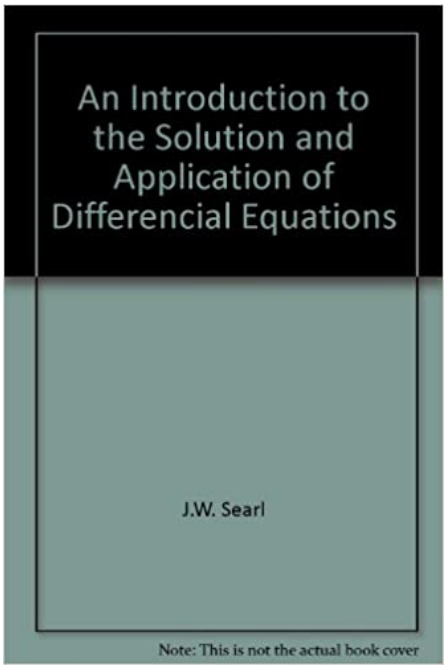
A Solution Manual For

An introduction to the solution and applications of differential equations, J.W. Searl, 1966



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Contents

1 Chapter 4, Ex. 4.1

1.1 problem 1

Internal problem ID [3134]

Book: An introduction to the solution and applications of differential equations, J.W. Searl, 1966
Section: Chapter 4, Ex. 4.1
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x^2 + 2yx = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(2*x*y(x)+x^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^2}$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 16

```
DSolve[2*x*y[x]+x^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x^2}$$
$$y(x) \to 0$$

1.2 problem 2

Internal problem ID [3135]

Book: An introduction to the solution and applications of differential equations, J.W. Searl, 1966
Section: Chapter 4, Ex. 4.1
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, [_Abel, '2nd ty`

$$\boxed{y + (-y + x)\, y' = -x}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 22

```
dsolve([(x+y(x))+(x-y(x))*diff(y(x),x)=0,y(0) = 0],y(x), singsol=all)
```

$$y(x) = x\left(1 + \sqrt{2}\right)$$
$$y(x) = -x\left(\sqrt{2} - 1\right)$$

✓ Solution by Mathematica
Time used: 0.482 (sec). Leaf size: 40

```
DSolve[{(x+y[x])+(x-y[x])*y'[x]==0,y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - \sqrt{2}\sqrt{x^2}$$
$$y(x) \to \sqrt{2}\sqrt{x^2} + x$$

1.3 problem 3

Internal problem ID [3136]

Book: An introduction to the solution and applications of differential equations, J.W. Searl, 1966
Section: Chapter 4, Ex. 4.1
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' \ln(x) + \frac{y+x}{x} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(ln(x)*diff(y(x),x)+(x+y(x))/x=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 - x}{\ln(x)}$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 16

```
DSolve[Log[x]*y'[x]+(x+y[x])/x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-x + c_1}{\log(x)}$$

1.4 problem 4

Internal problem ID [3137]

Book: An introduction to the solution and applications of differential equations, J.W. Searl, 1966
Section: Chapter 4, Ex. 4.1
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{\cos(y) - x\sin(y)\,y' = \sec(x)^2}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.985 (sec). Leaf size: 23

```
dsolve([cos(y(x))-x*sin(y(x))*diff(y(x),x)=sec(x)^2,y(0) = 0],y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{\tan(x)}{x}\right)$$
$$y(x) = -\arccos\left(\frac{\tan(x)}{x}\right)$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{Cos[y[x]]-x*Sin[y[x]]*y'[x]==Sec[x]^2,y[0]==0},y[x],x,IncludeSingularSolutions -> Tr
```

{}

1.5 problem 5

Internal problem ID [3138]

Book: An introduction to the solution and applications of differential equations, J.W. Searl, 1966
Section: Chapter 4, Ex. 4.1
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{y \sin\left(\frac{x}{y}\right) + x \cos\left(\frac{x}{y}\right) + \left(x \sin\left(\frac{x}{y}\right) - \frac{x^2 \cos\left(\frac{x}{y}\right)}{y}\right) y' = 1}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 23

```
dsolve((y(x)*sin(x/y(x))+x*cos(x/y(x))-1)+(x*sin(x/y(x))-x^2/y(x)*cos(x/y(x)))*diff(y(x),x)=
```

$$y(x) = \frac{x}{\text{RootOf}\left(x^2 \sin\left(\_Z\right) + \_Z c_1 - x\_Z\right)}$$

✓ Solution by Mathematica
Time used: 0.444 (sec). Leaf size: 20

```
DSolve[(y[x]*Sin[x/y[x]]+x*Cos[x/y[x]]-1)+(x*Sin[x/y[x]]-x^2/y[x]*Cos[x/y[x]])*y'[x]==0,y[x]
```

$$\text{Solve}\left[x - x y(x) \sin\left(\frac{x}{y(x)}\right) = c_1, y(x)\right]$$

1.6 problem 6

Internal problem ID [3139]

Book: An introduction to the solution and applications of differential equations, J.W. Searl, 1966
Section: Chapter 4, Ex. 4.1
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, _dAlembert]`

$$\boxed{\frac{x}{y^2+x^2}+\frac{y}{x^2}+\left(\frac{y}{y^2+x^2}-\frac{1}{x}\right)y'=0}$$

With initial conditions

$$[y(1)=0]$$

✓ Solution by Maple
Time used: 0.266 (sec). Leaf size: 33

```
dsolve([(x/(x^2+y(x)^2)+y(x)/x^2)+(y(x)/(x^2+y(x)^2)-1/x)*diff(y(x),x)=0,y(1) = 0],y(x), sin
```

$$y(x)=\frac{x\left(\text{RootOf}\left(4+4\ln(x)^2+4\ln(x)\_Z+\_Z^2-4\,\mathrm{e}^{-Z}\right)+2\ln(x)\right)}{2}$$

✓ Solution by Mathematica
Time used: 0.175 (sec). Leaf size: 28

```
DSolve[{(x/(x^2+y[x]^2)+y[x]/x^2)+(y[x]/(x^2+y[x]^2)-1/x)*y'[x]==0,y[1]==0},y[x],x,IncludeSi
```

$$\text{Solve}\left[\frac{y(x)}{x}-\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right)=\log(x),y(x)\right]$$

2 Chapter 4, Ex. 4.2

2.1 problem 1

Internal problem ID [3140]

Book: An introduction to the solution and applications of differential equations, J.W. Searl, 1966
Section: Chapter 4, Ex. 4.2
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2\left(y^2+1\right)y'+y^2\left(x^2+1\right)=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 93

```
dsolve(x^2*(1+y(x)^2)*diff(y(x),x)+y(x)^2*(x^2+1)=0,y(x), singsol=all)
```

$$y(x)=\frac{-x^2-c_1x+\sqrt{1+x^4+2c_1x^3+\left(c_1^2+2\right)x^2-2c_1x}+1}{2x}$$
$$y(x)=\frac{-x^2-c_1x-\sqrt{1+x^4+2c_1x^3+\left(c_1^2+2\right)x^2-2c_1x}+1}{2x}$$

✓ Solution by Mathematica
Time used: 1.162 (sec). Leaf size: 95

```
DSolve[x^2*(1+y[x]^2)*y'[x]+y[x]^2*(x^2+1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{x^2+\sqrt{4x^2+\left(-x^2+c_1x+1\right)^2}-c_1x-1}{2x}$$
$$y(x)\to \frac{-x^2+\sqrt{4x^2+\left(-x^2+c_1x+1\right)^2}+c_1x+1}{2x}$$
$$y(x)\to 0$$

2.2 problem 2

Internal problem ID [3141]

Book: An introduction to the solution and applications of differential equations, J.W. Searl, 1966
Section: Chapter 4, Ex. 4.2
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x(x-1)\,y' - \cot(y) = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 15

```
dsolve(x*(x-1)*diff(y(x),x)=cot(y(x)),y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{x}{c_1\,(x-1)}\right)$$

✓ Solution by Mathematica
Time used: 52.823 (sec). Leaf size: 59

```
DSolve[x*(x-1)*y'[x]==Cot[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arccos\left(-\frac{e^{-c_1}x}{x-1}\right)$$
$$y(x) \to \arccos\left(-\frac{e^{-c_1}x}{x-1}\right)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

2.3 problem 3

Internal problem ID [3142]

Book: An introduction to the solution and applications of differential equations, J.W. Searl, 1966
Section: Chapter 4, Ex. 4.2
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{ry' - \frac{(a^2 - r^2)\tan(y)}{a^2 + r^2} = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve(r*diff(y(r),r)= (a^2-r^2)/(a^2+r^2)*tan(y(r)),y(r), singsol=all)
```

$$y(r) = \arcsin\left(\frac{rc_1}{a^2 + r^2}\right)$$

✓ Solution by Mathematica
Time used: 23.337 (sec). Leaf size: 26

```
DSolve[r*y'[r]== (a^2-r^2)/(a^2+r^2)*Tan[y[r]],y[r],r,IncludeSingularSolutions -> True]
```

$$y(r) \to \arcsin\left(\frac{e^{c_1} r}{a^2 + r^2}\right)$$
$$y(r) \to 0$$

2.4 problem 4

Internal problem ID [3143]

Book: An introduction to the solution and applications of differential equations, J.W. Searl, 1966
Section: Chapter 4, Ex. 4.2
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sqrt{x^2+1}\,y' + \sqrt{y^2+1} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(sqrt(1+x^2)*diff(y(x),x)+sqrt(1+y(x)^2)=0,y(x), singsol=all)
```

$$y(x) = -\sinh\left(\operatorname{arcsinh}(x) + c_1\right)$$

✓ Solution by Mathematica
Time used: 0.349 (sec). Leaf size: 59

```
DSolve[Sqrt[1+x^2]*y'[x]+Sqrt[1+y[x]^2]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-c_1}\left(\left(-1+e^{2c_1}\right)\sqrt{x^2+1} - \left(1+e^{2c_1}\right)x\right)$$
$$y(x) \to -i$$
$$y(x) \to i$$

2.5 problem 5

Internal problem ID [3144]

Book: An introduction to the solution and applications of differential equations, J.W. Searl, 1966
Section: Chapter 4, Ex. 4.2
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{x(y^2+1)}{y\,(x^2+1)} = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)=(x*(1+y(x)^2))/(y(x)*(1+x^2)),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \sqrt{2x^2+1}$$

✓ Solution by Mathematica
Time used: 0.549 (sec). Leaf size: 16

```
DSolve[{y'[x]==(x*(1+y[x]^2))/(y[x]*(1+x^2)),y[0]==1},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \sqrt{2x^2+1}$$

2.6 problem 6

Internal problem ID [3145]

Book: An introduction to the solution and applications of differential equations, J.W. Searl, 1966
Section: Chapter 4, Ex. 4.2
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y^2 y' - 3y^6 = 2}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.235 (sec). Leaf size: 77

```
dsolve([y(x)^2*diff(y(x),x)=2+3*y(x)^6,y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{3^{\frac{5}{6}} 2^{\frac{1}{6}} \tan\left(3\sqrt{6}\, x\right)^{\frac{1}{3}}}{3}$$
$$y(x) = \frac{\tan\left(3\sqrt{6}\, x\right)^{\frac{1}{3}} \left(3i3^{\frac{1}{6}} - 3^{\frac{2}{3}}\right) 6^{\frac{1}{6}}}{6}$$
$$y(x) = -\frac{\tan\left(3\sqrt{6}\, x\right)^{\frac{1}{3}} \left(3i3^{\frac{1}{6}} + 3^{\frac{2}{3}}\right) 6^{\frac{1}{6}}}{6}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 87

```
DSolve[{y[x]^2*y'[x]==2+3*y[x]^6,y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[6]{\frac{2}{3}} \sqrt[3]{\tan\left(3\sqrt{6}x\right)}$$

$$y(x) \to -\sqrt[3]{-1} \sqrt[6]{\frac{2}{3}} \sqrt[3]{\tan\left(3\sqrt{6}x\right)}$$

$$y(x) \to (-1)^{2/3} \sqrt[6]{\frac{2}{3}} \sqrt[3]{\tan\left(3\sqrt{6}x\right)}$$