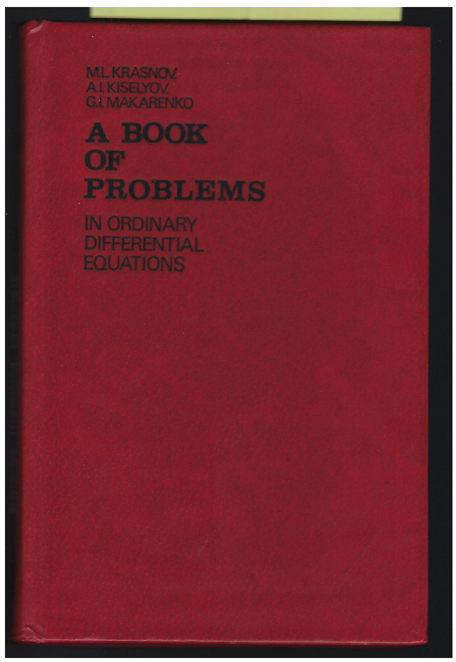
A Solution Manual For

A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983



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Contents

1 Section 1. Basic concepts and definitions. Exercises page 18

1.1 problem 2

Internal problem ID [14934]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 1. Basic concepts and definitions. Exercises page 18
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{y' - y^2 = x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x)=x^2+y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{x\left(\text{BesselJ}\left(-\frac{3}{4}, \frac{x^2}{2}\right) c_1 + \text{BesselY}\left(-\frac{3}{4}, \frac{x^2}{2}\right)\right)}{c_1 \,\text{BesselJ}\left(\frac{1}{4}, \frac{x^2}{2}\right) + \text{BesselY}\left(\frac{1}{4}, \frac{x^2}{2}\right)}$$

✓ Solution by Mathematica
Time used: 0.132 (sec). Leaf size: 169

```
DSolve[y'[x]==x^2+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2\left(-2\,\text{BesselJ}\left(-\frac{3}{4}, \frac{x^2}{2}\right) + c_1\left(\text{BesselJ}\left(\frac{3}{4}, \frac{x^2}{2}\right) - \text{BesselJ}\left(-\frac{5}{4}, \frac{x^2}{2}\right)\right)\right) - c_1\,\text{BesselJ}\left(-\frac{1}{4}, \frac{x^2}{2}\right)}{2x\left(\text{BesselJ}\left(\frac{1}{4}, \frac{x^2}{2}\right) + c_1\,\text{BesselJ}\left(-\frac{1}{4}, \frac{x^2}{2}\right)\right)}$$

$$y(x) \to -\frac{x^2\,\text{BesselJ}\left(-\frac{5}{4}, \frac{x^2}{2}\right) - x^2\,\text{BesselJ}\left(\frac{3}{4}, \frac{x^2}{2}\right) + \text{BesselJ}\left(-\frac{1}{4}, \frac{x^2}{2}\right)}{2x\,\text{BesselJ}\left(-\frac{1}{4}, \frac{x^2}{2}\right)}$$

1.2 problem 3

Internal problem ID [14935]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 1. Basic concepts and definitions. Exercises page 18
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' - \frac{x}{y} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)=x/y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{x^2 + c_1}$$
$$y(x) = -\sqrt{x^2 + c_1}$$

✓ Solution by Mathematica
Time used: 0.106 (sec). Leaf size: 35

```
DSolve[y'[x]==x/y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x^2 + 2c_1}$$
$$y(x) \to \sqrt{x^2 + 2c_1}$$

1.3 problem 4

Internal problem ID [14936]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 1. Basic concepts and definitions. Exercises page 18
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y - 3y^{\frac{1}{3}} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=y(x)+3*y(x)^(1/3),y(x), singsol=all)
```

$$3 + y(x)^{\frac{2}{3}} - e^{\frac{2x}{3}} c_1 = 0$$

✓ Solution by Mathematica
Time used: 2.285 (sec). Leaf size: 39

```
DSolve[y'[x]==y[x]+3*y[x]^(1/3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(-3 + e^{\frac{2(x+c_1)}{3}}\right)^{3/2}$$
$$y(x) \to 0$$
$$y(x) \to -3i\sqrt{3}$$

1.4 problem 5

Internal problem ID [14937]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 1. Basic concepts and definitions. Exercises page 18
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - \sqrt{-y + x} = 0}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 50

```
dsolve(diff(y(x),x)=sqrt(x-y(x)),y(x), singsol=all)
```

$$x + \ln(-y(x) + x - 1) + 2\sqrt{-y(x) + x} + \ln\left(-1 + \sqrt{-y(x) + x}\right) - \ln\left(1 + \sqrt{-y(x) + x}\right) - c_1 = 0$$

✓ Solution by Mathematica
Time used: 7.657 (sec). Leaf size: 53

```
DSolve[y'[x]==Sqrt[x-y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -W\left(e^{-\frac{x}{2}-1-\frac{c_1}{2}}\right)^2 - 2W\left(e^{-\frac{x}{2}-1-\frac{c_1}{2}}\right) + x - 1$$
$$y(x) \to x - 1$$

1.5 problem 6

Internal problem ID [14938]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 1. Basic concepts and definitions. Exercises page 18
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y' - \sqrt{x^2 - y} = -x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 171

```
dsolve(diff(y(x),x)=sqrt(x^2-y(x))-x,y(x), singsol=all)
```

$$\frac{250\left(x^6c_1y(x)^2 + \frac{12x^4c_1y(x)^3}{5} + \frac{48x^2c_1y(x)^4}{25} + \frac{64c_1y(x)^5}{125} - \frac{1}{125}\right)(x^2 - y(x))^{\frac{3}{2}}(x^2 + 4y(x)) - 250\left(x^6c_1y(x)^2 + \frac{1}{}\right.}{(5x^2 + 4y(x))^3 y(x)^2 \left(-\sqrt{x^2 - y(x)} + x\right)^2 \left(3x + 2\sqrt{}\right.}$$

$$= 0$$

✓ Solution by Mathematica

Time used: 4.748 (sec). Leaf size: 416

```
DSolve[y'[x]==Sqrt[x^2-y[x]]-x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{Root}\left[64\#1^5+240\#1^4x^2+300\#1^3x^4+\#1^2\left(125x^6-40e^{5c_1}x\right)-10\#1e^{5c_1}x^3 - 4e^{5c_1}x^5+e^{10c_1}\&,1\right]$$

$$y(x) \to \text{Root}\left[64\#1^5+240\#1^4x^2+300\#1^3x^4+\#1^2\left(125x^6-40e^{5c_1}x\right)-10\#1e^{5c_1}x^3 - 4e^{5c_1}x^5+e^{10c_1}\&,2\right]$$

$$y(x) \to \text{Root}\left[64\#1^5+240\#1^4x^2+300\#1^3x^4+\#1^2\left(125x^6-40e^{5c_1}x\right)-10\#1e^{5c_1}x^3 - 4e^{5c_1}x^5+e^{10c_1}\&,3\right]$$

$$y(x) \to \text{Root}\left[64\#1^5+240\#1^4x^2+300\#1^3x^4+\#1^2\left(125x^6-40e^{5c_1}x\right)-10\#1e^{5c_1}x^3 - 4e^{5c_1}x^5+e^{10c_1}\&,4\right]$$

$$y(x) \to \text{Root}\left[64\#1^5+240\#1^4x^2+300\#1^3x^4+\#1^2\left(125x^6-40e^{5c_1}x\right)-10\#1e^{5c_1}x^3 - 4e^{5c_1}x^5+e^{10c_1}\&,5\right]$$

$$y(x) \to 0$$

1.6 problem 7

Internal problem ID [14939]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 1. Basic concepts and definitions. Exercises page 18
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \sqrt{1 - y^2} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x)=sqrt(1-y(x)^2),y(x), singsol=all)
```

$$y(x) = \sin(c_1 + x)$$

✓ Solution by Mathematica
Time used: 0.216 (sec). Leaf size: 28

```
DSolve[y'[x]==Sqrt[1-y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to \cos(x + c_1) \\ y(x) &\to -1 \\ y(x) &\to 1 \\ y(x) &\to \text{Interval}[\{-1, 1\}] \end{aligned}$$

1.7 problem 8

Internal problem ID [14940]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 1. Basic concepts and definitions. Exercises page 18
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{y+1}{-y+x} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 34

```
dsolve(diff(y(x),x)=(y(x)+1)/(x-y(x)),y(x), singsol=all)
```

$$y(x) = \frac{-1 - x - \text{LambertW}\left(-(1+x)\,\mathrm{e}^{-c_1}\right)}{\text{LambertW}\left(-\,(1+x)\,\mathrm{e}^{-c_1}\right)}$$

✓ Solution by Mathematica
Time used: 0.132 (sec). Leaf size: 34

```
DSolve[y'[x]==(y[x]+1)/(x-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x = (y(x)+1)\left(-\frac{1}{y(x)+1} - \log(y(x)+1)\right) + c_1(y(x)+1), y(x)\right]$$

1.8 problem 9

Internal problem ID [14941]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISE-LYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 1. Basic concepts and definitions. Exercises page 18
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - \sin(y) = -\cos(x)}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=sin(y(x))-cos(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==Sin[y[x]]-Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.9 problem 10

Internal problem ID [14942]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 1. Basic concepts and definitions. Exercises page 18
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + \cot(y) = 1}$$

✓ Solution by Maple
Time used: 0.14 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)=1-cot(y(x)),y(x), singsol=all)
```

$$x + \frac{\ln\left(\csc(y(x))^2\right)}{4} + \frac{\pi}{4} - \frac{\ln(-1+\cot(y(x)))}{2} - \frac{y(x)}{2} + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.376 (sec). Leaf size: 69

```
DSolve[y'[x]==1-Cot[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\left(\frac{1}{4}+\frac{i}{4}\right)\log(-\tan(\#1)+i) - \frac{1}{2}\log(1-\tan(\#1)) + \left(\frac{1}{4}-\frac{i}{4}\right)\log(\tan(\#1)+i)\&\right][-x+c_1]$$

$$y(x) \to \frac{\pi}{4}$$

1.10 problem 11

Internal problem ID [14943]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 1. Basic concepts and definitions. Exercises page 18
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - (3x - y)^{\frac{1}{3}} = -1}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 86

```
dsolve(diff(y(x),x)=(3*x-y(x))^(1/3)-1,y(x), singsol=all)
```

$$x + \frac{3(3x - y(x))^{\frac{2}{3}}}{2} + 32\ln\left(-4 + (3x - y(x))^{\frac{1}{3}}\right) - 16\ln\left((3x - y(x))^{\frac{2}{3}} + 4(3x - y(x))^{\frac{1}{3}} + 16\right) + 16\ln(-64 + 3x - y(x)) + 12(3x - y(x))^{\frac{1}{3}} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.203 (sec). Leaf size: 55

```
DSolve[y'[x]==(3*x-y[x])^(1/3)-1,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{3}{2}(3x - y(x))^{2/3} + 12\sqrt[3]{3x - y(x)} + 48\log\left(\sqrt[3]{3x - y(x)} - 4\right) + x = c_1, y(x)\right]$$

1.11 problem 13

Internal problem ID [14944]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 1. Basic concepts and definitions. Exercises page 18
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - \sin(yx) = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=sin(x*y(x)),y(0) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 6

```
DSolve[{y'[x]==Sin[x*y[x]],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 0$$

1.12 problem 14

Internal problem ID [14945]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 1. Basic concepts and definitions. Exercises page 18
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + y = \cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x)+y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = \frac{\sin(x) + c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 14

```
DSolve[x*y'[x]+y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sin(x) + c_1}{x}$$

1.13 problem 15

Internal problem ID [14946]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 1. Basic concepts and definitions. Exercises page 18
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + 2y = e^x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)+2*y(x)=exp(x),y(x), singsol=all)
```

$$y(x) = \frac{(e^{3x} + 3c_1)\, e^{-2x}}{3}$$

✓ Solution by Mathematica
Time used: 0.049 (sec). Leaf size: 21

```
DSolve[y'[x]+2*y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x}{3} + c_1 e^{-2x}$$

1.14 problem 16

Internal problem ID [14947]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 1. Basic concepts and definitions. Exercises page 18
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{\left(-x^2+1\right) y'+yx=2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve((1-x^2)*diff(y(x),x)+x*y(x)=2*x,y(x), singsol=all)
```

$$y(x)=\sqrt{-1+x}\,\sqrt{1+x}\,c_1+2$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 24

```
DSolve[(1-x^2)*y'[x]+x*y[x]==2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to 2+c_1\sqrt{x^2-1}$$
$$y(x)\to 2$$

2 Section 2. The method of isoclines. Exercises page 27

2.1 problem 21

Internal problem ID [14948]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 2. The method of isoclines. Exercises page 27
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = x + 1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=x+1,y(x), singsol=all)
```

$$y(x) = \frac{1}{2}x^2 + x + c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 16

```
DSolve[y'[x]==x+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2} + x + c_1$$

2.2 problem 22

Internal problem ID [14949]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 2. The method of isoclines. Exercises page 27
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **`[[_linear, ‘class A‘]]`**

$$\boxed{y' - y = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)=x+y(x),y(x), singsol=all)
```

$$y(x) = -x - 1 + c_1 e^x$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 16

```
DSolve[y'[x]==x+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + c_1 e^x - 1$$

2.3 problem 23

Internal problem ID [14950]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 2. The method of isoclines. Exercises page 27
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - y = -x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)=y(x)-x,y(x), singsol=all)
```

$$y(x) = x + 1 + c_1 e^x$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 14

```
DSolve[y'[x]==y[x]-x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + c_1 e^x + 1$$

2.4 problem 24

Internal problem ID [14951]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 2. The method of isoclines. Exercises page 27
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + y = \frac{x}{2} + \frac{3}{2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=1/2*(x-2*y(x)+3),y(x), singsol=all)
```

$$y(x) = \frac{x}{2} + 1 + c_1 \mathrm{e}^{-x}$$

✓ Solution by Mathematica
Time used: 0.057 (sec). Leaf size: 20

```
DSolve[y'[x]==1/2*(x-2*y[x]+3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{2} + c_1 e^{-x} + 1$$

2.5 problem 25

Internal problem ID [14952]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 2. The method of isoclines. Exercises page 27
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - (y-1)^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)=(y(x)-1)^2,y(x), singsol=all)
```

$$y(x) = \frac{c_1 + x - 1}{c_1 + x}$$

✓ Solution by Mathematica
Time used: 0.107 (sec). Leaf size: 22

```
DSolve[y'[x]==(y[x]-1)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x - 1 + c_1}{x + c_1}$$
$$y(x) \to 1$$

2.6 problem 26

Internal problem ID [14953]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 2. The method of isoclines. Exercises page 27
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - (y - 1)\,x = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)=(y(x)-1)*x,y(x), singsol=all)
```

$$y(x) = 1 + c_1 \mathrm{e}^{\frac{x^2}{2}}$$

✓ Solution by Mathematica
Time used: 0.058 (sec). Leaf size: 24

```
DSolve[y'[x]==(y[x]-1)*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 + c_1 e^{\frac{x^2}{2}}$$
$$y(x) \to 1$$

2.7 problem 27

Internal problem ID [14954]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 2. The method of isoclines. Exercises page 27
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + y^2 = x^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 44

```
dsolve(diff(y(x),x)=x^2-y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x\left(\operatorname{BesselI}\left(-\frac{3}{4}, \frac{x^2}{2}\right) c_1 - \operatorname{BesselK}\left(\frac{3}{4}, \frac{x^2}{2}\right)\right)}{c_1 \operatorname{BesselI}\left(\frac{1}{4}, \frac{x^2}{2}\right) + \operatorname{BesselK}\left(\frac{1}{4}, \frac{x^2}{2}\right)}$$

✓ Solution by Mathematica
Time used: 0.114 (sec). Leaf size: 197

```
DSolve[y'[x]==x^2-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{-ix^2\left(2\operatorname{BesselJ}\left(-\frac{3}{4}, \frac{ix^2}{2}\right) + c_1\left(\operatorname{BesselJ}\left(-\frac{5}{4}, \frac{ix^2}{2}\right) - \operatorname{BesselJ}\left(\frac{3}{4}, \frac{ix^2}{2}\right)\right)\right) - c_1 \operatorname{BesselJ}\left(-\frac{1}{4}, \frac{ix^2}{2}\right)}{2x\left(\operatorname{BesselJ}\left(\frac{1}{4}, \frac{ix^2}{2}\right) + c_1 \operatorname{BesselJ}\left(-\frac{1}{4}, \frac{ix^2}{2}\right)\right)}$$

$$y(x) \to \frac{ix^2 \operatorname{BesselJ}\left(-\frac{5}{4}, \frac{ix^2}{2}\right) - ix^2 \operatorname{BesselJ}\left(\frac{3}{4}, \frac{ix^2}{2}\right) + \operatorname{BesselJ}\left(-\frac{1}{4}, \frac{ix^2}{2}\right)}{2x \operatorname{BesselJ}\left(-\frac{1}{4}, \frac{ix^2}{2}\right)}$$

2.8 problem 28

Internal problem ID [14955]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 2. The method of isoclines. Exercises page 27
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - \cos(-y + x) = 0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)=cos(x-y(x)),y(x), singsol=all)
```

$$y(x) = x - 2 \operatorname{arccot}(c_1 - x)$$

✓ Solution by Mathematica
Time used: 0.45 (sec). Leaf size: 40

```
DSolve[y'[x]==Cos[x-y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to x + 2\cot^{-1}\left(x - \frac{c_1}{2}\right) \\ y(x) &\to x + 2\cot^{-1}\left(x - \frac{c_1}{2}\right) \\ y(x) &\to x \end{aligned}$$

2.9 problem 29

Internal problem ID [14956]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 2. The method of isoclines. Exercises page 27
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - y = -x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=y(x)-x^2,y(x), singsol=all)
```

$$y(x) = x^2 + 2x + 2 + c_1 e^x$$

✓ Solution by Mathematica
Time used: 0.045 (sec). Leaf size: 19

```
DSolve[y'[x]==y[x]-x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + 2x + c_1 e^x + 2$$

2.10 problem 30

Internal problem ID [14957]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 2. The method of isoclines. Exercises page 27
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = x^2 + 2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)=x^2+2*x-y(x),y(x), singsol=all)
```

$$y(x) = x^2 + c_1 e^{-x}$$

✓ Solution by Mathematica
Time used: 0.048 (sec). Leaf size: 17

```
DSolve[y'[x]==x^2+2*x-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + c_1 e^{-x}$$

2.11 problem 31

Internal problem ID [14958]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 2. The method of isoclines. Exercises page 27
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{y+1}{x-1} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)=(y(x)+1)/(x-1),y(x), singsol=all)
```

$$y(x) = -1 + (-1 + x) c_1$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 18

```
DSolve[y'[x]==(y[x]+1)/(x-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -1 + c_1(x-1)$$
$$y(x) \to -1$$

2.12 problem 32

Internal problem ID [14959]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 2. The method of isoclines. Exercises page 27
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{y+x}{-y+x} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)=(x+y(x))/(x-y(x)),y(x), singsol=all)
```

$$y(x) = \tan\left(\operatorname{RootOf}\left(-2\_Z + \ln\left(\sec\left(\_Z\right)^2\right) + 2\ln\left(x\right) + 2c_1\right)\right)x$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 36

```
DSolve[y'[x]==(x+y[x])/(x-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right) - \arctan\left(\frac{y(x)}{x}\right) = -\log(x) + c_1, y(x)\right]$$

2.13 problem 33

Internal problem ID [14960]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 2. The method of isoclines. Exercises page 27
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = 1 - x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=1-x,y(x), singsol=all)
```

$$y(x) = -\frac{1}{2}x^2 + x + c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 16

```
DSolve[y'[x]==1-x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2}{2} + x + c_1$$

2.14 problem 34

Internal problem ID [14961]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 2. The method of isoclines. Exercises page 27
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = 2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=2*x-y(x),y(x), singsol=all)
```

$$y(x) = 2x - 2 + c_1 \mathrm{e}^{-x}$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 18

```
DSolve[y'[x]==2*x-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x + c_1 e^{-x} - 2$$

2.15 problem 35

Internal problem ID [14962]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 2. The method of isoclines. Exercises page 27
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - y = x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)=x^2+y(x),y(x), singsol=all)
```

$$y(x) = -x^2 - 2x - 2 + c_1 e^x$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 21

```
DSolve[y'[x]==x^2+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^2 - 2x + c_1 e^x - 2$$

2.16 problem 36

Internal problem ID [14963]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 2. The method of isoclines. Exercises page 27
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \frac{y}{x} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)=-y(x)/x,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 16

```
DSolve[y'[x]==-y[x]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x}$$
$$y(x) \to 0$$

2.17 problem 37

Internal problem ID [14964]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 2. The method of isoclines. Exercises page 27
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 7

```
dsolve(diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x) = c_1 + x$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 9

```
DSolve[y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + c_1$$

2.18 problem 38

Internal problem ID [14965]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 2. The method of isoclines. Exercises page 27
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{1}{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x)=1/x,y(x), singsol=all)
```

$$y(x) = \ln(x) + c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 10

```
DSolve[y'[x]==1/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(x) + c_1$$

2.19 problem 39

Internal problem ID [14966]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 2. The method of isoclines. Exercises page 27
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x)=y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^x$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 16

```
DSolve[y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x$$
$$y(x) \to 0$$

2.20 problem 40

Internal problem ID [14967]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 2. The method of isoclines. Exercises page 27
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)=y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{1}{c_1 - x}$$

✓ Solution by Mathematica
Time used: 0.088 (sec). Leaf size: 18

```
DSolve[y'[x]==y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{x + c_1}$$
$$y(x) \to 0$$

3 Section 3. The method of successive approximation. Exercises page 31

3.1 problem 41

Internal problem ID [14968]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 3. The method of successive approximation. Exercises page 31
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + y^2 = x^2}$$

With initial conditions

$$[y(-1) = 0]$$

✓ Solution by Maple
Time used: 0.141 (sec). Leaf size: 55

```
dsolve([diff(y(x),x)=x^2-y(x)^2,y(-1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{x\left(\text{BesselI}\left(-\frac{3}{4}, \frac{x^2}{2}\right)\text{BesselK}\left(\frac{3}{4}, \frac{1}{2}\right) - \text{BesselK}\left(\frac{3}{4}, \frac{x^2}{2}\right)\text{BesselI}\left(-\frac{3}{4}, \frac{1}{2}\right)\right)}{\text{BesselK}\left(\frac{1}{4}, \frac{x^2}{2}\right)\text{BesselI}\left(-\frac{3}{4}, \frac{1}{2}\right) + \text{BesselK}\left(\frac{3}{4}, \frac{1}{2}\right)\text{BesselI}\left(\frac{1}{4}, \frac{x^2}{2}\right)}$$

✓ Solution by Mathematica
Time used: 0.115 (sec). Leaf size: 211

```
DSolve[{y'[x]==x^2-y[x]^2,{y[-1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{i\left(x^2\left(-\text{BesselJ}\left(-\frac{5}{4}, \frac{i}{2}\right) + i\,\text{BesselJ}\left(-\frac{1}{4}, \frac{i}{2}\right) + \text{BesselJ}\left(\frac{3}{4}, \frac{i}{2}\right)\right)\text{BesselJ}\left(-\frac{3}{4}, \frac{ix^2}{2}\right) + x^2\,\text{BesselJ}\left(-\frac{3}{4}, \frac{i}{2}\right)\text{B}\right.}{x\left(2\,\text{BesselJ}\left(-\frac{3}{4}, \frac{i}{2}\right)\text{BesselJ}\left(-\frac{1}{4}, \frac{ix^2}{2}\right) + \left(-\text{BesselJ}\left(-\frac{5}{4}, \frac{i}{2}\right) + i\,\text{Be}\right.\right.}$$

3.2 problem 42

Internal problem ID [14969]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 3. The method of successive approximation. Exercises page 31
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{y' - y^2 = x}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 35

```
dsolve([diff(y(x),x)=x+y(x)^2,y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{3}\,\mathrm{AiryAi}(1, -x) + \mathrm{AiryBi}(1, -x)}{\sqrt{3}\,\mathrm{AiryAi}(-x) + \mathrm{AiryBi}(-x)}$$

✓ Solution by Mathematica
Time used: 1.269 (sec). Leaf size: 80

```
DSolve[{y'[x]==x+y[x]^2,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^{3/2}\,\mathrm{BesselJ}\left(-\frac{4}{3}, \frac{2x^{3/2}}{3}\right) - x^{3/2}\,\mathrm{BesselJ}\left(\frac{2}{3}, \frac{2x^{3/2}}{3}\right) + \mathrm{BesselJ}\left(-\frac{1}{3}, \frac{2x^{3/2}}{3}\right)}{2x\,\mathrm{BesselJ}\left(-\frac{1}{3}, \frac{2x^{3/2}}{3}\right)}$$

3.3 problem 43

Internal problem ID [14970]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 3. The method of successive approximation. Exercises page 31
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - y = x}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)=x+y(x),y(0) = 1],y(x), singsol=all)
```

$$y(x) = -x - 1 + 2\,\mathrm{e}^{x}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 15

```
DSolve[{y'[x]==x+y[x],{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + 2e^{x} - 1$$

3.4 problem 44

Internal problem ID [14971]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 3. The method of successive approximation. Exercises page 31
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - 2y = -2x^2 - 3}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve([diff(y(x),x)=2*y(x)-2*x^2-3,y(0) = 2],y(x), singsol=all)
```

$$y(x) = x^2 + x + 2$$

✓ Solution by Mathematica
Time used: 0.091 (sec). Leaf size: 11

```
DSolve[{y'[x]==2*y[x]-2*x^2-3,{y[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + x + 2$$

3.5 problem 45

Internal problem ID [14972]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 3. The method of successive approximation. Exercises page 31
Problem number: 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + y = 2x}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve([x*diff(y(x),x)=2*x-y(x),y(1) = 2],y(x), singsol=all)
```

$$y(x) = x + \frac{1}{x}$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 10

```
DSolve[{x*y'[x]==2*x-y[x],{y[1]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + \frac{1}{x}$$

4 Section 4. Equations with variables separable and equations reducible to them. Exercises page 38

4.1 problem 46

Internal problem ID [14973]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^2 + (x^2 + 1)\, y' = -1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve((1+y(x)^2)+(1+x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\tan(\arctan(x) + c_1)$$

✓ Solution by Mathematica
Time used: 0.234 (sec). Leaf size: 29

```
DSolve[(1+y[x]^2)+(1+x^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\tan(\arctan(x) - c_1)$$
$$y(x) \to -i$$
$$y(x) \to i$$

4.2 problem 47

Internal problem ID [14974]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^2 + xyy' = -1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
dsolve((1+y(x)^2)+(x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-x^2 + c_1}}{x}$$
$$y(x) = -\frac{\sqrt{-x^2 + c_1}}{x}$$

✓ Solution by Mathematica

Time used: 0.347 (sec). Leaf size: 96

```
DSolve[(1+y[x]^2)+(x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-x^2 + e^{2c_1}}}{x}$$
$$y(x) \to \frac{\sqrt{-x^2 + e^{2c_1}}}{x}$$
$$y(x) \to -i$$
$$y(x) \to i$$
$$y(x) \to \frac{x}{\sqrt{-x^2}}$$
$$y(x) \to \frac{\sqrt{-x^2}}{x}$$

4.3 problem 48

Internal problem ID [14975]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' \sin (x)-y \cos (x)=0}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right)=1\right]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 6

```
dsolve([diff(y(x),x)*sin(x)-y(x)*cos(x)=0,y(1/2*Pi) = 1],y(x), singsol=all)
```

$$y(x)=\sin (x)$$

✓ Solution by Mathematica
Time used: 0.044 (sec). Leaf size: 7

```
DSolve[{y'[x]*Sin[x]-y[x]*Cos[x]==0,{y[Pi/2]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \sin (x)$$

4.4 problem 49

Internal problem ID [14976]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^2 - y'x = -1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve((1+y(x)^2)=x*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \tan(\ln(x) + c_1)$$

✓ Solution by Mathematica
Time used: 0.167 (sec). Leaf size: 25

```
DSolve[(1+y[x]^2)==x*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan(\log(x) + c_1)$$
$$y(x) \to -i$$
$$y(x) \to i$$

4.5 problem 50

Internal problem ID [14977]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_separable]

$$\boxed{x\sqrt{1+y^2}+yy'\sqrt{x^2+1}=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x*sqrt(1+y(x)^2)+y(x)*diff(y(x),x)*sqrt(1+x^2)=0,y(x), singsol=all)
```

$$\sqrt{x^2+1}+\sqrt{1+y(x)^2}+c_1=0$$

✓ Solution by Mathematica
Time used: 0.29 (sec). Leaf size: 75

```
DSolve[x*Sqrt[1+y[x]^2]+y[x]*y'[x]*Sqrt[1+x^2]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\sqrt{x^2+c_1\left(-2\sqrt{x^2+1}+c_1\right)}$$
$$y(x)\to \sqrt{x^2+c_1\left(-2\sqrt{x^2+1}+c_1\right)}$$
$$y(x)\to -i$$
$$y(x)\to i$$

4.6 problem 51

Internal problem ID [14978]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 51.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$x\sqrt{1-y^2}+y\sqrt{-x^2+1}\,y'=0$$

With initial conditions

$$[y(0)=1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve([x*sqrt(1-y(x)^2)+y(x)*sqrt(1-x^2)*diff(y(x),x)=0,y(0) = 1],y(x), singsol=all)
```

$$y(x)=1$$

✓ Solution by Mathematica
Time used: 3.582 (sec). Leaf size: 32

```
DSolve[{x*Sqrt[1-y[x]^2]+y[x]*Sqrt[1-x^2]*y'[x]==0,{y[0]==1}},y[x],x,IncludeSingularSolution
```

$$y(x)\to 1$$
$$y(x)\to\sqrt{x^2+2\sqrt{1-x^2}-1}$$

4.7 problem 52

Internal problem ID [14979]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{e^{-y}y' = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(exp(-y(x))*diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x) = \ln\left(-\frac{1}{c_1 + x}\right)$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 16

```
DSolve[Exp[-y[x]]*y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\log(-x - c_1)$$

4.8 problem 53

Internal problem ID [14980]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 53.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\ln(y)\, y + y'x = 1}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple
Time used: 0.922 (sec). Leaf size: 38

```
dsolve([y(x)*ln(y(x))+x*diff(y(x),x)=1,y(1) = 1],y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(\int_{1}^{\_Z} \frac{1}{\ln(\_a)\_a - 1} d\_a + \ln(x)\right)$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y[x]*Log[y[x]]+x*y'[x]==1,{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

{}

4.9 problem 54

Internal problem ID [14981]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 54.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - a^{y+x} = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 22

```
dsolve(diff(y(x),x)=a^(x+y(x)),y(x), singsol=all)
```

$$y(x) = \frac{\ln\left(-\frac{1}{c_1 \ln(a)+a^x}\right)}{\ln(a)}$$

✓ Solution by Mathematica
Time used: 3.796 (sec). Leaf size: 24

```
DSolve[y'[x]==a^(x+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\log(-a^x - c_1 \log(a))}{\log(a)}$$

4.10 problem 55

Internal problem ID [14982]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 55.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{e^{y}\left(x^{2}+1\right)y'-2x(e^{y}+1)=0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 13

```
dsolve(exp(y(x))*(1+x^2)*diff(y(x),x)-2*x*(1+exp(y(x)))=0,y(x), singsol=all)
```

$$y(x)=\ln\left(c_{1}x^{2}+c_{1}-1\right)$$

✓ Solution by Mathematica
Time used: 0.638 (sec). Leaf size: 27

```
DSolve[Exp[y[x]]*(1+x^2)*y'[x]-2*x*(1+Exp[y[x]])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\log\left(-1+e^{c_{1}}\left(x^{2}+1\right)\right)$$
$$y(x)\to i\pi$$

4.11 problem 56

Internal problem ID [14983]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 56.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2x\sqrt{1-y^2}-(x^2+1)\,y'=0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 15

```
dsolve(2*x*sqrt(1-y(x)^2)=diff(y(x),x)*(1+x^2),y(x), singsol=all)
```

$$y(x)=\sin\left(\ln\left(x^2+1\right)+2c_1\right)$$

✓ Solution by Mathematica
Time used: 0.271 (sec). Leaf size: 33

```
DSolve[2*x*Sqrt[1-y[x]^2]==y'[x]*(1+x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}
y(x) &\to \cos\left(\log\left(x^2+1\right)+c_1\right)\\
y(x) &\to -1\\
y(x) &\to 1\\
y(x) &\to \text{Interval}[\{-1,1\}]
\end{aligned}$$

4.12 problem 57

Internal problem ID [14984]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 57.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{e^{x} \sin \left(y\right)^{3}+\left(e^{2 x}+1\right) \cos \left(y\right) y^{\prime}=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve(exp(x)*sin(y(x))^3+(1+exp(2*x))*cos(y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{c_1+\arctan(e^{x})}}}{2}\right)$$
$$y(x) = -\arctan\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{c_1+\arctan(e^{x})}}}{2}\right)$$

✓ Solution by Mathematica

Time used: 1.83 (sec). Leaf size: 56

```
DSolve[Exp[x]*Sin[y[x]]^3+(1+Exp[2*x])*Cos[y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\csc^{-1}\left(\sqrt{2}\sqrt{\arctan\left(e^{x}\right)-4c_1}\right)$$
$$y(x) \to \csc^{-1}\left(\sqrt{2}\sqrt{\arctan\left(e^{x}\right)-4c_1}\right)$$
$$y(x) \to 0$$

4.13 problem 58

Internal problem ID [14985]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 58.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sin (x) y^2 + \cos (x)^2 \ln (y) y' = 0}$$

✓ Solution by Maple
Time used: 0.36 (sec). Leaf size: 21

```
dsolve(y(x)^2*sin(x)+cos(x)^2*ln(y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\text{LambertW}\left(-(\sec (x) + c_1) \,\mathrm{e}^{-1}\right)}{\sec (x) + c_1}$$

✓ Solution by Mathematica
Time used: 60.174 (sec). Leaf size: 29

```
DSolve[y[x]^2*Sin[x]+Cos[x]^2*Log[y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\cos(x) W\left(\frac{-\sec(x)+c_1}{e}\right)}{-1 + c_1 \cos(x)}$$

4.14 problem 59

Internal problem ID [14986]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 59.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - \sin(-y + x) = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)=sin(x-y(x)),y(x), singsol=all)
```

$$y(x) = x - 2\arctan\left(\frac{2 - x + c_1}{c_1 - x}\right)$$

✓ Solution by Mathematica
Time used: 0.415 (sec). Leaf size: 64

```
DSolve[y'[x]==Sin[x-y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[y(x) - \sec(x - y(x))\left(2\sqrt{\cos^2(x - y(x))}\arcsin\left(\frac{\sqrt{1 - \sin(x - y(x))}}{\sqrt{2}}\right) + \sin(x - y(x)) + 1\right) = c_1, y(x)\right]$$

4.15 problem 60

Internal problem ID [14987]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 60.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - yb = ax + c}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x)=a*x+b*y(x)+c,y(x), singsol=all)
```

$$y(x) = \frac{e^{bx} c_1 b^2 + (-ax - c) b - a}{b^2}$$

✓ Solution by Mathematica
Time used: 0.105 (sec). Leaf size: 28

```
DSolve[y'[x]==a*x+b*y[x]+c,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{abx + a + bc}{b^2} + c_1 e^{bx}$$

4.16 problem 61

Internal problem ID [14988]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$\boxed{(y + x)^2 y' = a^2}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 24

```
dsolve((x+y(x))^2*diff(y(x),x)=a^2,y(x), singsol=all)
```

$$y(x) = a \operatorname{RootOf}(\tan(\_Z) a - \_Za + c_1 - x) - c_1$$

✓ Solution by Mathematica
Time used: 0.113 (sec). Leaf size: 21

```
DSolve[(x+y[x])^2*y'[x]==a^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[y(x) - a\arctan\left(\frac{y(x) + x}{a}\right) = c_1, y(x)\right]$$

4.17 problem 62

Internal problem ID [14989]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 62.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + y - a(yx + 1) = 0}$$

With initial conditions

$$\left[y\left(\frac{1}{a}\right) = -a\right]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve([y(x)+x*diff(y(x),x)=a*(1+x*y(x)),y(1/a) = -a],y(x), singsol=all)
```

$$y(x) = -\frac{1}{x}$$

✓ Solution by Mathematica
Time used: 0.048 (sec). Leaf size: 10

```
DSolve[{y[x]+x*y'[x]==a*(1+x*y[x]),{y[1/a]==-a}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{x}$$

4.18 problem 63

Internal problem ID [14990]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 63.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^2 + 2x\sqrt{ax - x^2}\, y' = -a^2}$$

With initial conditions

$$[y(a) = 0]$$

✓ Solution by Maple
Time used: 0.562 (sec). Leaf size: 22

```
dsolve([(a^2+y(x)^2)+2*x*sqrt(a*x-x^2)*diff(y(x),x)=0,y(a) = 0],y(x), singsol=all)
```

$$y(x) = \tan\left(\frac{a - x}{\sqrt{x\,(a - x)}}\right) a$$

✓ Solution by Mathematica
Time used: 31.916 (sec). Leaf size: 23

```
DSolve[{(a^2+y[x]^2)+2*x*Sqrt[a*x-x^2]*y'[x]==0,{y[a]==0}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to a \tan\left(\frac{\sqrt{x(a - x)}}{x}\right)$$

4.19 problem 81

Internal problem ID [14991]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 81.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{y}{x} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 7

```
dsolve([diff(y(x),x)=y(x)/x,y(0) = 0],y(x), singsol=all)
```

$$y(x) = c_1 x$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y'[x]==y[x]/x,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

4.20 problem 85

Internal problem ID [14992]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 85.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{\cos(y') = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(cos(diff(y(x),x))=0,y(x), singsol=all)
```

$$y(x) = \frac{\pi x}{2} + c_1$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 27

```
DSolve[Cos[y'[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\pi x}{2} + c_1$$
$$y(x) \to \frac{\pi x}{2} + c_1$$

4.21 problem 86

Internal problem ID [14993]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 86.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{e^{y'} = 1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 5

```
dsolve(exp(diff(y(x),x))=1,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 7

```
DSolve[Exp[y'[x]]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1$$

4.22 problem 87

Internal problem ID [14994]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 87.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{\sin(y') = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(sin(diff(y(x),x))=x,y(x), singsol=all)
```

$$y(x) = x \arcsin(x) + \sqrt{-x^2+1} + c_1$$

✓ Solution by Mathematica
Time used: 0.008 (sec). Leaf size: 23

```
DSolve[Sin[y'[x]]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \arcsin(x) + \sqrt{1-x^2} + c_1$$

4.23 problem 88

Internal problem ID [14995]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 88.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{\ln(y') = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 8

```
dsolve(ln(diff(y(x),x))=x,y(x), singsol=all)
```

$$y = e^{x} + c_1$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 11

```
DSolve[Log[y'[x]]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x} + c_1$$

4.24 problem 89

Internal problem ID [14996]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 89.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{\tan(y') = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve(tan(diff(y(x),x))=0,y(x), singsol=all)
```

$$y = c_1$$

✓ Solution by Mathematica
Time used: 0.109 (sec). Leaf size: 7

```
DSolve[Tan[y'[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1$$

4.25 problem 90

Internal problem ID [14997]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 90.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{e^{y'} = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(exp(diff(y(x),x))=x,y(x), singsol=all)
```

$$y = x\ln(x) - x + c_1$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 15

```
DSolve[Exp[y'[x]]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + x\log(x) + c_1$$

4.26 problem 91

Internal problem ID [14998]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 91.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{\tan(y') = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(tan(diff(y(x),x))=x,y(x), singsol=all)
```

$$y = x\arctan(x) - \frac{\ln(x^2+1)}{2} + c_1$$

✓ Solution by Mathematica
Time used: 0.069 (sec). Leaf size: 163

```
DSolve[Tan[y'[x]]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}
y(x) &\to -\frac{\sqrt{x^2}\log(x^2+1)}{2x} - x\cos^{-1}\left(-\frac{1}{\sqrt{x^2+1}}\right) + c_1 \\
y(x) &\to -\frac{\sqrt{x^2}\log(x^2+1)}{2x} + x\cos^{-1}\left(\frac{1}{\sqrt{x^2+1}}\right) + c_1 \\
y(x) &\to \frac{\sqrt{x^2}\log(x^2+1)}{2x} + x\cos^{-1}\left(-\frac{1}{\sqrt{x^2+1}}\right) + c_1 \\
y(x) &\to \frac{\sqrt{x^2}\log(x^2+1)}{2x} - x\cos^{-1}\left(\frac{1}{\sqrt{x^2+1}}\right) + c_1
\end{aligned}$$

4.27 problem 92

Internal problem ID [14999]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 92.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2y'\cos(y) = -1}$$

With initial conditions

$$\left[y(\infty) = \frac{16\pi}{3}\right]$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 21

```
dsolve([x^2*diff(y(x),x)*cos(y(x))+1=0,y(infinity) = 16/3*Pi],y(x), singsol=all)
```

$$y = \arcsin\left(\frac{\sqrt{3}\,x - 2}{2x}\right) + 5\pi$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x^2*y'[x]*Cos[y[x]]+1==0,{y[Infinity]==16/3*Pi}},y[x],x,IncludeSingularSolutions ->
```

{}

4.28 problem 93

Internal problem ID [15000]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 93.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2y' + \cos(2y) = 1}$$

With initial conditions

$$\left[y(\infty) = \frac{10\pi}{3}\right]$$

✓ Solution by Maple
Time used: 0.375 (sec). Leaf size: 23

```
dsolve([x^2*diff(y(x),x)+cos(2*y(x))=1,y(infinity) = 10/3*Pi],y(x), singsol=all)
```

$$y = \frac{7\pi}{2} - \arctan\left(\frac{\sqrt{3}\,x + 6}{3x}\right)$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x^2*y'[x]+Cos[2*y[x]]==1,{y[Infinity]==10/3*Pi}},y[x],x,IncludeSingularSolutions ->
```

{}

4.29 problem 94

Internal problem ID [15001]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 94.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' x^3 - \sin(y) = 1}$$

With initial conditions

$$[y(\infty) = 5\pi]$$

✗ Solution by Maple

```
dsolve([x^3*diff(y(x),x)-sin(y(x))=1,y(infinity) = 5*Pi],y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x^3*y'[x]-Sin[y[x]]==1,{y[Infinity]==5*Pi}},y[x],x,IncludeSingularSolutions -> True]
```

{}

4.30 problem 95

Internal problem ID [15002]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 95.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2+1)\,y' - \frac{\cos(2y)^2}{2} = 0}$$

With initial conditions

$$\left[y(-\infty) = \frac{7\pi}{2}\right]$$

✓ Solution by Maple
Time used: 0.437 (sec). Leaf size: 17

```
dsolve([(1+x^2)*diff(y(x),x)-1/2*cos(2*y(x))^2=0,y(-infinity) = 7/2*Pi],y(x), singsol=all)
```

$$y = \frac{\arctan\left(\arctan(x) + \frac{\pi}{2}\right)}{2} + \frac{7\pi}{2}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{(1+x^2)*y'[x]-1/2*Cos[2*y[x]]^2==0,{y[-Infinity]==7/2*Pi}},y[x],x,IncludeSingularSol
```

{}

4.31 problem 96

Internal problem ID [15003]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 96.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{e^{y} - e^{4y} y' = 1}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 33

```
dsolve(exp(y(x))=exp(4*y(x))*diff(y(x),x)+1,y(x), singsol=all)
```

$$x - \frac{e^{3y}}{3} - \frac{e^{2y}}{2} - e^{y} - \ln\left(e^{y} - 1\right) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.366 (sec). Leaf size: 48

```
DSolve[Exp[y[x]]==Exp[4*y[x]]*y'[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{1}{6}e^{\#1}\left(3e^{\#1} + 2e^{2\#1} + 6\right) + \log\left(e^{\#1} - 1\right) \&\right][x + c_1]$$
$$y(x) \to 0$$

4.32 problem 97

Internal problem ID [15004]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x+1)\,y'-y=-1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve((x+1)*diff(y(x),x)=y(x)-1,y(x), singsol=all)
```

$$y=c_1x+c_1+1$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 18

```
DSolve[(x+1)*y'[x]==y[x]-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to 1+c_1(x+1)$$
$$y(x)\to 1$$

4.33 problem 98

Internal problem ID [15005]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 98.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 2x(\pi + y) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)=2*x*(Pi+y(x)),y(x), singsol=all)
```

$$y = -\pi + c_1 e^{x^2}$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 24

```
DSolve[y'[x]==2*x*(Pi+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\pi + c_1 e^{x^2}$$
$$y(x) \to -\pi$$

4.34 problem 99

Internal problem ID [15006]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 4. Equations with variables separable and equations reducible to them. Exercises page 38
Problem number: 99.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2y' + \sin(2y) = 1}$$

With initial conditions

$$\left[y(\infty) = \frac{11\pi}{4}\right]$$

✓ Solution by Maple
Time used: 0.219 (sec). Leaf size: 20

```
dsolve([x^2*diff(y(x),x)+sin(2*y(x))=1,y(infinity) = 11/4*Pi],y(x), singsol=all)
```

$$y = -\arctan\left(\frac{x+2}{x-2}\right) + 3\pi$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x^2*y'[x]+Sin[2*y[x]]==1,{y[Infinity]==11/4*Pi}},y[x],x,IncludeSingularSolutions ->
```

$$\{\}$$

5 Section 5. Homogeneous equations. Exercises page 44

5.1 problem 100

Internal problem ID [15007]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 5. Homogeneous equations. Exercises page 44
Problem number: 100.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y'x - y - x\cos\left(\frac{y}{x}\right)^2 = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 11

```
dsolve(x*diff(y(x),x)=y(x)+x*cos(y(x)/x)^2,y(x), singsol=all)
```

$$y = \arctan(\ln(x) + c_1)\, x$$

✓ Solution by Mathematica
Time used: 0.434 (sec). Leaf size: 35

```
DSolve[x*y'[x]==y[x]+x*Cos[y[x]/x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\tan^{-1}(\log(x) + 2c_1)$$
$$y(x) \to -\frac{\pi x}{2}$$
$$y(x) \to \frac{\pi x}{2}$$

5.2 problem 101

Internal problem ID [15008]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 5. Homogeneous equations. Exercises page 44
Problem number: 101.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - y = -x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x-y(x))+x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = x(c_1 - \ln(x))$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 14

```
DSolve[(x-y[x])+x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(-\log(x) + c_1)$$

5.3 problem 102

Internal problem ID [15009]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 5. Homogeneous equations. Exercises page 44
Problem number: 102.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{y'x - y(\ln(y) - \ln(x)) = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x)=y(x)*( ln(y(x))-ln(x) ),y(x), singsol=all)
```

$$y = \mathrm{e}^{c_1 x+1} x$$

✓ Solution by Mathematica
Time used: 0.226 (sec). Leaf size: 24

```
DSolve[x*y'[x]==y[x]*( Log[y[x]]-Log[x] ),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to xe^{1+e^{c_1 x}}$$
$$y(x) \to ex$$

5.4 problem 103

Internal problem ID [15010]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 5. Homogeneous equations. Exercises page 44
Problem number: 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Riccati]`

$$\boxed{x^2y' - y^2 + yx = x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x)=y(x)^2-x*y(x)+x^2,y(x), singsol=all)
```

$$y = \frac{x(\ln(x) + c_1 - 1)}{\ln(x) + c_1}$$

✓ Solution by Mathematica
Time used: 0.15 (sec). Leaf size: 25

```
DSolve[x^2*y'[x]==y[x]^2-x*y[x]+x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x(\log(x) - 1 + c_1)}{\log(x) + c_1}$$
$$y(x) \to x$$

5.5 problem 104

Internal problem ID [15011]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 5. Homogeneous equations. Exercises page 44
Problem number: 104.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y'x - y - \sqrt{y^2 - x^2} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(x*diff(y(x),x)=y(x)+sqrt(y(x)^2-x^2),y(x), singsol=all)
```

$$\frac{-c_1x^2 + y + \sqrt{y^2 - x^2}}{x^2} = 0$$

✓ Solution by Mathematica
Time used: 0.409 (sec). Leaf size: 14

```
DSolve[x*y'[x]==y[x]+Sqrt[y[x]^2-x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\cosh(\log(x) + c_1)$$

5.6 problem 105

Internal problem ID [15012]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 5. Homogeneous equations. Exercises page 44
Problem number: 105.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Riccati]`

$$\boxed{2x^2y' - y^2 = x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(2*x^2*diff(y(x),x)=x^2+y(x)^2,y(x), singsol=all)
```

$$y = \frac{x(\ln(x) + c_1 - 2)}{\ln(x) + c_1}$$

✓ Solution by Mathematica
Time used: 0.122 (sec). Leaf size: 29

```
DSolve[2*x^2*y'[x]==x^2+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x(\log(x) - 2 + 2c_1)}{\log(x) + 2c_1}$$
$$y(x) \to x$$

5.7 problem 106

Internal problem ID [15013]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 5. Homogeneous equations. Exercises page 44
Problem number: 106.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{-3y + (2y - 3x)\, y' = -4x}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 51

```
dsolve((4*x-3*y(x))+(2*y(x)-3*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{3c_1x - \sqrt{c_1^2x^2 + 4}}{2c_1}$$
$$y = \frac{3c_1x + \sqrt{c_1^2x^2 + 4}}{2c_1}$$

✓ Solution by Mathematica
Time used: 0.388 (sec). Leaf size: 95

```
DSolve[(4*x-3*y[x])+(2*y[x]-3*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(3x - \sqrt{x^2 + 4e^{c_1}}\right)$$
$$y(x) \to \frac{1}{2}\left(3x + \sqrt{x^2 + 4e^{c_1}}\right)$$
$$y(x) \to \frac{1}{2}\left(3x - \sqrt{x^2}\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{x^2} + 3x\right)$$

5.8 problem 107

Internal problem ID [15014]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 5. Homogeneous equations. Exercises page 44
Problem number: 107.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{y + (y + x)\, y' = x}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 51

```
dsolve((y(x)-x)+(y(x)+x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{-c_1 x - \sqrt{2c_1^2 x^2 + 1}}{c_1}$$
$$y = \frac{-c_1 x + \sqrt{2c_1^2 x^2 + 1}}{c_1}$$

✓ Solution by Mathematica
Time used: 0.427 (sec). Leaf size: 94

```
DSolve[(y[x]-x)+(y[x]+x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x - \sqrt{2x^2 + e^{2c_1}}$$
$$y(x) \to -x + \sqrt{2x^2 + e^{2c_1}}$$
$$y(x) \to -\sqrt{2}\sqrt{x^2} - x$$
$$y(x) \to \sqrt{2}\sqrt{x^2} - x$$

5.9 problem 108

Internal problem ID [15015]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 5. Homogeneous equations. Exercises page 44
Problem number: 108.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y + (1 - x)\, y' = -x + 2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x+y(x)-2+(1-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = (-1 + x) \ln (-1 + x) + 1 + c_1(-1 + x)$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 21

```
DSolve[x+y[x]-2+(1-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x - 1) \left(\frac{1}{x - 1} + \log(x - 1) + c_1\right)$$

5.10 problem 109

Internal problem ID [15016]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 5. Homogeneous equations. Exercises page 44
Problem number: 109.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$3y - (3x - 7y - 3)\, y' = 7x - 7$$

✓ Solution by Maple
Time used: 0.203 (sec). Leaf size: 1814

```
dsolve((3*y(x)-7*x+7)-(3*x-7*y(x)-3)*diff(y(x),x)=0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica
Time used: 60.866 (sec). Leaf size: 7785

```
DSolve[(3*y[x]-7*x+7)-(3*x-7*y[x]-3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

5.11 problem 110

Internal problem ID [15017]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 5. Homogeneous equations. Exercises page 44
Problem number: 110.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{y + (x - y + 4)\, y' = -x + 2}$$

✓ Solution by Maple
Time used: 0.11 (sec). Leaf size: 30

```
dsolve((x+y(x)-2)+(x-y(x)+4)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{-\sqrt{2\,(x+1)^2\, c_1^2 + 1} + (x+4)\, c_1}{c_1}$$

✓ Solution by Mathematica
Time used: 0.124 (sec). Leaf size: 59

```
DSolve[(x+y[x]-2)+(x-y[x]+4)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -i\sqrt{-2x^2 - 4x - 16 - c_1} + x + 4$$
$$y(x) \to i\sqrt{-2x^2 - 4x - 16 - c_1} + x + 4$$

5.12 problem 111

Internal problem ID [15018]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 5. Homogeneous equations. Exercises page 44
Problem number: 111.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd ty`

$$y + (x - y - 2)\, y' = -x$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 30

```
dsolve((x+y(x))+(x-y(x)-2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{-\sqrt{2\,(-1+x)^2\, c_1^2 + 1} + (x-2)\, c_1}{c_1}$$

✓ Solution by Mathematica
Time used: 0.117 (sec). Leaf size: 59

```
DSolve[(x+y[x])+(x-y[x]-2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -i\sqrt{-2x^2 + 4x - 4 - c_1} + x - 2$$
$$y(x) \to i\sqrt{-2x^2 + 4x - 4 - c_1} + x - 2$$

5.13 problem 112

Internal problem ID [15019]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 5. Homogeneous equations. Exercises page 44
Problem number: 112.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{3y + (3x + 2y - 5)\, y' = -2x + 5}$$

✓ Solution by Maple
Time used: 0.109 (sec). Leaf size: 33

```
dsolve((2*x+3*y(x)-5)+(3*x+2*y(x)-5)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{-\sqrt{5\,(-1+x)^2\, c_1^2 + 4} + (-3x+5)\, c_1}{2c_1}$$

✓ Solution by Mathematica
Time used: 0.129 (sec). Leaf size: 65

```
DSolve[(2*x+3*y[x]-5)+(3*x+2*y[x]-5)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-\sqrt{5x^2 - 10x + 25 + 4c_1} - 3x + 5\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{5x^2 - 10x + 25 + 4c_1} - 3x + 5\right)$$

5.14 problem 113

Internal problem ID [15020]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 5. Homogeneous equations. Exercises page 44
Problem number: 113.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd ty`

$$4y + (4x + 2y + 1)\, y' = -8x - 1$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 41

```
dsolve((8*x+4*y(x)+1)+(4*x+2*y(x)+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = -2x - \frac{1}{2} - \frac{\sqrt{-4c_1 + 4x + 1}}{2}$$
$$y = -2x - \frac{1}{2} + \frac{\sqrt{-4c_1 + 4x + 1}}{2}$$

✓ Solution by Mathematica
Time used: 0.105 (sec). Leaf size: 55

```
DSolve[(8*x+4*y[x]+1)+(4*x+2*y[x]+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-4x - \sqrt{4x + 1 + 4c_1} - 1\right)$$
$$y(x) \to \frac{1}{2}\left(-4x + \sqrt{4x + 1 + 4c_1} - 1\right)$$

5.15 problem 114

Internal problem ID [15021]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 5. Homogeneous equations. Exercises page 44
Problem number: 114.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$-2y + (3x - 6y + 2)\, y' = 1 - x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve((x-2*y(x)-1)+(3*x-6*y(x)+2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = -\frac{\text{LambertW}\left(-3\,\mathrm{e}^{\frac{5x}{2}-\frac{5c_1}{2}}\right)}{3} + \frac{x}{2}$$

✓ Solution by Mathematica

Time used: 4.144 (sec). Leaf size: 38

```
DSolve[(x-2*y[x]-1)+(3*x-6*y[x]+2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}\left(3x - 2W\left(-e^{\frac{5x}{2}-1+c_1}\right)\right)$$
$$y(x) \to \frac{x}{2}$$

5.16 problem 115

Internal problem ID [15022]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 5. Homogeneous equations. Exercises page 44
Problem number: 115.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{y + (y - 1 + x)\, y' = -x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 39

```
dsolve((x+y(x))+(x+y(x)-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = 1 - x - \sqrt{2c_1 - 2x + 1}$$
$$y = 1 - x + \sqrt{2c_1 - 2x + 1}$$

✓ Solution by Mathematica
Time used: 0.123 (sec). Leaf size: 43

```
DSolve[(x+y[x])+(x+y[x]-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x - \sqrt{-2x + 1 + c_1} + 1$$
$$y(x) \to -x + \sqrt{-2x + 1 + c_1} + 1$$

5.17 problem 116

Internal problem ID [15023]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 5. Homogeneous equations. Exercises page 44
Problem number: 116.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$\boxed{2xy'(-y^2+x)+y^3=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 28

```
dsolve(2*x*diff(y(x),x)*(x-y(x)^2)+y(x)^3=0,y(x), singsol=all)
```

$$y(x)=\frac{e^{\frac{c_1}{2}}}{\sqrt{-\frac{e^{c_1}}{x\,\mathrm{LambertW}\left(-\frac{e^{c_1}}{x}\right)}}}$$

✓ Solution by Mathematica
Time used: 6.48 (sec). Leaf size: 60

```
DSolve[2*x*y'[x]*(x-y[x]^2)+y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -i\sqrt{x}\sqrt{W\left(-\frac{e^{c_1}}{x}\right)}$$
$$y(x)\to i\sqrt{x}\sqrt{W\left(-\frac{e^{c_1}}{x}\right)}$$
$$y(x)\to 0$$

5.18 problem 117

Internal problem ID [15024]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 5. Homogeneous equations. Exercises page 44
Problem number: 117.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$\boxed{4y^6 - 6y^5xy' = -x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 127

```
dsolve(4*y(x)^6+x^3=6*x*y(x)^5*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \left(x^3(c_1x-1)\right)^{\frac{1}{6}}$$
$$y(x) = -\left(x^3(c_1x-1)\right)^{\frac{1}{6}}$$
$$y(x) = -\frac{\left(1+i\sqrt{3}\right)\left(x^3(c_1x-1)\right)^{\frac{1}{6}}}{2}$$
$$y(x) = \frac{\left(i\sqrt{3}-1\right)\left(x^3(c_1x-1)\right)^{\frac{1}{6}}}{2}$$
$$y(x) = -\frac{\left(i\sqrt{3}-1\right)\left(x^3(c_1x-1)\right)^{\frac{1}{6}}}{2}$$
$$y(x) = \frac{\left(1+i\sqrt{3}\right)\left(x^3(c_1x-1)\right)^{\frac{1}{6}}}{2}$$

✓ Solution by Mathematica

Time used: 0.276 (sec). Leaf size: 144

```
DSolve[4*y[x]^6+x^3==6*x*y[x]^5*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) &\to -\sqrt{x}\sqrt[6]{-1+c_1x} \\
y(x) &\to \sqrt{x}\sqrt[6]{-1+c_1x} \\
y(x) &\to -\sqrt[3]{-1}\sqrt{x}\sqrt[6]{-1+c_1x} \\
y(x) &\to \sqrt[3]{-1}\sqrt{x}\sqrt[6]{-1+c_1x} \\
y(x) &\to -(-1)^{2/3}\sqrt{x}\sqrt[6]{-1+c_1x} \\
y(x) &\to (-1)^{2/3}\sqrt{x}\sqrt[6]{-1+c_1x}
\end{aligned}
$$

5.19 problem 118

Internal problem ID [15025]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 5. Homogeneous equations. Exercises page 44
Problem number: 118.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G']]`

$$
\boxed{y\left(1+\sqrt{y^4x^2+1}\right)+2y'x=0}
$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(y(x)*(1+sqrt(x^2*y(x)^4+1))+2*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$
y(x)=\frac{\operatorname{RootOf}\left(-\ln(x)+c_1-2\left(\int^{\_Z}\frac{1}{\_a\sqrt{\_a^4+1}}d\_a\right)\right)}{\sqrt{x}}
$$

✓ Solution by Mathematica
Time used: 0.624 (sec). Leaf size: 80

```
DSolve[y[x]*(1+Sqrt[x^2*y[x]^4+1])+2*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x)&\to -\frac{i\sqrt{2}e^{\frac{c_1}{2}}}{\sqrt{-x^2+e^{2c_1}}}\\
y(x)&\to \frac{i\sqrt{2}e^{\frac{c_1}{2}}}{\sqrt{-x^2+e^{2c_1}}}\\
y(x)&\to 0
\end{aligned}
$$

5.20 problem 119

Internal problem ID [15026]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 5. Homogeneous equations. Exercises page 44
Problem number: 119.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{y^3 + 3(y^3 - x)\, y^2 y' = -x}$$

✓ Solution by Maple
Time used: 0.046 (sec). Leaf size: 35

```
dsolve((x+y(x)^3)+3*(y(x)^3-x)*y(x)^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$\ln(x) - c_1 + \frac{\ln\left(\frac{y(x)^6+x^2}{x^2}\right)}{2} - \arctan\left(\frac{y(x)^3}{x}\right) = 0$$

✓ Solution by Mathematica
Time used: 0.154 (sec). Leaf size: 27

```
DSolve[(x+y[x]^3)+3*(y[x]^3-x)*y[x]^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\arctan\left(\frac{x}{y(x)^3}\right) + \frac{1}{2}\log\left(x^2 + y(x)^6\right) = c_1, y(x)\right]$$

6 Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54

6.1 problem 125

Internal problem ID [15027]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 125.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + 2y = \mathrm{e}^{-x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)+2*y(x)=exp(-x),y(x), singsol=all)
```

$$y(x) = (\mathrm{e}^{x} + c_1)\,\mathrm{e}^{-2x}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 17

```
DSolve[y'[x]+2*y[x]==Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}(e^{x} + c_1)$$

6.2 problem 126

Internal problem ID [15028]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 126.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{-y'x - y = -x^2}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve([x^2-x*diff(y(x),x)=y(x),y(1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{x^3 - 1}{3x}$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 17

```
DSolve[{x^2-x*y'[x]==y[x],{y[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3 - 1}{3x}$$

6.3 problem 127

Internal problem ID [15029]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 127.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - 2yx = 2x\,\mathrm{e}^{x^2}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)-2*x*y(x)=2*x*exp(x^2),y(x), singsol=all)
```

$$y(x) = \left(x^2 + c_1\right)\mathrm{e}^{x^2}$$

✓ Solution by Mathematica
Time used: 0.049 (sec). Leaf size: 17

```
DSolve[y'[x]-2*x*y[x]==2*x*Exp[x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{x^2}\left(x^2 + c_1\right)$$

6.4 problem 128

Internal problem ID [15030]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 128.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + 2yx = \mathrm{e}^{-x^2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+2*x*y(x)=exp(-x^2),y(x), singsol=all)
```

$$y(x) = (x + c_1)\,\mathrm{e}^{-x^2}$$

✓ Solution by Mathematica
Time used: 0.055 (sec). Leaf size: 17

```
DSolve[y'[x]+2*x*y[x]==Exp[-x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x^2}(x + c_1)$$

6.5 problem 129

Internal problem ID [15031]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 129.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\cos(x)\, y' - \sin(x)\, y = 2x}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve([diff(y(x),x)*cos(x)-y(x)*sin(x)=2*x,y(0) = 0],y(x), singsol=all)
```

$$y(x) = x^2 \sec(x)$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 11

```
DSolve[{y'[x]*Cos[x]-y[x]*Sin[x]==2*x,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 \sec(x)$$

6.6 problem 130

Internal problem ID [15032]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 130.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - 2y = \cos(x)\, x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x)-2*y(x)=x^3*cos(x),y(x), singsol=all)
```

$$y(x) = (\sin(x) + c_1)\, x^2$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 14

```
DSolve[x*y'[x]-2*y[x]==x^3*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(\sin(x) + c_1)$$

6.7 problem 131

Internal problem ID [15033]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 131.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \tan(x) y = \frac{1}{\cos(x)^3}}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve([diff(y(x),x)-y(x)*tan(x)=1/cos(x)^3,y(0) = 0],y(x), singsol=all)
```

$$y(x) = \sec(x) \tan(x)$$

✓ Solution by Mathematica
Time used: 0.055 (sec). Leaf size: 10

```
DSolve[{y'[x]-y[x]*Tan[x]==1/Cos[x]^3,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan(x) \sec(x)$$

6.8 problem 132

Internal problem ID [15034]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 132.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x \ln(x) y' - y = 3x^3 \ln(x)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)*x*ln(x)-y(x)=3*x^3*(ln(x))^2,y(x), singsol=all)
```

$$y(x) = (x^3 + c_1) \ln(x)$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 14

```
DSolve[y'[x]*x*Log[x]-y[x]==3*x^3*(Log[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x^3 + c_1) \log(x)$$

6.9 problem 133

Internal problem ID [15035]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 133.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{(-y^2 + 2x)\, y' - 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve((2*x-y(x)^2)*diff(y(x),x)=2*y(x),y(x), singsol=all)
```

$$y(x) = c_1 - \sqrt{c_1^2 - 2x}$$
$$y(x) = c_1 + \sqrt{c_1^2 - 2x}$$

✓ Solution by Mathematica
Time used: 0.253 (sec). Leaf size: 46

```
DSolve[(2*x-y[x]^2)*y'[x]==2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 - \sqrt{-2x + c_1{}^2}$$
$$y(x) \to \sqrt{-2x + c_1{}^2} + c_1$$
$$y(x) \to 0$$

6.10 problem 134

Internal problem ID [15036]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 134.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + y\cos(x) = \cos(x)}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)+y(x)*cos(x)=cos(x),y(0) = 1],y(x), singsol=all)
```

$$y(x) = 1$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 6

```
DSolve[{y'[x]+y[x]*Cos[x]==Cos[x],{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1$$

6.11 problem 135

Internal problem ID [15037]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 135.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y' - \frac{y}{2\ln(y)\,y + y - x} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)=y(x)/(2*y(x)*ln(y(x))+y(x)-x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{RootOf}\left(\_Z\,\mathrm{e}^{2\_Z} - x\,\mathrm{e}^{\_Z} + c_1\right)}$$

✓ Solution by Mathematica
Time used: 0.141 (sec). Leaf size: 19

```
DSolve[y'[x]==y[x]/(2*y[x]*Log[y[x]]+y[x]-x),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x = y(x)\log(y(x)) + \frac{c_1}{y(x)}, y(x)\right]$$

6.12 problem 136

Internal problem ID [15038]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 136.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, `_with_symmetry_[F(x)*G(y),0]`]]`

$$\boxed{\left(\frac{e^{-y^2}}{2} - yx\right) y' = 1}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 34

```
dsolve((exp(-(y(x)^2))/2-x*y(x))*diff(y(x),x)-1=0,y(x), singsol=all)
```

$$\frac{\left(-\sqrt{\pi}\,\sqrt{2}\,\operatorname{erf}\left(\frac{\sqrt{2}\,y(x)}{2}\right) - 4c_1\right) e^{-\frac{y(x)^2}{2}}}{4} + x = 0$$

✓ Solution by Mathematica

Time used: 0.211 (sec). Leaf size: 32

```
DSolve[(Exp[-(y[x]^2)/2]-x*y[x])*y'[x]-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x = e^{-\frac{1}{2}y(x)^2}y(x) + c_1 e^{-\frac{1}{2}y(x)^2}, y(x)\right]$$

6.13 problem 137

Internal problem ID [15039]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 137.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - y\,e^{x} = 2x\,e^{e^{x}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)-y(x)*exp(x)=2*x*exp( exp(x) ),y(x), singsol=all)
```

$$y(x) = (x^2 + c_1)\,e^{e^{x}}$$

✓ Solution by Mathematica
Time used: 0.053 (sec). Leaf size: 17

```
DSolve[y'[x]-y[x]*Exp[x]==2*x*Exp[ Exp[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{e^{x}}(x^2 + c_1)$$

6.14 problem 138

Internal problem ID [15040]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 138.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + yx\,e^{x} = e^{(1-x)e^{x}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)+x*y(x)*exp(x)=exp( (1-x)*exp(x) ),y(x), singsol=all)
```

$$y(x) = (x + c_1)\,e^{-(-1+x)e^{x}}$$

✓ Solution by Mathematica
Time used: 0.109 (sec). Leaf size: 20

```
DSolve[y'[x]+x*y[x]*Exp[x]==Exp[ (1-x)*Exp[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-e^{x}(x-1)}(x + c_1)$$

6.15 problem 148

Internal problem ID [15041]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 148.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - y \ln (2) = 2^{\sin(x)} (\cos (x) - 1) \ln (2)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)-y(x)*ln(2)=2^(sin(x))*(cos(x)-1)*ln(2),y(x), singsol=all)
```

$$y(x) = 2^x c_1 + 2^{\sin(x)}$$

✓ Solution by Mathematica
Time used: 0.368 (sec). Leaf size: 16

```
DSolve[y'[x]-y[x]*Log[2]==2^(Sin[x])*(Cos[x]-1)*Log[2],y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to 2^{\sin(x)} + c_1 2^x$$

6.16 problem 149

Internal problem ID [15042]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 149.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - y = -2\,\mathrm{e}^{-x}}$$

With initial conditions

$$[y(\infty) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 8

```
dsolve([diff(y(x),x)-y(x)=-2*exp(-x),y(infinity) = 0],y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.068 (sec). Leaf size: 10

```
DSolve[{y'[x]-y[x]==-2*Exp[-x],{y[Infinity]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}$$

6.17 problem 150

Internal problem ID [15043]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 150.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' \sin(x) - y \cos(x) = -\frac{\sin(x)^2}{x^2}$$

With initial conditions

$$[y(\infty) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 10

```
dsolve([diff(y(x),x)*sin(x)-y(x)*cos(x)=-sin(x)^2/x^2,y(infinity) = 0],y(x), singsol=all)
```

$$y(x) = \frac{\sin(x)}{x}$$

✓ Solution by Mathematica
Time used: 0.075 (sec). Leaf size: 19

```
DSolve[{y'[x]*Sin[x]-y[x]*Cos[x]==-Sin[x]^2/x^2,{y[Infinity]==0}},y[x],x,IncludeSingularSolu
```

$$y(x) \to \sin(x) \left(\text{Interval}[\{0, \text{Indeterminate}\}, \{\text{Indeterminate}, 0\}] + \frac{1}{x} \right)$$

6.18 problem 151

Internal problem ID [15044]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISE-LYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 151.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$x^2 y' \cos\left(\frac{1}{x}\right) - y \sin\left(\frac{1}{x}\right) = -1$$

With initial conditions

$$[y(\infty) = 1]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 13

```
dsolve([x^2*diff(y(x),x)*cos(1/x)-y(x)*sin(1/x)=-1,y(infinity) = 1],y(x), singsol=all)
```

$$y(x) = \sin\left(\frac{1}{x}\right) + \cos\left(\frac{1}{x}\right)$$

✓ Solution by Mathematica
Time used: 0.113 (sec). Leaf size: 14

```
DSolve[{x^2*y'[x]*Cos[1/x]-y[x]*Sin[1/x]==-1,{y[Infinity]==1}},y[x],x,IncludeSingularSolutio
```

$$y(x) \to \sin\left(\frac{1}{x}\right) + \cos\left(\frac{1}{x}\right)$$

6.19 problem 152

Internal problem ID [15045]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 152.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2y'x - y = 1 - \frac{2}{\sqrt{x}}}$$

With initial conditions

$$[y(\infty) = -1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve([2*x*diff(y(x),x)-y(x)=1-2/sqrt(x),y(infinity) = -1],y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{x}-1}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.061 (sec). Leaf size: 12

```
DSolve[{2*x*y'[x]-y[x]==1-2/Sqrt[x],{y[Infinity]==-1}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{1}{\sqrt{x}} - 1$$

6.20 problem 153

Internal problem ID [15046]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISE-LYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 153.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2y'x + y = (x^2 + 1)\,\mathrm{e}^x}$$

With initial conditions

$$[y(-\infty) = 1]$$

✓ Solution by Maple
Time used: 0.188 (sec). Leaf size: 11

```
dsolve([2*x*diff(y(x),x)+y(x)=(x^2+1)*exp(x),y(-infinity) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\infty i}{\sqrt{\operatorname{signum}(x)}}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{2*x*y'[x]+y[x]==(x^2+1)*Exp[x],{y[-Infinity]==1}},y[x],x,IncludeSingularSolutions ->
```

{}

6.21 problem 154

Internal problem ID [15047]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 154.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + y = 2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x*diff(y(x),x)+y(x)=2*x,y(x), singsol=all)
```

$$y(x) = x + \frac{c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 13

```
DSolve[x*y'[x]+y[x]==2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + \frac{c_1}{x}$$

6.22 problem 155

Internal problem ID [15048]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 155.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' \sin(x) + y \cos(x) = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(sin(x)*diff(y(x),x)+y(x)*cos(x)=1,y(x), singsol=all)
```

$$y(x) = (x + c_1) \csc(x)$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 12

```
DSolve[Sin[x]*y'[x]+y[x]*Cos[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_1) \csc(x)$$

6.23 problem 156

Internal problem ID [15049]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 156.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\cos(x)\, y' - \sin(x)\, y = -\sin(2x)}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = 0\right]$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 6

```
dsolve([cos(x)*diff(y(x),x)-y(x)*sin(x)=-sin(2*x),y(1/2*Pi) = 0],y(x), singsol=all)
```

$$y(x) = \cos(x)$$

✓ Solution by Mathematica
Time used: 0.133 (sec). Leaf size: 7

```
DSolve[{Cos[x]*y'[x]-y[x]*Sin[x]==-Sin[2*x],{y[Pi/2]==0}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \cos(x)$$

6.24 problem 157

Internal problem ID [15050]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 157.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + 2yx - 2y^2x = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+2*x*y(x)=2*x*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{1}{1 + e^{x^2} c_1}$$

✓ Solution by Mathematica
Time used: 0.204 (sec). Leaf size: 27

```
DSolve[y'[x]+2*x*y[x]==2*x*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{1 + e^{x^2 + c_1}}$$
$$y(x) \to 0$$
$$y(x) \to 1$$

6.25 problem 158

Internal problem ID [15051]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 158.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{3y^2xy' - 2y^3 = x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 57

```
dsolve(3*x*y(x)^2*diff(y(x),x)-2*y(x)^3=x^3,y(x), singsol=all)
```

$$y(x) = \left((x + c_1)\, x^2\right)^{\frac{1}{3}}$$
$$y(x) = -\frac{\left((x + c_1)\, x^2\right)^{\frac{1}{3}}\left(1 + i\sqrt{3}\right)}{2}$$
$$y(x) = \frac{\left((x + c_1)\, x^2\right)^{\frac{1}{3}}\left(i\sqrt{3} - 1\right)}{2}$$

✓ Solution by Mathematica
Time used: 0.18 (sec). Leaf size: 66

```
DSolve[3*x*y[x]^2*y'[x]-2*y[x]^3==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^{2/3}\sqrt[3]{x + c_1}$$
$$y(x) \to -\sqrt[3]{-1}x^{2/3}\sqrt[3]{x + c_1}$$
$$y(x) \to (-1)^{2/3}x^{2/3}\sqrt[3]{x + c_1}$$

6.26 problem 159

Internal problem ID [15052]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 159.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{(x^3 + e^y)\, y' = 3x^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve((x^3+exp(y(x)))*diff(y(x),x)=3*x^2,y(x), singsol=all)
```

$$y(x) = \ln\left(\frac{x^3}{\text{LambertW}\left(\frac{x^3}{c_1}\right)}\right)$$

✓ Solution by Mathematica
Time used: 3.536 (sec). Leaf size: 19

```
DSolve[(x^3+Exp[y[x]])*y'[x]==3*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to W\left(e^{-c_1}x^3\right) + c_1$$

6.27 problem 160

Internal problem ID [15053]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 160.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + 3yx - y\,\mathrm{e}^{x^2} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)+3*x*y(x)=y(x)*exp(x^2),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{3x^2}{2}+\frac{\sqrt{\pi}\,\mathrm{erfi}(x)}{2}}$$

✓ Solution by Mathematica
Time used: 0.075 (sec). Leaf size: 33

```
DSolve[y'[x]+3*x*y[x]==y[x]*Exp[x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{\frac{1}{2}\left(\sqrt{\pi}\mathrm{erfi}(x)-3x^2\right)}$$
$$y(x) \to 0$$

6.28 problem 161

Internal problem ID [15054]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 161.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y' - 2y\,\mathrm{e}^{x} - 2\sqrt{y\,\mathrm{e}^{x}} = 0}$$

✓ Solution by Maple
Time used: 0.156 (sec). Leaf size: 53

```
dsolve(diff(y(x),x)-2*y(x)*exp(x)=2*sqrt(y(x)*exp(x)),y(x), singsol=all)
```

$$\frac{y(x)\,\mathrm{e}^{\frac{x}{2}-\mathrm{e}^{x}} - \left(\int \mathrm{e}^{\frac{x}{2}-\mathrm{e}^{x}}dx\right)\sqrt{y\left(x\right)\mathrm{e}^{x}} + c_1\sqrt{y\left(x\right)\mathrm{e}^{x}}}{\sqrt{y\left(x\right)\mathrm{e}^{x}}} = 0$$

✓ Solution by Mathematica
Time used: 0.215 (sec). Leaf size: 56

```
DSolve[y'[x]-2*y[x]*Exp[x]==2*Sqrt[y[x]*Exp[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2\left(\sqrt{\pi}\sqrt{y(x)}\text{erf}\left(\frac{\sqrt{e^{x}y(x)}}{\sqrt{y(x)}}\right) - e^{-e^{x}}y(x)\right)}{\sqrt{y(x)}} = c_1, y(x)\right]$$

6.29 problem 162

Internal problem ID [15055]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 162.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{2y' \ln(x) + \frac{y}{x} - \frac{\cos(x)}{y} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 36

```
dsolve(2*diff(y(x),x)*ln(x)+y(x)/x=cos(x)/y(x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\ln(x)(\sin(x) + c_1)}}{\ln(x)}$$
$$y(x) = -\frac{\sqrt{\ln(x)(\sin(x) + c_1)}}{\ln(x)}$$

✓ Solution by Mathematica
Time used: 0.311 (sec). Leaf size: 42

```
DSolve[2*y'[x]*Log[x]+y[x]/x==Cos[x]/y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\sin(x) + c_1}}{\sqrt{\log(x)}}$$
$$y(x) \to \frac{\sqrt{\sin(x) + c_1}}{\sqrt{\log(x)}}$$

6.30 problem 163

Internal problem ID [15056]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 163.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{2y' \sin(x) + y \cos(x) - \sin(x)^2 y^3 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(2*diff(y(x),x)*sin(x)+y(x)*cos(x)=y(x)^3*sin(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{(-x+c_1)\sin(x)}}$$
$$y(x) = -\frac{1}{\sqrt{(-x+c_1)\sin(x)}}$$

✓ Solution by Mathematica
Time used: 0.516 (sec). Leaf size: 43

```
DSolve[2*y'[x]*Sin[x]+y[x]*Cos[x]==y[x]^3*Sin[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{(-x+c_1)\sin(x)}}$$
$$y(x) \to \frac{1}{\sqrt{-((x-c_1)\sin(x))}}$$
$$y(x) \to 0$$

6.31 problem 164

Internal problem ID [15057]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 164.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{(1 + x^2 + y^2)\, y' + yx = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 113

```
dsolve((x^2+y(x)^2+1)*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{-x^2 - 1 - \sqrt{x^4 + 2x^2 - 4c_1}}$$
$$y(x) = \sqrt{-x^2 - 1 + \sqrt{x^4 + 2x^2 - 4c_1}}$$
$$y(x) = -\sqrt{-x^2 - 1 - \sqrt{x^4 + 2x^2 - 4c_1}}$$
$$y(x) = -\sqrt{-x^2 - 1 + \sqrt{x^4 + 2x^2 - 4c_1}}$$

✓ Solution by Mathematica

Time used: 2.437 (sec). Leaf size: 146

```
DSolve[(x^2+y[x]^2+1)*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^2 - \sqrt{x^4 + 2x^2 + 1 + 4c_1} - 1}$$
$$y(x) \to \sqrt{-x^2 - \sqrt{x^4 + 2x^2 + 1 + 4c_1} - 1}$$
$$y(x) \to -\sqrt{-x^2 + \sqrt{x^4 + 2x^2 + 1 + 4c_1} - 1}$$
$$y(x) \to \sqrt{-x^2 + \sqrt{x^4 + 2x^2 + 1 + 4c_1} - 1}$$
$$y(x) \to 0$$

6.32 problem 165

Internal problem ID [15058]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 165.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y\cos(x) - y^2\cos(x) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)-y(x)*cos(x)=y(x)^2*cos(x),y(x), singsol=all)
```

$$y(x) = \frac{1}{\mathrm{e}^{-\sin(x)}c_1 - 1}$$

✓ Solution by Mathematica
Time used: 0.359 (sec). Leaf size: 35

```
DSolve[y'[x]-y[x]*Cos[x]==y[x]^2*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{\sin(x)+c_1}}{-1 + e^{\sin(x)+c_1}}$$
$$y(x) \to -1$$
$$y(x) \to 0$$

6.33 problem 166

Internal problem ID [15059]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 166.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - \tan(y) - \frac{e^x}{\cos(y)} = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)-tan(y(x))=exp(x)/cos(y(x)),y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica
Time used: 11.451 (sec). Leaf size: 14

```
DSolve[y'[x]-Tan[y[x]]==Exp[x]/Cos[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin\left(e^x(x + c_1)\right)$$

6.34 problem 167

Internal problem ID [15060]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 167.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y' - y(\mathrm{e}^{x} + \ln(y)) = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)=y(x)*(exp(x)+ln(y(x))),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{e}^{x}(x+c_1)}$$

✓ Solution by Mathematica
Time used: 0.372 (sec). Leaf size: 15

```
DSolve[y'[x]==y[x]*(Exp[x]+Log[y[x]]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{e^{x}(x+c_1)}$$

6.35 problem 168

Internal problem ID [15061]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 168.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{\cos(y)\, y' + \sin(y) = x + 1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)*cos(y(x))+sin(y(x))=x+1,y(x), singsol=all)
```

$$y(x) = -\arcsin\left(-x + c_1 e^{-x}\right)$$

✓ Solution by Mathematica
Time used: 13.261 (sec). Leaf size: 17

```
DSolve[y'[x]*Cos[y[x]]+Sin[y[x]]==x+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin\left(x - c_1 e^{-x}\right)$$

6.36 problem 169

Internal problem ID [15062]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 169.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$yy' - (x-1)\,\mathrm{e}^{-\frac{y^2}{2}} = -1$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(y(x)*diff(y(x),x)+1=(x-1)*exp(-y(x)^2/2),y(x), singsol=all)
```

$$y(x) = \sqrt{2}\,\sqrt{\ln\left(-c_1\mathrm{e}^{-x} + x - 2\right)}$$
$$y(x) = -\sqrt{2}\,\sqrt{\ln\left(-c_1\mathrm{e}^{-x} + x - 2\right)}$$

✓ Solution by Mathematica

Time used: 7.375 (sec). Leaf size: 60

```
DSolve[y[x]*y'[x]+1==(x-1)*Exp[-y[x]^2/2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}\sqrt{-x + \log\left(e^x(x-2) + c_1\right)}$$
$$y(x) \to \sqrt{2}\sqrt{-x + \log\left(e^x(x-2) + c_1\right)}$$

6.37 problem 170

Internal problem ID [15063]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 6. Linear equations of the first order. The Bernoulli equation. Exercises page 54
Problem number: 170.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$y' + x\sin(2y) - 2x\,\mathrm{e}^{-x^2}\cos(y)^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)+x*sin(2*y(x))=2*x*exp(-x^2)*cos(y(x))^2,y(x), singsol=all)
```

$$y(x) = \arctan\left(\left(x^2 + 2c_1\right)\mathrm{e}^{-x^2}\right)$$

✓ Solution by Mathematica

Time used: 10.038 (sec). Leaf size: 70

```
DSolve[y'[x]+x*Sin[2*y[x]]==2*x*Exp[-x^2]*Cos[y[x]]^2,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \arctan\left(e^{-x^2}\left(x^2 + c_1\right)\right)$$
$$y(x) \to -\frac{1}{2}\pi e^{x^2}\sqrt{e^{-2x^2}}$$
$$y(x) \to \frac{1}{2}\pi e^{x^2}\sqrt{e^{-2x^2}}$$

7 Section 7, Total differential equations. The integrating factor. Exercises page 61

7.1 problem 175

Internal problem ID [15064]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 7, Total differential equations. The integrating factor. Exercises page 61
Problem number: 175.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$\boxed{x(2x^2+y^2)+y(x^2+2y^2)\,y'=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 125

```
dsolve(x*(2*x^2+y(x)^2)+y(x)*(x^2+2*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2c_1x^2-2\sqrt{-3c_1^2x^4+4}}}{2\sqrt{c_1}}$$
$$y(x) = \frac{\sqrt{-2c_1x^2-2\sqrt{-3c_1^2x^4+4}}}{2\sqrt{c_1}}$$
$$y(x) = -\frac{\sqrt{-2c_1x^2+2\sqrt{-3c_1^2x^4+4}}}{2\sqrt{c_1}}$$
$$y(x) = \frac{\sqrt{-2c_1x^2+2\sqrt{-3c_1^2x^4+4}}}{2\sqrt{c_1}}$$

✓ Solution by Mathematica

Time used: 23.583 (sec). Leaf size: 303

```
DSolve[x*(2*x^2+y[x]^2)+y[x]*(x^2+2*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -\frac{\sqrt{-x^2 - \sqrt{-3x^4 + 4e^{2c_1}}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-x^2 - \sqrt{-3x^4 + 4e^{2c_1}}}}{\sqrt{2}}$$
$$y(x) \to -\frac{\sqrt{-x^2 + \sqrt{-3x^4 + 4e^{2c_1}}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-x^2 + \sqrt{-3x^4 + 4e^{2c_1}}}}{\sqrt{2}}$$
$$y(x) \to -\frac{\sqrt{-\sqrt{3}\sqrt{-x^4} - x^2}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-\sqrt{3}\sqrt{-x^4} - x^2}}{\sqrt{2}}$$
$$y(x) \to -\frac{\sqrt{\sqrt{3}\sqrt{-x^4} - x^2}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{\sqrt{3}\sqrt{-x^4} - x^2}}{\sqrt{2}}$$

7.2 problem 176

Internal problem ID [15065]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 7, Total differential equations. The integrating factor. Exercises page 61
Problem number: 176.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{6y^2x + (6x^2y + 4y^3)\, y' = -3x^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 125

```
dsolve((3*x^2+6*x*y(x)^2)+(6*x^2*y(x)+4*y(x)^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-6x^2 - 2\sqrt{9x^4 - 4x^3 - 4c_1}}}{2}$$
$$y(x) = \frac{\sqrt{-6x^2 - 2\sqrt{9x^4 - 4x^3 - 4c_1}}}{2}$$
$$y(x) = -\frac{\sqrt{-6x^2 + 2\sqrt{9x^4 - 4x^3 - 4c_1}}}{2}$$
$$y(x) = \frac{\sqrt{-6x^2 + 2\sqrt{9x^4 - 4x^3 - 4c_1}}}{2}$$

✓ Solution by Mathematica

Time used: 5.982 (sec). Leaf size: 163

```
DSolve[(3*x^2+6*x*y[x]^2)+(6*x^2*y[x]+4*y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{\sqrt{-3x^2 - \sqrt{9x^4 - 4x^3 + 4c_1}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-3x^2 - \sqrt{9x^4 - 4x^3 + 4c_1}}}{\sqrt{2}}$$
$$y(x) \to -\frac{\sqrt{-3x^2 + \sqrt{9x^4 - 4x^3 + 4c_1}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-3x^2 + \sqrt{9x^4 - 4x^3 + 4c_1}}}{\sqrt{2}}$$

7.3 problem 177

Internal problem ID [15066]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISE-LYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 7, Total differential equations. The integrating factor. Exercises page 61
Problem number: 177.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{\frac{x}{\sqrt{x^2+y^2}}+\frac{1}{y}+\left(\frac{y}{\sqrt{x^2+y^2}}+\frac{1}{y}-\frac{x}{y^2}\right)y'=-\frac{1}{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve((x/sqrt(x^2+y(x)^2)+1/x+1/y(x))+(y(x)/sqrt(x^2+y(x)^2)+1/y(x)-x/y(x)^2)*diff(y(x),x)=
```

$$\frac{y(x)\ln(y(x))+\left(\sqrt{x^2+y(x)^2}+c_1+\ln(x)\right)y(x)+x}{y(x)}=0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(x/Sqrt[x^2+y[x]^2]+1/x+1/y[x])+(y[x]/Sqrt[x^2+y[x]^2]+1/y[x]-x/y[x]^2)*y'[x]==0,y[x]
```

Not solved

7.4 problem 178

Internal problem ID [15067]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 7, Total differential equations. The integrating factor. Exercises page 61
Problem number: 178.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$3x^2 \tan(y) - \frac{2y^3}{x^3} + \left(x^3 \sec(y)^2 + 4y^3 + \frac{3y^2}{x^2}\right) y' = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve((3*x^2*tan(y(x))-2*y(x)^3/x^3  )+( x^3*sec(y(x))^2+4*y(x)^3+ 3*y(x)^2/x^2 )*diff(y(x)
```

$$x^3 \tan(y(x)) + \frac{y(x)^3}{x^2} + y(x)^4 + c_1 = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(3*x^2*Tan[y[x]]-2*y[x]^3/x^3  )+( x^3*Sec[y[x]]^2+4*y[x]^3+ 3*y[x]^2/x^2 )*y'[x]==0,
```

Not solved

7.5 problem 179

Internal problem ID [15068]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 7, Total differential equations. The integrating factor. Exercises page 61
Problem number: 179.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _exact, _rational]`

$$\boxed{\frac{x^2+y^2}{x^2y}-\frac{(x^2+y^2)\,y'}{y^2x}=-2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 65

```
dsolve((2*x+ (x^2+y(x)^2)/(x^2*y(x))  )=( (x^2+y(x)^2)/(x*y(x)^2) )*diff(y(x),x),y(x), sings
```

$$y(x)=-\frac{\left(-x^2+\sqrt{x^4+4c_1x^2+4c_1^2+4}-2c_1\right)x}{2}$$
$$y(x)=\frac{\left(x^2+2c_1+\sqrt{x^4+4c_1x^2+4c_1^2+4}\right)x}{2}$$

✓ Solution by Mathematica

Time used: 0.364 (sec). Leaf size: 78

```
DSolve[(2*x+ (x^2+y[x]^2)/(x^2*y[x])  )==( (x^2+y[x]^2)/(x*y[x]^2) )*y'[x],y[x],x,IncludeSin
```

$$y(x)\to\frac{1}{2}x\left(x^2-\sqrt{x^4+2c_1x^2+4+c_1{}^2}+c_1\right)$$
$$y(x)\to\frac{1}{2}x\left(x^2+\sqrt{x^4+2c_1x^2+4+c_1{}^2}+c_1\right)$$
$$y(x)\to 0$$

7.6 problem 180

Internal problem ID [15069]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 7, Total differential equations. The integrating factor. Exercises page 61
Problem number: 180.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\frac{\sin(2x)}{y} + \left(y - \frac{\sin(x)^2}{y^2}\right) y' = -x$$

✓ Solution by Maple
Time used: 0.141 (sec). Leaf size: 401

```
dsolve(( sin(2*x)/y(x)+x )+( y(x)-sin(x)^2/y(x)^2 )*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(-108 + 108\cos(2x) + 12\sqrt{12x^6 + 72c_1x^4 + 144x^2c_1^2 + 96c_1^3 + 81 - 162\cos(2x) + 81\cos(2x)^2}\right)^{\frac{2}{3}} - 1}{6\left(-108 + 108\cos(2x) + 12\sqrt{12x^6 + 72c_1x^4 + 144x^2c_1^2 + 96c_1^3 + 81 - 162\cos(2x) + 81\cos(2x)}\right.}$$

$$y(x) = -\frac{\left(\frac{i\sqrt{3}}{12} + \frac{1}{12}\right)\left(-108 + 108\cos(2x) + 12\sqrt{12x^6 + 72c_1x^4 + 144x^2c_1^2 + 96c_1^3 + 81 - 162\cos(2x) + 81\,\mathrm{co}\right.}{\left(-108 + 108\cos(2x) + 12\sqrt{12x^6 + 72c_1x^4 + 144x^2c_1^2 + 96c_1^3 + 81 - 162\cos(2x) +\right.}$$

$$y(x) = \frac{\frac{\left(-108+108\cos(2x)+12\sqrt{12x^6+72c_1x^4+144x^2c_1^2+96c_1^3+81-162\cos(2x)+81\cos(2x)^2}\right)^{\frac{2}{3}}\left(i\sqrt{3}-1\right)}{12} + \left(x^2 + 2c_1\right)\left(1 + i\sqrt{3}\right)}{\left(-108 + 108\cos(2x) + 12\sqrt{12x^6 + 72c_1x^4 + 144x^2c_1^2 + 96c_1^3 + 81 - 162\cos(2x) + 81\cos(2x)^2}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 6.373 (sec). Leaf size: 394

```
DSolve[( Sin[2*x]/y[x]+x  )+( y[x]-Sin[x]^2/y[x]^2 )*y'[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{\sqrt[3]{2}\left(2\sqrt{3}\sqrt{27\sin^4(x)+(x^2-c_1)^3}+9\cos(2x)-9\right)^{2/3}-2\sqrt[3]{3}(x^2-c_1)}{6^{2/3}\sqrt[3]{2\sqrt{3}\sqrt{27\sin^4(x)+(x^2-c_1)^3}+9\cos(2x)-9}}$$

$$y(x) \to \frac{6\sqrt[3]{2}(1+i\sqrt{3})(x^2-c_1)+i6^{2/3}(\sqrt{3}+i)\left(2\sqrt{3}\sqrt{27\sin^4(x)+(x^2-c_1)^3}+9\cos(2x)-9\right)^{2/3}}{12\sqrt[3]{3}\sqrt[3]{2\sqrt{3}\sqrt{27\sin^4(x)+(x^2-c_1)^3}+9\cos(2x)-9}}$$

$$y(x) \to \frac{6\sqrt[3]{2}(1-i\sqrt{3})(x^2-c_1)-6^{2/3}(1+i\sqrt{3})\left(2\sqrt{3}\sqrt{27\sin^4(x)+(x^2-c_1)^3}+9\cos(2x)-9\right)^{2/3}}{12\sqrt[3]{3}\sqrt[3]{2\sqrt{3}\sqrt{27\sin^4(x)+(x^2-c_1)^3}+9\cos(2x)-9}}$$

$$y(x) \to 0$$

7.7 problem 181

Internal problem ID [15070]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 7, Total differential equations. The integrating factor. Exercises page 61
Problem number: 181.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$-y + \left(2y - x + 3y^2\right) y' = -3x^2 + 2x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 637

```
dsolve(( 3*x^2-2*x-y(x)  )+( 2*y(x)-x+3*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(-36x - 108x^3 + 108x^2 - 108c_1 - 8 + 12\sqrt{81x^6 - 162x^5 + 162c_1x^3 + 135x^4 - 162c_1x^2 - 54x^3 + 81c_1^2}\right.}{6} + \frac{2x + \frac{2}{3}}{\left(-36x - 108x^3 + 108x^2 - 108c_1 - 8 + 12\sqrt{81x^6 - 162x^5 + 162c_1x^3 + 135x^4 - 162c_1x^2 - 54x^3 + 81}\right.} - \frac{1}{3}$$

$$y(x) = \frac{i\left(4 - \left(-36x - 108x^3 + 108x^2 - 108c_1 - 8 + 12\sqrt{81x^6 - 162x^5 + 135x^4 + (162c_1 - 54)x^3 + (-162c_1\right.\right.}{12\left(-36x - 108x^3 +\right.}$$

$$y(x) = \frac{i\left(\left(-36x - 108x^3 + 108x^2 - 108c_1 - 8 + 12\sqrt{81x^6 - 162x^5 + 135x^4 + (162c_1 - 54)x^3 + (-162c_1 - 1\right.\right.}{12\left(-36x - 108x^3 +\right.}$$

✓ Solution by Mathematica

Time used: 5.636 (sec). Leaf size: 478

```
DSolve[( 3*x^2-2*x-y[x]  )+( 2*y[x]-x+3*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{6}\left(-\frac{2\sqrt[3]{2}(3x+1)}{\sqrt[3]{27x^3-27x^2+\sqrt{-4(3x+1)^3+(27x^3-27x^2+9x+2+27c_1)^2}+9x+2+27c_1}} - 2^{2/3}\sqrt[3]{27x^3-27x^2+\sqrt{-4(3x+1)^3+(27x^3-27x^2+9x+2+27c_1)^2}+9x+2+27c_1} - 2\right)$$

$$y(x) \to \frac{1}{12}\left(\frac{2\sqrt[3]{2}(1+i\sqrt{3})(3x+1)}{\sqrt[3]{27x^3-27x^2+\sqrt{-4(3x+1)^3+(27x^3-27x^2+9x+2+27c_1)^2}+9x+2+27c_1}} + 2^{2/3}\left(1-i\sqrt{3}\right)\sqrt[3]{27x^3-27x^2+\sqrt{-4(3x+1)^3+(27x^3-27x^2+9x+2+27c_1)^2}+9x+2+27c_1} - 4\right)$$

$$y(x) \to \frac{1}{12}\left(\frac{2\sqrt[3]{2}(1-i\sqrt{3})(3x+1)}{\sqrt[3]{27x^3-27x^2+\sqrt{-4(3x+1)^3+(27x^3-27x^2+9x+2+27c_1)^2}+9x+2+27c_1}} + 2^{2/3}\left(1+i\sqrt{3}\right)\sqrt[3]{27x^3-27x^2+\sqrt{-4(3x+1)^3+(27x^3-27x^2+9x+2+27c_1)^2}+9x+2+27c_1} - 4\right)$$

7.8 problem 182

Internal problem ID [15071]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 7, Total differential equations. The integrating factor. Exercises page 61
Problem number: 182.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\frac{xy}{\sqrt{x^2+1}} + 2yx - \frac{y}{x} + \left(\sqrt{x^2+1} + x^2 - \ln(x)\right) y' = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 64

```
dsolve(( x*y(x)/sqrt(1+x^2) + 2*x*y(x) -y(x)/x  )+(  sqrt(1+x^2) + x^2-ln(x) )*diff(y(x),x)=
```

$$y(x) = c_1 \mathrm{e}^{-\left(\int \frac{2\sqrt{x^2+1}\,x^2+x^2-\sqrt{x^2+1}}{\sqrt{x^2+1}\,x\left(\sqrt{x^2+1}+x^2-\ln(x)\right)}dx\right)}$$

✓ Solution by Mathematica
Time used: 7.409 (sec). Leaf size: 94

```
DSolve[( x*y[x]/Sqrt[1+x^2] + 2*x*y[x] -y[x]/x  )+(  Sqrt[1+x^2] + x^2-Log[x] )*y'[x]==0,y[x
```

$$y(x) \to c_1 \exp\left(\int_1^x \frac{\sqrt{K[1]^2+1} - K[1]^2\left(2\sqrt{K[1]^2+1}+1\right)}{K[1]\left(\left(\sqrt{K[1]^2+1}+1\right)K[1]^2 - \sqrt{K[1]^2+1}\log(K[1])+1\right)}dK[1]\right)$$
$$y(x) \to 0$$

7.9 problem 184

Internal problem ID [15072]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 7, Total differential equations. The integrating factor. Exercises page 61
Problem number: 184.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{\sin(y) + \sin(x)\, y + \left(\cos(y)\, x - \cos(x) + \frac{1}{y}\right) y' = -\frac{1}{x}}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 21

```
dsolve(( sin(y(x))+y(x)*sin(x)+1/x )+( x*cos(y(x))-cos(x)+1/y(x) )*diff(y(x),x)=0,y(x), sing
```

$$-y(x)\cos(x) + \sin(y(x))\, x + \ln(x) + \ln(y(x)) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.332 (sec). Leaf size: 23

```
DSolve[( Sin[y[x]]+y[x]*Sin[x]+1/x )+( x*Cos[y[x]]-Cos[x]+1/y[x] )*y'[x]==0,y[x],x,IncludeSi
```

$$\text{Solve}[\log(y(x)) + x\sin(y(x)) - y(x)\cos(x) + \log(x) = c_1, y(x)]$$

7.10 problem 185

Internal problem ID [15073]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 7, Total differential equations. The integrating factor. Exercises page 61
Problem number: 185.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\frac{y+\sin(x)\cos(yx)^2}{\cos(yx)^2}+\left(\frac{x}{\cos(yx)^2}+\sin(y)\right)y'=0$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 19

```
dsolve(( (y(x)+sin(x)*cos(x*y(x))^2 )/cos(x*y(x))^2 )+( x/cos(x*y(x))^2+sin(y(x)) )*diff(y(x
```

$$\tan(xy(x))-\cos(x)-\cos(y(x))+c_1=0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[( (y[x]+Sin[x]*Cos[x*y[x]]^2 )/Cos[x*y[x]]^2 )+( x/Cos[x*y[x]]^2+Sin[y[x]] )*y'[x]==0
```

Not solved

7.11 problem 186

Internal problem ID [15074]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 7, Total differential equations. The integrating factor. Exercises page 61
Problem number: 186.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, _dAlembert]`

$$\boxed{\frac{2x}{y^3}+\frac{(y^2-3x^2)\,y'}{y^4}=0}$$

With initial conditions

$$[y(1)=1]$$

✓ Solution by Maple

Time used: 1.5 (sec). Leaf size: 5

```
dsolve([( 2*x/y(x)^3)+( (y(x)^2-3*x^2)/y(x)^4 )*diff(y(x),x)=0,y(1) = 1],y(x), singsol=all)
```

$$y(x)=x$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{( 2*x/y[x]^3)+( (y[x]^2-3*x^2)/y[x]^4 )*y'[x]==0,{y[1]==1}},y[x],x,IncludeSingularSo
```

Timed out

7.12 problem 187

Internal problem ID [15075]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 7, Total differential equations. The integrating factor. Exercises page 61
Problem number: 187.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$y(x^2 + y^2 + a^2)\, y' + x(y^2 - a^2 + x^2) = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 129

```
dsolve(( y(x)*(x^2+y(x)^2+a^2))*diff(y(x),x)+x*(x^2+y(x)^2-a^2)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{-a^2 - x^2 - 2\sqrt{a^2x^2 - c_1}}$$
$$y(x) = \sqrt{-a^2 - x^2 + 2\sqrt{a^2x^2 - c_1}}$$
$$y(x) = -\sqrt{-a^2 - x^2 - 2\sqrt{a^2x^2 - c_1}}$$
$$y(x) = -\sqrt{-a^2 - x^2 + 2\sqrt{a^2x^2 - c_1}}$$

✓ Solution by Mathematica
Time used: 2.374 (sec). Leaf size: 165

```
DSolve[( y[x]*(x^2+y[x]^2+a^2))*y'[x]+x*(x^2+y[x]^2-a^2)==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to -\sqrt{-a^2 - \sqrt{a^4 + 4a^2x^2 + 4c_1} - x^2}$$
$$y(x) \to \sqrt{-a^2 - \sqrt{a^4 + 4a^2x^2 + 4c_1} - x^2}$$
$$y(x) \to -\sqrt{-a^2 + \sqrt{a^4 + 4a^2x^2 + 4c_1} - x^2}$$
$$y(x) \to \sqrt{-a^2 + \sqrt{a^4 + 4a^2x^2 + 4c_1} - x^2}$$

7.13 problem 188

Internal problem ID [15076]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISE-LYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 7, Total differential equations. The integrating factor. Exercises page 61
Problem number: 188.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$\boxed{3x^2y + y^3 + \left(x^3 + 3y^2x\right)y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 274

```
dsolve(( 3*x^2*y(x)+y(x)^3)+(x^3+3*x*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{12^{\frac{1}{3}}\left(x^4c_1^2 12^{\frac{1}{3}} - \left(\left(\sqrt{3}\,\sqrt{4c_1^4x^8+27}+9\right)x^2c_1\right)^{\frac{2}{3}}\right)}{6c_1x\left(\left(\sqrt{3}\,\sqrt{4c_1^4x^8+27}+9\right)x^2c_1\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{3^{\frac{1}{3}}2^{\frac{2}{3}}\left(\left(1+i\sqrt{3}\right)\left(\left(\sqrt{3}\,\sqrt{4c_1^4x^8+27}+9\right)x^2c_1\right)^{\frac{2}{3}} + c_1^2 2^{\frac{2}{3}}x^4\left(i3^{\frac{5}{6}} - 3^{\frac{1}{3}}\right)\right)}{12\left(\left(\sqrt{3}\,\sqrt{4c_1^4x^8+27}+9\right)x^2c_1\right)^{\frac{1}{3}}xc_1}$$

$$y(x) = \frac{\left(\left(i\sqrt{3}-1\right)\left(\left(\sqrt{3}\,\sqrt{4c_1^4x^8+27}+9\right)x^2c_1\right)^{\frac{2}{3}} + c_1^2\left(i3^{\frac{5}{6}}+3^{\frac{1}{3}}\right)2^{\frac{2}{3}}x^4\right)3^{\frac{1}{3}}2^{\frac{2}{3}}}{12\left(\left(\sqrt{3}\,\sqrt{4c_1^4x^8+27}+9\right)x^2c_1\right)^{\frac{1}{3}}xc_1}$$

✓ Solution by Mathematica

Time used: 60.214 (sec). Leaf size: 338

```
DSolve[( 3*x^2*y[x]+y[x]^3)+(x^3+3*x*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{-2\sqrt[3]{3}x^2 + \sqrt[3]{2}\left(\frac{\sqrt{12x^8+81e^{2c_1}}+9e^{c_1}}{x}\right)^{2/3}}{6^{2/3}\sqrt[3]{\frac{\sqrt{12x^8+81e^{2c_1}}+9e^{c_1}}{x}}}$$

$$y(x) \to \frac{i2^{2/3}\sqrt[3]{3}\left(\sqrt{3}+i\right)\left(\frac{\sqrt{12x^8+81e^{2c_1}}+9e^{c_1}}{x}\right)^{2/3} + 2\sqrt[3]{2}\sqrt[6]{3}\left(\sqrt{3}+3i\right)x^2}{12\sqrt[3]{\frac{\sqrt{12x^8+81e^{2c_1}}+9e^{c_1}}{x}}}$$

$$y(x) \to \frac{2\sqrt[3]{2}\sqrt[6]{3}\left(\sqrt{3}-3i\right)x^2 - i2^{2/3}\sqrt[3]{3}\left(\sqrt{3}-i\right)\left(\frac{\sqrt{12x^8+81e^{2c_1}}+9e^{c_1}}{x}\right)^{2/3}}{12\sqrt[3]{\frac{\sqrt{12x^8+81e^{2c_1}}+9e^{c_1}}{x}}}$$

7.14 problem 189

Internal problem ID [15077]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISE-LYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 7, Total differential equations. The integrating factor. Exercises page 61
Problem number: 189.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], [_Abel`

$$\boxed{-x^2y + x^2(y - x)\, y' = -1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 51

```
dsolve(( 1-x^2*y(x))+x^2*(y(x)-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x^2 + \sqrt{x\,(x^3 - 2c_1x + 2)}}{x}$$
$$y(x) = \frac{x^2 - \sqrt{x\,(x^3 - 2c_1x + 2)}}{x}$$

✓ Solution by Mathematica
Time used: 0.493 (sec). Leaf size: 66

```
DSolve[( 1-x^2*y[x])+x^2*(y[x]-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + \sqrt{-\frac{1}{x^2}}\sqrt{-x\,(x^3 + c_1x + 2)}$$
$$y(x) \to x - \sqrt{-\frac{1}{x^2}}\sqrt{-x\,(x^3 + c_1x + 2)}$$

7.15 problem 190

Internal problem ID [15078]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 7, Total differential equations. The integrating factor. Exercises page 61
Problem number: 190.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y - y'x = -x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(( x^2+y(x))-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = (x + c_1)\, x$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 11

```
DSolve[( x^2+y[x])-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(x + c_1)$$

7.16 problem 191

Internal problem ID [15079]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 7, Total differential equations. The integrating factor. Exercises page 61
Problem number: 191.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$\boxed{y^2 - 2xyy' = -x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(( x+y(x)^2)-2*x*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{(c_1 + \ln(x))\, x}$$
$$y(x) = -\sqrt{(c_1 + \ln(x))\, x}$$

✓ Solution by Mathematica
Time used: 0.178 (sec). Leaf size: 40

```
DSolve[( x+y[x]^2)-2*x*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x}\sqrt{\log(x) + c_1}$$
$$y(x) \to \sqrt{x}\sqrt{\log(x) + c_1}$$

7.17 problem 192

Internal problem ID [15080]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 7, Total differential equations. The integrating factor. Exercises page 61
Problem number: 192.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2x^2y + 2y + (2x^3 + 2x)\, y' = -5}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(( 2*x^2*y(x)+2*y(x)+5)+(2*x^3+2*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-\frac{5\arctan(x)}{2} + c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 21

```
DSolve[( 2*x^2*y[x]+2*y[x]+5)+(2*x^3+2*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-5\arctan(x) + 2c_1}{2x}$$

7.18 problem 193

Internal problem ID [15081]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 7, Total differential equations. The integrating factor. Exercises page 61
Problem number: 193.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{-2y^3x + 3y'y^2x^2 = -x^4\ln(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 84

```
dsolve(( x^4*ln(x)-2*x*y(x)^3)+(3*x^2*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$
\begin{aligned}
y(x) &= \left(-x^2(x\ln(x) - c_1 - x)\right)^{\frac{1}{3}} \\
y(x) &= -\frac{\left(-x^2(x\ln(x) - c_1 - x)\right)^{\frac{1}{3}}\left(1 + i\sqrt{3}\right)}{2} \\
y(x) &= \frac{\left(-x^2(x\ln(x) - c_1 - x)\right)^{\frac{1}{3}}\left(i\sqrt{3} - 1\right)}{2}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.485 (sec). Leaf size: 77

```
DSolve[( x^4*Log[x]-2*x*y[x]^3)+(3*x^2*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$
\begin{aligned}
y(x) &\to \sqrt[3]{x^2(x + x(-\log(x)) + c_1)} \\
y(x) &\to -\sqrt[3]{-1}\sqrt[3]{x^2(x + x(-\log(x)) + c_1)} \\
y(x) &\to (-1)^{2/3}\sqrt[3]{x^2(x + x(-\log(x)) + c_1)}
\end{aligned}
$$

7.19 problem 194

Internal problem ID [15082]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 7, Total differential equations. The integrating factor. Exercises page 61
Problem number: 194.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{\cos(y)\, y' + \sin(y) = -\sin(x) - x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 24

```
dsolve(( x+sin(x)+sin(y(x)))+( cos(y(x)) )*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\arcsin\left(x + \frac{\sin(x)}{2} - \frac{\cos(x)}{2} - 1 + c_1 \mathrm{e}^{-x}\right)$$

✓ Solution by Mathematica
Time used: 33.179 (sec). Leaf size: 61

```
DSolve[( x+Sin[x]+Sin[y[x]])+( Cos[y[x]] )*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin\left(\frac{1}{2}(-2x - \sin(x) + \cos(x) + 2c_1 e^{-x} + 2)\right)$$
$$y(x) \to -\arcsin\left(\frac{1}{2}(2x + \sin(x) - \cos(x) - 2c_1 e^{-x} - 2)\right)$$

7.20 problem 195

Internal problem ID [15083]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 7, Total differential equations. The integrating factor. Exercises page 61
Problem number: 195.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{2y^2x - 3y^3 + (7 - 3y^2x)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 65

```
dsolve(( 2*x*y(x)^2-3*y(x)^3)+( 7-3*x*y(x)^2 )*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x^2 + c_1 + \sqrt{x^4 + 2c_1x^2 + c_1^2 - 84x}}{6x}$$
$$y(x) = \frac{x^2 - \sqrt{x^4 + 2c_1x^2 + c_1^2 - 84x} + c_1}{6x}$$

✓ Solution by Mathematica
Time used: 0.406 (sec). Leaf size: 86

```
DSolve[( 2*x*y[x]^2-3*y[x]^3)+( 7-3*x*y[x]^2 )*y'[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{x^2 - \sqrt{x^4 + 2c_1x^2 - 84x + c_1{}^2} + c_1}{6x}$$
$$y(x) \to \frac{x^2 + \sqrt{x^4 + 2c_1x^2 - 84x + c_1{}^2} + c_1}{6x}$$
$$y(x) \to 0$$

7.21 problem 196

Internal problem ID [15084]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 7, Total differential equations. The integrating factor. Exercises page 61
Problem number: 196.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$\boxed{3y^2 + \left(2y^3 - 6yx\right) y' = x}$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 101

```
dsolve(( 3*y(x)^2-x)+( 2*y(x)^3-6*x*y(x) )*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2\sqrt{c_1 (c_1 - 8x)} + 2c_1 - 4x}}{2}$$
$$y(x) = \frac{\sqrt{-2\sqrt{c_1 (c_1 - 8x)} + 2c_1 - 4x}}{2}$$
$$y(x) = -\frac{\sqrt{2\sqrt{c_1 (c_1 - 8x)} + 2c_1 - 4x}}{2}$$
$$y(x) = \frac{\sqrt{2\sqrt{c_1 (c_1 - 8x)} + 2c_1 - 4x}}{2}$$

✓ Solution by Mathematica

Time used: 11.553 (sec). Leaf size: 185

```
DSolve[( 3*y[x]^2-x)+( 2*y[x]^3-6*x*y[x] )*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{\sqrt{-2x-e^{\frac{c_1}{2}}\sqrt{8x+e^{c_1}}-e^{c_1}}}{\sqrt{2}}$$
$$y(x)\to \frac{\sqrt{-2x-e^{\frac{c_1}{2}}\sqrt{8x+e^{c_1}}-e^{c_1}}}{\sqrt{2}}$$
$$y(x)\to -\frac{\sqrt{-2x+e^{\frac{c_1}{2}}\sqrt{8x+e^{c_1}}-e^{c_1}}}{\sqrt{2}}$$
$$y(x)\to \frac{\sqrt{-2x+e^{\frac{c_1}{2}}\sqrt{8x+e^{c_1}}-e^{c_1}}}{\sqrt{2}}$$

7.22 problem 197

Internal problem ID [15085]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 7, Total differential equations. The integrating factor. Exercises page 61
Problem number: 197.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{y^2 - 2xyy' = -x^2 - 1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 29

```
dsolve(( x^2+y(x)^2+1)-( 2*x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{c_1 x + x^2 - 1}$$
$$y(x) = -\sqrt{c_1 x + x^2 - 1}$$

✓ Solution by Mathematica
Time used: 0.26 (sec). Leaf size: 37

```
DSolve[( x^2+y[x]^2+1)-( 2*x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x^2 + c_1 x - 1}$$
$$y(x) \to \sqrt{x^2 + c_1 x - 1}$$

7.23 problem 198

Internal problem ID [15086]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 7, Total differential equations. The integrating factor. Exercises page 61
Problem number: 198.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]'], [_Ab`

$$\boxed{-yx + \left(x^2 + y\right) y' = -x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
dsolve(( x -x*y(x) )+( y(x)+x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{2c_1 + 1 - \sqrt{2c_1 x^2 + 2c_1 + 1}}{2c_1}$$
$$y(x) = \frac{2c_1 + 1 + \sqrt{2c_1 x^2 + 2c_1 + 1}}{2c_1}$$

✓ Solution by Mathematica

Time used: 4.513 (sec). Leaf size: 295

```
DSolve[( x -x*y[x] )+( y[x]+x^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^2 + \frac{1}{\frac{1}{x^2+1} - \frac{1+i}{(x^2+1)\sqrt{-2(x^2+1)\cosh\left(\frac{2c_1}{9}\right) - 2(x^2+1)\sinh\left(\frac{2c_1}{9}\right) + 2i}}}$$
$$y(x) \to -x^2 + \frac{1}{\frac{1}{x^2+1} + \frac{1+i}{(x^2+1)\sqrt{-2(x^2+1)\cosh\left(\frac{2c_1}{9}\right) - 2(x^2+1)\sinh\left(\frac{2c_1}{9}\right) + 2i}}}$$
$$y(x) \to -x^2 + \frac{1}{\frac{1}{x^2+1} - \frac{1+i}{\sqrt{2}(x^2+1)\sqrt{(x^2+1)\cosh\left(\frac{2c_1}{9}\right) + (x^2+1)\sinh\left(\frac{2c_1}{9}\right) + i}}}$$
$$y(x) \to -x^2 + \frac{1}{\frac{1}{x^2+1} + \frac{1+i}{\sqrt{2}(x^2+1)\sqrt{(x^2+1)\cosh\left(\frac{2c_1}{9}\right) + (x^2+1)\sinh\left(\frac{2c_1}{9}\right) + i}}}$$
$$y(x) \to 1$$
$$y(x) \to \frac{1}{2}(1 - x^2)$$

8 Section 8. First order not solved for the derivative. Exercises page 67

8.1 problem 199

Internal problem ID [15087]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8. First order not solved for the derivative. Exercises page 67
Problem number: 199.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type **[_quadrature]**

$$\boxed{4y'^2 = 9x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(4*diff(y(x),x)^2-9*x=0,y(x), singsol=all)
```

$$y(x) = -x^{\frac{3}{2}} + c_1$$
$$y(x) = x^{\frac{3}{2}} + c_1$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 27

```
DSolve[4*y'[x]^2-9*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^{3/2} + c_1$$
$$y(x) \to x^{3/2} + c_1$$

8.2 problem 200

Internal problem ID [15088]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8. First order not solved for the derivative. Exercises page 67
Problem number: 200.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'^2 - 2yy' - y^2(\mathrm{e}^{2x} - 1) = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 27

```
dsolve(diff(y(x),x)^2-2*y(x)*diff(y(x),x)=y(x)^2*(exp(2*x)-1),y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = c_1 \mathrm{e}^{x-\mathrm{e}^x}$$
$$y(x) = c_1 \mathrm{e}^{x+\mathrm{e}^x}$$

✓ Solution by Mathematica
Time used: 0.103 (sec). Leaf size: 36

```
DSolve[y'[x]^2-2*y[x]*y'[x]==y[x]^2*(Exp[2*x]-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x-e^x}$$
$$y(x) \to c_1 e^{x+e^x}$$
$$y(x) \to 0$$

8.3 problem 201

Internal problem ID [15089]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8. First order not solved for the derivative. Exercises page 67
Problem number: 201.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - 2y'x = 8x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)^2-2*x*diff(y(x),x)-8*x^2=0,y(x), singsol=all)
```

$$y(x) = 2x^2 + c_1$$
$$y(x) = -x^2 + c_1$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 25

```
DSolve[y'[x]^2-2*x*y'[x]-8*x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^2 + c_1$$
$$y(x) \to 2x^2 + c_1$$

8.4 problem 202

Internal problem ID [15090]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8. First order not solved for the derivative. Exercises page 67
Problem number: 202.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2 {y'}^2 + 3xyy' + 2y^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x)^2+3*x*y(x)*diff(y(x),x)+2*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x}$$
$$y(x) = \frac{c_1}{x^2}$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 26

```
DSolve[x^2*y'[x]^2+3*x*y[x]*y'[x]+2*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x^2}$$
$$y(x) \to \frac{c_1}{x}$$
$$y(x) \to 0$$

8.5 problem 203

Internal problem ID [15091]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8. First order not solved for the derivative. Exercises page 67
Problem number: 203.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type **[_quadrature]**

$$\boxed{y'^2 - (2x+y)\,y' + yx = -x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)^2-(2*x+y(x))*diff(y(x),x)+x^2+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} + c_1$$
$$y(x) = -x - 1 + \mathrm{e}^x c_1$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 30

```
DSolve[y'[x]^2-(2*x+y[x])*y'[x]+x^2+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2} + c_1$$
$$y(x) \to -x + c_1 e^x - 1$$

8.6 problem 204

Internal problem ID [15092]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8. First order not solved for the derivative. Exercises page 67
Problem number: 204.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_exponential_symmetries]]`

$$\boxed{y'^3 + (x+2)\,\mathrm{e}^{y} = 0}$$

✓ Solution by Maple
Time used: 0.203 (sec). Leaf size: 87

```
dsolve(diff(y(x),x)^3+(x+2)*exp(y(x))=0,y(x), singsol=all)
```

$$y(x) = 3\ln(12) - 3\ln\left((6+3x)(2+x)^{\frac{1}{3}} + 4c_1\right)$$
$$y(x) = 3\ln(24) - 3\ln\left(-3\left(1+i\sqrt{3}\right)(2+x)^{\frac{4}{3}} + 8c_1\right)$$
$$y(x) = 3\ln(24) - 3\ln\left(3\left(i\sqrt{3}-1\right)(2+x)^{\frac{4}{3}} + 8c_1\right)$$

✓ Solution by Mathematica
Time used: 6.699 (sec). Leaf size: 126

```
DSolve[y'[x]^3+(x+2)*Exp[y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -3\log\left(\frac{1}{12}\left(3\sqrt[3]{x+2}x + 6\sqrt[3]{x+2} - 4c_1\right)\right)$$
$$y(x) \to -3\log\left(\frac{1}{12}\left(-3\sqrt[3]{-1}\sqrt[3]{x+2}x - 6\sqrt[3]{-1}\sqrt[3]{x+2} - 4c_1\right)\right)$$
$$y(x) \to -3\log\left(\frac{1}{12}\left(3(-1)^{2/3}\sqrt[3]{x+2}x + 6(-1)^{2/3}\sqrt[3]{x+2} - 4c_1\right)\right)$$

8.7 problem 205

Internal problem ID [15093]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8. First order not solved for the derivative. Exercises page 67
Problem number: 205.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 - yy'^2 + x^2y' - x^2y = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 30

```
dsolve(diff(y(x),x)^3=y(x)*diff(y(x),x)^2-x^2*diff(y(x),x)+x^2*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{ix^2}{2} + c_1$$
$$y(x) = \frac{ix^2}{2} + c_1$$
$$y(x) = e^x c_1$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 43

```
DSolve[y'[x]^3==y[x]*y'[x]^2-x^2*y'[x]+x^2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x$$
$$y(x) \to c_1 - \frac{ix^2}{2}$$
$$y(x) \to \frac{ix^2}{2} + c_1$$

8.8 problem 206

Internal problem ID [15094]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8. First order not solved for the derivative. Exercises page 67
Problem number: 206.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y'^2 - yy' = -\mathrm{e}^x$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 34

```
dsolve(diff(y(x),x)^2-y(x)*diff(y(x),x)+exp(x)=0,y(x), singsol=all)
```

$$y(x) = -2\,\mathrm{e}^{\frac{x}{2}}$$
$$y(x) = 2\,\mathrm{e}^{\frac{x}{2}}$$
$$y(x) = \frac{\mathrm{e}^x c_1^2 + 1}{c_1}$$

✓ Solution by Mathematica

Time used: 60.203 (sec). Leaf size: 59

```
DSolve[y'[x]^2-y[x]*y'[x]+Exp[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-e^{-c_1}\left(-e^x + e^{c_1}\right)^2}$$
$$y(x) \to \sqrt{-e^{-c_1}\left(e^x - e^{c_1}\right)^2}$$

8.9 problem 207

Internal problem ID [15095]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8. First order not solved for the derivative. Exercises page 67
Problem number: 207.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$y'^2 - 4y'x + 2y = -2x^2$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 77

```
dsolve(diff(y(x),x)^2-4*x*diff(y(x),x)+2*y(x)+2*x^2=0,y(x), singsol=all)
```

$$y(x) = x^2$$
$$y(x) = \frac{1}{2}x^2 + c_1 x - \frac{1}{2}c_1^2$$
$$y(x) = \frac{1}{2}x^2 - c_1 x - \frac{1}{2}c_1^2$$
$$y(x) = \frac{1}{2}x^2 - c_1 x - \frac{1}{2}c_1^2$$
$$y(x) = \frac{1}{2}x^2 + c_1 x - \frac{1}{2}c_1^2$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]^2-4*x*y'[x]+2*y[x]+2*x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

8.10 problem 208

Internal problem ID [15096]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8. First order not solved for the derivative. Exercises page 67
Problem number: 208.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$y - {y'}^2 e^{y'} = 0$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 38

```
dsolve(y(x)=diff(y(x),x)^2*exp(diff(y(x),x)),y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{(x - c_1)\left(\text{LambertW}\left((x - c_1)\,e\right) - 1\right)^2}{\text{LambertW}\left((x - c_1)\,e\right)}$$

✓ Solution by Mathematica
Time used: 0.283 (sec). Leaf size: 102

```
DSolve[y[x]==y'[x]^2*Exp[y'[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{\#1}{W\left(-\frac{\sqrt{\#1}}{2}\right)} + \frac{\#1}{2W\left(-\frac{\sqrt{\#1}}{2}\right)^2}\&\right][2x + c_1]$$
$$y(x) \to \text{InverseFunction}\left[\frac{\#1}{W\left(\frac{\sqrt{\#1}}{2}\right)} + \frac{\#1}{2W\left(\frac{\sqrt{\#1}}{2}\right)^2}\&\right][2x + c_1]$$
$$y(x) \to 0$$

8.11 problem 209

Internal problem ID [15097]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8. First order not solved for the derivative. Exercises page 67
Problem number: 209.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$y' = e^{\frac{y'}{y}}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)=exp(diff(y(x),x)/y(x)),y(x), singsol=all)
```

$$y(x) = -\mathrm{LambertW}\left(c_1 e^{-x}\right) e^{-\frac{1}{\mathrm{LambertW}\left(c_1 e^{-x}\right)}}$$

✓ Solution by Mathematica
Time used: 0.066 (sec). Leaf size: 33

```
DSolve[y'[x]==Exp[y'[x]/y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{1}{W\left(-\frac{1}{\#1}\right)} - \log\left(W\left(-\frac{1}{\#1}\right)\right)\&\right][-x + c_1]$$

8.12 problem 210

Internal problem ID [15098]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8. First order not solved for the derivative. Exercises page 67
Problem number: 210.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{-\ln(y') - \sin(y') = -x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve(x=ln(diff(y(x),x))+sin(diff(y(x),x)),y(x), singsol=all)
```

$$y(x) = \int \operatorname{RootOf}(-x + \ln(\_Z) + \sin(\_Z))\, dx + c_1$$

✓ Solution by Mathematica
Time used: 0.078 (sec). Leaf size: 33

```
DSolve[x==Log[y'[x]]+Sin[y'[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}[\{y(x) = K[1] + K[1]\sin(K[1]) + \cos(K[1]) + c_1, x = \log(K[1]) + \sin(K[1])\}, \{y(x), K[1]\}]$$

8.13 problem 211

Internal problem ID [15099]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8. First order not solved for the derivative. Exercises page 67
Problem number: 211.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{-y'^2 + 2y' = -x + 2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 37

```
dsolve(x=diff(y(x),x)^2-2*diff(y(x),x)+2,y(x), singsol=all)
```

$$y(x) = \frac{(-2x+2)\sqrt{-1+x}}{3} + x + c_1$$
$$y(x) = \frac{(2x-2)\sqrt{-1+x}}{3} + x + c_1$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 39

```
DSolve[x==y'[x]^2-2*y'[x]+2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{3}(x-1)^{3/2} + x + c_1$$
$$y(x) \to \frac{2}{3}(x-1)^{3/2} + x + c_1$$

8.14 problem 212

Internal problem ID [15100]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8. First order not solved for the derivative. Exercises page 67
Problem number: 212.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y - y' \ln(y') = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 63

```
dsolve(y(x)=diff(y(x),x)*ln(diff(y(x),x)),y(x), singsol=all)
```

$$y(x) = \left(-1-\sqrt{1-2c_1+2x}\right) e^{-1-\sqrt{1-2c_1+2x}}$$
$$y(x) = \left(-1+\sqrt{1-2c_1+2x}\right) e^{-1+\sqrt{1-2c_1+2x}}$$

✓ Solution by Mathematica

Time used: 4.166 (sec). Leaf size: 83

```
DSolve[y[x]==y'[x]*Log[y'[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{-1-\sqrt{2x+1+2c_1}}\left(1+\sqrt{2x+1+2c_1}\right)$$
$$y(x) \to e^{-1+\sqrt{2x+1+2c_1}}\left(-1+\sqrt{2x+1+2c_1}\right)$$
$$y(x) \to 0$$

8.15 problem 213

Internal problem ID [15101]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8. First order not solved for the derivative. Exercises page 67
Problem number: 213.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y - (y' - 1)\,\mathrm{e}^{y'} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 22

```
dsolve(y(x)=(diff(y(x),x)-1)*exp(diff(y(x),x)),y(x), singsol=all)
```

$$y(x) = -1$$
$$y(x) = (\ln(x - c_1) - 1)(x - c_1)$$

✓ Solution by Mathematica
Time used: 0.435 (sec). Leaf size: 22

```
DSolve[y[x]==(y'[x]-1)*Exp[y'[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_1)(-1 + \log(x + c_1))$$
$$y(x) \to -1$$

8.16 problem 214

Internal problem ID [15102]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8. First order not solved for the derivative. Exercises page 67
Problem number: 214.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x{y'}^2 - \mathrm{e}^{\frac{1}{y'}} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 71

```
dsolve(diff(y(x),x)^2*x=exp(1/diff(y(x),x)),y(x), singsol=all)
```

$$y(x) = \frac{4c_1 \operatorname{LambertW}\left(-\frac{\sqrt{x}}{2}\right)^2 + 2x \operatorname{LambertW}\left(-\frac{\sqrt{x}}{2}\right) + x}{4 \operatorname{LambertW}\left(-\frac{\sqrt{x}}{2}\right)^2}$$
$$y(x) = \frac{4c_1 \operatorname{LambertW}\left(\frac{\sqrt{x}}{2}\right)^2 + 2x \operatorname{LambertW}\left(\frac{\sqrt{x}}{2}\right) + x}{4 \operatorname{LambertW}\left(\frac{\sqrt{x}}{2}\right)^2}$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 67

```
DSolve[y'[x]^2*x==Exp[1/y'[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \int_1^x \frac{1}{2W\left(-\frac{1}{2\sqrt{\frac{1}{K[1]}}}\right)} dK[1] + c_1$$
$$y(x) \to \int_1^x \frac{1}{2W\left(\frac{1}{2\sqrt{\frac{1}{K[2]}}}\right)} dK[2] + c_1$$

8.17 problem 215

Internal problem ID [15103]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8. First order not solved for the derivative. Exercises page 67
Problem number: 215.
ODE order: 1.
ODE degree: 6.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x\left(y'^2+1\right)^{\frac{3}{2}}=a}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 233

```
dsolve(x*(1+diff(y(x),x)^2)^(3/2)=a,y(x), singsol=all)
```

$$y(x)=\int \frac{\sqrt{(a\,x^2)^{\frac{2}{3}}-x^2}}{x}dx+c_1$$

$$y(x)=-\frac{\left(\int \frac{\sqrt{-2i\sqrt{3}\,(a\,x^2)^{\frac{2}{3}}-2(a\,x^2)^{\frac{2}{3}}-4x^2}}{x}dx\right)}{2}+c_1$$

$$y(x)=\frac{\left(\int \frac{\sqrt{-2i\sqrt{3}\,(a\,x^2)^{\frac{2}{3}}-2(a\,x^2)^{\frac{2}{3}}-4x^2}}{x}dx\right)}{2}+c_1$$

$$y(x)=-\left(\int \frac{\sqrt{(a\,x^2)^{\frac{2}{3}}-x^2}}{x}dx\right)+c_1$$

$$y(x)=-\frac{\sqrt{2}\left(\int \frac{\sqrt{i\sqrt{3}\,(a\,x^2)^{\frac{2}{3}}-(a\,x^2)^{\frac{2}{3}}-2x^2}}{x}dx\right)}{2}+c_1$$

$$y(x)=\frac{\sqrt{2}\left(\int \frac{\sqrt{i\sqrt{3}\,(a\,x^2)^{\frac{2}{3}}-(a\,x^2)^{\frac{2}{3}}-2x^2}}{x}dx\right)}{2}+c_1$$

✓ Solution by Mathematica

Time used: 19.313 (sec). Leaf size: 375

```
DSolve[x*(1+y'[x]^2)^(3/2)==a,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[3]{x}\sqrt{\frac{a^{2/3}}{x^{2/3}} - 1}\left(x^{2/3} - a^{2/3}\right) + c_1$$
$$y(x) \to \sqrt[3]{x}\sqrt{\frac{a^{2/3}}{x^{2/3}} - 1}\left(a^{2/3} - x^{2/3}\right) + c_1$$
$$y(x) \to c_1 - \frac{1}{2}\sqrt[3]{x}\sqrt{-1 + \frac{i\left(\sqrt{3}+i\right)a^{2/3}}{2x^{2/3}}}\left(2x^{2/3} + \left(1 - i\sqrt{3}\right)a^{2/3}\right)$$
$$y(x) \to \frac{1}{2}\sqrt[3]{x}\sqrt{-1 + \frac{i\left(\sqrt{3}+i\right)a^{2/3}}{2x^{2/3}}}\left(2x^{2/3} + \left(1 - i\sqrt{3}\right)a^{2/3}\right) + c_1$$
$$y(x) \to c_1 - \frac{1}{2}\sqrt[3]{x}\sqrt{-1 - \frac{i\left(\sqrt{3}-i\right)a^{2/3}}{2x^{2/3}}}\left(2x^{2/3} + \left(1 + i\sqrt{3}\right)a^{2/3}\right)$$
$$y(x) \to \frac{1}{2}\sqrt[3]{x}\sqrt{-1 - \frac{i\left(\sqrt{3}-i\right)a^{2/3}}{2x^{2/3}}}\left(2x^{2/3} + \left(1 + i\sqrt{3}\right)a^{2/3}\right) + c_1$$

8.18 problem 216

Internal problem ID [15104]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8. First order not solved for the derivative. Exercises page 67
Problem number: 216.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y^{\frac{2}{5}}+y'^{\frac{2}{5}}=a^{\frac{2}{5}}}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 26

```
dsolve(y(x)^(2/5)+diff(y(x),x)^(2/5)=a^(2/5),y(x), singsol=all)
```

$$x-\left(\int^{y(x)}\frac{1}{\left(a^{\frac{2}{5}}-\_a^{\frac{2}{5}}\right)^{\frac{5}{2}}}d\_a\right)-c_1=0$$

✓ Solution by Mathematica
Time used: 0.746 (sec). Leaf size: 89

```
DSolve[y[x]^(2/5)+y'[x]^(2/5)==a^(2/5),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \text{InverseFunction}\left[5\arctan\left(\frac{\sqrt[5]{\#1}}{\sqrt{a^{2/5}-\#1^{2/5}}}\right)+\frac{5\sqrt[5]{\#1}\left(4\#1^{2/5}-3a^{2/5}\right)}{3\left(a^{2/5}-\#1^{2/5}\right)^{3/2}}\&\right][x+c_1]$$
$$y(x)\to a$$

8.19 problem 217

Internal problem ID [15105]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8. First order not solved for the derivative. Exercises page 67
Problem number: 217.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{-y' - \sin(y') = -x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 16

```
dsolve(x=diff(y(x),x)+sin(diff(y(x),x)),y(x), singsol=all)
```

$$y(x) = \int \operatorname{RootOf}(-x + \_Z + \sin(\_Z))\,dx + c_1$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 38

```
DSolve[x==y'[x]+Sin[y'[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x = K[1] + \sin(K[1]), y(x) = \frac{K[1]^2}{2} + K[1]\sin(K[1]) + \cos(K[1]) + c_1\right\}, \{y(x), K[1]\}\right]$$

8.20 problem 218

Internal problem ID [15106]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8. First order not solved for the derivative. Exercises page 67
Problem number: 218.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$y - y'(1 + y' \cos(y')) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(y(x)=diff(y(x),x)*(1+diff(y(x),x)*cos(diff(y(x),x))),y(x), singsol=all)
```

$$y(x) = 0$$

$$x - \left(\int^{y(x)} \frac{1}{\mathrm{RootOf}\left(\cos(\_Z)\_Z^2 - \_a + \_Z\right)} d\_a\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.154 (sec). Leaf size: 38

```
DSolve[y[x]==y'[x]*(1+y'[x]*Cos[y'[x]]),y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\{x = \log(K[1]) + \sin(K[1]) + K[1]\cos(K[1]) + c_1, y(x) = K[1] + K[1]^2\cos(K[1])\}, \{y(x), K[1]\}\right]$$

8.21 problem 219

Internal problem ID [15107]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8. First order not solved for the derivative. Exercises page 67
Problem number: 219.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y - \arcsin(y') - \ln\left(y'^2 + 1\right) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
dsolve(y(x)=arcsin(diff(y(x),x))+ln(1+diff(y(x),x)^2),y(x), singsol=all)
```

$$x - \left(\int^{y(x)} \csc\left(\operatorname{RootOf}\left(-\_a + \_Z + \ln\left(2 - \cos\left(\_Z\right)^2\right)\right)\right) d\_a\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.245 (sec). Leaf size: 46

```
DSolve[y[x]==ArcSin[y'[x]]+Log[1+y'[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x = 2\arctan(K[1]) - \operatorname{arctanh}\left(\sqrt{1 - K[1]^2}\right) + c_1, y(x) = \arcsin(K[1]) + \log\left(K[1]^2 + 1\right)\right\}, \{y(x), K[1]\}\right]$$

9 Section 8.3. The Lagrange and Clairaut equations. Exercises page 72

9.1 problem 220

Internal problem ID [15108]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8.3. The Lagrange and Clairaut equations. Exercises page 72
Problem number: 220.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y - 2y'x - \ln(y') = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 69

```
dsolve(y(x)=2*x*diff(y(x),x)+ln(diff(y(x),x)),y(x), singsol=all)
```

$$y(x) = -1 + \sqrt{4c_1x + 1} - \ln(2) + \ln\left(\frac{-1 + \sqrt{4c_1x + 1}}{x}\right)$$
$$y(x) = -1 - \sqrt{4c_1x + 1} - \ln(2) + \ln\left(\frac{-1 - \sqrt{4c_1x + 1}}{x}\right)$$

✓ Solution by Mathematica
Time used: 0.096 (sec). Leaf size: 32

```
DSolve[y[x]==2*x*y'[x]+Log[y'[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[W\left(2xe^{y(x)}\right) - \log\left(W\left(2xe^{y(x)}\right) + 2\right) - y(x) = c_1, y(x)\right]$$

9.2 problem 221

Internal problem ID [15109]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8.3. The Lagrange and Clairaut equations. Exercises page 72
Problem number: 221.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$y - x(1 + y') - y'^2 = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 36

```
dsolve(y(x)=x*(1+diff(y(x),x))+diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x) = x - \frac{x^2}{4} + \operatorname{LambertW}\left(\frac{c_1 e^{-1+\frac{x}{2}}}{2}\right)^2 + 2\operatorname{LambertW}\left(\frac{c_1 e^{-1+\frac{x}{2}}}{2}\right) + 1$$

✓ Solution by Mathematica
Time used: 2.322 (sec). Leaf size: 177

```
DSolve[y[x]==x*(1+y'[x])+y'[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}
&\text{Solve}\Big[-\sqrt{x^2 + 4y(x) - 4x} + 2\log\left(\sqrt{x^2 + 4y(x) - 4x} - x + 2\right) \\
&\quad - 2\log\left(-x\sqrt{x^2 + 4y(x) - 4x} + x^2 + 4y(x) - 2x - 4\right) + x = c_1, y(x)\Big] \\
&\text{Solve}\left[-4\operatorname{arctanh}\left(\frac{(x-5)\sqrt{x^2 + 4y(x) - 4x} - x^2 - 4y(x) + 7x - 6}{(x-3)\sqrt{x^2 + 4y(x) - 4x} - x^2 - 4y(x) + 5x - 2}\right)\right. \\
&\quad \left. + \sqrt{x^2 + 4y(x) - 4x} + x = c_1, y(x)\right]
\end{aligned}$$

9.3 problem 222

Internal problem ID [15110]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8.3. The Lagrange and Clairaut equations. Exercises page 72
Problem number: 222.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_dAlembert]`

$$\boxed{y - 2y'x - \sin(y') = 0}$$

✓ Solution by Maple
Time used: 0.11 (sec). Leaf size: 44

```
dsolve(y(x)=2*x*diff(y(x),x)+sin(diff(y(x),x)),y(x), singsol=all)
```

$$y(x) = 0$$
$$\left[x(\_T) = \frac{-\_T\sin(\_T) - \cos(\_T) + c_1}{\_T^2}, y(\_T) = \frac{-\_T\sin(\_T) - 2\cos(\_T) + 2c_1}{\_T}\right]$$

✓ Solution by Mathematica
Time used: 0.059 (sec). Leaf size: 47

```
DSolve[y[x]==2*x*y'[x]+Sin[y'[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x = \frac{-K[1]\sin(K[1]) - \cos(K[1])}{K[1]^2} + \frac{c_1}{K[1]^2}, y(x) = 2xK[1] + \sin(K[1])\right\}, \{y(x), K[1]\}\right]$$

9.4 problem 223

Internal problem ID [15111]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8.3. The Lagrange and Clairaut equations. Exercises page 72
Problem number: 223.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_dAlembert]`

$$\boxed{y - x{y'}^2 + \frac{1}{y'} = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 1835

```
dsolve(y(x)=x*diff(y(x),x)^2-1/diff(y(x),x),y(x), singsol=all)
```

$$\frac{12x^3\left(2\left(\left(\sqrt{3}\sqrt{\frac{-4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{2}{3}}y(x)+x\left(\frac{2^{\frac{1}{3}}\left(3^{\frac{1}{6}}\sqrt{\frac{-4y(x)^3+27x}{x}}+3\,3^{\frac{2}{3}}\right)\left(\left(\sqrt{3}\sqrt{\frac{-4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{1}{3}}}{2}+2^{\frac{2}{3}}3^{\frac{1}{3}}\cdots\right.\right.}{\left(2^{\frac{2}{3}}3^{\frac{1}{3}}\left(\left(\sqrt{3}\sqrt{\frac{-4y(x)^3+27x}{x}}+9\right)^2x^4\right)^{\frac{1}{3}}+2x\left(2^{\frac{1}{3}}3^{\frac{2}{3}}y\left(x\right)-3\left(\left(\sqrt{3}\sqrt{\frac{-4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{1}{3}}\right)\right)^2\left(y\left(x\right)\cdots\right.}$$

$$+x$$

$$-\frac{18x^4\left(2^{\frac{2}{3}}3^{\frac{5}{6}}\sqrt{\frac{-4y(x)^3+27x}{x}}\,x+2y(x)\,3^{\frac{2}{3}}2^{\frac{1}{3}}\left(\left(\sqrt{3}\sqrt{\frac{-4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{1}{3}}+9\,3^{\frac{1}{3}}2^{\frac{2}{3}}x-3\left(\left(\sqrt{3}\sqrt{\frac{-4y(x)^3\cdots}{x}}\right.\right.\right.}{\left(2y\left(x\right)3^{\frac{2}{3}}2^{\frac{1}{3}}x+2^{\frac{2}{3}}3^{\frac{1}{3}}\left(\left(\sqrt{3}\sqrt{\frac{-4y(x)^3+27x}{x}}+9\right)^2x^4\right)^{\frac{1}{3}}-6x\left(\left(\sqrt{3}\sqrt{\frac{-4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{1}{3}}\right)^2\left(\cdots\right.}$$

$$=0$$

$$-\frac{3x^3\left(\frac{8\left(\left(\sqrt{3}\sqrt{\frac{-4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{2}{3}}y(x)}{9}+x\left(\left(\left(\frac{i3^{\frac{2}{3}}}{9}-\frac{3^{\frac{1}{6}}}{9}\right)\sqrt{\frac{-4y(x)^3+27x}{x}}+i3^{\frac{1}{6}}-\frac{3^{\frac{2}{3}}}{3}\right)2^{\frac{1}{3}}\left(\left(\sqrt{3}\sqrt{\frac{-4y(x)^3\cdots}{x}}\right.\right.\right.\right.}{2\left(\left(i-\sqrt{3}\right)\left(\left(\sqrt{3}\sqrt{\frac{-4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{2}{3}}+\left(i3^{\frac{1}{3}}+3^{\frac{5}{6}}\right)2^{\frac{2}{3}}y\left(x\right)x\right)^2\left(-\frac{\left(i3^{\frac{5}{6}}+3^{\frac{1}{3}}\right)2^{\frac{2}{3}}\left(\left(\sqrt{3}\sqrt{\frac{-4y(x)^3+27x}{x}}+\cdots\right.\right.}{6}\right.}$$

$$+x$$

$$+\frac{216x^4\left(\left(\sqrt{3}\sqrt{\frac{-4y(x)^3+27x}{x}}+9\right)^2x^4\right)^{\frac{1}{3}}3^{\frac{1}{3}}2^{\frac{2}{3}}\left(-\left(\left(\sqrt{3}\sqrt{\frac{-4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{2}{3}}+y(x)\left(i3^{\frac{1}{6}}-\frac{3^{\frac{2}{3}}}{3}\right)2^{\frac{1}{3}}\left(\cdots\right.\right.}{\left(\left(1+i\sqrt{3}\right)\left(\left(\sqrt{3}\sqrt{\frac{-4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{2}{3}}+\left(-i3^{\frac{5}{6}}+3^{\frac{1}{3}}\right)xy\left(x\right)2^{\frac{2}{3}}\right)^2\left(\frac{\left(-3^{\frac{5}{6}}+i3^{\frac{1}{3}}\right)2^{\frac{2}{3}}\left(\left(\sqrt{3}\sqrt{\frac{-4y(x)^3+27x}{x}}\cdots\right.\right.}{2}\right.}$$

$$=0$$

$$\frac{3x^3\left(\left(\sqrt{3}\sqrt{\frac{-4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{2}{3}}c_1\left(-\frac{8\left(\left(\sqrt{3}\sqrt{\frac{-4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{2}{3}}y(x)}{9}+x\left(\left(\left(\frac{i3^{\frac{2}{3}}}{9}+\frac{3^{\frac{1}{6}}}{9}\right)\sqrt{\frac{-4y(x)^3+27x}{x}}\cdots\right.\right.\right.}{2\left(-\frac{\left(i3^{\frac{5}{6}}-3^{\frac{1}{3}}\right)2^{\frac{2}{3}}\left(\left(\sqrt{3}\sqrt{\frac{-4y(x)^3+27x}{x}}+9\right)^2x^4\right)^{\frac{1}{3}}}{6}+\left(2\left(\left(\sqrt{3}\sqrt{\frac{-4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{1}{3}}+2^{\frac{1}{3}}y\left(x\right)\left(i3^{\frac{1}{6}}+\frac{3^{\frac{2}{3}}}{3}\right)\right)x\cdots\right.}$$

$$+x$$

$$216x^4\left(\left(\sqrt{3}\sqrt{\frac{-4y(x)^3+27x}{x}}+9\right)^2x^4\right)^{\frac{1}{3}}3^{\frac{1}{3}}2^{\frac{2}{3}}\left(\left(\left(\sqrt{3}\sqrt{\frac{-4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{2}{3}}+y(x)\left(i3^{\frac{1}{6}}+\frac{3^{\frac{2}{3}}}{3}\right)2^{\frac{1}{3}}\left(\cdots\right.\right.$$

✓ <u>Solution by Mathematica</u>

Time used: 145.256 (sec). Leaf size: 19969

```
DSolve[y[x]==x*y'[x]^2-1/y'[x],y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

9.5 problem 224

Internal problem ID [15112]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8.3. The Lagrange and Clairaut equations. Exercises page 72
Problem number: 224.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_dAlembert]`

$$y - \frac{3xy'}{2} - e^{y'} = 0$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 201

```
dsolve(y(x)=3/2*x*diff(y(x),x)+exp(diff(y(x),x)),y(x), singsol=all)
```

$$y(x) = 1$$

$$\frac{27\left(\left(-2x^2 \operatorname{LambertW}\left(\frac{2\,e^{\frac{2y(x)}{3x}}}{3x}\right)^2 - 4\left(x - \frac{2y(x)}{3}\right) x \operatorname{LambertW}\left(\frac{2\,e^{\frac{2y(x)}{3x}}}{3x}\right) - 4x^2 + \frac{8xy(x)}{3} - \frac{8y(x)^2}{9}\right) e^{\frac{-3x \operatorname{La}}{}}\right.}{\left(3x \operatorname{Lan}\right.}$$

$$= 0$$

✓ Solution by Mathematica
Time used: 0.567 (sec). Leaf size: 52

```
DSolve[y[x]==3/2*x*y'[x]+Exp[y'[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x = -\frac{2e^{K[1]}(K[1]^2 - 2K[1] + 2)}{K[1]^3} + \frac{c_1}{K[1]^3}, y(x) = \frac{3}{2}xK[1] + e^{K[1]}\right\}, \{y(x), K[1]\}\right]$$

9.6 problem 225

Internal problem ID [15113]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8.3. The Lagrange and Clairaut equations. Exercises page 72
Problem number: 225.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y - xy' - \frac{a}{y'^2} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 76

```
dsolve(y(x)=x*diff(y(x),x)+a/diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x) = \frac{3\,2^{\frac{1}{3}} (a\,x^2)^{\frac{1}{3}}}{2}$$
$$y(x) = -\frac{3\,2^{\frac{1}{3}} (a\,x^2)^{\frac{1}{3}} \left(1 + i\sqrt{3}\right)}{4}$$
$$y(x) = \frac{3\,2^{\frac{1}{3}} (a\,x^2)^{\frac{1}{3}} \left(i\sqrt{3} - 1\right)}{4}$$
$$y(x) = \frac{c_1^3 x + a}{c_1^2}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 89

```
DSolve[y[x]==x*y'[x]+a/y'[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{a}{c_1{}^2} + c_1 x$$
$$y(x) \to \frac{3\sqrt[3]{a}x^{2/3}}{2^{2/3}}$$
$$y(x) \to -\frac{3\sqrt[3]{-1}\sqrt[3]{a}x^{2/3}}{2^{2/3}}$$
$$y(x) \to \frac{3(-1)^{2/3}\sqrt[3]{a}x^{2/3}}{2^{2/3}}$$

9.7 problem 226

Internal problem ID [15114]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8.3. The Lagrange and Clairaut equations. Exercises page 72
Problem number: 226.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y - xy' - y'^2 = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve(y(x)=x*diff(y(x),x)+diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{4}$$
$$y(x) = c_1(x + c_1)$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 23

```
DSolve[y[x]==x*y'[x]+y'[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x + c_1)$$
$$y(x) \to -\frac{x^2}{4}$$

9.8 problem 227

Internal problem ID [15115]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8.3. The Lagrange and Clairaut equations. Exercises page 72
Problem number: 227.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Clairaut]`

$$xy'^2 - yy' - y' = -1$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 38

```
dsolve(x*diff(y(x),x)^2-y(x)*diff(y(x),x)-diff(y(x),x)+1=0,y(x), singsol=all)
```

$$y(x) = -1 - 2\sqrt{x}$$
$$y(x) = -1 + 2\sqrt{x}$$
$$y(x) = \frac{c_1^2 x - c_1 + 1}{c_1}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 46

```
DSolve[x*y'[x]^2-y[x]*y'[x]-y'[x]+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x - 1 + \frac{1}{c_1}$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to -2\sqrt{x} - 1$$
$$y(x) \to 2\sqrt{x} - 1$$

9.9 problem 228

Internal problem ID [15116]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8.3. The Lagrange and Clairaut equations. Exercises page 72
Problem number: 228.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$y - xy' - a\sqrt{1 + y'^2} = 0$$

✓ Solution by Maple
Time used: 0.391 (sec). Leaf size: 17

```
dsolve(y(x)=x*diff(y(x),x)+a*sqrt(1+diff(y(x),x)^2),y(x), singsol=all)
```

$$y(x) = c_1 x + a\sqrt{c_1^2 + 1}$$

✓ Solution by Mathematica
Time used: 0.061 (sec). Leaf size: 27

```
DSolve[y[x]==x*y'[x]+a*Sqrt[1+y'[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to a\sqrt{1 + c_1{}^2} + c_1 x$$
$$y(x) \to a$$

9.10 problem 229

Internal problem ID [15117]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 8.3. The Lagrange and Clairaut equations. Exercises page 72
Problem number: 229.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Clairaut]`

$$-\frac{1}{y'^2} = -x + \frac{y}{y'}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 33

```
dsolve(x=y(x)/diff(y(x),x)+1/diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x) = -2\sqrt{-x}$$
$$y(x) = 2\sqrt{-x}$$
$$y(x) = c_1 x - \frac{1}{c_1}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 47

```
DSolve[x==y[x]/y'[x]+1/y'[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x - \frac{1}{c_1}$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to -2i\sqrt{x}$$
$$y(x) \to 2i\sqrt{x}$$

10 Section 9. The Riccati equation. Exercises page 75

10.1 problem 232

Internal problem ID [15118]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISE-LYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 9. The Riccati equation. Exercises page 75
Problem number: 232.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$
\boxed{y' \mathrm{e}^{-x} + y^2 - 2y\, \mathrm{e}^{x} = 1 - \mathrm{e}^{2x}}
$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)*exp(-x)+y(x)^2-2*y(x)*exp(x)=1-exp(2*x),y(x), singsol=all)
```

$$
y(x) = \frac{\mathrm{e}^{x} + \mathrm{e}^{2x} c_1 + c_1}{\mathrm{e}^{x} c_1 + 1}
$$

✓ Solution by Mathematica

Time used: 0.311 (sec). Leaf size: 24

```
DSolve[y'[x]*Exp[-x]+y[x]^2-2*y[x]*Exp[x]==1-Exp[2*x],y[x],x,IncludeSingularSolutions -> Tru
```

$$
\begin{aligned}
y(x) &\to e^x + \frac{1}{e^x + c_1} \\
y(x) &\to e^x
\end{aligned}
$$

10.2 problem 233

Internal problem ID [15119]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 9. The Riccati equation. Exercises page 75
Problem number: 233.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$\boxed{y' + y^2 - 2y\sin(x) = -\sin(x)^2 + \cos(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+y(x)^2-2*y(x)*sin(x)+sin(x)^2-cos(x)=0,y(x), singsol=all)
```

$$y(x) = \sin(x) + \frac{1}{x - c_1}$$

✓ Solution by Mathematica

Time used: 0.257 (sec). Leaf size: 20

```
DSolve[y'[x]+y[x]^2-2*y[x]*Sin[x]+Sin[x]^2-Cos[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \sin(x) + \frac{1}{x + c_1}$$
$$y(x) \to \sin(x)$$

10.3 problem 234

Internal problem ID [15120]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 9. The Riccati equation. Exercises page 75
Problem number: 234.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Riccati]`

$$\boxed{xy' - y^2 + (2x + 1)\, y = x^2 + 2x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 22

```
dsolve(x*diff(y(x),x)-y(x)^2+(2*x+1)*y(x)=x^2+2*x,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^2 - x - 1}{c_1 x - 1}$$

✓ Solution by Mathematica
Time used: 0.161 (sec). Leaf size: 34

```
DSolve[x*y'[x]-y[x]^2+(2*x+1)*y[x]==x^2+2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2 - c_1 x - c_1}{x - c_1}$$
$$y(x) \to x + 1$$

10.4 problem 235

Internal problem ID [15121]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 9. The Riccati equation. Exercises page 75
Problem number: 235.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Riccati]`

$$\boxed{x^2y' - y^2x^2 - yx = 1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x)=x^2*y(x)^2+x*y(x)+1,y(x), singsol=all)
```

$$y(x) = \frac{-\ln(x) + c_1 - 1}{x(-c_1 + \ln(x))}$$

✓ Solution by Mathematica
Time used: 0.163 (sec). Leaf size: 33

```
DSolve[x^2*y'[x]==x^2*y[x]^2+x*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\log(x) + 1 + c_1}{x\log(x) + c_1 x}$$
$$y(x) \to -\frac{1}{x}$$

11 Section 11. Singular solutions of differential equations. Exercises page 92

11.1 problem 260

Internal problem ID [15122]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 11. Singular solutions of differential equations. Exercises page 92
Problem number: 260.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{\left(1+y'^2\right) y^2 - 4yy' = 4x}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 71

```
dsolve((1+diff(y(x),x)^2)*y(x)^2-4*y(x)*diff(y(x),x)-4*x=0,y(x), singsol=all)
```

$$
\begin{aligned}
y(x) &= -2\sqrt{x+1} \\
y(x) &= 2\sqrt{x+1} \\
y(x) &= \sqrt{-c_1^2+2c_1x-x^2+4x+4} \\
y(x) &= -\sqrt{-x^2+(2c_1+4)\,x-c_1^2+4}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.459 (sec). Leaf size: 65

```
DSolve[(1+y'[x]^2)*y[x]^2-4*y[x]*y'[x]-4*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) &\to -\frac{1}{2}\sqrt{-4x^2-4(-4+c_1)x+16-c_1{}^2} \\
y(x) &\to \frac{1}{2}\sqrt{-4x^2-4(-4+c_1)x+16-c_1{}^2}
\end{aligned}
$$

11.2 problem 261

Internal problem ID [15123]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 11. Singular solutions of differential equations. Exercises page 92
Problem number: 261.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{{y'}^{2}-4y=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)^2-4*y(x)=0,y(x), singsol=all)
```

$$
\begin{aligned}
y(x) &= 0 \\
y(x) &= (x-c_1)^2
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.045 (sec). Leaf size: 38

```
DSolve[y'[x]^2-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) &\to \frac{1}{4}(-2x+c_1)^2 \\
y(x) &\to \frac{1}{4}(2x+c_1)^2 \\
y(x) &\to 0
\end{aligned}
$$

11.3 problem 262

Internal problem ID [15124]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 11. Singular solutions of differential equations. Exercises page 92
Problem number: 262.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y'^3 - 4xyy' + 8y^2 = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)^3-4*x*y(x)*diff(y(x),x)+8*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{4x^3}{27}$$
$$y(x) = 0$$
$$y(x) = \frac{(4c_1x - 1)^2}{64c_1^3}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]^3-4*x*y[x]*y'[x]+8*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

11.4 problem 263

Internal problem ID [15125]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 11. Singular solutions of differential equations. Exercises page 92
Problem number: 263.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - y^2 = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)^2-y(x)^2=0,y(x), singsol=all)
```

$$y(x) = e^x c_1$$
$$y(x) = c_1 e^{-x}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]^2-y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

11.5 problem 264

Internal problem ID [15126]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 11. Singular solutions of differential equations. Exercises page 92
Problem number: 264.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^{\frac{2}{3}} = a}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)=y(x)^(2/3)+a,y(x), singsol=all)
```

$$x - 3y(x)^{\frac{1}{3}} + 2\sqrt{a}\, \arctan\left(\frac{y(x)^{\frac{1}{3}}}{\sqrt{a}}\right) - \sqrt{a}\, \arctan\left(\frac{\sqrt{3}\,\sqrt{a} - 2y(x)^{\frac{1}{3}}}{\sqrt{a}}\right) + \sqrt{a}\, \arctan\left(\frac{2y(x)^{\frac{1}{3}} + \sqrt{3}\,\sqrt{a}}{\sqrt{a}}\right) - \sqrt{a}\, \arctan\left(\frac{y(x)}{a^{\frac{3}{2}}}\right) + c_1 = 0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]=y[x]^(2/3)+a,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

11.6 problem 265

Internal problem ID [15127]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 11. Singular solutions of differential equations. Exercises page 92
Problem number: 265.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G']]`

$$\boxed{(y'x+y)^2+3x^5(y'x-2y)=0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 65

```
dsolve((x*diff(y(x),x)+y(x))^2+3*x^5*(x*diff(y(x),x)-2*y(x))=0,y(x), singsol=all)
```

$$y(x) = -\frac{x^5}{4}$$
$$y(x) = \frac{c_1(x^3+c_1)}{x}$$
$$y(x) = \frac{c_1(-x^3+c_1)}{x}$$
$$y(x) = \frac{c_1(-x^3+c_1)}{x}$$
$$y(x) = \frac{c_1(x^3+c_1)}{x}$$

✓ Solution by Mathematica

Time used: 1.645 (sec). Leaf size: 94

```
DSolve[(x*y'[x]+y[x])^2+3*x^5*(x*y'[x]-2*y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{i(\cosh(3c_1) + \sinh(3c_1))\left(x^3 - i\cosh(3c_1) - i\sinh(3c_1)\right)}{x}$$
$$y(x) \to \frac{i(\cosh(3c_1) + \sinh(3c_1))\left(x^3 + i\cosh(3c_1) + i\sinh(3c_1)\right)}{x}$$
$$y(x) \to 0$$

11.7 problem 266

Internal problem ID [15128]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 11. Singular solutions of differential equations. Exercises page 92
Problem number: 266.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$y(y - 2xy')^2 - 2y' = 0$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 99

```
dsolve(y(x)*(y(x)-2*x*diff(y(x),x))^2=2*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = -\frac{1}{2\sqrt{-x}}$$
$$y(x) = \frac{1}{2\sqrt{-x}}$$
$$y(x) = 0$$
$$y(x) = \frac{\sqrt{(x + c_1)\, x}}{c_1 \sqrt{x}}$$
$$y(x) = \frac{\sqrt{x\,(x - c_1)}}{c_1 \sqrt{x}}$$
$$y(x) = -\frac{\sqrt{(x + c_1)\, x}}{c_1 \sqrt{x}}$$
$$y(x) = -\frac{\sqrt{x\,(x - c_1)}}{c_1 \sqrt{x}}$$

✓ Solution by Mathematica

Time used: 1.935 (sec). Leaf size: 158

```
DSolve[y[x]*(y[x]-2*x*y'[x])^2==2*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}\sqrt{e^{-2c_1}\left(2x - e^{c_1}\right)}$$
$$y(x) \to \sqrt{2}\sqrt{e^{-2c_1}\left(2x - e^{c_1}\right)}$$
$$y(x) \to -\sqrt{2}\sqrt{e^{-2c_1}\left(2x + e^{c_1}\right)}$$
$$y(x) \to \sqrt{2}\sqrt{e^{-2c_1}\left(2x + e^{c_1}\right)}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{i}{2\sqrt{x}}$$
$$y(x) \to \frac{i}{2\sqrt{x}}$$

11.8 problem 267

Internal problem ID [15129]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 11. Singular solutions of differential equations. Exercises page 92
Problem number: 267.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{8y'^3 - 12y'^2 - 27y = -27x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 37

```
dsolve(8*diff(y(x),x)^3-12*diff(y(x),x)^2=27*(y(x)-x),y(x), singsol=all)
```

$$y(x) = x - \frac{4}{27}$$
$$y(x) = (-x + c_1)\sqrt{x - c_1} + c_1$$
$$y(x) = (x - c_1)^{\frac{3}{2}} + c_1$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[8*y'[x]^3-12*y'[x]^2==27*(y[x]-x),y[x],x,IncludeSingularSolutions -> True]
```

Timed out

11.9 problem 268

Internal problem ID [15130]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 11. Singular solutions of differential equations. Exercises page 92
Problem number: 268.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type **[_quadrature]**

$$\boxed{(y' - 1)^2 - y^2 = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve((diff(y(x),x)-1)^2=y(x)^2,y(x), singsol=all)
```

$$y(x) = -1 + e^x c_1$$
$$y(x) = 1 + c_1 e^{-x}$$

✓ Solution by Mathematica
Time used: 0.074 (sec). Leaf size: 37

```
DSolve[(y'[x]-1)^2==y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 + c_1 e^{-x}$$
$$y(x) \to -1 + c_1 e^x$$
$$y(x) \to -1$$
$$y(x) \to 1$$

11.10 problem 269

Internal problem ID [15131]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 11. Singular solutions of differential equations. Exercises page 92
Problem number: 269.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y - y'^2 + xy' = x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 845

```
dsolve(y(x)=diff(y(x),x)^2-x*diff(y(x),x)+x^2/x,y(x), singsol=all)
```

$$y(x) = -2\left(\frac{-1+x}{\left(-12c_1+4\sqrt{-4x^3+9c_1^2+12x^2-12x+4}\right)^{\frac{1}{3}}} + \frac{\left(-12c_1+4\sqrt{-4x^3+9c_1^2+12x^2-12x+4}\right)^{\frac{1}{3}}}{4} - \frac{1}{2}\right)\left(\frac{-1+x}{\left(-12c_1+4\sqrt{-4x^3+9c_1^2+12x^2-12x+4}\right)^{\frac{1}{3}}} + \frac{\left(-12c_1+4\sqrt{-4x^3+9c_1^2+12x^2-12x+4}\right)^{\frac{1}{3}}}{4} + \frac{1}{2}\right)x + \frac{\left(1+\left(\frac{\left(-12c_1+4\sqrt{-4x^3+9c_1^2+12x^2-12x+4}\right)^{\frac{1}{3}}}{2} - \frac{2(1-x)}{\left(-12c_1+4\sqrt{-4x^3+9c_1^2+12x^2-12x+4}\right)^{\frac{1}{3}}}\right)^2\right)^2}{4}$$

$$y(x) = \frac{\left(1+\left(-\frac{i\left(-12c_1+4\sqrt{-4x^3+9c_1^2+12x^2-12x+4}\right)^{\frac{1}{3}}}{4} + \frac{i(1-x)}{\left(-12c_1+4\sqrt{-4x^3+9c_1^2+12x^2-12x+4}\right)^{\frac{1}{3}}} + \frac{\sqrt{3}\left(\frac{\left(-12c_1+4\sqrt{-4x^3+9c_1^2+12x^2\cdots}\right.}{2}\right.}{\cdots}\right.\right.}{2} + \frac{\left(1+\left(\frac{\left(-12c_1+4\sqrt{-4x^3+9c_1^2+12x^2-12x+4}\right)^{\frac{1}{3}}}{4} - \frac{1-x}{\left(-12c_1+4\sqrt{-4x^3+9c_1^2+12x^2-12x+4}\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(-12c_1+4\sqrt{-4x^3+9c_1^2+12x\cdots}\right.}{2}\right.}{\cdots}\right.\right.}{4}$$

$$y(x) = \frac{\left(1+\left(-\frac{i\left(-12c_1+4\sqrt{-4x^3+9c_1^2+12x^2-12x+4}\right)^{\frac{1}{3}}}{4} + \frac{i(1-x)}{\left(-12c_1+4\sqrt{-4x^3+9c_1^2+12x^2-12x+4}\right)^{\frac{1}{3}}} - \frac{\sqrt{3}\left(\frac{\left(-12c_1+4\sqrt{-4x^3+9c_1^2+12x^2\cdots}\right.}{2}\right.}{\cdots}\right.\right.}{2}$$

✓ Solution by Mathematica

Time used: 61.116 (sec). Leaf size: 2409

```
DSolve[y[x]==y'[x]^2-x*y'[x]+x^2/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2\sqrt[3]{2}x^4 - 8\sqrt[3]{2}x^3 + 12\sqrt[3]{2}x^2 + 4x^2\sqrt[3]{-2x^6 + 12x^5 - 30x^4 + 40x^3 - 10e^{3c_1}x^3 - 30x^2 + 30e^{3c_1}x^2 + 12x -}}{}$$

$$y(x) \to \frac{1}{4}(2x^2 - 2x + 3) + \frac{(1 + i\sqrt{3})(x-1)(-(x-1)^3 + 2e^{3c_1})}{4\,2^{2/3}\sqrt[3]{-2x^6 + 12x^5 - 30x^4 + 40x^3 - 10e^{3c_1}x^3 - 30x^2 + 30e^{3c_1}x^2 + 12x - 30e^{3c_1}x + \sqrt{e^{3c_1}(4(x-1)}}} + \frac{i(\sqrt{3}+i)\sqrt[3]{-2x^6 + 12x^5 - 30x^4 + 40x^3 - 10e^{3c_1}x^3 - 30x^2 + 30e^{3c_1}x^2 + 12x - 30e^{3c_1}x + \sqrt{e^{3c_1}(4(x}}}{8\sqrt[3]{2}}$$

$$y(x) \to \frac{1}{4}(2x^2 - 2x + 3) + \frac{i(\sqrt{3}+i)(x-1)((x-1)^3 - 2e^{3c_1})}{4\,2^{2/3}\sqrt[3]{-2x^6 + 12x^5 - 30x^4 + 40x^3 - 10e^{3c_1}x^3 - 30x^2 + 30e^{3c_1}x^2 + 12x - 30e^{3c_1}x + \sqrt{e^{3c_1}(4(x-1)}}} - \frac{i(\sqrt{3}-i)\sqrt[3]{-2x^6 + 12x^5 - 30x^4 + 40x^3 - 10e^{3c_1}x^3 - 30x^2 + 30e^{3c_1}x^2 + 12x - 30e^{3c_1}x + \sqrt{e^{3c_1}(4(x}}}{8\sqrt[3]{2}}$$

$$y(x) \to \frac{2\sqrt[3]{2}x^4 - 8\sqrt[3]{2}x^3 + 12\sqrt[3]{2}x^2 + 4x^2\sqrt[3]{-2x^6 + 12x^5 - 30x^4 + 40x^3 + 10e^{3c_1}x^3 - 30x^2 - 30e^{3c_1}x^2 + 12x +}}{}$$

$$y(x) \to \frac{1}{4}(2x^2 - 2x + 3) - \frac{i(\sqrt{3}-i)(x-1)((x-1)^3 + 2e^{3c_1})}{4\,2^{2/3}\sqrt[3]{-2x^6 + 12x^5 - 30x^4 + 40x^3 + 10e^{3c_1}x^3 - 30x^2 - 30e^{3c_1}x^2 + 12x + 30e^{3c_1}x + \sqrt{e^{3c_1}(-4(x-}}} + \frac{i(\sqrt{3}+i)\sqrt[3]{-2x^6 + 12x^5 - 30x^4 + 40x^3 + 10e^{3c_1}x^3 - 30x^2 - 30e^{3c_1}x^2 + 12x + 30e^{3c_1}x + \sqrt{e^{3c_1}(-4}}}{8\sqrt[3]{2}}$$

$$y(x) \to \frac{1}{4}(2x^2 - 2x + 3) + \frac{i(\sqrt{3}+i)(x-1)((x-1)^3 + 2e^{3c_1})}{4\,2^{2/3}\sqrt[3]{-2x^6 + 12x^5 - 30x^4 + 40x^3 + 10e^{3c_1}x^3 - 30x^2 - 30e^{3c_1}x^2 + 12x + 30e^{3c_1}x + \sqrt{e^{3c_1}(-4(x-}}} - \frac{i(\sqrt{3}-i)\sqrt[3]{-2x^6 + 12x^5 - 30x^4 + 40x^3 + 10e^{3c_1}x^3 - 30x^2 - 30e^{3c_1}x^2 + 12x + 30e^{3c_1}x + \sqrt{e^{3c_1}(-4}}}{8\sqrt[3]{2}}$$

11.11 problem 270

Internal problem ID [15132]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 11. Singular solutions of differential equations. Exercises page 92
Problem number: 270.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{(xy'+y)^2-y^2y'=0}$$

✓ Solution by Maple
Time used: 0.234 (sec). Leaf size: 124

```
dsolve((x*diff(y(x),x)+y(x))^2=y(x)^2*diff(y(x),x),y(x), singsol=all)
```

$$
\begin{aligned}
y(x) &= 4x \\
y(x) &= 0 \\
y(x) &= -\frac{2c_1^2\left(-\sqrt{2}\, c_1+x\right)}{-2c_1^2+x^2} \\
y(x) &= -\frac{2c_1^2\left(\sqrt{2}\, c_1+x\right)}{-2c_1^2+x^2} \\
y(x) &= \frac{c_1^3\sqrt{2}-2c_1^2x}{-2c_1^2+4x^2} \\
y(x) &= \frac{c_1^2\left(\sqrt{2}\, c_1+2x\right)}{2c_1^2-4x^2}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.635 (sec). Leaf size: 62

```
DSolve[(x*y'[x]+y[x])^2==y[x]^2*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{4e^{-2c_1}}{2 + e^{2c_1}x}$$
$$y(x) \to -\frac{e^{-2c_1}}{2 + 4e^{2c_1}x}$$
$$y(x) \to 0$$
$$y(x) \to 4x$$

11.12 problem 271

Internal problem ID [15133]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 11. Singular solutions of differential equations. Exercises page 92
Problem number: 271.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y^2 {y'}^2 + y^2 = 1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 51

```
dsolve(y(x)^2*diff(y(x),x)^2+y(x)^2=1,y(x), singsol=all)
```

$$y(x) = -1$$
$$y(x) = 1$$
$$y(x) = \sqrt{-c_1^2 + 2c_1 x - x^2 + 1}$$
$$y(x) = -\sqrt{-(x - c_1 + 1)(x - c_1 - 1)}$$

✓ Solution by Mathematica
Time used: 0.185 (sec). Leaf size: 119

```
DSolve[y[x]^2*y'[x]^2+y[x]^2==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^2 - 2c_1 x + 1 - c_1{}^2}$$
$$y(x) \to \sqrt{-x^2 - 2c_1 x + 1 - c_1{}^2}$$
$$y(x) \to -\sqrt{-x^2 + 2c_1 x + 1 - c_1{}^2}$$
$$y(x) \to \sqrt{-x^2 + 2c_1 x + 1 - c_1{}^2}$$
$$y(x) \to -1$$
$$y(x) \to 1$$

11.13 problem 272

Internal problem ID [15134]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 11. Singular solutions of differential equations. Exercises page 92
Problem number: 272.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y'^2 - yy' = -\mathrm{e}^x$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 34

```
dsolve(diff(y(x),x)^2-y(x)*diff(y(x),x)+exp(x)=0,y(x), singsol=all)
```

$$y(x) = -2\,\mathrm{e}^{\frac{x}{2}}$$
$$y(x) = 2\,\mathrm{e}^{\frac{x}{2}}$$
$$y(x) = \frac{\mathrm{e}^x c_1^2 + 1}{c_1}$$

✓ Solution by Mathematica

Time used: 60.179 (sec). Leaf size: 59

```
DSolve[y'[x]^2-y[x]*y'[x]+Exp[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-e^{-c_1}\left(-e^x + e^{c_1}\right)^2}$$
$$y(x) \to \sqrt{-e^{-c_1}\left(e^x - e^{c_1}\right)^2}$$

11.14 problem 273

Internal problem ID [15135]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 11. Singular solutions of differential equations. Exercises page 92
Problem number: 273.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{3xy'^2 - 6yy' + 2y = -x}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 32

```
dsolve(3*x*diff(y(x),x)^2-6*y(x)*diff(y(x),x)+x+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = x$$
$$y(x) = -\frac{x}{3}$$
$$y(x) = \frac{4c_1^2 + 2c_1 x + x^2}{6c_1}$$

✓ Solution by Mathematica
Time used: 0.307 (sec). Leaf size: 67

```
DSolve[3*x*y'[x]^2-6*y[x]*y'[x]+x+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{3}x\left(-1 + 2\cosh\left(-\log(x) + \sqrt{3}c_1\right)\right)$$
$$y(x) \to -\frac{1}{3}x\left(-1 + 2\cosh\left(\log(x) + \sqrt{3}c_1\right)\right)$$
$$y(x) \to -\frac{x}{3}$$
$$y(x) \to x$$

11.15 problem 274

Internal problem ID [15136]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 11. Singular solutions of differential equations. Exercises page 92
Problem number: 274.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Clairaut]`

$$y - xy' - \sqrt{a^2 y'^2 + b^2} = 0$$

✓ Solution by Maple
Time used: 0.266 (sec). Leaf size: 21

```
dsolve(y(x)=x*diff(y(x),x)+sqrt(a^2*diff(y(x),x)^2+b^2),y(x), singsol=all)
```

$$y(x) = c_1 x + \sqrt{a^2 c_1^2 + b^2}$$

✓ Solution by Mathematica
Time used: 0.359 (sec). Leaf size: 37

```
DSolve[y[x]==x*y'[x]+Sqrt[a^2*y'[x]^2+b^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{b^2 + a^2 c_1{}^2} + c_1 x$$
$$y(x) \to \sqrt{b^2}$$

12 Section 12. Miscellaneous problems. Exercises page 93

12.1 problem 275

Internal problem ID [15137]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 275.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Riccati]`

$$\boxed{y' - (-y+x)^2 = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)=(x-y(x))^2+1,y(x), singsol=all)
```

$$y(x) = \frac{c_1x + x^2 - 1}{x + c_1}$$

✓ Solution by Mathematica
Time used: 0.151 (sec). Leaf size: 20

```
DSolve[y'[x]==(x-y[x])^2+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + \frac{1}{-x + c_1}$$
$$y(x) \to x$$

12.2 problem 276

Internal problem ID [15138]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 276.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$x \sin(x) y' + (\sin(x) - \cos(x) x) y = \sin(x) \cos(x) - x$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve(x*sin(x)*diff(y(x),x)+(sin(x)-x*cos(x))*y(x)=sin(x)*cos(x)-x,y(x), singsol=all)
```

$$y(x) = \frac{\sin(x) c_1}{x} + \cos(x)$$

✓ Solution by Mathematica
Time used: 0.127 (sec). Leaf size: 16

```
DSolve[x*Sin[x]*y'[x]+(Sin[x]-x*Cos[x])*y[x]==Sin[x]*Cos[x]-x,y[x],x,IncludeSingularSolution
```

$$y(x) \to \cos(x) + \frac{c_1 \sin(x)}{x}$$

12.3 problem 277

Internal problem ID [15139]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 277.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + \cos(x)\, y - y^n \sin(2x) = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 49

```
dsolve(diff(y(x),x)+y(x)*cos(x)=y(x)^n*sin(2*x),y(x), singsol=all)
```

$$y(x) = \left(\frac{e^{\sin(x)(n-1)} c_1 n - e^{\sin(x)(n-1)} c_1 + 2\sin(x)\, n - 2\sin(x) + 2}{n-1}\right)^{-\frac{1}{n-1}}$$

✓ Solution by Mathematica
Time used: 6.877 (sec). Leaf size: 36

```
DSolve[y'[x]+y[x]*Cos[x]==y[x]^n*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(c_1 e^{(n-1)\sin(x)} + \frac{2}{n-1} + 2\sin(x)\right)^{\frac{1}{1-n}}$$

12.4 problem 278

Internal problem ID [15140]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 278.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$-3y^2x + (y^3 - 3yx^2)\, y' = -x^3$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 119

```
dsolve((x^3-3*x*y(x)^2)+(y(x)^3-3*x^2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{3c_1x^2 - \sqrt{8c_1^2x^4 + 1}}}{\sqrt{c_1}}$$
$$y(x) = \frac{\sqrt{3c_1x^2 + \sqrt{8c_1^2x^4 + 1}}}{\sqrt{c_1}}$$
$$y(x) = -\frac{\sqrt{3c_1x^2 - \sqrt{8c_1^2x^4 + 1}}}{\sqrt{c_1}}$$
$$y(x) = -\frac{\sqrt{3c_1x^2 + \sqrt{8c_1^2x^4 + 1}}}{\sqrt{c_1}}$$

✓ Solution by Mathematica

Time used: 7.89 (sec). Leaf size: 245

```
DSolve[(x^3-3*x*y[x]^2)+(y[x]^3-3*x^2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -\sqrt{3x^2 - \sqrt{8x^4 + e^{4c_1}}}$$
$$y(x) \to \sqrt{3x^2 - \sqrt{8x^4 + e^{4c_1}}}$$
$$y(x) \to -\sqrt{3x^2 + \sqrt{8x^4 + e^{4c_1}}}$$
$$y(x) \to \sqrt{3x^2 + \sqrt{8x^4 + e^{4c_1}}}$$
$$y(x) \to -\sqrt{3x^2 - 2\sqrt{2}\sqrt{x^4}}$$
$$y(x) \to \sqrt{3x^2 - 2\sqrt{2}\sqrt{x^4}}$$
$$y(x) \to -\sqrt{2\sqrt{2}\sqrt{x^4} + 3x^2}$$
$$y(x) \to \sqrt{2\sqrt{2}\sqrt{x^4} + 3x^2}$$

12.5 problem 279

Internal problem ID [15141]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 279.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, _dAlembert]`

$$5yx - 4y^2 + \left(y^2 - 8yx + \frac{5x^2}{2}\right) y' = 6x^2$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 443

```
dsolve((5*x*y(x)-4*y(x)^2-6*x^2)+(y(x)^2-8*x*y(x)+25/10*x^2)*diff(y(x),x)=0,y(x), singsol=al
```

$$y = \frac{\frac{\left(416x^3c_1^3+2+2\sqrt{3898c_1^6x^6+416x^3c_1^3+1}\right)^{\frac{1}{3}}}{2} + \frac{27x^2c_1^2}{\left(416x^3c_1^3+2+2\sqrt{3898c_1^6x^6+416x^3c_1^3+1}\right)^{\frac{1}{3}}} + 4c_1x}{c_1}$$

$$y = \frac{54i\sqrt{3}\,c_1^2x^2 - i\left(416x^3c_1^3+2+2\sqrt{3898c_1^6x^6+416x^3c_1^3+1}\right)^{\frac{2}{3}}\sqrt{3} - 54x^2c_1^2 + 16c_1x\left(416x^3c_1^3+2+2\sqrt{3\right.}}{4\left(416x^3c_1^3+2+2\sqrt{3898c_1^6x^6+416x^3\right.}}$$

$$y = -\frac{54i\sqrt{3}\,c_1^2x^2 - i\left(416x^3c_1^3+2+2\sqrt{3898c_1^6x^6+416x^3c_1^3+1}\right)^{\frac{2}{3}}\sqrt{3} + 54x^2c_1^2 - 16c_1x\left(416x^3c_1^3+2+2\right.}{4\left(416x^3c_1^3+2+2\sqrt{3898c_1^6x^6+416\right.}}$$

✓ Solution by Mathematica

Time used: 29.95 (sec). Leaf size: 741

```
DSolve[(5*x*y[x]-4*y[x]^2-6*x^2)+(y[x]^2-8*x*y[x]+25/10*x^2)*y'[x]==0,y[x],x,IncludeSingular
```

$$y(x) \to \frac{\sqrt[3]{208x^3 + \sqrt{3898x^6 + 416e^{3c_1}x^3 + e^{6c_1}} + e^{3c_1}}}{2^{2/3}} + \frac{27x^2}{\sqrt[3]{2}\sqrt[3]{208x^3 + \sqrt{3898x^6 + 416e^{3c_1}x^3 + e^{6c_1}} + e^{3c_1}}} + 4x$$

$$y(x) \to -\frac{\left(1 - i\sqrt{3}\right)\sqrt[3]{208x^3 + \sqrt{3898x^6 + 416e^{3c_1}x^3 + e^{6c_1}} + e^{3c_1}}}{2\ 2^{2/3}} - \frac{27\left(1 + i\sqrt{3}\right)x^2}{2\sqrt[3]{2}\sqrt[3]{208x^3 + \sqrt{3898x^6 + 416e^{3c_1}x^3 + e^{6c_1}} + e^{3c_1}}} + 4x$$

$$y(x) \to -\frac{\left(1 + i\sqrt{3}\right)\sqrt[3]{208x^3 + \sqrt{3898x^6 + 416e^{3c_1}x^3 + e^{6c_1}} + e^{3c_1}}}{2\ 2^{2/3}} - \frac{27\left(1 - i\sqrt{3}\right)x^2}{2\sqrt[3]{2}\sqrt[3]{208x^3 + \sqrt{3898x^6 + 416e^{3c_1}x^3 + e^{6c_1}} + e^{3c_1}}} + 4x$$

$$y(x) \to \frac{27\ 2^{2/3}x^2 + 8\sqrt[3]{\sqrt{3898}\sqrt{x^6} + 208x^3}x + \sqrt[3]{2}\left(\sqrt{3898}\sqrt{x^6} + 208x^3\right)^{2/3}}{2\sqrt[3]{\sqrt{3898}\sqrt{x^6} + 208x^3}}$$

$$y(x) \to \frac{27i2^{2/3}\sqrt{3}x^2 - 27\ 2^{2/3}x^2 + 16\sqrt[3]{\sqrt{3898}\sqrt{x^6} + 208x^3}x - i\sqrt[3]{2}\sqrt{3}\left(\sqrt{3898}\sqrt{x^6} + 208x^3\right)^{2/3} - \sqrt[3]{2}\left(\sqrt{3898}\right.}{4\sqrt[3]{\sqrt{3898}\sqrt{x^6} + 208x^3}}$$

$$y(x) \to \frac{\left(\sqrt{3898}\sqrt{x^6} + 208x^3\right)^{2/3}\text{Root}\left[\#1^3 - 16\&, 3\right] - 54\sqrt[3]{-1}2^{2/3}x^2 + 16\sqrt[3]{\sqrt{3898}\sqrt{x^6} + 208x^3}x}{4\sqrt[3]{\sqrt{3898}\sqrt{x^6} + 208x^3}}$$

12.6 problem 280

Internal problem ID [15142]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 280.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$3y^2x + \left(3yx^2 - 6y^2 - 1\right) y' = x^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 771

```
dsolve((3*x*y(x)^2-x^2)+(3*x^2*y(x)-6*y(x)^2-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{\left(-108x^2 - 144x^3 + 432c_1 + 27x^6 + 12\sqrt{-54x^9 + 162c_1x^6 + 144x^6 + 216x^5 - 864c_1x^3 - 27x^4 - 648c_1}}{12} + \frac{3x^4 - 8}{4\left(-108x^2 - 144x^3 + 432c_1 + 27x^6 + 12\sqrt{-54x^9 + 162c_1x^6 + 144x^6 + 216x^5 - 864c_1x^3 - 27x^4 - 64}\right.} + \frac{x^2}{4}$$

$$y = \frac{24 + i\left(-24 + 9x^4 - \left(-108x^2 - 144x^3 + 432c_1 + 27x^6 + 12\sqrt{-54x^9 + (162c_1 + 144)x^6 + 216x^5 - 27x\right.\right.}{}$$

$$y = \frac{24 + i\left(-9x^4 + \left(-108x^2 - 144x^3 + 432c_1 + 27x^6 + 12\sqrt{-54x^9 + (162c_1 + 144)x^6 + 216x^5 - 27x^4 - 8\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 7.603 (sec). Leaf size: 570

```
DSolve[(3*x*y[x]^2-x^2)+(3*x^2*y[x]-6*y[x]^2-1)*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{x^2}{4} - \frac{\sqrt[3]{-\frac{27x^6}{4} + 36x^3 + 27x^2 + \sqrt{4\left(6 - \frac{9x^4}{4}\right)^3 + \left(-\frac{27x^6}{4} + 36x^3 + 27x^2 + 108c_1\right)^2} + 108c_1}}{6\sqrt[3]{2}} + \frac{6 - \frac{9x^4}{4}}{3\ 2^{2/3}\sqrt[3]{-\frac{27x^6}{4} + 36x^3 + 27x^2 + \sqrt{4\left(6 - \frac{9x^4}{4}\right)^3 + \left(-\frac{27x^6}{4} + 36x^3 + 27x^2 + 108c_1\right)^2} + 108c_1}}$$

$$y(x) \to \frac{x^2}{4} + \frac{\left(1 - i\sqrt{3}\right)\sqrt[3]{-\frac{27x^6}{4} + 36x^3 + 27x^2 + \sqrt{4\left(6 - \frac{9x^4}{4}\right)^3 + \left(-\frac{27x^6}{4} + 36x^3 + 27x^2 + 108c_1\right)^2} + 108c_1}}{12\sqrt[3]{2}} - \frac{\left(1 + i\sqrt{3}\right)\left(6 - \frac{9x^4}{4}\right)}{6\ 2^{2/3}\sqrt[3]{-\frac{27x^6}{4} + 36x^3 + 27x^2 + \sqrt{4\left(6 - \frac{9x^4}{4}\right)^3 + \left(-\frac{27x^6}{4} + 36x^3 + 27x^2 + 108c_1\right)^2} + 108c_1}}$$

$$y(x) \to \frac{x^2}{4} + \frac{\left(1 + i\sqrt{3}\right)\sqrt[3]{-\frac{27x^6}{4} + 36x^3 + 27x^2 + \sqrt{4\left(6 - \frac{9x^4}{4}\right)^3 + \left(-\frac{27x^6}{4} + 36x^3 + 27x^2 + 108c_1\right)^2} + 108c_1}}{12\sqrt[3]{2}} - \frac{\left(1 - i\sqrt{3}\right)\left(6 - \frac{9x^4}{4}\right)}{6\ 2^{2/3}\sqrt[3]{-\frac{27x^6}{4} + 36x^3 + 27x^2 + \sqrt{4\left(6 - \frac{9x^4}{4}\right)^3 + \left(-\frac{27x^6}{4} + 36x^3 + 27x^2 + 108c_1\right)^2} + 108c_1}}$$

12.7 problem 281

Internal problem ID [15143]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 281.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y - xy^2 \ln(x) + xy' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve((y(x)-x*y(x)^2*ln(x))+(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = -\frac{2}{\left(\ln(x)^2 - 2c_1\right) x}$$

✓ Solution by Mathematica
Time used: 0.153 (sec). Leaf size: 27

```
DSolve[(y[x]-x*y[x]^2*Log[x])+x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{-x \log^2(x) + 2c_1 x}$$
$$y(x) \to 0$$

12.8 problem 282

Internal problem ID [15144]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 282.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$
\boxed{2xy\,\mathrm{e}^{x^2}+\mathrm{e}^{x^2}y'=x\sin(x)}
$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve((2*x*y(x)*exp(x^2)-x*sin(x))+(exp(x^2))*diff(y(x),x)=0,y(x), singsol=all)
```

$$
y=(\sin(x)-x\cos(x)+c_1)\,\mathrm{e}^{-x^2}
$$

✓ Solution by Mathematica
Time used: 0.092 (sec). Leaf size: 23

```
DSolve[(2*x*y[x]*Exp[x^2]-x*Sin[x])+Exp[x^2]*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$
y(x)\to e^{-x^2}(\sin(x)-x\cos(x)+c_1)
$$

12.9 problem 283

Internal problem ID [15145]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISE-LYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 283.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_exponential_symmetries]]`

$$\boxed{y' - \frac{1}{2x - y^2} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)=1/(2*x-y(x)^2),y(x), singsol=all)
```

$$x - \frac{y^2}{2} - \frac{y}{2} - \frac{1}{4} - e^{2y} c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.135 (sec). Leaf size: 31

```
DSolve[y'[x]==1/(2*x-y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x = \frac{1}{4}\left(2y(x)^2 + 2y(x) + 1\right) + c_1 e^{2y(x)}, y(x)\right]$$

12.10 problem 284

Internal problem ID [15146]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 284.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xy' - y' = -x^2 + 3x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^2+x*diff(y(x),x)=3*x+diff(y(x),x),y(x), singsol=all)
```

$$y = -\frac{x^2}{2} + 2x + 2\ln(x-1) + c_1$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 24

```
DSolve[x^2+x*y'[x]==3*x+y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}(x-1)^2 + x + 2\log(x-1) + c_1$$

12.11 problem 285

Internal problem ID [15147]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 285.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{y'yx - y^2 = x^4}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(x*y(x)*diff(y(x),x)-y(x)^2=x^4,y(x), singsol=all)
```

$$y = \sqrt{x^2 + c_1}\, x$$
$$y = -\sqrt{x^2 + c_1}\, x$$

✓ Solution by Mathematica
Time used: 0.166 (sec). Leaf size: 34

```
DSolve[x*y[x]*y'[x]-y[x]^2==x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\sqrt{x^2 + c_1}$$
$$y(x) \to x\sqrt{x^2 + c_1}$$

12.12 problem 286

Internal problem ID [15148]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 286.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{\frac{1}{x^2 - yx + y^2} - \frac{y'}{2y^2 - yx} = 0}$$

✓ Solution by Maple

Time used: 0.938 (sec). Leaf size: 40

```
dsolve(1/(x^2-x*y(x)+y(x)^2)=diff(y(x),x)/(2*y(x)^2-x*y(x)),y(x), singsol=all)
```

$$y = \left(\operatorname{RootOf}\left(\_Z^8 c_1 x^2 + 2\_Z^6 c_1 x^2 - \_Z^4 - 2\_Z^2 - 1\right)^2 + 2\right) x$$

✓ Solution by Mathematica

Time used: 60.201 (sec). Leaf size: 1805

```
DSolve[1/(x^2-x*y[x]+y[x]^2)==y'[x]/(2*y[x]^2-x*y[x]),y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{6}\left(-\sqrt{3}\sqrt{\sqrt[3]{54e^{2c_1}x^4+6\sqrt{3}\sqrt{e^{4c_1}x^4\left(27x^4+e^{4c_1}\right)}+e^{6c_1}}+\frac{e^{4c_1}}{\sqrt[3]{54e^{2c_1}x^4+6\sqrt{3}\sqrt{e^{4c_1}x^4\left(27x^4+e^{4c_1}\right)}+}}}\right.$$
$$-\sqrt{3}\sqrt{-\sqrt[3]{54e^{2c_1}x^4+6\sqrt{3}\sqrt{e^{4c_1}x^4\left(27x^4+e^{4c_1}\right)}+e^{6c_1}}-\frac{e^{4c_1}}{\sqrt[3]{54e^{2c_1}x^4+6\sqrt{3}\sqrt{e^{4c_1}x^4\left(27x^4+e^{4c_1}\right)}+e^{}}}}$$
$$\left.+9x\right)$$

$$y(x) \to \frac{1}{6}\left(-\sqrt{3}\sqrt{\sqrt[3]{54e^{2c_1}x^4+6\sqrt{3}\sqrt{e^{4c_1}x^4\left(27x^4+e^{4c_1}\right)}+e^{6c_1}}+\frac{e^{4c_1}}{\sqrt[3]{54e^{2c_1}x^4+6\sqrt{3}\sqrt{e^{4c_1}x^4\left(27x^4+e^{4c_1}\right)}+}}}\right.$$
$$+\sqrt{3}\sqrt{-\sqrt[3]{54e^{2c_1}x^4+6\sqrt{3}\sqrt{e^{4c_1}x^4\left(27x^4+e^{4c_1}\right)}+e^{6c_1}}-\frac{e^{4c_1}}{\sqrt[3]{54e^{2c_1}x^4+6\sqrt{3}\sqrt{e^{4c_1}x^4\left(27x^4+e^{4c_1}\right)}+e^{}}}}$$
$$\left.+9x\right)$$

$y(x)$

$$\frac{1}{\ } \quad e^{4c_1}$$

12.13 problem 287

Internal problem ID [15149]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 287.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(2x-1)\,y'-2y=\frac{1-4x}{x^2}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve((2*x-1)*diff(y(x),x)-2*y(x)=(1-4*x)/x^2,y(x), singsol=all)
```

$$y=(2x-1)\,c_1+\frac{1}{x}$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 18

```
DSolve[(2*x-1)*y'[x]-2*y[x]==(1-4*x)/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{x}+2c_1x-c_1$$

12.14 problem 288

Internal problem ID [15150]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 288.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{-y+(3x+y+1)\,y'=-x-3}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 29

```
dsolve((x-y(x)+3)+(3*x+y(x)+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y=2-\frac{(x+1)\,(\text{LambertW}\,(-2c_1(x+1))-2)}{\text{LambertW}\,(-2c_1\,(x+1))}$$

✓ Solution by Mathematica
Time used: 0.771 (sec). Leaf size: 163

```
DSolve[(x-y[x]+3)+(3*x+y[x]+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2^{2/3}\left(x\left(-\log\left(\frac{3\ 2^{2/3}(y(x)+x-1)}{y(x)+3x+1}\right)\right)+(x-1)\log\left(\frac{6\ 2^{2/3}(x+1)}{y(x)+3x+1}\right)+\log\left(\frac{3\ 2^{2/3}(y(x)+x-1)}{y(x)+3x+1}\right)+y(x)\left(\log\left(\frac{6}{y(}\right.\right.\right.}{9(y(x)+x-1)}\right.$$

12.15 problem 289

Internal problem ID [15151]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 289.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' + \cos\left(\frac{y}{2} + \frac{x}{2}\right) - \cos\left(-\frac{y}{2} + \frac{x}{2}\right) = 0$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 62

```
dsolve(diff(y(x),x)+cos((x+y(x))/2)=cos((x-y(x))/2),y(x), singsol=all)
```

$$y = 2\arctan\left(\frac{2\,\mathrm{e}^{-2\cos\left(\frac{x}{2}\right)}c_1}{\mathrm{e}^{-4\cos\left(\frac{x}{2}\right)}c_1^2 + 1}, \frac{-\mathrm{e}^{-4\cos\left(\frac{x}{2}\right)}c_1^2 + 1}{\mathrm{e}^{-4\cos\left(\frac{x}{2}\right)}c_1^2 + 1}\right)$$

✓ Solution by Mathematica
Time used: 0.486 (sec). Leaf size: 70

```
DSolve[y'[x]+Cos[(x+y[x])/2]==Cos[(x-y[x])/2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2\arccos\left(\tanh\left(\frac{1}{2}\left(4\cos\left(\frac{x}{2}\right) - c_1\right)\right)\right)$$
$$y(x) \to 2\arccos\left(\tanh\left(\frac{1}{2}\left(4\cos\left(\frac{x}{2}\right) - c_1\right)\right)\right)$$
$$y(x) \to 0$$
$$y(x) \to -2\pi$$
$$y(x) \to 2\pi$$

12.16 problem 290

Internal problem ID [15152]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 290.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'(3x^2 - 2x) - y(6x - 2) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)*(3*x^2-2*x)-y(x)*(6*x-2)=0,y(x), singsol=all)
```

$$y = c_1 x(3x - 2)$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 19

```
DSolve[y'[x]*(3*x^2-2*x)-y[x]*(6*x-2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(2 - 3x)x$$
$$y(x) \to 0$$

12.17 problem 291

Internal problem ID [15153]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 291.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _rational, _Bernoulli]`

$$
\boxed{x y^2 y' - y^3 = \frac{x^4}{3}}
$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 49

```
dsolve(x*y(x)^2*diff(y(x),x)-y(x)^3=1/3*x^4,y(x), singsol=all)
```

$$
\begin{aligned}
y &= (x+c_1)^{\frac{1}{3}} x \\
y &= -\frac{(x+c_1)^{\frac{1}{3}} \left(1+i\sqrt{3}\right) x}{2} \\
y &= \frac{(x+c_1)^{\frac{1}{3}} \left(i\sqrt{3}-1\right) x}{2}
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.174 (sec). Leaf size: 54

```
DSolve[x*y[x]^2*y'[x]-y[x]^3==1/3*x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) &\to x\sqrt[3]{x+c_1} \\
y(x) &\to -\sqrt[3]{-1}x\sqrt[3]{x+c_1} \\
y(x) &\to (-1)^{2/3}x\sqrt[3]{x+c_1}
\end{aligned}
$$

12.18 problem 292

Internal problem ID [15154]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 292.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _dAlembert]`

$$\boxed{e^{\frac{x}{y}} + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right) y' = -1}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple
Time used: 0.312 (sec). Leaf size: 21

```
dsolve([(1+exp(x/y(x)))+(exp(x/y(x))*(1-x/y(x)))*diff(y(x),x)=0,y(1) = 1],y(x), singsol=all)
```

$$y = -\frac{x}{\text{LambertW}\left(\frac{x}{-1+x-e}\right)}$$

✓ Solution by Mathematica
Time used: 1.228 (sec). Leaf size: 21

```
DSolve[{(1+Exp[x/y[x]])+(Exp[x/y[x]]*(1-x/y[x]))*y'[x]==0,{y[1]==1}},y[x],x,IncludeSingularS
```

$$y(x) \to -\frac{x}{W\left(\frac{x}{x-e-1}\right)}$$

12.19 problem 293

Internal problem ID [15155]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 293.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{y^2 - y'yx = -x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve((x^2+y(x)^2)-x*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \sqrt{2\ln(x) + c_1}\, x$$
$$y = -\sqrt{2\ln(x) + c_1}\, x$$

✓ Solution by Mathematica
Time used: 0.159 (sec). Leaf size: 36

```
DSolve[(x^2+y[x]^2)-x*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\sqrt{2\log(x) + c_1}$$
$$y(x) \to x\sqrt{2\log(x) + c_1}$$

12.20 problem 294

Internal problem ID [15156]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 294.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$-y + (x - y + 3)\, y' = -x - 2$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 19

```
dsolve((x-y(x)+2)+(x-y(x)+3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = x - \frac{\mathrm{LambertW}\left(-c_1 \mathrm{e}^{5+4x}\right)}{2} + \frac{5}{2}$$

✓ Solution by Mathematica
Time used: 3.14 (sec). Leaf size: 35

```
DSolve[(x-y[x]+2)+(x-y[x]+3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2} W\left(-e^{4x-1+c_1}\right) + x + \frac{5}{2}$$
$$y(x) \to x + \frac{5}{2}$$

12.21 problem 295

Internal problem ID [15157]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 295.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{y^2x + y - xy' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve((x*y(x)^2+y(x))-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = -\frac{2x}{x^2 - 2c_1}$$

✓ Solution by Mathematica
Time used: 0.124 (sec). Leaf size: 23

```
DSolve[(x*y[x]^2+y[x])-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2x}{x^2 - 2c_1}$$
$$y(x) \to 0$$

12.22 problem 296

Internal problem ID [15158]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 296.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{y^2 + 2yy' = -x^2 - 2x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 37

```
dsolve((x^2+y(x)^2+2*x)+(2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \sqrt{c_1 \mathrm{e}^{-x} - x^2}$$
$$y = -\sqrt{c_1 \mathrm{e}^{-x} - x^2}$$

✓ Solution by Mathematica
Time used: 5.559 (sec). Leaf size: 47

```
DSolve[(x^2+y[x]^2+2*x)+(2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^2 + c_1 e^{-x}}$$
$$y(x) \to \sqrt{-x^2 + c_1 e^{-x}}$$

12.23 problem 297

Internal problem ID [15159]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 297.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$(x-1)\left(y^2-y+1\right)-(y-1)\left(x^2+x+1\right)y'=0$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 2397

```
dsolve(((x-1)*(y(x)^2-y(x)+1))=((y(x)-1)*(x^2+x+1))*diff(y(x),x),y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica
Time used: 0.578 (sec). Leaf size: 96

```
DSolve[((x-1)*(y[x]^2-y[x]+1))==((y[x]-1)*(x^2+x+1))*y'[x],y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \text{InverseFunction}\left[\frac{1}{2}\log\left(\#1^2-\#1+1\right) - \frac{\arctan\left(\frac{2\#1-1}{\sqrt{3}}\right)}{\sqrt{3}}\&\right]\left[-\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(x^2+x+1\right)+c_1\right]$$
$$y(x) \to \sqrt[3]{-1}$$
$$y(x) \to -(-1)^{2/3}$$

12.24 problem 298

Internal problem ID [15160]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 298.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{(x - 2yx - y^2)\, y' + y^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve((x-2*x*y(x)-y(x)^2)*diff(y(x),x)+y(x)^2=0,y(x), singsol=all)
```

$$y = \frac{1}{\operatorname{RootOf}\left(-\_Z^2 x + e^{-\_Z} c_1 + 1\right)}$$

✓ Solution by Mathematica
Time used: 0.143 (sec). Leaf size: 23

```
DSolve[(x-2*x*y[x]-y[x]^2)*y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x = y(x)^2 + c_1 e^{\frac{1}{y(x)}} y(x)^2, y(x)\right]$$

12.25 problem 299

Internal problem ID [15161]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 299.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]'], [_Abel, '2nd type`

$$\boxed{\cos(x)\, y + (2y - \sin(x))\, y' = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve(y(x)*cos(x)+(2*y(x)-sin(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = -\frac{\sin(x)}{2\,\mathrm{LambertW}\left(-\frac{\sin(x)\mathrm{e}^{\frac{c_1}{2}}}{2}\right)}$$

✓ Solution by Mathematica

Time used: 10.969 (sec). Leaf size: 349

```
DSolve[y[x]*Cos[x]+(2*y[x]-Sin[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[-\frac{\sqrt[3]{-2}\left(\frac{2^{2/3}\cos(x)(4y(x)+\sin(x))}{\sqrt[3]{-\cos^3(x)}(\sin(x)-2y(x))}+(-2)^{2/3}\right)\left(\frac{\left(-\cos^3(x)\right)^{2/3}\sec^2(x)(4y(x)+\sin(x))}{\sqrt[3]{2}(\sin(x)-2y(x))}+(-2)^{2/3}\right)\left(-\log\left(\frac{\ldots}{\sqrt[3]{\ldots}}\right.\right.}{\ldots}\right.$$

12.26 problem 300

Internal problem ID [15162]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 300.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - e^{x+2y} = 1}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 30

```
dsolve(diff(y(x),x)-1=exp(x+2*y(x)),y(x), singsol=all)
```

$$y = -\frac{x}{2} + \frac{\ln(3)}{2} + \frac{\ln\left(\frac{e^{3x}}{-2\,e^{3x}+c_1}\right)}{2}$$

✓ Solution by Mathematica
Time used: 0.931 (sec). Leaf size: 26

```
DSolve[y'[x]-1==Exp[x+2*y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - \frac{1}{2}\log\left(-\frac{2}{3}\left(e^{3x} + 3c_1\right)\right)$$

12.27 problem 301

Internal problem ID [15163]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 301.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$4yx^3 - 2y^2x + (y^2 + 2yx^2 - x^4)\, y' = -2x^5$$

✓ Solution by Maple
Time used: 0.109 (sec). Leaf size: 53

```
dsolve(2*(x^5+2*x^3*y(x)-y(x)^2*x)+(y(x)^2+2*x^2*y(x)-x^4)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{c_1}{2} - \frac{\sqrt{-4x^4 + 4c_1x^2 + c_1^2}}{2}$$
$$y = \frac{c_1}{2} + \frac{\sqrt{-4x^4 + 4c_1x^2 + c_1^2}}{2}$$

✓ Solution by Mathematica
Time used: 15.349 (sec). Leaf size: 87

```
DSolve[2*(x^5+2*x^3*y[x]-y[x]^2*x)+(y[x]^2+2*x^2*y[x]-x^4)*y'[x]==0,y[x],x,IncludeSingularSo
```

$$y(x) \to \frac{1}{2}\left(e^{2c_1} - \sqrt{-4x^4 + 4e^{2c_1}x^2 + e^{4c_1}}\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{-4x^4 + 4e^{2c_1}x^2 + e^{4c_1}} + e^{2c_1}\right)$$

12.28 problem 302

Internal problem ID [15164]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 302.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{x^2y^ny' - 2xy' + y = 0}$$

✓ Solution by Maple
Time used: 0.203 (sec). Leaf size: 32

```
dsolve(x^2*y(x)^n*diff(y(x),x)=2*x*diff(y(x),x)-y(x),y(x), singsol=all)
```

$$y^{2n}(y^n x - n - 2)^n x^{-n} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.18 (sec). Leaf size: 41

```
DSolve[x^2*y[x]^n*y'[x]==2*x*y'[x]-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{n(\log(x) - \log(-xy(x)^n + n + 2))}{n+2} - \frac{2n\log(y(x))}{n+2} = c_1, y(x)\right]$$

12.29 problem 303

Internal problem ID [15165]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 303.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(3y + 3x + a^2)\, y' - 4y = b^2 + 4x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 79

```
dsolve((3*(x+y(x))+a^2)*diff(y(x),x)=4*(x+y(x))+b^2,y(x), singsol=all)
```

$$y = \frac{(4a^2 - 3b^2)\,\mathrm{LambertW}\left(\frac{3\,\mathrm{e}^{\frac{3a^2+3b^2-49c_1+49x}{4a^2-3b^2}}}{4a^2-3b^2}\right)}{21} - \frac{a^2}{7} - \frac{b^2}{7} - x$$

✓ Solution by Mathematica
Time used: 60.042 (sec). Leaf size: 97

```
DSolve[(3*(x+y[x])+a^2)*y'[x]==4*(x+y[x])+b^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{21}\left(-3\left(a^2+b^2+7x\right) + \left(4a^2-3b^2\right) W\left(-4\left(2^{\frac{3b^2}{2a^2}-2} e^{\frac{49x-3b^2(-1+c_1)}{4a^2}-1+c_1}\right)^{\frac{4a^2}{4a^2-3b^2}}\right)\right)$$

12.30 problem 304

Internal problem ID [15166]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 304.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$\boxed{-y^2 + 2y'yx = -x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve((x-y(x)^2)+2*x*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \sqrt{-x(\ln(x) - c_1)}$$
$$y = -\sqrt{(-\ln(x) + c_1)\, x}$$

✓ Solution by Mathematica
Time used: 0.196 (sec). Leaf size: 44

```
DSolve[(x-y[x]^2)+2*x*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x}\sqrt{-\log(x) + c_1}$$
$$y(x) \to \sqrt{x}\sqrt{-\log(x) + c_1}$$

12.31 problem 305

Internal problem ID [15167]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 305.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{xy' + y - y^2 \ln(x) = 0}$$

With initial conditions

$$\left[y(1) = \frac{1}{2}\right]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 11

```
dsolve([x*diff(y(x),x)+y(x)=y(x)^2*ln(x),y(1) = 1/2],y(x), singsol=all)
```

$$y = \frac{1}{1 + x + \ln(x)}$$

✓ Solution by Mathematica
Time used: 0.142 (sec). Leaf size: 12

```
DSolve[{x*y'[x]+y[x]==y[x]^2*Log[x],{y[1]==1/2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{x + \log(x) + 1}$$

12.32 problem 306

Internal problem ID [15168]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 306.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{-\cos(\ln(y))\, y' = -\sin(\ln(x))}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 81

```
dsolve(sin(ln(x))-cos(ln(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \mathrm{e}^{\mathrm{RootOf}\left(-2\cos(\ln(x))x^2\sin(\ln(x))-2\sin(\ln(x))\sin(\_Z)\mathrm{e}^{\_Z}x+2\sin(\_Z)\cos(\ln(x))\mathrm{e}^{\_Z}x-2\,\mathrm{e}^{2\_Z}\cos(\_Z)^2+4c_1x\sin(\ln(x))-4\cos(\ln(x))c_1x-\right.}$$

✓ Solution by Mathematica

Time used: 0.386 (sec). Leaf size: 47

```
DSolve[Sin[Log[x]]-Cos[Log[y[x]]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{1}{2}\#1\sin(\log(\#1))+\frac{1}{2}\#1\cos(\log(\#1))\&\right]\left[\frac{1}{2}x\sin(\log(x)) - \frac{1}{2}x\cos(\log(x))+c_1\right]$$

12.33 problem 307

Internal problem ID [15169]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 307.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y' - \sqrt{\frac{9y^2 - 6y + 2}{x^2 - 2x + 5}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 71

```
dsolve(diff(y(x),x)=sqrt(  (9*y(x)^2-6*y(x)+2)/ (x^2-2*x+5)   ),y(x), singsol=all)
```

$$-\frac{\sqrt{\frac{9y^2-6y+2}{x^2-2x+5}}\,\sqrt{x^2-2x+5}\,\operatorname{arcsinh}\left(\frac{x}{2}-\frac{1}{2}\right)}{\sqrt{9y^2-6y+2}} + \frac{\operatorname{arcsinh}(3y-1)}{3} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 5.598 (sec). Leaf size: 160

```
DSolve[y'[x]==Sqrt[  (9*y[x]^2-6*y[x]+2)/ (x^2-2*x+5)   ],y[x],x,IncludeSingularSolutions ->
```

$$\begin{aligned} y(x) \to \frac{1}{96}\Big(& e^{3c_1}\left(x^3 + \left(\sqrt{x^2-2x+5}-3\right)x^2 - 2\left(\sqrt{x^2-2x+5}-3\right)x \right.\\ &\left.+ 2\left(\sqrt{x^2-2x+5}-2\right)\right) - 64e^{-3c_1}\left(-x^3 + \left(\sqrt{x^2-2x+5}+3\right)x^2 \right.\\ &\left. - 2\left(\sqrt{x^2-2x+5}+3\right)x + 2\left(\sqrt{x^2-2x+5}+2\right)\right) + 32\Big) \end{aligned}$$

$$y(x) \to \frac{1}{3} - \frac{i}{3}$$

$$y(x) \to \frac{1}{3} + \frac{i}{3}$$

12.34 problem 308

Internal problem ID [15170]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 308.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$
(5x - 7y + 1)\, y' + y = 1 - x
$$

✓ Solution by Maple
Time used: 1.734 (sec). Leaf size: 92

```
dsolve((5*x-7*y(x)+1)*diff(y(x),x)+(x+y(x)-1)=0,y(x), singsol=all)
```

$$
y = \frac{-\operatorname{RootOf}\left(7\_Z^{16} + \left(-128c_1x^4 + 256c_1x^3 - 192c_1x^2 + 64c_1x - 8c_1\right)\_Z^4 - 16c_1x^4 + 32c_1x^3 - 24c_1x^2 +}{2c_1\left(2x-1\right)^3}
$$

✓ Solution by Mathematica
Time used: 60.319 (sec). Leaf size: 8165

```
DSolve[(5*x-7*y[x]+1)*y'[x]+(x+y[x]-1)==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

12.35 problem 309

Internal problem ID [15171]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 309.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{y + (2x + 2y - 1)\, y' = -1 - x}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 20

```
dsolve([(x+y(x)+1)+(2*x+2*y(x)-1)*diff(y(x),x)=0,y(1) = 2],y(x), singsol=all)
```

$$y = -x + \frac{3\,\mathrm{LambertW}\left(\frac{2\,\mathrm{e}^{\frac{1}{3}+\frac{x}{3}}}{3}\right)}{2} + 2$$

✓ Solution by Mathematica
Time used: 3.539 (sec). Leaf size: 28

```
DSolve[{(x+y[x]+1)+(2*x+2*y[x]-1)*y'[x]==0,{y[1]==2}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{3}{2} W\left(\frac{2}{3} e^{\frac{x+1}{3}}\right) - x + 2$$

12.36 problem 310

Internal problem ID [15172]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 310.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{y^3 + 2(x^2 - y^2 x)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 28

```
dsolve(y(x)^3+2*(x^2-x*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y = \frac{e^{\frac{c_1}{2}}}{\sqrt{-\frac{e^{c_1}}{x \operatorname{LambertW}\left(-\frac{e^{c_1}}{x}\right)}}}$$

✓ Solution by Mathematica
Time used: 2.795 (sec). Leaf size: 60

```
DSolve[y[x]^3+2*(x^2-x*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -i\sqrt{x}\sqrt{W\left(-\frac{e^{c_1}}{x}\right)}$$
$$y(x) \to i\sqrt{x}\sqrt{W\left(-\frac{e^{c_1}}{x}\right)}$$
$$y(x) \to 0$$

12.37 problem 311

Internal problem ID [15173]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 311.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational]`

$$y' - \frac{2(y+2)^2}{(x+y-1)^2} = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 26

```
dsolve(diff(y(x),x)=2*(  (y(x)+2)/(x+y(x)-1) )^2,y(x), singsol=all)
```

$$y = -2 + (-x+3)\tan\left(\mathrm{RootOf}\left(-2\_Z + \ln\left(\tan\left(\_Z\right)\right) + \ln\left(x-3\right) + c_1\right)\right)$$

✓ Solution by Mathematica
Time used: 0.15 (sec). Leaf size: 27

```
DSolve[y'[x]==2*(  (y[x]+2)/(x+y[x]-1) )^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[2\arctan\left(\frac{3-x}{y(x)+2}\right) + \log(y(x)+2) = c_1, y(x)\right]$$

12.38 problem 312

Internal problem ID [15174]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 312.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G']]`

$$\boxed{4x^2y'^2 - y^2 - y^3x = 0}$$

✓ Solution by Maple
Time used: 1.187 (sec). Leaf size: 1759

```
dsolve(4*x^2*diff(y(x),x)^2-y(x)^2=x*y(x)^3,y(x), singsol=all)
```

$y = 0$
Expression too large to display
Expression too large to display

✓ Solution by Mathematica

Time used: 105.55 (sec). Leaf size: 1401

```
DSolve[4*x^2*y'[x]^2-y[x]^2==x*y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{Root}\big[\#1^8\left(x^9-2e^{3c_1}x^6+e^{6c_1}x^3\right)+\#1^7\left(-24x^8-120e^{3c_1}x^5\right)+\#1^6\left(252x^7-444e^{3c_1}x^4\right)+\#1^5\left(-1512x^6+56e^{3c_1}x^3\right)+\#1^4\left(5670x^5-66e^{3c_1}x^2\right)+\#1^3\left(-13608x^4+48e^{3c_1}x\right)+\#1^2\left(20412x^3-16e^{3c_1}\right)-17496\#1x^2+6561x\&,1\big]$$

$$y(x) \to \text{Root}\big[\#1^8\left(x^9-2e^{3c_1}x^6+e^{6c_1}x^3\right)+\#1^7\left(-24x^8-120e^{3c_1}x^5\right)+\#1^6\left(252x^7-444e^{3c_1}x^4\right)+\#1^5\left(-1512x^6+56e^{3c_1}x^3\right)+\#1^4\left(5670x^5-66e^{3c_1}x^2\right)+\#1^3\left(-13608x^4+48e^{3c_1}x\right)+\#1^2\left(20412x^3-16e^{3c_1}\right)-17496\#1x^2+6561x\&,2\big]$$

$$y(x) \to \text{Root}\big[\#1^8\left(x^9-2e^{3c_1}x^6+e^{6c_1}x^3\right)+\#1^7\left(-24x^8-120e^{3c_1}x^5\right)+\#1^6\left(252x^7-444e^{3c_1}x^4\right)+\#1^5\left(-1512x^6+56e^{3c_1}x^3\right)+\#1^4\left(5670x^5-66e^{3c_1}x^2\right)+\#1^3\left(-13608x^4+48e^{3c_1}x\right)+\#1^2\left(20412x^3-16e^{3c_1}\right)-17496\#1x^2+6561x\&,3\big]$$

$$y(x) \to \text{Root}\big[\#1^8\left(x^9-2e^{3c_1}x^6+e^{6c_1}x^3\right)+\#1^7\left(-24x^8-120e^{3c_1}x^5\right)+\#1^6\left(252x^7-444e^{3c_1}x^4\right)+\#1^5\left(-1512x^6+56e^{3c_1}x^3\right)+\#1^4\left(5670x^5-66e^{3c_1}x^2\right)+\#1^3\left(-13608x^4+48e^{3c_1}x\right)+\#1^2\left(20412x^3-16e^{3c_1}\right)-17496\#1x^2+6561x\&,4\big]$$

$$y(x) \to \text{Root}\big[\#1^8\left(x^9-2e^{3c_1}x^6+e^{6c_1}x^3\right)+\#1^7\left(-24x^8-120e^{3c_1}x^5\right)+\#1^6\left(252x^7-444e^{3c_1}x^4\right)+\#1^5\left(-1512x^6+56e^{3c_1}x^3\right)+\#1^4\left(5670x^5-66e^{3c_1}x^2\right)+\#1^3\left(-13608x^4+48e^{3c_1}x\right)+\#1^2\left(20412x^3-16e^{3c_1}\right)-17496\#1x^2+6561x\&,5\big]$$

$$y(x) \to \text{Root}\big[\#1^8\left(x^9-2e^{3c_1}x^6+e^{6c_1}x^3\right)+\#1^7\left(-24x^8-120e^{3c_1}x^5\right)+\#1^6\left(252x^7-444e^{3c_1}x^4\right)+\#1^5\left(-1512x^6+56e^{3c_1}x^3\right)+\#1^4\left(5670x^5-66e^{3c_1}x^2\right)+\#1^3\left(-13608x^4+48e^{3c_1}x\right)+\#1^2\left(20412x^3-16e^{3c_1}\right)-17496\#1x^2+6561x\&,6\big]$$

$$y(x) \to \text{Root}\big[\#1^8\left(x^9-2e^{3c_1}x^6+e^{6c_1}x^3\right)+\#1^7\left(-24x^8-120e^{3c_1}x^5\right)+\#1^6\left(252x^7-444e^{3c_1}x^4\right)+\#1^5\left(-1512x^6+56e^{3c_1}x^3\right)+\#1^4\left(5670x^5-66e^{3c_1}x^2\right)+\#1^3\left(-13608x^4+48e^{3c_1}x\right)+\#1^2\left(20412x^3-16e^{3c_1}\right)-17496\#1x^2+6561x\&,7\big]$$

$$y(x) \to \text{Root}\big[\#1^8\left(x^9-2e^{3c_1}x^6+e^{6c_1}x^3\right)+\#1^7\left(-24x^8-120e^{3c_1}x^5\right)+\#1^6\left(252x^7-444e^{3c_1}x^4\right)+\#1^5\left(-1512x^6+56e^{3c_1}x^3\right)+\#1^4\left(5670x^5-66e^{3c_1}x^2\right)+\#1^3\left(-13608x^4+48e^{3c_1}x\right)+\#1^2\left(20412x^3-16e^{3c_1}\right)-17496\#1x^2+6561x\&,8\big]$$

12.39 problem 313

Internal problem ID [15175]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Section 12. Miscellaneous problems. Exercises page 93
Problem number: 313.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, _dAlembert]`

$$\boxed{y' + xy'^2 - y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 59

```
dsolve(diff(y(x),x)+x*diff(y(x),x)^2-y(x)=0,y(x), singsol=all)
```

$$y = 2\,\mathrm{e}^{\mathrm{RootOf}\left(-x\,\mathrm{e}^{2\_Z}+2\,\mathrm{e}^{\_Z}x+\_Z+c_1-x-\mathrm{e}^{\_Z}\right)}x + \mathrm{RootOf}\left(-x\,\mathrm{e}^{2\_Z}+2\,\mathrm{e}^{\_Z}x+\_Z+c_1-x-\mathrm{e}^{\_Z}\right)+c_1-x$$

✓ Solution by Mathematica

Time used: 0.881 (sec). Leaf size: 46

```
DSolve[y'[x]+x*y'[x]^2-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x=\frac{\log(K[1])-K[1]}{(K[1]-1)^2}+\frac{c_1}{(K[1]-1)^2},y(x)=xK[1]^2+K[1]\right\},\{y(x),K[1]\}\right]$$

13 Chapter 2 (Higher order ODE's). Section 13. Basic concepts and definitions. Exercises page 98

13.1 problem 318

Internal problem ID [15176]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 13. Basic concepts and definitions. Exercises page 98
Problem number: 318.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = 2\cos(x) + 2\sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)+y(x)=2*(cos(x)+sin(x)),y(x), singsol=all)
```

$$y = (c_1 - x + 1)\cos(x) + \sin(x)(x + c_2)$$

✓ Solution by Mathematica
Time used: 0.109 (sec). Leaf size: 23

```
DSolve[y''[x]+y[x]==2*(Cos[x]+Sin[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (-x + 1 + c_1)\cos(x) + (x + c_2)\sin(x)$$

13.2 problem 319

Internal problem ID [15177]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 13. Basic concepts and definitions. Exercises page 98
Problem number: 319.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _quadrature]]`

$$\boxed{xy''' = 2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x$3)=2,y(x), singsol=all)
```

$$y = x^2 \ln (x) + \frac{(c_1 - 3)\, x^2}{2} + c_2 x + c_3$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 28

```
DSolve[x*y'''[x]==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 \log(x) + \left(-\frac{3}{2} + c_3\right) x^2 + c_2 x + c_1$$

13.3 problem 320

Internal problem ID [15178]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 13. Basic concepts and definitions. Exercises page 98
Problem number: 320.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{y'' - y'^2 = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)=diff(y(x),x)^2,y(x), singsol=all)
```

$$y = -\ln(-c_1 x - c_2)$$

✓ Solution by Mathematica
Time used: 0.165 (sec). Leaf size: 15

```
DSolve[y''[x]==y'[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \log(x + c_1)$$

13.4 problem 321

Internal problem ID [15179]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 13. Basic concepts and definitions. Exercises page 98
Problem number: 321.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{(x-1)\,y''=1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve((x-1)*diff(y(x),x$2)=1,y(x), singsol=all)
```

$$y=\ln(x-1)\,(x-1)+(c_1-1)\,x+c_2+1$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 22

```
DSolve[(x-1)*y''[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to(x-1)\log(x-1)+(-1+c_2)x+c_1$$

13.5 problem 322

Internal problem ID [15180]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 13. Basic concepts and definitions. Exercises page 98
Problem number: 322.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^4 = 1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 33

```
dsolve(diff(y(x),x)*diff(y(x),x)^3=1,y(x), singsol=all)
```

$$y = -ix + c_1$$
$$y = ix + c_1$$
$$y = x + c_1$$
$$y = -x + c_1$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 43

```
DSolve[y'[x]*y'[x]^3==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + c_1$$
$$y(x) \to c_1 - ix$$
$$y(x) \to ix + c_1$$
$$y(x) \to x + c_1$$

13.6 problem 323

Internal problem ID [15181]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 13. Basic concepts and definitions. Exercises page 98
Problem number: 323.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x$2)+y(x)=0,y(x), singsol=all)
```

$$y = c_1 \sin(x) + c_2 \cos(x)$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 16

```
DSolve[y''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(x) + c_2 \sin(x)$$

13.7 problem 324

Internal problem ID [15182]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 13. Basic concepts and definitions. Exercises page 98
Problem number: 324.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 3y' + 2y = 2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=2,y(x), singsol=all)
```

$$y = \mathrm{e}^{2x}c_1 + c_2\mathrm{e}^{x} + 1$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 21

```
DSolve[y''[x]-3*y'[x]+2*y[x]==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1e^{x} + c_2e^{2x} + 1$$

13.8 problem 325

Internal problem ID [15183]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 13. Basic concepts and definitions. Exercises page 98
Problem number: 325.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - \left(1 + y'^2\right)^{\frac{3}{2}} = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 49

```
dsolve(diff(y(x),x$2)=(1+diff(y(x),x)^2)^(3/2),y(x), singsol=all)
```

$$y = -ix + c_1$$
$$y = ix + c_1$$
$$y = (c_1 + x + 1)(c_1 + x - 1)\sqrt{-\frac{1}{(c_1 + x + 1)(c_1 + x - 1)}} + c_2$$

✓ Solution by Mathematica
Time used: 0.253 (sec). Leaf size: 59

```
DSolve[y''[x]==(1+y'[x]^2)^(3/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - i\sqrt{x^2 + 2c_1x - 1 + c_1^2}$$
$$y(x) \to i\sqrt{x^2 + 2c_1x - 1 + c_1^2} + c_2$$

13.9 problem 326

Internal problem ID [15184]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 13. Basic concepts and definitions. Exercises page 98
Problem number: 326.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], [`

$$
\boxed{y'^2 + yy'' = 1}
$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 35

```
dsolve(diff(y(x),x)^2+y(x)*diff(y(x),x$2)=1,y(x), singsol=all)
```

$$
y = \sqrt{-2c_1x + x^2 + 2c_2}
$$
$$
y = -\sqrt{-2c_1x + x^2 + 2c_2}
$$

✓ Solution by Mathematica
Time used: 0.593 (sec). Leaf size: 79

```
DSolve[y'[x]^2+y[x]*y''[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x) \to -\sqrt{(x + c_2)^2 - e^{2c_1}}
$$
$$
y(x) \to \sqrt{(x + c_2)^2 - e^{2c_1}}
$$
$$
y(x) \to -\sqrt{(x + c_2)^2}
$$
$$
y(x) \to \sqrt{(x + c_2)^2}
$$

14 Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107

14.1 problem 327

Internal problem ID [15185]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 327.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _quadrature]]`

$$\boxed{y'''' = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$4)=x,y(x), singsol=all)
```

$$y = \frac{x^5}{120} + \frac{c_1 x^3}{6} + \frac{c_2 x^2}{2} + \frac{(3c_1^2 + 2c_3)\, x}{2} + c_4$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 31

```
DSolve[y''''[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^5}{120} + c_4 x^3 + c_3 x^2 + c_2 x + c_1$$

14.2 problem 328

Internal problem ID [15186]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 328.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _quadrature]]`

$$\boxed{y''' = x + \cos(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$3)=x+cos(x),y(x), singsol=all)
```

$$y = \frac{x^4}{24} + \frac{c_1 x^2}{2} - \sin(x) + c_2 x + c_3$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 29

```
DSolve[y'''[x]==x+Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^4}{24} + c_3 x^2 - \sin(x) + c_2 x + c_1$$

14.3 problem 329

Internal problem ID [15187]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 329.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y''(x+2)^5 = 1}$$

With initial conditions

$$\left[y(-1) = \frac{1}{12}, y'(-1) = -\frac{1}{4}\right]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve([diff(y(x),x$2)*(x+2)^5=1,y(-1) = 1/12, D(y)(-1) = -1/4],y(x), singsol=all)
```

$$y = \frac{1}{12\,(x+2)^3}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 14

```
DSolve[{y''[x]*(x+2)^5==1,{y[-1]==1/12,y'[-1]==-1/4}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{12(x+2)^3}$$

14.4 problem 330

Internal problem ID [15188]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 330.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = \mathrm{e}^{x} x}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve([diff(y(x),x$2)=x*exp(x),y(0) = 0, D(y)(0) = 0],y(x), singsol=all)
```

$$y = (x - 2)\,\mathrm{e}^{x} + x + 2$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 15

```
DSolve[{y''[x]==x*Exp[x],{y[0]==0,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(x - 2) + x + 2$$

14.5 problem 331

Internal problem ID [15189]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 331.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = 2\ln(x)\,x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)=2*x*ln(x),y(x), singsol=all)
```

$$y = -\frac{5x^3}{18} + \frac{x^3\ln(x)}{3} + c_1x + c_2$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 28

```
DSolve[y''[x]==2*x*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{5x^3}{18} + \frac{1}{3}x^3\log(x) + c_2x + c_1$$

14.6 problem 332

Internal problem ID [15190]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 332.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' - y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x*diff(y(x),x$2)=diff(y(x),x),y(x), singsol=all)
```

$$y = c_2x^2 + c_1$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 17

```
DSolve[x*y''[x]==y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1x^2}{2} + c_2$$

14.7 problem 333

Internal problem ID [15191]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 333.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' + y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(x*diff(y(x),x$2)+diff(y(x),x)=0,y(x), singsol=all)
```

$$y = c_2 \ln(x) + c_1$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 13

```
DSolve[x*y''[x]+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \log(x) + c_2$$

14.8 problem 334

Internal problem ID [15192]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 334.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' - (2x^2 + 1)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x$2)=(1+2*x^2)*diff(y(x),x),y(x), singsol=all)
```

$$y = c_1 + e^{x^2} c_2$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 19

```
DSolve[x*y''[x]==(1+2*x^2)*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 e^{x^2}}{2} + c_2$$

14.9 problem 335

Internal problem ID [15193]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 335.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' - y' = x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x$2)=diff(y(x),x)+x^2,y(x), singsol=all)
```

$$y = \frac{1}{3}x^3 + \frac{1}{2}c_1x^2 + c_2$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 24

```
DSolve[x*y''[x]==y'[x]+x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{3} + \frac{c_1x^2}{2} + c_2$$

14.10 problem 336

Internal problem ID [15194]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 336.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x \ln (x) y'' - y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*ln(x)*diff(y(x),x$2)=diff(y(x),x),y(x), singsol=all)
```

$$y = \ln (x) c_2 x - c_2 x + c_1$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 19

```
DSolve[x*Log[x]*y''[x]==y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(-x) + c_1 x \log(x) + c_2$$

14.11 problem 337

Internal problem ID [15195]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 337.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_separable]`

$$yx - y' \ln\left(\frac{y'}{x}\right) = 0$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 63

```
dsolve(x*y(x)=diff(y(x),x)*ln(diff(y(x),x)/x),y(x), singsol=all)
```

$$y = \left(-1-\sqrt{x^2-2c_1+1}\right) e^{-1-\sqrt{x^2-2c_1+1}}$$
$$y = \left(-1+\sqrt{x^2-2c_1+1}\right) e^{-1+\sqrt{x^2-2c_1+1}}$$

✓ Solution by Mathematica
Time used: 4.223 (sec). Leaf size: 83

```
DSolve[x*y[x]==y'[x]*Log[y'[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{-1-\sqrt{x^2+1+2c_1}}\left(1+\sqrt{x^2+1+2c_1}\right)$$
$$y(x) \to e^{-1+\sqrt{x^2+1+2c_1}}\left(-1+\sqrt{x^2+1+2c_1}\right)$$
$$y(x) \to 0$$

14.12 problem 338

Internal problem ID [15196]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 338.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_poly_y`

$$\boxed{2y'' - \frac{y'}{x} - \frac{x^2}{y'} = 0}$$

With initial conditions

$$\left[y(1) = \frac{\sqrt{2}}{5}, y'(1) = \frac{\sqrt{2}}{2}\right]$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 12

```
dsolve([2*diff(y(x),x$2)=diff(y(x),x)/x+x^2/diff(y(x),x),y(1) = 1/5*2^(1/2), D(y)(1) = 1/2*2
```

$$y = \frac{\sqrt{2}\, x^{\frac{5}{2}}}{5}$$

✓ Solution by Mathematica
Time used: 0.12 (sec). Leaf size: 26

```
DSolve[{2*y''[x]==y'[x]/x+x^2/y'[x],{y[1]==Sqrt[2]/5,y'[1]==Sqrt[2]/2}},y[x],x,IncludeSingul
```

$$y(x) \to \frac{1}{5}\sqrt{2}x^{3/2}\sqrt{x^2}$$

14.13 problem 339

Internal problem ID [15197]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISE-LYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 339.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x], [_3rd_order, _missing_y], [_3rd_ord`

$$\frac{d^3}{dx^3}y(x) = \sqrt{1 - \left(\frac{d^2}{dx^2}y(x)\right)^2}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$3)=sqrt(1-diff(y(x),x$2)^2),y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica
Time used: 0.287 (sec). Leaf size: 34

```
DSolve[y'''[x]==Sqrt[1-y''[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3x - \cos(x + c_1) + c_2$$
$$y(x) \to \text{Interval}[\{-1, 1\}] + c_3x + c_2$$

14.14 problem 340

Internal problem ID [15198]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 340.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{xy''' - y'' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x*diff(y(x),x$3)-diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y = c_3x^3 + c_2x + c_1$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 21

```
DSolve[x*y'''[x]-y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1x^3}{6} + c_3x + c_2$$

14.15 problem 341

Internal problem ID [15199]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 341.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - \sqrt{1 + y'^2} = 0}$$

✓ Solution by Maple
Time used: 0.188 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)=sqrt(1+diff(y(x),x)^2),y(x), singsol=all)
```

$$y = -ix + c_1$$
$$y = ix + c_1$$
$$y = \cosh(x + c_1) + c_2$$

✓ Solution by Mathematica
Time used: 0.231 (sec). Leaf size: 29

```
DSolve[y''[x]==Sqrt[1+y'[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(e^{-x-c_1} + e^{x+c_1}\right) + c_2$$

14.16 problem 342

Internal problem ID [15200]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 342.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{y'' - y'^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)=diff(y(x),x)^2,y(x), singsol=all)
```

$$y = -\ln(-c_1 x - c_2)$$

✓ Solution by Mathematica
Time used: 1.667 (sec). Leaf size: 16

```
DSolve[y''[x]==1+y'[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \log(\cos(x + c_1))$$

14.17 problem 343

Internal problem ID [15201]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 343.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$
\boxed{y'' - \sqrt{-y'^2 + 1} = 0}
$$

✓ Solution by Maple
Time used: 2.297 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)=sqrt(1-diff(y(x),x)^2),y(x), singsol=all)
```

$$
\begin{aligned}
y &= -x + c_1 \\
y &= x + c_1 \\
y &= -\cos(x + c_1) + c_2
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.166 (sec). Leaf size: 24

```
DSolve[y''[x]==Sqrt[1-y'[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) &\to \sin(x + c_1) + c_2 \\
y(x) &\to \text{Interval}[\{-1, 1\}] + c_2
\end{aligned}
$$

14.18 problem 344

Internal problem ID [15202]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 344.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{y'' - {y'}^2 = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)=1+diff(y(x),x)^2,y(x), singsol=all)
```

$$y = -\ln(-c_2 \cos(x) + c_1 \sin(x))$$

✓ Solution by Mathematica

Time used: 1.595 (sec). Leaf size: 16

```
DSolve[y''[x]==1+y'[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \log(\cos(x + c_1))$$

14.19 problem 345

Internal problem ID [15203]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 345.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - \sqrt{y' + 1} = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)=sqrt(1+diff(y(x),x)),y(x), singsol=all)
```

$$y = -x + c_1$$
$$y = \frac{1}{12}x^3 + \frac{1}{4}c_1 x^2 + \frac{1}{4}c_1^2 x - x + c_2$$

✓ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 30

```
DSolve[y''[x]==Sqrt[1+y'[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}x\left(x^2 + 3c_1 x + 3\left(-4 + c_1{}^2\right)\right) + c_2$$

14.20 problem 346

Internal problem ID [15204]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 346.
ODE order: 2.
ODE degree: 0.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' \ln(y') = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$2)=diff(y(x),x)*ln(diff(y(x),x)),y(0) = 0, D(y)(0) = 1],y(x), singsol=al
```

$$y = -\operatorname{expIntegral}_1(-2i\pi\_Z5\,\mathrm{e}^x) + \operatorname{expIntegral}_1(-2i\pi\_Z5)$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y''[x]==y'[x]*Log[y'[x]],{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True
```

{}

14.21 problem 347

Internal problem ID [15205]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 347.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' = -2}$$

With initial conditions

$$[y(0) = 0, y'(0) = -2]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 7

```
dsolve([diff(y(x),x$2)+diff(y(x),x)+2=0,y(0) = 0, D(y)(0) = -2],y(x), singsol=all)
```

$$y = -2x$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 8

```
DSolve[{y''[x]+y'[x]+2==0,{y[0]==0,y'[0]==-2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2x$$

14.22 problem 348

Internal problem ID [15206]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 348.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{y'' - y'(y' + 1) = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$2)=diff(y(x),x)*(1+diff(y(x),x)),y(x), singsol=all)
```

$$y = -\ln\left(-c_1 e^x - c_2\right)$$

✓ Solution by Mathematica
Time used: 1.619 (sec). Leaf size: 31

```
DSolve[y''[x]==y'[x]*(1+y'[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \log\left(-1 + e^{x+c_1}\right)$$
$$y(x) \to c_2 - i\pi$$

14.23 problem 349

Internal problem ID [15207]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 349.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$3y'' - \left(1 + y'^2\right)^{\frac{3}{2}} = 0$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 49

```
dsolve(3*diff(y(x),x$2)=(1+diff(y(x),x)^2)^(3/2),y(x), singsol=all)
```

$$y = -ix + c_1$$
$$y = ix + c_1$$
$$y = (c_1 + x + 3)(c_1 + x - 3)\sqrt{-\frac{1}{(c_1 + x + 3)(c_1 + x - 3)}} + c_2$$

✓ Solution by Mathematica
Time used: 0.26 (sec). Leaf size: 63

```
DSolve[3*y''[x]==(1+y'[x]^2)^(3/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - i\sqrt{x^2 + 6c_1 x - 9 + 9{c_1}^2}$$
$$y(x) \to i\sqrt{x^2 + 6c_1 x - 9 + 9{c_1}^2} + c_2$$

14.24 problem 350

Internal problem ID [15208]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 350.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x], [_3rd_order, _missing_y], [_3rd_ord`

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$3)+diff(y(x),x$2)^2=0,y(x), singsol=all)
```

$$y = \ln(x + c_1)(x + c_1) + (c_2 - 1)x - c_1 + c_3$$

✓ Solution by Mathematica
Time used: 0.315 (sec). Leaf size: 28

```
DSolve[y'''[x]+y''[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (-1 + c_3)x + (x - c_1)\log(x - c_1) + c_2$$

14.25 problem 351

Internal problem ID [15209]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 351.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{yy'' - y'^2 = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve(y(x)*diff(y(x),x$2)=diff(y(x),x)^2,y(x), singsol=all)
```

$$y = 0$$
$$y = e^{c_1 x} c_2$$

✓ Solution by Mathematica
Time used: 0.101 (sec). Leaf size: 14

```
DSolve[y[x]*y''[x]==y'[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 e^{c_1 x}$$

14.26 problem 352

Internal problem ID [15210]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 352.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _`

$$\boxed{y'' - 2yy' = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.046 (sec). Leaf size: 11

```
dsolve([diff(y(x),x$2)=2*y(x)*diff(y(x),x),y(0) = 1, D(y)(0) = 1],y(x), singsol=all)
```

$$y = -\frac{1}{x - 1}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y''[x]==2*y[x]*y'[x],{y[0]==1,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

{}

14.27 problem 353

Internal problem ID [15211]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 353.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{3y'y'' - 2y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 11

```
dsolve([3*diff(y(x),x)*diff(y(x),x$2)=2*y(x),y(0) = 1, D(y)(0) = 1],y(x), singsol=all)
```

$$y = \frac{(x+3)^3}{27}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{3*y'[x]*y''[x]==2*y[x],{y[0]==1,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

{}

14.28 problem 354

Internal problem ID [15212]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 354.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{2y'' - 3y^2 = 0}$$

With initial conditions

$$[y(-2) = 1, y'(-2) = -1]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 11

```
dsolve([2*diff(y(x),x$2)=3*y(x)^2,y(-2) = 1, D(y)(-2) = -1],y(x), singsol=all)
```

$$y = \frac{4}{(x + 4)^2}$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 12

```
DSolve[{2*y''[x]==3*y[x]^2,{y[-2]==1,y'[-2]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4}{(x + 4)^2}$$

14.29 problem 355

Internal problem ID [15213]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 355.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _`

$$\boxed{yy'' + y'^2 = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 33

```
dsolve(y(x)*diff(y(x),x$2)+diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y = 0$$
$$y = \sqrt{2c_1x + 2c_2}$$
$$y = -\sqrt{2c_1x + 2c_2}$$

✓ Solution by Mathematica
Time used: 0.154 (sec). Leaf size: 20

```
DSolve[y[x]*y''[x]+y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2\sqrt{2x - c_1}$$

14.30 problem 356

Internal problem ID [15214]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 356.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1],`

$$\boxed{yy'' - y' - y'^2 = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve(y(x)*diff(y(x),x$2)=diff(y(x),x)+diff(y(x),x)^2,y(x), singsol=all)
```

$$y = 0$$
$$y = \frac{e^{c_1(x+c_2)} + 1}{c_1}$$

✓ Solution by Mathematica
Time used: 1.452 (sec). Leaf size: 26

```
DSolve[y[x]*y''[x]==y'[x]+y'[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1 + e^{c_1(x+c_2)}}{c_1}$$
$$y(x) \to \text{Indeterminate}$$

14.31 problem 357

Internal problem ID [15215]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 357.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{yy'' - y'^2 = 1}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 55

```
dsolve(y(x)*diff(y(x),x$2)=1+diff(y(x),x)^2,y(x), singsol=all)
```

$$y = \frac{c_1\left(\mathrm{e}^{\frac{x+c_2}{c_1}} + \mathrm{e}^{\frac{-x-c_2}{c_1}}\right)}{2}$$
$$y = \frac{c_1\left(\mathrm{e}^{\frac{x+c_2}{c_1}} + \mathrm{e}^{\frac{-x-c_2}{c_1}}\right)}{2}$$

✓ Solution by Mathematica
Time used: 60.132 (sec). Leaf size: 80

```
DSolve[y[x]*y''[x]==1+y'[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{-c_1} \tanh\left(e^{c_1}(x+c_2)\right)}{\sqrt{-\operatorname{sech}^2\left(e^{c_1}(x+c_2)\right)}}$$
$$y(x) \to \frac{e^{-c_1} \tanh\left(e^{c_1}(x+c_2)\right)}{\sqrt{-\operatorname{sech}^2\left(e^{c_1}(x+c_2)\right)}}$$

14.32 problem 358

Internal problem ID [15216]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 358.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{2yy'' - y'^2 = 1}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 22

```
dsolve(2*y(x)*diff(y(x),x$2)=1+diff(y(x),x)^2,y(x), singsol=all)
```

$$y = \frac{(c_1^2 + 1)\, x^2}{4c_2} + c_1 x + c_2$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 34

```
DSolve[2*y[x]*y''[x]==1+y'[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(1 + c_1{}^2)\, x^2}{4c_2} + c_1 x + c_2$$
$$y(x) \to \text{Indeterminate}$$

14.33 problem 359

Internal problem ID [15217]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 359.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y^3y'' = -1}$$

With initial conditions

$$[y(1) = 1, y'(1) = 0]$$

✗ Solution by Maple

```
dsolve([y(x)^3*diff(y(x),x$2)=-1,y(1) = 1, D(y)(1) = 0],y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 15

```
DSolve[{y[x]^3*y''[x]==-1,{y[1]==1,y'[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{-((x-2)x)}$$

14.34 problem 360

Internal problem ID [15218]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 360.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _with_potential_symmet`

$$\boxed{yy'' - y'^2 - y'y^2 = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 27

```
dsolve(y(x)*diff(y(x),x$2)-diff(y(x),x)^2=y(x)^2*diff(y(x),x),y(x), singsol=all)
```

$$y = 0$$
$$y = -\frac{c_1 e^{c_1(x+c_2)}}{-1 + e^{c_1(x+c_2)}}$$

✓ Solution by Mathematica
Time used: 1.39 (sec). Leaf size: 43

```
DSolve[y[x]*y''[x]-y'[x]^2==y[x]^2*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{c_1 e^{c_1(x+c_2)}}{-1 + e^{c_1(x+c_2)}}$$
$$y(x) \to -\frac{1}{x + c_2}$$

14.35 problem 361

Internal problem ID [15219]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 361.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' - e^{2y} = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 12

```
dsolve([diff(y(x),x$2)=exp(2*y(x)),y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y = -\frac{\ln\left((x-1)^2\right)}{2}$$

✓ Solution by Mathematica
Time used: 0.148 (sec). Leaf size: 13

```
DSolve[{y''[x]==Exp[2*y[x]],{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\log(1-x)$$

14.36 problem 362

Internal problem ID [15220]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 362.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{2yy'' - 3{y'}^2 - 4y^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(2*y(x)*diff(y(x),x$2)-3*diff(y(x),x)^2=4*y(x)^2,y(x), singsol=all)
```

$$y = 0$$
$$y = \frac{4}{(c_2 \cos(x) - c_1 \sin(x))^2}$$

✓ Solution by Mathematica

Time used: 0.637 (sec). Leaf size: 17

```
DSolve[2*y[x]*y''[x]-3*y'[x]^2==4*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \sec^2(x + 2c_1)$$

14.37 problem 363

Internal problem ID [15221]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 14. Differential equations admitting of depression of their order. Exercises page 107
Problem number: 363.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x], [_3rd_order, _exact, _nonlinear], [`

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 11

```
dsolve([diff(y(x),x$3)=3*y(x)*diff(y(x),x),y(0) = 1, D(y)(0) = 1, (D@@2)(y)(0) = 3/2],y(x),
```

$$y = \frac{4}{(x-2)^2}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'''[x]==3*y[x]*y'[x],{y[0]==1,y'[0]==1,y''[0]==3/2}},y[x],x,IncludeSingularSolution
```

{}

15 Chapter 2 (Higher order ODE's). Section 15.2 Homogeneous differential equations with constant coefficients. Exercises page 121

15.1 problem 432

Internal problem ID [15222]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.2 Homogeneous differential equations with constant coefficients. Exercises page 121
Problem number: 432.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)-y(x)=0,y(x), singsol=all)
```

$$y = c_1 e^{-x} + c_2 e^{x}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 20

```
DSolve[y''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x + c_2 e^{-x}$$

15.2 problem 433

Internal problem ID [15223]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.2 Homogeneous differential equations with constant coefficients. Exercises page 121
Problem number: 433.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{3y'' - 2y' - 8y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(3*diff(y(x),x$2)-2*diff(y(x),x)-8*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_2 \mathrm{e}^{\frac{10x}{3}} + c_1\right) \mathrm{e}^{-\frac{4x}{3}}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 24

```
DSolve[3*y''[x]-2*y'[x]-8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-4x/3} + c_2 e^{2x}$$

15.3 problem 434

Internal problem ID [15224]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.2 Homogeneous differential equations with constant coefficients. Exercises page 121
Problem number: 434.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 3y'' + 3y' - y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 2, y''(0) = 3]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 10

```
dsolve([diff(y(x),x$3)-3*diff(y(x),x$2)+3*diff(y(x),x)-y(x)=0,y(0) = 1, D(y)(0) = 2, (D@@2)(
```

$$y(x) = e^x(1 + x)$$

✓ Solution by Mathematica
Time used: 0.007 (sec). Leaf size: 12

```
DSolve[{y'''[x]-3*y''[x]+3*y'[x]-y[x]==0,{y[0]==1,y'[0]==2,y''[0]==3}},y[x],x,IncludeSingula
```

$$y(x) \to e^x(x + 1)$$

15.4 problem 435

Internal problem ID [15225]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.2 Homogeneous differential equations with constant coefficients. Exercises page 121
Problem number: 435.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}(c_2x + c_1)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 18

```
DSolve[y''[x]+2*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_2x + c_1)$$

15.5 problem 436

Internal problem ID [15226]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.2 Homogeneous differential equations with constant coefficients. Exercises page 121
Problem number: 436.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y' + 3y = 0}$$

With initial conditions

$$[y(0) = 6, y'(0) = 10]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(x),x$2)-4*diff(y(x),x)+3*y(x)=0,y(0) = 6, D(y)(0) = 10],y(x), singsol=all)
```

$$y(x) = 2\,\mathrm{e}^{3x} + 4\,\mathrm{e}^{x}$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 17

```
DSolve[{y''[x]-4*y'[x]+3*y[x]==0,{y[0]==6,y'[0]==10}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to 2e^{x}\left(e^{2x} + 2\right)$$

15.6 problem 437

Internal problem ID [15227]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.2 Homogeneous differential equations with constant coefficients. Exercises page 121
Problem number: 437.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 6y'' + 11y' + 6y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$3)+6*diff(y(x),x$2)+11*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-3x} + c_2 e^{-x} + c_3 e^{-2x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 27

```
DSolve[y'''[x]+6*y''[x]+11*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}(e^x(c_3e^x + c_2) + c_1)$$

15.7 problem 438

Internal problem ID [15228]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.2 Homogeneous differential equations with constant coefficients. Exercises page 121
Problem number: 438.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' - 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\left(1+\sqrt{3}\right)x} + c_2 \mathrm{e}^{-\left(\sqrt{3}-1\right)x}$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 34

```
DSolve[y''[x]-2*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x-\sqrt{3}x}\left(c_2 e^{2\sqrt{3}x} + c_1\right)$$

15.8 problem 439

Internal problem ID [15229]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.2 Homogeneous differential equations with constant coefficients. Exercises page 121
Problem number: 439.
ODE order: 6.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(6)} + 2y^{(5)} + y'''' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$6)+2*diff(y(x),x$5)+diff(y(x),x$4)=0,y(x), singsol=all)
```

$$y(x) = (c_6x + c_5)\,\mathrm{e}^{-x} + c_4x^3 + c_3x^2 + c_2x + c_1$$

✓ Solution by Mathematica
Time used: 0.135 (sec). Leaf size: 37

```
DSolve[y''''''[x]+2*y'''''[x]+y''''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_2(x+4) + c_1) + x(x(c_6x + c_5) + c_4) + c_3$$

15.9 problem 440

Internal problem ID [15230]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.2 Homogeneous differential equations with constant coefficients. Exercises page 121
Problem number: 440.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' - 8y' + 5y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(4*diff(y(x),x$2)-8*diff(y(x),x)+5*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{x}\left(c_1 \sin\left(\frac{x}{2}\right) + c_2 \cos\left(\frac{x}{2}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 28

```
DSolve[4*y''[x]-8*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}\left(c_2 \cos\left(\frac{x}{2}\right) + c_1 \sin\left(\frac{x}{2}\right)\right)$$

15.10 problem 441

Internal problem ID [15231]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.2 Homogeneous differential equations with constant coefficients. Exercises page 121
Problem number: 441.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 8y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$3)-8*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{2x} + c_2 e^{-x} \sin\left(x\sqrt{3}\right) + c_3 e^{-x} \cos\left(x\sqrt{3}\right)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 42

```
DSolve[y'''[x]-8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_1 e^{3x} + c_2 \cos\left(\sqrt{3}x\right) + c_3 \sin\left(\sqrt{3}x\right)\right)$$

15.11 problem 442

Internal problem ID [15232]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.2 Homogeneous differential equations with constant coefficients. Exercises page 121
Problem number: 442.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' + 4y''' + 10y'' + 12y' + 5y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$4)+4*diff(y(x),x$3)+10*diff(y(x),x$2)+12*diff(y(x),x)+5*y(x)=0,y(x), sing
```

$$y(x) = e^{-x}(c_1 + c_2x + c_3 \sin(2x) + c_4 \cos(2x))$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 32

```
DSolve[y''''[x]+4*y'''[x]+10*y''[x]+12*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to e^{-x}(c_4x + c_2 \cos(2x) + c_1 \sin(2x) + c_3)$$

15.12 problem 443

Internal problem ID [15233]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.2 Homogeneous differential equations with constant coefficients. Exercises page 121
Problem number: 443.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + 2y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+2*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = e^{x} \sin(x)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 11

```
DSolve[{y''[x]-2*y'[x]+2*y[x]==0,{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to e^{x} \sin(x)$$

15.13 problem 444

Internal problem ID [15234]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.2 Homogeneous differential equations with constant coefficients. Exercises page 121
Problem number: 444.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type **[[_2nd_order, _missing_x]]**

$$\boxed{y'' - 2y' + 3y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 3]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 24

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+3*y(x)=0,y(0) = 1, D(y)(0) = 3],y(x), singsol=all)
```

$$y(x) = e^x \left(\sqrt{2}\, \sin\left(\sqrt{2}\, x\right) + \cos\left(\sqrt{2}\, x\right)\right)$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 32

```
DSolve[{y''[x]-2*y'[x]+3*y[x]==0,{y[0]==1,y'[0]==3}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to e^x \left(\sqrt{2} \sin\left(\sqrt{2}x\right) + \cos\left(\sqrt{2}x\right)\right)$$

15.14 problem 445

Internal problem ID [15235]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.2 Homogeneous differential equations with constant coefficients. Exercises page 121
Problem number: 445.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 2y''' + 4y'' - 2y' - 5y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$3)+4*diff(y(x),x$2)-2*diff(y(x),x)-5*y(x)=0,y(x), singso
```

$$y(x) = (c_3 \sin(2x) + c_4 \cos(2x) + c_1) e^{-x} + c_2 e^{x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 36

```
DSolve[y''''[x]+2*y'''[x]+4*y''[x]-2*y'[x]-5*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to e^{-x} \left(c_4 e^{2x} + c_2 \cos(2x) + c_1 \sin(2x) + c_3\right)$$

15.15 problem 446

Internal problem ID [15236]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.2 Homogeneous differential equations with constant coefficients. Exercises page 121
Problem number: 446.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type **[[_high_order, _missing_x]]**

$$y^{(5)} + 4y'''' + 5y''' - 6y' - 4y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$5)+4*diff(y(x),x$4)+5*diff(y(x),x$3)-6*diff(y(x),x)-4*y(x)=0,y(x), singso
```

$$y(x) = e^{-2x}\left(c_2 e^{3x} + (\sin(x)\, c_4 + \cos(x)\, c_5 + c_1)\, e^x + c_3\right)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 44

```
DSolve[y'''''[x]+4*y''''[x]+5*y'''[x]-6*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to e^{-2x}\left(c_4 e^x + c_5 e^{3x} + c_2 e^x \cos(x) + c_1 e^x \sin(x) + c_3\right)$$

15.16 problem 447

Internal problem ID [15237]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.2 Homogeneous differential equations with constant coefficients. Exercises page 121
Problem number: 447.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 2y'' - y' - 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)+2*diff(y(x),x$2)-diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_2 e^{3x} + c_1 e^{x} + c_3\right) e^{-2x}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 28

```
DSolve[y'''[x]+2*y''[x]-y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(c_2 e^{x} + c_3 e^{3x} + c_1\right)$$

15.17 problem 448

Internal problem ID [15238]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.2 Homogeneous differential equations with constant coefficients. Exercises page 121
Problem number: 448.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 2y'' + 2y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$3)-2*diff(y(x),x$2)+2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 e^x \sin(x) + c_3 e^x \cos(x)$$

✓ Solution by Mathematica
Time used: 0.126 (sec). Leaf size: 34

```
DSolve[y'''[x]-2*y''[x]+2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^x ((c_2 - c_1)\cos(x) + (c_1 + c_2)\sin(x)) + c_3$$

15.18 problem 449

Internal problem ID [15239]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.2 Homogeneous differential equations with constant coefficients. Exercises page 121
Problem number: 449.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$4)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x} + c_2 e^{x} + c_3 \sin(x) + c_4 \cos(x)$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 30

```
DSolve[y''''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x} + c_3 e^{-x} + c_2 \cos(x) + c_4 \sin(x)$$

15.19 problem 450

Internal problem ID [15240]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.2 Homogeneous differential equations with constant coefficients. Exercises page 121
Problem number: 450.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _quadrature]]`

$$\boxed{y^{(5)} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$5)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{24}c_1x^4 + \frac{1}{6}c_2x^3 + \frac{1}{2}c_3x^2 + c_4x + c_5$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 27

```
DSolve[y'''''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(x(x(c_5x + c_4) + c_3) + c_2) + c_1$$

15.20 problem 451

Internal problem ID [15241]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.2 Homogeneous differential equations with constant coefficients. Exercises page 121
Problem number: 451.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 3y' - 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = (c_3x + c_2)\,e^{-x} + c_1e^{2x}$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 26

```
DSolve[y'''[x]-3*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_2x + c_3e^{3x} + c_1\right)$$

15.21 problem 452

Internal problem ID [15242]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.2 Homogeneous differential equations with constant coefficients. Exercises page 121
Problem number: 452.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{2y''' - 3y'' + y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(2*diff(y(x),x$3)-3*diff(y(x),x$2)+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 e^{\frac{x}{2}} + c_3 e^{x}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 25

```
DSolve[2*y'''[x]-3*y''[x]+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2c_1 e^{x/2} + c_2 e^{x} + c_3$$

15.22 problem 453

Internal problem ID [15243]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.2 Homogeneous differential equations with constant coefficients. Exercises page 121
Problem number: 453.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + y'' = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0, y''(0) = 1]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 10

```
dsolve([diff(y(x),x$3)+diff(y(x),x$2)=0,y(0) = 1, D(y)(0) = 0, (D@@2)(y)(0) = 1],y(x), sings
```

$$y = e^{-x} + x$$

✓ Solution by Mathematica
Time used: 0.096 (sec). Leaf size: 12

```
DSolve[{y'''[x]+y''[x]==0,{y[0]==1,y'[0]==0,y''[0]==1}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to x + e^{-x}$$

16 Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132

16.1 problem 474

Internal problem ID [15244]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 474.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 3y' = 3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)=3,y(x), singsol=all)
```

$$y(x) = -\frac{c_1 \mathrm{e}^{-3x}}{3} + x + c_2$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 20

```
DSolve[y''[x]+3*y'[x]==3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - \frac{1}{3} c_1 e^{-3x} + c_2$$

16.2 problem 475

Internal problem ID [15245]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 475.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - 7y' = (x-1)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)-7*diff(y(x),x)=(x-1)^2,y(x), singsol=all)
```

$$y(x) = \frac{6x^2}{49} - \frac{x^3}{21} + \frac{e^{7x}c_1}{7} - \frac{37x}{343} + c_2$$

✓ Solution by Mathematica
Time used: 0.159 (sec). Leaf size: 38

```
DSolve[y''[x]-7*y'[x]==(x-1)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^3}{21} + \frac{6x^2}{49} - \frac{37x}{343} + \frac{1}{7}c_1e^{7x} + c_2$$

16.3 problem 476

Internal problem ID [15246]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 476.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + 3y' = e^{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)=exp(x),y(x), singsol=all)
```

$$y(x) = -\frac{e^{-3x}\left(-3c_2e^{3x} + c_1 - \frac{3\,e^{4x}}{4}\right)}{3}$$

✓ Solution by Mathematica
Time used: 0.062 (sec). Leaf size: 26

```
DSolve[y''[x]+3*y'[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x}{4} - \frac{1}{3}c_1e^{-3x} + c_2$$

16.4 problem 477

Internal problem ID [15247]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 477.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + 7y' = e^{-7x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)+7*diff(y(x),x)=exp(-7*x),y(x), singsol=all)
```

$$y(x) = \frac{(-7x - 7c_1 - 1)\, e^{-7x}}{49} + c_2$$

✓ Solution by Mathematica
Time used: 0.077 (sec). Leaf size: 26

```
DSolve[y''[x]+7*y'[x]==Exp[-7*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{1}{49} e^{-7x}(7x + 1 + 7c_1)$$

16.5 problem 478

Internal problem ID [15248]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 478.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 8y' + 16y = (1 - x)\,\mathrm{e}^{4x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)-8*diff(y(x),x)+16*y(x)=(1-x)*exp(4*x),y(x), singsol=all)
```

$$y(x) = -\frac{(x^3 - 3x^2 + (-6c_1 + 2)\,x - 6c_2)\,\mathrm{e}^{4x}}{6}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 34

```
DSolve[y''[x]-8*y'[x]+16*y[x]==(1-x)*Exp[4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}e^{4x}(-x^3 + 3x^2 + 6c_2x + 6c_1)$$

16.6 problem 479

Internal problem ID [15249]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 479.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 10y' + 25y = e^{5x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)-10*diff(y(x),x)+25*y(x)=exp(5*x),y(x), singsol=all)
```

$$y(x) = e^{5x}\left(c_2 + c_1 x + \frac{1}{2}x^2\right)$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 27

```
DSolve[y''[x]-10*y'[x]+25*y[x]==Exp[5*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{5x}\left(x^2 + 2c_2 x + 2c_1\right)$$

16.7 problem 480

Internal problem ID [15250]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 480.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{4y'' - 3y' = x\,\mathrm{e}^{\frac{3x}{4}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(4*diff(y(x),x$2)-3*diff(y(x),x)=x*exp(3/4*x),y(x), singsol=all)
```

$$y(x) = \frac{(9x^2 + 72c_1 - 24x + 32)\,\mathrm{e}^{\frac{3x}{4}}}{54} + c_2$$

✓ Solution by Mathematica
Time used: 0.155 (sec). Leaf size: 35

```
DSolve[y''[x]-3*y'[x]==x*Exp[3/4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{16}{243}e^{3x/4}(9x - 8) + \frac{1}{3}c_1e^{3x} + c_2$$

16.8 problem 481

Internal problem ID [15251]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 481.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - 4y' = \mathrm{e}^{4x} x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)=x*exp(4*x),y(x), singsol=all)
```

$$y(x) = \frac{(8x^2 + 16c_1 - 4x + 1)\,\mathrm{e}^{4x}}{64} + c_2$$

✓ Solution by Mathematica
Time used: 0.072 (sec). Leaf size: 31

```
DSolve[y''[x]-4*y'[x]==x*Exp[4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{64} e^{4x} \left(8x^2 - 4x + 1 + 16c_1\right) + c_2$$

16.9 problem 482

Internal problem ID [15252]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 482.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 25y = \cos(5x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+25*y(x)=cos(5*x),y(x), singsol=all)
```

$$y(x) = \frac{(50c_1 + 1)\cos(5x)}{50} + \frac{\sin(5x)(x + 10c_2)}{10}$$

✓ Solution by Mathematica
Time used: 0.097 (sec). Leaf size: 31

```
DSolve[y''[x]+25*y[x]==Cos[5*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\frac{1}{100} + c_1\right)\cos(5x) + \frac{1}{10}(x + 10c_2)\sin(5x)$$

16.10 problem 483

Internal problem ID [15253]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 483.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = -\cos(x) + \sin(x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+y(x)=sin(x)-cos(x),y(x), singsol=all)
```

$$y(x) = \frac{(2c_1 - x - 1)\cos(x)}{2} - \frac{\sin(x)(x - 2c_2)}{2}$$

✓ Solution by Mathematica
Time used: 0.093 (sec). Leaf size: 31

```
DSolve[y''[x]+y[x]==Sin[x]-Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(-((x + 1 - 2c_1)\cos(x)) - (x - 2c_2)\sin(x))$$

16.11 problem 484

Internal problem ID [15254]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 484.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 16y = \sin(4x + \alpha)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)+16*y(x)=sin(4*x+alpha),y(x), singsol=all)
```

$$y(x) = \sin(4x)\, c_2 + \cos(4x)\, c_1 - \frac{x\cos(4x + \alpha)}{8} + \frac{\sin(4x + \alpha)}{64}$$

✓ Solution by Mathematica

Time used: 0.099 (sec). Leaf size: 41

```
DSolve[y''[x]+16*y[x]==Sin[4*x+\[Alpha]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{64}\sin(\alpha + 4x) - \frac{1}{8}x\cos(\alpha + 4x) + c_1\cos(4x) + c_2\sin(4x)$$

16.12 problem 485

Internal problem ID [15255]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 485.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 8y = e^{2x}(\sin(2x) + \cos(2x))}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+8*y(x)=exp(2*x)*(sin(2*x)+cos(2*x)),y(x), singsol=all)
```

$$y(x) = \frac{(16c_2 e^{-2x} + e^{2x}) \sin(2x)}{16} + e^{-2x} \cos(2x) c_1$$

✓ Solution by Mathematica

Time used: 0.287 (sec). Leaf size: 38

```
DSolve[y''[x]+4*y'[x]+8*y[x]==Exp[2*x]*(Sin[2*x]+Cos[2*x]),y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{1}{16} e^{-2x} \left(16c_2 \cos(2x) + \left(e^{4x} + 16c_1\right) \sin(2x)\right)$$

16.13 problem 486

Internal problem ID [15256]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 486.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 4y' + 8y = e^{2x}(-\cos(2x) + \sin(2x))$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+8*y(x)=exp(2*x)*(sin(2*x)-cos(2*x)),y(x), singsol=all)
```

$$y(x) = -\frac{e^{2x}\left(\left(x - 4c_1 + \frac{1}{2}\right)\cos(2x) + \sin(2x)(x - 4c_2)\right)}{4}$$

✓ Solution by Mathematica

Time used: 0.187 (sec). Leaf size: 41

```
DSolve[y''[x]-4*y'[x]+8*y[x]==Exp[2*x]*(Sin[2*x]-Cos[2*x]),y[x],x,IncludeSingularSolutions -
```

$$y(x) \to -\frac{1}{8}e^{2x}((2x + 1 - 8c_2)\cos(2x) + 2(x - 4c_1)\sin(2x))$$

16.14 problem 487

Internal problem ID [15257]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 487.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 6y' + 13y = e^{-3x} \cos(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+6*diff(y(x),x)+13*y(x)=exp(-3*x)*cos(2*x),y(x), singsol=all)
```

$$y(x) = \frac{e^{-3x}\left(\sin(2x)(x + 4c_2) + 4\cos(2x)\left(c_1 + \frac{1}{8}\right)\right)}{4}$$

✓ Solution by Mathematica
Time used: 0.094 (sec). Leaf size: 38

```
DSolve[y''[x]+6*y'[x]+13*y[x]==Exp[-3*x]*Cos[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{16} e^{-3x}((1 + 16c_2)\cos(2x) + 4(x + 4c_1)\sin(2x))$$

16.15 problem 488

Internal problem ID [15258]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 488.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + k^2 y = k \sin(kx + \alpha)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$2)+k^2*y(x)=k*sin(k*x+alpha),y(x), singsol=all)
```

$$y(x) = \frac{-2kx \cos(kx + \alpha) + 4 \sin(kx) c_2 k + 4 \cos(kx) c_1 k + \sin(kx + \alpha)}{4k}$$

✓ Solution by Mathematica
Time used: 0.109 (sec). Leaf size: 44

```
DSolve[y''[x]+k^2*y[x]==k*Sin[k*x+\[Alpha]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sin(\alpha + kx)}{4k} - \frac{1}{2} x \cos(\alpha + kx) + c_1 \cos(kx) + c_2 \sin(kx)$$

16.16 problem 489

Internal problem ID [15259]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 489.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + k^2 y = k}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)+k^2*y(x)=k,y(x), singsol=all)
```

$$y(x) = \sin(kx)\, c_2 + \cos(kx)\, c_1 + \frac{1}{k}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 23

```
DSolve[y''[x]+k^2*y[x]==k,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(kx) + c_2 \sin(kx) + \frac{1}{k}$$

16.17 problem 490

Internal problem ID [15260]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 490.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' + y = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 41

```
dsolve(diff(y(x),x$3)+y(x)=x,y(x), singsol=all)
```

$$y(x) = \left(c_2 e^{\frac{3x}{2}} \cos\left(\frac{x\sqrt{3}}{2}\right) + c_3 e^{\frac{3x}{2}} \sin\left(\frac{x\sqrt{3}}{2}\right) + e^x x + c_1\right) e^{-x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 57

```
DSolve[y'''[x]+y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + c_1 e^{-x} + c_3 e^{x/2} \cos\left(\frac{\sqrt{3}x}{2}\right) + c_2 e^{x/2} \sin\left(\frac{\sqrt{3}x}{2}\right)$$

16.18 problem 491

Internal problem ID [15261]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 491.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 6y'' + 11y' + 6y = 1}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$3)+6*diff(y(x),x$2)+11*diff(y(x),x)+6*y(x)=1,y(x), singsol=all)
```

$$y(x) = \frac{1}{6} + c_1 \mathrm{e}^{-3x} + c_2 \mathrm{e}^{-2x} + c_3 \mathrm{e}^{-x}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 33

```
DSolve[y'''[x]+6*y''[x]+11*y'[x]+6*y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-3x} + c_2 e^{-2x} + c_3 e^{-x} + \frac{1}{6}$$

16.19 problem 492

Internal problem ID [15262]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 492.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + y' = 2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$3)+diff(y(x),x)=2,y(x), singsol=all)
```

$$y(x) = \sin(x) c_1 - \cos(x) c_2 + 2x + c_3$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 22

```
DSolve[y'''[x]+y'[x]==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x - c_2 \cos(x) + c_1 \sin(x) + c_3$$

16.20 problem 493

Internal problem ID [15263]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 493.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + y'' = 3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$3)+diff(y(x),x$2)=3,y(x), singsol=all)
```

$$y(x) = \frac{3x^2}{2} + c_1 e^{-x} + c_2 x + c_3$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 27

```
DSolve[y'''[x]+y''[x]==3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3x^2}{2} + c_3 x + c_1 e^{-x} + c_2$$

16.21 problem 494

Internal problem ID [15264]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 494.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - y = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$4)-y(x)=1,y(x), singsol=all)
```

$$y(x) = -1 + \cos(x) c_1 + c_2 e^{x} + c_3 \sin(x) + c_4 e^{-x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 31

```
DSolve[y''''[x]-y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x + c_3 e^{-x} + c_2 \cos(x) + c_4 \sin(x) - 1$$

16.22 problem 495

Internal problem ID [15265]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 495.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - y' = 2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 55

```
dsolve(diff(y(x),x$4)-diff(y(x),x)=2,y(x), singsol=all)
```

$$y(x) = -\frac{e^{-\frac{x}{2}}\left(\sqrt{3}\,c_3 + c_2\right)\cos\left(\frac{x\sqrt{3}}{2}\right)}{2} + \frac{e^{-\frac{x}{2}}\left(\sqrt{3}\,c_2 - c_3\right)\sin\left(\frac{x\sqrt{3}}{2}\right)}{2} + c_1 e^{x} - 2x + c_4$$

✓ Solution by Mathematica
Time used: 0.219 (sec). Leaf size: 85

```
DSolve[y''''[x]-y'[x]==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2x + c_1 e^x - \frac{1}{2}\left(c_2 + \sqrt{3}c_3\right)e^{-x/2}\cos\left(\frac{\sqrt{3}x}{2}\right) + \frac{1}{2}\left(\sqrt{3}c_2 - c_3\right)e^{-x/2}\sin\left(\frac{\sqrt{3}x}{2}\right) + c_4$$

16.23 problem 496

Internal problem ID [15266]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 496.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - y'' = 3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$4)-diff(y(x),x$2)=3,y(x), singsol=all)
```

$$y(x) = c_2 \mathrm{e}^{x} - \frac{3x^2}{2} + c_1 \mathrm{e}^{-x} + c_3 x + c_4$$

✓ Solution by Mathematica
Time used: 0.045 (sec). Leaf size: 33

```
DSolve[y''''[x]-y''[x]==3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{3x^2}{2} + c_4 x + c_1 e^{x} + c_2 e^{-x} + c_3$$

16.24 problem 497

Internal problem ID [15267]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 497.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - y''' = 4}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$4)-diff(y(x),x$3)=4,y(x), singsol=all)
```

$$y(x) = c_1 e^x + \frac{c_2 x^2}{2} - \frac{2x^3}{3} + c_3 x + c_4$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 31

```
DSolve[y''''[x]-y'''[x]==4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2x^3}{3} + c_4 x^2 + c_3 x + c_1 e^x + c_2$$

16.25 problem 498

Internal problem ID [15268]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 498.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 4y''' + 4y'' = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$4)+4*diff(y(x),x$3)+4*diff(y(x),x$2)=1,y(x), singsol=all)
```

$$y(x) = \frac{(2c_1x + 2c_1 + 2c_2)\,\mathrm{e}^{-2x}}{8} + \frac{x^2}{8} + c_3x + c_4$$

✓ Solution by Mathematica
Time used: 0.166 (sec). Leaf size: 37

```
DSolve[y''''[x]+4*y'''[x]+4*y''[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{8} + c_4x + \frac{1}{4}e^{-2x}(c_2(x+1) + c_1) + c_3$$

16.26 problem 499

Internal problem ID [15269]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 499.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y'''' + 2y''' + y'' = e^{4x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$3)+diff(y(x),x$2)=exp(4*x),y(x), singsol=all)
```

$$y(x) = (c_1(x+2)+c_2)\,e^{-x} + c_3x + c_4 + \frac{e^{4x}}{400}$$

✓ Solution by Mathematica
Time used: 0.166 (sec). Leaf size: 36

```
DSolve[y''''[x]+2*y'''[x]+y''[x]==Exp[4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{4x}}{400} + e^{-x}(c_2(x+2)+c_1) + c_4x + c_3$$

16.27 problem 500

Internal problem ID [15270]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 500.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y'''' + 2y''' + y'' = \mathrm{e}^{-x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$3)+diff(y(x),x$2)=exp(-x),y(x), singsol=all)
```

$$y(x) = \frac{(x^2 + (2c_1 + 4)\,x + 4c_1 + 2c_2 + 6)\,\mathrm{e}^{-x}}{2} + c_3x + c_4$$

✓ Solution by Mathematica
Time used: 0.111 (sec). Leaf size: 47

```
DSolve[y''''[x]+2*y'''[x]+y''[x]==Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(x^2 + 2x(c_4e^x + 2 + c_2) + 2(c_3e^x + 3 + c_1 + 2c_2)\right)$$

16.28 problem 501

Internal problem ID [15271]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 501.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y'''' + 2y''' + y'' = x\,\mathrm{e}^{-x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$3)+diff(y(x),x$2)=x*exp(-x),y(x), singsol=all)
```

$$y(x) = \frac{(24 + x^3 + 6x^2 + 6(3 + c_1)\,x + 12c_1 + 6c_2)\,\mathrm{e}^{-x}}{6} + c_3x + c_4$$

✓ Solution by Mathematica
Time used: 0.127 (sec). Leaf size: 52

```
DSolve[y''''[x]+2*y'''[x]+y''[x]==x*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}e^{-x}\left(x^3 + 6x^2 + 6x(c_4e^x + 3 + c_2) + 6(c_3e^x + 4 + c_1 + 2c_2)\right)$$

16.29 problem 502

Internal problem ID [15272]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 502.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' + 4y'' + 4y = \sin(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$4)+4*diff(y(x),x$2)+4*y(x)=sin(2*x),y(x), singsol=all)
```

$$y(x) = (c_3x + c_1)\cos\left(\sqrt{2}\,x\right) + (c_4x + c_2)\sin\left(\sqrt{2}\,x\right) + \frac{\sin(2x)}{4}$$

✓ Solution by Mathematica
Time used: 0.521 (sec). Leaf size: 46

```
DSolve[y''''[x]+4*y''[x]+4*y[x]==Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\sin(2x) + (c_2x + c_1)\cos\left(\sqrt{2}x\right) + (c_4x + c_3)\sin\left(\sqrt{2}x\right)$$

16.30 problem 503

Internal problem ID [15273]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 503.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' + 4y'' + 4y = \cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$4)+4*diff(y(x),x$2)+4*y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = (c_3x + c_1)\cos\left(\sqrt{2}\,x\right) + (c_4x + c_2)\sin\left(\sqrt{2}\,x\right) + \cos(x)$$

✓ Solution by Mathematica
Time used: 0.288 (sec). Leaf size: 40

```
DSolve[y''''[x]+4*y''[x]+4*y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x) + (c_2x + c_1)\cos\left(\sqrt{2}x\right) + (c_4x + c_3)\sin\left(\sqrt{2}x\right)$$

16.31 problem 504

Internal problem ID [15274]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 504.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' + 4y'' + 4y = x \sin(2x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$4)+4*diff(y(x),x$2)+4*y(x)=x*sin(2*x),y(x), singsol=all)
```

$$y(x) = (c_3x + c_1)\cos\left(\sqrt{2}\,x\right) + (c_4x + c_2)\sin\left(\sqrt{2}\,x\right) + \frac{x\sin(2x)}{4} + \cos(2x)$$

✓ Solution by Mathematica
Time used: 0.007 (sec). Leaf size: 58

```
DSolve[y''''[x]+4*y''[x]+4*y[x]==x*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}x\sin(2x) + \cos(2x) + (c_2x + c_1)\cos\left(\sqrt{2}x\right) + c_3\sin\left(\sqrt{2}x\right) + c_4x\sin\left(\sqrt{2}x\right)$$

16.32 problem 505

Internal problem ID [15275]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 505.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' + 2n^2y'' + n^4y = a\sin(nx + \alpha)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 67

```
dsolve(diff(y(x),x$4)+2*n^2*diff(y(x),x$2)+n^4*y(x)=a*sin(n*x+alpha),y(x), singsol=all)
```

$$y(x) = \frac{a(-n^2x^2+2)\sin(nx+\alpha) - 2(ax\cos(nx+\alpha) - 4((c_3x+c_1)\cos(nx) + \sin(nx)(c_4x+c_2))n^3)n}{8n^4}$$

✓ Solution by Mathematica

Time used: 0.193 (sec). Leaf size: 79

```
DSolve[y''''[x]+2*n^2*y''[x]+n^4*y[x]==a*Sin[n*x+\[Alpha]],y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{3a\sin(\alpha+nx)}{16n^4} - \frac{ax\cos(\alpha+nx)}{4n^3} - \frac{ax^2\sin(\alpha+nx)}{8n^2} + (c_2x+c_1)\cos(nx) + c_4x\sin(nx) + c_3\sin(nx)$$

16.33 problem 506

Internal problem ID [15276]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 506.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' - 2n^2y'' + n^4y = \cos(nx + \alpha)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 49

```
dsolve(diff(y(x),x$4)-2*n^2*diff(y(x),x$2)+n^4*y(x)=cos(n*x+alpha),y(x), singsol=all)
```

$$y(x) = \frac{\cos(nx + \alpha) + (4c_4x + 4c_2)\, n^4 \mathrm{e}^{-nx} + (4c_3x + 4c_1)\, n^4 \mathrm{e}^{nx}}{4n^4}$$

✓ Solution by Mathematica
Time used: 0.234 (sec). Leaf size: 52

```
DSolve[y''''[x]-2*n^2*y''[x]+n^4*y[x]==Cos[n*x+\[Alpha]],y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{\cos(\alpha + nx)}{4n^4} + e^{-nx}\left(c_3e^{2nx} + c_4xe^{2nx} + c_2x + c_1\right)$$

16.34 problem 507

Internal problem ID [15277]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 507.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' + 4y''' + 6y'' + 4y' + y = \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$4)+4*diff(y(x),x$3)+6*diff(y(x),x$2)+4*diff(y(x),x)+y(x)=sin(x),y(x), sin
```

$$y(x) = \left(c_3x^3 + c_2x^2 + c_4x + c_1\right) e^{-x} - \frac{\sin(x)}{4}$$

✓ Solution by Mathematica
Time used: 0.102 (sec). Leaf size: 35

```
DSolve[y''''[x]+4*y'''[x]+6*y''[x]+4*y'[x]+y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to -\frac{\sin(x)}{4} + e^{-x}(x(x(c_4x + c_3) + c_2) + c_1)$$

16.35 problem 508

Internal problem ID [15278]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 508.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$\boxed{y'''' - 4y''' + 6y'' - 4y' + y = e^x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$4)-4*diff(y(x),x$3)+6*diff(y(x),x$2)-4*diff(y(x),x)+y(x)=exp(x),y(x), sin
```

$$y(x) = e^x \left(\frac{1}{24}x^4 + c_1 + c_2x + c_3x^2 + c_4x^3\right)$$

✓ Solution by Mathematica
Time used: 0.007 (sec). Leaf size: 39

```
DSolve[y''''[x]-4*y'''[x]+6*y''[x]-4*y'[x]+y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{1}{24}e^x \left(x^4 + 24c_4x^3 + 24c_3x^2 + 24c_2x + 24c_1\right)$$

16.36 problem 509

Internal problem ID [15279]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 509.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' - 4y''' + 6y'' - 4y' + y = x\,\mathrm{e}^{x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$4)-4*diff(y(x),x$3)+6*diff(y(x),x$2)-4*diff(y(x),x)+y(x)=x*exp(x),y(x), s
```

$$y(x) = \mathrm{e}^{x}\left(\frac{1}{120}x^{5} + c_1 + c_2 x + c_3 x^{2} + c_4 x^{3}\right)$$

✓ Solution by Mathematica
Time used: 0.007 (sec). Leaf size: 39

```
DSolve[y''''[x]-4*y'''[x]+6*y''[x]-4*y'[x]+y[x]==x*Exp[x],y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{120}e^{x}\left(x^{5} + 120c_4x^{3} + 120c_3x^{2} + 120c_2x + 120c_1\right)$$

16.37 problem 510

Internal problem ID [15280]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 510.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + y = -2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=-2,y(x), singsol=all)
```

$$y(x) = -2 + (c_1x + c_2)\,\mathrm{e}^{-x}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 23

```
DSolve[y''[x]+2*y'[x]+y[x]==-2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(-2e^x + c_2x + c_1)$$

16.38 problem 511

Internal problem ID [15281]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 511.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' = -2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)=-2,y(x), singsol=all)
```

$$y(x) = -\frac{e^{-2x}c_1}{2} - x + c_2$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 22

```
DSolve[y''[x]+2*y'[x]==-2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x - \frac{1}{2}c_1e^{-2x} + c_2$$

16.39 problem 512

Internal problem ID [15282]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 512.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 9y = 9}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)+9*y(x)=9,y(x), singsol=all)
```

$$y(x) = \sin(3x) c_2 + \cos(3x) c_1 + 1$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 21

```
DSolve[y''[x]+9*y[x]==9,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(3x) + c_2 \sin(3x) + 1$$

16.40 problem 513

Internal problem ID [15283]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISE-LYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 513.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + y'' = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$3)+diff(y(x),x$2)=1,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} + c_1 e^{-x} + c_2 x + c_3$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 27

```
DSolve[y'''[x]+y''[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2} + c_3 x + c_1 e^{-x} + c_2$$

16.41 problem 514

Internal problem ID [15284]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 514.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{5y''' - 7y'' = 3}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 21

```
dsolve(5*diff(y(x),x$3)-7*diff(y(x),x$2)=3,y(x), singsol=all)
```

$$y(x) = -\frac{3x^2}{14} + \frac{25\,\mathrm{e}^{\frac{7x}{5}} c_1}{49} + c_2 x + c_3$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 30

```
DSolve[y'''[x]-7*y''[x]==3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{3x^2}{14} + c_3 x + \frac{1}{49} c_1 e^{7x} + c_2$$

16.42 problem 515

Internal problem ID [15285]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 515.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 6y''' = -6}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$4)-6*diff(y(x),x$3)=-6,y(x), singsol=all)
```

$$y(x) = \frac{e^{6x}c_1}{216} + \frac{x^3}{6} + \frac{c_2x^2}{2} + c_3x + c_4$$

✓ Solution by Mathematica
Time used: 0.064 (sec). Leaf size: 36

```
DSolve[y''''[x]-6*y'''[x]==-6,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{6} + c_4x^2 + c_3x + \frac{1}{216}c_1e^{6x} + c_2$$

16.43 problem 516

Internal problem ID [15286]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 516.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{3y'''' + y''' = 2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(3*diff(y(x),x$4)+diff(y(x),x$3)=2,y(x), singsol=all)
```

$$y(x) = \frac{x^3}{3} + \frac{c_2 x^2}{2} - 27\,\mathrm{e}^{-\frac{x}{3}} c_1 + c_3 x + c_4$$

✓ Solution by Mathematica
Time used: 0.074 (sec). Leaf size: 36

```
DSolve[3*y''''[x]+y'''[x]==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{3} + c_4 x^2 + c_3 x - 27 c_1 e^{-x/3} + c_2$$

16.44 problem 517

Internal problem ID [15287]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 517.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 2y''' + 2y'' - 2y' + y = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$4)-2*diff(y(x),x$3)+2*diff(y(x),x$2)-2*diff(y(x),x)+y(x)=1,y(x), singsol=
```

$$y(x) = (c_4 x + c_2)\,\mathrm{e}^{x} + \cos(x)\, c_1 + c_3 \sin(x) + 1$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 30

```
DSolve[y''''[x]-2*y'''[x]+2*y''[x]-2*y'[x]+y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3 e^x + c_4 e^x x + c_1 \cos(x) + c_2 \sin(x) + 1$$

16.45 problem 518

Internal problem ID [15288]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 518.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y' + 4y = x^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=x^2,y(x), singsol=all)
```

$$y(x) = \frac{3}{8} + (c_1 x + c_2)\,e^{2x} + \frac{x^2}{4} + \frac{x}{2}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 37

```
DSolve[y''[x]-4*y'[x]+4*y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}\left(2x^2 + 4x + 3\right) + c_1 e^{2x} + c_2 e^{2x} x$$

16.46 problem 519

Internal problem ID [15289]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 519.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + 8y' = 8x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+8*diff(y(x),x)=8*x,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} - \frac{e^{-8x}c_1}{8} - \frac{x}{8} + c_2$$

✓ Solution by Mathematica
Time used: 0.07 (sec). Leaf size: 31

```
DSolve[y''[x]+8*y'[x]==8*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2} - \frac{x}{8} - \frac{1}{8}c_1e^{-8x} + c_2$$

16.47 problem 520

Internal problem ID [15290]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 520.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'k + k^2y = \mathrm{e}^x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)-2*k*diff(y(x),x)+k^2*y(x)=exp(x),y(x), singsol=all)
```

$$y(x) = \frac{(k-1)^2 (c_1 x + c_2) \mathrm{e}^{kx} + \mathrm{e}^x}{(k-1)^2}$$

✓ Solution by Mathematica
Time used: 0.14 (sec). Leaf size: 28

```
DSolve[y''[x]-2*k*y'[x]+k^2*y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x}{(k-1)^2} + (c_2 x + c_1) e^{kx}$$

16.48 problem 521

Internal problem ID [15291]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 521.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y' + 4y = 8\,\mathrm{e}^{-2x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+4*y(x)=8*exp(-2*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-2x}\left(c_1 x + 4x^2 + c_2\right)$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 23

```
DSolve[y''[x]+4*y'[x]+4*y[x]==8*Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(4x^2 + c_2 x + c_1\right)$$

16.49 problem 522

Internal problem ID [15292]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 522.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y' + 3y = 9\,\mathrm{e}^{-3x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+3*y(x)=9*exp(-3*x),y(x), singsol=all)
```

$$y(x) = \frac{(-9x + 2c_2)\,\mathrm{e}^{-3x}}{2} + c_1\mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 32

```
DSolve[y''[x]+4*y'[x]+3*y[x]==9*Exp[-3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{-3x}\left(-18x + 4c_2e^{2x} - 9 + 4c_1\right)$$

16.50 problem 523

Internal problem ID [15293]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 523.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{7y'' - y' = 14x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(7*diff(y(x),x$2)-diff(y(x),x)=14*x,y(x), singsol=all)
```

$$y(x) = 7\,\mathrm{e}^{\frac{x}{7}} c_1 - 7x^2 - 98x + c_2$$

✓ Solution by Mathematica
Time used: 0.067 (sec). Leaf size: 27

```
DSolve[7*y''[x]-y'[x]==14*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -7x^2 - 98x + 7c_1 e^{x/7} + c_2$$

16.51 problem 524

Internal problem ID [15294]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 524.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + 3y' = 3x\,\mathrm{e}^{-3x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)=3*x*exp(-3*x),y(x), singsol=all)
```

$$y(x) = \frac{(-9x^2 - 6c_1 - 6x - 2)\,\mathrm{e}^{-3x}}{18} + c_2$$

✓ Solution by Mathematica
Time used: 0.088 (sec). Leaf size: 31

```
DSolve[y''[x]+3*y'[x]==3*x*Exp[-3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{1}{18}e^{-3x}(9x^2 + 6x + 2 + 6c_1)$$

16.52 problem 525

Internal problem ID [15295]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 525.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 5y' + 6y = 10(1 - x)\,\mathrm{e}^{-2x}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+5*diff(y(x),x)+6*y(x)=10*(1-x)*exp(-2*x),y(x), singsol=all)
```

$$y(x) = \left(-5x^2 + c_1 + 20x\right)\mathrm{e}^{-2x} + \mathrm{e}^{-3x}c_2$$

✓ Solution by Mathematica
Time used: 0.06 (sec). Leaf size: 30

```
DSolve[y''[x]+5*y'[x]+6*y[x]==10*(1-x)*Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(e^x\left(-5x^2 + 20x - 20 + c_2\right) + c_1\right)$$

16.53 problem 526

Internal problem ID [15296]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 526.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y' + 2y = x + 1}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+2*y(x)=1+x,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} \sin(x) c_2 + \mathrm{e}^{-x} \cos(x) c_1 + \frac{x}{2}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 32

```
DSolve[y''[x]+2*y'[x]+2*y[x]==1+x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-x} (e^x x + 2c_2 \cos(x) + 2c_1 \sin(x))$$

16.54 problem 527

Internal problem ID [15297]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 527.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' + y = (x^2 + x)\,\mathrm{e}^x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=(x+x^2)*exp(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{2}} \cos\left(\frac{x\sqrt{3}}{2}\right) c_1 + \mathrm{e}^{-\frac{x}{2}} \sin\left(\frac{x\sqrt{3}}{2}\right) c_2 + \frac{\mathrm{e}^x \left(x^2 - x + \frac{1}{3}\right)}{3}$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 65

```
DSolve[y''[x]+y'[x]+y[x]==(x+x^2)*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{9} e^{-x/2} \left(e^{3x/2}(3x^2 - 3x + 1) + 9c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + 9c_1 \sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

16.55 problem 528

Internal problem ID [15298]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 528.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' - 2y = 8\sin(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)-2*y(x)=8*sin(2*x),y(x), singsol=all)
```

$$y(x) = e^{\left(-2+\sqrt{6}\right)x} c_2 + e^{-\left(2+\sqrt{6}\right)x} c_1 - \frac{16\cos(2x)}{25} - \frac{12\sin(2x)}{25}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 52

```
DSolve[y''[x]+4*y'[x]-2*y[x]==8*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\left(\left(2+\sqrt{6}\right)x\right)} + c_2 e^{\left(\sqrt{6}-2\right)x} - \frac{4}{25}(3\sin(2x) + 4\cos(2x))$$

16.56 problem 529

Internal problem ID [15299]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 529.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = 4\cos(x)\, x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)+y(x)=4*x*cos(x),y(x), singsol=all)
```

$$y(x) = (x^2 + c_2 - 1)\sin(x) + \cos(x)(x + c_1)$$

✓ Solution by Mathematica
Time used: 0.047 (sec). Leaf size: 30

```
DSolve[y''[x]+y[x]==4*x*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(2x^2 - 1 + 2c_2)\sin(x) + (x + c_1)\cos(x)$$

16.57 problem 530

Internal problem ID [15300]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 530.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2my' + m^2 y = \sin(nx)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 56

```
dsolve(diff(y(x),x$2)-2*m*diff(y(x),x)+m^2*y(x)=sin(n*x),y(x), singsol=all)
```

$$y(x) = \frac{(m^2+n^2)^2 (c_1 x + c_2) e^{mx} + (m^2-n^2)\sin(nx) + 2\cos(nx) mn}{(m^2+n^2)^2}$$

✓ Solution by Mathematica
Time used: 0.182 (sec). Leaf size: 56

```
DSolve[y''[x]-2*m*y'[x]+m^2*y[x]==Sin[n*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(m^2-n^2)\sin(nx) + 2mn\cos(nx)}{(m^2+n^2)^2} + c_1 e^{mx} + c_2 x e^{mx}$$

16.58 problem 531

Internal problem ID [15301]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 531.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + 5y = e^{-x} \sin(2x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+5*y(x)=exp(-x)*sin(2*x),y(x), singsol=all)
```

$$y(x) = -\frac{e^{-x}((x - 4c_1)\cos(2x) - 4\sin(2x)c_2)}{4}$$

✓ Solution by Mathematica

Time used: 0.083 (sec). Leaf size: 38

```
DSolve[y''[x]+2*y'[x]+5*y[x]==Exp[-x]*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{16}e^{-x}((1 + 16c_1)\sin(2x) - 4(x - 4c_2)\cos(2x))$$

16.59 problem 532

Internal problem ID [15302]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 532.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + a^2 y = 2\cos(mx) + 3\sin(mx)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$2)+a^2*y(x)=2*cos(m*x)+3*sin(m*x),y(x), singsol=all)
```

$$y(x) = \sin(ax)\, c_2 + \cos(ax)\, c_1 + \frac{2\cos(mx) + 3\sin(mx)}{a^2 - m^2}$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 45

```
DSolve[y''[x]+a^2*y[x]==2*Cos[m*x]+3*Sin[m*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3\sin(mx) + 2\cos(mx)}{a^2 - m^2} + c_1\cos(ax) + c_2\sin(ax)$$

16.60 problem 533

Internal problem ID [15303]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 533.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - y' = e^{x} \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)-diff(y(x),x)=exp(x)*sin(x),y(x), singsol=all)
```

$$y(x) = \frac{(2c_1 - \cos(x) - \sin(x))\, e^{x}}{2} + c_2$$

✓ Solution by Mathematica
Time used: 0.106 (sec). Leaf size: 24

```
DSolve[y''[x]-y'[x]==Exp[x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{1}{2} e^{x} (\sin(x) + \cos(x) - 2c_1)$$

16.61 problem 534

Internal problem ID [15304]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 534.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + 2y' = 4\,e^{x}(\sin(x) + \cos(x))}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)=4*exp(x)*(sin(x)+cos(x)),y(x), singsol=all)
```

$$y(x) = -\frac{\left(\frac{4(\cos(x)-3\sin(x))e^{3x}}{5} - 2c_2e^{2x} + c_1\right)e^{-2x}}{2}$$

✓ Solution by Mathematica
Time used: 0.229 (sec). Leaf size: 37

```
DSolve[y''[x]+2*y'[x]==4*Exp[x]*(Sin[x]+Cos[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{6}{5}e^x\sin(x) - \frac{2}{5}e^x\cos(x) - \frac{1}{2}c_1e^{-2x} + c_2$$

16.62 problem 535

Internal problem ID [15305]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 535.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y' + 5y = 10\,\mathrm{e}^{-2x} \cos(x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+5*y(x)=10*exp(-2*x)*cos(x),y(x), singsol=all)
```

$$y(x) = ((c_2 + 5x)\sin(x) + \cos(x)\,c_1)\,\mathrm{e}^{-2x}$$

✓ Solution by Mathematica
Time used: 0.09 (sec). Leaf size: 34

```
DSolve[y''[x]+4*y'[x]+5*y[x]==10*Exp[-2*x]*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-2x}((5 + 2c_2)\cos(x) + 2(5x + c_1)\sin(x))$$

16.63 problem 536

Internal problem ID [15306]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 536.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{4y'' + 8y' = \sin(x)\, x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(4*diff(y(x),x$2)+8*diff(y(x),x)=x*sin(x),y(x), singsol=all)
```

$$y(x) = -\frac{e^{-2x} c_1}{2} + \frac{(-1 - 5x) \cos(x)}{50} + \frac{(-5x + 14) \sin(x)}{100} + c_2$$

✓ Solution by Mathematica
Time used: 0.266 (sec). Leaf size: 42

```
DSolve[4*y''[x]+8*y'[x]==x*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\frac{7}{50} - \frac{x}{20}\right) \sin(x) - \frac{1}{50}(5x + 1) \cos(x) - \frac{1}{2} c_1 e^{-2x} + c_2$$

16.64 problem 537

Internal problem ID [15307]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 537.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 3y' + 2y = x\,\mathrm{e}^{x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=x*exp(x),y(x), singsol=all)
```

$$y(x) = -\frac{(-2c_1\mathrm{e}^{x} + x^2 - 2c_2 + 2x)\,\mathrm{e}^{x}}{2}$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 33

```
DSolve[y''[x]-3*y'[x]+2*y[x]==x*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^x\left(-x^2 - 2x + 2(c_2e^x - 1 + c_1)\right)$$

16.65 problem 538

Internal problem ID [15308]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 538.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' - 2y = x^2 \mathrm{e}^{4x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-2*y(x)=x^2*exp(4*x),y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(\frac{7}{18} + x^2 - x\right) \mathrm{e}^{6x} + 18c_1 \mathrm{e}^{3x} + 18c_2\right) \mathrm{e}^{-2x}}{18}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 39

```
DSolve[y''[x]+y'[x]-2*y[x]==x^2*Exp[4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{324} e^{4x} \left(18x^2 - 18x + 7\right) + c_1 e^{-2x} + c_2 e^x$$

16.66 problem 539

Internal problem ID [15309]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 539.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 3y' + 2y = (x^2 + x)\,\mathrm{e}^{3x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=(x+x^2)*exp(3*x),y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{x}\left(\left(x^2 - 2x + 2\right)\mathrm{e}^{2x} + 2c_1\mathrm{e}^{x} + 2c_2\right)}{2}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 37

```
DSolve[y''[x]-3*y'[x]+2*y[x]==(x+x^2)*Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{3x}\left(x^2 - 2x + 2\right) + c_1e^{x} + c_2e^{2x}$$

16.67 problem 540

Internal problem ID [15310]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 540.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - y'' + y' - y = x^2 + x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)+diff(y(x),x)-y(x)=x+x^2,y(x), singsol=all)
```

$$y(x) = -x^2 - 3x - 1 + \cos(x)\, c_1 + c_2 e^x + c_3 \sin(x)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 31

```
DSolve[y'''[x]-y''[x]+y'[x]-y[x]==x+x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^2 - 3x + c_3 e^x + c_1 \cos(x) + c_2 \sin(x) - 1$$

16.68 problem 541

Internal problem ID [15311]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 541.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$\boxed{y'''' - 2y''' + 2y'' - 2y' + y = e^{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$4)-2*diff(y(x),x$3)+2*diff(y(x),x$2)-2*diff(y(x),x)+y(x)=exp(x),y(x), sin
```

$$y(x) = \frac{(4c_4x + x^2 + 4c_2)\, e^{x}}{4} + \cos(x)\, c_1 + c_3 \sin(x)$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 40

```
DSolve[y''''[x]-2*y'''[x]+2*y''[x]-2*y'[x]+y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{1}{4}e^{x}\left(x^2 - 2x + 4c_4x + 1 + 4c_3\right) + c_1\cos(x) + c_2\sin(x)$$

16.69 problem 542

Internal problem ID [15312]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 542.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2y' + y = x^3$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=x^3,y(x), singsol=all)
```

$$y(x) = (c_1 x + c_2)\, e^x + x^3 + 6x^2 + 18x + 24$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 31

```
DSolve[y''[x]-2*y'[x]+y[x]==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^3 + 6x^2 + x(18 + c_2 e^x) + c_1 e^x + 24$$

16.70 problem 543

Internal problem ID [15313]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 543.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y'''' + y'' = x^2 + x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$4)+diff(y(x),x$2)=x^2+x,y(x), singsol=all)
```

$$y(x) = \frac{x^3}{6} - x^2 + \frac{x^4}{12} - \cos(x)\, c_1 - \sin(x)\, c_2 + c_3 x + c_4$$

✓ Solution by Mathematica
Time used: 0.073 (sec). Leaf size: 43

```
DSolve[y''''[x]+y''[x]==x^2+x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^4}{12} + \frac{x^3}{6} - x^2 + c_4 x - c_1 \cos(x) - c_2 \sin(x) + c_3$$

16.71 problem 544

Internal problem ID [15314]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 544.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = x^2 \sin(x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)+y(x)=x^2*sin(x),y(x), singsol=all)
```

$$y(x) = \frac{(-2x^3 + 12c_1 + 3x)\cos(x)}{12} + \frac{\sin(x)(x^2 + 4c_2 - 1)}{4}$$

✓ Solution by Mathematica
Time used: 0.102 (sec). Leaf size: 41

```
DSolve[y''[x]+y[x]==x^2*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(-\frac{x^3}{6} + \frac{x}{4} + c_1\right)\cos(x) + \frac{1}{8}(2x^2 - 1 + 8c_2)\sin(x)$$

16.72 problem 545

Internal problem ID [15315]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 545.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + y = x^2 e^{-x} \cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=x^2*exp(-x)*cos(x),y(x), singsol=all)
```

$$y(x) = -\left(\left(x^2 - 6\right)\cos(x) - c_1 x - 4\sin(x)\, x - c_2\right) e^{-x}$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 32

```
DSolve[y''[x]+2*y'[x]+y[x]==x^2*Exp[-x]*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(-\left(x^2 - 6\right)\cos(x) + 4x\sin(x) + c_2 x + c_1\right)$$

16.73 problem 546

Internal problem ID [15316]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 546.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - y = \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x$3)-y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = c_2 e^{-\frac{x}{2}} \cos\left(\frac{x\sqrt{3}}{2}\right) + c_3 e^{-\frac{x}{2}} \sin\left(\frac{x\sqrt{3}}{2}\right) + c_1 e^{x} - \frac{\sin(x)}{2} + \frac{\cos(x)}{2}$$

✓ Solution by Mathematica
Time used: 0.419 (sec). Leaf size: 66

```
DSolve[y'''[x]-y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sin(x)}{2} + \frac{\cos(x)}{2} + c_1 e^{x} + c_2 e^{-x/2} \cos\left(\frac{\sqrt{3}x}{2}\right) + c_3 e^{-x/2} \sin\left(\frac{\sqrt{3}x}{2}\right)$$

16.74 problem 547

Internal problem ID [15317]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 547.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' - 2y'' + y = \cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$4)-2*diff(y(x),x$2)+y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = (c_4x + c_2)\,\mathrm{e}^{-x} + (c_3x + c_1)\,\mathrm{e}^{x} + \frac{\cos(x)}{4}$$

✓ Solution by Mathematica
Time used: 0.074 (sec). Leaf size: 42

```
DSolve[y''''[x]-2*y''[x]+y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\cos(x)}{4} + e^{-x}\left(c_2x + c_3e^{2x} + c_4e^{2x}x + c_1\right)$$

16.75 problem 548

Internal problem ID [15318]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 548.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 3y'' + 3y' - y = e^{x} \cos(2x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)+3*diff(y(x),x)-y(x)=exp(x)*cos(2*x),y(x), singsol=all
```

$$y(x) = -\frac{e^{x}(-8c_3x^2 - 8c_2x + \sin(2x) - 8c_1 - 2x)}{8}$$

✓ Solution by Mathematica
Time used: 0.069 (sec). Leaf size: 33

```
DSolve[y'''[x]-3*y''[x]+3*y'[x]-y[x]==Exp[x]*Cos[2*x],y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{8}e^{x}(-\sin(2x) + 8(x(c_3x + c_2) + c_1))$$

16.76 problem 549

Internal problem ID [15319]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Trial and error method. Exercises page 132
Problem number: 549.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 5y = e^{2x}(\sin(x) + 2\cos(x))}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+5*y(x)=exp(2*x)*(sin(x)+2*cos(x)),y(x), singsol=all)
```

$$y(x) = -\frac{((x - 2c_1 - 2)\cos(x) - 2\sin(x)(c_2 + x))\, e^{2x}}{2}$$

✓ Solution by Mathematica
Time used: 0.047 (sec). Leaf size: 36

```
DSolve[y''[x]-4*y'[x]+5*y[x]==Exp[2*x]*(Sin[x]+Cos[x]),y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{1}{2}e^{2x}((-x + 1 + 2c_2)\cos(x) + (x + 2c_1)\sin(x))$$

17 Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137

17.1 problem 551

Internal problem ID [15320]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 551.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y' - 2y = \mathrm{e}^{x} + \mathrm{e}^{-2x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-2*y(x)=exp(x)+exp(-2*x),y(x), singsol=all)
```

$$y(x) = \frac{(4c_1\mathrm{e}^{4x} - 2\,\mathrm{e}^{3x} + 4c_2\mathrm{e}^{x} + 1)\,\mathrm{e}^{-2x}}{4}$$

✓ Solution by Mathematica

Time used: 0.114 (sec). Leaf size: 39

```
DSolve[y''[x]-y'[x]-2*y[x]==Exp[x]+Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{-2x}\left(-2e^{3x} + 4c_1e^{x} + 4c_2e^{4x} + 1\right)$$

17.2 problem 552

Internal problem ID [15321]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 552.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + 4y' = x + e^{-4x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)=x+exp(-4*x),y(x), singsol=all)
```

$$y(x) = \frac{(-4x - 4c_1 - 1)\, e^{-4x}}{16} + \frac{x^2}{8} - \frac{x}{16} + c_2$$

✓ Solution by Mathematica
Time used: 0.35 (sec). Leaf size: 38

```
DSolve[y''[x]+4*y'[x]==x+Exp[-4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{8} - \frac{x}{16} - \frac{1}{16} e^{-4x}(4x + 1 + 4c_1) + c_2$$

17.3 problem 553

Internal problem ID [15322]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 553.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y = x + \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)-y(x)=x+sin(x),y(x), singsol=all)
```

$$y(x) = c_2 e^{-x} + c_1 e^{x} - \frac{\sin(x)}{2} - x$$

✓ Solution by Mathematica
Time used: 0.112 (sec). Leaf size: 29

```
DSolve[y''[x]-y[x]==x+Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x - \frac{\sin(x)}{2} + c_1 e^{x} + c_2 e^{-x}$$

17.4 problem 554

Internal problem ID [15323]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 554.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + 2y = (1 + \sin(x))\, e^x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+2*y(x)=(1+sin(x))*exp(x),y(x), singsol=all)
```

$$y(x) = -\frac{e^x((x - 2c_1)\cos(x) - 2 + (-2c_2 - 1)\sin(x))}{2}$$

✓ Solution by Mathematica

Time used: 0.194 (sec). Leaf size: 32

```
DSolve[y''[x]-2*y'[x]+2*y[x]==(1+Sin[x])*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^x(-((x - 2c_2)\cos(x)) + 2(1 + c_1)\sin(x) + 2)$$

17.5 problem 555

Internal problem ID [15324]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 555.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - y'' = 1 + e^x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)=1+exp(x),y(x), singsol=all)
```

$$y(x) = (c_1 + x - 2)\, e^x - \frac{x^2}{2} + c_2 x + c_3$$

✓ Solution by Mathematica
Time used: 0.115 (sec). Leaf size: 28

```
DSolve[y'''[x]-y''[x]==1+Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2}{2} + c_3 x + e^x (x - 2 + c_1) + c_2$$

17.6 problem 556

Internal problem ID [15325]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 556.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$y''' + 4y' = e^{2x} + \sin(2x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$3)+4*diff(y(x),x)=exp(2*x)+sin(2*x),y(x), singsol=all)
```

$$y(x) = \frac{(-8c_2 - 1)\cos(2x)}{16} + \frac{(-x + 4c_1)\sin(2x)}{8} + c_3 + \frac{e^{2x}}{16}$$

✓ Solution by Mathematica
Time used: 0.836 (sec). Leaf size: 44

```
DSolve[y'''[x]+4*y'[x]==Exp[2*x]+Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{32}\left(2e^{2x} - ((3 + 16c_2)\cos(2x)) - 4(x - 4c_1)\sin(2x)\right) + c_3$$

17.7 problem 557

Internal problem ID [15326]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 557.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \sin(x)\sin(2x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+4*y(x)=sin(x)*sin(2*x),y(x), singsol=all)
```

$$y(x) = \sin(2x)\,c_2 + \cos(2x)\,c_1 - \frac{2\cos(x)}{15} + \frac{2\cos(x)^3}{5}$$

✓ Solution by Mathematica
Time used: 0.178 (sec). Leaf size: 34

```
DSolve[y''[x]+4*y[x]==Sin[x]*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\cos(x)}{6} + \frac{1}{10}\cos(3x) + c_1\cos(2x) + c_2\sin(2x)$$

17.8 problem 558

Internal problem ID [15327]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 558.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - 4y' = 2\cos(4x)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)=2*cos(4*x)^2,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{4x}}{4} - \frac{\sin(8x)}{160} - \frac{\cos(8x)}{80} - \frac{x}{4} + c_2$$

✓ Solution by Mathematica
Time used: 0.243 (sec). Leaf size: 40

```
DSolve[y''[x]-4*y'[x]==2*Cos[4*x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{4} - \frac{1}{160}\sin(8x) - \frac{1}{80}\cos(8x) + \frac{1}{4}c_1 e^{4x} + c_2$$

17.9 problem 559

Internal problem ID [15328]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 559.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y' - 2y = 4x - 2\,\mathrm{e}^{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-2*y(x)=4*x-2*exp(x),y(x), singsol=all)
```

$$y(x) = c_2\mathrm{e}^{-x} + c_1\mathrm{e}^{2x} + \mathrm{e}^{x} - 2x + 1$$

✓ Solution by Mathematica
Time used: 0.111 (sec). Leaf size: 29

```
DSolve[y''[x]-y'[x]-2*y[x]==4*x-2*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2x + e^{x} + c_1e^{-x} + c_2e^{2x} + 1$$

17.10 problem 560

Internal problem ID [15329]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 560.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - 3y' = 18x - 10\cos(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)=18*x-10*cos(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{3x}}{3} - 3x^2 + 3\sin(x) + \cos(x) - 2x + c_2$$

✓ Solution by Mathematica

Time used: 0.162 (sec). Leaf size: 33

```
DSolve[y''[x]-3*y'[x]==18*x-10*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -3x^2 - 2x + 3\sin(x) + \cos(x) + \frac{1}{3}c_1 e^{3x} + c_2$$

17.11 problem 561

Internal problem ID [15330]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 561.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + y = 2 + e^{x} \sin(x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=2+exp(x)*sin(x),y(x), singsol=all)
```

$$y(x) = 2 + (c_1 x + c_2 - \sin(x)) e^{x}$$

✓ Solution by Mathematica
Time used: 0.145 (sec). Leaf size: 25

```
DSolve[y''[x]-2*y'[x]+y[x]==2+Exp[x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{x} \sin(x) + e^{x}(c_2 x + c_1) + 2$$

17.12 problem 562

Internal problem ID [15331]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 562.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' + 2y = (5x + 4)\,e^{x} + e^{-x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+2*y(x)=(5*x+4)*exp(x)+exp(-x),y(x), singsol=all)
```

$$y(x) = e^{-x}\sin(x)\,c_2 + e^{-x}\cos(x)\,c_1 + e^{x}x + e^{-x}$$

✓ Solution by Mathematica

Time used: 0.193 (sec). Leaf size: 30

```
DSolve[y''[x]+2*y'[x]+2*y[x]==(5*x+4)*Exp[x]+Exp[-x],y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to e^{-x}\left(e^{2x}x + c_2\cos(x) + c_1\sin(x) + 1\right)$$

17.13 problem 563

Internal problem ID [15332]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 563.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + 5y = 4\,\mathrm{e}^{-x} + 17\sin(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+5*y(x)=4*exp(-x)+17*sin(2*x),y(x), singsol=all)
```

$$y(x) = ((c_1+1)\cos(2x) + \sin(2x)\,c_2 + 1)\,\mathrm{e}^{-x} - 4\cos(2x) + \sin(2x)$$

✓ Solution by Mathematica
Time used: 0.335 (sec). Leaf size: 37

```
DSolve[y''[x]+2*y'[x]+5*y[x]==4*Exp[-x]+17*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}((-4e^x + c_2)\cos(2x) + (e^x + c_1)\sin(2x) + 1)$$

17.14 problem 564

Internal problem ID [15333]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 564.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{2y'' - 3y' - 2y = 5\,\mathrm{e}^{x}\cosh(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(2*diff(y(x),x$2)-3*diff(y(x),x)-2*y(x)=5*exp(x)*cosh(x),y(x), singsol=all)
```

$$y(x) = -\frac{5}{4} + \mathrm{e}^{-\frac{x}{2}} c_2 + \frac{(-2 + 5x + 10c_1)\,\mathrm{e}^{2x}}{10}$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 36

```
DSolve[2*y''[x]-3*y'[x]-2*y[x]==5*Exp[x]*Cosh[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x/2} + e^{2x}\left(\frac{x}{2} - \frac{1}{5} + c_2\right) - \frac{5}{4}$$

17.15 problem 565

Internal problem ID [15334]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 565.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = x \sin(x)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)+4*y(x)=x*sin(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{(-8x^2 + 128c_2 + 1)\sin(2x)}{128} + \frac{(-x + 32c_1)\cos(2x)}{32} + \frac{x}{8}$$

✓ Solution by Mathematica
Time used: 0.245 (sec). Leaf size: 41

```
DSolve[y''[x]+4*y[x]==x*Sin[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{128}\left((-8x^2 + 1 + 128c_2)\sin(2x) + 16x - 4(x - 32c_1)\cos(2x)\right)$$

17.16 problem 566

Internal problem ID [15335]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 566.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' + 2y''' + 2y'' + 2y' + y = x\,\mathrm{e}^{x} + \frac{\cos(x)}{2}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 43

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$3)+2*diff(y(x),x$2)+2*diff(y(x),x)+y(x)=x*exp(x)+1/2*cos
```

$$y(x) = (c_4x + c_3)\,\mathrm{e}^{-x} + \frac{(-x + 8c_1 + 1)\cos(x)}{8} + \frac{(x-2)\,\mathrm{e}^{x}}{8} + \frac{\sin(x)\,(4c_2 + 1)}{4}$$

✓ Solution by Mathematica

Time used: 0.234 (sec). Leaf size: 52

```
DSolve[y''''[x]+2*y'''[x]+2*y''[x]+2*y'[x]+y[x]==x*Exp[x]+1/2*Cos[x],y[x],x,IncludeSingularS
```

$$y(x) \to \frac{1}{16}\left(2e^{x}(x-2) + 16e^{-x}(c_4x + c_3) - 2(x - 1 - 8c_1)\cos(x) + (3 + 16c_2)\sin(x)\right)$$

17.17 problem 567

Internal problem ID [15336]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 567.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$y'' + y' = \cos(x)^2 + e^x + x^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=cos(x)^2+exp(x)+x^2,y(x), singsol=all)
```

$$y(x) = -x^2 + \frac{x^3}{3} - c_1 e^{-x} + \frac{e^x}{2} - \frac{\cos(2x)}{10} + \frac{\sin(2x)}{20} + \frac{5x}{2} + c_2$$

✓ Solution by Mathematica

Time used: 0.529 (sec). Leaf size: 55

```
DSolve[y''[x]+y'[x]==Cos[x]^2+Exp[x]+x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}\left(x(2x^2 - 6x + 15) + 3e^x\right) + \frac{1}{20}\sin(2x) - \frac{1}{10}\cos(2x) - c_1 e^{-x} + c_2$$

17.18 problem 568

Internal problem ID [15337]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 568.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y'''' + 4y''' = e^{x} + 3\sin(2x) + 1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 60

```
dsolve(diff(y(x),x$4)+4*diff(y(x),x$3)=exp(x)+3*sin(2*x)+1,y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(-\frac{18\sin(x)^2}{5} + \frac{9\sin(x)\cos(x)}{5} + x^3 + \left(12c_2 - \frac{18}{5}\right)x^2 + \left(24c_3 - \frac{9}{5}\right)x + 24c_4\right)e^{4x} + \frac{24\,e^{5x}}{5} - \frac{3c_1}{8}\right)e^{-4x}}{24}$$

✓ Solution by Mathematica

Time used: 0.877 (sec). Leaf size: 59

```
DSolve[y''''[x]+4*y'''[x]==Exp[x]+3*Sin[2*x]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{24} + c_4x^2 + \frac{e^x}{5} + \frac{3}{80}\sin(2x) + \frac{3}{40}\cos(2x) + c_3x - \frac{1}{64}c_1e^{-4x} + c_2$$

17.19 problem 569

Internal problem ID [15338]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 569.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + 5y = 10\sin(x) + 17\sin(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+5*y(x)=10*sin(x)+17*sin(2*x),y(x), singsol=all)
```

$$y(x) = (c_1 e^x + 4)\cos(2x) + e^x \sin(2x) c_2 + \cos(x) + 2\sin(x) + \sin(2x)$$

✓ Solution by Mathematica
Time used: 0.538 (sec). Leaf size: 37

```
DSolve[y''[x]-2*y'[x]+5*y[x]==10*Sin[x]+17*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x) + (4 + c_2 e^x)\cos(2x) + 2\sin(x)(\cos(x) + c_1 e^x \cos(x) + 1)$$

17.20 problem 570

Internal problem ID [15339]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 570.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + y' = x^2 - e^{-x} + e^x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=x^2-exp(-x)+exp(x),y(x), singsol=all)
```

$$y(x) = (1 + x - c_1) e^{-x} + \frac{x^3}{3} - x^2 + 2x + c_2 + \frac{e^x}{2}$$

✓ Solution by Mathematica
Time used: 0.159 (sec). Leaf size: 43

```
DSolve[y''[x]+y'[x]==x^2-Exp[-x]+Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{3} - x^2 + 2x + \frac{e^x}{2} + e^{-x}(x + 1 - c_1) + c_2$$

17.21 problem 571

Internal problem ID [15340]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 571.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' - 3y = 2x + e^{-x} - 2\,e^{3x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-3*y(x)=2*x+exp(-x)-2*exp(3*x),y(x), singsol=all)
```

$$y(x) = \frac{4}{9} + \frac{(-1 - 4x + 16c_1)\,e^{-x}}{16} + \frac{(1 - 4x + 8c_2)\,e^{3x}}{8} - \frac{2x}{3}$$

✓ Solution by Mathematica
Time used: 0.501 (sec). Leaf size: 51

```
DSolve[y''[x]-2*y'[x]-3*y[x]==2*x+Exp[-x]-2*Exp[3*x],y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{1}{144}e^{-x}\left(e^x(64 - 96x) - 9(4x + 1 - 16c_1) - 18e^{4x}(4x - 1 - 8c_2)\right)$$

17.22 problem 572

Internal problem ID [15341]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 572.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = e^x + 4\sin(2x) + 2\cos(x)^2 - 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+4*y(x)=exp(x)+4*sin(2*x)+2*cos(x)^2-1,y(x), singsol=all)
```

$$y(x) = \frac{(2 + x + 4c_2)\sin(2x)}{4} + (c_1 - x)\cos(2x) + \frac{e^x}{5}$$

✓ Solution by Mathematica

Time used: 0.435 (sec). Leaf size: 42

```
DSolve[y''[x]+4*y[x]==Exp[x]+4*Sin[2*x]+2*Cos[x]^2-1,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{e^x}{5} + \left(-x + \frac{1}{16} + c_1\right)\cos(2x) + \frac{1}{4}(x + 1 + 4c_2)\sin(2x)$$

17.23 problem 573

Internal problem ID [15342]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 573.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 3y' + 2y = 6x\,\mathrm{e}^{-x}\left(1 - \mathrm{e}^{-x}\right)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=6*x*exp(-x)*(1-exp(-x)),y(x), singsol=all)
```

$$y(x) = 3\left(\left(x^2 + 2x - \frac{1}{3}c_1 + 2\right)\mathrm{e}^{-x} + x^2 - 2x + \frac{c_2}{3}\right)\mathrm{e}^{-x}$$

✓ Solution by Mathematica
Time used: 0.186 (sec). Leaf size: 39

```
DSolve[y''[x]+3*y'[x]+2*y[x]==6*x*Exp[-x]*(1-Exp[-x]),y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^{-2x}\left(3x^2 + e^x\left(3x^2 - 6x + 6 + c_2\right) + 6x + 6 + c_1\right)$$

17.24 problem 574

Internal problem ID [15343]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 574.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y = \cos(2x)^2 + \sin\left(\frac{x}{2}\right)^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)+y(x)=cos(2*x)^2+sin(x/2)^2,y(x), singsol=all)
```

$$y(x) = 1 - \frac{\cos(4x)}{30} + \frac{(-1 + 8c_1)\cos(x)}{8} + \frac{(-x + 4c_2)\sin(x)}{4}$$

✓ Solution by Mathematica
Time used: 0.103 (sec). Leaf size: 36

```
DSolve[y''[x]+y[x]==Cos[2*x]^2+Sin[x/2]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{4}x\sin(x) - \frac{1}{30}\cos(4x) + \left(-\frac{1}{4} + c_1\right)\cos(x) + c_2\sin(x) + 1$$

17.25 problem 575

Internal problem ID [15344]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 575.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 5y = 1 + 8\cos(x) + e^{2x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+5*y(x)=1+8*cos(x)+exp(2*x),y(x), singsol=all)
```

$$y(x) = e^{2x}\sin(x)\,c_2 + e^{2x}\cos(x)\,c_1 - \sin(x) + \cos(x) + \frac{1}{5} + e^{2x}$$

✓ Solution by Mathematica
Time used: 0.341 (sec). Leaf size: 40

```
DSolve[y''[x]-4*y'[x]+5*y[x]==1+8*Cos[x]+Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x} + \left(1 + c_2e^{2x}\right)\cos(x) + \left(-1 + c_1e^{2x}\right)\sin(x) + \frac{1}{5}$$

17.26 problem 576

Internal problem ID [15345]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 576.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2y' + 2y = e^{x} \sin\left(\frac{x}{2}\right)^{2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+2*y(x)=exp(x)*sin(x/2)^2,y(x), singsol=all)
```

$$y(x) = -\frac{\left(\left(-4c_1 + \frac{1}{2}\right)\cos(x) - 2 + (x - 4c_2)\sin(x)\right)e^{x}}{4}$$

✓ Solution by Mathematica
Time used: 0.056 (sec). Leaf size: 33

```
DSolve[y''[x]-2*y'[x]+2*y[x]==Exp[x]*Sin[x/2]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{8}e^{x}((1 - 8c_2)\cos(x) + 2(x - 4c_1)\sin(x) - 4)$$

17.27 problem 577

Internal problem ID [15346]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 577.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - 3y' = 1 + \mathrm{e}^{x} + \cos(x) + \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)=1+exp(x)+cos(x)+sin(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{3x}}{3} - \frac{2\sin(x)}{5} - \frac{\mathrm{e}^{x}}{2} + \frac{\cos(x)}{5} - \frac{x}{3} + c_2$$

✓ Solution by Mathematica
Time used: 0.261 (sec). Leaf size: 43

```
DSolve[y''[x]-3*y'[x]==1+Exp[x]+Cos[x]+Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{3} - \frac{e^{x}}{2} - \frac{2\sin(x)}{5} + \frac{\cos(x)}{5} + \frac{1}{3}c_1 e^{3x} + c_2$$

17.28 problem 578

Internal problem ID [15347]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 578.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2y' + 5y = e^{x}\left(1 - 2\sin(x)^{2}\right) + 10x + 1$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+5*y(x)=exp(x)*(1-2*sin(x)^2)+10*x+1,y(x), singsol=all)
```

$$y(x) = \frac{e^{x}(x + 4c_2)\sin(2x)}{4} + e^{x}\cos(2x)\,c_1 + 2x + 1$$

✓ Solution by Mathematica

Time used: 1.163 (sec). Leaf size: 44

```
DSolve[y''[x]-2*y'[x]+5*y[x]==Exp[x]*(1-2*Sin[x]^2)+10*x+1,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to 2x + \frac{1}{16}(1 + 16c_2)e^{x}\cos(2x) + \frac{1}{4}e^{x}(x + 4c_1)\sin(2x) + 1$$

17.29 problem 579

Internal problem ID [15348]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 579.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 4y = 4x + \sin(x) + \sin(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=4*x+sin(x)+sin(2*x),y(x), singsol=all)
```

$$y(x) = 1 + (c_1 x + c_2)\,\mathrm{e}^{2x} + x + \frac{4\cos(x)}{25} + \frac{3\sin(x)}{25} + \frac{\cos(2x)}{8}$$

✓ Solution by Mathematica
Time used: 0.324 (sec). Leaf size: 45

```
DSolve[y''[x]-4*y'[x]+4*y[x]==4*x+Sin[x]+Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + \frac{3\sin(x)}{25} + \frac{4\cos(x)}{25} + \frac{1}{8}\cos(2x) + c_2 e^{2x} x + c_1 e^{2x} + 1$$

17.30 problem 580

Internal problem ID [15349]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 580.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' + y = 1 + 2\cos(x) + \cos(2x) - \sin(2x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=1+2*cos(x)+cos(2*x)-sin(2*x),y(x), singsol=all)
```

$$y(x) = 1 + (c_1 x + c_2)\,\mathrm{e}^{-x} + \sin(x) + \frac{\cos(2x)}{25} + \frac{7\sin(2x)}{25}$$

✓ Solution by Mathematica
Time used: 1.184 (sec). Leaf size: 42

```
DSolve[y''[x]+2*y'[x]+y[x]==1+2*Cos[x]+Cos[2*x]-Sin[2*x],y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \sin(x) + \frac{7}{25}\sin(2x) + \frac{1}{25}\cos(2x) + c_1 e^{-x} + c_2 e^{-x} x + 1$$

17.31 problem 581

Internal problem ID [15350]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 581.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' + y = -1 + \sin(x) + x + x^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)+1=sin(x)+x+x^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{2}} \sin\left(\frac{x\sqrt{3}}{2}\right) c_2 + \mathrm{e}^{-\frac{x}{2}} \cos\left(\frac{x\sqrt{3}}{2}\right) c_1 - 2 + x^2 - \cos(x) - x$$

✓ Solution by Mathematica
Time used: 2.943 (sec). Leaf size: 59

```
DSolve[y''[x]+y'[x]+y[x]+1==Sin[x]+x+x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 - x - \cos(x) + c_2 e^{-x/2} \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 e^{-x/2} \sin\left(\frac{\sqrt{3}x}{2}\right) - 2$$

17.32 problem 582

Internal problem ID [15351]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 582.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 6y' + 9y = 18\,\mathrm{e}^{-3x} + 8\sin(x) + 6\cos(x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+6*diff(y(x),x)+9*y(x)=18*exp(-3*x)+8*sin(x)+6*cos(x),y(x), singsol=all
```

$$y(x) = \left(c_1 x + 9x^2 + c_2\right)\mathrm{e}^{-3x} + \sin(x)$$

✓ Solution by Mathematica
Time used: 0.257 (sec). Leaf size: 31

```
DSolve[y''[x]+6*y'[x]+9*y[x]==18*Exp[-3*x]+8*Sin[x]+6*Cos[x],y[x],x,IncludeSingularSolutions
```

$$y(x) \to e^{-3x}\left(9x^2 + e^{3x}\sin(x) + c_2 x + c_1\right)$$

17.33 problem 583

Internal problem ID [15352]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 583.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + 2y' = -1 + 3\sin(2x) + \cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+1=3*sin(2*x)+cos(x),y(x), singsol=all)
```

$$y(x) = -\frac{e^{-2x}c_1}{2} + \frac{2\sin(x)}{5} - \frac{3\sin(2x)}{8} - \frac{\cos(x)}{5} - \frac{3\cos(2x)}{8} - \frac{x}{2} + c_2$$

✓ Solution by Mathematica
Time used: 0.316 (sec). Leaf size: 52

```
DSolve[y''[x]+2*y'[x]+1==3*Sin[2*x]+Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{2} + \frac{2\sin(x)}{5} - \frac{3}{8}\sin(2x) - \frac{\cos(x)}{5} - \frac{3}{8}\cos(2x) - \frac{1}{2}c_1e^{-2x} + c_2$$

17.34 problem 584

Internal problem ID [15353]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 584.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - 2y'' + y' = 2x + e^x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$3)-2*diff(y(x),x$2)+diff(y(x),x)=2*x+exp(x),y(x), singsol=all)
```

$$y(x) = \frac{(x^2 + (2c_1 - 2)x - 2c_1 + 2c_2 + 2)e^x}{2} + x^2 + 4x + c_3$$

✓ Solution by Mathematica
Time used: 0.268 (sec). Leaf size: 39

```
DSolve[y'''[x]-2*y''[x]+y'[x]==2*x+Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + e^x\left(\frac{x^2}{2} + (-1 + c_2)x + 1 + c_1 - c_2\right) + 4x + c_3$$

17.35 problem 585

Internal problem ID [15354]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 585.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = 2\sin(x)\sin(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+y(x)=2*sin(x)*sin(2*x),y(x), singsol=all)
```

$$y(x) = -\frac{\cos(x)\sin(x)^2}{2} + \frac{(2c_2 + x)\sin(x)}{2} + \cos(x)\,c_1$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 33

```
DSolve[y''[x]+y[x]==2*Sin[x]*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}(\cos(3x) + (-1 + 8c_1)\cos(x) + 4(x + 2c_2)\sin(x))$$

17.36 problem 586

Internal problem ID [15355]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 586.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - y'' - 2y' = 4x + 3\sin(x) + \cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)-2*diff(y(x),x)=4*x+3*sin(x)+cos(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{2x}}{2} - c_2 e^{-x} - x^2 + \cos(x) + x + c_3$$

✓ Solution by Mathematica
Time used: 0.338 (sec). Leaf size: 46

```
DSolve[y'''[x]-y''[x]-2*y'[x]==4*x+2*Sin[x]+Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^2 + x - \frac{\sin(x)}{10} + \frac{7\cos(x)}{10} - c_1 e^{-x} + \frac{1}{2} c_2 e^{2x} + c_3$$

17.37 problem 587

Internal problem ID [15356]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 587.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - 4y' = x\,\mathrm{e}^{2x} + \sin(x) + x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 44

```
dsolve(diff(y(x),x$3)-4*diff(y(x),x)=x*exp(2*x)+sin(x)+x^2,y(x), singsol=all)
```

$$y(x) = \frac{(8x^2 + 64c_1 - 12x + 7)\,\mathrm{e}^{2x}}{128} - \frac{x^3}{12} - \frac{c_2\mathrm{e}^{-2x}}{2} - \frac{x}{8} + c_3 + \frac{\cos(x)}{5}$$

✓ Solution by Mathematica
Time used: 0.617 (sec). Leaf size: 60

```
DSolve[y'''[x]-4*y'[x]==x*Exp[2*x]+Sin[x]+x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^3}{12} + \frac{1}{128}e^{2x}\left(8x^2 - 12x + 7 + 64c_1\right) - \frac{x}{8} + \frac{\cos(x)}{5} - \frac{1}{2}c_2e^{-2x} + c_3$$

17.38 problem 588

Internal problem ID [15357]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 588.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y^{(5)} - y'''' = x\,\mathrm{e}^{x} - 1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 41

```
dsolve(diff(y(x),x$5)-diff(y(x),x$4)=x*exp(x)-1,y(x), singsol=all)
```

$$y(x) = \frac{(x^2 + 2c_1 - 8x + 20)\,\mathrm{e}^{x}}{2} + \frac{x^4}{24} + \frac{c_2 x^3}{6} + \frac{c_3 x^2}{2} + c_4 x + c_5$$

✓ Solution by Mathematica
Time used: 0.337 (sec). Leaf size: 49

```
DSolve[y'''''[x]-y''''[x]==x*Exp[x]-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^4}{24} + c_5 x^3 + c_4 x^2 + e^x\left(\frac{x^2}{2} - 4x + 10 + c_1\right) + c_3 x + c_2$$

17.39 problem 589

Internal problem ID [15358]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Superposition principle. Exercises page 137
Problem number: 589.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$y^{(5)} - y''' = x + 2\,\mathrm{e}^{-x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x$5)-diff(y(x),x$3)=x+2*exp(-x),y(x), singsol=all)
```

$$y(x) = \frac{(7 + 2x - 2c_1)\,\mathrm{e}^{-x}}{2} - \frac{x^4}{24} + \frac{c_3 x^2}{2} + c_4 x + c_2 \mathrm{e}^{x} + c_5$$

✓ Solution by Mathematica
Time used: 0.394 (sec). Leaf size: 46

```
DSolve[y'''''[x]-y'''[x]==x+2*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^4}{24} + c_5 x^2 + c_4 x + c_1 e^x + e^{-x}\left(x + \frac{7}{2} - c_2\right) + c_3$$

18 Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140

18.1 problem 590

Internal problem ID [15359]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140
Problem number: 590.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y = 2 - 2x}$$

With initial conditions

$$[y(0) = 2, y'(0) = -2]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 9

```
dsolve([diff(y(x),x$2)+y(x)=2*(1-x),y(0) = 2, D(y)(0) = -2],y(x), singsol=all)
```

$$y(x) = 2 - 2x$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 10

```
DSolve[{y''[x]+y[x]==2*(1-x),{y[0]==2,y'[0]==-2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2 - 2x$$

18.2 problem 591

Internal problem ID [15360]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140
Problem number: 591.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 6y' + 9y = 9x^2 - 12x + 2}$$

With initial conditions

$$[y(0) = 1, y'(0) = 3]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve([diff(y(x),x$2)-6*diff(y(x),x)+9*y(x)=9*x^2-12*x+2,y(0) = 1, D(y)(0) = 3],y(x), sings
```

$$y(x) = e^{3x} + x^2$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 14

```
DSolve[{y''[x]-6*y'[x]+9*y[x]==9*x^2-12*x+2,{y[0]==1,y'[0]==3}},y[x],x,IncludeSingularSoluti
```

$$y(x) \to x^2 + e^{3x}$$

18.3 problem 592

Internal problem ID [15361]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140
Problem number: 592.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 9y = 36\,\mathrm{e}^{3x}}$$

With initial conditions

$$[y(0) = 2, y'(0) = 6]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 10

```
dsolve([diff(y(x),x$2)+9*y(x)=36*exp(3*x),y(0) = 2, D(y)(0) = 6],y(x), singsol=all)
```

$$y(x) = 2\,\mathrm{e}^{3x}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 12

```
DSolve[{y''[x]+9*y[x]==36*Exp[3*x],{y[0]==2,y'[0]==6}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to 2e^{3x}$$

18.4 problem 593

Internal problem ID [15362]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140
Problem number: 593.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y' + 4y = 2\,e^{2x}}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve([diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=2*exp(2*x),y(0) = 0, D(y)(0) = 0],y(x), singsol
```

$$y(x) = e^{2x}x^2$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 14

```
DSolve[{y''[x]-4*y'[x]+4*y[x]==2*Exp[2*x],{y[0]==0,y'[0]==0}},y[x],x,IncludeSingularSolution
```

$$y(x) \to e^{2x}x^2$$

18.5 problem 594

Internal problem ID [15363]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140
Problem number: 594.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 5y' + 6y = (12x - 7)\,e^{-x}}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve([diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=(12*x-7)*exp(-x),y(0) = 0, D(y)(0) = 0],y(x), s
```

$$y(x) = e^{2x} - e^{3x} + e^{-x}x$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 25

```
DSolve[{y''[x]-5*y'[x]+6*y[x]==(12*x-7)*Exp[-x],{y[0]==0,y'[0]==0}},y[x],x,IncludeSingularSo
```

$$y(x) \to e^{-x}\left(x + e^{3x} - e^{4x}\right)$$

18.6 problem 595

Internal problem ID [15364]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140
Problem number: 595.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + y' = e^{-x}}$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 13

```
dsolve([diff(y(x),x$2)+diff(y(x),x)=exp(-x),y(0) = 1, D(y)(0) = -1],y(x), singsol=all)
```

$$y(x) = -e^{-x}x + 1$$

✓ Solution by Mathematica
Time used: 0.069 (sec). Leaf size: 15

```
DSolve[{y''[x]+y'[x]==Exp[-x],{y[0]==1,y'[0]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 - e^{-x}x$$

18.7 problem 596

Internal problem ID [15365]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140
Problem number: 596.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 6y' + 9y = 10\sin(x)$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve([diff(y(x),x$2)+6*diff(y(x),x)+9*y(x)=10*sin(x),y(0) = 0, D(y)(0) = 0],y(x), singsol=
```

$$y(x) = \frac{3\,\mathrm{e}^{-3x}}{5} + x\,\mathrm{e}^{-3x} - \frac{3\cos(x)}{5} + \frac{4\sin(x)}{5}$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 33

```
DSolve[{y''[x]+6*y'[x]+9*y[x]==10*Sin[x],{y[0]==0,y'[0]==0}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{5}\left(5e^{-3x}x + 3e^{-3x} + 4\sin(x) - 3\cos(x)\right)$$

18.8 problem 597

Internal problem ID [15366]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140
Problem number: 597.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = 2\cos(x)}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve([diff(y(x),x$2)+y(x)=2*cos(x),y(0) = 1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \cos(x) + \sin(x)\,x$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 12

```
DSolve[{y''[x]+y[x]==2*Cos[x],{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\sin(x) + \cos(x)$$

18.9 problem 598

Internal problem ID [15367]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140
Problem number: 598.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \sin(x)}$$

With initial conditions

$$[y(0) = 1, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$2)+4*y(x)=sin(x),y(0) = 1, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\sin(2x)}{3} + \cos(2x) + \frac{\sin(x)}{3}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 22

```
DSolve[{y''[x]+4*y[x]==Sin[x],{y[0]==1,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}(\sin(x) + \sin(2x) + 3\cos(2x))$$

18.10 problem 599

Internal problem ID [15368]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140
Problem number: 599.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = 4\cos(x)\, x}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve([diff(y(x),x$2)+y(x)=4*x*cos(x),y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = x(\cos(x) + \sin(x)\, x)$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 14

```
DSolve[{y''[x]+y[x]==4*x*Cos[x],{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(x\sin(x) + \cos(x))$$

18.11 problem 600

Internal problem ID [15369]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140
Problem number: 600.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 4y' + 5y = 2\,\mathrm{e}^{x} x^2$$

With initial conditions

$$[y(0) = 2, y'(0) = 3]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 25

```
dsolve([diff(y(x),x$2)-4*diff(y(x),x)+5*y(x)=2*x^2*exp(x),y(0) = 2, D(y)(0) = 3],y(x), sings
```

$$y(x) = (\cos(x) - 2\sin(x))\,\mathrm{e}^{2x} + (1 + x)^2\,\mathrm{e}^{x}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 28

```
DSolve[{y''[x]-4*y'[x]+5*y[x]==2*x^2*Exp[x],{y[0]==2,y'[0]==3}},y[x],x,IncludeSingularSoluti
```

$$y(x) \to e^x\left((x+1)^2 - 2e^x \sin(x) + e^x \cos(x)\right)$$

18.12 problem 601

Internal problem ID [15370]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140
Problem number: 601.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 6y' + 9y = 16\,\mathrm{e}^{-x} + 9x - 6}$$

With initial conditions

$$[y(0) = 1, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve([diff(y(x),x$2)-6*diff(y(x),x)+9*y(x)=16*exp(-x)+9*x-6,y(0) = 1, D(y)(0) = 1],y(x), s
```

$$y(x) = \mathrm{e}^{3x}x + x + \mathrm{e}^{-x}$$

✓ Solution by Mathematica
Time used: 0.28 (sec). Leaf size: 19

```
DSolve[{y''[x]-6*y'[x]+9*y[x]==16*Exp[-x]+9*x-6,{y[0]==1,y'[0]==1}},y[x],x,IncludeSingularSo
```

$$y(x) \to e^{3x}x + x + e^{-x}$$

18.13 problem 602

Internal problem ID [15371]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140
Problem number: 602.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - y' = -5\,e^{-x}(\sin(x) + \cos(x))}$$

With initial conditions

$$[y(0) = -4, y'(0) = 5]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 22

```
dsolve([diff(y(x),x$2)-diff(y(x),x)=-5*exp(-x)*(sin(x)+cos(x)),y(0) = -4, D(y)(0) = 5],y(x),
```

$$y(x) = 2\,e^{x} - 4 + e^{-x}(-2\cos(x) + \sin(x))$$

✓ Solution by Mathematica
Time used: 0.173 (sec). Leaf size: 28

```
DSolve[{y''[x]-y'[x]==-5*Exp[-x]*(Sin[x]+Cos[x]),{y[0]==-4,y'[0]==5}},y[x],x,IncludeSingular
```

$$y(x) \to e^{-x}(2e^{x}(e^{x} - 2) + \sin(x) - 2\cos(x))$$

18.14 problem 603

Internal problem ID [15372]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140
Problem number: 603.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + 2y = 4\,\mathrm{e}^{x}\cos(x)}$$

With initial conditions

$$[y(\pi) = \pi\,\mathrm{e}^{\pi}, y'(\pi) = \mathrm{e}^{\pi}]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+2*y(x)=4*exp(x)*cos(x),y(Pi) = Pi*exp(Pi), D(y)(Pi) =
```

$$y(x) = \mathrm{e}^{x}(2x - \pi - 1)\sin(x) - \mathrm{e}^{x}\cos(x)\,\pi$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 24

```
DSolve[{y''[x]-2*y'[x]+2*y[x]==4*Exp[x]*Cos[x],{y[Pi]==Pi*Exp[Pi],y'[Pi]==Exp[Pi]}},y[x],x,I
```

$$y(x) \to -e^{x}((-2x + \pi + 1)\sin(x) + \pi\cos(x))$$

18.15 problem 604

Internal problem ID [15373]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISE-LYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140
Problem number: 604.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - y' = -2x}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1, y''(0) = 2]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$3)-diff(y(x),x)=-2*x,y(0) = 0, D(y)(0) = 1, (D@@2)(y)(0) = 2],y(x), sing
```

$$y(x) = -\frac{e^{-x}}{2} + \frac{e^{x}}{2} + x^2$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 25

```
DSolve[{y'''[x]-y'[x]==-2*x,{y[0]==0,y'[0]==1,y''[0]==2}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to x^2 - \frac{e^{-x}}{2} + \frac{e^{x}}{2}$$

18.16 problem 605

Internal problem ID [15374]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140
Problem number: 605.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$\boxed{y'''' - y = 8\,\mathrm{e}^{x}}$$

With initial conditions

$$[y(0) = -1, y'(0) = 0, y''(0) = 1, y'''(0) = 0]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 24

```
dsolve([diff(y(x),x$4)-y(x)=8*exp(x),y(0) = -1, D(y)(0) = 0, (D@@2)(y)(0) = 1, (D@@3)(y)(0)
```

$$y(x) = \mathrm{e}^{-x} + (2x-3)\,\mathrm{e}^{x} + \cos(x) + 2\sin(x)$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 28

```
DSolve[{y''''[x]-y[x]==8*Exp[x],{y[0]==-1,y'[0]==0,y''[0]==1,y'''[0]==0}},y[x],x,IncludeSing
```

$$y(x) \to 2e^{x}x + e^{-x} - 3e^{x} + 2\sin(x) + \cos(x)$$

18.17 problem 606

Internal problem ID [15375]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140
Problem number: 606.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - y = 2x}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0, y''(0) = 2]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 25

```
dsolve([diff(y(x),x$3)-y(x)=2*x,y(0) = 0, D(y)(0) = 0, (D@@2)(y)(0) = 2],y(x), singsol=all)
```

$$y(x) = -2x + \frac{4\,\mathrm{e}^{x}}{3} - \frac{4\,\mathrm{e}^{-\frac{x}{2}} \cos\left(\frac{x\sqrt{3}}{2}\right)}{3}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 38

```
DSolve[{y'''[x]-y[x]==2*x,{y[0]==0,y'[0]==0,y''[0]==2}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{1}{3}\left(-6x + 4e^{x} - 4e^{-x/2} \cos\left(\frac{\sqrt{3}x}{2}\right)\right)$$

18.18 problem 607

Internal problem ID [15376]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140
Problem number: 607.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$\boxed{y'''' - y = 8\,e^{x}}$$

With initial conditions

$$[y(0) = 0, y'(0) = 2, y''(0) = 4, y'''(0) = 6]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve([diff(y(x),x$4)-y(x)=8*exp(x),y(0) = 0, D(y)(0) = 2, (D@@2)(y)(0) = 4, (D@@3)(y)(0) =
```

$$y(x) = 2\,e^{x}x$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 11

```
DSolve[{y''''[x]-y[x]==8*Exp[x],{y[0]==0,y'[0]==2,y''[0]==4,y'''[0]==6}},y[x],x,IncludeSingu
```

$$y(x) \to 2e^{x}x$$

18.19 problem 608

Internal problem ID [15377]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140
Problem number: 608.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 5y = \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+5*y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = e^{2x} \sin(x) c_2 + e^{2x} \cos(x) c_1 + \frac{\cos(x)}{8} + \frac{\sin(x)}{8}$$

✓ Solution by Mathematica
Time used: 0.073 (sec). Leaf size: 36

```
DSolve[y''[x]-4*y'[x]+5*y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}\left(\sin(x) + \cos(x) + 8c_2 e^{2x} \cos(x) + 8c_1 e^{2x} \sin(x)\right)$$

18.20 problem 609

Internal problem ID [15378]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140
Problem number: 609.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + 5y = 4\cos(2x) + \sin(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+5*y(x)=4*cos(2*x)+sin(2*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}\sin(2x)\,c_2 + \mathrm{e}^{-x}\cos(2x)\,c_1 + \sin(2x)$$

✓ Solution by Mathematica
Time used: 0.338 (sec). Leaf size: 30

```
DSolve[y''[x]+2*y'[x]+5*y[x]==4*Cos[2*x]+Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_2\cos(2x) + (e^x + c_1)\sin(2x))$$

18.21 problem 610

Internal problem ID [15379]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140
Problem number: 610.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$2)-y(x)=1,y(x), singsol=all)
```

$$y(x) = c_2 e^{-x} + c_1 e^{x} - 1$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 21

```
DSolve[y''[x]-y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x + c_2 e^{-x} - 1$$

18.22 problem 611

Internal problem ID [15380]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140
Problem number: 611.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y = -2\cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-y(x)=-2*cos(x),y(x), singsol=all)
```

$$y(x) = c_2 e^{-x} + c_1 e^{x} + \cos(x)$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 22

```
DSolve[y''[x]-y[x]==-2*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x) + c_1 e^{x} + c_2 e^{-x}$$

18.23 problem 612

Internal problem ID [15381]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140
Problem number: 612.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y' + y = 4\,\mathrm{e}^{-x}}$$

With initial conditions

$$[y(\infty) = 0]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 12

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+y(x)=4*exp(-x),y(infinity) = 0],y(x), singsol=all)
```

$$y(x) = -\operatorname{signum}\left(c_1\mathrm{e}^{x}\right)\infty$$

✓ Solution by Mathematica
Time used: 0.053 (sec). Leaf size: 10

```
DSolve[{y''[x]-2*y'[x]+y[x]==4*Exp[-x],{y[Infinity]==0}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to e^{-x}$$

18.24 problem 613

Internal problem ID [15382]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140
Problem number: 613.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y' + 3y = 8\,\mathrm{e}^{x} + 9}$$

With initial conditions

$$[y(-\infty) = 3]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 14

```
dsolve([diff(y(x),x$2)+4*diff(y(x),x)+3*y(x)=8*exp(x)+9,y(-infinity) = 3],y(x), singsol=all)
```

$$y(x) = -\operatorname{signum}\left(c_1 \mathrm{e}^{-x}\right)\infty$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y''[x]+4*y'[x]+3*y[x]==8*Exp[x]+9,{y[-Infinity]==3}},y[x],x,IncludeSingularSolutions
```

{}

18.25 problem 614

Internal problem ID [15383]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140
Problem number: 614.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' - 5y = 1}$$

With initial conditions

$$\left[y(\infty) = -\frac{1}{5}\right]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$2)-diff(y(x),x)-5*y(x)=1,y(infinity) = -1/5],y(x), singsol=all)
```

$$y(x) = -\operatorname{signum}\left(c_2 \mathrm{e}^{-\frac{\left(-1+\sqrt{21}\right)x}{2}}\right)\infty$$

✓ Solution by Mathematica

Time used: 0.559 (sec). Leaf size: 26

```
DSolve[{y''[x]-y'[x]-5*y[x]==1,{y[Infinity]==-1/5}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{5} + c_1 e^{-\frac{1}{2}\left(\sqrt{21}-1\right)x}$$

18.26 problem 615

Internal problem ID [15384]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISE-LYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140
Problem number: 615.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 4y = 2\,\mathrm{e}^{x}(\sin(x) + 7\cos(x))}$$

With initial conditions

$$[y(-\infty) = 0]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 13

```
dsolve([diff(y(x),x$2)+4*diff(y(x),x)+4*y(x)=2*exp(x)*(sin(x)+7*cos(x)),y(-infinity) = 0],y(
```

$$y(x) = \operatorname{signum}\left(\mathrm{e}^{-2x} c_1\right) \infty$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y''[x]+4*y'[x]+4*y[x]==2*Exp[x]*(Sin[x]+7*Cos[x]),{y[-Infinity]==0}},y[x],x,IncludeS
```

Not solved

18.27 problem 616

Internal problem ID [15385]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140
Problem number: 616.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 5y' + 6y = 2\,\mathrm{e}^{-2x}(9\sin(2x) + 4\cos(2x))}$$

With initial conditions

$$[y(\infty) = 0]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 31

```
dsolve([diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=2*exp(-2*x)*(9*sin(2*x)+4*cos(2*x)),y(infinity)
```

$$y(x) = c_2\mathrm{e}^{2x} + \frac{(113\cos(2x) + 36\sin(2x))\,\mathrm{e}^{-2x}}{145}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y''[x]-5*y'[x]+6*y[x]==2*Exp[-2*x]*(9*Sin[2*x]+4*Cos[2*x]),{y[Infinity]==0}},y[x],x,
```

Not solved

18.28 problem 617

Internal problem ID [15386]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.3 Nonhomogeneous linear equations with constant coefficients. Initial value problem. Exercises page 140
Problem number: 617.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 4y = e^{-x}(9x^2 + 5x - 12)}$$

With initial conditions

$$[y(\infty) = 0]$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 14

```
dsolve([diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=exp(-x)*(9*x^2+5*x-12),y(infinity) = 0],y(x), s
```

$$y(x) = -\operatorname{signum}\left(c_1 e^{2x}\right)\infty$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y''[x]-4*y'[x]+4*y[x]==Exp[-x]*(9*x^2+5*x-12),{y[Infinity]==0}},y[x],x,IncludeSingul
```

{}

19 Chapter 2 (Higher order ODE's). Section 15.4 Nonhomogeneous linear equations with constant coefficients. The Euler equations. Exercises page 143

19.1 problem 618

Internal problem ID [15387]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.4 Nonhomogeneous linear equations with constant coefficients. The Euler equations. Exercises page 143
Problem number: 618.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2y'' + xy' - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2 x^2 + c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 16

```
DSolve[x^2*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x} + c_2 x$$

19.2 problem 619

Internal problem ID [15388]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.4 Nonhomogeneous linear equations with constant coefficients. The Euler equations. Exercises page 143
Problem number: 619.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2y'' + 3xy' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2 \ln(x) + c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 17

```
DSolve[x^2*y''[x]+3*x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 \log(x) + c_1}{x}$$

19.3 problem 620

Internal problem ID [15389]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.4 Nonhomogeneous linear equations with constant coefficients. The Euler equations. Exercises page 143
Problem number: 620.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 2xy' + 6y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x^2*diff(y(x),x$2)+2*x*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin\left(\frac{\sqrt{23}\,\ln(x)}{2}\right) + c_2 \cos\left(\frac{\sqrt{23}\,\ln(x)}{2}\right)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 42

```
DSolve[x^2*y''[x]+2*x*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 \cos\left(\frac{1}{2}\sqrt{23}\log(x)\right) + c_1 \sin\left(\frac{1}{2}\sqrt{23}\log(x)\right)}{\sqrt{x}}$$

19.4 problem 621

Internal problem ID [15390]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.4 Nonhomogeneous linear equations with constant coefficients. The Euler equations. Exercises page 143
Problem number: 621.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' + y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(x*diff(y(x),x$2)+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_2 \ln(x) + c_1$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 13

```
DSolve[x*y''[x]+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \log(x) + c_2$$

19.5 problem 622

Internal problem ID [15391]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.4 Nonhomogeneous linear equations with constant coefficients. The Euler equations. Exercises page 143
Problem number: 622.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2+x)^2 y'' + 3(2+x) y' - 3y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve((x+2)^2*diff(y(x),x$2)+3*(x+2)*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 + c_2(x+2)^4}{(x+2)^3}$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 20

```
DSolve[(x+2)^2*y''[x]+3*(x+2)*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x+2) + \frac{c_2}{(x+2)^3}$$

19.6 problem 623

Internal problem ID [15392]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.4 Nonhomogeneous linear equations with constant coefficients. The Euler equations. Exercises page 143
Problem number: 623.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1+2x)^2 y'' - 2(1+2x) y' + 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve((2*x+1)^2*diff(y(x),x$2)-2*(2*x+1)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{(2x+1)(-c_2 \ln(2) + c_2 \ln(2x+1) + c_1)}{2}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 23

```
DSolve[(2*x+1)^2*y''[x]-2*(2*x+1)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (2x+1)(c_2 \log(2x+1) + c_1)$$

19.7 problem 624

Internal problem ID [15393]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.4 Nonhomogeneous linear equations with constant coefficients. The Euler equations. Exercises page 143
Problem number: 624.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{x^2y''' - 3xy'' + 3y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve(x^2*diff(y(x),x$3)-3*x*diff(y(x),x$2)+3*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_3x^4 + c_2x^2 + c_1$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 26

```
DSolve[x^2*y'''[x]-3*x*y''[x]+3*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^4}{4} + \frac{c_1x^2}{2} + c_3$$

19.8 problem 625

Internal problem ID [15394]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.4 Nonhomogeneous linear equations with constant coefficients. The Euler equations. Exercises page 143
Problem number: 625.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{x^2y''' - 2y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$3)=2*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = c_1 + c_2 \ln(x) + c_3x^3$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 22

```
DSolve[x^2*y'''[x]==2*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^3}{3} + c_1 \log(x) + c_3$$

19.9 problem 626

Internal problem ID [15395]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.4 Nonhomogeneous linear equations with constant coefficients. The Euler equations. Exercises page 143
Problem number: 626.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{(x+1)^2 y''' - 12y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve((x+1)^2*diff(y(x),x$3)-12*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + \frac{c_2}{(1+x)^2} + c_3(1+x)^5$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 30

```
DSolve[(x+1)^2*y'''[x]-12*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{5}c_1(x+1)^5 - \frac{c_2}{2(x+1)^2} + c_3$$

19.10 problem 627

Internal problem ID [15396]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.4 Nonhomogeneous linear equations with constant coefficients. The Euler equations. Exercises page 143
Problem number: 627.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{(1+2x)^2 y''' + 2(1+2x)\, y'' + y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 50

```
dsolve((2*x+1)^2*diff(y(x),x$3)+2*(2*x+1)*diff(y(x),x$2)+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + \frac{c_2(2x+1)\sin\left(-\frac{\ln(2)}{2} + \frac{\ln(2x+1)}{2}\right)}{2} + \frac{c_3(2x+1)\cos\left(-\frac{\ln(2)}{2} + \frac{\ln(2x+1)}{2}\right)}{2}$$

✓ Solution by Mathematica
Time used: 0.153 (sec). Leaf size: 58

```
DSolve[(2*x+1)^2*y'''[x]+2*(2*x+1)*y''[x]+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{5}(2x+1)\left((2c_1 - c_2)\cos\left(\frac{1}{2}\log(2x+1)\right) + (c_1 + 2c_2)\sin\left(\frac{1}{2}\log(2x+1)\right)\right) + c_3$$

19.11 problem 628

Internal problem ID [15397]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.4 Nonhomogeneous linear equations with constant coefficients. The Euler equations. Exercises page 143
Problem number: 628.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + xy' + y = x(6 - \ln(x))}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=x*(6-ln(x)),y(x), singsol=all)
```

$$y(x) = \sin(\ln(x))\, c_2 + \cos(\ln(x))\, c_1 - \frac{x(\ln(x) - 7)}{2}$$

✓ Solution by Mathematica
Time used: 0.138 (sec). Leaf size: 27

```
DSolve[x^2*y''[x]+x*y'[x]+y[x]==x*(6-Log[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}x(\log(x) - 7) + c_1\cos(\log(x)) + c_2\sin(\log(x))$$

19.12 problem 629

Internal problem ID [15398]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.4 Nonhomogeneous linear equations with constant coefficients. The Euler equations. Exercises page 143
Problem number: 629.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' - 2y = \sin(\ln(x))}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 25

```
dsolve(x^2*diff(y(x),x$2)-2*y(x)=sin(ln(x)),y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + c_2x^2 + \frac{\cos(\ln(x))}{10} - \frac{3\sin(\ln(x))}{10}$$

✓ Solution by Mathematica
Time used: 0.044 (sec). Leaf size: 31

```
DSolve[x^2*y''[x]-2*y[x]==Sin[Log[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x^2 + \frac{c_1}{x} + \frac{1}{10}(\cos(\log(x)) - 3\sin(\log(x)))$$

19.13 problem 630

Internal problem ID [15399]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.4 Nonhomogeneous linear equations with constant coefficients. The Euler equations. Exercises page 143
Problem number: 630.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$x^2y'' - xy' - 3y = -\frac{16\ln(x)}{x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)-3*y(x)=-16*ln(x)/x,y(x), singsol=all)
```

$$y(x) = \frac{4c_2x^4 + 8\ln(x)^2 + 4\ln(x) + 4c_1 + 1}{4x}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 35

```
DSolve[x^2*y''[x]-x*y'[x]-3*y[x]==-16*Log[x]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4c_2x^4 + 8\log^2(x) + 4\log(x) + 1 + 4c_1}{4x}$$

19.14 problem 631

Internal problem ID [15400]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.4 Nonhomogeneous linear equations with constant coefficients. The Euler equations. Exercises page 143
Problem number: 631.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2xy' - 2y = x^2 - 2x + 2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)-2*y(x)=x^2-2*x+2,y(x), singsol=all)
```

$$y(x) = x^{\frac{3}{2}+\frac{\sqrt{17}}{2}}c_2 + x^{\frac{3}{2}-\frac{\sqrt{17}}{2}}c_1 - \frac{x^2}{4} + \frac{x}{2} - 1$$

✓ Solution by Mathematica
Time used: 0.398 (sec). Leaf size: 53

```
DSolve[x^2*y''[x]-2*x*y'[x]-2*y[x]==x^2-2*x+2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x^{\frac{1}{2}\left(3+\sqrt{17}\right)} + c_1x^{\frac{3}{2}-\frac{\sqrt{17}}{2}} - \frac{x^2}{4} + \frac{x}{2} - 1$$

19.15 problem 632

Internal problem ID [15401]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.4 Nonhomogeneous linear equations with constant coefficients. The Euler equations. Exercises page 143
Problem number: 632.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + xy' - y = x^m}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=x^m,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + c_2 x + \frac{x^m}{(m-1)(m+1)}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 27

```
DSolve[x^2*y''[x]+x*y'[x]-y[x]==x^m,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^m}{m^2-1} + c_2 x + \frac{c_1}{x}$$

19.16 problem 633

Internal problem ID [15402]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.4 Nonhomogeneous linear equations with constant coefficients. The Euler equations. Exercises page 143
Problem number: 633.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + 4xy' + 2y = 2\ln(x)^2 + 12x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+2*y(x)=2*(ln(x))^2+12*x,y(x), singsol=all)
```

$$y(x) = \frac{c_2}{x^2} + 2x + \frac{7}{2} + \frac{c_1}{x} - 3\ln(x) + \ln(x)^2$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 32

```
DSolve[x^2*y''[x]+4*x*y'[x]+2*y[x]==2*(Log[x])^2+12*x,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{c_1}{x^2} + 2x + \log^2(x) - 3\log(x) + \frac{c_2}{x} + \frac{7}{2}$$

19.17 problem 634

Internal problem ID [15403]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.4 Nonhomogeneous linear equations with constant coefficients. The Euler equations. Exercises page 143
Problem number: 634.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$(x+1)^3 y'' + 3(x+1)^2 y' + (x+1) y = 6 \ln (x+1)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve((x+1)^3*diff(y(x),x$2)+3*(x+1)^2*diff(y(x),x)+(x+1)*y(x)=6*ln(x+1),y(x), singsol=all)
```

$$y(x) = \frac{c_1 \ln (1+x) + \ln (1+x)^3 + c_2}{1+x}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 27

```
DSolve[(x+1)^3*y''[x]+3*(x+1)^2*y'[x]+(x+1)*y[x]==6*Log[x+1],y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{\log^3(x+1) + c_2 \log(x+1) + c_1}{x+1}$$

19.18 problem 635

Internal problem ID [15404]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.4 Nonhomogeneous linear equations with constant coefficients. The Euler equations. Exercises page 143
Problem number: 635.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-2)^2 y'' - 3(x-2) y' + 4y = x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 25

```
dsolve((x-2)^2*diff(y(x),x$2)-3*(x-2)*diff(y(x),x)+4*y(x)=x,y(x), singsol=all)
```

$$y(x) = (x-2)^2 c_2 + (x-2)^2 \ln(x-2) c_1 + x - \frac{3}{2}$$

✓ Solution by Mathematica
Time used: 0.061 (sec). Leaf size: 31

```
DSolve[(x-2)^2*y''[x]-3*(x-2)*y'[x]+4*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + c_1(x-2)^2 + 2c_2(x-2)^2 \log(x-2) - \frac{3}{2}$$

20 Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148

20.1 problem 636

Internal problem ID [15405]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 636.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(1+2x)\, y''+(4x-2)\, y'-8y=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve((2*x+1)*diff(y(x),x$2)+(4*x-2)*diff(y(x),x)-8*y(x)=0,y(x), singsol=all)
```

$$y(x)=4c_1x^2+c_2\mathrm{e}^{-2x}+c_1$$

✓ Solution by Mathematica
Time used: 0.192 (sec). Leaf size: 27

```
DSolve[(2*x+1)*y''[x]+(4*x-2)*y'[x]-8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{1}{2}c_2(4x^2+1)+c_1e^{-2x}$$

20.2 problem 637

Internal problem ID [15406]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 637.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$\boxed{(x^2 - x)\, y'' + (2x - 3)\, y' - 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2-x)*diff(y(x),x$2)+(2*x-3)*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^2} + c_2\left(x - \frac{3}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 23

```
DSolve[(x^2-x)*y''[x]+(2*x-3)*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x^2} + \frac{1}{6}c_2(3 - 2x)$$

20.3 problem 638

Internal problem ID [15407]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 638.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x^2 + 3x)\, y'' - 6y'(x + 1) + 6y = 6}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve((3*x+2*x^2)*diff(y(x),x$2)-6*(1+x)*diff(y(x),x)+6*y(x)=6,y(x), singsol=all)
```

$$y(x) = c_2 x^3 + c_1 x + c_1 + 1$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 20

```
DSolve[(3*x+2*x^2)*y''[x]-6*(1+x)*y'[x]+6*y[x]==6,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x^3 - c_2(x + 1) + 1$$

20.4 problem 639

Internal problem ID [15408]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 639.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(\ln(x)-1)\, y''-xy'+y=0}$$

Given that one solution of the ode is

$$y_1=x$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 12

```
dsolve([x^2*(ln(x)-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,x],singsol=all)
```

$$y(x)=c_1x+c_2\ln(x)$$

✓ Solution by Mathematica
Time used: 0.104 (sec). Leaf size: 16

```
DSolve[x^2*(Log[x]-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1x-c_2\log(x)$$

20.5 problem 640

Internal problem ID [15409]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 640.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (\tan(x) - 2\cot(x))\, y' + 2\cot(x)^2\, y = 0}$$

Given that one solution of the ode is

$$y_1 = \sin(x)$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 13

```
dsolve([diff(y(x),x$2)+(tan(x)-2*cot(x))*diff(y(x),x)+2*cot(x)^2*y(x)=0,sin(x)],singsol=all)
```

$$y(x) = \sin(x)\,(\sin(x)\, c_2 + c_1)$$

✓ Solution by Mathematica
Time used: 2.22 (sec). Leaf size: 27

```
DSolve[y''[x]+(Tan[x]-2*Cot[x])*y'[x]+2*Cot[x]^2*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to c_1\sqrt{-\sin^2(x)} - c_2\sin^2(x)$$

20.6 problem 641

Internal problem ID [15410]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 641.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' \tan(x) + \cos(x)^2 y = 0}$$

Given that one solution of the ode is

$$y_1 = \cos(\sin(x))$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 15

```
dsolve([diff(y(x),x$2)+tan(x)*diff(y(x),x)+cos(x)^2*y(x)=0,cos(sin(x))],singsol=all)
```

$$y(x) = c_1 \sin(\sin(x)) + c_2 \cos(\sin(x))$$

✓ Solution by Mathematica

Time used: 0.066 (sec). Leaf size: 18

```
DSolve[y''[x]+Tan[x]*y'[x]+Cos[x]^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \sin(\sin(x)) + c_1 \cos(\sin(x))$$

20.7 problem 642

Internal problem ID [15411]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 642.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{(x^2+1)\, y''+xy'-y=1}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve([(1+x^2)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=1,x],singsol=all)
```

$$y(x) = \sqrt{x^2+1}\, c_2 + c_1 x - 1$$

✓ Solution by Mathematica
Time used: 0.069 (sec). Leaf size: 80

```
DSolve[(1+x^2)*y''[x]+x*y'[x]-y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-\sqrt{x^2+1}+(c_1-ic_2)x^2+x\left(c_1\left(-\sqrt{x^2+1}\right)+ic_2\sqrt{x^2+1}+1\right)+c_1}{\sqrt{x^2+1}-x}$$

20.8 problem 643

Internal problem ID [15412]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 643.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' - xy' - 3y = 5x^4}$$

Given that one solution of the ode is

$$y_1 = \frac{1}{x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve([x^2*diff(y(x),x$2)-x*diff(y(x),x)-3*y(x)=5*x^4,1/x],singsol=all)
```

$$y(x) = \frac{c_2x^4 + x^5 + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 21

```
DSolve[x^2*y''[x]-x*y'[x]-3*y[x]==5*x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^5 + c_2x^4 + c_1}{x}$$

20.9 problem 644

Internal problem ID [15413]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 644.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-1)\,y''-xy'+y=(x-1)^2\,\mathrm{e}^x}$$

Given that one solution of the ode is

$$y_1=\mathrm{e}^x$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 22

```
dsolve([(x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=(x-1)^2*exp(x),exp(x)],singsol=all)
```

$$y(x)=\frac{(x^2+2c_1-2x)\,\mathrm{e}^x}{2}+c_2x$$

✓ Solution by Mathematica
Time used: 0.067 (sec). Leaf size: 28

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==(x-1)^2*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^x\left(\frac{x^2}{2}-x+c_1\right)-c_2x$$

20.10 problem 645

Internal problem ID [15414]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISE-LYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 645.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' + \mathrm{e}^{-2x} y = \mathrm{e}^{-3x}}$$

Given that one solution of the ode is

$$y_1 = \cos\left(\mathrm{e}^{-x}\right)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 28

```
dsolve([diff(y(x),x$2)+diff(y(x),x)+exp(-2*x)*y(x)=exp(-3*x),cos(exp(-x))],singsol=all)
```

$$y(x) = \sin\left(\mathrm{e}^{-x}\right) c_2 + \cos\left(\mathrm{e}^{-x}\right) c_1 + \sin\left(\mathrm{e}^{-x}\right) + \mathrm{e}^{-x}$$

✓ Solution by Mathematica
Time used: 0.131 (sec). Leaf size: 30

```
DSolve[y''[x]+y'[x]+Exp[-2*x]*y[x]==Exp[-3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x} + c_1 \cos\left(e^{-x}\right) - c_2 \sin\left(e^{-x}\right)$$

20.11 problem 646

Internal problem ID [15415]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 646.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$(x^4 - x^3)\, y'' + (2x^3 - 2x^2 - x)\, y' - y = \frac{(x-1)^2}{x}$$

Given that one solution of the ode is

$$y_1 = \frac{1}{x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve([(x^4-x^3)*diff(y(x),x$2)+(2*x^3-2*x^2-x)*diff(y(x),x)-y(x)=(x-1)^2/x,1/x],singsol=al
```

$$y(x) = \frac{e^{\frac{1}{x}} c_1 x - \ln(x) + c_2 + x}{x}$$

✓ Solution by Mathematica
Time used: 0.102 (sec). Leaf size: 27

```
DSolve[(x^4-x^3)*y''[x]+(2*x^3-2*x^2-x)*y'[x]-y[x]==(x-1)^2/x,y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{x - \log(x) + c_2 \left(-e^{\frac{1}{x}}\right) x + c_1}{x}$$

20.12 problem 647

Internal problem ID [15416]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 647.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$
\boxed{y'' - y' + e^{2x}y = x\,e^{2x} - 1}
$$

Given that one solution of the ode is

$$
y_1 = \sin\left(e^x\right)
$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 16

```
dsolve([diff(y(x),x$2)-diff(y(x),x)+exp(2*x)*y(x)=x*exp(2*x)-1,sin(exp(x))],singsol=all)
```

$$
y(x) = \sin\left(e^x\right) c_2 + \cos\left(e^x\right) c_1 + x
$$

✓ Solution by Mathematica
Time used: 0.341 (sec). Leaf size: 21

```
DSolve[y''[x]-y'[x]+Exp[2*x]*y[x]==x*Exp[2*x]-1,y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x) \to x + c_1 \cos\left(e^x\right) + c_2 \sin\left(e^x\right)
$$

20.13 problem 648

Internal problem ID [15417]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 648.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(x-1)\,y''-(2x-1)\,y'+2y=(2x-3)\,x^2}$$

Given that one solution of the ode is

$$y_1 = x^2$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve([x*(x-1)*diff(y(x),x$2)-(2*x-1)*diff(y(x),x)+2*y(x)=x^2*(2*x-3),x^2],singsol=all)
```

$$y(x) = c_2x^2 + x^3 - 2c_1x + c_1$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 40

```
DSolve[x*(x-1)*y''[x]-(2*x-1)*y'[x]+2*y[x]==x^2*(2*x-3),y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to x^3 + \left(-\frac{1}{2}+c_1\right)x^2 + (1-2c_1+c_2)x - \frac{1}{2} + c_1 - \frac{c_2}{2}$$

20.14 problem 653

Internal problem ID [15418]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 653.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \frac{1}{\sin(x)}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+y(x)=1/sin(x),y(x), singsol=all)
```

$$y(x) = \ln(\sin(x))\sin(x) + (c_1 - x)\cos(x) + \sin(x)\, c_2$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 24

```
DSolve[y''[x]+y[x]==1/Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (-x + c_1)\cos(x) + \sin(x)(\log(\sin(x)) + c_2)$$

20.15 problem 654

Internal problem ID [15419]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 654.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y''+y'=\frac{1}{1+\mathrm{e}^{x}}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=1/(1+exp(x)),y(x), singsol=all)
```

$$y(x)=\left(-\mathrm{e}^{-x}-1\right)\ln\left(1+\mathrm{e}^{x}\right)-c_1\mathrm{e}^{-x}+c_2+\ln\left(\mathrm{e}^{x}\right)$$

✓ Solution by Mathematica
Time used: 0.098 (sec). Leaf size: 33

```
DSolve[y''[x]+y'[x]==1/(1+Exp[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x-\log\left(e^x+1\right)-e^{-x}(\log\left(e^x+1\right)+c_1)+c_2$$

20.16 problem 655

Internal problem ID [15420]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 655.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \frac{1}{\cos(x)^3}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)+y(x)=1/cos(x)^3,y(x), singsol=all)
```

$$y(x) = (-1 + c_1)\cos(x) + \sin(x)\, c_2 + \frac{\sec(x)}{2}$$

✓ Solution by Mathematica
Time used: 0.066 (sec). Leaf size: 25

```
DSolve[y''[x]+y[x]==1/Cos[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sec(x)}{2} + c_1\cos(x) + \sin(x)(\tan(x) + c_2)$$

20.17 problem 656

Internal problem ID [15421]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 656.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \frac{1}{\sqrt{\sin(x)^5 \cos(x)}}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 48

```
dsolve(diff(y(x),x$2)+y(x)=1/sqrt(sin(x)^5*cos(x)),y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 + \left(\int \frac{\cos(x)}{\sqrt{\sin(x)^5 \cos(x)}} dx\right) \sin(x) - \left(\int \frac{\sin(x)}{\sqrt{\sin(x)^5 \cos(x)}} dx\right) \cos(x)$$

✓ Solution by Mathematica
Time used: 0.162 (sec). Leaf size: 35

```
DSolve[y''[x]+y[x]==1/Sqrt[Sin[x]^5*Cos[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(x) + c_2 \sin(x) + \frac{4}{3} \csc^8(x) \left(\sin^5(x) \cos(x)\right)^{3/2}$$

20.18 problem 657

Internal problem ID [15422]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 657.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + y = \frac{e^x}{x^2 + 1}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=exp(x)/(x^2+1),y(x), singsol=all)
```

$$y(x) = e^x \left(c_2 + c_1 x - \frac{\ln(x^2 + 1)}{2} + x \arctan(x)\right)$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 35

```
DSolve[y''[x]-2*y'[x]+y[x]==Exp[x]/(1+x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^x \left(2x \arctan(x) - \log\left(x^2 + 1\right) + 2(c_2 x + c_1)\right)$$

20.19 problem 658

Internal problem ID [15423]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 658.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''+2y'+2y=\frac{e^{-x}}{\sin(x)}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+2*y(x)=1/(exp(x)*sin(x)),y(x), singsol=all)
```

$$y(x)=-(-\ln(\sin(x))\sin(x)+(x-c_1)\cos(x)-\sin(x)\,c_2)\,e^{-x}$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 30

```
DSolve[y''[x]+2*y'[x]+2*y[x]==1/(Exp[x]*Sin[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{-x}((-x+c_2)\cos(x)+\sin(x)(\log(\sin(x))+c_1))$$

20.20 problem 659

Internal problem ID [15424]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 659.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \frac{2}{\sin(x)^3}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+y(x)=2/sin(x)^3,y(x), singsol=all)
```

$$y(x) = (c_1 + 2\cot(x))\cos(x) + \sin(x)\,c_2 - \csc(x)$$

✓ Solution by Mathematica
Time used: 0.055 (sec). Leaf size: 25

```
DSolve[y''[x]+y[x]==2/Sin[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\csc(x) + c_2\sin(x) + \cos(x)(2\cot(x) + c_1)$$

20.21 problem 660

Internal problem ID [15425]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 660.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + y' = e^{2x} \cos(e^x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=exp(2*x)*cos(exp(x)),y(x), singsol=all)
```

$$y(x) = (-c_1 + 2\sin(e^x))\,e^{-x} + c_2 - \cos(e^x) - 1$$

✓ Solution by Mathematica
Time used: 0.258 (sec). Leaf size: 32

```
DSolve[y''[x]+y'[x]==Exp[2*x]*Cos[Exp[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - e^{-x}(-2\sin(e^x) + e^x\cos(e^x) + c_1)$$

20.22 problem 661

Internal problem ID [15426]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 661.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + y'' = \frac{x-1}{x^3}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x$3)+diff(y(x),x$2)=(x-1)/x^3,y(x), singsol=all)
```

$$y(x) = -\frac{\left(\int \int \frac{\mathrm{e}^{-x} \operatorname{expIntegral}_1(-x) x^2 - 2\,\mathrm{e}^{-x} c_1 x^2 + x - 1}{x^2} dx dx\right)}{2} + c_2 x + c_3$$

✓ Solution by Mathematica
Time used: 0.238 (sec). Leaf size: 35

```
DSolve[y'''[x]+y''[x]==(x-1)/x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x} \operatorname{ExpIntegralEi}(x)}{2} - \log(x) + c_1 e^{-x} + c_3 x + c_2$$

20.23 problem 662

Internal problem ID [15427]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 662.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$xy'' - (2x^2+1)\, y' = 4x^3 e^{x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x*diff(y(x),x$2)-(1+2*x^2)*diff(y(x),x)=4*x^3*exp(x^2),y(x), singsol=all)
```

$$y(x) = \frac{(2x^2 + c_1 - 2)\, e^{x^2}}{2} + c_2$$

✓ Solution by Mathematica
Time used: 0.074 (sec). Leaf size: 25

```
DSolve[x*y''[x]-(1+2*x^2)*y'[x]==4*x^3*Exp[x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x^2}\left(x^2 - 1 + \frac{c_1}{2}\right) + c_2$$

20.24 problem 663

Internal problem ID [15428]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 663.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - 2y' \tan(x) = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-2*tan(x)*diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x) = -\frac{\ln(1+\cos(2x))}{4} + \frac{\ln(\cos(x))}{2} + \frac{(4c_1+2x)\tan(x)}{4} + c_2$$

✓ Solution by Mathematica
Time used: 0.047 (sec). Leaf size: 19

```
DSolve[y''[x]-2*Tan[x]*y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\frac{x}{2} + c_1\right)\tan(x) + c_2$$

20.25 problem 664

Internal problem ID [15429]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 664.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x \ln(x) y'' - y' = \ln(x)^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 26

```
dsolve(x*ln(x)*diff(y(x),x$2)-diff(y(x),x)=ln(x)^2,y(x), singsol=all)
```

$$y(x) = \ln(x)^2 x + x(c_1 - 2)\ln(x) + (-c_1 + 2)x + c_2$$

✓ Solution by Mathematica
Time used: 0.049 (sec). Leaf size: 29

```
DSolve[x*Log[x]*y''[x]-y'[x]==Log[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\log^2(x) - (-2 + c_1)x + (-2 + c_1)x\log(x) + c_2$$

20.26 problem 665

Internal problem ID [15430]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 665.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' + (2x-1)\,y' = -4x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x$2)+(2*x-1)*diff(y(x),x)=-4*x^2,y(x), singsol=all)
```

$$y(x) = \frac{(-2x-1)\,c_1 \mathrm{e}^{-2x}}{4} - x^2 + c_2$$

✓ Solution by Mathematica
Time used: 0.126 (sec). Leaf size: 35

```
DSolve[x*y''[x]+(2*x-1)*y'[x]==-4*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{1}{4}e^{-2x}\left(4e^{2x}x^2 + 2c_1 x + c_1\right)$$

20.27 problem 666

Internal problem ID [15431]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 666.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + y' \tan(x) = \cot(x) \cos(x)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)+tan(x)*diff(y(x),x)=cos(x)*cot(x),y(x), singsol=all)
```

$$y(x) = c_2 + \sin(x)(-1 + \ln(\sin(x)) + c_1)$$

✓ Solution by Mathematica
Time used: 0.14 (sec). Leaf size: 39

```
DSolve[y''[x]+Tan[x]*y'[x]==Cos[x]*Cot[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\sqrt{\sin^2(x)} \log\left(\sin^2(x)\right) - (1 + c_2)\sqrt{\sin^2(x)} + c_1$$

20.28 problem 667

Internal problem ID [15432]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 667.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4xy'' + 2y' + y = 1}$$

With initial conditions

$$[y(\infty) = 1]$$

✗ Solution by Maple

```
dsolve([4*x*diff(y(x),x$2)+2*diff(y(x),x)+y(x)=1,y(infinity) = 1],y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica
Time used: 0.045 (sec). Leaf size: 25

```
DSolve[{4*x*y''[x]+2*y'[x]+y[x]==1,{y[Infinity]==1}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to c_1 \cos\left(\sqrt{x}\right) + c_2 \sin\left(\sqrt{x}\right) + 1$$

20.29 problem 668

Internal problem ID [15433]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 668.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{4xy'' + 2y' + y = \frac{6+x}{x^2}}$$

With initial conditions

$$[y(\infty) = 0]$$

✓ Solution by Maple

Time used: 0.75 (sec). Leaf size: 41

```
dsolve([4*x*diff(y(x),x$2)+2*diff(y(x),x)+y(x)=(6+x)/x^2,y(infinity) = 0],y(x), singsol=all)
```

$$y(x) = undefined$$

✓ Solution by Mathematica

Time used: 0.192 (sec). Leaf size: 27

```
DSolve[{4*x*y''[x]+2*y'[x]+y[x]==(6+x)/x^2,{y[Infinity]==0}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{x} + c_1 \cos\left(\sqrt{x}\right) + c_2 \sin\left(\sqrt{x}\right)$$

20.30 problem 669

Internal problem ID [15434]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 669.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{(x^2+1)\,y''+2xy'=\frac{1}{x^2+1}}$$

With initial conditions

$$\left[y(\infty)=\frac{\pi^2}{8}, y'(0)=0\right]$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 10

```
dsolve([(1+x^2)*diff(y(x),x$2)+2*x*diff(y(x),x)=1/(1+x^2),y(infinity) = 1/8*Pi^2, D(y)(0) =
```

$$y(x)=\frac{\arctan(x)^2}{2}$$

✓ Solution by Mathematica
Time used: 0.044 (sec). Leaf size: 13

```
DSolve[{(1+x^2)*y''[x]+2*x*y'[x]==1/(1+x^2),{y[Infinity]==Pi^2/8,y'[0]==0}},y[x],x,IncludeSi
```

$$y(x)\to\frac{\arctan(x)^2}{2}$$

20.31 problem 670

Internal problem ID [15435]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 670.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(1-x)\,y''+xy'-y=(x-1)^2\,\mathrm{e}^x$$

With initial conditions

$$[y(-\infty)=0, y'(0)=1]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 16

```
dsolve([(1-x)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=(x-1)^2*exp(x),y(-infinity) = 0, D(y)(0) =
```

$$y(x)=-\frac{x(x-2)\,\mathrm{e}^x}{2}$$

✓ Solution by Mathematica
Time used: 0.083 (sec). Leaf size: 16

```
DSolve[{(1-x)*y''[x]+x*y'[x]-y[x]==(x-1)^2*Exp[x],{y[-Infinity]==0,y'[0]==1}},y[x],x,Include
```

$$y(x)\to -\frac{1}{2}e^x(x-2)x$$

20.32 problem 671

Internal problem ID [15436]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 671.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$2x^2(2-\ln(x))\, y''+x(4-\ln(x))\, y'-y=\frac{(2-\ln(x))^2}{\sqrt{x}}$$

With initial conditions

$$[y(\infty)=0]$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 21

```
dsolve([2*x^2*(2-ln(x))*diff(y(x),x$2)+x*(4-ln(x))*diff(y(x),x)-y(x)=(2-ln(x))^2/sqrt(x),y(i
```

$$y(x)=\frac{\sqrt{x}\,\ln(x)\, c_2-\ln(x)+1}{\sqrt{x}}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{2*x^2*(2-Log[x])*y''[x]+x*(4-Log[x])*y'[x]-y[x]==(2-Log[x])^2/Sqrt[x],{y[Infinity]==
```

Not solved

20.33 problem 672

Internal problem ID [15437]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 672.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + \frac{2y'}{x} - y = 4\,\mathrm{e}^{x}$$

With initial conditions

$$[y(-\infty) = 0, y'(-1) = -\mathrm{e}^{-1}]$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 13

```
dsolve([diff(y(x),x$2)+2/x*diff(y(x),x)-y(x)=4*exp(x),y(-infinity) = 0, D(y)(-1) = -1/exp(1)
```

$$y(x) = (x - 1)\,\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.079 (sec). Leaf size: 12

```
DSolve[{y''[x]+2/x*y'[x]-y[x]==4*Exp[x],{y[-Infinity]==0,y'[-1]==-1/Exp[1]}},y[x],x,IncludeS
```

$$y(x) \to e^{x}(x - 1)$$

20.34 problem 673

Internal problem ID [15438]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 673.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^3(\ln(x)-1)\,y''-x^2y'+yx=2\ln(x)$$

With initial conditions

$$[y(\infty)=0]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 16

```
dsolve([x^3*(ln(x)-1)*diff(y(x),x$2)-x^2*diff(y(x),x)+x*y(x)=2*ln(x),y(infinity) = 0],y(x),
```

$$y(x)=\frac{-c_1\ln(x)\,x+1}{x}$$

✓ Solution by Mathematica
Time used: 0.116 (sec). Leaf size: 8

```
DSolve[{x^3*(Log[x]-1)*y''[x]-x^2*y'[x]+x*y[x]==2*Log[x],{y[Infinity]==0}},y[x],x,IncludeSin
```

$$y(x)\to\frac{1}{x}$$

20.35 problem 674

Internal problem ID [15439]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 15.5 Linear equations with variable coefficients. The Lagrange method. Exercises page 148
Problem number: 674.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 - 2x) y'' + (-x^2 + 2) y' - 2(1 - x) y = 2x - 2$$

With initial conditions

$$[y(\infty) = 1]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 13

```
dsolve([(x^2-2*x)*diff(y(x),x$2)+(2-x^2)*diff(y(x),x)-2*(1-x)*y(x)=2*(x-1),y(infinity) = 1],
```

$$y(x) = -\operatorname{signum}(c_1 x^2) \infty$$

✓ Solution by Mathematica

Time used: 0.314 (sec). Leaf size: 6

```
DSolve[{(x^2-2*x)*y''[x]+(2-x^2)*y'[x]-2*(1-x)*y[x]==2*(x-1),{y[Infinity]==1}},y[x],x,Includ
```

$$y(x) \to 1$$

21 Chapter 2 (Higher order ODE's). Section 16. The method of isoclines for differential equations of the second order. Exercises page 158

21.1 problem 696

Internal problem ID [15440]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 16. The method of isoclines for differential equations of the second order. Exercises page 158
Problem number: 696.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + x' + x = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 28

```
dsolve(diff(x(t),t$2)+diff(x(t),t)+x(t)=0,x(t), singsol=all)
```

$$x(t) = \mathrm{e}^{-\frac{t}{2}} \left(c_1 \sin\left(\frac{\sqrt{3}\,t}{2}\right) + c_2 \cos\left(\frac{\sqrt{3}\,t}{2}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 42

```
DSolve[x''[t]+x'[t]+x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{-t/2} \left(c_2 \cos\left(\frac{\sqrt{3}t}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}t}{2}\right)\right)$$

21.2 problem 697

Internal problem ID [15441]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 16. The method of isoclines for differential equations of the second order. Exercises page 158
Problem number: 697.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + 2x' + 6x = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(x(t),t$2)+2*diff(x(t),t)+6*x(t)=0,x(t), singsol=all)
```

$$x(t) = \mathrm{e}^{-t}\left(c_1 \sin\left(\sqrt{5}\, t\right) + c_2 \cos\left(\sqrt{5}\, t\right)\right)$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 34

```
DSolve[x''[t]+2*x'[t]+6*x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{-t}\left(c_2 \cos\left(\sqrt{5}t\right) + c_1 \sin\left(\sqrt{5}t\right)\right)$$

21.3 problem 698

Internal problem ID [15442]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 16. The method of isoclines for differential equations of the second order. Exercises page 158
Problem number: 698.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + 2x' + x = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve(diff(x(t),t$2)+2*diff(x(t),t)+x(t)=0,x(t), singsol=all)
```

$$x(t) = \mathrm{e}^{-t}(c_2 t + c_1)$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 18

```
DSolve[x''[t]+2*x'[t]+x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{-t}(c_2 t + c_1)$$

21.4 problem 699

Internal problem ID [15443]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 16. The method of isoclines for differential equations of the second order. Exercises page 158
Problem number: 699.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + x'^2 + x = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 61

```
dsolve(diff(x(t),t$2)+diff(x(t),t)^2+x(t)=0,x(t), singsol=all)
```

$$-2\left(\int^{x(t)} \frac{1}{\sqrt{2 + 4\,\mathrm{e}^{-2\_a} c_1 - 4\_a}} d\_a\right) - t - c_2 = 0$$
$$2\left(\int^{x(t)} \frac{1}{\sqrt{2 + 4\,\mathrm{e}^{-2\_a} c_1 - 4\_a}} d\_a\right) - t - c_2 = 0$$

✓ Solution by Mathematica

Time used: 0.81 (sec). Leaf size: 272

```
DSolve[x''[t]+x'[t]^2+x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{\sqrt{2}}{\sqrt{2e^{-2K[1]}c_1 - 2K[1] + 1}}dK[1]\&\right][t + c_2]$$
$$x(t) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sqrt{2}}{\sqrt{2e^{-2K[2]}c_1 - 2K[2] + 1}}dK[2]\&\right][t + c_2]$$
$$x(t) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{\sqrt{2}}{\sqrt{2e^{-2K[1]}(-c_1) - 2K[1] + 1}}dK[1]\&\right][t + c_2]$$
$$x(t) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{\sqrt{2}}{\sqrt{2e^{-2K[1]}c_1 - 2K[1] + 1}}dK[1]\&\right][t + c_2]$$
$$x(t) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sqrt{2}}{\sqrt{2e^{-2K[2]}(-c_1) - 2K[2] + 1}}dK[2]\&\right][t + c_2]$$
$$x(t) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sqrt{2}}{\sqrt{2e^{-2K[2]}c_1 - 2K[2] + 1}}dK[2]\&\right][t + c_2]$$

21.5 problem 700

Internal problem ID [15444]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 16. The method of isoclines for differential equations of the second order. Exercises page 158
Problem number: 700.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' - 2x'^2 + x' - 2x = 0}$$

✗ Solution by Maple

```
dsolve(diff(x(t),t$2)-2*diff(x(t),t)^2+diff(x(t),t)-2*x(t)=0,x(t), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x''[t]-2*x'[t]^2+x'[t]-2*x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

Not solved

21.6 problem 701

Internal problem ID [15445]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 16. The method of isoclines for differential equations of the second order. Exercises page 158
Problem number: 701.
ODE order: 2.
ODE degree: 0.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$x'' - x\,e^{x'} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(diff(x(t),t$2)-x(t)*exp(diff(x(t),t))=0,x(t), singsol=all)
```

$$-\left(\int^{x(t)} \frac{1}{\operatorname{LambertW}\left(\frac{(\_a^2+2c_1)\,\mathrm{e}^{-1}}{2}\right)+1}d\_a\right)-t-c_2=0$$

✓ Solution by Mathematica

Time used: 0.389 (sec). Leaf size: 126

```
DSolve[x''[t]-x[t]*Exp[x'[t]]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{-W\left(\frac{K[1]^2+2c_1}{2e}\right)-1}dK[1]\&\right][t+c_2]$$
$$x(t) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{-W\left(\frac{K[1]^2+2(-1)c_1}{2e}\right)-1}dK[1]\&\right][t+c_2]$$
$$x(t) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{-W\left(\frac{K[1]^2+2c_1}{2e}\right)-1}dK[1]\&\right][t+c_2]$$

21.7 problem 702

Internal problem ID [15446]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISE-LYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 16. The method of isoclines for differential equations of the second order. Exercises page 158
Problem number: 702.
ODE order: 2.
ODE degree: 0.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + e^{-x'} - x = 0}$$

✗ Solution by Maple

```
dsolve(diff(x(t),t$2)+exp(-diff(x(t),t))-x(t)=0,x(t), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x''[t]+Exp[-x'[t]]-x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

Not solved

21.8 problem 703

Internal problem ID [15447]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 16. The method of isoclines for differential equations of the second order. Exercises page 158
Problem number: 703.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$x'' + x{x'}^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(x(t),t$2)+x(t)*diff(x(t),t)^2=0,x(t), singsol=all)
```

$$x(t) = -i \operatorname{RootOf}\left(i\sqrt{2}\, c_1 t + i\sqrt{2}\, c_2 - \operatorname{erf}\left(\_Z\right)\sqrt{\pi}\right)\sqrt{2}$$

✓ Solution by Mathematica

Time used: 1.757 (sec). Leaf size: 34

```
DSolve[x''[t]+x[t]*x'[t]^2==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -i\sqrt{2}\operatorname{erf}^{-1}\left(i\sqrt{\frac{2}{\pi}}c_1(t+c_2)\right)$$

21.9 problem 704

Internal problem ID [15448]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 16. The method of isoclines for differential equations of the second order. Exercises page 158
Problem number: 704.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], [`

$$\boxed{x'' + (x+2)\,x' = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 32

```
dsolve(diff(x(t),t$2)+(x(t)+2)*diff(x(t),t)=0,x(t), singsol=all)
```

$$x(t) = -\frac{\left(\sqrt{2}\,c_1 - \tanh\left(\frac{(t+c_2)\sqrt{2}}{2c_1}\right)\right)\sqrt{2}}{c_1}$$

✓ Solution by Mathematica
Time used: 60.064 (sec). Leaf size: 40

```
DSolve[x''[t]+(x[t]+2)*x'[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -2 + \sqrt{2}\sqrt{2+c_1}\tanh\left(\frac{\sqrt{2+c_1}(t+c_2)}{\sqrt{2}}\right)$$

21.10 problem 705

Internal problem ID [15449]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 16. The method of isoclines for differential equations of the second order. Exercises page 158
Problem number: 705.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' - x' + x - x^2 = 0}$$

✗ Solution by Maple

```
dsolve(diff(x(t),t$2)-diff(x(t),t)+x(t)-x(t)^2=0,x(t), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x''[t]-x'[t]+x[t]-x[t]^2==0,x[t],t,IncludeSingularSolutions -> True]
```

Not solved

22 Chapter 2 (Higher order ODE's). Section 17. Boundary value problems. Exercises page 163

22.1 problem 706 (a)

Internal problem ID [15450]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 17. Boundary value problems. Exercises page 163
Problem number: 706 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + \lambda y = 0}$$

With initial conditions

$$[y'(0) = 0, y'(\pi) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 5

```
dsolve([diff(y(x),x$2)+lambda*y(x)=0,D(y)(0) = 0, D(y)(Pi) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 32

```
DSolve[{y''[x]+\[Lambda]*y[x]==0,{y'[0]==0,y'[Pi]==0}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \begin{cases} c_1 \cos\left(x\sqrt{\lambda}\right) & \underset{\cdot}{n} \in \mathbb{Z} \wedge \underset{\cdot}{n} \geq 0 \wedge \lambda = \underset{\cdot}{n}^2 \\ 0 & \text{True} \end{cases}$$

22.2 problem 707

Internal problem ID [15451]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 17. Boundary value problems. Exercises page 163
Problem number: 707.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + \lambda y = 0}$$

With initial conditions

$$[y(0) = 0, y(1) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 5

```
dsolve([diff(y(x),x$2)+lambda*y(x)=0,y(0) = 0, y(1) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 36

```
DSolve[{y''[x]+\[Lambda]*y[x]==0,{y[0]==0,y[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \begin{cases} c_1 \sin\left(x\sqrt{\lambda}\right) & n \in \mathbb{Z} \land n \geq 1 \land \lambda = n^2\pi^2 \\ 0 & \text{True} \end{cases}$$

22.3 problem 708 (a)

Internal problem ID [15452]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 17. Boundary value problems. Exercises page 163
Problem number: 708 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

With initial conditions

$$[y(0) = 0, y(2\pi) = 1]$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 27

```
dsolve([diff(y(x),x$2)-y(x)=0,y(0) = 0, y(2*Pi) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{-x+2\pi}(\mathrm{e}^{2x} - 1)}{\mathrm{e}^{4\pi} - 1}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 31

```
DSolve[{y''[x]-y[x]==0,{y[0]==0,y[2*Pi]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{2\pi-x}(e^{2x} - 1)}{e^{4\pi} - 1}$$

22.4 problem 708 (b)

Internal problem ID [15453]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 17. Boundary value problems. Exercises page 163
Problem number: 708 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

With initial conditions

$$[y(0) = 0, y(2\pi) = 1]$$

✗ Solution by Maple

```
dsolve([diff(y(x),x$2)+y(x)=0,y(0) = 0, y(2*Pi) = 1],y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y''[x]+y[x]==0,{y[0]==0,y[2*Pi]==1}},y[x],x,IncludeSingularSolutions -> True]
```

{}

22.5 problem 710

Internal problem ID [15454]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 17. Boundary value problems. Exercises page 163
Problem number: 710.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], [`

$$\boxed{yy'' + {y'}^2 = -1}$$

With initial conditions

$$[y(0) = 1, y(1) = 2]$$

✓ Solution by Maple
Time used: 0.781 (sec). Leaf size: 16

```
dsolve([y(x)*diff(y(x),x$2)+diff(y(x),x)^2+1=0,y(0) = 1, y(1) = 2],y(x), singsol=all)
```

$$y(x) = \sqrt{-x^2 + 4x + 1}$$

✓ Solution by Mathematica
Time used: 12.271 (sec). Leaf size: 19

```
DSolve[{y[x]*y''[x]+y'[x]^2+1==0,{y[0]==1,y[1]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{-x^2 + 4x + 1}$$

22.6 problem 711

Internal problem ID [15455]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 17. Boundary value problems. Exercises page 163
Problem number: 711.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

With initial conditions

$$\left[y(0) = 0, y\left(\frac{\pi}{2}\right) = \alpha\right]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 8

```
dsolve([diff(y(x),x$2)+y(x)=0,y(0) = 0, y(1/2*Pi) = alpha],y(x), singsol=all)
```

$$y(x) = \sin(x)\,\alpha$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 9

```
DSolve[{y''[x]+y[x]==0,{y[0]==0,y[Pi/2]==\[Alpha]}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \alpha \sin(x)$$

22.7 problem 712

Internal problem ID [15456]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 17. Boundary value problems. Exercises page 163
Problem number: 712.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

With initial conditions

$$[y(0) = 0, y'(1) = 1]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$2)-y(x)=0,y(0) = 0, D(y)(1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{e^{1-x}(e^{2x} - 1)}{e^2 + 1}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 27

```
DSolve[{y''[x]-y[x]==0,{y[0]==0,y'[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{1-x}(e^{2x} - 1)}{1 + e^2}$$

22.8 problem 713

Internal problem ID [15457]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 17. Boundary value problems. Exercises page 163
Problem number: 713.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + 2y = 0}$$

With initial conditions

$$[y(0) = 0, y'(\pi) = e^{\pi}]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 10

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+2*y(x)=0,y(0) = 0, D(y)(Pi) = exp(Pi)],y(x), singsol=a
```

$$y(x) = -e^x \sin(x)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 12

```
DSolve[{y''[x]-2*y'[x]+2*y[x]==0,{y[0]==0,y'[Pi]==Exp[Pi]}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to -e^x \sin(x)$$

22.9 problem 714

Internal problem ID [15458]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 17. Boundary value problems. Exercises page 163
Problem number: 714.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + \alpha y' = 0}$$

With initial conditions

$$[y(0) = \mathrm{e}^{\alpha}, y'(1) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 6

```
dsolve([diff(y(x),x$2)+alpha*diff(y(x),x)=0,y(0) = exp(alpha), D(y)(1) = 0],y(x), singsol=al
```

$$y(x) = \mathrm{e}^{\alpha}$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 8

```
DSolve[{y''[x]+\[Alpha]*y'[x]==0,{y[0]==Exp[\[Alpha]],y'[1]==0}},y[x],x,IncludeSingularSolut
```

$$y(x) \to e^{\alpha}$$

22.10 problem 715

Internal problem ID [15459]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 17. Boundary value problems. Exercises page 163
Problem number: 715.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + \alpha^2 y = 1}$$

With initial conditions

$$[y'(0) = \alpha, y'(\pi) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve([diff(y(x),x$2)+alpha^2*y(x)=1,D(y)(0) = alpha, D(y)(Pi) = 0],y(x), singsol=all)
```

$$y(x) = \sin(\alpha x) + \cos(\alpha x)\cot(\alpha\pi) + \frac{1}{\alpha^2}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y''[x]+\[Alpha]^2*y'[x]==1,{y'[0]==\[Alpha],y'[Pi]==0}},y[x],x,IncludeSingularSoluti
```

{}

22.11 problem 716

Internal problem ID [15460]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 17. Boundary value problems. Exercises page 163
Problem number: 716.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 1}$$

With initial conditions

$$[y(0) = 0, y'(\pi) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 10

```
dsolve([diff(y(x),x$2)+y(x)=1,y(0) = 0, D(y)(Pi) = 0],y(x), singsol=all)
```

$$y(x) = 1 - \cos(x)$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 11

```
DSolve[{y''[x]+y[x]==1,{y[0]==0,y'[Pi]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 - \cos(x)$$

22.12 problem 717

Internal problem ID [15461]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 17. Boundary value problems. Exercises page 163
Problem number: 717.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + \lambda^2 y = 0}$$

With initial conditions

$$[y'(0) = 0, y'(\pi) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x$2)+lambda^2*y(x)=0,D(y)(0) = 0, D(y)(Pi) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 36

```
DSolve[{y''[x]+\[Lambda]^2*y[x]==0,{y'[0]==0,y'[Pi]==0}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \left\{ \begin{array}{cc} c_1 \cos\left(x\sqrt{\lambda^2}\right) & n \in \mathbb{Z} \land n \geq 0 \land \lambda^2 = n^2 \\ 0 & \text{True} \end{array} \right.$$

22.13 problem 718

Internal problem ID [15462]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 17. Boundary value problems. Exercises page 163
Problem number: 718.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + \lambda^2 y = 0}$$

With initial conditions

$$[y(0) = 0, y'(\pi) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x$2)+lambda^2*y(x)=0,y(0) = 0, D(y)(Pi) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 40

```
DSolve[{y''[x]+\[Lambda]^2*y[x]==0,{y[0]==0,y'[Pi]==0}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \begin{cases} c_1 \sin\left(x\sqrt{\lambda^2}\right) & n \in \mathbb{Z} \land n \geq 1 \land \lambda^2 = \left(n - \frac{1}{2}\right)^2 \\ 0 & \text{True} \end{cases}$$

22.14 problem 719

Internal problem ID [15463]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 17. Boundary value problems. Exercises page 163
Problem number: 719.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(x),x$3)+diff(y(x),x$2)-diff(y(x),x)-y(x)=0,y(0) = -1, y(1) = 0, D(y)(0) = 2],
```

$$y(x) = e^{-x}(x - 1)$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 14

```
DSolve[{y'''[x]+y''[x]-y'[x]-y[x]==0,{y[0]==-1,y[1]==0,y'[0]==2}},y[x],x,IncludeSingularSolu
```

$$y(x) \to e^{-x}(x - 1)$$

22.15 problem 720

Internal problem ID [15464]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 17. Boundary value problems. Exercises page 163
Problem number: 720.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 5

```
dsolve([diff(y(x),x$4)-lambda^4*y(x)=0,y(0) = 0, (D@@2)(y)(0) = 0, y(Pi) = 0, (D@@2)(y)(Pi)
```

$$y(x) = 0$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 6

```
DSolve[{y''''[x]-\[Lambda]^4*y[x]==0,{y[0]==0,y''[0]==0,y[Pi]==0,y''[Pi]==0}},y[x],x,Include
```

$$y(x) \to 0$$

22.16 problem 721

Internal problem ID [15465]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 17. Boundary value problems. Exercises page 163
Problem number: 721.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' + y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(x*diff(y(x),x$2)+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_2 \ln(x) + c_1$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 13

```
DSolve[x*y''[x]+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \log(x) + c_2$$

22.17 problem 722

Internal problem ID [15466]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 17. Boundary value problems. Exercises page 163
Problem number: 722.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$x^2y'''' + 4xy''' + 2y'' = 0$$

With initial conditions

$$[y(1) = 0, y'(1) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 22

```
dsolve([x^2*diff(y(x),x$4)+4*x*diff(y(x),x$3)+2*diff(y(x),x$2)=0,y(1) = 0, D(y)(1) = 0],y(x)
```

$$y(x) = (-c_3 + (x-1)\,c_4)\ln(x) + c_3(x-1)$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 29

```
DSolve[{x^2*y''''[x]+4*x*y'''[x]+2*y''[x]==0,{y[1]==0,y'[1]==0}},y[x],x,IncludeSingularSolut
```

$$y(x) \to (c_1 - c_2)(x-1) + (c_2x - c_1)\log(x)$$

22.18 problem 723

Internal problem ID [15467]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 17. Boundary value problems. Exercises page 163
Problem number: 723.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{x^3y'''' + 6x^2y''' + 6xy'' = 0}$$

With initial conditions

$$[y(1) = 0, y'(1) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve([x^3*diff(y(x),x$4)+6*x^2*diff(y(x),x$3)+6*x*diff(y(x),x$2)=0,y(1) = 0, D(y)(1) = 0],
```

$$y(x) = -c_3 - c_4 + (c_3 - c_4)\ln(x) + \frac{c_3}{x} + c_4x$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 34

```
DSolve[{x^3*y''''[x]+6*x^2*y'''[x]+6*x*y''[x]==0,{y[1]==0,y'[1]==0}},y[x],x,IncludeSingularS
```

$$y(x) \to \frac{(x-1)(c_1(x-1) + 2c_2x)}{2x} - c_2\log(x)$$

23 Chapter 2 (Higher order ODE's). Section 18.1 Integration of differential equation in series. Power series. Exercises page 171

23.1 problem 724

Internal problem ID [15468]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.1 Integration of differential equation in series. Power series. Exercises page 171
Problem number: 724.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + yx = 1}$$

With initial conditions

$$[y(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([diff(y(x),x)=1-x*y(x),y(0) = 0],y(x),type='series',x=0);
```

$$y(x) = x - \frac{1}{3}x^3 + \frac{1}{15}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{y'[x]==1-x*y[x],{y[0]==0}},y[x],{x,0,5}]
```

$$y(x) \to \frac{x^5}{15} - \frac{x^3}{3} + x$$

23.2 problem 725

Internal problem ID [15469]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.1 Integration of differential equation in series. Power series. Exercises page 171
Problem number: 725.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$y' - \frac{y-x}{y+x} = 0$$

With initial conditions

$$[y(0) = 1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([diff(y(x),x)=(y(x)-x)/(y(x)+x),y(0) = 1],y(x),type='series',x=0);
```

$$y(x) = 1 + x - x^2 + \frac{4}{3}x^3 - \frac{5}{2}x^4 + \frac{16}{3}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 32

```
AsymptoticDSolveValue[{y'[x]==(y[x]-x)/(y[x]+x),{y[0]==1}},y[x],{x,0,5}]
```

$$y(x) \to \frac{16x^5}{3} - \frac{5x^4}{2} + \frac{4x^3}{3} - x^2 + x + 1$$

23.3 problem 726

Internal problem ID [15470]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.1 Integration of differential equation in series. Power series. Exercises page 171
Problem number: 726.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \sin(x)\, y = 0}$$

With initial conditions

$$[y(0) = 1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([diff(y(x),x)=sin(x)*y(x),y(0) = 1],y(x),type='series',x=0);
```

$$y(x) = 1 + \frac{1}{2}x^2 + \frac{1}{12}x^4 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{y'[x]==Sin[x]*y[x],{y[0]==1}},y[x],{x,0,5}]
```

$$y(x) \to \frac{x^4}{12} + \frac{x^2}{2} + 1$$

23.4 problem 727

Internal problem ID [15471]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.1 Integration of differential equation in series. Power series. Exercises page 171
Problem number: 727.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + yx = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
Order:=6;
dsolve([diff(y(x),x$2)+x*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x),type='series',x=0);
```

$$y(x) = x - \frac{1}{12}x^4 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 12

```
AsymptoticDSolveValue[{y''[x]+x*y[x]==0,{y[0]==0,y'[0]==1}},y[x],{x,0,5}]
```

$$y(x) \to x - \frac{x^4}{12}$$

23.5 problem 728

Internal problem ID [15472]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.1 Integration of differential equation in series. Power series. Exercises page 171
Problem number: 728.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - y' \sin(x) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([diff(y(x),x$2)-diff(y(x),x)*sin(x)=0,y(0) = 0, D(y)(0) = 1],y(x),type='series',x=0);
```

$$y(x) = x + \frac{1}{6}x^3 + \frac{1}{60}x^5 + \mathrm{O}(x^6)$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{y''[x]+Sin[x]*y'[x]==0,{y[0]==0,y'[0]==1}},y[x],{x,0,5}]
```

$$y(x) \to \frac{x^5}{30} - \frac{x^3}{6} + x$$

23.6 problem 729

Internal problem ID [15473]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISE-LYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.1 Integration of differential equation in series. Power series. Exercises page 171
Problem number: 729.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{xy'' + \sin(x)\, y = x}$$

With initial conditions

$$[y(\pi) = 1, y'(\pi) = 0]$$

With the expansion point for the power series method at $x = \pi$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
Order:=6;
dsolve([x*diff(y(x),x$2)+y(x)*sin(x)=x,y(Pi) = 1, D(y)(Pi) = 0],y(x),type='series',x=Pi);
```

$$y(x) = 1 + \frac{1}{2}(-\pi + x)^2 + \frac{1}{6}\frac{1}{\pi}(-\pi + x)^3 - \frac{1}{12}\frac{1}{\pi^2}(-\pi + x)^4 + \frac{1}{60}\frac{\pi^2 + 3}{\pi^3}(-\pi + x)^5 + \mathrm{O}\left((-\pi + x)^6\right)$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 75

```
AsymptoticDSolveValue[{x*y''[x]+Sin[x]*y[x]==x,{y[Pi]==1,y'[Pi]==0}},y[x],{x,Pi,5}]
```

$$y(x) \to \frac{1}{60}\left(\frac{3}{2\pi} - \frac{\pi^2 - 6}{2\pi^3}\right)(x - \pi)^5 - \frac{(x - \pi)^4}{12\pi^2} + \frac{(x - \pi)^3}{6\pi} + \frac{1}{2}(x - \pi)^2 + 1$$

23.7 problem 730

Internal problem ID [15474]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.1 Integration of differential equation in series. Power series. Exercises page 171
Problem number: 730.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\ln (x) y'' - y \sin (x) = 0}$$

With initial conditions

$$[y(\mathrm{e}) = \mathrm{e}^{-1}, y'(\mathrm{e}) = 0]$$

With the expansion point for the power series method at $x = \mathrm{e}$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 142

```
Order:=6;
dsolve([ln(x)*diff(y(x),x$2)-y(x)*sin(x)=0,y(exp(1)) = 1/exp(1), D(y)(exp(1)) = 0],y(x),type
```

$$y(x) = \mathrm{e}^{-1} + \frac{1}{2}\sin(\mathrm{e})\,\mathrm{e}^{-1}(x-\mathrm{e})^2 + \frac{1}{6}(\cos(\mathrm{e})\,\mathrm{e} - \sin(\mathrm{e}))\,\mathrm{e}^{-2}(x-\mathrm{e})^3 + \left(\frac{\mathrm{e}^{-3}\mathrm{e}^2\sin(\mathrm{e})^2}{24} - \frac{(\mathrm{e}^2-3)\,\mathrm{e}^{-3}\sin(\mathrm{e})}{24} - \frac{\mathrm{e}^{-3}\cos(\mathrm{e})\,\mathrm{e}}{12}\right)(x-\mathrm{e})^4 + \left(-\frac{\mathrm{e}^{-4}\mathrm{e}^2\sin(\mathrm{e})^2}{30} + \frac{(4\cos(\mathrm{e})\,\mathrm{e}^3 + 3\,\mathrm{e}^2 - 14)\,\mathrm{e}^{-4}\sin(\mathrm{e})}{120} + \frac{3\cos(\mathrm{e})\,\mathrm{e}^{-4}\left(\mathrm{e} - \frac{\mathrm{e}^3}{9}\right)}{40}\right)(x-\mathrm{e})^5 + O\left((x-\mathrm{e})^6\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
AsymptoticDSolveValue[{Log[x]*y''[x]-Sin[x]*y[x]==0,{y[Exp[1]]==1/Exp[1],y'[Exp[1]]==0}},y[x
```

Not solved

23.8 problem 731

Internal problem ID [15475]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.1 Integration of differential equation in series. Power series. Exercises page 171
Problem number: 731.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [NONE]

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 14

```
Order:=6;
dsolve([diff(y(x),x$3)+x*sin(y(x))=0,y(0) = 1/2*Pi, D(y)(0) = 0, (D@@2)(y)(0) = 0],y(x),type
```

$$y = \frac{\pi}{2} - \frac{1}{24}x^4 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.055 (sec). Leaf size: 16

```
AsymptoticDSolveValue[{y'''[x]+x*Sin[y[x]]==0,{y[0]==Pi/2,y'[0]==0,y''[0]==0}},y[x],{x,0,5}]
```

$$y(x) \to \frac{\pi}{2} - \frac{x^4}{24}$$

23.9 problem 732

Internal problem ID [15476]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.1 Integration of differential equation in series. Power series. Exercises page 171
Problem number: 732.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 2yx = 0}$$

With initial conditions

$$[y(0) = 1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([diff(y(x),x)-2*x*y(x)=0,y(0) = 1],y(x),type='series',x=0);
```

$$y = 1 + x^2 + \frac{1}{2}x^4 + O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 15

```
AsymptoticDSolveValue[{y'[x]-2*x*y[x]==0,{y[0]==1}},y[x],{x,0,5}]
```

$$y(x) \to \frac{x^4}{2} + x^2 + 1$$

23.10 problem 733

Internal problem ID [15477]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.1 Integration of differential equation in series. Power series. Exercises page 171
Problem number: 733.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + y'x + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y = \left(1 - \frac{1}{2}x^2 + \frac{1}{8}x^4\right) y(0) + \left(x - \frac{1}{3}x^3 + \frac{1}{15}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]+x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{15} - \frac{x^3}{3} + x\right) + c_1\left(\frac{x^4}{8} - \frac{x^2}{2} + 1\right)$$

23.11 problem 734

Internal problem ID [15478]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.1 Integration of differential equation in series. Power series. Exercises page 171
Problem number: 734.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x + y = 1}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 12

```
Order:=6;
dsolve([diff(y(x),x$2)-x*diff(y(x),x)+y(x)=1,y(0) = 0, D(y)(0) = 0],y(x),type='series',x=0);
```

$$y = \frac{1}{2}x^2 + \frac{1}{24}x^4 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 18

```
AsymptoticDSolveValue[{y''[x]-x*y'[x]+y[x]==1,{y[0]==0,y'[0]==0}},y[x],{x,0,5}]
```

$$y(x) \to \frac{x^4}{24} + \frac{x^2}{2}$$

23.12 problem 735

Internal problem ID [15479]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.1 Integration of differential equation in series. Power series. Exercises page 171
Problem number: 735.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - (x^2 + 1)\, y = 0}$$

With initial conditions

$$[y(0) = -2, y'(0) = 2]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([diff(y(x),x$2)-(1+x^2)*y(x)=0,y(0) = -2, D(y)(0) = 2],y(x),type='series',x=0);
```

$$y = -2 + 2x - x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{7}{60}x^5 + \mathrm{O}(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[{y''[x]-(1+x^2)*y[x]==0,{y[0]==-2,y'[0]==2}},y[x],{x,0,5}]
```

$$y(x) \to \frac{7x^5}{60} - \frac{x^4}{4} + \frac{x^3}{3} - x^2 + 2x - 2$$

23.13 problem 736

Internal problem ID [15480]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.1 Integration of differential equation in series. Power series. Exercises page 171
Problem number: 736.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y x^2 + y' = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([diff(y(x),x$2)=x^2*y(x)-diff(y(x),x),y(0) = 1, D(y)(0) = 0],y(x),type='series',x=0);
```

$$y = 1 + \frac{1}{12}x^4 - \frac{1}{60}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{y''[x]==x^2*y[x]-y'[x],{y[0]==1,y'[0]==0}},y[x],{x,0,5}]
```

$$y(x) \to -\frac{x^5}{60} + \frac{x^4}{12} + 1$$

23.14 problem 737

Internal problem ID [15481]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.1 Integration of differential equation in series. Power series. Exercises page 171
Problem number: 737.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y\,\mathrm{e}^{x} = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve(diff(y(x),x$2)-y(x)*exp(x)=0,y(x),type='series',x=0);
```

$$y = \left(1+\frac{1}{2}x^2+\frac{1}{6}x^3+\frac{1}{12}x^4+\frac{1}{24}x^5\right)y(0)+\left(x+\frac{1}{6}x^3+\frac{1}{12}x^4+\frac{1}{30}x^5\right)D(y)(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[y''[x]-y[x]*Exp[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{30}+\frac{x^4}{12}+\frac{x^3}{6}+x\right)+c_1\left(\frac{x^5}{24}+\frac{x^4}{12}+\frac{x^3}{6}+\frac{x^2}{2}+1\right)$$

23.15 problem 738

Internal problem ID [15482]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.1 Integration of differential equation in series. Power series. Exercises page 171
Problem number: 738.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - e^y - yx = 0}$$

With initial conditions

$$[y(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
Order:=6;
dsolve([diff(y(x),x)=exp(y(x))+x*y(x),y(0) = 0],y(x),type='series',x=0);
```

$$y = x + \frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{11}{24}x^4 + \frac{53}{120}x^5 + O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 33

```
AsymptoticDSolveValue[{y'[x]==Exp[y[x]]+x*y[x],{y[0]==0}},y[x],{x,0,5}]
```

$$y(x) \to \frac{53x^5}{120} + \frac{11x^4}{24} + \frac{2x^3}{3} + \frac{x^2}{2} + x$$

24 Chapter 2 (Higher order ODE's). Section 18.2. Expanding a solution in generalized power series. Bessels equation. Exercises page 177

24.1 problem 739

Internal problem ID [15483]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.2. Expanding a solution in generalized power series. Bessels equation. Exercises page 177
Problem number: 739.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$4xy'' + 2y' + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 44

```
Order:=6;
dsolve(4*x*diff(y(x),x$2)+2*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y = c_1\sqrt{x}\left(1 - \frac{1}{6}x + \frac{1}{120}x^2 - \frac{1}{5040}x^3 + \frac{1}{362880}x^4 - \frac{1}{39916800}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(1 - \frac{1}{2}x + \frac{1}{24}x^2 - \frac{1}{720}x^3 + \frac{1}{40320}x^4 - \frac{1}{3628800}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 85

```
AsymptoticDSolveValue[4*x*y''[x]+2*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(-\frac{x^5}{39916800} + \frac{x^4}{362880} - \frac{x^3}{5040} + \frac{x^2}{120} - \frac{x}{6} + 1\right) + c_2\left(-\frac{x^5}{3628800} + \frac{x^4}{40320} - \frac{x^3}{720} + \frac{x^2}{24} - \frac{x}{2} + 1\right)$$

24.2 problem 740

Internal problem ID [15484]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.2. Expanding a solution in generalized power series. Bessels equation. Exercises page 177
Problem number: 740.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x+1)\,y'-ny=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 114

```
Order:=6;
dsolve((1+x)*diff(y(x),x)-n*y(x)=0,y(x),type='series',x=0);
```

$$y=\left(1+nx+\frac{n(-1+n)\,x^2}{2}+\frac{n(n^2-3n+2)\,x^3}{6}+\frac{n(n^3-6n^2+11n-6)\,x^4}{24}+\frac{n(n^4-10n^3+35n^2-50n+24)\,x^5}{120}\right)y(0)+O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 143

```
AsymptoticDSolveValue[(1+x)*y'[x]-n*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{n^5x^5}{120}-\frac{n^4x^5}{12}+\frac{n^4x^4}{24}+\frac{7n^3x^5}{24}-\frac{n^3x^4}{4}+\frac{n^3x^3}{6}-\frac{5n^2x^5}{12}+\frac{11n^2x^4}{24}-\frac{n^2x^3}{2}+\frac{n^2x^2}{2}+\frac{nx^5}{5}-\frac{nx^4}{4}+\frac{nx^3}{3}-\frac{nx^2}{2}+nx+1\right)$$

24.3 problem 741

Internal problem ID [15485]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.2. Expanding a solution in generalized power series. Bessels equation. Exercises page 177
Problem number: 741.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$\boxed{9x(1-x)\,y''-12y'+4y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 44

```
Order:=6;
dsolve(9*x*(1-x)*diff(y(x),x$2)-12*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y = c_1 x^{\frac{7}{3}}\left(1+\frac{4}{5}x+\frac{44}{65}x^2+\frac{77}{130}x^3+\frac{1309}{2470}x^4+\frac{119}{247}x^5+\mathrm{O}\left(x^6\right)\right) + c_2\left(1+\frac{1}{3}x+\frac{2}{9}x^2+\frac{14}{81}x^3+\frac{35}{243}x^4+\frac{91}{729}x^5+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 85

```
AsymptoticDSolveValue[9*x*(1-x)*y''[x]-12*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{91x^5}{729}+\frac{35x^4}{243}+\frac{14x^3}{81}+\frac{2x^2}{9}+\frac{x}{3}+1\right) + c_1\left(\frac{119x^5}{247}+\frac{1309x^4}{2470}+\frac{77x^3}{130}+\frac{44x^2}{65}+\frac{4x}{5}+1\right)x^{7/3}$$

24.4 problem 744

Internal problem ID [15486]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.2. Expanding a solution in generalized power series. Bessels equation. Exercises page 177
Problem number: 744.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(4x^2 - \frac{1}{9}\right) y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(4*x^2-1/9)*y(x)=0,y(x), singsol=all)
```

$$y = c_1 \operatorname{BesselJ}\left(\frac{1}{3}, 2x\right) + c_2 \operatorname{BesselY}\left(\frac{1}{3}, 2x\right)$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 26

```
DSolve[x^2*y''[x]+x*y'[x]+(4*x^2-1/9)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}\left(\frac{1}{3}, 2x\right) + c_2 \operatorname{BesselY}\left(\frac{1}{3}, 2x\right)$$

24.5 problem 745

Internal problem ID [15487]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.2. Expanding a solution in generalized power series. Bessels equation. Exercises page 177
Problem number: 745.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(x^2 - \frac{1}{4}\right)y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x)=0,y(x), singsol=all)
```

$$y = \frac{c_1 \sin(x) + c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2e^{2ix})}{2\sqrt{x}}$$

24.6 problem 746

Internal problem ID [15488]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.2. Expanding a solution in generalized power series. Bessels equation. Exercises page 177
Problem number: 746.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{y'}{x} + \frac{y}{9} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)+1/x*diff(y(x),x)+1/9*y(x)=0,y(x), singsol=all)
```

$$y = c_1 \operatorname{BesselJ}\left(0, \frac{x}{3}\right) + c_2 \operatorname{BesselY}\left(0, \frac{x}{3}\right)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 26

```
DSolve[y''[x]+1/x*y'[x]+1/9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}\left(0, \frac{x}{3}\right) + c_2 \operatorname{BesselY}\left(0, \frac{x}{3}\right)$$

24.7 problem 747

Internal problem ID [15489]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.2. Expanding a solution in generalized power series. Bessels equation. Exercises page 177
Problem number: 747.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{y'}{x} + 4y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)+1/x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y = c_1 \operatorname{BesselJ}(0, 2x) + c_2 \operatorname{BesselY}(0, 2x)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 22

```
DSolve[y''[x]+1/x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \text{BesselJ}(0, 2x) + c_2 \text{BesselY}(0, 2x)$$

24.8 problem 748

Internal problem ID [15490]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.2. Expanding a solution in generalized power series. Bessels equation. Exercises page 177
Problem number: 748.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2y'x + 4(x^4 - 1)\,y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 45

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+4*(x^4-1)*y(x)=0,y(x), singsol=all)
```

$$y = -\frac{-\frac{\text{BesselY}\left(\frac{1}{4},x^2\right)c_2}{2} - \frac{\text{BesselJ}\left(\frac{1}{4},x^2\right)c_1}{2} + x^2\left(c_1 \,\text{BesselJ}\left(-\frac{3}{4}, x^2\right) + \text{BesselY}\left(-\frac{3}{4}, x^2\right) c_2\right)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.158 (sec). Leaf size: 46

```
DSolve[x^2*y''[x]-2*x*y'[x]+4*(x^4-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{3/2}\left(c_2 \,\text{Gamma}\left(\frac{9}{4}\right) \text{BesselJ}\left(\frac{5}{4}, x^2\right) - 4c_1 \,\text{Gamma}\left(\frac{3}{4}\right) \text{BesselJ}\left(-\frac{5}{4}, x^2\right)\right)}{2^{3/4}}$$

24.9 problem 749

Internal problem ID [15491]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.2. Expanding a solution in generalized power series. Bessels equation. Exercises page 177
Problem number: 749.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{y''x + \frac{y'}{2} + \frac{y}{4} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x$2)+1/2*diff(y(x),x)+1/4*y(x)=0,y(x), singsol=all)
```

$$y = c_1 \sin\left(\sqrt{x}\right) + c_2 \cos\left(\sqrt{x}\right)$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 24

```
DSolve[x*y''[x]+1/2*y'[x]+1/4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \cos\left(\sqrt{x}\right) + c_2 \sin\left(\sqrt{x}\right)$$

24.10 problem 750

Internal problem ID [15492]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.2. Expanding a solution in generalized power series. Bessels equation. Exercises page 177
Problem number: 750.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{y'' + \frac{5y'}{x} + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)+5/x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y = \frac{-\operatorname{BesselY}(0,x)\, c_2 x - \operatorname{BesselJ}(0,x)\, c_1 x + 2\operatorname{BesselY}(1,x)\, c_2 + 2\operatorname{BesselJ}(1,x)\, c_1}{x^3}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 22

```
DSolve[y''[x]+5/x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 \operatorname{BesselJ}(2,x) + c_2 \operatorname{BesselY}(2,x)}{x^2}$$

24.11 problem 751

Internal problem ID [15493]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.2. Expanding a solution in generalized power series. Bessels equation. Exercises page 177
Problem number: 751.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{3y'}{x} + 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+3/x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y = \frac{c_1 \operatorname{BesselJ}(1, 2x) + c_2 \operatorname{BesselY}(1, 2x)}{x}$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 26

```
DSolve[y''[x]+3/x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 \operatorname{BesselJ}(1, 2x) + c_2 \operatorname{BesselY}(1, 2x)}{x}$$

25 Chapter 2 (Higher order ODE's). Section 18.3. Finding periodic solutions of linear differential equations. Exercises page 187

25.1 problem 757

Internal problem ID [15494]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.3. Finding periodic solutions of linear differential equations. Exercises page 187
Problem number: 757.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \cos(x)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)+4*y(x)=cos(x)^2,y(x), singsol=all)
```

$$y = \frac{(8c_1 + 1)\cos(2x)}{8} + \frac{1}{8} + \frac{(x + 8c_2)\sin(2x)}{8}$$

✓ Solution by Mathematica
Time used: 0.097 (sec). Leaf size: 33

```
DSolve[y''[x]+4*y[x]==Cos[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}((1 + 8c_1)\cos(2x) + (x + 8c_2)\sin(2x) + 1)$$

25.2 problem 758

Internal problem ID [15495]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.3. Finding periodic solutions of linear differential equations. Exercises page 187
Problem number: 758.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y' + 4y = \pi^2 - x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=Pi^2-x^2,y(x), singsol=all)
```

$$y = -\frac{3}{8} + (c_1 x + c_2)\,\mathrm{e}^{2x} - \frac{x^2}{4} + \frac{\pi^2}{4} - \frac{x}{2}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 42

```
DSolve[y''[x]-4*y'[x]+4*y[x]==Pi^2-x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}\left(-2x^2 - 4x + 2\pi^2 - 3\right) + c_1 e^{2x} + c_2 e^{2x} x$$

25.3 problem 759

Internal problem ID [15496]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.3. Finding periodic solutions of linear differential equations. Exercises page 187
Problem number: 759.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y = \cos(\pi x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 41

```
dsolve(diff(y(x),x$2)-4*y(x)=cos(Pi*x),y(x), singsol=all)
```

$$y = \frac{c_1(\pi^2+4)\,\mathrm{e}^{-2x} + c_2(\pi^2+4)\,\mathrm{e}^{2x} - \cos(\pi x)}{\pi^2+4}$$

✓ Solution by Mathematica
Time used: 0.097 (sec). Leaf size: 35

```
DSolve[y''[x]-4*y[x]==Cos[Pi*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\cos(\pi x)}{4+\pi^2} + c_1 e^{2x} + c_2 e^{-2x}$$

25.4 problem 760

Internal problem ID [15497]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.3. Finding periodic solutions of linear differential equations. Exercises page 187
Problem number: 760.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 4y = \arcsin(\sin(x))}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=arcsin(sin(x)),y(x), singsol=all)
```

$$y = e^{2x}\left(c_2 + c_1 x - \left(\int \arcsin(\sin(x))\, x\, e^{-2x} dx\right) + x\left(\int \arcsin(\sin(x))\, e^{-2x} dx\right)\right)$$

✓ Solution by Mathematica

Time used: 1.363 (sec). Leaf size: 38

```
DSolve[y''[x]-4*y'[x]+4*y[x]==ArcSin[Sin[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(\arcsin(\sin(x)) + 4e^{2x}(c_2 x + c_1) + \sqrt{\cos^2(x)}\sec(x)\right)$$

25.5 problem 761

Internal problem ID [15498]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 2 (Higher order ODE's). Section 18.3. Finding periodic solutions of linear differential equations. Exercises page 187
Problem number: 761.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = \sin(x)^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+9*y(x)=sin(x)^3,y(x), singsol=all)
```

$$y = \frac{(x + 24c_1)\cos(3x)}{24} + \frac{(144c_2 - 1)\sin(3x)}{144} + \frac{3\sin(x)}{32}$$

✓ Solution by Mathematica
Time used: 0.205 (sec). Leaf size: 40

```
DSolve[y''[x]+9*y[x]==Sin[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3\sin(x)}{32} - \frac{1}{144}\sin(3x) + \left(\frac{x}{24} + c_1\right)\cos(3x) + c_2\sin(3x)$$

26 Chapter 3 (Systems of differential equations). Section 19. Basic concepts and definitions. Exercises page 199

26.1 problem 767

Internal problem ID [15499]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 19. Basic concepts and definitions. Exercises page 199
Problem number: 767.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -2tx_1(t)^2$$
$$x_2'(t) = \frac{x_2(t)}{t} + 1$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 23

```
dsolve([diff(x__1(t),t)=-2*t*x__1(t)^2,diff(x__2(t),t)=(x__2(t)+t)/t],singsol=all)
```

$$\left\{x_1(t) = \frac{1}{t^2 + c_2}\right\}$$
$$\{x_2(t) = (\ln(t) + c_1)\, t\}$$

✓ Solution by Mathematica
Time used: 0.143 (sec). Leaf size: 40

```
DSolve[{x1'[t]==-2*t*x1[t]^2,x2'[t]==(x2[t]+t)/t},{x1[t],x2[t]},t,IncludeSingularSolutions -
```

$$\mathrm{x1}(t) \to \frac{1}{t^2 - c_1}$$
$$\mathrm{x2}(t) \to t(\log(t) + c_2)$$
$$\mathrm{x1}(t) \to 0$$
$$\mathrm{x2}(t) \to t(\log(t) + c_2)$$

26.2 problem 768

Internal problem ID [15500]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 19. Basic concepts and definitions. Exercises page 199
Problem number: 768.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = e^t e^{-x_1(t)}$$
$$x_2'(t) = 2\, e^{x_1(t)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 24

```
dsolve([diff(x__1(t),t)=exp(t-x__1(t)),diff(x__2(t),t)=2*exp(x__1(t))],singsol=all)
```

$$\{x_1(t) = \ln\left(e^t + c_2\right)\}$$
$$\{x_2(t) = \int 2\, e^{x_1(t)} dt + c_1\}$$

✓ Solution by Mathematica
Time used: 0.15 (sec). Leaf size: 28

```
DSolve[{x1'[t]==Exp[t-x1[t]],x2'[t]==2*Exp[x1[t]]},{x1[t],x2[t]},t,IncludeSingularSolutions
```

$$\text{x1}(t) \rightarrow \log\left(e^t + c_1\right)$$
$$\text{x2}(t) \rightarrow 2e^t + 2c_1 t + c_2$$

26.3 problem 769

Internal problem ID [15501]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 19. Basic concepts and definitions. Exercises page 199
Problem number: 769.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = y(t)$$
$$y'(t) = \frac{y(t)^2}{x(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve([diff(x(t),t)=y(t),diff(y(t),t)=y(t)^2/x(t)],singsol=all)
```

$$\{x(t) = \mathrm{e}^{c_1 t} c_2\}$$
$$\{y(t) = \frac{d}{dt} x(t)\}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 28

```
DSolve[{x'[t]==y[t],y'[t]==y[t]^2/x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 c_2 e^{c_1 t}$$
$$x(t) \to c_2 e^{c_1 t}$$

26.4 problem 771

Internal problem ID [15502]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 19. Basic concepts and definitions. Exercises page 199
Problem number: 771.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= \frac{x_1(t)^2}{x_2(t)} \\
x_2'(t) &= x_2(t) - x_1(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 66

```
dsolve([diff(x__1(t),t)=x__1(t)^2/x__2(t),diff(x__2(t),t)=x__2(t)-x__1(t)],singsol=all)
```

$$
[\{x_1(t) = 0\}, \{x_2(t) = c_1 \mathrm{e}^t\}]
$$

$$
\left[\left\{x_1(t) = \frac{1}{\sqrt{2\,\mathrm{e}^{-t} c_1 - 2c_2}}, x_1(t) = -\frac{1}{\sqrt{2\,\mathrm{e}^{-t} c_1 - 2c_2}}\right\}, \left\{x_2(t) = \frac{x_1(t)^2}{\frac{d}{dt} x_1(t)}\right\}\right]
$$

✓ Solution by Mathematica

Time used: 0.102 (sec). Leaf size: 143

```
DSolve[{x1'[t]==x1[t]^2/x2[t],x2'[t]==x2[t]-x1[t]},{x1[t],x2[t]},t,IncludeSingularSolutions
```

$$
\begin{aligned}
\mathrm{x2}(t) &\to 2ie^{\frac{t}{2}+c_2}\sqrt{-1+2c_1e^{t+2c_2}} \\
\mathrm{x1}(t) &\to -\frac{ie^{\frac{t}{2}+c_2}}{\sqrt{-1+2c_1e^{t+2c_2}}} \\
\mathrm{x2}(t) &\to -2ie^{\frac{t}{2}+c_2}\sqrt{-1+2c_1e^{t+2c_2}} \\
\mathrm{x1}(t) &\to \frac{ie^{\frac{t}{2}+c_2}}{\sqrt{-1+2c_1e^{t+2c_2}}}
\end{aligned}
$$

26.5 problem 772

Internal problem ID [15503]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 19. Basic concepts and definitions. Exercises page 199
Problem number: 772.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= \frac{\mathrm{e}^{-x(t)}}{t} \\
y'(t) &= \frac{x(t)\,\mathrm{e}^{-y(t)}}{t}
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 26

```
dsolve([diff(x(t),t)=exp(-x(t))/t,diff(y(t),t)=x(t)/t*exp(-y(t))],singsol=all)
```

$$
\begin{aligned}
&\{x(t) = \ln(\ln(t) + c_2)\} \\
&\left\{y(t) = \ln\left(\int \frac{x(t)}{t} dt + c_1\right)\right\}
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.107 (sec). Leaf size: 41

```
DSolve[{x'[t]==Exp[-x[t]],y'[t]==x[t]/t*Exp[-y[t]]},{x[t],y[t]},t,IncludeSingularSolutions -
```

$$
\begin{aligned}
x(t) &\to \log(t + c_1) \\
y(t) &\to \log\left(\text{PolyLog}\left(2, \frac{t}{c_1} + 1\right) + \log\left(-\frac{t}{c_1}\right)\log(t + c_1) + c_2\right)
\end{aligned}
$$

26.6 problem 773

Internal problem ID [15504]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 19. Basic concepts and definitions. Exercises page 199
Problem number: 773.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = \frac{y(t)}{x(t) + y(t)} + \frac{t}{x(t) + y(t)}$$
$$y'(t) = \frac{x(t)}{x(t) + y(t)} - \frac{t}{x(t) + y(t)}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 61

```
dsolve([diff(x(t),t)=(y(t)+t)/(x(t)+y(t)),diff(y(t),t)=(x(t)-t)/(x(t)+y(t))],singsol=all)
```

$$[\{x(t) = t\}, \{y(t) = c_1\}]$$
$$\left[\left\{x(t) = \frac{c_1 t^2 - c_2 t + 1}{c_1 t - c_2}\right\}, \left\{y(t) = \frac{-x(t)\left(\frac{d}{dt}x(t)\right) + t}{\frac{d}{dt}x(t) - 1}\right\}\right]$$

✓ Solution by Mathematica

Time used: 67.434 (sec). Leaf size: 45

```
DSolve[{x'[t]==(y[t]+t)/(x[t]+y[t]),y'[t]==(x[t]-t)/(x[t]+y[t])},{x[t],y[t]},t,IncludeSingul
```

$$x(t) \to \frac{t^2 + c_1 t + c_2}{t + c_1}$$
$$y(t) \to \frac{c_1 t + c_1^2 - c_2}{t + c_1}$$

26.7 problem 774

Internal problem ID [15505]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 19. Basic concepts and definitions. Exercises page 199
Problem number: 774.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= \frac{t}{y(t)-x(t)} - \frac{y(t)}{y(t)-x(t)} \\
y'(t) &= \frac{x(t)}{y(t)-x(t)} - \frac{t}{y(t)-x(t)}
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.422 (sec). Leaf size: 132

```
dsolve([diff(x(t),t)=(t-y(t))/(y(t)-x(t)),diff(y(t),t)=(x(t)-t)/(y(t)-x(t))],singsol=all)
```

$$
\left\{x(t) = t + \mathrm{RootOf}\left(-t + \int^{\_Z} -\frac{2\left(e^{c_1}\_f^2 - 1\right)}{-4 + 3\,e^{c_1}\_f^2 - \sqrt{-3\,e^{c_1}\_f^2 + 4}\,e^{\frac{c_1}{2}}\_f} d\_f + c_2\right), x(t) = t + \mathrm{RootOf}\left(-t + \int^{\_Z} -\frac{2\left(e^{c_1}\_f^2 - 1\right)}{3\,e^{c_1}\_f^2 + \sqrt{-3\,e^{c_1}\_f^2 + 4}\,e^{\frac{c_1}{2}}\_f - 4} d\_f + c_2\right)\right\}
$$

$$
\left\{y(t) = \frac{x(t)\left(\frac{d}{dt}x(t)\right) + t}{\frac{d}{dt}x(t) + 1}\right\}
$$

✓ Solution by Mathematica

Time used: 14.351 (sec). Leaf size: 151

```
DSolve[{x'[t]==(t-y[t])/(y[t]-x[t]),y'[t]==(x[t]-t)/(y[t]-x[t])},{x[t],y[t]},t,IncludeSingul
```

$$
\begin{aligned}
x(t) &\to \frac{1}{2}\left(-\sqrt{-3t^2+2c_1t+c_1{}^2+4c_2}-t+c_1\right) \\
y(t) &\to \frac{1}{2}\left(\sqrt{-3t^2+2c_1t+c_1{}^2+4c_2}-t+c_1\right) \\
x(t) &\to \frac{1}{2}\left(\sqrt{-3t^2+2c_1t+c_1{}^2+4c_2}-t+c_1\right) \\
y(t) &\to \frac{1}{2}\left(-\sqrt{-3t^2+2c_1t+c_1{}^2+4c_2}-t+c_1\right)
\end{aligned}
$$

26.8 problem 775

Internal problem ID [15506]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 19. Basic concepts and definitions. Exercises page 199
Problem number: 775.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= \frac{y(t)}{x(t)+y(t)} + \frac{t}{x(t)+y(t)} \\
y'(t) &= \frac{t}{x(t)+y(t)} + \frac{x(t)}{x(t)+y(t)}
\end{aligned}
$$

✓ Solution by Maple

Time used: 1.656 (sec). Leaf size: 3853

```
dsolve([diff(x(t),t)=(t+y(t))/(y(t)+x(t)),diff(y(t),t)=(t+x(t))/(y(t)+x(t))],singsol=all)
```

Expression too large to display
Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x'[t]==(t+y[t])/(y[t]+x[t]),y'[t]==(x[t]+t)/(y[t]+x[t])},{x[t],y[t]},t,IncludeSingul
```

Not solved

27 Chapter 3 (Systems of differential equations). Section 20. The method of elimination. Exercises page 212

27.1 problem 776

Internal problem ID [15507]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 20. The method of elimination. Exercises page 212
Problem number: 776.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -9y(t)$$
$$y'(t) = x(t)$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=-9*y(t),diff(y(t),t)=x(t)],singsol=all)
```

$$x(t) = c_1 \sin(3t) + c_2 \cos(3t)$$
$$y(t) = -\frac{c_1 \cos(3t)}{3} + \frac{c_2 \sin(3t)}{3}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 42

```
DSolve[{x'[t]==-9*y[t],y'[t]==x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 \cos(3t) - 3c_2 \sin(3t)$$
$$y(t) \to c_2 \cos(3t) + \frac{1}{3} c_1 \sin(3t)$$

27.2 problem 777

Internal problem ID [15508]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 20. The method of elimination. Exercises page 212
Problem number: 777.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= y(t) + t \\
y'(t) &= x(t) - t
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 37

```
dsolve([diff(x(t),t)=y(t)+t,diff(y(t),t)=x(t)-t],singsol=all)
```

$$
\begin{aligned}
x(t) &= c_2 e^t + e^{-t} c_1 + t - 1 \\
y(t) &= c_2 e^t - e^{-t} c_1 + 1 - t
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 78

```
DSolve[{x'[t]==y[t]+t,y'[t]==x[t]-t},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
x(t) &\to \frac{1}{2}e^{-t}\left(2e^t(t-1) + (c_1 + c_2)e^{2t} + c_1 - c_2\right) \\
y(t) &\to \frac{1}{2}e^{-t}\left(-2e^t(t-1) + (c_1 + c_2)e^{2t} - c_1 + c_2\right)
\end{aligned}
$$

27.3 problem 778

Internal problem ID [15509]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 20. The method of elimination. Exercises page 212
Problem number: 778.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -3x(t) - 4y(t)$$
$$y'(t) = -2x(t) - 5y(t)$$

With initial conditions

$$[x(0) = 1, y(0) = 4]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 32

```
dsolve([diff(x(t),t)+3*x(t)+4*y(t) = 0, diff(y(t),t)+2*x(t)+5*y(t) = 0, x(0) = 1, y(0) = 4],
```

$$x(t) = 3\,\mathrm{e}^{-7t} - 2\,\mathrm{e}^{-t}$$
$$y(t) = 3\,\mathrm{e}^{-7t} + \mathrm{e}^{-t}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 36

```
DSolve[{x'[t]+3*x[t]+4*y[t]==0,y'[t]+2*x[t]+5*y[t]==0},{x[0]==1,y[0]==4},{x[t],y[t]},t,Inclu
```

$$x(t) \to e^{-7t}(3 - 2e^{6t})$$
$$y(t) \to e^{-7t}(e^{6t} + 3)$$

27.4 problem 779

Internal problem ID [15510]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 20. The method of elimination. Exercises page 212
Problem number: 779.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) + 5y(t)$$
$$y'(t) = -x(t) - 3y(t)$$

With initial conditions

$$[x(0) = -2, y(0) = 1]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 27

```
dsolve([diff(x(t),t) = x(t)+5*y(t), diff(y(t),t) = -x(t)-3*y(t), x(0) = -2, y(0) = 1], sings
```

$$x(t) = e^{-t}(\sin(t) - 2\cos(t))$$
$$y(t) = e^{-t}\cos(t)$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 20

```
DSolve[{x'[t]+3*x[t]+4*y[t]==0,y'[t]+2*x[t]+5*y[t]==0},{x[0]==-2,y[0]==1},{x[t],y[t]},t,Incl
```

$$x(t) \to -2e^{-t}$$
$$y(t) \to e^{-t}$$

27.5 problem 780

Internal problem ID [15511]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 20. The method of elimination. Exercises page 212
Problem number: 780.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -y(t) + \cos(t)$$
$$y'(t) = -4y(t) + 4\cos(t) + 3x(t) - \sin(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 37

```
dsolve([4*diff(x(t),t)-diff(y(t),t)+3*x(t)=sin(t),diff(x(t),t)+y(t)=cos(t)],singsol=all)
```

$$x(t) = \frac{c_2 e^{-3t}}{3} + e^{-t} c_1$$
$$y(t) = c_2 e^{-3t} + e^{-t} c_1 + \cos(t)$$

✓ Solution by Mathematica

Time used: 0.218 (sec). Leaf size: 76

```
DSolve[{4*x'[t]-y'[t]+3*x[t]==Sin[t],x'[t]+y[t]==Cos[t]},{x[t],y[t]},t,IncludeSingularSoluti
```

$$x(t) \to \frac{1}{2} e^{-3t} \left(c_1 \left(3e^{2t} - 1\right) - c_2 \left(e^{2t} - 1\right)\right)$$
$$y(t) \to \cos(t) + \frac{1}{2} e^{-3t} \left(3c_1 \left(e^{2t} - 1\right) - c_2 \left(e^{2t} - 3\right)\right)$$

27.6 problem 781

Internal problem ID [15512]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 20. The method of elimination. Exercises page 212
Problem number: 781.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= -y(t) + z(t) \\
y'(t) &= z(t) \\
z'(t) &= -x(t) + z(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 56

```
dsolve([diff(x(t),t)=-y(t)+z(t),diff(y(t),t)=z(t),diff(z(t),t)=-x(t)+z(t)],singsol=all)
```

$$
\begin{aligned}
x(t) &= c_2 \sin(t) + c_3 \sin(t) - c_2 \cos(t) + c_3 \cos(t) \\
y(t) &= c_1 \mathrm{e}^t - c_2 \cos(t) + c_3 \sin(t) \\
z(t) &= c_1 \mathrm{e}^t + c_2 \sin(t) + c_3 \cos(t)
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 112

```
DSolve[{x'[t]==-y[t]+z[t],y'[t]==z[t],z'[t]==-x[t]+z[t]},{x[t],y[t],z[t]},t,IncludeSingularS
```

$$
\begin{aligned}
x(t) &\to c_1 \cos(t) + (c_3 - c_2) \sin(t) \\
y(t) &\to \frac{1}{2}\left((-c_1 + c_2 + c_3)e^t + (c_1 + c_2 - c_3)\cos(t) + (c_1 - c_2 + c_3)\sin(t)\right) \\
z(t) &\to \frac{1}{2}\left((-c_1 + c_2 + c_3)e^t + (c_1 - c_2 + c_3)\cos(t) - (c_1 + c_2 - c_3)\sin(t)\right)
\end{aligned}
$$

27.7 problem 782

Internal problem ID [15513]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 20. The method of elimination. Exercises page 212
Problem number: 782.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= y(t) + z(t)\\
y'(t) &= x(t) + z(t)\\
z'(t) &= x(t) + y(t)
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 64

```
dsolve([diff(x(t),t)=y(t)+z(t),diff(y(t),t)=x(t)+z(t),diff(z(t),t)=x(t)+y(t)],singsol=all)
```

$$
\begin{aligned}
x(t) &= c_2 e^{2t} + c_3 e^{-t}\\
y(t) &= c_2 e^{2t} + c_3 e^{-t} + e^{-t} c_1\\
z(t) &= c_2 e^{2t} - 2 c_3 e^{-t} - e^{-t} c_1
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 124

```
DSolve[{x'[t]==y[t]+z[t],y'[t]==x[t]+z[t],z'[t]==x[t]+y[t]},{x[t],y[t],z[t]},t,IncludeSingul
```

$$
\begin{aligned}
x(t) &\to \frac{1}{3} e^{-t} \left(c_1 \left(e^{3t} + 2\right) + (c_2 + c_3) \left(e^{3t} - 1\right)\right)\\
y(t) &\to \frac{1}{3} e^{-t} \left(c_1 \left(e^{3t} - 1\right) + c_2 \left(e^{3t} + 2\right) + c_3 \left(e^{3t} - 1\right)\right)\\
z(t) &\to \frac{1}{3} e^{-t} \left(c_1 \left(e^{3t} - 1\right) + c_2 \left(e^{3t} - 1\right) + c_3 \left(e^{3t} + 2\right)\right)
\end{aligned}
$$

27.8 problem 783

Internal problem ID [15514]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 20. The method of elimination. Exercises page 212
Problem number: 783.
ODE order: 2.
ODE degree: 1.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 48

```
dsolve([diff(x(t),t$2)=y(t),diff(y(t),t$2)=x(t)],singsol=all)
```

$$x(t) = c_1 e^t + c_2 e^{-t} - c_3 \sin(t) - c_4 \cos(t)$$
$$y(t) = c_1 e^t + c_2 e^{-t} + c_3 \sin(t) + c_4 \cos(t)$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 172

```
DSolve[{x''[t]==y[t],y''[t]==x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{4}e^{-t}\left(c_1e^{2t} + c_2e^{2t} + c_3e^{2t} + c_4e^{2t} + 2(c_1 - c_3)e^t\cos(t) + 2(c_2 - c_4)e^t\sin(t) + c_1 - c_2 + c_3 - c_4\right)$$
$$y(t) \to \frac{1}{4}e^{-t}\left(c_1e^{2t} + c_2e^{2t} + c_3e^{2t} + c_4e^{2t} - 2(c_1 - c_3)e^t\cos(t) - 2(c_2 - c_4)e^t\sin(t) + c_1 - c_2 + c_3 - c_4\right)$$

27.9 problem 784

Internal problem ID [15515]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 20. The method of elimination. Exercises page 212
Problem number: 784.
ODE order: 2.
ODE degree: 1.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 40

```
dsolve([diff(x(t),t$2)+diff(y(t),t)+x(t)=0,diff(x(t),t)+diff(y(t),t$2)=0],singsol=all)
```

$$x(t) = c_1 - \frac{1}{2}t^2c_1 - c_2t - c_3$$
$$y(t) = \frac{1}{6}t^3c_1 + \frac{1}{2}c_2t^2 + c_3t + c_4$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 61

```
DSolve[{x''[t]+y'[t]+x[t]==0,x'[t]+y''[t]==0},{x[t],y[t]},t,IncludeSingularSolutions -> True
```

$$x(t) \to -\frac{c_1t^2}{2} - \frac{c_4t^2}{2} + c_2t + c_1$$
$$y(t) \to \frac{1}{6}(c_1 + c_4)t^3 - \frac{c_2t^2}{2} + c_4t + c_3$$

27.10 problem 785

Internal problem ID [15516]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 20. The method of elimination. Exercises page 212
Problem number: 785.
ODE order: 2.
ODE degree: 1.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 47

```
dsolve([diff(x(t),t$2)=3*x(t)+y(t),diff(y(t),t)=-2*x(t)],singsol=all)
```

$$x(t) = c_1 e^{-2t} - \frac{c_2 e^t}{2} - \frac{c_3 e^t t}{2} - \frac{c_3 e^t}{2}$$
$$y(t) = c_1 e^{-2t} + c_2 e^t + c_3 e^t t$$

✓ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 125

```
DSolve[{x''[t]==3*x[t]+y[t],y'[t]==-2*x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{9} e^{-2t} \left(c_1 \left(e^{3t}(3t+5)+4\right) + c_2 \left(e^{3t}(3t+2)-2\right) + c_3 \left(e^{3t}(3t-1)+1\right)\right)$$
$$y(t) \to \frac{1}{9} e^{-2t} \left(c_1 \left(4-2e^{3t}(3t+2)\right) + c_2 \left(e^{3t}(2-6t)-2\right) + c_3 \left(e^{3t}(8-6t)+1\right)\right)$$

27.11 problem 786

Internal problem ID [15517]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 20. The method of elimination. Exercises page 212
Problem number: 786.
ODE order: 2.
ODE degree: 1.

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 19

```
dsolve([diff(diff(x(t),t),t) = x(t)^2+y(t), diff(y(t),t) = -2*x(t)*diff(x(t),t)+x(t), x(0) =
```

$$x(t) = e^t$$
$$y(t) = -e^{2t} + e^t$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x''[t]==x[t]^2+y[t],y'[t]==-2*x[t]*x'[t]+x[t]},{x[0]==1,x'[0]==1,y[0]==0},{x[t],y[t]
```

Not solved

28 Chapter 3 (Systems of differential equations). Section 21. Finding integrable combinations. Exercises page 219

28.1 problem 787

Internal problem ID [15518]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 21. Finding integrable combinations. Exercises page 219
Problem number: 787.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t)^2 + y(t)^2$$
$$y'(t) = 2x(t)\,y(t)$$

✓ Solution by Maple
Time used: 0.36 (sec). Leaf size: 65

```
dsolve([diff(x(t),t)=x(t)^2+y(t)^2,diff(y(t),t)=2*x(t)*y(t)],singsol=all)
```

$$\left[\{y(t) = 0\}, \left\{x(t) = \frac{1}{-t + c_1}\right\}\right]$$
$$\left[\left\{y(t) = \frac{4c_1}{c_1^2 c_2^2 + 2c_1^2 c_2 t + c_1^2 t^2 - 16}\right\}, \left\{x(t) = \frac{\frac{d}{dt}y(t)}{2y(t)}\right\}\right]$$

✓ Solution by Mathematica
Time used: 41.052 (sec). Leaf size: 3516

```
DSolve[{x'[t]==x[t]^2+y[t]^2,y'[t]==-2*x[t]*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

Too large to display

28.2 problem 788

Internal problem ID [15519]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 21. Finding integrable combinations. Exercises page 219
Problem number: 788.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= -\frac{1}{y(t)} \\
y'(t) &= \frac{1}{x(t)}
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 24

```
dsolve([diff(x(t),t)=-1/y(t),diff(y(t),t)=1/x(t)],singsol=all)
```

$$
\begin{gathered}
\{x(t) = \mathrm{e}^{c_1 t} c_2\} \\
\left\{y(t) = -\frac{1}{\frac{d}{dt}x(t)}\right\}
\end{gathered}
$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 35

```
DSolve[{x'[t]==-1/y[t],y'[t]==1/x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(t) &\to \frac{c_1 e^{\frac{t}{c_1}}}{c_2} \\
x(t) &\to c_2 e^{-\frac{t}{c_1}}
\end{aligned}
$$

28.3 problem 789

Internal problem ID [15520]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 21. Finding integrable combinations. Exercises page 219
Problem number: 789.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = \frac{x(t)}{y(t)}$$
$$y'(t) = \frac{y(t)}{x(t)}$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 34

```
dsolve([diff(x(t),t)=x(t)/y(t),diff(y(t),t)=y(t)/x(t)],singsol=all)
```

$$\left\{x(t) = \frac{-1 + e^{c_2 c_1} e^{c_1 t}}{c_1}\right\}$$
$$\left\{y(t) = \frac{x(t)}{\frac{d}{dt} x(t)}\right\}$$

✓ Solution by Mathematica
Time used: 0.076 (sec). Leaf size: 45

```
DSolve[{x'[t]==x[t]/y[t],y'[t]==y[t]/x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{e^{c_1 t} + c_1 c_2}{c_1{}^2 c_2}$$
$$x(t) \to c_2 e^{c_1(-t)} + \frac{1}{c_1}$$

28.4 problem 790

Internal problem ID [15521]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 21. Finding integrable combinations. Exercises page 219
Problem number: 790.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -\frac{y(t)}{y(t) - x(t)}$$
$$y'(t) = -\frac{x(t)}{y(t) - x(t)}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 48

```
dsolve([diff(x(t),t)=y(t)/(x(t)-y(t)),diff(y(t),t)=x(t)/(x(t)-y(t))],singsol=all)
```

$$\left\{x(t) = \frac{-c_1 t^2 - 2c_2 t - 2}{2c_1 t + 2c_2}\right\}$$
$$\left\{y(t) = \frac{x(t)\left(\frac{d}{dt}x(t)\right)}{\frac{d}{dt}x(t) + 1}\right\}$$

✓ Solution by Mathematica

Time used: 0.106 (sec). Leaf size: 145

```
DSolve[{x'[t]==y[t]/(x[t]-y[t]),y'[t]==x[t]/(x[t]-y[t])},{x[t],y[t]},t,IncludeSingularSoluti
```

$$y(t) \to -\frac{1}{2}\sqrt{\frac{(t^2 - 2c_2t + c_2{}^2 + 2c_1)^2}{(t - c_2)^2}}$$

$$x(t) \to -\frac{t^2 - 2c_2t + c_2{}^2 - 2c_1}{2t - 2c_2}$$

$$y(t) \to \frac{1}{2}\sqrt{\frac{(t^2 - 2c_2t + c_2{}^2 + 2c_1)^2}{(t - c_2)^2}}$$

$$x(t) \to -\frac{t^2 - 2c_2t + c_2{}^2 - 2c_1}{2t - 2c_2}$$

28.5 problem 791

Internal problem ID [15522]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 21. Finding integrable combinations. Exercises page 219
Problem number: 791.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = \sin(x(t))\cos(y(t))$$
$$y'(t) = \cos(x(t))\sin(y(t))$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 38

```
dsolve([diff(x(t),t)=sin(x(t))*cos(y(t)),diff(y(t),t)=cos(x(t))*sin(y(t))],singsol=all)
```

$$\left\{y(t) = \operatorname{arccot}\left(\frac{(c_1 e^{2t} - c_2)\, e^{-t}}{2}\right)\right\}$$
$$\left\{x(t) = \arccos\left(\frac{\frac{d}{dt}y(t)}{\sin(y(t))}\right)\right\}$$

✓ Solution by Mathematica
Time used: 0.492 (sec). Leaf size: 121

```
DSolve[{x'[t]==Sin[x[t]]*Cos[y[t]],y'[t]==Cos[x[t]]*Sin[y[t]]},{x[t],y[t]},t,IncludeSingular
```

$$y(t) \to \arcsin\left(e^{c_1}\sin\left(\text{InverseFunction}\left[-\operatorname{arctanh}\left(\frac{\sqrt{2}\cos(\#1)}{\sqrt{-e^{2c_1}\cos\left(2\left(\frac{\pi}{2}-\#1\right)\right)+2-e^{2c_1}}}\right)\&\right][t+c_2]\right)\right)$$

$$x(t) \to \text{InverseFunction}\left[-\operatorname{arctanh}\left(\frac{\sqrt{2}\cos(\#1)}{\sqrt{-e^{2c_1}\cos\left(2\left(\frac{\pi}{2}-\#1\right)\right)+2-e^{2c_1}}}\right)\&\right][t+c_2]$$

28.6 problem 792

Internal problem ID [15523]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 21. Finding integrable combinations. Exercises page 219
Problem number: 792.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = \frac{e^{-t}}{y(t)}$$
$$y'(t) = \frac{e^{-t}}{x(t)}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 52

```
dsolve([exp(t)*diff(x(t),t)=1/y(t),exp(t)*diff(y(t),t)=1/x(t)],singsol=all)
```

$$\left\{x(t) = \sqrt{-2\,e^{-t}c_1 + 2c_2}, x(t) = -\sqrt{-2\,e^{-t}c_1 + 2c_2}\right\}$$
$$\left\{y(t) = \frac{e^{-t}}{\frac{d}{dt}x(t)}\right\}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 125

```
DSolve[{Exp[t]*x'[t]==1/y[t],Exp[t]*y'[t]==1/x[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$y(t) \to -\sqrt{2}\sqrt{c_1}\sqrt{-e^{-t} + c_1c_2}$$
$$x(t) \to -\frac{\sqrt{-2e^{-t} + 2c_1c_2}}{\sqrt{c_1}}$$
$$y(t) \to \sqrt{2}\sqrt{c_1}\sqrt{-e^{-t} + c_1c_2}$$
$$x(t) \to \frac{\sqrt{-2e^{-t} + 2c_1c_2}}{\sqrt{c_1}}$$

28.7 problem 793

Internal problem ID [15524]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 21. Finding integrable combinations. Exercises page 219
Problem number: 793.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = \cos(x(t))^2 \cos(y(t))^2 + \sin(x(t))^2 \cos(y(t))^2$$
$$y'(t) = -2\sin(x(t))\cos(x(t))\sin(y(t))\cos(y(t))$$

With initial conditions

$$[x(0) = 0, y(0) = 0]$$

✗ Solution by Maple

```
dsolve([diff(x(t),t) = cos(x(t))^2*cos(y(t))^2+sin(x(t))^2*cos(y(t))^2, diff(y(t),t) = -1/2*
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x'[t]==Cos[x[t]]^2*Cos[y[t]]^2+Sin[x[t]]^2*Cos[y[t]]^2,y'[t]==-1/2*Sin[2*x[t]]*Sin[2
```

{}

29 Chapter 3 (Systems of differential equations). Section 22. Integration of homogeneous linear systems with constant coefficients. Eulers method. Exercises page 230

29.1 problem 802

Internal problem ID [15525]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 22. Integration of homogeneous linear systems with constant coefficients. Eulers method. Exercises page 230
Problem number: 802.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 8y(t) - x(t)$$
$$y'(t) = x(t) + y(t)$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=8*y(t)-x(t),diff(y(t),t)=x(t)+y(t)],singsol=all)
```

$$x(t) = c_1 e^{3t} + c_2 e^{-3t}$$
$$y(t) = \frac{c_1 e^{3t}}{2} - \frac{c_2 e^{-3t}}{4}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 72

```
DSolve[{x'[t]==8*y[t]-x[t],y'[t]==x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{3} e^{-3t} \left(c_1 \left(e^{6t} + 2\right) + 4c_2 \left(e^{6t} - 1\right)\right)$$
$$y(t) \to \frac{1}{6} e^{-3t} \left(c_1 \left(e^{6t} - 1\right) + 2c_2 \left(2e^{6t} + 1\right)\right)$$

29.2 problem 803

Internal problem ID [15526]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 22. Integration of homogeneous linear systems with constant coefficients. Eulers method. Exercises page 230
Problem number: 803.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) - y(t)$$
$$y'(t) = y(t) - x(t)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 25

```
dsolve([diff(x(t),t)=x(t)-y(t),diff(y(t),t)=y(t)-x(t)],singsol=all)
```

$$x(t) = c_1 + c_2 e^{2t}$$
$$y(t) = -c_2 e^{2t} + c_1$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 60

```
DSolve[{x'[t]==x[t]-y[t],y'[t]==y[t]-x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{2}\left(c_1 e^{2t} - c_2 e^{2t} + c_1 + c_2\right)$$
$$y(t) \to \frac{1}{2}\left(c_1\left(-e^{2t}\right) + c_2 e^{2t} + c_1 + c_2\right)$$

29.3 problem 804

Internal problem ID [15527]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 22. Integration of homogeneous linear systems with constant coefficients. Eulers method. Exercises page 230
Problem number: 804.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 2x(t) + y(t)$$
$$y'(t) = x(t) - 3y(t)$$

With initial conditions

$$[x(0) = 0, y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve([diff(x(t),t) = 2*x(t)+y(t), diff(y(t),t) = x(t)-3*y(t), x(0) = 0, y(0) = 0], singsol
```

$$x(t) = 0$$
$$y(t) = 0$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 10

```
DSolve[{x'[t]==2*x[t]+y[t],y'[t]==x[t]-3*y[t]},{x[0]==0,y[0]==0},{x[t],y[t]},t,IncludeSingul
```

$$x(t) \to 0$$
$$y(t) \to 0$$

29.4 problem 805

Internal problem ID [15528]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 22. Integration of homogeneous linear systems with constant coefficients. Eulers method. Exercises page 230
Problem number: 805.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) + y(t)$$
$$y'(t) = 4y(t) - 2x(t)$$

With initial conditions

$$[x(0) = 0, y(0) = -1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 30

```
dsolve([diff(x(t),t) = x(t)+y(t), diff(y(t),t) = 4*y(t)-2*x(t), x(0) = 0, y(0) = -1], singso
```

$$x(t) = -e^{3t} + e^{2t}$$
$$y(t) = -2\,e^{3t} + e^{2t}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 33

```
DSolve[{x'[t]==x[t]+y[t],y'[t]==4*y[t]-2*x[t]},{x[0]==0,y[0]==-1},{x[t],y[t]},t,IncludeSingu
```

$$x(t) \to -e^{2t}(e^t - 1)$$
$$y(t) \to e^{2t} - 2e^{3t}$$

29.5 problem 806

Internal problem ID [15529]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 22. Integration of homogeneous linear systems with constant coefficients. Eulers method. Exercises page 230
Problem number: 806.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 4x(t) - 5y(t)$$
$$y'(t) = x(t)$$

With initial conditions

$$[x(0) = 0, y(0) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 28

```
dsolve([diff(x(t),t) = 4*x(t)-5*y(t), diff(y(t),t) = x(t), x(0) = 0, y(0) = 1], singsol=all)
```

$$x(t) = -5\,e^{2t}\sin(t)$$
$$y(t) = e^{2t}(-2\sin(t) + \cos(t))$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 27

```
DSolve[{x'[t]==4*x[t]-4*y[t],y'[t]==x[t]},{x[0]==0,y[0]==1},{x[t],y[t]},t,IncludeSingularSol
```

$$x(t) \to -4e^{2t}t$$
$$y(t) \to e^{2t}(1 - 2t)$$

29.6 problem 807

Internal problem ID [15530]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 22. Integration of homogeneous linear systems with constant coefficients. Eulers method. Exercises page 230
Problem number: 807.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= -x(t) + y(t) + z(t) \\
y'(t) &= x(t) - y(t) + z(t) \\
z'(t) &= x(t) + y(t) - z(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 58

```
dsolve([diff(x(t),t)=-x(t)+y(t)+z(t),diff(y(t),t)=x(t)-y(t)+z(t),diff(z(t),t)=x(t)+y(t)-z(t)
```

$$
\begin{aligned}
x(t) &= c_2 e^{t} + c_3 e^{-2t} \\
y(t) &= c_2 e^{t} + c_3 e^{-2t} + c_1 e^{-2t} \\
z(t) &= c_2 e^{t} - 2c_3 e^{-2t} - c_1 e^{-2t}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 124

```
DSolve[{x'[t]==-x[t]+y[t]+z[t],y'[t]==x[t]-y[t]+z[t],z'[t]==x[t]+y[t]-z[t]},{x[t],y[t],z[t]}
```

$$
\begin{aligned}
x(t) &\to \frac{1}{3}e^{-2t}\left(c_1\left(e^{3t}+2\right)+\left(c_2+c_3\right)\left(e^{3t}-1\right)\right) \\
y(t) &\to \frac{1}{3}e^{-2t}\left(c_1\left(e^{3t}-1\right)+c_2\left(e^{3t}+2\right)+c_3\left(e^{3t}-1\right)\right) \\
z(t) &\to \frac{1}{3}e^{-2t}\left(c_1\left(e^{3t}-1\right)+c_2\left(e^{3t}-1\right)+c_3\left(e^{3t}+2\right)\right)
\end{aligned}
$$

29.7 problem 808

Internal problem ID [15531]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 22. Integration of homogeneous linear systems with constant coefficients. Eulers method. Exercises page 230
Problem number: 808.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 2x(t) - y(t) + z(t)$$
$$y'(t) = x(t) + 2y(t) - z(t)$$
$$z'(t) = x(t) - y(t) + 2z(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 52

```
dsolve([diff(x(t),t)=2*x(t)-y(t)+z(t),diff(y(t),t)=x(t)+2*y(t)-z(t),diff(z(t),t)=x(t)-y(t)+2
```

$$x(t) = c_2 e^{3t} + c_3 e^{2t}$$
$$y(t) = c_3 e^{2t} + c_1 e^{t}$$
$$z(t) = c_3 e^{2t} + c_2 e^{3t} + c_1 e^{t}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 99

```
DSolve[{x'[t]==2*x[t]-y[t]+z[t],y'[t]==x[t]+2*y[t]-z[t],z'[t]==x[t]-y[t]+2*z[t]},{x[t],y[t],
```

$$x(t) \to e^{2t}(c_1 - (c_2 - c_3)(e^t - 1))$$
$$y(t) \to e^t(c_1(e^t - 1) + (c_2 - c_3)e^t + c_3)$$
$$z(t) \to e^t(c_1(e^t - 1) + (c_2 - c_3)e^t + (c_3 - c_2)e^{2t} + c_3)$$

29.8 problem 809

Internal problem ID [15532]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 22. Integration of homogeneous linear systems with constant coefficients. Eulers method. Exercises page 230
Problem number: 809.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= 2x(t) - y(t) + z(t) \\
y'(t) &= x(t) + z(t) \\
z'(t) &= y(t) - 2z(t) - 3x(t)
\end{aligned}
$$

With initial conditions

$$[x(0) = 0, y(0) = 0, z(0) = 1]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 35

```
dsolve([diff(x(t),t) = 2*x(t)-y(t)+z(t), diff(y(t),t) = x(t)+z(t), diff(z(t),t) = y(t)-2*z(t
```

$$
\begin{aligned}
x(t) &= 1 - e^{-t} \\
y(t) &= 1 - e^{-t} \\
z(t) &= 2\,e^{-t} - 1
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 38

```
DSolve[{x'[t]==2*x[t]-y[t]+z[t],y'[t]==x[t]+z[t],z'[t]==y[t]-2*z[t]-3*x[t]},{x[0]==0,y[0]==0
```

$$
\begin{aligned}
x(t) &\to 1 - e^{-t} \\
y(t) &\to 1 - e^{-t} \\
z(t) &\to 2e^{-t} - 1
\end{aligned}
$$

30 Chapter 3 (Systems of differential equations). Section 23. Methods of integrating nonhomogeneous linear systems with constant coefficients. Exercises page 234

30.1 problem 810

Internal problem ID [15533]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 23. Methods of integrating nonhomogeneous linear systems with constant coefficients. Exercises page 234
Problem number: 810.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -2x(t) + y(t) - e^{2t}$$
$$y'(t) = -3x(t) + 2y(t) + 6\,e^{2t}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 43

```
dsolve([diff(x(t),t)+2*x(t)-y(t)=-exp(2*t),diff(y(t),t)+3*x(t)-2*y(t)=6*exp(2*t)],singsol=al
```

$$x(t) = c_2 e^{t} + e^{-t} c_1 + 2\,e^{2t}$$
$$y(t) = 3c_2 e^{t} + e^{-t} c_1 + 9\,e^{2t}$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 85

```
DSolve[{x'[t]+2*x[t]-y[t]==-Exp[2*t],y'[t]+3*x[t]-2*y[t]==6*Exp[2*t]},{x[t],y[t]},t,IncludeS
```

$$x(t) \to \frac{1}{2}e^{-t}\left(4e^{3t} + (c_2 - c_1)e^{2t} + 3c_1 - c_2\right)$$
$$y(t) \to \frac{1}{2}e^{-t}\left(18e^{3t} - 3(c_1 - c_2)e^{2t} + 3c_1 - c_2\right)$$

30.2 problem 811

Internal problem ID [15534]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 23. Methods of integrating nonhomogeneous linear systems with constant coefficients. Exercises page 234
Problem number: 811.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) + y(t) - \cos(t)$$
$$y'(t) = -y(t) - 2x(t) + \cos(t) + \sin(t)$$

With initial conditions

$$[x(0) = 1, y(0) = -2]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 33

```
dsolve([diff(x(t),t) = x(t)+y(t)-cos(t), diff(y(t),t) = -y(t)-2*x(t)+cos(t)+sin(t), x(0) = 1
```

$$x(t) = -\sin(t) + \cos(t) - \cos(t)\, t$$
$$y(t) = -2\cos(t) + \sin(t)\, t + \cos(t)\, t$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 31

```
DSolve[{x'[t]==x[t]+y[t]-Cos[t],y'[t]==-y[t]-2*x[t]+Cos[t]+Sin[t]},{x[0]==1,y[0]==-2},{x[t],
```

$$x(t) \to -\sin(t) - t\cos(t) + \cos(t)$$
$$y(t) \to t\sin(t) + (t-2)\cos(t)$$

30.3 problem 812

Internal problem ID [15535]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 23. Methods of integrating nonhomogeneous linear systems with constant coefficients. Exercises page 234
Problem number: 812.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = y(t) + \tan(t)^2 - 1$$
$$y'(t) = \tan(t) - x(t)$$

✓ Solution by Maple
Time used: 0.14 (sec). Leaf size: 30

```
dsolve([diff(x(t),t)=y(t)+tan(t)^2-1,diff(y(t),t)=tan(t)-x(t)],singsol=all)
```

$$x(t) = c_2 \sin(t) + c_1 \cos(t) + \tan(t)$$
$$y(t) = c_2 \cos(t) - c_1 \sin(t) + 2$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 34

```
DSolve[{x'[t]==y[t]+Tan[t]^2-1,y'[t]==Tan[t]-x[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to \tan(t) + c_1 \cos(t) + c_2 \sin(t)$$
$$y(t) \to c_2 \cos(t) - c_1 \sin(t) + 2$$

30.4 problem 813

Internal problem ID [15536]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 23. Methods of integrating nonhomogeneous linear systems with constant coefficients. Exercises page 234
Problem number: 813.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= -\frac{4x(t)\,\mathrm{e}^t}{\mathrm{e}^t-1} - \frac{2y(t)\,\mathrm{e}^t}{\mathrm{e}^t-1} + \frac{4x(t)}{\mathrm{e}^t-1} + \frac{2y(t)}{\mathrm{e}^t-1} + \frac{2}{\mathrm{e}^t-1} \\
y'(t) &= \frac{6x(t)\,\mathrm{e}^t}{\mathrm{e}^t-1} + \frac{3y(t)\,\mathrm{e}^t}{\mathrm{e}^t-1} - \frac{6x(t)}{\mathrm{e}^t-1} - \frac{3y(t)}{\mathrm{e}^t-1} - \frac{3}{\mathrm{e}^t-1}
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 86

```
dsolve([diff(x(t),t)=-4*x(t)-2*y(t)+2/(exp(t)-1),diff(y(t),t)=6*x(t)+3*y(t)-3/(exp(t)-1)],si
```

$$
\begin{aligned}
x(t) &= 2\,\mathrm{e}^{-t}\ln\left(\mathrm{e}^t-1\right) - \mathrm{e}^{-t}c_1 + 2\,\mathrm{e}^{-t} + c_2 \\
y(t) &= \frac{6\,\mathrm{e}^{-t}\ln\left(\mathrm{e}^t-1\right) - 4c_2\mathrm{e}^t - 3\,\mathrm{e}^{-t}c_1 - 6\ln\left(\mathrm{e}^t-1\right) + 6\,\mathrm{e}^{-t} + 3c_1 + 4c_2 - 6}{2\,\mathrm{e}^t - 2}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 76

```
DSolve[{x'[t]==-4*x[t]-2*y[t]+2/(Exp[t]-1),y'[t]==6*x[t]+3*y[t]-3/(Exp[t]-1)},{x[t],y[t]},t,
```

$$
\begin{aligned}
x(t) &\to e^{-t}\left(2\log\left(e^t-1\right) + c_1\left(4-3e^t\right) - 2c_2\left(e^t-1\right)\right) \\
y(t) &\to e^{-t}\left(-3\log\left(e^t-1\right) + 6c_1\left(e^t-1\right) + c_2\left(4e^t-3\right)\right)
\end{aligned}
$$

30.5 problem 814

Internal problem ID [15537]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 23. Methods of integrating nonhomogeneous linear systems with constant coefficients. Exercises page 234
Problem number: 814.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = y(t)$$
$$y'(t) = -x(t) + \frac{1}{\cos(t)}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 48

```
dsolve([diff(x(t),t)=y(t),diff(y(t),t)=-x(t)+1/cos(t)],singsol=all)
```

$$x(t) = c_2 \sin(t) + c_1 \cos(t) + \sin(t)\, t + \cos(t) \ln(\cos(t))$$
$$y(t) = c_2 \cos(t) - c_1 \sin(t) + \cos(t)\, t - \sin(t) \ln(\cos(t))$$

✓ Solution by Mathematica
Time used: 0.007 (sec). Leaf size: 43

```
DSolve[{x'[t]==y[t],y'[t]==-x[t]+1/Cos[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to (t + c_2)\sin(t) + \cos(t)(\log(\cos(t)) + c_1)$$
$$y(t) \to (t + c_2)\cos(t) - \sin(t)(\log(\cos(t)) + c_1)$$

31 Chapter 3 (Systems of differential equations). Section 23.2 The method of undetermined coefficients. Exercises page 239

31.1 problem 815

Internal problem ID [15538]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 23.2 The method of undetermined coefficients. Exercises page 239
Problem number: 815.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = y(t)$$
$$y'(t) = 1 - x(t)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 28

```
dsolve([diff(x(t),t)=y(t),diff(y(t),t)=1-x(t)],singsol=all)
```

$$x(t) = c_2 \sin(t) + c_1 \cos(t) + 1$$
$$y(t) = c_2 \cos(t) - c_1 \sin(t)$$

✓ Solution by Mathematica
Time used: 0.007 (sec). Leaf size: 32

```
DSolve[{x'[t]==y[t],y'[t]==1-x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 \cos(t) + c_2 \sin(t) + 1$$
$$y(t) \to c_2 \cos(t) - c_1 \sin(t)$$

31.2 problem 816

Internal problem ID [15539]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 23.2 The method of undetermined coefficients. Exercises page 239
Problem number: 816.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= 3 - 2y(t) \\
y'(t) &= 2x(t) - 2t
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 37

```
dsolve([diff(x(t),t)=3-2*y(t),diff(y(t),t)=2*x(t)-2*t],singsol=all)
```

$$
\begin{aligned}
x(t) &= c_2 \sin(2t) + c_1 \cos(2t) + t \\
y(t) &= -c_2 \cos(2t) + c_1 \sin(2t) + 1
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.126 (sec). Leaf size: 41

```
DSolve[{x'[t]==3-2*y[t],y'[t]==2*x[t]-2*t},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
x(t) &\to t + c_1 \cos(2t) - c_2 \sin(2t) \\
y(t) &\to c_2 \cos(2t) + c_1 \sin(2t) + 1
\end{aligned}
$$

31.3 problem 817

Internal problem ID [15540]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 23.2 The method of undetermined coefficients. Exercises page 239
Problem number: 817.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -y(t) + \sin(t)$$
$$y'(t) = x(t) + \cos(t)$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 29

```
dsolve([diff(x(t),t)=-y(t)+sin(t),diff(y(t),t)=x(t)+cos(t)],singsol=all)
```

$$x(t) = c_1 \sin(t) + c_2 \cos(t)$$
$$y(t) = -c_1 \cos(t) + c_2 \sin(t) + \sin(t)$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 41

```
DSolve[{x'[t]==-y[t]+Sin[t],y'[t]==x[t]+Cos[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tr
```

$$x(t) \to \left(-\frac{1}{2} + c_1\right) \cos(t) - c_2 \sin(t)$$
$$y(t) \to \frac{\sin(t)}{2} + c_2 \cos(t) + c_1 \sin(t)$$

31.4 problem 818

Internal problem ID [15541]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 23.2 The method of undetermined coefficients. Exercises page 239
Problem number: 818.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) + y(t) + e^t$$
$$y'(t) = x(t) + y(t) - e^t$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 34

```
dsolve([diff(x(t),t)=x(t)+y(t)+exp(t),diff(y(t),t)=x(t)+y(t)-exp(t)],singsol=all)
```

$$x(t) = \frac{c_1 e^{2t}}{2} + e^t + c_2$$
$$y(t) = \frac{c_1 e^{2t}}{2} - e^t - c_2$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 62

```
DSolve[{x'[t]==x[t]+y[t]+Exp[t],y'[t]==x[t]+y[t]-Exp[t]},{x[t],y[t]},t,IncludeSingularSoluti
```

$$x(t) \to \frac{1}{2}(2e^t + (c_1 + c_2)e^{2t} + c_1 - c_2)$$
$$y(t) \to \frac{1}{2}(-2e^t + (c_1 + c_2)e^{2t} - c_1 + c_2)$$

31.5 problem 819

Internal problem ID [15542]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 23.2 The method of undetermined coefficients. Exercises page 239
Problem number: 819.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 4x(t) - 5y(t) + 4t - 1$$
$$y'(t) = x(t) - 2y(t) + t$$

With initial conditions

$$[x(0) = 0, y(0) = 0]$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 12

```
dsolve([diff(x(t),t) = 4*x(t)-5*y(t)+4*t-1, diff(y(t),t) = x(t)-2*y(t)+t, x(0) = 0, y(0) = 0
```

$$x(t) = -t$$
$$y(t) = 0$$

✓ Solution by Mathematica
Time used: 0.148 (sec). Leaf size: 12

```
DSolve[{x'[t]==4*x[t]-5*y[t]+4*t-1,y'[t]==x[t]-2*y[t]+t},{x[0]==0,y[0]==0},{x[t],y[t]},t,Inc
```

$$x(t) \to -t$$
$$y(t) \to 0$$

31.6 problem 820

Internal problem ID [15543]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 23.2 The method of undetermined coefficients. Exercises page 239
Problem number: 820.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = y(t) - x(t) + e^t$$
$$y'(t) = x(t) - y(t) + e^t$$

With initial conditions

$$[x(0) = 0, y(0) = 1]$$

✓ Solution by Maple
Time used: 0.046 (sec). Leaf size: 28

```
dsolve([diff(x(t),t) = y(t)-x(t)+exp(t), diff(y(t),t) = x(t)-y(t)+exp(t), x(0) = 0, y(0) = 1
```

$$x(t) = -\frac{e^{-2t}}{2} + e^t - \frac{1}{2}$$
$$y(t) = \frac{e^{-2t}}{2} + e^t - \frac{1}{2}$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 40

```
DSolve[{x'[t]==y[t]-x[t]+Exp[t],y'[t]==x[t]-y[t]+Exp[t]},{x[0]==0,y[0]==1},{x[t],y[t]},t,Inc
```

$$x(t) \to -\frac{e^{-2t}}{2} + e^t - \frac{1}{2}$$
$$y(t) \to \frac{e^{-2t}}{2} + e^t - \frac{1}{2}$$

31.7 problem 821

Internal problem ID [15544]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 23.2 The method of undetermined coefficients. Exercises page 239
Problem number: 821.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = t^2 - y(t)$$
$$y'(t) = x(t) + t$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 32

```
dsolve([diff(x(t),t)+y(t)=t^2,diff(y(t),t)-x(t)=t],singsol=all)
```

$$x(t) = c_2 \sin(t) + c_1 \cos(t) + t$$
$$y(t) = t^2 - c_2 \cos(t) + c_1 \sin(t) - 1$$

✓ Solution by Mathematica
Time used: 0.061 (sec). Leaf size: 36

```
DSolve[{x'[t]+y[t]==t^2,y'[t]-x[t]==t},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to t + c_1 \cos(t) - c_2 \sin(t)$$
$$y(t) \to t^2 + c_2 \cos(t) + c_1 \sin(t) - 1$$

31.8 problem 822

Internal problem ID [15545]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 23.2 The method of undetermined coefficients. Exercises page 239
Problem number: 822.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = \sin(t) - e^{-t} - y(t)$$
$$y'(t) = -\sin(t) + 2e^{-t}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 39

```
dsolve([diff(x(t),t)+diff(y(t),t)+y(t)=exp(-t),2*diff(x(t),t)+diff(y(t),t)+2*y(t)=sin(t)],si
```

$$x(t) = -\sin(t) - e^{-t} - \cos(t) + c_1 t + c_2$$
$$y(t) = \cos(t) - 2e^{-t} - c_1$$

✓ Solution by Mathematica
Time used: 0.244 (sec). Leaf size: 43

```
DSolve[{x'[t]+y'[t]+y[t]==Exp[-t],2*x'[t]+y'[t]+2*y[t]==Sin[t]},{x[t],y[t]},t,IncludeSingula
```

$$x(t) \to -e^{-t} - \sin(t) - \cos(t) - c_2 t + c_1$$
$$y(t) \to -2e^{-t} + \cos(t) + c_2$$

31.9 problem 823

Internal problem ID [15546]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 23.2 The method of undetermined coefficients. Exercises page 239
Problem number: 823.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= 2x(t) + y(t) - 2z(t) + 2 - t \\
y'(t) &= 1 - x(t) \\
z'(t) &= x(t) + y(t) - z(t) + 1 - t
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 51

```
dsolve([diff(x(t),t)=2*x(t)+y(t)-2*z(t)+2-t,diff(y(t),t)=1-x(t),diff(z(t),t)=x(t)+y(t)-z(t)+
```

$$
\begin{aligned}
x(t) &= c_1 \sin(t) - c_2 e^t - c_3 \cos(t) \\
y(t) &= t + c_1 \cos(t) + c_2 e^t + c_3 \sin(t) \\
z(t) &= 1 + c_1 \sin(t) - c_3 \cos(t)
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.129 (sec). Leaf size: 92

```
DSolve[{x'[t]==2*x[t]+y[t]-2*z[t]+2-t,y'[t]==1-x[t],z'[t]==x[t]+y[t]-z[t]+1-t},{x[t],y[t],z[
```

$$
\begin{aligned}
x(t) &\to (c_1 - c_3)e^t + c_3 \cos(t) + (c_1 + c_2 - c_3)\sin(t) \\
y(t) &\to t - c_1 e^t + c_3 e^t + (c_1 + c_2 - c_3)\cos(t) - c_3 \sin(t) \\
z(t) &\to c_3 \cos(t) + (c_1 + c_2 - c_3)\sin(t) + 1
\end{aligned}
$$

31.10 problem 824

Internal problem ID [15547]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 23.2 The method of undetermined coefficients. Exercises page 239
Problem number: 824.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -x(t) - 2y(t) + 2\,e^{-t}$$
$$y'(t) = -y(t) - z(t) + 1$$
$$z'(t) = -z(t) + 1$$

With initial conditions

$$[x(0) = 1, y(0) = 1, z(0) = 1]$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 20

```
dsolve([diff(x(t),t)+x(t)+2*y(t) = 2*exp(-t), diff(y(t),t)+y(t)+z(t) = 1, diff(z(t),t)+z(t)
```

$$x(t) = e^{-t}$$
$$y(t) = e^{-t}$$
$$z(t) = 1$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 22

```
DSolve[{x'[t]+x[t]+2*y[t]==2*Exp[-t],y'[t]+y[t]+z[t]==1,z'[t]+z[t]==1},{x[0]==1,y[0]==1,z[0]
```

$$x(t) \to e^{-t}$$
$$y(t) \to e^{-t}$$
$$z(t) \to 1$$

32 Chapter 3 (Systems of differential equations). Section 23.3 dAlemberts method. Exercises page 243

32.1 problem 825

Internal problem ID [15548]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISE-LYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 23.3 dAlemberts method. Exercises page 243
Problem number: 825.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= 5x(t) + 4y(t) \\
y'(t) &= x(t) + 2y(t)
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 32

```
dsolve([diff(x(t),t)=5*x(t)+4*y(t),diff(y(t),t)=x(t)+2*y(t)],singsol=all)
```

$$
\begin{aligned}
x(t) &= c_1 \mathrm{e}^{t} + c_2 \mathrm{e}^{6t} \\
y(t) &= -c_1 \mathrm{e}^{t} + \frac{c_2 \mathrm{e}^{6t}}{4}
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 67

```
DSolve[{x'[t]==5*x[t]+4*y[t],y'[t]==x[t]+2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> T
```

$$
\begin{aligned}
x(t) &\to \frac{1}{5} e^{t} \left(c_1 \left(4e^{5t} + 1\right) + 4c_2 \left(e^{5t} - 1\right)\right) \\
y(t) &\to \frac{1}{5} e^{t} \left(c_1 \left(e^{5t} - 1\right) + c_2 \left(e^{5t} + 4\right)\right)
\end{aligned}
$$

32.2 problem 826

Internal problem ID [15549]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 23.3 dAlemberts method. Exercises page 243
Problem number: 826.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= 6x(t) + y(t) \\
y'(t) &= 4x(t) + 3y(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 35

```
dsolve([diff(x(t),t)=6*x(t)+y(t),diff(y(t),t)=4*x(t)+3*y(t)],singsol=all)
```

$$
\begin{aligned}
x(t) &= c_1 e^{7t} + c_2 e^{2t} \\
y(t) &= c_1 e^{7t} - 4c_2 e^{2t}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 71

```
DSolve[{x'[t]==6*x[t]+y[t],y'[t]==4*x[t]+3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> T
```

$$
\begin{aligned}
x(t) &\to \frac{1}{5}e^{2t}\left(c_1\left(4e^{5t}+1\right)+c_2\left(e^{5t}-1\right)\right) \\
y(t) &\to \frac{1}{5}e^{2t}\left(4c_1\left(e^{5t}-1\right)+c_2\left(e^{5t}+4\right)\right)
\end{aligned}
$$

32.3 problem 827

Internal problem ID [15550]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 23.3 dAlemberts method. Exercises page 243
Problem number: 827.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 2x(t) - 4y(t) + 1$$
$$y'(t) = -x(t) + 5y(t)$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 34

```
dsolve([diff(x(t),t)=2*x(t)-4*y(t)+1,diff(y(t),t)=-x(t)+5*y(t)],singsol=all)
```

$$x(t) = c_2 e^t + e^{6t} c_1 - \frac{5}{6}$$
$$y(t) = \frac{c_2 e^t}{4} - e^{6t} c_1 - \frac{1}{6}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 67

```
DSolve[{x'[t]==2*x[t]-4*y[t],y'[t]==-x[t]+5*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{1}{5} e^t \left(c_1 \left(e^{5t} + 4\right) - 4c_2 \left(e^{5t} - 1\right)\right)$$
$$y(t) \to \frac{1}{5} e^t \left(c_1 \left(-e^{5t}\right) + 4c_2 e^{5t} + c_1 + c_2\right)$$

32.4 problem 828

Internal problem ID [15551]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 23.3 dAlemberts method. Exercises page 243
Problem number: 828.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 3x(t) + y(t) + e^t$$
$$y'(t) = x(t) + 3y(t) - e^t$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 43

```
dsolve([diff(x(t),t)=3*x(t)+y(t)+exp(t),diff(y(t),t)=x(t)+3*y(t)-exp(t)],singsol=all)
```

$$x(t) = \frac{c_1 e^{4t}}{2} - e^t + c_2 e^{2t}$$
$$y(t) = \frac{c_1 e^{4t}}{2} + e^t - c_2 e^{2t}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 70

```
DSolve[{x'[t]==3*x[t]+y[t]+Exp[t],y'[t]==x[t]+3*y[t]-Exp[t]},{x[t],y[t]},t,IncludeSingularSo
```

$$x(t) \to \frac{1}{2}e^t\left((c_1 - c_2)e^t + (c_1 + c_2)e^{3t} - 2\right)$$
$$y(t) \to \frac{1}{2}e^t\left((c_2 - c_1)e^t + (c_1 + c_2)e^{3t} + 2\right)$$

32.5 problem 829

Internal problem ID [15552]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3 (Systems of differential equations). Section 23.3 dAlemberts method. Exercises page 243
Problem number: 829.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 2x(t) + 4y(t) + \cos(t)$$
$$y'(t) = -x(t) - 2y(t) + \sin(t)$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=2*x(t)+4*y(t)+cos(t),diff(y(t),t)=-x(t)-2*y(t)+sin(t)],singsol=all)
```

$$x(t) = -2\cos(t) - 3\sin(t) + c_1 t + c_2$$
$$y(t) = 2\sin(t) + \frac{c_1}{4} - \frac{c_1 t}{2} - \frac{c_2}{2}$$

✓ Solution by Mathematica
Time used: 0.049 (sec). Leaf size: 46

```
DSolve[{x'[t]==2*x[t]+4*y[t]+Cos[t],y'[t]==-x[t]-2*y[t]+Sin[t]},{x[t],y[t]},t,IncludeSingula
```

$$x(t) \to -3\sin(t) - 2\cos(t) + 2c_1 t + 4c_2 t + c_1$$
$$y(t) \to 2\sin(t) - (c_1 + 2c_2)t + c_2$$

33 Chapter 3. Section 24.2. Solving the Cauchy problem for linear differential equation with constant coefficients. Exercises page 249

33.1 problem 830

Internal problem ID [15553]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISE-LYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3. Section 24.2. Solving the Cauchy problem for linear differential equation with constant coefficients. Exercises page 249
Problem number: 830.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{x' + 3x = e^{-2t}}$$

With initial conditions

$$[x(0) = 0]$$

✓ Solution by Maple
Time used: 0.454 (sec). Leaf size: 15

```
dsolve([diff(x(t),t)+3*x(t)=exp(-2*t),x(0) = 0],x(t), singsol=all)
```

$$x(t) = e^{-2t} - e^{-3t}$$

✓ Solution by Mathematica
Time used: 0.096 (sec). Leaf size: 16

```
DSolve[{x'[t]+3*x[t]==Exp[-2*t],{x[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{-3t}(e^t - 1)$$

33.2 problem 831

Internal problem ID [15554]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3. Section 24.2. Solving the Cauchy problem for linear differential equation with constant coefficients. Exercises page 249
Problem number: 831.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{x' - 3x = 3t^3 + 3t^2 + 2t + 1}$$

With initial conditions

$$[x(0) = -1]$$

✓ Solution by Maple
Time used: 0.359 (sec). Leaf size: 15

```
dsolve([diff(x(t),t)-3*x(t)=3*t^3+3*t^2+2*t+1,x(0) = -1],x(t), singsol=all)
```

$$x(t) = -(t + 1)\left(t^2 + t + 1\right)$$

✓ Solution by Mathematica
Time used: 0.136 (sec). Leaf size: 20

```
DSolve[{x'[t]-3*x[t]==3*t^3+3*t^2+2*t+1,{x[0]==-1}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -t^3 - 2t^2 - 2t - 1$$

33.3 problem 832

Internal problem ID [15555]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3. Section 24.2. Solving the Cauchy problem for linear differential equation with constant coefficients. Exercises page 249
Problem number: 832.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{x' - x = \cos(t) - \sin(t)}$$

With initial conditions

$$[x(0) = 0]$$

✓ Solution by Maple
Time used: 0.375 (sec). Leaf size: 6

```
dsolve([diff(x(t),t)-x(t)=cos(t)-sin(t),x(0) = 0],x(t), singsol=all)
```

$$x(t) = \sin(t)$$

✓ Solution by Mathematica
Time used: 0.045 (sec). Leaf size: 7

```
DSolve[{x'[t]-x[t]==Cos[t]-Sin[t],{x[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \sin(t)$$

33.4 problem 833

Internal problem ID [15556]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3. Section 24.2. Solving the Cauchy problem for linear differential equation with constant coefficients. Exercises page 249
Problem number: 833.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{2x' + 6x = t\,\mathrm{e}^{-3t}}$$

With initial conditions

$$\left[x(0) = -\frac{1}{2}\right]$$

✓ Solution by Maple
Time used: 0.391 (sec). Leaf size: 15

```
dsolve([2*diff(x(t),t)+6*x(t)=t*exp(-3*t),x(0) = -1/2],x(t), singsol=all)
```

$$x(t) = \frac{\mathrm{e}^{-3t}(t^2 - 2)}{4}$$

✓ Solution by Mathematica
Time used: 0.063 (sec). Leaf size: 19

```
DSolve[{2*x'[t]+6*x[t]==t*Exp[-3*t],{x[0]==-1/2}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{4}e^{-3t}\left(t^2 - 2\right)$$

33.5 problem 834

Internal problem ID [15557]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3. Section 24.2. Solving the Cauchy problem for linear differential equation with constant coefficients. Exercises page 249
Problem number: 834.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{x' + x = 2\sin(t)}$$

With initial conditions

$$[x(0) = 0]$$

✓ Solution by Maple
Time used: 0.406 (sec). Leaf size: 15

```
dsolve([diff(x(t),t)+x(t)=2*sin(t),x(0) = 0],x(t), singsol=all)
```

$$x(t) = -\cos(t) + \sin(t) + e^{-t}$$

✓ Solution by Mathematica
Time used: 0.055 (sec). Leaf size: 17

```
DSolve[{x'[t]+x[t]==2*Sin[t],{x[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{-t} + \sin(t) - \cos(t)$$

33.6 problem 835

Internal problem ID [15558]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3. Section 24.2. Solving the Cauchy problem for linear differential equation with constant coefficients. Exercises page 249
Problem number: 835.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{x'' = 0}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple
Time used: 0.266 (sec). Leaf size: 5

```
dsolve([diff(x(t),t$2)=0,x(0) = 0, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = 0$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 6

```
DSolve[{x''[t]==0,{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow 0$$

33.7 problem 836

Internal problem ID [15559]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3. Section 24.2. Solving the Cauchy problem for linear differential equation with constant coefficients. Exercises page 249
Problem number: 836.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{x'' = 1}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple
Time used: 0.343 (sec). Leaf size: 9

```
dsolve([diff(x(t),t$2)=1,x(0) = 0, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = \frac{t^2}{2}$$

✓ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 12

```
DSolve[{x''[t]==1,{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{t^2}{2}$$

33.8 problem 837

Internal problem ID [15560]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3. Section 24.2. Solving the Cauchy problem for linear differential equation with constant coefficients. Exercises page 249
Problem number: 837.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{x'' = \cos(t)}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple
Time used: 0.344 (sec). Leaf size: 10

```
dsolve([diff(x(t),t$2)=cos(t),x(0) = 0, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = 1 - \cos(t)$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 11

```
DSolve[{x''[t]==Cos[t],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to 1 - \cos(t)$$

33.9 problem 838

Internal problem ID [15561]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3. Section 24.2. Solving the Cauchy problem for linear differential equation with constant coefficients. Exercises page 249
Problem number: 838.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + x' = 0}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple
Time used: 0.375 (sec). Leaf size: 5

```
dsolve([diff(x(t),t$2)+diff(x(t),t)=0,x(0) = 0, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = 0$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 6

```
DSolve[{x''[t]+x'[t]==0,{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to 0$$

33.10 problem 839

Internal problem ID [15562]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3. Section 24.2. Solving the Cauchy problem for linear differential equation with constant coefficients. Exercises page 249
Problem number: 839.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + x' = 0}$$

With initial conditions

$$[x(0) = 1, x'(0) = -1]$$

✓ Solution by Maple
Time used: 0.282 (sec). Leaf size: 8

```
dsolve([diff(x(t),t$2)+diff(x(t),t)=0,x(0) = 1, D(x)(0) = -1],x(t), singsol=all)
```

$$x(t) = \mathrm{e}^{-t}$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 10

```
DSolve[{x''[t]+x'[t]==0,{x[0]==1,x'[0]==-1}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{-t}$$

33.11 problem 840

Internal problem ID [15563]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3. Section 24.2. Solving the Cauchy problem for linear differential equation with constant coefficients. Exercises page 249
Problem number: 840.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' - x' = 1}$$

With initial conditions

$$[x(0) = -1, x'(0) = -1]$$

✓ Solution by Maple
Time used: 0.36 (sec). Leaf size: 9

```
dsolve([diff(x(t),t$2)-diff(x(t),t)=1,x(0) = -1, D(x)(0) = -1],x(t), singsol=all)
```

$$x(t) = -t - 1$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 10

```
DSolve[{x''[t]-x'[t]==1,{x[0]==-1,x'[0]==-1}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -t - 1$$

33.12 problem 841

Internal problem ID [15564]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3. Section 24.2. Solving the Cauchy problem for linear differential equation with constant coefficients. Exercises page 249
Problem number: 841.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x'' + x = t}$$

With initial conditions

$$[x(0) = 0, x'(0) = 1]$$

✓ Solution by Maple
Time used: 0.329 (sec). Leaf size: 5

```
dsolve([diff(x(t),t$2)+x(t)=t,x(0) = 0, D(x)(0) = 1],x(t), singsol=all)
```

$$x(t) = t$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 6

```
DSolve[{x''[t]+x[t]==t,{x[0]==0,x'[0]==1}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to t$$

33.13 problem 842

Internal problem ID [15565]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3. Section 24.2. Solving the Cauchy problem for linear differential equation with constant coefficients. Exercises page 249
Problem number: 842.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x'' + 6x' = 12t + 2}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 7

```
dsolve([diff(x(t),t$2)+6*diff(x(t),t)=12*t+2,x(0) = 0, D(x)(0) = 0],x(t), singsol=all)
```

$$x = t^2$$

✓ Solution by Mathematica
Time used: 0.126 (sec). Leaf size: 8

```
DSolve[{x''[t]+6*x'[t]==12*t+2,{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to t^2$$

33.14 problem 843

Internal problem ID [15566]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3. Section 24.2. Solving the Cauchy problem for linear differential equation with constant coefficients. Exercises page 249
Problem number: 843.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' - 2x' + 2x = 2}$$

With initial conditions

$$[x(0) = 1, x'(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 5

```
dsolve([diff(x(t),t$2)-2*diff(x(t),t)+2*x(t)=2,x(0) = 1, D(x)(0) = 0],x(t), singsol=all)
```

$$x = 1$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 6

```
DSolve[{x''[t]-2*x'[t]+2*x[t]==2,{x[0]==1,x'[0]==0}},x[t],t,IncludeSingularSolutions -> True
```

$$x(t) \to 1$$

33.15 problem 844

Internal problem ID [15567]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISE-LYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3. Section 24.2. Solving the Cauchy problem for linear differential equation with constant coefficients. Exercises page 249
Problem number: 844.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + 4x' + 4x = 4}$$

With initial conditions

$$[x(0) = 1, x'(0) = -4]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 13

```
dsolve([diff(x(t),t$2)+4*diff(x(t),t)+4*x(t)=4,x(0) = 1, D(x)(0) = -4],x(t), singsol=all)
```

$$x = 1 - 4t\,\mathrm{e}^{-2t}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 15

```
DSolve[{x''[t]+4*x'[t]+4*x[t]==4,{x[0]==1,x'[0]==-4}},x[t],t,IncludeSingularSolutions -> Tru
```

$$x(t) \to 1 - 4e^{-2t}t$$

33.16 problem 845

Internal problem ID [15568]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3. Section 24.2. Solving the Cauchy problem for linear differential equation with constant coefficients. Exercises page 249
Problem number: 845.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{2x'' - 2x' = (1+t)\,\mathrm{e}^{t}}$$

With initial conditions

$$\left[x(0) = \frac{1}{2}, x'(0) = \frac{1}{2}\right]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 13

```
dsolve([2*diff(x(t),t$2)-2*diff(x(t),t)=(1+t)*exp(t),x(0) = 1/2, D(x)(0) = 1/2],x(t), singso
```

$$x = \frac{\mathrm{e}^{t}(t^{2}+2)}{4}$$

✓ Solution by Mathematica
Time used: 0.059 (sec). Leaf size: 17

```
DSolve[{2*x''[t]-2*x'[t]==(1+t)*Exp[t],{x[0]==1/2,x'[0]==1/2}},x[t],t,IncludeSingularSolutio
```

$$x(t) \to \frac{1}{4}e^{t}\left(t^{2}+2\right)$$

33.17 problem 846

Internal problem ID [15569]

Book: A book of problems in ordinary differential equations. M.L. KRASNOV, A.L. KISELYOV, G.I. MARKARENKO. MIR, MOSCOW. 1983
Section: Chapter 3. Section 24.2. Solving the Cauchy problem for linear differential equation with constant coefficients. Exercises page 249
Problem number: 846.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + x = 2\cos(t)}$$

With initial conditions

$$[x(0) = -1, x'(0) = 1]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 15

```
dsolve([diff(x(t),t$2)+x(t)=2*cos(t),x(0) = -1, D(x)(0) = 1],x(t), singsol=all)
```

$$x = -\cos(t) + \sin(t)(1+t)$$

✓ Solution by Mathematica
Time used: 0.047 (sec). Leaf size: 16

```
DSolve[{x''[t]+x[t]==2*Cos[t],{x[0]==-1,x'[0]==1}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to (t+1)\sin(t) - \cos(t)$$