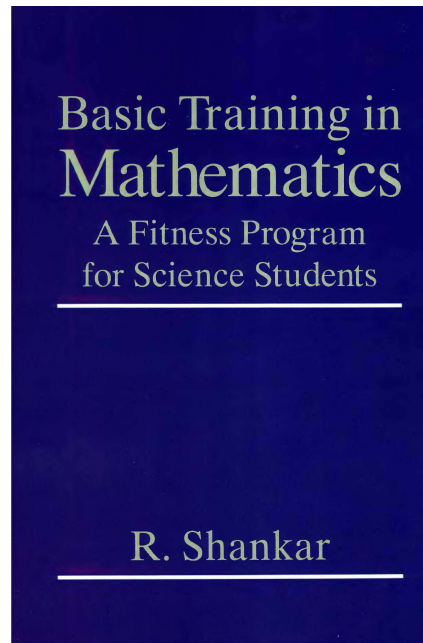


**A Solution Manual For**

**Basic Training in Mathematics. By R.  
Shankar. Plenum Press. NY. 1995**



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# 1 Chapter 10, Differential equations. Section 10.2, ODEs with constant Coefficients. page 307

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## 1.1 problem 10.2.4

Internal problem ID [5045]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.2, ODEs with constant Coefficients.  
page 307

**Problem number:** 10.2.4.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$x'' - \omega^2 x = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(x(t),t$2)-omega^2*x(t)=0,x(t), singsol=all)
```

$$x(t) = c_1 e^{\omega t} + c_2 e^{-\omega t}$$

### ✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 23

```
DSolve[x''[t]-\[Omega]^2*x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow c_1 e^{t\omega} + c_2 e^{-t\omega}$$

## 1.2 problem 10.2.5

Internal problem ID [5046]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.2, ODEs with constant Coefficients.  
page 307

**Problem number:** 10.2.5.

**ODE order:** 3.

**ODE degree:** 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$x''' - x'' + x' - x = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(diff(x(t),t$3)-diff(x(t),t$2)+diff(x(t),t)-x(t)=0,x(t), singsol=all)
```

$$x(t) = c_1 e^t + c_2 \sin(t) + c_3 \cos(t)$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 22

```
DSolve[x'''[t]-x''[t]+x'[t]-x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow c_3 e^t + c_1 \cos(t) + c_2 \sin(t)$$

### 1.3 problem 10.2.8 part(1)

Internal problem ID [5047]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.2, ODEs with constant Coefficients.  
page 307

**Problem number:** 10.2.8 part(1).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$x'' + 42x' + x = 0$$

With initial conditions

$$[x(0) = 1, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 43

```
dsolve([diff(x(t),t$2)+42*diff(x(t),t)+x(t)=0,x(0) = 1, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = \frac{(220 + 21\sqrt{110}) e^{(-21+2\sqrt{110})t}}{440} + \frac{(220 - 21\sqrt{110}) e^{(-21-2\sqrt{110})t}}{440}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 53

```
DSolve[{x'[t]+42*x'[t]+x[t]==0,{x[0]==1,x'[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow \frac{e^{-((21+2\sqrt{110})t)} \left( (881 + 84\sqrt{110}) e^{4\sqrt{110}t} - 1 \right)}{880 + 84\sqrt{110}}$$

## 1.4 problem 10.2.8 part(2)

Internal problem ID [5048]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.2, ODEs with constant Coefficients.  
page 307

**Problem number:** 10.2.8 part(2).

**ODE order:** 4.

**ODE degree:** 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$x'''' + x = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 61

```
dsolve(diff(x(t),t$4)+x(t)=0,x(t), singsol=all)
```

$$x(t) = \left(-c_1 e^{-\frac{\sqrt{2}t}{2}} - c_2 e^{\frac{\sqrt{2}t}{2}}\right) \sin\left(\frac{\sqrt{2}t}{2}\right) + \left(c_3 e^{-\frac{\sqrt{2}t}{2}} + c_4 e^{\frac{\sqrt{2}t}{2}}\right) \cos\left(\frac{\sqrt{2}t}{2}\right)$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 65

```
DSolve[x''''[t]+x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow e^{-\frac{t}{\sqrt{2}}} \left( (c_1 e^{\sqrt{2}t} + c_2) \cos\left(\frac{t}{\sqrt{2}}\right) + (c_4 e^{\sqrt{2}t} + c_3) \sin\left(\frac{t}{\sqrt{2}}\right) \right)$$

## 1.5 problem 10.2.8 part(3)

Internal problem ID [5049]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.2, ODEs with constant Coefficients.  
page 307

**Problem number:** 10.2.8 part(3).

**ODE order:** 3.

**ODE degree:** 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$x''' - 3x'' - 9x' - 5x = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(x(t),t$3)-3*diff(x(t),t$2)-9*diff(x(t),t)-5*x(t)=0,x(t), singsol=all)
```

$$x(t) = (c_3 t + c_2) e^{-t} + c_1 e^{5t}$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 26

```
DSolve[x'''[t]-3*x''[t]-9*x'[t]-5*x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow e^{-t}(c_2 t + c_3 e^{6t} + c_1)$$



## 1.6 problem 10.2.10

Internal problem ID [5050]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.2, ODEs with constant Coefficients.  
page 307

**Problem number:** 10.2.10.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x'' + 2\gamma x' + \omega_0 x = F \cos(\omega t)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 111

```
dsolve(diff(x(t),t$2)+2*gamma*diff(x(t),t)+omega_0*x(t)=F*cos(omega*t),x(t), singsol=all)
```

$x(t)$

$$= \frac{-F(\omega^2 - \omega_0) \cos(\omega t) + 2F \sin(\omega t) \gamma \omega + 4\left(\frac{\omega^4}{4} + \left(\gamma^2 - \frac{\omega_0}{2}\right) \omega^2 + \frac{\omega_0^2}{4}\right) \left( e^{-(\gamma + \sqrt{\gamma^2 - \omega_0})t} c_1 + e^{(-\gamma + \sqrt{\gamma^2 - \omega_0})t} c_2 \right)}{\omega^4 + (4\gamma^2 - 2\omega_0) \omega^2 + \omega_0^2}$$

✓ Solution by Mathematica

Time used: 0.509 (sec). Leaf size: 108

```
DSolve[x''[t]+2*\[Gamma]*x'[t]+Subscript[\[Omega],0]*x[t]==F*Cos[\[Omega]*t],x[t],t,IncludeS
```

$$x(t) \rightarrow \frac{F(\omega(2\gamma \sin(t\omega) - \omega \cos(t\omega)) + \omega_0 \cos(t\omega))}{4\gamma^2\omega^2 + \omega^4 - 2\omega_0\omega^2 + \omega_0^2} + c_1 e^{-t(\sqrt{\gamma^2 - \omega_0} + \gamma)} + c_2 e^{t(\sqrt{\gamma^2 - \omega_0} - \gamma)}$$

## 1.7 problem 10.2.11 (i)

Internal problem ID [5051]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.2, ODEs with constant Coefficients.  
page 307

**Problem number:** 10.2.11 (i).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - y' - 2y = e^{2x}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve([diff(y(x),x$2)-diff(y(x),x)-2*y(x)=exp(2*x),y(0) = 1, D(y)(0) = 0],y(x), singsol=all
```

$$y(x) = \frac{(3x + 2)e^{2x}}{9} + \frac{7e^{-x}}{9}$$

### ✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 27

```
DSolve[{y'[x]-y[x]-2*y[x]==Exp[2*x],{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow \frac{1}{9}e^{-x}(e^{3x}(3x + 2) + 7)$$

## 1.8 problem 10.2.11 (ii)

Internal problem ID [5052]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.2, ODEs with constant Coefficients.  
page 307

**Problem number:** 10.2.11 (ii).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2y' + y = 2 \cos(x)$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 11

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+y(x)=2*cos(x),y(0) = 1, D(y)(0) = 0],y(x), singsol=all
```

$$y(x) = e^x - \sin(x)$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 13

```
DSolve[{y''[x]-2*y'[x]+y[x]==2*Cos[x],{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow e^x - \sin(x)$$

## 1.9 problem 10.2.11 (iii)

Internal problem ID [5053]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.2, ODEs with constant Coefficients.  
page 307

**Problem number:** 10.2.11 (iii).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 16y = 16 \cos(4x)$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve([diff(y(x),x$2)+16*y(x)=16*cos(4*x),y(0) = 1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \cos(4x) + 2 \sin(4x)x$$

### ✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 17

```
DSolve[{y'[x]+16*y[x]==16*Cos[4*x],{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \rightarrow 2x \sin(4x) + \cos(4x)$$

## 1.10 problem 10.2.11 (iv)

Internal problem ID [5054]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.2, ODEs with constant Coefficients.  
page 307

**Problem number:** 10.2.11 (iv).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - y = \cosh(x)$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$2)-y(x)=cosh(x),y(0) = 1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{(-x + 2)e^{-x}}{4} + \frac{e^x(x + 2)}{4}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 28

```
DSolve[{y'[x]-y[x]==Cosh[x],{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{4}e^{-x}(-x + e^{2x}(x + 2) + 2)$$

**2 Chapter 10, Differential equations. Section 10.3,  
ODEs with variable Coefficients. First order.**

**page 315**

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## 2.1 problem 10.3.2

Internal problem ID [5055]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.3, ODEs with variable Coefficients.

First order. page 315

**Problem number:** 10.3.2.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' - y = e^{2x}$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)-y(x)=exp(2*x),y(x), singsol=all)
```

$$y(x) = (e^x + c_1) e^x$$

### ✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 15

```
DSolve[y'[x]-y[x]==Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x(e^x + c_1)$$

## 2.2 problem 10.3.3

Internal problem ID [5056]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.3, ODEs with variable Coefficients.

First order. page 315

**Problem number:** 10.3.3.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$x^2y' + 2xy = x - 1$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve([x^2*diff(y(x),x)+2*x*y(x)-x+1=0,y(1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{(x-1)^2}{2x^2}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 17

```
DSolve[{x^2*y'[x]+2*x*y[x]-x+1==0},{y[1]==0}],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{(x-1)^2}{2x^2}$$



## 2.3 problem 10.3.4

Internal problem ID [5057]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.3, ODEs with variable Coefficients.

First order. page 315

**Problem number:** 10.3.4.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y + y' = (x + 1)^2$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve([diff(y(x),x)+y(x)=(x+1)^2,y(0) = 0],y(x), singsol=all)
```

$$y(x) = x^2 + 1 - e^{-x}$$

✓ Solution by Mathematica

Time used: 0.102 (sec). Leaf size: 17

```
DSolve[{y'[x]+y[x]==(x+1)^2,{y[0]==0}],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^2 - e^{-x} + 1$$

## 2.4 problem 10.3.5

Internal problem ID [5058]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.3, ODEs with variable Coefficients.

First order. page 315

**Problem number:** 10.3.5.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$x^2y' + 2xy = \sinh(x)$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 16

```
dsolve([x^2*diff(y(x),x)+2*x*y(x)=sinh(x),y(1) = 2],y(x), singsol=all)
```

$$y(x) = \frac{\cosh(x) + 2 - \cosh(1)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 17

```
DSolve[{x^2*y'[x]+2*x*y[x]==Sinh[x],{y[1]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{\cosh(x) + 2 - \cosh(1)}{x^2}$$

## 2.5 problem 10.3.6

Internal problem ID [5059]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.3, ODEs with variable Coefficients.

First order. page 315

**Problem number:** 10.3.6.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$y' + \frac{y}{1-x} = x^2 - 2x$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)+y(x)/(1-x)+2*x-x^2=0,y(x), singsol=all)
```

$$y(x) = \frac{(x^2 - 2x - 2 \ln(x - 1) + 2c_1)(x - 1)}{2}$$

### ✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 27

```
DSolve[y'[x]+y[x]/(1-x)+2*x-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow (x - 1) \left( \frac{1}{2}(x - 1)^2 - \log(x - 1) + c_1 \right)$$

## 2.6 problem 10.3.7

Internal problem ID [5060]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.3, ODEs with variable Coefficients.

First order. page 315

**Problem number:** 10.3.7.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$y' + \frac{y}{1-x} = x^2 - x$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)+y(x)/(1-x)+x-x^2=0,y(x), singsol=all)
```

$$y(x) = \frac{(x^2 + 2c_1)(x - 1)}{2}$$

### ✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 20

```
DSolve[y'[x]+y[x]/(1-x)+x-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2}(x - 1)(x^2 + 2c_1)$$

## 2.7 problem 10.3.8

Internal problem ID [5061]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.3, ODEs with variable Coefficients.

First order. page 315

**Problem number:** 10.3.8.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$(x^2 + 1) y' - xy = 1$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((1+x^2)*diff(y(x),x)=1+x*y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{x^2 + 1} c_1 + x$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 19

```
DSolve[(1+x^2)*y'[x]==1+x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x + c_1 \sqrt{x^2 + 1}$$

## 2.8 problem 10.3.9 (a)

Internal problem ID [5062]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.3, ODEs with variable Coefficients.

First order. page 315

**Problem number:** 10.3.9 (a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$y' + xy - xy^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)+x*y(x)=x*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{1}{1 + e^{\frac{x^2}{2}} c_1}$$

✓ Solution by Mathematica

Time used: 0.25 (sec). Leaf size: 31

```
DSolve[y'[x]+x*y[x]==x*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}y(x) &\rightarrow \frac{1}{1 + e^{\frac{x^2}{2} + c_1}} \\y(x) &\rightarrow 0 \\y(x) &\rightarrow 1\end{aligned}$$

## 2.9 problem 10.3.9 (b)

Internal problem ID [5063]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.3, ODEs with variable Coefficients.

First order. page 315

**Problem number:** 10.3.9 (b).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$3xy' + y + y^4x^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 88

```
dsolve(3*x*diff(y(x),x)+y(x)+x^2*y(x)^4=0,y(x), singsol=all)
```

$$y(x) = \frac{((x + c_1)^2 x^2)^{\frac{1}{3}}}{(x + c_1) x}$$
$$y(x) = -\frac{((x + c_1)^2 x^2)^{\frac{1}{3}} (1 + i\sqrt{3})}{2(x + c_1) x}$$
$$y(x) = \frac{((x + c_1)^2 x^2)^{\frac{1}{3}} (i\sqrt{3} - 1)}{2(x + c_1) x}$$

✓ Solution by Mathematica

Time used: 0.3 (sec). Leaf size: 61

```
DSolve[3*x*y'[x]+y[x]+x^2*y[x]^4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{\sqrt[3]{x(x+c_1)}}$$

$$y(x) \rightarrow -\frac{\sqrt[3]{-1}}{\sqrt[3]{x(x+c_1)}}$$

$$y(x) \rightarrow \frac{(-1)^{2/3}}{\sqrt[3]{x(x+c_1)}}$$

$$y(x) \rightarrow 0$$



**3 Chapter 10, Differential equations. Section 10.4,  
ODEs with variable Coefficients. Second order  
and Homogeneous. page 318**

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### 3.1 problem 10.4.8 (a)

Internal problem ID [5064]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.4, ODEs with variable Coefficients.

Second order and Homogeneous. page 318

**Problem number:** 10.4.8 (a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(x+1)^2 y'' + (-x^2 + 1) y' + (x-1) y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve(x*(x+1)^2*diff(y(x),x$2)+(1-x^2)*diff(y(x),x)+(x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = (x + 1)(c_2 \ln(x) + c_1)$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 17

```
DSolve[x*(x+1)^2*y'[x]+(1-x^2)*y'[x]+(x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow (x + 1)(c_2 \log(x) + c_1)$$

## 3.2 problem 10.4.8 (b)

Internal problem ID [5065]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.4, ODEs with variable Coefficients.

Second order and Homogeneous. page 318

**Problem number:** 10.4.8 (b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x(1-x)y'' + 2(1-2x)y' - 2y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x*(1-x)*diff(y(x),x$2)+2*(1-2*x)*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1x + c_2}{x(x-1)}$$

### ✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 22

```
DSolve[x*(1-x)*y'[x]+2*(1-2*x)*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_2x + c_1}{x - x^2}$$

### 3.3 problem 10.4.8 (c)

Internal problem ID [5066]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.4, ODEs with variable Coefficients.

Second order and Homogeneous. page 318

**Problem number:** 10.4.8 (c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$x^2 y'' + xy' - 9y = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-9*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2 x^6 + c_1}{x^3}$$

#### ✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+x*y'[x]-9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_2 x^6 + c_1}{x^3}$$

### 3.4 problem 10.4.8 (d)

Internal problem ID [5067]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.4, ODEs with variable Coefficients.

Second order and Homogeneous. page 318

**Problem number:** 10.4.8 (d).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$xy'' + \frac{y'}{2} + 2y = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x$2)+1/2*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(2\sqrt{x}\sqrt{2}\right) + c_2 \cos\left(2\sqrt{x}\sqrt{2}\right)$$

#### ✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 38

```
DSolve[x*y''[x]+1/2*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \cos\left(2\sqrt{2}\sqrt{x}\right) + c_2 \sin\left(2\sqrt{2}\sqrt{x}\right)$$

### 3.5 problem 10.4.8 (e)

Internal problem ID [5068]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.4, ODEs with variable Coefficients.

Second order and Homogeneous. page 318

**Problem number:** 10.4.8 (e).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^2y'' - xy' + y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = x(c_2 \ln(x) + c_1)$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 15

```
DSolve[x^2*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x(c_2 \log(x) + c_1)$$

### 3.6 problem 10.4.8 (f)

Internal problem ID [5069]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.4, ODEs with variable Coefficients.  
Second order and Homogeneous. page 318

**Problem number:** 10.4.8 (f).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$2xy'' - y' + 2y = 0$$

#### ✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 36

```
dsolve(2*x*diff(y(x),x$2)-diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = (2\sqrt{x}c_1 + c_2) \cos(2\sqrt{x}) - \sin(2\sqrt{x}) (-2\sqrt{x}c_2 + c_1)$$

#### ✓ Solution by Mathematica

Time used: 0.199 (sec). Leaf size: 59

```
DSolve[2*x*y'[x]-y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{2i\sqrt{x}} (2\sqrt{x} + i) + \frac{1}{8} c_2 e^{-2i\sqrt{x}} (1 + 2i\sqrt{x})$$

### 3.7 problem 10.4.8 (g)

Internal problem ID [5070]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.4, ODEs with variable Coefficients. Second order and Homogeneous. page 318

**Problem number:** 10.4.8 (g).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' + xy' - 2y = 0$$

#### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 28

```
dsolve(x*diff(y(x),x$2)+x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{(x+1)c_2e^{-x}}{2} + (x+2)x\left(c_1 + \frac{\text{expIntegral}_1(x)c_2}{2}\right)$$

#### ✓ Solution by Mathematica

Time used: 0.125 (sec). Leaf size: 39

```
DSolve[x*y''[x]+x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1x(x+2) - \frac{1}{2}c_2e^{-x}(e^x(x+2)x \text{ExpIntegralEi}(-x) + x + 1)$$



### 3.8 problem 10.4.8 (h)

Internal problem ID [5071]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.4, ODEs with variable Coefficients.

Second order and Homogeneous. page 318

**Problem number:** 10.4.8 (h).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(x-1)^2 y'' - 2y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*(x-1)^2*diff(y(x),x$2)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{2c_2 x \ln(x) - c_2 x^2 + c_1 x + c_2}{x-1}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 33

```
DSolve[x*(x-1)^2*y''[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{-c_2 x^2 - c_1 x + 2c_2 x \log(x) + c_2}{x-1}$$

### 3.9 problem 10.4.9 (i)

Internal problem ID [5072]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.4, ODEs with variable Coefficients.  
Second order and Homogeneous. page 318

**Problem number:** 10.4.9 (i).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear]`

$$y' - \frac{2y}{x} = x^2$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)-2*y(x)/x-x^2=0,y(x), singsol=all)
```

$$y(x) = (x + c_1) x^2$$

#### ✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 13

```
DSolve[y'[x]-2*y[x]/x-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^2(x + c_1)$$

### 3.10 problem 10.4.9 (ii)

Internal problem ID [5073]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.4, ODEs with variable Coefficients.  
Second order and Homogeneous. page 318

**Problem number:** 10.4.9 (ii).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$y' + \frac{2y}{x} = x^3$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)+2*y(x)/x-x^3=0,y(x), singsol=all)
```

$$y(x) = \frac{x^6 + 6c_1}{6x^2}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 13

```
DSolve[y'[x]-2*y[x]/x-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^2(x + c_1)$$

### 3.11 problem 10.4.10

Internal problem ID [5074]

**Book:** Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995

**Section:** Chapter 10, Differential equations. Section 10.4, ODEs with variable Coefficients.  
Second order and Homogeneous. page 318

**Problem number:** 10.4.10.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type [Laguerre]

$$xy'' + (1-x)y' + my = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 21

```
dsolve(x*diff(y(x),x$2)+(1-x)*diff(y(x),x)+m*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \text{KummerM}(-m, 1, x) + c_2 \text{KummerU}(-m, 1, x)$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 21

```
DSolve[x*y''[x]+(1-x)*y'[x]+m*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \text{HypergeometricU}(-m, 1, x) + c_2 \text{LaguerreL}(m, x)$$