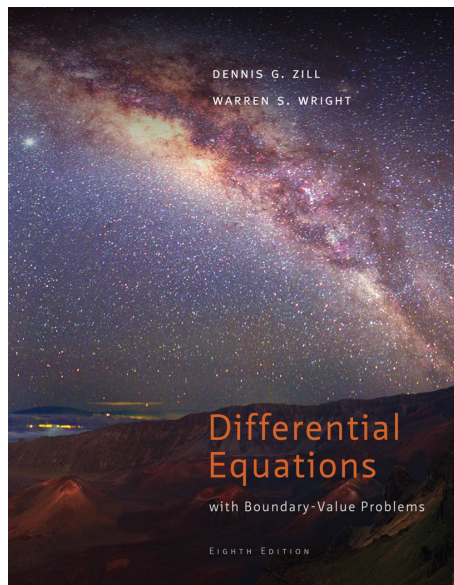


A Solution Manual For

**DIFFERENTIAL EQUATIONS with
Boundary Value Problems. DENNIS G.
ZILL, WARREN S. WRIGHT,
MICHAEL R. CULLEN. Brooks/Cole.
Boston, MA. 2013. 8th edition.**



Nasser M. Abbasi

May 16, 2024

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1.1 problem 3. series method

Internal problem ID [6550]

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Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 3. series method.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;  
dsolve(diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6\right) y(0) + \left(x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[y''[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left(-\frac{x^7}{5040} + \frac{x^5}{120} - \frac{x^3}{6} + x \right) + c_1 \left(-\frac{x^6}{720} + \frac{x^4}{24} - \frac{x^2}{2} + 1 \right)$$

1.2 problem 3. direct method

Internal problem ID [6551]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 3. direct method.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x$2)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(x) + \cos(x) c_2$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 16

```
DSolve[y''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \cos(x) + c_2 \sin(x)$$

1.3 problem 4. series method

Internal problem ID [6552]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 4. series method.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;  
dsolve(diff(y(x),x$2)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 + \frac{1}{24}x^4 + \frac{1}{720}x^6\right) y(0) + \left(x + \frac{1}{6}x^3 + \frac{1}{120}x^5 + \frac{1}{5040}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[y''[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left(\frac{x^7}{5040} + \frac{x^5}{120} + \frac{x^3}{6} + x \right) + c_1 \left(\frac{x^6}{720} + \frac{x^4}{24} + \frac{x^2}{2} + 1 \right)$$

1.4 problem 4. direct method

Internal problem ID [6553]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 4. direct method.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x} + e^x c_2$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 20

```
DSolve[y''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^x + c_2 e^{-x}$$

1.5 problem 5. series method

Internal problem ID [6554]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 5. series method.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - y' = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;  
dsolve(diff(y(x),x$2)-diff(y(x),x)=0,y(x),type='series',x=0);
```

$$y(x) = y(0) + \left(x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \frac{1}{5040}x^7 \right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 53

```
AsymptoticDSolveValue[y''[x]-y'[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left(\frac{x^7}{5040} + \frac{x^6}{720} + \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x \right) + c_1$$

1.6 problem 5. direct method

Internal problem ID [6555]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 5. direct method.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x$2)-diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = e^x c_1 + c_2$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 14

```
DSolve[y''[x]-y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^x + c_2$$

1.7 problem 6. series method

Internal problem ID [6556]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 6. series method.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 2y' = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;  
dsolve(diff(y(x),x$2)+2*diff(y(x),x)=0,y(x),type='series',x=0);
```

$$y(x) = y(0) + \left(x - x^2 + \frac{2}{3}x^3 - \frac{1}{3}x^4 + \frac{2}{15}x^5 - \frac{2}{45}x^6 + \frac{4}{315}x^7 \right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 51

```
AsymptoticDSolveValue[y''[x]+2*y'[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left(\frac{4x^7}{315} - \frac{2x^6}{45} + \frac{2x^5}{15} - \frac{x^4}{3} + \frac{2x^3}{3} - x^2 + x \right) + c_1$$

1.8 problem 6. direct method

Internal problem ID [6557]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 6. direct method.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 2y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 e^{-2x}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 19

```
DSolve[y''[x]+2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_2 - \frac{1}{2}c_1 e^{-2x}$$

1.9 problem 7

Internal problem ID [6558]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 7.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$y'' - xy = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=8;  
dsolve(diff(y(x),x$2)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{6}x^3 + \frac{1}{180}x^6\right) y(0) + \left(x + \frac{1}{12}x^4 + \frac{1}{504}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y'[x]-x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left(\frac{x^7}{504} + \frac{x^4}{12} + x \right) + c_1 \left(\frac{x^6}{180} + \frac{x^3}{6} + 1 \right)$$

1.10 problem 8

Internal problem ID [6559]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 8.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$y'' + yx^2 = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
Order:=8;  
dsolve(diff(y(x),x$2)+x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^4}{12}\right) y(0) + \left(x - \frac{1}{20}x^5\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]+x^2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left(x - \frac{x^5}{20}\right) + c_1 \left(1 - \frac{x^4}{12}\right)$$

1.11 problem 9

Internal problem ID [6560]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 9.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [Lienard]

$$y'' - 2xy' + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;  
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 - \frac{7}{240}x^6\right) y(0) + \left(x + \frac{1}{6}x^3 + \frac{1}{24}x^5 + \frac{1}{112}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[y''[x]-2*x*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left(\frac{x^7}{112} + \frac{x^5}{24} + \frac{x^3}{6} + x \right) + c_1 \left(-\frac{7x^6}{240} - \frac{x^4}{8} - \frac{x^2}{2} + 1 \right)$$

1.12 problem 10

Internal problem ID [6561]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 10.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [Hermite]

$$y'' - xy' + 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
Order:=8;  
dsolve(diff(y(x),x$2)-x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (-x^2 + 1)y(0) + \left(x - \frac{1}{6}x^3 - \frac{1}{120}x^5 - \frac{1}{1680}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[y'[x]-x*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1(1 - x^2) + c_2\left(-\frac{x^7}{1680} - \frac{x^5}{120} - \frac{x^3}{6} + x\right)$$

1.13 problem 11

Internal problem ID [6562]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 11.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + x^2 y' + xy = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
Order:=8;  
dsolve(diff(y(x),x$2)+x^2*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{6}x^3 + \frac{1}{45}x^6\right) y(0) + \left(x - \frac{1}{6}x^4 + \frac{5}{252}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]+x^2*y'[x]+x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left(\frac{5x^7}{252} - \frac{x^4}{6} + x \right) + c_1 \left(\frac{x^6}{45} - \frac{x^3}{6} + 1 \right)$$

1.14 problem 12

Internal problem ID [6563]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 12.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$y'' + 2xy' + 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;  
dsolve(diff(y(x),x$2)+2*x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x^2 + \frac{1}{2}x^4 - \frac{1}{6}x^6\right) y(0) + \left(x - \frac{2}{3}x^3 + \frac{4}{15}x^5 - \frac{8}{105}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 54

```
AsymptoticDSolveValue[y''[x]+2*x*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left(-\frac{8x^7}{105} + \frac{4x^5}{15} - \frac{2x^3}{3} + x \right) + c_1 \left(-\frac{x^6}{6} + \frac{x^4}{2} - x^2 + 1 \right)$$

1.15 problem 13

Internal problem ID [6564]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 13.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$(x - 1)y'' + y' = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;  
dsolve((x-1)*diff(y(x),x$2)+diff(y(x),x)=0,y(x),type='series',x=0);
```

$$y(x) = y(0) + \left(x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \frac{1}{5}x^5 + \frac{1}{6}x^6 + \frac{1}{7}x^7 \right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 53

```
AsymptoticDSolveValue[(x-1)*y''[x]+y'[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left(\frac{x^7}{7} + \frac{x^6}{6} + \frac{x^5}{5} + \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x \right) + c_1$$

1.16 problem 14

Internal problem ID [6565]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 14.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x + 2)y'' + xy' - y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
Order:=8;  
dsolve((x+2)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{4}x^2 - \frac{1}{24}x^3 + \frac{1}{480}x^5 - \frac{1}{1440}x^6 + \frac{1}{6720}x^7\right) y(0) + D(y)(0)x + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 91

```
AsymptoticDSolveValue[(x+2)*y''[x]+x*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left(\frac{29x^7}{20160} - \frac{7x^6}{1440} + \frac{x^5}{240} + \frac{x^4}{24} - \frac{x^3}{6} + x \right) + c_1 \left(\frac{x^7}{8064} + \frac{x^6}{576} - \frac{x^5}{96} + \frac{x^4}{48} + \frac{x^3}{24} - \frac{x^2}{4} + 1 \right)$$

1.17 problem 15

Internal problem ID [6566]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 15.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$y'' - (1 + x)y' - y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 74

```
Order:=8;
dsolve(diff(y(x),x$2)-(x+1)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{6}x^4 + \frac{1}{15}x^5 + \frac{7}{180}x^6 + \frac{19}{1260}x^7\right) y(0) \\ + \left(x + \frac{1}{2}x^2 + \frac{1}{2}x^3 + \frac{1}{4}x^4 + \frac{3}{20}x^5 + \frac{1}{15}x^6 + \frac{13}{420}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 98

```
AsymptoticDSolveValue[y''[x]-(x+1)*y'[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left(\frac{19x^7}{1260} + \frac{7x^6}{180} + \frac{x^5}{15} + \frac{x^4}{6} + \frac{x^3}{6} + \frac{x^2}{2} + 1 \right) \\ + c_2 \left(\frac{13x^7}{420} + \frac{x^6}{15} + \frac{3x^5}{20} + \frac{x^4}{4} + \frac{x^3}{2} + \frac{x^2}{2} + x \right)$$

1.18 problem 16

Internal problem ID [6567]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 16.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$(x^2 + 1)y'' - 6y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
Order:=8;  
dsolve((x^2+1)*diff(y(x),x$2)-6*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + 3x^2 + x^4 - \frac{1}{5}x^6\right) y(0) + (x^3 + x) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 32

```
AsymptoticDSolveValue[(x^2+1)*y''[x]-6*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2(x^3 + x) + c_1\left(-\frac{x^6}{5} + x^4 + 3x^2 + 1\right)$$

1.19 problem 17

Internal problem ID [6568]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 17.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 + 2)y'' + 3xy' - y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 44

```
Order:=8;  
dsolve((x^2+2)*diff(y(x),x$2)+3*x*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{4}x^2 - \frac{7}{96}x^4 + \frac{161}{5760}x^6\right) y(0) + \left(x - \frac{1}{6}x^3 + \frac{7}{120}x^5 - \frac{17}{720}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[(x^2+2)*y''[x]+3*x*y'[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left(-\frac{17x^7}{720} + \frac{7x^5}{120} - \frac{x^3}{6} + x \right) + c_1 \left(\frac{161x^6}{5760} - \frac{7x^4}{96} + \frac{x^2}{4} + 1 \right)$$

1.20 problem 18

Internal problem ID [6569]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 18.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(x^2 - 1)y'' + xy' - y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
Order:=8;  
dsolve((x^2-1)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 - \frac{1}{16}x^6\right)y(0) + D(y)(0)x + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[(x^2-1)*y''[x]+x*y'[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left(-\frac{x^6}{16} - \frac{x^4}{8} - \frac{x^2}{2} + 1\right) + c_2x$$

1.21 problem 19

Internal problem ID [6570]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 19.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x - 1)y'' - xy' + y = 0$$

With initial conditions

$$[y(0) = -2, y'(0) = 6]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=8;
```

```
dsolve([(x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(0) = -2, D(y)(0) = 6],y(x),type='series
```

$$y(x) = -2 + 6x - x^2 - \frac{1}{3}x^3 - \frac{1}{12}x^4 - \frac{1}{60}x^5 - \frac{1}{360}x^6 - \frac{1}{2520}x^7 + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 48

```
AsymptoticDSolveValue[{(x-1)*y''[x]-x*y'[x]+y[x]==0,{y[0]==-2,y'[0]==6}},y[x],{x,0,7}]
```

$$y(x) \rightarrow -\frac{x^7}{2520} - \frac{x^6}{360} - \frac{x^5}{60} - \frac{x^4}{12} - \frac{x^3}{3} - x^2 + 6x - 2$$

1.22 problem 20

Internal problem ID [6571]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 20.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(1 + x)y'' - (2 - x)y' + y = 0$$

With initial conditions

$$[y(0) = 2, y'(0) = -1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=8;
```

```
dsolve([(x+1)*diff(y(x),x$2)-(2-x)*diff(y(x),x)+y(x)=0,y(0) = 2, D(y)(0) = -1],y(x),type='se
```

$$y(x) = 2 - x - 2x^2 - \frac{1}{3}x^3 + \frac{1}{2}x^4 - \frac{1}{30}x^5 - \frac{13}{180}x^6 + \frac{1}{28}x^7 + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 48

```
AsymptoticDSolveValue[{{(x+1)*y''[x]-(2-x)*y'[x]+y[x]==0,{y[0]==2,y'[0]==-1}},y[x],{x,0,7}]
```

$$y(x) \rightarrow \frac{x^7}{28} - \frac{13x^6}{180} - \frac{x^5}{30} + \frac{x^4}{2} - \frac{x^3}{3} - 2x^2 - x + 2$$

1.23 problem 21

Internal problem ID [6572]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 21.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 2xy' + 8y = 0$$

With initial conditions

$$[y(0) = 3, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
Order:=8;  
dsolve([diff(y(x),x$2)-2*x*diff(y(x),x)+8*y(x)=0,y(0) = 3, D(y)(0) = 0],y(x),type='series',x
```

$$y(x) = 4x^4 - 12x^2 + 3$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 15

```
AsymptoticDSolveValue[{y'[x]-2*x*y'[x]+8*y[x]==0,{y[0]==3,y'[0]==0}},y[x],{x,0,7}]
```

$$y(x) \rightarrow 4x^4 - 12x^2 + 3$$

1.24 problem 22

Internal problem ID [6573]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 22.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$(x^2 + 1)y'' + 2xy' = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
Order:=8;
```

```
dsolve([(x^2+1)*diff(y(x),x$2)+2*x*diff(y(x),x)=0,y(0) = 0, D(y)(0) = 1],y(x),type='series',
```

$$y(x) = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 26

```
AsymptoticDSolveValue[{(x^2+1)*y''[x]+2*x*y'[x]==0,{y[0]==0,y'[0]==1}},y[x],{x,0,7}]
```

$$y(x) \rightarrow -\frac{x^7}{7} + \frac{x^5}{5} - \frac{x^3}{3} + x$$

1.25 problem 23

Internal problem ID [6574]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 23.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \sin(x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=8;  
dsolve(diff(y(x),x$2)+sin(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{6}x^3 + \frac{1}{120}x^5 + \frac{1}{180}x^6 - \frac{1}{5040}x^7\right) y(0) \\ + \left(x - \frac{1}{12}x^4 + \frac{1}{180}x^6 + \frac{1}{504}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[y''[x]+Sin[x]*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left(\frac{x^7}{504} + \frac{x^6}{180} - \frac{x^4}{12} + x \right) + c_1 \left(-\frac{x^7}{5040} + \frac{x^6}{180} + \frac{x^5}{120} - \frac{x^3}{6} + 1 \right)$$

1.26 problem 24

Internal problem ID [6575]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 24.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + y'e^x - y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 69

```
Order:=8;  
dsolve(diff(y(x),x$2)+exp(x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 - \frac{1}{6}x^3 - \frac{1}{120}x^5 + \frac{1}{240}x^6 + \frac{1}{840}x^7\right) y(0) \\ + \left(x - \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{24}x^4 + \frac{1}{120}x^5 - \frac{1}{720}x^6 + \frac{1}{5040}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 91

```
AsymptoticDSolveValue[y''[x]+Exp[x]*y'[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left(\frac{x^7}{840} + \frac{x^6}{240} - \frac{x^5}{120} - \frac{x^3}{6} + \frac{x^2}{2} + 1 \right) + c_2 \left(\frac{x^7}{5040} - \frac{x^6}{720} + \frac{x^5}{120} - \frac{x^4}{24} + \frac{x^3}{6} - \frac{x^2}{2} + x \right)$$

1.27 problem 25 expansion at 0

Internal problem ID [6576]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 25 expansion at 0.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\cos(x) y'' + y' + 5y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 69

```
Order:=8;
```

```
dsolve(cos(x)*diff(y(x),x$2)+diff(y(x),x)+5*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{5}{2}x^2 + \frac{5}{6}x^3 + \frac{5}{8}x^4 - \frac{5}{24}x^5 + \frac{1}{16}x^6 - \frac{13}{336}x^7\right) y(0) \\ + \left(x - \frac{1}{2}x^2 - \frac{2}{3}x^3 + \frac{1}{3}x^4 + \frac{1}{80}x^6 + \frac{11}{5040}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 91

```
AsymptoticDSolveValue[Cos[x]*y''[x]+y'[x]+5*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left(\frac{11x^7}{5040} + \frac{x^6}{80} + \frac{x^4}{3} - \frac{2x^3}{3} - \frac{x^2}{2} + x \right) \\ + c_1 \left(-\frac{13x^7}{336} + \frac{x^6}{16} - \frac{5x^5}{24} + \frac{5x^4}{8} + \frac{5x^3}{6} - \frac{5x^2}{2} + 1 \right)$$

1.28 problem 25 expansion at 1

Internal problem ID [6577]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 25 expansion at 1.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\cos(x) y'' + y' + 5y = 0$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 860

```
Order:=8;  
dsolve(cos(x)*diff(y(x),x$2)+diff(y(x),x)+5*y(x)=0,y(x),type='series',x=1);
```

Expression too large to display

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 1808

```
AsymptoticDSolveValue[Cos[x]*y''[x]+y'[x]+5*y[x]==0,y[x],{x,1,7}]
```

Too large to display

1.29 problem 26 (a)

Internal problem ID [6578]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 26 (a).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - xy = 1$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
Order:=8;  
dsolve(diff(y(x),x$2)-x*y(x)=1,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{6}x^3 + \frac{1}{180}x^6\right)y(0) + \left(x + \frac{1}{12}x^4 + \frac{1}{504}x^7\right)D(y)(0) + \frac{x^2}{2} + \frac{x^5}{40} + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 56

```
AsymptoticDSolveValue[y'[x]-x*y[x]==1,y[x],{x,0,7}]
```

$$y(x) \rightarrow \frac{x^5}{40} + \frac{x^2}{2} + c_2 \left(\frac{x^7}{504} + \frac{x^4}{12} + x \right) + c_1 \left(\frac{x^6}{180} + \frac{x^3}{6} + 1 \right)$$

1.30 problem 26 (b)

Internal problem ID [6579]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 26 (b).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$y'' - 4xy' - 4y = e^x$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 56

```
Order:=8;
dsolve(diff(y(x),x$2)-4*x*diff(y(x),x)-4*y(x)=exp(x),y(x),type='series',x=0);
```

$$y(x) = \left(1 + 2x^2 + 2x^4 + \frac{4}{3}x^6\right) y(0) + \left(x + \frac{4}{3}x^3 + \frac{16}{15}x^5 + \frac{64}{105}x^7\right) D(y)(0) \\ + \frac{x^2}{2} + \frac{x^3}{6} + \frac{13x^4}{24} + \frac{17x^5}{120} + \frac{29x^6}{80} + \frac{409x^7}{5040} + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 94

```
AsymptoticDSolveValue[y'[x]-4*x*y'[x]-4*y[x]==Exp[x],y[x],{x,0,7}]
```

$$y(x) \rightarrow \frac{409x^7}{5040} + \frac{29x^6}{80} + \frac{17x^5}{120} + \frac{13x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} \\ + c_2 \left(\frac{64x^7}{105} + \frac{16x^5}{15} + \frac{4x^3}{3} + x \right) + c_1 \left(\frac{4x^6}{3} + 2x^4 + 2x^2 + 1 \right)$$

1.31 problem 27

Internal problem ID [6580]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 27.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' + \sin(x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;
```

```
dsolve(x*diff(y(x),x$2)+sin(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{18}x^4 - \frac{53}{10800}x^6\right) y(0) \\ + \left(x - \frac{1}{6}x^3 + \frac{1}{60}x^5 - \frac{19}{15120}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[x*y''[x]+Sin[x]*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left(-\frac{19x^7}{15120} + \frac{x^5}{60} - \frac{x^3}{6} + x \right) + c_1 \left(-\frac{53x^6}{10800} + \frac{x^4}{18} - \frac{x^2}{2} + 1 \right)$$

1.32 problem 28

Internal problem ID [6581]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 28.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 5xy' + y\sqrt{x} = 0$$

With the expansion point for the power series method at $x = 0$.

X Solution by Maple

```
Order:=8;  
dsolve(diff(y(x),x$2)+5*x*diff(y(x),x)+sqrt(x)*y(x)=0,y(x),type='series',x=0);
```

No solution found

X Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
AsymptoticDSolveValue[y''[x]+5*x*y'[x]+Sqrt[x]*y[x]==0,y[x],{x,0,7}]
```

Not solved

1.33 problem 29 (a)

Internal problem ID [6582]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 29 (a).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$y'' + xy' + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;  
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{8}x^4 - \frac{1}{48}x^6\right) y(0) + \left(x - \frac{1}{3}x^3 + \frac{1}{15}x^5 - \frac{1}{105}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[y''[x]+x*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left(-\frac{x^7}{105} + \frac{x^5}{15} - \frac{x^3}{3} + x \right) + c_1 \left(-\frac{x^6}{48} + \frac{x^4}{8} - \frac{x^2}{2} + 1 \right)$$

1.34 problem 30 (b)

Internal problem ID [6583]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

Problem number: 30 (b).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + y \cos(x) = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = 1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=8;  
dsolve([diff(y(x),x$2)+cos(x)*y(x)=0,y(0) = 1, D(y)(0) = 1],y(x),type='series',x=0);
```

$$y(x) = 1 + x - \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{12}x^4 + \frac{1}{30}x^5 - \frac{1}{80}x^6 - \frac{19}{5040}x^7 + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 48

```
AsymptoticDSolveValue[{y'[x]+Cos[x]*y[x]==0,{y[0]==1,y'[0]==1}},y[x],{x,0,7}]
```

$$y(x) \rightarrow -\frac{19x^7}{5040} - \frac{x^6}{80} + \frac{x^5}{30} + \frac{x^4}{12} - \frac{x^3}{6} - \frac{x^2}{2} + x + 1$$

2 CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3.

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2.1 problem 1

Internal problem ID [6584]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 1.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^3y'' + 4x^2y' + 3y = 0$$

With the expansion point for the power series method at $x = 0$.

 Solution by Maple

Order:=8;

```
dsolve(x^3*diff(y(x),x$2)+4*x^2*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=0);
```

No solution found

 Solution by Mathematica

Time used: 0.089 (sec). Leaf size: 374

```
AsymptoticDSolveValue[x^3*y''[x]+4*x^2*y'[x]+3*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow \frac{c_1 e^{-\frac{2i\sqrt{3}}{\sqrt{x}} \left(-\frac{9447234753831875i\sqrt{3}x^{13/2}}{4611686018427387904} + \frac{3806522094375i\sqrt{3}x^{11/2}}{4503599627370496} - \frac{14315125825ix^{9/2}}{8796093022208\sqrt{3}} + \frac{8083075ix^{7/2}}{4294967296\sqrt{3}} - \frac{15015i\sqrt{3}x^{5/2}}{8388608} + 3 \right)}}{c_2 e^{\frac{2i\sqrt{3}}{\sqrt{x}} \left(\frac{9447234753831875i\sqrt{3}x^{13/2}}{4611686018427387904} - \frac{3806522094375i\sqrt{3}x^{11/2}}{4503599627370496} + \frac{14315125825ix^{9/2}}{8796093022208\sqrt{3}} - \frac{8083075ix^{7/2}}{4294967296\sqrt{3}} + \frac{15015i\sqrt{3}x^{5/2}}{8388608} - 3 \right)}} +$$

2.2 problem 2

Internal problem ID [6585]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 2.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(x+3)^2 y'' - y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 70

```
Order:=8;
dsolve(x*(x+3)^2*diff(y(x),x$2)-y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = & c_1 x \left(1 + \frac{1}{18}x - \frac{11}{972}x^2 + \frac{277}{104976}x^3 - \frac{12539}{18895680}x^4 + \frac{893821}{5101833600}x^5 \right. \\ & \left. - \frac{13183337}{275499014400}x^6 + \frac{265861081}{19835929036800}x^7 + O(x^8) \right) \\ & + c_2 \left(\ln(x) \left(\frac{1}{9}x + \frac{1}{162}x^2 - \frac{11}{8748}x^3 + \frac{277}{944784}x^4 - \frac{12539}{170061120}x^5 + \frac{893821}{45916502400}x^6 \right. \right. \\ & \left. \left. - \frac{13183337}{2479491129600}x^7 + O(x^8) \right) + \left(1 - \frac{5}{108}x^2 + \frac{167}{26244}x^3 - \frac{13583}{11337408}x^4 \right. \right. \\ & \left. \left. + \frac{1327279}{5101833600}x^5 - \frac{21146863}{344373768000}x^6 + \frac{379766273}{24794911296000}x^7 + O(x^8) \right) \right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.77 (sec). Leaf size: 121

```
AsymptoticDSolveValue[x*(x+3)^2*y'[x]-y[x]==0,y[x],{x,0,7}]
```

$$\begin{aligned} y(x) &\rightarrow c_1 \left(\frac{x(893821x^5 - 3385530x^4 + 13462200x^3 - 57736800x^2 + 283435200x + 5101833600) \log(x)}{45916502400} \right. \\ &\quad \left. + \frac{24742849x^6 - 74732085x^5 + 184497750x^4 + 52488000x^3 - 10628820000x^2 + 382637520000x + 688747536000}{688747536000} \right) \\ &\quad + c_2 \left(-\frac{13183337x^7}{275499014400} + \frac{893821x^6}{5101833600} - \frac{12539x^5}{18895680} + \frac{277x^4}{104976} - \frac{11x^3}{972} + \frac{x^2}{18} + x \right) \end{aligned}$$

2.3 problem 3

Internal problem ID [6586]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 3.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 - 9)^2 y'' + (x + 3) y' + 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 74

```
Order:=8;  
dsolve((x^2-9)^2*diff(y(x),x$2)+(x+3)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{81}x^2 + \frac{1}{6561}x^3 - \frac{289}{708588}x^4 + \frac{304}{23914845}x^5 - \frac{194981}{7748409780}x^6 + \frac{1732937}{1464449448420}x^7 \right) y(0) + \left(x - \frac{1}{54}x^2 - \frac{13}{2187}x^3 - \frac{131}{236196}x^4 - \frac{596}{1594323}x^5 - \frac{78469}{2582803260}x^6 - \frac{13738871}{488149816140}x^7 \right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 98

```
AsymptoticDSolveValue[(x^2-9)^2*y'[x]+(x+3)*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left(\frac{1732937x^7}{1464449448420} - \frac{194981x^6}{7748409780} + \frac{304x^5}{23914845} - \frac{289x^4}{708588} + \frac{x^3}{6561} - \frac{x^2}{81} + 1 \right) \\ + c_2 \left(-\frac{13738871x^7}{488149816140} - \frac{78469x^6}{2582803260} - \frac{596x^5}{1594323} - \frac{131x^4}{236196} - \frac{13x^3}{2187} - \frac{x^2}{54} + x \right)$$

2.4 problem 4

Internal problem ID [6587]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 4.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{y'}{x} + \frac{y}{(x-1)^3} = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 64

```
Order:=8;  
dsolve(diff(y(x),x$2)-1/x*diff(y(x),x)+1/(x-1)^3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^2 \left(1 + \frac{1}{8}x^2 + \frac{1}{5}x^3 + \frac{49}{192}x^4 + \frac{423}{1400}x^5 + \frac{15941}{46080}x^6 + \frac{30511}{78400}x^7 + O(x^8) \right) \\ + c_2 \left(\ln(x) \left(-x^2 - \frac{1}{8}x^4 - \frac{1}{5}x^5 - \frac{49}{192}x^6 - \frac{423}{1400}x^7 + O(x^8) \right) \right. \\ \left. + \left(-2 - 2x^3 - \frac{45}{32}x^4 - \frac{34}{25}x^5 - \frac{1673}{1152}x^6 - \frac{313337}{196000}x^7 + O(x^8) \right) \right)$$

✓ Solution by Mathematica

Time used: 0.373 (sec). Leaf size: 107

```
AsymptoticDSolveValue[y''[x]-1/x*y'[x]+1/(x-1)^3*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left(\frac{(245x^4 + 192x^3 + 120x^2 + 960)x^2 \log(x)}{1920} + \frac{-25025x^6 - 16416x^5 - 2250x^4 + 28800x^3 - 180000x^2 + 28800}{28800} \right) + c_2 \left(\frac{15941x^8}{46080} + \frac{423x^7}{1400} + \frac{49x^6}{192} + \frac{x^5}{5} + \frac{x^4}{8} + x^2 \right)$$

2.5 problem 5

Internal problem ID [6588]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 5.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^3 + 4x)y'' - 2xy' + 6y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 72

```
Order:=8;
```

```
dsolve((x^3+4*x)*diff(y(x),x$2)-2*x*diff(y(x),x)+6*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = & c_1 x \left(1 - \frac{1}{2}x + \frac{1}{24}x^2 + \frac{1}{48}x^3 - \frac{1}{384}x^4 - \frac{5}{2304}x^5 + \frac{5}{21504}x^6 + \frac{15}{50176}x^7 + O(x^8) \right) \\ & + \left(-\frac{3}{2}x + \frac{3}{4}x^2 - \frac{1}{16}x^3 - \frac{1}{32}x^4 + \frac{1}{256}x^5 + \frac{5}{1536}x^6 - \frac{5}{14336}x^7 + O(x^8) \right) \ln(x) c_2 \\ & + \left(1 + \frac{1}{2}x - \frac{7}{4}x^2 + \frac{31}{96}x^3 + \frac{1}{24}x^4 - \frac{67}{3072}x^5 - \frac{43}{10240}x^6 + \frac{43061}{18063360}x^7 + O(x^8) \right) c_2 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.237 (sec). Leaf size: 121

```
AsymptoticDSolveValue[(x^3+4*x)*y'[x]-2*x*y'[x]+6*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left(\frac{x(5x^5 + 6x^4 - 48x^3 - 96x^2 + 1152x - 2304) \log(x)}{1536} + \frac{-229x^6 - 790x^5 + 2240x^4 + 11840x^3 - 76800x^2 + 61440x + 30720}{30720} \right) + c_2 \left(\frac{5x^7}{21504} - \frac{5x^6}{2304} - \frac{x^5}{384} + \frac{x^4}{48} + \frac{x^3}{24} - \frac{x^2}{2} + x \right)$$

2.6 problem 6

Internal problem ID [6589]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 6.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x - 5)^2 y'' + 4xy' + (x^2 - 25)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 2223

Order:=8;

dsolve(x^2*(x-5)^2*diff(y(x),x\$2)+4*x*diff(y(x),x)+(x^2-25)*y(x)=0,y(x),type='series',x=0);

$$\begin{aligned}
 & y(x) \\
 &= x^{\frac{21}{50}} \left(c_1 x^{-\frac{\sqrt{2941}}{50}} \left(1 + \frac{-1166 - 4\sqrt{2941}}{-3125 + 125\sqrt{2941}} x - \frac{9}{15625} \frac{879\sqrt{2941} - 79709}{(-25 + \sqrt{2941})(-50 + \sqrt{2941})} x^2 \right. \right. \\
 &\quad + \frac{\frac{15291084\sqrt{2941}}{1953125} - \frac{906742764}{1953125}}{(-25 + \sqrt{2941})(-50 + \sqrt{2941})(-75 + \sqrt{2941})} x^3 \\
 &\quad - \frac{12}{244140625} \frac{-122814219551 + 2200649681\sqrt{2941}}{(-25 + \sqrt{2941})(-50 + \sqrt{2941})(-75 + \sqrt{2941})(-100 + \sqrt{2941})} x^4 \\
 &\quad + \frac{-\frac{10008934775328384}{152587890625} + \frac{181292058002304\sqrt{2941}}{152587890625}}{(-25 + \sqrt{2941})(-50 + \sqrt{2941})(-75 + \sqrt{2941})(-100 + \sqrt{2941})(-125 + \sqrt{2941})} x^5 \\
 &\quad + \frac{-\frac{13371141904684696752}{19073486328125} + \frac{250187169310576512\sqrt{2941}}{19073486328125}}{(-25 + \sqrt{2941})(-50 + \sqrt{2941})(-75 + \sqrt{2941})(-100 + \sqrt{2941})(-125 + \sqrt{2941})(-150 + \sqrt{2941})} x^6 \\
 &\quad - \frac{96}{16689300537109375} \frac{-20689947387639015669859 + 38182014559665663}{(-25 + \sqrt{2941})(-50 + \sqrt{2941})(-75 + \sqrt{2941})(-100 + \sqrt{2941})(-125 + \sqrt{2941})(-150 + \sqrt{2941})} x^7 \\
 &\quad \left. + O(x^8) \right) + c_2 x^{\frac{\sqrt{2941}}{50}} \left(1 + \frac{1166 - 4\sqrt{2941}}{125\sqrt{2941} + 3125} x + \frac{\frac{7911\sqrt{2941}}{15625} + \frac{717381}{15625}}{(\sqrt{2941} + 25)(50 + \sqrt{2941})} x^2 \right. \\
 &\quad + \frac{\frac{15291084\sqrt{2941}}{1953125} + \frac{906742764}{1953125}}{(\sqrt{2941} + 25)(50 + \sqrt{2941})(\sqrt{2941} + 75)} x^3 \\
 &\quad + \frac{\frac{1473770634612}{244140625} + \frac{26407796172\sqrt{2941}}{244140625}}{(\sqrt{2941} + 25)(50 + \sqrt{2941})(\sqrt{2941} + 75)(100 + \sqrt{2941})} x^4 \\
 &\quad + \frac{\frac{10008934775328384}{152587890625} + \frac{181292058002304\sqrt{2941}}{152587890625}}{(\sqrt{2941} + 25)(50 + \sqrt{2941})(\sqrt{2941} + 75)(100 + \sqrt{2941})(125 + \sqrt{2941})} x^5 \\
 &\quad - \frac{48}{19073486328125} \frac{278565456347597849 + 5212232693970344\sqrt{2941}}{(\sqrt{2941} + 25)(50 + \sqrt{2941})(\sqrt{2941} + 75)(100 + \sqrt{2941})(125 + \sqrt{2941})(150 + \sqrt{2941})} x^6 \\
 &\quad - \frac{96}{16689300537109375} \frac{20689947387639015669859 + 381820145596656632404\sqrt{2941}}{(\sqrt{2941} + 25)(50 + \sqrt{2941})(\sqrt{2941} + 75)(100 + \sqrt{2941})(125 + \sqrt{2941})(150 + \sqrt{2941})} x^7 \\
 &\quad \left. + O(x^8) \right) \Big)
 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 22488

```
AsymptoticDSolveValue[x^2*(x-5)^2*y'[x]+4*x*y'[x]+(x^2-25)*y[x]==0,y[x],{x,0,7}]
```

Too large to display

2.7 problem 7

Internal problem ID [6590]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 7.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 + x - 6)y'' + (x + 3)y' + (-2 + x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 74

```
Order:=8;
```

```
dsolve((x^2+x-6)*diff(y(x),x$2)+(x+3)*diff(y(x),x)+(x-2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{6}x^2 - \frac{1}{108}x^3 - \frac{17}{2592}x^4 - \frac{7}{2160}x^5 - \frac{139}{116640}x^6 - \frac{5377}{9797760}x^7\right) y(0) \\ + \left(x + \frac{1}{4}x^2 + \frac{1}{36}x^3 + \frac{23}{864}x^4 + \frac{13}{1440}x^5 + \frac{619}{155520}x^6 + \frac{689}{408240}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 98

```
AsymptoticDSolveValue[(x^2+x-6)*y'[x]+(x+3)*y'[x]+(x-2)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left(-\frac{5377x^7}{9797760} - \frac{139x^6}{116640} - \frac{7x^5}{2160} - \frac{17x^4}{2592} - \frac{x^3}{108} - \frac{x^2}{6} + 1 \right) \\ + c_2 \left(\frac{689x^7}{408240} + \frac{619x^6}{155520} + \frac{13x^5}{1440} + \frac{23x^4}{864} + \frac{x^3}{36} + \frac{x^2}{4} + x \right)$$

2.8 problem 8

Internal problem ID [6591]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 8.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(x^2 + 1)^2 y'' + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 70

```
Order:=8;
```

```
dsolve(x*(x^2+1)^2*diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = & c_1 x \left(1 - \frac{1}{2}x + \frac{1}{12}x^2 + \frac{23}{144}x^3 - \frac{167}{2880}x^4 - \frac{7993}{86400}x^5 + \frac{23599}{518400}x^6 + \frac{1860281}{29030400}x^7 \right. \\ & \left. + O(x^8) \right) + c_2 \left(\ln(x) \left(-x + \frac{1}{2}x^2 - \frac{1}{12}x^3 - \frac{23}{144}x^4 + \frac{167}{2880}x^5 + \frac{7993}{86400}x^6 \right. \right. \\ & \left. \left. - \frac{23599}{518400}x^7 + O(x^8) \right) \right) \\ & + \left(1 - \frac{3}{4}x^2 + \frac{19}{36}x^3 + \frac{85}{1728}x^4 - \frac{21907}{86400}x^5 + \frac{787}{81000}x^6 + \frac{5987917}{36288000}x^7 + O(x^8) \right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.081 (sec). Leaf size: 121

```
AsymptoticDSolveValue[x*(x^2+1)^2*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left(\frac{x(7993x^5 + 5010x^4 - 13800x^3 - 7200x^2 + 43200x - 86400) \log(x)}{86400} \right. \\ \left. + \frac{-107303x^6 - 403755x^5 + 270750x^4 + 792000x^3 - 1620000x^2 + 1296000x + 1296000}{1296000} \right) \\ + c_2 \left(\frac{23599x^7}{518400} - \frac{7993x^6}{86400} - \frac{167x^5}{2880} + \frac{23x^4}{144} + \frac{x^3}{12} - \frac{x^2}{2} + x \right)$$

2.9 problem 9

Internal problem ID [6592]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 9.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^3(x^2 - 25)(-2 + x)^2 y'' + 3x(-2 + x)y' + 7(5 + x)y = 0$$

With the expansion point for the power series method at $x = 0$.

X Solution by Maple

```
Order:=8;
```

```
dsolve(x^3*(x^2-25)*(x-2)^2*diff(y(x),x$2)+3*x*(x-2)*diff(y(x),x)+7*(x+5)*y(x)=0,y(x),type='
```

No solution found

✓ Solution by Mathematica

Time used: 0.149 (sec). Leaf size: 127

AsymptoticDSolveValue[x^3*(x^2-25)*(x-2)^2*y'[x]+3*x*(x-2)*y'[x]+7*(x+5)*y[x]==0,y[x],{x,0,

$$\begin{aligned}
 y(x) \rightarrow & c_2 \left(-\frac{8443721376476278698619699192242145x^7}{24679069470425088} \right. \\
 & + \frac{256276439033972389997207276999x^6}{228509902503936} - \frac{1337698720169782190618881x^5}{352638738432} \\
 & + \frac{42840301537653264505x^4}{3265173504} - \frac{344729362309955x^3}{7558272} + \frac{3590248795x^2}{23328} - \frac{50309x}{108} \\
 & \left. + 1 \right) x^{35/6} \\
 & + c_1 e^{\frac{3}{50}/x} \left(\frac{27670480145177700385838149741665715823829792301x^7}{411876407368125000000000000000000000000} + \frac{3104172516869718247583976968553108060901x^6}{435848050125000000000000000000000} - \frac{379071980}{5380} \right)
 \end{aligned}$$

2.10 problem 10

Internal problem ID [6593]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 10.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^3 - 2x^2 + 3x)^2 y'' + x(x - 3)^2 y' - (1 + x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 55

```
Order:=8;
```

```
dsolve((x^3-2*x^2+3*x)^2*diff(y(x),x$2)+x*(x-3)^2*diff(y(x),x)-(x+1)*y(x)=0,y(x),type='series')
```

$$y(x) = c_2 x^{\frac{2}{3}} \left(1 + \frac{1}{45}x + \frac{149}{3240}x^2 + \frac{2701}{192456}x^3 + \frac{236933}{121247280}x^4 - \frac{67092967}{92754169200}x^5 - \frac{30839263691}{50087251368000}x^6 - \frac{14846109458423}{72576427232232000}x^7 + \dots \right)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 118

AsymptoticDSolveValue[(x^3-2*x^2+3*x)^2*y'[x]+x*(x-3)^2*y'[x]-(x+1)*y[x]==0,y[x],{x,0,7}]

$$y(x) \rightarrow c_1 \sqrt[3]{x} \left(-\frac{14846109458423x^7}{72576427232232000} - \frac{30839263691x^6}{50087251368000} - \frac{67092967x^5}{92754169200} + \frac{236933x^4}{121247280} \right. \\ \left. + \frac{2701x^3}{192456} + \frac{149x^2}{3240} + \frac{x}{45} + 1 \right) \\ + \frac{c_2 \left(-\frac{2917066898x^7}{2604972321315} - \frac{70024699x^6}{43525017900} + \frac{7435523x^5}{3224075400} + \frac{106583x^4}{5511240} + \frac{1591x^3}{30618} - \frac{5x^2}{162} + \frac{13x}{9} + 1 \right)}{\sqrt[3]{x}}$$

2.11 problem 11

Internal problem ID [6594]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 11.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 - 1)y'' + 5(1 + x)y' + (x^2 - x)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 69

`Order:=8;`

```
dsolve((x^2-1)*diff(y(x),x$2)+5*(x+1)*diff(y(x),x)+(x^2-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{6}x^3 - \frac{1}{8}x^4 - \frac{3}{10}x^5 - \frac{17}{45}x^6 - \frac{199}{336}x^7\right)y(0) \\ + \left(x + \frac{5}{2}x^2 + 5x^3 + \frac{26}{3}x^4 + \frac{1661}{120}x^5 + \frac{4967}{240}x^6 + \frac{14881}{504}x^7\right)D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 89

```
AsymptoticDSolveValue[(x^2-1)*y''[x]+5*(x+1)*y'[x]+(x^2-x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left(-\frac{199x^7}{336} - \frac{17x^6}{45} - \frac{3x^5}{10} - \frac{x^4}{8} - \frac{x^3}{6} + 1 \right) \\ + c_2 \left(\frac{14881x^7}{504} + \frac{4967x^6}{240} + \frac{1661x^5}{120} + \frac{26x^4}{3} + 5x^3 + \frac{5x^2}{2} + x \right)$$

2.12 problem 12

Internal problem ID [6595]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 12.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' + (x + 3)y' + 7yx^2 = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 68

```
Order:=8;
```

```
dsolve(x*diff(y(x),x$2)+(x+3)*diff(y(x),x)+7*x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 \left(1 - \frac{7}{15}x^3 + \frac{7}{120}x^4 - \frac{1}{150}x^5 + \frac{11}{160}x^6 - \frac{197}{15120}x^7 + O(x^8) \right) + \frac{c_2 (\ln(x) (2x^2 - \frac{14}{15}x^5 + \frac{7}{60}x^6 - \frac{1}{75}x^7 + O(x^8)) + (-2 + 4x - 3x^2 + 4x^3 - 4x^4 + \frac{547}{225}x^5 - \frac{5329}{3600}x^6 + \frac{764}{7875}x^7 + O(x^8)))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.057 (sec). Leaf size: 96

```
AsymptoticDSolveValue[x*y''[x]+(x+3)*y'[x]+7*x^2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left(\frac{11x^6}{160} - \frac{x^5}{150} + \frac{7x^4}{120} - \frac{7x^3}{15} + 1 \right) + c_1 \left(\frac{5539x^6 - 10432x^5 + 14400x^4 - 14400x^3 + 14400x^2 - 14400x + 7200}{7200x^2} - \frac{1}{120} (7x^4 - 56x^3 + 120) \log(x) \right)$$

2.13 problem 13

Internal problem ID [6596]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 13.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + \left(\frac{5}{3}x + x^2\right) y' - \frac{y}{3} = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
Order:=8;
```

```
dsolve(x^2*diff(y(x),x$2)+(5/3*x+x^2)*diff(y(x),x)-1/3*y(x)=0,y(x),type='series',x=0);
```

$y(x)$

$$= \frac{c_2 x^{\frac{4}{3}} \left(1 - \frac{1}{7}x + \frac{1}{35}x^2 - \frac{1}{195}x^3 + \frac{1}{1248}x^4 - \frac{1}{9120}x^5 + \frac{1}{75240}x^6 - \frac{1}{693000}x^7 + O(x^8)\right) + c_1(1 - 3x + O(x^8))}{x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 72

```
AsymptoticDSolveValue[x^2*y''[x]+(5/3*x+x^2)*y'[x]-1/3*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \sqrt[3]{x} \left(-\frac{x^7}{693000} + \frac{x^6}{75240} - \frac{x^5}{9120} + \frac{x^4}{1248} - \frac{x^3}{195} + \frac{x^2}{35} - \frac{x}{7} + 1 \right) + \frac{c_2(1 - 3x)}{x}$$

2.14 problem 14

Internal problem ID [6597]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 14.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$xy'' + y' + 10y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 71

```
Order:=8;  
dsolve(x*diff(y(x),x$2)+diff(y(x),x)+10*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1) \left(1 - 10x + 25x^2 - \frac{250}{9}x^3 + \frac{625}{36}x^4 - \frac{125}{18}x^5 + \frac{625}{324}x^6 - \frac{3125}{7938}x^7 + O(x^8) \right) + \left(20x - 75x^2 + \frac{2750}{27}x^3 - \frac{15625}{216}x^4 + \frac{3425}{108}x^5 - \frac{6125}{648}x^6 + \frac{75625}{37044}x^7 + O(x^8) \right) c_2$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 147

```
AsymptoticDSolveValue[x*y''[x]+y'[x]+10*y[x]==0,y[x],{x,0,7}]
```

$$\begin{aligned} y(x) \rightarrow & c_1 \left(-\frac{3125x^7}{7938} + \frac{625x^6}{324} - \frac{125x^5}{18} + \frac{625x^4}{36} - \frac{250x^3}{9} + 25x^2 - 10x + 1 \right) \\ & + c_2 \left(\frac{75625x^7}{37044} - \frac{6125x^6}{648} + \frac{3425x^5}{108} - \frac{15625x^4}{216} + \frac{2750x^3}{27} - 75x^2 \right. \\ & \left. + \left(-\frac{3125x^7}{7938} + \frac{625x^6}{324} - \frac{125x^5}{18} + \frac{625x^4}{36} - \frac{250x^3}{9} + 25x^2 - 10x + 1 \right) \log(x) + 20x \right) \end{aligned}$$

2.15 problem 15

Internal problem ID [6598]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 15.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [[_Emden, _Fowler]]

$$2xy'' - y' + 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 52

```
Order:=8;  
dsolve(2*x*diff(y(x),x$2)-diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{3}{2}} \left(1 - \frac{2}{5}x + \frac{2}{35}x^2 - \frac{4}{945}x^3 + \frac{2}{10395}x^4 - \frac{4}{675675}x^5 + \frac{4}{30405375}x^6 - \frac{8}{3618239625}x^7 + O(x^8) \right) \\ + c_2 \left(1 + 2x - 2x^2 + \frac{4}{9}x^3 - \frac{2}{45}x^4 + \frac{4}{1575}x^5 - \frac{4}{42525}x^6 + \frac{8}{3274425}x^7 + O(x^8) \right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 109

```
AsymptoticDSolveValue[2*x*y'[x]-y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left(\frac{8x^7}{3274425} - \frac{4x^6}{42525} + \frac{4x^5}{1575} - \frac{2x^4}{45} + \frac{4x^3}{9} - 2x^2 + 2x + 1 \right) \\ + c_1 \left(-\frac{8x^7}{3618239625} + \frac{4x^6}{30405375} - \frac{4x^5}{675675} + \frac{2x^4}{10395} - \frac{4x^3}{945} + \frac{2x^2}{35} - \frac{2x}{5} + 1 \right) x^{3/2}$$

2.16 problem 16

Internal problem ID [6599]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 16.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2xy'' + 5y' + xy = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 36

```
Order:=8;  
dsolve(2*x*diff(y(x),x$2)+5*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 \left(1 - \frac{1}{2}x^2 + \frac{1}{40}x^4 - \frac{1}{2160}x^6 + O(x^8)\right)}{x^{\frac{3}{2}}} + c_2 \left(1 - \frac{1}{14}x^2 + \frac{1}{616}x^4 - \frac{1}{55440}x^6 + O(x^8)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 61

```
AsymptoticDSolveValue[2*x*y''[x]+5*y'[x]+x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left(-\frac{x^6}{55440} + \frac{x^4}{616} - \frac{x^2}{14} + 1\right) + \frac{c_2 \left(-\frac{x^6}{2160} + \frac{x^4}{40} - \frac{x^2}{2} + 1\right)}{x^{3/2}}$$

2.17 problem 17

Internal problem ID [6600]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 17.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$4xy'' + \frac{y'}{2} + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 52

```
Order:=8;  
dsolve(4*x*diff(y(x),x$2)+1/2*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{7}{8}} \left(1 - \frac{2}{15}x + \frac{2}{345}x^2 - \frac{4}{32085}x^3 + \frac{2}{1251315}x^4 - \frac{4}{294059025}x^5 + \frac{4}{48519739125}x^6 - \frac{8}{21397204954125}x^7 + O(x^8) \right) + c_2 \left(1 - 2x + \frac{2}{9}x^2 - \frac{4}{459}x^3 + \frac{2}{11475}x^4 - \frac{4}{1893375}x^5 + \frac{4}{232885125}x^6 - \frac{8}{79879597875}x^7 + O(x^8) \right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 111

```
AsymptoticDSolveValue[4*x*y''[x]+1/2*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left(-\frac{8x^7}{79879597875} + \frac{4x^6}{232885125} - \frac{4x^5}{1893375} + \frac{2x^4}{11475} - \frac{4x^3}{459} + \frac{2x^2}{9} - 2x + 1 \right) \\ + c_1 x^{7/8} \left(-\frac{8x^7}{21397204954125} + \frac{4x^6}{48519739125} - \frac{4x^5}{294059025} + \frac{2x^4}{1251315} - \frac{4x^3}{32085} \right. \\ \left. + \frac{2x^2}{345} - \frac{2x}{15} + 1 \right)$$

2.18 problem 18

Internal problem ID [6601]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 18.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2y'' - xy' + (x^2 + 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

```
Order:=8;
```

```
dsolve(2*x^2*diff(y(x),x$2)-x*diff(y(x),x)+(x^2+1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x} \left(1 - \frac{1}{6}x^2 + \frac{1}{168}x^4 - \frac{1}{11088}x^6 + O(x^8) \right) \\ + c_2x \left(1 - \frac{1}{10}x^2 + \frac{1}{360}x^4 - \frac{1}{28080}x^6 + O(x^8) \right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 62

```
AsymptoticDSolveValue[2*x^2*y'[x]-x*y'[x]+(x^2+1)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1x \left(-\frac{x^6}{28080} + \frac{x^4}{360} - \frac{x^2}{10} + 1 \right) + c_2\sqrt{x} \left(-\frac{x^6}{11088} + \frac{x^4}{168} - \frac{x^2}{6} + 1 \right)$$

2.19 problem 19

Internal problem ID [6602]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 19.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$3xy'' + (2 - x)y' - y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 52

```
Order:=8;  
dsolve(3*x*diff(y(x),x$2)+(2-x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{1}{3}} \left(1 + \frac{1}{3}x + \frac{1}{18}x^2 + \frac{1}{162}x^3 + \frac{1}{1944}x^4 + \frac{1}{29160}x^5 + \frac{1}{524880}x^6 + \frac{1}{11022480}x^7 + O(x^8) \right) + c_2 \left(1 + \frac{1}{2}x + \frac{1}{10}x^2 + \frac{1}{80}x^3 + \frac{1}{880}x^4 + \frac{1}{12320}x^5 + \frac{1}{209440}x^6 + \frac{1}{4188800}x^7 + O(x^8) \right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 113

```
AsymptoticDSolveValue[3*x*y''[x]+(2-x)*y'[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \sqrt[3]{x} \left(\frac{x^7}{11022480} + \frac{x^6}{524880} + \frac{x^5}{29160} + \frac{x^4}{1944} + \frac{x^3}{162} + \frac{x^2}{18} + \frac{x}{3} + 1 \right) \\ + c_2 \left(\frac{x^7}{4188800} + \frac{x^6}{209440} + \frac{x^5}{12320} + \frac{x^4}{880} + \frac{x^3}{80} + \frac{x^2}{10} + \frac{x}{2} + 1 \right)$$

2.20 problem 20

Internal problem ID [6603]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 20.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' - \left(x - \frac{2}{9}\right) y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 55

```
Order:=8;  
dsolve(x^2*diff(y(x),x$2)-(x-2/9)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{1}{3}} \left(1 + \frac{3}{2}x + \frac{9}{20}x^2 + \frac{9}{160}x^3 + \frac{27}{7040}x^4 + \frac{81}{492800}x^5 + \frac{81}{16755200}x^6 + \frac{243}{2345728000}x^7 + O(x^8) \right) + c_2 x^{\frac{2}{3}} \left(1 + \frac{3}{4}x + \frac{9}{56}x^2 + \frac{9}{560}x^3 + \frac{27}{29120}x^4 + \frac{81}{2329600}x^5 + \frac{81}{88524800}x^6 + \frac{243}{13632819200}x^7 + O(x^8) \right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 118

```
AsymptoticDSolveValue[x^2*y''[x]-(x-2/9)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \sqrt[3]{x} \left(\frac{243x^7}{2345728000} + \frac{81x^6}{16755200} + \frac{81x^5}{492800} + \frac{27x^4}{7040} + \frac{9x^3}{160} + \frac{9x^2}{20} + \frac{3x}{2} + 1 \right) \\ + c_1 x^{2/3} \left(\frac{243x^7}{13632819200} + \frac{81x^6}{88524800} + \frac{81x^5}{2329600} + \frac{27x^4}{29120} + \frac{9x^3}{560} + \frac{9x^2}{56} + \frac{3x}{4} + 1 \right)$$

2.21 problem 21

Internal problem ID [6604]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 21.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [_Laguerre]

$$2xy'' - (2x + 3)y' + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 52

```
Order:=8;
```

```
dsolve(2*x*dif(y(x),x$2)-(3+2*x)*dif(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{5}{2}} \left(1 + \frac{4}{7}x + \frac{4}{21}x^2 + \frac{32}{693}x^3 + \frac{80}{9009}x^4 + \frac{64}{45045}x^5 + \frac{64}{328185}x^6 + \frac{1024}{43648605}x^7 + O(x^8) \right) + c_2 \left(1 + \frac{1}{3}x - \frac{1}{6}x^2 - \frac{1}{6}x^3 - \frac{5}{72}x^4 - \frac{7}{360}x^5 - \frac{1}{240}x^6 - \frac{11}{15120}x^7 + O(x^8) \right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 113

```
AsymptoticDSolveValue[2*x*y''[x]-(3+2*x)*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left(-\frac{11x^7}{15120} - \frac{x^6}{240} - \frac{7x^5}{360} - \frac{5x^4}{72} - \frac{x^3}{6} - \frac{x^2}{6} + \frac{x}{3} + 1 \right) + c_1 \left(\frac{1024x^7}{43648605} + \frac{64x^6}{328185} + \frac{64x^5}{45045} + \frac{80x^4}{9009} + \frac{32x^3}{693} + \frac{4x^2}{21} + \frac{4x}{7} + 1 \right) x^{5/2}$$

2.22 problem 22

Internal problem ID [6605]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 22.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + xy' + \left(x^2 - \frac{4}{9}\right) y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 39

```
Order:=8;
```

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-4/9)*y(x)=0,y(x),type='series',x=0);
```

$y(x)$

$$= \frac{c_2 x^{\frac{4}{3}} \left(1 - \frac{3}{20}x^2 + \frac{9}{1280}x^4 - \frac{9}{56320}x^6 + O(x^8)\right) + c_1 \left(1 - \frac{3}{4}x^2 + \frac{9}{128}x^4 - \frac{9}{3584}x^6 + O(x^8)\right)}{x^{\frac{2}{3}}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 66

```
AsymptoticDSolveValue[x^2*y'[x]+x*y'[x]+(x^2-4/9)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 x^{2/3} \left(-\frac{9x^6}{56320} + \frac{9x^4}{1280} - \frac{3x^2}{20} + 1 \right) + \frac{c_2 \left(-\frac{9x^6}{3584} + \frac{9x^4}{128} - \frac{3x^2}{4} + 1 \right)}{x^{2/3}}$$

2.23 problem 23

Internal problem ID [6606]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 23.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2y'' + 9x^2y' + 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 55

```
Order:=8;
```

```
dsolve(9*x^2*diff(y(x),x$2)+9*x^2*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{1}{3}} \left(1 - \frac{1}{2}x + \frac{1}{5}x^2 - \frac{7}{120}x^3 + \frac{7}{528}x^4 - \frac{13}{5280}x^5 + \frac{13}{33660}x^6 - \frac{247}{4712400}x^7 + O(x^8) \right) \\ + c_2 x^{\frac{2}{3}} \left(1 - \frac{1}{2}x + \frac{5}{28}x^2 - \frac{1}{21}x^3 + \frac{11}{1092}x^4 - \frac{11}{6240}x^5 + \frac{187}{711360}x^6 - \frac{17}{497952}x^7 \right. \\ \left. + O(x^8) \right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 118

```
AsymptoticDSolveValue[9*x^2*y''[x]+9*x^2*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \sqrt[3]{x} \left(-\frac{247x^7}{4712400} + \frac{13x^6}{33660} - \frac{13x^5}{5280} + \frac{7x^4}{528} - \frac{7x^3}{120} + \frac{x^2}{5} - \frac{x}{2} + 1 \right) \\ + c_1 x^{2/3} \left(-\frac{17x^7}{497952} + \frac{187x^6}{711360} - \frac{11x^5}{6240} + \frac{11x^4}{1092} - \frac{x^3}{21} + \frac{5x^2}{28} - \frac{x}{2} + 1 \right)$$

2.24 problem 24

Internal problem ID [6607]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 24.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2y'' + 3xy' + (2x - 1)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 55

Order:=8;

`dsolve(2*x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+(2*x-1)*y(x)=0,y(x),type='series',x=0);`

$y(x)$

$$= \frac{c_2 x^{\frac{3}{2}} \left(1 - \frac{2}{5}x + \frac{2}{35}x^2 - \frac{4}{945}x^3 + \frac{2}{10395}x^4 - \frac{4}{675675}x^5 + \frac{4}{30405375}x^6 - \frac{8}{3618239625}x^7 + O(x^8) \right) + c_1(1 + 2x - 2x^2)}{x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 112

`AsymptoticDSolveValue[2*x^2*y'[x]+3*x*y'[x]+(2*x-1)*y[x]==0,y[x],{x,0,7}]`

$$y(x) \rightarrow c_1 \sqrt{x} \left(-\frac{8x^7}{3618239625} + \frac{4x^6}{30405375} - \frac{4x^5}{675675} + \frac{2x^4}{10395} - \frac{4x^3}{945} + \frac{2x^2}{35} - \frac{2x}{5} + 1 \right) + \frac{c_2 \left(\frac{8x^7}{3274425} - \frac{4x^6}{42525} + \frac{4x^5}{1575} - \frac{2x^4}{45} + \frac{4x^3}{9} - 2x^2 + 2x + 1 \right)}{x}$$

2.25 problem 25

Internal problem ID [6608]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 25.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' + 2y' - xy = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 36

```
Order:=8;  
dsolve(x*dif(y(x),x$2)+2*dif(y(x),x)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 \left(1 + \frac{1}{6}x^2 + \frac{1}{120}x^4 + \frac{1}{5040}x^6 + O(x^8) \right) + \frac{c_2 \left(1 + \frac{1}{2}x^2 + \frac{1}{24}x^4 + \frac{1}{720}x^6 + O(x^8) \right)}{x}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 56

```
AsymptoticDSolveValue[x*y''[x]+2*y'[x]-x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left(\frac{x^5}{720} + \frac{x^3}{24} + \frac{x}{2} + \frac{1}{x} \right) + c_2 \left(\frac{x^6}{5040} + \frac{x^4}{120} + \frac{x^2}{6} + 1 \right)$$

2.26 problem 26

Internal problem ID [6609]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 26.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + xy' + \left(x^2 - \frac{1}{4}\right) y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 39

```
Order:=8;
```

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x \left(1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 - \frac{1}{5040}x^6 + O(x^8)\right) + c_2 \left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + O(x^8)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 76

```
AsymptoticDSolveValue[x^2*y'[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left(-\frac{x^{11/2}}{720} + \frac{x^{7/2}}{24} - \frac{x^{3/2}}{2} + \frac{1}{\sqrt{x}} \right) + c_2 \left(-\frac{x^{13/2}}{5040} + \frac{x^{9/2}}{120} - \frac{x^{5/2}}{6} + \sqrt{x} \right)$$

2.27 problem 27

Internal problem ID [6610]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 27.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_Laguerre, [_2nd_order, _linear, ' _with_symmetry_[0,F(x)] ']]`

$$xy'' - xy' + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 46

```
Order:=8;  
dsolve(x*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x(1 + O(x^8)) + (-x + O(x^8)) \ln(x) c_2 \\ + \left(1 + x - \frac{1}{2}x^2 - \frac{1}{12}x^3 - \frac{1}{72}x^4 - \frac{1}{480}x^5 - \frac{1}{3600}x^6 - \frac{1}{30240}x^7 + O(x^8)\right) c_2$$

✓ Solution by Mathematica

Time used: 0.085 (sec). Leaf size: 51

```
AsymptoticDSolveValue[x*y''[x]-x*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left(\frac{-2x^6 - 15x^5 - 100x^4 - 600x^3 - 3600x^2 + 14400x + 7200}{7200} - x \log(x) \right) + c_2 x$$

2.28 problem 28

Internal problem ID [6611]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 28.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{3y'}{x} - 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 52

```
Order:=8;  
dsolve(diff(y(x),x$2)+3/x*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 \left(1 + \frac{1}{4}x^2 + \frac{1}{48}x^4 + \frac{1}{1152}x^6 + O(x^8)\right) x^2 + c_2 \left(\ln(x) \left((-2)x^2 - \frac{1}{2}x^4 - \frac{1}{24}x^6 + O(x^8)\right) + \left(-2 + \frac{3}{8}x^4 + \frac{7}{144}x^6 + O(x^8)\right)\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 74

```
AsymptoticDSolveValue[y''[x]+3/x*y'[x]-2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left(\frac{x^6}{1152} + \frac{x^4}{48} + \frac{x^2}{4} + 1 \right) + c_1 \left(\frac{1}{48} (x^4 + 12x^2 + 48) \log(x) - \frac{5x^6 + 45x^4 + 72x^2 - 144}{144x^2} \right)$$

2.29 problem 29

Internal problem ID [6612]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 29.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$xy'' + (1 - x)y' - y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 71

```
Order:=8;
```

```
dsolve(x*diff(y(x),x$2)+(1-x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1) \left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \frac{1}{5040}x^7 + O(x^8) \right) \\ + \left(-x - \frac{3}{4}x^2 - \frac{11}{36}x^3 - \frac{25}{288}x^4 - \frac{137}{7200}x^5 - \frac{49}{14400}x^6 - \frac{121}{235200}x^7 + O(x^8) \right) c_2$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 149

```
AsymptoticDSolveValue[x*y''[x]+(1-x)*y'[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left(\frac{x^7}{5040} + \frac{x^6}{720} + \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1 \right) + c_2 \left(-\frac{121x^7}{235200} - \frac{49x^6}{14400} - \frac{137x^5}{7200} \right. \\ \left. - \frac{25x^4}{288} - \frac{11x^3}{36} - \frac{3x^2}{4} + \left(\frac{x^7}{5040} + \frac{x^6}{720} + \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1 \right) \log(x) - x \right)$$

2.30 problem 30

Internal problem ID [6613]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 30.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$xy'' + y' + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 71

```
Order:=8;  
dsolve(x*diff(y(x),x$2)+diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1) \left(1 - x + \frac{1}{4}x^2 - \frac{1}{36}x^3 + \frac{1}{576}x^4 - \frac{1}{14400}x^5 + \frac{1}{518400}x^6 - \frac{1}{25401600}x^7 + O(x^8) \right) + \left(2x - \frac{3}{4}x^2 + \frac{11}{108}x^3 - \frac{25}{3456}x^4 + \frac{137}{432000}x^5 - \frac{49}{5184000}x^6 + \frac{121}{592704000}x^7 + O(x^8) \right) c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 153

```
AsymptoticDSolveValue[x*y''[x]+y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left(-\frac{x^7}{25401600} + \frac{x^6}{518400} - \frac{x^5}{14400} + \frac{x^4}{576} - \frac{x^3}{36} + \frac{x^2}{4} - x + 1 \right) \\ + c_2 \left(\frac{121x^7}{592704000} - \frac{49x^6}{5184000} + \frac{137x^5}{432000} - \frac{25x^4}{3456} + \frac{11x^3}{108} - \frac{3x^2}{4} \right. \\ \left. + \left(-\frac{x^7}{25401600} + \frac{x^6}{518400} - \frac{x^5}{14400} + \frac{x^4}{576} - \frac{x^3}{36} + \frac{x^2}{4} - x + 1 \right) \log(x) + 2x \right)$$

2.31 problem 31

Internal problem ID [6614]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 31.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' + (x - 6)y' - 3y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 46

```
Order:=8;
```

```
dsolve(x*diff(y(x),x$2)+(x-6)*diff(y(x),x)-3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^7 \left(1 - \frac{1}{2}x + \frac{5}{36}x^2 - \frac{1}{36}x^3 + \frac{7}{1584}x^4 - \frac{7}{11880}x^5 + \frac{7}{102960}x^6 - \frac{1}{144144}x^7 + O(x^8) \right) + c_2 (3628800 - 1814400x + 362880x^2 - 30240x^3 + 36x^7 + O(x^8))$$

✓ Solution by Mathematica

Time used: 0.107 (sec). Leaf size: 77

```
AsymptoticDSolveValue[x*y''[x]+(x-6)*y'[x]-3*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left(-\frac{x^3}{120} + \frac{x^2}{10} - \frac{x}{2} + 1 \right) + c_2 \left(\frac{7x^{13}}{102960} - \frac{7x^{12}}{11880} + \frac{7x^{11}}{1584} - \frac{x^{10}}{36} + \frac{5x^9}{36} - \frac{x^8}{2} + x^7 \right)$$

2.32 problem 32

Internal problem ID [6615]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 32.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x(x-1)y'' + 3y' - 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 50

```
Order:=8;
```

```
dsolve(x*(x-1)*diff(y(x),x$2)+3*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^4 (1 + 2x + 3x^2 + 4x^3 + 5x^4 + 6x^5 + 7x^6 + 8x^7 + O(x^8)) \\ + c_2 (-144 - 96x - 48x^2 + 48x^4 + 96x^5 + 144x^6 + 192x^7 + O(x^8))$$

✓ Solution by Mathematica

Time used: 0.376 (sec). Leaf size: 77

```
AsymptoticDSolveValue[x*(x-1)*y''[x]+3*y'[x]-2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left(-x^6 - \frac{2x^5}{3} - \frac{x^4}{3} + \frac{x^2}{3} + \frac{2x}{3} + 1 \right) + c_2 (7x^{10} + 6x^9 + 5x^8 + 4x^7 + 3x^6 + 2x^5 + x^4)$$

2.33 problem 33(b)

Internal problem ID [6616]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 33(b).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{2y'}{t} + \lambda y = 0$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 56

```
Order:=8;  
dsolve(diff(y(t),t$2)+2/t*diff(y(t),t)+lambda*y(t)=0,y(t),type='series',t=0);
```

$$y(t) = c_1 \left(1 - \frac{1}{6} \lambda t^2 + \frac{1}{120} \lambda^2 t^4 - \frac{1}{5040} \lambda^3 t^6 + O(t^8) \right) \\ + \frac{c_2 \left(1 - \frac{1}{2} \lambda t^2 + \frac{1}{24} \lambda^2 t^4 - \frac{1}{720} \lambda^3 t^6 + O(t^8) \right)}{t}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 70

```
AsymptoticDSolveValue[y''[t]+2/t*y'[t]+\[Lambda]*y[t]==0,y[t],{t,0,7}]
```

$$y(t) \rightarrow c_1 \left(-\frac{1}{720} \lambda^3 t^5 + \frac{\lambda^2 t^3}{24} - \frac{\lambda t}{2} + \frac{1}{t} \right) + c_2 \left(-\frac{\lambda^3 t^6}{5040} + \frac{\lambda^2 t^4}{120} - \frac{\lambda t^2}{6} + 1 \right)$$

2.34 problem 36 (a)

Internal problem ID [6617]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 36 (a).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^3y'' + y = 0$$

With the expansion point for the power series method at $x = 0$.

X Solution by Maple

```
Order:=8;  
dsolve(x^3*diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 294

```
AsymptoticDSolveValue[x^3*y''[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 e^{-\frac{2i}{\sqrt{x}}x^{3/4}} \left(-\frac{11100458801337530625ix^{13/2}}{4611686018427387904} + \frac{1327867167401775ix^{11/2}}{4503599627370496} \right. \\ \left. - \frac{468131288625ix^{9/2}}{8796093022208} + \frac{66891825ix^{7/2}}{4294967296} - \frac{72765ix^{5/2}}{8388608} + \frac{105ix^{3/2}}{8192} \right. \\ \left. + \frac{1149690375852815671875x^7}{147573952589676412928} - \frac{232376754295310625x^6}{288230376151711744} + \frac{33424574007825x^5}{281474976710656} \right. \\ \left. - \frac{14783093325x^4}{549755813888} + \frac{2837835x^3}{268435456} - \frac{4725x^2}{524288} + \frac{15x}{512} - \frac{3i\sqrt{x}}{16} \right. \\ \left. + 1 \right) + c_2 e^{\frac{2i}{\sqrt{x}}x^{3/4}} \left(\frac{11100458801337530625ix^{13/2}}{4611686018427387904} - \frac{1327867167401775ix^{11/2}}{4503599627370496} + \frac{468131288625ix^{9/2}}{8796093022208} - \frac{668}{42} \right)$$

2.35 problem 36(b)

Internal problem ID [6618]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

Problem number: 36(b).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x^2y'' + (3x - 1)y' + y = 0$$

With the expansion point for the power series method at $x = 0$.

 Solution by Maple

```
Order:=8;  
dsolve(x^2*diff(y(x),x$2)+(3*x-1)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

No solution found

 Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 53

```
AsymptoticDSolveValue[x^2*y''[x]+(3*x-1)*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1(5040x^7 + 720x^6 + 120x^5 + 24x^4 + 6x^3 + 2x^2 + x + 1) + \frac{c_2 e^{-1/x}}{x}$$

3 CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267

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3.1 problem 1

Internal problem ID [6619]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267

Problem number: 1.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + x y' + \left(x^2 - \frac{1}{9}\right) y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/9)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \text{BesselJ}\left(\frac{1}{3}, x\right) + c_2 \text{BesselY}\left(\frac{1}{3}, x\right)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1/9)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \text{BesselJ}\left(\frac{1}{3}, x\right) + c_2 \text{BesselY}\left(\frac{1}{3}, x\right)$$

3.2 problem 2

Internal problem ID [6620]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267

Problem number: 2.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [Bessel]

$$x^2 y'' + x y' + (x^2 - 1) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \text{BesselJ}(1, x) + c_2 \text{BesselY}(1, x)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 18

```
DSolve[x^2*y'[x]+x*y'[x]+(x^2-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \text{BesselJ}(1, x) + c_2 \text{BesselY}(1, x)$$

3.3 problem 3

Internal problem ID [6621]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267

Problem number: 3.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + 4xy' + (4x^2 - 25)y = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 47

```
dsolve(4*x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2-25)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-3c_2(ix - \frac{1}{3}x^2 + 1)e^{-ix} + 3e^{ix}(ix + \frac{1}{3}x^2 - 1)c_1}{x^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.113 (sec). Leaf size: 59

```
DSolve[4*x^2*y'[x]+4*x*y'[x]+(4*x^2-25)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{\sqrt{\frac{2}{\pi}}((-c_2x^2 + 3c_1x + 3c_2)\cos(x) + (c_1(x^2 - 3) + 3c_2x)\sin(x))}{x^{5/2}}$$

3.4 problem 4

Internal problem ID [6622]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267

Problem number: 4.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$16x^2y'' + 16xy' + (16x^2 - 1)y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(16*x^2*diff(y(x),x$2)+16*x*diff(y(x),x)+(16*x^2-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \text{BesselJ}\left(\frac{1}{4}, x\right) + c_2 \text{BesselY}\left(\frac{1}{4}, x\right)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 22

```
DSolve[16*x^2*y'[x]+16*x*y'[x]+(16*x^2-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \text{BesselJ}\left(\frac{1}{4}, x\right) + c_2 \text{BesselY}\left(\frac{1}{4}, x\right)$$

3.5 problem 5

Internal problem ID [6623]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267

Problem number: 5.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [Lienard]

$$xy'' + y' + xy = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x$2)+diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \text{BesselJ}(0, x) + c_2 \text{BesselY}(0, x)$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 18

```
DSolve[x*y''[x]+y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \text{BesselJ}(0, x) + c_2 \text{BesselY}(0, x)$$

3.6 problem 6

Internal problem ID [6624]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267

Problem number: 6.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [Bessel]

$$xy'' + y' + \left(x - \frac{4}{x}\right)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(x*diff(y(x),x),x)+(x-4/x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-\text{BesselJ}(0, x) c_1 x - \text{BesselY}(0, x) c_2 x + 2c_1 \text{BesselJ}(1, x) + 2c_2 \text{BesselY}(1, x)}{x}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 18

```
DSolve[D[x*y'[x],x]+(x-4/x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \text{BesselJ}(2, x) + c_2 \text{BesselY}(2, x)$$

3.7 problem 7

Internal problem ID [6625]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267

Problem number: 7.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + xy' + (9x^2 - 4)y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(9*x^2-4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-3 \text{BesselY}(0, 3x) c_2 x - 3 \text{BesselJ}(0, 3x) c_1 x + 2 \text{BesselY}(1, 3x) c_2 + 2 \text{BesselJ}(1, 3x) c_1}{3x}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]+x*y'[x]+(9*x^2-4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \text{BesselJ}(2, 3x) + c_2 \text{BesselY}(2, 3x)$$

3.8 problem 8

Internal problem ID [6626]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267

Problem number: 8.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + xy' + \left(36x^2 - \frac{1}{4}\right) y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(36*x^2-1/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(6x) + c_2 \cos(6x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(36*x^2-1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^{-6ix}(12c_1 - ic_2 e^{12ix})}{12\sqrt{x}}$$

3.9 problem 9

Internal problem ID [6627]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267

Problem number: 9.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + xy' + \left(25x^2 - \frac{4}{9}\right) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(25*x^2-4/9)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \text{BesselJ}\left(\frac{2}{3}, 5x\right) + c_2 \text{BesselY}\left(\frac{2}{3}, 5x\right)$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 26

```
DSolve[x^2*y''[x]+x*y'[x]+(25*x^2-4/9)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \text{BesselJ}\left(\frac{2}{3}, 5x\right) + c_2 \text{BesselY}\left(\frac{2}{3}, 5x\right)$$

3.10 problem 10

Internal problem ID [6628]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267

Problem number: 10.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + x y' + (2x^2 - 64) y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 101

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(2*x^2-64)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-16\sqrt{2} c_1 (x^6 - 75x^4 + 1080x^2 - 2520) \text{BesselJ}(1, \sqrt{2}x) - 16\sqrt{2} c_2 (x^6 - 75x^4 + 1080x^2 - 2520) \text{BesselY}(1, \sqrt{2}x)}{x^7}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 30

```
DSolve[x^2*y''[x]+x*y'[x]+(2*x^2-64)*y[x]==0,y[x],x,IncludeSingularSolutions->True]
```

$$y(x) \rightarrow c_1 \text{BesselJ}\left(8, \sqrt{2}x\right) + c_2 \text{BesselY}\left(8, \sqrt{2}x\right)$$

3.11 problem 13

Internal problem ID [6629]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267

Problem number: 13.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$xy'' + 2y' + 4y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2 \text{BesselY}(1, 4\sqrt{x}) + c_1 \text{BesselJ}(1, 4\sqrt{x})}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 42

```
DSolve[x*y''[x]+2*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_1 \text{BesselJ}(1, 4\sqrt{x}) - 2ic_2 \text{BesselY}(1, 4\sqrt{x})}{2\sqrt{x}}$$

3.12 problem 14

Internal problem ID [6630]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267

Problem number: 14.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [Lienard]

$$xy'' + 3y' + xy = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)+3*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \text{BesselJ}(1, x) + c_2 \text{BesselY}(1, x)}{x}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 22

```
DSolve[x*y''[x]+3*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_1 \text{BesselJ}(1, x) + c_2 \text{BesselY}(1, x)}{x}$$

3.13 problem 15

Internal problem ID [6631]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267

Problem number: 15.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [Lienard]

$$xy'' - y' + xy = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x$2)-diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = x(c_1 \text{BesselJ}(1, x) + c_2 \text{BesselY}(1, x))$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 20

```
DSolve[x*y''[x]-y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x(c_1 \text{BesselJ}(1, x) + c_2 \text{BesselY}(1, x))$$

3.14 problem 16

Internal problem ID [6632]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267

Problem number: 16.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [_Lienard]

$$xy'' - 5y' + xy = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve(x*diff(y(x),x$2)-5*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = -(c_1(x^2 - 8) \text{BesselJ}(1, x) + c_2(x^2 - 8) \text{BesselY}(1, x) + 4x(c_1 \text{BesselJ}(0, x) + c_2 \text{BesselY}(0, x))) x$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 22

```
DSolve[x*y''[x]-5*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^3(c_1 \text{BesselJ}(3, x) + c_2 \text{BesselY}(3, x))$$

3.15 problem 17

Internal problem ID [6633]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267

Problem number: 17.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + (x^2 - 2) y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 27

```
dsolve(x^2*diff(y(x),x$2)+(x^2-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{(c_1 x + c_2) \cos(x) + \sin(x) (c_2 x - c_1)}{x}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 21

```
DSolve[x^2*y''[x]+(x^2-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x(c_1 j_1(x) - c_2 y_1(x))$$

3.16 problem 18

Internal problem ID [6634]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267

Problem number: 18.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + (16x^2 + 1)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(4*x^2*diff(y(x),x$2)+(16*x^2+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = (\text{BesselJ}(0, 2x) c_1 + \text{BesselY}(0, 2x) c_2) \sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 28

```
DSolve[4*x^2*y''[x]+(16*x^2+1)*y[x]==0,y[x],x,IncludeSingularSolutions->True]
```

$$y(x) \rightarrow \sqrt{x}(c_1 \text{BesselJ}(0, 2x) + c_2 \text{BesselY}(0, 2x))$$

3.17 problem 19

Internal problem ID [6635]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267

Problem number: 19.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$xy'' + 3y' + yx^3 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(x*diff(y(x),x$2)+3*diff(y(x),x)+x^3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin\left(\frac{x^2}{2}\right) + c_2 \cos\left(\frac{x^2}{2}\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.077 (sec). Leaf size: 43

```
DSolve[x*y''[x]+3*y'[x]+x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^{-\frac{ix^2}{2}} \left(2c_1 - ic_2 e^{ix^2}\right)}{2x^2}$$

3.18 problem 20

Internal problem ID [6636]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267

Problem number: 20.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2y'' + 9xy' + (x^6 - 36)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(9*x^2*diff(y(x),x$2)+9*x*diff(y(x),x)+(x^6-36)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \text{BesselJ}\left(\frac{2}{3}, \frac{x^3}{9}\right) + c_2 \text{BesselY}\left(\frac{2}{3}, \frac{x^3}{9}\right)$$

✓ Solution by Mathematica

Time used: 0.151 (sec). Leaf size: 43

```
DSolve[9*x^2*y'[x]+9*x*y'[x]+(x^6-36)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 3c_1 \text{Gamma}\left(\frac{4}{3}\right) \text{BesselJ}\left(-\frac{2}{3}, \frac{x^3}{9}\right) + c_2 \text{Gamma}\left(\frac{5}{3}\right) \text{BesselJ}\left(\frac{2}{3}, \frac{x^3}{9}\right)$$

3.19 problem 22(a)

Internal problem ID [6637]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267

Problem number: 22(a).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$y'' - yx^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(\text{BesselK} \left(\frac{1}{4}, \frac{x^2}{2} \right) c_2 + \text{BesselI} \left(\frac{1}{4}, \frac{x^2}{2} \right) c_1 \right) \sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 37

```
DSolve[y''[x]-x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_2 \text{ParabolicCylinderD} \left(-\frac{1}{2}, i\sqrt{2}x \right) + c_1 \text{ParabolicCylinderD} \left(-\frac{1}{2}, \sqrt{2}x \right)$$

3.20 problem 22 (b)

Internal problem ID [6638]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267

Problem number: 22 (b).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$xy'' + y' - 7yx^3 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x*diff(y(x),x$2)+diff(y(x),x)-7*x^3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \text{BesselI}\left(0, \frac{\sqrt{7}x^2}{2}\right) + c_2 \text{BesselK}\left(0, \frac{\sqrt{7}x^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.089 (sec). Leaf size: 41

```
DSolve[x*y''[x]+y'[x]-7*x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \text{BesselI}\left(0, \frac{\sqrt{7}x^2}{2}\right) + 2c_2 K_0\left(\frac{\sqrt{7}x^2}{2}\right)$$

3.21 problem 23

Internal problem ID [6639]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267

Problem number: 23.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x$2)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(x) + \cos(x) c_2$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 16

```
DSolve[y''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \cos(x) + c_2 \sin(x)$$

3.22 problem 24

Internal problem ID [6640]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267

Problem number: 24.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + 4xy' + y(x^2 + 2) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x) + \cos(x) c_2}{x^2}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 37

```
DSolve[x^2*y''[x]+4*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x^2}$$

3.23 problem 25

Internal problem ID [6641]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267

Problem number: 25.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$16x^2y'' + 32xy' + (x^4 - 12)y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 25

```
dsolve(16*x^2*diff(y(x),x$2)+32*x*diff(y(x),x)+(x^4-12)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin\left(\frac{x^2}{8}\right) + c_2 \cos\left(\frac{x^2}{8}\right)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.086 (sec). Leaf size: 42

```
DSolve[16*x^2*y''[x]+32*x*y'[x]+(x^4-12)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^{-\frac{ix^2}{8}} \left(c_1 - 2ic_2 e^{\frac{ix^2}{4}} \right)}{x^{3/2}}$$

3.24 problem 26

Internal problem ID [6642]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267

Problem number: 26.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' - 4xy' + (16x^2 + 3)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(16*x^2+3)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x} (c_1 \sin(2x) + c_2 \cos(2x))$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 39

```
DSolve[4*x^2*y''[x]-4*x*y'[x]+(16*x^2+3)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{4} e^{-2ix} \sqrt{x} (4c_1 - ic_2 e^{4ix})$$

**4 CHAPTER 6 SERIES SOLUTIONS OF
LINEAR EQUATIONS. CHAPTER 6 IN
REVIEW. Page 271**

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4.1 problem 9

Internal problem ID [6643]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. CHAPTER 6 IN REVIEW. Page 271

Problem number: 9.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$2xy'' + y' + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 52

```
Order:=8;  
dsolve(2*x*diff(y(x),x$2)+diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 \sqrt{x} \left(1 - \frac{1}{3}x + \frac{1}{30}x^2 - \frac{1}{630}x^3 + \frac{1}{22680}x^4 - \frac{1}{1247400}x^5 + \frac{1}{97297200}x^6 - \frac{1}{10216206000}x^7 + O(x^8) \right) + c_2 \left(1 - x + \frac{1}{6}x^2 - \frac{1}{90}x^3 + \frac{1}{2520}x^4 - \frac{1}{113400}x^5 + \frac{1}{7484400}x^6 - \frac{1}{681080400}x^7 + O(x^8) \right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 111

```
AsymptoticDSolveValue[2*x*y'[x]+y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \sqrt{x} \left(-\frac{x^7}{10216206000} + \frac{x^6}{97297200} - \frac{x^5}{1247400} + \frac{x^4}{22680} - \frac{x^3}{630} + \frac{x^2}{30} - \frac{x}{3} + 1 \right) \\ + c_2 \left(-\frac{x^7}{681080400} + \frac{x^6}{7484400} - \frac{x^5}{113400} + \frac{x^4}{2520} - \frac{x^3}{90} + \frac{x^2}{6} - x + 1 \right)$$

4.2 problem 10

Internal problem ID [6644]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. CHAPTER 6 IN REVIEW. Page 271

Problem number: 10.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$y'' - xy' - y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;  
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 + \frac{1}{8}x^4 + \frac{1}{48}x^6\right) y(0) + \left(x + \frac{1}{3}x^3 + \frac{1}{15}x^5 + \frac{1}{105}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[y''[x]-x*y'[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left(\frac{x^7}{105} + \frac{x^5}{15} + \frac{x^3}{3} + x \right) + c_1 \left(\frac{x^6}{48} + \frac{x^4}{8} + \frac{x^2}{2} + 1 \right)$$

4.3 problem 11

Internal problem ID [6645]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. CHAPTER 6 IN REVIEW. Page 271

Problem number: 11.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$(x - 1)y'' + 3y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

```
Order:=8;  
dsolve((x-1)*diff(y(x),x$2)+3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{3}{2}x^2 + \frac{1}{2}x^3 + \frac{5}{8}x^4 + \frac{9}{20}x^5 + \frac{29}{80}x^6 + \frac{163}{560}x^7\right) y(0) \\ + \left(x + \frac{1}{2}x^3 + \frac{1}{4}x^4 + \frac{9}{40}x^5 + \frac{7}{40}x^6 + \frac{79}{560}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 91

```
AsymptoticDSolveValue[(x-1)*y''[x]+3*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left(\frac{79x^7}{560} + \frac{7x^6}{40} + \frac{9x^5}{40} + \frac{x^4}{4} + \frac{x^3}{2} + x \right) \\ + c_1 \left(\frac{163x^7}{560} + \frac{29x^6}{80} + \frac{9x^5}{20} + \frac{5x^4}{8} + \frac{x^3}{2} + \frac{3x^2}{2} + 1 \right)$$

4.4 problem 12

Internal problem ID [6646]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. CHAPTER 6 IN REVIEW. Page 271

Problem number: 12.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - x^2y' + xy = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=8;  
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{6}x^3 - \frac{1}{90}x^6\right)y(0) + D(y)(0)x + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 27

```
AsymptoticDSolveValue[y''[x]-x^2*y'[x]+x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left(-\frac{x^6}{90} - \frac{x^3}{6} + 1\right) + c_2x$$

4.5 problem 13

Internal problem ID [6647]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. CHAPTER 6 IN REVIEW. Page 271

Problem number: 13.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [_Laguerre]

$$xy'' - (x + 2)y' + 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 52

```
Order:=8;  
dsolve(x*diff(y(x),x$2)-(x+2)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^3 \left(1 + \frac{1}{4}x + \frac{1}{20}x^2 + \frac{1}{120}x^3 + \frac{1}{840}x^4 + \frac{1}{6720}x^5 + \frac{1}{60480}x^6 + \frac{1}{604800}x^7 + O(x^8) \right) \\ + c_2 \left(12 + 12x + 6x^2 + 2x^3 + \frac{1}{2}x^4 + \frac{1}{10}x^5 + \frac{1}{60}x^6 + \frac{1}{420}x^7 + O(x^8) \right)$$

✓ Solution by Mathematica

Time used: 0.084 (sec). Leaf size: 94

```
AsymptoticDSolveValue[x*y''[x]-(x+2)*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left(\frac{x^6}{720} + \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1 \right) \\ + c_2 \left(\frac{x^9}{60480} + \frac{x^8}{6720} + \frac{x^7}{840} + \frac{x^6}{120} + \frac{x^5}{20} + \frac{x^4}{4} + x^3 \right)$$

4.6 problem 14

Internal problem ID [6648]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. CHAPTER 6 IN REVIEW. Page 271

Problem number: 14.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\cos(x)y'' + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
Order:=8;  
dsolve(cos(x)*diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{720}x^6\right)y(0) + \left(x - \frac{1}{6}x^3 - \frac{1}{60}x^5 - \frac{13}{5040}x^7\right)D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 49

```
AsymptoticDSolveValue[Cos[x]*y''[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left(\frac{x^6}{720} - \frac{x^2}{2} + 1 \right) + c_2 \left(-\frac{13x^7}{5040} - \frac{x^5}{60} - \frac{x^3}{6} + x \right)$$

4.7 problem 15

Internal problem ID [6649]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. CHAPTER 6 IN REVIEW. Page 271

Problem number: 15.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + xy' + 2y = 0$$

With initial conditions

$$[y(0) = 3, y'(0) = -2]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=8;
```

```
dsolve([diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(0) = 3, D(y)(0) = -2],y(x),type='series',x=
```

$$y(x) = 3 - 2x - 3x^2 + x^3 + x^4 - \frac{1}{4}x^5 - \frac{1}{5}x^6 + \frac{1}{24}x^7 + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[{y'[x]+x*y'[x]+2*y[x]==0,{y[0]==3,y'[0]==-2}},y[x],{x,0,7}]
```

$$y(x) \rightarrow \frac{x^7}{24} - \frac{x^6}{5} - \frac{x^5}{4} + x^4 + x^3 - 3x^2 - 2x + 3$$

4.8 problem 16

Internal problem ID [6650]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. CHAPTER 6 IN REVIEW. Page 271

Problem number: 16.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$(x + 2)y'' + 3y = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
Order:=8;
dsolve([(x+2)*diff(y(x),x$2)+3*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x),type='series',x=0);
```

$$y(x) = x - \frac{1}{4}x^3 + \frac{1}{16}x^4 - \frac{1}{320}x^6 + \frac{1}{896}x^7 + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 33

```
AsymptoticDSolveValue[{(x+2)*y''[x]+3*y[x]==0,{y[0]==0,y'[0]==1}},y[x],{x,0,7}]
```

$$y(x) \rightarrow \frac{x^7}{896} - \frac{x^6}{320} + \frac{x^4}{16} - \frac{x^3}{4} + x$$

4.9 problem 17

Internal problem ID [6651]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. CHAPTER 6 IN REVIEW. Page 271

Problem number: 17.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(1 - 2 \sin(x)) y'' + xy = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
Order:=8;
```

```
dsolve((1-2*sin(x))*diff(y(x),x$2)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{6}x^3 - \frac{1}{6}x^4 - \frac{1}{5}x^5 - \frac{1}{4}x^6 - \frac{85}{252}x^7\right) y(0) \\ + \left(x - \frac{1}{12}x^4 - \frac{1}{10}x^5 - \frac{2}{15}x^6 - \frac{13}{72}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 77

```
AsymptoticDSolveValue[(1-2*SIN[x])*y'[x]+x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left(-\frac{13x^7}{72} - \frac{2x^6}{15} - \frac{x^5}{10} - \frac{x^4}{12} + x \right) + c_1 \left(-\frac{85x^7}{252} - \frac{x^6}{4} - \frac{x^5}{5} - \frac{x^4}{6} - \frac{x^3}{6} + 1 \right)$$

4.10 problem 18

Internal problem ID [6652]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. CHAPTER 6 IN REVIEW. Page 271

Problem number: 18.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$y'' + xy' + y = 0$$

With initial conditions

$$[y(1) = -6, y'(1) = 3]$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
Order:=8;
```

```
dsolve([diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(1) = -6, D(y)(1) = 3],y(x),type='series',x=1)
```

$$y(x) = -6 + 3(x-1) + \frac{3}{2}(x-1)^2 - \frac{3}{2}(x-1)^3 + \frac{3}{10}(x-1)^5 - \frac{1}{20}(x-1)^6 - \frac{1}{28}(x-1)^7 + O((x-1)^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 55

```
AsymptoticDSolveValue[{y'[x]+x*y'[x]+y[x]==0,{y[1]==-6,y'[1]==3}},y[x],{x,1,7}]
```

$$y(x) \rightarrow -\frac{1}{28}(x-1)^7 - \frac{1}{20}(x-1)^6 + \frac{3}{10}(x-1)^5 - \frac{3}{2}(x-1)^3 + \frac{3}{2}(x-1)^2 + 3(x-1) - 6$$

4.11 problem 19

Internal problem ID [6653]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. CHAPTER 6 IN REVIEW. Page 271

Problem number: 19.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$xy'' + (-\cos(x) + 1)y' + yx^2 = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 59

```
Order:=8;  
dsolve(x*diff(y(x),x$2)+(1-cos(x))*diff(y(x),x)+x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{6}x^3 + \frac{1}{80}x^5 + \frac{1}{180}x^6 - \frac{5}{4032}x^7\right) y(0) \\ + \left(x - \frac{1}{12}x^3 - \frac{1}{12}x^4 + \frac{1}{120}x^5 + \frac{1}{120}x^6 + \frac{73}{60480}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 36

```
AsymptoticDSolveValue[x*y''[x]+(1-Cos[x])*y'[x]+x^2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow -\frac{53x^7}{8640} + \frac{x^5}{48} + \frac{x^4}{6} - \frac{x^3}{3} - 2x + 3$$

4.12 problem 20

Internal problem ID [6654]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. CHAPTER 6 IN REVIEW. Page 271

Problem number: 20.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(e^x - 1 - x)y'' + xy = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.312 (sec). Leaf size: 70

```
Order:=8;
```

```
dsolve((exp(x)-1-x)*diff(y(x),x$2)+x*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = & c_1 x \left(1 - x + \frac{4}{9}x^2 - \frac{29}{216}x^3 + \frac{37}{1200}x^4 - \frac{58}{10125}x^5 + \frac{14209}{15876000}x^6 - \frac{107329}{889056000}x^7 \right. \\ & \left. + O(x^8) \right) + c_2 \left(\ln(x) \left((-2)x + 2x^2 - \frac{8}{9}x^3 + \frac{29}{108}x^4 - \frac{37}{600}x^5 + \frac{116}{10125}x^6 \right. \right. \\ & \left. \left. - \frac{14209}{7938000}x^7 + O(x^8) \right) + \left(1 - \frac{8}{3}x^2 + \frac{175}{108}x^3 - \frac{3727}{6480}x^4 + \frac{47531}{324000}x^5 \right. \right. \\ & \left. \left. - \frac{3003737}{102060000}x^6 + \frac{48833381}{10001880000}x^7 + O(x^8) \right) \right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.366 (sec). Leaf size: 133

```
AsymptoticDSolveValue[(Exp[x]-1-x)*y'[x]+x*y[x]==0,y[x],{x,0,7}]
```

$$\begin{aligned} y(x) \rightarrow & c_1 \left(x^6 \left(\frac{116 \log(x)}{10125} - \frac{3003737}{102060000} \right) + x^5 \left(\frac{47531}{324000} - \frac{37 \log(x)}{600} \right) \right. \\ & + x^4 \left(\frac{29 \log(x)}{108} - \frac{3727}{6480} \right) + x^3 \left(\frac{175}{108} - \frac{8 \log(x)}{9} \right) + x^2 \left(2 \log(x) - \frac{8}{3} \right) - 2x \log(x) \\ & \left. + 1 \right) + c_2 x \left(-\frac{107329x^7}{889056000} + \frac{14209x^6}{15876000} - \frac{58x^5}{10125} + \frac{37x^4}{1200} - \frac{29x^3}{216} + \frac{4x^2}{9} - x + 1 \right) \end{aligned}$$

4.13 problem 21

Internal problem ID [6655]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. CHAPTER 6 IN REVIEW. Page 271

Problem number: 21.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$y'' + x^2 y' + 2xy = 10x^3 - 2x + 5$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
Order:=8;
```

```
dsolve(diff(y(x),x$2)+x^2*diff(y(x),x)+2*x*y(x)=5-2*x+10*x^3,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{3}x^3 + \frac{1}{18}x^6\right) y(0) + \left(x - \frac{1}{4}x^4 + \frac{1}{28}x^7\right) D(y)(0) + \frac{5x^2}{2} - \frac{x^3}{3} + \frac{x^6}{18} + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 63

```
AsymptoticDSolveValue[y''[x]+x^2*y'[x]+2*x*y[x]==5-2*x+10*x^3,y[x],{x,0,7}]
```

$$y(x) \rightarrow \frac{x^6}{18} - \frac{x^3}{3} + \frac{5x^2}{2} + c_2 \left(\frac{x^7}{28} - \frac{x^4}{4} + x \right) + c_1 \left(\frac{x^6}{18} - \frac{x^3}{3} + 1 \right)$$

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289

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5.1 problem 31

Internal problem ID [6656]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.2.2 TRANSFORMS OF DERIVATIVES Page 289

Problem number: 31.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' - y = 1$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 1.672 (sec). Leaf size: 8

```
dsolve([diff(y(t),t)-y(t)=1,y(0) = 0],y(t), singsol=all)
```

$$y(t) = -1 + e^t$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 10

```
DSolve[{y'[t]-y[t]==1,{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow e^t - 1$$

5.2 problem 32

Internal problem ID [6657]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.2.2 TRANSFORMS OF DERIVATIVES Page 289

Problem number: 32.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$2y' + y = 0$$

With initial conditions

$$[y(0) = -3]$$

✓ Solution by Maple

Time used: 1.438 (sec). Leaf size: 10

```
dsolve([2*diff(y(t),t)+y(t)=0,y(0) = -3],y(t), singsol=all)
```

$$y(t) = -3e^{-\frac{t}{2}}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 14

```
DSolve[{2*y'[t]+y[t]==0,{y[0]==-3}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow -3e^{-t/2}$$

5.3 problem 33

Internal problem ID [6658]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.2.2 TRANSFORMS OF DERIVATIVES Page 289

Problem number: 33.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' + 6y = e^{4t}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 1.719 (sec). Leaf size: 17

```
dsolve([diff(y(t),t)+6*y(t)=exp(4*t),y(0) = 2],y(t), singsol=all)
```

$$y(t) = \frac{(e^{10t} + 19)e^{-6t}}{10}$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 21

```
DSolve[{y'[t]+6*y[t]==Exp[4*t],{y[0]==2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{10}e^{-6t}(e^{10t} + 19)$$

5.4 problem 34

Internal problem ID [6659]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.2.2 TRANSFORMS OF DERIVATIVES Page 289

Problem number: 34.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' - y = 2 \cos(5t)$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 1.703 (sec). Leaf size: 21

```
dsolve([diff(y(t),t)-y(t)=2*cos(5*t),y(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{e^t}{13} - \frac{\cos(5t)}{13} + \frac{5 \sin(5t)}{13}$$

✓ Solution by Mathematica

Time used: 0.102 (sec). Leaf size: 25

```
DSolve[{y'[t]-y[t]==2*Cos[5*t],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{13}(e^t + 5 \sin(5t) - \cos(5t))$$

5.5 problem 35

Internal problem ID [6660]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.2.2 TRANSFORMS OF DERIVATIVES Page 289

Problem number: 35.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 5y' + 4y = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 1.688 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)+5*diff(y(t),t)+4*y(t)=0,y(0) = 1, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = -\frac{e^{-4t}}{3} + \frac{4e^{-t}}{3}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 23

```
DSolve[{y''[t]+5*y'[t]+4*y[t]==0,{y[0]==1,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \rightarrow \frac{1}{3}e^{-4t}(4e^{3t} - 1)$$

5.6 problem 36

Internal problem ID [6661]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.2.2 TRANSFORMS OF DERIVATIVES Page 289

Problem number: 36.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$y'' - 4y' = 6e^{3t} - 3e^{-t}$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

✓ Solution by Maple

Time used: 1.812 (sec). Leaf size: 24

```
dsolve([diff(y(t),t$2)-4*diff(y(t),t)=6*exp(3*t)-3*exp(-t),y(0) = 1, D(y)(0) = -1],y(t), sin
```

$$y(t) = \frac{5}{2} + \frac{11e^{4t}}{10} - 2e^{3t} - \frac{3e^{-t}}{5}$$

✓ Solution by Mathematica

Time used: 0.139 (sec). Leaf size: 34

```
DSolve[{y'[t]-4*y'[t]==6*Exp[3*t]-3*Exp[-t],{y[0]==1,y'[0]==-1}},y[t],t,IncludeSingularSolu
```

$$y(t) \rightarrow -\frac{3e^{-t}}{5} - 2e^{3t} + \frac{11e^{4t}}{10} + \frac{5}{2}$$

5.7 problem 37

Internal problem ID [6662]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.2.2 TRANSFORMS OF DERIVATIVES Page 289

Problem number: 37.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y = \sqrt{2} \sin(t\sqrt{2})$$

With initial conditions

$$[y(0) = 10, y'(0) = 0]$$

✓ Solution by Maple

Time used: 1.765 (sec). Leaf size: 24

```
dsolve([diff(y(t),t$2)+y(t)=sqrt(2)*sin(sqrt(2)*t),y(0) = 10, D(y)(0) = 0],y(t), singsol=all
```

$$y(t) = -\sqrt{2} \sin(\sqrt{2}t) + 10 \cos(t) + 2 \sin(t)$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 29

```
DSolve[{y'[t]+y[t]==Sqrt[2]*Sin[Sqrt[2]*t],{y[0]==10,y'[0]==0}},y[t],t,IncludeSingularSolut
```

$$y(t) \rightarrow 2 \sin(t) - \sqrt{2} \sin(\sqrt{2}t) + 10 \cos(t)$$

5.8 problem 38

Internal problem ID [6663]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.2.2 TRANSFORMS OF DERIVATIVES Page 289

Problem number: 38.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 9y = e^t$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 1.828 (sec). Leaf size: 21

```
dsolve([diff(y(t),t$2)+9*y(t)=exp(t),y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = -\frac{\cos(3t)}{10} - \frac{\sin(3t)}{30} + \frac{e^t}{10}$$

✓ Solution by Mathematica

Time used: 0.145 (sec). Leaf size: 27

```
DSolve[{y'[t]+9*y[t]==Exp[t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{30}(3e^t - \sin(3t) - 3\cos(3t))$$

5.9 problem 39

Internal problem ID [6664]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.2.2 TRANSFORMS OF DERIVATIVES Page 289

Problem number: 39.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$2y''' + 3y'' - 3y' - 2y = e^{-t}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0, y''(0) = 1]$$

✓ Solution by Maple

Time used: 1.859 (sec). Leaf size: 25

```
dsolve([2*dif(y(t),t$3)+3*dif(y(t),t$2)-3*dif(y(t),t)-2*y(t)=exp(-t),y(0) = 0, D(y)(0) =
```

$$y(t) = \frac{7 \cosh(t)}{9} - \frac{2 \sinh(t)}{9} + \frac{e^{-2t}}{9} - \frac{8e^{-\frac{t}{2}}}{9}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 37

```
DSolve[{2*y'''[t]+3*y''[t]-3*y'[t]-2*y[t]==Exp[-t],{y[0]==0,y'[0]==0,y''[0]==1}},y[t],t,Incl
```

$$y(t) \rightarrow \frac{1}{18}e^{-2t}(9e^t - 16e^{3t/2} + 5e^{3t} + 2)$$

5.10 problem 40

Internal problem ID [6665]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.2.2 TRANSFORMS OF DERIVATIVES Page 289

Problem number: 40.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' + 2y'' - y' - 2y = \sin(3t)$$

With initial conditions

$$[y(0) = 0, y'(0) = 0, y''(0) = 1]$$

✓ Solution by Maple

Time used: 1.875 (sec). Leaf size: 31

```
dsolve([diff(y(t),t$3)+2*diff(y(t),t$2)-diff(y(t),t)-2*y(t)=sin(3*t),y(0) = 0, D(y)(0) = 0,
```

$$y(t) = \frac{3 \cos(3t)}{130} - \frac{\sin(3t)}{65} - \frac{13 \cosh(t)}{30} + \frac{13 \sinh(t)}{15} + \frac{16 e^{-2t}}{39}$$

✓ Solution by Mathematica

Time used: 0.072 (sec). Leaf size: 42

```
DSolve[{y'''[t]+2*y''[t]-y'[t]-2*y[t]==Sin[3*t],{y[0]==0,y'[0]==0,y''[0]==1}},y[t],t,Include
```

$$y(t) \rightarrow \frac{1}{780} (e^{-2t} (-507e^t + 169e^{3t} + 320) - 12 \sin(3t) + 18 \cos(3t))$$

5.11 problem 41

Internal problem ID [6666]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.2.2 TRANSFORMS OF DERIVATIVES Page 289

Problem number: 41.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' + y = e^{-3t} \cos(2t)$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 2.0 (sec). Leaf size: 28

```
dsolve([diff(y(t),t)+y(t)=exp(-3*t)*cos(2*t),y(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{e^{-t}}{4} + \frac{e^{-3t}(-\cos(2t) + \sin(2t))}{4}$$

✓ Solution by Mathematica

Time used: 0.124 (sec). Leaf size: 30

```
DSolve[{y'[t]+y[t]==Exp[-3*t]*Cos[2*t],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{4}e^{-3t}(e^{2t} + \sin(2t) - \cos(2t))$$

5.12 problem 42

Internal problem ID [6667]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.2.2 TRANSFORMS OF DERIVATIVES Page 289

Problem number: 42.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 2y' + 5y = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = 3]$$

✓ Solution by Maple

Time used: 1.765 (sec). Leaf size: 16

```
dsolve([diff(y(t),t$2)-2*diff(y(t),t)+5*y(t)=0,y(0) = 1, D(y)(0) = 3],y(t), singsol=all)
```

$$y(t) = e^t(\cos(2t) + \sin(2t))$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 18

```
DSolve[{y'[t]-2*y'[t]+5*y[t]==0,{y[0]==1,y'[0]==3}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \rightarrow e^t(\sin(2t) + \cos(2t))$$

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297

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6.1 problem 21

Internal problem ID [6668]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297

Problem number: 21.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' + 4y = e^{-4t}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 1.672 (sec). Leaf size: 12

```
dsolve([diff(y(t),t)+4*y(t)=exp(-4*t),y(0) = 2],y(t), singsol=all)
```

$$y(t) = (t + 2)e^{-4t}$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 14

```
DSolve[{y'[t]+4*y[t]==Exp[-4*t],{y[0]==2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow e^{-4t}(t + 2)$$

6.2 problem 22

Internal problem ID [6669]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297

Problem number: 22.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' - y = 1 + te^t$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 1.719 (sec). Leaf size: 15

```
dsolve([diff(y(t),t)-y(t)=1+t*exp(t),y(0) = 0],y(t), singsol=all)
```

$$y(t) = -1 + \frac{e^t t^2}{2} + e^t$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 19

```
DSolve[{y'[t]-y[t]==1+t*Exp[t],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{2}e^t(t^2 + 2) - 1$$

6.3 problem 23

Internal problem ID [6670]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297

Problem number: 23.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 2y' + y = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = 1]$$

✓ Solution by Maple

Time used: 1.687 (sec). Leaf size: 14

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+y(t)=0,y(0) = 1, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = (2t + 1)e^{-t}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 16

```
DSolve[{y'[t]+2*y'[t]+y[t]==0,{y[0]==1,y'[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow e^{-t}(2t + 1)$$

6.4 problem 24

Internal problem ID [6671]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297

Problem number: 24.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 4y' + 4y = t^3 e^{2t}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 1.829 (sec). Leaf size: 13

```
dsolve([diff(y(t),t$2)-4*diff(y(t),t)+4*y(t)=t^3*exp(2*t),y(0) = 0, D(y)(0) = 0],y(t), sings
```

$$y(t) = \frac{t^5 e^{2t}}{20}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 17

```
DSolve[{y''[t]-4*y'[t]+4*y[t]==t^3*Exp[2*t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSoluti
```

$$y(t) \rightarrow \frac{1}{20} e^{2t} t^5$$

6.5 problem 25

Internal problem ID [6672]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297

Problem number: 25.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 6y' + 9y = t$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 1.641 (sec). Leaf size: 20

```
dsolve([diff(y(t),t$2)-6*diff(y(t),t)+9*y(t)=t,y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = \frac{(30t - 2)e^{3t}}{27} + \frac{t}{9} + \frac{2}{27}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 25

```
DSolve[{y'[t]-6*y'[t]+9*y[t]==t,{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \rightarrow \frac{1}{27}(3t + e^{3t}(30t - 2) + 2)$$

6.6 problem 26

Internal problem ID [6673]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297

Problem number: 26.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 4y' + 4y = t^3$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 1.688 (sec). Leaf size: 30

```
dsolve([diff(y(t),t$2)-4*diff(y(t),t)+4*y(t)=t^3,y(0) = 1, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{(2 - 13t) e^{2t}}{8} + \frac{t^3}{4} + \frac{3t^2}{4} + \frac{9t}{8} + \frac{3}{4}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 35

```
DSolve[{y''[t]-4*y'[t]+4*y[t]==t^3,{y[0]==1,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{8}(2t^3 + 6t^2 + 9t + e^{2t}(2 - 13t) + 6)$$

6.7 problem 27

Internal problem ID [6674]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297

Problem number: 27.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 6y' + 13y = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = -3]$$

✓ Solution by Maple

Time used: 1.735 (sec). Leaf size: 14

```
dsolve([diff(y(t),t$2)-6*diff(y(t),t)+13*y(t)=0,y(0) = 0, D(y)(0) = -3],y(t), singsol=all)
```

$$y(t) = -\frac{3e^{3t} \sin(2t)}{2}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 16

```
DSolve[{y'[t]-6*y'[t]+13*y[t]==0,{y[0]==0,y'[0]==-3}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow -3e^{3t} \sin(t) \cos(t)$$

6.8 problem 28

Internal problem ID [6675]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297

Problem number: 28.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$2y'' + 20y' + 51y = 0$$

With initial conditions

$$[y(0) = 2, y'(0) = 0]$$

✓ Solution by Maple

Time used: 1.938 (sec). Leaf size: 30

```
dsolve([2*diff(y(t),t$2)+20*diff(y(t),t)+51*y(t)=0,y(0) = 2, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = 2e^{-5t} \left(\cos\left(\frac{\sqrt{2}t}{2}\right) + 5\sqrt{2} \sin\left(\frac{\sqrt{2}t}{2}\right) \right)$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 36

```
DSolve[{2*y'[t]+20*y'[t]+51*y[t]==0,{y[0]==2,y'[0]==0}},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \rightarrow 2e^{-5t} \left(5\sqrt{2} \sin\left(\frac{t}{\sqrt{2}}\right) + \cos\left(\frac{t}{\sqrt{2}}\right) \right)$$

6.9 problem 29

Internal problem ID [6676]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297

Problem number: 29.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - y = e^t \cos(t)$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 2.0 (sec). Leaf size: 24

```
dsolve([diff(y(t),t$2)-y(t)=exp(t)*cos(t),y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{e^{-t}}{5} + \frac{e^t(-\cos(t) + 2\sin(t))}{5}$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 29

```
DSolve[{y''[t]-y[t]==Exp[t]*Cos[t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{5}(e^{-t} + 2e^t \sin(t) - e^t \cos(t))$$

6.10 problem 30

Internal problem ID [6677]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297

Problem number: 30.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 2y' + 5y = t + 1$$

With initial conditions

$$[y(0) = 0, y'(0) = 4]$$

✓ Solution by Maple

Time used: 1.797 (sec). Leaf size: 26

```
dsolve([diff(y(t),t$2)-2*diff(y(t),t)+5*y(t)=1+t,y(0) = 0, D(y)(0) = 4],y(t), singsol=all)
```

$$y(t) = -\frac{7e^t \cos(2t)}{25} + \frac{51e^t \sin(2t)}{25} + \frac{t}{5} + \frac{7}{25}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 32

```
DSolve[{y'[t]-2*y'[t]+5*y[t]==1+t,{y[0]==0,y'[0]==4}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{25}(5t + 51e^t \sin(2t) - 7e^t \cos(2t) + 7)$$

6.11 problem 31

Internal problem ID [6678]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297

Problem number: 31.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 2y' + y = 0$$

With initial conditions

$$[y(1) = 2, y'(0) = 2]$$

✓ Solution by Maple

Time used: 1.547 (sec). Leaf size: 26

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+y(t)=0,y(1) = 2, D(y)(0) = 2],y(t), singsol=all)
```

$$y(t) = e^{-t}(t - 1 + et + e)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 18

```
DSolve[{y'[t]+2*y'[t]+y[t]==0,{y[1]==2,y'[0]==2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow e^{-t}(et + t + e - 1)$$

6.12 problem 32

Internal problem ID [6679]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297

Problem number: 32.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 8y' + 20y = 0$$

With initial conditions

$$[y(0) = 0, y'(\pi) = 0]$$

✓ Solution by Maple

Time used: 1.782 (sec). Leaf size: 5

```
dsolve([diff(y(t),t$2)+8*diff(y(t),t)+20*y(t)=0,y(0) = 0, D(y)(Pi) = 0],y(t), singsol=all)
```

$$y(t) = 0$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 6

```
DSolve[{y'[t]+8*y'[t]+20*y[t]==0,{y[0]==0,y'[Pi]==0}},y[t],t,IncludeSingularSolutions -> Tr
```

$$y(t) \rightarrow 0$$

6.13 problem 63

Internal problem ID [6680]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297

Problem number: 63.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' + y = \begin{cases} 0 & 0 \leq t < 1 \\ 5 & 1 \leq t \end{cases}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 2.625 (sec). Leaf size: 22

```
dsolve([diff(y(t),t)+y(t)=piecewise(0<=t and t<1,0,t>=1,5),y(0) = 0],y(t), singsol=all)
```

$$y(t) = \begin{cases} 0 & t < 1 \\ 5 - 5e^{1-t} & 1 \leq t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 23

```
DSolve[{y'[t]+y[t]==Piecewise[{{0,0<=t<1},{5,t>=1}}],{y[0]==0}],y[t],t,IncludeSingularSoluti
```

$$y(t) \rightarrow \begin{cases} 0 & t \leq 1 \\ 5 - 5e^{1-t} & \text{True} \end{cases}$$

6.14 problem 64

Internal problem ID [6681]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297

Problem number: 64.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' + y = \begin{cases} 1 & 0 \leq t < 1 \\ -1 & 1 \leq t \end{cases}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 2.782 (sec). Leaf size: 35

```
dsolve([diff(y(t),t)+y(t)=piecewise(0<=t and t<1,1,t>=1,-1),y(0) = 0],y(t), singsol=all)
```

$$y(t) = -e^{-t} - \left(\begin{cases} -1 & t < 1 \\ 1 - 2e^{1-t} & 1 \leq t \end{cases} \right)$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 43

```
DSolve[{y'[t]+y[t]==Piecewise[{{1,0<=t<1},{-1,t>=1}}],{y[0]==0}],y[t],t,IncludeSingularSolut
```

$$y(t) \rightarrow \begin{cases} 0 & t \leq 0 \\ 1 - e^{-t} & 0 < t \leq 1 \\ -e^{-t}(1 - 2e + e^t) & \text{True} \end{cases}$$

6.15 problem 65

Internal problem ID [6682]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297

Problem number: 65.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' + y = \begin{cases} t & 0 \leq t < 1 \\ 0 & 1 \leq t \end{cases}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 3.313 (sec). Leaf size: 51

```
dsolve([diff(y(t),t)+y(t)=piecewise(0<=t and t<1,t,t>=1,0),y(0) = 0],y(t), singsol=all)
```

$$y(t) = \begin{cases} -1 + e^{-t} + t & t < 1 \\ 1 + e^{-1} & t = 1 \\ e^{-t} & 1 < t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 32

```
DSolve[{y'[t]+y[t]==Piecewise[{{t,0<=t<1},{0,t>=1}}],{y[0]==0}],y[t],t,IncludeSingularSoluti
```

$$y(t) \rightarrow \begin{cases} 0 & t \leq 0 \\ t + e^{-t} - 1 & 0 < t \leq 1 \\ e^{-t} & \text{True} \end{cases}$$

6.16 problem 66

Internal problem ID [6683]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297

Problem number: 66.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y = \begin{cases} 1 & 0 \leq t < 1 \\ 0 & 1 \leq t \end{cases}$$

With initial conditions

$$[y(0) = 0, y'(0) = -1]$$

✓ Solution by Maple

Time used: 2.734 (sec). Leaf size: 47

```
dsolve([diff(y(t),t$2)+4*y(t)=piecewise(0<=t and t<1,1,t>=1,0),y(0) = 0, D(y)(0) = -1],y(t),
```

$$y(t) = -\frac{\sin(2t)}{2} - \frac{\cos(2t)}{4} + \frac{\left(\begin{cases} 1 & t < 1 \\ \cos(2t - 2) & 1 \leq t \end{cases} \right)}{4}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 65

```
DSolve[{y''[t]+4*y[t]==Piecewise[{{1,0<=t<1},{0,t>=1}}],{y[0]==0,y'[0]==-1}},y[t],t,IncludeS
```

$$y(t) \rightarrow \begin{cases} -\cos(t)\sin(t) & t \leq 0 \\ \frac{1}{4}(-\cos(2t) - 2\sin(2t) + 1) & 0 < t \leq 1 \\ \frac{1}{4}(\cos(2 - 2t) - \cos(2t) - 2\sin(2t)) & \text{True} \end{cases}$$

6.17 problem 67

Internal problem ID [6684]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297

Problem number: 67.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y = \sin(t) \operatorname{Heaviside}(t - 2\pi)$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 2.031 (sec). Leaf size: 28

```
dsolve([diff(y(t),t$2)+4*y(t)=sin(t)*Heaviside(t-2*Pi),y(0) = 1, D(y)(0) = 0],y(t), singsol=
```

$$y(t) = -\frac{(\cos(t) - 1) \sin(t) \operatorname{Heaviside}(t - 2\pi)}{3} + 2 \cos(t)^2 - 1$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 36

```
DSolve[{y'[t]+4*y[t]==Sin[t]*UnitStep[t-2*Pi],{y[0]==1,y'[0]==0}},y[t],t,IncludeSingularSol
```

$$y(t) \rightarrow \begin{cases} \cos(2t) & t \leq 2\pi \\ \frac{1}{3}(3 \cos(2t) - \cos(t) \sin(t) + \sin(t)) & \text{True} \end{cases}$$

6.18 problem 68

Internal problem ID [6685]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297

Problem number: 68.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 5y' + 6y = \text{Heaviside}(t - 1)$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 1.906 (sec). Leaf size: 62

```
dsolve([diff(y(t),t$2)-5*diff(y(t),t)+6*y(t)=Heaviside(t-1),y(0) = 0, D(y)(0) = 1],y(t), sin
```

$$y(t) = \frac{\text{Heaviside}(t - 1)}{6} - e^{2t} + e^{3t} - \frac{\text{Heaviside}(t - 1) e^{2t-2}}{2} + \frac{\text{Heaviside}(t - 1) e^{3t-3}}{3}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 60

```
DSolve[{y'[t]-5*y'[t]+6*y[t]==UnitStep[t-1],{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolut
```

$$y(t) \rightarrow \begin{cases} e^{2t}(-1 + e^t) & t \leq 1 \\ \frac{1}{6} - e^{2t} + e^{3t} - \frac{1}{2}e^{2t-2} + \frac{1}{3}e^{3t-3} & \text{True} \end{cases}$$

6.19 problem 69

Internal problem ID [6686]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297

Problem number: 69.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y = \begin{cases} 0 & 0 \leq t < \pi \\ 1 & \pi \leq t < 2\pi \\ 0 & 2\pi \leq t \end{cases}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 2.968 (sec). Leaf size: 50

```
dsolve([diff(y(t),t$2)+y(t)=piecewise(0<=t and t<Pi,0,Pi<=t and t<2*Pi,1,t>=2*Pi,0),y(0) = 0
```

$$y(t) = \sin(t) + \begin{pmatrix} 0 & t < \pi \\ \cos(t) + 1 & t < 2\pi \\ 2 \cos(t) & 2\pi \leq t \end{pmatrix}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 35

```
DSolve[{y''[t]+y[t]==Piecewise[{{0,0<=t<Pi},{1,Pi<=t<2*Pi},{0,t>=2*Pi}}],{y[0]==0,y'[0]==1}]
```

$$y(t) \rightarrow \begin{cases} \sin(t) & t \leq \pi \\ \cos(t) + \sin(t) + 1 & \pi < t \leq 2\pi \\ 2 \cos(t) + \sin(t) & \text{True} \end{cases}$$

6.20 problem 70

Internal problem ID [6687]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297

Problem number: 70.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y' + 3y = 1 - \text{Heaviside}(-2 + t) - \text{Heaviside}(t - 4) + \text{Heaviside}(t - 6)$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 2.109 (sec). Leaf size: 94

```
dsolve([diff(y(t),t$2)+4*diff(y(t),t)+3*y(t)=1-Heaviside(t-2)-Heaviside(t-4)+Heaviside(t-6),
```

$$y(t) = \frac{1}{3} - \frac{\text{Heaviside}(t-4)}{3} + \frac{e^{-3t}}{6} - \frac{e^{-t}}{2} - \frac{\text{Heaviside}(t-6)e^{-t+6}}{2} \\ + \frac{\text{Heaviside}(t-6)e^{-3t+18}}{6} + \frac{\text{Heaviside}(t-6)}{3} \\ + \frac{\text{Heaviside}(t-4)e^{-t+4}}{2} - \frac{\text{Heaviside}(t-4)e^{-3t+12}}{6} \\ + \frac{\text{Heaviside}(t-2)e^{-t+2}}{2} - \frac{\text{Heaviside}(t-2)e^{-3t+6}}{6} - \frac{\text{Heaviside}(t-2)}{3}$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 175

`DSolve[{y'[t]+4*y'[t]+3*y[t]==1-UnitStep[t-2]-UnitStep[t-4]+UnitStep[t-6],{y[0]==0,y'[0]==0}`

$y(t)$

$$\rightarrow \left\{ \begin{array}{ll} \frac{1}{6}e^{-3t}(-1+e^t)^2(1+2e^t) & t \leq 2 \\ -\frac{1}{6}e^{-3t}(-1+e^2)(1+e^2+e^4-3e^{2t}) & 2 < t \leq 4 \\ \frac{1}{6}e^{-3t}(-1+e^2)^2(1+e^2)(1+e^2+2e^4+e^6+2e^8+e^{10}+e^{12}-3e^{2t}) & t > 6 \\ -\frac{1}{6}e^{-3t}(-1+e^6+e^{12}+3e^{2t}+2e^{3t}-3e^{2t+2}-3e^{2t+4}) & \text{True} \end{array} \right.$$

7 CHAPTER 7 THE LAPLACE TRANSFORM.

7.4.1 DERIVATIVES OF A TRANSFORM.

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7.1 problem 9

Internal problem ID [6688]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.4.1 DERIVATIVES OF A TRANSFORM. Page 309

Problem number: 9.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' + y = \sin(t)t$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 1.703 (sec). Leaf size: 23

```
dsolve([diff(y(t),t)+y(t)=t*sin(t),y(0) = 0],y(t), singsol=all)
```

$$y(t) = -\frac{e^{-t}}{2} + \frac{(1-t)\cos(t)}{2} + \frac{t\sin(t)}{2}$$

✓ Solution by Mathematica

Time used: 0.086 (sec). Leaf size: 28

```
DSolve[{y'[t]+y[t]==t*Sin[t],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{2}(-e^{-t} + t\sin(t) - t\cos(t) + \cos(t))$$

7.2 problem 10

Internal problem ID [6689]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.4.1 DERIVATIVES OF A TRANSFORM. Page 309

Problem number: 10.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' - y = e^t \sin(t) t$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 1.922 (sec). Leaf size: 15

```
dsolve([diff(y(t),t)-y(t)=t*exp(t)*sin(t),y(0) = 0],y(t), singsol=all)
```

$$y(t) = -e^t(-\sin(t) + \cos(t) t)$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 17

```
DSolve[{y'[t]-y[t]==t*Exp[t]*Sin[t],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow e^t(\sin(t) - t \cos(t))$$

7.3 problem 11

Internal problem ID [6690]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.4.1 DERIVATIVES OF A TRANSFORM. Page 309

Problem number: 11.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 9y = \cos(3t)$$

With initial conditions

$$[y(0) = 2, y'(0) = 5]$$

✓ Solution by Maple

Time used: 1.718 (sec). Leaf size: 20

```
dsolve([diff(y(t),t$2)+9*y(t)=cos(3*t),y(0) = 2, D(y)(0) = 5],y(t), singsol=all)
```

$$y(t) = 2 \cos(3t) + \frac{\sin(3t)(10+t)}{6}$$

✓ Solution by Mathematica

Time used: 0.104 (sec). Leaf size: 23

```
DSolve[{y'[t]+9*y[t]==Cos[3*t],{y[0]==2,y'[0]==5}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{6}(t+10)\sin(3t) + 2\cos(3t)$$

7.4 problem 12

Internal problem ID [6691]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.4.1 DERIVATIVES OF A TRANSFORM. Page 309

Problem number: 12.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y = \sin(t)$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

✓ Solution by Maple

Time used: 1.735 (sec). Leaf size: 16

```
dsolve([diff(y(t),t$2)+y(t)=sin(t),y(0) = 1, D(y)(0) = -1],y(t), singsol=all)
```

$$y(t) = -\frac{\sin(t)}{2} - \frac{\cos(t)(t-2)}{2}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 21

```
DSolve[{y'[t]+y[t]==Sin[t],{y[0]==1,y'[0]==-1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow -\frac{\sin(t)}{2} - \frac{1}{2}t \cos(t) + \cos(t)$$

7.5 problem 13

Internal problem ID [6692]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.4.1 DERIVATIVES OF A TRANSFORM. Page 309

Problem number: 13.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 16y = \begin{cases} \cos(4t) & 0 \leq t < \pi \\ 0 & \pi \leq t \end{cases}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 2.703 (sec). Leaf size: 29

```
dsolve([diff(y(t),t$2)+16*y(t)=piecewise(0<=t and t<Pi,cos(4*t),t>= Pi,0),y(0) = 0, D(y)(0)
```

$$y(t) = \frac{\sin(4t) \left(2 + \begin{pmatrix} t & t < \pi \\ \pi & \pi \leq t \end{pmatrix} \right)}{8}$$

✓ Solution by Mathematica

Time used: 0.135 (sec). Leaf size: 60

```
DSolve[{y'[t]+16*y[t]==Piecewise[{{Cos[4*t],0<=t<Pi},{0,t>=Pi}}],{y[0]==1,y'[0]==1}},y[t],t
```

$$y(t) \rightarrow \begin{cases} \cos(4t) + \frac{1}{4} \sin(4t) & t \leq 0 \\ \cos(4t) + \frac{1}{8}(2 + \pi) \sin(4t) & t > \pi \\ \cos(4t) + \frac{1}{8}(t + 2) \sin(4t) & \text{True} \end{cases}$$

7.6 problem 14

Internal problem ID [6693]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.4.1 DERIVATIVES OF A TRANSFORM. Page 309

Problem number: 14.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y = \begin{cases} 1 & 0 \leq t < \frac{\pi}{2} \\ \sin(t) & \frac{\pi}{2} \leq t \end{cases}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 2.734 (sec). Leaf size: 30

```
dsolve([diff(y(t),t$2)+y(t)=piecewise(0<=t and t<Pi/2,1,t>= Pi/2,sin(t)),y(0) = 1, D(y)(0) =
```

$$y(t) = \begin{cases} 1 & t < \frac{\pi}{2} \\ \frac{(-2t+\pi)\cos(t)}{4} + \sin(t) & \frac{\pi}{2} \leq t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 38

```
DSolve[{y''[t]+y[t]==Piecewise[{{1,0<=t<Pi/2},{Sin[t],t>=Pi/2}},{y[0]==1,y'[0]==0}],y[t],t,
```

$$y(t) \rightarrow \begin{cases} \cos(t) & t \leq 0 \\ 1 & t > 0 \wedge 2t \leq \pi \\ \frac{1}{4}(\pi - 2t) \cos(t) + \sin(t) & \text{True} \end{cases}$$

7.7 problem 17

Internal problem ID [6694]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.4.1 DERIVATIVES OF A TRANSFORM. Page 309

Problem number: 17.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$ty'' - y' = 2t^2$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 16

```
dsolve([t*dif(y(t),t$2)-dif(y(t),t)=2*t^2,y(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{t^2(4t + 3c_1)}{6}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 29

```
DSolve[{y''[t]-y'[t]==2*t^2,{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow -\frac{2t^3}{3} - 2t^2 - 4t + c_1(e^t - 1)$$

7.8 problem 18

Internal problem ID [6695]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.4.1 DERIVATIVES OF A TRANSFORM. Page 309

Problem number: 18.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2y'' + ty' - 2y = 10$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 9

```
dsolve([2*dif(y(t),t$2)+t*dif(y(t),t)-2*y(t)=10,y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{5t^2}{2}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 10

```
DSolve[{y'[t]+t*y'[t]-2*y[t]==10,{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow 5t^2$$

7.9 problem 36

Internal problem ID [6696]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.4.1 DERIVATIVES OF A TRANSFORM. Page 309

Problem number: 36.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y = \sin(t) + \sin(t)t$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 1.797 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)+y(t)=sin(t)+t*sin(t),y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = -\frac{(-\sin(t) + \cos(t)t)(t+2)}{4}$$

✓ Solution by Mathematica

Time used: 0.109 (sec). Leaf size: 21

```
DSolve[{y'[t]+y[t]==Sin[t]+t*Sin[t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \rightarrow -\frac{1}{4}(t+2)(t \cos(t) - \sin(t))$$

**8 CHAPTER 7 THE LAPLACE TRANSFORM.
EXERCISES 7.5. Page 315**

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8.1 problem 1

Internal problem ID [6697]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315

Problem number: 1.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear, 'class A']`

$$y' - 3y = \delta(-2 + t)$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 1.968 (sec). Leaf size: 15

```
dsolve([diff(y(t),t)-3*y(t)=Dirac(t-2),y(0) = 0],y(t), singsol=all)
```

$$y(t) = \text{Heaviside}(t - 2) e^{3t-6}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 17

```
DSolve[{y'[t]-3*y[t]==DiracDelta[t-2],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow e^{3t-6}\theta(t - 2)$$

8.2 problem 2

Internal problem ID [6698]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315

Problem number: 2.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear, 'class A']`

$$y' + y = \delta(t - 1)$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 1.938 (sec). Leaf size: 22

```
dsolve([diff(y(t),t)+y(t)=Dirac(t-1),y(0) = 2],y(t), singsol=all)
```

$$y(t) = \text{Heaviside}(t - 1) e^{1-t} + 2 e^{-t}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 19

```
DSolve[{y'[t]+y[t]==DiracDelta[t-1],{y[0]==2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow e^{-t}(e\theta(t - 1) + 2)$$

8.3 problem 3

Internal problem ID [6699]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315

Problem number: 3.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y = \delta(t - 2\pi)$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 1.953 (sec). Leaf size: 15

```
dsolve([diff(y(t),t$2)+y(t)=Dirac(t-2*Pi),y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = \sin(t) (\text{Heaviside}(t - 2\pi) + 1)$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 16

```
DSolve[{y''[t]+y[t]==DiracDelta[t-2*Pi],{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions
```

$$y(t) \rightarrow (\theta(t - 2\pi) + 1) \sin(t)$$

8.4 problem 4

Internal problem ID [6700]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315

Problem number: 4.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 16y = \delta(t - 2\pi)$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 1.891 (sec). Leaf size: 16

```
dsolve([diff(y(t),t$2)+16*y(t)=Dirac(t-2*Pi),y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{\text{Heaviside}(t - 2\pi) \sin(4t)}{4}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 19

```
DSolve[{y''[t]+16*y[t]==DiracDelta[t-2*Pi],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions->True]
```

$$y(t) \rightarrow \frac{1}{4}\theta(t - 2\pi) \sin(4t)$$

8.5 problem 5

Internal problem ID [6701]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315

Problem number: 5.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y = \delta\left(t - \frac{\pi}{2}\right) + \delta\left(t - \frac{3\pi}{2}\right)$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 2.062 (sec). Leaf size: 22

```
dsolve([diff(y(t),t$2)+y(t)=Dirac(t-1/2*Pi)+Dirac(t-3/2*Pi),y(0) = 0, D(y)(0) = 0],y(t), sin
```

$$y(t) = \left(\text{Heaviside}\left(t - \frac{3\pi}{2}\right) - \text{Heaviside}\left(t - \frac{\pi}{2}\right) \right) \cos(t)$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 27

```
DSolve[{y''[t]+y[t]==DiracDelta[t-1/2*Pi]+DiracDelta[t-3/2*Pi],{y[0]==0,y'[0]==0}},y[t],t,In
```

$$y(t) \rightarrow (\theta(2t - 3\pi) - \theta(2t - \pi)) \cos(t)$$

8.6 problem 6

Internal problem ID [6702]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315

Problem number: 6.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y = \delta(t - 2\pi) + \delta(t - 4\pi)$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 2.078 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$2)+y(t)=Dirac(t-2*Pi)+Dirac(t-4*Pi),y(0) = 1, D(y)(0) = 0],y(t), singsol
```

$$y(t) = \sin(t) \operatorname{Heaviside}(t - 2\pi) + \sin(t) \operatorname{Heaviside}(t - 4\pi) + \cos(t)$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 26

```
DSolve[{y''[t]+y[t]==DiracDelta[t-2*Pi]+DiracDelta[t-4*Pi],{y[0]==1,y'[0]==0}},y[t],t,Includ
```

$$y(t) \rightarrow \theta(t - 4\pi) \sin(t) + \theta(t - 2\pi) \sin(t) + \cos(t)$$

8.7 problem 7

Internal problem ID [6703]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315

Problem number: 7.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$y'' + 2y' = \delta(t - 1)$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 1.984 (sec). Leaf size: 28

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)=Dirac(t-1),y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = -\frac{\text{Heaviside}(t-1)e^{-2t+2}}{2} + \frac{\text{Heaviside}(t-1)}{2} + \frac{1}{2} - \frac{e^{-2t}}{2}$$

✓ Solution by Mathematica

Time used: 0.144 (sec). Leaf size: 37

```
DSolve[{y''[t]+2*y'[t]==DiracDelta[t-1],{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions
```

$$y(t) \rightarrow \frac{1}{2}e^{-2t}((e^{2t} - e^2)\theta(t-1) + e^{2t} - 1)$$

8.8 problem 8

Internal problem ID [6704]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315

Problem number: 8.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$y'' - 2y' = 1 + \delta(-2 + t)$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 2.016 (sec). Leaf size: 39

```
dsolve([diff(y(t),t$2)-2*diff(y(t),t)=1+Dirac(t-2),y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = -\frac{3}{4} - \frac{\text{Heaviside}(t-2)}{2} + \frac{3e^{2t}}{4} - \frac{t}{2} + \frac{\text{Heaviside}(t-2)e^{2t-4}}{2}$$

✓ Solution by Mathematica

Time used: 0.116 (sec). Leaf size: 37

```
DSolve[{y''[t]-2*y'[t]==1+DiracDelta[t-2],{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolution
```

$$y(t) \rightarrow \frac{1}{4}((2e^{2t-4} - 2)\theta(t-2) - 2t + 3e^{2t} - 3)$$

8.9 problem 9

Internal problem ID [6705]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315

Problem number: 9.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y' + 5y = \delta(t - 2\pi)$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 2.109 (sec). Leaf size: 21

```
dsolve([diff(y(t),t$2)+4*diff(y(t),t)+5*y(t)=Dirac(t-2*Pi),y(0) = 0, D(y)(0) = 0],y(t), sing
```

$$y(t) = \sin(t) \operatorname{Heaviside}(t - 2\pi) e^{4\pi - 2t}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 23

```
DSolve[{y'[t]+4*y'[t]+5*y[t]==DiracDelta[t-2*Pi],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingular
```

$$y(t) \rightarrow e^{4\pi - 2t} \theta(t - 2\pi) \sin(t)$$

8.10 problem 10

Internal problem ID [6706]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315

Problem number: 10.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' + y = \delta(t - 1)$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 1.937 (sec). Leaf size: 18

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+y(t)=Dirac(t-1),y(0) = 0, D(y)(0) = 0],y(t), singsol=a
```

$$y(t) = (t - 1) \text{Heaviside}(t - 1) e^{1-t}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 20

```
DSolve[{y'[t]+2*y'[t]+y[t]==DiracDelta[t-1],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolut
```

$$y(t) \rightarrow e^{1-t}(t - 1)\theta(t - 1)$$

8.11 problem 11

Internal problem ID [6707]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315

Problem number: 11.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y' + 13y = \delta(t - \pi) + \delta(t - 3\pi)$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 2.297 (sec). Leaf size: 59

```
dsolve([diff(y(t),t$2)+4*diff(y(t),t)+13*y(t)=Dirac(t-Pi)+Dirac(t-3*Pi),y(0) = 1, D(y)(0) =
```

$$y(t) = \left(\cos(3t) + \frac{2 \sin(3t)}{3} \right) e^{-2t} - \frac{\sin(3t) e^{-2t+6\pi} \text{Heaviside}(t - 3\pi)}{3} - \frac{\sin(3t) e^{-2t+2\pi} \text{Heaviside}(t - \pi)}{3}$$

✓ Solution by Mathematica

Time used: 0.084 (sec). Leaf size: 59

```
DSolve[{y''[t]+4*y'[t]+13*y[t]==DiracDelta[t-Pi]+DiracDelta[t-3*Pi],{y[0]==1,y'[0]==0}},y[t]
```

$$y(t) \rightarrow -\frac{1}{3}e^{-2t}(e^{6\pi}\theta(t - 3\pi)\sin(3t) + e^{2\pi}\theta(t - \pi)\sin(3t) - 2\sin(3t) - 3\cos(3t))$$

8.12 problem 12

Internal problem ID [6708]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315

Problem number: 12.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 7y' + 6y = e^t + \delta(-2 + t) + \delta(t - 4)$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 2.266 (sec). Leaf size: 90

```
dsolve([diff(y(t),t$2)-7*diff(y(t),t)+6*y(t)=exp(t)+Dirac(t-2)+Dirac(t-4),y(0) = 0, D(y)(0)
```

$$y(t) = \frac{e^{-24+6t} \operatorname{Heaviside}(t-4)}{5} + \frac{e^{-12+6t} \operatorname{Heaviside}(t-2)}{5} - \frac{e^{t-4} \operatorname{Heaviside}(t-4)}{5} - \frac{e^{t-2} \operatorname{Heaviside}(t-2)}{5} + \frac{e^{6t}}{25} + \frac{(-5t-1)e^t}{25}$$

✓ Solution by Mathematica

Time used: 0.115 (sec). Leaf size: 67

```
DSolve[{y'[t]-7*y'[t]+6*y[t]==Exp[t]+DiracDelta[t-2]+DiracDelta[t-4],{y[0]==9,y'[0]==0}},y[t]
```

$$y(t) \rightarrow \frac{1}{25} e^{t-24} (5(e^{5t} - e^{20}) \theta(t-4) + 5(e^{5t+12} - e^{22}) \theta(t-2) + e^{24} (-5t - 44e^{5t} + 269))$$

8.13 problem 15(a)

Internal problem ID [6709]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315

Problem number: 15(a).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 2y' + 10y = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 1.688 (sec). Leaf size: 14

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+10*y(t)=0,y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = \frac{e^{-t} \sin(3t)}{3}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 18

```
DSolve[{y'[t]+2*y'[t]+10*y[t]==0,{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{3}e^{-t} \sin(3t)$$

8.14 problem 15(b)

Internal problem ID [6710]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315

Problem number: 15(b).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_2nd_order, _linear, _nonhomogeneous]`

$$y'' + 2y' + 10y = \delta(t)$$

✓ Solution by Maple

Time used: 1.86 (sec). Leaf size: 30

```
dsolve(diff(y(t),t$2)+2*diff(y(t),t)+10*y(t)=Dirac(t),y(t), singsol=all)
```

$$y(t) = \frac{e^{-t}(3y(0) \cos(3t) + \sin(3t)(D(y)(0) + y(0) + 1))}{3}$$

✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 38

```
DSolve[y''[t]+2*y'[t]+10*y[t]==DiracDelta[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{3}e^{-t}(\theta(t) \sin(3t) + 3c_2 \cos(3t) + 3c_1 \sin(3t))$$

**9 CHAPTER 8 SYSTEMS OF LINEAR
FIRST-ORDER DIFFERENTIAL EQUATIONS.
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9.1 problem 1

Internal problem ID [6711]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332

Problem number: 1.

ODE order: 1.

ODE degree: 1.

Solve

$$x'(t) = 3x(t) - 5y$$

$$y' = 4x(t) + 8y$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 84

```
dsolve([diff(x(t),t)=3*x(t)-5*y(t),diff(y(t),t)=4*x(t)+8*y(t)],singsol=all)
```

$$x(t) = e^{\frac{11t}{2}} \left(\sin\left(\frac{\sqrt{55}t}{2}\right) c_1 + \cos\left(\frac{\sqrt{55}t}{2}\right) c_2 \right)$$
$$y(t) = \frac{e^{\frac{11t}{2}} \left(\sin\left(\frac{\sqrt{55}t}{2}\right) \sqrt{55} c_2 - \cos\left(\frac{\sqrt{55}t}{2}\right) \sqrt{55} c_1 - 5 \sin\left(\frac{\sqrt{55}t}{2}\right) c_1 - 5 \cos\left(\frac{\sqrt{55}t}{2}\right) c_2 \right)}{10}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 113

```
DSolve[{x'[t]==3*x[t]-5*y[t],y'[t]==4*x[t]+8*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \rightarrow \frac{1}{11} e^{11t/2} \left(11c_1 \cos\left(\frac{\sqrt{55}t}{2}\right) - \sqrt{55}(c_1 + 2c_2) \sin\left(\frac{\sqrt{55}t}{2}\right) \right)$$
$$y(t) \rightarrow \frac{1}{55} e^{11t/2} \left(55c_2 \cos\left(\frac{\sqrt{55}t}{2}\right) + \sqrt{55}(8c_1 + 5c_2) \sin\left(\frac{\sqrt{55}t}{2}\right) \right)$$

9.2 problem 2

Internal problem ID [6712]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332

Problem number: 2.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= 4x(t) - 7y \\ y' &= 5x(t)\end{aligned}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 78

```
dsolve([diff(x(t),t)=4*x(t)-7*y(t),diff(y(t),t)=5*x(t)],singsol=all)
```

$$\begin{aligned}x(t) &= \frac{e^{2t}(\sqrt{31} \sin(\sqrt{31}t) c_2 - \sqrt{31} \cos(\sqrt{31}t) c_1 - 2 \sin(\sqrt{31}t) c_1 - 2 \cos(\sqrt{31}t) c_2)}{5} \\ y(t) &= e^{2t}(\sin(\sqrt{31}t) c_1 + \cos(\sqrt{31}t) c_2)\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 98

```
DSolve[{x'[t]==4*x[t]-7*y[t],y'[t]==5*x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$\begin{aligned}x(t) &\rightarrow c_1 e^{2t} \cos(\sqrt{31}t) + \frac{(2c_1 - 7c_2)e^{2t} \sin(\sqrt{31}t)}{\sqrt{31}} \\ y(t) &\rightarrow c_2 e^{2t} \cos(\sqrt{31}t) + \frac{(5c_1 - 2c_2)e^{2t} \sin(\sqrt{31}t)}{\sqrt{31}}\end{aligned}$$

9.3 problem 3

Internal problem ID [6713]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332

Problem number: 3.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= -3x(t) + 4y - 9z(t) \\y' &= 6x(t) - y \\z'(t) &= 10x(t) + 4y + 3z(t)\end{aligned}$$

✓ Solution by Maple

Time used: 0.406 (sec). Leaf size: 2255

```
dsolve([diff(x(t),t)=-3*x(t)+4*y(t)-9*z(t),diff(y(t),t)=6*x(t)-y(t),diff(z(t),t)=10*x(t)+4*y
```

Expression too large to display

$y(t)$

$$\begin{aligned} &= \cos \left(\frac{\left((4726 + 306\sqrt{291})^{\frac{2}{3}} + 170 \right) t\sqrt{3} 1156^{\frac{1}{3}}}{204 (139 + 9\sqrt{291})^{\frac{1}{3}}} \right) e^{\frac{\left(-170 + (4726 + 306\sqrt{291})^{\frac{2}{3}} - 2(4726 + 306\sqrt{291})^{\frac{1}{3}} \right) t}{6(4726 + 306\sqrt{291})^{\frac{1}{3}}}} c_3 \\ &+ e^{\frac{\left(-170 + (4726 + 306\sqrt{291})^{\frac{2}{3}} - 2(4726 + 306\sqrt{291})^{\frac{1}{3}} \right) t}{6(4726 + 306\sqrt{291})^{\frac{1}{3}}}} \sin \left(\frac{\left((4726 + 306\sqrt{291})^{\frac{2}{3}} + 170 \right) t\sqrt{3} 1156^{\frac{1}{3}}}{204 (139 + 9\sqrt{291})^{\frac{1}{3}}} \right) c_2 \\ &- \frac{\left((4726 + 306\sqrt{291})^{\frac{2}{3}} + (4726 + 306\sqrt{291})^{\frac{1}{3}} - 170 \right) t}{3(4726 + 306\sqrt{291})^{\frac{1}{3}}} \\ &+ c_1 e \end{aligned}$$

Expression too large to display

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 510

`DSolve[{x'[t]==-3*x[t]+4*y[t]-9*z[t],y'[t]==6*x[t]-y[t],z'[t]==10*x[t]+4*y[t]+3*z[t]},{x[t],`

$$\begin{aligned}
 x(t) &\rightarrow 4c_2\text{RootSum}\left[\#1^3 + \#1^2 + 57\#1 + 369\&, \frac{\#1e^{\#1t} - 12e^{\#1t}}{3\#1^2 + 2\#1 + 57}\&\right] \\
 &\quad - 9c_3\text{RootSum}\left[\#1^3 + \#1^2 + 57\#1 + 369\&, \frac{\#1e^{\#1t} + e^{\#1t}}{3\#1^2 + 2\#1 + 57}\&\right] \\
 &\quad + c_1\text{RootSum}\left[\#1^3 + \#1^2 + 57\#1 + 369\&, \frac{\#1^2e^{\#1t} - 2\#1e^{\#1t} - 3e^{\#1t}}{3\#1^2 + 2\#1 + 57}\&\right] \\
 y(t) &\rightarrow -54c_3\text{RootSum}\left[\#1^3 + \#1^2 + 57\#1 + 369\&, \frac{e^{\#1t}}{3\#1^2 + 2\#1 + 57}\&\right] \\
 &\quad + 6c_1\text{RootSum}\left[\#1^3 + \#1^2 + 57\#1 + 369\&, \frac{\#1e^{\#1t} - 3e^{\#1t}}{3\#1^2 + 2\#1 + 57}\&\right] \\
 &\quad + c_2\text{RootSum}\left[\#1^3 + \#1^2 + 57\#1 + 369\&, \frac{\#1^2e^{\#1t} + 81e^{\#1t}}{3\#1^2 + 2\#1 + 57}\&\right] \\
 z(t) &\rightarrow 4c_2\text{RootSum}\left[\#1^3 + \#1^2 + 57\#1 + 369\&, \frac{\#1e^{\#1t} + 13e^{\#1t}}{3\#1^2 + 2\#1 + 57}\&\right] \\
 &\quad + 2c_1\text{RootSum}\left[\#1^3 + \#1^2 + 57\#1 + 369\&, \frac{5\#1e^{\#1t} + 17e^{\#1t}}{3\#1^2 + 2\#1 + 57}\&\right] \\
 &\quad + c_3\text{RootSum}\left[\#1^3 + \#1^2 + 57\#1 + 369\&, \frac{\#1^2e^{\#1t} + 4\#1e^{\#1t} - 21e^{\#1t}}{3\#1^2 + 2\#1 + 57}\&\right]
 \end{aligned}$$

9.4 problem 4

Internal problem ID [6714]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332

Problem number: 4.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= x(t) - y \\y' &= x(t) + 2z(t) \\z'(t) &= -x(t) + z(t)\end{aligned}$$

✓ Solution by Maple

Time used: 0.313 (sec). Leaf size: 2266

```
dsolve([diff(x(t),t)=x(t)-y(t),diff(y(t),t)=x(t)+2*z(t),diff(z(t),t)=-x(t)+z(t)],singsol=all
```

$$\begin{aligned}x(t) = & e^{-\frac{\left(-8+(244+12\sqrt{417})^{\frac{2}{3}}-8(244+12\sqrt{417})^{\frac{1}{3}}\right)t}{12(244+12\sqrt{417})^{\frac{1}{3}}}} \cos\left(\frac{\left(\left(244+12\sqrt{417}\right)^{\frac{2}{3}}+8\right)t\sqrt{3}2^{\frac{1}{3}}}{24(61+3\sqrt{417})^{\frac{1}{3}}}\right) c_3 \\ & - e^{-\frac{\left(-8+(244+12\sqrt{417})^{\frac{2}{3}}-8(244+12\sqrt{417})^{\frac{1}{3}}\right)t}{12(244+12\sqrt{417})^{\frac{1}{3}}}} \sin\left(\frac{\left(\left(244+12\sqrt{417}\right)^{\frac{2}{3}}+8\right)t\sqrt{3}2^{\frac{1}{3}}}{24(61+3\sqrt{417})^{\frac{1}{3}}}\right) c_2 \\ & + c_1 e^{\frac{\left(\left(244+12\sqrt{417}\right)^{\frac{2}{3}}+4(244+12\sqrt{417})^{\frac{1}{3}}-8\right)t}{6(244+12\sqrt{417})^{\frac{1}{3}}}}\end{aligned}$$

Expression too large to display

Expression too large to display

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 503

`DSolve[{x'[t]==x[t]-y[t],y'[t]==x[t]+2*z[t],z'[t]==-x[t]+z[t]},{x[t],y[t],z[t]},t,IncludeSin`

$$\begin{aligned}
 x(t) &\rightarrow -2c_3 \text{RootSum} \left[\#1^3 - 2\#1^2 + 2\#1 - 3\&, \frac{e^{\#1t}}{3\#1^2 - 4\#1 + 2} \& \right] \\
 &\quad - c_2 \text{RootSum} \left[\#1^3 - 2\#1^2 + 2\#1 - 3\&, \frac{\#1e^{\#1t} - e^{\#1t}}{3\#1^2 - 4\#1 + 2} \& \right] \\
 &\quad + c_1 \text{RootSum} \left[\#1^3 - 2\#1^2 + 2\#1 - 3\&, \frac{\#1^2e^{\#1t} - \#1e^{\#1t}}{3\#1^2 - 4\#1 + 2} \& \right] \\
 y(t) &\rightarrow c_1 \text{RootSum} \left[\#1^3 - 2\#1^2 + 2\#1 - 3\&, \frac{\#1e^{\#1t} - 3e^{\#1t}}{3\#1^2 - 4\#1 + 2} \& \right] \\
 &\quad + 2c_3 \text{RootSum} \left[\#1^3 - 2\#1^2 + 2\#1 - 3\&, \frac{\#1e^{\#1t} - e^{\#1t}}{3\#1^2 - 4\#1 + 2} \& \right] \\
 &\quad + c_2 \text{RootSum} \left[\#1^3 - 2\#1^2 + 2\#1 - 3\&, \frac{\#1^2e^{\#1t} - 2\#1e^{\#1t} + e^{\#1t}}{3\#1^2 - 4\#1 + 2} \& \right] \\
 z(t) &\rightarrow c_2 \text{RootSum} \left[\#1^3 - 2\#1^2 + 2\#1 - 3\&, \frac{e^{\#1t}}{3\#1^2 - 4\#1 + 2} \& \right] \\
 &\quad - c_1 \text{RootSum} \left[\#1^3 - 2\#1^2 + 2\#1 - 3\&, \frac{\#1e^{\#1t}}{3\#1^2 - 4\#1 + 2} \& \right] \\
 &\quad + c_3 \text{RootSum} \left[\#1^3 - 2\#1^2 + 2\#1 - 3\&, \frac{\#1^2e^{\#1t} - \#1e^{\#1t} + e^{\#1t}}{3\#1^2 - 4\#1 + 2} \& \right]
 \end{aligned}$$

9.5 problem 5

Internal problem ID [6715]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332

Problem number: 5.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= x(t) - y + z(t) + t - 1 \\y' &= 2x(t) + y - z(t) - 3t^2 \\z'(t) &= x(t) + y + z(t) + t^2 - t + 2\end{aligned}$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 172

```
dsolve([diff(x(t),t)=x(t)-y(t)+z(t)+t-1,diff(y(t),t)=2*x(t)+y(t)-z(t)-3*t^2,diff(z(t),t)=x(t)
```

$$\begin{aligned}x(t) &= t^2 - \frac{1}{6} + c_1 e^{2t} + c_2 e^{\frac{t}{2}} \cos\left(\frac{\sqrt{11}t}{2}\right) + c_3 e^{\frac{t}{2}} \sin\left(\frac{\sqrt{11}t}{2}\right) \\y(t) &= -\frac{t^2}{2} - \frac{7}{4} + \frac{c_1 e^{2t}}{2} - \frac{c_2 e^{\frac{t}{2}} \cos\left(\frac{\sqrt{11}t}{2}\right)}{2} + \frac{c_2 e^{\frac{t}{2}} \sqrt{11} \sin\left(\frac{\sqrt{11}t}{2}\right)}{2} \\&\quad - \frac{c_3 e^{\frac{t}{2}} \sin\left(\frac{\sqrt{11}t}{2}\right)}{2} - \frac{c_3 e^{\frac{t}{2}} \sqrt{11} \cos\left(\frac{\sqrt{11}t}{2}\right)}{2} - \frac{3t}{2} \\z(t) &= -\frac{t}{2} + \frac{3c_1 e^{2t}}{2} - c_2 e^{\frac{t}{2}} \cos\left(\frac{\sqrt{11}t}{2}\right) - c_3 e^{\frac{t}{2}} \sin\left(\frac{\sqrt{11}t}{2}\right) - \frac{3t^2}{2} - \frac{7}{12}\end{aligned}$$

✓ Solution by Mathematica

Time used: 15.906 (sec). Leaf size: 304

`DSolve[{x'[t]==x[t]-y[t]+z[t]+t-1,y'[t]==2*x[t]+y[t]-z[t]-3*t^2,z'[t]==x[t]+y[t]+z[t]+t^2-t+1},{x[t],y[t],z[t]},t]`

$$x(t) \rightarrow t^2 + \frac{2}{5}c_1e^{2t} + \frac{2}{5}c_3e^{2t} + \frac{1}{5}(3c_1 - 2c_3)e^{t/2} \cos\left(\frac{\sqrt{11}t}{2}\right) - \frac{(c_1 + 10c_2 - 4c_3)e^{t/2} \sin\left(\frac{\sqrt{11}t}{2}\right)}{5\sqrt{11}} - \frac{1}{6}$$

$$y(t) \rightarrow \frac{1}{220}\left(-11(10t^2 + 30t - 4(c_1 + c_3)e^{2t} + 35) - 44(c_1 - 5c_2 + c_3)e^{t/2} \cos\left(\frac{\sqrt{11}t}{2}\right) + 4\sqrt{11}(17c_1 + 5c_2 - 13c_3)e^{t/2} \sin\left(\frac{\sqrt{11}t}{2}\right)\right)$$

$$z(t) \rightarrow -\frac{3t^2}{2} - \frac{t}{2} + \frac{3}{5}(c_1 + c_3)e^{2t} - \frac{1}{5}(3c_1 - 2c_3)e^{t/2} \cos\left(\frac{\sqrt{11}t}{2}\right) + \frac{(c_1 + 10c_2 - 4c_3)e^{t/2} \sin\left(\frac{\sqrt{11}t}{2}\right)}{5\sqrt{11}} - \frac{7}{12}$$

9.6 problem 6

Internal problem ID [6716]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332

Problem number: 6.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= -3x(t) + 4y(t) + 2e^{-t} \sin(t) \cos(t) \\y'(t) &= 5x(t) + 9z(t) + 8e^{-t} \cos(t)^2 - 4e^{-t} \\z'(t) &= y(t) + 6z(t) - e^{-t}\end{aligned}$$

 Solution by Maple

```
dsolve([diff(x(t),t)=-3*x(t)+4*y(t)+exp(-t)*sin(2*t),diff(y(t),t)=5*x(t)+9*z(t)+4*exp(-t)*co
```

No solution found

 Solution by Mathematica

Time used: 0.649 (sec). Leaf size: 2949

```
DSolve[{x'[t]==-3*x[t]+4*y[t]+Exp[-t]*Sin[2*t],y'[t]==5*x[t]+9*z[t]+4*Exp[-t]*Cos[2*t],z'[t]
```

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9.7 problem 7

Internal problem ID [6717]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332

Problem number: 7.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= 4x(t) + 2y + e^t \\y' &= -x(t) + 3y - e^t\end{aligned}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 106

```
dsolve([diff(x(t),t)=4*x(t)+2*y(t)+exp(t),diff(y(t),t)=-x(t)+3*y(t)-exp(t)],singsol=all)
```

$$\begin{aligned}x(t) &= e^{\frac{7t}{2}} \sin\left(\frac{\sqrt{7}t}{2}\right) c_2 + e^{\frac{7t}{2}} \cos\left(\frac{\sqrt{7}t}{2}\right) c_1 - \frac{e^t}{2} \\y(t) &= -\frac{e^{\frac{7t}{2}} \sin\left(\frac{\sqrt{7}t}{2}\right) c_2}{4} + \frac{e^{\frac{7t}{2}} \sqrt{7} \cos\left(\frac{\sqrt{7}t}{2}\right) c_2}{4} \\&\quad - \frac{e^{\frac{7t}{2}} \cos\left(\frac{\sqrt{7}t}{2}\right) c_1}{4} - \frac{e^{\frac{7t}{2}} \sqrt{7} \sin\left(\frac{\sqrt{7}t}{2}\right) c_1}{4} + \frac{e^t}{4}\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.463 (sec). Leaf size: 129

```
DSolve[{x'[t]==4*x[t]+2*y[t]+Exp[t],y'[t]==-x[t]+3*y[t]-Exp[t]},{x[t],y[t]},t,IncludeSingular
```

$$x(t) \rightarrow -\frac{e^t}{2} + c_1 e^{7t/2} \cos\left(\frac{\sqrt{7}t}{2}\right) + \frac{(c_1 + 4c_2)e^{7t/2} \sin\left(\frac{\sqrt{7}t}{2}\right)}{\sqrt{7}}$$

$$y(t) \rightarrow \frac{e^t}{4} + c_2 e^{7t/2} \cos\left(\frac{\sqrt{7}t}{2}\right) - \frac{(2c_1 + c_2)e^{7t/2} \sin\left(\frac{\sqrt{7}t}{2}\right)}{\sqrt{7}}$$

9.8 problem 8

Internal problem ID [6718]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332

Problem number: 8.

ODE order: 1.

ODE degree: 1.

Solve

$$x'(t) = 7x(t) + 5y(t) - 9z(t) - 8e^{-2t}$$

$$y'(t) = 4x(t) + y(t) + z(t) + 2e^{5t}$$

$$z'(t) = -2y(t) + 3z(t) + e^{5t} - 3e^{-2t}$$

✓ Solution by Maple

Time used: 5.688 (sec). Leaf size: 9313

```
dsolve([diff(x(t),t)=7*x(t)+5*y(t)-9*z(t)-8*exp(-2*t),diff(y(t),t)=4*x(t)+y(t)+z(t)+2*exp(5*t),diff(z(t),t)=-2*y(t)+3*z(t)+exp(5*t)-3*exp(-2*t)],t)
```

Expression too large to display

Expression too large to display

Expression too large to display

✓ Solution by Mathematica

Time used: 0.206 (sec). Leaf size: 3002

```
DSolve[{x'[t]==7*x[t]+5*y[t]-9*z[t]-8*Exp[-2*t],y'[t]==4*x[t]+y[t]+z[t]+2*Exp[5*t],z'[t]==-2*y[t]+3*z[t]+Exp[5*t]-3*Exp[-2*t]},x[t],y[t],z[t],t]
```

Too large to display

9.9 problem 9

Internal problem ID [6719]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332

Problem number: 9.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= x(t) - y(t) + 2z(t) + e^{-t} - 3t \\y'(t) &= 3x(t) - 4y(t) + z(t) + 2e^{-t} + t \\z'(t) &= -2x(t) + 5y(t) + 6z(t) + 2e^{-t} - t\end{aligned}$$

 Solution by Maple

```
dsolve([diff(x(t),t)=x(t)-y(t)+2*z(t)+exp(-t)-3*t,diff(y(t),t)=3*x(t)-4*y(t)+z(t)+2*exp(-t)+
```

No solution found

 Solution by Mathematica

Time used: 0.191 (sec). Leaf size: 3251

```
DSolve[{x'[t]==x[t]-y[t]+2*z[t]+Exp[-t]-3*t,y'[t]==3*x[t]-4*y[t]+z[t]+2*Exp[-t]+t,z'[t]==-2*
```

Too large to display

9.10 problem 10

Internal problem ID [6720]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332

Problem number: 10.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= t e^{4t} + 4 \sin(t) - 4 e^{4t} - 7y + 3x(t) \\y' &= 2t e^{4t} + e^{4t} + 8 \sin(t) + y + x(t)\end{aligned}$$

✓ Solution by Maple

Time used: 1.156 (sec). Leaf size: 134

```
dsolve([diff(x(t),t)=3*x(t)-7*y(t)+4*sin(t)+(t-4)*exp(4*t),diff(y(t),t)=x(t)+y(t)+8*sin(t)+
```

$$\begin{aligned}x(t) &= e^{2t} \sin(\sqrt{6}t) c_2 + e^{2t} \cos(\sqrt{6}t) c_1 - \frac{11 e^{4t} t}{10} - \frac{34 e^{4t}}{25} - \frac{204 \cos(t)}{97} - \frac{556 \sin(t)}{97} \\y(t) &= \frac{3 e^{4t} t}{10} + \frac{e^{2t} \sqrt{6} \sin(\sqrt{6}t) c_1}{7} - \frac{e^{2t} \sqrt{6} \cos(\sqrt{6}t) c_2}{7} - \frac{11 e^{4t}}{50} \\&\quad + \frac{e^{2t} \sin(\sqrt{6}t) c_2}{7} + \frac{e^{2t} \cos(\sqrt{6}t) c_1}{7} - \frac{8 \cos(t)}{97} - \frac{212 \sin(t)}{97}\end{aligned}$$

✓ Solution by Mathematica

Time used: 11.331 (sec). Leaf size: 190

```
DSolve[{x'[t]==3*x[t]-7*y[t]+4*Sin[t]+(t-4)*Exp[4*t],y'[t]==x[t]+y[t]+8*Sin[t]+(2*t+1)*Exp[4
```

$$\begin{aligned}x(t) &\rightarrow -\frac{11}{10}e^{4t}t - \frac{34e^{4t}}{25} - \frac{556 \sin(t)}{97} - \frac{204 \cos(t)}{97} \\ &\quad + c_1 e^{2t} \cos(\sqrt{6}t) + \frac{c_1 e^{2t} \sin(\sqrt{6}t)}{\sqrt{6}} - \frac{7c_2 e^{2t} \sin(\sqrt{6}t)}{\sqrt{6}} \\ y(t) &\rightarrow \frac{3}{10}e^{4t}t - \frac{11e^{4t}}{50} - \frac{212 \sin(t)}{97} - \frac{8 \cos(t)}{97} \\ &\quad + c_2 e^{2t} \cos(\sqrt{6}t) + \frac{c_1 e^{2t} \sin(\sqrt{6}t)}{\sqrt{6}} - \frac{c_2 e^{2t} \sin(\sqrt{6}t)}{\sqrt{6}}\end{aligned}$$

9.11 problem 11

Internal problem ID [6721]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332

Problem number: 11.

ODE order: 1.

ODE degree: 1.

Solve

$$x'(t) = 3x(t) - 4y$$

$$y' = 4x(t) - 7y$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve([diff(x(t),t)=3*x(t)-4*y(t),diff(y(t),t)=4*x(t)-7*y(t)],singsol=all)
```

$$x(t) = c_1 e^{-5t} + c_2 e^t$$

$$y(t) = 2c_1 e^{-5t} + \frac{c_2 e^t}{2}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 73

```
DSolve[{x'[t]==3*x[t]-4*y[t],y'[t]==4*x[t]-7*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \rightarrow \frac{1}{3} e^{-5t} (c_1 (4e^{6t} - 1) - 2c_2 (e^{6t} - 1))$$

$$y(t) \rightarrow \frac{1}{3} e^{-5t} (2c_1 (e^{6t} - 1) - c_2 (e^{6t} - 4))$$

9.12 problem 12

Internal problem ID [6722]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332

Problem number: 12.

ODE order: 1.

ODE degree: 1.

Solve

$$x'(t) = -2x(t) + 5y$$

$$y' = -2x(t) + 4y$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 44

```
dsolve([diff(x(t),t)=-2*x(t)+5*y(t),diff(y(t),t)=-2*x(t)+4*y(t)],singsol=all)
```

$$x(t) = e^t(c_1 \sin(t) + c_2 \cos(t))$$
$$y(t) = \frac{e^t(3c_1 \sin(t) - c_2 \sin(t) + c_1 \cos(t) + 3c_2 \cos(t))}{5}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 51

```
DSolve[{x'[t]==-2*x[t]+5*y[t],y'[t]==-2*x[t]+4*y[t]},{x[t],y[t]},t,IncludeSingularSolutions
```

$$x(t) \rightarrow e^t(c_1 \cos(t) + (5c_2 - 3c_1) \sin(t))$$
$$y(t) \rightarrow e^t(c_2(3 \sin(t) + \cos(t)) - 2c_1 \sin(t))$$

9.13 problem 13

Internal problem ID [6723]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332

Problem number: 13.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= -x(t) + \frac{y}{4} \\ y' &= x(t) - y\end{aligned}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=-x(t)+1/4*y(t),diff(y(t),t)=x(t)-y(t)],singsol=all)
```

$$\begin{aligned}x(t) &= c_1 e^{-\frac{t}{2}} + c_2 e^{-\frac{3t}{2}} \\ y(t) &= 2c_1 e^{-\frac{t}{2}} - 2c_2 e^{-\frac{3t}{2}}\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 66

```
DSolve[{x'[t]==-x[t]+1/4*y[t],y'[t]==x[t]-y[t]},{x[t],y[t]},t,IncludeSingularSolutions->True]
```

$$\begin{aligned}x(t) &\rightarrow \frac{1}{4}e^{-3t/2}(2c_1(e^t + 1) + c_2(e^t - 1)) \\ y(t) &\rightarrow \frac{1}{2}e^{-3t/2}(2c_1(e^t - 1) + c_2(e^t + 1))\end{aligned}$$

9.14 problem 14

Internal problem ID [6724]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332

Problem number: 14.

ODE order: 1.

ODE degree: 1.

Solve

$$x'(t) = 2x(t) + y$$

$$y' = -x(t)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve([diff(x(t),t)=2*x(t)+y(t),diff(y(t),t)=-x(t)],singsol=all)
```

$$x(t) = -e^t(c_2t + c_1 + c_2)$$

$$y(t) = e^t(c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 38

```
DSolve[{x'[t]==2*x[t]+y[t],y'[t]==-x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow e^t(c_1(t+1) + c_2t)$$

$$y(t) \rightarrow e^t(c_2 - (c_1 + c_2)t)$$

9.15 problem 15

Internal problem ID [6725]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332

Problem number: 15.

ODE order: 1.

ODE degree: 1.

Solve

$$x'(t) = x(t) + 2y + z(t)$$

$$y' = 6x(t) - y$$

$$z'(t) = -x(t) - 2y - z(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 61

```
dsolve([diff(x(t),t)=x(t)+2*y(t)+z(t),diff(y(t),t)=6*x(t)-y(t),diff(z(t),t)=-x(t)-2*y(t)-z(t)
```

$$\begin{aligned}x(t) &= -\frac{c_2 e^{-4t}}{2} + \frac{2c_3 e^{3t}}{3} + \frac{c_1}{6} \\y(t) &= c_1 + c_2 e^{-4t} + c_3 e^{3t} \\z(t) &= \frac{c_2 e^{-4t}}{2} - \frac{2c_3 e^{3t}}{3} - \frac{13c_1}{6}\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 190

```
DSolve[{x'[t]==x[t]+2*y[t]+z[t],y'[t]==6*x[t]-y[t],z'[t]==-x[t]-2*y[t]-z[t]},{x[t],y[t],z[t]
```

$$\begin{aligned}x(t) &\rightarrow \frac{1}{84}e^{-4t}(c_1(-7e^{4t} + 64e^{7t} + 27) + 24c_2(e^{7t} - 1) + c_3(-7e^{4t} + 16e^{7t} - 9)) \\y(t) &\rightarrow \frac{1}{14}e^{-4t}(c_1(-7e^{4t} + 16e^{7t} - 9) + c_2(6e^{7t} + 8) + c_3(-7e^{4t} + 4e^{7t} + 3)) \\z(t) &\rightarrow \frac{1}{84}e^{-4t}(c_1(91e^{4t} - 64e^{7t} - 27) - 24c_2(e^{7t} - 1) - c_3(-91e^{4t} + 16e^{7t} - 9))\end{aligned}$$

9.16 problem 16

Internal problem ID [6726]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332

Problem number: 16.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= x(t) + z(t) \\y' &= x(t) + y \\z'(t) &= -2x(t) - z(t)\end{aligned}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 65

```
dsolve([diff(x(t),t)=x(t)+z(t),diff(y(t),t)=x(t)+y(t),diff(z(t),t)=-2*x(t)-z(t)],singsol=all
```

$$\begin{aligned}x(t) &= c_2 \sin(t) + c_3 \cos(t) \\y(t) &= -\frac{c_2 \cos(t)}{2} - \frac{c_3 \cos(t)}{2} - \frac{c_2 \sin(t)}{2} + \frac{c_3 \sin(t)}{2} + c_1 e^t \\z(t) &= -c_3 \sin(t) + c_2 \cos(t) - c_2 \sin(t) - c_3 \cos(t)\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 76

```
DSolve[{x'[t]==x[t]+z[t],y'[t]==x[t]+y[t],z'[t]==-2*x[t]-z[t]},{x[t],y[t],z[t]},t,IncludeSin
```

$$\begin{aligned}x(t) &\rightarrow c_1 \cos(t) + (c_1 + c_3) \sin(t) \\y(t) &\rightarrow c_2 e^t + c_1 (e^t - \cos(t)) - \frac{1}{2} c_3 (-e^t + \sin(t) + \cos(t)) \\z(t) &\rightarrow c_3 \cos(t) - (2c_1 + c_3) \sin(t)\end{aligned}$$

10 CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

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10.1 problem 1

Internal problem ID [6727]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 1.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= x(t) + 2y \\ y' &= 4x(t) + 3y\end{aligned}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=x(t)+2*y(t),diff(y(t),t)=4*x(t)+3*y(t)],singsol=all)
```

$$\begin{aligned}x(t) &= e^{-t}c_1 + c_2e^{5t} \\ y(t) &= -e^{-t}c_1 + 2c_2e^{5t}\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 71

```
DSolve[{x'[t]==x[t]+2*y[t],y'[t]==4*x[t]+3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions->T
```

$$\begin{aligned}x(t) &\rightarrow \frac{1}{3}e^{-t}(c_1(e^{6t} + 2) + c_2(e^{6t} - 1)) \\ y(t) &\rightarrow \frac{1}{3}e^{-t}(2c_1(e^{6t} - 1) + c_2(2e^{6t} + 1))\end{aligned}$$

10.2 problem 2

Internal problem ID [6728]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 2.

ODE order: 1.

ODE degree: 1.

Solve

$$x'(t) = 2x(t) + 2y$$

$$y' = x(t) + 3y$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve([diff(x(t),t)=2*x(t)+2*y(t),diff(y(t),t)=x(t)+3*y(t)],singsol=all)
```

$$x(t) = c_1 e^t + c_2 e^{4t}$$

$$y(t) = -\frac{c_1 e^t}{2} + c_2 e^{4t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 67

```
DSolve[{x'[t]==2*x[t]+2*y[t],y'[t]==x[t]+3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> T
```

$$x(t) \rightarrow \frac{1}{3}e^t(c_1(e^{3t} + 2) + 2c_2(e^{3t} - 1))$$

$$y(t) \rightarrow \frac{1}{3}e^t(c_1(e^{3t} - 1) + c_2(2e^{3t} + 1))$$

10.3 problem 3

Internal problem ID [6729]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 3.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= -4x(t) + 2y \\y' &= -\frac{5x(t)}{2} + 2y\end{aligned}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
dsolve([diff(x(t),t)=-4*x(t)+2*y(t),diff(y(t),t)=-5/2*x(t)+2*y(t)],singsol=all)
```

$$\begin{aligned}x(t) &= c_1 e^t + c_2 e^{-3t} \\y(t) &= \frac{5c_1 e^t}{2} + \frac{c_2 e^{-3t}}{2}\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 149

```
DSolve[{x'[t]==-4*x[t]+2*y[t],y'[t]==5/2*x[t]+2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions
```

$$\begin{aligned}x(t) &\rightarrow \frac{1}{28} e^{-((1+\sqrt{14})t)} \left(c_1 \left((14 - 3\sqrt{14}) e^{2\sqrt{14}t} + 14 + 3\sqrt{14} \right) + 2\sqrt{14} c_2 \left(e^{2\sqrt{14}t} - 1 \right) \right) \\y(t) &\rightarrow \frac{1}{56} e^{-((1+\sqrt{14})t)} \left(5\sqrt{14} c_1 \left(e^{2\sqrt{14}t} - 1 \right) + 2c_2 \left((14 + 3\sqrt{14}) e^{2\sqrt{14}t} + 14 - 3\sqrt{14} \right) \right)\end{aligned}$$

10.4 problem 4

Internal problem ID [6730]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 4.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= -\frac{5x(t)}{2} + 2y \\y' &= \frac{3x(t)}{4} - 2y\end{aligned}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=-5/2*x(t)+2*y(t),diff(y(t),t)=3/4*x(t)-2*y(t)],singsol=all)
```

$$\begin{aligned}x(t) &= e^{-\frac{7t}{2}}c_1 + c_2e^{-t} \\y(t) &= -\frac{e^{-\frac{7t}{2}}c_1}{2} + \frac{3c_2e^{-t}}{4}\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 165

```
DSolve[{x'[t]==5/2*x[t]+2*y[t],y'[t]==3/4*x[t]-2*y[t]},{x[t],y[t]},t,IncludeSingularSolution
```

$$\begin{aligned}x(t) &\rightarrow \frac{1}{210}e^{\frac{1}{4}(t-\sqrt{105}t)}\left(3c_1\left(\left(35+3\sqrt{105}\right)e^{\frac{\sqrt{105}t}{2}}+35-3\sqrt{105}\right)+8\sqrt{105}c_2\left(e^{\frac{\sqrt{105}t}{2}}-1\right)\right) \\y(t) &\rightarrow \frac{1}{70}e^{\frac{1}{4}(t-\sqrt{105}t)}\left(\sqrt{105}c_1\left(e^{\frac{\sqrt{105}t}{2}}-1\right)-c_2\left(\left(3\sqrt{105}-35\right)e^{\frac{\sqrt{105}t}{2}}-35-3\sqrt{105}\right)\right)\end{aligned}$$

10.5 problem 5

Internal problem ID [6731]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 5.

ODE order: 1.

ODE degree: 1.

Solve

$$x'(t) = 10x(t) - 5y$$

$$y' = 8x(t) - 12y$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=10*x(t)-5*y(t),diff(y(t),t)=8*x(t)-12*y(t)],singsol=all)
```

$$x(t) = c_1 e^{8t} + c_2 e^{-10t}$$

$$y(t) = \frac{2c_1 e^{8t}}{5} + 4c_2 e^{-10t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 73

```
DSolve[{x'[t]==10*x[t]-5*y[t],y'[t]==8*x[t]-12*y[t]},{x[t],y[t]},t,IncludeSingularSolutions
```

$$x(t) \rightarrow \frac{1}{18} e^{-10t} (c_1 (20e^{18t} - 2) - 5c_2 (e^{18t} - 1))$$

$$y(t) \rightarrow \frac{1}{9} e^{-10t} (4c_1 (e^{18t} - 1) - c_2 (e^{18t} - 10))$$

10.6 problem 6

Internal problem ID [6732]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 6.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= -6x(t) + 2y \\ y' &= -3x(t) + y\end{aligned}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve([diff(x(t),t)=-6*x(t)+2*y(t),diff(y(t),t)=-3*x(t)+y(t)],singsol=all)
```

$$\begin{aligned}x(t) &= c_1 + c_2 e^{-5t} \\ y(t) &= \frac{c_2 e^{-5t}}{2} + 3c_1\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 73

```
DSolve[{x'[t]==-6*x[t]+2*y[t],y'[t]==-3*x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$\begin{aligned}x(t) &\rightarrow \frac{1}{5}e^{-5t}(2c_2(e^{5t} - 1) - c_1(e^{5t} - 6)) \\ y(t) &\rightarrow \frac{1}{5}e^{-5t}(c_2(6e^{5t} - 1) - 3c_1(e^{5t} - 1))\end{aligned}$$

10.7 problem 7

Internal problem ID [6733]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 7.

ODE order: 1.

ODE degree: 1.

Solve

$$x'(t) = x(t) + y - z(t)$$

$$y' = 2y$$

$$z'(t) = y - z(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 50

```
dsolve([diff(x(t),t)=x(t)+y(t)-z(t),diff(y(t),t)=2*y(t),diff(z(t),t)=y(t)-z(t)],singsol=all)
```

$$x(t) = \frac{2c_3e^{2t}}{3} + c_1e^t + \frac{c_2e^{-t}}{2}$$
$$y(t) = c_3e^{2t}$$
$$z(t) = \frac{c_3e^{2t}}{3} + c_2e^{-t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 88

```
DSolve[{x'[t]==x[t]+y[t]-z[t],y'[t]==2*y[t],z'[t]==y[t]-z[t]},{x[t],y[t],z[t]},t,IncludeSing
```

$$x(t) \rightarrow \frac{1}{6}e^{-t}(4c_2e^{3t} + (6c_1 - 3(c_2 + c_3))e^{2t} - c_2 + 3c_3)$$
$$y(t) \rightarrow c_2e^{2t}$$
$$z(t) \rightarrow \frac{1}{3}e^{-t}(c_2(e^{3t} - 1) + 3c_3)$$

10.8 problem 8

Internal problem ID [6734]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 8.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= 2x(t) - 7y \\y' &= 5x(t) + 10y + 4z(t) \\z'(t) &= 5y + 2z(t)\end{aligned}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 66

```
dsolve([diff(x(t),t)=2*x(t)-7*y(t),diff(y(t),t)=5*x(t)+10*y(t)+4*z(t),diff(z(t),t)=5*y(t)+2*z(t)),{x(t),y(t),z(t)}
```

$$\begin{aligned}x(t) &= -\frac{7c_3e^{7t}}{5} - \frac{7c_2e^{5t}}{3} + c_1e^{2t} \\y(t) &= c_2e^{5t} + c_3e^{7t} \\z(t) &= \frac{5c_2e^{5t}}{3} + c_3e^{7t} - \frac{5c_1e^{2t}}{4}\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 190

```
DSolve[{x'[t]==2*x[t]-7*y[t],y'[t]==5*x[t]+10*y[t]+4*z[t],z'[t]==5*y[t]+2*z[t]},{x[t],y[t],z[t]}
```

$$\begin{aligned}x(t) &\rightarrow -\frac{1}{30}e^{2t}(5c_1(-35e^{3t} + 21e^{5t} + 8) + 7(-5(3c_2 + 4c_3)e^{3t} + 3(5c_2 + 4c_3)e^{5t} + 8c_3)) \\y(t) &\rightarrow \frac{1}{2}e^{5t}(5c_1(e^{2t} - 1) + c_2(5e^{2t} - 3) + 4c_3(e^{2t} - 1)) \\z(t) &\rightarrow \frac{1}{6}e^{2t}(5c_1(-5e^{3t} + 3e^{5t} + 2) - 5(3c_2 + 4c_3)e^{3t} + 3(5c_2 + 4c_3)e^{5t} + 14c_3)\end{aligned}$$

10.9 problem 9

Internal problem ID [6735]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 9.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= -x(t) + y \\y' &= x(t) + 2y + z(t) \\z'(t) &= 3y - z(t)\end{aligned}$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 67

```
dsolve([diff(x(t),t)=-x(t)+y(t),diff(y(t),t)=x(t)+2*y(t)+z(t),diff(z(t),t)=3*y(t)-z(t)],sing
```

$$\begin{aligned}x(t) &= \frac{c_2 e^{3t}}{4} + e^{-t} c_1 - c_3 e^{-2t} \\y(t) &= c_2 e^{3t} + c_3 e^{-2t} \\z(t) &= \frac{3c_2 e^{3t}}{4} - e^{-t} c_1 - 3c_3 e^{-2t}\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 158

```
DSolve[{x'[t]==-x[t]+y[t],y'[t]==x[t]+2*y[t]+z[t],z'[t]==3*y[t]-z[t]},{x[t],y[t],z[t]},t,Inc
```

$$\begin{aligned}x(t) &\rightarrow \frac{1}{20} e^{-2t} (c_1 (15e^t + e^{5t} + 4) + 4c_2 (e^{5t} - 1) + c_3 (-5e^t + e^{5t} + 4)) \\y(t) &\rightarrow \frac{1}{5} e^{-2t} (c_1 (e^{5t} - 1) + c_2 (4e^{5t} + 1) + c_3 (e^{5t} - 1)) \\z(t) &\rightarrow \frac{1}{20} e^{-2t} (3c_1 (-5e^t + e^{5t} + 4) + 12c_2 (e^{5t} - 1) + c_3 (5e^t + 3e^{5t} + 12))\end{aligned}$$

10.10 problem 10

Internal problem ID [6736]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 10.

ODE order: 1.

ODE degree: 1.

Solve

$$x'(t) = x(t) + z(t)$$

$$y' = y$$

$$z'(t) = x(t) + z(t)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve([diff(x(t),t)=x(t)+z(t),diff(y(t),t)=y(t),diff(z(t),t)=x(t)+z(t)],singsol=all)
```

$$x(t) = c_1 + c_2 e^{2t}$$

$$y(t) = c_3 e^t$$

$$z(t) = c_2 e^{2t} - c_1$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 128

```
DSolve[{x'[t]==x[t]+z[t],y'[t]==y[t],z'[t]==x[t]+z[t]},{x[t],y[t],z[t]},t,IncludeSingularSol
```

$$x(t) \rightarrow \frac{1}{2}(c_1(e^{2t} + 1) + c_2(e^{2t} - 1))$$

$$z(t) \rightarrow \frac{1}{2}(c_1(e^{2t} - 1) + c_2(e^{2t} + 1))$$

$$y(t) \rightarrow c_3 e^t$$

$$x(t) \rightarrow \frac{1}{2}(c_1(e^{2t} + 1) + c_2(e^{2t} - 1))$$

$$z(t) \rightarrow \frac{1}{2}(c_1(e^{2t} - 1) + c_2(e^{2t} + 1))$$

$$y(t) \rightarrow 0$$

10.11 problem 11

Internal problem ID [6737]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 11.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= -x(t) - y \\y' &= \frac{3x(t)}{4} - \frac{3y}{2} + 3z(t) \\z'(t) &= \frac{x(t)}{8} + \frac{y}{4} - \frac{z(t)}{2}\end{aligned}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 67

```
dsolve([diff(x(t),t)=-x(t)-y(t),diff(y(t),t)=3/4*x(t)-3/2*y(t)+3*z(t),diff(z(t),t)=1/8*x(t)+
```

$$\begin{aligned}x(t) &= c_1 e^{-\frac{3t}{2}} + c_2 e^{-t} + c_3 e^{-\frac{t}{2}} \\y(t) &= \frac{c_1 e^{-\frac{3t}{2}}}{2} - \frac{c_3 e^{-\frac{t}{2}}}{2} \\z(t) &= -\frac{c_1 e^{-\frac{3t}{2}}}{4} - \frac{c_2 e^{-t}}{4} - \frac{5c_3 e^{-\frac{t}{2}}}{12}\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 168

```
DSolve[{x'[t]==-x[t]-y[t],y'[t]==3/4*x[t]-3/2*y[t]+3*z[t],z'[t]==1/8x[t]+1/4*y[t]-1/2*z[t]},
```

$$x(t) \rightarrow \frac{1}{2}e^{-3t/2}(c_1(8e^{t/2} - 3e^t - 3) - 4(e^{t/2} - 1)(3c_3(e^{t/2} - 1) + c_2))$$

$$y(t) \rightarrow \frac{1}{4}e^{-3t/2}(3c_1(e^t - 1) + 4(3c_3(e^t - 1) + c_2))$$

$$z(t) \rightarrow \frac{1}{8}e^{-3t/2}(c_1(-8e^{t/2} + 5e^t + 3) + 4c_2(e^{t/2} - 1) + 4c_3(-6e^{t/2} + 5e^t + 3))$$

10.12 problem 11

Internal problem ID [6738]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 11.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= -x(t) - y \\y' &= \frac{3x(t)}{4} - \frac{3y}{2} + 3z(t) \\z'(t) &= \frac{x(t)}{8} + \frac{y}{4} - \frac{z(t)}{2}\end{aligned}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 67

```
dsolve([diff(x(t),t)=-x(t)-y(t),diff(y(t),t)=3/4*x(t)-3/2*y(t)+3*z(t),diff(z(t),t)=1/8*x(t)+
```

$$\begin{aligned}x(t) &= c_1 e^{-\frac{3t}{2}} + c_2 e^{-t} + c_3 e^{-\frac{t}{2}} \\y(t) &= \frac{c_1 e^{-\frac{3t}{2}}}{2} - \frac{c_3 e^{-\frac{t}{2}}}{2} \\z(t) &= -\frac{c_1 e^{-\frac{3t}{2}}}{4} - \frac{c_2 e^{-t}}{4} - \frac{5c_3 e^{-\frac{t}{2}}}{12}\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 168

```
DSolve[{x'[t]==-x[t]-y[t],y'[t]==3/4*x[t]-3/2*y[t]+3*z[t],z'[t]==1/8x[t]+1/4*y[t]-1/2*z[t]},
```

$$x(t) \rightarrow \frac{1}{2}e^{-3t/2}(c_1(8e^{t/2} - 3e^t - 3) - 4(e^{t/2} - 1)(3c_3(e^{t/2} - 1) + c_2))$$

$$y(t) \rightarrow \frac{1}{4}e^{-3t/2}(3c_1(e^t - 1) + 4(3c_3(e^t - 1) + c_2))$$

$$z(t) \rightarrow \frac{1}{8}e^{-3t/2}(c_1(-8e^{t/2} + 5e^t + 3) + 4c_2(e^{t/2} - 1) + 4c_3(-6e^{t/2} + 5e^t + 3))$$

10.13 problem 12

Internal problem ID [6739]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 12.

ODE order: 1.

ODE degree: 1.

Solve

$$x'(t) = -x(t) + 4y + 2z(t)$$

$$y' = 4x(t) - y - 2z(t)$$

$$z'(t) = 6z(t)$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 58

```
dsolve([diff(x(t),t)=-x(t)+4*y(t)+2*z(t),diff(y(t),t)=4*x(t)-y(t)-2*z(t),diff(z(t),t)=6*z(t))
```

$$x(t) = c_2 e^{3t} + c_1 e^{-5t} + \frac{2c_3 e^{6t}}{11}$$
$$y(t) = -\frac{2c_3 e^{6t}}{11} - c_1 e^{-5t} + c_2 e^{3t}$$
$$z(t) = c_3 e^{6t}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 105

```
DSolve[{x'[t]==-x[t]+4*y[t]+2*z[t],y'[t]==4*x[t]-y[t]-2*z[t],z'[t]==6*z[t]},{x[t],y[t],z[t]}
```

$$x(t) \rightarrow \frac{1}{22} e^{-5t} (11c_1 (e^{8t} + 1) + 11c_2 (e^{8t} - 1) + 4c_3 (e^{11t} - 1))$$
$$y(t) \rightarrow \frac{1}{22} e^{-5t} (11c_1 (e^{8t} - 1) + 11c_2 (e^{8t} + 1) - 4c_3 (e^{11t} - 1))$$
$$z(t) \rightarrow c_3 e^{6t}$$

10.14 problem 13

Internal problem ID [6740]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 13.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= \frac{x(t)}{2} \\y' &= x(t) - \frac{y}{2}\end{aligned}$$

With initial conditions

$$[x(0) = 4, y(0) = 5]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve([diff(x(t),t) = 1/2*x(t), diff(y(t),t) = x(t)-1/2*y(t), x(0) = 4, y(0) = 5], singsol=
```

$$\begin{aligned}x(t) &= 4e^{\frac{t}{2}} \\y(t) &= 4e^{\frac{t}{2}} + e^{-\frac{t}{2}}\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 32

```
DSolve[{x'[t]==1/2*x[t], y'[t]==x[t]-1/2*y[t]},{x[0]==4,y[0]==5},{x[t],y[t]},t,IncludeSingular
```

$$\begin{aligned}x(t) &\rightarrow 4e^{t/2} \\y(t) &\rightarrow e^{-t/2}(4e^t + 1)\end{aligned}$$

10.15 problem 14

Internal problem ID [6741]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 14.

ODE order: 1.

ODE degree: 1.

Solve

$$x'(t) = x(t) + y + 4z(t)$$

$$y' = 2y$$

$$z'(t) = x(t) + y + z(t)$$

With initial conditions

$$[x(0) = 1, y(0) = 3, z(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 53

```
dsolve([diff(x(t),t) = x(t)+y(t)+4*z(t), diff(y(t),t) = 2*y(t), diff(z(t),t) = x(t)+y(t)+z(t)
```

$$x(t) = e^{-t} + 5e^{3t} - 5e^{2t}$$

$$y(t) = 3e^{2t}$$

$$z(t) = -\frac{e^{-t}}{2} + \frac{5e^{3t}}{2} - 2e^{2t}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 63

```
DSolve[{x'[t]==x[t]+y[t]+4*z[t],y'[t]==2*y[t],z'[t]==x[t]+y[t]+z[t]},{x[0]==1,y[0]==3,z[0]==
```

$$x(t) \rightarrow e^{-t} - 5e^{2t} + 5e^{3t}$$

$$y(t) \rightarrow 3e^{2t}$$

$$z(t) \rightarrow \frac{1}{2}e^{-t}(-4e^{3t} + 5e^{4t} - 1)$$

10.16 problem 15

Internal problem ID [6742]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 15.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= \frac{9x(t)}{10} + \frac{21y(t)}{10} + \frac{16z(t)}{5} \\y'(t) &= \frac{7x(t)}{10} + \frac{13y(t)}{2} + \frac{21z(t)}{5} \\z'(t) &= \frac{11x(t)}{10} + \frac{17y(t)}{10} + \frac{17z(t)}{5}\end{aligned}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 1014

`dsolve([diff(x(t),t)=9/10*x(t)+21/10*y(t)+32/10*z(t),diff(y(t),t)=7/10*x(t)+65/10*y(t)+42/10`

$$\begin{aligned}
 x(t) &= c_1 e^{-\frac{\left(i\sqrt{3}(329940+60i\sqrt{29760999})^{\frac{2}{3}}-6000i\sqrt{3}+(329940+60i\sqrt{29760999})^{\frac{2}{3}}-216(329940+60i\sqrt{29760999})^{\frac{1}{3}}+6000\right)t}{60(329940+60i\sqrt{29760999})^{\frac{1}{3}}}} \\
 &+ c_2 e^{\frac{\left(i\sqrt{3}(329940+60i\sqrt{29760999})^{\frac{2}{3}}-6000i\sqrt{3}-(329940+60i\sqrt{29760999})^{\frac{2}{3}}+216(329940+60i\sqrt{29760999})^{\frac{1}{3}}-6000\right)t}{60(329940+60i\sqrt{29760999})^{\frac{1}{3}}}} \\
 y(t) &+ c_3 e^{\frac{\left((329940+60i\sqrt{29760999})^{\frac{2}{3}}+108(329940+60i\sqrt{29760999})^{\frac{1}{3}}+6000\right)t}{30(329940+60i\sqrt{29760999})^{\frac{1}{3}}}}
 \end{aligned}$$

$$= \frac{\left(8i\sqrt{3}(329940+60i\sqrt{29760999})^{\frac{4}{3}}-8(329940+60i\sqrt{29760999})^{\frac{4}{3}}+207000i\sqrt{3}(329940+60i\sqrt{29760999})^{\frac{1}{3}}\right)}{88731(329940+60i\sqrt{29760999})^{\frac{1}{3}}}$$

$$- \frac{\left(8i\sqrt{3}(329940+60i\sqrt{29760999})^{\frac{4}{3}}+8(329940+60i\sqrt{29760999})^{\frac{4}{3}}+207000i\sqrt{3}(329940+60i\sqrt{29760999})^{\frac{1}{3}}\right)}{88731(329940+60i\sqrt{29760999})^{\frac{1}{3}}}$$

$$+ \frac{\left(16(329940+60i\sqrt{29760999})^{\frac{4}{3}}+20757(329940+60i\sqrt{29760999})^{\frac{2}{3}}+4140i\sqrt{29760999}+414000(329940+60i\sqrt{29760999})^{\frac{1}{3}}\right)}{88731(329940+60i\sqrt{29760999})^{\frac{1}{3}}}$$

$$z(t) =$$

$$- \frac{\left(7i\sqrt{3}(329940+60i\sqrt{29760999})^{\frac{4}{3}}-7(329940+60i\sqrt{29760999})^{\frac{4}{3}}-3516000i\sqrt{3}(329940+60i\sqrt{29760999})^{\frac{1}{3}}\right)}{59154(329940+60i\sqrt{29760999})^{\frac{1}{3}}}$$

$$+ \frac{\left(7i\sqrt{3}(329940+60i\sqrt{29760999})^{\frac{4}{3}}+7(329940+60i\sqrt{29760999})^{\frac{4}{3}}-3516000i\sqrt{3}(329940+60i\sqrt{29760999})^{\frac{1}{3}}\right)}{59154(329940+60i\sqrt{29760999})^{\frac{1}{3}}}$$

$$- \frac{\left(7(329940+60i\sqrt{29760999})^{\frac{4}{3}}-40830(329940+60i\sqrt{29760999})^{\frac{2}{3}}-35160i\sqrt{29760999}-3516000(329940+60i\sqrt{29760999})^{\frac{1}{3}}\right)}{59154(329940+60i\sqrt{29760999})^{\frac{1}{3}}}$$

$$59154(329940+60i\sqrt{29760999})^{\frac{1}{3}}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 616

`DSolve[{x'[t]==9/10*x[t]+21/10*y[t]+32/10*z[t],y'[t]==7/10*x[t]+65/10*y[t]+42/10*z[t],z'[t]=`

$$\begin{aligned}
 x(t) \rightarrow & 2c_3 \text{RootSum} \left[\#1^3 - 108\#1^2 + 1888\#1 + 904\&, \frac{16\#1e^{\frac{\#1t}{10}} - 599e^{\frac{\#1t}{10}}}{3\#1^2 - 216\#1 + 1888}\& \right] \\
 & + c_2 \text{RootSum} \left[\#1^3 - 108\#1^2 + 1888\#1 + 904\&, \frac{21\#1e^{\frac{\#1t}{10}} - 170e^{\frac{\#1t}{10}}}{3\#1^2 - 216\#1 + 1888}\& \right] \\
 & + c_1 \text{RootSum} \left[\#1^3 - 108\#1^2 + 1888\#1 \right. \\
 & \left. + 904\&, \frac{\#1^2e^{\frac{\#1t}{10}} - 99\#1e^{\frac{\#1t}{10}} + 1496e^{\frac{\#1t}{10}}}{3\#1^2 - 216\#1 + 1888}\& \right] \\
 y(t) \rightarrow & 7c_1 \text{RootSum} \left[\#1^3 - 108\#1^2 + 1888\#1 + 904\&, \frac{\#1e^{\frac{\#1t}{10}} + 32e^{\frac{\#1t}{10}}}{3\#1^2 - 216\#1 + 1888}\& \right] \\
 & + 14c_3 \text{RootSum} \left[\#1^3 - 108\#1^2 + 1888\#1 + 904\&, \frac{3\#1e^{\frac{\#1t}{10}} - 11e^{\frac{\#1t}{10}}}{3\#1^2 - 216\#1 + 1888}\& \right] \\
 & + c_2 \text{RootSum} \left[\#1^3 - 108\#1^2 + 1888\#1 \right. \\
 & \left. + 904\&, \frac{\#1^2e^{\frac{\#1t}{10}} - 43\#1e^{\frac{\#1t}{10}} - 46e^{\frac{\#1t}{10}}}{3\#1^2 - 216\#1 + 1888}\& \right] \\
 z(t) \rightarrow & c_1 \text{RootSum} \left[\#1^3 - 108\#1^2 + 1888\#1 + 904\&, \frac{11\#1e^{\frac{\#1t}{10}} - 596e^{\frac{\#1t}{10}}}{3\#1^2 - 216\#1 + 1888}\& \right] \\
 & + c_2 \text{RootSum} \left[\#1^3 - 108\#1^2 + 1888\#1 + 904\&, \frac{17\#1e^{\frac{\#1t}{10}} + 78e^{\frac{\#1t}{10}}}{3\#1^2 - 216\#1 + 1888}\& \right] \\
 & + c_3 \text{RootSum} \left[\#1^3 - 108\#1^2 + 1888\#1 \right. \\
 & \left. + 904\&, \frac{\#1^2e^{\frac{\#1t}{10}} - 74\#1e^{\frac{\#1t}{10}} + 438e^{\frac{\#1t}{10}}}{3\#1^2 - 216\#1 + 1888}\& \right]
 \end{aligned}$$

10.17 problem 16

Internal problem ID [6743]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 16.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x_1'(t) &= x_1(t) + 2x_3(t) - \frac{9x_4(t)}{5} \\x_2'(t) &= \frac{51x_2(t)}{10} - x_4(t) + 3x_5(t) \\x_3'(t) &= x_1(t) + 2x_2(t) - 3x_3(t) \\x_4'(t) &= x_2(t) - \frac{31x_3(t)}{10} + 4x_4(t) \\x_5'(t) &= -\frac{14x_1(t)}{5} + \frac{3x_4(t)}{2} - x_5(t)\end{aligned}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 1389

`dsolve([diff(x__1(t),t)=x__1(t)+2*x__3(t)-18/10*x__4(t),diff(x__2(t),t)=51/10*x__2(t)-x__4(t)`

$x_1(t)$

$$\begin{aligned}
 & \frac{1334393 \left(\sum_{a=1}^5 \text{RootOf} (500_Z^5 - 3050_Z^4 - 4450_Z^3 + 35110_Z^2 + 20779_Z - 81879, \text{index} = _a) \right)}{1052220} \\
 & + \frac{3565 \left(\sum_{a=1}^5 \text{RootOf} (500_Z^5 - 3050_Z^4 - 4450_Z^3 + 35110_Z^2 + 20779_Z - 81879, \text{index} = _a) \right)^4}{105222} \\
 & - \frac{29663 \left(\sum_{a=1}^5 \text{RootOf} (500_Z^5 - 3050_Z^4 - 4450_Z^3 + 35110_Z^2 + 20779_Z - 81879, \text{index} = _a) \right)^5}{210444} \\
 & - \frac{49145 \left(\sum_{a=1}^5 \text{RootOf} (500_Z^5 - 3050_Z^4 - 4450_Z^3 + 35110_Z^2 + 20779_Z - 81879, \text{index} = _a) \right)^6}{105222} \\
 & + \frac{20174057 \left(\sum_{a=1}^5 e^{\text{RootOf}(500_Z^5 - 3050_Z^4 - 4450_Z^3 + 35110_Z^2 + 20779_Z - 81879, \text{index} = _a)t} C_a \right)}{10522200}
 \end{aligned}$$

$x_2(t) =$

$$\begin{aligned}
 & \frac{16027 \left(\sum_{a=1}^5 \text{RootOf} (500_Z^5 - 3050_Z^4 - 4450_Z^3 + 35110_Z^2 + 20779_Z - 81879, \text{index} = _a) \right)^7}{420888} \\
 & + \frac{10385 \left(\sum_{a=1}^5 \text{RootOf} (500_Z^5 - 3050_Z^4 - 4450_Z^3 + 35110_Z^2 + 20779_Z - 81879, \text{index} = _a) \right)^8}{210444} \\
 & - \frac{210025 \left(\sum_{a=1}^5 \text{RootOf} (500_Z^5 - 3050_Z^4 - 4450_Z^3 + 35110_Z^2 + 20779_Z - 81879, \text{index} = _a) \right)^9}{210444} \\
 & - \frac{371003 \left(\sum_{a=1}^5 \text{RootOf} (500_Z^5 - 3050_Z^4 - 4450_Z^3 + 35110_Z^2 + 20779_Z - 81879, \text{index} = _a) \right)^{10}}{2104440} \\
 & + \frac{92973853 \left(\sum_{a=1}^5 e^{\text{RootOf}(500_Z^5 - 3050_Z^4 - 4450_Z^3 + 35110_Z^2 + 20779_Z - 81879, \text{index} = _a)t} C_a \right)}{21044400}
 \end{aligned}$$

$x_3(t) =$

$$\begin{aligned}
 & \frac{33875 \left(\sum_{a=1}^5 \text{RootOf} (500_Z^5 - 3050_Z^4 - 4450_Z^3 + 35110_Z^2 + 20779_Z - 81879, \text{index} = _a) \right)^{11}}{105222} \\
 & - \frac{2585 \left(\sum_{a=1}^5 \text{RootOf} (500_Z^5 - 3050_Z^4 - 4450_Z^3 + 35110_Z^2 + 20779_Z - 81879, \text{index} = _a) \right)^{12}}{246} \\
 & - \frac{79853 \left(\sum_{a=1}^5 \text{RootOf} (500_Z^5 - 3050_Z^4 - 4450_Z^3 + 35110_Z^2 + 20779_Z - 81879, \text{index} = _a) \right)^{13}}{210444}
 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 2839

```
DSolve[{x1'[t]==x1[t]+2*x3[t]-18/10*x4[t],x2'[t]==51/10*x2[t]-x4[t]+3*x5[t],x3'[t]==x1[t]+2*
```

Too large to display

10.18 problem 19

Internal problem ID [6744]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 19.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= 3x(t) - y \\y' &= 9x(t) - 3y\end{aligned}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve([diff(x(t),t)=3*x(t)-y(t),diff(y(t),t)=9*x(t)-3*y(t)],singsol=all)
```

$$\begin{aligned}x(t) &= c_1t + c_2 \\y(t) &= 3c_1t - c_1 + 3c_2\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 34

```
DSolve[{x'[t]==3*x[t]-y[t],y'[t]==9*x[t]-3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> T
```

$$\begin{aligned}x(t) &\rightarrow 3c_1t - c_2t + c_1 \\y(t) &\rightarrow 9c_1t - 3c_2t + c_2\end{aligned}$$

10.19 problem 20

Internal problem ID [6745]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 20.

ODE order: 1.

ODE degree: 1.

Solve

$$x'(t) = -6x(t) + 5y$$

$$y' = -5x(t) + 4y$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
dsolve([diff(x(t),t)=-6*x(t)+5*y(t),diff(y(t),t)=-5*x(t)+4*y(t)],singsol=all)
```

$$x(t) = e^{-t}(c_2t + c_1)$$

$$y(t) = \frac{e^{-t}(5c_2t + 5c_1 + c_2)}{5}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 46

```
DSolve[{x'[t]==-6*x[t]+5*y[t],y'[t]==-5*x[t]+4*y[t]},{x[t],y[t]},t,IncludeSingularSolutions
```

$$x(t) \rightarrow e^{-t}(-5c_1t + 5c_2t + c_1)$$

$$y(t) \rightarrow e^{-t}(-5c_1t + 5c_2t + c_2)$$

10.20 problem 21

Internal problem ID [6746]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 21.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= -x(t) + 3y \\ y' &= -3x(t) + 5y\end{aligned}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve([diff(x(t),t)=-x(t)+3*y(t),diff(y(t),t)=-3*x(t)+5*y(t)],singsol=all)
```

$$\begin{aligned}x(t) &= e^{2t}(c_2t + c_1) \\ y(t) &= \frac{e^{2t}(3c_2t + 3c_1 + c_2)}{3}\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 46

```
DSolve[{x'[t]==-x[t]+3*y[t],y'[t]==-3*x[t]+5*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$\begin{aligned}x(t) &\rightarrow e^{2t}(-3c_1t + 3c_2t + c_1) \\ y(t) &\rightarrow e^{2t}(-3c_1t + 3c_2t + c_2)\end{aligned}$$

10.21 problem 22

Internal problem ID [6747]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 22.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= 12x(t) - 9y \\ y' &= 4x(t)\end{aligned}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve([diff(x(t),t)=12*x(t)-9*y(t),diff(y(t),t)=4*x(t)],singsol=all)
```

$$\begin{aligned}x(t) &= \frac{e^{6t}(6c_2t + 6c_1 + c_2)}{4} \\ y(t) &= e^{6t}(c_2t + c_1)\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 46

```
DSolve[{x'[t]==12*x[t]-9*y[t],y'[t]==4*x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$\begin{aligned}x(t) &\rightarrow e^{6t}(6c_1t - 9c_2t + c_1) \\ y(t) &\rightarrow e^{6t}(4c_1t - 6c_2t + c_2)\end{aligned}$$

10.22 problem 23

Internal problem ID [6748]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 23.

ODE order: 1.

ODE degree: 1.

Solve

$$x'(t) = 3x(t) - y - z(t)$$

$$y' = x(t) + y - z(t)$$

$$z'(t) = x(t) - y + z(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 51

```
dsolve([diff(x(t),t)=3*x(t)-y(t)-z(t),diff(y(t),t)=x(t)+y(t)-z(t),diff(z(t),t)=x(t)-y(t)+z(t)
```

$$\begin{aligned}x(t) &= c_2 e^t + c_3 e^{2t} \\y(t) &= c_2 e^t + c_3 e^{2t} + c_1 e^{2t} \\z(t) &= c_2 e^t - c_1 e^{2t}\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 86

```
DSolve[{x'[t]==3*x[t]-y[t]-z[t],y'[t]==x[t]+y[t]-z[t],z'[t]==x[t]-y[t]+z[t]},{x[t],y[t],z[t]
```

$$\begin{aligned}x(t) &\rightarrow e^t(c_1(2e^t - 1) - (c_2 + c_3)(e^t - 1)) \\y(t) &\rightarrow e^t(c_1(e^t - 1) - c_3 e^t + c_2 + c_3) \\z(t) &\rightarrow e^t(c_1(e^t - 1) - c_2 e^t + c_2 + c_3)\end{aligned}$$

10.23 problem 24

Internal problem ID [6749]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 24.

ODE order: 1.

ODE degree: 1.

Solve

$$x'(t) = 3x(t) + 2y + 4z(t)$$

$$y' = 2x(t) + 2z(t)$$

$$z'(t) = 4x(t) + 2y + 3z(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 67

```
dsolve([diff(x(t),t)=3*x(t)+2*y(t)+4*z(t),diff(y(t),t)=2*x(t)+2*z(t),diff(z(t),t)=4*x(t)+2*y
```

$$\begin{aligned}x(t) &= 2c_2e^{8t} + 2c_3e^{-t} + e^{-t}c_1 \\y(t) &= c_2e^{8t} + c_3e^{-t} \\z(t) &= 2c_2e^{8t} - \frac{5c_3e^{-t}}{2} - e^{-t}c_1\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 135

```
DSolve[{x'[t]==3*x[t]+2*y[t]+4*z[t],y'[t]==2*x[t]+2*z[t],z'[t]==4*x[t]+2*y[t]+3*z[t]},{x[t],
```

$$\begin{aligned}x(t) &\rightarrow \frac{1}{9}e^{-t}(c_1(4e^{9t} + 5) + 2(c_2 + 2c_3)(e^{9t} - 1)) \\y(t) &\rightarrow \frac{1}{9}e^{-t}(2c_1(e^{9t} - 1) + c_2(e^{9t} + 8) + 2c_3(e^{9t} - 1)) \\z(t) &\rightarrow \frac{1}{9}e^{-t}(4c_1(e^{9t} - 1) + 2c_2(e^{9t} - 1) + c_3(4e^{9t} + 5))\end{aligned}$$

10.24 problem 25

Internal problem ID [6750]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 25.

ODE order: 1.

ODE degree: 1.

Solve

$$x'(t) = 5x(t) - 4y$$

$$y' = x(t) + 2z(t)$$

$$z'(t) = 2y + 5z(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 63

```
dsolve([diff(x(t),t)=5*x(t)-4*y(t),diff(y(t),t)=x(t)+2*z(t),diff(z(t),t)=2*y(t)+5*z(t)],sing
```

$$\begin{aligned}x(t) &= -4e^{5t}c_3t + e^{5t}c_1 + \frac{4c_2}{5} \\y(t) &= c_2 + c_3e^{5t} \\z(t) &= 2e^{5t}c_3t - \frac{e^{5t}c_1}{2} + \frac{5c_3e^{5t}}{2} - \frac{2c_2}{5}\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 141

```
DSolve[{x'[t]==5*x[t]-4*y[t],y'[t]==x[t]+2*z[t],z'[t]==2*y[t]+5*z[t]},{x[t],y[t],z[t]},t,Inc
```

$$\begin{aligned}x(t) &\rightarrow \frac{1}{25}(c_1(e^{5t}(29 - 20t) - 4) - 4(5c_2(e^{5t} - 1) + 2c_3(e^{5t}(5t - 1) + 1))) \\y(t) &\rightarrow \frac{1}{5}c_1(e^{5t} - 1) + \frac{2}{5}c_3(e^{5t} - 1) + c_2 \\z(t) &\rightarrow \frac{1}{25}(2c_1(e^{5t}(5t - 1) + 1) + 10c_2(e^{5t} - 1) + c_3(e^{5t}(20t + 21) + 4))\end{aligned}$$

10.25 problem 26

Internal problem ID [6751]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 26.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= x(t) \\y' &= 3y + z(t) \\z'(t) &= -y + z(t)\end{aligned}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 39

```
dsolve([diff(x(t),t)=x(t),diff(y(t),t)=3*y(t)+z(t),diff(z(t),t)=-y(t)+z(t)],singsol=all)
```

$$\begin{aligned}x(t) &= c_3 e^t \\y(t) &= e^{2t}(c_2 t + c_1) \\z(t) &= -e^{2t}(c_2 t + c_1 - c_2)\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 96

```
DSolve[{x'[t]==x[t],y'[t]==3*y[t]+z[t],z'[t]==-y[t]+z[t]},{x[t],y[t],z[t]},t,IncludeSingular
```

$$\begin{aligned}x(t) &\rightarrow c_1 e^t \\y(t) &\rightarrow e^{2t}(c_2(t+1) + c_3 t) \\z(t) &\rightarrow e^{2t}(c_3 - (c_2 + c_3)t) \\x(t) &\rightarrow 0 \\y(t) &\rightarrow e^{2t}(c_2(t+1) + c_3 t) \\z(t) &\rightarrow e^{2t}(c_3 - (c_2 + c_3)t)\end{aligned}$$

10.26 problem 27

Internal problem ID [6752]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 27.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= x(t) \\y' &= 2x(t) + 2y - z(t) \\z'(t) &= y\end{aligned}$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 46

```
dsolve([diff(x(t),t)=x(t),diff(y(t),t)=2*x(t)+2*y(t)-z(t),diff(z(t),t)=y(t)],singsol=all)
```

$$\begin{aligned}x(t) &= c_3 e^t \\y(t) &= e^t (c_3 t^2 + c_1 t + 2c_3 t + c_1 + c_2) \\z(t) &= e^t (c_3 t^2 + c_1 t + c_2)\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 65

```
DSolve[{x'[t]==x[t],y'[t]==2*x[t]+2*y[t]-z[t],z'[t]==y[t]},{x[t],y[t],z[t]},t,IncludeSingularSolutions->True]
```

$$\begin{aligned}x(t) &\rightarrow c_1 e^t \\y(t) &\rightarrow e^t (c_1 t^2 + (2c_1 + c_2 - c_3)t + c_2) \\z(t) &\rightarrow e^t (c_1 t^2 + (c_2 - c_3)t + c_3)\end{aligned}$$

10.27 problem 28

Internal problem ID [6753]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 28.

ODE order: 1.

ODE degree: 1.

Solve

$$x'(t) = 4x(t) + y$$

$$y' = 4y + z(t)$$

$$z'(t) = 4z(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 46

```
dsolve([diff(x(t),t)=4*x(t)+y(t),diff(y(t),t)=4*y(t)+z(t),diff(z(t),t)=4*z(t)],singsol=all)
```

$$x(t) = \frac{(c_3 t^2 + 2c_2 t + 2c_1) e^{4t}}{2}$$

$$y(t) = (c_3 t + c_2) e^{4t}$$

$$z(t) = c_3 e^{4t}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 57

```
DSolve[{x'[t]==4*x[t]+y[t],y'[t]==4*y[t]+z[t],z'[t]==4*z[t]},{x[t],y[t],z[t]},t,IncludeSingu
```

$$x(t) \rightarrow \frac{1}{2} e^{4t} (t(c_3 t + 2c_2) + 2c_1)$$

$$y(t) \rightarrow e^{4t} (c_3 t + c_2)$$

$$z(t) \rightarrow c_3 e^{4t}$$

10.28 problem 29

Internal problem ID [6754]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 29.

ODE order: 1.

ODE degree: 1.

Solve

$$x'(t) = 2x(t) + 4y$$

$$y' = -x(t) + 6y$$

With initial conditions

$$[x(0) = -1, y(0) = 6]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 29

```
dsolve([diff(x(t),t) = 2*x(t)+4*y(t), diff(y(t),t) = -x(t)+6*y(t), x(0) = -1, y(0) = 6], sin
```

$$x(t) = e^{4t}(26t - 1)$$

$$y(t) = \frac{e^{4t}(52t + 24)}{4}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[{x'[t]==2*x[t]+4*y[t],y'[t]==-x[t]+6*y[t]},{x[0]==-1,y[0]==6},{x[t],y[t]},t,IncludeSi
```

$$x(t) \rightarrow e^{4t}(26t - 1)$$

$$y(t) \rightarrow e^{4t}(13t + 6)$$

10.29 problem 30

Internal problem ID [6755]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 30.

ODE order: 1.

ODE degree: 1.

Solve

$$x'(t) = z(t)$$

$$y' = y$$

$$z'(t) = x(t)$$

With initial conditions

$$[x(0) = 1, y(0) = 2, z(0) = 5]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 37

```
dsolve([diff(x(t),t) = z(t), diff(y(t),t) = y(t), diff(z(t),t) = x(t), x(0) = 1, y(0) = 2, z(0) = 5])
```

$$x(t) = 3e^t - 2e^{-t}$$

$$y(t) = 2e^t$$

$$z(t) = 3e^t + 2e^{-t}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 42

```
DSolve[{x'[t]==z[t],y'[t]==y[t],z'[t]==x[t]},{x[0]==1,y[0]==2,z[0]==5},{x[t],y[t],z[t]},t,In
```

$$x(t) \rightarrow 3e^t - 2e^{-t}$$

$$z(t) \rightarrow 2e^{-t} + 3e^t$$

$$y(t) \rightarrow 2e^t$$

10.30 problem 33

Internal problem ID [6756]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 33.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= 6x(t) - y \\y' &= 5x(t) + 2y\end{aligned}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
dsolve([diff(x(t),t)=6*x(t)-y(t),diff(y(t),t)=5*x(t)+2*y(t)],singsol=all)
```

$$\begin{aligned}x(t) &= e^{4t}(c_1 \sin(t) + c_2 \cos(t)) \\y(t) &= e^{4t}(-c_1 \cos(t) + c_2 \sin(t) + 2c_1 \sin(t) + 2c_2 \cos(t))\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 55

```
DSolve[{x'[t]==6*x[t]-y[t],y'[t]==5*x[t]+2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> T
```

$$\begin{aligned}x(t) &\rightarrow e^{4t}(c_1(2 \sin(t) + \cos(t)) - c_2 \sin(t)) \\y(t) &\rightarrow e^{4t}(c_2 \cos(t) + (5c_1 - 2c_2) \sin(t))\end{aligned}$$

10.31 problem 34

Internal problem ID [6757]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 34.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= x(t) + y \\ y' &= -2x(t) - y\end{aligned}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve([diff(x(t),t)=x(t)+y(t),diff(y(t),t)=-2*x(t)-y(t)],singsol=all)
```

$$\begin{aligned}x(t) &= c_1 \sin(t) + c_2 \cos(t) \\ y(t) &= c_1 \cos(t) - c_2 \sin(t) - c_1 \sin(t) - c_2 \cos(t)\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 39

```
DSolve[{x'[t]==x[t]+y[t],y'[t]==-2*x[t]-y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True
```

$$\begin{aligned}x(t) &\rightarrow c_1 \cos(t) + (c_1 + c_2) \sin(t) \\ y(t) &\rightarrow c_2 \cos(t) - (2c_1 + c_2) \sin(t)\end{aligned}$$

10.32 problem 35

Internal problem ID [6758]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 35.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= 5x(t) + y \\ y' &= -2x(t) + 3y\end{aligned}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 46

```
dsolve([diff(x(t),t)=5*x(t)+y(t),diff(y(t),t)=-2*x(t)+3*y(t)],singsol=all)
```

$$\begin{aligned}x(t) &= e^{4t}(c_1 \sin(t) + c_2 \cos(t)) \\ y(t) &= -e^{4t}(c_1 \sin(t) + c_2 \sin(t) - c_1 \cos(t) + c_2 \cos(t))\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 51

```
DSolve[{x'[t]==5*x[t]+y[t],y'[t]==-2*x[t]+3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$\begin{aligned}x(t) &\rightarrow e^{4t}(c_1 \cos(t) + (c_1 + c_2) \sin(t)) \\ y(t) &\rightarrow e^{4t}(c_2 \cos(t) - (2c_1 + c_2) \sin(t))\end{aligned}$$

10.33 problem 36

Internal problem ID [6759]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 36.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= 4x(t) + 5y \\ y' &= -2x(t) + 6y\end{aligned}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 59

```
dsolve([diff(x(t),t)=4*x(t)+5*y(t),diff(y(t),t)=-2*x(t)+6*y(t)],singsol=all)
```

$$\begin{aligned}x(t) &= e^{5t}(c_1 \sin(3t) + c_2 \cos(3t)) \\ y(t) &= \frac{e^{5t}(c_1 \sin(3t) - 3c_2 \sin(3t) + 3c_1 \cos(3t) + c_2 \cos(3t))}{5}\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 69

```
DSolve[{x'[t]==4*x[t]+5*y[t],y'[t]==-2*x[t]+6*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -
```

$$\begin{aligned}x(t) &\rightarrow \frac{1}{3}e^{5t}(3c_1 \cos(3t) - (c_1 - 5c_2) \sin(3t)) \\ y(t) &\rightarrow \frac{1}{3}e^{5t}(3c_2 \cos(3t) + (c_2 - 2c_1) \sin(3t))\end{aligned}$$

10.34 problem 37

Internal problem ID [6760]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 37.

ODE order: 1.

ODE degree: 1.

Solve

$$x'(t) = 4x(t) - 5y$$

$$y' = 5x(t) - 4y$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 50

```
dsolve([diff(x(t),t)=4*x(t)-5*y(t),diff(y(t),t)=5*x(t)-4*y(t)],singsol=all)
```

$$x(t) = c_1 \sin(3t) + c_2 \cos(3t)$$
$$y(t) = -\frac{3c_1 \cos(3t)}{5} + \frac{3c_2 \sin(3t)}{5} + \frac{4c_1 \sin(3t)}{5} + \frac{4c_2 \cos(3t)}{5}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 58

```
DSolve[{x'[t]==4*x[t]-5*y[t],y'[t]==5*x[t]-4*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \rightarrow c_1 \cos(3t) + \frac{1}{3}(4c_1 - 5c_2) \sin(3t)$$
$$y(t) \rightarrow c_2 \cos(3t) + \frac{1}{3}(5c_1 - 4c_2) \sin(3t)$$

10.35 problem 38

Internal problem ID [6761]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 38.

ODE order: 1.

ODE degree: 1.

Solve

$$x'(t) = x(t) - 8y$$

$$y' = x(t) - 3y$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 58

```
dsolve([diff(x(t),t)=x(t)-8*y(t),diff(y(t),t)=x(t)-3*y(t)],singsol=all)
```

$$x(t) = e^{-t}(c_1 \sin(2t) + c_2 \cos(2t))$$

$$y(t) = \frac{e^{-t}(c_1 \sin(2t) + c_2 \sin(2t) - c_1 \cos(2t) + c_2 \cos(2t))}{4}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 64

```
DSolve[{x'[t]==x[t]-8*y[t],y'[t]==x[t]-3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions->True]
```

$$x(t) \rightarrow e^{-t}(c_1 \cos(2t) + (c_1 - 4c_2) \sin(2t))$$

$$y(t) \rightarrow \frac{1}{2}e^{-t}(2c_2 \cos(2t) + (c_1 - 2c_2) \sin(2t))$$

10.36 problem 39

Internal problem ID [6762]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 39.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= z(t) \\ y' &= -z(t) \\ z'(t) &= y\end{aligned}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 41

```
dsolve([diff(x(t),t)=z(t),diff(y(t),t)=-z(t),diff(z(t),t)=y(t)],singsol=all)
```

$$\begin{aligned}x(t) &= c_3 \sin(t) - c_2 \cos(t) + c_1 \\ y(t) &= -c_3 \sin(t) + c_2 \cos(t) \\ z(t) &= c_2 \sin(t) + c_3 \cos(t)\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 50

```
DSolve[{x'[t]==z[t],y'[t]==-z[t],z'[t]==y[t]},{x[t],y[t],z[t]},t,IncludeSingularSolutions ->
```

$$\begin{aligned}x(t) &\rightarrow -c_2 \cos(t) + c_3 \sin(t) + c_1 + c_2 \\ y(t) &\rightarrow c_2 \cos(t) - c_3 \sin(t) \\ z(t) &\rightarrow c_3 \cos(t) + c_2 \sin(t)\end{aligned}$$

10.37 problem 40

Internal problem ID [6763]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 40.

ODE order: 1.

ODE degree: 1.

Solve

$$x'(t) = 2x(t) + y + 2z(t)$$

$$y' = 3x(t) + 6z(t)$$

$$z'(t) = -4x(t) - 3z(t)$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 92

```
dsolve([diff(x(t),t)=2*x(t)+y(t)+2*z(t),diff(y(t),t)=3*x(t)+6*z(t),diff(z(t),t)=-4*x(t)-3*z(t)
```

$$x(t) = -\frac{e^t(2c_2 \sin(2t) - c_3 \sin(2t) + c_2 \cos(2t) + 2c_3 \cos(2t))}{2}$$
$$y(t) = -2c_1 e^{-3t} - \frac{3c_2 e^t \cos(2t)}{2} + \frac{3 \sin(2t) e^t c_3}{2}$$
$$z(t) = c_1 e^{-3t} + c_2 e^t \sin(2t) + e^t c_3 \cos(2t)$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 176

```
DSolve[{x'[t]==2*x[t]+y[t]+2*z[t],y'[t]==3*x[t]+6*z[t],z'[t]==-4*x[t]-3*z[t]},{x[t],y[t],z[t]
```

$$x(t) \rightarrow \frac{1}{2}e^t(2c_1 \cos(2t) + (c_1 + c_2 + 2c_3) \sin(2t))$$

$$y(t) \rightarrow \frac{2}{5}(-3c_1 + c_2 - 3c_3)e^{-3t} + \frac{3}{5}(2c_1 + c_2 + 2c_3)e^t \cos(2t) - \frac{3}{5}(3c_1 - c_2 - 2c_3)e^t \sin(t) \cos(t)$$

$$z(t) \rightarrow \frac{1}{5}e^{-3t}(-(3c_1 - c_2 - 2c_3)e^{4t} \cos(2t) - 2(2c_1 + c_2 + 2c_3)e^{4t} \sin(2t) + 3c_1 - c_2 + 3c_3)$$

10.38 problem 45

Internal problem ID [6764]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

Problem number: 45.

ODE order: 1.

ODE degree: 1.

Solve

$$x'(t) = x(t) - 12y - 14z(t)$$

$$y' = x(t) + 2y - 3z(t)$$

$$z'(t) = x(t) + y - 2z(t)$$

With initial conditions

$$[x(0) = 4, y(0) = 6, z(0) = -7]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 62

```
dsolve([diff(x(t),t) = x(t)-12*y(t)-14*z(t), diff(y(t),t) = x(t)+2*y(t)-3*z(t), diff(z(t),t)
```

$$x(t) = -25e^t + 11\sin(5t) + 29\cos(5t)$$

$$y(t) = 7e^t + 6\sin(5t) - \cos(5t)$$

$$z(t) = -6e^t - \cos(5t) + 6\sin(5t)$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 65

```
DSolve[{x'[t]==x[t]-12*y[t]-14*z[t], y'[t]==x[t]+2*y[t]-3*z[t], z'[t]==x[t]+y[t]-2*z[t]}, {x[0]
```

$$x(t) \rightarrow -25e^t + 11\sin(5t) + 29\cos(5t)$$

$$y(t) \rightarrow 7e^t + 6\sin(5t) - \cos(5t)$$

$$z(t) \rightarrow -6e^t + 6\sin(5t) - \cos(5t)$$

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FIRST-ORDER DIFFERENTIAL
EQUATIONS. EXERCISES 8.3. Page 354**

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11.1 problem 1

Internal problem ID [6765]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.3. Page 354

Problem number: 1.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= 2x(t) + 3y - 7 \\y' &= -x(t) - 2y + 5\end{aligned}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve([diff(x(t),t)=2*x(t)+3*y(t)-7,diff(y(t),t)=-x(t)-2*y(t)+5],singsol=all)
```

$$\begin{aligned}x(t) &= c_2 e^t + e^{-t} c_1 - 1 \\y(t) &= -\frac{c_2 e^t}{3} - e^{-t} c_1 + 3\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 76

```
DSolve[{x'[t]==2*x[t]+3*y[t]-7,y'[t]==-x[t]-2*y[t]+5},{x[t],y[t]},t,IncludeSingularSolutions
```

$$\begin{aligned}x(t) &\rightarrow \frac{1}{2} e^{-t} (-2e^t + 3(c_1 + c_2)e^{2t} - c_1 - 3c_2) \\y(t) &\rightarrow \frac{1}{2} e^{-t} (6e^t - (c_1 + c_2)e^{2t} + c_1 + 3c_2)\end{aligned}$$

11.2 problem 2

Internal problem ID [6766]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.

Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.3. Page 354

Problem number: 2.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= 5x(t) + 9y + 2 \\y' &= -x(t) + 11y + 6\end{aligned}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve([diff(x(t),t)=5*x(t)+9*y(t)+2,diff(y(t),t)=-x(t)+11*y(t)+6],singsol=all)
```

$$\begin{aligned}x(t) &= \frac{1}{2} + e^{8t}(c_1t + c_2) \\y(t) &= -\frac{1}{2} + \frac{e^{8t}(3c_1t + c_1 + 3c_2)}{9}\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.073 (sec). Leaf size: 54

```
DSolve[{x'[t]==5*x[t]+9*y[t]+2,y'[t]==-x[t]+11*y[t]+6},{x[t],y[t]},t,IncludeSingularSolution
```

$$\begin{aligned}x(t) &\rightarrow \frac{1}{2} + e^{8t}(-3c_1t + 9c_2t + c_1) \\y(t) &\rightarrow -\frac{1}{2} + e^{8t}(c_1(-t) + 3c_2t + c_2)\end{aligned}$$