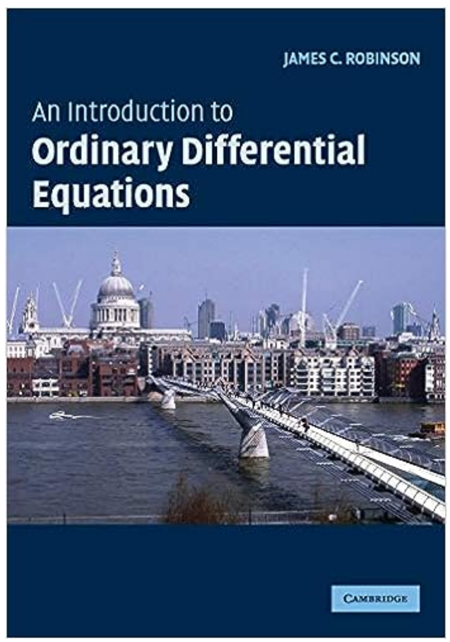
A Solution Manual For

AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004



Nasser M. Abbasi

May 16, 2024

Contents

1 Chapter 5, Trivial differential equations. Exercises page 33

1.1 problem 5.1 (i)

Internal problem ID [11968]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 5, Trivial differential equations. Exercises page 33
Problem number: 5.1 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' = \cos(t) + \sin(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(x(t),t)=sin(t)+cos(t),x(t), singsol=all)
```

$$x(t) = -\cos(t) + \sin(t) + c_1$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 14

```
DSolve[x'[t]==Sin[t]+Cos[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \sin(t) - \cos(t) + c_1$$

1.2 problem 5.1 (ii)

Internal problem ID [11969]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 5, Trivial differential equations. Exercises page 33
Problem number: 5.1 (ii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{1}{x^2 - 1}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=1/(x^2-1),y(x), singsol=all)
```

$$y(x) = -\operatorname{arctanh}(x) + c_1$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 26

```
DSolve[y'[x]==1/(x^2-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(\log(1 - x) - \log(x + 1) + 2c_1)$$

1.3 problem 5.1 (iii)

Internal problem ID [11970]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 5, Trivial differential equations. Exercises page 33
Problem number: 5.1 (iii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{u' = 4t\ln(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(u(t),t)=4*t*ln(t),u(t), singsol=all)
```

$$u(t) = 2\ln(t)\,t^2 - t^2 + c_1$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 20

```
DSolve[u'[t]==4*t*Log[t],u[t],t,IncludeSingularSolutions -> True]
```

$$u(t) \to -t^2 + 2t^2\log(t) + c_1$$

1.4 problem 5.1 (iv)

Internal problem ID [11971]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 5, Trivial differential equations. Exercises page 33
Problem number: 5.1 (iv).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{z' = x\,\mathrm{e}^{-2x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(z(x),x)=x*exp(-2*x),z(x), singsol=all)
```

$$z(x) = \frac{(-2x-1)\,\mathrm{e}^{-2x}}{4} + c_1$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 22

```
DSolve[z'[x]==x*Exp[-2*x],z[x],x,IncludeSingularSolutions -> True]
```

$$z(x) \to -\frac{1}{4}e^{-2x}(2x+1) + c_1$$

1.5 problem 5.1 (v)

Internal problem ID [11972]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 5, Trivial differential equations. Exercises page 33
Problem number: 5.1 (v).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{T' = \mathrm{e}^{-t} \sin (2t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(T(t),t)=exp(-t)*sin(2*t),T(t), singsol=all)
```

$$T(t) = \frac{\mathrm{e}^{-t}(-2\cos (2t) - \sin (2t))}{5} + c_1$$

✓ Solution by Mathematica
Time used: 0.055 (sec). Leaf size: 28

```
DSolve[T'[t]==Exp[-t]*Sin[2*t],T[t],t,IncludeSingularSolutions -> True]
```

$$T(t) \to -\frac{1}{5}e^{-t}(\sin(2t) + 2\cos(2t)) + c_1$$

1.6 problem 5.4 (i)

Internal problem ID [11973]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 5, Trivial differential equations. Exercises page 33
Problem number: 5.4 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' = \sec(t)^2}$$

With initial conditions

$$\left[x\left(\frac{\pi}{4}\right) = 0\right]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 8

```
dsolve([diff(x(t),t)=sec(t)^2,x(1/4*Pi) = 0],x(t), singsol=all)
```

$$x(t) = \tan(t) - 1$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 9

```
DSolve[{x'[t]==Sec[t]^2,{x[Pi/4]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \tan(t) - 1$$

1.7 problem 5.4 (ii)

Internal problem ID [11974]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 5, Trivial differential equations. Exercises page 33
Problem number: 5.4 (ii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = x - \frac{1}{3}x^3}$$

With initial conditions

$$[y(-1) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(x),x)=x-1/3*x^3,y(-1) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{(x^2-3)^2}{12} + \frac{4}{3}$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 21

```
DSolve[{y'[x]==x-1/3*x^3,{y[-1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}(-x^4 + 6x^2 + 7)$$

1.8 problem 5.4 (iii)

Internal problem ID [11975]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 5, Trivial differential equations. Exercises page 33
Problem number: 5.4 (iii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' = 2\sin(t)^2}$$

With initial conditions

$$\left[x\left(\frac{\pi}{4}\right) = \frac{\pi}{4}\right]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 13

```
dsolve([diff(x(t),t)=2*sin(t)^2,x(1/4*Pi) = 1/4*Pi],x(t), singsol=all)
```

$$x(t) = t + \frac{1}{2} - \frac{\sin(2t)}{2}$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 16

```
DSolve[{x'[t]==2*Sin[t]^2,{x[Pi/4]==Pi/4}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to t - \sin(t)\cos(t) + \frac{1}{2}$$

1.9 problem 5.4 (iv)

Internal problem ID [11976]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 5, Trivial differential equations. Exercises page 33
Problem number: 5.4 (iv).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xV' = x^2 + 1}$$

With initial conditions

$$[V(1) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 13

```
dsolve([x*diff(V(x),x)=1+x^2,V(1) = 1],V(x), singsol=all)
```

$$V(x) = \frac{x^2}{2} + \ln(x) + \frac{1}{2}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 18

```
DSolve[{x*V'[x]==1+x^2,{V[1]==1}},V[x],x,IncludeSingularSolutions -> True]
```

$$V(x) \to \frac{1}{2}\left(x^2 + 2\log(x) + 1\right)$$

1.10 problem 5.4 (v)

Internal problem ID [11977]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 5, Trivial differential equations. Exercises page 33
Problem number: 5.4 (v).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{x' e^{3t} + 3x\, e^{3t} = e^{-t}}$$

With initial conditions

$$[x(0) = 3]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve([diff(x(t)*exp(3*t),t)=exp(-t),x(0) = 3],x(t), singsol=all)
```

$$x(t) = -\left(e^{-t} - 4\right) e^{-3t}$$

✓ Solution by Mathematica
Time used: 0.092 (sec). Leaf size: 18

```
DSolve[{D[x[t]*Exp[3*t],t]==Exp[-t],{x[0]==3}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{-4t}\left(4e^t - 1\right)$$

2 Chapter 7, Scalar autonomous ODEs. Exercises page 56

2.1 problem 7.1 (i)

Internal problem ID [11978]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 7, Scalar autonomous ODEs. Exercises page 56
Problem number: 7.1 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' + x = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(x(t),t)=-x(t)+1,x(t), singsol=all)
```

$$x(t) = 1 + \mathrm{e}^{-t} c_1$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 20

```
DSolve[x'[t]==-x[t]+1,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to 1 + c_1 e^{-t}$$
$$x(t) \to 1$$

2.2 problem 7.1 (ii)

Internal problem ID [11979]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 7, Scalar autonomous ODEs. Exercises page 56
Problem number: 7.1 (ii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - x(-x+2) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(x(t),t)=x(t)*(2-x(t)),x(t), singsol=all)
```

$$x(t) = \frac{2}{1 + 2\,\mathrm{e}^{-2t} c_1}$$

✓ Solution by Mathematica
Time used: 0.503 (sec). Leaf size: 36

```
DSolve[x'[t]==x[t]*(2-x[t]),x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{2e^{2t}}{e^{2t} + e^{2c_1}}$$
$$x(t) \to 0$$
$$x(t) \to 2$$

2.3 problem 7.1 (iii)

Internal problem ID [11980]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 7, Scalar autonomous ODEs. Exercises page 56
Problem number: 7.1 (iii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - (x+1)(-x+2)\sin(x) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(x(t),t)=(1+x(t))*(2-x(t))*sin(x(t)),x(t), singsol=all)
```

$$t + \int^{x(t)} \frac{\csc(\_a)}{(\_a+1)(\_a-2)} d\_a + c_1 = 0$$

✓ Solution by Mathematica
Time used: 15.593 (sec). Leaf size: 52

```
DSolve[x'[t]==(1+x[t])*(2-x[t])*Sin[x[t]],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\csc(K[1])}{(K[1]-2)(K[1]+1)} dK[1]\&\right][-t+c_1]$$
$$x(t) \to -1$$
$$x(t) \to 0$$
$$x(t) \to 2$$

2.4 problem 7.1 (iv)

Internal problem ID [11981]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 7, Scalar autonomous ODEs. Exercises page 56
Problem number: 7.1 (iv).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' + x(1-x)(-x+2) = 0}$$

✓ Solution by Maple
Time used: 0.079 (sec). Leaf size: 34

```
dsolve(diff(x(t),t)=-x(t)*(1-x(t))*(2-x(t)),x(t), singsol=all)
```

$$x(t) = \frac{c_1 \mathrm{e}^t + \sqrt{-1 + \mathrm{e}^{2t} c_1^2}}{\sqrt{-1 + \mathrm{e}^{2t} c_1^2}}$$

✓ Solution by Mathematica
Time used: 19.885 (sec). Leaf size: 159

```
DSolve[x'[t]==-x[t]*(1-x[t])*(2-x[t]),x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{e^{2t} - \sqrt{e^{4t} + e^{2(t+c_1)}} + e^{2c_1}}{e^{2t} + e^{2c_1}}$$
$$x(t) \to \frac{e^{2t} + \sqrt{e^{4t} + e^{2(t+c_1)}} + e^{2c_1}}{e^{2t} + e^{2c_1}}$$
$$x(t) \to 0$$
$$x(t) \to 1$$
$$x(t) \to 2$$
$$x(t) \to 1 - e^{-2t}\sqrt{e^{4t}}$$
$$x(t) \to e^{-2t}\sqrt{e^{4t}} + 1$$

2.5 problem 7.1 (v)

Internal problem ID [11982]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 7, Scalar autonomous ODEs. Exercises page 56
Problem number: 7.1 (v).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - x^2 + x^4 = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 47

```
dsolve(diff(x(t),t)=x(t)^2-x(t)^4,x(t), singsol=all)
```

$$x(t) = \mathrm{e}^{\mathrm{RootOf}\left(\ln\left(\mathrm{e}^{\_Z}-2\right)\mathrm{e}^{\_Z}+2c_1\mathrm{e}^{\_Z}-\_Z\,\mathrm{e}^{\_Z}+2t\,\mathrm{e}^{\_Z}-\ln\left(\mathrm{e}^{\_Z}-2\right)-2c_1+\_Z-2t+2\right)} - 1$$

✓ Solution by Mathematica

Time used: 0.414 (sec). Leaf size: 53

```
DSolve[x'[t]==x[t]^2-x[t]^4,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \text{InverseFunction}\left[\frac{1}{\#1} + \frac{1}{2}\log(1-\#1) - \frac{1}{2}\log(\#1+1)\&\right][-t+c_1]$$
$$x(t) \to -1$$
$$x(t) \to 0$$
$$x(t) \to 1$$

3 Chapter 8, Separable equations. Exercises page 72

3.1 problem 8.1 (i)

Internal problem ID [11983]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 8, Separable equations. Exercises page 72
Problem number: 8.1 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x' - t^3(1-x) = 0}$$

With initial conditions

$$[x(0) = 3]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve([diff(x(t),t)=t^3*(1-x(t)),x(0) = 3],x(t), singsol=all)
```

$$x(t) = 1 + 2\,\mathrm{e}^{-\frac{t^4}{4}}$$

✓ Solution by Mathematica

Time used: 0.074 (sec). Leaf size: 18

```
DSolve[{x'[t]==t^3*(1-x[t]),{x[0]==3}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to 2e^{-\frac{t^4}{4}} + 1$$

3.2 problem 8.1 (ii)

Internal problem ID [11984]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 8, Separable equations. Exercises page 72
Problem number: 8.1 (ii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - (1 + y^2) \tan(x) = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)=(1+y(x)^2)*tan(x),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \cot\left(\frac{\pi}{4} + \ln(\cos(x))\right)$$

✓ Solution by Mathematica
Time used: 0.472 (sec). Leaf size: 15

```
DSolve[{y'[x]==(1+y[x]^2)*Tan[x],{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cot\left(\log(\cos(x)) + \frac{\pi}{4}\right)$$

3.3 problem 8.1 (iii)

Internal problem ID [11985]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 8, Separable equations. Exercises page 72
Problem number: 8.1 (iii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x' - x t^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(x(t),t)=t^2*x(t),x(t), singsol=all)
```

$$x(t) = c_1 \mathrm{e}^{\frac{t^3}{3}}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 22

```
DSolve[x'[t]==t^2*x[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 e^{\frac{t^3}{3}}$$
$$x(t) \to 0$$

3.4 problem 8.1 (iv)

Internal problem ID [11986]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 8, Separable equations. Exercises page 72
Problem number: 8.1 (iv).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' + x^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(x(t),t)=-x(t)^2,x(t), singsol=all)
```

$$x(t) = \frac{1}{t + c_1}$$

✓ Solution by Mathematica
Time used: 0.169 (sec). Leaf size: 18

```
DSolve[x'[t]==-x[t]^2,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{t - c_1}$$
$$x(t) \to 0$$

3.5 problem 8.1 (v)

Internal problem ID [11987]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 8, Separable equations. Exercises page 72
Problem number: 8.1 (v).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y^2 \mathrm{e}^{-t^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(t),t)=exp(-t^2)*y(t)^2,y(t), singsol=all)
```

$$y(t) = -\frac{2}{\sqrt{\pi}\,\operatorname{erf}(t) - 2c_1}$$

✓ Solution by Mathematica

Time used: 0.347 (sec). Leaf size: 27

```
DSolve[y'[t]==Exp[-t^2]*y[t]^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{2}{\sqrt{\pi}\operatorname{erf}(t) + 2c_1}$$
$$y(t) \to 0$$

3.6 problem 8.2

Internal problem ID [11988]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 8, Separable equations. Exercises page 72
Problem number: 8.2 .
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' + px = q}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(x(t),t)+p*x(t)=q,x(t), singsol=all)
```

$$x(t) = \frac{e^{-pt}c_1p + q}{p}$$

✓ Solution by Mathematica
Time used: 0.063 (sec). Leaf size: 29

```
DSolve[x'[t]+p*x[t]==q,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{q}{p} + c_1e^{-pt}$$
$$x(t) \to \frac{q}{p}$$

3.7 problem 8.3

Internal problem ID [11989]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 8, Separable equations. Exercises page 72
Problem number: 8.3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x - yk = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(x*diff(y(x),x)=k*y(x),y(x), singsol=all)
```

$$y(x) = c_1 x^k$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 16

```
DSolve[x*y'[x]==k*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x^k$$
$$y(x) \to 0$$

3.8 problem 8.4

Internal problem ID [11990]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 8, Separable equations. Exercises page 72
Problem number: 8.4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{i' - p(t)\, i = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(i(t),t)=p(t)*i(t),i(t), singsol=all)
```

$$i(t) = c_1 \mathrm{e}^{\int p(t)dt}$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 25

```
DSolve[i'[t]==p[t]*i[t],i[t],t,IncludeSingularSolutions -> True]
```

$$i(t) \to c_1 \exp\left(\int_1^t p(K[1])dK[1]\right)$$
$$i(t) \to 0$$

3.9 problem 8.5

Internal problem ID [11991]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 8, Separable equations. Exercises page 72
Problem number: 8.5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - \lambda x = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(x(t),t)=lambda*x(t),x(t), singsol=all)
```

$$x(t) = c_1 \mathrm{e}^{\lambda t}$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 18

```
DSolve[x'[t]==\[Lambda]*x[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 e^{\lambda t}$$
$$x(t) \to 0$$

3.10 problem 8.6

Internal problem ID [11992]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 8, Separable equations. Exercises page 72
Problem number: 8.6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{mv' - kv^2 = -mg}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(m*diff(v(t),t)=-m*g+k*v(t)^2,v(t), singsol=all)
```

$$v(t) = -\frac{\tanh\left(\frac{\sqrt{mgk}\,(t+c_1)}{m}\right)\sqrt{mgk}}{k}$$

✓ Solution by Mathematica
Time used: 14.167 (sec). Leaf size: 87

```
DSolve[m*v'[t]==-m*g+k*v[t]^2,v[t],t,IncludeSingularSolutions -> True]
```

$$v(t) \to \frac{\sqrt{g}\sqrt{m}\tanh\left(\frac{\sqrt{g}\sqrt{k}(-t+c_1m)}{\sqrt{m}}\right)}{\sqrt{k}}$$
$$v(t) \to -\frac{\sqrt{g}\sqrt{m}}{\sqrt{k}}$$
$$v(t) \to \frac{\sqrt{g}\sqrt{m}}{\sqrt{k}}$$

3.11 problem 8.7

Internal problem ID [11993]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 8, Separable equations. Exercises page 72
Problem number: 8.7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - kx + x^2 = 0}$$

With initial conditions

$$[x(0) = x_0]$$

✓ Solution by Maple
Time used: 0.046 (sec). Leaf size: 22

```
dsolve([diff(x(t),t)=k*x(t)-x(t)^2,x(0) = x__0],x(t), singsol=all)
```

$$x(t) = \frac{kx_0}{(-x_0 + k)\,e^{-kt} + x_0}$$

✓ Solution by Mathematica
Time used: 1.052 (sec). Leaf size: 26

```
DSolve[{x'[t]==k*x[t]-x[t]^2,{x[0]==x0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{k\text{x0}e^{kt}}{\text{x0}\,(e^{kt} - 1) + k}$$

3.12 problem 8.8

Internal problem ID [11994]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 8, Separable equations. Exercises page 72
Problem number: 8.8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' + x(k^2 + x^2) = 0}$$

With initial conditions

$$[x(0) = x_0]$$

✗ Solution by Maple

```
dsolve([diff(x(t),t)=-x(t)*(k^2+x(t)^2),x(0) = x__0],x(t), singsol=all)
```

No solution found

✓ Solution by Mathematica
Time used: 1.848 (sec). Leaf size: 62

```
DSolve[{x'[t]==-x[t]*(k^2+x[t]^2),{x[0]==x0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{k}{\sqrt{e^{2k^2t}\left(\frac{k^2}{\mathrm{x0}^2}+1\right)-1}}$$
$$x(t) \to \frac{k}{\sqrt{e^{2k^2t}\left(\frac{k^2}{\mathrm{x0}^2}+1\right)-1}}$$

4 Chapter 9, First order linear equations and the integrating factor. Exercises page 86

4.1 problem 9.1 (i)

Internal problem ID [11995]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 9, First order linear equations and the integrating factor. Exercises page 86
Problem number: 9.1 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{y}{x} = x^2}$$

With initial conditions

$$[y(0) = y_0]$$

✗ Solution by Maple

```
dsolve([diff(y(x),x)+y(x)/x=x^2,y(0) = y__0],y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'[x]+y[x]/x==x^2,{y[0]==y0}},y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.2 problem 9.1 (ii)

Internal problem ID [11996]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 9, First order linear equations and the integrating factor. Exercises page 86
Problem number: 9.1 (ii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x' + xt = 4t}$$

With initial conditions

$$[x(0) = 2]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve([diff(x(t),t)+t*x(t)=4*t,x(0) = 2],x(t), singsol=all)
```

$$x(t) = 4 - 2\,\mathrm{e}^{-\frac{t^2}{2}}$$

✓ Solution by Mathematica
Time used: 0.066 (sec). Leaf size: 18

```
DSolve[{x'[t]+t*x[t]==4*t,{x[0]==2}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to 4 - 2e^{-\frac{t^2}{2}}$$

4.3 problem 9.1 (iii)

Internal problem ID [11997]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 9, First order linear equations and the integrating factor. Exercises page 86
Problem number: 9.1 (iii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{z' - z\tan(y) = \sin(y)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(z(y),y)=z(y)*tan(y)+sin(y),z(y), singsol=all)
```

$$z(y) = -\frac{\cos(y)}{2} + \sec(y)\,c_1 + \frac{\sec(y)}{4}$$

✓ Solution by Mathematica
Time used: 0.075 (sec). Leaf size: 17

```
DSolve[z'[y]==z[y]*Tan[y]+Sin[y],z[y],y,IncludeSingularSolutions -> True]
```

$$z(y) \to -\frac{\cos(y)}{2} + c_1\sec(y)$$

4.4 problem 9.1 (iv)

Internal problem ID [11998]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 9, First order linear equations and the integrating factor. Exercises page 86
Problem number: 9.1 (iv).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \mathrm{e}^{-x} y = 1}$$

With initial conditions

$$[y(0) = \mathrm{e}]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 23

```
dsolve([diff(y(x),x)+exp(-x)*y(x)=1,y(0) = exp(1)],y(x), singsol=all)
```

$$y(x) = \left(\mathrm{expIntegral}_1\left(\mathrm{e}^{-x}\right) + 1 - \mathrm{expIntegral}_1(1)\right) \mathrm{e}^{\mathrm{e}^{-x}}$$

✓ Solution by Mathematica
Time used: 0.104 (sec). Leaf size: 27

```
DSolve[{y'[x]+Exp[-x]*y[x]==1,{y[0]==Exp[1]}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{e^{-x}}\left(-\operatorname{ExpIntegralEi}\left(-e^{-x}\right) + \operatorname{ExpIntegralEi}(-1) + 1\right)$$

4.5 problem 9.1 (v)

Internal problem ID [11999]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 9, First order linear equations and the integrating factor. Exercises page 86
Problem number: 9.1 (v).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x' + x \tanh(t) = 3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(x(t),t)+x(t)*tanh(t)=3,x(t), singsol=all)
```

$$x(t) = 3\tanh(t) + \operatorname{sech}(t)\, c_1$$

✓ Solution by Mathematica

Time used: 0.094 (sec). Leaf size: 15

```
DSolve[x'[t]+x[t]*Tanh[t]==3,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \operatorname{sech}(t)(3\sinh(t) + c_1)$$

4.6 problem 9.1 (vi)

Internal problem ID [12000]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 9, First order linear equations and the integrating factor. Exercises page 86
Problem number: 9.1 (vi).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + 2y\cot(x) = 5}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = 1\right]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 29

```
dsolve([diff(y(x),x)+2*y(x)*cot(x)=5,y(1/2*Pi) = 1],y(x), singsol=all)
```

$$y(x) = \frac{-10x + 5\sin(2x) - 4 + 5\pi}{-2 + 2\cos(2x)}$$

✓ Solution by Mathematica
Time used: 0.082 (sec). Leaf size: 27

```
DSolve[{y'[x]+2*y[x]*Cot[x]==5,{y[Pi/2]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}(10x - 5\sin(2x) - 5\pi + 4)\csc^2(x)$$

4.7 problem 9.1 (vii)

Internal problem ID [12001]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 9, First order linear equations and the integrating factor. Exercises page 86
Problem number: 9.1 (vii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{x' + 5x = t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(x(t),t)+5*x(t)=t,x(t), singsol=all)
```

$$x(t) = \frac{t}{5} - \frac{1}{25} + \mathrm{e}^{-5t}c_1$$

✓ Solution by Mathematica
Time used: 0.071 (sec). Leaf size: 22

```
DSolve[x'[t]+5*x[t]==t,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{t}{5} + c_1 e^{-5t} - \frac{1}{25}$$

4.8 problem 9.1 (viii)

Internal problem ID [12002]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 9, First order linear equations and the integrating factor. Exercises page 86
Problem number: 9.1 (viii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x' + \left(a + \frac{1}{t}\right) x = b}$$

With initial conditions

$$[x(1) = x_0]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 38

```
dsolve([diff(x(t),t)+(a+1/t)*x(t)=b,x(1) = x__0],x(t), singsol=all)
```

$$x(t) = \frac{(x_0 a^2 - ab + b)\,\mathrm{e}^{-a(t-1)} + b(at-1)}{t\,a^2}$$

✓ Solution by Mathematica
Time used: 0.101 (sec). Leaf size: 48

```
DSolve[{x'[t]+(a+1/t)*x[t]==b,{x[1]==x0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{e^{-at}(e^a a^2 \mathrm{x0} + be^{at}(at-1) - (a-1)e^a b)}{a^2 t}$$

4.9 problem 9.4

Internal problem ID [12003]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 9, First order linear equations and the integrating factor. Exercises page 86
Problem number: 9.4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{T' + k(T - \mu - a\cos(\omega(t - \phi))) = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 58

```
dsolve(diff(T(t),t)=-k*(T(t)- (mu+a*cos( omega*(t-phi)))),T(t), singsol=all)
```

$$T(t) = \frac{\cos(\omega(-t + \phi))\, a\, k^2 - \sin(\omega(-t + \phi))\, a k \omega + (k^2 + \omega^2)\left(e^{-kt} c_1 + \mu\right)}{k^2 + \omega^2}$$

✓ Solution by Mathematica
Time used: 0.511 (sec). Leaf size: 60

```
DSolve[T'[t]==-k*(T[t]- (mu+a*Cos[ omega*(t-phi)])),T[t],t,IncludeSingularSolutions -> True]
```

$$T(t) \to -\frac{ak\omega \sin(\omega(\phi - t))}{k^2 + \omega^2} + \frac{ak^2 \cos(\omega(\phi - t))}{k^2 + \omega^2} + c_1 e^{-kt} + \mu$$

5 Chapter 10, Two tricks for nonlinear equations. Exercises page 97

5.1 problem 10.1 (i)

Internal problem ID [12004]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 10, Two tricks for nonlinear equations. Exercises page 97
Problem number: 10.1 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x),G(x)]'], [_Abel, '`

$$\boxed{2yx + (x^2 + 2y)\, y' = \sec(x)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 49

```
dsolve((2*x*y(x)- sec(x)^2)+(x^2+2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{2} - \frac{\sqrt{x^4 + 4\tan(x) - 4c_1}}{2}$$
$$y(x) = -\frac{x^2}{2} + \frac{\sqrt{x^4 + 4\tan(x) - 4c_1}}{2}$$

✓ Solution by Mathematica
Time used: 26.886 (sec). Leaf size: 90

```
DSolve[(2*x*y[x]- Sec[x]^2)+(x^2+2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-x^2 - \sqrt{\sec^2(x)}\sqrt{\cos^2(x)\,(x^4 + 4\tan(x) + 4c_1)}\right)$$
$$y(x) \to \frac{1}{2}\left(-x^2 + \sqrt{\sec^2(x)}\sqrt{\cos^2(x)\,(x^4 + 4\tan(x) + 4c_1)}\right)$$

5.2 problem 10.1 (ii)

Internal problem ID [12005]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 10, Two tricks for nonlinear equations. Exercises page 97
Problem number: 10.1 (ii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y\,e^{x} + yx\,e^{x} + (x\,e^{x} + 2)\,y' = -1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve((1+exp(x)*y(x)+x*exp(x)*y(x))+(x*exp(x)+2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 - x}{e^{x}x + 2}$$

✓ Solution by Mathematica
Time used: 0.142 (sec). Leaf size: 21

```
DSolve[(1+Exp[x]*y[x]+x*Exp[x]*y[x])+(x*Exp[x]+2)*y'[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{-x + c_1}{e^{x}x + 2}$$

5.3 problem 10.1 (iii)

Internal problem ID [12006]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 10, Two tricks for nonlinear equations. Exercises page 97
Problem number: 10.1 (iii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{(\cos(y)\,x + \cos(x))\,y' + \sin(y) - \sin(x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 15

```
dsolve((x*cos(y(x))+cos(x))*diff(y(x),x)+sin(y(x))-y(x)*sin(x)=0,y(x), singsol=all)
```

$$\cos(x)\,y(x) + x\sin(y(x)) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.254 (sec). Leaf size: 17

```
DSolve[(x*Cos[y[x]]+Cos[x])*y'[x]+Sin[y[x]]-y[x]*Sin[x]==0,y[x],x,IncludeSingularSolutions -
```

$$\text{Solve}[x\sin(y(x)) + y(x)\cos(x) = c_1, y(x)]$$

5.4 problem 10.1 (iv)

Internal problem ID [12007]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 10, Two tricks for nonlinear equations. Exercises page 97
Problem number: 10.1 (iv).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{e^{x} \sin \left(y\right)+y+\left(e^{x} \cos \left(y\right)+x+e^{y}\right) y^{\prime}=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(exp(x)*sin(y(x))+y(x)+ (exp(x)*cos(y(x))+x+exp(y(x)))*diff(y(x),x)=0,y(x), singsol=al
```

$$y(x)\, x+e^{x} \sin \left(y(x)\right)+e^{y(x)}+c_{1}=0$$

✓ Solution by Mathematica
Time used: 0.637 (sec). Leaf size: 22

```
DSolve[Exp[x]*Sin[y[x]]+y[x]+ (Exp[x]*Cos[y[x]]+x+Exp[y[x]])*y'[x]==0,y[x],x,IncludeSingular
```

$$\text{Solve}\left[e^{y(x)}+xy(x)+e^x\sin(y(x))=c_1,y(x)\right]$$

5.5 problem 10.2

Internal problem ID [12008]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 10, Two tricks for nonlinear equations. Exercises page 97
Problem number: 10.2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$\boxed{e^{-y} \sec(x) - e^{-y} y' = -2\cos(x)}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 45

```
dsolve(exp(-y(x))*sec(x)+2*cos(x)-exp(-y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \ln\left(-\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2}{\left(-4\cos\left(\frac{x}{2}\right)^2+c_1+2x\right)\left(2\cos\left(\frac{x}{2}\right)^2-1\right)}\right)$$

✓ Solution by Mathematica
Time used: 2.559 (sec). Leaf size: 33

```
DSolve[Exp[-y[x]]*Sec[x]+2*Cos[x]-Exp[-y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \log\left(\frac{e^{2\operatorname{arctanh}\left(\tan\left(\frac{x}{2}\right)\right)}}{2(-x+\cos(x)-2c_1)}\right)$$

5.6 problem 10.3 (i)

Internal problem ID [12009]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 10, Two tricks for nonlinear equations. Exercises page 97
Problem number: 10.3 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2yy' = -V'(x)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 25

```
dsolve(diff(V(x),x)+2*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{-V(x) + c_1}$$
$$y(x) = -\sqrt{-V(x) + c_1}$$

✓ Solution by Mathematica
Time used: 0.126 (sec). Leaf size: 37

```
DSolve[V'[x]+2*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-V(x) + 2c_1}$$
$$y(x) \to \sqrt{-V(x) + 2c_1}$$

5.7 problem 10.3 (ii)

Internal problem ID [12010]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 10, Two tricks for nonlinear equations. Exercises page 97
Problem number: 10.3 (ii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\left(\frac{1}{y} - a\right) y' = -\frac{2}{x} + b}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve((1/y(x)-a)*diff(y(x),x)+2/x-b=0,y(x), singsol=all)
```

$$y(x) = -\frac{\mathrm{LambertW}\left(-\frac{a\,\mathrm{e}^{bx} c_1}{x^2}\right)}{a}$$

✓ Solution by Mathematica
Time used: 6.296 (sec). Leaf size: 32

```
DSolve[(1/y[x]-a)*y'[x]+2/x-b==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{W\left(-\frac{ae^{bx-c_1}}{x^2}\right)}{a}$$
$$y(x) \to 0$$

5.8 problem 10.4 (i)

Internal problem ID [12011]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 10, Two tricks for nonlinear equations. Exercises page 97
Problem number: 10.4 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Riccati]`

$$\boxed{yx + y^2 - x^2y' = -x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x*y(x)+y(x)^2+x^2-x^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \tan(\ln(x) + c_1) x$$

✓ Solution by Mathematica
Time used: 0.314 (sec). Leaf size: 13

```
DSolve[x*y[x]+y[x]^2+x^2-x^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \tan(\log(x) + c_1)$$

5.9 problem 10.4 (ii)

Internal problem ID [12012]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 10, Two tricks for nonlinear equations. Exercises page 97
Problem number: 10.4 (ii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{x' - \frac{x^2 + t\sqrt{x^2 + t^2}}{xt} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(x(t),t)=(x(t)^2+t*sqrt(t^2+x(t)^2))/(t*x(t)),x(t), singsol=all)
```

$$\frac{t\ln(t) - c_1 t - \sqrt{t^2 + x(t)^2}}{t} = 0$$

✓ Solution by Mathematica

Time used: 0.512 (sec). Leaf size: 54

```
DSolve[x'[t]==(x[t]^2+t*Sqrt[t^2+x[t]^2])/(t*x[t]),x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -t\sqrt{\log^2(t) + 2c_1\log(t) - 1 + c_1{}^2}$$
$$x(t) \to t\sqrt{\log^2(t) + 2c_1\log(t) - 1 + c_1{}^2}$$

5.10 problem 10.5

Internal problem ID [12013]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 10, Two tricks for nonlinear equations. Exercises page 97
Problem number: 10.5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - kx + x^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(x(t),t)=k*x(t)-x(t)^2,x(t), singsol=all)
```

$$x(t) = \frac{k}{1 + \mathrm{e}^{-kt} c_1 k}$$

✓ Solution by Mathematica
Time used: 0.963 (sec). Leaf size: 37

```
DSolve[x'[t]==k*x[t]-x[t]^2,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{k e^{k(t+c_1)}}{-1 + e^{k(t+c_1)}}$$
$$x(t) \to 0$$
$$x(t) \to k$$

6 Chapter 12, Homogeneous second order linear equations. Exercises page 118

6.1 problem 12.1 (i)

Internal problem ID [12014]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' - 3x' + 2x = 0}$$

With initial conditions

$$[x(0) = 2, x'(0) = 6]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve([diff(x(t),t$2)-3*diff(x(t),t)+2*x(t)=0,x(0) = 2, D(x)(0) = 6],x(t), singsol=all)
```

$$x(t) = -2\,e^{t} + 4\,e^{2t}$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 17

```
DSolve[{x''[t]-3*x'[t]+2*x[t]==0,{x[0]==2,x'[0]==6}},x[t],t,IncludeSingularSolutions -> True
```

$$x(t) \to 2e^{t}\left(2e^{t} - 1\right)$$

6.2 problem 12.1 (ii)

Internal problem ID [12015]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (ii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y' + 4y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 3]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve([diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=0,y(0) = 0, D(y)(0) = 3],y(x), singsol=all)
```

$$y(x) = 3x\,\mathrm{e}^{2x}$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 13

```
DSolve[{y''[x]-4*y'[x]+4*y[x]==0,{y[0]==0,y'[0]==3}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to 3e^{2x}x$$

6.3 problem 12.1 (iii)

Internal problem ID [12016]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (iii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{z'' - 4z' + 13z = 0}$$

With initial conditions

$$[z(0) = 7, z'(0) = 42]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve([diff(z(t),t$2)-4*diff(z(t),t)+13*z(t)=0,z(0) = 7, D(z)(0) = 42],z(t), singsol=all)
```

$$z(t) = \frac{7\,\mathrm{e}^{2t}(4\sin(3t) + 3\cos(3t))}{3}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 27

```
DSolve[{z''[t]-4*z'[t]+13*z[t]==0,{z[0]==7,z'[0]==42}},z[t],t,IncludeSingularSolutions -> Tr
```

$$z(t) \to \frac{7}{3}e^{2t}(4\sin(3t) + 3\cos(3t))$$

6.4 problem 12.1 (iv)

Internal problem ID [12017]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (iv).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - 6y = 0}$$

With initial conditions

$$[y(0) = -1, y'(0) = 8]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve([diff(y(t),t$2)+diff(y(t),t)-6*y(t)=0,y(0) = -1, D(y)(0) = 8],y(t), singsol=all)
```

$$y(t) = \left(\mathrm{e}^{5t} - 2\right)\mathrm{e}^{-3t}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 18

```
DSolve[{y''[t]+y'[t]-6*y[t]==0,{y[0]==-1,y'[0]==8}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-3t}\left(e^{5t} - 2\right)$$

6.5 problem 12.1 (v)

Internal problem ID [12018]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (v).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y' = 0}$$

With initial conditions

$$[y(0) = 13, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 5

```
dsolve([diff(y(t),t$2)-4*diff(y(t),t)=0,y(0) = 13, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = 13$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 6

```
DSolve[{y''[t]-4*y'[t]==0,{y[0]==13,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 13$$

6.6 problem 12.1 (vi)

Internal problem ID [12019]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (vi).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{\theta'' + 4\theta = 0}$$

With initial conditions

$$[\theta(0) = 0, \theta'(0) = 10]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 10

```
dsolve([diff(theta(t),t$2)+4*theta(t)=0,theta(0) = 0, D(theta)(0) = 10],theta(t), singsol=al
```

$$\theta(t) = 5\sin(2t)$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 11

```
DSolve[{\[Theta]''[t]+4*\[Theta][t]==0,{\[Theta][0]==0,\[Theta]'[0]==10}},\[Theta][t],t,Incl
```

$$\theta(t) \to 5\sin(2t)$$

6.7 problem 12.1 (vii)

Internal problem ID [12020]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (vii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + 10y = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+10*y(t)=0,y(0) = 3, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = e^{-t}(\sin(3t) + 3\cos(3t))$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 22

```
DSolve[{y''[t]+2*y'[t]+10*y[t]==0,{y[0]==3,y'[0]==0}},y[t],t,IncludeSingularSolutions -> Tru
```

$$y(t) \to e^{-t}(\sin(3t) + 3\cos(3t))$$

6.8 problem 12.1 (viii)

Internal problem ID [12021]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (viii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2z'' + 7z' - 4z = 0}$$

With initial conditions

$$[z(0) = 0, z'(0) = 9]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve([2*diff(z(t),t$2)+7*diff(z(t),t)-4*z(t)=0,z(0) = 0, D(z)(0) = 9],z(t), singsol=all)
```

$$z(t) = 2\left(e^{\frac{9t}{2}} - 1\right) e^{-4t}$$

✓ Solution by Mathematica
Time used: 0.047 (sec). Leaf size: 49

```
DSolve[{z''[t]+7*z'[t]-4*z[t]==0,{z[0]==3,z'[0]==9}},z[t],t,IncludeSingularSolutions -> True
```

$$z(t) \to \frac{3}{10} e^{-\frac{1}{2}\left(7+\sqrt{65}\right)t} \left(\left(5+\sqrt{65}\right) e^{\sqrt{65}t} + 5 - \sqrt{65}\right)$$

6.9 problem 12.1 (ix)

Internal problem ID [12022]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (ix).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = -1]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 11

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+y(t)=0,y(0) = 0, D(y)(0) = -1],y(t), singsol=all)
```

$$y(t) = -t\,e^{-t}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 13

```
DSolve[{y''[t]+2*y'[t]+y[t]==0,{y[0]==0,y'[0]==-1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -e^{-t}t$$

6.10 problem 12.1 (x)

Internal problem ID [12023]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (x).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + 6x' + 10x = 0}$$

With initial conditions

$$[x(0) = 3, x'(0) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve([diff(x(t),t$2)+6*diff(x(t),t)+10*x(t)=0,x(0) = 3, D(x)(0) = 1],x(t), singsol=all)
```

$$x(t) = \mathrm{e}^{-3t}(10\sin(t) + 3\cos(t))$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 20

```
DSolve[{x''[t]+6*x'[t]+10*x[t]==0,{x[0]==3,x'[0]==1}},x[t],t,IncludeSingularSolutions -> Tru
```

$$x(t) \to e^{-3t}(10\sin(t) + 3\cos(t))$$

6.11 problem 12.1 (xi)

Internal problem ID [12024]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (xi).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4x'' - 20x' + 21x = 0}$$

With initial conditions

$$[x(0) = -4, x'(0) = -12]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([4*diff(x(t),t$2)-20*diff(x(t),t)+21*x(t)=0,x(0) = -4, D(x)(0) = -12],x(t), singsol=a
```

$$x(t) = -\mathrm{e}^{\frac{3t}{2}} - 3\,\mathrm{e}^{\frac{7t}{2}}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 23

```
DSolve[{4*x''[t]-20*x'[t]+21*x[t]==0,{x[0]==-4,x'[0]==-12}},x[t],t,IncludeSingularSolutions
```

$$x(t) \to -e^{3t/2}(3e^{2t} + 1)$$

6.12 problem 12.1 (xii)

Internal problem ID [12025]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (xii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - 2y = 0}$$

With initial conditions

$$[y(0) = 4, y'(0) = -4]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 16

```
dsolve([diff(y(t),t$2)+diff(y(t),t)-2*y(t)=0,y(0) = 4, D(y)(0) = -4],y(t), singsol=all)
```

$$y(t) = \frac{4(\mathrm{e}^{3t} + 2)\,\mathrm{e}^{-2t}}{3}$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 21

```
DSolve[{y''[t]+y'[t]-2*y[t]==0,{y[0]==4,y'[0]==-4}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{4}{3}e^{-2t}(e^{3t} + 2)$$

6.13 problem 12.1 (xiii)

Internal problem ID [12026]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (xiii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y = 0}$$

With initial conditions

$$[y(0) = 10, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)-4*y(t)=0,y(0) = 10, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = 5\,\mathrm{e}^{2t} + 5\,\mathrm{e}^{-2t}$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 19

```
DSolve[{y''[t]-4*y[t]==0,{y[0]==10,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 5e^{-2t}\left(e^{4t} + 1\right)$$

6.14 problem 12.1 (xiv)

Internal problem ID [12027]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (xiv).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y' + 4y = 0}$$

With initial conditions

$$[y(0) = 27, y'(0) = -54]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 10

```
dsolve([diff(y(t),t$2)+4*diff(y(t),t)+4*y(t)=0,y(0) = 27, D(y)(0) = -54],y(t), singsol=all)
```

$$y(t) = 27\,e^{-2t}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 12

```
DSolve[{y''[t]+4*y'[t]+4*y[t]==0,{y[0]==27,y'[0]==-54}},y[t],t,IncludeSingularSolutions -> T
```

$$y(t) \to 27e^{-2t}$$

6.15 problem 12.1 (xv)

Internal problem ID [12028]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (xv).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + \omega^2 y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 12

```
dsolve([diff(y(t),t$2)+omega^2*y(t)=0,y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = \frac{\sin(\omega t)}{\omega}$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 13

```
DSolve[{y''[t]+w^2*y[t]==0,{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{\sin(tw)}{w}$$

7 Chapter 14, Inhomogeneous second order linear equations. Exercises page 140

7.1 problem 14.1 (i)

Internal problem ID [12029]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 14, Inhomogeneous second order linear equations. Exercises page 140
Problem number: 14.1 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x'' - 4x = t^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(x(t),t$2)-4*x(t)=t^2,x(t), singsol=all)
```

$$x(t) = c_2 e^{2t} + e^{-2t} c_1 - \frac{t^2}{4} - \frac{1}{8}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 32

```
DSolve[x''[t]-4*x[t]==t^2,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{t^2}{4} + c_1 e^{2t} + c_2 e^{-2t} - \frac{1}{8}$$

7.2 problem 14.1 (ii)

Internal problem ID [12030]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 14, Inhomogeneous second order linear equations. Exercises page 140
Problem number: 14.1 (ii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x'' - 4x' = t^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(x(t),t$2)-4*diff(x(t),t)=t^2,x(t), singsol=all)
```

$$x(t) = -\frac{t^2}{16} - \frac{t^3}{12} + \frac{c_1 e^{4t}}{4} - \frac{t}{32} + c_2$$

✓ Solution by Mathematica
Time used: 0.096 (sec). Leaf size: 36

```
DSolve[x''[t]-4*x'[t]==t^2,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{96}\left(-8t^3 - 6t^2 - 3t + 24c_1 e^{4t} + 96c_2\right)$$

7.3 problem 14.1 (iii)

Internal problem ID [12031]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 14, Inhomogeneous second order linear equations. Exercises page 140
Problem number: 14.1 (iii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x'' + x' - 2x = 3\,\mathrm{e}^{-t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(x(t),t$2)+diff(x(t),t)-2*x(t)=3*exp(-t),x(t), singsol=all)
```

$$x(t) = -\frac{(-2c_2\mathrm{e}^{3t} + 3\,\mathrm{e}^{t} - 2c_1)\,\mathrm{e}^{-2t}}{2}$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 29

```
DSolve[x''[t]+x'[t]-2*x[t]==3*Exp[-t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{3e^{-t}}{2} + c_1e^{-2t} + c_2e^{t}$$

7.4 problem 14.1 (iv)

Internal problem ID [12032]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 14, Inhomogeneous second order linear equations. Exercises page 140
Problem number: 14.1 (iv).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x'' + x' - 2x = \mathrm{e}^{t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(x(t),t$2)+diff(x(t),t)-2*x(t)=exp(t),x(t), singsol=all)
```

$$x(t) = \frac{\mathrm{e}^{-2t}\left((t + 3c_2)\,\mathrm{e}^{3t} + 3c_1\right)}{3}$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 29

```
DSolve[x''[t]+x'[t]-2*x[t]==Exp[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 e^{-2t} + e^{t}\left(\frac{t}{3} - \frac{1}{9} + c_2\right)$$

7.5 problem 14.1 (v)

Internal problem ID [12033]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 14, Inhomogeneous second order linear equations. Exercises page 140
Problem number: 14.1 (v).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x'' + 2x' + x = \mathrm{e}^{-t}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(x(t),t$2)+2*diff(x(t),t)+x(t)=exp(-t),x(t), singsol=all)
```

$$x(t) = \mathrm{e}^{-t}\left(c_2 + c_1 t + \frac{1}{2}t^2\right)$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 27

```
DSolve[x''[t]+2*x'[t]+x[t]==Exp[-t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{2}e^{-t}(t^2 + 2c_2 t + 2c_1)$$

7.6 problem 14.1 (vi)

Internal problem ID [12034]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 14, Inhomogeneous second order linear equations. Exercises page 140
Problem number: 14.1 (vi).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + \omega^2 x = \sin(\alpha t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(x(t),t$2)+omega^2*x(t)=sin(alpha*t),x(t), singsol=all)
```

$$x(t) = \sin(\omega t)\, c_2 + \cos(\omega t)\, c_1 + \frac{\sin(\alpha t)}{-\alpha^2 + \omega^2}$$

✓ Solution by Mathematica
Time used: 0.359 (sec). Leaf size: 56

```
DSolve[x''[t]+w^2*x[t]==Sin[a*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{-(c_1(a^2 - w^2)\cos(tw)) + c_2(w^2 - a^2)\sin(tw) + \sin(at)}{(w-a)(a+w)}$$

7.7 problem 14.1 (vii)

Internal problem ID [12035]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 14, Inhomogeneous second order linear equations. Exercises page 140
Problem number: 14.1 (vii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + \omega^2 x = \sin(\omega t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(x(t),t$2)+omega^2*x(t)=sin(omega*t),x(t), singsol=all)
```

$$x(t) = \frac{\sin(\omega t)\left(2c_2\omega^2 + 1\right) - \omega\cos(\omega t)\left(-2c_1\omega + t\right)}{2\omega^2}$$

✓ Solution by Mathematica
Time used: 0.081 (sec). Leaf size: 29

```
DSolve[x''[t]+w^2*x[t]==Sin[w*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \left(-\frac{t}{2w} + c_1\right)\cos(tw) + c_2\sin(tw)$$

7.8 problem 14.1 (viii)

Internal problem ID [12036]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 14, Inhomogeneous second order linear equations. Exercises page 140
Problem number: 14.1 (viii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x'' + 2x' + 10x = e^{-t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(x(t),t$2)+2*diff(x(t),t)+10*x(t)=exp(-t),x(t), singsol=all)
```

$$x(t) = \frac{e^{-t}(9c_2 \sin(3t) + 9c_1 \cos(3t) + 1)}{9}$$

✓ Solution by Mathematica
Time used: 0.086 (sec). Leaf size: 32

```
DSolve[x''[t]+2*x'[t]+10*x[t]==Exp[-t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{9}e^{-t}(9c_2 \cos(3t) + 9c_1 \sin(3t) + 1)$$

7.9 problem 14.1 (ix)

Internal problem ID [12037]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 14, Inhomogeneous second order linear equations. Exercises page 140
Problem number: 14.1 (ix).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 2x' + 10x = e^{-t} \cos(3t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(x(t),t$2)+2*diff(x(t),t)+10*x(t)=exp(-t)*cos(3*t),x(t), singsol=all)
```

$$x(t) = \frac{\left(\left(6c_1 + \frac{1}{3}\right)\cos(3t) + \sin(3t)(t + 6c_2)\right) e^{-t}}{6}$$

✓ Solution by Mathematica
Time used: 0.084 (sec). Leaf size: 38

```
DSolve[x''[t]+2*x'[t]+10*x[t]==Exp[-t]*Cos[3*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{36}e^{-t}((1 + 36c_2)\cos(3t) + 6(t + 6c_1)\sin(3t))$$

7.10 problem 14.1 (x)

Internal problem ID [12038]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 14, Inhomogeneous second order linear equations. Exercises page 140
Problem number: 14.1 (x).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 6x' + 10x = \cos(t)\, e^{-2t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(x(t),t$2)+6*diff(x(t),t)+10*x(t)=exp(-2*t)*cos(t),x(t), singsol=all)
```

$$x(t) = (\sin(t)\, c_2 + \cos(t)\, c_1)\, e^{-3t} + \frac{e^{-2t}(\cos(t) + 2\sin(t))}{5}$$

✓ Solution by Mathematica
Time used: 0.087 (sec). Leaf size: 33

```
DSolve[x''[t]+6*x'[t]+10*x[t]==Exp[-3*t]*Cos[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{2} e^{-3t}((1 + 2c_2)\cos(t) + (t + 2c_1)\sin(t))$$

7.11 problem 14.1 (xi)

Internal problem ID [12039]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 14, Inhomogeneous second order linear equations. Exercises page 140
Problem number: 14.1 (xi).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x'' + 4x' + 4x = e^{2t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(x(t),t$2)+4*diff(x(t),t)+4*x(t)=exp(2*t),x(t), singsol=all)
```

$$x(t) = (c_1 t + c_2)\, e^{-2t} + \frac{e^{2t}}{16}$$

✓ Solution by Mathematica
Time used: 0.064 (sec). Leaf size: 28

```
DSolve[x''[t]+4*x'[t]+4*x[t]==Exp[2*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{e^{2t}}{16} + e^{-2t}(c_2 t + c_1)$$

7.12 problem 14.2

Internal problem ID [12040]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 14, Inhomogeneous second order linear equations. Exercises page 140
Problem number: 14.2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + x' - 2x = 12\,\mathrm{e}^{-t} - 6\,\mathrm{e}^{t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(x(t),t$2)+diff(x(t),t)-2*x(t)=12*exp(-t)-6*exp(t),x(t), singsol=all)
```

$$x(t) = -2\left(\left(t - \frac{c_2}{2} - \frac{1}{3}\right)\mathrm{e}^{3t} - \frac{c_1}{2} + 3\,\mathrm{e}^{t}\right)\mathrm{e}^{-2t}$$

✓ Solution by Mathematica
Time used: 0.093 (sec). Leaf size: 34

```
DSolve[x''[t]+x'[t]-2*x[t]==12*Exp[-t]-6*Exp[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{-2t}\left(-6e^{t} + e^{3t}\left(-2t + \frac{2}{3} + c_2\right) + c_1\right)$$

7.13 problem 14.3

Internal problem ID [12041]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 14, Inhomogeneous second order linear equations. Exercises page 140
Problem number: 14.3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 4x = 289t\,\mathrm{e}^{t}\sin(2t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(x(t),t$2)+4*x(t)=289*t*exp(t)*sin(2*t),x(t), singsol=all)
```

$$x(t) = \left((-68t + 76)\,\mathrm{e}^{t} + c_1\right)\cos(2t) + 17\sin(2t)\left(\mathrm{e}^{t}\left(t - \frac{2}{17}\right) + \frac{c_2}{17}\right)$$

✓ Solution by Mathematica
Time used: 0.05 (sec). Leaf size: 40

```
DSolve[x''[t]+4*x[t]==289*t*Exp[t]*Sin[2*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \left(e^{t}(76 - 68t) + c_1\right)\cos(2t) + \left(e^{t}(17t - 2) + c_2\right)\sin(2t)$$

8 Chapter 15, Resonance. Exercises page 148

8.1 problem 15.1

Internal problem ID [12042]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 15, Resonance. Exercises page 148
Problem number: 15.1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + \omega^2 x = \cos(\alpha t)}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 27

```
dsolve([diff(x(t),t$2)+omega^2*x(t)=cos(alpha*t),x(0) = 0, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = \frac{\cos(\omega t) - \cos(\alpha t)}{\alpha^2 - \omega^2}$$

✓ Solution by Mathematica
Time used: 0.25 (sec). Leaf size: 28

```
DSolve[{x''[t]+w^2*x[t]==Cos[a*t],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolutions -> Tru
```

$$x(t) \to \frac{\cos(tw) - \cos(at)}{a^2 - w^2}$$

8.2 problem 15.3

Internal problem ID [12043]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 15, Resonance. Exercises page 148
Problem number: 15.3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + \omega^2 x = \cos(\omega t)}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve([diff(x(t),t$2)+omega^2*x(t)=cos(omega*t),x(0) = 0, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = \frac{\sin(\omega t)\, t}{2\omega}$$

✓ Solution by Mathematica
Time used: 0.099 (sec). Leaf size: 17

```
DSolve[{x''[t]+w^2*x[t]==Cos[w*t],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolutions -> Tru
```

$$x(t) \to \frac{t \sin(tw)}{2w}$$

9 Chapter 16, Higher order linear equations with constant coefficients. Exercises page 153

9.1 problem 16.1 (i)

Internal problem ID [12044]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 16, Higher order linear equations with constant coefficients. Exercises page 153
Problem number: 16.1 (i).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x''' - 6x'' + 11x' - 6x = \mathrm{e}^{-t}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(x(t),t$3)-6*diff(x(t),t$2)+11*diff(x(t),t)-6*x(t)=exp(-t),x(t), singsol=all)
```

$$x(t) = -\frac{\mathrm{e}^{-t}}{24} + c_1\mathrm{e}^{t} + c_2\mathrm{e}^{2t} + c_3\mathrm{e}^{3t}$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 37

```
DSolve[x'''[t]-6*x''[t]+11*x'[t]-6*x[t]==Exp[-t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{e^{-t}}{24} + c_1e^{t} + c_2e^{2t} + c_3e^{3t}$$

9.2 problem 16.1 (ii)

Internal problem ID [12045]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 16, Higher order linear equations with constant coefficients. Exercises page 153
Problem number: 16.1 (ii).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 3y'' + 2y = \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)+2*y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = c_3 \mathrm{e}^{-\left(\sqrt{3}-1\right)x} + c_1 \mathrm{e}^{x} + c_2 \mathrm{e}^{\left(1+\sqrt{3}\right)x} + \frac{5\sin(x)}{26} + \frac{\cos(x)}{26}$$

✓ Solution by Mathematica
Time used: 0.206 (sec). Leaf size: 49

```
DSolve[y'''[x]-3*y''[x]+2*y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{26}\left(5\sin(x) + \cos(x) + 26e^{x}\left(c_1 e^{-\sqrt{3}x} + c_2 e^{\sqrt{3}x} + c_3\right)\right)$$

9.3 problem 16.1 (iii)

Internal problem ID [12046]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 16, Higher order linear equations with constant coefficients. Exercises page 153
Problem number: 16.1 (iii).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$x'''' - 4x''' + 8x'' - 8x' + 4x = \sin(t)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(x(t),t$4)-4*diff(x(t),t$3)+8*diff(x(t),t$2)-8*diff(x(t),t)+4*x(t)=sin(t),x(t), s
```

$$x(t) = ((c_3t + c_1)\cos(t) + \sin(t)(c_4t + c_2))\,\mathrm{e}^t + \frac{4\cos(t)}{25} - \frac{3\sin(t)}{25}$$

✓ Solution by Mathematica
Time used: 0.258 (sec). Leaf size: 42

```
DSolve[x''''[t]-4*x'''[t]+8*x''[t]-8*x'[t]+4*x[t]==Sin[t],x[t],t,IncludeSingularSolutions ->
```

$$x(t) \to \left(\frac{4}{25} + e^t(c_4t + c_3)\right)\cos(t) + \left(-\frac{3}{25} + e^t(c_2t + c_1)\right)\sin(t)$$

9.4 problem 16.1 (iv)

Internal problem ID [12047]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 16, Higher order linear equations with constant coefficients. Exercises page 153
Problem number: 16.1 (iv).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$\boxed{x'''' - 5x'' + 4x = e^t}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 36

```
dsolve(diff(x(t),t$4)-5*diff(x(t),t$2)+4*x(t)=exp(t),x(t), singsol=all)
```

$$x(t) = -\frac{e^{-2t}\left((t-6c_1)\,e^{3t}-6c_3e^t-6c_4e^{4t}-6c_2\right)}{6}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 45

```
DSolve[x''''[t]-5*x''[t]+4*x[t]==Exp[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{-2t}\left(c_2e^t + e^{3t}\left(-\frac{t}{6}-\frac{1}{36}+c_3\right)+c_4e^{4t}+c_1\right)$$

10 Chapter 17, Reduction of order. Exercises page 162

10.1 problem 17.1

Internal problem ID [12048]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 17, Reduction of order. Exercises page 162
Problem number: 17.1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2y'' - (t^2 + 2t)\, y' + (t + 2)\, y = 0}$$

Given that one solution of the ode is

$$y_1 = t$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve([t^2*diff(y(t),t$2)-(t^2+2*t)*diff(y(t),t)+(t+2)*y(t)=0,t],singsol=all)
```

$$y(t) = t(c_1 + c_2 e^t)$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 16

```
DSolve[t^2*y''[t]-(t^2+2*t)*y'[t]+(t+2)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t(c_2 e^t + c_1)$$

10.2 problem 17.2

Internal problem ID [12049]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 17, Reduction of order. Exercises page 162
Problem number: 17.2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-1)\,y''-y'x+y=0}$$

Given that one solution of the ode is

$$y_1 = \mathrm{e}^x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve([(x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,exp(x)],singsol=all)
```

$$y(x) = c_2\mathrm{e}^x + c_1x$$

✓ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 17

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1e^x - c_2x$$

10.3 problem 17.3

Internal problem ID [12050]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 17, Reduction of order. Exercises page 162
Problem number: 17.3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(\cos(t)\,t - \sin(t))\,x'' - x't\sin(t) - x\sin(t) = 0}$$

Given that one solution of the ode is

$$x_1 = t$$

✗ Solution by Maple

```
dsolve([(t*cos(t)-sin(t))*diff(x(t),t$2)-diff(x(t),t)*t*sin(t)-x(t)*sin(t)=0,t],singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(t*Cos[t]-Sin[t])*x''[t]-x'[t]*t*Sin[t]-x[t]*Sin[t]==0,x[t],t,IncludeSingularSolution
```

Not solved

10.4 problem 17.4

Internal problem ID [12051]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 17, Reduction of order. Exercises page 162
Problem number: 17.4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-t^2+t)\,x''+(-t^2+2)\,x'+(-t+2)\,x=0}$$

Given that one solution of the ode is

$$x_1 = \mathrm{e}^{-t}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve([(t-t^2)*diff(x(t),t$2)+(2-t^2)*diff(x(t),t)+(2-t)*x(t)=0,exp(-t)],singsol=all)
```

$$x(t) = \frac{c_2 \mathrm{e}^{-t} t + c_1}{t}$$

✓ Solution by Mathematica
Time used: 0.104 (sec). Leaf size: 42

```
DSolve[(t-t^2)*x''[t]+(2-t^2)*x'[t]+(2-t)*x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{e^{-t}\sqrt{1-t}(c_1 e^t - c_2 t)}{\sqrt{t-1}t}$$

10.5 problem 17.5

Internal problem ID [12052]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 17, Reduction of order. Exercises page 162
Problem number: 17.5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Hermite]`

$$\boxed{y'' - y'x + y = 0}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 38

```
dsolve([diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,x],singsol=all)
```

$$y(x) = c_2 \mathrm{e}^{\frac{x^2}{2}} + \frac{\left(i c_2 \sqrt{2}\, \sqrt{\pi}\, \operatorname{erf}\left(\frac{i\sqrt{2}\, x}{2}\right) + 2c_1\right) x}{2}$$

✓ Solution by Mathematica
Time used: 0.091 (sec). Leaf size: 61

```
DSolve[y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\frac{\pi}{2}} c_2 \sqrt{x^2} \operatorname{erfi}\left(\frac{\sqrt{x^2}}{\sqrt{2}}\right) + c_2 e^{\frac{x^2}{2}} + \sqrt{2} c_1 x$$

10.6 problem 17.6

Internal problem ID [12053]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 17, Reduction of order. Exercises page 162
Problem number: 17.6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\tan(t)\, x'' - 3x' + (\tan(t) + 3\cot(t))\, x = 0}$$

Given that one solution of the ode is

$$x_1 = \sin(t)$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 13

```
dsolve([tan(t)*diff(x(t),t$2)-3*diff(x(t),t)+(tan(t)+3*cot(t))*x(t)=0,sin(t)],singsol=all)
```

$$x(t) = \sin(t)\,(c_1 + c_2\cos(t))$$

✓ Solution by Mathematica
Time used: 0.374 (sec). Leaf size: 24

```
DSolve[Tan[t]*x''[t]-3*x'[t]+(Tan[t]+3*Cot[t])*x[t]==0,x[t],t,IncludeSingularSolutions -> Tr
```

$$x(t) \to \sqrt{-\sin^2(t)}(c_2\cos(t) + c_1)$$

11 Chapter 18, The variation of constants formula. Exercises page 168

11.1 problem 18.1 (i)

Internal problem ID [12054]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 18, The variation of constants formula. Exercises page 168
Problem number: 18.1 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y' - 6y = e^{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-6*y(x)=exp(x),y(x), singsol=all)
```

$$y(x) = \frac{(6c_1 e^{5x} - e^{3x} + 6c_2) e^{-2x}}{6}$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 29

```
DSolve[y''[x]-y'[x]-6*y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{x}}{6} + c_1 e^{-2x} + c_2 e^{3x}$$

11.2 problem 18.1 (ii)

Internal problem ID [12055]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 18, The variation of constants formula. Exercises page 168
Problem number: 18.1 (ii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' - x = \frac{1}{t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(x(t),t$2)-x(t)=1/t,x(t), singsol=all)
```

$$x(t) = \frac{\operatorname{expIntegral}_1(-t)\,\mathrm{e}^{-t}}{2} + c_2\mathrm{e}^{-t} + \mathrm{e}^{t}\left(c_1 - \frac{\operatorname{expIntegral}_1(t)}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 42

```
DSolve[x''[t]-x[t]==1/t,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{2}e^{-t}\left(e^{2t}\,\text{ExpIntegralEi}(-t) - \text{ExpIntegralEi}(t) + 2\left(c_1e^{2t} + c_2\right)\right)$$

11.3 problem 18.1 (iii)

Internal problem ID [12056]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 18, The variation of constants formula. Exercises page 168
Problem number: 18.1 (iii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \cot(2x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)+4*y(x)=cot(2*x),y(x), singsol=all)
```

$$y(x) = c_2 \sin(2x) + \cos(2x) c_1 + \frac{\sin(2x) \ln(\csc(2x) - \cot(2x))}{4}$$

✓ Solution by Mathematica
Time used: 0.188 (sec). Leaf size: 34

```
DSolve[y''[x]+4*y[x]==Cot[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(2x) + \frac{1}{4} \sin(2x)(\log(\sin(x)) - \log(\cos(x)) + 4c_2)$$

11.4 problem 18.1 (iv)

Internal problem ID [12057]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 18, The variation of constants formula. Exercises page 168
Problem number: 18.1 (iv).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{t^2x'' - 2x = t^3}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve(t^2*diff(x(t),t$2)-2*x(t)=t^3,x(t), singsol=all)
```

$$x(t) = c_2t^2 + \frac{t^3}{4} + \frac{c_1}{t}$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 25

```
DSolve[t^2*x''[t]-2*x[t]==t^3,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{t^3}{4} + c_2t^2 + \frac{c_1}{t}$$

11.5 problem 18.1 (v)

Internal problem ID [12058]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 18, The variation of constants formula. Exercises page 168
Problem number: 18.1 (v).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x'' - 4x' = \tan(t)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(x(t),t$2)-4*diff(x(t),t)=tan(t),x(t), singsol=all)
```

$$x(t) = \int \left(\int \tan(t)\, \mathrm{e}^{-4t} dt + c_1 \right) \mathrm{e}^{4t} dt + c_2$$

✓ Solution by Mathematica

Time used: 60.232 (sec). Leaf size: 82

```
DSolve[x''[t]-4*x'[t]==Tan[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \int_1^t \left(e^{4K[1]} c_1 + \frac{1}{20} \left(-5i \operatorname{Hypergeometric2F1}\left(2i, 1, 1 + 2i, -e^{2iK[1]}\right) - (2-4i) e^{2iK[1]} \operatorname{Hypergeometric2F1}\left(1, 1 + 2i, 2 + 2i, -e^{2iK[1]}\right)\right)\right) dK[1] + c_2$$

11.6 problem 18.1 (vi)

Internal problem ID [12059]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 18, The variation of constants formula. Exercises page 168
Problem number: 18.1 (vi).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\left(\tan(x)^2-1\right)y''-4y'\tan(x)^3+2y\sec(x)^4=\left(\tan(x)^2-1\right)\left(1-2\sin(x)^2\right)$$

Given that one solution of the ode is

$$y_1=\sec(x)^2$$

✓ Solution by Maple
Time used: 0.25 (sec). Leaf size: 29

```
dsolve([(tan(x)^2-1)*diff(y(x),x$2)-4*tan(x)^3*diff(y(x),x)+2*y(x)*sec(x)^4=(tan(x)^2-1)*(1-
```

$$y(x)=\frac{(4c_1+2x)\tan(x)}{4}+\sec(x)^2c_2-\frac{\cos(x)^2}{4}+\frac{1}{2}$$

✓ Solution by Mathematica
Time used: 0.764 (sec). Leaf size: 66

```
DSolve[(Tan[x]^2-1)*y''[x]-4*Tan[x]^3*y'[x]+2*y[x]*Sec[x]^4==(Tan[x]^2-1)*(1-2*Sin[x]^2),y[x
```

$$y(x)\to\sqrt{\sin^2(x)}\sec(x)\arctan\left(\frac{\cos(x)}{1-\sqrt{\sin^2(x)}}\right)-\frac{1}{4}\cos^2(x)+c_1\sec^2(x)+c_2\sqrt{\sin^2(x)}\sec(x)+\frac{1}{2}$$

12 Chapter 19, CauchyEuler equations. Exercises page 174

12.1 problem 19.1 (i)

Internal problem ID [12060]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 19, CauchyEuler equations. Exercises page 174
Problem number: 19.1 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' - 4y'x + 6y = 0}$$

With initial conditions

$$[y(1) = 0, y'(1) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve([x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=0,y(1) = 0, D(y)(1) = 1],y(x), singsol=al
```

$$y(x) = x^2(-1 + x)$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 12

```
DSolve[{x^2*y''[x]-4*x*y'[x]+6*y[x]==0,{y[1]==0,y'[1]==1}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to (x - 1)x^2$$

12.2 problem 19.1 (ii)

Internal problem ID [12061]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 19, CauchyEuler equations. Exercises page 174
Problem number: 19.1 (ii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{4x^2y'' + y = 0}$$

With initial conditions

$$[y(1) = 1, y'(1) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 15

```
dsolve([4*x^2*diff(y(x),x$2)+y(x)=0,y(1) = 1, D(y)(1) = 0],y(x), singsol=all)
```

$$y(x) = \sqrt{x}\left(1 - \frac{\ln(x)}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 47

```
DSolve[{x^2*y''[x]+y[x]==0,{y[1]==1,y'[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{3}\sqrt{x}\left(\sqrt{3}\sin\left(\frac{1}{2}\sqrt{3}\log(x)\right) - 3\cos\left(\frac{1}{2}\sqrt{3}\log(x)\right)\right)$$

12.3 problem 19.1 (iii)

Internal problem ID [12062]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 19, CauchyEuler equations. Exercises page 174
Problem number: 19.1 (iii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2x'' - 5tx' + 10x = 0}$$

With initial conditions

$$[x(1) = 2, x'(1) = 1]$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 19

```
dsolve([t^2*diff(x(t),t$2)-5*t*diff(x(t),t)+10*x(t)=0,x(1) = 2, D(x)(1) = 1],x(t), singsol=a
```

$$x(t) = t^3(-5\sin(\ln(t)) + 2\cos(\ln(t)))$$

✓ Solution by Mathematica
Time used: 0.197 (sec). Leaf size: 256

```
DSolve[{t^2*x''[t]-5*t*x[t]+10*x[t]==0,{x[1]==2,x'[1]==1}},x[t],t,IncludeSingularSolutions -
```

$$x(t) \to \frac{2\sqrt{t}\left(\left(\text{BesselI}\left(-1-i\sqrt{39}, 2\sqrt{5}\right) + \text{BesselI}\left(1-i\sqrt{39}, 2\sqrt{5}\right)\right)\text{BesselI}\left(i\sqrt{39}, 2\sqrt{5}\sqrt{t}\right) - \left(\text{BesselI}\left(-1+\right.\right.\right.}{\text{BesselI}\left(i\sqrt{39}, 2\sqrt{5}\right)\left(\text{BesselI}\left(-1-i\sqrt{39}, 2\sqrt{5}\right) + \text{BesselI}\left(1-i\sqrt{39}, 2\sqrt{5}\right)\right) - \text{BesselI}\left(-i\sqrt{39}, 2\right.}$$

12.4 problem 19.1 (iv)

Internal problem ID [12063]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 19, CauchyEuler equations. Exercises page 174
Problem number: 19.1 (iv).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{t^2x'' + tx' - x = 0}$$

With initial conditions

$$[x(1) = 1, x'(1) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([t^2*diff(x(t),t$2)+t*diff(x(t),t)-x(t)=0,x(1) = 1, D(x)(1) = 1],x(t), singsol=all)
```

$$x(t) = t$$

✓ Solution by Mathematica

Time used: 0.119 (sec). Leaf size: 172

```
DSolve[{t^2*x''[t]+t*x[t]-x[t]==0,{x[1]==1,x'[1]==1}},x[t],t,IncludeSingularSolutions -> Tru
```

$$x(t) \to \frac{\sqrt{t}\left(\left(\text{BesselJ}\left(\sqrt{5},2\right) - \text{BesselJ}\left(-1+\sqrt{5},2\right) + \text{BesselJ}\left(1+\sqrt{5},2\right)\right)\text{BesselJ}\left(-\sqrt{5},2\sqrt{t}\right) - \left(\text{BesselJ}\left(\right.\right.\right.}{\text{BesselJ}\left(\sqrt{5},2\right)\left(\text{BesselJ}\left(-1-\sqrt{5},2\right) - \text{BesselJ}\left(1-\sqrt{5},2\right)\right) + \text{BesselJ}\left(-\sqrt{\right.}}$$

12.5 problem 19.1 (v)

Internal problem ID [12064]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 19, CauchyEuler equations. Exercises page 174
Problem number: 19.1 (v).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2z'' + 3xz' + 4z = 0}$$

With initial conditions

$$[z(1) = 0, z'(1) = 5]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 19

```
dsolve([x^2*diff(z(x),x$2)+3*x*diff(z(x),x)+4*z(x)=0,z(1) = 0, D(z)(1) = 5],z(x), singsol=al
```

$$z(x) = \frac{5\sqrt{3}\,\sin\left(\sqrt{3}\,\ln(x)\right)}{3x}$$

✓ Solution by Mathematica
Time used: 0.145 (sec). Leaf size: 220

```
DSolve[{x^2*z''[x]+3*x*z[x]+4*z[x]==0,{z[1]==0,z'[1]==5}},z[x],x,IncludeSingularSolutions ->
```

$$z(x) \to \frac{10\sqrt{x}\left(\text{BesselJ}\left(i\sqrt{15}, 2\sqrt{3}\right)\text{BesselJ}\left(-i\sqrt{15}, 2\sqrt{3}\sqrt{x}\right) - \text{BesselJ}\left(-i\sqrt{15}\right.\right.}{\sqrt{3}\left(\text{BesselJ}\left(i\sqrt{15}, 2\sqrt{3}\right)\left(\text{BesselJ}\left(-1 - i\sqrt{15}, 2\sqrt{3}\right) - \text{BesselJ}\left(1 - i\sqrt{15}, 2\sqrt{3}\right)\right) + \text{BesselJ}\left(-i\sqrt{15}, \right.\right.}$$

12.6 problem 19.1 (vi)

Internal problem ID [12065]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 19, CauchyEuler equations. Exercises page 174
Problem number: 19.1 (vi).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2y'' - y'x - 3y = 0}$$

With initial conditions

$$[y(1) = 1, y'(1) = -1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 7

```
dsolve([x^2*diff(y(x),x$2)-x*diff(y(x),x)-3*y(x)=0,y(1) = 1, D(y)(1) = -1],y(x), singsol=all
```

$$y(x) = \frac{1}{x}$$

✓ Solution by Mathematica

Time used: 0.138 (sec). Leaf size: 169

```
DSolve[{x^2*y''[x]-x*y[x]-3*y[x]==0,{y[1]==1,y'[1]==-1}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{\sqrt{x}\left(\left(3\,\text{BesselI}\left(-\sqrt{13},2\right) + \text{BesselI}\left(-1-\sqrt{13},2\right) + \text{BesselI}\left(1-\sqrt{13},2\right)\right)\text{BesselI}\left(\sqrt{13},2\sqrt{x}\right) - (3\,\text{Be}}{\text{BesselI}\left(\sqrt{13},2\right)\left(\text{BesselI}\left(-1-\sqrt{13},2\right) + \text{BesselI}\left(1-\sqrt{13},2\right)\right) - \text{BesselI}}$$

12.7 problem 19.1 (vii)

Internal problem ID [12066]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 19, CauchyEuler equations. Exercises page 174
Problem number: 19.1 (vii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{4t^2x'' + 8tx' + 5x = 0}$$

With initial conditions

$$[x(1) = 2, x'(1) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve([4*t^2*diff(x(t),t$2)+8*t*diff(x(t),t)+5*x(t)=0,x(1) = 2, D(x)(1) = 0],x(t), singsol=
```

$$x(t) = \frac{\sin(\ln(t)) + 2\cos(\ln(t))}{\sqrt{t}}$$

✓ Solution by Mathematica

Time used: 0.101 (sec). Leaf size: 232

```
DSolve[{4*t^2*x''[t]+8*t*x[t]+5*x[t]==0,{x[1]==2,x'[1]==0}},x[t],t,IncludeSingularSolutions
```

$$x(t) \to \frac{\sqrt{t}\left(\left(2\,\text{BesselJ}\left(-1+2i,2\sqrt{2}\right)+\sqrt{2}\,\text{BesselJ}\left(2i,2\sqrt{2}\right)-2\,\text{BesselJ}\left(1+2i,2\sqrt{2}\right)\right)\text{BesselJ}\left(-2i,2\sqrt{2}\sqrt{t}\right.\right.}{\text{BesselJ}\left(-1+2i,2\sqrt{2}\right)\text{BesselJ}\left(-2i,2\sqrt{2}\right)-\text{BesselJ}\left(-1-2i,2\sqrt{2}\right)\text{BesselJ}\left(2i,2\sqrt{2}\right.}$$

12.8 problem 19.1 (viii)

Internal problem ID [12067]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 19, CauchyEuler equations. Exercises page 174
Problem number: 19.1 (viii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 5y'x + 5y = 0}$$

With initial conditions

$$[y(1) = -2, y'(1) = 1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve([x^2*diff(y(x),x$2)-5*x*diff(y(x),x)+5*y(x)=0,y(1) = -2, D(y)(1) = 1],y(x), singsol=a
```

$$y(x) = \frac{3}{4}x^5 - \frac{11}{4}x$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 17

```
DSolve[{x^2*y''[x]-5*x*y'[x]+5*y[x]==0,{y[1]==-2,y'[1]==1}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{4}x(3x^4 - 11)$$

12.9 problem 19.1 (ix)

Internal problem ID [12068]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 19, CauchyEuler equations. Exercises page 174
Problem number: 19.1 (ix).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{3x^2z'' + 5xz' - z = 0}$$

With initial conditions

$$[z(1) = 2, z'(1) = -1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([3*x^2*diff(z(x),x$2)+5*x*diff(z(x),x)-z(x)=0,z(1) = 2, D(z)(1) = -1],z(x), singsol=a
```

$$z(x) = \frac{3x^{\frac{4}{3}} + 5}{4x}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 21

```
DSolve[{3*x^2*z''[x]+5*x*z'[x]-z[x]==0,{z[1]==2,z'[1]==-1}},z[x],x,IncludeSingularSolutions
```

$$z(x) \to \frac{3x^{4/3} + 5}{4x}$$

12.10 problem 19.1 (x)

Internal problem ID [12069]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 19, CauchyEuler equations. Exercises page 174
Problem number: 19.1 (x).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2x'' + 3tx' + 13x = 0}$$

With initial conditions

$$[x(1) = -1, x'(1) = 2]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 32

```
dsolve([t^2*diff(x(t),t$2)+3*t*diff(x(t),t)+13*x(t)=0,x(1) = -1, D(x)(1) = 2],x(t), singsol=
```

$$x(t) = \frac{\sqrt{3}\,\sin\left(2\sqrt{3}\,\ln\left(t\right)\right) - 6\cos\left(2\sqrt{3}\,\ln\left(t\right)\right)}{6t}$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 41

```
DSolve[{t^2*x''[t]+3*t*x'[t]+13*x[t]==0,{x[1]==-1,x'[1]==2}},x[t],t,IncludeSingularSolutions
```

$$x(t) \to \frac{\sqrt{3}\sin\left(2\sqrt{3}\log(t)\right) - 6\cos\left(2\sqrt{3}\log(t)\right)}{6t}$$

12.11 problem 19.2

Internal problem ID [12070]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 19, CauchyEuler equations. Exercises page 174
Problem number: 19.2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{ay'' + (b-a)\,y' + cy = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 69

```
dsolve(a*diff(y(z),z$2)+(b-a)*diff(y(z),z)+c*y(z)=0,y(z), singsol=all)
```

$$y(z) = c_1 \mathrm{e}^{\frac{\left(a-b+\sqrt{a^2+(-2b-4c)a+b^2}\right)z}{2a}} + \mathrm{e}^{-\frac{\left(-a+b+\sqrt{a^2+(-2b-4c)a+b^2}\right)z}{2a}} c_2$$

✓ Solution by Mathematica
Time used: 0.073 (sec). Leaf size: 72

```
DSolve[a*y''[z]+(b-a)*y'[z]+c*y[z]==0,y[z],z,IncludeSingularSolutions -> True]
```

$$y(z) \to \left(c_2 e^{\frac{z\sqrt{a^2-2a(b+2c)+b^2}}{a}} + c_1\right) \exp\left(-\frac{z\left(\sqrt{a^2-2a(b+2c)+b^2}-a+b\right)}{2a}\right)$$

13 Chapter 20, Series solutions of second order linear equations. Exercises page 195

13.1 problem 20.1

Internal problem ID [12071]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 20, Series solutions of second order linear equations. Exercises page 195
Problem number: 20.1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-x^2+1)\,y''-2y'x+n(n+1)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 101

```
Order:=6;
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+n*(n+1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{n(n+1)\,x^2}{2}+\frac{n(n^3+2n^2-5n-6)\,x^4}{24}\right)y(0)+\left(x-\frac{(n^2+n-2)\,x^3}{6}+\frac{(n^4+2n^3-13n^2-14n+24)\,x^5}{120}\right)D(y)\,(0)+O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 120

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-2*x*y'[x]+n*(n+1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{1}{120}\left(n^2+n\right)^2x^5+\frac{7}{60}\left(-n^2-n\right)x^5+\frac{1}{6}\left(-n^2-n\right)x^3+\frac{x^5}{5}+\frac{x^3}{3}+x\right)+c_1\left(\frac{1}{24}\left(n^2+n\right)^2x^4+\frac{1}{4}\left(-n^2-n\right)x^4+\frac{1}{2}\left(-n^2-n\right)x^2+1\right)$$

13.2 problem 20.2 (i)

Internal problem ID [12072]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 20, Series solutions of second order linear equations. Exercises page 195
Problem number: 20.2 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Hermite]`

$$\boxed{y'' - y'x + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 - \frac{1}{24}x^4\right) y(0) + xD(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 27

```
AsymptoticDSolveValue[y''[x]-x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{x^4}{24} - \frac{x^2}{2} + 1\right) + c_2 x$$

13.3 problem 20.2 (ii)

Internal problem ID [12073]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 20, Series solutions of second order linear equations. Exercises page 195
Problem number: 20.2 (ii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{(x^2+1)\,y''+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((1+x^2)*diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{1}{2}x^2+\frac{1}{8}x^4\right)y(0)+\left(x-\frac{1}{6}x^3+\frac{7}{120}x^5\right)D(y)(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[(1+x^2)*y''[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{7x^5}{120}-\frac{x^3}{6}+x\right)+c_1\left(\frac{x^4}{8}-\frac{x^2}{2}+1\right)$$

13.4 problem 20.2 (iii)

Internal problem ID [12074]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 20, Series solutions of second order linear equations. Exercises page 195
Problem number: 20.2 (iii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$2xy'' + y' - 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 44

```
Order:=6;
dsolve(2*x*diff(y(x),x$2)+diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1 + \frac{2}{3}x + \frac{2}{15}x^2 + \frac{4}{315}x^3 + \frac{2}{2835}x^4 + \frac{4}{155925}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(1 + 2x + \frac{2}{3}x^2 + \frac{4}{45}x^3 + \frac{2}{315}x^4 + \frac{4}{14175}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 83

```
AsymptoticDSolveValue[2*x*y''[x]+y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{4x^5}{155925} + \frac{2x^4}{2835} + \frac{4x^3}{315} + \frac{2x^2}{15} + \frac{2x}{3} + 1\right) + c_2\left(\frac{4x^5}{14175} + \frac{2x^4}{315} + \frac{4x^3}{45} + \frac{2x^2}{3} + 2x + 1\right)$$

13.5 problem 20.2 (iv) (k=-2)

Internal problem ID [12075]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 20, Series solutions of second order linear equations. Exercises page 195
Problem number: 20.2 (iv) (k=-2).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'x - 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
Order:=6;
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)-4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + 2x^2 + \frac{4}{3}x^4\right) y(0) + \left(x + x^3 + \frac{1}{2}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[y''[x]-2*x*y'[x]-4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{2} + x^3 + x\right) + c_1\left(\frac{4x^4}{3} + 2x^2 + 1\right)$$

13.6 problem 20.2 (iv) (k=2)

Internal problem ID [12076]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 20, Series solutions of second order linear equations. Exercises page 195
Problem number: 20.2 (iv) (k=2).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'x + 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
Order:=6;
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(-2x^2 + 1\right) y(0) + \left(x - \frac{1}{3}x^3 - \frac{1}{30}x^5\right) D(y)(0) + O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 33

```
AsymptoticDSolveValue[y''[x]-2*x*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1\left(1 - 2x^2\right) + c_2\left(-\frac{x^5}{30} - \frac{x^3}{3} + x\right)$$

13.7 problem 20.3

Internal problem ID [12077]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 20, Series solutions of second order linear equations. Exercises page 195
Problem number: 20.3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x(1-x)\,y''-3y'x-y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 60

```
Order:=6;
dsolve(x*(1-x)*diff(y(x),x$2)-3*x*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned}y(x) = {} & \ln(x)\left(x+2x^2+3x^3+4x^4+5x^5+\mathrm{O}\left(x^6\right)\right)c_2 \\ & + c_1 x\left(1+2x+3x^2+4x^3+5x^4+6x^5+\mathrm{O}\left(x^6\right)\right) \\ & + \left(1+3x+5x^2+7x^3+9x^4+11x^5+\mathrm{O}\left(x^6\right)\right)c_2\end{aligned}$$

✓ Solution by Mathematica
Time used: 0.059 (sec). Leaf size: 63

```
AsymptoticDSolveValue[x*(1-x)*y''[x]-3*x*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned}y(x) \to {} & c_1\left(x^4+x^3+x^2+\left(4x^3+3x^2+2x+1\right)x\log(x)+x+1\right) \\ & + c_2\left(5x^5+4x^4+3x^3+2x^2+x\right)\end{aligned}$$

13.8 problem 20.4

Internal problem ID [12078]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 20, Series solutions of second order linear equations. Exercises page 195
Problem number: 20.4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x - x^2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 41

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_1 + c_2 \ln(x)) \left(1 + \frac{1}{4}x^2 + \frac{1}{64}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(-\frac{1}{4}x^2 - \frac{3}{128}x^4 + \mathrm{O}\left(x^6\right)\right) c_2$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 60

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]-x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^4}{64} + \frac{x^2}{4} + 1\right) + c_2\left(-\frac{3x^4}{128} - \frac{x^2}{4} + \left(\frac{x^4}{64} + \frac{x^2}{4} + 1\right)\log(x)\right)$$

13.9 problem 20.5

Internal problem ID [12079]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 20, Series solutions of second order linear equations. Exercises page 195
Problem number: 20.5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Bessel]`

$$\boxed{x^2y'' + y'x + y(x^2 - 1) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^2\left(1 - \frac{1}{8}x^2 + \frac{1}{192}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2\left(\ln\left(x\right)\left(x^2 - \frac{1}{8}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(-2 + \frac{3}{32}x^4 + \mathrm{O}\left(x^6\right)\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 58

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{192} - \frac{x^3}{8} + x\right) + c_1\left(\frac{1}{16}x\left(x^2 - 8\right)\log(x) - \frac{5x^4 - 16x^2 - 64}{64x}\right)$$

13.10 problem 20.7

Internal problem ID [12080]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 20, Series solutions of second order linear equations. Exercises page 195
Problem number: 20.7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Bessel]`

$$\boxed{x^2y'' + y'x + (-n^2 + x^2)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 77

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-n^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x^{-n}\left(1 + \frac{1}{4n-4}x^2 + \frac{1}{32}\frac{1}{(n-2)(n-1)}x^4 + \mathrm{O}\left(x^6\right)\right)c_1 + c_2x^n\left(1 - \frac{1}{4n+4}x^2 + \frac{1}{32}\frac{1}{(n+2)(n+1)}x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 160

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-n^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^4}{(-n^2-n+(1-n)(2-n)+2)\,(-n^2-n+(3-n)(4-n)+4)} - \frac{x^2}{-n^2-n+(1-n)(2-n)+2}+1\right)x^{-n} + c_1\left(\frac{x^4}{(-n^2+n+(n+1)(n+2)+2)\,(-n^2+n+(n+3)(n+4)+4)} - \frac{x^2}{-n^2+n+(n+1)(n+2)+2}+1\right)x^n$$

14 Chapter 26, Explicit solutions of coupled linear systems. Exercises page 257

14.1 problem 26.1 (i)

Internal problem ID [12081]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 26, Explicit solutions of coupled linear systems. Exercises page 257
Problem number: 26.1 (i).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 4x(t) - y(t)$$
$$y'(t) = 2x(t) + y(t) + t^2$$

With initial conditions

$$[x(0) = 0, y(0) = 1]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 52

```
dsolve([diff(x(t),t) = 4*x(t)-y(t), diff(y(t),t) = 2*x(t)+y(t)+t^2, x(0) = 0, y(0) = 1], sin
```

$$x(t) = -\frac{29\,e^{3t}}{27} + \frac{5\,e^{2t}}{4} - \frac{t^2}{6} - \frac{5t}{18} - \frac{19}{108}$$
$$y(t) = -\frac{29\,e^{3t}}{27} + \frac{5\,e^{2t}}{2} - \frac{7t}{9} - \frac{23}{54} - \frac{2t^2}{3}$$

✓ Solution by Mathematica
Time used: 0.128 (sec). Leaf size: 64

```
DSolve[{x'[t]==4*x[t]-y[t],y'[t]==2*x[t]+y[t]+t^2},{x[0]==0,y[0]==1},{x[t],y[t]},t,IncludeSi
```

$$x(t) \to \frac{1}{108}\left(-18t^2 - 30t + 135e^{2t} - 116e^{3t} - 19\right)$$
$$y(t) \to \frac{1}{54}\left(-36t^2 - 42t + 135e^{2t} - 58e^{3t} - 23\right)$$

14.2 problem 26.1 (ii)

Internal problem ID [12082]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 26, Explicit solutions of coupled linear systems. Exercises page 257
Problem number: 26.1 (ii).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) - 4y(t) + 2\cos(t)^2 - 1$$
$$y'(t) = x(t) + y(t)$$

With initial conditions

$$[x(0) = 1, y(0) = 1]$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 66

```
dsolve([diff(x(t),t) = x(t)-4*y(t)+cos(2*t), diff(y(t),t) = x(t)+y(t), x(0) = 1, y(0) = 1],
```

$$x(t) = \frac{26\,e^t\cos(2t)}{17} - \frac{32\,e^t\sin(2t)}{17} + \frac{2\sin(2t)}{17} - \frac{9\cos(2t)}{17}$$
$$y(t) = \frac{13\,e^t\sin(2t)}{17} + \frac{16\,e^t\cos(2t)}{17} + \frac{\cos(2t)}{17} - \frac{4\sin(2t)}{17}$$

✓ Solution by Mathematica

Time used: 0.122 (sec). Leaf size: 67

```
DSolve[{x'[t]==x[t]-4*y[t]+Cos[2*t],y'[t]==x[t]+y[t]},{x[0]==1,y[0]==1},{x[t],y[t]},t,Includ
```

$$x(t) \to \frac{1}{17}\left((26e^t - 9)\cos(2t) - 2(16e^t - 1)\sin(2t)\right)$$
$$y(t) \to \frac{1}{17}\left((13e^t - 4)\sin(2t) + (16e^t + 1)\cos(2t)\right)$$

14.3 problem 26.1 (iii)

Internal problem ID [12083]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 26, Explicit solutions of coupled linear systems. Exercises page 257
Problem number: 26.1 (iii).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 2x(t) + 2y(t)$$
$$y'(t) = 6x(t) + 3y(t) + e^t$$

With initial conditions

$$[x(0) = 0, y(0) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 42

```
dsolve([diff(x(t),t) = 2*x(t)+2*y(t), diff(y(t),t) = 6*x(t)+3*y(t)+exp(t), x(0) = 0, y(0) =
```

$$x(t) = \frac{12 e^{6t}}{35} - \frac{e^{-t}}{7} - \frac{e^{t}}{5}$$
$$y(t) = \frac{24 e^{6t}}{35} + \frac{3 e^{-t}}{14} + \frac{e^{t}}{10}$$

✓ Solution by Mathematica

Time used: 0.101 (sec). Leaf size: 58

```
DSolve[{x'[t]==2*x[t]+2*y[t],y'[t]==6*x[t]+3*y[t]+Exp[t]},{x[0]==0,y[0]==1},{x[t],y[t]},t,In
```

$$x(t) \to \frac{1}{35} e^{-t} \left(-7e^{2t} + 12e^{7t} - 5\right)$$
$$y(t) \to \frac{1}{70} e^{-t} \left(7e^{2t} + 48e^{7t} + 15\right)$$

14.4 problem 26.1 (iv)

Internal problem ID [12084]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 26, Explicit solutions of coupled linear systems. Exercises page 257
Problem number: 26.1 (iv).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 5x(t) - 4y(t) + e^{3t}$$
$$y'(t) = x(t) + y(t)$$

With initial conditions

$$[x(0) = 1, y(0) = -1]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 35

```
dsolve([diff(x(t),t) = 5*x(t)-4*y(t)+exp(3*t), diff(y(t),t) = x(t)+y(t), x(0) = 1, y(0) = -1
```

$$x(t) = e^{3t}(t^2 + 7t + 1)$$
$$y(t) = \frac{e^{3t}(t^2 + 6t - 2)}{2}$$

✓ Solution by Mathematica
Time used: 0.007 (sec). Leaf size: 39

```
DSolve[{x'[t]==5*x[t]-4*y[t]+Exp[3*t],y'[t]==x[t]+y[t]},{x[0]==1,y[0]==-1},{x[t],y[t]},t,Inc
```

$$x(t) \to e^{3t}(t^2 + 7t + 1)$$
$$y(t) \to \frac{1}{2}e^{3t}(t^2 + 6t - 2)$$

14.5 problem 26.1 (v)

Internal problem ID [12085]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 26, Explicit solutions of coupled linear systems. Exercises page 257
Problem number: 26.1 (v).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 2x(t) + 5y(t)$$
$$y'(t) = -2x(t) + 4\cos(t)^3 - 3\cos(t)$$

With initial conditions

$$[x(0) = 2, y(0) = -1]$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 66

```
dsolve([diff(x(t),t) = 2*x(t)+5*y(t), diff(y(t),t) = -2*x(t)+cos(3*t), x(0) = 2, y(0) = -1],
```

$$x(t) = -\frac{16\,e^{t}\sin(3t)}{111} + \frac{69\,e^{t}\cos(3t)}{37} + \frac{5\cos(3t)}{37} - \frac{30\sin(3t)}{37}$$
$$y(t) = -\frac{121\,e^{t}\sin(3t)}{111} - \frac{17\,e^{t}\cos(3t)}{37} + \frac{9\sin(3t)}{37} - \frac{20\cos(3t)}{37}$$

✓ Solution by Mathematica

Time used: 0.363 (sec). Leaf size: 70

```
DSolve[{x'[t]==2*x[t]+5*y[t],y'[t]==-2*x[t]+Cos[3*t]},{x[0]==2,y[0]==-1},{x[t],y[t]},t,Inclu
```

$$x(t) \to \frac{1}{111}\left(3\left(69e^{t}+5\right)\cos(3t) - 2\left(8e^{t}+45\right)\sin(3t)\right)$$
$$y(t) \to \frac{1}{111}\left(-\left(121e^{t}-27\right)\sin(3t) - 3\left(17e^{t}+20\right)\cos(3t)\right)$$

14.6 problem 26.1 (vi)

Internal problem ID [12086]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 26, Explicit solutions of coupled linear systems. Exercises page 257
Problem number: 26.1 (vi).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) + y(t) + \mathrm{e}^{-t}$$
$$y'(t) = 4x(t) - 2y(t) + \mathrm{e}^{2t}$$

With initial conditions

$$[x(0) = 1, y(0) = -1]$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 60

```
dsolve([diff(x(t),t) = x(t)+y(t)+exp(-t), diff(y(t),t) = 4*x(t)-2*y(t)+exp(2*t), x(0) = 1, y
```

$$x(t) = \frac{62\,\mathrm{e}^{2t}}{75} + \frac{17\,\mathrm{e}^{-3t}}{50} + \frac{\mathrm{e}^{2t}t}{5} - \frac{\mathrm{e}^{-t}}{6}$$
$$y(t) = \frac{77\,\mathrm{e}^{2t}}{75} - \frac{34\,\mathrm{e}^{-3t}}{25} + \frac{\mathrm{e}^{2t}t}{5} - \frac{2\,\mathrm{e}^{-t}}{3}$$

✓ Solution by Mathematica
Time used: 0.69 (sec). Leaf size: 67

```
DSolve[{x'[t]==x[t]+y[t]+Exp[-t],y'[t]==4*x[t]-2*y[t]+Exp[2*t]},{x[0]==1,y[0]==-1},{x[t],y[t
```

$$x(t) \to \frac{1}{150}e^{-3t}\left(2e^{5t}(15t + 62) - 25e^{2t} + 51\right)$$
$$y(t) \to \frac{1}{75}e^{-3t}\left(e^{5t}(15t + 77) - 50e^{2t} - 102\right)$$

14.7 problem 26.1 (vii)

Internal problem ID [12087]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 26, Explicit solutions of coupled linear systems. Exercises page 257
Problem number: 26.1 (vii).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 8x(t) + 14y(t)$$
$$y'(t) = 7x(t) + y(t)$$

With initial conditions

$$[x(0) = 1, y(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
dsolve([diff(x(t),t) = 8*x(t)+14*y(t), diff(y(t),t) = 7*x(t)+y(t), x(0) = 1, y(0) = 1], sing
```

$$x(t) = -\frac{e^{-6t}}{3} + \frac{4\,e^{15t}}{3}$$
$$y(t) = \frac{e^{-6t}}{3} + \frac{2\,e^{15t}}{3}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 44

```
DSolve[{x'[t]==8*x[t]+14*y[t],y'[t]==7*x[t]+y[t]},{x[0]==1,y[0]==1},{x[t],y[t]},t,IncludeSin
```

$$x(t) \to \frac{1}{3}e^{-6t}\left(4e^{21t} - 1\right)$$
$$y(t) \to \frac{1}{3}e^{-6t}\left(2e^{21t} + 1\right)$$

15 Chapter 28, Distinct real eigenvalues. Exercises page 282

15.1 problem 28.2 (i)

Internal problem ID [12097]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 28, Distinct real eigenvalues. Exercises page 282
Problem number: 28.2 (i).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 8x(t) + 14y(t)$$
$$y'(t) = 7x(t) + y(t)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=8*x(t)+14*y(t),diff(y(t),t)=7*x(t)+y(t)],singsol=all)
```

$$x(t) = \mathrm{e}^{-6t} c_1 + c_2 \mathrm{e}^{15t}$$
$$y(t) = -\mathrm{e}^{-6t} c_1 + \frac{c_2 \mathrm{e}^{15t}}{2}$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 71

```
DSolve[{x'[t]==8*x[t]+14*y[t],y'[t]==7*x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{1}{3} e^{-6t} \left(c_1 \left(2e^{21t} + 1\right) + 2c_2 \left(e^{21t} - 1\right)\right)$$
$$y(t) \to \frac{1}{3} e^{-6t} \left(c_1 \left(e^{21t} - 1\right) + c_2 \left(e^{21t} + 2\right)\right)$$

15.2 problem 28.2 (ii)

Internal problem ID [12098]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 28, Distinct real eigenvalues. Exercises page 282
Problem number: 28.2 (ii).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 2x(t)$$
$$y'(t) = -5x(t) - 3y(t)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 28

```
dsolve([diff(x(t),t)=2*x(t),diff(y(t),t)=-5*x(t)-3*y(t)],singsol=all)
```

$$x(t) = c_2 \mathrm{e}^{2t}$$
$$y(t) = -c_2 \mathrm{e}^{2t} + c_1 \mathrm{e}^{-3t}$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 36

```
DSolve[{x'[t]==2*x[t],y'[t]==-5*x[t]-3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 e^{2t}$$
$$y(t) \to e^{-3t}\left(c_1\left(-e^{5t}\right) + c_1 + c_2\right)$$

15.3 problem 28.2 (iii)

Internal problem ID [12099]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 28, Distinct real eigenvalues. Exercises page 282
Problem number: 28.2 (iii).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 11x(t) - 2y(t)$$
$$y'(t) = 3x(t) + 4y(t)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=11*x(t)-2*y(t),diff(y(t),t)=3*x(t)+4*y(t)],singsol=all)
```

$$x(t) = e^{5t}c_1 + c_2e^{10t}$$
$$y(t) = 3\,e^{5t}c_1 + \frac{c_2e^{10t}}{2}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 95

```
DSolve[{x'[t]==2*x[t]-2*y[t],y'[t]==3*x[t]+4*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{1}{5}e^{3t}\left(5c_1\cos\left(\sqrt{5}t\right) - \sqrt{5}(c_1 + 2c_2)\sin\left(\sqrt{5}t\right)\right)$$
$$y(t) \to \frac{1}{5}e^{3t}\left(5c_2\cos\left(\sqrt{5}t\right) + \sqrt{5}(3c_1 + c_2)\sin\left(\sqrt{5}t\right)\right)$$

15.4 problem 28.2 (iv)

Internal problem ID [12100]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 28, Distinct real eigenvalues. Exercises page 282
Problem number: 28.2 (iv).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) + 20y(t)$$
$$y'(t) = 40x(t) - 19y(t)$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 35

```
dsolve([diff(x(t),t)=x(t)+20*y(t),diff(y(t),t)=40*x(t)-19*y(t)],singsol=all)
```

$$x(t) = c_1 e^{21t} + c_2 e^{-39t}$$
$$y(t) = c_1 e^{21t} - 2c_2 e^{-39t}$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 71

```
DSolve[{x'[t]==x[t]+20*y[t],y'[t]==40*x[t]-19*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -
```

$$x(t) \to \frac{1}{3}e^{-39t}\left(c_1\left(2e^{60t}+1\right)+c_2\left(e^{60t}-1\right)\right)$$
$$y(t) \to \frac{1}{3}e^{-39t}\left(2c_1\left(e^{60t}-1\right)+c_2\left(e^{60t}+2\right)\right)$$

15.5 problem 28.6 (iii)

Internal problem ID [12101]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 28, Distinct real eigenvalues. Exercises page 282
Problem number: 28.6 (iii).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -2x(t) + 2y(t)$$
$$y'(t) = x(t) - y(t)$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 25

```
dsolve([diff(x(t),t)=-2*x(t)+2*y(t),diff(y(t),t)=x(t)-y(t)],singsol=all)
```

$$x(t) = c_1 + c_2 e^{-3t}$$
$$y(t) = -\frac{c_2 e^{-3t}}{2} + c_1$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 71

```
DSolve[{x'[t]==-2*x[t]+2*y[t],y'[t]==x[t]-y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tr
```

$$x(t) \to \frac{1}{3}e^{-3t}\left(c_1\left(e^{3t}+2\right)+2c_2\left(e^{3t}-1\right)\right)$$
$$y(t) \to \frac{1}{3}e^{-3t}\left(c_1\left(e^{3t}-1\right)+c_2\left(2e^{3t}+1\right)\right)$$

16 Chapter 29, Complex eigenvalues. Exercises page 292

16.1 problem 29.3 (i)

Internal problem ID [12102]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 29, Complex eigenvalues. Exercises page 292
Problem number: 29.3 (i).
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= -y(t) \\
y'(t) &= x(t) - y(t)
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 82

```
dsolve([diff(x(t),t)=-y(t),diff(y(t),t)=x(t)-y(t)],singsol=all)
```

$$
\begin{aligned}
x(t) &= \mathrm{e}^{-\frac{t}{2}}\left(\sin\left(\frac{\sqrt{3}\,t}{2}\right)c_1 + \cos\left(\frac{\sqrt{3}\,t}{2}\right)c_2\right) \\
y(t) &= \frac{\mathrm{e}^{-\frac{t}{2}}\left(\sqrt{3}\,\sin\left(\frac{\sqrt{3}\,t}{2}\right)c_2 - \sqrt{3}\,\cos\left(\frac{\sqrt{3}\,t}{2}\right)c_1 + \sin\left(\frac{\sqrt{3}\,t}{2}\right)c_1 + \cos\left(\frac{\sqrt{3}\,t}{2}\right)c_2\right)}{2}
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 112

```
DSolve[{x'[t]==-y[t],y'[t]==x[t]-y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
x(t) &\to \frac{1}{3}e^{-t/2}\left(3c_1\cos\left(\frac{\sqrt{3}t}{2}\right) + \sqrt{3}(c_1 - 2c_2)\sin\left(\frac{\sqrt{3}t}{2}\right)\right) \\
y(t) &\to \frac{1}{3}e^{-t/2}\left(3c_2\cos\left(\frac{\sqrt{3}t}{2}\right) + \sqrt{3}(2c_1 - c_2)\sin\left(\frac{\sqrt{3}t}{2}\right)\right)
\end{aligned}
$$

16.2 problem 29.3 (ii)

Internal problem ID [12103]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 29, Complex eigenvalues. Exercises page 292
Problem number: 29.3 (ii).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -2x(t) + 3y(t)$$
$$y'(t) = -6x(t) + 4y(t)$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 53

```
dsolve([diff(x(t),t)=-2*x(t)+3*y(t),diff(y(t),t)=-6*x(t)+4*y(t)],singsol=all)
```

$$x(t) = e^{t}(c_1 \sin(3t) + c_2 \cos(3t))$$
$$y(t) = e^{t}(c_1 \cos(3t) + c_2 \cos(3t) + c_1 \sin(3t) - c_2 \sin(3t))$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 56

```
DSolve[{x'[t]==-2*x[t]+3*y[t],y'[t]==-6*x[t]+4*y[t]},{x[t],y[t]},t,IncludeSingularSolutions
```

$$x(t) \to e^{t}(c_1 \cos(3t) + (c_2 - c_1)\sin(3t))$$
$$y(t) \to e^{t}(c_2 \cos(3t) + (c_2 - 2c_1)\sin(3t))$$

16.3 problem 29.3 (iii)

Internal problem ID [12104]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 29, Complex eigenvalues. Exercises page 292
Problem number: 29.3 (iii).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -11x(t) - 2y(t)$$
$$y'(t) = 13x(t) - 9y(t)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 59

```
dsolve([diff(x(t),t)=-11*x(t)-2*y(t),diff(y(t),t)=13*x(t)-9*y(t)],singsol=all)
```

$$x(t) = e^{-10t}(\sin(5t) c_1 + c_2 \cos(5t))$$
$$y(t) = -\frac{e^{-10t}(\sin(5t) c_1 - 5c_2 \sin(5t) + 5\cos(5t) c_1 + c_2 \cos(5t))}{2}$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 69

```
DSolve[{x'[t]==-11*x[t]-2*y[t],y'[t]==13*x[t]-9*y[t]},{x[t],y[t]},t,IncludeSingularSolutions
```

$$x(t) \to \frac{1}{5} e^{-10t}(5c_1 \cos(5t) - (c_1 + 2c_2)\sin(5t))$$
$$y(t) \to \frac{1}{5} e^{-10t}(5c_2 \cos(5t) + (13c_1 + c_2)\sin(5t))$$

16.4 problem 29.3 (iv)

Internal problem ID [12105]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 29, Complex eigenvalues. Exercises page 292
Problem number: 29.3 (iv).
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= 7x(t) - 5y(t) \\
y'(t) &= 10x(t) - 3y(t)
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 57

```
dsolve([diff(x(t),t)=7*x(t)-5*y(t),diff(y(t),t)=10*x(t)-3*y(t)],singsol=all)
```

$$
\begin{aligned}
x(t) &= e^{2t}(\sin(5t)\, c_1 + c_2 \cos(5t)) \\
y(t) &= e^{2t}(\sin(5t)\, c_1 + c_2 \sin(5t) - \cos(5t)\, c_1 + c_2 \cos(5t))
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.008 (sec). Leaf size: 62

```
DSolve[{x'[t]==7*x[t]-5*y[t],y'[t]==10*x[t]-3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -
```

$$
\begin{aligned}
x(t) &\to e^{2t}(c_1 \cos(5t) + (c_1 - c_2)\sin(5t)) \\
y(t) &\to e^{2t}(c_2 \cos(5t) + (2c_1 - c_2)\sin(5t))
\end{aligned}
$$

17 Chapter 30, A repeated real eigenvalue. Exercises page 299

17.1 problem 30.1 (i)

Internal problem ID [12106]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 30, A repeated real eigenvalue. Exercises page 299
Problem number: 30.1 (i).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 5x(t) - 4y(t)$$
$$y'(t) = x(t) + y(t)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve([diff(x(t),t)=5*x(t)-4*y(t),diff(y(t),t)=x(t)+y(t)],singsol=all)
```

$$x(t) = e^{3t}(c_2 t + c_1)$$
$$y(t) = \frac{e^{3t}(2c_2 t + 2c_1 - c_2)}{4}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 45

```
DSolve[{x'[t]==5*x[t]-4*y[t],y'[t]==x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tru
```

$$x(t) \to e^{3t}(2c_1 t - 4c_2 t + c_1)$$
$$y(t) \to e^{3t}((c_1 - 2c_2)t + c_2)$$

17.2 problem 30.1 (ii)

Internal problem ID [12107]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 30, A repeated real eigenvalue. Exercises page 299
Problem number: 30.1 (ii).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -6x(t) + 2y(t)$$
$$y'(t) = -2x(t) - 2y(t)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
dsolve([diff(x(t),t)=-6*x(t)+2*y(t),diff(y(t),t)=-2*x(t)-2*y(t)],singsol=all)
```

$$x(t) = \mathrm{e}^{-4t}(c_2 t + c_1)$$
$$y(t) = \frac{\mathrm{e}^{-4t}(2c_2 t + 2c_1 + c_2)}{2}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 46

```
DSolve[{x'[t]==-6*x[t]+2*y[t],y'[t]==-2*x[t]-2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions
```

$$x(t) \to e^{-4t}(-2c_1 t + 2c_2 t + c_1)$$
$$y(t) \to e^{-4t}(-2c_1 t + 2c_2 t + c_2)$$

17.3 problem 30.1 (iii)

Internal problem ID [12108]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 30, A repeated real eigenvalue. Exercises page 299
Problem number: 30.1 (iii).
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= -3x(t) - y(t) \\
y'(t) &= x(t) - 5y(t)
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve([diff(x(t),t)=-3*x(t)-y(t),diff(y(t),t)=x(t)-5*y(t)],singsol=all)
```

$$
\begin{aligned}
x(t) &= \mathrm{e}^{-4t}(c_2 t + c_1) \\
y(t) &= \mathrm{e}^{-4t}(c_2 t + c_1 - c_2)
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 44

```
DSolve[{x'[t]==-3*x[t]-y[t],y'[t]==x[t]-5*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tr
```

$$
\begin{aligned}
x(t) &\to e^{-4t}(c_1(t+1) - c_2 t) \\
y(t) &\to e^{-4t}((c_1 - c_2)t + c_2)
\end{aligned}
$$

17.4 problem 30.1 (iv)

Internal problem ID [12109]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 30, A repeated real eigenvalue. Exercises page 299
Problem number: 30.1 (iv).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 13x(t)$$
$$y'(t) = 13y(t)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve([diff(x(t),t)=13*x(t),diff(y(t),t)=13*y(t)],singsol=all)
```

$$x(t) = c_2 e^{13t}$$
$$y(t) = e^{13t} c_1$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 65

```
DSolve[{x'[t]==13*x[t],y'[t]==13*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 e^{13t}$$
$$y(t) \to c_2 e^{13t}$$
$$x(t) \to c_1 e^{13t}$$
$$y(t) \to 0$$
$$x(t) \to 0$$
$$y(t) \to c_2 e^{13t}$$
$$x(t) \to 0$$
$$y(t) \to 0$$

17.5 problem 30.1 (v)

Internal problem ID [12110]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 30, A repeated real eigenvalue. Exercises page 299
Problem number: 30.1 (v).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 7x(t) - 4y(t)$$
$$y'(t) = x(t) + 3y(t)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve([diff(x(t),t)=7*x(t)-4*y(t),diff(y(t),t)=x(t)+3*y(t)],singsol=all)
```

$$x(t) = \mathrm{e}^{5t}(c_2 t + c_1)$$
$$y(t) = \frac{\mathrm{e}^{5t}(2c_2 t + 2c_1 - c_2)}{4}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 45

```
DSolve[{x'[t]==7*x[t]-4*y[t],y'[t]==x[t]+3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> T
```

$$x(t) \to e^{5t}(2c_1 t - 4c_2 t + c_1)$$
$$y(t) \to e^{5t}((c_1 - 2c_2)t + c_2)$$

17.6 problem 30.5 (iii)

Internal problem ID [12111]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 30, A repeated real eigenvalue. Exercises page 299
Problem number: 30.5 (iii).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -x(t) + y(t)$$
$$y'(t) = -x(t) + y(t)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve([diff(x(t),t)=-x(t)+y(t),diff(y(t),t)=-x(t)+y(t)],singsol=all)
```

$$x(t) = c_1 t + c_2$$
$$y(t) = c_1 t + c_1 + c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 32

```
DSolve[{x'[t]==-x[t]+y[t],y'[t]==-x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1(-t) + c_2 t + c_1$$
$$y(t) \to (c_2 - c_1)t + c_2$$