A Solution Manual For

Collection of Kovacic problems

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1.1 problem 1

Internal problem ID [7491]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(x^2-1)\,y''-2xy'+2y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2-1)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+c_2(x^2+1)$$

✓ Solution by Mathematica
Time used: 0.092 (sec). Leaf size: 39

```
DSolve[(x^2-1)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{\sqrt{x^2-1}(c_1(x-1)^2+c_2x)}{\sqrt{1-x^2}}$$

1.2 problem 2

Internal problem ID [7492]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(x^2-1)\,y''-6xy'+12y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve((x^2-1)*diff(y(x),x$2)-6*x*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(x^3+x)+c_2(x^4+6x^2+1)$$

✓ Solution by Mathematica
Time used: 0.132 (sec). Leaf size: 45

```
DSolve[(x^2-1)*y''[x]-6*x*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\frac{\sqrt{x^2-1}(c_2x(x^2+1)+c_1(x-1)^4)}{\sqrt{1-x^2}}$$

1.3 problem 3

Internal problem ID [7493]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+3)\, y''-7xy'+16y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 101

```
dsolve((x^2+3)*diff(y(x),x$2)-7*x*diff(y(x),x)+16*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x^4-9x^2+\frac{27}{8}\right)+c_2\left(\frac{\ln\left(\sqrt{x^2+3}-x\right)x^4}{64}+\frac{25\sqrt{x^2+3}\,x^3}{768}+\frac{25x^4}{768}-\frac{9\ln\left(\sqrt{x^2+3}-x\right)x^2}{64}-\frac{55\sqrt{x^2+3}\,x}{512}-\frac{75x^2}{256}+\frac{27\ln\left(\sqrt{x^2+3}-x\right)}{512}+\frac{225}{2048}\right)$$

✓ Solution by Mathematica

Time used: 0.523 (sec). Leaf size: 492

```
DSolve[(x^2+3)*y''[x]-7*x*y'[x]+16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{24} c_2 \Big(12960 x^2 \text{RootSum}\Big[7838208000\#1^4 - 188281584000\#1^2 - 241544908800\#1$$

$$+18453344881\&, \#1 \log \Big(-41175721196870400\#1^3 - 16606327460698080\#1^2 + 101387038251671139$$

$$+ 5248800 x^2 \text{RootSum}\Big[21088072057248000000\#1^4 - 30882886815600000\#1^2$$

$$+ 97825688064000\#1$$

$$+18453344881\&, \#1 \log \Big(273530830607325028080000000\#1^3 - 272385286174100257200000\#1^2 - 410617$$

$$- 4860 \text{RootSum}\Big[7838208000\#1^4 - 188281584000\#1^2 - 241544908800\#1$$

$$+18453344881\&, \#1 \log \Big(-41175721196870400\#1^3 - 16606327460698080\#1^2 + 101387038251671139$$

$$- 1968300 \text{RootSum}\Big[21088072057248000000\#1^4 - 30882886815600000\#1^2$$

$$+ 97825688064000\#1$$

$$+18453344881\&, \#1 \log \Big(273530830607325028080000000\#1^3 - 272385286174100257200000\#1^2 - 410617$$

$$- 1440 x^4 \text{RootSum}\Big[7838208000\#1^4 - 188281584000\#1^2 - 241544908800\#1$$

$$+18453344881\&, \#1 \log \Big(-41175721196870400\#1^3 - 16606327460698080\#1^2 + 101387038251671139$$

$$- 583200 x^4 \text{RootSum}\Big[21088072057248000000\#1^4 - 30882886815600000\#1^2$$

$$+ 97825688064000\#1$$

$$+18453344881\&, \#1 \log \Big(273530830607325028080000000\#1^3 - 272385286174100257200000\#1^2 - 410617$$

$$+ 165\sqrt{x^2+3}x + 216x^2 \log\left(\sqrt{x^2+3} - x\right) - 81 \log\left(\sqrt{x^2+3} - x\right)$$

$$- 24x^4 \log\left(\sqrt{x^2+3} - x\right) - 50\sqrt{x^2+3}x^3 \Big) + c_1 \left(x^4 - 9x^2 + \frac{27}{8} \right)$$

1.4 problem 4

Internal problem ID [7494]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(x^2-1)\,y''+8xy'+12y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve((x^2-1)*diff(y(x),x$2)+8*x*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(3x^2+1)}{(x-1)^3\,(x+1)^3}+\frac{c_2(x^3+3x)}{(x-1)^3\,(x+1)^3}$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 37

```
DSolve[(x^2-1)*y''[x]+8*x*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{3c_1(x-1)^3-c_2(3x^2+1)}{3\,(x^2-1)^3}$$

1.5 problem 5

Internal problem ID [7495]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3y'' + xy' - 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve(3*diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x^4 + 18x^2 + 27) + c_2(x^4 + 18x^2 + 27)\left(\int \frac{e^{-\frac{x^2}{6}}}{(x^4 + 18x^2 + 27)^2}dx\right)$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 43

```
DSolve[3*y''[x]+x*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\frac{x^2}{6}} \operatorname{HermiteH}\left(-5, \frac{x}{\sqrt{6}}\right) + \frac{1}{27}c_2(x^4 + 18x^2 + 27)$$

1.6 problem 6

Internal problem ID [7496]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{5y'' - 2xy' + 10y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 59

```
dsolve(5*diff(y(x),x$2)-2*x*diff(y(x),x)+10*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(x^5 - 25x^3 + \frac{375}{4}x\right) + c_2\left(x^5 - 25x^3 + \frac{375}{4}x\right)\left(\int \frac{e^{\frac{x^2}{5}}}{\left(4x^4 - 100x^2 + 375\right)^2 x^2} dx\right)$$

✓ Solution by Mathematica
Time used: 0.174 (sec). Leaf size: 138

```
DSolve[5*y''[x]-2*x*y'[x]+10*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{200}\sqrt{\frac{\pi}{5}}c_2\sqrt{x^2}\left(4x^4 - 100x^2 + 375\right)\operatorname{erfi}\left(\frac{\sqrt{x^2}}{\sqrt{5}}\right) + \frac{32c_1x^5}{25\sqrt{5}} - \frac{32c_1x^3}{\sqrt{5}} - \frac{9}{20}c_2e^{\frac{x^2}{5}}x^2 + c_2e^{\frac{x^2}{5}} + \frac{1}{50}c_2e^{\frac{x^2}{5}}x^4 + 24\sqrt{5}c_1x$$

1.7 problem 7

Internal problem ID [7497]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - x^2y' - 3yx = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 76

```
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)-3*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{x^3}{3}} x - \frac{9c_2 e^{\frac{x^3}{3}} 3^{\frac{2}{3}} e^{-\frac{x^3}{6}} \left(x^6 \operatorname{WhittakerM}\left(\frac{1}{3}, \frac{5}{6}, \frac{x^3}{3}\right) + 5 \operatorname{WhittakerM}\left(\frac{4}{3}, \frac{5}{6}, \frac{x^3}{3}\right) x^3 + 10 \operatorname{WhittakerM}\left(\frac{4}{3}, \frac{5}{6}, \frac{x^3}{3}\right)\right)}{10x^3 \left(x^3\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica
Time used: 0.139 (sec). Leaf size: 51

```
DSolve[y''[x]-x^2*y'[x]-3*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{9} e^{\frac{x^3}{3}} \left(9c_1 x - 3^{2/3} c_2 \sqrt[3]{x^3} \Gamma\left(-\frac{1}{3}, \frac{x^3}{3}\right)\right)$$

1.8 problem 8

Internal problem ID [7498]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+1\right)y''+2xy'-2y=0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 16

```
dsolve((1+x^2)*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+c_2(\arctan(x)\,x+1)$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 48

```
DSolve[(1+x^2)*y''[x]+2*x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}i(2c_1x-c_2x\log(1-ix)+c_2x\log(1+ix)+2ic_2)$$

1.9 problem 9

Internal problem ID [7499]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + xy' - 2y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x^2+1) + c_2(x^2+1)\left(\int \frac{\mathrm{e}^{-\frac{x^2}{2}}}{(x^2+1)^2}dx\right)$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 35

```
DSolve[y''[x]+x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\frac{x^2}{2}} \operatorname{HermiteH}\left(-3, \frac{x}{\sqrt{2}}\right) + c_2(x^2+1)$$

1.10 problem 10

Internal problem ID [7500]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2 - 6x + 10)\, y'' - 4(-3 + x)\, y' + 6y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve((x^2-6*x+10)*diff(y(x),x$2)-4*(x-3)*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\frac{26}{3} + x^2 - 6x\right) + c_2(x^3 - 30x + 60)$$

✓ Solution by Mathematica

Time used: 0.086 (sec). Leaf size: 36

```
DSolve[(x^2-6*x+10)*y''[x]-4*(x-3)*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{3} i\left(c_2(3x^2 - 18x + 26) + 3c_1(x - (3 + i))^3\right)$$

1.11 problem 11

Internal problem ID [7501]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+6x)\,y''+(3x+9)\,y'-3y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve((x^2+6*x)*diff(y(x),x$2)+(3*x+9)*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(x+3)+\frac{c_2(2x^2+12x+9)}{\sqrt{x^2+6x}}$$

✓ Solution by Mathematica
Time used: 0.072 (sec). Leaf size: 82

```
DSolve[(x^2+6*x)*y''[x]+(3*x+9)*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{9\sqrt{\pi}c_2\sqrt[4]{-x(x+6)}Q_{\frac{3}{2}}^{\frac{1}{2}}\left(\frac{x}{3}+1\right)+\sqrt{6}c_1(2x^2+12x+9)}{9\sqrt{\pi}\sqrt[4]{-x^2}\sqrt{x+6}}$$

1.12 problem 12

Internal problem ID [7502]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{ty'' + (t^2 - 1)\, y' + t^2 y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve(t*diff(y(t),t$2)+ (t^2-1)*diff(y(t),t)+t^2*y(t) = 0,y(t), singsol=all)
```

$$y(t) = c_1 e^{t-\frac{1}{2}t^2}(t-1) + c_2 e^{t-\frac{1}{2}t^2}(t-1)\left(\int \frac{t\, e^{\frac{1}{2}t^2-2t}}{(t-1)^2} dt\right)$$

✓ Solution by Mathematica

Time used: 0.434 (sec). Leaf size: 70

```
DSolve[t*y''[t]+(t^2-1)*y'[t]+t^2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2} e^{-\frac{t^2}{2}+t-2}\left(\sqrt{2\pi} c_2 (t-1) \operatorname{erfi}\left(\frac{t-2}{\sqrt{2}}\right) + 2e^2 c_1 (t-1) - 2c_2 e^{\frac{1}{2}(t-2)^2}\right)$$

1.13 problem 13

Internal problem ID [7503]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2y'' - t(t+2)\,y' + (t+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(t^2*diff(y(t),t$2)-t*(t+2)*diff(y(t),t)+(t+2)*y(t) = 0,y(t), singsol=all)
```

$$y(t) = c_1 t + c_2 t\,\mathrm{e}^t$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 16

```
DSolve[t^2*y''[t]-t*(t+2)*y'[t]+(t+2)*y[t] == 0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t\left(c_2 e^t + c_1\right)$$

1.14 problem 14

Internal problem ID [7504]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{ty'' - (t+1)\,y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(t*diff(y(t),t$2)-(1+t)*diff(y(t),t)+y(t) = 0,y(t), singsol=all)
```

$$y(t) = c_1(t+1) + c_2\mathrm{e}^t$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 19

```
DSolve[t*y''[t]-(1+t)*y'[t]+y[t] == 0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^t - c_2(t+1)$$

1.15 problem 15

Internal problem ID [7505]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-t+1)\, y'' + t y' - y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve((1-t)*diff(y(t),t$2)+t*diff(y(t),t)-y(t) = 0,y(t), singsol=all)
```

$$y(t) = c_1 t + c_2 \mathrm{e}^{t}$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 17

```
DSolve[(1-t)*y''[t]+t*y'[t]-y[t] == 0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^t - c_2 t$$

1.16 problem 16

Internal problem ID [7506]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-25/100)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-25/100)*y[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2e^{2ix})}{2\sqrt{x}}$$

1.17 problem 17

Internal problem ID [7507]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{ty'' - (t+1)\,y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(t*diff(y(t),t$2)-(1+t)*diff(y(t),t)+y(t) = 0,y(t), singsol=all)
```

$$y(t) = c_1(t+1) + c_2\mathrm{e}^t$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 19

```
DSolve[t*y''[t]-(1+t)*y'[t]+y[t] ==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^t - c_2(t+1)$$

1.18 problem 18

Internal problem ID [7508]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-t+1)\, y'' + t y' - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve((1-t)*diff(y(t),t$2)+t*diff(y(t),t)-y(t) = 0,y(t), singsol=all)
```

$$y(t) = c_1 t + c_2 \mathrm{e}^t$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 17

```
DSolve[(1-t)*y''[t]+t*y'[t]-y[t] ==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^t - c_2 t$$

1.19 problem 19

Internal problem ID [7509]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + xy' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 57

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x\,\mathrm{e}^{-\frac{x^2}{2}} + \frac{c_2\sqrt{2}\,\mathrm{e}^{-\frac{x^2}{2}}\left(i\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{\frac{x^2}{2}} - \pi\,\mathrm{erf}\left(\frac{i\sqrt{2}\,x}{2}\right)x\right)}{2\sqrt{\pi}}$$

✓ Solution by Mathematica
Time used: 0.06 (sec). Leaf size: 69

```
DSolve[y''[x]+x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\frac{\pi}{2}}c_2 e^{-\frac{x^2}{2}}\sqrt{x^2}\mathrm{erfi}\left(\frac{\sqrt{x^2}}{\sqrt{2}}\right) + \sqrt{2}c_1 e^{-\frac{x^2}{2}}x + c_2$$

1.20 problem 20

Internal problem ID [7510]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''-4xy'+6y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve((1+x^2)*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(-3x^2+1)+c_2(x^3-3x)$$

✓ Solution by Mathematica

Time used: 0.08 (sec). Leaf size: 33

```
DSolve[(1+x^2)*y''[x]-4*x*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{1}{3}i\left(c_2(3x^2-1)+3c_1(x-i)^3\right)$$

1.21 problem 21

Internal problem ID [7511]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1-x)\,y''+xy'-y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve((1-x)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+\mathrm{e}^x c_2$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 17

```
DSolve[(1-x)*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1e^x-c_2x$$

1.22 problem 22

Internal problem ID [7512]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2y'' + xy' + 3y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 47

```
dsolve(2*diff(y(x),x$2)+x*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x^2}{4}} (x^2 - 2) + c_2 \mathrm{e}^{-\frac{x^2}{4}} (x^2 - 2) \left(\int \frac{\mathrm{e}^{\frac{x^2}{4}}}{(x^2 - 2)^2} dx \right)$$

✓ Solution by Mathematica
Time used: 0.333 (sec). Leaf size: 61

```
DSolve[2*y''[x]+x*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8} e^{-\frac{x^2}{4}} \left(\sqrt{\pi} c_2 (x^2 - 2) \operatorname{erfi}\left(\frac{x}{2}\right) + 8c_1 (x^2 - 2) - 2c_2 e^{\frac{x^2}{4}} x \right)$$

1.23 problem 23

Internal problem ID [7513]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + xy' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 57

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x\, \mathrm{e}^{-\frac{x^2}{2}} + \frac{c_2 \sqrt{2}\, \mathrm{e}^{-\frac{x^2}{2}} \left(i\sqrt{2}\,\sqrt{\pi}\, \mathrm{e}^{\frac{x^2}{2}} - \pi \operatorname{erf}\left(\frac{i\sqrt{2}\, x}{2}\right) x\right)}{2\sqrt{\pi}}$$

✓ Solution by Mathematica
Time used: 0.062 (sec). Leaf size: 69

```
DSolve[y''[x]+x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\frac{\pi}{2}} c_2 e^{-\frac{x^2}{2}} \sqrt{x^2} \operatorname{erfi}\left(\frac{\sqrt{x^2}}{\sqrt{2}}\right) + \sqrt{2} c_1 e^{-\frac{x^2}{2}} x + c_2$$

1.24 problem 24

Internal problem ID [7514]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1-x)\,y''+xy'-y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve((1-x)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+\mathrm{e}^{x}c_2$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 17

```
DSolve[(1-x)*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1e^x-c_2x$$

1.25 problem 25

Internal problem ID [7515]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + xy' + 2y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 57

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x\,\mathrm{e}^{-\frac{x^2}{2}} + \frac{c_2\sqrt{2}\,\mathrm{e}^{-\frac{x^2}{2}}\left(i\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{\frac{x^2}{2}} - \pi\ \mathrm{erf}\left(\frac{i\sqrt{2}\,x}{2}\right)x\right)}{2\sqrt{\pi}}$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 69

```
DSolve[y''[x]+x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\frac{\pi}{2}}c_2 e^{-\frac{x^2}{2}}\sqrt{x^2}\mathrm{erfi}\left(\frac{\sqrt{x^2}}{\sqrt{2}}\right) + \sqrt{2}c_1 e^{-\frac{x^2}{2}}x + c_2$$

1.26 problem 26

Internal problem ID [7516]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(-x^2+4\right) y''+xy'+2y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 69

```
dsolve((4-x^2)*diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\sqrt{x^2-6}\,\sin\left(\int\frac{\sqrt{-x^2+4}\,\sqrt{3}}{x^2-6}dx\right)+c_2\sqrt{x^2-6}\,\cos\left(\int\frac{\sqrt{-x^2+4}\,\sqrt{3}}{x^2-6}dx\right)$$

✓ Solution by Mathematica
Time used: 0.055 (sec). Leaf size: 58

```
DSolve[(4-x^2)*y''[x]+x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\left(x^2-4\right)^{3/4}\left(c_1P_{-\frac{1}{2}+\sqrt{3}}^{\frac{3}{2}}\left(\frac{x}{2}\right)+c_2Q_{-\frac{1}{2}+\sqrt{3}}^{\frac{3}{2}}\left(\frac{x}{2}\right)\right)$$

1.27 problem 27

Internal problem ID [7517]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' - 4xy' + (-16x^2 + 3)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(3-16*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\sinh(2x) + c_2\sqrt{x}\,\cosh(2x)$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 32

```
DSolve[4*x^2*y''[x]-4*x*y'[x]+(3-16*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{-2x}\sqrt{x}\left(c_2e^{4x} + 4c_1\right)$$

1.28 problem 28

Internal problem ID [7518]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-1)\,y''-xy'+y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+\mathrm{e}^{x}c_2$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 17

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1e^x-c_2x$$

1.29 problem 29

Internal problem ID [7519]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2xy' + (x^2+2)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sin(x) + c_2 \cos(x)\, x$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-ix} x - \frac{1}{2} i c_2 e^{ix} x$$

1.30 problem 31

Internal problem ID [7520]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 - 2x)\,y'' + (-x^2 + 2)\,y' + (2x - 2)\,y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve((x^2-2*x)*diff(y(x),x$2)+(2-x^2)*diff(y(x),x)+(2*x-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^2 + e^x c_2$$

✓ Solution by Mathematica
Time used: 0.078 (sec). Leaf size: 18

```
DSolve[(x^2-2*x)*y''[x]+(2-x^2)*y'[x]+(2*x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to c_2 x^2 + c_1 e^x$$

1.31 problem 32

Internal problem ID [7521]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + (-8x^2 + 4x)\, y' + (4x^2 - 4x - 1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(4*x^2*diff(y(x),x$2)+(4*x-8*x^2)*diff(y(x),x)+(4*x^2-4*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^x}{\sqrt{x}} + c_2\sqrt{x}\, e^x$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 21

```
DSolve[4*x^2*y''[x]+(4*x-8*x^2)*y'[x]+(4*x^2-4*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{e^x(c_2x + c_1)}{\sqrt{x}}$$

1.32 problem 33

Internal problem ID [7522]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4xy' + (4x^2 + 2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x^2} + c_2 e^{-x^2} x$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 20

```
DSolve[y''[x]+4*x*y'[x]+(4*x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x^2}(c_2 x + c_1)$$

1.33 problem 34

Internal problem ID [7523]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x+1)\,y''-2y'-(3+2x)\,y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve((2*x+1)*diff(y(x),x$2)-2*diff(y(x),x)-(2*x+3)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-x}c_1+c_2x\,\mathrm{e}^{x}$$

✓ Solution by Mathematica
Time used: 0.085 (sec). Leaf size: 29

```
DSolve[(2*x+1)*y''[x]-2*y'[x]-(2*x+3)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{-x-\frac{1}{2}}\left(c_2e^{2x+1}x+c_1\right)$$

1.34 problem 35

Internal problem ID [7524]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (2x+2)\,y' + (x+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x$2)-(2*x+2)*diff(y(x),x)+(x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2 e^x x^3$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 23

```
DSolve[x*y''[x]-(2*x+2)*y'[x]+(x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}e^x\left(c_2 x^3 + 3c_1\right)$$

1.35 problem 36

Internal problem ID [7525]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2xy' + (x^2 + 2)\, y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sin(x) + c_2 \cos(x)\, x$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-ix} x - \frac{1}{2} i c_2 e^{ix} x$$

1.36 problem 38

Internal problem ID [7526]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' - 4xy' + (-16x^2 + 3)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(3-16*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\, \sinh(2x) + c_2\sqrt{x}\, \cosh(2x)$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 32

```
DSolve[4*x^2*y''[x]-4*x*y'[x]+(3-16*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{-2x}\sqrt{x}\left(c_2e^{4x} + 4c_1\right)$$

1.37 problem 39

Internal problem ID [7527]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' - 4xy' + \left(4x^2+3\right)y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2+3)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\sin(x) + c_2\sqrt{x}\,\cos(x)$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 39

```
DSolve[4*x^2*y''[x]-4*x*y'[x]+(4*x^2+3)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-ix}\sqrt{x}\left(2c_1 - ic_2e^{2ix}\right)$$

1.38 problem 40

Internal problem ID [7528]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2xy' - (x^2 - 2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)-(x^2-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sinh(x) + c_2 x \cosh(x)$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 25

```
DSolve[x^2*y''[x]-2*x*y'[x]-(x^2-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x} x + \frac{1}{2} c_2 e^x x$$

1.39 problem 41

Internal problem ID [7529]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 41.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - 2x(1+x)\,y' + (x^2+2x+2)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)-2*x*(x+1)*diff(y(x),x)+(x^2+2*x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x\,\mathrm{e}^x + c_2\mathrm{e}^x x^2$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 17

```
DSolve[x^2*y''[x]-2*x*(x+1)*y'[x]+(x^2+2*x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^x x(c_2x + c_1)$$

1.40 problem 42

Internal problem ID [7530]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 42.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - 2x(x+2)\,y' + \left(x^2+4x+6\right)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)-2*x*(x+2)*diff(y(x),x)+(x^2+4*x+6)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\mathrm{e}^x x^2 + c_2\mathrm{e}^x x^3$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 19

```
DSolve[x^2*y''[x]-2*x*(x+2)*y'[x]+(x^2+4*x+6)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^x x^2(c_2x + c_1)$$

1.41 problem 43

Internal problem ID [7531]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 43.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 4xy' + (x^2 + 6)\, y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(x^2+6)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^2 \sin(x) + c_2 \cos(x)\, x^2$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 37

```
DSolve[x^2*y''[x]-4*x*y'[x]+(x^2+6)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-ix} x^2 \left(2c_1 - ic_2 e^{2ix}\right)$$

1.42 problem 44

Internal problem ID [7532]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 44.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-1)\,y''-xy'+y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+\mathrm{e}^{x}c_2$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 17

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1e^x-c_2x$$

1.43 problem 45

Internal problem ID [7533]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 45.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' - 4x(1+x)\,y' + (3+2x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*(x+1)*diff(y(x),x)+(2*x+3)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x} + c_2\sqrt{x}\,\mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 20

```
DSolve[4*x^2*y''[x]-4*x*(x+1)*y'[x]+(2*x+3)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}(c_2e^x + c_1)$$

1.44 problem 46

Internal problem ID [7534]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 46.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(3x-1)\,y''-(3x+2)\,y'-(6x-8)\,y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve((3*x-1)*diff(y(x),x$2)-(3*x+2)*diff(y(x),x)-(6*x-8)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{2x} + \mathrm{e}^{-x} c_2 x$$

✓ Solution by Mathematica

Time used: 0.157 (sec). Leaf size: 35

```
DSolve[(3*x-1)*y''[x]-(3*x+2)*y'[x]-(6*x-8)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^{-x-\frac{1}{2}}(c_1 e^{3x} + 2ec_2 x)}{\sqrt{2}}$$

1.45 problem 47

Internal problem ID [7535]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 47.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x+2)\,y''+xy'+3y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 115

```
dsolve((2+x)*diff(y(x),x$2)+x*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\mathrm{e}^{-x}\left(x^5-20x^3-40x^2+32\right)-\frac{c_2(\mathrm{e}^{-2}\,\mathrm{expIntegral}_1\left(-2-x\right)x^5+\mathrm{e}^{x}x^4-20\,\mathrm{e}^{-2}\,\mathrm{expIntegral}_1\left(-2-x\right)x^3-\mathrm{e}^{x}x^3-40\,\mathrm{e}^{-2}\,\mathrm{expIntegral}}{240}$$

✓ Solution by Mathematica

Time used: 0.68 (sec). Leaf size: 81

```
DSolve[(2+x)*y''[x]+x*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{960}e^{-x-1}\left(c_2\left(x^2-6x+4\right)(x+2)^3\,\mathrm{ExpIntegralEi}(x+2)+3840c_1\left(x^2-6x+4\right)(x+2)^3-c_2e^{x+2}\left(x^4-x^3-18x^2-22x+8\right)\right)$$

1.46 problem 48

Internal problem ID [7536]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 48.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-x)\,y''+x(4+x)\,y'+(-x+2)\,y=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 53

```
dsolve(x^2*(1-x)*diff(y(x),x$2)+x*(4+x)*diff(y(x),x)+(2-x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(x^2+6x+3)}{x}+\frac{c_2(3\ln(x)\,x^3+18x^2\ln(x)+9x\ln(x)+51x^2+48x+1)}{3x^2}$$

✓ Solution by Mathematica
Time used: 0.081 (sec). Leaf size: 53

```
DSolve[x^2*(1-x)*y''[x]+x*(4+x)*y'[x]+(2-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{3c_1x(x^2+6x+3)-c_2(51x^2+3(x^2+6x+3)\,x\log(x)+48x+1)}{3x^2}$$

1.47 problem 49

Internal problem ID [7537]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 49.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1+x)\,y''+x(2x+1)\,y'-(4+6x)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(x^2*(1+x)*diff(y(x),x$2)+x*(1+2*x)*diff(y(x),x)-(4+6*x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x^2+\frac{c_2(12x^4\ln(x)-12\ln(x+1)\,x^4+12x^3-6x^2+4x-3)}{12x^2}$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 52

```
DSolve[x^2*(1+x)*y''[x]+x*(1+2*x)*y'[x]-(4+6*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\to c_1x^2+\frac{c_2(12x^4\log(x)-12x^4\log(x+1)+12x^3-6x^2+4x-3)}{12x^2}$$

1.48 problem 50

Internal problem ID [7538]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 50.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^{2}\left(2x^{2}+1\right)y''+x\left(2x^{2}+4\right)y'+2\left(1-x^{2}\right)y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(x^2*(1+2*x^2)*diff(y(x),x$2)+x*(4+2*x^2)*diff(y(x),x)+2*(1-x^2)*y(x)=0,y(x), singsol=
```

$$y(x)=\frac{c_{1}}{x}+\frac{c_{2}\sqrt{2}\left(\sqrt{2}\sqrt{2x^{2}+1}\,x^{2}+3\operatorname{arcsinh}\left(\sqrt{2}\,x\right)x-\sqrt{2}\sqrt{2x^{2}+1}\right)}{2x^{2}}$$

✓ Solution by Mathematica

Time used: 0.196 (sec). Leaf size: 77

```
DSolve[x^2*(1+2*x^2)*y''[x]+x*(4+2*x^2)*y'[x]+2*(1-x^2)*y[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x)\to -\frac{c_{2}\sqrt{2x^{2}+1}}{x^{2}}+c_{2}\sqrt{2x^{2}+1}-\frac{3c_{2}\log\left(\sqrt{2x^{2}+1}-\sqrt{2}x\right)}{\sqrt{2}x}+\frac{c_{1}}{x}$$

1.49 problem 51

Internal problem ID [7539]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 51.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+2)\,y''+2x(x^2+5)\,y'+2(-x^2+3)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 85

```
dsolve(x^2*(2+x^2)*diff(y(x),x$2)+2*x*(x^2+5)*diff(y(x),x)+2*(3-x^2)*y(x)=0,y(x), singsol=al
```

$$y(x)=\frac{c_1(x^2+8)}{x}-\frac{c_2\sqrt{2}\left(\operatorname{arctanh}\left(\frac{\sqrt{2}}{\sqrt{x^2+2}}\right)x^4-\sqrt{2}\,\sqrt{x^2+2}\,x^2+8\operatorname{arctanh}\left(\frac{\sqrt{2}}{\sqrt{x^2+2}}\right)x^2+4\sqrt{2}\,\sqrt{x^2+2}\right)}{64x^3}$$

✓ Solution by Mathematica

Time used: 0.288 (sec). Leaf size: 88

```
DSolve[x^2*(2+x^2)*y''[x]+2*x*(x^2+5)*y'[x]+2*(3-x^2)*y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x)\to\frac{-\sqrt{2}c_2(x^2+8)\,x^2\operatorname{arctanh}\left(\frac{\sqrt{x^2+2}}{\sqrt{2}}\right)+64c_1x^4+2x^2\left(c_2\sqrt{x^2+2}+256c_1\right)-8c_2\sqrt{x^2+2}}{64x^3}$$

1.50 problem 52

Internal problem ID [7540]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 52.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''+6xy'+6y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve((1+x^2)*diff(y(x),x$2)+6*x*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x}{(x^2+1)^2}+\frac{c_2(x^2-1)}{(x^2+1)^2}$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 29

```
DSolve[(1+x^2)*y''[x]+6*x*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_2x-c_1(x-i)^2}{(x^2+1)^2}$$

1.51 problem 53

Internal problem ID [7541]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 53.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\, y''+2xy'-2y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve((1+x^2)*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+c_2(\arctan(x)\, x+1)$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 48

```
DSolve[(1+x^2)*y''[x]+2*x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{1}{2}i(2c_1x-c_2x\log(1-ix)+c_2x\log(1+ix)+2ic_2)$$

1.52 problem 54

Internal problem ID [7542]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 54.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\, y''-8xy'+20y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve((1+x^2)*diff(y(x),x$2)-8*x*diff(y(x),x)+20*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(5x^4-10x^2+1)+c_2(x^5-10x^3+5x)$$

✓ Solution by Mathematica
Time used: 0.075 (sec). Leaf size: 38

```
DSolve[(1+x^2)*y''[x]-8*x*y'[x]+20*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{1}{5}ic_2(5x^4-10x^2+1)+c_1(1+ix)^5$$

1.53 problem 55

Internal problem ID [7543]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 55.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(1-x^2)\,y''-8xy'-12y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 43

```
dsolve((1-x^2)*diff(y(x),x$2)-8*x*diff(y(x),x)-12*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(3x^2+1)}{(x-1)^3\,(x+1)^3}+\frac{c_2(x^3+3x)}{(x-1)^3\,(x+1)^3}$$

✓ Solution by Mathematica
Time used: 0.045 (sec). Leaf size: 37

```
DSolve[(1-x^2)*y''[x]-8*x*y'[x]-12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{3c_1(x-1)^3-c_2(3x^2+1)}{3\,(x^2-1)^3}$$

1.54 problem 56

Internal problem ID [7544]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 56.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(2x^2+1)\,y''+7xy'+2y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve((1+2*x^2)*diff(y(x),x$2)+7*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x}{(2x^2+1)^{\frac{3}{4}}}+\frac{c_2x\left(\int \frac{1}{(2x^2+1)^{\frac{1}{4}}x^2}dx\right)}{(2x^2+1)^{\frac{3}{4}}}$$

✓ Solution by Mathematica

Time used: 0.063 (sec). Leaf size: 66

```
DSolve[(1+2*x^2)*y''[x]+7*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_2Q_{\frac{1}{4}}^{\frac{3}{4}}\left(i\sqrt{2}x\right)}{(2x^2+1)^{3/8}}+\frac{2i\sqrt[4]{2}c_1x}{(2x^2+1)^{3/4}\,\text{Gamma}\left(\frac{1}{4}\right)}$$

1.55 problem 57

Internal problem ID [7545]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 57.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(1-x^2)\,y''-5xy'-4y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 46

```
dsolve((1-x^2)*diff(y(x),x$2)-5*x*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x}{(x^2-1)^{\frac{3}{2}}}+\frac{c_2\left(\ln\left(x+\sqrt{x^2-1}\right)x-\sqrt{x^2-1}\right)}{(x^2-1)^{\frac{3}{2}}}$$

✓ Solution by Mathematica
Time used: 0.114 (sec). Leaf size: 52

```
DSolve[(1-x^2)*y''[x]-5*x*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{-c_2\sqrt{x^2-1}-c_2x\log\left(\sqrt{x^2-1}-x\right)+c_1x}{(x^2-1)^{3/2}}$$

1.56 problem 58

Internal problem ID [7546]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 58.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''-10xy'+28y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 38

```
dsolve((1+x^2)*diff(y(x),x$2)-10*x*diff(y(x),x)+28*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(1+\frac{35}{3}x^4-14x^2\right)+c_2\left(x^7+21x^5-105x^3+35x\right)$$

✓ Solution by Mathematica
Time used: 0.082 (sec). Leaf size: 40

```
DSolve[(1+x^2)*y''[x]-10*x*y'[x]+28*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{105}c_2\left(35x^4-42x^2+3\right)-c_1(x-i)^6(x+6i)$$

1.57 problem 59

Internal problem ID [7547]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 59.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + xy' + 2y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 57

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \,\mathrm{e}^{-\frac{x^2}{2}} + \frac{c_2 \sqrt{2}\,\mathrm{e}^{-\frac{x^2}{2}} \left(i\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{\frac{x^2}{2}} - \pi \,\mathrm{erf}\left(\frac{i\sqrt{2}\,x}{2}\right) x\right)}{2\sqrt{\pi}}$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 69

```
DSolve[y''[x]+x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\frac{\pi}{2}} c_2 e^{-\frac{x^2}{2}} \sqrt{x^2}\,\mathrm{erfi}\left(\frac{\sqrt{x^2}}{\sqrt{2}}\right) + \sqrt{2} c_1 e^{-\frac{x^2}{2}} x + c_2$$

1.58 problem 60

Internal problem ID [7548]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 60.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x^2+1)\,y''-9xy'-6y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 76

```
dsolve((1+2*x^2)*diff(y(x),x$2)-9*x*diff(y(x),x)-6*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(3x^6+5x^4+3x^2+1)\\+c_2(3x^6+5x^4+3x^2+1)\left(\int\frac{(2x^2+1)^{\frac{9}{4}}}{(3x^4+2x^2+1)^2\,(x^2+1)^2}dx\right)$$

✓ Solution by Mathematica

Time used: 0.349 (sec). Leaf size: 71

```
DSolve[(1+2*x^2)*y''[x]-9*x*y'[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_2(2x^2+1)^{13/8}\,Q_{\frac{11}{4}}^{\frac{13}{4}}\left(i\sqrt{2}x\right)+\frac{64\sqrt[4]{2}c_1(3x^6+5x^4+3x^2+1)}{3\,\mathrm{Gamma}\left(-\frac{9}{4}\right)}$$

1.59 problem 61

Internal problem ID [7549]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 61.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x^2-8x+11)\,y''-16(x-2)\,y'+36y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 45

```
dsolve((11-8*x+2*x^2)*diff(y(x),x$2)-16*(x-2)*diff(y(x),x)+36*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(-\frac{31}{5}+x^3-6x^2+\frac{111}{10}x\right)+c_2\left(x^6-12x^5+\frac{165}{2}x^4-\frac{16577}{8}-\frac{5445}{4}x^2+3267x\right)$$

✓ Solution by Mathematica
Time used: 0.998 (sec). Leaf size: 91

```
DSolve[(11-8*x+2*x^2)*y''[x]-16*(x-2)*y'[x]+36*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x)\to\frac{1}{15}ic_2\left(10x^3-60x^2+111x-62\right)+\frac{c_1\left(2x+5i\sqrt{6}-4\right)\left(2(x-4)x+11\right)^2\left(2ix+\sqrt{6}-4i\right)^3}{2\left(-2ix+\sqrt{6}+4i\right)^2}$$

1.60 problem 62

Internal problem ID [7550]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 62.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (-3 + x) y' + 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 68

```
dsolve(diff(y(x),x$2)+(x-3)*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{1}{2}x^2+3x}\left(x^2-6x+8\right) + c_2 \mathrm{e}^{-\frac{1}{2}x^2+3x}\left(x^2-6x+8\right)\left(\int \frac{\mathrm{e}^{\frac{1}{2}x^2-3x}}{(x-2)^2 (x-4)^2} dx\right)$$

✓ Solution by Mathematica

Time used: 0.588 (sec). Leaf size: 90

```
DSolve[y''[x]+(x-3)*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{-\frac{1}{2}(x-6)x-8}\left(e^{7/2}\sqrt{2\pi}c_2\left(x^2-6x+8\right)\operatorname{erfi}\left(\frac{x-3}{\sqrt{2}}\right) + 4e^8 c_1\left(x^2-6x+8\right) - 2c_2 e^{\frac{1}{2}(x-4)^2+x}(x-3)\right)$$

1.61 problem 63

Internal problem ID [7551]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 63.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-8x+14)\,y''-8(x-4)\,y'+20y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve((x^2-8*x+14)*diff(y(x),x$2)-8*(x-4)*diff(y(x),x)+20*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(\frac{1604}{5}+x^4-16x^3+100x^2-288x\right)+c_2\left(x^5-140x^3+1120x^2-3500x+4032\right)$$

✓ Solution by Mathematica

Time used: 0.114 (sec). Leaf size: 77

```
DSolve[(x^2-8*x+14)*y''[x]+8*(x-4)*y'[x]+20*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_1P^3_{\frac{1}{2}i\left(i+\sqrt{31}\right)}\left(\frac{x-4}{\sqrt{2}}\right)+c_2Q^3_{\frac{1}{2}i\left(i+\sqrt{31}\right)}\left(\frac{x-4}{\sqrt{2}}\right)}{\left(x^2-8x+14\right)^{3/2}}$$

1.62 problem 64

Internal problem ID [7552]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 64.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x^2+4x+5)\,y''-20(1+x)\,y'+60y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 55

```
dsolve((2*x^2+4*x+5)*diff(y(x),x$2)-20*(x+1)*diff(y(x),x)+60*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(-\frac{7}{4}+x^5+5x^4+5x^3-5x^2-\frac{31}{4}x\right)+c_2\left(x^6+\frac{155}{8}-\frac{75}{2}x^4-100x^3-\frac{225}{4}x^2+30x\right)$$

✓ Solution by Mathematica
Time used: 1.003 (sec). Leaf size: 83

```
DSolve[(2*x^2+4*x+5)*y''[x]-20*(x+1)*y'[x]+60*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to\frac{(2x^2+4x+5)^{5/2}\left(4c_2(4x^5+20x^4+20x^3-20x^2-31x-7)+c_1\left(2ix+\sqrt{6}+2i\right)^6\right)}{(4x^2+8x+10)^{5/2}}$$

1.63 problem 65

Internal problem ID [7553]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 65.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^3+1)\,y''+7x^2y'+9yx=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve((1+x^3)*diff(y(x),x$2)+7*x^2*diff(y(x),x)+9*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x}{(x^3+1)^{\frac{4}{3}}}+\frac{c_2x\left(\int\frac{((x+1)(x^2-x+1))^{\frac{1}{3}}}{x^2}dx\right)}{(x^3+1)^{\frac{4}{3}}}$$

✓ Solution by Mathematica

Time used: 1.109 (sec). Leaf size: 118

```
DSolve[(1+x^3)*y''[x]+7*x^2*y'[x]+9*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-2\sqrt{3}c_2x\arctan\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)-6c_2\sqrt[3]{x^3+1}-2c_2x\log\left(\sqrt[3]{x^3+1}-x\right)+c_2x\log\left(\sqrt[3]{x^3+1}x+(x^3\right.}{6\,(x^3+1)^{4/3}}$$

1.64 problem 66

Internal problem ID [7554]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 66.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x^5+1)\, y''+14y'x^4+10yx^3=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
dsolve((1+2*x^5)*diff(y(x),x$2)+14*x^4*diff(y(x),x)+10*x^3*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x}{(2x^5+1)^{\frac{2}{5}}}+\frac{c_2x\left(\int \frac{1}{(2x^5+1)^{\frac{3}{5}}x^2}dx\right)}{(2x^5+1)^{\frac{2}{5}}}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(1+2*x^5)*y''[x]+14*x^4*y'[x]+10*x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

1.65 problem 67

Internal problem ID [7555]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 67.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x^6 + 7yx^5 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 69

```
dsolve(diff(y(x),x$2)+x^6*diff(y(x),x)+7*x^5*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x^7}{7}} x + \frac{7c_2(-1)^{\frac{6}{7}} \mathrm{e}^{-\frac{x^7}{7}} \left(-\Gamma\left(\frac{6}{7}\right) x^7 + (-x^7)^{\frac{6}{7}} 7^{\frac{1}{7}} \mathrm{e}^{\frac{x^7}{7}} + \Gamma\left(\frac{6}{7}, -\frac{x^7}{7}\right) x^7\right)}{(-x^7)^{\frac{6}{7}}}$$

✓ Solution by Mathematica
Time used: 0.201 (sec). Leaf size: 53

```
DSolve[y''[x]+x^6*y'[x]+7*x^5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{49} e^{-\frac{x^7}{7}} \left(49c_1 x - 7^{6/7} c_2 \sqrt[7]{-x^7} \Gamma\left(-\frac{1}{7}, -\frac{x^7}{7}\right)\right)$$

1.66 problem 68

Internal problem ID [7556]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 68.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$
\boxed{(x^8+1)\, y''-16y'x^7+72yx^6=0}
$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve((1+x^8)*diff(y(x),x$2)-16*x^7*diff(y(x),x)+72*x^6*y(x)=0,y(x), singsol=all)
```

$$
y(x)=c_1\left(-\frac{7}{9}+x^8\right)+c_2\left(x^9-\frac{9}{7}x\right)
$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(1+x^8)*y''[x]-16*x^7*y'[x]+72*x^6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

1.67 problem 69

Internal problem ID [7557]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 69.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x^5 + 6yx^4 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 81

```
dsolve(diff(y(x),x$2)+x^5*diff(y(x),x)+6*x^4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x^6}{6}} x - \frac{2c_2 \mathrm{e}^{-\frac{x^6}{6}} \left(\left(-x^6\right)^{\frac{5}{6}} 6^{\frac{2}{3}} \sqrt{3}\, \sqrt{2}\, \mathrm{e}^{\frac{x^6}{6}} - 6\Gamma\left(\frac{5}{6}\right) x^6 + 6\Gamma\left(\frac{5}{6}, -\frac{x^6}{6}\right) x^6 \right)}{\left(-x^6\right)^{\frac{5}{6}} \left(\sqrt{3} + i\right)}$$

✓ Solution by Mathematica
Time used: 0.207 (sec). Leaf size: 53

```
DSolve[y''[x]+x^5*y'[x]+6*x^4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{36} e^{-\frac{x^6}{6}} \left(36 c_1 x - 6^{5/6} c_2 \sqrt[6]{-x^6}\, \Gamma\left(-\frac{1}{6}, -\frac{x^6}{6}\right)\right)$$

1.68 problem 70

Internal problem ID [7558]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 70.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(3x+1)\,y''+xy'+2y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 56

```
dsolve((1+3*x)*diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(3x+1)^{\frac{10}{9}}\,\mathrm{e}^{-\frac{x}{3}}(x-6)+c_2(3x+1)^{\frac{10}{9}}\,\mathrm{e}^{-\frac{x}{3}}(x-6)\left(\int\frac{\mathrm{e}^{\frac{x}{3}}}{(x-6)^2\,(3x+1)^{\frac{19}{9}}}dx\right)$$

✓ Solution by Mathematica
Time used: 3.176 (sec). Leaf size: 124

```
DSolve[(1+3*x)*y''[x]+x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{e^{-\frac{x}{3}-\frac{1}{9}}\left(1520c_1\sqrt[9]{3x+1}(3x^2-17x-6)-2^{8/9}c_2e^{\frac{x}{3}+\frac{1}{9}}(9x^2-48x-26)+2^{8/9}3^{7/9}c_2\sqrt[9]{-3x-1}(3x^2-17\right.}{380\;2^{17/18}}$$

1.69 problem 71

Internal problem ID [7559]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 71.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(3x^2+x+1\right)y''+(2+15x)\,y'+12y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 143

```
dsolve((1+x+3*x^2)*diff(y(x),x$2)+(2+15*x)*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\left(\frac{i\sqrt{11}-6x-1}{i\sqrt{11}+6x+1}\right)^{-\frac{i\sqrt{11}}{22}}x}{\left(3x^2+x+1\right)^{\frac{3}{2}}}+\frac{c_2\left(\frac{i\sqrt{11}-6x-1}{i\sqrt{11}+6x+1}\right)^{-\frac{i\sqrt{11}}{22}}x\left(\int\frac{\sqrt{3x^2+x+1}\left(\frac{i\sqrt{11}+6x+1}{i\sqrt{11}-6x-1}\right)^{-\frac{i\sqrt{11}}{22}}}{x^2}dx\right)}{\left(3x^2+x+1\right)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 3.347 (sec). Leaf size: 93

```
DSolve[(1+x+3*x^2)*y''[x]+(2+15*x)*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{xe^{\frac{\arctan\left(\frac{6x+1}{\sqrt{11}}\right)}{\sqrt{11}}}\left(c_2\int_1^x\frac{e^{-\frac{\arctan\left(\frac{6K[1]+1}{\sqrt{11}}\right)}{\sqrt{11}}}\sqrt{3K[1]^2+K[1]+1}}{K[1]^2}dK[1]+c_1\right)}{\left(3x^2+x+1\right)^{3/2}}$$

1.70 problem 72

Internal problem ID [7560]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 72.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x+2)\,y''+(1+x)\,y'+3y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 89

```
dsolve((2+x)*diff(y(x),x$2)+(1+x)*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\mathrm{e}^{-x}x\left(x^3-12x-16\right)-\frac{c_2\left(\mathrm{e}^{-2}\operatorname{expIntegral}_1\left(-2-x\right)x^4+\mathrm{e}^x x^3-12\,\mathrm{e}^{-2}\operatorname{expIntegral}_1\left(-2-x\right)x^2-\mathrm{e}^x x^2-16\,\mathrm{e}^{-2}\operatorname{expIntegral}\right.}{48}$$

✓ Solution by Mathematica
Time used: 0.359 (sec). Leaf size: 99

```
DSolve[(2+x)*y''[x]+(1+x)*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{e^{-x-1}\left(c_2(x-4)(x+2)^2x\operatorname{ExpIntegralEi}(x+2)+384c_1x^4-c_2e^{x+2}x^3+x^2\left(c_2e^{x+2}-4608c_1\right)+x(10c_2e\right.}{96\sqrt{2}}$$

1.71 problem 73

Internal problem ID [7561]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 73.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(4+x)\,y'' + (x+2)\,y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 108

```
dsolve((4+x)*diff(y(x),x$2)+(2+x)*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x} x\left(x^3 + 12x^2 + 48x + 64\right) - \frac{c_2\left(\mathrm{e}^{-4}\,\mathrm{expIntegral}_1\left(-x-4\right)x^4 + 12\,\mathrm{e}^{-4}\,\mathrm{expIntegral}_1\left(-x-4\right)x^3 + \mathrm{e}^{x}x^3 + 48\,\mathrm{e}^{-4}\,\mathrm{expIntegral}_1\left(-x-\right.\right.}{24}$$

✓ Solution by Mathematica

Time used: 0.191 (sec). Leaf size: 97

```
DSolve[(4+x)*y''[x]+(2+x)*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{24}e^{-x-4}\left(c_2x(x+4)^3\,\mathrm{ExpIntegralEi}(x+4) + e^4\left(24c_1x^4 + x^3(288c_1 - c_2e^x) + 9x^2(128c_1 - c_2e^x) + 2x(768c_1 - 11c_2e^x) - 6c_2e^x\right)\right)$$

1.72 problem 74

Internal problem ID [7562]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 74.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(2x^2+3x\right)y''+10(1+x)\,y'+8y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
dsolve((3*x+2*x^2)*diff(y(x),x$2)+10*(1+x)*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(x+2)}{x^{\frac{7}{3}}\left(2x+3\right)^{\frac{2}{3}}}+\frac{c_2(x+2)\left(\int \frac{x^{\frac{4}{3}}}{(x+2)^2(2x+3)^{\frac{1}{3}}}dx\right)}{x^{\frac{7}{3}}\left(2x+3\right)^{\frac{2}{3}}}$$

✓ Solution by Mathematica

Time used: 0.887 (sec). Leaf size: 245

```
DSolve[(3*x+2*x^2)*y''[x]+10*(1+x)*y'[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{2\,2^{2/3}\sqrt{3}c_2(x+2)\arctan\left(\frac{\sqrt{3}\sqrt[3]{x}}{\sqrt[3]{x}+2^{2/3}\sqrt[3]{2x+3}}\right)+2^{2/3}c_2x\log\left(2x^{2/3}+2^{2/3}\sqrt[3]{2x+3}\sqrt[3]{x}+\sqrt[3]{2}(2x+3)^{2/3}\right.}{}$$

1.73 problem 75

Internal problem ID [7563]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 75.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - (6-7x)\,y' + 8y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 81

```
dsolve(x^2*diff(y(x),x$2)-(6-7*x)*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{-\frac{6}{x}}(x-2)}{x^5} + \frac{c_2\left(x^3\mathrm{e}^{\frac{6}{x}} + 12x^2\mathrm{e}^{\frac{6}{x}} + 108 \operatorname{expIntegral}_1\left(-\frac{6}{x}\right)x - 36x\,\mathrm{e}^{\frac{6}{x}} - 216 \operatorname{expIntegral}_1\left(-\frac{6}{x}\right)\right)\mathrm{e}^{-\frac{6}{x}}}{2x^5}$$

✓ Solution by Mathematica

Time used: 0.179 (sec). Leaf size: 59

```
DSolve[x^2*y''[x]-(6-7*x)*y'[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-6/x}\left(-108c_2(x-2)\operatorname{ExpIntegralEi}\left(\frac{6}{x}\right) + c_2e^{6/x}x(x^2+12x-36) + 2c_1(x-2)\right)}{2x^5}$$

1.74 problem 76

Internal problem ID [7564]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 76.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x^2+x+1)\,y''+(1+7x)\,y'+2y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 149

```
dsolve((1+x+2*x^2)*diff(y(x),x$2)+(1+7*x)*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\left(\frac{i\sqrt{7}-4x-1}{i\sqrt{7}+4x+1}\right)^{-\frac{3i\sqrt{7}}{28}}(x+1)}{(2x^2+x+1)^{\frac{3}{4}}}+\frac{c_2\left(\frac{i\sqrt{7}-4x-1}{i\sqrt{7}+4x+1}\right)^{-\frac{3i\sqrt{7}}{28}}(x+1)\left(\int\frac{\left(\frac{i\sqrt{7}+4x+1}{i\sqrt{7}-4x-1}\right)^{-\frac{3i\sqrt{7}}{28}}}{(x+1)^2(2x^2+x+1)^{\frac{1}{4}}}dx\right)}{(2x^2+x+1)^{\frac{3}{4}}}$$

✓ Solution by Mathematica

Time used: 2.389 (sec). Leaf size: 102

```
DSolve[(1+x+2*x^2)*y''[x]+(1+7*x)*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{(x+1)e^{\frac{3\arctan\left(\frac{4x+1}{\sqrt{7}}\right)}{2\sqrt{7}}}\left(c_2\int_1^x\frac{e^{-\frac{3\arctan\left(\frac{4K[1]+1}{\sqrt{7}}\right)}{2\sqrt{7}}}}{(K[1]+1)^2\sqrt[4]{2K[1]^2+K[1]+1}}dK[1]+c_1\right)}{(2x^2+x+1)^{3/4}}$$

1.75 problem 77

Internal problem ID [7565]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 77.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x+3)\,y''+(2x+1)\,y'-(-x+2)\,y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 44

```
dsolve((3+x)*diff(y(x),x$2)+(1+2*x)*diff(y(x),x)-(2-x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-x}c_1+c_2\mathrm{e}^{-x}\left(x^6+18x^5+135x^4+540x^3+1215x^2+1458x\right)$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 29

```
DSolve[(3+x)*y''[x]+(1+2*x)*y'[x]-(2-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{6}e^{-x-3}\left(c_2(x+3)^6+6c_1\right)$$

1.76 problem 78

Internal problem ID [7566]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 78.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 3xy' + (2x^2 + 4)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 50

```
dsolve(diff(y(x),x$2)+3*x*diff(y(x),x)+(4+2*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^2}(x^2 - 1) + c_2 \mathrm{e}^{-x^2}(x^2 - 1) \left(\int \frac{\mathrm{e}^{\frac{x^2}{2}}}{(x-1)^2 (x+1)^2} dx \right)$$

✓ Solution by Mathematica
Time used: 0.427 (sec). Leaf size: 63

```
DSolve[y''[x]+3*x*y'[x]+(4+2*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} e^{-x^2} \left(\sqrt{2\pi} c_2 (x^2 - 1) \operatorname{erfi}\left(\frac{x}{\sqrt{2}} \right) + 4c_1 (x^2 - 1) - 2c_2 e^{\frac{x^2}{2}} x \right)$$

1.77 problem 79

Internal problem ID [7567]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 79.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(4x+2)\,y''-4y'-(6+4x)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve((2+4*x)*diff(y(x),x$2)-4*diff(y(x),x)-(6+4*x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-x}c_1+c_2x\,\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 29

```
DSolve[(2+4*x)*y''[x]-4*y'[x]-(6+4*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{-x-\frac{1}{2}}\left(c_2e^{2x+1}x+c_1\right)$$

1.78 problem 80

Internal problem ID [7568]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 80.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 3xy' + (2x^2 + 5)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)-3*x*diff(y(x),x)+(5+2*x^2)*y(x)=0,y(x), singsol=all)
```

$$\begin{aligned} y(x) &= c_1 \mathrm{e}^{\frac{x^2}{2}} (x^6 - 15x^4 + 45x^2 - 15) \\ &\quad + c_2 \mathrm{e}^{\frac{x^2}{2}} (x^6 - 15x^4 + 45x^2 - 15) \left(\int \frac{\mathrm{e}^{\frac{x^2}{2}}}{(x^6 - 15x^4 + 45x^2 - 15)^2} dx \right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.901 (sec). Leaf size: 95

```
DSolve[y''[x]-3*x*y'[x]+(5+2*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{\frac{x^2}{2}} \left(\sqrt{2\pi} c_2 (x^6 - 15x^4 + 45x^2 - 15) \operatorname{erfi}\left(\frac{x}{\sqrt{2}}\right) - 2c_2 e^{\frac{x^2}{2}} x (x^4 - 14x^2 + 33) + 1440 c_1 (x^6 - 15x^4 + 45x^2 -\right.}{1440}$$

1.79 problem 81

Internal problem ID [7569]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 81.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2y'' + 5xy' + (2x^2 + 4)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(2*diff(y(x),x$2)+5*x*diff(y(x),x)+(4+2*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x^2} + c_2 e^{-x^2} \operatorname{erf}\left(\frac{i\sqrt{3}\, x}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.097 (sec). Leaf size: 42

```
DSolve[2*y''[x]+5*x*y'[x]+(4+2*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} e^{-x^2} \left(\sqrt{3\pi} c_2 \operatorname{erfi}\left(\frac{\sqrt{3}x}{2}\right) + 3c_1\right)$$

1.80 problem 82

Internal problem ID [7570]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 82.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4xy' + (4x^2 + 2)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+4*x*diff(y(x),x)+(2+4*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^2} + c_2 \mathrm{e}^{-x^2} x$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 20

```
DSolve[y''[x]+4*x*y'[x]+(2+4*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x^2}(c_2 x + c_1)$$

1.81 problem 83

Internal problem ID [7571]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 83.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4xy' + (4x^2 + 2)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+4*x*diff(y(x),x)+(2+4*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x^2} + c_2 e^{-x^2} x$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 20

```
DSolve[y''[x]+4*x*y'[x]+(2+4*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x^2}(c_2 x + c_1)$$

1.82 problem 84

Internal problem ID [7572]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 84.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x^2+x+1)y''+x(11x^2+11x+9)y'+(7x^2+10x+6)y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 141

```
dsolve(2*x^2*(1+x+x^2)*diff(y(x),x$2)+x*(9+11*x+11*x^2)*diff(y(x),x)+(6+10*x+7*x^2)*y(x)=0,y
```

$$y(x)=\frac{c_1\sqrt{x^2+x+1}\left(\frac{i\sqrt{3}+2x+1}{i\sqrt{3}-2x-1}\right)^{-\frac{i\sqrt{3}}{6}}}{x^2}+\frac{c_2\sqrt{x^2+x+1}\left(\frac{i\sqrt{3}+2x+1}{i\sqrt{3}-2x-1}\right)^{-\frac{i\sqrt{3}}{6}}\left(\int\frac{\left(\frac{i\sqrt{3}-2x-1}{i\sqrt{3}+2x+1}\right)^{-\frac{i\sqrt{3}}{6}}}{(x^2+x+1)^{\frac{3}{2}}\sqrt{x}}dx\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 1.375 (sec). Leaf size: 93

```
DSolve[2*x^2*(1+x+x^2)*y''[x]+x*(9+11*x+11*x^2)*y'[x]+(6+10*x+7*x^2)*y[x]==0,y[x],x,IncludeS
```

$$y(x)\to\frac{\sqrt{x^2+x+1}e^{-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}}\left(c_2\int_1^x\frac{e^{\frac{\arctan\left(\frac{2K[1]+1}{\sqrt{3}}\right)}{\sqrt{3}}}}{\sqrt{K[1]}(K[1]^2+K[1]+1)^{3/2}}dK[1]+c_1\right)}{x^2}$$

1.83 problem 85

Internal problem ID [7573]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 85.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$3x^2y'' + 2x(-2x^2 + x + 1)y' + (-8x^2 + 2x)y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
dsolve(3*x^2*diff(y(x),x$2)+2*x*(1+x-2*x^2)*diff(y(x),x)+(2*x-8*x^2)*y(x)=0,y(x), singsol=al
```

$$y(x) = c_1 x^{\frac{1}{3}} e^{\frac{2}{3}x^2-\frac{2}{3}x} + c_2 x^{\frac{1}{3}} e^{\frac{2}{3}x^2-\frac{2}{3}x} \left(\int \frac{e^{-\frac{2}{3}x^2+\frac{2}{3}x}}{x^{\frac{4}{3}}} dx\right)$$

✓ Solution by Mathematica

Time used: 9.303 (sec). Leaf size: 53

```
DSolve[3*x^2*y''[x]+2*x*(1+x-2*x^2)*y'[x]+(2*x-8*x^2)*y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to e^{\frac{2}{3}(x-1)x} \sqrt[3]{x} \left(c_2 \int_1^x \frac{e^{-\frac{2}{3}(K[1]-1)K[1]}}{K[1]^{4/3}} dK[1] + c_1\right)$$

1.84 problem 86

Internal problem ID [7574]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 86.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$12x^2(1+x)\,y''+x\left(3x^2+35x+11\right)y'-\left(-5x^2-10x+1\right)y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 48

```
dsolve(12*x^2*(1+x)*diff(y(x),x$2)+x*(11+35*x+3*x^2)*diff(y(x),x)-(1-10*x-5*x^2)*y(x)=0,y(x)
```

$$y(x)=\frac{c_1 \mathrm{e}^{-\frac{x}{4}}}{(x+1)^{\frac{3}{4}}\,x^{\frac{1}{4}}}+\frac{c_2 \mathrm{e}^{-\frac{x}{4}}\left(\int \frac{\mathrm{e}^{\frac{x}{4}}}{(x+1)^{\frac{1}{4}}\,x^{\frac{5}{12}}}dx\right)}{(x+1)^{\frac{3}{4}}\,x^{\frac{1}{4}}}$$

✓ Solution by Mathematica

Time used: 20.702 (sec). Leaf size: 61

```
DSolve[12*x^2*(1+x)*y''[x]+x*(11+35*x+3*x^2)*y'[x]-(1-10*x-5*x^2)*y[x]==0,y[x],x,IncludeSing
```

$$y(x)\to \frac{e^{-x/4}\left(c_2\int_1^x \frac{e^{\frac{K[1]}{4}}}{K[1]^{5/12}\sqrt[4]{K[1]+1}}dK[1]+c_1\right)}{\sqrt[4]{x}(x+1)^{3/4}}$$

1.85 problem 87

Internal problem ID [7575]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 87.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$x^2(10x^2+x+5)\,y''+x(48x^2+3x+4)\,y'+(36x^2+x)\,y=0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 137

```
dsolve(x^2*(5+x+10*x^2)*diff(y(x),x$2)+x*(4+3*x+48*x^2)*diff(y(x),x)+(x+36*x^2)*y(x)=0,y(x),
```

$$y(x)=\frac{c_1x^{\frac{1}{5}}\left(\frac{i\sqrt{199}+20x+1}{i\sqrt{199}-20x-1}\right)^{-\frac{i\sqrt{199}}{995}}}{10x^2+x+5}+\frac{c_2x^{\frac{1}{5}}\left(\frac{i\sqrt{199}+20x+1}{i\sqrt{199}-20x-1}\right)^{-\frac{i\sqrt{199}}{995}}\left(\int\frac{\left(\frac{i\sqrt{199}-20x-1}{i\sqrt{199}+20x+1}\right)^{-\frac{i\sqrt{199}}{995}}}{x^{\frac{6}{5}}}dx\right)}{10x^2+x+5}$$

✓ Solution by Mathematica
Time used: 2.147 (sec). Leaf size: 88

```
DSolve[x^2*(5+x+10*x^2)*y''[x]+x*(4+3*x+48*x^2)*y'[x]+(x+36*x^2)*y[x]==0,y[x],x,IncludeSingu
```

$$y(x)\to\frac{\sqrt[5]{x}e^{-\frac{2\arctan\left(\frac{20x+1}{\sqrt{199}}\right)}{5\sqrt{199}}}\left(c_2\int_1^x\frac{e^{\frac{2\arctan\left(\frac{20K[1]+1}{\sqrt{199}}\right)}{5\sqrt{199}}}}{K[1]^{6/5}}dK[1]+c_1\right)}{10x^2+x+5}$$

1.86 problem 88

Internal problem ID [7576]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 88.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$18x^2(1+x)\,y''+3x\left(x^2+11x+5\right)y'-\left(-5x^2-2x+1\right)y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve(18*x^2*(1+x)*diff(y(x),x$2)+3*x*(5+11*x+x^2)*diff(y(x),x)-(1-2*x-5*x^2)*y(x)=0,y(x),
```

$$y(x)=c_1\mathrm{e}^{-\frac{x}{6}}\left(\frac{x+1}{x}\right)^{\frac{1}{6}}+c_2\mathrm{e}^{-\frac{x}{6}}\left(\frac{x+1}{x}\right)^{\frac{1}{6}}\left(\int\frac{\mathrm{e}^{\frac{x}{6}}}{(x+1)^{\frac{7}{6}}\sqrt{x}}dx\right)$$

✓ Solution by Mathematica

Time used: 3.726 (sec). Leaf size: 73

```
DSolve[18*x^2*(1+x)*y''[x]+3*x*(5+11*x+x^2)*y'[x]-(1-2*x-5*x^2)*y[x]==0,y[x],x,IncludeSingul
```

$$y(x)\to\frac{e^{-x/6}\left(c_2\int_1^x\frac{e^{\frac{K[1]}{6}}\sqrt[3]{\frac{K[1]}{K[1]+1}}}{K[1]^{5/6}(K[1]+1)^{5/6}}dK[1]+c_1\right)}{\sqrt[6]{\frac{x}{x+1}}}$$

1.87 problem 89

Internal problem ID [7577]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 89.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' + x(3+2x)\,y' - (1-x)\,y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(2*x^2*diff(y(x),x$2)+x*(3+2*x)*diff(y(x),x)-(1-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{-x}}{x} + \frac{c_2 \mathrm{e}^{-x} \left(\int \sqrt{x}\, \mathrm{e}^{x} dx\right)}{x}$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 33

```
DSolve[2*x^2*y''[x]+x*(3+2*x)*y'[x]-(1-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}\left(c_2 x^{3/2} L_{-\frac{3}{2}}^{\frac{3}{2}}(x) + c_1\right)}{x}$$

1.88 problem 90

Internal problem ID [7578]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 90.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' + x(x+5)\,y' - (2-3x)\,y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 33

```
dsolve(2*x^2*diff(y(x),x$2)+x*(5+x)*diff(y(x),x)-(2-3*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\mathrm{e}^{-\frac{x}{2}} + c_2\sqrt{x}\,\mathrm{e}^{-\frac{x}{2}}\left(\int \frac{\mathrm{e}^{\frac{x}{2}}}{x^{\frac{7}{2}}}dx\right)$$

✓ Solution by Mathematica
Time used: 0.23 (sec). Leaf size: 70

```
DSolve[2*x^2*y''[x]+x*(5+x)*y'[x]-(2-3*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{15}\left(-\frac{2c_2(x^2+x+3)}{x^2} + 15c_1e^{-x/2}\sqrt{x} + \sqrt{2}c_2e^{-x/2}\sqrt{-x}\Gamma\left(\frac{1}{2}, -\frac{x}{2}\right)\right)$$

1.89 problem 91

Internal problem ID [7579]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 91.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3x^2y'' + x(1+x)\,y' - y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(3*x^2*diff(y(x),x$2)+x*(1+x)*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{-\frac{x}{3}}}{x^{\frac{1}{3}}} + \frac{c_2 \mathrm{e}^{-\frac{x}{3}} \left(\int x^{\frac{1}{3}} \mathrm{e}^{\frac{x}{3}} dx\right)}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 50

```
DSolve[3*x^2*y''[x]+x*(1+x)*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x/3}\left(c_2 x^{2/3} - 3\sqrt[3]{3}c_1(-x)^{2/3}\Gamma\left(\frac{4}{3}, -\frac{x}{3}\right)\right)}{x}$$

1.90 problem 92

Internal problem ID [7580]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 92.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' - xy' + (1 - 2x)\,y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(2*x^2*diff(y(x),x$2)-x*diff(y(x),x)+(1-2*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\sinh\left(2\sqrt{x}\right) + c_2\sqrt{x}\,\cosh\left(2\sqrt{x}\right)$$

✓ Solution by Mathematica
Time used: 0.047 (sec). Leaf size: 41

```
DSolve[2*x^2*y''[x]-x*y'[x]+(1-2*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-2\sqrt{x}}\sqrt{x}\left(2c_1e^{4\sqrt{x}} - c_2\right)$$

1.91 problem 93

Internal problem ID [7581]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 93.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3x^2y'' + x(1+x)\,y' - (3x+1)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(3*x^2*diff(y(x),x$2)+x*(1+x)*diff(y(x),x)-(1+3*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x(x^2+20x+70) + c_2x(x^2+20x+70)\left(\int \frac{e^{-\frac{x}{3}}}{x^{\frac{7}{3}}\,(x^2+20x+70)^2}dx\right)$$

✓ Solution by Mathematica

Time used: 1.549 (sec). Leaf size: 78

```
DSolve[3*x^2*y''[x]+x*(1+x)*y'[x]-(1+3*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1x(x^2+20x+70) - \frac{c_2x(x^2+20x+70)\,\Gamma\left(\frac{2}{3},\frac{x}{3}\right)}{1680\sqrt[3]{3}} + \frac{c_2e^{-x/3}(x^3+19x^2+54x-18)}{1680\sqrt[3]{x}}$$

1.92 problem 94

Internal problem ID [7582]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 94.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(x+3)\,y''+x(1+5x)\,y'+(1+x)\,y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 36

```
dsolve(2*x^2*(3+x)*diff(y(x),x$2)+x*(1+5*x)*diff(y(x),x)+(1+x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x^{\frac{1}{3}}}{(x+3)^{\frac{4}{3}}}+\frac{c_2x^{\frac{1}{3}}\left(\int\frac{(x+3)^{\frac{1}{3}}}{x^{\frac{5}{6}}}dx\right)}{(x+3)^{\frac{4}{3}}}$$

✓ Solution by Mathematica

Time used: 20.076 (sec). Leaf size: 50

```
DSolve[2*x^2*(3+x)*y''[x]+x*(1+5*x)*y'[x]+(1+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\to\frac{\sqrt[3]{x}\left(6\sqrt[3]{3}c_2\sqrt[6]{x}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{6},\frac{7}{6},-\frac{x}{3}\right)+c_1\right)}{(x+3)^{4/3}}$$

1.93 problem 95

Internal problem ID [7583]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 95.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(4+x)\,y'' - x(-3x+1)\,y' + y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 36

```
dsolve(x^2*(4+x)*diff(y(x),x$2)-x*(1-3*x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^{\frac{1}{4}}}{(x+4)^{\frac{9}{4}}} + \frac{c_2 x^{\frac{1}{4}} \left(\int \frac{(x+4)^{\frac{5}{4}}}{x^{\frac{1}{4}}} dx\right)}{(x+4)^{\frac{9}{4}}}$$

✓ Solution by Mathematica

Time used: 0.399 (sec). Leaf size: 89

```
DSolve[x^2*(4+x)*y''[x]-x*(1-3*x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[4]{x}\left(-10c_2 \arctan\left(\sqrt[4]{\frac{x}{x+4}}\right) + 10c_2 \operatorname{arctanh}\left(\sqrt[4]{\frac{x}{x+4}}\right) + c_2\sqrt[4]{x+4}x^{7/4} + 9c_2\sqrt[4]{x+4}x^{3/4} + 2c_1\right)}{2(x+4)^{9/4}}$$

1.94 problem 96

Internal problem ID [7584]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 96.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' + 5xy' + (1 + x)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(2*x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+(1+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin\left(\sqrt{x}\,\sqrt{2}\right)}{x} + \frac{c_2 \cos\left(\sqrt{x}\,\sqrt{2}\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.091 (sec). Leaf size: 60

```
DSolve[2*x^2*y''[x]+5*x*y'[x]+(1+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{i\sqrt{2}\sqrt{x}} + i\sqrt{2}c_2 e^{-i\sqrt{2}\sqrt{x}}}{2x}$$

1.95 problem 97

Internal problem ID [7585]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 97.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{6x^2y'' + x(10-x)\,y' - (x+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 25

```
dsolve(6*x^2*diff(y(x),x$2)+x*(10-x)*diff(y(x),x)-(2+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + \frac{c_2\left(\int x^{\frac{1}{3}} \mathrm{e}^{\frac{x}{6}} dx\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 38

```
DSolve[6*x^2*y''[x]+x*(10-x)*y'[x]-(2+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2\sqrt[3]{x}L_{-\frac{4}{3}}^{\frac{4}{3}}\left(\frac{x}{6}\right) + \frac{6\sqrt[3]{6}c_1}{x}$$

1.96 problem 98

Internal problem ID [7586]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 98.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(3+4x)\,y''+x(11+4x)\,y'-(3+4x)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 60

```
dsolve(x^2*(3+4*x)*diff(y(x),x$2)+x*(11+4*x)*diff(y(x),x)-(3+4*x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(48x^2+32x+7)}{x^3}+\frac{c_2(48x^2+32x+7)\left(\int\frac{(4x+3)^{\frac{8}{3}}x^{\frac{7}{3}}}{(48x^2+32x+7)^2}dx\right)}{x^3}$$

✓ Solution by Mathematica

Time used: 1.197 (sec). Leaf size: 339

```
DSolve[x^2*(3+4*x)*y''[x]+x*(11+4*x)*y'[x]-(3+4*x)*y[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x)\to\frac{-12\sqrt[3]{2}\sqrt{3}c_2(48x^2+32x+7)\arctan\left(\frac{\sqrt{3}\sqrt[3]{x}}{\sqrt[3]{x}+\sqrt[3]{8x+6}}\right)+384c_2(4x+3)^{2/3}x^{10/3}+576c_2(4x+3)^{2/3}x^{7/3}}{}$$

1.97 problem 99

Internal problem ID [7587]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 99.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(3x+2)\,y''+x(4+11x)\,y'-(1-x)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(2*x^2*(2+3*x)*diff(y(x),x$2)+x*(4+11*x)*diff(y(x),x)-(1-x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1}{\sqrt{x}}+\frac{c_2(3x+2)^{\frac{1}{6}}}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 32

```
DSolve[2*x^2*(2+3*x)*y''[x]+x*(4+11*x)*y'[x]-(1-x)*y[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x)\to\frac{c_2\sqrt[6]{6x+4}+2^{5/6}c_1}{\sqrt{x}}$$

1.98 problem 100

Internal problem ID [7588]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 100.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x+2)\,y''+5x(1-x)\,y'-(-8x+2)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 88

```
dsolve(x^2*(2+x)*diff(y(x),x$2)+5*x*(1-x)*diff(y(x),x)-(2-8*x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(40x^4-160x^3+60x^2+8x+1)}{x^2}+\frac{c_2(40x^4-160x^3+60x^2+8x+1)\left(\int\frac{x^{\frac{3}{2}}(x+2)^{\frac{15}{2}}}{(40x^4-160x^3+60x^2+8x+1)^2}dx\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 48.622 (sec). Leaf size: 1347

```
DSolve[x^2*(2+x)*y''[x]+5*x*(1-x)*y'[x]-(2-8*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

Too large to display

1.99 problem 101

Internal problem ID [7589]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 101.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$8x^2(1-x^2)\,y'' + 2x(-13x^2+1)\,y' + (-9x^2+1)\,y = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 60

```
dsolve(8*x^2*(1-x^2)*diff(y(x),x$2)+2*x*(1-13*x^2)*diff(y(x),x)+(1-9*x^2)*y(x)=0,y(x), sings
```

$$y(x) = c_1\sqrt{\frac{1}{(x-1)(x+1)}}\,x^{\frac{1}{4}} + c_2\sqrt{\frac{1}{(x-1)(x+1)}}\,x^{\frac{1}{4}}\left(\int \frac{\sqrt{\frac{1}{(x-1)(x+1)}}}{x^{\frac{3}{4}}}dx\right)$$

✓ Solution by Mathematica
Time used: 20.097 (sec). Leaf size: 47

```
DSolve[8*x^2*(1-x^2)*y''[x]+2*x*(1-13*x^2)*y'[x]+(1-9*x^2)*y[x]==0,y[x],x,IncludeSingularSol
```

$$y(x) \to \frac{\sqrt[4]{x}\left(4c_2\sqrt[4]{x}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{1}{2},\frac{9}{8},x^2\right)+c_1\right)}{\sqrt{1-x^2}}$$

1.100 problem 102

Internal problem ID [7590]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 102.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+1)\,y''-2x(-x^2+2)\,y'+4y=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-2*x*(2-x^2)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x(3x^2+1)}{(x^2+1)^2}+\frac{c_2x^4}{(x^2+1)^2}$$

✓ Solution by Mathematica
Time used: 0.049 (sec). Leaf size: 35

```
DSolve[x^2*(1+x^2)*y''[x]-2*x*(2-x^2)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to -\frac{-3c_1x^4+3c_2x^3+c_2x}{3(x^2+1)^2}$$

1.101 problem 103

Internal problem ID [7591]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 103.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x(x^2+3)\,y''+(-x^2+2)\,y'-8yx=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x*(3+x^2)*diff(y(x),x$2)+(2-x^2)*diff(y(x),x)-8*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x^4+\frac{11}{2}x^2+\frac{55}{8}\right)+c_2(x^2+3)^{\frac{11}{6}}\,x^{\frac{1}{3}}$$

✓ Solution by Mathematica

Time used: 1.51 (sec). Leaf size: 41

```
DSolve[x*(3+x^2)*y''[x]+(2-x^2)*y'[x]-8*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\sqrt[3]{x}(x^2+3)^{11/6}-\frac{1}{55}c_2(8x^4+44x^2+55)$$

1.102 problem 104

Internal problem ID [7592]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 104.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(1-x^2)\,y''+x(-19x^2+7)\,y'-(14x^2+1)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 60

```
dsolve(4*x^2*(1-x^2)*diff(y(x),x$2)+x*(7-19*x^2)*diff(y(x),x)-(1+14*x^2)*y(x)=0,y(x), singso
```

$$y(x)=\frac{c_1\sqrt{\frac{1}{(x-1)(x+1)}}}{x}+\frac{c_2\sqrt{\frac{1}{(x-1)(x+1)}}\left(\int\sqrt{\frac{1}{(x-1)(x+1)}}\,x^{\frac{1}{4}}dx\right)}{x}$$

✓ Solution by Mathematica

Time used: 20.113 (sec). Leaf size: 50

```
DSolve[4*x^2*(1-x^2)*y''[x]+x*(7-19*x^2)*y'[x]-(1+14*x^2)*y[x]==0,y[x],x,IncludeSingularSolu
```

$$y(x)\to\frac{4c_2x^{5/4}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{8},\frac{13}{8},x^2\right)+5c_1}{5x\sqrt{1-x^2}}$$

1.103 problem 105

Internal problem ID [7593]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 105.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3x^2(-x^2+2)\,y''+x(-11x^2+1)\,y'+(-5x^2+1)\,y=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 42

```
dsolve(3*x^2*(2-x^2)*diff(y(x),x$2)+x*(1-11*x^2)*diff(y(x),x)+(1-5*x^2)*y(x)=0,y(x), singsol
```

$$y(x)=\frac{c_1\sqrt{x}}{(x^2-2)^{\frac{3}{4}}}+\frac{c_2\sqrt{x}\left(\int\frac{1}{(x^2-2)^{\frac{1}{4}}x^{\frac{7}{6}}}dx\right)}{(x^2-2)^{\frac{3}{4}}}$$

✓ Solution by Mathematica
Time used: 20.112 (sec). Leaf size: 57

```
DSolve[3*x^2*(2-x^2)*y''[x]+x*(1-11*x^2)*y'[x]+(1-5*x^2)*y[x]==0,y[x],x,IncludeSingularSolut
```

$$y(x)\to\frac{c_1\sqrt{x}-3\;2^{3/4}c_2\sqrt[3]{x}\,\text{Hypergeometric2F1}\left(-\frac{1}{12},\frac{1}{4},\frac{11}{12},\frac{x^2}{2}\right)}{(2-x^2)^{3/4}}$$

1.104 problem 106

Internal problem ID [7594]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 106.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x^2+2)\,y'' - x(-7x^2+12)\,y' + (3x^2+7)\,y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve(2*x^2*(2+x^2)*diff(y(x),x$2)-x*(12-7*x^2)*diff(y(x),x)+(7+3*x^2)*y(x)=0,y(x), singsol
```

$$y(x) = \frac{c_1 x^{\frac{7}{2}}}{(x^2+2)^{\frac{9}{4}}} + \frac{c_2 x^{\frac{7}{2}} \left(\int \frac{(x^2+2)^{\frac{5}{4}}}{x^4} dx\right)}{(x^2+2)^{\frac{9}{4}}}$$

✓ Solution by Mathematica

Time used: 20.114 (sec). Leaf size: 57

```
DSolve[2*x^2*(2+x^2)*y''[x]-x*(12-7*x^2)*y'[x]+(7+3*x^2)*y[x]==0,y[x],x,IncludeSingularSolut
```

$$y(x) \to \frac{\sqrt{x}\left(3c_1x^3 - 2\sqrt[4]{2}c_2 \,\text{Hypergeometric2F1}\left(-\frac{3}{2}, -\frac{5}{4}, -\frac{1}{2}, -\frac{x^2}{2}\right)\right)}{3\,(x^2+2)^{9/4}}$$

1.105 problem 107

Internal problem ID [7595]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 107.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x^2+2)\,y''+x(7x^2+4)\,y'-(-3x^2+1)\,y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 38

```
dsolve(2*x^2*(2+x^2)*diff(y(x),x$2)+x*(4+7*x^2)*diff(y(x),x)-(1-3*x^2)*y(x)=0,y(x), singsol=
```

$$y(x)=\frac{c_1}{(x^2+2)^{\frac{1}{4}}\sqrt{x}}+\frac{c_2\left(\int\frac{1}{(x^2+2)^{\frac{3}{4}}}dx\right)}{(x^2+2)^{\frac{1}{4}}\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.074 (sec). Leaf size: 68

```
DSolve[2*x^2*(2+x^2)*y''[x]+x*(4+7*x^2)*y'[x]-(1-3*x^2)*y[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x)\to\frac{c_2\sqrt[8]{x^2+2}\,\text{Gamma}\left(\frac{3}{4}\right)Q_{-\frac{1}{4}}^{\frac{1}{4}}\left(\frac{ix}{\sqrt{2}}\right)+2^{3/8}c_1}{\sqrt{x}\sqrt[4]{x^2+2}\,\text{Gamma}\left(\frac{3}{4}\right)}$$

1.106 problem 108

Internal problem ID [7596]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 108.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(2x^2+1)\,y''+5x(6x^2+1)\,y'-(-40x^2+2)\,y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 48

```
dsolve(2*x^2*(1+2*x^2)*diff(y(x),x$2)+5*x*(1+6*x^2)*diff(y(x),x)-(2-40*x^2)*y(x)=0,y(x), sin
```

$$y(x)=\frac{c_1\sqrt{x}}{(2x^2+1)^{\frac{3}{2}}}+\frac{c_2\sqrt{x}\left(\int\frac{\sqrt{2x^2+1}}{x^{\frac{7}{2}}}dx\right)}{(2x^2+1)^{\frac{3}{2}}}$$

✓ Solution by Mathematica
Time used: 20.118 (sec). Leaf size: 52

```
DSolve[2*x^2*(1+2*x^2)*y''[x]+5*x*(1+6*x^2)*y'[x]-(2-40*x^2)*y[x]==0,y[x],x,IncludeSingularS
```

$$y(x)\to\frac{5c_1x^{5/2}-2c_2\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},-2x^2\right)}{5x^2\left(2x^2+1\right)^{3/2}}$$

1.107 problem 109

Internal problem ID [7597]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 109.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x(x^2+1)\,y''+(7x^2+4)\,y'+8yx=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
dsolve(x*(1+x^2)*diff(y(x),x$2)+(4+7*x^2)*diff(y(x),x)+8*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1}{\sqrt{x^2+1}\,x^3}+\frac{c_2\left(\frac{x\sqrt{x^2+1}}{2}-\frac{\operatorname{arcsinh}(x)}{2}\right)}{\sqrt{x^2+1}\,x^3}$$

✓ Solution by Mathematica

Time used: 0.123 (sec). Leaf size: 56

```
DSolve[x*(1+x^2)*y''[x]+(4+7*x^2)*y'[x]+8*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_2x\sqrt{x^2+1}+c_2\log\left(\sqrt{x^2+1}-x\right)+2c_1}{2x^3\sqrt{x^2+1}}$$

1.108 problem 110

Internal problem ID [7598]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 110.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x^2+1)\,y''+x(8x^2+3)\,y'-(-4x^2+3)\,y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 42

```
dsolve(2*x^2*(1+x^2)*diff(y(x),x$2)+x*(3+8*x^2)*diff(y(x),x)-(3-4*x^2)*y(x)=0,y(x), singsol=
```

$$y(x)=\frac{c_1}{(x^2+1)^{\frac{1}{4}}x^{\frac{3}{2}}}+\frac{c_2\left(\int\frac{x^{\frac{3}{2}}}{(x^2+1)^{\frac{3}{4}}}dx\right)}{(x^2+1)^{\frac{1}{4}}x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 20.041 (sec). Leaf size: 60

```
DSolve[2*x^2*(1+x^2)*y''[x]+x*(3+8*x^2)*y'[x]-(3-4*x^2)*y[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x)\to-\frac{c_2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-x^2\right)}{x\sqrt[4]{x^2+1}}+\frac{c_1}{x^{3/2}\sqrt[4]{x^2+1}}+\frac{c_2}{x}$$

1.109 problem 111

Internal problem ID [7599]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 111.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2y'' + 3x(x^2+3)\,y' - (-5x^2+1)\,y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 39

```
dsolve(9*x^2*diff(y(x),x$2)+3*x*(3+x^2)*diff(y(x),x)-(1-5*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{-\frac{x^2}{6}}}{x^{\frac{1}{3}}} + \frac{c_2 e^{-\frac{x^2}{6}} \left(\int \frac{e^{\frac{x^2}{6}}}{x^{\frac{1}{3}}} dx\right)}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica
Time used: 0.225 (sec). Leaf size: 61

```
DSolve[9*x^2*y''[x]+3*x*(3+x^2)*y'[x]-(1-5*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{e^{-\frac{x^2}{6}} \left(2c_1 x^{4/3} + \sqrt[3]{6} c_2 (-x^2)^{2/3}\,\Gamma\left(\frac{1}{3}, -\frac{x^2}{6}\right)\right)}{2x^{5/3}}$$

1.110 problem 112

Internal problem ID [7600]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 112.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$6x^2y'' + x(6x^2+1)\,y' + (9x^2+1)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(6*x^2*diff(y(x),x$2)+x*(1+6*x^2)*diff(y(x),x)+(1+9*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{\frac{1}{3}} e^{-\frac{x^2}{2}} + c_2 x^{\frac{1}{3}} e^{-\frac{x^2}{2}} \left(\int \frac{e^{\frac{x^2}{2}}}{x^{\frac{5}{6}}} dx \right)$$

✓ Solution by Mathematica

Time used: 0.351 (sec). Leaf size: 61

```
DSolve[6*x^2*y''[x]+x*(1+6*x^2)*y'[x]+(1+9*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{e^{-\frac{x^2}{2}} \left(2c_1 x^{11/6} + \sqrt[12]{2} c_2 (-x^2)^{11/12} \Gamma\left(\frac{1}{12}, -\frac{x^2}{2}\right)\right)}{2x^{3/2}}$$

1.111 problem 113

Internal problem ID [7601]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 113.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2(x^2+1)\,y'' + 3x(13x^2+3)\,y' - (-25x^2+1)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(9*x^2*(1+x^2)*diff(y(x),x$2)+3*x*(3+13*x^2)*diff(y(x),x)-(1-25*x^2)*y(x)=0,y(x), sing
```

$$y(x) = \frac{c_1}{(x^2+1)^{\frac{2}{3}}\, x^{\frac{1}{3}}} + \frac{c_2\left(\int \frac{1}{(x^3+x)^{\frac{1}{3}}}dx\right)}{(x^2+1)^{\frac{2}{3}}\, x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.878 (sec). Leaf size: 124

```
DSolve[9*x^2*(1+x^2)*y''[x]+3*x*(3+13*x^2)*y'[x]-(1-25*x^2)*y[x]==0,y[x],x,IncludeSingularSo
```

$y(x)$

$$\to \frac{2\sqrt{3}c_2 \arctan\left(\frac{\sqrt{3}x^{2/3}}{x^{2/3}+2\sqrt[3]{x^2+1}}\right) - 2c_2\log\left(\sqrt[3]{x^2+1}-x^{2/3}\right) + c_2\log\left(x^{4/3}+(x^2+1)^{2/3}+\sqrt[3]{x^2+1}x^{2/3}\right.}{4\sqrt[3]{x}\,(x^2+1)^{2/3}}$$

1.112 problem 114

Internal problem ID [7602]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 114.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x^2+1)\,y''+4x(6x^2+1)\,y'-(-25x^2+1)\,y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
dsolve(4*x^2*(1+x^2)*diff(y(x),x$2)+4*x*(1+6*x^2)*diff(y(x),x)-(1-25*x^2)*y(x)=0,y(x), sings
```

$$y(x)=\frac{c_1\sqrt{x}}{(x^2+1)^{\frac{3}{2}}}+\frac{c_2\left(\operatorname{arcsinh}(x)\,x-\sqrt{x^2+1}\right)}{\sqrt{x}\,(x^2+1)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.12 (sec). Leaf size: 57

```
DSolve[4*x^2*(1+x^2)*y''[x]+4*x*(1+6*x^2)*y'[x]-(1-25*x^2)*y[x]==0,y[x],x,IncludeSingularSol
```

$$y(x)\to\frac{-c_2\sqrt{x^2+1}-c_2x\log\left(\sqrt{x^2+1}-x\right)+c_1x}{\sqrt{x}\,(x^2+1)^{3/2}}$$

1.113 problem 115

Internal problem ID [7603]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 115.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$8x^2\left(2x^2+1\right)y''+2x\left(34x^2+5\right)y'-\left(-30x^2+1\right)y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(8*x^2*(1+2*x^2)*diff(y(x),x$2)+2*x*(5+34*x^2)*diff(y(x),x)-(1-30*x^2)*y(x)=0,y(x), si
```

$$y(x)=\frac{c_1x^{\frac{1}{4}}}{\sqrt{2x^2+1}}+\frac{c_2x^{\frac{1}{4}}\left(\int\frac{1}{\sqrt{2x^2+1}\,x^{\frac{7}{4}}}dx\right)}{\sqrt{2x^2+1}}$$

✓ Solution by Mathematica

Time used: 20.109 (sec). Leaf size: 54

```
DSolve[8*x^2*(1+2*x^2)*y''[x]+2*x*(5+34*x^2)*y'[x]-(1-30*x^2)*y[x]==0,y[x],x,IncludeSingular
```

$$y(x)\to\frac{3c_1x^{3/4}-4c_2\,\text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{1}{2},\frac{5}{8},-2x^2\right)}{3\sqrt{x}\sqrt{2x^2+1}}$$

1.114 problem 116

Internal problem ID [7604]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 116.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(1+x)\,y'' - x(-3x+1)\,y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(2*x^2*(1+x)*diff(y(x),x$2)-x*(1-3*x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x}{x+1} + \frac{c_2\sqrt{x}}{x+1}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 25

```
DSolve[2*x^2*(1+x)*y''[x]-x*(1-3*x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1\sqrt{x} + 2c_2 x}{x+1}$$

1.115 problem 117

Internal problem ID [7605]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 117.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$6x^2(2x^2+1)\,y''+x(50x^2+1)\,y'+(30x^2+1)\,y=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(6*x^2*(1+2*x^2)*diff(y(x),x$2)+x*(1+50*x^2)*diff(y(x),x)+(1+30*x^2)*y(x)=0,y(x), sing
```

$$y(x)=\frac{c_1\sqrt{x}}{2x^2+1}+\frac{c_2x^{\frac{1}{3}}}{2x^2+1}$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 32

```
DSolve[6*x^2*(1+2*x^2)*y''[x]+x*(1+50*x^2)*y'[x]+(1+30*x^2)*y[x]==0,y[x],x,IncludeSingularSo
```

$$y(x)\to\frac{\sqrt[3]{x}\left(6c_2\sqrt[6]{x}+c_1\right)}{2x^2+1}$$

1.116 problem 118

Internal problem ID [7606]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 118.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{28x^2(-3x+1)\, y'' - 7x(5+9x)\, y' + 7(2+9x)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(28*x^2*(1-3*x)*diff(y(x),x$2)-7*x*(5+9*x)*diff(y(x),x)+7*(2+9*x)*y(x)=0,y(x), singsol
```

$$y(x) = \frac{c_1 x^2}{3x-1} + \frac{c_2 x^{\frac{1}{4}}}{3x-1}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 30

```
DSolve[28*x^2*(1-3*x)*y''[x]-7*x*(5+9*x)*y'[x]+7*(2+9*x)*y[x]==0,y[x],x,IncludeSingularSolut
```

$$y(x) \to \frac{4c_2 x^2 + 7c_1 \sqrt[4]{x}}{7-21x}$$

1.117 problem 119

Internal problem ID [7607]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 119.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$8x^2\left(-x^2+2\right)y''+2x\left(-21x^2+10\right)y'-\left(35x^2+2\right)y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(8*x^2*(2-x^2)*diff(y(x),x$2)+2*x*(10-21*x^2)*diff(y(x),x)-(2+35*x^2)*y(x)=0,y(x), sin
```

$$y(x)=\frac{c_1}{(x^2-2)\sqrt{x}}+\frac{c_2x^{\frac{1}{4}}}{x^2-2}$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 34

```
DSolve[8*x^2*(2-x^2)*y''[x]+2*x*(10-21*x^2)*y'[x]-(2+35*x^2)*y[x]==0,y[x],x,IncludeSingularS
```

$$y(x)\to\frac{\frac{3c_1}{\sqrt{x}}+4c_2\sqrt[4]{x}}{6-3x^2}$$

1.118 problem 120

Internal problem ID [7608]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 120.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x^2+3x+1)\,y''-4x(-3x^2-3x+1)\,y'+3(x^2-x+1)\,y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(4*x^2*(1+3*x+x^2)*diff(y(x),x$2)-4*x*(1-3*x-3*x^2)*diff(y(x),x)+3*(1-x+x^2)*y(x)=0,y(
```

$$y(x)=\frac{c_1\sqrt{x}}{x^2+3x+1}+\frac{c_2x^{\frac{3}{2}}}{x^2+3x+1}$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 28

```
DSolve[4*x^2*(1+3*x+x^2)*y''[x]-4*x*(1-3*x-3*x^2)*y'[x]+3*(1-x+x^2)*y[x]==0,y[x],x,IncludeSi
```

$$y(x)\to\frac{\sqrt{x}(c_2x+c_1)}{x^2+3x+1}$$

1.119 problem 121

Internal problem ID [7609]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 121.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3x^2(1+x)^2 y'' - x(-11x^2 - 10x + 1) y' + (5x^2 + 1) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(3*x^2*(1+x)^2*diff(y(x),x$2)-x*(1-10*x-11*x^2)*diff(y(x),x)+(1+5*x^2)*y(x)=0,y(x), si
```

$$y(x) = \frac{c_1 x}{(x+1)^2} + \frac{c_2 x^{\frac{1}{3}}}{(x+1)^2}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 29

```
DSolve[3*x^2*(1+x)^2*y''[x]-x*(1-10*x-11*x^2)*y'[x]+(1+5*x^2)*y[x]==0,y[x],x,IncludeSingular
```

$$y(x) \to \frac{2c_1 \sqrt[3]{x} + 3c_2 x}{2(x+1)^2}$$

1.120 problem 122

Internal problem ID [7610]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 122.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x^2+2x+3)\,y''-x(-15x^2-14x+3)\,y'+(7x^2+3)\,y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
dsolve(4*x^2*(3+2*x+x^2)*diff(y(x),x$2)-x*(3-14*x-15*x^2)*diff(y(x),x)+(3+7*x^2)*y(x)=0,y(x)
```

$$y(x)=\frac{c_1x}{x^2+2x+3}+\frac{c_2x^{\frac{1}{4}}}{x^2+2x+3}$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 33

```
DSolve[4*x^2*(3+2*x+x^2)*y''[x]-x*(3-14*x-15*x^2)*y'[x]+(3+7*x^2)*y[x]==0,y[x],x,IncludeSing
```

$$y(x)\to\frac{3c_1\sqrt[4]{x}+4c_2x}{3x^2+6x+9}$$

1.121 problem 123

Internal problem ID [7611]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 123.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2-2x+1)\,y''-x(x+3)\,y'+(4+x)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 52

```
dsolve(x^2*(1-2*x+x^2)*diff(y(x),x$2)-x*(3+x)*diff(y(x),x)+(4+x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x^2\mathrm{e}^{-\frac{4}{x-1}}}{x-1}+\frac{c_2x^2\,\mathrm{expIntegral}_1\left(-\frac{4x}{x-1}\right)\mathrm{e}^{-\frac{4x}{x-1}}}{x-1}$$

✓ Solution by Mathematica

Time used: 0.236 (sec). Leaf size: 54

```
DSolve[x^2*(1-2*x+x^2)*y''[x]-x*(3+x)*y'[x]+(4+x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{e^{-\frac{4x}{x-1}}\sqrt{1-x}\,x^2\left(c_2\,\mathrm{ExpIntegralEi}\left(\frac{4x}{x-1}\right)+e^4c_1\right)}{(x-1)^{3/2}}$$

1.122 problem 124

Internal problem ID [7612]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 124.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x+2)\,y''+5x^2y'+(1+x)\,y=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 53

```
dsolve(2*x^2*(2+x)*diff(y(x),x$2)+5*x^2*diff(y(x),x)+(1+x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\sqrt{x}}{(x+2)^{\frac{3}{2}}}-\frac{c_2\sqrt{2}\left(-2\sqrt{2}\,\sqrt{x+2}+4\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,\sqrt{x+2}}{2}\right)\right)\sqrt{x}}{2\,(x+2)^{\frac{3}{2}}}$$

✓ Solution by Mathematica
Time used: 0.113 (sec). Leaf size: 55

```
DSolve[2*x^2*(2+x)*y''[x]+5*x^2*y'[x]+(1+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{\sqrt{x}\left(-2\sqrt{2}c_2\mathrm{arctanh}\left(\frac{\sqrt{x+2}}{\sqrt{2}}\right)+2c_2\sqrt{x+2}+c_1\right)}{(x+2)^{3/2}}$$

1.123 problem 125

Internal problem ID [7613]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 125.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(-x^2+2)\,y''-2x(2x^2+1)\,y'+(-2x^2+2)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
dsolve(x^2*(2-x^2)*diff(y(x),x$2)-2*x*(1+2*x^2)*diff(y(x),x)+(2-2*x^2)*y(x)=0,y(x), singsol=
```

$$y(x)=\frac{c_1 x}{(x^2-2)^{\frac{3}{2}}}+\frac{c_2\sqrt{2}\,x\left(2\arctan\left(\frac{\sqrt{2}}{\sqrt{x^2-2}}\right)+\sqrt{2}\,\sqrt{x^2-2}\right)}{2\,(x^2-2)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.152 (sec). Leaf size: 58

```
DSolve[x^2*(2-x^2)*y''[x]-2*x*(1+2*x^2)*y'[x]+(2-2*x^2)*y[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x)\to\frac{x\left(-\sqrt{2}c_2\text{arctanh}\left(\sqrt{1-\frac{x^2}{2}}\right)+c_2\sqrt{2-x^2}+c_1\right)}{(2-x^2)^{3/2}}$$

1.124 problem 126

Internal problem ID [7614]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 126.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - x(5-x)\,y' + (9-4x)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(x^2*diff(y(x),x$2)-x*(5-x)*diff(y(x),x)+(9-4*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^3(x+1) + c_2x^3\left(\text{expIntegral}_1\,(x)\,x + \text{expIntegral}_1\,(x) - \mathrm{e}^{-x}\right)$$

✓ Solution by Mathematica

Time used: 0.229 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]-x*(5-x)*y'[x]+(9-4*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}x^3(c_2e^x(x+1)\,\text{ExpIntegralEi}(-x) + c_1e^x(x+1) + c_2)$$

1.125 problem 127

Internal problem ID [7615]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 127.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x^2+x+1)\,y''+12x^2(1+x)\,y'+(3x^2+3x+1)\,y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 143

```
dsolve(4*x^2*(1+x+x^2)*diff(y(x),x$2)+12*x^2*(1+x)*diff(y(x),x)+(1+3*x+3*x^2)*y(x)=0,y(x), s
```

$$y(x)=c_1\sqrt{\frac{x}{x^2+x+1}}\left(\frac{i\sqrt{3}+2x+1}{i\sqrt{3}-2x-1}\right)^{-\frac{i\sqrt{3}}{2}} + c_2\sqrt{\frac{x}{x^2+x+1}}\left(\frac{i\sqrt{3}+2x+1}{i\sqrt{3}-2x-1}\right)^{-\frac{i\sqrt{3}}{2}}\left(\int\frac{\left(\frac{i\sqrt{3}-2x-1}{i\sqrt{3}+2x+1}\right)^{-\frac{i\sqrt{3}}{2}}}{x\sqrt{x^2+x+1}}dx\right)$$

✓ Solution by Mathematica

Time used: 1.028 (sec). Leaf size: 93

```
DSolve[4*x^2*(1+x+x^2)*y''[x]+12*x^2*(1+x)*y'[x]+(1+3*x+3*x^2)*y[x]==0,y[x],x,IncludeSingula
```

$$y(x)\to\frac{\sqrt{x}e^{-\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}\left(c_2\int_1^x\frac{e^{\sqrt{3}\arctan\left(\frac{2K[1]+1}{\sqrt{3}}\right)}}{K[1]\sqrt{K[1]^2+K[1]+1}}dK[1]+c_1\right)}{\sqrt{x^2+x+1}}$$

1.126 problem 128

Internal problem ID [7616]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 128.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x^2+x+1)\,y''-x(-2x^2-4x+1)\,y'+y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 137

```
dsolve(x^2*(1+x+x^2)*diff(y(x),x$2)-x*(1-4*x-2*x^2)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1 x\left(\frac{i\sqrt{3}+2x+1}{i\sqrt{3}-2x-1}\right)^{-\frac{7i\sqrt{3}}{6}}}{\sqrt{x^2+x+1}}+\frac{c_2 x\left(\frac{i\sqrt{3}+2x+1}{i\sqrt{3}-2x-1}\right)^{-\frac{7i\sqrt{3}}{6}}\left(\int\frac{\left(\frac{i\sqrt{3}-2x-1}{i\sqrt{3}+2x+1}\right)^{-\frac{7i\sqrt{3}}{6}}}{x\sqrt{x^2+x+1}}dx\right)}{\sqrt{x^2+x+1}}$$

✓ Solution by Mathematica

Time used: 1.035 (sec). Leaf size: 90

```
DSolve[x^2*(1+x+x^2)*y''[x]-x*(1-4*x-2*x^2)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{xe^{-\frac{7\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}}\left(c_2\int_1^x\frac{e^{\frac{7\arctan\left(\frac{2K[1]+1}{\sqrt{3}}\right)}{\sqrt{3}}}}{K[1]\sqrt{K[1]^2+K[1]+1}}dK[1]+c_1\right)}{\sqrt{x^2+x+1}}$$

1.127 problem 129

Internal problem ID [7617]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 129.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2y'' + 3x(-2x^2 + 3x + 5)\,y' + (-14x^2 + 12x + 1)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve(9*x^2*diff(y(x),x$2)+3*x*(5+3*x-2*x^2)*diff(y(x),x)+(1+12*x-14*x^2)*y(x)=0,y(x), sing
```

$$y(x) = \frac{c_1 e^{\frac{1}{3}x^2 - x}}{x^{\frac{1}{3}}} + \frac{c_2 e^{\frac{1}{3}x^2 - x}\left(\int \frac{e^{-\frac{1}{3}x^2 + x}}{x}dx\right)}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.552 (sec). Leaf size: 52

```
DSolve[9*x^2*y''[x]+3*x*(5+3*x-2*x^2)*y'[x]+(1+12*x-14*x^2)*y[x]==0,y[x],x,IncludeSingularSo
```

$$y(x) \to \frac{e^{\frac{1}{3}(x-3)x}\left(c_2 \int_1^x \frac{e^{K[1]-\frac{K[1]^2}{3}}}{K[1]}dK[1] + c_1\right)}{\sqrt[3]{x}}$$

1.128 problem 130

Internal problem ID [7618]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 130.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(2x+1)\,y''+x\left(3x^2+14x+5\right)y'+\left(12x^2+18x+4\right)y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve(x^2*(1+2*x)*diff(y(x),x$2)+x*(5+14*x+3*x^2)*diff(y(x),x)+(4+18*x+12*x^2)*y(x)=0,y(x),
```

$$y(x)=\frac{c_1\mathrm{e}^{-\frac{3x}{2}}}{(2x+1)^{\frac{1}{4}}\,x^2}+\frac{c_2\mathrm{e}^{-\frac{3x}{2}}\left(\int\frac{\mathrm{e}^{\frac{3x}{2}}}{(2x+1)^{\frac{3}{4}}x}dx\right)}{(2x+1)^{\frac{1}{4}}\,x^2}$$

✓ Solution by Mathematica

Time used: 14.745 (sec). Leaf size: 61

```
DSolve[x^2*(1+2*x)*y''[x]+x*(5+14*x+3*x^2)*y'[x]+(4+18*x+12*x^2)*y[x]==0,y[x],x,IncludeSingu
```

$$y(x)\to\frac{e^{-3x/2}\left(c_2\int_1^x\frac{e^{\frac{3K[1]}{2}}}{K[1](2K[1]+1)^{3/4}}dK[1]+c_1\right)}{x^2\sqrt[4]{2x+1}}$$

1.129 problem 131

Internal problem ID [7619]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 131.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$16x^2y'' + 4x(2x^2 + x + 6)\, y' + (18x^2 + 5x + 1)\, y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 51

```
dsolve(16*x^2*diff(y(x),x$2)+4*x*(6+x+2*x^2)*diff(y(x),x)+(1+5*x+18*x^2)*y(x)=0,y(x), singso
```

$$y(x) = \frac{c_1 \mathrm{e}^{-\frac{1}{4}x^2-\frac{1}{4}x}}{x^{\frac{1}{4}}} + \frac{c_2 \mathrm{e}^{-\frac{1}{4}x^2-\frac{1}{4}x} \left(\int \frac{\mathrm{e}^{\frac{1}{4}x^2+\frac{1}{4}x}}{x} dx\right)}{x^{\frac{1}{4}}}$$

✓ Solution by Mathematica

Time used: 0.404 (sec). Leaf size: 51

```
DSolve[16*x^2*y''[x]+4*x*(6+x+2*x^2)*y'[x]+(1+5*x+18*x^2)*y[x]==0,y[x],x,IncludeSingularSolu
```

$$y(x) \to \frac{e^{-\frac{1}{4}x(x+1)} \left(c_2 \int_1^x \frac{e^{\frac{1}{4}K[1](K[1]+1)}}{K[1]} dK[1] + c_1\right)}{\sqrt[4]{x}}$$

1.130 problem 132

Internal problem ID [7620]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 132.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2(1+x)\,y''+3x\left(-x^2+11x+5\right)y'+\left(-7x^2+16x+1\right)y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(9*x^2*(1+x)*diff(y(x),x$2)+3*x*(5+11*x-x^2)*diff(y(x),x)+(1+16*x-7*x^2)*y(x)=0,y(x),
```

$$y(x)=\frac{c_1 \mathrm{e}^{\frac{x}{3}}}{(x+1)^{\frac{4}{3}}\,x^{\frac{1}{3}}}+\frac{c_2 \mathrm{e}^{\frac{x}{3}}\left(\int \frac{(x+1)^{\frac{1}{3}}\,\mathrm{e}^{-\frac{x}{3}}}{x}dx\right)}{(x+1)^{\frac{4}{3}}\,x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 7.827 (sec). Leaf size: 50

```
DSolve[9*x^2*(1+x)*y''[x]+3*x*(5+11*x-x^2)*y'[x]+(1+16*x-7*x^2)*y[x]==0,y[x],x,IncludeSingul
```

$$y(x)\to \frac{e^{x/3}\left(c_1-\sqrt[3]{3e}c_2\Gamma\left(\frac{1}{3},\frac{x+1}{3}\right)\right)}{\sqrt[3]{x}(x+1)^{4/3}}$$

1.131 problem 133

Internal problem ID [7621]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 133.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$36x^2(1-2x)\,y''+24x(1-9x)\,y'+(1-70x)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(36*x^2*(1-2*x)*diff(y(x),x$2)+24*x*(1-9*x)*diff(y(x),x)+(1-70*x)*y(x)=0,y(x), singsol
```

$$y(x)=\frac{c_1x^{\frac{1}{6}}}{(-1+2x)^{\frac{4}{3}}}+\frac{c_2x^{\frac{1}{6}}\left(\int\frac{(-1+2x)^{\frac{1}{3}}}{x}dx\right)}{(-1+2x)^{\frac{4}{3}}}$$

✓ Solution by Mathematica

Time used: 0.195 (sec). Leaf size: 111

```
DSolve[36*x^2*(1-2*x)*y''[x]+24*x*(1-9*x)*y'[x]+(1-70*x)*y[x]==0,y[x],x,IncludeSingularSolut
```

$$y(x)\to\frac{\sqrt[6]{x}\left(-2\sqrt{3}c_2\arctan\left(\frac{2\sqrt[3]{1-2x}+1}{\sqrt{3}}\right)+6c_2\sqrt[3]{1-2x}+2c_2\log\left(\sqrt[3]{1-2x}-1\right)-c_2\log\left((1-2x)^{2/3}+\sqrt[3]{1\right.\right.}{2(1-2x)^{4/3}}$$

1.132 problem 134

Internal problem ID [7622]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 134.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1+x)\,y'' - x(3-x)\,y' + 4y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(x^2*(1+x)*diff(y(x),x$2)-x*(3-x)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^2(x-1)}{(x+1)^3} + \frac{c_2 x^2(x\ln(x) - \ln(x) - 4)}{(x+1)^3}$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 33

```
DSolve[x^2*(1+x)*y''[x]-x*(3-x)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2(c_1(x-1) + c_2(x-1)\log(x) - 4c_2)}{(x+1)^3}$$

1.133 problem 135

Internal problem ID [7623]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 135.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1-2x)\,y''-x(5-4x)\,y'+(9-4x)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(x^2*(1-2*x)*diff(y(x),x$2)-x*(5-4*x)*diff(y(x),x)+(9-4*x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x^3}{(-1+2x)^2}+\frac{c_2x^3(2x-\ln(x))}{(-1+2x)^2}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 29

```
DSolve[x^2*(1-2*x)*y''[x]-x*(5-4*x)*y'[x]+(9-4*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{x^3(-2c_2x+c_2\log(x)+c_1)}{(1-2x)^2}$$

1.134 problem 136

Internal problem ID [7624]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 136.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x+2)\, y'' + x^2 y' + (1-x)\, y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 71

```
dsolve(2*x^2*(2+x)*diff(y(x),x$2)+x^2*diff(y(x),x)+(1-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x^2+2x} + \frac{c_2\sqrt{2}\left(\operatorname{arctanh}\left(\frac{\sqrt{2}\,\sqrt{x+2}}{2}\right)x - \sqrt{2}\,\sqrt{x+2} + 2\operatorname{arctanh}\left(\frac{\sqrt{2}\,\sqrt{x+2}}{2}\right)\right)\sqrt{x\,(x+2)}}{2x+4}$$

✓ Solution by Mathematica
Time used: 0.151 (sec). Leaf size: 65

```
DSolve[2*x^2*(2+x)*y''[x]+x^2*y'[x]+(1-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{x}\left(2\left(c_1\sqrt{x+2}+c_2\right) - \sqrt{2}c_2\sqrt{x+2}\operatorname{arctanh}\left(\frac{\sqrt{x+2}}{\sqrt{2}}\right)\right)}{2\sqrt[4]{2}}$$

1.135 problem 137

Internal problem ID [7625]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 137.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(1+x)\,y'' - x(-x+6)\,y' + (8-x)\,y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 59

```
dsolve(2*x^2*(1+x)*diff(y(x),x$2)-x*(6-x)*diff(y(x),x)+(8-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^2}{(x+1)^{\frac{5}{2}}} + \frac{c_2 x^2 \left(\frac{2\sqrt{x+1}\,x}{3} + \frac{8\sqrt{x+1}}{3} + \ln\left(\sqrt{x+1}-1\right) - \ln\left(\sqrt{x+1}+1\right)\right)}{(x+1)^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.097 (sec). Leaf size: 50

```
DSolve[2*x^2*(1+x)*y''[x]-x*(6-x)*y'[x]+(8-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{x^2\left(-6c_2\operatorname{arctanh}\left(\sqrt{x+1}\right) + 2c_2\sqrt{x+1}(x+4) + 3c_1\right)}{3(x+1)^{5/2}}$$

1.136 problem 138

Internal problem ID [7626]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 138.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(2x+1)\,y''+x(5+9x)\,y'+(4+3x)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 130

```
dsolve(x^2*(1+2*x)*diff(y(x),x$2)+x*(5+9*x)*diff(y(x),x)+(4+3*x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(2x+1)^{\frac{3}{2}}}{x^2}+\frac{c_2\left(-12\ln\left(\sqrt{2x+1}+1\right)x^2+12\ln\left(\sqrt{2x+1}-1\right)x^2+12\sqrt{2x+1}\,x-12\ln\left(\sqrt{2x+1}+1\right)x+12\ln}{3x^2\sqrt{2x+1}}$$

✓ Solution by Mathematica

Time used: 0.136 (sec). Leaf size: 56

```
DSolve[x^2*(1+2*x)*y''[x]+x*(5+9*x)*y'[x]+(4+3*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{2c_2\left(-3(2x+1)^{3/2}\operatorname{arctanh}\left(\sqrt{2x+1}\right)+6x+4\right)+3c_1(2x+1)^{3/2}}{3x^2}$$

1.137 problem 139

Internal problem ID [7627]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 139.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-2x)\,y''-x(4x+5)\,y'+(9+4x)\,y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 63

```
dsolve(x^2*(1-2*x)*diff(y(x),x$2)-x*(5+4*x)*diff(y(x),x)+(9+4*x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x^3(8x+1)}{(-1+2x)^6}+\frac{c_2x^3\left(\frac{4x^4}{3}-\frac{16x^3}{3}-8x\ln(x)+12x^2-\ln(x)+\frac{203x}{128}-\frac{3125}{1024}\right)}{(-1+2x)^6}$$

✓ Solution by Mathematica

Time used: 0.091 (sec). Leaf size: 63

```
DSolve[x^2*(1-2*x)*y''[x]-x*(5+4*x)*y'[x]+(9+4*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to -\frac{x^3(c_2(4096x^4-16384x^3+36864x^2+4872x-9375)-48c_1(8x+1)-3072c_2(8x+1)\log(x))}{384(1-2x)^6}$$

1.138 problem 140

Internal problem ID [7628]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 140.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1-x)\,y''+x(7+x)\,y'+(9-x)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 75

```
dsolve(x^2*(1-x)*diff(y(x),x$2)+x*(7+x)*diff(y(x),x)+(9-x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(x^4+16x^3+36x^2+16x+1)}{x^3}+\frac{c_2\left(x^4\ln(x)+16x^3\ln(x)+36x^2\ln(x)+40x^3+16x\ln(x)+150x^2+\ln(x)+\frac{280x}{3}+\frac{25}{3}\right)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.095 (sec). Leaf size: 78

```
DSolve[x^2*(1-x)*y''[x]+x*(7+x)*y'[x]+(9-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{5c_2(24x^3+90x^2+56x+5)+3c_1(x^4+16x^3+36x^2+16x+1)+3c_2(x^4+16x^3+36x^2+16x+1)\log}{3x^3}$$

1.139 problem 141

Internal problem ID [7629]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 141.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(1-x^2)\,y' + (x^2+1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(x^2*diff(y(x),x$2)-x*(1-x^2)*diff(y(x),x)+(1+x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x\,\mathrm{e}^{-\frac{x^2}{2}} + c_2 x\,\mathrm{e}^{-\frac{x^2}{2}}\,\mathrm{expIntegral}_1\left(-\frac{x^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 35

```
DSolve[x^2*y''[x]-x*(1-x^2)*y'[x]+(1+x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-\frac{x^2}{2}}x\left(c_1\,\mathrm{ExpIntegralEi}\left(\frac{x^2}{2}\right) + 2c_2\right)$$

1.140 problem 142

Internal problem ID [7630]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 142.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x^2+1)\, y''-3x(1-x^2)\, y'+4y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 37

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-3*x*(1-x^2)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x^2}{(x^2+1)^2}+\frac{c_2x^2\left(\frac{x^2}{2}+\ln(x)\right)}{(x^2+1)^2}$$

✓ Solution by Mathematica
Time used: 0.05 (sec). Leaf size: 36

```
DSolve[x^2*(1+x^2)*y''[x]-3*x*(1-x^2)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to\frac{x^2(c_2x^2+2c_2\log(x)+2c_1)}{2(x^2+1)^2}$$

1.141 problem 143

Internal problem ID [7631]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 143.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + 2y'x^3 + (3x^2 + 1)\, y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve(4*x^2*diff(y(x),x$2)+2*x^3*diff(y(x),x)+(1+3*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\mathrm{e}^{-\frac{x^2}{4}} + c_2\sqrt{x}\,\mathrm{e}^{-\frac{x^2}{4}}\,\mathrm{expIntegral}_1\left(-\frac{x^2}{4}\right)$$

✓ Solution by Mathematica

Time used: 0.121 (sec). Leaf size: 39

```
DSolve[4*x^2*y''[x]+2*x^3*y'[x]+(1+3*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-\frac{x^2}{4}}\sqrt{x}\left(c_2\,\mathrm{ExpIntegralEi}\left(\frac{x^2}{4}\right) + 2c_1\right)$$

1.142 problem 144

Internal problem ID [7632]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 144.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+1)\,y'' - x(-2x^2+1)\,y' + y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(1-2*x^2)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x}{\sqrt{x^2+1}} + \frac{c_2 x \operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)}{\sqrt{x^2+1}}$$

✓ Solution by Mathematica

Time used: 0.085 (sec). Leaf size: 33

```
DSolve[x^2*(1+x^2)*y''[x]-x*(1-2*x^2)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x\left(c_1 - c_2 \operatorname{arctanh}\left(\sqrt{x^2+1}\right)\right)}{\sqrt{x^2+1}}$$

1.143 problem 145

Internal problem ID [7633]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 145.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x^2+2)\,y''+7y'x^3+(3x^2+1)\,y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve(2*x^2*(2+x^2)*diff(y(x),x$2)+7*x^3*diff(y(x),x)+(1+3*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\sqrt{x}}{(x^2+2)^{\frac{3}{4}}}+\frac{c_2\sqrt{x}\left(\int\frac{1}{(x^2+2)^{\frac{1}{4}}x}dx\right)}{(x^2+2)^{\frac{3}{4}}}$$

✓ Solution by Mathematica

Time used: 0.189 (sec). Leaf size: 77

```
DSolve[2*x^2*(2+x^2)*y''[x]+7*x^3*y'[x]+(1+3*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{\sqrt{x}\left(2^{3/4}c_2\arctan\left(\frac{\sqrt[4]{x^2+2}}{\sqrt[4]{2}}\right)-2^{3/4}c_2\text{arctanh}\left(\frac{\sqrt[4]{x^2+2}}{\sqrt[4]{2}}\right)+2c_1\right)}{2(x^2+2)^{3/4}}$$

1.144 problem 146

Internal problem ID [7634]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 146.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+1)\,y''-x(-4x^2+1)\,y'+(2x^2+1)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(1-4*x^2)*diff(y(x),x)+(1+2*x^2)*y(x)=0,y(x), singsol=al
```

$$y(x)=\frac{c_1x}{(x^2+1)^{\frac{3}{2}}}+\frac{c_2x\left(\sqrt{x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)\right)}{(x^2+1)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.099 (sec). Leaf size: 45

```
DSolve[x^2*(1+x^2)*y''[x]-x*(1-4*x^2)*y'[x]+(1+2*x^2)*y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x)\to\frac{x\left(-c_2\operatorname{arctanh}\left(\sqrt{x^2+1}\right)+c_2\sqrt{x^2+1}+c_1\right)}{(x^2+1)^{3/2}}$$

1.145 problem 147

Internal problem ID [7635]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 147.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x^2+4)\,y''+3x(3x^2+8)\,y'+(-9x^2+1)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(4*x^2*(4+x^2)*diff(y(x),x$2)+3*x*(8+3*x^2)*diff(y(x),x)+(1-9*x^2)*y(x)=0,y(x), singso
```

$$y(x)=\frac{c_1(x^2+4)^{\frac{5}{8}}}{x^{\frac{1}{4}}}+\frac{c_2(x^2+4)^{\frac{5}{8}}\left(\int\frac{1}{(x^2+4)^{\frac{13}{8}}x}dx\right)}{x^{\frac{1}{4}}}$$

✓ Solution by Mathematica

Time used: 0.526 (sec). Leaf size: 198

```
DSolve[4*x^2*(4+x^2)*y''[x]+3*x*(8+3*x^2)*y'[x]+(1-9*x^2)*y[x]==0,y[x],x,IncludeSingularSolu
```

$$y(x)\rightarrow\frac{c_2\left(5\,2^{3/4}(x^2+4)^{5/8}\arctan\left(\frac{\sqrt[8]{x^2+4}}{\sqrt[4]{2}}\right)+5\sqrt[4]{2}(x^2+4)^{5/8}\arctan\left(\frac{\sqrt{2}-\sqrt[4]{x^2+4}}{2^{3/4}\sqrt[8]{x^2+4}}\right)-5\,2^{3/4}(x^2+4)^{5/8}\,\text{a}}{80\sqrt[4]{x}}$$

1.146 problem 148

Internal problem ID [7636]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 148.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3x^2(x^2+3)\,y''+x(11x^2+3)\,y'+(5x^2+1)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(3*x^2*(3+x^2)*diff(y(x),x$2)+x*(3+11*x^2)*diff(y(x),x)+(1+5*x^2)*y(x)=0,y(x), singsol
```

$$y(x)=\frac{c_1x^{\frac{1}{3}}}{(x^2+3)^{\frac{2}{3}}}+\frac{c_2x^{\frac{1}{3}}\left(\int\frac{1}{(x^2+3)^{\frac{1}{3}}x}dx\right)}{(x^2+3)^{\frac{2}{3}}}$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 94

```
DSolve[3*x^2*(3+x^2)*y''[x]+x*(3+11*x^2)*y'[x]+(1+5*x^2)*y[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x)\to\frac{c_1\exp\left(\frac{1}{3}\text{RootSum}\left[3\#1^3+11\#1^2+9\#1+3\&,\frac{3\#1^2\log(x-\#1)-4\#1\log(x-\#1)+9\log(x-\#1)}{9\#1^2+22\#1+9}\&\right]\right)}{\sqrt[3]{x}}$$

$$y(x)\to 0$$

1.147 problem 149

Internal problem ID [7637]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 149.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2y'' - 3x(-2x^2+7)\,y' + (2x^2+25)\,y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve(9*x^2*diff(y(x),x$2)-3*x*(7-2*x^2)*diff(y(x),x)+(25+2*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{\frac{5}{3}} \mathrm{e}^{-\frac{x^2}{3}} + c_2 x^{\frac{5}{3}} \mathrm{e}^{-\frac{x^2}{3}} \operatorname{expIntegral}_1\left(-\frac{x^2}{3}\right)$$

✓ Solution by Mathematica

Time used: 0.112 (sec). Leaf size: 39

```
DSolve[9*x^2*y''[x]-3*x*(7-2*x^2)*y'[x]+(25+2*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{1}{2} e^{-\frac{x^2}{3}} x^{5/3} \left(c_2 \operatorname{ExpIntegralEi}\left(\frac{x^2}{3}\right) + 2c_1\right)$$

1.148 problem 150

Internal problem ID [7638]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 150.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(1-x^2)\, y' + (x^2+1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(x^2*diff(y(x),x$2)-x*(1-x^2)*diff(y(x),x)+(1+x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x\, \mathrm{e}^{-\frac{x^2}{2}} + c_2 x\, \mathrm{e}^{-\frac{x^2}{2}} \operatorname{expIntegral}_1\left(-\frac{x^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 35

```
DSolve[x^2*y''[x]-x*(1-x^2)*y'[x]+(1+x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-\frac{x^2}{2}} x \left(c_1 \operatorname{ExpIntegralEi}\left(\frac{x^2}{2}\right) + 2c_2\right)$$

1.149 problem 151

Internal problem ID [7639]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 151.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-2x)\,y''+3xy'+(1+4x)\,y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve(x^2*(1-2*x)*diff(y(x),x$2)+3*x*diff(y(x),x)+(1+4*x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1}{x}-\frac{c_2(-8x^3+18x^2+3\ln(x)-18x)}{3x}$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 36

```
DSolve[x^2*(1-2*x)*y''[x]+3*x*y'[x]+(1+4*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{2}{3}c_2\left(4x^2-9x+9\right)+\frac{c_1}{x}+\frac{c_2\log(x)}{x}$$

1.150 problem 152

Internal problem ID [7640]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 152.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(1+x)\,y''+(1-x)\,y'+y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x*(1+x)*diff(y(x),x$2)+(1-x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(x-1)+c_2(x\ln(x)-\ln(x)-4)$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 23

```
DSolve[x*(1+x)*y''[x]+(1-x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1(x-1)+c_2((x-1)\log(x)-4)$$

1.151 problem 153

Internal problem ID [7641]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 153.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1-x)\,y''-x(3-5x)\,y'+(4-5x)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 89

```
dsolve(x^2*(1-x)*diff(y(x),x$2)-x*(3-5*x)*diff(y(x),x)+(4-5*x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x^2\left(x^3-3x^2+3x-1\right)+c_2x^2\left(-\ln\left(x-1\right)x^3+x^3\ln\left(x\right)+3\ln\left(x-1\right)x^2-3x^2\ln\left(x\right)-3\ln\left(x-1\right)x+3x\ln\left(x\right)-x^2+\ln\left(x-1\right)-\ln\left(x\right)+\frac{5x}{2}-\frac{11}{6}\right)$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 76

```
DSolve[x^2*(1-x)*y''[x]-x*(3-5*x)*y'[x]+(4-5*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\to-\frac{1}{6}x^2\left(6c_1x^3-18c_1x^2-6c_2x^2+18c_1x+15c_2x-6c_2(x-1)^3\log(x-1)+6c_2(x-1)^3\log(x)-6c_1-11c_2\right)$$

1.152 problem 154

Internal problem ID [7642]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 154.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+1)\,y''-x(9x^2+1)\,y'+(25x^2+1)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(1+9*x^2)*diff(y(x),x)+(1+25*x^2)*y(x)=0,y(x), singsol=a
```

$$y(x)=c_1x(x^4-4x^2+1)+c_2(x^4\ln(x)-4x^2\ln(x)-6x^2+\ln(x)+3)\,x$$

✓ Solution by Mathematica

Time used: 0.085 (sec). Leaf size: 43

```
DSolve[x^2*(1+x^2)*y''[x]-x*(1+9*x^2)*y'[x]+(1+25*x^2)*y[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x)\to c_1(x^5-4x^3+x)+c_2x(-6x^2+(x^4-4x^2+1)\log(x)+3)$$

1.153 problem 155

Internal problem ID [7643]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 155.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2y'' + 3x(1 - x^2)\,y' + (7x^2 + 1)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(9*x^2*diff(y(x),x$2)+3*x*(1-x^2)*diff(y(x),x)+(1+7*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{\frac{1}{3}}(x^2 - 6) + c_2 x^{\frac{1}{3}}(x^2 - 6)\left(\int \frac{\mathrm{e}^{\frac{x^2}{6}}}{(x^2 - 6)^2\, x}dx\right)$$

✓ Solution by Mathematica

Time used: 3.367 (sec). Leaf size: 53

```
DSolve[9*x^2*y''[x]+3*x*(1-x^2)*y'[x]+(1+7*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{1}{72}\sqrt[3]{x}\left(c_2(x^2 - 6)\,\text{ExpIntegralEi}\left(\frac{x^2}{6}\right) + 72c_1(x^2 - 6) - 6c_2 e^{\frac{x^2}{6}}\right)$$

1.154 problem 156

Internal problem ID [7644]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 156.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x(x^2+1)\,y''+(1-x^2)\,y'-8yx=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 71

```
dsolve(x*(1+x^2)*diff(y(x),x$2)+(1-x^2)*diff(y(x),x)-8*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(x^4+2x^2+1)+c_2\left(-\frac{\ln(x^2+1)\,x^4}{2}+x^4\ln(x)-\ln(x^2+1)\,x^2+2x^2\ln(x)+\frac{x^2}{2}-\frac{\ln(x^2+1)}{2}+\ln(x)+\frac{3}{4}\right)$$

✓ Solution by Mathematica

Time used: 0.072 (sec). Leaf size: 55

```
DSolve[x*(1+x^2)*y''[x]+(1-x^2)*y'[x]-8*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1(x^2+1)^2+\frac{1}{4}c_2\left(2x^2+4(x^2+1)^2\log(x)-2(x^2+1)^2\log(x^2+1)+3\right)$$

1.155 problem 157

Internal problem ID [7645]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 157.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 2x(-x^2+4)\,y' + (7x^2+1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(4*x^2*diff(y(x),x$2)+2*x*(4-x^2)*diff(y(x),x)+(1+7*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^4-16x^2+32)}{\sqrt{x}} + \frac{c_2(x^4-16x^2+32)\left(\int \frac{e^{\frac{x^2}{4}}}{x(x^4-16x^2+32)^2}dx\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.442 (sec). Leaf size: 68

```
DSolve[4*x^2*y''[x]+2*x*(4-x^2)*y'[x]+(1+7*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{c_2(x^4-16x^2+32)\,\text{ExpIntegralEi}\left(\frac{x^2}{4}\right) - 4c_2e^{\frac{x^2}{4}}(x^2-12) + 2048c_1(x^4-16x^2+32)}{2048\sqrt{x}}$$

1.156 problem 158

Internal problem ID [7646]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 158.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(1+x)\,y''+8x^2y'+(1+x)\,y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(4*x^2*(1+x)*diff(y(x),x$2)+8*x^2*diff(y(x),x)+(1+x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\sqrt{x}}{x+1}+\frac{c_2\sqrt{x}\,\ln(x)}{x+1}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 24

```
DSolve[4*x^2*(1+x)*y''[x]+8*x^2*y'[x]+(1+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{\sqrt{x}(c_2\log(x)+c_1)}{x+1}$$

1.157 problem 159

Internal problem ID [7647]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 159.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2(x+3)\,y'' + 3x(3+7x)\,y' + (3+4x)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(9*x^2*(3+x)*diff(y(x),x$2)+3*x*(3+7*x)*diff(y(x),x)+(3+4*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^{\frac{1}{3}}}{x+3} + \frac{c_2 x^{\frac{1}{3}} \ln(x)}{x+3}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 24

```
DSolve[9*x^2*(3+x)*y''[x]+3*x*(3+7*x)*y'[x]+(3+4*x)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{\sqrt[3]{x}(c_2 \log(x) + c_1)}{x+3}$$

1.158 problem 160

Internal problem ID [7648]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 160.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(-x^2+2)\,y''-x(3x^2+2)\,y'+(-x^2+2)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x^2*(2-x^2)*diff(y(x),x$2)-x*(2+3*x^2)*diff(y(x),x)+(2-x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x}{x^2-2}+\frac{c_2x\ln(x)}{x^2-2}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 23

```
DSolve[x^2*(2-x^2)*y''[x]-x*(2+3*x^2)*y'[x]+(2-x^2)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to-\frac{x(c_2\log(x)+c_1)}{x^2-2}$$

1.159 problem 161

Internal problem ID [7649]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 161.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{16x^2(x^2+1)\,y''+8x(9x^2+1)\,y'+(49x^2+1)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(16*x^2*(1+x^2)*diff(y(x),x$2)+8*x*(1+9*x^2)*diff(y(x),x)+(1+49*x^2)*y(x)=0,y(x), sing
```

$$y(x)=\frac{c_1x^{\frac{1}{4}}}{x^2+1}+\frac{c_2x^{\frac{1}{4}}\ln(x)}{x^2+1}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 26

```
DSolve[16*x^2*(1+x^2)*y''[x]+8*x*(1+9*x^2)*y'[x]+(1+49*x^2)*y[x]==0,y[x],x,IncludeSingularSo
```

$$y(x)\to\frac{\sqrt[4]{x}(c_2\log(x)+c_1)}{x^2+1}$$

1.160 problem 162

Internal problem ID [7650]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 162.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(4+3x)\,y''-x(4-3x)\,y'+4y=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x^2*(4+3*x)*diff(y(x),x$2)-x*(4-3*x)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x}{3x+4}+\frac{c_2x\ln(x)}{3x+4}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 22

```
DSolve[x^2*(4+3*x)*y''[x]-x*(4-3*x)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x(c_2\log(x)+c_1)}{3x+4}$$

1.161 problem 163

Internal problem ID [7651]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 163.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2(x^2+3x+1)\,y''+8x^2(3+2x)\,y'+(9x^2+3x+1)\,y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 37

```
dsolve(4*x^2*(1+3*x+x^2)*diff(y(x),x$2)+8*x^2*(3+2*x)*diff(y(x),x)+(1+3*x+9*x^2)*y(x)=0,y(x)
```

$$y(x)=\frac{c_1\sqrt{x}}{x^2+3x+1}+\frac{c_2\sqrt{x}\,\ln(x)}{x^2+3x+1}$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 29

```
DSolve[4*x^2*(1+3*x+x^2)*y''[x]+8*x^2*(3+2*x)*y'[x]+(1+3*x+9*x^2)*y[x]==0,y[x],x,IncludeSing
```

$$y(x)\to\frac{\sqrt{x}(c_2\log(x)+c_1)}{x^2+3x+1}$$

1.162 problem 164

Internal problem ID [7652]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 164.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-x)^2 y'' - x(-3x^2+2x+1) y' + (x^2+1) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*(1-x)^2*diff(y(x),x$2)-x*(1+2*x-3*x^2)*diff(y(x),x)+(1+x^2)*y(x)=0,y(x), singsol=
```

$$y(x) = \frac{c_1 x}{(x-1)^2} + \frac{c_2 x \ln(x)}{(x-1)^2}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 20

```
DSolve[x^2*(1-x)^2*y''[x]-x*(1+2*x-3*x^2)*y'[x]+(1+x^2)*y[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{x(c_2 \log(x) + c_1)}{(x-1)^2}$$

1.163 problem 165

Internal problem ID [7653]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 165.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2(x^2+x+1)\,y''+3x(13x^2+7x+1)\,y'+(25x^2+4x+1)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(9*x^2*(1+x+x^2)*diff(y(x),x$2)+3*x*(1+7*x+13*x^2)*diff(y(x),x)+(1+4*x+25*x^2)*y(x)=0,
```

$$y(x)=\frac{c_1x^{\frac{1}{3}}}{x^2+x+1}+\frac{c_2x^{\frac{1}{3}}\ln(x)}{x^2+x+1}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 27

```
DSolve[9*x^2*(1+x+x^2)*y''[x]+3*x*(1+7*x+13*x^2)*y'[x]+(1+4*x+25*x^2)*y[x]==0,y[x],x,Include
```

$$y(x)\to\frac{\sqrt[3]{x}(c_2\log(x)+c_1)}{x^2+x+1}$$

1.164 problem 166

Internal problem ID [7654]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 166.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(x+2)\,y''-x(4-7x)\,y'-(5-3x)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 80

```
dsolve(2*x^2*(2+x)*diff(y(x),x$2)-x*(4-7*x)*diff(y(x),x)-(5-3*x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x^{\frac{5}{2}}}{(x+2)^{\frac{7}{2}}}-\frac{c_2\sqrt{2}\left(33\sqrt{2}\,\sqrt{x+2}\,x^2+15\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,\sqrt{x+2}}{2}\right)x^3+52\sqrt{2}\,\sqrt{x+2}\,x+32\sqrt{2}\,\sqrt{x+2}\right)}{48\sqrt{x}\,(x+2)^{\frac{7}{2}}}$$

✓ Solution by Mathematica

Time used: 0.331 (sec). Leaf size: 92

```
DSolve[2*x^2*(2+x)*y''[x]-x*(4-7*x)*y'[x]-(5-3*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to -\frac{15\sqrt{2}c_2x^3\mathrm{arctanh}\left(\frac{\sqrt{x+2}}{\sqrt{2}}\right)-48c_1x^3+66c_2\sqrt{x+2}x^2+104c_2\sqrt{x+2}x+64c_2\sqrt{x+2}}{48\sqrt{x}(x+2)^{7/2}}$$

1.165 problem 167

Internal problem ID [7655]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 167.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-2x)\,y''+x(8-9x)\,y'+(6-3x)\,y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 42

```
dsolve(x^2*(1-2*x)*diff(y(x),x$2)+x*(8-9*x)*diff(y(x),x)+(6-3*x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(231x^3-198x^2+66x-8)}{x^6}+\frac{c_2(3x+4)\,(-1+2x)^{\frac{9}{2}}}{x^6}$$

✓ Solution by Mathematica

Time used: 0.224 (sec). Leaf size: 49

```
DSolve[x^2*(1-2*x)*y''[x]+x*(8-9*x)*y'[x]+(6-3*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{c_2(231x^3-198x^2+66x-8)+385c_1(3x+4)(1-2x)^{9/2}}{1155x^6}$$

1.166 problem 168

Internal problem ID [7656]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 168.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+1)\,y''+x(10x^2+3)\,y'-(-14x^2+15)\,y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 89

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)+x*(3+10*x^2)*diff(y(x),x)-(15-14*x^2)*y(x)=0,y(x), singsol
```

$$y(x)=\frac{c_1x^3}{(x^2+1)^{\frac{5}{2}}}-\frac{c_2\left(-3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)x^8+3\sqrt{x^2+1}\,x^6-2x^4\sqrt{x^2+1}-24\sqrt{x^2+1}\,x^2-16\sqrt{x^2+1}\right)}{128x^5\,(x^2+1)^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.166 (sec). Leaf size: 75

```
DSolve[x^2*(1+x^2)*y''[x]+x*(3+10*x^2)*y'[x]-(15-14*x^2)*y[x]==0,y[x],x,IncludeSingularSolut
```

$$y(x)\to\frac{c_2\left(\sqrt{x^2+1}(3x^6-2x^4-24x^2-16)-3x^8\mathrm{arctanh}\left(\sqrt{x^2+1}\right)\right)+128c_1x^8}{128x^5\,(x^2+1)^{5/2}}$$

1.167 problem 169

Internal problem ID [7657]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 169.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(-2x^2+1)\,y''+x(-13x^2+7)\,y'-14x^2y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
dsolve(x^2*(1-2*x^2)*diff(y(x),x$2)+x*(7-13*x^2)*diff(y(x),x)-14*x^2*y(x)=0,y(x), singsol=al
```

$$y(x)=\frac{c_1(5x^4-20x^2+8)}{x^6}+\frac{c_2(2x^2-1)^{\frac{5}{4}}}{x^6}$$

✓ Solution by Mathematica

Time used: 0.126 (sec). Leaf size: 43

```
DSolve[x^2*(1-2*x^2)*y''[x]+x*(7-13*x^2)*y'[x]-14*x^2*y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x)\to\frac{15c_1(1-2x^2)^{5/4}+c_2(-5x^4+20x^2-8)}{15x^6}$$

1.168 problem 170

Internal problem ID [7658]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 170.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(1+x)\,y'' + 4x(2x+1)\,y' - (3x+1)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(4*x^2*(1+x)*diff(y(x),x$2)+4*x*(1+2*x)*diff(y(x),x)-(1+3*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x} + \frac{c_2(\ln (x+1)\, x - x\ln (x) - 1)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 32

```
DSolve[4*x^2*(1+x)*y''[x]+4*x*(1+2*x)*y'[x]-(1+3*x)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{c_1 x + c_2(-x\log(x) + x\log(x+1) - 1)}{\sqrt{x}}$$

1.169 problem 171

Internal problem ID [7659]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 171.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(3x+2)\,y''+x(4+21x)\,y'-(1-9x)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 61

```
dsolve(2*x^2*(2+3*x)*diff(y(x),x$2)+x*(4+21*x)*diff(y(x),x)-(1-9*x)*y(x)=0,y(x), singsol=all
```

$$y(x)=\frac{c_1\sqrt{x}}{(3x+2)^{\frac{3}{2}}}+\frac{c_2\sqrt{2}\left(\sqrt{2}\,\sqrt{3x+2}+3\ \operatorname{arctanh}\left(\frac{\sqrt{2}\,\sqrt{3x+2}}{2}\right)x\right)}{2\sqrt{x}\ (3x+2)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.191 (sec). Leaf size: 64

```
DSolve[2*x^2*(2+3*x)*y''[x]+x*(4+21*x)*y'[x]-(1-9*x)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to -\frac{3\sqrt{2}c_2x\text{arctanh}\left(\sqrt{\frac{3x}{2}+1}\right)-2c_1x+2c_2\sqrt{3x+2}}{2\sqrt{x}(3x+2)^{3/2}}$$

1.170 problem 172

Internal problem ID [7660]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 172.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(x+2)\,y' - (2-3x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 46

```
dsolve(x^2*diff(y(x),x$2)+x*(2+x)*diff(y(x),x)-(2-3*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x} x + \frac{c_2 \mathrm{e}^{-x} (\mathrm{expIntegral}_1\,(-x)\, x^3 + \mathrm{e}^x x^2 + x\,\mathrm{e}^x + 2\,\mathrm{e}^x)}{6x^2}$$

✓ Solution by Mathematica

Time used: 0.083 (sec). Leaf size: 46

```
DSolve[x^2*y''[x]+x*(2+x)*y'[x]-(2-3*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}(c_2(x^3\,\mathrm{ExpIntegralEi}(x) - e^x(x^2+x+2)) + 6c_1x^3)}{6x^2}$$

1.171 problem 173

Internal problem ID [7661]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 173.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2(1+x)\,y''+4x(3+8x)\,y'-(5-49x)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(4*x^2*(1+x)*diff(y(x),x$2)+4*x*(3+8*x)*diff(y(x),x)-(5-49*x)*y(x)=0,y(x), singsol=all
```

$$y(x)=\frac{c_1\sqrt{x}}{(x+1)^4}+\frac{c_2(6x^3\ln(x)-18x^2-9x-2)}{6\,(x+1)^4\,x^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.061 (sec). Leaf size: 52

```
DSolve[4*x^2*(1+x)*y''[x]+4*x*(3+8*x)*y'[x]-(5-49*x)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to\frac{6c_1x^3+6c_2x^3\log(x)-18c_2x^2-9c_2x-2c_2}{6x^{5/2}(x+1)^4}$$

1.172 problem 174

Internal problem ID [7662]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 174.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1+x)\,y''-x(3+10x)\,y'+30yx=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 62

```
dsolve(x^2*(1+x)*diff(y(x),x$2)-x*(3+10*x)*diff(y(x),x)+30*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x^5-\frac{5}{2}x^4\right)+c_2\left(3x^5\ln(x)+\frac{x^6}{4}-\frac{15x^4\ln(x)}{2}-\frac{5x^5}{8}-\frac{299x^4}{16}+5x^3+\frac{5x^2}{4}+\frac{x}{4}+\frac{1}{40}\right)$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 68

```
DSolve[x^2*(1+x)*y''[x]-x*(3+10*x)*y'[x]+30*x*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to c_1\left(x^5-\frac{5x^4}{2}\right)+\frac{1}{20}c_2\left(20x^6-50x^5-1495x^4+120(2x-5)x^4\log(x)+400x^3+100x^2+20x+2\right)$$

1.173 problem 175

Internal problem ID [7663]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 175.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(1+x)\,y' - 3(x+3)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 73

```
dsolve(x^2*diff(y(x),x$2)+x*(1+x)*diff(y(x),x)-3*(3+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^3 - \frac{c_2(-\operatorname{expIntegral}_1(x)\,x^6 + \mathrm{e}^{-x}x^5 - \mathrm{e}^{-x}x^4 + 2\,\mathrm{e}^{-x}x^3 - 6x^2\mathrm{e}^{-x} + 24\,\mathrm{e}^{-x}x - 120\,\mathrm{e}^{-x})}{720x^3}$$

✓ Solution by Mathematica

Time used: 0.103 (sec). Leaf size: 60

```
DSolve[x^2*y''[x]+x*(1+x)*y'[x]-3*(3+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2e^{-x}(e^x x^6\,\text{ExpIntegralEi}(-x) + x^5 - x^4 + 2x^3 - 6x^2 + 24x - 120)}{720x^3} + c_1x^3$$

1.174 problem 176

Internal problem ID [7664]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 176.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(2x+1)\,y''+x(9+13x)\,y'+(5x+7)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(x^2*(1+2*x)*diff(y(x),x$2)+x*(9+13*x)*diff(y(x),x)+(7+5*x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(143x^2+104x+20)}{x^7}+\frac{c_2(35x^3-45x^2+36x-20)\,(2x+1)^{\frac{7}{2}}}{x^7}$$

✓ Solution by Mathematica

Time used: 1.731 (sec). Leaf size: 58

```
DSolve[x^2*(1+2*x)*y''[x]+x*(9+13*x)*y'[x]+(7+5*x)*y[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x)\to\frac{c_1(13x(11x+8)+20)}{143x^7}+\frac{c_2(35x^3-45x^2+36x-20)\,(2x+1)^{7/2}}{315x^7}$$

1.175 problem 177

Internal problem ID [7665]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 177.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(2x+1)\,y'' - 2x(-x+4)\,y' - (5x+7)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(4*x^2*(1+2*x)*diff(y(x),x$2)-2*x*(4-x)*diff(y(x),x)-(7+5*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{x}} + \frac{c_2(5x^3 - 10x^2 - 40x - 16)}{(2x+1)^{\frac{5}{4}}\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.096 (sec). Leaf size: 47

```
DSolve[4*x^2*(1+2*x)*y''[x]-2*x*(4-x)*y'[x]-(7+5*x)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{\frac{2c_2(5x^3-10x^2-40x-16)}{(2x+1)^{5/4}} + 35c_1}{35\sqrt{x}}$$

1.176 problem 178

Internal problem ID [7666]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 178.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3x^2(x+3)\,y''-x(15+x)\,y'-20y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(3*x^2*(3+x)*diff(y(x),x$2)-x*(15+x)*diff(y(x),x)-20*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(x^2-36x-243)}{x^{\frac{2}{3}}}+\frac{c_2(7x+27)}{x^{\frac{2}{3}}\,(x+3)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.263 (sec). Leaf size: 43

```
DSolve[3*x^2*(3+x)*y''[x]-x*(15+x)*y'[x]-20*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{21c_2(x^2-36x-243)+\frac{4c_1(7x+27)}{\sqrt[3]{x+3}}}{28x^{2/3}}$$

1.177 problem 179

Internal problem ID [7667]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 179.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1+x)\,y''+x(1-10x)\,y'-(9-10x)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 74

```
dsolve(x^2*(1+x)*diff(y(x),x$2)+x*(1-10*x)*diff(y(x),x)-(9-10*x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(715x^4+572x^3+234x^2+52x+5)}{x^3}+\frac{c_2\left(x^{13}+\frac{91}{8}x^{12}+\frac{117}{2}x^{11}+\frac{715}{4}x^{10}+\frac{715}{2}x^9+\frac{3861}{8}x^8+429x^7+\frac{429}{2}x^6\right)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.088 (sec). Leaf size: 51

```
DSolve[x^2*(1+x)*y''[x]+x*(1-10*x)*y'[x]-(9-10*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{6435c_1(x+1)^{12}(8x-5)-8c_2(715x^4+572x^3+234x^2+52x+5)}{51480x^3}$$

1.178 problem 180

Internal problem ID [7668]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 180.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1+x)\,y''+3x^2y'-(-x+6)\,y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 78

```
dsolve(x^2*(1+x)*diff(y(x),x$2)+3*x^2*diff(y(x),x)-(6-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^3+6x^2+9x+4)}{x^2\,(x+1)^2} + \frac{c_2\left(\ln\,(x+1)\,x^3+6\ln\,(x+1)\,x^2+9\ln\,(x+1)\,x+10x^2+4\ln\,(x+1)+\frac{43x}{2}+\frac{34}{3}\right)}{x^2\,(x+1)^2}$$

✓ Solution by Mathematica

Time used: 0.076 (sec). Leaf size: 49

```
DSolve[x^2*(1+x)*y''[x]+3*x^2*y'[x]-(6-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{\frac{c_2(60x^2+129x+68)}{(x+1)^2}+6c_1(x+4)+6c_2(x+4)\log(x+1)}{6x^2}$$

1.179 problem 181

Internal problem ID [7669]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 181.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(2x+1)\,y'' - 2x(3+14x)\,y' + (6+100x)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
dsolve(x^2*(1+2*x)*diff(y(x),x$2)-2*x*(3+14*x)*diff(y(x),x)+(6+100*x)*y(x)=0,y(x), singsol=a
```

$$y(x) = c_1 x\left(2016x^4 + 672x^3 + 144x^2 + 18x + 1\right) + c_2 x\left(x^9 + \frac{9}{2}x^8 + 9x^7 + \frac{21}{2}x^6 + \frac{63}{8}x^5\right)$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 44

```
DSolve[x^2*(1+2*x)*y''[x]-2*x*(3+14*x)*y'[x]+(6+100*x)*y[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to c_1 x(2x+1)^9 - \frac{c_2 x(2016x^4 + 672x^3 + 144x^2 + 18x + 1)}{20160}$$

1.180 problem 182

Internal problem ID [7670]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 182.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1+x)\,y''-x(6+11x)\,y'+(6+32x)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(x^2*(1+x)*diff(y(x),x$2)-x*(6+11*x)*diff(y(x),x)+(6+32*x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x(35x^3+42x^2+21x+4)+c_2x\left(x^7+\frac{14}{3}x^6+7x^5\right)$$

✓ Solution by Mathematica

Time used: 0.074 (sec). Leaf size: 45

```
DSolve[x^2*(1+x)*y''[x]-x*(6+11*x)*y'[x]+(6+32*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{1}{3}c_1x(x+1)^6(3x-4)-\frac{1}{140}c_2x(35x^3+42x^2+21x+4)$$

1.181 problem 183

Internal problem ID [7671]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 183.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2(1+x)\,y''+4x(1+4x)\,y'-(49+27x)\,y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(4*x^2*(1+x)*diff(y(x),x$2)+4*x*(1+4*x)*diff(y(x),x)-(49+27*x)*y(x)=0,y(x), singsol=al
```

$$y(x)=\frac{c_1(7x+6)}{(x+1)^2\,x^{\frac{7}{2}}}+\frac{c_2x^{\frac{7}{2}}}{(x+1)^2}$$

✓ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 36

```
DSolve[4*x^2*(1+x)*y''[x]+4*x*(1+4*x)*y'[x]-(49+27*x)*y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x)\to\frac{42c_1x^7-7c_2x-6c_2}{42x^{7/2}(x+1)^2}$$

1.182 problem 184

Internal problem ID [7672]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 184.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x^2+1)\,y''-x(-2x^2+7)\,y'+12y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 77

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(7-2*x^2)*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^6}{(x^2+1)^{\frac{7}{2}}}+\frac{c_2 x^2\left(8x^4\sqrt{x^2+1}-15x^4\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)-9\sqrt{x^2+1}\,x^2-2\sqrt{x^2+1}\right)}{8\,(x^2+1)^{\frac{7}{2}}}$$

✓ Solution by Mathematica

Time used: 0.147 (sec). Leaf size: 88

```
DSolve[x^2*(1+x^2)*y''[x]-x*(7-2*x^2)*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x)\to \frac{-15c_2x^6\text{arctanh}\left(\sqrt{x^2+1}\right)-2c_2\sqrt{x^2+1}x^2+8x^6\left(c_2\sqrt{x^2+1}+c_1\right)-9c_2\sqrt{x^2+1}x^4}{8\left(x^2+1\right)^{7/2}}$$

1.183 problem 185

Internal problem ID [7673]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 185.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(-x^2+7)\,y' + 12y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(x^2*diff(y(x),x$2)-x*(7-x^2)*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^6\mathrm{e}^{-\frac{x^2}{2}} + \frac{c_2x^2\mathrm{e}^{-\frac{x^2}{2}}\left(\mathrm{expIntegral}_1\left(-\frac{x^2}{2}\right)x^4 + 2\,\mathrm{e}^{\frac{x^2}{2}}x^2 + 4\,\mathrm{e}^{\frac{x^2}{2}}\right)}{16}$$

✓ Solution by Mathematica

Time used: 0.121 (sec). Leaf size: 61

```
DSolve[x^2*y''[x]-x*(7-x^2)*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{16}c_2e^{-\frac{x^2}{2}}x^6\,\mathrm{ExpIntegralEi}\left(\frac{x^2}{2}\right) - \frac{1}{8}c_2\left(x^2+2\right)x^2 + c_1e^{-\frac{x^2}{2}}x^6$$

1.184 problem 186

Internal problem ID [7674]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 186.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x(2x^2 + 1)\, y' - (-10x^2 + 1)\, y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve(x^2*diff(y(x),x$2)+x*(1+2*x^2)*diff(y(x),x)-(1-10*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x\, \mathrm{e}^{-x^2}(x^2 - 2) + c_2 x\, \mathrm{e}^{-x^2}(x^2 - 2)\left(\int \frac{\mathrm{e}^{x^2}}{(x^2 - 2)^2\, x^3}dx\right)$$

✓ Solution by Mathematica

Time used: 0.223 (sec). Leaf size: 68

```
DSolve[x^2*y''[x]+x*(1+2*x^2)*y'[x]-(1-10*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{e^{-x^2}\left(c_2(x^2 - 2)\, x^2\, \mathrm{ExpIntegralEi}\,(x^2) + 4c_1x^4 - x^2\left(c_2e^{x^2} + 8c_1\right) + c_2e^{x^2}\right)}{4x}$$

1.185 problem 187

Internal problem ID [7675]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 187.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x(-2x^2+1)\,y' - 4(2x^2+1)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
dsolve(x^2*diff(y(x),x$2)+x*(1-2*x^2)*diff(y(x),x)-4*(1+2*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^2\mathrm{e}^{x^2} - \frac{c_2\mathrm{e}^{x^2}\left(-\operatorname{expIntegral}_1\left(x^2\right)x^4 + \mathrm{e}^{-x^2}x^2 - \mathrm{e}^{-x^2}\right)}{4x^2}$$

✓ Solution by Mathematica

Time used: 0.1 (sec). Leaf size: 46

```
DSolve[x^2*y''[x]+x*(1-2*x^2)*y'[x]-4*(1+2*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{c_2\left(e^{x^2}x^4\,\text{ExpIntegralEi}\left(-x^2\right) + x^2 - 1\right)}{4x^2} + c_1e^{x^2}x^2$$

1.186 problem 188

Internal problem ID [7676]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 188.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x(-3x^2 + 1)\, y' - 4(-3x^2 + 1)\, y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(x^2*diff(y(x),x$2)+x*(1-3*x^2)*diff(y(x),x)-4*(1-3*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^2(x^2 - 2) + c_2x^2(x^2 - 2)\left(\int \frac{e^{\frac{3x^2}{2}}}{(x^2 - 2)^2 x^5}dx\right)$$

✓ Solution by Mathematica

Time used: 0.26 (sec). Leaf size: 89

```
DSolve[x^2*y''[x]+x*(1-3*x^2)*y'[x]-4*(1-3*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{1}{64}\left(27c_2(x^2 - 2)\, x^2 \operatorname{ExpIntegralEi}\left(\frac{3x^2}{2}\right) + 64c_1x^4 - 2x^2\left(9c_2e^{\frac{3x^2}{2}} + 64c_1\right) + 24c_2e^{\frac{3x^2}{2}} + \frac{8c_2e^{\frac{3x^2}{2}}}{x^2}\right)$$

1.187 problem 189

Internal problem ID [7677]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 189.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+1)\,y''+x(11x^2+5)\,y'+24x^2y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)+x*(5+11*x^2)*diff(y(x),x)+24*x^2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(2x^2+1)}{(x^2+1)^2\,x^4}+\frac{c_2}{(x^2+1)^2}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 36

```
DSolve[x^2*(1+x^2)*y''[x]+x*(5+11*x^2)*y'[x]+24*x^2*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to-\frac{-4c_1x^4+2c_2x^2+c_2}{4x^4\,(x^2+1)^2}$$

1.188 problem 190

Internal problem ID [7678]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 190.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2(x^2+1)\,y''+8xy'-(-x^2+35)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(4*x^2*(1+x^2)*diff(y(x),x$2)+8*x*diff(y(x),x)-(35-x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(x^4+2x^2+1)}{x^{\frac{7}{2}}}+\frac{c_2\left(\frac{\ln(x^2+1)x^4}{2}+\ln(x^2+1)\,x^2+x^2+\frac{\ln(x^2+1)}{2}+\frac{3}{4}\right)}{x^{\frac{7}{2}}}$$

✓ Solution by Mathematica

Time used: 0.075 (sec). Leaf size: 53

```
DSolve[4*x^2*(1+x^2)*y''[x]+8*x*y'[x]-(35-x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x)\to\frac{4c_1(x^2+1)^2+c_2(4x^2+3)+2c_2(x^2+1)^2\log(x^2+1)}{4x^{7/2}}$$

1.189 problem 191

Internal problem ID [7679]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 191.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+1)\,y''-x(-x^2+5)\,y'-(25x^2+7)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(5-x^2)*diff(y(x),x)-(7+25*x^2)*y(x)=0,y(x), singsol=all
```

$$y(x)=\frac{c_1}{(x^2+1)^2\,x}+\frac{c_2\left(x^{10}+\frac{5}{4}x^8\right)}{(x^2+1)^2\,x}$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 37

```
DSolve[x^2*(1+x^2)*y''[x]-x*(5-x^2)*y'[x]-(7+25*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to\frac{c_2(4x^2+5)\,x^8+40c_1}{40x\,(x^2+1)^2}$$

1.190 problem 192

Internal problem ID [7680]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 192.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x^2+1)\,y''+x(2x^2+5)\,y'-21y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)+x*(5+2*x^2)*diff(y(x),x)-21*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(35x^6+140x^4+168x^2+64)}{x^7}+\frac{c_2(x^2+1)^{\frac{5}{2}}\,(x^2+8)}{x^7}$$

✓ Solution by Mathematica

Time used: 0.169 (sec). Leaf size: 52

```
DSolve[x^2*(1+x^2)*y''[x]+x*(5+2*x^2)*y'[x]-21*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x)\to\frac{35c_1(x^2+1)^{5/2}\,(x^2+8)-c_2(35x^6+140x^4+168x^2+64)}{35x^7}$$

1.191 problem 193

Internal problem ID [7681]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 193.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2\left(x^2+1\right)y''+4x\left(x^2+2\right)y'-\left(x^2+15\right)y=0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 29

```
dsolve(4*x^2*(1+x^2)*diff(y(x),x$2)+4*x*(2+x^2)*diff(y(x),x)-(15+x^2)*y(x)=0,y(x), singsol=a
```

$$y(x)=\frac{c_1(3x^2+2)}{x^{\frac{5}{2}}}+\frac{c_2(x^2+1)^{\frac{3}{2}}}{x^{\frac{5}{2}}}$$

✓ Solution by Mathematica
Time used: 0.102 (sec). Leaf size: 39

```
DSolve[4*x^2*(1+x^2)*y''[x]+4*x*(2+x^2)*y'[x]-(15+x^2)*y[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x)\to\frac{3c_1(x^2+1)^{3/2}-c_2(3x^2+2)}{3x^{5/2}}$$

1.192 problem 194

Internal problem ID [7682]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 194.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \frac{2(t+1)\,y'}{t^2+2t-1} + \frac{2y}{t^2+2t-1} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(t),t$2)-2*(t+1)/(t^2+2*t-1)*diff(y(t),t)+2/(t^2+2*t-1)*y(t)=0,y(t), singsol=al
```

$$y(t) = c_1(t+1) + c_2(t^2+1)$$

✓ Solution by Mathematica

Time used: 0.21 (sec). Leaf size: 64

```
DSolve[y''[t]-2*(t+1)/(t^2+2*t-1)*y'[t]+2/(t^2+2*t-1)*y[t]==0,y[t],t,IncludeSingularSolution
```

$$y(t) \to \frac{\sqrt{t^2+2t-1}\left(c_1\left(t^2 - 2\left(\sqrt{2}-1\right)t - 2\sqrt{2} + 3\right) + c_2(t+1)\right)}{\sqrt{-t^2-2t+1}}$$

1.193 problem 195

Internal problem ID [7683]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 195.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 4ty' + (4t^2 - 2)\, y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(t),t$2)-4*t*diff(y(t),t)+(4*t^2-2)*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 e^{t^2} + c_2 e^{t^2} t$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 18

```
DSolve[y''[t]-4*t*y'[t]+(4*t^2-2)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{t^2}(c_2 t + c_1)$$

1.194 problem 196

Internal problem ID [7684]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 196.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$(-t^2+1)\,y''-2ty'+2y=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve((1-t^2)*diff(y(t),t$2)-2*t*diff(y(t),t)+2*y(t)=0,y(t), singsol=all)
```

$$y(t)=c_1t+c_2\left(-\frac{\ln\left(t+1\right)t}{2}+\frac{\ln\left(t-1\right)t}{2}+1\right)$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 33

```
DSolve[(1-t^2)*y''[t]-2*t*y'[t]+2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to c_1t-\frac{1}{2}c_2(t\log(1-t)-t\log(t+1)+2)$$

1.195 problem 197

Internal problem ID [7685]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 197.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(t^2+1)\,y''-2ty'+2y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve((1+t^2)*diff(y(t),t$2)-2*t*diff(y(t),t)+2*y(t)=0,y(t), singsol=all)
```

$$y(t)=c_1t+c_2(t^2-1)$$

✓ Solution by Mathematica
Time used: 0.048 (sec). Leaf size: 21

```
DSolve[(1+t^2)*y''[t]-2*t*y'[t]+2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to c_2t-c_1(t-i)^2$$

1.196 problem 198

Internal problem ID [7686]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 198.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$(-t^2+1)\,y''-2ty'+6y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve((1-t^2)*diff(y(t),t$2)-2*t*diff(y(t),t)+6*y(t)=0,y(t), singsol=all)
```

$$y(t)=c_1(-3t^2+1)+c_2\left(-\frac{3\ln(t+1)\,t^2}{8}+\frac{3\ln(t-1)\,t^2}{8}+\frac{\ln(t+1)}{8}-\frac{\ln(t-1)}{8}+\frac{3t}{4}\right)$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 55

```
DSolve[(1-t^2)*y''[t]-2*t*y'[t]+6*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to\frac{1}{2}c_1(3t^2-1)-\frac{1}{4}c_2\left((3t^2-1)\log(1-t)+(1-3t^2)\log(t+1)+6t\right)$$

1.197 problem 199

Internal problem ID [7687]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 199.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2t+1)\,y''-4(t+1)\,y'+4y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve((2*t+1)*diff(y(t),t$2)-4*(t+1)*diff(y(t),t)+4*y(t)=0,y(t), singsol=all)
```

$$y(t)=c_1(t+1)+c_2\mathrm{e}^{2t}$$

✓ Solution by Mathematica

Time used: 0.138 (sec). Leaf size: 23

```
DSolve[(2*t+1)*y''[t]-4*(t+1)*y'[t]+4*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to c_1e^{2t+1}-c_2(t+1)$$

1.198 problem 200

Internal problem ID [7688]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 200.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$t^2y'' + ty' + \left(t^2 - \frac{1}{4}\right) y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(t^2*diff(y(t),t$2)+t*diff(y(t),t)+(t^2-1/4)*y(t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_1 \sin(t)}{\sqrt{t}} + \frac{c_2 \cos(t)}{\sqrt{t}}$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 39

```
DSolve[t^2*y''[t]+t*y'[t]+(t^2-1/4)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{e^{-it}(2c_1 - ic_2e^{2it})}{2\sqrt{t}}$$

1.199 problem 201

Internal problem ID [7689]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 201.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{2ty'}{t^2 + 1} + \frac{2y}{t^2 + 1} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(t),t$2)-2*t/(1+t^2)*diff(y(t),t)+2/(1+t^2)*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 t + c_2(t^2 - 1)$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 21

```
DSolve[y''[t]-2*t/(1+t^2)*y'[t]+2/(1+t^2)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_2 t - c_1(t - i)^2$$

1.200 problem 202

Internal problem ID [7690]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 202.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + (t^2 + 2t + 1)\, y' - (4t + 4)\, y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 83

```
dsolve(diff(y(t),t$2)+(t^2+2*t+1)*diff(y(t),t)-(4+4*t)*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1(t^4 + 4t^3 + 6t^2 + 8t + 5) + c_2(t^4 + 4t^3 + 6t^2 + 8t + 5)\left(\int \frac{e^{-\frac{1}{3}t^3 - t^2 - t}}{(t+1)^2 (t^3 + 3t^2 + 3t + 5)^2} dt\right)$$

✓ Solution by Mathematica

Time used: 3.024 (sec). Leaf size: 132

```
DSolve[y''[t]+(t^2+2*t+1)*y'[t]-(4+4*t)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{36} e^{-\frac{1}{3}t(t^2+3t+3)} \left(-3c_2(t^3 + 3t^2 + 3t + 4) + 3^{2/3} c_2 e^{\frac{1}{3}(t+1)^3} \sqrt[3]{(t+1)^3}(t^3 + 3t^2 + 3t + 5)\, \Gamma\left(\frac{2}{3}, \frac{1}{3}(t+1)^3\right) + 36 c_1 e^{\frac{t^3}{3} + t^2 + t}(t^4 + 4t^3 + 6t^2 + 8t + 5)\right)$$

1.201 problem 204

Internal problem ID [7691]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 204.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{2ty'' + (1-2t)\,y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(2*t*diff(y(t),t$2)+(1-2*t)*diff(y(t),t)-y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 e^t + c_2 e^t \left(\int \frac{e^{-t}}{\sqrt{t}} dt\right)$$

✓ Solution by Mathematica

Time used: 0.109 (sec). Leaf size: 21

```
DSolve[2*t*y''[t]+(1-2*t)*y'[t]-y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^t \left(c_1 - c_2 \Gamma\left(\frac{1}{2}, t\right)\right)$$

1.202 problem 205

Internal problem ID [7692]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 205.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2ty'' + (t+1)\,y' - 2y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 45

```
dsolve(2*t*diff(y(t),t$2)+(1+t)*diff(y(t),t)-2*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1\left(t^2+6t+3\right) + c_2\left(t^2+6t+3\right)\left(\int \frac{e^{-\frac{t}{2}}}{\left(t^2+6t+3\right)^2\sqrt{t}}dt\right)$$

✓ Solution by Mathematica
Time used: 10.591 (sec). Leaf size: 71

```
DSolve[2*t*y''[t]+(1+t)*y'[t]-2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{24}\left(\sqrt{2\pi}c_2\left(t^2+6t+3\right)\operatorname{erf}\left(\frac{\sqrt{t}}{\sqrt{2}}\right) + 24c_1\left(t^2+6t+3\right) + 2c_2e^{-t/2}\sqrt{t}(t+5)\right)$$

1.203 problem 206

Internal problem ID [7693]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 206.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2t^2y'' - ty' + (t+1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(2*t^2*diff(y(t),t$2)-t*diff(y(t),t)+(1+t)*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \sin\left(\sqrt{2}\,\sqrt{t}\right)\sqrt{t} + c_2\sqrt{t}\,\cos\left(\sqrt{2}\,\sqrt{t}\right)$$

✓ Solution by Mathematica

Time used: 0.083 (sec). Leaf size: 62

```
DSolve[2*t^2*y''[t]-t*y'[t]+(1+t)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}e^{-i\sqrt{2}\sqrt{t}}\sqrt{t}\left(2c_1e^{2i\sqrt{2}\sqrt{t}} + i\sqrt{2}c_2\right)$$

1.204 problem 207

Internal problem ID [7694]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 207.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2t^2y'' + (t^2 - t)\, y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(2*t^2*diff(y(t),t$2)+(t^2-t)*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1\sqrt{t}\,\mathrm{e}^{-\frac{t}{2}} + c_2\sqrt{t}\,\mathrm{e}^{-\frac{t}{2}}\left(\int \frac{\mathrm{e}^{\frac{t}{2}}}{\sqrt{t}}dt\right)$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 46

```
DSolve[2*t^2*y''[t]+(t^2-t)*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t/2}\left(c_2\sqrt{t} + \sqrt{2}c_1\sqrt{-t}\Gamma\left(\frac{1}{2}, -\frac{t}{2}\right)\right)$$

1.205 problem 208

Internal problem ID [7695]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 208.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2y'' + \left(-t^2 + t\right) y' - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(t^2*diff(y(t),t$2)+(t-t^2)*diff(y(t),t)-y(t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_1(t+1)}{t} + \frac{c_2 e^t}{t}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 23

```
DSolve[t^2*y''[t]+(t-t^2)*y'[t]-y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_2 e^t - c_1(t+1)}{t}$$

1.206 problem 209

Internal problem ID [7696]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 209.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{ty'' - (t^2+2)\, y' + yt = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 47

```
dsolve(t*diff(y(t),t$2)-(t^2+2)*diff(y(t),t)+t*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 e^{\frac{t^2}{2}} + \frac{c_2 e^{\frac{t^2}{2}} \left(-\sqrt{\pi}\,\sqrt{2}\,\operatorname{erf}\left(\frac{\sqrt{2}\,t}{2}\right) + 2t\, e^{-\frac{t^2}{2}}\right)}{2}$$

✓ Solution by Mathematica
Time used: 0.13 (sec). Leaf size: 52

```
DSolve[t*y''[t]-(t^2+2)*y'[t]+t*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \sqrt{\frac{\pi}{2}} c_2 e^{\frac{t^2}{2}} \operatorname{erf}\left(\frac{t}{\sqrt{2}}\right) + c_1 e^{\frac{t^2}{2}} - c_2 t$$

1.207 problem 210

Internal problem ID [7697]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 210.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2y'' + t(t+1)\,y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(t^2*diff(y(t),t$2)+t*(t+1)*diff(y(t),t)-y(t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_1(t-1)}{t} + \frac{c_2 \mathrm{e}^{-t}}{t}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 26

```
DSolve[t^2*y''[t]+t*(t+1)*y'[t]-y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{e^{-t}(c_1e^t(t-1) + c_2)}{t}$$

1.208 problem 211

Internal problem ID [7698]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 211.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{ty'' - (t+4)\,y' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(t*diff(y(t),t$2)-(4+t)*diff(y(t),t)+2*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1\left(t^2+6t+12\right)+c_2\mathrm{e}^t\left(t^2-6t+12\right)$$

✓ Solution by Mathematica
Time used: 0.064 (sec). Leaf size: 85

```
DSolve[t*y''[t]-(4+t)*y'[t]+2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{2e^{t/2}\sqrt{t}\left((c_2t^2-6ic_1t+12c_2)\cosh\left(\frac{t}{2}\right)+i(c_1(t^2+12)+6ic_2t)\sinh\left(\frac{t}{2}\right)\right)}{\sqrt{\pi}\sqrt{-it}}$$

1.209 problem 212

Internal problem ID [7699]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 212.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2y'' + (t^2 - 3t)\,y' + 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(t^2*diff(y(t),t$2)+(t^2-3*t)*diff(y(t),t)+3*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 t^3 \mathrm{e}^{-t} + \frac{c_2 t\, \mathrm{e}^{-t}(\mathrm{expIntegral}_1\,(-t)\, t^2 + \mathrm{e}^t t + \mathrm{e}^t)}{2}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 41

```
DSolve[t^2*y''[t]+(t^2-3*t)*y'[t]+3*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}e^{-t}(c_1 t^3\,\mathrm{ExpIntegralEi}(t) + 2c_2 t^3 - c_1 e^t (t+1)t)$$

1.210 problem 213

Internal problem ID [7700]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 213.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{ty'' + ty' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(t*diff(y(t),t$2)+t*diff(y(t),t)+2*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 e^{-t}(t-2)\,t + \frac{c_2(\operatorname{expIntegral}_1(-t)\,t^2 + e^t t - 2\operatorname{expIntegral}_1(-t)\,t - e^t)\,e^{-t}}{2}$$

✓ Solution by Mathematica

Time used: 0.137 (sec). Leaf size: 51

```
DSolve[t*y''[t]+t*y'[t]+2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}e^{-t}\left(c_2(t-2)t\,\text{ExpIntegralEi}(t) + 2c_1t^2 - t\left(c_2e^t + 4c_1\right) + c_2e^t\right)$$

1.211 problem 214

Internal problem ID [7701]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 214.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{ty'' + \left(-t^2 + 1\right) y' + 4yt = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
dsolve(t*diff(y(t),t$2)+(1-t^2)*diff(y(t),t)+4*t*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1\left(t^4 - 8t^2 + 8\right) + c_2\left(t^4 - 8t^2 + 8\right)\left(\int \frac{e^{\frac{t^2}{2}}}{\left(t^4 - 8t^2 + 8\right)^2 t} dt\right)$$

✓ Solution by Mathematica

Time used: 0.42 (sec). Leaf size: 61

```
DSolve[t*y''[t]+(1-t^2)*y'[t]+4*t*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{128}c_2\left(\left(t^4 - 8t^2 + 8\right) \text{ExpIntegralEi}\left(\frac{t^2}{2}\right) - 2e^{\frac{t^2}{2}}\left(t^2 - 6\right)\right) + c_1\left(t^4 - 8t^2 + 8\right)$$

1.212 problem 215

Internal problem ID [7702]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 215.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2y'' - t(t+1)\,y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(t^2*diff(y(t),t$2)-t*(1+t)*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{t} c_1 t + c_2 \mathrm{e}^{t} t \,\mathrm{expIntegral}_1\,(t)$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 20

```
DSolve[t^2*y''[t]-t*(1+t)*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^t t(c_1 \,\mathrm{ExpIntegralEi}(-t) + c_2)$$

1.213 problem 216

Internal problem ID [7703]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 216.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4xy' + (4x^2 + 6)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2+6)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^2} \cos(2x) + c_2 \mathrm{e}^{-x^2} \sin(2x)$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 37

```
DSolve[y''[x]+4*x*y'[x]+(4*x^2+6)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} e^{-x(x+2i)} \left(4c_1 - ic_2 e^{4ix}\right)$$

1.214 problem 217

Internal problem ID [7704]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 217.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-z^2+1)\,y''-3zy'+\lambda y=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 55

```
dsolve((1-z^2)*diff(y(z),z$2)-3*z*diff(y(z),z)+lambda*y(z)=0,y(z), singsol=all)
```

$$y(z)=\frac{c_1\left(z+\sqrt{z^2-1}\right)^{\sqrt{\lambda+1}}}{\sqrt{z^2-1}}+\frac{c_2\left(z+\sqrt{z^2-1}\right)^{-\sqrt{\lambda+1}}}{\sqrt{z^2-1}}$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 54

```
DSolve[(1-z^2)*y''[z]-3*z*y'[z]+\[Lambda]*y[z]==0,y[z],z,IncludeSingularSolutions -> True]
```

$$y(z)\to\frac{c_1P_{\sqrt{\lambda+1}-\frac{1}{2}}^{\frac{1}{2}}(z)+c_2Q_{\sqrt{\lambda+1}-\frac{1}{2}}^{\frac{1}{2}}(z)}{\sqrt[4]{z^2-1}}$$

1.215 problem 218

Internal problem ID [7705]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 218.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4zy'' + 2(1 - z) y' - y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(4*z*diff(y(z),z$2)+2*(1-z)*diff(y(z),z)-y(z)=0,y(z), singsol=all)
```

$$y(z) = c_1 e^{\frac{z}{2}} + c_2 e^{\frac{z}{2}} \left(\int \frac{e^{-\frac{z}{2}}}{\sqrt{z}} dz \right)$$

✓ Solution by Mathematica
Time used: 0.133 (sec). Leaf size: 34

```
DSolve[4*z*y''[z]+2*(1-z)*y'[z]-y[z]==0,y[z],z,IncludeSingularSolutions -> True]
```

$$y(z) \to e^{z/2} \left(c_1 - \sqrt{2} c_2 \Gamma\left(\frac{1}{2}, \frac{z}{2} \right) \right)$$

1.216 problem 219

Internal problem ID [7706]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 219.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{f'' + 2(z-1)\,f' + 4f = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 74

```
dsolve(diff(f(z),z$2)+2*(z-1)*diff(f(z),z)+4*f(z)=0,f(z), singsol=all)
```

$$f(z) = c_1 \mathrm{e}^{-z^2+2z}(z-1) + c_2 \mathrm{e}^{-z^2+2z}\left(-\mathrm{e}^{-1}\sqrt{\pi}\ \mathrm{erf}\,(iz-i)\,z + \mathrm{e}^{-1}\sqrt{\pi}\ \mathrm{erf}\,(iz-i) + i\mathrm{e}^{z^2-2z}\right)$$

✓ Solution by Mathematica

Time used: 0.176 (sec). Leaf size: 72

```
DSolve[f''[z]+2*(z-a)*f'[z]+4*f[z]==0,f[z],z,IncludeSingularSolutions -> True]
```

$$f(z) \to e^{z(2a-z)}\left(-\sqrt{\pi}c_2\sqrt{(a-z)^2}\mathrm{erfi}\left(\sqrt{(a-z)^2}\right) + c_2 e^{(a-z)^2} - 2ac_1 + 2c_1 z\right)$$

1.217 problem 220

Internal problem ID [7707]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 220.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{zy'' - 2y' + zy = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(z*diff(y(z),z$2)-2*diff(y(z),z)+z*y(z)=0,y(z), singsol=all)
```

$$y(z) = c_1(\cos(z)\,z - \sin(z)) + c_2(\cos(z) + \sin(z)\,z)$$

✓ Solution by Mathematica
Time used: 0.065 (sec). Leaf size: 39

```
DSolve[z*y''[z]-2*y'[z]+z*y[z]==0,y[z],z,IncludeSingularSolutions -> True]
```

$$y(z) \to -\sqrt{\frac{2}{\pi}}((c_1 z + c_2)\cos(z) + (c_2 z - c_1)\sin(z))$$

1.218 problem 221

Internal problem ID [7708]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 221.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{zy'' + (2z-3)\,y' + \frac{4y}{z} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(z*diff(y(z),z$2)+(2*z-3)*diff(y(z),z)+4/z*y(z)=0,y(z), singsol=all)
```

$$y(z) = c_1 z^2 \mathrm{e}^{-2z}(2z-1) + c_2 z^2 \left(2 \operatorname{expIntegral}_1(-2z)\, z - \operatorname{expIntegral}_1(-2z) + \mathrm{e}^{2z}\right) \mathrm{e}^{-2z}$$

✓ Solution by Mathematica

Time used: 0.614 (sec). Leaf size: 47

```
DSolve[z*y''[z]+(2*z-3)*y'[z]+4/z*y[z]==0,y[z],z,IncludeSingularSolutions -> True]
```

$$y(z) \to -\frac{1}{2} e^{-2z} z^2 \left(4c_2(1-2z)\,\text{ExpIntegralEi}(2z) - 2c_1 z + 4c_2 e^{2z} + c_1\right)$$

1.219 problem 222

Internal problem ID [7709]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 222.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_erf]`

$$\boxed{y'' + 2xy' + 4y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)+2*x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \,\mathrm{e}^{-x^2} + c_2 \mathrm{e}^{-x^2} \left(-\sqrt{\pi}\, \mathrm{erfi}\,(x)\, x + \mathrm{e}^{x^2}\right)$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 51

```
DSolve[y''[x]+2*x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x^2} \left(-\sqrt{\pi} c_2 \sqrt{x^2} \mathrm{erfi}\left(\sqrt{x^2}\right) + c_2 e^{x^2} + 2 c_1 x\right)$$

1.220 problem 223

Internal problem ID [7710]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 223.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + xy' + 3y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 50

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x^2}{2}} \left(x^2 - 1\right) + c_2 \mathrm{e}^{-\frac{x^2}{2}} \left(x^2 - 1\right) \left(\int \frac{\mathrm{e}^{\frac{x^2}{2}}}{\left(x-1\right)^2 \left(x+1\right)^2} dx\right)$$

✓ Solution by Mathematica
Time used: 0.131 (sec). Leaf size: 65

```
DSolve[y''[x]+x*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} e^{-\frac{x^2}{2}} \left(\sqrt{2\pi} c_2 (x^2 - 1) \operatorname{erfi}\left(\frac{x}{\sqrt{2}}\right) + 4c_1 (x^2 - 1) - 2c_2 e^{\frac{x^2}{2}} x\right)$$

1.221 problem 224

Internal problem ID [7711]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 224.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - x^2y' - 3yx = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 76

```
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)-3*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{x^3}{3}} x - \frac{9c_2 e^{\frac{x^3}{3}} 3^{\frac{2}{3}} e^{-\frac{x^3}{6}} \left(x^6 \,\mathrm{WhittakerM}\left(\frac{1}{3}, \frac{5}{6}, \frac{x^3}{3}\right) + 5 \,\mathrm{WhittakerM}\left(\frac{4}{3}, \frac{5}{6}, \frac{x^3}{3}\right) x^3 + 10 \,\mathrm{WhittakerM}\left(\frac{4}{3}, \frac{5}{6}, \frac{x^3}{3}\right)\right)}{10x^3 \left(x^3\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.079 (sec). Leaf size: 51

```
DSolve[y''[x]-x^2*y'[x]-3*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{9} e^{\frac{x^3}{3}} \left(9c_1 x - 3^{2/3} c_2 \sqrt[3]{x^3} \Gamma\left(-\frac{1}{3}, \frac{x^3}{3}\right)\right)$$

1.222 problem 225

Internal problem ID [7712]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 225.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\left(-4x^2+1\right)y''-20xy'-16y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 57

```
dsolve((1-4*x^2)*diff(y(x),x$2)-20*x*diff(y(x),x)-16*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x}{\left(4x^2-1\right)^{\frac{3}{2}}}+\frac{c_2\left(2\ln\left(2x+\sqrt{4x^2-1}\right)x-\sqrt{4x^2-1}\right)}{\left(4x^2-1\right)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.172 (sec). Leaf size: 73

```
DSolve[(1-4*x^2)*y''[x]-20*x*y'[x]-16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{4c_2x\arctan\left(\frac{\sqrt{1-4x^2}}{2x+1}\right)-c_2\sqrt{1-4x^2}+c_1x}{\sqrt[4]{1-4x^2}\left(4x^2-1\right)^{5/4}}$$

1.223 problem 226

Internal problem ID [7713]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 226.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(x^2-1)\, y''-6xy'+12y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve((x^2-1)*diff(y(x),x$2)-6*x*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(x^3+x)+c_2(x^4+6x^2+1)$$

✓ Solution by Mathematica
Time used: 0.121 (sec). Leaf size: 45

```
DSolve[(x^2-1)*y''[x]-6*x*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{\sqrt{x^2-1}(c_2x(x^2+1)+c_1(x-1)^4)}{\sqrt{1-x^2}}$$

1.224 problem 227

Internal problem ID [7714]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 227.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + xy' + (x + 2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 64

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+(2+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{1}{2}x^2+x}\left(x^2 - 4x + 3\right) + c_2 e^{-\frac{1}{2}x^2+x}\left(x^2 - 4x + 3\right)\left(\int \frac{e^{\frac{1}{2}x^2-2x}}{\left(x-1\right)^2\left(x-3\right)^2}dx\right)$$

✓ Solution by Mathematica

Time used: 0.526 (sec). Leaf size: 94

```
DSolve[y''[x]+x*y'[x]+(2+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{-\frac{x^2}{2}+x-\frac{9}{2}}\left(e^{5/2}\sqrt{2\pi}c_2\left(x^2-4x+3\right)\operatorname{erfi}\left(\frac{x-2}{\sqrt{2}}\right) + 4e^{9/2}c_1\left(x^2-4x+3\right) - 2c_2 e^{\frac{1}{2}(x-3)^2+x}(x-2)\right)$$

1.225 problem 228

Internal problem ID [7715]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 228.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(2x^2+1\right)y''+7xy'+2y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 44

```
dsolve((1+2*x^2)*diff(y(x),x$2)+7*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x}{\left(2x^2+1\right)^{\frac{3}{4}}}+\frac{c_2x\left(\int\frac{1}{\left(2x^2+1\right)^{\frac{1}{4}}x^2}dx\right)}{\left(2x^2+1\right)^{\frac{3}{4}}}$$

✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 66

```
DSolve[(1+2*x^2)*y''[x]+7*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_2Q_{\frac{1}{4}}^{\frac{3}{4}}\left(i\sqrt{2}x\right)}{\left(2x^2+1\right)^{3/8}}+\frac{2i\sqrt[4]{2}c_1x}{\left(2x^2+1\right)^{3/4}\operatorname{Gamma}\left(\frac{1}{4}\right)}$$

1.226 problem 229

Internal problem ID [7716]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 229.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{4y'' + xy' + 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 52

```
dsolve(4*diff(y(x),x$2)+x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x^2}{8}} x(x^2-12) + c_2 \mathrm{e}^{-\frac{x^2}{8}} x(x^2-12) \left(\int \frac{\mathrm{e}^{\frac{x^2}{8}}}{(x^2-12)^2 x^2} dx \right)$$

✓ Solution by Mathematica
Time used: 0.109 (sec). Leaf size: 122

```
DSolve[4*y''[x]+x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{x^2}{8}} \left(\sqrt{2\pi} c_2 (x^2-12) x^2 \operatorname{erfi}\left(\frac{\sqrt{x^2}}{2\sqrt{2}} \right) + 4\sqrt{x^2} \left(2\sqrt{2} c_1 x^3 - c_2 e^{\frac{x^2}{8}} x^2 + 8 c_2 e^{\frac{x^2}{8}} - 24\sqrt{2} c_1 x \right) \right)}{32\sqrt{x^2}}$$

1.227 problem 230

Internal problem ID [7717]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 230.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + xy' - 4y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 50

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x^4 + 6x^2 + 3) + c_2(x^4 + 6x^2 + 3)\left(\int \frac{e^{-\frac{x^2}{2}}}{(x^4 + 6x^2 + 3)^2}dx\right)$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 43

```
DSolve[y''[x]+x*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\frac{x^2}{2}} \text{HermiteH}\left(-5, \frac{x}{\sqrt{2}}\right) + \frac{1}{3}c_2(x^4 + 6x^2 + 3)$$

1.228 problem 231

Internal problem ID [7718]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4xy'' - xy' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 48

```
dsolve(4*x*diff(y(x),x$2)-x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x^2 - 8x) + c_2\left(\frac{\text{expIntegral}_1\left(-\frac{x}{4}\right)x^2}{128} + \frac{e^{\frac{x}{4}}x}{32} - \frac{\text{expIntegral}_1\left(-\frac{x}{4}\right)x}{16} - \frac{e^{\frac{x}{4}}}{8}\right)$$

✓ Solution by Mathematica
Time used: 0.133 (sec). Leaf size: 43

```
DSolve[4*x*y''[x]-x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{128}c_2\left((x-8)x \operatorname{ExpIntegralEi}\left(\frac{x}{4}\right) - 4e^{x/4}(x-4)\right) + c_1(x-8)x$$

1.229 problem 232

Internal problem ID [7719]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 232.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$6x^2y'' + x(1+18x)\,y' + (12x+1)\,y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
dsolve(6*x^2*diff(y(x),x$2)+x*(1+18*x)*diff(y(x),x)+(1+12*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\mathrm{e}^{-3x} + c_2\sqrt{x}\,\mathrm{e}^{-3x}\left(\int \frac{\mathrm{e}^{3x}}{x^{\frac{7}{6}}}dx\right)$$

✓ Solution by Mathematica

Time used: 0.237 (sec). Leaf size: 47

```
DSolve[6*x^2*y''[x]+x*(1+18*x)*y'[x]+(1+12*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^{-3x}\left(\frac{\sqrt[6]{3}c_2x^{4/3}\Gamma\left(-\frac{1}{6},-3x\right)}{(-x)^{5/6}} + c_1\sqrt{x}\right)$$

1.230 problem 233

Internal problem ID [7720]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 233.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3x^2y'' - x(x+8)\,y' + 6y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 59

```
dsolve(3*x^2*diff(y(x),x$2)-x*(x+8)*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{\frac{2}{3}} \mathrm{e}^{\frac{x}{3}} \left(x^2 - 2x + 4\right) + c_2 x^{\frac{2}{3}} \mathrm{e}^{\frac{x}{3}} \left(x^2 - 2x + 4\right) \left(\int \frac{x^{\frac{4}{3}} \mathrm{e}^{-\frac{x}{3}}}{\left(x^2 - 2x + 4\right)^2} dx\right)$$

✓ Solution by Mathematica
Time used: 1.152 (sec). Leaf size: 79

```
DSolve[3*x^2*y''[x]-x*(x+8)*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x/3} x^{2/3} (x^2 - 2x + 4) - \frac{c_2 e^{x/3} x^{2/3} (x^2 - 2x + 4)\, \Gamma\left(\frac{1}{3}, \frac{x}{3}\right)}{6\ 3^{2/3}} + \frac{1}{6} c_2 (x - 4) x$$

1.231 problem 234

Internal problem ID [7721]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 234.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2y'' - x(2x+1)\,y' + 2(4x-1)\,y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 55

```
dsolve(2*x^2*diff(y(x),x$2)-x*(1+2*x)*diff(y(x),x)+2*(4*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^2(4x^2-36x+63) + c_2x^2(4x^2-36x+63)\left(\int \frac{e^x}{(4x^2-36x+63)^2\,x^{\frac{7}{2}}}dx\right)$$

✓ Solution by Mathematica
Time used: 1.738 (sec). Leaf size: 89

```
DSolve[2*x^2*y''[x]-x*(1+2*x)*y'[x]+2*(4*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to c_1\left(x^4 - 9x^3 + \frac{63x^2}{4}\right) - \frac{4c_2\left(\sqrt{\pi}(-4x^2+36x-63)\,x^{5/2}\text{erfi}\left(\sqrt{x}\right) + 2e^x(2x^4-17x^3+24x^2+6x+3)\right)}{945\sqrt{x}}$$

1.232 problem 235

Internal problem ID [7722]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 235.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' - 4x^2y' + (2x+1)\,y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve(4*x^2*diff(y(x),x$2)-4*x^2*diff(y(x),x)+(1+2*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x} + c_2\sqrt{x}\ \mathrm{expIntegral}_1\,(-x)$$

✓ Solution by Mathematica
Time used: 0.044 (sec). Leaf size: 19

```
DSolve[4*x^2*y''[x]-4*x^2*y'[x]+(1+2*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}(c_2\,\mathrm{ExpIntegralEi}(x) + c_1)$$

1.233 problem 236

Internal problem ID [7723]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 236.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(-2x+3)\,y' + (1-2x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)+x*(3-2*x)*diff(y(x),x)+(1-2*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + \frac{c_2 \operatorname{expIntegral}_1(-2x)}{x}$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 19

```
DSolve[x^2*y''[x]+x*(3-2*x)*y'[x]+(1-2*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 \operatorname{ExpIntegralEi}(2x) + c_1}{x}$$

1.234 problem 237

Internal problem ID [7724]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 237.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - x(x+3)\,y' + (-x+4)\,y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 60

```
dsolve(x^2*diff(y(x),x$2)-x*(3+x)*diff(y(x),x)+(4-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x x^2(x^2+4x+2) - \frac{c_2 x^2(-x^2 \operatorname{expIntegral}_1(x) + e^{-x}x - 4\operatorname{expIntegral}_1(x)\,x + 3\,e^{-x} - 2\operatorname{expIntegral}_1(x))\,e^x}{4}$$

✓ Solution by Mathematica

Time used: 0.213 (sec). Leaf size: 52

```
DSolve[x^2*y''[x]-x*(3+x)*y'[x]+(4-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}x^2(c_2e^x(x^2+4x+2)\operatorname{ExpIntegralEi}(-x) + 4c_1e^x(x^2+4x+2) + c_2(x+3))$$

1.235 problem 238

Internal problem ID [7725]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 238.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(3-x)\,y' + y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 35

```
dsolve(x^2*diff(y(x),x$2)+x*(3-x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x-1)}{x} + \frac{c_2(\operatorname{expIntegral}_1(-x)\,x - \operatorname{expIntegral}_1(-x) + \mathrm{e}^x)}{x}$$

✓ Solution by Mathematica
Time used: 0.21 (sec). Leaf size: 31

```
DSolve[x^2*y''[x]+x*(3-x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2(x-1)\operatorname{ExpIntegralEi}(x) + c_1(x-1) - c_2e^x}{x}$$

1.236 problem 239

Internal problem ID [7726]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 239.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - \left(2\sqrt{5}-1\right)xy' + \left(\frac{19}{4} - 3x^2\right)y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve(x^2*diff(y(x),x$2)-(2*sqrt(5)-1)*x*diff(y(x),x)+(19/4-3*x^2)*y(x)=0,y(x), singsol=all
```

$$y(x) = \frac{c_1 x^{\sqrt{5}} \sinh\left(\sqrt{3}\,x\right)}{\sqrt{x}} + \frac{c_2 x^{\sqrt{5}} \cosh\left(\sqrt{3}\,x\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.1 (sec). Leaf size: 53

```
DSolve[x^2*y''[x]-(2*Sqrt[5]-1)*x*y'[x]+(19/4-3*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{6} e^{-\sqrt{3}x} x^{\sqrt{5}-\frac{1}{2}} \left(\sqrt{3} c_2 e^{2\sqrt{3}x} + 6c_1\right)$$

1.237 problem 240

Internal problem ID [7727]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 240.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x(-3 + x)\,y' + (-x + 4)\,y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 28

```
dsolve(x^2*diff(y(x),x$2)+x*(x-3)*diff(y(x),x)+(4-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}c_1x^2 + c_2x^2\mathrm{e}^{-x}\,\mathrm{expIntegral}_1\left(-x\right)$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]+x*(x-3)*y'[x]+(4-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}x^2(c_2\,\mathrm{ExpIntegralEi}(x) + c_1)$$

1.238 problem 241

Internal problem ID [7728]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 241.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x^2y' - (x+2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x^2*diff(y(x),x$2)+x^2*diff(y(x),x)-(2+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2 - 2x + 2)}{x} + \frac{c_2 e^{-x}}{x}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 31

```
DSolve[x^2*y''[x]+x^2*y'[x]-(2+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}(c_2 e^x (x^2 - 2x + 2) + c_1)}{x}$$

1.239 problem 242

Internal problem ID [7729]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 242.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + 2x^2y' + \left(x - \frac{3}{4}\right)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x^2*diff(y(x),x$2)+2*x^2*diff(y(x),x)+(x-3/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{x}} + \frac{c_2 e^{-2x}(2x+1)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]+2*x^2*y'[x]+(x-3/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4c_1 - c_2 e^{-2x}(2x+1)}{4\sqrt{x}}$$

1.240 problem 243

Internal problem ID [7730]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 243.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1+x)\,y''+x^2y'-2y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(x^2*(1+x)*diff(y(x),x$2)+x^2*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(x+2)}{x}+\frac{c_2(\ln(x+1)\,x+2\ln(x+1)+4)}{x}$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 30

```
DSolve[x^2*(1+x)*y''[x]+x^2*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_1(x+2)+c_2(x+2)\log(x+1)+4c_2}{x}$$

1.241 problem 244

Internal problem ID [7731]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 244.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x(x^2+6)\,y' + 6y = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 44

```
dsolve(x^2*diff(y(x),x$2)+x*(6+x^2)*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2+3)}{x^2} + \frac{c_2(x^2+3)\left(\int \frac{e^{-\frac{x^2}{2}}}{x^2(x^2+3)^2}dx\right)}{x^2}$$

✓ Solution by Mathematica
Time used: 0.532 (sec). Leaf size: 65

```
DSolve[x^2*y''[x]+x*(6+x^2)*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{2\pi}c_2x(x^2+3)\operatorname{erf}\left(\frac{x}{\sqrt{2}}\right) - 12c_1x(x^2+3) + 2c_2e^{-\frac{x^2}{2}}(x^2+2)}{12x^3}$$

1.242 problem 245

Internal problem ID [7732]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 245.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' + x(1-x)\, y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)+x*(1-x)*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x+1)}{x} + \frac{c_2 \mathrm{e}^x}{x}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 23

```
DSolve[x^2*y''[x]+x*(1-x)*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 e^x - c_1(x+1)}{x}$$

1.243 problem 246

Internal problem ID [7733]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 246.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - x(x+3)\,y' + 4y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(x^2*diff(y(x),x$2)-x*(x+3)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^x x^2 (x+1) + c_2 x^2 \left(-\operatorname{expIntegral}_1(x)\, x - \operatorname{expIntegral}_1(x) + \mathrm{e}^{-x}\right) \mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 34

```
DSolve[x^2*y''[x]-x*(x+3)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 (c_2 e^x (x+1) \operatorname{ExpIntegralEi}(-x) + c_1 e^x (x+1) + c_2)$$

1.244 problem 247

Internal problem ID [7734]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 247.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x^2y' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)-x^2*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x+2)}{x} + \frac{c_2 e^x (x-2)}{x}$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 72

```
DSolve[x^2*y''[x]-x^2*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2e^{x/2}\left((c_1x + 2ic_2)\cosh\left(\frac{x}{2}\right) - (ic_2x + 2c_1)\sinh\left(\frac{x}{2}\right)\right)}{\sqrt{\pi}\sqrt{-ix}\sqrt{x}}$$

1.245 problem 248

Internal problem ID [7735]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 248.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x^2y' - (3x+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 70

```
dsolve(x^2*diff(y(x),x$2)-x^2*diff(y(x),x)-(3*x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^2\mathrm{e}^x(x+4) - \frac{c_2(-\operatorname{expIntegral}_1(x)\,x^4 + \mathrm{e}^{-x}x^3 - 4\operatorname{expIntegral}_1(x)\,x^3 + 3x^2\mathrm{e}^{-x} - 2\,\mathrm{e}^{-x}x + 2\,\mathrm{e}^{-x})\,\mathrm{e}^x}{24x}$$

✓ Solution by Mathematica

Time used: 0.16 (sec). Leaf size: 59

```
DSolve[x^2*y''[x]-x^2*y'[x]-(3*x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{24}c_2e^x(x+4)x^2\,\text{ExpIntegralEi}(-x) + c_1e^x(x+4)x^2 - \frac{c_2(x^3+3x^2-2x+2)}{24x}$$

1.246 problem 249

Internal problem ID [7736]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 249.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' + x(5-x)\, y' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 58

```
dsolve(x^2*diff(y(x),x$2)+x*(5-x)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2-4x+2)}{x^2} + \frac{c_2\left(\frac{x^2\,\mathrm{expIntegral}_1(-x)}{4} + \frac{x\,\mathrm{e}^x}{4} - \mathrm{expIntegral}_1\,(-x)\,x - \frac{3\,\mathrm{e}^x}{4} + \frac{\mathrm{expIntegral}_1(-x)}{2}\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.197 (sec). Leaf size: 48

```
DSolve[x^2*y''[x]+x*(5-x)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2(x^2-4x+2)\,\mathrm{ExpIntegralEi}(x) + 4c_1(x^2-4x+2) - c_2 e^x(x-3)}{4x^2}$$

1.247 problem 250

Internal problem ID [7737]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 250.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + 4x(1-x)\,y' + (2x-9)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(4*x^2*diff(y(x),x$2)+4*x*(1-x)*diff(y(x),x)+(2*x-9)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2+2x+2)}{x^{\frac{3}{2}}} + \frac{c_2\mathrm{e}^x}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 30

```
DSolve[4*x^2*y''[x]+4*x*(1-x)*y'[x]+(2*x-9)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1e^x - c_2(x^2+2x+2)}{x^{3/2}}$$

1.248 problem 251

Internal problem ID [7738]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 251.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + 2x(x+2)\,y' + 2(1+x)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(x^2*diff(y(x),x$2)+2*x*(2+x)*diff(y(x),x)+2*(1+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + \frac{c_2(2\,\mathrm{expIntegral}_1\,(2x)\,x - \mathrm{e}^{-2x})}{x^2}$$

✓ Solution by Mathematica

Time used: 0.072 (sec). Leaf size: 32

```
DSolve[x^2*y''[x]+2*x*(2+x)*y'[x]+2*(1+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-2c_2x\,\mathrm{ExpIntegralEi}(-2x) + c_1x - c_2e^{-2x}}{x^2}$$

1.249 problem 252

Internal problem ID [7739]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 252.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - x(1-x)\, y' + (1-x)\, y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x^2*diff(y(x),x$2)-x*(1-x)*diff(y(x),x)+(1-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x + \text{expIntegral}_1\,(x)\, xc_2$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 17

```
DSolve[x^2*y''[x]-x*(1-x)*y'[x]+(1-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(c_2\,\text{ExpIntegralEi}(-x) + c_1)$$

1.250 problem 253

Internal problem ID [7740]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 253.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + 4x(2x+1)\,y' + (4x-1)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(4*x^2*diff(y(x),x$2)+4*x*(1+2*x)*diff(y(x),x)+(4*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{x}} + \frac{c_2 e^{-2x}}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 26

```
DSolve[4*x^2*y''[x]+4*x*(1+2*x)*y'[x]+(4*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{2c_1 e^{-2x} + c_2}{2\sqrt{x}}$$

1.251 problem 254

Internal problem ID [7741]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 254.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(4+x)\,y' + (x+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(x^2*diff(y(x),x$2)+x*(4+x)*diff(y(x),x)+(2+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + \frac{c_2(-\operatorname{expIntegral}_1(x)\,x + e^{-x})}{x^2}$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 32

```
DSolve[x^2*y''[x]+x*(4+x)*y'[x]+(2+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-c_2x\,\text{ExpIntegralEi}(-x) + c_1x - c_2e^{-x}}{x^2}$$

1.252 problem 255

Internal problem ID [7742]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 255.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' + \left(x^2 - \frac{9}{4}\right)y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-9/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{ix}(x+i)}{x^{\frac{3}{2}}} + \frac{c_2 e^{-ix}(x-i)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.072 (sec). Leaf size: 44

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-9/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}}((c_1x + c_2)\cos(x) + (c_2x - c_1)\sin(x))}{x^{3/2}}$$

1.253 problem 256

Internal problem ID [7743]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 256.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + 2y' + yx = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{x} + \frac{c_2 \cos(x)}{x}$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 37

```
DSolve[x*y''[x]+2*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x}$$

1.254 problem 257

Internal problem ID [7744]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 257.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + 5(1-2x)\,y' - 5y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 42

```
dsolve(2*x*diff(y(x),x$2)+5*(1-2*x)*diff(y(x),x)-5*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(10x+1)}{x^{\frac{3}{2}}} + \frac{c_2(10x+1)\left(\int \frac{\sqrt{x}\,e^{5x}}{(10x+1)^2}dx\right)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 40

```
DSolve[2*x*y''[x]+5*(1-2*x)*y'[x]-5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 L_{-\frac{1}{2}}^{\frac{3}{2}}(5x) + \frac{c_1(10x+1)}{10\sqrt{5}x^{3/2}}$$

1.255 problem 258

Internal problem ID [7745]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 258.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2e^{2ix})}{2\sqrt{x}}$$

1.256 problem 259

Internal problem ID [7746]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 259.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x y'' + (x + n) y' + (n + 1) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(x*diff(y(x),x$2)+(x+n)*diff(y(x),x)+(n+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(-x+n) + c_2 \mathrm{e}^{-x}(-x+n)\left(\int \frac{\mathrm{e}^{x} x^{-n}}{(-x+n)^2} dx\right)$$

✓ Solution by Mathematica

Time used: 1.051 (sec). Leaf size: 48

```
DSolve[x*y''[x]+(x+n)*y'[x]+(n+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(n-x)\left(c_2 \int_1^x \frac{e^{K[1]} K[1]^{-n}}{(n-K[1])^2} dK[1] + c_1\right)$$

1.257 problem 260

Internal problem ID [7747]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 260.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^4 y'' + x y' + y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
dsolve(x^4*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2-1)}{x} + \frac{c_2(x^2-1)\left(\int \frac{x^2 e^{\frac{1}{2x^2}}}{(x+1)^2(x-1)^2} dx\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.218 (sec). Leaf size: 61

```
DSolve[x^4*y''[x]+x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{2\pi}c_2(x^2-1)\operatorname{erfi}\left(\frac{1}{\sqrt{2}x}\right) - 4c_1(x^2-1) + 2c_2 e^{\frac{1}{2x^2}} x}{4x}$$

1.258 problem 261

Internal problem ID [7748]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 261.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + (2x^2 + x)\, y' - 4y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(x^2*diff(y(x),x$2)+(x+2*x^2)*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(2x^2 - 4x + 3)}{x^2} + \frac{c_2 e^{-2x}(2x + 3)}{2x^2}$$

✓ Solution by Mathematica

Time used: 0.362 (sec). Leaf size: 44

```
DSolve[x^2*y''[x]+(x+2*x^2)*y'[x]-4*y[x]==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(\frac{2c_1 e^{-2x}(2x + 3)}{x^2} + \frac{c_2(2x^2 - 4x + 3)}{x^2} - 2\right)$$

1.259 problem 262

Internal problem ID [7749]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 262.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(4x^3-14x^2-2x\right)y''-\left(6x^2-7x+1\right)y'+\left(6x-1\right)y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve((4*x^3-14*x^2-2*x)*diff(y(x),x$2)-(6*x^2-7*x+1)*diff(y(x),x)+(6*x-1)*y(x)=0,y(x), sin
```

$$y(x)=c_1(x-1)+c_2\sqrt{x}\,(2x+1)$$

✓ Solution by Mathematica
Time used: 6.298 (sec). Leaf size: 26

```
DSolve[(4*x^3-14*x^2-2*x)*y''[x]-(6*x^2-7*x+1)*y'[x]+(6*x-1)*y[x]==0,y[x],x,IncludeSingularS
```

$$y(x)\to c_1(x-1)-2c_2\sqrt{x}(2x+1)$$

1.260 problem 263

Internal problem ID [7750]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 263.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' + x^2 y' + (x - 2) y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x^2*diff(y(x),x$2)+x^2*diff(y(x),x)+(x-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + \frac{c_2 e^{-x}(x^2 + 2x + 2)}{x}$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 29

```
DSolve[x^2*y''[x]+x^2*y'[x]+(x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 - c_2 e^{-x}(x^2 + 2x + 2)}{x}$$

1.261 problem 264

Internal problem ID [7751]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 264.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - x^2y' + (x-2)\,y = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 25

```
dsolve(x^2*diff(y(x),x$2)-x^2*diff(y(x),x)+(x-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2+2x+2)}{x} + \frac{c_2 e^x}{x}$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 28

```
DSolve[x^2*y''[x]-x^2*y'[x]+(x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 e^x - c_2(x^2+2x+2)}{x}$$

1.262 problem 265

Internal problem ID [7752]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 265.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-4x)\,y''-\frac{xy'}{2}-\frac{3yx}{4}=0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 131

```
dsolve(x^2*(1-4*x)*diff(y(x),x$2)+((1-(3/2))*x-(6-4*(3/2))*x^2)*diff(y(x),x)+(3/2)*(1-(3/2))
```

$$y(x)=c_1\sqrt{3x-1}\left(\frac{4\sqrt{(-1+4x)\,x}\,x+8x^2-2\sqrt{(-1+4x)\,x}-5x+1}{5x-1+2\sqrt{(-1+4x)\,x}}\right)^{\frac{1}{4}}+c_2\sqrt{3x-1}\left(\frac{5x-1+2\sqrt{(-1+4x)\,x}}{4\sqrt{(-1+4x)\,x}\,x+8x^2-2\sqrt{(-1+4x)\,x}-5x+1}\right)^{\frac{1}{4}}$$

✓ Solution by Mathematica
Time used: 0.385 (sec). Leaf size: 111

```
DSolve[x^2*(1-4*x)*y''[x]+((1-(3/2))*x-(6-4*(3/2))*x^2)*y'[x]+(3/2)*(1-(3/2))*x*y[x]==0,y[x]
```

$$y(x)\to\frac{\sqrt[4]{x}\sqrt[4]{4x-1}\left(6c_1\left(\sqrt{4x-1}-i\right)^{3/2}+ic_2\left(\sqrt{4x-1}+i\right)^{3/2}\right)}{6\sqrt[4]{1-4x}\sqrt[4]{\sqrt{4x-1}-i}\sqrt[4]{\sqrt{4x-1}+i}}$$

1.263 problem 266

Internal problem ID [7753]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 266.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + (x^2 + x)\, y' + (-9 + x)\, y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 40

```
dsolve(x^2*diff(y(x),x$2)+(x+x^2)*diff(y(x),x)+(x-9)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2 - 8x + 20)}{x^3} + \frac{c_2\mathrm{e}^{-x}(x^3 + 9x^2 + 36x + 60)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.209 (sec). Leaf size: 42

```
DSolve[x^2*y''[x]+(x+x^2)*y'[x]+(x-9)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1((x-8)x + 20) - c_2e^{-x}(x^3 + 9x^2 + 36x + 60)}{x^3}$$

1.264 problem 267

Internal problem ID [7754]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 267.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(1+x)\,y' + (3x-1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 60

```
dsolve(x^2*diff(y(x),x$2)+x*(x+1)*diff(y(x),x)+(3*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x\,\mathrm{e}^{-x}(x-3) + \frac{c_2(\mathrm{expIntegral}_1\,(-x)\,x^3 + \mathrm{e}^x x^2 - 3x^2\,\mathrm{expIntegral}_1\,(-x) - 2x\,\mathrm{e}^x - \mathrm{e}^x)\,\mathrm{e}^{-x}}{6x}$$

✓ Solution by Mathematica

Time used: 0.162 (sec). Leaf size: 66

```
DSolve[x^2*y''[x]+x*(x+1)*y'[x]+(3*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}(c_2(x-3)x^2\,\mathrm{ExpIntegralEi}(x) + 6c_1x^3 - x^2(c_2e^x + 18c_1) + 2c_2e^x x + c_2e^x)}{6x}$$

1.265 problem 268

Internal problem ID [7755]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 268.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - (x^2 + 4x)\, y' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(x^2*diff(y(x),x$2)-(x^2+4*x)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x x^4 - \frac{c_2 x\, e^x(-\operatorname{expIntegral}_1(x)\, x^3 + x^2 e^{-x} - e^{-x} x + 2\, e^{-x})}{6}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 41

```
DSolve[x^2*y''[x]-(x^2+4*x)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 e^x x^4 - \frac{1}{6} c_1 x\left(e^x x^3 \operatorname{ExpIntegralEi}(-x) + x^2 - x + 2\right)$$

1.266 problem 269

Internal problem ID [7756]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 269.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2y'' - (3x+2)\,y' + \frac{(2x-1)\,y}{x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(2*x^2*diff(y(x),x$2)-(3*x+2)*diff(y(x),x)+(2*x-1)/x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(5x+2)}{\sqrt{x}} + \frac{c_2(5x+2)\left(\int \frac{x^{\frac{5}{2}}\,\mathrm{e}^{-\frac{1}{x}}}{(5x+2)^2}dx\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.225 (sec). Leaf size: 70

```
DSolve[2*x^2*y''[x]-(3*x+2)*y'[x]+(2*x-1)/x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2\sqrt{\pi}c_2(5x+2)\text{erf}\left(\frac{1}{\sqrt{x}}\right)}{3\sqrt{x}} + \frac{2}{3}c_2e^{-1/x}\left(x^2-4x-2\right) + \frac{c_1(5x+2)}{5\sqrt{x}}$$

1.267 problem 270

Internal problem ID [7757]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 270.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$x(1-x)\,y'' + \left(-2x+\frac{3}{2}\right)y' - \frac{y}{4} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(x*(1-x)*diff(y(x),x$2)+(3/2-2*x)*diff(y(x),x)-1/4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{x}} + \frac{c_2 \ln\left(x - \frac{1}{2} + \sqrt{x\,(x-1)}\right)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.111 (sec). Leaf size: 51

```
DSolve[x*(1-x)*y''[x]+(3/2-2*x)*y'[x]-1/4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{\sqrt{x}} - \frac{2c_2\sqrt{x-1}\log\left(\sqrt{x-1}-\sqrt{x}\right)}{\sqrt{-((x-1)x)}}$$

1.268 problem 271

Internal problem ID [7758]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 271.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x(1-x)\,y''+xy'-y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(2*x*(1-x)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+c_2\left(\arctan\left(\sqrt{x-1}\right)x-\sqrt{x-1}\right)$$

✓ Solution by Mathematica

Time used: 0.11 (sec). Leaf size: 43

```
DSolve[2*x*(1-x)*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\sqrt[4]{2}\left(c_2x\text{arctanh}\left(\sqrt{1-x}\right)+c_1x-c_2\sqrt{1-x}\right)$$

1.269 problem 272

Internal problem ID [7759]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 272.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$\boxed{2x(1-x)\,y''+(1-11x)\,y'-10y=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 33

```
dsolve(2*x*(1-x)*diff(y(x),x$2)+(1-11*x)*diff(y(x),x)-10*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(x^2+6x+1)}{(x-1)^4}+\frac{c_2\sqrt{x}\,(x+1)}{(x-1)^4}$$

✓ Solution by Mathematica
Time used: 0.103 (sec). Leaf size: 35

```
DSolve[2*x*(1-x)*y''[x]+(1-11*x)*y'[x]-10*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_1\sqrt{x}(x+1)-2c_2(x^2+6x+1)}{(x-1)^4}$$

1.270 problem 273

Internal problem ID [7760]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 273.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$x(1-x)\,y''+\frac{(1-2x)\,y'}{3}+\frac{20y}{9}=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*(1-x)*diff(y(x),x$2)+1/3*(1-2*x)*diff(y(x),x)+20/9*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(6x-5)\,x^{\frac{2}{3}}+c_2(6x-1)\,(x-1)^{\frac{2}{3}}$$

✓ Solution by Mathematica

Time used: 0.057 (sec). Leaf size: 51

```
DSolve[x*(1-x)*y''[x]+1/3*(1-2*x)*y'[x]+20/9*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x)\to c_2\sqrt[3]{-((x-1)x)}Q_1^{\frac{2}{3}}(2x-1)+\frac{c_1x^{2/3}(6x-5)}{3\,\text{Gamma}\left(\frac{4}{3}\right)}$$

1.271 problem 274

Internal problem ID [7761]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 274.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4y'' + \frac{3(-x^2+2)\,y}{(1-x^2)^2} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(4*diff(y(x),x$2)+3*(2-x^2)/(1-x^2)^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(x^2-1\right)^{\frac{1}{4}} x + c_2\left(x^2-1\right)^{\frac{3}{4}}$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 51

```
DSolve[4*y''[x]+3*(2-x^2)/(1-x^2)^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x^2-1}\left(c_2 Q_{\frac{1}{2}}^{\frac{1}{2}}(x) + \frac{\sqrt{\frac{2}{\pi}}c_1 x}{\sqrt[4]{1-x^2}}\right)$$

1.272 problem 275

Internal problem ID [7762]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 275.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$u'' - \frac{2u'}{x} - a^2 u = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(u(x),x$2)-2/x*diff(u(x),x)-a^2*u(x)=0,u(x), singsol=all)
```

$$u(x) = c_1 e^{ax}(ax - 1) + \frac{c_2 e^{-ax}(ax + 1)}{a}$$

✓ Solution by Mathematica
Time used: 0.146 (sec). Leaf size: 68

```
DSolve[u''[x]-2/x*u'[x]-a^2*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to \frac{\sqrt{\frac{2}{\pi}}\sqrt{x}((iac_2x + c_1)\sinh(ax) - (ac_1x + ic_2)\cosh(ax))}{a\sqrt{-iax}}$$

1.273 problem 276

Internal problem ID [7763]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 276.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{u'' + \frac{2u'}{x} - a^2 u = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 23

```
dsolve(diff(u(x),x$2)+2/x*diff(u(x),x)-a^2*u(x)=0,u(x), singsol=all)
```

$$u(x) = \frac{c_1 \sinh(ax)}{x} + \frac{c_2 \cosh(ax)}{x}$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 35

```
DSolve[u''[x]+2/x*u'[x]-a^2*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to \frac{2ac_1 e^{-ax} + c_2 e^{ax}}{2ax}$$

1.274 problem 277

Internal problem ID [7764]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 277.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{u'' + \frac{2u'}{x} + a^2 u = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(u(x),x$2)+2/x*diff(u(x),x)+a^2*u(x)=0,u(x), singsol=all)
```

$$u(x) = \frac{c_1 \sin(ax)}{x} + \frac{c_2 \cos(ax)}{x}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 42

```
DSolve[u''[x]+2/x*u'[x]+a^2*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to \frac{e^{-iax}\left(2c_1 - \frac{ic_2 e^{2iax}}{a}\right)}{2x}$$

1.275 problem 278

Internal problem ID [7765]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 278.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{u'' + \frac{4u'}{x} - a^2 u = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve(diff(u(x),x$2)+4/x*diff(u(x),x)-a^2*u(x)=0,u(x), singsol=all)
```

$$u(x) = \frac{c_1 \mathrm{e}^{ax}(ax - 1)}{x^3} + \frac{c_2 \mathrm{e}^{-ax}(ax + 1)}{x^3 a}$$

✓ Solution by Mathematica

Time used: 0.107 (sec). Leaf size: 68

```
DSolve[u''[x]+4/x*u'[x]-a^2*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to \frac{\sqrt{\frac{2}{\pi}}((iac_2x + c_1)\sinh(ax) - (ac_1x + ic_2)\cosh(ax))}{ax^{5/2}\sqrt{-iax}}$$

1.276 problem 279

Internal problem ID [7766]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 279.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{u'' + \frac{4u'}{x} + a^2u = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 41

```
dsolve(diff(u(x),x$2)+4/x*diff(u(x),x)+a^2*u(x)=0,u(x), singsol=all)
```

$$u(x) = \frac{c_1(\cos(ax)\,ax - \sin(ax))}{x^3} + \frac{c_2(\cos(ax) + \sin(ax)\,ax)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.097 (sec). Leaf size: 57

```
DSolve[u''[x]+4/x*u'[x]+a^2*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to -\frac{\sqrt{\frac{2}{\pi}}((ac_1x + c_2)\cos(ax) + (ac_2x - c_1)\sin(ax))}{x^{3/2}(ax)^{3/2}}$$

1.277 problem 280

Internal problem ID [7767]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 280.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - a^2 y - \frac{6y}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-a^2*y(x)=6*y(x)/x^2,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{-ax}(a^2x^2 + 3ax + 3)}{x^2a^2} + \frac{c_2 e^{ax}(a^2x^2 - 3ax + 3)}{3x^2}$$

✓ Solution by Mathematica

Time used: 0.145 (sec). Leaf size: 90

```
DSolve[y''[x]-a^2*y[x]==6*y[x]/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{\frac{2}{\pi}}((a^2c_2x^2 - 3iac_1x + 3c_2)\cosh(ax) + i(c_1(a^2x^2 + 3) + 3iac_2x)\sinh(ax))}{a^2x^{3/2}\sqrt{-iax}}$$

1.278 problem 281

Internal problem ID [7768]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 281.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + n^2 y - \frac{6y}{x^2} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 67

```
dsolve(diff(y(x),x$2)+n^2*y(x)=6*y(x)/x^2,y(x), singsol=all)
```

$$y(x) = \frac{c_1(\cos(nx)\, x^2n^2 - 3\sin(nx)\, nx - 3\cos(nx))}{x^2} + \frac{c_2(\sin(nx)\, x^2n^2 + 3\cos(nx)\, nx - 3\sin(nx))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.148 (sec). Leaf size: 79

```
DSolve[y''[x]+n^2*y[x]==6*y[x]/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}}\sqrt{x}((c_2(-n^2)\, x^2 + 3c_1nx + 3c_2)\cos(nx) + (c_1(n^2x^2 - 3) + 3c_2nx)\sin(nx))}{(nx)^{5/2}}$$

1.279 problem 282

Internal problem ID [7769]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 282.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' - \left(x^2 + \frac{1}{4}\right) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-(x^2+1/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{c_1 \sinh(x)}{\sqrt{x}} + \frac{c_2 \cosh(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 32

```
DSolve[x^2*y''[x]+x*y'[x]-(x^2+1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}(c_2e^{2x} + 2c_1)}{2\sqrt{x}}$$

1.280 problem 283

Internal problem ID [7770]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 283.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' + x y' + \frac{(-9a^2 + 4x^2)\, y}{4a^2} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 43

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(4*x^2-9*a^2)/(4*a^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{\frac{ix}{a}} (-ix + a)}{x^{\frac{3}{2}}} + \frac{c_2 e^{-\frac{ix}{a}} (ix + a)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.112 (sec). Leaf size: 62

```
DSolve[x^2*y''[x]+x*y'[x]+(4*x^2-9*a^2)/(4*a^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}} \left((a c_2 + c_1 x) \cos\left(\frac{x}{a}\right) + (c_2 x - a c_1) \sin\left(\frac{x}{a}\right) \right)}{x \sqrt{\frac{x}{a}}}$$

1.281 problem 284

Internal problem ID [7771]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 284.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + xy' + \left(x^2 - \frac{25}{4}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-25/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{ix}(x^2 + 3ix - 3)}{x^{\frac{5}{2}}} + \frac{c_2 e^{-ix}(x^2 - 3ix - 3)}{x^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.121 (sec). Leaf size: 59

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-25/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}}((-c_2x^2 + 3c_1x + 3c_2)\cos(x) + (c_1(x^2 - 3) + 3c_2x)\sin(x))}{x^{5/2}}$$

1.282 problem 285

Internal problem ID [7772]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 285.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + qy' - \frac{2y}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)+q*diff(y(x),x)=2*y(x)/x^2,y(x), singsol=all)
```

$$y(x) = \frac{c_1(qx - 2)}{x} + \frac{c_2 e^{-qx}(qx + 2)}{qx}$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 80

```
DSolve[y''[x]+q*y'[x]==2*y[x]/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{q x^{3/2} e^{-\frac{qx}{2}} \left(2(ic_2 qx + 2c_1)\sinh\left(\frac{qx}{2}\right) - 2(c_1 qx + 2ic_2)\cosh\left(\frac{qx}{2}\right)\right)}{\sqrt{\pi}(-iqx)^{5/2}}$$

1.283 problem 286

Internal problem ID [7773]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 286.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + 3y' + 4yx^3 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x$2)+3*diff(y(x),x)+4*x^3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x^2)}{x^2} + \frac{c_2 \cos(x^2)}{x^2}$$

✓ Solution by Mathematica
Time used: 0.075 (sec). Leaf size: 41

```
DSolve[x*y''[x]+3*y'[x]+4*x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4c_1 e^{-ix^2} - ic_2 e^{ix^2}}{4x^2}$$

1.284 problem 287

Internal problem ID [7774]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 287.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2 + 2x)\, y'' - 2(1 + x)\, y' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2+2*x)*diff(y(x),x$2)-2*(x+1)*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x + 1) + c_2x^2$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 19

```
DSolve[(x^2+2*x)*y''[x]-2*(x+1)*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1x^2 - c_2(x + 1)$$

1.285 problem 288

Internal problem ID [7775]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 288.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2 + 2x)\, y'' - 2(1 + x)\, y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2+2*x)*diff(y(x),x$2)-2*(x+1)*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x + 1) + c_2x^2$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 19

```
DSolve[(x^2+2*x)*y''[x]-2*(x+1)*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1x^2 - c_2(x + 1)$$

1.286 problem 289

Internal problem ID [7776]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 289.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''-2xy'+2y=0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 15

```
dsolve((x^2+1)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+c_2(x^2-1)$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 21

```
DSolve[(x^2+1)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_2x-c_1(x-i)^2$$

1.287 problem 290

Internal problem ID [7777]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 290.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''-2xy'+2y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2+1)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+c_2(x^2-1)$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 21

```
DSolve[(x^2+1)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_2x-c_1(x-i)^2$$

1.288 problem 291

Internal problem ID [7778]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 291.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4xy' + (4x^2 - 2)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{x^2} + c_2 x\, e^{x^2}$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 18

```
DSolve[y''[x]-4*x*y'[x]+(4*x^2-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x^2} (c_2 x + c_1)$$

1.289 problem 292

Internal problem ID [7779]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 292.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4xy' + (4x^2 - 2)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{x^2} + c_2 x\, e^{x^2}$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 18

```
DSolve[y''[x]-4*x*y'[x]+(4*x^2-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x^2}(c_2 x + c_1)$$

1.290 problem 293

Internal problem ID [7780]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 293.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x-3)\,y''-xy'+y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve((2*x-3)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+c_2x\left(\int\frac{(-3+2x)^{\frac{3}{4}}\,\mathrm{e}^{\frac{x}{2}}}{x^2}dx\right)$$

✓ Solution by Mathematica

Time used: 0.08 (sec). Leaf size: 63

```
DSolve[(2*x-3)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to 2\ \ 2^{3/4}(2x-3)\left(c_2(2x-3)^{3/4}L_{-\frac{3}{4}}^{\frac{7}{4}}\left(\frac{x}{2}-\frac{3}{4}\right)+\frac{4\sqrt{2}c_1x}{2x-3}\right)$$

1.291 problem 294

Internal problem ID [7781]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 294.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Hermite]`

$$\boxed{y'' - xy' - 3y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 47

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{x^2}{2}} \left(x^2+1\right) + c_2 \mathrm{e}^{\frac{x^2}{2}} \left(x^2+1\right) \left(\int \frac{\mathrm{e}^{-\frac{x^2}{2}}}{\left(x^2+1\right)^2} dx\right)$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 35

```
DSolve[y''[x]-x*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{HermiteH}\left(-3, \frac{x}{\sqrt{2}}\right) + c_2 e^{\frac{x^2}{2}} \left(x^2+1\right)$$

1.292 problem 295

Internal problem ID [7782]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 295.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''-xy'+y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve((1+x^2)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+c_2\left(\operatorname{arcsinh}(x)\,x-\sqrt{x^2+1}\right)$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 42

```
DSolve[(1+x^2)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -c_2\sqrt{x^2+1}-c_2x\log\left(\sqrt{x^2+1}-x\right)+c_1x$$

1.293 problem 296

Internal problem ID [7783]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 296.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Hermite]`

$$\boxed{y'' - xy' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x^2 - 1) + c_2(x^2 - 1)\left(\int \frac{e^{\frac{x^2}{2}}}{(x-1)^2(x+1)^2}dx\right)$$

✓ Solution by Mathematica
Time used: 0.107 (sec). Leaf size: 54

```
DSolve[y''[x]-x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}c_2\left(\sqrt{2\pi}(x^2 - 1)\operatorname{erfi}\left(\frac{x}{\sqrt{2}}\right) - 2e^{\frac{x^2}{2}}x\right) + c_1(x^2 - 1)$$

1.294 problem 297

Internal problem ID [7784]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 297.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1-x^2)\,y''-y'+y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 177

```
dsolve((1-x^2)*diff(y(x),x$2)-diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\sqrt{2x+3}\left(\frac{3\sqrt{5}\,x+2\sqrt{5}-5\sqrt{x^2-1}}{3\sqrt{5}\,x+2\sqrt{5}+5\sqrt{x^2-1}}\right)^{\frac{1}{4}}\left(x+\sqrt{x^2-1}\right)^{\frac{3\sqrt{5}}{10}}\left(x+\sqrt{x^2-1}\right)^{\frac{\sqrt{5}}{5}}+c_2\sqrt{2x+3}\left(\frac{3\sqrt{5}\,x+2\sqrt{5}+5\sqrt{x^2-1}}{3\sqrt{5}\,x+2\sqrt{5}-5\sqrt{x^2-1}}\right)^{\frac{1}{4}}\left(x+\sqrt{x^2-1}\right)^{-\frac{3\sqrt{5}}{10}}\left(x+\sqrt{x^2-1}\right)^{-\frac{\sqrt{5}}{5}}$$

✓ Solution by Mathematica

Time used: 30.669 (sec). Leaf size: 198

```
DSolve[(1-x^2)*y''[x]-y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{\sqrt[4]{x+1}\left(\sqrt{x+1}-\sqrt{x-1}\right)^{-1-\sqrt{5}}\left(-2x+2\sqrt{x-1}\sqrt{x+1}+\sqrt{5}-3\right)e^{-\operatorname{arctanh}\left(x-\sqrt{x-1}\sqrt{x+1}\right)}\left(c_2\int_1^x \frac{e^{2\operatorname{arct}\cdots}}{\cdots}\right.}{\sqrt[4]{1-x}}$$

1.295 problem 298

Internal problem ID [7785]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 298.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(1+x)^2 y'' + (1-x^2) y' + (x-1) y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*(x+1)^2*diff(y(x),x$2)+(1-x^2)*diff(y(x),x)+(x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x+1) + c_2(x+1)\ln(x)$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 17

```
DSolve[x*(x+1)^2*y''[x]+(1-x^2)*y'[x]+(x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x+1)(c_2 \log(x) + c_1)$$

1.296 problem 299

Internal problem ID [7786]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 299.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2xy'' - y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 75

```
dsolve(2*x*diff(y(x),x$2)-diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{2i\sqrt{x}} \sqrt{\frac{(1+4x)\,(2i\sqrt{x}-1)}{1+2i\sqrt{x}}} + c_2 e^{-2i\sqrt{x}} \sqrt{\frac{(1+4x)\,(1+2i\sqrt{x})}{2i\sqrt{x}-1}}$$

✓ Solution by Mathematica

Time used: 0.21 (sec). Leaf size: 59

```
DSolve[2*x*y''[x]-y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{2i\sqrt{x}}(2\sqrt{x}+i) + \frac{1}{8} c_2 e^{-2i\sqrt{x}}(1+2i\sqrt{x})$$

1.297 problem 300

Internal problem ID [7787]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 300.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + xy' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
dsolve(x*diff(y(x),x$2)+x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x^2 + 2x) + c_2\left(\frac{x^2 \,\mathrm{expIntegral}_1(x)}{2} - \frac{\mathrm{e}^{-x}x}{2} + \mathrm{expIntegral}_1(x)\, x - \frac{\mathrm{e}^{-x}}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.131 (sec). Leaf size: 39

```
DSolve[x*y''[x]+x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x(x+2) - \frac{1}{2}c_2 e^{-x}(e^x(x+2)x \,\mathrm{ExpIntegralEi}(-x) + x + 1)$$

1.298 problem 301

Internal problem ID [7788]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 301.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(x-1)^2 y'' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
dsolve(x*(x-1)^2*diff(y(x),x$2)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x}{x-1} + \frac{c_2(2x\ln(x) - x^2 + 1)}{x-1}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 33

```
DSolve[x*(x-1)^2*y''[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-c_2 x^2 - c_1 x + 2c_2 x \log(x) + c_2}{x-1}$$

1.299 problem 302

Internal problem ID [7789]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 302.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2xy' + x^2 y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{x^2}{2}} \cos(x) + c_2 e^{\frac{x^2}{2}} \sin(x)$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 39

```
DSolve[y''[x]-2*x*y'[x]+x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{\frac{1}{2}x(x-2i)} \left(2c_1 - ic_2 e^{2ix}\right)$$

1.300 problem 303

Internal problem ID [7790]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 303.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(-x^2+2)\,y''-(x^2+4x+2)\,((1-x)\,y'+y)=0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve(x*(2-x^2)*diff(y(x),x$2)-(x^2+4*x+2)*((1-x)*diff(y(x),x)+y(x))=0,y(x), singsol=all)
```

$$y(x)=c_1(x-1)+c_2\mathrm{e}^x x^2$$

✓ Solution by Mathematica
Time used: 0.14 (sec). Leaf size: 21

```
DSolve[x*(2-x^2)*y''[x]-(x^2+4*x+2)*((1-x)*y'[x]+y[x])==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to c_1e^x x^2+c_2(x-1)$$

1.301 problem 304

Internal problem ID [7791]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 304.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1+x)\,y''-(2x+1)\,(xy'-y)=0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*(1+x)*diff(y(x),x$2)-(1+2*x)*(x*diff(y(x),x)-y(x))=0,y(x), singsol=all)
```

$$y(x)=c_1x+c_2x(x+\ln(x))$$

✓ Solution by Mathematica
Time used: 0.188 (sec). Leaf size: 132

```
DSolve[x^2*(1+x)*y''[x]-(1+2*x)*(x*y'[x]+y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_2x^{1+\sqrt{2}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}+\sqrt{2}-\frac{\sqrt{17}}{2},-\frac{1}{2}+\sqrt{2}+\frac{\sqrt{17}}{2},1+2\sqrt{2},-x\right)+c_1x^{1-\sqrt{2}}\,\text{Hypergeometric2F1}\left(\frac{1}{2}\left(-1-2\sqrt{2}-\sqrt{17}\right),\frac{1}{2}\left(-1-2\sqrt{2}+\sqrt{17}\right),1-2\sqrt{2},-x\right)$$

1.302 problem 305

Internal problem ID [7792]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 305.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2(-x+2)\,x^2y''-(-x+4)\,xy'+(3-x)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(2*(2-x)*x^2*diff(y(x),x$2)-(4-x)*x*diff(y(x),x)+(3-x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\sqrt{x}+c_2\sqrt{x^2-2x}$$

✓ Solution by Mathematica

Time used: 0.064 (sec). Leaf size: 41

```
DSolve[2*(2-x)*x^2*y''[x]-(4-x)*x*y'[x]+(3-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to\frac{\sqrt[4]{x-2}\sqrt{x}\left(2c_2\sqrt{x-2}+c_1\right)}{\sqrt[4]{2-x}}$$

1.303 problem 306

Internal problem ID [7793]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 306.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(1-x)\,x^2y'' + (5x-4)\,xy' + (6-9x)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve((1-x)*x^2*diff(y(x),x$2)+(5*x-4)*x*diff(y(x),x)+(6-9*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^3 + c_2x^2(x\ln(x) + 1)$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 24

```
DSolve[(1-x)*x^2*y''[x]+(5*x-4)*x*y'[x]+(6-9*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to x^2(c_1x - c_2(x\log(x) + 1))$$

1.304 problem 307

Internal problem ID [7794]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 307.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' + (4x^2 + 1)\, y' + 4x(x^2 + 1)\, y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x$2)+(4*x^2+1)*diff(y(x),x)+4*x*(x^2+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x^2} + c_2 e^{-x^2} \ln(x)$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 21

```
DSolve[x*y''[x]+(4*x^2+1)*y'[x]+4*x*(x^2+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x^2}(c_2 \log(x) + c_1)$$

1.305 problem 308

Internal problem ID [7795]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 308.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2xy' + 8y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\frac{4}{3}x^4 - 4x^2 + 1\right) + c_2\left(\frac{4}{3}x^4 - 4x^2 + 1\right)\left(\int \frac{\mathrm{e}^{x^2}}{(4x^4 - 12x^2 + 3)^2}dx\right)$$

✓ Solution by Mathematica
Time used: 1.113 (sec). Leaf size: 63

```
DSolve[y''[x]-2*x*y'[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\left(x^4 - 3x^2 + \frac{3}{4}\right) - \frac{1}{12}c_2\left(\sqrt{\pi}(-4x^4 + 12x^2 - 3)\operatorname{erfi}(x) + 2e^{x^2}x(2x^2 - 5)\right)$$

1.306 problem 309

Internal problem ID [7796]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 309.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2xy' + 8y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\frac{4}{3}x^4 - 4x^2 + 1\right) + c_2\left(\frac{4}{3}x^4 - 4x^2 + 1\right)\left(\int \frac{e^{x^2}}{\left(4x^4 - 12x^2 + 3\right)^2}dx\right)$$

✓ Solution by Mathematica
Time used: 0.075 (sec). Leaf size: 63

```
DSolve[y''[x]-2*x*y'[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\left(x^4 - 3x^2 + \frac{3}{4}\right) - \frac{1}{12}c_2\left(\sqrt{\pi}\left(-4x^4 + 12x^2 - 3\right)\operatorname{erfi}(x) + 2e^{x^2}x\left(2x^2 - 5\right)\right)$$

1.307 problem 310

Internal problem ID [7797]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 310.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(1-x^2)\,y''-2xy'+12y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(-\frac{5}{3}x^3+x\right)+c_2\left(-\frac{5\ln(x+1)\,x^3}{24}+\frac{5\ln(x-1)\,x^3}{24}+\frac{\ln(x+1)\,x}{8}-\frac{\ln(x-1)\,x}{8}+\frac{5x^2}{12}-\frac{1}{9}\right)$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 59

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}c_1x(5x^2-3)+c_2\left(-\frac{5x^2}{2}-\frac{1}{4}(5x^2-3)\,x(\log(1-x)-\log(x+1))+\frac{2}{3}\right)$$

1.308 problem 311

Internal problem ID [7798]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 311.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(x+2)\,y''+2(1+x)\,y'-2y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
dsolve(x*(x+2)*diff(y(x),x$2)+2*(x+1)*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(x+1)+c_2\left(\frac{x\ln(x)}{2}-\frac{\ln(x+2)\,x}{2}+\frac{\ln(x)}{2}-\frac{\ln(x+2)}{2}+1\right)$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 37

```
DSolve[x*(x+2)*y''[x]+2*(x+1)*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1(x+1)-\frac{1}{2}c_2((x+1)\log(-x)-(x+1)\log(x+2)+2)$$

1.309 problem 313

Internal problem ID [7799]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 313.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$x(x+2)\,y''+(1+x)\,y'-4y=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x*(x+2)*diff(y(x),x$2)+(x+1)*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(2x^2+4x+1\right)+c_2(x+1)\sqrt{x\,(x+2)}$$

✓ Solution by Mathematica
Time used: 0.101 (sec). Leaf size: 53

```
DSolve[x*(x+2)*y''[x]+(x+1)*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\cosh\left(4\log\left(\sqrt{x+2}-\sqrt{x}\right)\right)-ic_2\sinh\left(4\log\left(\sqrt{x+2}-\sqrt{x}\right)\right)$$

1.310 problem 314

Internal problem ID [7800]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 314.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-1)\,y''-xy'+y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+\mathrm{e}^xc_2$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 17

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1e^x-c_2x$$

1.311 problem 315

Internal problem ID [7801]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 315.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''-2xy'+2y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((1+x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+c_2(x^2-1)$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 21

```
DSolve[(1+x^2)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_2x-c_1(x-i)^2$$

1.312 problem 316

Internal problem ID [7802]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 316.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2 - 2x + 10)\, y'' + xy' - 4y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 50

```
dsolve((x^2-2*x+10)*diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(x^2 - \frac{4}{3}x + 5\right) + c_2(3x-4)\sqrt{x^2-2x+10}\left(\frac{-x+1+3i}{x-1+3i}\right)^{\frac{i}{6}}$$

✓ Solution by Mathematica

Time used: 0.842 (sec). Leaf size: 92

```
DSolve[(x^2-2*x+10)*y''[x]+x*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}(3x - 4)\sqrt{x^2-2x+10}e^{-\frac{1}{3}\arctan\left(\frac{x-1}{3}\right)}\left(c_2\int_1^x \frac{9e^{\frac{1}{3}\arctan\left(\frac{1}{3}(K[1]-1)\right)}}{(4-3K[1])^2\,(K[1]^2-2K[1]+10)^{3/2}}dK[1] + c_1\right)$$

1.313 problem 317

Internal problem ID [7803]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 317.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-2x+10)\,y''+xy'-4y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 50

```
dsolve((x^2-2*x+10)*diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x^2-\frac{4}{3}x+5\right)+c_2(3x-4)\sqrt{x^2-2x+10}\left(\frac{-x+1+3i}{x-1+3i}\right)^{\frac{i}{6}}$$

✓ Solution by Mathematica

Time used: 0.571 (sec). Leaf size: 92

```
DSolve[(x^2-2*x+10)*y''[x]+x*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{3}(3x-4)\sqrt{x^2-2x+10}e^{-\frac{1}{3}\arctan\left(\frac{x-1}{3}\right)}\left(c_2\int_1^x\frac{9e^{\frac{1}{3}\arctan\left(\frac{1}{3}(K[1]-1)\right)}}{(4-3K[1])^2\,(K[1]^2-2K[1]+10)^{3/2}}dK[1]+c_1\right)$$

1.314 problem 318

Internal problem ID [7804]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 318.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Hermite]`

$$y'' - xy' + 2y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x^2 - 1) + c_2(x^2 - 1)\left(\int \frac{e^{\frac{x^2}{2}}}{(x-1)^2 (x+1)^2} dx\right)$$

✓ Solution by Mathematica
Time used: 0.114 (sec). Leaf size: 54

```
DSolve[y''[x]-x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} c_2 \left(\sqrt{2\pi}(x^2 - 1) \operatorname{erfi}\left(\frac{x}{\sqrt{2}}\right) - 2e^{\frac{x^2}{2}} x\right) + c_1(x^2 - 1)$$

1.315 problem 319

Internal problem ID [7805]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 319.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x+2)\,y''+xy'-y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve((x+2)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+c_2\mathrm{e}^{-x}(x+4)$$

✓ Solution by Mathematica
Time used: 0.126 (sec). Leaf size: 72

```
DSolve[(x+2)*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{2\sqrt{\frac{2}{\pi}}e^{-x-2}\sqrt{x+2}(c_1(e^{x+2}x+x+4)-ic_2((e^{x+2}-1)\,x-4))}{\sqrt{-i(x+2)}}$$

1.316 problem 320

Internal problem ID [7806]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 320.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{(x^2+1)\,y''-6y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve((x^2+1)*diff(y(x),x$2)-6*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x^3+x\right)+c_2\left(\frac{3\arctan\left(x\right)x^3}{2}+\frac{3\arctan\left(x\right)x}{2}+\frac{3x^2}{2}+1\right)$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 36

```
DSolve[(x^2+1)*y''[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\left(x^3+x\right)-\frac{1}{2}c_2\left(3\left(x^3+x\right)\arctan(x)+3x^2+2\right)$$

1.317 problem 321

Internal problem ID [7807]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 321.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2+2)\,y''+3xy'-y=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 67

```
dsolve((x^2+2)*diff(y(x),x$2)+3*x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\left(\sqrt{2}\,x+\sqrt{2}\,\sqrt{x^2+2}\right)^{\sqrt{2}}}{\sqrt{x^2+2}}+\frac{c_2\left(\frac{\sqrt{2}}{2\sqrt{x^2+2}+2x}\right)^{\sqrt{2}}}{\sqrt{x^2+2}}$$

✓ Solution by Mathematica
Time used: 0.108 (sec). Leaf size: 92

```
DSolve[(x^2+2)*y''[x]+3*x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{2^{3/4}c_1\cos\left(2\sqrt{2}\arcsin\left(\frac{1}{2}\sqrt{2-i\sqrt{2}x}\right)\right)}{\sqrt{\pi}\sqrt{x^2+2}}+\frac{c_2Q_{-\frac{1}{2}+\sqrt{2}}^{\frac{1}{2}}\left(\frac{ix}{\sqrt{2}}\right)}{\sqrt[4]{x^2+2}}$$

1.318 problem 322

Internal problem ID [7808]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 322.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-1)\,y''-xy'+y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+\mathrm{e}^{x}c_2$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 17

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1e^x-c_2x$$

1.319 problem 323

Internal problem ID [7809]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 323.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2xy' + 8y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\frac{4}{3}x^4 - 4x^2 + 1\right) + c_2\left(\frac{4}{3}x^4 - 4x^2 + 1\right)\left(\int \frac{e^{x^2}}{(4x^4 - 12x^2 + 3)^2}dx\right)$$

✓ Solution by Mathematica
Time used: 0.047 (sec). Leaf size: 49

```
DSolve[y''[x]-2*x*y''[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{4x-2}\left(c_1 \operatorname{BesselI}\left(1, 4\sqrt{x-\frac{1}{2}}\right) - c_2 K_1\left(4\sqrt{x-\frac{1}{2}}\right)\right)$$

1.320 problem 325

Internal problem ID [7810]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 325.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \left(\frac{5}{3}x + x^2\right)y' - \frac{y}{3} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(x^2*diff(y(x),x$2)+(5/3*x+x^2)*diff(y(x),x)-1/3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(3x-1)}{x} + \frac{c_2(3x-1)\left(\int \frac{x^{\frac{1}{3}}\mathrm{e}^{-x}}{(3x-1)^2}dx\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.355 (sec). Leaf size: 47

```
DSolve[x^2*y''[x]+(5/3*x+x^2)*y'[x]-1/3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{-3c_1x + 3c_2e^{-x}\sqrt[3]{x} + c_2(1-3x)\Gamma\left(\frac{1}{3},x\right) + c_1}{3x}$$

1.321 problem 326

Internal problem ID [7811]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 326.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2xy'' - y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 75

```
dsolve(2*x*diff(y(x),x$2)-diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{2i\sqrt{x}} \sqrt{\frac{(1+4x)\left(2i\sqrt{x}-1\right)}{1+2i\sqrt{x}}} + c_2 \mathrm{e}^{-2i\sqrt{x}} \sqrt{\frac{(1+4x)\left(1+2i\sqrt{x}\right)}{2i\sqrt{x}-1}}$$

✓ Solution by Mathematica

Time used: 0.091 (sec). Leaf size: 59

```
DSolve[2*x*y''[x]-y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{2i\sqrt{x}}\left(2\sqrt{x}+i\right) + \frac{1}{8} c_2 e^{-2i\sqrt{x}}\left(1+2i\sqrt{x}\right)$$

1.322 problem 327

Internal problem ID [7812]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 327.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{2xy'' - (3 + 2x)\, y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 41

```
dsolve(2*x*diff(y(x),x$2)-(3+2*x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{x}(-3 + 2x)}{2} + c_2 \mathrm{e}^{x}(-3 + 2x)\left(\int \frac{x^{\frac{3}{2}} \mathrm{e}^{-x}}{(-3 + 2x)^2} dx\right)$$

✓ Solution by Mathematica
Time used: 0.575 (sec). Leaf size: 54

```
DSolve[2*x*y''[x]-(3+2*x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-\sqrt{\pi} c_2 e^x (2x - 3)\operatorname{erf}\left(\sqrt{x}\right) + 2c_1 e^x (2x - 3) - 6c_2\sqrt{x}\right)$$

1.323 problem 328

Internal problem ID [7813]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 328.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2y'' + 3xy' + (2x - 1)\,y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 81

```
dsolve(2*x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+(2*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{2i\sqrt{x}}\sqrt{\frac{(1+4x)(2i\sqrt{x}-1)}{1+2i\sqrt{x}}}}{x} + \frac{c_2 e^{-2i\sqrt{x}}\sqrt{\frac{(1+4x)(1+2i\sqrt{x})}{2i\sqrt{x}-1}}}{x}$$

✓ Solution by Mathematica

Time used: 0.09 (sec). Leaf size: 64

```
DSolve[2*x^2*y''[x]+3*x*y'[x]+(2*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-2i\sqrt{x}}\left(8c_1 e^{4i\sqrt{x}}\left(2\sqrt{x}+i\right) + c_2\left(1+2i\sqrt{x}\right)\right)}{8x}$$

1.324 problem 329

Internal problem ID [7814]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 329.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + 2y' - yx = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sinh(x)}{x} + \frac{c_2 \cosh(x)}{x}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 28

```
DSolve[x*y''[x]+2*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-x} + c_2 e^x}{2x}$$

1.325 problem 330

Internal problem ID [7815]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 330.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2e^{2ix})}{2\sqrt{x}}$$

1.326 problem 331

Internal problem ID [7816]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 331.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' + (x - 6)\,y' - 3y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 39

```
dsolve(x*diff(y(x),x$2)+(x-6)*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(x^3 - 12x^2 + 60x - 120\right) + c_2\mathrm{e}^{-x}\left(x^3 + 12x^2 + 60x + 120\right)$$

✓ Solution by Mathematica
Time used: 0.073 (sec). Leaf size: 98

```
DSolve[x*y''[x]+(x-6)*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2e^{-x/2}\sqrt{x}\left((c_1x^3 + 12ic_2x^2 + 60c_1x + 120ic_2)\cosh\left(\frac{x}{2}\right) - (12c_1(x^2+10) + ic_2x(x^2+60))\sinh\left(\frac{x}{2}\right)\right)}{\sqrt{\pi}\sqrt{-ix}}$$

1.327 problem 332

Internal problem ID [7817]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 332.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^4 y'' + \lambda y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(x^4*diff(y(x),x$2)+lambda*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sinh\left(\frac{\sqrt{-\lambda}}{x}\right) + c_2 x \cosh\left(\frac{\sqrt{-\lambda}}{x}\right)$$

✓ Solution by Mathematica
Time used: 0.136 (sec). Leaf size: 52

```
DSolve[x^4*y''[x]+\[Lambda]*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 x e^{\frac{i\sqrt{\lambda}}{x}} - \frac{i c_2 x e^{-\frac{i\sqrt{\lambda}}{x}}}{2\sqrt{\lambda}}$$

1.328 problem 333

Internal problem ID [7818]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 333.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 4xy' + (4x^2 - 25)\,y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 43

```
dsolve(4*x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2-25)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{ix}(x^2 + 3ix - 3)}{x^{\frac{5}{2}}} + \frac{c_2 e^{-ix}(x^2 - 3ix - 3)}{x^{\frac{5}{2}}}$$

✓ Solution by Mathematica
Time used: 0.126 (sec). Leaf size: 59

```
DSolve[4*x^2*y''[x]+4*x*y'[x]+(4*x^2-25)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}}((-c_2x^2 + 3c_1x + 3c_2)\cos(x) + (c_1(x^2 - 3) + 3c_2x)\sin(x))}{x^{5/2}}$$

1.329 problem 334

Internal problem ID [7819]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 334.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' + \left(36x^2 - \frac{1}{4}\right)y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(36*x^2-1/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(6x)}{\sqrt{x}} + \frac{c_2 \cos(6x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(36*x^2-1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-6ix}(12c_1 - ic_2e^{12ix})}{12\sqrt{x}}$$

1.330 problem 335

Internal problem ID [7820]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 335.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y(x^2 - 2) = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 31

```
dsolve(x^2*diff(y(x),x$2)+(x^2-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(-\sin(x) + \cos(x)\, x)}{x} + \frac{c_2(\cos(x) + x\sin(x))}{x}$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 21

```
DSolve[x^2*y''[x]+(x^2-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(c_1 j_1(x) - c_2 y_1(x))$$

1.331 problem 336

Internal problem ID [7821]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 336.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + 3y' + yx^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x$2)+3*diff(y(x),x)+x^3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin\left(\frac{x^2}{2}\right)}{x^2} + \frac{c_2 \cos\left(\frac{x^2}{2}\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 43

```
DSolve[x*y''[x]+3*y'[x]+x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{ix^2}{2}}\left(2c_1 - ic_2e^{ix^2}\right)}{2x^2}$$

1.332 problem 337

Internal problem ID [7822]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 337.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 4xy' + (x^2 + 2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{x^2} + \frac{c_2 \cos(x)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 37

```
DSolve[x^2*y''[x]+4*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x^2}$$

1.333 problem 338

Internal problem ID [7823]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 338.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{16x^2y'' + 32xy' + (x^4 - 12)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(16*x^2*diff(y(x),x$2)+32*x*diff(y(x),x)+(x^4-12)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin\left(\frac{x^2}{8}\right)}{x^{\frac{3}{2}}} + \frac{c_2 \cos\left(\frac{x^2}{8}\right)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 42

```
DSolve[16*x^2*y''[x]+32*x*y'[x]+(x^4-12)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{ix^2}{8}}\left(c_1 - 2ic_2 e^{\frac{ix^2}{4}}\right)}{x^{3/2}}$$

1.334 problem 339

Internal problem ID [7824]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 339.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - x^2 y' + yx = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 63

```
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x - \frac{c_2 3^{\frac{1}{3}} \left(6(-x^3)^{\frac{1}{3}} \Gamma\left(\frac{2}{3}\right) 3^{\frac{2}{3}} - 6(-x^3)^{\frac{1}{3}} \Gamma\left(\frac{2}{3}, -\frac{x^3}{3}\right) 3^{\frac{2}{3}} + 18\, \mathrm{e}^{\frac{x^3}{3}}\right)}{3\left(1 + \sqrt{-3}\right)}$$

✓ Solution by Mathematica
Time used: 0.068 (sec). Leaf size: 41

```
DSolve[y''[x]-x^2*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x - \frac{c_2 \sqrt[3]{-x^3}\,\Gamma\left(-\frac{1}{3}, -\frac{x^3}{3}\right)}{3\sqrt[3]{3}}$$

1.335 problem 340

Internal problem ID [7825]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 340.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{xy'' - (x+2)\, y' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)-(x+2)*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x^2 + 2x + 2) + \mathrm{e}^x c_2$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 24

```
DSolve[x*y''[x]-(x+2)*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x - c_2(x^2 + 2x + 2)$$

1.336 problem 341

Internal problem ID [7826]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 341.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + xy' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 50

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x\,\mathrm{e}^{-\frac{x^2}{2}} - \frac{c_2 \mathrm{e}^{-\frac{x^2}{2}} \left(i\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,x}{2}\right) x + 2\,\mathrm{e}^{\frac{x^2}{2}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 69

```
DSolve[y''[x]+x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\frac{\pi}{2}} c_2 e^{-\frac{x^2}{2}} \sqrt{x^2}\,\mathrm{erfi}\left(\frac{\sqrt{x^2}}{\sqrt{2}}\right) + \sqrt{2} c_1 e^{-\frac{x^2}{2}} x + c_2$$

1.337 problem 342

Internal problem ID [7827]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 342.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(1-x^2)\, y''-2xy'+2y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 26

```
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+c_2\left(-\frac{\ln\left(x+1\right)x}{2}+\frac{\ln\left(x-1\right)x}{2}+1\right)$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 33

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1x-\frac{1}{2}c_2(x\log(1-x)-x\log(x+1)+2)$$

1.338 problem 343

Internal problem ID [7828]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 343.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4xy' + (4x^2 - 2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{x^2} + c_2 x\, e^{x^2}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 18

```
DSolve[y''[x]-4*x*y'[x]+(4*x^2-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x^2}(c_2 x + c_1)$$

1.339 problem 344

Internal problem ID [7829]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 344.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$(1-x^2)\, y'' - 2xy' + 30y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 83

```
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+30*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\frac{21}{5}x^5 - \frac{14}{3}x^3 + x\right) + c_2\left(-\frac{21\ln(x+1)\, x^5}{640} + \frac{21\ln(x-1)\, x^5}{640} + \frac{7\ln(x+1)\, x^3}{192} - \frac{7\ln(x-1)\, x^3}{192} + \frac{21x^4}{320} - \frac{\ln(x+1)\, x}{128} + \frac{\ln(x-1)\, x}{128} - \frac{49x^2}{960} + \frac{1}{225}\right)$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 76

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+30*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}c_1x(63x^4 - 70x^2 + 15) + c_2\left(-\frac{63x^4}{8} + \frac{49x^2}{8} - \frac{1}{16}(63x^4 - 70x^2 + 15)\, x(\log(1-x) - \log(x+1)) - \frac{8}{15}\right)$$

1.340 problem 345

Internal problem ID [7830]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 345.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + 2y' + yx = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{x} + \frac{c_2 \cos(x)}{x}$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 37

```
DSolve[x*y''[x]+2*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x}$$

1.341 problem 346

Internal problem ID [7831]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 346.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (2x+1)\,y' + (1+x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)+(2*x+1)*diff(y(x),x)+(x+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} c_1 + c_2 \mathrm{e}^{-x} \ln(x)$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 19

```
DSolve[x*y''[x]+(2*x+1)*y'[x]+(x+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_2 \log(x) + c_1)$$

1.342 problem 347

Internal problem ID [7832]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 347.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$\boxed{2x(x-1)\,y''-(1+x)\,y'+y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(2*x*(x-1)*diff(y(x),x$2)-(x+1)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(x+1)+c_2\sqrt{x}$$

✓ Solution by Mathematica
Time used: 0.056 (sec). Leaf size: 21

```
DSolve[2*x*(x-1)*y''[x]-(x+1)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\sqrt{x}-2c_2(x+1)$$

1.343 problem 348

Internal problem ID [7833]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 348.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + 2y' + 4yx = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+4*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(2x)}{x} + \frac{c_2 \cos(2x)}{x}$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 37

```
DSolve[x*y''[x]+2*y'[x]+4*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4c_1 e^{-2ix} - ic_2 e^{2ix}}{4x}$$

1.344 problem 349

Internal problem ID [7834]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 349.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (-2x+2)\, y' + (x-2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x$2)+(2-2*x)*diff(y(x),x)+(x-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^x}{x} + \mathrm{e}^x c_2$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 19

```
DSolve[x*y''[x]+(2-2*x)*y'[x]+(x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x (c_2 x + c_1)}{x}$$

1.345 problem 350

Internal problem ID [7835]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 350.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 6xy' + (4x^2 + 6)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)+6*x*diff(y(x),x)+(4*x^2+6)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(2x)}{x^3} + \frac{c_2 \cos(2x)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 37

```
DSolve[x^2*y''[x]+6*x*y'[x]+(4*x^2+6)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4c_1 e^{-2ix} - ic_2 e^{2ix}}{4x^3}$$

1.346 problem 351

Internal problem ID [7836]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 351.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (1 - 2x)\, y' + (x - 1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x$2)+(1-2*x)*diff(y(x),x)+(x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2 e^x \ln(x)$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 17

```
DSolve[x*y''[x]+(1-2*x)*y'[x]+(x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x(c_2 \log(x) + c_1)$$

1.347 problem 352

Internal problem ID [7837]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 352.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$x(1-x)\,y'' + \left(2x+\frac{1}{2}\right)y' - 2y = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 61

```
dsolve(x*(1-x)*diff(y(x),x$2)+(1/2+2*x)*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(1+4x) + c_2\left(4\sqrt{x\,(x-1)}\,x - 12\ln\left(x-\frac{1}{2}+\sqrt{x\,(x-1)}\right)x + 26\sqrt{x\,(x-1)} - 3\ln\left(x-\frac{1}{2}+\sqrt{x\,(x-1)}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.416 (sec). Leaf size: 64

```
DSolve[x*(1-x)*y''[x]+(1/2+2*x)*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}c_2\left(\sqrt{-((x-1)x)}(2x+13) - 6(4x+1)\arctan\left(\frac{\sqrt{1-x}}{\sqrt{x}+1}\right)\right) + c_1\left(x+\frac{1}{4}\right)$$

1.348 problem 353

Internal problem ID [7838]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 353.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4(t^2-3t+2)\,y''-2y'+y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 62

```
dsolve(4*(t^2-3*t+2)*diff(y(t),t$2)-2*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t)=c_1\sqrt{t-1}+\frac{c_2\sqrt{t-2}\,(t-1)\left(\ln\left(t-\frac{3}{2}+\sqrt{t^2-3t+2}\right)\sqrt{t^2-3t+2}-2t+4\right)}{t^2-3t+2}$$

✓ Solution by Mathematica

Time used: 0.257 (sec). Leaf size: 53

```
DSolve[4*(t^2-3*t+2)*y''[t]-2*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to\sqrt{1-t}\left(-2c_2\text{arctanh}\left(\frac{1}{\sqrt{\frac{t-1}{t-2}}}\right)+\frac{2c_2}{\sqrt{\frac{t-1}{t-2}}}+c_1\right)$$

1.349 problem 354

Internal problem ID [7839]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 354.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2(t^2-5t+6)\,y''+(2t-3)\,y'-8y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 32

```
dsolve(2*(t^2-5*t+6)*diff(y(t),t$2)+(2*t-3)*diff(y(t),t)-8*y(t)=0,y(t), singsol=all)
```

$$y(t)=c_1\left(t^2-\frac{13}{3}t+\frac{37}{8}\right)+\frac{c_2(6t-17)\,(t-2)^{\frac{3}{2}}}{\sqrt{t-3}}$$

✓ Solution by Mathematica

Time used: 0.267 (sec). Leaf size: 84

```
DSolve[2*(t^2-5*t+6)*y''[t]+(2*t-3)*y'[t]-8*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to\frac{\sqrt[4]{2-t}\left(5c_1\sqrt[4]{t-3}\sqrt{t-2}(6t^2-29t+34)+24c_2(t-3)^{3/4}\left(24t^2-104t+111\right)\right)}{30(3-t)^{3/4}\sqrt[4]{t-2}}$$

1.350 problem 355

Internal problem ID [7840]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 355.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3t(t+1)\,y'' + ty' - y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(3*t*(1+t)*diff(y(t),t$2)+t*diff(y(t),t)-y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 t + c_2 t \left(\int \frac{1}{(t+1)^{\frac{1}{3}}\, t^2} dt \right)$$

✓ Solution by Mathematica

Time used: 0.23 (sec). Leaf size: 93

```
DSolve[3*t*(1+t)*y''[t]+t*y'[t]-y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{6c_1 t - c_2 \left(2\sqrt{3}t \arctan\left(\frac{2\sqrt[3]{t+1}+1}{\sqrt{3}} \right) + 6(t+1)^{2/3} + 2t \log\left(\sqrt[3]{t+1} - 1 \right) - t \log\left((t+1)^{2/3} + \sqrt[3]{t+1} \right.\right.}{6\sqrt[6]{3}}$$

1.351 problem 356

Internal problem ID [7841]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 356.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' + \frac{\left(\frac{3}{4} + x\right) y}{4} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)+1/4*(x+3/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(\sqrt{x}\right) x^{\frac{1}{4}} + c_2 x^{\frac{1}{4}} \cos\left(\sqrt{x}\right)$$

✓ Solution by Mathematica

Time used: 0.068 (sec). Leaf size: 43

```
DSolve[x^2*y''[x]+1/4*(x+3/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-i\sqrt{x}} \sqrt[4]{x} \left(c_1 e^{2i\sqrt{x}} + i c_2\right)$$

1.352 problem 357

Internal problem ID [7842]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 357.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + xy' + \frac{(x^2 - 1)\, y}{4} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+1/4*(x^2-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin\left(\frac{x}{2}\right)}{\sqrt{x}} + \frac{c_2 \cos\left(\frac{x}{2}\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 36

```
DSolve[x^2*y''[x]+x*y'[x]+1/4*(x^2-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{ix}{2}}\left(c_1 - ic_2 e^{ix}\right)}{\sqrt{x}}$$

1.353 problem 358

Internal problem ID [7843]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 358.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (1-2x)\, y' + (x-1)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x$2)+(1-2*x)*diff(y(x),x)+(x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2 e^x \ln(x)$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 17

```
DSolve[x*y''[x]+(1-2*x)*y'[x]+(x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x (c_2 \log(x) + c_1)$$

1.354 problem 359

Internal problem ID [7844]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 359.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{xy'' - (1 + x)\, y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x*diff(y(x),x$2)-(x+1)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x + 1) + e^x c_2$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 19

```
DSolve[x*y''[x]-(x+1)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x - c_2(x + 1)$$

1.355 problem 360

Internal problem ID [7845]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 360.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + 3y' + 4yx^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x$2)+3*diff(y(x),x)+4*x^3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x^2)}{x^2} + \frac{c_2 \cos(x^2)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.061 (sec). Leaf size: 41

```
DSolve[x*y''[x]+3*y'[x]+4*x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4c_1 e^{-ix^2} - ic_2 e^{ix^2}}{4x^2}$$

1.356 problem 361

Internal problem ID [7846]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 361.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-x^2)\, y'' + 2x(1-x^2)\, y' - 2y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 54

```
dsolve(x^2*(1-x^2)*diff(y(x),x$2)+2*x*(1-x^2)*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2-1)}{x^2} + \frac{c_2\left(-\frac{\ln(x+1)x^2}{4} + \frac{\ln(x-1)x^2}{4} + \frac{\ln(x+1)}{4} - \frac{\ln(x-1)}{4} - \frac{x}{2}\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 56

```
DSolve[x^2*(1-x^2)*y''[x]+2*x*(1-x^2)*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{-4c_1x^2 - c_2(x^2-1)\log(1-x) + c_2(x^2-1)\log(x+1) + 2c_2x + 4c_1}{4x^2}$$

1.357 problem 362

Internal problem ID [7847]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 362.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + (x-2)\,y' - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(2*x*diff(y(x),x$2)+(x-2)*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x-2) + c_2 e^{-\frac{x}{2}}$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 23

```
DSolve[2*x*y''[x]+(x-2)*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x/2} + 2c_2(x-2)$$

1.358 problem 363

Internal problem ID [7848]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 363.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + 2y' + yx = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{x} + \frac{c_2 \cos(x)}{x}$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 37

```
DSolve[x*y''[x]+2*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x}$$

1.359 problem 364

Internal problem ID [7849]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 364.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 2x^2y' + (x^4 + 2x - 1)\, y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+2*x^2*diff(y(x),x)+(x^4+2*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{1}{3}x^3 - x} + c_2 e^{-\frac{1}{3}x^3 + x}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 34

```
DSolve[y''[x]+2*x^2*y'[x]+(x^4+2*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-\frac{1}{3}x(x^2+3)} \left(c_2 e^{2x} + 2c_1\right)$$

1.360 problem 365

Internal problem ID [7850]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 365.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{u'' + \frac{u}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(diff(u(x),x$2)+1/x^2*u(x)=0,u(x), singsol=all)
```

$$u(x) = c_1 \sqrt{x}\, x^{\frac{\sqrt{-3}}{2}} + c_2 \sqrt{x}\, x^{-\frac{\sqrt{-3}}{2}}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 42

```
DSolve[u''[x]+1/x^2*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to \sqrt{x}\left(c_1 \cos\left(\frac{1}{2}\sqrt{3}\log(x)\right) + c_2 \sin\left(\frac{1}{2}\sqrt{3}\log(x)\right)\right)$$

1.361 problem 366

Internal problem ID [7851]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 366.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$u'' - (2x+1)\,u' + \left(x^2 + x - 1\right) u = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(u(x),x$2)-(2*x+1)*diff(u(x),x)+(x^2+x-1)*u(x)=0,u(x), singsol=all)
```

$$u(x) = c_1 \mathrm{e}^{\frac{x^2}{2}} + c_2 \mathrm{e}^{\frac{1}{2}x^2+x}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 24

```
DSolve[u''[x]-(2*x+1)*u'[x]+(x^2+x-1)*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to e^{\frac{x^2}{2}} \left(c_2 e^x + c_1\right)$$

1.362 problem 367

Internal problem ID [7852]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 367.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 2y' + \left(1 + \frac{2}{(3x+1)^2}\right) y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+(1+2/(1+3*x)^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(3x+1)^{\frac{1}{3}} e^{-x} + c_2(3x+1)^{\frac{2}{3}} e^{-x}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 35

```
DSolve[y''[x]+2*y'[x]+(1+2/(1+3*x)^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x} \sqrt[3]{3x+1} \left(c_2 \sqrt[3]{3x+1} + c_1\right)$$

1.363 problem 368

Internal problem ID [7853]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 368.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - 2xy' + (x^2 + 2)\, y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sin(x) + c_2 \cos(x)\, x$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-ix} x - \frac{1}{2} i c_2 e^{ix} x$$

1.364 problem 369

Internal problem ID [7854]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 369.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{2y'}{x} - \frac{2y}{(1+x)^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)+2/x*diff(y(x),x)-2/(1+x)^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x\,(x+1)} + \frac{c_2(x^3+3x^2+3x)}{x\,(x+1)}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 34

```
DSolve[y''[x]+2/x*y'[x]-2/(1+x)^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 x(x^2+3x+3)+3c_1}{3x(x+1)}$$

1.365 problem 370

Internal problem ID [7855]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 370.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + \frac{y}{2x^4} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+1/(2*x^4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sin\left(\frac{\sqrt{2}}{2x}\right) + c_2 x \cos\left(\frac{\sqrt{2}}{2x}\right)$$

✓ Solution by Mathematica

Time used: 0.096 (sec). Leaf size: 50

```
DSolve[y''[x]+1/(2*x^4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{\frac{i}{\sqrt{2}x}} x - \frac{i c_2 e^{-\frac{i}{\sqrt{2}x}} x}{\sqrt{2}}$$

1.366 problem 371

Internal problem ID [7856]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 371.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - xy' - yx = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + \frac{c_2\sqrt{2}\left(\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)x - i\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{\frac{1}{2}x^2+2x} + 2\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\right)\mathrm{e}^{-x}}{2\sqrt{\pi}}$$

✓ Solution by Mathematica
Time used: 0.181 (sec). Leaf size: 78

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(-\sqrt{2\pi}c_2\sqrt{(x+2)^2}\mathrm{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) + 2\sqrt{2}c_1(x+2) + 2c_2e^{\frac{1}{2}(x+2)^2}\right)$$

1.367 problem 372

Internal problem ID [7857]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 372.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - xy' - yx = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x}(x+2) + \frac{c_2\sqrt{2}\left(\pi\, e^{-2}\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)x - i\sqrt{2}\,\sqrt{\pi}\, e^{\frac{1}{2}x^2+2x} + 2\pi\, e^{-2}\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\right)e^{-x}}{2\sqrt{\pi}}$$

✓ Solution by Mathematica

Time used: 0.105 (sec). Leaf size: 78

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(-\sqrt{2\pi}c_2\sqrt{(x+2)^2}\operatorname{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) + 2\sqrt{2}c_1(x+2) + 2c_2 e^{\frac{1}{2}(x+2)^2}\right)$$

1.368 problem 373

Internal problem ID [7858]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 373.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - xy' - yx = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + \frac{c_2\sqrt{2}\left(\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)x - i\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{\frac{1}{2}x^2+2x} + 2\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\right)\mathrm{e}^{-x}}{2\sqrt{\pi}}$$

✓ Solution by Mathematica
Time used: 0.108 (sec). Leaf size: 78

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(-\sqrt{2\pi}c_2\sqrt{(x+2)^2}\mathrm{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) + 2\sqrt{2}c_1(x+2) + 2c_2e^{\frac{1}{2}(x+2)^2}\right)$$

1.369 problem 374

Internal problem ID [7859]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 374.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - xy' - yx = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x}(x+2) + \frac{c_2\sqrt{2}\left(\pi\, e^{-2}\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)x - i\sqrt{2}\,\sqrt{\pi}\,e^{\frac{1}{2}x^2+2x} + 2\pi\, e^{-2}\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\right)e^{-x}}{2\sqrt{\pi}}$$

✓ Solution by Mathematica

Time used: 0.098 (sec). Leaf size: 78

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(-\sqrt{2\pi}c_2\sqrt{(x+2)^2}\operatorname{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) + 2\sqrt{2}c_1(x+2) + 2c_2 e^{\frac{1}{2}(x+2)^2}\right)$$

1.370 problem 375

Internal problem ID [7860]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 375.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - xy' - yx = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + \frac{c_2\sqrt{2}\left(\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)x - i\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{\frac{1}{2}x^2+2x} + 2\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\right)\mathrm{e}^{-x}}{2\sqrt{\pi}}$$

✓ Solution by Mathematica

Time used: 0.109 (sec). Leaf size: 78

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(-\sqrt{2\pi}c_2\sqrt{(x+2)^2}\mathrm{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) + 2\sqrt{2}c_1(x+2) + 2c_2e^{\frac{1}{2}(x+2)^2}\right)$$

1.371 problem 376

Internal problem ID [7861]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 376.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - xy' - yx = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + \frac{c_2\sqrt{2}\left(\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)x - i\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{\frac{1}{2}x^2+2x} + 2\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\right)\mathrm{e}^{-x}}{2\sqrt{\pi}}$$

✓ Solution by Mathematica
Time used: 0.11 (sec). Leaf size: 78

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(-\sqrt{2\pi}c_2\sqrt{(x+2)^2}\text{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) + 2\sqrt{2}c_1(x+2) + 2c_2e^{\frac{1}{2}(x+2)^2}\right)$$

1.372 problem 377

Internal problem ID [7862]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 377.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - xy' - yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x}(x+2) + \frac{c_2\sqrt{2}\left(\pi\,e^{-2}\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)x - i\sqrt{2}\,\sqrt{\pi}\,e^{\frac{1}{2}x^2+2x} + 2\pi\,e^{-2}\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\right)e^{-x}}{2\sqrt{\pi}}$$

✓ Solution by Mathematica

Time used: 0.11 (sec). Leaf size: 78

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(-\sqrt{2\pi}c_2\sqrt{(x+2)^2}\operatorname{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) + 2\sqrt{2}c_1(x+2) + 2c_2e^{\frac{1}{2}(x+2)^2}\right)$$

1.373 problem 378

Internal problem ID [7863]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 378.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - xy' - yx = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + \frac{c_2\sqrt{2}\left(\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)x - i\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{\frac{1}{2}x^2+2x} + 2\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\right)\mathrm{e}^{-x}}{2\sqrt{\pi}}$$

✓ Solution by Mathematica

Time used: 0.109 (sec). Leaf size: 78

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(-\sqrt{2\pi}c_2\sqrt{(x+2)^2}\mathrm{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) + 2\sqrt{2}c_1(x+2) + 2c_2e^{\frac{1}{2}(x+2)^2}\right)$$

1.374 problem 379

Internal problem ID [7864]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 379.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - xy' - yx = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x}(x+2) + \frac{c_2\sqrt{2}\left(\pi\, e^{-2}\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)x - i\sqrt{2}\,\sqrt{\pi}\, e^{\frac{1}{2}x^2+2x} + 2\pi\, e^{-2}\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\right)e^{-x}}{2\sqrt{\pi}}$$

✓ Solution by Mathematica
Time used: 0.108 (sec). Leaf size: 78

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(-\sqrt{2\pi}c_2\sqrt{(x+2)^2}\operatorname{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) + 2\sqrt{2}c_1(x+2) + 2c_2e^{\frac{1}{2}(x+2)^2}\right)$$

1.375 problem 380

Internal problem ID [7865]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 380.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$
\boxed{y'' - xy' - yx = 0}
$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$
y(x) = c_1 \mathrm{e}^{-x}(x+2) + \frac{c_2\sqrt{2}\left(\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)x - i\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{\frac{1}{2}x^2+2x} + 2\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\right)\mathrm{e}^{-x}}{2\sqrt{\pi}}
$$

✓ Solution by Mathematica
Time used: 0.1 (sec). Leaf size: 78

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x) \to \frac{1}{2}e^{-x}\left(-\sqrt{2\pi}c_2\sqrt{(x+2)^2}\mathrm{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) + 2\sqrt{2}c_1(x+2) + 2c_2e^{\frac{1}{2}(x+2)^2}\right)
$$

1.376 problem 381

Internal problem ID [7866]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 381.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - xy' - yx = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + \frac{c_2\sqrt{2}\left(\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)x - i\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{\frac{1}{2}x^2+2x} + 2\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\right)\mathrm{e}^{-x}}{2\sqrt{\pi}}$$

✓ Solution by Mathematica

Time used: 0.098 (sec). Leaf size: 78

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(-\sqrt{2\pi}c_2\sqrt{(x+2)^2}\text{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) + 2\sqrt{2}c_1(x+2) + 2c_2e^{\frac{1}{2}(x+2)^2}\right)$$

1.377 problem 382

Internal problem ID [7867]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 382.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + 2y' + yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin (x)}{x} + \frac{c_2 \cos (x)}{x}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 37

```
DSolve[x*y''[x]+2*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x}$$

1.378 problem 383

Internal problem ID [7868]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 383.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2x^2y'' + 3xy' - yx = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 31

```
dsolve(2*x^2*diff(y(x),x$2)+3*x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sinh\left(\sqrt{x}\,\sqrt{2}\right)}{\sqrt{x}} + \frac{c_2 \cosh\left(\sqrt{x}\,\sqrt{2}\right)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.089 (sec). Leaf size: 56

```
DSolve[2*x^2*y''[x]+3*x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\sqrt{2}\sqrt{x}}\left(2c_1 e^{2\sqrt{2}\sqrt{x}} - \sqrt{2}c_2\right)}{2\sqrt{x}}$$

1.379 problem 384

Internal problem ID [7869]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 384.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + (3x^2 + 2x)\, y' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x^2*diff(y(x), x, x) + (2*x+3*x^2)*diff(y(x),x)-2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(9x^2 - 6x + 2)}{x^2} + \frac{c_2 e^{-3x}}{x^2}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 35

```
DSolve[x^2*y''[x]+(2*x+3*x^2)*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1(9x^2 - 6x + 2) + 27c_2 e^{-3x}}{27x^2}$$

1.380 problem 385

Internal problem ID [7870]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 385.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x^2+x+1)y''+x(11x^2+11x+9)y'+(7x^2+10x+6)y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 141

```
dsolve(2*x^2*(1+x+x^2)*diff(y(x), x$2) + x*(9+11*x+11*x^2)*diff(y(x), x) + (6+10*x+7*x^2)*y(
```

$$y(x)=\frac{c_1\sqrt{x^2+x+1}\left(\frac{i\sqrt{3}+2x+1}{i\sqrt{3}-2x-1}\right)^{-\frac{i\sqrt{3}}{6}}}{x^2}+\frac{c_2\sqrt{x^2+x+1}\left(\frac{i\sqrt{3}+2x+1}{i\sqrt{3}-2x-1}\right)^{-\frac{i\sqrt{3}}{6}}\left(\int\frac{\left(\frac{i\sqrt{3}-2x-1}{i\sqrt{3}+2x+1}\right)^{-\frac{i\sqrt{3}}{6}}}{(x^2+x+1)^{\frac{3}{2}}\sqrt{x}}dx\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.708 (sec). Leaf size: 93

```
DSolve[2*x^2*(1+x+x^2)*y''[x] + x*(9+11*x+11*x^2)*y'[x] + (6+10*x+7*x^2)*y[x] == 0,y[x],x,In
```

$$y(x)\to\frac{\sqrt{x^2+x+1}e^{-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}}\left(c_2\int_1^x\frac{e^{\frac{\arctan\left(\frac{2K[1]+1}{\sqrt{3}}\right)}{\sqrt{3}}}}{\sqrt{K[1]}(K[1]^2+K[1]+1)^{3/2}}dK[1]+c_1\right)}{x^2}$$

1.381 problem 388

Internal problem ID [7871]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 388.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (1 + x)\, y' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 37

```
dsolve(x*diff(y(x), x$2) +(1+x)*diff(y(x),x)+2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x-1) + c_2(\mathrm{expIntegral}_1(-x)\, x - \mathrm{expIntegral}_1(-x) + \mathrm{e}^{x})\, \mathrm{e}^{-x}$$

✓ Solution by Mathematica
Time used: 0.066 (sec). Leaf size: 33

```
DSolve[x*y''[x] +(1+x)*y'[x]+2*y[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_2(x-1)\, \mathrm{ExpIntegralEi}(x) + c_1(x-1) - c_2 e^{x})$$

1.382 problem 389

Internal problem ID [7872]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 389.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2 - 2x + 1)\,y'' - x(x + 3)\,y' + (4 + x)\,y = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 52

```
dsolve(x^2*(1-2*x+x^2)*diff(y(x), x$2) -x*(3+x)*diff(y(x),x)+(4+x)*y(x) = 0,y(x), singsol=al
```

$$y(x) = \frac{c_1 x^2 \mathrm{e}^{-\frac{4}{x-1}}}{x - 1} + \frac{c_2 x^2 \,\mathrm{expIntegral}_1\left(-\frac{4x}{x-1}\right) \mathrm{e}^{-\frac{4x}{x-1}}}{x - 1}$$

✓ Solution by Mathematica
Time used: 0.213 (sec). Leaf size: 54

```
DSolve[x^2*(1-2*x+x^2)*y''[x] -x*(3+x)*y'[x]+(4+x)*y[x] == 0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{e^{-\frac{4x}{x-1}} \sqrt{1 - x}\, x^2 \left(c_2 \,\mathrm{ExpIntegralEi}\left(\frac{4x}{x-1}\right) + e^4 c_1\right)}{(x - 1)^{3/2}}$$

1.383 problem 390

Internal problem ID [7873]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 390.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x+2)\,y''+5x^2y'+(1+x)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
dsolve(2*x^2*(2+x)*diff(y(x), x$2) +5*x^2*diff(y(x),x)+(1+x)*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\sqrt{x}}{(x+2)^{\frac{3}{2}}}+\frac{c_2\sqrt{2}\left(2\sqrt{2}\,\sqrt{x+2}-4\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,\sqrt{x+2}}{2}\right)\right)\sqrt{x}}{2\,(x+2)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.074 (sec). Leaf size: 55

```
DSolve[2*x^2*(2+x)*y''[x] +5*x^2*y'[x]+(1+x)*y[x] == 0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x)\to\frac{\sqrt{x}\left(-2\sqrt{2}c_2\mathrm{arctanh}\left(\frac{\sqrt{x+2}}{\sqrt{2}}\right)+2c_2\sqrt{x+2}+c_1\right)}{(x+2)^{3/2}}$$

1.384 problem 391

Internal problem ID [7874]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 391.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 4xy' + (x^2 + 2) y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x), x, x) + 4*x*diff(y(x), x) + (x^2+2)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{x^2} + \frac{c_2 \cos(x)}{x^2}$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 37

```
DSolve[x^2*y''[x]+4*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1e^{-ix} - ic_2e^{ix}}{2x^2}$$

1.385 problem 392

Internal problem ID [7875]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 392.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2e^{2ix})}{2\sqrt{x}}$$

1.386 problem 394

Internal problem ID [7876]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 394.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - xy' - \left(x^2 + \frac{5}{4}\right)y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)-(x^2+5/4)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^x (x-1)}{\sqrt{x}} + \frac{c_2 e^{-x}(x+1)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.084 (sec). Leaf size: 53

```
DSolve[x^2*y''[x]-x*y'[x]-(x^2+5/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{\frac{2}{\pi}}((ic_2x + c_1)\sinh(x) - (c_1x + ic_2)\cosh(x))}{\sqrt{-ix}}$$

1.387 problem 395

Internal problem ID [7877]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 395.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2e^{2ix})}{2\sqrt{x}}$$

1.388 problem 396

Internal problem ID [7878]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 396.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 3xy' + 4yx^4 = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+4*x^4*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x^2)}{x^2} + \frac{c_2 \cos(x^2)}{x^2}$$

✓ Solution by Mathematica
Time used: 0.056 (sec). Leaf size: 41

```
DSolve[x^2*y''[x]+3*x*y'[x]+4*x^4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4c_1 e^{-ix^2} - ic_2 e^{ix^2}}{4x^2}$$

1.389 problem 398

Internal problem ID [7879]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 398.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - (x^2 + 3)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)=(x^2+3)*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{x^2}{2}} x + c_2 \mathrm{e}^{\frac{x^2}{2}} \left(\sqrt{\pi}\, \mathrm{erf}\,(x)\, x + \mathrm{e}^{-x^2}\right)$$

✓ Solution by Mathematica

Time used: 0.076 (sec). Leaf size: 46

```
DSolve[y''[x]==(x^2+3)*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{x^2}{2}} \left(-\sqrt{\pi} c_2 e^{x^2} x \mathrm{erf}(x) + c_1 e^{x^2} x - c_2\right)$$

1.390 problem 399

Internal problem ID [7880]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 399.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2xy' + (x^2 + 1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+2*x*diff(y(x),x)+(x^2+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{x^2}{2}} + c_2 e^{-\frac{x^2}{2}} x$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 22

```
DSolve[y''[x]+2*x*y'[x]+(x^2+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{x^2}{2}}(c_2 x + c_1)$$

1.391 problem 400

Internal problem ID [7881]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 400.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^3y'' + y' - \frac{y}{x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x^3*diff(y(x),x$2)+ diff(y(x),x)-1/x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1x + c_2x \operatorname{erf}\left(\frac{i\sqrt{2}}{2x}\right)$$

✓ Solution by Mathematica

Time used: 0.078 (sec). Leaf size: 34

```
DSolve[x^3*y''[x]+ y'[x]-1/x*y[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1x - \sqrt{\frac{\pi}{2}}c_2x\text{erfi}\left(\frac{1}{\sqrt{2}x}\right)$$

1.392 problem 401

Internal problem ID [7882]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 401.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(diff(y(x),x),x)+x*diff(y(x),x)+(x^2-1/4)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2e^{2ix})}{2\sqrt{x}}$$

1.393 problem 402

Internal problem ID [7883]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 402.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + \left(-8x^2 + 4x\right) y' + \left(4x^2 - 4x - 1\right) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(4*x^2*diff(diff(y(x),x),x)+(-8*x^2+4*x)*diff(y(x),x)+(4*x^2-4*x-1)*y(x) = 0,y(x), sin
```

$$y(x) = \frac{c_1 e^x}{\sqrt{x}} + c_2\sqrt{x}\, e^x$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 21

```
DSolve[4*x^2*y''[x]+(-8*x^2+4*x)*y'[x]+(4*x^2-4*x-1)*y[x] == 0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to \frac{e^x(c_2x + c_1)}{\sqrt{x}}$$

1.394 problem 404

Internal problem ID [7884]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 404.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' + y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{x}{2}} \cos\left(\frac{\sqrt{3}\, x}{2}\right) + c_2 e^{\frac{x}{2}} \sin\left(\frac{\sqrt{3}\, x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 42

```
DSolve[y''[x]-y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x/2}\left(c_1 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_2 \sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

1.395 problem 405

Internal problem ID [7885]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 405.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(x^2-1)\,y''-2xy'+2y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2-1)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+c_2(x^2+1)$$

✓ Solution by Mathematica
Time used: 0.083 (sec). Leaf size: 39

```
DSolve[(x^2-1)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{\sqrt{x^2-1}(c_1(x-1)^2+c_2x)}{\sqrt{1-x^2}}$$

1.396 problem 406

Internal problem ID [7886]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 406.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(x+2)\,y' + (x+2)\,y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x^2*diff(y(x),x$2)-x*(x+2)*diff(y(x),x)+(x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x + e^x c_2 x$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 16

```
DSolve[x^2*y''[x]-x*(x+2)*y'[x]+(x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(c_2 e^x + c_1)$$

1.397 problem 407

Internal problem ID [7887]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 407.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1+x)\,y'' - (x+2)\,y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve((x+1)*diff(y(x),x$2)-(x+2)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x+2) + e^x c_2$$

✓ Solution by Mathematica

Time used: 0.15 (sec). Leaf size: 29

```
DSolve[(x+1)*y''[x]-(x+2)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 e^{x+1} - 2c_2(x+2)}{\sqrt{2e}}$$

1.398 problem 408

Internal problem ID [7888]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 408.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(1-x^2)\, y''+2xy'-2y=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve((1-x^2)*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+c_2(x^2+1)$$

✓ Solution by Mathematica
Time used: 0.062 (sec). Leaf size: 39

```
DSolve[(1-x^2)*y''[x]+2*x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{\sqrt{x^2-1}(c_1(x-1)^2+c_2x)}{\sqrt{1-x^2}}$$

1.399 problem 409

Internal problem ID [7889]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 409.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(1-x^2)\,y''-2xy'+2y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+c_2\left(-\frac{\ln(x+1)\,x}{2}+\frac{\ln(x-1)\,x}{2}+1\right)$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 33

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1x-\frac{1}{2}c_2(x\log(1-x)-x\log(x+1)+2)$$

1.400 problem 410

Internal problem ID [7890]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 410.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 33

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x - \frac{1}{2}c_2(x\log(1-x) - x\log(x+1) + 2)$$

1.401 problem 411

Internal problem ID [7891]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 411.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(x^2-1)\, y''-6xy'+12y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve((x^2-1)*diff(y(x),x$2)-6*x*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(x^3+x)+c_2(x^4+6x^2+1)$$

✓ Solution by Mathematica
Time used: 0.12 (sec). Leaf size: 45

```
DSolve[(x^2-1)*y''[x]-6*x*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{\sqrt{x^2-1}(c_2x(x^2+1)+c_1(x-1)^4)}{\sqrt{1-x^2}}$$

1.402 problem 412

Internal problem ID [7892]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 412.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(x^2+3\right)y''-7xy'+16y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 101

```
dsolve((x^2+3)*diff(y(x),x$2)-7*x*diff(y(x),x)+16*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x^4-9x^2+\frac{27}{8}\right)+c_2\left(\frac{\ln\left(\sqrt{x^2+3}-x\right)x^4}{64}+\frac{25\sqrt{x^2+3}\,x^3}{768}+\frac{25x^4}{768}-\frac{9\ln\left(\sqrt{x^2+3}-x\right)x^2}{64}-\frac{55\sqrt{x^2+3}\,x}{512}-\frac{75x^2}{256}+\frac{27\ln\left(\sqrt{x^2+3}-x\right)}{512}+\frac{225}{2048}\right)$$

✓ Solution by Mathematica

Time used: 0.23 (sec). Leaf size: 492

```
DSolve[(x^2+3)*y''[x]-7*x*y'[x]+16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{24}c_2\Big(12960x^2\text{RootSum}\Big[7838208000\#1^4 - 188281584000\#1^2 - 241544908800\#1 + 18453344881\&, \#1\log\Big(-411757211968704000\#1^3 - 166063274606980800\#1^2 + 10138703825167113
+ 5248800x^2\text{RootSum}\Big[210880720572480000000\#1^4 - 30882886815600000\#1^2 + 97825688064000\#1 + 18453344881\&, \#1\log\Big(273530830607325028080000000\#1^3 - 2723852861741002572000\#1^2 - 410617
- 4860\text{RootSum}\Big[7838208000\#1^4 - 188281584000\#1^2 - 241544908800\#1 + 18453344881\&, \#1\log\Big(-411757211968704000\#1^3 - 166063274606980800\#1^2 + 10138703825167113
- 1968300\text{RootSum}\Big[210880720572480000000\#1^4 - 30882886815600000\#1^2 + 97825688064000\#1 + 18453344881\&, \#1\log\Big(273530830607325028080000000\#1^3 - 2723852861741002572000\#1^2 - 410617
- 1440x^4\text{RootSum}\Big[7838208000\#1^4 - 188281584000\#1^2 - 241544908800\#1 + 18453344881\&, \#1\log\Big(-411757211968704000\#1^3 - 166063274606980800\#1^2 + 10138703825167113
- 583200x^4\text{RootSum}\Big[210880720572480000000\#1^4 - 30882886815600000\#1^2 + 97825688064000\#1 + 18453344881\&, \#1\log\Big(273530830607325028080000000\#1^3 - 2723852861741002572000\#1^2 - 410617
+ 165\sqrt{x^2+3}x + 216x^2\log\left(\sqrt{x^2+3} - x\right) - 81\log\left(\sqrt{x^2+3} - x\right) - 24x^4\log\left(\sqrt{x^2+3} - x\right) - 50\sqrt{x^2+3}x^3\Big) + c_1\left(x^4 - 9x^2 + \frac{27}{8}\right)$$

1.403 problem 413

Internal problem ID [7893]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 413.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(x^2-1)\,y''+8xy'+12y=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 43

```
dsolve((x^2-1)*diff(y(x),x$2)+8*x*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(3x^2+1)}{(x-1)^3\,(x+1)^3}+\frac{c_2(x^3+3x)}{(x-1)^3\,(x+1)^3}$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 37

```
DSolve[(x^2-1)*y''[x]+8*x*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{3c_1(x-1)^3-c_2(3x^2+1)}{3\,(x^2-1)^3}$$

1.404 problem 414

Internal problem ID [7894]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 414.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3y'' + xy' - 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 50

```
dsolve(3*diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x^4 + 18x^2 + 27) + c_2(x^4 + 18x^2 + 27)\left(\int \frac{e^{-\frac{x^2}{6}}}{(x^4 + 18x^2 + 27)^2} dx\right)$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 43

```
DSolve[3*y''[x]+x*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\frac{x^2}{6}} \operatorname{HermiteH}\left(-5, \frac{x}{\sqrt{6}}\right) + \frac{1}{27} c_2(x^4 + 18x^2 + 27)$$

1.405 problem 415

Internal problem ID [7895]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 415.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{5y'' - 2xy' + 10y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(5*diff(y(x),x$2)-2*x*diff(y(x),x)+10*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(x^5 - 25x^3 + \frac{375}{4}x\right) + c_2\left(x^5 - 25x^3 + \frac{375}{4}x\right)\left(\int \frac{e^{\frac{x^2}{5}}}{\left(4x^4 - 100x^2 + 375\right)^2 x^2}dx\right)$$

✓ Solution by Mathematica

Time used: 0.168 (sec). Leaf size: 138

```
DSolve[5*y''[x]-2*x*y'[x]+10*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{200}\sqrt{\frac{\pi}{5}}c_2\sqrt{x^2}\left(4x^4 - 100x^2 + 375\right)\operatorname{erfi}\left(\frac{\sqrt{x^2}}{\sqrt{5}}\right) + \frac{32c_1x^5}{25\sqrt{5}} - \frac{32c_1x^3}{\sqrt{5}} - \frac{9}{20}c_2e^{\frac{x^2}{5}}x^2 + c_2e^{\frac{x^2}{5}} + \frac{1}{50}c_2e^{\frac{x^2}{5}}x^4 + 24\sqrt{5}c_1x$$

1.406 problem 416

Internal problem ID [7896]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 416.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - x^2 y' - 3yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 76

```
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)-3*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{x^3}{3}} x - \frac{9 c_2 e^{\frac{x^3}{3}} 3^{\frac{2}{3}} e^{-\frac{x^3}{6}} \left(x^6 \operatorname{WhittakerM}\left(\frac{1}{3}, \frac{5}{6}, \frac{x^3}{3}\right) + 5 \operatorname{WhittakerM}\left(\frac{4}{3}, \frac{5}{6}, \frac{x^3}{3}\right) x^3 + 10 \operatorname{WhittakerM}\left(\frac{4}{3}, \frac{5}{6}, \frac{x^3}{3}\right)\right)}{10 x^3 \left(x^3\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.08 (sec). Leaf size: 51

```
DSolve[y''[x]-x^2*y'[x]-3*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{9} e^{\frac{x^3}{3}} \left(9 c_1 x - 3^{2/3} c_2 \sqrt[3]{x^3} \Gamma\left(-\frac{1}{3}, \frac{x^3}{3}\right)\right)$$

1.407 problem 417

Internal problem ID [7897]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 417.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''+2xy'-2y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve((1+x^2)*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+c_2(\arctan(x)\,x+1)$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 48

```
DSolve[(1+x^2)*y''[x]+2*x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}i(2c_1x-c_2x\log(1-ix)+c_2x\log(1+ix)+2ic_2)$$

1.408 problem 418

Internal problem ID [7898]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 418.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + xy' - 2y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x^2+1) + c_2(x^2+1)\left(\int \frac{e^{-\frac{x^2}{2}}}{(x^2+1)^2}dx\right)$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 35

```
DSolve[y''[x]+x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\frac{x^2}{2}} \operatorname{HermiteH}\left(-3, \frac{x}{\sqrt{2}}\right) + c_2(x^2+1)$$

1.409 problem 419

Internal problem ID [7899]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 419.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(x^2-6x+10\right)y''-4(-3+x)\,y'+6y=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve((x^2-6*x+10)*diff(y(x),x$2)-4*(x-3)*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(\frac{26}{3}+x^2-6x\right)+c_2\left(x^3-30x+60\right)$$

✓ Solution by Mathematica
Time used: 0.08 (sec). Leaf size: 36

```
DSolve[(x^2-6*x+10)*y''[x]-4*(x-3)*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{1}{3}i\left(c_2\left(3x^2-18x+26\right)+3c_1(x-(3+i))^3\right)$$

1.410 problem 420

Internal problem ID [7900]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 420.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+6x)\,y''+(3x+9)\,y'-3y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve((x^2+6*x)*diff(y(x),x$2)+(3*x+9)*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(x+3)+\frac{c_2(2x^2+12x+9)}{\sqrt{x^2+6x}}$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 82

```
DSolve[(x^2+6*x)*y''[x]+(3*x+9)*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{9\sqrt{\pi}c_2\sqrt[4]{-x(x+6)}Q_{\frac{3}{2}}^{\frac{1}{2}}\left(\frac{x}{3}+1\right)+\sqrt{6}c_1(2x^2+12x+9)}{9\sqrt{\pi}\sqrt[4]{-x^2}\sqrt{x+6}}$$

1.411 problem 421

Internal problem ID [7901]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 421.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{ty'' + (t^2 - 1)\, y' + t^3 y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 39

```
dsolve(t*diff(y(t),t$2)+ (t^2-1)*diff(y(t),t)+t^3*y(t) = 0,y(t), singsol=all)
```

$$y(t) = c_1 \mathrm{e}^{-\frac{t^2}{4}} \cos\left(\frac{t^2\sqrt{3}}{4}\right) + c_2 \mathrm{e}^{-\frac{t^2}{4}} \sin\left(\frac{t^2\sqrt{3}}{4}\right)$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 48

```
DSolve[t*y''[t]+(t^2-1)*y'[t]+t^3*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-\frac{t^2}{4}} \left(c_2 \cos\left(\frac{\sqrt{3}t^2}{4}\right) + c_1 \sin\left(\frac{\sqrt{3}t^2}{4}\right)\right)$$

1.412 problem 422

Internal problem ID [7902]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 422.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2y'' - t(t+2)\,y' + (t+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(t^2*diff(y(t),t$2)-t*(t+2)*diff(y(t),t)+(t+2)*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 t + c_2 t\,e^t$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 16

```
DSolve[t^2*y''[t]-t*(t+2)*y'[t]+(t+2)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t\left(c_2 e^t + c_1\right)$$

1.413 problem 423

Internal problem ID [7903]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 423.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-1)\,y''-xy'+y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+\mathrm{e}^x c_2$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 17

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1e^x-c_2x$$

1.414 problem 424

Internal problem ID [7904]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 424.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - \left(x - \frac{3}{16}\right)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x^2*diff(y(x),x$2)-(x-1875/10000)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{\frac{1}{4}} \sinh\left(2\sqrt{x}\right) + c_2 x^{\frac{1}{4}} \cosh\left(2\sqrt{x}\right)$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 41

```
DSolve[x^2*y''[x]-(x-1875/10000)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-2\sqrt{x}}\sqrt[4]{x}\left(2c_1e^{4\sqrt{x}} - c_2\right)$$

1.415 problem 425

Internal problem ID [7905]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 425.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-25/100)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-25/100)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2e^{2ix})}{2\sqrt{x}}$$

1.416 problem 426

Internal problem ID [7906]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 426.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2y'' - t(t+2)\,y' + (t+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(t^2*diff(y(t),t$2)-t*(t+2)*diff(y(t),t)+(t+2)*y(t) = 0,y(t), singsol=all)
```

$$y(t) = c_1t + c_2t\,\mathrm{e}^t$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 16

```
DSolve[t^2*y''[t]-t*(t+2)*y'[t]+(t+2)*y[t] == 0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t\left(c_2e^t + c_1\right)$$

1.417 problem 427

Internal problem ID [7907]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 427.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{ty'' - (t+1)\,y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(t*diff(y(t),t$2)-(1+t)*diff(y(t),t)+y(t) = 0,y(t), singsol=all)
```

$$y(t) = c_1(t+1) + c_2e^t$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 19

```
DSolve[t*y''[t]-(1+t)*y'[t]+y[t] == 0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1e^t - c_2(t+1)$$

1.418 problem 428

Internal problem ID [7908]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 428.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-t+1)\,y''+ty'-y=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve((1-t)*diff(y(t),t$2)+t*diff(y(t),t)-y(t) = 0,y(t), singsol=all)
```

$$y(t)=c_1t+c_2\mathrm{e}^t$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 17

```
DSolve[(1-t)*y''[t]+t*y'[t]-y[t] == 0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to c_1e^t-c_2t$$

1.419 problem 429

Internal problem ID [7909]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 429.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-25/100)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-25/100)*y[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2e^{2ix})}{2\sqrt{x}}$$

1.420 problem 430

Internal problem ID [7910]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 430.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{ty'' - (t+1)\,y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(t*diff(y(t),t$2)-(1+t)*diff(y(t),t)+y(t) = 0,y(t), singsol=all)
```

$$y(t) = c_1(t+1) + c_2\mathrm{e}^t$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 19

```
DSolve[t*y''[t]-(1+t)*y'[t]+y[t] ==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1e^t - c_2(t+1)$$

1.421 problem 431

Internal problem ID [7911]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 431.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-t+1)\, y'' + t y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((1-t)*diff(y(t),t$2)+t*diff(y(t),t)-y(t) = 0,y(t), singsol=all)
```

$$y(t) = c_1 t + c_2 \mathrm{e}^{t}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 17

```
DSolve[(1-t)*y''[t]+t*y'[t]-y[t] ==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^t - c_2 t$$

1.422 problem 432

Internal problem ID [7912]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 432.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + xy' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 50

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \,\mathrm{e}^{-\frac{x^2}{2}} - \frac{c_2 \mathrm{e}^{-\frac{x^2}{2}} \left(i\sqrt{\pi}\,\sqrt{2}\,\operatorname{erf}\left(\frac{i\sqrt{2}\,x}{2}\right) x + 2\,\mathrm{e}^{\frac{x^2}{2}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 69

```
DSolve[y''[x]+x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\frac{\pi}{2}} c_2 e^{-\frac{x^2}{2}} \sqrt{x^2} \operatorname{erfi}\left(\frac{\sqrt{x^2}}{\sqrt{2}}\right) + \sqrt{2} c_1 e^{-\frac{x^2}{2}} x + c_2$$

1.423 problem 433

Internal problem ID [7913]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 433.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''-4xy'+6y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve((1+x^2)*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(-3x^2+1)+c_2(x^3-3x)$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 33

```
DSolve[(1+x^2)*y''[x]-4*x*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{1}{3}i\left(c_2(3x^2-1)+3c_1(x-i)^3\right)$$

1.424 problem 434

Internal problem ID [7914]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 434.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1-x)\, y'' + x y' - y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve((1-x)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x + \mathrm{e}^{x} c_2$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 17

```
DSolve[(1-x)*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x} - c_2 x$$

1.425 problem 435

Internal problem ID [7915]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 435.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2y'' + xy' + 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(2*diff(y(x),x$2)+x*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{x^2}{4}} (x^2 - 2) + c_2 e^{-\frac{x^2}{4}} (x^2 - 2) \left(\int \frac{e^{\frac{x^2}{4}}}{(x^2 - 2)^2} dx \right)$$

✓ Solution by Mathematica

Time used: 0.101 (sec). Leaf size: 61

```
DSolve[2*y''[x]+x*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8} e^{-\frac{x^2}{4}} \left(\sqrt{\pi} c_2 (x^2 - 2) \operatorname{erfi}\left(\frac{x}{2}\right) + 8c_1 (x^2 - 2) - 2c_2 e^{\frac{x^2}{4}} x \right)$$

1.426 problem 436

Internal problem ID [7916]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 436.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + xy' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \,\mathrm{e}^{-\frac{x^2}{2}} - \frac{c_2 \mathrm{e}^{-\frac{x^2}{2}} \left(i\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,x}{2}\right) x + 2\,\mathrm{e}^{\frac{x^2}{2}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 69

```
DSolve[y''[x]+x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\frac{\pi}{2}} c_2 e^{-\frac{x^2}{2}} \sqrt{x^2}\,\mathrm{erfi}\left(\frac{\sqrt{x^2}}{\sqrt{2}}\right) + \sqrt{2} c_1 e^{-\frac{x^2}{2}} x + c_2$$

1.427 problem 437

Internal problem ID [7917]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 437.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1-x)\,y''+xy'-y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve((1-x)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+\mathrm{e}^{x}c_2$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 17

```
DSolve[(1-x)*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1e^x-c_2x$$

1.428 problem 438

Internal problem ID [7918]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 438.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + xy' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 50

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \, \mathrm{e}^{-\frac{x^2}{2}} - \frac{c_2 \mathrm{e}^{-\frac{x^2}{2}} \left(i\sqrt{\pi}\, \sqrt{2}\, \operatorname{erf}\left(\frac{i\sqrt{2}\, x}{2}\right) x + 2\,\mathrm{e}^{\frac{x^2}{2}}\right)}{2}$$

✓ Solution by Mathematica
Time used: 0.045 (sec). Leaf size: 69

```
DSolve[y''[x]+x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\frac{\pi}{2}} c_2 e^{-\frac{x^2}{2}} \sqrt{x^2} \operatorname{erfi}\left(\frac{\sqrt{x^2}}{\sqrt{2}}\right) + \sqrt{2} c_1 e^{-\frac{x^2}{2}} x + c_2$$

1.429 problem 439

Internal problem ID [7919]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 439.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(-x^2+4)\,y''+xy'+2y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 69

```
dsolve((4-x^2)*diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\sqrt{x^2-6}\,\sin\left(\int\frac{\sqrt{-x^2+4}\,\sqrt{3}}{x^2-6}dx\right)+c_2\sqrt{x^2-6}\,\cos\left(\int\frac{\sqrt{-x^2+4}\,\sqrt{3}}{x^2-6}dx\right)$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 58

```
DSolve[(4-x^2)*y''[x]+x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\left(x^2-4\right)^{3/4}\left(c_1P^{\frac{3}{2}}_{-\frac{1}{2}+\sqrt{3}}\left(\frac{x}{2}\right)+c_2Q^{\frac{3}{2}}_{-\frac{1}{2}+\sqrt{3}}\left(\frac{x}{2}\right)\right)$$

1.430 problem 440

Internal problem ID [7920]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 440.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' - 4xy' + \left(-16x^2 + 3\right) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(3-16*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\sinh(2x) + c_2\sqrt{x}\,\cosh(2x)$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 32

```
DSolve[4*x^2*y''[x]-4*x*y'[x]+(3-16*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{-2x}\sqrt{x}\left(c_2e^{4x} + 4c_1\right)$$

1.431 problem 441

Internal problem ID [7921]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 441.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-1)\,y''-xy'+y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+\mathrm{e}^{x}c_2$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 17

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1e^x-c_2x$$

1.432 problem 442

Internal problem ID [7922]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 442.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - 2xy' + (x^2 + 2) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sin(x) + c_2 \cos(x) x$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-ix} x - \frac{1}{2} i c_2 e^{ix} x$$

1.433 problem 444

Internal problem ID [7923]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 444.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(x^2-2x\right)y''+\left(-x^2+2\right)y'+(2x-2)\,y=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve((x^2-2*x)*diff(y(x),x$2)+(2-x^2)*diff(y(x),x)+(2*x-2)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x^2+\mathrm{e}^x c_2$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 18

```
DSolve[(x^2-2*x)*y''[x]+(2-x^2)*y'[x]+(2*x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to c_2x^2+c_1e^x$$

1.434 problem 445

Internal problem ID [7924]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 445.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x+1)\,y''-2y'-(3+2x)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve((2*x+1)*diff(y(x),x$2)-2*diff(y(x),x)-(2*x+3)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-x}c_1+\mathrm{e}^{x}c_2x$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 29

```
DSolve[(2*x+1)*y''[x]-2*y'[x]-(2*x+3)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{-x-\frac{1}{2}}\left(c_2e^{2x+1}x+c_1\right)$$

1.435 problem 446

Internal problem ID [7925]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 446.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + (-8x^2 + 4x)\, y' + (4x^2 - 4x - 1)\, y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(4*x^2*diff(y(x),x$2)+(4*x-8*x^2)*diff(y(x),x)+(4*x^2-4*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^x}{\sqrt{x}} + c_2\sqrt{x}\, e^x$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 21

```
DSolve[4*x^2*y''[x]+(4*x-8*x^2)*y'[x]+(4*x^2-4*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{e^x(c_2x + c_1)}{\sqrt{x}}$$

1.436 problem 447

Internal problem ID [7926]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 447.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4xy' + (4x^2 + 2)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^2} + c_2 \mathrm{e}^{-x^2} x$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 20

```
DSolve[y''[x]+4*x*y'[x]+(4*x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x^2}(c_2 x + c_1)$$

1.437 problem 448

Internal problem ID [7927]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 448.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 2x(x-1)\,y' + \left(x^2-2x+2\right)y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x^2*diff(y(x),x$2)+2*x*(x-1)*diff(y(x),x)+(x^2-2*x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x} x + \mathrm{e}^{-x} c_2 x^2$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 19

```
DSolve[x^2*y''[x]+2*x*(x-1)*y'[x]+(x^2-2*x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^{-x} x (c_2 x + c_1)$$

1.438 problem 449

Internal problem ID [7928]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 449.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - x(2x-1)\,y' + \left(x^2 - x - 1\right)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)-x*(2*x-1)*diff(y(x),x)+(x^2-x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^x}{x} + e^x c_2 x$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 23

```
DSolve[x^2*y''[x]-x*(2*x-1)*y'[x]+(x^2-x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x\left(\frac{c_1}{x} + \frac{c_2 x}{2}\right)$$

1.439 problem 450

Internal problem ID [7929]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 450.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1-2x)\,y''+2y'+(2x-3)\,y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve((1-2*x)*diff(y(x),x$2)+2*diff(y(x),x)+(2*x-3)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\mathrm{e}^{x}+\mathrm{e}^{-x}c_2x$$

✓ Solution by Mathematica

Time used: 0.169 (sec). Leaf size: 48

```
DSolve[(1-2*x)*y''[x]+2*y'[x]+(2*x-3)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{e^{-x-\frac{1}{2}}\sqrt{1-2x}(c_1e^{2x}-ec_2x)}{\sqrt{2x-1}}$$

1.440 problem 451

Internal problem ID [7930]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 451.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + (1 + 4x)\, y' + (2x + 1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(2*x*diff(y(x),x$2)+(4*x+1)*diff(y(x),x)+(2*x+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} c_1 + c_2 \sqrt{x}\, \mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 23

```
DSolve[2*x*y''[x]+(4*x+1)*y'[x]+(2*x+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x} \left(2c_2 \sqrt{x} + c_1\right)$$

1.441 problem 452

Internal problem ID [7931]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 452.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (2x+1)\,y' + (1+x)\,y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x$2)-(2*x+1)*diff(y(x),x)+(x+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2 e^x x^2$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 23

```
DSolve[x*y''[x]-(2*x+1)*y'[x]+(x+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^x\left(c_2x^2 + 2c_1\right)$$

1.442 problem 453

Internal problem ID [7932]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 453.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' - 4x(1+x)\,y' + (3+2x)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*(x+1)*diff(y(x),x)+(2*x+3)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x} + c_2\sqrt{x}\,e^x$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 20

```
DSolve[4*x^2*y''[x]-4*x*(x+1)*y'[x]+(2*x+3)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sqrt{x}(c_2e^x + c_1)$$

1.443 problem 454

Internal problem ID [7933]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 454.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (-2x + 2) y' + (x - 2) y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x$2)+(2-2*x)*diff(y(x),x)+(x-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^x}{x} + e^x c_2$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 19

```
DSolve[x*y''[x]+(2-2*x)*y'[x]+(x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x (c_2 x + c_1)}{x}$$

1.444 problem 455

Internal problem ID [7934]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 455.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, ‘_with_symmetry_[0,F`

$$\boxed{x^2y'' - 2xy' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 13

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_2x^2 + c_1x$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 14

```
DSolve[x^2*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(c_2x + c_1)$$

1.445 problem 456

Internal problem ID [7935]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 456.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (2x+2)\,y' + (x+2)\,y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x$2)-(2*x+2)*diff(y(x),x)+(x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^x + c_2 \mathrm{e}^x x^3$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 23

```
DSolve[x*y''[x]-(2*x+2)*y'[x]+(x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} e^x \left(c_2 x^3 + 3c_1\right)$$

1.446 problem 457

Internal problem ID [7936]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 457.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2xy' + \left(x^2 + 2\right) y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sin(x) + c_2 \cos(x) x$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-ix} x - \frac{1}{2} i c_2 e^{ix} x$$

1.447 problem 458

Internal problem ID [7937]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 458.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (1 + 4x)\, y' + (4x + 2)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x*diff(y(x),x$2)-(4*x+1)*diff(y(x),x)+(4*x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{2x} + \mathrm{e}^{2x} c_2 x^2$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 25

```
DSolve[x*y''[x]-(4*x+1)*y'[x]+(4*x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{2x} \left(c_2 x^2 + 2c_1\right)$$

1.448 problem 460

Internal problem ID [7938]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 460.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' - 4xy' + \left(-16x^2 + 3\right) y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(3-16*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\sinh(2x) + c_2\sqrt{x}\,\cosh(2x)$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 32

```
DSolve[4*x^2*y''[x]-4*x*y'[x]+(3-16*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{-2x}\sqrt{x}\left(c_2e^{4x} + 4c_1\right)$$

1.449 problem 461

Internal problem ID [7939]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 461.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(2x+1)\,xy''-2\left(2x^2-1\right)y'-4(1+x)\,y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve((2*x+1)*x*diff(y(x),x$2)-2*(2*x^2-1)*diff(y(x),x)-4*(x+1)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1}{x}+c_2\mathrm{e}^{2x}$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 28

```
DSolve[(2*x+1)*x*y''[x]-2*(2*x^2-1)*y'[x]-4*(x+1)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{c_2e^{2x+1}x+c_1}{\sqrt{e}x}$$

1.450 problem 462

Internal problem ID [7940]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 462.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2-2x)\,y''+(-x^2+2)\,y'+(2x-2)\,y=0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 14

```
dsolve((x^2-2*x)*diff(y(x),x$2)+(2-x^2)*diff(y(x),x)+(2*x-2)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x^2+\mathrm{e}^xc_2$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 18

```
DSolve[(x^2-2*x)*y''[x]+(2-x^2)*y'[x]+(2*x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to c_2x^2+c_1e^x$$

1.451 problem 463

Internal problem ID [7941]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 463.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (1 + 4x)\, y' + (4x + 2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x*diff(y(x),x$2)-(4*x+1)*diff(y(x),x)+(4*x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{2x} + \mathrm{e}^{2x} c_2 x^2$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 25

```
DSolve[x*y''[x]-(4*x+1)*y'[x]+(4*x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{2x} (c_2 x^2 + 2c_1)$$

1.452 problem 464

Internal problem ID [7942]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 464.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(3x-1)\,y''-(3x+2)\,y'-(6x-8)\,y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve((3*x-1)*diff(y(x),x$2)-(3*x+2)*diff(y(x),x)-(6*x-8)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\mathrm{e}^{2x}+\mathrm{e}^{-x}c_2x$$

✓ Solution by Mathematica
Time used: 0.137 (sec). Leaf size: 35

```
DSolve[(3*x-1)*y''[x]-(3*x+2)*y'[x]-(6*x-8)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{e^{-x-\frac{1}{2}}\left(c_1e^{3x}+2ec_2x\right)}{\sqrt{2}}$$

1.453 problem 465

Internal problem ID [7943]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 465.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1+x)^2 y'' - 2(1+x) y' - (x^2+2x-1) y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve((x+1)^2*diff(y(x),x$2)-2*(x+1)*diff(y(x),x)-(x^2+2*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sinh(x)(x+1) + c_2 \cosh(x)(x+1)$$

✓ Solution by Mathematica
Time used: 0.117 (sec). Leaf size: 147

```
DSolve[(x+1)^2*y''[x]-2*(x+1)*x*y'[x]-(x^2+2*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to 2^{\frac{1}{2}i\left(\sqrt{7}+i\right)} e^{-\left(\left(\sqrt{2}-1\right)(x+1)\right)} (x+1)^{\frac{1}{2}i\left(\sqrt{7}+i\right)} \left(c_1 \text{HypergeometricU}\left(\frac{1}{2}\left(1-\sqrt{2}+i\sqrt{7}\right), 1+i\sqrt{7}, 2\sqrt{2}(x+1)\right) + c_2 L_{\frac{1}{2}\left(-1+\sqrt{2}-i\sqrt{7}\right)}^{i\sqrt{7}}\left(2\sqrt{2}(x+1)\right)\right)$$

1.454 problem 466

Internal problem ID [7944]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 466.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + \left(-8x^2 + 4x\right) y' + \left(4x^2 - 4x - 1\right) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(4*x^2*diff(y(x),x$2)+(4*x-8*x^2)*diff(y(x),x)+(4*x^2-4*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^x}{\sqrt{x}} + c_2\sqrt{x}\, e^x$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 21

```
DSolve[4*x^2*y''[x]+(4*x-8*x^2)*y'[x]+(4*x^2-4*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{e^x(c_2x + c_1)}{\sqrt{x}}$$

1.455 problem 467

Internal problem ID [7945]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 467.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4xy' + (4x^2 + 2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^2} + c_2 \mathrm{e}^{-x^2} x$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 21

```
DSolve[4*x^2*y''[x]+(4*x-8*x^2)*y'[x]+(4*x^2-4*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{e^x (c_2 x + c_1)}{\sqrt{x}}$$

1.456 problem 468

Internal problem ID [7946]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 468.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x+1)\,y''-2y'-(3+2x)\,y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve((2*x+1)*diff(y(x),x$2)-2*diff(y(x),x)-(2*x+3)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-x}c_1+\mathrm{e}^{x}c_2x$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 29

```
DSolve[(2*x+1)*y''[x]-2*y'[x]-(2*x+3)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{-x-\frac{1}{2}}\left(c_2e^{2x+1}x+c_1\right)$$

1.457 problem 469

Internal problem ID [7947]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 469.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (2x+2)\,y' + (x+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x$2)-(2*x+2)*diff(y(x),x)+(x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^x + c_2 \mathrm{e}^x x^3$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 29

```
DSolve[x*y''[x]-(2*x+2)*y'[x]+(x+2)*y[x]==6*x^3*Exp[x],y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{1}{6} e^x \left(9x^4 + 2c_2 x^3 + 6c_1\right)$$

1.458 problem 470

Internal problem ID [7948]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 470.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2xy' + (x^2+2)\,y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x\sin(x) + c_2\cos(x)\,x$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1e^{-ix}x - \frac{1}{2}ic_2e^{ix}x$$

1.459 problem 472

Internal problem ID [7949]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 472.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' - 4xy' + (-16x^2 + 3)\,y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(3-16*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\sinh(2x) + c_2\sqrt{x}\,\cosh(2x)$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 32

```
DSolve[4*x^2*y''[x]-4*x*y'[x]+(3-16*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{-2x}\sqrt{x}\left(c_2e^{4x} + 4c_1\right)$$

1.460 problem 473

Internal problem ID [7950]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 473.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' - 4xy' + (4x^2 + 3)\, y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2+3)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\sin(x) + c_2\sqrt{x}\,\cos(x)$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 39

```
DSolve[4*x^2*y''[x]-4*x*y'[x]+(4*x^2+3)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-ix}\sqrt{x}\left(2c_1 - ic_2e^{2ix}\right)$$

1.461 problem 474

Internal problem ID [7951]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 474.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2xy' - y(x^2 - 2) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)-(x^2-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sinh(x) + c_2 x \cosh(x)$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 25

```
DSolve[x^2*y''[x]-2*x*y'[x]-(x^2-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x} x + \frac{1}{2} c_2 e^x x$$

1.462 problem 475

Internal problem ID [7952]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 475.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - 2x(1+x)\,y' + \left(x^2+2x+2\right)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)-2*x*(x+1)*diff(y(x),x)+(x^2+2*x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^x c_1 x + c_2 e^x x^2$$

✓ Solution by Mathematica

Time used: 0.132 (sec). Leaf size: 41

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2*x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{ix}x(c_1 \operatorname{HypergeometricU}(-i,0,-2ix) + c_2 L_i^{-1}(-2ix))$$

1.463 problem 476

Internal problem ID [7953]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 476.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2x(x+2)\,y' + \left(x^2+4x+6\right)y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)-2*x*(x+2)*diff(y(x),x)+(x^2+4*x+6)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\mathrm{e}^x x^2 + c_2\mathrm{e}^x x^3$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 19

```
DSolve[x^2*y''[x]-2*x*(x+2)*y'[x]+(x^2+4*x+6)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^x x^2(c_2x + c_1)$$

1.464 problem 477

Internal problem ID [7954]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 477.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 4xy' + (x^2 + 6)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(x^2+6)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^2 \sin(x) + c_2 \cos(x)\, x^2$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 37

```
DSolve[x^2*y''[x]-4*x*y'[x]+(x^2+6)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-ix} x^2 \left(2c_1 - ic_2 e^{2ix}\right)$$

1.465 problem 478

Internal problem ID [7955]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 478.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-1)\,y''-xy'+y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+\mathrm{e}^{x}c_2$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 17

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1e^{x}-c_2x$$

1.466 problem 479

Internal problem ID [7956]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 479.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' - 4x(1+x)\,y' + (3+2x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*(x+1)*diff(y(x),x)+(2*x+3)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x} + c_2\sqrt{x}\,\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 20

```
DSolve[4*x^2*y''[x]-4*x*(x+1)*y'[x]+(2*x+3)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}(c_2e^x + c_1)$$

1.467 problem 480

Internal problem ID [7957]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 480.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(3x-1)\,y''-(3x+2)\,y'-(6x-8)\,y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve((3*x-1)*diff(y(x),x$2)-(3*x+2)*diff(y(x),x)-(6*x-8)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\mathrm{e}^{2x}+\mathrm{e}^{-x}c_2x$$

✓ Solution by Mathematica

Time used: 0.08 (sec). Leaf size: 35

```
DSolve[(3*x-1)*y''[x]-(3*x+2)*y'[x]-(6*x-8)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{e^{-x-\frac{1}{2}}(c_1e^{3x}+2ec_2x)}{\sqrt{2}}$$

1.468 problem 481

Internal problem ID [7958]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 481.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x+2)\,y''+xy'+3y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 115

```
dsolve((2+x)*diff(y(x),x$2)+x*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\mathrm{e}^{-x}\left(x^5-20x^3-40x^2+32\right)\\ -\frac{c_2(\mathrm{expIntegral}_1\left(-2-x\right)\mathrm{e}^{-2}x^5+\mathrm{e}^x x^4-20\,\mathrm{e}^{-2}\,\mathrm{expIntegral}_1\left(-2-x\right)x^3-\mathrm{e}^x x^3-40\,\mathrm{e}^{-2}\,\mathrm{expIntegral}}{240}$$

✓ Solution by Mathematica

Time used: 0.148 (sec). Leaf size: 81

```
DSolve[(2+x)*y''[x]+x*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{1}{960}e^{-x-1}\left(c_2\left(x^2-6x+4\right)(x+2)^3\,\mathrm{ExpIntegralEi}(x+2)\right.\\ \left.+3840c_1\left(x^2-6x+4\right)(x+2)^3-c_2e^{x+2}\left(x^4-x^3-18x^2-22x+8\right)\right)$$

1.469 problem 482

Internal problem ID [7959]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 482.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-x)\,y''+x(4+x)\,y'+(-x+2)\,y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
dsolve(x^2*(1-x)*diff(y(x),x$2)+x*(4+x)*diff(y(x),x)+(2-x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(x^2+6x+3)}{x}+\frac{c_2(3x^3\ln(x)+18x^2\ln(x)+9x\ln(x)+51x^2+48x+1)}{3x^2}$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 53

```
DSolve[x^2*(1-x)*y''[x]+x*(4+x)*y'[x]+(2-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{3c_1x(x^2+6x+3)-c_2(51x^2+3(x^2+6x+3)\,x\log(x)+48x+1)}{3x^2}$$

1.470 problem 483

Internal problem ID [7960]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 483.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1+x)\,y''+x(2x+1)\,y'-(6x+4)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(x^2*(1+x)*diff(y(x),x$2)+x*(1+2*x)*diff(y(x),x)-(4+6*x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x^2-\frac{c_2(12\ln(x+1)\,x^4-12x^4\ln(x)-12x^3+6x^2-4x+3)}{12x^2}$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 52

```
DSolve[x^2*(1+x)*y''[x]+x*(1+2*x)*y'[x]-(4+6*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\to c_1x^2+\frac{c_2(12x^4\log(x)-12x^4\log(x+1)+12x^3-6x^2+4x-3)}{12x^2}$$

1.471 problem 484

Internal problem ID [7961]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 484.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(2x^2+1)\,y''+x(2x^2+4)\,y'+2(1-x^2)\,y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 59

```
dsolve(x^2*(1+2*x^2)*diff(y(x),x$2)+x*(4+2*x^2)*diff(y(x),x)+2*(1-x^2)*y(x)=0,y(x), singsol=
```

$$y(x)=\frac{c_1}{x}+\frac{c_2\sqrt{2}\left(\sqrt{2}\,\sqrt{2x^2+1}\,x^2+3\,\mathrm{arcsinh}\left(\sqrt{2}\,x\right)x-\sqrt{2}\,\sqrt{2x^2+1}\right)}{2x^2}$$

✓ Solution by Mathematica

Time used: 0.091 (sec). Leaf size: 77

```
DSolve[x^2*(1+2*x^2)*y''[x]+x*(4+2*x^2)*y'[x]+2*(1-x^2)*y[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x)\to -\frac{c_2\sqrt{2x^2+1}}{x^2}+c_2\sqrt{2x^2+1}-\frac{3c_2\log\left(\sqrt{2x^2+1}-\sqrt{2}x\right)}{\sqrt{2}x}+\frac{c_1}{x}$$

1.472 problem 485

Internal problem ID [7962]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 485.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+2)\,y''+2x(x^2+5)\,y'+2(-x^2+3)\,y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 85

```
dsolve(x^2*(2+x^2)*diff(y(x),x$2)+2*x*(x^2+5)*diff(y(x),x)+2*(3-x^2)*y(x)=0,y(x), singsol=al
```

$$y(x)=\frac{c_1(x^2+8)}{x}-\frac{c_2\sqrt{2}\left(\operatorname{arctanh}\left(\frac{\sqrt{2}}{\sqrt{x^2+2}}\right)x^4-\sqrt{2}\,\sqrt{x^2+2}\,x^2+8\operatorname{arctanh}\left(\frac{\sqrt{2}}{\sqrt{x^2+2}}\right)x^2+4\sqrt{2}\,\sqrt{x^2+2}\right)}{64x^3}$$

✓ Solution by Mathematica

Time used: 0.135 (sec). Leaf size: 88

```
DSolve[x^2*(2+x^2)*y''[x]+2*x*(x^2+5)*y'[x]+2*(3-x^2)*y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x)\to\frac{-\sqrt{2}c_2(x^2+8)\,x^2\operatorname{arctanh}\left(\frac{\sqrt{x^2+2}}{\sqrt{2}}\right)+64c_1x^4+2x^2\left(c_2\sqrt{x^2+2}+256c_1\right)-8c_2\sqrt{x^2+2}}{64x^3}$$

1.473 problem 486

Internal problem ID [7963]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 486.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''+6xy'+6y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve((1+x^2)*diff(y(x),x$2)+6*x*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x}{(x^2+1)^2}+\frac{c_2(x^2-1)}{(x^2+1)^2}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 29

```
DSolve[(1+x^2)*y''[x]+6*x*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_2x-c_1(x-i)^2}{(x^2+1)^2}$$

1.474 problem 487

Internal problem ID [7964]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 487.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\, y''+2xy'-2y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve((1+x^2)*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+c_2(\arctan(x)\, x+1)$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 48

```
DSolve[(1+x^2)*y''[x]+2*x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{1}{2}i(2c_1x-c_2x\log(1-ix)+c_2x\log(1+ix)+2ic_2)$$

1.475 problem 488

Internal problem ID [7965]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 488.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''-8xy'+20y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve((1+x^2)*diff(y(x),x$2)-8*x*diff(y(x),x)+20*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(5x^4-10x^2+1)+c_2(x^5-10x^3+5x)$$

✓ Solution by Mathematica
Time used: 0.073 (sec). Leaf size: 38

```
DSolve[(1+x^2)*y''[x]-8*x*y'[x]+20*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{5}ic_2(5x^4-10x^2+1)+c_1(1+ix)^5$$

1.476 problem 489

Internal problem ID [7966]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 489.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(1 - x^2)\, y'' - 8xy' - 12y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 43

```
dsolve((1-x^2)*diff(y(x),x$2)-8*x*diff(y(x),x)-12*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(3x^2 + 1)}{(x - 1)^3 (x + 1)^3} + \frac{c_2(x^3 + 3x)}{(x - 1)^3 (x + 1)^3}$$

✓ Solution by Mathematica
Time used: 0.045 (sec). Leaf size: 37

```
DSolve[(1-x^2)*y''[x]-8*x*y'[x]-12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3c_1(x - 1)^3 - c_2(3x^2 + 1)}{3\,(x^2 - 1)^3}$$

1.477 problem 490

Internal problem ID [7967]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 490.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x^2+1)\,y''+7xy'+2y=0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 44

```
dsolve((1+2*x^2)*diff(y(x),x$2)+7*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1 x}{(2x^2+1)^{\frac{3}{4}}}+\frac{c_2 x\left(\int \frac{1}{(2x^2+1)^{\frac{1}{4}}x^2}dx\right)}{(2x^2+1)^{\frac{3}{4}}}$$

✓ Solution by Mathematica
Time used: 0.062 (sec). Leaf size: 66

```
DSolve[(1+2*x^2)*y''[x]+7*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{c_2 Q_{\frac{1}{4}}^{\frac{3}{4}}\left(i\sqrt{2}x\right)}{(2x^2+1)^{3/8}}+\frac{2i\sqrt[4]{2}c_1 x}{(2x^2+1)^{3/4}\,\text{Gamma}\left(\frac{1}{4}\right)}$$

1.478 problem 491

Internal problem ID [7968]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 491.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(1-x^2)\,y''-5xy'-4y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 46

```
dsolve((1-x^2)*diff(y(x),x$2)-5*x*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x}{(x^2-1)^{\frac{3}{2}}}+\frac{c_2\left(\ln\left(x+\sqrt{x^2-1}\right)x-\sqrt{x^2-1}\right)}{(x^2-1)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 52

```
DSolve[(1-x^2)*y''[x]-5*x*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{-c_2\sqrt{x^2-1}-c_2x\log\left(\sqrt{x^2-1}-x\right)+c_1x}{(x^2-1)^{3/2}}$$

1.479 problem 492

Internal problem ID [7969]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 492.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\, y'' - 10xy' + 28y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve((1+x^2)*diff(y(x),x$2)-10*x*diff(y(x),x)+28*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(1 + \frac{35}{3}x^4 - 14x^2\right) + c_2\left(x^7 + 21x^5 - 105x^3 + 35x\right)$$

✓ Solution by Mathematica

Time used: 0.075 (sec). Leaf size: 40

```
DSolve[(1+x^2)*y''[x]-10*x*y'[x]+28*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{105}c_2\left(35x^4 - 42x^2 + 3\right) - c_1(x-i)^6(x+6i)$$

1.480 problem 493

Internal problem ID [7970]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 493.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + xy' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \,\mathrm{e}^{-\frac{x^2}{2}} - \frac{c_2 \mathrm{e}^{-\frac{x^2}{2}} \left(i\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,x}{2}\right) x + 2\,\mathrm{e}^{\frac{x^2}{2}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 69

```
DSolve[y''[x]+x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\frac{\pi}{2}} c_2 e^{-\frac{x^2}{2}} \sqrt{x^2} \mathrm{erfi}\left(\frac{\sqrt{x^2}}{\sqrt{2}}\right) + \sqrt{2} c_1 e^{-\frac{x^2}{2}} x + c_2$$

1.481 problem 495

Internal problem ID [7971]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 495.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x^2 - 8x + 11)\, y'' - 16(x - 2)\, y' + 36y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve((11-8*x+2*x^2)*diff(y(x),x$2)-16*(x-2)*diff(y(x),x)+36*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(-\frac{31}{5} + x^3 - 6x^2 + \frac{111}{10}x\right) + c_2\left(x^6 - 12x^5 + \frac{165}{2}x^4 - \frac{16577}{8} - \frac{5445}{4}x^2 + 3267x\right)$$

✓ Solution by Mathematica

Time used: 0.898 (sec). Leaf size: 91

```
DSolve[(11-8*x+2*x^2)*y''[x]-16*(x-2)*y'[x]+36*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{1}{15}ic_2\left(10x^3 - 60x^2 + 111x - 62\right) + \frac{c_1\left(2x + 5i\sqrt{6} - 4\right)\left(2(x-4)x + 11\right)^2\left(2ix + \sqrt{6} - 4i\right)^3}{2\left(-2ix + \sqrt{6} + 4i\right)^2}$$

1.482 problem 496

Internal problem ID [7972]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 496.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (-3 + x) y' + 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 68

```
dsolve(diff(y(x),x$2)+(x-3)*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{1}{2}x^2+3x} (x^2 - 6x + 8) + c_2 \mathrm{e}^{-\frac{1}{2}x^2+3x} (x^2 - 6x + 8) \left(\int \frac{\mathrm{e}^{\frac{1}{2}x^2-3x}}{(x-2)^2 (x-4)^2} dx \right)$$

✓ Solution by Mathematica

Time used: 0.275 (sec). Leaf size: 90

```
DSolve[y''[x]+(x-3)*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} e^{-\frac{1}{2}(x-6)x-8} \left(e^{7/2} \sqrt{2\pi} c_2 (x^2 - 6x + 8) \operatorname{erfi}\left(\frac{x-3}{\sqrt{2}} \right) + 4e^8 c_1 (x^2 - 6x + 8) - 2c_2 e^{\frac{1}{2}(x-4)^2+x} (x-3) \right)$$

1.483 problem 497

Internal problem ID [7973]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 497.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 - 8x + 14)\, y'' - 8(x - 4)\, y' + 20y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve((x^2-8*x+14)*diff(y(x),x$2)-8*(x-4)*diff(y(x),x)+20*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\frac{1604}{5} + x^4 - 16x^3 + 100x^2 - 288x\right) + c_2\left(x^5 - 140x^3 + 1120x^2 - 3500x + 4032\right)$$

✓ Solution by Mathematica

Time used: 0.105 (sec). Leaf size: 77

```
DSolve[(x^2-8*x+14)*y''[x]+8*(x-4)*y'[x]+20*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 P^3_{\frac{1}{2}i\left(i+\sqrt{31}\right)}\left(\frac{x-4}{\sqrt{2}}\right) + c_2 Q^3_{\frac{1}{2}i\left(i+\sqrt{31}\right)}\left(\frac{x-4}{\sqrt{2}}\right)}{\left(x^2 - 8x + 14\right)^{3/2}}$$

1.484 problem 498

Internal problem ID [7974]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 498.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(2x^2+4x+5)\,y''-20(1+x)\,y'+60y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
dsolve((2*x^2+4*x+5)*diff(y(x),x$2)-20*(x+1)*diff(y(x),x)+60*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(-\frac{7}{4}+x^5+5x^4+5x^3-5x^2-\frac{31}{4}x\right)+c_2\left(x^6+\frac{155}{8}-\frac{75}{2}x^4-100x^3-\frac{225}{4}x^2+30x\right)$$

✓ Solution by Mathematica

Time used: 0.917 (sec). Leaf size: 83

```
DSolve[(2*x^2+4*x+5)*y''[x]-20*(x+1)*y'[x]+60*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to\frac{(2x^2+4x+5)^{5/2}\left(4c_2(4x^5+20x^4+20x^3-20x^2-31x-7)+c_1\left(2ix+\sqrt{6}+2i\right)^6\right)}{(4x^2+8x+10)^{5/2}}$$

1.485 problem 499

Internal problem ID [7975]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 499.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^3+1)\,y''+7x^2y'+9yx=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve((1+x^3)*diff(y(x),x$2)+7*x^2*diff(y(x),x)+9*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x}{(x^3+1)^{\frac{4}{3}}}+\frac{c_2x\left(\int\frac{((x+1)(x^2-x+1))^{\frac{1}{3}}}{x^2}dx\right)}{(x^3+1)^{\frac{4}{3}}}$$

✓ Solution by Mathematica

Time used: 0.798 (sec). Leaf size: 118

```
DSolve[(1+x^3)*y''[x]+7*x^2*y'[x]+9*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{-2\sqrt{3}c_2x\arctan\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)-6c_2\sqrt[3]{x^3+1}-2c_2x\log\left(\sqrt[3]{x^3+1}-x\right)+c_2x\log\left(\sqrt[3]{x^3+1}x+(x^3\right.}{6\,(x^3+1)^{4/3}}$$

1.486 problem 500

Internal problem ID [7976]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 500.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x^5 + 1) y'' + 14y'x^4 + 10yx^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve((1+2*x^5)*diff(y(x),x$2)+14*x^4*diff(y(x),x)+10*x^3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x}{(2x^5 + 1)^{\frac{2}{5}}} + \frac{c_2 x \left(\int \frac{1}{(2x^5+1)^{\frac{3}{5}} x^2} dx\right)}{(2x^5 + 1)^{\frac{2}{5}}}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(1+2*x^5)*y''[x]+14*x^4*y'[x]+10*x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

1.487 problem 501

Internal problem ID [7977]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 501.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x^6 + 7yx^5 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 69

```
dsolve(diff(y(x),x$2)+x^6*diff(y(x),x)+7*x^5*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x^7}{7}} x + \frac{7c_2(-1)^{\frac{6}{7}} \mathrm{e}^{-\frac{x^7}{7}} \left(-\Gamma\left(\frac{6}{7}\right) x^7 + (-x^7)^{\frac{6}{7}} 7^{\frac{1}{7}} \mathrm{e}^{\frac{x^7}{7}} + \Gamma\left(\frac{6}{7}, -\frac{x^7}{7}\right) x^7\right)}{(-x^7)^{\frac{6}{7}}}$$

✓ Solution by Mathematica

Time used: 0.113 (sec). Leaf size: 53

```
DSolve[y''[x]+x^6*y'[x]+7*x^5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{49} e^{-\frac{x^7}{7}} \left(49c_1 x - 7^{6/7} c_2 \sqrt[7]{-x^7} \Gamma\left(-\frac{1}{7}, -\frac{x^7}{7}\right)\right)$$

1.488 problem 502

Internal problem ID [7978]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 502.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^8+1)\,y''-16y'x^7+72yx^6=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve((1+x^8)*diff(y(x),x$2)-16*x^7*diff(y(x),x)+72*x^6*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(-\frac{7}{9}+x^8\right)+c_2\left(x^9-\frac{9}{7}x\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(1+x^8)*y''[x]-16*x^7*y'[x]+72*x^6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

1.489 problem 503

Internal problem ID [7979]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 503.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x^5 + 6yx^4 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 81

```
dsolve(diff(y(x),x$2)+x^5*diff(y(x),x)+6*x^4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x^6}{6}} x - \frac{2c_2 \mathrm{e}^{-\frac{x^6}{6}} \left(6^{\frac{2}{3}} \sqrt{3}\, \sqrt{2}\, (-x^6)^{\frac{5}{6}} \mathrm{e}^{\frac{x^6}{6}} + 6\Gamma\left(\frac{5}{6}, -\frac{x^6}{6}\right) x^6 - 6\Gamma\left(\frac{5}{6}\right) x^6\right)}{(-x^6)^{\frac{5}{6}} \left(\sqrt{3} + i\right)}$$

✓ Solution by Mathematica

Time used: 0.117 (sec). Leaf size: 53

```
DSolve[y''[x]+x^5*y'[x]+6*x^4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{36} e^{-\frac{x^6}{6}} \left(36 c_1 x - 6^{5/6} c_2 \sqrt[6]{-x^6}\, \Gamma\left(-\frac{1}{6}, -\frac{x^6}{6}\right)\right)$$

1.490 problem 504

Internal problem ID [7980]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 504.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(3x+1)\,y''+xy'+2y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 56

```
dsolve((1+3*x)*diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(3x+1)^{\frac{10}{9}}\,\mathrm{e}^{-\frac{x}{3}}(x-6)+c_2(3x+1)^{\frac{10}{9}}\,\mathrm{e}^{-\frac{x}{3}}(x-6)\left(\int\frac{\mathrm{e}^{\frac{x}{3}}}{(x-6)^2\,(3x+1)^{\frac{19}{9}}}dx\right)$$

✓ Solution by Mathematica

Time used: 0.9 (sec). Leaf size: 124

```
DSolve[(1+3*x)*y''[x]+x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{e^{-\frac{x}{3}-\frac{1}{9}}\left(1520c_1\sqrt[9]{3x+1}(3x^2-17x-6)-2^{8/9}c_2e^{\frac{x}{3}+\frac{1}{9}}(9x^2-48x-26)+2^{8/9}3^{7/9}c_2\sqrt[9]{-3x-1}(3x^2-17\right.}{380\;2^{17/18}}$$

1.491 problem 505

Internal problem ID [7981]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 505.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(3x^2+x+1)\,y''+(2+15x)\,y'+12y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 143

```
dsolve((1+x+3*x^2)*diff(y(x),x$2)+(2+15*x)*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\left(\frac{i\sqrt{11}-6x-1}{i\sqrt{11}+6x+1}\right)^{-\frac{i\sqrt{11}}{22}}x}{(3x^2+x+1)^{\frac{3}{2}}}+\frac{c_2\left(\frac{i\sqrt{11}-6x-1}{i\sqrt{11}+6x+1}\right)^{-\frac{i\sqrt{11}}{22}}x\left(\int\frac{\sqrt{3x^2+x+1}\left(\frac{i\sqrt{11}+6x+1}{i\sqrt{11}-6x-1}\right)^{-\frac{i\sqrt{11}}{22}}}{x^2}dx\right)}{(3x^2+x+1)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 3.09 (sec). Leaf size: 93

```
DSolve[(1+x+3*x^2)*y''[x]+(2+15*x)*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{xe^{\frac{\arctan\left(\frac{6x+1}{\sqrt{11}}\right)}{\sqrt{11}}}\left(c_2\int_1^x\frac{e^{-\frac{\arctan\left(\frac{6K[1]+1}{\sqrt{11}}\right)}{\sqrt{11}}}\sqrt{3K[1]^2+K[1]+1}}{K[1]^2}dK[1]+c_1\right)}{(3x^2+x+1)^{3/2}}$$

1.492 problem 506

Internal problem ID [7982]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 506.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x+2)\,y''+(1+x)\,y'+3y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 89

```
dsolve((2+x)*diff(y(x),x$2)+(1+x)*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\mathrm{e}^{-x}x\left(x^3-12x-16\right)-\frac{c_2(\mathrm{e}^{-2}\,\mathrm{expIntegral}_1\left(-2-x\right)x^4+\mathrm{e}^x x^3-12\,\mathrm{e}^{-2}\,\mathrm{expIntegral}_1\left(-2-x\right)x^2-\mathrm{e}^x x^2-16\,\mathrm{e}^{-2}\,\mathrm{expIntegral}}{48}$$

✓ Solution by Mathematica

Time used: 0.189 (sec). Leaf size: 99

```
DSolve[(2+x)*y''[x]+(1+x)*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{e^{-x-1}(c_2(x-4)(x+2)^2x\,\mathrm{ExpIntegralEi}(x+2)+384c_1x^4-c_2e^{x+2}x^3+x^2(c_2e^{x+2}-4608c_1)+x(10c_2e}{96\sqrt{2}}$$

1.493 problem 507

Internal problem ID [7983]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 507.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(4+x)\,y''+(x+2)\,y'+2y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 108

```
dsolve((4+x)*diff(y(x),x$2)+(2+x)*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\mathrm{e}^{-x}x\left(x^3+12x^2+48x+64\right)$$
$$-\frac{c_2(\mathrm{e}^{-4}\operatorname{expIntegral}_1\left(-x-4\right)x^4+12\,\mathrm{e}^{-4}\operatorname{expIntegral}_1\left(-x-4\right)x^3+\mathrm{e}^{x}x^3+48\,\mathrm{e}^{-4}\operatorname{expIntegral}_1\left(-x-\right.}{24}$$

✓ Solution by Mathematica
Time used: 0.108 (sec). Leaf size: 97

```
DSolve[(4+x)*y''[x]+(2+x)*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{24}e^{-x-4}\left(c_2x(x+4)^3\operatorname{ExpIntegralEi}(x+4)\right.$$
$$\left.+e^4\left(24c_1x^4+x^3(288c_1-c_2e^x)+9x^2(128c_1-c_2e^x)+2x(768c_1-11c_2e^x)-6c_2e^x\right)\right)$$

1.494 problem 508

Internal problem ID [7984]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 508.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(2x^2+3x)\,y''+10(1+x)\,y'+8y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
dsolve((3*x+2*x^2)*diff(y(x),x$2)+10*(1+x)*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(x+2)}{x^{\frac{7}{3}}\,(2x+3)^{\frac{2}{3}}}+\frac{c_2(x+2)\left(\int\frac{x^{\frac{4}{3}}}{(x+2)^2(2x+3)^{\frac{1}{3}}}dx\right)}{x^{\frac{7}{3}}\,(2x+3)^{\frac{2}{3}}}$$

✓ Solution by Mathematica

Time used: 0.28 (sec). Leaf size: 245

```
DSolve[(3*x+2*x^2)*y''[x]+10*(1+x)*y'[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{2\,2^{2/3}\sqrt{3}c_2(x+2)\arctan\left(\frac{\sqrt{3}\sqrt[3]{x}}{\sqrt[3]{x}+2^{2/3}\sqrt[3]{2x+3}}\right)+2^{2/3}c_2x\log\left(2x^{2/3}+2^{2/3}\sqrt[3]{2x+3}\sqrt[3]{x}+\sqrt[3]{2}(2x+3)^{2/3}\right.}{}$$

1.495 problem 509

Internal problem ID [7985]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 509.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - (6 - 7x)\,y' + 8y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 81

```
dsolve(x^2*diff(y(x),x$2)-(6-7*x)*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{-\frac{6}{x}}(x-2)}{x^5} + \frac{c_2 \left(x^3 \mathrm{e}^{\frac{6}{x}} + 12x^2 \mathrm{e}^{\frac{6}{x}} + 108 \,\mathrm{expIntegral}_1\left(-\frac{6}{x}\right) x - 36x\,\mathrm{e}^{\frac{6}{x}} - 216 \,\mathrm{expIntegral}_1\left(-\frac{6}{x}\right)\right) \mathrm{e}^{-\frac{6}{x}}}{2x^5}$$

✓ Solution by Mathematica

Time used: 0.146 (sec). Leaf size: 59

```
DSolve[x^2*y''[x]-(6-7*x)*y'[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-6/x}\left(-108c_2(x-2)\,\mathrm{ExpIntegralEi}\left(\frac{6}{x}\right) + c_2 e^{6/x} x(x^2+12x-36) + 2c_1(x-2)\right)}{2x^5}$$

1.496 problem 510

Internal problem ID [7986]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 510.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(2x^2+x+1)\,y''+(1+7x)\,y'+2y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 149

```
dsolve((1+x+2*x^2)*diff(y(x),x$2)+(1+7*x)*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\left(\frac{i\sqrt{7}-4x-1}{i\sqrt{7}+4x+1}\right)^{-\frac{3i\sqrt{7}}{28}}(x+1)}{(2x^2+x+1)^{\frac{3}{4}}}+\frac{c_2\left(\frac{i\sqrt{7}-4x-1}{i\sqrt{7}+4x+1}\right)^{-\frac{3i\sqrt{7}}{28}}(x+1)\left(\int\frac{\left(\frac{i\sqrt{7}+4x+1}{i\sqrt{7}-4x-1}\right)^{-\frac{3i\sqrt{7}}{28}}}{(x+1)^2(2x^2+x+1)^{\frac{1}{4}}}dx\right)}{(2x^2+x+1)^{\frac{3}{4}}}$$

✓ Solution by Mathematica

Time used: 2.132 (sec). Leaf size: 102

```
DSolve[(1+x+2*x^2)*y''[x]+(1+7*x)*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{(x+1)e^{\frac{3\arctan\left(\frac{4x+1}{\sqrt{7}}\right)}{2\sqrt{7}}}\left(c_2\int_1^x\frac{e^{-\frac{3\arctan\left(\frac{4K[1]+1}{\sqrt{7}}\right)}{2\sqrt{7}}}}{(K[1]+1)^2\sqrt[4]{2K[1]^2+K[1]+1}}dK[1]+c_1\right)}{(2x^2+x+1)^{3/4}}$$

1.497 problem 511

Internal problem ID [7987]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 511.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x+3)\,y''+(2x+1)\,y'-(-x+2)\,y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 44

```
dsolve((3+x)*diff(y(x),x$2)+(1+2*x)*diff(y(x),x)-(2-x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-x}c_1+c_2\mathrm{e}^{-x}\left(x^6+18x^5+135x^4+540x^3+1215x^2+1458x\right)$$

✓ Solution by Mathematica
Time used: 0.055 (sec). Leaf size: 29

```
DSolve[(3+x)*y''[x]+(1+2*x)*y'[x]-(2-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{6}e^{-x-3}\left(c_2(x+3)^6+6c_1\right)$$

1.498 problem 512

Internal problem ID [7988]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 512.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 3xy' + (2x^2 + 4)\, y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve(diff(y(x),x$2)+3*x*diff(y(x),x)+(4+2*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x^2 - 1)\, e^{-x^2} + c_2 e^{-x^2}(x^2 - 1)\left(\int \frac{e^{\frac{x^2}{2}}}{(x-1)^2 (x+1)^2} dx\right)$$

✓ Solution by Mathematica

Time used: 0.151 (sec). Leaf size: 63

```
DSolve[y''[x]+3*x*y'[x]+(4+2*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} e^{-x^2}\left(\sqrt{2\pi} c_2 (x^2 - 1)\, \text{erfi}\left(\frac{x}{\sqrt{2}}\right) + 4c_1(x^2 - 1) - 2c_2 e^{\frac{x^2}{2}} x\right)$$

1.499 problem 513

Internal problem ID [7989]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 513.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(4x+2)\,y''-4y'-(4x+6)\,y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve((2+4*x)*diff(y(x),x$2)-4*diff(y(x),x)-(6+4*x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-x}c_1+\mathrm{e}^{x}c_2x$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 29

```
DSolve[(2+4*x)*y''[x]-4*y'[x]-(6+4*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{-x-\frac{1}{2}}\left(c_2e^{2x+1}x+c_1\right)$$

1.500 problem 514

Internal problem ID [7990]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 514.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 3xy' + (2x^2 + 5)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)-3*x*diff(y(x),x)+(5+2*x^2)*y(x)=0,y(x), singsol=all)
```

$$\begin{aligned} y(x) = {} & c_1 \mathrm{e}^{\frac{x^2}{2}} (x^6 - 15x^4 + 45x^2 - 15) \\ & + c_2 \mathrm{e}^{\frac{x^2}{2}} (x^6 - 15x^4 + 45x^2 - 15) \left(\int \frac{\mathrm{e}^{\frac{x^2}{2}}}{(x^6 - 15x^4 + 45x^2 - 15)^2} dx \right) \end{aligned}$$

✓ Solution by Mathematica
Time used: 0.269 (sec). Leaf size: 95

```
DSolve[y''[x]-3*x*y'[x]+(5+2*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{\frac{x^2}{2}} \left(\sqrt{2\pi} c_2 (x^6 - 15x^4 + 45x^2 - 15) \operatorname{erfi}\left(\frac{x}{\sqrt{2}}\right) - 2c_2 e^{\frac{x^2}{2}} x(x^4 - 14x^2 + 33) + 1440 c_1 (x^6 - 15x^4 + 45x^2 - \right.}{1440}$$

1.501 problem 515

Internal problem ID [7991]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 515.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2y'' + 5xy' + (2x^2 + 4)\, y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(2*diff(y(x),x$2)+5*x*diff(y(x),x)+(4+2*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x^2} + c_2 e^{-x^2} \operatorname{erf}\left(\frac{i\sqrt{3}\, x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.074 (sec). Leaf size: 42

```
DSolve[2*y''[x]+5*x*y'[x]+(4+2*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} e^{-x^2} \left(\sqrt{3\pi} c_2 \operatorname{erfi}\left(\frac{\sqrt{3}x}{2}\right) + 3c_1\right)$$

1.502 problem 516

Internal problem ID [7992]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 516.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4xy' + (4x^2 + 2)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+4*x*diff(y(x),x)+(2+4*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^2} + c_2 \mathrm{e}^{-x^2} x$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 20

```
DSolve[y''[x]+4*x*y'[x]+(2+4*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x^2}(c_2 x + c_1)$$

1.503 problem 517

Internal problem ID [7993]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 517.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4xy' + (4x^2 + 2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+4*x*diff(y(x),x)+(2+4*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^2} + c_2 \mathrm{e}^{-x^2} x$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 20

```
DSolve[y''[x]+4*x*y'[x]+(2+4*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x^2}(c_2 x + c_1)$$

1.504 problem 518

Internal problem ID [7994]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 518.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x^2+x+1)\,y''+x(11x^2+11x+9)\,y'+(7x^2+10x+6)\,y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 141

```
dsolve(2*x^2*(1+x+x^2)*diff(y(x),x$2)+x*(9+11*x+11*x^2)*diff(y(x),x)+(6+10*x+7*x^2)*y(x)=0,y
```

$$y(x)=\frac{c_1\sqrt{x^2+x+1}\left(\frac{i\sqrt{3}+2x+1}{i\sqrt{3}-2x-1}\right)^{-\frac{i\sqrt{3}}{6}}}{x^2}+\frac{c_2\sqrt{x^2+x+1}\left(\frac{i\sqrt{3}+2x+1}{i\sqrt{3}-2x-1}\right)^{-\frac{i\sqrt{3}}{6}}\left(\int\frac{\left(\frac{i\sqrt{3}-2x-1}{i\sqrt{3}+2x+1}\right)^{-\frac{i\sqrt{3}}{6}}}{(x^2+x+1)^{\frac{3}{2}}\sqrt{x}}dx\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.77 (sec). Leaf size: 93

```
DSolve[2*x^2*(1+x+x^2)*y''[x]+x*(9+11*x+11*x^2)*y'[x]+(6+10*x+7*x^2)*y[x]==0,y[x],x,IncludeS
```

$$y(x)\to\frac{\sqrt{x^2+x+1}e^{-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}}\left(c_2\int_1^x\frac{e^{\frac{\arctan\left(\frac{2K[1]+1}{\sqrt{3}}\right)}{\sqrt{3}}}}{\sqrt{K[1]}(K[1]^2+K[1]+1)^{3/2}}dK[1]+c_1\right)}{x^2}$$

1.505 problem 519

Internal problem ID [7995]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 519.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$3x^2y'' + 2x(-2x^2 + x + 1)y' + (-8x^2 + 2x)y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 51

```
dsolve(3*x^2*diff(y(x),x$2)+2*x*(1+x-2*x^2)*diff(y(x),x)+(2*x-8*x^2)*y(x)=0,y(x), singsol=al
```

$$y(x) = c_1 x^{\frac{1}{3}} e^{\frac{2}{3}x^2 - \frac{2}{3}x} + c_2 x^{\frac{1}{3}} e^{\frac{2}{3}x^2 - \frac{2}{3}x} \left(\int \frac{e^{-\frac{2}{3}x^2 + \frac{2}{3}x}}{x^{\frac{4}{3}}} dx \right)$$

✓ Solution by Mathematica
Time used: 0.379 (sec). Leaf size: 53

```
DSolve[3*x^2*y''[x]+2*x*(1+x-2*x^2)*y'[x]+(2*x-8*x^2)*y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to e^{\frac{2}{3}(x-1)x} \sqrt[3]{x} \left(c_2 \int_1^x \frac{e^{-\frac{2}{3}(K[1]-1)K[1]}}{K[1]^{4/3}} dK[1] + c_1 \right)$$

1.506 problem 520

Internal problem ID [7996]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 520.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$12x^2(1+x)\,y''+x\left(3x^2+35x+11\right)y'-\left(-5x^2-10x+1\right)y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(12*x^2*(1+x)*diff(y(x),x$2)+x*(11+35*x+3*x^2)*diff(y(x),x)-(1-10*x-5*x^2)*y(x)=0,y(x)
```

$$y(x)=\frac{c_1 \mathrm{e}^{-\frac{x}{4}}}{(x+1)^{\frac{3}{4}}\,x^{\frac{1}{4}}}+\frac{c_2 \mathrm{e}^{-\frac{x}{4}}\left(\int \frac{\mathrm{e}^{\frac{x}{4}}}{(x+1)^{\frac{1}{4}}\,x^{\frac{5}{12}}}dx\right)}{(x+1)^{\frac{3}{4}}\,x^{\frac{1}{4}}}$$

✓ Solution by Mathematica

Time used: 0.418 (sec). Leaf size: 61

```
DSolve[12*x^2*(1+x)*y''[x]+x*(11+35*x+3*x^2)*y'[x]-(1-10*x-5*x^2)*y[x]==0,y[x],x,IncludeSing
```

$$y(x)\to \frac{e^{-x/4}\left(c_2\int_1^x \frac{e^{\frac{K[1]}{4}}}{K[1]^{5/12}\sqrt[4]{K[1]+1}}dK[1]+c_1\right)}{\sqrt[4]{x}(x+1)^{3/4}}$$

1.507 problem 521

Internal problem ID [7997]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 521.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y''+3y'+4y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\mathrm{e}^{-\frac{3x}{2}}\cos\left(\frac{\sqrt{7}\,x}{2}\right)+c_2\mathrm{e}^{-\frac{3x}{2}}\sin\left(\frac{\sqrt{7}\,x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 42

```
DSolve[y''[x]+3*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{-3x/2}\left(c_2\cos\left(\frac{\sqrt{7}x}{2}\right)+c_1\sin\left(\frac{\sqrt{7}x}{2}\right)\right)$$

1.508 problem 522

Internal problem ID [7998]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 522.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$18x^2(1+x)\,y''+3x\left(x^2+11x+5\right)y'-\left(-5x^2-2x+1\right)y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 50

```
dsolve(18*x^2*(1+x)*diff(y(x),x$2)+3*x*(5+11*x+x^2)*diff(y(x),x)-(1-2*x-5*x^2)*y(x)=0,y(x),
```

$$y(x)=c_1\mathrm{e}^{-\frac{x}{6}}\left(\frac{x+1}{x}\right)^{\frac{1}{6}}+c_2\mathrm{e}^{-\frac{x}{6}}\left(\frac{x+1}{x}\right)^{\frac{1}{6}}\left(\int\frac{\mathrm{e}^{\frac{x}{6}}}{(x+1)^{\frac{7}{6}}\sqrt{x}}dx\right)$$

✓ Solution by Mathematica

Time used: 0.555 (sec). Leaf size: 73

```
DSolve[18*x^2*(1+x)*y''[x]+3*x*(5+11*x+x^2)*y'[x]-(1-2*x-5*x^2)*y[x]==0,y[x],x,IncludeSingul
```

$$y(x)\to\frac{e^{-x/6}\left(c_2\int_1^x\frac{e^{\frac{K[1]}{6}}\sqrt[3]{\frac{K[1]}{K[1]+1}}}{K[1]^{5/6}(K[1]+1)^{5/6}}dK[1]+c_1\right)}{\sqrt[6]{\frac{x}{x+1}}}$$

1.509 problem 523

Internal problem ID [7999]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 523.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2y'' + x(3+2x)\,y' - (1-x)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(2*x^2*diff(y(x),x$2)+x*(3+2*x)*diff(y(x),x)-(1-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{-x}}{x} + \frac{c_2 \mathrm{e}^{-x} \left(\int \sqrt{x}\, \mathrm{e}^{x} dx\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 33

```
DSolve[2*x^2*y''[x]+x*(3+2*x)*y'[x]-(1-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}\left(c_2 x^{3/2} L_{-\frac{3}{2}}^{\frac{3}{2}}(x) + c_1\right)}{x}$$

1.510 problem 524

Internal problem ID [8000]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 524.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' + x(x+5)\,y' - (2-3x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(2*x^2*diff(y(x),x$2)+x*(5+x)*diff(y(x),x)-(2-3*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\mathrm{e}^{-\frac{x}{2}} + c_2\sqrt{x}\,\mathrm{e}^{-\frac{x}{2}}\left(\int \frac{\mathrm{e}^{\frac{x}{2}}}{x^{\frac{7}{2}}}dx\right)$$

✓ Solution by Mathematica

Time used: 0.094 (sec). Leaf size: 70

```
DSolve[2*x^2*y''[x]+x*(5+x)*y'[x]-(2-3*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{15}\left(-\frac{2c_2(x^2+x+3)}{x^2} + 15c_1e^{-x/2}\sqrt{x} + \sqrt{2}c_2e^{-x/2}\sqrt{-x}\Gamma\left(\frac{1}{2},-\frac{x}{2}\right)\right)$$

1.511 problem 525

Internal problem ID [8001]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 525.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3x^2y'' + x(1+x)\,y' - y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
dsolve(3*x^2*diff(y(x),x$2)+x*(1+x)*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{-\frac{x}{3}}}{x^{\frac{1}{3}}} + \frac{c_2 \mathrm{e}^{-\frac{x}{3}} \left(\int x^{\frac{1}{3}} \mathrm{e}^{\frac{x}{3}} dx\right)}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 50

```
DSolve[3*x^2*y''[x]+x*(1+x)*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x/3}\left(c_2 x^{2/3} - 3\sqrt[3]{3}c_1(-x)^{2/3}\Gamma\left(\frac{4}{3}, -\frac{x}{3}\right)\right)}{x}$$

1.512 problem 526

Internal problem ID [8002]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 526.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' - xy' + (1-2x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(2*x^2*diff(y(x),x$2)-x*diff(y(x),x)+(1-2*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\sinh\left(2\sqrt{x}\right) + c_2\sqrt{x}\,\cosh\left(2\sqrt{x}\right)$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 41

```
DSolve[2*x^2*y''[x]-x*y'[x]+(1-2*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-2\sqrt{x}}\sqrt{x}\left(2c_1e^{4\sqrt{x}} - c_2\right)$$

1.513 problem 527

Internal problem ID [8003]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 527.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3x^2y'' + x(1+x)\,y' - (3x+1)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(3*x^2*diff(y(x),x$2)+x*(1+x)*diff(y(x),x)-(1+3*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x(x^2+20x+70) + c_2x(x^2+20x+70)\left(\int \frac{e^{-\frac{x}{3}}}{x^{\frac{7}{3}}\,(x^2+20x+70)^2}dx\right)$$

✓ Solution by Mathematica

Time used: 0.156 (sec). Leaf size: 78

```
DSolve[3*x^2*y''[x]+x*(1+x)*y'[x]-(1+3*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1x(x^2+20x+70) - \frac{c_2x(x^2+20x+70)\,\Gamma\left(\frac{2}{3},\frac{x}{3}\right)}{1680\sqrt[3]{3}} + \frac{c_2e^{-x/3}(x^3+19x^2+54x-18)}{1680\sqrt[3]{x}}$$

1.514 problem 528

Internal problem ID [8004]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 528.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x+3)\,y''+x(1+5x)\,y'+(1+x)\,y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
dsolve(2*x^2*(3+x)*diff(y(x),x$2)+x*(1+5*x)*diff(y(x),x)+(1+x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x^{\frac{1}{3}}}{(x+3)^{\frac{4}{3}}}+\frac{c_2x^{\frac{1}{3}}\left(\int\frac{(x+3)^{\frac{1}{3}}}{x^{\frac{5}{6}}}dx\right)}{(x+3)^{\frac{4}{3}}}$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 50

```
DSolve[2*x^2*(3+x)*y''[x]+x*(1+5*x)*y'[x]+(1+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\to\frac{\sqrt[3]{x}\left(6\sqrt[3]{3}c_2\sqrt[6]{x}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{6},\frac{7}{6},-\frac{x}{3}\right)+c_1\right)}{(x+3)^{4/3}}$$

1.515 problem 529

Internal problem ID [8005]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 529.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(4+x)\,y'' - x(-3x+1)\,y' + y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(x^2*(4+x)*diff(y(x),x$2)-x*(1-3*x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^{\frac{1}{4}}}{(x+4)^{\frac{9}{4}}} + \frac{c_2 x^{\frac{1}{4}} \left(\int \frac{(x+4)^{\frac{5}{4}}}{x^{\frac{1}{4}}} dx\right)}{(x+4)^{\frac{9}{4}}}$$

✓ Solution by Mathematica

Time used: 0.102 (sec). Leaf size: 89

```
DSolve[x^2*(4+x)*y''[x]-x*(1-3*x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[4]{x}\left(-10c_2 \arctan\left(\sqrt[4]{\frac{x}{x+4}}\right) + 10c_2 \operatorname{arctanh}\left(\sqrt[4]{\frac{x}{x+4}}\right) + c_2\sqrt[4]{x+4}\,x^{7/4} + 9c_2\sqrt[4]{x+4}\,x^{3/4} + 2c_1\right)}{2(x+4)^{9/4}}$$

1.516 problem 530

Internal problem ID [8006]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 530.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' + 5xy' + (1+x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(2*x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+(1+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin\left(\sqrt{x}\,\sqrt{2}\right)}{x} + \frac{c_2 \cos\left(\sqrt{x}\,\sqrt{2}\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 60

```
DSolve[2*x^2*y''[x]+5*x*y'[x]+(1+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{i\sqrt{2}\sqrt{x}} + i\sqrt{2}c_2 e^{-i\sqrt{2}\sqrt{x}}}{2x}$$

1.517 problem 531

Internal problem ID [8007]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 531.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{6x^2y'' + x(10-x)\,y' - (x+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(6*x^2*diff(y(x),x$2)+x*(10-x)*diff(y(x),x)-(2+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + \frac{c_2\left(\int x^{\frac{1}{3}}\mathrm{e}^{\frac{x}{6}}dx\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 38

```
DSolve[6*x^2*y''[x]+x*(10-x)*y'[x]-(2+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2\sqrt[3]{x}L_{-\frac{4}{3}}^{\frac{4}{3}}\left(\frac{x}{6}\right) + \frac{6\sqrt[3]{6}c_1}{x}$$

1.518 problem 532

Internal problem ID [8008]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 532.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(3+4x)\,y''+x(11+4x)\,y'-(3+4x)\,y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 60

```
dsolve(x^2*(3+4*x)*diff(y(x),x$2)+x*(11+4*x)*diff(y(x),x)-(3+4*x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(48x^2+32x+7)}{x^3}+\frac{c_2(48x^2+32x+7)\left(\int \frac{(4x+3)^{\frac{8}{3}}x^{\frac{7}{3}}}{(48x^2+32x+7)^2}dx\right)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.313 (sec). Leaf size: 339

```
DSolve[x^2*(3+4*x)*y''[x]+x*(11+4*x)*y'[x]-(3+4*x)*y[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{-12\sqrt[3]{2}\sqrt{3}c_2(48x^2+32x+7)\arctan\left(\frac{\sqrt{3}\sqrt[3]{x}}{\sqrt[3]{x}+\sqrt[3]{8x+6}}\right)+384c_2(4x+3)^{2/3}x^{10/3}+576c_2(4x+3)^{2/3}x^{7/3}}{}$$

1.519 problem 533

Internal problem ID [8009]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 533.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(3x+2)\,y'' + x(4+11x)\,y' - (1-x)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(2*x^2*(2+3*x)*diff(y(x),x$2)+x*(4+11*x)*diff(y(x),x)-(1-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{x}} + \frac{c_2(3x+2)^{\frac{1}{6}}}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 32

```
DSolve[2*x^2*(2+3*x)*y''[x]+x*(4+11*x)*y'[x]-(1-x)*y[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{c_2\sqrt[6]{6x+4} + 2^{5/6}c_1}{\sqrt{x}}$$

1.520 problem 534

Internal problem ID [8010]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 534.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x+2)\,y''+5x(1-x)\,y'-(-8x+2)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 88

```
dsolve(x^2*(2+x)*diff(y(x),x$2)+5*x*(1-x)*diff(y(x),x)-(2-8*x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(40x^4-160x^3+60x^2+8x+1)}{x^2}+\frac{c_2(40x^4-160x^3+60x^2+8x+1)\left(\int\frac{x^{\frac{3}{2}}(x+2)^{\frac{15}{2}}}{(40x^4-160x^3+60x^2+8x+1)^2}dx\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 47.99 (sec). Leaf size: 1347

```
DSolve[x^2*(2+x)*y''[x]+5*x*(1-x)*y'[x]-(2-8*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

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1.521 problem 535

Internal problem ID [8011]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 535.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$8x^2(1-x^2)\,y'' + 2x(-13x^2+1)\,y' + (-9x^2+1)\,y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 60

```
dsolve(8*x^2*(1-x^2)*diff(y(x),x$2)+2*x*(1-13*x^2)*diff(y(x),x)+(1-9*x^2)*y(x)=0,y(x), sings
```

$$y(x) = c_1\sqrt{\frac{1}{(x-1)(x+1)}}\,x^{\frac{1}{4}} + c_2\sqrt{\frac{1}{(x-1)(x+1)}}\,x^{\frac{1}{4}}\left(\int\frac{\sqrt{\frac{1}{(x-1)(x+1)}}}{x^{\frac{3}{4}}}dx\right)$$

✓ Solution by Mathematica

Time used: 0.077 (sec). Leaf size: 47

```
DSolve[8*x^2*(1-x^2)*y''[x]+2*x*(1-13*x^2)*y'[x]+(1-9*x^2)*y[x]==0,y[x],x,IncludeSingularSol
```

$$y(x) \to \frac{\sqrt[4]{x}\left(4c_2\sqrt[4]{x}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{1}{2},\frac{9}{8},x^2\right)+c_1\right)}{\sqrt{1-x^2}}$$

1.522 problem 536

Internal problem ID [8012]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 536.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+1)\,y''-2x(-x^2+2)\,y'+4y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-2*x*(2-x^2)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x(3x^2+1)}{(x^2+1)^2}+\frac{x^4c_2}{(x^2+1)^2}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 35

```
DSolve[x^2*(1+x^2)*y''[x]-2*x*(2-x^2)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to -\frac{-3c_1x^4+3c_2x^3+c_2x}{3\,(x^2+1)^2}$$

1.523 problem 537

Internal problem ID [8013]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 537.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x(x^2+3)\,y''+(-x^2+2)\,y'-8yx=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x*(3+x^2)*diff(y(x),x$2)+(2-x^2)*diff(y(x),x)-8*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x^4+\frac{11}{2}x^2+\frac{55}{8}\right)+c_2(x^2+3)^{\frac{11}{6}}\,x^{\frac{1}{3}}$$

✓ Solution by Mathematica

Time used: 0.091 (sec). Leaf size: 41

```
DSolve[x*(3+x^2)*y''[x]+(2-x^2)*y'[x]-8*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\sqrt[3]{x}(x^2+3)^{11/6}-\frac{1}{55}c_2(8x^4+44x^2+55)$$

1.524 problem 538

Internal problem ID [8014]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 538.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(1-x^2)\,y'' + x(-19x^2+7)\,y' - (14x^2+1)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 60

```
dsolve(4*x^2*(1-x^2)*diff(y(x),x$2)+x*(7-19*x^2)*diff(y(x),x)-(1+14*x^2)*y(x)=0,y(x), singso
```

$$y(x) = \frac{c_1\sqrt{\frac{1}{(x-1)(x+1)}}}{x} + \frac{c_2\sqrt{\frac{1}{(x-1)(x+1)}}\left(\int\sqrt{\frac{1}{(x-1)(x+1)}}\,x^{\frac{1}{4}}dx\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.075 (sec). Leaf size: 50

```
DSolve[4*x^2*(1-x^2)*y''[x]+x*(7-19*x^2)*y'[x]-(1+14*x^2)*y[x]==0,y[x],x,IncludeSingularSolu
```

$$y(x) \to \frac{4c_2x^{5/4}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{8},\frac{13}{8},x^2\right)+5c_1}{5x\sqrt{1-x^2}}$$

1.525 problem 539

Internal problem ID [8015]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 539.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3x^2(-x^2+2)\,y''+x(-11x^2+1)\,y'+(-5x^2+1)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(3*x^2*(2-x^2)*diff(y(x),x$2)+x*(1-11*x^2)*diff(y(x),x)+(1-5*x^2)*y(x)=0,y(x), singsol
```

$$y(x)=\frac{c_1\sqrt{x}}{(x^2-2)^{\frac{3}{4}}}+\frac{c_2\sqrt{x}\left(\int\frac{1}{(x^2-2)^{\frac{1}{4}}x^{\frac{7}{6}}}dx\right)}{(x^2-2)^{\frac{3}{4}}}$$

✓ Solution by Mathematica

Time used: 0.095 (sec). Leaf size: 57

```
DSolve[3*x^2*(2-x^2)*y''[x]+x*(1-11*x^2)*y'[x]+(1-5*x^2)*y[x]==0,y[x],x,IncludeSingularSolut
```

$$y(x)\to\frac{c_1\sqrt{x}-3\,2^{3/4}c_2\sqrt[3]{x}\,\text{Hypergeometric2F1}\left(-\frac{1}{12},\frac{1}{4},\frac{11}{12},\frac{x^2}{2}\right)}{(2-x^2)^{3/4}}$$

1.526 problem 540

Internal problem ID [8016]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 540.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x^2+2)\,y''-x(-7x^2+12)\,y'+(3x^2+7)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(2*x^2*(2+x^2)*diff(y(x),x$2)-x*(12-7*x^2)*diff(y(x),x)+(7+3*x^2)*y(x)=0,y(x), singsol
```

$$y(x)=\frac{c_1x^{\frac{7}{2}}}{(x^2+2)^{\frac{9}{4}}}+\frac{c_2x^{\frac{7}{2}}\left(\int\frac{(x^2+2)^{\frac{5}{4}}}{x^4}dx\right)}{(x^2+2)^{\frac{9}{4}}}$$

✓ Solution by Mathematica

Time used: 0.111 (sec). Leaf size: 57

```
DSolve[2*x^2*(2+x^2)*y''[x]-x*(12-7*x^2)*y'[x]+(7+3*x^2)*y[x]==0,y[x],x,IncludeSingularSolut
```

$$y(x)\to\frac{\sqrt{x}\left(3c_1x^3-2\sqrt[4]{2}c_2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{5}{4},-\frac{1}{2},-\frac{x^2}{2}\right)\right)}{3\,(x^2+2)^{9/4}}$$

1.527 problem 541

Internal problem ID [8017]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 541.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x^2+2)\,y''+x(7x^2+4)\,y'-(-3x^2+1)\,y=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 38

```
dsolve(2*x^2*(2+x^2)*diff(y(x),x$2)+x*(4+7*x^2)*diff(y(x),x)-(1-3*x^2)*y(x)=0,y(x), singsol=
```

$$y(x)=\frac{c_1}{(x^2+2)^{\frac{1}{4}}\sqrt{x}}+\frac{c_2\left(\int\frac{1}{(x^2+2)^{\frac{3}{4}}}dx\right)}{(x^2+2)^{\frac{1}{4}}\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.074 (sec). Leaf size: 68

```
DSolve[2*x^2*(2+x^2)*y''[x]+x*(4+7*x^2)*y'[x]-(1-3*x^2)*y[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x)\to\frac{c_2\sqrt[8]{x^2+2}\,\text{Gamma}\left(\frac{3}{4}\right)Q_{-\frac{1}{4}}^{\frac{1}{4}}\left(\frac{ix}{\sqrt{2}}\right)+2^{3/8}c_1}{\sqrt{x}\sqrt[4]{x^2+2}\,\text{Gamma}\left(\frac{3}{4}\right)}$$

1.528 problem 542

Internal problem ID [8018]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 542.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(2x^2+1)\,y'' + 5x(6x^2+1)\,y' - (-40x^2+2)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(2*x^2*(1+2*x^2)*diff(y(x),x$2)+5*x*(1+6*x^2)*diff(y(x),x)-(2-40*x^2)*y(x)=0,y(x), sin
```

$$y(x) = \frac{c_1\sqrt{x}}{(2x^2+1)^{\frac{3}{2}}} + \frac{c_2\sqrt{x}\left(\int \frac{\sqrt{2x^2+1}}{x^{\frac{7}{2}}}dx\right)}{(2x^2+1)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.103 (sec). Leaf size: 52

```
DSolve[2*x^2*(1+2*x^2)*y''[x]+5*x*(1+6*x^2)*y'[x]-(2-40*x^2)*y[x]==0,y[x],x,IncludeSingularS
```

$$y(x) \to \frac{5c_1x^{5/2} - 2c_2\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},-2x^2\right)}{5x^2\left(2x^2+1\right)^{3/2}}$$

1.529 problem 543

Internal problem ID [8019]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 543.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x(x^2+1)\,y''+(7x^2+4)\,y'+8yx=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(x*(1+x^2)*diff(y(x),x$2)+(4+7*x^2)*diff(y(x),x)+8*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1}{\sqrt{x^2+1}\,x^3}+\frac{c_2\left(\frac{x\sqrt{x^2+1}}{2}-\frac{\operatorname{arcsinh}(x)}{2}\right)}{\sqrt{x^2+1}\,x^3}$$

✓ Solution by Mathematica

Time used: 0.076 (sec). Leaf size: 56

```
DSolve[x*(1+x^2)*y''[x]+(4+7*x^2)*y'[x]+8*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_2x\sqrt{x^2+1}+c_2\log\left(\sqrt{x^2+1}-x\right)+2c_1}{2x^3\sqrt{x^2+1}}$$

1.530 problem 544

Internal problem ID [8020]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 544.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x^2+1)\,y''+x(8x^2+3)\,y'-(-4x^2+3)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(2*x^2*(1+x^2)*diff(y(x),x$2)+x*(3+8*x^2)*diff(y(x),x)-(3-4*x^2)*y(x)=0,y(x), singsol=
```

$$y(x)=\frac{c_1}{(x^2+1)^{\frac{1}{4}}\,x^{\frac{3}{2}}}+\frac{c_2\left(\int\frac{x^{\frac{3}{2}}}{(x^2+1)^{\frac{3}{4}}}dx\right)}{(x^2+1)^{\frac{1}{4}}\,x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.081 (sec). Leaf size: 60

```
DSolve[2*x^2*(1+x^2)*y''[x]+x*(3+8*x^2)*y'[x]-(3-4*x^2)*y[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x)\to-\frac{c_2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-x^2\right)}{x\sqrt[4]{x^2+1}}+\frac{c_1}{x^{3/2}\sqrt[4]{x^2+1}}+\frac{c_2}{x}$$

1.531 problem 545

Internal problem ID [8021]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 545.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2y'' + 3x(x^2+3)\,y' - (-5x^2+1)\,y = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 39

```
dsolve(9*x^2*diff(y(x),x$2)+3*x*(3+x^2)*diff(y(x),x)-(1-5*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{-\frac{x^2}{6}}}{x^{\frac{1}{3}}} + \frac{c_2 e^{-\frac{x^2}{6}} \left(\int \frac{e^{\frac{x^2}{6}}}{x^{\frac{1}{3}}} dx\right)}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica
Time used: 0.093 (sec). Leaf size: 61

```
DSolve[9*x^2*y''[x]+3*x*(3+x^2)*y'[x]-(1-5*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{e^{-\frac{x^2}{6}} \left(2c_1 x^{4/3} + \sqrt[3]{6} c_2 (-x^2)^{2/3} \Gamma\left(\frac{1}{3}, -\frac{x^2}{6}\right)\right)}{2x^{5/3}}$$

1.532 problem 546

Internal problem ID [8022]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 546.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$6x^2y'' + x(6x^2+1)\,y' + (9x^2+1)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(6*x^2*diff(y(x),x$2)+x*(1+6*x^2)*diff(y(x),x)+(1+9*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{\frac{1}{3}} \mathrm{e}^{-\frac{x^2}{2}} + c_2 x^{\frac{1}{3}} \mathrm{e}^{-\frac{x^2}{2}} \left(\int \frac{\mathrm{e}^{\frac{x^2}{2}}}{x^{\frac{5}{6}}} dx\right)$$

✓ Solution by Mathematica

Time used: 0.094 (sec). Leaf size: 61

```
DSolve[6*x^2*y''[x]+x*(1+6*x^2)*y'[x]+(1+9*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{e^{-\frac{x^2}{2}}\left(2c_1 x^{11/6} + \sqrt[12]{2}c_2(-x^2)^{11/12}\,\Gamma\left(\frac{1}{12}, -\frac{x^2}{2}\right)\right)}{2x^{3/2}}$$

1.533 problem 547

Internal problem ID [8023]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 547.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2(x^2+1)\,y''+3x(13x^2+3)\,y'-(-25x^2+1)\,y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 38

```
dsolve(9*x^2*(1+x^2)*diff(y(x),x$2)+3*x*(3+13*x^2)*diff(y(x),x)-(1-25*x^2)*y(x)=0,y(x), sing
```

$$y(x)=\frac{c_1}{(x^2+1)^{\frac{2}{3}}\,x^{\frac{1}{3}}}+\frac{c_2\left(\int\frac{1}{(x^3+x)^{\frac{1}{3}}}dx\right)}{(x^2+1)^{\frac{2}{3}}\,x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.115 (sec). Leaf size: 124

```
DSolve[9*x^2*(1+x^2)*y''[x]+3*x*(3+13*x^2)*y'[x]-(1-25*x^2)*y[x]==0,y[x],x,IncludeSingularSo
```

$y(x)$

$$\rightarrow \frac{2\sqrt{3}c_2\arctan\left(\frac{\sqrt{3}x^{2/3}}{x^{2/3}+2\sqrt[3]{x^2+1}}\right)-2c_2\log\left(\sqrt[3]{x^2+1}-x^{2/3}\right)+c_2\log\left(x^{4/3}+(x^2+1)^{2/3}+\sqrt[3]{x^2+1}x^{2/3}\right.}{4\sqrt[3]{x}\,(x^2+1)^{2/3}}$$

1.534 problem 548

Internal problem ID [8024]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 548.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x^2+1)\,y''+4x(6x^2+1)\,y'-(-25x^2+1)\,y=0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 43

```
dsolve(4*x^2*(1+x^2)*diff(y(x),x$2)+4*x*(1+6*x^2)*diff(y(x),x)-(1-25*x^2)*y(x)=0,y(x), sings
```

$$y(x)=\frac{c_1\sqrt{x}}{(x^2+1)^{\frac{3}{2}}}+\frac{c_2\left(\operatorname{arcsinh}(x)\,x-\sqrt{x^2+1}\right)}{\sqrt{x}\,(x^2+1)^{\frac{3}{2}}}$$

✓ Solution by Mathematica
Time used: 0.079 (sec). Leaf size: 57

```
DSolve[4*x^2*(1+x^2)*y''[x]+4*x*(1+6*x^2)*y'[x]-(1-25*x^2)*y[x]==0,y[x],x,IncludeSingularSol
```

$$y(x)\to\frac{-c_2\sqrt{x^2+1}-c_2x\log\left(\sqrt{x^2+1}-x\right)+c_1x}{\sqrt{x}\,(x^2+1)^{3/2}}$$

1.535 problem 549

Internal problem ID [8025]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 549.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$8x^2(2x^2+1)\,y''+2x(34x^2+5)\,y'-(-30x^2+1)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(8*x^2*(1+2*x^2)*diff(y(x),x$2)+2*x*(5+34*x^2)*diff(y(x),x)-(1-30*x^2)*y(x)=0,y(x), si
```

$$y(x)=\frac{c_1x^{\frac{1}{4}}}{\sqrt{2x^2+1}}+\frac{c_2x^{\frac{1}{4}}\left(\int\frac{1}{\sqrt{2x^2+1}\,x^{\frac{7}{4}}}dx\right)}{\sqrt{2x^2+1}}$$

✓ Solution by Mathematica

Time used: 0.101 (sec). Leaf size: 54

```
DSolve[8*x^2*(1+2*x^2)*y''[x]+2*x*(5+34*x^2)*y'[x]-(1-30*x^2)*y[x]==0,y[x],x,IncludeSingular
```

$$y(x)\to\frac{3c_1x^{3/4}-4c_2\,\text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{1}{2},\frac{5}{8},-2x^2\right)}{3\sqrt{x}\sqrt{2x^2+1}}$$

1.536 problem 550

Internal problem ID [8026]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 550.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(1+x)\,y'' - x(-3x+1)\,y' + y = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 23

```
dsolve(2*x^2*(1+x)*diff(y(x),x$2)-x*(1-3*x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x}{x+1} + \frac{c_2\sqrt{x}}{x+1}$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 25

```
DSolve[2*x^2*(1+x)*y''[x]-x*(1-3*x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1\sqrt{x} + 2c_2 x}{x+1}$$

1.537 problem 551

Internal problem ID [8027]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 551.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$6x^2(2x^2+1)\,y''+x(50x^2+1)\,y'+(30x^2+1)\,y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
dsolve(6*x^2*(1+2*x^2)*diff(y(x),x$2)+x*(1+50*x^2)*diff(y(x),x)+(1+30*x^2)*y(x)=0,y(x), sing
```

$$y(x)=\frac{c_1\sqrt{x}}{2x^2+1}+\frac{c_2x^{\frac{1}{3}}}{2x^2+1}$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 32

```
DSolve[6*x^2*(1+2*x^2)*y''[x]+x*(1+50*x^2)*y'[x]+(1+30*x^2)*y[x]==0,y[x],x,IncludeSingularSo
```

$$y(x)\to\frac{\sqrt[3]{x}\left(6c_2\sqrt[6]{x}+c_1\right)}{2x^2+1}$$

1.538 problem 552

Internal problem ID [8028]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 552.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$28x^2(-3x+1)\,y'' - 7x(5+9x)\,y' + 7(2+9x)\,y = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 29

```
dsolve(28*x^2*(1-3*x)*diff(y(x),x$2)-7*x*(5+9*x)*diff(y(x),x)+7*(2+9*x)*y(x)=0,y(x), singsol
```

$$y(x) = \frac{c_1 x^2}{3x-1} + \frac{c_2 x^{\frac{1}{4}}}{3x-1}$$

✓ Solution by Mathematica
Time used: 0.045 (sec). Leaf size: 30

```
DSolve[28*x^2*(1-3*x)*y''[x]-7*x*(5+9*x)*y'[x]+7*(2+9*x)*y[x]==0,y[x],x,IncludeSingularSolut
```

$$y(x) \to \frac{4c_2 x^2 + 7c_1 \sqrt[4]{x}}{7-21x}$$

1.539 problem 553

Internal problem ID [8029]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 553.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$8x^2(-x^2+2)\,y''+2x(-21x^2+10)\,y'-(35x^2+2)\,y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(8*x^2*(2-x^2)*diff(y(x),x$2)+2*x*(10-21*x^2)*diff(y(x),x)-(2+35*x^2)*y(x)=0,y(x), sin
```

$$y(x)=\frac{c_1}{(x^2-2)\sqrt{x}}+\frac{c_2x^{\frac{1}{4}}}{x^2-2}$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 34

```
DSolve[8*x^2*(2-x^2)*y''[x]+2*x*(10-21*x^2)*y'[x]-(2+35*x^2)*y[x]==0,y[x],x,IncludeSingularS
```

$$y(x)\to\frac{\frac{3c_1}{\sqrt{x}}+4c_2\sqrt[4]{x}}{6-3x^2}$$

1.540 problem 554

Internal problem ID [8030]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 554.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x^2+3x+1)\,y''-4x(-3x^2-3x+1)\,y'+3(x^2-x+1)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(4*x^2*(1+3*x+x^2)*diff(y(x),x$2)-4*x*(1-3*x-3*x^2)*diff(y(x),x)+3*(1-x+x^2)*y(x)=0,y(
```

$$y(x)=\frac{c_1\sqrt{x}}{x^2+3x+1}+\frac{c_2x^{\frac{3}{2}}}{x^2+3x+1}$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 28

```
DSolve[4*x^2*(1+3*x+x^2)*y''[x]-4*x*(1-3*x-3*x^2)*y'[x]+3*(1-x+x^2)*y[x]==0,y[x],x,IncludeSi
```

$$y(x)\to\frac{\sqrt{x}(c_2x+c_1)}{x^2+3x+1}$$

1.541 problem 555

Internal problem ID [8031]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 555.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3x^2(1+x)^2 y'' - x(-11x^2 - 10x + 1) y' + (5x^2 + 1) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(3*x^2*(1+x)^2*diff(y(x),x$2)-x*(1-10*x-11*x^2)*diff(y(x),x)+(1+5*x^2)*y(x)=0,y(x), si
```

$$y(x) = \frac{c_1 x}{(x+1)^2} + \frac{c_2 x^{\frac{1}{3}}}{(x+1)^2}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 29

```
DSolve[3*x^2*(1+x)^2*y''[x]-x*(1-10*x-11*x^2)*y'[x]+(1+5*x^2)*y[x]==0,y[x],x,IncludeSingular
```

$$y(x) \to \frac{2c_1 \sqrt[3]{x} + 3c_2 x}{2(x+1)^2}$$

1.542 problem 556

Internal problem ID [8032]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 556.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x^2+2x+3)\,y''-x(-15x^2-14x+3)\,y'+(7x^2+3)\,y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(4*x^2*(3+2*x+x^2)*diff(y(x),x$2)-x*(3-14*x-15*x^2)*diff(y(x),x)+(3+7*x^2)*y(x)=0,y(x)
```

$$y(x)=\frac{c_1x}{x^2+2x+3}+\frac{c_2x^{\frac{1}{4}}}{x^2+2x+3}$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 33

```
DSolve[4*x^2*(3+2*x+x^2)*y''[x]-x*(3-14*x-15*x^2)*y'[x]+(3+7*x^2)*y[x]==0,y[x],x,IncludeSing
```

$$y(x)\to\frac{3c_1\sqrt[4]{x}+4c_2x}{3x^2+6x+9}$$

1.543 problem 557

Internal problem ID [8033]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 557.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2-2x+1)\,y''-x(x+3)\,y'+(4+x)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 52

```
dsolve(x^2*(1-2*x+x^2)*diff(y(x),x$2)-x*(3+x)*diff(y(x),x)+(4+x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x^2\mathrm{e}^{-\frac{4}{x-1}}}{x-1}+\frac{c_2x^2\,\mathrm{expIntegral}_1\left(-\frac{4x}{x-1}\right)\mathrm{e}^{-\frac{4x}{x-1}}}{x-1}$$

✓ Solution by Mathematica

Time used: 0.207 (sec). Leaf size: 54

```
DSolve[x^2*(1-2*x+x^2)*y''[x]-x*(3+x)*y'[x]+(4+x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{e^{-\frac{4x}{x-1}}\sqrt{1-x}x^2\left(c_2\,\mathrm{ExpIntegralEi}\left(\frac{4x}{x-1}\right)+e^4c_1\right)}{(x-1)^{3/2}}$$

1.544 problem 558

Internal problem ID [8034]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 558.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x+2)\,y''+5x^2y'+(1+x)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
dsolve(2*x^2*(2+x)*diff(y(x),x$2)+5*x^2*diff(y(x),x)+(1+x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\sqrt{x}}{(x+2)^{\frac{3}{2}}}+\frac{c_2\sqrt{2}\left(2\sqrt{2}\,\sqrt{x+2}-4\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,\sqrt{x+2}}{2}\right)\right)\sqrt{x}}{2\,(x+2)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.074 (sec). Leaf size: 55

```
DSolve[2*x^2*(2+x)*y''[x]+5*x^2*y'[x]+(1+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{\sqrt{x}\left(-2\sqrt{2}c_2\mathrm{arctanh}\left(\frac{\sqrt{x+2}}{\sqrt{2}}\right)+2c_2\sqrt{x+2}+c_1\right)}{(x+2)^{3/2}}$$

1.545 problem 559

Internal problem ID [8035]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 559.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(-x^2+2)\,y''-2x(2x^2+1)\,y'+(-2x^2+2)\,y=0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 55

```
dsolve(x^2*(2-x^2)*diff(y(x),x$2)-2*x*(1+2*x^2)*diff(y(x),x)+(2-2*x^2)*y(x)=0,y(x), singsol=
```

$$y(x)=\frac{c_1x}{(x^2-2)^{\frac{3}{2}}}+\frac{c_2\sqrt{2}\,x\left(2\arctan\left(\frac{\sqrt{2}}{\sqrt{x^2-2}}\right)+\sqrt{2}\,\sqrt{x^2-2}\right)}{2\,(x^2-2)^{\frac{3}{2}}}$$

✓ Solution by Mathematica
Time used: 0.098 (sec). Leaf size: 58

```
DSolve[x^2*(2-x^2)*y''[x]-2*x*(1+2*x^2)*y'[x]+(2-2*x^2)*y[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x)\to\frac{x\left(-\sqrt{2}c_2\text{arctanh}\left(\sqrt{1-\frac{x^2}{2}}\right)+c_2\sqrt{2-x^2}+c_1\right)}{(2-x^2)^{3/2}}$$

1.546 problem 560

Internal problem ID [8036]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 560.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - x(5-x)\,y' + (9-4x)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(x^2*diff(y(x),x$2)-x*(5-x)*diff(y(x),x)+(9-4*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^3(x+1) + c_2x^3\left(\text{expIntegral}_1\,(x)\,x + \text{expIntegral}_1\,(x) - \mathrm{e}^{-x}\right)$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]-x*(5-x)*y'[x]+(9-4*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}x^3(c_2e^x(x+1)\,\text{ExpIntegralEi}(-x) + c_1e^x(x+1) + c_2)$$

1.547 problem 561

Internal problem ID [8037]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 561.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x^2+x+1)\,y''+12x^2(1+x)\,y'+(3x^2+3x+1)\,y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 143

```
dsolve(4*x^2*(1+x+x^2)*diff(y(x),x$2)+12*x^2*(1+x)*diff(y(x),x)+(1+3*x+3*x^2)*y(x)=0,y(x), s
```

$$y(x)=c_1\sqrt{\frac{x}{x^2+x+1}}\left(\frac{i\sqrt{3}+2x+1}{i\sqrt{3}-2x-1}\right)^{-\frac{i\sqrt{3}}{2}}+c_2\sqrt{\frac{x}{x^2+x+1}}\left(\frac{i\sqrt{3}+2x+1}{i\sqrt{3}-2x-1}\right)^{-\frac{i\sqrt{3}}{2}}\left(\int\frac{\left(\frac{i\sqrt{3}-2x-1}{i\sqrt{3}+2x+1}\right)^{-\frac{i\sqrt{3}}{2}}}{x\sqrt{x^2+x+1}}dx\right)$$

✓ Solution by Mathematica

Time used: 0.914 (sec). Leaf size: 93

```
DSolve[4*x^2*(1+x+x^2)*y''[x]+12*x^2*(1+x)*y'[x]+(1+3*x+3*x^2)*y[x]==0,y[x],x,IncludeSingula
```

$$y(x)\to\frac{\sqrt{x}e^{-\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}\left(c_2\int_1^x\frac{e^{\sqrt{3}\arctan\left(\frac{2K[1]+1}{\sqrt{3}}\right)}}{K[1]\sqrt{K[1]^2+K[1]+1}}dK[1]+c_1\right)}{\sqrt{x^2+x+1}}$$

1.548 problem 562

Internal problem ID [8038]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 562.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+x+1)\,y''-x(-2x^2-4x+1)\,y'+y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 137

```
dsolve(x^2*(1+x+x^2)*diff(y(x),x$2)-x*(1-4*x-2*x^2)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x\left(\frac{i\sqrt{3}+2x+1}{i\sqrt{3}-2x-1}\right)^{-\frac{7i\sqrt{3}}{6}}}{\sqrt{x^2+x+1}}+\frac{c_2x\left(\frac{i\sqrt{3}+2x+1}{i\sqrt{3}-2x-1}\right)^{-\frac{7i\sqrt{3}}{6}}\left(\int\frac{\left(\frac{i\sqrt{3}-2x-1}{i\sqrt{3}+2x+1}\right)^{-\frac{7i\sqrt{3}}{6}}}{x\sqrt{x^2+x+1}}dx\right)}{\sqrt{x^2+x+1}}$$

✓ Solution by Mathematica

Time used: 0.914 (sec). Leaf size: 90

```
DSolve[x^2*(1+x+x^2)*y''[x]-x*(1-4*x-2*x^2)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{xe^{-\frac{7\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}}\left(c_2\int_1^x\frac{e^{\frac{7\arctan\left(\frac{2K[1]+1}{\sqrt{3}}\right)}{\sqrt{3}}}}{K[1]\sqrt{K[1]^2+K[1]+1}}dK[1]+c_1\right)}{\sqrt{x^2+x+1}}$$

1.549 problem 563

Internal problem ID [8039]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 563.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2y'' + 3x(-2x^2 + 3x + 5)\,y' + (-14x^2 + 12x + 1)\,y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 49

```
dsolve(9*x^2*diff(y(x),x$2)+3*x*(5+3*x-2*x^2)*diff(y(x),x)+(1+12*x-14*x^2)*y(x)=0,y(x), sing
```

$$y(x) = \frac{c_1 \mathrm{e}^{\frac{1}{3}x^2-x}}{x^{\frac{1}{3}}} + \frac{c_2 \mathrm{e}^{\frac{1}{3}x^2-x}\left(\int \frac{\mathrm{e}^{-\frac{1}{3}x^2+x}}{x}dx\right)}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.369 (sec). Leaf size: 52

```
DSolve[9*x^2*y''[x]+3*x*(5+3*x-2*x^2)*y'[x]+(1+12*x-14*x^2)*y[x]==0,y[x],x,IncludeSingularSo
```

$$y(x) \to \frac{e^{\frac{1}{3}(x-3)x}\left(c_2 \int_1^x \frac{e^{K[1]-\frac{K[1]^2}{3}}}{K[1]}dK[1] + c_1\right)}{\sqrt[3]{x}}$$

1.550 problem 564

Internal problem ID [8040]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 564.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(2x+1)\,y'' + x\left(3x^2+14x+5\right)y' + \left(12x^2+18x+4\right)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve(x^2*(1+2*x)*diff(y(x),x$2)+x*(5+14*x+3*x^2)*diff(y(x),x)+(4+18*x+12*x^2)*y(x)=0,y(x),
```

$$y(x) = \frac{c_1 \mathrm{e}^{-\frac{3x}{2}}}{(2x+1)^{\frac{1}{4}} x^2} + \frac{c_2 \mathrm{e}^{-\frac{3x}{2}} \left(\int \frac{\mathrm{e}^{\frac{3x}{2}}}{(2x+1)^{\frac{3}{4}} x} dx\right)}{(2x+1)^{\frac{1}{4}} x^2}$$

✓ Solution by Mathematica

Time used: 0.442 (sec). Leaf size: 61

```
DSolve[x^2*(1+2*x)*y''[x]+x*(5+14*x+3*x^2)*y'[x]+(4+18*x+12*x^2)*y[x]==0,y[x],x,IncludeSingu
```

$$y(x) \to \frac{e^{-3x/2}\left(c_2 \int_1^x \frac{e^{\frac{3K[1]}{2}}}{K[1](2K[1]+1)^{3/4}} dK[1] + c_1\right)}{x^2 \sqrt[4]{2x+1}}$$

1.551 problem 565

Internal problem ID [8041]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 565.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$16x^2y'' + 4x(2x^2 + x + 6)\,y' + (18x^2 + 5x + 1)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 51

```
dsolve(16*x^2*diff(y(x),x$2)+4*x*(6+x+2*x^2)*diff(y(x),x)+(1+5*x+18*x^2)*y(x)=0,y(x), singso
```

$$y(x) = \frac{c_1 \mathrm{e}^{-\frac{1}{4}x^2-\frac{1}{4}x}}{x^{\frac{1}{4}}} + \frac{c_2 \mathrm{e}^{-\frac{1}{4}x^2-\frac{1}{4}x}\left(\int \frac{\mathrm{e}^{\frac{1}{4}x^2+\frac{1}{4}x}}{x}dx\right)}{x^{\frac{1}{4}}}$$

✓ Solution by Mathematica

Time used: 0.348 (sec). Leaf size: 51

```
DSolve[16*x^2*y''[x]+4*x*(6+x+2*x^2)*y'[x]+(1+5*x+18*x^2)*y[x]==0,y[x],x,IncludeSingularSolu
```

$$y(x) \to \frac{e^{-\frac{1}{4}x(x+1)}\left(c_2 \int_1^x \frac{e^{\frac{1}{4}K[1](K[1]+1)}}{K[1]}dK[1] + c_1\right)}{\sqrt[4]{x}}$$

1.552 problem 566

Internal problem ID [8042]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 566.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2(1+x)\,y''+3x\left(-x^2+11x+5\right)y'+\left(-7x^2+16x+1\right)y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(9*x^2*(1+x)*diff(y(x),x$2)+3*x*(5+11*x-x^2)*diff(y(x),x)+(1+16*x-7*x^2)*y(x)=0,y(x),
```

$$y(x)=\frac{c_1\,\mathrm{e}^{\frac{x}{3}}}{(x+1)^{\frac{4}{3}}\,x^{\frac{1}{3}}}+\frac{c_2\,\mathrm{e}^{\frac{x}{3}}\left(\int\frac{(x+1)^{\frac{1}{3}}\,\mathrm{e}^{-\frac{x}{3}}}{x}dx\right)}{(x+1)^{\frac{4}{3}}\,x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.133 (sec). Leaf size: 50

```
DSolve[9*x^2*(1+x)*y''[x]+3*x*(5+11*x-x^2)*y'[x]+(1+16*x-7*x^2)*y[x]==0,y[x],x,IncludeSingul
```

$$y(x)\to\frac{e^{x/3}\left(c_1-\sqrt[3]{3}ec_2\Gamma\left(\frac{1}{3},\frac{x+1}{3}\right)\right)}{\sqrt[3]{x}(x+1)^{4/3}}$$

1.553 problem 567

Internal problem ID [8043]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 567.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$36x^2(1-2x)\,y'' + 24x(1-9x)\,y' + (1-70x)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(36*x^2*(1-2*x)*diff(y(x),x$2)+24*x*(1-9*x)*diff(y(x),x)+(1-70*x)*y(x)=0,y(x), singsol
```

$$y(x) = \frac{c_1 x^{\frac{1}{6}}}{(-1+2x)^{\frac{4}{3}}} + \frac{c_2 x^{\frac{1}{6}} \left(\int \frac{(-1+2x)^{\frac{1}{3}}}{x} dx\right)}{(-1+2x)^{\frac{4}{3}}}$$

✓ Solution by Mathematica

Time used: 0.15 (sec). Leaf size: 111

```
DSolve[36*x^2*(1-2*x)*y''[x]+24*x*(1-9*x)*y'[x]+(1-70*x)*y[x]==0,y[x],x,IncludeSingularSolut
```

$$y(x) \to \frac{\sqrt[6]{x}\left(-2\sqrt{3}c_2 \arctan\left(\frac{2\sqrt[3]{1-2x}+1}{\sqrt{3}}\right) + 6c_2\sqrt[3]{1-2x} + 2c_2 \log\left(\sqrt[3]{1-2x}-1\right) - c_2 \log\left((1-2x)^{2/3} + \sqrt[3]{1\right.}}{2(1-2x)^{4/3}}$$

1.554 problem 568

Internal problem ID [8044]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 568.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1+x)\,y''-x(3-x)\,y'+4y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 38

```
dsolve(x^2*(1+x)*diff(y(x),x$2)-x*(3-x)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x^2(x-1)}{(x+1)^3}+\frac{c_2x^2(x\ln(x)-\ln(x)-4)}{(x+1)^3}$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 33

```
DSolve[x^2*(1+x)*y''[x]-x*(3-x)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x^2(c_1(x-1)+c_2(x-1)\log(x)-4c_2)}{(x+1)^3}$$

1.555 problem 569

Internal problem ID [8045]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 569.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-2x)\,y''-x(5-4x)\,y'+(9-4x)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(x^2*(1-2*x)*diff(y(x),x$2)-x*(5-4*x)*diff(y(x),x)+(9-4*x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x^3}{(-1+2x)^2}+\frac{c_2x^3(2x-\ln(x))}{(-1+2x)^2}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 29

```
DSolve[x^2*(1-2*x)*y''[x]-x*(5-4*x)*y'[x]+(9-4*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{x^3(-2c_2x+c_2\log(x)+c_1)}{(1-2x)^2}$$

1.556 problem 570

Internal problem ID [8046]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 570.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x+2)\,y''+x^2y'+(1-x)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 71

```
dsolve(2*x^2*(2+x)*diff(y(x),x$2)+x^2*diff(y(x),x)+(1-x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\sqrt{x^2+2x}-\frac{c_2\sqrt{2}\left(\operatorname{arctanh}\left(\frac{\sqrt{2}\,\sqrt{x+2}}{2}\right)x-\sqrt{2}\,\sqrt{x+2}+2\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,\sqrt{x+2}}{2}\right)\right)\sqrt{x\,(x+2)}}{2\,(x+2)}$$

✓ Solution by Mathematica

Time used: 0.1 (sec). Leaf size: 65

```
DSolve[2*x^2*(2+x)*y''[x]+x^2*y'[x]+(1-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{\sqrt{x}\left(2\left(c_1\sqrt{x+2}+c_2\right)-\sqrt{2}c_2\sqrt{x+2}\operatorname{arctanh}\left(\frac{\sqrt{x+2}}{\sqrt{2}}\right)\right)}{2\sqrt[4]{2}}$$

1.557 problem 571

Internal problem ID [8047]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 571.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(1+x)\,y'' - x(-x+6)\,y' + (8-x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 59

```
dsolve(2*x^2*(1+x)*diff(y(x),x$2)-x*(6-x)*diff(y(x),x)+(8-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^2}{(x+1)^{\frac{5}{2}}} + \frac{c_2 x^2 \left(\frac{2\sqrt{x+1}\,x}{3} + \frac{8\sqrt{x+1}}{3} + \ln\left(\sqrt{x+1}-1\right) - \ln\left(\sqrt{x+1}+1\right)\right)}{(x+1)^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.072 (sec). Leaf size: 50

```
DSolve[2*x^2*(1+x)*y''[x]-x*(6-x)*y'[x]+(8-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{x^2\left(-6c_2 \operatorname{arctanh}\left(\sqrt{x+1}\right) + 2c_2\sqrt{x+1}(x+4) + 3c_1\right)}{3(x+1)^{5/2}}$$

1.558 problem 572

Internal problem ID [8048]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 572.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(2x+1)\,y''+x(5+9x)\,y'+(4+3x)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 130

```
dsolve(x^2*(1+2*x)*diff(y(x),x$2)+x*(5+9*x)*diff(y(x),x)+(4+3*x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(2x+1)^{\frac{3}{2}}}{x^2}+\frac{c_2\left(-12\ln\left(\sqrt{2x+1}+1\right)x^2-12\ln\left(\sqrt{2x+1}+1\right)x+12\ln\left(\sqrt{2x+1}-1\right)x^2+12\ln\left(\sqrt{2x+1}-1\right)\right.}{3x^2\sqrt{2x+1}}$$

✓ Solution by Mathematica

Time used: 0.094 (sec). Leaf size: 56

```
DSolve[x^2*(1+2*x)*y''[x]+x*(5+9*x)*y'[x]+(4+3*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{2c_2\left(-3(2x+1)^{3/2}\text{arctanh}\left(\sqrt{2x+1}\right)+6x+4\right)+3c_1(2x+1)^{3/2}}{3x^2}$$

1.559 problem 573

Internal problem ID [8049]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 573.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-2x)\,y''-x(4x+5)\,y'+(9+4x)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 63

```
dsolve(x^2*(1-2*x)*diff(y(x),x$2)-x*(5+4*x)*diff(y(x),x)+(9+4*x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x^3(8x+1)}{(-1+2x)^6}+\frac{c_2x^3\left(\frac{4x^4}{3}-\frac{16x^3}{3}-8x\ln(x)+12x^2-\ln(x)+\frac{203x}{128}-\frac{3125}{1024}\right)}{(-1+2x)^6}$$

✓ Solution by Mathematica

Time used: 0.085 (sec). Leaf size: 63

```
DSolve[x^2*(1-2*x)*y''[x]-x*(5+4*x)*y'[x]+(9+4*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to -\frac{x^3(c_2(4096x^4-16384x^3+36864x^2+4872x-9375)-48c_1(8x+1)-3072c_2(8x+1)\log(x))}{384(1-2x)^6}$$

1.560 problem 574

Internal problem ID [8050]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 574.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-x)\,y''+x(7+x)\,y'+(9-x)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 75

```
dsolve(x^2*(1-x)*diff(y(x),x$2)+x*(7+x)*diff(y(x),x)+(9-x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(x^4+16x^3+36x^2+16x+1)}{x^3}+\frac{c_2\left(x^4\ln(x)+16x^3\ln(x)+36x^2\ln(x)+40x^3+16x\ln(x)+150x^2+\ln(x)+\frac{280x}{3}+\frac{25}{3}\right)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.094 (sec). Leaf size: 78

```
DSolve[x^2*(1-x)*y''[x]+x*(7+x)*y'[x]+(9-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{5c_2(24x^3+90x^2+56x+5)+3c_1(x^4+16x^3+36x^2+16x+1)+3c_2(x^4+16x^3+36x^2+16x+1)\log}{3x^3}$$

1.561 problem 575

Internal problem ID [8051]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 575.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(1-x^2)\, y' + (x^2+1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(x^2*diff(y(x),x$2)-x*(1-x^2)*diff(y(x),x)+(1+x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \, \mathrm{e}^{-\frac{x^2}{2}} + c_2 x \, \mathrm{e}^{-\frac{x^2}{2}} \, \mathrm{expIntegral}_1\left(-\frac{x^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 35

```
DSolve[x^2*y''[x]-x*(1-x^2)*y'[x]+(1+x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-\frac{x^2}{2}} x \left(c_1 \, \mathrm{ExpIntegralEi}\left(\frac{x^2}{2}\right) + 2c_2\right)$$

1.562 problem 576

Internal problem ID [8052]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 576.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+1)\,y''-3x(1-x^2)\,y'+4y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-3*x*(1-x^2)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x^2}{(x^2+1)^2}+\frac{c_2x^2\left(\frac{x^2}{2}+\ln(x)\right)}{(x^2+1)^2}$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 36

```
DSolve[x^2*(1+x^2)*y''[x]-3*x*(1-x^2)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to\frac{x^2(c_2x^2+2c_2\log(x)+2c_1)}{2(x^2+1)^2}$$

1.563 problem 577

Internal problem ID [8053]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 577.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 2y'x^3 + (3x^2 + 1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(4*x^2*diff(y(x),x$2)+2*x^3*diff(y(x),x)+(1+3*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\mathrm{e}^{-\frac{x^2}{4}} + c_2\sqrt{x}\,\mathrm{e}^{-\frac{x^2}{4}}\,\mathrm{expIntegral}_1\left(-\frac{x^2}{4}\right)$$

✓ Solution by Mathematica

Time used: 0.072 (sec). Leaf size: 39

```
DSolve[4*x^2*y''[x]+2*x^3*y'[x]+(1+3*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-\frac{x^2}{4}}\sqrt{x}\left(c_2\,\mathrm{ExpIntegralEi}\left(\frac{x^2}{4}\right) + 2c_1\right)$$

1.564 problem 578

Internal problem ID [8054]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 578.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x^2+1)\,y''-x(-2x^2+1)\,y'+y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(1-2*x^2)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x}{\sqrt{x^2+1}}+\frac{c_2x\,\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)}{\sqrt{x^2+1}}$$

✓ Solution by Mathematica

Time used: 0.061 (sec). Leaf size: 33

```
DSolve[x^2*(1+x^2)*y''[x]-x*(1-2*x^2)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x\left(c_1-c_2\operatorname{arctanh}\left(\sqrt{x^2+1}\right)\right)}{\sqrt{x^2+1}}$$

1.565 problem 579

Internal problem ID [8055]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 579.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x^2+2)\,y''+7y'x^3+(3x^2+1)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(2*x^2*(2+x^2)*diff(y(x),x$2)+7*x^3*diff(y(x),x)+(1+3*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\sqrt{x}}{(x^2+2)^{\frac{3}{4}}}+\frac{c_2\sqrt{x}\left(\int\frac{1}{(x^2+2)^{\frac{1}{4}}x}dx\right)}{(x^2+2)^{\frac{3}{4}}}$$

✓ Solution by Mathematica

Time used: 0.114 (sec). Leaf size: 77

```
DSolve[2*x^2*(2+x^2)*y''[x]+7*x^3*y'[x]+(1+3*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{\sqrt{x}\left(2^{3/4}c_2\arctan\left(\frac{\sqrt[4]{x^2+2}}{\sqrt[4]{2}}\right)-2^{3/4}c_2\operatorname{arctanh}\left(\frac{\sqrt[4]{x^2+2}}{\sqrt[4]{2}}\right)+2c_1\right)}{2(x^2+2)^{3/4}}$$

1.566 problem 580

Internal problem ID [8056]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 580.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+1)\,y''-x(-4x^2+1)\,y'+(2x^2+1)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(1-4*x^2)*diff(y(x),x)+(1+2*x^2)*y(x)=0,y(x), singsol=al
```

$$y(x)=\frac{c_1x}{(x^2+1)^{\frac{3}{2}}}+\frac{c_2x\left(\sqrt{x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)\right)}{(x^2+1)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.074 (sec). Leaf size: 45

```
DSolve[x^2*(1+x^2)*y''[x]-x*(1-4*x^2)*y'[x]+(1+2*x^2)*y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x)\to\frac{x\left(-c_2\operatorname{arctanh}\left(\sqrt{x^2+1}\right)+c_2\sqrt{x^2+1}+c_1\right)}{(x^2+1)^{3/2}}$$

1.567 problem 581

Internal problem ID [8057]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 581.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x^2+4)\,y''+3x(3x^2+8)\,y'+(-9x^2+1)\,y=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 42

```
dsolve(4*x^2*(4+x^2)*diff(y(x),x$2)+3*x*(8+3*x^2)*diff(y(x),x)+(1-9*x^2)*y(x)=0,y(x), singso
```

$$y(x)=\frac{c_1(x^2+4)^{\frac{5}{8}}}{x^{\frac{1}{4}}}+\frac{c_2(x^2+4)^{\frac{5}{8}}\left(\int\frac{1}{(x^2+4)^{\frac{13}{8}}x}dx\right)}{x^{\frac{1}{4}}}$$

✓ Solution by Mathematica
Time used: 0.28 (sec). Leaf size: 198

```
DSolve[4*x^2*(4+x^2)*y''[x]+3*x*(8+3*x^2)*y'[x]+(1-9*x^2)*y[x]==0,y[x],x,IncludeSingularSolu
```

$$y(x)\to\frac{c_2\left(5\,2^{3/4}(x^2+4)^{5/8}\arctan\left(\frac{\sqrt[8]{x^2+4}}{\sqrt[4]{2}}\right)+5\sqrt[4]{2}(x^2+4)^{5/8}\arctan\left(\frac{\sqrt{2}-\sqrt[4]{x^2+4}}{2^{3/4}\sqrt[8]{x^2+4}}\right)-5\,2^{3/4}(x^2+4)^{5/8}\,\mathrm{a}}{80\sqrt[4]{x}}$$

1.568 problem 582

Internal problem ID [8058]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 582.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3x^2\left(x^2+3\right)y''+x\left(11x^2+3\right)y'+\left(5x^2+1\right)y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 42

```
dsolve(3*x^2*(3+x^2)*diff(y(x),x$2)+x*(3+11*x^2)*diff(y(x),x)+(1+5*x^2)*y(x)=0,y(x), singsol
```

$$y(x)=\frac{c_1x^{\frac{1}{3}}}{\left(x^2+3\right)^{\frac{2}{3}}}+\frac{c_2x^{\frac{1}{3}}\left(\int\frac{1}{\left(x^2+3\right)^{\frac{1}{3}}x}dx\right)}{\left(x^2+3\right)^{\frac{2}{3}}}$$

✓ Solution by Mathematica

Time used: 0.061 (sec). Leaf size: 94

```
DSolve[3*x^2*(3+x^2)*y''[x]+x*(3+11*x^2)*y'[x]+(1+5*x^2)*y[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x)\to\frac{c_1\exp\left(\frac{1}{3}\text{RootSum}\left[3\#1^3+11\#1^2+9\#1+3\&,\frac{3\#1^2\log(x-\#1)-4\#1\log(x-\#1)+9\log(x-\#1)}{9\#1^2+22\#1+9}\&\right]\right)}{\sqrt[3]{x}}$$

$$y(x)\to 0$$

1.569 problem 583

Internal problem ID [8059]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 583.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9x^2y'' - 3x(-2x^2+7)\,y' + (2x^2+25)\,y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(9*x^2*diff(y(x),x$2)-3*x*(7-2*x^2)*diff(y(x),x)+(25+2*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{\frac{5}{3}} \mathrm{e}^{-\frac{x^2}{3}} + c_2 x^{\frac{5}{3}} \mathrm{e}^{-\frac{x^2}{3}} \operatorname{expIntegral}_1\left(-\frac{x^2}{3}\right)$$

✓ Solution by Mathematica
Time used: 0.076 (sec). Leaf size: 39

```
DSolve[9*x^2*y''[x]-3*x*(7-2*x^2)*y'[x]+(25+2*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{1}{2} e^{-\frac{x^2}{3}} x^{5/3} \left(c_2 \operatorname{ExpIntegralEi}\left(\frac{x^2}{3}\right) + 2c_1\right)$$

1.570 problem 584

Internal problem ID [8060]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 584.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - x(1-x^2)\,y' + (x^2+1)\,y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 30

```
dsolve(x^2*diff(y(x),x$2)-x*(1-x^2)*diff(y(x),x)+(1+x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x\,\mathrm{e}^{-\frac{x^2}{2}} + c_2 x\,\mathrm{e}^{-\frac{x^2}{2}}\,\mathrm{expIntegral}_1\left(-\frac{x^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 35

```
DSolve[x^2*y''[x]-x*(1-x^2)*y'[x]+(1+x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-\frac{x^2}{2}}x\left(c_1\,\mathrm{ExpIntegralEi}\left(\frac{x^2}{2}\right) + 2c_2\right)$$

1.571 problem 585

Internal problem ID [8061]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 585.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-2x)\,y''+3xy'+(1+4x)\,y=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(x^2*(1-2*x)*diff(y(x),x$2)+3*x*diff(y(x),x)+(1+4*x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1}{x}-\frac{c_2(-8x^3+18x^2+3\ln(x)-18x)}{3x}$$

✓ Solution by Mathematica
Time used: 0.044 (sec). Leaf size: 36

```
DSolve[x^2*(1-2*x)*y''[x]+3*x*y'[x]+(1+4*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{2}{3}c_2\left(4x^2-9x+9\right)+\frac{c_1}{x}+\frac{c_2\log(x)}{x}$$

1.572 problem 586

Internal problem ID [8062]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 586.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(1+x)\,y''+(1-x)\,y'+y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(x*(1+x)*diff(y(x),x$2)+(1-x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(x-1)+c_2(x\ln(x)-\ln(x)-4)$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 23

```
DSolve[x*(1+x)*y''[x]+(1-x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1(x-1)+c_2((x-1)\log(x)-4)$$

1.573 problem 587

Internal problem ID [8063]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 587.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1-x)\,y''-x(3-5x)\,y'+(4-5x)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 89

```
dsolve(x^2*(1-x)*diff(y(x),x$2)-x*(3-5*x)*diff(y(x),x)+(4-5*x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x^2(x^3-3x^2+3x-1)+c_2x^2\left(x^3\ln(x)-\ln(x-1)\,x^3-3x^2\ln(x)+3\ln(x-1)\,x^2+3x\ln(x)-3\ln(x-1)\,x-x^2-\ln(x)+\ln(x-1)+\frac{5x}{2}-\frac{11}{6}\right)$$

✓ Solution by Mathematica

Time used: 0.075 (sec). Leaf size: 76

```
DSolve[x^2*(1-x)*y''[x]-x*(3-5*x)*y'[x]+(4-5*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\to-\frac{1}{6}x^2(6c_1x^3-18c_1x^2-6c_2x^2+18c_1x+15c_2x-6c_2(x-1)^3\log(x-1)+6c_2(x-1)^3\log(x)-6c_1-11c_2)$$

1.574 problem 588

Internal problem ID [8064]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 588.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+1)\,y''-x(9x^2+1)\,y'+(25x^2+1)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(1+9*x^2)*diff(y(x),x)+(1+25*x^2)*y(x)=0,y(x), singsol=a
```

$$y(x)=c_1x(x^4-4x^2+1)+c_2(x^4\ln(x)-4x^2\ln(x)-6x^2+\ln(x)+3)\,x$$

✓ Solution by Mathematica

Time used: 0.084 (sec). Leaf size: 43

```
DSolve[x^2*(1+x^2)*y''[x]-x*(1+9*x^2)*y'[x]+(1+25*x^2)*y[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x)\to c_1(x^5-4x^3+x)+c_2x(-6x^2+(x^4-4x^2+1)\log(x)+3)$$

1.575 problem 589

Internal problem ID [8065]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 589.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2y'' + 3x(1-x^2)\,y' + (7x^2+1)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(9*x^2*diff(y(x),x$2)+3*x*(1-x^2)*diff(y(x),x)+(1+7*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{\frac{1}{3}}(x^2-6) + c_2 x^{\frac{1}{3}}(x^2-6)\left(\int \frac{\mathrm{e}^{\frac{x^2}{6}}}{(x^2-6)^2\, x}dx\right)$$

✓ Solution by Mathematica

Time used: 0.309 (sec). Leaf size: 53

```
DSolve[9*x^2*y''[x]+3*x*(1-x^2)*y'[x]+(1+7*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{1}{72}\sqrt[3]{x}\left(c_2(x^2-6)\,\text{ExpIntegralEi}\left(\frac{x^2}{6}\right) + 72c_1(x^2-6) - 6c_2e^{\frac{x^2}{6}}\right)$$

1.576 problem 590

Internal problem ID [8066]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 590.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$
\boxed{x(x^2+1)\,y''+(1-x^2)\,y'-8yx=0}
$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 71

```
dsolve(x*(1+x^2)*diff(y(x),x$2)+(1-x^2)*diff(y(x),x)-8*x*y(x)=0,y(x), singsol=all)
```

$$
y(x)=c_1(x^4+2x^2+1)+c_2\left(-\frac{\ln(x^2+1)\,x^4}{2}+x^4\ln(x)-\ln(x^2+1)\,x^2+2x^2\ln(x)+\frac{x^2}{2}-\frac{\ln(x^2+1)}{2}+\ln(x)+\frac{3}{4}\right)
$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 55

```
DSolve[x*(1+x^2)*y''[x]+(1-x^2)*y'[x]-8*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x)\to c_1(x^2+1)^2+\frac{1}{4}c_2\left(2x^2+4(x^2+1)^2\log(x)-2(x^2+1)^2\log(x^2+1)+3\right)
$$

1.577 problem 591

Internal problem ID [8067]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 591.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 2x(-x^2+4)\,y' + (7x^2+1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(4*x^2*diff(y(x),x$2)+2*x*(4-x^2)*diff(y(x),x)+(1+7*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^4-16x^2+32)}{\sqrt{x}} + \frac{c_2(x^4-16x^2+32)\left(\int \frac{e^{\frac{x^2}{4}}}{x(x^4-16x^2+32)^2}dx\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.15 (sec). Leaf size: 68

```
DSolve[4*x^2*y''[x]+2*x*(4-x^2)*y'[x]+(1+7*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{c_2(x^4-16x^2+32)\,\text{ExpIntegralEi}\left(\frac{x^2}{4}\right) - 4c_2e^{\frac{x^2}{4}}(x^2-12) + 2048c_1(x^4-16x^2+32)}{2048\sqrt{x}}$$

1.578 problem 592

Internal problem ID [8068]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 592.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(1+x)\,y''+8x^2y'+(1+x)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(4*x^2*(1+x)*diff(y(x),x$2)+8*x^2*diff(y(x),x)+(1+x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\sqrt{x}}{x+1}+\frac{c_2\sqrt{x}\,\ln(x)}{x+1}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 24

```
DSolve[4*x^2*(1+x)*y''[x]+8*x^2*y'[x]+(1+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{\sqrt{x}(c_2\log(x)+c_1)}{x+1}$$

1.579 problem 593

Internal problem ID [8069]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 593.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9x^2(x+3)\,y''+3x(3+7x)\,y'+(3+4x)\,y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(9*x^2*(3+x)*diff(y(x),x$2)+3*x*(3+7*x)*diff(y(x),x)+(3+4*x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x^{\frac{1}{3}}}{x+3}+\frac{c_2x^{\frac{1}{3}}\ln(x)}{x+3}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 24

```
DSolve[9*x^2*(3+x)*y''[x]+3*x*(3+7*x)*y'[x]+(3+4*x)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to\frac{\sqrt[3]{x}(c_2\log(x)+c_1)}{x+3}$$

1.580 problem 594

Internal problem ID [8070]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 594.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(-x^2+2)\,y''-x(3x^2+2)\,y'+(-x^2+2)\,y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(x^2*(2-x^2)*diff(y(x),x$2)-x*(2+3*x^2)*diff(y(x),x)+(2-x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x}{x^2-2}+\frac{c_2x\ln(x)}{x^2-2}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 23

```
DSolve[x^2*(2-x^2)*y''[x]-x*(2+3*x^2)*y'[x]+(2-x^2)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to-\frac{x(c_2\log(x)+c_1)}{x^2-2}$$

1.581 problem 595

Internal problem ID [8071]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 595.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$16x^2(x^2+1)\,y''+8x(9x^2+1)\,y'+(49x^2+1)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(16*x^2*(1+x^2)*diff(y(x),x$2)+8*x*(1+9*x^2)*diff(y(x),x)+(1+49*x^2)*y(x)=0,y(x), sing
```

$$y(x)=\frac{c_1 x^{\frac{1}{4}}}{x^2+1}+\frac{c_2 x^{\frac{1}{4}}\ln(x)}{x^2+1}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 26

```
DSolve[16*x^2*(1+x^2)*y''[x]+8*x*(1+9*x^2)*y'[x]+(1+49*x^2)*y[x]==0,y[x],x,IncludeSingularSo
```

$$y(x)\to\frac{\sqrt[4]{x}(c_2\log(x)+c_1)}{x^2+1}$$

1.582 problem 596

Internal problem ID [8072]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 596.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(4+3x)\,y''-x(4-3x)\,y'+4y=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x^2*(4+3*x)*diff(y(x),x$2)-x*(4-3*x)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x}{3x+4}+\frac{c_2x\ln(x)}{3x+4}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 22

```
DSolve[x^2*(4+3*x)*y''[x]-x*(4-3*x)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x(c_2\log(x)+c_1)}{3x+4}$$

1.583 problem 597

Internal problem ID [8073]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 597.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2\left(x^2+3x+1\right)y''+8x^2(3+2x)\,y'+\left(9x^2+3x+1\right)y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve(4*x^2*(1+3*x+x^2)*diff(y(x),x$2)+8*x^2*(3+2*x)*diff(y(x),x)+(1+3*x+9*x^2)*y(x)=0,y(x)
```

$$y(x)=\frac{c_1\sqrt{x}}{x^2+3x+1}+\frac{c_2\sqrt{x}\,\ln(x)}{x^2+3x+1}$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 29

```
DSolve[4*x^2*(1+3*x+x^2)*y''[x]+8*x^2*(3+2*x)*y'[x]+(1+3*x+9*x^2)*y[x]==0,y[x],x,IncludeSing
```

$$y(x)\to\frac{\sqrt{x}(c_2\log(x)+c_1)}{x^2+3x+1}$$

1.584 problem 598

Internal problem ID [8074]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 598.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1-x)^2 y'' - x(-3x^2+2x+1)\, y' + (x^2+1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*(1-x)^2*diff(y(x),x$2)-x*(1+2*x-3*x^2)*diff(y(x),x)+(1+x^2)*y(x)=0,y(x), singsol=
```

$$y(x) = \frac{c_1 x}{(x-1)^2} + \frac{c_2 x \ln(x)}{(x-1)^2}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 20

```
DSolve[x^2*(1-x)^2*y''[x]-x*(1+2*x-3*x^2)*y'[x]+(1+x^2)*y[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{x(c_2 \log(x) + c_1)}{(x-1)^2}$$

1.585 problem 599

Internal problem ID [8075]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 599.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2(x^2+x+1)y''+3x(13x^2+7x+1)y'+(25x^2+4x+1)y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(9*x^2*(1+x+x^2)*diff(y(x),x$2)+3*x*(1+7*x+13*x^2)*diff(y(x),x)+(1+4*x+25*x^2)*y(x)=0,
```

$$y(x)=\frac{c_1x^{\frac{1}{3}}}{x^2+x+1}+\frac{c_2x^{\frac{1}{3}}\ln(x)}{x^2+x+1}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 27

```
DSolve[9*x^2*(1+x+x^2)*y''[x]+3*x*(1+7*x+13*x^2)*y'[x]+(1+4*x+25*x^2)*y[x]==0,y[x],x,Include
```

$$y(x)\to\frac{\sqrt[3]{x}(c_2\log(x)+c_1)}{x^2+x+1}$$

1.586 problem 600

Internal problem ID [8076]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 600.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x+2)\,y'' - x(4-7x)\,y' - (5-3x)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 80

```
dsolve(2*x^2*(2+x)*diff(y(x),x$2)-x*(4-7*x)*diff(y(x),x)-(5-3*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^{\frac{5}{2}}}{(x+2)^{\frac{7}{2}}} - \frac{c_2\sqrt{2}\left(33\sqrt{2}\,\sqrt{x+2}\,x^2 + 15\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,\sqrt{x+2}}{2}\right)x^3 + 52\sqrt{2}\,\sqrt{x+2}\,x + 32\sqrt{2}\,\sqrt{x+2}\right)}{48\sqrt{x}\,(x+2)^{\frac{7}{2}}}$$

✓ Solution by Mathematica

Time used: 0.117 (sec). Leaf size: 92

```
DSolve[2*x^2*(2+x)*y''[x]-x*(4-7*x)*y'[x]-(5-3*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{15\sqrt{2}c_2x^3\mathrm{arctanh}\left(\frac{\sqrt{x+2}}{\sqrt{2}}\right) - 48c_1x^3 + 66c_2\sqrt{x+2}x^2 + 104c_2\sqrt{x+2}x + 64c_2\sqrt{x+2}}{48\sqrt{x}(x+2)^{7/2}}$$

1.587 problem 601

Internal problem ID [8077]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 601.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-2x)\,y''+x(8-9x)\,y'+(6-3x)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(x^2*(1-2*x)*diff(y(x),x$2)+x*(8-9*x)*diff(y(x),x)+(6-3*x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(231x^3-198x^2+66x-8)}{x^6}+\frac{c_2(3x+4)\,(-1+2x)^{\frac{9}{2}}}{x^6}$$

✓ Solution by Mathematica

Time used: 0.118 (sec). Leaf size: 49

```
DSolve[x^2*(1-2*x)*y''[x]+x*(8-9*x)*y'[x]+(6-3*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{c_2(231x^3-198x^2+66x-8)+385c_1(3x+4)(1-2x)^{9/2}}{1155x^6}$$

1.588 problem 602

Internal problem ID [8078]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 602.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+1)\,y''+x(10x^2+3)\,y'-(-14x^2+15)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 89

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)+x*(3+10*x^2)*diff(y(x),x)-(15-14*x^2)*y(x)=0,y(x), singsol
```

$$y(x)=\frac{c_1x^3}{(x^2+1)^{\frac{5}{2}}} - \frac{c_2\left(-3\,\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)x^8+3\sqrt{x^2+1}\,x^6-2x^4\sqrt{x^2+1}-24\sqrt{x^2+1}\,x^2-16\sqrt{x^2+1}\right)}{128x^5\,(x^2+1)^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.111 (sec). Leaf size: 75

```
DSolve[x^2*(1+x^2)*y''[x]+x*(3+10*x^2)*y'[x]-(15-14*x^2)*y[x]==0,y[x],x,IncludeSingularSolut
```

$$y(x)\to\frac{c_2\left(\sqrt{x^2+1}(3x^6-2x^4-24x^2-16)-3x^8\operatorname{arctanh}\left(\sqrt{x^2+1}\right)\right)+128c_1x^8}{128x^5\,(x^2+1)^{5/2}}$$

1.589 problem 603

Internal problem ID [8079]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 603.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(-2x^2+1)\,y''+x(-13x^2+7)\,y'-14x^2y=0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 36

```
dsolve(x^2*(1-2*x^2)*diff(y(x),x$2)+x*(7-13*x^2)*diff(y(x),x)-14*x^2*y(x)=0,y(x), singsol=al
```

$$y(x)=\frac{c_1(5x^4-20x^2+8)}{x^6}+\frac{c_2(2x^2-1)^{\frac{5}{4}}}{x^6}$$

✓ Solution by Mathematica
Time used: 0.092 (sec). Leaf size: 43

```
DSolve[x^2*(1-2*x^2)*y''[x]+x*(7-13*x^2)*y'[x]-14*x^2*y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x)\to\frac{15c_1(1-2x^2)^{5/4}+c_2(-5x^4+20x^2-8)}{15x^6}$$

1.590 problem 604

Internal problem ID [8080]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 604.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2(1+x)\,y''+4x(2x+1)\,y'-(3x+1)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(4*x^2*(1+x)*diff(y(x),x$2)+4*x*(1+2*x)*diff(y(x),x)-(1+3*x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\sqrt{x}+\frac{c_2(x\ln(x)-\ln(x+1)\,x+1)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 32

```
DSolve[4*x^2*(1+x)*y''[x]+4*x*(1+2*x)*y'[x]-(1+3*x)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to\frac{c_1x+c_2(-x\log(x)+x\log(x+1)-1)}{\sqrt{x}}$$

1.591 problem 605

Internal problem ID [8081]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 605.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(3x+2)\,y''+x(4+21x)\,y'-(1-9x)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 61

```
dsolve(2*x^2*(2+3*x)*diff(y(x),x$2)+x*(4+21*x)*diff(y(x),x)-(1-9*x)*y(x)=0,y(x), singsol=all
```

$$y(x)=\frac{c_1\sqrt{x}}{(3x+2)^{\frac{3}{2}}}+\frac{c_2\sqrt{2}\left(\sqrt{2}\,\sqrt{3x+2}+3\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,\sqrt{3x+2}}{2}\right)x\right)}{2\sqrt{x}\,(3x+2)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.091 (sec). Leaf size: 64

```
DSolve[2*x^2*(2+3*x)*y''[x]+x*(4+21*x)*y'[x]-(1-9*x)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to -\frac{3\sqrt{2}c_2x\operatorname{arctanh}\left(\sqrt{\frac{3x}{2}+1}\right)-2c_1x+2c_2\sqrt{3x+2}}{2\sqrt{x}(3x+2)^{3/2}}$$

1.592 problem 606

Internal problem ID [8082]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 606.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x(x+2)\,y' - (2-3x)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 46

```
dsolve(x^2*diff(y(x),x$2)+x*(2+x)*diff(y(x),x)-(2-3*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x} x + \frac{c_2 \mathrm{e}^{-x} (\mathrm{expIntegral}_1\,(-x)\,x^3 + \mathrm{e}^x x^2 + x\,\mathrm{e}^x + 2\,\mathrm{e}^x)}{6x^2}$$

✓ Solution by Mathematica

Time used: 0.057 (sec). Leaf size: 46

```
DSolve[x^2*y''[x]+x*(2+x)*y'[x]-(2-3*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}(c_2(x^3\,\mathrm{ExpIntegralEi}(x) - e^x(x^2+x+2)) + 6c_1x^3)}{6x^2}$$

1.593 problem 607

Internal problem ID [8083]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 607.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(1+x)\,y''+4x(3+8x)\,y'-(5-49x)\,y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
dsolve(4*x^2*(1+x)*diff(y(x),x$2)+4*x*(3+8*x)*diff(y(x),x)-(5-49*x)*y(x)=0,y(x), singsol=all
```

$$y(x)=\frac{c_1\sqrt{x}}{(x+1)^4}+\frac{c_2(6x^3\ln(x)-18x^2-9x-2)}{6\,(x+1)^4\,x^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 52

```
DSolve[4*x^2*(1+x)*y''[x]+4*x*(3+8*x)*y'[x]-(5-49*x)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to\frac{6c_1x^3+6c_2x^3\log(x)-18c_2x^2-9c_2x-2c_2}{6x^{5/2}(x+1)^4}$$

1.594 problem 608

Internal problem ID [8084]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 608.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1+x)\,y'' - x(3+10x)\,y' + 30yx = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 62

```
dsolve(x^2*(1+x)*diff(y(x),x$2)-x*(3+10*x)*diff(y(x),x)+30*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(x^5 - \frac{5}{2}x^4\right) + c_2\left(3x^5\ln(x) + \frac{x^6}{4} - \frac{15x^4\ln(x)}{2} - \frac{5x^5}{8} - \frac{299x^4}{16} + 5x^3 + \frac{5x^2}{4} + \frac{x}{4} + \frac{1}{40}\right)$$

✓ Solution by Mathematica

Time used: 0.064 (sec). Leaf size: 68

```
DSolve[x^2*(1+x)*y''[x]-x*(3+10*x)*y'[x]+30*x*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to c_1\left(x^5 - \frac{5x^4}{2}\right) + \frac{1}{20}c_2\left(20x^6 - 50x^5 - 1495x^4 + 120(2x-5)x^4\log(x) + 400x^3 + 100x^2 + 20x + 2\right)$$

1.595 problem 609

Internal problem ID [8085]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 609.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(1+x)\,y' - 3(x+3)\,y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 73

```
dsolve(x^2*diff(y(x),x$2)+x*(1+x)*diff(y(x),x)-3*(3+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^3 - \frac{c_2(-\operatorname{expIntegral}_1(x)\,x^6 + \mathrm{e}^{-x}x^5 - \mathrm{e}^{-x}x^4 + 2\,\mathrm{e}^{-x}x^3 - 6x^2\mathrm{e}^{-x} + 24\,\mathrm{e}^{-x}x - 120\,\mathrm{e}^{-x})}{720x^3}$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 60

```
DSolve[x^2*y''[x]+x*(1+x)*y'[x]-3*(3+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2e^{-x}(e^x x^6\,\text{ExpIntegralEi}(-x) + x^5 - x^4 + 2x^3 - 6x^2 + 24x - 120)}{720x^3} + c_1x^3$$

1.596 problem 610

Internal problem ID [8086]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 610.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(2x+1)\,y'' + x(9+13x)\,y' + (5x+7)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(x^2*(1+2*x)*diff(y(x),x$2)+x*(9+13*x)*diff(y(x),x)+(7+5*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(143x^2+104x+20)}{x^7} + \frac{c_2(35x^3-45x^2+36x-20)\,(2x+1)^{\frac{7}{2}}}{x^7}$$

✓ Solution by Mathematica

Time used: 0.104 (sec). Leaf size: 58

```
DSolve[x^2*(1+2*x)*y''[x]+x*(9+13*x)*y'[x]+(7+5*x)*y[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{c_1(13x(11x+8)+20)}{143x^7} + \frac{c_2(35x^3-45x^2+36x-20)\,(2x+1)^{7/2}}{315x^7}$$

1.597 problem 611

Internal problem ID [8087]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 611.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(2x+1)\,y'' - 2x(-x+4)\,y' - (5x+7)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(4*x^2*(1+2*x)*diff(y(x),x$2)-2*x*(4-x)*diff(y(x),x)-(7+5*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{x}} + \frac{c_2(5x^3 - 10x^2 - 40x - 16)}{(2x+1)^{\frac{5}{4}}\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 47

```
DSolve[4*x^2*(1+2*x)*y''[x]-2*x*(4-x)*y'[x]-(7+5*x)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{\frac{2c_2(5x^3-10x^2-40x-16)}{(2x+1)^{5/4}} + 35c_1}{35\sqrt{x}}$$

1.598 problem 612

Internal problem ID [8088]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 612.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3x^2(x+3)\,y''-x(15+x)\,y'-20y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(3*x^2*(3+x)*diff(y(x),x$2)-x*(15+x)*diff(y(x),x)-20*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(x^2-36x-243)}{x^{\frac{2}{3}}}+\frac{c_2(7x+27)}{x^{\frac{2}{3}}\,(x+3)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.083 (sec). Leaf size: 43

```
DSolve[3*x^2*(3+x)*y''[x]-x*(15+x)*y'[x]-20*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{21c_2(x^2-36x-243)+\frac{4c_1(7x+27)}{\sqrt[3]{x+3}}}{28x^{2/3}}$$

1.599 problem 613

Internal problem ID [8089]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 613.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1+x)\,y''+x(1-10x)\,y'-(9-10x)\,y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 74

```
dsolve(x^2*(1+x)*diff(y(x),x$2)+x*(1-10*x)*diff(y(x),x)-(9-10*x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(715x^4+572x^3+234x^2+52x+5)}{x^3}+\frac{c_2\left(x^{13}+\frac{91}{8}x^{12}+\frac{117}{2}x^{11}+\frac{715}{4}x^{10}+\frac{715}{2}x^9+\frac{3861}{8}x^8+429x^7+\frac{429}{2}x^6\right)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.088 (sec). Leaf size: 51

```
DSolve[x^2*(1+x)*y''[x]+x*(1-10*x)*y'[x]-(9-10*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{6435c_1(x+1)^{12}(8x-5)-8c_2(715x^4+572x^3+234x^2+52x+5)}{51480x^3}$$

1.600 problem 614

Internal problem ID [8090]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 614.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1+x)\,y''+3x^2y'-(-x+6)\,y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 78

```
dsolve(x^2*(1+x)*diff(y(x),x$2)+3*x^2*diff(y(x),x)-(6-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^3+6x^2+9x+4)}{x^2\,(x+1)^2} + \frac{c_2\left(\ln(x+1)\,x^3+6\ln(x+1)\,x^2+9\ln(x+1)\,x+10x^2+4\ln(x+1)+\frac{43x}{2}+\frac{34}{3}\right)}{x^2\,(x+1)^2}$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 49

```
DSolve[x^2*(1+x)*y''[x]+3*x^2*y'[x]-(6-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{\frac{c_2(60x^2+129x+68)}{(x+1)^2}+6c_1(x+4)+6c_2(x+4)\log(x+1)}{6x^2}$$

1.601 problem 615

Internal problem ID [8091]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 615.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(2x+1)\,y'' - 2x(3+14x)\,y' + (6+100x)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
dsolve(x^2*(1+2*x)*diff(y(x),x$2)-2*x*(3+14*x)*diff(y(x),x)+(6+100*x)*y(x)=0,y(x), singsol=a
```

$$y(x) = c_1 x\left(2016x^4 + 672x^3 + 144x^2 + 18x + 1\right) + c_2 x\left(x^9 + \frac{9}{2}x^8 + 9x^7 + \frac{21}{2}x^6 + \frac{63}{8}x^5\right)$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 44

```
DSolve[x^2*(1+2*x)*y''[x]-2*x*(3+14*x)*y'[x]+(6+100*x)*y[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to c_1 x(2x+1)^9 - \frac{c_2 x(2016x^4 + 672x^3 + 144x^2 + 18x + 1)}{20160}$$

1.602 problem 616

Internal problem ID [8092]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 616.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1+x)\,y'' - x(6+11x)\,y' + (6+32x)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(x^2*(1+x)*diff(y(x),x$2)-x*(6+11*x)*diff(y(x),x)+(6+32*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x(35x^3 + 42x^2 + 21x + 4) + c_2 x\left(x^7 + \frac{14}{3}x^6 + 7x^5\right)$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 45

```
DSolve[x^2*(1+x)*y''[x]-x*(6+11*x)*y'[x]+(6+32*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{3}c_1 x(x+1)^6(3x-4) - \frac{1}{140}c_2 x(35x^3 + 42x^2 + 21x + 4)$$

1.603 problem 617

Internal problem ID [8093]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 617.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2(1+x)\,y''+4x(1+4x)\,y'-(49+27x)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(4*x^2*(1+x)*diff(y(x),x$2)+4*x*(1+4*x)*diff(y(x),x)-(49+27*x)*y(x)=0,y(x), singsol=al
```

$$y(x)=\frac{c_1(7x+6)}{(x+1)^2\,x^{\frac{7}{2}}}+\frac{c_2x^{\frac{7}{2}}}{(x+1)^2}$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 36

```
DSolve[4*x^2*(1+x)*y''[x]+4*x*(1+4*x)*y'[x]-(49+27*x)*y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x)\to\frac{42c_1x^7-7c_2x-6c_2}{42x^{7/2}(x+1)^2}$$

1.604 problem 618

Internal problem ID [8094]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 618.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+1)\,y''-x(-2x^2+7)\,y'+12y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 77

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(7-2*x^2)*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1x^6}{(x^2+1)^{\frac{7}{2}}}+\frac{c_2x^2\left(8x^4\sqrt{x^2+1}-15x^4\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)-9\sqrt{x^2+1}\,x^2-2\sqrt{x^2+1}\right)}{8\,(x^2+1)^{\frac{7}{2}}}$$

✓ Solution by Mathematica

Time used: 0.098 (sec). Leaf size: 88

```
DSolve[x^2*(1+x^2)*y''[x]-x*(7-2*x^2)*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x)\to\frac{-15c_2x^6\operatorname{arctanh}\left(\sqrt{x^2+1}\right)-2c_2\sqrt{x^2+1}x^2+8x^6\left(c_2\sqrt{x^2+1}+c_1\right)-9c_2\sqrt{x^2+1}x^4}{8\,(x^2+1)^{7/2}}$$

1.605 problem 619

Internal problem ID [8095]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 619.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' - x(-x^2+7)\, y' + 12y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 59

```
dsolve(x^2*diff(y(x),x$2)-x*(7-x^2)*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^6 \mathrm{e}^{-\frac{x^2}{2}} + \frac{c_2 x^2 \mathrm{e}^{-\frac{x^2}{2}} \left(\mathrm{expIntegral}_1\left(-\frac{x^2}{2}\right) x^4 + 2\,\mathrm{e}^{\frac{x^2}{2}} x^2 + 4\,\mathrm{e}^{\frac{x^2}{2}}\right)}{16}$$

✓ Solution by Mathematica
Time used: 0.089 (sec). Leaf size: 61

```
DSolve[x^2*y''[x]-x*(7-x^2)*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{16} c_2 e^{-\frac{x^2}{2}} x^6 \,\mathrm{ExpIntegralEi}\left(\frac{x^2}{2}\right) - \frac{1}{8} c_2 (x^2+2)\, x^2 + c_1 e^{-\frac{x^2}{2}} x^6$$

1.606 problem 620

Internal problem ID [8096]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 620.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x(2x^2 + 1)\, y' - (-10x^2 + 1)\, y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 50

```
dsolve(x^2*diff(y(x),x$2)+x*(1+2*x^2)*diff(y(x),x)-(1-10*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x\, \mathrm{e}^{-x^2}(x^2 - 2) + c_2 x\, \mathrm{e}^{-x^2}(x^2 - 2)\left(\int \frac{\mathrm{e}^{x^2}}{(x^2 - 2)^2\, x^3} dx\right)$$

✓ Solution by Mathematica
Time used: 0.109 (sec). Leaf size: 68

```
DSolve[x^2*y''[x]+x*(1+2*x^2)*y'[x]-(1-10*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{e^{-x^2}\left(c_2(x^2 - 2)\, x^2\, \text{ExpIntegralEi}\,(x^2) + 4c_1x^4 - x^2\left(c_2e^{x^2} + 8c_1\right) + c_2e^{x^2}\right)}{4x}$$

1.607 problem 621

Internal problem ID [8097]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 621.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x(-2x^2+1)\,y' - 4(2x^2+1)\,y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
dsolve(x^2*diff(y(x),x$2)+x*(1-2*x^2)*diff(y(x),x)-4*(1+2*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^2\mathrm{e}^{x^2} + \frac{c_2\mathrm{e}^{x^2}\left(\mathrm{expIntegral}_1\left(x^2\right)x^4 - \mathrm{e}^{-x^2}x^2 + \mathrm{e}^{-x^2}\right)}{4x^2}$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 46

```
DSolve[x^2*y''[x]+x*(1-2*x^2)*y'[x]-4*(1+2*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{c_2\left(e^{x^2}x^4\,\mathrm{ExpIntegralEi}\left(-x^2\right) + x^2 - 1\right)}{4x^2} + c_1e^{x^2}x^2$$

1.608 problem 622

Internal problem ID [8098]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 622.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x(-3x^2+1)\,y' - 4(-3x^2+1)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(x^2*diff(y(x),x$2)+x*(1-3*x^2)*diff(y(x),x)-4*(1-3*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^2(x^2-2) + c_2x^2(x^2-2)\left(\int \frac{e^{\frac{3x^2}{2}}}{(x^2-2)^2\,x^5}dx\right)$$

✓ Solution by Mathematica

Time used: 0.11 (sec). Leaf size: 89

```
DSolve[x^2*y''[x]+x*(1-3*x^2)*y'[x]-4*(1-3*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{1}{64}\left(27c_2(x^2-2)\,x^2\,\text{ExpIntegralEi}\left(\frac{3x^2}{2}\right) + 64c_1x^4 - 2x^2\left(9c_2e^{\frac{3x^2}{2}} + 64c_1\right) + 24c_2e^{\frac{3x^2}{2}} + \frac{8c_2e^{\frac{3x^2}{2}}}{x^2}\right)$$

1.609 problem 623

Internal problem ID [8099]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 623.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+1)\,y''+x(11x^2+5)\,y'+24x^2y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)+x*(5+11*x^2)*diff(y(x),x)+24*x^2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(2x^2+1)}{(x^2+1)^2\,x^4}+\frac{c_2}{(x^2+1)^2}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 36

```
DSolve[x^2*(1+x^2)*y''[x]+x*(5+11*x^2)*y'[x]+24*x^2*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to -\frac{-4c_1x^4+2c_2x^2+c_2}{4x^4\,(x^2+1)^2}$$

1.610 problem 624

Internal problem ID [8100]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 624.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x^2+1)\,y''+8xy'-(-x^2+35)\,y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 59

```
dsolve(4*x^2*(1+x^2)*diff(y(x),x$2)+8*x*diff(y(x),x)-(35-x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(x^4+2x^2+1)}{x^{\frac{7}{2}}}+\frac{c_2\left(\frac{\ln(x^2+1)x^4}{2}+\ln(x^2+1)\,x^2+x^2+\frac{\ln(x^2+1)}{2}+\frac{3}{4}\right)}{x^{\frac{7}{2}}}$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 53

```
DSolve[4*x^2*(1+x^2)*y''[x]+8*x*y'[x]-(35-x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x)\to\frac{4c_1(x^2+1)^2+c_2(4x^2+3)+2c_2(x^2+1)^2\log(x^2+1)}{4x^{7/2}}$$

1.611 problem 625

Internal problem ID [8101]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 625.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+1)\,y''-x(-x^2+5)\,y'-(25x^2+7)\,y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 38

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(5-x^2)*diff(y(x),x)-(7+25*x^2)*y(x)=0,y(x), singsol=all
```

$$y(x)=\frac{c_1}{(x^2+1)^2\,x}+\frac{c_2\left(x^{10}+\frac{5}{4}x^8\right)}{(x^2+1)^2\,x}$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 37

```
DSolve[x^2*(1+x^2)*y''[x]-x*(5-x^2)*y'[x]-(7+25*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to\frac{c_2(4x^2+5)\,x^8+40c_1}{40x\,(x^2+1)^2}$$

1.612 problem 626

Internal problem ID [8102]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 626.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^{2}\left(x^{2}+1\right) y''+x\left(2x^{2}+5\right) y'-21y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)+x*(5+2*x^2)*diff(y(x),x)-21*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_{1}(35x^{6}+140x^{4}+168x^{2}+64)}{x^{7}}+\frac{c_{2}(x^{2}+1)^{\frac{5}{2}}\left(x^{2}+8\right)}{x^{7}}$$

✓ Solution by Mathematica

Time used: 0.107 (sec). Leaf size: 52

```
DSolve[x^2*(1+x^2)*y''[x]+x*(5+2*x^2)*y'[x]-21*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x)\to \frac{35c_{1}(x^{2}+1)^{5/2}\left(x^{2}+8\right)-c_{2}(35x^{6}+140x^{4}+168x^{2}+64)}{35x^{7}}$$

1.613 problem 627

Internal problem ID [8103]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 627.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x^2+1)\,y''+4x(x^2+2)\,y'-(x^2+15)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(4*x^2*(1+x^2)*diff(y(x),x$2)+4*x*(2+x^2)*diff(y(x),x)-(15+x^2)*y(x)=0,y(x), singsol=a
```

$$y(x)=\frac{c_1(3x^2+2)}{x^{\frac{5}{2}}}+\frac{c_2(x^2+1)^{\frac{3}{2}}}{x^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.076 (sec). Leaf size: 39

```
DSolve[4*x^2*(1+x^2)*y''[x]+4*x*(2+x^2)*y'[x]-(15+x^2)*y[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x)\to\frac{3c_1(x^2+1)^{3/2}-c_2(3x^2+2)}{3x^{5/2}}$$

1.614 problem 628

Internal problem ID [8104]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 628.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{2(t+1)\,y'}{t^2+2t-1} + \frac{2y}{t^2+2t-1} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(t),t$2)-2*(t+1)/(t^2+2*t-1)*diff(y(t),t)+2/(t^2+2*t-1)*y(t)=0,y(t), singsol=al
```

$$y(t) = c_1(t+1) + c_2(t^2+1)$$

✓ Solution by Mathematica

Time used: 0.198 (sec). Leaf size: 64

```
DSolve[y''[t]-2*(t+1)/(t^2+2*t-1)*y'[t]+2/(t^2+2*t-1)*y[t]==0,y[t],t,IncludeSingularSolution
```

$$y(t) \to \frac{\sqrt{t^2+2t-1}\left(c_1\left(t^2-2\left(\sqrt{2}-1\right)t-2\sqrt{2}+3\right)+c_2(t+1)\right)}{\sqrt{-t^2-2t+1}}$$

1.615 problem 629

Internal problem ID [8105]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 629.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4ty' + (4t^2 - 2)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(t),t$2)-4*t*diff(y(t),t)+(4*t^2-2)*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 e^{t^2} + c_2 e^{t^2} t$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 18

```
DSolve[y''[t]-4*t*y'[t]+(4*t^2-2)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{t^2}(c_2 t + c_1)$$

1.616 problem 630

Internal problem ID [8106]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 630.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-t^2+1)\,y''-2ty'+2y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve((1-t^2)*diff(y(t),t$2)-2*t*diff(y(t),t)+2*y(t)=0,y(t), singsol=all)
```

$$y(t)=c_1t+c_2\left(-\frac{\ln(t+1)\,t}{2}+\frac{\ln(t-1)\,t}{2}+1\right)$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 33

```
DSolve[(1-t^2)*y''[t]-2*t*y'[t]+2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to c_1t-\frac{1}{2}c_2(t\log(1-t)-t\log(t+1)+2)$$

1.617 problem 631

Internal problem ID [8107]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 631.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(t^2 + 1) y'' - 2ty' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((1+t^2)*diff(y(t),t$2)-2*t*diff(y(t),t)+2*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 t + c_2(t^2 - 1)$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 21

```
DSolve[(1+t^2)*y''[t]-2*t*y'[t]+2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_2 t - c_1(t - i)^2$$

1.618 problem 632

Internal problem ID [8108]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 632.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$(-t^2+1)\,y''-2ty'+6y=0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 50

```
dsolve((1-t^2)*diff(y(t),t$2)-2*t*diff(y(t),t)+6*y(t)=0,y(t), singsol=all)
```

$$y(t)=c_1(-3t^2+1)+c_2\left(-\frac{3\ln(t+1)\,t^2}{8}+\frac{3\ln(t-1)\,t^2}{8}+\frac{\ln(t+1)}{8}-\frac{\ln(t-1)}{8}+\frac{3t}{4}\right)$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 55

```
DSolve[(1-t^2)*y''[t]-2*t*y'[t]+6*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to\frac{1}{2}c_1(3t^2-1)-\frac{1}{4}c_2\left((3t^2-1)\log(1-t)+(1-3t^2)\log(t+1)+6t\right)$$

1.619 problem 633

Internal problem ID [8109]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 633.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(2t+1)\,y''-4(t+1)\,y'+4y=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve((2*t+1)*diff(y(t),t$2)-4*(t+1)*diff(y(t),t)+4*y(t)=0,y(t), singsol=all)
```

$$y(t)=c_1(t+1)+c_2\mathrm{e}^{2t}$$

✓ Solution by Mathematica
Time used: 0.047 (sec). Leaf size: 23

```
DSolve[(2*t+1)*y''[t]-4*(t+1)*y'[t]+4*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to c_1e^{2t+1}-c_2(t+1)$$

1.620 problem 634

Internal problem ID [8110]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 634.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$t^2y'' + ty' + \left(t^2 - \frac{1}{4}\right)y = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(t^2*diff(y(t),t$2)+t*diff(y(t),t)+(t^2-1/4)*y(t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_1 \sin(t)}{\sqrt{t}} + \frac{c_2 \cos(t)}{\sqrt{t}}$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 39

```
DSolve[t^2*y''[t]+t*y'[t]+(t^2-1/4)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{e^{-it}(2c_1 - ic_2e^{2it})}{2\sqrt{t}}$$

1.621 problem 635

Internal problem ID [8111]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 635.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{2ty'}{t^2+1} + \frac{2y}{t^2+1} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(t),t$2)-2*t/(1+t^2)*diff(y(t),t)+2/(1+t^2)*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 t + c_2(t^2 - 1)$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 21

```
DSolve[y''[t]-2*t/(1+t^2)*y'[t]+2/(1+t^2)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_2 t - c_1(t - i)^2$$

1.622 problem 636

Internal problem ID [8112]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 636.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + (t^2 + 2t + 1)\, y' - (4 + 4t)\, y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 83

```
dsolve(diff(y(t),t$2)+(t^2+2*t+1)*diff(y(t),t)-(4+4*t)*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 (t^4 + 4t^3 + 6t^2 + 8t + 5) + c_2 (t^4 + 4t^3 + 6t^2 + 8t + 5) \left(\int \frac{e^{-\frac{1}{3}t^3 - t^2 - t}}{(t+1)^2 (t^3 + 3t^2 + 3t + 5)^2} dt \right)$$

✓ Solution by Mathematica

Time used: 0.369 (sec). Leaf size: 132

```
DSolve[y''[t]+(t^2+2*t+1)*y'[t]-(4+4*t)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{36} e^{-\frac{1}{3}t(t^2+3t+3)} \left(-3c_2 (t^3 + 3t^2 + 3t + 4) + 3^{2/3} c_2 e^{\frac{1}{3}(t+1)^3} \sqrt[3]{(t+1)^3} (t^3 + 3t^2 + 3t + 5)\, \Gamma\left(\frac{2}{3}, \frac{1}{3}(t+1)^3\right) + 36 c_1 e^{\frac{t^3}{3} + t^2 + t} (t^4 + 4t^3 + 6t^2 + 8t + 5) \right)$$

1.623 problem 638

Internal problem ID [8113]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 638.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$2ty'' + (1 - 2t)\, y' - y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(2*t*diff(y(t),t$2)+(1-2*t)*diff(y(t),t)-y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \mathrm{e}^t + c_2 \mathrm{e}^t \left(\int \frac{\mathrm{e}^{-t}}{\sqrt{t}} dt \right)$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 21

```
DSolve[2*t*y''[t]+(1-2*t)*y'[t]-y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^t \left(c_1 - c_2 \Gamma\left(\frac{1}{2}, t\right) \right)$$

1.624 problem 639

Internal problem ID [8114]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 639.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2ty'' + (t+1)\, y' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
dsolve(2*t*diff(y(t),t$2)+(1+t)*diff(y(t),t)-2*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 (t^2 + 6t + 3) + c_2 (t^2 + 6t + 3) \left(\int \frac{e^{-\frac{t}{2}}}{(t^2 + 6t + 3)^2 \sqrt{t}} dt \right)$$

✓ Solution by Mathematica

Time used: 0.175 (sec). Leaf size: 71

```
DSolve[2*t*y''[t]+(1+t)*y'[t]-2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{24} \left(\sqrt{2\pi} c_2 (t^2 + 6t + 3) \operatorname{erf}\left(\frac{\sqrt{t}}{\sqrt{2}} \right) + 24 c_1 (t^2 + 6t + 3) + 2 c_2 e^{-t/2} \sqrt{t} (t+5) \right)$$

1.625 problem 640

Internal problem ID [8115]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 640.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2t^2y'' - ty' + (t+1)\,y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 31

```
dsolve(2*t^2*diff(y(t),t$2)-t*diff(y(t),t)+(1+t)*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \sin\left(\sqrt{2}\,\sqrt{t}\right)\sqrt{t} + c_2\sqrt{t}\,\cos\left(\sqrt{2}\,\sqrt{t}\right)$$

✓ Solution by Mathematica
Time used: 0.079 (sec). Leaf size: 62

```
DSolve[2*t^2*y''[t]-t*y'[t]+(1+t)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}e^{-i\sqrt{2}\sqrt{t}}\sqrt{t}\left(2c_1e^{2i\sqrt{2}\sqrt{t}} + i\sqrt{2}c_2\right)$$

1.626 problem 641

Internal problem ID [8116]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 641.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2t^2y'' + (t^2 - t)\, y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(2*t^2*diff(y(t),t$2)+(t^2-t)*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1\sqrt{t}\,\mathrm{e}^{-\frac{t}{2}} + c_2\sqrt{t}\,\mathrm{e}^{-\frac{t}{2}}\left(\int \frac{\mathrm{e}^{\frac{t}{2}}}{\sqrt{t}}dt\right)$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 46

```
DSolve[2*t^2*y''[t]+(t^2-t)*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t/2}\left(c_2\sqrt{t} + \sqrt{2}c_1\sqrt{-t}\Gamma\left(\frac{1}{2}, -\frac{t}{2}\right)\right)$$

1.627 problem 642

Internal problem ID [8117]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 642.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2y'' + \left(-t^2 + t\right) y' - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(t^2*diff(y(t),t$2)+(t-t^2)*diff(y(t),t)-y(t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_1(t+1)}{t} + \frac{c_2 e^t}{t}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 23

```
DSolve[t^2*y''[t]+(t-t^2)*y'[t]-y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_2 e^t - c_1(t+1)}{t}$$

1.628 problem 643

Internal problem ID [8118]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 643.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$
\boxed{ty'' - (t^2 + 2)\, y' + yt = 0}
$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(t*diff(y(t),t$2)-(t^2+2)*diff(y(t),t)+t*y(t)=0,y(t), singsol=all)
```

$$
y(t) = c_1 \mathrm{e}^{\frac{t^2}{2}} + \frac{c_2 \mathrm{e}^{\frac{t^2}{2}} \left(-\sqrt{\pi}\, \sqrt{2}\, \mathrm{erf}\left(\frac{\sqrt{2}\, t}{2}\right) + 2t\, \mathrm{e}^{-\frac{t^2}{2}}\right)}{2}
$$

✓ Solution by Mathematica

Time used: 0.095 (sec). Leaf size: 52

```
DSolve[t*y''[t]-(t^2+2)*y'[t]+t*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$
y(t) \to \sqrt{\frac{\pi}{2}} c_2 e^{\frac{t^2}{2}} \mathrm{erf}\left(\frac{t}{\sqrt{2}}\right) + c_1 e^{\frac{t^2}{2}} - c_2 t
$$

1.629 problem 644

Internal problem ID [8119]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 644.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2y'' + t(t+1)\,y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(t^2*diff(y(t),t$2)+t*(t+1)*diff(y(t),t)-y(t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_1(t-1)}{t} + \frac{c_2 e^{-t}}{t}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 26

```
DSolve[t^2*y''[t]+t*(t+1)*y'[t]-y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{e^{-t}(c_1 e^t(t-1) + c_2)}{t}$$

1.630 problem 645

Internal problem ID [8120]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 645.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{ty'' - (t+4)\,y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(t*diff(y(t),t$2)-(4+t)*diff(y(t),t)+2*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1\left(t^2 + 6t + 12\right) + c_2 \mathrm{e}^t \left(t^2 - 6t + 12\right)$$

✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 85

```
DSolve[t*y''[t]-(4+t)*y'[t]+2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{2e^{t/2}\sqrt{t}\left((c_2t^2 - 6ic_1t + 12c_2)\cosh\left(\frac{t}{2}\right) + i(c_1(t^2+12) + 6ic_2t)\sinh\left(\frac{t}{2}\right)\right)}{\sqrt{\pi}\sqrt{-it}}$$

1.631 problem 646

Internal problem ID [8121]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 646.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2y'' + (t^2 - 3t)\,y' + 3y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 38

```
dsolve(t^2*diff(y(t),t$2)+(t^2-3*t)*diff(y(t),t)+3*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1t^3\mathrm{e}^{-t} + \frac{c_2t\,\mathrm{e}^{-t}(\mathrm{expIntegral}_1\,(-t)\,t^2 + \mathrm{e}^t t + \mathrm{e}^t)}{2}$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 41

```
DSolve[t^2*y''[t]+(t^2-3*t)*y'[t]+3*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}e^{-t}(c_1t^3\,\mathrm{ExpIntegralEi}(t) + 2c_2t^3 - c_1e^t(t+1)t)$$

1.632 problem 647

Internal problem ID [8122]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 647.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{ty'' + ty' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(t*diff(y(t),t$2)+t*diff(y(t),t)+2*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 e^{-t}(t-2)\,t + \frac{c_2(\text{expIntegral}_1\,(-t)\,t^2 + e^t t - 2\,\text{expIntegral}_1\,(-t)\,t - e^t)\,e^{-t}}{2}$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 51

```
DSolve[t*y''[t]+t*y'[t]+2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}e^{-t}\left(c_2(t-2)t\,\text{ExpIntegralEi}(t) + 2c_1t^2 - t\left(c_2e^t + 4c_1\right) + c_2e^t\right)$$

1.633 problem 648

Internal problem ID [8123]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 648.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$ty'' + (-t^2 + 1)\, y' + 4yt = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
dsolve(t*diff(y(t),t$2)+(1-t^2)*diff(y(t),t)+4*t*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1(t^4 - 8t^2 + 8) + c_2(t^4 - 8t^2 + 8)\left(\int \frac{e^{\frac{t^2}{2}}}{(t^4 - 8t^2 + 8)^2\, t} dt\right)$$

✓ Solution by Mathematica

Time used: 0.138 (sec). Leaf size: 61

```
DSolve[t*y''[t]+(1-t^2)*y'[t]+4*t*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{128} c_2 \left((t^4 - 8t^2 + 8)\, \text{ExpIntegralEi}\left(\frac{t^2}{2}\right) - 2e^{\frac{t^2}{2}}(t^2 - 6)\right) + c_1(t^4 - 8t^2 + 8)$$

1.634 problem 649

Internal problem ID [8124]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 649.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2y'' - t(t+1)\,y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(t^2*diff(y(t),t$2)-t*(1+t)*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 e^t t + c_2 e^t t \operatorname{expIntegral}_1(t)$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 20

```
DSolve[t^2*y''[t]-t*(1+t)*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^t t(c_1 \operatorname{ExpIntegralEi}(-t) + c_2)$$

1.635 problem 650

Internal problem ID [8125]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 650.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4xy' + (4x^2 + 6)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2+6)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^2} \cos(2x) + c_2 \mathrm{e}^{-x^2} \sin(2x)$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 37

```
DSolve[y''[x]+4*x*y'[x]+(4*x^2+6)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} e^{-x(x+2i)} \left(4c_1 - ic_2 e^{4ix}\right)$$

1.636 problem 651

Internal problem ID [8126]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 651.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-z^2+1)\, y'' - 3zy' + y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 51

```
dsolve((1-z^2)*diff(y(z),z$2)-3*z*diff(y(z),z)+y(z)=0,y(z), singsol=all)
```

$$y(z) = \frac{c_1\left(z+\sqrt{z^2-1}\right)^{\sqrt{2}}}{\sqrt{z^2-1}} + \frac{c_2\left(z+\sqrt{z^2-1}\right)^{-\sqrt{2}}}{\sqrt{z^2-1}}$$

✓ Solution by Mathematica
Time used: 0.071 (sec). Leaf size: 90

```
DSolve[(1-z^2)*y''[z]-3*z*y'[z]+y[z]==0,y[z],z,IncludeSingularSolutions -> True]
```

$$y(z) \to \frac{\sqrt{2}c_1 \cos\left(2\sqrt{2}\arcsin\left(\frac{\sqrt{1-z}}{\sqrt{2}}\right)\right) + \sqrt{\pi}c_2\sqrt[4]{1-z^2}Q^{\frac{1}{2}}_{-\frac{1}{2}+\sqrt{2}}(z)}{\sqrt{\pi}\sqrt[4]{-\left(z^2-1\right)^2}}$$

1.637 problem 652

Internal problem ID [8127]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 652.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4zy'' + 2(1 - z)\, y' - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(4*z*diff(y(z),z$2)+2*(1-z)*diff(y(z),z)-y(z)=0,y(z), singsol=all)
```

$$y(z) = c_1 e^{\frac{z}{2}} + c_2 e^{\frac{z}{2}} \left(\int \frac{e^{-\frac{z}{2}}}{\sqrt{z}} dz \right)$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 34

```
DSolve[4*z*y''[z]+2*(1-z)*y'[z]-y[z]==0,y[z],z,IncludeSingularSolutions -> True]
```

$$y(z) \to e^{z/2} \left(c_1 - \sqrt{2} c_2 \Gamma\left(\frac{1}{2}, \frac{z}{2}\right) \right)$$

1.638 problem 653

Internal problem ID [8128]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 653.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$f'' + 2(z-1)\, f' + 4f = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 74

```
dsolve(diff(f(z),z$2)+2*(z-1)*diff(f(z),z)+4*f(z)=0,f(z), singsol=all)
```

$$f(z) = c_1 \mathrm{e}^{-z^2+2z}(z-1) + c_2 \mathrm{e}^{-z^2+2z}\left(\sqrt{\pi}\,\mathrm{e}^{-1}\operatorname{erf}(iz-i)\, z - \sqrt{\pi}\,\mathrm{e}^{-1}\operatorname{erf}(iz-i) - i\mathrm{e}^{z^2-2z}\right)$$

✓ Solution by Mathematica

Time used: 0.174 (sec). Leaf size: 72

```
DSolve[f''[z]+2*(z-a)*f'[z]+4*f[z]==0,f[z],z,IncludeSingularSolutions -> True]
```

$$f(z) \to e^{z(2a-z)}\left(-\sqrt{\pi}c_2\sqrt{(a-z)^2}\operatorname{erfi}\left(\sqrt{(a-z)^2}\right) + c_2 e^{(a-z)^2} - 2ac_1 + 2c_1 z\right)$$

1.639 problem 654

Internal problem ID [8129]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 654.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{zy'' - 2y' + zy = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 25

```
dsolve(z*diff(y(z),z$2)-2*diff(y(z),z)+z*y(z)=0,y(z), singsol=all)
```

$$y(z) = c_1(\cos(z)\, z - \sin(z)) + c_2(\cos(z) + \sin(z)\, z)$$

✓ Solution by Mathematica
Time used: 0.069 (sec). Leaf size: 39

```
DSolve[z*y''[z]-2*y'[z]+z*y[z]==0,y[z],z,IncludeSingularSolutions -> True]
```

$$y(z) \to -\sqrt{\frac{2}{\pi}}((c_1z + c_2)\cos(z) + (c_2z - c_1)\sin(z))$$

1.640 problem 655

Internal problem ID [8130]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 655.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{zy'' + (2z-3)\,y' + \frac{4y}{z} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(z*diff(y(z),z$2)+(2*z-3)*diff(y(z),z)+4/z*y(z)=0,y(z), singsol=all)
```

$$y(z) = c_1 z^2 \mathrm{e}^{-2z}(2z-1) + c_2 z^2 \left(2 \operatorname{expIntegral}_1(-2z)\, z - \operatorname{expIntegral}_1(-2z) + \mathrm{e}^{2z}\right) \mathrm{e}^{-2z}$$

✓ Solution by Mathematica

Time used: 0.087 (sec). Leaf size: 47

```
DSolve[z*y''[z]+(2*z-3)*y'[z]+4/z*y[z]==0,y[z],z,IncludeSingularSolutions -> True]
```

$$y(z) \to -\frac{1}{2} e^{-2z} z^2 \left(4c_2(1-2z)\,\text{ExpIntegralEi}(2z) - 2c_1 z + 4c_2 e^{2z} + c_1\right)$$

1.641 problem 656

Internal problem ID [8131]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 656.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (1 - 2x)\, y' + (x - 1)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x$2)+(1-2*x)*diff(y(x),x)+(x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2 e^x \ln(x)$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 17

```
DSolve[x*y''[x]+(1-2*x)*y'[x]+(x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x (c_2 \log(x) + c_1)$$

1.642 problem 657

Internal problem ID [8132]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 657.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2xy' + (x^2 + 2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sin(x) + c_2 \cos(x)\, x$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-ix} x - \frac{1}{2} i c_2 e^{ix} x$$

1.643 problem 658

Internal problem ID [8133]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 658.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(1-x^2)\, y''-2xy'+2y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+c_2\left(-\frac{\ln(x+1)\,x}{2}+\frac{\ln(x-1)\,x}{2}+1\right)$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 33

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1x-\frac{1}{2}c_2(x\log(1-x)-x\log(x+1)+2)$$

1.644 problem 659

Internal problem ID [8134]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 659.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 4xy' + (4x^2 - 1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(4*x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 39

```
DSolve[4*x^2*y''[x]+4*x*y'[x]+(4*x^2-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2e^{2ix})}{2\sqrt{x}}$$

1.645 problem 660

Internal problem ID [8135]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 660.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (2x+1)\,y' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x*diff(y(x),x$2)-(2*x+1)*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(2x+1) + c_2\mathrm{e}^{2x}$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 25

```
DSolve[x*y''[x]-(2*x+1)*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{2x} - \frac{1}{4}c_2(2x+1)$$

1.646 problem 661

Internal problem ID [8136]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 661.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_erf]`

$$\boxed{y'' + 2xy' + 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)+2*x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x\,\mathrm{e}^{-x^2} + c_2 \mathrm{e}^{-x^2}\left(\sqrt{\pi}\,\mathrm{erfi}\,(x)\,x - \mathrm{e}^{x^2}\right)$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 51

```
DSolve[y''[x]+2*x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x^2}\left(-\sqrt{\pi}c_2\sqrt{x^2}\mathrm{erfi}\left(\sqrt{x^2}\right) + c_2 e^{x^2} + 2c_1 x\right)$$

1.647 problem 662

Internal problem ID [8137]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 662.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + xy' + 3y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 50

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{x^2}{2}} \left(x^2 - 1\right) + c_2 e^{-\frac{x^2}{2}} \left(x^2 - 1\right) \left(\int \frac{e^{\frac{x^2}{2}}}{\left(x - 1\right)^2 \left(x + 1\right)^2} dx\right)$$

✓ Solution by Mathematica
Time used: 0.131 (sec). Leaf size: 65

```
DSolve[y''[x]+x*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} e^{-\frac{x^2}{2}} \left(\sqrt{2\pi} c_2 \left(x^2 - 1\right) \operatorname{erfi}\left(\frac{x}{\sqrt{2}}\right) + 4c_1 \left(x^2 - 1\right) - 2c_2 e^{\frac{x^2}{2}} x\right)$$

1.648 problem 663

Internal problem ID [8138]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 663.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - x^2y' - 3yx = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 76

```
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)-3*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{x^3}{3}} x - \frac{9c_2 e^{\frac{x^3}{3}} 3^{\frac{2}{3}} e^{-\frac{x^3}{6}} \left(x^6 \operatorname{WhittakerM}\left(\frac{1}{3}, \frac{5}{6}, \frac{x^3}{3}\right) + 5 \operatorname{WhittakerM}\left(\frac{4}{3}, \frac{5}{6}, \frac{x^3}{3}\right) x^3 + 10 \operatorname{WhittakerM}\left(\frac{4}{3}, \frac{5}{6}, \frac{x^3}{3}\right)\right)}{10x^3 (x^3)^{\frac{1}{3}}}$$

✓ Solution by Mathematica
Time used: 0.078 (sec). Leaf size: 51

```
DSolve[y''[x]-x^2*y'[x]-3*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{9} e^{\frac{x^3}{3}} \left(9c_1 x - 3^{2/3} c_2 \sqrt[3]{x^3} \Gamma\left(-\frac{1}{3}, \frac{x^3}{3}\right)\right)$$

1.649 problem 664

Internal problem ID [8139]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 664.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-4x^2+1)\,y''-20xy'-16y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 57

```
dsolve((1-4*x^2)*diff(y(x),x$2)-20*x*diff(y(x),x)-16*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x}{(4x^2-1)^{\frac{3}{2}}}+\frac{c_2\left(2\ln\left(2x+\sqrt{4x^2-1}\right)x-\sqrt{4x^2-1}\right)}{(4x^2-1)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.109 (sec). Leaf size: 73

```
DSolve[(1-4*x^2)*y''[x]-20*x*y'[x]-16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{4c_2x\arctan\left(\frac{\sqrt{1-4x^2}}{2x+1}\right)-c_2\sqrt{1-4x^2}+c_1x}{\sqrt[4]{1-4x^2}\,(4x^2-1)^{5/4}}$$

1.650 problem 665

Internal problem ID [8140]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 665.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(x^2-1)\, y''-6xy'+12y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve((x^2-1)*diff(y(x),x$2)-6*x*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(x^3+x)+c_2(x^4+6x^2+1)$$

✓ Solution by Mathematica
Time used: 0.124 (sec). Leaf size: 45

```
DSolve[(x^2-1)*y''[x]-6*x*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{\sqrt{x^2-1}(c_2x(x^2+1)+c_1(x-1)^4)}{\sqrt{1-x^2}}$$

1.651 problem 666

Internal problem ID [8141]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 666.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + xy' + (x + 2)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 64

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+(2+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{1}{2}x^2+x}(x^2 - 4x + 3) + c_2 \mathrm{e}^{-\frac{1}{2}x^2+x}(x^2 - 4x + 3)\left(\int \frac{\mathrm{e}^{\frac{1}{2}x^2-2x}}{(x-1)^2\,(x-3)^2}dx\right)$$

✓ Solution by Mathematica

Time used: 0.273 (sec). Leaf size: 94

```
DSolve[y''[x]+x*y'[x]+(2+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{-\frac{x^2}{2}+x-\frac{9}{2}}\left(e^{5/2}\sqrt{2\pi}c_2\,(x^2 - 4x + 3)\,\mathrm{erfi}\left(\frac{x-2}{\sqrt{2}}\right) + 4e^{9/2}c_1\,(x^2 - 4x + 3) - 2c_2 e^{\frac{1}{2}(x-3)^2+x}(x-2)\right)$$

1.652 problem 667

Internal problem ID [8142]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 667.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(2x^2+1)\,y''+7xy'+2y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve((1+2*x^2)*diff(y(x),x$2)+7*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x}{(2x^2+1)^{\frac{3}{4}}}+\frac{c_2x\left(\int\frac{1}{(2x^2+1)^{\frac{1}{4}}x^2}dx\right)}{(2x^2+1)^{\frac{3}{4}}}$$

✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 66

```
DSolve[(1+2*x^2)*y''[x]+7*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_2Q_{\frac{1}{4}}^{\frac{3}{4}}(i\sqrt{2}x)}{(2x^2+1)^{3/8}}+\frac{2i\sqrt[4]{2}c_1x}{(2x^2+1)^{3/4}\,\text{Gamma}\left(\frac{1}{4}\right)}$$

1.653 problem 668

Internal problem ID [8143]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 668.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{4y'' + xy' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 52

```
dsolve(4*diff(y(x),x$2)+x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x^2}{8}} x(x^2 - 12) + c_2 \mathrm{e}^{-\frac{x^2}{8}} x(x^2 - 12) \left(\int \frac{\mathrm{e}^{\frac{x^2}{8}}}{(x^2 - 12)^2 x^2} dx \right)$$

✓ Solution by Mathematica

Time used: 0.111 (sec). Leaf size: 122

```
DSolve[4*y''[x]+x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{x^2}{8}} \left(\sqrt{2\pi} c_2 (x^2 - 12) x^2 \operatorname{erfi}\left(\frac{\sqrt{x^2}}{2\sqrt{2}} \right) + 4\sqrt{x^2} \left(2\sqrt{2} c_1 x^3 - c_2 e^{\frac{x^2}{8}} x^2 + 8 c_2 e^{\frac{x^2}{8}} - 24\sqrt{2} c_1 x \right) \right)}{32\sqrt{x^2}}$$

1.654 problem 669

Internal problem ID [8144]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 669.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + xy' - 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 50

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x^4 + 6x^2 + 3) + c_2(x^4 + 6x^2 + 3)\left(\int \frac{e^{-\frac{x^2}{2}}}{(x^4 + 6x^2 + 3)^2} dx\right)$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 43

```
DSolve[y''[x]+x*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\frac{x^2}{2}} \operatorname{HermiteH}\left(-5, \frac{x}{\sqrt{2}}\right) + \frac{1}{3} c_2(x^4 + 6x^2 + 3)$$

1.655 problem 670

Internal problem ID [8145]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 670.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4xy'' - xy' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 48

```
dsolve(4*x*diff(y(x),x$2)-x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x^2 - 8x) + c_2\left(\frac{\text{expIntegral}_1\left(-\frac{x}{4}\right)x^2}{128} + \frac{e^{\frac{x}{4}}x}{32} - \frac{\text{expIntegral}_1\left(-\frac{x}{4}\right)x}{16} - \frac{e^{\frac{x}{4}}}{8}\right)$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 43

```
DSolve[4*x*y''[x]-x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{128}c_2\left((x-8)x\,\text{ExpIntegralEi}\left(\frac{x}{4}\right) - 4e^{x/4}(x-4)\right) + c_1(x-8)x$$

1.656 problem 671

Internal problem ID [8146]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 671.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$6x^2y'' + x(1+18x)\,y' + (12x+1)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(6*x^2*diff(y(x),x$2)+x*(1+18*x)*diff(y(x),x)+(1+12*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\mathrm{e}^{-3x} + c_2\sqrt{x}\,\mathrm{e}^{-3x}\left(\int \frac{\mathrm{e}^{3x}}{x^{\frac{7}{6}}}dx\right)$$

✓ Solution by Mathematica

Time used: 0.061 (sec). Leaf size: 47

```
DSolve[6*x^2*y''[x]+x*(1+18*x)*y'[x]+(1+12*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^{-3x}\left(\frac{\sqrt[6]{3}c_2x^{4/3}\Gamma\left(-\frac{1}{6}, -3x\right)}{(-x)^{5/6}} + c_1\sqrt{x}\right)$$

1.657 problem 672

Internal problem ID [8147]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 672.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3x^2y'' - x(x+8)\,y' + 6y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 59

```
dsolve(3*x^2*diff(y(x),x$2)-x*(x+8)*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{\frac{2}{3}} e^{\frac{x}{3}} \left(x^2 - 2x + 4\right) + c_2 x^{\frac{2}{3}} e^{\frac{x}{3}} \left(x^2 - 2x + 4\right) \left(\int \frac{x^{\frac{4}{3}} e^{-\frac{x}{3}}}{\left(x^2 - 2x + 4\right)^2} dx\right)$$

✓ Solution by Mathematica
Time used: 0.172 (sec). Leaf size: 79

```
DSolve[3*x^2*y''[x]-x*(x+8)*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x/3} x^{2/3} (x^2 - 2x + 4) - \frac{c_2 e^{x/3} x^{2/3} (x^2 - 2x + 4)\, \Gamma\left(\frac{1}{3}, \frac{x}{3}\right)}{6\ 3^{2/3}} + \frac{1}{6} c_2 (x - 4) x$$

1.658 problem 673

Internal problem ID [8148]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 673.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2y'' - x(2x+1)\,y' + 2(4x-1)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
dsolve(2*x^2*diff(y(x),x$2)-x*(1+2*x)*diff(y(x),x)+2*(4*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^2(4x^2-36x+63) + c_2x^2(4x^2-36x+63)\left(\int \frac{e^x}{(4x^2-36x+63)^2\,x^{\frac{7}{2}}}dx\right)$$

✓ Solution by Mathematica

Time used: 0.098 (sec). Leaf size: 89

```
DSolve[2*x^2*y''[x]-x*(1+2*x)*y'[x]+2*(4*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to c_1\left(x^4 - 9x^3 + \frac{63x^2}{4}\right) - \frac{4c_2\left(\sqrt{\pi}(-4x^2+36x-63)\,x^{5/2}\text{erfi}\left(\sqrt{x}\right) + 2e^x(2x^4-17x^3+24x^2+6x+3)\right)}{945\sqrt{x}}$$

1.659 problem 674

Internal problem ID [8149]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 674.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' - 4x^2y' + (2x + 1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(4*x^2*diff(y(x),x$2)-4*x^2*diff(y(x),x)+(1+2*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x} + c_2\sqrt{x}\ \mathrm{expIntegral}_1\left(-x\right)$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 19

```
DSolve[4*x^2*y''[x]-4*x^2*y'[x]+(1+2*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}(c_2\,\mathrm{ExpIntegralEi}(x) + c_1)$$

1.660 problem 675

Internal problem ID [8150]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 675.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' + x(-2x+3) y' + (1-2x) y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)+x*(3-2*x)*diff(y(x),x)+(1-2*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + \frac{c_2 \,\mathrm{expIntegral}_1(-2x)}{x}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 19

```
DSolve[x^2*y''[x]+x*(3-2*x)*y'[x]+(1-2*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 \,\mathrm{ExpIntegralEi}(2x) + c_1}{x}$$

1.661 problem 676

Internal problem ID [8151]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 676.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - x(x+3)\,y' + (-x+4)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 60

```
dsolve(x^2*diff(y(x),x$2)-x*(3+x)*diff(y(x),x)+(4-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^x x^2 (x^2 + 4x + 2) - \frac{c_2 x^2 (-x^2 \,\mathrm{expIntegral}_1(x) + \mathrm{e}^{-x} x - 4 \,\mathrm{expIntegral}_1(x)\, x + 3\,\mathrm{e}^{-x} - 2 \,\mathrm{expIntegral}_1(x))\, \mathrm{e}^x}{4}$$

✓ Solution by Mathematica

Time used: 0.117 (sec). Leaf size: 52

```
DSolve[x^2*y''[x]-x*(3+x)*y'[x]+(4-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} x^2 \left(c_2 e^x (x^2 + 4x + 2) \,\mathrm{ExpIntegralEi}(-x) + 4c_1 e^x (x^2 + 4x + 2) + c_2 (x+3)\right)$$

1.662 problem 677

Internal problem ID [8152]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 677.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x(3-x)\,y' + y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve(x^2*diff(y(x),x$2)+x*(3-x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x-1)}{x} + \frac{c_2(\operatorname{expIntegral}_1(-x)\,x - \operatorname{expIntegral}_1(-x) + \mathrm{e}^x)}{x}$$

✓ Solution by Mathematica
Time used: 0.053 (sec). Leaf size: 31

```
DSolve[x^2*y''[x]+x*(3-x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2(x-1)\,\text{ExpIntegralEi}(x) + c_1(x-1) - c_2e^x}{x}$$

1.663 problem 678

Internal problem ID [8153]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 678.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - \left(2\sqrt{5}-1\right)xy' + \left(\frac{19}{4} - 3x^2\right)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(x^2*diff(y(x),x$2)-(2*sqrt(5)-1)*x*diff(y(x),x)+(19/4-3*x^2)*y(x)=0,y(x), singsol=all
```

$$y(x) = \frac{c_1 x^{\sqrt{5}} \sinh\left(\sqrt{3}\,x\right)}{\sqrt{x}} + \frac{c_2 x^{\sqrt{5}} \cosh\left(\sqrt{3}\,x\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.086 (sec). Leaf size: 53

```
DSolve[x^2*y''[x]-(2*Sqrt[5]-1)*x*y'[x]+(19/4-3*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{6} e^{-\sqrt{3}x} x^{\sqrt{5}-\frac{1}{2}} \left(\sqrt{3} c_2 e^{2\sqrt{3}x} + 6c_1\right)$$

1.664 problem 679

Internal problem ID [8154]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 679.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(-3+x)\,y' + (-x+4)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(x^2*diff(y(x),x$2)+x*(x-3)*diff(y(x),x)+(4-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}c_1x^2 + c_2x^2\mathrm{e}^{-x}\,\mathrm{expIntegral}_1\,(-x)$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]+x*(x-3)*y'[x]+(4-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}x^2(c_2\,\mathrm{ExpIntegralEi}(x) + c_1)$$

1.665 problem 680

Internal problem ID [8155]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 680.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x^2y' - (x+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(x^2*diff(y(x),x$2)+x^2*diff(y(x),x)-(2+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2 - 2x + 2)}{x} + \frac{c_2 \mathrm{e}^{-x}}{x}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 31

```
DSolve[x^2*y''[x]+x^2*y'[x]-(2+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}(c_2 e^x (x^2 - 2x + 2) + c_1)}{x}$$

1.666 problem 681

Internal problem ID [8156]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 681.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + 2x^2y' + \left(x - \frac{3}{4}\right)y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve(x^2*diff(y(x),x$2)+2*x^2*diff(y(x),x)+(x-3/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{x}} + \frac{c_2 \mathrm{e}^{-2x}(2x+1)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]+2*x^2*y'[x]+(x-3/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4c_1 - c_2 e^{-2x}(2x+1)}{4\sqrt{x}}$$

1.667 problem 682

Internal problem ID [8157]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 682.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1+x)\,y''+x^2y'-2y=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 32

```
dsolve(x^2*(1+x)*diff(y(x),x$2)+x^2*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(x+2)}{x}+\frac{c_2(\ln(x+1)\,x+2\ln(x+1)+4)}{x}$$

✓ Solution by Mathematica
Time used: 0.053 (sec). Leaf size: 30

```
DSolve[x^2*(1+x)*y''[x]+x^2*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_1(x+2)+c_2(x+2)\log(x+1)+4c_2}{x}$$

1.668 problem 683

Internal problem ID [8158]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 683.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x(x^2+6)\,y' + 6y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(x^2*diff(y(x),x$2)+x*(6+x^2)*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2+3)}{x^2} + \frac{c_2(x^2+3)\left(\int \frac{e^{-\frac{x^2}{2}}}{x^2(x^2+3)^2}dx\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.159 (sec). Leaf size: 65

```
DSolve[x^2*y''[x]+x*(6+x^2)*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{2\pi}c_2x(x^2+3)\,\mathrm{erf}\left(\frac{x}{\sqrt{2}}\right) - 12c_1x(x^2+3) + 2c_2e^{-\frac{x^2}{2}}(x^2+2)}{12x^3}$$

1.669 problem 684

Internal problem ID [8159]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 684.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(1-x)\,y' - y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)+x*(1-x)*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x+1)}{x} + \frac{c_2\mathrm{e}^x}{x}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 23

```
DSolve[x^2*y''[x]+x*(1-x)*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2e^x - c_1(x+1)}{x}$$

1.670 problem 685

Internal problem ID [8160]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 685.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(x+3)\,y' + 4y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 38

```
dsolve(x^2*diff(y(x),x$2)-x*(x+3)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^x x^2 (x+1) + c_2 x^2 \left(-\operatorname{expIntegral}_1(x)\, x - \operatorname{expIntegral}_1(x) + \mathrm{e}^{-x}\right) \mathrm{e}^x$$

✓ Solution by Mathematica
Time used: 0.068 (sec). Leaf size: 34

```
DSolve[x^2*y''[x]-x*(x+3)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 (c_2 e^x (x+1) \operatorname{ExpIntegralEi}(-x) + c_1 e^x (x+1) + c_2)$$

1.671 problem 686

Internal problem ID [8161]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 686.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x^2y' - 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)-x^2*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x+2)}{x} + \frac{c_2 e^x (x-2)}{x}$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 72

```
DSolve[x^2*y''[x]-x^2*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2e^{x/2}\left((c_1x + 2ic_2)\cosh\left(\frac{x}{2}\right) - (ic_2x + 2c_1)\sinh\left(\frac{x}{2}\right)\right)}{\sqrt{\pi}\sqrt{-ix}\sqrt{x}}$$

1.672 problem 687

Internal problem ID [8162]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 687.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x^2y' - (3x+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 70

```
dsolve(x^2*diff(y(x),x$2)-x^2*diff(y(x),x)-(3*x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^2\mathrm{e}^x(x+4) - \frac{c_2(-\operatorname{expIntegral}_1(x)\,x^4 + \mathrm{e}^{-x}x^3 - 4\operatorname{expIntegral}_1(x)\,x^3 + 3x^2\mathrm{e}^{-x} - 2\,\mathrm{e}^{-x}x + 2\,\mathrm{e}^{-x})\,\mathrm{e}^x}{24x}$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 59

```
DSolve[x^2*y''[x]-x^2*y'[x]-(3*x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{24}c_2e^x(x+4)x^2\,\mathrm{ExpIntegralEi}(-x) + c_1e^x(x+4)x^2 - \frac{c_2(x^3+3x^2-2x+2)}{24x}$$

1.673 problem 688

Internal problem ID [8163]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 688.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(5-x)\,y' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 58

```
dsolve(x^2*diff(y(x),x$2)+x*(5-x)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2-4x+2)}{x^2} + \frac{c_2\left(\frac{x^2 \operatorname{expIntegral}_1(-x)}{4} + \frac{x\,\mathrm{e}^x}{4} - \operatorname{expIntegral}_1(-x)\,x - \frac{3\,\mathrm{e}^x}{4} + \frac{\operatorname{expIntegral}_1(-x)}{2}\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.104 (sec). Leaf size: 48

```
DSolve[x^2*y''[x]+x*(5-x)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2(x^2-4x+2)\,\text{ExpIntegralEi}(x) + 4c_1(x^2-4x+2) - c_2e^x(x-3)}{4x^2}$$

1.674 problem 689

Internal problem ID [8164]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 689.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + 4x(1-x)\,y' + (2x-9)\,y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(4*x^2*diff(y(x),x$2)+4*x*(1-x)*diff(y(x),x)+(2*x-9)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2+2x+2)}{x^{\frac{3}{2}}} + \frac{c_2\mathrm{e}^x}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 30

```
DSolve[4*x^2*y''[x]+4*x*(1-x)*y'[x]+(2*x-9)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1e^x - c_2(x^2+2x+2)}{x^{3/2}}$$

1.675 problem 690

Internal problem ID [8165]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 690.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 2x(x+2)\,y' + 2(1+x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(x^2*diff(y(x),x$2)+2*x*(2+x)*diff(y(x),x)+2*(1+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + \frac{c_2(2\ \mathrm{expIntegral}_1\,(2x)\,x - \mathrm{e}^{-2x})}{x^2}$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 32

```
DSolve[x^2*y''[x]+2*x*(2+x)*y'[x]+2*(1+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-2c_2x\,\mathrm{ExpIntegralEi}(-2x) + c_1x - c_2e^{-2x}}{x^2}$$

1.676 problem 691

Internal problem ID [8166]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 691.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(1-x)\,y' + (1-x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x^2*diff(y(x),x$2)-x*(1-x)*diff(y(x),x)+(1-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x + \mathrm{expIntegral}_1\,(x)\,xc_2$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 17

```
DSolve[x^2*y''[x]-x*(1-x)*y'[x]+(1-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x(c_2\,\mathrm{ExpIntegralEi}(-x) + c_1)$$

1.677 problem 692

Internal problem ID [8167]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 692.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 4x(2x+1)\,y' + (4x-1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(4*x^2*diff(y(x),x$2)+4*x*(1+2*x)*diff(y(x),x)+(4*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{x}} + \frac{c_2 \mathrm{e}^{-2x}}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 26

```
DSolve[4*x^2*y''[x]+4*x*(1+2*x)*y'[x]+(4*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{2c_1 e^{-2x} + c_2}{2\sqrt{x}}$$

1.678 problem 693

Internal problem ID [8168]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 693.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' + x(4+x)\, y' + (x+2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(x^2*diff(y(x),x$2)+x*(4+x)*diff(y(x),x)+(2+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + \frac{c_2(-\operatorname{expIntegral}_1(x)\, x + \mathrm{e}^{-x})}{x^2}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 32

```
DSolve[x^2*y''[x]+x*(4+x)*y'[x]+(2+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{-c_2 x \operatorname{ExpIntegralEi}(-x) + c_1 x - c_2 e^{-x}}{x^2}$$

1.679 problem 694

Internal problem ID [8169]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 694.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' + \left(x^2 - \frac{9}{4}\right)y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-9/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{ix}(x+i)}{x^{\frac{3}{2}}} + \frac{c_2 e^{-ix}(x-i)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.078 (sec). Leaf size: 44

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-9/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}}((c_1x + c_2)\cos(x) + (c_2x - c_1)\sin(x))}{x^{3/2}}$$

1.680 problem 695

Internal problem ID [8170]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 695.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + 2y' + yx = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{x} + \frac{c_2 \cos(x)}{x}$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 37

```
DSolve[x*y''[x]+2*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x}$$

1.681 problem 696

Internal problem ID [8171]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 696.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + 5(1-2x)\,y' - 5y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 42

```
dsolve(2*x*diff(y(x),x$2)+5*(1-2*x)*diff(y(x),x)-5*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(10x+1)}{x^{\frac{3}{2}}} + \frac{c_2(10x+1)\left(\int \frac{\sqrt{x}\,e^{5x}}{(10x+1)^2}dx\right)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 40

```
DSolve[2*x*y''[x]+5*(1-2*x)*y'[x]-5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 L_{-\frac{1}{2}}^{\frac{3}{2}}(5x) + \frac{c_1(10x+1)}{10\sqrt{5}x^{3/2}}$$

1.682 problem 697

Internal problem ID [8172]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 697.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right) y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2e^{2ix})}{2\sqrt{x}}$$

1.683 problem 698

Internal problem ID [8173]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 698.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' + (x+n)\,y' + (n+1)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(x*diff(y(x),x$2)+(x+n)*diff(y(x),x)+(n+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(-x+n) + c_2 \mathrm{e}^{-x}(-x+n)\left(\int \frac{\mathrm{e}^{x} x^{-n}}{(-x+n)^2} dx\right)$$

✓ Solution by Mathematica

Time used: 0.524 (sec). Leaf size: 48

```
DSolve[x*y''[x]+(x+n)*y'[x]+(n+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(n-x)\left(c_2 \int_1^x \frac{e^{K[1]} K[1]^{-n}}{(n-K[1])^2} dK[1] + c_1\right)$$

1.684 problem 699

Internal problem ID [8174]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 699.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^4y'' + xy' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 47

```
dsolve(x^4*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2-1)}{x} + \frac{c_2(x^2-1)\left(\int \frac{x^2 e^{\frac{1}{2x^2}}}{(x+1)^2(x-1)^2}dx\right)}{x}$$

✓ Solution by Mathematica
Time used: 0.142 (sec). Leaf size: 61

```
DSolve[x^4*y''[x]+x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{2\pi}c_2(x^2-1)\,\text{erfi}\left(\frac{1}{\sqrt{2}x}\right) - 4c_1(x^2-1) + 2c_2e^{\frac{1}{2x^2}}x}{4x}$$

1.685 problem 700

Internal problem ID [8175]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 700.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + (2x^2 + x)\, y' - 4y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 35

```
dsolve(x^2*diff(y(x),x$2)+(x+2*x^2)*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(2x^2 - 4x + 3)}{x^2} + \frac{c_2 \mathrm{e}^{-2x}(2x + 3)}{2x^2}$$

✓ Solution by Mathematica
Time used: 0.07 (sec). Leaf size: 44

```
DSolve[x^2*y''[x]+(x+2*x^2)*y'[x]-4*y[x]==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(\frac{2c_1 e^{-2x}(2x + 3)}{x^2} + \frac{c_2(2x^2 - 4x + 3)}{x^2} - 2\right)$$

1.686 problem 701

Internal problem ID [8176]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 701.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(4x^3 - 14x^2 - 2x)\, y'' - (6x^2 - 7x + 1)\, y' + (6x - 1)\, y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve((4*x^3-14*x^2-2*x)*diff(y(x),x$2)-(6*x^2-7*x+1)*diff(y(x),x)+(6*x-1)*y(x)=0,y(x), sin
```

$$y(x) = c_1(x-1) + c_2\sqrt{x}\,(2x+1)$$

✓ Solution by Mathematica

Time used: 6.075 (sec). Leaf size: 26

```
DSolve[(4*x^3-14*x^2-2*x)*y''[x]-(6*x^2-7*x+1)*y'[x]+(6*x-1)*y[x]==0,y[x],x,IncludeSingularS
```

$$y(x) \to c_1(x-1) - 2c_2\sqrt{x}(2x+1)$$

1.687 problem 702

Internal problem ID [8177]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 702.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x^2y' + (x - 2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x^2*diff(y(x),x$2)+x^2*diff(y(x),x)+(x-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + \frac{c_2 \mathrm{e}^{-x}(x^2 + 2x + 2)}{x}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 29

```
DSolve[x^2*y''[x]+x^2*y'[x]+(x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 - c_2 e^{-x}(x^2 + 2x + 2)}{x}$$

1.688 problem 703

Internal problem ID [8178]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 703.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x^2y' + (x-2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(x^2*diff(y(x),x$2)-x^2*diff(y(x),x)+(x-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2+2x+2)}{x} + \frac{c_2 \mathrm{e}^x}{x}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 28

```
DSolve[x^2*y''[x]-x^2*y'[x]+(x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 e^x - c_2(x^2+2x+2)}{x}$$

1.689 problem 704

Internal problem ID [8179]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 704.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-4x)\,y''+\left(-\frac{1}{4}x-x^2\right)y'-\frac{5yx}{16}=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 69

```
dsolve(x^2*(1-4*x)*diff(y(x),x$2)+((1-(5/4))*x-(6-4*(5/4))*x^2)*diff(y(x),x)+(5/4)*(1-(5/4))
```

$$y(x)=c_1\left(\frac{x\left(1+i\sqrt{-1+4x}\right)}{i\sqrt{-1+4x}-1}\right)^{\frac{5}{8}}+c_2\left(\frac{x\left(i\sqrt{-1+4x}-1\right)}{1+i\sqrt{-1+4x}}\right)^{\frac{5}{8}}$$

✓ Solution by Mathematica

Time used: 0.355 (sec). Leaf size: 111

```
DSolve[x^2*(1-4*x)*y''[x]+((1-(5/4))*x-(6-4*(5/4))*x^2)*y'[x]+(5/4)*(1-(5/4))*x*y[x]==0,y[x]
```

$$y(x)\to\frac{\sqrt[8]{x}\sqrt[4]{4x-1}\left(5c_1\left(\sqrt{4x-1}-i\right)^{5/4}+ic_2\left(\sqrt{4x-1}+i\right)^{5/4}\right)}{5\sqrt[4]{1-4x}\sqrt[8]{\sqrt{4x-1}-i}\sqrt[8]{\sqrt{4x-1}+i}}$$

1.690 problem 705

Internal problem ID [8180]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 705.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + (x^2 + x)\,y' + (-9 + x)\,y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 40

```
dsolve(x^2*diff(y(x),x$2)+(x+x^2)*diff(y(x),x)+(x-9)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2 - 8x + 20)}{x^3} + \frac{c_2 e^{-x}(x^3 + 9x^2 + 36x + 60)}{x^3}$$

✓ Solution by Mathematica
Time used: 0.067 (sec). Leaf size: 42

```
DSolve[x^2*y''[x]+(x+x^2)*y'[x]+(x-9)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1((x-8)x+20) - c_2 e^{-x}(x^3 + 9x^2 + 36x + 60)}{x^3}$$

1.691 problem 706

Internal problem ID [8181]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 706.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x(1 + x)\, y' + (3x - 1)\, y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 60

```
dsolve(x^2*diff(y(x),x$2)+x*(x+1)*diff(y(x),x)+(3*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x\, \mathrm{e}^{-x}(x - 3) + \frac{c_2(\mathrm{expIntegral}_1(-x)\, x^3 + \mathrm{e}^x x^2 - 3x^2\, \mathrm{expIntegral}_1(-x) - 2x\, \mathrm{e}^x - \mathrm{e}^x)\, \mathrm{e}^{-x}}{6x}$$

✓ Solution by Mathematica

Time used: 0.086 (sec). Leaf size: 66

```
DSolve[x^2*y''[x]+x*(x+1)*y'[x]+(3*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}(c_2(x - 3)x^2\, \mathrm{ExpIntegralEi}(x) + 6c_1x^3 - x^2(c_2e^x + 18c_1) + 2c_2e^x x + c_2e^x)}{6x}$$

1.692 problem 707

Internal problem ID [8182]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 707.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - (x^2 + 4x)\, y' + 4y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(x^2*diff(y(x),x$2)-(x^2+4*x)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^x x^4 - \frac{c_2 x\, \mathrm{e}^x (-\operatorname{expIntegral}_1(x)\, x^3 + x^2 \mathrm{e}^{-x} - \mathrm{e}^{-x} x + 2\, \mathrm{e}^{-x})}{6}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 41

```
DSolve[x^2*y''[x]-(x^2+4*x)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 e^x x^4 - \frac{1}{6} c_1 x \left(e^x x^3 \operatorname{ExpIntegralEi}(-x) + x^2 - x + 2\right)$$

1.693 problem 708

Internal problem ID [8183]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 708.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2y'' - (3x+2)\,y' + \frac{(2x-1)\,y}{x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(2*x^2*diff(y(x),x$2)-(3*x+2)*diff(y(x),x)+(2*x-1)/x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(5x+2)}{\sqrt{x}} + \frac{c_2(5x+2)\left(\int \frac{x^{\frac{5}{2}}\,\mathrm{e}^{-\frac{1}{x}}}{(5x+2)^2}dx\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.159 (sec). Leaf size: 70

```
DSolve[2*x^2*y''[x]-(3*x+2)*y'[x]+(2*x-1)/x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2\sqrt{\pi}c_2(5x+2)\text{erf}\left(\frac{1}{\sqrt{x}}\right)}{3\sqrt{x}} + \frac{2}{3}c_2e^{-1/x}\left(x^2-4x-2\right) + \frac{c_1(5x+2)}{5\sqrt{x}}$$

1.694 problem 709

Internal problem ID [8184]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 709.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$x(1-x)\,y'' + \left(-2x + \frac{3}{2}\right) y' - \frac{y}{4} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(x*(1-x)*diff(y(x),x$2)+(3/2-2*x)*diff(y(x),x)-1/4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{x}} + \frac{c_2 \ln\left(x - \frac{1}{2} + \sqrt{x\,(x-1)}\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 51

```
DSolve[x*(1-x)*y''[x]+(3/2-2*x)*y'[x]-1/4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{\sqrt{x}} - \frac{2c_2\sqrt{x-1}\log\left(\sqrt{x-1} - \sqrt{x}\right)}{\sqrt{-((x-1)x)}}$$

1.695 problem 710

Internal problem ID [8185]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 710.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x(1-x)\,y''+xy'-y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(2*x*(1-x)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+c_2\left(\arctan\left(\sqrt{x-1}\right)x-\sqrt{x-1}\right)$$

✓ Solution by Mathematica

Time used: 0.064 (sec). Leaf size: 43

```
DSolve[2*x*(1-x)*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\sqrt[4]{2}\left(c_2x\operatorname{arctanh}\left(\sqrt{1-x}\right)+c_1x-c_2\sqrt{1-x}\right)$$

1.696 problem 711

Internal problem ID [8186]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 711.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$2x(1-x)\, y'' + (1-11x)\, y' - 10y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(2*x*(1-x)*diff(y(x),x$2)+(1-11*x)*diff(y(x),x)-10*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2+6x+1)}{(x-1)^4} + \frac{c_2\sqrt{x}\,(x+1)}{(x-1)^4}$$

✓ Solution by Mathematica
Time used: 0.061 (sec). Leaf size: 35

```
DSolve[2*x*(1-x)*y''[x]+(1-11*x)*y'[x]-10*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1\sqrt{x}(x+1) - 2c_2(x^2+6x+1)}{(x-1)^4}$$

1.697 problem 712

Internal problem ID [8187]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 712.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$x(1-x)\,y'' + \frac{(1-2x)\,y'}{3} + \frac{20y}{9} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*(1-x)*diff(y(x),x$2)+1/3*(1-2*x)*diff(y(x),x)+20/9*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(6x-5)\,x^{\frac{2}{3}} + c_2(6x-1)\,(x-1)^{\frac{2}{3}}$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 51

```
DSolve[x*(1-x)*y''[x]+1/3*(1-2*x)*y'[x]+20/9*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to c_2\sqrt[3]{-((x-1)x)}Q_1^{\frac{2}{3}}(2x-1) + \frac{c_1x^{2/3}(6x-5)}{3\,\text{Gamma}\left(\frac{4}{3}\right)}$$

1.698 problem 713

Internal problem ID [8188]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 713.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4y'' + \frac{3(-x^2+2)\,y}{(1-x^2)^2} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve(4*diff(y(x),x$2)+3*(2-x^2)/(1-x^2)^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x^2-1)^{\frac{1}{4}}\,x + c_2(x^2-1)^{\frac{3}{4}}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 51

```
DSolve[4*y''[x]+3*(2-x^2)/(1-x^2)^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x^2-1}\left(c_2 Q_{\frac{1}{2}}^{\frac{1}{2}}(x) + \frac{\sqrt{\frac{2}{\pi}}c_1 x}{\sqrt[4]{1-x^2}}\right)$$

1.699 problem 714

Internal problem ID [8189]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 714.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{u'' - \frac{2u'}{x} - a^2 u = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(u(x),x$2)-2/x*diff(u(x),x)-a^2*u(x)=0,u(x), singsol=all)
```

$$u(x) = c_1 \mathrm{e}^{ax}(ax-1) + \frac{c_2 \mathrm{e}^{-ax}(ax+1)}{a}$$

✓ Solution by Mathematica
Time used: 0.136 (sec). Leaf size: 68

```
DSolve[u''[x]-2/x*u'[x]-a^2*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to \frac{\sqrt{\frac{2}{\pi}}\sqrt{x}((iac_2x + c_1)\sinh(ax) - (ac_1x + ic_2)\cosh(ax))}{a\sqrt{-iax}}$$

1.700 problem 715

Internal problem ID [8190]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 715.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$u'' + \frac{2u'}{x} - a^2 u = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(u(x),x$2)+2/x*diff(u(x),x)-a^2*u(x)=0,u(x), singsol=all)
```

$$u(x) = \frac{c_1 \sinh (ax)}{x} + \frac{c_2 \cosh (ax)}{x}$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 35

```
DSolve[u''[x]+2/x*u'[x]-a^2*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to \frac{2ac_1 e^{-ax} + c_2 e^{ax}}{2ax}$$

1.701 problem 716

Internal problem ID [8191]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 716.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{u'' + \frac{2u'}{x} + a^2 u = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(u(x),x$2)+2/x*diff(u(x),x)+a^2*u(x)=0,u(x), singsol=all)
```

$$u(x) = \frac{c_1 \sin(ax)}{x} + \frac{c_2 \cos(ax)}{x}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 42

```
DSolve[u''[x]+2/x*u'[x]+a^2*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to \frac{e^{-iax}\left(2c_1 - \frac{ic_2 e^{2iax}}{a}\right)}{2x}$$

1.702 problem 717

Internal problem ID [8192]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 717.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{u'' + \frac{4u'}{x} - a^2u = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(u(x),x$2)+4/x*diff(u(x),x)-a^2*u(x)=0,u(x), singsol=all)
```

$$u(x) = \frac{c_1 e^{ax}(ax - 1)}{x^3} + \frac{c_2 e^{-ax}(ax + 1)}{x^3 a}$$

✓ Solution by Mathematica

Time used: 0.11 (sec). Leaf size: 68

```
DSolve[u''[x]+4/x*u'[x]-a^2*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to \frac{\sqrt{\frac{2}{\pi}}((iac_2x + c_1)\sinh(ax) - (ac_1x + ic_2)\cosh(ax))}{ax^{5/2}\sqrt{-iax}}$$

1.703 problem 718

Internal problem ID [8193]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 718.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$u'' + \frac{4u'}{x} + a^2 u = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
dsolve(diff(u(x),x$2)+4/x*diff(u(x),x)+a^2*u(x)=0,u(x), singsol=all)
```

$$u(x) = \frac{c_1(\cos(ax)\,ax - \sin(ax))}{x^3} + \frac{c_2(\cos(ax) + \sin(ax)\,ax)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.119 (sec). Leaf size: 57

```
DSolve[u''[x]+4/x*u'[x]+a^2*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to -\frac{\sqrt{\frac{2}{\pi}}((ac_1x + c_2)\cos(ax) + (ac_2x - c_1)\sin(ax))}{x^{3/2}(ax)^{3/2}}$$

1.704 problem 719

Internal problem ID [8194]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 719.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - a^2 y - \frac{6y}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-a^2*y(x)=6*y(x)/x^2,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{-ax}(a^2x^2 + 3ax + 3)}{x^2a^2} + \frac{c_2 e^{ax}(a^2x^2 - 3ax + 3)}{3x^2}$$

✓ Solution by Mathematica

Time used: 0.145 (sec). Leaf size: 90

```
DSolve[y''[x]-a^2*y[x]==6*y[x]/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{\frac{2}{\pi}}((a^2c_2x^2 - 3iac_1x + 3c_2)\cosh(ax) + i(c_1(a^2x^2 + 3) + 3iac_2x)\sinh(ax))}{a^2x^{3/2}\sqrt{-iax}}$$

1.705 problem 720

Internal problem ID [8195]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 720.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + n^2 y - \frac{6y}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 67

```
dsolve(diff(y(x),x$2)+n^2*y(x)=6*y(x)/x^2,y(x), singsol=all)
```

$$y(x) = \frac{c_1(\cos(nx)\, x^2 n^2 - 3\sin(nx)\, nx - 3\cos(nx))}{x^2} + \frac{c_2(\sin(nx)\, x^2 n^2 + 3\cos(nx)\, nx - 3\sin(nx))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.15 (sec). Leaf size: 79

```
DSolve[y''[x]+n^2*y[x]==6*y[x]/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}}\sqrt{x}((c_2(-n^2)\, x^2 + 3c_1 nx + 3c_2)\cos(nx) + (c_1(n^2x^2 - 3) + 3c_2 nx)\sin(nx))}{(nx)^{5/2}}$$

1.706 problem 721

Internal problem ID [8196]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 721.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' - \left(x^2 + \frac{1}{4}\right)y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-(x^2+1/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sinh(x)}{\sqrt{x}} + \frac{c_2 \cosh(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 32

```
DSolve[x^2*y''[x]+x*y'[x]-(x^2+1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}(c_2 e^{2x} + 2c_1)}{2\sqrt{x}}$$

1.707 problem 722

Internal problem ID [8197]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 722.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' + \frac{(-9a^2 + 4x^2)\, y}{4a^2} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 43

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(4*x^2-9*a^2)/(4*a^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{\frac{ix}{a}} (-ix + a)}{x^{\frac{3}{2}}} + \frac{c_2 e^{-\frac{ix}{a}} (ix + a)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica
Time used: 0.121 (sec). Leaf size: 62

```
DSolve[x^2*y''[x]+x*y'[x]+(4*x^2-9*a^2)/(4*a^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}} \left((ac_2 + c_1 x) \cos\left(\frac{x}{a}\right) + (c_2 x - ac_1) \sin\left(\frac{x}{a}\right) \right)}{x\sqrt{\frac{x}{a}}}$$

1.708 problem 723

Internal problem ID [8198]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 723.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' + \left(x^2 - \frac{25}{4}\right)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-25/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{ix}(x^2 + 3ix - 3)}{x^{\frac{5}{2}}} + \frac{c_2 e^{-ix}(x^2 - 3ix - 3)}{x^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.116 (sec). Leaf size: 59

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-25/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}}((-c_2x^2 + 3c_1x + 3c_2)\cos(x) + (c_1(x^2 - 3) + 3c_2x)\sin(x))}{x^{5/2}}$$

1.709 problem 724

Internal problem ID [8199]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 724.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + qy' - \frac{2y}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)+q*diff(y(x),x)=2*y(x)/x^2,y(x), singsol=all)
```

$$y(x) = \frac{c_1(qx-2)}{x} + \frac{c_2 e^{-qx}(qx+2)}{qx}$$

✓ Solution by Mathematica

Time used: 0.066 (sec). Leaf size: 80

```
DSolve[y''[x]+q*y'[x]==2*y[x]/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{qx^{3/2}e^{-\frac{qx}{2}}\left(2(ic_2qx+2c_1)\sinh\left(\frac{qx}{2}\right)-2(c_1qx+2ic_2)\cosh\left(\frac{qx}{2}\right)\right)}{\sqrt{\pi}(-iqx)^{5/2}}$$

1.710 problem 725

Internal problem ID [8200]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 725.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + 3y' + 4yx^3 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x$2)+3*diff(y(x),x)+4*x^3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x^2)}{x^2} + \frac{c_2 \cos(x^2)}{x^2}$$

✓ Solution by Mathematica
Time used: 0.056 (sec). Leaf size: 41

```
DSolve[x*y''[x]+3*y'[x]+4*x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4c_1 e^{-ix^2} - ic_2 e^{ix^2}}{4x^2}$$

1.711 problem 726

Internal problem ID [8201]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 726.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(-x+2)\, y'' + 2xy' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve(x^2*(2-x)*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x-1)}{x} + xc_2$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 24

```
DSolve[x^2*(2-x)*y''[x]+2*x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1(x-2)^2 + c_2(x-1)}{x}$$

1.712 problem 727

Internal problem ID [8202]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 727.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\, y''-2xy'+2y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2+1)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+c_2(x^2-1)$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 21

```
DSolve[(x^2+1)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_2x-c_1(x-i)^2$$

1.713 problem 728

Internal problem ID [8203]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 728.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - 2(1+x)\,y' + (x+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x$2)-2*(x+1)*diff(y(x),x)+(x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2 e^x x^3$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 23

```
DSolve[x*y''[x]-2*(x+1)*y'[x]+(x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} e^x \left(c_2 x^3 + 3c_1\right)$$

1.714 problem 729

Internal problem ID [8204]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 729.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3xy'' - 2(3x - 1)\, y' + (3x - 2)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(3*x*diff(y(x),x$2)-2*(3*x-1)*diff(y(x),x)+(3*x-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x} + c_2 x^{\frac{1}{3}} \mathrm{e}^{x}$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 21

```
DSolve[3*x*y''[x]-2*(3*x-1)*y'[x]+(3*x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x} \left(3c_2 \sqrt[3]{x} + c_1\right)$$

1.715 problem 730

Internal problem ID [8205]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 730.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(1+x)\,y'' - (x-1)\,y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x*(x+1)*diff(y(x),x$2)-(x-1)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x-1) + c_2(x\ln(x) - \ln(x) - 4)$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 23

```
DSolve[x*(x+1)*y''[x]-(x-1)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x-1) + c_2((x-1)\log(x) - 4)$$

1.716 problem 731

Internal problem ID [8206]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 731.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(x^2+2x\right) y''-2(1+x)\, y'+2y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2+2*x)*diff(y(x),x$2)-2*(x+1)*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(x+1)+c_2x^2$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 19

```
DSolve[(x^2+2*x)*y''[x]-2*(x+1)*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1x^2-c_2(x+1)$$

1.717 problem 732

Internal problem ID [8207]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 732.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 + 2x)\, y'' - 2(1 + x)\, y' + 2y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2+2*x)*diff(y(x),x$2)-2*(x+1)*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x + 1) + c_2x^2$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 19

```
DSolve[(x^2+2*x)*y''[x]-2*(x+1)*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1x^2 - c_2(x + 1)$$

1.718 problem 733

Internal problem ID [8208]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 733.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''-2xy'+2y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2+1)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+c_2(x^2-1)$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 21

```
DSolve[(x^2+1)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_2x-c_1(x-i)^2$$

1.719 problem 734

Internal problem ID [8209]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 734.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''-2xy'+2y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2+1)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+c_2(x^2-1)$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 21

```
DSolve[(x^2+1)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_2x-c_1(x-i)^2$$

1.720 problem 735

Internal problem ID [8210]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 735.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 4xy' + (4x^2 - 2)\, y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x^2} + c_2 x\, \mathrm{e}^{x^2}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 18

```
DSolve[y''[x]-4*x*y'[x]+(4*x^2-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x^2}(c_2 x + c_1)$$

1.721 problem 736

Internal problem ID [8211]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 736.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4xy' + (4x^2 - 2)\, y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{x^2} + c_2 x\, e^{x^2}$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 18

```
DSolve[y''[x]-4*x*y'[x]+(4*x^2-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x^2}(c_2 x + c_1)$$

1.722 problem 737

Internal problem ID [8212]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 737.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(2x-3)\,y''-xy'+y=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve((2*x-3)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+c_2x\left(\int \frac{(-3+2x)^{\frac{3}{4}}\,\mathrm{e}^{\frac{x}{2}}}{x^2}dx\right)$$

✓ Solution by Mathematica
Time used: 0.087 (sec). Leaf size: 63

```
DSolve[(2*x-3)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to 2\ 2^{3/4}(2x-3)\left(c_2(2x-3)^{3/4}L_{-\frac{3}{4}}^{\frac{7}{4}}\left(\frac{x}{2}-\frac{3}{4}\right)+\frac{4\sqrt{2}c_1x}{2x-3}\right)$$

1.723 problem 738

Internal problem ID [8213]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 738.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Hermite]`

$$\boxed{y'' - xy' - 3y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 47

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{x^2}{2}} \left(x^2+1\right) + c_2 \mathrm{e}^{\frac{x^2}{2}} \left(x^2+1\right) \left(\int \frac{\mathrm{e}^{-\frac{x^2}{2}}}{\left(x^2+1\right)^2} dx\right)$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 35

```
DSolve[y''[x]-x*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{HermiteH}\left(-3, \frac{x}{\sqrt{2}}\right) + c_2 e^{\frac{x^2}{2}} \left(x^2+1\right)$$

1.724 problem 739

Internal problem ID [8214]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 739.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''-xy'+y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve((1+x^2)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+c_2\left(\operatorname{arcsinh}(x)\,x-\sqrt{x^2+1}\right)$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 42

```
DSolve[(1+x^2)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -c_2\sqrt{x^2+1}-c_2x\log\left(\sqrt{x^2+1}-x\right)+c_1x$$

1.725 problem 740

Internal problem ID [8215]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 740.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Hermite]`

$$\boxed{y'' - xy' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x^2 - 1) + c_2(x^2 - 1)\left(\int \frac{e^{\frac{x^2}{2}}}{(x-1)^2 (x+1)^2} dx\right)$$

✓ Solution by Mathematica
Time used: 0.11 (sec). Leaf size: 54

```
DSolve[y''[x]-x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}c_2\left(\sqrt{2\pi}(x^2 - 1)\,\mathrm{erfi}\left(\frac{x}{\sqrt{2}}\right) - 2e^{\frac{x^2}{2}}x\right) + c_1(x^2 - 1)$$

1.726 problem 741

Internal problem ID [8216]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 741.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1-x^2)\, y'' - y' + y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 177

```
dsolve((1-x^2)*diff(y(x),x$2)-diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{2x+3}\left(\frac{3\sqrt{5}\,x+2\sqrt{5}-5\sqrt{x^2-1}}{3\sqrt{5}\,x+2\sqrt{5}+5\sqrt{x^2-1}}\right)^{\frac{1}{4}}\left(x+\sqrt{x^2-1}\right)^{\frac{3\sqrt{5}}{10}}\left(x+\sqrt{x^2-1}\right)^{\frac{\sqrt{5}}{5}} + c_2\sqrt{2x+3}\left(\frac{3\sqrt{5}\,x+2\sqrt{5}+5\sqrt{x^2-1}}{3\sqrt{5}\,x+2\sqrt{5}-5\sqrt{x^2-1}}\right)^{\frac{1}{4}}\left(x+\sqrt{x^2-1}\right)^{-\frac{3\sqrt{5}}{10}}\left(x+\sqrt{x^2-1}\right)^{-\frac{\sqrt{5}}{5}}$$

✓ Solution by Mathematica

Time used: 2.8 (sec). Leaf size: 198

```
DSolve[(1-x^2)*y''[x]-y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[4]{x+1}\left(\sqrt{x+1}-\sqrt{x-1}\right)^{-1-\sqrt{5}}\left(-2x+2\sqrt{x-1}\sqrt{x+1}+\sqrt{5}-3\right)e^{-\text{arctanh}\left(x-\sqrt{x-1}\sqrt{x+1}\right)}\left(c_2\int_1^x \frac{e^{2\text{arct}}}{}\right.}{\sqrt[4]{1-x}}$$

1.727 problem 742

Internal problem ID [8217]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 742.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(1+x)^2 y'' + (1-x^2) y' + (x-1) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*(x+1)^2*diff(y(x),x$2)+(1-x^2)*diff(y(x),x)+(x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x+1) + c_2(x+1)\ln(x)$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 17

```
DSolve[x*(x+1)^2*y''[x]+(1-x^2)*y'[x]+(x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x+1)(c_2 \log(x) + c_1)$$

1.728 problem 743

Internal problem ID [8218]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 743.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2xy'' - y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 75

```
dsolve(2*x*diff(y(x),x$2)-diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{2i\sqrt{x}} \sqrt{\frac{(1+4x)\left(2i\sqrt{x}-1\right)}{1+2i\sqrt{x}}} + c_2 e^{-2i\sqrt{x}} \sqrt{\frac{(1+4x)\left(1+2i\sqrt{x}\right)}{2i\sqrt{x}-1}}$$

✓ Solution by Mathematica

Time used: 0.09 (sec). Leaf size: 59

```
DSolve[2*x*y''[x]-y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{2i\sqrt{x}} \left(2\sqrt{x}+i\right) + \frac{1}{8} c_2 e^{-2i\sqrt{x}} \left(1+2i\sqrt{x}\right)$$

1.729 problem 744

Internal problem ID [8219]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 744.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + xy' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(x*diff(y(x),x$2)+x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x^2 + 2x) + c_2\left(\frac{x^2 \operatorname{expIntegral}_1(x)}{2} - \frac{e^{-x}x}{2} + \operatorname{expIntegral}_1(x)\, x - \frac{e^{-x}}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 39

```
DSolve[x*y''[x]+x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x(x+2) - \frac{1}{2} c_2 e^{-x}(e^x(x+2)x \operatorname{ExpIntegralEi}(-x) + x + 1)$$

1.730 problem 745

Internal problem ID [8220]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 745.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(x-1)^2 y'' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(x*(x-1)^2*diff(y(x),x$2)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x}{x-1} + \frac{c_2(2x\ln(x) - x^2 + 1)}{x-1}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 33

```
DSolve[x*(x-1)^2*y''[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-c_2x^2 - c_1x + 2c_2x\log(x) + c_2}{x-1}$$

1.731 problem 746

Internal problem ID [8221]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 746.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2xy' + x^2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{x^2}{2}} \cos(x) + c_2 e^{\frac{x^2}{2}} \sin(x)$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 39

```
DSolve[y''[x]-2*x*y'[x]+x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{\frac{1}{2}x(x-2i)} \left(2c_1 - ic_2 e^{2ix}\right)$$

1.732 problem 747

Internal problem ID [8222]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 747.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(-x^2+2)\,y''-(x^2+4x+2)\,((1-x)\,y'+y)=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*(2-x^2)*diff(y(x),x$2)-(x^2+4*x+2)*((1-x)*diff(y(x),x)+y(x))=0,y(x), singsol=all)
```

$$y(x)=c_1(x-1)+c_2\mathrm{e}^x x^2$$

✓ Solution by Mathematica

Time used: 0.072 (sec). Leaf size: 21

```
DSolve[x*(2-x^2)*y''[x]-(x^2+4*x+2)*((1-x)*y'[x]+y[x])==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to c_1e^x x^2+c_2(x-1)$$

1.733 problem 748

Internal problem ID [8223]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 748.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1+x)\,y'' - (2x+1)\,(xy' - y) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*(1+x)*diff(y(x),x$2)-(1+2*x)*(x*diff(y(x),x)-y(x))=0,y(x), singsol=all)
```

$$y(x) = c_1 x + c_2 x(x + \ln(x))$$

✓ Solution by Mathematica

Time used: 0.188 (sec). Leaf size: 132

```
DSolve[x^2*(1+x)*y''[x]-(1+2*x)*(x*y'[x]+y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 x^{1+\sqrt{2}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}+\sqrt{2}-\frac{\sqrt{17}}{2}, -\frac{1}{2}+\sqrt{2}+\frac{\sqrt{17}}{2}, 1+2\sqrt{2}, -x\right) + c_1 x^{1-\sqrt{2}}\,\text{Hypergeometric2F1}\left(\frac{1}{2}\left(-1-2\sqrt{2}-\sqrt{17}\right), \frac{1}{2}\left(-1-2\sqrt{2}+\sqrt{17}\right), 1 - 2\sqrt{2}, -x\right)$$

1.734 problem 749

Internal problem ID [8224]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 749.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2(-x+2)\, x^2 y'' - (-x+4)\, x y' + (3-x)\, y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(2*(2-x)*x^2*diff(y(x),x$2)-(4-x)*x*diff(y(x),x)+(3-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x} + c_2\sqrt{x^2-2x}$$

✓ Solution by Mathematica

Time used: 0.061 (sec). Leaf size: 41

```
DSolve[2*(2-x)*x^2*y''[x]-(4-x)*x*y'[x]+(3-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{\sqrt[4]{x-2}\sqrt{x}\left(2c_2\sqrt{x-2}+c_1\right)}{\sqrt[4]{2-x}}$$

1.735 problem 750

Internal problem ID [8225]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 750.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1-x)\,x^2y''+(5x-4)\,xy'+(6-9x)\,y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve((1-x)*x^2*diff(y(x),x$2)+(5*x-4)*x*diff(y(x),x)+(6-9*x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x^3+c_2x^2(x\ln(x)+1)$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 24

```
DSolve[(1-x)*x^2*y''[x]+(5*x-4)*x*y'[x]+(6-9*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\to x^2(c_1x-c_2(x\log(x)+1))$$

1.736 problem 751

Internal problem ID [8226]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 751.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x y'' + \left(4x^2+1\right) y' + 4x\left(x^2+1\right) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x$2)+(4*x^2+1)*diff(y(x),x)+4*x*(x^2+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^2} + c_2 \mathrm{e}^{-x^2} \ln(x)$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 21

```
DSolve[x*y''[x]+(4*x^2+1)*y'[x]+4*x*(x^2+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x^2}(c_2 \log(x) + c_1)$$

1.737 problem 752

Internal problem ID [8227]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 752.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2xy' + 8y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\frac{4}{3}x^4 - 4x^2 + 1\right) + c_2\left(\frac{4}{3}x^4 - 4x^2 + 1\right)\left(\int \frac{\mathrm{e}^{x^2}}{(4x^4 - 12x^2 + 3)^2}dx\right)$$

✓ Solution by Mathematica
Time used: 0.103 (sec). Leaf size: 63

```
DSolve[y''[x]-2*x*y'[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\left(x^4 - 3x^2 + \frac{3}{4}\right) - \frac{1}{12}c_2\left(\sqrt{\pi}(-4x^4 + 12x^2 - 3)\,\mathrm{erfi}(x) + 2e^{x^2}x(2x^2 - 5)\right)$$

1.738 problem 753

Internal problem ID [8228]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 753.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2xy' + 8y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\frac{4}{3}x^4 - 4x^2 + 1\right) + c_2\left(\frac{4}{3}x^4 - 4x^2 + 1\right)\left(\int \frac{\mathrm{e}^{x^2}}{\left(4x^4 - 12x^2 + 3\right)^2}dx\right)$$

✓ Solution by Mathematica

Time used: 0.075 (sec). Leaf size: 63

```
DSolve[y''[x]-2*x*y'[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\left(x^4 - 3x^2 + \frac{3}{4}\right) - \frac{1}{12}c_2\left(\sqrt{\pi}\left(-4x^4 + 12x^2 - 3\right)\operatorname{erfi}(x) + 2e^{x^2}x\left(2x^2 - 5\right)\right)$$

1.739 problem 754

Internal problem ID [8229]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 754.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$(1-x^2)\,y''-2xy'+12y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(-\frac{5}{3}x^3+x\right)+c_2\left(-\frac{5\ln(x+1)\,x^3}{24}+\frac{5\ln(x-1)\,x^3}{24}+\frac{\ln(x+1)\,x}{8}-\frac{\ln(x-1)\,x}{8}+\frac{5x^2}{12}-\frac{1}{9}\right)$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 59

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}c_1x(5x^2-3)+c_2\left(-\frac{5x^2}{2}-\frac{1}{4}(5x^2-3)\,x(\log(1-x)-\log(x+1))+\frac{2}{3}\right)$$

1.740 problem 755

Internal problem ID [8230]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 755.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(x+2)\,y''+2(1+x)\,y'-2y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 36

```
dsolve(x*(x+2)*diff(y(x),x$2)+2*(x+1)*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(x+1)+c_2\left(\frac{x\ln(x)}{2}-\frac{\ln(x+2)\,x}{2}+\frac{\ln(x)}{2}-\frac{\ln(x+2)}{2}+1\right)$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 37

```
DSolve[x*(x+2)*y''[x]+2*(x+1)*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1(x+1)-\frac{1}{2}c_2((x+1)\log(-x)-(x+1)\log(x+2)+2)$$

1.741 problem 757

Internal problem ID [8231]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 757.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{x(x+2)\,y''+(1+x)\,y'-4y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x*(x+2)*diff(y(x),x$2)+(x+1)*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(2x^2+4x+1\right)+c_2(x+1)\sqrt{x\,(x+2)}$$

✓ Solution by Mathematica
Time used: 0.06 (sec). Leaf size: 53

```
DSolve[x*(x+2)*y''[x]+(x+1)*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\cosh\left(4\log\left(\sqrt{x+2}-\sqrt{x}\right)\right)-ic_2\sinh\left(4\log\left(\sqrt{x+2}-\sqrt{x}\right)\right)$$

1.742 problem 758

Internal problem ID [8232]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 758.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-1)\,y''-xy'+y=0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 12

```
dsolve((x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+\mathrm{e}^{x}c_2$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 17

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1e^x-c_2x$$

1.743 problem 759

Internal problem ID [8233]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 759.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\, y''-2xy'+2y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve((1+x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+c_2(x^2-1)$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 21

```
DSolve[(1+x^2)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_2x-c_1(x-i)^2$$

1.744 problem 760

Internal problem ID [8234]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 760.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2 - 2x + 10) y'' + xy' - 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve((x^2-2*x+10)*diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \left(x^2 - \frac{4}{3}x + 5\right) + c_2(3x - 4)\sqrt{x^2 - 2x + 10}\left(\frac{-x + 1 + 3i}{x - 1 + 3i}\right)^{\frac{i}{6}}$$

✓ Solution by Mathematica

Time used: 0.672 (sec). Leaf size: 92

```
DSolve[(x^2-2*x+10)*y''[x]+x*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}(3x - 4)\sqrt{x^2 - 2x + 10}e^{-\frac{1}{3}\arctan\left(\frac{x-1}{3}\right)}\left(c_2 \int_1^x \frac{9e^{\frac{1}{3}\arctan\left(\frac{1}{3}(K[1]-1)\right)}}{(4 - 3K[1])^2 (K[1]^2 - 2K[1] + 10)^{3/2}} dK[1] + c_1\right)$$

1.745 problem 761

Internal problem ID [8235]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 761.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2 - 2x + 10)\, y'' + xy' - 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve((x^2-2*x+10)*diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(x^2 - \frac{4}{3}x + 5\right) + c_2(3x-4)\sqrt{x^2-2x+10}\left(\frac{-x+1+3i}{x-1+3i}\right)^{\frac{i}{6}}$$

✓ Solution by Mathematica

Time used: 0.579 (sec). Leaf size: 92

```
DSolve[(x^2-2*x+10)*y''[x]+x*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}(3x - 4)\sqrt{x^2-2x+10}e^{-\frac{1}{3}\arctan\left(\frac{x-1}{3}\right)}\left(c_2\int_1^x \frac{9e^{\frac{1}{3}\arctan\left(\frac{1}{3}(K[1]-1)\right)}}{(4-3K[1])^2\left(K[1]^2-2K[1]+10\right)^{3/2}}dK[1] + c_1\right)$$

1.746 problem 762

Internal problem ID [8236]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 762.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Hermite]`

$$\boxed{y'' - xy' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x^2 - 1) + c_2(x^2 - 1)\left(\int \frac{e^{\frac{x^2}{2}}}{(x-1)^2(x+1)^2}dx\right)$$

✓ Solution by Mathematica

Time used: 0.116 (sec). Leaf size: 54

```
DSolve[y''[x]-x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}c_2\left(\sqrt{2\pi}(x^2 - 1)\operatorname{erfi}\left(\frac{x}{\sqrt{2}}\right) - 2e^{\frac{x^2}{2}}x\right) + c_1(x^2 - 1)$$

1.747 problem 763

Internal problem ID [8237]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 763.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x+2)\, y'' + x y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve((x+2)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x + c_2 \mathrm{e}^{-x}(x+4)$$

✓ Solution by Mathematica

Time used: 0.14 (sec). Leaf size: 72

```
DSolve[(x+2)*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2\sqrt{\frac{2}{\pi}} e^{-x-2}\sqrt{x+2}\left(c_1\left(e^{x+2}x + x + 4\right) - i c_2\left(\left(e^{x+2} - 1\right)x - 4\right)\right)}{\sqrt{-i(x+2)}}$$

1.748 problem 764

Internal problem ID [8238]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 764.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{(x^2+1)\,y''-6y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve((x^2+1)*diff(y(x),x$2)-6*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(x^3+x)+c_2\left(\frac{3\arctan(x)\,x^3}{2}+\frac{3\arctan(x)\,x}{2}+\frac{3x^2}{2}+1\right)$$

✓ Solution by Mathematica
Time used: 0.049 (sec). Leaf size: 36

```
DSolve[(x^2+1)*y''[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1(x^3+x)-\frac{1}{2}c_2\left(3(x^3+x)\arctan(x)+3x^2+2\right)$$

1.749 problem 765

Internal problem ID [8239]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 765.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+2)\,y''+3xy'-y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 67

```
dsolve((x^2+2)*diff(y(x),x$2)+3*x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\left(\sqrt{2}\,x+\sqrt{2}\,\sqrt{x^2+2}\right)^{\sqrt{2}}}{\sqrt{x^2+2}}+\frac{c_2\left(\frac{\sqrt{2}}{2\sqrt{x^2+2}+2x}\right)^{\sqrt{2}}}{\sqrt{x^2+2}}$$

✓ Solution by Mathematica

Time used: 0.107 (sec). Leaf size: 92

```
DSolve[(x^2+2)*y''[x]+3*x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{2^{3/4}c_1\cos\left(2\sqrt{2}\arcsin\left(\frac{1}{2}\sqrt{2-i\sqrt{2}x}\right)\right)}{\sqrt{\pi}\sqrt{x^2+2}}+\frac{c_2Q_{-\frac{1}{2}+\sqrt{2}}^{\frac{1}{2}}\left(\frac{ix}{\sqrt{2}}\right)}{\sqrt[4]{x^2+2}}$$

1.750 problem 766

Internal problem ID [8240]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 766.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-1)\,y''-xy'+y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+\mathrm{e}^{x}c_2$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 17

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1e^x-c_2x$$

1.751 problem 767

Internal problem ID [8241]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 767.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 2xy' + 8y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\frac{4}{3}x^4 - 4x^2 + 1\right) + c_2\left(\frac{4}{3}x^4 - 4x^2 + 1\right)\left(\int \frac{e^{x^2}}{(4x^4 - 12x^2 + 3)^2}dx\right)$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 49

```
DSolve[y''[x]-2*x*y''[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{4x-2}\left(c_1 \,\text{BesselI}\left(1, 4\sqrt{x-\frac{1}{2}}\right) - c_2K_1\left(4\sqrt{x-\frac{1}{2}}\right)\right)$$

1.752 problem 769

Internal problem ID [8242]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 769.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + \left(\frac{5}{3}x + x^2\right) y' - \frac{y}{3} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve(x^2*diff(y(x),x$2)+(5/3*x+x^2)*diff(y(x),x)-1/3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(3x-1)}{x} + \frac{c_2(3x-1)\left(\int \frac{x^{\frac{1}{3}} e^{-x}}{(3x-1)^2} dx\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.09 (sec). Leaf size: 47

```
DSolve[x^2*y''[x]+(5/3*x+x^2)*y'[x]-1/3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{-3c_1x + 3c_2e^{-x}\sqrt[3]{x} + c_2(1-3x)\Gamma\left(\frac{1}{3}, x\right) + c_1}{3x}$$

1.753 problem 770

Internal problem ID [8243]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 770.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2xy'' - y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 75

```
dsolve(2*x*diff(y(x),x$2)-diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{2i\sqrt{x}} \sqrt{\frac{(1+4x)\left(2i\sqrt{x}-1\right)}{1+2i\sqrt{x}}} + c_2 \mathrm{e}^{-2i\sqrt{x}} \sqrt{\frac{(1+4x)\left(1+2i\sqrt{x}\right)}{2i\sqrt{x}-1}}$$

✓ Solution by Mathematica

Time used: 0.085 (sec). Leaf size: 59

```
DSolve[2*x*y''[x]-y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{2i\sqrt{x}}\left(2\sqrt{x}+i\right) + \frac{1}{8} c_2 e^{-2i\sqrt{x}}\left(1+2i\sqrt{x}\right)$$

1.754 problem 771

Internal problem ID [8244]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 771.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{2xy'' - (3 + 2x)\, y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 41

```
dsolve(2*x*diff(y(x),x$2)-(3+2*x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{x}(-3+2x)}{2} + c_2 \mathrm{e}^{x}(-3+2x)\left(\int \frac{x^{\frac{3}{2}}\mathrm{e}^{-x}}{(-3+2x)^{2}}dx\right)$$

✓ Solution by Mathematica
Time used: 0.094 (sec). Leaf size: 54

```
DSolve[2*x*y''[x]-(3+2*x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-\sqrt{\pi}c_2 e^{x}(2x-3)\operatorname{erf}\left(\sqrt{x}\right) + 2c_1 e^{x}(2x-3) - 6c_2\sqrt{x}\right)$$

1.755 problem 772

Internal problem ID [8245]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 772.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2y'' + 3xy' + (2x - 1)\,y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 81

```
dsolve(2*x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+(2*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{2i\sqrt{x}}\sqrt{\frac{(1+4x)(2i\sqrt{x}-1)}{1+2i\sqrt{x}}}}{x} + \frac{c_2 e^{-2i\sqrt{x}}\sqrt{\frac{(1+4x)(1+2i\sqrt{x})}{2i\sqrt{x}-1}}}{x}$$

✓ Solution by Mathematica

Time used: 0.09 (sec). Leaf size: 64

```
DSolve[2*x^2*y''[x]+3*x*y'[x]+(2*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-2i\sqrt{x}}\left(8c_1 e^{4i\sqrt{x}}\left(2\sqrt{x}+i\right) + c_2\left(1+2i\sqrt{x}\right)\right)}{8x}$$

1.756 problem 773

Internal problem ID [8246]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 773.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + 2y' - yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sinh(x)}{x} + \frac{c_2 \cosh(x)}{x}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 28

```
DSolve[x*y''[x]+2*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-x} + c_2 e^{x}}{2x}$$

1.757 problem 774

Internal problem ID [8247]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 774.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2e^{2ix})}{2\sqrt{x}}$$

1.758 problem 775

Internal problem ID [8248]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 775.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' + (x-6)\,y' - 3y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 39

```
dsolve(x*diff(y(x),x$2)+(x-6)*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(x^3 - 12x^2 + 60x - 120\right) + c_2 \mathrm{e}^{-x}\left(x^3 + 12x^2 + 60x + 120\right)$$

✓ Solution by Mathematica
Time used: 0.069 (sec). Leaf size: 98

```
DSolve[x*y''[x]+(x-6)*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2e^{-x/2}\sqrt{x}\left((c_1x^3 + 12ic_2x^2 + 60c_1x + 120ic_2)\cosh\left(\frac{x}{2}\right) - (12c_1(x^2+10) + ic_2x(x^2+60))\sinh\left(\frac{x}{2}\right)\right)}{\sqrt{\pi}\sqrt{-ix}}$$

1.759 problem 776

Internal problem ID [8249]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 776.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^4 y'' + \lambda y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(x^4*diff(y(x),x$2)+lambda*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sinh\left(\frac{\sqrt{-\lambda}}{x}\right) + c_2 x \cosh\left(\frac{\sqrt{-\lambda}}{x}\right)$$

✓ Solution by Mathematica
Time used: 0.131 (sec). Leaf size: 52

```
DSolve[x^4*y''[x]+\[Lambda]*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x e^{\frac{i\sqrt{\lambda}}{x}} - \frac{i c_2 x e^{-\frac{i\sqrt{\lambda}}{x}}}{2\sqrt{\lambda}}$$

1.760 problem 777

Internal problem ID [8250]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 777.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 4xy' + (4x^2 - 25)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
dsolve(4*x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2-25)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{ix}(x^2 + 3ix - 3)}{x^{\frac{5}{2}}} + \frac{c_2 e^{-ix}(x^2 - 3ix - 3)}{x^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.127 (sec). Leaf size: 59

```
DSolve[4*x^2*y''[x]+4*x*y'[x]+(4*x^2-25)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}}((-c_2x^2 + 3c_1x + 3c_2)\cos(x) + (c_1(x^2 - 3) + 3c_2x)\sin(x))}{x^{5/2}}$$

1.761 problem 778

Internal problem ID [8251]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 778.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' + \left(36x^2 - \frac{1}{4}\right)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(36*x^2-1/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(6x)}{\sqrt{x}} + \frac{c_2 \cos(6x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(36*x^2-1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-6ix}(12c_1 - ic_2 e^{12ix})}{12\sqrt{x}}$$

1.762 problem 779

Internal problem ID [8252]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 779.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y(x^2 - 2) = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 31

```
dsolve(x^2*diff(y(x),x$2)+(x^2-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(-\sin(x) + \cos(x)\,x)}{x} + \frac{c_2(\cos(x) + x\sin(x))}{x}$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 21

```
DSolve[x^2*y''[x]+(x^2-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(c_1 j_1(x) - c_2 y_1(x))$$

1.763 problem 780

Internal problem ID [8253]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 780.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$xy'' + 3y' + yx^3 = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x$2)+3*diff(y(x),x)+x^3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin\left(\frac{x^2}{2}\right)}{x^2} + \frac{c_2 \cos\left(\frac{x^2}{2}\right)}{x^2}$$

✓ Solution by Mathematica
Time used: 0.059 (sec). Leaf size: 43

```
DSolve[x*y''[x]+3*y'[x]+x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{ix^2}{2}}\left(2c_1 - ic_2 e^{ix^2}\right)}{2x^2}$$

1.764 problem 781

Internal problem ID [8254]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 781.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 4xy' + (x^2 + 2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{x^2} + \frac{c_2 \cos(x)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 37

```
DSolve[x^2*y''[x]+4*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x^2}$$

1.765 problem 782

Internal problem ID [8255]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 782.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{16x^2y'' + 32xy' + (x^4 - 12)\,y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(16*x^2*diff(y(x),x$2)+32*x*diff(y(x),x)+(x^4-12)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin\left(\frac{x^2}{8}\right)}{x^{\frac{3}{2}}} + \frac{c_2 \cos\left(\frac{x^2}{8}\right)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica
Time used: 0.063 (sec). Leaf size: 42

```
DSolve[16*x^2*y''[x]+32*x*y'[x]+(x^4-12)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{ix^2}{8}}\left(c_1 - 2ic_2 e^{\frac{ix^2}{4}}\right)}{x^{3/2}}$$

1.766 problem 783

Internal problem ID [8256]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 783.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - x^2 y' + yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 63

```
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x - \frac{c_2 3^{\frac{1}{3}} \left(6(-x^3)^{\frac{1}{3}} \Gamma\left(\frac{2}{3}\right) 3^{\frac{2}{3}} - 6(-x^3)^{\frac{1}{3}} \Gamma\left(\frac{2}{3}, -\frac{x^3}{3}\right) 3^{\frac{2}{3}} + 18 \,\mathrm{e}^{\frac{x^3}{3}}\right)}{3 \left(1 + \sqrt{-3}\right)}$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 41

```
DSolve[y''[x]-x^2*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x - \frac{c_2 \sqrt[3]{-x^3} \Gamma\left(-\frac{1}{3}, -\frac{x^3}{3}\right)}{3\sqrt[3]{3}}$$

1.767 problem 784

Internal problem ID [8257]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 784.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{xy'' - (x + 2) y' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)-(x+2)*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(x^2 + 2x + 2\right) + \mathrm{e}^x c_2$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 24

```
DSolve[x*y''[x]-(x+2)*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x - c_2\left(x^2 + 2x + 2\right)$$

1.768 problem 785

Internal problem ID [8258]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 785.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + xy' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 50

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \,\mathrm{e}^{-\frac{x^2}{2}} - \frac{c_2 \mathrm{e}^{-\frac{x^2}{2}} \left(i\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,x}{2}\right) x + 2\,\mathrm{e}^{\frac{x^2}{2}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 69

```
DSolve[y''[x]+x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\frac{\pi}{2}} c_2 e^{-\frac{x^2}{2}} \sqrt{x^2} \mathrm{erfi}\left(\frac{\sqrt{x^2}}{\sqrt{2}}\right) + \sqrt{2} c_1 e^{-\frac{x^2}{2}} x + c_2$$

1.769 problem 786

Internal problem ID [8259]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 786.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(1-x^2)\,y''-2xy'+2y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+c_2\left(-\frac{\ln(x+1)\,x}{2}+\frac{\ln(x-1)\,x}{2}+1\right)$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 33

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1x-\frac{1}{2}c_2(x\log(1-x)-x\log(x+1)+2)$$

1.770 problem 787

Internal problem ID [8260]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 787.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4xy' + (4x^2 - 2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x^2} + c_2 x\, \mathrm{e}^{x^2}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 18

```
DSolve[y''[x]-4*x*y'[x]+(4*x^2-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x^2}(c_2 x + c_1)$$

1.771 problem 788

Internal problem ID [8261]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 788.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(1-x^2)\,y''-2xy'+30y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 83

```
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+30*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(\frac{21}{5}x^5-\frac{14}{3}x^3+x\right)+c_2\left(-\frac{21\ln(x+1)\,x^5}{640}+\frac{21\ln(x-1)\,x^5}{640}+\frac{7\ln(x+1)\,x^3}{192}-\frac{7\ln(x-1)\,x^3}{192}+\frac{21x^4}{320}-\frac{\ln(x+1)\,x}{128}+\frac{\ln(x-1)\,x}{128}-\frac{49x^2}{960}+\frac{1}{225}\right)$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 76

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+30*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{8}c_1x(63x^4-70x^2+15)+c_2\left(-\frac{63x^4}{8}+\frac{49x^2}{8}-\frac{1}{16}(63x^4-70x^2+15)\,x(\log(1-x)-\log(x+1))-\frac{8}{15}\right)$$

1.772 problem 789

Internal problem ID [8262]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 789.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + 2y' + yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{x} + \frac{c_2 \cos(x)}{x}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 37

```
DSolve[x*y''[x]+2*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x}$$

1.773 problem 790

Internal problem ID [8263]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 790.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (2x+1)\,y' + (1+x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)+(2*x+1)*diff(y(x),x)+(x+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} c_1 + c_2 \mathrm{e}^{-x} \ln(x)$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 19

```
DSolve[x*y''[x]+(2*x+1)*y'[x]+(x+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_2 \log(x) + c_1)$$

1.774 problem 791

Internal problem ID [8264]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 791.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$\boxed{2x(x-1)\,y''-(1+x)\,y'+y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(2*x*(x-1)*diff(y(x),x$2)-(x+1)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(x+1)+c_2\sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 21

```
DSolve[2*x*(x-1)*y''[x]-(x+1)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\sqrt{x}-2c_2(x+1)$$

1.775 problem 792

Internal problem ID [8265]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 792.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + 2y' + 4yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+4*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(2x)}{x} + \frac{c_2 \cos(2x)}{x}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 37

```
DSolve[x*y''[x]+2*y'[x]+4*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4c_1 e^{-2ix} - ic_2 e^{2ix}}{4x}$$

1.776 problem 793

Internal problem ID [8266]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 793.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (-2x + 2)\, y' + (x - 2)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x$2)+(2-2*x)*diff(y(x),x)+(x-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^x}{x} + \mathrm{e}^x c_2$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 19

```
DSolve[x*y''[x]+(2-2*x)*y'[x]+(x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x (c_2 x + c_1)}{x}$$

1.777 problem 794

Internal problem ID [8267]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 794.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + 6xy' + (4x^2 + 6)\, y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)+6*x*diff(y(x),x)+(4*x^2+6)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(2x)}{x^3} + \frac{c_2 \cos(2x)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 37

```
DSolve[x^2*y''[x]+6*x*y'[x]+(4*x^2+6)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4c_1 e^{-2ix} - ic_2 e^{2ix}}{4x^3}$$

1.778 problem 795

Internal problem ID [8268]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 795.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (1 - 2x)\, y' + (x - 1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x$2)+(1-2*x)*diff(y(x),x)+(x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2 e^x \ln(x)$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 17

```
DSolve[x*y''[x]+(1-2*x)*y'[x]+(x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x (c_2 \log(x) + c_1)$$

1.779 problem 796

Internal problem ID [8269]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 796.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$x(1-x)\,y''+\left(2x+\frac{1}{2}\right)y'-2y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 61

```
dsolve(x*(1-x)*diff(y(x),x$2)+(1/2+2*x)*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(1+4x)+c_2\left(4\sqrt{x\,(x-1)}\,x-12\ln\left(x-\frac{1}{2}+\sqrt{x\,(x-1)}\right)x+26\sqrt{x\,(x-1)}-3\ln\left(x-\frac{1}{2}+\sqrt{x\,(x-1)}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.124 (sec). Leaf size: 64

```
DSolve[x*(1-x)*y''[x]+(1/2+2*x)*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}c_2\left(\sqrt{-((x-1)x)}(2x+13)-6(4x+1)\arctan\left(\frac{\sqrt{1-x}}{\sqrt{x}+1}\right)\right)+c_1\left(x+\frac{1}{4}\right)$$

1.780 problem 797

Internal problem ID [8270]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 797.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4(t^2-3t+2)\,y''-2y'+y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 62

```
dsolve(4*(t^2-3*t+2)*diff(y(t),t$2)-2*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t)=c_1\sqrt{t-1}+\frac{c_2\sqrt{t-2}\,(t-1)\left(\ln\left(t-\frac{3}{2}+\sqrt{t^2-3t+2}\right)\sqrt{t^2-3t+2}-2t+4\right)}{t^2-3t+2}$$

✓ Solution by Mathematica

Time used: 0.155 (sec). Leaf size: 53

```
DSolve[4*(t^2-3*t+2)*y''[t]-2*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to\sqrt{1-t}\left(-2c_2\operatorname{arctanh}\left(\frac{1}{\sqrt{\frac{t-1}{t-2}}}\right)+\frac{2c_2}{\sqrt{\frac{t-1}{t-2}}}+c_1\right)$$

1.781 problem 798

Internal problem ID [8271]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 798.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2(t^2 - 5t + 6)\, y'' + (2t - 3)\, y' - 8y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(2*(t^2-5*t+6)*diff(y(t),t$2)+(2*t-3)*diff(y(t),t)-8*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1\left(t^2 - \frac{13}{3}t + \frac{37}{8}\right) + \frac{c_2(6t - 17)\,(t - 2)^{\frac{3}{2}}}{\sqrt{t - 3}}$$

✓ Solution by Mathematica

Time used: 0.162 (sec). Leaf size: 84

```
DSolve[2*(t^2-5*t+6)*y''[t]+(2*t-3)*y'[t]-8*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{\sqrt[4]{2 - t}\left(5c_1\sqrt[4]{t - 3}\sqrt{t - 2}(6t^2 - 29t + 34) + 24c_2(t - 3)^{3/4}\,(24t^2 - 104t + 111)\right)}{30(3 - t)^{3/4}\sqrt[4]{t - 2}}$$

1.782 problem 799

Internal problem ID [8272]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 799.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3t(t+1)\,y''+ty'-y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(3*t*(1+t)*diff(y(t),t$2)+t*diff(y(t),t)-y(t)=0,y(t), singsol=all)
```

$$y(t)=c_1t+c_2t\left(\int\frac{1}{(t+1)^{\frac{1}{3}}\,t^2}dt\right)$$

✓ Solution by Mathematica

Time used: 0.118 (sec). Leaf size: 93

```
DSolve[3*t*(1+t)*y''[t]+t*y'[t]-y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to\frac{6c_1t-c_2\left(2\sqrt{3}t\arctan\left(\frac{2\sqrt[3]{t+1}+1}{\sqrt{3}}\right)+6(t+1)^{2/3}+2t\log\left(\sqrt[3]{t+1}-1\right)-t\log\left((t+1)^{2/3}+\sqrt[3]{t+1}\right.\right.}{6\sqrt[6]{3}}$$

1.783 problem 800

Internal problem ID [8273]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 800.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + \frac{\left(\frac{3}{4}+x\right)y}{4} = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)+1/4*(x+3/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(\sqrt{x}\right) x^{\frac{1}{4}} + c_2 x^{\frac{1}{4}} \cos\left(\sqrt{x}\right)$$

✓ Solution by Mathematica
Time used: 0.048 (sec). Leaf size: 43

```
DSolve[x^2*y''[x]+1/4*(x+3/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-i\sqrt{x}}\sqrt[4]{x}\left(c_1 e^{2i\sqrt{x}} + ic_2\right)$$

1.784 problem 801

Internal problem ID [8274]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 801.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + xy' + \frac{(x^2-1)\,y}{4} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+1/4*(x^2-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin\left(\frac{x}{2}\right)}{\sqrt{x}} + \frac{c_2 \cos\left(\frac{x}{2}\right)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 36

```
DSolve[x^2*y''[x]+x*y'[x]+1/4*(x^2-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{ix}{2}}\left(c_1 - ic_2e^{ix}\right)}{\sqrt{x}}$$

1.785 problem 802

Internal problem ID [8275]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 802.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (1-2x)\,y' + (x-1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x$2)+(1-2*x)*diff(y(x),x)+(x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2 e^x \ln(x)$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 17

```
DSolve[x*y''[x]+(1-2*x)*y'[x]+(x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x (c_2 \log(x) + c_1)$$

1.786 problem 803

Internal problem ID [8276]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 803.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{xy'' - (1+x)\,y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x*diff(y(x),x$2)-(x+1)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x+1) + \mathrm{e}^x c_2$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 19

```
DSolve[x*y''[x]-(x+1)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x - c_2(x+1)$$

1.787 problem 804

Internal problem ID [8277]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 804.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + 3y' + 4yx^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x$2)+3*diff(y(x),x)+4*x^3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x^2)}{x^2} + \frac{c_2 \cos(x^2)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 41

```
DSolve[x*y''[x]+3*y'[x]+4*x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{4c_1 e^{-ix^2} - ic_2 e^{ix^2}}{4x^2}$$

1.788 problem 805

Internal problem ID [8278]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 805.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1-x^2)\,y''+2x(1-x^2)\,y'-2y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve(x^2*(1-x^2)*diff(y(x),x$2)+2*x*(1-x^2)*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(x^2-1)}{x^2}+\frac{c_2\left(-\frac{\ln(x+1)x^2}{4}+\frac{\ln(x-1)x^2}{4}+\frac{\ln(x+1)}{4}-\frac{\ln(x-1)}{4}-\frac{x}{2}\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 56

```
DSolve[x^2*(1-x^2)*y''[x]+2*x*(1-x^2)*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to\frac{-4c_1x^2-c_2(x^2-1)\log(1-x)+c_2(x^2-1)\log(x+1)+2c_2x+4c_1}{4x^2}$$

1.789 problem 806

Internal problem ID [8279]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 806.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + (x-2)\,y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(2*x*diff(y(x),x$2)+(x-2)*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x-2) + c_2\mathrm{e}^{-\frac{x}{2}}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 23

```
DSolve[2*x*y''[x]+(x-2)*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1e^{-x/2} + 2c_2(x-2)$$

1.790 problem 807

Internal problem ID [8280]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 807.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + 2y' + yx = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{x} + \frac{c_2 \cos(x)}{x}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 37

```
DSolve[x*y''[x]+2*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x}$$

1.791 problem 808

Internal problem ID [8281]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 808.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 2x^2y' + (x^4 + 2x - 1)\,y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+2*x^2*diff(y(x),x)+(x^4+2*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{1}{3}x^3 - x} + c_2 e^{-\frac{1}{3}x^3 + x}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 34

```
DSolve[y''[x]+2*x^2*y'[x]+(x^4+2*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-\frac{1}{3}x(x^2+3)}\left(c_2e^{2x} + 2c_1\right)$$

1.792 problem 809

Internal problem ID [8282]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 809.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{u'' + 2u' + u = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve(diff(u(x),x$2)+2*diff(u(x),x)+u(x)=0,u(x), singsol=all)
```

$$u(x) = \mathrm{e}^{-x} c_1 + \mathrm{e}^{-x} c_2 x$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 18

```
DSolve[u''[x]+2*u'[x]+u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to e^{-x}(c_2 x + c_1)$$

1.793 problem 810

Internal problem ID [8283]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 810.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$u'' - (2x+1)\,u' + (x^2+x-1)\,u = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(u(x),x$2)-(2*x+1)*diff(u(x),x)+(x^2+x-1)*u(x)=0,u(x), singsol=all)
```

$$u(x) = c_1 e^{\frac{x^2}{2}} + c_2 e^{\frac{1}{2}x^2+x}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 24

```
DSolve[u''[x]-(2*x+1)*u'[x]+(x^2+x-1)*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to e^{\frac{x^2}{2}}(c_2 e^x + c_1)$$

1.794 problem 811

Internal problem ID [8284]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 811.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 2y' + \left(1 + \frac{2}{(3x+1)^2}\right) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+(1+2/(1+3*x)^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(3x+1)^{\frac{1}{3}} \mathrm{e}^{-x} + c_2(3x+1)^{\frac{2}{3}} \mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 35

```
DSolve[y''[x]+2*y'[x]+(1+2/(1+3*x)^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x} \sqrt[3]{3x+1} \left(c_2 \sqrt[3]{3x+1} + c_1\right)$$

1.795 problem 812

Internal problem ID [8285]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 812.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - 2xy' + (x^2 + 2) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sin(x) + c_2 \cos(x) x$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-ix} x - \frac{1}{2} i c_2 e^{ix} x$$

1.796 problem 813

Internal problem ID [8286]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 813.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{2y'}{x} - \frac{2y}{(1+x)^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)+2/x*diff(y(x),x)-2/(1+x)^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x(x+1)} + \frac{c_2(x^3+3x^2+3x)}{x(x+1)}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 34

```
DSolve[y''[x]+2/x*y'[x]-2/(1+x)^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 x(x^2+3x+3)+3c_1}{3x(x+1)}$$

1.797 problem 815

Internal problem ID [8287]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 815.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - xy' - yx = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + \frac{c_2\sqrt{2}\left(\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)x - i\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{\frac{1}{2}x^2+2x} + 2\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\right)\mathrm{e}^{-x}}{2\sqrt{\pi}}$$

✓ Solution by Mathematica

Time used: 0.163 (sec). Leaf size: 78

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(-\sqrt{2\pi}c_2\sqrt{(x+2)^2}\mathrm{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) + 2\sqrt{2}c_1(x+2) + 2c_2e^{\frac{1}{2}(x+2)^2}\right)$$

1.798 problem 816

Internal problem ID [8288]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 816.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - xy' - yx = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + \frac{c_2\sqrt{2}\left(\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)x - i\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{\frac{1}{2}x^2+2x} + 2\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\right)\mathrm{e}^{-x}}{2\sqrt{\pi}}$$

✓ Solution by Mathematica

Time used: 0.097 (sec). Leaf size: 78

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(-\sqrt{2\pi}c_2\sqrt{(x+2)^2}\mathrm{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) + 2\sqrt{2}c_1(x+2) + 2c_2e^{\frac{1}{2}(x+2)^2}\right)$$

1.799 problem 817

Internal problem ID [8289]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 817.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - xy' - yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x}(x+2) + \frac{c_2\sqrt{2}\left(\pi\, e^{-2} \operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right) x - i\sqrt{2}\,\sqrt{\pi}\, e^{\frac{1}{2}x^2+2x} + 2\pi\, e^{-2} \operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\right) e^{-x}}{2\sqrt{\pi}}$$

✓ Solution by Mathematica

Time used: 0.106 (sec). Leaf size: 78

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-x}\left(-\sqrt{2\pi} c_2 \sqrt{(x+2)^2} \operatorname{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) + 2\sqrt{2} c_1 (x+2) + 2 c_2 e^{\frac{1}{2}(x+2)^2}\right)$$

1.800 problem 818

Internal problem ID [8290]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 818.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - xy' - yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + \frac{c_2\sqrt{2}\left(\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)x - i\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{\frac{1}{2}x^2+2x} + 2\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\right)\mathrm{e}^{-x}}{2\sqrt{\pi}}$$

✓ Solution by Mathematica

Time used: 0.108 (sec). Leaf size: 78

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(-\sqrt{2\pi}c_2\sqrt{(x+2)^2}\mathrm{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) + 2\sqrt{2}c_1(x+2) + 2c_2e^{\frac{1}{2}(x+2)^2}\right)$$

1.801 problem 819

Internal problem ID [8291]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 819.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - xy' - yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + \frac{c_2\sqrt{2}\left(\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)x - i\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{\frac{1}{2}x^2+2x} + 2\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\right)\mathrm{e}^{-x}}{2\sqrt{\pi}}$$

✓ Solution by Mathematica

Time used: 0.113 (sec). Leaf size: 78

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(-\sqrt{2\pi}c_2\sqrt{(x+2)^2}\mathrm{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) + 2\sqrt{2}c_1(x+2) + 2c_2e^{\frac{1}{2}(x+2)^2}\right)$$

1.802 problem 820

Internal problem ID [8292]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 820.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - xy' - yx = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + \frac{c_2\sqrt{2}\left(\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)x - i\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{\frac{1}{2}x^2+2x} + 2\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\right)\mathrm{e}^{-x}}{2\sqrt{\pi}}$$

✓ Solution by Mathematica
Time used: 0.112 (sec). Leaf size: 78

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(-\sqrt{2\pi}c_2\sqrt{(x+2)^2}\mathrm{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) + 2\sqrt{2}c_1(x+2) + 2c_2e^{\frac{1}{2}(x+2)^2}\right)$$

1.803 problem 821

Internal problem ID [8293]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 821.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - xy' - yx = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + \frac{c_2\sqrt{2}\left(\pi\,\mathrm{e}^{-2}\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)x - i\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{\frac{1}{2}x^2+2x} + 2\pi\,\mathrm{e}^{-2}\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\right)\mathrm{e}^{-x}}{2\sqrt{\pi}}$$

✓ Solution by Mathematica

Time used: 0.109 (sec). Leaf size: 78

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(-\sqrt{2\pi}c_2\sqrt{(x+2)^2}\operatorname{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) + 2\sqrt{2}c_1(x+2) + 2c_2e^{\frac{1}{2}(x+2)^2}\right)$$

1.804 problem 822

Internal problem ID [8294]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 822.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - xy' - yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + \frac{c_2\sqrt{2}\left(\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)x - i\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{\frac{1}{2}x^2+2x} + 2\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\right)\mathrm{e}^{-x}}{2\sqrt{\pi}}$$

✓ Solution by Mathematica

Time used: 0.11 (sec). Leaf size: 78

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(-\sqrt{2\pi}c_2\sqrt{(x+2)^2}\mathrm{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) + 2\sqrt{2}c_1(x+2) + 2c_2e^{\frac{1}{2}(x+2)^2}\right)$$

1.805 problem 823

Internal problem ID [8295]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 823.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - xy' - yx = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + \frac{c_2\sqrt{2}\left(\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)x - i\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{\frac{1}{2}x^2+2x} + 2\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\right)\mathrm{e}^{-x}}{2\sqrt{\pi}}$$

✓ Solution by Mathematica
Time used: 0.102 (sec). Leaf size: 78

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(-\sqrt{2\pi}c_2\sqrt{(x+2)^2}\mathrm{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) + 2\sqrt{2}c_1(x+2) + 2c_2e^{\frac{1}{2}(x+2)^2}\right)$$

1.806 problem 824

Internal problem ID [8296]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 824.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - xy' - yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + \frac{c_2\sqrt{2}\left(\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)x - i\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{\frac{1}{2}x^2+2x} + 2\pi\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\right)\mathrm{e}^{-x}}{2\sqrt{\pi}}$$

✓ Solution by Mathematica

Time used: 0.098 (sec). Leaf size: 78

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(-\sqrt{2\pi}c_2\sqrt{(x+2)^2}\mathrm{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) + 2\sqrt{2}c_1(x+2) + 2c_2e^{\frac{1}{2}(x+2)^2}\right)$$

1.807 problem 825

Internal problem ID [8297]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 825.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - xy' - yx = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x}(x+2) + \frac{c_2\sqrt{2}\left(\pi e^{-2} \operatorname{erf}\left(\frac{i\sqrt{2}(x+2)}{2}\right) x - i\sqrt{2}\sqrt{\pi} e^{\frac{1}{2}x^2+2x} + 2\pi e^{-2} \operatorname{erf}\left(\frac{i\sqrt{2}(x+2)}{2}\right)\right) e^{-x}}{2\sqrt{\pi}}$$

✓ Solution by Mathematica
Time used: 0.104 (sec). Leaf size: 78

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(-\sqrt{2\pi}c_2\sqrt{(x+2)^2}\operatorname{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) + 2\sqrt{2}c_1(x+2) + 2c_2 e^{\frac{1}{2}(x+2)^2}\right)$$

1.808 problem 826

Internal problem ID [8298]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 826.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$xy'' + 2y' + yx = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{x} + \frac{c_2 \cos(x)}{x}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 37

```
DSolve[x*y''[x]+2*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x}$$

1.809 problem 827

Internal problem ID [8299]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 827.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$2x^2y'' + 3xy' - yx = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 31

```
dsolve(2*x^2*diff(y(x),x$2)+3*x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sinh\left(\sqrt{x}\,\sqrt{2}\right)}{\sqrt{x}} + \frac{c_2 \cosh\left(\sqrt{x}\,\sqrt{2}\right)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.061 (sec). Leaf size: 56

```
DSolve[2*x^2*y''[x]+3*x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\sqrt{2}\sqrt{x}}\left(2c_1 e^{2\sqrt{2}\sqrt{x}} - \sqrt{2}c_2\right)}{2\sqrt{x}}$$

1.810 problem 828

Internal problem ID [8300]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 828.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \left(3x^2 + 2x\right) y' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x^2*diff(y(x), x, x) + (2*x+3*x^2)*diff(y(x),x)-2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(9x^2 - 6x + 2)}{x^2} + \frac{c_2 e^{-3x}}{x^2}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 35

```
DSolve[x^2*y''[x]+(2*x+3*x^2)*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1(9x^2 - 6x + 2) + 27c_2 e^{-3x}}{27x^2}$$

1.811 problem 829

Internal problem ID [8301]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 829.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x^2+x+1)\,y''+x(11x^2+11x+9)\,y'+(7x^2+10x+6)\,y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 141

```
dsolve(2*x^2*(1+x+x^2)*diff(y(x), x$2) + x*(9+11*x+11*x^2)*diff(y(x), x) + (6+10*x+7*x^2)*y(
```

$$y(x)=\frac{c_1\sqrt{x^2+x+1}\left(\frac{i\sqrt{3}+2x+1}{i\sqrt{3}-2x-1}\right)^{-\frac{i\sqrt{3}}{6}}}{x^2}+\frac{c_2\sqrt{x^2+x+1}\left(\frac{i\sqrt{3}+2x+1}{i\sqrt{3}-2x-1}\right)^{-\frac{i\sqrt{3}}{6}}\left(\int\frac{\left(\frac{i\sqrt{3}-2x-1}{i\sqrt{3}+2x+1}\right)^{-\frac{i\sqrt{3}}{6}}}{(x^2+x+1)^{\frac{3}{2}}\sqrt{x}}dx\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.718 (sec). Leaf size: 93

```
DSolve[2*x^2*(1+x+x^2)*y''[x] + x*(9+11*x+11*x^2)*y'[x] + (6+10*x+7*x^2)*y[x] == 0,y[x],x,In
```

$$y(x)\to\frac{\sqrt{x^2+x+1}\,e^{-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}}\left(c_2\int_1^x\frac{e^{\frac{\arctan\left(\frac{2K[1]+1}{\sqrt{3}}\right)}{\sqrt{3}}}}{\sqrt{K[1]}(K[1]^2+K[1]+1)^{3/2}}dK[1]+c_1\right)}{x^2}$$

1.812 problem 830

Internal problem ID [8302]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 830.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (1 + x)\, y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(x*diff(y(x), x$2) +(1+x)*diff(y(x),x)+2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x-1) + c_2(\mathrm{expIntegral}_1\left(-x\right)x - \mathrm{expIntegral}_1\left(-x\right) + \mathrm{e}^{x})\,\mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 33

```
DSolve[x*y''[x] +(1+x)*y'[x]+2*y[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_2(x-1)\,\mathrm{ExpIntegralEi}(x) + c_1(x-1) - c_2 e^{x})$$

1.813 problem 831

Internal problem ID [8303]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 831.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2-2x+1)\,y''-x(x+3)\,y'+(4+x)\,y=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 52

```
dsolve(x^2*(1-2*x+x^2)*diff(y(x), x$2) -x*(3+x)*diff(y(x),x)+(4+x)*y(x) = 0,y(x), singsol=al
```

$$y(x)=\frac{c_1x^2\mathrm{e}^{-\frac{4}{x-1}}}{x-1}+\frac{c_2x^2\,\mathrm{expIntegral}_1\left(-\frac{4x}{x-1}\right)\mathrm{e}^{-\frac{4x}{x-1}}}{x-1}$$

✓ Solution by Mathematica
Time used: 0.214 (sec). Leaf size: 54

```
DSolve[x^2*(1-2*x+x^2)*y''[x] -x*(3+x)*y'[x]+(4+x)*y[x] == 0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to\frac{e^{-\frac{4x}{x-1}}\sqrt{1-x}x^2\left(c_2\,\mathrm{ExpIntegralEi}\left(\frac{4x}{x-1}\right)+e^4c_1\right)}{(x-1)^{3/2}}$$

1.814 problem 832

Internal problem ID [8304]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 832.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x+2)\,y'' + 5x^2y' + (1+x)\,y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
dsolve(2*x^2*(2+x)*diff(y(x), x$2) +5*x^2*diff(y(x),x)+(1+x)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1\sqrt{x}}{(x+2)^{\frac{3}{2}}} - \frac{c_2\sqrt{2}\left(-2\sqrt{2}\,\sqrt{x+2} + 4\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,\sqrt{x+2}}{2}\right)\right)\sqrt{x}}{2\,(x+2)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 55

```
DSolve[2*x^2*(2+x)*y''[x] +5*x^2*y'[x]+(1+x)*y[x] == 0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{\sqrt{x}\left(-2\sqrt{2}c_2\text{arctanh}\left(\frac{\sqrt{x+2}}{\sqrt{2}}\right) + 2c_2\sqrt{x+2} + c_1\right)}{(x+2)^{3/2}}$$

1.815 problem 833

Internal problem ID [8305]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 833.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 4xy' + (x^2 + 2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x), x, x) + 4*x*diff(y(x), x) + (x^2+2)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{x^2} + \frac{c_2 \cos(x)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 37

```
DSolve[x^2*y''[x]+4*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x^2}$$

1.816 problem 834

Internal problem ID [8306]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 834.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2e^{2ix})}{2\sqrt{x}}$$

1.817 problem 835

Internal problem ID [8307]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 835.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - xy' - \left(x^2 + \frac{5}{4}\right)y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)-(x^2+5/4)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^x (x-1)}{\sqrt{x}} + \frac{c_2 e^{-x}(x+1)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.087 (sec). Leaf size: 53

```
DSolve[x^2*y''[x]-x*y'[x]-(x^2+5/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{\frac{2}{\pi}}((ic_2x + c_1)\sinh(x) - (c_1x + ic_2)\cosh(x))}{\sqrt{-ix}}$$

1.818 problem 836

Internal problem ID [8308]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 836.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2e^{2ix})}{2\sqrt{x}}$$

1.819 problem 837

Internal problem ID [8309]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 837.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 3xy' + 4yx^4 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+4*x^4*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x^2)}{x^2} + \frac{c_2 \cos(x^2)}{x^2}$$

✓ Solution by Mathematica
Time used: 0.054 (sec). Leaf size: 41

```
DSolve[x^2*y''[x]+3*x*y'[x]+4*x^4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4c_1 e^{-ix^2} - ic_2 e^{ix^2}}{4x^2}$$

1.820 problem 838

Internal problem ID [8310]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 838.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - (x^2 + 3)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)=(x^2+3)*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{x^2}{2}} x + c_2 \mathrm{e}^{\frac{x^2}{2}} \left(\sqrt{\pi}\, \mathrm{erf}\,(x)\, x + \mathrm{e}^{-x^2}\right)$$

✓ Solution by Mathematica
Time used: 0.081 (sec). Leaf size: 46

```
DSolve[y''[x]==(x^2+3)*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{x^2}{2}} \left(-\sqrt{\pi} c_2 e^{x^2} x \mathrm{erf}(x) + c_1 e^{x^2} x - c_2\right)$$

1.821 problem 839

Internal problem ID [8311]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 839.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2xy' + (x^2 + 1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+2*x*diff(y(x),x)+(x^2+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{x^2}{2}} + c_2 e^{-\frac{x^2}{2}} x$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 22

```
DSolve[y''[x]+2*x*y'[x]+(x^2+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{x^2}{2}} (c_2 x + c_1)$$

1.822 problem 840

Internal problem ID [8312]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 840.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(diff(y(x),x),x)+x*diff(y(x),x)+(x^2-1/4)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2e^{2ix})}{2\sqrt{x}}$$

1.823 problem 841

Internal problem ID [8313]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 841.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + \left(-8x^2 + 4x\right)y' + \left(4x^2 - 4x - 1\right)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(4*x^2*diff(diff(y(x),x),x)+(-8*x^2+4*x)*diff(y(x),x)+(4*x^2-4*x-1)*y(x) = 0,y(x), sin
```

$$y(x) = \frac{c_1 e^x}{\sqrt{x}} + c_2\sqrt{x}\, e^x$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 21

```
DSolve[4*x^2*y''[x]+(-8*x^2+4*x)*y'[x]+(4*x^2-4*x-1)*y[x] == 0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to \frac{e^x(c_2x + c_1)}{\sqrt{x}}$$

1.824 problem 843

Internal problem ID [8314]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 843.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 9

```
dsolve(diff(y(x),x$2)=((4*(1/2)^2-1)/(4*x^2))*y(x),y(x), singsol=all)
```

$$y(x) = xc_2 + c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 12

```
DSolve[y''[x]==((4*(1/2)^2-1)/(4*x^2))*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x + c_1$$

1.825 problem 844

Internal problem ID [8315]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 844.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{y'' - \frac{2y}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)=((4*(3/2)^2-1)/(4*x^2))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + c_2 x^2$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 18

```
DSolve[y''[x]==((4*(3/2)^2-1)/(4*x^2))*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 x^3 + c_1}{x}$$

1.826 problem 845

Internal problem ID [8316]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 845.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - \frac{6y}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)=((4*(5/2)^2-1)/(4*x^2))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^2} + c_2 x^3$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 18

```
DSolve[y''[x]==((4*(5/2)^2-1)/(4*x^2))*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 x^5 + c_1}{x^2}$$

2 section 2. Solution found using all possible Kovacic cases

2.1 problem 1 853
2.2 problem 2 855
2.3 problem 3 856
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2.1 problem 1

Internal problem ID [8317]

Book: Collection of Kovacic problems
Section: section 2. Solution found using all possible Kovacic cases
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \left(-\frac{3}{16x^2} - \frac{2}{9(x-1)^2} + \frac{3}{16x(x-1)}\right) y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 209

```
dsolve(diff(y(x),x$2)= ( -3/(16*x^2)- 2/(9*(x-1)^2) + 3/(16*x*(x-1))) *y(x),y(x), singsol=al
```

$$y(x) = c_1 \sqrt{(x-1)^{\frac{1}{3}}+1}\left((x-1)^{\frac{2}{3}}-(x-1)^{\frac{1}{3}}+1\right)^{\frac{1}{4}}(x-1)^{\frac{1}{3}}\left(\frac{\sqrt{3}\,(x-1)^{\frac{1}{3}}+2\sqrt{(x-1)^{\frac{2}{3}}-(x-1)^{\frac{1}{3}}+1}-\sqrt{3}}{-\sqrt{3}\,(x-1)^{\frac{1}{3}}+2\sqrt{(x-1)^{\frac{2}{3}}-(x-1)^{\frac{1}{3}}+1}+\sqrt{3}}\right)^{\frac{1}{8}} + \frac{c_2 \sqrt{(x-1)^{\frac{1}{3}}+1}\left((x-1)^{\frac{2}{3}}-(x-1)^{\frac{1}{3}}+1\right)^{\frac{1}{4}}(x-1)^{\frac{1}{3}}}{\left(\frac{\sqrt{3}\,(x-1)^{\frac{1}{3}}+2\sqrt{(x-1)^{\frac{2}{3}}-(x-1)^{\frac{1}{3}}+1}-\sqrt{3}}{-\sqrt{3}\,(x-1)^{\frac{1}{3}}+2\sqrt{(x-1)^{\frac{2}{3}}-(x-1)^{\frac{1}{3}}+1}+\sqrt{3}}\right)^{\frac{1}{8}}}$$

✓ Solution by Mathematica

Time used: 0.245 (sec). Leaf size: 550

```
DSolve[y''[x]== ( -3/(16*x^2)- 2/(9*(x-1)^2) + 3/(16*x*(x-1))) *y[x],y[x],x,IncludeSingularS
```

$$
\begin{aligned}
y(x) \to{} & c_1 \exp\left(\int_1^x \operatorname{Root}\left[2048K[1]^4 - 3484K[1]^3 + 2313K[1]^2 - 702K[1]\right.\right. \\
& + \left(20736K[1]^8 - 82944K[1]^7 + 124416K[1]^6 - 82944K[1]^5 + 20736K[1]^4\right)\#1^4 \\
& + \left(-48384K[1]^7 + 165888K[1]^6 - 207360K[1]^5 + 110592K[1]^4 - 20736K[1]^3\right)\#1^3 \\
& + \left(41472K[1]^6 - 118368K[1]^5 + 120096K[1]^4 - 50976K[1]^3 + 7776K[1]^2\right)\#1^2 \\
& + \left(-15360K[1]^5 + 34992K[1]^4 - 28272K[1]^3 + 9936K[1]^2 - 1296K[1]\right)\#1 \\
& \left.\left. + 81\&, 1\right] dK[1]\right) + c_2 \exp\left(\int_1^x \operatorname{Root}\left[2048K[1]^4 - 3484K[1]^3 + 2313K[1]^2 - 702K[1]\right.\right. \\
& + \left(20736K[1]^8 - 82944K[1]^7 + 124416K[1]^6 - 82944K[1]^5 + 20736K[1]^4\right)\#1^4 \\
& + \left(-48384K[1]^7 + 165888K[1]^6 - 207360K[1]^5 + 110592K[1]^4 - 20736K[1]^3\right)\#1^3 \\
& + \left(41472K[1]^6 - 118368K[1]^5 + 120096K[1]^4 - 50976K[1]^3 + 7776K[1]^2\right)\#1^2 \\
& + \left(-15360K[1]^5 + 34992K[1]^4 - 28272K[1]^3 + 9936K[1]^2 - 1296K[1]\right)\#1 \\
& \left.\left. + 81\&, 1\right] dK[1]\right) \int_1^x \exp\left(-2\int_1^{K[2]} \operatorname{Root}\left[2048K[1]^4 - 3484K[1]^3 + 2313K[1]^2\right.\right. \\
& - 702K[1] + \left(20736K[1]^8 - 82944K[1]^7 + 124416K[1]^6 - 82944K[1]^5 + 20736K[1]^4\right)\#1^4 \\
& + \left(-48384K[1]^7 + 165888K[1]^6 - 207360K[1]^5 + 110592K[1]^4 - 20736K[1]^3\right)\#1^3 + \left(41472K[1]^6 - 11836\right. \\
& \left.\left. + \left(-15360K[1]^5 + 34992K[1]^4 - 28272K[1]^3 + 9936K[1]^2 - 1296K[1]\right)\#1 + 81\&, 1\right] dK[1]\right) dK[2]
\end{aligned}
$$

2.2 problem 2

Internal problem ID [8318]

Book: Collection of Kovacic problems
Section: section 2. Solution found using all possible Kovacic cases
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - \frac{20y}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)=((4*(9/2)^2-1)/(4*x^2))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^4} + c_2 x^5$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 18

```
DSolve[y''[x]==((4*(9/2)^2-1)/(4*x^2))*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 x^9 + c_1}{x^4}$$

2.3 problem 3

Internal problem ID [8319]

Book: Collection of Kovacic problems
Section: section 2. Solution found using all possible Kovacic cases
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - \frac{12y}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)=((4*(7/2)^2-1)/(4*x^2))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^3} + x^4 c_2$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 18

```
DSolve[y''[x]==((4*(7/2)^2-1)/(4*x^2))*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 x^7 + c_1}{x^3}$$

2.4 problem 4

Internal problem ID [8320]

Book: Collection of Kovacic problems
Section: section 2. Solution found using all possible Kovacic cases
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - \frac{y}{4x^2} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)-1/(4*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,x^{\frac{\sqrt{2}}{2}} + c_2\sqrt{x}\,x^{-\frac{\sqrt{2}}{2}}$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 32

```
DSolve[y''[x]-1/(4*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^{\frac{1}{2}-\frac{1}{\sqrt{2}}}\left(c_2x^{\sqrt{2}} + c_1\right)$$

2.5 problem 5

Internal problem ID [8321]

Book: Collection of Kovacic problems
Section: section 2. Solution found using all possible Kovacic cases
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (2x+2)\,y' + (x+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(diff(y(x),x),x)-(2*x+2)*diff(y(x),x)+(2+x)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x} + c_2 \mathrm{e}^{x} x^3$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 23

```
DSolve[x*y''[x]-(2*x+2)*y'[x]+(2+x)*y[x] ==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} e^{x} \left(c_2 x^3 + 3c_1\right)$$

2.6 problem 6

Internal problem ID [8322]

Book: Collection of Kovacic problems
Section: section 2. Solution found using all possible Kovacic cases
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + \frac{y}{x^2} = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+1/x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,x^{\frac{\sqrt{-3}}{2}} + c_2\sqrt{x}\,x^{-\frac{\sqrt{-3}}{2}}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 42

```
DSolve[y''[x]+1/x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}\left(c_1 \cos\left(\frac{1}{2}\sqrt{3}\log(x)\right) + c_2 \sin\left(\frac{1}{2}\sqrt{3}\log(x)\right)\right)$$

2.7 problem 7

Internal problem ID [8323]

Book: Collection of Kovacic problems
Section: section 2. Solution found using all possible Kovacic cases
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1-x^2)\,y''+y'+y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 177

```
dsolve((1-x^2)*diff(y(x),x$2)+diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\sqrt{-3+2x}\left(\frac{3\sqrt{5}\,x-2\sqrt{5}-5\sqrt{x^2-1}}{3\sqrt{5}\,x-2\sqrt{5}+5\sqrt{x^2-1}}\right)^{\frac{1}{4}}\left(x+\sqrt{x^2-1}\right)^{\frac{3\sqrt{5}}{10}}\left(x+\sqrt{x^2-1}\right)^{\frac{\sqrt{5}}{5}}+c_2\sqrt{-3+2x}\left(\frac{3\sqrt{5}\,x-2\sqrt{5}+5\sqrt{x^2-1}}{3\sqrt{5}\,x-2\sqrt{5}-5\sqrt{x^2-1}}\right)^{\frac{1}{4}}\left(x+\sqrt{x^2-1}\right)^{-\frac{3\sqrt{5}}{10}}\left(x+\sqrt{x^2-1}\right)^{-\frac{\sqrt{5}}{5}}$$

✓ Solution by Mathematica

Time used: 36.335 (sec). Leaf size: 171

```
DSolve[(1-x^2)*y''[x]+y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{\sqrt[4]{1-x}\left(\sqrt{5}\sqrt{x-1}-\sqrt{x+1}\right)e^{2\sqrt{5}\text{arctanh}\left(\frac{\sqrt{x+1}+\sqrt{2}}{\sqrt{x-1}}\right)}\left(c_2\int_1^x\frac{2e^{-4\sqrt{5}\text{arctanh}\left(\frac{\sqrt{K[1]+1}+\sqrt{2}}{\sqrt{K[1]-1}}\right)}\sqrt{\frac{K[1]-1}{K[1]+1}}}{\left(\sqrt{K[1]+1}-\sqrt{5}\sqrt{K[1]-1}\right)^2}dK[1]+c_1\right)}{\sqrt{2}\sqrt[4]{x-1}}$$

2.8 problem 8

Internal problem ID [8324]

Book: Collection of Kovacic problems
Section: section 2. Solution found using all possible Kovacic cases
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2 - x)\, y'' - xy' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve((x^2-x)*diff(y(x), x$2)-x*diff(y(x), x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 x + c_2(x \ln (x) + 1)$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 20

```
DSolve[(x^2-x)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x - c_2(x \log(x) + 1)$$

2.9 problem 9

Internal problem ID [8325]

Book: Collection of Kovacic problems
Section: section 2. Solution found using all possible Kovacic cases
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(-x^2+2)\,y''-x(4x^2+3)\,y'+(-2x^2+2)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(x^2*(2-x^2)*diff(y(x), x$2) - x*(3+4*x^2)*diff(y(x), x) + (2-2*x^2)*y(x) = 0,y(x), si
```

$$y(x)=\frac{c_1\sqrt{x}\,(x^2+1)}{(x^2-2)^{\frac{7}{4}}}+\frac{c_2\sqrt{x}\,(x^2+1)\left(\int\frac{(x^2-2)^{\frac{3}{4}}\sqrt{x}}{(x^2+1)^2}dx\right)}{(x^2-2)^{\frac{7}{4}}}$$

✓ Solution by Mathematica

Time used: 20.212 (sec). Leaf size: 86

```
DSolve[x^2*(2-x^2)*y''[x] - x*(3+4*x^2)*y'[x] + (2-2*x^2)*y[x] == 0,y[x],x,IncludeSingularSo
```

$$y(x)\to\frac{2^{3/4}c_2(x^2+1)\,x^2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{x^2}{2}\right)+3c_2(2-x^2)^{3/4}\,x^2+6c_1(x^2+1)\sqrt{x}}{6\,(2-x^2)^{7/4}}$$

3 section 3. Problems from Kovacic related papers

3.1 problem Kovacic 1985 paper. page 13. section 3.2, example 1 864
3.2 problem Kovacic 1985 paper. page 14. section 3.2, example 2 865
3.3 problem Kovacic 1985 paper. page 15. Weber equation 866
3.4 problem Kovacic 1985 paper. page 19. section 4.2. Example 1 867
3.5 problem Kovacic 1985 paper. page 23. section 5.2. Example 1 868
3.6 problem Kovacic 1985 paper. page 25. section 5.2. Example 2 870
3.7 problem Kovacic 2005 paper. Example 2 871
3.8 problem David Saunders 1981 paper. Example 1 872
3.9 problem David Saunders 1981 paper. Example 3 873
3.10 problem Carolyn J. Smith 1984 paper. Appendix B examples and tests. Example 1 874
3.11 problem Carolyn J. Smith 1984 paper. Appendix B examples and tests. Example 2 875
3.12 problem Carolyn J. Smith 1984 paper. Appendix B examples and tests. Example 3 876

3.1 problem Kovacic 1985 paper. page 13. section 3.2, example 1

Internal problem ID [8326]

Book: Collection of Kovacic problems
Section: section 3. Problems from Kovacic related papers
Problem number: Kovacic 1985 paper. page 13. section 3.2, example 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \frac{(4x^6 - 8x^5 + 12x^4 + 4x^3 + 7x^2 - 20x + 4)\, y}{4x^4} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 89

```
dsolve(diff(y(x),x$2)= (4*x^6-8*x^5+12*x^4+4*x^3+7*x^2-20*x+4)/(4*x^4)*y(x),y(x), singsol=al
```

$$y(x) = \frac{c_1 \mathrm{e}^{\frac{x^3-2x^2-2}{2x}}(x^2-1)}{x^{\frac{3}{2}}} + \frac{c_2 \mathrm{e}^{\frac{x^3-2x^2-2}{2x}}(x^2-1)\left(\int \frac{x^3 \mathrm{e}^{-\frac{x^3-2x^2-2}{x}}}{(x-1)^2 (x+1)^2} dx\right)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.629 (sec). Leaf size: 79

```
DSolve[y''[x]== (4*x^6-8*x^5+12*x^4+4*x^3+7*x^2-20*x+4)/(4*x^4)*y[x],y[x],x,IncludeSingularS
```

$$y(x) \to \frac{e^{\frac{x^2}{2}-x-\frac{1}{x}}(x^2-1)\left(c_2 \int_1^x \frac{e^{-K[1]^2+2K[1]+\frac{2}{K[1]}} K[1]^3}{(K[1]^2-1)^2} dK[1] + c_1\right)}{x^{3/2}}$$

3.2 problem Kovacic 1985 paper. page 14. section 3.2, example 2

Internal problem ID [8327]

Book: Collection of Kovacic problems
Section: section 3. Problems from Kovacic related papers
Problem number: Kovacic 1985 paper. page 14. section 3.2, example 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \left(-1 + \frac{6}{x^2}\right) y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 47

```
dsolve(diff(y(x),x$2)= ( (4*(5/2)^2-1)/(4*x^2)-1)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1(\cos(x)\,x^2 - 3x\sin(x) - 3\cos(x))}{x^2} + \frac{c_2(x^2\sin(x) + 3\cos(x)\,x - 3\sin(x))}{x^2}$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 21

```
DSolve[y''[x]== ( (4*(5/2)^2-1)/(4*x^2)-1)*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(c_1 j_2(x) - c_2 y_2(x))$$

3.3 problem Kovacic 1985 paper. page 15. Weber equation

Internal problem ID [8328]

Book: Collection of Kovacic problems
Section: section 3. Problems from Kovacic related papers
Problem number: Kovacic 1985 paper. page 15. Weber equation.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \left(\frac{x^2}{4} - \frac{11}{2}\right) y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 67

```
dsolve(diff(y(x),x$2)= (1/4*x^2-1/2-5)*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x^2}{4}} x(x^4 - 10x^2 + 15) + c_2 \mathrm{e}^{-\frac{x^2}{4}} x(x^4 - 10x^2 + 15) \left(\int \frac{\mathrm{e}^{\frac{x^2}{2}}}{(x^4 - 10x^2 + 15)^2 x^2} dx\right)$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 22

```
DSolve[y''[x]== (1/4*x^2-1/2-5)*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \,\mathrm{ParabolicCylinderD}(-6, ix) + c_1 \,\mathrm{ParabolicCylinderD}(5, x)$$

3.4 problem Kovacic 1985 paper. page 19. section 4.2. Example 1

Internal problem ID [8329]

Book: Collection of Kovacic problems
Section: section 3. Problems from Kovacic related papers
Problem number: Kovacic 1985 paper. page 19. section 4.2. Example 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \left(\frac{1}{x} - \frac{3}{16x^2}\right) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)= (1/x-3/(16*x^2))*y(x),y(x), singsol=all)
```

$$y(x) = c_1 x^{\frac{1}{4}} \sinh\left(2\sqrt{x}\right) + c_2 x^{\frac{1}{4}} \cosh\left(2\sqrt{x}\right)$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 41

```
DSolve[y''[x]== (1/x-3/(16*x^2))*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-2\sqrt{x}} \sqrt[4]{x} \left(2c_1 e^{4\sqrt{x}} - c_2\right)$$

3.5 problem Kovacic 1985 paper. page 23. section 5.2. Example 1

Internal problem ID [8330]

Book: Collection of Kovacic problems
Section: section 3. Problems from Kovacic related papers
Problem number: Kovacic 1985 paper. page 23. section 5.2. Example 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \left(-\frac{3}{16x^2} - \frac{2}{9(x-1)^2} + \frac{3}{16x(x-1)}\right) y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 209

```
dsolve(diff(y(x),x$2)= ( -3/(16*x^2) - 2/(9*(x-1)^2) + 3/(16*x*(x-1))  )*y(x),y(x), singsol=
```

$$y(x) = c_1 \sqrt{(x-1)^{\frac{1}{3}}+1}\left((x-1)^{\frac{2}{3}}-(x-1)^{\frac{1}{3}}+1\right)^{\frac{1}{4}}(x-1)^{\frac{1}{3}}\left(\frac{\sqrt{3}\,(x-1)^{\frac{1}{3}}+2\sqrt{(x-1)^{\frac{2}{3}}-(x-1)^{\frac{1}{3}}+1}-\sqrt{3}}{-\sqrt{3}\,(x-1)^{\frac{1}{3}}+2\sqrt{(x-1)^{\frac{2}{3}}-(x-1)^{\frac{1}{3}}+1}+\sqrt{3}}\right)^{\frac{1}{8}} + \frac{c_2\sqrt{(x-1)^{\frac{1}{3}}+1}\left((x-1)^{\frac{2}{3}}-(x-1)^{\frac{1}{3}}+1\right)^{\frac{1}{4}}(x-1)^{\frac{1}{3}}}{\left(\frac{\sqrt{3}\,(x-1)^{\frac{1}{3}}+2\sqrt{(x-1)^{\frac{2}{3}}-(x-1)^{\frac{1}{3}}+1}-\sqrt{3}}{-\sqrt{3}\,(x-1)^{\frac{1}{3}}+2\sqrt{(x-1)^{\frac{2}{3}}-(x-1)^{\frac{1}{3}}+1}+\sqrt{3}}\right)^{\frac{1}{8}}}$$

✓ Solution by Mathematica

Time used: 0.235 (sec). Leaf size: 550

```
DSolve[y''[x]== ( -3/(16*x^2) - 2/(9*(x-1)^2) + 3/(16*x*(x-1))  )*y[x],y[x],x,IncludeSingula
```

$$
\begin{aligned}
y(x) \to\ & c_1 \exp\left(\int_1^x \operatorname{Root}\big[2048K[1]^4 - 3484K[1]^3 + 2313K[1]^2 - 702K[1] \right. \\
& + \left(20736K[1]^8 - 82944K[1]^7 + 124416K[1]^6 - 82944K[1]^5 + 20736K[1]^4\right)\#1^4 \\
& + \left(-48384K[1]^7 + 165888K[1]^6 - 207360K[1]^5 + 110592K[1]^4 - 20736K[1]^3\right)\#1^3 \\
& + \left(41472K[1]^6 - 118368K[1]^5 + 120096K[1]^4 - 50976K[1]^3 + 7776K[1]^2\right)\#1^2 \\
& + \left(-15360K[1]^5 + 34992K[1]^4 - 28272K[1]^3 + 9936K[1]^2 - 1296K[1]\right)\#1 \\
& \left. + 81\&, 1\big]\, dK[1]\right) + c_2 \exp\left(\int_1^x \operatorname{Root}\big[2048K[1]^4 - 3484K[1]^3 + 2313K[1]^2 - 702K[1] \right. \\
& + \left(20736K[1]^8 - 82944K[1]^7 + 124416K[1]^6 - 82944K[1]^5 + 20736K[1]^4\right)\#1^4 \\
& + \left(-48384K[1]^7 + 165888K[1]^6 - 207360K[1]^5 + 110592K[1]^4 - 20736K[1]^3\right)\#1^3 \\
& + \left(41472K[1]^6 - 118368K[1]^5 + 120096K[1]^4 - 50976K[1]^3 + 7776K[1]^2\right)\#1^2 \\
& + \left(-15360K[1]^5 + 34992K[1]^4 - 28272K[1]^3 + 9936K[1]^2 - 1296K[1]\right)\#1 \\
& \left. + 81\&, 1\big]\, dK[1]\right) \int_1^x \exp\left(-2\int_1^{K[2]} \operatorname{Root}\big[2048K[1]^4 - 3484K[1]^3 + 2313K[1]^2 \right. \\
& - 702K[1] + \left(20736K[1]^8 - 82944K[1]^7 + 124416K[1]^6 - 82944K[1]^5 + 20736K[1]^4\right)\#1^4 \\
& + \left(-48384K[1]^7 + 165888K[1]^6 - 207360K[1]^5 + 110592K[1]^4 - 20736K[1]^3\right)\#1^3 + \left(41472K[1]^6 - 11836\right. \\
& \left. + \left(-15360K[1]^5 + 34992K[1]^4 - 28272K[1]^3 + 9936K[1]^2 - 1296K[1]\right)\#1 + 81\&, 1\big]\, dK[1]\right) dK[2]
\end{aligned}
$$

3.6 problem Kovacic 1985 paper. page 25. section 5.2. Example 2

Internal problem ID [8331]

Book: Collection of Kovacic problems
Section: section 3. Problems from Kovacic related papers
Problem number: Kovacic 1985 paper. page 25. section 5.2. Example 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{(5x^2 + 27)\, y}{36\,(x^2 - 1)^2} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 103

```
dsolve(diff(y(x),x$2)= -(5*x^2+27)/(36*(x^2-1)^2)*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \left(x^2 - 1\right)^{\frac{1}{3}} e^{\int \mathrm{RootOf}\left(-1+\left(432x^4-864x^2+432\right)\_Z^4+\left(-72x^2+72\right)\_Z^2+16x\_Z,\mathrm{index}=1\right)dx} + c_2 \left(x^2 - 1\right)^{\frac{1}{3}} e^{\int \mathrm{RootOf}\left(-1+\left(432x^4-864x^2+432\right)\_Z^4+\left(-72x^2+72\right)\_Z^2+16x\_Z,\mathrm{index}=2\right)dx}$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 38

```
DSolve[y''[x]== -(5*x^2+27)/(36*(x^2-1)^2)*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x^2 - 1}\left(c_1 P_{-\frac{1}{6}}^{\frac{1}{3}}(x) + c_2 Q_{-\frac{1}{6}}^{\frac{1}{3}}(x)\right)$$

3.7 problem Kovacic 2005 paper. Example 2

Internal problem ID [8332]

Book: Collection of Kovacic problems
Section: section 3. Problems from Kovacic related papers
Problem number: Kovacic 2005 paper. Example 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + \frac{y}{4x^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)= -1/(4*x^2)*y(x),y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x} + c_2\sqrt{x}\,\ln(x)$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 24

```
DSolve[y''[x]== -1/(4*x^2)*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\sqrt{x}(c_2\log(x) + 2c_1)$$

3.8 problem David Saunders 1981 paper. Example 1

Internal problem ID [8333]

Book: Collection of Kovacic problems
Section: section 3. Problems from Kovacic related papers
Problem number: David Saunders 1981 paper. Example 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - (x^2+3)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)= (x^2+3)*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{x^2}{2}} x + c_2 \mathrm{e}^{\frac{x^2}{2}} \left(\sqrt{\pi}\, \operatorname{erf}(x)\, x + \mathrm{e}^{-x^2}\right)$$

✓ Solution by Mathematica
Time used: 0.077 (sec). Leaf size: 46

```
DSolve[y''[x]== (x^2+3)*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{x^2}{2}} \left(-\sqrt{\pi} c_2 e^{x^2} x \operatorname{erf}(x) + c_1 e^{x^2} x - c_2\right)$$

3.9 problem David Saunders 1981 paper. Example 3

Internal problem ID [8334]

Book: Collection of Kovacic problems
Section: section 3. Problems from Kovacic related papers
Problem number: David Saunders 1981 paper. Example 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2y'' - 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)= 2*y(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + c_2x^2$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]== 2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^3 + c_1}{x}$$

3.10 problem Carolyn J. Smith 1984 paper. Appendix B examples and tests. Example 1

Internal problem ID [8335]

Book: Collection of Kovacic problems
Section: section 3. Problems from Kovacic related papers
Problem number: Carolyn J. Smith 1984 paper. Appendix B examples and tests. Example 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4xy' + (4x^2 + 2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x^2} + c_2 x\, e^{-x^2}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 20

```
DSolve[y''[x]+4*x*y'[x]+(4*x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x^2}(c_2 x + c_1)$$

3.11 problem Carolyn J. Smith 1984 paper. Appendix B examples and tests. Example 2

Internal problem ID [8336]

Book: Collection of Kovacic problems
Section: section 3. Problems from Kovacic related papers
Problem number: Carolyn J. Smith 1984 paper. Appendix B examples and tests. Example 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2xy' + (x^2+2)\,y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sin(x) + c_2 \cos(x)\, x$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-ix} x - \frac{1}{2} i c_2 e^{ix} x$$

3.12 problem Carolyn J. Smith 1984 paper. Appendix B examples and tests. Example 3

Internal problem ID [8337]

Book: Collection of Kovacic problems
Section: section 3. Problems from Kovacic related papers
Problem number: Carolyn J. Smith 1984 paper. Appendix B examples and tests. Example 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x-2)^2 y'' - (x-2) y' - 3y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
dsolve((x-2)^2*diff(y(x),x$2)-(x-2)*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x-2} + \frac{c_2(x^4 - 8x^3 + 24x^2 - 32x)}{x-2}$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 22

```
DSolve[(x-2)^2*y''[x]-(x-2)*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x-2)^3 + \frac{c_2}{x-2}$$