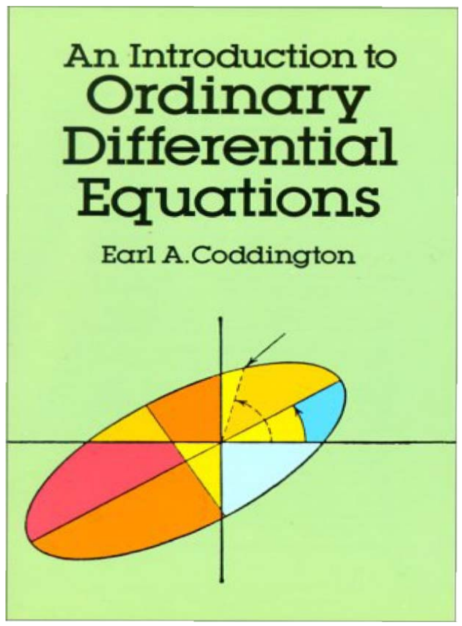
A Solution Manual For

An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961



Nasser M. Abbasi

May 16, 2024

Contents

1 Chapter 1.3 Introduction– Linear equations of First Order. Page 38

1.1 problem 1 (a)

Internal problem ID [5912]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.3 Introduction– Linear equations of First Order. Page 38
Problem number: 1 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = e^{3x} + \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=exp(3*x)+sin(x),y(x), singsol=all)
```

$$y(x) = \frac{e^{3x}}{3} - \cos(x) + c_1$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 21

```
DSolve[y'[x]==Exp[3*x]+Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{3x}}{3} - \cos(x) + c_1$$

1.2 problem 1 (b)

Internal problem ID [5913]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.3 Introduction– Linear equations of First Order. Page 38
Problem number: 1 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = x + 2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)=2+x,y(x), singsol=all)
```

$$y(x) = \frac{1}{6}x^3 + x^2 + c_1x + c_2$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 22

```
DSolve[y''[x]==2+x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{6} + x^2 + c_2x + c_1$$

1.3 problem 1 (d)

Internal problem ID [5914]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.3 Introduction– Linear equations of First Order. Page 38
Problem number: 1 (d).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _quadrature]]`

$$\boxed{y''' = x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$3)=x^2,y(x), singsol=all)
```

$$y(x) = \frac{1}{60}x^5 + \frac{1}{2}c_1x^2 + c_2x + c_3$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 25

```
DSolve[y'''[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^5}{60} + c_3x^2 + c_2x + c_1$$

1.4 problem 2 (a)

Internal problem ID [5915]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.3 Introduction– Linear equations of First Order. Page 38
Problem number: 2 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + y\cos(x) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)+cos(x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\sin(x)}$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 19

```
DSolve[y'[x]+Cos[x]*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\sin(x)}$$
$$y(x) \to 0$$

1.5 problem 2 (b)

Internal problem ID [5916]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.3 Introduction– Linear equations of First Order. Page 38
Problem number: 2 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\cos(x) = \cos(x)\sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+cos(x)*y(x)=sin(x)*cos(x),y(x), singsol=all)
```

$$y(x) = \sin(x) - 1 + c_1 \mathrm{e}^{-\sin(x)}$$

✓ Solution by Mathematica
Time used: 0.049 (sec). Leaf size: 18

```
DSolve[y'[x]+Cos[x]*y[x]==Sin[x]*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x) + c_1 e^{-\sin(x)} - 1$$

1.6 problem 2 (c)

Internal problem ID [5917]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.3 Introduction– Linear equations of First Order. Page 38
Problem number: 2 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x} + \mathrm{e}^{x} c_2$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 20

```
DSolve[y''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x} + c_2 e^{-x}$$

1.7 problem 2 (f)

Internal problem ID [5918]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.3 Introduction– Linear equations of First Order. Page 38
Problem number: 2 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(2x) + c_2 \cos(2x)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 20

```
DSolve[y''[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(2x) + c_2 \sin(2x)$$

1.8 problem 2 (h)

Internal problem ID [5919]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.3 Introduction– Linear equations of First Order. Page 38
Problem number: 2 (h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + k^2 y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+k^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(kx) + c_2 \cos(kx)$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 20

```
DSolve[y''[x]+k^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(kx) + c_2 \sin(kx)$$

1.9 problem 3(a)

Internal problem ID [5920]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.3 Introduction– Linear equations of First Order. Page 38
Problem number: 3(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + 5y = 2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)+5*y(x)=2,y(x), singsol=all)
```

$$y(x) = \frac{2}{5} + \mathrm{e}^{-5x} c_1$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 24

```
DSolve[y'[x]+5*y[x]==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{5} + c_1 e^{-5x}$$
$$y(x) \to \frac{2}{5}$$

1.10 problem 4(a)

Internal problem ID [5921]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.3 Introduction– Linear equations of First Order. Page 38
Problem number: 4(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = 3x + 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)=3*x+1,y(x), singsol=all)
```

$$y(x) = \frac{1}{2}x^3 + \frac{1}{2}x^2 + c_1x + c_2$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 25

```
DSolve[y''[x]==3*x+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(x^3 + x^2 + 2c_2x + 2c_1)$$

1.11 problem 5(a)

Internal problem ID [5922]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.3 Introduction– Linear equations of First Order. Page 38
Problem number: 5(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_quadrature]**

$$\boxed{y' - yk = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=k*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{kx}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 18

```
DSolve[y'[x]==k*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{kx}$$
$$y(x) \to 0$$

2 Chapter 1.6 Introduction– Linear equations of First Order. Page 41

2.1 problem 1(a)

Internal problem ID [5923]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.6 Introduction– Linear equations of First Order. Page 41
Problem number: 1(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 2y = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)-2*y(x)=1,y(x), singsol=all)
```

$$y(x) = -\frac{1}{2} + e^{2x}c_1$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 24

```
DSolve[y'[x]-2*y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2} + c_1e^{2x}$$
$$y(x) \to -\frac{1}{2}$$

2.2 problem 1(b)

Internal problem ID [5924]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.6 Introduction– Linear equations of First Order. Page 41
Problem number: 1(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y + y' = e^x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+y(x)=exp(x),y(x), singsol=all)
```

$$y(x) = \frac{e^x}{2} + c_1 e^{-x}$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 21

```
DSolve[y'[x]+y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x}{2} + c_1 e^{-x}$$

2.3 problem 1(c)

Internal problem ID [5925]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.6 Introduction– Linear equations of First Order. Page 41
Problem number: 1(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - 2y = x^2 + x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)-2*y(x)=x^2+x,y(x), singsol=all)
```

$$y(x) = e^{2x}c_1 - \frac{(x+1)^2}{2}$$

✓ Solution by Mathematica
Time used: 0.073 (sec). Leaf size: 23

```
DSolve[y'[x]-2*y[x]==x^2+x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}(x+1)^2 + c_1 e^{2x}$$

2.4 problem 1(d)

Internal problem ID [5926]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.6 Introduction– Linear equations of First Order. Page 41
Problem number: 1(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y + 3y' = 2\,\mathrm{e}^{-x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(3*diff(y(x),x)+y(x)=2*exp(-x),y(x), singsol=all)
```

$$y(x) = -\mathrm{e}^{-x} + \mathrm{e}^{-\frac{x}{3}} c_1$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 23

```
DSolve[3*y'[x]+y[x]==2*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(-1 + c_1 e^{2x/3}\right)$$

2.5 problem 1(e)

Internal problem ID [5927]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.6 Introduction– Linear equations of First Order. Page 41
Problem number: 1(e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + 3y = e^{ix}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)+3*y(x)=exp(I*x),y(x), singsol=all)
```

$$y(x) = -\frac{e^{-3x}\left((-3+i)\,e^{(3+i)x} - 10c_1\right)}{10}$$

✓ Solution by Mathematica
Time used: 0.054 (sec). Leaf size: 29

```
DSolve[y'[x]+3*y[x]==Exp[I*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\frac{3}{10} - \frac{i}{10}\right)e^{ix} + c_1 e^{-3x}$$

2.6 problem 2

Internal problem ID [5928]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.6 Introduction– Linear equations of First Order. Page 41
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + iy = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+I*y(x)=x,y(x), singsol=all)
```

$$y(x) = -ix + 1 + e^{-ix}c_1$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 22

```
DSolve[y'[x]+I*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -ix + c_1e^{-ix} + 1$$

2.7 problem 3

Internal problem ID [5929]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.6 Introduction– Linear equations of First Order. Page 41
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{Ly' + Ry = E}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(L*diff(y(x),x)+R*y(x)=E,y(x), singsol=all)
```

$$y(x) = \frac{e^{-\frac{Rx}{L}} c_1 R + E}{R}$$

✓ Solution by Mathematica
Time used: 0.056 (sec). Leaf size: 23

```
DSolve[L*y'[x]+R*y[x]==E0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\text{E0} - \text{E0}e^{-\frac{Rx}{L}}}{R}$$

2.8 problem 4

Internal problem ID [5930]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.6 Introduction– Linear equations of First Order. Page 41
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{Ly' + Ry = E\sin(\omega x)}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 45

```
dsolve([L*diff(y(x),x)+R*y(x)=E*sin(omega*x),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{E\left(e^{-\frac{Rx}{L}}L\omega - L\cos(\omega x)\,\omega + \sin(\omega x)\,R\right)}{\omega^2L^2 + R^2}$$

✓ Solution by Mathematica
Time used: 0.115 (sec). Leaf size: 47

```
DSolve[{L*y'[x]+R*y[x]==E0*Sin[\[Omega]*x],{y[0]==0}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{\mathrm{E0}\left(L\omega e^{-\frac{Rx}{L}} - L\omega\cos(x\omega) + R\sin(x\omega)\right)}{L^2\omega^2 + R^2}$$

2.9 problem 5

Internal problem ID [5931]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.6 Introduction– Linear equations of First Order. Page 41
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{Ly' + Ry = E\,\mathrm{e}^{i\omega x}}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 38

```
dsolve([L*diff(y(x),x)+R*y(x)=E*exp(I*omega*x),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{E\left(\mathrm{e}^{\frac{x(iL\omega+R)}{L}} - 1\right)\mathrm{e}^{-\frac{Rx}{L}}}{iL\omega + R}$$

✓ Solution by Mathematica
Time used: 0.101 (sec). Leaf size: 43

```
DSolve[{L*y'[x]+R*y[x]==E0*Exp[I*\[Omega]*x],{y[0]==0}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{\mathrm{E0}e^{-\frac{Rx}{L}}\left(-1 + e^{\frac{x(R+iL\omega)}{L}}\right)}{R + iL\omega}$$

2.10 problem 7

Internal problem ID [5932]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.6 Introduction– Linear equations of First Order. Page 41
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + ya = b(x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)+a*y(x)=b(x),y(x), singsol=all)
```

$$y(x) = \left(\int b(x)\,\mathrm{e}^{ax}dx + c_1\right)\mathrm{e}^{-ax}$$

✓ Solution by Mathematica
Time used: 0.048 (sec). Leaf size: 32

```
DSolve[y'[x]+a*y[x]==b[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-ax}\left(\int_1^x e^{aK[1]}b(K[1])dK[1] + c_1\right)$$

3 Chapter 1. Introduction– Linear equations of First Order. Page 45

3.1 problem 1(a)

Internal problem ID [5933]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1. Introduction– Linear equations of First Order. Page 45
Problem number: 1(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2xy + y' = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+2*x*y(x)=x,y(x), singsol=all)
```

$$y(x) = \frac{1}{2} + \mathrm{e}^{-x^2} c_1$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 26

```
DSolve[y'[x]+2*x*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} + c_1 e^{-x^2}$$
$$y(x) \to \frac{1}{2}$$

3.2 problem 1(b)

Internal problem ID [5934]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1. Introduction– Linear equations of First Order. Page 45
Problem number: 1(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' + y = 3x^3 - 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x*diff(y(x),x)+y(x)=3*x^3-1,y(x), singsol=all)
```

$$y(x) = \frac{\frac{3}{4}x^4 - x + c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 20

```
DSolve[x*y'[x]+y[x]==3*x^3-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3x^3}{4} + \frac{c_1}{x} - 1$$

3.3 problem 1(c)

Internal problem ID [5935]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1. Introduction– Linear equations of First Order. Page 45
Problem number: 1(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{y' + \mathrm{e}^{x} y = 3\,\mathrm{e}^{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)+exp(x)*y(x)=3*exp(x),y(x), singsol=all)
```

$$y(x) = 3 + \mathrm{e}^{-\mathrm{e}^{x}} c_1$$

✓ Solution by Mathematica
Time used: 0.057 (sec). Leaf size: 22

```
DSolve[y'[x]+Exp[x]*y[x]==3*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3 + c_1 e^{-e^{x}}$$
$$y(x) \to 3$$

3.4 problem 1(d)

Internal problem ID [5936]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1. Introduction– Linear equations of First Order. Page 45
Problem number: 1(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - y\tan(x) = \mathrm{e}^{\sin(x)}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)-tan(x)*y(x)=exp(sin(x)),y(x), singsol=all)
```

$$y(x) = \sec(x)\left(\mathrm{e}^{\sin(x)} + c_1\right)$$

✓ Solution by Mathematica
Time used: 0.149 (sec). Leaf size: 15

```
DSolve[y'[x]-Tan[x]*y[x]==Exp[Sin[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sec(x)\left(e^{\sin(x)} + c_1\right)$$

3.5 problem 1(e)

Internal problem ID [5937]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1. Introduction– Linear equations of First Order. Page 45
Problem number: 1(e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2xy + y' = x\,\mathrm{e}^{-x^2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)+2*x*y(x)=x*exp(-x^2),y(x), singsol=all)
```

$$y(x) = \frac{(x^2 + 2c_1)\,\mathrm{e}^{-x^2}}{2}$$

✓ Solution by Mathematica
Time used: 0.057 (sec). Leaf size: 24

```
DSolve[y'[x]+2*x*y[x]==x*Exp[-x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x^2}\left(x^2 + 2c_1\right)$$

3.6 problem 2

Internal problem ID [5938]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1. Introduction– Linear equations of First Order. Page 45
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\cos(x) = \mathrm{e}^{-\sin(x)}}$$

With initial conditions

$$[y(\pi) = \pi]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 11

```
dsolve([diff(y(x),x)+cos(x)*y(x)=exp(-sin(x)),y(Pi) = Pi],y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\sin(x)}x$$

✓ Solution by Mathematica

Time used: 0.138 (sec). Leaf size: 13

```
DSolve[{y'[x]+Cos[x]*y[x]==Exp[-Sin[x]],{y[Pi]==Pi}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to xe^{-\sin(x)}$$

3.7 problem 3

Internal problem ID [5939]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1. Introduction– Linear equations of First Order. Page 45
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x^2y' + 2xy = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x^2*diff(y(x),x)+2*x*y(x)=1,y(x), singsol=all)
```

$$y(x) = \frac{x + c_1}{x^2}$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 13

```
DSolve[x^2*y'[x]+2*x*y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x + c_1}{x^2}$$

3.8 problem 8

Internal problem ID [5940]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1. Introduction– Linear equations of First Order. Page 45
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + 2y = b(x)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)+2*y(x)=b(x),y(x), singsol=all)
```

$$y(x) = \left(\int b(x)\,\mathrm{e}^{2x}dx + c_1\right)\mathrm{e}^{-2x}$$

✓ Solution by Mathematica
Time used: 0.045 (sec). Leaf size: 31

```
DSolve[y'[x]+2*y[x]==b[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(\int_1^x e^{2K[1]}b(K[1])dK[1] + c_1\right)$$

3.9 problem 14(a)

Internal problem ID [5941]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1. Introduction– Linear equations of First Order. Page 45
Problem number: 14(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y = 1}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 8

```
dsolve([diff(y(x),x)=1+y(x),y(0) = 0],y(x), singsol=all)
```

$$y(x) = -1 + e^x$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 10

```
DSolve[{y'[x]==1+y[x],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x - 1$$

3.10 problem 14(b)

Internal problem ID [5942]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1. Introduction– Linear equations of First Order. Page 45
Problem number: 14(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^2 = 1}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.046 (sec). Leaf size: 6

```
dsolve([diff(y(x),x)=1+y(x)^2,y(0) = 0],y(x), singsol=all)
```

$$y(x) = \tan(x)$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 7

```
DSolve[{y'[x]==1+y[x]^2,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan(x)$$

3.11 problem 14(b)

Internal problem ID [5943]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1. Introduction– Linear equations of First Order. Page 45
Problem number: 14(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^2 = 1}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 6

```
dsolve([diff(y(x),x)=1+y(x)^2,y(0) = 0],y(x), singsol=all)
```

$$y(x) = \tan(x)$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 7

```
DSolve[{y'[x]==1+y[x]^2,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan(x)$$

4 Chapter 2. Linear equations with constant coefficients. Page 52

4.1 problem 1(a)

Internal problem ID [5944]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 52
Problem number: 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2x} c_1 + c_2 \mathrm{e}^{-2x}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 22

```
DSolve[y''[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(c_1 e^{4x} + c_2\right)$$

4.2 problem 1(b)

Internal problem ID [5945]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 52
Problem number: 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{3y'' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(3*diff(y(x),x$2)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(\frac{\sqrt{6}\,x}{3}\right) + c_2 \cos\left(\frac{\sqrt{6}\,x}{3}\right)$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 32

```
DSolve[3*y''[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos\left(\sqrt{\frac{2}{3}}x\right) + c_2 \sin\left(\sqrt{\frac{2}{3}}x\right)$$

4.3 problem 1(c)

Internal problem ID [5946]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 52
Problem number: 1(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 16y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+16*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(4x) + c_2 \cos(4x)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 20

```
DSolve[y''[x]+16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(4x) + c_2 \sin(4x)$$

4.4 problem 1(d)

Internal problem ID [5947]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 52
Problem number: 1(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = c_1 x + c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 12

```
DSolve[y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 x + c_1$$

4.5 problem 1(e)

Internal problem ID [5948]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 52
Problem number: 1(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2iy' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+2*I*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-ix}\left(c_1 \sin\left(\sqrt{2}\,x\right) + c_2 \cos\left(\sqrt{2}\,x\right)\right)$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 38

```
DSolve[y''[x]+2*I*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-i\left(1+\sqrt{2}\right)x}\left(c_2 e^{2i\sqrt{2}x} + c_1\right)$$

4.6 problem 1(f)

Internal problem ID [5949]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 52
Problem number: 1(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y' + 5y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+5*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{2x}(c_1 \sin(x) + \cos(x) c_2)$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 22

```
DSolve[y''[x]-4*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}(c_2 \cos(x) + c_1 \sin(x))$$

4.7 problem 1(g)

Internal problem ID [5950]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 52
Problem number: 1(g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + (-1 + 3i)\, y' - 3iy = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$2)+(3*I-1)*diff(y(x),x)-3*I*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x} c_1 + c_2 \mathrm{e}^{-3ix}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 22

```
DSolve[y''[x]+(3*I-1)*y'[x]-3*I*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-3ix} + c_2 e^{x}$$

4.8 problem 2(a)

Internal problem ID [5951]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 52
Problem number: 2(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - 6y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$2)+diff(y(x),x)-6*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{(3\,e^{5x} + 2)\,e^{-3x}}{5}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 23

```
DSolve[{y''[x]+y'[x]-6*y[x]==0,{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{5}e^{-3x}(3e^{5x} + 2)$$

4.9 problem 2(b)

Internal problem ID [5952]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 52
Problem number: 2(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - 6y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 16

```
dsolve([diff(y(x),x$2)+diff(y(x),x)-6*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{(e^{5x} - 1)\,e^{-3x}}{5}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 21

```
DSolve[{y''[x]+y'[x]-6*y[x]==0,{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{5}e^{-3x}\left(e^{5x} - 1\right)$$

4.10 problem 3(a)

Internal problem ID [5953]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 52
Problem number: 3(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

With initial conditions

$$\left[y(0) = 1, y\left(\frac{\pi}{2}\right) = 2\right]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 11

```
dsolve([diff(y(x),x$2)+y(x)=0,y(0) = 1, y(1/2*Pi) = 2],y(x), singsol=all)
```

$$y(x) = 2\sin(x) + \cos(x)$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 12

```
DSolve[{y''[x]+y[x]==0,{y[0]==1,y[Pi/2]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\sin(x) + \cos(x)$$

4.11 problem 3(b)

Internal problem ID [5954]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 52
Problem number: 3(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

With initial conditions

$$[y(0) = 0, y(\pi) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 8

```
dsolve([diff(y(x),x$2)+y(x)=0,y(0) = 0, y(Pi) = 0],y(x), singsol=all)
```

$$y(x) = c_1 \sin(x)$$

✓ Solution by Mathematica
Time used: 0.007 (sec). Leaf size: 10

```
DSolve[{y''[x]+y[x]==0,{y[0]==0,y[Pi]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \sin(x)$$

4.12 problem 3(c)

Internal problem ID [5955]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 52
Problem number: 3(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type **[[_2nd_order, _missing_x]]**

$$\boxed{y'' + y = 0}$$

With initial conditions

$$\left[y(0) = 0, y'\left(\frac{\pi}{2}\right) = 0\right]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 8

```
dsolve([diff(y(x),x$2)+y(x)=0,y(0) = 0, D(y)(1/2*Pi) = 0],y(x), singsol=all)
```

$$y(x) = c_1 \sin(x)$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 10

```
DSolve[{y''[x]+y[x]==0,{y[0]==0,y'[Pi/2]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \sin(x)$$

4.13 problem 3(d)

Internal problem ID [5956]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 52
Problem number: 3(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type **`[[_2nd_order, _missing_x]]`**

$$\boxed{y'' + y = 0}$$

With initial conditions

$$\left[y(0) = 0, y\left(\frac{\pi}{2}\right) = 0\right]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 5

```
dsolve([diff(y(x),x$2)+y(x)=0,y(0) = 0, y(1/2*Pi) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 6

```
DSolve[{y''[x]+y[x]==0,{y[0]==0,y[Pi/2]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

5 Chapter 2. Linear equations with constant coefficients. Page 59

5.1 problem 1(a)

Internal problem ID [5957]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 59
Problem number: 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' - 3y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)-3*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{e^{3x}}{4} - \frac{e^{-x}}{4}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 21

```
DSolve[{y''[x]-2*y'[x]-3*y[x]==0,{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{1}{4}e^{-x}(e^{4x} - 1)$$

5.2 problem 1(b)

Internal problem ID [5958]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 59
Problem number: 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type **[[_2nd_order, _missing_x]]**

$$\boxed{y'' + (1+4i)\, y' + y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 5

```
dsolve([diff(y(x),x$2)+(4*I+1)*diff(y(x),x)+y(x)=0,y(0) = 0, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 6

```
DSolve[{y''[x]+(4*I+1)*y'[x]+y[x]==0,{y[0]==0,y'[0]==0}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to 0$$

5.3 problem 1(c)

Internal problem ID [5959]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 59
Problem number: 1(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + (-1+3i)\, y' - 3iy = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 20

```
dsolve([diff(y(x),x$2)+(3*I-1)*diff(y(x),x)-3*I*y(x)=0,y(0) = 2, D(y)(0) = 0],y(x), singsol=
```

$$y(x) = \left(\frac{9}{5} + \frac{3i}{5}\right) \mathrm{e}^{x} + \left(\frac{1}{5} - \frac{3i}{5}\right) \mathrm{e}^{-3ix}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 31

```
DSolve[{y''[x]+(3*I-1)*y'[x]-3*I*y[x]==0,{y[0]==2,y'[0]==0}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{5} e^{-3ix}\left((9+3i)e^{(1+3i)x} + (1-3i)\right)$$

5.4 problem 1(d)

Internal problem ID [5960]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 59
Problem number: 1(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 10y = 0}$$

With initial conditions

$$[y(0) = \pi, y'(0) = \pi^2]$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 27

```
dsolve([diff(y(x),x$2)+10*y(x)=0,y(0) = Pi, D(y)(0) = Pi^2],y(x), singsol=all)
```

$$y(x) = \frac{\pi\left(\pi\sqrt{10}\,\sin\left(\sqrt{10}\,x\right) + 10\cos\left(\sqrt{10}\,x\right)\right)}{10}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 33

```
DSolve[{y''[x]+10*y[x]==0,{y[0]==Pi,y'[0]==Pi^2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\pi^2 \sin\left(\sqrt{10}x\right)}{\sqrt{10}} + \pi\cos\left(\sqrt{10}x\right)$$

6 Chapter 2. Linear equations with constant coefficients. Page 69

6.1 problem 1(a)

Internal problem ID [5961]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 69
Problem number: 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+4*y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = \sin(2x)\, c_2 + \cos(2x)\, c_1 + \frac{\cos(x)}{3}$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 26

```
DSolve[y''[x]+4*y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\cos(x)}{3} + c_1 \cos(2x) + c_2 \sin(2x)$$

6.2 problem 1(b)

Internal problem ID [5962]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 69
Problem number: 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = \sin(3x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+9*y(x)=sin(3*x),y(x), singsol=all)
```

$$y(x) = \frac{(-x + 6c_1)\cos(3x)}{6} + \sin(3x)\, c_2$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 33

```
DSolve[y''[x]+9*y[x]==Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(-\frac{x}{6} + c_1\right)\cos(3x) + \frac{1}{36}(1 + 36c_2)\sin(3x)$$

6.3 problem 1(c)

Internal problem ID [5963]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 69
Problem number: 1(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \tan(x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+y(x)=tan(x),y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 - \cos(x) \ln(\sec(x) + \tan(x))$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 23

```
DSolve[y''[x]+y[x]==Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x)(-\text{arctanh}(\sin(x))) + c_1 \cos(x) + c_2 \sin(x)$$

6.4 problem 1(d)

Internal problem ID [5964]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 69
Problem number: 1(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2iy' + y = x}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)+2*I*diff(y(x),x)+y(x)=x,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-ix} \sin\left(\sqrt{2}\, x\right) c_2 + \mathrm{e}^{-ix} \cos\left(\sqrt{2}\, x\right) c_1 + x - 2i$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 44

```
DSolve[y''[x]+2*I*y'[x]+y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + c_1 e^{-i\left(1+\sqrt{2}\right)x} + c_2 e^{i\left(\sqrt{2}-1\right)x} - 2i$$

6.5 problem 1(e)

Internal problem ID [5965]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 69
Problem number: 1(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 5y = 3\,\mathrm{e}^{-x} + 2x^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+5*y(x)=3*exp(-x)+2*x^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2x} \sin(x)\, c_2 + \mathrm{e}^{2x} \cos(x)\, c_1 + \frac{3\,\mathrm{e}^{-x}}{10} + \frac{2x^2}{5} + \frac{16x}{25} + \frac{44}{125}$$

✓ Solution by Mathematica
Time used: 0.316 (sec). Leaf size: 47

```
DSolve[y''[x]-4*y'[x]+5*y[x]==3*Exp[-x]+2*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{250}\left(100x^2 + 160x + 75e^{-x} + 88\right) + c_2 e^{2x} \cos(x) + c_1 e^{2x} \sin(x)$$

6.6 problem 1(f)

Internal problem ID [5966]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 69
Problem number: 1(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 7y' + 6y = \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)-7*diff(y(x),x)+6*y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = e^{6x}c_2 + e^{x}c_1 + \frac{7\cos(x)}{74} + \frac{5\sin(x)}{74}$$

✓ Solution by Mathematica
Time used: 0.063 (sec). Leaf size: 32

```
DSolve[y''[x]-7*y'[x]+6*y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{5\sin(x)}{74} + \frac{7\cos(x)}{74} + c_1e^{x} + c_2e^{6x}$$

6.7 problem 1(g)

Internal problem ID [5967]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 69
Problem number: 1(g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = 2 \sin(x) \sin(2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+y(x)=2*sin(x)*sin(2*x),y(x), singsol=all)
```

$$y(x) = -\frac{\cos(x)\sin(x)^2}{2} + \frac{(2c_2 + x)\sin(x)}{2} + \cos(x)\, c_1$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 33

```
DSolve[y''[x]+y[x]==2*Sin[x]*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}(\cos(3x) + (-1 + 8c_1)\cos(x) + 4(x + 2c_2)\sin(x))$$

6.8 problem 1(h)

Internal problem ID [5968]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 69
Problem number: 1(h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sec(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+y(x)=sec(x),y(x), singsol=all)
```

$$y(x) = -\ln(\sec(x))\cos(x) + \cos(x) c_1 + \sin(x)(x + c_2)$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 22

```
DSolve[y''[x]+y[x]==Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_2)\sin(x) + \cos(x)(\log(\cos(x)) + c_1)$$

6.9 problem 1(i)

Internal problem ID [5969]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 69
Problem number: 1(i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4y'' - y = e^x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve(4*diff(y(x),x$2)-y(x)=exp(x),y(x), singsol=all)
```

$$y(x) = e^{\frac{x}{2}} c_2 + c_1 e^{-\frac{x}{2}} + \frac{e^x}{3}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 33

```
DSolve[4*y''[x]-y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x}{3} + c_1 e^{x/2} + c_2 e^{-x/2}$$

6.10 problem 1(j)

Internal problem ID [5970]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 69
Problem number: 1(j).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{6y'' + 5y' - 6y = x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(6*diff(y(x),x$2)+5*diff(y(x),x)-6*y(x)=x,y(x), singsol=all)
```

$$y(x) = -\frac{\left(\left(x+\frac{5}{6}\right) e^{\frac{3x}{2}} - 6\,e^{\frac{13x}{6}} c_2 - 6c_1\right) e^{-\frac{3x}{2}}}{6}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 34

```
DSolve[6*y''[x]+5*y'[x]-6*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{6} + c_1 e^{2x/3} + c_2 e^{-3x/2} - \frac{5}{36}$$

6.11 problem 4(c)

Internal problem ID [5971]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 69
Problem number: 4(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + \omega^2 y = A\cos(\omega x)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$2)+omega^2*y(x)=A*cos(omega*x),y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\sin(\omega x)\left(1 + \frac{Ax}{2}\right)}{\omega}$$

✓ Solution by Mathematica
Time used: 0.058 (sec). Leaf size: 21

```
DSolve[{y''[x]+\[Omega]^2*y[x]==A*Cos[\[Omega]*x],{y[0]==0,y'[0]==1}},y[x],x,IncludeSingular
```

$$y(x) \to \frac{(Ax + 2)\sin(x\omega)}{2\omega}$$

7 Chapter 2. Linear equations with constant coefficients. Page 74

7.1 problem 4(a)

Internal problem ID [5972]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 74
Problem number: 4(a).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 8y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$3)-8*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{2x}c_1 + c_2e^{-x}\sin\left(\sqrt{3}\,x\right) + c_3e^{-x}\cos\left(\sqrt{3}\,x\right)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 42

```
DSolve[y'''[x]-8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_1e^{3x} + c_2\cos\left(\sqrt{3}x\right) + c_3\sin\left(\sqrt{3}x\right)\right)$$

7.2 problem 4(b)

Internal problem ID [5973]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 74
Problem number: 4(b).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 16y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 65

```
dsolve(diff(y(x),x$4)+16*y(x)=0,y(x), singsol=all)
```

$$y(x) = -c_1 e^{-\sqrt{2}\,x} \sin\left(\sqrt{2}\,x\right) - c_2 e^{\sqrt{2}\,x} \sin\left(\sqrt{2}\,x\right) + c_3 e^{-\sqrt{2}\,x} \cos\left(\sqrt{2}\,x\right) + c_4 e^{\sqrt{2}\,x} \cos\left(\sqrt{2}\,x\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 67

```
DSolve[y''''[x]+16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\sqrt{2}x}\left(\left(c_1 e^{2\sqrt{2}x} + c_2\right)\cos\left(\sqrt{2}x\right) + \left(c_4 e^{2\sqrt{2}x} + c_3\right)\sin\left(\sqrt{2}x\right)\right)$$

7.3 problem 4(c)

Internal problem ID [5974]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 74
Problem number: 4(c).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 5y'' + 6y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$3)-5*diff(y(x),x$2)+6*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 e^{2x} + c_3 e^{3x}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 30

```
DSolve[y'''[x]-5*y''[x]+6*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}c_1 e^{2x} + \frac{1}{3}c_2 e^{3x} + c_3$$

7.4 problem 4(d)

Internal problem ID [5975]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 74
Problem number: 4(d).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type **[[_3rd_order, _missing_x]]**

$$\boxed{y''' - iy'' + 4y' - 4iy = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$3)-I*diff(y(x),x$2)+4*diff(y(x),x)-4*I*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{ix} + c_2 e^{2ix} + c_3 e^{-2ix}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 36

```
DSolve[y'''[x]-I*y''[x]+4*y'[x]-4*I*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2ix} \left(c_2 e^{4ix} + c_3 e^{3ix} + c_1\right)$$

7.5 problem 4(f)

Internal problem ID [5976]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 74
Problem number: 4(f).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 5y'' + 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$4)+5*diff(y(x),x$2)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = 2c_2 \cos(x)^2 + (2c_1 \sin(x) + c_4) \cos(x) + c_3 \sin(x) - c_2$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 30

```
DSolve[y''''[x]+5*y''[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(2x) + c_4 \sin(x) + \cos(x)(2c_2 \sin(x) + c_3)$$

7.6 problem 4(g)

Internal problem ID [5977]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 74
Problem number: 4(g).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 16y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$4)-16*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{2x}c_1 + c_2 e^{-2x} + c_3 \sin(2x) + c_4 \cos(2x)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 36

```
DSolve[y''''[x]-16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{2x} + c_3 e^{-2x} + c_2 \cos(2x) + c_4 \sin(2x)$$

7.7 problem 4(h)

Internal problem ID [5978]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 74
Problem number: 4(h).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 3y' - 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = (c_3 x + c_2)\,e^{-x} + e^{2x} c_1$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 26

```
DSolve[y'''[x]-3*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_2 x + c_3 e^{3x} + c_1\right)$$

7.8 problem 4(i)

Internal problem ID [5979]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 74
Problem number: 4(i).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' - 3iy'' - 3y' + iy = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$3)-3*I*diff(y(x),x$2)-3*diff(y(x),x)+I*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{ix}(c_3x^2 + c_2x + c_1)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 25

```
DSolve[y'''[x]-3*I*y''[x]-3*y'[x]+I*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{ix}(x(c_3x + c_2) + c_1)$$

8 Chapter 2. Linear equations with constant coefficients. Page 79

8.1 problem 1(c)

Internal problem ID [5980]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 79
Problem number: 1(c).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 4y' = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1, y''(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$3)-4*diff(y(x),x)=0,y(0) = 0, D(y)(0) = 1, (D@@2)(y)(0) = 0],y(x), sings
```

$$y(x) = \frac{e^{2x}}{4} - \frac{e^{-2x}}{4}$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 69

```
DSolve[{y'''[x]-4*y[x]==0,{y[0]==0,y'[0]==1,y''[0]==0}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{e^{-\frac{x}{\sqrt[3]{2}}}\left(e^{\frac{3x}{\sqrt[3]{2}}} + \sqrt{3}\sin\left(\frac{\sqrt{3}x}{\sqrt[3]{2}}\right) - \cos\left(\frac{\sqrt{3}x}{\sqrt[3]{2}}\right)\right)}{3\ 2^{2/3}}$$

8.2 problem 2(c)

Internal problem ID [5981]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 79
Problem number: 2(c).
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(5)} - y'''' - y' + y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0, y''(0) = 0, y'''(0) = 0, y''''(0) = 0]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 28

```
dsolve([diff(y(x),x$5)-diff(y(x),x$4)-diff(y(x),x)+y(x)=0,y(0) = 1, D(y)(0) = 0, (D@@2)(y)(0
```

$$y(x) = \frac{e^{-x}}{8} + \frac{(-2x+5)\,e^{x}}{8} + \frac{\cos(x)}{4} - \frac{\sin(x)}{4}$$

✓ Solution by Mathematica
Time used: 0.007 (sec). Leaf size: 34

```
DSolve[{y'''''[x]-y''''[x]-y'[x]+y[x]==0,{y[0]==1,y'[0]==0,y''[0]==0,y'''[0]==0,y''''[0]==0}
```

$$y(x) \to \frac{1}{8}\left(-2e^{x}x + e^{-x} + 5e^{x} - 2\sin(x) + 2\cos(x)\right)$$

9 Chapter 2. Linear equations with constant coefficients. Page 83

9.1 problem 1(a)

Internal problem ID [5982]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 83
Problem number: 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x$2)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(x) + \cos(x) c_2$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 16

```
DSolve[y''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(x) + c_2 \sin(x)$$

9.2 problem 1(b)

Internal problem ID [5983]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 83
Problem number: 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x} + \mathrm{e}^{x} c_2$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 20

```
DSolve[y''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x} + c_2 e^{-x}$$

9.3 problem 1(c)

Internal problem ID [5984]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 83
Problem number: 1(c).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$4)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x} + \mathrm{e}^{x} c_2 + c_3 \sin(x) + c_4 \cos(x)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 30

```
DSolve[y''''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x + c_3 e^{-x} + c_2 \cos(x) + c_4 \sin(x)$$

9.4 problem 1(d)

Internal problem ID [5985]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 83
Problem number: 1(d).
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y^{(5)} + 2y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 140

```
dsolve(diff(y(x),x$5)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{\left(-i 2^{\frac{7}{10}} \sqrt{5-\sqrt{5}}+2^{\frac{1}{5}} \sqrt{5}+2^{\frac{1}{5}}\right)x}{4}} + c_2 e^{-\frac{x\left(i\left(\sqrt{5}+1\right) 2^{\frac{7}{10}} \sqrt{5-\sqrt{5}}+2\, 2^{\frac{1}{5}}\left(\sqrt{5}-1\right)\right)}{8}} + c_3 e^{-2^{\frac{1}{5}} x} + c_4 e^{\frac{\left(i\left(\sqrt{5}+1\right) 2^{\frac{7}{10}} \sqrt{5-\sqrt{5}}-2\, 2^{\frac{1}{5}}\left(\sqrt{5}-1\right)\right)x}{8}} + c_5 e^{2^{\frac{1}{5}}\left(\cos\left(\frac{\pi}{5}\right)+i \sin\left(\frac{\pi}{5}\right)\right)x}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 180

```
DSolve[y'''''[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{\left(\sqrt{5}-1\right)x}{2\ 2^{4/5}}}\left(c_5 e^{\frac{\left(\sqrt{5}-5\right)x}{2\ 2^{4/5}}} + c_3 e^{\frac{\sqrt{5}x}{2^{4/5}}} \cos\left(\frac{\sqrt{5-\sqrt{5}}x}{2\ 2^{3/10}}\right) + c_4 \cos\left(\frac{\sqrt{5+\sqrt{5}}x}{2\ 2^{3/10}}\right) + c_2 e^{\frac{\sqrt{5}x}{2^{4/5}}} \sin\left(\frac{\sqrt{5-\sqrt{5}}x}{2\ 2^{3/10}}\right) + c_1 \sin\left(\frac{\sqrt{5+\sqrt{5}}x}{2\ 2^{3/10}}\right)\right)$$

9.5 problem 1(e)

Internal problem ID [5986]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 83
Problem number: 1(e).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 5y'' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$4)-5*diff(y(x),x$2)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(e^{4x}c_1 + c_4e^{3x} + e^x c_2 + c_3\right)e^{-2x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 35

```
DSolve[y''''[x]-5*y''[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(c_2e^x + e^{3x}(c_4e^x + c_3) + c_1\right)$$

9.6 problem 2

Internal problem ID [5987]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 83
Problem number: 2.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1, y''(0) = 0]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 39

```
dsolve([diff(y(x),x$3)+y(x)=0,y(0) = 0, D(y)(0) = 1, (D@@2)(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{\left(\mathrm{e}^{\frac{3x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right)\sqrt{3} + \mathrm{e}^{\frac{3x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right) - 1\right)\mathrm{e}^{-x}}{3}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 59

```
DSolve[{y'''[x]+y[x]==0,{y[0]==0,y'[0]==1,y''[0]==0}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{3}e^{-x}\left(\sqrt{3}e^{3x/2}\sin\left(\frac{\sqrt{3}x}{2}\right) + e^{3x/2}\cos\left(\frac{\sqrt{3}x}{2}\right) - 1\right)$$

9.7 problem 3(a)

Internal problem ID [5988]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 83
Problem number: 3(a).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - iy'' + y' - iy = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$3)-I*diff(y(x),x$2)+diff(y(x),x)-I*y(x)=0,y(x), singsol=all)
```

$$y(x) = (c_3 x + c_2)\,\mathrm{e}^{ix} + \mathrm{e}^{-ix} c_1$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 31

```
DSolve[y'''[x]-I*y''[x]+y'[x]-I*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-ix}\left(e^{2ix}(c_3 x + c_2) + c_1\right)$$

9.8 problem 3(b)

Internal problem ID [5989]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 83
Problem number: 3(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2iy' - y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)-2*I*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{ix}(c_2x + c_1)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 20

```
DSolve[y''[x]-2*I*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{ix}(c_2x + c_1)$$

9.9 problem 5(b)

Internal problem ID [5990]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 83
Problem number: 5(b).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - k^4 y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0, y(1) = 0, y'(1) = 0]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 5

```
dsolve([diff(y(x),x$4)-k^4*y(x)=0,y(0) = 0, D(y)(0) = 0, y(1) = 0, D(y)(1) = 0],y(x), singso
```

$$y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 6

```
DSolve[{y''''[x]-k^4*y[x]==0,{y[0]==0,y[1]==0,y'[0]==0,y'[1]==0}},y[x],x,IncludeSingularSolu
```

$$y(x) \to 0$$

10 Chapter 2. Linear equations with constant coefficients. Page 89

10.1 problem 1(a)

Internal problem ID [5991]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 89
Problem number: 1(a).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - y = x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x$3)-y(x)=x,y(x), singsol=all)
```

$$y(x) = -x + e^{x} c_1 + c_2 e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\, x}{2}\right) + c_3 e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\, x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 57

```
DSolve[y'''[x]-y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + c_1 e^{x} + c_2 e^{-x/2} \cos\left(\frac{\sqrt{3}x}{2}\right) + c_3 e^{-x/2} \sin\left(\frac{\sqrt{3}x}{2}\right)$$

10.2 problem 1(b)

Internal problem ID [5992]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 89
Problem number: 1(b).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$y''' - 8y = e^{ix}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 44

```
dsolve(diff(y(x),x$3)-8*y(x)=exp(I*x),y(x), singsol=all)
```

$$y(x) = \left(-\frac{8}{65} + \frac{i}{65}\right) e^{ix} + e^{2x} c_1 + c_2 e^{-x} \cos\left(\sqrt{3}\, x\right) + c_3 e^{-x} \sin\left(\sqrt{3}\, x\right)$$

✓ Solution by Mathematica
Time used: 0.472 (sec). Leaf size: 59

```
DSolve[y'''[x]-8*y[x]==Exp[I*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{65} e^{-x} \left(-(8-i) e^{(1+i)x} + 65 c_1 e^{3x} + 65 c_2 \cos\left(\sqrt{3}x\right) + 65 c_3 \sin\left(\sqrt{3}x\right)\right)$$

10.3 problem 1(c)

Internal problem ID [5993]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 89
Problem number: 1(c).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' + 16y = \cos(x)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 67

```
dsolve(diff(y(x),x$4)+16*y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = c_4 \mathrm{e}^{-\sqrt{2}\,x} \sin\left(\sqrt{2}\,x\right) + c_2 \mathrm{e}^{\sqrt{2}\,x} \sin\left(\sqrt{2}\,x\right) + c_3 \mathrm{e}^{-\sqrt{2}\,x} \cos\left(\sqrt{2}\,x\right) + c_1 \mathrm{e}^{\sqrt{2}\,x} \cos\left(\sqrt{2}\,x\right) + \frac{\cos(x)}{17}$$

✓ Solution by Mathematica
Time used: 0.762 (sec). Leaf size: 74

```
DSolve[y''''[x]+16*y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\cos(x)}{17} + e^{-\sqrt{2}x}\left(\left(c_1 e^{2\sqrt{2}x} + c_2\right)\cos\left(\sqrt{2}x\right) + \left(c_4 e^{2\sqrt{2}x} + c_3\right)\sin\left(\sqrt{2}x\right)\right)$$

10.4 problem 1(d)

Internal problem ID [5994]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 89
Problem number: 1(d).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$y'''' - 4y''' + 6y'' - 4y' + y = e^x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$4)-4*diff(y(x),x$3)+6*diff(y(x),x$2)-4*diff(y(x),x)+y(x)=exp(x),y(x), sin
```

$$y(x) = e^x \left(\frac{1}{24}x^4 + c_1 + c_2 x + c_3 x^2 + x^3 c_4\right)$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 39

```
DSolve[y''''[x]-4*y'''[x]+6*y''[x]-4*y'[x]+y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{1}{24} e^x \left(x^4 + 24c_4 x^3 + 24c_3 x^2 + 24c_2 x + 24c_1\right)$$

10.5 problem 1(e)

Internal problem ID [5995]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 89
Problem number: 1(e).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' - y = \cos(x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$4)-y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = c_4 e^{-x} + \frac{(4c_1 - 1)\cos(x)}{4} + \frac{(-x + 4c_3)\sin(x)}{4} + e^x c_2$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 40

```
DSolve[y''''[x]-y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x + c_3 e^{-x} + \left(-\frac{1}{2} + c_2\right)\cos(x) + \left(-\frac{x}{4} + c_4\right)\sin(x)$$

10.6 problem 1(f)

Internal problem ID [5996]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 89
Problem number: 1(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2iy' - y = e^{ix} - 2\,e^{-ix}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 58

```
dsolve(diff(y(x),x$2)-2*I*diff(y(x),x)-y(x)=exp(I*x)-2*exp(-I*x),y(x), singsol=all)
```

$$y(x) = -1+\cos\left(\frac{x}{2}\right)^2 \left(x^2+2ix+2\right)+\sin\left(\frac{x}{2}\right) x(ix-2)\cos\left(\frac{x}{2}\right)+(c_1x+c_2)\,e^{ix}-ix-\frac{x^2}{2}$$

✓ Solution by Mathematica
Time used: 0.177 (sec). Leaf size: 39

```
DSolve[y''[x]-2*I*y'[x]-y[x]==Exp[I*x]-2*Exp[-I*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-ix}\left(1+e^{2ix}\left(x^2+2c_2x+2c_1\right)\right)$$

11 Chapter 2. Linear equations with constant coefficients. Page 93

11.1 problem 1(a)

Internal problem ID [5997]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 93
Problem number: 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \cos(x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+4*y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = \sin(2x)\, c_2 + \cos(2x)\, c_1 + \frac{\cos(x)}{3}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 26

```
DSolve[y''[x]+4*y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\cos(x)}{3} + c_1 \cos(2x) + c_2 \sin(2x)$$

11.2 problem 1(b)

Internal problem ID [5998]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 93
Problem number: 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \sin(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+4*y(x)=sin(2*x),y(x), singsol=all)
```

$$y(x) = \frac{(-x + 4c_1)\cos(2x)}{4} + \sin(2x)\, c_2$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 33

```
DSolve[y''[x]+4*y[x]==Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(-\frac{x}{4} + c_1\right)\cos(2x) + \frac{1}{8}(1 + 16c_2)\sin(x)\cos(x)$$

11.3 problem 1(c)

Internal problem ID [5999]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 93
Problem number: 1(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y = 3\,e^{2x} + 4\,e^{-x}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)-4*y(x)=3*exp(2*x)+4*exp(-x),y(x), singsol=all)
```

$$y(x) = e^{-2x}\left(\frac{(12x + 16c_2 - 3)\,e^{4x}}{16} + c_1 - \frac{4\,e^{x}}{3}\right)$$

✓ Solution by Mathematica
Time used: 0.345 (sec). Leaf size: 86

```
DSolve[y''[x]-4*y[x]==3*exp[2*x]+4*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(e^{4x}\int_1^x \frac{1}{4}e^{-3K[1]}\left(3e^{K[1]}\exp(2K[1]) + 4\right)dK[1] + \int_1^x -\frac{1}{4}e^{K[2]}\left(3e^{K[2]}\exp(2K[2]) + 4\right)dK[2] + c_1e^{4x} + c_2\right)$$

11.4 problem 1(d)

Internal problem ID [6000]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 93
Problem number: 1(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y' - 2y = x^2 + \cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-2*y(x)=x^2+cos(x),y(x), singsol=all)
```

$$y(x) = c_2 e^{2x} + c_1 e^{-x} - \frac{x^2}{2} - \frac{3\cos(x)}{10} - \frac{\sin(x)}{10} + \frac{x}{2} - \frac{3}{4}$$

✓ Solution by Mathematica
Time used: 0.142 (sec). Leaf size: 44

```
DSolve[y''[x]-y'[x]-2*y[x]==x^2+Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{20}\left(-10x^2 + 10x - 2\sin(x) - 6\cos(x) - 15\right) + c_1 e^{-x} + c_2 e^{2x}$$

11.5 problem 1(e)

Internal problem ID [6001]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 93
Problem number: 1(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 9y = x^2 e^{3x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)+9*y(x)=x^2*exp(3*x),y(x), singsol=all)
```

$$y(x) = \frac{\left(x - \frac{1}{3}\right)^2 e^{3x}}{18} + \cos(3x)\, c_1 + \sin(3x)\, c_2$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 36

```
DSolve[y''[x]+9*y[x]==x^2*Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{162} e^{3x} (1 - 3x)^2 + c_1 \cos(3x) + c_2 \sin(3x)$$

11.6 problem 1(f)

Internal problem ID [6002]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 93
Problem number: 1(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y = x\,\mathrm{e}^{x} \cos(2x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$2)+y(x)=x*exp(x)*cos(2*x),y(x), singsol=all)
```

$$y(x) = \frac{\left((-10x+22)\cos(x)^2 + (20x-4)\sin(x)\cos(x) + 5x - 11\right)\mathrm{e}^{x}}{50} + \cos(x)\,c_1 + \sin(x)\,c_2$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 45

```
DSolve[y''[x]+y[x]==x*Exp[x]*Cos[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{50}e^{x}(2(1-5x)\sin(2x) + (5x-11)\cos(2x)) + c_1\cos(x) + c_2\sin(x)$$

11.7 problem 1(g)

Internal problem ID [6003]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 93
Problem number: 1(g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + iy' + 2y = 2\cosh(2x) + e^{-2x}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)+I*diff(y(x),x)+2*y(x)=2*cosh(2*x)+exp(-2*x),y(x), singsol=all)
```

$$y(x) = c_2 e^{ix} + e^{-2ix} c_1 + \left(\frac{3}{10} + \frac{i}{10}\right) e^{-2x} + \left(\frac{3}{20} - \frac{i}{20}\right) e^{2x}$$

✓ Solution by Mathematica
Time used: 0.157 (sec). Leaf size: 48

```
DSolve[y''[x]+I*y'[x]+2*y[x]==2*Cosh[2*x]+Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{20} e^{-2x}\left((3-i)e^{4x} + (6+2i)\right) + c_1 e^{-2ix} + c_2 e^{ix}$$

11.8 problem 1(h)

Internal problem ID [6004]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 93
Problem number: 1(h).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _quadrature]]`

$$\boxed{y''' = x^2 + e^{-x} \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$3)=x^2+exp(-x)*sin(x),y(x), singsol=all)
```

$$y(x) = \frac{e^{-x}(-\cos(x) + \sin(x))}{4} + \frac{x^5}{60} + \frac{c_1 x^2}{2} + c_2 x + c_3$$

✓ Solution by Mathematica
Time used: 0.114 (sec). Leaf size: 47

```
DSolve[y'''[x]==x^2+Exp[-x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^5}{60} + c_3 x^2 + \frac{1}{4} e^{-x} \sin(x) - \frac{1}{4} e^{-x} \cos(x) + c_2 x + c_1$$

11.9 problem 1(i)

Internal problem ID [6005]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 93
Problem number: 1(i).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' + 3y'' + 3y' + y = x^2 \mathrm{e}^{-x}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$3)+3*diff(y(x),x$2)+3*diff(y(x),x)+y(x)=x^2*exp(-x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}\left(\frac{1}{60}x^5 + c_1 + c_2 x + c_3 x^2\right)$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 34

```
DSolve[y'''[x]+3*y''[x]+3*y'[x]+y[x]==x^2*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{60}e^{-x}\left(x^5 + 60c_3x^2 + 60c_2x + 60c_1\right)$$

12 Chapter 3. Linear equations with variable coefficients. Page 108

12.1 problem 1(c.1)

Internal problem ID [6006]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 108
Problem number: 1(c.1).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$y'' + \frac{y'}{x} - \frac{y}{x^2} = 0$$

With initial conditions

$$[y(1) = 1, y'(1) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 13

```
dsolve([diff(y(x),x$2)+1/x*diff(y(x),x)-1/x^2*y(x)=0,y(1) = 1, D(y)(1) = 0],y(x), singsol=al
```

$$y(x) = \frac{1}{2x} + \frac{x}{2}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 17

```
DSolve[{y''[x]+1/x*y'[x]-1/x^2*y[x]==0,{y[1]==1,y'[1]==0}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{x^2 + 1}{2x}$$

12.2 problem 1(c.2)

Internal problem ID [6007]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 108
Problem number: 1(c.2).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$y'' + \frac{y'}{x} - \frac{y}{x^2} = 0$$

With initial conditions

$$[y(1) = 0, y'(1) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 13

```
dsolve([diff(y(x),x$2)+1/x*diff(y(x),x)-1/x^2*y(x)=0,y(1) = 0, D(y)(1) = 1],y(x), singsol=al
```

$$y(x) = -\frac{1}{2x} + \frac{x}{2}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 17

```
DSolve[{y''[x]+1/x*y'[x]-1/x^2*y[x]==0,{y[1]==0,y'[1]==1}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{x^2 - 1}{2x}$$

12.3 problem 2

Internal problem ID [6008]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 108
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(3x-1)^2 y'' + (9x-3) y' - 9y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve((3*x-1)^2*diff(y(x),x$2)+(9*x-3)*diff(y(x),x)-9*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{9\left(x-\frac{1}{3}\right)^2 c_2 + 9c_1}{9x-3}$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 39

```
DSolve[(3*x-1)^2*y''[x]+(9*x-3)*y'[x]-9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1(-9x^2+6x-2) - 3ic_2x(3x-2)}{6x-2}$$

13 Chapter 3. Linear equations with variable coefficients. Page 121

13.1 problem 1(a)

Internal problem ID [6009]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 121
Problem number: 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 7xy' + 15y = 0}$$

Given that one solution of the ode is

$$y_1 = x^3$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)-7*x*diff(y(x),x)+15*y(x)=0,x^3],singsol=all)
```

$$y(x) = x^3(c_1x^2 + c_2)$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]-7*x*y'[x]+15*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^3(c_2x^2 + c_1)$$

13.2 problem 1(b)

Internal problem ID [6010]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 121
Problem number: 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - xy' + y = 0}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve([x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,x],singsol=all)
```

$$y(x) = x(c_2 \ln (x) + c_1)$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 15

```
DSolve[x^2*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(c_2 \log(x) + c_1)$$

13.3 problem 1(c)

Internal problem ID [6011]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 121
Problem number: 1(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 4xy' + (4x^2 - 2)\, y = 0$$

Given that one solution of the ode is

$$y_1 = e^{x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve([diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2-2)*y(x)=0,exp(x^2)],singsol=all)
```

$$y(x) = e^{x^2}(c_2 x + c_1)$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 18

```
DSolve[y''[x]-4*x*y'[x]+(4*x^2-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x^2}(c_2 x + c_1)$$

13.4 problem 1(d)

Internal problem ID [6012]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 121
Problem number: 1(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{xy'' - (1+x)\,y' + y = 0}$$

Given that one solution of the ode is

$$y_1 = e^x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve([x*diff(y(x),x$2)-(x+1)*diff(y(x),x)+y(x)=0,exp(x)],singsol=all)
```

$$y(x) = e^x c_2 + c_1 x + c_1$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 19

```
DSolve[x*y''[x]-(x+1)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x - c_2(x+1)$$

13.5 problem 1(e)

Internal problem ID [6013]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 121
Problem number: 1(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(-x^{2}+1\right) y''-2 x y'+2 y=0}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve([(1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,x],singsol=all)
```

$$y(x) = -\frac{c_2 \ln\left(x+1\right) x}{2}+\frac{c_2 \ln\left(x-1\right) x}{2}+c_1 x+c_2$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 33

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x-\frac{1}{2} c_2 (x \log(1-x)-x \log(x+1)+2)$$

13.6 problem 1(f)

Internal problem ID [6014]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 121
Problem number: 1(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2xy' + 2y = 0}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,x],singsol=all)
```

$$y(x) = \mathrm{e}^{x^2} c_2 + x\left(-\sqrt{\pi}\, c_2 \operatorname{erfi}(x) + c_1\right)$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 43

```
DSolve[y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\pi} c_2 \sqrt{x^2} \operatorname{erfi}\left(\sqrt{x^2}\right) + c_2 e^{x^2} + 2c_1 x$$

13.7 problem 2

Internal problem ID [6015]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 121
Problem number: 2.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^3y''' - 3x^2y'' + 6xy' - 6y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve([x^3*diff(y(x),x$3)-3*x^2*diff(y(x),x$2)+6*x*diff(y(x),x)-6*y(x)=0,x],singsol=all)
```

$$y(x) = x(c_2x^2 + c_1x + c_3)$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 19

```
DSolve[x^3*y'''[x]-3*x^2*y''[x]+6*x*y'[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(x(c_3x + c_2) + c_1)$$

14 Chapter 3. Linear equations with variable coefficients. Page 124

14.1 problem 1

Internal problem ID [6016]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 124
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2y'' - 2y = 0}$$

Given that one solution of the ode is

$$y_1 = x^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)-2*y(x)=0,x^2],singsol=all)
```

$$y(x) = \frac{c_1 x^3 + c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 x^3 + c_1}{x}$$

14.2 problem 2

Internal problem ID [6017]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 124
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - xy' + y = 0}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve([x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,x],singsol=all)
```

$$y(x) = x(c_2 \ln(x) + c_1)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 15

```
DSolve[x^2*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(c_2 \log(x) + c_1)$$

14.3 problem 3

Internal problem ID [6018]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 124
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 4xy' + y(x^2+2) = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+(2+x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x) + \cos(x) c_2}{x^2}$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 37

```
DSolve[x^2*y''[x]+4*x*y'[x]+(2+x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x^2}$$

15 Chapter 3. Linear equations with variable coefficients. Page 130

15.1 problem 1(a)

Internal problem ID [6019]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 130
Problem number: 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Hermite]`

$$\boxed{y'' - xy' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 - \frac{1}{24}x^4\right) y(0) + D(y)(0)\,x + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 27

```
AsymptoticDSolveValue[y''[x]-x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{x^4}{24} - \frac{x^2}{2} + 1\right) + c_2 x$$

15.2 problem 1(b)

Internal problem ID [6020]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 130
Problem number: 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 3x^2y' - xy = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)+3*x^2*diff(y(x),x)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{x^3}{6}\right) y(0) + \left(x - \frac{1}{6}x^4\right) D(y)\left(0\right) + O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]+3*x^2*y'[x]-x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x - \frac{x^4}{6}\right) + c_1\left(\frac{x^3}{6} + 1\right)$$

15.3 problem 1(c)

Internal problem ID [6021]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 130
Problem number: 1(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - yx^2 = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)-x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{x^4}{12}\right) y(0) + \left(x + \frac{1}{20}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]-x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{20} + x\right) + c_1\left(\frac{x^4}{12} + 1\right)$$

15.4 problem 1(d)

Internal problem ID [6022]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 130
Problem number: 1(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x^3 + yx^2 = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)+x^3*diff(y(x),x)+x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^4}{12}\right) y(0) + \left(x - \frac{1}{10}x^5\right) D(y)\,(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]+x^3*y'[x]+x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x - \frac{x^5}{10}\right) + c_1\left(1 - \frac{x^4}{12}\right)$$

15.5 problem 1(e)

Internal problem ID [6023]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 130
Problem number: 1(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4\right) y(0) + \left(x - \frac{1}{6}x^3 + \frac{1}{120}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{120} - \frac{x^3}{6} + x\right) + c_1\left(\frac{x^4}{24} - \frac{x^2}{2} + 1\right)$$

15.6 problem 2

Internal problem ID [6024]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 130
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (x-1)^2 y' - (x-1)\, y = 0}$$

With initial conditions

$$[y(1) = 1, y'(1) = 0]$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
Order:=6;
dsolve([diff(y(x),x$2)+(x-1)^2*diff(y(x),x)-(x-1)*y(x)=0,y(1) = 1, D(y)(1) = 0],y(x),type='s
```

$$y(x) = 1 + \frac{1}{6}(x-1)^3 + O\left((x-1)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 14

```
AsymptoticDSolveValue[{y''[x]+(x-1)^2*y'[x]-(x-1)*y[x]==0,{y[1]==1,y'[1]==0}},y[x],{x,1,5}]
```

$$y(x) \to \frac{1}{6}(x-1)^3 + 1$$

15.7 problem 3

Internal problem ID [6025]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 130
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{(x^2+1)\,y''+y=0}$$

With initial conditions

$$[y(0)=0, y'(0)=1]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([(1+x^2)*diff(y(x),x$2)+y(x)=0,y(0) = 0, D(y)(0) = 1],y(x),type='series',x=0);
```

$$y(x)=x-\frac{1}{6}x^3+\frac{7}{120}x^5+\mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{(1+x^2)*y''[x]+y[x]==0,{y[0]==0,y'[0]==1}},y[x],{x,0,5}]
```

$$y(x)\to\frac{7x^5}{120}-\frac{x^3}{6}+x$$

15.8 problem 4

Internal problem ID [6026]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 130
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + e^{x} y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
Order:=6;
dsolve([diff(y(x),x$2)+exp(x)*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x),type='series',x=0);
```

$$y(x) = 1 - \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{40}x^5 + O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[{y''[x]+Exp[x]*y[x]==0,{}},y[x],{x,0,5}]
```

$$y(x) \to c_2\left(-\frac{x^5}{60} - \frac{x^4}{12} - \frac{x^3}{6} + x\right) + c_1\left(\frac{x^5}{40} - \frac{x^3}{6} - \frac{x^2}{2} + 1\right)$$

15.9 problem 5

Internal problem ID [6027]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 130
Problem number: 5.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - xy = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0, y''(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 14

```
dsolve([diff(y(x),x$3)-x*y(x)=0,y(0) = 1, D(y)(0) = 0, (D@@2)(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \text{hypergeom}\left([\,],\left[\frac{1}{2},\frac{3}{4}\right],\frac{x^4}{64}\right)$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 21

```
DSolve[{y'''[x]-x*y[x]==0,{y[0]==1,y'[0]==0,y''[0]==0}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to {}_0F_2\left(;\frac{1}{2},\frac{3}{4};\frac{x^4}{64}\right)$$

15.10 problem 6

Internal problem ID [6028]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 130
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type **[_Gegenbauer]**

$$\boxed{(-x^2+1)\,y''-2xy'+\alpha(\alpha+1)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 101

```
Order:=6;
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+alpha*(alpha+1)*y(x)=0,y(x),type='series',x=0
```

$$y(x)=\left(1-\frac{\alpha(\alpha+1)\,x^2}{2}+\frac{\alpha(\alpha^3+2\alpha^2-5\alpha-6)\,x^4}{24}\right)y(0)+\left(x-\frac{(\alpha^2+\alpha-2)\,x^3}{6}+\frac{(\alpha^4+2\alpha^3-13\alpha^2-14\alpha+24)\,x^5}{120}\right)D(y)\,(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 127

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-2*x*y'[x]+\[Alpha]*(\[Alpha]+1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{1}{60}(-\alpha^2-\alpha)\,x^5-\frac{1}{120}(-\alpha^2-\alpha)\,(\alpha^2+\alpha)\,x^5-\frac{1}{10}(\alpha^2+\alpha)\,x^5+\frac{x^5}{5}-\frac{1}{6}(\alpha^2+\alpha)\,x^3+\frac{x^3}{3}+x\right)+c_1\left(\frac{1}{24}(\alpha^2+\alpha)^2\,x^4-\frac{1}{4}(\alpha^2+\alpha)\,x^4-\frac{1}{2}(\alpha^2+\alpha)\,x^2+1\right)$$

15.11 problem 7

Internal problem ID [6029]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 130
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']`

$$\boxed{(-x^2+1)\,y''-xy'+\alpha^2y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve((1-x^2)*diff(y(x),x$2)-x*diff(y(x),x)+alpha^2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x+\sqrt{x^2-1}\right)^{-\alpha}+c_2\left(x+\sqrt{x^2-1}\right)^{\alpha}$$

✓ Solution by Mathematica

Time used: 0.089 (sec). Leaf size: 91

```
DSolve[(1-x^2)*y''[x]-x*y'[x]+\[Alpha]^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\cosh\left(\frac{1}{2}\alpha\left(\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)-\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)\right)\right)$$
$$-ic_2\sinh\left(\frac{1}{2}\alpha\left(\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)-\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)\right)\right)$$

15.12 problem 8

Internal problem ID [6030]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 130
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2xy' + 2\alpha y = 0}$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+2*alpha*y(x)=0,y(x), singsol=all)
```

$$y(x) = x\left(\text{KummerM}\left(\frac{1}{2} - \frac{\alpha}{2}, \frac{3}{2}, x^2\right) c_1 + \text{KummerU}\left(\frac{1}{2} - \frac{\alpha}{2}, \frac{3}{2}, x^2\right) c_2\right)$$

✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 91

```
DSolve[(1-x^2)*y''[x]-x*y'[x]+\[Alpha]^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cosh\left(\frac{1}{2}\alpha\left(\log\left(1 - \frac{x}{\sqrt{x^2-1}}\right) - \log\left(\frac{x}{\sqrt{x^2-1}} + 1\right)\right)\right) - ic_2 \sinh\left(\frac{1}{2}\alpha\left(\log\left(1 - \frac{x}{\sqrt{x^2-1}}\right) - \log\left(\frac{x}{\sqrt{x^2-1}} + 1\right)\right)\right)$$

16 Chapter 4. Linear equations with Regular Singular Points. Page 149

16.1 problem 1(a)

Internal problem ID [6031]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 149
Problem number: 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' + 2xy' - 6y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-6*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1x^5 + c_2}{x^3}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+2*x*y'[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^5 + c_1}{x^3}$$

16.2 problem 1(b)

Internal problem ID [6032]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 149
Problem number: 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2x^2y'' + xy' - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(2*x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x + \frac{c_2}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 18

```
DSolve[2*x^2*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{\sqrt{x}} + c_2x$$

16.3 problem 1(c)

Internal problem ID [6033]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 149
Problem number: 1(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' + xy' - 4y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1x^4 + c_2}{x^2}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+x*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^4 + c_1}{x^2}$$

16.4 problem 1(d)

Internal problem ID [6034]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 149
Problem number: 1(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - 5xy' + 9y = x^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^2*diff(y(x),x$2)-5*x*diff(y(x),x)+9*y(x)=x^2,y(x), singsol=all)
```

$$y(x) = x^2(\ln(x)\, c_1x + c_2x + 1)$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]-5*x*y'[x]+9*y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(c_1x + 3c_2x\log(x) + 1)$$

16.5 problem 1(e)

Internal problem ID [6035]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 149
Problem number: 1(e).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _homogeneous]]`

$$x^3y''' + 2x^2y'' - xy' + y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x^3*diff(y(x),x$3)+2*x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_3 \ln(x) x^2 + c_2 x^2 + c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 22

```
DSolve[x^3*y'''[x]+2*x^2*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x} + c_2 x + c_3 x \log(x)$$

16.6 problem 2(a)

Internal problem ID [6036]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 149
Problem number: 2(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + xy' + 4y = 1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+4*y(x)=1,y(x), singsol=all)
```

$$y(x) = \sin(2\ln(x))\, c_2 + \cos(2\ln(x))\, c_1 + \frac{1}{4}$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 25

```
DSolve[x^2*y''[x]+x*y'[x]+4*y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(2\log(x)) + c_2 \sin(2\log(x)) + \frac{1}{4}$$

16.7 problem 2(b)

Internal problem ID [6037]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 149
Problem number: 2(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^2y'' - 3xy' + 5y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+5*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^2(c_1 \sin(\ln(x)) + c_2 \cos(\ln(x)))$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]-3*x*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(c_2 \cos(\log(x)) + c_1 \sin(\log(x)))$$

16.8 problem 2(c)

Internal problem ID [6038]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 149
Problem number: 2(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + (-2-i)\,xy' + 3iy = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x^2*diff(y(x),x$2)-(2+I)*x*diff(y(x),x)+3*I*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^3 + c_2x^i$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 20

```
DSolve[x^2*y''[x]-(2+I)*x*y'[x]+3*I*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1x^i + c_2x^3$$

16.9 problem 2(d)

Internal problem ID [6039]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 149
Problem number: 2(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + xy' - 4\pi y = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 44

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-4*Pi*y(x)=x,y(x), singsol=all)
```

$$y(x) = \frac{c_2(4\pi - 1)\,x^{-2\sqrt{\pi}} + c_1(4\pi - 1)\,x^{2\sqrt{\pi}} - x}{4\pi - 1}$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]-4*Pi*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x^{2\sqrt{\pi}} + c_1x^{-2\sqrt{\pi}} + \frac{x}{1 - 4\pi}$$

17 Chapter 4. Linear equations with Regular Singular Points. Page 154

17.1 problem 1(a)

Internal problem ID [6040]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 154
Problem number: 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + (x^2 + x)\, y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 53

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+(x+x^2)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x\left(1 - \frac{1}{3}x + \frac{1}{12}x^2 - \frac{1}{60}x^3 + \frac{1}{360}x^4 - \frac{1}{2520}x^5 + \frac{1}{20160}x^6 - \frac{1}{181440}x^7 + \mathrm{O}\left(x^8\right)\right) + \frac{c_2\left(-2 + 2x - x^2 + \frac{1}{3}x^3 - \frac{1}{12}x^4 + \frac{1}{60}x^5 - \frac{1}{360}x^6 + \frac{1}{2520}x^7 + \mathrm{O}\left(x^8\right)\right)}{x}$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 92

```
AsymptoticDSolveValue[x^2*y''[x]+(x+x^2)*y'[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{x^5}{720} - \frac{x^4}{120} + \frac{x^3}{24} - \frac{x^2}{6} + \frac{x}{2} + \frac{1}{x} - 1\right) + c_2\left(\frac{x^7}{20160} - \frac{x^6}{2520} + \frac{x^5}{360} - \frac{x^4}{60} + \frac{x^3}{12} - \frac{x^2}{3} + x\right)$$

17.2 problem 1(b)

Internal problem ID [6041]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 154
Problem number: 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3x^2y'' + y'x^6 + 2xy = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 70

```
Order:=8;
dsolve(3*x^2*diff(y(x),x$2)+x^6*diff(y(x),x)+2*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x\left(1 - \frac{1}{3}x + \frac{1}{27}x^2 - \frac{1}{486}x^3 + \frac{1}{14580}x^4 - \frac{7291}{656100}x^5 + \frac{225991}{41334300}x^6 - \frac{2522341}{3472081200}x^7 + O\left(x^8\right)\right) + c_2\left(\ln\left(x\right)\left(-\frac{2}{3}x + \frac{2}{9}x^2 - \frac{2}{81}x^3 + \frac{1}{729}x^4 - \frac{1}{21870}x^5 + \frac{7291}{984150}x^6 - \frac{225991}{62001450}x^7 + O\left(x^8\right)\right) + \left(1 - \frac{1}{3}x^2 + \frac{14}{243}x^3 - \frac{35}{8748}x^4 + \frac{101}{656100}x^5 + \frac{69199}{14762250}x^6 + \frac{19882543}{4340101500}x^7 + O\left(x^8\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 121

```
AsymptoticDSolveValue[3*x^2*y''[x]+x^6*y'[x]+2*x*y[x]==0,y[x],{x,0,7}]
```

$$
\begin{aligned}
y(x) \to\; & c_1\left(\frac{x(7291x^5 - 45x^4 + 1350x^3 - 24300x^2 + 218700x - 656100)\log(x)}{984150}\right. \\
& \left. + \frac{-80332x^6 + 5895x^5 - 158625x^4 + 2430000x^3 - 16402500x^2 + 19683000x + 29524500}{29524500}\right) \\
& + c_2\left(\frac{225991x^7}{41334300} - \frac{7291x^6}{656100} + \frac{x^5}{14580} - \frac{x^4}{486} + \frac{x^3}{27} - \frac{x^2}{3} + x\right)
\end{aligned}
$$

17.3 problem 1(c)

Internal problem ID [6042]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 154
Problem number: 1(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - 5y' + 3yx^2 = 0$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)-5*diff(y(x),x)+3*x^2*y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 106

```
AsymptoticDSolveValue[x^2*y''[x]-5*y'[x]+3*x^2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 \left(\frac{339x^7}{8750} + \frac{49x^6}{1250} + \frac{18x^5}{625} + \frac{3x^4}{50} + \frac{x^3}{5} + 1 \right) + c_2 e^{-5/x} \left(-\frac{302083x^7}{218750} + \frac{5243x^6}{6250} - \frac{357x^5}{625} + \frac{113x^4}{250} - \frac{49x^3}{125} + \frac{6x^2}{25} - \frac{2x}{5} + 1 \right) x^2$$

17.4 problem 1(d)

Internal problem ID [6043]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 154
Problem number: 1(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y''x + 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 70

```
Order:=8;
dsolve(x*diff(y(x),x$2)+4*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned}
y(x) = {} & c_1 x \left(1 - 2x + \frac{4}{3}x^2 - \frac{4}{9}x^3 + \frac{4}{45}x^4 - \frac{8}{675}x^5 + \frac{16}{14175}x^6 - \frac{8}{99225}x^7 + \mathrm{O}\left(x^8\right)\right) \\
& + c_2 \left(\ln\left(x\right)\left(\left(-4\right)x + 8x^2 - \frac{16}{3}x^3 + \frac{16}{9}x^4 - \frac{16}{45}x^5 + \frac{32}{675}x^6 - \frac{64}{14175}x^7 + \mathrm{O}\left(x^8\right)\right)\right. \\
& \left. + \left(1 - 12x^2 + \frac{112}{9}x^3 - \frac{140}{27}x^4 + \frac{808}{675}x^5 - \frac{1792}{10125}x^6 + \frac{9056}{496125}x^7 + \mathrm{O}\left(x^8\right)\right)\right)
\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 119

```
AsymptoticDSolveValue[x*y''[x]+4*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{4}{675}x\left(8x^5-60x^4+300x^3-900x^2+1350x-675\right)\log(x)+\frac{-2272x^6+15720x^5-70500x^4+180000x^3-202500x^2+40500x+10125}{10125}\right)+c_2\left(\frac{16x^7}{14175}-\frac{8x^6}{675}+\frac{4x^5}{45}-\frac{4x^4}{9}+\frac{4x^3}{3}-2x^2+x\right)$$

17.5 problem 1(e)

Internal problem ID [6044]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 154
Problem number: 1(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-x^2+1)\, y'' - 2xy' + 2y = 0}$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 49

```
Order:=8;
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=1);
```

$$y(x) = \left(-\frac{5}{2}(x-1) - \frac{3}{8}(x-1)^2 + \frac{1}{12}(x-1)^3 - \frac{5}{192}(x-1)^4 + \frac{3}{320}(x-1)^5 - \frac{7}{1920}(x-1)^6 + \frac{1}{672}(x-1)^7 + \mathrm{O}\left((x-1)^8\right)\right) c_2 + \left(1 + (x-1) + \mathrm{O}\left((x-1)^8\right)\right)(c_2 \ln(x-1) + c_1)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 86

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],{x,1,7}]
```

$$y(x) \to c_1 x + c_2\left(\frac{1}{672}(x-1)^7 - \frac{7(x-1)^6}{1920} + \frac{3}{320}(x-1)^5 - \frac{5}{192}(x-1)^4 + \frac{1}{12}(x-1)^3 - \frac{3}{8}(x-1)^2 - 2(x-1) + \frac{1-x}{2} + x\log(x-1)\right)$$

17.6 problem 1(f)

Internal problem ID [6045]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 154
Problem number: 1(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2+x-2)^2 y''+3(x+2)\,y'+(x-1)\,y=0$$

With the expansion point for the power series method at $x=-2$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 57

```
Order:=8;
dsolve((x^2+x-2)^2*diff(y(x),x$2)+3*(x+2)*diff(y(x),x)+(x-1)*y(x)=0,y(x),type='series',x=-2)
```

$$y(x) = \frac{c_1\left(1-\frac{5}{9}(x+2)+\frac{23}{324}(x+2)^2+\frac{271}{43740}(x+2)^3+\frac{10517}{12597120}(x+2)^4+\frac{778801}{6235574400}(x+2)^5+\frac{16965493}{942818849280}(x+\cdots\right.}{\cdots}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 148

```
AsymptoticDSolveValue[(x^2+x-2)^2*y''[x]+3*(x+2)*y'[x]+(x-1)*y[x]==0,y[x],{x,-2,7}]
```

$$y(x)\to c_1(x+2)\left(-\frac{52991201(x+2)^7}{11727918720000}-\frac{5797423(x+2)^6}{290405606400}-\frac{709507(x+2)^5}{8066822400}-\frac{11093(x+2)^4}{28304640}-\frac{53(x+2)^3}{29484}-\frac{11(x+2)^2}{1260}+\frac{1}{21}(-x-2)+1\right)+\frac{c_2\left(\frac{899971067(x+2)^7}{458981357990400}+\frac{16965493(x+2)^6}{942818849280}+\frac{778801(x+2)^5}{6235574400}+\frac{10517(x+2)^4}{12597120}+\frac{271(x+2)^3}{43740}+\frac{23}{324}(x+2)^2-\frac{5(x+2)}{9}+1\right)}{\sqrt[3]{x+2}}$$

17.7 problem 1(g)

Internal problem ID [6046]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 154
Problem number: 1(g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \sin(x)\, y' + y\cos(x) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.203 (sec). Leaf size: 53

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+sin(x)*diff(y(x),x)+cos(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{-i}\left(1 + \left(\frac{1}{12} + \frac{i}{24}\right)x^2 + \left(\frac{29}{28800} + \frac{67i}{28800}\right)x^4 + \left(-\frac{893}{14515200} - \frac{17i}{4838400}\right)x^6 + \mathrm{O}\left(x^8\right)\right) + c_2 x^{i}\left(1 + \left(\frac{1}{12} - \frac{i}{24}\right)x^2 + \left(\frac{29}{28800} - \frac{67i}{28800}\right)x^4 + \left(-\frac{893}{14515200} + \frac{17i}{4838400}\right)x^6 + \mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 112

```
AsymptoticDSolveValue[x^2*y''[x]+Sin[x]*y'[x]+Cos[x]*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 x^{-i}\left(\left(-\frac{26459}{59222016000}-\frac{12449i}{7402752000}\right)x^8-\left(\frac{893}{14515200}+\frac{17i}{4838400}\right)x^6+\left(\frac{29}{28800}+\frac{67i}{28800}\right)x^4+\left(\frac{1}{12}+\frac{i}{24}\right)x^2+1\right)+c_2 x^{i}\left(\left(-\frac{26459}{59222016000}+\frac{12449i}{7402752000}\right)x^8-\left(\frac{893}{14515200}-\frac{17i}{4838400}\right)x^6+\left(\frac{29}{28800}-\frac{67i}{28800}\right)x^4+\left(\frac{1}{12}-\frac{i}{24}\right)x^2+1\right)$$

17.8 problem 2(b)

Internal problem ID [6047]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 154
Problem number: 2(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 39

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x\left(1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 - \frac{1}{5040}x^6 + \mathrm{O}\left(x^8\right)\right) + c_2\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \mathrm{O}\left(x^8\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 76

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{x^{11/2}}{720} + \frac{x^{7/2}}{24} - \frac{x^{3/2}}{2} + \frac{1}{\sqrt{x}}\right) + c_2\left(-\frac{x^{13/2}}{5040} + \frac{x^{9/2}}{120} - \frac{x^{5/2}}{6} + \sqrt{x}\right)$$

17.9 problem 2(c)

Internal problem ID [6048]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 154
Problem number: 2(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + (4x^4 - 5x)\,y' + y(x^2 + 2) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 51

```
Order:=8;
dsolve(4*x^2*diff(y(x),x$2)+(4*x^4-5*x)*diff(y(x),x)+(x^2+2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{1}{4}}\left(1-\frac{1}{2}x^2-\frac{1}{15}x^3+\frac{1}{72}x^4+\frac{137}{1950}x^5+\frac{307}{36720}x^6-\frac{7169}{3439800}x^7+\mathrm{O}\left(x^8\right)\right)+c_2x^2\left(1 - \frac{1}{30}x^2 - \frac{8}{57}x^3 + \frac{1}{2760}x^4 + \frac{64}{12825}x^5 + \frac{147181}{9753840}x^6 - \frac{4037}{72268875}x^7 + \mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 106

```
AsymptoticDSolveValue[4*x^2*y''[x]+(4*x^4-5*x)*y'[x]+(x^2+2)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{4037x^7}{72268875} + \frac{147181x^6}{9753840} + \frac{64x^5}{12825} + \frac{x^4}{2760} - \frac{8x^3}{57} - \frac{x^2}{30} + 1\right)x^2 + c_2\left(-\frac{7169x^7}{3439800} + \frac{307x^6}{36720} + \frac{137x^5}{1950} + \frac{x^4}{72} - \frac{x^3}{15} - \frac{x^2}{2} + 1\right)\sqrt[4]{x}$$

17.10 problem 2(d)

Internal problem ID [6049]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 154
Problem number: 2(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + (-3x^2 + x)\, y' + \mathrm{e}^x y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 85

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+(x-3*x^2)*diff(y(x),x)+exp(x)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = {} & c_1 x^{-i}\left(1 + (1-i)\,x + \left(\frac{7}{16} - \frac{13i}{16}\right)x^2 + \left(\frac{7}{39} - \frac{395i}{936}\right)x^3 + \left(\frac{2117}{29952} - \frac{5197i}{29952}\right)x^4 \right. \\ & + \left(\frac{5521}{217152} - \frac{642043i}{10857600}\right)x^5 + \left(\frac{782461}{97718400} - \frac{8813057i}{521164800}\right)x^6 \\ & \left. + \left(\frac{1238071931}{580056422400} - \frac{3271304833i}{812078991360}\right)x^7 + \mathrm{O}\left(x^8\right)\right) \\ & + c_2 x^{i}\left(1 + (1+i)\,x + \left(\frac{7}{16} + \frac{13i}{16}\right)x^2 + \left(\frac{7}{39} + \frac{395i}{936}\right)x^3 + \left(\frac{2117}{29952} + \frac{5197i}{29952}\right)x^4 \right. \\ & + \left(\frac{5521}{217152} + \frac{642043i}{10857600}\right)x^5 + \left(\frac{782461}{97718400} + \frac{8813057i}{521164800}\right)x^6 \\ & \left. + \left(\frac{1238071931}{580056422400} + \frac{3271304833i}{812078991360}\right)x^7 + \mathrm{O}\left(x^8\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 122

```
AsymptoticDSolveValue[x^2*y''[x]+(x-3*x^2)*y'[x]+Exp[x]*y[x]==0,y[x],{x,0,7}]
```

$$
\begin{aligned}
y(x) \to & \left(\frac{1}{97718400}+\frac{11 i}{1563494400}\right) c_1 x^i ((1302761+756800 i) x^6 \\
& +(4384656+2763936 i) x^5+(12605400+8289000 i) x^4 \\
& +(31161600+19814400 i) x^3+(66096000+33955200 i) x^2 \\
& +(111974400+20736000 i) x+(66355200-45619200 i)) \\
& -\left(\frac{11}{1563494400}+\frac{i}{97718400}\right) c_2 x^{-i} ((756800+1302761 i) x^6 \\
& +(2763936+4384656 i) x^5+(8289000+12605400 i) x^4 \\
& +(19814400+31161600 i) x^3+(33955200+66096000 i) x^2 \\
& +(20736000+111974400 i) x-(45619200-66355200 i))
\end{aligned}
$$

18 Chapter 4. Linear equations with Regular Singular Points. Page 159

18.1 problem 1(a)

Internal problem ID [6050]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 159
Problem number: 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$3x^2y'' + 5xy' + 3xy = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 52

```
Order:=8;
dsolve(3*x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+3*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 - 3x + \frac{9}{8}x^2 - \frac{9}{56}x^3 + \frac{27}{2240}x^4 - \frac{81}{145600}x^5 + \frac{81}{4659200}x^6 - \frac{243}{619673600}x^7 + \mathrm{O}\left(x^8\right)\right)}{x^{\frac{2}{3}}} + c_2\left(1 - \frac{3}{5}x + \frac{9}{80}x^2 - \frac{9}{880}x^3 + \frac{27}{49280}x^4 - \frac{81}{4188800}x^5 + \frac{81}{167552000}x^6 - \frac{243}{26975872000}x^7 + \mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 111

```
AsymptoticDSolveValue[3*x^2*y''[x]+5*x*y'[x]+3*x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{243x^7}{26975872000} + \frac{81x^6}{167552000} - \frac{81x^5}{4188800} + \frac{27x^4}{49280} - \frac{9x^3}{880} + \frac{9x^2}{80} - \frac{3x}{5} + 1\right) + \frac{c_2\left(-\frac{243x^7}{619673600} + \frac{81x^6}{4659200} - \frac{81x^5}{145600} + \frac{27x^4}{2240} - \frac{9x^3}{56} + \frac{9x^2}{8} - 3x + 1\right)}{x^{2/3}}$$

18.2 problem 1(b)

Internal problem ID [6051]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 159
Problem number: 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{x^2y'' + xy' + yx^2 = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 47

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln (x) + c_1) \left(1 - \frac{1}{4}x^2 + \frac{1}{64}x^4 - \frac{1}{2304}x^6 + \mathrm{O}\left(x^8\right)\right) + \left(\frac{1}{4}x^2 - \frac{3}{128}x^4 + \frac{11}{13824}x^6 + \mathrm{O}\left(x^8\right)\right) c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 81

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+x^2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{x^6}{2304} + \frac{x^4}{64} - \frac{x^2}{4} + 1\right) + c_2\left(\frac{11x^6}{13824} - \frac{3x^4}{128} + \frac{x^2}{4} + \left(-\frac{x^6}{2304} + \frac{x^4}{64} - \frac{x^2}{4} + 1\right) \log(x)\right)$$

18.3 problem 2

Internal problem ID [6052]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 159
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^2y'' + y'e^x x + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 85

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*exp(x)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = {} & c_1 x^{-i}\left(1 + \left(-\frac{2}{5} + \frac{i}{5}\right)x + \left(\frac{3}{80} + \frac{i}{80}\right)x^2 + \left(\frac{67}{9360} - \frac{9i}{1040}\right)x^3 \right. \\ & + \left(-\frac{103}{149760} - \frac{229i}{149760}\right)x^4 + \left(-\frac{2831}{7238400} + \frac{607i}{4343040}\right)x^5 \\ & + \left(-\frac{59077}{1563494400} + \frac{26063i}{260582400}\right)x^6 + \left(\frac{22952047}{2030197478400} + \frac{8634893i}{580056422400}\right)x^7 \\ & \left. + \mathrm{O}\left(x^8\right)\right) + c_2 x^{i}\left(1 + \left(-\frac{2}{5} - \frac{i}{5}\right)x + \left(\frac{3}{80} - \frac{i}{80}\right)x^2 + \left(\frac{67}{9360} + \frac{9i}{1040}\right)x^3 \right. \\ & + \left(-\frac{103}{149760} + \frac{229i}{149760}\right)x^4 + \left(-\frac{2831}{7238400} - \frac{607i}{4343040}\right)x^5 \\ & + \left(-\frac{59077}{1563494400} - \frac{26063i}{260582400}\right)x^6 + \left(\frac{22952047}{2030197478400} - \frac{8634893i}{580056422400}\right)x^7 \\ & \left. + \mathrm{O}\left(x^8\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 122

```
AsymptoticDSolveValue[x^2*y''[x]+x*Exp[x]*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$
\begin{aligned}
y(x) \to & \left(\frac{11}{1563494400}+\frac{i}{97718400}\right)c_2x^{-i}\big((4913+7070i)x^6-(8568-32328i)x^5 \\
& -(132840+24120i)x^4-(247680+869760i)x^3+(2540160-1918080i)x^2 \\
& -(4976640-35665920i)x+(45619200-66355200i)\big) \\
& -\left(\frac{1}{97718400}+\frac{11i}{1563494400}\right)c_1x^{i}\big((7070+4913i)x^6+(32328-8568i)x^5 \\
& -(24120+132840i)x^4-(869760+247680i)x^3-(1918080-2540160i)x^2 \\
& +(35665920-4976640i)x-(66355200-45619200i)\big)
\end{aligned}
$$

19 Chapter 4. Linear equations with Regular Singular Points. Page 166

19.1 problem 1(i)

Internal problem ID [6053]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 166
Problem number: 1(i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2y'' + (x^2 + 5x)\, y' + (x^2 - 2)\, y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 55

```
Order:=8;
dsolve(2*x^2*diff(y(x),x$2)+(5*x+x^2)*diff(y(x),x)+(x^2-2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{5}{2}}\left(1 - \frac{1}{14}x - \frac{25}{504}x^2 + \frac{197}{33264}x^3 + \frac{1921}{3459456}x^4 - \frac{11653}{103783680}x^5 + \frac{12923}{21171870720}x^6 + \frac{917285}{1126343522304}x^7 + \mathrm{O}\left(x^8\right)\right) + c_1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 116

```
AsymptoticDSolveValue[2*x^2*y''[x]+(5*x+x^2)*y'[x]+(x^2-2)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{917285x^7}{1126343522304} + \frac{12923x^6}{21171870720} - \frac{11653x^5}{103783680} + \frac{1921x^4}{3459456} + \frac{197x^3}{33264} - \frac{25x^2}{504} - \frac{x}{14} + 1\right) + \frac{c_2\left(-\frac{4x^7}{35721} + \frac{101x^6}{45360} - \frac{x^5}{540} - \frac{19x^4}{216} + \frac{2x^3}{9} + \frac{5x^2}{6} - \frac{2x}{3} + 1\right)}{x^2}$$

19.2 problem 1(ii)

Internal problem ID [6054]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 166
Problem number: 1(ii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' - 4y'\mathrm{e}^x x + 3y\cos(x) = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 81

```
Order:=8;
dsolve(4*x^2*diff(y(x),x$2)-4*x*exp(x)*diff(y(x),x)+3*cos(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(x\left(1+\frac{3}{4}x+\frac{1}{2}x^2+\frac{103}{384}x^3+\frac{669}{5120}x^4+\frac{54731}{921600}x^5+\frac{123443}{4838400}x^6+\frac{30273113}{2890137600}x^7+\mathrm{O}\left(x^8\right)\right)c_1 + c_2\left(\ln(x)\left(\frac{1}{2}x+\frac{3}{8}x^2+\frac{1}{4}x^3+\frac{103}{768}x^4+\frac{669}{10240}x^5+\frac{54731}{1843200}x^6+\frac{123443}{9676800}x^7+\mathrm{O}\left(x^8\right)\right) + \left(1+x+\frac{3}{4}x^2+\frac{59}{144}x^3+\frac{5701}{27648}x^4+\frac{17519}{184320}x^5+\frac{6852157}{165888000}x^6+\frac{417496453}{24385536000}x^7+\mathrm{O}\left(x^8\right)\right)\right)\right)\sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.146 (sec). Leaf size: 146

```
AsymptoticDSolveValue[4*x^2*y''[x]-4*x*Exp[x]*y'[x]+3*Cos[x]*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(\frac{123443x^{15/2}}{4838400} + \frac{54731x^{13/2}}{921600} + \frac{669x^{11/2}}{5120} + \frac{103x^{9/2}}{384} + \frac{x^{7/2}}{2} + \frac{3x^{5/2}}{4} + x^{3/2}\right) + c_1\left(\frac{(54731x^5 + 120420x^4 + 247200x^3 + 460800x^2 + 691200x + 921600)\,x^{3/2}\log(x)}{1843200} + \frac{(192636\right.$$

19.3 problem 1(iii)

Internal problem ID [6055]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 166
Problem number: 1(iii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(-x^2+1)\,x^2y''+3(x^2+x)\,y'+y=0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 81

```
Order:=8;
dsolve((1-x^2)*x^2*diff(y(x),x$2)+3*(x+x^2)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{(c_2\ln(x)+c_1)\left(1+3x+\frac{1}{2}x^2-\frac{1}{6}x^3+\frac{1}{16}x^4-\frac{43}{1200}x^5+\frac{161}{7200}x^6-\frac{1837}{117600}x^7+\mathrm{O}(x^8)\right)+\left((-9)x-\frac{7}{2}x^2+\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 84

```
AsymptoticDSolveValue[(1-x^2)*y''[x]+3*(x+x^2)*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_2\left(\frac{53x^7}{630}+\frac{5x^6}{24}+\frac{2x^5}{15}-\frac{x^4}{4}-\frac{2x^3}{3}+x\right)+c_1\left(-\frac{19x^7}{420}-\frac{x^6}{144}+\frac{3x^5}{20}+\frac{5x^4}{24}-\frac{x^2}{2}+1\right)$$

19.4 problem 3(a)

Internal problem ID [6056]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 166
Problem number: 3(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type **[[_2nd_order, _with_linear_symmetries]]**

$$\boxed{x^2y'' + 3xy' + (1+x)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 81

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{(c_2\ln(x) + c_1)\left(1 - x + \frac{1}{4}x^2 - \frac{1}{36}x^3 + \frac{1}{576}x^4 - \frac{1}{14400}x^5 + \frac{1}{518400}x^6 - \frac{1}{25401600}x^7 + \mathrm{O}(x^8)\right) + \left(2x - \frac{3}{4}x^2 + \right.}{x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 164

```
AsymptoticDSolveValue[x^2*y''[x]+3*x*y'[x]+(1+x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to \frac{c_1\left(-\frac{x^7}{25401600}+\frac{x^6}{518400}-\frac{x^5}{14400}+\frac{x^4}{576}-\frac{x^3}{36}+\frac{x^2}{4}-x+1\right)}{x} + c_2\left(\frac{\frac{121x^7}{592704000}-\frac{49x^6}{5184000}+\frac{137x^5}{432000}-\frac{25x^4}{3456}+\frac{11x^3}{108}-\frac{3x^2}{4}+2x}{x} + \frac{\left(-\frac{x^7}{25401600}+\frac{x^6}{518400}-\frac{x^5}{14400}+\frac{x^4}{576}-\frac{x^3}{36}+\frac{x^2}{4}-x+1\right)\log(x)}{x}\right)$$

19.5 problem 3(b)

Internal problem ID [6057]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 166
Problem number: 3(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + 2x^2y' - 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 53

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+2*x^2*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^2\left(1 - x + \frac{3}{5}x^2 - \frac{4}{15}x^3 + \frac{2}{21}x^4 - \frac{1}{35}x^5 + \frac{1}{135}x^6 - \frac{8}{4725}x^7 + \mathrm{O}\left(x^8\right)\right) + \frac{c_2\left(12 - 12x + 8x^3 - 8x^4 + \frac{24}{5}x^5 - \frac{32}{15}x^6 + \frac{16}{21}x^7 + \mathrm{O}\left(x^8\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 87

```
AsymptoticDSolveValue[x^2*y''[x]+2*x^2*y'[x]-2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{8x^5}{45} + \frac{2x^4}{5} - \frac{2x^3}{3} + \frac{2x^2}{3} + \frac{1}{x} - 1\right) + c_2\left(\frac{x^8}{135} - \frac{x^7}{35} + \frac{2x^6}{21} - \frac{4x^5}{15} + \frac{3x^4}{5} - x^3 + x^2\right)$$

19.6 problem 3(c)

Internal problem ID [6058]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 166
Problem number: 3(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 5xy' + (-x^3 + 3)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 35

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+(3-x^3)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 + \frac{1}{15}x^3 + \frac{1}{720}x^6 + \mathrm{O}\left(x^8\right)\right)}{x} + \frac{c_2\left(-2 - \frac{2}{3}x^3 - \frac{1}{36}x^6 + \mathrm{O}\left(x^8\right)\right)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 40

```
AsymptoticDSolveValue[x^2*y''[x]+5*x*y'[x]+(3-3*x^3)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{x^3}{8} + \frac{1}{x^3} + 1\right) + c_2\left(\frac{x^5}{80} + \frac{x^2}{5} + \frac{1}{x}\right)$$

19.7 problem 3(d)

Internal problem ID [6059]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 166
Problem number: 3(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2x(1+x)\,y' + 2(1+x)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 53

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)-2*x*(x+1)*diff(y(x),x)+2*(x+1)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned}y(x) = {} & c_1x^2\left(1 + x + \frac{2}{3}x^2 + \frac{1}{3}x^3 + \frac{2}{15}x^4 + \frac{2}{45}x^5 + \frac{4}{315}x^6 + \frac{1}{315}x^7 + \mathrm{O}\left(x^8\right)\right) \\ & + c_2x\left(1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4 + \frac{4}{15}x^5 + \frac{4}{45}x^6 + \frac{8}{315}x^7 + \mathrm{O}\left(x^8\right)\right)\end{aligned}$$

✓ Solution by Mathematica
Time used: 0.086 (sec). Leaf size: 92

```
AsymptoticDSolveValue[x^2*y''[x]-2*x*(x+1)*y'[x]+2*(1+x)*y[x]==0,y[x],{x,0,7}]
```

$$\begin{aligned}y(x) \to {} & c_1\left(\frac{4x^7}{45} + \frac{4x^6}{15} + \frac{2x^5}{3} + \frac{4x^4}{3} + 2x^3 + 2x^2 + x\right) \\ & + c_2\left(\frac{4x^8}{315} + \frac{2x^7}{45} + \frac{2x^6}{15} + \frac{x^5}{3} + \frac{2x^4}{3} + x^3 + x^2\right)\end{aligned}$$

19.8 problem 3(e)

Internal problem ID [6060]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 166
Problem number: 3(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Bessel]`

$$\boxed{x^2y'' + xy' + (x^2 - 1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 53

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^2\left(1 - \frac{1}{8}x^2 + \frac{1}{192}x^4 - \frac{1}{9216}x^6 + \mathrm{O}\left(x^8\right)\right) + c_2\left(\ln\left(x\right)\left(x^2 - \frac{1}{8}x^4 + \frac{1}{192}x^6 + \mathrm{O}\left(x^8\right)\right) + \left(-2 + \frac{3}{32}x^4 - \frac{7}{1152}x\right.\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 75

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-1)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(-\frac{x^7}{9216} + \frac{x^5}{192} - \frac{x^3}{8} + x\right) + c_1\left(\frac{5x^6 - 90x^4 + 288x^2 + 1152}{1152x} - \frac{1}{384}x\left(x^4 - 24x^2 + 192\right)\log(x)\right)$$

19.9 problem 3(f)

Internal problem ID [6061]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 166
Problem number: 3(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{x^2y'' - 2x^2y' + (4x-2)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 55

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)-2*x^2*diff(y(x),x)+(4*x-2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^2\left(1 + \mathrm{O}\left(x^8\right)\right) + \frac{c_2\left(\ln\left(x\right)\left(\left(-48\right)x^3 + \mathrm{O}\left(x^8\right)\right) + \left(12 + 36x + 72x^2 + 88x^3 - 24x^4 - \frac{24}{5}x^5 - \frac{16}{15}x^6 - \frac{8}{35}x^7 + \mathrm{O}\left(x^8\right)\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.1 (sec). Leaf size: 58

```
AsymptoticDSolveValue[x^2*y''[x]-2*x^2*y'[x]+(4*x-2)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2x^2 + c_1\left(-4x^2\log(x) - \frac{4x^6 + 18x^5 + 90x^4 - 390x^3 - 270x^2 - 135x - 45}{45x}\right)$$

20 Chapter 4. Linear equations with Regular Singular Points. Page 182

20.1 problem 4

Internal problem ID [6062]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 182
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-x^2+1)\,y''-2xy'+2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
Order:=8;
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-x^2-\frac{1}{3}x^4-\frac{1}{5}x^6\right)y(0)+D(y)\,(0)\,x+O(x^8)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 32

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_1\left(-\frac{x^6}{5}-\frac{x^4}{3}-x^2+1\right)+c_2x$$

21 Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190

21.1 problem 1(a)

Internal problem ID [6063]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 1(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{y' - yx^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=x^2*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{x^3}{3}}$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 22

```
DSolve[y'[x]==x^2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{\frac{x^3}{3}}$$
$$y(x) \to 0$$

21.2 problem 1(b)

Internal problem ID [6064]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 1(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'y = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(y(x)*diff(y(x),x)=x,y(x), singsol=all)
```

$$y(x) = \sqrt{x^2 + c_1}$$
$$y(x) = -\sqrt{x^2 + c_1}$$

✓ Solution by Mathematica
Time used: 0.081 (sec). Leaf size: 35

```
DSolve[y[x]*y'[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x^2 + 2c_1}$$
$$y(x) \to \sqrt{x^2 + 2c_1}$$

21.3 problem 1(c)

Internal problem ID [6065]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 1(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' - \frac{x^2 + x}{y - y^2} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 498

```
dsolve(diff(y(x),x)=(x+x^2)/(y(x)-y(x)^2),y(x), singsol=all)
```

$$y(x) = \frac{\left(1 - 4x^3 - 6x^2 - 12c_1 + 2\sqrt{4x^6 + 12x^5 + 24c_1x^3 + 9x^4 + 36c_1x^2 - 2x^3 + 36c_1^2 - 3x^2 - 6c_1}\right)^{\frac{1}{3}}}{2} + \frac{1}{2\left(1 - 4x^3 - 6x^2 - 12c_1 + 2\sqrt{4x^6 + 12x^5 + 24c_1x^3 + 9x^4 + 36c_1x^2 - 2x^3 + 36c_1^2 - 3x^2 - 6c_1}\right)^{\frac{1}{3}}} + \frac{1}{2}$$

$$y(x) = -\frac{(1 + i\sqrt{3})\left(-4x^3 - 6x^2 + 2\sqrt{(2x^3 + 3x^2 + 6c_1)(2x^3 + 3x^2 + 6c_1 - 1)} - 12c_1 + 1\right)^{\frac{2}{3}} - i\sqrt{3} - 2\left(-4x\right.}{4\left(-4x^3 - 6x^2 + 2\sqrt{(2x^3 + 3x^2 + 6c_1)(2x^3 + 3x^2 +}\right.}$$

$$y(x) = \frac{(i\sqrt{3} - 1)\left(-4x^3 - 6x^2 + 2\sqrt{(2x^3 + 3x^2 + 6c_1)(2x^3 + 3x^2 + 6c_1 - 1)} - 12c_1 + 1\right)^{\frac{2}{3}} - i\sqrt{3} + 2\left(-4x^3\right.}{4\left(-4x^3 - 6x^2 + 2\sqrt{(2x^3 + 3x^2 + 6c_1)(2x^3 + 3x^2 + 6}\right.}$$

✓ Solution by Mathematica

Time used: 4.147 (sec). Leaf size: 346

```
DSolve[y'[x]==(x+x^2)/(y[x]-y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(\sqrt[3]{-4x^3-6x^2+\sqrt{-1+(-4x^3-6x^2+1+12c_1)^2}+1+12c_1} + \frac{1}{\sqrt[3]{-4x^3-6x^2+\sqrt{-1+(-4x^3-6x^2+1+12c_1)^2}+1+12c_1}}+1\right)$$

$$y(x) \to \frac{1}{8}\left(2i\left(\sqrt{3}+i\right)\sqrt[3]{-4x^3-6x^2+\sqrt{-1+(-4x^3-6x^2+1+12c_1)^2}+1+12c_1} + \frac{-2-2i\sqrt{3}}{\sqrt[3]{-4x^3-6x^2+\sqrt{-1+(-4x^3-6x^2+1+12c_1)^2}+1+12c_1}}+4\right)$$

$$y(x) \to \frac{1}{8}\left(-2\left(1+i\sqrt{3}\right)\sqrt[3]{-4x^3-6x^2+\sqrt{-1+(-4x^3-6x^2+1+12c_1)^2}+1+12c_1} + \frac{2i(\sqrt{3}+i)}{\sqrt[3]{-4x^3-6x^2+\sqrt{-1+(-4x^3-6x^2+1+12c_1)^2}+1+12c_1}}+4\right)$$

21.4 problem 1(d)

Internal problem ID [6066]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 1(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{e^{x-y}}{1 + e^{x}} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=exp(x-y(x))/(1+exp(x)),y(x), singsol=all)
```

$$y(x) = \ln\left(\ln\left(e^{x} + 1\right) + c_1\right)$$

✓ Solution by Mathematica
Time used: 0.465 (sec). Leaf size: 15

```
DSolve[y'[x]==Exp[x-y[x]]/(1+Exp[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(\log\left(e^{x} + 1\right) + c_1\right)$$

21.5 problem 1(e)

Internal problem ID [6067]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 1(e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y^2x^2 = -4x^2}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 28

```
dsolve(diff(y(x),x)=x^2*y(x)^2-4*x^2,y(x), singsol=all)
```

$$y(x) = \frac{-2 - 2\,\mathrm{e}^{\frac{4x^3}{3}} c_1}{\mathrm{e}^{\frac{4x^3}{3}} c_1 - 1}$$

✓ Solution by Mathematica
Time used: 0.258 (sec). Leaf size: 52

```
DSolve[y'[x]==x^2*y[x]^2-4*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2 - 2e^{\frac{4x^3}{3}+4c_1}}{1 + e^{\frac{4x^3}{3}+4c_1}}$$
$$y(x) \to -2$$
$$y(x) \to 2$$

21.6 problem 2(a)

Internal problem ID [6068]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 2(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^2 = 0}$$

With initial conditions

$$[y(x_0) = y_0]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 18

```
dsolve([diff(y(x),x)=y(x)^2,y(x__0) = y__0],y(x), singsol=all)
```

$$y(x) = -\frac{y_0}{-1 + (x - x_0)\, y_0}$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 16

```
DSolve[{y'[x]==x2*y[x],{y[x0]==y0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{y0} e^{\text{x2}(x - \text{x0})}$$

21.7 problem 3(a)

Internal problem ID [6069]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 3(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 2\sqrt{y} = 0}$$

With initial conditions

$$[y(x_0) = y_0]$$

✓ Solution by Maple
Time used: 0.156 (sec). Leaf size: 28

```
dsolve([diff(y(x),x)=2*sqrt(y(x)),y(x__0) = y__0],y(x), singsol=all)
```

$$y(x) = (2x - 2x_0)\sqrt{y_0} + x^2 - 2xx_0 + x_0^2 + y_0$$

✓ Solution by Mathematica
Time used: 0.108 (sec). Leaf size: 33

```
DSolve[{y'[x]==2*Sqrt[y[x]],{y[x0]==y0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(x - \text{x0} + \sqrt{\text{y0}}\right)^2$$
$$y(x) \to \left(-x + \text{x0} + \sqrt{\text{y0}}\right)^2$$

21.8 problem 3(b)

Internal problem ID [6070]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 3(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 2\sqrt{y} = 0}$$

With initial conditions

$$[y(x_0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=2*sqrt(y(x)),y(x__0) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y'[x]==2*Sqrt[y[x]],{y[x0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

21.9 problem 4(a)

Internal problem ID [6071]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 4(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$y' - \frac{x+y}{x-y} = 0$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)=(x+y(x))/(x-y(x)),y(x), singsol=all)
```

$$y(x) = \tan\left(\operatorname{RootOf}\left(-2\_Z + \ln\left(\sec\left(\_Z\right)^2\right) + 2\ln\left(x\right) + 2c_1\right)\right)x$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 36

```
DSolve[y'[x]==(x+y[x])/(x-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right) - \arctan\left(\frac{y(x)}{x}\right) = -\log(x) + c_1, y(x)\right]$$

21.10 problem 4(b)

Internal problem ID [6072]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 4(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{y^2}{xy + x^2} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=y(x)^2/(x*y(x)+x^2),y(x), singsol=all)
```

$$y(x) = x \,\mathrm{LambertW}\left(\frac{\mathrm{e}^{-c_1}}{x}\right)$$

✓ Solution by Mathematica

Time used: 2.317 (sec). Leaf size: 21

```
DSolve[y'[x]==y[x]^2/(x*y[x]+x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to xW\left(\frac{e^{c_1}}{x}\right)$$
$$y(x) \to 0$$

21.11 problem 4(c)

Internal problem ID [6073]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 4(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Riccati]`

$$\boxed{y' - \frac{x^2 + xy + y^2}{x^2} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)=(x^2+x*y(x)+y(x)^2)/x^2,y(x), singsol=all)
```

$$y(x) = \tan(\ln(x) + c_1)\, x$$

✓ Solution by Mathematica
Time used: 0.188 (sec). Leaf size: 13

```
DSolve[y'[x]==(x^2+x*y[x]+y[x]^2)/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \tan(\log(x) + c_1)$$

21.12 problem 4(d)

Internal problem ID [6074]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 4(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y' - \frac{y + x\,\mathrm{e}^{-\frac{2y}{x}}}{x} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=(y(x)+x*exp(-2*y(x)/x))/x,y(x), singsol=all)
```

$$y(x) = \frac{(\ln(2) + \ln(\ln(x) + c_1))\,x}{2}$$

✓ Solution by Mathematica
Time used: 0.412 (sec). Leaf size: 18

```
DSolve[y'[x]==(y[x]+x*Exp[-2*y[x]/x])/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}x\log(2(\log(x) + c_1))$$

21.13 problem 5(a)

Internal problem ID [6075]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 5(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$y' - \frac{x - y + 2}{-1 + y + x} = 0$$

✓ Solution by Maple
Time used: 0.391 (sec). Leaf size: 33

```
dsolve(diff(y(x),x)=(x-y(x)+2)/(x+y(x)-1),y(x), singsol=all)
```

$$y(x) = \frac{-\sqrt{1 + 8\left(x + \frac{1}{2}\right)^2 c_1^2} + (-2x + 2)\, c_1}{2c_1}$$

✓ Solution by Mathematica
Time used: 0.154 (sec). Leaf size: 53

```
DSolve[y'[x]==(x-y[x]+2)/(x+y[x]-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2x^2 + 2x + 1 + c_1} - x + 1$$
$$y(x) \to \sqrt{2x^2 + 2x + 1 + c_1} - x + 1$$

21.14 problem 5(b)

Internal problem ID [6076]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 5(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$y' - \frac{2x + 3y + 1}{x - 2y - 1} = 0$$

✓ Solution by Maple
Time used: 0.312 (sec). Leaf size: 61

```
dsolve(diff(y(x),x)=(2*x+3*y(x)+1)/(x-2*y(x)-1),y(x), singsol=all)
```

$$y(x) = -\frac{5}{14} - \frac{x}{2} + \frac{\sqrt{3}\,(7x-1)\tan\left(\mathrm{RootOf}\left(-2\sqrt{3}\,\ln(2) + \sqrt{3}\,\ln\left(\sec(\_Z)^2\,(7x-1)^2\right) + \sqrt{3}\,\ln(3) + 2\sqrt{3}\,c_1 - 4\_Z\right)\right)}{14}$$

✓ Solution by Mathematica
Time used: 0.12 (sec). Leaf size: 85

```
DSolve[y'[x]==(2*x+3*y[x]+1)/(x-2*y[x]-1),y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[32\sqrt{3}\arctan\left(\frac{4y(x)+5x+1}{\sqrt{3}(-2y(x)+x-1)}\right) = 3\left(8\log\left(\frac{4(7x^2+7y(x)^2+(7x+5)y(x)+x+1)}{(1-7x)^2}\right) + 16\log(7x-1) + 7c_1\right), y(x)\right]$$

21.15 problem 5(c)

Internal problem ID [6077]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 5(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$y' - \frac{y + x + 1}{2x + 2y - 1} = 0$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)=(x+y(x)+1)/(2*x+2*y(x)-1),y(x), singsol=all)
```

$$y(x) = -\frac{\text{LambertW}\left(-2\,\mathrm{e}^{-3x+3c_1}\right)}{2} - x$$

✓ Solution by Mathematica
Time used: 4.2 (sec). Leaf size: 32

```
DSolve[y'[x]==(x+y[x]+1)/(2*x+2*y[x]-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x - \frac{1}{2}W\left(-e^{-3x-1+c_1}\right)$$
$$y(x) \to -x$$

21.16 problem 6(b)

Internal problem ID [6078]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 6(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, _Riccati]`

$$y' - \frac{(-1+y+x)^2}{2(x+2)^2} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)=1/2*((x+y(x)-1)/(x+2))^2,y(x), singsol=all)
```

$$y(x) = 3 + \tan\left(\frac{\ln(x+2)}{2} + \frac{c_1}{2}\right)(x+2)$$

✓ Solution by Mathematica

Time used: 0.411 (sec). Leaf size: 99

```
DSolve[y'[x]==1/2*((x+y[x]-1)/(x+2))^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2^i(x+2)^i x + (2+3i)2^i(x+2)^i - 2ic_1x - (6+4i)c_1}{i2^i(x+2)^i - 2c_1}$$
$$y(x) \to ix + (3+2i)$$
$$y(x) \to ix + (3+2i)$$

22 Chapter 5. Existence and uniqueness of solutions to first order equations. Page 198

22.1 problem 1(a)

Internal problem ID [6079]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 198
Problem number: 1(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, _dAlembert]`

$$\boxed{2xy + (x^2 + 3y^2)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 189

```
dsolve(2*x*y(x)+(x^2+3*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-12c_1x^2 + \left(108 + 12\sqrt{12c_1^3x^6 + 81}\right)^{\frac{2}{3}}}{6\left(108 + 12\sqrt{12c_1^3x^6 + 81}\right)^{\frac{1}{3}}\sqrt{c_1}}$$

$$y(x) = -\frac{(1 + i\sqrt{3})\left(108 + 12\sqrt{12c_1^3x^6 + 81}\right)^{\frac{1}{3}}}{12\sqrt{c_1}} - \frac{x^2(i\sqrt{3} - 1)\sqrt{c_1}}{\left(108 + 12\sqrt{12c_1^3x^6 + 81}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{(i\sqrt{3} - 1)\left(108 + 12\sqrt{12c_1^3x^6 + 81}\right)^{\frac{1}{3}}}{12\sqrt{c_1}} + \frac{(1 + i\sqrt{3})\, x^2\sqrt{c_1}}{\left(108 + 12\sqrt{12c_1^3x^6 + 81}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 27.686 (sec). Leaf size: 442

```
DSolve[2*x*y[x]+(x^2+3*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-2\sqrt[3]{3}x^2 + \sqrt[3]{2}\left(\sqrt{12x^6 + 81e^{2c_1}} + 9e^{c_1}\right)^{2/3}}{6^{2/3}\sqrt[3]{\sqrt{12x^6 + 81e^{2c_1}} + 9e^{c_1}}}$$

$$y(x) \to \frac{i2^{2/3}\sqrt[3]{3}(\sqrt{3} + i)\left(\sqrt{12x^6 + 81e^{2c_1}} + 9e^{c_1}\right)^{2/3} + 2\sqrt[3]{2}\sqrt[6]{3}(\sqrt{3} + 3i)\,x^2}{12\sqrt[3]{\sqrt{12x^6 + 81e^{2c_1}} + 9e^{c_1}}}$$

$$y(x) \to \frac{2^{2/3}\sqrt[3]{3}(-1 - i\sqrt{3})\left(\sqrt{12x^6 + 81e^{2c_1}} + 9e^{c_1}\right)^{2/3} + 2\sqrt[3]{2}\sqrt[6]{3}(\sqrt{3} - 3i)\,x^2}{12\sqrt[3]{\sqrt{12x^6 + 81e^{2c_1}} + 9e^{c_1}}}$$

$$y(x) \to 0$$

$$y(x) \to \frac{\sqrt[3]{x^6} - x^2}{\sqrt{3}\sqrt[6]{x^6}}$$

$$y(x) \to \frac{(\sqrt{3} - 3i)\,x^2 - (\sqrt{3} + 3i)\sqrt[3]{x^6}}{6\sqrt[6]{x^6}}$$

$$y(x) \to \frac{(\sqrt{3} + 3i)\,x^2 - (\sqrt{3} - 3i)\sqrt[3]{x^6}}{6\sqrt[6]{x^6}}$$

22.2 problem 1(b)

Internal problem ID [6080]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 198
Problem number: 1(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xy + (x + y)\, y' = -x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve((x^2+x*y(x))+(x+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -x$$
$$y(x) = -\frac{x^2}{2} + c_1$$

✓ Solution by Mathematica
Time used: 0.132 (sec). Leaf size: 53

```
DSolve[(x^2+y[x])+(x+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x - \sqrt{-\frac{2x^3}{3} + x^2 + c_1}$$
$$y(x) \to -x + \sqrt{-\frac{2x^3}{3} + x^2 + c_1}$$

22.3 problem 1(c)

Internal problem ID [6081]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 198
Problem number: 1(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{e^{y}(1+y)\,y' = -e^{x}}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 13

```
dsolve(exp(x)+(exp(y(x))*(y(x)+1))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{LambertW}\left(-c_1 - e^{x}\right)$$

✓ Solution by Mathematica
Time used: 60.161 (sec). Leaf size: 14

```
DSolve[Exp[x]+(Exp[y[x]]*(y[x]+1))*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to W(-e^{x} + c_1)$$

22.4 problem 1(d)

Internal problem ID [6082]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 198
Problem number: 1(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\cos(x)\cos(y)^2 - \sin(x)\sin(2y)\,y' = 0}$$

✓ Solution by Maple
Time used: 0.219 (sec). Leaf size: 25

```
dsolve(cos(x)*cos(y(x))^2-sin(x)*sin(2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{1}{\sqrt{c_1 \sin(x)}}\right)$$
$$y(x) = \frac{\pi}{2} + \arcsin\left(\frac{1}{\sqrt{c_1 \sin(x)}}\right)$$

✓ Solution by Mathematica

Time used: 6.536 (sec). Leaf size: 73

```
DSolve[Cos[x]*Cos[y[x]]^2-Sin[x]*Sin[2*y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$
$$y(x) \to -\arccos\left(-\frac{c_1}{4\sqrt{\sin(x)}}\right)$$
$$y(x) \to \arccos\left(-\frac{c_1}{4\sqrt{\sin(x)}}\right)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

22.5 problem 1(e)

Internal problem ID [6083]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 198
Problem number: 1(e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^3x^2 - x^3y^2y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x^2*y(x)^3-x^3*y(x)^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = c_1x$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 19

```
DSolve[x^2*y[x]^3-x^3*y[x]^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$
$$y(x) \to c_1x$$
$$y(x) \to 0$$

22.6 problem 1(f)

Internal problem ID [6084]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 198
Problem number: 1(f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$y + (x - y)\,y' = -x$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 49

```
dsolve((x+y(x))+(x-y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x - \sqrt{2x^2 c_1^2 + 1}}{c_1}$$
$$y(x) = \frac{c_1 x + \sqrt{2x^2 c_1^2 + 1}}{c_1}$$

✓ Solution by Mathematica
Time used: 0.449 (sec). Leaf size: 86

```
DSolve[(x+y[x])+(x-y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - \sqrt{2x^2 + e^{2c_1}}$$
$$y(x) \to x + \sqrt{2x^2 + e^{2c_1}}$$
$$y(x) \to x - \sqrt{2}\sqrt{x^2}$$
$$y(x) \to \sqrt{2}\sqrt{x^2} + x$$

22.7 problem 1(g)

Internal problem ID [6085]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 198
Problem number: 1(g).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{2\,e^{2x}y+2\cos\left(y\right)x+\left(e^{2x}-x^{2}\sin\left(y\right)\right)y'=0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 19

```
dsolve((2*y(x)*exp(2*x)+2*x*cos(y(x)))+(exp(2*x)-x^2*sin(y(x)))*diff(y(x),x)=0,y(x), singsol
```

$$\cos\left(y(x)\right)x^{2}+y(x)\,e^{2x}+c_{1}=0$$

✓ Solution by Mathematica
Time used: 0.414 (sec). Leaf size: 30

```
DSolve[(2*y[x]*Exp[2*x]+2*x*Cos[y[x]])+(Exp[2*x]-x^2*Sin[y[x]])*y'[x]==0,y[x],x,IncludeSingu
```

$$\text{Solve}\left[2\left(\frac{1}{2}x^{2}\cos(y(x))+\frac{1}{2}e^{2x}y(x)\right)=c_{1},y(x)\right]$$

22.8 problem 1(h)

Internal problem ID [6086]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 198
Problem number: 1(h).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' + y = -3\ln(x)\,x^2 - x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve((3*x^2*ln(x)+x^2+y(x))+x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-x^3\ln(x) + c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 19

```
DSolve[(3*x^2*Log[x]+x^2+y[x])+x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-x^3\log(x) + c_1}{x}$$

22.9 problem 2(a)

Internal problem ID [6087]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 198
Problem number: 2(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{2y^3 + 3xy^2y' = -2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 73

```
dsolve((2*y(x)^3+2)+(3*x*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{((-x^2 + c_1)\,x)^{\frac{1}{3}}}{x}$$
$$y(x) = -\frac{((-x^2 + c_1)\,x)^{\frac{1}{3}}\,(1 + i\sqrt{3})}{2x}$$
$$y(x) = \frac{((-x^2 + c_1)\,x)^{\frac{1}{3}}\,(i\sqrt{3} - 1)}{2x}$$

✓ Solution by Mathematica

Time used: 0.281 (sec). Leaf size: 215

```
DSolve[(3*y[x]^3+2)+(3*x*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[3]{-\frac{1}{3}}\sqrt[3]{-2x^3+e^{9c_1}}}{x}$$

$$y(x) \to \frac{\sqrt[3]{-2x^3+e^{9c_1}}}{\sqrt[3]{3}x}$$

$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{-2x^3+e^{9c_1}}}{\sqrt[3]{3}x}$$

$$y(x) \to \sqrt[3]{-\frac{2}{3}}$$

$$y(x) \to -\sqrt[3]{\frac{2}{3}}$$

$$y(x) \to -(-1)^{2/3}\sqrt[3]{\frac{2}{3}}$$

$$y(x) \to \frac{\sqrt[3]{-\frac{2}{3}}x^2}{(-x^3)^{2/3}}$$

$$y(x) \to \frac{\sqrt[3]{\frac{2}{3}}\sqrt[3]{-x^3}}{x}$$

$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{\frac{2}{3}}\sqrt[3]{-x^3}}{x}$$

22.10 problem 2(b)

Internal problem ID [6088]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 198
Problem number: 2(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$-2y' \sin(y) \sin(x) + \cos(x) \cos(y) = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(cos(x)*cos(y(x))-2*sin(x)*sin(y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{1}{\sqrt{c_1 \sin(x)}}\right)$$
$$y(x) = \frac{\pi}{2} + \arcsin\left(\frac{1}{\sqrt{c_1 \sin(x)}}\right)$$

✓ Solution by Mathematica
Time used: 0.491 (sec). Leaf size: 43

```
DSolve[Cos[x]*cos[y[x]]-(2*Sin[x]*Sin[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sin(K[1])}{\cos(K[1])} dK[1]\&\right]\left[\frac{1}{2}\log(\sin(x)) + c_1\right]$$
$$y(x) \to \cos^{(-1)}(0)$$

22.11 problem 2(c)

Internal problem ID [6089]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 198
Problem number: 2(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{5x^3y^2 + 2y + (3yx^4 + 2x)\,y' = 0}$$

✓ Solution by Maple

Time used: 0.515 (sec). Leaf size: 350

```
dsolve((5*x^3*y(x)^2+2*y(x))+(3*x^4*y(x)+2*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\frac{12^{\frac{2}{3}}\left(12^{\frac{1}{3}}c_1^2+\left(\left(9x^2+\sqrt{-12c_1^4+81x^4}\right)c_1\right)^{\frac{2}{3}}\right)^2}{36c_1^2\left(\left(9x^2+\sqrt{-12c_1^4+81x^4}\right)c_1\right)^{\frac{2}{3}}}-1}{x^3}$$

$$y(x) = \frac{-\frac{c_1\left(\left(9x^2+\sqrt{-12c_1^4+81x^4}\right)c_1\right)^{\frac{2}{3}}}{3}+\frac{3\,2^{\frac{1}{3}}\left(x^2+\frac{\sqrt{-12c_1^4+81x^4}}{9}\right)\left(i3^{\frac{1}{6}}-\frac{3^{\frac{2}{3}}}{3}\right)\left(\left(9x^2+\sqrt{-12c_1^4+81x^4}\right)c_1\right)^{\frac{1}{3}}}{4}-\frac{\left(i3^{\frac{5}{6}}+3^{\frac{1}{3}}\right)2^{\frac{2}{3}}c_1^3}{6}}{c_1\left(\left(9x^2+\sqrt{-12c_1^4+81x^4}\right)c_1\right)^{\frac{2}{3}}x^3}$$

$$y(x) = -\frac{3\left(\frac{4c_1\left(\left(9x^2+\sqrt{-12c_1^4+81x^4}\right)c_1\right)^{\frac{2}{3}}}{9}+2^{\frac{1}{3}}\left(x^2+\frac{\sqrt{-12c_1^4+81x^4}}{9}\right)\left(i3^{\frac{1}{6}}+\frac{3^{\frac{2}{3}}}{3}\right)\left(\left(9x^2+\sqrt{-12c_1^4+81x^4}\right)c_1\right)^{\frac{1}{3}} -\right.}{4\left(\left(9x^2+\sqrt{-12c_1^4+81x^4}\right)c_1\right)^{\frac{2}{3}}x^3c_1}$$

✓ Solution by Mathematica

Time used: 49.208 (sec). Leaf size: 400

```
DSolve[(5*x^3*y[x]^2+2*y[x])+(3*x^4*y[x]+2*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{-2x^2 + \frac{2x^4}{\sqrt[3]{\frac{27c_1x^{10}}{2} - x^6 + \frac{3}{2}\sqrt{3}\sqrt{c_1x^{16}\left(-4+27c_1x^4\right)}}} + 2^{2/3}\sqrt[3]{27c_1x^{10} - 2x^6 + 3\sqrt{3}\sqrt{c_1x^{16}\left(-4+27c\right.}}}{6x^5}$$

$$y(x) \to \frac{-4x^2 - \frac{2\left(1+i\sqrt{3}\right)x^4}{\sqrt[3]{\frac{27c_1x^{10}}{2} - x^6 + \frac{3}{2}\sqrt{3}\sqrt{c_1x^{16}\left(-4+27c_1x^4\right)}}} + i2^{2/3}\left(\sqrt{3}+i\right)\sqrt[3]{27c_1x^{10} - 2x^6 + 3\sqrt{3}\sqrt{c_1x^{16}}}}{12x^5}$$

$$y(x) \to -\frac{4x^2 - \frac{2i\left(\sqrt{3}+i\right)x^4}{\sqrt[3]{\frac{27c_1x^{10}}{2} - x^6 + \frac{3}{2}\sqrt{3}\sqrt{c_1x^{16}\left(-4+27c_1x^4\right)}}} + 2^{2/3}\left(1+i\sqrt{3}\right)\sqrt[3]{27c_1x^{10} - 2x^6 + 3\sqrt{3}\sqrt{c_1x^{16}}}}{12x^5}$$

22.12 problem 2(d)

Internal problem ID [6090]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 198
Problem number: 2(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{e^{y} + x\,e^{y} + x\,e^{y} y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 13

```
dsolve((exp(y(x))+x*exp(y(x)))+(x*exp(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -x - \ln(x) + c_1$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 15

```
DSolve[(Exp[y[x]]+x*Exp[y[x]])+(x*Exp[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to -x - \log(x) + c_1$$

23 Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 238

23.1 problem 1(a)

Internal problem ID [6091]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 238
Problem number: 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x) = -c_1 e^{-x} + x + c_2$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 18

```
DSolve[y''[x]+y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - c_1 e^{-x} + c_2$$

23.2 problem 1(b)

Internal problem ID [6092]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 238
Problem number: 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + y' e^x = e^x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)+exp(x)*diff(y(x),x)=exp(x),y(x), singsol=all)
```

$$y(x) = -c_1 \operatorname{expIntegral}_1(e^x) + x + c_2$$

✓ Solution by Mathematica
Time used: 0.081 (sec). Leaf size: 18

```
DSolve[y''[x]+Exp[x]*y'[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{ExpIntegralEi}(-e^x) + x + c_2$$

23.3 problem 1(c)

Internal problem ID [6093]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 238
Problem number: 1(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$yy'' + 4y'^2 = 0$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 158

```
dsolve(y(x)*diff(y(x),x$2)+4*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = (5c_1x + 5c_2)^{\frac{1}{5}}$$
$$y(x) = -\frac{\left(i\sqrt{2}\,\sqrt{5-\sqrt{5}} + \sqrt{5} + 1\right)(5c_1x + 5c_2)^{\frac{1}{5}}}{4}$$
$$y(x) = \frac{\left(i\sqrt{2}\,\sqrt{5-\sqrt{5}} - \sqrt{5} - 1\right)(5c_1x + 5c_2)^{\frac{1}{5}}}{4}$$
$$y(x) = -\frac{\left(i\sqrt{2}\,\sqrt{5+\sqrt{5}} - \sqrt{5} + 1\right)(5c_1x + 5c_2)^{\frac{1}{5}}}{4}$$
$$y(x) = \frac{\left(i\sqrt{2}\,\sqrt{5+\sqrt{5}} + \sqrt{5} - 1\right)(5c_1x + 5c_2)^{\frac{1}{5}}}{4}$$

✓ Solution by Mathematica

Time used: 0.178 (sec). Leaf size: 20

```
DSolve[y[x]*y''[x]+4*(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \sqrt[5]{5x - c_1}$$

23.4 problem 1(d)

Internal problem ID [6094]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 238
Problem number: 1(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + k^2 y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+k^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(kx) + c_2 \cos(kx)$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 20

```
DSolve[y''[x]+k^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(kx) + c_2 \sin(kx)$$

23.5 problem 1(e)

Internal problem ID [6095]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 238
Problem number: 1(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _`

$$\boxed{y'' - y'y = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)=y(x)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \frac{\tan\left(\frac{(x+c_2)\sqrt{2}}{2c_1}\right)\sqrt{2}}{c_1}$$

✓ Solution by Mathematica
Time used: 16.739 (sec). Leaf size: 34

```
DSolve[y''[x]==y[x]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{2}\sqrt{c_1}\tan\left(\frac{\sqrt{c_1}(x+c_2)}{\sqrt{2}}\right)$$

23.6 problem 1(f)

Internal problem ID [6096]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 238
Problem number: 1(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' - 2y' = x^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x$2)-2*diff(y(x),x)=x^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{4}x^4 + \frac{1}{3}c_1x^3 + c_2$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 24

```
DSolve[x*y''[x]-2*y'[x]==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^4}{4} + \frac{c_1x^3}{3} + c_2$$

23.7 problem 2

Internal problem ID [6097]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 238
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{y'' - y'^2 = 1}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 7

```
dsolve([diff(y(x),x$2)=1+diff(y(x),x)^2,y(0) = 0, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \ln (\sec (x))$$

✓ Solution by Mathematica
Time used: 2.581 (sec). Leaf size: 27

```
DSolve[{y''[x]==1+(y'[x])^2,{y[0]==0,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\log(-\cos(x)) + i\pi$$
$$y(x) \rightarrow -\log(\cos(x))$$

23.8 problem 3

Internal problem ID [6098]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 238
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_poly_y`

$$\boxed{y'' + \frac{1}{2y'^2} = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

✓ Solution by Maple
Time used: 0.469 (sec). Leaf size: 26

```
dsolve([diff(y(x),x$2)=-1/(2*diff(y(x),x)^2),y(0) = 1, D(y)(0) = -1],y(x), singsol=all)
```

$$y(x) = \frac{3\left(x + \frac{2}{3}\right)\left(-12x - 8\right)^{\frac{1}{3}}\left(i\sqrt{3} - 1\right)}{16} + \frac{3}{2}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 27

```
DSolve[{y''[x]==-1/(2*(y'[x])^2),{y[0]==1,y'[0]==-1}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{8}\left(12 - (-2)^{2/3}(-3x - 2)^{4/3}\right)$$

23.9 problem 5(b)

Internal problem ID [6099]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 238
Problem number: 5(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' + \sin(y) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = \beta]$$

✓ Solution by Maple
Time used: 1.062 (sec). Leaf size: 53

```
dsolve([diff(y(x),x$2)+sin(y(x))=0,y(0) = 0, D(y)(0) = beta],y(x), singsol=all)
```

$$y(x) = \text{RootOf}\left(-\left(\int_0^{\_Z} \frac{1}{\sqrt{2\cos(\_a) + \beta^2 - 2}} d\_a\right) + x\right)$$
$$y(x) = \text{RootOf}\left(\int_0^{\_Z} \frac{1}{\sqrt{2\cos(\_a) + \beta^2 - 2}} d\_a + x\right)$$

✓ Solution by Mathematica
Time used: 0.621 (sec). Leaf size: 19

```
DSolve[{y''[x]+Sin[y[x]]==0,{y[0]==0,y'[0]==\[Beta]}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to 2\,\text{JacobiAmplitude}\left(\frac{x\beta}{2}, \frac{4}{\beta^2}\right)$$

23.10 problem 5(c)

Internal problem ID [6100]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 238
Problem number: 5(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' + \sin(y) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 2]$$

✓ Solution by Maple
Time used: 1.296 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$2)+sin(y(x))=0,y(0) = 0, D(y)(0) = 2],y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(-\left(\int_0^{\_Z} \sec\left(\frac{\_a}{2}\right) \operatorname{csgn}\left(\cos\left(\frac{\_a}{2}\right)\right) d\_a\right) + 2x\right)$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y''[x]+Sin[y[x]]==0,{y[0]==0,y'[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

{}

24 Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 250

24.1 problem 3

Internal problem ID [6101]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 250
Problem number: 3.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(x) = y_1(x)$$
$$y_2'(x) = y_1(x) + y_2(x)$$

With initial conditions

$$[y_1(0) = 1, y_2(0) = 2]$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 16

```
dsolve([diff(y__1(x),x) = y__1(x), diff(y__2(x),x) = y__1(x)+y__2(x), y__1(0) = 1, y__2(0) =
```

$$y_1(x) = e^x$$
$$y_2(x) = (x+2)\,e^x$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 18

```
DSolve[{y1'[x]==y1[x],y2'[x]==y1[x]+y2[x]},{y1[0]==1,y2[0]==2},{y1[x],y2[x]},x,IncludeSingul
```

$$\text{y1}(x) \to e^x$$
$$\text{y2}(x) \to e^x(x+2)$$

24.2 problem 4

Internal problem ID [6102]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 250
Problem number: 4.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(x) = y_2(x)$$
$$y_2'(x) = 6y_1(x) + y_2(x)$$

With initial conditions

$$[y_1(0) = 1, y_2(0) = -1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve([diff(y__1(x),x) = y__2(x), diff(y__2(x),x) = 6*y__1(x)+y__2(x), y__1(0) = 1, y__2(0)
```

$$y_1(x) = \frac{4\,\mathrm{e}^{-2x}}{5} + \frac{\mathrm{e}^{3x}}{5}$$
$$y_2(x) = -\frac{8\,\mathrm{e}^{-2x}}{5} + \frac{3\,\mathrm{e}^{3x}}{5}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 42

```
DSolve[{y1'[x]==y2[x],y2'[x]==6*y1[x]+y2[x]},{y1[0]==1,y2[0]==-1},{y1[x],y2[x]},x,IncludeSin
```

$$\text{y1}(x) \to \frac{1}{5}e^{-2x}\left(e^{5x} + 4\right)$$
$$\text{y2}(x) \to \frac{1}{5}e^{-2x}\left(3e^{5x} - 8\right)$$

24.3 problem 5

Internal problem ID [6103]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 250
Problem number: 5.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(x) = y_1(x) + y_2(x)$$
$$y_2'(x) = y_1(x) + y_2(x) + e^{3x}$$

With initial conditions

$$[y_1(0) = 0, y_2(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 36

```
dsolve([diff(y__1(x),x) = y__1(x)+y__2(x), diff(y__2(x),x) = y__1(x)+y__2(x)+exp(3*x), y__1(
```

$$y_1(x) = -\frac{e^{2x}}{2} + \frac{e^{3x}}{3} + \frac{1}{6}$$
$$y_2(x) = -\frac{e^{2x}}{2} + \frac{2\,e^{3x}}{3} - \frac{1}{6}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 46

```
DSolve[{y1'[x]==y1[x]+y2[x],y2'[x]==y1[x]+y2[x]+Exp[3*x]},{y1[0]==0,y2[0]==0},{y1[x],y2[x]},
```

$$\text{y1}(x) \to \frac{1}{6}(e^x - 1)^2 (2e^x + 1)$$
$$\text{y2}(x) \to \frac{1}{6}\left(-3e^{2x} + 4e^{3x} - 1\right)$$

25 Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 254

25.1 problem 2

Internal problem ID [6104]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 254
Problem number: 2.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(x) = 3y_1(x) + xy_3(x)$$
$$y_2'(x) = y_2(x) + x^3 y_3(x)$$
$$y_3'(x) = 2xy_2(x) - y_2(x) + e^x y_3(x)$$

✗ Solution by Maple

```
dsolve([diff(y__1(x),x)=3*y__1(x)+x*y__3(x),diff(y__2(x),x)=y__2(x)+x^3*y__3(x),diff(y__3(x)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y1'[x]==3*y1[x]+x*y3[x],y2'[x]==y2[x]+x^3*y3[x],y3'[x]==2*x*y1[x]-y2[x]+Exp[x]*y3[x]
```

Not solved