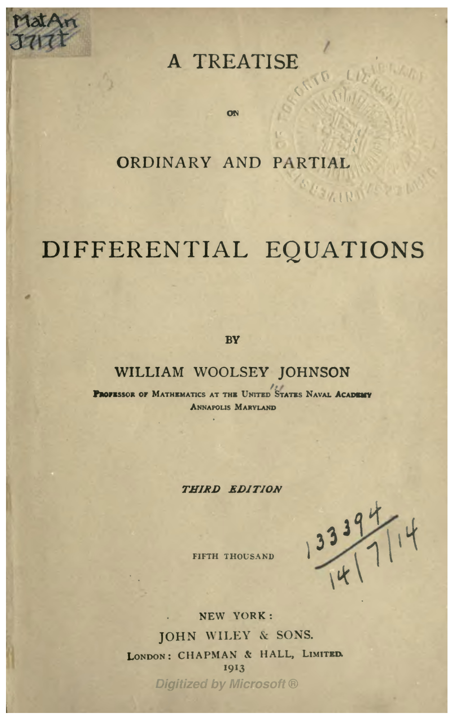
A Solution Manual For

A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913



A TREATISE

ON

ORDINARY AND PARTIAL

DIFFERENTIAL EQUATIONS

BY

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FIFTH THOUSAND

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1 Chapter 1, Nature and meaning of a differential equation between two variables. page 12

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1.1 problem 1

Internal problem ID [4681]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter 1, Nature and meaning of a differential equation between two variables. page 12
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + y\tan(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x)+y(x)*tan(x)=0,y(x), singsol=all)
```

$$y(x) = \cos(x)\, c_1$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 15

```
DSolve[y'[x]+y[x]*Tan[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(x)$$
$$y(x) \to 0$$

1.2 problem 2

Internal problem ID [4682]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter 1, Nature and meaning of a differential equation between two variables. page 12
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' - 2xy' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = x(c_1x + c_2)$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 14

```
DSolve[x^2*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(c_2x + c_1)$$

1.3 problem 3

Internal problem ID [4683]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter 1, Nature and meaning of a differential equation between two variables. page 12
Problem number: 3.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{yy'^2 + 2xy' - y = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 71

```
dsolve(y(x)*diff(y(x),x)^2+2*x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$
\begin{aligned}
y(x) &= -ix \\
y(x) &= ix \\
y(x) &= 0 \\
y(x) &= \sqrt{c_1 (c_1 - 2x)} \\
y(x) &= \sqrt{c_1 (c_1 + 2x)} \\
y(x) &= -\sqrt{c_1 (c_1 - 2x)} \\
y(x) &= -\sqrt{c_1 (c_1 + 2x)}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.462 (sec). Leaf size: 126

```
DSolve[y[x]*(y'[x])^2+2*x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) &\to -e^{\frac{c_1}{2}}\sqrt{-2x+e^{c_1}} \\
y(x) &\to e^{\frac{c_1}{2}}\sqrt{-2x+e^{c_1}} \\
y(x) &\to -e^{\frac{c_1}{2}}\sqrt{2x+e^{c_1}} \\
y(x) &\to e^{\frac{c_1}{2}}\sqrt{2x+e^{c_1}} \\
y(x) &\to 0 \\
y(x) &\to -ix \\
y(x) &\to ix
\end{aligned}
$$

1.4 problem 4

Internal problem ID [4684]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter 1, Nature and meaning of a differential equation between two variables. page 12
Problem number: 4.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2(-x^2+1) = -1}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 33

```
dsolve(diff(y(x),x)^2*(1-x^2)+1=0,y(x), singsol=all)
```

$$y(x) = \ln\left(x+\sqrt{x^2-1}\right)+c_1$$
$$y(x) = -\ln\left(x+\sqrt{x^2-1}\right)+c_1$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 89

```
DSolve[y'[x]^2*(1-x^2)+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)-\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)+2c_1\right)$$
$$y(x) \to \frac{1}{2}\left(-\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)+\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)+2c_1\right)$$

1.5 problem 5

Internal problem ID [4685]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter 1, Nature and meaning of a differential equation between two variables. page 12
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - ya = e^{ax}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=exp(a*x)+a*y(x),y(x), singsol=all)
```

$$y(x) = (x + c_1)\, e^{ax}$$

✓ Solution by Mathematica
Time used: 0.044 (sec). Leaf size: 15

```
DSolve[y'[x]==Exp[a*x]+a*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{ax}(x + c_1)$$

1.6 problem 9

Internal problem ID [4686]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter 1, Nature and meaning of a differential equation between two variables. page 12
Problem number: 9.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\left(1+y'^2\right)^3-a^2y''^2=0$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 93

```
dsolve((diff(y(x),x)^2+1)^3=a^2*(diff(y(x),x$2))^2,y(x), singsol=all)
```

$$\begin{aligned} y(x) &= -ix+c_1 \\ y(x) &= ix+c_1 \\ y(x) &= \frac{(a+x+c_1)(-a+x+c_1)}{\sqrt{a^2-c_1^2-2c_1x-x^2}}+c_2 \\ y(x) &= \frac{(a+x+c_1)(a-x-c_1)}{\sqrt{a^2-c_1^2-2c_1x-x^2}}+c_2 \end{aligned}$$

✓ Solution by Mathematica
Time used: 0.658 (sec). Leaf size: 141

```
DSolve[(y'[x]^2+1)^3==a^2*(y''[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to c_2-i\sqrt{a^2(-1+c_1{}^2)-2ac_1x+x^2} \\ y(x) &\to i\sqrt{a^2(-1+c_1{}^2)-2ac_1x+x^2}+c_2 \\ y(x) &\to c_2-i\sqrt{a^2(-1+c_1{}^2)+2ac_1x+x^2} \\ y(x) &\to i\sqrt{a^2(-1+c_1{}^2)+2ac_1x+x^2}+c_2 \end{aligned}$$

2 Chapter 2, Equations of the first order and degree. page 20

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2.5 problem 5 16
2.6 problem 6 18
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2.8 problem 8 20
2.9 problem 9 21

2.1 problem 1

Internal problem ID [4687]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter 2, Equations of the first order and degree. page 20
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x+1)\,y+(1-y)\,xy'=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve((1+x)*y(x)+(1-y(x))*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\operatorname{LambertW}\left(-\frac{e^{-x}}{c_1 x}\right)$$

✓ Solution by Mathematica
Time used: 3.094 (sec). Leaf size: 28

```
DSolve[(1+x)*y[x]+(1-y[x])*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to -W\left(-\frac{e^{-x-c_1}}{x}\right) \\ y(x) &\to 0 \end{aligned}$$

2.2 problem 2

Internal problem ID [4688]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter 2, Equations of the first order and degree. page 20
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y^2 a x = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=a*y(x)^2*x,y(x), singsol=all)
```

$$y(x) = -\frac{2}{a\,x^2 - 2c_1}$$

✓ Solution by Mathematica
Time used: 0.127 (sec). Leaf size: 24

```
DSolve[y'[x]==a*y[x]^2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{ax^2 + 2c_1}$$
$$y(x) \to 0$$

2.3 problem 3

Internal problem ID [4689]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter 2, Equations of the first order and degree. page 20
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{y^2 + xy^2 + (x^2 - yx^2)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 35

```
dsolve((y(x)^2+x*y(x)^2)+(x^2-y(x)*x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = x\,\mathrm{e}^{\frac{\mathrm{LambertW}\left(-\frac{\mathrm{e}^{\frac{-c_1 x+1}{x}}}{x}\right)x + c_1 x - 1}{x}}$$

✓ Solution by Mathematica
Time used: 5.302 (sec). Leaf size: 30

```
DSolve[(y[x]^2+x*y[x]^2)+(x^2-y[x]*x^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{W\left(-\frac{e^{\frac{1}{x}-c_1}}{x}\right)}$$
$$y(x) \to 0$$

2.4 problem 4

Internal problem ID [4690]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter 2, Equations of the first order and degree. page 20
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy(x^2+1)\,y'-y^2=1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 54

```
dsolve(x*y(x)*(1+x^2)*diff(y(x),x)=1+y(x)^2,y(x), singsol=all)
```

$$y(x)=\frac{\sqrt{(x^2+1)\,(c_1x^2-1)}}{x^2+1}$$
$$y(x)=-\frac{\sqrt{(x^2+1)\,(c_1x^2-1)}}{x^2+1}$$

✓ Solution by Mathematica

Time used: 1.206 (sec). Leaf size: 131

```
DSolve[x*y[x]*(1+x^2)*y'[x]==1+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) &\to -\frac{\sqrt{-1+(-1+e^{2c_1})\,x^2}}{\sqrt{x^2+1}} \\
y(x) &\to \frac{\sqrt{-1+(-1+e^{2c_1})\,x^2}}{\sqrt{x^2+1}} \\
y(x) &\to -i \\
y(x) &\to i \\
y(x) &\to -\frac{\sqrt{-x^2-1}}{\sqrt{x^2+1}} \\
y(x) &\to \frac{\sqrt{-x^2-1}}{\sqrt{x^2+1}}
\end{aligned}
$$

2.5 problem 5

Internal problem ID [4691]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter 2, Equations of the first order and degree. page 20
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\frac{x}{1+y} - \frac{yy'}{x+1} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 498

```
dsolve(x/(1+y(x))=y(x)/(1+x)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \frac{\left(-1+4x^3+6x^2+12c_1+2\sqrt{4x^6+12x^5+24c_1x^3+9x^4+36c_1x^2-2x^3+36c_1^2-3x^2-6c_1}\right)^{\frac{1}{3}}}{2} + \frac{1}{2\left(-1+4x^3+6x^2+12c_1+2\sqrt{4x^6+12x^5+24c_1x^3+9x^4+36c_1x^2-2x^3+36c_1^2-3x^2-6c_1}\right)^{\frac{1}{3}}} - \frac{1}{2}$$

$$y(x) = -\frac{\left(1+i\sqrt{3}\right)\left(4x^3+6x^2+2\sqrt{(2x^3+3x^2+6c_1)(2x^3+3x^2+6c_1-1)}+12c_1-1\right)^{\frac{2}{3}} - i\sqrt{3}+2\left(4x^3+\right.}{4\left(4x^3+6x^2+2\sqrt{(2x^3+3x^2+6c_1)(2x^3+3x^2+6c\right.}}$$

$$y(x) = \frac{\left(i\sqrt{3}-1\right)\left(4x^3+6x^2+2\sqrt{(2x^3+3x^2+6c_1)(2x^3+3x^2+6c_1-1)}+12c_1-1\right)^{\frac{2}{3}} - i\sqrt{3}-2\left(4x^3+6x\right.}{4\left(4x^3+6x^2+2\sqrt{(2x^3+3x^2+6c_1)(2x^3+3x^2+6c_1\right.}}$$

✓ Solution by Mathematica

Time used: 4.125 (sec). Leaf size: 346

```
DSolve[x/(1+y[x])==y[x]/(1+x)*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(\sqrt[3]{4x^3+6x^2+\sqrt{-1+(4x^3+6x^2-1+12c_1)^2}-1+12c_1} + \frac{1}{\sqrt[3]{4x^3+6x^2+\sqrt{-1+(4x^3+6x^2-1+12c_1)^2}-1+12c_1}}-1\right)$$

$$y(x) \to \frac{1}{8}\left(2i\left(\sqrt{3}+i\right)\sqrt[3]{4x^3+6x^2+\sqrt{-1+(4x^3+6x^2-1+12c_1)^2}-1+12c_1} + \frac{-2-2i\sqrt{3}}{\sqrt[3]{4x^3+6x^2+\sqrt{-1+(4x^3+6x^2-1+12c_1)^2}-1+12c_1}}-4\right)$$

$$y(x) \to \frac{1}{8}\left(-2\left(1+i\sqrt{3}\right)\sqrt[3]{4x^3+6x^2+\sqrt{-1+(4x^3+6x^2-1+12c_1)^2}-1+12c_1} + \frac{2i\left(\sqrt{3}+i\right)}{\sqrt[3]{4x^3+6x^2+\sqrt{-1+(4x^3+6x^2-1+12c_1)^2}-1+12c_1}}-4\right)$$

2.6 problem 6

Internal problem ID [4692]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter 2, Equations of the first order and degree. page 20
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + y^2 b^2 = a^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x)+b^2*y(x)^2=a^2,y(x), singsol=all)
```

$$y(x) = -\frac{a\left(e^{-2ba(x+c_1)} + 1\right)}{b\left(e^{-2ba(x+c_1)} - 1\right)}$$

✓ Solution by Mathematica
Time used: 3.208 (sec). Leaf size: 37

```
DSolve[y'[x]+b^2*y[x]^2==a^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{a \tanh(ab(x + c_1))}{b}$$
$$y(x) \to -\frac{a}{b}$$
$$y(x) \to \frac{a}{b}$$

2.7 problem 7

Internal problem ID [4693]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter 2, Equations of the first order and degree. page 20
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{1 + y^2}{x^2 + 1} = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)=(y(x)^2+1)/(x^2+1),y(x), singsol=all)
```

$$y(x) = \tan\left(\arctan\left(x\right) + c_1\right)$$

✓ Solution by Mathematica
Time used: 0.237 (sec). Leaf size: 25

```
DSolve[y'[x]==(y[x]^2+1)/(x^2+1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan(\arctan(x) + c_1)$$
$$y(x) \to -i$$
$$y(x) \to i$$

2.8 problem 8

Internal problem ID [4694]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter 2, Equations of the first order and degree. page 20
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sin(x)\cos(y) - \cos(x)\sin(y)\,y' = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 11

```
dsolve(sin(x)*cos(y(x))=cos(x)*sin(y(x))*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{\cos(x)}{c_1}\right)$$

✓ Solution by Mathematica
Time used: 5.183 (sec). Leaf size: 47

```
DSolve[Sin[x]*Cos[y[x]]==Cos[x]*Sin[y[x]]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arccos\left(\frac{1}{2}c_1\cos(x)\right)$$
$$y(x) \to \arccos\left(\frac{1}{2}c_1\cos(x)\right)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

2.9 problem 9

Internal problem ID [4695]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter 2, Equations of the first order and degree. page 20
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{axy' + 2y - xyy' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 44

```
dsolve(a*x*diff(y(x),x)+2*y(x)=x*y(x)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = x^{-\frac{2}{a}} \mathrm{e}^{\frac{-a \operatorname{LambertW}\left(-\frac{x^{-\frac{2}{a}} \mathrm{e}^{-\frac{2c_1}{a}}}{a}\right) - 2c_1}{a}}$$

✓ Solution by Mathematica
Time used: 60.019 (sec). Leaf size: 29

```
DSolve[a*x*y'[x]+2*y[x]==x*y[x]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -aW\left(-\frac{e^{\frac{c_1}{a}} x^{-2/a}}{a}\right)$$

3 Chapter VII, Solutions in series. Examples XIV. page 177

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3.1 problem 1

Internal problem ID [4696]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XIV. page 177
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' + (x+n)\,y' + (1+n)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 248

```
Order:=6;
dsolve(x*diff(y(x),x$2)+(x+n)*diff(y(x),x)+(n+1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{1-n}\left(1 + 2\frac{1}{n-2}x + 3\frac{1}{(-3+n)(n-2)}x^2 + 4\frac{1}{(-4+n)(-3+n)(n-2)}x^3 + 5\frac{1}{(-5+n)(-4+n)(-3+n)(n-2)}x^4 + 6\frac{1}{(-6+n)(-5+n)(-4+n)(-3+n)(n-2)}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(1 + \frac{-1-n}{n}x + \frac{1}{2}\frac{n+2}{n}x^2 - \frac{1}{6}\frac{n+3}{n}x^3 + \frac{1}{24}\frac{n+4}{n}x^4 - \frac{1}{120}\frac{n+5}{n}x^5 + \mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 519

```
AsymptoticDSolveValue[x*y''[x]+(x+n)*y'[x]+(n+1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{(-n-1)(n+2)(n+3)(n+4)(n+5)x^5}{n(2n+2)(3n+6)(4n+12)(5n+20)} - \frac{(-n-1)(n+2)(n+3)(n+4)x^4}{n(2n+2)(3n+6)(4n+12)} + \frac{(-n-1)(n+2)(n+3)x^3}{n(2n+2)(3n+6)} - \frac{(-n-1)(n+2)x^2}{n(2n+2)} + \frac{(-n-1)x}{n} + 1\right)$$

$$+c_1\left(-\frac{720x^5}{((1-n)(2-n)+n(2-n))((2-n)(3-n)+n(3-n))((3-n)(4-n)+n(4-n))((4-n)(5}\right.$$

$$+\frac{120x^4}{((1-n)(2-n)+n(2-n))((2-n)(3-n)+n(3-n))((3-n)(4-n)+n(4-n))((4-n)(5-n)}$$

$$-\frac{24x^3}{((1-n)(2-n)+n(2-n))((2-n)(3-n)+n(3-n))((3-n)(4-n)+n(4-n))}$$

$$+\frac{6x^2}{((1-n)(2-n)+n(2-n))((2-n)(3-n)+n(3-n))}$$

$$\left.-\frac{2x}{(1-n)(2-n)+n(2-n)} + 1\right)x^{1-n}$$

3.2 problem 2

Internal problem ID [4697]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XIV. page 177
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + xy = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^3}{6}\right) y(0) + \left(x - \frac{1}{12}x^4\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x - \frac{x^4}{12}\right) + c_1\left(1 - \frac{x^3}{6}\right)$$

3.3 problem 3

Internal problem ID [4698]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XIV. page 177
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$2x^2y'' - xy' + (-x^2+1)\,y = x^2$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)-x*diff(y(x),x)+(1-x^2)*y(x)=x^2,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1+\frac{1}{6}x^2+\frac{1}{168}x^4+\mathrm{O}\left(x^6\right)\right) + c_2x\left(1+\frac{1}{10}x^2+\frac{1}{360}x^4+\mathrm{O}\left(x^6\right)\right)+x^2\left(\frac{1}{3}+\frac{1}{63}x^2+\mathrm{O}\left(x^4\right)\right)$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 160

```
AsymptoticDSolveValue[2*x^2*y''[x]-x*y'[x]+(1-x^2)*y[x]==x^2,y[x],{x,0,5}]
```

$$y(x)\to c_2x\left(\frac{x^6}{28080}+\frac{x^4}{360}+\frac{x^2}{10}+1\right)+c_1\sqrt{x}\left(\frac{x^6}{11088}+\frac{x^4}{168}+\frac{x^2}{6}+1\right)+\sqrt{x}\left(-\frac{x^{11/2}}{1980}-\frac{x^{7/2}}{35}-\frac{2x^{3/2}}{3}\right)\left(\frac{x^6}{11088}+\frac{x^4}{168}+\frac{x^2}{6}+1\right)+x\left(\frac{x^5}{840}+\frac{x^3}{18}+x\right)\left(\frac{x^6}{28080}+\frac{x^4}{360}+\frac{x^2}{10}+1\right)$$

3.4 problem 4

Internal problem ID [4699]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XIV. page 177
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{xy'' + 2y' + a^3x^2y = 2}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 50

```
Order:=6;
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+a^3*x^2*y(x)=2,y(x),type='series',x=0);
```

$$y(x) = c_1\left(1 - \frac{1}{12}a^3x^3 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(1 - \frac{1}{6}a^3x^3 + \mathrm{O}\left(x^6\right)\right)}{x} + x\left(1 - \frac{1}{20}a^3x^3 + \mathrm{O}\left(x^5\right)\right)$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 136

```
AsymptoticDSolveValue[x*y''[x]+2*y'[x]+a^3*x^2*y[x]==2,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{a^6x^6}{504} - \frac{a^3x^3}{12} + 1\right) + \frac{c_2\left(\frac{a^6x^6}{180} - \frac{a^3x^3}{6} + 1\right)}{x} + \left(2x - \frac{a^3x^4}{12}\right)\left(\frac{a^6x^6}{504} - \frac{a^3x^3}{12} + 1\right) + \frac{\left(\frac{a^3x^5}{30} - x^2\right)\left(\frac{a^6x^6}{180} - \frac{a^3x^3}{6} + 1\right)}{x}$$

3.5 problem 5

Internal problem ID [4700]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XIV. page 177
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + a\,x^2 y = x + 1}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
Order:=6;
dsolve(diff(y(x),x$2)+a*x^2*y(x)=1+x,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{a\,x^4}{12}\right) y(0) + \left(x - \frac{1}{20} a\,x^5\right) D(y)\,(0) + \frac{x^2}{2} + \frac{x^3}{6} + O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 44

```
AsymptoticDSolveValue[y''[x]+a*x^2*y[x]==1+x,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x - \frac{ax^5}{20}\right) + c_1\left(1 - \frac{ax^4}{12}\right) + \frac{x^3}{6} + \frac{x^2}{2}$$

3.6 problem 7

Internal problem ID [4701]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XIV. page 177
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^4y'' + xy' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(x^4*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 49

```
AsymptoticDSolveValue[x^4*y''[x]+x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \frac{c_1(1-x^2)}{x} + c_2 e^{\frac{1}{2x^2}} \left(420x^6 + 45x^4 + 6x^2 + 1\right) x^4$$

3.7 problem 8

Internal problem ID [4702]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XIV. page 177
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + (2x^2 + x)\, y' - 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+(x+2*x^2)*diff(y(x),x)-4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^2\left(1 - \frac{4}{5}x + \frac{2}{5}x^2 - \frac{16}{105}x^3 + \frac{1}{21}x^4 - \frac{4}{315}x^5 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(-144 + 192x - 96x^2 + 32x^4 - \frac{128}{5}x^5 + \mathrm{O}\left(x^6\right)\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 208

```
AsymptoticDSolveValue[x^2*y''[x]+(x+2*x^2)*y'[x]-4*y[x]==2,y[x],{x,0,5}]
```

$$y(x) \to \frac{c_1\left(\frac{2x^2}{3}-\frac{4x}{3}+1\right)}{x^2}+c_2\left(-\frac{4x^5}{315}+\frac{x^4}{21}-\frac{16x^3}{105}+\frac{2x^2}{5}-\frac{4x}{5}+1\right)x^2 + \left(-\frac{4x^5}{315}+\frac{x^4}{21}-\frac{16x^3}{105}+\frac{2x^2}{5}-\frac{4x}{5}+1\right)\left(\frac{7x^6}{2430}+\frac{19x^5}{2025}+\frac{5x^4}{216}+\frac{2x^3}{45}+\frac{x^2}{18}-\frac{1}{4x^2}-\frac{1}{3x}\right)x^2+\frac{\left(\frac{2x^2}{3}-\frac{4x}{3}+1\right)\left(-\frac{x^6}{84}-\frac{4x^5}{105}-\frac{x^4}{10}-\frac{x^3}{5}-\frac{x^2}{4}\right)}{x^2}$$

3.8 problem 9

Internal problem ID [4703]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XIV. page 177
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$
\boxed{(-x^2 + x)\, y'' + 3y' + 2y = 0}
$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 36

```
Order:=6;
dsolve((x-x^2)*diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$
y(x) = c_1 \left(1 - \frac{2}{3}x + \frac{1}{6}x^2 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2(-2 + 8x - 12x^2 + 8x^3 - 2x^4 + \mathrm{O}\left(x^6\right))}{x^2}
$$

✓ Solution by Mathematica
Time used: 0.05 (sec). Leaf size: 40

```
AsymptoticDSolveValue[(x-x^2)*y''[x]+3*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$
y(x) \to c_1 \left(x^2 + \frac{1}{x^2} - 4x - \frac{4}{x} + 6\right) + c_2 \left(\frac{x^2}{6} - \frac{2x}{3} + 1\right)
$$

3.9 problem 10

Internal problem ID [4704]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XIV. page 177
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(4x^3 - 14x^2 - 2x)\, y'' - (6x^2 - 7x + 1)\, y' + (-1 + 6x)\, y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 28

```
Order:=6;
dsolve((4*x^3-14*x^2-2*x)*diff(y(x),x$2)-(6*x^2-7*x+1)*diff(y(x),x)+(6*x-1)*y(x)=0,y(x),type
```

$$y(x) = c_1\sqrt{x}\left(1 + 2x + \mathrm{O}\left(x^6\right)\right) + c_2\left(1 - x + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 25

```
AsymptoticDSolveValue[(4*x^3-14*x^2-2*x)*y''[x]-(6*x^2-7*x+1)*y'[x]+(6*x-1)*y[x]==0,y[x],{x,
```

$$y(x) \to c_1\sqrt{x}(2x + 1) + c_2(1 - x)$$

3.10 problem 11

Internal problem ID [4705]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XIV. page 177
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x^2y' + (-2+x)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 43

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x^2*diff(y(x),x)+(x-2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^2\left(1 - \frac{3}{4}x + \frac{3}{10}x^2 - \frac{1}{12}x^3 + \frac{1}{56}x^4 - \frac{1}{320}x^5 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(12 - 2x^3 + \frac{3}{2}x^4 - \frac{3}{5}x^5 + \mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 60

```
AsymptoticDSolveValue[x^2*y''[x]+x^2*y'[x]+(x-2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^3}{8} - \frac{x^2}{6} + \frac{1}{x}\right) + c_2\left(\frac{x^6}{56} - \frac{x^5}{12} + \frac{3x^4}{10} - \frac{3x^3}{4} + x^2\right)$$

3.11 problem 13

Internal problem ID [4706]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XIV. page 177
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - x^2y' + (-2 + x)\, y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x^2*diff(y(x),x)+(x-2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^2\left(1 + \frac{1}{4}x + \frac{1}{20}x^2 + \frac{1}{120}x^3 + \frac{1}{840}x^4 + \frac{1}{6720}x^5 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(12 + 12x + 6x^2 + 2x^3 + \frac{1}{2}x^4 + \frac{1}{10}x^5 + \mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 66

```
AsymptoticDSolveValue[x^2*y''[x]-x^2*y'[x]+(x-2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^3}{24} + \frac{x^2}{6} + \frac{x}{2} + \frac{1}{x} + 1\right) + c_2\left(\frac{x^6}{840} + \frac{x^5}{120} + \frac{x^4}{20} + \frac{x^3}{4} + x^2\right)$$

3.12 problem 14

Internal problem ID [4707]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XIV. page 177
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-4x)\,y'' + \left((-n+1)\,x - (6-4n)\,x^2\right) y' + n(-n+1)\,xy = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 471

```
Order:=6;
dsolve(x^2*(1-4*x)*diff(y(x),x$2)+((1-n)*x-(6-4*n)*x^2)*diff(y(x),x)+n*(1-n)*x*y(x)=0,y(x),t
```

$$y(x) = c_1 x^n \left(1 + nx + \frac{1}{2}n(n+3)\,x^2 + \frac{1}{6}(n+5)\,(n+4)\,nx^3 + \frac{1}{24}n(n+5)\,(n+7)\,(n+6)\,x^4 + \frac{1}{120}(n+9)\,(n+8)\,(n+7)\,(n+6)\,nx^5 + \mathrm{O}\left(x^6\right)\right) + c_2 \left(1 - nx + \frac{1}{2}n(-3+n)\,x^2 - \frac{1}{6}(-4+n)\,(-5+n)\,nx^3 + \frac{1}{24}n(-5+n)\,(-6+n)\,(n-7)\,x^4 - \frac{1}{120}(-6+n)\,(n-7)\,(n-8)\,(n-9)\,nx^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 2114

```
AsymptoticDSolveValue[x^2*(1-4*x)*y''[x]+((1-n)*x-(6-4*n)*x^2)*y'[x]+n*(1-n)*x*y[x]==0,y[x],
```

$y(x)$

$$\rightarrow \left(\frac{\left(512n - 256(n-n^2) - \frac{(n^2+n)\left(64(n-n^2)-128(n+1)\right)}{(1-n)(n+1)+n(n+1)} - \frac{\left(16(n-n^2)-32(n+2)\right)\left(8n-4(n-n^2)-\frac{(n^2+n)\left(-n^2+n-2(n+1)\right)}{(1-n)(n+1)+n(n+1)}\right)}{(1-n)(n+2)+(n+1)(n+2)} \right.}{} \right.$$

$$+ \frac{\left(128n - 64(n-n^2) - \frac{(n^2+n)\left(16(n-n^2)-32(n+1)\right)}{(1-n)(n+1)+n(n+1)} - \frac{\left(4(n-n^2)-8(n+2)\right)\left(8n-4(n-n^2)-\frac{(n^2+n)\left(-n^2+n-2(n+1)\right)}{(1-n)(n+1)+n(n+1)}\right)}{(1-n)(n+2)+(n+1)(n+2)} - \right.}{(1-n)(n+4)+}$$

$$+ \frac{\left(32n - 16(n-n^2) - \frac{(n^2+n)\left(4(n-n^2)-8(n+1)\right)}{(1-n)(n+1)+n(n+1)} - \frac{\left(-n^2+n-2(n+2)\right)\left(8n-4(n-n^2)-\frac{(n^2+n)\left(-n^2+n-2(n+1)\right)}{(1-n)(n+1)+n(n+1)}\right)}{(1-n)(n+2)+(n+1)(n+2)} \right) x^3}{(1-n)(n+3)+(n+2)(n+3)}$$

3.13 problem 15

Internal problem ID [4708]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XIV. page 177
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + (x^2 + x)\, y' + (x - 9)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 41

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+(x+x^2)*diff(y(x),x)+(x-9)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^3\left(1 - \frac{4}{7}x + \frac{5}{28}x^2 - \frac{5}{126}x^3 + \frac{1}{144}x^4 - \frac{1}{990}x^5 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2(-86400 + 34560x - 4320x^2 + \mathrm{O}\left(x^6\right))}{x^3}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 60

```
AsymptoticDSolveValue[x^2*y''[x]+(x+x^2)*y'[x]+(x-9)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1\left(\frac{1}{x^3} - \frac{2}{5x^2} + \frac{1}{20x}\right) + c_2\left(\frac{x^7}{144} - \frac{5x^6}{126} + \frac{5x^5}{28} - \frac{4x^4}{7} + x^3\right)$$

3.14 problem 16

Internal problem ID [4709]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XIV. page 177
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$(a^2 + x^2)\, y'' + xy' - yn^2 = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 72

```
Order:=6;
dsolve((a^2+x^2)*diff(y(x),x$2)+x*diff(y(x),x)-n^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{n^2x^2}{2a^2} + \frac{n^2(n^2-4)\, x^4}{24a^4}\right) y(0) + \left(x + \frac{(n^2-1)\, x^3}{6a^2} + \frac{(n^4 - 10n^2 + 9)\, x^5}{120a^4}\right) D(y)\,(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 112

```
AsymptoticDSolveValue[(a^2+x^2)*y''[x]+x*y'[x]-n^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{n^4x^5}{120a^4} - \frac{n^2x^5}{12a^4} + \frac{3x^5}{40a^4} + \frac{n^2x^3}{6a^2} - \frac{x^3}{6a^2} + x\right) + c_1\left(\frac{n^4x^4}{24a^4} - \frac{n^2x^4}{6a^4} + \frac{n^2x^2}{2a^2} + 1\right)$$

3.15 problem 18

Internal problem ID [4710]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XIV. page 177
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']`

$$(-x^2+1)\,y''-xy'+ya^2=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 71

```
Order:=6;
dsolve((1-x^2)*diff(y(x),x$2)-x*diff(y(x),x)+a^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{x^2a^2}{2}+\frac{a^2(a^2-4)\,x^4}{24}\right)y(0)+\left(x-\frac{(a^2-1)\,x^3}{6}+\frac{(a^4-10a^2+9)\,x^5}{120}\right)D(y)\,(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 88

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-x*y'[x]+a^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{a^4x^5}{120}-\frac{a^2x^5}{12}-\frac{a^2x^3}{6}+\frac{3x^5}{40}+\frac{x^3}{6}+x\right)+c_1\left(\frac{a^4x^4}{24}-\frac{a^2x^4}{6}-\frac{a^2x^2}{2}+1\right)$$

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4.1 problem 1

Internal problem ID [4711]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XV. page 194
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
Order:=6;
dsolve(x*diff(y(x),x$2)+diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln (x) + c_1) \left(1 - x + \frac{1}{4}x^2 - \frac{1}{36}x^3 + \frac{1}{576}x^4 - \frac{1}{14400}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(2x - \frac{3}{4}x^2 + \frac{11}{108}x^3 - \frac{25}{3456}x^4 + \frac{137}{432000}x^5 + \mathrm{O}\left(x^6\right)\right) c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 111

```
AsymptoticDSolveValue[x*y''[x]+y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(-\frac{x^5}{14400} + \frac{x^4}{576} - \frac{x^3}{36} + \frac{x^2}{4} - x + 1\right) + c_2 \left(\frac{137x^5}{432000} - \frac{25x^4}{3456} + \frac{11x^3}{108} - \frac{3x^2}{4} + \left(-\frac{x^5}{14400} + \frac{x^4}{576} - \frac{x^3}{36} + \frac{x^2}{4} - x + 1\right) \log(x) + 2x\right)$$

4.2 problem 2

Internal problem ID [4712]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XV. page 194
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + y' + pxy = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 59

```
Order:=6;
dsolve(x*diff(y(x),x$2)+diff(y(x),x)+p*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1)\left(1 - \frac{1}{4}p\,x^2 + \frac{1}{64}p^2x^4 + \mathrm{O}\left(x^6\right)\right) + \left(\frac{p}{4}x^2 - \frac{3}{128}p^2x^4 + \mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 72

```
AsymptoticDSolveValue[x*y''[x]+y'[x]+p*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{p^2x^4}{64} - \frac{px^2}{4} + 1\right) + c_2\left(-\frac{3}{128}p^2x^4 + \left(\frac{p^2x^4}{64} - \frac{px^2}{4} + 1\right)\log(x) + \frac{px^2}{4}\right)$$

4.3 problem 3

Internal problem ID [4713]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XV. page 194
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 58

```
Order:=6;
dsolve(x*diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x \left(1 - \frac{1}{2}x + \frac{1}{12}x^2 - \frac{1}{144}x^3 + \frac{1}{2880}x^4 - \frac{1}{86400}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2 \left(\ln\left(x\right)\left(-x + \frac{1}{2}x^2 - \frac{1}{12}x^3 + \frac{1}{144}x^4 - \frac{1}{2880}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(1 - \frac{3}{4}x^2 + \frac{7}{36}x^3 - \frac{35}{1728}x^4 + \frac{101}{86400}x^5 + \mathrm{O}\left(x^6\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 85

```
AsymptoticDSolveValue[x*y''[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{1}{144}x\left(x^3 - 12x^2 + 72x - 144\right)\log(x) + \frac{-47x^4 + 480x^3 - 2160x^2 + 1728x + 1728}{1728}\right) + c_2 \left(\frac{x^5}{2880} - \frac{x^4}{144} + \frac{x^3}{12} - \frac{x^2}{2} + x\right)$$

4.4 problem 4

Internal problem ID [4714]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XV. page 194
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^3y'' - (2x-1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(x^3*diff(y(x),x$2)-(2*x-1)*y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 222

```
AsymptoticDSolveValue[x^3*y''[x]-(2*x-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 e^{-\frac{2i}{\sqrt{x}}} x^{3/4} \left(-\frac{1159525191825ix^{9/2}}{8796093022208} + \frac{218243025ix^{7/2}}{4294967296} - \frac{405405ix^{5/2}}{8388608} + \frac{3465ix^{3/2}}{8192} + \frac{75369137468625x^5}{281474976710656} - \frac{41247931725x^4}{549755813888} + \frac{11486475x^3}{268435456} - \frac{45045x^2}{524288} - \frac{945x}{512} - \frac{35i\sqrt{x}}{16} + 1 \right) + c_2 e^{\frac{2i}{\sqrt{x}}} x^{3/4} \left(\frac{1159525191825ix^{9/2}}{8796093022208} - \frac{218243025ix^{7/2}}{4294967296} + \frac{405405ix^{5/2}}{8388608} - \frac{3465ix^{3/2}}{8192} + \frac{75369137468625}{28147497671065} \right.$$

4.5 problem 5

Internal problem ID [4715]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XV. page 194
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x(x+1)\,y' + (3x-1)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 65

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(x+1)*diff(y(x),x)+(3*x-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^2\left(1-\frac{4}{3}x+\frac{5}{6}x^2-\frac{1}{3}x^3+\frac{7}{72}x^4-\frac{1}{45}x^5+\mathrm{O}\left(x^6\right)\right)+c_2\left(\ln\left(x\right)\left(6x^2-8x^3+5x^4-2x^5+\mathrm{O}\left(x^6\right)\right)+\left(-2\right.\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 85

```
AsymptoticDSolveValue[x^2*y''[x]+x*(x+1)*y'[x]+(3*x-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{13x^4-12x^3-4x^2+8x+4}{4x}-\frac{1}{2}x\left(5x^2-8x+6\right)\log(x)\right) + c_2\left(\frac{7x^5}{72}-\frac{x^4}{3}+\frac{5x^3}{6}-\frac{4x^2}{3}+x\right)$$

4.6 problem 6

Internal problem ID [4716]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XV. page 194
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-x^2 + x)\, y'' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 58

```
Order:=6;
dsolve((x-x^2)*diff(y(x),x$2)-y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = {} & c_1 x\left(1 + \frac{1}{2}x + \frac{1}{4}x^2 + \frac{7}{48}x^3 + \frac{91}{960}x^4 + \frac{637}{9600}x^5 + \mathrm{O}\left(x^6\right)\right) \\ & + c_2\left(\ln(x)\left(x + \frac{1}{2}x^2 + \frac{1}{4}x^3 + \frac{7}{48}x^4 + \frac{91}{960}x^5 + \mathrm{O}\left(x^6\right)\right)\right. \\ & \left. + \left(1 - \frac{1}{4}x^2 - \frac{1}{12}x^3 - \frac{17}{576}x^4 - \frac{311}{28800}x^5 + \mathrm{O}\left(x^6\right)\right)\right) \end{aligned}$$

✓ Solution by Mathematica
Time used: 0.044 (sec). Leaf size: 87

```
AsymptoticDSolveValue[(x-x^2)*y''[x]-y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned} y(x) \to {} & c_1\left(\frac{1}{48}x\left(7x^3 + 12x^2 + 24x + 48\right)\log(x)\right. \\ & \left. + \frac{1}{576}\left(-185x^4 - 336x^3 - 720x^2 - 1152x + 576\right)\right) + c_2\left(\frac{91x^5}{960} + \frac{7x^4}{48} + \frac{x^3}{4} + \frac{x^2}{2} + x\right) \end{aligned}$$

4.7 problem 7

Internal problem ID [4717]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XV. page 194
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_elliptic, _class_I]]`

$$\boxed{x(-x^2+1)\,y''+(-3x^2+1)\,y'-xy=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 41

```
Order:=6;
dsolve(x*(1-x^2)*diff(y(x),x$2)+(1-3*x^2)*diff(y(x),x)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(c_2\ln(x)+c_1)\left(1+\frac{1}{4}x^2+\frac{9}{64}x^4+\mathrm{O}\left(x^6\right)\right)+\left(\frac{1}{4}x^2+\frac{21}{128}x^4+\mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 60

```
AsymptoticDSolveValue[x*(1-x^2)*y''[x]+(1-3*x^2)*y'[x]-x*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{9x^4}{64}+\frac{x^2}{4}+1\right)+c_2\left(\frac{21x^4}{128}+\frac{x^2}{4}+\left(\frac{9x^4}{64}+\frac{x^2}{4}+1\right)\log(x)\right)$$

4.8 problem 8

Internal problem ID [4718]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XV. page 194
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$y'' + \frac{ay}{x^{\frac{3}{2}}} = 0$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(diff(y(x),x$2)+a/x^(3/2)*y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.244 (sec). Leaf size: 576

```
AsymptoticDSolveValue[y''[x]+a/x^(3/2)*y[x]==0,y[x],{x,0,5}]
```

$$
\begin{aligned}
y(x) \rightarrow & -\frac{16x^5(126a^{10}c_2\log(x) - 252\pi a^{10}c_1 + 504\gamma a^{10}c_2 - 1423a^{10}c_2 + 252a^{10}c_2\log(a) + 504a^{10}c_2\log(2))}{281302875\pi} \\
& + \frac{32x^{9/2}(1260a^9c_2\log(x) - 2520\pi a^9c_1 + 5040\gamma a^9c_2 - 13663a^9c_2 + 2520a^9c_2\log(a) + 5040a^9c_2\log(2))}{281302875\pi} \\
& - \frac{8x^4(140a^8c_2\log(x) - 280\pi a^8c_1 + 560\gamma a^8c_2 - 1447a^8c_2 + 280a^8c_2\log(a) + 560a^8c_2\log(2))}{496125\pi} \\
& + \frac{128x^{7/2}(105a^7c_2\log(x) - 210\pi a^7c_1 + 420\gamma a^7c_2 - 1024a^7c_2 + 210a^7c_2\log(a) + 420a^7c_2\log(2))}{496125\pi} \\
& - \frac{32x^3(15a^6c_2\log(x) - 30\pi a^6c_1 + 60\gamma a^6c_2 - 136a^6c_2 + 30a^6c_2\log(a) + 60a^6c_2\log(2))}{2025\pi} \\
& + \frac{32x^{5/2}(30a^5c_2\log(x) - 60\pi a^5c_1 + 120\gamma a^5c_2 - 247a^5c_2 + 60a^5c_2\log(a) + 120a^5c_2\log(2))}{675\pi} \\
& - \frac{8x^2(6a^4c_2\log(x) - 12\pi a^4c_1 + 24\gamma a^4c_2 - 43a^4c_2 + 12a^4c_2\log(a) + 24a^4c_2\log(2))}{9\pi} \\
& + \frac{32x^{3/2}(3a^3c_2\log(x) - 6\pi a^3c_1 + 12\gamma a^3c_2 - 17a^3c_2 + 6a^3c_2\log(a) + 12a^3c_2\log(2))}{9\pi} \\
& - \frac{8x(a^2c_2\log(x) - 2\pi a^2c_1 + 4\gamma a^2c_2 - 3a^2c_2 + 2a^2c_2\log(a) + 4a^2c_2\log(2))}{\pi} \\
& + \frac{8ac_2\sqrt{x}}{\pi} + \frac{2c_2}{\pi}
\end{aligned}
$$

4.9 problem 9

Internal problem ID [4719]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XV. page 194
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - (x^2+4x)\,y' + 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 61

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-(x^2+4*x)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(c_1x^3\left(1+x+\frac{1}{2}x^2+\frac{1}{6}x^3+\frac{1}{24}x^4+\frac{1}{120}x^5+\mathrm{O}\left(x^6\right)\right) + c_2\left(\ln(x)\left(6x^3+6x^4+3x^5+\mathrm{O}\left(x^6\right)\right) + \left(12-6x+6x^2+11x^3+5x^4+x^5+\mathrm{O}\left(x^6\right)\right)\right)\right)x$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 74

```
AsymptoticDSolveValue[x^2*y''[x]-(x^2+4*x)*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{1}{2}(x+1)x^4\log(x)+\frac{1}{4}\left(x^4+3x^3+2x^2-2x+4\right)x\right)+c_2\left(\frac{x^8}{24}+\frac{x^7}{6}+\frac{x^6}{2}+x^5+x^4\right)$$

4.10 problem 10

Internal problem ID [4720]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XV. page 194
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_elliptic, _class_II]]`

$$\boxed{x(-x^2+1)\,y''+(-x^2+1)\,y'+xy=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 41

```
Order:=6;
dsolve(x*(1-x^2)*diff(y(x),x$2)+(1-x^2)*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(c_2\ln(x)+c_1)\left(1-\frac{1}{4}x^2-\frac{3}{64}x^4+\mathrm{O}\left(x^6\right)\right)+\left(\frac{1}{4}x^2+\frac{1}{128}x^4+\mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 60

```
AsymptoticDSolveValue[x*(1-x^2)*y''[x]+(1-x^2)*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(-\frac{3x^4}{64}-\frac{x^2}{4}+1\right)+c_2\left(\frac{x^4}{128}+\frac{x^2}{4}+\left(-\frac{3x^4}{64}-\frac{x^2}{4}+1\right)\log(x)\right)$$

4.11 problem 11

Internal problem ID [4721]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XV. page 194
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$4x(1-x)\,y''-4y'-y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 60

```
Order:=6;
dsolve(4*x*(1-x)*diff(y(x),x$2)-4*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1x^2\left(1+\frac{3}{4}x+\frac{75}{128}x^2+\frac{245}{512}x^3+\frac{6615}{16384}x^4+\frac{22869}{65536}x^5+\mathrm{O}\left(x^6\right)\right)+c_2\left(\ln\left(x\right)\left(\frac{1}{16}x^2+\frac{3}{64}x^3+\frac{75}{2048}x^4+\frac{245}{8192}x^5+\mathrm{O}\left(x^6\right)\right)+\left(-2+\frac{1}{2}x+\frac{1}{2}x^2+\frac{3}{8}x^3+\frac{2415}{8192}x^4+\frac{23779}{98304}x^5+\mathrm{O}\left(x^6\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 86

```
AsymptoticDSolveValue[4*x*(1-x)*y''[x]-4*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{135x^4+192x^3+256x^2-4096x+16384}{16384}-\frac{x^2(75x^2+96x+128)\log(x)}{4096}\right)+c_2\left(\frac{6615x^6}{16384}+\frac{245x^5}{512}+\frac{75x^4}{128}+\frac{3x^3}{4}+x^2\right)$$

4.12 problem 12

Internal problem ID [4722]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XV. page 194
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x^3y'' + y = x^{\frac{3}{2}}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(x^3*diff(y(x),x$2)+y(x)=x^(3/2),y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.251 (sec). Leaf size: 688

```
AsymptoticDSolveValue[x^3*y''[x]+y[x]==x^(3/2),y[x],{x,0,5}]
```

$$y(x) \to \frac{e^{\frac{2i}{\sqrt{x}}}x^{3/4}\left(\frac{468131288625ix^{9/2}}{8796093022208} - \frac{66891825ix^{7/2}}{4294967296} + \frac{72765ix^{5/2}}{8388608} - \frac{105ix^{3/2}}{8192} + \frac{33424574007825x^5}{281474976710656} - \frac{14783093325x^4}{549755813888} + \frac{2837835x^3}{268435456}\right.}{}$$

$$+ \frac{e^{-\frac{2i}{\sqrt{x}}}x^{3/4}\left(-\frac{468131288625ix^{9/2}}{8796093022208} + \frac{66891825ix^{7/2}}{4294967296} - \frac{72765ix^{5/2}}{8388608} + \frac{105ix^{3/2}}{8192} + \frac{33424574007825x^5}{281474976710656} - \frac{14783093325x^4}{549755813888} + \frac{28378}{26843}\right.}{}$$

$$+ c_1e^{-\frac{2i}{\sqrt{x}}}x^{3/4}\left(-\frac{468131288625ix^{9/2}}{8796093022208} + \frac{66891825ix^{7/2}}{4294967296} - \frac{72765ix^{5/2}}{8388608} + \frac{105ix^{3/2}}{8192} + \frac{33424574007825x^5}{281474976710656} - \frac{14}{54}\right.$$

4.13 problem 13

Internal problem ID [4723]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XV. page 194
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$2x^2y'' - (2+3x)\,y' + \frac{(2x-1)\,y}{x} = \sqrt{x}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)-(3*x+2)*diff(y(x),x)+(2*x-1)/x*y(x)=x^(1/2),y(x),type='series',x
```

No solution found

✓ Solution by Mathematica

Time used: 0.158 (sec). Leaf size: 222

```
AsymptoticDSolveValue[2*x^2*y''[x]-(3*x+2)*y'[x]+(2*x-1)/x*y[x]==x^(1/2),y[x],{x,0,5}]
```

$$y(x) \to \frac{1}{256}e^{-1/x}\left(-\frac{405405x^5}{16} + \frac{45045x^4}{16} - \frac{693x^3}{2} + \frac{189x^2}{4} - 7x + 1\right)x^4\left(\frac{2e^{\frac{1}{x}}(15663375x^7 + 20072325x^6 + 10329540x^5 + 4131816x^4 + 2754544x^3 + 5509088x^2 - 64x}{x^{3/2}} - 11018112\sqrt{\pi}\mathrm{erfi}\left(\frac{1}{\sqrt{x}}\right)\right)$$

$$+c_2e^{-1/x}\left(-\frac{405405x^5}{16} + \frac{45045x^4}{16} - \frac{693x^3}{2} + \frac{189x^2}{4} - 7x + 1\right)x^4 + \frac{\left(\frac{5x}{2}+1\right)\left(-\frac{15015x^6}{64} + \frac{693x^5}{20} - \frac{189x^4}{32} + 7\right.}{\sqrt{x}}$$

4.14 problem 14

Internal problem ID [4724]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XV. page 194
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{(-x^2 + x)\, y'' + 3y' + 2y = 3x^2}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 50

```
Order:=6;
dsolve((x-x^2)*diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=3*x^2,y(x),type='series',x=0);
```

$$y(x) = c_1\left(1 - \frac{2}{3}x + \frac{1}{6}x^2 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2(-2 + 8x - 12x^2 + 8x^3 - 2x^4 + \mathrm{O}(x^6))}{x^2} + x^3\left(\frac{1}{5} + \frac{1}{30}x + \frac{1}{105}x^2 + \mathrm{O}\left(x^3\right)\right)$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 91

```
AsymptoticDSolveValue[(x-x^2)*y''[x]+3*y'[x]+2*y[x]==3*x^2,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^2}{6} - \frac{2x}{3} + 1\right) + \frac{c_2(1 - 4x)}{x^2} + \frac{(1 - 4x)\left(-\frac{5x^6}{6} - \frac{3x^5}{10}\right)}{x^2} + \left(\frac{x^2}{6} - \frac{2x}{3} + 1\right)\left(-5x^6 - \frac{9x^5}{5} + \frac{x^3}{2}\right)$$

5 Chapter VII, Solutions in series. Examples XVI. page 220

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5.1 problem 5

Internal problem ID [4725]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XVI. page 220
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$x(1-x)\,y'' + \left(\frac{3}{2} - 2x\right)y' - \frac{y}{4} = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 34

```
Order:=6;
dsolve(x*(1-x)*diff(y(x),x$2)+(3/2-2*x)*diff(y(x),x)-1/4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1(1 + \mathrm{O}(x^6))}{\sqrt{x}} + c_2\left(1 + \frac{1}{6}x + \frac{3}{40}x^2 + \frac{5}{112}x^3 + \frac{35}{1152}x^4 + \frac{63}{2816}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 50

```
AsymptoticDSolveValue[x*(1-x)*y''[x]+(3/2-2*x)*y'[x]-1/4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{63x^5}{2816} + \frac{35x^4}{1152} + \frac{5x^3}{112} + \frac{3x^2}{40} + \frac{x}{6} + 1\right) + \frac{c_2}{\sqrt{x}}$$

5.2 problem 6

Internal problem ID [4726]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XVI. page 220
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x(1-x)\,y''+xy'-y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
Order:=6;
dsolve(2*x*(1-x)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(\frac{1}{2}x+\mathrm{O}\left(x^{6}\right)\right)\ln\left(x\right)c_2+c_1x\left(1+\mathrm{O}\left(x^{6}\right)\right)+\left(1-\frac{1}{2}x+\frac{1}{8}x^{2}+\frac{1}{32}x^{3}+\frac{5}{384}x^{4}+\frac{7}{1024}x^{5}+\mathrm{O}\left(x^{6}\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 43

```
AsymptoticDSolveValue[2*x*(1-x)*y''[x]+x*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{1}{384}\left(5x^{4}+12x^{3}+48x^{2}-768x+384\right)+\frac{1}{2}x\log(x)\right)+c_2x$$

5.3 problem 8

Internal problem ID [4727]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XVI. page 220
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$\boxed{2x(1-x)\,y''+(1-11x)\,y'-10y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 44

```
Order:=6;
dsolve(2*x*(1-x)*diff(y(x),x$2)+(1-11*x)*diff(y(x),x)-10*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1\sqrt{x}\left(1+5x+14x^2+30x^3+55x^4+91x^5+\mathrm{O}\left(x^6\right)\right)\\+c_2\left(1+10x+35x^2+84x^3+165x^4+286x^5+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 65

```
AsymptoticDSolveValue[2*x*(1-x)*y''[x]+(1-11*x)*y'[x]-10*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\sqrt{x}\left(91x^5+55x^4+30x^3+14x^2+5x+1\right)\\+c_2\left(286x^5+165x^4+84x^3+35x^2+10x+1\right)$$

5.4 problem 9

Internal problem ID [4728]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XVI. page 220
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$x(1-x)\,y'' + \frac{(1-2x)\,y'}{3} + \frac{20y}{9} = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 36

```
Order:=6;
dsolve(x*(1-x)*diff(y(x),x$2)+1/3*(1-2*x)*diff(y(x),x)+20/9*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{2}{3}}\left(1-\frac{6}{5}x+\mathrm{O}\left(x^6\right)\right)+c_2\left(1-\frac{20}{3}x+\frac{35}{9}x^2+\frac{50}{81}x^3+\frac{65}{243}x^4+\frac{112}{729}x^5+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 57

```
AsymptoticDSolveValue[x*(1-x)*y''[x]+1/3*(1-2*x)*y'[x]+20/9*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(1-\frac{6x}{5}\right)x^{2/3}+c_2\left(\frac{112x^5}{729}+\frac{65x^4}{243}+\frac{50x^3}{81}+\frac{35x^2}{9}-\frac{20x}{3}+1\right)$$

5.5 problem 10

Internal problem ID [4729]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XVI. page 220
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{2x(1-x)\,y''+y'+4y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 38

```
Order:=6;
dsolve(2*x*(1-x)*diff(y(x),x$2)+diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1\sqrt{x}\left(1-\frac{3}{2}x+\frac{3}{8}x^2+\frac{1}{16}x^3+\frac{3}{128}x^4+\frac{3}{256}x^5+\mathrm{O}\left(x^6\right)\right)+c_2\left(1-4x+\frac{8}{3}x^2+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 62

```
AsymptoticDSolveValue[2*x*(1-x)*y''[x]+y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{8x^2}{3}-4x+1\right)+c_1\sqrt{x}\left(\frac{3x^5}{256}+\frac{3x^4}{128}+\frac{x^3}{16}+\frac{3x^2}{8}-\frac{3x}{2}+1\right)$$

5.6 problem 11

Internal problem ID [4730]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter VII, Solutions in series. Examples XVI. page 220
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4y'' + \frac{3(-x^2+2)\,y}{(-x^2+1)^2} = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(4*diff(y(x),x$2)+3*(2-x^2)/(1-x^2)^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{3}{4}x^2 - \frac{3}{32}x^4\right) y(0) + \left(x - \frac{1}{4}x^3 - \frac{3}{32}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[4*y''[x]+3*(2-x^2)/(1-x^2)^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(-\frac{3x^5}{32} - \frac{x^3}{4} + x\right) + c_1\left(-\frac{3x^4}{32} - \frac{3x^2}{4} + 1\right)$$

6 Chapter IX, Special forms of differential equations. Examples XVII. page 247

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6.1 problem 1

Internal problem ID [4731]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter IX, Special forms of differential equations. Examples XVII. page 247
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y' + y^2 = \frac{a^2}{x^4}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x)+y(x)^2=a^2/x^4,y(x), singsol=all)
```

$$y(x) = \frac{-\sqrt{-a^2}\tan\left(\frac{\sqrt{-a^2}\,(c_1x-1)}{x}\right) + x}{x^2}$$

✓ Solution by Mathematica

Time used: 0.384 (sec). Leaf size: 71

```
DSolve[y'[x]+y[x]^2==a^2/x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-2a^2c_1e^{\frac{2a}{x}} + 2ac_1xe^{\frac{2a}{x}} + a + x}{x^2\left(1 + 2ac_1e^{\frac{2a}{x}}\right)}$$

$$y(x) \to \frac{x-a}{x^2}$$

6.2 problem 2

Internal problem ID [4732]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter IX, Special forms of differential equations. Examples XVII. page 247
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{u'' - \frac{a^2 u}{x^{\frac{2}{3}}} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 41

```
dsolve(diff(u(x),x$2)-a^2*x^(-2/3)*u(x)=0,u(x), singsol=all)
```

$$u(x) = \sqrt{x}\left(\text{BesselY}\left(\frac{3}{4}, \frac{3\sqrt{-a^2}\, x^{\frac{2}{3}}}{2}\right) c_2 + \text{BesselJ}\left(\frac{3}{4}, \frac{3\sqrt{-a^2}\, x^{\frac{2}{3}}}{2}\right) c_1\right)$$

✓ Solution by Mathematica
Time used: 0.05 (sec). Leaf size: 79

```
DSolve[u''[x]-a^2*x^(-2/3)*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to \frac{3^{3/4} a^{3/4} \sqrt{x}\left(16 c_1 \text{Gamma}\left(\frac{5}{4}\right) \text{BesselI}\left(-\frac{3}{4}, \frac{3}{2} a x^{2/3}\right) + 3(-1)^{3/4} c_2 \text{Gamma}\left(\frac{3}{4}\right) \text{BesselI}\left(\frac{3}{4}, \frac{3}{2} a x^{2/3}\right)\right)}{8\sqrt{2}}$$

6.3 problem 3

Internal problem ID [4733]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter IX, Special forms of differential equations. Examples XVII. page 247
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{u'' - \frac{2u'}{x} - a^2 u = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(u(x),x$2)-2/x*diff(u(x),x)-a^2*u(x)=0,u(x), singsol=all)
```

$$u(x) = c_1 e^{ax}(ax - 1) + c_2 e^{-ax}(ax + 1)$$

✓ Solution by Mathematica
Time used: 0.13 (sec). Leaf size: 68

```
DSolve[u''[x]-2/x*u'[x]-a^2*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to \frac{\sqrt{\frac{2}{\pi}}\sqrt{x}((iac_2x + c_1)\sinh(ax) - (ac_1x + ic_2)\cosh(ax))}{a\sqrt{-iax}}$$

6.4 problem 4

Internal problem ID [4734]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter IX, Special forms of differential equations. Examples XVII. page 247
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{u'' + \frac{2u'}{x} - a^2 u = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(u(x),x$2)+2/x*diff(u(x),x)-a^2*u(x)=0,u(x), singsol=all)
```

$$u(x) = \frac{c_1 \sinh(ax) + c_2 \cosh(ax)}{x}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 35

```
DSolve[u''[x]+2/x*u'[x]-a^2*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to \frac{2ac_1 e^{-ax} + c_2 e^{ax}}{2ax}$$

6.5 problem 5

Internal problem ID [4735]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter IX, Special forms of differential equations. Examples XVII. page 247
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{u'' + \frac{2u'}{x} + a^2 u = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(u(x),x$2)+2/x*diff(u(x),x)+a^2*u(x)=0,u(x), singsol=all)
```

$$u(x) = \frac{c_1 \sin(ax) + c_2 \cos(ax)}{x}$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 42

```
DSolve[u''[x]+2/x*u'[x]+a^2*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to \frac{e^{-iax}\left(2c_1 - \frac{ic_2 e^{2iax}}{a}\right)}{2x}$$

6.6 problem 6

Internal problem ID [4736]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter IX, Special forms of differential equations. Examples XVII. page 247
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{u'' + \frac{4u'}{x} - a^2 u = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(u(x),x$2)+4/x*diff(u(x),x)-a^2*u(x)=0,u(x), singsol=all)
```

$$u(x) = \frac{c_1 e^{ax}(ax-1) + c_2 e^{-ax}(ax+1)}{x^3}$$

✓ Solution by Mathematica
Time used: 0.091 (sec). Leaf size: 68

```
DSolve[u''[x]+4/x*u'[x]-a^2*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to \frac{\sqrt{\frac{2}{\pi}}\left((iac_2x + c_1)\sinh(ax) - (ac_1x + ic_2)\cosh(ax)\right)}{ax^{5/2}\sqrt{-iax}}$$

6.7 problem 7

Internal problem ID [4737]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter IX, Special forms of differential equations. Examples XVII. page 247
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{u'' + \frac{4u'}{x} + a^2 u = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(diff(u(x),x$2)+4/x*diff(u(x),x)+a^2*u(x)=0,u(x), singsol=all)
```

$$u(x) = \frac{(a c_1 x + c_2) \cos (a x) + \sin (a x) (a c_2 x - c_1)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.094 (sec). Leaf size: 57

```
DSolve[u''[x]+4/x*u'[x]+a^2*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to -\frac{\sqrt{\frac{2}{\pi}}((a c_1 x + c_2) \cos(a x) + (a c_2 x - c_1) \sin(a x))}{x^{3/2}(a x)^{3/2}}$$

6.8 problem 8

Internal problem ID [4738]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter IX, Special forms of differential equations. Examples XVII. page 247
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y a^2 - \frac{6y}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(diff(y(x),x$2)-a^2*y(x)=6*y(x)/x^2,y(x), singsol=all)
```

$$y(x) = \frac{c_2 e^{-ax}(x^2a^2 + 3ax + 3) + c_1 e^{ax}(x^2a^2 - 3ax + 3)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.138 (sec). Leaf size: 90

```
DSolve[y''[x]-a^2*y[x]==6*y[x]/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{\frac{2}{\pi}}\left((a^2c_2x^2 - 3iac_1x + 3c_2)\cosh(ax) + i(c_1(a^2x^2 + 3) + 3iac_2x)\sinh(ax)\right)}{a^2x^{3/2}\sqrt{-iax}}$$

6.9 problem 9

Internal problem ID [4739]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter IX, Special forms of differential equations. Examples XVII. page 247
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + yn^2 - \frac{6y}{x^2} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 53

```
dsolve(diff(y(x),x$2)+n^2*y(x)=6*y(x)/x^2,y(x), singsol=all)
```

$$y(x) = \frac{(c_1 n^2 x^2 + 3c_2 nx - 3c_1)\cos(nx) + \sin(nx)(c_2 n^2 x^2 - 3c_1 nx - 3c_2)}{x^2}$$

✓ Solution by Mathematica
Time used: 0.143 (sec). Leaf size: 79

```
DSolve[y''[x]+n^2*y[x]==6*y[x]/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}}\sqrt{x}((c_2(-n^2)x^2 + 3c_1 nx + 3c_2)\cos(nx) + (c_1(n^2x^2 - 3) + 3c_2 nx)\sin(nx))}{(nx)^{5/2}}$$

6.10 problem 10

Internal problem ID [4740]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter IX, Special forms of differential equations. Examples XVII. page 247
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + xy' - \left(x^2 + \frac{1}{4}\right)y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-(x^2+1/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sinh(x) + c_2 \cosh(x)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 32

```
DSolve[x^2*y''[x]+x*y'[x]-(x^2+1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}(c_2 e^{2x} + 2c_1)}{2\sqrt{x}}$$

6.11 problem 11

Internal problem ID [4741]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter IX, Special forms of differential equations. Examples XVII. page 247
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' + \frac{(-9a^2 + 4x^2)\, y}{4a^2} = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 41

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(4*x^2-9*a^2)/(4*a^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{(ix + a)\, c_2 \mathrm{e}^{-\frac{ix}{a}} + (-ix + a)\, \mathrm{e}^{\frac{ix}{a}} c_1}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica
Time used: 0.101 (sec). Leaf size: 62

```
DSolve[x^2*y''[x]+x*y'[x]+(4*x^2-9*a^2)/(4*a^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}}\left(\left(ac_2 + c_1x\right)\cos\left(\frac{x}{a}\right) + \left(c_2x - ac_1\right)\sin\left(\frac{x}{a}\right)\right)}{x\sqrt{\frac{x}{a}}}$$

6.12 problem 12

Internal problem ID [4742]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter IX, Special forms of differential equations. Examples XVII. page 247
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + xy' + \left(x^2 - \frac{25}{4}\right)y = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 47

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-25/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-3\left(ix - \frac{1}{3}x^2 + 1\right)c_2 e^{-ix} + 3\left(ix + \frac{1}{3}x^2 - 1\right)c_1 e^{ix}}{x^{\frac{5}{2}}}$$

✓ Solution by Mathematica
Time used: 0.111 (sec). Leaf size: 59

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-25/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}}\left(\left(-c_2x^2 + 3c_1x + 3c_2\right)\cos(x) + \left(c_1\left(x^2-3\right) + 3c_2x\right)\sin(x)\right)}{x^{5/2}}$$

6.13 problem 15

Internal problem ID [4743]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter IX, Special forms of differential equations. Examples XVII. page 247
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + qy' - \frac{2y}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)+q*diff(y(x),x)=2*y(x)/x^2,y(x), singsol=all)
```

$$y(x) = \frac{c_2 \mathrm{e}^{-qx}(qx+2) + c_1(qx-2)}{x}$$

✓ Solution by Mathematica

Time used: 0.064 (sec). Leaf size: 80

```
DSolve[y''[x]+q*y'[x]==2*y[x]/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{qx^{3/2}e^{-\frac{qx}{2}}\left(2(ic_2qx+2c_1)\sinh\left(\frac{qx}{2}\right) - 2(c_1qx+2ic_2)\cosh\left(\frac{qx}{2}\right)\right)}{\sqrt{\pi}(-iqx)^{5/2}}$$

6.14 problem 18

Internal problem ID [4744]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter IX, Special forms of differential equations. Examples XVII. page 247
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + e^{2x}y - yn^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+exp(2*x)*y(x)=n^2*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}(n, e^x) + c_2 \operatorname{BesselY}(n, e^x)$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 46

```
DSolve[y''[x]+Exp[2*x]*y[x]==n^2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{Gamma}(1-n) \operatorname{BesselJ}\left(-n, \sqrt{e^{2x}}\right) + c_2 \operatorname{Gamma}(n+1) \operatorname{BesselJ}\left(n, \sqrt{e^{2x}}\right)$$

6.15 problem 19

Internal problem ID [4745]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter IX, Special forms of differential equations. Examples XVII. page 247
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + \frac{y}{4x} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+y(x)/(4*x)=0,y(x), singsol=all)
```

$$y(x) = \left(\operatorname{BesselY}\left(1, \sqrt{x}\right) c_2 + \operatorname{BesselJ}\left(1, \sqrt{x}\right) c_1\right) \sqrt{x}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 38

```
DSolve[y''[x]+y[x]/(4*x)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\sqrt{x}\left(c_1 \operatorname{BesselJ}\left(1, \sqrt{x}\right) + 2ic_2 \operatorname{BesselY}\left(1, \sqrt{x}\right)\right)$$

6.16 problem 20

Internal problem ID [4746]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter IX, Special forms of differential equations. Examples XVII. page 247
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y''x + y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x$2)+diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}\left(0, 2\sqrt{x}\right) + c_2 \operatorname{BesselY}\left(0, 2\sqrt{x}\right)$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 31

```
DSolve[x*y''[x]+y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}\left(0, 2\sqrt{x}\right) + 2c_2 \operatorname{BesselY}\left(0, 2\sqrt{x}\right)$$

6.17 problem 21

Internal problem ID [4747]

Book: A treatise on ordinary and partial differential equations by William Woolsey Johnson. 1913
Section: Chapter IX, Special forms of differential equations. Examples XVII. page 247
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y''x + 3y' + 4yx^3 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(x*diff(y(x),x$2)+3*diff(y(x),x)+4*x^3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x^2) + c_2 \cos(x^2)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.08 (sec). Leaf size: 41

```
DSolve[x*y''[x]+3*y'[x]+4*x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4c_1 e^{-ix^2} - ic_2 e^{ix^2}}{4x^2}$$