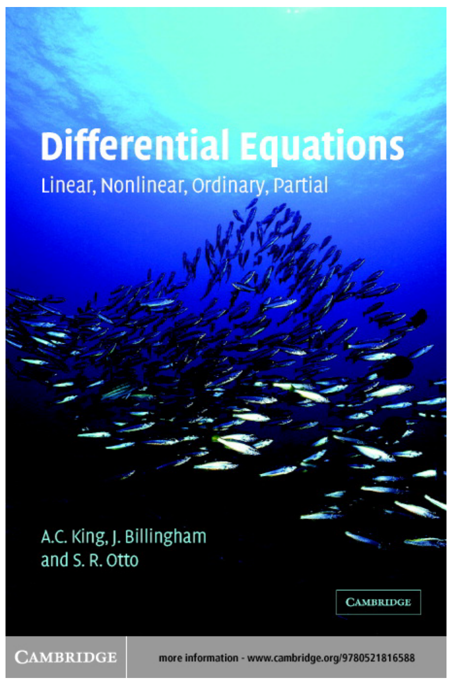
A Solution Manual For

Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003



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May 16, 2024

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1.1 problem Problem 1.1(a)

Internal problem ID [12394]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
Problem number: Problem 1.1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-1)\,y''-y'x+y=0}$$

Given that one solution of the ode is

$$y_1 = \mathrm{e}^x$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 12

```
dsolve([(x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,exp(x)],singsol=all)
```

$$y(x) = c_2\mathrm{e}^x + c_1 x$$

✓ Solution by Mathematica
Time used: 0.055 (sec). Leaf size: 17

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x - c_2 x$$

1.2 problem Problem 1.1(b)

Internal problem ID [12395]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
Problem number: Problem 1.1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + 2y' + yx = 0}$$

Given that one solution of the ode is

$$y_1 = \frac{\sin(x)}{x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve([x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,sin(x)/x],singsol=all)
```

$$y(x) = \frac{c_1 \sin(x) + c_2 \cos(x)}{x}$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 37

```
DSolve[x*y''[x]+2*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x}$$

1.3 problem Problem 1.3(a)

Internal problem ID [12396]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
Problem number: Problem 1.3(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + y = x^{\frac{3}{2}} \mathrm{e}^{x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=x^(3/2)*exp(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x} \left(c_2 + c_1 x + \frac{4x^{\frac{7}{2}}}{35}\right)$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 29

```
DSolve[y''[x]-2*y'[x]+y[x]==x^(3/2)*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{35} e^x \left(4x^{7/2} + 35c_2 x + 35c_1\right)$$

1.4 problem Problem 1.3(b)

Internal problem ID [12397]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
Problem number: Problem 1.3(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = 2\sec(2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+4*y(x)=2*sec(2*x),y(x), singsol=all)
```

$$y(x) = -\frac{\ln(\sec(2x))\cos(2x)}{2} + \cos(2x)\,c_1 + \sin(2x)\,(c_2 + x)$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 32

```
DSolve[y''[x]+4*y[x]==2*Sec[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_2)\sin(2x) + \cos(2x)\left(\frac{1}{2}\log(\cos(2x)) + c_1\right)$$

1.5 problem Problem 1.3(c)

Internal problem ID [12398]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
Problem number: Problem 1.3(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + \frac{y'}{x} + \left(1 - \frac{1}{4x^2}\right) y = x$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 62

```
dsolve(diff(y(x),x$2)+1/x*diff(y(x),x)+(1-1/(4*x^2))*y(x)=x,y(x), singsol=all)
```

$$y(x) = \frac{\sin(x) c_2 + c_1 \cos(x) + \frac{3\cos(x)\sqrt{\pi}\ \mathrm{FresnelS}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}}{4} - \frac{3\sin(x)\sqrt{\pi}\ \mathrm{FresnelC}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}}{4} + x^{\frac{3}{2}}}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.443 (sec). Leaf size: 111

```
DSolve[y''[x]+1/x*y'[x]+(1-1/(4*x^2))*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}\left(-\frac{e^{2ix}x^{3/2}\Gamma\left(\frac{5}{2}, ix\right)}{\sqrt{-ix}} + \sqrt{x^2}\left(2c_1 - ic_2 e^{2ix}\right) + \frac{(ix)^{3/2}\Gamma\left(\frac{5}{2}, -ix\right)}{\sqrt{x}}\right)}{2\sqrt{x}\sqrt{x^2}}$$

1.6 problem Problem 1.3(d)

Internal problem ID [12399]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
Problem number: Problem 1.3(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = f(x)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 34

```
dsolve([diff(y(x),x$2)+y(x)=f(x),y(0) = 0, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \left(\int_0^x \cos(\_z1)\, f(\_z1)\, d\_z1\right)\sin(x) - \left(\int_0^x \sin(\_z1)\, f(\_z1)\, d\_z1\right)\cos(x)$$

✓ Solution by Mathematica
Time used: 0.105 (sec). Leaf size: 77

```
DSolve[{y''[x]+y[x]==f[x],{y[0]==0,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \to & -\sin(x)\int_1^0 \cos(K[2])f(K[2])dK[2] + \sin(x)\int_1^x \cos(K[2])f(K[2])dK[2] \\ & + \cos(x)\left(\int_1^x -f(K[1])\sin(K[1])dK[1] - \int_1^0 -f(K[1])\sin(K[1])dK[1]\right) \end{aligned}$$

1.7 problem Problem 1.6(a)

Internal problem ID [12400]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
Problem number: Problem 1.6(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y''x^2 + x\left(x - \frac{1}{2}\right)y' + \frac{y}{2} = 0$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 39

```
dsolve(x^2*diff(y(x),x$2)+x*(x-1/2)*diff(y(x),x)+1/2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{e^{-x}\left(\operatorname{erf}\left(\sqrt{-x}\right)\sqrt{\pi}\, c_1 x + 2c_2\sqrt{x}\,\sqrt{-x}\right)}{2\sqrt{-x}}$$

✓ Solution by Mathematica
Time used: 0.084 (sec). Leaf size: 37

```
DSolve[x^2*y''[x]+x*(x-1/2)*y'[x]+1/2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_2\sqrt{x} + c_1\sqrt{-x}\Gamma\left(\frac{1}{2}, -x\right)\right)$$

1.8 problem Problem 1.6(b)

Internal problem ID [12401]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
Problem number: Problem 1.6(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x^2 + x(x+1)\, y' - y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)+x*(1+x)*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2 \mathrm{e}^{-x} + c_1(-1+x)}{x}$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 26

```
DSolve[x^2*y''[x]+x*(1+x)*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}(c_1 e^x (x-1) + c_2)}{x}$$

1.9 problem Problem 1.7

Internal problem ID [12402]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
Problem number: Problem 1.7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$\boxed{x(1-x)\,y''+(-5x+1)\,y'-4y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 59

```
Order:=6;
dsolve(x*(1-x)*diff(y(x),x$2)+(1-5*x)*diff(y(x),x)-4*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned}y(x)=&\,(c_1+c_2\ln(x))\left(1+4x+9x^2+16x^3+25x^4+36x^5+\mathrm{O}\left(x^6\right)\right)\\&+\left((-4)\,x-12x^2-24x^3-40x^4-60x^5+\mathrm{O}\left(x^6\right)\right)c_2\end{aligned}$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 87

```
AsymptoticDSolveValue[x*(1-x)*y''[x]+(1-5*x)*y'[x]-4*y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned}y(x)\to&\,c_1\left(36x^5+25x^4+16x^3+9x^2+4x+1\right)\\&+c_2\left(-60x^5-40x^4-24x^3-12x^2+\left(36x^5+25x^4+16x^3+9x^2+4x+1\right)\log(x)-4x\right)\end{aligned}$$

1.10 problem Problem 1.8(a)

Internal problem ID [12403]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
Problem number: Problem 1.8(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-1)^2 y'' + (x+1) y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve((x^2-1)^2*diff(y(x),x$2)+(x+1)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1+\frac{1}{2}x^2-\frac{1}{6}x^3+\frac{1}{6}x^4-\frac{7}{60}x^5\right)y(0) + \left(x-\frac{1}{2}x^2+\frac{1}{6}x^3-\frac{1}{6}x^4+\frac{7}{60}x^5\right)D(y)(0)+O(x^6)$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 70

```
AsymptoticDSolveValue[(x^2-1)^2*y''[x]+(x+1)*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{7x^5}{60}+\frac{x^4}{6}-\frac{x^3}{6}+\frac{x^2}{2}+1\right)+c_2\left(\frac{7x^5}{60}-\frac{x^4}{6}+\frac{x^3}{6}-\frac{x^2}{2}+x\right)$$

1.11 problem Problem 1.8(b)

Internal problem ID [12404]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
Problem number: Problem 1.8(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + 4y' - yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 32

```
Order:=6;
dsolve(x*diff(y(x),x$2)+4*diff(y(x),x)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\left(1 + \frac{1}{10}x^2 + \frac{1}{280}x^4 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(12 - 6x^2 - \frac{3}{2}x^4 + \mathrm{O}\left(x^6\right)\right)}{x^3}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 42

```
AsymptoticDSolveValue[x*y''[x]+4*y'[x]-x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{1}{x^3} - \frac{x}{8} - \frac{1}{2x}\right) + c_2\left(\frac{x^4}{280} + \frac{x^2}{10} + 1\right)$$

1.12 problem Problem 1.9

Internal problem ID [12405]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
Problem number: Problem 1.9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + (x+1)\,y' - yk = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 132

```
Order:=6;
dsolve(2*x*diff(y(x),x$2)+(1+x)*diff(y(x),x)-k*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x}\,c_1\left(1+\left(\frac{k}{3}-\frac{1}{6}\right)x+\left(\frac{1}{30}k^2-\frac{1}{15}k+\frac{1}{40}\right)x^2 + \frac{1}{5040}(2k-5)\,(2k-3)\,(-1+2k)\,x^3 + \frac{1}{362880}(2k-7)\,(2k-5)\,(2k-3)\,(-1+2k)\,x^4 + \frac{1}{39916800}(2k-9)\,(2k-7)\,(2k-5)\,(2k-3)\,(-1+2k)\,x^5+\mathrm{O}\left(x^6\right)\right)+c_2\left(1+kx + \frac{1}{6}(-1+k)\,kx^2+\frac{1}{90}(-2+k)\,(-1+k)\,kx^3+\frac{1}{2520}(k-3)\,(-2+k)\,(-1+k)\,kx^4 + \frac{1}{113400}(-4+k)\,(k-3)\,(-2+k)\,(-1+k)\,kx^5+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 304

```
AsymptoticDSolveValue[2*x*y''[x]+(1+x)*y'[x]-k*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{4\left(\frac{3}{4}-\frac{k}{2}\right)\left(\frac{5}{4}-\frac{k}{2}\right)\left(\frac{7}{4}-\frac{k}{2}\right)\left(\frac{9}{4}-\frac{k}{2}\right)\left(\frac{k}{2}-\frac{1}{4}\right)x^5}{155925} - \frac{2\left(\frac{3}{4}-\frac{k}{2}\right)\left(\frac{5}{4}-\frac{k}{2}\right)\left(\frac{7}{4}-\frac{k}{2}\right)\left(\frac{k}{2}-\frac{1}{4}\right)x^4}{2835} + \frac{4}{315}\left(\frac{3}{4}-\frac{k}{2}\right)\left(\frac{5}{4}-\frac{k}{2}\right)\left(\frac{k}{2}-\frac{1}{4}\right)x^3 - \frac{2}{15}\left(\frac{3}{4}-\frac{k}{2}\right)\left(\frac{k}{2}-\frac{1}{4}\right)x^2 + \frac{2}{3}\left(\frac{k}{2}-\frac{1}{4}\right)x + 1\right) + c_2\left(\frac{2\left(\frac{1}{2}-\frac{k}{2}\right)\left(1-\frac{k}{2}\right)\left(\frac{3}{2}-\frac{k}{2}\right)\left(2-\frac{k}{2}\right)kx^5}{14175} - \frac{1}{315}\left(\frac{1}{2}-\frac{k}{2}\right)\left(1-\frac{k}{2}\right)\left(\frac{3}{2}-\frac{k}{2}\right)kx^4 + \frac{2}{45}\left(\frac{1}{2}-\frac{k}{2}\right)\left(1-\frac{k}{2}\right)kx^3 - \frac{1}{3}\left(\frac{1}{2}-\frac{k}{2}\right)kx^2 + kx + 1\right)$$

1.13 problem Problem 1.11(a)

Internal problem ID [12406]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
Problem number: Problem 1.11(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^3y'' + x^2y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(x^3*diff(y(x),x$2)+x^2*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.057 (sec). Leaf size: 222

```
AsymptoticDSolveValue[x^3*y''[x]+x^2*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 e^{-\frac{2i}{\sqrt{x}}} \sqrt[4]{x} \left(\frac{418854310875ix^{9/2}}{8796093022208} - \frac{57972915ix^{7/2}}{4294967296} + \frac{59535ix^{5/2}}{8388608} - \frac{75ix^{3/2}}{8192} - \frac{30241281245175x^5}{281474976710656} + \frac{13043905875x^4}{549755813888} - \frac{2401245x^3}{268435456} + \frac{3675x^2}{524288} - \frac{9x}{512} + \frac{i\sqrt{x}}{16} + 1 \right) + c_2 e^{\frac{2i}{\sqrt{x}}} \sqrt[4]{x} \left(-\frac{418854310875ix^{9/2}}{8796093022208} + \frac{57972915ix^{7/2}}{4294967296} - \frac{59535ix^{5/2}}{8388608} + \frac{75ix^{3/2}}{8192} - \frac{30241281245175x^5}{281474976710656} + \right.$$

1.14 problem Problem 1.11(b)

Internal problem ID [12407]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
Problem number: Problem 1.11(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y''x^2 + y' - 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 28

```
AsymptoticDSolveValue[x^2*y''[x]+y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 e^{\frac{1}{x}} x^2 + c_1 \left(2x^2 + 2x + 1\right)$$

1.15 problem Problem 1.12

Internal problem ID [12408]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
Problem number: Problem 1.12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2y''x^2 + x(1-x)\,y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 45

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)+x*(1-x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 + \frac{1}{2}x + \frac{1}{8}x^2 + \frac{1}{48}x^3 + \frac{1}{384}x^4 + \frac{1}{3840}x^5 + \mathrm{O}\left(x^6\right)\right)}{\sqrt{x}} + c_2 x\left(1 + \frac{1}{5}x + \frac{1}{35}x^2 + \frac{1}{315}x^3 + \frac{1}{3465}x^4 + \frac{1}{45045}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 86

```
AsymptoticDSolveValue[2*x^2*y''[x]+x*(1-x)*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 x\left(\frac{x^5}{45045} + \frac{x^4}{3465} + \frac{x^3}{315} + \frac{x^2}{35} + \frac{x}{5} + 1\right) + \frac{c_2\left(\frac{x^5}{3840} + \frac{x^4}{384} + \frac{x^3}{48} + \frac{x^2}{8} + \frac{x}{2} + 1\right)}{\sqrt{x}}$$

1.16 problem Problem 1.13

Internal problem ID [12409]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
Section: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
Problem number: Problem 1.13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y''x(x-1)+3y'x+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 60

```
Order:=6;
dsolve(x*(x-1)*diff(y(x),x$2)+3*x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned}y(x) = {} & c_1x\left(1+2x+3x^2+4x^3+5x^4+6x^5+\mathrm{O}\left(x^6\right)\right)\\ & +\left(x+2x^2+3x^3+4x^4+5x^5+\mathrm{O}\left(x^6\right)\right)\ln\left(x\right)c_2\\ & +\left(1+3x+5x^2+7x^3+9x^4+11x^5+\mathrm{O}\left(x^6\right)\right)c_2\end{aligned}$$

✓ Solution by Mathematica
Time used: 0.06 (sec). Leaf size: 63

```
AsymptoticDSolveValue[x*(x-1)*y''[x]+3*x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned}y(x)\to {} & c_1\left(x^4+x^3+x^2+\left(4x^3+3x^2+2x+1\right)x\log(x)+x+1\right)\\ & +c_2\left(5x^5+4x^4+3x^3+2x^2+x\right)\end{aligned}$$

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2.1 problem Problem 3.7(a)

Internal problem ID [12410]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
Section: Chapter 3 Bessel functions. Problems page 89
Problem number: Problem 3.7(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - x^2 y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(\operatorname{BesselK}\left(\frac{1}{4}, \frac{x^2}{2}\right) c_2 + \operatorname{BesselI}\left(\frac{1}{4}, \frac{x^2}{2}\right) c_1\right) \sqrt{x}$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 37

```
DSolve[y''[x]-x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \operatorname{ParabolicCylinderD}\left(-\frac{1}{2}, i\sqrt{2}x\right) + c_1 \operatorname{ParabolicCylinderD}\left(-\frac{1}{2}, \sqrt{2}x\right)$$

2.2 problem Problem 3.7(b)

Internal problem ID [12411]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
Section: Chapter 3 Bessel functions. Problems page 89
Problem number: Problem 3.7(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x$2)+diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}\left(0, 2\sqrt{x}\right) + c_2 \operatorname{BesselY}\left(0, 2\sqrt{x}\right)$$

✓ Solution by Mathematica
Time used: 0.05 (sec). Leaf size: 31

```
DSolve[x*y''[x]+y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}\left(0, 2\sqrt{x}\right) + 2c_2 \operatorname{BesselY}\left(0, 2\sqrt{x}\right)$$

2.3 problem Problem 3.7(c)

Internal problem ID [12412]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
Section: Chapter 3 Bessel functions. Problems page 89
Problem number: Problem 3.7(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (x+1)^2 y = 0}$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x$2)+(x+1)^2*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]+(x+1)^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.4 problem Problem 3.7(d)

Internal problem ID [12413]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
Section: Chapter 3 Bessel functions. Problems page 89
Problem number: Problem 3.7(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + \alpha^2 y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+alpha^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(\alpha x) + c_2 \cos(\alpha x)$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 20

```
DSolve[y''[x]+a^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(ax) + c_2 \sin(ax)$$

2.5 problem Problem 3.7(e)

Internal problem ID [12414]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
Section: Chapter 3 Bessel functions. Problems page 89
Problem number: Problem 3.7(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - \alpha^2 y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-alpha^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\alpha x} + c_2 \mathrm{e}^{\alpha x}$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 23

```
DSolve[y''[x]-a^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{ax} + c_2 e^{-ax}$$

2.6 problem Problem 3.7(f)

Internal problem ID [12415]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
Section: Chapter 3 Bessel functions. Problems page 89
Problem number: Problem 3.7(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + \beta y' + y\gamma = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 41

```
dsolve(diff(y(x),x$2)+beta*diff(y(x),x)+gamma*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{\left(-\beta+\sqrt{\beta^2-4\gamma}\right)x}{2}} + c_2 \mathrm{e}^{-\frac{\left(\beta+\sqrt{\beta^2-4\gamma}\right)x}{2}}$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 47

```
DSolve[y''[x]+\[Beta]*y'[x]+\[Gamma]*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-\frac{1}{2}x\left(\sqrt{\beta^2-4\gamma}+\beta\right)}\left(c_2 e^{x\sqrt{\beta^2-4\gamma}} + c_1\right)$$

2.7 problem Problem 3.7(g)

Internal problem ID [12416]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
Section: Chapter 3 Bessel functions. Problems page 89
Problem number: Problem 3.7(g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-x^2+1)\, y'' - 2y'x + n(n+1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 15

```
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+n*(n+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{LegendreP}(n, x) + c_2 \operatorname{LegendreQ}(n, x)$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 18

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+n*(n+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{LegendreP}(n, x) + c_2 \operatorname{LegendreQ}(n, x)$$

2.8 problem Problem 3.12

Internal problem ID [12417]

Book: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
Section: Chapter 3 Bessel functions. Problems page 89
Problem number: Problem 3.12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y''x^2 + y'x + \left(-\nu^2 + x^2\right) y = \sin(x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 158

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-nu^2)*y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = -\frac{x^{1-\nu}2^{\nu-1}\,\mathrm{BesselJ}(\nu,x)\,\Gamma(\nu+2)\,\mathrm{hypergeom}\left(\left[\frac{1}{2}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2}\right],\left[\frac{3}{2},1-\nu,\frac{3}{2}-\nu,\frac{3}{2}-\frac{\nu}{2}\right],-x^2\right)}{\nu(\nu-1)(\nu+1)} + \mathrm{BesselJ}(\nu,x)\,c_2 + \mathrm{BesselY}(\nu,x)\,c_1 - \frac{\pi 2^{-1-\nu}x^{\nu+1}\left(\mathrm{BesselJ}(\nu,x)\cot(\pi\nu) - \mathrm{BesselY}(\nu,x)\right)\mathrm{hypergeom}\left(\left[\frac{\nu}{2}+\frac{1}{2},\frac{5}{4}+\frac{\nu}{2},\frac{3}{4}+\frac{\nu}{2}\right],\left[\frac{3}{2},\nu+1,\frac{3}{2}+\right.\right.}{\Gamma(\nu+2)}$$

✓ Solution by Mathematica
Time used: 1.228 (sec). Leaf size: 205

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-\[Nu]^2)*y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -\frac{\pi 2^{\nu-1}\csc(\pi\nu)x^{1-\nu}\,\mathrm{BesselJ}(\nu,x)\,{}_3F_4\left(\frac{1}{2}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2};\frac{3}{2},1-\nu,\frac{3}{2}-\nu,\frac{3}{2}-\frac{\nu}{2};-x^2\right)}{(\nu-1)\,\mathrm{Gamma}(1-\nu)} + \frac{\pi 2^{-\nu-1}x^{\nu+1}\left(\mathrm{BesselY}(\nu,x) - \cot(\pi\nu)\,\mathrm{BesselJ}(\nu,x)\right){}_3F_4\left(\frac{\nu}{2}+\frac{1}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2},\frac{\nu}{2}+\frac{3}{2},\nu+1,\nu+\frac{3}{2};-x\right.}{(\nu+1)\,\mathrm{Gamma}(\nu+1)} + c_1\,\mathrm{BesselJ}(\nu,x) + c_2\,\mathrm{BesselY}(\nu,x)$$