''[x]-2*y'[x]+5*y[x]==Cos[x]+2*DiracDelta[x-Pi],{y[0]==1,y'[0]==0}},y[x],x,IncludeS
```

$$y(x) \to \frac{1}{10}\left(10e^{x-\pi}\theta(x - \pi)\sin(2x) - \sin(x) + 8e^{x}\cos(2x) + (2 - 7e^{x}\sin(x))\cos(x)\right)$$

16.6 problem 6

Internal problem ID [12499]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.5, page 273
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = (\delta(x - \pi)) \cos(x)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 16

```
dsolve([diff(y(x),x$2)+4*y(x)=cos(x)*Dirac(x-Pi),y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{\sin(2x)(\text{Heaviside}(x - \pi) - 1)}{2}$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 19

```
DSolve[{y''[x]+4*y[x]==Cos[x]*DiracDelta[x-Pi],{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSol
```

$$y(x) \to (\theta(x - \pi) - 1)\sin(x)(-\cos(x))$$

16.7 problem 7

Internal problem ID [12500]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 5. The Laplace Transform Method. Exercises 5.5, page 273
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y a^2 = (\delta(x - \pi)) f(x)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 24

```
dsolve([diff(y(x),x$2)+a^2*y(x)=Dirac(x-Pi)*f(x),y(0) = 0, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{\text{Heaviside}\,(x - \pi)\, f(\pi) \sin\left(a(x - \pi)\right)}{a}$$

✓ Solution by Mathematica

Time used: 0.398 (sec). Leaf size: 26

```
DSolve[{y''[x]+a^2*y[x]==DiracDelta[x-Pi]*f[x],{y[0]==0,y'[0]==0}},y[x],x,IncludeSingularSol
```

$$y(x) \to -\frac{f(\pi)\theta(x - \pi)\sin(a(\pi - x))}{a}$$

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17.1 problem 1

Internal problem ID [12501]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 7. Systems of First-Order Differential Equations. Exercises page 329
Problem number: 1.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(x) &= 2y_1(x) - 3y_2(x) \\
y_2'(x) &= y_1(x) - 2y_2(x)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 31

```
dsolve([diff(y__1(x),x)=2*y__1(x)-3*y__2(x),diff(y__2(x),x)=y__1(x)-2*y__2(x)],[y__1(x), y__
```

$$y_1(x) = c_1 e^{-x} + 3c_2 e^{x}$$

$$y_2(x) = c_1 e^{-x} + c_2 e^{x}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 72

```
DSolve[{y1'[x]==2*y1[x]-3*y2[x],y2'[x]==y1[x]-2*y2[x]},{y1[x],y2[x]},x,IncludeSingularSoluti
```

$$
\begin{aligned}
\text{y1}(x) &\to \frac{1}{2}e^{-x}\left(c_1\left(3e^{2x}-1\right)-3c_2\left(e^{2x}-1\right)\right) \\
\text{y2}(x) &\to \frac{1}{2}e^{-x}\left(c_1\left(e^{2x}-1\right)-c_2\left(e^{2x}-3\right)\right)
\end{aligned}
$$

17.2 problem 3

Internal problem ID [12502]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 7. Systems of First-Order Differential Equations. Exercises page 329
Problem number: 3.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(x) = y_1(x) - 2y_2(x)$$
$$y_2'(x) = y_1(x) + 3y_2(x)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
dsolve([diff(y__1(x),x)=y__1(x)-2*y__2(x),diff(y__2(x),x)=y__1(x)+3*y__2(x)],[y__1(x), y__2(
```

$$y_1(x) = e^{2x}(\cos(x) c_1 - \cos(x) c_2 - \sin(x) c_1 - \sin(x) c_2)$$

$$y_2(x) = e^{2x}(\cos(x) c_2 + \sin(x) c_1)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 51

```
DSolve[{y1'[x]==y1[x]-2*y2[x],y2'[x]==y1[x]+3*y2[x]},{y1[x],y2[x]},x,IncludeSingularSolution
```

$$\mathrm{y1}(x) \to e^{2x}(c_1 \cos(x) - (c_1 + 2c_2)\sin(x))$$
$$\mathrm{y2}(x) \to e^{2x}(c_2 \cos(x) + (c_1 + c_2)\sin(x))$$

17.3 problem 4

Internal problem ID [12503]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 7. Systems of First-Order Differential Equations. Exercises page 329
Problem number: 4.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(x) &= y_1(x) + 2y_2(x) + x - 1 \\
y_2'(x) &= 3y_1(x) + 2y_2(x) - 5x - 2
\end{aligned}
$$

With initial conditions

$$[y_1(0) = -2, y_2(0) = 3]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 18

```
dsolve([diff(y__1(x),x) = y__1(x)+2*y__2(x)+x-1, diff(y__2(x),x) = 3*y__1(x)+2*y__2(x)-5*x-2
```

$$y_1(x) = -2 + 3x$$

$$y_2(x) = 3 - 2x$$

✓ Solution by Mathematica

Time used: 0.316 (sec). Leaf size: 18

```
DSolve[{y1'[x]==y1[x]+2*y2[x]+x-1,y2'[x]==3*y1[x]+2*y2[x]-5*x-2},{y1[0]==-2,y2[0]==3},{y1[x]
```

$$
\begin{aligned}
\mathrm{y1}(x) &\to 3x - 2 \\
\mathrm{y2}(x) &\to 3 - 2x
\end{aligned}
$$

17.4 problem 5

Internal problem ID [12504]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 7. Systems of First-Order Differential Equations. Exercises page 329
Problem number: 5.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(x) &= \frac{2y_1(x)}{x} - \frac{y_2(x)}{x^2} - 3 + \frac{1}{x} - \frac{1}{x^2} \\
y_2'(x) &= 2y_1(x) + 1 - 6x
\end{aligned}
$$

With initial conditions

$$[y_1(1) = -2, y_2(1) = -5]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve([diff(y__1(x),x) = 2*y__1(x)/x-y__2(x)/x^2-3+1/x-1/x^2, diff(y__2(x),x) = 2*y__1(x)+1
```

$$y_1(x) = -2x$$

$$y_2(x) = -1 + x(-5x + 1)$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 19

```
DSolve[{y1'[x]==2*y1[x]/x-y2[x]/x^2-3+1/x-1/x^2,y2'[x]==2*y1[x]+1-6*x},{y1[1]==-2,y2[1]==-5}
```

$$
\begin{aligned}
\mathrm{y1}(x) &\to -2x \\
\mathrm{y2}(x) &\to -5x^2 + x - 1
\end{aligned}
$$

17.5 problem 6

Internal problem ID [12505]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 7. Systems of First-Order Differential Equations. Exercises page 329
Problem number: 6.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(x) = \frac{5y_1(x)}{x} + \frac{4y_2(x)}{x} - 2x$$
$$y_2'(x) = -\frac{6y_1(x)}{x} - \frac{5y_2(x)}{x} + 5x$$

With initial conditions

$$[y_1(-1) = 3, y_2(-1) = -3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
dsolve([diff(y__1(x),x) = 5*y__1(x)/x+4*y__2(x)/x-2*x, diff(y__2(x),x) = -6*y__1(x)/x-5*y__2
```

$$y_1(x) = -\frac{-6x^3 - 3x^2 + 6}{3x}$$

$$y_2(x) = \frac{-x^3 - x^2 + 3}{x}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 33

```
DSolve[{y1'[x]==5*y1[x]/x+4*y2[x]/x-2*x,y2'[x]==-6*y1[x]/x-5*y2[x]/x+5*x},{y1[-1]==3,y2[-1]=
```

$$\text{y1}(x) \rightarrow 2x^2 + x - \frac{2}{x}$$
$$\text{y2}(x) \rightarrow -\frac{x^3 + x^2 - 3}{x}$$

17.6 problem 13 (a)

Internal problem ID [12506]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 7. Systems of First-Order Differential Equations. Exercises page 329
Problem number: 13 (a).
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(x) = 3y_1(x) - 2y_2(x)$$
$$y_2'(x) = -y_1(x) + y_2(x)$$

With initial conditions

$$[y_1(0) = 1, y_2(0) = -1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 79

```
dsolve([diff(y__1(x),x) = 3*y__1(x)-2*y__2(x), diff(y__2(x),x) = -y__1(x)+y__2(x), y__1(0) =
```

$$y_1(x) = \frac{\mathrm{e}^{\left(2+\sqrt{3}\right)x}\sqrt{3}}{2} - \frac{\mathrm{e}^{-\left(-2+\sqrt{3}\right)x}\sqrt{3}}{2} + \frac{\mathrm{e}^{\left(2+\sqrt{3}\right)x}}{2} + \frac{\mathrm{e}^{-\left(-2+\sqrt{3}\right)x}}{2}$$

$$y_2(x) = -\frac{\mathrm{e}^{\left(2+\sqrt{3}\right)x}}{2} - \frac{\mathrm{e}^{-\left(-2+\sqrt{3}\right)x}}{2}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 79

```
DSolve[{y1'[x]==3*y1[x]-2*y2[x],y2'[x]==-y1[x]+y2[x]},{y1[0]==1,y2[0]==-1},{y1[x],y2[x]},x,I
```

$$\mathrm{y1}(x) \to \frac{1}{2}e^{-\left(\left(\sqrt{3}-2\right)x\right)}\left(\left(1+\sqrt{3}\right)e^{2\sqrt{3}x} + 1 - \sqrt{3}\right)$$
$$\mathrm{y2}(x) \to -\frac{1}{2}e^{-\left(\left(\sqrt{3}-2\right)x\right)}\left(e^{2\sqrt{3}x} + 1\right)$$

17.7 problem 13 (b(i))

Internal problem ID [12507]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 7. Systems of First-Order Differential Equations. Exercises page 329
Problem number: 13 (b(i)).
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(x) = \sin(x)\, y_1(x) + \sqrt{x}\, y_2(x) + \ln(x)$$
$$y_2'(x) = \tan(x)\, y_1(x) - e^x y_2(x) + 1$$

With initial conditions

$$[y_1(1) = 1, y_2(1) = -1]$$

✗ Solution by Maple

```
dsolve([diff(y__1(x),x) = sin(x)*y__1(x)+x^(1/2)*y__2(x)+ln(x), diff(y__2(x),x) = tan(x)*y__
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y1'[x]==Sin[x]*y1[x]+Sqrt[x]*y2[x]+Log[x],y2'[x]==Tan[x]*y1[x]-Exp[x]*y2[x]+1},{y1[1
```

Not solved

17.8 problem 13 (b(ii))

Internal problem ID [12508]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 7. Systems of First-Order Differential Equations. Exercises page 329
Problem number: 13 (b(ii)).
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(x) = \sin(x)\, y_1(x) + \sqrt{x}\, y_2(x) + \ln(x)$$
$$y_2'(x) = \tan(x)\, y_1(x) - e^x y_2(x) + 1$$

With initial conditions

$$[y_1(2) = 1, y_2(2) = -1]$$

✗ Solution by Maple

```
dsolve([diff(y__1(x),x) = sin(x)*y__1(x)+x^(1/2)*y__2(x)+ln(x), diff(y__2(x),x) = tan(x)*y__
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y1'[x]==Sin[x]*y1[x]+Sqrt[x]*y2[x]+Log[x],y2'[x]==Tan[x]*y1[x]-Exp[x]*y2[x]+1},{y1[2
```

Not solved

17.9 problem 13 (c(i))

Internal problem ID [12509]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 7. Systems of First-Order Differential Equations. Exercises page 329
Problem number: 13 (c(i)).
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(x) = e^{-x} y_1(x) - \sqrt{x+1}\, y_2(x) + x^2$$
$$y_2'(x) = \frac{y_1(x)}{x^2 - 4x + 4}$$

With initial conditions

$$[y_1(0) = 0, y_2(0) = 1]$$

✗ Solution by Maple

```
dsolve([diff(y__1(x),x) = exp(-x)*y__1(x)-(1+x)^(1/2)*y__2(x)+x^2, diff(y__2(x),x) = y__1(x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y1'[x]==Exp[-x]*y1[x]-Sqrt[x+1]*y2[x]+x^2,y2'[x]==y1[x]/(x-2)^2},{y1[0]==0,y2[0]==1}
```

Not solved

17.10 problem 13 (c(ii))

Internal problem ID [12510]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 7. Systems of First-Order Differential Equations. Exercises page 329
Problem number: 13 (c(ii)).
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(x) = \mathrm{e}^{-x} y_1(x) - \sqrt{x+1}\, y_2(x) + x^2$$
$$y_2'(x) = \frac{y_1(x)}{x^2 - 4x + 4}$$

With initial conditions

$$[y_1(3) = 1, y_2(3) = 0]$$

✗ Solution by Maple

```
dsolve([diff(y__1(x),x) = exp(-x)*y__1(x)-(1+x)^(1/2)*y__2(x)+x^2, diff(y__2(x),x) = y__1(x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y1'[x]==Exp[-x]*y1[x]-Sqrt[x+1]*y2[x]+x^2,y2'[x]==y1[x]/(x-2)^2},{y1[3]==1,y2[3]==0}
```

Not solved

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18.1 problem 1

Internal problem ID [12519]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 8. Linear Systems of First-Order Differential Equations. Exercises 8.3 page 379
Problem number: 1.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(x) &= 2y_1(x) - 3y_2(x) + 5\,\mathrm{e}^{x} \\
y_2'(x) &= y_1(x) + 4y_2(x) - 2\,\mathrm{e}^{-x}
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 111

```
dsolve([diff(y__1(x),x)=2*y__1(x)-3*y__2(x)+5*exp(x),diff(y__2(x),x)=y__1(x)+4*y__2(x)-2*exp
```

$$
\begin{aligned}
y_1(x) = {} & \mathrm{e}^{3x}\cos\left(\sqrt{2}\,x\right)\sqrt{2}\,c_1 - \mathrm{e}^{3x}\sqrt{2}\,\sin\left(\sqrt{2}\,x\right)c_2 \\
& - \mathrm{e}^{3x}\cos\left(\sqrt{2}\,x\right)c_2 - \mathrm{e}^{3x}\sin\left(\sqrt{2}\,x\right)c_1 - \frac{5\,\mathrm{e}^{x}}{2} + \frac{\mathrm{e}^{-x}}{3}
\end{aligned}
$$

$$
y_2(x) = \mathrm{e}^{3x}\cos\left(\sqrt{2}\,x\right)c_2 + \mathrm{e}^{3x}\sin\left(\sqrt{2}\,x\right)c_1 + \frac{\mathrm{e}^{-x}}{3} + \frac{5\,\mathrm{e}^{x}}{6}
$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 108

```
DSolve[{y1'[x]==2*y1[x]-3*y2[x]+5*Exp[x],y2'[x]==y1[x]+4*y2[x]-2*Exp[-x]},{y1[x],y2[x]},x,In
```

$$
\begin{aligned}
\text{y1}(x) &\to -\frac{1}{2}e^{x}\left(-2c_1e^{2x}\cos\left(\sqrt{2}x\right) + \sqrt{2}(c_1+3c_2)e^{2x}\sin\left(\sqrt{2}x\right) + 5\right) \\
\text{y2}(x) &\to \frac{5e^{x}}{6} + c_2e^{3x}\cos\left(\sqrt{2}x\right) + \frac{(c_1+c_2)e^{3x}\sin\left(\sqrt{2}x\right)}{\sqrt{2}}
\end{aligned}
$$

18.2 problem 2

Internal problem ID [12520]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 8. Linear Systems of First-Order Differential Equations. Exercises 8.3 page 379
Problem number: 2.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(x) &= y_2(x) - 2y_1(x) + 2\cos(x)\sin(x) \\
y_2'(x) &= -3y_1(x) + y_2(x) - 8\cos(x)^3 + 6\cos(x)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.891 (sec). Leaf size: 146

```
dsolve([diff(y__1(x),x)=y__2(x)-2*y__1(x)+sin(2*x),diff(y__2(x),x)=-3*y__1(x)+y__2(x)-2*cos(
```

$$
\begin{aligned}
y_1(x) = {} & \frac{c_2 e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right)}{2} - \frac{c_2\sqrt{3}\, e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right)}{6} + \frac{c_1 e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right)}{2} \\
& + \frac{c_1\sqrt{3}\, e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right)}{6} - \frac{4\cos(2x)}{13} + \frac{7\sin(2x)}{13} - \frac{6\sin(3x)}{73} + \frac{16\cos(3x)}{73}
\end{aligned}
$$

$$
\begin{aligned}
y_2(x) = {} & c_2 e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right) + c_1 e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right) \\
& + \frac{9\sin(2x)}{13} + \frac{6\cos(2x)}{13} + \frac{14\cos(3x)}{73} - \frac{60\sin(3x)}{73}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 4.455 (sec). Leaf size: 223

```
DSolve[{y1'[x]==y2[x]-2*y1[x]+Sin[2*x],y2'[x]==-3*y1[x]+y2[x]-2*Cos[3*x]},{y1[x],y2[x]},x,In
```

$$
\begin{aligned}
\mathrm{y1}(x) \to & \frac{7}{13}\sin(2x) - \frac{6}{73}\sin(3x) - \frac{4}{13}\cos(2x) + \frac{16}{73}\cos(3x) \\
& + c_1 e^{-x/2}\cos\left(\frac{\sqrt{3}x}{2}\right) - \sqrt{3}c_1 e^{-x/2}\sin\left(\frac{\sqrt{3}x}{2}\right) + \frac{2c_2 e^{-x/2}\sin\left(\frac{\sqrt{3}x}{2}\right)}{\sqrt{3}} \\
\mathrm{y2}(x) \to & \frac{9}{13}\sin(2x) - \frac{60}{73}\sin(3x) + \frac{6}{13}\cos(2x) + \frac{14}{73}\cos(3x) \\
& + c_2 e^{-x/2}\cos\left(\frac{\sqrt{3}x}{2}\right) - 2\sqrt{3}c_1 e^{-x/2}\sin\left(\frac{\sqrt{3}x}{2}\right) + \sqrt{3}c_2 e^{-x/2}\sin\left(\frac{\sqrt{3}x}{2}\right)
\end{aligned}
$$

18.3 problem 3

Internal problem ID [12521]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 8. Linear Systems of First-Order Differential Equations. Exercises 8.3 page 379
Problem number: 3.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(x) &= 2y_2(x) \\
y_2'(x) &= 3y_1(x) \\
y_3'(x) &= 2y_3(x) - y_1(x)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 123

```
dsolve([diff(y__1(x),x)=2*y__2(x),diff(y__2(x),x)=3*y__1(x),diff(y__3(x),x)=2*y__3(x)-y__1(x
```

$$y_1(x) = -c_2 e^{\sqrt{6}\,x}\sqrt{6} + c_3 e^{-\sqrt{6}\,x}\sqrt{6} + 2c_2 e^{\sqrt{6}\,x} + 2c_3 e^{-\sqrt{6}\,x}$$

$$y_2(x) = c_2 e^{\sqrt{6}\,x}\sqrt{6} - c_3 e^{-\sqrt{6}\,x}\sqrt{6} - 3c_2 e^{\sqrt{6}\,x} - 3c_3 e^{-\sqrt{6}\,x}$$

$$y_3(x) = c_1 e^{2x} + c_2 e^{\sqrt{6}\,x} + c_3 e^{-\sqrt{6}\,x}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 232

```
DSolve[{y1'[x]==2*y2[x],y2'[x]==3*y1[x],y3'[x]==2*y3[x]-y1[x]},{y1[x],y2[x],y3[x]},x,Include
```

$$\mathrm{y1}(x) \to \frac{1}{6}e^{-\sqrt{6}x}\left(3c_1\left(e^{2\sqrt{6}x}+1\right)+\sqrt{6}c_2\left(e^{2\sqrt{6}x}-1\right)\right)$$

$$\mathrm{y2}(x) \to \frac{1}{4}e^{-\sqrt{6}x}\left(\sqrt{6}c_1\left(e^{2\sqrt{6}x}-1\right)+2c_2\left(e^{2\sqrt{6}x}+1\right)\right)$$

$$\mathrm{y3}(x) \to \frac{1}{12}e^{-\sqrt{6}x}\left(2\left(c_2\left(-\left(3+\sqrt{6}\right)e^{2\sqrt{6}x}+6e^{\left(2+\sqrt{6}\right)x}-3+\sqrt{6}\right)+6c_3e^{\left(2+\sqrt{6}\right)x}\right)\right.$$
$$\left.-3c_1\left(\left(2+\sqrt{6}\right)e^{2\sqrt{6}x}-4e^{\left(2+\sqrt{6}\right)x}+2-\sqrt{6}\right)\right)$$

18.4 problem 4

Internal problem ID [12522]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 8. Linear Systems of First-Order Differential Equations. Exercises 8.3 page 379
Problem number: 4.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(x) &= 2xy_1(x) - x^2y_2(x) + 4x \\
y_2'(x) &= y_1(x)\,\mathrm{e}^x + 3\,\mathrm{e}^{-x}y_2(x) - 4\cos(x)^3 + 3\cos(x)
\end{aligned}
$$

✗ Solution by Maple

```
dsolve([diff(y__1(x),x)=2*x*y__1(x)-x^2*y__2(x)+4*x,diff(y__2(x),x)=exp(x)*y__1(x)+3*exp(-x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y1'[x]==2*x*y1[x]-x^2*y2[x]+4*x,y2'[x]==Exp[x]*y1[x]+3*Exp[-x]*y2[x]-Cos[3*x]},{y1[x
```

Not solved

18.5 problem 5 a

Internal problem ID [12523]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 8. Linear Systems of First-Order Differential Equations. Exercises 8.3 page 379
Problem number: 5 a.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(x) &= 2y_1(x) - 3y_2(x) \\
y_2'(x) &= y_1(x) - 2y_2(x)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve([diff(y__1(x),x)=2*y__1(x)-3*y__2(x),diff(y__2(x),x)=y__1(x)-2*y__2(x)],[y__1(x), y__
```

$$y_1(x) = 3c_1\mathrm{e}^{x} + c_2\mathrm{e}^{-x}$$

$$y_2(x) = c_1\mathrm{e}^{x} + c_2\mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 81

```
DSolve[{y1'[x]==-2*y1[x]-3*y2[x],y2'[x]==y1[x]-2*y2[x]},{y1[x],y2[x]},x,IncludeSingularSolut
```

$$
\begin{aligned}
\mathrm{y1}(x) &\to e^{-2x}\left(c_1\cos\left(\sqrt{3}x\right) - \sqrt{3}c_2\sin\left(\sqrt{3}x\right)\right) \\
\mathrm{y2}(x) &\to \frac{1}{3}e^{-2x}\left(3c_2\cos\left(\sqrt{3}x\right) + \sqrt{3}c_1\sin\left(\sqrt{3}x\right)\right)
\end{aligned}
$$

18.6 problem 5 c

Internal problem ID [12524]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 8. Linear Systems of First-Order Differential Equations. Exercises 8.3 page 379
Problem number: 5 c.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(x) = 2y_1(x) - 3y_2(x) + 4x - 2$$
$$y_2'(x) = y_1(x) - 2y_2(x) + 3x$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 36

```
dsolve([diff(y__1(x),x)=2*y__1(x)-3*y__2(x)+4*x-2,diff(y__2(x),x)=y__1(x)-2*y__2(x)+3*x],[y_
```

$$y_1(x) = 3c_2 e^x + c_1 e^{-x} + x$$

$$y_2(x) = c_2 e^x + c_1 e^{-x} - 1 + 2x$$

✓ Solution by Mathematica

Time used: 3.724 (sec). Leaf size: 101

```
DSolve[{y1'[x]==-2*y1[x]-3*y2[x]+4*x-2,y2'[x]==y1[x]-2*y2[x]+3*x},{y1[x],y2[x]},x,IncludeSin
```

$$\text{y1}(x) \to -\frac{x}{7} + c_1 e^{-2x} \cos\left(\sqrt{3}x\right) - \sqrt{3}c_2 e^{-2x} \sin\left(\sqrt{3}x\right) + \frac{4}{49}$$
$$\text{y2}(x) \to \frac{10x}{7} + c_2 e^{-2x} \cos\left(\sqrt{3}x\right) + \frac{c_1 e^{-2x} \sin\left(\sqrt{3}x\right)}{\sqrt{3}} - \frac{33}{49}$$

18.7 problem 6 a

Internal problem ID [12525]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 8. Linear Systems of First-Order Differential Equations. Exercises 8.3 page 379
Problem number: 6 a.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(x) &= \frac{5y_1(x)}{x} + \frac{4y_2(x)}{x} \\
y_2'(x) &= -\frac{6y_1(x)}{x} - \frac{5y_2(x)}{x}
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve([diff(y__1(x),x)=5/x*y__1(x)+4/x*y__2(x),diff(y__2(x),x)=-6/x*y__1(x)-5/x*y__2(x)],[y
```

$$y_1(x) = -\frac{3c_1x^2 + 2c_2}{3x}$$

$$y_2(x) = \frac{c_1x^2 + c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 34

```
DSolve[{y1'[x]==5/x*y1[x]+4/x*y2[x],y2'[x]==-6/x*y1[x]-5/x*y2[x]},{y1[x],y2[x]},x,IncludeSin
```

$$
\begin{aligned}
\text{y1}(x) &\to \frac{c_1}{x} + c_2x \\
\text{y2}(x) &\to -\frac{3c_1}{2x} - c_2x
\end{aligned}
$$

18.8 problem 6 c

Internal problem ID [12526]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 8. Linear Systems of First-Order Differential Equations. Exercises 8.3 page 379
Problem number: 6 c.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(x) &= \frac{5y_1(x)}{x} + \frac{4y_2(x)}{x} - 2x \\
y_2'(x) &= -\frac{6y_1(x)}{x} - \frac{5y_2(x)}{x} + 5x
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 44

```
dsolve([diff(y__1(x),x)=5/x*y__1(x)+4/x*y__2(x)-2*x,diff(y__2(x),x)=-6/x*y__1(x)-5/x*y__2(x)
```

$$
y_1(x) = -\frac{3c_1x^2 - 6x^3 + 2c_2}{3x}
$$

$$
y_2(x) = \frac{c_1x^2 - x^3 + c_2}{x}
$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 44

```
DSolve[{y1'[x]==5/x*y1[x]+4/x*y2[x]-2*x,y2'[x]==-6/x*y1[x]-5/x*y2[x]+5*x},{y1[x],y2[x]},x,In
```

$$
\begin{aligned}
\mathrm{y1}(x) &\to 2x^2 + c_2x + \frac{c_1}{x} \\
\mathrm{y2}(x) &\to -x^2 - c_2x - \frac{3c_1}{2x}
\end{aligned}
$$

18.9 problem 7

Internal problem ID [12527]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 8. Linear Systems of First-Order Differential Equations. Exercises 8.3 page 379
Problem number: 7.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(x) &= 2y_1(x) + y_2(x) - 2y_3(x) \\
y_2'(x) &= 3y_2(x) - 2y_3(x) \\
y_3'(x) &= 3y_1(x) + y_2(x) - 3y_3(x)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 65

```
dsolve([diff(y__1(x),x)=2*y__1(x)+y__2(x)-2*y__3(x),diff(y__2(x),x)=3*y__2(x)-2*y__3(x),diff
```

$$y_1(x) = c_1 e^x + c_2 e^{2x} + \frac{c_3 e^{-x}}{2}$$

$$y_2(x) = c_1 e^x + 2c_2 e^{2x} + \frac{c_3 e^{-x}}{2}$$

$$y_3(x) = c_1 e^x + c_2 e^{2x} + c_3 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 159

```
DSolve[{y1'[x]==2*y1[x]+y2[x]-2*y3[x],y2'[x]==3*y2[x]-2*y3[x],y3'[x]==3*y1[x]+y2[x]-3*y3[x]}
```

$$y1(x) \to e^{-x}\left((e^x-1)\left(c_2e^{2x}-c_3e^x-c_3\right)-c_1\left(-3e^{2x}+e^{3x}+1\right)\right)$$

$$y2(x) \to e^{-x}\left(-\left(c_1(2e^x+1)\left(e^x-1\right)^2\right)+2c_2e^{3x}-(c_2+c_3)e^{2x}+c_3\right)$$

$$y3(x) \to e^{-x}\left(-\left(c_1\left(-3e^{2x}+e^{3x}+2\right)\right)+c_2e^{3x}-(c_2+c_3)e^{2x}+2c_3\right)$$

18.10 problem 8

Internal problem ID [12528]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 8. Linear Systems of First-Order Differential Equations. Exercises 8.3 page 379
Problem number: 8.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(x) &= 5y_1(x) - 5y_2(x) - 5y_3(x) \\
y_2'(x) &= -y_1(x) + 4y_2(x) + 2y_3(x) \\
y_3'(x) &= 3y_1(x) - 5y_2(x) - 3y_3(x)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 69

```
dsolve([diff(y__1(x),x)=5*y__1(x)-5*y__2(x)-5*y__3(x),diff(y__2(x),x)=-1*y__1(x)+4*y__2(x)+2
```

$$y_1(x) = \mathrm{e}^{2x}(\sin(x)\, c_2 + \cos(x)\, c_3)$$

$$y_2(x) = -\frac{\mathrm{e}^{2x}(2\sin(x)\, c_2 - \sin(x)\, c_3 + \cos(x)\, c_2 + 2\cos(x)\, c_3 + 5c_1)}{5}$$

$$y_3(x) = \mathrm{e}^{2x}(\sin(x)\, c_2 + \cos(x)\, c_3 + c_1)$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 109

```
DSolve[{y1'[x]==5*y1[x]-5*y2[x]-5*y3[x],y2'[x]==-1*y1[x]+4*y2[x]+2*y3[x],y3'[x]==3*y1[x]-5*y
```

$$y1(x) \to e^{2x}(c_1 \cos(x) + (3c_1 - 5(c_2 + c_3)) \sin(x))$$
$$y2(x) \to e^{2x}(-c_1(\sin(x) + \cos(x) - 1) + c_3(2\sin(x) + \cos(x) - 1) + c_2(2\sin(x) + \cos(x)))$$
$$y3(x) \to e^{2x}(c_1 \cos(x) + (3c_1 - 5(c_2 + c_3)) \sin(x) - c_1 + c_3)$$

18.11 problem 9

Internal problem ID [12529]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 8. Linear Systems of First-Order Differential Equations. Exercises 8.3 page 379
Problem number: 9.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(x) &= 4y_1(x) + 6y_2(x) + 6y_3(x)\\
y_2'(x) &= y_1(x) + 3y_2(x) + 2y_3(x)\\
y_3'(x) &= -y_1(x) - 4y_2(x) - 3y_3(x)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 63

```
dsolve([diff(y__1(x),x)=4*y__1(x)+6*y__2(x)+6*y__3(x),diff(y__2(x),x)=1*y__1(x)+3*y__2(x)+2*
```

$$y_1(x) = -3c_2 e^{4x} - \frac{6c_3 e^{-x}}{7}$$

$$y_2(x) = -c_1 e^{x} - c_2 e^{4x} - \frac{2c_3 e^{-x}}{7}$$

$$y_3(x) = c_1 e^{x} + c_2 e^{4x} + c_3 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 145

```
DSolve[{y1'[x]==4*y1[x]+6*y2[x]+6*y3[x],y2'[x]==1*y1[x]+3*y2[x]+2*y3[x],y3'[x]==-1*y1[x]-4*y
```

$$\text{y1}(x) \to \frac{1}{5}e^{-x}\left((5c_1+6(c_2+c_3))e^{5x}-6(c_2+c_3)\right)$$
$$\text{y2}(x) \to \frac{1}{15}e^{-x}\left(-5(c_1-3c_2)e^{2x}+(5c_1+6(c_2+c_3))e^{5x}-6(c_2+c_3)\right)$$
$$\text{y3}(x) \to \frac{1}{3}(c_1-3c_2)e^{x}+\frac{7}{5}(c_2+c_3)e^{-x}-\frac{1}{15}(5c_1+6(c_2+c_3))e^{4x}$$

18.12 problem 10

Internal problem ID [12530]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 8. Linear Systems of First-Order Differential Equations. Exercises 8.3 page 379
Problem number: 10.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(x) &= y_1(x) + 2y_2(x) - 3y_3(x)\\
y_2'(x) &= -3y_1(x) + 4y_2(x) - 2y_3(x)\\
y_3'(x) &= 2y_1(x) + y_3(x)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 102

```
dsolve([diff(y__1(x),x)=1*y__1(x)+2*y__2(x)-3*y__3(x),diff(y__2(x),x)=-3*y__1(x)+4*y__2(x)-2
```

$$y_1(x) = \frac{e^{2x}(\sin(3x)\,c_2 - 3\sin(3x)\,c_3 + 3\cos(3x)\,c_2 + \cos(3x)\,c_3 + c_1)}{2}$$

$$y_2(x) = -\frac{e^{2x}(2\sin(3x)\,c_2 + 6\sin(3x)\,c_3 - 6\cos(3x)\,c_2 + 2\cos(3x)\,c_3 - 7c_1)}{4}$$

$$y_3(x) = e^{2x}(\sin(3x)\,c_2 + \cos(3x)\,c_3 + c_1)$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 176

```
DSolve[{y1'[x]==1*y1[x]+2*y2[x]-3*y3[x],y2'[x]==-3*y1[x]+4*y2[x]-2*y3[x],y3'[x]==2*y1[x]+0*y
```

$$\text{y1}(x) \to \frac{1}{9}e^{2x}((11c_1 - 2(c_2 + c_3))\cos(3x) - 3(c_1 - 2c_2 + 3c_3)\sin(3x) + 2(-c_1 + c_2 + c_3))$$

$$\text{y2}(x) \to \frac{1}{9}e^{2x}((7c_1 + 2c_2 - 7c_3)\cos(3x) + (-9c_1 + 6c_2 - 6c_3)\sin(3x) + 7(-c_1 + c_2 + c_3))$$

$$\text{y3}(x) \to \frac{1}{9}e^{2x}((4c_1 - 4c_2 + 5c_3)\cos(3x) + (6c_1 - 3c_3)\sin(3x) + 4(-c_1 + c_2 + c_3))$$

18.13 problem 11

Internal problem ID [12531]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 8. Linear Systems of First-Order Differential Equations. Exercises 8.3 page 379
Problem number: 11.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(x) &= -2y_1(x) - y_2(x) + y_3(x) \\
y_2'(x) &= -y_1(x) - 2y_2(x) - y_3(x) \\
y_3'(x) &= y_1(x) - y_2(x) - 2y_3(x)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 51

```
dsolve([diff(y__1(x),x)=-2*y__1(x)-1*y__2(x)+1*y__3(x),diff(y__2(x),x)=-1*y__1(x)-2*y__2(x)-
```

$$
y_1(x) = -2c_3\mathrm{e}^{-3x} + c_2 + \mathrm{e}^{-3x}c_1
$$

$$
y_2(x) = -c_2 - c_3\mathrm{e}^{-3x} + \mathrm{e}^{-3x}c_1
$$

$$
y_3(x) = c_2 + c_3\mathrm{e}^{-3x}
$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 130

```
DSolve[{y1'[x]==-2*y1[x]-1*y2[x]+1*y3[x],y2'[x]==-1*y1[x]-2*y2[x]-1*y3[x],y3'[x]==1*y1[x]-1*
```

$$
\begin{aligned}
\mathrm{y1}(x) &\to \frac{1}{3}e^{-3x}\left(c_1\left(e^{3x}+2\right)-(c_2-c_3)\left(e^{3x}-1\right)\right) \\
\mathrm{y2}(x) &\to \frac{1}{3}e^{-3x}\left(-\left(c_1\left(e^{3x}-1\right)\right)+c_2\left(e^{3x}+2\right)-c_3\left(e^{3x}-1\right)\right) \\
\mathrm{y3}(x) &\to \frac{1}{3}e^{-3x}\left(c_1\left(e^{3x}-1\right)-c_2\left(e^{3x}-1\right)+c_3\left(e^{3x}+2\right)\right)
\end{aligned}
$$

18.14 problem 12

Internal problem ID [12532]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 8. Linear Systems of First-Order Differential Equations. Exercises 8.3 page 379
Problem number: 12.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(x) &= y_1(x) + y_2(x) + 2y_3(x) \\
y_2'(x) &= y_1(x) + y_2(x) + 2y_3(x) \\
y_3'(x) &= 2y_1(x) + 2y_2(x) + 4y_3(x)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 45

```
dsolve([diff(y__1(x),x)=1*y__1(x)+1*y__2(x)+2*y__3(x),diff(y__2(x),x)=1*y__1(x)+1*y__2(x)+2*
```

$$y_1(x) = \frac{c_3 e^{6x}}{2} - \frac{5c_2}{2} - c_1$$

$$y_2(x) = \frac{c_2}{2} + \frac{c_3 e^{6x}}{2} + c_1$$

$$y_3(x) = c_2 + c_3 e^{6x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 114

```
DSolve[{y1'[x]==1*y1[x]+1*y2[x]+2*y3[x],y2'[x]==1*y1[x]+1*y2[x]+2*y3[x],y3'[x]==2*y1[x]+2*y2
```

$$
\begin{aligned}
\mathrm{y1}(x) &\to \frac{1}{6}\left(c_1\left(e^{6x}+5\right)+(c_2+2c_3)\left(e^{6x}-1\right)\right) \\
\mathrm{y2}(x) &\to \frac{1}{6}\left(c_1\left(e^{6x}-1\right)+c_2\left(e^{6x}+5\right)+2c_3\left(e^{6x}-1\right)\right) \\
\mathrm{y3}(x) &\to \frac{1}{3}\left(c_1\left(e^{6x}-1\right)+c_2\left(e^{6x}-1\right)+c_3\left(2e^{6x}+1\right)\right)
\end{aligned}
$$

18.15 problem 13

Internal problem ID [12533]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 8. Linear Systems of First-Order Differential Equations. Exercises 8.3 page 379
Problem number: 13.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(x) &= 2y_1(x) + y_2(x) \\
y_2'(x) &= -y_1(x) + 2y_2(x) \\
y_3'(x) &= 3y_3(x) - 4y_4(x) \\
y_4'(x) &= 4y_3(x) + 3y_4(x)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 80

```
dsolve([diff(y__1(x),x)=2*y__1(x)+1*y__2(x)+0*y__3(x)+0*y__4(x),diff(y__2(x),x)=-1*y__1(x)+2
```

$$y_1(x) = e^{2x}(\sin(x) c_2 - \cos(x) c_1)$$

$$y_2(x) = e^{2x}(\sin(x) c_1 + \cos(x) c_2)$$

$$y_3(x) = e^{3x}(\cos(4x) c_3 - \sin(4x) c_4)$$

$$y_4(x) = e^{3x}(c_4 \cos(4x) + c_3 \sin(4x))$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 92

```
DSolve[{y1'[x]==2*y1[x]+1*y2[x]+0*y3[x]+0*y4[x],y2'[x]==-1*y1[x]+2*y2[x]+0*y3[x]+0*y4[x],y3'
```

$$
\begin{aligned}
\text{y1}(x) &\to e^{2x}(c_1\cos(x) + c_2\sin(x)) \\
\text{y2}(x) &\to e^{2x}(c_2\cos(x) - c_1\sin(x)) \\
\text{y3}(x) &\to e^{3x}(c_3\cos(4x) - c_4\sin(4x)) \\
\text{y4}(x) &\to e^{3x}(c_4\cos(4x) + c_3\sin(4x))
\end{aligned}
$$

18.16 problem 14

Internal problem ID [12534]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 8. Linear Systems of First-Order Differential Equations. Exercises 8.3 page 379
Problem number: 14.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(x) &= y_2(x) \\
y_2'(x) &= -3y_1(x) + 2y_3(x) \\
y_3'(x) &= y_4(x) \\
y_4'(x) &= 2y_1(x) - 5y_3(x)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 548

```
dsolve([diff(y__1(x),x)=0*y__1(x)+1*y__2(x)+0*y__3(x)+0*y__4(x),diff(y__2(x),x)=-3*y__1(x)+0
```

$$
\begin{aligned}
y_1(x) = &-\frac{5c_1(4+\sqrt{5})^{\frac{3}{2}}\cos\left(\sqrt{4+\sqrt{5}}\,x\right)}{22}-\frac{5c_2(4-\sqrt{5})^{\frac{3}{2}}\cos\left(\sqrt{4-\sqrt{5}}\,x\right)}{22} \\
&-\frac{5c_3(4+\sqrt{5})^{\frac{3}{2}}\sin\left(\sqrt{4+\sqrt{5}}\,x\right)}{22}-\frac{5c_4(4-\sqrt{5})^{\frac{3}{2}}\sin\left(\sqrt{4-\sqrt{5}}\,x\right)}{22} \\
&+\frac{29c_1\sqrt{4+\sqrt{5}}\,\cos\left(\sqrt{4+\sqrt{5}}\,x\right)}{22}+\frac{29c_2\sqrt{4-\sqrt{5}}\,\cos\left(\sqrt{4-\sqrt{5}}\,x\right)}{22} \\
&+\frac{29c_3\sqrt{4+\sqrt{5}}\,\sin\left(\sqrt{4+\sqrt{5}}\,x\right)}{22}+\frac{29c_4\sqrt{4-\sqrt{5}}\,\sin\left(\sqrt{4-\sqrt{5}}\,x\right)}{22}
\end{aligned}
$$

$$
\begin{aligned}
y_2(x) = &-\frac{c_3\cos\left(\sqrt{4+\sqrt{5}}\,x\right)\sqrt{5}}{2}+\frac{c_4\cos\left(\sqrt{4-\sqrt{5}}\,x\right)\sqrt{5}}{2} \\
&+\frac{\sqrt{5}\,\sin\left(\sqrt{4+\sqrt{5}}\,x\right)c_1}{2}-\frac{c_2\sin\left(\sqrt{4-\sqrt{5}}\,x\right)\sqrt{5}}{2}+\frac{c_3\cos\left(\sqrt{4+\sqrt{5}}\,x\right)}{2} \\
&+\frac{c_4\cos\left(\sqrt{4-\sqrt{5}}\,x\right)}{2}-\frac{c_1\sin\left(\sqrt{4+\sqrt{5}}\,x\right)}{2}-\frac{c_2\sin\left(\sqrt{4-\sqrt{5}}\,x\right)}{2}
\end{aligned}
$$

$$
\begin{aligned}
y_3(x) = &\frac{8c_1\sqrt{4+\sqrt{5}}\,\cos\left(\sqrt{4+\sqrt{5}}\,x\right)}{11}+\frac{8c_2\sqrt{4-\sqrt{5}}\,\cos\left(\sqrt{4-\sqrt{5}}\,x\right)}{11} \\
&+\frac{8c_3\sqrt{4+\sqrt{5}}\,\sin\left(\sqrt{4+\sqrt{5}}\,x\right)}{11}+\frac{8c_4\sqrt{4-\sqrt{5}}\,\sin\left(\sqrt{4-\sqrt{5}}\,x\right)}{11} \\
&-\frac{c_1(4+\sqrt{5})^{\frac{3}{2}}\cos\left(\sqrt{4+\sqrt{5}}\,x\right)}{11}-\frac{c_2(4-\sqrt{5})^{\frac{3}{2}}\cos\left(\sqrt{4-\sqrt{5}}\,x\right)}{11} \\
&-\frac{c_3(4+\sqrt{5})^{\frac{3}{2}}\sin\left(\sqrt{4+\sqrt{5}}\,x\right)}{11}-\frac{c_4(4-\sqrt{5})^{\frac{3}{2}}\sin\left(\sqrt{4-\sqrt{5}}\,x\right)}{11}
\end{aligned}
$$

$$
\begin{aligned}
y_4(x) = &-c_1\sin\left(\sqrt{4+\sqrt{5}}\,x\right)-c_2\sin\left(\sqrt{4-\sqrt{5}}\,x\right) \\
&+c_3\cos\left(\sqrt{4+\sqrt{5}}\,x\right)+c_4\cos\left(\sqrt{4-\sqrt{5}}\,x\right)
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.099 (sec). Leaf size: 730

```
DSolve[{y1'[x]==0*y1[x]+1*y2[x]+0*y3[x]+0*y4[x],y2'[x]==-3*y1[x]+0*y2[x]+2*y3[x]+0*y4[x],y3'
```

$$
\begin{aligned}
\mathrm{y1}(x) \to\ & \frac{1}{2}c_3\mathrm{RootSum}\left[\#1^4+8\#1^2+11\&,\frac{e^{\#1x}}{\#1^2+4}\&\right] \\
& + \frac{1}{4}c_1\mathrm{RootSum}\left[\#1^4+8\#1^2+11\&,\frac{\#1^2e^{\#1x}+5e^{\#1x}}{\#1^2+4}\&\right] \\
& + \frac{1}{2}c_4\mathrm{RootSum}\left[\#1^4+8\#1^2+11\&,\frac{e^{\#1x}}{\#1^3+4\#1}\&\right] \\
& + \frac{1}{4}c_2\mathrm{RootSum}\left[\#1^4+8\#1^2+11\&,\frac{\#1^2e^{\#1x}+5e^{\#1x}}{\#1^3+4\#1}\&\right] \\
\mathrm{y2}(x) \to\ & \frac{1}{2}c_4\mathrm{RootSum}\left[\#1^4+8\#1^2+11\&,\frac{e^{\#1x}}{\#1^2+4}\&\right] \\
& + \frac{1}{2}c_3\mathrm{RootSum}\left[\#1^4+8\#1^2+11\&,\frac{\#1e^{\#1x}}{\#1^2+4}\&\right] \\
& + \frac{1}{4}c_2\mathrm{RootSum}\left[\#1^4+8\#1^2+11\&,\frac{\#1^2e^{\#1x}+5e^{\#1x}}{\#1^2+4}\&\right] \\
& - \frac{1}{4}c_1\mathrm{RootSum}\left[\#1^4+8\#1^2+11\&,\frac{3\#1^2e^{\#1x}+11e^{\#1x}}{\#1^3+4\#1}\&\right] \\
\mathrm{y3}(x) \to\ & \frac{1}{2}c_1\mathrm{RootSum}\left[\#1^4+8\#1^2+11\&,\frac{e^{\#1x}}{\#1^2+4}\&\right] \\
& + \frac{1}{4}c_3\mathrm{RootSum}\left[\#1^4+8\#1^2+11\&,\frac{\#1^2e^{\#1x}+3e^{\#1x}}{\#1^2+4}\&\right] \\
& + \frac{1}{2}c_2\mathrm{RootSum}\left[\#1^4+8\#1^2+11\&,\frac{e^{\#1x}}{\#1^3+4\#1}\&\right] \\
& + \frac{1}{4}c_4\mathrm{RootSum}\left[\#1^4+8\#1^2+11\&,\frac{\#1^2e^{\#1x}+3e^{\#1x}}{\#1^3+4\#1}\&\right] \\
\mathrm{y4}(x) \to\ & \frac{1}{2}c_2\mathrm{RootSum}\left[\#1^4+8\#1^2+11\&,\frac{e^{\#1x}}{\#1^2+4}\&\right] \\
& + \frac{1}{2}c_1\mathrm{RootSum}\left[\#1^4+8\#1^2+11\&,\frac{\#1e^{\#1x}}{\#1^2+4}\&\right] \\
& + \frac{1}{4}c_4\mathrm{RootSum}\left[\#1^4+8\#1^2+11\&,\frac{\#1^2e^{\#1x}+3e^{\#1x}}{\#1^2+4}\&\right] \\
& - \frac{1}{4}c_3\mathrm{RootSum}\left[\#1^4+8\#1^2+11\&,\frac{5\#1^2e^{\#1x}+11e^{\#1x}}{\#1^3+4\#1}\&\right]
\end{aligned}
$$

18.17 problem 15

Internal problem ID [12535]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 8. Linear Systems of First-Order Differential Equations. Exercises 8.3 page 379
Problem number: 15.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(x) &= 3y_1(x) + 2y_2(x)\\
y_2'(x) &= -2y_1(x) + 3y_2(x)\\
y_3'(x) &= y_3(x)\\
y_4'(x) &= 2y_4(x)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 61

```
dsolve([diff(y__1(x),x)=3*y__1(x)+2*y__2(x)+0*y__3(x)+0*y__4(x),diff(y__2(x),x)=-2*y__1(x)+3
```

$$y_1(x) = e^{3x}(\sin(2x)\, c_2 - \cos(2x)\, c_1)$$

$$y_2(x) = e^{3x}(\sin(2x)\, c_1 + \cos(2x)\, c_2)$$

$$y_3(x) = c_3 e^{x}$$

$$y_4(x) = c_4 e^{2x}$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 255

```
DSolve[{y1'[x]==3*y1[x]+2*y2[x]+0*y3[x]+0*y4[x],y2'[x]==-2*y1[x]+3*y2[x]+0*y3[x]+0*y4[x],y3'
```

$$
\begin{aligned}
\mathrm{y1}(x) &\to e^{3x}(c_1\cos(2x) + c_2\sin(2x)) \\
\mathrm{y2}(x) &\to e^{3x}(c_2\cos(2x) - c_1\sin(2x)) \\
\mathrm{y3}(x) &\to c_3e^{x} \\
\mathrm{y4}(x) &\to c_4e^{2x} \\
\mathrm{y1}(x) &\to e^{3x}(c_1\cos(2x) + c_2\sin(2x)) \\
\mathrm{y2}(x) &\to e^{3x}(c_2\cos(2x) - c_1\sin(2x)) \\
\mathrm{y3}(x) &\to c_3e^{x} \\
\mathrm{y4}(x) &\to 0 \\
\mathrm{y1}(x) &\to e^{3x}(c_1\cos(2x) + c_2\sin(2x)) \\
\mathrm{y2}(x) &\to e^{3x}(c_2\cos(2x) - c_1\sin(2x)) \\
\mathrm{y3}(x) &\to 0 \\
\mathrm{y4}(x) &\to c_4e^{2x} \\
\mathrm{y1}(x) &\to e^{3x}(c_1\cos(2x) + c_2\sin(2x)) \\
\mathrm{y2}(x) &\to e^{3x}(c_2\cos(2x) - c_1\sin(2x)) \\
\mathrm{y3}(x) &\to 0 \\
\mathrm{y4}(x) &\to 0
\end{aligned}
$$

18.18 problem 16

Internal problem ID [12536]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 8. Linear Systems of First-Order Differential Equations. Exercises 8.3 page 379
Problem number: 16.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(x) &= y_2(x) + y_4(x)\\
y_2'(x) &= y_1(x) - y_3(x)\\
y_3'(x) &= y_4(x)\\
y_4'(x) &= y_3(x)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 71

```
dsolve([diff(y__1(x),x)=0*y__1(x)+1*y__2(x)+0*y__3(x)+1*y__4(x),diff(y__2(x),x)=1*y__1(x)+0*
```

$$y_1(x) = c_1 e^x - c_2 e^{-x} + c_3 e^x - c_4 e^{-x}$$

$$y_2(x) = c_1 e^x + c_2 e^{-x}$$

$$y_3(x) = c_3 e^x - c_4 e^{-x}$$

$$y_4(x) = c_3 e^x + c_4 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 148

```
DSolve[{y1'[x]==0*y1[x]+1*y2[x]+0*y3[x]+1*y4[x],y2'[x]==1*y1[x]+0*y2[x]-1*y3[x]+0*y4[x],y3'[
```

$$y1(x) \to \frac{1}{2}e^{-x}\left(c_1\left(e^{2x}+1\right)+\left(c_2+c_4\right)\left(e^{2x}-1\right)\right)$$

$$y2(x) \to \frac{1}{2}e^{-x}\left(c_1\left(e^{2x}-1\right)+c_2e^{2x}-c_3e^{2x}+c_2+c_3\right)$$

$$y3(x) \to \frac{1}{2}e^{-x}\left(c_3\left(e^{2x}+1\right)+c_4\left(e^{2x}-1\right)\right)$$

$$y4(x) \to \frac{1}{2}e^{-x}\left(c_3\left(e^{2x}-1\right)+c_4\left(e^{2x}+1\right)\right)$$

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19.1 problem 1

Internal problem ID [12537]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 10. Applications of Systems of Equations. Exercises 10.2 page 432
Problem number: 1.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -2x(t) + 3y(t)$$
$$y'(t) = -x(t) + 2y(t)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve([diff(x(t),t)=-2*x(t)+3*y(t),diff(y(t),t)=-x(t)+2*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = c_1\mathrm{e}^{t} + 3c_2\mathrm{e}^{-t}$$

$$y(t) = c_1\mathrm{e}^{t} + c_2\mathrm{e}^{-t}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 72

```
DSolve[{x'[t]==-2*x[t]+3*y[t],y'[t]==-x[t]+2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{1}{2}e^{-t}\left(3c_2\left(e^{2t}-1\right) - c_1\left(e^{2t}-3\right)\right)$$
$$y(t) \to -\frac{1}{2}e^{-t}\left(c_1\left(e^{2t}-1\right) + c_2\left(1-3e^{2t}\right)\right)$$

19.2 problem 2

Internal problem ID [12538]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 10. Applications of Systems of Equations. Exercises 10.2 page 432
Problem number: 2.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -x(t) + 2y(t)$$
$$y'(t) = -2x(t) + 3y(t)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve([diff(x(t),t)=-x(t)+2*y(t),diff(y(t),t)=-2*x(t)+3*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = \frac{e^t(2c_2t + 2c_1 - c_2)}{2}$$

$$y(t) = e^t(c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 42

```
DSolve[{x'[t]==-x[t]+2*y[t],y'[t]==-2*x[t]+3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to e^t(-2c_1t + 2c_2t + c_1)$$
$$y(t) \to e^t(-2c_1t + 2c_2t + c_2)$$

19.3 problem 3

Internal problem ID [12539]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 10. Applications of Systems of Equations. Exercises 10.2 page 432
Problem number: 3.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -x(t) - 2y(t)$$
$$y'(t) = 2x(t) - 3y(t)$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 78

```
dsolve([diff(x(t),t)=-x(t)-2*y(t),diff(y(t),t)=2*x(t)-3*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{e^{-2t}\left(\sqrt{3}\,\sin\left(\sqrt{3}\,t\right)c_2 - \sqrt{3}\,\cos\left(\sqrt{3}\,t\right)c_1 - \sin\left(\sqrt{3}\,t\right)c_1 - \cos\left(\sqrt{3}\,t\right)c_2\right)}{2}$$

$$y(t) = e^{-2t}\left(\sin\left(\sqrt{3}\,t\right)c_1 + \cos\left(\sqrt{3}\,t\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 96

```
DSolve[{x'[t]==-x[t]-2*y[t],y'[t]==2*x[t]-3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{1}{3}e^{-2t}\left(3c_1\cos\left(\sqrt{3}t\right) + \sqrt{3}(c_1 - 2c_2)\sin\left(\sqrt{3}t\right)\right)$$
$$y(t) \to \frac{1}{3}e^{-2t}\left(3c_2\cos\left(\sqrt{3}t\right) + \sqrt{3}(2c_1 - c_2)\sin\left(\sqrt{3}t\right)\right)$$

19.4 problem 4

Internal problem ID [12540]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 10. Applications of Systems of Equations. Exercises 10.2 page 432
Problem number: 4.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= -x(t) - 2y(t) \\
y'(t) &= 5x(t) + y(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 50

```
dsolve([diff(x(t),t)=-x(t)-2*y(t),diff(y(t),t)=5*x(t)+1*y(t)],[x(t), y(t)], singsol=all)
```

$$
x(t) = \frac{3c_1 \cos(3t)}{5} - \frac{3c_2 \sin(3t)}{5} - \frac{c_1 \sin(3t)}{5} - \frac{c_2 \cos(3t)}{5}
$$

$$
y(t) = c_1 \sin(3t) + c_2 \cos(3t)
$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 54

```
DSolve[{x'[t]==-x[t]-2*y[t],y'[t]==5*x[t]+1*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$
\begin{aligned}
x(t) &\to c_1 \cos(3t) - \frac{1}{3}(c_1 + 2c_2) \sin(3t) \\
y(t) &\to c_2 \cos(3t) + \frac{1}{3}(5c_1 + c_2) \sin(3t)
\end{aligned}
$$

19.5 problem 5

Internal problem ID [12541]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 10. Applications of Systems of Equations. Exercises 10.2 page 432
Problem number: 5.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= -x(t) + 2y(t) \\
y'(t) &= -2x(t) - y(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 46

```
dsolve([diff(x(t),t)=-x(t)+2*y(t),diff(y(t),t)=-2*x(t)-1*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\mathrm{e}^{-t}(\cos(2t)\, c_1 - \sin(2t)\, c_2)$$

$$y(t) = \mathrm{e}^{-t}(\cos(2t)\, c_2 + \sin(2t)\, c_1)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 51

```
DSolve[{x'[t]==-x[t]+2*y[t],y'[t]==-2*x[t]-1*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$
\begin{aligned}
x(t) &\to e^{-t}(c_1 \cos(2t) + c_2 \sin(2t)) \\
y(t) &\to e^{-t}(c_2 \cos(2t) - c_1 \sin(2t))
\end{aligned}
$$

19.6 problem 6

Internal problem ID [12542]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 10. Applications of Systems of Equations. Exercises 10.2 page 432
Problem number: 6.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) - 2y(t)$$
$$y'(t) = 2x(t) + y(t)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
dsolve([diff(x(t),t)=x(t)-2*y(t),diff(y(t),t)=2*x(t)+1*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = e^{t}(\cos(2t)\, c_1 - \sin(2t)\, c_2)$$

$$y(t) = e^{t}(\cos(2t)\, c_2 + \sin(2t)\, c_1)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 47

```
DSolve[{x'[t]==x[t]-2*y[t],y'[t]==2*x[t]+1*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> T
```

$$x(t) \to e^t(c_1\cos(2t) - c_2\sin(2t))$$
$$y(t) \to e^t(c_2\cos(2t) + c_1\sin(2t))$$

19.7 problem 7

Internal problem ID [12543]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 10. Applications of Systems of Equations. Exercises 10.2 page 432
Problem number: 7.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= -5x(t) - y(t) + 2 \\
y'(t) &= 3x(t) - y(t) - 3
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 39

```
dsolve([diff(x(t),t)=-5*x(t)-y(t)+2,diff(y(t),t)=3*x(t)-1*y(t)-3],[x(t), y(t)], singsol=all)
```

$$
x(t) = \frac{e^{-4t}c_1}{2} - \frac{e^{-2t}c_2}{3} + \frac{5}{8}
$$

$$
y(t) = -\frac{9}{8} - \frac{e^{-4t}c_1}{2} + e^{-2t}c_2
$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 93

```
DSolve[{x'[t]==-5*x[t]-y[t]+2,y'[t]==3*x[t]-1*y[t]-3},{x[t],y[t]},t,IncludeSingularSolutions
```

$$
\begin{aligned}
x(t) &\to \frac{1}{48}e^{-4t}\left(30e^{4t} - (1 + 24c_1 + 24c_2)e^{2t} + 3 + 72c_1 + 24c_2\right) \\
y(t) &\to \frac{1}{16}e^{-4t}\left(-18e^{4t} + (1 + 24c_1 + 24c_2)e^{2t} - 1 - 24c_1 - 8c_2\right)
\end{aligned}
$$

19.8 problem 8

Internal problem ID [12544]

Book: Ordinary Differential Equations by Charles E. Roberts, Jr. CRC Press. 2010
Section: Chapter 10. Applications of Systems of Equations. Exercises 10.2 page 432
Problem number: 8.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 3x(t) - 2y(t) - 6$$
$$y'(t) = 4x(t) - y(t) + 2$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 60

```
dsolve([diff(x(t),t)=3*x(t)-2*y(t)-6,diff(y(t),t)=4*x(t)-1*y(t)+2],[x(t), y(t)], singsol=all
```

$$x(t) = -2 - \frac{e^{t}(\sin(2t)\, c_1 - \sin(2t)\, c_2 - \cos(2t)\, c_1 - \cos(2t)\, c_2)}{2}$$

$$y(t) = -6 + e^{t}(\sin(2t)\, c_2 + \cos(2t)\, c_1)$$

✓ Solution by Mathematica

Time used: 0.358 (sec). Leaf size: 64

```
DSolve[{x'[t]==3*x[t]-2*y[t]-6,y'[t]==4*x[t]-1*y[t]+2},{x[t],y[t]},t,IncludeSingularSolution
```

$$x(t) \to c_1 e^t \cos(2t) + (c_1 - c_2)e^t \sin(2t) - 2$$
$$y(t) \to c_2 e^t \cos(2t) + (2c_1 - c_2)e^t \sin(2t) - 6$$