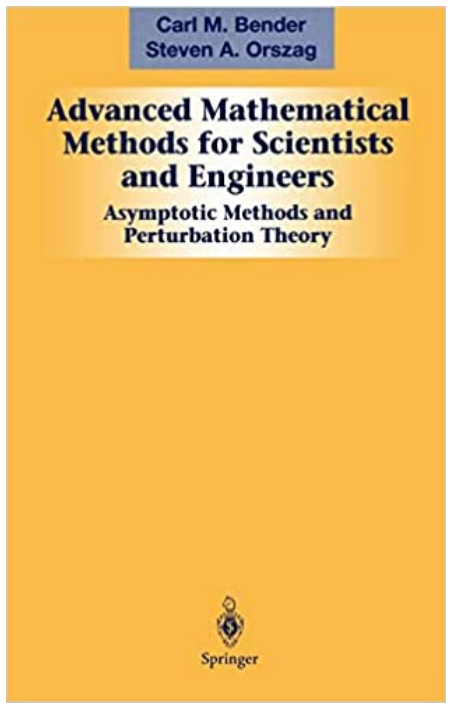
A Solution Manual For

Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag. Springer October 29, 1999



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May 16, 2024

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1 Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136

1.1 problem 3.5

Internal problem ID [5480]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag. Springer October 29, 1999
Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136
Problem number: 3.5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x-1)(-2+x)\,y''+(4x-6)\,y'+2y=0}$$

With initial conditions

$$[y(0)=2, y'(0)=1]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(x-1)*(x-2)*diff(y(x),x$2)+(4*x-6)*diff(y(x),x)+2*y(x)=0,y(0) = 2, D(y)(0) = 1],y(x)
```

$$y(x)=2+x+\frac{1}{2}x^2+\frac{1}{4}x^3+\frac{1}{8}x^4+\frac{1}{16}x^5+\mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[{(x-1)*(x-2)*y''[x]+(4*x-6)*y'[x]+2*y[x]==0,{y[0]==2,y'[0]==1}},y[x],{
```

$$y(x)\to \frac{x^5}{16}+\frac{x^4}{8}+\frac{x^3}{4}+\frac{x^2}{2}+x+2$$

1.2 problem 3.6 (a)

Internal problem ID [5481]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag. Springer October 29, 1999
Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136
Problem number: 3.6 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2xy' + 8y = 0}$$

With initial conditions

$$[y(0) = 4, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([diff(y(x),x$2)-2*x*diff(y(x),x)+8*y(x)=0,y(0) = 4, D(y)(0) = 0],y(x),type='series',x
```

$$y(x) = 4 - 16x^2 + \frac{16}{3}x^4 + O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 17

```
AsymptoticDSolveValue[{y''[x]-2*x*y'[x]+8*y[x]==0,{y[0]==4,y'[0]==0}},y[x],{x,0,5}]
```

$$y(x) \to \frac{16x^4}{3} - 16x^2 + 4$$

1.3 problem 3.6 (b)

Internal problem ID [5482]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag. Springer October 29, 1999
Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136
Problem number: 3.6 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2xy' + 8y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 4]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([diff(y(x),x$2)-2*x*diff(y(x),x)+8*y(x)=0,y(0) = 0, D(y)(0) = 4],y(x),type='series',x
```

$$y(x) = 4x - 4x^3 + \frac{2}{5}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{y''[x]-2*x*y'[x]+8*y[x]==0,{y[0]==0,y'[0]==4}},y[x],{x,0,5}]
```

$$y(x) \to \frac{2x^5}{5} - 4x^3 + 4x$$

1.4 problem 3.6 (c)

Internal problem ID [5483]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag. Springer October 29, 1999
Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136
Problem number: 3.6 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-x^2+1)\,y''-2xy'+12y=0}$$

With initial conditions

$$[y(0)=0, y'(0)=3]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
Order:=6;
dsolve([(1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+12*y(x)=0,y(0) = 0, D(y)(0) = 3],y(x),type='
```

$$y(x)=-5x^3+3x$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 12

```
AsymptoticDSolveValue[{(1-x^2)*y''[x]-2*x*y'[x]+12*y[x]==0,{y[0]==0,y'[0]==3}},y[x],{x,0,5}]
```

$$y(x)\to 3x-5x^3$$

1.5 problem 3.6 (d)

Internal problem ID [5484]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag. Springer October 29, 1999
Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136
Problem number: 3.6 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - (x-1)\, y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
Order:=6;
dsolve([diff(y(x),x$2)=(x-1)*y(x),y(0) = 1, D(y)(0) = 0],y(x),type='series',x=0);
```

$$y(x) = 1 - \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{30}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 33

```
AsymptoticDSolveValue[{y''[x]==(x-1)*y[x],{y[0]==1,y'[0]==0}},y[x],{x,0,5}]
```

$$y(x) \to -\frac{x^5}{30} + \frac{x^4}{24} + \frac{x^3}{6} - \frac{x^2}{2} + 1$$

1.6 problem 3.24 (a)

Internal problem ID [5485]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag. Springer October 29, 1999
Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136
Problem number: 3.24 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(x+2)\,y''+2(1+x)\,y'-2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 43

```
Order:=6;
dsolve(x*(x+2)*diff(y(x),x$2)+2*(x+1)*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(c_2\ln(x)+c_1)\left(1+x+\mathrm{O}\left(x^6\right)\right)\\+\left(-\frac{5}{2}x-\frac{3}{8}x^2+\frac{1}{12}x^3-\frac{5}{192}x^4+\frac{3}{320}x^5+\mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 53

```
AsymptoticDSolveValue[x*(x+2)*y''[x]+2*(x+1)*y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{3x^5}{320}-\frac{5x^4}{192}+\frac{x^3}{12}-\frac{3x^2}{8}-\frac{5x}{2}+(x+1)\log(x)\right)+c_1(x+1)$$

1.7 problem 3.24 (b)

Internal problem ID [5486]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag. Springer October 29, 1999
Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136
Problem number: 3.24 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 58

```
Order:=6;
dsolve(x*diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x \left(1 - \frac{1}{2}x + \frac{1}{12}x^2 - \frac{1}{144}x^3 + \frac{1}{2880}x^4 - \frac{1}{86400}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2 \left(\ln(x)\left(-x + \frac{1}{2}x^2 - \frac{1}{12}x^3 + \frac{1}{144}x^4 - \frac{1}{2880}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(1 - \frac{3}{4}x^2 + \frac{7}{36}x^3 - \frac{35}{1728}x^4 + \frac{101}{86400}x^5 + \mathrm{O}\left(x^6\right)\right)\right)$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 85

```
AsymptoticDSolveValue[x*y''[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{1}{144}x\left(x^3 - 12x^2 + 72x - 144\right)\log(x) + \frac{-47x^4 + 480x^3 - 2160x^2 + 1728x + 1728}{1728}\right) + c_2\left(\frac{x^5}{2880} - \frac{x^4}{144} + \frac{x^3}{12} - \frac{x^2}{2} + x\right)$$

1.8 problem 3.24 (c)

Internal problem ID [5487]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag. Springer October 29, 1999
Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136
Problem number: 3.24 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (e^x - 1)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
Order:=6;
dsolve(diff(y(x),x$2)+(exp(x)-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{6}x^3 - \frac{1}{24}x^4 - \frac{1}{120}x^5\right) y(0) + \left(x - \frac{1}{12}x^4 - \frac{1}{40}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 49

```
AsymptoticDSolveValue[y''[x]+(Exp[x]-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(-\frac{x^5}{40} - \frac{x^4}{12} + x\right) + c_1\left(-\frac{x^5}{120} - \frac{x^4}{24} - \frac{x^3}{6} + 1\right)$$

1.9 problem 3.24 (d)

Internal problem ID [5488]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag. Springer October 29, 1999
Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136
Problem number: 3.24 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x(1-x)\,y''-3xy'-y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 60

```
Order:=6;
dsolve(x*(1-x)*diff(y(x),x$2)-3*x*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned}y(x)=&\ln(x)\left(x+2x^2+3x^3+4x^4+5x^5+\mathrm{O}\left(x^6\right)\right)c_2\\&+c_1x\left(1+2x+3x^2+4x^3+5x^4+6x^5+\mathrm{O}\left(x^6\right)\right)\\&+\left(1+3x+5x^2+7x^3+9x^4+11x^5+\mathrm{O}\left(x^6\right)\right)c_2\end{aligned}$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 63

```
AsymptoticDSolveValue[x*(1-x)*y''[x]-3*x*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned}y(x)\to\ &c_1\left(x^4+x^3+x^2+\left(4x^3+3x^2+2x+1\right)x\log(x)+x+1\right)\\&+c_2\left(5x^5+4x^4+3x^3+2x^2+x\right)\end{aligned}$$

1.10 problem 3.24 (e)

Internal problem ID [5489]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag. Springer October 29, 1999
Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136
Problem number: 3.24 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{2xy'' - y' + yx^2 = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 28

```
Order:=6;
dsolve(2*x*diff(y(x),x$2)-diff(y(x),x)+x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{3}{2}} \left(1 - \frac{1}{27} x^3 + \mathrm{O}\left(x^6\right)\right) + c_2 \left(1 - \frac{1}{9} x^3 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 33

```
AsymptoticDSolveValue[2*x*y''[x]-y'[x]+x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 \left(1 - \frac{x^3}{9}\right) + c_1 \left(1 - \frac{x^3}{27}\right) x^{3/2}$$

1.11 problem 3.24 (f)

Internal problem ID [5490]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag. Springer October 29, 1999
Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136
Problem number: 3.24 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\sin(x)\, y'' - 2\cos(x)\, y' - \sin(x)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 32

```
Order:=6;
dsolve(sin(x)*diff(y(x),x$2)-2*cos(x)*diff(y(x),x)-sin(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^3 \left(1 - \frac{1}{10}x^2 + \frac{1}{280}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2 \left(12 - 6x^2 + \frac{1}{2}x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 44

```
AsymptoticDSolveValue[Sin[x]*y''[x]-2*Cos[x]*y'[x]-Sin[x]*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{x^4}{24} - \frac{x^2}{2} + 1\right) + c_2 \left(\frac{x^7}{280} - \frac{x^5}{10} + x^3\right)$$

1.12 problem 3.24 (g)

Internal problem ID [5491]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag. Springer October 29, 1999
Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136
Problem number: 3.24 (g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - yx^2 = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)-x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{x^4}{12}\right) y(0) + \left(x + \frac{1}{20}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]-x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{20} + x\right) + c_1\left(\frac{x^4}{12} + 1\right)$$

1.13 problem 3.24 (h)

Internal problem ID [5492]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag. Springer October 29, 1999
Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136
Problem number: 3.24 (h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{x(x+2)\,y''+(1+x)\,y'-4y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 38

```
Order:=6;
dsolve(x*(x+2)*diff(y(x),x$2)+(x+1)*diff(y(x),x)-4*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1\sqrt{x}\left(1+\frac{5}{4}x+\frac{7}{32}x^2-\frac{3}{128}x^3+\frac{11}{2048}x^4-\frac{13}{8192}x^5+\mathrm{O}\left(x^6\right)\right)\\+c_2\left(1+4x+2x^2+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 60

```
AsymptoticDSolveValue[x*(x+2)*y''[x]+(x+1)*y'[x]-4*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(2x^2+4x+1\right)+c_1\sqrt{x}\left(-\frac{13x^5}{8192}+\frac{11x^4}{2048}-\frac{3x^3}{128}+\frac{7x^2}{32}+\frac{5x}{4}+1\right)$$

1.14 problem 3.24 (i)

Internal problem ID [5493]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag. Springer October 29, 1999
Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136
Problem number: 3.24 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{xy'' + \left(\frac{1}{2} - x\right) y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 44

```
Order:=6;
dsolve(x*diff(y(x),x$2)+(1/2-x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(1 + 2x + \frac{4}{3}x^2 + \frac{8}{15}x^3 + \frac{16}{105}x^4 + \frac{32}{945}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 79

```
AsymptoticDSolveValue[x*y''[x]+(1/2-x)*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1\right) + c_2\left(\frac{32x^5}{945} + \frac{16x^4}{105} + \frac{8x^3}{15} + \frac{4x^2}{3} + 2x + 1\right)$$

1.15 problem 3.25 v=1/2

Internal problem ID [5494]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag. Springer October 29, 1999
Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136
Problem number: 3.25 v=1/2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' + \left(x^2 + \frac{1}{4}\right)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2+(1/2)^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{-\frac{i}{2}}\left(1 + \left(-\frac{1}{5} - \frac{i}{10}\right)x^2 + \left(\frac{7}{680} + \frac{3i}{340}\right)x^4 + \mathrm{O}\left(x^6\right)\right) + c_2 x^{\frac{i}{2}}\left(1 + \left(-\frac{1}{5} + \frac{i}{10}\right)x^2 + \left(\frac{7}{680} - \frac{3i}{340}\right)x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 66

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2+1/4)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \left(\frac{7}{680} + \frac{3i}{340}\right)c_2 x^{-\frac{i}{2}}\left(x^4 - (16 - 4i)x^2 + (56 - 48i)\right) + \left(\frac{7}{680} - \frac{3i}{340}\right)c_1 x^{\frac{i}{2}}\left(x^4 - (16 + 4i)x^2 + (56 + 48i)\right)$$

1.16 problem 3.25 v=3/2

Internal problem ID [5495]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag. Springer October 29, 1999
Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136
Problem number: 3.25 v=3/2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + xy' + \left(x^2 + \frac{9}{4}\right)y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2+(3/2)^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{-\frac{3i}{2}}\left(1 + \left(-\frac{1}{13} - \frac{3i}{26}\right)x^2 + \left(-\frac{1}{2600} + \frac{9i}{1300}\right)x^4 + \mathrm{O}\left(x^6\right)\right) + c_2 x^{\frac{3i}{2}}\left(1 + \left(-\frac{1}{13} + \frac{3i}{26}\right)x^2 + \left(-\frac{1}{2600} - \frac{9i}{1300}\right)x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 66

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2+9/4)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \left(-\frac{1}{2600} - \frac{9i}{1300}\right)c_1 x^{\frac{3i}{2}}\left(x^4 - (16 + 12i)x^2 - (8 - 144i)\right) - \left(\frac{1}{2600} - \frac{9i}{1300}\right)c_2 x^{-\frac{3i}{2}}\left(x^4 - (16 - 12i)x^2 - (8 + 144i)\right)$$

1.17 problem 3.25 v=5/2

Internal problem ID [5496]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag. Springer October 29, 1999
Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136
Problem number: 3.25 v=5/2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' + \left(x^2 + \frac{25}{4}\right) y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2+(5/2)^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{-\frac{5i}{2}} \left(1 + \left(-\frac{1}{29} - \frac{5i}{58}\right) x^2 + \left(-\frac{17}{9512} + \frac{15i}{4756}\right) x^4 + \mathrm{O}\left(x^6\right)\right) + c_2 x^{\frac{5i}{2}} \left(1 + \left(-\frac{1}{29} + \frac{5i}{58}\right) x^2 + \left(-\frac{17}{9512} - \frac{15i}{4756}\right) x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 66

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2+(5/2)^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \left(-\frac{17}{9512} - \frac{15i}{4756}\right) c_1 x^{\frac{5i}{2}} \left(x^4 - (16 + 20i)x^2 - (136 - 240i)\right) - \left(\frac{17}{9512} - \frac{15i}{4756}\right) c_2 x^{-\frac{5i}{2}} \left(x^4 - (16 - 20i)x^2 - (136 + 240i)\right)$$

1.18 problem 3.26

Internal problem ID [5497]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag. Springer October 29, 1999
Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136
Problem number: 3.26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-1)\,y''-xy'+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+\frac{1}{2}x^{2}+\frac{1}{6}x^{3}+\frac{1}{24}x^{4}+\frac{1}{120}x^{5}\right)y(0)+D(y)\,(0)\,x+O\left(x^{6}\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 41

```
AsymptoticDSolveValue[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{x^5}{120}+\frac{x^4}{24}+\frac{x^3}{6}+\frac{x^2}{2}+1\right)+c_2x$$

1.19 problem 3.48 (a)

Internal problem ID [5498]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag. Springer October 29, 1999
Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136
Problem number: 3.48 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + xy = \cos(x)}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
Order:=6;
dsolve(diff(y(x),x)+x*y(x)=cos(x),y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{8}x^4\right)y(0) + x - \frac{x^3}{2} + \frac{13x^5}{120} + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 38

```
AsymptoticDSolveValue[y'[x]+x*y[x]==Cos[x],y[x],{x,0,5}]
```

$$y(x) \to \frac{13x^5}{120} - \frac{x^3}{2} + c_1\left(\frac{x^4}{8} - \frac{x^2}{2} + 1\right) + x$$

1.20 problem 3.48 (b)

Internal problem ID [5499]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag. Springer October 29, 1999
Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136
Problem number: 3.48 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + xy = \frac{1}{x^3}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 41

```
dsolve(diff(y(x),x)+x*y(x)=1/x^3,y(x), singsol=all)
```

$$y(x) = \frac{4c_1 x^2 \mathrm{e}^{-\frac{x^2}{2}} - \mathrm{expIntegral}_1\left(-\frac{x^2}{2}\right) x^2 \mathrm{e}^{-\frac{x^2}{2}} - 2}{4x^2}$$

✓ Solution by Mathematica
Time used: 0.067 (sec). Leaf size: 46

```
DSolve[y'[x]+x*y[x]==1/x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} e^{-\frac{x^2}{2}} \operatorname{ExpIntegralEi}\left(\frac{x^2}{2}\right) - \frac{1}{2x^2} + c_1 e^{-\frac{x^2}{2}}$$

1.21 problem 3.48 (c)

Internal problem ID [5500]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag. Springer October 29, 1999
Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136
Problem number: 3.48 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^3y'' + y = \frac{1}{x^4}}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(x^3*diff(y(x),x$2)+y(x)=1/x^4,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.364 (sec). Leaf size: 800

```
AsymptoticDSolveValue[x^3*y''[x]+y[x]==1/x^4,y[x],{x,0,5}]
```

$$y(x) \to e^{-\frac{2i}{\sqrt{x}}}x^{3/4}\left(\frac{33424574007825x^5}{281474976710656} - \frac{468131288625ix^{9/2}}{8796093022208} - \frac{14783093325x^4}{549755813888} + \frac{66891825ix^{7/2}}{4294967296} + \frac{2837835x^3}{268435456} - \frac{72765ix^{5/2}}{8388608} - \frac{4725x^2}{524288} + \frac{105ix^{3/2}}{8192} + \frac{15x}{512} - \frac{3i\sqrt{x}}{16} + 1\right)c_1 + e^{\frac{2i}{\sqrt{x}}}x^{3/4}\left(\frac{33424574007825x^5}{281474976710656} + \frac{468131288625ix^{9/2}}{8796093022208} - \frac{14783093325x^4}{549755813888} - \frac{66891825ix^{7/2}}{4294967296} + \frac{28378}{26843}\right.$$

1.22 problem 3.48 (d)

Internal problem ID [5501]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag. Springer October 29, 1999
Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136
Problem number: 3.48 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{xy'' - 2y' + y = \cos(x)}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(x*diff(y(x),x$2)-2*diff(y(x),x)+y(x)=cos(x),y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.161 (sec). Leaf size: 312

```
AsymptoticDSolveValue[x*y''[x]-2*y'[x]+y[x]==Cos[x],y[x],{x,0,5}]
```

$$y(x) \to c_1\left(x^4\left(\frac{\log(x)}{48}-\frac{5}{192}\right)-\frac{1}{12}x^3\log(x)+\frac{x^2}{4}+\frac{x}{2}+1\right)+c_2\left(-\frac{x^5}{806400}+\frac{x^4}{20160}-\frac{x^3}{720}+\frac{x^2}{40}-\frac{x}{4}+1\right)x^3+\left(-\frac{x^5}{806400}+\frac{x^4}{20160}-\frac{x^3}{720}+\frac{x^2}{40}-\frac{x}{4}+1\right)x^3\left(\frac{x^6\left(-20160\log^2(x)+141222\log(x)-201569\right)}{3135283200}+\frac{x^5(22277-114360\log(x))}{435456000}+\frac{x^4(69541-29064\log(x))}{34836480}+\frac{x^3(1860\log(x)+193)}{388800}-\frac{1}{6x^2}+\frac{x^2(4\log(x)-23)}{1152}-\frac{1}{6x}+\frac{1}{36}x(-\log(x)-2)-\frac{\log(x)}{12}\right)+\left(\frac{x^6(5791-672\log(x))}{8709120}-\frac{589x^5}{302400}-\frac{89x^4}{8640}+\frac{19x^3}{360}+\frac{x^2}{24}-\frac{x}{3}\right)\left(x^4\left(\frac{\log(x)}{48}-\frac{5}{192}\right)-\frac{1}{12}x^3\log(x)+\frac{x^2}{4}+\frac{x}{2}+1\right)$$

1.23 problem 3.50

Internal problem ID [5502]

Book: Advanced Mathematical Methods for Scientists and Engineers, Bender and Orszag. Springer October 29, 1999
Section: Chapter 3. APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS. page 136
Problem number: 3.50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{y}{x} = \cos(x)}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(diff(y(x),x)-y(x)/x=cos(x),y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica
Time used: 0.056 (sec). Leaf size: 34

```
AsymptoticDSolveValue[y'[x]-y[x]/x==Cos[x],y[x],{x,0,5}]
```

$$y(x) \to x\left(-\frac{x^6}{4320} + \frac{x^4}{96} - \frac{x^2}{4} + \log(x)\right) + c_1 x$$