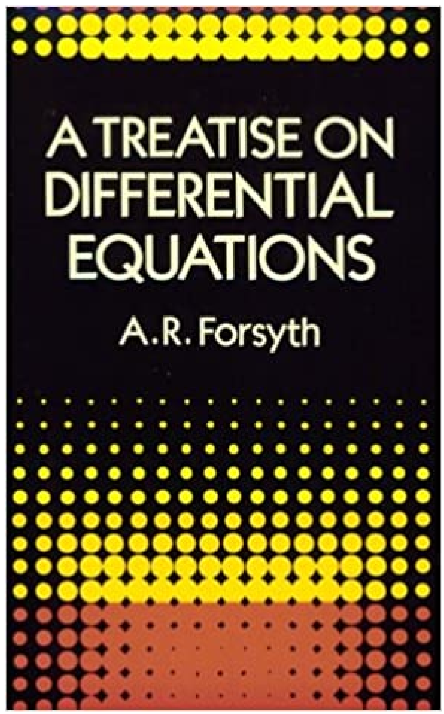
A Solution Manual For

A treatise on Differential Equations by A. R. Forsyth. 6th edition. 1929. Macmillan Co. ltd. New York, reprinted 1956



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May 16, 2024

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1 Chapter VI. Note I. Integration of linear equations in series by the method of Frobenius. page 243

1.1 problem Ex. 5, page 256

Internal problem ID [5471]

Book: A treatise on Differential Equations by A. R. Forsyth. 6th edition. 1929. Macmillan Co. ltd. New York, reprinted 1956
Section: Chapter VI. Note I. Integration of linear equations in series by the method of Frobenius. page 243
Problem number: Ex. 5, page 256.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(-x^2+2)\,y''-(x^2+4x+2)\,((1-x)\,y'+y)=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 44

```
Order:=6;
dsolve(x*(2-x^2)*diff(y(x),x$2)-(x^2+4*x+2)*((1-x)*diff(y(x),x)+y(x))=0,y(x),type='series',x
```

$$y(x)=c_1x^2\left(1+x+\frac{1}{2}x^2+\frac{1}{6}x^3+\frac{1}{24}x^4+\frac{1}{120}x^5+\mathrm{O}\left(x^6\right)\right)+c_2\left(-2+2x+4x^2+4x^3+2x^4+\frac{2}{3}x^5+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 64

```
AsymptoticDSolveValue[x*(2-x^2)*y''[x]-(x^2+4*x+2)*((1-x)*y'[x]+y[x])==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(-\frac{5x^4}{4}-\frac{5x^3}{2}-\frac{5x^2}{2}-x+1\right)+c_2\left(\frac{x^6}{24}+\frac{x^5}{6}+\frac{x^4}{2}+x^3+x^2\right)$$

1.2 problem Ex. 6(i), page 257

Internal problem ID [5472]

Book: A treatise on Differential Equations by A. R. Forsyth. 6th edition. 1929. Macmillan Co. ltd. New York, reprinted 1956
Section: Chapter VI. Note I. Integration of linear equations in series by the method of Frobenius. page 243
Problem number: Ex. 6(i), page 257.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1+x)\,y'' - (1+2x)\,(-y+xy') = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*(1+x)*diff(y(x),x$2)-(1+2*x)*(x*diff(y(x),x)-y(x))=0,y(x),type='series',x=0);
```

$$y(x) = x\left((c_2 \ln(x) + c_1)\left(1 + \mathrm{O}\left(x^6\right)\right) + \left(x + \mathrm{O}\left(x^6\right)\right) c_2\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 2760

```
AsymptoticDSolveValue[x^2*(1+x)*y''[x]-(1+2*x)*(x*y'[x]+y[x])==0,y[x],{x,0,5}]
```

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1.3 problem Ex. 6(ii), page 257

Internal problem ID [5473]

Book: A treatise on Differential Equations by A. R. Forsyth. 6th edition. 1929. Macmillan Co. ltd. New York, reprinted 1956
Section: Chapter VI. Note I. Integration of linear equations in series by the method of Frobenius. page 243
Problem number: Ex. 6(ii), page 257.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

Solve

$$x^3(1+x)\,y''' - (2+4x)\,x^2y'' + (4+10x)\,xy' - (4+12x)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 79

```
Order:=6;
dsolve(x^3*(1+x)*diff(y(x),x$3)-(2+4*x)*x^2*diff(y(x),x$2)+(4+10*x)*x*diff(y(x),x)-(4+12*x)*
```

$$y(x) = x\left(\left(2x+\mathrm{O}\left(x^6\right)\right)\ln\left(x\right)^2 c_3 + \ln\left(x\right)\left(2+\mathrm{O}\left(x^6\right)\right)c_2x + 2\left(\left(-4\right)x+\mathrm{O}\left(x^6\right)\right)\ln\left(x\right)c_3 + \left(5+\mathrm{O}\left(x^6\right)\right)c_2x + c_1x\left(1+\mathrm{O}\left(x^6\right)\right) + \left(2+4x+2x^2+\mathrm{O}\left(x^6\right)\right)c_3\right)$$

✓ Solution by Mathematica
Time used: 0.514 (sec). Leaf size: 49

```
AsymptoticDSolveValue[x^3*(1+x)*y'''[x]-(2+4*x)*x^2*y''[x]+(4+10*x)*x*y'[x]-(4+12*x)*y[x]==0
```

$$y(x) \to c_2x^2 + c_1\left(2\left(x^2+11x+1\right)x + 2x^2\log^2(x) - 14x^2\log(x)\right) + c_3x^2\log(x)$$

1.4 problem Ex. 6(iii), page 257

Internal problem ID [5474]

Book: A treatise on Differential Equations by A. R. Forsyth. 6th edition. 1929. Macmillan Co. ltd. New York, reprinted 1956
Section: Chapter VI. Note I. Integration of linear equations in series by the method of Frobenius. page 243
Problem number: Ex. 6(iii), page 257.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

Solve

$$x^3(x^2+1)\,y''' - (4x^2+2)\,x^2y'' + (10x^2+4)\,xy' - (12x^2+4)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 52

```
Order:=6;
dsolve(x^3*(1+x^2)*diff(y(x),x$3)-(2+4*x^2)*x^2*diff(y(x),x$2)+(4+10*x^2)*x*diff(y(x),x)-(4+
```

$$y(x) = \left(c_3\left(2+2x^2+O\left(x^6\right)\right) + \left(\left(1+O\left(x^6\right)\right)c_1 + c_2\left(\ln\left(x\right)\left(2+O\left(x^6\right)\right) + \left(5+O\left(x^6\right)\right)\right)\right)x\right)x$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 30

```
AsymptoticDSolveValue[x^3*(1+x^2)*y'''[x]-(2+4*x^2)*x^2*y''[x]+(4+10*x^2)*x*y'[x]-(4+12*x^2)
```

$$y(x) \to c_1\left(2x^3+2x\right) + c_2x^2 + c_3x^2\log(x)$$

1.5 problem Ex. 6(iv), page 257

Internal problem ID [5475]

Book: A treatise on Differential Equations by A. R. Forsyth. 6th edition. 1929. Macmillan Co. ltd. New York, reprinted 1956
Section: Chapter VI. Note I. Integration of linear equations in series by the method of Frobenius. page 243
Problem number: Ex. 6(iv), page 257.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2(2-x)\, x^2 y'' - (4-x)\, x y' + (-x+3)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
dsolve(2*(2-x)*x^2*diff(y(x),x$2)-(4-x)*x*diff(y(x),x)+(3-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x}\left(x\left(1+\frac{1}{8}x+\frac{1}{32}x^2+\frac{5}{512}x^3+\frac{7}{2048}x^4+\frac{21}{16384}x^5+\mathrm{O}\left(x^6\right)\right)c_1 + \left(1+\frac{1}{4}x+\frac{1}{32}x^2+\frac{1}{128}x^3+\frac{5}{2048}x^4+\frac{7}{8192}x^5+\mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica
Time used: 0.055 (sec). Leaf size: 94

```
AsymptoticDSolveValue[2*(2-x)*x^2*y''[x]-(4-x)*x*y'[x]+(3-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{5x^{9/2}}{2048}-\frac{x^{7/2}}{128}-\frac{x^{5/2}}{32}-\frac{x^{3/2}}{4}+\sqrt{x}\right)+c_2\left(\frac{7x^{11/2}}{2048}+\frac{5x^{9/2}}{512}+\frac{x^{7/2}}{32}+\frac{x^{5/2}}{8}+x^{3/2}\right)$$

1.6 problem Ex. 6(v), page 257

Internal problem ID [5476]

Book: A treatise on Differential Equations by A. R. Forsyth. 6th edition. 1929. Macmillan Co. ltd. New York, reprinted 1956
Section: Chapter VI. Note I. Integration of linear equations in series by the method of Frobenius. page 243
Problem number: Ex. 6(v), page 257.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1-x)\,x^2y''+(5x-4)\,xy'+(6-9x)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 43

```
Order:=6;
dsolve((1-x)*x^2*diff(y(x),x$2)+(5*x-4)*x*diff(y(x),x)+(6-9*x)*y(x)=0,y(x),type='series',x=0
```

$$y(x)=x^2\left(\ln(x)\left(x+\mathrm{O}\left(x^6\right)\right)c_2+c_1x\left(1+\mathrm{O}\left(x^6\right)\right)+\left(1-x+\mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 30

```
AsymptoticDSolveValue[(1-x)*x^2*y''[x]+(5*x-4)*x*y'[x]+(6-9*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2x^3+c_1\left(x^3\log(x)-x^2(3x-1)\right)$$

1.7 problem Ex. 6(vi), page 257

Internal problem ID [5477]

Book: A treatise on Differential Equations by A. R. Forsyth. 6th edition. 1929. Macmillan Co. ltd. New York, reprinted 1956
Section: Chapter VI. Note I. Integration of linear equations in series by the method of Frobenius. page 243
Problem number: Ex. 6(vi), page 257.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (4x^2 + 1)\, y' + 4xy(x^2 + 1) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
Order:=6;
dsolve(x*diff(y(x),x$2)+(4*x^2+1)*diff(y(x),x)+4*x*(x^2+1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x^2 + \frac{1}{2}x^4\right)(c_2 \ln(x) + c_1) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 40

```
AsymptoticDSolveValue[x*y''[x]+(4*x^2+1)*y'[x]+4*x*(x^2+1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^4}{2} - x^2 + 1\right) + c_2\left(\frac{x^4}{2} - x^2 + 1\right)\log(x)$$

1.8 problem Ex. 8(i), page 258

Internal problem ID [5478]

Book: A treatise on Differential Equations by A. R. Forsyth. 6th edition. 1929. Macmillan Co. ltd. New York, reprinted 1956
Section: Chapter VI. Note I. Integration of linear equations in series by the method of Frobenius. page 243
Problem number: Ex. 8(i), page 258.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 4(a + x)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 947

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+4*(x+a)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x}\left(c_1 x^{-\frac{\sqrt{1-16a}}{2}}\left(1+4\frac{1}{-1+\sqrt{1-16a}}x+8\frac{1}{\left(-1+\sqrt{1-16a}\right)\left(-2+\sqrt{1-16a}\right)}x^2 + \frac{32}{3}\frac{1}{\left(-1+\sqrt{1-16a}\right)\left(-2+\sqrt{1-16a}\right)\left(-3+\sqrt{1-16a}\right)}x^3 + \frac{32}{3}\frac{1}{\left(-1+\sqrt{1-16a}\right)\left(-2+\sqrt{1-16a}\right)\left(-3+\sqrt{1-16a}\right)\left(-4+\sqrt{1-16a}\right)}x^4 + \frac{128}{15}\frac{1}{\left(-1+\sqrt{1-16a}\right)\left(-2+\sqrt{1-16a}\right)\left(-3+\sqrt{1-16a}\right)\left(-4+\sqrt{1-16a}\right)\left(-5+\sqrt{1-16a}\right)}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2 x^{\frac{\sqrt{1-16a}}{2}}\left(1-4\frac{1}{1+\sqrt{1-16a}}x+8\frac{1}{\left(1+\sqrt{1-16a}\right)\left(2+\sqrt{1-16a}\right)}x^2 - \frac{32}{3}\frac{1}{\left(1+\sqrt{1-16a}\right)\left(2+\sqrt{1-16a}\right)\left(3+\sqrt{1-16a}\right)}x^3 + \frac{32}{3}\frac{1}{\left(1+\sqrt{1-16a}\right)\left(2+\sqrt{1-16a}\right)\left(3+\sqrt{1-16a}\right)\left(4+\sqrt{1-16a}\right)}x^4 - \frac{128}{15}\frac{1}{\left(1+\sqrt{1-16a}\right)\left(2+\sqrt{1-16a}\right)\left(3+\sqrt{1-16a}\right)\left(4+\sqrt{1-16a}\right)\left(5+\sqrt{1-16a}\right)}x^5 + \mathrm{O}\left(x^6\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 1356

```
AsymptoticDSolveValue[x^2*y''[x]+4*(x+a)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \left(-\frac{}{\left(\left(\frac{1}{2}\left(1-\sqrt{1-16a}\right)+1\right)\left(\frac{1}{2}\left(1-\sqrt{1-16a}\right)+2\right)+4a\right)\left(\left(\frac{1}{2}\left(1-\sqrt{1-16a}\right)+2\right)\left(\frac{1}{2}\left(1-\sqrt{1-16a}\right.\right.\right.}\right.$$

$$+\frac{}{\left(\left(\frac{1}{2}\left(1-\sqrt{1-16a}\right)+1\right)\left(\frac{1}{2}\left(1-\sqrt{1-16a}\right)+2\right)+4a\right)\left(\left(\frac{1}{2}\left(1-\sqrt{1-16a}\right)+2\right)\left(\frac{1}{2}\left(1-\sqrt{1-16a}\right)\right.\right.}$$

$$-\frac{64x^3}{\left(\left(\frac{1}{2}\left(1-\sqrt{1-16a}\right)+1\right)\left(\frac{1}{2}\left(1-\sqrt{1-16a}\right)+2\right)+4a\right)\left(\left(\frac{1}{2}\left(1-\sqrt{1-16a}\right)+2\right)\left(\frac{1}{2}\left(1-\sqrt{1-16a}\right)\right.\right.}$$

$$+\frac{16x^2}{\left(\left(\frac{1}{2}\left(1-\sqrt{1-16a}\right)+1\right)\left(\frac{1}{2}\left(1-\sqrt{1-16a}\right)+2\right)+4a\right)\left(\frac{1}{2}\left(\frac{1}{2}\left(1-\sqrt{1-16a}\right)+1\right)\left(1-\sqrt{1-16a}\right)\right.}$$

$$\left.-\frac{4x}{\frac{1}{2}\left(\frac{1}{2}\left(1-\sqrt{1-16a}\right)+1\right)\left(1-\sqrt{1-16a}\right)+4a}+1\right)c_2x^{\frac{1}{2}\left(1-\sqrt{1-16a}\right)}$$

$$+\left(-\frac{}{\left(\left(\frac{1}{2}\left(\sqrt{1-16a}+1\right)+1\right)\left(\frac{1}{2}\left(\sqrt{1-16a}+1\right)+2\right)+4a\right)\left(\left(\frac{1}{2}\left(\sqrt{1-16a}+1\right)+2\right)\left(\frac{1}{2}\left(\sqrt{1-16a}+\right.\right.\right.}\right.$$

$$+\frac{}{\left(\left(\frac{1}{2}\left(\sqrt{1-16a}+1\right)+1\right)\left(\frac{1}{2}\left(\sqrt{1-16a}+1\right)+2\right)+4a\right)\left(\left(\frac{1}{2}\left(\sqrt{1-16a}+1\right)+2\right)\left(\frac{1}{2}\left(\sqrt{1-16a}+1\right)\right.\right.}$$

$$-\frac{64x^3}{\left(\left(\frac{1}{2}\left(\sqrt{1-16a}+1\right)+1\right)\left(\frac{1}{2}\left(\sqrt{1-16a}+1\right)+2\right)+4a\right)\left(\left(\frac{1}{2}\left(\sqrt{1-16a}+1\right)+2\right)\left(\frac{1}{2}\left(\sqrt{1-16a}+1\right)\right.\right.}$$

$$+\frac{16x^2}{\left(\left(\frac{1}{2}\left(\sqrt{1-16a}+1\right)+1\right)\left(\frac{1}{2}\left(\sqrt{1-16a}+1\right)+2\right)+4a\right)\left(\frac{1}{2}\left(\frac{1}{2}\left(\sqrt{1-16a}+1\right)+1\right)\left(\sqrt{1-16a}+1\right)\right.}$$

$$\left.-\frac{4x}{\frac{1}{2}\left(\frac{1}{2}\left(\sqrt{1-16a}+1\right)+1\right)\left(\sqrt{1-16a}+1\right)+4a}+1\right)c_1x^{\frac{1}{2}\left(\sqrt{1-16a}+1\right)}$$

1.9 problem Ex. 8(ii), page 258

Internal problem ID [5479]

Book: A treatise on Differential Equations by A. R. Forsyth. 6th edition. 1929. Macmillan Co. ltd. New York, reprinted 1956
Section: Chapter VI. Note I. Integration of linear equations in series by the method of Frobenius. page 243
Problem number: Ex. 8(ii), page 258.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' + (x^3 + 1) y' + bxy = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 73

```
Order:=6;
dsolve(x*diff(y(x),x$2)+(1+x*x^2)*diff(y(x),x)+b*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1) \left(1 - \frac{1}{4} b x^2 + \frac{1}{64} b^2 x^4 + \frac{1}{50} b x^5 + \mathrm{O}(x^6)\right) + \left(\frac{b}{4} x^2 - \frac{1}{9} x^3 - \frac{3}{128} b^2 x^4 - \frac{61}{4500} b x^5 + \mathrm{O}(x^6)\right) c_2$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 103

```
AsymptoticDSolveValue[x*y''[x]+(1+x*x^2)*y'[x]+b*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{b^2 x^4}{64} + \frac{b x^5}{50} - \frac{b x^2}{4} + 1\right) + c_2 \left(-\frac{3 b^2 x^4}{128} + \left(\frac{b^2 x^4}{64} + \frac{b x^5}{50} - \frac{b x^2}{4} + 1\right) \log(x) - \frac{61 b x^5}{4500} + \frac{b x^2}{4} - \frac{x^3}{9}\right)$$