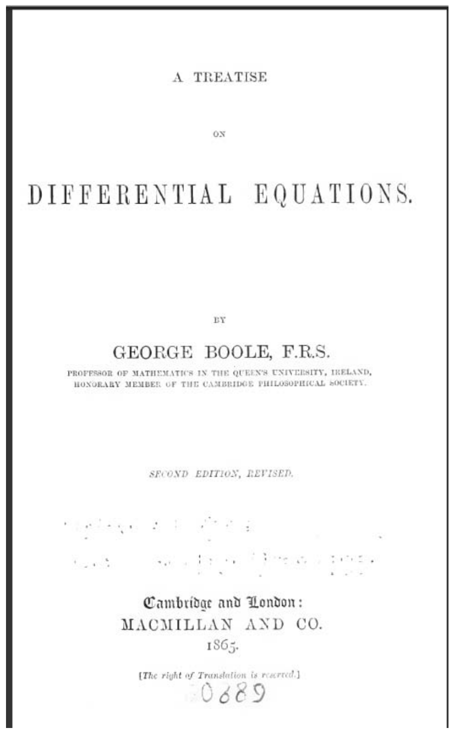
A Solution Manual For

Differential Equations, By George Boole F.R.S. 1865


A TREATISE

ON

DIFFERENTIAL EQUATIONS.

BY

GEORGE BOOLE, F.R.S.

PROFESSOR OF MATHEMATICS IN THE QUEEN'S UNIVERSITY, IRELAND,
HONORARY MEMBER OF THE CAMBRIDGE PHILOSOPHICAL SOCIETY.

SECOND EDITION, REVISED.

Cambridge and London:
MACMILLAN AND CO.
1865.

[*The right of Translation is reserved.*]

0689

Nasser M. Abbasi

May 16, 2024

Contents

1 Chapter 2

1.1 problem 1.1

Internal problem ID [4355]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 2
Problem number: 1.1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x+1)\,y+(1-y)\,xy'=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve((1+x)*y(x)+(1-y(x))*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\operatorname{LambertW}\left(-\frac{\mathrm{e}^{-x}}{c_1 x}\right)$$

✓ Solution by Mathematica
Time used: 3.139 (sec). Leaf size: 28

```
DSolve[(1+x)*y[x]+(1-y[x])*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -W\left(-\frac{e^{-x-c_1}}{x}\right)$$
$$y(x) \to 0$$

1.2 problem 1.2

Internal problem ID [4356]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 2
Problem number: 1.2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^2 + xy^2 + (x^2 - yx^2)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve((y(x)^2+x*y(x)^2)+(x^2-y(x)*x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = x\, \mathrm{e}^{\frac{\mathrm{LambertW}\left(-\frac{\mathrm{e}^{\frac{-c_1 x+1}{x}}}{x}\right) x + c_1 x - 1}{x}}$$

✓ Solution by Mathematica

Time used: 5.328 (sec). Leaf size: 30

```
DSolve[(y[x]^2+x*y[x]^2)+(x^2-y[x]*x^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{W\left(-\frac{e^{\frac{1}{x}-c_1}}{x}\right)}$$
$$y(x) \to 0$$

1.3 problem 1.3

Internal problem ID [4357]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 2
Problem number: 1.3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy(x^2+1)\,y' - y^2 = 1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 54

```
dsolve(x*y(x)*(1+x^2)*diff(y(x),x)-(1+y(x)^2)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(x^2+1)(c_1x^2-1)}}{x^2+1}$$
$$y(x) = -\frac{\sqrt{(x^2+1)(c_1x^2-1)}}{x^2+1}$$

✓ Solution by Mathematica

Time used: 1.211 (sec). Leaf size: 131

```
DSolve[x*y[x]*(1+x^2)*y'[x]-(1+y[x]^2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-1 + (-1 + e^{2c_1})\, x^2}}{\sqrt{x^2+1}}$$
$$y(x) \to \frac{\sqrt{-1 + (-1 + e^{2c_1})\, x^2}}{\sqrt{x^2+1}}$$
$$y(x) \to -i$$
$$y(x) \to i$$
$$y(x) \to -\frac{\sqrt{-x^2-1}}{\sqrt{x^2+1}}$$
$$y(x) \to \frac{\sqrt{-x^2-1}}{\sqrt{x^2+1}}$$

1.4 problem 1.4

Internal problem ID [4358]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 2
Problem number: 1.4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y^2 - \left(y + \sqrt{1+y^2}\right)\left(x^2+1\right)^{\frac{3}{2}} y' = -1$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 28

```
dsolve((1+y(x)^2)-(y(x)+sqrt(1+y(x)^2))*(1+x^2)^(3/2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$\frac{x}{\sqrt{x^2+1}} - \operatorname{arcsinh}(y(x)) - \frac{\ln\left(1+y(x)^2\right)}{2} + c_1 = 0$$

✓ Solution by Mathematica
Time used: 15.063 (sec). Leaf size: 115

```
DSolve[(1+y[x]^2)-(y[x]+Sqrt[1+y[x]^2])*(1+x^2)^(3/2)*y'[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to -\frac{i\left(1+e^{\frac{x}{\sqrt{x^2+1}}+c_1}\right)}{\sqrt{1+2e^{\frac{x}{\sqrt{x^2+1}}+c_1}}}$$
$$y(x) \to \frac{i\left(1+e^{\frac{x}{\sqrt{x^2+1}}+c_1}\right)}{\sqrt{1+2e^{\frac{x}{\sqrt{x^2+1}}+c_1}}}$$
$$y(x) \to -i$$
$$y(x) \to i$$

1.5 problem 1.5

Internal problem ID [4359]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 2
Problem number: 1.5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sin(x)\cos(y) - \cos(x)\sin(y)\,y' = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 11

```
dsolve(sin(x)*cos(y(x))-cos(x)*sin(y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{\cos(x)}{c_1}\right)$$

✓ Solution by Mathematica
Time used: 5.43 (sec). Leaf size: 47

```
DSolve[Sin[x]*Cos[y[x]]-Cos[x]*Sin[y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arccos\left(\frac{1}{2}c_1\cos(x)\right)$$
$$y(x) \to \arccos\left(\frac{1}{2}c_1\cos(x)\right)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

1.6 problem 1.6

Internal problem ID [4360]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 2
Problem number: 1.6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\sec(x)^2 \tan(y) + \sec(y)^2 \tan(x) y' = 0$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 47

```
dsolve(sec(x)^2*tan(y(x))+sec(y(x))^2*tan(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\arctan\left(\frac{2\tan(x)c_1}{c_1^2\tan(x)^2+1}, \frac{c_1^2\tan(x)^2-1}{c_1^2\tan(x)^2+1}\right)}{2}$$

✓ Solution by Mathematica
Time used: 0.457 (sec). Leaf size: 68

```
DSolve[Sec[x]^2*Tan[y[x]]+Sec[y[x]]^2*Tan[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to -\frac{1}{2}\arccos(-\tanh(\operatorname{arctanh}(\cos(2x)) + 2c_1))$$
$$y(x) \to \frac{1}{2}\arccos(-\tanh(\operatorname{arctanh}(\cos(2x)) + 2c_1))$$
$$y(x) \to 0$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

1.7 problem 3.1

Internal problem ID [4361]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 2
Problem number: 3.1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(-x+y)\,y'+y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve((y(x)-x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=-\frac{x}{\mathrm{LambertW}\left(-x\,\mathrm{e}^{-c_1}\right)}$$

✓ Solution by Mathematica

Time used: 3.943 (sec). Leaf size: 25

```
DSolve[(y[x]-x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\frac{x}{W\left(-e^{-c_1}x\right)}$$
$$y(x)\to 0$$

1.8 problem 3.2

Internal problem ID [4362]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 2
Problem number: 3.2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{(2\sqrt{xy} - x)\, y' + y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve((2*sqrt(x*y(x))-x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$\ln\left(y(x)\right) + \frac{x}{\sqrt{xy\left(x\right)}} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.244 (sec). Leaf size: 33

```
DSolve[(2*Sqrt[x*y[x]]-x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2}{\sqrt{\frac{y(x)}{x}}} + 2\log\left(\frac{y(x)}{x}\right) = -2\log(x) + c_1, y(x)\right]$$

1.9 problem 3.3

Internal problem ID [4363]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 2
Problem number: 3.3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{xy' - y - \sqrt{x^2 + y^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve(x*diff(y(x),x)-y(x)-sqrt(x^2+y(x)^2)=0,y(x), singsol=all)
```

$$\frac{-c_1 x^2 + \sqrt{x^2 + y(x)^2} + y(x)}{x^2} = 0$$

✓ Solution by Mathematica

Time used: 0.327 (sec). Leaf size: 27

```
DSolve[x*y'[x]-y[x]-Sqrt[x^2+y[x]^2]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-c_1} \left(-1 + e^{2c_1} x^2\right)$$

1.10 problem 3.4

Internal problem ID [4364]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 2
Problem number: 3.4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{-y\cos\left(\frac{y}{x}\right)+x\cos\left(\frac{y}{x}\right)y'=-x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x-y(x)*cos(y(x)/x))+x*cos(y(x)/x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=-\arcsin\left(\ln\left(x\right)+c_1\right)x$$

✓ Solution by Mathematica

Time used: 0.359 (sec). Leaf size: 15

```
DSolve[(x-y[x]*Cos[y[x]/x])+x*Cos[y[x]/x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x\arcsin(-\log(x)+c_1)$$

1.11 problem 3.5

Internal problem ID [4365]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 2
Problem number: 3.5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$8y + (5y + 7x)\, y' = -10x$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 38

```
dsolve((8*y(x)+10*x)+(5*y(x)+7*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = x\left(\operatorname{RootOf}\left(\_Z^{25}c_1x^5 - 2\_Z^{20}c_1x^5 + \_Z^{15}c_1x^5 - 1\right)^5 - 2\right)$$

✓ Solution by Mathematica

Time used: 2.163 (sec). Leaf size: 276

```
DSolve[(8*y[x]+10*x)+(5*y[x]+7*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{Root}\left[\#1^5 + 8\#1^4x + 25\#1^3x^2 + 38\#1^2x^3 + 28\#1x^4 + 8x^5 - e^{c_1}\&, 1\right]$$
$$y(x) \to \text{Root}\left[\#1^5 + 8\#1^4x + 25\#1^3x^2 + 38\#1^2x^3 + 28\#1x^4 + 8x^5 - e^{c_1}\&, 2\right]$$
$$y(x) \to \text{Root}\left[\#1^5 + 8\#1^4x + 25\#1^3x^2 + 38\#1^2x^3 + 28\#1x^4 + 8x^5 - e^{c_1}\&, 3\right]$$
$$y(x) \to \text{Root}\left[\#1^5 + 8\#1^4x + 25\#1^3x^2 + 38\#1^2x^3 + 28\#1x^4 + 8x^5 - e^{c_1}\&, 4\right]$$
$$y(x) \to \text{Root}\left[\#1^5 + 8\#1^4x + 25\#1^3x^2 + 38\#1^2x^3 + 28\#1x^4 + 8x^5 - e^{c_1}\&, 5\right]$$

1.12 problem 4.1

Internal problem ID [4366]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 2
Problem number: 4.1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{-y + (2y - 1)\, y' = -2x - 1}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 67

```
dsolve((2*x-y(x)+1)+(2*y(x)-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{15}\,\tan\left(\mathrm{RootOf}\left(\sqrt{15}\,\ln\left((1+4x)^2\sec(\_Z)^2\right) - 3\sqrt{15}\,\ln(2) + \sqrt{15}\,\ln(3) + \sqrt{15}\,\ln(5) + 2\sqrt{15}\,c_1 -\right.\right.}{16} + \frac{x}{4} + \frac{9}{16}$$

✓ Solution by Mathematica

Time used: 0.139 (sec). Leaf size: 85

```
DSolve[(2*x-y[x]+1)+(2*y[x]-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[2\sqrt{15}\arctan\left(\frac{-2y(x)+8x+3}{\sqrt{15}(2y(x)-1)}\right) = 15\left(\log\left(\frac{2(8x^2+8y(x)^2-(4x+9)y(x)+6x+3)}{(4x+1)^2}\right) + 2\log(4x+1) + 8c_1\right), y(x)\right]$$

1.13 problem 4.2

Internal problem ID [4367]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 2
Problem number: 4.2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$3y + (7y - 3x + 3)\, y' = 7x - 7$$

✓ Solution by Maple
Time used: 0.297 (sec). Leaf size: 1814

```
dsolve((3*y(x)-7*x+7)+(7*y(x)-3*x+3)*diff(y(x),x)=0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica
Time used: 60.698 (sec). Leaf size: 7785

```
DSolve[(3*y[x]-7*x+7)+(7*y[x]-3*x+3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

1.14 problem 6.1

Internal problem ID [4368]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 2
Problem number: 6.1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{xy}{x^2+1} = \frac{1}{2x\,(x^2+1)}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x)+x/(1+x^2)*y(x)=1/(2*x*(1+x^2)),y(x), singsol=all)
```

$$y(x) = \frac{-\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right) + 2c_1}{2\sqrt{x^2+1}}$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 33

```
DSolve[y'[x]+x/(1+x^2)*y[x]==1/(2*x*(1+x^2)),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\operatorname{arctanh}\left(\sqrt{x^2+1}\right) - 2c_1}{2\sqrt{x^2+1}}$$

1.15 problem 6.2

Internal problem ID [4369]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 2
Problem number: 6.2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x(-x^2+1)\,y'+(2x^2-1)\,y=a\,x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x*(1-x^2)*diff(y(x),x)+(2*x^2-1)*y(x)=a*x^3,y(x), singsol=all)
```

$$y(x)=x\left(\sqrt{x-1}\,\sqrt{1+x}\,c_1+a\right)$$

✓ Solution by Mathematica
Time used: 0.047 (sec). Leaf size: 23

```
DSolve[x*(1-x^2)*y'[x]+(2*x^2-1)*y[x]==a*x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x\left(a+c_1\sqrt{1-x^2}\right)$$

1.16 problem 6.3

Internal problem ID [4370]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 2
Problem number: 6.3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' + \frac{y}{(-x^2+1)^{\frac{3}{2}}} = \frac{x + \sqrt{-x^2+1}}{(-x^2+1)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 56

```
dsolve(diff(y(x),x)+y(x)/(1-x^2)^(3/2)=(x+sqrt(1-x^2))/(1-x^2)^2,y(x), singsol=all)
```

$$y(x) = \left(\int \frac{e^{\frac{x}{\sqrt{-x^2+1}}}\left(x+\sqrt{-x^2+1}\right)}{(x-1)^2(1+x)^2}dx + c_1\right)e^{-\frac{x}{\sqrt{-x^2+1}}}$$

✓ Solution by Mathematica
Time used: 0.169 (sec). Leaf size: 38

```
DSolve[y'[x]+y[x]/(1-x^2)^(3/2)==(x+Sqrt[1-x^2])/(1-x^2)^2,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{x}{\sqrt{1-x^2}} + c_1 e^{-\frac{x}{\sqrt{1-x^2}}}$$

1.17 problem 6.4

Internal problem ID [4371]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 2
Problem number: 6.4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\cos(x) = \frac{\sin(2x)}{2}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+y(x)*cos(x)=1/2*sin(2*x),y(x), singsol=all)
```

$$y(x) = \sin(x) - 1 + e^{-\sin(x)}c_1$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 18

```
DSolve[y'[x]+y[x]*Cos[x]==1/2*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x) + c_1e^{-\sin(x)} - 1$$

1.18 problem 6.5

Internal problem ID [4372]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 2
Problem number: 6.5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2+1)\,y'+y=\arctan(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve((1+x^2)*diff(y(x),x)+y(x)=arctan(x),y(x), singsol=all)
```

$$y(x)=\arctan(x)-1+\mathrm{e}^{-\arctan(x)}c_1$$

✓ Solution by Mathematica
Time used: 0.143 (sec). Leaf size: 18

```
DSolve[(1+x^2)*y'[x]+y[x]==ArcTan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\arctan(x)+c_1e^{-\arctan(x)}-1$$

1.19 problem 10.1

Internal problem ID [4373]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 2
Problem number: 10.1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_separable]

$$\boxed{(-x^2+1)\, z' - xz - axz^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve((1-x^2)*diff(z(x),x)-x*z(x)=a*x*z(x)^2,z(x), singsol=all)
```

$$z(x) = \frac{1}{\sqrt{x-1}\,\sqrt{1+x}\,c_1 - a}$$

✓ Solution by Mathematica
Time used: 3.943 (sec). Leaf size: 47

```
DSolve[(1-x^2)*z'[x]-x*z[x]==a*x*z[x]^2,z[x],x,IncludeSingularSolutions -> True]
```

$$z(x) \to -\frac{e^{c_1}}{-\sqrt{1-x^2} + ae^{c_1}}$$
$$z(x) \to 0$$
$$z(x) \to -\frac{1}{a}$$

1.20 problem 10.2

Internal problem ID [4374]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 2
Problem number: 10.2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{3z^2z' - az^3 = x + 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 106

```
dsolve(3*z(x)^2*diff(z(x),x)-a*z(x)^3=x+1,z(x), singsol=all)
```

$$z(x) = \frac{\left(\left(\mathrm{e}^{ax}c_1a^2 - 1 + (-1-x)\,a\right)a\right)^{\frac{1}{3}}}{a}$$
$$z(x) = -\frac{\left(\left(\mathrm{e}^{ax}c_1a^2 - 1 + (-1-x)\,a\right)a\right)^{\frac{1}{3}}\left(1 + i\sqrt{3}\right)}{2a}$$
$$z(x) = \frac{\left(\left(\mathrm{e}^{ax}c_1a^2 - 1 + (-1-x)\,a\right)a\right)^{\frac{1}{3}}\left(i\sqrt{3} - 1\right)}{2a}$$

✓ Solution by Mathematica
Time used: 14.566 (sec). Leaf size: 111

```
DSolve[3*z[x]^2*z'[x]-a*z[x]^3==x+1,z[x],x,IncludeSingularSolutions -> True]
```

$$z(x) \to \frac{\sqrt[3]{a^2c_1e^{ax} - a(x+1) - 1}}{a^{2/3}}$$
$$z(x) \to -\frac{\sqrt[3]{-1}\sqrt[3]{a^2c_1e^{ax} - a(x+1) - 1}}{a^{2/3}}$$
$$z(x) \to \frac{(-1)^{2/3}\sqrt[3]{a^2c_1e^{ax} - a(x+1) - 1}}{a^{2/3}}$$

1.21 problem 10.3

Internal problem ID [4375]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 2
Problem number: 10.3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{z' + 2xz - 2a\,x^3 z^3 = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 53

```
dsolve(diff(z(x),x)+2*x*z(x)=2*a*x^3*z(x)^3,z(x), singsol=all)
```

$$z(x) = -\frac{2}{\sqrt{4a\,x^2 + 4\,\mathrm{e}^{2x^2} c_1 + 2a}}$$
$$z(x) = \frac{2}{\sqrt{4a\,x^2 + 4\,\mathrm{e}^{2x^2} c_1 + 2a}}$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 29

```
DSolve[z'[x]+2*x*z[x]==2*a*x^3*z[x],z[x],x,IncludeSingularSolutions -> True]
```

$$z(x) \to c_1 e^{\frac{ax^4}{2} - x^2}$$
$$z(x) \to 0$$

1.22 problem 10.4

Internal problem ID [4376]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 2
Problem number: 10.4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$z' + z\cos(x) - z^n \sin(2x) = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 49

```
dsolve(diff(z(x),x)+z(x)*cos(x)=z(x)^n*sin(2*x),z(x), singsol=all)
```

$$z(x) = \left(\frac{e^{\sin(x)(n-1)}c_1 n - e^{\sin(x)(n-1)}c_1 + 2\sin(x)\,n - 2\sin(x) + 2}{n-1}\right)^{-\frac{1}{n-1}}$$

✓ Solution by Mathematica
Time used: 6.964 (sec). Leaf size: 36

```
DSolve[z'[x]+z[x]*Cos[x]==z[x]^n*Sin[2*x],z[x],x,IncludeSingularSolutions -> True]
```

$$z(x) \to \left(c_1 e^{(n-1)\sin(x)} + \frac{2}{n-1} + 2\sin(x)\right)^{\frac{1}{1-n}}$$

1.23 problem 10.5

Internal problem ID [4377]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 2
Problem number: 10.5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{xy' + y - \ln(x)\, y^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x*diff(y(x),x)+y(x)=y(x)^2*ln(x),y(x), singsol=all)
```

$$y(x) = \frac{1}{1 + c_1 x + \ln(x)}$$

✓ Solution by Mathematica
Time used: 0.157 (sec). Leaf size: 20

```
DSolve[x*y'[x]+y[x]==y[x]^2*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{\log(x) + c_1 x + 1}$$
$$y(x) \to 0$$

2 Chapter 3

2.1 problem 1

Internal problem ID [4378]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 3
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$3xy^2 + \left(y^3 + 3yx^2\right) y' = -x^3$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 119

```
dsolve((x^3+3*x*y(x)^2)+(y(x)^3+3*x^2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-3c_1x^2 - \sqrt{8c_1^2x^4 + 1}}}{\sqrt{c_1}}$$
$$y(x) = \frac{\sqrt{-3c_1x^2 + \sqrt{8c_1^2x^4 + 1}}}{\sqrt{c_1}}$$
$$y(x) = -\frac{\sqrt{-3c_1x^2 - \sqrt{8c_1^2x^4 + 1}}}{\sqrt{c_1}}$$
$$y(x) = -\frac{\sqrt{-3c_1x^2 + \sqrt{8c_1^2x^4 + 1}}}{\sqrt{c_1}}$$

✓ Solution by Mathematica

Time used: 8.383 (sec). Leaf size: 245

```
DSolve[(x^3+3*x*y[x]^2)+(y[x]^3+3*x^2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -\sqrt{-3x^2 - \sqrt{8x^4 + e^{4c_1}}}$$
$$y(x) \to \sqrt{-3x^2 - \sqrt{8x^4 + e^{4c_1}}}$$
$$y(x) \to -\sqrt{-3x^2 + \sqrt{8x^4 + e^{4c_1}}}$$
$$y(x) \to \sqrt{-3x^2 + \sqrt{8x^4 + e^{4c_1}}}$$
$$y(x) \to -\sqrt{-2\sqrt{2}\sqrt{x^4} - 3x^2}$$
$$y(x) \to \sqrt{-2\sqrt{2}\sqrt{x^4} - 3x^2}$$
$$y(x) \to -\sqrt{2\sqrt{2}\sqrt{x^4} - 3x^2}$$
$$y(x) \to \sqrt{2\sqrt{2}\sqrt{x^4} - 3x^2}$$

2.2 problem 2

Internal problem ID [4379]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 3
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, _Bernoulli]`

$$\boxed{\frac{y^2}{x^2} - \frac{2yy'}{x} = -1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve((1+y(x)^2/x^2)-2*y(x)/x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{(x+c_1)\,x}$$
$$y(x) = -\sqrt{(x+c_1)\,x}$$

✓ Solution by Mathematica
Time used: 0.186 (sec). Leaf size: 38

```
DSolve[(1+y[x]^2/x^2)-2*y[x]/x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x}\sqrt{x+c_1}$$
$$y(x) \to \sqrt{x}\sqrt{x+c_1}$$

2.3 problem 3

Internal problem ID [4380]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 3
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{\frac{3x}{y^3} + \left(\frac{1}{y^2} - \frac{3x^2}{y^4}\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 22

```
dsolve((3*x/y(x)^3)+(1/y(x)^2-3*x^2/y(x)^4)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{3}\,\sqrt{-\frac{1}{\text{LambertW}\left(-3c_1x^2\right)}}\,x$$

✓ Solution by Mathematica

Time used: 6.543 (sec). Leaf size: 66

```
DSolve[(3*x/y[x]^3)+(1/y[x]^2-3*x^2/y[x]^4)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -\frac{i\sqrt{3}x}{\sqrt{W\left(-3e^{-2c_1}x^2\right)}}$$
$$y(x) \to \frac{i\sqrt{3}x}{\sqrt{W\left(-3e^{-2c_1}x^2\right)}}$$
$$y(x) \to 0$$

2.4 problem 4

Internal problem ID [4381]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 3
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _exact, _rational]`

$$\boxed{y'y + \frac{xy'}{x^2 + y^2} - \frac{y}{x^2 + y^2} = -x}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 26

```
dsolve(x+y(x)*diff(y(x),x)+x/(x^2+y(x)^2)*diff(y(x),x)- y(x)/(x^2+y(x)^2)=0,y(x), singsol=al
```

$$y(x) = \cot\left(\text{RootOf}\left(2c_1 \sin\left(\_Z\right)^2 - 2\_Z \sin\left(\_Z\right)^2 + x^2\right)\right) x$$

✓ Solution by Mathematica

Time used: 0.108 (sec). Leaf size: 31

```
DSolve[x+y[x]*y'[x]+x/(x^2+y[x]^2)*y'[x]- y[x]/(x^2+y[x]^2)==0,y[x],x,IncludeSingularSolutio
```

$$\text{Solve}\left[-\arctan\left(\frac{x}{y(x)}\right) + \frac{x^2}{2} + \frac{y(x)^2}{2} = c_1, y(x)\right]$$

2.5 problem 5

Internal problem ID [4382]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 3
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _dAlembert]`

$$\boxed{e^{\frac{x}{y}} + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right) y' = -1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve((1+exp(x/y(x)))+exp(x/y(x))*(1-x/y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{x}{\text{LambertW}\left(\frac{xc_1}{c_1x-1}\right)}$$

✓ Solution by Mathematica

Time used: 1.182 (sec). Leaf size: 34

```
DSolve[(1+Exp[x/y[x]])+Exp[x/y[x]]*(1-x/y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -\frac{x}{W\left(\frac{x}{x-e^{c_1}}\right)}$$
$$y(x) \to -\frac{x}{W(1)}$$

2.6 problem 6

Internal problem ID [4383]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 3
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _exact, _rational, _Bernoulli]`

$$\boxed{e^{x}\left(x^{2}+y^{2}+2x\right)+2y\,e^{x}y'=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(exp(x)*(x^2+y(x)^2+2*x)+2*y(x)*exp(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\sqrt{e^{-x}c_{1}-x^{2}}$$
$$y(x)=-\sqrt{e^{-x}c_{1}-x^{2}}$$

✓ Solution by Mathematica

Time used: 5.67 (sec). Leaf size: 47

```
DSolve[Exp[x]*(x^2+y[x]^2+2*x)+2*y[x]*Exp[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to -\sqrt{-x^{2}+c_{1}e^{-x}}$$
$$y(x)\to \sqrt{-x^{2}+c_{1}e^{-x}}$$

2.7 problem 7

Internal problem ID [4384]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 3
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$n\cos(nx+my)-m\sin(mx+ny)+(m\cos(nx+my)-n\sin(mx+ny))\,y'=0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 43

```
dsolve((n*cos(n*x+m*y(x))-m*sin(m*x+n*y(x)))+(m*cos(n*x+m*y(x))-n*sin(m*x+n*y(x)))*diff(y(x)
```

$$y(x)=\frac{-nx+\operatorname{RootOf}\left(2m^2x-2n^2x-2\arcsin\left(\sin\left(\_Z\right)+c_1\right)m-m\pi+2\_Zn\right)}{m}$$

✓ Solution by Mathematica
Time used: 0.741 (sec). Leaf size: 50

```
DSolve[(n*Cos[n*x+m*y[x]]-m*Sin[m*x+n*y[x]])+(m*Cos[n*x+m*y[x]]-n*Sin[m*x+n*y[x]])*y'[x]==0,
```

$$\begin{aligned}&\text{Solve}[\sin(mx)\sin(ny(x))-\cos(mx)\cos(ny(x))\\&\quad-\sin(nx)\cos(my(x))-\cos(nx)\sin(my(x))=c_1,y(x)]\end{aligned}$$

2.8 problem 8.1

Internal problem ID [4385]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 3
Problem number: 8.1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _exact]`

$$\boxed{\frac{x}{\sqrt{1+x^2+y^2}}+\frac{yy'}{\sqrt{1+x^2+y^2}}+\frac{y}{x^2+y^2}-\frac{xy'}{x^2+y^2}=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 25

```
dsolve( x/sqrt(1+x^2+y(x)^2) + y(x)/sqrt(1+x^2+y(x)^2)*diff(y(x),x)+  y(x)/(x^2+y(x)^2) - x/
```

$$\arctan\left(\frac{x}{y\left(x\right)}\right)+\sqrt{1+x^2+y\left(x\right)^2}-c_1=0$$

✓ Solution by Mathematica

Time used: 0.264 (sec). Leaf size: 27

```
DSolve[ x/Sqrt[1+x^2+y[x]^2] + y[x]/Sqrt[1+x^2+y[x]^2]*y'[x]+y[x]/(x^2+y[x]^2) - x/(x^2+y[x]
```

$$\text{Solve}\left[\arctan\left(\frac{x}{y(x)}\right)+\sqrt{x^2+y(x)^2+1}=c_1,y(x)\right]$$

2.9 problem 10

Internal problem ID [4386]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 3
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{\frac{x^n y'}{b y^2 - c\, x^{2a}} - \frac{a y x^{a-1}}{b y^2 - c\, x^{2a}} = -x^{a-1}}$$

✗ Solution by Maple

```
dsolve( x^n/(b*y(x)^2-c*x^(2*a))*diff(y(x),x) - a*y(x)*x^(a-1)/(b*y(x)^2-c*x^(2*a)) + x^(a-1
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^n/(b*y[x]^2-c*x^(2*a))*y'[x] - a*y[x]*x^(a-1)/(b*y[x]^2-c*x^(2*a)) + x^(a-1)==0,y[x
```

Not solved

3 Chapter 4

3.1 problem 2

Internal problem ID [4387]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 4
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{2xy + (y^2 - 2x^2)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 22

```
dsolve(2*x*y(x)+(y(x)^2-2*x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{2}\,\sqrt{-\frac{1}{\mathrm{LambertW}\left(-2c_1x^2\right)}}\,x$$

✓ Solution by Mathematica
Time used: 7.214 (sec). Leaf size: 66

```
DSolve[2*x*y[x]+(y[x]^2-2*x^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{i\sqrt{2}x}{\sqrt{W\left(-2e^{-2c_1}x^2\right)}}$$
$$y(x) \to \frac{i\sqrt{2}x}{\sqrt{W\left(-2e^{-2c_1}x^2\right)}}$$
$$y(x) \to 0$$

3.2 problem 4

Internal problem ID [4388]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 4
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{\frac{2}{y} - \frac{2y'}{x} = -\frac{1}{x} - \frac{y'}{y}}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 51

```
dsolve(1/x+1/y(x)*diff(y(x),x)+2*(1/y(x)-1/x*diff(y(x),x))=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x - \sqrt{5x^2 c_1^2 + 4}}{2c_1}$$
$$y(x) = \frac{c_1 x + \sqrt{5x^2 c_1^2 + 4}}{2c_1}$$

✓ Solution by Mathematica

Time used: 0.46 (sec). Leaf size: 102

```
DSolve[1/x+1/y[x]*y'[x]+2*(1/y[x]-1/x*y'[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(x - \sqrt{5x^2 - 4e^{c_1}}\right)$$
$$y(x) \to \frac{1}{2}\left(x + \sqrt{5x^2 - 4e^{c_1}}\right)$$
$$y(x) \to \frac{1}{2}\left(x - \sqrt{5}\sqrt{x^2}\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{5}\sqrt{x^2} + x\right)$$

3.3 problem 5.1

Internal problem ID [4389]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 4
Problem number: 5.1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{xy' - y - \sqrt{x^2 + y^2} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(x*diff(y(x),x)-y(x)=sqrt(x^2+y(x)^2),y(x), singsol=all)
```

$$\frac{-c_1x^2 + \sqrt{x^2 + y(x)^2} + y(x)}{x^2} = 0$$

✓ Solution by Mathematica
Time used: 0.336 (sec). Leaf size: 27

```
DSolve[x*y'[x]-y[x]==Sqrt[x^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-c_1}\left(-1 + e^{2c_1}x^2\right)$$

3.4 problem 5.2

Internal problem ID [4390]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 4
Problem number: 5.2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{8y+(5y+7x)\,y'=-10x}$$

✓ Solution by Maple
Time used: 0.187 (sec). Leaf size: 38

```
dsolve((8*y(x)+10*x)+(5*y(x)+7*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=x\left(\operatorname{RootOf}\left(\_Z^{25}c_1x^5-2\_Z^{20}c_1x^5+\_Z^{15}c_1x^5-1\right)^5-2\right)$$

✓ Solution by Mathematica
Time used: 2.162 (sec). Leaf size: 276

```
DSolve[(8*y[x]+10*x)+(5*y[x]+7*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \operatorname{Root}\left[\#1^5+8\#1^4x+25\#1^3x^2+38\#1^2x^3+28\#1x^4+8x^5-e^{c_1}\&,1\right]$$
$$y(x)\to \operatorname{Root}\left[\#1^5+8\#1^4x+25\#1^3x^2+38\#1^2x^3+28\#1x^4+8x^5-e^{c_1}\&,2\right]$$
$$y(x)\to \operatorname{Root}\left[\#1^5+8\#1^4x+25\#1^3x^2+38\#1^2x^3+28\#1x^4+8x^5-e^{c_1}\&,3\right]$$
$$y(x)\to \operatorname{Root}\left[\#1^5+8\#1^4x+25\#1^3x^2+38\#1^2x^3+28\#1x^4+8x^5-e^{c_1}\&,4\right]$$
$$y(x)\to \operatorname{Root}\left[\#1^5+8\#1^4x+25\#1^3x^2+38\#1^2x^3+28\#1x^4+8x^5-e^{c_1}\&,5\right]$$

3.5 problem 5.3

Internal problem ID [4391]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 4
Problem number: 5.3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{2xy - y^2 + (y^2 + 2xy - x^2)\, y' = -x^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 55

```
dsolve((x^2+2*x*y(x)-y(x)^2)+(y(x)^2+2*x*y(x)-x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1 - \sqrt{-4x^2c_1^2 + 4c_1x + 1}}{2c_1}$$
$$y(x) = \frac{1 + \sqrt{-4x^2c_1^2 + 4c_1x + 1}}{2c_1}$$

✓ Solution by Mathematica

Time used: 1.304 (sec). Leaf size: 75

```
DSolve[(x^2+2*x*y[x]-y[x]^2)+(y[x]^2+2*x*y[x]-x^2)*y'[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{2}\left(e^{c_1} - \sqrt{-4x^2 + 4e^{c_1}x + e^{2c_1}}\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{-4x^2 + 4e^{c_1}x + e^{2c_1}} + e^{c_1}\right)$$

3.6 problem 5.4

Internal problem ID [4392]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 4
Problem number: 5.4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y^2 + (xy + x^2)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 45

```
dsolve(y(x)^2+(x*y(x)+x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1 + \sqrt{c_1 x^2 + 1}}{c_1 x}$$
$$y(x) = \frac{1 - \sqrt{c_1 x^2 + 1}}{c_1 x}$$

✓ Solution by Mathematica
Time used: 2.31 (sec). Leaf size: 80

```
DSolve[y[x]^2+(x*y[x]+x^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{2c_1} - \sqrt{e^{2c_1}(x^2 + e^{2c_1})}}{x}$$
$$y(x) \to \frac{\sqrt{e^{2c_1}(x^2 + e^{2c_1})} + e^{2c_1}}{x}$$
$$y(x) \to 0$$

3.7 problem 5.4

Internal problem ID [4393]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 4
Problem number: 5.4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{\left(x\cos\left(\frac{y}{x}\right)+y\sin\left(\frac{y}{x}\right)\right)y+\left(x\cos\left(\frac{y}{x}\right)-y\sin\left(\frac{y}{x}\right)\right)xy'=0}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 18

```
dsolve((x*cos(y(x)/x)+y(x)*sin(y(x)/x))*y(x)+(x*cos(y(x)/x)-y(x)*sin(y(x)/x))*x*diff(y(x),x)
```

$$y(x)=x\,\mathrm{RootOf}\left(\_Z\cos\left(\_Z\right)x^2-c_1\right)$$

✓ Solution by Mathematica
Time used: 0.347 (sec). Leaf size: 31

```
DSolve[(x*Cos[y[x]/x]+y[x]*Sin[y[x]/x])*y[x]+(x*Cos[y[x]/x]-y[x]*Sin[y[x]/x])*x*y'[x]==0,y[x
```

$$\mathrm{Solve}\left[-\log\left(\frac{y(x)}{x}\right)-\log\left(\cos\left(\frac{y(x)}{x}\right)\right)=2\log(x)+c_1,y(x)\right]$$

3.8 problem 7.1

Internal problem ID [4394]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 4
Problem number: 7.1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(y^2x^2 + xy)\, y + (y^2x^2 - 1)\, xy' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 25

```
dsolve((x^2*y(x)^2+x*y(x))*y(x)+(x^2*y(x)^2-1)*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{x}$$
$$y(x) = -\frac{\mathrm{LambertW}\left(-x\,\mathrm{e}^{-c_1}\right)}{x}$$

✓ Solution by Mathematica

Time used: 2.043 (sec). Leaf size: 43

```
DSolve[(x^2*y[x]^2+x*y[x])*y[x]+(x^2*y[x]^2-1)*x*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{1}{x}$$
$$y(x) \to -\frac{W(-e^{-c_1}x)}{x}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{1}{x}$$

3.9 problem 7.1

Internal problem ID [4395]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 4
Problem number: 7.1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{(x^3y^3 + y^2x^2 + xy + 1)\, y + (x^3y^3 - y^2x^2 - xy + 1)\, xy' = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 42

```
dsolve((x^3*y(x)^3+x^2*y(x)^2+x*y(x)+1)*y(x)+(x^3*y(x)^3-x^2*y(x)^2-x*y(x)+1)*x*diff(y(x),x)
```

$$y(x) = -\frac{1}{x}$$
$$y(x) = \frac{\mathrm{e}^{\mathrm{RootOf}\left(-\mathrm{e}^{2\_Z}-2\ln(x)\mathrm{e}^{\_Z}+2c_1\mathrm{e}^{\_Z}+2\_Z\mathrm{e}^{\_Z}+1\right)}}{x}$$

✓ Solution by Mathematica

Time used: 0.219 (sec). Leaf size: 35

```
DSolve[(x^3*y[x]^3+x^2*y[x]^2+x*y[x]+1)*y[x]+(x^3*y[x]^3-x^2*y[x]^2-x*y[x]+1)*x*y'[x]==0,y[x
```

$$y(x) \to -\frac{1}{x}$$
$$\text{Solve}\left[xy(x) - \frac{1}{xy(x)} - 2\log(y(x)) = c_1, y(x)\right]$$

4 Chapter 5

4.1 problem 1.1

Internal problem ID [4396]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 5
Problem number: 1.1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{y^2 + 2y'y = -x^2 - 2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve((x^2+y(x)^2+2*x)+2*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{e^{-x} c_1 - x^2}$$
$$y(x) = -\sqrt{e^{-x} c_1 - x^2}$$

✓ Solution by Mathematica

Time used: 5.675 (sec). Leaf size: 47

```
DSolve[(x^2+y[x]^2+2*x)+2*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^2 + c_1 e^{-x}}$$
$$y(x) \to \sqrt{-x^2 + c_1 e^{-x}}$$

4.2 problem 1.2

Internal problem ID [4397]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 5
Problem number: 1.2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y^2 - 2xyy' = -x^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve((x^2+y(x)^2)-2*x*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{(x + c_1)\, x}$$
$$y(x) = -\sqrt{(x + c_1)\, x}$$

✓ Solution by Mathematica
Time used: 0.209 (sec). Leaf size: 38

```
DSolve[(x^2+y[x]^2)-2*x*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x}\sqrt{x + c_1}$$
$$y(x) \to \sqrt{x}\sqrt{x + c_1}$$

4.3 problem 2

Internal problem ID [4398]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 5
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{2xy + \left(y^2 - 3x^2\right) y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 317

```
dsolve((2*x*y(x))+(y(x)^2-3*x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1 + \frac{\left(12\sqrt{3}\,x\sqrt{27x^2c_1^2-4}\,c_1-108x^2c_1^2+8\right)^{\frac{1}{3}}}{2} + \frac{2}{\left(12\sqrt{3}\,x\sqrt{27x^2c_1^2-4}\,c_1-108x^2c_1^2+8\right)^{\frac{1}{3}}}}{3c_1}$$

$$y(x) = -\frac{\left(1+i\sqrt{3}\right)\left(12\sqrt{3}\,x\sqrt{27x^2c_1^2-4}\,c_1-108x^2c_1^2+8\right)^{\frac{2}{3}} - 4i\sqrt{3} - 4\left(12\sqrt{3}\,x\sqrt{27x^2c_1^2-4}\,c_1-108x^2c_1^2 +}{12\left(12\sqrt{3}\,x\sqrt{27x^2c_1^2-4}\,c_1-108x^2c_1^2+8\right)^{\frac{1}{3}} c_1}$$

$$y(x) = \frac{\left(i\sqrt{3}-1\right)\left(12\sqrt{3}\,x\sqrt{27x^2c_1^2-4}\,c_1-108x^2c_1^2+8\right)^{\frac{2}{3}} - 4i\sqrt{3} + 4\left(12\sqrt{3}\,x\sqrt{27x^2c_1^2-4}\,c_1-108x^2c_1^2+8\right.}{12\left(12\sqrt{3}\,x\sqrt{27x^2c_1^2-4}\,c_1-108x^2c_1^2+8\right)^{\frac{1}{3}} c_1}$$

✓ Solution by Mathematica

Time used: 60.189 (sec). Leaf size: 458

```
DSolve[(2*x*y[x])+(y[x]^2-3*x^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(\frac{\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}}{\sqrt[3]{2}} + \frac{\sqrt[3]{2}e^{2c_1}}{\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}} - e^{c_1}\right)$$

$$y(x) \to \frac{i(\sqrt{3}+i)\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}}{6\sqrt[3]{2}} - \frac{i(\sqrt{3}-i)e^{2c_1}}{3\ 2^{2/3}\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}} - \frac{e^{c_1}}{3}$$

$$y(x) \to -\frac{i(\sqrt{3}-i)\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}}{6\sqrt[3]{2}} + \frac{i(\sqrt{3}+i)e^{2c_1}}{3\ 2^{2/3}\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}} - \frac{e^{c_1}}{3}$$

4.4 problem 3

Internal problem ID [4399]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 5
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{y + (-x + 2y)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(y(x)+(2*y(x)-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{x}{2\,\mathrm{LambertW}\left(-\frac{x\,\mathrm{e}^{-\frac{c_1}{2}}}{2}\right)}$$

✓ Solution by Mathematica

Time used: 4.711 (sec). Leaf size: 31

```
DSolve[y[x]+(2*y[x]-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{2W\left(-\frac{1}{2}e^{-\frac{c_1}{2}}x\right)}$$
$$y(x) \to 0$$

5 Chapter 6

5.1 problem 1

Internal problem ID [4400]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 6
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{xy' - ya + y^2 = x^{-2a}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 74

```
dsolve(x*diff(y(x),x)-a*y(x)+y(x)^2=x^(-2*a),y(x), singsol=all)
```

$$y(x) = \frac{(-x^{-a}c_1 + a)\sinh\left(\frac{x^{-a}}{a}\right) + (c_1 a - x^{-a})\cosh\left(\frac{x^{-a}}{a}\right)}{\cosh\left(\frac{x^{-a}}{a}\right)c_1 + \sinh\left(\frac{x^{-a}}{a}\right)}$$

✓ Solution by Mathematica
Time used: 0.393 (sec). Leaf size: 112

```
DSolve[x*y'[x]-a*y[x]+y[x]^2==x^(-2*a),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{-a}\left((ax^a + ic_1)\cosh\left(\frac{x^{-a}}{a}\right) - i(ac_1x^a - i)\sinh\left(\frac{x^{-a}}{a}\right)\right)}{\cosh\left(\frac{x^{-a}}{a}\right) - ic_1\sinh\left(\frac{x^{-a}}{a}\right)}$$

$$y(x) \to a - x^{-a}\coth\left(\frac{x^{-a}}{a}\right)$$

5.2 problem 2

Internal problem ID [4401]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 6
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{xy' - ya + y^2 = x^{-\frac{2a}{3}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 119

```
dsolve(x*diff(y(x),x)-a*y(x)+y(x)^2=x^(-2*a/3),y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(-2a+3x^{-\frac{a}{3}}\right)\sqrt{x^{-\frac{2a}{3}}}+a\left(a-x^{-\frac{a}{3}}\right)\right)e^{\frac{6x^{-\frac{a}{3}}}{a}}+\left(\left(2a+3x^{-\frac{a}{3}}\right)\sqrt{x^{-\frac{2a}{3}}}+a\left(a+x^{-\frac{a}{3}}\right)\right)c_1}{\left(-3\sqrt{x^{-\frac{2a}{3}}}+a\right)e^{\frac{6x^{-\frac{a}{3}}}{a}}+c_1\left(3\sqrt{x^{-\frac{2a}{3}}}+a\right)}$$

✓ Solution by Mathematica
Time used: 0.427 (sec). Leaf size: 270

```
DSolve[x*y'[x]-a*y[x]+y[x]^2==x^(-2*a/3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{-a/3}\left(\left(a^2x^{2a/3}-3iac_1x^{a/3}+3\right)\cosh\left(\frac{3x^{-a/3}}{a}\right)+i\left(a^2c_1x^{2a/3}+3iax^{a/3}+3c_1\right)\sinh\left(\frac{3x^{-a/3}}{a}\right)\right)}{\left(ax^{a/3}-3ic_1\right)\cosh\left(\frac{3x^{-a/3}}{a}\right)+i\left(ac_1x^{a/3}+3i\right)\sinh\left(\frac{3x^{-a/3}}{a}\right)}$$

$$y(x) \to \frac{\left(a^2x^{2a/3}+3\right)\sinh\left(\frac{3x^{-a/3}}{a}\right)-3ax^{a/3}\cosh\left(\frac{3x^{-a/3}}{a}\right)}{ax^{2a/3}\sinh\left(\frac{3x^{-a/3}}{a}\right)-3x^{a/3}\cosh\left(\frac{3x^{-a/3}}{a}\right)}$$

5.3 problem 3

Internal problem ID [4402]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 6
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Riccati, _special]]`

$$\boxed{u' + u^2 = \frac{c}{x^{\frac{4}{3}}}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(u(x),x)+u(x)^2=c*x^(-4/3),u(x), singsol=all)
```

$$u(x) = -\frac{3c}{x^{\frac{1}{3}}\left(3x^{\frac{1}{3}}\tan\left(3\sqrt{-c}\left(x^{\frac{1}{3}}-c_1\right)\right)\sqrt{-c}+1\right)}$$

✓ Solution by Mathematica

Time used: 0.286 (sec). Leaf size: 183

```
DSolve[u'[x]+u[x]^2==c*x^(-4/3),u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to \frac{3c\left(3i\sinh\left(3\sqrt{c}\sqrt[3]{x}\right)+8c_1\cosh\left(3\sqrt{c}\sqrt[3]{x}\right)\right)}{\sqrt[3]{x}\left(\left(9i\sqrt{c}\sqrt[3]{x}-8c_1\right)\cosh\left(3\sqrt{c}\sqrt[3]{x}\right)+3\left(8\sqrt{c}c_1\sqrt[3]{x}-i\right)\sinh\left(3\sqrt{c}\sqrt[3]{x}\right)\right)}$$

$$u(x) \to -\frac{3c\cosh\left(3\sqrt{c}\sqrt[3]{x}\right)}{\sqrt[3]{x}\left(\cosh\left(3\sqrt{c}\sqrt[3]{x}\right)-3\sqrt{c}\sqrt[3]{x}\sinh\left(3\sqrt{c}\sqrt[3]{x}\right)\right)}$$

5.4 problem 4

Internal problem ID [4403]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 6
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Riccati, _special]]`

$$\boxed{u' + bu^2 = \frac{c}{x^4}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(u(x),x)+b*u(x)^2=c*x^(-4),u(x), singsol=all)
```

$$u(x) = \frac{-\sqrt{-bc}\,\tan\left(\frac{\sqrt{-bc}\,(c_1x-1)}{x}\right) + x}{b\,x^2}$$

✓ Solution by Mathematica

Time used: 0.308 (sec). Leaf size: 98

```
DSolve[u'[x]+b*u[x]^2==x^(-4),u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to \frac{-2bc_1e^{\frac{2\sqrt{b}}{x}} + \sqrt{b}\left(1 + 2c_1xe^{\frac{2\sqrt{b}}{x}}\right) + x}{x^2\left(b + 2b^{3/2}c_1e^{\frac{2\sqrt{b}}{x}}\right)}$$

$$u(x) \to \frac{x - \sqrt{b}}{bx^2}$$

5.5 problem 5

Internal problem ID [4404]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 6
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Riccati, _special]]`

$$\boxed{u' - u^2 = \frac{2}{x^{\frac{8}{3}}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 78

```
dsolve(diff(u(x),x)-u(x)^2=2*x^(-8/3),u(x), singsol=all)
```

$$u(x) = -\frac{3\left(\tan\left(3\sqrt{2}\left(\left(\frac{1}{x}\right)^{\frac{1}{3}} - c_1\right)\right)\sqrt{2}\,x\left(\frac{1}{x}\right)^{\frac{2}{3}} + \frac{x\left(\frac{1}{x}\right)^{\frac{1}{3}}}{3} - 2\right)}{\left(\frac{1}{x}\right)^{\frac{1}{3}} x^2\left(3\left(\frac{1}{x}\right)^{\frac{1}{3}}\sqrt{2}\tan\left(3\sqrt{2}\left(\left(\frac{1}{x}\right)^{\frac{1}{3}} - c_1\right)\right) + 1\right)}$$

✓ Solution by Mathematica

Time used: 0.266 (sec). Leaf size: 215

```
DSolve[u'[x]-u[x]^2==x^(-8/3),u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to -\frac{\left(-9\sqrt[3]{\frac{1}{x}}+c_1\left(8-24\left(\frac{1}{x}\right)^{2/3}\right)\right)\cos\left(3\sqrt[3]{\frac{1}{x}}\right)+3\left(-3\left(\frac{1}{x}\right)^{2/3}+8c_1\sqrt[3]{\frac{1}{x}}+1\right)\sin\left(3\sqrt[3]{\frac{1}{x}}\right)}{x\left(\left(-9\sqrt[3]{\frac{1}{x}}+8c_1\right)\cos\left(3\sqrt[3]{\frac{1}{x}}\right)+3\left(1+8c_1\sqrt[3]{\frac{1}{x}}\right)\sin\left(3\sqrt[3]{\frac{1}{x}}\right)\right)}$$

$$u(x) \to \frac{\left(3\left(\frac{1}{x}\right)^{2/3}-1\right)\cos\left(3\sqrt[3]{\frac{1}{x}}\right)-3\sqrt[3]{\frac{1}{x}}\sin\left(3\sqrt[3]{\frac{1}{x}}\right)}{x\left(3\sqrt[3]{\frac{1}{x}}\sin\left(3\sqrt[3]{\frac{1}{x}}\right)+\cos\left(3\sqrt[3]{\frac{1}{x}}\right)\right)}$$

5.6 problem 12

Internal problem ID [4405]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 6
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\frac{\sqrt{f\,x^{4}+c\,x^{3}+c\,x^{2}+bx+a}\,y'}{\sqrt{a+by+cy^{2}+cy^{3}+fy^{4}}}=-1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 56

```
dsolve((sqrt(a+b*x+c*x^2+c*x^3+f*x^4))/(sqrt(a+b*y(x)+c*y(x)^2+c*y(x)^3+f*y(x)^4))*diff(y(x)
```

$$\int \frac{1}{\sqrt{f\,x^{4}+x^{3}c+x^{2}c+xb+a}}dx+\int^{y(x)}\frac{1}{\sqrt{\_a^{4}f+\_a^{3}c+\_a^{2}c+\_ab+a}}d\_a+c_{1}=0$$

✓ Solution by Mathematica
Time used: 21.472 (sec). Leaf size: 2239

```
DSolve[Sqrt[a+b*x+c*x^2+c*x^3+f*x^4]/Sqrt[a+b*y[x]+c*y[x]^2+c*y[x]^3+f*y[x]^4]*y'[x]==-1,y[x
```

Too large to display

6 Chapter 7

6.1 problem 1

Internal problem ID [4406]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 7
Problem number: 1.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{{y'}^2 - 5y' = -6}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve((diff(y(x),x))^2-5*diff(y(x),x)+6=0,y(x), singsol=all)
```

$$y(x) = 3x + c_1$$
$$y(x) = 2x + c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 21

```
DSolve[(y'[x])^2-5*y'[x]+6==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x + c_1$$
$$y(x) \to 3x + c_1$$

6.2 problem 2

Internal problem ID [4407]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 7
Problem number: 2.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 = \frac{a^2}{x^2}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve((diff(y(x),x))^2-a^2/x^2=0,y(x), singsol=all)
```

$$y(x) = a \ln (x) + c_1$$
$$y(x) = -a \ln (x) + c_1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 24

```
DSolve[(y'[x])^2-a^2/x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -a \log(x) + c_1$$
$$y(x) \to a \log(x) + c_1$$

6.3 problem 3

Internal problem ID [4408]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 7
Problem number: 3.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 = \frac{1-x}{x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 45

```
dsolve((diff(y(x),x))^2=(1-x)/x,y(x), singsol=all)
```

$$y(x) = \sqrt{-x(x-1)} + \frac{\arcsin(2x-1)}{2} + c_1$$
$$y(x) = -\sqrt{-x(x-1)} - \frac{\arcsin(2x-1)}{2} + c_1$$

✓ Solution by Mathematica
Time used: 0.058 (sec). Leaf size: 81

```
DSolve[(y'[x])^2==(1-x)/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2\arctan\left(\frac{\sqrt{1-x}}{\sqrt{x}+1}\right) + \sqrt{-((x-1)x)} + c_1$$
$$y(x) \to 2\arctan\left(\frac{\sqrt{1-x}}{\sqrt{x}+1}\right) - \sqrt{-((x-1)x)} + c_1$$

6.4 problem 4

Internal problem ID [4409]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 7
Problem number: 4.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{y'^2 = -\frac{2xy'}{y} + 1}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 45

```
dsolve((diff(y(x),x))^2+2*x/y(x)*diff(y(x),x)-1=0,y(x), singsol=all)
```

$$y(x) = -ix$$
$$y(x) = ix$$
$$y(x) = -\frac{2\sqrt{c_1 x + 1}}{c_1}$$
$$y(x) = \frac{2\sqrt{c_1 x + 1}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.466 (sec). Leaf size: 126

```
DSolve[(y'[x])^2+2*x/y[x]*y'[x]-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{\frac{c_1}{2}}\sqrt{-2x + e^{c_1}}$$
$$y(x) \to e^{\frac{c_1}{2}}\sqrt{-2x + e^{c_1}}$$
$$y(x) \to -e^{\frac{c_1}{2}}\sqrt{2x + e^{c_1}}$$
$$y(x) \to e^{\frac{c_1}{2}}\sqrt{2x + e^{c_1}}$$
$$y(x) \to 0$$
$$y(x) \to -ix$$
$$y(x) \to ix$$

6.5 problem 5

Internal problem ID [4410]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 7
Problem number: 5.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y - ay' - by'^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 207

```
dsolve(y(x)=a*diff(y(x),x)+b*(diff(y(x),x))^2,y(x), singsol=all)
```

$$y(x) = e^{\frac{-a \,\mathrm{LambertW}\left(\frac{2 e^{\frac{-c_1 - a + x}{a}}}{a\sqrt{\frac{1}{b}}}\right) - a + x - c_1}{a}} \left(a\sqrt{\frac{1}{b}} + e^{\frac{-a \,\mathrm{LambertW}\left(\frac{2 e^{\frac{-c_1 - a + x}{a}}}{a\sqrt{\frac{1}{b}}}\right) - a + x - c_1}{a}}\right)$$

$$y(x) = \frac{a^2 \left(\mathrm{LambertW}\left(-\frac{2\sqrt{b}\, e^{\frac{-c_1 - a + x}{a}}}{a}\right) + 2\right) \mathrm{LambertW}\left(-\frac{2\sqrt{b}\, e^{\frac{-c_1 - a + x}{a}}}{a}\right)}{4b}$$

$$y(x) = \frac{a^2 \left(\mathrm{LambertW}\left(\frac{2\sqrt{b}\, e^{\frac{-c_1 - a + x}{a}}}{a}\right) + 2\right) \mathrm{LambertW}\left(\frac{2\sqrt{b}\, e^{\frac{-c_1 - a + x}{a}}}{a}\right)}{4b}$$

✓ Solution by Mathematica

Time used: 0.803 (sec). Leaf size: 123

```
DSolve[y[x]==a*y'[x]+b*(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{\sqrt{4\#1b + a^2} + a\log\left(b\left(a - \sqrt{4\#1b + a^2}\right)\right)}{2b}\&\right]\left[\frac{x}{2b} + c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\sqrt{4\#1b + a^2} - a\log\left(\sqrt{4\#1b + a^2} + a\right)}{2b}\&\right]\left[-\frac{x}{2b} + c_1\right]$$

$$y(x) \to 0$$

6.6 problem 6

Internal problem ID [4411]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 7
Problem number: 6.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{-ay' - by'^2 = -x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 80

```
dsolve(x=a*diff(y(x),x)+b*(diff(y(x),x))^2,y(x), singsol=all)
```

$$y(x) = \frac{(a^2 + 4xb)^{\frac{3}{2}} + 12c_1 b^2 - 6axb}{12b^2}$$
$$y(x) = \frac{12c_1 b^2 - a^2\sqrt{a^2 + 4xb} - 6axb - 4bx\sqrt{a^2 + 4xb}}{12b^2}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 74

```
DSolve[x==a*y'[x]+b*(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(a^2 + 4bx)^{3/2} - 6abx + 12b^2 c_1}{12b^2}$$
$$y(x) \to -\frac{\frac{(a^2+4bx)^{3/2}}{6b} + ax}{2b} + c_1$$

6.7 problem 7

Internal problem ID [4412]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 7
Problem number: 7.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y - \sqrt{1 + y'^2} - ay' = 0$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 112

```
dsolve(y(x)=a*diff(y(x),x)+sqrt(1+(diff(y(x),x))^2),y(x), singsol=all)
```

$$-\left(\int^{y(x)} \frac{1}{a\_a + \sqrt{\_a^2 + a^2 - 1}} d\_a\right) a^2 + \int^{y(x)} \frac{1}{a\_a + \sqrt{\_a^2 + a^2 - 1}} d\_a - c_1 + x = 0$$

$$\left(\int^{y(x)} \frac{1}{-a\_a + \sqrt{\_a^2 + a^2 - 1}} d\_a\right) a^2 - \left(\int^{y(x)} \frac{1}{-a\_a + \sqrt{\_a^2 + a^2 - 1}} d\_a\right) - c_1 + x = 0$$

✓ Solution by Mathematica

Time used: 0.597 (sec). Leaf size: 210

```
DSolve[y[x]==a*y'[x]+Sqrt[1+(y'[x])^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{a\left(\log\left(\sqrt{\#1^2+a^2-1}-\#1-a+1\right)+\log\left(\sqrt{\#1^2+a^2-1}-\#1+a-1\right)\right)-(a}{a^2-1}+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[\frac{a\left(\log\left(\sqrt{\#1^2+a^2-1}-\#1-a-1\right)+\log\left(\sqrt{\#1^2+a^2-1}-\#1+a+1\right)\right)-(a}{a^2-1}+c_1\right]$$

$$y(x) \to 1$$

6.8 problem 8

Internal problem ID [4413]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 7
Problem number: 8.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$-\sqrt{1+y'^2}-ay'=-x$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 113

```
dsolve(x=a*diff(y(x),x)+sqrt(1+(diff(y(x),x))^2),y(x), singsol=all)
```

$$y(x)=\frac{a\,x^2+x\sqrt{a^2+x^2-1}+\left(\ln\left(x+\sqrt{a^2+x^2-1}\right)+2c_1\right)(1+a)\,(a-1)}{2a^2-2}$$
$$y(x)=\frac{a\,x^2-x\sqrt{a^2+x^2-1}-(1+a)\,(a-1)\left(\ln\left(x+\sqrt{a^2+x^2-1}\right)-2c_1\right)}{2a^2-2}$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 113

```
DSolve[x==a*y'[x]+Sqrt[1+(y'[x])^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}\left(\frac{x\left(ax-\sqrt{a^2+x^2-1}\right)}{a^2-1}+\log\left(\sqrt{a^2+x^2-1}-x\right)\right)+c_1$$
$$y(x)\to\frac{1}{2}\left(\frac{x\left(\sqrt{a^2+x^2-1}+ax\right)}{a^2-1}-\log\left(\sqrt{a^2+x^2-1}-x\right)\right)+c_1$$

6.9 problem 9

Internal problem ID [4414]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 7
Problem number: 9.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \frac{\sqrt{1+y'^2}}{x} = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 33

```
dsolve(diff(y(x),x)-1/x*sqrt(1+(diff(y(x),x))^2)=0,y(x), singsol=all)
```

$$y(x) = \ln\left(x + \sqrt{x^2-1}\right) + c_1$$
$$y(x) = -\ln\left(x + \sqrt{x^2-1}\right) + c_1$$

✓ Solution by Mathematica
Time used: 0.008 (sec). Leaf size: 89

```
DSolve[y'[x]-1/x*Sqrt[1+(y'[x])^2]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(\log\left(1 - \frac{x}{\sqrt{x^2-1}}\right) - \log\left(\frac{x}{\sqrt{x^2-1}} + 1\right) + 2c_1\right)$$
$$y(x) \to \frac{1}{2}\left(-\log\left(1 - \frac{x}{\sqrt{x^2-1}}\right) + \log\left(\frac{x}{\sqrt{x^2-1}} + 1\right) + 2c_1\right)$$

6.10 problem 10

Internal problem ID [4415]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 7
Problem number: 10.
ODE order: 1.
ODE degree: 6.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x^2\left(1+{y'}^2\right)^3 = a^2}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 605

```
dsolve(x^2*(1+(diff(y(x),x))^2)^3-a^2=0,y(x), singsol=all)
```

$$y(x) = \frac{-\sqrt{\frac{x(a^2x)^{\frac{1}{3}}\left(a^2-(a^2x)^{\frac{2}{3}}\right)}{a^2}}\, a^2 + c_1(a^2x)^{\frac{2}{3}} + \sqrt{\frac{x(a^2x)^{\frac{1}{3}}\left(a^2-(a^2x)^{\frac{2}{3}}\right)}{a^2}}\,(a^2x)^{\frac{2}{3}}}{(a^2x)^{\frac{2}{3}}}$$

$$y(x) = \frac{\left(a^2-(a^2x)^{\frac{2}{3}}\right)\sqrt{\frac{x(a^2x)^{\frac{1}{3}}\left(a^2-(a^2x)^{\frac{2}{3}}\right)}{a^2}} + c_1(a^2x)^{\frac{2}{3}}}{(a^2x)^{\frac{2}{3}}}$$

$$y(x) = -\frac{\sqrt{2}\sqrt{-x\left(i\sqrt{3}\,(a^2x)^{\frac{1}{3}}+(a^2x)^{\frac{1}{3}}+2x\right)}\sqrt{\frac{\left(2i(a^2x)^{\frac{2}{3}}+ia^2-\sqrt{3}\,a^2\right)x(a^2x)^{\frac{1}{3}}}{a^2}}\left(2(a^2x)^{\frac{2}{3}}+a^2+i\sqrt{3}\,a^2\right)}{4\sqrt{\left(i\,(a^2x)^{\frac{1}{3}}+2ix-\sqrt{3}\,(a^2x)^{\frac{1}{3}}\right)x}\,(a^2x)^{\frac{2}{3}}} + c_1$$

$$y(x) = \frac{\sqrt{2}\sqrt{-x\left(i\sqrt{3}\,(a^2x)^{\frac{1}{3}}+(a^2x)^{\frac{1}{3}}+2x\right)}\sqrt{\frac{\left(2i(a^2x)^{\frac{2}{3}}+ia^2-\sqrt{3}\,a^2\right)x(a^2x)^{\frac{1}{3}}}{a^2}}\left(2(a^2x)^{\frac{2}{3}}+a^2+i\sqrt{3}\,a^2\right)}{4\sqrt{\left(i\,(a^2x)^{\frac{1}{3}}+2ix-\sqrt{3}\,(a^2x)^{\frac{1}{3}}\right)x}\,(a^2x)^{\frac{2}{3}}} + c_1$$

$$y(x) = \frac{\left(-2(a^2x)^{\frac{2}{3}}\sqrt{2}+\left(i\sqrt{6}-\sqrt{2}\right)a^2\right)\sqrt{\frac{\left(\left(i\sqrt{3}-1\right)a^2-2(a^2x)^{\frac{2}{3}}\right)x(a^2x)^{\frac{1}{3}}}{a^2}} + 4c_1(a^2x)^{\frac{2}{3}}}{4\,(a^2x)^{\frac{2}{3}}}$$

$$y(x) = -\frac{\left(-2(a^2x)^{\frac{2}{3}}\sqrt{2}+\left(i\sqrt{6}-\sqrt{2}\right)a^2\right)\sqrt{\frac{\left(\left(i\sqrt{3}-1\right)a^2-2(a^2x)^{\frac{2}{3}}\right)x(a^2x)^{\frac{1}{3}}}{a^2}} - 4c_1(a^2x)^{\frac{2}{3}}}{4\,(a^2x)^{\frac{2}{3}}}$$

✓ Solution by Mathematica

Time used: 18.927 (sec). Leaf size: 375

```
DSolve[x^2*(1+(y'[x])^2)^3-a^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[3]{x}\sqrt{\frac{a^{2/3}}{x^{2/3}} - 1}\left(x^{2/3} - a^{2/3}\right) + c_1$$
$$y(x) \to \sqrt[3]{x}\sqrt{\frac{a^{2/3}}{x^{2/3}} - 1}\left(a^{2/3} - x^{2/3}\right) + c_1$$
$$y(x) \to c_1 - \frac{1}{2}\sqrt[3]{x}\sqrt{-1 + \frac{i\left(\sqrt{3}+i\right)a^{2/3}}{2x^{2/3}}}\left(2x^{2/3} + \left(1 - i\sqrt{3}\right)a^{2/3}\right)$$
$$y(x) \to \frac{1}{2}\sqrt[3]{x}\sqrt{-1 + \frac{i\left(\sqrt{3}+i\right)a^{2/3}}{2x^{2/3}}}\left(2x^{2/3} + \left(1 - i\sqrt{3}\right)a^{2/3}\right) + c_1$$
$$y(x) \to c_1 - \frac{1}{2}\sqrt[3]{x}\sqrt{-1 - \frac{i\left(\sqrt{3}-i\right)a^{2/3}}{2x^{2/3}}}\left(2x^{2/3} + \left(1 + i\sqrt{3}\right)a^{2/3}\right)$$
$$y(x) \to \frac{1}{2}\sqrt[3]{x}\sqrt{-1 - \frac{i\left(\sqrt{3}-i\right)a^{2/3}}{2x^{2/3}}}\left(2x^{2/3} + \left(1 + i\sqrt{3}\right)a^{2/3}\right) + c_1$$

6.11 problem 11

Internal problem ID [4416]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 7
Problem number: 11.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 = -1 + \frac{(a+x)^2}{2ax + x^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve(1+(diff(y(x),x))^2=(x+a)^2/(x^2+2*a*x),y(x), singsol=all)
```

$$y(x) = a \ln \left(x + a + \sqrt{x\,(2a + x)}\right) + c_1$$
$$y(x) = -a \ln \left(x + a + \sqrt{x\,(2a + x)}\right) + c_1$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 107

```
DSolve[1+(y'[x])^2==(x+a)^2/(x^2+2*a*x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2a\sqrt{x}\sqrt{2a+x}\log\left(\sqrt{2a+x}-\sqrt{x}\right)}{\sqrt{x(2a+x)}} + c_1$$
$$y(x) \to \frac{2a\sqrt{x}\sqrt{2a+x}\log\left(\sqrt{2a+x}-\sqrt{x}\right)}{\sqrt{x(2a+x)}} + c_1$$

6.12 problem 12

Internal problem ID [4417]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 7
Problem number: 12.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$y - xy' - y' + y'^2 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 22

```
dsolve(y(x)=x*diff(y(x),x)+diff(y(x),x)-(diff(y(x),x))^2,y(x), singsol=all)
```

$$y(x) = \frac{(1+x)^2}{4}$$
$$y(x) = c_1(-c_1 + x + 1)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 28

```
DSolve[y[x]==x*y'[x]+y'[x]-(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x + 1 - c_1)$$
$$y(x) \to \frac{1}{4}(x+1)^2$$

6.13 problem 13

Internal problem ID [4418]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 7
Problem number: 13.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Clairaut]`

$$\boxed{y - xy' - \sqrt{b^2 - a^2 y'^2} = 0}$$

✓ Solution by Maple

Time used: 0.297 (sec). Leaf size: 22

```
dsolve(y(x)=x*diff(y(x),x)+sqrt(b^2-a^2*(diff(y(x),x))^2),y(x), singsol=all)
```

$$y(x) = c_1 x + \sqrt{-a^2 c_1^2 + b^2}$$

✓ Solution by Mathematica

Time used: 0.349 (sec). Leaf size: 38

```
DSolve[y[x]==x*y'[x]+Sqrt[b^2-a^2*(y'[x])^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{b^2 - a^2 c_1{}^2} + c_1 x$$
$$y(x) \to \sqrt{b^2}$$

6.14 problem 14

Internal problem ID [4419]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 7
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$y - xy' - x\sqrt{1 + y'^2} = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 97

```
dsolve(y(x)=x*diff(y(x),x)+x*sqrt(1+(diff(y(x),x))^2),y(x), singsol=all)
```

$$y(x) = \frac{\left(\sqrt{-\frac{c_1^2}{x(-2c_1+x)}}\,\sqrt{-x\left(-2c_1+x\right)} - x + c_1\right)x}{\sqrt{-x\left(-2c_1+x\right)}}$$
$$y(x) = \frac{\left(\sqrt{-\frac{c_1^2}{x(-2c_1+x)}}\,\sqrt{-x\left(-2c_1+x\right)} + x - c_1\right)x}{\sqrt{-x\left(-2c_1+x\right)}}$$

✓ Solution by Mathematica

Time used: 0.269 (sec). Leaf size: 37

```
DSolve[y[x]==x*y'[x]+x*Sqrt[1+(y'[x])^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x(x-c_1)}$$
$$y(x) \to \sqrt{-x(x-c_1)}$$

6.15 problem 15

Internal problem ID [4420]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 7
Problem number: 15.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$y - xy' - ax\sqrt{1 + y'^2} = 0$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 340

```
dsolve(y(x)=x*diff(y(x),x)+a*x*sqrt(1+(diff(y(x),x))^2),y(x), singsol=all)
```

$$\frac{x\sqrt{\frac{-x^2a^2+y(x)^2a^2+2\sqrt{-x^2a^2+x^2+y(x)^2}\,ay(x)+x^2+y(x)^2}{(a^2-1)^2x^2}} - \mathrm{e}^{\frac{\operatorname{arcsinh}\left(\frac{\sqrt{-x^2a^2+x^2+y(x)^2}\,a+y(x)}{(a^2-1)x}\right)}{a}}c_1}{\sqrt{\frac{-x^2a^2+y(x)^2a^2+2\sqrt{-x^2a^2+x^2+y(x)^2}\,ay(x)+x^2+y(x)^2}{(a^2-1)^2x^2}}} = 0$$

$$\frac{x\sqrt{\frac{-x^2a^2+y(x)^2a^2-2\sqrt{-x^2a^2+x^2+y(x)^2}\,ay(x)+x^2+y(x)^2}{(a^2-1)^2x^2}} - \mathrm{e}^{\frac{\operatorname{arcsinh}\left(\frac{-\sqrt{-x^2a^2+x^2+y(x)^2}\,a+y(x)}{(a^2-1)x}\right)}{a}}c_1}{\sqrt{\frac{-x^2a^2+y(x)^2a^2-2\sqrt{-x^2a^2+x^2+y(x)^2}\,ay(x)+x^2+y(x)^2}{(a^2-1)^2x^2}}} = 0$$

✓ Solution by Mathematica

Time used: 0.993 (sec). Leaf size: 223

```
DSolve[y[x]==x*y'[x]+a*x*Sqrt[1+(y'[x])^2],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2i\arctan\left(\frac{y(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)-2ia\arctan\left(\frac{ay(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)+a\log\left(\frac{y(x)^2}{x^2}+1\right)}{2a^2-2}=\frac{a\log\left(x-a^2x\right)}{1-a^2}+c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{-2i\arctan\left(\frac{y(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)+2ia\arctan\left(\frac{ay(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)+a\log\left(\frac{y(x)^2}{x^2}+1\right)}{2a^2-2}=\frac{a\log\left(x-a^2x\right)}{1-a^2}+c_1,y(x)\right]$$

6.16 problem 16

Internal problem ID [4421]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 7
Problem number: 16.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_dAlembert]`

$$\boxed{-yy' - ay'^2 = -x}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 396

```
dsolve(x-y(x)*diff(y(x),x)=a*(diff(y(x),x))^2,y(x), singsol=all)
```

$$\frac{c_1\left(y(x)-\sqrt{4ax+y(x)^2}\right)}{\sqrt{\frac{-y(x)+\sqrt{4ax+y(x)^2}-2a}{a}}\sqrt{\frac{-y(x)+\sqrt{4ax+y(x)^2}+2a}{a}}}+x-\frac{\left(y(x)-\sqrt{4ax+y(x)^2}\right)\left(-3\ln(2)+2\ln\left(\frac{2\sqrt{\frac{y(x)^2-y(x)\sqrt{4ax+y(x)^2}-2a^2+2ax}{a^2}}\,a-\left(y(x)-\sqrt{4ax+y(x)^2}\right)\sqrt{2}}{a}\right)\right)\sqrt{2}}{4\sqrt{\frac{y(x)^2-y(x)\sqrt{4ax+y(x)^2}-2a^2+2ax}{a^2}}}=0$$

$$\frac{c_1\left(y(x)+\sqrt{4ax+y(x)^2}\right)}{2\sqrt{\frac{-y(x)-\sqrt{4ax+y(x)^2}-2a}{a}}\sqrt{\frac{-y(x)-\sqrt{4ax+y(x)^2}+2a}{a}}}+x-\frac{\left(-\frac{3\ln(2)}{2}+\ln\left(\frac{2\sqrt{\frac{y(x)\sqrt{4ax+y(x)^2}-2a^2+2ax+y(x)^2}{a^2}}\,a-\left(y(x)+\sqrt{4ax+y(x)^2}\right)\sqrt{2}}{a}\right)\right)\left(y(x)+\sqrt{4ax+y(x)^2}\right)\sqrt{2}}{2\sqrt{\frac{y(x)\sqrt{4ax+y(x)^2}-2a^2+2ax+y(x)^2}{a^2}}}=0$$

✓ Solution by Mathematica

Time used: 0.55 (sec). Leaf size: 79

```
DSolve[x-y[x]*y'[x]==a*(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x=-\frac{2aK[1]\arctan\left(\frac{\sqrt{1-K[1]^2}}{K[1]+1}\right)}{\sqrt{1-K[1]^2}}+\frac{c_1K[1]}{\sqrt{1-K[1]^2}},y(x)=\frac{x}{K[1]}-aK[1]\right\},\{y(x),K[1]\}\right]$$

6.17 problem 17

Internal problem ID [4422]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 7
Problem number: 17.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$yy' - a\sqrt{1 + y'^2} = -x$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 237

```
dsolve(x+y(x)*diff(y(x),x)=a*sqrt(1+(diff(y(x),x))^2),y(x), singsol=all)
```

$$y(x) = \csc\left(\operatorname{RootOf}\left(\left(\sin(\_Z)\_Za + \sin(\_Z)c_1 - \cos(\_Z)a - x\right)\left(\sin(\_Z)\_Za + \sin(\_Z)c_1 + \cos(\_Z)a - x\right)\right)\right.$$
$$-\cot\left(\operatorname{RootOf}\left(\left(\sin(\_Z)\_Za + \sin(\_Z)c_1 - \cos(\_Z)a - x\right)\left(\sin(\_Z)\_Za + \sin(\_Z)c_1 + \cos(\_Z)a - x\right)\right.\right.$$

$$y(x) = \csc\left(\operatorname{RootOf}\left(\left(\sin(\_Z)\_Za + \sin(\_Z)c_1 + \cos(\_Z)a + x\right)\left(\sin(\_Z)\_Za + \sin(\_Z)c_1 - \cos(\_Z)a + x\right)\right)\right.$$
$$-\cot\left(\operatorname{RootOf}\left(\left(\sin(\_Z)\_Za + \sin(\_Z)c_1 + \cos(\_Z)a + x\right)\left(\sin(\_Z)\_Za + \sin(\_Z)c_1 - \cos(\_Z)a + x\right)\right.\right.$$

✓ Solution by Mathematica

Time used: 3.538 (sec). Leaf size: 388

```
DSolve[x+y[x]*y'[x]==a*Sqrt[1+(y'[x])^2],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\frac{2a\sqrt{a^2y(x)^2-a^4}\arctan\left(\frac{ax\sqrt{y(x)^2-a^2}}{y(x)\left(\sqrt{a^2(y(x)^2-a^2)}-\sqrt{a^2(-a^2+x^2+y(x)^2)}\right)+a^2x}\right)}{\sqrt{y(x)^2-a^2}}-\sqrt{a^2(-a^2+x^2+y(x)^2)}}{a^2}\right.$$

$$\left.-\frac{a\sqrt{y(x)^2-a^2}\arctan\left(\frac{\sqrt{y(x)^2-a^2}}{a}\right)}{\sqrt{a^2(y(x)^2-a^2)}}=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{a\sqrt{y(x)^2-a^2}\arctan\left(\frac{\sqrt{y(x)^2-a^2}}{a}\right)}{\sqrt{a^2(y(x)^2-a^2)}}\right.$$

$$\left.+\frac{\sqrt{a^2(-a^2+x^2+y(x)^2)}-\frac{2a\sqrt{a^2y(x)^2-a^4}\arctan\left(\frac{ax\sqrt{y(x)^2-a^2}}{y(x)\left(\sqrt{a^2(-a^2+x^2+y(x)^2)}-\sqrt{a^2(y(x)^2-a^2)}\right)+a^2x}\right)}{\sqrt{y(x)^2-a^2}}}{a^2}=c_1,y(x)\right]$$

6.18 problem 18

Internal problem ID [4423]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 7
Problem number: 18.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{y'y - y^2 + y^2 y'^2 = x}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 77

```
dsolve(y(x)*diff(y(x),x)=x+(y(x)^2-y(x)^2*(diff(y(x),x))^2),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-1-4x}}{2}$$
$$y(x) = \frac{\sqrt{-1-4x}}{2}$$
$$y(x) = -\frac{\sqrt{4x^2 + (-8c_1 - 4)\,x + 4c_1^2 - 1}}{2}$$
$$y(x) = \frac{\sqrt{4x^2 + (-8c_1 - 4)\,x + 4c_1^2 - 1}}{2}$$

✓ Solution by Mathematica
Time used: 0.236 (sec). Leaf size: 69

```
DSolve[y[x]*y'[x]==x+(y[x]^2-y[x]^2*(y'[x])^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}\sqrt{4x^2 - 4(1 + 4c_1)x - 1 + 16c_1{}^2}$$
$$y(x) \to \frac{1}{2}\sqrt{4x^2 - 4(1 + 4c_1)x - 1 + 16c_1{}^2}$$

6.19 problem 19

Internal problem ID [4424]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 7
Problem number: 19.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y-\frac{1}{\sqrt{1+y'^2}}-\frac{y'}{\sqrt{1+y'^2}}=x}$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 49

```
dsolve(y(x)-1/sqrt(1+(diff(y(x),x))^2)=(x+diff(y(x),x)/sqrt(1+(diff(y(x),x))^2)),y(x), sings
```

$$y(x)=\frac{c_1\sqrt{-\frac{1}{(-c_1+x+1)(x-c_1-1)}}+1}{\sqrt{-\frac{1}{(-c_1+x+1)(x-c_1-1)}}}$$

✓ Solution by Mathematica

Time used: 42.598 (sec). Leaf size: 15753

```
DSolve[y[x]-1/Sqrt[1+(y'[x])^2]==(x+y'[x]/Sqrt[1+(y'[x])^2]),y[x],x,IncludeSingularSolutions
```

Too large to display

6.20 problem 20

Internal problem ID [4425]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 7
Problem number: 20.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$y - 2xy' - xy'^2 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
dsolve(y(x)-2*x*diff(y(x),x)=(x*(diff(y(x),x))^2),y(x), singsol=all)
```

$$y(x) = -x$$
$$y(x) = c_1 + 2\sqrt{c_1 x}$$
$$y(x) = c_1 - 2\sqrt{c_1 x}$$

✓ Solution by Mathematica

Time used: 0.116 (sec). Leaf size: 63

```
DSolve[y[x]-2*x*y'[x]==(x*(y'[x])^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{c_1} - 2e^{\frac{c_1}{2}}\sqrt{x}$$
$$y(x) \to 2e^{-\frac{c_1}{2}}\sqrt{x} + e^{-c_1}$$
$$y(x) \to 0$$
$$y(x) \to -x$$

6.21 problem 21

Internal problem ID [4426]

Book: Differential Equations, By George Boole F.R.S. 1865
Section: Chapter 7
Problem number: 21.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{\frac{y - xy'}{y^2 + y'} - \frac{y - xy'}{1 + x^2 y'} = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 19

```
dsolve((y(x)-x*diff(y(x),x))/(y(x)^2+diff(y(x),x))=(y(x)-x*diff(y(x),x))/(1+x^2*diff(y(x),x)
```

$$y(x) = c_1 x$$
$$y(x) = -\tanh\left(-\operatorname{arctanh}(x) + c_1\right)$$

✓ Solution by Mathematica

Time used: 60.122 (sec). Leaf size: 45

```
DSolve[(y[x]-x*y'[x])/(y[x]^2+y'[x])==(y[x]-x*y'[x])/(1+x^2*y'[x]),y[x],x,IncludeSingularSol
```

$$y(x) \to -\frac{x + e^{2c_1}(x-1) + 1}{-x + e^{2c_1}(x-1) - 1}$$
$$y(x) \to c_1 x$$