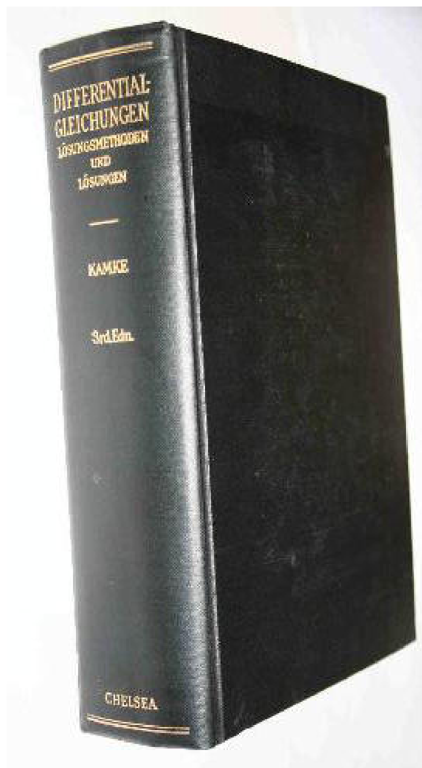
A Solution Manual For

Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948



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Contents

1 Chapter 1, linear first order

1.1 problem 1

Internal problem ID [8338]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{1}{\sqrt{\mathrm{a4}\ x^4 + \mathrm{a3}\ x^3 + \mathrm{a2}\ x^2 + \mathrm{a1}\, x + \mathrm{a0}}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x) - (a4*x^4+a3*x^3+a2*x^2+a1*x+a0)^(-1/2)=0,y(x), singsol=all)
```

$$y(x) = \int \frac{1}{\sqrt{\mathrm{a4}\ x^4 + \mathrm{a3}\ x^3 + \mathrm{a2}\ x^2 + \mathrm{a1}\, x + \mathrm{a0}}} dx + c_1$$

✓ Solution by Mathematica
Time used: 10.268 (sec). Leaf size: 1117

```
DSolve[y'[x] - (a4*x^4+a3*x^3+a2*x^2+a1*x+a0)^(-1/2)==0,y[x],x,IncludeSingularSolutions -> T
```

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1.2 problem 2

Internal problem ID [8339]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + ay = c\,\mathrm{e}^{xb}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x) + a*y(x) - c*exp(b*x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(c\,\mathrm{e}^{x(a+b)} + c_1(a+b)\right)\mathrm{e}^{-ax}}{a+b}$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 33

```
DSolve[y'[x]+ a*y[x] - c*Exp[b*x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ax}\left(ce^{x(a+b)} + c_1(a+b)\right)}{a+b}$$

1.3 problem 3

Internal problem ID [8340]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + ay = b\sin(cx)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x) + a*y(x) - b*sin(c*x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{-ax} c_1 (a^2 + c^2) + b(-c\cos(cx) + \sin(cx)\, a)}{a^2 + c^2}$$

✓ Solution by Mathematica

Time used: 0.086 (sec). Leaf size: 40

```
DSolve[y'[x] + a*y[x] - b*Sin[c*x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{b(a\sin(cx) - c\cos(cx))}{a^2 + c^2} + c_1 e^{-ax}$$

1.4 problem 4

Internal problem ID [8341]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + 2yx = x\,\mathrm{e}^{-x^2}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x) + 2*x*y(x) - x*exp(-x^2)=0,y(x), singsol=all)
```

$$y(x) = \frac{(x^2 + 2c_1)\,\mathrm{e}^{-x^2}}{2}$$

✓ Solution by Mathematica

Time used: 0.057 (sec). Leaf size: 24

```
DSolve[y'[x] + 2*x*y[x] - x*Exp[-x^2]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x^2}\left(x^2 + 2c_1\right)$$

1.5 problem 5

Internal problem ID [8342]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\cos(x) = \mathrm{e}^{2x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x) + y(x)*cos(x) - exp(2*x)=0,y(x), singsol=all)
```

$$y(x) = \left(\int \mathrm{e}^{2x+\sin(x)}dx + c_1\right)\mathrm{e}^{-\sin(x)}$$

✓ Solution by Mathematica
Time used: 0.748 (sec). Leaf size: 32

```
DSolve[y'[x] + y[x]*Cos[x] - Exp[2*x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\sin(x)}\left(\int_1^x e^{2K[1]+\sin(K[1])}dK[1] + c_1\right)$$

1.6 problem 6

Internal problem ID [8343]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\cos(x) = \frac{\sin(2x)}{2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x) + y(x)*cos(x) - sin(2*x)/2=0,y(x), singsol=all)
```

$$y(x) = \sin(x) - 1 + e^{-\sin(x)}c_1$$

✓ Solution by Mathematica
Time used: 0.053 (sec). Leaf size: 18

```
DSolve[y'[x] + y[x]*Cos[x] - Sin[2*x]/2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x) + c_1 e^{-\sin(x)} - 1$$

1.7 problem 7

Internal problem ID [8344]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\cos(x) = e^{-\sin(x)}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x) + y(x)*cos(x) - exp(-sin(x))=0,y(x), singsol=all)
```

$$y(x) = (x + c_1)\, e^{-\sin(x)}$$

✓ Solution by Mathematica

Time used: 0.128 (sec). Leaf size: 16

```
DSolve[y'[x] + y[x]*Cos[x] - Exp[-Sin[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_1)e^{-\sin(x)}$$

1.8 problem 8

Internal problem ID [8345]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' + \tan(x) y = \sin(2x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x) + y(x)*tan(x) - sin(2*x)=0,y(x), singsol=all)
```

$$y(x) = (-2\cos(x) + c_1)\cos(x)$$

✓ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 15

```
DSolve[y'[x]+ y[x]*Tan[x] - Sin[2*x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x)(-2\cos(x) + c_1)$$

1.9 problem 9

Internal problem ID [8346]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - (\sin(\ln(x)) + \cos(\ln(x)) + a)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x) - (sin(ln(x)) + cos(ln(x)) +a)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x(\sin(\ln(x))+a)}$$

✓ Solution by Mathematica
Time used: 0.059 (sec). Leaf size: 22

```
DSolve[y'[x] - (Sin[Log[x]] + Cos[Log[x]] +a)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to c_1 e^{x(a+\sin(\log(x)))}$$
$$y(x) \to 0$$

1.10 problem 10

Internal problem ID [8347]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + f'(x)\, y = f(x)\, f'(x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve(diff(y(x),x) + diff(f(x),x)*y(x) - f(x)*diff(f(x),x)=0,y(x), singsol=all)
```

$$y(x) = f(x) - 1 + e^{-f(x)} c_1$$

✓ Solution by Mathematica
Time used: 0.049 (sec). Leaf size: 18

```
DSolve[y'[x] + f'[x]*y[x] - f[x]*f'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to f(x) + c_1 e^{-f(x)} - 1$$

1.11 problem 11

Internal problem ID [8348]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + f(x)\, y = g(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)  + f(x)*y(x) - g(x)=0,y(x), singsol=all)
```

$$y(x) = \left(\int g(x)\, \mathrm{e}^{\int f(x)dx}dx + c_1\right) \mathrm{e}^{-\left(\int f(x)dx\right)}$$

✓ Solution by Mathematica
Time used: 0.054 (sec). Leaf size: 51

```
DSolve[y'[x]  + f[x]*y[x] - g[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \exp\left(\int_1^x -f(K[1])dK[1]\right) \left(\int_1^x \exp\left(-\int_1^{K[2]} -f(K[1])dK[1]\right) g(K[2])dK[2] + c_1\right)$$

1.12 problem 12

Internal problem ID [8349]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + y^2 = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x) + y(x)^2 - 1=0,y(x), singsol=all)
```

$$y(x) = \tanh(x + c_1)$$

✓ Solution by Mathematica

Time used: 0.638 (sec). Leaf size: 44

```
DSolve[y'[x] + y[x]^2 - 1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^{2x} - e^{2c_1}}{e^{2x} + e^{2c_1}}$$
$$y(x) \rightarrow -1$$
$$y(x) \rightarrow 1$$

1.13 problem 13

Internal problem ID [8350]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + y^2 = xa + b}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 79

```
dsolve(diff(y(x),x) + y(x)^2 - a*x - b=0,y(x), singsol=all)
```

$$y(x) = -\frac{i(-ia)^{\frac{1}{3}}\left(\text{AiryAi}\left(1, -\frac{ax+b}{(-ia)^{\frac{2}{3}}}\right)c_1 + \text{AiryBi}\left(1, -\frac{ax+b}{(-ia)^{\frac{2}{3}}}\right)\right)}{\text{AiryAi}\left(-\frac{ax+b}{(-ia)^{\frac{2}{3}}}\right)c_1 + \text{AiryBi}\left(-\frac{ax+b}{(-ia)^{\frac{2}{3}}}\right)}$$

✓ Solution by Mathematica
Time used: 0.185 (sec). Leaf size: 105

```
DSolve[y'[x] + y[x]^2 - a*x - b==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{a}\left(\text{AiryBiPrime}\left(\frac{b+ax}{a^{2/3}}\right) + c_1 \text{AiryAiPrime}\left(\frac{b+ax}{a^{2/3}}\right)\right)}{\text{AiryBi}\left(\frac{b+ax}{a^{2/3}}\right) + c_1 \text{AiryAi}\left(\frac{b+ax}{a^{2/3}}\right)}$$

$$y(x) \to \frac{\sqrt[3]{a}\,\text{AiryAiPrime}\left(\frac{b+ax}{a^{2/3}}\right)}{\text{AiryAi}\left(\frac{b+ax}{a^{2/3}}\right)}$$

1.14 problem 14

Internal problem ID [8351]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$y' + y^2 = -a\,x^m$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 187

```
dsolve(diff(y(x),x) + y(x)^2 + a*x^m=0,y(x), singsol=all)
```

$$y(x) = \frac{-\sqrt{a}\,x^{\frac{m}{2}+1}\,\mathrm{BesselJ}\left(\frac{m+3}{m+2}, \frac{2\sqrt{a}\,x^{\frac{m}{2}+1}}{m+2}\right) c_1 - \mathrm{BesselY}\left(\frac{m+3}{m+2}, \frac{2\sqrt{a}\,x^{\frac{m}{2}+1}}{m+2}\right)\sqrt{a}\,x^{\frac{m}{2}+1} + c_1\,\mathrm{BesselJ}\left(\frac{1}{m+2}, \frac{2\sqrt{a}\,x^{\frac{m}{2}+}}{m+2}\cdots\right.}{x\left(c_1\,\mathrm{BesselJ}\left(\frac{1}{m+2}, \frac{2\sqrt{a}\,x^{\frac{m}{2}+1}}{m+2}\right) + \mathrm{BesselY}\left(\frac{1}{m+2}, \frac{2\sqrt{a}\,x^{\frac{m}{2}+1}}{m+2}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.402 (sec). Leaf size: 639

```
DSolve[y'[x] + y[x]^2 + a*x^m==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a}x^{\frac{m}{2}+1}\operatorname{Gamma}\left(1+\frac{1}{m+2}\right)\operatorname{BesselJ}\left(\frac{1}{m+2}-1,\frac{2\sqrt{a}x^{\frac{m}{2}+1}}{m+2}\right)-\sqrt{a}x^{\frac{m}{2}+1}\operatorname{Gamma}\left(1+\frac{1}{m+2}\right)\operatorname{BesselJ}\left(1+\frac{}{m}\right.}{}$$

$$y(x) \to \frac{1}{2}\left(\frac{\sqrt{a}x^{m/2}\left(\operatorname{BesselJ}\left(-\frac{m+3}{m+2},\frac{2\sqrt{a}x^{\frac{m}{2}+1}}{m+2}\right)-\operatorname{BesselJ}\left(\frac{m+1}{m+2},\frac{2\sqrt{a}x^{\frac{m}{2}+1}}{m+2}\right)\right)}{\operatorname{BesselJ}\left(-\frac{1}{m+2},\frac{2\sqrt{a}x^{\frac{m}{2}+1}}{m+2}\right)}+\frac{1}{x}\right)$$

$$y(x) \to \frac{1}{2}\left(\frac{\sqrt{a}x^{m/2}\left(\operatorname{BesselJ}\left(-\frac{m+3}{m+2},\frac{2\sqrt{a}x^{\frac{m}{2}+1}}{m+2}\right)-\operatorname{BesselJ}\left(\frac{m+1}{m+2},\frac{2\sqrt{a}x^{\frac{m}{2}+1}}{m+2}\right)\right)}{\operatorname{BesselJ}\left(-\frac{1}{m+2},\frac{2\sqrt{a}x^{\frac{m}{2}+1}}{m+2}\right)}+\frac{1}{x}\right)$$

1.15 problem 15

Internal problem ID [8352]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Riccati]`

$$\boxed{y' + y^2 - 2x^2y = -x^4 + 2x + 1}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 35

```
dsolve(diff(y(x),x) + y(x)^2 - 2*x^2*y(x) + x^4 -2*x-1=0,y(x), singsol=all)
```

$$y(x) = \frac{x^2 e^{2x} - c_1 x^2 + e^{2x} + c_1}{e^{2x} - c_1}$$

✓ Solution by Mathematica
Time used: 0.137 (sec). Leaf size: 34

```
DSolve[y'[x] + y[x]^2 - 2*x^2*y[x] + x^4 -2*x-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 - \frac{2}{1 + 2c_1 e^{2x}} + 1$$
$$y(x) \to x^2 + 1$$

1.16 problem 16

Internal problem ID [8353]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + y^2 + (yx - 1) f(x) = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 75

```
dsolve(diff(y(x),x) + y(x)^2 +(x*y(x)-1)*f(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{e^{-\left(\int \frac{f(x)x^2+2}{x}dx\right)}x + \int e^{-\left(\int \frac{f(x)x^2+2}{x}dx\right)}dx - c_1}{\left(-c_1 + \int e^{-\left(\int \frac{f(x)x^2+2}{x}dx\right)}dx\right)x}$$

✓ Solution by Mathematica
Time used: 0.134 (sec). Leaf size: 114

```
DSolve[y'[x] + y[x]^2 +(x*y[x]-1)*f[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x \exp\left(-\int_1^x \left(f(K[1])K[1] + \frac{2}{K[1]}\right) dK[1]\right) + \int_1^x \exp\left(-\int_1^{K[2]} \left(f(K[1])K[1] + \frac{2}{K[1]}\right) dK[1]\right) dK[2] + c_1}{x\left(\int_1^x \exp\left(-\int_1^{K[2]} \left(f(K[1])K[1] + \frac{2}{K[1]}\right) dK[1]\right) dK[2] + c_1\right)}$$

$$y(x) \to \frac{1}{x}$$

1.17 problem 17

Internal problem ID [8354]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^2 - 3y = -4}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 24

```
dsolve(diff(y(x),x) - y(x)^2 -3*y(x) + 4=0,y(x), singsol=all)
```

$$y(x) = \frac{-4c_1 e^{5x} - 1}{-1 + c_1 e^{5x}}$$

✓ Solution by Mathematica
Time used: 0.49 (sec). Leaf size: 40

```
DSolve[y'[x] - y[x]^2 -3*y[x] + 4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-1 - 4e^{5(x+c_1)}}{-1 + e^{5(x+c_1)}}$$
$$y(x) \to -4$$
$$y(x) \to 1$$

1.18 problem 18

Internal problem ID [8355]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - yx = x - 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 66

```
dsolve(diff(y(x),x) - y(x)^2 - x*y(x) - x + 1=0,y(x), singsol=all)
```

$$y(x) = \frac{-i\sqrt{\pi}\,\mathrm{e}^{-2}\sqrt{2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x-2)}{2}\right) + 2\,\mathrm{e}^{\frac{x(x-4)}{2}} - 2c_1}{i\sqrt{\pi}\,\mathrm{e}^{-2}\sqrt{2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x-2)}{2}\right) + 2c_1}$$

✓ Solution by Mathematica
Time used: 0.176 (sec). Leaf size: 54

```
DSolve[y'[x]- y[x]^2 - x*y[x] - x + 1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -1 + \frac{2e^{\frac{1}{2}(x-2)^2}}{-\sqrt{2\pi}\mathrm{erfi}\left(\frac{x-2}{\sqrt{2}}\right) + 2e^2c_1}$$

$$y(x) \to -1$$

1.19 problem 19

Internal problem ID [8356]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Riccati]`

$$\boxed{y' - (x + y)^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x) - (y(x) + x)^2=0,y(x), singsol=all)
```

$$y(x) = -x - \tan(-x + c_1)$$

✓ Solution by Mathematica

Time used: 0.463 (sec). Leaf size: 14

```
DSolve[y'[x] - (y[x] + x)^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + \tan(x + c_1)$$

1.20 problem 20

Internal problem ID [8357]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 + (x^2 + 1)\, y = 2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 67

```
dsolve(diff(y(x),x) - y(x)^2 +(x^2 + 1)*y(x) - 2*x=0,y(x), singsol=all)
```

$$y(x) = \frac{-x^2\left(\int e^{\frac{x(x^2+3)}{3}}dx\right) + c_1x^2 + e^{\frac{x(x^2+3)}{3}} - \left(\int e^{\frac{x(x^2+3)}{3}}dx\right) + c_1}{c_1 - \left(\int e^{\frac{x(x^2+3)}{3}}dx\right)}$$

✓ Solution by Mathematica
Time used: 0.294 (sec). Leaf size: 58

```
DSolve[y'[x] - y[x]^2 +(x^2 + 1)*y[x] - 2*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{\frac{x^3}{3}+x}}{-\int_1^x e^{\frac{K[1]^3}{3}+K[1]}dK[1] + c_1} + x^2 + 1$$
$$y(x) \to x^2 + 1$$

1.21 problem 21

Internal problem ID [8358]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 + y\sin(x) = \cos(x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 38

```
dsolve(diff(y(x),x) - y(x)^2 +y(x)*sin(x) - cos(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sin(x)\left(\int \mathrm{e}^{-\cos(x)}dx\right) + c_1\sin(x) - \mathrm{e}^{-\cos(x)}}{c_1 + \int \mathrm{e}^{-\cos(x)}dx}$$

✓ Solution by Mathematica

Time used: 42.767 (sec). Leaf size: 158

```
DSolve[y'[x] - y[x]^2 +y[x]*Sin[x] - Cos[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1\sin(x)\int_1^x e^{-\cos(K[1])}dK[1] + \sin(x) + c_1\left(-e^{-\cos(x)}\right)}{1 + c_1\int_1^x e^{-\cos(K[1])}dK[1]}$$

$$y(x) \to \sin(x)$$

$$y(x) \to \frac{\sin^3(x)e^{\cos(x)}\int_1^{\cos(x)}\frac{e^{-K[1]}K[1]}{(1-K[1]^2)^{3/2}}dK[1]}{\sin^2(x)e^{\cos(x)}\int_1^{\cos(x)}\frac{e^{-K[1]}K[1]}{(1-K[1]^2)^{3/2}}dK[1] - \sqrt{\sin^2(x)}}$$

1.22 problem 22

Internal problem ID [8359]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - y\sin(2x) = \cos(2x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 96

```
dsolve(diff(y(x),x) - y(x)^2 -y(x)*sin(2*x) - cos(2*x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sin(x)\left(\operatorname{HeunC}\left(1, \frac{1}{2}, -\frac{1}{2}, -1, \frac{7}{8}, \frac{\cos(2x)}{2} + \frac{1}{2}\right)c_1 + 2\cos(x)\left(\cos(x)\operatorname{HeunCPrime}\left(1, \frac{1}{2}, -\frac{1}{2}, -1, \frac{7}{8}, \frac{\cos(2x)}{2}\right.\right.\right.}{c_1\cos(x)\operatorname{HeunC}\left(1, \frac{1}{2}, -\frac{1}{2}, -1, \frac{7}{8}, \frac{\cos(2x)}{2} + \frac{1}{2}\right) + \operatorname{HeunC}\left(1, -\frac{1}{2}, -\right.}$$

✓ Solution by Mathematica
Time used: 2.235 (sec). Leaf size: 111

```
DSolve[y'[x] - y[x]^2 -y[x]*Sin[2*x] - Cos[2*x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sec(x)\left(\sin(x)\int_1^{\cos(x)} \frac{e^{-K[1]^2}}{K[1]^2\sqrt{K[1]^2-1}}dK[1] + c_1\sin(x) + \frac{e^{-\cos^2(x)}\tan(x)}{\sqrt{-\sin^2(x)}}\right)}{\int_1^{\cos(x)} \frac{e^{-K[1]^2}}{K[1]^2\sqrt{K[1]^2-1}}dK[1] + c_1}$$

$$y(x) \to \tan(x)$$

1.23 problem 23

Internal problem ID [8360]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + ay^2 = b}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x) + a*y(x)^2 - b=0,y(x), singsol=all)
```

$$y(x) = \frac{\tanh\left(\sqrt{ab}\,(x + c_1)\right)\sqrt{ab}}{a}$$

✓ Solution by Mathematica
Time used: 5.188 (sec). Leaf size: 63

```
DSolve[y'[x] + a*y[x]^2 - b==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{b}\tanh\left(\sqrt{a}\sqrt{b}(x + c_1)\right)}{\sqrt{a}}$$
$$y(x) \to -\frac{\sqrt{b}}{\sqrt{a}}$$
$$y(x) \to \frac{\sqrt{b}}{\sqrt{a}}$$

1.24 problem 24

Internal problem ID [8361]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{y' + a y^2 = b\, x^\nu}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 214

```
dsolve(diff(y(x),x) + a*y(x)^2 - b*x^nu=0,y(x), singsol=all)
```

$$y(x) = \frac{-\sqrt{-ab}\, x^{\frac{\nu}{2}+1} \operatorname{BesselJ}\left(\frac{3+\nu}{\nu+2}, \frac{2\sqrt{-ab}\, x^{\frac{\nu}{2}+1}}{\nu+2}\right) c_1 - \operatorname{BesselY}\left(\frac{3+\nu}{\nu+2}, \frac{2\sqrt{-ab}\, x^{\frac{\nu}{2}+1}}{\nu+2}\right) \sqrt{-ab}\, x^{\frac{\nu}{2}+1} + c_1 \operatorname{BesselJ}\left(\frac{1}{\nu+2}, 2\right.}{xa \left(c_1 \operatorname{BesselJ}\left(\frac{1}{\nu+2}, \frac{2\sqrt{-ab}\, x^{\frac{\nu}{2}+1}}{\nu+2}\right) + \operatorname{BesselY}\left(\frac{1}{\nu+2}, \frac{2\sqrt{-ab}\, x^{\frac{\nu}{2}+1}}{\nu+2}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.453 (sec). Leaf size: 770

```
DSolve[y'[x] + a*y[x]^2 - b*x^nu == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a}\sqrt{b}(-1)^{\frac{1}{\nu+2}}x^{\frac{\nu}{2}+1}\,\text{Gamma}\left(1+\frac{1}{\nu+2}\right)\text{BesselI}\left(\frac{1}{\nu+2}-1,\frac{2\sqrt{a}\sqrt{b}x^{\frac{\nu}{2}+1}}{\nu+2}\right)+\sqrt{a}\sqrt{b}(-1)^{\frac{1}{\nu+2}}x^{\frac{\nu}{2}+1}\,\text{Gamma}\,(1+}{}$$

$$y(x) \to \frac{\frac{\sqrt{a}\sqrt{b}x^{\nu/2}\left(\text{BesselI}\left(\frac{\nu+1}{\nu+2},\frac{2\sqrt{a}\sqrt{b}x^{\frac{\nu}{2}+1}}{\nu+2}\right)+\text{BesselI}\left(-\frac{\nu+3}{\nu+2},\frac{2\sqrt{a}\sqrt{b}x^{\frac{\nu}{2}+1}}{\nu+2}\right)\right)}{\text{BesselI}\left(-\frac{1}{\nu+2},\frac{2\sqrt{a}\sqrt{b}x^{\frac{\nu}{2}+1}}{\nu+2}\right)}+\frac{1}{x}}{2a}$$

$$y(x) \to \frac{\frac{\sqrt{a}\sqrt{b}x^{\nu/2}\left(\text{BesselI}\left(\frac{\nu+1}{\nu+2},\frac{2\sqrt{a}\sqrt{b}x^{\frac{\nu}{2}+1}}{\nu+2}\right)+\text{BesselI}\left(-\frac{\nu+3}{\nu+2},\frac{2\sqrt{a}\sqrt{b}x^{\frac{\nu}{2}+1}}{\nu+2}\right)\right)}{\text{BesselI}\left(-\frac{1}{\nu+2},\frac{2\sqrt{a}\sqrt{b}x^{\frac{\nu}{2}+1}}{\nu+2}\right)}+\frac{1}{x}}{2a}$$

1.25 problem 25

Internal problem ID [8362]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + ay^2 = b\,x^{2\nu} + c\,x^{-1+\nu}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 347

```
dsolve(diff(y(x),x) + a*y(x)^2 - b*x^(2*nu) - c*x^(nu-1)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(\frac{\nu}{2}+1\right)\sqrt{b}-\frac{\sqrt{a}\,c}{2}\right)\mathrm{WhittakerM}\left(-\frac{(-2\nu-2)\sqrt{b}+\sqrt{a}\,c}{\sqrt{b}\,(2\nu+2)}, \frac{1}{2\nu+2}, \frac{2\sqrt{a}\,\sqrt{b}\,x^{\nu+1}}{\nu+1}\right) - c_1\sqrt{b}\,(\nu+1)\,\mathrm{WhittakerW}\left(-\frac{(}{}\right.}{\sqrt{b}\left(\mathrm{WhittakerW}\left(-\frac{\nu}{\sqrt{b}\,(}\right.\right.}$$

✓ Solution by Mathematica

Time used: 1.092 (sec). Leaf size: 722

```
DSolve[y'[x] + a*y[x]^2 - b*x^(2*nu) - c*x^(nu-1)==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -\frac{x^\nu \left(\sqrt{b}c_1(\nu+1)\sqrt{(\nu+1)^2}\,\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{a}c}{\sqrt{b}\sqrt{(\nu+1)^2}}+\frac{\nu}{\nu+1}\right),\frac{\nu}{\nu+1},\frac{2\sqrt{a}\sqrt{b}x^{\nu+1}}{\sqrt{(\nu+1)^2}}\right)+c_1\left(\sqrt{a}c(\nu+\right.\right.}{\sqrt{a}(\nu+1)^2\left(L\right.}$$

$$y(x) \to \frac{x^\nu\left(-\frac{\left(\sqrt{a}c(\nu+1)+\sqrt{b}\sqrt{(\nu+1)^2}\nu\right)\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{a}c}{\sqrt{b}\sqrt{(\nu+1)^2}}+\frac{\nu}{\nu+1}+2\right),\frac{\nu}{\nu+1}+1,\frac{2\sqrt{a}\sqrt{b}x^{\nu+1}}{\sqrt{(\nu+1)^2}}\right)}{\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{a}c}{\sqrt{b}\sqrt{(\nu+1)^2}}+\frac{\nu}{\nu+1}\right),\frac{\nu}{\nu+1},\frac{2\sqrt{a}\sqrt{b}x^{\nu+1}}{\sqrt{(\nu+1)^2}}\right)}-\sqrt{b}\sqrt{(\nu+1)^2}(\nu+1)\right)}{\sqrt{a}(\nu+1)^2}$$

1.26 problem 26

Internal problem ID [8363]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - (Ay - a)(By - b) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x) - (A*y(x) - a)*(B*y(x) - b)=0,y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{(x+c_1)(Ab-Ba)}a - b}{A\,\mathrm{e}^{(x+c_1)(Ab-Ba)} - B}$$

✓ Solution by Mathematica
Time used: 2.605 (sec). Leaf size: 74

```
DSolve[y'[x] - (A*y[x] - a)*(B*y[x] - b)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{ae^{Ab(x+c_1)} - be^{aB(x+c_1)}}{Ae^{Ab(x+c_1)} - Be^{aB(x+c_1)}}$$
$$y(x) \to \frac{a}{A}$$
$$y(x) \to \frac{b}{B}$$

1.27 problem 27

Internal problem ID [8364]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + ay(y - x) = 1}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 73

```
dsolve(diff(y(x),x) + a*y(x)*(y(x)-x) - 1=0,y(x), singsol=all)
```

$$y(x) = \frac{2a^{\frac{3}{2}}c_1x + \operatorname{erf}\left(\frac{\sqrt{2}\,\sqrt{a}\,x}{2}\right)\sqrt{\pi}\,\sqrt{2}\,ax + 2\sqrt{a}\,e^{-\frac{a\,x^2}{2}}}{a\left(\sqrt{\pi}\,\sqrt{2}\,\operatorname{erf}\left(\frac{\sqrt{2}\,\sqrt{a}\,x}{2}\right) + 2c_1\sqrt{a}\right)}$$

✓ Solution by Mathematica
Time used: 2.078 (sec). Leaf size: 93

```
DSolve[y'[x] + a*y[x]*(y[x]-x) - 1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{2\pi}c_1x\operatorname{erf}\left(\frac{\sqrt{a}x}{\sqrt{2}}\right) + \frac{2\left(ax+c_1e^{-\frac{ax^2}{2}}\right)}{\sqrt{a}}}{2\sqrt{a} + \sqrt{2\pi}c_1\operatorname{erf}\left(\frac{\sqrt{a}x}{\sqrt{2}}\right)}$$

$$y(x) \to x$$

1.28 problem 28

Internal problem ID [8365]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + xy^2 - yx^3 = 2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 51

```
dsolve(diff(y(x),x) + x*y(x)^2 -x^3*y(x) - 2*x=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\pi}\, \operatorname{erf}\left(\frac{x^2}{2}\right) c_1 x^2 + \sqrt{\pi}\, x^2 + 2\, \mathrm{e}^{-\frac{x^4}{4}} c_1}{\sqrt{\pi}\, \left(\operatorname{erf}\left(\frac{x^2}{2}\right) c_1 + 1\right)}$$

✓ Solution by Mathematica
Time used: 0.317 (sec). Leaf size: 70

```
DSolve[y'[x] + x*y[x]^2 -x^3*y[x] - 2*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{\pi} x^2 \text{erf}\left(\frac{x^2}{2}\right) + 2e^{-\frac{x^4}{4}} + 2c_1 x^2}{\sqrt{\pi}\text{erf}\left(\frac{x^2}{2}\right) + 2c_1}$$
$$y(x) \to x^2$$

1.29 problem 29

Internal problem ID [8366]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - xy^2 - 3yx = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(diff(y(x),x) - x*y(x)^2 - 3*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{3}{-1 + 3\,\mathrm{e}^{-\frac{3x^2}{2}} c_1}$$

✓ Solution by Mathematica

Time used: 0.215 (sec). Leaf size: 49

```
DSolve[y'[x] - x*y[x]^2 - 3*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{3e^{\frac{3x^2}{2}+3c_1}}{-1 + e^{\frac{3x^2}{2}+3c_1}}$$
$$y(x) \to -3$$
$$y(x) \to 0$$

1.30 problem 30

Internal problem ID [8367]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + x^{-a-1}y^2 = x^a}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 51

```
dsolve(diff(y(x),x) + x^(-a-1)*y(x)^2 - x^a=0,y(x), singsol=all)
```

$$y(x) = \frac{x^{\frac{1}{2}+a}\left(-\operatorname{BesselK}\left(a+1, 2\sqrt{x}\right) c_1 + \operatorname{BesselI}\left(a+1, 2\sqrt{x}\right)\right)}{\operatorname{BesselK}\left(a, 2\sqrt{x}\right) c_1 + \operatorname{BesselI}\left(a, 2\sqrt{x}\right)}$$

✓ Solution by Mathematica
Time used: 0.363 (sec). Leaf size: 265

```
DSolve[y'[x] + x^(-a-1)*y[x]^2 - x^a==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^a\left(\sqrt{x}\operatorname{Gamma}(1-a)\operatorname{BesselI}\left(-a-1, 2\sqrt{x}\right) + \sqrt{x}\operatorname{Gamma}(1-a)\operatorname{BesselI}\left(1-a, 2\sqrt{x}\right) - a\operatorname{Gamma}(1\right.}{2\left(\operatorname{Ga}\right.}$$

$$y(x) \to \frac{x^a\left(\sqrt{x}\operatorname{BesselI}\left(a-1, 2\sqrt{x}\right) - a\operatorname{BesselI}\left(a, 2\sqrt{x}\right) + \sqrt{x}\operatorname{BesselI}\left(a+1, 2\sqrt{x}\right)\right)}{2\operatorname{BesselI}\left(a, 2\sqrt{x}\right)}$$

1.31 problem 31

Internal problem ID [8368]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - a\,x^n\left(1 + y^2\right) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x) - a*x^n*(y(x)^2+1)=0,y(x), singsol=all)
```

$$y(x) = \tan\left(\frac{a(x^{1+n} + (1+n)\,c_1)}{1+n}\right)$$

✓ Solution by Mathematica
Time used: 0.365 (sec). Leaf size: 35

```
DSolve[y'[x] - a*x^n*(y[x]^2+1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan\left(\frac{ax^{n+1}}{n+1} + c_1\right)$$
$$y(x) \to -i$$
$$y(x) \to i$$

1.32 problem 32

Internal problem ID [8369]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + y^2 \sin(x) = \frac{2\sin(x)}{\cos(x)^2}}$$

✓ Solution by Maple
Time used: 0.25 (sec). Leaf size: 27

```
dsolve(diff(y(x),x) + y(x)^2*sin(x) - 2*sin(x)/cos(x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{-2\cos(x)^2 c_1 - 2\sec(x)}{\cos(x)^3 c_1 - 2}$$

✓ Solution by Mathematica
Time used: 0.926 (sec). Leaf size: 32

```
DSolve[y'[x] + y[x]^2*Sin[x] - 2*Sin[x]/Cos[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sec(x)\left(-2\cos^3(x) + c_1\right)}{\cos^3(x) + c_1}$$
$$y(x) \to \sec(x)$$

1.33 problem 33

Internal problem ID [8370]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \frac{y^2 f'(x)}{g(x)} = -\frac{g'(x)}{f(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 58

```
dsolve(diff(y(x),x) - y(x)^2*diff(f(x),x)/g(x) + diff(g(x),x)/f(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-g(x)\, f(x) \left(\int \frac{\frac{d}{dx} f(x)}{g(x) f(x)^2} dx\right) - g(x)\, f(x)\, c_1 - 1}{f(x)^2 \left(\int \frac{\frac{d}{dx} f(x)}{g(x) f(x)^2} dx + c_1\right)}$$

✓ Solution by Mathematica

Time used: 0.353 (sec). Leaf size: 160

```
DSolve[y'[x] - y[x]^2*f'[x]/g[x] + g'[x]/f[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} \left(\frac{1}{(g(x)+f(x)K[2])^2} - \int_1^x \left(\frac{2(f(K[1])K[2]^2 f'(K[1]) - g(K[1])g'(K[1]))}{g(K[1])(g(K[1])+f(K[1])K[2])^3} - \frac{2K[2]f'(K[1])}{g(K[1])(g(K[1])+f(K[1])K[2])^2}\right) dK[1]\right) dK[2] + \int_1^x -\frac{f(K[1])y(x)^2 f'(K[1]) - g(K[1])g'(K[1])}{f(K[1])g(K[1])(g(K[1])+f(K[1])y(x))^2} dK[1] = c_1, y(x)\right]$$

1.34 problem 34

Internal problem ID [8371]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + f(x)\, y^2 + g(x)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x) + f(x)*y(x)^2 + g(x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{e^{-\left(\int g(x)dx\right)}}{\int e^{-\left(\int g(x)dx\right)} f(x)\, dx + c_1}$$

✓ Solution by Mathematica

Time used: 0.178 (sec). Leaf size: 59

```
DSolve[y'[x] + f[x]*y[x]^2 + g[x]*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\exp\left(\int_1^x -g(K[1])dK[1]\right)}{-\int_1^x -\exp\left(\int_1^{K[2]} -g(K[1])dK[1]\right) f(K[2])dK[2] + c_1}$$
$$y(x) \to 0$$

1.35 problem 35

Internal problem ID [8372]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' + f(x)\left(y^2 + 2ay + b\right) = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 35

```
dsolve(diff(y(x),x) + f(x)*(y(x)^2 + 2*a*y(x) +b)=0,y(x), singsol=all)
```

$$y(x) = -a + \tanh\left(\sqrt{a^2-b}\left(\int f(x)\,dx + c_1\right)\right)\sqrt{a^2-b}$$

✓ Solution by Mathematica

Time used: 0.243 (sec). Leaf size: 89

```
DSolve[y'[x] + f[x]*(y[x]^2 + 2*a*y[x] +b)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -a + \sqrt{b-a^2}\tan\left(\sqrt{b-a^2}\left(\int_1^x -f(K[1])dK[1] + c_1\right)\right)$$
$$y(x) \to -\sqrt{a^2-b} - a$$
$$y(x) \to \sqrt{a^2-b} - a$$

1.36 problem 36

Internal problem ID [8373]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$y' + y^3 + ay^2x = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 71

```
dsolve(diff(y(x),x) + y(x)^3 + a*x*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{2a}{a^2x^2 + 2\,\mathrm{RootOf}\left(\mathrm{AiryBi}\left(\_Z\right)2^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}}c_1x + 2^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}}x\,\mathrm{AiryAi}\left(\_Z\right) + 2\,\mathrm{AiryBi}\left(1,\_Z\right)c_1 + 2\,\mathrm{Airy}}$$

✓ Solution by Mathematica

Time used: 0.253 (sec). Leaf size: 195

```
DSolve[y'[x] + y[x]^3 + a*x*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\text{AiryAiPrime}\left(\frac{\sqrt[3]{-\frac{1}{2}}\sqrt[3]{a}}{y(x)}-\frac{1}{2}\sqrt[3]{-\frac{1}{2}}a^{4/3}x^2\right)-\left(-\frac{1}{2}\right)^{2/3}a^{2/3}x\,\text{AiryAi}\left(\frac{\sqrt[3]{-\frac{1}{2}}\sqrt[3]{a}}{y(x)}-\frac{1}{2}\sqrt[3]{-\frac{1}{2}}a^{4/3}x^2\right)}{\text{AiryBiPrime}\left(\frac{\sqrt[3]{-\frac{1}{2}}\sqrt[3]{a}}{y(x)}-\frac{1}{2}\sqrt[3]{-\frac{1}{2}}a^{4/3}x^2\right)-\left(-\frac{1}{2}\right)^{2/3}a^{2/3}x\,\text{AiryBi}\left(\frac{\sqrt[3]{-\frac{1}{2}}\sqrt[3]{a}}{y(x)}-\frac{1}{2}\sqrt[3]{-\frac{1}{2}}a^{4/3}x^2\right)}\right.$$

$$\left.+c_1=0,y(x)\right]$$

1.37 problem 37

Internal problem ID [8374]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$\boxed{y' - y^3 - a\,\mathrm{e}^x y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 62

```
dsolve(diff(y(x),x) - y(x)^3 - a*exp(x)*y(x)^2=0,y(x), singsol=all)
```

$$\frac{a\ \mathrm{erf}\left(\frac{(\mathrm{e}^x a y(x)+1)\sqrt{2}}{2y(x)}\right)\sqrt{2}\,\sqrt{\pi}+2c_1 a+2\,\mathrm{e}^{-x-\frac{(\mathrm{e}^x a y(x)+1)^2}{2y(x)^2}}}{2a}=0$$

✓ Solution by Mathematica

Time used: 0.737 (sec). Leaf size: 78

```
DSolve[y'[x] - y[x]^3 - a*Exp[x]*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-iae^x=\frac{2e^{\frac{1}{2}\left(-iae^x-\frac{i}{y(x)}\right)^2}}{\sqrt{2\pi}\text{erfi}\left(\frac{-iae^x-\frac{i}{y(x)}}{\sqrt{2}}\right)+2c_1},y(x)\right]$$

1.38 problem 38

Internal problem ID [8375]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Abel]`

$$\boxed{y' - a y^3 = \frac{b}{x^{\frac{3}{2}}}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x) - a*y(x)^3 - b*x^(-3/2)=0,y(x), singsol=all)
```

$$y(x) = \frac{\operatorname{RootOf}\left(-\ln(x) + c_1 + 2\left(\int^{\_Z} \frac{1}{2a\_a^3 + \_a + 2b} d\_a\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.326 (sec). Leaf size: 320

```
DSolve[y'[x] - a*y[x]^3 - b*x^(-3/2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2}{3}ab^2\text{RootSum}\left[8\#1^9ab^2+24\#1^6ab^2+24\#1^3ab^2+\#1^3\right.\right.$$

$$+8ab^2\&,\frac{4\#1^6\log\left(y(x)\sqrt[3]{\frac{ax^{3/2}}{b}}-\#1\right)+2\#1^4\sqrt[3]{-\frac{1}{ab^2}}\log\left(y(x)\sqrt[3]{\frac{ax^{3/2}}{b}}-\#1\right)+8\#1^3\log\left(y(x)\sqrt[3]{\frac{ax}{}}\right.}{24\#1^8}$$

$$\left.+c_1,y(x)\right]$$

1.39 problem 39

Internal problem ID [8376]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \mathrm{a3}\, y^3 - \mathrm{a2}\, y^2 - \mathrm{a1}\, y = \mathrm{a0}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x) - a3*y(x)^3 - a2*y(x)^2 - a1*y(x) - a0=0,y(x), singsol=all)
```

$$x - \left(\int^{y(x)} \frac{1}{\_a^3\, \mathrm{a3} + \_a^2\, \mathrm{a2} + \_a\, \mathrm{a1} + \mathrm{a0}} d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.109 (sec). Leaf size: 54

```
DSolve[y'[x] - a3*y[x]^3 - a2*y[x]^2 - a1*y[x] - a0==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$\mathrm{Solve}\left[\mathrm{RootSum}\left[\#1^3\mathrm{a3}+\#1^2\mathrm{a2}+\#1\mathrm{a1}+\mathrm{a0}\&, \frac{\log(y(x)-\#1)}{3\#1^2\mathrm{a3}+2\#1\mathrm{a2}+\mathrm{a1}}\&\right] = x + c_1, y(x)\right]$$

1.40 problem 40

Internal problem ID [8377]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$\boxed{y' + 3ay^3 + 6ay^2x = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 57

```
dsolve(diff(y(x),x) + 3*a*y(x)^3 + 6*a*x*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{3a\,x^2 + \operatorname{RootOf}\left(3^{\frac{1}{3}}\left(-a\right)^{\frac{1}{3}} \operatorname{AiryBi}\left(\_Z\right) c_1 x + 3^{\frac{1}{3}}\left(-a\right)^{\frac{1}{3}} x \operatorname{AiryAi}\left(\_Z\right) + \operatorname{AiryBi}\left(1, \_Z\right) c_1 + \operatorname{AiryAi}\left(1, \right.\right.}$$

✓ Solution by Mathematica
Time used: 0.314 (sec). Leaf size: 185

```
DSolve[y'[x] + 3*a*y[x]^3 + 6*a*x*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\sqrt[3]{-3}\sqrt[3]{a}x \operatorname{AiryAi}\left((-3)^{2/3}a^{2/3}x^2 - \frac{(-1)^{2/3}}{\sqrt[3]{3}\sqrt[3]{a}y(x)}\right) + \operatorname{AiryAiPrime}\left((-3)^{2/3}a^{2/3}x^2 - \frac{(-1)^{2/3}}{\sqrt[3]{3}\sqrt[3]{a}y(x)}\right)}{\sqrt[3]{-3}\sqrt[3]{a}x \operatorname{AiryBi}\left((-3)^{2/3}a^{2/3}x^2 - \frac{(-1)^{2/3}}{\sqrt[3]{3}\sqrt[3]{a}y(x)}\right) + \operatorname{AiryBiPrime}\left((-3)^{2/3}a^{2/3}x^2 - \frac{(-1)^{2/3}}{\sqrt[3]{3}\sqrt[3]{a}y(x)}\right)} + c_1 = 0, y(x)\right]$$

1.41 problem 41

Internal problem ID [8378]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _Abel]`

$$\boxed{y' + axy^3 + by^2 = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 103

```
dsolve(diff(y(x),x) + a*x*y(x)^3 + b*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{e^{\operatorname{RootOf}\left(2\sqrt{b^2+4a}\,b \operatorname{arctanh}\left(\frac{2a\,e^{\_Z}+b}{\sqrt{b^2+4a}}\right)-\ln\left(x^2\left(a\,e^{2\_Z}+b\,e^{\_Z}-1\right)\right)b^2+2c_1b^2+2\_Z b^2-4\ln\left(x^2\left(a\,e^{2\_Z}+b\,e^{\_Z}-1\right)\right)a+8c_1a+8a\_Z\right)}}{x}$$

✓ Solution by Mathematica

Time used: 0.196 (sec). Leaf size: 103

```
DSolve[y'[x] + a*x*y[x]^3 + b*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\operatorname{Solve}\left[-\frac{b^2\left(\frac{2\arctan\left(\frac{-2axy(x)-b}{b\sqrt{-\frac{4a}{b^2}-1}}\right)}{\sqrt{-\frac{4a}{b^2}-1}}-\log\left(\frac{a(-x)y(x)(-axy(x)-b)-a}{a^2x^2y(x)^2}\right)\right)}{2a}=-\frac{b^2\log(x)}{a}+c_1,y(x)\right]$$

1.42 problem 42

Internal problem ID [8379]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$\boxed{y' - x(x+2)\,y^3 - (x+3)\,y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
dsolve(diff(y(x),x) - x*(x+2)*y(x)^3 - (x+3)*y(x)^2=0,y(x), singsol=all)
```

$$\frac{\frac{\sqrt{2+(x^2+2x)y(x)}}{2} + \left(\operatorname{arctanh}\left(\frac{\sqrt{y(x)}\,x}{\sqrt{2+(x^2+2x)y(x)}}\right) + c_1\right)\sqrt{y(x)}}{\sqrt{y(x)}} = 0$$

✓ Solution by Mathematica

Time used: 0.764 (sec). Leaf size: 485

```
DSolve[y'[x] - x*(x+2)*y[x]^3 - (x+3)*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[c_1=-\frac{\frac{i\sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{2y(x)}+\frac{1}{4}(x+1)^2-\frac{1}{4}}\left(\frac{\sinh\left(\sqrt{\frac{1}{2y(x)}+\frac{1}{4}(x+1)^2-\frac{1}{4}}\right)}{\sqrt{\frac{1}{2y(x)}+\frac{1}{4}(x+1)^2-\frac{1}{4}}}-\cosh\left(\sqrt{\frac{1}{2y(x)}+\frac{1}{4}(x+1)^2-\frac{1}{4}}\right)\right)}{\sqrt{-i\sqrt{\frac{1}{2y(x)}+\frac{1}{4}(x+1)^2-\frac{1}{4}}}}-\frac{i\sqrt{\frac{2}{\pi}}\left(\frac{x+1}{2}+\frac{1}{2}\right)\sinh\left(\sqrt{\frac{1}{2y(x)}+\frac{1}{4}(x+1)^2-\frac{1}{4}}\right)}{\sqrt{-i\sqrt{\frac{1}{2y(x)}+\frac{1}{4}(x+1)^2-\frac{1}{4}}}}}{\frac{i\sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{2y(x)}+\frac{1}{4}(x+1)^2-\frac{1}{4}}\left(i\sinh\left(\sqrt{\frac{1}{2y(x)}+\frac{1}{4}(x+1)^2-\frac{1}{4}}\right)-\frac{i\cosh\left(\sqrt{\frac{1}{2y(x)}+\frac{1}{4}(x+1)^2-\frac{1}{4}}\right)}{\sqrt{\frac{1}{2y(x)}+\frac{1}{4}(x+1)^2-\frac{1}{4}}}\right)}{\sqrt{-i\sqrt{\frac{1}{2y(x)}+\frac{1}{4}(x+1)^2-\frac{1}{4}}}}-\frac{\sqrt{\frac{2}{\pi}}\left(\frac{x+1}{2}+\frac{1}{2}\right)\cosh\left(\sqrt{\frac{1}{2y(x)}+\frac{1}{4}(x+1)^2-\frac{1}{4}}\right)}{\sqrt{-i\sqrt{\frac{1}{2y(x)}+\frac{1}{4}(x+1)^2-\frac{1}{4}}}}},\right.$$

1.43 problem 43

Internal problem ID [8380]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$y' + (4a^2x + 3a\,x^2 + b)\,y^3 + 3xy^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 470

```
dsolve(diff(y(x),x) + (3*a*x^2 + 4*a^2*x + b)*y(x)^3 + 3*x*y(x)^2=0,y(x), singsol=all)
```

$$\frac{a\sqrt{3}\left(\operatorname{BesselI}\left(1+\frac{\sqrt{\frac{4a^3-3b}{a^3}}}{2}, -\frac{\sqrt{3}\sqrt{\frac{(4a^2x+3a\,x^2+b)y(x)-2a}{y(x)a^3}}}{2}\right)c_1 - \operatorname{BesselK}\left(1+\frac{\sqrt{\frac{4a^3-3b}{a^3}}}{2}, -\frac{\sqrt{3}\sqrt{\frac{(4a^2x+3a\,x^2+b)y(x)-}{y(x)a^3}}}{2}\right.\right.}{\operatorname{BesselI}\left(1+\frac{\sqrt{\frac{4a^3-3b}{a^3}}}{2}, -\frac{\sqrt{3}\sqrt{\frac{(4a^2x+3a\,x^2+b)y(x)-2a}{y(x)a^3}}}{2}\right)}$$

$$= 0$$

✓ Solution by Mathematica

Time used: 4.252 (sec). Leaf size: 490

```
DSolve[y'[x] + (3*a*x^2 + 4*a^2*x + b)*y[x]^3 + 3*x*y[x]^2==0,y[x],x,IncludeSingularSolution
```

$$\text{Solve}\left[c_1 = -\frac{i\sqrt{-\frac{4a^3-3b}{4a^3}-\frac{3}{2a^2y(x)}+\frac{(-2a-3x)^2}{4a^2}}\text{BesselJ}\left(\frac{1}{2}\sqrt{\frac{4a^3-3b}{a^3}}+1,-i\sqrt{\frac{(-2a-3x)^2}{4a^2}-\frac{4a^3-3b}{4a^3}-\frac{3}{2a^2y(x)}}\right)+\left(\frac{1}{2}\sqrt{\frac{4a^3-}{a^3}}\right.}{i\sqrt{-\frac{4a^3-3b}{4a^3}-\frac{3}{2a^2y(x)}+\frac{(-2a-3x)^2}{4a^2}}\text{BesselY}\left(\frac{1}{2}\sqrt{\frac{4a^3-3b}{a^3}}+1,-i\sqrt{\frac{(-2a-3x)^2}{4a^2}-\frac{4a^3-3b}{4a^3}-\frac{3}{2a^2y(x)}}\right)+\left(\frac{1}{2}\sqrt{\frac{4a^3-}{a^3}}\right.}\right.$$

1.44 problem 44

Internal problem ID [8381]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + 2a\, x^3 y^3 + 2yx = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 53

```
dsolve(diff(y(x),x) + 2*a*x^3*y(x)^3 + 2*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{2}{\sqrt{-4a\, x^2 + 4\, \mathrm{e}^{2x^2} c_1 - 2a}}$$
$$y(x) = \frac{2}{\sqrt{-4a\, x^2 + 4\, \mathrm{e}^{2x^2} c_1 - 2a}}$$

✓ Solution by Mathematica
Time used: 7.17 (sec). Leaf size: 70

```
DSolve[y'[x] + 2*a*x^3*y[x]^3 + 2*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{-\frac{1}{2}a\,(2x^2+1) + c_1 e^{2x^2}}}$$
$$y(x) \to \frac{1}{\sqrt{-\frac{1}{2}a\,(2x^2+1) + c_1 e^{2x^2}}}$$
$$y(x) \to 0$$

1.45 problem 45

Internal problem ID [8382]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$y' + 2(a^2x^3 - x\,b^2)\,y^3 + 3by^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 113

```
dsolve(diff(y(x),x) + 2*(a^2*x^3 - b^2*x)*y(x)^3 + 3*b*y(x)^2=0,y(x), singsol=all)
```

$$c_1 + \frac{\left(\frac{a^2y(x)^2x^4 - y(x)^2b^2x^2 + 2bxy(x) - 1}{(bxy(x)-1)^2}\right)^{\frac{1}{4}} ax}{\sqrt{\frac{a\,x^2y(x)}{bxy(x)-1}}\,b\,(bxy\,(x)-1)} - \left(\int^{\frac{a\,x^2y(x)}{bxy(x)-1}} \frac{(\_a^2-1)^{\frac{1}{4}}}{\sqrt{\_a}} d\_a\right) = 0$$

✓ Solution by Mathematica

Time used: 0.442 (sec). Leaf size: 133

```
DSolve[y'[x] + 2*(a^2*x^3 - b^2*x)*y[x]^3 + 3*b*y[x]^2==0,y[x],x,IncludeSingularSolutions ->
```

$$\text{Solve}\left[c_1 = \sqrt[4]{\left(\frac{b}{ax} - \frac{1}{ax^2y(x)}\right)^2 - 1}\left(-\frac{\left(\frac{b}{ax} - \frac{1}{ax^2y(x)}\right)\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \left(\frac{b}{ax} - \frac{1}{ax^2y(x)}\right)^2\right)}{2\sqrt[4]{1 - \left(\frac{b}{ax} - \frac{1}{ax^2y(x)}\right)^2}} - \frac{ax}{b}\right), y(x)\right]$$

1.46 problem 46

Internal problem ID [8383]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$\boxed{y' - x^a y^3 + 3y^2 - x^{-a} y = x^{-2a} - a\, x^{-a-1}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 2084

```
dsolve(diff(y(x),x) - x^a*y(x)^3 + 3*y(x)^2 - x^(-a)*y(x) -x^(-2*a) + a*x^(-a-1)=0,y(x), sin
```

Expression too large to display
Expression too large to display

✓ Solution by Mathematica

Time used: 13.471 (sec). Leaf size: 231

```
DSolve[y'[x] - x^a*y[x]^3 + 3*y[x]^2 - x^(-a)*y[x] -x^(-2*a) + a*x^(-a-1)==0,y[x],x,IncludeS
```

$$y(x) \to x^{-a} - \frac{e^{\frac{2x^{1-a}}{a-1}}}{\sqrt{-\frac{2^{\frac{3a+1}{a-1}} x^{a+1} \left(\frac{x^{1-a}}{1-a}\right)^{\frac{a+1}{a-1}} \Gamma\left(\frac{a+1}{1-a}, -\frac{4x^{1-a}}{a-1}\right)}{a-1} + c_1}}$$

$$y(x) \to x^{-a} + \frac{e^{\frac{2x^{1-a}}{a-1}}}{\sqrt{-\frac{2^{\frac{3a+1}{a-1}} x^{a+1} \left(\frac{x^{1-a}}{1-a}\right)^{\frac{a+1}{a-1}} \Gamma\left(\frac{a+1}{1-a}, -\frac{4x^{1-a}}{a-1}\right)}{a-1} + c_1}}$$

$$y(x) \to x^{-a}$$

1.47 problem 47

Internal problem ID [8384]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$y' - a(x^n - x)\,y^3 - y^2 = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) - a*(x^n - x)*y(x)^3 - y(x)^2=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x] - a*(x^n - x)*y[x]^3 - y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.48 problem 48

Internal problem ID [8385]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$y' - (a\,x^n + xb)\,y^3 - cy^2 = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) - (a*x^n + b*x)*y(x)^3 - c*y(x)^2=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x] - (a*x^n + b*x)*y[x]^3 - c*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.49 problem 49

Internal problem ID [8386]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 49.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$y' + a\phi'(x)\, y^3 + 6a\phi(x)\, y^2 + \frac{(1+2a)\, y\phi''(x)}{\phi'(x)} = -2 - 2a$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) + a*diff(phi(x),x)*y(x)^3 + 6*a*phi(x)*y(x)^2 +(2*a+1)*y(x)*diff(phi(x),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x] + a*phi'[x]*y[x]^3 + 6*a*phi[x]*y[x]^2 +(2*a+1)*y[x]*phi''[x]/phi'[x] +2*(a+1)=
```

Not solved

1.50 problem 50

Internal problem ID [8387]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$y' - f_3(x)\, y^3 - f_2(x)\, y^2 - f_1(x)\, y = f_0(x)$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) - f__3(x)*y(x)^3 - f__2(x)*y(x)^2 - f__1(x)*y(x) - f__0(x)=0,y(x), sings
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x] - f3[x]*y[x]^3 - f2[x]*y[x]^2 - f1[x]*y[x] - f0[x]==0,y[x],x,IncludeSingularSol
```

Not solved

1.51 problem 51

Internal problem ID [8388]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 51.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$y' - (y - f(x))(y - g(x))\left(y - \frac{af(x) + bg(x)}{a+b}\right)h(x) - \frac{f'(x)(y - g(x))}{f(x) - g(x)} - \frac{g'(x)(y - f(x))}{g(x) - f(x)} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 649

```
dsolve(diff(y(x),x) - (y(x)-f(x))*(y(x)-g(x))*(y(x)-(a*f(x)+b*g(x))/(a+b))*h(x)- diff(f(x),x
```

$y(x)$

$$= \frac{2(f(x) - g(x))\left(a + \frac{b}{2}\right) e^{\text{RootOf}\left(-2a^3b\left(\int g(x)f(x)h(x)dx\right) - 2a^2b^2\left(\int g(x)f(x)h(x)dx\right) - 2a\,b^3\left(\int g(x)f(x)h(x)dx\right) + a^3b\left(\int f(x)^2h(x\right.\right.}}{}$$

✓ Solution by Mathematica

Time used: 1.168 (sec). Leaf size: 355

```
DSolve[y'[x] - (y[x]-f[x])*(y[x]-g[x])*(y[x]-(a*f[x]+b*g[x])/(a+b))*h[x]- f'[x]*(y[x]-g[x])/
```

$$\text{Solve}\left[-\frac{1}{3}(a\right.$$

$$-b)^{2/3}(2a+b)^{2/3}(a+2b)^{2/3}\text{RootSum}\left[\#1^3(a-b)^{2/3}(2a+b)^{2/3}(a+2b)^{2/3}-3\#1a^2-3\#1ab-3\#1b^2+(a-b)^{2/}\right.$$

1.52 problem 52

Internal problem ID [8389]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _Chini]`

$$\boxed{y' - a y^n = b\, x^{\frac{n}{-n+1}}}$$

✓ Solution by Maple
Time used: 0.141 (sec). Leaf size: 65

```
dsolve(diff(y(x),x) - a*y(x)^n - b*x^(n/(1-n))=0,y(x), singsol=all)
```

$$x^{\frac{n}{n-1}} \left(\int_{\_b}^{y(x)} \frac{1}{a\_a^n (n-1)\, x^{\frac{2n-1}{n-1}} + x^{\frac{n}{n-1}}\_a + bx(n-1)} d\_a\right) - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.32 (sec). Leaf size: 117

```
DSolve[y'[x] - a*y[x]^n - b*x^(n/(1-n))==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{\left(\frac{ax^{-\frac{n}{1-n}}}{b}\right)^{\frac{1}{n}} y(x)} \frac{1}{K[1]^n - \left(\frac{(-1)^n b^{1-n}(n-1)^{-n}}{a}\right)^{\frac{1}{n}} K[1] + 1} dK[1] = \int_1^x bK[2]^{\frac{n}{1-n}} \left(\frac{aK[2]^{-\frac{n}{1-n}}}{b}\right)^{\frac{1}{n}} dK[2 \right.$$
$$\left. + c_1, y(x)\right]$$

1.53 problem 53

Internal problem ID [8390]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 53.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Chini, [_1st_order, ‘_with_symmetry_[F(x),G(x)*y+H(x)]‘]]`

$$y' - f(x)^{-n+1}\, g'(x)\, y^n (ag(x)+b)^{-n} - \frac{f'(x)\, y}{f(x)} = g'(x)\, f(x)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 214

```
dsolve(diff(y(x),x) - f(x)^(1-n)*diff(g(x),x)*y(x)^n/(a*g(x)+b)^n - diff(f(x),x)*y(x)/f(x) -
```

$$y(x) = \frac{\operatorname{RootOf}\left(-f(x)^n \left(\left(\frac{d}{dx}g(x)\right)^3 f(x)^{-n+2} na(ag(x)+b)^{-1-n}\right)^n (ag(x)+b)^n \left(\int^{\_Z} \frac{}{\_a f(x)^n (ag(x)+b)^n \left(\left(\frac{d}{dx}g(x\right.\right.}\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 0.405 (sec). Leaf size: 96

```
DSolve[y'[x] - f[x]^(1-n)*g'[x]*y[x]^n/(a*g[x]+b)^n - f'[x]*y[x]/f[x] - f[x]*g'[x]==0,y[x],x
```

$$\text{Solve}\left[\int_1^{\left(f(x)^{-n}(b+ag(x))^{-n}\right)^{\frac{1}{n}} y(x)} \frac{1}{K[1]^n - (a^n)^{\frac{1}{n}} K[1] + 1} dK[1] = \frac{f(x)(ag(x)+b)\log(ag(x)+b)\left(f(x)^{-n}(ag\right.}{a} + c_1, y(x)\right]$$

1.54 problem 54

Internal problem ID [8391]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 54.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Chini, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y' - a^n f(x)^{-n+1} g'(x)\, y^n - \frac{f'(x)\, y}{f\,(x)} = g'(x)\, f(x)}$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 38

```
dsolve(diff(y(x),x) - a^n*f(x)^(1-n)*diff(g(x),x)*y(x)^n - diff(f(x),x)*y(x)/f(x) - f(x)*dif
```

$$\frac{ay(x) \operatorname{LerchPhi}\left(-\left(\frac{ay(x)}{f(x)}\right)^n, 1, \frac{1}{n}\right)}{nf\,(x)} - ag(x) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.336 (sec). Leaf size: 74

```
DSolve[y'[x] - a^n*f[x]^(1-n)*g'[x]*y[x]^n - f'[x]*y[x]/f[x] - f[x]*g'[x]==0,y[x],x,IncludeS
```

$$\text{Solve}\left[y(x)\left(a^n f(x)^{-n}\right)^{\frac{1}{n}} \text{Hypergeometric2F1}\left(1, \frac{1}{n}, 1 + \frac{1}{n}, -\left(\left(a^n f(x)^{-n}\right)^{\frac{1}{n}} y(x)\right)^n\right) = f(x)g(x)\left(a^n f(x)^{-n}\right)^{\frac{1}{n}} + c_1, y(x)\right]$$

1.55 problem 55

Internal problem ID [8392]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 55.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Chini]`

$$\boxed{y' - f(x)\, y^n - g(x)\, y = h(x)}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) - f(x)*y(x)^n - g(x)*y(x) - h(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x] - f[x]*y[x]^n - g[x]*y[x] - h[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.56 problem 56

Internal problem ID [8393]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 56.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$\boxed{y' - f(x)\, y^a - g(x)\, y^b = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) - f(x)*y(x)^a - g(x)*y(x)^b=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x] - f[x]*y[x]^a - g[x]*y[x]^b==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.57 problem 57

Internal problem ID [8394]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 57.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \sqrt{|y|} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x) - sqrt(abs(y(x)))=0,y(x), singsol=all)
```

$$x + 2\left(\begin{cases} \sqrt{-y(x)} & y(x) \leq 0 \\ -\sqrt{y(x)} & 0 < y(x) \end{cases}\right) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.163 (sec). Leaf size: 31

```
DSolve[y'[x] - Sqrt[Abs[y[x]]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{\sqrt{|K[1]|}} dK[1]\&\right][x + c_1]$$
$$y(x) \to 0$$

1.58 problem 58

Internal problem ID [8395]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 58.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _Chini]`

$$\boxed{y' - a\sqrt{y} = xb}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 68

```
dsolve(diff(y(x),x) - a*sqrt(y(x)) - b*x=0,y(x), singsol=all)
```

$$-\frac{\ln\left(\sqrt{y(x)}\, ax + b\, x^2 - 2y(x)\right)}{2} + \frac{a\sqrt{y(x)}\, \operatorname{arctanh}\left(\frac{a\sqrt{y(x)}+2bx}{\sqrt{y(x)(a^2+8b)}}\right)}{\sqrt{y(x)\,(a^2+8b)}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.273 (sec). Leaf size: 119

```
DSolve[y'[x] - a*Sqrt[y[x]] - b*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{a^2\left(-\frac{2a\,\text{arctanh}\left(\frac{a^2-4b\sqrt{\frac{a^2y(x)}{b^2x^2}}}{a\sqrt{a^2+8b}}\right)}{\sqrt{a^2+8b}}-\log\left(a^2\left(\sqrt{\frac{a^2y(x)}{b^2x^2}}+1\right)-\frac{2a^2y(x)}{bx^2}\right)\right)}{2b}=\frac{a^2\log(x)}{b}+c_1,y(x)\right]$$

1.59 problem 59

Internal problem ID [8396]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 59.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - a\sqrt{1 + y^2} = b}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x) - a*sqrt(y(x)^2+1) - b=0,y(x), singsol=all)
```

$$x - \left(\int^{y(x)} \frac{1}{a\sqrt{\_a^2 + 1} + b} d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.624 (sec). Leaf size: 127

```
DSolve[y'[x] - a*Sqrt[y[x]^2+1] - b==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{\frac{2b\arctan\left(\frac{\left(\sqrt{\#1^2+1}-\#1\right)a+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\log\left(\sqrt{\#1^2+1}-\#1\right)}{a}\&\right][x+c_1]$$

$$y(x) \to -\frac{\sqrt{b^2-a^2}}{a}$$

$$y(x) \to \frac{\sqrt{b^2-a^2}}{a}$$

1.60 problem 60

Internal problem ID [8397]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 60.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' - \frac{\sqrt{y^2 - 1}}{\sqrt{x^2 - 1}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x) - sqrt(y(x)^2-1)/sqrt(x^2-1)=0,y(x), singsol=all)
```

$$\ln\left(x + \sqrt{x^2 - 1}\right) - \ln\left(y(x) + \sqrt{y(x)^2 - 1}\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 4.959 (sec). Leaf size: 153

```
DSolve[y'[x] - Sqrt[y[x]^2-1]/Sqrt[x^2-1]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}e^{-c_1}\sqrt{2x^2 - 2\sqrt{x^2 - 1}x + e^{4c_1}\left(2x^2 + 2\sqrt{x^2 - 1}x - 1\right) - 1 + 2e^{2c_1}}$$
$$y(x) \to \frac{1}{2}e^{-c_1}\sqrt{2x^2 - 2\sqrt{x^2 - 1}x + e^{4c_1}\left(2x^2 + 2\sqrt{x^2 - 1}x - 1\right) - 1 + 2e^{2c_1}}$$
$$y(x) \to -1$$
$$y(x) \to 1$$

1.61 problem 61

Internal problem ID [8398]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' - \frac{\sqrt{x^2-1}}{\sqrt{y^2-1}} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 50

```
dsolve(diff(y(x),x) - sqrt(x^2-1)/sqrt(y(x)^2-1)=0,y(x), singsol=all)
```

$$c_1 + x\sqrt{x^2-1} - \ln\left(x+\sqrt{x^2-1}\right) - y(x)\sqrt{y(x)^2-1} + \ln\left(y(x)+\sqrt{y(x)^2-1}\right) = 0$$

✓ Solution by Mathematica

Time used: 0.599 (sec). Leaf size: 79

```
DSolve[y'[x] - Sqrt[x^2-1]/Sqrt[y[x]^2-1]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{1}{2}\#1\sqrt{\#1^2-1} - \operatorname{arctanh}\left(\frac{\sqrt{\#1^2-1}}{\#1-1}\right)\&\right]\left[\operatorname{arctanh}\left(\frac{\sqrt{x^2-1}}{1-x}\right) + \frac{1}{2}\sqrt{x^2-1}x + c_1\right]$$

1.62 problem 62

Internal problem ID [8399]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 62.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y' - \frac{y - x^2\sqrt{x^2 - y^2}}{xy\sqrt{x^2 - y^2} + x} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 34

```
dsolve(diff(y(x),x) - (y(x)-x^2*sqrt(x^2-y(x)^2))/(x*y(x)*sqrt(x^2-y(x)^2)+x)=0,y(x), singso
```

$$\frac{y(x)^2}{2} + \arctan\left(\frac{y(x)}{\sqrt{x^2 - y(x)^2}}\right) + \frac{x^2}{2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.836 (sec). Leaf size: 44

```
DSolve[y'[x] - (y[x]-x^2*Sqrt[x^2-y[x]^2])/(x*y[x]*Sqrt[x^2-y[x]^2]+x)==0,y[x],x,IncludeSing
```

$$\text{Solve}\left[-\arctan\left(\frac{\sqrt{x^2 - y(x)^2}}{y(x)}\right) + \frac{x^2}{2} + \frac{y(x)^2}{2} = c_1, y(x)\right]$$

1.63 problem 63

Internal problem ID [8400]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 63.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' - \frac{1+y^2}{|y+\sqrt{1+y}|\,(x+1)^{\frac{3}{2}}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x) - (1+ y(x)^2)/(abs(y(x)+sqrt(1+y(x)))*sqrt(1+x)^3)=0,y(x), singsol=all)
```

$$-\frac{2}{\sqrt{x+1}} - \left(\int^{y(x)} \frac{|\_a+\sqrt{\_a+1}|}{\_a^2+1} d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.287 (sec). Leaf size: 62

```
DSolve[y'[x] - (1+ y[x]^2)/(Abs[y[x]+Sqrt[1+y[x]]]*Sqrt[1+x]^3)==0,y[x],x,IncludeSingularSol
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\left|K[1]+\sqrt{K[1]+1}\right|}{K[1]^2+1} dK[1]\&\right]\left[-\frac{2}{\sqrt{x+1}}+c_1\right]$$

$$y(x) \to -i$$

$$y(x) \to i$$

1.64 problem 64

Internal problem ID [8401]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 64.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' - \sqrt{\frac{a y^2 + b y + c}{a\,x^2 + b x + c}} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 153

```
dsolve(diff(y(x),x) - sqrt((a*y(x)^2+b*y(x)+c)/(a*x^2+b*x+c))=0,y(x), singsol=all)
```

$$-\frac{\sqrt{a\,x^2+bx+c}\left(-\ln(2)+\ln\left(\frac{2\sqrt{a\,x^2+bx+c}\,\sqrt{a}+2ax+b}{\sqrt{a}}\right)\right)\sqrt{\frac{ay(x)^2+by(x)+c}{a\,x^2+bx+c}}-\left(c_1\sqrt{a}+\ln\left(\frac{2\sqrt{ay(x)^2+by(x)+c}\,\sqrt{a}}{\sqrt{a}}\right.\right.}{\sqrt{ay\left(x\right)^2+by\left(x\right)+c}\,\sqrt{a}}$$
$$= 0$$

✓ Solution by Mathematica

Time used: 5.542 (sec). Leaf size: 142

```
DSolve[y'[x]- Sqrt[(a*y[x]^2+b*y[x]+c)/(a*x^2+b*x+c)]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{e^{-\sqrt{a}c_1}\left(2\sqrt{a}\left(-1+e^{2\sqrt{a}c_1}\right)\sqrt{x(ax+b)+c}+b\left(-1+e^{\sqrt{a}c_1}\right)^2+2ax\left(1+e^{2\sqrt{a}c_1}\right)\right)}{4a}$$

$$y(x) \to -\frac{\sqrt{b^2-4ac}+b}{2a}$$

$$y(x) \to \frac{\sqrt{b^2-4ac}-b}{2a}$$

1.65 problem 65

Internal problem ID [8402]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 65.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' - \sqrt{\frac{y^3+1}{x^3+1}} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 47

```
dsolve(diff(y(x),x) - sqrt((y(x)^3+1)/(x^3+1))=0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{\sqrt{\_a^3+1}} d\_a - \frac{\int^x \sqrt{\frac{y(x)^3+1}{\_a^3+1}} d\_a}{\sqrt{y(x)^3+1}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 96.558 (sec). Leaf size: 337

```
DSolve[y'[x] - Sqrt[(y[x]^3+1)/(x^3+1)]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{i(\#1+1)\sqrt{1+\frac{6i}{\left(\sqrt{3}-3i\right)(\#1+1)}}\sqrt{\frac{2}{3}-\frac{4i}{\left(\sqrt{3}+3i\right)(\#1+1)}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{\#1+1}}\right),\right.}{\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{\#1^2-\#1+1}} + c_1\right]$$

$$y(x) \to -1$$
$$y(x) \to \sqrt[3]{-1}$$
$$y(x) \to -(-1)^{2/3}$$

1.66 problem 66

Internal problem ID [8403]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 66.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{\sqrt{|y(-1+y)(-1+ay)|}}{\sqrt{|x(x-1)(xa-1)|}} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 40

```
dsolve(diff(y(x),x) - sqrt(abs(y(x)*(1-y(x))*(1-a*y(x))))/sqrt(abs(x*(1-x)*(1-a*x)))=0,y(x),
```

$$\int \frac{1}{\sqrt{|x(x-1)(ax-1)|}}dx - \left(\int^{y(x)} \frac{1}{\sqrt{|\_a(\_a-1)(\_aa-1)|}}d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.415 (sec). Leaf size: 81

```
DSolve[y'[x] - Sqrt[Abs[y[x]*(1-y[x])*(1-a*y[x])]]/Sqrt[Abs[x*(1-x)*(1-a*x)]]==0,y[x],x,Incl
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{\sqrt{|(1-K[1])K[1](1-aK[1])|}}dK[1]\&\right]\left[\int_1^x \frac{1}{\sqrt{|(K[2]-1)K[2](aK[2]-1)|}}dK\right.$$

$$y(x) \to 0$$

$$y(x) \to 1$$

$$y(x) \to \frac{1}{a}$$

1.67 problem 67

Internal problem ID [8404]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 67.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' - \frac{\sqrt{1-y^4}}{\sqrt{-x^4+1}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x) - sqrt(1-y(x)^4)/sqrt(1-x^4)=0,y(x), singsol=all)
```

$$\int \frac{1}{\sqrt{-x^4+1}}dx - \left(\int^{y(x)} \frac{1}{\sqrt{-\_a^4+1}}d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 40.367 (sec). Leaf size: 38

```
DSolve[y'[x] - Sqrt[1-y[x]^4]/Sqrt[1-x^4]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \operatorname{sn}(c_1 + \operatorname{EllipticF}(\arcsin(x), -1)|-1)$$
$$y(x) \to -1$$
$$y(x) \to -i$$
$$y(x) \to i$$
$$y(x) \to 1$$

1.68 problem 68

Internal problem ID [8405]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 68.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y' - \sqrt{\frac{ay^4 + by^2 + 1}{a\,x^4 + x^2b + 1}} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 77

```
dsolve(diff(y(x),x) - sqrt((a*y(x)^4+b*y(x)^2+1)/(a*x^4+b*x^2+1))=0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{\sqrt{a\_a^4 + \_a^2 b + 1}} d\_a - \frac{\int^x \sqrt{\frac{ay(x)^4 + by(x)^2 + 1}{a\_a^4 + \_a^2 b + 1}} d\_a}{\sqrt{ay\left(x\right)^4 + by\left(x\right)^2 + 1}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 46.307 (sec). Leaf size: 505

```
DSolve[y'[x] - Sqrt[(a*y[x]^4+b*y[x]^2+1)/(a*x^4+b*x^2+1)]==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to \text{InverseFunction}\left[-\frac{i\sqrt{\frac{2\#1^2a+\sqrt{b^2-4a}+b}{\sqrt{b^2-4a}+b}}\sqrt{\frac{2\#1^2a}{b-\sqrt{b^2-4a}}+1}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{a}{b+\sqrt{b^2-4a}}}\#1\right),\frac{b+\sqrt{b^2-4a}}{b-\sqrt{b^2-4a}}\right)}{\sqrt{2}\sqrt{\frac{a}{\sqrt{b^2-4a}+b}}\sqrt{\#1^4a+\#1^2b+1}}\right.$$
$$\left.-\frac{i\sqrt{\frac{\sqrt{b^2-4a}+2ax^2+b}{\sqrt{b^2-4a}+b}}\sqrt{\frac{2ax^2}{b-\sqrt{b^2-4a}}+1}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{a}{b+\sqrt{b^2-4a}}}x\right),\frac{b+\sqrt{b^2-4a}}{b-\sqrt{b^2-4a}}\right)}{\sqrt{2}\sqrt{\frac{a}{\sqrt{b^2-4a}+b}}\sqrt{ax^4+bx^2+1}}\right]$$

$$y(x) \to -\frac{\sqrt{-\frac{\sqrt{b^2-4a}+b}{a}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{-\frac{\sqrt{b^2-4a}+b}{a}}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{\frac{\sqrt{b^2-4a}-b}{a}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{\frac{\sqrt{b^2-4a}-b}{a}}}{\sqrt{2}}$$

1.69 problem 69

Internal problem ID [8406]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' - \sqrt{(b_4y^4 + b_3y^3 + b_2y^2 + b_1y + b_0)(a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0)} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 111

```
dsolve(diff(y(x),x) - sqrt((b__4*y(x)^4+b__3*y(x)^3+b__2*y(x)^2+b__1*y(x)+b__0)*(a__4*x^4+a_
```

$$\int^{y(x)} \frac{1}{\sqrt{\_a^4b_4 + \_a^3b_3 + \_a^2b_2 + \_ab_1 + b_0}}d\_a - \frac{\int^x \sqrt{(b_4y(x)^4 + b_3y(x)^3 + b_2y(x)^2 + b_1y(x) + b_0)(\_a^4a_4 + \_a^3a_3 + \_a^2a_2 + \_aa_1 + a_0)}d\_a}{\sqrt{b_4y(x)^4 + b_3y(x)^3 + b_2y(x)^2 + b_1y(x) + b_0}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 27.368 (sec). Leaf size: 1163

```
DSolve[y'[x] - Sqrt[(b4*y[x]^4+b3*y[x]^3+b2*y[x]^2+b1*y[x]+b0)*(a4*x^4+a3*x^3+a2*x^2+a1*x+a0
```

Too large to display

1.70 problem 70

Internal problem ID [8407]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 70.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' - \sqrt{\frac{a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0}{b_4y^4 + b_3y^3 + b_2y^2 + b_1y + b_0}} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 113

```
dsolve(diff(y(x),x) - sqrt((a__4*x^4+a__3*x^3+a__2*x^2+a__1*x+a__0)/(b__4*y(x)^4+b__3*y(x)^3
```

$$\int^{y(x)} \sqrt{\_a^4b_4 + \_a^3b_3 + \_a^2b_2 + \_ab_1 + b_0}d\_a - \sqrt{b_4y(x)^4 + b_3y(x)^3 + b_2y(x)^2 + b_1y(x) + b_0}\left(\int^x \sqrt{\frac{\_a^4a_4 + \_a^3a_3 + \_a^2a_2 + \_aa_1 + a_0}{b_4y(x)^4 + b_3y(x)^3 + b_2y(x)^2 + b_1y(x) + b_0}}d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 55.285 (sec). Leaf size: 78

```
DSolve[y'[x] - Sqrt[(a4*x^4+a3*x^3+a2*x^2+a1*x+a0)/(b4*y[x]^4+b3*y[x]^3+b2*y[x]^2+b1*y[x]+b0
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \sqrt{\text{b4}K[1]^4 + \text{b3}K[1]^3 + \text{b2}K[1]^2 + \text{b1}K[1] + \text{b0}}dK[1]\&\right]\left[\int_1^x \sqrt{\text{a0} + K[2](\text{a1} + \right. \left. + c_1\right]$$

1.71 problem 71

Internal problem ID [8408]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 71.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' - \sqrt{\frac{b_4y^4 + b_3y^3 + b_2y^2 + b_1y + b_0}{a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 113

```
dsolve(diff(y(x),x) - sqrt((b__4*y(x)^4+b__3*y(x)^3+b__2*y(x)^2+b__1*y(x)+b__0)/(a__4*x^4+a_
```

$$\int^{y(x)} \frac{1}{\sqrt{\_a^4b_4 + \_a^3b_3 + \_a^2b_2 + \_ab_1 + b_0}}d\_a - \frac{\int^x \sqrt{\frac{b_4y(x)^4+b_3y(x)^3+b_2y(x)^2+b_1y(x)+b_0}{\_a^4a_4+\_a^3a_3+\_a^2a_2+\_aa_1+a_0}}d\_a}{\sqrt{b_4y\left(x\right)^4 + b_3y\left(x\right)^3 + b_2y\left(x\right)^2 + b_1y\left(x\right) + b_0}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 2.236 (sec). Leaf size: 2237

```
DSolve[y'[x] - Sqrt[(b4*y[x]^4+b3*y[x]^3+b2*y[x]^2+b1*y[x]+b0)/(a4*x^4+a3*x^3+a2*x^2+a1*x+a0
```

Too large to display

1.72 problem 72

Internal problem ID [8409]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 72.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' - \mathrm{R1}\left(x, \sqrt{a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0}\right) \mathrm{R2}\left(y, \sqrt{b_4y^4 + b_3y^3 + b_2y^2 + b_1y + b_0}\right) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 64

```
dsolve(diff(y(x),x) - R1(x,sqrt(a__4*x^4+a__3*x^3+a__2*x^2+a__1*x+a__0))*R2(y(x),sqrt(b__4*y
```

$$\int \mathrm{R1}\left(x, \sqrt{a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0}\right) dx - \left(\int^{y(x)} \frac{1}{\mathrm{R2}\left(\_a, \sqrt{\_a^4b_4 + \_a^3b_3 + \_a^2b_2 + \_ab_1 + b_0}\right)} d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.445 (sec). Leaf size: 86

```
DSolve[y'[x] - R1[x,Sqrt[a4*x^4+a3*x^3+a2*x^2+a1*x+a0]]*R2[y[x],Sqrt[b4*y[x]^4+b3*y[x]^3+b2*
```

$$y(x) \to \mathrm{InverseFunction}\left[\int_1^{\#1} \frac{1}{\mathrm{R2}\left(K[1], \sqrt{\mathrm{b4}K[1]^4 + \mathrm{b3}K[1]^3 + \mathrm{b2}K[1]^2 + \mathrm{b1}K[1] + \mathrm{b0}}\right)} dK[1]\&\right]\left[\int_1^x \mathrm{R1}\left(\right.\right. + c_1\Big]$$

1.73 problem 73

Internal problem ID [8410]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 73.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' - \left(\frac{a_3x^3 + a_2x^2 + a_1x + a_0}{a_3y^3 + a_2y^2 + a_1y + a_0}\right)^{\frac{2}{3}} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 91

```
dsolve(diff(y(x),x) - ((a__3*x^3+a__2*x^2+a__1*x+a__0)/(a__3*y(x)^3+a__2*y(x)^2+a__1*y(x)+a_
```

$$\int^{y(x)} \left(\_a^3a_3 + \_a^2a_2 + \_aa_1 + a_0\right)^{\frac{2}{3}} d\_a - \left(a_3y(x)^3 + a_2y(x)^2 + a_1y(x) + a_0\right)^{\frac{2}{3}} \left(\int^x \left(\frac{\_a^3a_3 + \_a^2a_2 + \_aa_1 + a_0}{a_3y(x)^3 + a_2y(x)^2 + a_1y(x) + a_0}\right)^{\frac{2}{3}} d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.149 (sec). Leaf size: 733

```
DSolve[y'[x] - ((a3*x^3+a2*x^2+a1*x+a0)/(a3*y[x]^3+a2*y[x]^2+a1*y[x]+a0))^(2/3)==0,y[x],x,In
```

$$\text{Solve}\left[\frac{3(\text{a0}+y(x)(\text{a1}+y(x)(\text{a2}+\text{a3}y(x))))^{2/3}\left(y(x)-\text{Root}\left[\#1^3\text{a3}+\#1^2\text{a2}+\#1\text{a1}+\text{a0}\&,1\right]\right)\text{Appell}}{5\left(\frac{y(x)-\text{Root}\left[\#1^3\text{a3}+\#1^2\text{a2}+\#\right.}{\text{Root}\left[\#1^3\text{a3}+\#1^2\text{a2}+\#1\text{a1}+\text{a0}\&,1\right]-\text{Root}\left[\#\right.}\right.}\right.$$

$$\left.+c_1,y(x)\right]$$

1.74 problem 74

Internal problem ID [8411]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 74.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - f(x)\,(y - g(x))\,\sqrt{(y-a)\,(y-b)} = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) - f(x)*(y(x)-g(x))*sqrt((y(x)-a)*(y(x)-b))=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x] - f[x]*(y[x]-g[x])*Sqrt[(y[x]-a)*(y[x]-b)]==0,y[x],x,IncludeSingularSolutions -
```

Not solved

1.75 problem 75

Internal problem ID [8412]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 75.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - e^{x-y} = -e^x}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 20

```
dsolve(diff(y(x),x) - exp(x-y(x)) + exp(x)=0,y(x), singsol=all)
```

$$y(x) = -e^x + \ln\left(-1 + e^{e^x + c_1}\right) - c_1$$

✓ Solution by Mathematica
Time used: 2.171 (sec). Leaf size: 23

```
DSolve[y'[x] - Exp[x-y[x]] + Exp[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(1 + e^{-e^x + c_1}\right)$$
$$y(x) \to 0$$

1.76 problem 76

Internal problem ID [8413]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 76.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - a\cos(y) = -b}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 41

```
dsolve(diff(y(x),x) - a*cos(y(x)) + b=0,y(x), singsol=all)
```

$$y(x) = 2\arctan\left(\frac{\tanh\left(\frac{\sqrt{a^2-b^2}\,(x+c_1)}{2}\right)\sqrt{a^2-b^2}}{a+b}\right)$$

✓ Solution by Mathematica
Time used: 60.136 (sec). Leaf size: 51

```
DSolve[y'[x] - a*Cos[y[x]] + b==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\arctan\left(\frac{(a-b)\tanh\left(\frac{1}{2}\sqrt{a^2-b^2}(x-c_1)\right)}{\sqrt{a^2-b^2}}\right)$$

1.77 problem 77

Internal problem ID [8414]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 77.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$\boxed{y' - \cos(ay + xb) = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 54

```
dsolve(diff(y(x),x) - cos(a*y(x)+b*x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-bx - 2\arctan\left(\frac{\tanh\left(\frac{\sqrt{a^2-b^2}\,(-x+c_1)}{2}\right)\sqrt{a^2-b^2}}{a-b}\right)}{a}$$

✓ Solution by Mathematica

Time used: 60.355 (sec). Leaf size: 58

```
DSolve[y'[x] - Cos[a*y[x]+b*x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-bx + 2\arctan\left(\frac{(a+b)\tanh\left(\frac{1}{2}\sqrt{a^2-b^2}(x-c_1)\right)}{\sqrt{a^2-b^2}}\right)}{a}$$

1.78 problem 78

Internal problem ID [8415]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 78.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' + a\sin(\alpha y + \beta x) = -b}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 88

```
dsolve(diff(y(x),x) + a*sin(alpha*y(x)+beta*x) + b=0,y(x), singsol=all)
```

$$y(x) = \frac{-\beta x + 2\arctan\left(\frac{\tan\left(\frac{\sqrt{(-a^2+b^2)\alpha^2-2\alpha b\beta+\beta^2}\,(-x+c_1)}{2}\right)\sqrt{(-a^2+b^2)\alpha^2-2\alpha b\beta+\beta^2}-a\alpha}{b\alpha-\beta}\right)}{\alpha}$$

✓ Solution by Mathematica
Time used: 60.949 (sec). Leaf size: 86

```
DSolve[y'[x]+ a*Sin[\[Alpha]*y[x]+\[Beta]*x] + b==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-\beta x + 2\arctan\left(\frac{-a\alpha+\sqrt{(\beta-\alpha b)^2-a^2\alpha^2}\tan\left(\frac{1}{2}(-x+c_1)\sqrt{(\beta-\alpha b)^2-a^2\alpha^2}\right)}{\alpha b-\beta}\right)}{\alpha}$$

1.79 problem 79

Internal problem ID [8416]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 79.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [`y=_G(x,y')`]

$$\boxed{y' + f(x)\cos(ay) + g(x)\sin(ay) = -h(x)}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) + f(x)*cos(a*y(x)) + g(x)*sin(a*y(x)) + h(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x] + f[x]*Cos[a*y[x]] + g[x]*Sin[a*y[x]] + h[x]==0,y[x],x,IncludeSingularSolutions
```

Not solved

1.80 problem 80

Internal problem ID [8417]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 80.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' + f(x)\sin(y) + (1 - f'(x))\cos(y) = f'(x) + 1}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 41

```
dsolve(diff(y(x),x) + f(x)*sin(y(x)) + (1-diff(f(x),x))*cos(y(x)) - diff(f(x),x) - 1=0,y(x),
```

$$y(x) = -2\arctan\left(\frac{e^{\int f(x)dx} - \left(\int e^{\int f(x)dx}dx\right) f(x) - c_1 f(x)}{c_1 + \int e^{\int f(x)dx}dx}\right)$$

✓ Solution by Mathematica
Time used: 7.14 (sec). Leaf size: 68

```
DSolve[y'[x] + f[x]*Sin[y[x]] + (1-f'[x])*Cos[y[x]] - f'[x]- 1==0,y[x],x,IncludeSingularSolu
```

$$y(x) \to 2\arctan\left(f(x) + \frac{\exp\left(-\int_1^x -f(K[1])dK[1]\right)}{\int_1^x -\exp\left(-\int_1^{K[2]} -f(K[1])dK[1]\right) dK[2] + c_1}\right)$$
$$y(x) \to 2\arctan(f(x))$$

1.81 problem 81

Internal problem ID [8418]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 81.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [`y=_G(x,y')`]

$$\boxed{y' + 2\tan(y)\tan(x) = 1}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 78

```
dsolve(diff(y(x),x) + 2*tan(y(x))*tan(x) - 1=0,y(x), singsol=all)
```

$$c_1 + \frac{\tan(x)}{\left(\frac{(1+\tan(y(x))^2)(1+\tan(x)^2)}{(\tan(y(x))\tan(x)-1)^2}\right)^{\frac{1}{4}}} + \frac{(\tan(y(x)) + \tan(x))\,\mathrm{hypergeom}\left(\left[\frac{1}{2}, \frac{5}{4}\right], \left[\frac{3}{2}\right], -\frac{(\tan(y(x))+\tan(x))^2}{(\tan(y(x))\tan(x)-1)^2}\right)}{2\tan(y(x))\tan(x) - 2} = 0$$

✓ Solution by Mathematica

Time used: 1.262 (sec). Leaf size: 220

```
DSolve[y'[x] + 2*Tan[y[x]]*Tan[x] - 1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[c_1 = \frac{\frac{1}{2}\left(\frac{1}{\frac{i\tan(x)}{\tan^2(x)+1} - \frac{i\tan^2(x)\tan(y(x))}{\tan^2(x)+1}} + i\cot(x)\right)\sqrt[4]{1 - \left(\frac{1}{\frac{i\tan(x)}{\tan^2(x)+1} - \frac{i\tan^2(x)\tan(y(x))}{\tan^2(x)+1}} + i\cot(x)\right)^2}\,\mathrm{Hyperge}}{\sqrt[4]{-1 + \left(\frac{1}{\frac{i\tan(x)}{\tan^2(x)+1} - \frac{i\tan^2(x)\tan(y(x))}{\tan^2(x)+1}}\right.}}\right.$$

1.82 problem 82

Internal problem ID [8419]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 82.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - a\left(1 + \tan\left(y\right)^2\right) + \tan\left(y\right)\tan\left(x\right) = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) - a*(1+tan(y(x))^2) + tan(y(x))*tan(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]- a*(1+Tan[y[x]]^2) + Tan[y[x]]*Tan[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

Not solved

1.83 problem 83

Internal problem ID [8420]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 83.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - \tan(yx) = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 44

```
dsolve(diff(y(x),x) - tan(x*y(x))=0,y(x), singsol=all)
```

$$y(x) = -i\,\mathrm{RootOf}\left(-\operatorname{erf}\left(\frac{(-x+\_Z)\sqrt{2}}{2}\right)\sqrt{\pi} - \sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{2}\,(x+\_Z)}{2}\right) + c_1\sqrt{2}\right)$$

✓ Solution by Mathematica

Time used: 0.283 (sec). Leaf size: 69

```
DSolve[y'[x] - Tan[x*y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\frac{1}{2}\sqrt{\frac{\pi}{2}}e^{\frac{x^2}{2}}\left(\operatorname{erfi}\left(\frac{y(x)-ix}{\sqrt{2}}\right)+\operatorname{erfi}\left(\frac{y(x)+ix}{\sqrt{2}}\right)\right) = c_1e^{\frac{x^2}{2}}, y(x)\right]$$

1.84 problem 84

Internal problem ID [8421]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 84.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - f(xa + by) = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 37

```
dsolve(diff(y(x),x) - f(a*x + b*y(x))=0,y(x), singsol=all)
```

$$y(x) = \frac{\operatorname{RootOf}\left(\left(\int^{\_Z} \frac{1}{f(\_a b) b + a} d\_a\right) b - x + c_1\right) b - ax}{b}$$

✓ Solution by Mathematica
Time used: 0.211 (sec). Leaf size: 248

```
DSolve[y'[x] - f[a*x + b*y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} -\frac{f(ax + bK[2]) \int_1^x \left(\frac{b^2 f'(aK[1]+bK[2])}{a+bf(aK[1]+bK[2])} - \frac{b^3 f(aK[1]+bK[2]) f'(aK[1]+bK[2])}{(a+bf(aK[1]+bK[2]))^2}\right) dK[1] b + b + a \int_1^x \left(\frac{b^2 f'(aK[1]+bK[2])}{a+bf(aK[1]+bK[2])} - \right.}{a + bf(ax + bK[2])}\right.$$
$$\left. + \int_1^x \frac{bf(aK[1] + by(x))}{a + bf(aK[1] + by(x))} dK[1] = c_1, y(x)\right]$$

1.85 problem 85

Internal problem ID [8422]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 85.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, ‘_with_symmetry_[F(x),G(y)]‘]]`

$$y' - x^{a-1}y^{1-b}f\left(\frac{x^a}{a} + \frac{y^b}{b}\right) = 0$$

✓ Solution by Maple

Time used: 0.235 (sec). Leaf size: 152

```
dsolve(diff(y(x),x) - x^(a-1)*y(x)^(1-b)*f(x^a/a + y(x)^b/b)=0,y(x), singsol=all)
```

$$y(x) = \left(\frac{\mathrm{RootOf}\left(\left(\left(\int^{\_Z} \frac{1}{-f\left(\frac{\left(a^{\frac{1}{a}}\right)^a b+\left((\_a-b)^{\frac{1}{b}}\right)^b a}{ab}\right)\left(a^{\frac{1}{a}}\right)^a\left((\_a-b)^{\frac{1}{b}}\right)^{-b} b+f\left(\frac{\left(a^{\frac{1}{a}}\right)^a b+\left((\_a-b)^{\frac{1}{b}}\right)^b a}{ab}\right)\left(a^{\frac{1}{a}}\right)^a\left((\_a-b)^{\frac{1}{b}}\right)^{-b}\_ \cdots}\right.\right.\right.}{a}\right.$$

✓ Solution by Mathematica

Time used: 0.322 (sec). Leaf size: 238

```
DSolve[y'[x] - x^(a-1)*y[x]^(1-b)*f[x^a/a + y[x]^b/b]==0,y[x],x,IncludeSingularSolutions ->
```

$$\text{Solve}\left[\int_1^{y(x)}\left(-\frac{K[2]^{b-1}}{f\left(\frac{x^a}{a}+\frac{K[2]^b}{b}\right)+1}\right.\right.$$
$$\left.-\int_1^x\left(\frac{K[1]^{a-1}K[2]^{b-1}f'\left(\frac{K[1]^a}{a}+\frac{K[2]^b}{b}\right)}{f\left(\frac{K[1]^a}{a}+\frac{K[2]^b}{b}\right)+1}-\frac{f\left(\frac{K[1]^a}{a}+\frac{K[2]^b}{b}\right)K[1]^{a-1}K[2]^{b-1}f'\left(\frac{K[1]^a}{a}+\frac{K[2]^b}{b}\right)}{\left(f\left(\frac{K[1]^a}{a}+\frac{K[2]^b}{b}\right)+1\right)^2}\right)dK[1]\right)d$$
$$\left.+\int_1^x\frac{f\left(\frac{K[1]^a}{a}+\frac{y(x)^b}{b}\right)K[1]^{a-1}}{f\left(\frac{K[1]^a}{a}+\frac{y(x)^b}{b}\right)+1}dK[1]=c_1,y(x)\right]$$

1.86 problem 86

Internal problem ID [8423]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 86.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y' - \frac{y - x f(x^2 + a y^2)}{x + a y f(x^2 + a y^2)} = 0$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 52

```
dsolve(diff(y(x),x) - (y(x)-x*f(x^2+a*y(x)^2))/(x+a*y(x)*f(x^2+a*y(x)^2))=0,y(x), singsol=al
```

$$\frac{\arctan\left(\frac{\sqrt{a}\,x}{\sqrt{a^2 y(x)^2}}\right)}{\sqrt{a}} - \frac{\left(\int^{y(x)^2+\frac{x^2}{a}} \frac{f(\_a a)}{\_a} d\_a\right)}{2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.476 (sec). Leaf size: 184

```
DSolve[y'[x]- (y[x]-x*f[x^2+a*y[x]^2])/(x+a*y[x]*f[x^2+a*y[x]^2])==0,y[x],x,IncludeSingularS
```

$$\text{Solve}\Bigg[\int_1^{y(x)} \Bigg(\frac{-f(x^2 + aK[2]^2)\,K[2]a^2 - xa}{x^2 + aK[2]^2} - \int_1^x \left(\frac{a - 2a^2K[1]K[2]f'(K[1]^2 + aK[2]^2)}{K[1]^2 + aK[2]^2} - \frac{2aK[2]\,(aK[2] - af(K[1]^2 + aK[2]^2)\,K[1])}{(K[1]^2 + aK[2]^2)^2}\right) dK[1]\Bigg) dK[2] + \int_1^x \frac{ay(x) - af(K[1]^2 + ay(x)^2)\,K[1]}{K[1]^2 + ay(x)^2} dK[1] = c_1, y(x)\Bigg]$$

1.87 problem 87

Internal problem ID [8424]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 87.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y' - \frac{ya f(x^c y) + c x^a y^b}{xb f(x^c y) - x^a y^b} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) - (y(x)*a*f(x^c*y(x))+c*x^a*y(x)^b)/(x*b*f(x^c*y(x))-x^a*y(x)^b)=0,y(x),
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x] - (y[x]*a*f[x^c*y[x]]+c*x^a*y[x]^b)/(x*b*f[x^c*y[x]]-x^a*y[x]^b)==0,y[x],x,Incl
```

Not solved

1.88 problem 88

Internal problem ID [8425]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 88.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$2y' - 3y^2 - 4ay = b + c\,\mathrm{e}^{-2xa}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 256

```
dsolve(2*diff(y(x),x) - 3*y(x)^2 - 4*a*y(x) - b - c*exp(-2*a*x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-\sqrt{3}\left(\mathrm{BesselY}\left(-\frac{\sqrt{4a^2-3b}-2a}{2a}, \frac{\sqrt{3}\sqrt{c}\,\mathrm{e}^{-ax}}{2a}\right)c_1 + \mathrm{BesselJ}\left(-\frac{\sqrt{4a^2-3b}-2a}{2a}, \frac{\sqrt{3}\sqrt{c}\,\mathrm{e}^{-ax}}{2a}\right)\right)\mathrm{e}^{-ax}\sqrt{c} - \left(\sqrt{4a^2-3b}\right.}{3\,\mathrm{BesselY}\left(-\frac{\sqrt{4a^2-3b}}{2a}, \frac{\sqrt{3}\sqrt{c}\,\mathrm{e}^{-ax}}{2a}\right)c_1 + 3\,\mathrm{BesselJ}\left(\right.}$$

✓ Solution by Mathematica
Time used: 2.009 (sec). Leaf size: 2746

```
DSolve[2*y'[x] - 3*y[x]^2 - 4*a*y[x] - b - c*Exp[-2*a*x]==0,y[x],x,IncludeSingularSolutions
```

Too large to display

1.89 problem 89

Internal problem ID [8426]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 89.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xy' = \sqrt{a^2 - x^2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 51

```
dsolve(x*diff(y(x),x) - sqrt(a^2 - x^2)=0,y(x), singsol=all)
```

$$y(x) = -a\ \mathrm{csgn}\,(a)\ln{(2)} - a\ \mathrm{csgn}\,(a)\ln\left(\frac{a\left(\mathrm{csgn}\,(a)\sqrt{a^2-x^2}+a\right)}{x}\right) + \sqrt{a^2-x^2} + c_1$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 42

```
DSolve[x*y'[x] - Sqrt[a^2 - x^2]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -a\,\mathrm{arctanh}\left(\frac{\sqrt{a^2-x^2}}{a}\right) + \sqrt{a^2-x^2} + c_1$$

1.90 problem 90

Internal problem ID [8427]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' + y = x\sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x) + y(x) - x*sin(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-\cos(x)\,x + \sin(x) + c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 19

```
DSolve[x*y'[x]+ y[x] - x*Sin[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sin(x) - x\cos(x) + c_1}{x}$$

1.91 problem 91

Internal problem ID [8428]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 91.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' - y = \frac{x}{\ln (x)}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x*diff(y(x),x) - y(x) - x/ln(x)=0,y(x), singsol=all)
```

$$y(x) = (\ln (\ln (x)) + c_1) x$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 13

```
DSolve[x*y'[x] - y[x] - x/Log[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(\log(\log(x)) + c_1)$$

1.92 problem 92

Internal problem ID [8429]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 92.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' - y = \sin(x)\, x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x) - y(x) - x^2*sin(x)=0,y(x), singsol=all)
```

$$y(x) = (-\cos(x) + c_1)\, x$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 14

```
DSolve[x*y'[x] - y[x] - x^2*Sin[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(-\cos(x) + c_1)$$

1.93 problem 93

Internal problem ID [8430]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 93.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' - y = \frac{x \cos (\ln (\ln (x)))}{\ln (x)}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x) - y(x) - x*cos(ln(ln(x)))/ln(x)=0,y(x), singsol=all)
```

$$y(x) = (\sin (\ln (\ln (x))) + c_1) x$$

✓ Solution by Mathematica
Time used: 0.074 (sec). Leaf size: 14

```
DSolve[x*y'[x] - y[x] - x*Cos[Log[Log[x]]]/Log[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to x(\sin(\log(\log(x))) + c_1)$$

1.94 problem 94

Internal problem ID [8431]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 94.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' + ay = -b\,x^n}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x) +a*y(x) + b*x^n=0,y(x), singsol=all)
```

$$y(x) = -\frac{b\,x^n}{a+n} + x^{-a}c_1$$

✓ Solution by Mathematica
Time used: 0.061 (sec). Leaf size: 25

```
DSolve[x*y'[x] +a*y[x] + b*x^n==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{bx^n}{a+n} + c_1x^{-a}$$

1.95 problem 95

Internal problem ID [8432]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 95.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{xy' + y^2 = -x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x) + y(x)^2 + x^2=0,y(x), singsol=all)
```

$$y(x) = -\frac{(c_1 \operatorname{BesselY}(1, x) + \operatorname{BesselJ}(1, x))\, x}{c_1 \operatorname{BesselY}(0, x) + \operatorname{BesselJ}(0, x)}$$

✓ Solution by Mathematica
Time used: 0.152 (sec). Leaf size: 45

```
DSolve[x*y'[x] + y[x]^2 + x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x(\operatorname{BesselY}(1, x) + c_1 \operatorname{BesselJ}(1, x))}{\operatorname{BesselY}(0, x) + c_1 \operatorname{BesselJ}(0, x)}$$
$$y(x) \to -\frac{x \operatorname{BesselJ}(1, x)}{\operatorname{BesselJ}(0, x)}$$

1.96 problem 96

Internal problem ID [8433]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 96.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy' - y^2 = -1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x*diff(y(x),x) - y(x)^2 + 1=0,y(x), singsol=all)
```

$$y(x) = -\tanh(\ln(x) + c_1)$$

✓ Solution by Mathematica
Time used: 0.492 (sec). Leaf size: 43

```
DSolve[x*y'[x] - y[x]^2 + 1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1 - e^{2c_1}x^2}{1 + e^{2c_1}x^2}$$
$$y(x) \to -1$$
$$y(x) \to 1$$

1.97 problem 97

Internal problem ID [8434]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Riccati]`

$$\boxed{xy' + ay^2 - y = -x^2 b}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 25

```
dsolve(x*diff(y(x),x) + a*y(x)^2 - y(x) + b*x^2=0,y(x), singsol=all)
```

$$y(x) = -\frac{\tan\left(\sqrt{ab}\,(x + c_1)\right) x\sqrt{ab}}{a}$$

✓ Solution by Mathematica
Time used: 16.893 (sec). Leaf size: 36

```
DSolve[x*y'[x] + a*y[x]^2 - y[x] + b*x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{b}x\tan\left(\sqrt{a}\sqrt{b}(x - c_1)\right)}{\sqrt{a}}$$

1.98 problem 98

Internal problem ID [8435]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 98.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{xy' + ay^2 - by = -c\,x^{2b}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
dsolve(x*diff(y(x),x) + a*y(x)^2 - b*y(x) + c*x^(2*b)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\tan\left(\frac{x^b\sqrt{a}\,\sqrt{c}+c_1 b}{b}\right)\sqrt{c}\,x^b}{\sqrt{a}}$$

✓ Solution by Mathematica

Time used: 0.356 (sec). Leaf size: 211

```
DSolve[x*y'[x] + a*y[x]^2 - b*y[x] + c*x^(2*b)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{-c}x^b\left(-\cos\left(\frac{\sqrt{-a}\sqrt{-c}x^b}{b}\right)+c_1\sin\left(\frac{\sqrt{-a}\sqrt{-c}x^b}{b}\right)\right)}{\sqrt{-a}\left(\sin\left(\frac{\sqrt{-a}\sqrt{-c}x^b}{b}\right)+c_1\cos\left(\frac{\sqrt{-a}\sqrt{-c}x^b}{b}\right)\right)}$$

$$y(x) \to \frac{\sqrt{-c}x^b\tan\left(\frac{\sqrt{-a}\sqrt{-c}x^b}{b}\right)}{\sqrt{-a}}$$

$$y(x) \to \frac{\sqrt{-c}x^b\tan\left(\frac{\sqrt{-a}\sqrt{-c}x^b}{b}\right)}{\sqrt{-a}}$$

1.99 problem 99

Internal problem ID [8436]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 99.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{xy' + ay^2 - by = c\,x^\beta}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 171

```
dsolve(x*diff(y(x),x) + a*y(x)^2 - b*y(x) - c*x^beta=0,y(x), singsol=all)
```

$$y(x) = \frac{-\sqrt{-ac}\left(\text{BesselY}\left(\frac{b}{\beta}+1, \frac{2\sqrt{-ac}\,x^{\frac{\beta}{2}}}{\beta}\right)c_1 + \text{BesselJ}\left(\frac{b}{\beta}+1, \frac{2\sqrt{-ac}\,x^{\frac{\beta}{2}}}{\beta}\right)\right)x^{\frac{\beta}{2}} + b\left(\text{BesselY}\left(\frac{b}{\beta}, \frac{2\sqrt{-ac}\,x^{\frac{\beta}{2}}}{\beta}\right)\right.}{a\left(\text{BesselY}\left(\frac{b}{\beta}, \frac{2\sqrt{-ac}\,x^{\frac{\beta}{2}}}{\beta}\right)c_1 + \text{BesselJ}\left(\frac{b}{\beta}, \frac{2\sqrt{-ac}\,x^{\frac{\beta}{2}}}{\beta}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.322 (sec). Leaf size: 428

```
DSolve[x*y'[x] + a*y[x]^2 - b*y[x] - c*x^\[Beta]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-a}\sqrt{c}x^{\beta/2}\left(-2\,\text{BesselJ}\left(\frac{b}{\beta}-1, \frac{2\sqrt{-a}\sqrt{c}x^{\beta/2}}{\beta}\right) + c_1\left(\text{BesselJ}\left(1-\frac{b}{\beta}, \frac{2\sqrt{-a}\sqrt{c}x^{\beta/2}}{\beta}\right) - \text{BesselJ}\left(-\frac{b+\beta}{\beta}, \frac{2\sqrt{}}{}\right.\right.\right.}{2a\left(\text{BesselJ}\left(\frac{b}{\beta}, \frac{2\sqrt{-a}\sqrt{c}x^{\beta/2}}{\beta}\right) + c_1\,\text{BesselJ}\left(-\frac{b}{\beta}, \frac{2\sqrt{-a}\sqrt{c}x^{\beta/2}}{\beta}\right)\right.}$$

$$y(x) \to \frac{-\sqrt{-a}\sqrt{c}x^{\beta/2}\,\text{BesselJ}\left(1-\frac{b}{\beta}, \frac{2\sqrt{-a}\sqrt{c}x^{\beta/2}}{\beta}\right) + \sqrt{-a}\sqrt{c}x^{\beta/2}\,\text{BesselJ}\left(-\frac{b+\beta}{\beta}, \frac{2\sqrt{-a}\sqrt{c}x^{\beta/2}}{\beta}\right) + b\,\text{BesselJ}\left(-\frac{b}{\beta}\right.}{2a\,\text{BesselJ}\left(-\frac{b}{\beta}, \frac{2\sqrt{-a}\sqrt{c}x^{\beta/2}}{\beta}\right)}$$

1.100 problem 100

Internal problem ID [8437]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 100.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Riccati, _special]]`

$$\boxed{xy' + xy^2 = -a}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(x*diff(y(x),x) + x*y(x)^2 + a=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{a}\left(\operatorname{BesselJ}\left(0, 2\sqrt{a}\,\sqrt{x}\right) c_1 + \operatorname{BesselY}\left(0, 2\sqrt{a}\,\sqrt{x}\right)\right)}{\sqrt{x}\,\left(c_1 \operatorname{BesselJ}\left(1, 2\sqrt{a}\,\sqrt{x}\right) + \operatorname{BesselY}\left(1, 2\sqrt{a}\,\sqrt{x}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.224 (sec). Leaf size: 289

```
DSolve[x*y'[x] + x*y[x]^2 + a==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2\sqrt{a}\sqrt{x}\operatorname{BesselY}\left(0, 2\sqrt{a}\sqrt{x}\right) + 2\operatorname{BesselY}\left(1, 2\sqrt{a}\sqrt{x}\right) - 2\sqrt{a}\sqrt{x}\operatorname{BesselY}\left(2, 2\sqrt{a}\sqrt{x}\right) - i\sqrt{a}c_1\sqrt{x}\operatorname{Bess}}{4x\operatorname{BesselY}\left(1, 2\sqrt{a}\sqrt{x}\right) - 2ic_1 x\operatorname{BesselJ}}$$

$$y(x) \to \frac{\sqrt{a}\sqrt{x}\operatorname{BesselJ}\left(0, 2\sqrt{a}\sqrt{x}\right) + \operatorname{BesselJ}\left(1, 2\sqrt{a}\sqrt{x}\right) - \sqrt{a}\sqrt{x}\operatorname{BesselJ}\left(2, 2\sqrt{a}\sqrt{x}\right)}{2x\operatorname{BesselJ}\left(1, 2\sqrt{a}\sqrt{x}\right)}$$

1.101 problem 101

Internal problem ID [8438]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 101.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{xy' + xy^2 - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x) + x*y(x)^2 - y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{2x}{x^2 + 2c_1}$$

✓ Solution by Mathematica
Time used: 0.14 (sec). Leaf size: 23

```
DSolve[x*y'[x] + x*y[x]^2 - y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x}{x^2 + 2c_1}$$
$$y(x) \to 0$$

1.102 problem 102

Internal problem ID [8439]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 102.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Riccati]`

$$\boxed{xy' + xy^2 - y = x^3a}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve(x*diff(y(x),x) + x*y(x)^2 - y(x) - a*x^3=0,y(x), singsol=all)
```

$$y(x) = \tanh\left(\sqrt{a}\left(\frac{x^2}{2} + c_1\right)\right) x\sqrt{a}$$

✓ Solution by Mathematica
Time used: 3.825 (sec). Leaf size: 30

```
DSolve[x*y'[x] + x*y[x]^2 - y[x] - a*x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{a}x \tanh\left(\frac{1}{2}\sqrt{a}(x^2 + 2c_1)\right)$$

1.103 problem 103

Internal problem ID [8440]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Riccati]`

$$\boxed{xy' + xy^2 - (2x^2+1)\,y = x^3}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 29

```
dsolve(x*diff(y(x),x) + x*y(x)^2 - (2*x^2+1)*y(x) - x^3=0,y(x), singsol=all)
```

$$y(x) = \frac{x\left(\sqrt{2}+2\tanh\left(\frac{(x^2+2c_1)\sqrt{2}}{2}\right)\right)\sqrt{2}}{2}$$

✓ Solution by Mathematica
Time used: 0.523 (sec). Leaf size: 99

```
DSolve[x*y'[x] + x*y[x]^2 - (2*x^2+1)*y[x] - x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x\left(\left(1+\sqrt{2}\right)e^{\sqrt{2}x^2} - \left(\sqrt{2}-1\right)e^{2\sqrt{2}c_1}\right)}{e^{\sqrt{2}x^2} + e^{2\sqrt{2}c_1}}$$
$$y(x) \to \left(1+\sqrt{2}\right)x$$
$$y(x) \to x - \sqrt{2}x$$

1.104 problem 104

Internal problem ID [8441]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 104.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Ricca`

$$\boxed{xy' + ay^2x + 2y = -xb}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 86

```
dsolve(x*diff(y(x),x) + a*x*y(x)^2 + 2*y(x) + b*x=0,y(x), singsol=all)
```

$$y(x) = \frac{-2bac_1x - i\sqrt{b}\,\sqrt{a}\,\mathrm{e}^{-2i\sqrt{a}\,\sqrt{b}\,x}x - 2ic_1\sqrt{a}\,\sqrt{b} - \mathrm{e}^{-2i\sqrt{a}\,\sqrt{b}\,x}}{xa\left(2ic_1\sqrt{a}\,\sqrt{b} + \mathrm{e}^{-2i\sqrt{a}\,\sqrt{b}\,x}\right)}$$

✓ Solution by Mathematica
Time used: 2.922 (sec). Leaf size: 43

```
DSolve[x*y'[x] + a*x*y[x]^2 + 2*y[x] + b*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{ax} - \sqrt{\frac{b}{a}}\tan\left(ax\sqrt{\frac{b}{a}} - c_1\right)$$

1.105 problem 105

Internal problem ID [8442]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 105.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{xy' + ay^2x + by = -cx - d}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 403

```
dsolve(x*diff(y(x),x) + a*x*y(x)^2 + b*y(x) + c*x + d=0,y(x), singsol=all)
```

$$y(x) = -\frac{4c\left(a\,c^3\left(ad-b\sqrt{-ac}\right)\operatorname{KummerM}\left(\frac{\sqrt{-ac}\,d+c(b+2)}{2c}, b+1, 2x\sqrt{-ac}\right) - \frac{c_1\left(a\,d^2+b^2c\right)\operatorname{KummerU}\left(\cdots\right.}{\cdots}\right.}{4a^2c^3\left(\sqrt{-ac}\,d+bc\right)\operatorname{KummerM}\left(\frac{\sqrt{-ac}\,d+c(b+2)}{2c}, b+1, 2x\sqrt{-ac}\right) - c_1\sqrt{-ac}\,\left(a\,d^2+b^2c\right)\operatorname{KummerU}\left(\cdots\right.}$$

✓ Solution by Mathematica

Time used: 0.453 (sec). Leaf size: 421

```
DSolve[x*y'[x] + a*x*y[x]^2 + b*y[x] + c*x + d==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{i\left(\sqrt{c}c_1\operatorname{HypergeometricU}\left(\frac{1}{2}\left(b+\frac{i\sqrt{a}d}{\sqrt{c}}\right), b, 2i\sqrt{a}\sqrt{c}x\right) + c_1\left(b\sqrt{c}+i\sqrt{a}d\right)\operatorname{HypergeometricU}\left(\frac{1}{2}\left(b+\cdots\right.\right.\right.}{\sqrt{a}\left(c_1\operatorname{HypergeometricU}\left(\frac{1}{2}\left(b+\frac{i\sqrt{a}d}{\sqrt{c}}\right), b, 2i\cdots\right.\right.}$$

$$y(x) \to \frac{\frac{\left(\sqrt{a}d-ib\sqrt{c}\right)\operatorname{HypergeometricU}\left(\frac{1}{2}\left(b+\frac{i\sqrt{a}d}{\sqrt{c}}+2\right), b+1, 2i\sqrt{a}\sqrt{c}x\right)}{\operatorname{HypergeometricU}\left(\frac{1}{2}\left(b+\frac{i\sqrt{a}d}{\sqrt{c}}\right), b, 2i\sqrt{a}\sqrt{c}x\right)} - i\sqrt{c}}{\sqrt{a}}$$

1.106 problem 106

Internal problem ID [8443]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 106.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{xy' + x^a y^2 + \frac{(a-b)\,y}{2} = -x^b}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve(x*diff(y(x),x) + x^a*y(x)^2 + (a-b)*y(x)/2 + x^b=0,y(x), singsol=all)
```

$$y(x) = -\tan\left(\frac{2x^{\frac{a}{2}+\frac{b}{2}} + c_1(a+b)}{a+b}\right) x^{-\frac{a}{2}+\frac{b}{2}}$$

✓ Solution by Mathematica

Time used: 0.539 (sec). Leaf size: 40

```
DSolve[x*y'[x] + x^a*y[x]^2 + (a-b)*y[x]/2 + x^b==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^{\frac{b-a}{2}} \tan\left(\frac{2x^{\frac{a+b}{2}}}{a+b} - c_1\right)$$

1.107 problem 107

Internal problem ID [8444]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 107.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{xy' + a\,x^{\alpha}y^2 + by = c\,x^{\beta}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 240

```
dsolve(x*diff(y(x),x) + a*x^alpha*y(x)^2 + b*y(x) - c*x^beta=0,y(x), singsol=all)
```

$y(x)$

$$= \frac{c\,x^{\beta}\left(\mathrm{BesselY}\left(\frac{b+\beta}{\alpha+\beta}, \frac{2\sqrt{-ac}\,x^{\frac{\alpha}{2}+\frac{\beta}{2}}}{\alpha+\beta}\right)c_1 + \mathrm{BesselJ}\left(\frac{b+\beta}{\alpha+\beta}, \frac{2\sqrt{-c}}{\alpha}\right.\right.}{-\left(\mathrm{BesselY}\left(\frac{b+2\beta+\alpha}{\alpha+\beta}, \frac{2\sqrt{-ac}\,x^{\frac{\alpha}{2}+\frac{\beta}{2}}}{\alpha+\beta}\right)c_1 + \mathrm{BesselJ}\left(\frac{b+2\beta+\alpha}{\alpha+\beta}, \frac{2\sqrt{-ac}\,x^{\frac{\alpha}{2}+\frac{\beta}{2}}}{\alpha+\beta}\right)\right)x^{\frac{\alpha}{2}+\frac{\beta}{2}}\sqrt{-ac} + (b+\beta)\left(\mathrm{Bessel}\right.}$$

✓ Solution by Mathematica

Time used: 0.977 (sec). Leaf size: 1474

```
DSolve[x*y'[x] + a*x^\[Alpha]*y[x]^2 + b*y[x] - c*x^\[Beta]==0,y[x],x,IncludeSingularSolutio
```

Too large to display

1.108 problem 108

Internal problem ID [8445]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 108.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{xy' - y^2 \ln(x) + y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 13

```
dsolve(x*diff(y(x),x) - y(x)^2*ln(x) + y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{1 + c_1 x + \ln(x)}$$

✓ Solution by Mathematica
Time used: 0.143 (sec). Leaf size: 20

```
DSolve[x*y'[x] - y[x]^2*Log[x] + y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{\log(x) + c_1 x + 1}$$
$$y(x) \to 0$$

1.109 problem 109

Internal problem ID [8446]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 109.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{xy' - y(2y\ln(x) - 1) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x) - y(x)*(2*y(x)*ln(x)-1)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{2 + c_1 x + 2\ln(x)}$$

✓ Solution by Mathematica
Time used: 0.138 (sec). Leaf size: 22

```
DSolve[x*y'[x] - y[x]*(2*y[x]*Log[x]-1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2\log(x) + c_1 x + 2}$$
$$y(x) \to 0$$

1.110 problem 110

Internal problem ID [8447]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 110.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$xy' + f(x)\left(y^2 - x^2\right) = 0$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x) + f(x)*(y(x)^2-x^2)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x] + f[x]*(y[x]^2-x^2)==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.111 problem 111

Internal problem ID [8448]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 111.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Abel]`

$$\boxed{xy' + y^3 + 3xy^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 59

```
dsolve(x*diff(y(x),x) + y(x)^3 + 3*x*y(x)^2=0,y(x), singsol=all)
```

$$\frac{3\,\mathrm{erf}\left(\frac{i(3xy(x)-1)\sqrt{2}}{2y(x)}\right)\sqrt{2}\,\sqrt{\pi}\,x - 2i\mathrm{e}^{\frac{(3xy(x)-1)^2}{2y(x)^2}} + 6c_1x}{6x} = 0$$

✓ Solution by Mathematica
Time used: 0.333 (sec). Leaf size: 55

```
DSolve[x*y'[x] + y[x]^3 + 3*x*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[-3x = \frac{2e^{\frac{1}{2}\left(\frac{1}{y(x)}-3x\right)^2}}{\sqrt{2\pi}\mathrm{erfi}\left(\frac{\frac{1}{y(x)}-3x}{\sqrt{2}}\right)+2c_1}, y(x)\right]$$

1.112 problem 112

Internal problem ID [8449]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 112.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{xy' - \sqrt{x^2 + y^2} - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(x*diff(y(x),x) - sqrt(y(x)^2 + x^2) - y(x)=0,y(x), singsol=all)
```

$$\frac{-c_1x^2 + y(x) + \sqrt{y(x)^2 + x^2}}{x^2} = 0$$

✓ Solution by Mathematica

Time used: 0.356 (sec). Leaf size: 27

```
DSolve[x*y'[x] - Sqrt[y[x]^2 + x^2] - y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-c_1}\left(-1 + e^{2c_1}x^2\right)$$

1.113 problem 113

Internal problem ID [8450]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 113.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{xy' + a\sqrt{x^2+y^2} - y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
dsolve(x*diff(y(x),x) + a*sqrt(y(x)^2 + x^2) - y(x)=0,y(x), singsol=all)
```

$$\frac{x^a y(x) + x^a \sqrt{y(x)^2 + x^2} - c_1 x}{x} = 0$$

✓ Solution by Mathematica

Time used: 0.253 (sec). Leaf size: 36

```
DSolve[x*y'[x] + a*Sqrt[y[x]^2 + x^2] - y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-c_1} x^{1-a} \left(-x^{2a} + e^{2c_1}\right)$$

1.114 problem 114

Internal problem ID [8451]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 114.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{xy' - \sqrt{x^2+y^2}\,x - y = 0}$$

✓ Solution by Maple

Time used: 0.516 (sec). Leaf size: 28

```
dsolve(x*diff(y(x),x) - x*sqrt(y(x)^2 + x^2) - y(x)=0,y(x), singsol=all)
```

$$\ln\left(\sqrt{y(x)^2+x^2}+y(x)\right) - x - \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.261 (sec). Leaf size: 30

```
DSolve[x*y'[x] - x*Sqrt[y[x]^2 + x^2] - y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}xe^{-x-c_1}\left(-1+e^{2(x+c_1)}\right)$$

1.115 problem 115

Internal problem ID [8452]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 115.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{xy' - x(y - x)\sqrt{x^2 + y^2} - y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 50

```
dsolve(x*diff(y(x),x) - x*(y(x)-x)*sqrt(y(x)^2 + x^2) - y(x)=0,y(x), singsol=all)
```

$$\ln(2) + \ln\left(\frac{x\left(\sqrt{2y(x)^2 + 2x^2} + y(x) + x\right)}{y(x) - x}\right) + \frac{\sqrt{2}\,x^2}{2} - \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.405 (sec). Leaf size: 84

```
DSolve[x*y'[x] - x*(y[x]-x)*Sqrt[y[x]^2 + x^2] - y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{x\tanh\left(\frac{x^2+2c_1}{2\sqrt{2}}\right)\left(2 + \sqrt{2}\tanh\left(\frac{x^2+2c_1}{2\sqrt{2}}\right)\right)}{\sqrt{2} + 2\tanh\left(\frac{x^2+2c_1}{2\sqrt{2}}\right)}$$

$$y(x) \to x$$

1.116 problem 117

Internal problem ID [8453]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 117.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{xy' - x\,\mathrm{e}^{\frac{y}{x}} - y = x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve(x*diff(y(x),x) - x*exp(y(x)/x) - y(x) - x=0,y(x), singsol=all)
```

$$y(x) = \left(\ln\left(-\frac{x}{-1 + x\,\mathrm{e}^{c_1}}\right) + c_1\right)x$$

✓ Solution by Mathematica
Time used: 4.535 (sec). Leaf size: 38

```
DSolve[x*y'[x] - x*Exp[y[x]/x] - y[x] - x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\log\left(\frac{1}{2}\left(-1 + \tanh\left(\frac{1}{2}(-\log(x) - c_1)\right)\right)\right)$$
$$y(x) \to i\pi x$$

1.117 problem 118

Internal problem ID [8454]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 118.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy' - y\ln(y) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 8

```
dsolve(x*diff(y(x),x) - y(x)*ln(y(x))=0,y(x), singsol=all)
```

$$y(x) = e^{c_1 x}$$

✓ Solution by Mathematica
Time used: 0.174 (sec). Leaf size: 18

```
DSolve[x*y'[x] - y[x]*Log[y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{e^{c_1} x}$$
$$y(x) \to 1$$

1.118 problem 119

Internal problem ID [8455]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 119.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{xy' - y(\ln(yx) - 1) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x*diff(y(x),x) - y(x)*(ln(x*y(x))-1)=0,y(x), singsol=all)
```

$$y(x) = \frac{e^{\frac{x}{c_1}}}{x}$$

✓ Solution by Mathematica

Time used: 0.179 (sec). Leaf size: 24

```
DSolve[x*y'[x] - y[x]*(Log[x*y[x]]-1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{e^{c_1} x}}{x}$$
$$y(x) \to \frac{1}{x}$$

1.119 problem 120

Internal problem ID [8456]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 120.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, ‘_with_symmetry_[F(x),G(y)]‘]]`

$$x y' - y\left(x \ln\left(\frac{x^2}{y}\right) + 2\right) = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x) - y(x)*(x*ln(x^2/y(x))+2)=0,y(x), singsol=all)
```

$$y(x) = x^2 \mathrm{e}^{-\mathrm{e}^{-x} c_1}$$

✓ Solution by Mathematica
Time used: 0.25 (sec). Leaf size: 20

```
DSolve[x*y'[x] - y[x]*(x*Log[x^2/y[x]]+2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 e^{-2c_1 e^{-x}}$$

1.120 problem 121

Internal problem ID [8457]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 121.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{xy' - \sin(x - y) = 0}$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x) + sin(y(x)-x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x] + Sin[y[x]-x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.121 problem 122

Internal problem ID [8458]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 122.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `['y=_G(x,y')']`

$$\boxed{xy' + (\sin(y) - 3x^2 \cos(y)) \cos(y) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x) + (sin(y(x))-3*x^2*cos(y(x)))*cos(y(x))=0,y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{x^3 + 2c_1}{x}\right)$$

✓ Solution by Mathematica
Time used: 2.063 (sec). Leaf size: 53

```
DSolve[x*y'[x] + (Sin[y[x]]-3*x^2*Cos[y[x]])*Cos[y[x]]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \arctan\left(x^2 + \frac{c_1}{2x}\right)$$
$$y(x) \to -\frac{1}{2}\pi\sqrt{\frac{1}{x^2}}x$$
$$y(x) \to \frac{1}{2}\pi\sqrt{\frac{1}{x^2}}x$$

1.122 problem 123

Internal problem ID [8459]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 123.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$
\boxed{xy' - x\sin\left(\frac{y}{x}\right) - y = 0}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
dsolve(x*diff(y(x),x) - x*sin(y(x)/x) - y(x)=0,y(x), singsol=all)
```

$$
y(x) = \arctan\left(\frac{2xc_1}{c_1^2x^2+1}, \frac{-c_1^2x^2+1}{c_1^2x^2+1}\right)x
$$

✓ Solution by Mathematica

Time used: 0.345 (sec). Leaf size: 52

```
DSolve[x*y'[x] - x*Sin[y[x]/x] - y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) &\to -x\arccos(-\tanh(\log(x)+c_1)) \\
y(x) &\to x\arccos(-\tanh(\log(x)+c_1)) \\
y(x) &\to 0 \\
y(x) &\to -\pi x \\
y(x) &\to \pi x
\end{aligned}
$$

1.123 problem 124

Internal problem ID [8460]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 124.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{xy' + x\cos\left(\frac{y}{x}\right) - y = -x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x) + x*cos(y(x)/x) - y(x) + x=0,y(x), singsol=all)
```

$$y(x) = -2\arctan\left(\ln\left(x\right) + c_1\right)x$$

✓ Solution by Mathematica
Time used: 0.362 (sec). Leaf size: 31

```
DSolve[x*y'[x] + x*Cos[y[x]/x] - y[x] + x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x\arctan(-\log(x) + c_1)$$
$$y(x) \to -\pi x$$
$$y(x) \to \pi x$$

1.124 problem 125

Internal problem ID [8461]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 125.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{xy' + x\tan\left(\frac{y}{x}\right) - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x*diff(y(x),x) + x*tan(y(x)/x) - y(x)=0,y(x), singsol=all)
```

$$y(x) = x \arcsin\left(\frac{1}{c_1 x}\right)$$

✓ Solution by Mathematica
Time used: 12.833 (sec). Leaf size: 21

```
DSolve[x*y'[x] + x*Tan[y[x]/x] - y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \arcsin\left(\frac{e^{c_1}}{x}\right)$$
$$y(x) \to 0$$

1.125 problem 126

Internal problem ID [8462]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 126.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{xy' - yf(yx) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x*diff(y(x),x) - y(x)*f(x*y(x))=0,y(x), singsol=all)
```

$$y(x) = \frac{\operatorname{RootOf}\left(-\ln(x) + c_1 + \int^{\_Z} \frac{1}{\_a(1+f(\_a))} d\_a\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.225 (sec). Leaf size: 115

```
DSolve[x*y'[x] - y[x]*f[x*y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} \left(\frac{1}{(-f(xK[2]) - 1)K[2]} - \int_1^x \left(\frac{f'(K[1]K[2])}{f(K[1]K[2]) + 1} - \frac{f(K[1]K[2])f'(K[1]K[2])}{(f(K[1]K[2]) + 1)^2}\right) dK[1]\right) dK[2] + \int_1^x \frac{f(K[1]y(x))}{(f(K[1]y(x)) + 1)K[1]} dK[1] = c_1, y(x)\right]$$

1.126 problem 127

Internal problem ID [8463]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 127.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{xy' - yf(x^a y^b) = 0}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 39

```
dsolve(x*diff(y(x),x) - y(x)*f(x^a*y(x)^b)=0,y(x), singsol=all)
```

$$\int_{\_b}^{y(x)} \frac{1}{(f(x^a \_a^b)\, b + a)\, \_a} d\_a - \frac{\ln(x)}{b} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.344 (sec). Leaf size: 186

```
DSolve[x*y'[x] - y[x]*f[x^a*y[x]^b]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} \left(-\frac{b}{(a+bf(x^a K[2]^b))\, K[2]} - \int_1^x \left(\frac{b^2 K[1]^{a-1} K[2]^{b-1} f'(K[1]^a K[2]^b)}{a+bf(K[1]^a K[2]^b)} - \frac{b^3 f(K[1]^a K[2]^b)\, K[1]^{a-1} K[2]^{b-1} f'(K[1]^a K[2]^b)}{(a+bf(K[1]^a K[2]^b))^2}\right) dK[1]\right) dK + \int_1^x \frac{bf(K[1]^a y(x)^b)}{(a+bf(K[1]^a y(x)^b))\, K[1]} dK[1] = c_1, y(x)\right]$$

1.127 problem 128

Internal problem ID [8464]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 128.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, `_with_symmetry_[F(x),G(x)*y+H(x)]`]]`

$$\boxed{xy' + ay - f(x)\, g(x^a y) = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 33

```
dsolve(x*diff(y(x),x) + a*y(x) - f(x)*g(x^a*y(x))=0,y(x), singsol=all)
```

$$y(x) = \text{RootOf}\left(-\left(\int f(x)\, x^{a-1} dx\right) + \int^{\_Z} \frac{1}{g(\_a)} d\_a + c_1\right) x^{-a}$$

✓ Solution by Mathematica

Time used: 1.892 (sec). Leaf size: 41

```
DSolve[x*y'[x] + a*y[x] - f[x]*g[x^a*y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{x^a y(x)} \frac{1}{g(K[1])} dK[1] = \int_1^x f(K[2]) K[2]^{a-1} dK[2] + c_1, y(x)\right]$$

1.128 problem 129

Internal problem ID [8465]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 129.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{(1+x)\,y' + y(y-x) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve((x+1)*diff(y(x),x) + y(x)*(y(x)-x)=0,y(x), singsol=all)
```

$$y(x) = \frac{e^{x}}{-e^{-1}\,(x+1)\,\mathrm{expIntegral}_{1}\,(-x-1) - e^{x} + c_{1}\,(x+1)}$$

✓ Solution by Mathematica
Time used: 0.333 (sec). Leaf size: 42

```
DSolve[(x+1)*y'[x]+ y[x]*(y[x]-x)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{x+1}}{-(x+1)\,\mathrm{ExpIntegralEi}(x+1) + e\,(e^{x} - c_{1}(x+1))}$$
$$y(x) \to 0$$

1.129 problem 130

Internal problem ID [8466]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 130.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2xy' - y = 2x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(2*x*diff(y(x),x) - y(x) -2*x^3=0,y(x), singsol=all)
```

$$y(x) = \frac{2x^3}{5} + c_1\sqrt{x}$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 21

```
DSolve[2*x*y'[x] - y[x] -2*x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x^3}{5} + c_1\sqrt{x}$$

1.130 problem 131

Internal problem ID [8467]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 131.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(2x+1)\,y' - 4\,\mathrm{e}^{-y} = -2}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 31

```
dsolve((2*x+1)*diff(y(x),x) - 4*exp(-y(x)) + 2=0,y(x), singsol=all)
```

$$y(x) = -\ln\left(\frac{2x+1}{-1+(4x+2)\,\mathrm{e}^{2c_1}}\right) - 2c_1$$

✓ Solution by Mathematica
Time used: 0.661 (sec). Leaf size: 26

```
DSolve[(2*x+1)*y'[x] - 4*Exp[-y[x]] + 2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(2 + \frac{e^{c_1}}{2x+1}\right)$$
$$y(x) \to \log(2)$$

1.131 problem 132

Internal problem ID [8468]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 132.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{3xy' - 3x\ln(x)\, y^4 - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 162

```
dsolve(3*x*diff(y(x),x) - 3*x*ln(x)*y(x)^4 - y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{2^{\frac{2}{3}}\left(-x(6x^2\ln(x) - 3x^2 - 4c_1)^2\right)^{\frac{1}{3}}}{6x^2\ln(x) - 3x^2 - 4c_1}$$

$$y(x) = -\frac{2^{\frac{2}{3}}\left(-x(6x^2\ln(x) - 3x^2 - 4c_1)^2\right)^{\frac{1}{3}}(1 + i\sqrt{3})}{12x^2\ln(x) - 6x^2 - 8c_1}$$

$$y(x) = \frac{2^{\frac{2}{3}}\left(-x(6x^2\ln(x) - 3x^2 - 4c_1)^2\right)^{\frac{1}{3}}(i\sqrt{3} - 1)}{12x^2\ln(x) - 6x^2 - 8c_1}$$

✓ Solution by Mathematica

Time used: 0.215 (sec). Leaf size: 120

```
DSolve[3*x*y'[x] - 3*x*Log[x]*y[x]^4 - y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(-2)^{2/3}\sqrt[3]{x}}{\sqrt[3]{3x^2 - 6x^2\log(x) + 4c_1}}$$
$$y(x) \to \frac{2^{2/3}\sqrt[3]{x}}{\sqrt[3]{3x^2 - 6x^2\log(x) + 4c_1}}$$
$$y(x) \to -\frac{\sqrt[3]{-1}2^{2/3}\sqrt[3]{x}}{\sqrt[3]{3x^2 - 6x^2\log(x) + 4c_1}}$$
$$y(x) \to 0$$

1.132 problem 133

Internal problem ID [8469]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 133.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x^2y' + y = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x^2*diff(y(x),x) + y(x) - x=0,y(x), singsol=all)
```

$$y(x) = \left(\text{expIntegral}_1\left(\frac{1}{x}\right) + c_1\right) e^{\frac{1}{x}}$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 22

```
DSolve[x^2*y'[x] + y[x] - x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{1}{x}}\left(-\text{ExpIntegralEi}\left(-\frac{1}{x}\right) + c_1\right)$$

1.133 problem 134

Internal problem ID [8470]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 134.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x^2y' - y = -x^2\mathrm{e}^{x-\frac{1}{x}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x) - y(x) + x^2*exp(x-1/x)=0,y(x), singsol=all)
```

$$y(x) = (-\mathrm{e}^x + c_1)\,\mathrm{e}^{-\frac{1}{x}}$$

✓ Solution by Mathematica
Time used: 0.064 (sec). Leaf size: 21

```
DSolve[x^2*y'[x] - y[x] + x^2*Exp[x-1/x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-1/x}(-e^x + c_1)$$

1.134 problem 135

Internal problem ID [8471]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 135.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2y' - (x-1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x^2*diff(y(x),x) - (x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x\,\mathrm{e}^{\frac{1}{x}}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 19

```
DSolve[x^2*y'[x] - (x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{\frac{1}{x}} x$$
$$y(x) \to 0$$

1.135 problem 136

Internal problem ID [8472]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 136.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Riccati]`

$$\boxed{x^2y' + y^2 + yx = -x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^2*diff(y(x),x) + y(x)^2 + x*y(x) + x^2=0,y(x), singsol=all)
```

$$y(x) = -\frac{x(\ln(x) + c_1 - 1)}{\ln(x) + c_1}$$

✓ Solution by Mathematica
Time used: 0.137 (sec). Leaf size: 31

```
DSolve[x^2*y'[x] + y[x]^2 + x*y[x] + x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x(\log(x) - 1 - c_1)}{-\log(x) + c_1}$$
$$y(x) \to -x$$

1.136 problem 137

Internal problem ID [8473]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 137.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{x^2y' - y^2 - yx = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x^2*diff(y(x),x) - y(x)^2 - x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x}{-\ln(x) + c_1}$$

✓ Solution by Mathematica
Time used: 0.13 (sec). Leaf size: 21

```
DSolve[x^2*y'[x] - y[x]^2 - x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{-\log(x) + c_1}$$
$$y(x) \to 0$$

1.137 problem 138

Internal problem ID [8474]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 138.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Riccati]`

$$\boxed{x^2y' - y^2 - yx = x^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 11

```
dsolve(x^2*diff(y(x),x) - y(x)^2 - x*y(x) - x^2=0,y(x), singsol=all)
```

$$y(x) = \tan(\ln(x) + c_1)\, x$$

✓ Solution by Mathematica
Time used: 0.175 (sec). Leaf size: 13

```
DSolve[x^2*y'[x] - y[x]^2 - x*y[x] - x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\tan(\log(x) + c_1)$$

1.138 problem 139

Internal problem ID [8475]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 139.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$x^2 (y' + y^2) = -a\,x^k + b(b-1)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 229

```
dsolve(x^2*(diff(y(x),x)+y(x)^2) + a*x^k - b*(b-1)=0,y(x), singsol=all)
```

$$y(x) = \frac{-x^{\frac{k}{2}}\left(\operatorname{BesselY}\left(\frac{\operatorname{csgn}(2b-1)(2b-1)+k}{k}, \frac{2\sqrt{a}\,x^{\frac{k}{2}}}{k}\right)c_1 + \operatorname{BesselJ}\left(\frac{\operatorname{csgn}(2b-1)(2b-1)+k}{k}, \frac{2\sqrt{a}\,x^{\frac{k}{2}}}{k}\right)\right)\sqrt{a} + \left(\frac{1}{2} + \operatorname{csgn}(2\right.}{x\left(\operatorname{BesselY}\left(\frac{\operatorname{csgn}(2b-1)(2b-1)}{k}, \frac{2\sqrt{a}\,x^{\frac{k}{2}}}{k}\right)c_1 + \right.}$$

✓ Solution by Mathematica

Time used: 0.496 (sec). Leaf size: 627

```
DSolve[x^2*(y'[x]+y[x]^2) + a*x^k - b*(b-1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a}x^k \operatorname{Gamma}\left(\frac{2b+k-1}{k}\right) \operatorname{BesselJ}\left(-\frac{-2b+k+1}{k}, \frac{2\sqrt{a}\sqrt{x^k}}{k}\right) - \sqrt{a}x^k \operatorname{Gamma}\left(\frac{2b+k-1}{k}\right) \operatorname{BesselJ}\left(\frac{2b+k-1}{k}, \frac{2\sqrt{a}\sqrt{x^k}}{k}\right.}{}$$

$$y(x) \to \frac{\frac{\sqrt{a}\sqrt{x^k}\left(\operatorname{BesselJ}\left(-\frac{2b+k-1}{k}, \frac{2\sqrt{a}\sqrt{x^k}}{k}\right) - \operatorname{BesselJ}\left(\frac{-2b+k+1}{k}, \frac{2\sqrt{a}\sqrt{x^k}}{k}\right)\right)}{\operatorname{BesselJ}\left(\frac{1-2b}{k}, \frac{2\sqrt{a}\sqrt{x^k}}{k}\right)} + 1}{2x}$$

$$y(x) \to \frac{\frac{\sqrt{a}\sqrt{x^k}\left(\operatorname{BesselJ}\left(-\frac{2b+k-1}{k}, \frac{2\sqrt{a}\sqrt{x^k}}{k}\right) - \operatorname{BesselJ}\left(\frac{-2b+k+1}{k}, \frac{2\sqrt{a}\sqrt{x^k}}{k}\right)\right)}{\operatorname{BesselJ}\left(\frac{1-2b}{k}, \frac{2\sqrt{a}\sqrt{x^k}}{k}\right)} + 1}{2x}$$

1.139 problem 140

Internal problem ID [8476]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 140.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Riccati]`

$$\boxed{x^2(y' + y^2) + 4yx = -2}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve(x^2*(diff(y(x),x)+y(x)^2) + 4*x*y(x) + 2=0,y(x), singsol=all)
```

$$y(x) = \frac{-2c_1 + x}{x(-x + c_1)}$$

✓ Solution by Mathematica

Time used: 0.151 (sec). Leaf size: 26

```
DSolve[x^2*(y'[x]+y[x]^2) + 4*x*y[x] + 2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{x} + \frac{1}{x + c_1}$$
$$y(x) \to -\frac{2}{x}$$

1.140 problem 141

Internal problem ID [8477]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 141.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Riccati]`

$$\boxed{x^2(y' + y^2) + axy = -b}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 50

```
dsolve(x^2*(diff(y(x),x)+y(x)^2) + a*x*y(x) + b=0,y(x), singsol=all)
```

$$y(x) = \frac{1 - a + \tanh\left(\frac{\sqrt{a^2-2a-4b+1}\,(\ln(x)-c_1)}{2}\right)\sqrt{a^2 - 2a - 4b + 1}}{2x}$$

✓ Solution by Mathematica

Time used: 0.227 (sec). Leaf size: 90

```
DSolve[x^2*(y'[x]+y[x]^2) + a*x*y[x] + b==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a^2 - 2a - 4b + 1}\left(1 - \frac{2c_1}{x^{\sqrt{a^2-2a-4b+1}} + c_1}\right) - a + 1}{2x}$$

$$y(x) \to -\frac{\sqrt{a^2 - 2a - 4b + 1} + a - 1}{2x}$$

1.141 problem 142

Internal problem ID [8478]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 142.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{x^2(y' - y^2) - a\,x^2 y = -xa - 2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 52

```
dsolve(x^2*(diff(y(x),x)-y(x)^2) - a*x^2*y(x) + a*x + 2=0,y(x), singsol=all)
```

$$y(x) = \frac{-(ax-1)\,(a^2x^2+2)\,e^{ax} + c_1}{((a^2x^2-2ax+2)\,e^{ax} + c_1)\,x}$$

✓ Solution by Mathematica

Time used: 0.375 (sec). Leaf size: 78

```
DSolve[x^2*(y'[x]-y[x]^2) - a*x^2*y[x] + a*x + 2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{ax}(-a^3x^3 + a^2x^2 - 2ax + 2) + a^3c_1}{x\,(e^{ax}\,(a^2x^2 - 2ax + 2) + a^3c_1)}$$
$$y(x) \to \frac{1}{x}$$

1.142 problem 143

Internal problem ID [8479]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 143.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Riccati, _special]]`

$$\boxed{x^2(y' + ay^2) = b}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(x^2*(diff(y(x),x)+a*y(x)^2) - b=0,y(x), singsol=all)
```

$$y(x) = \frac{1 + \tanh\left(\frac{\sqrt{4ab+1}\,(\ln(x)-c_1)}{2}\right)\sqrt{4ab+1}}{2ax}$$

✓ Solution by Mathematica

Time used: 0.18 (sec). Leaf size: 77

```
DSolve[x^2*(y'[x]+a*y[x]^2) - b==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{-1 + \sqrt{4ab+1}\left(-1 + \frac{2c_1}{x^{\sqrt{4ab+1}}+c_1}\right)}{2ax}$$

$$y(x) \to -\frac{\sqrt{4ab+1} - 1}{2ax}$$

1.143 problem 144

Internal problem ID [8480]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 144.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$x^2(y' + ay^2) = -b\,x^\alpha - c$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 219

```
dsolve(x^2*(diff(y(x),x)+a*y(x)^2) + b*x^alpha + c=0,y(x), singsol=all)
```

$$y(x) = \frac{-2\sqrt{ab}\left(\mathrm{BesselY}\left(\frac{\sqrt{-4ac+1}}{\alpha}+1, \frac{2\sqrt{ab}\,x^{\frac{\alpha}{2}}}{\alpha}\right)c_1 + \mathrm{BesselJ}\left(\frac{\sqrt{-4ac+1}}{\alpha}+1, \frac{2\sqrt{ab}\,x^{\frac{\alpha}{2}}}{\alpha}\right)\right)x^{\frac{\alpha}{2}} + \left(\sqrt{-4ac+1}+1\right)\left(\right.}{2xa\left(\mathrm{BesselY}\left(\frac{\sqrt{-4ac+1}}{\alpha}, \frac{2\sqrt{ab}\,x^{\frac{\alpha}{2}}}{\alpha}\right)c_1 + \mathrm{BesselJ}\left(\frac{\sqrt{-4ac+}}{\alpha}\right.\right.}$$

✓ Solution by Mathematica

Time used: 1.108 (sec). Leaf size: 1777

```
DSolve[x^2*(y'[x]+a*y[x]^2) + b*x^\[Alpha] + c==0,y[x],x,IncludeSingularSolutions -> True]
```

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1.144 problem 145

Internal problem ID [8481]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 145.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Abel]`

$$x^2y' + ay^3 - x^2ay^2 = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 148

```
dsolve(x^2*diff(y(x),x) + a*y(x)^3 - a*x^2*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{-ax - 2^{\frac{2}{3}}(-a)^{\frac{2}{3}} \operatorname{RootOf}\left(\operatorname{AiryBi}\left(\frac{\left(\_Z^2 2^{\frac{1}{3}}(-a)^{\frac{1}{3}}x-1\right)2^{\frac{2}{3}}}{2(-a)^{\frac{1}{3}}x}\right)c_1\_Z + \_Z\operatorname{AiryAi}\left(\frac{\left(\_Z^2 2^{\frac{1}{3}}(-a)^{\frac{1}{3}}x-1\right)2^{\frac{2}{3}}}{2(-a)^{\frac{1}{3}}x}\right) + \mathrm{Ai}\right.}$$

✓ Solution by Mathematica
Time used: 0.436 (sec). Leaf size: 267

```
DSolve[x^2*y'[x] + a*y[x]^3 - a*x^2*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\operatorname{Solve}\left[\frac{\left(-\frac{1}{2^{2/3}a^{2/3}y(x)} - \frac{\sqrt[3]{a}x}{2^{2/3}}\right)\operatorname{AiryAi}\left(\left(-\frac{\sqrt[3]{a}x}{2^{2/3}} - \frac{1}{2^{2/3}a^{2/3}y(x)}\right)^2 + \frac{1}{\sqrt[3]{2}\sqrt[3]{a}x}\right) + \operatorname{AiryAiPrime}\left(\left(-\frac{\sqrt[3]{a}x}{2^{2/3}}\right.\right.}{\left(-\frac{1}{2^{2/3}a^{2/3}y(x)} - \frac{\sqrt[3]{a}x}{2^{2/3}}\right)\operatorname{AiryBi}\left(\left(-\frac{\sqrt[3]{a}x}{2^{2/3}} - \frac{1}{2^{2/3}a^{2/3}y(x)}\right)^2 + \frac{1}{\sqrt[3]{2}\sqrt[3]{a}x}\right) + \operatorname{AiryBiPrime}\left(\left(-\frac{\sqrt[3]{a}x}{2^{2/3}}\right.\right.} + c_1 = 0, y(x)\right]$$

1.145 problem 146

Internal problem ID [8482]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 146.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Abel]`

$$\boxed{x^2y' + y^3x + ay^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 65

```
dsolve(x^2*diff(y(x),x) + x*y(x)^3 + a*y(x)^2=0,y(x), singsol=all)
```

$$c_1 + \mathrm{e}^{-\frac{((-x+a)y(x)+x)((a+x)y(x)+x)}{2y(x)^2x^2}}x + \frac{\operatorname{erf}\left(\frac{\sqrt{2}\,(ay(x)+x)}{2y(x)x}\right)\sqrt{2}\,\sqrt{\pi}\,a\,\mathrm{e}^{\frac{1}{2}}}{2} = 0$$

✓ Solution by Mathematica
Time used: 0.607 (sec). Leaf size: 78

```
DSolve[x^2*y'[x] + x*y[x]^3 + a*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{ia}{x} = \frac{2e^{\frac{1}{2}\left(-\frac{ia}{x}-\frac{i}{y(x)}\right)^2}}{\sqrt{2\pi}\text{erfi}\left(\frac{-\frac{ia}{x}-\frac{i}{y(x)}}{\sqrt{2}}\right)+2c_1}, y(x)\right]$$

1.146 problem 147

Internal problem ID [8483]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 147.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Abel]`

$$x^2y' + x^2y^3a + by^2 = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 182

```
dsolve(x^2*diff(y(x),x) + a*x^2*y(x)^3 + b*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = -\frac{2^{\frac{1}{3}}abx}{2^{\frac{1}{3}}a\,b^2 - 2\,(b^2a^2)^{\frac{2}{3}}\operatorname{RootOf}\left(\operatorname{AiryBi}\left(\frac{-a2^{\frac{2}{3}}x+2\_Z^2(b^2a^2)^{\frac{1}{3}}}{2(b^2a^2)^{\frac{1}{3}}}\right)c_1\_Z + \_Z\operatorname{AiryAi}\left(\frac{-a2^{\frac{2}{3}}x+2\_Z^2(b^2a^2)^{\frac{1}{3}}}{2(b^2a^2)^{\frac{1}{3}}}\right) + \right.}$$

✓ Solution by Mathematica
Time used: 0.581 (sec). Leaf size: 343

```
DSolve[x^2*y'[x] + a*x^2*y[x]^3 + b*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\left(\frac{b^{2/3}}{2^{2/3}\sqrt[3]{a}x}+\frac{1}{2^{2/3}\sqrt[3]{a}\sqrt[3]{b}y(x)}\right)\text{AiryAi}\left(\left(\frac{b^{2/3}}{2^{2/3}\sqrt[3]{a}x}+\frac{1}{2^{2/3}\sqrt[3]{a}y(x)\sqrt[3]{b}}\right)^2-\frac{\sqrt[3]{a}x}{\sqrt[3]{2}b^{2/3}}\right)+\text{AiryAiPrime}\left(\left(\right.\right.}{\left(\frac{b^{2/3}}{2^{2/3}\sqrt[3]{a}x}+\frac{1}{2^{2/3}\sqrt[3]{a}\sqrt[3]{b}y(x)}\right)\text{AiryBi}\left(\left(\frac{b^{2/3}}{2^{2/3}\sqrt[3]{a}x}+\frac{1}{2^{2/3}\sqrt[3]{a}y(x)\sqrt[3]{b}}\right)^2-\frac{\sqrt[3]{a}x}{\sqrt[3]{2}b^{2/3}}\right)+\text{AiryBiPrime}\left(\left(\right.\right.}\right.$$

$$\left. + c_1 = 0, y(x)\right]$$

1.147 problem 148

Internal problem ID [8484]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 148.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2+1)\,y' + yx = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve((x^2+1)*diff(y(x),x) + x*y(x) - 1=0,y(x), singsol=all)
```

$$y(x) = \frac{\operatorname{arcsinh}(x) + c_1}{\sqrt{x^2+1}}$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 34

```
DSolve[(x^2+1)*y'[x] + x*y[x] - 1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-\log\left(\sqrt{x^2+1} - x\right) + c_1}{\sqrt{x^2+1}}$$

1.148 problem 149

Internal problem ID [8485]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 149.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2+1)\,y'+yx=x(x^2+1)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve((x^2+1)*diff(y(x),x) + x*y(x) - x*(x^2+1)=0,y(x), singsol=all)
```

$$y(x)=\frac{x^2}{3}+\frac{1}{3}+\frac{c_1}{\sqrt{x^2+1}}$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 27

```
DSolve[(x^2+1)*y'[x] + x*y[x] - x*(x^2+1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{3}(x^2+1)+\frac{c_1}{\sqrt{x^2+1}}$$

1.149 problem 150

Internal problem ID [8486]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 150.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2+1)\,y'+2yx=2x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve((x^2+1)*diff(y(x),x) + 2*x*y(x) - 2*x^2=0,y(x), singsol=all)
```

$$y(x)=\frac{2x^3+3c_1}{3x^2+3}$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 25

```
DSolve[(x^2+1)*y'[x] + 2*x*y[x] - 2*x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{2x^3+3c_1}{3x^2+3}$$

1.150 problem 151

Internal problem ID [8487]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 151.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Abel]`

$$\boxed{(x^2+1)\,y' + (1+y^2)\,(2xy-1) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 65

```
dsolve((x^2+1)*diff(y(x),x) + (y(x)^2+1)*(2*x*y(x) - 1)=0,y(x), singsol=all)
```

$$c_1 + \frac{x}{\left(\frac{(x^2+1)\left(y(x)^2+1\right)}{(xy(x)-1)^2}\right)^{\frac{1}{4}}} + \frac{(x+y(x))\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{5}{4}\right],\left[\frac{3}{2}\right],-\frac{(x+y(x))^2}{(xy(x)-1)^2}\right)}{2xy(x)-2} = 0$$

✓ Solution by Mathematica
Time used: 0.421 (sec). Leaf size: 203

```
DSolve[(x^2+1)*y'[x] + (y[x]^2+1)*(2*x*y[x] - 1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[c_1 = \frac{\frac{1}{2}\left(\frac{1}{\frac{ix}{x^2+1}-\frac{ix^2y(x)}{x^2+1}}+\frac{i}{x}\right)\sqrt[4]{1-\left(\frac{1}{\frac{ix}{x^2+1}-\frac{ix^2y(x)}{x^2+1}}+\frac{i}{x}\right)^2}\ \mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{3}{2},\left(\frac{1}{\frac{ix}{x^2+1}-\frac{ix^2y(x)}{x^2+1}}\right.\right.}{\sqrt[4]{-1+\left(\frac{1}{\frac{ix}{x^2+1}-\frac{ix^2y(x)}{x^2+1}}+\frac{i}{x}\right)^2}}\right.$$

1.151 problem 152

Internal problem ID [8488]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 152.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `['y=_G(x,y')']`

$$\left(x^{2}+1\right) y'+x \sin \left(y\right) \cos \left(y\right)-x\left(x^{2}+1\right) \cos \left(y\right)^{2}=0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 142

```
dsolve((x^2+1)*diff(y(x),x) + x*sin(y(x))*cos(y(x)) - x*(x^2+1)*cos(y(x))^2=0,y(x), singsol=
```

$$y(x)=\frac{\arctan\left(\frac{6\sqrt{x^{2}+1}\left(\sqrt{x^{2}+1}\,x^{2}+\sqrt{x^{2}+1}+3c_{1}\right)}{10+6c_{1}\left(x^{2}+1\right)^{\frac{3}{2}}+x^{6}+3x^{4}+12x^{2}+9c_{1}^{2}},\frac{8+6\left(-x^{2}-1\right)c_{1}\sqrt{x^{2}+1}-x^{6}-3x^{4}+6x^{2}-9c_{1}^{2}}{10+6c_{1}\left(x^{2}+1\right)^{\frac{3}{2}}+x^{6}+3x^{4}+12x^{2}+9c_{1}^{2}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 8.84 (sec). Leaf size: 97

```
DSolve[(x^2+1)*y'[x] + x*Sin[y[x]]*Cos[y[x]] - x*(x^2+1)*Cos[y[x]]^2==0,y[x],x,IncludeSingul
```

$$y(x)\to \arctan\left(\frac{x^{4}+2x^{2}-6c_{1}\sqrt{x^{2}+1}+1}{3x^{2}+3}\right)$$
$$y(x)\to -\frac{1}{2}\pi\sqrt{\frac{1}{x^{2}+1}}\sqrt{x^{2}+1}$$
$$y(x)\to \frac{1}{2}\pi\sqrt{\frac{1}{x^{2}+1}}\sqrt{x^{2}+1}$$

1.152 problem 153

Internal problem ID [8489]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 153.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2-1)\,y'-yx=-a}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve((x^2-1)*diff(y(x),x) - x*y(x) + a=0,y(x), singsol=all)
```

$$y(x)=\sqrt{x-1}\,\sqrt{x+1}\,c_1+ax$$

✓ Solution by Mathematica
Time used: 0.074 (sec). Leaf size: 21

```
DSolve[(x^2-1)*y'[x] - x*y[x] + a==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to ax+c_1\sqrt{x^2-1}$$

1.153 problem 154

Internal problem ID [8490]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 154.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2-1)\,y'+2yx=\cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve((x^2-1)*diff(y(x),x) + 2*x*y(x) - cos(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{\sin(x)+c_1}{x^2-1}$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 18

```
DSolve[(x^2-1)*y'[x] + 2*x*y[x] - Cos[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{\sin(x)+c_1}{x^2-1}$$

1.154 problem 155

Internal problem ID [8491]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 155.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Ricca`

$$(x^2 - 1) y' + y^2 - 2yx = -1$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve((x^2-1)*diff(y(x),x) + y(x)^2 - 2*x*y(x) + 1=0,y(x), singsol=all)
```

$$y(x) = x + \frac{1}{c_1 - \operatorname{arctanh}(x)}$$

✓ Solution by Mathematica
Time used: 0.224 (sec). Leaf size: 52

```
DSolve[(x^2-1)*y'[x]+ y[x]^2 - 2*x*y[x] + 1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x \log(1-x) - x \log(x+1) + 2c_1 x + 2}{\log(1-x) - \log(x+1) + 2c_1}$$
$$y(x) \to x$$

1.155 problem 156

Internal problem ID [8492]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 156.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{(x^2-1)\, y' - y(y-x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve((x^2-1)*diff(y(x),x) - y(x)*(y(x)-x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{x-1}\,\sqrt{x+1}\,c_1 + x}$$

✓ Solution by Mathematica

Time used: 0.165 (sec). Leaf size: 26

```
DSolve[(x^2-1)*y'[x] - y[x]*(y[x]-x)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{x + c_1\sqrt{x^2-1}}$$
$$y(x) \to 0$$

1.156 problem 157

Internal problem ID [8493]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 157.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$(x^2-1)\,y' + a\left(y^2 - 2yx + 1\right) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 280

```
dsolve((x^2-1)*diff(y(x),x) + a*(y(x)^2-2*x*y(x)+1)=0,y(x), singsol=all)
```

$$y(x) = \frac{2\left(-\frac{x}{2}-\frac{1}{2}\right)^{2a}\left(-\frac{\left(-\frac{x}{2}-\frac{1}{2}\right)^{-2a}(x+1)(x-1)^2\operatorname{HeunCPrime}\left(0,2a-1,0,0,a^2-a+\frac{1}{2},\frac{2}{x+1}\right)}{8} - (x-1)^2\operatorname{HeunCPrime}\left(0,-2a+\right.\right.}{a\,(x+1)^2\left(c_1\,\operatorname{hyperg}\right.}$$

✓ Solution by Mathematica

Time used: 0.355 (sec). Leaf size: 47

```
DSolve[(x^2-1)*y'[x] + a*(y[x]^2-2*x*y[x]+1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\operatorname{LegendreQ}(a,x) + c_1 \operatorname{LegendreP}(a,x)}{\operatorname{LegendreQ}(a-1,x) + c_1 \operatorname{LegendreP}(a-1,x)}$$

$$y(x) \to \frac{\operatorname{LegendreP}(a,x)}{\operatorname{LegendreP}(a-1,x)}$$

1.157 problem 158

Internal problem ID [8494]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 158.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2-1)\,y'+ay^2x+yx=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve((x^2-1)*diff(y(x),x) + a*x*y(x)^2 + x*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{1}{\sqrt{x-1}\,\sqrt{x+1}\,c_1-a}$$

✓ Solution by Mathematica
Time used: 4.042 (sec). Leaf size: 45

```
DSolve[(x^2-1)*y'[x] + a*x*y[x]^2 + x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{e^{c_1}}{-\sqrt{x^2-1}+ae^{c_1}}$$
$$y(x)\to 0$$
$$y(x)\to -\frac{1}{a}$$

1.158 problem 159

Internal problem ID [8495]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 159.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2-1)\,y'-2xy\ln(y)=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve((x^2-1)*diff(y(x),x) - 2*x*y(x)*ln(y(x))=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{c_1(x+1)(x-1)}$$

✓ Solution by Mathematica
Time used: 0.22 (sec). Leaf size: 22

```
DSolve[(x^2-1)*y'[x] - 2*x*y[x]*Log[y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{e^{c_1}(x^2-1)}$$
$$y(x)\to 1$$

1.159 problem 160

Internal problem ID [8496]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 160.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{(x^2-4)\,y'+(x+2)\,y^2-4y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve((x^2-4)*diff(y(x),x) + (x+2)*y(x)^2 - 4*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{x-2}{(\ln(x+2)+c_1)\,(x+2)}$$

✓ Solution by Mathematica
Time used: 0.206 (sec). Leaf size: 32

```
DSolve[(x^2-4)*y'[x] + (x+2)*y[x]^2 - 4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{2-x}{(x+2)(-\log(x+2)+c_1)}$$
$$y(x)\to 0$$

1.160 problem 161

Internal problem ID [8497]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 161.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2 - 5x + 6)\, y' + 3yx - 8y = -x^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 27

```
dsolve((x^2-5*x+6)*diff(y(x),x) + 3*x*y(x) - 8*y(x) + x^2=0,y(x), singsol=all)
```

$$y(x) = \frac{-\frac{1}{4}x^4 + \frac{2}{3}x^3 + c_1}{(x-2)^2 (x-3)}$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 34

```
DSolve[(x^2-5*x+6)*y'[x] + 3*x*y[x] - 8*y[x] + x^2==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{-3x^4 + 8x^3 - 12c_1}{12(x-3)(x-2)^2}$$

1.161 problem 162

Internal problem ID [8498]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 162.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Ricca`

$$(x-a)(x-b)y' + y^2 + k(y+x-a)(y+x-b) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve((x-a)*(x-b)*diff(y(x),x) + y(x)^2 + k*(y(x)+x-a)*(y(x)+x-b)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left((-x+b)^{1+k} + (-x+a)^k c_1(-x+a)\right)k}{(1+k)\left(c_1(-x+a)^k + (-x+b)^k\right)}$$

✓ Solution by Mathematica

Time used: 60.296 (sec). Leaf size: 99

```
DSolve[(x-a)*(x-b)*y'[x] + y[x]^2 + k*(y[x]+x-a)*(y[x]+x-b)==0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to \frac{1}{2}\left(\frac{k(a+b-2x)}{k+1} + \sqrt{-\frac{k^2(a-b)^2}{(k+1)^2}}\tan\left(\frac{(k+1)\sqrt{-\frac{k^2(a-b)^2}{(k+1)^2}}(\log(x-b)-\log(x-a))}{2(a-b)} + c_1\right)\right)$$

1.162 problem 163

Internal problem ID [8499]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 163.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{2x^2y' - 2y^2 - yx = -2a^2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(2*x^2*diff(y(x),x) - 2*y(x)^2 - x*y(x) + 2*a^2*x=0,y(x), singsol=all)
```

$$y(x) = \tanh\left(\frac{ic_1\sqrt{x} + 2a}{\sqrt{x}}\right)\sqrt{x}\,a$$

✓ Solution by Mathematica
Time used: 0.374 (sec). Leaf size: 43

```
DSolve[2*x^2*y'[x] - 2*y[x]^2 - x*y[x] + 2*a^2*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-a^2}\sqrt{x}\tan\left(\frac{2\sqrt{-a^2}}{\sqrt{x}} - c_1\right)$$

1.163 problem 164

Internal problem ID [8500]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 164.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$2x^2y' - 2y^2 - 3yx = -2a^2x$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 102

```
dsolve(2*x^2*diff(y(x),x) - 2*y(x)^2 - 3*x*y(x) + 2*a^2*x=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(-2xc_1\sqrt{-\frac{a^2}{x}} - x\right)\sin\left(2\sqrt{-\frac{a^2}{x}}\right) - x\left(c_1 - 2\sqrt{-\frac{a^2}{x}}\right)\cos\left(2\sqrt{-\frac{a^2}{x}}\right)}{2\cos\left(2\sqrt{-\frac{a^2}{x}}\right)c_1 + 2\sin\left(2\sqrt{-\frac{a^2}{x}}\right)}$$

✓ Solution by Mathematica
Time used: 0.286 (sec). Leaf size: 94

```
DSolve[2*x^2*y'[x]- 2*y[x]^2 - 3*x*y[x] + 2*a^2*x==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{4a^2c_1\sqrt{x} + 2a\sqrt{x}e^{\frac{4a}{\sqrt{x}}} - xe^{\frac{4a}{\sqrt{x}}} + 2ac_1x}{2e^{\frac{4a}{\sqrt{x}}} - 4ac_1}$$
$$y(x) \to a(-\sqrt{x}) - \frac{x}{2}$$

1.164 problem 165

Internal problem ID [8501]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 165.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Ricca`

$$x(2x-1)\,y' + y^2 - (1+4x)\,y = -4x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*(2*x-1)*diff(y(x),x) + y(x)^2 - (4*x+1)*y(x) + 4*x=0,y(x), singsol=all)
```

$$y(x) = \frac{2x^2 + c_1}{x + c_1}$$

✓ Solution by Mathematica
Time used: 0.207 (sec). Leaf size: 27

```
DSolve[x*(2*x-1)*y'[x] + y[x]^2 - (4*x+1)*y[x] + 4*x==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to 1 + \frac{x(2x-1)}{x - c_1}$$
$$y(x) \to 1$$

1.165 problem 166

Internal problem ID [8502]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 166.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{2x(x-1)\,y' + (x-1)\,y^2 = x}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 97

```
dsolve(2*x*(x-1)*diff(y(x),x) + (x-1)*y(x)^2 - x=0,y(x), singsol=all)
```

$$y(x) = \frac{x\left(\operatorname{LegendreQ}\left(-\frac{1}{2},1,\frac{2-x}{x}\right)c_1 - \operatorname{LegendreQ}\left(\frac{1}{2},1,\frac{2-x}{x}\right)c_1 + \operatorname{LegendreP}\left(-\frac{1}{2},1,\frac{2-x}{x}\right) - \operatorname{LegendreP}\left(\frac{1}{2},1,2\right.\right.}{2\left(\operatorname{LegendreQ}\left(-\frac{1}{2},1,\frac{2-x}{x}\right)c_1 + \operatorname{LegendreP}\left(-\frac{1}{2},1,\frac{2-x}{x}\right)\right)(x-1)}$$

✓ Solution by Mathematica
Time used: 34.239 (sec). Leaf size: 77

```
DSolve[2*x*(x-1)*y'[x] + (x-1)*y[x]^2 - x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2\left(\pi G_{2,2}^{2,0}\left(x\middle|\begin{array}{c}\frac{1}{2},\frac{3}{2}\\0,1\end{array}\right) + c_1(\operatorname{EllipticK}(x) - \operatorname{EllipticE}(x))\right)}{\pi G_{2,2}^{2,0}\left(x\middle|\begin{array}{c}\frac{1}{2},\frac{3}{2}\\0,0\end{array}\right) + 2c_1\operatorname{EllipticE}(x)}$$

$$y(x) \to 1 - \frac{\operatorname{EllipticK}(x)}{\operatorname{EllipticE}(x)}$$

1.166 problem 167

Internal problem ID [8503]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 167.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Riccati]`

$$\boxed{3x^2y' - 7y^2 - 3yx = x^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve(3*x^2*diff(y(x),x) - 7*y(x)^2 - 3*x*y(x) - x^2=0,y(x), singsol=all)
```

$$y(x) = \frac{\tan\left(\frac{(\ln(x)+c_1)\sqrt{7}}{3}\right)x\sqrt{7}}{7}$$

✓ Solution by Mathematica
Time used: 0.297 (sec). Leaf size: 29

```
DSolve[3*x^2*y'[x] - 7*y[x]^2 - 3*x*y[x] - x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x\tan\left(\frac{1}{3}\sqrt{7}(\log(x)+3c_1)\right)}{\sqrt{7}}$$

1.167 problem 168

Internal problem ID [8504]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 168.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$3(x^2-4)\,y'+y^2-yx=3$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 167

```
dsolve(3*(x^2-4)*diff(y(x),x) + y(x)^2 - x*y(x) - 3=0,y(x), singsol=all)
```

$y(x) =$

$$-\frac{\left((x-2)\left(-2x-4\right)^{\frac{1}{3}}\text{hypergeom}\left(\right.\right.}{\left(-2x-4\right)^{\frac{1}{3}}(x-2)(x+2)^2\,\text{hypergeom}\left(\left[-\frac{1}{6},\frac{1}{6}\right],\left[-\frac{1}{3}\right],-\frac{4}{x-2}\right)+32\left(x-\frac{5}{4}\right)c_1(x+2)^2\,\text{hypergeom}\left(\right.}$$

✓ Solution by Mathematica
Time used: 0.404 (sec). Leaf size: 135

```
DSolve[3*(x^2-4)*y'[x] + y[x]^2 - x*y[x] - 3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{-2c_1xP_{-\frac{1}{6}}^{\frac{1}{3}}\left(\frac{x}{2}\right)+3c_1P_{\frac{5}{6}}^{\frac{1}{3}}\left(\frac{x}{2}\right)-2xQ_{-\frac{1}{6}}^{\frac{1}{3}}\left(\frac{x}{2}\right)+3Q_{\frac{5}{6}}^{\frac{1}{3}}\left(\frac{x}{2}\right)}{Q_{-\frac{1}{6}}^{\frac{1}{3}}\left(\frac{x}{2}\right)+c_1P_{-\frac{1}{6}}^{\frac{1}{3}}\left(\frac{x}{2}\right)}$$

$$y(x)\to\frac{3P_{\frac{5}{6}}^{\frac{1}{3}}\left(\frac{x}{2}\right)}{P_{-\frac{1}{6}}^{\frac{1}{3}}\left(\frac{x}{2}\right)}-2x$$

1.168 problem 169

Internal problem ID [8505]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 169.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Abel]`

$$(xa+b)^2\, y' + (xa+b)\, y^3 + cy^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 126

```
dsolve((a*x+b)^2*diff(y(x),x) + (a*x+b)*y(x)^3 + c*y(x)^2=0,y(x), singsol=all)
```

$$\frac{\left(\sqrt{a}\, b + a^{\frac{3}{2}} x\right) e^{-\frac{((ax+b+c)y(x)+a(ax+b))((-ax-b+c)y(x)+a(ax+b))}{2y(x)^2(ax+b)^2 a}} + \frac{c\sqrt{2}\,\sqrt{\pi}\, e^{\frac{1}{2a}} \operatorname{erf}\left(\frac{(y(x)c+a(ax+b))\sqrt{2}}{2\sqrt{a}\, y(x)(ax+b)}\right)}{2} + a^{\frac{3}{2}} c_1}{a^{\frac{3}{2}}} = 0$$

✓ Solution by Mathematica

Time used: 1.441 (sec). Leaf size: 149

```
DSolve[(a*x+b)^2*y'[x] + (a*x+b)*y[x]^3 + c*y[x]^2==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$\text{Solve}\left[-\frac{c}{\sqrt{-a(ax+b)^2}} = \frac{2\exp\left(\frac{1}{2}\left(-\frac{c}{\sqrt{-a(ax+b)^2}} - \frac{\left(-a(ax+b)^2\right)^{3/2}}{ay(x)(ax+b)^3}\right)^2\right)}{\sqrt{2\pi}\operatorname{erfi}\left(\frac{-\frac{c}{\sqrt{-a(ax+b)^2}} - \frac{\left(-a(ax+b)^2\right)^{3/2}}{ay(x)(ax+b)^3}}{\sqrt{2}}\right) + 2c_1}, y(x)\right]$$

1.169 problem 170

Internal problem ID [8506]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 170.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Riccati]`

$$\boxed{y'x^3 - y^2 = x^4}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^3*diff(y(x),x) - y(x)^2 - x^4=0,y(x), singsol=all)
```

$$y(x) = \frac{x^2(\ln(x) - c_1 - 1)}{\ln(x) - c_1}$$

✓ Solution by Mathematica

Time used: 0.164 (sec). Leaf size: 29

```
DSolve[x^3*y'[x] - y[x]^2 - x^4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2(\log(x) - 1 + c_1)}{\log(x) + c_1}$$
$$y(x) \to x^2$$

1.170 problem 171

Internal problem ID [8507]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 171.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{y'x^3 - y^2 - x^2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^3*diff(y(x),x) - y(x)^2 - x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{c_1x + 1}$$

✓ Solution by Mathematica

Time used: 0.135 (sec). Leaf size: 22

```
DSolve[x^3*y'[x] - y[x]^2 - x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{1 + c_1x}$$
$$y(x) \to 0$$

1.171 problem 172

Internal problem ID [8508]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 172.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Riccati]`

$$\boxed{y'x^3 - x^4y^2 + x^2y = -20}$$

✓ Solution by Maple
Time used: 0.203 (sec). Leaf size: 26

```
dsolve(x^3*diff(y(x),x) - x^4*y(x)^2 + x^2*y(x) + 20=0,y(x), singsol=all)
```

$$y(x) = \frac{5x^9 + 4c_1}{(-x^9 + c_1)\, x^2}$$

✓ Solution by Mathematica
Time used: 0.16 (sec). Leaf size: 36

```
DSolve[x^3*y'[x] - x^4*y[x]^2 + x^2*y[x] + 20==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-5x^9 + 4c_1}{x^2\,(x^9 + c_1)}$$
$$y(x) \to \frac{4}{x^2}$$

1.172 problem 173

Internal problem ID [8509]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 173.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x^3 - y^2x^6 - (2x-3)\,x^2y = -3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x^3*diff(y(x),x) - x^6*y(x)^2 - (2*x-3)*x^2*y(x) + 3=0,y(x), singsol=all)
```

$$y(x) = \frac{-3\,\mathrm{e}^{4x}c_1 - 3}{x^3\left(\mathrm{e}^{4x}c_1 - 3\right)}$$

✓ Solution by Mathematica
Time used: 0.165 (sec). Leaf size: 34

```
DSolve[x^3*y'[x] - x^6*y[x]^2 - (2*x-3)*x^2*y[x] + 3==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{-3 + \frac{1}{\frac{1}{4}+c_1e^{4x}}}{x^3}$$
$$y(x) \to -\frac{3}{x^3}$$

1.173 problem 174

Internal problem ID [8510]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 174.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x(x^2+1)\,y'+x^2y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x*(x^2+1)*diff(y(x),x) + x^2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1}{\sqrt{x^2+1}}$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 22

```
DSolve[x*(x^2+1)*y'[x] + x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_1}{\sqrt{x^2+1}}$$
$$y(x)\to 0$$

1.174 problem 175

Internal problem ID [8511]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 175.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$x(x^2-1)\,y' - (2x^2-1)\,y = -x^3a$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x*(x^2-1)*diff(y(x),x) - (2*x^2-1)*y(x) + a*x^3=0,y(x), singsol=all)
```

$$y(x) = x\left(\sqrt{x-1}\,\sqrt{x+1}\,c_1 + a\right)$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 23

```
DSolve[x*(x^2-1)*y'[x] - (2*x^2-1)*y[x] + a*x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(a + c_1\sqrt{1-x^2}\right)$$

1.175 problem 176

Internal problem ID [8512]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 176.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{x(x^2-1)\,y'+(x^2-1)\,y^2=x^2}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 30

```
dsolve(x*(x^2-1)*diff(y(x),x) + (x^2-1)*y(x)^2 - x^2=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\,\mathrm{EllipticCE}(x)+\mathrm{EllipticE}(x)-\mathrm{EllipticK}(x)}{c_1\,\mathrm{EllipticCE}(x)-c_1\,\mathrm{EllipticCK}(x)+\mathrm{EllipticE}(x)}$$

✓ Solution by Mathematica

Time used: 0.9 (sec). Leaf size: 91

```
DSolve[x*(x^2-1)*y'[x] + (x^2-1)*y[x]^2 - x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\frac{2\left(\pi G_{2,2}^{2,0}\left(x^2\middle|\begin{matrix}\frac{1}{2},\frac{3}{2}\\0,1\end{matrix}\right)+c_1(\mathrm{EllipticK}(x^2)-\mathrm{EllipticE}(x^2))\right)}{\pi G_{2,2}^{2,0}\left(x^2\middle|\begin{matrix}\frac{1}{2},\frac{3}{2}\\0,0\end{matrix}\right)+2c_1\,\mathrm{EllipticE}(x^2)}$$

$$y(x)\to1-\frac{\mathrm{EllipticK}(x^2)}{\mathrm{EllipticE}(x^2)}$$

1.176 problem 177

Internal problem ID [8513]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 177.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{x^2(x-1)\,y'-y^2-x(x-2)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*(x-1)*diff(y(x),x) - y(x)^2 - x*(x-2)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{x^2}{1+c_1\,(x-1)}$$

✓ Solution by Mathematica

Time used: 0.207 (sec). Leaf size: 25

```
DSolve[x^2*(x-1)*y'[x] - y[x]^2 - x*(x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x^2}{c_1(-x)+1+c_1}$$
$$y(x)\to 0$$

1.177 problem 178

Internal problem ID [8514]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 178.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{2x(x^2-1)\,y'+2(x^2-1)\,y^2-(3x^2-5)\,y=-x^2+3}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 105

```
dsolve(2*x*(x^2-1)*diff(y(x),x) + 2*(x^2-1)*y(x)^2 - (3*x^2-5)*y(x) + x^2 - 3=0,y(x), singso
```

$$y(x)=\frac{2\sqrt{2}\,\mathrm{EllipticF}\left(\sqrt{x+1},\frac{\sqrt{2}}{2}\right)\sqrt{-x}\,\sqrt{1-x}\,\sqrt{x+1}-\sqrt{x-1}\,\sqrt{x}\,\sqrt{x+1}\,c_1+2x}{\sqrt{x+1}\,\left(2\,\mathrm{EllipticF}\left(\sqrt{x+1},\frac{\sqrt{2}}{2}\right)\sqrt{-x}\,\sqrt{2}\,\sqrt{1-x}-c_1\sqrt{x}\,\sqrt{x-1}\right)}$$

✓ Solution by Mathematica

Time used: 20.302 (sec). Leaf size: 54

```
DSolve[2*x*(x^2-1)*y'[x] + 2*(x^2-1)*y[x]^2 - (3*x^2-5)*y[x] + x^2 - 3==0,y[x],x,IncludeSing
```

$$y(x)\to 1+\frac{\sqrt{x}}{\sqrt{1-x^2}\left(2\sqrt{x}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},x^2\right)+c_1\right)}$$
$$y(x)\to 1$$

1.178 problem 179

Internal problem ID [8515]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 179.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{3x(x^2-1)\,y'+xy^2-(x^2+1)\,y=3x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 190

```
dsolve(3*x*(x^2-1)*diff(y(x),x) + x*y(x)^2 - (x^2+1)*y(x) - 3*x=0,y(x), singsol=all)
```

$$y(x) = \frac{80c_1\sqrt{3}\,\pi\left(x^2-\frac{2}{5}\right)\operatorname{LegendreP}\left(-\frac{1}{6},-\frac{1}{3},\frac{-x^2-1}{x^2-1}\right)+315\Gamma\left(\frac{2}{3}\right)\left(\frac{24(x^2)^{\frac{1}{3}}\operatorname{LegendreP}\left(-\frac{1}{6},\frac{1}{3},\frac{-x^2-1}{x^2-1}\right)\Gamma\left(\frac{2}{3}\right)x^{\frac{4}{3}}}{35}+(x^2)^{\frac{1}{6}}\left(\right.\right.}{x^{\frac{1}{3}}\left(16x^{\frac{2}{3}}\pi\sqrt{3}\operatorname{LegendreP}\left(-\frac{1}{6},-\frac{1}{3},\frac{-x^2-1}{x^2-1}\right)c_1+72\left(\right.\right.}$$

✓ Solution by Mathematica

Time used: 4.513 (sec). Leaf size: 3149

```
DSolve[3*x*(x^2-1)*y'[x] + x*y[x]^2 - (x^2+1)*y[x] - 3*x==0,y[x],x,IncludeSingularSolutions
```

Too large to display

1.179 problem 180

Internal problem ID [8516]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 180.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Riccati]`

$$(a\,x^2 + xb + c)\,(xy' - y) - y^2 = -x^2$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 58

```
dsolve((a*x^2+b*x+c)*(x*diff(y(x),x)-y(x)) - y(x)^2 + x^2=0,y(x), singsol=all)
```

$$y(x) = -\tanh\left(\frac{c_1\sqrt{4ac-b^2} + 2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}\right)x$$

✓ Solution by Mathematica

Time used: 1.181 (sec). Leaf size: 116

```
DSolve[(a*x^2+b*x+c)*(x*y'[x]-y[x]) - y[x]^2 + x^2==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to -\frac{x\left(-1+\exp\left(\frac{4\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+2c_1\right)\right)}{1+\exp\left(\frac{4\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+2c_1\right)}$$

$$y(x) \to -x$$

$$y(x) \to x$$

1.180 problem 181

Internal problem ID [8517]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 181.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Riccati, _special]]`

$$\boxed{x^4(y' + y^2) = -a}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(x^4*(diff(y(x),x)+y(x)^2) + a=0,y(x), singsol=all)
```

$$y(x) = \frac{-\tan\left(\frac{\sqrt{a}\,(c_1 x-1)}{x}\right)\sqrt{a}+x}{x^2}$$

✓ Solution by Mathematica

Time used: 0.36 (sec). Leaf size: 111

```
DSolve[x^4*(y'[x]+y[x]^2) + a==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-2iac_1e^{\frac{2i\sqrt{a}}{x}}+\sqrt{a}\left(1+2c_1xe^{\frac{2i\sqrt{a}}{x}}\right)-ix}{x^2\left(2\sqrt{a}c_1e^{\frac{2i\sqrt{a}}{x}}-i\right)}$$

$$y(x) \to \frac{x-i\sqrt{a}}{x^2}$$

1.181 problem 182

Internal problem ID [8518]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 182.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{x(x^3-1)\,y' - 2xy^2 + y = -x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x*(x^3-1)*diff(y(x),x) - 2*x*y(x)^2 + y(x) + x^2=0,y(x), singsol=all)
```

$$y(x) = \frac{x(x+c_1)}{c_1x^2+1}$$

✓ Solution by Mathematica

Time used: 0.368 (sec). Leaf size: 31

```
DSolve[x*(x^3-1)*y'[x] - 2*x*y[x]^2 + y[x] + x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x(1+2c_1x)}{x^2+2c_1}$$
$$y(x) \to x^2$$

1.182 problem 183

Internal problem ID [8519]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 183.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(2x^4 - x)\, y' - 2(x^3 - 1)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve((2*x^4-x)*diff(y(x),x) - 2*(x^3-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^2}{(2x^3 - 1)^{\frac{1}{3}}}$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 27

```
DSolve[(2*x^4-x)*y'[x] - 2*(x^3-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 x^2}{\sqrt[3]{1 - 2x^3}}$$
$$y(x) \to 0$$

1.183 problem 184

Internal problem ID [8520]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 184.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{\left(a\,x^{2}+xb+c\right)^{2}\left(y'+y^{2}\right)=-A}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 493

```
dsolve((a*x^2+b*x+c)^2*(diff(y(x),x)+y(x)^2) + A=0,y(x), singsol=all)
```

$$y(x) = \frac{2\left(c_1\left(i\sqrt{\frac{-4ac+b^2-4A}{a^2}}\,a\sqrt{4ac-b^2}-2\sqrt{-4ac+b^2}\left(ax+\frac{b}{2}\right)\right)\left(\frac{-b+i\sqrt{4ac-b^2}-2ax}{b+i\sqrt{4ac-b^2}+2ax}\right)^{-\frac{a\sqrt{\frac{-4ac+b^2-4A}{a^2}}}{2\sqrt{-4ac+b^2}}}-\left(\frac{-b+i\sqrt{4}}{b+i\sqrt{4a}}\right.\cdots}{\sqrt{-4ac+b^2}\left(b+i\sqrt{4ac-b^2}+2ax\right)\left(-b+i\sqrt{4ac-b^2}-2ax\right)\left(c_1\left(\frac{-b+i\sqrt{4ac-}}{b+i\sqrt{4ac-b}}\right.\cdots\right.}$$

✓ Solution by Mathematica

Time used: 3.439 (sec). Leaf size: 743

```
DSolve[(a*x^2+b*x+c)^2*(y'[x]+y[x]^2) + A==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{b^2 c_1 \left(-\exp\left(\frac{2\sqrt{4ac-b^2}\sqrt{1-\frac{4A}{b^2-4ac}}\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{b^2-4ac}}\right)\right) + bc_1\sqrt{b^2-4ac}\sqrt{1-\frac{4A}{b^2-4ac}}\exp\left(\frac{2\sqrt{4ac-b^2}\sqrt{1-\frac{4A}{b^2-4ac}}\,\mathrm{a}}{\sqrt{b^2-4a}}\right.}{}$$

$$y(x) \to \frac{2ax\sqrt{b^2-4ac}\sqrt{1-\frac{4A}{b^2-4ac}} + b\sqrt{b^2-4ac}\sqrt{1-\frac{4A}{b^2-4ac}} + 4ac + 4A - b^2}{2\sqrt{b^2-4ac}\sqrt{1-\frac{4A}{b^2-4ac}}(x(ax+b)+c)}$$

1.184 problem 185

Internal problem ID [8521]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 185.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Abel]`

$$\boxed{x^7y' + 2(x^2+1)\,y^3 + 5x^3y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 78

```
dsolve(x^7*diff(y(x),x) + 2*(x^2+1)*y(x)^3 + 5*x^3*y(x)^2=0,y(x), singsol=all)
```

$$c_1 + \frac{x}{\left(\frac{x^6+y(x)^2x^2+2x^3y(x)+y(x)^2}{y(x)^2x^2}\right)^{\frac{1}{4}}} + \frac{(x^3+y(x))\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{5}{4}\right],\left[\frac{3}{2}\right],-\frac{(x^3+y(x))^2}{x^2y(x)^2}\right)}{2xy(x)} = 0$$

✓ Solution by Mathematica

Time used: 0.319 (sec). Leaf size: 123

```
DSolve[x^7*y'[x] + 2*(x^2+1)*y[x]^3 + 5*x^3*y[x]^2==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$\mathrm{Solve}\left[c_1 = \frac{\frac{1}{2}\sqrt[4]{1-\left(\frac{ix^2}{y(x)}+\frac{i}{x}\right)^2}\left(\frac{ix^2}{y(x)}+\frac{i}{x}\right)\mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{3}{2},\left(\frac{ix^2}{y(x)}+\frac{i}{x}\right)^2\right)+ix}{\sqrt[4]{-1+\left(\frac{ix^2}{y(x)}+\frac{i}{x}\right)^2}},y(x)\right]$$

1.185 problem 186

Internal problem ID [8522]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 186.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _Riccati]`

$$\boxed{x^n y' + y^2 - (n-1)\, x^{n-1} y = -x^{2n-2}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(x^n*diff(y(x),x) + y(x)^2 -(n-1)*x^(n-1)*y(x) + x^(2*n-2)=0,y(x), singsol=all)
```

$$y(x) = \tan\left(-\ln\left(x\right) + c_1\right) x^{n-1}$$

✓ Solution by Mathematica

Time used: 0.603 (sec). Leaf size: 19

```
DSolve[x^n*y'[x] + y[x]^2 -(n-1)*x^(n-1)*y[x] + x^(2*n-2)==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to x^{n-1} \tan(-\log(x) + c_1)$$

1.186 problem 187

Internal problem ID [8523]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 187.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _Riccati]`

$$x^n y' - a y^2 = b\, x^{2n-2}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 59

```
dsolve(x^n*diff(y(x),x) - a*y(x)^2 - b*x^(2*n-2)=0,y(x), singsol=all)
```

$$y(x) = \frac{x^{n-1}\left(n-1+\tan\left(\frac{\sqrt{4ab-n^2+2n-1}\,(\ln(x)-c_1)}{2}\right)\sqrt{4ab-n^2+2n-1}\right)}{2a}$$

✓ Solution by Mathematica

Time used: 0.537 (sec). Leaf size: 202

```
DSolve[x^n*y'[x]- a*y[x]^2 - b*x^(2*n-2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{n-1}\left(\left(-\sqrt{a}\sqrt{b}\sqrt{\frac{(n-1)^2}{ab}-4}+n-1\right)x^{\sqrt{a}\sqrt{b}\sqrt{\frac{(n-1)^2}{ab}-4}}+c_1\left(\sqrt{a}\sqrt{b}\sqrt{\frac{(n-1)^2}{ab}-4}+n-1\right)\right)}{2a\left(x^{\sqrt{a}\sqrt{b}\sqrt{\frac{(n-1)^2}{ab}-4}}+c_1\right)}$$

$$y(x) \to \frac{x^{n-1}\left(\sqrt{a}\sqrt{b}\sqrt{\frac{(n-1)^2}{ab}-4}+n-1\right)}{2a}$$

1.187 problem 188

Internal problem ID [8524]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 188.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _Abel]`

$$\boxed{x^{1+2n}y' - ay^3 = b\,x^{3n}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(x^(2*n+1)*diff(y(x),x) - a*y(x)^3 - b*x^(3*n)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{RootOf}\left(-\ln(x) + c_1 + \int^{\_Z} \frac{1}{\_a^3 a - \_an + b} d\_a\right) x^n$$

✓ Solution by Mathematica

Time used: 0.315 (sec). Leaf size: 331

```
DSolve[x^(2*n+1)*y'[x] - a*y[x]^3 - b*x^(3*n)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{3}ab^2\text{RootSum}\left[\#1^9ab^2+3\#1^6ab^2+3\#1^3ab^2-\#1^3n^3\right.\right.$$

$$+ab^2\&,\frac{\#1^6\log\left(y(x)\sqrt[3]{\frac{ax^{-3n}}{b}}-\#1\right)+\#1^4\sqrt[3]{\frac{n^3}{ab^2}}\log\left(y(x)\sqrt[3]{\frac{ax^{-3n}}{b}}-\#1\right)+2\#1^3\log\left(y(x)\sqrt[3]{\frac{ax^{-3n}}{b}}\right.}{3\#1^8ab^2}$$

$$\left.+c_1,y(x)\right]$$

1.188 problem 189

Internal problem ID [8525]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 189.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$x^{m(n-1)+n}y' - ay^n = b\,x^{n(m+1)}$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 62

```
dsolve(x^(m*(n-1)+n)*diff(y(x),x) - a*y(x)^n - b*x^(n*(m+1))=0,y(x), singsol=all)
```

$$-x^{n(m+1)}\left(\int_{\_b}^{y(x)}\frac{1}{b\,x^{(1+n)(m+1)} - \_a\,(m+1)\,x^{n(m+1)} + a\,x^{m+1}\_a^{n}}d\_a\right) + \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.386 (sec). Leaf size: 91

```
DSolve[x^(m*(n-1)+n)*y'[x] - a*y[x]^n - b*x^(n*(m+1))==0,y[x],x,IncludeSingularSolutions ->
```

$$\text{Solve}\left[\int_1^{\left(\frac{ax^{-((m+1)n)}}{b}\right)^{\frac{1}{n}}y(x)}\frac{1}{K[1]^n - \left(\frac{b^{1-n}(m+1)^n}{a}\right)^{\frac{1}{n}}K[1]+1}dK[1] = bx^{m+1}\log(x)\left(\frac{ax^{-((m+1)n)}}{b}\right)^{\frac{1}{n}} + c_1, y(x)\right]$$

1.189 problem 190

Internal problem ID [8526]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 190.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sqrt{x^2-1}\,y' - \sqrt{y^2-1} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(sqrt(x^2-1)*diff(y(x),x) - sqrt(y(x)^2-1)=0,y(x), singsol=all)
```

$$\ln\left(x+\sqrt{x^2-1}\right) - \ln\left(y(x)+\sqrt{y\left(x\right)^2-1}\right) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 6.494 (sec). Leaf size: 153

```
DSolve[Sqrt[x^2-1]*y'[x] - Sqrt[y[x]^2-1]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}e^{-c_1}\sqrt{2x^2-2\sqrt{x^2-1}x+e^{4c_1}\left(2x^2+2\sqrt{x^2-1}x-1\right)-1+2e^{2c_1}}$$
$$y(x) \to \frac{1}{2}e^{-c_1}\sqrt{2x^2-2\sqrt{x^2-1}x+e^{4c_1}\left(2x^2+2\sqrt{x^2-1}x-1\right)-1+2e^{2c_1}}$$
$$y(x) \to -1$$
$$y(x) \to 1$$

1.190 problem 191

Internal problem ID [8527]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 191.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\sqrt{1-x^2}\,y' - y\sqrt{y^2-1} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(sqrt(1-x^2)*diff(y(x),x) - y(x)*sqrt(y(x)^2-1)=0,y(x), singsol=all)
```

$$\arcsin(x) + \arctan\left(\frac{1}{\sqrt{y(x)^2-1}}\right) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 2.105 (sec). Leaf size: 90

```
DSolve[Sqrt[1-x^2]*y'[x] - y[x]*Sqrt[y[x]^2-1]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\sec^2\left(2\arctan\left(\frac{\sqrt{1-x^2}}{x+1}\right) - c_1\right)}$$
$$y(x) \to \sqrt{\sec^2\left(2\arctan\left(\frac{\sqrt{1-x^2}}{x+1}\right) - c_1\right)}$$
$$y(x) \to -1$$
$$y(x) \to 0$$
$$y(x) \to 1$$

1.191 problem 192

Internal problem ID [8528]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 192.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\sqrt{a^2+x^2}\,y'+y=\sqrt{a^2+x^2}-x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(sqrt(x^2+a^2)*diff(y(x),x) + y(x) - sqrt(x^2+a^2) + x=0,y(x), singsol=all)
```

$$y(x)=\frac{a^2\ln\left(x+\sqrt{a^2+x^2}\right)+c_1}{x+\sqrt{a^2+x^2}}$$

✓ Solution by Mathematica

Time used: 8.215 (sec). Leaf size: 103

```
DSolve[Sqrt[x^2+a^2]*y'[x] + y[x] - Sqrt[x^2+a^2] + x==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{1}{2}\left(x-\sqrt{a^2+x^2}\right)\left(\log\left(1-\frac{x}{\sqrt{a^2+x^2}}\right)-\log\left(\frac{x}{\sqrt{a^2+x^2}}+1\right)\right)+\frac{c_1\sqrt{1-\frac{x}{\sqrt{a^2+x^2}}}}{\sqrt{\frac{x}{\sqrt{a^2+x^2}}+1}}$$

1.192 problem 193

Internal problem ID [8529]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 193.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$xy' \ln(x) + y = ax(1 + \ln(x))$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve(x*diff(y(x),x)*ln(x) + y(x) - a*x*(ln(x)+1)=0,y(x), singsol=all)
```

$$y(x) = ax + \frac{c_1}{\ln(x)}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 16

```
DSolve[x*y'[x]*Log[x] + y[x] - a*x*(Log[x]+1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to ax + \frac{c_1}{\log(x)}$$

1.193 problem 194

Internal problem ID [8530]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 194.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{xy'\ln(x) - y^2\ln(x) - \left(2\ln(x)^2 + 1\right)y = \ln(x)^3}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x)*ln(x) - y(x)^2*ln(x) - (2*ln(x)^2+1)*y(x) - ln(x)^3=0,y(x), singsol=al
```

$$y(x) = -\frac{\ln(x)\left(\ln(x)^2 + c_1 + 2\right)}{\ln(x)^2 + c_1}$$

✓ Solution by Mathematica

Time used: 0.332 (sec). Leaf size: 38

```
DSolve[x*y'[x]*Log[x] - y[x]^2*Log[x] - (2*Log[x]^2+1)*y[x] - Log[x]^3==0,y[x],x,IncludeSing
```

$$y(x) \to -\frac{\log(x)\left(\log^2(x) + 2 + 2c_1\right)}{\log^2(x) + 2c_1}$$
$$y(x) \to -\log(x)$$

1.194 problem 195

Internal problem ID [8531]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 195.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' \sin(x) - y^2 \sin(x)^2 + (\cos(x) - 3\sin(x))\, y = -4}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 26

```
dsolve(sin(x)*diff(y(x),x) - y(x)^2*sin(x)^2 + (cos(x) - 3*sin(x))*y(x) + 4=0,y(x), singsol=
```

$$y(x) = -\frac{4\csc(x)\,(c_1 e^{5x} + 1)}{c_1 e^{5x} - 4}$$

✓ Solution by Mathematica

Time used: 0.267 (sec). Leaf size: 32

```
DSolve[Sin[x]*y'[x] - y[x]^2*Sin[x]^2 + (Cos[x] - 3*Sin[x])*y[x] + 4==0,y[x],x,IncludeSingul
```

$$y(x) \to \left(-4 + \frac{1}{\frac{1}{5} + c_1 e^{5x}}\right)\csc(x)$$
$$y(x) \to -4\csc(x)$$

1.195 problem 196

Internal problem ID [8532]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 196.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' \cos(x) + y = -(1+\sin(x))\cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(cos(x)*diff(y(x),x) + y(x) + (1 + sin(x))*cos(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-2\ln(\sec(x)+\tan(x)) + 2\ln(\cos(x)) + \sin(x) + c_1}{\sec(x)+\tan(x)}$$

✓ Solution by Mathematica
Time used: 0.705 (sec). Leaf size: 40

```
DSolve[Cos[x]*y'[x] + y[x] + (1 + Sin[x])*Cos[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2\operatorname{arctanh}\left(\tan\left(\frac{x}{2}\right)\right)}\left(\sin(x) + 4\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + c_1\right)$$

1.196 problem 197

Internal problem ID [8533]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 197.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' \cos(x) - y^4 - y \sin(x) = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 181

```
dsolve(cos(x)*diff(y(x),x) - y(x)^4 - y(x)*sin(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sec(x)\left(\cos(x)^3\left(-\cos(x)^3 c_1 + 2\sin(x)\cos(x)^2 + \sin(x)\right)^2\right)^{\frac{1}{3}}}{\cos(x)^3 c_1 - 2\sin(x)\cos(x)^2 - \sin(x)}$$
$$y(x) = \frac{\sec(x)\left(\cos(x)^3\left(-\cos(x)^3 c_1 + 2\sin(x)\cos(x)^2 + \sin(x)\right)^2\right)^{\frac{1}{3}}\left(1 + i\sqrt{3}\right)}{-2\cos(x)^3 c_1 + 4\sin(x)\cos(x)^2 + 2\sin(x)}$$
$$y(x) = \frac{\sec(x)\left(\cos(x)^3\left(-\cos(x)^3 c_1 + 2\sin(x)\cos(x)^2 + \sin(x)\right)^2\right)^{\frac{1}{3}}\left(i\sqrt{3} - 1\right)}{2\cos(x)^3 c_1 - 4\sin(x)\cos(x)^2 - 2\sin(x)}$$

✓ Solution by Mathematica

Time used: 0.829 (sec). Leaf size: 109

```
DSolve[Cos[x]*y'[x] - y[x]^4 - y[x]*Sin[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{\sqrt[3]{-\sin^3(x) + c_1 \cos^3(x) - 3\sin(x)\cos^2(x)}}$$
$$y(x) \to -\frac{\sqrt[3]{-1}}{\sqrt[3]{-\sin^3(x) + c_1 \cos^3(x) - 3\sin(x)\cos^2(x)}}$$
$$y(x) \to \frac{(-1)^{2/3}}{\sqrt[3]{-\sin^3(x) + c_1 \cos^3(x) - 3\sin(x)\cos^2(x)}}$$
$$y(x) \to 0$$

1.197 problem 198

Internal problem ID [8534]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 198.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\cos(x)\sin(x)\,y' - y = \sin(x)^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(sin(x)*cos(x)*diff(y(x),x) - y(x) - sin(x)^3=0,y(x), singsol=all)
```

$$y(x) = -\sin(x) + c_1\tan(x)$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 15

```
DSolve[Sin[x]*Cos[x]*y'[x] - y[x] - Sin[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sin(x) + c_1\tan(x)$$

1.198 problem 199

Internal problem ID [8535]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 199.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sin(2x)\,y' + \sin(2y) = 0}$$

✓ Solution by Maple

Time used: 0.485 (sec). Leaf size: 80

```
dsolve(sin(2*x)*diff(y(x),x) + sin(2*y(x))=0,y(x), singsol=all)
```

$$y(x) = \frac{\arctan\left(-\frac{2\sin(2x)c_1}{\cos(2x)c_1^2-c_1^2-\cos(2x)-1}, \frac{\cos(2x)c_1^2-c_1^2+\cos(2x)+1}{\cos(2x)c_1^2-c_1^2-\cos(2x)-1}\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.462 (sec). Leaf size: 68

```
DSolve[Sin[2*x]*y'[x] + Sin[2*y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}\arccos(-\tanh(\operatorname{arctanh}(\cos(2x)) + 2c_1))$$
$$y(x) \to \frac{1}{2}\arccos(-\tanh(\operatorname{arctanh}(\cos(2x)) + 2c_1))$$
$$y(x) \to 0$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

1.199 problem 200

Internal problem ID [8536]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 200.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\left(\sin \left(x\right)^{2} a+b\right) y'+ay\sin \left(2x\right)=-Ax\left(\sin \left(x\right)^{2} a+c\right)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 53

```
dsolve((a*sin(x)^2+b)*diff(y(x),x) + a*y(x)*sin(2*x) + A*x*(a*sin(x)^2+c)=0,y(x), singsol=al
```

$$y(x)=\frac{-\cos \left(2x\right) Aa-2Axa\sin \left(2x\right)+2x^{2}(a+2c)\,A-8c_{1}}{4a\cos \left(2x\right)-4a-8b}$$

✓ Solution by Mathematica
Time used: 0.383 (sec). Leaf size: 59

```
DSolve[(a*Sin[x]^2+b)*y'[x] + a*y[x]*Sin[2*x] + A*x*(a*Sin[x]^2+c)==0,y[x],x,IncludeSingular
```

$$y(x)\to \frac{2aAx^{2}-2aAx\sin(2x)-aA\cos(2x)+4Acx^{2}+4c_{1}}{4a\cos(2x)-4(a+2b)}$$

1.200 problem 201

Internal problem ID [8537]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 201.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{2f(x)\,y' + 2f(x)\,y^2 - f'(x)\,y = 2f(x)^2}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 23

```
dsolve(2*f(x)*diff(y(x),x)+2*f(x)*y(x)^2-diff(f(x),x)*y(x)-2*f(x)^2=0,y(x), singsol=all)
```

$$y(x) = i\tan\left(-i\left(\int \sqrt{f\left(x\right)}dx\right) + c_1\right)\sqrt{f\left(x\right)}$$

✓ Solution by Mathematica

Time used: 0.256 (sec). Leaf size: 39

```
DSolve[2*f[x]*y'[x]+2*f[x]*y[x]^2-f'[x]*y[x]-2*f[x]^2==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to i\sqrt{f(x)}\tan\left(i\int_1^x -\sqrt{f(K[1])}dK[1] + c_1\right)$$

1.201 problem 202

Internal problem ID [8538]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 202.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$\boxed{f(x)\, y' + g(x)\, s(y) = -h(x)}$$

✗ Solution by Maple

```
dsolve(f(x)*diff(y(x),x)+g(x)*s(y(x))+h(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[f[x]*y'[x]+g[x]*s[y[x]]+h[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.202 problem 203

Internal problem ID [8539]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 203.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' + y = -x^3}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+y(x)+x^3=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+y[x]+x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.203 problem 204

Internal problem ID [8540]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 204.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$yy' + ay = -x$$

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 56

```
dsolve(y(x)*diff(y(x),x)+a*y(x)+x=0,y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(\_Z^2 - e^{\operatorname{RootOf}\left(\left(4\,e^{\_Z}\cosh\left(\frac{\sqrt{a^2-4}\,(2c_1+\_Z+2\ln(x))}{2a}\right)^2+a^2-4\right)x^2\right)} + 1 + a\_Z\right)x$$

✓ Solution by Mathematica

Time used: 0.097 (sec). Leaf size: 70

```
DSolve[y[x]*y'[x]+a*y[x]+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\log\left(\frac{ay(x)}{x} + \frac{y(x)^2}{x^2} + 1\right) - \frac{a\arctan\left(\frac{a+\frac{2y(x)}{x}}{\sqrt{4-a^2}}\right)}{\sqrt{4-a^2}} = -\log(x) + c_1, y(x)\right]$$

1.204 problem 205

Internal problem ID [8541]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 205.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class A‘]]`

$$
yy' + ay = -\frac{(a^2 - 1)\, x}{4} - b\, x^n
$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+a*y(x)+(a^2-1)/(4)*x+b*x^n=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+a*y[x]+(a^2-1)/(4)*x+b*x^n==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.205 problem 206

Internal problem ID [8542]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 206.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' + ay = -b\,\mathrm{e}^{x} + 2a}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+a*y(x)+b*exp(x)-2*a=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+a*y[x]+b*Exp[x]-2*a==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.206 problem 207

Internal problem ID [8543]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 207.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{yy' + y^2 = -4x(1+x)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 37

```
dsolve(y(x)*diff(y(x),x)+y(x)^2+4*x*(x+1)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{\mathrm{e}^{-2x}c_1 - 4x^2}$$
$$y(x) = -\sqrt{\mathrm{e}^{-2x}c_1 - 4x^2}$$

✓ Solution by Mathematica
Time used: 6.025 (sec). Leaf size: 47

```
DSolve[y[x]*y'[x]+y[x]^2+4*x*(x+1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-4x^2 + c_1e^{-2x}}$$
$$y(x) \to \sqrt{-4x^2 + c_1e^{-2x}}$$

1.207 problem 208

Internal problem ID [8544]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 208.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{yy' + ay^2 = b\cos(c+x)}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 106

```
dsolve(y(x)*diff(y(x),x)+a*y(x)^2-b*cos(x+c)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{16c_1\left(a^2+\frac{1}{4}\right)^2 \mathrm{e}^{-2ax} + 16\left(a\cos(x+c) + \frac{\sin(x+c)}{2}\right)\left(a^2+\frac{1}{4}\right)b}}{4a^2+1}$$
$$y(x) = -\frac{\sqrt{16c_1\left(a^2+\frac{1}{4}\right)^2 \mathrm{e}^{-2ax} + 16\left(a\cos(x+c) + \frac{\sin(x+c)}{2}\right)\left(a^2+\frac{1}{4}\right)b}}{4a^2+1}$$

✓ Solution by Mathematica

Time used: 4.754 (sec). Leaf size: 120

```
DSolve[y[x]*y'[x]+a*y[x]^2-b*Cos[x+c]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{4ab\cos(c+x) + e^{-2ax}(4a^2c_1 + 2be^{2ax}\sin(c+x) + c_1)}}{\sqrt{4a^2+1}}$$
$$y(x) \to \frac{\sqrt{4ab\cos(c+x) + e^{-2ax}(4a^2c_1 + 2be^{2ax}\sin(c+x) + c_1)}}{\sqrt{4a^2+1}}$$

1.208 problem 209

Internal problem ID [8545]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 209.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{yy' - \sqrt{a y^2 + b} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(y(x)*diff(y(x),x)-sqrt(a*y(x)^2+b)=0,y(x), singsol=all)
```

$$\frac{-\sqrt{a y(x)^2 + b} + (x + c_1) a}{a} = 0$$

✓ Solution by Mathematica
Time used: 0.531 (sec). Leaf size: 94

```
DSolve[y[x]*y'[x]-Sqrt[a*y[x]^2+b]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-b + a^2(x + c_1)^2}}{\sqrt{a}}$$
$$y(x) \to \frac{\sqrt{-b + a^2(x + c_1)^2}}{\sqrt{a}}$$
$$y(x) \to -\frac{i\sqrt{b}}{\sqrt{a}}$$
$$y(x) \to \frac{i\sqrt{b}}{\sqrt{a}}$$

1.209 problem 210

Internal problem ID [8546]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 210.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yy' + xy^2 = 4x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 33

```
dsolve(y(x)*diff(y(x),x)+x*y(x)^2-4*x=0,y(x), singsol=all)
```

$$y(x) = \sqrt{c_1 e^{-x^2} + 4}$$
$$y(x) = -\sqrt{c_1 e^{-x^2} + 4}$$

✓ Solution by Mathematica
Time used: 1.932 (sec). Leaf size: 57

```
DSolve[y[x]*y'[x]+x*y[x]^2-4*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{4 + e^{-x^2+2c_1}}$$
$$y(x) \to \sqrt{4 + e^{-x^2+2c_1}}$$
$$y(x) \to -2$$
$$y(x) \to 2$$

1.210 problem 211

Internal problem ID [8547]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 211.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{yy' - x\,\mathrm{e}^{\frac{x}{y}} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(y(x)*diff(y(x),x)-x*exp(x/y(x))=0,y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(-\left(\int^{\_Z} \frac{\_a}{-\_a^2 + \mathrm{e}^{\frac{1}{\_a}}} d\_a\right) + \ln(x) + c_1\right) x$$

✓ Solution by Mathematica
Time used: 0.218 (sec). Leaf size: 41

```
DSolve[y[x]*y'[x]-x*Exp[x/y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{\frac{y(x)}{x}} \frac{K[1]}{K[1]^2 - e^{\frac{1}{K[1]}}} dK[1] = -\log(x) + c_1, y(x)\right]$$

1.211 problem 212

Internal problem ID [8548]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 212.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$yy' + f(x^2 + y^2)\, g(x) = -x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(y(x)*diff(y(x),x)+f(x^2+y(x)^2)*g(x)+x=0,y(x), singsol=all)
```

$$\int_{\_b}^{y(x)} \frac{\_a}{f(\_a^2 + x^2)} d\_a + \int g(x)\, dx - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.267 (sec). Leaf size: 95

```
DSolve[y[x]*y'[x]+f[x^2+y[x]^2]*g[x]+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} \left(\frac{K[2]}{f(x^2 + K[2]^2)} - \int_1^x -\frac{2K[1]K[2]f'(K[1]^2 + K[2]^2)}{f(K[1]^2 + K[2]^2)^2} dK[1]\right) dK[2] + \int_1^x \left(g(K[1]) + \frac{K[1]}{f(K[1]^2 + y(x)^2)}\right) dK[1] = c_1, y(x)\right]$$

1.212 problem 213

Internal problem ID [8549]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 213.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(y+1)\,y'-y=x}$$

✓ Solution by Maple
Time used: 0.984 (sec). Leaf size: 66

```
dsolve((y(x)+1)*diff(y(x),x)-y(x)-x=0,y(x), singsol=all)
```

$$-\frac{\ln\left(\frac{y(x)^2+(-x+3)y(x)-x^2+x+1}{(x-1)^2}\right)}{2}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(-2y(x)-3+x)\sqrt{5}}{5x-5}\right)}{5}-\ln(x-1)-c_1=0$$

✓ Solution by Mathematica
Time used: 0.102 (sec). Leaf size: 71

```
DSolve[(y[x]+1)*y'[x]-y[x]-x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\log\left(\frac{x^2-y(x)^2+(x-3)y(x)-x-1}{(x-1)^2}\right)+\log(1-x)=\frac{\operatorname{arctanh}\left(\frac{y(x)+2x-1}{\sqrt{5}(y(x)+1)}\right)}{\sqrt{5}}+c_1,y(x)\right]$$

1.213 problem 214

Internal problem ID [8550]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 214.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$(x+y-1)\,y'-y=-2x-3$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 51

```
dsolve((y(x)+x-1)*diff(y(x),x)-y(x)+2*x+3=0,y(x), singsol=all)
```

$$y(x)=\frac{5}{3}+\frac{\tan\left(\operatorname{RootOf}\left(\sqrt{2}\,\ln(2)+\sqrt{2}\,\ln\left(\sec(\_Z)^2(3x+2)^2\right)+2c_1\sqrt{2}-2\_Z\right)\right)\sqrt{2}\,(-3x-2)}{3}$$

✓ Solution by Mathematica

Time used: 0.119 (sec). Leaf size: 78

```
DSolve[(y[x]+x-1)*y'[x]-y[x]+2*x+3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2\sqrt{2}\arctan\left(\frac{-y(x)+2x+3}{\sqrt{2}(y(x)+x-1)}\right)=2\log\left(\frac{6x^2+3y(x)^2-10y(x)+8x+11}{(3x+2)^2}\right)+4\log(3x+2)+3c_1,y(x)\right]$$

1.214 problem 215

Internal problem ID [8551]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 215.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$(y+2x-2)\,y'-y=-x-1$$

✓ Solution by Maple

Time used: 0.281 (sec). Leaf size: 61

```
dsolve((y(x)+2*x-2)*diff(y(x),x)-y(x)+x+1=0,y(x), singsol=all)
```

$$y(x)=\frac{3}{2}-\frac{x}{2}+\frac{\sqrt{3}\,(3x-1)\tan\left(\operatorname{RootOf}\left(\sqrt{3}\,\ln(3)-2\sqrt{3}\,\ln(2)+\sqrt{3}\,\ln\left(\sec(\_Z)^2\,(3x-1)^2\right)+2\sqrt{3}\,c_1+6\_Z\right)\right)}{6}$$

✓ Solution by Mathematica

Time used: 0.113 (sec). Leaf size: 80

```
DSolve[(y[x]+2*x-2)*y'[x]-y[x]+x+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[6\sqrt{3}\arctan\left(\frac{4-3y(x)}{\sqrt{3}(y(x)+2x-2)}\right)=3\log\left(\frac{3x^2+3y(x)^2+3(x-3)y(x)-6x+7}{(1-3x)^2}\right)+6\log(3x-1)+2c_1,y(x)\right]$$

1.215 problem 216

Internal problem ID [8552]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 216.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(y-2x+1)\,y'+y=-x}$$

✓ Solution by Maple

Time used: 0.297 (sec). Leaf size: 61

```
dsolve((y(x)-2*x+1)*diff(y(x),x)+y(x)+x=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{3}\,\tan\left(\operatorname{RootOf}\left(\sqrt{3}\,\ln\left(3\right)-2\sqrt{3}\,\ln\left(2\right)+\sqrt{3}\,\ln\left(\sec\left(\_Z\right)^{2}\left(3x-1\right)^{2}\right)+2\sqrt{3}\,c_{1}+6\_Z\right)\right)\left(-3x+1\right)}{6} + \frac{x}{2}-\frac{1}{2}$$

✓ Solution by Mathematica

Time used: 0.107 (sec). Leaf size: 82

```
DSolve[(y[x]-2*x+1)*y'[x]+y[x]+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[6\sqrt{3}\arctan\left(\frac{3y(x)+1}{\sqrt{3}(-y(x)+2x-1)}\right)=3\log\left(\frac{3x^2+3y(x)^2-3(x-1)y(x)-3x+1}{(1-3x)^2}\right) + 6\log(3x-1)+2c_1,y(x)\right]$$

1.216 problem 217

Internal problem ID [8553]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 217.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]'], [_Ab`

$$\boxed{(-x^2+y)\,y' = x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve((y(x)-x^2)*diff(y(x),x)-x=0,y(x), singsol=all)
```

$$y(x) = x^2 + \frac{\operatorname{LambertW}\left(-4c_1 e^{-2x^2-1}\right)}{2} + \frac{1}{2}$$

✓ Solution by Mathematica
Time used: 5.05 (sec). Leaf size: 40

```
DSolve[(y[x]-x^2)*y'[x]-x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + \frac{1}{2}\left(1 + W\left(-e^{-2x^2-1+c_1}\right)\right)$$
$$y(x) \to x^2 + \frac{1}{2}$$

1.217 problem 218

Internal problem ID [8554]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 218.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\left(-x^2+y\right)y'+4yx=0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 57

```
dsolve((y(x)-x^2)*diff(y(x),x)+4*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=-\frac{c_1\sqrt{c_1^2-4x^2}}{2}+\frac{c_1^2}{2}-x^2$$
$$y(x)=\frac{c_1\sqrt{c_1^2-4x^2}}{2}+\frac{c_1^2}{2}-x^2$$

✓ Solution by Mathematica

Time used: 2.598 (sec). Leaf size: 246

```
DSolve[(y[x]-x^2)*y'[x]+4*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 \left(1 + \frac{2-2i}{\frac{i\sqrt{2}}{\sqrt{x^2\cosh\left(\frac{2c_1}{9}\right)+x^2\sinh\left(\frac{2c_1}{9}\right)-i}} - (1-i)}\right)$$

$$y(x) \to x^2 \left(1 + \frac{2-2i}{(-1+i) - \frac{i\sqrt{2}}{\sqrt{x^2\cosh\left(\frac{2c_1}{9}\right)+x^2\sinh\left(\frac{2c_1}{9}\right)-i}}}\right)$$

$$y(x) \to x^2 \left(1 + \frac{2-2i}{(-1+i) - \frac{\sqrt{2}}{\sqrt{x^2\cosh\left(\frac{2c_1}{9}\right)+x^2\sinh\left(\frac{2c_1}{9}\right)+i}}}\right)$$

$$y(x) \to x^2 \left(1 + \frac{2-2i}{\frac{\sqrt{2}}{\sqrt{x^2\cosh\left(\frac{2c_1}{9}\right)+x^2\sinh\left(\frac{2c_1}{9}\right)+i}} - (1-i)}\right)$$

$$y(x) \to 0$$

$$y(x) \to -x^2$$

1.218 problem 219

Internal problem ID [8555]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 219.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{(y+g(x))\,y' - f_2(x)\,y^2 - f_1(x)\,y = f_0(x)}$$

✗ Solution by Maple

```
dsolve((y(x)+g(x))*diff(y(x),x)-f__2(x)*y(x)^2-f__1(x)*y(x)-f__0(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y[x]+g[x])*y'[x]-f2[x]*y[x]^2-f1[x]*y[x]-f0[x]==0,y[x],x,IncludeSingularSolutions ->
```

Timed out

1.219 problem 220

Internal problem ID [8556]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 220.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{2yy' - xy^2 = x^3}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 43

```
dsolve(2*y(x)*diff(y(x),x)-x*y(x)^2-x^3=0,y(x), singsol=all)
```

$$y(x) = \sqrt{\mathrm{e}^{\frac{x^2}{2}} c_1 - x^2 - 2}$$
$$y(x) = -\sqrt{\mathrm{e}^{\frac{x^2}{2}} c_1 - x^2 - 2}$$

✓ Solution by Mathematica
Time used: 7.217 (sec). Leaf size: 57

```
DSolve[2*y[x]*y'[x]-x*y[x]^2-x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^2 + c_1 e^{\frac{x^2}{2}} - 2}$$
$$y(x) \to \sqrt{-x^2 + c_1 e^{\frac{x^2}{2}} - 2}$$

1.220 problem 221

Internal problem ID [8557]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 221.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(2y + x + 1)\, y' - 2y = x - 1}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 21

```
dsolve((2*y(x)+x+1)*diff(y(x),x)-(2*y(x)+x-1)=0,y(x), singsol=all)
```

$$y(x) = -\frac{x}{2} + \frac{2\,\text{LambertW}\left(\frac{c_1 e^{\frac{9x}{4}-\frac{1}{4}}}{4}\right)}{3} + \frac{1}{6}$$

✓ Solution by Mathematica
Time used: 4.843 (sec). Leaf size: 43

```
DSolve[(2*y[x]+x+1)*y'[x]-(2*y[x]+x-1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}\left(4W\left(-e^{\frac{9x}{4}-1+c_1}\right) - 3x + 1\right)$$
$$y(x) \to \frac{1}{6}(1 - 3x)$$

1.221 problem 222

Internal problem ID [8558]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 222.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(2y + x + 7)\, y' - y = -2x - 4}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 32

```
dsolve((2*y(x)+x+7)*diff(y(x),x)-y(x)+2*x+4=0,y(x), singsol=all)
```

$$y(x) = -2 + \tan\left(\operatorname{RootOf}\left(\ln\left(\sec\left(\_Z\right)^2\right) - \_Z + 2\ln\left(x+3\right) + 2c_1\right)\right)\left(-x-3\right)$$

✓ Solution by Mathematica
Time used: 0.062 (sec). Leaf size: 65

```
DSolve[(2*y[x]+x+7)*y'[x]-y[x]+2*x+4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2\arctan\left(\frac{y(x)-2(x+2)}{2y(x)+x+7}\right) + 2\log\left(\frac{4(x^2+y(x)^2+4y(x)+6x+13)}{5(x+3)^2}\right) + 4\log(x+3) + 5c_1 = 0, y(x)\right]$$

1.222 problem 223

Internal problem ID [8559]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 223.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{(2y - x)\, y' - y = 2x}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 51

```
dsolve((2*y(x)-x)*diff(y(x),x)-y(x)-2*x=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x - \sqrt{5c_1^2 x^2 + 4}}{2c_1}$$
$$y(x) = \frac{c_1 x + \sqrt{5c_1^2 x^2 + 4}}{2c_1}$$

✓ Solution by Mathematica

Time used: 0.469 (sec). Leaf size: 102

```
DSolve[(2*y[x]-x)*y'[x]-y[x]-2*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(x - \sqrt{5x^2 - 4e^{c_1}}\right)$$
$$y(x) \to \frac{1}{2}\left(x + \sqrt{5x^2 - 4e^{c_1}}\right)$$
$$y(x) \to \frac{1}{2}\left(x - \sqrt{5}\sqrt{x^2}\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{5}\sqrt{x^2} + x\right)$$

1.223 problem 224

Internal problem ID [8560]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 224.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$(2y - 6x)\, y' - y = -3x - 2$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve((2*y(x)-6*x)*diff(y(x),x)-y(x)+3*x+2=0,y(x), singsol=all)
```

$$y(x) = -\frac{2\,\text{LambertW}\left(-\frac{e^{\frac{25x}{4}-1-\frac{25c_1}{4}}}{2}\right)}{5} + 3x - \frac{2}{5}$$

✓ Solution by Mathematica

Time used: 3.708 (sec). Leaf size: 40

```
DSolve[(2*y[x]-6*x)*y'[x]-y[x]+3*x+2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3x - \frac{2}{5}\left(1 + W\left(-e^{\frac{25x}{4}-1+c_1}\right)\right)$$
$$y(x) \to 3x - \frac{2}{5}$$

1.224 problem 225

Internal problem ID [8561]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 225.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(4y + 2x + 3)\, y' - 2y = 1 + x}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 20

```
dsolve((4*y(x)+2*x+3)*diff(y(x),x)-2*y(x)-x-1=0,y(x), singsol=all)
```

$$y(x) = -\frac{x}{2} + \frac{\mathrm{LambertW}\left(c_1 \mathrm{e}^{5+8x}\right)}{8} - \frac{5}{8}$$

✓ Solution by Mathematica

Time used: 4.688 (sec). Leaf size: 39

```
DSolve[(4*y[x]+2*x+3)*y'[x]-2*y[x]-x-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}\left(W\left(-e^{8x-1+c_1}\right) - 4x - 5\right)$$
$$y(x) \to \frac{1}{8}(-4x - 5)$$

1.225 problem 226

Internal problem ID [8562]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 226.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$(4y - 2x - 3)\, y' + 2y = 1 + x$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 21

```
dsolve((4*y(x)-2*x-3)*diff(y(x),x)+2*y(x)-x-1=0,y(x), singsol=all)
```

$$y(x) = \frac{x}{2} - \frac{\text{LambertW}\left(-c_1 e^{5+8x}\right)}{8} + \frac{5}{8}$$

✓ Solution by Mathematica

Time used: 1.461 (sec). Leaf size: 41

```
DSolve[(4*y[x]-2*x-3)*y'[x]+2*y[x]-x-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}\left(-W\left(-e^{8x-1+c_1}\right) + 4x + 5\right)$$
$$y(x) \to \frac{1}{8}(4x + 5)$$

1.226 problem 227

Internal problem ID [8563]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 227.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd ty`

$$(4y - 3x - 5)\, y' - 3y = -7x - 2$$

✓ Solution by Maple
Time used: 0.297 (sec). Leaf size: 33

```
dsolve((4*y(x)-3*x-5)*diff(y(x),x)-3*y(x)+7*x+2=0,y(x), singsol=all)
```

$$y(x) = \frac{-\sqrt{4 - 6859\left(x - \frac{7}{19}\right)^2 c_1^2} + (57x + 95)\, c_1}{76 c_1}$$

✓ Solution by Mathematica
Time used: 0.129 (sec). Leaf size: 71

```
DSolve[(4*y[x]-3*x-5)*y'[x]-3*y[x]+7*x+2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-i\sqrt{19x^2 - 14x - 25 - 16c_1} + 3x + 5\right)$$
$$y(x) \to \frac{1}{4}\left(i\sqrt{19x^2 - 14x - 25 - 16c_1} + 3x + 5\right)$$

1.227 problem 228

Internal problem ID [8564]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 228.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$(4y + 11x - 11)\, y' - 25y = 8x - 62$$

✓ Solution by Maple

Time used: 0.515 (sec). Leaf size: 218

```
dsolve((4*y(x)+11*x-11) *diff(y(x),x)-25*y(x)-8*x+62=0,y(x), singsol=all)
```

$$y(x) = \frac{4\left(x+\frac{1}{2}\right)\left(i\sqrt{3}-1\right)\left(708588\sqrt{\left(-\frac{32}{177147}+\left(x-\frac{1}{9}\right)^2 c_1\right)\left(x-\frac{1}{9}\right)^2 c_1}+64-70858\right.}{i\sqrt{3}\left(708588\sqrt{\left(-\frac{32}{177147}+\left(x-\frac{1}{9}\right)^2 c_1\right)\left(x-\frac{1}{9}\right)^2 c_1}+64-708588\left(x-\frac{1}{9}\right)^2 c_1\right)^{\frac{2}{3}}-16i\sqrt{3}-\left(708588\right.}$$

✓ Solution by Mathematica

Time used: 60.17 (sec). Leaf size: 1677

```
DSolve[(4*y[x]+11*x-11)*y'[x]-25*y[x]-8*x+62==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

1.228 problem 229

Internal problem ID [8565]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 229.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd ty`

$$(12y - 5x - 8)\,y' - 5y = -2x - 3$$

✓ Solution by Maple
Time used: 0.297 (sec). Leaf size: 32

```
dsolve((12*y(x)-5*x-8)*diff(y(x),x)-5*y(x)+2*x+3=0,y(x), singsol=all)
```

$$y(x) = \frac{-\sqrt{(x+4)^2\, c_1^2 + 24} + (5x+8)\, c_1}{12c_1}$$

✓ Solution by Mathematica
Time used: 0.141 (sec). Leaf size: 77

```
DSolve[(12*y[x]-5*x-8)*y'[x]-5*y[x]+2*x+3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}\left(-i\sqrt{-x^2 - 8x - 16(4 + 9c_1)} + 5x + 8\right)$$
$$y(x) \to \frac{1}{12}\left(i\sqrt{-x^2 - 8x - 16(4 + 9c_1)} + 5x + 8\right)$$

1.229 problem 230

Internal problem ID [8566]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 230.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$a y y' + b y^2 = -f(x)$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 98

```
dsolve(a*y(x)*diff(y(x),x)+b*y(x)^2+f(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{e^{\frac{2bx}{a}} a \left(c_1 a - 2\left(\int e^{\frac{2bx}{a}} f(x)\, dx\right)\right)}\, e^{-\frac{2bx}{a}}}{a}$$

$$y(x) = -\frac{\sqrt{e^{\frac{2bx}{a}} a \left(c_1 a - 2\left(\int e^{\frac{2bx}{a}} f(x)\, dx\right)\right)}\, e^{-\frac{2bx}{a}}}{a}$$

✓ Solution by Mathematica
Time used: 0.334 (sec). Leaf size: 98

```
DSolve[a*y[x]*y'[x]+b*y[x]^2+f[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{-\frac{bx}{a}} \sqrt{2 \int_1^x -\frac{e^{\frac{2bK[1]}{a}} f(K[1])}{a} dK[1] + c_1}$$

$$y(x) \to e^{-\frac{bx}{a}} \sqrt{2 \int_1^x -\frac{e^{\frac{2bK[1]}{a}} f(K[1])}{a} dK[1] + c_1}$$

1.230 problem 231

Internal problem ID [8567]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 231.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$(ay + xb + c)\, y' + \alpha y = -\beta x - \gamma$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 236

```
dsolve((a*y(x)+b*x+c)*diff(y(x),x)+alpha*y(x)+beta*x+gamma=0,y(x), singsol=all)
```

$$y(x) = \frac{-((\beta x + \gamma) a + (-bx - c)\alpha)\sqrt{4a\beta - \alpha^2 - 2b\alpha - b^2}\tan\left(\mathrm{RootOf}\left(-2\sqrt{4a\beta - \alpha^2 - 2b\alpha - b^2}\ln(2) + \right.\right.}{}$$

✓ Solution by Mathematica

Time used: 1.756 (sec). Leaf size: 260

```
DSolve[(a*y[x]+b*x+c)*y'[x]+\[Alpha]*y[x]+\[Beta]*x+\[Gamma]==0,y[x],x,IncludeSingularSoluti
```

$$\text{Solve}\left[\frac{(b-\alpha)^2\left(-\frac{2\arctan\left(\frac{\frac{2(a(\gamma+\beta x)-\alpha bx+\alpha(-c))}{ay(x)+bx+c}+\alpha-b}{(\alpha-b)\sqrt{\frac{4(a\beta-\alpha b)}{(b-\alpha)^2}-1}}\right)}{\sqrt{\frac{4(a\beta-\alpha b)}{(b-\alpha)^2}-1}}-\log\left(\frac{(ay(x)+bx+c)\left((a(\gamma+\beta x)-\alpha bx+\alpha(-c))\left(\frac{a(\gamma+\beta x)-\alpha bx+\alpha(-c)}{ay(x)+bx+c}+c\right.\right.}{(-a(\gamma+\beta x)+\alpha bx+\alpha c)^2}\right.\right.}{2(a\beta-\alpha b)}\right.$$

$$\left.+c_1,y(x)\right]$$

1.231 problem 232

Internal problem ID [8568]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 232.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{xy'y + y^2 = -x^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 39

```
dsolve(x*y(x)*diff(y(x),x)+y(x)^2+x^2=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2x^4 + 4c_1}}{2x}$$
$$y(x) = \frac{\sqrt{-2x^4 + 4c_1}}{2x}$$

✓ Solution by Mathematica
Time used: 0.215 (sec). Leaf size: 46

```
DSolve[x*y[x]*y'[x]+y[x]^2+x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-\frac{x^4}{2} + c_1}}{x}$$
$$y(x) \to \frac{\sqrt{-\frac{x^4}{2} + c_1}}{x}$$

1.232 problem 233

Internal problem ID [8569]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 233.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _Bernoulli]`

$$\boxed{xy'y - y^2 = -a\,x^3\cos(x)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 30

```
dsolve(x*y(x)*diff(y(x),x)-y(x)^2+a*x^3*cos(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{-2a\sin(x) + c_1}\,x$$
$$y(x) = -\sqrt{-2a\sin(x) + c_1}\,x$$

✓ Solution by Mathematica
Time used: 0.351 (sec). Leaf size: 38

```
DSolve[x*y[x]*y'[x]-y[x]^2+a*x^3*Cos[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\sqrt{-2a\sin(x) + c_1}$$
$$y(x) \to x\sqrt{-2a\sin(x) + c_1}$$

1.233 problem 234

Internal problem ID [8570]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 234.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{xy'y - y^2 + yx = -x^3 + 2x^2}$$

✗ Solution by Maple

```
dsolve(x*y(x)*diff(y(x),x)-y(x)^2+x*y(x)+x^3-2*x^2=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y[x]*y'[x]-y[x]^2+x*y[x]+x^3-2*x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.234 problem 235

Internal problem ID [8571]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 235.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_exponential_symmetries], _rational, [_Abel`

$$\boxed{(yx + a)\, y' + by = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 55

```
dsolve((x*y(x)+a)*diff(y(x),x)+b*y(x)=0,y(x), singsol=all)
```

$$\frac{-\mathrm{e}^{\frac{y(x)}{b}} c_1 b x + \mathrm{expIntegral}_1\left(-\frac{y(x)}{b}\right) c_1 a + 1}{-\mathrm{e}^{\frac{y(x)}{b}} b x + a\ \mathrm{expIntegral}_1\left(-\frac{y(x)}{b}\right)} = 0$$

✓ Solution by Mathematica

Time used: 0.081 (sec). Leaf size: 40

```
DSolve[(x*y[x]+a)*y'[x]+b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[x = -\frac{a e^{-\frac{y(x)}{b}}\ \mathrm{ExpIntegralEi}\left(\frac{y(x)}{b}\right)}{b} + c_1 e^{-\frac{y(x)}{b}}, y(x)\right]$$

1.235 problem 236

Internal problem ID [8572]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 236.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$x(y+4)\,y' - y^2 - 2y = 2x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 121

```
dsolve(x*(y(x)+4)*diff(y(x),x)-y(x)^2-2*y(x)-2*x=0,y(x), singsol=all)
```

$$y(x) = \frac{-\sqrt{\frac{(x+4)c_1-4}{x+4}}\,x\sqrt{x+4} - 4\sqrt{x}}{-\sqrt{x+4}\,\sqrt{\frac{(x+4)c_1-4}{x+4}} + \sqrt{x}}$$
$$y(x) = \frac{\sqrt{\frac{(x+4)c_1-4}{x+4}}\,x\sqrt{x+4} - 4\sqrt{x}}{\sqrt{x+4}\,\sqrt{\frac{(x+4)c_1-4}{x+4}} + \sqrt{x}}$$

✓ Solution by Mathematica
Time used: 1.047 (sec). Leaf size: 89

```
DSolve[x*(y[x]+4)*y'[x]-y[x]^2-2*y[x]-2*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -4 + \frac{1}{\frac{1}{x+4} - \frac{\sqrt{x}}{(x+4)^{3/2}\sqrt{-\frac{4}{x+4}+c_1}}}$$
$$y(x) \to -4 + \frac{1}{\frac{1}{x+4} + \frac{\sqrt{x}}{(x+4)^{3/2}\sqrt{-\frac{4}{x+4}+c_1}}}$$
$$y(x) \to x$$

1.236 problem 237

Internal problem ID [8573]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 237.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$\boxed{x(y+a)\,y'+by=-cx}$$

✗ Solution by Maple

```
dsolve(x*(y(x)+a)*diff(y(x),x)+b*y(x)+c*x=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*(y[x]+a)*y'[x]+b*y[x]+c*x==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.237 problem 238

Internal problem ID [8574]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 238.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, [_Abel, ‘2n`

$$\boxed{(x(x+y)+a)\,y'-y(x+y)=b}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 91

```
dsolve((x*(y(x)+x)+a)*diff(y(x),x)-y(x)*(y(x)+x)-b=0,y(x), singsol=all)
```

$$y(x)=\frac{bac_1x+x+\sqrt{(a+b)\left(-1+\left(a\,x^2+b\,x^2+a^2\right)c_1\right)}}{a^2c_1-1}$$
$$y(x)=\frac{bac_1x+x-\sqrt{(a+b)\left(-1+\left(a\,x^2+b\,x^2+a^2\right)c_1\right)}}{a^2c_1-1}$$

✓ Solution by Mathematica

Time used: 5.281 (sec). Leaf size: 186

```
DSolve[(x*(y[x]+x)+a)*y'[x]-y[x]*(y[x]+x)-b==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\frac{-\frac{1}{\frac{a}{a^2+ax^2+bx^2}-\frac{x}{\left(a^2+ax^2+bx^2\right)^{3/2}\sqrt{-\frac{1}{(a+b)\left(a^2+ax^2+bx^2\right)}+c_1}}}+a+x^2}{x}$$
$$y(x)\to-\frac{-\frac{1}{\frac{a}{a^2+ax^2+bx^2}+\frac{x}{\left(a^2+ax^2+bx^2\right)^{3/2}\sqrt{-\frac{1}{(a+b)\left(a^2+ax^2+bx^2\right)}+c_1}}}+a+x^2}{x}$$
$$y(x)\to\frac{bx}{a}$$

1.238 problem 239

Internal problem ID [8575]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 239.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(yx - x^2)\, y' + y^2 - 3yx = 2x^2}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 59

```
dsolve((x*y(x)-x^2)*diff(y(x),x)+y(x)^2-3*x*y(x)-2*x^2=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^2 - \sqrt{2c_1^2 x^4 + 1}}{c_1 x}$$
$$y(x) = \frac{c_1 x^2 + \sqrt{2c_1^2 x^4 + 1}}{c_1 x}$$

✓ Solution by Mathematica
Time used: 0.681 (sec). Leaf size: 99

```
DSolve[(x*y[x]-x^2)*y'[x]+y[x]^2-3*x*y[x]-2*x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - \frac{\sqrt{2x^4 + e^{2c_1}}}{x}$$
$$y(x) \to x + \frac{\sqrt{2x^4 + e^{2c_1}}}{x}$$
$$y(x) \to x - \frac{\sqrt{2}\sqrt{x^4}}{x}$$
$$y(x) \to \frac{\sqrt{2}\sqrt{x^4}}{x} + x$$

1.239 problem 240

Internal problem ID [8576]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 240.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{2xy'y - y^2 = -xa}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
dsolve(2*x*y(x)*diff(y(x),x)-y(x)^2+a*x=0,y(x), singsol=all)
```

$$y(x) = \sqrt{-x\left(a\ln\left(x\right) - c_1\right)}$$
$$y(x) = -\sqrt{-x\left(a\ln\left(x\right) - c_1\right)}$$

✓ Solution by Mathematica

Time used: 0.413 (sec). Leaf size: 39

```
DSolve[2*x*y[x]*y'[x]-y[x]^2+a*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x(-a\log(x) + c_1)}$$
$$y(x) \to \sqrt{x(-a\log(x) + c_1)}$$

1.240 problem 241

Internal problem ID [8577]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 241.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{2xy'y - y^2 = -a\,x^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 29

```
dsolve(2*x*y(x)*diff(y(x),x)-y(x)^2+a*x^2=0,y(x), singsol=all)
```

$$y(x) = \sqrt{(-ax + c_1)\,x}$$
$$y(x) = -\sqrt{(-ax + c_1)\,x}$$

✓ Solution by Mathematica
Time used: 0.376 (sec). Leaf size: 37

```
DSolve[2*x*y[x]*y'[x]-y[x]^2+a*x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x(-ax + c_1)}$$
$$y(x) \to \sqrt{x(-ax + c_1)}$$

1.241 problem 242

Internal problem ID [8578]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 242.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2xy'y + 2y^2 = -1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 39

```
dsolve(2*x*y(x)*diff(y(x),x)+2*y(x)^2+1=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2x^2 + 4c_1}}{2x}$$
$$y(x) = \frac{\sqrt{-2x^2 + 4c_1}}{2x}$$

✓ Solution by Mathematica

Time used: 0.337 (sec). Leaf size: 128

```
DSolve[2*x*y[x]*y'[x]+2*y[x]^2+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-x^2 + e^{4c_1}}}{\sqrt{2}x}$$
$$y(x) \to \frac{\sqrt{-x^2 + e^{4c_1}}}{\sqrt{2}x}$$
$$y(x) \to -\frac{i}{\sqrt{2}}$$
$$y(x) \to \frac{i}{\sqrt{2}}$$
$$y(x) \to \frac{x}{\sqrt{2}\sqrt{-x^2}}$$
$$y(x) \to \frac{\sqrt{-x^2}}{\sqrt{2}x}$$

1.242 problem 243

Internal problem ID [8579]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 243.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{x(2y + x - 1)\, y' - y(y + 2x + 1) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 389

```
dsolve(x*(2*y(x)+x-1)*diff(y(x),x)-y(x)*(y(x)+2*x+1)=0,y(x), singsol=all)
```

$$y(x) = \frac{3\, 5^{\frac{1}{3}} \left(x\left(\sqrt{5}\, \sqrt{\frac{80c_1x^2-160c_1x+80c_1-x}{c_1}} + 20x - 20\right) c_1^2\right)^{\frac{1}{3}}}{40c_1} + \frac{3x 5^{\frac{2}{3}}}{40\left(x\left(\sqrt{5}\, \sqrt{\frac{80c_1x^2-160c_1x+80c_1-x}{c_1}} + 20x - 20\right) c_1^2\right)^{\frac{1}{3}}} - 1 + x$$

$$y(x) = \frac{\frac{3\, 5^{\frac{1}{3}} \left(-i\sqrt{3}-1\right) \left(x\left(\sqrt{5}\, \sqrt{\frac{80(x-1)^2c_1-x}{c_1}}+20x-20\right) c_1^2\right)^{\frac{2}{3}}}{80} + \frac{3c_1 \left(\frac{80(x-1)\left(x\left(\sqrt{5}\, \sqrt{\frac{80(x-1)^2c_1-x}{c_1}}+20x-20\right) c_1^2\right)^{\frac{1}{3}}}{3} + \left(i\sqrt{3}-1\right) x 5^{\frac{2}{3}}\right)}{80}}{c_1\left(x\left(\sqrt{5}\, \sqrt{\frac{80(x-1)^2c_1-x}{c_1}} + 20x - 20\right) c_1^2\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{3\left(5^{\frac{1}{3}} (1 - i\sqrt{3}) \left(x\left(\sqrt{5}\, \sqrt{\frac{80(x-1)^2c_1-x}{c_1}} + 20x - 20\right) c_1^2\right)^{\frac{2}{3}} + c_1 \left(\frac{80(1-x)\left(x\left(\sqrt{5}\, \sqrt{\frac{80(x-1)^2c_1-x}{c_1}}+20x-20\right) c_1^2\right)}{3}\right.\right.}{80\left(x\left(\sqrt{5}\, \sqrt{\frac{80(x-1)^2c_1-x}{c_1}} + 20x - 20\right) c_1^2\right)^{\frac{1}{3}} c_1}$$

✓ Solution by Mathematica

Time used: 43.09 (sec). Leaf size: 463

```
DSolve[x*(2*y[x]+x-1)*y'[x]-y[x]*(y[x]+2*x+1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[3]{2}x}{\sqrt[3]{-27{c_1}^2x^2+\sqrt{108{c_1}^3x^3+(27{c_1}^2x-27{c_1}^2x^2)^2}+27{c_1}^2x}} + \frac{\sqrt[3]{-27{c_1}^2x^2+\sqrt{108{c_1}^3x^3+(27{c_1}^2x-27{c_1}^2x^2)^2}+27{c_1}^2x}}{3\sqrt[3]{2}c_1} + x - 1$$

$$y(x) \to \frac{(1+i\sqrt{3})\,x}{2^{2/3}\sqrt[3]{-27{c_1}^2x^2+\sqrt{108{c_1}^3x^3+(27{c_1}^2x-27{c_1}^2x^2)^2}+27{c_1}^2x}} - \frac{(1-i\sqrt{3})\sqrt[3]{-27{c_1}^2x^2+\sqrt{108{c_1}^3x^3+(27{c_1}^2x-27{c_1}^2x^2)^2}+27{c_1}^2x}}{6\sqrt[3]{2}c_1} + x - 1$$

$$y(x) \to \frac{(1-i\sqrt{3})\,x}{2^{2/3}\sqrt[3]{-27{c_1}^2x^2+\sqrt{108{c_1}^3x^3+(27{c_1}^2x-27{c_1}^2x^2)^2}+27{c_1}^2x}} - \frac{(1+i\sqrt{3})\sqrt[3]{-27{c_1}^2x^2+\sqrt{108{c_1}^3x^3+(27{c_1}^2x-27{c_1}^2x^2)^2}+27{c_1}^2x}}{6\sqrt[3]{2}c_1} + x - 1$$

$$y(x) \to \text{Indeterminate}$$

$$y(x) \to x - 1$$

1.243 problem 244

Internal problem ID [8580]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 244.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$\boxed{x(2y - x - 1)\, y' + y(-y + 2x - 1) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 391

```
dsolve(x*(2*y(x)-x-1)*diff(y(x),x)+y(x)*(2*x-y(x)-1)=0,y(x), singsol=all)
```

$$y(x) = \frac{3\,5^{\frac{1}{3}}\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2+160c_1x+80c_1-x}{c_1}}-20x-20\right)c_1^2\right)^{\frac{1}{3}}}{40c_1} + \frac{3x5^{\frac{2}{3}}}{40\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2+160c_1x+80c_1-x}{c_1}}-20x-20\right)c_1^2\right)^{\frac{1}{3}}} - x - 1$$

$$y(x) = \frac{\frac{3\,5^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\left(-20\left(-\frac{\sqrt{5}\,\sqrt{\frac{80(x+1)^2c_1-x}{c_1}}}{20}+x+1\right)x\,c_1^2\right)^{\frac{2}{3}}}{80} + \frac{3c_1\left(\frac{80(-x-1)\left(-20\left(-\frac{\sqrt{5}\,\sqrt{\frac{80(x+1)^2c_1-x}{c_1}}}{20}+x+1\right)x\,c_1^2\right)^{\frac{1}{3}}}{3}+\left(i\sqrt{3}-1\right)x5^{\frac{2}{3}}\right)}{80}}{\left(-20\left(-\frac{\sqrt{5}\,\sqrt{\frac{80(x+1)^2c_1-x}{c_1}}}{20}+x+1\right)x\,c_1^2\right)^{\frac{1}{3}}c_1}$$

$$y(x) = -\frac{3\left(5^{\frac{1}{3}}\left(1-i\sqrt{3}\right)\left(-20\left(-\frac{\sqrt{5}\,\sqrt{\frac{80(x+1)^2c_1-x}{c_1}}}{20}+x+1\right)x\,c_1^2\right)^{\frac{2}{3}} + c_1\left(\frac{80(x+1)\left(-20\left(-\frac{\sqrt{5}\,\sqrt{\frac{80(x+1)^2c_1-x}{c_1}}}{20}+x+1\right)\right.}{3}\right.\right.}{80\left(-20\left(-\frac{\sqrt{5}\,\sqrt{\frac{80(x+1)^2c_1-x}{c_1}}}{20}+x+1\right)x\,c_1^2\right)^{\frac{1}{3}}c_1}$$

✓ Solution by Mathematica

Time used: 42.104 (sec). Leaf size: 471

```
DSolve[x*(2*y[x]-x-1)*y'[x]+y[x]*(2*x-y[x]-1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[3]{2}x}{\sqrt[3]{27c_1{}^2x^2+\sqrt{(27c_1{}^2x^2+27c_1{}^2x)^2-108c_1{}^3x^3}+27c_1{}^2x}} - \frac{\sqrt[3]{27c_1{}^2x^2+\sqrt{(27c_1{}^2x^2+27c_1{}^2x)^2-108c_1{}^3x^3}+27c_1{}^2x}}{3\sqrt[3]{2}c_1} - x - 1$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)x}{2^{2/3}\sqrt[3]{27c_1{}^2x^2+\sqrt{(27c_1{}^2x^2+27c_1{}^2x)^2-108c_1{}^3x^3}+27c_1{}^2x}} + \frac{\left(1-i\sqrt{3}\right)\sqrt[3]{27c_1{}^2x^2+\sqrt{(27c_1{}^2x^2+27c_1{}^2x)^2-108c_1{}^3x^3}+27c_1{}^2x}}{6\sqrt[3]{2}c_1} - x - 1$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)x}{2^{2/3}\sqrt[3]{27c_1{}^2x^2+\sqrt{(27c_1{}^2x^2+27c_1{}^2x)^2-108c_1{}^3x^3}+27c_1{}^2x}} + \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{27c_1{}^2x^2+\sqrt{(27c_1{}^2x^2+27c_1{}^2x)^2-108c_1{}^3x^3}+27c_1{}^2x}}{6\sqrt[3]{2}c_1} - x - 1$$

$$y(x) \to \text{Indeterminate}$$

$$y(x) \to -x - 1$$

1.244 problem 245

Internal problem ID [8581]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 245.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(2yx + 4x^3)\, y' + y^2 + 112x^2 y = 0}$$

✓ Solution by Maple

Time used: 1.219 (sec). Leaf size: 31

```
dsolve((2*x*y(x)+4*x^3)*diff(y(x),x)+y(x)^2+112*x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^{28}\, \mathrm{RootOf}\left(x^{30}\_Z^{360} - 24x^{30}\_Z^{330} - c_1\right)^{330}}$$

✓ Solution by Mathematica

Time used: 14.016 (sec). Leaf size: 1453

```
DSolve[(2*x*y[x]+4*x^3)*y'[x]+y[x]^2+112*x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

Too large to display

1.245 problem 246

Internal problem ID [8582]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 246.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$x(3y + 2x)\, y' + 3(x + y)^2 = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 63

```
dsolve(x*(3*y(x)+2*x)*diff(y(x),x)+3*(y(x)+x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{-4c_1x^2 - \sqrt{-2c_1^2x^4 + 6}}{6c_1x}$$
$$y(x) = \frac{-4c_1x^2 + \sqrt{-2c_1^2x^4 + 6}}{6c_1x}$$

✓ Solution by Mathematica

Time used: 1.769 (sec). Leaf size: 135

```
DSolve[x*(3*y[x]+2*x)*y'[x]+3*(y[x]+x)^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{4x^2 + \sqrt{-2x^4 + 6e^{4c_1}}}{6x}$$
$$y(x) \to \frac{-4x^2 + \sqrt{-2x^4 + 6e^{4c_1}}}{6x}$$
$$y(x) \to -\frac{\sqrt{2}\sqrt{-x^4} + 4x^2}{6x}$$
$$y(x) \to \frac{\sqrt{2}\sqrt{-x^4} - 4x^2}{6x}$$

1.246 problem 247

Internal problem ID [8583]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 247.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$(3x+2)(y-2x-1)y'-y^2+yx=7x^2+9x+3$$

✓ Solution by Maple

Time used: 0.485 (sec). Leaf size: 392

```
dsolve((3*x+2)*(y(x)-2*x-1)*diff(y(x),x)-y(x)^2+x*y(x)-7*x^2-9*x-3=0,y(x), singsol=all)
```

$$y(x) = \frac{\frac{14\left(x+\frac{2}{3}\right)\left(-\frac{4}{1701}+\left(x^2+\frac{26}{21}x+\frac{8}{21}\right)c_1^2\right)\left(2\sqrt{-2187\left(-\frac{1}{243}+\left(x+\frac{2}{3}\right)^2c_1^2\right)\left(x+\frac{2}{3}\right)^2c_1^2}+\left(-729x^3-1458x^2-972x-216\right)c_1^3+(6x+4)c_1\right)^{\frac{2}{3}}}{27}+21\left(\left(\cdots\right.\right.}{\left(\frac{\left(1-i\sqrt{3}\right)\left(2\sqrt{-2187\left(-\cdots\right.}\right.}{\cdots}\right.}$$

✓ Solution by Mathematica

Time used: 66.883 (sec). Leaf size: 590

```
DSolve[(3*x+2)*(y[x]-2*x-1)*y'[x]-y[x]^2+x*y[x]-7*x^2-9*x-3==0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to \frac{9x^2 + x\left(12 + \sqrt[3]{27x^3 + 54x^2 + 36x - 2e^{2c_1}(3x+2) + 2\sqrt{e^{2c_1}(3x+2)^2\left(-(3x+2)^2 + e^{2c_1}\right)} + 8}\right) + \left(2\right.}{2\sqrt[3]{27x^3 + 54x^2 + 36x - 2e^{2c_1}(3x+2) + 2\sqrt{e^{2c_1}\left(\right.}}}$$

$$y(x) \to -\frac{i\left(\sqrt{3}-i\right)(3x+2)^2}{4\sqrt[3]{27x^3 + 54x^2 + 36x - 2e^{2c_1}(3x+2) + 2\sqrt{e^{2c_1}(3x+2)^2\left(-(3x+2)^2 + e^{2c_1}\right)} + 8}} + \frac{1}{4}i\left(\sqrt{3} + i\right)\sqrt[3]{27x^3 + 54x^2 + 36x - 2e^{2c_1}(3x+2) + 2\sqrt{e^{2c_1}(3x+2)^2\left(-(3x+2)^2 + e^{2c_1}\right)} + 8} + \frac{x}{2}$$

$$y(x) \to \frac{i\left(\sqrt{3}+i\right)(3x+2)^2}{4\sqrt[3]{27x^3 + 54x^2 + 36x - 2e^{2c_1}(3x+2) + 2\sqrt{e^{2c_1}(3x+2)^2\left(-(3x+2)^2 + e^{2c_1}\right)} + 8}} - \frac{1}{4}\left(1 + i\sqrt{3}\right)\sqrt[3]{27x^3 + 54x^2 + 36x - 2e^{2c_1}(3x+2) + 2\sqrt{e^{2c_1}(3x+2)^2\left(-(3x+2)^2 + e^{2c_1}\right)} + 8} + \frac{x}{2}$$

1.247 problem 248

Internal problem ID [8584]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 248.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$(6yx + x^2 + 3)\, y' + 3y^2 + 2yx = -2x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 75

```
dsolve((6*x*y(x)+x^2+3)*diff(y(x),x)+3*y(x)^2+2*x*y(x)+2*x=0,y(x), singsol=all)
```

$$y(x) = \frac{-x^2 - 3 + \sqrt{x^4 - 12x^3 - 12c_1x + 6x^2 + 9}}{6x}$$
$$y(x) = \frac{-x^2 - 3 - \sqrt{x^4 - 12x^3 - 12c_1x + 6x^2 + 9}}{6x}$$

✓ Solution by Mathematica
Time used: 0.522 (sec). Leaf size: 83

```
DSolve[(6*x*y[x]+x^2+3)*y'[x]+3*y[x]^2+2*x*y[x]+2*x==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -\frac{x^2 + \sqrt{x^4 - 12x^3 + 6x^2 + 36c_1x + 9} + 3}{6x}$$
$$y(x) \to -\frac{x^2 - \sqrt{x^4 - 12x^3 + 6x^2 + 36c_1x + 9} + 3}{6x}$$

1.248 problem 249

Internal problem ID [8585]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 249.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]'], [_Ab`

$$(axy + b\,x^n)\,y' + \alpha y^3 + \beta y^2 = 0$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 202

```
dsolve((a*x*y(x)+b*x^n)*diff(y(x),x)+alpha*y(x)^3+beta*y(x)^2=0,y(x), singsol=all)
```

$y(x)$

$$= \overline{\text{RootOf}\left(-\_Z^{\frac{a(n-1)}{\beta}} x^{-n+1} a^2 \beta n + c_1 a^2 b\, n^2 - \_Z^{\frac{an-a+\beta}{\beta}} \beta b a n + \_Z^{\frac{a(n-1)}{\beta}} x^{-n+1} a^2 \beta - \_Z^{\frac{a(n-1)}{\beta}} x^{-n+1} a\, \beta^2 \right.}$$

✓ Solution by Mathematica

Time used: 2.447 (sec). Leaf size: 115

```
DSolve[(a*x*y[x]+b*x^n)*y'[x]+\[Alpha]*y[x]^3+\[Beta]*y[x]^2==0,y[x],x,IncludeSingularSoluti
```

$$\text{Solve}\left[\frac{(a(-n) + a + \alpha y(x))y(x)^{\frac{a-an}{\beta}-1}(\beta + \alpha y(x))^{\frac{a(n-1)}{\beta}}}{a^2(n-1)^2(a(n-1)+\beta)} + \frac{x^{1-n}\exp\left(-\frac{a(n-1)(\log(y(x))-\log(\beta+\alpha y(x)))}{\beta}\right)}{ab(1-n)(n-1)} = c_1, y(x)\right]$$

1.249 problem 250

Internal problem ID [8586]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 250.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$\left(Bxy + A\,x^2 + xa + by + c\right) y' + Axy + \beta y = Bg(x)^2 - x\alpha - \gamma$$

✗ Solution by Maple

```
dsolve((B*x*y(x)+A*x^2+a*x+b*y(x)+c)*diff(y(x),x)-B*g(x)^2+A*x*y(x)+alpha*x+beta*y(x)+gamma=
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(B*x*y[x]+A*x^2+a*x+b*y[x]+c)*y'[x]-B*g[x]^2+A*x*y[x]+\[Alpha]*x+\[Beta]*y[x]+\[Gamma
```

Timed out

1.250 problem 251

Internal problem ID [8587]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 251.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{(x^2y-1)\,y'+xy^2=1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 51

```
dsolve((x^2*y(x)-1)*diff(y(x),x)+x*y(x)^2-1=0,y(x), singsol=all)
```

$$y(x)=\frac{1+\sqrt{-2c_1x^2+2x^3+1}}{x^2}$$
$$y(x)=\frac{1-\sqrt{-2c_1x^2+2x^3+1}}{x^2}$$

✓ Solution by Mathematica
Time used: 0.535 (sec). Leaf size: 57

```
DSolve[(x^2*y[x]-1)*y'[x]+x*y[x]^2-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1-\sqrt{2x^3+c_1x^2+1}}{x^2}$$
$$y(x)\to\frac{1+\sqrt{2x^3+c_1x^2+1}}{x^2}$$

1.251 problem 252

Internal problem ID [8588]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 252.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\left(x^2y-1\right)y'-xy^2=-1$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 963

```
dsolve((x^2*y(x)-1)*diff(y(x),x)-(x*y(x)^2-1)=0,y(x), singsol=all)
```

$$y(x) = \frac{4^{\frac{2}{3}}\left(\left(\sqrt{5}\,\sqrt{-\frac{(x^3-1)^2}{c_1x^6-80x^6+160x^3-80}}-\frac{1}{4}\right)c_1(-80+(c_1-80)\,x^6+160x^3)^2\right)^{\frac{2}{3}}+((-c_1+80)\,x^7-160x^4+8}{x^24^{\frac{2}{3}}\left(\left(\sqrt{5}\,\sqrt{-\frac{(x^3-1)^2}{c_1x^6-80x^6+160x^3-80}}-\frac{1}{4}\right)c_1\,(-80+(c_1-80)\,x^6+160x^3)^2\right)^{\frac{2}{3}}+\left(c_1x^4-4^{\frac{1}{3}}\right(}$$

$$y(x) = \frac{4^{\frac{2}{3}}\left(\left(\sqrt{5}\,\sqrt{-\frac{(x^3-1)^2}{c_1x^6-80x^6+160x^3-80}}-\frac{1}{4}\right)c_1(-80+(c_1-80)\,x^6+160x^3)^2\right)^{\frac{2}{3}}(\sqrt{3}+i)+\left(2i4^{\frac{1}{3}}\left(\left(\sqrt{5}\sqrt{-}\right.\right.\right.}{x^24^{\frac{2}{3}}\left(\left(\sqrt{5}\,\sqrt{-\frac{(x^3-1)^2}{c_1x^6-80x^6+160x^3-80}}-\frac{1}{4}\right)c_1\,(-80+(c_1-80)\,x^6+160x^3)^2\right)^{\frac{2}{3}}(\sqrt{3}+i)+\left(2i4^{\frac{1}{3}}\left(\left(\sqrt{5}\right.\right.\right.}$$

$$y(x) = \frac{(i-\sqrt{3})\,4^{\frac{2}{3}}\left(\left(\sqrt{5}\,\sqrt{-\frac{(x^3-1)^2}{c_1x^6-80x^6+160x^3-80}}-\frac{1}{4}\right)c_1(-80+(c_1-80)\,x^6+160x^3)^2\right)^{\frac{2}{3}}+x(-80+(c_1-8}{(i-\sqrt{3})\,x^24^{\frac{2}{3}}\left(\left(\sqrt{5}\,\sqrt{-\frac{(x^3-1)^2}{c_1x^6-80x^6+160x^3-80}}-\frac{1}{4}\right)c_1\,(-80+(c_1-80)\,x^6+160x^3)^2\right)^{\frac{2}{3}}+(-80+(c_1-8}$$

✓ Solution by Mathematica

Time used: 36.312 (sec). Leaf size: 506

```
DSolve[(x^2*y[x]-1)*y'[x]-(x*y[x]^2-1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{-(1-6c_1)^2x^3+\sqrt{(-1+6c_1)^3\left(6c_1x^6+(2-12c_1)x^3-1+6c_1\right)}+1+36c_1{}^2-12c_1}}{-1+6c_1} - \frac{x^2}{\sqrt[3]{-(1-6c_1)^2x^3+\sqrt{(-1+6c_1)^3\left(6c_1x^6+(2-12c_1)x^3-1+6c_1\right)}+1+36c_1{}^2-12c_1}} + x$$

$$y(x) \to \frac{i\left(\sqrt{3}+i\right)\sqrt[3]{-(1-6c_1)^2x^3+\sqrt{(-1+6c_1)^3\left(6c_1x^6+(2-12c_1)x^3-1+6c_1\right)}+1+36c_1{}^2-12c_1}}{-2+12c_1} + \frac{\left(1+i\sqrt{3}\right)x^2}{2\sqrt[3]{-(1-6c_1)^2x^3+\sqrt{(-1+6c_1)^3\left(6c_1x^6+(2-12c_1)x^3-1+6c_1\right)}+1+36c_1{}^2-12c_1}} + x$$

$$y(x) \to -\frac{i\left(\sqrt{3}-i\right)\sqrt[3]{-(1-6c_1)^2x^3+\sqrt{(-1+6c_1)^3\left(6c_1x^6+(2-12c_1)x^3-1+6c_1\right)}+1+36c_1{}^2-12c_1}}{-2+12c_1} + \frac{\left(1-i\sqrt{3}\right)x^2}{2\sqrt[3]{-(1-6c_1)^2x^3+\sqrt{(-1+6c_1)^3\left(6c_1x^6+(2-12c_1)x^3-1+6c_1\right)}+1+36c_1{}^2-12c_1}} + x$$

$$y(x) \to x$$

1.252 problem 253

Internal problem ID [8589]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 253.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{(x^2y-1)\,y'+8xy^2=8}$$

✗ Solution by Maple

```
dsolve((x^2*y(x)-1)*diff(y(x),x)+8*(x*y(x)^2-1)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(x^2*y[x]-1)*y'[x]+8*(x*y[x]^2-1)==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.253 problem 254

Internal problem ID [8590]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 254.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$x(yx-2)\,y'+y^3x^2+xy^2-2y=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 59

```
dsolve(x*(x*y(x)-2)*diff(y(x),x)+x^2*y(x)^3+x*y(x)^2-2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{-1+\sqrt{1-4\ln(x)+4c_1}}{2\left(-\ln(x)+c_1\right)x}$$
$$y(x)=\frac{1+\sqrt{1-4\ln(x)+4c_1}}{2\left(\ln(x)-c_1\right)x}$$

✓ Solution by Mathematica
Time used: 1.179 (sec). Leaf size: 86

```
DSolve[x*(x*y[x]-2)*y'[x]+x^2*y[x]^3+x*y[x]^2-2*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\to\frac{2}{x+\sqrt{-\frac{1}{x^3}}x^2\sqrt{-x(-4\log(x)+1+4c_1)}}$$
$$y(x)\to\frac{2}{x+\left(-\frac{1}{x^3}\right)^{3/2}x^5\sqrt{-x(-4\log(x)+1+4c_1)}}$$
$$y(x)\to 0$$

1.254 problem 255

Internal problem ID [8591]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 255.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$x(yx-3)\,y'+xy^2-y=0$$

✓ Solution by Maple

Time used: 0.235 (sec). Leaf size: 74

```
dsolve(x*(x*y(x)-3)*diff(y(x),x)+x*y(x)^2-y(x)=0,y(x), singsol=all)
```

$$y(x)=-\frac{3\,\mathrm{LambertW}\left(\frac{(-x^2)^{\frac{1}{3}}c_1}{3}\right)}{x}$$
$$y(x)=-\frac{3\,\mathrm{LambertW}\left(-\frac{(-x^2)^{\frac{1}{3}}c_1\left(1+i\sqrt{3}\right)}{6}\right)}{x}$$
$$y(x)=-\frac{3\,\mathrm{LambertW}\left(\frac{(-x^2)^{\frac{1}{3}}c_1\left(i\sqrt{3}-1\right)}{6}\right)}{x}$$

✓ Solution by Mathematica

Time used: 15.505 (sec). Leaf size: 35

```
DSolve[x*(x*y[x]-3)*y'[x]+x*y[x]^2-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{3W\left(e^{-1+\frac{9c_1}{2^{2/3}}}x^{2/3}\right)}{x}$$
$$y(x)\to 0$$

1.255 problem 256

Internal problem ID [8592]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 256.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2(-1+y)\,y'+(x-1)\,y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 33

```
dsolve(x^2*(y(x)-1)*diff(y(x),x)+(x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x)=x\,\mathrm{e}^{\frac{-\operatorname{LambertW}\left(-x\,\mathrm{e}^{\frac{c_1x+1}{x}}\right)x+c_1x+1}{x}}$$

✓ Solution by Mathematica

Time used: 2.975 (sec). Leaf size: 26

```
DSolve[x^2*(y[x]-1)*y'[x]+(x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -W\left(x\left(-e^{\frac{1}{x}-c_1}\right)\right)$$
$$y(x)\to 0$$

1.256 problem 257

Internal problem ID [8593]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 257.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$x\left(yx+x^4-1\right)y'-y\left(yx-x^4-1\right)=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 98

```
dsolve(x*(x*y(x)+x^4-1)*diff(y(x),x)-y(x)*(x*y(x)-x^4-1)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(-c_1+\mathrm{e}^{\mathrm{RootOf}\left(-2\_Zx^4\mathrm{e}^{2\_Z}+2x^4\mathrm{e}^{2\_Z}-2\,\mathrm{e}^{\_Z}c_1x^4+\mathrm{e}^{2\_Z}-2c_1\mathrm{e}^{\_Z}+c_1^2\right)}\right)\mathrm{e}^{-\mathrm{RootOf}\left(-2\_Zx^4\mathrm{e}^{2\_Z}+2x^4\mathrm{e}^{2\_Z}-2\,\mathrm{e}^{\_Z}c_1x^4+\mathrm{e}^{2\_Z}-2c_1\mathrm{e}\right.}}{x}$$

✓ Solution by Mathematica

Time used: 0.297 (sec). Leaf size: 39

```
DSolve[x*(x*y[x]+x^4-1)*y'[x]-y[x]*(x*y[x]-x^4-1)==0,y[x],x,IncludeSingularSolutions -> True
```

$$\mathrm{Solve}\left[2x^2+\frac{y(x)}{x}+\frac{x\left(-2\log\left(\frac{1}{1-xy(x)}\right)-2+c_1\right)}{y(x)}=0,y(x)\right]$$

1.257 problem 258

Internal problem ID [8594]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 258.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{2x^2y'y + y^2 = 2x^3 + x^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 33

```
dsolve(2*x^2*y(x)*diff(y(x),x)+y(x)^2-2*x^3-x^2=0,y(x), singsol=all)
```

$$y(x) = \sqrt{\mathrm{e}^{\frac{1}{x}} c_1 + x^2}$$
$$y(x) = -\sqrt{\mathrm{e}^{\frac{1}{x}} c_1 + x^2}$$

✓ Solution by Mathematica
Time used: 7.128 (sec). Leaf size: 43

```
DSolve[2*x^2*y[x]*y'[x]+y[x]^2-2*x^3-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x^2 + c_1 e^{\frac{1}{x}}}$$
$$y(x) \to \sqrt{x^2 + c_1 e^{\frac{1}{x}}}$$

1.258 problem 259

Internal problem ID [8595]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 259.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{2x^2y'y - y^2 = x^2 \mathrm{e}^{x-\frac{1}{x}}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
dsolve(2*x^2*y(x)*diff(y(x),x)-y(x)^2-x^2*exp(x-1/x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{\mathrm{e}^{-\frac{1}{x}} c_1 + \mathrm{e}^{\frac{(x-1)(x+1)}{x}}}$$
$$y(x) = -\sqrt{\mathrm{e}^{-\frac{1}{x}} c_1 + \mathrm{e}^{\frac{(x-1)(x+1)}{x}}}$$

✓ Solution by Mathematica

Time used: 0.964 (sec). Leaf size: 50

```
DSolve[2*x^2*y[x]*y'[x]-y[x]^2-x^2*Exp[x-1/x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{-\frac{1}{2}/x}\sqrt{e^x + c_1}$$
$$y(x) \to e^{-\frac{1}{2}/x}\sqrt{e^x + c_1}$$

1.259 problem 260

Internal problem ID [8596]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 260.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(2x^2y + x)\, y' - y^3x^2 + 2xy^2 + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve((2*x^2*y(x)+x)*diff(y(x),x)-x^2*y(x)^3+2*x*y(x)^2+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-2 + \sqrt{4 - 2\ln(x) + 2c_1}}{2(\ln(x) - c_1)\, x}$$
$$y(x) = \frac{2 + \sqrt{4 - 2\ln(x) + 2c_1}}{2(-\ln(x) + c_1)\, x}$$

✓ Solution by Mathematica

Time used: 0.756 (sec). Leaf size: 79

```
DSolve[(2*x^2*y[x]+x)*y'[x]-x^2*y[x]^3+2*x*y[x]^2+y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{x}{-2x^2 + \frac{\sqrt{x(-2\log(x)+4+c_1)}}{\sqrt{\frac{1}{x^3}}}}$$
$$y(x) \to -\frac{x}{2x^2 + \frac{\sqrt{x(-2\log(x)+4+c_1)}}{\sqrt{\frac{1}{x^3}}}}$$
$$y(x) \to 0$$

1.260 problem 261

Internal problem ID [8597]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 261.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(2x^2y - x)\, y' - 2xy^2 - y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 18

```
dsolve((2*x^2*y(x)-x)*diff(y(x),x)-2*x*y(x)^2-y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{2\,\mathrm{LambertW}\left(-\frac{c_1}{2x^2}\right) x}$$

✓ Solution by Mathematica

Time used: 5.765 (sec). Leaf size: 37

```
DSolve[(2*x^2*y[x]-x)*y'[x]-2*x*y[x]^2-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2xW\left(\frac{e^{-1+\frac{9c_1}{2^{2/3}}}}{x^2}\right)}$$

$$y(x) \to 0$$

1.261 problem 262

Internal problem ID [8598]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 262.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(2x^2y - x^3)\, y' + y^3 - 4xy^2 = -2x^3}$$

✓ Solution by Maple

Time used: 0.391 (sec). Leaf size: 65

```
dsolve((2*x^2*y(x)-x^3)*diff(y(x),x)+y(x)^3-4*x*y(x)^2+2*x^3=0,y(x), singsol=all)
```

$$y(x) = \frac{x(2c_1x^2 - \sqrt{3c_1x^2 + 1})}{c_1x^2 - 1}$$
$$y(x) = \frac{x(2c_1x^2 + \sqrt{3c_1x^2 + 1})}{c_1x^2 - 1}$$

✓ Solution by Mathematica

Time used: 14.03 (sec). Leaf size: 132

```
DSolve[(2*x^2*y[x]-x^3)*y'[x]+y[x]^3-4*x*y[x]^2+2*x^3==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{2x^3 - \sqrt{e^{2c_1}x^2(-3x^2 + e^{2c_1})}}{x^2 + e^{2c_1}}$$
$$y(x) \to \frac{2x^3 + \sqrt{e^{2c_1}x^2(-3x^2 + e^{2c_1})}}{x^2 + e^{2c_1}}$$
$$y(x) \to 2x$$
$$y(x) \to -\sqrt{x^2}$$
$$y(x) \to \sqrt{x^2}$$

1.262 problem 263

Internal problem ID [8599]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 263.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{yy' + 3y^2x^2 = -2x^3 - 7}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 169

```
dsolve(2*x^3+y(x)*diff(y(x),x)+3*x^2*y(x)^2+7=0,y(x), singsol=all)
```

$$y(x) = -\frac{2^{\frac{5}{6}}\sqrt{3}\sqrt{-80\left(\frac{9\Gamma\left(\frac{2}{3}\right)\left(-\frac{3\,\mathrm{e}^{-2x^3}c_1}{2}+x\right)2^{\frac{1}{3}}(-x^3)^{\frac{1}{3}}}{40}+x\,\mathrm{e}^{-2x^3}\left(\pi\sqrt{3}-\frac{3\Gamma\left(\frac{1}{3},-2x^3\right)\Gamma\left(\frac{2}{3}\right)}{2}\right)\right)(-x^3)^{\frac{1}{3}}}}{18\,(-x^3)^{\frac{1}{3}}\sqrt{\Gamma\left(\frac{2}{3}\right)}}$$

$$y(x) = \frac{2^{\frac{5}{6}}\sqrt{3}\sqrt{-80\left(\frac{9\Gamma\left(\frac{2}{3}\right)\left(-\frac{3\,\mathrm{e}^{-2x^3}c_1}{2}+x\right)2^{\frac{1}{3}}(-x^3)^{\frac{1}{3}}}{40}+x\,\mathrm{e}^{-2x^3}\left(\pi\sqrt{3}-\frac{3\Gamma\left(\frac{1}{3},-2x^3\right)\Gamma\left(\frac{2}{3}\right)}{2}\right)\right)(-x^3)^{\frac{1}{3}}}}{18\,(-x^3)^{\frac{1}{3}}\sqrt{\Gamma\left(\frac{2}{3}\right)}}$$

✓ Solution by Mathematica

Time used: 4.884 (sec). Leaf size: 166

```
DSolve[2*x^3+y[x]*y'[x]+3*x^2*y[x]^2+7==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{e^{-2x^3}\left(-7\ 2^{2/3}(-x^3)^{2/3}\Gamma\left(\frac{1}{3},-2x^3\right)+2^{2/3}(-x^3)^{2/3}\Gamma\left(\frac{4}{3},-2x^3\right)+3c_1x^2\right)}{x^2}}}{\sqrt{3}}$$

$$y(x) \to \frac{\sqrt{\frac{e^{-2x^3}\left(-7\ 2^{2/3}(-x^3)^{2/3}\Gamma\left(\frac{1}{3},-2x^3\right)+2^{2/3}(-x^3)^{2/3}\Gamma\left(\frac{4}{3},-2x^3\right)+3c_1x^2\right)}{x^2}}}{\sqrt{3}}$$

1.263 problem 264

Internal problem ID [8600]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 264.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$2x(yx^3+1)\,y'+(3yx^3-1)\,y=0$$

✓ Solution by Maple

Time used: 0.515 (sec). Leaf size: 37

```
dsolve(2*x*(x^3*y(x)+1)*diff(y(x),x)+(3*x^3*y(x)-1)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{\operatorname{RootOf}\left(\_Z^{98}c_1-14\_Z^{77}c_1+49\_Z^{56}c_1-9x^7\right)^{21}-7}{3x^3}$$

✓ Solution by Mathematica

Time used: 6.016 (sec). Leaf size: 680

```
DSolve[2*x*(x^3*y[x]+1)*y'[x]+(3*x^3*y[x]-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \text{Root}\left[81\#1^7 e^{\frac{21c_1}{2}} x^{12} + 756\#1^6 e^{\frac{21c_1}{2}} x^9 + 2646\#1^5 e^{\frac{21c_1}{2}} x^6 + 4116\#1^4 e^{\frac{21c_1}{2}} x^3 + 2401\#1^3 e^{\frac{21c_1}{2}} - x^{3/2}\&, 1\right]$$

$$y(x) \to \text{Root}\left[81\#1^7 e^{\frac{21c_1}{2}} x^{12} + 756\#1^6 e^{\frac{21c_1}{2}} x^9 + 2646\#1^5 e^{\frac{21c_1}{2}} x^6 + 4116\#1^4 e^{\frac{21c_1}{2}} x^3 + 2401\#1^3 e^{\frac{21c_1}{2}} - x^{3/2}\&, 2\right]$$

$$y(x) \to \text{Root}\left[81\#1^7 e^{\frac{21c_1}{2}} x^{12} + 756\#1^6 e^{\frac{21c_1}{2}} x^9 + 2646\#1^5 e^{\frac{21c_1}{2}} x^6 + 4116\#1^4 e^{\frac{21c_1}{2}} x^3 + 2401\#1^3 e^{\frac{21c_1}{2}} - x^{3/2}\&, 3\right]$$

$$y(x) \to \text{Root}\left[81\#1^7 e^{\frac{21c_1}{2}} x^{12} + 756\#1^6 e^{\frac{21c_1}{2}} x^9 + 2646\#1^5 e^{\frac{21c_1}{2}} x^6 + 4116\#1^4 e^{\frac{21c_1}{2}} x^3 + 2401\#1^3 e^{\frac{21c_1}{2}} - x^{3/2}\&, 4\right]$$

$$y(x) \to \text{Root}\left[81\#1^7 e^{\frac{21c_1}{2}} x^{12} + 756\#1^6 e^{\frac{21c_1}{2}} x^9 + 2646\#1^5 e^{\frac{21c_1}{2}} x^6 + 4116\#1^4 e^{\frac{21c_1}{2}} x^3 + 2401\#1^3 e^{\frac{21c_1}{2}} - x^{3/2}\&, 5\right]$$

$$y(x) \to \text{Root}\left[81\#1^7 e^{\frac{21c_1}{2}} x^{12} + 756\#1^6 e^{\frac{21c_1}{2}} x^9 + 2646\#1^5 e^{\frac{21c_1}{2}} x^6 + 4116\#1^4 e^{\frac{21c_1}{2}} x^3 + 2401\#1^3 e^{\frac{21c_1}{2}} - x^{3/2}\&, 6\right]$$

$$y(x) \to \text{Root}\left[81\#1^7 e^{\frac{21c_1}{2}} x^{12} + 756\#1^6 e^{\frac{21c_1}{2}} x^9 + 2646\#1^5 e^{\frac{21c_1}{2}} x^6 + 4116\#1^4 e^{\frac{21c_1}{2}} x^3 + 2401\#1^3 e^{\frac{21c_1}{2}} - x^{3/2}\&, 7\right]$$

1.264 problem 265

Internal problem ID [8601]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 265.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\left(x^{n(n+1)}y-1\right)y'+2(n+1)^2\,x^{n-1}\left(x^{n^2}y^2-1\right)=0$$

✗ Solution by Maple

```
dsolve((x^(n*(n+1))*y(x)-1)*diff(y(x),x)+2*(n+1)^2*x^(n-1)*(x^(n^2)*y(x)^2-1)=0,y(x), singso
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(x^(n*(n+1))*y[x]-1)*y'[x]+2*(n+1)^2*x^(n-1)*(x^(n^2)*y[x]^2-1)==0,y[x],x,IncludeSing
```

Timed out

1.265 problem 266

Internal problem ID [8602]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 266.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [`x=_G(y,y')`]

$$(y-x)\sqrt{x^2+1}\,y' - a\sqrt{\left(1+y^2\right)^3} = 0$$

✓ Solution by Maple

Time used: 0.234 (sec). Leaf size: 187

```
dsolve((y(x)-x)*sqrt(x^2+1)*diff(y(x),x)-a*sqrt((y(x)^2+1)^3)=0,y(x), singsol=all)
```

$$y(x) = \frac{-x+\sqrt{-a^2\left(x^2+1\right)^2\left(a^2-1\right)}}{a^2x^2+a^2-1}$$

$$y(x) = \frac{-x-\sqrt{-a^2\left(x^2+1\right)^2\left(a^2-1\right)}}{a^2x^2+a^2-1}$$

$$\frac{\sqrt{2}\sqrt{\frac{a^2}{1+\cos(2\arctan(x)-2\arctan(y(x)))}}\cos\left(\arctan\left(x\right)-\arctan\left(y(x)\right)\right)\arctan\left(\frac{\cos(\arctan(x)-\arctan(y(x)))}{\sqrt{a^2-1}}\right)+\text{arctan}}{\sqrt{a^2-1}}$$

$= 0$

✓ Solution by Mathematica

Time used: 2.935 (sec). Leaf size: 69

```
DSolve[(y[x]-x)*Sqrt[x^2+1]*y'[x]-a*Sqrt[(y[x]^2+1)^3]==0,y[x],x,IncludeSingularSolutions ->
```

$$\text{Solve}\left[\left\{\frac{2a\arctan\left(\frac{1-a\tan\left(\frac{K[1]}{2}\right)}{\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}}+K[1]+\arctan(x)=c_1,y(x)=\frac{\tan(K[1])+x}{1-x\tan(K[1])}\right\},\{K[1],y(x)\}\right]$$

1.266 problem 267

Internal problem ID [8603]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 267.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _Bernoulli]`

$$\boxed{yy' \sin(x)^2 + \sin(x)\cos(x)\, y^2 = 1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 28

```
dsolve(y(x)*diff(y(x),x)*sin(x)^2+y(x)^2*cos(x)*sin(x)-1=0,y(x), singsol=all)
```

$$y(x) = \csc(x)\sqrt{2x + c_1}$$
$$y(x) = -\csc(x)\sqrt{2x + c_1}$$

✓ Solution by Mathematica
Time used: 0.481 (sec). Leaf size: 36

```
DSolve[y[x]*y'[x]*Sin[x]^2+y[x]^2*Cos[x]*Sin[x]-1==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -\sqrt{2x + c_1}\csc(x)$$
$$y(x) \to \sqrt{2x + c_1}\csc(x)$$

1.267 problem 268

Internal problem ID [8604]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 268.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{f(x)\, yy' + g(x)\, y^2 = -h(x)}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 114

```
dsolve(f(x)*y(x)*diff(y(x),x)+g(x)*y(x)^2+h(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{\mathrm{e}^{\int \frac{2g(x)}{f(x)}dx} \left(-2 \left(\int \frac{\mathrm{e}^{\int \frac{2g(x)}{f(x)}dx} h(x)}{f(x)}dx\right) + c_1\right)}\, \mathrm{e}^{-2\left(\int \frac{g(x)}{f(x)}dx\right)}$$

$$y(x) = -\sqrt{\mathrm{e}^{\int \frac{2g(x)}{f(x)}dx} \left(-2 \left(\int \frac{\mathrm{e}^{\int \frac{2g(x)}{f(x)}dx} h(x)}{f(x)}dx\right) + c_1\right)}\, \mathrm{e}^{-2\left(\int \frac{g(x)}{f(x)}dx\right)}$$

✓ Solution by Mathematica

Time used: 0.329 (sec). Leaf size: 146

```
DSolve[f[x]*y[x]*y'[x]+g[x]*y[x]^2+h[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\exp\left(\int_1^x -\frac{g(K[1])}{f(K[1])}dK[1]\right)\sqrt{2\int_1^x -\frac{\exp\left(-2\int_1^{K[2]} -\frac{g(K[1])}{f(K[1])}dK[1]\right)h(K[2])}{f(K[2])}dK[2]+c_1}$$

$$y(x) \to \exp\left(\int_1^x -\frac{g(K[1])}{f(K[1])}dK[1]\right)\sqrt{2\int_1^x -\frac{\exp\left(-2\int_1^{K[2]} -\frac{g(K[1])}{f(K[1])}dK[1]\right)h(K[2])}{f(K[2])}dK[2]+c_1}$$

1.268 problem 269

Internal problem ID [8605]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 269.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class C']]`

$$(g_1(x)\,y + g_0(x))\,y' - f_1(x)\,y - f_2(x)\,y^2 - f_3(x)\,y^3 = f_0(x)$$

✗ Solution by Maple

```
dsolve((g__1(x)*y(x)+g__0(x))*diff(y(x),x)-f__1(x)*y(x)-f__2(x)*y(x)^2-f__3(x)*y(x)^3-f__0(x
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(g1[x]*y[x]+g0[x])*y'[x]-f1[x]*y[x]-f2[x]*y[x]^2-f3[x]*y[x]^3-f0[x]==0,y[x],x,Include
```

Timed out

1.269 problem 270

Internal problem ID [8606]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 270.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{(y^2 - x)\, y' - y = -x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 318

```
dsolve((y(x)^2-x)*diff(y(x),x)-y(x)+x^2=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(-4x^3 - 12c_1 + 4\sqrt{x^6 + (6c_1 - 4)\, x^3 + 9c_1^2}\right)^{\frac{2}{3}} + 4x}{2\left(-4x^3 - 12c_1 + 4\sqrt{x^6 + (6c_1 - 4)\, x^3 + 9c_1^2}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{i\left(-\left(-4x^3 - 12c_1 + 4\sqrt{x^6 + (6c_1 - 4)\, x^3 + 9c_1^2}\right)^{\frac{2}{3}} + 4x\right)\sqrt{3} - \left(-4x^3 - 12c_1 + 4\sqrt{x^6 + (6c_1 - 4)\, x^3 -}\right.}{4\left(-4x^3 - 12c_1 + 4\sqrt{x^6 + (6c_1 - 4)\, x^3 + 9c_1^2}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{i\left(\left(-4x^3 - 12c_1 + 4\sqrt{x^6 + (6c_1 - 4)\, x^3 + 9c_1^2}\right)^{\frac{2}{3}} - 4x\right)\sqrt{3} - \left(-4x^3 - 12c_1 + 4\sqrt{x^6 + (6c_1 - 4)\, x^3 + 9}\right.}{4\left(-4x^3 - 12c_1 + 4\sqrt{x^6 + (6c_1 - 4)\, x^3 + 9c_1^2}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 3.718 (sec). Leaf size: 326

```
DSolve[(y[x]^2-x)*y'[x]-y[x]+x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2x + \sqrt[3]{2}\left(x^3 + \sqrt{x^6 + (-4+6c_1)x^3 + 9{c_1}^2} + 3c_1\right)^{2/3}}{2^{2/3}\sqrt[3]{x^3 + \sqrt{x^6 + (-4+6c_1)x^3 + 9{c_1}^2} + 3c_1}}$$

$$y(x) \to \frac{2^{2/3}(1 - i\sqrt{3})\left(x^3 + \sqrt{x^6 + (-4+6c_1)x^3 + 9{c_1}^2} + 3c_1\right)^{2/3} + \sqrt[3]{2}(2 + 2i\sqrt{3})\,x}{4\sqrt[3]{x^3 + \sqrt{x^6 + (-4+6c_1)x^3 + 9{c_1}^2} + 3c_1}}$$

$$y(x) \to \frac{2^{2/3}(1 + i\sqrt{3})\left(x^3 + \sqrt{x^6 + (-4+6c_1)x^3 + 9{c_1}^2} + 3c_1\right)^{2/3} + \sqrt[3]{2}(2 - 2i\sqrt{3})\,x}{4\sqrt[3]{x^3 + \sqrt{x^6 + (-4+6c_1)x^3 + 9{c_1}^2} + 3c_1}}$$

1.270 problem 271

Internal problem ID [8607]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 271.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$\boxed{(x^2 + y^2)\, y' + 2x(2x + y) = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 321

```
dsolve((y(x)^2+x^2)*diff(y(x),x)+2*x*(y(x)+2*x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{2\left(c_1 x^2 - \frac{\left(4-16c_1^{\frac{3}{2}}x^3+4\sqrt{20c_1^3x^6-8c_1^{\frac{3}{2}}x^3+1}\right)^{\frac{2}{3}}}{4}\right)}{\sqrt{c_1}\left(4-16c_1^{\frac{3}{2}}x^3+4\sqrt{20c_1^3x^6-8c_1^{\frac{3}{2}}x^3+1}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{(1+i\sqrt{3})\left(4-16c_1^{\frac{3}{2}}x^3+4\sqrt{20c_1^3x^6-8c_1^{\frac{3}{2}}x^3+1}\right)^{\frac{1}{3}}}{4\sqrt{c_1}} - \frac{x^2\sqrt{c_1}\,(i\sqrt{3}-1)}{\left(4-16c_1^{\frac{3}{2}}x^3+4\sqrt{20c_1^3x^6-8c_1^{\frac{3}{2}}x^3+1}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{4i\sqrt{3}\,c_1x^2 + i\sqrt{3}\left(4-16c_1^{\frac{3}{2}}x^3+4\sqrt{20c_1^3x^6-8c_1^{\frac{3}{2}}x^3+1}\right)^{\frac{2}{3}} + 4c_1x^2 - \left(4-16c_1^{\frac{3}{2}}x^3+4\sqrt{20c_1^3x^6-8c_1^{\frac{3}{2}}x \cdots}\right.}{4\left(4-16c_1^{\frac{3}{2}}x^3+4\sqrt{20c_1^3x^6-8c_1^{\frac{3}{2}}x^3+1}\right)^{\frac{1}{3}}\sqrt{c_1}}$$

✓ Solution by Mathematica

Time used: 19.146 (sec). Leaf size: 593

```
DSolve[(y[x]^2+x^2)*y'[x]+2*x*(y[x]+2*x)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{-4x^3+\sqrt{20x^6-8e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}{\sqrt[3]{2}}-\frac{\sqrt[3]{2}x^2}{\sqrt[3]{-4x^3+\sqrt{20x^6-8e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}$$

$$y(x) \to \frac{\sqrt[3]{2}\left(2+2i\sqrt{3}\right)x^2+i2^{2/3}\left(\sqrt{3}+i\right)\left(-4x^3+\sqrt{20x^6-8e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}\right)^{2/3}}{4\sqrt[3]{-4x^3+\sqrt{20x^6-8e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)x^2}{2^{2/3}\sqrt[3]{-4x^3+\sqrt{20x^6-8e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}-\frac{\left(1+i\sqrt{3}\right)\sqrt[3]{-4x^3+\sqrt{20x^6-8e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}{2\sqrt[3]{2}}$$

$$y(x) \to \sqrt[3]{\sqrt{5}\sqrt{x^6}-2x^3}-\frac{x^2}{\sqrt[3]{\sqrt{5}\sqrt{x^6}-2x^3}}$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)x^2+\left(-1-i\sqrt{3}\right)\left(\sqrt{5}\sqrt{x^6}-2x^3\right)^{2/3}}{2\sqrt[3]{\sqrt{5}\sqrt{x^6}-2x^3}}$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)x^2+i\left(\sqrt{3}+i\right)\left(\sqrt{5}\sqrt{x^6}-2x^3\right)^{2/3}}{2\sqrt[3]{\sqrt{5}\sqrt{x^6}-2x^3}}$$

1.271 problem 272

Internal problem ID [8608]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 272.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{(x^2 + y^2)\, y' - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 42

```
dsolve((y(x)^2+x^2)*diff(y(x),x)-y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{2\sqrt{3}\ \mathrm{RootOf}\left(-2\sqrt{3}\,\mathrm{e}^{\frac{2\_Z\sqrt{3}}{3}-c_1}+\sqrt{3}\,x-3\tan\left(\_Z\right)x\right)}{3}-c_1}$$

✓ Solution by Mathematica

Time used: 0.123 (sec). Leaf size: 42

```
DSolve[(y[x]^2+x^2)*y'[x]-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2\arctan\left(\frac{\frac{2y(x)}{x}-1}{\sqrt{3}}\right)}{\sqrt{3}} + \log\left(\frac{y(x)}{x}\right) = -\log(x) + c_1, y(x)\right]$$

1.272 problem 273

Internal problem ID [8609]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 273.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]`

$$\left(y^{2}+x^{2}+a\right) y'+2yx=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 313

```
dsolve((y(x)^2+x^2+a)*diff(y(x),x)+2*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{\left(-12c_{1}+4\sqrt{4x^{6}+12x^{4}a+12a^{2}x^{2}+4a^{3}+9c_{1}^{2}}\right)^{\frac{2}{3}}-4x^{2}-4a}{2\left(-12c_{1}+4\sqrt{4x^{6}+12x^{4}a+12a^{2}x^{2}+4a^{3}+9c_{1}^{2}}\right)^{\frac{1}{3}}}$$

$$y(x)=-\frac{\left(\frac{i\sqrt{3}}{4}+\frac{1}{4}\right)\left(-12c_{1}+4\sqrt{4x^{6}+12x^{4}a+12a^{2}x^{2}+4a^{3}+9c_{1}^{2}}\right)^{\frac{2}{3}}+\left(i\sqrt{3}-1\right)\left(x^{2}+a\right)}{\left(-12c_{1}+4\sqrt{4x^{6}+12x^{4}a+12a^{2}x^{2}+4a^{3}+9c_{1}^{2}}\right)^{\frac{1}{3}}}$$

$$y(x)=\frac{\frac{\left(i\sqrt{3}-1\right)\left(-12c_{1}+4\sqrt{4x^{6}+12x^{4}a+12a^{2}x^{2}+4a^{3}+9c_{1}^{2}}\right)^{\frac{2}{3}}}{4}+\left(x^{2}+a\right)\left(1+i\sqrt{3}\right)}{\left(-12c_{1}+4\sqrt{4x^{6}+12x^{4}a+12a^{2}x^{2}+4a^{3}+9c_{1}^{2}}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 4.409 (sec). Leaf size: 299

```
DSolve[(y[x]^2+x^2+a)*y'[x]+2*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{2}\left(\sqrt{4(a+x^2)^3+9c_1^2}+3c_1\right)^{2/3}-2a-2x^2}{2^{2/3}\sqrt[3]{\sqrt{4(a+x^2)^3+9c_1^2}+3c_1}}$$

$$y(x) \to \frac{(1+i\sqrt{3})(a+x^2)}{2^{2/3}\sqrt[3]{\sqrt{4(a+x^2)^3+9c_1^2}+3c_1}}+\frac{i(\sqrt{3}+i)\sqrt[3]{\sqrt{4(a+x^2)^3+9c_1^2}+3c_1}}{2\sqrt[3]{2}}$$

$$y(x) \to \frac{(1-i\sqrt{3})(a+x^2)}{2^{2/3}\sqrt[3]{\sqrt{4(a+x^2)^3+9c_1^2}+3c_1}}-\frac{i(\sqrt{3}-i)\sqrt[3]{\sqrt{4(a+x^2)^3+9c_1^2}+3c_1}}{2\sqrt[3]{2}}$$

$$y(x) \to 0$$

1.273 problem 274

Internal problem ID [8610]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 274.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{(y^2 + x^2 + a)\, y' + 2yx = -x^2 - b}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 505

```
dsolve((y(x)^2+x^2+a)*diff(y(x),x)+2*x*y(x)+x^2+b=0,y(x), singsol=all)
```

$$y(x) = \frac{-4x^2 - 4a + \left(-4x^3 - 12bx - 12c_1 + 4\sqrt{5x^6 + 6\,(2a+b)\,x^4 + 6c_1x^3 + 3\,(4a^2+3b^2)\,x^2 + 18c_1bx + 4a^3}\right.}{2\left(-4x^3 - 12bx - 12c_1 + 4\sqrt{5x^6 + 6\,(2a+b)\,x^4 + 6c_1x^3 + 3\,(4a^2+3b^2)\,x^2 + 18c_1bx + 4a^3 + 9c_1^2}\right.}$$

$$y(x) = -\frac{\left(\frac{i\sqrt{3}}{4} + \frac{1}{4}\right)\left(-4x^3 - 12bx - 12c_1 + 4\sqrt{5x^6 + 6\,(2a+b)\,x^4 + 6c_1x^3 + 3\,(4a^2+3b^2)\,x^2 + 18c_1bx + 4a^3}\right.}{\left(-4x^3 - 12bx - 12c_1 + 4\sqrt{5x^6 + 6\,(2a+b)\,x^4 + 6c_1x^3 + 3\,(4a^2+3b^2)\,x^2 + 18c_1bx}\right.}$$

$$y(x) = \frac{\frac{\left(i\sqrt{3}-1\right)\left(-4x^3-12bx-12c_1+4\sqrt{5x^6+6(2a+b)x^4+6c_1x^3+3(4a^2+3b^2)x^2+18c_1bx+4a^3+9c_1^2}\right)^{\frac{2}{3}}}{4} + (x^2+a)\left(1+i\sqrt{3}\right)}{\left(-4x^3 - 12bx - 12c_1 + 4\sqrt{5x^6 + 6\,(2a+b)\,x^4 + 6c_1x^3 + 3\,(4a^2+3b^2)\,x^2 + 18c_1bx + 4a^3 + 9c_1^2}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 6.695 (sec). Leaf size: 396

```
DSolve[(y[x]^2+x^2+a)*y'[x]+2*x*y[x]+x^2+b==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{2}\left(\sqrt{4(a+x^2)^3+(3bx+x^3-3c_1)^2}-3bx-x^3+3c_1\right)^{2/3}-2a-2x^2}{2^{2/3}\sqrt[3]{\sqrt{4(a+x^2)^3+(3bx+x^3-3c_1)^2}-3bx-x^3+3c_1}}$$

$$y(x) \to \frac{(1+i\sqrt{3})(a+x^2)}{2^{2/3}\sqrt[3]{\sqrt{4(a+x^2)^3+(3bx+x^3-3c_1)^2}-3bx-x^3+3c_1}} + \frac{i(\sqrt{3}+i)\sqrt[3]{\sqrt{4(a+x^2)^3+(3bx+x^3-3c_1)^2}-3bx-x^3+3c_1}}{2\sqrt[3]{2}}$$

$$y(x) \to \frac{(1-i\sqrt{3})(a+x^2)}{2^{2/3}\sqrt[3]{\sqrt{4(a+x^2)^3+(3bx+x^3-3c_1)^2}-3bx-x^3+3c_1}} - \frac{i(\sqrt{3}-i)\sqrt[3]{\sqrt{4(a+x^2)^3+(3bx+x^3-3c_1)^2}-3bx-x^3+3c_1}}{2\sqrt[3]{2}}$$

1.274 problem 275

Internal problem ID [8611]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 275.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{(y^2 + x + x^2)\, y' - y = 0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 40

```
dsolve((y(x)^2+x^2+x)*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$\frac{\mathrm{e}^{-2iy(x)}(y(x) + ix) + 2(x + iy(x))\, c_1}{2iy\,(x) + 2x} = 0$$

✓ Solution by Mathematica
Time used: 0.111 (sec). Leaf size: 18

```
DSolve[(y[x]^2+x^2+x)*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[y(x) - \arctan\left(\frac{x}{y(x)}\right) = c_1, y(x)\right]$$

1.275 problem 276

Internal problem ID [8612]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 276.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{(y^2 - x^2)\, y' + 2yx = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 47

```
dsolve((y(x)^2-x^2)*diff(y(x),x)+2*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1 - \sqrt{-4c_1^2x^2 + 1}}{2c_1}$$
$$y(x) = \frac{1 + \sqrt{-4c_1^2x^2 + 1}}{2c_1}$$

✓ Solution by Mathematica
Time used: 1.012 (sec). Leaf size: 66

```
DSolve[(y[x]^2-x^2)*y'[x]+2*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(e^{c_1} - \sqrt{-4x^2 + e^{2c_1}}\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{-4x^2 + e^{2c_1}} + e^{c_1}\right)$$
$$y(x) \to 0$$

1.276 problem 277

Internal problem ID [8613]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 277.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{(y^2 + x^4)\, y' - 4yx^3 = 0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 41

```
dsolve((y(x)^2+x^4)*diff(y(x),x)-4*x^3*y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{4x^4 + c_1^2}}{2} + \frac{c_1}{2}$$
$$y(x) = \frac{\sqrt{4x^4 + c_1^2}}{2} + \frac{c_1}{2}$$

✓ Solution by Mathematica

Time used: 0.285 (sec). Leaf size: 58

```
DSolve[(y[x]^2+x^4)*y'[x]-4*x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(c_1 - \sqrt{4x^4 + c_1{}^2}\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{4x^4 + c_1{}^2} + c_1\right)$$
$$y(x) \to 0$$

1.277 problem 278

Internal problem ID [8614]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 278.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{(y^2 + 4\sin(x))\, y' = \cos(x)}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 28

```
dsolve((y(x)^2+4*sin(x))*diff(y(x),x)-cos(x)=0,y(x), singsol=all)
```

$$\frac{(-8y(x)^2 - 4y(x) - 32\sin(x) - 1)\, e^{-4y(x)}}{32} + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.167 (sec). Leaf size: 39

```
DSolve[(y[x]^2+4*Sin[x])*y'[x]-Cos[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{1}{32}e^{-4y(x)}\left(8y(x)^2 + 4y(x) + 1\right) - e^{-4y(x)}\sin(x) = c_1, y(x)\right]$$

1.278 problem 279

Internal problem ID [8615]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 279.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\left(y^2+2y+x\right)y'+(x+y)^2\,y^2+y(y+1)=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 110

```
dsolve((y(x)^2+2*y(x)+x)*diff(y(x),x)+(y(x)+x)^2*y(x)^2+y(x)*(y(x)+1)=0,y(x), singsol=all)
```

$$y(x)=\frac{x^2-c_1x+\sqrt{x^4-2c_1x^3+\left(c_1^2-2\right)x^2+\left(4+2c_1\right)x-4c_1+1}-1}{2c_1-2x}$$
$$y(x)=\frac{-x^2+c_1x+\sqrt{x^4-2c_1x^3+\left(c_1^2-2\right)x^2+\left(4+2c_1\right)x-4c_1+1}+1}{2x-2c_1}$$

✓ Solution by Mathematica
Time used: 2.179 (sec). Leaf size: 146

```
DSolve[(y[x]^2+2*y[x]+x)*y'[x]+(y[x]+x)^2*y[x]^2+y[x]*(y[x]+1)==0,y[x],x,IncludeSingularSolu
```

$$y(x)\to-\frac{x^2+\sqrt{\left(-x^2+c_1x+1\right)^2+4\left(x-c_1\right)}-c_1x-1}{2\left(x-c_1\right)}$$
$$y(x)\to\frac{-x^2+\sqrt{\left(-x^2+c_1x+1\right)^2+4\left(x-c_1\right)}+c_1x+1}{2\left(x-c_1\right)}$$
$$y(x)\to\frac{1}{2}\left(-\sqrt{x^2}-x\right)$$
$$y(x)\to\frac{1}{2}\left(\sqrt{x^2}-x\right)$$

1.279 problem 280

Internal problem ID [8616]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 280.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{(x+y)^2 y' = a^2}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 24

```
dsolve((y(x)+x)^2*diff(y(x),x)-a^2=0,y(x), singsol=all)
```

$$y(x) = a \operatorname{RootOf}(\tan(\_Z) a - a\_Z + c_1 - x) - c_1$$

✓ Solution by Mathematica
Time used: 0.117 (sec). Leaf size: 21

```
DSolve[(y[x]+x)^2*y'[x]-a^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[y(x) - a \arctan\left(\frac{y(x)+x}{a}\right) = c_1, y(x)\right]$$

1.280 problem 281

Internal problem ID [8617]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 281.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{(y^2+2yx-x^2)\,y'-y^2+2yx=-x^2}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 55

```
dsolve((y(x)^2+2*x*y(x)-x^2)*diff(y(x),x)-y(x)^2+2*x*y(x)+x^2=0,y(x), singsol=all)
```

$$y(x)=\frac{1-\sqrt{-4c_1^2x^2+4c_1x+1}}{2c_1}$$
$$y(x)=\frac{1+\sqrt{-4c_1^2x^2+4c_1x+1}}{2c_1}$$

✓ Solution by Mathematica
Time used: 1.669 (sec). Leaf size: 75

```
DSolve[(y[x]^2+2*x*y[x]-x^2)*y'[x]-y[x]^2+2*x*y[x]+x^2==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{1}{2}\left(e^{c_1}-\sqrt{-4x^2+4e^{c_1}x+e^{2c_1}}\right)$$
$$y(x)\to\frac{1}{2}\left(\sqrt{-4x^2+4e^{c_1}x+e^{2c_1}}+e^{c_1}\right)$$

1.281 problem 282

Internal problem ID [8618]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 282.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational]`

$$\boxed{(y+3x-1)^2 y' - (2y-1)(4y+6x-3) = 0}$$

✓ Solution by Maple

Time used: 0.437 (sec). Leaf size: 75

```
dsolve((y(x)+3*x-1)^2*diff(y(x),x)-(2*y(x)-1)*(4*y(x)+6*x-3)=0,y(x), singsol=all)
```

$$-4\ln(2) - 3\ln\left(\frac{-y(x)+3x}{6x-1}\right) - \ln\left(\frac{2-3y(x)-3x}{6x-1}\right) + 3\ln\left(\frac{-2y(x)+1}{6x-1}\right) - \ln(6x-1) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.2 (sec). Leaf size: 1089

```
DSolve[(y[x]+3*x-1)^2*y'[x]-(2*y[x]-1)*(4*y[x]+6*x-3)==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{6}\left(-\sqrt{36x^2-12x+16e^{c_1}(6x-1)+3\,2^{2/3}\sqrt[3]{-e^{c_1}(6x-1)^4(6x-1+e^{c_1})}+1+16e^{2c_1}} -\frac{1}{2}\sqrt{-\frac{8\left(-(6x-1)^3+96e^{2c_1}(6x-1)+30e^{c_1}(1-6x)^2+64e^{3c_1}\right)}{\sqrt{36x^2-12x+16e^{c_1}(6x-1)+3\,2^{2/3}\sqrt[3]{-e^{c_1}(6x-1)^4(6x-1+e^{c_1})}+1+16e^{2c_1}}}+8(12x+} +12x+1+4e^{c_1}\right)$$

$$y(x) \to \frac{1}{6}\left(-\sqrt{36x^2-12x+16e^{c_1}(6x-1)+3\,2^{2/3}\sqrt[3]{-e^{c_1}(6x-1)^4(6x-1+e^{c_1})}+1+16e^{2c_1}} +\frac{1}{2}\sqrt{-\frac{8\left(-(6x-1)^3+96e^{2c_1}(6x-1)+30e^{c_1}(1-6x)^2+64e^{3c_1}\right)}{\sqrt{36x^2-12x+16e^{c_1}(6x-1)+3\,2^{2/3}\sqrt[3]{-e^{c_1}(6x-1)^4(6x-1+e^{c_1})}+1+16e^{2c_1}}}+8(12x+} +12x+1+4e^{c_1}\right)$$

$$y(x) \to \frac{1}{6}\left(\sqrt{36x^2-12x+16e^{c_1}(6x-1)+3\,2^{2/3}\sqrt[3]{-e^{c_1}(6x-1)^4(6x-1+e^{c_1})}+1+16e^{2c_1}} -\frac{1}{2}\sqrt{\frac{8\left(-(6x-1)^3+96e^{2c_1}(6x-1)+30e^{c_1}(1-6x)^2+64e^{3c_1}\right)}{\sqrt{36x^2-12x+16e^{c_1}(6x-1)+3\,2^{2/3}\sqrt[3]{-e^{c_1}(6x-1)^4(6x-1+e^{c_1})}+1+16e^{2c_1}}}+8(12x+1} +12x+1+4e^{c_1}\right)$$

$$y(x) \to \frac{1}{6}\left(\sqrt{36x^2-12x+16e^{c_1}(6x-1)+3\,2^{2/3}\sqrt[3]{-e^{c_1}(6x-1)^4(6x-1+e^{c_1})}+1+16e^{2c_1}} +\frac{1}{2}\sqrt{\frac{8\left(-(6x-1)^3+96e^{2c_1}(6x-1)+30e^{c_1}(1-6x)^2+64e^{3c_1}\right)}{\sqrt{36x^2-12x+16e^{c_1}(6x-1)+3\,2^{2/3}\sqrt[3]{-e^{c_1}(6x-1)^4(6x-1+e^{c_1})}+1+16e^{2c_1}}}+8(12x+1}\right.$$

1.282 problem 283

Internal problem ID [8619]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 283.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$3(y^2 - x^2)\, y' + 2y^3 - 6(1 + x)\, yx = 3\,\mathrm{e}^x$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 357

```
dsolve(3*(y(x)^2-x^2)*diff(y(x),x)+2*y(x)^3-6*x*(x+1)*y(x)-3*exp(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{2^{\frac{1}{3}}\left(2x^2\mathrm{e}^{4x} + 2^{\frac{1}{3}}\left(\left(\mathrm{e}^{3x} - c_1 + \sqrt{-4x^6\mathrm{e}^{4x} + \mathrm{e}^{6x} - 2c_1\mathrm{e}^{3x} + c_1^2}\right)\mathrm{e}^{4x}\right)^{\frac{2}{3}}\right)\mathrm{e}^{-2x}}{2\left(\left(\mathrm{e}^{3x} - c_1 + \sqrt{-4x^6\mathrm{e}^{4x} + \mathrm{e}^{6x} - 2c_1\mathrm{e}^{3x} + c_1^2}\right)\mathrm{e}^{4x}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(-2x^2\mathrm{e}^{4x}\left(i\sqrt{3} - 1\right) + 2^{\frac{1}{3}}\left(1 + i\sqrt{3}\right)\left(\left(\mathrm{e}^{3x} - c_1 + \sqrt{-4x^6\mathrm{e}^{4x} + \mathrm{e}^{6x} - 2c_1\mathrm{e}^{3x} + c_1^2}\right)\mathrm{e}^{4x}\right)^{\frac{2}{3}}\right)2^{\frac{1}{3}}\mathrm{e}^{-2x}}{4\left(\left(\mathrm{e}^{3x} - c_1 + \sqrt{-4x^6\mathrm{e}^{4x} + \mathrm{e}^{6x} - 2c_1\mathrm{e}^{3x} + c_1^2}\right)\mathrm{e}^{4x}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{2^{\frac{1}{3}}\mathrm{e}^{-2x}\left(-2x^2\mathrm{e}^{4x}\left(1 + i\sqrt{3}\right) + 2^{\frac{1}{3}}\left(i\sqrt{3} - 1\right)\left(\left(\mathrm{e}^{3x} - c_1 + \sqrt{-4x^6\mathrm{e}^{4x} + \mathrm{e}^{6x} - 2c_1\mathrm{e}^{3x} + c_1^2}\right)\mathrm{e}^{4x}\right)^{\frac{2}{3}}\right)}{4\left(\left(\mathrm{e}^{3x} - c_1 + \sqrt{-4x^6\mathrm{e}^{4x} + \mathrm{e}^{6x} - 2c_1\mathrm{e}^{3x} + c_1^2}\right)\mathrm{e}^{4x}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 60.3 (sec). Leaf size: 497

```
DSolve[3*(y[x]^2-x^2)*y'[x]+2*y[x]^3-6*x*(x+1)*y[x]-3*Exp[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to -\frac{e^{-2x}\sqrt[3]{\sqrt{e^{8x}\left(-4e^{4x}x^6+e^{6x}-2c_1e^{3x}+c_1{}^2\right)}-e^{7x}+c_1e^{4x}}}{\sqrt[3]{2}} - \frac{\sqrt[3]{2}e^{2x}x^2}{\sqrt[3]{\sqrt{e^{8x}\left(-4e^{4x}x^6+e^{6x}-2c_1e^{3x}+c_1{}^2\right)}-e^{7x}+c_1e^{4x}}}$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)e^{-2x}\sqrt[3]{\sqrt{e^{8x}\left(-4e^{4x}x^6+e^{6x}-2c_1e^{3x}+c_1{}^2\right)}-e^{7x}+c_1e^{4x}}}{2\sqrt[3]{2}} + \frac{\left(1+i\sqrt{3}\right)e^{2x}x^2}{2^{2/3}\sqrt[3]{\sqrt{e^{8x}\left(-4e^{4x}x^6+e^{6x}-2c_1e^{3x}+c_1{}^2\right)}-e^{7x}+c_1e^{4x}}}$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)e^{-2x}\sqrt[3]{\sqrt{e^{8x}\left(-4e^{4x}x^6+e^{6x}-2c_1e^{3x}+c_1{}^2\right)}-e^{7x}+c_1e^{4x}}}{2\sqrt[3]{2}} + \frac{\left(1-i\sqrt{3}\right)e^{2x}x^2}{2^{2/3}\sqrt[3]{\sqrt{e^{8x}\left(-4e^{4x}x^6+e^{6x}-2c_1e^{3x}+c_1{}^2\right)}-e^{7x}+c_1e^{4x}}}$$

1.283 problem 284

Internal problem ID [8620]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 284.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\left(4y^2+x^2\right)y'-yx=0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 32

```
dsolve((4*y(x)^2+x^2)*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{\mathrm{e}^{-c_1}\sqrt{\frac{\mathrm{e}^{2c_1}x^2}{\mathrm{LambertW}\left(\frac{\mathrm{e}^{2c_1}x^2}{4}\right)}}}{2}$$

✓ Solution by Mathematica
Time used: 9.932 (sec). Leaf size: 64

```
DSolve[(4*y[x]^2+x^2)*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\frac{x}{2\sqrt{W\left(\frac{1}{4}e^{-\frac{c_1}{2}}x^2\right)}}$$

$$y(x)\to\frac{x}{2\sqrt{W\left(\frac{1}{4}e^{-\frac{c_1}{2}}x^2\right)}}$$

$$y(x)\to0$$

1.284 problem 285

Internal problem ID [8621]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 285.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$\left(4y^2+2yx+3x^2\right)y'+y^2+6yx=-2x^2$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 432

```
dsolve((4*y(x)^2+2*x*y(x)+3*x^2)*diff(y(x),x)+y(x)^2+6*x*y(x)+2*x^2=0,y(x), singsol=all)
```

$$y(x)=\frac{\left(c_1^3x^3+8+2\sqrt{333c_1^6x^6+4c_1^3x^3+16}\right)^{\frac{1}{3}}-\frac{11c_1^2x^2}{\left(c_1^3x^3+8+2\sqrt{333c_1^6x^6+4c_1^3x^3+16}\right)^{\frac{1}{3}}}-c_1x}{4c_1}$$

$$y(x)=-\frac{11i\sqrt{3}\,c_1^2x^2+i\sqrt{3}\left(c_1^3x^3+8+2\sqrt{333c_1^6x^6+4c_1^3x^3+16}\right)^{\frac{2}{3}}-11c_1^2x^2+2c_1x\left(c_1^3x^3+8+2\sqrt{333c_1^6x^6\right.}{8\left(c_1^3x^3+8+2\sqrt{333c_1^6x^6+4c_1^3x^3+16}\right)^{\frac{1}{3}}c}$$

$$y(x)=\frac{11i\sqrt{3}\,c_1^2x^2+i\sqrt{3}\left(c_1^3x^3+8+2\sqrt{333c_1^6x^6+4c_1^3x^3+16}\right)^{\frac{2}{3}}+11c_1^2x^2-2c_1x\left(c_1^3x^3+8+2\sqrt{333c_1^6x^6+4\right.}{8\left(c_1^3x^3+8+2\sqrt{333c_1^6x^6+4c_1^3x^3+16}\right)^{\frac{1}{3}}c_1}$$

✓ Solution by Mathematica

Time used: 45.643 (sec). Leaf size: 612

```
DSolve[(4*y[x]^2+2*x*y[x]+3*x^2)*y'[x]+y[x]^2+6*x*y[x]+2*x^2==0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{1}{4}\left(\sqrt[3]{x^3+2\sqrt{333x^6+4e^{3c_1}x^3+16e^{6c_1}}+8e^{3c_1}} - \frac{11x^2}{\sqrt[3]{x^3+2\sqrt{333x^6+4e^{3c_1}x^3+16e^{6c_1}}+8e^{3c_1}}} - x\right)$$

$$y(x) \to \frac{1}{16}\left(2i\left(\sqrt{3}+i\right)\sqrt[3]{x^3+2\sqrt{333x^6+4e^{3c_1}x^3+16e^{6c_1}}+8e^{3c_1}} + \frac{22(1+i\sqrt{3})\,x^2}{\sqrt[3]{x^3+2\sqrt{333x^6+4e^{3c_1}x^3+16e^{6c_1}}+8e^{3c_1}}} - 4x\right)$$

$$y(x) \to \frac{1}{16}\left(-2\left(1+i\sqrt{3}\right)\sqrt[3]{x^3+2\sqrt{333x^6+4e^{3c_1}x^3+16e^{6c_1}}+8e^{3c_1}} + \frac{22(1-i\sqrt{3})\,x^2}{\sqrt[3]{x^3+2\sqrt{333x^6+4e^{3c_1}x^3+16e^{6c_1}}+8e^{3c_1}}} - 4x\right)$$

$$y(x) \to \frac{1}{4}\left(\sqrt[3]{6\sqrt{37}\sqrt{x^6}+x^3} - \frac{11x^2}{\sqrt[3]{6\sqrt{37}\sqrt{x^6}+x^3}} - x\right)$$

$$y(x) \to \frac{1}{8}\left(\left(-1-i\sqrt{3}\right)\sqrt[3]{6\sqrt{37}\sqrt{x^6}+x^3} + \frac{11(1-i\sqrt{3})\,x^2}{\sqrt[3]{6\sqrt{37}\sqrt{x^6}+x^3}} - 2x\right)$$

$$y(x) \to \frac{1}{8}\left(i\left(\sqrt{3}+i\right)\sqrt[3]{6\sqrt{37}\sqrt{x^6}+x^3} + \frac{11(1+i\sqrt{3})\,x^2}{\sqrt[3]{6\sqrt{37}\sqrt{x^6}+x^3}} - 2x\right)$$

1.285 problem 286

Internal problem ID [8622]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 286.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational]`

$$(2y - 3x + 1)^2 y' - (3y - 2x - 4)^2 = 0$$

✓ Solution by Maple

Time used: 1.953 (sec). Leaf size: 1337

```
dsolve((2*y(x)-3*x+1)^2*diff(y(x),x)-(3*y(x)-2*x-4)^2=0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 60.198 (sec). Leaf size: 3501

```
DSolve[(2*y[x]-3*x+1)^2*y'[x]-(3*y[x]-2*x-4)^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

1.286 problem 287

Internal problem ID [8623]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 287.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, _dAlembert]`

$$\boxed{(2y-4x+1)^2 y' - (-2x+y)^2 = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 56

```
dsolve((2*y(x)-4*x+1)^2*diff(y(x),x)-(y(x)-2*x)^2=0,y(x), singsol=all)
```

$$-\frac{x}{7}-\frac{9\sqrt{2}\,\operatorname{arctanh}\left(\frac{(7y(x)-14x+4)\sqrt{2}}{2}\right)}{98} - \frac{2\ln\left(7(y(x)-2x)^2+8y(x)-16x+2\right)}{49}+\frac{4y(x)}{7}-c_1=0$$

✓ Solution by Mathematica
Time used: 1.459 (sec). Leaf size: 77

```
DSolve[(2*y[x]-4*x+1)^2*y'[x]-(y[x]-2*x)^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{y(x)}{2}+\frac{1}{196}\left(14y(x)-\left(8-9\sqrt{2}\right)\log\left(-7y(x)+14x+\sqrt{2}-4\right) - \left(8+9\sqrt{2}\right)\log\left(7y(x)-14x+\sqrt{2}+4\right)-28x\right)=c_1,y(x)\right]$$

1.287 problem 288

Internal problem ID [8624]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 288.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]`

$$\left(6y^2 - 3x^2y + 1\right) y' - 3xy^2 = -x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 589

```
dsolve((6*y(x)^2-3*x^2*y(x)+1)*diff(y(x),x)-3*x*y(x)^2+x=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(-324x^2 - 432c_1 + 27x^6 + 12\sqrt{-81x^8 - 162c_1x^6 + 621x^4 + 1944c_1x^2 + 1296c_1^2 + 96}\right)^{\frac{1}{3}}}{12} + \frac{3x^4 - 8}{4\left(-324x^2 - 432c_1 + 27x^6 + 12\sqrt{-81x^8 - 162c_1x^6 + 621x^4 + 1944c_1x^2 + 1296c_1^2 + 96}\right)^{\frac{1}{3}}} + \frac{x^2}{4}$$

$$y(x) = \frac{9i\sqrt{3}\,x^4 - i\sqrt{3}\left(-324x^2 - 432c_1 + 27x^6 + 12\sqrt{-81x^8 - 162c_1x^6 + 621x^4 + 1944c_1x^2 + 1296c_1^2 + 96}\right)^{\frac{2}{3}}}{}$$

$$y(x) = -\frac{9i\sqrt{3}\,x^4 - i\sqrt{3}\left(-324x^2 - 432c_1 + 27x^6 + 12\sqrt{-81x^8 - 162c_1x^6 + 621x^4 + 1944c_1x^2 + 1296c_1^2 + 96}\right.}{}$$

✓ Solution by Mathematica

Time used: 2.446 (sec). Leaf size: 538

```
DSolve[(6*y[x]^2-3*x^2*y[x]+1)*y'[x]-3*x*y[x]^2+x==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{1}{36}\left(9x^2 - 3\sqrt[3]{3}\sqrt[3]{-9x^6+108x^2+4\sqrt{3}\sqrt{-27x^8-54c_1x^6+207x^4+648c_1x^2+32+432{c_1}^2}+144c_1} - \frac{3\,3^{2/3}(3x^4-8)}{\sqrt[3]{-9x^6+108x^2+4\sqrt{3}\sqrt{-27x^8-54c_1x^6+207x^4+648c_1x^2+32+432{c_1}^2}+144c_1}}\right)$$

$$y(x) \to \frac{1}{24}\left(6x^2 + \sqrt[3]{3}\left(1 - i\sqrt{3}\right)\sqrt[3]{-9x^6+108x^2+4\sqrt{3}\sqrt{-27x^8-54c_1x^6+207x^4+648c_1x^2+32+432{c_1}^2}+144c_1} + \frac{3^{2/3}\left(1+i\sqrt{3}\right)(3x^4-8)}{\sqrt[3]{-9x^6+108x^2+4\sqrt{3}\sqrt{-27x^8-54c_1x^6+207x^4+648c_1x^2+32+432{c_1}^2}+144c_1}}\right)$$

$$y(x) \to \frac{1}{24}\left(6x^2 + \sqrt[3]{3}\left(1 + i\sqrt{3}\right)\sqrt[3]{-9x^6+108x^2+4\sqrt{3}\sqrt{-27x^8-54c_1x^6+207x^4+648c_1x^2+32+432{c_1}^2}+144c_1} + \frac{3^{2/3}\left(1-i\sqrt{3}\right)(3x^4-8)}{\sqrt[3]{-9x^6+108x^2+4\sqrt{3}\sqrt{-27x^8-54c_1x^6+207x^4+648c_1x^2+32+432{c_1}^2}+144c_1}}\right)$$

$$y(x) \to -\frac{1}{\sqrt{3}}$$

$$y(x) \to \frac{1}{\sqrt{3}}$$

1.288 problem 289

Internal problem ID [8625]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 289.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x),G(x)]']`

$$(6y-x)^2 y' - 6y^2 + 2yx = -a$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 115

```
dsolve((6*y(x)-x)^2*diff(y(x),x)-6*y(x)^2+2*x*y(x)+a=0,y(x), singsol=all)
```

$$y(x) = \frac{(-x^3-18ax-18c_1)^{\frac{1}{3}}}{6} + \frac{x}{6}$$
$$y(x) = -\frac{(-x^3-18ax-18c_1)^{\frac{1}{3}}}{12} - \frac{i\sqrt{3}\,(-x^3-18ax-18c_1)^{\frac{1}{3}}}{12} + \frac{x}{6}$$
$$y(x) = -\frac{(-x^3-18ax-18c_1)^{\frac{1}{3}}}{12} + \frac{i\sqrt{3}\,(-x^3-18ax-18c_1)^{\frac{1}{3}}}{12} + \frac{x}{6}$$

✓ Solution by Mathematica

Time used: 0.65 (sec). Leaf size: 115

```
DSolve[(6*y[x]-x)^2*y'[x]-6*y[x]^2+2*x*y[x]+a==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}\left(x + \sqrt[3]{-18ax - x^3 + 18c_1}\right)$$
$$y(x) \to \frac{x}{6} + \frac{1}{12}i\left(\sqrt{3}+i\right)\sqrt[3]{-18ax - x^3 + 18c_1}$$
$$y(x) \to \frac{x}{6} - \frac{1}{12}\left(1+i\sqrt{3}\right)\sqrt[3]{-18ax - x^3 + 18c_1}$$

1.289 problem 290

Internal problem ID [8626]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 290.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$\left(ay^2+2bxy+c\,x^2\right)y'+by^2+2ycx=-d\,x^2$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 1117

```
dsolve((a*y(x)^2+2*b*x*y(x)+c*x^2)*diff(y(x),x)+b*y(x)^2+2*c*x*y(x)+d*x^2=0,y(x), singsol=al
```

$$y(x) = \frac{\frac{\left(-4c_1^3a^2d\,x^3+12x^3c\,c_1^3ba-8b^3x^3c_1^3+4\sqrt{a^2c_1^6d^2x^6-6abc\,c_1^6d\,x^6+4a\,c^3c_1^6x^6+4b^3c_1^6d\,x^6-3b^2c^2c_1^6x^6-2c_1^3a^2d\,x^3+6x^3c\,c_1^3ba-4b^3x^3c_1^3+a^2}\,a+4a^2\right.}{2}}{}$$

$$y(x) = -\frac{\left(\frac{i\sqrt{3}}{4}+\frac{1}{4}\right)\left(4\sqrt{x^6\left(a^2d^2+\left(-6bcd+4c^3\right)a+4b^3d-3b^2c^2\right)c_1^6-2x^3\left(a^2d-3acb+2b^3\right)c_1^3+a^2}\,a+\left(-\right.\right.}{\left(4\sqrt{x^6\left(a^2d^2\right.}\right.}$$

$$y(x) = \frac{\frac{\left(i\sqrt{3}-1\right)\left(4\sqrt{x^6(a^2d^2+(-6bcd+4c^3)a+4b^3d-3b^2c^2)c_1^6-2x^3(a^2d-3acb+2b^3)c_1^3+a^2}\,a+\left(-4c_1^3d\,x^3+4\right)a^2+12x^3c\,c_1^3ba-8b^3x^3c_1^3\right)^{\frac{2}{3}}}{4}+xc_1\left(\right.}{\left(4\sqrt{x^6\left(a^2d^2+\left(-6bcd+4c^3\right)a+4b^3\right.}\right.}$$

✓ Solution by Mathematica

Time used: 60.378 (sec). Leaf size: 744

```
DSolve[(a*y[x]^2+2*b*x*y[x]+c*x^2)*y'[x]+b*y[x]^2+2*c*x*y[x]+d*x^2==0,y[x],x,IncludeSingular
```

$$y(x) \to \frac{2^{2/3}\sqrt[3]{\sqrt{-4x^6\left(b^2-ac\right)^3+\left(a^2\left(-dx^3+e^{3c_1}\right)+3abcx^3-2b^3x^3\right)^2}-a^2dx^3+a^2e^{3c_1}+3abcx^3-2b^3x^3+}}{2}$$

$$y(x) \to \frac{9i2^{2/3}\left(\sqrt{3}+i\right)\sqrt[3]{\sqrt{-4x^6\left(b^2-ac\right)^3+\left(a^2\left(-dx^3+e^{3c_1}\right)+3abcx^3-2b^3x^3\right)^2}-a^2dx^3+a^2e^{3c_1}+3abcx^3}}{}$$

$$y(x) \to \frac{-9\ 2^{2/3}\left(1+i\sqrt{3}\right)\sqrt[3]{\sqrt{-4x^6\left(b^2-ac\right)^3+\left(a^2\left(-dx^3+e^{3c_1}\right)+3abcx^3-2b^3x^3\right)^2}-a^2dx^3+a^2e^{3c_1}+3ab}}{}$$

1.290 problem 291

Internal problem ID [8627]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 291.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$\left(b(\beta y + x\alpha)^2 - \beta(xa + by)\right) y' + a(\beta y + x\alpha)^2 - \alpha(xa + by) = 0$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 50

```
dsolve((b*(beta*y(x)+alpha*x)^2-beta*(b*y(x)+a*x))*diff(y(x),x)+a*(beta*y(x)+alpha*x)^2-alph
```

$$y(x) = \frac{-ax + e^{\operatorname{RootOf}\left(c_1 a\beta x - c_1\alpha b x - \_Z a\beta x + \_Z\alpha b x - c_1\beta\, e^{\_Z} + e^{\_Z}\_Z\beta + b\right)}}{b}$$

✓ Solution by Mathematica
Time used: 0.71 (sec). Leaf size: 39

```
DSolve[(b*(\[Beta]*y[x]+alpha*x)^2-\[Beta]*(b*y[x]+a*x))*y'[x]+a*(\[Beta]*y[x]+alpha*x)^2-al
```

$$\text{Solve}\left[\frac{a\beta\left(\log(ax + by(x)) + \frac{1}{\alpha x + \beta y(x)}\right)}{a\beta - \alpha b} = c_1, y(x)\right]$$

1.291 problem 292

Internal problem ID [8628]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 292.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational]`

$$\boxed{(ay + xb + c)^2 y' + (\alpha y + \beta x + \gamma)^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 115

```
dsolve((a*y(x)+b*x+c)^2*diff(y(x),x)+(alpha*y(x)+beta*x+gamma)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{((bx + c)\alpha - (\beta x + \gamma) a) \operatorname{RootOf}\left(\int^{-Z} \frac{(\_a a - b)^2}{\_a^3 a^2 - 2\_a^2 ab - \_a^2 \alpha^2 + 2\_a \alpha\beta + \_a b^2 - \beta^2} d\_a + \ln(a\beta x - \alpha bx + a\gamma}{a\beta - b\alpha}$$

✓ Solution by Mathematica

Time used: 22.46 (sec). Leaf size: 1653

```
DSolve[(a*y[x]+b*x+c)^2*y'[x]+(\[Alpha]*y[x]+\[Beta]*x+\[Gamma])^2==0,y[x],x,IncludeSingular
```

$$\text{Solve}\Big[\beta(b\alpha - a\beta)\text{RootSum}\Big[-\gamma^3b^3 - \alpha^3y(x)^3b^3 - \gamma\#1^2b^3 - 3\alpha^2\gamma y(x)^2b^3$$
$$+ 2\alpha^2\#1y(x)^2b^3 + 2\gamma^2\#1b^3 - 3\alpha\gamma^2y(x)b^3 - \alpha\#1^2y(x)b^3 + 4\alpha\gamma\#1y(x)b^3 + 3a\alpha^2\beta y(x)^3b^2$$
$$+ 3c\beta\gamma^2b^2 + c\beta\#1^2b^2 + 3c\alpha^2\beta y(x)^2b^2 + 6a\alpha\beta\gamma y(x)^2b^2 - 4a\alpha\beta\#1y(x)^2b^2 - 4c\beta\gamma\#1b^2$$
$$+ 3a\beta\gamma^2y(x)b^2 + a\beta\#1^2y(x)b^2 + 6c\alpha\beta\gamma y(x)b^2 - 4c\alpha\beta\#1y(x)b^2 - 4a\beta\gamma\#1y(x)b^2$$
$$- \alpha\beta\#1^3b - 3a^2\alpha\beta^2y(x)^3b + \alpha\beta\gamma\#1^2b - 6ac\alpha\beta^2y(x)^2b - 3a^2\beta^2\gamma y(x)^2b$$
$$+ 2a^2\beta^2\#1y(x)^2b - 3c^2\beta^2\gamma b + 2c^2\beta^2\#1b - 3c^2\alpha\beta^2y(x)b + \alpha^2\beta\#1^2y(x)b - 6ac\beta^2\gamma y(x)b$$
$$+ 4ac\beta^2\#1y(x)b + c^3\beta^3 + a\beta^2\#1^3 + a^3\beta^3y(x)^3 - c\alpha\beta^2\#1^2 + 3a^2c\beta^3y(x)^2 + 3ac^2\beta^3y(x)$$
$$- a\alpha\beta^2\#1^2y(x)\&, \frac{}{-2\gamma^2b^3 - 2\alpha^2y(x)^2b^3 + 2\gamma\#1b^3 - 4\alpha\gamma y(x)b^3 + 2\alpha\#1y(x)b^3 + 4a\alpha\beta y(x)^2b^2 + 4c\beta\gamma b^2 -}$$
$$+ \int_1^{y(x)} \Big(\frac{}{x^2\gamma b^3 + x^2\alpha K[1]b^3 + 2ax\alpha K[1]^2b^2 - cx^2\beta b^2 + 2cx\gamma b^2 + 2cx\alpha K[1]b^2 - ax^2\beta K[1]b^2 + 2ax\gamma K[1]b}$$
$$- \frac{}{-x^2\gamma b^3 - x^2\alpha K[1]b^3 - 2ax\alpha K[1]^2b^2 + cx^2\beta b^2 - 2cx\gamma b^2 - 2cx\alpha K[1]b^2 + ax^2\beta K[1]b^2 - 2ax\gamma K[1]b^2 - a}$$

1.292 problem 293

Internal problem ID [8629]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 293.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{x\left(y^2-3x\right)y'+2y^3-5yx=0}$$

✓ Solution by Maple
Time used: 0.079 (sec). Leaf size: 35

```
dsolve(x*(y(x)^2-3*x)*diff(y(x),x)+2*y(x)^3-5*x*y(x)=0,y(x), singsol=all)
```

$$\ln\left(x\right)-c_1-\frac{2\ln\left(\frac{5y(x)^2-13x}{x}\right)}{65}+\frac{6\ln\left(\frac{y(x)}{\sqrt{x}}\right)}{13}=0$$

✓ Solution by Mathematica

Time used: 6.936 (sec). Leaf size: 661

```
DSolve[x*(y[x]^2-3*x)*y'[x]+2*y[x]^3-5*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 1\right]$$
$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 2\right]$$
$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 3\right]$$
$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 4\right]$$
$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 5\right]$$
$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 6\right]$$
$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 7\right]$$
$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 8\right]$$
$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 9\right]$$
$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 10\right]$$
$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 11\right]$$
$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 12\right]$$
$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 13\right]$$
$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 14\right]$$
$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{350\,x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 15\right]$$

1.293 problem 294

Internal problem ID [8630]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 294.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$x(y^2+x^2-a)\,y'-(y^2+x^2+a)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 148

```
dsolve(x*(y(x)^2+x^2-a)*diff(y(x),x)-y(x)*(y(x)^2+x^2+a)=0,y(x), singsol=all)
```

$$\frac{y(x)^2(-x^2+a)}{-x^2-y(x)^2+a}=-\frac{\sqrt{x^2-a}\,x}{\sqrt{\frac{-c_1x^2+c_1a-4a}{-x^2+a}}}+\frac{x^2}{2}-\frac{a}{2}$$
$$\frac{y(x)^2(-x^2+a)}{-x^2-y(x)^2+a}=\frac{\sqrt{x^2-a}\,x}{\sqrt{\frac{-c_1x^2+c_1a-4a}{-x^2+a}}}+\frac{x^2}{2}-\frac{a}{2}$$

✓ Solution by Mathematica

Time used: 0.938 (sec). Leaf size: 65

```
DSolve[x*(y[x]^2+x^2-a)*y'[x]-y[x]*(y[x]^2+x^2+a)==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x)\to\frac{1}{2}\left(c_1x-\sqrt{-4a+(4+c_1{}^2)\,x^2}\right)$$
$$y(x)\to\frac{1}{2}\left(\sqrt{-4a+(4+c_1{}^2)\,x^2}+c_1x\right)$$

1.294 problem 295

Internal problem ID [8631]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 295.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$x\left(y^{2}+yx-x^{2}\right) y'-y^{3}+x y^{2}+x^{2} y=0$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 29

```
dsolve(x*(y(x)^2+x*y(x)-x^2)*diff(y(x),x)-y(x)^3+x*y(x)^2+x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{RootOf}\left(\mathrm{e}^{2\_Z}+2\,\mathrm{e}^{\_Z}\ln(x)+2c_1\mathrm{e}^{\_Z}+\mathrm{e}^{\_Z}\_Z+1\right)} x$$

✓ Solution by Mathematica

Time used: 0.195 (sec). Leaf size: 31

```
DSolve[x*(y[x]^2+x*y[x]-x^2)*y'[x]-y[x]^3+x*y[x]^2+x^2*y[x]==0,y[x],x,IncludeSingularSolutio
```

$$\text{Solve}\left[\frac{x}{y(x)}+\frac{y(x)}{x}+\log\left(\frac{y(x)}{x}\right)=-2\log(x)+c_1,y(x)\right]$$

1.295 problem 296

Internal problem ID [8632]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 296.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$x(y^2 + x^2y + x^2)\, y' - 2y^3 - 2y^2x^2 = -x^4$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 61

```
dsolve(x*(y(x)^2+x^2*y(x)+x^2)*diff(y(x),x)-2*y(x)^3-2*x^2*y(x)^2+x^4=0,y(x), singsol=all)
```

$$y(x) = -c_1x^2 - \sqrt{x^2 \left(1 + \left(c_1^2 - c_1\right) x^2\right)}$$
$$y(x) = -c_1x^2 + \sqrt{x^2 \left(1 + \left(c_1^2 - c_1\right) x^2\right)}$$

✓ Solution by Mathematica

Time used: 25.374 (sec). Leaf size: 88

```
DSolve[x*(y[x]^2+x^2*y[x]+x^2)*y'[x]-2*y[x]^3-2*x^2*y[x]^2+x^4==0,y[x],x,IncludeSingularSolu
```

$$y(x) \rightarrow -e^{-c_1}\left(x^2 + \sqrt{x^2 \left(x^2 - e^{c_1}x^2 + e^{2c_1}\right)}\right)$$
$$y(x) \rightarrow e^{-c_1}\left(-x^2 + \sqrt{x^2 \left(x^2 - e^{c_1}x^2 + e^{2c_1}\right)}\right)$$

1.296 problem 297

Internal problem ID [8633]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 297.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{2x(y^2+5x^2)\,y'+y^3-x^2y=0}$$

✓ Solution by Maple
Time used: 0.5 (sec). Leaf size: 29

```
dsolve(2*x*(y(x)^2+5*x^2)*diff(y(x),x)+y(x)^3-x^2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\mathrm{RootOf}\left(\_Z^{45}c_1x^9-\_Z^{18}-6\_Z^9-9\right)^{\frac{9}{2}}x$$

✓ Solution by Mathematica
Time used: 2.675 (sec). Leaf size: 216

```
DSolve[2*x*(y[x]^2+5*x^2)*y'[x]+y[x]^3-x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \mathrm{Root}\left[-\#1^5+\frac{\#1^2e^{3c_1}}{x^{3/2}}+3e^{3c_1}\sqrt{x}\&,1\right]$$
$$y(x)\to \mathrm{Root}\left[-\#1^5+\frac{\#1^2e^{3c_1}}{x^{3/2}}+3e^{3c_1}\sqrt{x}\&,2\right]$$
$$y(x)\to \mathrm{Root}\left[-\#1^5+\frac{\#1^2e^{3c_1}}{x^{3/2}}+3e^{3c_1}\sqrt{x}\&,3\right]$$
$$y(x)\to \mathrm{Root}\left[-\#1^5+\frac{\#1^2e^{3c_1}}{x^{3/2}}+3e^{3c_1}\sqrt{x}\&,4\right]$$
$$y(x)\to \mathrm{Root}\left[-\#1^5+\frac{\#1^2e^{3c_1}}{x^{3/2}}+3e^{3c_1}\sqrt{x}\&,5\right]$$

1.297 problem 298

Internal problem ID [8634]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 298.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact, _rational, _Bernoulli]`

$$\boxed{3y^2xy' + y^3 = 2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 73

```
dsolve(3*x*y(x)^2*diff(y(x),x)+y(x)^3-2*x=0,y(x), singsol=all)
```

$$y(x) = \frac{((x^2+c_1)\,x^2)^{\frac{1}{3}}}{x}$$
$$y(x) = -\frac{((x^2+c_1)\,x^2)^{\frac{1}{3}}\,(1+i\sqrt{3})}{2x}$$
$$y(x) = \frac{((x^2+c_1)\,x^2)^{\frac{1}{3}}\,(i\sqrt{3}-1)}{2x}$$

✓ Solution by Mathematica
Time used: 0.22 (sec). Leaf size: 72

```
DSolve[3*x*y[x]^2*y'[x]+y[x]^3-2*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{x^2+c_1}}{\sqrt[3]{x}}$$
$$y(x) \to -\frac{\sqrt[3]{-1}\,\sqrt[3]{x^2+c_1}}{\sqrt[3]{x}}$$
$$y(x) \to \frac{(-1)^{2/3}\,\sqrt[3]{x^2+c_1}}{\sqrt[3]{x}}$$

1.298 problem 299

Internal problem ID [8635]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 299.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact, _rational]`

$$\boxed{(3xy^2 - x^2)\, y' + y^3 - 2yx = 0}$$

✓ Solution by Maple
Time used: 0.297 (sec). Leaf size: 242

```
dsolve((3*x*y(x)^2-x^2)*diff(y(x),x)+y(x)^3-2*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{12^{\frac{1}{3}}\left(x^3 12^{\frac{1}{3}} + \left(\left(\sqrt{-12x^5 + 81c_1^2} + 9c_1\right)x^2\right)^{\frac{2}{3}}\right)}{6x\left(\left(\sqrt{-12x^5 + 81c_1^2} + 9c_1\right)x^2\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{3^{\frac{1}{3}}\left(\left(-i\sqrt{3} - 1\right)\left(\left(\sqrt{-12x^5 + 81c_1^2} + 9c_1\right)x^2\right)^{\frac{2}{3}} + \left(i3^{\frac{5}{6}} - 3^{\frac{1}{3}}\right)x^3 2^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{12\left(\left(\sqrt{-12x^5 + 81c_1^2} + 9c_1\right)x^2\right)^{\frac{1}{3}} x}$$

$$y(x) = -\frac{3^{\frac{1}{3}}2^{\frac{2}{3}}\left(\left(1 - i\sqrt{3}\right)\left(\left(\sqrt{-12x^5 + 81c_1^2} + 9c_1\right)x^2\right)^{\frac{2}{3}} + x^3 2^{\frac{2}{3}}\left(i3^{\frac{5}{6}} + 3^{\frac{1}{3}}\right)\right)}{12\left(\left(\sqrt{-12x^5 + 81c_1^2} + 9c_1\right)x^2\right)^{\frac{1}{3}} x}$$

✓ Solution by Mathematica

Time used: 34.309 (sec). Leaf size: 328

```
DSolve[(3*x*y[x]^2-x^2)*y'[x]+y[x]^3-2*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2\sqrt[3]{3}x^3 + \sqrt[3]{2}\left(9c_1x^2 + \sqrt{-12x^9 + 81c_1{}^2x^4}\right)^{2/3}}{6^{2/3}x\sqrt[3]{9c_1x^2 + \sqrt{-12x^9 + 81c_1{}^2x^4}}}$$

$$y(x) \to \frac{2\sqrt[3]{2}\sqrt[6]{3}\left(\sqrt{3} + 3i\right)x^3 + \sqrt[3]{3}\left(1 - i\sqrt{3}\right)\left(18c_1x^2 + 2\sqrt{-12x^9 + 81c_1{}^2x^4}\right)^{2/3}}{12x\sqrt[3]{9c_1x^2 + \sqrt{-12x^9 + 81c_1{}^2x^4}}}$$

$$y(x) \to \frac{2\sqrt[3]{2}\sqrt[6]{3}\left(\sqrt{3} - 3i\right)x^3 + \sqrt[3]{3}\left(1 + i\sqrt{3}\right)\left(18c_1x^2 + 2\sqrt{-12x^9 + 81c_1{}^2x^4}\right)^{2/3}}{12x\sqrt[3]{9c_1x^2 + \sqrt{-12x^9 + 81c_1{}^2x^4}}}$$

1.299 problem 300

Internal problem ID [8636]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 300.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact, _rational, _Bernoulli]`

$$\boxed{6y^2xy' + 2y^3 = -x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 92

```
dsolve(6*x*y(x)^2*diff(y(x),x)+2*y(x)^3+x=0,y(x), singsol=all)
```

$$y(x) = \frac{2^{\frac{1}{3}}\left(-\left(x^2-4c_1\right)x^2\right)^{\frac{1}{3}}}{2x}$$
$$y(x) = -\frac{2^{\frac{1}{3}}\left(-\left(x^2-4c_1\right)x^2\right)^{\frac{1}{3}}\left(1+i\sqrt{3}\right)}{4x}$$
$$y(x) = \frac{2^{\frac{1}{3}}\left(-\left(x^2-4c_1\right)x^2\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)}{4x}$$

✓ Solution by Mathematica

Time used: 0.228 (sec). Leaf size: 99

```
DSolve[6*x*y[x]^2*y'[x]+2*y[x]^3+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{-x^2+4c_1}}{2^{2/3}\sqrt[3]{x}}$$
$$y(x) \to -\frac{\sqrt[3]{-1}\sqrt[3]{-x^2+4c_1}}{2^{2/3}\sqrt[3]{x}}$$
$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{-x^2+4c_1}}{2^{2/3}\sqrt[3]{x}}$$

1.300 problem 301

Internal problem ID [8637]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 301.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{(6xy^2 + x^2)\, y' - y(3y^2 - x) = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 38

```
dsolve((6*x*y(x)^2+x^2)*diff(y(x),x)-y(x)*(3*y(x)^2-x)=0,y(x), singsol=all)
```

$$y(x) = \frac{e^{\frac{3c_1}{2}}\sqrt{6}}{6x\sqrt{\frac{e^{3c_1}}{x^3\,\mathrm{LambertW}\left(\frac{6\,e^{3c_1}}{x^3}\right)}}}$$

✓ Solution by Mathematica
Time used: 4.163 (sec). Leaf size: 69

```
DSolve[(6*x*y[x]^2+x^2)*y'[x]-y[x]*(3*y[x]^2-x)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{x}\sqrt{W\left(\frac{6e^{3c_1}}{x^3}\right)}}{\sqrt{6}}$$
$$y(x) \to \frac{\sqrt{x}\sqrt{W\left(\frac{6e^{3c_1}}{x^3}\right)}}{\sqrt{6}}$$
$$y(x) \to 0$$

1.301 problem 302

Internal problem ID [8638]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 302.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$(y^2x^2 + x)\, y' + y = 0$$

✓ Solution by Maple

Time used: 0.218 (sec). Leaf size: 137

```
dsolve((x^2*y(x)^2+x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{2}\,\sqrt{x c_1 \left(2c_1 + x - \sqrt{x\,(4c_1 + x)}\right)}}{2c_1 x}$$

$$y(x) = \frac{\sqrt{2}\,\sqrt{x c_1 \left(2c_1 + x - \sqrt{x\,(4c_1 + x)}\right)}}{2c_1 x}$$

$$y(x) = -\frac{\sqrt{2}\,\sqrt{x c_1 \left(2c_1 + x + \sqrt{x\,(4c_1 + x)}\right)}}{2c_1 x}$$

$$y(x) = \frac{\sqrt{2}\,\sqrt{x c_1 \left(2c_1 + x + \sqrt{x\,(4c_1 + x)}\right)}}{2c_1 x}$$

✓ Solution by Mathematica

Time used: 0.288 (sec). Leaf size: 65

```
DSolve[(x^2*y[x]^2+x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(c_1 - \frac{\sqrt{4 + c_1^2 x}}{\sqrt{x}}\right)$$
$$y(x) \to \frac{1}{2}\left(\frac{\sqrt{4 + c_1^2 x}}{\sqrt{x}} + c_1\right)$$
$$y(x) \to 0$$

1.302 problem 303

Internal problem ID [8639]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 303.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{(yx-1)^2 xy' + \left(y^2x^2+1\right) y = 0}$$

✓ Solution by Maple
Time used: 0.14 (sec). Leaf size: 34

```
dsolve((x*y(x)-1)^2*x*diff(y(x),x)+(x^2*y(x)^2+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{\mathrm{RootOf}\left(-2\,\mathrm{e}^{-\_Z}\ln(x)-\mathrm{e}^{2\_Z}+2c_1\mathrm{e}^{\_Z}+2\,\mathrm{e}^{\_Z}\_Z+1\right)}}{x}$$

✓ Solution by Mathematica
Time used: 0.097 (sec). Leaf size: 25

```
DSolve[(x*y[x]-1)^2*x*y'[x]+(x^2*y[x]^2+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[xy(x) - \frac{1}{xy(x)} - 2\log(y(x)) = c_1, y(x)\right]$$

1.303 problem 304

Internal problem ID [8640]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 304.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\left(10y^2x^3 + x^2y + 2x\right) y' + 5y^3x^2 + xy^2 = 0$$

✓ Solution by Maple

Time used: 0.343 (sec). Leaf size: 55

```
dsolve((10*x^3*y(x)^2+x^2*y(x)+2*x)*diff(y(x),x)+5*x^2*y(x)^3+x*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{\tan\left(\text{RootOf}\left(2\sqrt{10}\,\ln(2) + \sqrt{10}\,\ln\left(\frac{\tan(\_Z)^2 \sec(\_Z)^2}{x^2}\right) - \sqrt{10}\,\ln(5) + 2\sqrt{10}\,c_1 + 2\_Z\right)\right)\sqrt{10}}{5x}$$

✓ Solution by Mathematica

Time used: 0.294 (sec). Leaf size: 44

```
DSolve[(10*x^3*y[x]^2+x^2*y[x]+2*x)*y'[x]+5*x^2*y[x]^3+x*y[x]^2==0,y[x],x,IncludeSingularSol
```

$$\text{Solve}\left[y(x)\sqrt{5x^2y(x)^2 + 2}e^{\frac{\arctan\left(\sqrt{\frac{5}{2}}xy(x)\right)}{\sqrt{10}}} = c_1, y(x)\right]$$

1.304 problem 305

Internal problem ID [8641]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 305.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{(y^3 - 3x)\, y' - 3y = -x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve((y(x)^3-3*x)*diff(y(x),x)-3*y(x)+x^2=0,y(x), singsol=all)
```

$$\frac{x^3}{3} - 3xy(x) + \frac{y(x)^4}{4} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.164 (sec). Leaf size: 1211

```
DSolve[(y[x]^3-3*x)*y'[x]-3*y[x]+x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{4x^3+\left(243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}\right)^{2/3}+12c_1}{\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}}}}}{\sqrt{6}} - \frac{1}{2}\sqrt{-\frac{12\sqrt{6}x}{\sqrt{\frac{4x^3+\left(243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}\right)^{2/3}+12c_1}{\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}}}}} - \frac{2}{3}\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x}}}$$

$$y(x) \to \frac{1}{2}\sqrt{-\frac{12\sqrt{6}x}{\sqrt{\frac{4x^3+\left(243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}\right)^{2/3}+12c_1}{\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}}}}} - \frac{2}{3}\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4}}} - \frac{\sqrt{\frac{4x^3+\left(243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}\right)^{2/3}+12c_1}{\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}}}}}{\sqrt{6}}$$

$$y(x) \to \frac{\sqrt{\frac{4x^3+\left(243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}\right)^{2/3}+12c_1}{\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}}}}}{\sqrt{6}} - \frac{1}{2}\sqrt{\frac{12\sqrt{6}x}{\sqrt{\frac{4x^3+\left(243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}\right)^{2/3}+12c_1}{\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}}}}} - \frac{2}{3}\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4}}}$$

$$y(x) \to \frac{\sqrt{\frac{4x^3+\left(243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}\right)^{2/3}+12c_1}{\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}}}}}{\sqrt{6}} + \frac{1}{2}\sqrt{\frac{12\sqrt{6}x}{\sqrt{\frac{4x^3+\left(243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}\right)^{2/3}+12c_1}{\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}}}}} - \frac{2}{3}\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4}}}$$

1.305 problem 306

Internal problem ID [8642]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 306.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{(y^3 - x^3)\, y' - x^2 y = 0}$$

✓ Solution by Maple

Time used: 0.641 (sec). Leaf size: 389

```
dsolve((y(x)^3-x^3)*diff(y(x),x)-x^2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{x}{\left(x^3c_1\left(-c_1x^3+\sqrt{c_1^2x^6+1}\right)\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{x}{\left(-\left(c_1x^3+\sqrt{c_1^2x^6+1}\right)c_1x^3\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{4x}{\left(1+i\sqrt{3}\right)^2\left(x^3c_1\left(-c_1x^3+\sqrt{c_1^2x^6+1}\right)\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{4x}{\left(i\sqrt{3}-1\right)^2\left(x^3c_1\left(-c_1x^3+\sqrt{c_1^2x^6+1}\right)\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{4x}{\left(i\sqrt{3}-1\right)^2\left(x^3c_1\left(-c_1x^3+\sqrt{c_1^2x^6+1}\right)\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{4x}{\left(1+i\sqrt{3}\right)^2\left(x^3c_1\left(-c_1x^3+\sqrt{c_1^2x^6+1}\right)\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{4x}{\left(1+i\sqrt{3}\right)^2\left(-\left(c_1x^3+\sqrt{c_1^2x^6+1}\right)c_1x^3\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{4x}{\left(i\sqrt{3}-1\right)^2\left(-\left(c_1x^3+\sqrt{c_1^2x^6+1}\right)c_1x^3\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{4x}{\left(i\sqrt{3}-1\right)^2\left(-\left(c_1x^3+\sqrt{c_1^2x^6+1}\right)c_1x^3\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{4x}{\left(1+i\sqrt{3}\right)^2\left(-\left(c_1x^3+\sqrt{c_1^2x^6+1}\right)c_1x^3\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 6.356 (sec). Leaf size: 352

```
DSolve[-(x^2*y[x]) + (-x^3 + y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[3]{x^3 - \sqrt{x^6 - e^{6c_1}}}$$
$$y(x) \to -\sqrt[3]{-1}\sqrt[3]{x^3 - \sqrt{x^6 - e^{6c_1}}}$$
$$y(x) \to (-1)^{2/3}\sqrt[3]{x^3 - \sqrt{x^6 - e^{6c_1}}}$$
$$y(x) \to \sqrt[3]{x^3 + \sqrt{x^6 - e^{6c_1}}}$$
$$y(x) \to -\sqrt[3]{-1}\sqrt[3]{x^3 + \sqrt{x^6 - e^{6c_1}}}$$
$$y(x) \to (-1)^{2/3}\sqrt[3]{x^3 + \sqrt{x^6 - e^{6c_1}}}$$
$$y(x) \to 0$$
$$y(x) \to \sqrt[3]{x^3 - \sqrt{x^6}}$$
$$y(x) \to -\sqrt[3]{-1}\sqrt[3]{x^3 - \sqrt{x^6}}$$
$$y(x) \to (-1)^{2/3}\sqrt[3]{x^3 - \sqrt{x^6}}$$
$$y(x) \to \sqrt[3]{\sqrt{x^6} + x^3}$$
$$y(x) \to -\sqrt[3]{-1}\sqrt[3]{\sqrt{x^6} + x^3}$$
$$y(x) \to (-1)^{2/3}\sqrt[3]{\sqrt{x^6} + x^3}$$

1.306 problem 307

Internal problem ID [8643]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 307.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{(y^2 + x^2 + a)\, y y' + x(y^2 + x^2 - a) = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 113

```
dsolve((y(x)^2+x^2+a)*y(x)*diff(y(x),x)+(y(x)^2+x^2-a)*x = 0,y(x), singsol=all)
```

$$
\begin{aligned}
y(x) &= \sqrt{-x^2 - a - 2\sqrt{a\,x^2 - c_1}} \\
y(x) &= \sqrt{-x^2 - a + 2\sqrt{a\,x^2 - c_1}} \\
y(x) &= -\sqrt{-x^2 - a - 2\sqrt{a\,x^2 - c_1}} \\
y(x) &= -\sqrt{-x^2 - a + 2\sqrt{a\,x^2 - c_1}}
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 8.667 (sec). Leaf size: 149

```
DSolve[x*(-a + x^2 + y[x]^2) + y[x]*(a + x^2 + y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSoluti
```

$$
\begin{aligned}
y(x) &\to -\sqrt{-\sqrt{a^2 + 4ax^2 + 4c_1} - a - x^2} \\
y(x) &\to \sqrt{-\sqrt{a^2 + 4ax^2 + 4c_1} - a - x^2} \\
y(x) &\to -\sqrt{\sqrt{a^2 + 4ax^2 + 4c_1} - a - x^2} \\
y(x) &\to \sqrt{\sqrt{a^2 + 4ax^2 + 4c_1} - a - x^2}
\end{aligned}
$$

1.307 problem 308

Internal problem ID [8644]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 308.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2y^3y' + xy^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 37

```
dsolve(2*y(x)^3*diff(y(x),x)+x*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = -\frac{\sqrt{-2x^2 + 4c_1}}{2}$$
$$y(x) = \frac{\sqrt{-2x^2 + 4c_1}}{2}$$

✓ Solution by Mathematica
Time used: 0.076 (sec). Leaf size: 53

```
DSolve[x*y[x]^2 + 2*y[x]^3*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$
$$y(x) \to -\sqrt{-\frac{x^2}{2} + 2c_1}$$
$$y(x) \to \sqrt{-\frac{x^2}{2} + 2c_1}$$
$$y(x) \to 0$$

1.308 problem 309

Internal problem ID [8645]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 309.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(2y^3 + y)\, y' = 2x^3 + x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 113

```
dsolve((2*y(x)^3+y(x))*diff(y(x),x)-2*x^3-x = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2 - 2\sqrt{4x^4 + 4x^2 + 8c_1 + 1}}}{2}$$
$$y(x) = \frac{\sqrt{-2 - 2\sqrt{4x^4 + 4x^2 + 8c_1 + 1}}}{2}$$
$$y(x) = -\frac{\sqrt{-2 + 2\sqrt{4x^4 + 4x^2 + 8c_1 + 1}}}{2}$$
$$y(x) = \frac{\sqrt{-2 + 2\sqrt{4x^4 + 4x^2 + 8c_1 + 1}}}{2}$$

✓ Solution by Mathematica

Time used: 2.343 (sec). Leaf size: 151

```
DSolve[-x - 2*x^3 + (y[x] + 2*y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-1-\sqrt{4x^4+4x^2+1+8c_1}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-1-\sqrt{4x^4+4x^2+1+8c_1}}}{\sqrt{2}}$$
$$y(x) \to -\frac{\sqrt{-1+\sqrt{4x^4+4x^2+1+8c_1}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-1+\sqrt{4x^4+4x^2+1+8c_1}}}{\sqrt{2}}$$

1.309 problem 310

Internal problem ID [8646]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 310.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$\boxed{(2y^3+5x^2y)\, y' + 5xy^2 = -x^3}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 125

```
dsolve((2*y(x)^3+5*x^2*y(x))*diff(y(x),x)+5*x*y(x)^2+x^3 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-10c_1x^2 - 2\sqrt{23c_1^2x^4 + 2}}}{2\sqrt{c_1}}$$
$$y(x) = \frac{\sqrt{-10c_1x^2 - 2\sqrt{23c_1^2x^4 + 2}}}{2\sqrt{c_1}}$$
$$y(x) = -\frac{\sqrt{-10c_1x^2 + 2\sqrt{23c_1^2x^4 + 2}}}{2\sqrt{c_1}}$$
$$y(x) = \frac{\sqrt{-10c_1x^2 + 2\sqrt{23c_1^2x^4 + 2}}}{2\sqrt{c_1}}$$

✓ Solution by Mathematica

Time used: 21.205 (sec). Leaf size: 295

```
DSolve[x^3 + 5*x*y[x]^2 + (5*x^2*y[x] + 2*y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to -\frac{\sqrt{-5x^2 - \sqrt{23x^4 + 2e^{4c_1}}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{-5x^2 - \sqrt{23x^4 + 2e^{4c_1}}}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{-5x^2 + \sqrt{23x^4 + 2e^{4c_1}}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{-5x^2 + \sqrt{23x^4 + 2e^{4c_1}}}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{-\sqrt{23}\sqrt{x^4} - 5x^2}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{-\sqrt{23}\sqrt{x^4} - 5x^2}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{\sqrt{23}\sqrt{x^4} - 5x^2}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{\sqrt{23}\sqrt{x^4} - 5x^2}}{\sqrt{2}}$$

1.310 problem 311

Internal problem ID [8647]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 311.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$\boxed{(20y^3 - 3xy^2 + 6x^2y + 3x^3)\, y' - y^3 + 6xy^2 + 9x^2y = -4x^3}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 50

```
dsolve((20*y(x)^3-3*x*y(x)^2+6*x^2*y(x)+3*x^3)*diff(y(x),x)-y(x)^3+6*x*y(x)^2+9*x^2*y(x)+4*x
```

$$y(x) = \frac{\mathrm{RootOf}\left(c_1^4x^4 + 3\_Z c_1^3x^3 + 3\_Z^2c_1^2x^2 - \_Z^3c_1x + 5\_Z^4 - 1\right)}{c_1}$$

✓ Solution by Mathematica

Time used: 60.173 (sec). Leaf size: 2201

```
DSolve[4*x^3 + 9*x^2*y[x] + 6*x*y[x]^2 - y[x]^3 + (3*x^3 + 6*x^2*y[x] - 3*x*y[x]^2 + 20*y[x]
```

$$y(x) \to \frac{1}{2}\sqrt{-\frac{39x^2}{100} + \frac{\sqrt[3]{99x^6 + 351e^{c_1}x^2 + \sqrt{3}\sqrt{-67037x^{12} + 185406e^{c_1}x^8 - 83733e^{2c_1}x^4 + 32000e^{3c_1}}}}{5\sqrt[3]{2}3^{2/3}} + \frac{}{5\sqrt[3]{99}}}$$

$$-\frac{1}{2}\sqrt{-\frac{39x^2}{50} - \frac{\sqrt[3]{99x^6 + 351e^{c_1}x^2 + \sqrt{3}\sqrt{-67037x^{12} + 185406e^{c_1}x^8 - 83733e^{2c_1}x^4 + 32000e^{3c_1}}}}{5\sqrt[3]{2}3^{2/3}} + \frac{}{5\sqrt[3]{}}}$$

$$+\frac{x}{20}$$

$$y(x) \to \frac{1}{2}\sqrt{-\frac{39x^2}{100} + \frac{\sqrt[3]{99x^6 + 351e^{c_1}x^2 + \sqrt{3}\sqrt{-67037x^{12} + 185406e^{c_1}x^8 - 83733e^{2c_1}x^4 + 32000e^{3c_1}}}}{5\sqrt[3]{2}3^{2/3}} + \frac{}{5\sqrt[3]{99}}}$$

$$+\frac{1}{2}\sqrt{-\frac{39x^2}{50} - \frac{\sqrt[3]{99x^6 + 351e^{c_1}x^2 + \sqrt{3}\sqrt{-67037x^{12} + 185406e^{c_1}x^8 - 83733e^{2c_1}x^4 + 32000e^{3c_1}}}}{5\sqrt[3]{2}3^{2/3}} + \frac{}{5\sqrt[3]{}}}$$

$$+\frac{x}{20}$$

$$y(x) \to$$

$$-\frac{1}{2}\sqrt{-\frac{39x^2}{100} + \frac{\sqrt[3]{99x^6 + 351e^{c_1}x^2 + \sqrt{3}\sqrt{-67037x^{12} + 185406e^{c_1}x^8 - 83733e^{2c_1}x^4 + 32000e^{3c_1}}}}{5\sqrt[3]{2}3^{2/3}} + \frac{}{5\sqrt[3]{}}}$$

$$-\frac{1}{2}\sqrt{-\frac{39x^2}{50} - \frac{\sqrt[3]{99x^6 + 351e^{c_1}x^2 + \sqrt{3}\sqrt{-67037x^{12} + 185406e^{c_1}x^8 - 83733e^{2c_1}x^4 + 32000e^{3c_1}}}}{5\sqrt[3]{2}3^{2/3}} + \frac{}{5\sqrt[3]{}}}$$

$$+\frac{x}{20}$$

$$y(x) \to$$

1.311 problem 312

Internal problem ID [8648]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 312.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\left(\frac{y^2}{b}+\frac{x^2}{a}\right)(x+yy')+\frac{(a-b)(yy'-x)}{a+b}=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 244

```
dsolve((y(x)^2/b+x^2/a)*(y(x)*diff(y(x),x)+x)+(a-b)/(a+b)*(y(x)*diff(y(x),x)-x) = 0,y(x), si
```

$$y(x)=\frac{\sqrt{a\left(e^{\frac{-2\,\mathrm{LambertW}\left(\frac{(a+b)\,e^{\frac{\left(-x^2-b\right)a^2+\left(-b^2-2c_1\right)a+b^2x^2}{2a^2b}}}{2a^2b}\right)a^2b+\left(-x^2-b\right)a^2+\left(-b^2-2c_1\right)a+b^2x^2}{2a^2b}}+b\left(-x^2+a\right)\right)}}{a}$$

$$y(x)=-\frac{\sqrt{a\left(e^{\frac{-2\,\mathrm{LambertW}\left(\frac{(a+b)\,e^{\frac{\left(-x^2-b\right)a^2+\left(-b^2-2c_1\right)a+b^2x^2}{2a^2b}}}{2a^2b}\right)a^2b+\left(-x^2-b\right)a^2+\left(-b^2-2c_1\right)a+b^2x^2}{2a^2b}}+b\left(-x^2+a\right)\right)}}{a}$$

✓ Solution by Mathematica

Time used: 60.984 (sec). Leaf size: 178

```
DSolve[((a - b)*(-x + y[x]*y'[x]))/(a + b) + (x^2/a + y[x]^2/b)*(x + y[x]*y'[x])==0,y[x],x,I
```

$$y(x) \to -\frac{\sqrt{b}\sqrt{(a+b)(a-x^2)+2a^2W\left(\frac{c_1(a+b)\exp\left(-\frac{(a+b)\left(a\left(b+x^2\right)-bx^2\right)}{2a^2b}\right)}{2a^3b^2}\right)}}{\sqrt{a}\sqrt{a+b}}$$

$$y(x) \to \frac{\sqrt{b}\sqrt{(a+b)(a-x^2)+2a^2W\left(\frac{c_1(a+b)\exp\left(-\frac{(a+b)\left(a\left(b+x^2\right)-bx^2\right)}{2a^2b}\right)}{2a^3b^2}\right)}}{\sqrt{a}\sqrt{a+b}}$$

1.312 problem 313

Internal problem ID [8649]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 313.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\left(2a y^{3}+3a y^{2} x -x^{3} b +c \,x^{2}\right) y^{\prime }-a y^{3}+c y^{2}+3 y b \,x^{2} = -2 x^{3} b$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 595

```
dsolve((2*a*y(x)^3+3*a*x*y(x)^2-b*x^3+c*x^2)*diff(y(x),x)-a*y(x)^3+c*y(x)^2+3*b*x^2*y(x)+2*b
```

$$y(x) = -\frac{\left(-\left(\left(-9b\,x^{3}+9c_{1}x+\sqrt{\frac{81a\,b^{2}x^{6}-162abc_{1}x^{4}+12c^{3}x^{3}+81a\,c_{1}^{2}x^{2}-36c^{2}c_{1}x^{2}+36c\,c_{1}^{2}x-12c_{1}^{3}}{a}}\right)a^{2}\right)^{\frac{2}{3}}+\left(cx-c_{1}\right)a12^{\frac{1}{3}}}{6\left(\left(-9b\,x^{3}+9c_{1}x+\sqrt{\frac{81a\,b^{2}x^{6}-162abc_{1}x^{4}+12c^{3}x^{3}+81a\,c_{1}^{2}x^{2}-36c^{2}c_{1}x^{2}+36c\,c_{1}^{2}x-12c_{1}^{3}}{a}}\right)a^{2}\right)^{\frac{1}{3}}a}$$

$$y(x) = -\frac{3^{\frac{1}{3}}2^{\frac{2}{3}}\left(\left(1+i\sqrt{3}\right)\left(\left(-9b\,x^{3}+9c_{1}x+\sqrt{\frac{81a\,b^{2}x^{6}-162abc_{1}x^{4}+12c^{3}x^{3}+81a\,c_{1}^{2}x^{2}-36c^{2}c_{1}x^{2}+36c\,c_{1}^{2}x-12c_{1}^{3}}{a}}\right)a^{2}\right)^{\frac{2}{3}}+a}{12\left(\left(-9b\,x^{3}+9c_{1}x+\sqrt{\frac{81a\,b^{2}x^{6}-162abc_{1}x^{4}+12c^{3}x^{3}+81a\,c_{1}^{2}x^{2}-36c^{2}c_{1}x^{2}+36c\,c_{1}^{2}x-12c_{1}^{3}}{a}}\right)a^{2}}$$

$$y(x) = \frac{3^{\frac{1}{3}}\left(\left(i\sqrt{3}-1\right)\left(\left(-9b\,x^{3}+9c_{1}x+\sqrt{\frac{81a\,b^{2}x^{6}-162abc_{1}x^{4}+12c^{3}x^{3}+81a\,c_{1}^{2}x^{2}-36c^{2}c_{1}x^{2}+36c\,c_{1}^{2}x-12c_{1}^{3}}{a}}\right)a^{2}\right)^{\frac{2}{3}}+\left(i3^{\frac{5}{6}}-\right.}{12\left(\left(-9b\,x^{3}+9c_{1}x+\sqrt{\frac{81a\,b^{2}x^{6}-162abc_{1}x^{4}+12c^{3}x^{3}+81a\,c_{1}^{2}x^{2}-36c^{2}c_{1}x^{2}+36c\,c_{1}^{2}x-12c_{1}^{3}}{a}}\right)a^{2}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 60.244 (sec). Leaf size: 520

```
DSolve[2*b*x^3 + 3*b*x^2*y[x] + c*y[x]^2 - a*y[x]^3 + (c*x^2 - b*x^3 + 3*a*x*y[x]^2 + 2*a*y[
```

$$y(x) \to \frac{-\sqrt[3]{2}\left(\sqrt{3}\sqrt{a^3\left(27ax^2\left(bx^2+c_1\right)^2+4(cx+c_1)^3\right)}+9a^2bx^3+9a^2c_1x\right)^{2/3}+2\sqrt[3]{3}acx+2\sqrt[3]{3}ac_1}{6^{2/3}a\sqrt[3]{\sqrt{3}\sqrt{a^3\left(27ax^2\left(bx^2+c_1\right)^2+4(cx+c_1)^3\right)}+9a^2bx^3+9a^2c_1x}}$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)\sqrt[3]{\sqrt{\left(27a^2bx^3+27a^2c_1x\right)^2+4(3acx+3ac_1)^3}+27a^2bx^3+27a^2c_1x}}{6\sqrt[3]{2}a} - \frac{i\left(\sqrt{3}-i\right)(cx+c_1)}{2^{2/3}\sqrt[3]{\sqrt{\left(27a^2bx^3+27a^2c_1x\right)^2+4(3acx+3ac_1)^3}+27a^2bx^3+27a^2c_1x}}$$

$$y(x) \to \frac{i\left(\sqrt{3}+i\right)(cx+c_1)}{2^{2/3}\sqrt[3]{\sqrt{\left(27a^2bx^3+27a^2c_1x\right)^2+4(3acx+3ac_1)^3}+27a^2bx^3+27a^2c_1x}} + \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{\sqrt{\left(27a^2bx^3+27a^2c_1x\right)^2+4(3acx+3ac_1)^3}+27a^2bx^3+27a^2c_1x}}{6\sqrt[3]{2}a}$$

1.313 problem 314

Internal problem ID [8650]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 314.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{xy^3y' + y^4 = x\sin(x)}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 158

```
dsolve(x*y(x)^3*diff(y(x),x)+y(x)^4-x*sin(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{(4(-x^4 + 12x^2 - 24)\cos(x) + 16(x^3 - 6x)\sin(x) + c_1)^{\frac{1}{4}}}{x}$$
$$y(x) = -\frac{(4(-x^4 + 12x^2 - 24)\cos(x) + 16(x^3 - 6x)\sin(x) + c_1)^{\frac{1}{4}}}{x}$$
$$y(x) = -\frac{i(4(-x^4 + 12x^2 - 24)\cos(x) + 16(x^3 - 6x)\sin(x) + c_1)^{\frac{1}{4}}}{x}$$
$$y(x) = \frac{i(4(-x^4 + 12x^2 - 24)\cos(x) + 16(x^3 - 6x)\sin(x) + c_1)^{\frac{1}{4}}}{x}$$

✓ Solution by Mathematica

Time used: 0.637 (sec). Leaf size: 164

```
DSolve[-(x*Sin[x]) + y[x]^4 + x*y[x]^3*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[4]{16x\left(x^2-6\right)\sin(x)-4\left(x^4-12x^2+24\right)\cos(x)+c_1}}{x}$$
$$y(x) \to -\frac{i\sqrt[4]{16x\left(x^2-6\right)\sin(x)-4\left(x^4-12x^2+24\right)\cos(x)+c_1}}{x}$$
$$y(x) \to \frac{i\sqrt[4]{16x\left(x^2-6\right)\sin(x)-4\left(x^4-12x^2+24\right)\cos(x)+c_1}}{x}$$
$$y(x) \to \frac{\sqrt[4]{16x\left(x^2-6\right)\sin(x)-4\left(x^4-12x^2+24\right)\cos(x)+c_1}}{x}$$

1.314 problem 315

Internal problem ID [8651]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 315.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{(2y^3x - x^4)\,y' - y^4 + 2yx^3 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 317

```
dsolve((2*x*y(x)^3-x^4)*diff(y(x),x)-y(x)^4+2*x^3*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{12^{\frac{1}{3}}\left(x12^{\frac{1}{3}}c_1 + \left(x\left(-9c_1x^2 + \sqrt{3}\,\sqrt{\frac{27c_1^3x^4 - 4x}{c_1}}\right)c_1^2\right)^{\frac{2}{3}}\right)}{6c_1\left(x\left(-9c_1x^2 + \sqrt{3}\,\sqrt{\frac{27c_1^3x^4 - 4x}{c_1}}\right)c_1^2\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{3^{\frac{1}{3}}\left(\left(-i\sqrt{3} - 1\right)\left(x\left(-9c_1x^2 + \sqrt{3}\,\sqrt{\frac{27c_1^3x^4 - 4x}{c_1}}\right)c_1^2\right)^{\frac{2}{3}} + \left(i3^{\frac{5}{6}} - 3^{\frac{1}{3}}\right)x2^{\frac{2}{3}}c_1\right)2^{\frac{2}{3}}}{12\left(x\left(-9c_1x^2 + \sqrt{3}\,\sqrt{\frac{27c_1^3x^4 - 4x}{c_1}}\right)c_1^2\right)^{\frac{1}{3}}c_1}$$

$$y(x) = -\frac{3^{\frac{1}{3}}2^{\frac{2}{3}}\left(\left(1 - i\sqrt{3}\right)\left(x\left(-9c_1x^2 + \sqrt{3}\,\sqrt{\frac{27c_1^3x^4 - 4x}{c_1}}\right)c_1^2\right)^{\frac{2}{3}} + x2^{\frac{2}{3}}\left(i3^{\frac{5}{6}} + 3^{\frac{1}{3}}\right)c_1\right)}{12\left(x\left(-9c_1x^2 + \sqrt{3}\,\sqrt{\frac{27c_1^3x^4 - 4x}{c_1}}\right)c_1^2\right)^{\frac{1}{3}}c_1}$$

✓ Solution by Mathematica

Time used: 53.127 (sec). Leaf size: 440

```
DSolve[2*x^3*y[x] - y[x]^4 + (-x^4 + 2*x*y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{\sqrt[3]{2}\left(-9x^3+\sqrt{81x^6-12e^{3c_1}x^3}\right)^{2/3}+2\sqrt[3]{3}e^{c_1}x}{6^{2/3}\sqrt[3]{-9x^3+\sqrt{81x^6-12e^{3c_1}x^3}}}$$

$$y(x) \to \frac{i\sqrt[3]{2}\sqrt[6]{3}\left(\sqrt{3}+i\right)\left(-9x^3+\sqrt{81x^6-12e^{3c_1}x^3}\right)^{2/3}-2\left(\sqrt{3}+3i\right)e^{c_1}x}{2\ 2^{2/3}3^{5/6}\sqrt[3]{-9x^3+\sqrt{81x^6-12e^{3c_1}x^3}}}$$

$$y(x) \to \frac{\sqrt[3]{2}\sqrt[6]{3}\left(-1-i\sqrt{3}\right)\left(-9x^3+\sqrt{81x^6-12e^{3c_1}x^3}\right)^{2/3}-2\left(\sqrt{3}-3i\right)e^{c_1}x}{2\ 2^{2/3}3^{5/6}\sqrt[3]{-9x^3+\sqrt{81x^6-12e^{3c_1}x^3}}}$$

$$y(x) \to \frac{\sqrt[3]{\sqrt{x^6}-x^3}}{\sqrt[3]{2}}$$

$$y(x) \to -\frac{i\left(\sqrt{3}-i\right)\sqrt[3]{\sqrt{x^6}-x^3}}{2\sqrt[3]{2}}$$

$$y(x) \to \frac{i\left(\sqrt{3}+i\right)\sqrt[3]{\sqrt{x^6}-x^3}}{2\sqrt[3]{2}}$$

1.315 problem 316

Internal problem ID [8652]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 316.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$(2y^3x + y)\, y' + 2y^2 = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 60

```
dsolve((2*x*y(x)^3+y(x))*diff(y(x),x)+2*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \sqrt{2}\,\sqrt{-\operatorname{RootOf}\left(e^{-}\_Z\operatorname{expIntegral}_1(\_Z) + 4c_1e^{-}\_Z - 4x\right)}$$
$$y(x) = -\sqrt{2}\,\sqrt{-\operatorname{RootOf}\left(e^{-}\_Z\operatorname{expIntegral}_1(\_Z) + 4c_1e^{-}\_Z - 4x\right)}$$

✓ Solution by Mathematica

Time used: 0.319 (sec). Leaf size: 53

```
DSolve[2*y[x]^2 + (y[x] + 2*x*y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$
$$\text{Solve}\left[x = -\frac{1}{4}e^{-\frac{1}{2}y(x)^2}\,\text{ExpIntegralEi}\left(\frac{y(x)^2}{2}\right) + c_1e^{-\frac{1}{2}y(x)^2}, y(x)\right]$$
$$y(x) \to 0$$

1.316 problem 317

Internal problem ID [8653]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 317.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{(2y^3x + yx + x^2)\, y' + y^2 - yx = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 29

```
dsolve((2*x*y(x)^3+x*y(x)+x^2)*diff(y(x),x)+y(x)^2-x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = e^{\mathrm{RootOf}\left(-e^{3\_Z}-e^{\_Z}\ln(x)+c_1e^{\_Z}-e^{\_Z}\_Z+x\right)}$$

✓ Solution by Mathematica
Time used: 0.217 (sec). Leaf size: 23

```
DSolve[-(x*y[x]) + y[x]^2 + (x^2 + x*y[x] + 2*x*y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolut
```

$$\mathrm{Solve}\left[y(x)^2 - \frac{x}{y(x)} + \log(y(x)) + \log(x) = c_1, y(x)\right]$$

1.317 problem 318

Internal problem ID [8654]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 318.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{(3y^3x - 4yx + y)\, y' + y^2(y^2 - 2) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve((3*x*y(x)^3-4*x*y(x)+y(x))*diff(y(x),x)+y(x)^2*(y(x)^2-2) = 0,y(x), singsol=all)
```

$$y(x) = 0$$

$$x + \frac{1}{y(x)^2} - \frac{c_1}{\sqrt{y(x)^2 - 2}\, y(x)^2} = 0$$

✓ Solution by Mathematica

Time used: 60.173 (sec). Leaf size: 2353

```
DSolve[y[x]^2*(-2 + y[x]^2) + (y[x] - 4*x*y[x] + 3*x*y[x]^3)*y'[x]==0,y[x],x,IncludeSingular
```

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1.318 problem 319

Internal problem ID [8655]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 319.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{(7y^3x + y - 5x)\,y' + y^4 - 5y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 34

```
dsolve((7*x*y(x)^3+y(x)-5*x)*diff(y(x),x)+y(x)^4-5*y(x) = 0,y(x), singsol=all)
```

$$x + \frac{\frac{y(x)^5}{5} - \frac{5y(x)^2}{2} - c_1}{\left(y(x)^3 - 5\right)^2 y(x)} = 0$$

✓ Solution by Mathematica

Time used: 49.593 (sec). Leaf size: 342

```
DSolve[-5*y[x] + y[x]^4 + (-5*x + y[x] + 7*x*y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to \text{Root}\left[10\#1^7x + 2\#1^5 - 100\#1^4x - 25\#1^2 + 250\#1x - 10c_1\&, 1\right]$$
$$y(x) \to \text{Root}\left[10\#1^7x + 2\#1^5 - 100\#1^4x - 25\#1^2 + 250\#1x - 10c_1\&, 2\right]$$
$$y(x) \to \text{Root}\left[10\#1^7x + 2\#1^5 - 100\#1^4x - 25\#1^2 + 250\#1x - 10c_1\&, 3\right]$$
$$y(x) \to \text{Root}\left[10\#1^7x + 2\#1^5 - 100\#1^4x - 25\#1^2 + 250\#1x - 10c_1\&, 4\right]$$
$$y(x) \to \text{Root}\left[10\#1^7x + 2\#1^5 - 100\#1^4x - 25\#1^2 + 250\#1x - 10c_1\&, 5\right]$$
$$y(x) \to \text{Root}\left[10\#1^7x + 2\#1^5 - 100\#1^4x - 25\#1^2 + 250\#1x - 10c_1\&, 6\right]$$
$$y(x) \to \text{Root}\left[10\#1^7x + 2\#1^5 - 100\#1^4x - 25\#1^2 + 250\#1x - 10c_1\&, 7\right]$$
$$y(x) \to 0$$
$$y(x) \to -\sqrt[3]{-5}$$
$$y(x) \to \sqrt[3]{5}$$
$$y(x) \to (-1)^{2/3}\sqrt[3]{5}$$

1.319 problem 320

Internal problem ID [8656]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 320.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{(y^3x^2 + yx)\, y' = 1}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 78

```
dsolve((x^2*y(x)^3+x*y(x))*diff(y(x),x)-1 = 0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{2x^2 \operatorname{LambertW}\left(\frac{c_1 e^{-\frac{-1+2x}{2x}}}{2}\right) + 2x^2 - x}}{x}$$
$$y(x) = -\frac{\sqrt{2x^2 \operatorname{LambertW}\left(\frac{c_1 e^{-\frac{-1+2x}{2x}}}{2}\right) + 2x^2 - x}}{x}$$

✓ Solution by Mathematica
Time used: 0.113 (sec). Leaf size: 76

```
DSolve[-1 + (x*y[x] + x^2*y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{2xW\left(c_1 e^{\frac{1}{2x}-1}\right) + 2x - 1}}{\sqrt{x}}$$
$$y(x) \to \frac{\sqrt{2xW\left(c_1 e^{\frac{1}{2x}-1}\right) + 2x - 1}}{\sqrt{x}}$$

1.320 problem 321

Internal problem ID [8657]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 321.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{(2y^3x^2 + y^2x^2 - 2x)\, y' - 2y = 1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 46

```
dsolve((2*x^2*y(x)^3+x^2*y(x)^2-2*x)*diff(y(x),x)-2*y(x)-1 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{2}$$
$$y(x) = \frac{e^{\operatorname{RootOf}(x\,e^{3\_Z} - 4x\,e^{2\_Z} + 8c_1 x\,e^{\_Z} + 2\_Z x\,e^{\_Z} + 3x\,e^{\_Z} + 16)}}{2} - \frac{1}{2}$$

✓ Solution by Mathematica

Time used: 0.309 (sec). Leaf size: 47

```
DSolve[-1 - 2*y[x] + (-2*x + x^2*y[x]^2 + 2*x^2*y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolut
```

$$\text{Solve}\left[\frac{1}{64}\left(-4y(x)^2 + 4y(x) - 2\log(8y(x) + 4) + 3\right) - \frac{1}{4x(2y(x) + 1)} = c_1, y(x)\right]$$

1.321 problem 322

Internal problem ID [8658]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 322.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]`

$$\left(10y^3x^2 - 3y^2 - 2\right) y' + 5y^4x = -x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve((10*x^2*y(x)^3-3*y(x)^2-2)*diff(y(x),x)+5*x*y(x)^4+x = 0,y(x), singsol=all)
```

$$\frac{5y(x)^4 x^2}{2} + \frac{x^2}{2} - y(x)^3 - 2y(x) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.233 (sec). Leaf size: 2097

```
DSolve[x + 5*x*y[x]^4 + (-2 - 3*y[x]^2 + 10*x^2*y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolut
```

$y(x) \rightarrow$

$$-\frac{\sqrt{3}x^2\sqrt{\frac{5\sqrt[3]{6}x^2\sqrt[3]{189x^2+\sqrt{3}\sqrt{27\left(21x^2-2c_1\right)^2-16\left(5x^4-10c_1x^2-2\right)^3}-18c_1}+\frac{}{\sqrt[3]{189x^2+\sqrt{3}\sqrt{2}}}}{x^4}}}{}$$

$y(x)$

$$\rightarrow \frac{-\sqrt{3}x^2\sqrt{\frac{5\sqrt[3]{6}x^2\sqrt[3]{189x^2+\sqrt{3}\sqrt{27\left(21x^2-2c_1\right)^2-16\left(5x^4-10c_1x^2-2\right)^3}-18c_1}+\frac{}{\sqrt[3]{189x^2+\sqrt{3}\sqrt{2}}}}{x^4}}}{}$$

$y(x)$

$$\rightarrow \frac{\sqrt{3}x^2\sqrt{\frac{5\sqrt[3]{6}x^2\sqrt[3]{189x^2+\sqrt{3}\sqrt{27\left(21x^2-2c_1\right)^2-16\left(5x^4-10c_1x^2-2\right)^3}-18c_1}+\frac{}{\sqrt[3]{189x^2+\sqrt{3}\sqrt{27}}}}{x^4}}}{}$$

$y(x)$

$$\rightarrow \frac{\sqrt{3}x^2\sqrt{\frac{5\sqrt[3]{6}x^2\sqrt[3]{189x^2+\sqrt{3}\sqrt{27\left(21x^2-2c_1\right)^2-16\left(5x^4-10c_1x^2-2\right)^3}-18c_1}+\frac{}{\sqrt[3]{189x^2+\sqrt{3}\sqrt{27}}}}{x^4}}}{}$$

1.322 problem 323

Internal problem ID [8659]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 323.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$
\left(a x y^{3}+c\right) x y^{\prime}+\left(b\,x^{3} y+c\right) y=0
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 492

```
dsolve((a*x*y(x)^3+c)*x*diff(y(x),x)+(b*x^3*y(x)+c)*y(x) = 0,y(x), singsol=all)
```

$$
y(x) = \frac{3^{\frac{1}{3}}\left(-a\,x^{2}(b\,x^{2}-2c_{1})\,3^{\frac{1}{3}}+\left(\left(9c+\sqrt{\frac{3b^{3}x^{8}-18b^{2}c_{1}x^{6}+36b\,c_{1}^{2}x^{4}-24c_{1}^{3}x^{2}+81a\,c^{2}}{a}}\right)a^{2}x^{2}\right)^{\frac{2}{3}}\right)}{3\left(\left(9c+\sqrt{\frac{3b^{3}x^{8}-18b^{2}c_{1}x^{6}+36b\,c_{1}^{2}x^{4}-24c_{1}^{3}x^{2}+81a\,c^{2}}{a}}\right)a^{2}x^{2}\right)^{\frac{1}{3}}a x}
$$

$$
y(x) = -\frac{\left(\left(1+i\sqrt{3}\right)\left(\left(9c+\sqrt{\frac{3b^{3}x^{8}-18b^{2}c_{1}x^{6}+36b\,c_{1}^{2}x^{4}-24c_{1}^{3}x^{2}+81a\,c^{2}}{a}}\right)a^{2}x^{2}\right)^{\frac{2}{3}}+a\,x^{2}\left(i3^{\frac{5}{6}}-3^{\frac{1}{3}}\right)(b\,x^{2}-2c_{1})\right)3^{\frac{1}{3}}}{6\left(\left(9c+\sqrt{\frac{3b^{3}x^{8}-18b^{2}c_{1}x^{6}+36b\,c_{1}^{2}x^{4}-24c_{1}^{3}x^{2}+81a\,c^{2}}{a}}\right)a^{2}x^{2}\right)^{\frac{1}{3}}a x}
$$

$$
y(x) = \frac{\left(\left(i\sqrt{3}-1\right)\left(\left(9c+\sqrt{\frac{3b^{3}x^{8}-18b^{2}c_{1}x^{6}+36b\,c_{1}^{2}x^{4}-24c_{1}^{3}x^{2}+81a\,c^{2}}{a}}\right)a^{2}x^{2}\right)^{\frac{2}{3}}+\left(i3^{\frac{5}{6}}+3^{\frac{1}{3}}\right)a\,x^{2}(b\,x^{2}-2c_{1})\right)3^{\frac{1}{3}}}{6\left(\left(9c+\sqrt{\frac{3b^{3}x^{8}-18b^{2}c_{1}x^{6}+36b\,c_{1}^{2}x^{4}-24c_{1}^{3}x^{2}+81a\,c^{2}}{a}}\right)a^{2}x^{2}\right)^{\frac{1}{3}}a x}
$$

✓ Solution by Mathematica

Time used: 54.413 (sec). Leaf size: 484

```
DSolve[y[x]*(c + b*x^3*y[x]) + x*(c + a*x*y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{x(-bx^2+2c_1)}{\sqrt[3]{3}\sqrt[3]{9a^2cx^2+\sqrt{3}\sqrt{a^3x^4\left(27ac^2+x^2\left(bx^2-2c_1\right)^3\right)}}} + \frac{\sqrt[3]{9a^2cx^2+\sqrt{3}\sqrt{a^3x^4\left(27ac^2+x^2\left(bx^2-2c_1\right)^3\right)}}}{3^{2/3}ax}$$

$$y(x) \to \frac{i\sqrt[3]{3}\left(\sqrt{3}+i\right)\left(9a^2cx^2+\sqrt{3}\sqrt{a^3x^4\left(27ac^2+x^2\left(bx^2-2c_1\right)^3\right)}\right)^{2/3}+\sqrt[6]{3}\left(\sqrt{3}+3i\right)ax^2(bx^2-2c_1)}{6ax\sqrt[3]{9a^2cx^2+\sqrt{3}\sqrt{a^3x^4\left(27ac^2+x^2\left(bx^2-2c_1\right)^3\right)}}}$$

$$y(x) \to \frac{\sqrt[6]{3}\left(\sqrt{3}-3i\right)ax^2(bx^2-2c_1)-i\sqrt[3]{3}\left(\sqrt{3}-i\right)\left(9a^2cx^2+\sqrt{3}\sqrt{a^3x^4\left(27ac^2+x^2\left(bx^2-2c_1\right)^3\right)}\right)^{2/3}}{6ax\sqrt[3]{9a^2cx^2+\sqrt{3}\sqrt{a^3x^4\left(27ac^2+x^2\left(bx^2-2c_1\right)^3\right)}}}$$

1.323 problem 324

Internal problem ID [8660]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 324.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\left(2y^3x^3 - x\right)y' + 2y^3x^3 - y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 811

```
dsolve((2*x^3*y(x)^3-x)*diff(y(x),x)+2*x^3*y(x)^3-y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(c_1^3x^2 - 6c_1^2x^3 + 12c_1x^4 - 8x^5 + 3\sqrt{-6c_1^3x^2 + 36c_1^2x^3 - 72c_1x^4 + 48x^5 + 81} - 27\right)x\right)^{\frac{1}{3}}}{6x} + \frac{(-2x + c_1)^2 x}{6\left(\left(c_1^3x^2 - 6c_1^2x^3 + 12c_1x^4 - 8x^5 + 3\sqrt{-6c_1^3x^2 + 36c_1^2x^3 - 72c_1x^4 + 48x^5 + 81} - 27\right)x\right)^{\frac{1}{3}}} - \frac{x}{3} + \frac{c_1}{6}$$

$$y(x) = \frac{-2(-c_1x + 2x^2)\left(\left(c_1^3x^2 - 6c_1^2x^3 + 12c_1x^4 - 8x^5 + 3\sqrt{-6c_1^3x^2 + 36c_1^2x^3 - 72c_1x^4 + 48x^5 + 81} - 27\right)x\right)}{}$$

$$y(x) = \frac{2(c_1x - 2x^2)\left(\left(c_1^3x^2 - 6c_1^2x^3 + 12c_1x^4 - 8x^5 + 3\sqrt{-6c_1^3x^2 + 36c_1^2x^3 - 72c_1x^4 + 48x^5 + 81} - 27\right)x\right)^{\frac{1}{3}} +}{}$$

✓ Solution by Mathematica

Time used: 60.125 (sec). Leaf size: 672

```
DSolve[-y[x] + 2*x^3*y[x]^3 + (-x + 2*x^3*y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \rightarrow \frac{-2x^3 + c_1x^2 + \frac{x^4(-2x+c_1)^2}{\sqrt[3]{-8x^9 + 12c_1x^8 - 6c_1{}^2x^7 + c_1{}^3x^6 - 27x^4 + 3\sqrt{3}\sqrt{x^8\left(16x^5 - 24c_1x^4 + 12c_1{}^2x^3 - 2c_1{}^3\right.}}}}{}$$

$$y(x) \rightarrow \frac{2x^2(-2x+c_1) - \frac{i\left(\sqrt{3}-i\right)x^4(-2x+c_1)^2}{\sqrt[3]{-8x^9 + 12c_1x^8 - 6c_1{}^2x^7 + c_1{}^3x^6 - 27x^4 + 3\sqrt{3}\sqrt{x^8\left(16x^5 - 24c_1x^4 + 12c_1{}^2x^3 - 2c\right.}}}}{}$$

$$y(x) \rightarrow \frac{2x^2(-2x+c_1) + \frac{i\left(\sqrt{3}+i\right)x^4(-2x+c_1)^2}{\sqrt[3]{-8x^9 + 12c_1x^8 - 6c_1{}^2x^7 + c_1{}^3x^6 - 27x^4 + 3\sqrt{3}\sqrt{x^8\left(16x^5 - 24c_1x^4 + 12c_1{}^2x^3 - 2c\right.}}}}{}$$

1.324 problem 325

Internal problem ID [8661]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 325.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$y(y^3 - 2x^3)\, y' + x(2y^3 - x^3) = 0$$

✓ Solution by Maple

Time used: 0.922 (sec). Leaf size: 122

```
dsolve(y(x)*(y(x)^3-2*x^3)*diff(y(x),x)+(2*y(x)^3-x^3)*x = 0,y(x), singsol=all)
```

$$-\frac{4\ln(2)}{7} - \frac{2\ln\left(\frac{x^4+x^3y(x)+3y(x)^2x^2+xy(x)^3+y(x)^4}{x^4}\right)}{7} - \frac{2\sqrt{3}\arctan\left(\frac{(x+2y(x))\sqrt{3}}{3x}\right)}{7} + \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(x^3+4x^2y(x)+2xy(x)^2+2y(x)^3\right)}{3x^3}\right)}{7} + \frac{\ln\left(\frac{y(x)-x}{x}\right)}{7} - \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.165 (sec). Leaf size: 139

```
DSolve[x*(-x^3 + 2*y[x]^3) + y[x]*(-2*x^3 + y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions
```

$$\text{Solve}\left[\frac{1}{7}\text{RootSum}\left[\#1^4+\#1^3+3\#1^2+\#1 +1\&, \frac{8\#1^3\log\left(\frac{y(x)}{x}-\#1\right)+9\#1^2\log\left(\frac{y(x)}{x}-\#1\right)+12\#1\log\left(\frac{y(x)}{x}-\#1\right)-\log\left(\frac{y(x)}{x}-\#1\right)}{4\#1^3+3\#1^2+6\#1+1}\&\right] -\frac{1}{7}\log\left(1-\frac{y(x)}{x}\right) = -\log(x)+c_1, y(x)\right]$$

1.325 problem 326

Internal problem ID [8662]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 326.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$y\left((ay+xb)^3+x^3b\right)y'+x\left((ay+xb)^3+ay^3\right)=0$$

✓ Solution by Maple

Time used: 0.484 (sec). Leaf size: 160

```
dsolve(y(x)*((a*y(x)+b*x)^3+b*x^3)*diff(y(x),x)+x*((a*y(x)+b*x)^3+a*y(x)^3) = 0,y(x), singso
```

$$y(x) = \frac{x\left(c_1x-b\,\mathrm{RootOf}\left(b^2\_Z^4-2bxc_1\_Z^3+\left(a^2c_1^2x^2+b^2c_1^2x^2+c_1^2x^2-a^2\right)\_Z^2-2b\,x^3c_1^3\_Z+c_1^4x^4\right)\right)}{a\,\mathrm{RootOf}\left(b^2\_Z^4-2bxc_1\_Z^3+\left(a^2c_1^2x^2+b^2c_1^2x^2+c_1^2x^2-a^2\right)\_Z^2-2b\,x^3c_1^3\_Z+c_1^4x^4\right)}$$

✓ Solution by Mathematica

Time used: 61.479 (sec). Leaf size: 13289

```
DSolve[x*(a*y[x]^3 + (b*x + a*y[x])^3) + y[x]*(b*x^3 + (b*x + a*y[x])^3)*y'[x]==0,y[x],x,Inc
```

Too large to display

1.326 problem 327

Internal problem ID [8663]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 327.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\left(y^4x+2y^3x^2+2y+x\right)y'+y^5+y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 633

```
dsolve((x*y(x)^4+2*x^2*y(x)^3+2*y(x)+x)*diff(y(x),x)+y(x)^5+y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{-1+\frac{\left(108c_1^3x^2+12\sqrt{3}\,\sqrt{27c_1^4x^2+4c_1x^4+18c_1^2x^2-x^2-4c_1}\,c_1x+36c_1x^2-8\right)^{\frac{1}{3}}}{2}-\frac{2(3c_1x^2-1)}{\left(108c_1^3x^2+12\sqrt{3}\,\sqrt{27c_1^4x^2+4c_1x^4+18c_1^2x^2-x^2-4c_1}\,c_1x\right.}}{3c_1x}$$

$$y(x) = \frac{i\left(4-12c_1x^2-\left(108c_1^3x^2+12\sqrt{3}\,\sqrt{27c_1^4x^2+18c_1^2x^2+(4x^4-4)\,c_1-x^2}\,c_1x+36c_1x^2-8\right)^{\frac{2}{3}}\right)\sqrt{3}+12}{12\left(108c_1^3x^2+12\sqrt{3}\,\sqrt{27c_1^4x^2+18c_1^2x^2+(4x^4\right.}}$$

$$y(x) = \frac{12i\sqrt{3}\,c_1x^2+i\sqrt{3}\left(108c_1^3x^2+12\sqrt{3}\,\sqrt{27c_1^4x^2+18c_1^2x^2+(4x^4-4)\,c_1-x^2}\,c_1x+36c_1x^2-8\right)^{\frac{2}{3}}+12c_1x}{12xc_1}$$

✓ Solution by Mathematica

Time used: 10.131 (sec). Leaf size: 675

```
DSolve[y[x] + y[x]^5 + (x + 2*y[x] + 2*x^2*y[x]^3 + x*y[x]^4)*y'[x]==0,y[x],x,IncludeSingula
```

$$y(x) \to \frac{\frac{2c_1(3x^2+c_1)}{\sqrt[3]{\frac{9}{2}(3+c_1^2)x^2+\frac{3}{2}\sqrt{3}\sqrt{-4c_1^3x^6+(27-c_1^4+18c_1^2)x^4+4c_1^3x^2}+c_1^3}}+2^{2/3}\sqrt[3]{9(3+c_1^2)x^2+3\sqrt{3}}}{6x}$$

$$y(x) \to \frac{-\frac{2i(\sqrt{3}-i)c_1(3x^2+c_1)}{\sqrt[3]{\frac{9}{2}(3+c_1^2)x^2+\frac{3}{2}\sqrt{3}\sqrt{-4c_1^3x^6+(27-c_1^4+18c_1^2)x^4+4c_1^3x^2}+c_1^3}}+i2^{2/3}(\sqrt{3}+i)\sqrt[3]{9(3+c}}{12x}$$

$$y(x) \to \frac{\frac{2i(\sqrt{3}+i)c_1(3x^2+c_1)}{\sqrt[3]{\frac{9}{2}(3+c_1^2)x^2+\frac{3}{2}\sqrt{3}\sqrt{-4c_1^3x^6+(27-c_1^4+18c_1^2)x^4+4c_1^3x^2}+c_1^3}}-2^{2/3}(1+i\sqrt{3})\sqrt[3]{9(3+c_1}}{12x}$$

$$y(x) \to 0$$

$$y(x) \to -\sqrt[4]{-1}$$

$$y(x) \to \sqrt[4]{-1}$$

$$y(x) \to -(-1)^{3/4}$$

$$y(x) \to (-1)^{3/4}$$

$$y(x) \to \frac{1}{2}x\left(-1+\frac{ix^2}{\sqrt{-x^4}}\right)$$

$$y(x) \to -\frac{x}{2}+\frac{i\sqrt{-x^4}}{2x}$$

1.327 problem 328

Internal problem ID [8664]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 328.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{a\,x^2y^ny' - 2xy' + y = 0}$$

✓ Solution by Maple
Time used: 0.234 (sec). Leaf size: 33

```
dsolve(a*x^2*y(x)^n*diff(y(x),x)-2*x*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$(y(x)^n\,ax - n - 2)^n\,y(x)^{2n}\,x^{-n} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.21 (sec). Leaf size: 42

```
DSolve[y[x] - 2*x*y'[x] + a*x^2*y[x]^n*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{n(\log(x) - \log\left(-axy(x)^n + n + 2\right))}{n+2} - \frac{2n\log(y(x))}{n+2} = c_1, y(x)\right]$$

1.328 problem 329

Internal problem ID [8665]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 329.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{y^m x^n (axy' + by) + \alpha xy' + \beta y = 0}$$

✓ Solution by Maple
Time used: 0.484 (sec). Leaf size: 72

```
dsolve(y(x)^m*x^n*(a*x*diff(y(x),x)+b*y(x))+alpha*x*diff(y(x),x)+beta*y(x) = 0,y(x), singsol
```

$$x^{\beta m(an-bm)} \left(x^n (an-bm)\, y(x)^m - \beta m + \alpha n\right)^{-m(a\beta - b\alpha)} \left(y(x)^m\right)^{\alpha(an-bm)} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.582 (sec). Leaf size: 119

```
DSolve[\[Beta]*y[x] + \[Alpha]*x*y'[x] + x^n*y[x]^m*(b*y[x] + a*x*y'[x])==0,y[x],x,IncludeSi
```

$$\text{Solve}\left[\frac{m(\beta(bm-an)\log\left(nx^n(\alpha n-\beta m)\right) + n(a\beta-\alpha b)\log\left(x^n y(x)^m (bm-an) + \beta m - \alpha n\right))}{n(an-bm)(\alpha n-\beta m)} - \frac{\alpha m \log(\alpha n y(x) - \beta m y(x))}{\alpha n - \beta m} = c_1, y(x)\right]$$

1.329 problem 330

Internal problem ID [8666]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 330.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _dAlembert]`

$$\boxed{(f(x+y)+1)\,y'+f(x+y)=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve((f(x+y(x))+1)*diff(y(x),x)+f(x+y(x)) = 0,y(x), singsol=all)
```

$$y(x)=-x+\operatorname{RootOf}\left(-x+\int^{\_Z}(1+f(\_a))\,d\_a+c_1\right)$$

✓ Solution by Mathematica

Time used: 0.107 (sec). Leaf size: 52

```
DSolve[f[x + y[x]] + (1 + f[x + y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)}\left(f(x+K[2])-\int_1^x f'(K[1]+K[2])dK[1]+1\right)dK[2]\right.$$
$$\left.+\int_1^x f(K[1]+y(x))dK[1]=c_1,y(x)\right]$$

1.330 problem 331

Internal problem ID [8667]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 331.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\frac{y' f_{\nu}(x) \left(-y + y^{p+1}\right)}{-1 + y} - \frac{g_{\nu}(x) \left(-y + y^{q+1}\right)}{-1 + y} = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 78

```
dsolve(diff(y(x),x)*f[nu](x)*(-y(x)+y(x)^(p+1))/(-1+y(x))-g[nu](x)*(-y(x)+y(x)^(q+1))/(-1+y(
```

$$\frac{y(x)^{p+1} \operatorname{LerchPhi}\left(-y(x)^{q} (-1)^{\operatorname{csgn}(iy(x)^{q})}, 1, \frac{p+1}{q}\right) - y(x) \operatorname{LerchPhi}\left(-y(x)^{q} (-1)^{\operatorname{csgn}(iy(x)^{q})}, 1, \frac{1}{q}\right) + q\left(\int \frac{g_{\nu}}{f_{\nu}}\right.}{q}$$
$= 0$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-Sum[y[x]^nu*g[nu][x], {nu, 1, q}] + Sum[y[x]^nu*f[nu][x], {nu, 1, p}]*y'[x]==0,y[x],
```

Not solved

1.331 problem 332

Internal problem ID [8668]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 332.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{(\sqrt{yx}-1)\,xy'-(\sqrt{yx}+1)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(((x*y(x))^(1/2)-1)*x*diff(y(x),x)-((x*y(x))^(1/2)+1)*y(x) = 0,y(x), singsol=all)
```

$$-\frac{1+\left(c_1-\ln(x)+\frac{\ln(xy(x))}{2}\right)\sqrt{xy(x)}}{\sqrt{xy(x)}}=0$$

✓ Solution by Mathematica

Time used: 0.203 (sec). Leaf size: 29

```
DSolve[-(y[x]*(1 + Sqrt[x*y[x]])) + x*(-1 + Sqrt[x*y[x]])*y'[x]==0,y[x],x,IncludeSingularSol
```

$$\text{Solve}\left[\frac{2}{\sqrt{xy(x)}}+2\log(y(x))-\log(xy(x))=c_1,y(x)\right]$$

1.332 problem 333

Internal problem ID [8669]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 333.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$\left(2x^{\frac{5}{2}}y^{\frac{3}{2}}+yx^2-x\right)y'-x^{\frac{3}{2}}y^{\frac{5}{2}}+xy^2-y=0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 38

```
dsolve((2*x^(5/2)*y(x)^(3/2)+x^2*y(x)-x)*diff(y(x),x)-x^(3/2)*y(x)^(5/2)+x*y(x)^2-y(x) = 0,y
```

$$-\frac{3\left(\frac{\left(c_1+\frac{3\ln(x)}{2}-3\ln(y(x))\right)x^{\frac{3}{2}}y(x)^{\frac{3}{2}}}{3}+xy(x)-\frac{1}{3}\right)}{x^{\frac{3}{2}}y\left(x\right)^{\frac{3}{2}}}=0$$

✓ Solution by Mathematica

Time used: 0.313 (sec). Leaf size: 72

```
DSolve[-y[x] + x*y[x]^2 - x^(3/2)*y[x]^(5/2) + (-x + x^2*y[x] + 2*x^(5/2)*y[x]^(3/2))*y'[x]=
```

$$\text{Solve}\left[\frac{2\sqrt{xy(x)}\log(y(x))}{\sqrt{x}\sqrt{y(x)}}-\frac{\sqrt{xy(x)}\left(3x^{3/2}y(x)^{3/2}\log(x)+6xy(x)-2\right)}{3x^2y(x)^2}=c_1,y(x)\right]$$

1.333 problem 334

Internal problem ID [8670]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 334.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{\left(\sqrt{x+y}+1\right) y' = -1}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 19

```
dsolve(((x+y(x))^(1/2)+1)*diff(y(x),x)+1 = 0,y(x), singsol=all)
```

$$-y(x) - 2\sqrt{x+y(x)} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.097 (sec). Leaf size: 39

```
DSolve[1 + (1 + Sqrt[x + y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2\sqrt{x+1+c_1} + 2 + c_1$$
$$y(x) \to 2\sqrt{x+1+c_1} + 2 + c_1$$

1.334 problem 335

Internal problem ID [8671]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 335.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sqrt{y^2-1}\,y' = \sqrt{x^2-1}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 50

```
dsolve((y(x)^2-1)^(1/2)*diff(y(x),x)-(x^2-1)^(1/2) = 0,y(x), singsol=all)
```

$$c_1 + x\sqrt{x^2-1} - \ln\left(x+\sqrt{x^2-1}\right) - y(x)\sqrt{y(x)^2-1} + \ln\left(y(x)+\sqrt{y(x)^2-1}\right) = 0$$

✓ Solution by Mathematica
Time used: 0.506 (sec). Leaf size: 79

```
DSolve[-Sqrt[-1 + x^2] + Sqrt[-1 + y[x]^2]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{1}{2}\#1\sqrt{\#1^2-1} - \operatorname{arctanh}\left(\frac{\sqrt{\#1^2-1}}{\#1-1}\right)\&\right]\left[\operatorname{arctanh}\left(\frac{\sqrt{x^2-1}}{1-x}\right) + \frac{1}{2}\sqrt{x^2-1}x + c_1\right]$$

1.335 problem 336

Internal problem ID [8672]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 336.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\left(\sqrt{1+y^2}+xa\right)y'+ay=-\sqrt{x^2+1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 41

```
dsolve(((y(x)^2+1)^(1/2)+a*x)*diff(y(x),x)+(x^2+1)^(1/2)+a*y(x) = 0,y(x), singsol=all)
```

$$\frac{x\sqrt{x^2+1}}{2}+\frac{\operatorname{arcsinh}(x)}{2}+axy(x)+\frac{y(x)\sqrt{y(x)^2+1}}{2}+\frac{\operatorname{arcsinh}(y(x))}{2}+c_1=0$$

✓ Solution by Mathematica
Time used: 0.443 (sec). Leaf size: 80

```
DSolve[Sqrt[1 + x^2] + a*y[x] + (a*x + Sqrt[1 + y[x]^2])*y'[x]==0,y[x],x,IncludeSingularSolu
```

$$\text{Solve}\left[axy(x)+\frac{1}{2}\sqrt{x^2+1}x-\frac{1}{2}\log\left(\sqrt{x^2+1}-x\right)+\frac{1}{2}y(x)\sqrt{y(x)^2+1}-\frac{1}{2}\log\left(\sqrt{y(x)^2+1}-y(x)\right)=c_1,y(x)\right]$$

1.336 problem 337

Internal problem ID [8673]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 337.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{\left(\sqrt{x^2+y^2}+x\right)y'-y=0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 27

```
dsolve(((y(x)^2+x^2)^(1/2)+x)*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$\frac{-c_1 y(x)^2+\sqrt{y(x)^2+x^2}+x}{y(x)^2}=0$$

✓ Solution by Mathematica

Time used: 0.538 (sec). Leaf size: 57

```
DSolve[-y[x] + (x + Sqrt[x^2 + y[x]^2])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}
y(x) &\to -e^{\frac{c_1}{2}}\sqrt{2x+e^{c_1}} \\
y(x) &\to e^{\frac{c_1}{2}}\sqrt{2x+e^{c_1}} \\
y(x) &\to 0
\end{aligned}$$

1.337 problem 338

Internal problem ID [8674]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 338.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\left(\sqrt{x^2+y^2}\,y+\left(y^2-x^2\right)\sin\left(\alpha\right)-2xy\cos\left(\alpha\right)\right)y'+\sqrt{x^2+y^2}\,x+2xy\sin\left(\alpha\right)+\left(y^2-x^2\right)\cos\left(\alpha\right)=0$$

✓ Solution by Maple

Time used: 1.578 (sec). Leaf size: 132

```
dsolve((y(x)*(y(x)^2+x^2)^(1/2)+(y(x)^2-x^2)*sin(alpha)-2*x*y(x)*cos(alpha))*diff(y(x),x)+x*
```

$$y(x)=\mathrm{RootOf}\left(-\ln\left(x\right)-\left(\int^{\_Z}-\frac{-\_a^3\cos\left(2\alpha\right)-3\_a^2\sin\left(2\alpha\right)-\_a^3+3\_a\cos\left(2\alpha\right)+\sin\left(2\alpha\right)+\sqrt{2}\sqrt{\left(\_a^2+1\right)\left(\_a^2+1+\_a^2\cos\right.}}{\left(\_a^2+1\right)\left(\_a^2+1+\_a^2\cos\left(2\alpha\right)+2\_a\sin\left(2\alpha\right)-\cos\left(2\alpha\right)\right)}\right.\right.$$

$$\left.+c_1\right)x$$

✓ Solution by Mathematica

Time used: 5.901 (sec). Leaf size: 116

```
DSolve[2*x*Sin[\[Alpha]]*y[x] + Cos[\[Alpha]]*(-x^2 + y[x]^2) + x*Sqrt[x^2 + y[x]^2] + (-2*x
```

$$\text{Solve}\left[\sqrt{\cos^2(\alpha)}\sec(\alpha)\left(\log\left(\cos(\alpha)\left(\sin(\alpha)+\frac{\cos(\alpha)y(x)}{x}\right)\right)\right.\right.$$
$$\left.-\log\left(\frac{1}{2}\left(\cos(2\alpha)-2\sqrt{\cos^2(\alpha)}\sqrt{\frac{y(x)^2}{x^2}+1}-\frac{\sin(2\alpha)y(x)}{x}+1\right)\right)\right)$$
$$\left.+\log\left(\frac{y(x)^2}{x^2}+1\right)-\frac{1}{2}\log\left(\left(\sin(\alpha)+\frac{\cos(\alpha)y(x)}{x}\right)^2\right)=-\log(x)+c_1,y(x)\right]$$

1.338 problem 339

Internal problem ID [8675]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 339.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\left(x\sqrt{x^2+y^2+1}-y\left(x^2+y^2\right)\right)y'-y\sqrt{x^2+y^2+1}-x\left(x^2+y^2\right)=0$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 25

```
dsolve((x*(x^2+y(x)^2+1)^(1/2)-y(x)*(y(x)^2+x^2))*diff(y(x),x)-y(x)*(x^2+y(x)^2+1)^(1/2)-x*(
```

$$\arctan\left(\frac{x}{y(x)}\right)+\sqrt{x^2+y(x)^2+1}-c_1=0$$

✓ Solution by Mathematica

Time used: 0.284 (sec). Leaf size: 27

```
DSolve[-(x*(x^2 + y[x]^2)) - y[x]*Sqrt[1 + x^2 + y[x]^2] + (-(y[x]*(x^2 + y[x]^2)) + x*Sqrt[
```

$$\text{Solve}\left[\arctan\left(\frac{x}{y(x)}\right)+\sqrt{x^2+y(x)^2+1}=c_1,y(x)\right]$$

1.339 problem 340

Internal problem ID [8676]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 340.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `unknown`

$$\left(\frac{\mathrm{e1}\,(x+a)}{\left((x+a)^2+y^2\right)^{\frac{3}{2}}}+\frac{\mathrm{e2}\,(x-a)}{\left((x-a)^2+y^2\right)^{\frac{3}{2}}}\right)y'-y\left(\frac{\mathrm{e1}}{\left((x+a)^2+y^2\right)^{\frac{3}{2}}}+\frac{\mathrm{e2}}{\left((x-a)^2+y^2\right)^{\frac{3}{2}}}\right)=0$$

✗ Solution by Maple

```
dsolve((e1*(x+a)/((x+a)^2+y(x)^2)^(3/2)+e2*(x-a)/((x-a)^2+y(x)^2)^(3/2))*diff(y(x),x)-y(x)*(
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-(y[x]*(e2/((-a + x)^2 + y[x]^2)^(3/2) + e1/((a + x)^2 + y[x]^2)^(3/2))) + ((e2*(-a +
```

Not solved

1.340 problem 341

Internal problem ID [8677]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 341.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{(x\,\mathrm{e}^{y}+\mathrm{e}^{x})\,y'+\mathrm{e}^{y}+\mathrm{e}^{x}y=0}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 29

```
dsolve((x*exp(y(x))+exp(x))*diff(y(x),x)+exp(y(x))+y(x)*exp(x) = 0,y(x), singsol=all)
```

$$y(x)=-\operatorname{LambertW}\left(x\,\mathrm{e}^{-x-\mathrm{e}^{-x}c_1}\right)-\mathrm{e}^{-x}c_1$$

✓ Solution by Mathematica
Time used: 2.262 (sec). Leaf size: 33

```
DSolve[E^y[x] + E^x*y[x] + (E^x + E^y[x]*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x)\to c_1e^{-x}-W\left(xe^{-x+c_1e^{-x}}\right)$$

1.341 problem 342

Internal problem ID [8678]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 342.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$x(3\,\mathrm{e}^{yx} + 2\,\mathrm{e}^{-yx})\,(xy' + y) = -1$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve(x*(3*exp(x*y(x))+2*exp(-x*y(x)))*(x*diff(y(x),x)+y(x))+1 = 0,y(x), singsol=all)
```

$$y(x) = \frac{-\ln(5) + \ln(-\ln(x) + c_1)}{x}$$

✓ Solution by Mathematica
Time used: 60.44 (sec). Leaf size: 163

```
DSolve[1 + (2/E^(x*y[x]) + 3*E^(x*y[x]))*x*(y[x] + x*y'[x])==0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to -\frac{\operatorname{arccosh}\left(\frac{1}{24}\left(-5\sqrt{24 + \log^2\left(\frac{c_1}{x}\right)} - \log\left(\frac{c_1}{x}\right)\right)\right)}{x}$$
$$y(x) \to \frac{\operatorname{arccosh}\left(\frac{1}{24}\left(-5\sqrt{24 + \log^2\left(\frac{c_1}{x}\right)} - \log\left(\frac{c_1}{x}\right)\right)\right)}{x}$$
$$y(x) \to -\frac{\operatorname{arccosh}\left(\frac{1}{24}\left(5\sqrt{24 + \log^2\left(\frac{c_1}{x}\right)} - \log\left(\frac{c_1}{x}\right)\right)\right)}{x}$$
$$y(x) \to \frac{\operatorname{arccosh}\left(\frac{1}{24}\left(5\sqrt{24 + \log^2\left(\frac{c_1}{x}\right)} - \log\left(\frac{c_1}{x}\right)\right)\right)}{x}$$

1.342 problem 343

Internal problem ID [8679]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 343.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_exponential_symmetries]]`

$$\boxed{(\ln(y) + x)\, y' = 1}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 27

```
dsolve((ln(y(x))+x)*diff(y(x),x)-1 = 0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{RootOf}\left(-x - \_Z - \mathrm{e}^{\mathrm{e}^{-\_Z}} \mathrm{expIntegral}_1\left(\mathrm{e}^{-\_Z}\right) + c_1 \mathrm{e}^{\mathrm{e}^{-\_Z}}\right)}$$

✓ Solution by Mathematica
Time used: 0.132 (sec). Leaf size: 35

```
DSolve[-1 + (x + Log[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[x = e^{y(x)}\left(\mathrm{ExpIntegralEi}(-y(x)) - e^{-y(x)}\log(y(x))\right) + c_1 e^{y(x)}, y(x)\right]$$

1.343 problem 344

Internal problem ID [8680]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 344.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{(\ln(y) + 2x - 1)\, y' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve((ln(y(x))+2*x-1)*diff(y(x),x)-2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\mathrm{LambertW}\left(-2\, \mathrm{e}^{-2x} c_1\right)}{2c_1}$$

✓ Solution by Mathematica

Time used: 60.141 (sec). Leaf size: 23

```
DSolve[-2*y[x] + (-1 + 2*x + Log[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{W(-2c_1 e^{-2x})}{2c_1}$$

1.344 problem 345

Internal problem ID [8681]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 345.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{x\left(2x^2 y \ln(y) + 1\right) y' - 2y = 0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 36

```
dsolve(x*(2*x^2*y(x)*ln(y(x))+1)*diff(y(x),x)-2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = e^{\operatorname{RootOf}\left(2\_Z x^2 e^{2\_Z} - x^2 e^{2\_Z} + 2c_1 x^2 + 2\,e^{\_Z}\right)}$$

✓ Solution by Mathematica
Time used: 0.163 (sec). Leaf size: 35

```
DSolve[-2*y[x] + x*(1 + 2*x^2*Log[y[x]]*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$\text{Solve}\left[\frac{y(x)}{x^2} + 2\left(\frac{1}{2}y(x)^2\log(y(x)) - \frac{y(x)^2}{4}\right) = c_1, y(x)\right]$$

1.345 problem 346

Internal problem ID [8682]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 346.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{x(y\ln(yx)+y-xa)\,y'-y(ax\ln(yx)-y+xa)=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(x*(y(x)*ln(x*y(x))+y(x)-a*x)*diff(y(x),x)-y(x)*(a*x*ln(x*y(x))-y(x)+a*x) = 0,y(x), si
```

$$(xy(x))^{-ax+y(x)}-c_1=0$$

✓ Solution by Mathematica
Time used: 0.303 (sec). Leaf size: 24

```
DSolve[-((a*x + a*x*Log[x*y[x]] - y[x])*y[x]) + x*(-(a*x) + y[x] + Log[x*y[x]]*y[x])*y'[x]==
```

$$\text{Solve}[ax\log(xy(x))-y(x)\log(xy(x))=c_1,y(x)]$$

1.346 problem 347

Internal problem ID [8683]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 347.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y'(1+\sin(x))\sin(y)+\cos(x)(\cos(y)-1)=0$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)*(1+sin(x))*sin(y(x))+cos(x)*(cos(y(x))-1) = 0,y(x), singsol=all)
```

$$y(x)=\arccos(c_1\sin(x)+c_1+1)$$

✓ Solution by Mathematica
Time used: 3.429 (sec). Leaf size: 37

```
DSolve[Cos[x]*(-1 + Cos[y[x]]) + (1 + Sin[x])*Sin[y[x]]*y'[x]==0,y[x],x,IncludeSingularSolut
```

$$y(x)\to 0$$
$$y(x)\to 2\arcsin\left(\frac{1}{4}c_1\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)$$
$$y(x)\to 0$$

1.347 problem 348

Internal problem ID [8684]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 348.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$(x \cos(y) + \sin(x)) y' + y \cos(x) + \sin(y) = 0$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 15

```
dsolve((x*cos(y(x))+sin(x))*diff(y(x),x)+y(x)*cos(x)+sin(y(x)) = 0,y(x), singsol=all)
```

$$y(x) \sin(x) + x \sin(y(x)) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.163 (sec). Leaf size: 17

```
DSolve[Sin[y[x]] + Cos[x]*y[x] + (x*Cos[y[x]] + Sin[x])*y'[x]==0,y[x],x,IncludeSingularSolut
```

$$\text{Solve}[x \sin(y(x)) + y(x) \sin(x) = c_1, y(x)]$$

1.348 problem 349

Internal problem ID [8685]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 349.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘]]`

$$\boxed{xy' \cot\left(\frac{y}{x}\right) + 2x \sin\left(\frac{y}{x}\right) - y \cot\left(\frac{y}{x}\right) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x)*cot(y(x)/x)+2*x*sin(y(x)/x)-y(x)*cot(y(x)/x) = 0,y(x), singsol=all)
```

$$y(x) = \arcsin\left(\frac{1}{2\ln(x) + 2c_1}\right)x$$

✓ Solution by Mathematica
Time used: 0.48 (sec). Leaf size: 20

```
DSolve[2*x*Sin[y[x]/x] - Cot[y[x]/x]*y[x] + x*Cot[y[x]/x]*y'[x]==0,y[x],x,IncludeSingularSol
```

$$y(x) \to x \csc^{-1}(2(\log(x) + c_1))$$
$$y(x) \to 0$$

1.349 problem 350

Internal problem ID [8686]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 350.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `unknown`

$$\boxed{y' \cos(y) - \cos(x) \sin(y)^2 - \sin(y) = 0}$$

✓ Solution by Maple

Time used: 0.437 (sec). Leaf size: 266

```
dsolve(diff(y(x),x)*cos(y(x))-cos(x)*sin(y(x))^2-sin(y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \arctan\left(-\frac{2\,\mathrm{e}^{x}}{(\sin(x)+\cos(x))\,\mathrm{e}^{x}+2c_1}, \frac{\sqrt{(2\cos(x)\sin(x)\,\mathrm{e}^{2x}+4c_1\sin(x)\,\mathrm{e}^{x}+4\cos(x)\,c_1\mathrm{e}^{x}+4c_1^2+\mathrm{e}^{2x}}}{2\cos(x)\sin(x)\,\mathrm{e}^{2x}+4c_1\sin(x}\right.$$

$$y(x) = \arctan\left(-\frac{2\,\mathrm{e}^{x}}{(\sin(x)+\cos(x))\,\mathrm{e}^{x}+2c_1},\right.$$
$$-\frac{\sqrt{(2\cos(x)\sin(x)\,\mathrm{e}^{2x}+4c_1\sin(x)\,\mathrm{e}^{x}+4\cos(x)\,c_1\mathrm{e}^{x}+4c_1^2+\mathrm{e}^{2x})\,(2\cos(x)\sin(x)\,\mathrm{e}^{2x}+4c_1\sin(x)\,\mathrm{e}^{x}}}{2\cos(x)\sin(x)\,\mathrm{e}^{2x}+4c_1\sin(x)\,\mathrm{e}^{x}+4\cos(x)\,c_1\mathrm{e}^{x}+4c_1^2+\mathrm{e}^{2x}}$$

✓ Solution by Mathematica

Time used: 1.958 (sec). Leaf size: 58

```
DSolve[-Sin[y[x]] - Cos[x]*Sin[y[x]]^2 + Cos[y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \csc^{-1}\left(\frac{1}{2}(-\sin(x)-\cos(x)-2c_1e^{-x})\right)$$
$$y(x) \to -\csc^{-1}\left(\frac{1}{2}(\sin(x)+\cos(x)+2c_1e^{-x})\right)$$
$$y(x) \to 0$$

1.350 problem 351

Internal problem ID [8687]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 351.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' \cos(y) + x \sin(y) \cos(y)^2 - \sin(y)^3 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 55

```
dsolve(diff(y(x),x)*cos(y(x))+x*sin(y(x))*cos(y(x))^2-sin(y(x))^3 = 0,y(x), singsol=all)
```

$$y(x) = \arcsin\left(\frac{1}{\sqrt{1-\sqrt{\pi}\,\operatorname{erf}(x)\,\mathrm{e}^{x^2}-2c_1\mathrm{e}^{x^2}}}\right)$$
$$y(x) = -\arcsin\left(\frac{1}{\sqrt{1-\sqrt{\pi}\,\operatorname{erf}(x)\,\mathrm{e}^{x^2}-2c_1\mathrm{e}^{x^2}}}\right)$$

✓ Solution by Mathematica

Time used: 60.366 (sec). Leaf size: 61

```
DSolve[x*Cos[y[x]]^2*Sin[y[x]] - Sin[y[x]]^3 + Cos[y[x]]*y'[x]==0,y[x],x,IncludeSingularSolu
```

$$y(x) \to -\cot^{-1}\left(\sqrt{e^{x^2}\left(-\sqrt{\pi}\operatorname{erf}(x)+4c_1\right)}\right)$$
$$y(x) \to \cot^{-1}\left(\sqrt{e^{x^2}\left(-\sqrt{\pi}\operatorname{erf}(x)+4c_1\right)}\right)$$

1.351 problem 352

Internal problem ID [8688]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 352.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `unknown`

$$y'(\cos(y) - \sin(\alpha)\sin(x))\cos(y) + (\cos(x) - \sin(\alpha)\sin(y))\cos(x) = 0$$

✓ Solution by Maple
Time used: 0.14 (sec). Leaf size: 33

```
dsolve(diff(y(x),x)*(cos(y(x))-sin(alpha)*sin(x))*cos(y(x))+(cos(x)-sin(alpha)*sin(y(x)))*co
```

$$\frac{(-2\sin(\alpha)\sin(x) + \cos(y(x)))\sin(y(x))}{2} + \frac{\cos(x)\sin(x)}{2} + \frac{x}{2} + c_1 + \frac{y(x)}{2} = 0$$

✓ Solution by Mathematica
Time used: 0.468 (sec). Leaf size: 43

```
DSolve[Cos[x]*(Cos[x] - Sin[\[Alpha]]*Sin[y[x]]) + Cos[y[x]]*(Cos[y[x]] - Sin[\[Alpha]]*Sin[
```

$$\text{Solve}\left[4\sin(\alpha)\sin(x)\sin(y(x)) - 4\left(\frac{y(x)}{2} + \frac{1}{4}\sin(2y(x))\right) - 2x - \sin(2x) = c_1, y(x)\right]$$

1.352 problem 353

Internal problem ID [8689]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 353.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' \cos(y)\, x + \sin(y) = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x)*cos(y(x))+sin(y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \arcsin\left(\frac{1}{c_1 x}\right)$$

✓ Solution by Mathematica
Time used: 12.769 (sec). Leaf size: 19

```
DSolve[Sin[y[x]] + x*Cos[y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin\left(\frac{e^{c_1}}{x}\right)$$
$$y(x) \to 0$$

1.353 problem 354

Internal problem ID [8690]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 354.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{(x\sin(y)-1)\,y'+\cos(y)=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 108

```
dsolve((x*sin(y(x))-1)*diff(y(x),x)+cos(y(x)) = 0,y(x), singsol=all)
```

$$y(x)=\arctan\left(\frac{-\sqrt{c_1^2-x^2+1}\,c_1+x}{c_1^2+1},\frac{c_1x+\sqrt{c_1^2-x^2+1}}{c_1^2+1}\right)$$

$$y(x)=\arctan\left(\frac{\sqrt{c_1^2-x^2+1}\,c_1+x}{c_1^2+1},\frac{c_1x-\sqrt{c_1^2-x^2+1}}{c_1^2+1}\right)$$

✓ Solution by Mathematica

Time used: 1.14 (sec). Leaf size: 163

```
DSolve[Cos[y[x]] + (-1 + x*Sin[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arccos\left(\frac{c_1x - \sqrt{-x^2 + 1 + c_1{}^2}}{1 + c_1{}^2}\right)$$
$$y(x) \to \arccos\left(\frac{c_1x - \sqrt{-x^2 + 1 + c_1{}^2}}{1 + c_1{}^2}\right)$$
$$y(x) \to -\arccos\left(\frac{\sqrt{-x^2 + 1 + c_1{}^2} + c_1x}{1 + c_1{}^2}\right)$$
$$y(x) \to \arccos\left(\frac{\sqrt{-x^2 + 1 + c_1{}^2} + c_1x}{1 + c_1{}^2}\right)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

1.354 problem 355

Internal problem ID [8691]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 355.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$(x\cos(y)+\cos(x))\,y'-y\sin(x)+\sin(y)=0$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 15

```
dsolve((x*cos(y(x))+cos(x))*diff(y(x),x)-y(x)*sin(x)+sin(y(x)) = 0,y(x), singsol=all)
```

$$y(x)\cos(x)+x\sin(y(x))+c_1=0$$

✓ Solution by Mathematica
Time used: 0.154 (sec). Leaf size: 17

```
DSolve[Sin[y[x]] - Sin[x]*y[x] + (Cos[x] + x*Cos[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolut
```

$$\text{Solve}[x\sin(y(x))+y(x)\cos(x)=c_1,y(x)]$$

1.355 problem 356

Internal problem ID [8692]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 356.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\left(x^2 \cos(y) + 2y \sin(x)\right) y' + 2x \sin(y) + y^2 \cos(x) = 0$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 19

```
dsolve((x^2*cos(y(x))+2*y(x)*sin(x))*diff(y(x),x)+2*x*sin(y(x))+y(x)^2*cos(x) = 0,y(x), sing
```

$$y(x)^2 \sin(x) + x^2 \sin(y(x)) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.203 (sec). Leaf size: 21

```
DSolve[2*x*Sin[y[x]] + Cos[x]*y[x]^2 + (x^2*Cos[y[x]] + 2*Sin[x]*y[x])*y'[x]==0,y[x],x,Inclu
```

$$\text{Solve}\left[x^2 \sin(y(x)) + y(x)^2 \sin(x) = c_1, y(x)\right]$$

1.356 problem 357

Internal problem ID [8693]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 357.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$x y' \ln(x) \sin(y) + \cos(y)(1 - x\cos(y)) = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 13

```
dsolve(x*diff(y(x),x)*ln(x)*sin(y(x))+cos(y(x))*(1-x*cos(y(x))) = 0,y(x), singsol=all)
```

$$y(x) = \operatorname{arcsec}\left(\frac{x + c_1}{\ln(x)}\right)$$

✓ Solution by Mathematica

Time used: 1.07 (sec). Leaf size: 53

```
DSolve[Cos[y[x]]*(1 - x*Cos[y[x]]) + x*Log[x]*Sin[y[x]]*y'[x]==0,y[x],x,IncludeSingularSolut
```

$$y(x) \to -\sec^{-1}\left(\frac{x - c_1}{\log(x)}\right)$$
$$y(x) \to \sec^{-1}\left(\frac{x - c_1}{\log(x)}\right)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

1.357 problem 358

Internal problem ID [8694]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 358.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' \sin(y) \cos(x) + \cos(y) \sin(x) = 0}$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)*sin(y(x))*cos(x)+cos(y(x))*sin(x) = 0,y(x), singsol=all)
```

$$y(x) = \arccos(c_1 \sec(x))$$

✓ Solution by Mathematica

Time used: 6.074 (sec). Leaf size: 47

```
DSolve[Cos[y[x]]*Sin[x] + Cos[x]*Sin[y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arccos\left(\frac{1}{2}c_1 \sec(x)\right)$$
$$y(x) \to \arccos\left(\frac{1}{2}c_1 \sec(x)\right)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

1.358 problem 359

Internal problem ID [8695]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 359.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$3y' \sin(x) \sin(y) + 5 \cos(x)^4 y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 26

```
dsolve(3*diff(y(x),x)*sin(x)*sin(y(x))+5*cos(x)^4*y(x) = 0,y(x), singsol=all)
```

$$\frac{3 \operatorname{Si}(y(x))}{5} + c_1 + \ln(\csc(x) - \cot(x)) + \frac{\cos(x)^3}{3} + \cos(x) = 0$$

✓ Solution by Mathematica

Time used: 0.974 (sec). Leaf size: 47

```
DSolve[5*Cos[x]^4*y[x] + 3*Sin[x]*Sin[y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \text{SinIntegral}^{(-1)}\left(-\frac{5}{36}\left(15\cos(x) + \cos(3x) + 12\left(\log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right)\right)\right) + c_1\right)$$

$$y(x) \to 0$$

1.359 problem 360

Internal problem ID [8696]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 360.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$y' \cos(ay) - b(1 - c\cos(ay))\sqrt{\cos(ay)^2 - 1 + c\cos(ay)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve(diff(y(x),x)*cos(a*y(x))-b*(1-c*cos(a*y(x)))*(cos(a*y(x))^2-1+c*cos(a*y(x)))^(1/2) =
```

$$\frac{\int^{y(x)} \frac{\cos(\_a a)}{\sqrt{c\cos(\_a a) - \sin(\_a a)^2}\,(c\cos(\_a a) - 1)} d\_a + (x + c_1)\, b}{b} = 0$$

✓ Solution by Mathematica

Time used: 28.047 (sec). Leaf size: 504

```
DSolve[-(b*(1 - c*Cos[a*y[x]])*Sqrt[-1 + c*Cos[a*y[x]] + Cos[a*y[x]]^2]) + Cos[a*y[x]]*y'[x]
```

$$y(x) \rightarrow \text{InverseFunction}\left[-\frac{i(\cos(\#1a)+1)\sqrt{\frac{2c\cos(\#1a)+\cos(2\#1a)-1}{(\cos(\#1a)+1)^2}}\sqrt{\frac{c\tan^2\left(\frac{\#1a}{2}\right)+\sqrt{c^2+4}+2}{\sqrt{c^2+4}+2}}\sqrt{1-\frac{c\tan^2\left(\frac{\#1a}{2}\right)}{\sqrt{c^2+4}-2}}\left(\right.}{a\left(c^2-1\right)\sqrt{\frac{}{4-}}} + c_1\right]$$

$$y(x) \rightarrow -\frac{\arccos\left(\frac{1}{c}\right)}{a}$$

$$y(x) \rightarrow \frac{\arccos\left(\frac{1}{c}\right)}{a}$$

$$y(x) \rightarrow -\frac{\arccos\left(\frac{1}{2}\left(-\sqrt{c^2+4}-c\right)\right)}{a}$$

$$y(x) \rightarrow \frac{\arccos\left(\frac{1}{2}\left(-\sqrt{c^2+4}-c\right)\right)}{a}$$

$$y(x) \rightarrow -\frac{\arccos\left(\frac{1}{2}\left(\sqrt{c^2+4}-c\right)\right)}{a}$$

$$y(x) \rightarrow \frac{\arccos\left(\frac{1}{2}\left(\sqrt{c^2+4}-c\right)\right)}{a}$$

1.360 problem 361

Internal problem ID [8697]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 361.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$(x \sin(xy) + \cos(x+y) - \sin(y))\, y' + y \sin(xy) + \cos(x+y) = -\cos(x)$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 22

```
dsolve((x*sin(x*y(x))+cos(x+y(x))-sin(y(x)))*diff(y(x),x)+y(x)*sin(x*y(x))+cos(x+y(x))+cos(x
```

$$-\cos(xy(x)) + \sin(x) + \sin(x + y(x)) + \cos(y(x)) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.567 (sec). Leaf size: 31

```
DSolve[Cos[x] + Cos[x + y[x]] + Sin[x*y[x]]*y[x] + (Cos[x + y[x]] - Sin[y[x]] + x*Sin[x*y[x]
```

$$\text{Solve}[\cos(y(x)) - \cos(xy(x)) + \sin(x)\cos(y(x)) + \cos(x)\sin(y(x)) + \sin(x) = c_1, y(x)]$$

1.361 problem 362

Internal problem ID [8698]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 362.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{(x^2 y \sin(xy) - 4x) y' + x y^2 \sin(xy) - y = 0}$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 22

```
dsolve((x^2*y(x)*sin(x*y(x))-4*x)*diff(y(x),x)+x*y(x)^2*sin(x*y(x))-y(x) = 0,y(x), singsol=a
```

$$y(x) = \frac{\text{RootOf}\left(\_Z - e^{-\frac{\cos(\_Z)}{4}} c_1 x^{\frac{3}{4}}\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.26 (sec). Leaf size: 23

```
DSolve[-y[x] + x*Sin[x*y[x]]*y[x]^2 + (-4*x + x^2*Sin[x*y[x]]*y[x])*y'[x]==0,y[x],x,IncludeS
```

$$\text{Solve}[-4\log(y(x)) - \cos(xy(x)) - \log(x) = c_1, y(x)]$$

1.362 problem 363

Internal problem ID [8699]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 363.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{(xy'-y)\cos\left(\frac{y}{x}\right)^2=-x}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 32

```
dsolve((x*diff(y(x),x)-y(x))*cos(y(x)/x)^2+x = 0,y(x), singsol=all)
```

$$\frac{-x\sin\left(\frac{2y(x)}{x}\right)-2y(x)}{4x}-\ln(x)-c_1=0$$

✓ Solution by Mathematica
Time used: 0.261 (sec). Leaf size: 33

```
DSolve[x + Cos[y[x]/x]^2*(-y[x] + x*y'[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{y(x)}{2x}+\frac{1}{4}\sin\left(\frac{2y(x)}{x}\right)=-\log(x)+c_1,y(x)\right]$$

1.363 problem 364

Internal problem ID [8700]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 364.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\left(\sin\left(\frac{y}{x}\right) y - x\cos\left(\frac{y}{x}\right)\right) x y' - \left(x\cos\left(\frac{y}{x}\right) + \sin\left(\frac{y}{x}\right) y\right) y = 0$$

✓ Solution by Maple
Time used: 0.157 (sec). Leaf size: 18

```
dsolve((y(x)*sin(y(x)/x)-x*cos(y(x)/x))*x*diff(y(x),x)-(x*cos(y(x)/x)+y(x)*sin(y(x)/x))*y(x)
```

$$y(x) = x\,\mathrm{RootOf}\left(\_Z\cos\left(\_Z\right) x^2 - c_1\right)$$

✓ Solution by Mathematica
Time used: 0.333 (sec). Leaf size: 31

```
DSolve[-(y[x]*(x*Cos[y[x]/x] + Sin[y[x]/x]*y[x])) + x*(-(x*Cos[y[x]/x]) + Sin[y[x]/x]*y[x])*
```

$$\mathrm{Solve}\left[-\log\left(\frac{y(x)}{x}\right) - \log\left(\cos\left(\frac{y(x)}{x}\right)\right) = 2\log(x) + c_1, y(x)\right]$$

1.364 problem 365

Internal problem ID [8701]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 365.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{(yf(x^2+y^2)-x)\,y'+y+xf(x^2+y^2)=0}$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 34

```
dsolve((y(x)*f(y(x)^2+x^2)-x)*diff(y(x),x)+y(x)+x*f(y(x)^2+x^2) = 0,y(x), singsol=all)
```

$$y(x)=\cot\left(\mathrm{RootOf}\left(-2\_Z-\left(\int^{\csc(\_Z)^2x^2}\frac{f(\_a)}{\_a}d\_a\right)+2c_1\right)\right)x$$

✓ Solution by Mathematica

Time used: 0.294 (sec). Leaf size: 156

```
DSolve[x*f[x^2 + y[x]^2] + y[x] + (-x + f[x^2 + y[x]^2]*y[x])*y'[x]==0,y[x],x,IncludeSingula
```

$$\mathrm{Solve}\left[\int_1^{y(x)}\left(\frac{x-f(x^2+K[2]^2)\,K[2]}{x^2+K[2]^2}-\int_1^x\left(\frac{-2K[1]K[2]f'(K[1]^2+K[2]^2)-1}{K[1]^2+K[2]^2}-\frac{2(-f(K[1]^2+K[2]^2)\,K[1]-K[2])\,K[2]}{(K[1]^2+K[2]^2)^2}\right)dK[1]\right)dK[2]+\int_1^x\frac{-f(K[1]^2+y(x)^2)\,K[1]-y(x)}{K[1]^2+y(x)^2}dK[1]=c_1,y(x)\right]$$

1.365 problem 366

Internal problem ID [8702]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 366.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$f(x^2 + ay^2)(ayy' + x) - y - xy' = 0$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 45

```
dsolve(f(x^2+a*y(x)^2)*(a*y(x)*diff(y(x),x)+x)-y(x)-x*diff(y(x),x) = 0,y(x), singsol=all)
```

$$-\frac{ay(x)^2 x}{\sqrt{a^2 y(x)^2}} - \left(\int^{-\frac{ay(x)^2}{2}-\frac{x^2}{2}} f(-2\_a)\, d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.239 (sec). Leaf size: 91

```
DSolve[-y[x] - x*y'[x] + f[x^2 + a*y[x]^2]*(x + a*y[x]*y'[x])==0,y[x],x,IncludeSingularSolut
```

$$\text{Solve}\left[\int_1^{y(x)} \left(x - af(x^2 + aK[2]^2)K[2] - \int_1^x \left(1 - 2aK[1]K[2]f'(K[1]^2 + aK[2]^2)\right) dK[1]\right) dK[2] + \int_1^x \left(y(x) - f(K[1]^2 + ay(x)^2)K[1]\right) dK[1] = c_1, y(x)\right]$$

1.366 problem 367

Internal problem ID [8703]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 367.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$f(x^c y)\,(bxy' - a) - x^a y^b (xy' + cy) = 0$$

✗ Solution by Maple

```
dsolve(f(x^c*y(x))*(b*x*diff(y(x),x)-a)-x^a*y(x)^b*(x*diff(y(x),x)+c*y(x)) = 0,y(x), singsol
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-(x^a*y[x]^b*(c*y[x] + x*y'[x])) + f[x^c*y[x]]*(-a + b*x*y'[x])==0,y[x],x,IncludeSing
```

Not solved

1.367 problem 368

Internal problem ID [8704]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 368.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{y'^2 + ay = -x^2 b}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)^2+a*y(x)+b*x^2 = 0,y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 1.1 (sec). Leaf size: 581

```
DSolve[b*x^2 + a*y[x] + y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\text{RootSum}\left[\#1^4-\#1^3a+2\#1^2b+\#1ab+b^2\&,\frac{2\#1^3\log\left(\#1x-\sqrt{-ay(x)-bx^2}+\sqrt{-ay(x)}\right)-2\#1^3\log(x)-\#1^2a\log\left(\#1x-\sqrt{-ay(x)-bx\right.}}{}\right.\right.$$

$$\left.-\log\left(\sqrt{-ay(x)}\sqrt{-ay(x)-bx^2}+ay(x)\right)+\frac{1}{2}\log(y(x))+2\log(x)=c_1,y(x)\right]$$

$$\text{Solve}\left[\text{RootSum}\left[\#1^4+\#1^3a+2\#1^2b-\#1ab+b^2\&,\frac{-2\#1^3\log\left(\#1x-\sqrt{-ay(x)-bx^2}+\sqrt{-ay(x)}\right)+2\#1^3\log(x)-\#1^2a\log\left(\#1x-\sqrt{-ay(x)-\right.}}{}\right.\right.$$

$$\left.-\log\left(\sqrt{-ay(x)}\sqrt{-ay(x)-bx^2}+ay(x)\right)+\frac{1}{2}\log(y(x))+2\log(x)=c_1,y(x)\right]$$

1.368 problem 369

Internal problem ID [8705]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 369.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 + y^2 = a^2$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 60

```
dsolve(diff(y(x),x)^2+y(x)^2-a^2 = 0,y(x), singsol=all)
```

$$y(x) = -a$$
$$y(x) = a$$
$$y(x) = -\tan(-x + c_1)\sqrt{\cos(-x + c_1)^2 a^2}$$
$$y(x) = \tan(-x + c_1)\sqrt{\cos(-x + c_1)^2 a^2}$$

✓ Solution by Mathematica

Time used: 3.278 (sec). Leaf size: 111

```
DSolve[-a^2 + y[x]^2 + y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a\tan(x-c_1)}{\sqrt{\sec^2(x-c_1)}}$$
$$y(x) \to \frac{a\tan(x-c_1)}{\sqrt{\sec^2(x-c_1)}}$$
$$y(x) \to -\frac{a\tan(x+c_1)}{\sqrt{\sec^2(x+c_1)}}$$
$$y(x) \to \frac{a\tan(x+c_1)}{\sqrt{\sec^2(x+c_1)}}$$
$$y(x) \to -a$$
$$y(x) \to a$$

1.369 problem 370

Internal problem ID [8706]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 370.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y'^2 + y^2 = f(x)^2}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)^2+y(x)^2-f(x)^2 = 0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-f[x]^2 + y[x]^2 + y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.370 problem 371

Internal problem ID [8707]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 371.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 - y^3 + y^2 = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 22

```
dsolve(diff(y(x),x)^2-y(x)^3+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = 1$$
$$y(x) = 0$$
$$y(x) = \sec\left(-\frac{x}{2} + \frac{c_1}{2}\right)^2$$

✓ Solution by Mathematica

Time used: 1.066 (sec). Leaf size: 45

```
DSolve[y[x]^2 - y[x]^3 + y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sec^2\left(\frac{x - c_1}{2}\right)$$
$$y(x) \to 1 + \tan^2\left(\frac{x + c_1}{2}\right)$$
$$y(x) \to 0$$
$$y(x) \to 1$$

1.371 problem 372

Internal problem ID [8708]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 372.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{{y'}^2 - 4y^3 + ay = -b}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 233

```
dsolve(diff(y(x),x)^2-4*y(x)^3+a*y(x)+b = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(27b + 3\sqrt{-3a^3 + 81b^2}\right)^{\frac{2}{3}} + 3a}{6\left(27b + 3\sqrt{-3a^3 + 81b^2}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{-i\sqrt{3}\left(27b + 3\sqrt{-3a^3 + 81b^2}\right)^{\frac{2}{3}} + 3i\sqrt{3}\,a - \left(27b + 3\sqrt{-3a^3 + 81b^2}\right)^{\frac{2}{3}} - 3a}{12\left(27b + 3\sqrt{-3a^3 + 81b^2}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{-i\sqrt{3}\left(27b + 3\sqrt{-3a^3 + 81b^2}\right)^{\frac{2}{3}} + 3i\sqrt{3}\,a + \left(27b + 3\sqrt{-3a^3 + 81b^2}\right)^{\frac{2}{3}} + 3a}{12\left(27b + 3\sqrt{-3a^3 + 81b^2}\right)^{\frac{1}{3}}}$$

$$y(x) = \mathrm{WeierstrassP}\left(x + c_1, a, b\right)$$

✓ Solution by Mathematica

Time used: 0.421 (sec). Leaf size: 273

```
DSolve[b + a*y[x] - 4*y[x]^3 + y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \wp(x - c_1; a, b)$$
$$y(x) \to \wp(x + c_1; a, b)$$
$$y(x) \to \frac{\left(\sqrt{81b^2 - 3a^3} + 9b\right)^{2/3} + \sqrt[3]{3}a}{2\ 3^{2/3}\sqrt[3]{\sqrt{81b^2 - 3a^3} + 9b}}$$
$$y(x) \to \frac{i\sqrt[3]{3}\left(\sqrt{3} + i\right)\left(\sqrt{81b^2 - 3a^3} + 9b\right)^{2/3} - \sqrt[6]{3}\left(\sqrt{3} + 3i\right)a}{12\sqrt[3]{\sqrt{81b^2 - 3a^3} + 9b}}$$
$$y(x) \to \frac{\sqrt[3]{3}\left(-1 - i\sqrt{3}\right)\left(\sqrt{81b^2 - 3a^3} + 9b\right)^{2/3} - \sqrt[6]{3}\left(\sqrt{3} - 3i\right)a}{12\sqrt[3]{\sqrt{81b^2 - 3a^3} + 9b}}$$

1.372 problem 373

Internal problem ID [8709]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 373.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 + a^2y^2(\ln(y)^2 - 1) = 0$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 45

```
dsolve(diff(y(x),x)^2+a^2*y(x)^2*(ln(y(x))^2-1) = 0,y(x), singsol=all)
```

$$y(x) = e^{\mathrm{RootOf}\left(a^2 e^{2\_Z}(\_Z^2-1)\right)}$$
$$y(x) = e^{-\sin(a(-x+c_1))}$$
$$y(x) = e^{\sin(a(-x+c_1))}$$

✓ Solution by Mathematica
Time used: 11.771 (sec). Leaf size: 197

```
DSolve[a^2*(-1 + Log[y[x]]^2)*y[x]^2 + y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \exp\left(-\frac{1}{2}\sqrt{-e^{2iax-2c_1} - e^{2c_1-2iax} + 2}\right)$$
$$y(x) \to \exp\left(\frac{1}{2}\sqrt{-e^{2iax-2c_1} - e^{2c_1-2iax} + 2}\right)$$
$$y(x) \to \exp\left(-\frac{1}{2}\sqrt{-e^{-2iax-2c_1}\left(-1 + e^{2iax+2c_1}\right)^2}\right)$$
$$y(x) \to \exp\left(\frac{1}{2}\sqrt{-e^{-2iax-2c_1}\left(-1 + e^{2iax+2c_1}\right)^2}\right)$$
$$y(x) \to \frac{1}{e}$$
$$y(x) \to e$$

1.373 problem 374

Internal problem ID [8710]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 374.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - 2y' - y^2 = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 66

```
dsolve(diff(y(x),x)^2-2*diff(y(x),x)-y(x)^2 = 0,y(x), singsol=all)
```

$$\frac{-\sqrt{y(x)^2+1}+\operatorname{arcsinh}(y(x))\,y(x)-1+(x-c_1)\,y(x)}{y(x)}=0$$
$$\frac{\sqrt{y(x)^2+1}-\operatorname{arcsinh}(y(x))\,y(x)-1+(x-c_1)\,y(x)}{y(x)}=0$$

✓ Solution by Mathematica
Time used: 0.844 (sec). Leaf size: 104

```
DSolve[-y[x]^2 - 2*y'[x] + y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \text{InverseFunction}\left[-\frac{\sqrt{\#1^2+1}+\#1\log\left(\sqrt{\#1^2+1}-\#1\right)+1}{\#1}\&\right][-x+c_1]$$
$$y(x)\to \text{InverseFunction}\left[-\frac{\sqrt{\#1^2+1}}{\#1}-\log\left(\sqrt{\#1^2+1}-\#1\right)+\frac{1}{\#1}\&\right][x+c_1]$$
$$y(x)\to 0$$

1.374 problem 375

Internal problem ID [8711]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 375.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 + ay' = -xb}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 70

```
dsolve(diff(y(x),x)^2+a*diff(y(x),x)+b*x = 0,y(x), singsol=all)
```

$$y(x) = \frac{(a^2 - 4bx)^{\frac{3}{2}} - 6b(ax - 2c_1)}{12b}$$
$$y(x) = \frac{(-a^2 + 4bx)\sqrt{a^2 - 4bx} - 6b(ax - 2c_1)}{12b}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 68

```
DSolve[b*x + a*y'[x] + y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{(a^2 - 4bx)^{3/2} + 6abx}{12b} + c_1$$
$$y(x) \to \frac{1}{2}\left(\frac{(a^2 - 4bx)^{3/2}}{6b} - ax\right) + c_1$$

1.375 problem 376

Internal problem ID [8712]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 376.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 + ay' + by = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 245

```
dsolve(diff(y(x),x)^2+a*diff(y(x),x)+b*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{a^2\left(\operatorname{LambertW}\left(-\frac{2\sqrt{-b}\,\mathrm{e}^{\frac{(-x+c_1)b-a}{a}}}{a}\right)+2\right)\operatorname{LambertW}\left(-\frac{2\sqrt{-b}\,\mathrm{e}^{\frac{(-x+c_1)b-a}{a}}}{a}\right)}{4b}$$

$$y(x) = -\frac{a^2\left(\operatorname{LambertW}\left(\frac{2\sqrt{-b}\,\mathrm{e}^{\frac{(-x+c_1)b-a}{a}}}{a}\right)+2\right)\operatorname{LambertW}\left(\frac{2\sqrt{-b}\,\mathrm{e}^{\frac{(-x+c_1)b-a}{a}}}{a}\right)}{4b}$$

$$y(x) = \mathrm{e}^{\frac{-a\operatorname{LambertW}\left(\frac{2\,\mathrm{e}^{\frac{(-x+c_1)b-a}{a}}}{a\sqrt{-\frac{1}{b}}}\right)-a+(-x+c_1)b}{a}}\left(a\sqrt{-\frac{1}{b}}+\mathrm{e}^{\frac{-a\operatorname{LambertW}\left(\frac{2\,\mathrm{e}^{\frac{(-x+c_1)b-a}{a}}}{a\sqrt{-\frac{1}{b}}}\right)-a+(-x+c_1)b}{a}}\right)$$

✓ Solution by Mathematica

Time used: 0.809 (sec). Leaf size: 119

```
DSolve[b*y[x] + a*y'[x] + y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{\sqrt{a^2-4\#1b}+a\log\left(b\left(a-\sqrt{a^2-4\#1b}\right)\right)}{2b}\&\right]\left[\frac{x}{2}+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{\sqrt{a^2-4\#1b}-a\log\left(b\left(\sqrt{a^2-4\#1b}+a\right)\right)}{2b}\&\right]\left[-\frac{x}{2}+c_1\right]$$

$$y(x) \to 0$$

1.376 problem 377

Internal problem ID [8713]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 377.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y'^2 + (x-2)\,y' - y = -1}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)^2+(x-2)*diff(y(x),x)-y(x)+1 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{4}x^2 + x$$
$$y(x) = 1 + c_1^2 + (x-2)\,c_1$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 29

```
DSolve[1 - y[x] + (-2 + x)*y'[x] + y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x-2) + 1 + c_1{}^2$$
$$y(x) \to -\frac{1}{4}(x-4)x$$

1.377 problem 378

Internal problem ID [8714]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 378.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$y'^2 + (a + x)\, y' - y = 0$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)^2+(x+a)*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{(a + x)^2}{4}$$
$$y(x) = c_1(c_1 + a + x)$$

✓ Solution by Mathematica
Time used: 0.007 (sec). Leaf size: 26

```
DSolve[-y[x] + (a + x)*y'[x] + y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(a + x + c_1)$$
$$y(x) \to -\frac{1}{4}(a + x)^2$$

1.378 problem 379

Internal problem ID [8715]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 379.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$y'^2 - (1 + x)\, y' + y = 0$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 22

```
dsolve(diff(y(x),x)^2-(x+1)*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{(x+1)^2}{4}$$
$$y(x) = c_1(x + 1 - c_1)$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 28

```
DSolve[y[x] - (1 + x)*y'[x] + y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x + 1 - c_1)$$
$$y(x) \to \frac{1}{4}(x+1)^2$$

1.379 problem 380

Internal problem ID [8716]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 380.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{{y'}^2 + 2xy' - y = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 650

```
dsolve(diff(y(x),x)^2+2*x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(x^2 - x\left(-x^3 + 2\sqrt{3}\sqrt{-c_1\left(x^3 - 3c_1\right)} + 6c_1\right)^{\frac{1}{3}} + \left(-x^3 + 2\sqrt{3}\sqrt{-c_1\left(x^3 - 3c_1\right)} + 6c_1\right)^{\frac{2}{3}}\right)\left(x^2 + 3x\left(-\right.\right.}{4\left(-x^3 + 2\sqrt{3}\sqrt{-c_1\left(x^3 - 3c_1\right)}\right.}$$

$$y(x) = \frac{\left(i\left(-x^3 + 2\sqrt{3}\sqrt{-c_1\left(x^3 - 3c_1\right)} + 6c_1\right)^{\frac{2}{3}}\sqrt{3} - i\sqrt{3}\,x^2 + \left(-x^3 + 2\sqrt{3}\sqrt{-c_1\left(x^3 - 3c_1\right)} + 6c_1\right)^{\frac{2}{3}} + 2x\left(\right.\right.}{}$$

$$y(x) = \frac{\left(i\sqrt{3}\,x^2 - i\left(-x^3 + 2\sqrt{3}\sqrt{-c_1\left(x^3 - 3c_1\right)} + 6c_1\right)^{\frac{2}{3}}\sqrt{3} + x^2 + 2x\left(-x^3 + 2\sqrt{3}\sqrt{-c_1\left(x^3 - 3c_1\right)} + 6c_1\right)^{\frac{1}{3}}\right.}{}$$

✓ Solution by Mathematica

Time used: 60.156 (sec). Leaf size: 931

```
DSolve[-y[x] + 2*x*y'[x] + y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-x^2+\frac{x(x^3+8e^{3c_1})}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}+\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(-18x^2-\frac{9i(\sqrt{3}-i)\,x(x^3+8e^{3c_1})}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}+9i\left(\sqrt{3}+i\right)\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(-18x^2+\frac{9i(\sqrt{3}+i)\,x(x^3+8e^{3c_1})}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}-9\left(1+i\sqrt{3}\right)\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{4}\left(-x^2+\frac{x(x^3-8e^{3c_1})}{\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}+\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(-18x^2+\frac{9(1+i\sqrt{3})\,x(-x^3+8e^{3c_1})}{\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}+9i\left(\sqrt{3}+i\right)\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(-18x^2+\frac{9i(\sqrt{3}+i)\,x(x^3-8e^{3c_1})}{\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}-9\left(1+i\sqrt{3}\right)\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

1.380 problem 381

Internal problem ID [8717]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 381.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y'^2 - 2xy' + y = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 611

```
dsolve(diff(y(x),x)^2-2*x*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\left(x^2 + x\left(x^3 + 2\sqrt{3}\,\sqrt{-c_1\,(x^3 - 3c_1)} - 6c_1\right)^{\frac{1}{3}} + \left(x^3 + 2\sqrt{3}\,\sqrt{-c_1\,(x^3 - 3c_1)} - 6c_1\right)^{\frac{2}{3}}\right)\left(x^2 - 3x\left(x^3\right.\right.}{4\left(x^3 + 2\sqrt{3}\,\sqrt{-c_1\,(x^3 - 3c_1)} - \right.}$$

$$y(x) = -\frac{\left(i\left(x^3 + 2\sqrt{3}\,\sqrt{-c_1\,(x^3 - 3c_1)} - 6c_1\right)^{\frac{2}{3}}\sqrt{3} - i\sqrt{3}\,x^2 + \left(x^3 + 2\sqrt{3}\,\sqrt{-c_1\,(x^3 - 3c_1)} - 6c_1\right)^{\frac{2}{3}} - 2x\left(x\right.\right.}{}$$

$$y(x) = -\frac{\left(i\sqrt{3}\,x^2 - i\left(x^3 + 2\sqrt{3}\,\sqrt{-c_1\,(x^3 - 3c_1)} - 6c_1\right)^{\frac{2}{3}}\sqrt{3} + x^2 - 2x\left(x^3 + 2\sqrt{3}\,\sqrt{-c_1\,(x^3 - 3c_1)} - 6c_1\right)^{\frac{1}{3}}\right.}{}$$

✓ Solution by Mathematica

Time used: 60.176 (sec). Leaf size: 954

```
DSolve[y[x] - 2*x*y'[x] + y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(x^2 + \frac{x(x^3+8e^{3c_1})}{\sqrt[3]{x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}-8e^{6c_1}}} + \sqrt[3]{x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}-8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(18x^2 - \frac{9i(\sqrt{3}-i)\,x(x^3+8e^{3c_1})}{\sqrt[3]{x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}-8e^{6c_1}}} + 9i\left(\sqrt{3}+i\right)\sqrt[3]{x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}-8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(18x^2 + \frac{9i(\sqrt{3}+i)\,x(x^3+8e^{3c_1})}{\sqrt[3]{x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}-8e^{6c_1}}} - 9\left(1+i\sqrt{3}\right)\sqrt[3]{x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}-8e^{6c_1}}\right)$$

$$y(x) \to \frac{x^4 + \left(x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}-8e^{6c_1}\right)^{2/3} + x^2\sqrt[3]{x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}-8}}{4\sqrt[3]{x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}-8e^{6c_1}}}$$

$$y(x) \to \frac{1}{72}\left(18x^2 + \frac{9(1+i\sqrt{3})\,x(-x^3+8e^{3c_1})}{\sqrt[3]{x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}-8e^{6c_1}}} + 9i\left(\sqrt{3}+i\right)\sqrt[3]{x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}-8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(18x^2 + \frac{9i(\sqrt{3}+i)\,x(x^3-8e^{3c_1})}{\sqrt[3]{x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}-8e^{6c_1}}} - 9\left(1+i\sqrt{3}\right)\sqrt[3]{x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}-8e^{6c_1}}\right)$$

1.381 problem 382

Internal problem ID [8718]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 382.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 + axy' = x^2b + c}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 174

```
dsolve(diff(y(x),x)^2+a*x*diff(y(x),x)-b*x^2-c = 0,y(x), singsol=all)
```

$$y(x) = -\frac{4c\ln\left(\sqrt{a^2+4b}\,x+\sqrt{(a^2+4b)\,x^2+4c}\right)+\sqrt{a^2+4b}\left(a\,x^2+x\sqrt{(a^2+4b)\,x^2+4c}-4c_1\right)}{4\sqrt{a^2+4b}}$$

$$y(x) = -\frac{-4c\ln\left(\sqrt{a^2+4b}\,x+\sqrt{(a^2+4b)\,x^2+4c}\right)+\sqrt{a^2+4b}\left(a\,x^2-x\sqrt{(a^2+4b)\,x^2+4c}-4c_1\right)}{4\sqrt{a^2+4b}}$$

✓ Solution by Mathematica

Time used: 0.477 (sec). Leaf size: 197

```
DSolve[-c - b*x^2 + a*x*y'[x] + y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2c\arctan\left(\frac{x\sqrt{-a^2-4b}}{\sqrt{x^2(a^2+4b)+4c}-2\sqrt{c}}\right)}{\sqrt{-a^2-4b}} - \frac{1}{4}x\left(\sqrt{x^2\left(a^2+4b\right)+4c}+ax\right)+c_1$$

$$y(x) \to \frac{2c\arctan\left(\frac{x\sqrt{-a^2-4b}}{\sqrt{x^2(a^2+4b)+4c}-2\sqrt{c}}\right)}{\sqrt{-a^2-4b}} + \frac{1}{4}x\sqrt{x^2\left(a^2+4b\right)+4c}-\frac{ax^2}{4}+c_1$$

1.382 problem 383

Internal problem ID [8719]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 383.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{y'^2 + axy' + by = -c\,x^2}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)^2+a*x*diff(y(x),x)+b*y(x)+c*x^2 = 0,y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 2.085 (sec). Leaf size: 1085

```
DSolve[c*x^2 + b*y[x] + a*x*y'[x] + y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\text{RootSum}\left[\#1^4-2\#1^3b-2\#1^2a^2-4\#1^2ab+8\#1^2c-2\#1a^2b+8\#1bc+a^4-8a^2c\right.\right.$$
$$+16c^2\&,\frac{-\#1^3\log\left(\#1x-\sqrt{x^2\left(a^2-4c\right)-4by(x)}+2\sqrt{-by(x)}\right)+\#1^3\log(x)+\#1^2b\log\left(\#1x-\sqrt{x^2}\right.}{}$$
$$\left.-\log\left(\sqrt{-by(x)}\sqrt{x^2\left(a^2-4c\right)-4by(x)}+2by(x)\right)+\frac{1}{2}\log(y(x))+2\log(x)=c_1,y(x)\right]$$

$$\text{Solve}\left[\text{RootSum}\left[\#1^4+2\#1^3b-2\#1^2a^2-4\#1^2ab+8\#1^2c+2\#1a^2b-8\#1bc+a^4-8a^2c\right.\right.$$
$$+16c^2\&,\frac{\#1^3\log\left(\#1x-\sqrt{x^2\left(a^2-4c\right)-4by(x)}+2\sqrt{-by(x)}\right)+\#1^3(-\log(x))+\#1^2b\log\left(\#1x-\sqrt{x}\right.}{}$$
$$\left.-\log\left(\sqrt{-by(x)}\sqrt{x^2\left(a^2-4c\right)-4by(x)}+2by(x)\right)+\frac{1}{2}\log(y(x))+2\log(x)=c_1,y(x)\right]$$

1.383 problem 384

Internal problem ID [8720]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 384.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y'^2 + (xa + b)\, y' - ay = -c}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 50

```
dsolve(diff(y(x),x)^2+(a*x+b)*diff(y(x),x)-a*y(x)+c = 0,y(x), singsol=all)
```

$$y(x) = \frac{-a^2x^2 - 2axb - b^2 + 4c}{4a}$$
$$y(x) = \frac{c_1^2 + (ax + b)\, c_1 + c}{a}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 51

```
DSolve[c - a*y[x] + (b + a*x)*y'[x] + y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c + c_1(ax + b + c_1)}{a}$$
$$y(x) \to -\frac{a^2x^2 + 2abx + b^2 - 4c}{4a}$$

1.384 problem 385

Internal problem ID [8721]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 385.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{y'^2 - 2x^2y' + 2xy = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 161

```
dsolve(diff(y(x),x)^2-2*x^2*diff(y(x),x)+2*x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{x^4 - \operatorname{RootOf}\left(x^{16} - 12\_Z^2x^{12} + 16\_Z^3x^{10} + 30\_Z^4x^8 - 96\_Z^5x^6 + 100\_Z^6x^4 - 48\_Z^7x^2 + 9\_Z^8 - 1\right.}{2x}$$

$$y(x) = \frac{x^4 - \operatorname{RootOf}\left(x^{16} - 12\_Z^2x^{12} - 16\_Z^3x^{10} + 30\_Z^4x^8 + 96\_Z^5x^6 + 100\_Z^6x^4 + 48\_Z^7x^2 + 9\_Z^8 - 1\right.}{2x}$$

✓ Solution by Mathematica
Time used: 60.477 (sec). Leaf size: 4749

```
DSolve[2*x*y[x] - 2*x^2*y'[x] + y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

1.385 problem 386

Internal problem ID [8722]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 386.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y'^2 + a\,x^3 y' - 2a\,x^2 y = 0}$$

✓ Solution by Maple
Time used: 0.344 (sec). Leaf size: 27

```
dsolve(diff(y(x),x)^2+a*x^3*diff(y(x),x)-2*a*x^2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{a\,x^4}{8}$$
$$y(x) = \frac{c_1(a\,x^2 + 2c_1)}{a}$$

✓ Solution by Mathematica
Time used: 0.638 (sec). Leaf size: 78

```
DSolve[-2*a*x^2*y[x] + a*x^3*y'[x] + y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}e^{2c_1}\left(-2\sqrt{a}x^2 + e^{2c_1}\right)$$
$$y(x) \to 2\sqrt{a}e^{2c_1}x^2 + 8e^{4c_1}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{ax^4}{8}$$

1.386 problem 387

Internal problem ID [8723]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 387.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y'^2 = -(y' - y)\,\mathrm{e}^x$$

✓ Solution by Maple

Time used: 0.266 (sec). Leaf size: 119

```
dsolve(diff(y(x),x)^2+(diff(y(x),x)-y(x))*exp(x) = 0,y(x), singsol=all)
```

$$\frac{2y(x)\ln(y(x)) + 4\,\mathrm{arctanh}\left(\sqrt{\mathrm{e}^x\left(4y(x)+\mathrm{e}^x\right)}\,\mathrm{e}^{-x}\right)y(x) - c_1y(x) - \mathrm{e}^x + \sqrt{\mathrm{e}^x\left(4y(x)+\mathrm{e}^x\right)}}{y(x)} = 0$$

$$\frac{2y(x)\ln(y(x)) - 4\,\mathrm{arctanh}\left(\sqrt{\mathrm{e}^x\left(4y(x)+\mathrm{e}^x\right)}\,\mathrm{e}^{-x}\right)y(x) - c_1y(x) - \mathrm{e}^x - \sqrt{\mathrm{e}^x\left(4y(x)+\mathrm{e}^x\right)}}{y(x)} = 0$$

✓ Solution by Mathematica

Time used: 2.12 (sec). Leaf size: 143

```
DSolve[y'[x]^2 + E^x*(-y[x] + y'[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{e^{x/2}\sqrt{4y(x)+e^x} - 4y(x)\log\left(\sqrt{4y(x)+e^x} - e^{x/2}\right) + e^x}{2y(x)} = c_1, y(x)\right]$$

$$\text{Solve}\left[2\log(y(x)) - \frac{-e^{x/2}\sqrt{4y(x)+e^x} + 4y(x)\log\left(\sqrt{4y(x)+e^x} - e^{x/2}\right) + e^x}{2y(x)} = c_1, y(x)\right]$$

$$y(x) \to 0$$

1.387 problem 388

Internal problem ID [8724]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 388.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_dAlembert]`

$$\boxed{y'^2 - 2yy' = 2x}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 223

```
dsolve(diff(y(x),x)^2-2*y(x)*diff(y(x),x)-2*x = 0,y(x), singsol=all)
```

$$\frac{\frac{\left(y(x)-\sqrt{y(x)^2+2x}\right)\operatorname{arcsinh}\left(-y(x)+\sqrt{y(x)^2+2x}\right)}{2} + x\sqrt{2y\left(x\right)^2+2x-2y\left(x\right)\sqrt{y\left(x\right)^2+2x}+1}-2c_1y(x)+2c_1\sqrt{y}}{\sqrt{2y\left(x\right)^2+2x-2y\left(x\right)\sqrt{y\left(x\right)^2+2x}+1}}$$
$$= 0$$
$$\frac{\frac{\left(-y(x)-\sqrt{y(x)^2+2x}\right)\operatorname{arcsinh}\left(y(x)+\sqrt{y(x)^2+2x}\right)}{2} + x\sqrt{2y\left(x\right)^2+2x+2y\left(x\right)\sqrt{y\left(x\right)^2+2x}+1}+2c_1y(x)+2c_1\sqrt{y}}{\sqrt{2y\left(x\right)^2+2x+2y\left(x\right)\sqrt{y\left(x\right)^2+2x}+1}}$$
$$= 0$$

✓ Solution by Mathematica

Time used: 0.713 (sec). Leaf size: 74

```
DSolve[-2*x - 2*y[x]*y'[x] + y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x=-\frac{K[1]\log\left(\sqrt{K[1]^2+1}-K[1]\right)}{2\sqrt{K[1]^2+1}}+\frac{c_1K[1]}{\sqrt{K[1]^2+1}},y(x)=\frac{K[1]}{2}-\frac{x}{K[1]}\right\},\{y(x),K[1]\}\right]$$

1.388 problem 389

Internal problem ID [8725]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 389.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - (1+4y)\,y' + (1+4y)\,y = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 137

```
dsolve(diff(y(x),x)^2-(4*y(x)+1)*diff(y(x),x)+(4*y(x)+1)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{4}$$
$$y(x) = -\frac{\sqrt{-\mathrm{e}^{-2x}c_1}\,\mathrm{e}^{2x} + c_1}{c_1\sqrt{-\mathrm{e}^{-2x}c_1}}$$
$$y(x) = \frac{-\sqrt{-\mathrm{e}^{-2x}c_1}\,\mathrm{e}^{2x} + c_1}{\sqrt{-\mathrm{e}^{-2x}c_1}\,c_1}$$
$$y(x) = \frac{-\sqrt{-\mathrm{e}^{-2x}c_1}\,\mathrm{e}^{2x} + c_1}{\sqrt{-\mathrm{e}^{-2x}c_1}\,c_1}$$
$$y(x) = -\frac{\sqrt{-\mathrm{e}^{-2x}c_1}\,\mathrm{e}^{2x} + c_1}{c_1\sqrt{-\mathrm{e}^{-2x}c_1}}$$

✓ Solution by Mathematica

Time used: 0.208 (sec). Leaf size: 67

```
DSolve[y[x]*(1 + 4*y[x]) - (1 + 4*y[x])*y'[x] + y'[x]^2==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{1}{4}e^{x-4c_1}\left(e^x + 2e^{2c_1}\right)$$
$$y(x) \to \frac{1}{4}e^{x+2c_1}\left(-2 + e^{x+2c_1}\right)$$
$$y(x) \to -\frac{1}{4}$$
$$y(x) \to 0$$

1.389 problem 390

Internal problem ID [8726]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 390.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_dAlembert]`

$$y'^2 + ayy' = bx + c$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 291

```
dsolve(diff(y(x),x)^2+a*y(x)*diff(y(x),x)-b*x-c = 0,y(x), singsol=all)
```

$$y(x) = \frac{4\left(axb + ac - \frac{1}{2}b\right) e^{\mathrm{RootOf}\left(\sqrt{a}\, c_1 b\, e^{2\_Z} - e^{2\_Z} abx - e^{2\_Z}\_Zb - e^{2\_Z} ac + \sqrt{a}\, c_1 b^2 + a\, b^2 x - \_Z b^2 + acb\right)} - b^2 e^{-\mathrm{RootOf}\left(\sqrt{a}\, c_1 b\, e^{2\_Z} - e^{2}\right.}}{a^{\frac{3}{2}} \left(2\, e^{2\,\mathrm{RootOf}\left(\sqrt{a}\, c_1 b\, e^{2\_Z} - e^{2\_Z} abx - e^{2\_Z}\right.}\right.}$$

✓ Solution by Mathematica

Time used: 2.035 (sec). Leaf size: 161

```
DSolve[-c - b*x + a*y[x]*y'[x] + y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\left\{x = \left(\frac{a \log\left(\sqrt{b - aK[1]^2} - \sqrt{-a}K[1]\right)}{(-a)^{3/2}} - \frac{c\sqrt{b - aK[1]^2}}{bK[1]}\right) \exp\left(b\left(\frac{\log(K[1])}{b} - \frac{\log\left(b - aK[1]^2\right)}{2b}\right.\right.\right.\right.$$

1.390 problem 391

Internal problem ID [8727]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 391.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$
\boxed{y'^2 + (ay + xb)\, y' + yabx = 0}
$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x)^2+(a*y(x)+b*x)*diff(y(x),x)+a*b*x*y(x) = 0,y(x), singsol=all)
```

$$
y(x) = \mathrm{e}^{-ax} c_1
$$
$$
y(x) = -\frac{b\, x^2}{2} + c_1
$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 34

```
DSolve[a*b*x*y[x] + (b*x + a*y[x])*y'[x] + y'[x]^2==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$
y(x) \to c_1 e^{-ax}
$$
$$
y(x) \to -\frac{bx^2}{2} + c_1
$$
$$
y(x) \to 0
$$

1.391 problem 392

Internal problem ID [8728]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 392.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{y'^2 - xy'y + y^2 \ln(ay) = 0}$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 43

```
dsolve(diff(y(x),x)^2-x*y(x)*diff(y(x),x)+y(x)^2*ln(a*y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \frac{e^{\frac{x^2}{4}}}{a}$$
$$y(x) = \frac{e^{c_1(x-c_1)}}{a}$$
$$y(x) = \frac{e^{-c_1(x+c_1)}}{a}$$

✓ Solution by Mathematica
Time used: 0.317 (sec). Leaf size: 30

```
DSolve[Log[a*y[x]]*y[x]^2 - x*y[x]*y'[x] + y'[x]^2==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{e^{\frac{1}{4}c_1(2x-c_1)}}{a}$$
$$y(x) \to 0$$

1.392 problem 393

Internal problem ID [8729]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 393.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$y'^2 + 2yy' \cot(x) - y^2 = 0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 39

```
dsolve(diff(y(x),x)^2+2*y(x)*diff(y(x),x)*cot(x)-y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{\mathrm{csgn}(\sin(x))\, c_1}{\cos(x) + \mathrm{csgn}(\sec(x))}$$
$$y(x) = \csc(x)^2 (\cos(x) + \mathrm{csgn}(\sec(x)))\, \mathrm{csgn}(\sin(x))\, c_1$$

✓ Solution by Mathematica

Time used: 0.161 (sec). Leaf size: 36

```
DSolve[-y[x]^2 + 2*Cot[x]*y[x]*y'[x] + y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \csc^2\left(\frac{x}{2}\right)$$
$$y(x) \to c_1 \sec^2\left(\frac{x}{2}\right)$$
$$y(x) \to 0$$

1.393 problem 394

Internal problem ID [8730]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 394.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y'^2 + 2f(x)\,yy' + g(x)\,y^2 = \left(g(x) - f(x)^2\right)\mathrm{e}^{-2\left(\int_a^x f(\mathrm{xp})d\,\mathrm{xp}\right)}$$

✓ Solution by Maple

Time used: 0.906 (sec). Leaf size: 107

```
dsolve(diff(y(x),x)^2+2*f(x)*y(x)*diff(y(x),x)+g(x)*y(x)^2-(g(x)-f(x)^2)*exp(-2*int(f(xp),xp
```

$$y(x) = \tan\left(-\left(\int \mathrm{e}^{2\left(\int_a^x f(\mathrm{xp})d\,\mathrm{xp}\right)}\sqrt{-\mathrm{e}^{-4\left(\int_a^x f(\mathrm{xp})d\,\mathrm{xp}\right)}\left(f(x)^2 - g(x)\right)}dx\right) + c_1\right)\sqrt{\cos\left(-\left(\int \mathrm{e}^{2\left(\int_a^x f(\mathrm{xp})d\,\mathrm{xp}\right)}\sqrt{-\mathrm{e}^{-4\left(\int_a^x f(\mathrm{xp})d\,\mathrm{xp}\right)}\left(f(x)^2 - g(x)\right)}dx\right) + c_1\right)^2 \mathrm{e}^{-2\left(\int_a^x f(\mathrm{xp})d\,\mathrm{xp}\right)}}$$

✓ Solution by Mathematica

Time used: 60.339 (sec). Leaf size: 89

```
DSolve[-((-f[x]^2 + g[x])/E^(2*Integrate[f[xp], {xp, a, x}])) + g[x]*y[x]^2 + 2*f[x]*y[x]*y'
```

$$y(x) \to e^{-\int_a^x f(K[1])\,dK[1]}\left(\begin{cases} \sin\left(c_1 + \int_a^x \sqrt{g(K[1]) - f(K[1])^2}\,dK[1]\right) & g(x) > f(x)^2 \\ \cosh\left(c_1 + \int_a^x \sqrt{f(K[1])^2 - g(K[1])}\,dK[1]\right) & g(x) < f(x)^2 \\ c_1 & \text{True} \end{cases}\right)$$

1.394 problem 395

Internal problem ID [8731]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 395.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$y'^2 + 2f(x)\, yy' + g(x)\, y^2 = -h(x)$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)^2+2*f(x)*y(x)*diff(y(x),x)+g(x)*y(x)^2+h(x) = 0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[h[x] + g[x]*y[x]^2 + 2*f[x]*y[x]*y'[x] + y'[x]^2==0,y[x],x,IncludeSingularSolutions -
```

Not solved

1.395 problem 396

Internal problem ID [8732]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 396.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'^2 + y(y-x)\,y' - y^3x = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)^2+y(x)*(y(x)-x)*diff(y(x),x)-x*y(x)^3 = 0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{x^2}{2}} c_1$$
$$y(x) = \frac{1}{x + c_1}$$

✓ Solution by Mathematica
Time used: 0.13 (sec). Leaf size: 34

```
DSolve[-(x*y[x]^3) + y[x]*(-x + y[x])*y'[x] + y'[x]^2==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{x - c_1}$$
$$y(x) \to c_1 e^{\frac{x^2}{2}}$$
$$y(x) \to 0$$

1.396 problem 397

Internal problem ID [8733]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 397.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y'^2 - 2x^3y^2y' - 4y^3x^2 = 0$$

✓ Solution by Maple

Time used: 0.312 (sec). Leaf size: 132

```
dsolve(diff(y(x),x)^2-2*x^3*y(x)^2*diff(y(x),x)-4*x^2*y(x)^3 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{4}{x^4}$$
$$y(x) = 0$$
$$y(x) = \frac{\left(\sqrt{2}\,x^2c_1 - 2\right)c_1^2}{2c_1^2x^4 - 4}$$
$$y(x) = -\frac{\left(\sqrt{2}\,x^2c_1 + 2\right)c_1^2}{2c_1^2x^4 - 4}$$
$$y(x) = \frac{-2\sqrt{2}\,x^2 + 2c_1}{c_1\left(-2x^4 + c_1^2\right)}$$
$$y(x) = \frac{2\sqrt{2}\,x^2 + 2c_1}{c_1\left(-2x^4 + c_1^2\right)}$$

✓ Solution by Mathematica

Time used: 1.439 (sec). Leaf size: 177

```
DSolve[-4*x^2*y[x]^3 - 2*x^3*y[x]^2*y'[x] + y'[x]^2==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$\text{Solve}\left[-\frac{x\sqrt{x^4y(x)+4}y(x)^{3/2}\log\left(\sqrt{x^4y(x)+4}+x^2\sqrt{y(x)}\right)}{2\sqrt{x^2y(x)^3\left(x^4y(x)+4\right)}}-\frac{1}{4}\log(y(x))=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{xy(x)^{3/2}\sqrt{x^4y(x)+4}\log\left(\sqrt{x^4y(x)+4}+x^2\sqrt{y(x)}\right)}{2\sqrt{x^2y(x)^3\left(x^4y(x)+4\right)}}-\frac{1}{4}\log(y(x))=c_1,y(x)\right]$$

$$y(x)\to 0$$

$$y(x)\to -\frac{4}{x^4}$$

1.397 problem 398

Internal problem ID [8734]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 398.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y'^2 - 3xy^{\frac{2}{3}}y' + 9y^{\frac{5}{3}} = 0$$

✓ Solution by Maple

Time used: 0.343 (sec). Leaf size: 150

```
dsolve(diff(y(x),x)^2-3*x*y(x)^(2/3)*diff(y(x),x)+9*y(x)^(5/3) = 0,y(x), singsol=all)
```

$$y(x) = \frac{x^6}{64}$$
$$y(x) = 0$$
$$\ln(x) + \frac{\ln\left(\frac{64y(x)}{x^6} - 1\right)}{6} - \frac{\ln\left(4\left(\frac{y(x)}{x^6}\right)^{\frac{1}{3}} - 1\right)}{6} - \frac{\ln\left(16\left(\frac{y(x)}{x^6}\right)^{\frac{2}{3}} + 4\left(\frac{y(x)}{x^6}\right)^{\frac{1}{3}} + 1\right)}{6} + \frac{\ln\left(\frac{y(x)}{x^6}\right)}{6} - \frac{\sqrt{-\frac{y(x)\left(\frac{y(x)}{x^6}\right)^{\frac{1}{3}}\left(4\left(\frac{y(x)}{x^6}\right)^{\frac{1}{3}}-1\right)}{x^6}}\,\operatorname{arctanh}\left(\sqrt{-4\left(\frac{y(x)}{x^6}\right)^{\frac{1}{3}} + 1}\right)}{\left(\frac{y(x)}{x^6}\right)^{\frac{2}{3}}\sqrt{-4\left(\frac{y(x)}{x^6}\right)^{\frac{1}{3}} + 1}} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 17.354 (sec). Leaf size: 701

```
DSolve[9*y[x]^(5/3) - 3*x*y[x]^(2/3)*y'[x] + y'[x]^2==0,y[x],x,IncludeSingularSolutions -> T
```

$$\text{Solve}\left[\frac{8x^2\log(y(x)) - 6\sqrt{x^4}\log\left(x^2\sqrt{x^2-4\sqrt[3]{y(x)}}\right) - 3\sqrt{x^4}\log\left(4\sqrt[3]{y(x)}-x^2\right) + 6\left(\sqrt{x^4}-x^2\right)\log\left(1\right.}{}\right.$$

$$\left. - \frac{\sqrt{\left(x^2-4\sqrt[3]{y(x)}\right)y(x)^{4/3}}\log\left(\sqrt{x^2-4\sqrt[3]{y(x)}}-x\right)}{\sqrt{x^2-4\sqrt[3]{y(x)}}y(x)^{2/3}} = c_1, y(x)\right]$$

$$\text{Solve}\left[\frac{\sqrt{\left(x^2-4\sqrt[3]{y(x)}\right)y(x)^{4/3}}\log\left(\sqrt{x^2-4\sqrt[3]{y(x)}}-x\right)}{\sqrt{x^2-4\sqrt[3]{y(x)}}y(x)^{2/3}}\right.$$

$$+ \frac{8x^2\log(y(x)) + 6\sqrt{x^4}\log\left(x^2\sqrt{x^2-4\sqrt[3]{y(x)}}\right) + 3\sqrt{x^4}\log\left(4\sqrt[3]{y(x)}-x^2\right) + 6\left(x^2-\sqrt{x^4}\right)\log\left(16x^2\sqrt{\right.}{}$$

$$y(x) \to 0$$

1.398 problem 399

Internal problem ID [8735]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 399.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{2y'^2 + (x-1)\,y' - y = 0}$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 22

```
dsolve(2*diff(y(x),x)^2+(x-1)*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{(x-1)^2}{8}$$
$$y(x) = c_1(2c_1 + x - 1)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 28

```
DSolve[-y[x] + (-1 + x)*y'[x] + 2*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x - 1 + 2c_1)$$
$$y(x) \to -\frac{1}{8}(x-1)^2$$

1.399 problem 400

Internal problem ID [8736]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 400.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{2y'^2 - 2x^2y' + 3yx = 0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 77

```
dsolve(2*diff(y(x),x)^2-2*x^2*diff(y(x),x)+3*x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{x^3}{6}$$
$$y(x) = \frac{\sqrt{6}\,\sqrt{-c_1x}\,x}{3} + c_1$$
$$y(x) = -\frac{\sqrt{6}\,\sqrt{-c_1x}\,x}{3} + c_1$$
$$y(x) = -\frac{\sqrt{6}\,\sqrt{-c_1x}\,x}{3} + c_1$$
$$y(x) = \frac{\sqrt{6}\,\sqrt{-c_1x}\,x}{3} + c_1$$

✓ Solution by Mathematica

Time used: 2.612 (sec). Leaf size: 213

```
DSolve[3*x*y[x] - 2*x^2*y'[x] + 2*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{3}\left(1-\frac{\sqrt{x^4-6xy(x)}}{\sqrt{x}\sqrt{x^3-6y(x)}}\right)\log(y(x))+\frac{2\sqrt{x^4-6xy(x)}\log\left(x^{3/2}+\sqrt{x^3-6y(x)}\right)}{3\sqrt{x}\sqrt{x^3-6y(x)}}=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{3}\left(\frac{\sqrt{x^4-6xy(x)}}{\sqrt{x}\sqrt{x^3-6y(x)}}+1\right)\log(y(x))-\frac{2\sqrt{x^4-6xy(x)}\log\left(x^{3/2}+\sqrt{x^3-6y(x)}\right)}{3\sqrt{x}\sqrt{x^3-6y(x)}}=c_1,y(x)\right]$$

$$y(x)\to\frac{x^3}{6}$$

1.400 problem 401

Internal problem ID [8737]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 401.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$3y'^2 - 2xy' + y = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 611

```
dsolve(3*diff(y(x),x)^2-2*x*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\left(x^2 + x\left(x^3 + 6\sqrt{3}\sqrt{-c_1(x^3 - 27c_1)} - 54c_1\right)^{\frac{1}{3}} + \left(x^3 + 6\sqrt{3}\sqrt{-c_1(x^3 - 27c_1)} - 54c_1\right)^{\frac{2}{3}}\right)\left(x^2 - 3x\right.}{12\left(x^3 + 6\sqrt{3}\sqrt{-c_1(x^3 - 27\right.}}$$

$$y(x) = -\frac{\left(i\left(x^3 + 6\sqrt{3}\sqrt{-c_1(x^3 - 27c_1)} - 54c_1\right)^{\frac{2}{3}}\sqrt{3} - i\sqrt{3}\,x^2 + \left(x^3 + 6\sqrt{3}\sqrt{-c_1(x^3 - 27c_1)} - 54c_1\right)^{\frac{2}{3}} - 2\right.}{}$$

$$y(x) = -\frac{\left(i\sqrt{3}\,x^2 - i\left(x^3 + 6\sqrt{3}\sqrt{-c_1(x^3 - 27c_1)} - 54c_1\right)^{\frac{2}{3}}\sqrt{3} + x^2 - 2x\left(x^3 + 6\sqrt{3}\sqrt{-c_1(x^3 - 27c_1)} - 54c\right.}{}$$

✓ Solution by Mathematica

Time used: 60.179 (sec). Leaf size: 995

```
DSolve[y[x] - 2*x*y'[x] + 3*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}\left(x^2 + \frac{x(x^3+216e^{3c_1})}{\sqrt[3]{x^6-540e^{3c_1}x^3+24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3}-5832e^{6c_1}}} + \sqrt[3]{x^6-540e^{3c_1}x^3+24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3}-5832e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{24}\left(2x^2 - \frac{i(\sqrt{3}-i)\,x(x^3+216e^{3c_1})}{\sqrt[3]{x^6-540e^{3c_1}x^3+24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3}-5832e^{6c_1}}} + i\left(\sqrt{3}+i\right)\sqrt[3]{x^6-540e^{3c_1}x^3+24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3}-5832e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{24}\left(2x^2 + \frac{i(\sqrt{3}+i)\,x(x^3+216e^{3c_1})}{\sqrt[3]{x^6-540e^{3c_1}x^3+24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3}-5832e^{6c_1}}} - \left(1+i\sqrt{3}\right)\sqrt[3]{x^6-540e^{3c_1}x^3+24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3}-5832e^{6c_1}}\right)$$

$$y(x) \to \frac{x^4 + \left(x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}-8e^{6c_1}\right)^{2/3} + x^2\sqrt[3]{x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}-8}}{12\sqrt[3]{x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}-8e^{6c_1}}}$$

$$y(x) \to \frac{1}{24}\left(2x^2 + \frac{(1+i\sqrt{3})\,x(-x^3+8e^{3c_1})}{\sqrt[3]{x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}-8e^{6c_1}}} + i\left(\sqrt{3}+i\right)\sqrt[3]{x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}-8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{24}\left(2x^2 + \frac{i(\sqrt{3}+i)\,x(x^3-8e^{3c_1})}{\sqrt[3]{x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}-8e^{6c_1}}} - \left(1+i\sqrt{3}\right)\sqrt[3]{x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}-8e^{6c_1}}\right)$$

1.401 problem 402

Internal problem ID [8738]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 402.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$3y'^2 + 4xy' - y = -x^2$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 93

```
dsolve(3*diff(y(x),x)^2+4*x*diff(y(x),x)-y(x)+x^2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{3}$$
$$y(x) = -\frac{x^2}{4} + \frac{\sqrt{3}\, c_1 x}{6} + \frac{c_1^2}{4}$$
$$y(x) = -\frac{x^2}{4} - \frac{\sqrt{3}\, c_1 x}{6} + \frac{c_1^2}{4}$$
$$y(x) = -\frac{x^2}{4} - \frac{\sqrt{3}\, c_1 x}{6} + \frac{c_1^2}{4}$$
$$y(x) = -\frac{x^2}{4} + \frac{\sqrt{3}\, c_1 x}{6} + \frac{c_1^2}{4}$$

✓ Solution by Mathematica

Time used: 3.78 (sec). Leaf size: 121

```
DSolve[x^2 - y[x] + 4*x*y'[x] + 3*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}\left(-3x^2 + 2x - 2e^{c_1}(x+1) + 1 + e^{2c_1}\right)$$

$$y(x) \to \frac{-3x^2 - 3x^2\tanh^2\left(\frac{c_1}{2}\right) + 4x + 2(3x-2)x\tanh\left(\frac{c_1}{2}\right) + 4}{12\left(-1 + \tanh\left(\frac{c_1}{2}\right)\right)^2}$$

$$y(x) \to -\frac{x^2}{3}$$

$$y(x) \to \frac{1}{12}\left(-3x^2 + 2x + 1\right)$$

1.402 problem 403

Internal problem ID [8739]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 403.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{ay'^2 + by' - y = 0}$$

✓ Solution by Maple

Time used: 0.61 (sec). Leaf size: 207

```
dsolve(a*diff(y(x),x)^2+b*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = e^{\frac{-b \,\mathrm{LambertW}\left(\frac{2\,e^{\frac{-c_1-b+x}{b}}}{b\sqrt{\frac{1}{a}}}\right)-b+x-c_1}{b}}\left(b\sqrt{\frac{1}{a}}+e^{\frac{-b \,\mathrm{LambertW}\left(\frac{2\,e^{\frac{-c_1-b+x}{b}}}{b\sqrt{\frac{1}{a}}}\right)-b+x-c_1}{b}}\right)$$

$$y(x) = \frac{b^2\left(\mathrm{LambertW}\left(-\frac{2\sqrt{a}\,e^{\frac{-c_1-b+x}{b}}}{b}\right)+2\right)\mathrm{LambertW}\left(-\frac{2\sqrt{a}\,e^{\frac{-c_1-b+x}{b}}}{b}\right)}{4a}$$

$$y(x) = \frac{b^2\left(\mathrm{LambertW}\left(\frac{2\sqrt{a}\,e^{\frac{-c_1-b+x}{b}}}{b}\right)+2\right)\mathrm{LambertW}\left(\frac{2\sqrt{a}\,e^{\frac{-c_1-b+x}{b}}}{b}\right)}{4a}$$

✓ Solution by Mathematica

Time used: 0.797 (sec). Leaf size: 123

```
DSolve[-y[x] + b*y'[x] + a*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{\sqrt{4\#1a+b^2}+b\log\left(a\left(b-\sqrt{4\#1a+b^2}\right)\right)}{2a}\&\right]\left[\frac{x}{2a}+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\sqrt{4\#1a+b^2}-b\log\left(\sqrt{4\#1a+b^2}+b\right)}{2a}\&\right]\left[-\frac{x}{2a}+c_1\right]$$

$$y(x) \to 0$$

1.403 problem 404

Internal problem ID [8740]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 404.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{a{y'}^2 + b\,x^2 y' + ycx = 0}$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 479

```
dsolve(a*diff(y(x),x)^2+b*x^2*diff(y(x),x)+c*x*y(x) = 0,y(x), singsol=all)
```

$$-2a\left(\int^{y(x)} \frac{1+3\left(b\,x^3+6a\_f-\sqrt{-4\left(-\frac{b^2x^3}{4}+\_fac\right)x}\,x\right)\left(\int_{\_b}^{x} \frac{-\_a^4b^2+2\_a\_fac+\sqrt{\_a\left(\_a^3b^2-4\right.}}{\sqrt{\_a\left(\_a^3b^2-4\_fac\right)}\left(b\_a^3-\sqrt{\_a\left(\_a^3b^2-\right.}\right.}\right.}{b\,x^3+6a\_f-\sqrt{-4\left(-\frac{b^2x^3}{4}+\_fac\right)x}\,x}\right.$$

$$+\int_{\_b}^{x} \frac{-b\_a^2+\sqrt{-4\_a\left(-\frac{\_a^3b^2}{4}+y(x)\,ac\right)}}{b\_a^3+6ay(x)-\sqrt{-4\_a\left(-\frac{\_a^3b^2}{4}+y(x)\,ac\right)}\_a}d\_a+c_1=0$$

$$-2a\left(\int^{y(x)} \frac{1+3\left(b\,x^3+\sqrt{-4\left(-\frac{b^2x^3}{4}+\_fac\right)x}\,x+6a\_f\right)\left(\int_{\_b}^{x} \frac{\_a\left(\_a^3b^2+\sqrt{\_a\left(\_a^3b^2-4\_fac\right)}\_\right.}{\left(b\_a^3+\sqrt{\_a\left(\_a^3b^2-4\_fac\right)}\_a+6a\_f\right)^2\sqrt{}}\right.}{b\,x^3+\sqrt{-4\left(-\frac{b^2x^3}{4}+\_fac\right)x}\,x+6a\_f}\right.$$

$$-\left(\int_{\_b}^{x} \frac{b\_a^2+\sqrt{-4\_a\left(-\frac{\_a^3b^2}{4}+y(x)\,ac\right)}}{b\_a^3+\sqrt{-4\_a\left(-\frac{\_a^3b^2}{4}+y(x)\,ac\right)}\_a+6ay(x)}d\_a\right)+c_1=0$$

✓ Solution by Mathematica

Time used: 4.173 (sec). Leaf size: 313

```
DSolve[c*x*y[x] + b*x^2*y'[x] + a*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{-6b\,\text{arctanh}\left(\frac{bx\sqrt{b^2x^4-4acxy(x)}}{b^2x^3-4acy(x)}\right)+(6b+4c)\text{arctanh}\left(\frac{x^2(3b+2c)}{3\sqrt{b^2x^4-4acxy(x)}}\right)+(3b+2c)\log\left(9ay(x)+3bx^3\right.}{6(3b+c)}\right.$$

$$\left.+\frac{b\log(6by(x)+2cy(x))}{2(3b+c)}=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{6b\,\text{arctanh}\left(\frac{bx\sqrt{b^2x^4-4acxy(x)}}{b^2x^3-4acy(x)}\right)-2(3b+2c)\text{arctanh}\left(\frac{x^2(3b+2c)}{3\sqrt{b^2x^4-4acxy(x)}}\right)+(3b+2c)\log\left(9ay(x)+3bx^3\right.}{6(3b+c)}\right.$$

$$\left.+\frac{b\log(6by(x)+2cy(x))}{2(3b+c)}=c_1,y(x)\right]$$

$$y(x)\to 0$$

1.404 problem 405

Internal problem ID [8741]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 405.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_dAlembert]`

$$a{y'}^2 + y'y = x$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 396

```
dsolve(a*diff(y(x),x)^2+y(x)*diff(y(x),x)-x = 0,y(x), singsol=all)
```

$$\frac{c_1\left(y(x)-\sqrt{4ax+y(x)^2}\right)}{\sqrt{\frac{-y(x)+\sqrt{4ax+y(x)^2}+2a}{a}}\,\sqrt{\frac{-y(x)+\sqrt{4ax+y(x)^2}-2a}{a}}}+x$$
$$+\frac{\left(y(x)-\sqrt{4ax+y(x)^2}\right)\left(3\ln(2)-2\ln\left(\frac{2\sqrt{\frac{y(x)^2-y(x)\sqrt{4ax+y(x)^2}-2a^2+2ax}{a^2}}\,a-\left(y(x)-\sqrt{4ax+y(x)^2}\right)\sqrt{2}}{a}\right)\right)\sqrt{2}}{4\sqrt{\frac{y(x)^2-y(x)\sqrt{4ax+y(x)^2}-2a^2+2ax}{a^2}}}$$
$$=0$$

$$\frac{c_1\left(y(x)+\sqrt{4ax+y(x)^2}\right)}{2\sqrt{\frac{-y(x)-\sqrt{4ax+y(x)^2}+2a}{a}}\,\sqrt{\frac{-y(x)-\sqrt{4ax+y(x)^2}-2a}{a}}}+x$$
$$-\frac{\sqrt{2}\left(y(x)+\sqrt{4ax+y(x)^2}\right)\left(-\frac{3\ln(2)}{2}+\ln\left(\frac{2\sqrt{\frac{y(x)\sqrt{4ax+y(x)^2}-2a^2+2ax+y(x)^2}{a^2}}\,a-\left(y(x)+\sqrt{4ax+y(x)^2}\right)\sqrt{2}}{a}\right)\right)}{2\sqrt{\frac{y(x)\sqrt{4ax+y(x)^2}-2a^2+2ax+y(x)^2}{a^2}}}$$
$$=0$$

✓ Solution by Mathematica

Time used: 0.581 (sec). Leaf size: 79

```
DSolve[-x + y[x]*y'[x] + a*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x=-\frac{2aK[1]\arctan\left(\frac{\sqrt{1-K[1]^2}}{K[1]+1}\right)}{\sqrt{1-K[1]^2}}+\frac{c_1K[1]}{\sqrt{1-K[1]^2}},y(x)=\frac{x}{K[1]}-aK[1]\right\},\{y(x),K[1]\}\right]$$

1.405 problem 406

Internal problem ID [8742]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 406.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_dAlembert]`

$$\boxed{a{y'}^2 - yy' = x}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 269

```
dsolve(a*diff(y(x),x)^2-y(x)*diff(y(x),x)-x = 0,y(x), singsol=all)
```

$$\frac{-\frac{\sqrt{2}\left(y(x)+\sqrt{4ax+y(x)^2}\right)\operatorname{arcsinh}\left(\frac{y(x)+\sqrt{4ax+y(x)^2}}{2a}\right)}{2}+x\sqrt{\frac{y(x)\sqrt{4ax+y(x)^2}+2a^2+2ax+y(x)^2}{a^2}}+c_1y(x)+\sqrt{4ax+y\left(x\right)^2}\,c_1}{\sqrt{\frac{y(x)\sqrt{4ax+y(x)^2}+y(x)^2+2a(a+x)}{a^2}}}$$
$$=0$$
$$\frac{-\frac{\sqrt{2}\left(y(x)-\sqrt{4ax+y(x)^2}\right)\operatorname{arcsinh}\left(\frac{y(x)-\sqrt{4ax+y(x)^2}}{2a}\right)}{2}-\frac{c_1\sqrt{2}\,y(x)}{2}+\frac{c_1\sqrt{2}\,\sqrt{4ax+y(x)^2}}{2}+x\sqrt{\frac{y(x)^2-y(x)\sqrt{4ax+y(x)^2}+2a^2+2ax}{a^2}}}{\sqrt{\frac{-y(x)\sqrt{4ax+y(x)^2}+y(x)^2+2a(a+x)}{a^2}}}$$
$$=0$$

✓ Solution by Mathematica

Time used: 1.393 (sec). Leaf size: 71

```
DSolve[-x - y[x]*y'[x] + a*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x=-\frac{aK[1]\log\left(\sqrt{K[1]^2+1}-K[1]\right)}{\sqrt{K[1]^2+1}}+\frac{c_1K[1]}{\sqrt{K[1]^2+1}},y(x)=aK[1]-\frac{x}{K[1]}\right\},\{y(x),K[1]\}\right]$$

1.406 problem 407

Internal problem ID [8743]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 407.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{xy'^2 - y = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 39

```
dsolve(x*diff(y(x),x)^2-y(x) = 0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{\left(x + \sqrt{c_1 x}\right)^2}{x}$$
$$y(x) = \frac{\left(-x + \sqrt{c_1 x}\right)^2}{x}$$

✓ Solution by Mathematica
Time used: 0.05 (sec). Leaf size: 46

```
DSolve[-y[x] + x*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-2\sqrt{x} + c_1\right)^2$$
$$y(x) \to \frac{1}{4}\left(2\sqrt{x} + c_1\right)^2$$
$$y(x) \to 0$$

1.407 problem 408

Internal problem ID [8744]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 408.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$xy'^2 - 2y = -x$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 96

```
dsolve(x*diff(y(x),x)^2-2*y(x)+x = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(2\,\mathrm{LambertW}\left(\frac{\sqrt{c_1 x}}{c_1}\right)^2 + 2\,\mathrm{LambertW}\left(\frac{\sqrt{c_1 x}}{c_1}\right) + 1\right) x}{2\,\mathrm{LambertW}\left(\frac{\sqrt{c_1 x}}{c_1}\right)^2}$$
$$y(x) = \frac{\left(2\,\mathrm{LambertW}\left(-\frac{\sqrt{c_1 x}}{c_1}\right)^2 + 2\,\mathrm{LambertW}\left(-\frac{\sqrt{c_1 x}}{c_1}\right) + 1\right) x}{2\,\mathrm{LambertW}\left(-\frac{\sqrt{c_1 x}}{c_1}\right)^2}$$

✓ Solution by Mathematica

Time used: 0.626 (sec). Leaf size: 97

```
DSolve[x - 2*y[x] + x*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\frac{2}{\sqrt{\frac{2y(x)}{x}-1}-1} - 2\log\left(\sqrt{\frac{2y(x)}{x}-1}-1\right) = \log(x) + c_1, y(x)\right]$$
$$\mathrm{Solve}\left[\frac{2}{\sqrt{\frac{2y(x)}{x}-1}+1} + 2\log\left(\sqrt{\frac{2y(x)}{x}-1}+1\right) = -\log(x) + c_1, y(x)\right]$$

1.408 problem 409

Internal problem ID [8745]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 409.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, _dAlembert]`

$$xy'^2 - 2y' - y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 65

```
dsolve(x*diff(y(x),x)^2-2*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$\begin{aligned} y(x) = {} & 2x\,\mathrm{e}^{\mathrm{RootOf}\left(-x\,\mathrm{e}^{2\_Z}+2x\,\mathrm{e}^{\_Z}+2\,\mathrm{e}^{\_Z}+c_1-2\_Z-x\right)} \\ & -2\,\mathrm{RootOf}\left(-x\,\mathrm{e}^{2\_Z}+2x\,\mathrm{e}^{\_Z}+2\,\mathrm{e}^{\_Z}+c_1-2\_Z-x\right)+c_1-x \end{aligned}$$

✓ Solution by Mathematica

Time used: 1.441 (sec). Leaf size: 50

```
DSolve[-y[x] - 2*y'[x] + x*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\left\{x=\frac{2K[1]-2\log(K[1])}{(K[1]-1)^2}+\frac{c_1}{(K[1]-1)^2},y(x)=xK[1]^2-2K[1]\right\},\{y(x),K[1]\}\right]$$

1.409 problem 410

Internal problem ID [8746]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 410.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, _dAlembert]`

$$x{y'}^2 + 4y' - 2y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 67

```
dsolve(x*diff(y(x),x)^2+4*diff(y(x),x)-2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = 2x\,\mathrm{e}^{\mathrm{RootOf}\left(-x\,\mathrm{e}^{2\_Z}+4x\,\mathrm{e}^{\_Z}-4\,\mathrm{e}^{\_Z}+c_1+8\_Z-4x\right)} + 4\,\mathrm{RootOf}\left(-x\,\mathrm{e}^{2\_Z}+4x\,\mathrm{e}^{\_Z}-4\,\mathrm{e}^{\_Z}+c_1+8\_Z-4x\right) + \frac{c_1}{2} - 2x$$

✓ Solution by Mathematica

Time used: 30.799 (sec). Leaf size: 90

```
DSolve[-2*y[x] + 4*y'[x] + x*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\left\{x = -\frac{2(2K[1]-y(K[1]))}{K[1]^2}, y(x) = 4\left(\frac{2}{K[1]}+\log(K[1])\right)\exp\left(-4\left(\frac{1}{2}\log(2-K[1])-\frac{1}{2}\log(K[1])\right)\right) + c_1\exp\left(-4\left(\frac{1}{2}\log(2-K[1])-\frac{1}{2}\log(K[1])\right)\right)\right\}, \{y(x), K[1]\}\right]$$

1.410 problem 411

Internal problem ID [8747]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 411.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{xy'^2 + xy' - y = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 65

```
dsolve(x*diff(y(x),x)^2+x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(1 + 2\,\text{LambertW}\left(-\frac{1}{2\sqrt{\frac{c_1}{x}}}\right)\right)x}{4\,\text{LambertW}\left(-\frac{1}{2\sqrt{\frac{c_1}{x}}}\right)^2}$$
$$y(x) = \frac{\left(1 + 2\,\text{LambertW}\left(\frac{1}{2\sqrt{\frac{c_1}{x}}}\right)\right)x}{4\,\text{LambertW}\left(\frac{1}{2\sqrt{\frac{c_1}{x}}}\right)^2}$$

✓ Solution by Mathematica

Time used: 0.588 (sec). Leaf size: 102

```
DSolve[-y[x] + x*y'[x] + x*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{\sqrt{\frac{4y(x)}{x}+1}-1}-\log\left(\sqrt{\frac{4y(x)}{x}+1}-1\right)=\frac{\log(x)}{2}+c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{\sqrt{\frac{4y(x)}{x}+1}+1}+\log\left(\sqrt{\frac{4y(x)}{x}+1}+1\right)=-\frac{\log(x)}{2}+c_1,y(x)\right]$$

$$y(x)\to 0$$

1.411 problem 412

Internal problem ID [8748]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 412.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _dAlembert]`

$$\boxed{xy'^2 + yy' = -a}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 177

```
dsolve(x*diff(y(x),x)^2+y(x)*diff(y(x),x)+a = 0,y(x), singsol=all)
```

$$-\frac{8\left(-\frac{3c_1\left(y(x)-\sqrt{-4ax+y(x)^2}\right)\sqrt{\frac{-y(x)+\sqrt{-4ax+y(x)^2}}{x}}}{8}+ax-\frac{3y(x)^2}{4}+\frac{3y(x)\sqrt{-4ax+y(x)^2}}{4}\right)x}{3\left(y\left(x\right)-\sqrt{-4ax+y\left(x\right)^2}\right)^2}=0$$

$$-\frac{8\left(\frac{3c_1\left(y(x)+\sqrt{-4ax+y(x)^2}\right)\sqrt{\frac{-2y(x)-2\sqrt{-4ax+y(x)^2}}{x}}}{4}+ax-\frac{3y(x)^2}{4}-\frac{3y(x)\sqrt{-4ax+y(x)^2}}{4}\right)x}{3\left(y\left(x\right)+\sqrt{-4ax+y\left(x\right)^2}\right)^2}=0$$

✓ Solution by Mathematica

Time used: 60.298 (sec). Leaf size: 4845

```
DSolve[a + y[x]*y'[x] + x*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

1.412 problem 413

Internal problem ID [8749]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 413.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G']]`

$$\boxed{xy'^2 + yy' = x^2}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 272

```
dsolve(x*diff(y(x),x)^2+y(x)*diff(y(x),x)-x^2 = 0,y(x), singsol=all)
```

$$-\left(\int_{\_b}^{x}\frac{y(x)-\sqrt{4\_a^{3}+y\left(x\right)^{2}}}{\_a\left(4y\left(x\right)-\sqrt{4\_a^{3}+y\left(x\right)^{2}}\right)}d\_a\right)+2\left(\int^{y(x)}\frac{-1+\left(24\_f-6\sqrt{4x^{3}+\_f^{2}}\right)\left(\int_{\_b}^{x}\frac{\_a^{2}}{\left(-4\_f+\sqrt{4\_a^{3}+\_f^{2}}\right)^{2}\sqrt{4\_a^{3}+\_f^{2}}}d\_a\right)}{4\_f-\sqrt{4x^{3}+\_f^{2}}}d\_f\right)+c_{1}=0$$

$$-\left(\int_{\_b}^{x}\frac{y(x)+\sqrt{4\_a^{3}+y\left(x\right)^{2}}}{\left(\sqrt{4\_a^{3}+y\left(x\right)^{2}}+4y\left(x\right)\right)\_a}d\_a\right)-2\left(\int^{y(x)}\frac{1+6\left(\sqrt{4x^{3}+\_f^{2}}+4\_f\right)\left(\int_{\_b}^{x}\frac{\_a^{2}}{\left(\sqrt{4\_a^{3}+\_f^{2}}+4\_f\right)^{2}\sqrt{4\_a^{3}+\_f^{2}}}d\_a\right)}{\sqrt{4x^{3}+\_f^{2}}+4\_f}d\_f\right)+c_{1}=0$$

✓ Solution by Mathematica

Time used: 0.267 (sec). Leaf size: 673

```
DSolve[-x^2 + y[x]*y'[x] + x*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int\left(\frac{4\sqrt{4x^3+y(x)^2}x^2}{5y(x)\left(4x^3-15y(x)^2\right)}+\frac{16x^2}{5\left(4x^3-15y(x)^2\right)}-\frac{\sqrt{4x^3+y(x)^2}}{5y(x)x}+\frac{1}{5x}\right)dx\right.$$
$$+\int\left(\frac{8y(x)}{15y(x)^2-4x^3}\right.$$
$$-\int\left(-\frac{4\sqrt{4x^3+y(x)^2}x^2}{5y(x)^2\left(4x^3-15y(x)^2\right)}+\frac{24\sqrt{4x^3+y(x)^2}x^2}{\left(4x^3-15y(x)^2\right)^2}+\frac{4x^2}{5\left(4x^3-15y(x)^2\right)\sqrt{4x^3+y(x)^2}}+\frac{96y(x)x^2}{\left(4x^3-15y(x)^2\right)}\right.$$
$$\left.+\frac{2\sqrt{4x^3+y(x)^2}}{15y(x)^2-4x^3}\right)dy(x)=c_1,y(x)\Bigg]$$

$$\text{Solve}\left[\int\left(-\frac{4\sqrt{4x^3+y(x)^2}x^2}{5y(x)\left(4x^3-15y(x)^2\right)}+\frac{16x^2}{5\left(4x^3-15y(x)^2\right)}+\frac{\sqrt{4x^3+y(x)^2}}{5y(x)x}+\frac{1}{5x}\right)dx\right.$$
$$+\int\left(\frac{8y(x)}{15y(x)^2-4x^3}\right.$$
$$-\int\left(\frac{4\sqrt{4x^3+y(x)^2}x^2}{5y(x)^2\left(4x^3-15y(x)^2\right)}-\frac{24\sqrt{4x^3+y(x)^2}x^2}{\left(4x^3-15y(x)^2\right)^2}-\frac{4x^2}{5\left(4x^3-15y(x)^2\right)\sqrt{4x^3+y(x)^2}}+\frac{96y(x)x^2}{\left(4x^3-15y(x)^2\right)^2}\right.$$
$$\left.-\frac{2\sqrt{4x^3+y(x)^2}}{15y(x)^2-4x^3}\right)dy(x)=c_1,y(x)\Bigg]$$

1.413 problem 414

Internal problem ID [8750]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 414.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{xy'^2 + yy' = -x^3}$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 272

```
dsolve(x*diff(y(x),x)^2+y(x)*diff(y(x),x)+x^3 = 0,y(x), singsol=all)
```

$$-\left(\int_{\_b}^{x}\frac{y(x)-\sqrt{-4\_a^4+y(x)^2}}{\_a\left(5y(x)-\sqrt{-4\_a^4+y(x)^2}\right)}d\_a\right)$$
$$-2\left(\int^{y(x)}\frac{1+\left(40\_f-8\sqrt{-4x^4+\_f^2}\right)\left(\int_{\_b}^{x}\frac{\_a^3}{\left(-5\_f+\sqrt{-4\_a^4+\_f^2}\right)^2\sqrt{-4\_a^4+\_f^2}}d\_a\right)}{5\_f-\sqrt{-4x^4+\_f^2}}d\_f\right)$$
$$+c_1=0$$
$$-\left(\int_{\_b}^{x}\frac{y(x)+\sqrt{-4\_a^4+y(x)^2}}{\left(\sqrt{-4\_a^4+y(x)^2}+5y(x)\right)\_a}d\_a\right)$$
$$+2\left(\int^{y(x)}\frac{-1+8\left(\sqrt{-4x^4+\_f^2}+5\_f\right)\left(\int_{\_b}^{x}\frac{\_a^3}{\left(\sqrt{-4\_a^4+\_f^2}+5\_f\right)^2\sqrt{-4\_a^4+\_f^2}}d\_a\right)}{\sqrt{-4x^4+\_f^2}+5\_f}d\_f\right)$$
$$+c_1=0$$

✓ Solution by Mathematica

Time used: 0.71 (sec). Leaf size: 107

```
DSolve[x^3 + y[x]*y'[x] + x*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x^2\text{InverseFunction}\left[\int_1^{\#1}\frac{1}{5K[2]+\sqrt{K[2]^2-4}}dK[2]\&\right]\left[\int_1^x-\frac{1}{2K[3]}dK[3]+c_1\right]$$
$$y(x)\to x^2\text{InverseFunction}\left[\int_1^{\#1}\frac{1}{\sqrt{K[4]^2-4}-5K[4]}dK[4]\&\right]\left[\int_1^x\frac{1}{2K[5]}dK[5]+c_1\right]$$

1.414 problem 415

Internal problem ID [8751]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 415.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$x{y'}^2 + yy' - y^4 = 0$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 89

```
dsolve(x*diff(y(x),x)^2+y(x)*diff(y(x),x)-y(x)^4 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{2\sqrt{-x}}$$
$$y(x) = \frac{1}{2\sqrt{-x}}$$
$$y(x) = 0$$
$$y(x) = -\frac{\coth\left(-\frac{\ln(x)}{2} + \frac{c_1}{2}\right)\sqrt{\operatorname{sech}\left(-\frac{\ln(x)}{2} + \frac{c_1}{2}\right)^2 x}}{2x}$$
$$y(x) = \frac{\coth\left(-\frac{\ln(x)}{2} + \frac{c_1}{2}\right)\sqrt{\operatorname{sech}\left(-\frac{\ln(x)}{2} + \frac{c_1}{2}\right)^2 x}}{2x}$$

✓ Solution by Mathematica

Time used: 0.544 (sec). Leaf size: 84

```
DSolve[-y[x]^4 + y[x]*y'[x] + x*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{2e^{\frac{c_1}{2}}}{-4x+e^{c_1}}$$
$$y(x)\to \frac{2e^{\frac{c_1}{2}}}{-4x+e^{c_1}}$$
$$y(x)\to 0$$
$$y(x)\to -\frac{i}{2\sqrt{x}}$$
$$y(x)\to \frac{i}{2\sqrt{x}}$$

1.415 problem 416

Internal problem ID [8752]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 416.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$
\boxed{xy'^2 + (y - 3x)\, y' + y = 0}
$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 139

```
dsolve(x*diff(y(x),x)^2+(y(x)-3*x)*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$
y(x) = x
$$

$$
\frac{c_1\left(-5x + y(x) - \sqrt{9x^2 - 10xy(x) + y(x)^2}\right)}{x\left(\frac{3x - y(x) + \sqrt{9x^2 - 10xy(x) + y(x)^2}}{x}\right)^{\frac{3}{2}}} + x = 0
$$

$$
\frac{\left(-5x + y(x) + \sqrt{9x^2 - 10xy(x) + y(x)^2}\right) c_1 \sqrt{2}}{4x\left(\frac{-y(x) + 3x - \sqrt{9x^2 - 10xy(x) + y(x)^2}}{x}\right)^{\frac{3}{2}}} + x = 0
$$

✓ Solution by Mathematica

Time used: 60.334 (sec). Leaf size: 1225

```
DSolve[y[x] + (-3*x + y[x])*y'[x] + x*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{384}\left(\frac{\frac{4e^{8c_1}}{x^2}-6912e^{4c_1}}{\sqrt[3]{-\frac{-373248e^{4c_1}x^4+4320e^{8c_1}x^2-48\sqrt{6}x\sqrt{e^{8c_1}\left(216x^2+e^{4c_1}\right)^3}+e^{12c_1}}{x^3}}} + 4\sqrt[3]{\frac{373248e^{4c_1}x^4-4320e^{8c_1}x^2+48\sqrt{6}x\sqrt{e^{8c_1}\left(216x^2+e^{4c_1}\right)^3}-e^{12c_1}}{x^3}} - \frac{4e^{4c_1}}{x}\right)$$

$$y(x) \to \frac{1}{768}\left(\frac{\left(1+i\sqrt{3}\right)\left(6912e^{4c_1}-\frac{4e^{8c_1}}{x^2}\right)}{\sqrt[3]{-\frac{-373248e^{4c_1}x^4+4320e^{8c_1}x^2-48\sqrt{6}x\sqrt{e^{8c_1}\left(216x^2+e^{4c_1}\right)^3}+e^{12c_1}}{x^3}}} + 4i\left(\sqrt{3}+i\right)\sqrt[3]{-\frac{-373248e^{4c_1}x^4+4320e^{8c_1}x^2-48\sqrt{6}x\sqrt{e^{8c_1}\left(216x^2+e^{4c_1}\right)^3}+e^{12c_1}}{x^3}} - \frac{8e^{4c_1}}{x}\right)$$

$$y(x) \to \frac{1}{768}\left(\frac{\left(1-i\sqrt{3}\right)\left(6912e^{4c_1}-\frac{4e^{8c_1}}{x^2}\right)}{\sqrt[3]{-\frac{-373248e^{4c_1}x^4+4320e^{8c_1}x^2-48\sqrt{6}x\sqrt{e^{8c_1}\left(216x^2+e^{4c_1}\right)^3}+e^{12c_1}}{x^3}}} - 4\left(1+i\sqrt{3}\right)\sqrt[3]{-\frac{-373248e^{4c_1}x^4+4320e^{8c_1}x^2-48\sqrt{6}x\sqrt{e^{8c_1}\left(216x^2+e^{4c_1}\right)^3}+e^{12c_1}}{x^3}} - \frac{8e^{4c_1}}{x}\right)$$

1.416 problem 417

Internal problem ID [8753]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 417.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Clairaut]`

$$\boxed{xy'^2 - yy' = -a}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 35

```
dsolve(x*diff(y(x),x)^2-y(x)*diff(y(x),x)+a = 0,y(x), singsol=all)
```

$$y(x) = -2\sqrt{ax}$$
$$y(x) = 2\sqrt{ax}$$
$$y(x) = \frac{x\,c_1^2 + a}{c_1}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 53

```
DSolve[a - y[x]*y'[x] + x*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{a}{c_1} + c_1 x$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to -2\sqrt{a}\sqrt{x}$$
$$y(x) \to 2\sqrt{a}\sqrt{x}$$

1.417 problem 418

Internal problem ID [8754]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 418.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{xy'^2 - yy' + ay = 0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 42

```
dsolve(x*diff(y(x),x)^2-y(x)*diff(y(x),x)+a*y(x) = 0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = -\frac{\left(\operatorname{LambertW}\left(-\frac{x\,\mathrm{e}}{c_1 a}\right) - 1\right)^2 a x}{\operatorname{LambertW}\left(-\frac{x\,\mathrm{e}}{c_1 a}\right)}$$

✓ Solution by Mathematica

Time used: 2.88 (sec). Leaf size: 173

```
DSolve[a*y[x] - y[x]*y'[x] + x*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{-\sqrt{\frac{y(x)}{x}}\sqrt{\frac{y(x)}{x}-4a}-4a\log\left(\sqrt{\frac{y(x)}{x}-4a}-\sqrt{\frac{y(x)}{x}}\right)+\frac{y(x)}{x}}{4a}=-\frac{\log(x)}{2}+c_1,y(x)\right]$$

$$\text{Solve}\left[-\frac{\sqrt{\frac{y(x)}{x}}\sqrt{\frac{y(x)}{x}-4a}+4a\log\left(\sqrt{\frac{y(x)}{x}-4a}-\sqrt{\frac{y(x)}{x}}\right)+\frac{y(x)}{x}}{4a}=\frac{\log(x)}{2}+c_1,y(x)\right]$$

$$y(x)\to 0$$

1.418 problem 419

Internal problem ID [8755]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 419.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{xy'^2 + 2yy' = x}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 111

```
dsolve(x*diff(y(x),x)^2+2*y(x)*diff(y(x),x)-x = 0,y(x), singsol=all)
```

$$x + \frac{\left(y(x) - \sqrt{y(x)^2 + x^2}\right) 2^{\frac{1}{3}} c_1}{2 \left(\frac{3y(x)^2 - 3y(x)\sqrt{y(x)^2+x^2} + x^2}{x^2}\right)^{\frac{2}{3}} x} = 0$$

$$\frac{c_1 \left(\sqrt{y(x)^2 + x^2} + y(x)\right)}{x \left(\frac{3y(x)\sqrt{y(x)^2+x^2} + x^2 + 3y(x)^2}{x^2}\right)^{\frac{2}{3}}} + x = 0$$

✓ Solution by Mathematica

Time used: 60.68 (sec). Leaf size: 6977

```
DSolve[-x + 2*y[x]*y'[x] + x*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

1.419 problem 420

Internal problem ID [8756]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 420.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _dAlembert]`

$$\boxed{xy'^2 - 2yy' = -a}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 796

```
dsolve(x*diff(y(x),x)^2-2*y(x)*diff(y(x),x)+a = 0,y(x), singsol=all)
```

$$y(x) = \frac{x\left(\frac{4x^2}{\left(-36a\,c_1^2+8x^3+12\sqrt{a\left(9a\,c_1^2-4x^3\right)}\,c_1\right)^{\frac{1}{3}}}+2x+\left(-36a\,c_1^2+8x^3+12\sqrt{a\left(9a\,c_1^2-4x^3\right)}\,c_1\right)^{\frac{1}{3}}\right)}{12c_1} + \frac{3ac_1\left(-36a\,c_1^2+8x^3+12\sqrt{a\left(9a\,c_1^2-4x^3\right)}\,c_1\right)^{\frac{1}{3}}}{\left(-36a\,c_1^2+8x^3+12\sqrt{a\left(9a\,c_1^2-4x^3\right)}\,c_1\right)^{\frac{2}{3}}+2x\left(-36a\,c_1^2+8x^3+12\sqrt{a\left(9a\,c_1^2-4x^3\right)}\,c_1\right)^{\frac{1}{3}}+4x^2}$$

$$y(x) = -\frac{\left(\left(1+i\sqrt{3}\right)\left(-36a\,c_1^2+8x^3+12\sqrt{a\left(9a\,c_1^2-4x^3\right)}\,c_1\right)^{\frac{2}{3}}-4x\left(-36a\,c_1^2+8x^3+12\sqrt{a\left(9a\,c_1^2-4x^3\right)}\,c_1\right.}{24\left(-36a\,c_1^2+8x^3+12\sqrt{a\left(9a\,c_1^2-4x^3\right)}\,c_1\right)^{\frac{1}{3}}c_1} + \frac{6ac_1\left(-36a\,c_1^2+8x^3+12\sqrt{a\left(9a\,c_1^2-4x^3\right)}\,c_1\right.}{4i\sqrt{3}\,x^2-i\sqrt{3}\left(-36a\,c_1^2+8x^3+12\sqrt{a\left(9a\,c_1^2-4x^3\right)}\,c_1\right)^{\frac{2}{3}}-4x^2+4x\left(-36a\,c_1^2+8x^3+12\sqrt{a\left(9a\,c_1^2\right.}\right.}$$

$$y(x) = \frac{x\left(\left(i\sqrt{3}-1\right)\left(-36a\,c_1^2+8x^3+12\sqrt{a\left(9a\,c_1^2-4x^3\right)}\,c_1\right)^{\frac{2}{3}}+4x\left(-36a\,c_1^2+8x^3+12\sqrt{a\left(9a\,c_1^2-4x^3\right)}\,c_1\right)\right.}{24\left(-36a\,c_1^2+8x^3+12\sqrt{a\left(9a\,c_1^2-4x^3\right)}\,c_1\right)^{\frac{1}{3}}c_1} - \frac{6ac_1\left(-36a\,c_1^2+8x^3+12\sqrt{a\left(9a\,c_1^2-4x^3\right)}\,c_1\right.}{4i\sqrt{3}\,x^2-i\sqrt{3}\left(-36a\,c_1^2+8x^3+12\sqrt{a\left(9a\,c_1^2-4x^3\right)}\,c_1\right)^{\frac{2}{3}}+4x^2-4x\left(-36a\,c_1^2+8x^3+12\sqrt{a\left(9a\,c_1^2\right.}\right.}$$

✓ Solution by Mathematica

Time used: 60.166 (sec). Leaf size: 1553

```
DSolve[a - 2*y[x]*y'[x] + x*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{3c_1}{2}}\left(a^4x^4+\left(-a^6x^6+20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3+e^{3c_1}\right)^3}+8e^{6c_1}\right)^{2/3}-a^2x^2\sqrt[3]{-a^6x^6+20a^3e^{3c_1}x\cdots}\right.}{4\sqrt[3]{-a^6x^6+20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3+e^{3c_1}\right)^3}+8e^{6c\cdots}}}$$

$$y(x) \to \frac{ie^{-\frac{3c_1}{2}}\left(-\left(\left(\sqrt{3}-i\right)a^4x^4\right)+\left(\sqrt{3}+i\right)\left(-a^6x^6+20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3+e^{3c_1}\right)^3}+8e^{6c_1}\right)^{2/3}+2\cdots\right.}{8\sqrt[3]{-a^6x^6+20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3\cdots\right.}}}$$

$$y(x) \to \frac{e^{-\frac{3c_1}{2}}\left(i\left(\sqrt{3}+i\right)a^4x^4-i\left(\sqrt{3}-i\right)\left(-a^6x^6+20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3+e^{3c_1}\right)^3}+8e^{6c_1}\right)^{2/3}-2a^2x\cdots\right.}{8\sqrt[3]{-a^6x^6+20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x\cdots\right.}}}$$

$$y(x) \to \frac{e^{-\frac{3c_1}{2}}\left(a^4x^4+\left(a^6x^6-20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3+e^{3c_1}\right)^3}-8e^{6c_1}\right)^{2/3}+a^2x^2\sqrt[3]{a^6x^6-20a^3e^{3c_1}x^3+\cdots}\right.}{4\sqrt[3]{a^6x^6-20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3+e^{3c_1}\right)^3}-8e^{6c_1}}}$$

$$y(x) \to \frac{e^{-\frac{3c_1}{2}}\left(\left(-1-i\sqrt{3}\right)a^4x^4+i\left(\sqrt{3}+i\right)\left(a^6x^6-20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3+e^{3c_1}\right)^3}-8e^{6c_1}\right)^{2/3}+2a^2x\cdots\right.}{8\sqrt[3]{a^6x^6-20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3\cdots\right.}}}$$

$$y(x) \to \frac{e^{-\frac{3c_1}{2}}\left(i\left(\sqrt{3}+i\right)a^4x^4-i\left(\sqrt{3}-i\right)\left(a^6x^6-20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3+e^{3c_1}\right)^3}-8e^{6c_1}\right)^{2/3}+2a^2x^2\sqrt[3]{\cdots}\right.}{8\sqrt[3]{a^6x^6-20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3+\cdots\right.}}}$$

1.420 problem 421

Internal problem ID [8757]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 421.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{xy'^2 - 2yy' = x}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 32

```
dsolve(x*diff(y(x),x)^2-2*y(x)*diff(y(x),x)-x = 0,y(x), singsol=all)
```

$$y(x) = -ix$$
$$y(x) = ix$$
$$y(x) = \frac{-c_1^2 + x^2}{2c_1}$$

✓ Solution by Mathematica

Time used: 0.145 (sec). Leaf size: 71

```
DSolve[-x - 2*y[x]*y'[x] + x*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-c_1}\left(-x^2 + e^{2c_1}\right)$$
$$y(x) \to \frac{1}{2}e^{-c_1}\left(-1 + e^{2c_1}x^2\right)$$
$$y(x) \to -ix$$
$$y(x) \to ix$$

1.421 problem 422

Internal problem ID [8758]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 422.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{xy'^2 - 2yy' = -4x}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 30

```
dsolve(x*diff(y(x),x)^2-2*y(x)*diff(y(x),x)+4*x = 0,y(x), singsol=all)
```

$$y(x) = -2x$$
$$y(x) = 2x$$
$$y(x) = \frac{4c_1^2 + x^2}{2c_1}$$

✓ Solution by Mathematica
Time used: 0.279 (sec). Leaf size: 43

```
DSolve[4*x - 2*y[x]*y'[x] + x*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2x\cosh(-\log(x) + c_1)$$
$$y(x) \to -2x\cosh(\log(x) + c_1)$$
$$y(x) \to -2x$$
$$y(x) \to 2x$$

1.422 problem 423

Internal problem ID [8759]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 423.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{xy'^2 - 2yy' + 2y = -x}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 44

```
dsolve(x*diff(y(x),x)^2-2*y(x)*diff(y(x),x)+2*y(x)+x = 0,y(x), singsol=all)
```

$$y(x) = \left(1-\sqrt{2}\right)x$$
$$y(x) = \left(1+\sqrt{2}\right)x$$
$$y(x) = \frac{2c_1^2 + 2c_1x + x^2}{2c_1}$$

✓ Solution by Mathematica

Time used: 0.245 (sec). Leaf size: 78

```
DSolve[x + 2*y[x] - 2*y[x]*y'[x] + x*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}e^{-c_1}x^2 + x - e^{c_1}$$
$$y(x) \to -e^{c_1}x^2 + x - \frac{e^{-c_1}}{2}$$
$$y(x) \to x - \sqrt{2}x$$
$$y(x) \to \left(1+\sqrt{2}\right)x$$

1.423 problem 424

Internal problem ID [8760]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 424.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{xy'^2 + ayy' = -xb}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 217

```
dsolve(x*diff(y(x),x)^2+a*y(x)*diff(y(x),x)+b*x = 0,y(x), singsol=all)
```

$$\frac{-c_1 2^{\frac{a+2}{2a+2}} \left(ay(x) - \sqrt{a^2 y(x)^2 - 4b\,x^2}\right) \left(\frac{\left(-y(x)(a+1)\sqrt{a^2 y(x)^2 - 4b\,x^2} + (a^2+a)\,y(x)^2 - 2b\,x^2\right)a}{x^2}\right)^{\frac{-a-2}{2a+2}} + x^2}{x} = 0$$

$$\frac{c_1 2^{\frac{a+2}{2a+2}} \left(ay(x) + \sqrt{a^2 y(x)^2 - 4b\,x^2}\right) \left(\frac{a\left(y(x)(a+1)\sqrt{a^2 y(x)^2 - 4b\,x^2} + (a^2+a)\,y(x)^2 - 2b\,x^2\right)}{x^2}\right)^{\frac{-a-2}{2a+2}} + x^2}{x} = 0$$

✓ Solution by Mathematica

Time used: 2.076 (sec). Leaf size: 423

```
DSolve[b*x + a*y[x]*y'[x] + x*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{i\left(2\log\left(-i\sqrt{4b-\frac{a^2y(x)^2}{x^2}}+\frac{ay(x)}{x}+2i\sqrt{b}\right)+2(a+1)\log\left(i\sqrt{4b-\frac{a^2y(x)^2}{x^2}}+\frac{ay(x)}{x}-2i\sqrt{b}\right)-(a\right.}{4(a+1)}\right.$$

$$\left.-\frac{1}{2}i\log(x),y(x)\right]$$

$$\text{Solve}\left[\frac{i\left(2(a+1)\log\left(-i\sqrt{4b-\frac{a^2y(x)^2}{x^2}}+\frac{ay(x)}{x}+2i\sqrt{b}\right)+2\log\left(i\sqrt{4b-\frac{a^2y(x)^2}{x^2}}+\frac{ay(x)}{x}-2i\sqrt{b}\right)-(a+\right.}{4(a+1)}\right.$$

$$\left.+c_1,y(x)\right]$$

1.424 problem 425

Internal problem ID [8761]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 425.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _dAlembert]`

$$\boxed{(1+x)\,y'^2-(x+y)\,y'+y=0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 45

```
dsolve((x+1)*diff(y(x),x)^2-(x+y(x))*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x)=x+2-2\sqrt{x+1}$$
$$y(x)=x+2+2\sqrt{x+1}$$
$$y(x)=\frac{c_1(c_1x+c_1-x)}{c_1-1}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 51

```
DSolve[y[x] - (x + y[x])*y'[x] + (1 + x)*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\left(x+\frac{c_1}{-1+c_1}\right)$$
$$y(x)\to x-2\sqrt{x+1}+2$$
$$y(x)\to x+2\sqrt{x+1}+2$$

1.425 problem 426

Internal problem ID [8762]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 426.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$(3x+1)\, {y'}^{2}-3(y+2)\, y'=-9$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 51

```
dsolve((3*x+1)*diff(y(x),x)^2-3*(y(x)+2)*diff(y(x),x)+9 = 0,y(x), singsol=all)
```

$$y(x) = -2-2\sqrt{3x+1}$$
$$y(x) = -2+2\sqrt{3x+1}$$
$$y(x) = \frac{9+(3x+1)\, c_1^2-6c_1}{3c_1}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 60

```
DSolve[9 - 3*(2 + y[x])*y'[x] + (1 + 3*x)*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to c_1\left(x+\frac{1}{3}\right)-2+\frac{3}{c_1}$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to -2\left(\sqrt{3x+1}+1\right)$$
$$y(x) \to 2\left(\sqrt{3x+1}-1\right)$$

1.426 problem 427

Internal problem ID [8763]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 427.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _dAlembert]`

$$(3x+5)\, y'^2 - (x+3y)\, y' + y = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 60

```
dsolve((3*x+5)*diff(y(x),x)^2-(3*y(x)+x)*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{x}{3} + \frac{10}{9} - \frac{2\sqrt{15x+25}}{9}$$
$$y(x) = \frac{x}{3} + \frac{10}{9} + \frac{2\sqrt{15x+25}}{9}$$
$$y(x) = \frac{(3x+5)\, c_1^2 - c_1 x}{3c_1 - 1}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 80

```
DSolve[y[x] - (x + 3*y[x])*y'[x] + (5 + 3*x)*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to c_1\left(x + \frac{5c_1}{-1+3c_1}\right)$$
$$y(x) \to \frac{1}{9}\left(3x - 2\sqrt{5}\sqrt{3x+5} + 10\right)$$
$$y(x) \to \frac{1}{9}\left(3x + 2\sqrt{5}\sqrt{3x+5} + 10\right)$$

1.427 problem 428

Internal problem ID [8764]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 428.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _dAlembert]`

$$\boxed{axy'^2 + (xb - ay + c)\, y' - by = 0}$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 66

```
dsolve(a*x*diff(y(x),x)^2+(b*x-a*y(x)+c)*diff(y(x),x)-b*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{-bx + c - 2\sqrt{-bcx}}{a}$$
$$y(x) = \frac{-bx + c + 2\sqrt{-bcx}}{a}$$
$$y(x) = \frac{c_1(ac_1x + bx + c)}{ac_1 + b}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 80

```
DSolve[-(b*y[x]) + (c + b*x - a*y[x])*y'[x] + a*x*y'[x]^2==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to c_1\left(x + \frac{c}{b + ac_1}\right)$$
$$y(x) \to \frac{\left(\sqrt{c} - i\sqrt{b}\sqrt{x}\right)^2}{a}$$
$$y(x) \to \frac{\left(\sqrt{c} + i\sqrt{b}\sqrt{x}\right)^2}{a}$$

1.428 problem 429

Internal problem ID [8765]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 429.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _dAlembert]`

$$a x {y'}^2 - (a y + x b - a - b) y' + b y = 0$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 72

```
dsolve(a*x*diff(y(x),x)^2-(a*y(x)+b*x-a-b)*diff(y(x),x)+b*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{bx + a + b - 2\sqrt{bx\,(a+b)}}{a}$$
$$y(x) = \frac{bx + a + b + 2\sqrt{bx\,(a+b)}}{a}$$
$$y(x) = \frac{c_1(a c_1 x - bx + a + b)}{a c_1 - b}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 90

```
DSolve[b*y[x] - (-a - b + b*x + a*y[x])*y'[x] + a*x*y'[x]^2==0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to c_1\left(x + \frac{a+b}{-b + a c_1}\right)$$
$$y(x) \to \frac{-2\sqrt{a^2 b x (a+b)} + a^2 + ab(x+1)}{a^2}$$
$$y(x) \to \frac{2\sqrt{a^2 b x (a+b)} + a^2 + ab(x+1)}{a^2}$$

1.429 problem 430

Internal problem ID [8766]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 430.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, _dAlembert]`

$$\boxed{(\mathrm{a2}\,x+\mathrm{c2})\,{y'}^{2}+(\mathrm{a1}\,x+\mathrm{b1}\,y+\mathrm{c1})\,y'+\mathrm{b0}\,y=-\mathrm{a0}\,x-\mathrm{c0}}$$

✓ Solution by Maple

Time used: 0.36 (sec). Leaf size: 1983

```
dsolve((a2*x+c2)*diff(y(x),x)^2+(a1*x+b1*y(x)+c1)*diff(y(x),x)+a0*x+b0*y(x)+c0 = 0,y(x), sin
```

Expression too large to display
Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[c0 + a0*x + b0*y[x] + (c1 + a1*x + b1*y[x])*y'[x] + (c2 + a2*x)*y'[x]^2==0,y[x],x,Inc
```

Timed out

1.430 problem 431

Internal problem ID [8767]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 431.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$x^2 {y'}^2 - y^4 + y^2 = 0$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 52

```
dsolve(x^2*diff(y(x),x)^2-y(x)^4+y(x)^2 = 0,y(x), singsol=all)
```

$$\begin{aligned}
y(x) &= -1 \\
y(x) &= 1 \\
y(x) &= 0 \\
y(x) &= \csc\left(-\ln\left(x\right) + c_1\right) \operatorname{csgn}\left(\sec\left(-\ln\left(x\right) + c_1\right)\right) \\
y(x) &= -\csc\left(-\ln\left(x\right) + c_1\right) \operatorname{csgn}\left(\sec\left(-\ln\left(x\right) + c_1\right)\right)
\end{aligned}$$

✓ Solution by Mathematica

Time used: 1.514 (sec). Leaf size: 88

```
DSolve[y[x]^2 - y[x]^4 + x^2*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}
y(x) &\to -\sqrt{\sec^2(-\log(x) + c_1)} \\
y(x) &\to \sqrt{\sec^2(-\log(x) + c_1)} \\
y(x) &\to -\sqrt{\sec^2(\log(x) + c_1)} \\
y(x) &\to \sqrt{\sec^2(\log(x) + c_1)} \\
y(x) &\to -1 \\
y(x) &\to 0 \\
y(x) &\to 1
\end{aligned}$$

1.431 problem 432

Internal problem ID [8768]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 432.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational]`

$$\boxed{(xy' + a)^2 - 2ay = -x^2}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 78

```
dsolve((x*diff(y(x),x)+a)^2-2*a*y(x)+x^2 = 0,y(x), singsol=all)
```

$$y(x) - \mathrm{RootOf}\left(-a \operatorname{arcsinh}\left(\frac{\mathrm{RootOf}\left(-2ay(x) + a^2 + x^2 + 2a\_Z + \_Z^2\right)}{x}\right) - x\sqrt{\frac{a\left(-2\,\mathrm{RootOf}\left(-2ay\left(x\right) + a^2 + x^2 + 2a\_Z + \_Z^2\right) + 2\_Z - a\right)}{x^2}} + c_1\right) = 0$$

✓ Solution by Mathematica

Time used: 0.895 (sec). Leaf size: 82

```
DSolve[x^2 - 2*a*y[x] + (a + x*y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\left\{y(x) = \frac{2axK[1] + x^2K[1]^2 + a^2 + x^2}{2a}, x = \frac{a\log\left(\sqrt{K[1]^2+1} - K[1]\right)}{\sqrt{K[1]^2+1}} + \frac{c_1}{\sqrt{K[1]^2+1}}\right\}, \{y(x), K[1]\}\right]$$

1.432 problem 433

Internal problem ID [8769]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 433.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$(xy' + y + 2x)^2 - 4yx = 4x^2 + 4a$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 36

```
dsolve((x*diff(y(x),x)+y(x)+2*x)^2-4*x*y(x)-4*x^2-4*a = 0,y(x), singsol=all)
```

$$y(x) = \frac{-x^2 - a}{x}$$
$$y(x) = \frac{c_1^2 + 4c_1 x - 4a}{4x}$$

✓ Solution by Mathematica

Time used: 1.218 (sec). Leaf size: 44

```
DSolve[-4*a - 4*x^2 - 4*x*y[x] + (2*x + y[x] + x*y'[x])^2==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{-a + c_1(-2x + c_1)}{x}$$
$$y(x) \to -2\sqrt{a}$$
$$y(x) \to 2\sqrt{a}$$

1.433 problem 434

Internal problem ID [8770]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 434.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 7

```
dsolve(diff(y(x),x)-1 = 0,y(x), singsol=all)
```

$$y(x) = x + c_1$$

✓ Solution by Mathematica
Time used: 0.116 (sec). Leaf size: 71

```
DSolve[-x^2 - 2*x*y[x]*y'[x] + x^2*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to \frac{1}{2}e^{-c_1}\left(-x^2 + e^{2c_1}\right) \\ y(x) &\to \frac{1}{2}e^{-c_1}\left(-1 + e^{2c_1}x^2\right) \\ y(x) &\to -ix \\ y(x) &\to ix \end{aligned}$$

1.434 problem 435

Internal problem ID [8771]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 435.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$x^2 {y'}^2 - 2xy'y + y(y+1) = x$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x)+y(x)*(y(x)+1)-x = 0,y(x), singsol=all)
```

$$y(x) = x$$
$$y(x) = c_1\sqrt{x} - \frac{x\, c_1^2}{4} + x - 1$$

✓ Solution by Mathematica

Time used: 0.111 (sec). Leaf size: 55

```
DSolve[-x + y[x]*(1 + y[x]) - 2*x*y[x]*y'[x] + x^2*y'[x]^2==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to x + \frac{{c_1}^2 x}{4} - ic_1\sqrt{x} - 1$$
$$y(x) \to x + \frac{{c_1}^2 x}{4} + ic_1\sqrt{x} - 1$$

1.435 problem 436

Internal problem ID [8772]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 436.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$x^2y'^2 - 2xy'y + y^2(1 - x^2) = x^4$$

✓ Solution by Maple

Time used: 0.375 (sec). Leaf size: 58

```
dsolve(x^2*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x)+y(x)^2*(-x^2+1)-x^4 = 0,y(x), singsol=all)
```

$$y(x) = -ix$$
$$y(x) = ix$$
$$y(x) = -\frac{x(\mathrm{e}^{x} - c_1^2\mathrm{e}^{-x})}{2c_1}$$
$$y(x) = \frac{x(c_1^2\mathrm{e}^{x} - \mathrm{e}^{-x})}{2c_1}$$

✓ Solution by Mathematica

Time used: 0.216 (sec). Leaf size: 60

```
DSolve[-x^4 + (1 - x^2)*y[x]^2 - 2*x*y[x]*y'[x] + x^2*y'[x]^2==0,y[x],x,IncludeSingularSolut
```

$$y(x) \to \frac{1}{2}xe^{-x-c_1}\left(-1 + e^{2(x+c_1)}\right)$$
$$y(x) \to \frac{1}{2}\left(xe^{-x+c_1} - xe^{x-c_1}\right)$$

1.436 problem 437

Internal problem ID [8773]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 437.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Clairaut]`

$$\boxed{x^2 {y'}^2 - (2yx + a)\, y' + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 36

```
dsolve(x^2*diff(y(x),x)^2-(2*x*y(x)+a)*diff(y(x),x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{a}{4x}$$
$$y(x) = c_1 x - \sqrt{a c_1}$$
$$y(x) = c_1 x + \sqrt{a c_1}$$

✓ Solution by Mathematica

Time used: 0.335 (sec). Leaf size: 64

```
DSolve[y[x]^2 - (a + 2*x*y[x])*y'[x] + x^2*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{x - 2\sqrt{a} c_1}{4{c_1}^2}$$
$$y(x) \to \frac{x + 2\sqrt{a} c_1}{4{c_1}^2}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{a}{4x}$$

1.437 problem 438

Internal problem ID [8774]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 438.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2 {y'}^2 + 3xy'y + 2y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x)^2+3*x*y(x)*diff(y(x),x)+2*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x}$$
$$y(x) = \frac{c_1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 26

```
DSolve[2*y[x]^2 + 3*x*y[x]*y'[x] + x^2*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_1}{x^2}$$
$$y(x) \rightarrow \frac{c_1}{x}$$
$$y(x) \rightarrow 0$$

1.438 problem 439

Internal problem ID [8775]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 439.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2 y'^2 + 3xy'y + 3y^2 = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 35

```
dsolve(x^2*diff(y(x),x)^2+3*x*y(x)*diff(y(x),x)+3*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = c_1 x^{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}$$
$$y(x) = c_1 x^{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}$$

✓ Solution by Mathematica

Time used: 0.075 (sec). Leaf size: 54

```
DSolve[3*y[x]^2 + 3*x*y[x]*y'[x] + x^2*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x^{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}$$
$$y(x) \to c_1 x^{\frac{1}{2}i\left(\sqrt{3}+3i\right)}$$
$$y(x) \to 0$$

1.439 problem 440

Internal problem ID [8776]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 440.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2 {y'}^2 + 4xy'y - 5y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x)^2+4*x*y(x)*diff(y(x),x)-5*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = c_1 x$$
$$y(x) = \frac{c_1}{x^5}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 24

```
DSolve[-5*y[x]^2 + 4*x*y[x]*y'[x] + x^2*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x^5}$$
$$y(x) \to c_1 x$$
$$y(x) \to 0$$

1.440 problem 441

Internal problem ID [8777]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 441.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$x^2y'^2 - 4x(y+2)\,y' + 4y(y+2) = 0$$

✓ Solution by Maple

Time used: 1.266 (sec). Leaf size: 137

```
dsolve(x^2*diff(y(x),x)^2-4*x*(y(x)+2)*diff(y(x),x)+4*y(x)*(y(x)+2) = 0,y(x), singsol=all)
```

$$y(x) = -2$$
$$y(x) = \frac{-2\sqrt{2}\,\sqrt{c_1x^2} + x^2}{c_1}$$
$$y(x) = \frac{2\sqrt{2}\,\sqrt{c_1x^2} + x^2}{c_1}$$
$$y(x) = \frac{(-8c_1^2 + x^2)\left(-2c_1\sqrt{2} + x\right)x}{\left(-4c_1\sqrt{2}\,x + 8c_1^2 + x^2\right)c_1^2}$$
$$y(x) = \frac{(-8c_1^2 + x^2)\left(2c_1\sqrt{2} + x\right)x}{\left(4c_1\sqrt{2}\,x + 8c_1^2 + x^2\right)c_1^2}$$

✓ Solution by Mathematica

Time used: 0.198 (sec). Leaf size: 69

```
DSolve[4*y[x]*(2 + y[x]) - 4*x*(2 + y[x])*y'[x] + x^2*y'[x]^2==0,y[x],x,IncludeSingularSolut
```

$$y(x) \to e^{-c_1}x\left(x - 2\sqrt{2}e^{\frac{c_1}{2}}\right)$$
$$y(x) \to e^{c_1}x^2 - 2\sqrt{2}e^{\frac{c_1}{2}}x$$
$$y(x) \to -2$$
$$y(x) \to 0$$

1.441 problem 442

Internal problem ID [8778]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 442.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_linear]`

$$x^2{y'}^2 + (x^2y - 2yx + x^3)\, y' + (-x^2y + y^2)(1 - x) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(x^2*diff(y(x),x)^2+(x^2*y(x)-2*x*y(x)+x^3)*diff(y(x),x)+(y(x)^2-x^2*y(x))*(1-x) = 0,y
```

$$y(x) = (-x + c_1)\, x$$
$$y(x) = c_1 \mathrm{e}^{-x} x$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 26

```
DSolve[(1 - x)*(-(x^2*y[x]) + y[x]^2) + (x^3 - 2*x*y[x] + x^2*y[x])*y'[x] + x^2*y'[x]^2==0,y
```

$$y(x) \to c_1 e^{-x} x$$
$$y(x) \to x(-x + c_1)$$

1.442 problem 444

Internal problem ID [8779]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 444.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{x^2 {y'}^2 - y(-2x + y)\, y' + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 124

```
dsolve(x^2*diff(y(x),x)^2-y(x)*(y(x)-2*x)*diff(y(x),x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = 4x$$
$$y(x) = 0$$
$$y(x) = -\frac{2c_1^2\left(-c_1\sqrt{2} + x\right)}{-2c_1^2 + x^2}$$
$$y(x) = -\frac{2c_1^2\left(c_1\sqrt{2} + x\right)}{-2c_1^2 + x^2}$$
$$y(x) = \frac{\sqrt{2}\, c_1^3 - 2x\, c_1^2}{-2c_1^2 + 4x^2}$$
$$y(x) = \frac{c_1^2\left(c_1\sqrt{2} + 2x\right)}{2c_1^2 - 4x^2}$$

✓ Solution by Mathematica

Time used: 0.639 (sec). Leaf size: 62

```
DSolve[y[x]^2 - y[x]*(-2*x + y[x])*y'[x] + x^2*y'[x]^2==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{4e^{-2c_1}}{2 + e^{2c_1}x}$$
$$y(x) \to -\frac{e^{-2c_1}}{2 + 4e^{2c_1}x}$$
$$y(x) \to 0$$
$$y(x) \to 4x$$

1.443 problem 445

Internal problem ID [8780]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 445.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$x^2 {y'}^2 + \left(x^2 y^3 a + b\right) y' + y^3 a b = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
dsolve(x^2*diff(y(x),x)^2+(a*x^2*y(x)^3+b)*diff(y(x),x)+a*b*y(x)^3 = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{2ax + c_1}}$$
$$y(x) = -\frac{1}{\sqrt{2ax + c_1}}$$
$$y(x) = \frac{b}{x} + c_1$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 49

```
DSolve[a*b*y[x]^3 + (b + a*x^2*y[x]^3)*y'[x] + x^2*y'[x]^2==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to -\frac{1}{\sqrt{2ax - 2c_1}}$$
$$y(x) \to \frac{1}{\sqrt{2ax - 2c_1}}$$
$$y(x) \to \frac{b}{x} + c_1$$

1.444 problem 446

Internal problem ID [8781]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 446.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Clairaut]`

$$(x^2+1)\,y'^2 - 2xy'y + y^2 = 1$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 57

```
dsolve((x^2+1)*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x)+y(x)^2-1 = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{x^2+1}$$
$$y(x) = -\sqrt{x^2+1}$$
$$y(x) = c_1 x - \sqrt{-c_1^2+1}$$
$$y(x) = c_1 x + \sqrt{-c_1^2+1}$$

✓ Solution by Mathematica

Time used: 0.106 (sec). Leaf size: 73

```
DSolve[-1 + y[x]^2 - 2*x*y[x]*y'[x] + (1 + x^2)*y'[x]^2==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to c_1 x - \sqrt{1-c_1{}^2}$$
$$y(x) \to c_1 x + \sqrt{1-c_1{}^2}$$
$$y(x) \to -\sqrt{x^2+1}$$
$$y(x) \to \sqrt{x^2+1}$$

1.445 problem 447

Internal problem ID [8782]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 447.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(x^2-1)\, y'^2 = 1}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 33

```
dsolve((x^2-1)*diff(y(x),x)^2-1 = 0,y(x), singsol=all)
```

$$y(x) = \ln\left(x+\sqrt{x^2-1}\right)+c_1$$
$$y(x) = -\ln\left(x+\sqrt{x^2-1}\right)+c_1$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 89

```
DSolve[-1 + (-1 + x^2)*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)-\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)+2c_1\right)$$
$$y(x) \to \frac{1}{2}\left(-\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)+\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)+2c_1\right)$$

1.446 problem 448

Internal problem ID [8783]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 448.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{(x^2-1)\, y'^2 - y^2 = -1}$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 162

```
dsolve((x^2-1)*diff(y(x),x)^2-y(x)^2+1 = 0,y(x), singsol=all)
```

$$y(x) = -1$$
$$y(x) = 1$$
$$\frac{\sqrt{y(x)^2-1}\, \ln\left(y(x)+\sqrt{y(x)^2-1}\right)}{\sqrt{y(x)-1}\,\sqrt{y(x)+1}} - \frac{\int^x \frac{\sqrt{(\_a^2-1)\left(y(x)^2-1\right)}}{\_a^2-1} d\_a}{\sqrt{y(x)-1}\,\sqrt{y(x)+1}} + c_1 = 0$$
$$\frac{\sqrt{y(x)^2-1}\, \ln\left(y(x)+\sqrt{y(x)^2-1}\right)}{\sqrt{y(x)-1}\,\sqrt{y(x)+1}} + \frac{\int^x \frac{\sqrt{(\_a^2-1)\left(y(x)^2-1\right)}}{\_a^2-1} d\_a}{\sqrt{y(x)-1}\,\sqrt{y(x)+1}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 5.099 (sec). Leaf size: 297

```
DSolve[1 - y[x]^2 + (-1 + x^2)*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}e^{-c_1}\sqrt{2x^2 - 2\sqrt{x^2-1}x + e^{4c_1}\left(2x^2 + 2\sqrt{x^2-1}x - 1\right) - 1 + 2e^{2c_1}}$$
$$y(x) \to \frac{1}{2}e^{-c_1}\sqrt{2x^2 - 2\sqrt{x^2-1}x + e^{4c_1}\left(2x^2 + 2\sqrt{x^2-1}x - 1\right) - 1 + 2e^{2c_1}}$$
$$y(x) \to -\frac{1}{2}\sqrt{e^{-2c_1}\left(2x^2 + 2\sqrt{x^2-1}x + e^{4c_1}\left(2x^2 - 2\sqrt{x^2-1}x - 1\right) - 1 + 2e^{2c_1}\right)}$$
$$y(x) \to \frac{1}{2}\sqrt{e^{-2c_1}\left(2x^2 + 2\sqrt{x^2-1}x + e^{4c_1}\left(2x^2 - 2\sqrt{x^2-1}x - 1\right) - 1 + 2e^{2c_1}\right)}$$
$$y(x) \to -1$$
$$y(x) \to 1$$

1.447 problem 449

Internal problem ID [8784]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 449.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{(-a^2+x^2)\,{y'}^2+2xy'y+y^2=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve((-a^2+x^2)*diff(y(x),x)^2+2*x*y(x)*diff(y(x),x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x)=\frac{c_1}{-x+a}$$
$$y(x)=\frac{c_1}{a+x}$$

✓ Solution by Mathematica
Time used: 0.049 (sec). Leaf size: 32

```
DSolve[y[x]^2 + 2*x*y[x]*y'[x] + (-a^2 + x^2)*y'[x]^2==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{c_1}{a-x}$$
$$y(x)\to\frac{c_1}{a+x}$$
$$y(x)\to 0$$

1.448 problem 450

Internal problem ID [8785]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 450.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{(-a^2+x^2)\,y'^2-2xyy'=x^2}$$

✓ Solution by Maple
Time used: 0.891 (sec). Leaf size: 51

```
dsolve((-a^2+x^2)*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x)-x^2 = 0,y(x), singsol=all)
```

$$y(x)=\sqrt{a^2-x^2}$$
$$y(x)=-\sqrt{a^2-x^2}$$
$$y(x)=c_1x^2-a^2c_1-\frac{1}{4c_1}$$

✓ Solution by Mathematica
Time used: 0.413 (sec). Leaf size: 67

```
DSolve[-x^2 - 2*x*y[x]*y'[x] + (-a^2 + x^2)*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x)\to\frac{a^2-x^2+c_1{}^2}{2c_1}$$
$$y(x)\to\text{Indeterminate}$$
$$y(x)\to-\sqrt{a^2-x^2}$$
$$y(x)\to\sqrt{a^2-x^2}$$

1.449 problem 451

Internal problem ID [8786]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 451.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Clairaut]`

$$(x^2+a)\, y'^2 - 2xy'y + y^2 = -b$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 78

```
dsolve((x^2+a)*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x)+y(x)^2+b = 0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-ab\,(x^2+a)}}{a}$$
$$y(x) = -\frac{\sqrt{-ab\,(x^2+a)}}{a}$$
$$y(x) = c_1x - \sqrt{-a\,c_1^2 - b}$$
$$y(x) = c_1x + \sqrt{-a\,c_1^2 - b}$$

✓ Solution by Mathematica

Time used: 0.293 (sec). Leaf size: 96

```
DSolve[b + y[x]^2 - 2*x*y[x]*y'[x] + (a + x^2)*y'[x]^2==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to c_1x - \sqrt{-b - ac_1{}^2}$$
$$y(x) \to \sqrt{-b - ac_1{}^2} + c_1x$$
$$y(x) \to -\frac{\sqrt{-b\,(a+x^2)}}{\sqrt{a}}$$
$$y(x) \to \frac{\sqrt{-b\,(a+x^2)}}{\sqrt{a}}$$

1.450 problem 452

Internal problem ID [8787]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 452.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$
\boxed{(2x^2+1)\,y'^2+(y^2+2yx+x^2+2)\,y'+2y^2=-1}
$$

✗ Solution by Maple

```
dsolve((2*x^2+1)*diff(y(x),x)^2+(y(x)^2+2*x*y(x)+x^2+2)*diff(y(x),x)+2*y(x)^2+1 = 0,y(x), si
```

No solution found

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 49

```
DSolve[1 + 2*y[x]^2 + (2 + x^2 + 2*x*y[x] + y[x]^2)*y'[x] + (1 + 2*x^2)*y'[x]^2==0,y[x],x,In
```

$$
\begin{aligned}
y(x) &\to \frac{-c_1x+1+{c_1}^2}{x+c_1} \\
y(x) &\to -\frac{i}{\sqrt{2}} \\
y(x) &\to \frac{i}{\sqrt{2}}
\end{aligned}
$$

1.451 problem 453

Internal problem ID [8788]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 453.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$
\boxed{\left(a^{2}-1\right) x^{2} {y'}^{2}+2 x y y'-y^{2}=-a^{2} x^{2}}
$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 229

```
dsolve((a^2-1)*x^2*diff(y(x),x)^2+2*x*y(x)*diff(y(x),x)-y(x)^2+a^2*x^2 = 0,y(x), singsol=all
```

$$
\frac{2a\ln\left(x\right)-2\sqrt{-a^{2}}\,\arctan\left(\frac{a^{2}y(x)}{\sqrt{-a^{2}}\,\sqrt{\frac{-a^{2}x^{2}+x^{2}+y(x)^{2}}{x^{2}}}\,x}\right)+\ln\left(\frac{y(x)^{2}+x^{2}}{x^{2}}\right)a-2ac_{1}+2\ln\left(\frac{\sqrt{\frac{-a^{2}x^{2}+x^{2}+y(x)^{2}}{x^{2}}}\,x+y(x)}{x}\right.}{2a}
$$
$$
=0
$$

$$
\frac{2a\ln\left(x\right)+2\sqrt{-a^{2}}\,\arctan\left(\frac{a^{2}y(x)}{\sqrt{-a^{2}}\,\sqrt{\frac{-a^{2}x^{2}+x^{2}+y(x)^{2}}{x^{2}}}\,x}\right)+\ln\left(\frac{y(x)^{2}+x^{2}}{x^{2}}\right)a-2ac_{1}-2\ln\left(\frac{\sqrt{\frac{-a^{2}x^{2}+x^{2}+y(x)^{2}}{x^{2}}}\,x+y(x)}{x}\right.}{2a}
$$
$$
=0
$$

✓ Solution by Mathematica

Time used: 1.001 (sec). Leaf size: 223

```
DSolve[a^2*x^2 - y[x]^2 + 2*x*y[x]*y'[x] + (-1 + a^2)*x^2*y'[x]^2==0,y[x],x,IncludeSingularS
```

$$\text{Solve}\left[\frac{2i\arctan\left(\frac{y(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)-2ia\arctan\left(\frac{ay(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)+a\log\left(\frac{y(x)^2}{x^2}+1\right)}{2a^2-2}=\frac{a\log\left(x-a^2x\right)}{1-a^2}\right.$$

$$\left.+c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{-2i\arctan\left(\frac{y(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)+2ia\arctan\left(\frac{ay(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)+a\log\left(\frac{y(x)^2}{x^2}+1\right)}{2a^2-2}=\frac{a\log\left(x-a^2x\right)}{1-a^2}\right.$$

$$\left.+c_1,y(x)\right]$$

1.452 problem 454

Internal problem ID [8789]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 454.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$a\,x^2{y'}^2 - 2axyy' + y^2 = a(a-1)\,x^2$$

✓ Solution by Maple

Time used: 0.171 (sec). Leaf size: 106

```
dsolve(a*x^2*diff(y(x),x)^2-2*a*x*y(x)*diff(y(x),x)+y(x)^2-a*(a-1)*x^2 = 0,y(x), singsol=all
```

$$y(x) = \sqrt{-a}\,x$$
$$y(x) = -\sqrt{-a}\,x$$
$$y(x) = \operatorname{RootOf}\left(-\ln(x) - \left(\int^{\_Z} \frac{\sqrt{(a-1)\,(\_a^2+a)\,a}}{(a-1)\,(\_a^2+a)}d\_a\right) + c_1\right)x$$
$$y(x) = \operatorname{RootOf}\left(-\ln(x) + \int^{\_Z} \frac{\sqrt{(a-1)\,(\_a^2+a)\,a}}{(a-1)\,(\_a^2+a)}d\_a + c_1\right)x$$

✓ Solution by Mathematica

Time used: 0.61 (sec). Leaf size: 241

```
DSolve[-((-1 + a)*a*x^2) + y[x]^2 - 2*a*x*y[x]*y'[x] + a*x^2*y'[x]^2==0,y[x],x,IncludeSingul
```

$$y(x) \to \frac{1}{2}\sqrt{a}e^{-c_1}x^{1-\sqrt{\frac{a-1}{a}}}\left(x^{2\sqrt{\frac{a-1}{a}}} - e^{2c_1}\right)$$
$$y(x) \to \frac{1}{2}\sqrt{a}e^{-c_1}x^{1-\sqrt{\frac{a-1}{a}}}\left(-x^{2\sqrt{\frac{a-1}{a}}} + e^{2c_1}\right)$$
$$y(x) \to -\frac{1}{2}\sqrt{a}e^{-c_1}x^{1-\sqrt{\frac{a-1}{a}}}\left(-1 + e^{2c_1}x^{2\sqrt{\frac{a-1}{a}}}\right)$$
$$y(x) \to \frac{1}{2}\sqrt{a}e^{-c_1}x^{1-\sqrt{\frac{a-1}{a}}}\left(-1 + e^{2c_1}x^{2\sqrt{\frac{a-1}{a}}}\right)$$

1.453 problem 455

Internal problem ID [8790]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 455.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{x^3 {y'}^2 + x^2 y' y = -a}$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 66

```
dsolve(x^3*diff(y(x),x)^2+x^2*y(x)*diff(y(x),x)+a = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2\sqrt{ax}}{x}$$
$$y(x) = \frac{2\sqrt{ax}}{x}$$
$$y(x) = \frac{x\,c_1^2 + 4a}{2xc_1}$$
$$y(x) = \frac{4ax + c_1^2}{2xc_1}$$

✓ Solution by Mathematica

Time used: 0.79 (sec). Leaf size: 57

```
DSolve[a + x^2*y[x]*y'[x] + x^3*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{-\frac{c_1}{2}}(x + 4ae^{c_1})}{2x}$$
$$y(x) \to \frac{e^{-\frac{c_1}{2}}(x + 4ae^{c_1})}{2x}$$

1.454 problem 456

Internal problem ID [8791]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 456.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$x(x^2-1)\,y'^2+2(1-x^2)\,y'y+xy^2=x$$

✓ Solution by Maple

Time used: 0.36 (sec). Leaf size: 33

```
dsolve(x*(x^2-1)*diff(y(x),x)^2+2*(-x^2+1)*y(x)*diff(y(x),x)+x*y(x)^2-x = 0,y(x), singsol=al
```

$$y(x)=-x$$
$$y(x)=x$$
$$y(x)=\sqrt{-c_1^2+1}+\sqrt{x^2-1}\,c_1$$

✓ Solution by Mathematica

Time used: 0.585 (sec). Leaf size: 75

```
DSolve[-x + x*y[x]^2 + 2*(1 - x^2)*y[x]*y'[x] + x*(-1 + x^2)*y'[x]^2==0,y[x],x,IncludeSingul
```

$$y(x)\to -x\cos\left(2\arctan\left(\sqrt{\frac{x-1}{x+1}}\right)+ic_1\right)$$
$$y(x)\to -x\cos\left(2\arctan\left(\sqrt{\frac{x-1}{x+1}}\right)-ic_1\right)$$
$$y(x)\to -x$$
$$y(x)\to x$$

1.455 problem 457

Internal problem ID [8792]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 457.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$\boxed{x^4 {y'}^2 - x y' - y = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 81

```
dsolve(x^4*diff(y(x),x)^2-x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{4x^2}$$
$$y(x) = \frac{-c_1 i - x}{x\, c_1^2}$$
$$y(x) = \frac{c_1 i - x}{x\, c_1^2}$$
$$y(x) = \frac{c_1 i - x}{x\, c_1^2}$$
$$y(x) = \frac{-c_1 i - x}{x\, c_1^2}$$

✓ Solution by Mathematica

Time used: 0.498 (sec). Leaf size: 123

```
DSolve[-y[x] - x*y'[x] + x^4*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{x\sqrt{4x^2y(x)+1}\operatorname{arctanh}\left(\sqrt{4x^2y(x)+1}\right)}{\sqrt{4x^4y(x)+x^2}}-\frac{1}{2}\log(y(x))=c_1,y(x)\right]$$
$$\text{Solve}\left[\frac{x\sqrt{4x^2y(x)+1}\operatorname{arctanh}\left(\sqrt{4x^2y(x)+1}\right)}{\sqrt{4x^4y(x)+x^2}}-\frac{1}{2}\log(y(x))=c_1,y(x)\right]$$
$$y(x)\to 0$$

1.456 problem 458

Internal problem ID [8793]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 458.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$x^2(-a^2+x^2)\,y'^2=1$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 111

```
dsolve(x^2*(-a^2+x^2)*diff(y(x),x)^2-1 = 0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\sqrt{-a^2}-\ln(2)-\ln\left(\frac{\sqrt{-a^2}\,\sqrt{-a^2+x^2}-a^2}{x}\right)}{\sqrt{-a^2}}$$
$$y(x)=\frac{c_1\sqrt{-a^2}+\ln(2)+\ln\left(\frac{\sqrt{-a^2}\,\sqrt{-a^2+x^2}-a^2}{x}\right)}{\sqrt{-a^2}}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 120

```
DSolve[-1 + x^2*(-a^2 + x^2)*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\frac{x\sqrt{x^2-a^2}\arctan\left(\frac{\sqrt{x^2-a^2}}{a}\right)}{a\sqrt{x^4-a^2x^2}}+c_1$$
$$y(x)\to\frac{x\sqrt{x^2-a^2}\arctan\left(\frac{\sqrt{x^2-a^2}}{a}\right)}{a\sqrt{x^4-a^2x^2}}+c_1$$

1.457 problem 459

Internal problem ID [8794]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 459.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{e^{-2x}{y'}^2 - (y'-1)^2 + e^{-2y} = 0}$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 128

```
dsolve(exp(-2*x)*diff(y(x),x)^2-(diff(y(x),x)-1)^2+exp(-2*y(x)) = 0,y(x), singsol=all)
```

$$y(x) = c_1 - \ln\left(\frac{-\sqrt{e^{-2x+4c_1} - e^{-2x+2c_1}}\, e^{2x} - e^{2c_1}}{-e^{2c_1+2x} + e^{2c_1} + e^{2x}}\right)$$
$$y(x) = c_1 - \ln\left(\frac{\sqrt{e^{-2x+4c_1} - e^{-2x+2c_1}}\, e^{2x} - e^{2c_1}}{-e^{2c_1+2x} + e^{2c_1} + e^{2x}}\right)$$

✓ Solution by Mathematica

Time used: 24.762 (sec). Leaf size: 583

```
DSolve[E^(-2*y[x]) - (-1 + y'[x])^2 + y'[x]^2/E^(2*x)==0,y[x],x,IncludeSingularSolutions ->
```

$$\text{Solve}\Bigg[-\frac{\left(e^{2\operatorname{arctanh}(1-2e^x)+x}+e^x-1\right)\sqrt{e^{2y(x)}+e^{2x}-1}e^{y(x)-2\operatorname{arctanh}(1-2e^x)}\log\left(\sqrt{e^{2y(x)}+e^{2x}-1}+e^{y(x)}\right)}{\sqrt{e^{2(y(x)+x)}\left(e^{2y(x)}+e^{2x}-1\right)}}$$
$$-y(x)+\log\left(e^{y(x)}\right)-\frac{1}{2}\log\left(e^{y(x)}-1\right)-\frac{1}{2}\log\left(e^{y(x)}+1\right)$$
$$+\frac{1}{2}\log\left(\sqrt{e^{2y(x)+2x}\left(e^{2y(x)}+e^{2x}-1\right)}+e^{2y(x)+x}-e^x-e^{2x}\right)$$
$$+\frac{1}{2}\log\left(\sqrt{e^{2y(x)+2x}\left(e^{2y(x)}+e^{2x}-1\right)}+e^{2y(x)+x}-e^x+e^{2x}\right)$$
$$-x-\frac{1}{2}\log\left(1-e^x\right)-\frac{1}{2}\log\left(e^x-1\right)=c_1,y(x)\Bigg]$$

$$\text{Solve}\Bigg[\frac{\left(e^{2\operatorname{arctanh}(1-2e^x)+x}+e^x-1\right)\sqrt{e^{2y(x)}+e^{2x}-1}e^{y(x)-2\operatorname{arctanh}(1-2e^x)}\log\left(\sqrt{e^{2y(x)}+e^{2x}-1}+e^{y(x)}\right)}{\sqrt{e^{2(y(x)+x)}\left(e^{2y(x)}+e^{2x}-1\right)}}$$
$$-\frac{1}{2}\log\left(\sqrt{e^{2y(x)+2x}\left(e^{2y(x)}+e^{2x}-1\right)}+e^{2y(x)+x}-e^x-e^{2x}\right)$$
$$-\frac{1}{2}\log\left(\sqrt{e^{2y(x)+2x}\left(e^{2y(x)}+e^{2x}-1\right)}+e^{2y(x)+x}-e^x+e^{2x}\right)$$
$$+\frac{1}{2}\left(2y(x)-2\log\left(e^{y(x)}\right)+\log\left(e^{y(x)}-1\right)+\log\left(e^{y(x)}+1\right)\right)+x$$
$$-\frac{1}{2}\log\left(1-e^x\right)+\frac{1}{2}\log\left(e^x-1\right)+\log\left(e^x+1\right)=c_1,y(x)\Bigg]$$

$$y(x)\to\log\left(-\sqrt{1-e^{2x}}\right)$$
$$y(x)\to\frac{1}{2}\log\left(1-e^{2x}\right)$$

1.458 problem 460

Internal problem ID [8795]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 460.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{\left(y'^2 + y^2\right) \cos \left(x\right)^4 = a^2}$$

✗ Solution by Maple

```
dsolve((diff(y(x),x)^2+y(x)^2)*cos(x)^4-a^2 = 0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-a^2 + Cos[x]^4*(y[x]^2 + y'[x]^2)==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.459 problem 461

Internal problem ID [8796]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 461.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\mathrm{d0}(x)\, {y'}^2 + 2\,\mathrm{b0}(x)\, y y' + \mathrm{c0}(x)\, y^2 + 2\,\mathrm{d0}(x)\, y' + 2\,\mathrm{e0}(x)\, y = -\mathrm{f0}(x)$$

✗ Solution by Maple

```
dsolve(d0(x)*diff(y(x),x)^2+2*b0(x)*y(x)*diff(y(x),x)+c0(x)*y(x)^2+2*d0(x)*diff(y(x),x)+2*e0
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[f[x] + 2*e[x]*y[x] + c[x]*y[x]^2 + 2*d[x]*y'[x] + 2*b[x]*y[x]*y'[x] + a[x]*y'[x]^2==0
```

Timed out

1.460 problem 462

Internal problem ID [8797]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 462.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{yy'^2 = 1}$$

✓ Solution by Maple
Time used: 0.156 (sec). Leaf size: 27

```
dsolve(y(x)*diff(y(x),x)^2-1 = 0,y(x), singsol=all)
```

$$x - \frac{2y(x)^{\frac{3}{2}}}{3} - c_1 = 0$$
$$x + \frac{2y(x)^{\frac{3}{2}}}{3} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 43

```
DSolve[-1 + y[x]*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\frac{3}{2}\right)^{2/3} (-x + c_1)^{2/3}$$
$$y(x) \to \left(\frac{3}{2}\right)^{2/3} (x + c_1)^{2/3}$$

1.461 problem 463

Internal problem ID [8798]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 463.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{yy'^2 = e^{2x}}$$

✓ Solution by Maple
Time used: 0.187 (sec). Leaf size: 67

```
dsolve(y(x)*diff(y(x),x)^2-exp(2*x) = 0,y(x), singsol=all)
```

$$\frac{2y(x)^2 + 3c_1\sqrt{y(x)} - 3\sqrt{y(x)}\,e^{2x}}{3\sqrt{y(x)}} = 0$$
$$\frac{2y(x)^2 + 3c_1\sqrt{y(x)} + 3\sqrt{y(x)}\,e^{2x}}{3\sqrt{y(x)}} = 0$$

✓ Solution by Mathematica
Time used: 2.026 (sec). Leaf size: 47

```
DSolve[-E^(2*x) + y[x]*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\frac{3}{2}\right)^{2/3} (-e^x + c_1)^{2/3}$$
$$y(x) \to \left(\frac{3}{2}\right)^{2/3} (e^x + c_1)^{2/3}$$

1.462 problem 464

Internal problem ID [8799]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 464.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{yy'^2 + 2xy' - y = 0}$$

✓ Solution by Maple
Time used: 0.14 (sec). Leaf size: 71

```
dsolve(y(x)*diff(y(x),x)^2+2*x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -ix$$
$$y(x) = ix$$
$$y(x) = 0$$
$$y(x) = \sqrt{c_1 (-2x + c_1)}$$
$$y(x) = \sqrt{c_1 (2x + c_1)}$$
$$y(x) = -\sqrt{c_1 (-2x + c_1)}$$
$$y(x) = -\sqrt{c_1 (2x + c_1)}$$

✓ Solution by Mathematica

Time used: 0.487 (sec). Leaf size: 126

```
DSolve[-y[x] + 2*x*y'[x] + y[x]*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{\frac{c_1}{2}}\sqrt{-2x + e^{c_1}}$$
$$y(x) \to e^{\frac{c_1}{2}}\sqrt{-2x + e^{c_1}}$$
$$y(x) \to -e^{\frac{c_1}{2}}\sqrt{2x + e^{c_1}}$$
$$y(x) \to e^{\frac{c_1}{2}}\sqrt{2x + e^{c_1}}$$
$$y(x) \to 0$$
$$y(x) \to -ix$$
$$y(x) \to ix$$

1.463 problem 465

Internal problem ID [8800]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 465.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$yy'^2 + 2xy' - 9y = 0$$

✓ Solution by Maple

Time used: 0.328 (sec). Leaf size: 92

```
dsolve(y(x)*diff(y(x),x)^2+2*x*diff(y(x),x)-9*y(x) = 0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \text{RootOf}\left(-\ln(x) - \left(\int^{\_Z} \frac{\_a^2 + \sqrt{9\_a^2+1}+1}{\_a(\_a^2-7)}d\_a\right) + c_1\right)x$$

$$y(x) = \text{RootOf}\left(-\ln(x) + \int^{\_Z} -\frac{\_a^2 - \sqrt{9\_a^2+1}+1}{\_a(\_a^2-7)}d\_a + c_1\right)x$$

✓ Solution by Mathematica

Time used: 0.119 (sec). Leaf size: 112

```
DSolve[-9*y[x] + 2*x*y'[x] + y[x]*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int \frac{y(x)}{x\left(\frac{y(x)^2}{x^2} - \sqrt{\frac{9y(x)^2}{x^2}+1}+1\right)}d\frac{y(x)}{x} = -\log(x) + c_1, y(x)\right]$$

$$\text{Solve}\left[\int \frac{y(x)}{x\left(\frac{y(x)^2}{x^2} + \sqrt{\frac{9y(x)^2}{x^2}+1}+1\right)}d\frac{y(x)}{x} = -\log(x) + c_1, y(x)\right]$$

$$y(x) \to 0$$

1.464 problem 466

Internal problem ID [8801]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 466.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$yy'^2 - 2xy' + y = 0$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 71

```
dsolve(y(x)*diff(y(x),x)^2-2*x*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = -x$$
$$y(x) = x$$
$$y(x) = 0$$
$$y(x) = \sqrt{c_1 (-2ix + c_1)}$$
$$y(x) = \sqrt{c_1 (2ix + c_1)}$$
$$y(x) = -\sqrt{c_1 (-2ix + c_1)}$$
$$y(x) = -\sqrt{c_1 (2ix + c_1)}$$

✓ Solution by Mathematica

Time used: 2.634 (sec). Leaf size: 174

```
DSolve[y[x] - 2*x*y'[x] + y[x]*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{4}\left(\cosh\left(\frac{c_1}{2}\right)+\sinh\left(\frac{c_1}{2}\right)\right)\sqrt{-8ix+\cosh(c_1)+\sinh(c_1)}$$
$$y(x) \to \frac{1}{4}\left(\cosh\left(\frac{c_1}{2}\right)+\sinh\left(\frac{c_1}{2}\right)\right)\sqrt{-8ix+\cosh(c_1)+\sinh(c_1)}$$
$$y(x) \to -\frac{1}{4}\left(\cosh\left(\frac{c_1}{2}\right)+\sinh\left(\frac{c_1}{2}\right)\right)\sqrt{8ix+\cosh(c_1)+\sinh(c_1)}$$
$$y(x) \to \frac{1}{4}\left(\cosh\left(\frac{c_1}{2}\right)+\sinh\left(\frac{c_1}{2}\right)\right)\sqrt{8ix+\cosh(c_1)+\sinh(c_1)}$$
$$y(x) \to 0$$
$$y(x) \to -x$$
$$y(x) \to x$$

1.465 problem 467

Internal problem ID [8802]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 467.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{yy'^2 - 4xy' + y = 0}$$

✓ Solution by Maple

Time used: 0.312 (sec). Leaf size: 92

```
dsolve(y(x)*diff(y(x),x)^2-4*x*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \text{RootOf}\left(-\ln(x) - \left(\int^{\_Z} \frac{\_a^2 + \sqrt{-\_a^2+4} - 2}{\_a\,(\_a^2-3)} d\_a\right) + c_1\right) x$$
$$y(x) = \text{RootOf}\left(-\ln(x) + \int^{\_Z} -\frac{\_a^2 - \sqrt{-\_a^2+4} - 2}{\_a\,(\_a^2-3)} d\_a + c_1\right) x$$

✓ Solution by Mathematica

Time used: 60.178 (sec). Leaf size: 177

```
DSolve[y[x] - 4*x*y'[x] + y[x]*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\sqrt{\frac{8\,2^{2/3}x^4 + \sqrt[3]{2}\left(32x^6 - 40c_1{}^3x^3 + \sqrt{(c_1{}^4 - 16c_1x^3)^3} - c_1{}^6\right)^{2/3} + 4x^2\sqrt[3]{32x^6 - 40c_1{}^3x^3 + \sqrt{(c_1{}^4 - \cdots}}}{\sqrt[3]{32x^6 - 40c_1{}^3x^3 + \sqrt{(c_1{}^4 - 16c_1x^3)^3} - c_1{}^6}}}$$

1.466 problem 468

Internal problem ID [8803]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 468.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$yy'^2 - 4a^2xy' + a^2y = 0$$

✓ Solution by Maple
Time used: 0.344 (sec). Leaf size: 122

```
dsolve(y(x)*diff(y(x),x)^2-4*a^2*x*diff(y(x),x)+a^2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \operatorname{RootOf}\left(-\ln(x) - \left(\int^{\_Z} \frac{\_a^2 - 2a^2 + \sqrt{-\_a^2a^2 + 4a^4}}{\_a\,(\_a^2 - 3a^2)} d\_a\right) + c_1\right) x$$
$$y(x) = \operatorname{RootOf}\left(-\ln(x) + \int^{\_Z} -\frac{\_a^2 - 2a^2 - \sqrt{-\_a^2a^2 + 4a^4}}{\_a\,(\_a^2 - 3a^2)} d\_a + c_1\right) x$$

✓ Solution by Mathematica

Time used: 8.233 (sec). Leaf size: 758

```
DSolve[a^2*y[x] - 4*a^2*x*y'[x] + y[x]*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{8\left(4a^2-\frac{y(x)^2}{x^2}\right)^{3/2}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{y(x)}{x}-2a}}{2\sqrt{a}}\right)+\sqrt{a}\sqrt{\frac{y(x)}{ax}+2}\left(\sqrt{-\left(\frac{y(x)}{x}-2a\right)^2}\sqrt{2a+\frac{y(x)}{x}}\sqrt{4a^2-\frac{y(x)^2}{x^2}}\left(\right.\right.}{}\right.$$

$$\left.-\log(x)+c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{\sqrt{a}\sqrt{\frac{y(x)}{ax}+2}\left(\sqrt{-\left(\frac{y(x)}{x}-2a\right)^2}\sqrt{2a+\frac{y(x)}{x}}\sqrt{4a^2-\frac{y(x)^2}{x^2}}\left(\log\left(3a^2-\frac{y(x)^2}{x^2}\right)+8\arctan\left(\frac{\sqrt{2a-\frac{y(x)}{x}}}{\sqrt{2a+\frac{y(x)}{x}}}\right.\right.\right.}{}\right.$$

$$\left.-\log(x)+c_1,y(x)\right]$$

$$y(x)\to 0$$

1.467 problem 469

Internal problem ID [8804]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 469.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$y{y'}^2 + axy' + by = 0$$

✓ Solution by Maple

Time used: 0.313 (sec). Leaf size: 108

```
dsolve(y(x)*diff(y(x),x)^2+a*x*diff(y(x),x)+b*y(x) = 0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \mathrm{RootOf}\left(-2\ln(x) - \left(\int^{\_Z} \frac{2\_a^2 + \sqrt{-4b\_a^2 + a^2} + a}{\_a(\_a^2 + a + b)} d\_a\right) + 2c_1\right) x$$

$$y(x) = \mathrm{RootOf}\left(-2\ln(x) + \int^{\_Z} -\frac{2\_a^2 + a - \sqrt{-4b\_a^2 + a^2}}{\_a(\_a^2 + a + b)} d\_a + 2c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.597 (sec). Leaf size: 162

```
DSolve[b*y[x] + a*x*y'[x] + y[x]*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{a\log\left(\sqrt{a^2-\frac{4by(x)^2}{x^2}}+a\right)+(a+2b)\log\left(\sqrt{a^2-\frac{4by(x)^2}{x^2}}-a-2b\right)}{4(a+b)}=-\frac{\log(x)}{2}+c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{a\log\left(\sqrt{a^2-\frac{4by(x)^2}{x^2}}-a\right)+(a+2b)\log\left(\sqrt{a^2-\frac{4by(x)^2}{x^2}}+a+2b\right)}{4(a+b)}=-\frac{\log(x)}{2}+c_1,y(x)\right]$$

$$y(x)\to 0$$

1.468 problem 470

Internal problem ID [8805]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 470.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$yy'^2 + y'x^3 - x^2y = 0$$

✓ Solution by Maple

Time used: 0.281 (sec). Leaf size: 89

```
dsolve(y(x)*diff(y(x),x)^2+x^3*diff(y(x),x)-x^2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{i x^2}{2}$$
$$y(x) = \frac{i x^2}{2}$$
$$y(x) = 0$$
$$y(x) = -\frac{\sqrt{c_1 \left(-4x^2 + c_1\right)}}{4}$$
$$y(x) = \frac{\sqrt{c_1 \left(-4x^2 + c_1\right)}}{4}$$
$$y(x) = -\frac{2\sqrt{c_1 x^2 + 4}}{c_1}$$
$$y(x) = \frac{2\sqrt{c_1 x^2 + 4}}{c_1}$$

✓ Solution by Mathematica

Time used: 1.198 (sec). Leaf size: 244

```
DSolve[-(x^2*y[x]) + x^3*y'[x] + y[x]*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\sqrt{x^6+4x^2y(x)^2}\log\left(\sqrt{x^4+4y(x)^2}+x^2\right)}{2x\sqrt{x^4+4y(x)^2}} + \frac{1}{2}\left(1-\frac{\sqrt{x^6+4x^2y(x)^2}}{x\sqrt{x^4+4y(x)^2}}\right)\log(y(x)) = c_1, y(x)\right]$$

$$\text{Solve}\left[\frac{1}{2}\left(\frac{\sqrt{x^6+4x^2y(x)^2}}{x\sqrt{x^4+4y(x)^2}}+1\right)\log(y(x)) - \frac{\sqrt{x^6+4x^2y(x)^2}\log\left(\sqrt{x^4+4y(x)^2}+x^2\right)}{2x\sqrt{x^4+4y(x)^2}} = c_1, y(x)\right]$$

$$y(x) \to -\frac{ix^2}{2}$$

$$y(x) \to \frac{ix^2}{2}$$

1.469 problem 471

Internal problem ID [8806]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 471.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{yy'^2 - (y - x)\, y' = x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 33

```
dsolve(y(x)*diff(y(x),x)^2-(y(x)-x)*diff(y(x),x)-x = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{-x^2 + c_1}$$
$$y(x) = -\sqrt{-x^2 + c_1}$$
$$y(x) = x + c_1$$

✓ Solution by Mathematica
Time used: 0.117 (sec). Leaf size: 47

```
DSolve[-x - (-x + y[x])*y'[x] + y[x]*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + c_1$$
$$y(x) \to -\sqrt{-x^2 + 2c_1}$$
$$y(x) \to \sqrt{-x^2 + 2c_1}$$

1.470 problem 472

Internal problem ID [8807]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 472.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{(x+y)\,y'^2+2xy'-y=0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 121

```
dsolve((x+y(x))*diff(y(x),x)^2+2*x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x)=-\frac{x\left(1+i\sqrt{3}\right)}{2}$$

$$y(x)=\frac{x\left(i\sqrt{3}-1\right)}{2}$$

$$\ln\left(x\right)-\operatorname{arctanh}\left(\frac{y(x)+2x}{2x\sqrt{\frac{y(x)^2+xy(x)+x^2}{x^2}}}\right)+\ln\left(\frac{y(x)}{x}\right)-c_1=0$$

$$\ln\left(x\right)+\operatorname{arctanh}\left(\frac{y(x)+2x}{2x\sqrt{\frac{y(x)^2+xy(x)+x^2}{x^2}}}\right)+\ln\left(\frac{y(x)}{x}\right)-c_1=0$$

✓ <u>Solution by Mathematica</u>

Time used: 4.051 (sec). Leaf size: 166

```
DSolve[-y[x] + 2*x*y'[x] + (x + y[x])*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) &\to -\frac{2}{3}\sqrt{e^{c_1}\left(-3x+e^{c_1}\right)}-\frac{e^{c_1}}{3} \\
y(x) &\to \frac{2}{3}\sqrt{e^{c_1}\left(-3x+e^{c_1}\right)}-\frac{e^{c_1}}{3} \\
y(x) &\to e^{c_1}-2\sqrt{e^{c_1}\left(x+e^{c_1}\right)} \\
y(x) &\to 2\sqrt{e^{c_1}\left(x+e^{c_1}\right)}+e^{c_1} \\
y(x) &\to 0 \\
y(x) &\to -\frac{1}{2}i\left(\sqrt{3}-i\right)x \\
y(x) &\to \frac{1}{2}i\left(\sqrt{3}+i\right)x
\end{aligned}
$$

1.471 problem 473

Internal problem ID [8808]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 473.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$(-2x+y)\, y'^2 - 2(x-1)\, y' + y = 2$$

✓ Solution by Maple

Time used: 1.734 (sec). Leaf size: 71

```
dsolve((y(x)-2*x)*diff(y(x),x)^2-2*(x-1)*diff(y(x),x)+y(x)-2 = 0,y(x), singsol=all)
```

$$y(x) = -\sqrt{2}\, x + \sqrt{2} + x + 1$$
$$y(x) = (x-1)\sqrt{2} + x + 1$$
$$y(x) = 2 + \frac{c_1}{2} - \frac{\sqrt{c_1(-c_1+4x-4)}}{2}$$
$$y(x) = 2 + c_1 - \sqrt{c_1(-c_1+2x-2)}$$

✓ Solution by Mathematica

Time used: 3.644 (sec). Leaf size: 187

```
DSolve[-2 + y[x] - 2*(-1 + x)*y'[x] + (-2*x + y[x])*y'[x]^2==0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to -\frac{1}{2}\sqrt{-e^{c_1}(4x-4+e^{c_1})} + 2 - \frac{e^{c_1}}{2}$$
$$y(x) \to \frac{1}{2}\left(\sqrt{-e^{c_1}(4x-4+e^{c_1})} + 4 - e^{c_1}\right)$$
$$y(x) \to -\sqrt{-e^{c_1}(2x-2+e^{c_1})} + 2 - e^{c_1}$$
$$y(x) \to \sqrt{-e^{c_1}(2x-2+e^{c_1})} + 2 - e^{c_1}$$
$$y(x) \to 2$$
$$y(x) \to x - \sqrt{2}\sqrt{(x-1)^2} + 1$$
$$y(x) \to x + \sqrt{2}\sqrt{(x-1)^2} + 1$$

1.472 problem 474

Internal problem ID [8809]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 474.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, _dAlembert]`

$$2y{y'}^2 - (4x - 5)\, y' + 2y = 0$$

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 119

```
dsolve(2*y(x)*diff(y(x),x)^2-(4*x-5)*diff(y(x),x)+2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = x - \frac{5}{4}$$
$$y(x) = -x + \frac{5}{4}$$
$$y(x) = 0$$
$$y(x) = \frac{\sqrt{4c_1 + 2\sqrt{-c_1 (4x - 5)^2}}}{2}$$
$$y(x) = -\frac{\sqrt{4c_1 + 2\sqrt{-c_1 (4x - 5)^2}}}{2}$$
$$y(x) = \frac{\sqrt{4c_1 - 2\sqrt{-c_1 (4x - 5)^2}}}{2}$$
$$y(x) = -\frac{\sqrt{4c_1 - 2\sqrt{-c_1 (4x - 5)^2}}}{2}$$

✓ Solution by Mathematica

Time used: 0.659 (sec). Leaf size: 160

```
DSolve[2*y[x] - (-5 + 4*x)*y'[x] + 2*y[x]*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -i\sqrt{2}e^{\frac{c_1}{2}}\sqrt{4x-5+8e^{c_1}}$$
$$y(x) \to i\sqrt{2}e^{\frac{c_1}{2}}\sqrt{4x-5+8e^{c_1}}$$
$$y(x) \to -\frac{1}{4}ie^{\frac{c_1}{2}}\sqrt{8x-10+e^{c_1}}$$
$$y(x) \to \frac{1}{4}ie^{\frac{c_1}{2}}\sqrt{8x-10+e^{c_1}}$$
$$y(x) \to 0$$
$$y(x) \to \frac{5}{4}-x$$
$$y(x) \to x-\frac{5}{4}$$

1.473 problem 475

Internal problem ID [8810]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 475.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$4y{y'}^2 + 2xy' - y = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 67

```
dsolve(4*y(x)*diff(y(x),x)^2+2*x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{ix}{2}$$
$$y(x) = \frac{ix}{2}$$
$$y(x) = 0$$
$$y(x) = \sqrt{c_1 (-x + c_1)}$$
$$y(x) = \sqrt{c_1 (x + c_1)}$$
$$y(x) = -\sqrt{c_1 (-x + c_1)}$$
$$y(x) = -\sqrt{c_1 (x + c_1)}$$

✓ Solution by Mathematica

Time used: 0.527 (sec). Leaf size: 140

```
DSolve[-y[x] + 2*x*y'[x] + 4*y[x]*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}e^{2c_1}\sqrt{-2x + e^{4c_1}}$$
$$y(x) \to \frac{1}{2}e^{2c_1}\sqrt{-2x + e^{4c_1}}$$
$$y(x) \to -\frac{1}{2}e^{2c_1}\sqrt{2x + e^{4c_1}}$$
$$y(x) \to \frac{1}{2}e^{2c_1}\sqrt{2x + e^{4c_1}}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{ix}{2}$$
$$y(x) \to \frac{ix}{2}$$

1.474 problem 476

Internal problem ID [8811]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 476.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{9y{y'}^2+4y'x^3-4x^2y=0}$$

✓ Solution by Maple

Time used: 0.235 (sec). Leaf size: 89

```
dsolve(9*y(x)*diff(y(x),x)^2+4*x^3*diff(y(x),x)-4*x^2*y(x) = 0,y(x), singsol=all)
```

$$
\begin{aligned}
y(x) &= -\frac{i x^2}{3} \\
y(x) &= \frac{i x^2}{3} \\
y(x) &= 0 \\
y(x) &= -\frac{2\sqrt{c_1 x^2+9}}{c_1} \\
y(x) &= \frac{2\sqrt{c_1 x^2+9}}{c_1} \\
y(x) &= -\frac{\sqrt{c_1\left(-4x^2+c_1\right)}}{6} \\
y(x) &= \frac{\sqrt{c_1\left(-4x^2+c_1\right)}}{6}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 1.23 (sec). Leaf size: 244

```
DSolve[-4*x^2*y[x] + 4*x^3*y'[x] + 9*y[x]*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True
```

$$\text{Solve}\left[\frac{\sqrt{x^6+9x^2y(x)^2}\log\left(\sqrt{x^4+9y(x)^2}+x^2\right)}{2x\sqrt{x^4+9y(x)^2}}+\frac{1}{2}\left(1-\frac{\sqrt{x^6+9x^2y(x)^2}}{x\sqrt{x^4+9y(x)^2}}\right)\log(y(x))=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{2}\left(\frac{\sqrt{x^6+9x^2y(x)^2}}{x\sqrt{x^4+9y(x)^2}}+1\right)\log(y(x))-\frac{\sqrt{x^6+9x^2y(x)^2}\log\left(\sqrt{x^4+9y(x)^2}+x^2\right)}{2x\sqrt{x^4+9y(x)^2}}=c_1,y(x)\right]$$

$$y(x)\to -\frac{ix^2}{3}$$

$$y(x)\to \frac{ix^2}{3}$$

1.475 problem 477

Internal problem ID [8812]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 477.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, _dAlembert]`

$$\boxed{ayy'^2 + (2x - b)\, y' - y = 0}$$

✓ Solution by Maple

Time used: 0.312 (sec). Leaf size: 741

```
dsolve(a*y(x)*diff(y(x),x)^2+(2*x-b)*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{-2x+b}{2\sqrt{-a}}$$

$$y(x) = \frac{-2x+b}{2\sqrt{-a}}$$

$$y(x) = 0$$

$$-4a\left(\int^{y(x)} \frac{16\_f\left(\frac{1}{16}+\left(\left(\frac{b}{4}-\frac{x}{2}\right)\sqrt{4a\_f^2+b^2-4bx+4x^2}+a\_f^2+\frac{b^2}{4}-bx+x^2\right)\left(\int_{\_b}^{x} \frac{-}{\left(-4a\_f^2+2\sqrt{4a}\right.}\right.\right.}{(-2x+b)\sqrt{4a\_f^2+b^2-4bx+}}\right.$$

$$+2\left(\int_{\_b}^{x}\frac{-2\_a+b+\sqrt{4ay(x)^2+(-2\_a+b)^2}}{(-2\_a+b)\sqrt{4ay(x)^2+(-2\_a+b)^2}+4ay(x)^2+(-2\_a+b)^2}d\_a\right)+c_1=0$$

$$-4a\left(\int^{y(x)} \frac{16\_f\left(\frac{1}{16}+\left(\left(-\frac{b}{4}+\frac{x}{2}\right)\sqrt{4a\_f^2+b^2-4bx+4x^2}+a\_f^2+\frac{b^2}{4}-bx+x^2\right)\left(\int_{\_b}^{x} \frac{}{\left(4a\_f^2+2\sqrt{4a}\right.}\right.\right.}{(2x-b)\sqrt{4a\_f^2+b^2-4bx+}}\right.$$

$$-2\left(\int_{\_b}^{x}\frac{2\_a-b+\sqrt{4ay(x)^2+(-2\_a+b)^2}}{(2\_a-b)\sqrt{4ay(x)^2+(-2\_a+b)^2}+4ay(x)^2+(-2\_a+b)^2}d\_a\right)+c_1=0$$

✓ Solution by Mathematica

Time used: 0.873 (sec). Leaf size: 187

```
DSolve[-y[x] + (-b + 2*x)*y'[x] + a*y[x]*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) &\to -\sqrt{2}e^{\frac{c_1}{2}}\sqrt{2ae^{c_1}+b-2x} \\
y(x) &\to \sqrt{2}e^{\frac{c_1}{2}}\sqrt{2ae^{c_1}+b-2x} \\
y(x) &\to -\frac{e^{\frac{c_1}{2}}\sqrt{-2b+4x+e^{c_1}}}{2\sqrt{a}} \\
y(x) &\to \frac{e^{\frac{c_1}{2}}\sqrt{-2b+4x+e^{c_1}}}{2\sqrt{a}} \\
y(x) &\to 0 \\
y(x) &\to -\frac{i(b-2x)}{2\sqrt{a}} \\
y(x) &\to \frac{i(b-2x)}{2\sqrt{a}}
\end{aligned}
$$

1.476 problem 478

Internal problem ID [8813]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 478.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$(ay+b)\left(1+{y'}^2\right)=c$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 162

```
dsolve((a*y(x)+b)*(diff(y(x),x)^2+1)-c = 0,y(x), singsol=all)
```

$$y(x)=\frac{-b+c}{a}$$

$$\frac{-\arctan\left(\frac{2ay(x)+2b-c}{2\sqrt{-(ay(x)+b)(ay(x)+b-c)}}\right)c+2\sqrt{-\left(ay\left(x\right)+b\right)\left(ay\left(x\right)+b-c\right)}+\left(2x-2c_1\right)a}{2a}=0$$

$$\frac{\arctan\left(\frac{2ay(x)+2b-c}{2\sqrt{-(ay(x)+b)(ay(x)+b-c)}}\right)c-2\sqrt{-\left(ay\left(x\right)+b\right)\left(ay\left(x\right)+b-c\right)}+\left(2x-2c_1\right)a}{2a}=0$$

✓ Solution by Mathematica

Time used: 0.409 (sec). Leaf size: 154

```
DSolve[-c + (b + a*y[x])*(1 + y'[x]^2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{c\arctan\left(\frac{\sqrt{\#1a+b}}{\sqrt{-\#1a-b+c}}\right)-\sqrt{\#1a+b}\sqrt{-\#1a-b+c}}{a}\&\right][-x+c_1]$$

$$y(x) \to \text{InverseFunction}\left[\frac{c\arctan\left(\frac{\sqrt{\#1a+b}}{\sqrt{-\#1a-b+c}}\right)-\sqrt{\#1a+b}\sqrt{-\#1a-b+c}}{a}\&\right][x+c_1]$$

$$y(x) \to \frac{c-b}{a}$$

1.477 problem 479

Internal problem ID [8814]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 479.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, _dAlembert]`

$$(b_2y + a_2x + c_2)\, {y'}^2 + (a_1x + b_1y + c_1)\, y' + b_0y = -a_0x - c_0$$

✓ Solution by Maple

Time used: 1.234 (sec). Leaf size: 875

```
dsolve((b__2*y(x)+a__2*x+c__2)*diff(y(x),x)^2+(a__1*x+b__1*y(x)+c__1)*diff(y(x),x)+a__0*x+b_
```

$$x$$

$$-e^{\int \frac{-a_1x-b_1y(x)-c_1-\sqrt{\left(-4b_0b_2+b_1^2\right)y(x)^2+((-4a_0b_2+2a_1b_1-4a_2b_0)x-4b_2c_0+2c_1b_1-4c_2b_0)y(x)+\left(-4a_0a_2+a_1^2\right)x^2+(-4a_0c_2+2a_1c_1-4c_0a_2)x-4c_2c_0+c}}{2b_2y(x)+2c_2+2a_2x}}$$

$$+ c_3 \Bigg) = 0$$

$$x$$

$$-e^{\int \frac{-a_1x-b_1y(x)-c_1+\sqrt{\left(-4b_0b_2+b_1^2\right)y(x)^2+((-4a_0b_2+2a_1b_1-4a_2b_0)x-4b_2c_0+2c_1b_1-4c_2b_0)y(x)+\left(-4a_0a_2+a_1^2\right)x^2+(-4a_0c_2+2a_1c_1-4c_0a_2)x-4c_2c_0+c}}{2b_2y(x)+2c_2+2a_2x}}$$

$$+ c_3 \Bigg) = 0$$

✓ Solution by Mathematica

Time used: 4.811 (sec). Leaf size: 576

```
DSolve[c0 + a0*x + b0*y[x] + (c1 + a1*x + b1*y[x])*y'[x] + (c2 + a2*x + b2*y[x])*y'[x]^2==0,
```

$$\mathrm{Solve}\left[\left\{x = \right.\right.$$

$$-\frac{-(K[2](\mathrm{b2}K[2]+\mathrm{b1})+\mathrm{b0})\exp\left(\mathrm{RootSum}\left[\#1^3\mathrm{b2}+\#1^2\mathrm{a2}+\#1^2\mathrm{b1}+\#1\mathrm{a1}+\#1\mathrm{b0}+\mathrm{a0}\&,\frac{\#1^2\mathrm{b2}\,\mathrm{lo}}{}\right.\right.}{}$$

$$-\frac{K[2](K[2](\mathrm{c2}K[2]+\mathrm{c1})+\mathrm{c0})+(K[2](\mathrm{a2}K[2]+\mathrm{a1})+\mathrm{a0})\exp\left(\mathrm{RootSum}\left[\#1^3\mathrm{b2}+\#1^2\mathrm{a2}+\#1^2\mathrm{b1}+\right.\right.}{}$$

1.478 problem 480

Internal problem ID [8815]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 480.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational]`

$$\boxed{(ay - x^2)\, {y'}^2 + 2xy{y'}^2 - y^2 = 0}$$

✗ Solution by Maple

```
dsolve((a*y(x)-x^2)*diff(y(x),x)^2+2*x*y(x)*diff(y(x),x)^2-y(x)^2 = 0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-y[x]^2 + 2*x*y[x]*y'[x]^2 + (-x^2 + a*y[x])*y'[x]^2==0,y[x],x,IncludeSingularSolutio
```

Not solved

1.479 problem 481

Internal problem ID [8816]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 481.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$xyy'^2 + (x^2 + y^2)\, y' + yx = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(x*y(x)*diff(y(x),x)^2+(y(x)^2+x^2)*diff(y(x),x)+x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x}$$
$$y(x) = \sqrt{-x^2 + c_1}$$
$$y(x) = -\sqrt{-x^2 + c_1}$$

✓ Solution by Mathematica

Time used: 0.104 (sec). Leaf size: 54

```
DSolve[x*y[x] + (x^2 + y[x]^2)*y'[x] + x*y[x]*y'[x]^2==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{c_1}{x}$$
$$y(x) \to -\sqrt{-x^2 + 2c_1}$$
$$y(x) \to \sqrt{-x^2 + 2c_1}$$
$$y(x) \to 0$$

1.480 problem 482

Internal problem ID [8817]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 482.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational]`

$$xyy'^2 + (x^{22} - y^2 + a)\, y' - yx = 0$$

✗ Solution by Maple

```
dsolve(x*y(x)*diff(y(x),x)^2+(x^22-y(x)^2+a)*diff(y(x),x)-x*y(x) = 0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-(x*y[x]) + (a + x^22 - y[x]^2)*y'[x] + x*y[x]*y'[x]^2==0,y[x],x,IncludeSingularSolut
```

Not solved

1.481 problem 483

Internal problem ID [8818]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 483.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\left(2yx - x^2\right) {y'}^2 + 2xy'y + 2yx - y^2 = 0$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 103

```
dsolve((2*x*y(x)-x^2)*diff(y(x),x)^2+2*x*y(x)*diff(y(x),x)+2*x*y(x)-y(x)^2 = 0,y(x), singsol
```

$$y(x) = 0$$

$$y(x) = \operatorname{RootOf}\left(-2\ln(x) - \left(\int^{\_Z} \frac{2\_a^2 + \sqrt{2}\sqrt{\_a(\_a-1)^2}}{\_a(\_a^2+1)} d\_a\right) + 2c_1\right) x$$

$$y(x) = \operatorname{RootOf}\left(-2\ln(x) + \int^{\_Z} \frac{\sqrt{2}\sqrt{\_a(\_a-1)^2} - 2\_a^2}{\_a(\_a^2+1)} d\_a + 2c_1\right) x$$

✓ Solution by Mathematica

Time used: 4.343 (sec). Leaf size: 167

```
DSolve[2*x*y[x] - y[x]^2 + 2*x*y[x]*y'[x] + (-x^2 + 2*x*y[x])*y'[x]^2==0,y[x],x,IncludeSingu
```

$$y(x) \to -\sqrt{-x\left(x+2e^{\frac{c_1}{2}}\right)} - e^{\frac{c_1}{2}}$$
$$y(x) \to \sqrt{-x\left(x+2e^{\frac{c_1}{2}}\right)} - e^{\frac{c_1}{2}}$$
$$y(x) \to e^{\frac{c_1}{2}} - \sqrt{x\left(-x+2e^{\frac{c_1}{2}}\right)}$$
$$y(x) \to \sqrt{x\left(-x+2e^{\frac{c_1}{2}}\right)} + e^{\frac{c_1}{2}}$$
$$y(x) \to -\sqrt{-x^2}$$
$$y(x) \to \sqrt{-x^2}$$

1.482 problem 484

Internal problem ID [8819]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 484.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$(2xy - x^2)\, y'^2 - 6xyy' - y^2 + 2xy = 0$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 115

```
dsolve((2*x*y(x)-x^2)*diff(y(x),x)^2-6*x*y(x)*diff(y(x),x)-y(x)^2+2*x*y(x) = 0,y(x), singsol
```

$$y(x) = 0$$

$$y(x) = \mathrm{RootOf}\left(-2\ln(x) - \left(\int^{\_Z} \frac{2\_a^2 + \sqrt{2}\sqrt{\_a(\_a+1)^2} - 4\_a}{\_a(\_a^2 - 4\_a + 1)} d\_a\right) + 2c_1\right) x$$

$$y(x) = \mathrm{RootOf}\left(-2\ln(x) + \int^{\_Z} \frac{\sqrt{2}\sqrt{\_a(\_a+1)^2} - 2\_a^2 + 4\_a}{\_a(\_a^2 - 4\_a + 1)} d\_a + 2c_1\right) x$$

✓ Solution by Mathematica

Time used: 6.478 (sec). Leaf size: 196

```
DSolve[2*x*y[x] - y[x]^2 - 6*x*y[x]*y'[x] + (-x^2 + 2*x*y[x])*y'[x]^2==0,y[x],x,IncludeSingu
```

$$y(x) \to 2x - \sqrt{x\left(3x - 2e^{\frac{c_1}{2}}\right)} - e^{\frac{c_1}{2}}$$
$$y(x) \to 2x + \sqrt{x\left(3x - 2e^{\frac{c_1}{2}}\right)} - e^{\frac{c_1}{2}}$$
$$y(x) \to 2x - \sqrt{x\left(3x + 2e^{\frac{c_1}{2}}\right)} + e^{\frac{c_1}{2}}$$
$$y(x) \to 2x + \sqrt{x\left(3x + 2e^{\frac{c_1}{2}}\right)} + e^{\frac{c_1}{2}}$$
$$y(x) \to 2x - \sqrt{3}\sqrt{x^2}$$
$$y(x) \to \sqrt{3}\sqrt{x^2} + 2x$$

1.483 problem 485

Internal problem ID [8820]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 485.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational]`

$$a x y {y'}^2 - \left(a y^2 + b\, x^2 + c\right) y' + b x y = 0$$

✗ Solution by Maple

```
dsolve(a*x*y(x)*diff(y(x),x)^2-(a*y(x)^2+b*x^2+c)*diff(y(x),x)+b*x*y(x) = 0,y(x), singsol=al
```

No solution found

✓ Solution by Mathematica

Time used: 3.583 (sec). Leaf size: 155

```
DSolve[b*x*y[x] - (c + b*x^2 + a*y[x]^2)*y'[x] + a*x*y[x]*y'[x]^2==0,y[x],x,IncludeSingularS
```

$$y(x) \to \sqrt{c_1 \left(x^2 + \frac{c}{b - ac_1}\right)}$$

$$y(x) \to -\sqrt{-\frac{\left(\sqrt{c} + i\sqrt{b}x\right)^2}{a}}$$

$$y(x) \to \sqrt{-\frac{\left(\sqrt{c} + i\sqrt{b}x\right)^2}{a}}$$

$$y(x) \to -\sqrt{-\frac{\left(\sqrt{c} - i\sqrt{b}x\right)^2}{a}}$$

$$y(x) \to \sqrt{-\frac{\left(\sqrt{c} - i\sqrt{b}x\right)^2}{a}}$$

1.484 problem 486

Internal problem ID [8821]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 486.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y^2 {y'}^2 + y^2 = a^2$$

✓ Solution by Maple
Time used: 0.109 (sec). Leaf size: 54

```
dsolve(y(x)^2*diff(y(x),x)^2+y(x)^2-a^2 = 0,y(x), singsol=all)
```

$$y(x) = -a$$
$$y(x) = a$$
$$y(x) = \sqrt{a^2 - c_1^2 + 2c_1 x - x^2}$$
$$y(x) = -\sqrt{(a + x - c_1)(c_1 + a - x)}$$

✓ Solution by Mathematica
Time used: 0.209 (sec). Leaf size: 101

```
DSolve[-a^2 + y[x]^2 + y[x]^2*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{a^2 - (x + c_1)^2}$$
$$y(x) \to \sqrt{a^2 - (x + c_1)^2}$$
$$y(x) \to -\sqrt{a^2 - (x - c_1)^2}$$
$$y(x) \to \sqrt{a^2 - (x - c_1)^2}$$
$$y(x) \to -a$$
$$y(x) \to a$$

1.485 problem 487

Internal problem ID [8822]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 487.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y^2 {y'}^2 - 6y' x^3 + 4x^2 y = 0$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 107

```
dsolve(y(x)^2*diff(y(x),x)^2-6*x^3*diff(y(x),x)+4*x^2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{18^{\frac{1}{3}} x^{\frac{4}{3}}}{2}$$
$$y(x) = -\frac{18^{\frac{1}{3}} x^{\frac{4}{3}} (1 + i\sqrt{3})}{4}$$
$$y(x) = \frac{18^{\frac{1}{3}} x^{\frac{4}{3}} (i\sqrt{3} - 1)}{4}$$
$$y(x) = 0$$
$$y(x) = \operatorname{RootOf}\left(-4\ln(x) - 3\left(\int^{\_Z} \frac{4\_a^3 + 3\sqrt{-4\_a^3 + 9} - 9}{\_a(4\_a^3 - 9)} d\_a\right) + 4c_1\right) x^{\frac{4}{3}}$$

✓ Solution by Mathematica

Time used: 2.383 (sec). Leaf size: 304

```
DSolve[4*x^2*y[x] - 6*x^3*y'[x] + y[x]^2*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\sqrt{9x^6-4x^2y(x)^3}\log\left(\sqrt{9x^4-4y(x)^3}+3x^2\right)}{2x\sqrt{9x^4-4y(x)^3}} - \frac{3}{4}\left(\frac{\sqrt{9x^6-4x^2y(x)^3}\log(y(x))}{x\sqrt{9x^4-4y(x)^3}}-\log(y(x))\right)=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{3}{4}\left(\frac{\sqrt{9x^6-4x^2y(x)^3}\log(y(x))}{x\sqrt{9x^4-4y(x)^3}}+\log(y(x))\right) - \frac{\sqrt{9x^6-4x^2y(x)^3}\log\left(\sqrt{9x^4-4y(x)^3}+3x^2\right)}{2x\sqrt{9x^4-4y(x)^3}}=c_1,y(x)\right]$$

$$y(x)\to\left(-\frac{3}{2}\right)^{2/3}x^{4/3}$$

$$y(x)\to\left(\frac{3}{2}\right)^{2/3}x^{4/3}$$

$$y(x)\to-\sqrt[3]{-1}\left(\frac{3}{2}\right)^{2/3}x^{4/3}$$

1.486 problem 488

Internal problem ID [8823]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 488.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y^2y'^2 - 4ayy' + y^2 = -4a^2 + 4xa$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 72

```
dsolve(y(x)^2*diff(y(x),x)^2-4*a*y(x)*diff(y(x),x)+y(x)^2-4*a*x+4*a^2 = 0,y(x), singsol=all)
```

$$y(x) = -2\sqrt{ax}$$
$$y(x) = 2\sqrt{ax}$$
$$y(x) = \sqrt{4ax - c_1^2 + 2c_1x - x^2}$$
$$y(x) = -\sqrt{-x^2 + (4a + 2c_1)\,x - c_1^2}$$

✓ Solution by Mathematica

Time used: 0.644 (sec). Leaf size: 85

```
DSolve[4*a^2 - 4*a*x + y[x]^2 - 4*a*y[x]*y'[x] + y[x]^2*y'[x]^2==0,y[x],x,IncludeSingularSol
```

$$y(x) \to -\frac{\sqrt{16a^3x - 4a^2x^2 - 4ac_1x - c_1{}^2}}{2a}$$
$$y(x) \to \frac{\sqrt{16a^3x - 4a^2x^2 - 4ac_1x - c_1{}^2}}{2a}$$

1.487 problem 489

Internal problem ID [8824]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 489.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational]`

$$\boxed{y^2{y'}^2+2xy'y+ay^2=-xb-c}$$

✓ Solution by Maple

Time used: 0.328 (sec). Leaf size: 365

```
dsolve(y(x)^2*diff(y(x),x)^2+2*x*y(x)*diff(y(x),x)+a*y(x)^2+b*x+c = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2\sqrt{a\left(a\left(ax-\frac{1}{2}b+x\right)^2(a+1)^2\operatorname{RootOf}\left(-2b\ln\left(2ax-b+2x\right)+b\left(\int^{\_Z}-\frac{4\_a a^2-\sqrt{-(4\_a a^3+8\_a a^2}}{\_a(4\_a}\right.\right.\right.}}{a\left(a+1\right)}$$

$$y(x) = \frac{2\sqrt{a\left(a\left(ax-\frac{1}{2}b+x\right)^2(a+1)^2\operatorname{RootOf}\left(-2b\ln\left(2ax-b+2x\right)+b\left(\int^{\_Z}-\frac{4\_a a^2-\sqrt{-(4\_a a^3+8\_a a^2+4}}{\_a(4\_a a^2}\right.\right.\right.}}{a\left(a+1\right)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[c + b*x + a*y[x]^2 + 2*x*y[x]*y'[x] + y[x]^2*y'[x]^2==0,y[x],x,IncludeSingularSolutio
```

Timed out

1.488 problem 490

Internal problem ID [8825]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 490.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y^2 {y'}^2 - 2xy'y + 2y^2 = x^2 - a$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 83

```
dsolve(y(x)^2*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x)+2*y(x)^2-x^2+a = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{4x^2 - 2a}}{2}$$
$$y(x) = \frac{\sqrt{4x^2 - 2a}}{2}$$
$$y(x) = -\frac{\sqrt{-8c_1^2 + 16c_1 x - 4x^2 - 2a}}{2}$$
$$y(x) = \frac{\sqrt{-8c_1^2 + 16c_1 x - 4x^2 - 2a}}{2}$$

✓ Solution by Mathematica

Time used: 0.661 (sec). Leaf size: 63

```
DSolve[a - x^2 + 2*y[x]^2 - 2*x*y[x]*y'[x] + y[x]^2*y'[x]^2==0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to -\sqrt{-\frac{a}{2} - x^2 + 4c_1 x - 2c_1{}^2}$$
$$y(x) \to \sqrt{-\frac{a}{2} - x^2 + 4c_1 x - 2c_1{}^2}$$

1.489 problem 491

Internal problem ID [8826]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 491.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y^2 y'^2 + 2axyy' + (-a+1)\,y^2 = -a\,x^2 - (a-1)\,b$$

✓ Solution by Maple

Time used: 0.235 (sec). Leaf size: 88

```
dsolve(y(x)^2*diff(y(x),x)^2+2*a*x*y(x)*diff(y(x),x)+(1-a)*y(x)^2+a*x^2+(a-1)*b = 0,y(x), si
```

$$y(x) = \sqrt{-a\,x^2+b}$$
$$y(x) = -\sqrt{-a\,x^2+b}$$
$$y(x) = \sqrt{a\,c_1^2 - 2ac_1x - c_1^2 + 2c_1x - x^2 + b}$$
$$y(x) = -\sqrt{(a-1)\,c_1^2 - 2x\,(a-1)\,c_1 - x^2 + b}$$

✓ Solution by Mathematica

Time used: 1.059 (sec). Leaf size: 65

```
DSolve[(-1 + a)*b + a*x^2 + (1 - a)*y[x]^2 + 2*a*x*y[x]*y'[x] + y[x]^2*y'[x]^2==0,y[x],x,Inc
```

$$y(x) \to -\sqrt{-2(a-1)c_1x + (a-1)c_1{}^2 + b - x^2}$$
$$y(x) \to \sqrt{-2(a-1)c_1x + (a-1)c_1{}^2 + b - x^2}$$

1.490 problem 492

Internal problem ID [8827]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 492.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(y^2 - a^2)\, y'^2 + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 115

```
dsolve((y(x)^2-a^2)*diff(y(x),x)^2+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = 0$$

$$a \operatorname{csgn}(a) \ln(2) + a \operatorname{csgn}(a) \ln\left(\frac{a\left(\operatorname{csgn}(a)\sqrt{-y(x)^2 + a^2} + a\right)}{y(x)}\right) - \sqrt{-y(x)^2 + a^2} - c_1 + x = 0$$

$$-a \operatorname{csgn}(a) \ln(2) - a \operatorname{csgn}(a) \ln\left(\frac{a\left(\operatorname{csgn}(a)\sqrt{-y(x)^2 + a^2} + a\right)}{y(x)}\right) + \sqrt{-y(x)^2 + a^2} - c_1 + x = 0$$

✓ Solution by Mathematica

Time used: 0.317 (sec). Leaf size: 102

```
DSolve[y[x]^2 + (-a^2 + y[x]^2)*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\sqrt{a^2 - \#1^2} - a\text{arctanh}\left(\frac{\sqrt{a^2 - \#1^2}}{a}\right)\&\right][-x + c_1]$$
$$y(x) \to \text{InverseFunction}\left[\sqrt{a^2 - \#1^2} - a\text{arctanh}\left(\frac{\sqrt{a^2 - \#1^2}}{a}\right)\&\right][x + c_1]$$
$$y(x) \to 0$$

1.491 problem 493

Internal problem ID [8828]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 493.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `['y=_G(x,y')']`

$$(y^2 - 2xa + a^2)\, y'^2 + 2ayy' + y^2 = 0$$

✓ Solution by Maple

Time used: 0.453 (sec). Leaf size: 113

```
dsolve((y(x)^2-2*a*x+a^2)*diff(y(x),x)^2+2*a*y(x)*diff(y(x),x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = 0$$

$$\left[x(\_T) = \frac{\sqrt{\_T^2+1}\,\operatorname{arctanh}\left(\frac{1}{\sqrt{\_T^2+1}}\right)^2 a^2 + \left(-2ac_1\sqrt{\_T^2+1} - 2a^2\right)\operatorname{arctanh}\left(\frac{1}{\sqrt{\_T^2+1}}\right) + (a^2 + c}{2\sqrt{\_T^2+1}\,a}\right.$$

✓ Solution by Mathematica

Time used: 49.544 (sec). Leaf size: 408

```
DSolve[y[x]^2 + 2*a*y[x]*y'[x] + (a^2 - 2*a*x + y[x]^2)*y'[x]^2==0,y[x],x,IncludeSingularSol
```

$$\text{Solve}\left[\left\{y(x) = \frac{-\sqrt{-aK[1]^2\left(aK[1]^2 - 2xK[1]^2 - 2x\right)} - aK[1]}{K[1]^2+1}, x = \frac{aK[1]^2\text{arctanh}\left(\sqrt{K[1]^2+1}\right)^2 + aar}{}\right.\right.$$

$$\text{Solve}\left[\left\{y(x) = \frac{\sqrt{-aK[2]^2\left(aK[2]^2 - 2xK[2]^2 - 2x\right)} - aK[2]}{K[2]^2+1}, x = \frac{aK[2]^2\text{arctanh}\left(\sqrt{K[2]^2+1}\right)^2 + aarct}{}\right.\right.$$

$$y(x) \to 0$$

1.492 problem 494

Internal problem ID [8829]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 494.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$(y^2 - a^2x^2)\, y'^2 + 2xy'y = -(-a^2+1)\, x^2$$

✓ Solution by Maple
Time used: 0.328 (sec). Leaf size: 157

```
dsolve((y(x)^2-a^2*x^2)*diff(y(x),x)^2+2*x*y(x)*diff(y(x),x)+(-a^2+1)*x^2 = 0,y(x), singsol=
```

$$y(x) = \sqrt{a^2-1}\, x$$
$$y(x) = -\sqrt{a^2-1}\, x$$
$$y(x) = \mathrm{RootOf}\left(-\ln(x) + \int^{\_Z} -\frac{\_a^3 - \_a\, a^2 - \sqrt{a^2(\_a^2-a^2+1)} + \_a}{(\_a^2+1)(\_a^2-a^2+1)} d\_a + c_1\right) x$$
$$y(x) = \mathrm{RootOf}\left(-\ln(x) - \left(\int^{\_Z} \frac{\_a^3 - \_a\, a^2 + \sqrt{a^2(\_a^2-a^2+1)} + \_a}{(\_a^2+1)(\_a^2-a^2+1)} d\_a\right) + c_1\right) x$$

✓ Solution by Mathematica
Time used: 0.265 (sec). Leaf size: 80

```
DSolve[(1 - a^2)*x^2 + 2*x*y[x]*y'[x] + (-(a^2*x^2) + y[x]^2)*y'[x]^2==0,y[x],x,IncludeSingu
```

$$y(x) \to ac_1 - \sqrt{-x^2 + c_1^2}$$
$$y(x) \to ac_1 + \sqrt{-x^2 + c_1^2}$$
$$y(x) \to -\sqrt{a^2-1}x$$
$$y(x) \to \sqrt{a^2-1}x$$

1.493 problem 495

Internal problem ID [8830]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 495.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$(y^2 + (-a+1)\, x^2)\, y'^2 + 2axyy' + (-a+1)\, y^2 = -x^2$$

✓ Solution by Maple

Time used: 0.312 (sec). Leaf size: 75

```
dsolve((y(x)^2+(1-a)*x^2)*diff(y(x),x)^2+2*a*x*y(x)*diff(y(x),x)+(1-a)*y(x)^2+x^2 = 0,y(x),
```

$$y(x) = -ix$$
$$y(x) = ix$$
$$y(x) = \tan\left(\text{RootOf}\left(-2\_Z\sqrt{a-1} - \ln\left(\sec\left(\_Z\right)^2 x^2\right) + 2c_1\right)\right) x$$
$$y(x) = \tan\left(\text{RootOf}\left(2\_Z\sqrt{a-1} - \ln\left(\sec\left(\_Z\right)^2 x^2\right) + 2c_1\right)\right) x$$

✓ Solution by Mathematica

Time used: 0.319 (sec). Leaf size: 101

```
DSolve[x^2 + (1 - a)*y[x]^2 + 2*a*x*y[x]*y'[x] + ((1 - a)*x^2 + y[x]^2)*y'[x]^2==0,y[x],x,In
```

$$\text{Solve}\left[\sqrt{a-1}\arctan\left(\frac{y(x)}{x}\right) - \frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right) = \log(x) + c_1, y(x)\right]$$
$$\text{Solve}\left[\sqrt{a-1}\arctan\left(\frac{y(x)}{x}\right) + \frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right) = -\log(x) + c_1, y(x)\right]$$
$$y(x) \to -ix$$
$$y(x) \to ix$$

1.494 problem 496

Internal problem ID [8831]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 496.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$(y-x)^2\left(1+y'^2\right)-a^2(y'+1)^2=0$$

✓ Solution by Maple

Time used: 0.453 (sec). Leaf size: 135

```
dsolve((y(x)-x)^2*(diff(y(x),x)^2+1)-a^2*(diff(y(x),x)+1)^2 = 0,y(x), singsol=all)
```

$$y(x)=x-\sqrt{2}\,a$$
$$y(x)=x+\sqrt{2}\,a$$
$$y(x)=x+\operatorname{RootOf}\left(-2x-\left(\int^{\_Z}\frac{\_a^2-2a^2+\sqrt{-\_a^4+2\_a^2a^2}}{\_a^2-2a^2}d\_a\right)+2c_1\right)$$
$$y(x)=x+\operatorname{RootOf}\left(-2x+\int^{\_Z}-\frac{-2a^2+\_a^2-\sqrt{-\_a^4+2\_a^2a^2}}{\_a^2-2a^2}d\_a+2c_1\right)$$

✓ Solution by Mathematica

Time used: 50.68 (sec). Leaf size: 18407

```
DSolve[-(a^2*(1 + y'[x])^2) + (-x + y[x])^2*(1 + y'[x]^2)==0,y[x],x,IncludeSingularSolutions
```

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1.495 problem 497

Internal problem ID [8832]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 497.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{3y^2y'^2 - 2xy'y + 4y^2 = x^2}$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 105

```
dsolve(3*y(x)^2*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x)+4*y(x)^2-x^2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{3}\,x}{3}$$

$$y(x) = \frac{\sqrt{3}\,x}{3}$$

$$\ln(x) - \operatorname{arctanh}\left(\frac{\sqrt{\frac{x^2-3y(x)^2}{x^2}}}{2}\right) + \frac{\ln\left(\frac{y(x)^2+x^2}{x^2}\right)}{2} - c_1 = 0$$

$$\ln(x) + \operatorname{arctanh}\left(\frac{\sqrt{\frac{x^2-3y(x)^2}{x^2}}}{2}\right) + \frac{\ln\left(\frac{y(x)^2+x^2}{x^2}\right)}{2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.574 (sec). Leaf size: 179

```
DSolve[-x^2 + 4*y[x]^2 - 2*x*y[x]*y'[x] + 3*y[x]^2*y'[x]^2==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to -\frac{\sqrt{-3x^2 - 4ie^{3c_1}x + e^{6c_1}}}{\sqrt{3}}$$
$$y(x) \to \frac{\sqrt{-3x^2 - 4ie^{3c_1}x + e^{6c_1}}}{\sqrt{3}}$$
$$y(x) \to -\frac{\sqrt{-3x^2 + 4ie^{3c_1}x + e^{6c_1}}}{\sqrt{3}}$$
$$y(x) \to \frac{\sqrt{-3x^2 + 4ie^{3c_1}x + e^{6c_1}}}{\sqrt{3}}$$
$$y(x) \to -\sqrt{-x^2}$$
$$y(x) \to \sqrt{-x^2}$$

1.496 problem 498

Internal problem ID [8833]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 498.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(3y-2)\,{y'}^{2}+4y=4}$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 99

```
dsolve((3*y(x)-2)*diff(y(x),x)^2-4+4*y(x) = 0,y(x), singsol=all)
```

$$y(x)=1$$

$$y(x)=\frac{\sin\left(\operatorname{RootOf}\left(8\sqrt{3}\,c_1\_Z-8\sqrt{3}\,x\_Z+\cos\left(\_Z\right)^2-48c_1^2+96c_1x-48x^2-\_Z^2\right)\right)}{6}+\frac{5}{6}$$

$$y(x)=\frac{\sin\left(\operatorname{RootOf}\left(8\sqrt{3}\,c_1\_Z-8\sqrt{3}\,x\_Z-\cos\left(\_Z\right)^2+48c_1^2-96c_1x+48x^2+\_Z^2\right)\right)}{6}+\frac{5}{6}$$

✓ Solution by Mathematica

Time used: 0.367 (sec). Leaf size: 132

```
DSolve[-4 + 4*y[x] + (-2 + 3*y[x])*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{\arctan\left(\frac{\sqrt{3\#1-2}}{\sqrt{3-3\#1}}\right)}{\sqrt{3}} - \sqrt{1-\#1}\sqrt{3\#1-2}\&\right][-2x+c_1]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\arctan\left(\frac{\sqrt{3\#1-2}}{\sqrt{3-3\#1}}\right)}{\sqrt{3}} - \sqrt{1-\#1}\sqrt{3\#1-2}\&\right][2x+c_1]$$

$$y(x) \to 1$$

1.497 problem 499

Internal problem ID [8834]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 499.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\left(-a^2+1\right) y^2 {y'}^2 - 2a^2 x y y' + y^2 = a^2 x^2$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 191

```
dsolve((-a^2+1)*y(x)^2*diff(y(x),x)^2-2*a^2*x*y(x)*diff(y(x),x)+y(x)^2-a^2*x^2 = 0,y(x), sin
```

$$y(x) = \frac{xa}{\sqrt{-a^2+1}}$$
$$y(x) = -\frac{xa}{\sqrt{-a^2+1}}$$
$$y(x) = \mathrm{RootOf}\left(-\ln(x) + \int^{\_Z} -\frac{\left(\_a^2a^2 - \_a^2 + a^2 - \sqrt{\_a^2a^2 - \_a^2 + a^2}\right)\_a}{\left(\_a^2a^2 - \_a^2 + a^2\right)\left(\_a^2+1\right)} d\_a + c_1\right) x$$
$$y(x) = \mathrm{RootOf}\left(-\ln(x) - \left(\int^{\_Z} \frac{\left(\_a^2a^2 - \_a^2 + a^2 + \sqrt{\_a^2a^2 - \_a^2 + a^2}\right)\_a}{\left(\_a^2a^2 - \_a^2 + a^2\right)\left(\_a^2+1\right)} d\_a\right) + c_1\right) x$$

✓ Solution by Mathematica

Time used: 5.813 (sec). Leaf size: 251

```
DSolve[-(a^2*x^2) + y[x]^2 - 2*a^2*x*y[x]*y'[x] + (1 - a^2)*y[x]^2*y'[x]^2==0,y[x],x,Include
```

$$y(x) \to -\frac{\sqrt{(a^2-1)^3(-x^2) - 2(a^2-1)xe^{(a^2-1)c_1} + e^{2(a^2-1)c_1}}}{\sqrt{(a^2-1)^3}}$$

$$y(x) \to \frac{\sqrt{(a^2-1)^3(-x^2) - 2(a^2-1)xe^{(a^2-1)c_1} + e^{2(a^2-1)c_1}}}{\sqrt{(a^2-1)^3}}$$

$$y(x) \to -\frac{\sqrt{(a^2-1)^3(-x^2) + 2(a^2-1)xe^{(a^2-1)c_1} + e^{2(a^2-1)c_1}}}{\sqrt{(a^2-1)^3}}$$

$$y(x) \to \frac{\sqrt{(a^2-1)^3(-x^2) + 2(a^2-1)xe^{(a^2-1)c_1} + e^{2(a^2-1)c_1}}}{\sqrt{(a^2-1)^3}}$$

1.498 problem 500

Internal problem ID [8835]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 500.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, ‘_with_symmetry_[F(x),G(y)]‘]]`

$$\boxed{(a-b)\,y^2{y'}^2-2bxyy'+ay^2=x^2b+ab}$$

✓ Solution by Maple

Time used: 0.563 (sec). Leaf size: 766

```
dsolve((a-b)*y(x)^2*diff(y(x),x)^2-2*b*x*y(x)*diff(y(x),x)+a*y(x)^2-b*x^2-a*b = 0,y(x), sing
```

$$y(x) = \frac{\sqrt{b\left(x^2+a-b\right)\left(a-b\right)}}{a-b}$$
$$y(x) = -\frac{\sqrt{b\left(x^2+a-b\right)\left(a-b\right)}}{a-b}$$
$$\int_{\_b}^{x} \frac{-\_ab-\sqrt{a\left(\left(-a+b\right)y\left(x\right)^2+b\left(\_a^2+a-b\right)\right)}}{\sqrt{a\left(\left(-a+b\right)y\left(x\right)^2+b\left(\_a^2+a-b\right)\right)}\,\_a+\left(-a+b\right)y\left(x\right)^2+b\left(\_a^2+a-b\right)}d\_a$$
$$+\int^{y(x)} \frac{\_f\left(\left(\sqrt{a\left(-b^2+\left(\_f^2+x^2+a\right)b-a\_f^2\right)}\,x+\left(-a+b\right)\_f^2+b(x^2+a-b)\right)\left(\int_{\_b}^{x} \frac{(a-b)}{\sqrt{a\left(-b^2+\left(\_a^2+\right.\right.}}\right.\right.}{\sqrt{a\left(-b^2+\left(\_f^2+x^2+a\right)b-a\_f^2\right)}\,x}$$
$$+c_1 = 0$$
$$-\left(\int_{\_b}^{x} \frac{\_ab-\sqrt{a\left(\left(-a+b\right)y\left(x\right)^2+b\left(\_a^2+a-b\right)\right)}}{-\sqrt{a\left(\left(-a+b\right)y\left(x\right)^2+b\left(\_a^2+a-b\right)\right)}\,\_a+\left(-a+b\right)y\left(x\right)^2+b\left(\_a^2+a-b\right)}d\_a\right)$$
$$+\int^{y(x)} \frac{\left(\left(-\sqrt{a\left(-b^2+\left(\_f^2+x^2+a\right)b-a\_f^2\right)}\,x+\left(-a+b\right)\_f^2+b(x^2+a-b)\right)\left(\int_{\_b}^{x} -\frac{\left(-\right.}{\sqrt{a\left(-b^2+\left(\_a^2\right.\right.}}\right.\right.}{-\sqrt{a\left(-b^2+\left(\_f^2+x^2+a\right)b-a\_f^2\right.}}$$
$$+c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.424 (sec). Leaf size: 86

```
DSolve[-(a*b) - b*x^2 + a*y[x]^2 - 2*b*x*y[x]*y'[x] + (a - b)*y[x]^2*y'[x]^2==0,y[x],x,Inclu
```

$$y(x) \to -\frac{\sqrt{b\left(b-x^2\right)+a\left(-b+\left(x-c_1\right)^2\right)}}{\sqrt{b-a}}$$
$$y(x) \to \frac{\sqrt{b\left(b-x^2\right)+a\left(-b+\left(x-c_1\right)^2\right)}}{\sqrt{b-a}}$$

1.499 problem 501

Internal problem ID [8836]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 501.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `['y=_G(x,y')']`

$$(ay^2 + xb + c)\, y'^2 - by'y + dy^2 = 0$$

✓ Solution by Maple
Time used: 1.687 (sec). Leaf size: 50

```
dsolve((a*y(x)^2+b*x+c)*diff(y(x),x)^2-b*y(x)*diff(y(x),x)+d*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{(-bx - c)\sqrt{-ad}}{ab}$$
$$y(x) = \frac{\sqrt{-ad}\,(bx + c)}{ab}$$

✓ Solution by Mathematica
Time used: 71.894 (sec). Leaf size: 980

```
DSolve[d*y[x]^2 - b*y[x]*y'[x] + (c + b*x + a*y[x]^2)*y'[x]^2==0,y[x],x,IncludeSingularSolut
```

$$\text{Solve}\left[\left\{y(x) = \frac{bK[1] - \sqrt{-K[1]^2 (4abxK[1]^2 + 4acK[1]^2 - b^2 + 4bdx + 4cd)}}{2(aK[1]^2 + d)}, x = \frac{-2b^2c_1d^{5/2}\log\left(\sqrt{d}\sqrt{\right.}}{}\right.\right.$$

$$\text{Solve}\left[\left\{y(x) = \frac{\sqrt{-K[2]^2 (4abxK[2]^2 + 4acK[2]^2 - b^2 + 4bdx + 4cd)} + bK[2]}{2(aK[2]^2 + d)}, x = \frac{-2b^2c_1d^{5/2}\log\left(\sqrt{d}\sqrt{\right.}}{}\right.\right.$$

$$y(x) \to 0$$

1.500 problem 502

Internal problem ID [8837]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 502.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$
(ay - xb)^2 \left(a^2 {y'}^2 + b^2\right) - c^2 (ay' + b)^2 = 0
$$

✓ Solution by Maple

Time used: 0.703 (sec). Leaf size: 200

```
dsolve((a*y(x)-b*x)^2*(a^2*diff(y(x),x)^2+b^2)-c^2*(a*diff(y(x),x)+b)^2=0,y(x), singsol=all)
```

$$
\begin{aligned}
y(x) &= \frac{bx - \sqrt{2}\, c}{a} \\
y(x) &= \frac{bx + \sqrt{2}\, c}{a} \\
y(x) &= \frac{\operatorname{RootOf}\left(-a\left(\int^{\_Z} \frac{\_a^2 a^2 - 2c^2 + \sqrt{-a^2 \_a^2 (\_a^2 a^2 - 2c^2)}}{\_a^2 a^2 - 2c^2} d\_a\right) + 2c_1 b - 2bx\right) a + bx}{a} \\
y(x) &= \frac{\operatorname{RootOf}\left(a\left(\int^{\_Z} -\frac{\_a^2 a^2 - 2c^2 - \sqrt{-a^2 \_a^2 (\_a^2 a^2 - 2c^2)}}{\_a^2 a^2 - 2c^2} d\_a\right) + 2c_1 b - 2bx\right) a + bx}{a}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 2.306 (sec). Leaf size: 71

```
DSolve[-(c^2*(b + a*y'[x])^2) + (-(b*x) + a*y[x])^2*(b^2 + a^2*y'[x]^2)==0,y[x],x,IncludeSin
```

$$
\begin{aligned}
y(x) &\to \frac{bc_1 - \sqrt{c^2 - b^2 (x - c_1)^2}}{a} \\
y(x) &\to \frac{\sqrt{c^2 - b^2 (x - c_1)^2} + bc_1}{a}
\end{aligned}
$$

1.501 problem 503

Internal problem ID [8838]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 503.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational]`

$$(\mathrm{b2}\, y + \mathrm{a2}\, x + \mathrm{c2})^2 {y'}^2 + (\mathrm{a1}\, x + \mathrm{b1}\, y + \mathrm{c1})\, y' + \mathrm{b0}\, y = -\,\mathrm{c0} - \mathrm{a0}$$

✗ Solution by Maple

```
dsolve((b2*y(x)+a2*x+c2)^2*diff(y(x),x)^2+(a1*x+b1*y(x)+c1)*diff(y(x),x)+b0*y(x)+a0+c0=0,y(x
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[a0 + c0 + b0*y[x] + (c1 + a1*x + b1*y[x])*y'[x] + (c2 + a2*x + b2*y[x])^2*y'[x]^2==0,
```

Timed out

1.502 problem 504

Internal problem ID [8839]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 504.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational]`

$$x y^2 {y'}^2 - \left(y^3 + x^3 - a\right) y' + x^2 y = 0$$

✓ Solution by Maple
Time used: 0.297 (sec). Leaf size: 251

```
dsolve(x*y(x)^2*diff(y(x),x)^2-(y(x)^3+x^3-a)*diff(y(x),x)+x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(x^3 + a - 2x\sqrt{ax}\right)^{\frac{1}{3}}$$
$$y(x) = \left(x^3 + a + 2x\sqrt{ax}\right)^{\frac{1}{3}}$$
$$y(x) = -\frac{\left(x^3 + a - 2x\sqrt{ax}\right)^{\frac{1}{3}} \left(1 + i\sqrt{3}\right)}{2}$$
$$y(x) = \frac{\left(x^3 + a - 2x\sqrt{ax}\right)^{\frac{1}{3}} \left(i\sqrt{3} - 1\right)}{2}$$
$$y(x) = -\frac{\left(x^3 + a + 2x\sqrt{ax}\right)^{\frac{1}{3}} \left(1 + i\sqrt{3}\right)}{2}$$
$$y(x) = \frac{\left(x^3 + a + 2x\sqrt{ax}\right)^{\frac{1}{3}} \left(i\sqrt{3} - 1\right)}{2}$$
$$y(x) = 0$$
$$\int_{\_b}^{y(x)} \frac{\_a^2}{\sqrt{\_a^6 + \left(-2x^3 - 2a\right)\_a^3 + \left(-x^3 + a\right)^2}} d\_a + \frac{\ln(x)}{2} - c_1 = 0$$
$$\int_{\_b}^{y(x)} \frac{\_a^2}{\sqrt{\_a^6 + \left(-2x^3 - 2a\right)\_a^3 + \left(-x^3 + a\right)^2}} d\_a - \frac{\ln(x)}{2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.488 (sec). Leaf size: 194

```
DSolve[x^2*y[x] - (-a + x^3 + y[x]^3)*y'[x] + x*y[x]^2*y'[x]^2==0,y[x],x,IncludeSingularSolu
```

$$y(x) \to \frac{\sqrt[3]{a+(-1+c_1)x^3}}{\sqrt[3]{1-\frac{1}{c_1}}}$$
$$y(x) \to 0$$
$$y(x) \to \sqrt[3]{\left(\sqrt{a}-x^{3/2}\right)^2}$$
$$y(x) \to -\sqrt[3]{-1}\sqrt[3]{\left(\sqrt{a}-x^{3/2}\right)^2}$$
$$y(x) \to (-1)^{2/3}\sqrt[3]{\left(\sqrt{a}-x^{3/2}\right)^2}$$
$$y(x) \to \sqrt[3]{\left(\sqrt{a}+x^{3/2}\right)^2}$$
$$y(x) \to -\sqrt[3]{-1}\sqrt[3]{\left(\sqrt{a}+x^{3/2}\right)^2}$$
$$y(x) \to (-1)^{2/3}\sqrt[3]{\left(\sqrt{a}+x^{3/2}\right)^2}$$

1.503 problem 505

Internal problem ID [8840]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 505.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy^2{y'}^2 - 2y^3y' + 2xy^2 = x^3}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 52

```
dsolve(x*y(x)^2*diff(y(x),x)^2-2*y(x)^3*diff(y(x),x)+2*x*y(x)^2-x^3=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x^2 + c_1}$$
$$y(x) = -\sqrt{x^2 + c_1}$$
$$y(x) = \sqrt{c_1 x^2 + 1}\, x$$
$$y(x) = -\sqrt{c_1 x^2 + 1}\, x$$

✓ Solution by Mathematica

Time used: 0.561 (sec). Leaf size: 85

```
DSolve[-x^3 + 2*x*y[x]^2 - 2*y[x]^3*y'[x] + x*y[x]^2*y'[x]^2==0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to -\sqrt{x^2 + 2c_1}$$
$$y(x) \to \sqrt{x^2 + 2c_1}$$
$$y(x) \to -\sqrt{x^2 + c_1 x^4}$$
$$y(x) \to \sqrt{x^2 + c_1 x^4}$$
$$y(x) \to -x$$
$$y(x) \to x$$

1.504 problem 506

Internal problem ID [8841]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 506.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$x^2(xy^2-1)\,y'^2+2x^2y^2(y-x)\,y'-y^2(x^2y-1)=0$$

✗ Solution by Maple

```
dsolve(x^2*(x*y(x)^2-1)*diff(y(x),x)^2+2*x^2*y(x)^2*(y(x)-x)*diff(y(x),x)-y(x)^2*(x^2*y(x)-1
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-(y[x]^2*(-1 + x^2*y[x])) + 2*x^2*y[x]^2*(-x + y[x])*y'[x] + x^2*(-1 + x*y[x]^2)*y'[x
```

Not solved

1.505 problem 507

Internal problem ID [8842]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 507.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$(y^4 - a^2x^2)\, y'^2 + 2a^2xyy' + y^2(y^2 - a^2) = 0$$

✓ Solution by Maple

Time used: 1.735 (sec). Leaf size: 207

```
dsolve((y(x)^4-a^2*x^2)*diff(y(x),x)^2+2*a^2*x*y(x)*diff(y(x),x)+y(x)^2*(y(x)^2-a^2)=0,y(x),
```

$$y(x) = 0$$

$$y(x) - \mathrm{RootOf}\left(c_1\sqrt{\mathrm{RootOf}\left(\left(-y(x)^4 + a^2x^2\right)\_Z^2 - y(x)^2 + a^2 - 2\_Z a^2x\right)\_Z} + a\,\mathrm{hypergeom}\left(\left[-\frac{1}{4}, \frac{1}{4}\right], \left[\frac{3}{4}\right], \frac{\_Z^2\left(2\,\mathrm{RootOf}\left(\left(-y(x)^4 + a^2x^2\right)\_Z^2 - y(x)^2 + a^2 - 2\_Z a^2x\right) a^2x + \_Z\right.}{\_Z^4 - a^2x^2} + \_Z\left(-\frac{a^2\left(2\,\mathrm{RootOf}\left(\left(-y(x)^4 + a^2x^2\right)\_Z^2 - y(x)^2 + a^2 - 2\_Z a^2x\right)\_Z^2 x - \_Z^2 + x^2\right)}{\_Z^4 - a^2x^2}\right)^{\frac{1}{4}}\right) = 0$$

✓ Solution by Mathematica

Time used: 104.922 (sec). Leaf size: 395

```
DSolve[y[x]^2*(-a^2 + y[x]^2) + 2*a^2*x*y[x]*y'[x] + (-(a^2*x^2) + y[x]^4)*y'[x]^2==0,y[x],x
```

$$\text{Solve}\left[\left\{x=\frac{a^2K[1]y(K[1])-\sqrt{a^2K[1]^2\left(K[1]^2+1\right)y(K[1])^4}}{a^2K[1]^2},y(x)=\frac{-4a\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},\right.}{4\sqrt[4]{K[1]^2+1}}\right.\right.$$

$$\text{Solve}\left[\left\{x=\frac{a^2K[1]y(K[1])-\sqrt{a^2K[1]^2\left(K[1]^2+1\right)y(K[1])^4}}{a^2K[1]^2},y(x)=\frac{4a\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},-\right.}{4\sqrt[4]{K[1]^2+1}}\right.\right.$$

$$\text{Solve}\left[\left\{x=\frac{a^2K[1]y(K[1])+\sqrt{a^2K[1]^2\left(K[1]^2+1\right)y(K[1])^4}}{a^2K[1]^2},y(x)=\frac{-4a\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},\right.}{4\sqrt[4]{K[1]^2+1}}\right.\right.$$

$$\text{Solve}\left[\left\{x=\frac{a^2K[1]y(K[1])+\sqrt{a^2K[1]^2\left(K[1]^2+1\right)y(K[1])^4}}{a^2K[1]^2},y(x)=\frac{4a\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},-\right.}{4\sqrt[4]{K[1]^2+1}}\right.\right.$$

1.506 problem 508

Internal problem ID [8843]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 508.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\left(y^4+y^2x^2-x^2\right)y'^2+2xy'y-y^2=0$$

✓ Solution by Maple

Time used: 0.36 (sec). Leaf size: 64

```
dsolve((y(x)^4+x^2*y(x)^2-x^2)*diff(y(x),x)^2+2*x*y(x)*diff(y(x),x)-y(x)^2=0,y(x), singsol=a
```

$$y(x)=-ix$$
$$y(x)=ix$$
$$y(x)=0$$
$$y(x)=-\operatorname{arctanh}\left(\operatorname{RootOf}\left(\operatorname{arctanh}(\_Z)^2\_Z^2-2\operatorname{arctanh}(\_Z)c_1\_Z^2+c_1^2\_Z^2+x^2\_Z^2-x^2\right)\right)+c_1$$

✓ Solution by Mathematica

Time used: 1.461 (sec). Leaf size: 88

```
DSolve[-y[x]^2 + 2*x*y[x]*y'[x] + (-x^2 + x^2*y[x]^2 + y[x]^4)*y'[x]^2==0,y[x],x,IncludeSing
```

$$\text{Solve}\left[\frac{\sqrt{x^2+y(x)^2}\,y(x)\left(\log\left(\frac{x}{\sqrt{x^2+y(x)^2}}+1\right)-\log\left(1-\frac{x}{\sqrt{x^2+y(x)^2}}\right)\right)}{2x^2\sqrt{\frac{y(x)^2(x^2+y(x)^2)}{x^4}}}+y(x)=c_1,y(x)\right]$$

1.507 problem 509

Internal problem ID [8844]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 509.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$9y^4(x^2-1)\,y'^2-6xy^5y'=4x^2$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 231

```
dsolve(9*y(x)^4*(x^2-1)*diff(y(x),x)^2-6*x*y(x)^5*diff(y(x),x)-4*x^2=0,y(x), singsol=all)
```

$$y(x)=2^{\frac{1}{3}}(-x^2+1)^{\frac{1}{6}}$$
$$y(x)=-2^{\frac{1}{3}}(-x^2+1)^{\frac{1}{6}}$$
$$y(x)=-\frac{(1+i\sqrt{3})\,2^{\frac{1}{3}}(-x^2+1)^{\frac{1}{6}}}{2}$$
$$y(x)=\frac{(i\sqrt{3}-1)\,2^{\frac{1}{3}}(-x^2+1)^{\frac{1}{6}}}{2}$$
$$y(x)=-\frac{(i\sqrt{3}-1)\,2^{\frac{1}{3}}(-x^2+1)^{\frac{1}{6}}}{2}$$
$$y(x)=\frac{(1+i\sqrt{3})\,2^{\frac{1}{3}}(-x^2+1)^{\frac{1}{6}}}{2}$$
$$y(x)=\frac{2^{\frac{2}{3}}\left((-4c_1^2+x^2-1)\,c_1^2\right)^{\frac{1}{3}}}{2c_1}$$
$$y(x)=-\frac{2^{\frac{2}{3}}\left((-4c_1^2+x^2-1)\,c_1^2\right)^{\frac{1}{3}}(1+i\sqrt{3})}{4c_1}$$
$$y(x)=\frac{2^{\frac{2}{3}}\left((-4c_1^2+x^2-1)\,c_1^2\right)^{\frac{1}{3}}(i\sqrt{3}-1)}{4c_1}$$

✓ Solution by Mathematica

Time used: 0.429 (sec). Leaf size: 199

```
DSolve[-4*x^2 - 6*x*y[x]^5*y'[x] + 9*(-1 + x^2)*y[x]^4*y'[x]^2==0,y[x],x,IncludeSingularSolu
```

$$
\begin{aligned}
y(x) &\to -\frac{\sqrt[3]{-\frac{1}{2}}\sqrt[3]{-4x^2+4+c_1^2}}{\sqrt[3]{c_1}}\\
y(x) &\to -1\\
y(x) &\to 0\\
y(x) &\to \sqrt[3]{-\frac{1}{2}}\\
y(x) &\to \text{Indeterminate}\\
y(x) &\to -\sqrt[3]{-2}\sqrt[6]{1-x^2}\\
y(x) &\to \sqrt[3]{-2}\sqrt[6]{1-x^2}\\
y(x) &\to -\sqrt[3]{2}\sqrt[6]{1-x^2}\\
y(x) &\to \sqrt[3]{2}\sqrt[6]{1-x^2}\\
y(x) &\to -(-1)^{2/3}\sqrt[3]{2}\sqrt[6]{1-x^2}\\
y(x) &\to (-1)^{2/3}\sqrt[3]{2}\sqrt[6]{1-x^2}
\end{aligned}
$$

1.508 problem 510

Internal problem ID [8845]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 510.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$x^2(x^2y^4-1)\,y'^2+2x^3y^3(y^2-x^2)\,y'-y^2(x^4y^2-1)=0$$

✗ Solution by Maple

```
dsolve(x^2*(x^2*y(x)^4-1)*diff(y(x),x)^2+2*x^3*y(x)^3*(y(x)^2-x^2)*diff(y(x),x)-y(x)^2*(x^4*
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-(y[x]^2*(-1 + x^4*y[x]^2)) + 2*x^3*y[x]^3*(-x^2 + y[x]^2)*y'[x] + x^2*(-1 + x^2*y[x]
```

Not solved

1.509 problem 511

Internal problem ID [8846]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 511.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\left(a^2\sqrt{x^2+y^2}-x^2\right)y'^2+2xy'y+a^2\sqrt{x^2+y^2}-y^2=0$$

✓ Solution by Maple

Time used: 7.843 (sec). Leaf size: 311

```
dsolve((a^2*(y(x)^2+x^2)^(1/2)-x^2)*diff(y(x),x)^2+2*x*y(x)*diff(y(x),x)+a^2*(y(x)^2+x^2)^(1
```

$$y(x)=-ix$$
$$y(x)=ix$$
$$\frac{2\sqrt{-a^2+\sqrt{y(x)^2+x^2}}\sqrt{a^2\left(y(x)^2+x^2\right)^2\left(-a^2+\sqrt{y(x)^2+x^2}\right)}\arctan\left(\frac{\sqrt{-a^2+\sqrt{y(x)^2+x^2}}}{a}\right)-a\left(y(x)^2\right.}{a\left(y(x)^2+x^2\right)\left(a^2-\sqrt{y(x)^2+x^2}\right)}$$
$$=0$$
$$\frac{-2\sqrt{-a^2+\sqrt{y(x)^2+x^2}}\sqrt{a^2\left(y(x)^2+x^2\right)^2\left(-a^2+\sqrt{y(x)^2+x^2}\right)}\arctan\left(\frac{\sqrt{-a^2+\sqrt{y(x)^2+x^2}}}{a}\right)-a\left(y(x\right.}{a\left(y(x)^2+x^2\right)\left(a^2-\sqrt{y(x)^2+x^2}\right)}$$
$$=0$$

✓ Solution by Mathematica

Time used: 42.919 (sec). Leaf size: 229

```
DSolve[-y[x]^2 + a^2*Sqrt[x^2 + y[x]^2] + 2*x*y[x]*y'[x] + (-x^2 + a^2*Sqrt[x^2 + y[x]^2])*y
```

$$\text{Solve}\left[\arctan\left(\frac{x}{y(x)}\right) - \frac{2\sqrt{a^2\left(x^2+y(x)^2\right)\left(\sqrt{x^2+y(x)^2}-a^2\right)}\arctan\left(\frac{\sqrt{\sqrt{x^2+y(x)^2}-a^2}}{a}\right)}{a\sqrt{x^2+y(x)^2}\sqrt{\sqrt{x^2+y(x)^2}-a^2}} = c_1, y(x)\right]$$

$$\text{Solve}\left[\frac{2\sqrt{a^2\left(x^2+y(x)^2\right)\left(\sqrt{x^2+y(x)^2}-a^2\right)}\arctan\left(\frac{\sqrt{\sqrt{x^2+y(x)^2}-a^2}}{a}\right)}{a\sqrt{x^2+y(x)^2}\sqrt{\sqrt{x^2+y(x)^2}-a^2}} + \arctan\left(\frac{x}{y(x)}\right) = c_1, y(x)\right]$$

1.510 problem 512

Internal problem ID [8847]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 512.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\left(a\left(x^2+y^2\right)^{\frac{3}{2}}-x^2\right)y'^2+2xy'y+a\left(x^2+y^2\right)^{\frac{3}{2}}-y^2=0$$

✓ Solution by Maple

Time used: 14.375 (sec). Leaf size: 100

```
dsolve((a*(y(x)^2+x^2)^(3/2)-x^2)*diff(y(x),x)^2+2*x*y(x)*diff(y(x),x)+a*(y(x)^2+x^2)^(3/2)-
```

$$y(x) = -ix$$
$$y(x) = ix$$
$$y(x) = \cot\left(\operatorname{RootOf}\left(-4\_Z - 2\left(\int^{\csc(\_Z)^2x^2}\frac{\sqrt{-\_a^{\frac{17}{2}}\left(\sqrt{\_a}\,a-1\right)\left(2\sqrt{\_a}\,a+\cos(2)-1\right)^2a}}{\left(2\_a\,a^2-3\sqrt{\_a}\,a+1+\sqrt{\_a}\,a\cos(2)-\cos(2)\right)\_a^5}d\_a\right) + 4c_1\right)\right)x$$

✓ Solution by Mathematica

Time used: 49.818 (sec). Leaf size: 305

```
DSolve[-y[x]^2 + a*(x^2 + y[x]^2)^(3/2) + 2*x*y[x]*y'[x] + (-x^2 + a*(x^2 + y[x]^2)^(3/2))*y
```

$$\text{Solve}\left[\arctan\left(\frac{x}{y(x)}\right) - \frac{2\sqrt{a\left(x^2+y(x)^2\right)^2\left(-ax^2-ay(x)^2+\sqrt{x^2+y(x)^2}\right)}\arctan\left(\frac{\sqrt{a}\sqrt{x^2+y(x)^2}}{\sqrt{-ax^2-ay(x)^2+\sqrt{x^2+y(x)^2}}}\right)}{\sqrt{a}\left(x^2+y(x)^2\right)\sqrt{-ax^2-ay(x)^2+\sqrt{x^2+y(x)^2}}} = c_1, y(x)\right]$$

$$\text{Solve}\left[\frac{2\sqrt{a\left(x^2+y(x)^2\right)^2\left(-ax^2-ay(x)^2+\sqrt{x^2+y(x)^2}\right)}\arctan\left(\frac{\sqrt{a}\sqrt{x^2+y(x)^2}}{\sqrt{-ax^2-ay(x)^2+\sqrt{x^2+y(x)^2}}}\right)}{\sqrt{a}\left(x^2+y(x)^2\right)\sqrt{-ax^2-ay(x)^2+\sqrt{x^2+y(x)^2}}} + \arctan\left(\frac{x}{y(x)}\right) = c_1, y(x)\right]$$

1.511 problem 513

Internal problem ID [8848]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 513.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `['y=_G(x,y')']`

$$y'^2 \sin(y) + 2xy' \cos(y)^3 - \sin(y)\cos(y)^4 = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)^2*sin(y(x))+2*x*diff(y(x),x)*cos(y(x))^3-sin(y(x))*cos(y(x))^4=0,y(x), s
```

No solution found

✓ Solution by Mathematica

Time used: 1.829 (sec). Leaf size: 135

```
DSolve[-(Cos[y[x]]^4*Sin[y[x]]) + 2*x*Cos[y[x]]^3*y'[x] + Sin[y[x]]*y'[x]^2==0,y[x],x,Includ
```

$$\begin{aligned}
y(x) &\to -\arctan\left(2\sqrt{c_1}\sqrt{x+c_1}\right)\\
y(x) &\to \arctan\left(2\sqrt{c_1}\sqrt{x+c_1}\right)\\
y(x) &\to -\frac{\pi}{2}\\
y(x) &\to \frac{\pi}{2}\\
y(x) &\to -\arccos\left(-\frac{1}{\sqrt{1-x^2}}\right)\\
y(x) &\to \arccos\left(-\frac{1}{\sqrt{1-x^2}}\right)\\
y(x) &\to -\arccos\left(\frac{1}{\sqrt{1-x^2}}\right)\\
y(x) &\to \arccos\left(\frac{1}{\sqrt{1-x^2}}\right)
\end{aligned}$$

1.512 problem 514

Internal problem ID [8849]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 514.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$
{y'}^2(a\cos(y)+b)-c\cos(y)=-d
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 84

```
dsolve(diff(y(x),x)^2*(a*cos(y(x))+b)-c*cos(y(x))+d=0,y(x), singsol=all)
```

$$
y(x)=\arccos\left(\frac{d}{c}\right)
$$

$$
x-\left(\int^{y(x)}\frac{a\cos(\_a)+b}{\sqrt{(a\cos(\_a)+b)(c\cos(\_a)-d)}}d\_a\right)-c_1=0
$$

$$
x+\int^{y(x)}\frac{a\cos(\_a)+b}{\sqrt{(a\cos(\_a)+b)(c\cos(\_a)-d)}}d\_a-c_1=0
$$

✓ Solution by Mathematica

Time used: 14.351 (sec). Leaf size: 627

```
DSolve[d - c*Cos[y[x]] + (b + a*Cos[y[x]])*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \rightarrow \text{InverseFunction}\left[\frac{4\sin^2\left(\frac{\#1}{2}\right)\csc(\#1)\sqrt{a\cos(\#1)+b}\sqrt{\frac{\cot^2\left(\frac{\#1}{2}\right)(c-d)}{c+d}}\sqrt{\frac{\csc^2\left(\frac{\#1}{2}\right)(a+b)(d-c\cos(\#1))}{ad+bc}}\left(c(\right.}{} + c_1\right]$$

$$y(x) \rightarrow \text{InverseFunction}\left[\frac{4\sin^2\left(\frac{\#1}{2}\right)\csc(\#1)\sqrt{a\cos(\#1)+b}\sqrt{\frac{\cot^2\left(\frac{\#1}{2}\right)(c-d)}{c+d}}\sqrt{\frac{\csc^2\left(\frac{\#1}{2}\right)(a+b)(d-c\cos(\#1))}{ad+bc}}\left(c(\right.}{} + c_1\right]$$

$$y(x) \rightarrow -\arccos\left(\frac{d}{c}\right)$$

$$y(x) \rightarrow \arccos\left(\frac{d}{c}\right)$$

1.513 problem 515

Internal problem ID [8850]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 515.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$f\left(x^2+y^2\right)\left(1+y'^2\right)-\left(xy'-y\right)^2=0$$

✓ Solution by Maple
Time used: 0.265 (sec). Leaf size: 136

```
dsolve(f(y(x)^2+x^2)*(diff(y(x),x)^2+1)-(x*diff(y(x),x)-y(x))^2=0,y(x), singsol=all)
```

$$y(x)=-ix$$
$$y(x)=ix$$
$$y(x)=\operatorname{RootOf}\left(x^2+\_Z^2-f\left(\_Z^2+x^2\right)\right)$$
$$y(x)=\cot\left(\operatorname{RootOf}\left(-2\_Z-\left(\int^{\csc(\_Z)^2x^2}\frac{\sqrt{-f\left(\_a\right)\left(f\left(\_a\right)-\_a\right)}}{\_a\left(f\left(\_a\right)-\_a\right)}d\_a\right)+2c_1\right)\right)x$$
$$y(x)=\cot\left(\operatorname{RootOf}\left(-2\_Z+\int^{\csc(\_Z)^2x^2}\frac{\sqrt{-f\left(\_a\right)\left(f\left(\_a\right)-\_a\right)}}{\_a\left(f\left(\_a\right)-\_a\right)}d\_a+2c_1\right)\right)x$$

✓ Solution by Mathematica

Time used: 5.95 (sec). Leaf size: 1922

```
DSolve[-(-y[x] + x*y'[x])^2 + f[x^2 + y[x]^2]*(1 + y'[x]^2)==0,y[x],x,IncludeSingularSolutio
```

$$\text{Solve}\left[\int_1^x \left(\frac{\sqrt{f(K[1]^2+y(x)^2)(K[1]^2+y(x)^2-f(K[1]^2+y(x)^2))}K[1]}{f(K[1]^2+y(x)^2)(K[1]^2+y(x)^2)} - \frac{\sqrt{f(K[1]^2+y(x)^2)(K[1]^2+y(x)^2-f(K[1]^2+y(x)^2))}K[1]}{f(K[1]^2+y(x)^2)(K[1]^2+y(x)^2-f(K[1]^2+y(x)^2))} + \frac{y(x)}{K[1]^2+y(x)^2}\right) dK[1] + \int_1^{y(x)} \left(-\frac{x}{x^2+K[2]^2} - \int_1^x \left(-\frac{2K[2]^2}{(K[1]^2+K[2]^2)^2} - \frac{2K[1]\sqrt{f(K[1]^2+K[2]^2)(K[1]^2+K[2]^2-f(K[1]^2+K[2]^2))}f'(K[1]^2+K}{f(K[1]^2+K[2]^2)^2(K[1]^2+K[2]^2)}\right. + \frac{K[2]\sqrt{f(x^2+K[2]^2)(x^2+K[2]^2-f(x^2+K[2]^2))}}{f(x^2+K[2]^2)(x^2+K[2]^2)} - \frac{K[2]\sqrt{f(x^2+K[2]^2)(x^2+K[2]^2-f(x^2+K[2]^2))}}{f(x^2+K[2]^2)(x^2+K[2]^2-f(x^2+K[2]^2))}\right) dK[2] = c_1, y(x)\right]$$

$$\text{Solve}\left[\int_1^x \left(-\frac{\sqrt{f(K[3]^2+y(x)^2)(K[3]^2+y(x)^2-f(K[3]^2+y(x)^2))}K[3]}{f(K[3]^2+y(x)^2)(K[3]^2+y(x)^2)} + \frac{\sqrt{f(K[3]^2+y(x)^2)(K[3]^2+y(x)^2-f(K[3]^2+y(x)^2))}K[3]}{f(K[3]^2+y(x)^2)(K[3]^2+y(x)^2-f(K[3]^2+y(x)^2))} + \frac{y(x)}{K[3]^2+y(x)^2}\right) dK[3] + \int_1^{y(x)} \left(-\frac{x}{x^2+K[4]^2} - \int_1^x \left(-\frac{2K[4]^2}{(K[3]^2+K[4]^2)^2} + \frac{2K[3]\sqrt{f(K[3]^2+K[4]^2)(K[3]^2+K[4]^2-f(K[3]^2+K[4]^2))}f'(K[3]^2+K}{f(K[3]^2+K[4]^2)^2(K[3]^2+K[4]^2)}\right. - \frac{K[4]\sqrt{f(x^2+K[4]^2)(x^2+K[4]^2-f(x^2+K[4]^2))}}{f(x^2+K[4]^2)(x^2+K[4]^2)} + \frac{K[4]\sqrt{f(x^2+K[4]^2)(x^2+K[4]^2-f(x^2+K[4]^2))}}{f(x^2+K[4]^2)(x^2+K[4]^2-f(x^2+K[4]^2))}\right) dK[4] = c_1, y(x)\right]$$

1.514 problem 516

Internal problem ID [8851]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 516.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A']]`

$$(x^2+y^2)\, f\left(\frac{x}{\sqrt{x^2+y^2}}\right)\left(1+y'^2\right)-(xy'-y)^2=0$$

✓ Solution by Maple

Time used: 1.187 (sec). Leaf size: 72

```
dsolve((y(x)^2+x^2)*f(x/(y(x)^2+x^2)^(1/2))*(diff(y(x),x)^2+1)-(x*diff(y(x),x)-y(x))^2=0,y(x
```

$$y(x)=\operatorname{RootOf}\left(-\ln(x)-\left(\int^{\_Z}\frac{\_af\left(\frac{1}{\sqrt{\_a^2+1}}\right)-\sqrt{-f\left(\frac{1}{\sqrt{\_a^2+1}}\right)\left(f\left(\frac{1}{\sqrt{\_a^2+1}}\right)-1\right)}}{(\_a^2+1)\, f\left(\frac{1}{\sqrt{\_a^2+1}}\right)}d\_a\right)+c_1\right)x$$

✓ Solution by Mathematica

Time used: 2.053 (sec). Leaf size: 253

```
DSolve[-(-y[x] + x*y'[x])^2 + f[x/Sqrt[x^2 + y[x]^2]]*(x^2 + y[x]^2)*(1 + y'[x]^2)==0,y[x],x
```

$$\text{Solve}\left[\int_1^{\frac{y(x)}{x}} \frac{f\left(\frac{1}{\sqrt{K[1]^2+1}}\right)K[1]^2 + f\left(\frac{1}{\sqrt{K[1]^2+1}}\right) - 1}{\sqrt{f\left(\frac{1}{\sqrt{K[1]^2+1}}\right)}(K[1]-i)(K[1]+i)\left(\sqrt{f\left(\frac{1}{\sqrt{K[1]^2+1}}\right)}K[1] + i\sqrt{f\left(\frac{1}{\sqrt{K[1]^2+1}}\right)-1}\right)} dK[1] = -\log(x) + c_1, y(x)\right]$$

$$\text{Solve}\left[\int_1^{\frac{y(x)}{x}} \frac{f\left(\frac{1}{\sqrt{K[2]^2+1}}\right)K[2]^2 + f\left(\frac{1}{\sqrt{K[2]^2+1}}\right) - 1}{\sqrt{f\left(\frac{1}{\sqrt{K[2]^2+1}}\right)}(K[2]-i)(K[2]+i)\left(\sqrt{f\left(\frac{1}{\sqrt{K[2]^2+1}}\right)}K[2] - i\sqrt{f\left(\frac{1}{\sqrt{K[2]^2+1}}\right)-1}\right)} dK[2] = -\log(x) + c_1, y(x)\right]$$

1.515 problem 517

Internal problem ID [8852]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 517.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A']]`

$$(x^2+y^2)\, f\left(\frac{y}{\sqrt{x^2+y^2}}\right)\left(1+y'^2\right)-(xy'-y)^2=0$$

✓ Solution by Maple

Time used: 1.282 (sec). Leaf size: 79

```
dsolve((y(x)^2+x^2)*f(y(x)/(y(x)^2+x^2)^(1/2))*(diff(y(x),x)^2+1)-(x*diff(y(x),x)-y(x))^2=0,
```

$$y(x)=\operatorname{RootOf}\left(-\ln(x)+\int^{\_Z}-\frac{\_af\left(\frac{\_a}{\sqrt{\_a^2+1}}\right)-\sqrt{-f\left(\frac{\_a}{\sqrt{\_a^2+1}}\right)\left(f\left(\frac{\_a}{\sqrt{\_a^2+1}}\right)-1\right)}}{(\_a^2+1)\, f\left(\frac{\_a}{\sqrt{\_a^2+1}}\right)}d\_a+c_1\right)x$$

✓ Solution by Mathematica

Time used: 2.095 (sec). Leaf size: 283

```
DSolve[-(-y[x] + x*y'[x])^2 + f[y[x]/Sqrt[x^2 + y[x]^2]]*(x^2 + y[x]^2)*(1 + y'[x]^2)==0,y[x
```

$$\text{Solve}\left[\int_1^{\frac{y(x)}{x}} \frac{f\left(\frac{K[1]}{\sqrt{K[1]^2+1}}\right)K[1]^2+f\left(\frac{K[1]}{\sqrt{K[1]^2+1}}\right)-1}{\sqrt{f\left(\frac{K[1]}{\sqrt{K[1]^2+1}}\right)}(K[1]-i)(K[1]+i)\left(\sqrt{f\left(\frac{K[1]}{\sqrt{K[1]^2+1}}\right)}K[1]+i\sqrt{f\left(\frac{K[1]}{\sqrt{K[1]^2+1}}\right)-1}\right)}dK[1]= -\log(x)+c_1,y(x)\right]$$

$$\text{Solve}\left[\int_1^{\frac{y(x)}{x}} \frac{f\left(\frac{K[2]}{\sqrt{K[2]^2+1}}\right)K[2]^2+f\left(\frac{K[2]}{\sqrt{K[2]^2+1}}\right)-1}{\sqrt{f\left(\frac{K[2]}{\sqrt{K[2]^2+1}}\right)}(K[2]-i)(K[2]+i)\left(\sqrt{f\left(\frac{K[2]}{\sqrt{K[2]^2+1}}\right)}K[2]-i\sqrt{f\left(\frac{K[2]}{\sqrt{K[2]^2+1}}\right)-1}\right)}dK[2]= -\log(x)+c_1,y(x)\right]$$

1.516 problem 518

Internal problem ID [8853]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 518.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 - (y-a)^2 (y-b)^2 = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 146

```
dsolve(diff(y(x),x)^3-(y(x)-a)^2*(y(x)-b)^2=0,y(x), singsol=all)
```

$$y(x) = a$$
$$y(x) = b$$
$$x - \left(\int^{y(x)} \frac{1}{\left((\_a-a)^2 (\_a-b)^2\right)^{\frac{1}{3}}} d\_a\right) - c_1 = 0$$
$$\frac{2\left(\int^{y(x)} \frac{1}{\left((\_a-a)^2(\_a-b)^2\right)^{\frac{1}{3}}} d\_a\right) + i(x-c_1)\sqrt{3} + x - c_1}{1+i\sqrt{3}} = 0$$
$$\frac{-2\left(\int^{y(x)} \frac{1}{\left((\_a-a)^2(\_a-b)^2\right)^{\frac{1}{3}}} d\_a\right) + i(x-c_1)\sqrt{3} - x + c_1}{i\sqrt{3}-1} = 0$$

✓ Solution by Mathematica

Time used: 1.169 (sec). Leaf size: 246

```
DSolve[-((-a + y[x])^2*(-b + y[x])^2) + y'[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\&\right][x+c_1]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\&\right]\left[-\sqrt[3]{-1}x+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\&\right]\left[(-1)^{2/3}x+c_1\right]$$

$$y(x) \to a$$
$$y(x) \to b$$

1.517 problem 519

Internal problem ID [8854]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 519.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$
y'^3 - f(x)\left(a y^2 + b y + c\right)^2 = 0
$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 197

```
dsolve(diff(y(x),x)^3-f(x)*(a*y(x)^2+b*y(x)+c)^2=0,y(x), singsol=all)
```

$$
\int^{y(x)} \frac{1}{\left(\_a^2 a + \_a b + c\right)^{\frac{2}{3}}} d\_a - \frac{\int^x \left(f(\_a)\left(a y(x)^2 + b y(x) + c\right)^2\right)^{\frac{1}{3}} d\_a}{\left(a y(x)^2 + b y(x) + c\right)^{\frac{2}{3}}} + c_1 = 0
$$

$$
\int^{y(x)} \frac{1}{\left(\_a^2 a + \_a b + c\right)^{\frac{2}{3}}} d\_a + \frac{\left(1 + i\sqrt{3}\right)\left(\int^x \left(f(\_a)\left(a y(x)^2 + b y(x) + c\right)^2\right)^{\frac{1}{3}} d\_a\right)}{2\left(a y(x)^2 + b y(x) + c\right)^{\frac{2}{3}}} + c_1 = 0
$$

$$
\int^{y(x)} \frac{1}{\left(\_a^2 a + \_a b + c\right)^{\frac{2}{3}}} d\_a - \frac{\left(i\sqrt{3} - 1\right)\left(\int^x \left(f(\_a)\left(a y(x)^2 + b y(x) + c\right)^2\right)^{\frac{1}{3}} d\_a\right)}{2\left(a y(x)^2 + b y(x) + c\right)^{\frac{2}{3}}} + c_1 = 0
$$

✓ Solution by Mathematica

Time used: 21.249 (sec). Leaf size: 405

```
DSolve[-(f[x]*(c + b*y[x] + a*y[x]^2)^2) + y'[x]^3==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \text{InverseFunction}\left[\frac{\sqrt[3]{2}(2\#1a+b)\left(\frac{a(\#1(\#1a+b)+c)}{4ac-b^2}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{(b+2a\#1)^2}{b^2-4ac}\right)}{a(\#1(\#1a+b)+c)^{2/3}}\&\right]\left[\int_1^{x}\right.$$

$$y(x) \to \text{InverseFunction}\left[\frac{\sqrt[3]{2}(2\#1a+b)\left(\frac{a(\#1(\#1a+b)+c)}{4ac-b^2}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{(b+2a\#1)^2}{b^2-4ac}\right)}{a(\#1(\#1a+b)+c)^{2/3}}\&\right]\left[\int_1^{x}\right.$$

$$y(x) \to \text{InverseFunction}\left[\frac{\sqrt[3]{2}(2\#1a+b)\left(\frac{a(\#1(\#1a+b)+c)}{4ac-b^2}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{(b+2a\#1)^2}{b^2-4ac}\right)}{a(\#1(\#1a+b)+c)^{2/3}}\&\right]\left[\int_1^{x}\right.$$

$$y(x) \to -\frac{\sqrt{b^2-4ac}+b}{2a}$$

$$y(x) \to \frac{\sqrt{b^2-4ac}-b}{2a}$$

1.518 problem 520

Internal problem ID [8855]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 520.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^3 + y' - y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 221

```
dsolve(diff(y(x),x)^3+diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$x - 6\left(\int^{y(x)} \frac{\left(108\_a + 12\sqrt{81\_a^2 + 12}\right)^{\frac{1}{3}}}{\left(108\_a + 12\sqrt{81\_a^2 + 12}\right)^{\frac{2}{3}} - 12} d\_a\right) - c_1 = 0$$

$$\frac{-12\left(\int^{y(x)} \frac{\left(108\_a+12\sqrt{81\_a^2+12}\right)^{\frac{1}{3}}}{-6-6i\sqrt{3}-\left(108\_a+12\sqrt{81\_a^2+12}\right)^{\frac{2}{3}}} d\_a\right) + i(x - c_1)\sqrt{3} - c_1 + x}{1 + i\sqrt{3}} = 0$$

$$\frac{12\left(\int^{y(x)} \frac{\left(108\_a+12\sqrt{81\_a^2+12}\right)^{\frac{1}{3}}}{-\left(108\_a+12\sqrt{81\_a^2+12}\right)^{\frac{2}{3}} + \left(\sqrt{3}+3i\right)^2} d\_a\right) + i(x - c_1)\sqrt{3} + c_1 - x}{i\sqrt{3} - 1} = 0$$

✓ Solution by Mathematica

Time used: 0.358 (sec). Leaf size: 335

```
DSolve[-y[x] + y'[x] + y'[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int \frac{\sqrt[3]{\sqrt{729\#1^2+108}-27\#1}}{2^{2/3}\left(\sqrt{729\#1^2+108}-27\#1\right)^{2/3}-6\sqrt[3]{2}}d\#1\&\right]\left[-\frac{x}{6}+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[\int \frac{\sqrt[3]{\sqrt{729\#1^2+108}-27\#1}}{-i2^{2/3}\sqrt{3}\left(\sqrt{729\#1^2+108}-27\#1\right)^{2/3}+2^{2/3}\left(\sqrt{729\#1^2+108}-27\#1\right)^{2/3}-6}\right.$$

$$y(x) \to \text{InverseFunction}\left[\int \frac{\sqrt[3]{\sqrt{729\#1^2+108}-27\#1}}{i2^{2/3}\sqrt{3}\left(\sqrt{729\#1^2+108}-27\#1\right)^{2/3}+2^{2/3}\left(\sqrt{729\#1^2+108}-27\#1\right)^{2/3}+6i}\right.$$

$$y(x) \to 0$$

1.519 problem 521

Internal problem ID [8856]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 521.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y'^3 + xy' - y = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 37

```
dsolve(diff(y(x),x)^3+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{2\sqrt{3}\,(-x)^{\frac{3}{2}}}{9}$$
$$y(x) = -\frac{2\sqrt{3}\,(-x)^{\frac{3}{2}}}{9}$$
$$y(x) = c_1\left(c_1^2 + x\right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 54

```
DSolve[-y[x] + x*y'[x] + y'[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\left(x + c_1{}^2\right)$$
$$y(x) \to -\frac{2ix^{3/2}}{3\sqrt{3}}$$
$$y(x) \to \frac{2ix^{3/2}}{3\sqrt{3}}$$

1.520 problem 522

Internal problem ID [8857]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 522.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$y'^3 - (x+5)\, y' + y = 0$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 44

```
dsolve(diff(y(x),x)^3-(x+5)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{2\sqrt{3x+15}\,(x+5)}{9}$$
$$y(x) = \frac{2\sqrt{3x+15}\,(x+5)}{9}$$
$$y(x) = c_1\left(-c_1^2 + x + 5\right)$$

✓ Solution by Mathematica
Time used: 0.007 (sec). Leaf size: 57

```
DSolve[y[x] - (5 + x)*y'[x] + y'[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\left(x + 5 - c_1{}^2\right)$$
$$y(x) \to -\frac{2(x+5)^{3/2}}{3\sqrt{3}}$$
$$y(x) \to \frac{2(x+5)^{3/2}}{3\sqrt{3}}$$

1.521 problem 523

Internal problem ID [8858]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 523.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 - axy' = -x^3$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 252

```
dsolve(diff(y(x),x)^3-a*x*diff(y(x),x)+x^3=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(\int \left(\left(-108x^3+12\sqrt{3}\,\sqrt{-4a^3x^3+27x^6}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)-\frac{12\left(1+i\sqrt{3}\right)ax}{\left(-108x^3+12\sqrt{3}\,\sqrt{-4a^3x^3+27x^6}\right)^{\frac{1}{3}}}\right)dx\right)}{12} + c_1$$

$$y(x) = -\frac{\left(\int \left(\left(1+i\sqrt{3}\right)\left(-108x^3+12\sqrt{3}\,\sqrt{-4a^3x^3+27x^6}\right)^{\frac{1}{3}}-\frac{12\left(i\sqrt{3}-1\right)ax}{\left(-108x^3+12\sqrt{3}\,\sqrt{-4a^3x^3+27x^6}\right)^{\frac{1}{3}}}\right)dx\right)}{12} + c_1$$

$$y(x) = \frac{\left(\int \frac{\left(-108x^3+12\sqrt{3}\,\sqrt{-4a^3x^3+27x^6}\right)^{\frac{2}{3}}+12ax}{\left(-108x^3+12\sqrt{3}\,\sqrt{-4a^3x^3+27x^6}\right)^{\frac{1}{3}}}dx\right)}{6} + c_1$$

✓ Solution by Mathematica

Time used: 166.69 (sec). Leaf size: 349

```
DSolve[x^3 - a*x*y'[x] + y'[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \int_1^x \frac{2\sqrt[3]{3}aK[1] + \sqrt[3]{2}\left(\sqrt{81K[1]^6 - 12a^3K[1]^3} - 9K[1]^3\right)^{2/3}}{6^{2/3}\sqrt[3]{\sqrt{81K[1]^6 - 12a^3K[1]^3} - 9K[1]^3}}dK[1] + c_1$$

$$y(x) \to \int_1^x \frac{i\sqrt[3]{3}(i+\sqrt{3})\left(2\sqrt{81K[2]^6 - 12a^3K[2]^3} - 18K[2]^3\right)^{2/3} - 2\sqrt[3]{2}\sqrt[6]{3}(3i+\sqrt{3})\,aK[2]}{12\sqrt[3]{\sqrt{81K[2]^6 - 12a^3K[2]^3} - 9K[2]^3}}dK[2] + c_1$$

$$y(x) \to \int_1^x \frac{\sqrt[3]{3}(-1-i\sqrt{3})\left(2\sqrt{81K[3]^6 - 12a^3K[3]^3} - 18K[3]^3\right)^{2/3} - 2\sqrt[3]{2}\sqrt[6]{3}(-3i+\sqrt{3})\,aK[3]}{12\sqrt[3]{\sqrt{81K[3]^6 - 12a^3K[3]^3} - 9K[3]^3}}dK[3] + c_1$$

1.522 problem 524

Internal problem ID [8859]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 524.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 - 2yy' + y^2 = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 304

```
dsolve(diff(y(x),x)^3-2*y(x)*diff(y(x),x)+y(x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$-\sqrt{3}\,2^{\frac{1}{3}}\left(\int^{y(x)}\frac{\left(-3\sqrt{3}\,\_a^2+\sqrt{27\_a^4-32\_a^3}\right)^{\frac{1}{3}}}{2\,2^{\frac{2}{3}}\_a+\left(-3\sqrt{3}\,\_a^2+\sqrt{27\_a^4-32\_a^3}\right)^{\frac{2}{3}}}d\_a\right)+x-c_1=0$$

$$\frac{2i2^{\frac{1}{3}}3^{\frac{5}{6}}\left(\int^{y(x)}\frac{\left(-3\sqrt{3}\,\_a^2+\sqrt{27\_a^4-32\_a^3}\right)^{\frac{1}{3}}}{-3^{\frac{1}{3}}\left(-3\sqrt{3}\,\_a^2+\sqrt{27\_a^4-32\_a^3}\right)^{\frac{2}{3}}+\_a\left(i3^{\frac{5}{6}}+3^{\frac{1}{3}}\right)2^{\frac{2}{3}}}d\_a\right)+(x-c_1)\left(-i+\sqrt{3}\right)}{-i+\sqrt{3}}=0$$

$$\frac{2i2^{\frac{1}{3}}3^{\frac{5}{6}}\left(\int^{y(x)}\frac{\left(-3\sqrt{3}\,\_a^2+\sqrt{27\_a^4-32\_a^3}\right)^{\frac{1}{3}}}{3^{\frac{1}{3}}\left(-3\sqrt{3}\,\_a^2+\sqrt{27\_a^4-32\_a^3}\right)^{\frac{2}{3}}+\_a\left(i3^{\frac{5}{6}}-3^{\frac{1}{3}}\right)2^{\frac{2}{3}}}d\_a\right)+(x-c_1)\left(\sqrt{3}+i\right)}{\sqrt{3}+i}=0$$

✓ Solution by Mathematica

Time used: 0.402 (sec). Leaf size: 427

```
DSolve[y[x]^2 - 2*y[x]*y'[x] + y'[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int \frac{\sqrt[3]{\sqrt{3}\sqrt{\#1^3(27\#1-32)}-9\#1^2}}{\sqrt[3]{2}\left(\sqrt{3}\sqrt{\#1^3(27\#1-32)}-9\#1^2\right)^{2/3}+4\sqrt[3]{3}\#1}d\#1\&\right]\left[\frac{x}{6^{2/3}}+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[\int \frac{\sqrt[3]{\sqrt{3}\sqrt{\#1^3(27\#1-32)}-9\#1^2}}{\sqrt[3]{2}3^{2/3}\left(\sqrt{3}\sqrt{\#1^3(27\#1-32)}-9\#1^2\right)^{2/3}-\sqrt[3]{2}\sqrt[6]{3}i\left(\sqrt{3}\sqrt{\#1^3(27\#1-32)}-9\right.}\right.$$

$$y(x) \to \text{InverseFunction}\left[\int \frac{\sqrt[3]{\sqrt{3}\sqrt{\#1^3(27\#1-32)}-9\#1^2}}{\sqrt[3]{2}3^{2/3}\left(\sqrt{3}\sqrt{\#1^3(27\#1-32)}-9\#1^2\right)^{2/3}+\sqrt[3]{2}\sqrt[6]{3}i\left(\sqrt{3}\sqrt{\#1^3(27\#1-32)}-9\right.}\right.$$

$$y(x) \to 0$$

1.523 problem 525

Internal problem ID [8860]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 525.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$y'^2 - axyy' + 2ay^2 = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 98

```
dsolve(diff(y(x),x)^2-a*x*y(x)*diff(y(x),x)+2*a*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = c_1 \left(ax \operatorname{csgn}(a) + \sqrt{a(a x^2 - 8)}\right)^{-2 \operatorname{csgn}(a)} e^{\frac{x\left(ax+\sqrt{a(a x^2-8)}\right)}{4}}$$
$$y(x) = c_1 \left(ax \operatorname{csgn}(a) + \sqrt{a(a x^2 - 8)}\right)^{2 \operatorname{csgn}(a)} e^{-\frac{x\left(-ax+\sqrt{a(a x^2-8)}\right)}{4}}$$

✓ Solution by Mathematica

Time used: 0.229 (sec). Leaf size: 125

```
DSolve[2*a*y[x]^2 - a*x*y[x]*y'[x] + y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 e^{\frac{1}{4}\left(ax^2-\sqrt{a}x\sqrt{ax^2-8}\right)}}{\left(\sqrt{ax^2-8}-\sqrt{a}x\right)^2}$$
$$y(x) \to c_1 e^{\frac{1}{4}\left(ax^2+\sqrt{a}x\sqrt{ax^2-8}\right)}\left(\sqrt{ax^2-8}-\sqrt{a}x\right)^2$$
$$y(x) \to 0$$

1.524 problem 526

Internal problem ID [8861]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 526.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 - (y^2 + yx + x^2)\, y'^2 + (y^3x + y^2x^2 + yx^3)\, y' - y^3x^3 = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x)^3-(y(x)^2+x*y(x)+x^2)*diff(y(x),x)^2+(x*y(x)^3+x^2*y(x)^2+x^3*y(x))*diff
```

$$y(x) = \frac{x^3}{3} + c_1$$
$$y(x) = \frac{1}{-x + c_1}$$
$$y(x) = \mathrm{e}^{\frac{x^2}{2}} c_1$$

✓ Solution by Mathematica
Time used: 0.137 (sec). Leaf size: 48

```
DSolve[-(x^3*y[x]^3) + (x^3*y[x] + x^2*y[x]^2 + x*y[x]^3)*y'[x] - (x^2 + x*y[x] + y[x]^2)*y'
```

$$y(x) \to -\frac{1}{x + c_1}$$
$$y(x) \to c_1 e^{\frac{x^2}{2}}$$
$$y(x) \to \frac{x^3}{3} + c_1$$
$$y(x) \to 0$$

1.525 problem 527

Internal problem ID [8862]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 527.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y'^3 - xy^4y' - y^5 = 0}$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 47

```
dsolve(diff(y(x),x)^3-x*y(x)^4*diff(y(x),x)-y(x)^5=0,y(x), singsol=all)
```

$$y(x) = -\frac{3\sqrt{3}}{2x^{\frac{3}{2}}}$$
$$y(x) = \frac{3\sqrt{3}}{2x^{\frac{3}{2}}}$$
$$y(x) = 0$$
$$y(x) = c_1\sqrt{\frac{c_1^{10}}{(c_1^4x - 1)^2}}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 64

```
DSolve[-y[x]^5 - x*y[x]^4*y'[x] + y'[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{c_1 x - c_1{}^3}$$
$$y(x) \to 0$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to -\frac{3\sqrt{3}}{2x^{3/2}}$$
$$y(x) \to \frac{3\sqrt{3}}{2x^{3/2}}$$

1.526 problem 528

Internal problem ID [8863]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 528.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y'^3 + ay'^2 + by = -abx}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 127

```
dsolve(diff(y(x),x)^3+a*diff(y(x),x)^2+b*y(x)+a*b*x=0,y(x), singsol=all)
```

$$y(x) = \frac{-e^{3\,\mathrm{RootOf}\left(-2\_Z a^2-3\,e^{2\_Z}+8a\,e^{\_Z}+2c_1 b-5a^2-2bx\right)} + 2\,e^{2\,\mathrm{RootOf}\left(-2\_Z a^2-3\,e^{2\_Z}+8a\,e^{\_Z}+2c_1 b-5a^2-2bx\right)} a - e^{\mathrm{RootOf}\left(-2\_Z\right.}}{b}$$

✓ Solution by Mathematica

Time used: 0.546 (sec). Leaf size: 398

```
DSolve[a*b*x + b*y[x] + a*y'[x]^2 + y'[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x = -\frac{-a\left(\frac{\sqrt[3]{-2a^3+\sqrt{(-2a^3-27abx-27by(x))^2-4a^6}-27abx-27by(x)}}{3\sqrt[3]{2}} + \frac{}{3\sqrt[3]{-2a^3+\sqrt{(-2a^3-27ab\ldots}}}\right.}{} + c_1\right\}, y(x)\right]$$

1.527 problem 529

Internal problem ID [8864]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 529.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_dAlembert]`

$$y'^3 + xy'^2 - y = 0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 994

```
dsolve(diff(y(x),x)^3+x*diff(y(x),x)^2-y(x)=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \frac{\left(4x^2 - 2x\left(-36x^2 - 54x + 108c_1 - 8x^3 + 27 + 6\sqrt{-6\left(1+2c_1\right)\left(4x^3 + 18x^2 - 27c_1 + 27x\right)}\right)^{\frac{1}{3}} + 12x + \cdots}{\cdots}$$

$$y(x) = \frac{\left(\frac{\left(-i\sqrt{3}-1\right)\left(-36x^2-54x+108c_1-8x^3+27+6\sqrt{-6(1+2c_1)(4x^3+18x^2-27c_1+27x)}\right)^{\frac{2}{3}}}{4} + \left(2x + \frac{3}{2}\right)\left(-36x^2 - 54x + 108c_1 - \cdots\right.\right.}{\cdots}$$

$$y(x) = \frac{\left(\frac{\left(i\sqrt{3}-1\right)\left(-36x^2-54x+108c_1-8x^3+27+6\sqrt{-6(1+2c_1)(4x^3+18x^2-27c_1+27x)}\right)^{\frac{2}{3}}}{4} - \left(-2x - \frac{3}{2}\right)\left(-36x^2 - 54x + 108c_1 - \cdots\right.\right.}{\cdots}$$

✓ Solution by Mathematica

Time used: 84.456 (sec). Leaf size: 1516

```
DSolve[-y[x] + x*y'[x]^2 + y'[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-16x^4 + 8\left(\sqrt[3]{-8x^3 - 36x^2 - 54x + 108c_1 + 6\sqrt{6}\sqrt{-((4x^3 + 18x^2 + 27x - 27c_1)(2c_1 + 1)) + 27 - 12}}\right.}{}$$

$$y(x) \to \frac{1}{6}\left(-\frac{i(\sqrt{3} - i)x(2x + 3)^2}{\sqrt[3]{-8x^3 - 36x^2 + 6\sqrt{6}\sqrt{-((1 + 2c_1)(4x^3 + 18x^2 + 27x - 27c_1))} - 54x + 27 + 108c_1}} + \frac{1}{16}\left(-\frac{i(\sqrt{3} - i)(2x + 3)^2}{\sqrt[3]{-8x^3 - 36x^2 + 6\sqrt{6}\sqrt{-((1 + 2c_1)(4x^3 + 18x^2 + 27x - 27c_1))} - 54x + 27 + 108c_1}} + i(\sqrt{3} + i)\sqrt[3]{-8x^3 - 36x^2 + 6\sqrt{6}\sqrt{-((1 + 2c_1)(4x^3 + 18x^2 + 27x - 27c_1))} - 54x + 27 + 108c_1} - 4x + 6\right)^2 + i(\sqrt{3} + i)x\sqrt[3]{-8x^3 - 36x^2 + 6\sqrt{6}\sqrt{-((1 + 2c_1)(4x^3 + 18x^2 + 27x - 27c_1))} - 54x + 27 + 108c_1} + 2(3 - 2x)x - 6x + 6c_1\right)$$

$$y(x) \to \frac{1}{6}\left(\frac{i(\sqrt{3} + i)x(2x + 3)^2}{\sqrt[3]{-8x^3 - 36x^2 + 6\sqrt{6}\sqrt{-((1 + 2c_1)(4x^3 + 18x^2 + 27x - 27c_1))} - 54x + 27 + 108c_1}} + \frac{1}{16}\left(\frac{(1 - i\sqrt{3})(2x + 3)^2}{\sqrt[3]{-8x^3 - 36x^2 + 6\sqrt{6}\sqrt{-((1 + 2c_1)(4x^3 + 18x^2 + 27x - 27c_1))} - 54x + 27 + 108c_1}} + (1 + i\sqrt{3})\sqrt[3]{-8x^3 - 36x^2 + 6\sqrt{6}\sqrt{-((1 + 2c_1)(4x^3 + 18x^2 + 27x - 27c_1))} - 54x + 27 + 108c_1} + 4x - 6\right)^2 - (1 + i\sqrt{3})x\sqrt[3]{-8x^3 - 36x^2 + 6\sqrt{6}\sqrt{-((1 + 2c_1)(4x^3 + 18x^2 + 27x - 27c_1))} - 54x + 27 + 108c_1} + 2(3 - 2x)x - 6x + 6c_1\right)$$

1.528 problem 530

Internal problem ID [8865]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 530.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^3 - yy'^2 + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 420

```
dsolve(diff(y(x),x)^3-y(x)*diff(y(x),x)^2+y(x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$x - 6\left(\int^{y(x)} \frac{\left(8\_\_a^3 - 108\_\_a^2 + 12\sqrt{3}\,\sqrt{-4\_\_a^5 + 27\_\_a^4}\right)^{\frac{1}{3}}}{\left(8\_\_a^3 - 108\_\_a^2 + 12\sqrt{3}\,\sqrt{-4\_\_a^5 + 27\_\_a^4}\right)^{\frac{2}{3}} + 2\left(8\_\_a^3 - 108\_\_a^2 + 12\sqrt{3}\,\sqrt{-4\_\_a^5 + 27\_\_a^4\right.}}\right. - c_1 = 0$$

$$\frac{12\left(\int^{y(x)} \frac{\left(8\_\_a^3-108\_\_a^2+12\sqrt{3}\,\sqrt{-4\_\_a^5+27\_\_a^4}\right)^{\frac{1}{3}}}{\left(\left(8\_\_a^3-108\_\_a^2+12\sqrt{3}\,\sqrt{-4\_\_a^5+27\_\_a^4}\right)^{\frac{1}{3}}-2\_\_a\right)\left(i\sqrt{3}\_\_a+\left(8\_\_a^3-108\_\_a^2+12\sqrt{3}\,\sqrt{-4\_\_a^5+27\_\_a^4}\right)^{\frac{1}{3}}+\_\_a\right)} d\_\right.}{1 + i\sqrt{3}} = 0$$

$$\frac{12\left(\int^{y(x)} \frac{\left(8\_\_a^3-108\_\_a^2+12\sqrt{3}\,\sqrt{-4\_\_a^5+27\_\_a^4}\right)^{\frac{1}{3}}}{\left(-i\sqrt{3}\_\_a+\left(8\_\_a^3-108\_\_a^2+12\sqrt{3}\,\sqrt{-4\_\_a^5+27\_\_a^4}\right)^{\frac{1}{3}}+\_\_a\right)\left(-\left(8\_\_a^3-108\_\_a^2+12\sqrt{3}\,\sqrt{-4\_\_a^5+27\_\_a^4}\right)^{\frac{1}{3}}+2\_\_a\right.}\right.}{i\sqrt{3} - 1} = 0$$

✓ Solution by Mathematica

Time used: 56.542 (sec). Leaf size: 653

```
DSolve[y[x]^2 - y[x]*y'[x]^2 + y'[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sqrt[3]{2K[1]^3-27K[1]^2+3\sqrt{3}\sqrt{-K[1]^4(4K[1]-27)}}}{2\sqrt[3]{2}K[1]^2+2\sqrt[3]{2K[1]^3-27K[1]^2+3\sqrt{3}\sqrt{-K[1]^4(4K[1]-27)}}K[1]+2^{2/3}\left(2K\right.}\right.$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{}{2i\sqrt[3]{2}\sqrt{3}K[2]^2-2\sqrt[3]{2}K[2]^2+4\sqrt[3]{2K[2]^3-27K[2]^2+3\sqrt{3}\sqrt{-K[2]^4(4K[2]-2}}}\right.$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{}{-2i\sqrt[3]{2}\sqrt{3}K[3]^2-2\sqrt[3]{2}K[3]^2+4\sqrt[3]{2K[3]^3-27K[3]^2+3\sqrt{3}\sqrt{-K[3]^4(4K[3]-}}}\right.$$

$$y(x) \to 0$$

1.529 problem 531

Internal problem ID [8866]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 531.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$y'^2 - (y^4 + xy^2 + x^2)\, y'^2 + (xy^6 + x^2y^4 + y^2x^3)\, y' - y^6x^3 = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)^2-(y(x)^4+x*y(x)^2+x^2)*diff(y(x),x)^2+(x*y(x)^6+x^2*y(x)^4+x^3*y(x)^2)*
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-(x^3*y[x]^6) + (x^3*y[x]^2 + x^2*y[x]^4 + x*y[x]^6)*y'[x] + y'[x]^2 - (x^2 + x*y[x]^
```

Not solved

1.530 problem 532

Internal problem ID [8867]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 532.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$a{y'}^3 + b{y'}^2 + y'c - y = d$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 922

```
dsolve(a*diff(y(x),x)^3+b*diff(y(x),x)^2+c*diff(y(x),x)-y(x)-d=0,y(x), singsol=all)
```

$$3\sqrt{3}\,2^{\frac{1}{3}}a\left(\int^{y(x)}\frac{\left(9\sqrt{27\left(d+\_a\right.}\right.}{\sqrt{3}\,2^{\frac{1}{3}}\left(9\sqrt{27\left(d+\_a\right)^2a^2+18\left(\left(d+\_a\right)b+\frac{2c^2}{9}\right)ca+\left(-4\_a-4d\right)b^3-b^2c^2\,a+27\left(a^2\right.}\right.}\right.$$

$$+x-c_1=0$$

$$\frac{12\,2^{\frac{1}{3}}\sqrt{3}\,a\left(\int^{y(x)}\frac{\left(9\sqrt{27(d+\_a)^2a^2+18\left((d+\_a)b+\frac{2c^2}{9}\right)}\right.}{-3\,2^{\frac{1}{3}}\left(i-\frac{\sqrt{3}}{3}\right)b\left(9\sqrt{27(d+\_a)^2a^2+18\left((d+\_a)b+\frac{2c^2}{9}\right)ca+(-4\_a-4d)b^3-b^2c^2}\,a+27\left(a^2(d+\_a)+\frac{acb}{3}-\frac{2b^3}{27}\right)\sqrt{3}\right)^{\frac{1}{3}}}\right.}{}$$

$$=0$$

$$\frac{12\,2^{\frac{1}{3}}\sqrt{3}\,a\left(\int^{y(x)}\frac{\left(9\sqrt{27(d+\_a)^2a^2+18\left((d+\_a)b+\frac{2c^2}{9}\right)}\right.}{-3\,2^{\frac{1}{3}}\left(i+\frac{\sqrt{3}}{3}\right)b\left(9\sqrt{27(d+\_a)^2a^2+18\left((d+\_a)b+\frac{2c^2}{9}\right)ca+(-4\_a-4d)b^3-b^2c^2}\,a+27\left(a^2(d+\_a)+\frac{acb}{3}-\frac{2b^3}{27}\right)\sqrt{3}\right)^{\frac{1}{3}}}\right.}{}$$

$$=0$$

✓ Solution by Mathematica

Time used: 0.362 (sec). Leaf size: 1064

```
DSolve[-d - y[x] + c*y'[x] + b*y'[x]^2 + a*y'[x]^3==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \text{InverseFunction}\left[\int \frac{\sqrt[3]{2b^3 - 9acb - 27a^2d - \cdots}}{2\sqrt[3]{2}b^2 + 2\sqrt[3]{2b^3 - 9acb - 27a^2d - 27a^2\#1 + \sqrt{4\left(3ac - b^2\right)^3 + \left(2b^3 - 9acb - 27a^2 \cdots\right.}}}\right.$$

$$y(x) \to \text{InverseFunction}\left[\int \frac{\cdots}{2i\sqrt[3]{2}\sqrt{3}b^2 + 2\sqrt[3]{2}b^2 - 4\sqrt[3]{2b^3 - 9acb - 27a^2d - 27a^2\#1 + \sqrt{4\left(3ac - b^2\right)^3 + \left(2b^3 \cdots\right.}}}\right.$$

$$y(x) \to \text{InverseFunction}\left[\int \frac{\cdots}{-2i\sqrt[3]{2}\sqrt{3}b^2 + 2\sqrt[3]{2}b^2 - 4\sqrt[3]{2b^3 - 9acb - 27a^2d - 27a^2\#1 + \sqrt{4\left(3ac - b^2\right)^3 + \left(2b \cdots\right.}}}\right.$$

$$y(x) \to -d$$

1.531 problem 533

Internal problem ID [8868]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 533.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{xy'^3 - yy'^2 = -a}$$

✓ Solution by Maple
Time used: 0.14 (sec). Leaf size: 76

```
dsolve(x*diff(y(x),x)^3-y(x)*diff(y(x),x)^2+a=0,y(x), singsol=all)
```

$$y(x) = \frac{3\,2^{\frac{1}{3}}(a\,x^2)^{\frac{1}{3}}}{2}$$
$$y(x) = -\frac{3\,2^{\frac{1}{3}}(a\,x^2)^{\frac{1}{3}}\left(1+i\sqrt{3}\right)}{4}$$
$$y(x) = \frac{3\,2^{\frac{1}{3}}(a\,x^2)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)}{4}$$
$$y(x) = \frac{c_1^3x + a}{c_1^2}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 89

```
DSolve[a - y[x]*y'[x]^2 + x*y'[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{a}{{c_1}^2} + c_1 x$$
$$y(x) \to \frac{3\sqrt[3]{a}x^{2/3}}{2^{2/3}}$$
$$y(x) \to -\frac{3\sqrt[3]{-1}\sqrt[3]{a}x^{2/3}}{2^{2/3}}$$
$$y(x) \to \frac{3(-1)^{2/3}\sqrt[3]{a}x^{2/3}}{2^{2/3}}$$

1.532 problem 534

Internal problem ID [8869]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 534.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{4xy'^3 - 6yy'^2 + 3y = x}$$

✓ Solution by Maple
Time used: 0.109 (sec). Leaf size: 84

```
dsolve(4*x*diff(y(x),x)^3-6*y(x)*diff(y(x),x)^2+3*y(x)-x=0,y(x), singsol=all)
```

$$y(x) = -\frac{(1+\sqrt{3})\,x}{2}$$
$$y(x) = \frac{(\sqrt{3}-1)\,x}{2}$$
$$y(x) = x$$
$$y(x) = \frac{-(x+c_1)\sqrt{2}\,\sqrt{c_1\,(x+c_1)} - c_1^2}{3c_1}$$
$$y(x) = \frac{(x+c_1)\sqrt{2}\,\sqrt{c_1\,(x+c_1)} - c_1^2}{3c_1}$$

✓ Solution by Mathematica
Time used: 1.143 (sec). Leaf size: 79

```
DSolve[-x + 3*y[x] - 6*y[x]*y'[x]^2 + 4*x*y'[x]^3==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -\frac{\sqrt{2}\sqrt{c_1(x+c_1)^3} + c_1{}^2}{3c_1}$$
$$y(x) \to -\frac{c_1{}^2 - \sqrt{2}\sqrt{c_1(x+c_1)^3}}{3c_1}$$
$$y(x) \to \text{Indeterminate}$$

1.533 problem 535

Internal problem ID [8870]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 535.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$8xy'^3 - 12yy'^2 + 9y = 0$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 61

```
dsolve(8*x*diff(y(x),x)^3-12*y(x)*diff(y(x),x)^2+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{3x}{2}$$
$$y(x) = \frac{3x}{2}$$
$$y(x) = 0$$
$$y(x) = -\frac{(3c_1 + x)\sqrt{c_1(3c_1 + x)}}{3c_1}$$
$$y(x) = \frac{(3c_1 + x)\sqrt{c_1(3c_1 + x)}}{3c_1}$$

✓ Solution by Mathematica

Time used: 0.336 (sec). Leaf size: 77

```
DSolve[9*y[x] - 12*y[x]*y'[x]^2 + 8*x*y'[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{(x+3c_1)^{3/2}}{3\sqrt{c_1}}$$
$$y(x) \to \frac{(x+3c_1)^{3/2}}{3\sqrt{c_1}}$$
$$y(x) \to 0$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to -\frac{3x}{2}$$
$$y(x) \to \frac{3x}{2}$$

1.534 problem 536

Internal problem ID [8871]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 536.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$(-a^2 + x^2)\, y'^3 + bx(-a^2 + x^2)\, y'^2 + y' = -xb$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 52

```
dsolve((-a^2+x^2)*diff(y(x),x)^3+b*x*(-a^2+x^2)*diff(y(x),x)^2+diff(y(x),x)+b*x=0,y(x), sing
```

$$y(x) = -\frac{b\,x^2}{2} + c_1$$
$$y(x) = \arctan\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + c_1$$
$$y(x) = -\arctan\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + c_1$$

✓ Solution by Mathematica
Time used: 0.008 (sec). Leaf size: 64

```
DSolve[b*x + y'[x] + b*x*(-a^2 + x^2)*y'[x]^2 + (-a^2 + x^2)*y'[x]^3==0,y[x],x,IncludeSingul
```

$$y(x) \to -\frac{bx^2}{2} + c_1$$
$$y(x) \to -\arctan\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + c_1$$
$$y(x) \to \arctan\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + c_1$$

1.535 problem 537

Internal problem ID [8872]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 537.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$x^3 y'^3 - 3x^2 y y'^2 + (3xy^2 + x^6)\, y' - y^3 - 2yx^5 = 0$$

✗ Solution by Maple

```
dsolve(x^3*diff(y(x),x)^3-3*x^2*y(x)*diff(y(x),x)^2+(3*x*y(x)^2+x^6)*diff(y(x),x)-y(x)^3-2*x
```

No solution found

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 15

```
DSolve[-2*x^5*y[x] - y[x]^3 + (x^6 + 3*x*y[x]^2)*y'[x] - 3*x^2*y[x]*y'[x]^2 + x^3*y'[x]^3==0
```

$$y(x) \to c_1 x\left(x + c_1{}^2\right)$$

1.536 problem 538

Internal problem ID [8873]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 538.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$2(xy' + y)^3 - yy' = 0$$

✓ Solution by Maple

Time used: 3.578 (sec). Leaf size: 1755

```
dsolve(2*(x*diff(y(x),x)+y(x))^3-y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$y(x) = 0$
Expression too large to display
Expression too large to display
Expression too large to display

✓ Solution by Mathematica

Time used: 62.055 (sec). Leaf size: 179

```
DSolve[-(y[x]*y'[x]) + 2*(y[x] + x*y'[x])^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\int_1^x \frac{\text{InverseFunction}\left[-\frac{2\sqrt{\#1^2-8\#1^3}\arctan\left(\sqrt{8\#1-1}\right)}{\#1\sqrt{8\#1-1}} - 14\log\left(\#1^2(8\#1-1)\right) + \log\left(\#1^{14}(8\#1-1)^{15/2}\left(\#1-\sqrt{\#1^2-8\#\right.\right.}{K[1]}}{x}$$

1.537 problem 539

Internal problem ID [8874]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 539.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 \sin(x) - \left(y \sin(x) - \cos(x)^2\right) y'^2 - \left(y \cos(x)^2 + \sin(x)\right) y' + y \sin(x) = 0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 32

```
dsolve(diff(y(x),x)^3*sin(x)-(y(x)*sin(x)-cos(x)^2)*diff(y(x),x)^2-(y(x)*cos(x)^2+sin(x))*di
```

$$y(x) = c_1 e^x$$
$$y(x) = -\ln(\csc(x) - \cot(x)) + c_1$$
$$y(x) = -\cos(x) + c_1$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 32

```
DSolve[Sin[x]*y[x] - (Sin[x] + Cos[x]^2*y[x])*y'[x] - (-Cos[x]^2 + Sin[x]*y[x])*y'[x]^2 + Si
```

$$y(x) \to c_1 e^x$$
$$y(x) \to \operatorname{arctanh}(\cos(x)) + c_1$$
$$y(x) \to -\cos(x) + c_1$$

1.538 problem 540

Internal problem ID [8875]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 540.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$2yy'^3 - yy'^2 + 2xy' = x$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 111

```
dsolve(2*y(x)*diff(y(x),x)^3-y(x)*diff(y(x),x)^2+2*x*diff(y(x),x)-x=0,y(x), singsol=all)
```

$$x\left(1+\frac{c_1}{\left(\frac{-\sqrt{-xy(x)}+y(x)}{y(x)}\right)^{\frac{2}{3}}\left(\frac{-x+\sqrt{-xy(x)}+y(x)}{y(x)}\right)^{\frac{2}{3}}y(x)}\right)=0$$

$$x\left(1+\frac{c_1}{\left(\frac{\sqrt{-xy(x)}+y(x)}{y(x)}\right)^{\frac{2}{3}}\left(\frac{-x-\sqrt{-xy(x)}+y(x)}{y(x)}\right)^{\frac{2}{3}}y(x)}\right)=0$$

$$y(x)=\frac{x}{2}+c_1$$

✓ Solution by Mathematica
Time used: 3.408 (sec). Leaf size: 61

```
DSolve[-x + 2*x*y'[x] - y[x]*y'[x]^2 + 2*y[x]*y'[x]^3==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{x}{2}+c_1$$

$$y(x)\to\left(\frac{3c_1}{2}-ix^{3/2}\right)^{2/3}$$

$$y(x)\to\left(ix^{3/2}+\frac{3c_1}{2}\right)^{2/3}$$

1.539 problem 541

Internal problem ID [8876]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 541.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y^2 {y'}^3 + 2xy' - y = 0$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 97

```
dsolve(y(x)^2*diff(y(x),x)^3+2*x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{2(-x^3)^{\frac{1}{4}} 6^{\frac{1}{4}}}{3}$$
$$y(x) = \frac{2(-x^3)^{\frac{1}{4}} 6^{\frac{1}{4}}}{3}$$
$$y(x) = -\frac{2i(-x^3)^{\frac{1}{4}} 6^{\frac{1}{4}}}{3}$$
$$y(x) = \frac{2i(-x^3)^{\frac{1}{4}} 6^{\frac{1}{4}}}{3}$$
$$y(x) = 0$$
$$y(x) = \sqrt{c_1 (c_1^2 + 2x)}$$
$$y(x) = -\sqrt{c_1 (c_1^2 + 2x)}$$

✓ Solution by Mathematica

Time used: 0.116 (sec). Leaf size: 119

```
DSolve[-y[x] + 2*x*y'[x] + y[x]^2*y'[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2c_1x + c_1{}^3}$$
$$y(x) \to \sqrt{2c_1x + c_1{}^3}$$
$$y(x) \to (-1 - i)\left(\frac{2}{3}\right)^{3/4} x^{3/4}$$
$$y(x) \to (1 - i)\left(\frac{2}{3}\right)^{3/4} x^{3/4}$$
$$y(x) \to (-1 + i)\left(\frac{2}{3}\right)^{3/4} x^{3/4}$$
$$y(x) \to (1 + i)\left(\frac{2}{3}\right)^{3/4} x^{3/4}$$

1.540 problem 542

Internal problem ID [8877]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 542.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{16y^2y'^3 + 2xy' - y = 0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 104

```
dsolve(16*y(x)^2*diff(y(x),x)^3+2*x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{(-x^3)^{\frac{1}{4}} 6^{\frac{1}{4}}}{3}$$
$$y(x) = \frac{(-x^3)^{\frac{1}{4}} 6^{\frac{1}{4}}}{3}$$
$$y(x) = -\frac{i(-x^3)^{\frac{1}{4}} 6^{\frac{1}{4}}}{3}$$
$$y(x) = \frac{i(-x^3)^{\frac{1}{4}} 6^{\frac{1}{4}}}{3}$$
$$y(x) = 0$$
$$y(x) = \sqrt{2}\,\sqrt{c_1 (8c_1^2 + x)}$$
$$y(x) = -\sqrt{2}\,\sqrt{c_1 (8c_1^2 + x)}$$

✓ Solution by Mathematica

Time used: 0.115 (sec). Leaf size: 107

```
DSolve[-y[x] + 2*x*y'[x] + 16*y[x]^2*y'[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{c_1 (x + 2c_1{}^2)}$$
$$y(x) \to -\frac{\sqrt[4]{-2}x^{3/4}}{3^{3/4}}$$
$$y(x) \to \frac{(1-i)x^{3/4}}{\sqrt[4]{2}3^{3/4}}$$
$$y(x) \to \frac{i\sqrt[4]{-2}x^{3/4}}{3^{3/4}}$$
$$y(x) \to \frac{\sqrt[4]{-2}x^{3/4}}{3^{3/4}}$$

1.541 problem 543

Internal problem ID [8878]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 543.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$x y^2 {y'}^3 - y^3 {y'}^2 + x\left(x^2+1\right) y' - y x^2 = 0$$

✗ Solution by Maple

```
dsolve(x*y(x)^2*diff(y(x),x)^3-y(x)^3*diff(y(x),x)^2+x*(x^2+1)*diff(y(x),x)-x^2*y(x)=0,y(x),
```

No solution found

✓ Solution by Mathematica

Time used: 0.541 (sec). Leaf size: 399

```
DSolve[-(x^2*y[x]) + x*(1 + x^2)*y'[x] - y[x]^3*y'[x]^2 + x*y[x]^2*y'[x]^3==0,y[x],x,Include
```

$$y(x) \to -\sqrt{c_1\left(x^2 + \frac{1}{1 + c_1{}^2}\right)}$$

$$y(x) \to \sqrt{c_1\left(x^2 + \frac{1}{1 + c_1{}^2}\right)}$$

$$y(x) \to -\frac{\sqrt[4]{-8x^4 + 20x^2 - \sqrt{-\left(8x^2 - 1\right)^3} + 1}}{2^{3/4}}$$

$$y(x) \to -\frac{i\sqrt[4]{-8x^4 + 20x^2 - \sqrt{-\left(8x^2 - 1\right)^3} + 1}}{2^{3/4}}$$

$$y(x) \to \frac{i\sqrt[4]{-8x^4 + 20x^2 - \sqrt{-\left(8x^2 - 1\right)^3} + 1}}{2^{3/4}}$$

$$y(x) \to \frac{\sqrt[4]{-8x^4 + 20x^2 - \sqrt{-\left(8x^2 - 1\right)^3} + 1}}{2^{3/4}}$$

$$y(x) \to -\frac{\sqrt[4]{-8x^4 + 20x^2 + \sqrt{-\left(8x^2 - 1\right)^3} + 1}}{2^{3/4}}$$

$$y(x) \to -\frac{i\sqrt[4]{-8x^4 + 20x^2 + \sqrt{-\left(8x^2 - 1\right)^3} + 1}}{2^{3/4}}$$

$$y(x) \to \frac{i\sqrt[4]{-8x^4 + 20x^2 + \sqrt{-\left(8x^2 - 1\right)^3} + 1}}{2^{3/4}}$$

$$y(x) \to \frac{\sqrt[4]{-8x^4 + 20x^2 + \sqrt{-\left(8x^2 - 1\right)^3} + 1}}{2^{3/4}}$$

1.542 problem 544

Internal problem ID [8879]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 544.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$x^7y^2y'^3 - (3x^6y^3 - 1)\,y'^2 + 3x^5y^4y' - x^4y^5 = 0$$

✓ Solution by Maple

Time used: 0.937 (sec). Leaf size: 4638

```
dsolve(x^7*y(x)^2*diff(y(x),x)^3-(3*x^6*y(x)^3-1)*diff(y(x),x)^2+3*x^5*y(x)^4*diff(y(x),x)-x
```

$$y(x) = \frac{2^{\frac{2}{3}}}{3x^2}$$
$$y(x) = -\frac{2^{\frac{2}{3}}(1 + i\sqrt{3})}{6x^2}$$
$$y(x) = \frac{2^{\frac{2}{3}}(i\sqrt{3} - 1)}{6x^2}$$
$$y(x) = 0$$

Expression too large to display
Expression too large to display
Expression too large to display

✓ Solution by Mathematica

Time used: 2.117 (sec). Leaf size: 80

```
DSolve[-(x^4*y[x]^5) + 3*x^5*y[x]^4*y'[x] - (-1 + 3*x^6*y[x]^3)*y'[x]^2 + x^7*y[x]^2*y'[x]^3
```

$$y(x) \to \sqrt[3]{c_1x^3 + c_1{}^{2/3}}$$
$$y(x) \to 0$$
$$y(x) \to \frac{(-2)^{2/3}}{3x^2}$$
$$y(x) \to \frac{2^{2/3}}{3x^2}$$
$$y(x) \to -\frac{\sqrt[3]{-1}2^{2/3}}{3x^2}$$

1.543 problem 545

Internal problem ID [8880]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 545.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[_quadrature]`

$$y'^4 - (y-a)^3 (y-b)^2 = 0$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 133

```
dsolve(diff(y(x),x)^4-(y(x)-a)^3*(y(x)-b)^2=0,y(x), singsol=all)
```

$$y(x) = a$$
$$y(x) = b$$
$$x - \left(\int^{y(x)} \frac{1}{\left((\_a-a)^3 (\_a-b)^2\right)^{\frac{1}{4}}} d\_a\right) - c_1 = 0$$
$$x - i\left(\int^{y(x)} \frac{1}{\left((\_a-a)^3 (\_a-b)^2\right)^{\frac{1}{4}}} d\_a\right) - c_1 = 0$$
$$x + i\left(\int^{y(x)} \frac{1}{\left((\_a-a)^3 (\_a-b)^2\right)^{\frac{1}{4}}} d\_a\right) - c_1 = 0$$
$$x + \int^{y(x)} \frac{1}{\left((\_a-a)^3 (\_a-b)^2\right)^{\frac{1}{4}}} d\_a - c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.606 (sec). Leaf size: 333

```
DSolve[-((-a + y[x])^3*(-b + y[x])^2) + y'[x]^4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{4\sqrt[4]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[-\sqrt[4]{-1}x+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{4\sqrt[4]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\sqrt[4]{-1}x+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{4\sqrt[4]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[-(-1)^{3/4}x+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{4\sqrt[4]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[(-1)^{3/4}x+c_1\right]$$

$$y(x) \to a$$
$$y(x) \to b$$

1.544 problem 546

Internal problem ID [8881]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 546.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[_dAlembert]`

$$y'^4 + 3(x-1)\,y'^2 - 3(2y-1)\,y' = -3x$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 149

```
dsolve(diff(y(x),x)^4+3*(x-1)*diff(y(x),x)^2-3*(2*y(x)-1)*diff(y(x),x)+3*x=0,y(x), singsol=a
```

$$y(x) = x + \frac{1}{6}$$
$$y(x) = -x + \frac{5}{6}$$
$$y(x) = \frac{(3 - c_1^3 + (-5x+3)\,c_1)\sqrt{c_1^2+4x} - c_1^4 + (-7x+3)\,c_1^2 + 3c_1 - 8x^2}{6c_1 + 6\sqrt{c_1^2+4x}}$$
$$y(x) = \frac{(-3 + c_1^3 + (5x-3)\,c_1)\sqrt{c_1^2+4x} - c_1^4 + (-7x+3)\,c_1^2 + 3c_1 - 8x^2}{6c_1 - 6\sqrt{c_1^2+4x}}$$

✓ Solution by Mathematica

Time used: 0.496 (sec). Leaf size: 77

```
DSolve[3*x - 3*(-1 + 2*y[x])*y'[x] + 3*(-1 + x)*y'[x]^2 + y'[x]^4==0,y[x],x,IncludeSingularS
```

$$y(x) \to \frac{1}{12}\left(-6c_1(x-1) - \sqrt{(4x + c_1{}^2)^3} + 6 - c_1{}^3\right)$$
$$y(x) \to \frac{1}{12}\left(-6c_1(x-1) + \sqrt{(4x + c_1{}^2)^3} + 6 - c_1{}^3\right)$$

1.545 problem 547

Internal problem ID [8882]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 547.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$y'^4 - 4y(xy' - 2y)^2 = 0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 120

```
dsolve(diff(y(x),x)^4-4*y(x)*(x*diff(y(x),x)-2*y(x))^2=0,y(x), singsol=all)
```

$$y(x) = \frac{x^4}{16}$$
$$y(x) = 0$$
$$y(x)\left(\sqrt{x^2-4\sqrt{y(x)}}-x\right)^{-\frac{2\sqrt{x^2y(x)-4y(x)^{\frac{3}{2}}}}{\sqrt{x^2-4\sqrt{y(x)}}\sqrt{y(x)}}}\left(\sqrt{x^2-4\sqrt{y(x)}}+x\right)^{\frac{2\sqrt{x^2y(x)-4y(x)^{\frac{3}{2}}}}{\sqrt{x^2-4\sqrt{y(x)}}\sqrt{y(x)}}} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 32.391 (sec). Leaf size: 519

```
DSolve[y'[x]^4 - 4*y[x]*(-2*y[x] + x*y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\sqrt{\left(x^2+4\sqrt{y(x)}\right)y(x)}\log\left(\sqrt{x^2+4\sqrt{y(x)}}-x\right)}{\sqrt{x^2+4\sqrt{y(x)}}\sqrt{y(x)}}+\frac{1}{4}\left(\log(y(x))-\frac{\sqrt{x^2+4\sqrt{y(x)}}\sqrt{y(x)}\log(y(x))}{\sqrt{\left(x^2+4\sqrt{y(x)}\right)y(x)}}\right)=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{4}\left(\frac{\sqrt{x^2+4\sqrt{y(x)}}\sqrt{y(x)}\log(y(x))}{\sqrt{\left(x^2+4\sqrt{y(x)}\right)y(x)}}+\log(y(x))\right)-\frac{\sqrt{\left(x^2+4\sqrt{y(x)}\right)y(x)}\log\left(\sqrt{x^2+4\sqrt{y(x)}}-x\right)}{\sqrt{x^2+4\sqrt{y(x)}}\sqrt{y(x)}}=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{2}\left(\frac{\sqrt{x^2-4\sqrt{y(x)}}\sqrt{y(x)}\log(y(x))}{2\sqrt{x^2y(x)-4y(x)^{3/2}}}+\frac{1}{2}\log(y(x))\right)-\frac{\sqrt{\left(x^2-4\sqrt{y(x)}\right)y(x)}\log\left(\sqrt{x^2-4\sqrt{y(x)}}-x\right)}{\sqrt{x^2-4\sqrt{y(x)}}\sqrt{y(x)}}=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{\sqrt{\left(x^2-4\sqrt{y(x)}\right)y(x)}\log\left(\sqrt{x^2-4\sqrt{y(x)}}-x\right)}{\sqrt{x^2-4\sqrt{y(x)}}\sqrt{y(x)}}+\left(\frac{1}{4}-\frac{\sqrt{x^2y(x)-4y(x)^{3/2}}}{4\sqrt{x^2-4\sqrt{y(x)}}\sqrt{y(x)}}\right)\log(y(x))=c_1,y(x)\right]$$

$$y(x)\to 0$$

$$y(x)\to \frac{x^4}{16}$$

1.546 problem 548

Internal problem ID [8883]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 548.
ODE order: 1.
ODE degree: 6.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^6 - (y-a)^4 (y-b)^3 = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 281

```
dsolve(diff(y(x),x)^6-(y(x)-a)^4*(y(x)-b)^3=0,y(x), singsol=all)
```

$$y(x) = a$$
$$y(x) = b$$
$$x - \left(\int^{y(x)} \frac{1}{\left((\_a - a)^4 (\_a - b)^3\right)^{\frac{1}{6}}} d\_a\right) - c_1 = 0$$
$$\frac{2\left(\int^{y(x)} \frac{1}{\left((\_a-a)^4(\_a-b)^3\right)^{\frac{1}{6}}} d\_a\right) + i(x - c_1)\sqrt{3} + x - c_1}{1 + i\sqrt{3}} = 0$$
$$\frac{-2\left(\int^{y(x)} \frac{1}{\left((\_a-a)^4(\_a-b)^3\right)^{\frac{1}{6}}} d\_a\right) + i(x - c_1)\sqrt{3} - x + c_1}{i\sqrt{3} - 1} = 0$$
$$\frac{2\left(\int^{y(x)} \frac{1}{\left((\_a-a)^4(\_a-b)^3\right)^{\frac{1}{6}}} d\_a\right) + i(x - c_1)\sqrt{3} - x + c_1}{i\sqrt{3} - 1} = 0$$
$$\frac{-2\left(\int^{y(x)} \frac{1}{\left((\_a-a)^4(\_a-b)^3\right)^{\frac{1}{6}}} d\_a\right) + i(x - c_1)\sqrt{3} + x - c_1}{1 + i\sqrt{3}} = 0$$
$$x + \int^{y(x)} \frac{1}{\left((\_a - a)^4 (\_a - b)^3\right)^{\frac{1}{6}}} d\_a - c_1 = 0$$

✓ Solution by Mathematica

Time used: 2.119 (sec). Leaf size: 489

```
DSolve[-((-a + y[x])^4*(-b + y[x])^3) + y'[x]^6==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right][c_1 - ix]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right][ix + c_1]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[-\sqrt[6]{-1}x + c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\sqrt[6]{-1}x + c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[-(-1)^{5/6}x + c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[(-1)^{5/6}x + c_1\right]$$

$$y(x) \to a$$
$$y(x) \to b$$

1.547 problem 549

Internal problem ID [8884]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 549.
ODE order: 1.
ODE degree: 6.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x^2\left(1+y'^2\right)^3=a^2}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 601

```
dsolve(x^2*(diff(y(x),x)^2+1)^3-a^2=0,y(x), singsol=all)
```

$$y(x) = \frac{-\sqrt{\frac{x(a^2x)^{\frac{1}{3}}\left(a^2-(a^2x)^{\frac{2}{3}}\right)}{a^2}}\, a^2 + c_1(a^2x)^{\frac{2}{3}} + (a^2x)^{\frac{2}{3}}\sqrt{\frac{x(a^2x)^{\frac{1}{3}}\left(a^2-(a^2x)^{\frac{2}{3}}\right)}{a^2}}}{(a^2x)^{\frac{2}{3}}}$$

$$y(x) = \frac{\left(a^2-(a^2x)^{\frac{2}{3}}\right)\sqrt{\frac{x(a^2x)^{\frac{1}{3}}\left(a^2-(a^2x)^{\frac{2}{3}}\right)}{a^2}} + c_1(a^2x)^{\frac{2}{3}}}{(a^2x)^{\frac{2}{3}}}$$

$$y(x) = -\frac{\sqrt{2}\sqrt{-x\left(i\sqrt{3}\,(a^2x)^{\frac{1}{3}}+(a^2x)^{\frac{1}{3}}+2x\right)}\sqrt{\frac{x(a^2x)^{\frac{1}{3}}\left(2i(a^2x)^{\frac{2}{3}}+ia^2-\sqrt{3}\,a^2\right)}{a^2}}\left(2(a^2x)^{\frac{2}{3}}+a^2+i\sqrt{3}\,a^2\right)}{4\sqrt{\left(i\,(a^2x)^{\frac{1}{3}}+2ix-\sqrt{3}\,(a^2x)^{\frac{1}{3}}\right)x}\,(a^2x)^{\frac{2}{3}}} + c_1$$

$$y(x) = \frac{\sqrt{2}\sqrt{-x\left(i\sqrt{3}\,(a^2x)^{\frac{1}{3}}+(a^2x)^{\frac{1}{3}}+2x\right)}\sqrt{\frac{x(a^2x)^{\frac{1}{3}}\left(2i(a^2x)^{\frac{2}{3}}+ia^2-\sqrt{3}\,a^2\right)}{a^2}}\left(2(a^2x)^{\frac{2}{3}}+a^2+i\sqrt{3}\,a^2\right)}{4\sqrt{\left(i\,(a^2x)^{\frac{1}{3}}+2ix-\sqrt{3}\,(a^2x)^{\frac{1}{3}}\right)x}\,(a^2x)^{\frac{2}{3}}} + c_1$$

$$y(x) = \frac{\left(\left(i\sqrt{3}-1\right)a^2-2(a^2x)^{\frac{2}{3}}\right)\sqrt{2}\sqrt{\frac{\left(\left(i\sqrt{3}-1\right)a^2-2(a^2x)^{\frac{2}{3}}\right)x(a^2x)^{\frac{1}{3}}}{a^2}} + 4c_1(a^2x)^{\frac{2}{3}}}{4\,(a^2x)^{\frac{2}{3}}}$$

$$y(x) = -\frac{\left(-2(a^2x)^{\frac{2}{3}}\sqrt{2}+a^2\left(i\sqrt{6}-\sqrt{2}\right)\right)\sqrt{\frac{\left(\left(i\sqrt{3}-1\right)a^2-2(a^2x)^{\frac{2}{3}}\right)x(a^2x)^{\frac{1}{3}}}{a^2}} - 4c_1(a^2x)^{\frac{2}{3}}}{4\,(a^2x)^{\frac{2}{3}}}$$

✓ <u>Solution by Mathematica</u>

Time used: 19.706 (sec). Leaf size: 375

```
DSolve[-a^2 + x^2*(1 + y'[x]^2)^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[3]{x}\sqrt{\frac{a^{2/3}}{x^{2/3}}-1}\left(x^{2/3}-a^{2/3}\right)+c_1$$
$$y(x) \to \sqrt[3]{x}\sqrt{\frac{a^{2/3}}{x^{2/3}}-1}\left(a^{2/3}-x^{2/3}\right)+c_1$$
$$y(x) \to c_1-\frac{1}{2}\sqrt[3]{x}\sqrt{-1+\frac{i\left(\sqrt{3}+i\right)a^{2/3}}{2x^{2/3}}}\left(2x^{2/3}+\left(1-i\sqrt{3}\right)a^{2/3}\right)$$
$$y(x) \to \frac{1}{2}\sqrt[3]{x}\sqrt{-1+\frac{i\left(\sqrt{3}+i\right)a^{2/3}}{2x^{2/3}}}\left(2x^{2/3}+\left(1-i\sqrt{3}\right)a^{2/3}\right)+c_1$$
$$y(x) \to c_1-\frac{1}{2}\sqrt[3]{x}\sqrt{-1-\frac{i\left(\sqrt{3}-i\right)a^{2/3}}{2x^{2/3}}}\left(2x^{2/3}+\left(1+i\sqrt{3}\right)a^{2/3}\right)$$
$$y(x) \to \frac{1}{2}\sqrt[3]{x}\sqrt{-1-\frac{i\left(\sqrt{3}-i\right)a^{2/3}}{2x^{2/3}}}\left(2x^{2/3}+\left(1+i\sqrt{3}\right)a^{2/3}\right)+c_1$$

1.548 problem 550

Internal problem ID [8885]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 550.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{y'^{r} - a y^{s} = b\, x^{\frac{rs}{r-s}}}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 64

```
dsolve(diff(y(x),x)^r-a*y(x)^s-b*x^(r*s/(r-s))=0,y(x), singsol=all)
```

$$-\left(\int_{\_b}^{y(x)} \frac{1}{x\,(r-s)\left(a\_a^{s}+b\, x^{\frac{rs}{r-s}}\right)^{\frac{1}{r}}-r\_a}d\_a\right)+\frac{\ln(x)}{r-s}-c_1=0$$

✓ Solution by Mathematica

Time used: 0.932 (sec). Leaf size: 488

```
DSolve[-(b*x^((r*s)/(r - s))) - a*y[x]^s + y'[x]^r==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{r}{-rx\left(aK[2]^s+bx^{\frac{rs}{r-s}}\right)^{\frac{1}{r}}+sx\left(aK[2]^s+bx^{\frac{rs}{r-s}}\right)^{\frac{1}{r}}+rK[2]}\right.\right.$$

$$-\int_1^x\left(\frac{asK[2]^{s-1}\left(aK[2]^s+bK[1]^{\frac{rs}{r-s}}\right)^{\frac{1}{r}-1}}{rK[1]\left(aK[2]^s+bK[1]^{\frac{rs}{r-s}}\right)^{\frac{1}{r}}-sK[1]\left(aK[2]^s+bK[1]^{\frac{rs}{r-s}}\right)^{\frac{1}{r}}-rK[2]}-\frac{r\left(aK[2]^s+bK[1]^{\frac{rs}{r-s}}\right)^{\frac{1}{r}}\left(\right.}{\left(rK[1]\right.}\right.$$

$$\left.+\int_1^x\frac{r\left(ay(x)^s+bK[1]^{\frac{rs}{r-s}}\right)^{\frac{1}{r}}}{rK[1]\left(ay(x)^s+bK[1]^{\frac{rs}{r-s}}\right)^{\frac{1}{r}}-sK[1]\left(ay(x)^s+bK[1]^{\frac{rs}{r-s}}\right)^{\frac{1}{r}}-ry(x)}dK[1]=c_1,y(x)\right]$$

1.549 problem 551

Internal problem ID [8886]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 551.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_separable]`

$$y'^n - f(x)^n (y-a)^{n+1} (y-b)^{n-1} = 0$$

✓ Solution by Maple
Time used: 0.734 (sec). Leaf size: 55

```
dsolve(diff(y(x),x)^n-f(x)^n*(y(x)-a)^(n+1)*(y(x)-b)^(n-1)=0,y(x), singsol=all)
```

$$y(x) = \frac{b\left(-\frac{n}{(a-b)\left(\int f(x)dx+c_1\right)}\right)^n - a}{-1+\left(-\frac{n}{(a-b)\left(\int f(x)dx+c_1\right)}\right)^n}$$

✓ Solution by Mathematica
Time used: 6.298 (sec). Leaf size: 79

```
DSolve[-(f[x]^n*(-a + y[x])^(1 + n)*(-b + y[x])^(-1 + n)) + y'[x]^n==0,y[x],x,IncludeSingula
```

$$y(x) \to \frac{bn^n + a(a-b)^n \left(\int_1^x -(-1)^{\frac{1}{n}} f(K[1])dK[1] + c_1\right)^n}{n^n + (a-b)^n \left(\int_1^x -(-1)^{\frac{1}{n}} f(K[1])dK[1] + c_1\right)^n}$$

1.550 problem 552

Internal problem ID [8887]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 552.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y'^n - f(x)\, g(y) = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 43

```
dsolve(diff(y(x),x)^n-f(x)*g(y(x))=0,y(x), singsol=all)
```

$$\int^{y(x)} g(\_a)^{-\frac{1}{n}} d\_a - g(y(x))^{-\frac{1}{n}} \left(\int^{x} (f(\_a)\, g(y(x)))^{\frac{1}{n}} d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.239 (sec). Leaf size: 41

```
DSolve[-(f[x]*g[y[x]]) + y'[x]^n==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} g(K[1])^{-1/n} dK[1]\&\right]\left[\int_1^{x} f(K[2])^{\frac{1}{n}} dK[2] + c_1\right]$$

1.551 problem 553

Internal problem ID [8888]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 553.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$a{y'}^m + b{y'}^n - y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 36

```
dsolve(a*diff(y(x),x)^m+b*diff(y(x),x)^n-y(x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$x - \left(\int^{y(x)} \frac{1}{\text{RootOf}\left(a\_Z^m + b\_Z^n - \_a\right)} d\_a\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.199 (sec). Leaf size: 56

```
DSolve[-y[x] + a*y'[x]^m + b*y'[x]^n==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x = \frac{amK[1]^{m-1}}{m-1} + \frac{bnK[1]^{n-1}}{n-1} + c_1, y(x) = aK[1]^m + bK[1]^n\right\}, \{y(x), K[1]\}\right]$$

1.552 problem 554

Internal problem ID [8889]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 554.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{x^{n-1}{y'}^n - nxy' + y = 0}$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 40

```
dsolve(x^(n-1)*diff(y(x),x)^n-n*x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = -x^{n-1}\left(\frac{c_1\left(\frac{x}{c_1}\right)^{\frac{1}{n}}}{x}\right)^n + nc_1\left(\frac{x}{c_1}\right)^{\frac{1}{n}}$$

✓ Solution by Mathematica

Time used: 0.114 (sec). Leaf size: 54

```
DSolve[y[x] - n*x*y'[x] + x^(-1 + n)*y'[x]^n==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{y(x) = \frac{nx^2K[1] - x^nK[1]^n}{x}, x = c_1(K[1] - nK[1])^{\frac{n}{1-n}}\right\}, \{y(x), K[1]\}\right]$$

1.553 problem 555

Internal problem ID [8890]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 555.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Clairaut]`

$$\boxed{\sqrt{y'^2+1}+xy'-y=0}$$

✓ Solution by Maple
Time used: 0.266 (sec). Leaf size: 15

```
dsolve((diff(y(x),x)^2+1)^(1/2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x)=\sqrt{c_1^2+1}+c_1x$$

✓ Solution by Mathematica
Time used: 0.055 (sec). Leaf size: 25

```
DSolve[-y[x] + x*y'[x] + Sqrt[1 + y'[x]^2]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1x+\sqrt{1+c_1{}^2}$$
$$y(x)\to 1$$

1.554 problem 556

Internal problem ID [8891]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 556.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[_dAlembert]`

$$\boxed{\sqrt{y'^2+1}+xy'^2+y=0}$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 693

```
dsolve((diff(y(x),x)^2+1)^(1/2)+x*diff(y(x),x)^2+y(x)=0,y(x), singsol=all)
```

$$y(x) = -1$$

$$\frac{x\left(2\sqrt{2}\,\sqrt{\frac{2x^2-2xy(x)+\sqrt{4x^2-4xy(x)+1}+1}{x^2}}\,x - 4\,\operatorname{arcsinh}\left(\frac{\sqrt{-4xy(x)+2+2\sqrt{4x^2-4xy(x)+1}}}{2x}\right)x - 4xy(x) + 2 + 2\sqrt{4x^2 \cdots}\right.}{\left(\sqrt{-4xy\left(x\right)+2+2\sqrt{4x^2-4xy\left(x\right)+\cdots}}\right.}$$

$$= 0$$

$$\frac{2x^2\sqrt{2}\,\sqrt{\frac{2x^2-2xy(x)+\sqrt{4x^2-4xy(x)+1}+1}{x^2}} + 4x^3 + c_1x^2 - 4x^2y(x) + 4x^2\sqrt{-4xy\left(x\right)+2+2\sqrt{4x^2-4xy\left(x\right)+1}\cdots}}{\left(\sqrt{-4xy\left(x\right)+2+2\sqrt{4x^2-4xy\left(x\right)+\cdots}}\right.}$$

$$= 0$$

$$-\frac{x\left(4xy(x) + 4x\sqrt{-4xy\left(x\right)-2\sqrt{4x^2-4xy\left(x\right)+1}+2} + 4x\,\operatorname{arcsinh}\left(\frac{\sqrt{-4xy(x)-2\sqrt{4x^2-4xy(x)+1}+2}}{2x}\right) - 2x \cdots\right.}{\left(\sqrt{-4xy\left(x\right)-2\sqrt{4x^2-4xy\left(x\right)+1}+\cdots}\right.}$$

$$= 0$$

$$\frac{4x^3 + c_1x^2 - 4x^2y(x) + 4\sqrt{-4xy\left(x\right)-2\sqrt{4x^2-4xy\left(x\right)+1}+2}\,x^2 + 2x^2\sqrt{\frac{4x^2-4xy(x)-2\sqrt{4x^2-4xy(x)+1}+2}{x^2}} + \cdots}{\left(\sqrt{-4xy\left(x\right)-2\sqrt{4x^2-4xy\left(x\right)+1}+\cdots}\right.}$$

$$= 0$$

✓ Solution by Mathematica

Time used: 3.229 (sec). Leaf size: 78

```
DSolve[y[x] + x*y'[x]^2 + Sqrt[1 + y'[x]^2]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x = \frac{\log\left(\sqrt{K[1]^2+1}-K[1]\right)-\sqrt{K[1]^2+1}}{(K[1]+1)^2} + \frac{c_1}{(K[1]+1)^2}, y(x) = -xK[1]^2 - \sqrt{K[1]^2+1}\right\}, \{y(x), K[1]\}\right]$$

1.555 problem 557

Internal problem ID [8892]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 557.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$x\left(\sqrt{y'^2+1}+y'\right)-y=0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 97

```
dsolve(x*((diff(y(x),x)^2+1)^(1/2)+diff(y(x),x))-y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{x\left(\sqrt{-x\left(-2c_1+x\right)}\sqrt{-\frac{c_1^2}{x(-2c_1+x)}}-x+c_1\right)}{\sqrt{-x\left(-2c_1+x\right)}}$$
$$y(x)=\frac{x\left(\sqrt{-x\left(-2c_1+x\right)}\sqrt{-\frac{c_1^2}{x(-2c_1+x)}}+x-c_1\right)}{\sqrt{-x\left(-2c_1+x\right)}}$$

✓ Solution by Mathematica

Time used: 0.272 (sec). Leaf size: 37

```
DSolve[-y[x] + x*(y'[x] + Sqrt[1 + y'[x]^2])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\sqrt{-x(x-c_1)}$$
$$y(x)\to \sqrt{-x(x-c_1)}$$

1.556 problem 558

Internal problem ID [8893]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 558.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{ax\sqrt{y'^2+1}+xy'-y=0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 340

```
dsolve(a*x*(diff(y(x),x)^2+1)^(1/2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$\frac{x\sqrt{\frac{-a^2x^2+a^2y(x)^2+2\sqrt{-a^2x^2+x^2+y(x)^2}\,ay(x)+x^2+y(x)^2}{(a^2-1)^2x^2}}-\mathrm{e}^{\frac{\operatorname{arcsinh}\left(\frac{\sqrt{-a^2x^2+x^2+y(x)^2}\,a+y(x)}{(a^2-1)x}\right)}{a}}c_1}{\sqrt{\frac{-a^2x^2+a^2y(x)^2+2\sqrt{-a^2x^2+x^2+y(x)^2}\,ay(x)+x^2+y(x)^2}{(a^2-1)^2x^2}}}=0$$

$$\frac{x\sqrt{\frac{-a^2x^2+a^2y(x)^2-2\sqrt{-a^2x^2+x^2+y(x)^2}\,ay(x)+x^2+y(x)^2}{(a^2-1)^2x^2}}-\mathrm{e}^{\frac{\operatorname{arcsinh}\left(\frac{-\sqrt{-a^2x^2+x^2+y(x)^2}\,a+y(x)}{(a^2-1)x}\right)}{a}}c_1}{\sqrt{\frac{-a^2x^2+a^2y(x)^2-2\sqrt{-a^2x^2+x^2+y(x)^2}\,ay(x)+x^2+y(x)^2}{(a^2-1)^2x^2}}}=0$$

✓ Solution by Mathematica

Time used: 1.026 (sec). Leaf size: 223

```
DSolve[-y[x] + x*y'[x] + a*x*Sqrt[1 + y'[x]^2]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2i\arctan\left(\frac{y(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)-2ia\arctan\left(\frac{ay(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)+a\log\left(\frac{y(x)^2}{x^2}+1\right)}{2a^2-2}=\frac{a\log\left(x-a^2x\right)}{1-a^2}+c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{-2i\arctan\left(\frac{y(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)+2ia\arctan\left(\frac{ay(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)+a\log\left(\frac{y(x)^2}{x^2}+1\right)}{2a^2-2}=\frac{a\log\left(x-a^2x\right)}{1-a^2}+c_1,y(x)\right]$$

1.557 problem 559

Internal problem ID [8894]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 559.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$y\sqrt{y'^2+1} - ayy' = ax$$

✓ Solution by Maple

Time used: 0.516 (sec). Leaf size: 378

```
dsolve(y(x)*(diff(y(x),x)^2+1)^(1/2)-a*y(x)*diff(y(x),x)-a*x=0,y(x), singsol=all)
```

$$-\mathrm{e}^{a\left(\int^{\frac{-a^2x+\sqrt{y(x)^2(a^2-1)+a^2x^2}}{(a^2-1)y(x)}}\frac{a\sqrt{\_a^2+1}-\_a}{\sqrt{\_a^2+1}\left(-\sqrt{\_a^2+1}\,\_a+a(\_a^2+1)\right)\left(\_aa-\sqrt{\_a^2+1}\right)}d\_a\right)}c_1+x=0$$

$$-\mathrm{e}^{a\left(\int^{\frac{-a^2x-\sqrt{y(x)^2(a^2-1)+a^2x^2}}{(a^2-1)y(x)}}\frac{a\sqrt{\_a^2+1}-\_a}{\sqrt{\_a^2+1}\left(-\sqrt{\_a^2+1}\,\_a+a(\_a^2+1)\right)\left(\_aa-\sqrt{\_a^2+1}\right)}d\_a\right)}c_1+x=0$$

$$y(x)=\mathrm{RootOf}\left(-\ln(x)+\int^{\_Z}-\frac{\left(\_a^2a^2-\_a^2+a^2-\sqrt{\_a^2a^2-\_a^2+a^2}\right)\_a}{(\_a^2a^2-\_a^2+a^2)(\_a^2+1)}d\_a + c_1\right)x$$

$$y(x)=\mathrm{RootOf}\left(-\ln(x)-\left(\int^{\_Z}\frac{\left(\_a^2a^2-\_a^2+a^2+\sqrt{\_a^2a^2-\_a^2+a^2}\right)\_a}{(\_a^2a^2-\_a^2+a^2)(\_a^2+1)}d\_a\right) + c_1\right)x$$

✓ Solution by Mathematica

Time used: 6.694 (sec). Leaf size: 251

```
DSolve[-(a*x) - a*y[x]*y'[x] + y[x]*Sqrt[1 + y'[x]^2]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{\sqrt{(a^2-1)^3(-x^2) - 2(a^2-1)\, x e^{(a^2-1)c_1} + e^{2(a^2-1)c_1}}}{\sqrt{(a^2-1)^3}}$$

$$y(x) \to \frac{\sqrt{(a^2-1)^3(-x^2) - 2(a^2-1)\, x e^{(a^2-1)c_1} + e^{2(a^2-1)c_1}}}{\sqrt{(a^2-1)^3}}$$

$$y(x) \to -\frac{\sqrt{(a^2-1)^3(-x^2) + 2(a^2-1)\, x e^{(a^2-1)c_1} + e^{2(a^2-1)c_1}}}{\sqrt{(a^2-1)^3}}$$

$$y(x) \to \frac{\sqrt{(a^2-1)^3(-x^2) + 2(a^2-1)\, x e^{(a^2-1)c_1} + e^{2(a^2-1)c_1}}}{\sqrt{(a^2-1)^3}}$$

1.558 problem 560

Internal problem ID [8895]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 560.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational]`

$$a y\sqrt{y'^{2}+1}-2 y x y'+y^{2} = x^{2}$$

✓ Solution by Maple

Time used: 1.421 (sec). Leaf size: 1086

```
dsolve(a*y(x)*(diff(y(x),x)^2+1)^(1/2)-2*x*y(x)*diff(y(x),x)+y(x)^2-x^2=0,y(x), singsol=all)
```

$$-\left(\int_{\_b}^{x} \frac{-2 y(x)^{2} \_a+2 \_a^{3}+\sqrt{a^{2}\left(y(x)^{4}-\left(-2 \_a^{2}+a^{2}\right) y(x)^{2}+\_a^{4}\right)}}{\left(-\_a^{2}+y(x)^{2}\right) \sqrt{a^{2}\left(y(x)^{4}-\left(-2 \_a^{2}+a^{2}\right) y(x)^{2}+\_a^{4}\right)}-2 \_a y(x)^{4}+\left(-4 \_a^{3}+2 \_a a^{2}\right) y(x \cdots}\right.$$
$$+\int^{y(x)} \frac{4 \_f\left(\left(\frac{\left(\_f^{2}-x^{2}\right)\sqrt{a^{2}\left(\_f^{4}-\left(a^{2}-2 x^{2}\right)\_f^{2}+x^{4}\right)}}{2}-x^{5}-2 x^{3} \_f^{2}+\left(-\_f^{4}+\_f^{2} a^{2}\right) x\right)\left(\int_{\_b}^{x} \frac{\left(\left(-4 \_a^{2} \cdots\right.\right.}{\sqrt{a^{2}\left(\_f^{4}-\left(-2 \_ \cdots\right.}}\right.\right.}{\left(\_f^{2}-x^{2}\right)\sqrt{a^{2}\left(\_f^{4} \cdots\right.}}$$
$$+c_{1}=0$$

$$-\left(\int_{\_b}^{x} \frac{2 y(x)^{2} \_a-2 \_a^{3}+\sqrt{a^{2}\left(y(x)^{4}-\left(-2 \_a^{2}+a^{2}\right) y(x)^{2} \cdots\right.}}{2 \_a y(x)^{4}+4 \_a^{3} y(x)^{2}-2 \_a y(x)^{2} a^{2}+2 \_a^{5}+y(x)^{2} \sqrt{a^{2}\left(y(x)^{4}-\left(-2 \_a^{2}+a^{2}\right) y(x)^{2}+\_a^{4} \cdots\right.}}\right.$$
$$+\int^{y(x)} \frac{4 \_f\left(\left(\frac{\left(-\_f^{2}+x^{2}\right)\sqrt{a^{2}\left(\_f^{4}-\left(a^{2}-2 x^{2}\right)\_f^{2}+x^{4}\right)}}{2}-x^{5}-2 x^{3} \_f^{2}+\left(-\_f^{4}+\_f^{2} a^{2}\right) x\right)\left(\int_{\_b}^{x}-\frac{\left(\left(4 \_a \cdots\right.\right.}{\sqrt{a^{2}\left(\_f^{4}-\left(\cdots\right.\right.}}\right.\right.}{\left(-\_f^{2}+x^{2}\right)\sqrt{a^{2}\left(\_ \cdots\right.}}$$
$$+c_{1}=0$$

✓ Solution by Mathematica

Time used: 57.481 (sec). Leaf size: 135

```
DSolve[-x^2 + y[x]^2 - 2*x*y[x]*y'[x] + a*y[x]*Sqrt[1 + y'[x]^2]==0,y[x],x,IncludeSingularSo
```

$$y(x) \to -\frac{\sqrt{4x^2 - a^2(2 + c_1 x)^2}}{\sqrt{-4 + a^2 c_1{}^2}}$$
$$y(x) \to \frac{\sqrt{4x^2 - a^2(2 + c_1 x)^2}}{\sqrt{-4 + a^2 c_1{}^2}}$$
$$y(x) \to -\frac{\sqrt{-a^2 x^2}}{\sqrt{a^2}}$$
$$y(x) \to \frac{\sqrt{-a^2 x^2}}{\sqrt{a^2}}$$

1.559 problem 561

Internal problem ID [8896]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 561.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{f(y^2+x^2)\sqrt{y'^2+1}-xy'+y=0}$$

✓ Solution by Maple
Time used: 0.563 (sec). Leaf size: 42

```
dsolve(f(y(x)^2+x^2)*(diff(y(x),x)^2+1)^(1/2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=\cot\left(\operatorname{RootOf}\left(-2\_Z+\int^{\csc(\_Z)^2x^2}\frac{f(\_a)}{\sqrt{-f(\_a)^2+\_a}\,\_a}d\_a+2c_1\right)\right)x$$

✓ Solution by Mathematica
Time used: 5.569 (sec). Leaf size: 2138

```
DSolve[y[x] - x*y'[x] + f[x^2 + y[x]^2]*Sqrt[1 + y'[x]^2]==0,y[x],x,IncludeSingularSolutions
```

Too large to display

1.560 problem 562

Internal problem ID [8897]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 562.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_dAlembert]`

$$a\left(y'^3+1\right)^{\frac{1}{3}}+bxy'-y=0$$

✓ Solution by Maple

Time used: 0.344 (sec). Leaf size: 3321

```
dsolve(a*(diff(y(x),x)^3+1)^(1/3)+b*x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

Expression too large to display
Expression too large to display
Expression too large to display

✓ Solution by Mathematica

Time used: 0.097 (sec). Leaf size: 84

```
DSolve[-y[x] + b*x*y'[x] + a*(1 + y'[x]^3)^(1/3)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x=K[1]^{\frac{b}{1-b}}\left(\frac{a\int\frac{K[1]^{\frac{2b-1}{b-1}}}{(K[1]^3+1)^{2/3}}dK[1]}{1-b}+c_1\right),y(x)=a\sqrt[3]{K[1]^3+1}+bxK[1]\right\},\{K[1],y(x)\}\right]$$

1.561 problem 563

Internal problem ID [8898]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 563.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{\ln(y') + xy' + ya = -b}$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 73

```
dsolve(ln(diff(y(x),x))+x*diff(y(x),x)+a*y(x)+b=0,y(x), singsol=all)
```

$$\frac{-a\left(\left(\frac{\mathrm{LambertW}\left(x\,\mathrm{e}^{-ay(x)-b}\right)}{x}\right)^{-\frac{1}{a+1}}c_1 - x\right)\mathrm{LambertW}\left(x\,\mathrm{e}^{-ay(x)-b}\right) - x}{a\,\mathrm{LambertW}\left(x\,\mathrm{e}^{-ay(x)-b}\right)} = 0$$

✓ Solution by Mathematica
Time used: 0.143 (sec). Leaf size: 59

```
DSolve[b + Log[y'[x]] + a*y[x] + x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[a\left(\frac{(a+1)\log\left(1-aW\left(xe^{-ay(x)-b}\right)\right)}{a^2} + \frac{W\left(xe^{-ay(x)-b}\right)}{a}\right) + ay(x) = c_1, y(x)\right]$$

1.562 problem 564

Internal problem ID [8899]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 564.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{\ln(y') + a(xy' - y) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
dsolve(ln(diff(y(x),x))+a*(x*diff(y(x),x)-y(x))=0,y(x), singsol=all)
```

$$y(x) = \frac{\ln\left(-\frac{1}{ax}\right) - 1}{a}$$
$$y(x) = c_1 x + \frac{\ln(c_1)}{a}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 36

```
DSolve[Log[y'[x]] + a*(-y[x] + x*y'[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\log(c_1)}{a} + c_1 x$$
$$y(x) \to \frac{\log\left(-\frac{1}{ax}\right) - 1}{a}$$

1.563 problem 565

Internal problem ID [8900]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 565.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_separable]`

$$\boxed{y\ln(y') + y' - y\ln(y) - yx = 0}$$

✓ Solution by Maple
Time used: 0.109 (sec). Leaf size: 17

```
dsolve(y(x)*ln(diff(y(x),x))+diff(y(x),x)-y(x)*ln(y(x))-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{\mathrm{LambertW}(\mathrm{e}^x)(\mathrm{LambertW}(\mathrm{e}^x)+2)}{2}}$$

✓ Solution by Mathematica
Time used: 0.106 (sec). Leaf size: 24

```
DSolve[-(x*y[x]) - Log[y[x]]*y[x] + Log[y'[x]]*y[x] + y'[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to c_1 e^{\frac{1}{2}W(e^x)(W(e^x)+2)}$$

1.564 problem 566

Internal problem ID [8901]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 566.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{\sin(y') + y' = x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 16

```
dsolve(sin(diff(y(x),x))+diff(y(x),x)-x=0,y(x), singsol=all)
```

$$y(x) = \int \mathrm{RootOf}\left(\sin\left(\_Z\right) + \_Z - x\right) dx + c_1$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 38

```
DSolve[-x + Sin[y'[x]] + y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\left\{x = K[1] + \sin(K[1]), y(x) = \frac{K[1]^2}{2} + K[1]\sin(K[1]) + \cos(K[1]) + c_1\right\}, \{y(x), K[1]\}\right]$$

1.565 problem 567

Internal problem ID [8902]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 567.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{a\cos(y') + by' = -x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(a*cos(diff(y(x),x))+b*diff(y(x),x)+x=0,y(x), singsol=all)
```

$$y(x) = \int \operatorname{RootOf}(a\cos(\_Z) + b\_Z + x)\,dx + c_1$$

✓ Solution by Mathematica
Time used: 0.066 (sec). Leaf size: 49

```
DSolve[x + a*Cos[y'[x]] + b*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{y(x) = a\sin(K[1]) - aK[1]\cos(K[1]) - \frac{1}{2}bK[1]^2 + c_1, x = -a\cos(K[1]) - bK[1]\right\}, \{y(x), K[1]\}\right]$$

1.566 problem 568

Internal problem ID [8903]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 568.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 \sin(y') - y = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 32

```
dsolve(diff(y(x),x)^2*sin(diff(y(x),x))-y(x)=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$x - \left(\int^{y(x)} \frac{1}{\mathrm{RootOf}\left(\sin\left(\_Z\right)\_Z^2 - \_a\right)} d\_a\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.087 (sec). Leaf size: 34

```
DSolve[-y[x] + Sin[y'[x]]*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\left\{x = K[1]\sin(K[1]) - \cos(K[1]) + c_1, y(x) = K[1]^2 \sin(K[1])\right\}, \{y(x), K[1]\}\right]$$

1.567 problem 569

Internal problem ID [8904]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 569.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_Clairaut]`

$$\left(y'^2+1\right)\sin\left(xy'-y\right)^2=1$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 139

```
dsolve((diff(y(x),x)^2+1)*sin(x*diff(y(x),x)-y(x))^2-1=0,y(x), singsol=all)
```

$$y(x)=-x\sqrt{\frac{1}{x}}\,\sqrt{1-x}-\arcsin\left(\frac{1}{\sqrt{\frac{1}{x}}}\right)$$
$$y(x)=x\sqrt{\frac{1}{x}}\,\sqrt{1-x}+\arcsin\left(\frac{1}{\sqrt{\frac{1}{x}}}\right)$$
$$y(x)=-x\sqrt{-\frac{1}{x}}\,\sqrt{x+1}+\arcsin\left(\frac{1}{\sqrt{-\frac{1}{x}}}\right)$$
$$y(x)=x\sqrt{-\frac{1}{x}}\,\sqrt{x+1}-\arcsin\left(\frac{1}{\sqrt{-\frac{1}{x}}}\right)$$
$$y(x)=c_1x-\arcsin\left(\frac{1}{\sqrt{c_1^2+1}}\right)$$
$$y(x)=c_1x+\arcsin\left(\frac{1}{\sqrt{c_1^2+1}}\right)$$

✓ Solution by Mathematica

Time used: 0.334 (sec). Leaf size: 77

```
DSolve[-1 + Sin[y[x] - x*y'[x]]^2*(1 + y'[x]^2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x - \frac{1}{2}\arccos\left(\frac{-1 + {c_1}^2}{1 + {c_1}^2}\right)$$
$$y(x) \to \frac{1}{2}\arccos\left(\frac{-1 + {c_1}^2}{1 + {c_1}^2}\right) + c_1 x$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

1.568 problem 570

Internal problem ID [8905]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 570.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\left(y'^2+1\right)\left(\arctan\left(y'\right)+ax\right)+y'=0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve((diff(y(x),x)^2+1)*(arctan(diff(y(x),x))+a*x)+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\int \tan\left(\mathrm{RootOf}\left(ax+\sin\left(\_Z\right)\cos\left(\_Z\right)+\_Z\right)\right)dx+c_1$$

✓ Solution by Mathematica
Time used: 1.206 (sec). Leaf size: 58

```
DSolve[y'[x] + (a*x + ArcTan[y'[x]])*(1 + y'[x]^2)==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$\mathrm{Solve}\left[\left\{y(x)=\frac{1}{a\left(K[1]^2+1\right)}+c_1,x=\frac{K[1]^2(-\arctan(K[1]))-\arctan(K[1])-K[1]}{a\left(K[1]^2+1\right)}\right\},\{y(x),K[1]\}\right]$$

1.569 problem 571

Internal problem ID [8906]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 571.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type [`y=_G(x,y')`]

$$a\,x^n f(y') + xy' - y = 0$$

✓ Solution by Maple

Time used: 0.843 (sec). Leaf size: 169

```
dsolve(a*x^n*f(diff(y(x),x))+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$\left[y(\_T) = a\left(\left(\frac{(-n+1)\left(\int f(\_T)^{-\frac{1}{n}}d\_T\right)+c_1 an}{anf(\_T)}\right)^{\frac{1}{n-1}} f(\_T)^{\frac{1}{n(n-1)}}\right)^n f(\_T) + \left(\frac{(-n+1)\left(\int f(\_T)^{-\frac{1}{n}}d\_T\right)+c_1 an}{anf(\_T)}\right)^{\frac{1}{n-1}} f(\_T)^{\frac{1}{n(n-1)}}\_T, x(\_T) = \left(\frac{(-n+1)\left(\int f(\_T)^{-\frac{1}{n}}d\_T\right)+}{anf(\_T)}\right.\right.$$

✓ Solution by Mathematica

Time used: 0.117 (sec). Leaf size: 124

```
DSolve[a*x^n*f[y'[x]] - y[x] + x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{y(x) = ax^n f(K[1]) + xK[1], x = \left(nf(K[1])^{\frac{1}{n}-1}\int_1^{K[1]} -\frac{f(K[2])^{\frac{n-1}{n}-1}}{an}dK[2] - f(K[1])^{\frac{1}{n}-1}\int_1^{K[1]} -\frac{f(K[2])^{\frac{n-1}{n}-1}}{an}dK[2] + c_1 f(K\right.\right.\right.$$

1.570 problem 572

Internal problem ID [8907]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 572.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type [`x=_G(y,y')`]

$$(xy' - y)^n f(y') + yg(y') + xh(y') = 0$$

✗ Solution by Maple

```
dsolve((x*diff(y(x),x)-y(x))^n*f(diff(y(x),x))+y(x)*g(diff(y(x),x))+x*h(diff(y(x),x))=0,y(x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*h[y'[x]] + g[y'[x]]*y[x] + f[y'[x]]*(-y[x] + x*y'[x])^n==0,y[x],x,IncludeSingularSo
```

Not solved

1.571 problem 573

Internal problem ID [8908]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 573.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$f\left(x{y'}^2\right)+2xy'-y=0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 24

```
dsolve(f(x*diff(y(x),x)^2)+2*x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1+2\sqrt{x}\ \mathrm{RootOf}\left(-f\left(\_Z^2\right)-2\_Z+c_1+c_2\right)$$

✓ Solution by Mathematica

Time used: 0.504 (sec). Leaf size: 48

```
DSolve[f[x*y'[x]^2] - y[x] + 2*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to f(c_1)-2\sqrt{c_1}\sqrt{x}$$
$$y(x)\to f(c_1)+2\sqrt{c_1}\sqrt{x}$$
$$y(x)\to f(0)$$

1.572 problem 574

Internal problem ID [8909]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 574.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{f\left(x - \frac{3y'^2}{2}\right) + y'^3 - y = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 41

```
dsolve(f(x-3/2*diff(y(x),x)^2)+diff(y(x),x)^3-y(x)=0,y(x), singsol=all)
```

$$y(x) = f(c_1) - \frac{2\sqrt{6}\,\sqrt{(x-c_1)^3}}{9}$$
$$y(x) = f(c_1) + \frac{2\sqrt{6}\,\sqrt{(x-c_1)^3}}{9}$$

✓ Solution by Mathematica

Time used: 1.358 (sec). Leaf size: 62

```
DSolve[f[x - (3*y'[x]^2)/2] - y[x] + y'[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{9}\left(9f(c_1) + 2\sqrt{6}(x-c_1)^{3/2}\right)$$
$$y(x) \to \frac{1}{9}\left(9f(c_1) - 2\sqrt{6}(x-c_1)^{3/2}\right)$$

1.573 problem 575

Internal problem ID [8910]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 575.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[NONE]`

$$y' f(xyy' - y^2) - x^2 y' + yx = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)*f(x*y(x)*diff(y(x),x)-y(x)^2)-x^2*diff(y(x),x)+x*y(x)=0,y(x), singsol=al
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y[x] - x^2*y'[x] + f[-y[x]^2 + x*y[x]*y'[x]]*y'[x]==0,y[x],x,IncludeSingularSolutio
```

Not solved

1.574 problem 576

Internal problem ID [8911]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, linear first order
Problem number: 576.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[NONE]`

$$\phi(f(x, y, y'), g(x, y, y')) = 0$$

✗ Solution by Maple

```
dsolve(phi(f(x,y(x),diff(y(x),x)),g(x,y(x),diff(y(x),x)))=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[phi[f[x, y[x], y'[x]], g[x, y[x], y'[x]]]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2 Chapter 1, Additional non-linear first order

2.1 problem 577

Internal problem ID [8912]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 577.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - F\left(\frac{y}{x+a}\right) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 28

```
dsolve(diff(y(x),x) = F(y(x)/(x+a)),y(x), singsol=all)
```

$$y(x) = -\operatorname{RootOf}\left(\int^{\_Z} \frac{1}{F(-\_a) + \_a} d\_a + \ln(a+x) + c_1\right)(a+x)$$

✓ Solution by Mathematica

Time used: 0.342 (sec). Leaf size: 243

```
DSolve[y'[x] == F[y[x]/(a + x)],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{1}{-aF\left(\frac{K[2]}{a+x}\right)-xF\left(\frac{K[2]}{a+x}\right)+K[2]}\right.\right.$$

$$-\int_1^x\left(\frac{F'\left(\frac{K[2]}{a+K[1]}\right)}{(a+K[1])\left(aF\left(\frac{K[2]}{a+K[1]}\right)+K[1]F\left(\frac{K[2]}{a+K[1]}\right)-K[2]\right)}-\frac{F\left(\frac{K[2]}{a+K[1]}\right)\left(\frac{aF'\left(\frac{K[2]}{a+K[1]}\right)}{a+K[1]}+\frac{K[1]F'\left(\frac{K[2]}{a+K[1]}\right)}{a+K[1]}-\right.}{\left(aF\left(\frac{K[2]}{a+K[1]}\right)+K[1]F\left(\frac{K[2]}{a+K[1]}\right)-K[2]\right.}$$

$$\left.+\int_1^x\frac{F\left(\frac{y(x)}{a+K[1]}\right)}{aF\left(\frac{y(x)}{a+K[1]}\right)+K[1]F\left(\frac{y(x)}{a+K[1]}\right)-y(x)}dK[1]=c_1,y(x)\right]$$

2.2 problem 578

Internal problem ID [8913]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 578.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y' - F(y - x^2) = 2x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 22

```
dsolve(diff(y(x),x) = 2*x+F(y(x)-x^2),y(x), singsol=all)
```

$$y(x) = x^2 + \operatorname{RootOf}\left(-x + \int^{\_Z} \frac{1}{F(\_a)} d\_a + c_1\right)$$

✓ Solution by Mathematica
Time used: 0.192 (sec). Leaf size: 100

```
DSolve[y'[x] == 2*x + F[-x^2 + y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} -\frac{F(K[2]-x^2)\int_1^x -\frac{2K[1]F'(K[2]-K[1]^2)}{F(K[2]-K[1]^2)^2}dK[1]+1}{F(K[2]-x^2)}dK[2] + \int_1^x \left(\frac{2K[1]}{F(y(x)-K[1]^2)}+1\right)dK[1] = c_1, y(x)\right]$$

2.3 problem 579

Internal problem ID [8914]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 579.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y' - F\left(y + \frac{a\,x^2}{4} + \frac{bx}{2}\right) = -\frac{ax}{2}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 35

```
dsolve(diff(y(x),x) = -1/2*a*x+F(y(x)+1/4*a*x^2+1/2*b*x),y(x), singsol=all)
```

$$y(x) = -\frac{a\,x^2}{4} - \frac{bx}{2} + \text{RootOf}\left(-x + 2\left(\int^{\_Z} \frac{1}{2F\left(\_a\right) + b} d\_a\right) + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.227 (sec). Leaf size: 514

```
DSolve[y'[x] == -1/2*(a*x) + F[(b*x)/2 + (a*x^2)/4 + y[x]],y[x],x,IncludeSingularSolutions -
```

$$\text{Solve}\left[\int_1^{y(x)} -\frac{b\int_1^x\left(\frac{2aK[1]F'\left(\frac{1}{4}aK[1]^2+\frac{1}{2}bK[1]+K[2]\right)}{\left(b+2F\left(\frac{1}{4}aK[1]^2+\frac{1}{2}bK[1]+K[2]\right)\right)^2}+\frac{2F'\left(\frac{1}{4}aK[1]^2+\frac{1}{2}bK[1]+K[2]\right)}{b+2F\left(\frac{1}{4}aK[1]^2+\frac{1}{2}bK[1]+K[2]\right)}-\frac{4F\left(\frac{1}{4}aK[1]^2+\frac{1}{2}bK[1]+K[2]\right)F'\left(\frac{1}{4}aK[1]^2+\frac{1}{2}bK[1]+K[2\right.}{\left(b+2F\left(\frac{1}{4}aK[1]^2+\frac{1}{2}bK[1]+K[2]\right)\right)^2}}{} + \int_1^x\left(\frac{2F\left(\frac{1}{4}aK[1]^2+\frac{1}{2}bK[1]+y(x)\right)}{b+2F\left(\frac{1}{4}aK[1]^2+\frac{1}{2}bK[1]+y(x)\right)} - \frac{aK[1]}{b+2F\left(\frac{1}{4}aK[1]^2+\frac{1}{2}bK[1]+y(x)\right)}\right)dK[1]=c_1,y(x)\right]$$

2.4 problem 580

Internal problem ID [8915]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 580.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y' - F\left(y\,\mathrm{e}^{-bx}\right)\mathrm{e}^{bx} = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 28

```
dsolve(diff(y(x),x) = F(y(x)*exp(-b*x))*exp(b*x),y(x), singsol=all)
```

$$y(x) = \mathrm{RootOf}\left(-x + \int^{\_Z} \frac{1}{F(\_a) - \_ab} d\_a + c_1\right)\mathrm{e}^{bx}$$

✓ Solution by Mathematica
Time used: 0.265 (sec). Leaf size: 203

```
DSolve[y'[x] == E^(b*x)*F[y[x]/E^(b*x)],y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\int_1^{y(x)}\left(\frac{1}{bK[2] - e^{bx}F(e^{-bx}K[2])} - \int_1^x \left(\frac{F'(e^{-bK[1]}K[2])}{e^{bK[1]}F(e^{-bK[1]}K[2]) - bK[2]} - \frac{e^{bK[1]}F(e^{-bK[1]}K[2])\left(F'(e^{-bK[1]}K[2]) - b\right)}{\left(e^{bK[1]}F(e^{-bK[1]}K[2]) - bK[2]\right)^2}\right)dK[1]\right)dK[2] + \int_1^x \frac{e^{bK[1]}F(e^{-bK[1]}y(x))}{e^{bK[1]}F(e^{-bK[1]}y(x)) - by(x)}dK[1] = c_1, y(x)\right]$$

2.5 problem 581

Internal problem ID [8916]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 581.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, ‘_with_symmetry_[F(x),G(x)]‘]]`

$$y' - \frac{1 + 2F\left(\frac{4yx^2+1}{4x^2}\right) x}{2x^3} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 50

```
dsolve(diff(y(x),x) = 1/2*(1+2*F(1/4*(4*x^2*y(x)+1)/x^2)*x)/x^3,y(x), singsol=all)
```

$$y(x) = \frac{4\,\mathrm{RootOf}\left(F(\_Z)\right) x^2 - 1}{4x^2}$$
$$y(x) = \frac{4\,\mathrm{RootOf}\left(\left(\left(\int^{\_Z} \frac{1}{F(\_a)} d\_a\right) x + c_1 x + 1\right) x^2 - 1}{4x^2}$$

✓ Solution by Mathematica

Time used: 0.254 (sec). Leaf size: 144

```
DSolve[y'[x] == (1/2 + x*F[(1/4 + x^2*y[x])/x^2])/x^3,y[x],x,IncludeSingularSolutions -> Tru
```

$$\text{Solve}\left[\int_1^{y(x)} -\frac{F\left(\frac{K[2]x^2+\frac{1}{4}}{x^2}\right)\int_1^x -\frac{F'\left(\frac{K[2]K[1]^2+\frac{1}{4}}{K[1]^2}\right)}{2F\left(\frac{K[2]K[1]^2+\frac{1}{4}}{K[1]^2}\right)^2 K[1]^3}dK[1]+1}{F\left(\frac{K[2]x^2+\frac{1}{4}}{x^2}\right)}dK[2]\right.$$

$$\left.+\int_1^x\left(\frac{1}{K[1]^2}+\frac{1}{2K[1]^3F\left(\frac{y(x)K[1]^2+\frac{1}{4}}{K[1]^2}\right)}\right)dK[1]=c_1,y(x)\right]$$

2.6 problem 582

Internal problem ID [8917]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 582.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y' - \frac{1 + F\left(\frac{yax+1}{ax}\right) a\, x^2}{a\, x^2} = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 48

```
dsolve(diff(y(x),x) = (1+F((y(x)*a*x+1)/a/x)*a*x^2)/a/x^2,y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{RootOf}\left(F(\_Z)\right) ax - 1}{ax}$$

$$y(x) = \frac{\mathrm{RootOf}\left(-x + \int^{\_Z} \frac{1}{F(\_a)} d\_a + c_1\right) ax - 1}{ax}$$

✓ Solution by Mathematica

Time used: 0.292 (sec). Leaf size: 142

```
DSolve[y'[x] == (1 + a*x^2*F[(1 + a*x*y[x])/(a*x)])/(a*x^2),y[x],x,IncludeSingularSolutions
```

$$\mathrm{Solve}\left[\int_1^{y(x)} -\frac{F\left(\frac{axK[2]+1}{ax}\right) \int_1^x \frac{F'\left(\frac{aK[1]K[2]+1}{aK[1]}\right)}{aF\left(\frac{aK[1]K[2]+1}{aK[1]}\right)^2 K[1]^2} dK[1] - 1}{F\left(\frac{axK[2]+1}{ax}\right)} dK[2] + \int_1^x \left(-1 - \frac{1}{aK[1]^2 F\left(\frac{aK[1]y(x)+1}{aK[1]}\right)}\right) dK[1] = c_1, y(x)\right]$$

2.7 problem 583

Internal problem ID [8918]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 583.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' + \frac{\left(a\,x^2 - 2F\left(y + \frac{a\,x^4}{8}\right)\right)x}{2} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 44

```
dsolve(diff(y(x),x) = -1/2*(a*x^2-2*F(y(x)+1/8*a*x^4))*x,y(x), singsol=all)
```

$$y(x) = -\frac{x^4 a}{8} + \operatorname{RootOf}\left(F(\_Z)\right)$$

$$y(x) = -\frac{x^4 a}{8} + \operatorname{RootOf}\left(-x^2 + 2\left(\int^{\_Z} \frac{1}{F(\_a)} d\_a\right) + 2c_1\right)$$

✓ Solution by Mathematica

Time used: 0.265 (sec). Leaf size: 126

```
DSolve[y'[x] == -1/2*(x*(a*x^2 - 2*F[(a*x^4)/8 + y[x]])),y[x],x,IncludeSingularSolutions ->
```

$$\text{Solve}\left[\int_1^{y(x)} -\frac{F\left(\frac{ax^4}{8} + K[2]\right)\int_1^x \frac{aK[1]^3 F'\left(\frac{1}{8}aK[1]^4 + K[2]\right)}{2F\left(\frac{1}{8}aK[1]^4 + K[2]\right)^2} dK[1] + 1}{F\left(\frac{ax^4}{8} + K[2]\right)} dK[2] + \int_1^x \left(K[1] - \frac{aK[1]^3}{2F\left(\frac{1}{8}aK[1]^4 + y(x)\right)}\right) dK[1] = c_1, y(x)\right]$$

2.8 problem 584

Internal problem ID [8919]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 584.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y' - \frac{2a}{y + 2F(y^2 - 4ax)\,a} = 0$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 35

```
dsolve(diff(y(x),x) = 2*a/(y(x)+2*F(y(x)^2-4*a*x)*a),y(x), singsol=all)
```

$$\frac{y(x)}{2a} + \frac{\int^{-4ax+y(x)^2} \frac{1}{F(\_a)} d\_a}{8a^2} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.263 (sec). Leaf size: 115

```
DSolve[y'[x] == (2*a)/(2*a*F[-4*a*x + y[x]^2] + y[x]),y[x],x,IncludeSingularSolutions -> Tru
```

$$\text{Solve}\left[\int_1^{y(x)} \left(\frac{K[2]}{4a^2F(K[2]^2 - 4ax)} - \frac{2a\int_1^x \frac{K[2]F'(K[2]^2-4aK[1])}{aF(K[2]^2-4aK[1])^2} dK[1] - 1}{2a}\right) dK[2] + \int_1^x -\frac{1}{2aF(y(x)^2 - 4aK[1])} dK[1] = c_1, y(x)\right]$$

2.9 problem 585

Internal problem ID [8920]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 585.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{y' - F(\ln(\ln(y)) - \ln(x))\, y = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 136

```
dsolve(diff(y(x),x) = F(ln(ln(y(x)))-ln(x))*y(x),y(x), singsol=all)
```

$$\int_{\_b}^{x} \frac{F(\ln(\ln(y(x))) - \ln(\_a))}{\_aF(\ln(\ln(y(x))) - \ln(\_a)) - \ln(y(x))} d\_a - \left(\int^{y(x)} \frac{1 + \left(\int_{\_b}^{x} \frac{-D(F)\left(\ln\left(\ln\left(\_f\right)\right) - \ln(\_a)\right) + F\left(\ln\left(\ln\left(\_f\right)\right) - \ln(\_a)\right)}{\left(\_aF\left(\ln\left(\ln\left(\_f\right)\right) - \ln(\_a)\right) - \ln\left(\_f\right)\right)^{2}} d\_a\right)(xF(\ln(\ln(\_f)) - \ln(x)) - \ln(\_f}{\_f(xF(\ln(\ln(\_f)) - \ln(x)) - \ln(\_f))}\right. + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.22 (sec). Leaf size: 205

```
DSolve[y'[x] == F[-Log[x] + Log[Log[y[x]]]]*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{1}{K[2](xF(\log(\log(K[2]))-\log(x))-\log(K[2]))}\right.\right.$$
$$-\int_1^x\left(\frac{F(\log(\log(K[2]))-\log(K[1]))\left(\frac{K[1]F'(\log(\log(K[2]))-\log(K[1]))}{K[2]\log(K[2])}-\frac{1}{K[2]}\right)}{(F(\log(\log(K[2]))-\log(K[1]))K[1]-\log(K[2]))^2}-\frac{F'(\log(\log(K}{K[2](F(\log(\log(K[2]))-\log}\right.$$
$$\left.+\int_1^x-\frac{F(\log(\log(y(x)))-\log(K[1]))}{F(\log(\log(y(x)))-\log(K[1]))K[1]-\log(y(x))}dK[1]=c_1,y(x)\right]$$

2.10 problem 586

Internal problem ID [8921]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 586.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y' - \frac{F\left(\frac{y}{\sqrt{x^2+1}}\right) x}{\sqrt{x^2+1}} = 0$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 64

```
dsolve(diff(y(x),x) = F(y(x)/(x^2+1)^(1/2))*x/(x^2+1)^(1/2),y(x), singsol=all)
```

$$y(x) = \text{RootOf}\left(-F\left(\frac{\_Z}{\sqrt{x^2+1}}\right)\sqrt{x^2+1} + \_Z\right)$$

$$y(x) = \text{RootOf}\left(-\ln\left(x^2+1\right) + 2\left(\int^{\_Z} \frac{1}{F(\_a) - \_a} d\_a\right) + 2c_1\right)\sqrt{x^2+1}$$

✓ Solution by Mathematica

Time used: 0.933 (sec). Leaf size: 975

```
DSolve[y'[x] == (x*F[y[x]/Sqrt[1 + x^2]])/Sqrt[1 + x^2],y[x],x,IncludeSingularSolutions -> T
```

$$
\text{Solve}\left[\int_1^x\left(-\frac{K[1]\sqrt{K[1]^2+1}F\left(\frac{y(x)}{\sqrt{K[1]^2+1}}\right)^3}{y(x)\left(K[1]^2F\left(\frac{y(x)}{\sqrt{K[1]^2+1}}\right)^2+F\left(\frac{y(x)}{\sqrt{K[1]^2+1}}\right)^2-y(x)^2\right)}\right.\right.
$$

$$
\left.-\frac{K[1]F\left(\frac{y(x)}{\sqrt{K[1]^2+1}}\right)^2}{K[1]^2F\left(\frac{y(x)}{\sqrt{K[1]^2+1}}\right)^2+F\left(\frac{y(x)}{\sqrt{K[1]^2+1}}\right)^2-y(x)^2}+\frac{K[1]F\left(\frac{y(x)}{\sqrt{K[1]^2+1}}\right)}{\sqrt{K[1]^2+1}y(x)}\right)dK[1]
$$

$$
+\int_1^{y(x)}\left(-\frac{\sqrt{x^2+1}F\left(\frac{K[2]}{\sqrt{x^2+1}}\right)}{-x^2F\left(\frac{K[2]}{\sqrt{x^2+1}}\right)^2-F\left(\frac{K[2]}{\sqrt{x^2+1}}\right)^2+K[2]^2}\right.
$$

$$
-\int_1^x\left(\frac{K[1]\sqrt{K[1]^2+1}\left(\frac{2F\left(\frac{K[2]}{\sqrt{K[1]^2+1}}\right)F'\left(\frac{K[2]}{\sqrt{K[1]^2+1}}\right)K[1]^2}{\sqrt{K[1]^2+1}}-2K[2]+\frac{2F\left(\frac{K[2]}{\sqrt{K[1]^2+1}}\right)F'\left(\frac{K[2]}{\sqrt{K[1]^2+1}}\right)}{\sqrt{K[1]^2+1}}\right)F\left(\frac{K[2]}{\sqrt{K[1]^2+}}\right.}{K[2]\left(K[1]^2F\left(\frac{K[2]}{\sqrt{K[1]^2+1}}\right)^2+F\left(\frac{K[2]}{\sqrt{K[1]^2+1}}\right)^2-K[2]^2\right)^2}\right.
$$

$$
\left.\left.-\frac{K[2]}{-x^2F\left(\frac{K[2]}{\sqrt{x^2+1}}\right)^2-F\left(\frac{K[2]}{\sqrt{x^2+1}}\right)^2+K[2]^2}\right)dK[2]=c_1,y(x)\right]
$$

2.11 problem 587

Internal problem ID [8922]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 587.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' - \frac{\left(x^{\frac{3}{2}} + 2F\left(y - \frac{x^3}{6}\right)\right)\sqrt{x}}{2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x) = 1/2*(x^(3/2)+2*F(y(x)-1/6*x^3))*x^(1/2),y(x), singsol=all)
```

$$\int_{\_b}^{y(x)} \frac{1}{F\left(\_a - \frac{x^3}{6}\right)} d\_a - \frac{2x^{\frac{3}{2}}}{3} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.274 (sec). Leaf size: 123

```
DSolve[y'[x] == (Sqrt[x]*(x^(3/2) + 2*F[-1/6*x^3 + y[x]]))/2,y[x],x,IncludeSingularSolutions
```

$$\text{Solve}\left[\int_1^{y(x)} -\frac{F\left(K[2] - \frac{x^3}{6}\right) \int_1^x -\frac{K[1]^2 F'\left(K[2] - \frac{K[1]^3}{6}\right)}{2F\left(K[2] - \frac{K[1]^3}{6}\right)^2} dK[1] + 1}{F\left(K[2] - \frac{x^3}{6}\right)} dK[2] + \int_1^x \left(\frac{K[1]^2}{2F\left(y(x) - \frac{K[1]^3}{6}\right)} + \sqrt{K[1]}\right) dK[1] = c_1, y(x)\right]$$

2.12 problem 588

Internal problem ID [8923]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 588.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y' - \frac{x + F(-(x-y)(x+y))}{y} = 0$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 67

```
dsolve(diff(y(x),x) = (x+F(-(x-y(x))*(x+y(x))))/y(x),y(x), singsol=all)
```

$$y(x) = \mathrm{RootOf}\left(F\left(\_Z^2 - x^2\right)\right)$$
$$y(x) = \sqrt{x^2 + \mathrm{RootOf}\left(-2x + \int^{\_Z} \frac{1}{F(\_a)} d\_a + 2c_1\right)}$$
$$y(x) = -\sqrt{x^2 + \mathrm{RootOf}\left(-2x + \int^{\_Z} \frac{1}{F(\_a)} d\_a + 2c_1\right)}$$

✓ Solution by Mathematica
Time used: 0.211 (sec). Leaf size: 109

```
DSolve[y'[x] == (x + F[(-x + y[x])*(x + y[x])])/y[x],y[x],x,IncludeSingularSolutions -> True
```

$$\mathrm{Solve}\left[\int_1^{y(x)} \left(-\frac{K[2]}{F((K[2]-x)(x+K[2]))} - \int_1^x -\frac{2K[1]K[2]F'((K[2]-K[1])(K[1]+K[2]))}{F((K[2]-K[1])(K[1]+K[2]))^2} dK[1]\right) dK[2] + \int_1^x \left(\frac{K[1]}{F((y(x)-K[1])(K[1]+y(x)))} + 1\right) dK[1] = c_1, y(x)\right]$$

2.13 problem 589

Internal problem ID [8924]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 589.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y' - \frac{F\left(-\frac{-1+y\ln(x)}{y}\right)y^2}{x} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 51

```
dsolve(diff(y(x),x) = F(-(-1+y(x)*ln(x))/y(x))*y(x)^2/x,y(x), singsol=all)
```

$$y(x) = \frac{1}{\ln(x) + \operatorname{RootOf}(F(\_Z) + 1)}$$

$$\int_{\_b}^{y(x)} \frac{1}{\left(F\left(\frac{1-\_a\ln(x)}{\_a}\right)+1\right)\_a^2} d\_a - \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.244 (sec). Leaf size: 245

```
DSolve[y'[x] == (F[(1 - Log[x]*y[x])/y[x]]*y[x]^2)/x,y[x],x,IncludeSingularSolutions -> True
```

$$\text{Solve}\Bigg[\int_1^{y(x)}\Bigg(\frac{1}{\left(-F\left(\frac{1-K[2]\log(x)}{K[2]}\right)-1\right)K[2]^2} - \int_1^x\Bigg(\frac{\left(-\frac{\log(K[1])}{K[2]}-\frac{1-K[2]\log(K[1])}{K[2]^2}\right)F'\left(\frac{1-K[2]\log(K[1])}{K[2]}\right)}{\left(F\left(\frac{1-K[2]\log(K[1])}{K[2]}\right)+1\right)K[1]} - \frac{F\left(\frac{1-K[2]\log(K[1])}{K[2]}\right)\left(-\frac{\log(K[1])}{K[2]}-\frac{1-K[2]\log(K[1])}{K[2]^2}\right)F}{\left(F\left(\frac{1-K[2]\log(K[1])}{K[2]}\right)+1\right)^2 K[1]} + \int_1^x \frac{F\left(\frac{1-\log(K[1])y(x)}{y(x)}\right)}{\left(F\left(\frac{1-\log(K[1])y(x)}{y(x)}\right)+1\right)K[1]}dK[1] = c_1, y(x)\Bigg]$$

2.14 problem 590

Internal problem ID [8925]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 590.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y' - \frac{x}{-y + F(y^2 + x^2)} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 40

```
dsolve(diff(y(x),x) = x/(-y(x)+F(y(x)^2+x^2)),y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(F\left(\_Z^2 + x^2\right)\right)$$

$$-y(x) + \frac{\left(\int^{y(x)^2+x^2} \frac{1}{F(\_a)} d\_a\right)}{2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.254 (sec). Leaf size: 94

```
DSolve[y'[x] == x/(F[x^2 + y[x]^2] - y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} \left(-\frac{K[2]}{F(x^2 + K[2]^2)} - \int_1^x \frac{2K[1]K[2]F'(K[1]^2 + K[2]^2)}{F(K[1]^2 + K[2]^2)^2} dK[1] + 1\right) dK[2] + \int_1^x -\frac{K[1]}{F(K[1]^2 + y(x)^2)} dK[1] = c_1, y(x)\right]$$

2.15 problem 591

Internal problem ID [8926]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 591.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y' - \frac{F\left(\frac{y^2 a+b\,x^2}{a}\right) x}{\sqrt{a}\,y} = 0$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 126

```
dsolve(diff(y(x),x) = F((a*y(x)^2+b*x^2)/a)*x/a^(1/2)/y(x),y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(F\left(\frac{a\_Z^2 + b\,x^2}{a}\right)\sqrt{a} + b\right)$$

$$y(x) = \frac{\sqrt{a\left(-b\,x^2 + \operatorname{RootOf}\left(a\left(\int^{\_Z} \frac{1}{F(\_a)\sqrt{a}+b} d\_a\right) b - b\,x^2 + 2c_1 a\right) a\right)}}{a}$$

$$y(x) = -\frac{\sqrt{a\left(-b\,x^2 + \operatorname{RootOf}\left(a\left(\int^{\_Z} \frac{1}{F(\_a)\sqrt{a}+b} d\_a\right) b - b\,x^2 + 2c_1 a\right) a\right)}}{a}$$

✓ Solution by Mathematica

Time used: 0.531 (sec). Leaf size: 253

```
DSolve[y'[x] == (x*F[(b*x^2 + a*y[x]^2)/a])/(Sqrt[a]*y[x]),y[x],x,IncludeSingularSolutions -
```

$$\text{Solve}\left[\int_1^{y(x)}\left(-\frac{bK[2]}{b+\sqrt{a}F\left(\frac{bx^2+aK[2]^2}{a}\right)} - \int_1^x\left(\frac{2bK[1]K[2]F'\left(\frac{bK[1]^2+aK[2]^2}{a}\right)}{\sqrt{a}\left(b+\sqrt{a}F\left(\frac{bK[1]^2+aK[2]^2}{a}\right)\right)} - \frac{2bF\left(\frac{bK[1]^2+aK[2]^2}{a}\right)K[1]K[2]F'\left(\frac{bK[1]^2+aK[2]^2}{a}\right)}{\left(b+\sqrt{a}F\left(\frac{bK[1]^2+aK[2]^2}{a}\right)\right)^2}\right)dK[1]\right)dK[2] + \int_1^x\frac{bF\left(\frac{bK[1]^2+ay(x)^2}{a}\right)K[1]}{\sqrt{a}\left(b+\sqrt{a}F\left(\frac{bK[1]^2+ay(x)^2}{a}\right)\right)}dK[1] = c_1, y(x)\right]$$

2.16 problem 592

Internal problem ID [8927]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 592.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' - \frac{6x^3 + 5\sqrt{x} + 5F\left(y - \frac{2x^3}{5} - 2\sqrt{x}\right)}{5x} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(diff(y(x),x) = 1/5*(6*x^3+5*x^(1/2)+5*F(y(x)-2/5*x^3-2*x^(1/2)))/x,y(x), singsol=all)
```

$$\int_{\_b}^{y(x)} \frac{1}{F\left(\_a - \frac{2x^3}{5} - 2\sqrt{x}\right)} d\_a - \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.545 (sec). Leaf size: 241

```
DSolve[y'[x] == (Sqrt[x] + (6*x^3)/5 + F[-2*Sqrt[x] - (2*x^3)/5 + y[x]])/x,y[x],x,IncludeSin
```

$$\text{Solve}\left[\int_1^{y(x)} -\frac{F\left(-\frac{2x^3}{5}-2\sqrt{x}+K[2]\right)\int_1^x\left(-\frac{6F'\left(-\frac{2}{5}K[1]^3-2\sqrt{K[1]}+K[2]\right)K[1]^2}{5F\left(-\frac{2}{5}K[1]^3-2\sqrt{K[1]}+K[2]\right)^2}-\frac{F'\left(-\frac{2}{5}K[1]^3-2\sqrt{K[1]}+K[2]\right)}{F\left(-\frac{2}{5}K[1]^3-2\sqrt{K[1]}+K[2]\right)^2\sqrt{K[1]}}\right)dK[1]+1}{F\left(-\frac{2x^3}{5}-2\sqrt{x}+K[2]\right)} + \int_1^x\left(\frac{6K[1]^2}{5F\left(-\frac{2}{5}K[1]^3-2\sqrt{K[1]}+y(x)\right)}+\frac{1}{F\left(-\frac{2}{5}K[1]^3-2\sqrt{K[1]}+y(x)\right)\sqrt{K[1]}} + \frac{1}{K[1]}\right)dK[1]=c_1,y(x)\right]$$

2.17 problem 593

Internal problem ID [8928]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 593.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y' - \frac{F\left(y^{\frac{3}{2}} - \frac{3\,\mathrm{e}^{x}}{2}\right)\mathrm{e}^{x}}{\sqrt{y}} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(diff(y(x),x) = F(y(x)^(3/2)-3/2*exp(x))/y(x)^(1/2)*exp(x),y(x), singsol=all)
```

$$\int_{\_b}^{y(x)} \frac{\sqrt{\_a}}{F\left(\_a^{\frac{3}{2}} - \frac{3\,\mathrm{e}^{x}}{2}\right) - 1} d\_a - \mathrm{e}^{x} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.308 (sec). Leaf size: 221

```
DSolve[y'[x] == (E^x*F[(-3*E^x)/2 + y[x]^(3/2)])/Sqrt[y[x]],y[x],x,IncludeSingularSolutions
```

$$\text{Solve}\left[\int_1^{y(x)} \left(\frac{\sqrt{K[2]}}{F\left(K[2]^{3/2} - \frac{3e^x}{2}\right) - 1} - \int_1^x \left(\frac{3e^{K[1]}F\left(K[2]^{3/2} - \frac{3e^{K[1]}}{2}\right)\sqrt{K[2]}F'\left(K[2]^{3/2} - \frac{3e^{K[1]}}{2}\right)}{2\left(F\left(K[2]^{3/2} - \frac{3e^{K[1]}}{2}\right) - 1\right)^2} - \frac{3e^{K[1]}\sqrt{K[2]}F'\left(K[2]^{3/2} - \frac{3e^{K[1]}}{2}\right)}{2\left(F\left(K[2]^{3/2} - \frac{3e^{K[1]}}{2}\right) - 1\right)}\right) dK[1\right.\right.$$

2.18 problem 594

Internal problem ID [8929]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 594.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y' - \frac{F\left(-\frac{-y^2+b}{x^2}\right)x}{y} = 0$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 95

```
dsolve(diff(y(x),x) = F(-(-y(x)^2+b)/x^2)*x/y(x),y(x), singsol=all)
```

$$y(x) = \text{RootOf}\left(-F\left(\frac{\_Z^2 - b}{x^2}\right)x^2 + \_Z^2 - b\right)$$

$$y(x) = \sqrt{\text{RootOf}\left(-2\ln(x) + \int^{\_Z} \frac{1}{F(\_a) - \_a}d\_a + 2c_1\right)x^2 + b}$$

$$y(x) = -\sqrt{\text{RootOf}\left(-2\ln(x) + \int^{\_Z} \frac{1}{F(\_a) - \_a}d\_a + 2c_1\right)x^2 + b}$$

✓ Solution by Mathematica

Time used: 0.428 (sec). Leaf size: 236

```
DSolve[y'[x] == (x*F[(-b + y[x]^2)/x^2])/y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)}\left(-\frac{K[2]}{-F\left(\frac{K[2]^2-b}{x^2}\right)x^2+K[2]^2-b}\right.\right.$$
$$\left.-\int_1^x\left(\frac{F\left(\frac{K[2]^2-b}{K[1]^2}\right)K[1]\left(2K[2]F'\left(\frac{K[2]^2-b}{K[1]^2}\right)-2K[2]\right)}{\left(F\left(\frac{K[2]^2-b}{K[1]^2}\right)K[1]^2-K[2]^2+b\right)^2}-\frac{2K[2]F'\left(\frac{K[2]^2-b}{K[1]^2}\right)}{K[1]\left(F\left(\frac{K[2]^2-b}{K[1]^2}\right)K[1]^2-K[2]^2+b\right)}\right)dK[1]\right)$$
$$\left.+\int_1^x-\frac{F\left(\frac{y(x)^2-b}{K[1]^2}\right)K[1]}{F\left(\frac{y(x)^2-b}{K[1]^2}\right)K[1]^2-y(x)^2+b}dK[1]=c_1,y(x)\right]$$

2.19 problem 595

Internal problem ID [8930]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 595.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{y' - \frac{F\left(\frac{y^2x+1}{x}\right)}{y x^2} = 0}$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 92

```
dsolve(diff(y(x),x) = F((x*y(x)^2+1)/x)/y(x)/x^2,y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(2F\left(\frac{x\_Z^2+1}{x}\right)-1\right)$$

$$y(x) = \frac{\sqrt{x\left(\operatorname{RootOf}\left(\left(\left(\int^{\_Z}\frac{1}{-1+2F(\_a)}d\_a\right)x+c_1x+1\right)x-1\right)}}{x}$$

$$y(x) = -\frac{\sqrt{x\left(\operatorname{RootOf}\left(\left(\left(\int^{\_Z}\frac{1}{-1+2F(\_a)}d\_a\right)x+c_1x+1\right)x-1\right)}}{x}$$

✓ Solution by Mathematica

Time used: 0.251 (sec). Leaf size: 204

```
DSolve[y'[x] == F[(1 + x*y[x]^2)/x]/(x^2*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{K[2]}{2F\left(\frac{xK[2]^2+1}{x}\right)-1}-\int_1^x\left(\frac{4F\left(\frac{K[1]K[2]^2+1}{K[1]}\right)K[2]F'\left(\frac{K[1]K[2]^2+1}{K[1]}\right)}{\left(2F\left(\frac{K[1]K[2]^2+1}{K[1]}\right)-1\right)^2K[1]^2}-\frac{2K[2]F'\left(\frac{K[1]K[2]^2+1}{K[1]}\right)}{\left(2F\left(\frac{K[1]K[2]^2+1}{K[1]}\right)-1\right)K[1]^2}\right)dK[1]\right)dK[2]+\int_1^x-\frac{F\left(\frac{K[1]y(x)^2+1}{K[1]}\right)}{\left(2F\left(\frac{K[1]y(x)^2+1}{K[1]}\right)-1\right)K[1]^2}dK[1]=c_1,y(x)\right]$$

2.20 problem 596

Internal problem ID [8931]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 596.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' - \frac{-2x^2 + x + F(y + x^2 - x)}{x} = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 39

```
dsolve(diff(y(x),x) = (-2*x^2+x+F(y(x)+x^2-x))/x,y(x), singsol=all)
```

$$y(x) = -x^2 + x + \operatorname{RootOf}(F(\_Z))$$
$$y(x) = -x^2 + \operatorname{RootOf}\left(-\ln(x) + \int^{\_Z} \frac{1}{F(\_a)} d\_a + c_1\right) + x$$

✓ Solution by Mathematica
Time used: 0.267 (sec). Leaf size: 156

```
DSolve[y'[x] == (x - 2*x^2 + F[-x + x^2 + y[x]])/x,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} -\frac{F(x^2 - x + K[2]) \int_1^x \left(\frac{2K[1]F'(K[1]^2 - K[1] + K[2])}{F(K[1]^2 - K[1] + K[2])^2} - \frac{F'(K[1]^2 - K[1] + K[2])}{F(K[1]^2 - K[1] + K[2])^2}\right) dK[1] + 1}{F(x^2 - x + K[2])} dK[2] + \int_1^x \left(-\frac{2K[1]}{F(K[1]^2 - K[1] + y(x))} + \frac{1}{F(K[1]^2 - K[1] + y(x))} + \frac{1}{K[1]}\right) dK[1] = c_1, y(x)\right]$$

2.21 problem 597

Internal problem ID [8932]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 597.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y' - \frac{2a}{x^2\left(-y + 2F\left(\frac{y^2x-4a}{x}\right)a\right)} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 55

```
dsolve(diff(y(x),x) = 2*a/x^2/(-y(x)+2*F((x*y(x)^2-4*a)/x)*a),y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(F\left(\frac{x\_Z^2 - 4a}{x}\right)\right)$$

$$-\frac{y(x)}{2a} + \frac{\int^{y(x)^2-\frac{4a}{x}} \frac{1}{F(\_a)}d\_a}{8a^2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.347 (sec). Leaf size: 130

```
DSolve[y'[x] == (2*a)/(x^2*(2*a*F[(-4*a + x*y[x]^2)/x] - y[x])),y[x],x,IncludeSingularSoluti
```

$$\text{Solve}\left[\int_1^{y(x)}\left(-\frac{K[2]}{2aF\left(\frac{xK[2]^2-4a}{x}\right)} - \int_1^x \frac{2K[2]F'\left(\frac{K[1]K[2]^2-4a}{K[1]}\right)}{F\left(\frac{K[1]K[2]^2-4a}{K[1]}\right)^2 K[1]^2}dK[1] + 1\right)dK[2] + \int_1^x -\frac{1}{F\left(\frac{K[1]y(x)^2-4a}{K[1]}\right)K[1]^2}dK[1] = c_1, y(x)\right]$$

2.22 problem 598

Internal problem ID [8933]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 598.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘]]`

$$\boxed{y' - \frac{y + F\left(\frac{y}{x}\right)}{x-1} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x) = (y(x)+F(y(x)/x))/(x-1),y(x), singsol=all)
```

$$y(x) = \text{RootOf}\left(-\left(\int^{\_Z} \frac{1}{F(\_a) + \_a} d\_a\right) + \ln(x-1) - \ln(x) + c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.139 (sec). Leaf size: 37

```
DSolve[y'[x] == (F[y[x]/x] + y[x])/(-1 + x),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{\frac{y(x)}{x}} \frac{1}{F(K[1]) + K[1]} dK[1] = \log(1-x) - \log(x) + c_1, y(x)\right]$$

2.23 problem 599

Internal problem ID [8934]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 599.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{y' - \frac{-x + F(y^2 + x^2)}{y} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 69

```
dsolve(diff(y(x),x) = (-x+F(y(x)^2+x^2))/y(x),y(x), singsol=all)
```

$$y(x) = \text{RootOf}\left(F\left(\_Z^2 + x^2\right)\right)$$
$$y(x) = \sqrt{-x^2 + \text{RootOf}\left(-2x + \int^{\_Z} \frac{1}{F\left(\_a\right)} d\_a + 2c_1\right)}$$
$$y(x) = -\sqrt{-x^2 + \text{RootOf}\left(-2x + \int^{\_Z} \frac{1}{F\left(\_a\right)} d\_a + 2c_1\right)}$$

✓ Solution by Mathematica

Time used: 0.144 (sec). Leaf size: 95

```
DSolve[y'[x] == (-x + F[x^2 + y[x]^2])/y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} \left(-\frac{K[2]}{F\left(x^2 + K[2]^2\right)} - \int_1^x \frac{2K[1]K[2]F'\left(K[1]^2 + K[2]^2\right)}{F\left(K[1]^2 + K[2]^2\right)^2} dK[1]\right) dK[2] + \int_1^x \left(1 - \frac{K[1]}{F\left(K[1]^2 + y(x)^2\right)}\right) dK[1] = c_1, y(x)\right]$$

2.24 problem 600

Internal problem ID [8935]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 600.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y' - \frac{F\left(-\frac{-1+2y\ln(x)}{y}\right) y^2}{x} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 53

```
dsolve(diff(y(x),x) = F(-(-1+2*y(x)*ln(x))/y(x))*y(x)^2/x,y(x), singsol=all)
```

$$y(x) = \frac{1}{2\ln(x) + \text{RootOf}\left(F(\_Z) + 2\right)}$$

$$\int_{\_b}^{y(x)} \frac{1}{\left(F\left(\frac{1-2\_a\ln(x)}{\_a}\right) + 2\right)\_a^2} d\_a - \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.242 (sec). Leaf size: 246

```
DSolve[y'[x] == (F[(1 - 2*Log[x]*y[x])/y[x]]*y[x]^2)/x,y[x],x,IncludeSingularSolutions -> Tr
```

$$\text{Solve}\left[\int_1^{y(x)}\left(-\int_1^x\left(\frac{2\left(-\frac{2\log(K[1])}{K[2]}-\frac{1-2K[2]\log(K[1])}{K[2]^2}\right)F'\left(\frac{1-2K[2]\log(K[1])}{K[2]}\right)}{\left(F\left(\frac{1-2K[2]\log(K[1])}{K[2]}\right)+2\right)K[1]}-\frac{2F\left(\frac{1-2K[2]\log(K[1])}{K[2]}\right)\left(-\frac{2\log(K}{K[}\right.}{\left(F\left(\frac{1-2K}{}\right.\right.}\right.\right.$$

$$\left.\left.-\frac{2}{\left(F\left(\frac{1-2K[2]\log(x)}{K[2]}\right)+2\right)K[2]^2}\right)dK[2]$$

$$\left.+\int_1^x\frac{2F\left(\frac{1-2\log(K[1])y(x)}{y(x)}\right)}{\left(F\left(\frac{1-2\log(K[1])y(x)}{y(x)}\right)+2\right)K[1]}dK[1]=c_1,y(x)\right]$$

2.25 problem 601

Internal problem ID [8936]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 601.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{y' - \frac{F(-(x-y)(x+y))\,x}{y} = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 77

```
dsolve(diff(y(x),x) = F(-(x-y(x))*(x+y(x)))*x/y(x),y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(F\left(\_Z^2 - x^2\right) - 1\right)$$
$$y(x) = \sqrt{x^2 + \operatorname{RootOf}\left(-x^2 + \int^{\_Z} \frac{1}{F(\_a) - 1} d\_a + 2c_1\right)}$$
$$y(x) = -\sqrt{x^2 + \operatorname{RootOf}\left(-x^2 + \int^{\_Z} \frac{1}{F(\_a) - 1} d\_a + 2c_1\right)}$$

✓ Solution by Mathematica

Time used: 0.249 (sec). Leaf size: 182

```
DSolve[y'[x] == (x*F[(-x + y[x])*(x + y[x])])/y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} \left(\frac{K[2]}{F((K[2]-x)(x+K[2]))-1} - \int_1^x \left(\frac{2F((K[2]-K[1])(K[1]+K[2]))K[1]K[2]F'((K[2]-K[1])(K[1]+K[2]))}{(F((K[2]-K[1])(K[1]+K[2]))-1)^2} - \frac{2K[1]K[2]F'((K[2]-}{F((K[2]-K[1])}\right.\right.\right.$$
$$\left. + \int_1^x -\frac{F((y(x)-K[1])(K[1]+y(x)))K[1]}{F((y(x)-K[1])(K[1]+y(x)))-1} dK[1] = c_1, y(x)\right]$$

2.26 problem 602

Internal problem ID [8937]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 602.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y' - \frac{y^2\left(2 + F\left(\frac{x^2-y}{y x^2}\right) x^2\right)}{x^3} = 0$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 51

```
dsolve(diff(y(x),x) = 1/x^3*y(x)^2*(2+F((x^2-y(x))/y(x)/x^2)*x^2),y(x), singsol=all)
```

$$y(x) = \frac{x^2}{\operatorname{RootOf}\left(F\left(\_Z\right)\right) x^2 + 1}$$
$$y(x) = \frac{x^2}{\operatorname{RootOf}\left(-\ln\left(x\right) - \left(\int^{\_Z} \frac{1}{F(\_a)} d\_a\right) + c_1\right) x^2 + 1}$$

✓ Solution by Mathematica

Time used: 0.43 (sec). Leaf size: 167

```
DSolve[y'[x] == ((2 + x^2*F[(x^2 - y[x])/(x^2*y[x])])*y[x]^2)/x^3,y[x],x,IncludeSingularSolu
```

$$\text{Solve}\left[\int_1^{y(x)}\left(-\int_1^x -\frac{2\left(-\frac{K[1]^2-K[2]}{K[1]^2K[2]^2}-\frac{1}{K[1]^2K[2]}\right)F'\left(\frac{K[1]^2-K[2]}{K[1]^2K[2]}\right)}{F\left(\frac{K[1]^2-K[2]}{K[1]^2K[2]}\right)^2 K[1]^3}dK[1] - \frac{1}{F\left(\frac{x^2-K[2]}{x^2K[2]}\right)K[2]^2}\right)dK[2] + \int_1^x\left(\frac{1}{K[1]}+\frac{2}{K[1]^3F\left(\frac{K[1]^2-y(x)}{K[1]^2y(x)}\right)}\right)dK[1] = c_1, y(x)\right]$$

2.27 problem 603

Internal problem ID [8938]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 603.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y' - \frac{2F(y + \ln(2x+1))\,x + F(y + \ln(2x+1)) - 2}{2x+1} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 42

```
dsolve(diff(y(x),x) = 1/(2*x+1)*(2*F(y(x)+ln(2*x+1))*x+F(y(x)+ln(2*x+1))-2),y(x), singsol=al
```

$$y(x) = -\ln(2x+1) + \operatorname{RootOf}(F(\_Z))$$
$$y(x) = -\ln(2x+1) + \operatorname{RootOf}\left(-x + \int^{\_Z} \frac{1}{F(\_a)} d\_a + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.332 (sec). Leaf size: 117

```
DSolve[y'[x] == (-2 + F[Log[1 + 2*x] + y[x]] + 2*x*F[Log[1 + 2*x] + y[x]])/(1 + 2*x),y[x],x,
```

$$\text{Solve}\left[\int_1^{y(x)} -\frac{F(K[2]+\log(2x+1)) \int_1^x -\frac{2F'(K[2]+\log(2K[1]+1))}{F(K[2]+\log(2K[1]+1))^2(2K[1]+1)} dK[1] - 1}{F(K[2]+\log(2x+1))} dK[2] + \int_1^x \left(\frac{2}{F(\log(2K[1]+1)+y(x))(2K[1]+1)} - 1\right) dK[1] = c_1, y(x)\right]$$

2.28 problem 604

Internal problem ID [8939]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 604.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘x=_G(y,y’)‘]`

$$\boxed{y' - \frac{2y^3}{1 + 2F\left(\frac{1+4y^2x}{y^2}\right)y} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 47

```
dsolve(diff(y(x),x) = 2*y(x)^3/(1+2*F((1+4*x*y(x)^2)/y(x)^2)*y(x)),y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(F\left(\frac{4x\_Z^2+1}{\_Z^2}\right)\right)$$

$$-c_1 - \frac{1}{y(x)} - \frac{\left(\int^{4x+\frac{1}{y(x)^2}} \frac{1}{F(\_a)} d\_a\right)}{4} = 0$$

✓ Solution by Mathematica

Time used: 0.398 (sec). Leaf size: 143

```
DSolve[y'[x] == (2*y[x]^3)/(1 + 2*F[(1 + 4*x*y[x]^2)/y[x]^2]*y[x]),y[x],x,IncludeSingularSol
```

$$\text{Solve}\left[\int_1^{y(x)}\left(-\int_1^x \frac{\left(\frac{8K[1]}{K[2]} - \frac{2(4K[1]K[2]^2+1)}{K[2]^3}\right)F'\left(\frac{4K[1]K[2]^2+1}{K[2]^2}\right)}{F\left(\frac{4K[1]K[2]^2+1}{K[2]^2}\right)^2}dK[1] + \frac{1}{K[2]^2} + \frac{1}{2F\left(\frac{4xK[2]^2+1}{K[2]^2}\right)K[2]^3}\right)dK[2] + \int_1^x -\frac{1}{F\left(\frac{4K[1]y(x)^2+1}{y(x)^2}\right)}dK[1] = c_1, y(x)\right]$$

2.29 problem 605

Internal problem ID [8940]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 605.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y' + \frac{y^2\left(2x - F\left(-\frac{-2+yx}{2y}\right)\right)}{4x} = 0$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 43

```
dsolve(diff(y(x),x) = -1/4*y(x)^2*(2*x-F(-1/2*(-2+x*y(x))/y(x)))/x,y(x), singsol=all)
```

$$y(x) = \frac{2}{x + 2\,\mathrm{RootOf}\left(F\left(\_Z\right)\right)}$$
$$y(x) = \frac{2}{2\,\mathrm{RootOf}\left(-\ln\left(x\right) - 4\left(\int^{\_Z} \frac{1}{F(\_a)} d\_a\right) + c_1\right) + x}$$

✓ Solution by Mathematica
Time used: 0.788 (sec). Leaf size: 145

```
DSolve[y'[x] == -1/4*((2*x - F[(1 - (x*y[x])/2)/y[x]])*y[x]^2)/x,y[x],x,IncludeSingularSolut
```

$$\mathrm{Solve}\left[\int_1^{y(x)} \left(-\int_1^x \frac{2\left(-\frac{K[1]}{2K[2]} - \frac{1-\frac{1}{2}K[1]K[2]}{K[2]^2}\right) F'\left(\frac{1-\frac{1}{2}K[1]K[2]}{K[2]}\right)}{F\left(\frac{1-\frac{1}{2}K[1]K[2]}{K[2]}\right)^2} dK[1] - \frac{4}{F\left(\frac{1-\frac{1}{2}xK[2]}{K[2]}\right) K[2]^2}\right) dK[2] + \int_1^x \left(\frac{1}{K[1]} - \frac{2}{F\left(\frac{1-\frac{1}{2}K[1]y(x)}{y(x)}\right)}\right) dK[1] = c_1, y(x)\right]$$

2.30 problem 606

Internal problem ID [8941]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 606.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' + \left(-e^{-x^2} + x^2 e^{-x^2} - F\left(y - \frac{x^2 e^{-x^2}}{2}\right)\right) x = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve(diff(y(x),x) = -(-exp(-x^2)+x^2*exp(-x^2)-F(y(x)-1/2*x^2*exp(-x^2)))*x,y(x), singsol=
```

$$y(x) = \frac{e^{-x^2} x^2}{2} + \operatorname{RootOf}\left(x^2 - 2\left(\int^{\_Z} \frac{1}{F(\_a)} d\_a\right) + 2c_1\right)$$

✓ Solution by Mathematica

Time used: 0.482 (sec). Leaf size: 361

```
DSolve[y'[x] == x*(E^(-x^2) - x^2/E^x^2 + F[-1/2*x^2/E^x^2 + y[x]]),y[x],x,IncludeSingularSo
```

$$\text{Solve}\Bigg[\int_1^{y(x)} -\frac{F\left(K[2]-\frac{1}{2}e^{-x^2}x^2\right)\int_1^x\left(\frac{e^{-K[1]^2}F'\left(K[2]-\frac{1}{2}e^{-K[1]^2}K[1]^2\right)K[1]^3}{F\left(K[2]-\frac{1}{2}e^{-K[1]^2}K[1]^2\right)^2}-\frac{e^{-K[1]^2}\left(e^{K[1]^2}F\left(K[2]-\frac{1}{2}e^{-K[1]^2}K[1]^2\right)+1\right)F'\left(K[2]-\frac{1}{2}e^{-}\right.}{F\left(K[2]-\frac{1}{2}e^{-K[1]^2}K[1]^2\right)^2}}{F\left(K[2]-\frac{1}{2}e^{-x^2}x^2\right)}$$

$$+\int_1^x\left(\frac{e^{-K[1]^2}\left(e^{K[1]^2}F\left(y(x)-\frac{1}{2}e^{-K[1]^2}K[1]^2\right)+1\right)K[1]}{F\left(y(x)-\frac{1}{2}e^{-K[1]^2}K[1]^2\right)}-\frac{e^{-K[1]^2}K[1]^3}{F\left(y(x)-\frac{1}{2}e^{-K[1]^2}K[1]^2\right)}\right)dK[1]=c_1,y(x)\Bigg]$$

2.31 problem 607

Internal problem ID [8942]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 607.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y' - \frac{2y + F\left(\frac{y}{x^2}\right)x^3}{x} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 32

```
dsolve(diff(y(x),x) = (2*y(x)+F(1/x^2*y(x))*x^3)/x,y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(F(\_Z)\right)x^2$$
$$y(x) = \operatorname{RootOf}\left(-x + \int^{\_Z} \frac{1}{F(\_a)} d\_a + c_1\right)x^2$$

✓ Solution by Mathematica

Time used: 0.237 (sec). Leaf size: 121

```
DSolve[y'[x] == (x^3*F[y[x]/x^2] + 2*y[x])/x,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} -\frac{F\left(\frac{K[2]}{x^2}\right)\int_1^x\left(\frac{2}{F\left(\frac{K[2]}{K[1]^2}\right)K[1]^3} - \frac{2K[2]F'\left(\frac{K[2]}{K[1]^2}\right)}{F\left(\frac{K[2]}{K[1]^2}\right)^2K[1]^5}\right)dK[1]x^2 + 1}{x^2F\left(\frac{K[2]}{x^2}\right)}dK[2] + \int_1^x\left(\frac{2y(x)}{F\left(\frac{y(x)}{K[1]^2}\right)K[1]^3} + 1\right)dK[1] = c_1, y(x)\right]$$

2.32 problem 608

Internal problem ID [8943]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 608.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y' - \frac{\sqrt{y}}{\sqrt{y} + F\left(\frac{x-y}{\sqrt{y}}\right)} = 0$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 38

```
dsolve(diff(y(x),x) = y(x)^(1/2)/(y(x)^(1/2)+F((x-y(x))/y(x)^(1/2))),y(x), singsol=all)
```

$$\frac{\ln\left(y(x)\right)}{2} - \left(\int^{\frac{x-y(x)}{\sqrt{y(x)}}} \frac{1}{2F\left(\_a\right) - \_a} d\_a\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.508 (sec). Leaf size: 274

```
DSolve[y'[x] == Sqrt[y[x]]/(F[(x - y[x])/Sqrt[y[x]]] + Sqrt[y[x]]),y[x],x,IncludeSingularSol
```

$$\text{Solve}\left[\int_1^{y(x)}\left(-\frac{F\left(\frac{x-K[2]}{\sqrt{K[2]}}\right)}{x\sqrt{K[2]}}-\int_1^x -\frac{-\frac{F\left(\frac{K[1]-K[2]}{\sqrt{K[2]}}\right)}{\sqrt{K[2]}}-2\left(-\frac{K[1]-K[2]}{2K[2]^{3/2}}-\frac{1}{\sqrt{K[2]}}\right)\sqrt{K[2]}F'\left(\frac{K[1]-K[2]}{\sqrt{K[2]}}\right)-1}{\left(-2\sqrt{K[2]}F\left(\frac{K[1]-K[2]}{\sqrt{K[2]}}\right)+K[1]-K[2]\right)^2}dK[1]+\frac{2F\left(\frac{x-K[2]}{\sqrt{K[2]}}\right)^2+\sqrt{K[2]}F\left(\frac{x-K}{\sqrt{K}}\right.}{x\left(-x+K[2]+2F\left(\frac{x-K[2]}{\sqrt{K[2]}}\right)\right.}\right.\right.$$

$$\left.+\int_1^x\frac{1}{-2\sqrt{y(x)}F\left(\frac{K[1]-y(x)}{\sqrt{y(x)}}\right)+K[1]-y(x)}dK[1]=c_1,y(x)\right]$$

2.33 problem 609

Internal problem ID [8944]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 609.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y' - \frac{-3yx^2 + F(yx^3)}{x^3} = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 22

```
dsolve(diff(y(x),x) = (-3*x^2*y(x)+F(x^3*y(x)))/x^3,y(x), singsol=all)
```

$$y(x) = \frac{\operatorname{RootOf}\left(x - \left(\int^{\_Z} \frac{1}{F(\_a)} d\_a\right) + c_1\right)}{x^3}$$

✓ Solution by Mathematica
Time used: 0.278 (sec). Leaf size: 117

```
DSolve[y'[x] == (F[x^3*y[x]] - 3*x^2*y[x])/x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} -\frac{x^3 + F(x^3K[2]) \int_1^x \left(\frac{3K[1]^5K[2]F'(K[1]^3K[2])}{F(K[1]^3K[2])^2} - \frac{3K[1]^2}{F(K[1]^3K[2])}\right) dK[1]}{F(x^3K[2])} dK[2] + \int_1^x \left(1 - \frac{3K[1]^2y(x)}{F(K[1]^3y(x))}\right) dK[1] = c_1, y(x)\right]$$

2.34 problem 610

Internal problem ID [8945]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 610.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘]]`

$$y' - \frac{y + x^2 F\left(\frac{y}{x}\right)}{x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x) = (y(x)+F(y(x)/x)*x^2)/x,y(x), singsol=all)
```

$$y(x) = \text{RootOf}\left(x - \left(\int^{\_Z} \frac{1}{F\left(\_a\right)} d\_a\right) + c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.079 (sec). Leaf size: 25

```
DSolve[y'[x] == (x^2*F[y[x]/x] + y[x])/x,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{\frac{y(x)}{x}} \frac{1}{F(K[1])} dK[1] = x + c_1, y(x)\right]$$

2.35 problem 611

Internal problem ID [8946]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 611.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{y' - \frac{-2x - y + F((x+y)\,x)}{x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
dsolve(diff(y(x),x) = (-2*x-y(x)+F((x+y(x))*x))/x,y(x), singsol=all)
```

$$y(x) = \frac{-x^2 + \operatorname{RootOf}(F(\_Z))}{x}$$

$$y(x) = \frac{-x^2 + \operatorname{RootOf}\left(-x + \int^{\_Z} \frac{1}{F(\_a)} d\_a + c_1\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.296 (sec). Leaf size: 191

```
DSolve[y'[x] == (-2*x + F[x*(x + y[x])] - y[x])/x,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} -\frac{x + F(x(x+K[2])) \int_1^x \left(\frac{2F'(K[1](K[1]+K[2]))K[1]^2}{F(K[1](K[1]+K[2]))^2} + \frac{(K[2]-F(K[1](K[1]+K[2])))F'(K[1](K[1]+K[2]))K[1]}{F(K[1](K[1]+K[2]))^2} - \frac{1-K[1]F'(K[}{F(K[1](K}\right.}{F(x(x+K[2]))} + \int_1^x \left(-\frac{2K[1]}{F(K[1](K[1]+y(x)))} - \frac{y(x)-F(K[1](K[1]+y(x)))}{F(K[1](K[1]+y(x)))}\right) dK[1] = c_1, y(x)\right]$$

2.36 problem 612

Internal problem ID [8947]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 612.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y' - \frac{\left(y\,e^{-\frac{x^2}{4}}x + 2F\left(y\,e^{-\frac{x^2}{4}}\right)\right)e^{\frac{x^2}{4}}}{2} = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 38

```
dsolve(diff(y(x),x) = 1/2*(y(x)*exp(-1/4*x^2)*x+2*F(y(x)*exp(-1/4*x^2)))*exp(1/4*x^2),y(x),
```

$$y(x) = \mathrm{RootOf}\left(F\left(\_Z e^{-\frac{x^2}{4}}\right)\right)$$

$$y(x) = \mathrm{RootOf}\left(-x + \int^{\_Z} \frac{1}{F(\_a)} d\_a + c_1\right) e^{\frac{x^2}{4}}$$

✓ Solution by Mathematica

Time used: 0.349 (sec). Leaf size: 199

```
DSolve[y'[x] == (E^(x^2/4)*(2*F[y[x]/E^(x^2/4)] + (x*y[x])/E^(x^2/4)))/2,y[x],x,IncludeSingu
```

$$\text{Solve}\left[\int_1^{y(x)} -\frac{e^{-\frac{x^2}{4}}\left(e^{\frac{x^2}{4}}F\left(e^{-\frac{x^2}{4}}K[2]\right)\int_1^x\left(\frac{e^{-\frac{1}{4}K[1]^2}K[1]}{2F\left(e^{-\frac{1}{4}K[1]^2}K[2]\right)}-\frac{e^{-\frac{1}{2}K[1]^2}K[1]K[2]F'\left(e^{-\frac{1}{4}K[1]^2}K[2]\right)}{2F\left(e^{-\frac{1}{4}K[1]^2}K[2]\right)^2}\right)dK[1]+1\right)}{F\left(e^{-\frac{x^2}{4}}K[2]\right)}dK[2]+\int_1^x\left(\frac{e^{-\frac{1}{4}K[1]^2}K[1]y(x)}{2F\left(e^{-\frac{1}{4}K[1]^2}y(x)\right)}+1\right)dK[1]=c_1,y(x)\right]$$

2.37 problem 613

Internal problem ID [8948]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 613.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y' - \frac{x + y + F\left(-\frac{-y+x\ln(x)}{x}\right)x^2}{x} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 34

```
dsolve(diff(y(x),x) = (x+y(x)+F(-(-y(x)+x*ln(x))/x)*x^2)/x,y(x), singsol=all)
```

$$y(x) = x(\ln(x) + \operatorname{RootOf}(F(\_Z)))$$
$$y(x) = \left(\ln(x) + \operatorname{RootOf}\left(-x + \int^{\_Z} \frac{1}{F(\_a)} d\_a + c_1\right)\right) x$$

✓ Solution by Mathematica

Time used: 0.233 (sec). Leaf size: 226

```
DSolve[y'[x] == (x + x^2*F[(-(x*Log[x]) + y[x])/x] + y[x])/x,y[x],x,IncludeSingularSolutions
```

$$\text{Solve}\left[\int_1^{y(x)} -\frac{xF\left(\frac{K[2]-x\log(x)}{x}\right)\int_1^x\left(-\frac{K[2]F'\left(\frac{K[2]-K[1]\log(K[1])}{K[1]}\right)}{F\left(\frac{K[2]-K[1]\log(K[1])}{K[1]}\right)^2K[1]^3}-\frac{F'\left(\frac{K[2]-K[1]\log(K[1])}{K[1]}\right)}{F\left(\frac{K[2]-K[1]\log(K[1])}{K[1]}\right)^2K[1]^2}+\frac{1}{F\left(\frac{K[2]-K[1]\log(K[1])}{K[1]}\right)K[1]^2}\right)dK[1]}{xF\left(\frac{K[2]-x\log(x)}{x}\right)}\right.$$

$$\left.+\int_1^x\left(\frac{y(x)}{F\left(\frac{y(x)-K[1]\log(K[1])}{K[1]}\right)K[1]^2}+\frac{1}{F\left(\frac{y(x)-K[1]\log(K[1])}{K[1]}\right)K[1]}+1\right)dK[1]=c_1,y(x)\right]$$

2.38 problem 614

Internal problem ID [8949]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 614.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y' - \frac{x(a-1)(a+1)}{y + F\left(\frac{y^2}{2} - \frac{a^2x^2}{2} + \frac{x^2}{2}\right)a^2 - F\left(\frac{y^2}{2} - \frac{a^2x^2}{2} + \frac{x^2}{2}\right)} = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 93

```
dsolve(diff(y(x),x) = x*(a-1)*(a+1)/(y(x)+F(1/2*y(x)^2-1/2*a^2*x^2+1/2*x^2)*a^2-F(1/2*y(x)^2
```

$$y(x) = \mathrm{RootOf}\left(F\left(\frac{1}{2}\_Z^2 - \frac{1}{2}a^2x^2 + \frac{1}{2}x^2\right)\right)$$

$$\frac{\int^{-a^2x^2+x^2+y(x)^2} \frac{1}{F\left(\frac{\_a}{2}\right)}d\_a + (2a^2-2)\,y(x) - 2c_1a^4 + 4a^2c_1 - 2c_1}{2a^4 - 4a^2 + 2} = 0$$

✓ Solution by Mathematica

Time used: 0.323 (sec). Leaf size: 177

```
DSolve[y'[x] == ((-1 + a)*(1 + a)*x)/(-F[x^2/2 - (a^2*x^2)/2 + y[x]^2/2] + a^2*F[x^2/2 - (a^
```

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{K[2]}{(a-1)(a+1)F\left(-\frac{1}{2}a^2x^2+\frac{x^2}{2}+\frac{K[2]^2}{2}\right)} - \int_1^x \frac{K[1]K[2]F'\left(-\frac{1}{2}a^2K[1]^2+\frac{K[1]^2}{2}+\frac{K[2]^2}{2}\right)}{F\left(-\frac{1}{2}a^2K[1]^2+\frac{K[1]^2}{2}+\frac{K[2]^2}{2}\right)^2}dK[1]+1\right)dK[2] + \int_1^x -\frac{K[1]}{F\left(-\frac{1}{2}a^2K[1]^2+\frac{K[1]^2}{2}+\frac{y(x)^2}{2}\right)}dK[1]=c_1,y(x)\right]$$

2.39 problem 615

Internal problem ID [8950]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 615.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y' - \frac{y}{x\left(-1 + F\left(yx\right)y\right)} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 38

```
dsolve(diff(y(x),x) = y(x)/x/(-1+F(x*y(x))*y(x)),y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \operatorname{RootOf}\left(F(x\_Z)\right)$$
$$-y(x) + \int^{xy(x)} \frac{1}{F\left(\_a\right)\_a} d\_a - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.261 (sec). Leaf size: 77

```
DSolve[y'[x] == y[x]/(x*(-1 + F[x*y[x]]*y[x])),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} \left(-\int_1^x \frac{F'(K[1]K[2])}{F(K[1]K[2])^2} dK[1] - \frac{1}{F(xK[2])K[2]} + 1\right) dK[2] + \int_1^x -\frac{1}{F(K[1]y(x))K[1]} dK[1] = c_1, y(x)\right]$$

2.40 problem 616

Internal problem ID [8951]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 616.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y' + \frac{-x^2 + 2yx^3 - F((yx-1)x)}{x^4} = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 38

```
dsolve(diff(y(x),x) = -1/x^4*(-x^2+2*x^3*y(x)-F((x*y(x)-1)*x)),y(x), singsol=all)
```

$$y(x) = \frac{x + \operatorname{RootOf}(F(\_Z))}{x^2}$$
$$y(x) = \frac{\operatorname{RootOf}\left(\left(\int^{\_Z} \frac{1}{F(\_a)} d\_a\right) x + c_1 x + 1\right) + x}{x^2}$$

✓ Solution by Mathematica
Time used: 0.402 (sec). Leaf size: 177

```
DSolve[y'[x] == (x^2 + F[x*(-1 + x*y[x])] - 2*x^3*y[x])/x^4,y[x],x,IncludeSingularSolutions
```

$$\text{Solve}\left[\int_1^{y(x)} - \frac{x^2 + F(x(xK[2]-1)) \int_1^x \left(\frac{2K[2]F'(K[1](K[1]K[2]-1))K[1]^3}{F(K[1](K[1]K[2]-1))^2} - \frac{F'(K[1](K[1]K[2]-1))K[1]^2}{F(K[1](K[1]K[2]-1))^2} - \frac{2K[1]}{F(K[1](K[1]K[2]-1))}\right) dK[1}{F(x(xK[2]-1))} + \int_1^x \left(-\frac{2K[1]y(x)}{F(K[1](K[1]y(x)-1))} + \frac{1}{F(K[1](K[1]y(x)-1))} + \frac{1}{K[1]^2}\right) dK[1] = c_1, y(x)\right]$$

2.41 problem 617

Internal problem ID [8952]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 617.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y' - \frac{F\left(\frac{(3+y)\,e^{\frac{3x^2}{2}}}{3y}\right) x y^2 e^{3x^2} e^{-\frac{9x^2}{2}}}{9} = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 92

```
dsolve(diff(y(x),x) = 1/9*F(1/3*(3+y(x))*exp(3/2*x^2)/y(x))*x*y(x)^2*exp(3*x^2)/exp(9/2*x^2)
```

$$y(x) = \operatorname{RootOf}\left(F\left(\frac{(\_Z+3)\,e^{\frac{3x^2}{2}}}{3\_Z}\right)\_Z e^{3x^2} - 9\,e^{\frac{9x^2}{2}}\_Z - 27\,e^{\frac{9x^2}{2}}\right)$$

$$y(x) = -\frac{3\,e^{\frac{3x^2}{2}}}{e^{\frac{3x^2}{2}} - 3\operatorname{RootOf}\left(-x^2 - 18\left(\int^{\_Z} \frac{1}{F(\_a)-27\_a} d\_a\right) + 2c_1\right)}$$

✓ Solution by Mathematica

Time used: 0.892 (sec). Leaf size: 615

```
DSolve[y'[x] == (x*F[(E^((3*x^2)/2)*(3 + y[x]))/(3*y[x])]*y[x]^2)/(9*E^((3*x^2)/2)),y[x],x,I
```

$$
\text{Solve}\left[\int_1^{y(x)}\left(\frac{9e^{\frac{3x^2}{2}}-F\left(\frac{e^{\frac{3x^2}{2}}(K[2]+3)}{3K[2]}\right)}{3\left(\left(9e^{\frac{3x^2}{2}}-F\left(\frac{e^{\frac{3x^2}{2}}(K[2]+3)}{3K[2]}\right)\right)K[2]+27e^{\frac{3x^2}{2}}\right)}\right.\right.
$$

$$
-\int_1^x\left(-\frac{K[2]\left(\frac{e^{\frac{3K[1]^2}{2}}}{3K[2]}-\frac{e^{\frac{3K[1]^2}{2}}(K[2]+3)}{3K[2]^2}\right)F'\left(\frac{e^{\frac{3K[1]^2}{2}}(K[2]+3)}{3K[2]}\right)K[1]}{-9e^{\frac{3K[1]^2}{2}}K[2]+F\left(\frac{e^{\frac{3K[1]^2}{2}}(K[2]+3)}{3K[2]}\right)K[2]-27e^{\frac{3K[1]^2}{2}}}+\frac{F\left(\frac{e^{\frac{3K[1]^2}{2}}(K[2]+3)}{3K[2]}\right)K[2]\left(F\left(\frac{e^{\frac{3K[1]^2}{2}}(\ldots}{3K\ldots}\right.\right.}{\left(-9e^{3\ldots}\right.}\right.
$$

$$
\left.\left.-\frac{1}{3K[2]}\right)dK[2]+\int_1^x
$$

$$
\left.-\frac{F\left(\frac{e^{\frac{3K[1]^2}{2}}(y(x)+3)}{3y(x)}\right)K[1]y(x)}{-9e^{\frac{3K[1]^2}{2}}y(x)+F\left(\frac{e^{\frac{3K[1]^2}{2}}(y(x)+3)}{3y(x)}\right)y(x)-27e^{\frac{3K[1]^2}{2}}}dK[1]=c_1,y(x)\right]
$$

2.42 problem 618

Internal problem ID [8953]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 618.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$y' - \frac{(y+1)\left((y-\ln(y+1)-\ln(x))x+1\right)}{yx} = 0$$

✓ Solution by Maple

Time used: 0.594 (sec). Leaf size: 36

```
dsolve(diff(y(x),x) = (y(x)+1)*((y(x)-ln(y(x)+1)-ln(x))*x+1)/y(x)/x,y(x), singsol=all)
```

$$y(x) = -\operatorname{LambertW}\left(-\frac{e^{-1}}{x}\right) - 1$$
$$y(x) = -\operatorname{LambertW}\left(-\frac{e^{c_1 e^x - 1}}{x}\right) - 1$$

✓ Solution by Mathematica

Time used: 60.182 (sec). Leaf size: 25

```
DSolve[y'[x] == ((1 + y[x])*(1 + x*(-Log[x] - Log[1 + y[x]] + y[x])))/(x*y[x]),y[x],x,Includ
```

$$y(x) \to -1 - W\left(-\frac{e^{-1+c_1 e^x}}{x}\right)$$

2.43 problem 619

Internal problem ID [8954]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 619.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘x=_G(y,y’)‘]`

$$y' - \frac{6y}{8y^4 + 9y^3 + 12y^2 + 6y - F\left(-\frac{y^4}{3} - \frac{y^3}{2} - y^2 - y + x\right)} = 0$$

✓ Solution by Maple

Time used: 0.297 (sec). Leaf size: 81

```
dsolve(diff(y(x),x) = 6*y(x)/(8*y(x)^4+9*y(x)^3+12*y(x)^2+6*y(x)-F(-1/3*y(x)^4-1/2*y(x)^3-y(
```

$$\int_{\_b}^{y(x)} \frac{-8\_a^4 - 9\_a^3 - 12\_a^2 + F\left(-\frac{1}{3}\_a^4 - \frac{1}{2}\_a^3 - \_a^2 - \_a + x\right) - 6\_a}{F\left(-\frac{1}{3}\_a^4 - \frac{1}{2}\_a^3 - \_a^2 - \_a + x\right)\_a} d\_a - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.583 (sec). Leaf size: 330

```
DSolve[y'[x] == (6*y[x])/(-F[x - y[x] - y[x]^2 - y[x]^3/2 - y[x]^4/3] + 6*y[x] + 12*y[x]^2 +
```

$$\text{Solve}\left[\int_1^{y(x)}\left(-\frac{8K[2]^3}{F\left(-\frac{1}{3}K[2]^4-\frac{K[2]^3}{2}-K[2]^2-K[2]+x\right)}-\frac{9K[2]^2}{F\left(-\frac{1}{3}K[2]^4-\frac{K[2]^3}{2}-K[2]^2-K[2]+x\right)}-\frac{12K[2]}{F\left(-\frac{1}{3}K[2]^4-\frac{K[2]^3}{2}-K[2]^2-K[2]+x\right)}-\frac{F\left(-\frac{1}{3}K[2]^4-\frac{K[2]^3}{2}-K[2]^2-K[2]+x\right)\int_1^x-\frac{6\left(-\frac{4}{3}K[2]^3-\frac{3K[2]^2}{2}-2K[2]-1\right)F'\left(-\frac{1}{3}K[2]^4-\frac{K[2]^3}{2}-K[2]^2-K[2]+K[1]\right)}{F\left(-\frac{1}{3}K[2]^4-\frac{K[2]^3}{2}-K[2]^2-K[2]+K[1]\right)^2}d}{F\left(-\frac{1}{3}K[2]^4-\frac{K[2]^3}{2}-K[2]^2-K[2]+x\right)}+\frac{1}{K[2]}\right)dK[2]+\int_1^x\frac{6}{F\left(-\frac{1}{3}y(x)^4-\frac{y(x)^3}{2}-y(x)^2-y(x)+K[1]\right)}dK[1]=c_1,y(x)\right]$$

2.44 problem 620

Internal problem ID [8955]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 620.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$
y' - \frac{y^2 + 2yx + x^2 + \mathrm{e}^{2F(-(x-y)(x+y))}}{y^2 + 2yx + x^2 - \mathrm{e}^{2F(-(x-y)(x+y))}} = 0
$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 37

```
dsolve(diff(y(x),x) = (y(x)^2+2*x*y(x)+x^2+exp(2*F(-(x-y(x))*(x+y(x)))))/(y(x)^2+2*x*y(x)+x^
```

$$
y(x) = \mathrm{e}^{\mathrm{RootOf}\left(-\_Z + \int^{\mathrm{e}^{2\_Z} - 2x\,\mathrm{e}^{\_Z}} \frac{1}{\mathrm{e}^{2F(\_a)} + \_a} d\_a + c_1\right)} - x
$$

✓ Solution by Mathematica

Time used: 0.814 (sec). Leaf size: 205

```
DSolve[y'[x] == (E^(2*F[(-x + y[x])*(x + y[x])]) + x^2 + 2*x*y[x] + y[x]^2)/(-E^(2*F[(-x + y
```

$$
\mathrm{Solve}\left[\int_1^{y(x)} \left(-\frac{2K[2]}{-x^2 + e^{2F((K[2]-x)(x+K[2]))} + K[2]^2} - \int_1^x \left(\frac{2K[1]\left(-4e^{2F((K[2]-K[1])(K[1]+K[2]))}F'((K[2]-K[1])(K[1]+K[2]))K[2] - 2K[2]\right)}{\left(K[1]^2 - e^{2F((K[2]-K[1])(K[1]+K[2]))} - K[2]^2\right)^2} - \frac{1}{(K[1]+K[2])^2}\right) + \frac{1}{x+K[2]}\right) dK[2] + \int_1^x \left(\frac{1}{K[1]+y(x)} - \frac{2K[1]}{K[1]^2 - e^{2F((y(x)-K[1])(K[1]+y(x)))} - y(x)^2}\right) dK[1] = c_1, y(x)\right]
$$

2.45 problem 621

Internal problem ID [8956]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 621.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], [_Abel, ‘2nd type‘, ‘class C‘]]`

$$\boxed{y' - \frac{1}{y + \sqrt{x}} = 0}$$

✓ Solution by Maple

Time used: 0.531 (sec). Leaf size: 59

```
dsolve(diff(y(x),x) = 1/(y(x)+x^(1/2)),y(x), singsol=all)
```

$$y(x) = \frac{\operatorname{RootOf}\left(\_Z^{18} c_1 - 9x\_Z^6 - 6\_Z^3\sqrt{x} - 1\right)^3 \sqrt{x} + 1}{\operatorname{RootOf}\left(\_Z^{18} c_1 - 9x\_Z^6 - 6\_Z^3\sqrt{x} - 1\right)^3}$$

✓ Solution by Mathematica

Time used: 60.048 (sec). Leaf size: 445

```
DSolve[y'[x] == (Sqrt[x] + y[x])^(-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x} + \frac{1}{\text{Root}\left[\#1^6\left(16x^3+16e^{12c_1}\right)-24\#1^4x^2+8\#1^3x^{3/2}+9\#1^2x-6\#1\sqrt{x}+1\&,1\right]}$$

$$y(x) \to -\sqrt{x} + \frac{1}{\text{Root}\left[\#1^6\left(16x^3+16e^{12c_1}\right)-24\#1^4x^2+8\#1^3x^{3/2}+9\#1^2x-6\#1\sqrt{x}+1\&,2\right]}$$

$$y(x) \to -\sqrt{x} + \frac{1}{\text{Root}\left[\#1^6\left(16x^3+16e^{12c_1}\right)-24\#1^4x^2+8\#1^3x^{3/2}+9\#1^2x-6\#1\sqrt{x}+1\&,3\right]}$$

$$y(x) \to -\sqrt{x} + \frac{1}{\text{Root}\left[\#1^6\left(16x^3+16e^{12c_1}\right)-24\#1^4x^2+8\#1^3x^{3/2}+9\#1^2x-6\#1\sqrt{x}+1\&,4\right]}$$

$$y(x) \to -\sqrt{x} + \frac{1}{\text{Root}\left[\#1^6\left(16x^3+16e^{12c_1}\right)-24\#1^4x^2+8\#1^3x^{3/2}+9\#1^2x-6\#1\sqrt{x}+1\&,5\right]}$$

$$y(x) \to -\sqrt{x} + \frac{1}{\text{Root}\left[\#1^6\left(16x^3+16e^{12c_1}\right)-24\#1^4x^2+8\#1^3x^{3/2}+9\#1^2x-6\#1\sqrt{x}+1\&,6\right]}$$

2.46 problem 622

Internal problem ID [8957]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 622.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], [_Abel, '2nd type', 'c`

$$y' - \frac{1}{y + 2 + \sqrt{3x+1}} = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 70

```
dsolve(diff(y(x),x) = 1/(y(x)+2+(3*x+1)^(1/2)),y(x), singsol=all)
```

$$-2\sqrt{33}\,\operatorname{arctanh}\left(\frac{\left(\sqrt{3x+1}+2y(x)+4\right)\sqrt{33}}{11\sqrt{3x+1}}\right) + 11\ln\left(\left(3y(x)+6\right)\sqrt{3x+1}+3y(x)^2-6x+12y(x)+10\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.233 (sec). Leaf size: 140

```
DSolve[y'[x] == (2 + Sqrt[1 + 3*x] + y[x])^(-1),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[6\sqrt{33}\operatorname{arctanh}\left(\frac{3y(x)+7\sqrt{3x+1}+6}{\sqrt{33}\left(y(x)+\sqrt{3x+1}+2\right)}\right) + 44c_1 = 33\left(\log\left(\frac{-3\sqrt{3x+1}y(x)^2-3\left(3x+4\sqrt{3x+1}+1\right)y(x)+6x\left(\sqrt{3x+1}-3\right)-10\sqrt{3x+1}-6}{2(3x+1)^{3/2}}\right)\right.\right.$$

2.47 problem 623

Internal problem ID [8958]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 623.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, [_Abel, '2n`

$$\boxed{y' - \frac{x^2}{y + x^{\frac{3}{2}}} = 0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 51

```
dsolve(diff(y(x),x) = x^2/(y(x)+x^(3/2)),y(x), singsol=all)
```

$$-2\sqrt{33}\,\operatorname{arctanh}\left(\frac{\left(x^{\frac{3}{2}} + 2y(x)\right)\sqrt{33}}{11x^{\frac{3}{2}}}\right) + 11\ln\left(3y(x)\,x^{\frac{3}{2}} - 2x^3 + 3y(x)^2\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.156 (sec). Leaf size: 77

```
DSolve[y'[x] == x^2/(x^(3/2) + y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[6\sqrt{33}\operatorname{arctanh}\left(\frac{7x^{3/2} + 3y(x)}{\sqrt{33}\,(x^{3/2} + y(x))}\right) + 44c_1 = 33\left(\log\left(-\frac{3y(x)}{2x^{3/2}} - \frac{3y(x)^2}{2x^3} + 1\right) + 3\log(x)\right), y(x)\right]$$

2.48 problem 624

Internal problem ID [8959]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 624.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$y' - \frac{x^{\frac{5}{3}}}{y + x^{\frac{4}{3}}} = 0$$

✓ Solution by Maple
Time used: 0.579 (sec). Leaf size: 46

```
dsolve(diff(y(x),x) = x^(5/3)/(y(x)+x^(4/3)),y(x), singsol=all)
```

$$y(x) = \frac{\operatorname{RootOf}\left(\_Z^{192} + 12x^{\frac{4}{3}}\_Z^{176} + 48x^{\frac{8}{3}}\_Z^{160} + 64x^4\_Z^{144} - c_1\right)^{16}}{2} + \frac{x^{\frac{4}{3}}}{2}$$

✓ Solution by Mathematica
Time used: 79.305 (sec). Leaf size: 9837

```
DSolve[y'[x] == x^(5/3)/(x^(4/3) + y[x]),y[x],x,IncludeSingularSolutions -> True]
```

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2.49 problem 625

Internal problem ID [8960]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 625.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y' - \frac{ix^2\left(i - 2\sqrt{-x^3 + 6y}\right)}{2} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 55

```
dsolve(diff(y(x),x) = 1/2*I*x^2*(I-2*(-x^3+6*y(x))^(1/2)),y(x), singsol=all)
```

$$i \ln\left(x^3 - 6y(x) - 1\right) + 2\sqrt{-x^3 + 6y(x)} - 2\arctan\left(\sqrt{-x^3 + 6y(x)}\right) + 2ix^3 - c_1 = 0$$

✓ Solution by Mathematica
Time used: 11.298 (sec). Leaf size: 69

```
DSolve[y'[x] == (I/2)*x^2*(I - 2*Sqrt[-x^3 + 6*y[x]]),y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{6}\left(-W\left(-ie^{-x^3-1-6c_1}\right)^2 - 2W\left(-ie^{-x^3-1-6c_1}\right) + x^3 - 1\right)$$
$$y(x) \to \frac{1}{6}\left(x^3 - 1\right)$$

2.50 problem 626

Internal problem ID [8961]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 626.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class C']]`

$$y' - \frac{x}{y + \sqrt{x^2+1}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 118

```
dsolve(diff(y(x),x) = x/(y(x)+(x^2+1)^(1/2)),y(x), singsol=all)
```

$$-\frac{2\ln(11)}{3} + \frac{2\ln\left(\frac{-y(x)\sqrt{x^2+1}+x^2-y(x)^2+1}{\left(y(x)+\sqrt{x^2+1}\right)^2}\right)}{3} - \frac{4\sqrt{5}\,\mathrm{arctanh}\left(\frac{\left(3\sqrt{x^2+1}+y(x)\right)\sqrt{5}}{5y(x)+5\sqrt{x^2+1}}\right)}{15} - \frac{4\ln\left(\frac{\sqrt{x^2+1}}{y(x)+\sqrt{x^2+1}}\right)}{3} + \frac{2\ln(x^2+1)}{3} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.16 (sec). Leaf size: 88

```
DSolve[y'[x] == x/(Sqrt[1 + x^2] + y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\frac{1}{2}\left(\log\left(-\frac{y(x)^2}{x^2+1} - \frac{y(x)}{\sqrt{x^2+1}} + 1\right) + \log(x^2+1)\right) = \frac{\mathrm{arctanh}\left(\frac{3\sqrt{x^2+1}+y(x)}{\sqrt{5}\left(\sqrt{x^2+1}+y(x)\right)}\right)}{\sqrt{5}} + c_1, y(x)\right]$$

2.51 problem 627

Internal problem ID [8962]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 627.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \frac{(-1 + y\ln(x))^2}{x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x) = (-1+y(x)*ln(x))^2/x,y(x), singsol=all)
```

$$y(x) = \frac{\sin(\ln(x))\, c_1 + \cos(\ln(x))}{(\ln(x) + c_1)\cos(\ln(x)) + (c_1 \ln(x) - 1)\sin(\ln(x))}$$

✓ Solution by Mathematica

Time used: 1.298 (sec). Leaf size: 63

```
DSolve[y'[x] == (-1 + Log[x]*y[x])^2/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sin(\log(x)) + c_1 \cos(\log(x))}{(1 + c_1 \log(x))\cos(\log(x)) + (\log(x) - c_1)\sin(\log(x))}$$
$$y(x) \to \frac{\cos(\log(x))}{\log(x)\cos(\log(x)) - \sin(\log(x))}$$

2.52 problem 628

Internal problem ID [8963]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 628.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y' - \frac{x\left(-2+3\sqrt{x^2+3y}\right)}{3} = 0$$

✓ Solution by Maple
Time used: 0.11 (sec). Leaf size: 23

```
dsolve(diff(y(x),x) = 1/3*x*(-2+3*(x^2+3*y(x))^(1/2)),y(x), singsol=all)
```

$$c_1 + \frac{x^2}{3} + \frac{4}{27} - \frac{4\sqrt{x^2+3y(x)}}{9} = 0$$

✓ Solution by Mathematica
Time used: 0.275 (sec). Leaf size: 32

```
DSolve[y'[x] == (x*(-2 + 3*Sqrt[x^2 + 3*y[x]]))/3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{48}\left(9x^4 - 2(8+27c_1)x^2 + 81{c_1}^2\right)$$

2.53 problem 629

Internal problem ID [8964]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 629.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \frac{(-1 + 2y\ln(x))^2}{x} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 62

```
dsolve(diff(y(x),x) = (-1+2*y(x)*ln(x))^2/x,y(x), singsol=all)
```

$$y(x) = \frac{\sin\left(\sqrt{2}\,\ln(x)\right)c_1 + \cos\left(\sqrt{2}\,\ln(x)\right)}{\left(2c_1\ln(x) - \sqrt{2}\right)\sin\left(\sqrt{2}\,\ln(x)\right) + \left(c_1\sqrt{2} + 2\ln(x)\right)\cos\left(\sqrt{2}\,\ln(x)\right)}$$

✓ Solution by Mathematica
Time used: 1.353 (sec). Leaf size: 123

```
DSolve[y'[x] == (-1 + 2*Log[x]*y[x])^2/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sin\left(\sqrt{2}\log(x)\right) + c_1\cos\left(\sqrt{2}\log(x)\right)}{\left(\sqrt{2} + 2c_1\log(x)\right)\cos\left(\sqrt{2}\log(x)\right) + \left(2\log(x) - \sqrt{2}c_1\right)\sin\left(\sqrt{2}\log(x)\right)}$$
$$y(x) \to \frac{\cos\left(\sqrt{2}\log(x)\right)}{2\log(x)\cos\left(\sqrt{2}\log(x)\right) - \sqrt{2}\sin\left(\sqrt{2}\log(x)\right)}$$

2.54 problem 630

Internal problem ID [8965]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 630.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], [_Abel, '2nd type', 'c`

$$y' - \frac{e^{bx}}{y\,e^{-bx} + 1} = 0$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 58

```
dsolve(diff(y(x),x) = 1/(y(x)*exp(-b*x)+1)*exp(b*x),y(x), singsol=all)
```

$$y(x) = \mathrm{RootOf}\left(-e^{\mathrm{RootOf}\left(4\,e^{\_Z}\cosh\left(\frac{\sqrt{b(4+b)}\,(2c_1b-2bx-\_Z)}{2b}\right)^2+b+4\right)} - 1 + b\_Z + b\_Z^2\right)e^{bx}$$

✓ Solution by Mathematica

Time used: 0.343 (sec). Leaf size: 101

```
DSolve[y'[x] == E^(b*x)/(1 + y[x]/E^(b*x)),y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\frac{1}{2}b\left(\log\left(-be^{-2bx}y(x)^2 - be^{-bx}y(x) + 1\right) + 2bx\right) = \frac{b\arctan\left(\frac{(b+2)(-e^{bx})-by(x)}{b\sqrt{-\frac{b+4}{b}}(e^{bx}+y(x))}\right)}{\sqrt{-\frac{b+4}{b}}} + c_1, y(x)\right]$$

2.55 problem 631

Internal problem ID [8966]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 631.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y' - \frac{x^2\left(1 + 2\sqrt{x^3 - 6y}\right)}{2} = 0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 23

```
dsolve(diff(y(x),x) = 1/2*x^2*(1+2*(x^3-6*y(x))^(1/2)),y(x), singsol=all)
```

$$c_1 - x^3 - \frac{1}{4} - \sqrt{x^3 - 6y(x)} = 0$$

✓ Solution by Mathematica

Time used: 0.319 (sec). Leaf size: 31

```
DSolve[y'[x] == (x^2*(1 + 2*Sqrt[x^3 - 6*y[x]]))/2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}\left(-x^6 + (1 - 12c_1)x^3 - 36c_1{}^2\right)$$

2.56 problem 632

Internal problem ID [8967]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 632.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], [_Abel, '2nd type', 'c`

$$y' - \frac{e^{x}}{y\,e^{-x} + 1} = 0$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 52

```
dsolve(diff(y(x),x) = 1/(y(x)*exp(-x)+1)*exp(x),y(x), singsol=all)
```

$$x - \frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{2y(x)\sqrt{5}\,e^{-x}}{5} + \frac{\sqrt{5}}{5}\right)}{5} + \frac{\ln\left(y(x)^{2} e^{-2x} + y(x)\,e^{-x} - 1\right)}{2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.166 (sec). Leaf size: 65

```
DSolve[y'[x] == E^x/(1 + y[x]/E^x),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\log\left(-e^{-2x}y(x)^2 - e^{-x}y(x) + 1\right) + x = \frac{\operatorname{arctanh}\left(\frac{y(x)+3e^x}{\sqrt{5}(y(x)+e^x)}\right)}{\sqrt{5}} + c_1, y(x)\right]$$

2.57 problem 633

Internal problem ID [8968]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 633.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], [_Abel, '2nd type', 'c`

$$y' - \frac{e^{\frac{2x}{3}}}{y\,e^{-\frac{2x}{3}} + 1} = 0$$

✓ Solution by Maple

Time used: 0.718 (sec). Leaf size: 49

```
dsolve(diff(y(x),x) = 1/(y(x)*exp(-2/3*x)+1)*exp(2/3*x),y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(-e^{\operatorname{RootOf}\left(343\,\mathrm{sech}\left(\frac{(4c_1-4x-3\_Z)\sqrt{7}}{6}\right)^2+98\,e^{-\_Z}\right)} - 3 + 2\_Z + 2\_Z^2\right) e^{\frac{2x}{3}}$$

✓ Solution by Mathematica

Time used: 0.179 (sec). Leaf size: 85

```
DSolve[y'[x] == E^((2*x)/3)/(1 + y[x]/E^((2*x)/3)),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[7\left(3\log\left(-\frac{2}{3}e^{-4x/3}y(x)^2-\frac{2}{3}e^{-2x/3}y(x)+1\right)+4x-9c_1\right)=6\sqrt{7}\text{arctanh}\left(\frac{y(x)+4e^{2x/3}}{\sqrt{7}\left(y(x)+e^{2x/3}\right)}\right),y(x)\right]$$

2.58 problem 634

Internal problem ID [8969]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 634.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$
y' - \frac{1 + 2x^5\sqrt{4yx^2 + 1}}{2x^3} = 0
$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 29

```
dsolve(diff(y(x),x) = 1/2*(1+2*x^5*(4*x^2*y(x)+1)^(1/2))/x^3,y(x), singsol=all)
```

$$
\frac{x^5 + 2c_1x - 2\sqrt{4x^2y(x) + 1}}{2x} = 0
$$

✓ Solution by Mathematica

Time used: 0.393 (sec). Leaf size: 31

```
DSolve[y'[x] == (1/2 + x^5*Sqrt[1 + 4*x^2*y[x]])/x^3,y[x],x,IncludeSingularSolutions -> True
```

$$
y(x) \to \frac{1}{16}\left(x^8 - 8c_1x^4 - \frac{4}{x^2} + 16c_1{}^2\right)
$$

2.59 problem 635

Internal problem ID [8970]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 635.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$\boxed{y' - \frac{x\left(x + 2\sqrt{x^3 - 6y}\right)}{2} = 0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 22

```
dsolve(diff(y(x),x) = 1/2*x*(x+2*(x^3-6*y(x))^(1/2)),y(x), singsol=all)
```

$$c_1 - \frac{3x^2}{2} - \sqrt{x^3 - 6y\left(x\right)} = 0$$

✓ Solution by Mathematica

Time used: 0.307 (sec). Leaf size: 33

```
DSolve[y'[x] == (x*(x + 2*Sqrt[x^3 - 6*y[x]]))/2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{24}\left(-9x^4 + 4x^3 + 36c_1x^2 - 36{c_1}^2\right)$$

2.60 problem 636

Internal problem ID [8971]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 636.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y' - (-\ln(y) + x^2)\, y = 0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 19

```
dsolve(diff(y(x),x) = (-ln(y(x))+x^2)*y(x),y(x), singsol=all)
```

$$y(x) = e^{e^{-x} c_1 + x^2 - 2x + 2}$$

✓ Solution by Mathematica
Time used: 0.338 (sec). Leaf size: 24

```
DSolve[y'[x] == (x^2 - Log[y[x]])*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x^2 - 2x - 2c_1 e^{-x} + 2}$$

2.61 problem 637

Internal problem ID [8972]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 637.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, ‘2nd type‘, ‘class C‘], [_1st_order, ‘_with_symmetry_`

$$y' - \frac{e^{-x^2} x}{y\, e^{x^2} + 1} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 84

```
dsolve(diff(y(x),x) = 1/(y(x)*exp(x^2)+1)*exp(-x^2)*x,y(x), singsol=all)
```

$$y(x) = -\frac{\tan\left(\mathrm{RootOf}\left(2x^2 - \ln(2) + \ln(5) - \ln\left(\sec(\_Z)^2\right) + 2\ln(-1 + \tan(\_Z)) + 6c_1 - 2\_Z\right)\right) e^{-x^2}}{\tan\left(\mathrm{RootOf}\left(2x^2 - \ln(2) + \ln(5) - \ln\left(\sec(\_Z)^2\right) + 2\ln(-1 + \tan(\_Z)) + 6c_1 - 2\_Z\right)\right) - 1}$$

✓ Solution by Mathematica
Time used: 7.089 (sec). Leaf size: 62

```
DSolve[y'[x] == x/(E^x^2*(1 + E^x^2*y[x])),y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[-\frac{1}{2}\arctan\left(2e^{x^2}y(x) + 1\right) - \frac{1}{4}\log\left(2e^{2x^2}y(x)^2 + 2e^{x^2}y(x) + 1\right) + \frac{1}{2}\log\left(e^{x^2}\right) = c_1, y(x)\right]$$

2.62 problem 638

Internal problem ID [8973]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 638.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘x=_G(y,y’)‘]`

$$y' + (-\ln(\ln(y)) + \ln(x))\, y = 0$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 36

```
dsolve(diff(y(x),x) = -(-ln(ln(y(x)))+ln(x))*y(x),y(x), singsol=all)
```

$$-\left(\int_{\_b}^{y(x)} -\frac{1}{\_a\left(x\ln(x) - \ln(\ln(\_a))\, x + \ln(\_a)\right)} d\_a\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.109 (sec). Leaf size: 41

```
DSolve[y'[x] == (-Log[x] + Log[Log[y[x]]])*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} \frac{1}{K[1](x\log(x) + \log(K[1]) - x\log(\log(K[1])))} dK[1] = -\log(x) + c_1, y(x)\right]$$

2.63 problem 639

Internal problem ID [8974]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 639.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `['y=_G(x,y')']`

$$y' - (-\ln(\ln(y)) + \ln(x))^2 y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 46

```
dsolve(diff(y(x),x) = (-ln(ln(y(x)))+ln(x))^2*y(x),y(x), singsol=all)
```

$$\int_{\_b}^{y(x)} \frac{1}{\_a \left(\ln(x)^2 x - 2\ln(x)\ln(\ln(\_a))\, x + \ln(\ln(\_a))^2 x - \ln(\_a)\right)} d\_a - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.14 (sec). Leaf size: 53

```
DSolve[y'[x] == (Log[x] - Log[Log[y[x]]])^2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} \frac{1}{K[1]\left(x\log^2(x) - 2x\log(\log(K[1]))\log(x) + x\log^2(\log(K[1])) - \log(K[1])\right)} dK[1] = \log(x) + c_1, y(x)\right]$$

2.64 problem 640

Internal problem ID [8975]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 640.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$y' - \frac{y}{\ln(\ln(y)) - \ln(x) + 1} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
dsolve(diff(y(x),x) = 1/(ln(ln(y(x)))-ln(x)+1)*y(x),y(x), singsol=all)
```

$$\int_{\_b}^{y(x)} \frac{\ln(x) - \ln(\ln(\_a)) - 1}{(-\ln(\_a)\ln(\ln(\_a)) + (-1 + \ln(x))\ln(\_a) + x)\_a} d\_a - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.221 (sec). Leaf size: 53

```
DSolve[y'[x] == y[x]/(1 - Log[x] + Log[Log[y[x]]]),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} \frac{\log(x) - \log(\log(K[1])) - 1}{K[1](x + \log(x)\log(K[1]) - \log(K[1]) - \log(K[1])\log(\log(K[1])))} dK[1] = c_1, y(x)\right]$$

2.65 problem 641

Internal problem ID [8976]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 641.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' - \frac{1 + 2\sqrt{4yx^2 + 1}\,x^4}{2x^3} = 0$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 31

```
dsolve(diff(y(x),x) = 1/2*(1+2*(4*x^2*y(x)+1)^(1/2)*x^4)/x^3,y(x), singsol=all)
```

$$\frac{2x^4 + 3c_1x - 3\sqrt{4x^2y(x) + 1}}{3x} = 0$$

✓ Solution by Mathematica

Time used: 0.331 (sec). Leaf size: 33

```
DSolve[y'[x] == (1/2 + x^4*Sqrt[1 + 4*x^2*y[x]])/x^3,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{x^6}{9} - \frac{2c_1x^3}{3} - \frac{1}{4x^2} + c_1{}^2$$

2.66 problem 642

Internal problem ID [8977]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 642.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{\left(-y^2 + 4ax\right)^2}{y} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 227

```
dsolve(diff(y(x),x) = (-y(x)^2+4*a*x)^2/y(x),y(x), singsol=all)
```

$$y(x) = \frac{2\,e^{4a\,x^2+2\sqrt{2}\,\sqrt{a}\,x}\sqrt{\sqrt{a}\,\left(c_1 e^{4\sqrt{2}\,\sqrt{a}\,x}+1\right)\left(c_1\left(x\sqrt{a}-\frac{\sqrt{2}}{4}\right)e^{4\sqrt{2}\,\sqrt{a}\,x}+x\sqrt{a}+\frac{\sqrt{2}}{4}\right)e^{-8a\,x^2}e^{-4\sqrt{2}\,\sqrt{a}\,x}}}{c_1 e^{4\sqrt{2}\,\sqrt{a}\,x}+1}$$

$$y(x) = -\frac{2\,e^{4a\,x^2+2\sqrt{2}\,\sqrt{a}\,x}\sqrt{\sqrt{a}\,\left(c_1 e^{4\sqrt{2}\,\sqrt{a}\,x}+1\right)\left(c_1\left(x\sqrt{a}-\frac{\sqrt{2}}{4}\right)e^{4\sqrt{2}\,\sqrt{a}\,x}+x\sqrt{a}+\frac{\sqrt{2}}{4}\right)e^{-8a\,x^2}e^{-4\sqrt{2}\,\sqrt{a}\,x}}}{c_1 e^{4\sqrt{2}\,\sqrt{a}\,x}+1}$$

✓ Solution by Mathematica

Time used: 23.997 (sec). Leaf size: 95

```
DSolve[y'[x] == (4*a*x - y[x]^2)^2/y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{4ax - \sqrt{2}\sqrt{a}\tanh\left(\frac{\sqrt{2}(2ax - c_1)}{\sqrt{a}}\right)}$$

$$y(x) \to \sqrt{4ax - \sqrt{2}\sqrt{a}\tanh\left(\frac{\sqrt{2}(2ax - c_1)}{\sqrt{a}}\right)}$$

2.67 problem 643

Internal problem ID [8978]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 643.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$\boxed{y' - \frac{x\left(-2+3x\sqrt{x^2+3y}\right)}{3} = 0}$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 22

```
dsolve(diff(y(x),x) = 1/3*x*(-2+3*x*(x^2+3*y(x))^(1/2)),y(x), singsol=all)
```

$$c_1 + \frac{x^3}{2} - \sqrt{x^2+3y(x)} = 0$$

✓ Solution by Mathematica

Time used: 0.302 (sec). Leaf size: 31

```
DSolve[y'[x] == (x*(-2 + 3*x*Sqrt[x^2 + 3*y[x]]))/3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}\left(x^6 - 6c_1x^3 - 4x^2 + 9c_1{}^2\right)$$

2.68 problem 644

Internal problem ID [8979]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 644.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' + \frac{x^2\left(ax - 2\sqrt{a\,(a\,x^4 + 8y)}\right)}{2} = 0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 27

```
dsolve(diff(y(x),x) = -1/2*x^2*(a*x-2*(a*(a*x^4+8*y(x)))^(1/2)),y(x), singsol=all)
```

$$c_1 + \frac{4x^3 a}{3} - \sqrt{a\left(x^4 a + 8y\left(x\right)\right)} = 0$$

✓ Solution by Mathematica

Time used: 0.535 (sec). Leaf size: 34

```
DSolve[y'[x] == -1/2*(x^2*(a*x - 2*Sqrt[a*(a*x^4 + 8*y[x])])),y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{1}{72}a\left(16x^6 - 9x^4 - 96c_1x^3 + 144c_1{}^2\right)$$

2.69 problem 645

Internal problem ID [8980]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 645.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y' - (-\ln(y) + x) y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 14

```
dsolve(diff(y(x),x) = (-ln(y(x))+x)*y(x),y(x), singsol=all)
```

$$y(x) = e^{e^{-x} c_1 - 1 + x}$$

✓ Solution by Mathematica

Time used: 0.28 (sec). Leaf size: 20

```
DSolve[y'[x] == (x - Log[y[x]])*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x - e^{-x + c_1} - 1}$$

2.70 problem 646

Internal problem ID [8981]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 646.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' - \frac{x^3 + x^2 + 2\sqrt{x^3 - 6y}}{2(x+1)} = 0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 23

```
dsolve(diff(y(x),x) = 1/2*(x^3+x^2+2*(x^3-6*y(x))^(1/2))/(x+1),y(x), singsol=all)
```

$$c_1 - 3\ln(x+1) - \sqrt{x^3 - 6y(x)} = 0$$

✓ Solution by Mathematica

Time used: 0.452 (sec). Leaf size: 35

```
DSolve[y'[x] == (x^2/2 + x^3/2 + Sqrt[x^3 - 6*y[x]])/(1 + x),y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{6}\left(x^3 - 9\log^2(x+1) + 18c_1\log(x+1) - 9{c_1}^2\right)$$

2.71 problem 647

Internal problem ID [8982]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 647.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y' - \frac{\left(y^2 a + b\,x^2\right)^2 x}{a^{\frac{5}{2}} y} = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 460

```
dsolve(diff(y(x),x) = (a*y(x)^2+b*x^2)^2*x/a^(5/2)/y(x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-\left(\left(b\,x^2 - \sqrt{-\frac{b}{a^{\frac{3}{2}}}}\,a^{\frac{3}{2}}\right) \mathrm{e}^{\frac{x^2\left(-2\sqrt{-\frac{b}{a^{\frac{3}{2}}}}\,a^{\frac{3}{2}} + b\,x^2\right)}{2a^{\frac{3}{2}}}} + c_1\left(\sqrt{-\frac{b}{a^{\frac{3}{2}}}}\,a^{\frac{3}{2}} + b\,x^2\right) \mathrm{e}^{\frac{x^2\left(2\sqrt{-\frac{b}{a^{\frac{3}{2}}}}\,a^{\frac{3}{2}} + b\,x^2\right)}{2a^{\frac{3}{2}}}}\right)\left(c_1 \mathrm{e}^{\frac{x^2\left(2\sqrt{-\cdots}\right.}{2\cdots}}\right.}}{a\left(c_1 \mathrm{e}^{\frac{x^2\left(2\sqrt{-\frac{b}{a^{\frac{3}{2}}}}\,a^{\frac{3}{2}} + b\,x^2\right)}{2a^{\frac{3}{2}}}} + \mathrm{e}^{\frac{x^2\left(-2\sqrt{-\frac{b}{a^{\frac{3}{2}}}}\,a^{\frac{3}{2}} + b\,x^2\right)}{2a^{\frac{3}{2}}}}\right)}$$

$$y(x) = -\frac{\sqrt{-\left(\left(b\,x^2 - \sqrt{-\frac{b}{a^{\frac{3}{2}}}}\,a^{\frac{3}{2}}\right) \mathrm{e}^{\frac{x^2\left(-2\sqrt{-\frac{b}{a^{\frac{3}{2}}}}\,a^{\frac{3}{2}} + b\,x^2\right)}{2a^{\frac{3}{2}}}} + c_1\left(\sqrt{-\frac{b}{a^{\frac{3}{2}}}}\,a^{\frac{3}{2}} + b\,x^2\right) \mathrm{e}^{\frac{x^2\left(2\sqrt{-\frac{b}{a^{\frac{3}{2}}}}\,a^{\frac{3}{2}} + b\,x^2\right)}{2a^{\frac{3}{2}}}}\right)\left(c_1 \mathrm{e}^{\frac{x^2\left(2\sqrt{\cdots}\right.}{\cdots}}\right.}}{a\left(c_1 \mathrm{e}^{\frac{x^2\left(2\sqrt{-\frac{b}{a^{\frac{3}{2}}}}\,a^{\frac{3}{2}} + b\,x^2\right)}{2a^{\frac{3}{2}}}} + \mathrm{e}^{\frac{x^2\left(-2\sqrt{-\frac{b}{a^{\frac{3}{2}}}}\,a^{\frac{3}{2}} + b\,x^2\right)}{2a^{\frac{3}{2}}}}\right)}$$

✓ Solution by Mathematica

Time used: 14.816 (sec). Leaf size: 117

```
DSolve[y'[x] == (x*(b*x^2 + a*y[x]^2)^2)/(a^(5/2)*y[x]),y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to -\sqrt{\frac{-bx^2 + a^{3/4}\sqrt{b}\tan\left(\frac{a^{3/2}bx^2+2c_1}{a^{9/4}\sqrt{b}}\right)}{a}}$$
$$y(x) \to \sqrt{\frac{-bx^2 + a^{3/4}\sqrt{b}\tan\left(\frac{a^{3/2}bx^2+2c_1}{a^{9/4}\sqrt{b}}\right)}{a}}$$

2.72 problem 648

Internal problem ID [8983]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 648.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' + \frac{x^3\left(\sqrt{a}\,x + \sqrt{a} - 2\sqrt{a\,x^4 + 8y}\right)\sqrt{a}}{2x+2} = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 49

```
dsolve(diff(y(x),x) = -1/2*x^3*(a^(1/2)*x+a^(1/2)-2*(a*x^4+8*y(x))^(1/2))*a^(1/2)/(x+1),y(x)
```

$$-\sqrt{x^4a + 8y(x)} - 4\sqrt{a}\,\ln(x+1) + \frac{2(2x^3 - 3x^2 + 6x)\sqrt{a}}{3} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.823 (sec). Leaf size: 96

```
DSolve[y'[x] == -1/2*(Sqrt[a]*x^3*(Sqrt[a] + Sqrt[a]*x - 2*Sqrt[a*x^4 + 8*y[x]]))/(1 + x),y[
```

$$y(x) \to \frac{1}{72}a\left(16x^6 - 48x^5 + 123x^4 - 96c_1x^3 + 72(-1+2c_1)x^2 - 48\left(2x^3 - 3x^2 + 6x + 9 - 6c_1\right)\log(x+1) + 144\log^2(x+1) - 144(-3+2c_1)x + 36(3-2c_1)^2\right)$$

2.73 problem 649

Internal problem ID [8984]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 649.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' - x\sqrt{x^2 - 2x + 1 + 8y} = -\frac{x}{4} + \frac{1}{4}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 27

```
dsolve(diff(y(x),x) = -1/4*x+1/4+x*(x^2-2*x+1+8*y(x))^(1/2),y(x), singsol=all)
```

$$c_1 + \frac{x^2}{8} + \frac{17}{128} - \frac{\sqrt{x^2 - 2x + 1 + 8y(x)}}{16} = 0$$

✓ Solution by Mathematica
Time used: 0.569 (sec). Leaf size: 36

```
DSolve[y'[x] == 1/4 - x/4 + x*Sqrt[1 - 2*x + x^2 + 8*y[x]],y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{1}{8}\left(4x^4 - (1 + 16c_1)x^2 + 2x - 1 + 16{c_1}^2\right)$$

2.74 problem 650

Internal problem ID [8985]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 650.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$\boxed{y' - x\sqrt{x^2 + 2ax + a^2 + 4y} = -\frac{x}{2} - \frac{a}{2}}$$

✓ Solution by Maple
Time used: 0.109 (sec). Leaf size: 35

```
dsolve(diff(y(x),x) = -1/2*x-1/2*a+x*(x^2+2*a*x+a^2+4*y(x))^(1/2),y(x), singsol=all)
```

$$c_1 + \frac{a^2}{4} + \frac{x^2}{4} + \frac{1}{16} - \frac{\sqrt{x^2 + 2ax + a^2 + 4y(x)}}{4} = 0$$

✓ Solution by Mathematica
Time used: 0.658 (sec). Leaf size: 39

```
DSolve[y'[x] == -1/2*a - x/2 + x*Sqrt[a^2 + 2*a*x + x^2 + 4*y[x]],y[x],x,IncludeSingularSolu
```

$$y(x) \to \frac{1}{4}\left(-a^2 - 2ax + x^4 - (1 + 4c_1)x^2 + 4{c_1}^2\right)$$

2.75 problem 651

Internal problem ID [8986]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 651.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y' - \frac{(\ln(y) + x^2)\, y}{x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 10

```
dsolve(diff(y(x),x) = (ln(y(x))+x^2)*y(x)/x,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x(x+c_1)}$$

✓ Solution by Mathematica

Time used: 0.236 (sec). Leaf size: 15

```
DSolve[y'[x] == ((x^2 + Log[y[x]])*y[x])/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x(x+2c_1)}$$

2.76 problem 652

Internal problem ID [8987]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 652.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$y' - \frac{2a + x\sqrt{-y^2 + 4ax}}{y} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x) = (2*a+x*(-y(x)^2+4*a*x)^(1/2))/y(x),y(x), singsol=all)
```

$$-\sqrt{4ax - y(x)^2} - \frac{x^2}{2} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 6.547 (sec). Leaf size: 161

```
DSolve[y'[x] == (2*a + x*Sqrt[4*a*x - y[x]^2])/y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{256a^4x(16a - x^3) + 32a^2e^{c_1}x^2 - e^{2c_1}}}{32a^2}$$
$$y(x) \to \frac{\sqrt{256a^4x(16a - x^3) + 32a^2e^{c_1}x^2 - e^{2c_1}}}{32a^2}$$
$$y(x) \to -\frac{\sqrt{a^4x(16a - x^3)}}{2a^2}$$
$$y(x) \to \frac{\sqrt{a^4x(16a - x^3)}}{2a^2}$$

2.77 problem 653

Internal problem ID [8988]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 653.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' - x\sqrt{x^2 - 4x + 4y} = -\frac{x}{2} + 1$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 24

```
dsolve(diff(y(x),x) = -1/2*x+1+x*(x^2-4*x+4*y(x))^(1/2),y(x), singsol=all)
```

$$c_1 + x^2 + \frac{1}{4} - \sqrt{x^2 - 4x + 4y(x)} = 0$$

✓ Solution by Mathematica

Time used: 0.519 (sec). Leaf size: 31

```
DSolve[y'[x] == 1 - x/2 + x*Sqrt[-4*x + x^2 + 4*y[x]],y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{x^4}{4} - \frac{1}{4}(1 + 4c_1)x^2 + x + c_1{}^2$$

2.78 problem 654

Internal problem ID [8989]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 654.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' + \frac{2x^2 + 2x - 3\sqrt{x^2 + 3y}}{3x + 3} = 0$$

✓ Solution by Maple
Time used: 0.11 (sec). Leaf size: 23

```
dsolve(diff(y(x),x) = -1/3*(2*x^2+2*x-3*(x^2+3*y(x))^(1/2))/(x+1),y(x), singsol=all)
```

$$c_1 + \frac{3\ln(x+1)}{2} - \sqrt{x^2 + 3y(x)} = 0$$

✓ Solution by Mathematica
Time used: 0.453 (sec). Leaf size: 37

```
DSolve[y'[x] == ((-2*x)/3 - (2*x^2)/3 + Sqrt[x^2 + 3*y[x]])/(1 + x),y[x],x,IncludeSingularSo
```

$$y(x) \to \frac{1}{12}\left(-4x^2 + 9\log^2(x+1) - 18c_1\log(x+1) + 9c_1{}^2\right)$$

2.79 problem 655

Internal problem ID [8990]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 655.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], [_Abel, '2nd type', 'c`

$$y' - \frac{y^3 e^{-\frac{4x}{3}}}{y\,e^{-\frac{2x}{3}} + 1} = 0$$

✓ Solution by Maple

Time used: 0.563 (sec). Leaf size: 64

```
dsolve(diff(y(x),x) = y(x)^3/(y(x)*exp(-2/3*x)+1)*exp(-4/3*x),y(x), singsol=all)
```

$$x + \frac{3\sqrt{7}\,\operatorname{arctanh}\left(\frac{3y(x)\sqrt{7}\,e^{-\frac{2x}{3}}}{7} - \frac{\sqrt{7}}{7}\right)}{14} - \frac{3\ln\left(3y(x)^2 e^{-\frac{4x}{3}} - 2y(x)\,e^{-\frac{2x}{3}} - 2\right)}{4} + \frac{3\ln\left(y(x)\,e^{-\frac{2x}{3}}\right)}{2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 9.347 (sec). Leaf size: 89

```
DSolve[y'[x] == y[x]^3/(E^((4*x)/3)*(1 + y[x]/E^((2*x)/3))),y[x],x,IncludeSingularSolutions
```

$$\text{Solve}\left[\frac{3}{2}\log(y(x)) + \frac{3}{28}\left(-\left(7+\sqrt{7}\right)\log\left(-\sqrt{7}y(x)+y(x)+2e^{2x/3}\right)+\left(\sqrt{7}-7\right)\log\left(\sqrt{7}y(x)+y(x)+2e^{2x/3}\right)+14\log\left(e^{2x/3}\right)\right) =$$

2.80 problem 656

Internal problem ID [8991]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 656.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y' - \frac{(\ln(y) + x^3)\, y}{x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(diff(y(x),x) = (ln(y(x))+x^3)*y(x)/x,y(x), singsol=all)
```

$$y(x) = e^{\frac{x(x^2+2c_1)}{2}}$$

✓ Solution by Mathematica

Time used: 0.25 (sec). Leaf size: 20

```
DSolve[y'[x] == ((x^3 + Log[y[x]])*y[x])/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{x^3}{2}+3c_1x}$$

2.81 problem 657

Internal problem ID [8992]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 657.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, ‘_with_symmetry_[F(x),G(x)]‘]]`

$$y' - x^2\sqrt{x^2 - 2x + 1 + 8y} = -\frac{x}{4} + \frac{1}{4}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 26

```
dsolve(diff(y(x),x) = -1/4*x+1/4+x^2*(x^2-2*x+1+8*y(x))^(1/2),y(x), singsol=all)
```

$$c_1 + \frac{4x^3}{3} - \sqrt{x^2 - 2x + 1 + 8y(x)} = 0$$

✓ Solution by Mathematica

Time used: 0.741 (sec). Leaf size: 37

```
DSolve[y'[x] == 1/4 - x/4 + x^2*Sqrt[1 - 2*x + x^2 + 8*y[x]],y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{72}\left(16x^6 - 96c_1x^3 - 9x^2 + 18x - 9 + 144c_1^2\right)$$

2.82 problem 658

Internal problem ID [8993]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 658.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$\boxed{y' + \frac{x^2 - 1 - 4\sqrt{x^2 - 2x + 1 + 8y}}{4x + 4} = 0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 28

```
dsolve(diff(y(x),x) = -1/4*(x^2-1-4*(x^2-2*x+1+8*y(x))^(1/2))/(x+1),y(x), singsol=all)
```

$$c_1 + 4\ln(x+1) - \frac{1}{4} - \sqrt{x^2 - 2x + 1 + 8y(x)} = 0$$

✓ Solution by Mathematica

Time used: 1.196 (sec). Leaf size: 46

```
DSolve[y'[x] == (1/4 - x^2/4 + Sqrt[1 - 2*x + x^2 + 8*y[x]])/(1 + x),y[x],x,IncludeSingularS
```

$$y(x) \to \frac{1}{8}\left(-x^2 + 2x - 1 + 16{c_1}^2\right) + 2\log^2\left(\frac{1}{x+1}\right) + 4c_1 \log\left(\frac{1}{x+1}\right)$$

2.83 problem 659

Internal problem ID [8994]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 659.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, ‘_with_symmetry_[F(x),G(x)]‘]]`

$$y' - x\sqrt{a^2x^2 + 2abx + b^2 + 4ya - 4c} = -\frac{ax}{2} - \frac{b}{2}$$

✓ Solution by Maple

Time used: 0.562 (sec). Leaf size: 216

```
dsolve(diff(y(x),x) = -1/2*a*x-1/2*b+x*(a^2*x^2+2*a*b*x+b^2+4*a*y(x)-4*c)^(1/2),y(x), singso
```

$$y(x) = \frac{-a^2x^2 - 2axb - b^2 + 4c}{4a}$$

$$\frac{(1 - 4y(x)\,c_1a + c_1(x^4 - x^2)\,a^2 - 2bac_1x + (-b^2 + 4c)\,c_1)\,\sqrt{a^2x^2 + 2axb + b^2 + 4ay\,(x) - 4c} - a(-1 - 4y}{\left(a\,x^2 - \sqrt{a^2x^2 + 2axb + b^2 + 4ay\,(x) - 4c}\right)(-4ay\,(x) + (x^4 - x^2)\,a^2}$$
$$= 0$$

✓ Solution by Mathematica

Time used: 41.903 (sec). Leaf size: 70

```
DSolve[y'[x] == -1/2*b - (a*x)/2 + x*Sqrt[b^2 - 4*c + 2*a*b*x + a^2*x^2 + 4*a*y[x]],y[x],x,I
```

$$y(x) \to -\frac{a^2x^2 + b^2\left(-\log^2\left(\sinh\left(\frac{a(x^2-2c_1)}{b}\right) - \cosh\left(\frac{a(x^2-2c_1)}{b}\right)\right)\right) + 2abx + b^2 - 4c}{4a}$$

2.84 problem 660

Internal problem ID [8995]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 660.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' - x^2\sqrt{x^2 + 2ax + a^2 + 4y} = -\frac{x}{2} - \frac{a}{2}$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 29

```
dsolve(diff(y(x),x) = -1/2*x-1/2*a+x^2*(x^2+2*a*x+a^2+4*y(x))^(1/2),y(x), singsol=all)
```

$$c_1 + \frac{2x^3}{3} - \sqrt{x^2 + 2ax + a^2 + 4y(x)} = 0$$

✓ Solution by Mathematica

Time used: 0.439 (sec). Leaf size: 42

```
DSolve[y'[x] == -1/2*a - x/2 + x^2*Sqrt[a^2 + 2*a*x + x^2 + 4*y[x]],y[x],x,IncludeSingularSo
```

$$y(x) \to \frac{1}{36}\left(-9a^2 - 18ax + 4x^6 - 24c_1x^3 - 9x^2 + 36c_1{}^2\right)$$

2.85 problem 661

Internal problem ID [8996]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 661.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' - x^2\sqrt{a^2x^2 + 2abx + b^2 + 4ya - 4c} = -\frac{ax}{2} - \frac{b}{2}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 39

```
dsolve(diff(y(x),x) = -1/2*a*x-1/2*b+x^2*(a^2*x^2+2*a*b*x+b^2+4*a*y(x)-4*c)^(1/2),y(x), sing
```

$$c_1 + \frac{2x^3a}{3} - \sqrt{a^2x^2 + 2axb + b^2 + 4ay(x) - 4c} = 0$$

✓ Solution by Mathematica

Time used: 41.169 (sec). Leaf size: 76

```
DSolve[y'[x] == -1/2*b - (a*x)/2 + x^2*Sqrt[b^2 - 4*c + 2*a*b*x + a^2*x^2 + 4*a*y[x]],y[x],x
```

$$y(x) \to -\frac{a^2x^2 + b^2\left(-\log^2\left(\sinh\left(\frac{2a(x^3-3c_1)}{3b}\right) - \cosh\left(\frac{2a(x^3-3c_1)}{3b}\right)\right)\right) + 2abx + b^2 - 4c}{4a}$$

2.86 problem 662

Internal problem ID [8997]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 662.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$\boxed{y' - x^2\sqrt{x^2 + 2x + 1 - 4y} = \frac{x}{2} + \frac{1}{2}}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 26

```
dsolve(diff(y(x),x) = 1/2*x+1/2+x^2*(x^2+2*x+1-4*y(x))^(1/2),y(x), singsol=all)
```

$$c_1 - \frac{2x^3}{3} - \sqrt{x^2 + 2x + 1 - 4y(x)} = 0$$

✓ Solution by Mathematica

Time used: 0.723 (sec). Leaf size: 37

```
DSolve[y'[x] == 1/2 + x/2 + x^2*Sqrt[1 + 2*x + x^2 - 4*y[x]],y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{36}\left(-4x^6 + 24c_1x^3 + 9x^2 + 18x + 9 - 36{c_1}^2\right)$$

2.87 problem 663

Internal problem ID [8998]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 663.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `['y=_G(x,y')']`

$$y' - \frac{2a + x^2\sqrt{-y^2 + 4ax}}{y} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x) = (2*a+x^2*(-y(x)^2+4*a*x)^(1/2))/y(x),y(x), singsol=all)
```

$$-\sqrt{4ax - y(x)^2} - \frac{x^3}{3} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 5.05 (sec). Leaf size: 161

```
DSolve[y'[x] == (2*a + x^2*Sqrt[4*a*x - y[x]^2])/y[x],y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to -\frac{\sqrt{4096a^6x(36a - x^5) + 128a^3e^{c_1}x^3 - e^{2c_1}}}{192a^3}$$
$$y(x) \to \frac{\sqrt{4096a^6x(36a - x^5) + 128a^3e^{c_1}x^3 - e^{2c_1}}}{192a^3}$$
$$y(x) \to -\frac{\sqrt{a^6x(36a - x^5)}}{3a^3}$$
$$y(x) \to \frac{\sqrt{a^6x(36a - x^5)}}{3a^3}$$

2.88 problem 664

Internal problem ID [8999]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 664.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$\boxed{y' - x^2\sqrt{x^2 - 4x + 4y} = -\frac{x}{2} + 1}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 25

```
dsolve(diff(y(x),x) = -1/2*x+1+x^2*(x^2-4*x+4*y(x))^(1/2),y(x), singsol=all)
```

$$c_1 + \frac{2x^3}{3} - \sqrt{x^2 - 4x + 4y(x)} = 0$$

✓ Solution by Mathematica

Time used: 0.321 (sec). Leaf size: 34

```
DSolve[y'[x] == 1 - x/2 + x^2*Sqrt[-4*x + x^2 + 4*y[x]],y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{x^6}{9} - \frac{2c_1x^3}{3} - \frac{x^2}{4} + x + c_1{}^2$$

2.89 problem 665

Internal problem ID [9000]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 665.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' + \frac{\left(\sqrt{a}\, x^4 + x^3\sqrt{a} - 2\sqrt{a\, x^4 + 8y}\right)\sqrt{a}}{2x+2} = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 30

```
dsolve(diff(y(x),x) = -1/2*(a^(1/2)*x^4+a^(1/2)*x^3-2*(a*x^4+8*y(x))^(1/2))*a^(1/2)/(x+1),y(
```

$$4\sqrt{a}\,\ln\left(x+1\right) - \sqrt{x^4 a + 8y\left(x\right)} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.686 (sec). Leaf size: 39

```
DSolve[y'[x] == -1/2*(Sqrt[a]*(Sqrt[a]*x^3 + Sqrt[a]*x^4 - 2*Sqrt[a*x^4 + 8*y[x]]))/(1 + x),
```

$$y(x) \to -\frac{ax^4}{8} + 2a\log^2(x+1) - 4ac_1\log(x+1) + 2ac_1{}^2$$

2.90 problem 666

Internal problem ID [9001]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 666.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y' - \left(-\ln(y) + 1 + x^2 + x^3\right) y = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 24

```
dsolve(diff(y(x),x) = (-ln(y(x))+1+x^2+x^3)*y(x),y(x), singsol=all)
```

$$y(x) = e^{e^{-x}c_1 + x^3 - 2x^2 + 4x - 3}$$

✓ Solution by Mathematica

Time used: 0.411 (sec). Leaf size: 29

```
DSolve[y'[x] == (1 + x^2 + x^3 - Log[y[x]])*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x^3 - 2x^2 + 4x - c_1 e^{-x} - 3}$$

2.91 problem 667

Internal problem ID [9002]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 667.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], [_Abel, '2nd type', 'c`

$$y' - \frac{y^3 e^{-2bx}}{y\, e^{-bx} + 1} = 0$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 95

```
dsolve(diff(y(x),x) = y(x)^3/(y(x)*exp(-b*x)+1)*exp(-2*b*x),y(x), singsol=all)
```

$$-\frac{-2 \ln \left(y(x)\, e^{-bx}\right) \sqrt{b\,(4+b)} + \left(-2bx + \ln \left(-b y(x)\, e^{-bx} + y(x)^2\, e^{-2bx} - b\right) + 2c_1\right) \sqrt{b\,(4+b)} + 2b \operatorname{arcta}}{2\sqrt{b\,(4+b)}}$$
$$= 0$$

✓ Solution by Mathematica

Time used: 2.618 (sec). Leaf size: 95

```
DSolve[y'[x] == y[x]^3/(E^(2*b*x)*(1 + y[x]/E^(b*x))),y[x],x,IncludeSingularSolutions -> Tru
```

$$\text{Solve}\left[\frac{2\sqrt{\frac{b}{b+4}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{b}{b+4}}\left(2e^{bx}+y(x)\right)}{y(x)}\right) - \log\left(be^{bx}\left(e^{bx}+y(x)\right) - y(x)^2\right) + 2\log\left(e^{bx}\right)}{2b}\right.$$
$$\left. + \frac{\log(y(x))}{b} = c_1, y(x)\right]$$

2.92 problem 668

Internal problem ID [9003]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 668.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], [_Abel, '2nd type', 'c`

$$\boxed{y' - \frac{y^3 \mathrm{e}^{-2x}}{y\,\mathrm{e}^{-x} + 1} = 0}$$

✓ Solution by Maple

Time used: 0.906 (sec). Leaf size: 57

```
dsolve(diff(y(x),x) = 1/(y(x)*exp(-x)+1)*y(x)^3*exp(-2*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{RootOf}\left(2\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}\,\left(2\,\mathrm{e}^{-x+\_Z}-1\right)}{5}\right)-5\ln\left(\mathrm{e}^{2\_Z}-\mathrm{e}^{x+\_Z}-\mathrm{e}^{2x}\right)-10c_1+10\_Z+10x\right)}$$

✓ Solution by Mathematica

Time used: 0.545 (sec). Leaf size: 73

```
DSolve[y'[x] == y[x]^3/(E^(2*x)*(1 + y[x]/E^x)),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\log(y(x)) + \frac{1}{10}\left(-\left(5+\sqrt{5}\right)\log\left(-\sqrt{5}y(x)+y(x)+2e^x\right) + \left(\sqrt{5}-5\right)\log\left(\sqrt{5}y(x)+y(x)+2e^x\right)+10\log\left(e^x\right)\right) = c_1, y(x)\right]$$

2.93 problem 669

Internal problem ID [9004]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 669.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, ‘_with_symmetry_[F(x),G(y)]‘]]`

$$y' - \frac{\left(-2y^{\frac{3}{2}} + 3\,\mathrm{e}^{x}\right)^{2}\mathrm{e}^{x}}{4\sqrt{y}} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 116

```
dsolve(diff(y(x),x) = 1/4*(-2*y(x)^(3/2)+3*exp(x))^2*exp(x)/y(x)^(1/2),y(x), singsol=all)
```

$$\frac{\left(3\,\mathrm{e}^{2x-\frac{3\,\mathrm{e}^{x}}{2}-\frac{9\,\mathrm{e}^{2x}}{8}} + 3c_1\mathrm{e}^{2x+\frac{3\,\mathrm{e}^{x}}{2}-\frac{9\,\mathrm{e}^{2x}}{8}} + 2\left(1-y(x)^{\frac{3}{2}}\right)\mathrm{e}^{x-\frac{3\,\mathrm{e}^{x}}{2}-\frac{9\,\mathrm{e}^{2x}}{8}} + 2c_1\left(-1-y(x)^{\frac{3}{2}}\right)\mathrm{e}^{x+\frac{3\,\mathrm{e}^{x}}{2}-\frac{9\,\mathrm{e}^{2x}}{8}}\right)\mathrm{e}^{-x-\frac{3\,\mathrm{e}^{x}}{2}}}{-2y(x)^{\frac{3}{2}} + 3\,\mathrm{e}^{x} - 2} = 0$$

✓ Solution by Mathematica

Time used: 60.755 (sec). Leaf size: 222

```
DSolve[y'[x] == (E^x*(3*E^x - 2*y[x]^(3/2))^2)/(4*Sqrt[y[x]]),y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{\left(-e^{3e^{x}} + \frac{3}{2}e^{x+3e^{x}} + \frac{3}{2}e^{x+3c_1} + e^{3c_1}\right)^{2/3}}{\sqrt[3]{\left(e^{3e^{x}} + e^{3c_1}\right)^{2}}}$$

$$y(x) \to -\frac{\sqrt[3]{-1}\left(-e^{3e^{x}} + \frac{3}{2}e^{x+3e^{x}} + \frac{3}{2}e^{x+3c_1} + e^{3c_1}\right)^{2/3}}{\sqrt[3]{\left(e^{3e^{x}} + e^{3c_1}\right)^{2}}}$$

$$y(x) \to \frac{\left(-\frac{1}{2}\right)^{2/3}\left(-2e^{3e^{x}} + 3e^{x+3e^{x}} + 3e^{x+3c_1} + 2e^{3c_1}\right)^{2/3}}{\sqrt[3]{\left(e^{3e^{x}} + e^{3c_1}\right)^{2}}}$$

2.94 problem 670

Internal problem ID [9005]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 670.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, ‘_with_symmetry_[F(x),G(y)]‘]]`

$$y' - \frac{ix\left(i - 2\sqrt{-x^2 + 4\ln(a) + 4\ln(y)}\right)y}{2} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 72

```
dsolve(diff(y(x),x) = 1/2*I*x*(I-2*(-x^2+4*ln(a)+4*ln(y(x)))^(1/2))*y(x),y(x), singsol=all)
```

$$-\frac{i\ln(-x^2 + 4\ln(a) + 4\ln(y(x)) + 1)}{4} - \frac{\sqrt{-x^2 + 4\ln(a) + 4\ln(y(x))}}{2} + \frac{\arctan\left(\sqrt{-x^2 + 4\ln(a) + 4\ln(y(x))}\right)}{2} - \frac{ix^2}{2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 9.83 (sec). Leaf size: 86

```
DSolve[y'[x] == (I/2)*x*(I - 2*Sqrt[-x^2 + 4*Log[a] + 4*Log[y[x]]])*y[x],y[x],x,IncludeSingu
```

$$y(x) \to \exp\left(\frac{1}{4}\left(-4\log(a) - W\left(ie^{-x^2-1-4c_1}\right)^2 - 2W\left(ie^{-x^2-1-4c_1}\right) + x^2 - 1\right)\right)$$
$$y(x) \to 0$$
$$y(x) \to \frac{e^{\frac{1}{4}(x^2-1)}}{a}$$

2.95 problem 671

Internal problem ID [9006]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 671.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{(y^2x+1)^2}{yx^4} = 0$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 197

```
dsolve(diff(y(x),x) = (x*y(x)^2+1)^2/y(x)/x^4,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{2}\,e^{\frac{\sqrt{2}\,x+1}{x^2}}\sqrt{-\left(e^{\frac{2\sqrt{2}}{x}}+c_1\right)x\left(\left(2-\sqrt{2}\,x\right)e^{\frac{2\sqrt{2}}{x}}+c_1\left(\sqrt{2}\,x+2\right)\right)e^{-\frac{2}{x^2}}e^{-\frac{2\sqrt{2}}{x}}}}{2x\left(e^{\frac{2\sqrt{2}}{x}}+c_1\right)}$$

$$y(x) = \frac{\sqrt{2}\,e^{\frac{\sqrt{2}\,x+1}{x^2}}\sqrt{-\left(e^{\frac{2\sqrt{2}}{x}}+c_1\right)x\left(\left(2-\sqrt{2}\,x\right)e^{\frac{2\sqrt{2}}{x}}+c_1\left(\sqrt{2}\,x+2\right)\right)e^{-\frac{2}{x^2}}e^{-\frac{2\sqrt{2}}{x}}}}{2x\left(e^{\frac{2\sqrt{2}}{x}}+c_1\right)}$$

✓ Solution by Mathematica

Time used: 14.007 (sec). Leaf size: 206

```
DSolve[y'[x] == (1 + x*y[x]^2)^2/(x^4*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{-\sqrt{2}x+\left(\sqrt{2}x-2\right)e^{\frac{2\sqrt{2}(1+c_1x)}{x}}-2}{x}}}{\sqrt{2}\sqrt{1+e^{\frac{2\sqrt{2}(1+c_1x)}{x}}}}$$

$$y(x) \to \frac{\sqrt{\frac{-\sqrt{2}x+\left(\sqrt{2}x-2\right)e^{\frac{2\sqrt{2}(1+c_1x)}{x}}-2}{x}}}{\sqrt{2}\sqrt{1+e^{\frac{2\sqrt{2}(1+c_1x)}{x}}}}$$

$$y(x) \to -\sqrt{-\frac{1}{x}-\frac{1}{\sqrt{2}}}$$

$$y(x) \to \sqrt{-\frac{1}{x}-\frac{1}{\sqrt{2}}}$$

2.96 problem 672

Internal problem ID [9007]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 672.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$y' - \frac{x^2\left(3x + \sqrt{-9x^4 + 4y^3}\right)}{y^2} = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 36

```
dsolve(diff(y(x),x) = x^2*(3*x+(-9*x^4+4*y(x)^3)^(1/2))/y(x)^2,y(x), singsol=all)
```

$$\int_{\_b}^{y(x)} \frac{\_a^2}{\sqrt{-9x^4 + 4\_a^3}} d\_a - \frac{x^3}{3} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 12.374 (sec). Leaf size: 4512

```
DSolve[y'[x] == (x^2*(3*x + Sqrt[-9*x^4 + 4*y[x]^3]))/y[x]^2,y[x],x,IncludeSingularSolutions
```

Too large to display

2.97 problem 673

Internal problem ID [9008]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 673.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - \frac{-\sin(2y) + \cos(2y)\,x^2 + x^2}{2x} = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve(diff(y(x),x) = 1/2*(-sin(2*y(x))+cos(2*y(x))*x^2+x^2)/x,y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{x^3 + 6c_1}{3x}\right)$$

✓ Solution by Mathematica
Time used: 2.164 (sec). Leaf size: 57

```
DSolve[y'[x] == (x^2/2 + (x^2*Cos[2*y[x]])/2 - Sin[2*y[x]]/2)/x,y[x],x,IncludeSingularSoluti
```

$$y(x) \to \arctan\left(\frac{2x^3 + 3c_1}{6x}\right)$$
$$y(x) \to -\frac{1}{2}\pi\sqrt{\frac{1}{x^2}}x$$
$$y(x) \to \frac{1}{2}\pi\sqrt{\frac{1}{x^2}}x$$

2.98 problem 674

Internal problem ID [9009]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 674.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, ‘_with_symmetry_[F(x),G(x)]‘]]`

$$\boxed{y' + \frac{x^2 - x - 2 - 2\sqrt{x^2 - 4x + 4y}}{2x + 2} = 0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 27

```
dsolve(diff(y(x),x) = -1/2*(x^2-x-2-2*(x^2-4*x+4*y(x))^(1/2))/(x+1),y(x), singsol=all)
```

$$c_1 + 2\ln(x+1) - 1 - \sqrt{x^2 - 4x + 4y(x)} = 0$$

✓ Solution by Mathematica

Time used: 0.519 (sec). Leaf size: 32

```
DSolve[y'[x] == (1 + x/2 - x^2/2 + Sqrt[-4*x + x^2 + 4*y[x]])/(1 + x),y[x],x,IncludeSingular
```

$$y(x) \to -\frac{x^2}{4} + x + \log^2(x+1) - 2c_1 \log(x+1) + {c_1}^2$$

2.99 problem 675

Internal problem ID [9010]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 675.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _Riccati]`

$$y' - \frac{y + x^3 a\,\mathrm{e}^x + a\,x^4 + a\,x^3 - xy^2\mathrm{e}^x - y^2x^2 - y^2x}{x} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve(diff(y(x),x) = (y(x)+x^3*a*exp(x)+a*x^4+x^3*a-x*y(x)^2*exp(x)-x^2*y(x)^2-x*y(x)^2)/x,
```

$$y(x) = \tanh\left(\frac{\left((6x-6)\,\mathrm{e}^x + 2x^3 + 3x^2 + 6c_1\right)\sqrt{a}}{6}\right)\sqrt{a}\,x$$

✓ Solution by Mathematica

Time used: 12.255 (sec). Leaf size: 45

```
DSolve[y'[x] == (a*x^3 + a*E^x*x^3 + a*x^4 + y[x] - x*y[x]^2 - E^x*x*y[x]^2 - x^2*y[x]^2)/x,
```

$$y(x) \to \sqrt{a}x \tanh\left(\frac{1}{6}\sqrt{a}(2x^3 + 3x^2 + 6e^x(x-1) + 6c_1)\right)$$

2.100 problem 676

Internal problem ID [9011]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 676.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' - \frac{x + 1 + 2x^6\sqrt{4y x^2 + 1}}{2x^3 (x + 1)} = 0$$

✓ Solution by Maple

Time used: 0.266 (sec). Leaf size: 53

```
dsolve(diff(y(x),x) = 1/2*(x+1+2*x^6*(4*x^2*y(x)+1)^(1/2))/x^3/(x+1),y(x), singsol=all)
```

$$\frac{3x^5 - 4x^4 + 6x^3 + 12\ln(x + 1)x + 6c_1 x - 12x^2 - 6\sqrt{4x^2 y(x) + 1}}{6x} = 0$$

✓ Solution by Mathematica

Time used: 13.892 (sec). Leaf size: 83

```
DSolve[y'[x] == (1/2 + x/2 + x^6*Sqrt[1 + 4*x^2*y[x]])/(x^3*(1 + x)),y[x],x,IncludeSingularS
```

$$y(x) \to \frac{1}{4}\left(-\frac{1}{x^2} + \log^2\left((x+1)^2\left(\cosh\left(-\frac{x^4}{2}+\frac{2x^3}{3}-x^2+2x+2c_1\right)-\sinh\left(-\frac{x^4}{2}+\frac{2x^3}{3}-x^2+2x+2c_1\right)\right)\right)\right)$$

2.101 problem 677

Internal problem ID [9012]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 677.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _Riccati]`

$$y' - \frac{y + x^3 a \ln(x+1) + a\,x^4 + a\,x^3 - x y^2 \ln(x+1) - y^2 x^2 - y^2 x}{x} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 48

```
dsolve(diff(y(x),x) = (y(x)+x^3*a*ln(x+1)+a*x^4+x^3*a-x*y(x)^2*ln(x+1)-x^2*y(x)^2-x*y(x)^2)/
```

$$y(x) = \tanh\left(\frac{\sqrt{a}\,(6\ln(x+1)\,x^2 + 4x^3 + 3x^2 - 6\ln(x+1) + 12c_1 + 6x + 9)}{12}\right)\sqrt{a}\,x$$

✓ Solution by Mathematica

Time used: 11.983 (sec). Leaf size: 51

```
DSolve[y'[x] == (a*x^3 + a*x^4 + a*x^3*Log[1 + x] + y[x] - x*y[x]^2 - x^2*y[x]^2 - x*Log[1 +
```

$$y(x) \to \sqrt{a}x \tanh\left(\frac{1}{12}\sqrt{a}\left(4x^3 + 3x^2 + 6\left(x^2 - 1\right)\log(x+1) + 6x + 12c_1\right)\right)$$

2.102 problem 678

Internal problem ID [9013]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 678.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, ‘_with_symmetry_[F(x),G(x)]‘]]`

$$y' - \frac{x^2\left(x+1+2x\sqrt{x^3-6y}\right)}{2\,(x+1)} = 0$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 37

```
dsolve(diff(y(x),x) = 1/2*x^2*(x+1+2*x*(x^3-6*y(x))^(1/2))/(x+1),y(x), singsol=all)
```

$$c_1 - x^3 + \frac{3x^2}{2} - 3x + 3\ln(x+1) - \frac{1}{2} - \sqrt{x^3 - 6y(x)} = 0$$

✓ Solution by Mathematica

Time used: 4.061 (sec). Leaf size: 99

```
DSolve[y'[x] == (x^2*(1 + x + 2*x*Sqrt[x^3 - 6*y[x]]))/(2*(1 + x)),y[x],x,IncludeSingularSol
```

$$\begin{aligned} y(x) \to \frac{1}{24}&(-4x^6 + 12x^5 - 33x^4 + 4(-1+6c_1)x^3 - 6(-5+6c_1)x^2 \\ &+ 12\left(2x^3 - 3x^2 + 6x + 11 - 6c_1\right)\log(x+1) - 36\log^2(x+1) + 12(-11+6c_1)x \\ &- (11-6c_1)^2) \end{aligned}$$

2.103 problem 679

Internal problem ID [9014]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 679.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _Riccati]`

$$y' - \frac{y + x^3 \ln(x) + x^4 + x^3 + 7xy^2 \ln(x) + 7y^2x^2 + 7y^2x}{x} = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 37

```
dsolve(diff(y(x),x) = (y(x)+x^3*ln(x)+x^4+x^3+7*x*y(x)^2*ln(x)+7*x^2*y(x)^2+7*x*y(x)^2)/x,y(
```

$$y(x) = \frac{\tan\left(\frac{(6x^2 \ln(x)+4x^3+3x^2+12c_1)\sqrt{7}}{12}\right) x\sqrt{7}}{7}$$

✓ Solution by Mathematica
Time used: 0.419 (sec). Leaf size: 44

```
DSolve[y'[x] == (x^3 + x^4 + x^3*Log[x] + y[x] + 7*x*y[x]^2 + 7*x^2*y[x]^2 + 7*x*Log[x]*y[x]
```

$$y(x) \to \frac{x \tan\left(\frac{1}{12}\sqrt{7}(4x^3 + 3x^2 + 6x^2 \log(x) + 12c_1)\right)}{\sqrt{7}}$$

2.104 problem 680

Internal problem ID [9015]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 680.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' - \frac{x^2 + 2x + 1 + 2\sqrt{x^2 + 2x + 1 - 4y}}{2(x+1)} = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 28

```
dsolve(diff(y(x),x) = 1/2*(x^2+2*x+1+2*(x^2+2*x+1-4*y(x))^(1/2))/(x+1),y(x), singsol=all)
```

$$c_1 - 2\ln(x+1) - \frac{1}{2} - \sqrt{x^2 + 2x + 1 - 4y(x)} = 0$$

✓ Solution by Mathematica

Time used: 0.708 (sec). Leaf size: 39

```
DSolve[y'[x] == (1/2 + x + x^2/2 + Sqrt[1 + 2*x + x^2 - 4*y[x]])/(1 + x),y[x],x,IncludeSingu
```

$$y(x) \to \frac{1}{4}\left(x^2 + 2x - 4\log^2(x+1) + 8c_1\log(x+1) + 1 - 4c_1^2\right)$$

2.105 problem 681

Internal problem ID [9016]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 681.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _Riccati]`

$$y' - \frac{y + x^3 b \ln\left(\frac{1}{x}\right) + x^4 b + b\,x^3 + xay^2 \ln\left(\frac{1}{x}\right) + x^2 a y^2 + y^2 a x}{x} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
dsolve(diff(y(x),x) = (y(x)+x^3*b*ln(1/x)+x^4*b+b*x^3+x*a*y(x)^2*ln(1/x)+x^2*a*y(x)^2+a*x*y(
```

$$y(x) = \frac{\tan\left(\frac{\left(6x^2 \ln\left(\frac{1}{x}\right)+4x^3+9x^2+12c_1\right)\sqrt{ab}}{12}\right) x\sqrt{ab}}{a}$$

✓ Solution by Mathematica

Time used: 43.49 (sec). Leaf size: 54

```
DSolve[y'[x] == (b*x^3 + b*x^4 + b*x^3*Log[x^(-1)] + y[x] + a*x*y[x]^2 + a*x^2*y[x]^2 + a*x*
```

$$y(x) \to \frac{\sqrt{b}x \tan\left(\frac{1}{12}\sqrt{a}\sqrt{b}(4x^3 + 9x^2 - 6x^2\log(x) + 12c_1)\right)}{\sqrt{a}}$$

2.106 problem 682

Internal problem ID [9017]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 682.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{y' - \frac{2a}{x\left(-yx + 2y^2ax - 8a^2\right)} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(diff(y(x),x) = 2*a/x/(-x*y(x)+2*a*x*y(x)^2-8*a^2),y(x), singsol=all)
```

$$\frac{\left(-xy(x)^2 + 4a\right) e^{-4ay(x)} + c_1 x}{x} = 0$$

✓ Solution by Mathematica

Time used: 0.251 (sec). Leaf size: 39

```
DSolve[y'[x] == (2*a)/(x*(-8*a^2 - x*y[x] + 2*a*x*y[x]^2)),y[x],x,IncludeSingularSolutions -
```

$$\text{Solve}\left[\frac{y(x)^2 e^{-4ay(x)}}{8a} - \frac{e^{-4ay(x)}}{2x} = c_1, y(x)\right]$$

2.107 problem 683

Internal problem ID [9018]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 683.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$y' - \frac{y(-1 + \ln(x(x+1))\, yx^4 - \ln(x(x+1))\, x^3)}{x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x) = y(x)*(-1+ln(x*(x+1))*y(x)*x^4-ln(x*(x+1))*x^3)/x,y(x), singsol=all)
```

$$y(x) = \frac{1}{x\left((x(x+1))^{\frac{x^3}{3}} c_1 (x+1)^{\frac{1}{3}} e^{-\frac{2}{9}x^3+\frac{1}{6}x^2-\frac{1}{3}x} + 1\right)}$$

✓ Solution by Mathematica

Time used: 1.1 (sec). Leaf size: 77

```
DSolve[y'[x] == (y[x]*(-1 - x^3*Log[x*(1 + x)] + x^4*Log[x*(1 + x)]*y[x]))/x,y[x],x,IncludeS
```

$$y(x) \to \frac{e^{\frac{1}{9}x(2x^2+3)}}{x\left(e^{\frac{1}{9}x(2x^2+3)} + c_1 e^{\frac{x^2}{6}} \sqrt[3]{x+1}(x(x+1))^{\frac{x^3}{3}}\right)}$$

$$y(x) \to 0$$

2.108 problem 684

Internal problem ID [9019]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 684.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, ‘_with_symmetry_[F(x),G(x)*y+H(x)]‘]]`

$$y' - \frac{y + \sqrt{y^2 + x^2}\, x^2}{x} = 0$$

✓ Solution by Maple

Time used: 0.844 (sec). Leaf size: 30

```
dsolve(diff(y(x),x) = (y(x)+(y(x)^2+x^2)^(1/2)*x^2)/x,y(x), singsol=all)
```

$$\ln\left(\sqrt{y(x)^2 + x^2} + y(x)\right) - \frac{x^2}{2} - \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.309 (sec). Leaf size: 36

```
DSolve[y'[x] == (y[x] + x^2*Sqrt[x^2 + y[x]^2])/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} x e^{-\frac{x^2}{2} - c_1}\left(-1 + e^{x^2 + 2c_1}\right)$$

2.109 problem 685

Internal problem ID [9020]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 685.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _Riccati]`

$$y' - \frac{y + \ln\left((x-1)(x+1)\right)x^3 + 7\ln\left((x-1)(x+1)\right)xy^2}{x} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
dsolve(diff(y(x),x) = (y(x)+ln((x+1)*(x-1))*x^3+7*ln((x+1)*(x-1))*x*y(x)^2)/x,y(x), singsol=
```

$$y(x) = \frac{\tan\left(\frac{(x^2\ln(x^2-1)-x^2-\ln(x^2-1)+2c_1+1)\sqrt{7}}{2}\right)x\sqrt{7}}{7}$$

✓ Solution by Mathematica

Time used: 1.585 (sec). Leaf size: 62

```
DSolve[y'[x] == (x^3*Log[(-1 + x)*(1 + x)] + y[x] + 7*x*Log[(-1 + x)*(1 + x)]*y[x]^2)/x,y[x]
```

$$y(x) \to \frac{x\tan\left(\frac{1}{2}\sqrt{7}(-x^2+x^2\log(x-1)+x^2\log(x+1)-\log(1-x)-\log(x+1)+2c_1)\right)}{\sqrt{7}}$$

2.110 problem 686

Internal problem ID [9021]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 686.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class C'], [_1st_order, '_with_symmetry_`

$$y' - \frac{y^3 x \, \mathrm{e}^{2x^2}}{y \, \mathrm{e}^{x^2} + 1} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
dsolve(diff(y(x),x) = y(x)^3/(y(x)*exp(x^2)+1)*x*exp(2*x^2),y(x), singsol=all)
```

$$y(x) = \left(\cot\left(\mathrm{RootOf}\left(-2x^2 - \ln(2) + \ln(5) - \ln\left(\sec(\_Z)^2\right) + 2\ln(-1+\tan(\_Z)) + 6c_1 - 2\_Z\right)\right) - 1\right) \mathrm{e}^{-x^2}$$

✓ Solution by Mathematica

Time used: 7.286 (sec). Leaf size: 68

```
DSolve[y'[x] == (E^(2*x^2)*x*y[x]^3)/(1 + E^x^2*y[x]),y[x],x,IncludeSingularSolutions -> Tru
```

$$\mathrm{Solve}\left[\log(y(x)) - 2y(x)^2 \left(\frac{\log\left(e^{2x^2} y(x)^2 + 2e^{x^2} y(x) + 2\right)}{4y(x)^2} - \frac{\arctan\left(e^{x^2} y(x) + 1\right)}{2y(x)^2}\right) = c_1, y(x)\right]$$

2.111 problem 687

Internal problem ID [9022]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 687.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _Riccati]`

$$
\boxed{y' - \frac{y - \ln\left(\frac{x+1}{x-1}\right) x^3 + \ln\left(\frac{x+1}{x-1}\right) x y^2}{x} = 0}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve(diff(y(x),x) = (y(x)-ln((x+1)/(x-1))*x^3+ln((x+1)/(x-1))*x*y(x)^2)/x,y(x), singsol=al
```

$$
y(x) = -\tanh\left(\frac{x^2 \ln\left(\frac{x+1}{x-1}\right)}{2} - \frac{\ln\left(\frac{x+1}{x-1}\right)}{2} + c_1 + x - 1\right) x
$$

✓ Solution by Mathematica

Time used: 2.491 (sec). Leaf size: 123

```
DSolve[y'[x] == (-(x^3*Log[(1 + x)/(-1 + x)]) + y[x] + x*Log[(1 + x)/(-1 + x)]*y[x]^2)/x,y[x
```

$$
y(x) \to \frac{x\left((x-1)^{x^2} - (x+1)^{x^2} e^{2(x+c_1)} + x\left((x-1)^{x^2} + (x+1)^{x^2} e^{2(x+c_1)}\right)\right)}{(x-1)^{x^2} + (x+1)^{x^2} e^{2(x+c_1)} + x\left((x-1)^{x^2} - (x+1)^{x^2} e^{2(x+c_1)}\right)}
$$

$$
y(x) \to -x
$$

$$
y(x) \to x
$$

2.112 problem 688

Internal problem ID [9023]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 688.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _Riccati]`

$$y' - \frac{y + e^{\frac{x+1}{x-1}}x^3 + e^{\frac{x+1}{x-1}}xy^2}{x} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve(diff(y(x),x) = (y(x)+exp((x+1)/(x-1))*x^3+exp((x+1)/(x-1))*x*y(x)^2)/x,y(x), singsol=
```

$$y(x) = \tan\left(\frac{(x^2+2x-3)\,e^{\frac{x+1}{x-1}}}{2} + 4\,e\ \text{expIntegral}_1\left(-\frac{2}{x-1}\right) + c_1\right)x$$

✓ Solution by Mathematica

Time used: 5.536 (sec). Leaf size: 45

```
DSolve[y'[x] == (E^((1 + x)/(-1 + x))*x^3 + y[x] + E^((1 + x)/(-1 + x))*x*y[x]^2)/x,y[x],x,I
```

$$y(x) \to x\tan\left(-4e\,\text{ExpIntegralEi}\left(\frac{2}{x-1}\right) + \frac{1}{2}e^{\frac{x+1}{x-1}}(x^2+2x-3) + c_1\right)$$

2.113 problem 689

Internal problem ID [9024]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 689.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _Riccati]`

$$\boxed{y' - \frac{yx - y - e^{x+1}x^3 + e^{x+1}xy^2}{(x-1)\,x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(diff(y(x),x) = (x*y(x)-y(x)-exp(x+1)*x^3+exp(x+1)*x*y(x)^2)/(x-1)/x,y(x), singsol=all
```

$$y(x) = -\tanh\left(e^{x+1} - e^2 \operatorname{expIntegral}_1(1-x) + c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.853 (sec). Leaf size: 67

```
DSolve[y'[x] == (-(E^(1 + x)*x^3) - y[x] + x*y[x] + E^(1 + x)*x*y[x]^2)/((-1 + x)*x),y[x],x,
```

$$y(x) \to \frac{x - xe^{2\left(e^2 \text{ExpIntegralEi}(x-1)+e^{x+1}+c_1\right)}}{1 + e^{2\left(e^2 \text{ExpIntegralEi}(x-1)+e^{x+1}+c_1\right)}}$$
$$y(x) \to -x$$
$$y(x) \to x$$

2.114 problem 690

Internal problem ID [9025]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 690.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' - \frac{-x^2 + 1 + 4\sqrt{x^2 - 2x + 1 + 8y}\, x^3}{4\,(x+1)} = 0$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 40

```
dsolve(diff(y(x),x) = 1/4*(-x^2+1+4*x^3*(x^2-2*x+1+8*y(x))^(1/2))/(x+1),y(x), singsol=all)
```

$$c_1 + \frac{4x^3}{3} - 2x^2 - 4\ln(x+1) + 4x - \sqrt{x^2 - 2x + 1 + 8y(x)} = 0$$

✓ Solution by Mathematica

Time used: 1.35 (sec). Leaf size: 108

```
DSolve[y'[x] == (1/4 - x^2/4 + x^3*Sqrt[1 - 2*x + x^2 + 8*y[x]])/(1 + x),y[x],x,IncludeSingu
```

$$y(x) \to \frac{2x^6}{9} - \frac{2x^5}{3} + \frac{11x^4}{6} - \frac{2}{3}(3+2c_1)x^3 + \left(\frac{15}{8} + 2c_1\right)x^2 + \left(\frac{4x^3}{3} - 2x^2 + 4x - 4c_1\right)\log\left(\frac{1}{x+1}\right) + 2\log^2\left(\frac{1}{x+1}\right) + \left(\frac{1}{4} - 4c_1\right)x - \frac{1}{8} + 2c_1{}^2$$

2.115 problem 691

Internal problem ID [9026]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 691.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - \frac{-\sin(2y) + \cos(2y)\, x^3 + x^3}{2x} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(y(x),x) = 1/2*(-sin(2*y(x))+cos(2*y(x))*x^3+x^3)/x,y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{x^4 + 8c_1}{4x}\right)$$

✓ Solution by Mathematica
Time used: 3.896 (sec). Leaf size: 55

```
DSolve[y'[x] == (x^3/2 + (x^3*Cos[2*y[x]])/2 - Sin[2*y[x]]/2)/x,y[x],x,IncludeSingularSoluti
```

$$y(x) \to \arctan\left(\frac{x^4 + 2c_1}{4x}\right)$$
$$y(x) \to -\frac{1}{2}\pi\sqrt{\frac{1}{x^2}}x$$
$$y(x) \to \frac{1}{2}\pi\sqrt{\frac{1}{x^2}}x$$

2.116 problem 692

Internal problem ID [9027]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 692.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, ‘_with_symmetry_[F(x),G(x)*y+H(x)]‘]]`

$$\boxed{y' - \frac{y + x^3\sqrt{y^2 + x^2}}{x} = 0}$$

✓ Solution by Maple
Time used: 0.875 (sec). Leaf size: 30

```
dsolve(diff(y(x),x) = (y(x)+x^3*(y(x)^2+x^2)^(1/2))/x,y(x), singsol=all)
```

$$\ln\left(\sqrt{y(x)^2 + x^2} + y(x)\right) - \frac{x^3}{3} - \ln(x) - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.309 (sec). Leaf size: 40

```
DSolve[y'[x] == (y[x] + x^3*Sqrt[x^2 + y[x]^2])/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} x e^{-\frac{x^3}{3} - c_1} \left(-1 + e^{\frac{2x^3}{3} + 2c_1}\right)$$

2.117 problem 693

Internal problem ID [9028]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 693.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Abel]`

$$y' - \left(1 + y^2 \mathrm{e}^{-2bx} + \mathrm{e}^{-3bx} y^3\right) \mathrm{e}^{bx} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 33

```
dsolve(diff(y(x),x) = (1+y(x)^2*exp(-2*b*x)+y(x)^3*exp(-3*b*x))*exp(b*x),y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(-x + \int^{\_Z} \frac{1}{\_a^3 + \_a^2 - \_ab + 1} d\_a + c_1\right) \mathrm{e}^{bx}$$

✓ Solution by Mathematica

Time used: 2.849 (sec). Leaf size: 1121

```
DSolve[y'[x] == E^(b*x)*(1 + y[x]^2/E^(2*b*x) + y[x]^3/E^(3*b*x)),y[x],x,IncludeSingularSolu
```

$$\text{Solve}\left[\frac{1}{9}\text{RootSum}\left[-81b^2\#1^9 - 522b\#1^9 - 841\#1^9 - 243b^2\#1^6 - 1566b\#1^6\right.\right.$$

$$-2523\#1^6 + 729b^3\#1^3 + 486b^2\#1^3 - 1323b\#1^3 - 2496\#1^3 - 81b^2 - 522b$$

$$-841\&, \frac{81b^2 \log\left(\frac{3e^{-2bx}y(x)+e^{-bx}}{\sqrt[3]{(9b+29)e^{-3bx}}} - \#1\right)\#1^6 + 522b \log\left(\frac{3e^{-2bx}y(x)+e^{-bx}}{\sqrt[3]{(9b+29)e^{-3bx}}} - \#1\right)\#1^6 + 841 \log\left(\frac{3}{\sqrt[3]{}}\right.}{}$$

2.118 problem 694

Internal problem ID [9029]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 694.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y' - \frac{x+1+2\sqrt{4yx^2+1}\,x^3}{2x^3(x+1)} = 0}$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 36

```
dsolve(diff(y(x),x) = 1/2*(x+1+2*(4*x^2*y(x)+1)^(1/2)*x^3)/x^3/(x+1),y(x), singsol=all)
```

$$\frac{-2\ln(x+1)\,x + c_1 x + 2x^2 - \sqrt{4x^2 y(x)+1}}{x} = 0$$

✓ Solution by Mathematica

Time used: 1.237 (sec). Leaf size: 50

```
DSolve[y'[x] == (1/2 + x/2 + x^3*Sqrt[1 + 4*x^2*y[x]])/(x^3*(1 + x)),y[x],x,IncludeSingularS
```

$$y(x) \to x^2 - \frac{1}{4x^2} + \frac{1}{4}\log^2\left((x+1)^2\right) - 2c_1 x + (-x+c_1)\log\left((x+1)^2\right) + c_1{}^2$$

2.119 problem 695

Internal problem ID [9030]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 695.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _Riccati]`

$$y' - \frac{y \ln(x-1) + x^4 + x^3 + y^2x^2 + y^2x}{\ln(x-1)\, x} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(x),x) = (y(x)*ln(x-1)+x^4+x^3+x^2*y(x)^2+x*y(x)^2)/ln(x-1)/x,y(x), singsol=all
```

$$y(x) = \tan(-\operatorname{expIntegral}_1(-3\ln(x-1)) - 3\operatorname{expIntegral}_1(-2\ln(x-1)) - 2\operatorname{expIntegral}_1(-\ln(x-1)) + c_1)\, x$$

✓ Solution by Mathematica
Time used: 0.86 (sec). Leaf size: 34

```
DSolve[y'[x] == (x^3 + x^4 + Log[-1 + x]*y[x] + x*y[x]^2 + x^2*y[x]^2)/(x*Log[-1 + x]),y[x],
```

$$y(x) \to x \tan(2\,\text{ExpIntegralEi}(\log(x-1)) + 3\,\text{ExpIntegralEi}(2\log(x-1)) + \text{ExpIntegralEi}(3\log(x-1)) + c_1)$$

2.120 problem 696

Internal problem ID [9031]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 696.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _Riccati]`

$$y' - \frac{y \ln(x-1) + e^{x+1}x^3 + 7\,e^{x+1}xy^2}{\ln(x-1)\,x} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 32

```
dsolve(diff(y(x),x) = (y(x)*ln(x-1)+exp(x+1)*x^3+7*exp(x+1)*x*y(x)^2)/ln(x-1)/x,y(x), singso
```

$$y(x) = \frac{\tan\left(\left(e\left(\int \frac{x\,e^x}{\ln(x-1)}dx\right) + c_1\right)\sqrt{7}\right)x\sqrt{7}}{7}$$

✓ Solution by Mathematica

Time used: 1.53 (sec). Leaf size: 45

```
DSolve[y'[x] == (E^(1 + x)*x^3 + Log[-1 + x]*y[x] + 7*E^(1 + x)*x*y[x]^2)/(x*Log[-1 + x]),y[
```

$$y(x) \to \frac{x \tan\left(\sqrt{7}\left(\int_1^x \frac{e^{K[1]+1}K[1]}{\log(K[1]-1)}dK[1] + c_1\right)\right)}{\sqrt{7}}$$

2.121 problem 697

Internal problem ID [9032]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 697.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Abel]`

$$y' - \left(1 + y^2 \mathrm{e}^{-\frac{4x}{3}} + y^3 \mathrm{e}^{-2x}\right) \mathrm{e}^{\frac{2x}{3}} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 38

```
dsolve(diff(y(x),x) = (1+y(x)^2*exp(-4/3*x)+y(x)^3*exp(-2*x))*exp(2/3*x),y(x), singsol=all)
```

$$y(x) = \mathrm{RootOf}\left(-x + 3\left(\int^{\_Z} \frac{1}{3\_a^3 + 3\_a^2 - 2\_a + 3} d\_a\right) + c_1\right) \mathrm{e}^{\frac{2x}{3}}$$

✓ Solution by Mathematica

Time used: 0.243 (sec). Leaf size: 114

```
DSolve[y'[x] == E^((2*x)/3)*(1 + y[x]^2/E^((4*x)/3) + y[x]^3/E^(2*x)),y[x],x,IncludeSingular
```

$$\mathrm{Solve}\left[-\frac{35}{3}\mathrm{RootSum}\left[-35\#1^3 + 9\sqrt[3]{35}\#1 - 35\&, \frac{\log\left(\frac{3e^{-4x/3}y(x)+e^{-2x/3}}{\sqrt[3]{35}\sqrt[3]{e^{-2x}}} - \#1\right)}{3\sqrt[3]{35} - 35\#1^2}\&\right] = \frac{1}{9}35^{2/3}e^{4x/3}\left(e^{-2x}\right)^{2/3}x + c_1, y(x)\right]$$

2.122 problem 698

Internal problem ID [9033]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 698.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Abel]`

$$\boxed{y' - \left(1 + y^2 \mathrm{e}^{-2x} + y^3 \mathrm{e}^{-3x}\right) \mathrm{e}^x = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 30

```
dsolve(diff(y(x),x) = (1+y(x)^2*exp(-2*x)+y(x)^3*exp(-3*x))*exp(x),y(x), singsol=all)
```

$$y(x) = \mathrm{RootOf}\left(-x + \int^{\_Z} \frac{1}{\_a^3 + \_a^2 - \_a + 1} d\_a + c_1\right) \mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.238 (sec). Leaf size: 108

```
DSolve[y'[x] == E^x*(1 + y[x]^2/E^(2*x) + y[x]^3/E^(3*x)),y[x],x,IncludeSingularSolutions ->
```

$$\mathrm{Solve}\left[-\frac{19}{3}\mathrm{RootSum}\left[-19\#1^3 + 6\sqrt[3]{38}\#1 - 19\&, \frac{\log\left(\frac{3e^{-2x}y(x)+e^{-x}}{\sqrt[3]{38}\sqrt[3]{e^{-3x}}} - \#1\right)}{2\sqrt[3]{38} - 19\#1^2}\&\right] = \frac{1}{9}38^{2/3}e^{2x}\left(e^{-3x}\right)^{2/3}x + c_1, y(x)\right]$$

2.123 problem 699

Internal problem ID [9034]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 699.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' - \frac{x(-2x - 2 + 3x^2\sqrt{x^2 + 3y})}{3(x+1)} = 0$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 36

```
dsolve(diff(y(x),x) = 1/3*x*(-2*x-2+3*x^2*(x^2+3*y(x))^(1/2))/(x+1),y(x), singsol=all)
```

$$c_1 + \frac{x^3}{2} - \frac{3x^2}{4} - \frac{3\ln(x+1)}{2} + \frac{3x}{2} - \sqrt{x^2 + 3y(x)} = 0$$

✓ Solution by Mathematica

Time used: 0.915 (sec). Leaf size: 47

```
DSolve[y'[x] == (x*(-2 - 2*x + 3*x^2*Sqrt[x^2 + 3*y[x]]))/(3*(1 + x)),y[x],x,IncludeSingular
```

$$y(x) \to \frac{1}{3}\left(-x^2 + \frac{1}{16}\left(2x^3 - 3x^2 + 6x + 6\log\left(\frac{1}{x+1}\right) - 6c_1\right)^2\right)$$

2.124 problem 700

Internal problem ID [9035]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 700.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{y' - \frac{1}{x\,(y^2x + 1 + x)\,y} = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 62

```
dsolve(diff(y(x),x) = 1/x/(x*y(x)^2+1+x)/y(x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{x\left(2\,\mathrm{LambertW}\left(\frac{c_1 \mathrm{e}^{-\frac{x-1}{2x}}}{2}\right)x + x - 1\right)}}{x}$$

$$y(x) = -\frac{\sqrt{x\left(2\,\mathrm{LambertW}\left(\frac{c_1 \mathrm{e}^{-\frac{x-1}{2x}}}{2}\right)x + x - 1\right)}}{x}$$

✓ Solution by Mathematica

Time used: 60.147 (sec). Leaf size: 72

```
DSolve[y'[x] == 1/(x*y[x]*(1 + x + x*y[x]^2)),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{2xW\left(c_1 e^{\frac{1}{2}\left(\frac{1}{x}-1\right)}\right) + x - 1}}{\sqrt{x}}$$

$$y(x) \to \frac{\sqrt{2xW\left(c_1 e^{\frac{1}{2}\left(\frac{1}{x}-1\right)}\right) + x - 1}}{\sqrt{x}}$$

2.125 problem 701

Internal problem ID [9036]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 701.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - \frac{2x\,e^{x} - 2x - \ln(x) - 1 + x^{4}\ln(x) + x^{4} - 2yx^{2}\ln(x) - 2yx^{2} + \ln(x)\,y^{2} + y^{2}}{e^{x} - 1} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 71

```
dsolve(diff(y(x),x) = (2*x*exp(x)-2*x-ln(x)-1+x^4*ln(x)+x^4-2*y(x)*x^2*ln(x)-2*x^2*y(x)+y(x)
```

$$y(x) = \frac{-x^{2}e^{2\left(\int \frac{\ln(x)+1}{e^{x}-1}dx\right)} + c_1x^{2} + e^{2\left(\int \frac{\ln(x)+1}{e^{x}-1}dx\right)} + c_1}{-e^{2\left(\int \frac{\ln(x)+1}{e^{x}-1}dx\right)} + c_1}$$

✓ Solution by Mathematica

Time used: 2.447 (sec). Leaf size: 97

```
DSolve[y'[x] == (-1 - 2*x + 2*E^x*x + x^4 - Log[x] + x^4*Log[x] - 2*x^2*y[x] - 2*x^2*Log[x]*
```

$$y(x) \to \frac{\exp\left(\int_1^x \frac{2(\log(K[5])+1)}{-1+e^{K[5]}}dK[5]\right)}{-\int_1^x \frac{\exp\left(\int_1^{K[6]} \frac{2(\log(K[5])+1)}{-1+e^{K[5]}}dK[5]\right)(\log(K[6])+1)}{-1+e^{K[6]}}dK[6] + c_1} + x^2 + 1$$

$$y(x) \to x^2 + 1$$

2.126 problem 702

Internal problem ID [9037]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 702.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _Riccati]`

$$y' - \frac{-y\,e^{x} + yx - x^3\ln(x) - x^3 - xy^2\ln(x) - y^2x}{(-e^{x} + x)\,x} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(diff(y(x),x) = (-y(x)*exp(x)+x*y(x)-x^3*ln(x)-x^3-x*y(x)^2*ln(x)-x*y(x)^2)/(-exp(x)+x
```

$$y(x) = \tan\left(\int \frac{x\ln(x)}{e^{x} - x}dx + \int \frac{x}{e^{x} - x}dx + c_1\right)x$$

✓ Solution by Mathematica

Time used: 5.829 (sec). Leaf size: 37

```
DSolve[y'[x] == (-x^3 - x^3*Log[x] - E^x*y[x] + x*y[x] - x*y[x]^2 - x*Log[x]*y[x]^2)/(x*(-E^
```

$$y(x) \to x\tan\left(\int_1^x \frac{K[1](\log(K[1]) + 1)}{e^{K[1]} - K[1]}dK[1] + c_1\right)$$

2.127 problem 703

Internal problem ID [9038]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 703.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$y' - \frac{y(1 - x + yx^2 \ln(x) + yx^3 - x\ln(x) - x^2)}{(x-1)\,x} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x) = y(x)*(1-x+y(x)*x^2*ln(x)+x^3*y(x)-x*ln(x)-x^2)/(x-1)/x,y(x), singsol=a
```

$$y(x) = \frac{e^{\mathrm{dilog}(x)-x}}{x\left(-\left(\int \frac{e^{\mathrm{dilog}(x)-x}(\ln(x)+x)}{(x-1)^2}dx\right)+c_1\right)(x-1)}$$

✓ Solution by Mathematica
Time used: 1.215 (sec). Leaf size: 168

```
DSolve[y'[x] == (y[x]*(1 - x - x^2 - x*Log[x] + x^3*y[x] + x^2*Log[x]*y[x]))/((-1 + x)*x),y[
```

$$y(x) \to \frac{e^{-\mathrm{PolyLog}(2,x)-x}(1-x)^{-\log(x)-1}}{x\left(-\int_1^x e^{-K[1]-\mathrm{PolyLog}(2,K[1])}(1-K[1])^{-\log(K[1])-2}(-K[1]-\log(K[1]))dK[1]+c_1\right)}$$

$$y(x) \to 0$$

$$y(x) \to -\frac{e^{-\mathrm{PolyLog}(2,x)-x}(1-x)^{-\log(x)-1}}{x\int_1^x e^{-K[1]-\mathrm{PolyLog}(2,K[1])}(1-K[1])^{-\log(K[1])-2}(-K[1]-\log(K[1]))dK[1]}$$

2.128 problem 704

Internal problem ID [9039]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 704.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _Riccati]`

$$y' - \frac{y \ln(x) x - y + 2x^5 b + 2y^2 a x^3}{(x \ln(x) - 1) x} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 38

```
dsolve(diff(y(x),x) = (y(x)*ln(x)*x-y(x)+2*x^5*b+2*x^3*a*y(x)^2)/(x*ln(x)-1)/x,y(x), singsol
```

$$y(x) = \frac{\tan\left(2\sqrt{ab}\left(\int \frac{x^3}{x\ln(x)-1}dx + c_1\right)\right) x\sqrt{ab}}{a}$$

✓ Solution by Mathematica

Time used: 53.989 (sec). Leaf size: 55

```
DSolve[y'[x] == (2*b*x^5 - y[x] + x*Log[x]*y[x] + 2*a*x^3*y[x]^2)/(x*(-1 + x*Log[x])),y[x],x
```

$$y(x) \to \frac{\sqrt{b}x \tan\left(\sqrt{a}\sqrt{b}\left(\int_1^x \frac{2K[1]^3}{K[1]\log(K[1])-1}dK[1] + c_1\right)\right)}{\sqrt{a}}$$

2.129 problem 705

Internal problem ID [9040]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 705.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y' - \frac{(\ln(y) + x + x^3 + x^4)\, y}{x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x) = (ln(y(x))+x+x^3+x^4)*y(x)/x,y(x), singsol=all)
```

$$y(x) = x^x \mathrm{e}^{c_1 x + \frac{1}{2}x^3 + \frac{1}{3}x^4}$$

✓ Solution by Mathematica

Time used: 0.346 (sec). Leaf size: 30

```
DSolve[y'[x] == ((x + x^3 + x^4 + Log[y[x]])*y[x])/x,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to x^x e^{\frac{x^4}{3} + \frac{x^3}{2} + c_1 x}$$

2.130 problem 706

Internal problem ID [9041]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 706.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$y' + \frac{(-\ln(-1+y) + \ln(y+1) + 2\ln(x))\, x(y+1)^2}{8} = 0$$

✓ Solution by Maple

Time used: 5.203 (sec). Leaf size: 103

```
dsolve(diff(y(x),x) = -1/8*(-ln(-1+y(x))+ln(y(x)+1)+2*ln(x))*x*(y(x)+1)^2,y(x), singsol=all)
```

$$y(x) = e^{\operatorname{RootOf}\left(-x^2 e^{\_Z} \ln\left(\frac{e^{\_Z}-2}{x^2}\right) + \_Z x^2 e^{\_Z} + 8\, e^{\_Z} - 16\right)} - 1$$

$$-\left(\int^{y(x)}_{\_b} -\frac{1}{2(\_a+1)\left(-\frac{x^2(\_a+1)\ln(\_a-1)}{2} + \frac{x^2(\_a+1)\ln(\_a+1)}{2} + x^2(\_a+1)\ln(x) + 4\_a - 4\right)} d\_a\right) + \frac{\ln(x)}{8} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 4.545 (sec). Leaf size: 610

```
DSolve[y'[x] == -1/8*(x*(2*Log[x] - Log[-1 + y[x]] + Log[1 + y[x]])*(1 + y[x])^2),y[x],x,Inc
```

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{-2\log(x)x^2+\log(K[2]-1)x^2-\log(K[2]+1)x^2-8}{2\left(2\log(x)x^2-\log(K[2]-1)x^2+\log(K[2]+1)x^2+K[2]\left(2\log(x)x^2-\log(K[2]-1)x^2+\right.\right.}\right.\right.$$

$$-\int_1^x\left(-\frac{K[1](K[2]+1)\left(\frac{1}{K[2]+1}-\frac{1}{K[2]-1}\right)}{2K[2]\log(K[1])K[1]^2+2\log(K[1])K[1]^2-K[2]\log(K[2]-1)K[1]^2-\log(K[2]-1)K[1]^2+K}\right.$$

$$+\left.\frac{1}{2(K[2]+1)}\right)dK[2]+\int_1^x$$

$$-\frac{K[1](2\log(K[1])-\log(y(x)-1)+\log(y(x)+1))(y(x)+}{2\log(K[1])K[1]^2-\log(y(x)-1)K[1]^2+\log(y(x)+1)K[1]^2+2\log(K[1])y(x)K[1]^2-\log(y(x)-1)y(}$$

2.131 problem 707

Internal problem ID [9042]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 707.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$y' - \frac{\left(-\ln\left(-1+y\right)+\ln\left(y+1\right)+2\ln\left(x\right)\right)^2 x(y+1)^2}{16} = 0$$

✓ Solution by Maple

Time used: 0.437 (sec). Leaf size: 182

```
dsolve(diff(y(x),x) = 1/16*(-ln(-1+y(x))+ln(y(x)+1)+2*ln(x))^2*x*(y(x)+1)^2,y(x), singsol=al
```

$$y(x) = e^{\operatorname{RootOf}\left(x^2 e^{\_Z}\_Z^2 - 2x^2 e^{\_Z}\ln\left(\frac{e^{\_Z}-2}{x^2}\right)\_Z + 4\ln(x)^2 x^2 e^{\_Z} - 4\ln(x)\ln\left(e^{\_Z}-2\right)x^2 e^{\_Z} + \ln\left(e^{\_Z}-2\right)^2 x^2 e^{\_Z} - 16\, e^{\_Z} + 32\right)} - 1$$

$$\int_{\_b}^{y(x)} \frac{1}{4\left(\_a+1\right)\left(\frac{x^2(\_a+1)\ln(\_a-1)^2}{4} - \left(\ln\left(x\right)+\frac{\ln(\_a+1)}{2}\right)\left(\_a+1\right)x^2\ln\left(\_a-1\right) + \frac{x^2(\_a+1)\ln(\_a+1)^2}{4} + \right.}$$

$$-\frac{\ln\left(x\right)}{16} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 4.678 (sec). Leaf size: 1391

```
DSolve[y'[x] == (x*(2*Log[x] - Log[-1 + y[x]] + Log[1 + y[x]])^2*(1 + y[x])^2)/16,y[x],x,Inc
```

$$\text{Solve}\Bigg[\int_1^x$$

$$-\frac{}{4\log^2(K[1])K[1]^2+\log^2(y(x)-1)K[1]^2+\log^2(y(x)+1)K[1]^2-4\log(K[1])\log(y(x)-1)K[1]^2+4\log}$$

$$+\int_1^{y(x)}\left(\frac{-4\log^2(x)x^2-\log}{2\left(4\log^2(x)x^2+\log^2(K[2]-1)x^2+\log^2(K[2]+1)x^2-4\log(x)\log(K[2]-1)x^2+4\log(x)\log(\right.}\right.$$

$$-\int_1^x\left(-\frac{}{4K[2]\log^2(K[1])K[1]^2+4\log^2(K[1])K[1]^2+K[2]\log^2(K[2]-1)K[1]^2+\log^2(K[2]-1)K[1]^2}\right.$$

$$\left.+\frac{1}{2(K[2]+1)}\right)dK[2]=c_1,y(x)\Bigg]$$

2.132 problem 708

Internal problem ID [9043]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 708.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{(-y^2 + 4ax)^3}{(-y^2 + 4ax - 1)\, y} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 747

```
dsolve(diff(y(x),x) = (-y(x)^2+4*a*x)^3/(-y(x)^2+4*a*x-1)/y(x),y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 0.273 (sec). Leaf size: 89

```
DSolve[y'[x] == (4*a*x - y[x]^2)^3/(y[x]*(-1 + 4*a*x - y[x]^2)),y[x],x,IncludeSingularSoluti
```

$$\text{Solve}\left[2a\left(x - \frac{\text{RootSum}\left[-\#1^3 + 2\#1a - 2a\&, \frac{\#1a\log\left(-\#1+4ax-y(x)^2\right)-a\log\left(-\#1+4ax-y(x)^2\right)}{2a-3\#1^2}\&\right]}{2a}\right) = c_1, y(x)\right]$$

2.133 problem 709

Internal problem ID [9044]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 709.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `['y=_G(x,y')']`

$$y' - \frac{2ax + 2a + x^3\sqrt{-y^2 + 4ax}}{(x+1)\, y} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 39

```
dsolve(diff(y(x),x) = (2*a*x+2*a+x^3*(-y(x)^2+4*a*x)^(1/2))/(x+1)/y(x),y(x), singsol=all)
```

$$-\sqrt{4ax - y(x)^2} - \frac{x^3}{3} + \frac{x^2}{2} - x + \ln(x+1) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 3.881 (sec). Leaf size: 143

```
DSolve[y'[x] == (2*a + 2*a*x + x^3*Sqrt[4*a*x - y[x]^2])/((1 + x)*y[x]),y[x],x,IncludeSingul
```

$$y(x) \to -\frac{1}{6}\sqrt{144ax - (2x^3 - 3x^2 + 6x + 6c_1)^2 + 12(2x^3 - 3x^2 + 6x + 6c_1)\log(x+1) - 36\log^2(x+1)}$$

$$y(x) \to \frac{1}{6}\sqrt{144ax - (2x^3 - 3x^2 + 6x + 6c_1)^2 + 12(2x^3 - 3x^2 + 6x + 6c_1)\log(x+1) - 36\log^2(x+1)}$$

2.134 problem 710

Internal problem ID [9045]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 710.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - \frac{-\ln(x) + e^{\frac{1}{x}} + 4yx^2 + 2x + 2y^2x + 2x^3}{\ln(x) - e^{\frac{1}{x}}} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(diff(y(x),x) = (-ln(x)+exp(1/x)+4*x^2*y(x)+2*x+2*x*y(x)^2+2*x^3)/(ln(x)-exp(1/x)),y(x
```

$$y(x) = -x + \tan\left(2c_1 + 2\left(\int \frac{x}{\ln(x) - e^{\frac{1}{x}}} dx\right)\right)$$

✓ Solution by Mathematica

Time used: 1.529 (sec). Leaf size: 38

```
DSolve[y'[x] == (E^x^(-1) + 2*x + 2*x^3 - Log[x] + 4*x^2*y[x] + 2*x*y[x]^2)/(-E^x^(-1) + Log
```

$$y(x) \to -x + \tan\left(\int_1^x -\frac{2K[5]}{e^{\frac{1}{K[5]}} - \log(K[5])} dK[5] + c_1\right)$$

2.135 problem 711

Internal problem ID [9046]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 711.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y' + \frac{(\ln(y)\,x + \ln(y) - 1)\,y}{x+1} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 27

```
dsolve(diff(y(x),x) = -(ln(y(x))*x+ln(y(x))-1)*y(x)/(x+1),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{e}^{-x} c_1 - \mathrm{expIntegral}_1(-x-1)\mathrm{e}^{-x-1}}$$

✓ Solution by Mathematica

Time used: 0.501 (sec). Leaf size: 24

```
DSolve[y'[x] == ((1 - Log[y[x]] - x*Log[y[x]])*y[x])/(1 + x),y[x],x,IncludeSingularSolutions
```

$$y(x) \to e^{e^{-x-1}(\mathrm{ExpIntegralEi}(x+1) + e c_1)}$$

2.136 problem 712

Internal problem ID [9047]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 712.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$\boxed{y' - \frac{x^2 + 2x + 1 + 2\sqrt{x^2 + 2x + 1 - 4y}\, x^3}{2(x+1)} = 0}$$

✓ Solution by Maple
Time used: 0.156 (sec). Leaf size: 38

```
dsolve(diff(y(x),x) = 1/2*(x^2+2*x+1+2*x^3*(x^2+2*x+1-4*y(x))^(1/2))/(x+1),y(x), singsol=all
```

$$c_1 - \frac{2x^3}{3} + x^2 - 2x + 2\ln(x+1) - \sqrt{x^2 + 2x + 1 - 4y(x)} = 0$$

✓ Solution by Mathematica
Time used: 1.212 (sec). Leaf size: 49

```
DSolve[y'[x] == (1/2 + x + x^2/2 + x^3*Sqrt[1 + 2*x + x^2 - 4*y[x]])/(1 + x),y[x],x,IncludeS
```

$$y(x) \to \frac{1}{4}\left(x^2 - \frac{1}{9}\left(2x^3 - 3x^2 + 6x + 6\log\left(\frac{1}{x+1}\right) - 6c_1\right)^2 + 2x + 1\right)$$

2.137 problem 713

Internal problem ID [9048]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 713.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, [_Abel, '2n`

$$y' - \frac{-bya + b^2 + ab + b^2x - ba\sqrt{x} - a^2}{a\left(-ya + b + a + bx - a\sqrt{x}\right)} = 0$$

✓ Solution by Maple
Time used: 0.297 (sec). Leaf size: 110

```
dsolve(diff(y(x),x) = (-b*y(x)*a+b^2+a*b+b^2*x-b*a*x^(1/2)-a^2)/a/(-a*y(x)+b+a+b*x-a*x^(1/2)
```

$$y(x) = \frac{\text{RootOf}\left(-x^{\frac{3}{2}}ab + b^2x^2 - a^2\sqrt{x} - ba\sqrt{x} - 2a^2x + 2axb + 2b^2x + a^2 + 2ab + b^2 + \mathrm{e}^{\text{RootOf}\left(-4\,\mathrm{e}^{-Z}+9\,\text{sech}\left(-\right.\right.}\right.}{a}$$

✓ Solution by Mathematica

Time used: 60.086 (sec). Leaf size: 649

```
DSolve[y'[x] == (-a^2 + a*b + b^2 - a*b*Sqrt[x] + b^2*x - a*b*y[x])/(a*(a + b - a*Sqrt[x] +
```

$$y(x) \to -\frac{a\sqrt{x}-a-bx-b}{a} + \frac{1}{a^2\text{Root}\left[\#1^6\left(16x^3+16e^{12c_1}\right)-\frac{24\#1^4x^2}{a^4}+\frac{8\#1^3x^{3/2}}{a^6}+\frac{9\#1^2x}{a^8}-\frac{6\#1\sqrt{x}}{a^{10}}+\frac{1}{a^{12}}\&,1\right]}$$

$$y(x) \to -\frac{a\sqrt{x}-a-bx-b}{a} + \frac{1}{a^2\text{Root}\left[\#1^6\left(16x^3+16e^{12c_1}\right)-\frac{24\#1^4x^2}{a^4}+\frac{8\#1^3x^{3/2}}{a^6}+\frac{9\#1^2x}{a^8}-\frac{6\#1\sqrt{x}}{a^{10}}+\frac{1}{a^{12}}\&,2\right]}$$

$$y(x) \to -\frac{a\sqrt{x}-a-bx-b}{a} + \frac{1}{a^2\text{Root}\left[\#1^6\left(16x^3+16e^{12c_1}\right)-\frac{24\#1^4x^2}{a^4}+\frac{8\#1^3x^{3/2}}{a^6}+\frac{9\#1^2x}{a^8}-\frac{6\#1\sqrt{x}}{a^{10}}+\frac{1}{a^{12}}\&,3\right]}$$

$$y(x) \to -\frac{a\sqrt{x}-a-bx-b}{a} + \frac{1}{a^2\text{Root}\left[\#1^6\left(16x^3+16e^{12c_1}\right)-\frac{24\#1^4x^2}{a^4}+\frac{8\#1^3x^{3/2}}{a^6}+\frac{9\#1^2x}{a^8}-\frac{6\#1\sqrt{x}}{a^{10}}+\frac{1}{a^{12}}\&,4\right]}$$

$$y(x) \to -\frac{a\sqrt{x}-a-bx-b}{a} + \frac{1}{a^2\text{Root}\left[\#1^6\left(16x^3+16e^{12c_1}\right)-\frac{24\#1^4x^2}{a^4}+\frac{8\#1^3x^{3/2}}{a^6}+\frac{9\#1^2x}{a^8}-\frac{6\#1\sqrt{x}}{a^{10}}+\frac{1}{a^{12}}\&,5\right]}$$

$$y(x) \to -\frac{a\sqrt{x}-a-bx-b}{a} + \frac{1}{a^2\text{Root}\left[\#1^6\left(16x^3+16e^{12c_1}\right)-\frac{24\#1^4x^2}{a^4}+\frac{8\#1^3x^{3/2}}{a^6}+\frac{9\#1^2x}{a^8}-\frac{6\#1\sqrt{x}}{a^{10}}+\frac{1}{a^{12}}\&,6\right]}$$

2.138 problem 714

Internal problem ID [9049]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 714.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$y' + \frac{y\left(-\ln\left(\frac{1}{x}\right) + e^x + yx^2\ln(x) + yx^3 - x\ln(x) - x^2\right)}{\left(-\ln\left(\frac{1}{x}\right) + e^x\right)x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 96

```
dsolve(diff(y(x),x) = -y(x)*(-ln(1/x)+exp(x)+y(x)*x^2*ln(x)+x^3*y(x)-x*ln(x)-x^2)/(-ln(1/x)+
```

$$y(x) = \frac{e^{\int \frac{x\ln(x)+x^2-e^x+\ln\left(\frac{1}{x}\right)}{x\left(-\ln\left(\frac{1}{x}\right)+e^x\right)}dx}}{\int \frac{e^{\int \frac{x\ln(x)+x^2-e^x+\ln\left(\frac{1}{x}\right)}{x\left(-\ln\left(\frac{1}{x}\right)+e^x\right)}dx} x(\ln(x)+x)}{-\ln\left(\frac{1}{x}\right)+e^x}dx + c_1}$$

✓ Solution by Mathematica

Time used: 2.236 (sec). Leaf size: 290

```
DSolve[y'[x] == -((y[x]*(E^x - x^2 - Log[x^(-1)] - x*Log[x] + x^3*y[x] + x^2*Log[x]*y[x]))/(
```

$$y(x)\to \frac{\exp\left(\int_1^x \frac{K[1]^2+\log(K[1])K[1]-e^{K[1]}+\log\left(\frac{1}{K[1]}\right)}{K[1]\left(e^{K[1]}-\log\left(\frac{1}{K[1]}\right)\right)}dK[1]\right)}{-\int_1^x -\frac{\exp\left(\int_1^{K[2]} \frac{K[1]^2+\log(K[1])K[1]-e^{K[1]}+\log\left(\frac{1}{K[1]}\right)}{K[1]\left(e^{K[1]}-\log\left(\frac{1}{K[1]}\right)\right)}dK[1]\right)K[2](K[2]+\log(K[2]))}{e^{K[2]}-\log\left(\frac{1}{K[2]}\right)}dK[2]+c_1}$$

$$y(x)\to 0$$

$$y(x)\to -\frac{\exp\left(\int_1^x \frac{K[1]^2+\log(K[1])K[1]-e^{K[1]}+\log\left(\frac{1}{K[1]}\right)}{K[1]\left(e^{K[1]}-\log\left(\frac{1}{K[1]}\right)\right)}dK[1]\right)}{\int_1^x -\frac{\exp\left(\int_1^{K[2]} \frac{K[1]^2+\log(K[1])K[1]-e^{K[1]}+\log\left(\frac{1}{K[1]}\right)}{K[1]\left(e^{K[1]}-\log\left(\frac{1}{K[1]}\right)\right)}dK[1]\right)K[2](K[2]+\log(K[2]))}{e^{K[2]}-\log\left(\frac{1}{K[2]}\right)}dK[2]}$$

2.139 problem 715

Internal problem ID [9050]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 715.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, ‘_with_symmetry_[F(x),G(x)]‘]]`

$$y' - \frac{-x^2 + x + 2 + 2\sqrt{x^2 - 4x + 4y}\, x^3}{2(x+1)} = 0$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 39

```
dsolve(diff(y(x),x) = 1/2*(-x^2+x+2+2*x^3*(x^2-4*x+4*y(x))^(1/2))/(x+1),y(x), singsol=all)
```

$$c_1 + \frac{2x^3}{3} - x^2 - 2\ln(x+1) + 2x - \sqrt{x^2 - 4x + 4y(x)} = 0$$

✓ Solution by Mathematica

Time used: 1.264 (sec). Leaf size: 50

```
DSolve[y'[x] == (1 + x/2 - x^2/2 + x^3*Sqrt[-4*x + x^2 + 4*y[x]])/(1 + x),y[x],x,IncludeSing
```

$$y(x) \to \frac{1}{4}\left(-x^2 + \frac{1}{9}\left(2x^3 - 3x^2 + 6x + 6\log\left(\frac{1}{x+1}\right) - 6c_1\right)^2 + 4x\right)$$

2.140 problem 716

Internal problem ID [9051]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 716.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{3x^4 + 3x^3 + \sqrt{9x^4 - 4y^3}}{(x+1)\, y^2} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 37

```
dsolve(diff(y(x),x) = (3*x^4+3*x^3+(9*x^4-4*y(x)^3)^(1/2))/(x+1)/y(x)^2,y(x), singsol=all)
```

$$\int_{\_b}^{y(x)} \frac{\_a^2}{\sqrt{9x^4 - 4\_a^3}} d\_a - \ln(x+1) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 3.458 (sec). Leaf size: 133

```
DSolve[y'[x] == (3*x^3 + 3*x^4 + Sqrt[9*x^4 - 4*y[x]^3])/((1 + x)*y[x]^2),y[x],x,IncludeSing
```

$$y(x) \to \left(-\frac{3}{2}\right)^{2/3} \sqrt[3]{x^4 - 4\log^2(x+1) + 8c_1 \log(x+1) - 4{c_1}^2}$$
$$y(x) \to \left(\frac{3}{2}\right)^{2/3} \sqrt[3]{x^4 - 4\log^2(x+1) + 8c_1 \log(x+1) - 4{c_1}^2}$$
$$y(x) \to -\sqrt[3]{-1}\left(\frac{3}{2}\right)^{2/3} \sqrt[3]{x^4 - 4\log^2(x+1) + 8c_1 \log(x+1) - 4{c_1}^2}$$

2.141 problem 717

Internal problem ID [9052]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 717.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' + \frac{x^2 + x + ax + a - 2\sqrt{x^2 + 2ax + a^2 + 4y}}{2x + 2} = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 33

```
dsolve(diff(y(x),x) = -1/2*(x^2+x+a*x+a-2*(x^2+2*a*x+a^2+4*y(x))^(1/2))/(x+1),y(x), singsol=
```

$$c_1 + \frac{a}{2} + 2\ln(x+1) - \sqrt{x^2 + 2ax + a^2 + 4y(x)} = 0$$

✓ Solution by Mathematica

Time used: 0.662 (sec). Leaf size: 44

```
DSolve[y'[x] == (-1/2*a - x/2 - (a*x)/2 - x^2/2 + Sqrt[a^2 + 2*a*x + x^2 + 4*y[x]])/(1 + x),
```

$$y(x) \to -\frac{a^2}{4} - \frac{ax}{2} - \frac{x^2}{4} + \log^2(x+1) - 2c_1\log(x+1) + {c_1}^2$$

2.142 problem 718

Internal problem ID [9053]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 718.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$y' - \left(1 + y^2 e^{2x^2} + y^3 e^{3x^2}\right) e^{-x^2} x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve(diff(y(x),x) = (1+y(x)^2*exp(2*x^2)+y(x)^3*exp(3*x^2))*exp(-x^2)*x,y(x), singsol=all)
```

$$y(x) = -\frac{11\,e^{-x^2}\,\mathrm{RootOf}\left(-5x^2 + 20250\left(\int^{\_Z} \frac{1}{121\_a^3 + 3375\_a - 3375} d\_a\right) + 6c_1\right)}{45} - \frac{e^{-x^2}}{3}$$

✓ Solution by Mathematica

Time used: 0.249 (sec). Leaf size: 127

```
DSolve[y'[x] == (x*(1 + E^(2*x^2)*y[x]^2 + E^(3*x^2)*y[x]^3))/E^x^2,y[x],x,IncludeSingularSo
```

$$\mathrm{Solve}\left[\frac{11}{3}\mathrm{RootSum}\left[11\#1^3 + 15\sqrt[3]{11}\#1 + 11\&, \frac{\log\left(\frac{3e^{2x^2}xy(x)+e^{x^2}x}{\sqrt[3]{11}\sqrt[3]{e^{3x^2}x^3}} - \#1\right)}{11\#1^2 + 5\sqrt[3]{11}}\&\right] = \frac{11^{2/3}e^{x^2}x^3}{18\sqrt[3]{e^{3x^2}x^3}} + c_1, y(x)\right]$$

2.143 problem 719

Internal problem ID [9054]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 719.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$y' - \frac{y(-\mathrm{e}^{x} + \ln(2x)\,x^{2}y - \ln(2x)\,x)\,\mathrm{e}^{-x}}{x} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
dsolve(diff(y(x),x) = y(x)*(-exp(x)+ln(2*x)*x^2*y(x)-ln(2*x)*x)/x/exp(x),y(x), singsol=all)
```

$$y(x) = \frac{1}{x\left(1 + 2^{-\mathrm{e}^{-x}} x^{-\mathrm{e}^{-x}} \mathrm{e}^{-\operatorname{expIntegral}_1(x)} c_1\right)}$$

✓ Solution by Mathematica

Time used: 0.773 (sec). Leaf size: 49

```
DSolve[y'[x] == (y[x]*(-E^x - x*Log[2*x] + x^2*Log[2*x]*y[x]))/(E^x*x),y[x],x,IncludeSingula
```

$$y(x) \to \frac{2^{e^{-x}}}{x\left(2^{e^{-x}} + c_1 x^{-e^{-x}} e^{\text{ExpIntegralEi}(-x)}\right)}$$
$$y(x) \to 0$$

2.144 problem 720

Internal problem ID [9055]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 720.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `['y=_G(x,y')']`

$$y' - \frac{x^3\left(3x+3+\sqrt{9x^4-4y^3}\right)}{(x+1)\,y^2} = 0$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 48

```
dsolve(diff(y(x),x) = x^3*(3*x+3+(9*x^4-4*y(x)^3)^(1/2))/(x+1)/y(x)^2,y(x), singsol=all)
```

$$\int_{\_b}^{y(x)} \frac{\_a^2}{\sqrt{9x^4-4\_a^3}}d\_a - \frac{x^3}{3} + \frac{x^2}{2} - x + \ln(x+1) - c_1 = 0$$

✓ Solution by Mathematica
Time used: 4.423 (sec). Leaf size: 321

```
DSolve[y'[x] == (x^3*(3 + 3*x + Sqrt[9*x^4 - 4*y[x]^3]))/((1 + x)*y[x]^2),y[x],x,IncludeSing
```

$$y(x) \to \frac{\sqrt[3]{-4x^6+12x^5-24x^4+8(-1+3c_1)x^3-6(-5+6c_1)x^2+12\left(2x^3-3x^2+6x+11-6c_1\right)\log(x+1)}}{2^{2/3}}$$

$$y(x) \to -\frac{\sqrt[3]{-1}\sqrt[3]{-4x^6+12x^5-24x^4+8(-1+3c_1)x^3-6(-5+6c_1)x^2+12\left(2x^3-3x^2+6x+11-6c_1\right)\log(}}{2^{2/3}}$$

$$y(x) \to \left(-\frac{1}{2}\right)^{2/3}\sqrt[3]{-4x^6+12x^5-24x^4+8(-1+3c_1)x^3-6(-5+6c_1)x^2+12\left(2x^3-3x^2+6x+11-6c_1\right)l}$$

2.145 problem 721

Internal problem ID [9056]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 721.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - \frac{\left(18x^{\frac{3}{2}} + 36y^2 - 12yx^3 + x^6\right)\sqrt{x}}{36} = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x) = 1/36*(18*x^(3/2)+36*y(x)^2-12*x^3*y(x)+x^6)*x^(1/2),y(x), singsol=all)
```

$$y(x) = \frac{x^3}{6} - \frac{3}{2x^{\frac{3}{2}} - 3c_1}$$

✓ Solution by Mathematica
Time used: 0.199 (sec). Leaf size: 38

```
DSolve[y'[x] == (Sqrt[x]*(18*x^(3/2) + x^6 - 12*x^3*y[x] + 36*y[x]^2))/36,y[x],x,IncludeSing
```

$$y(x) \to \frac{x^3}{6} + \frac{1}{-\frac{2x^{3/2}}{3} + c_1}$$
$$y(x) \to \frac{x^3}{6}$$

2.146 problem 722

Internal problem ID [9057]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 722.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]'], [_Abel, '2nd ty`

$$y' + \frac{y^3}{(-1 + 2y\ln(x) - y)\,x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 78

```
dsolve(diff(y(x),x) = -y(x)^3/(-1+2*y(x)*ln(x)-y(x))/x,y(x), singsol=all)
```

$$y(x) = \frac{e^{\operatorname{RootOf}\left(e^{\_Z}\ln(2) - e^{\_Z}\ln\left(\frac{e^{\_Z}+2}{x^4}\right) + 3c_1 e^{\_Z} + e^{\_Z}\_Z + 2\right)}}{1 + (2\ln(x) - 1)\,e^{\operatorname{RootOf}\left(e^{\_Z}\ln(2) - e^{\_Z}\ln\left(\frac{e^{\_Z}+2}{x^4}\right) + 3c_1 e^{\_Z} + e^{\_Z}\_Z + 2\right)}}$$

✓ Solution by Mathematica

Time used: 17.707 (sec). Leaf size: 490

```
DSolve[y'[x] == -(y[x]^3/(x*(-1 - y[x] + 2*Log[x]*y[x]))),y[x],x,IncludeSingularSolutions ->
```

$$\text{Solve}\left[-\frac{\sqrt[3]{-2}\left((-2)^{2/3} - \frac{(1-2\log(x))^2\left(-\frac{1}{(2\log(x)-1)^3}\right)^{2/3}(y(x)(5-4\log(x))+2)}{2\sqrt[3]{2}(y(x)(2\log(x)-1)-1)}\right)\left(\frac{y(x)(4\log(x)-5)-2}{\sqrt[3]{2}\sqrt[3]{-\frac{1}{(2\log(x)-1)^3}(2\log(x)-1)(\cdots}}\right.}{\cdots}\right.$$

2.147 problem 723

Internal problem ID [9058]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 723.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y' - \frac{2a}{y + 2ay^4 - 16a^2xy^2 + 32a^3x^2} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 696

```
dsolve(diff(y(x),x) = 2*a/(y(x)+2*a*y(x)^4-16*a^2*x*y(x)^2+32*a^3*x^2),y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(64c_1^3a^4 - 576c_1a^3x + 3\sqrt{-12288a^7c_1^4x + 24576a^6c_1^2x^2 - 12288a^5x^3 + 384c_1^3a^4 - 3456c_1a^3x + 81} + 27\right.\right.}{6a} + \frac{8a^2(a\,c_1^2 + 3x)}{3\left(\left(64c_1^3a^4 - 576c_1a^3x + 3\sqrt{-12288a^7c_1^4x + 24576a^6c_1^2x^2 - 12288a^5x^3 + 384c_1^3a^4 - 3456c_1a^3x + 81}\right.\right.} + \frac{2c_1a}{3}$$

$$y(x) = \frac{\frac{\left(-i\sqrt{3}-1\right)\left(\left(64c_1^3a^4-576c_1a^3x+3\sqrt{-12288a^7c_1^4x+24576a^6c_1^2x^2-12288a^5x^3+384c_1^3a^4-3456c_1a^3x+81}+27\right)a^2\right)^{\frac{2}{3}}}{12} + \frac{4a^2\left(\frac{c_1\left(\left(64c_1^3a^4-5\right.\right.}{}\right.}{}}{a\left(\left(64c_1^3a^4 - 576c_1a^3x + 3\sqrt{-12288a^7c_1^4x + 24576a^6c_1^2x^2 - 12288a^5\right.\right.}$$

$$y(x) = \frac{\frac{\left(i\sqrt{3}-1\right)\left(\left(64c_1^3a^4-576c_1a^3x+3\sqrt{-12288a^7c_1^4x+24576a^6c_1^2x^2-12288a^5x^3+384c_1^3a^4-3456c_1a^3x+81}+27\right)a^2\right)^{\frac{2}{3}}}{12} + \frac{4a^2\left(\frac{c_1\left(\left(64c_1^3a^4-576\right.\right.}{}\right.}{}}{a\left(\left(64c_1^3a^4 - 576c_1a^3x + 3\sqrt{-12288a^7c_1^4x + 24576a^6c_1^2x^2 - 12288a^5\right.\right.}$$

✓ Solution by Mathematica

Time used: 19.546 (sec). Leaf size: 672

```
DSolve[y'[x] == (2*a)/(32*a^3*x^2 + y[x] - 16*a^2*x*y[x]^2 + 2*a*y[x]^4),y[x],x,IncludeSingu
```

$$y(x) \to -\frac{\sqrt[3]{-1024a^6c_1^3 + 9216a^5c_1x - 432a^2 + 16\sqrt{a^4\left((64a^4c_1^3 - 576a^3c_1x + 27)^2 - 4096a^5(3x + ac_1^2)^3\right)}}}{12\sqrt[3]{2}a} - \frac{8a^2(3x + ac_1^2)}{3\sqrt[3]{-64a^6c_1^3 + 576a^5c_1x - 27a^2 + 3\sqrt{3}\sqrt{-a^4\left(4096a^7c_1^4x - 8192a^6c_1^2x^2 + 4096a^5x^3 - 128a^4c_1^3 + 1\right.}}} + \frac{2ac_1}{3}$$

$$y(x) \to \frac{(1 - i\sqrt{3})\sqrt[3]{-1024a^6c_1^3 + 9216a^5c_1x - 432a^2 + 16\sqrt{a^4\left((64a^4c_1^3 - 576a^3c_1x + 27)^2 - 4096a^5(3x + a\right.}}}{24\sqrt[3]{2}a} + \frac{4(1 + i\sqrt{3})a^2(3x + ac_1^2)}{3\sqrt[3]{-64a^6c_1^3 + 576a^5c_1x - 27a^2 + 3\sqrt{3}\sqrt{-a^4\left(4096a^7c_1^4x - 8192a^6c_1^2x^2 + 4096a^5x^3 - 128a^4c_1^3 + 1\right.}}} + \frac{2ac_1}{3}$$

$$y(x) \to \frac{(1 + i\sqrt{3})\sqrt[3]{-1024a^6c_1^3 + 9216a^5c_1x - 432a^2 + 16\sqrt{a^4\left((64a^4c_1^3 - 576a^3c_1x + 27)^2 - 4096a^5(3x + a\right.}}}{24\sqrt[3]{2}a} + \frac{4(1 - i\sqrt{3})a^2(3x + ac_1^2)}{3\sqrt[3]{-64a^6c_1^3 + 576a^5c_1x - 27a^2 + 3\sqrt{3}\sqrt{-a^4\left(4096a^7c_1^4x - 8192a^6c_1^2x^2 + 4096a^5x^3 - 128a^4c_1^3 + 1\right.}}} + \frac{2ac_1}{3}$$

2.148 problem 724

Internal problem ID [9059]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 724.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]'], [_Abel, '2nd ty`

$$y' + \frac{y^3}{(-1 + y\ln(x) - y)\,x} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve(diff(y(x),x) = -y(x)^3/(-1+y(x)*ln(x)-y(x))/x,y(x), singsol=all)
```

$$y(x) = \frac{1}{-\operatorname{LambertW}(c_1 e^{-2} x) + \ln(x) - 2}$$

✓ Solution by Mathematica

Time used: 11.852 (sec). Leaf size: 422

```
DSolve[y'[x] == -(y[x]^3/(x*(-1 - y[x] + Log[x]*y[x]))),y[x],x,IncludeSingularSolutions -> T
```

$$\text{Solve}\left[-\frac{\sqrt[3]{-2}\left(\frac{1-y(x)(\log(x)-4)}{\sqrt[3]{2}\sqrt[3]{-\frac{1}{(\log(x)-1)^3}(\log(x)-1)(y(x)(\log(x)-1)-1)}} + (-2)^{2/3}\right)\left(\frac{2^{2/3}(y(x)(\log(x)-4)-1)}{\sqrt[3]{-\frac{1}{(\log(x)-1)^3}(\log(x)-1)(y(x)}} \right.}{}\right.$$

2.149 problem 725

Internal problem ID [9060]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 725.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - \frac{-\ln(x) + 2y\ln(2x)\,x + \ln(2x) + \ln(2x)\,y^2 + \ln(2x)\,x^2}{\ln(x)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x) = (-ln(x)+2*ln(2*x)*x*y(x)+ln(2*x)+ln(2*x)*y(x)^2+ln(2*x)*x^2)/ln(x),y(x
```

$$y(x) = -x - \tan(c_1 - x + \ln(2)\,\mathrm{expIntegral}_1(-\ln(x)))$$

✓ Solution by Mathematica

Time used: 0.693 (sec). Leaf size: 19

```
DSolve[y'[x] == (-Log[x] + Log[2*x] + x^2*Log[2*x] + 2*x*Log[2*x]*y[x] + Log[2*x]*y[x]^2)/Lo
```

$$y(x) \to -x + \tan(\log(2)\,\mathrm{LogIntegral}(x) + x + c_1)$$

2.150 problem 726

Internal problem ID [9061]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 726.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, [_Abel, '2n`

$$y' + \frac{bya - bc + b^2x + ba\sqrt{x} - a^2}{a\left(ya - c + bx + a\sqrt{x}\right)} = 0$$

✓ Solution by Maple
Time used: 0.328 (sec). Leaf size: 87

```
dsolve(diff(y(x),x) = -(b*y(x)*a-b*c+b^2*x+b*a*x^(1/2)-a^2)/a/(a*y(x)-c+b*x+a*x^(1/2)),y(x),
```

$$y(x) = \frac{\operatorname{RootOf}\left(x^{\frac{3}{2}}ab + b^2x^2 - \sqrt{x}\,ac - 2a^2x - 2bcx + c^2 - \mathrm{e}^{\operatorname{RootOf}\left(4\,\mathrm{e}^{-\_Z} + 9\,\mathrm{sech}\left(-\frac{3\_Z}{2} + \frac{c_1}{2}\right)^2 a^2 x\right)} + \left(a\sqrt{x} + 2bx - \right.\right.}{a}$$

✓ Solution by Mathematica

Time used: 60.087 (sec). Leaf size: 625

```
DSolve[y'[x] == (a^2 + b*c - a*b*Sqrt[x] - b^2*x - a*b*y[x])/(a*(-c + a*Sqrt[x] + b*x + a*y[
```

$$y(x) \to -\frac{a\sqrt{x}+bx-c}{a} + \frac{1}{a^2\text{Root}\left[\#1^6\left(16x^3+16e^{12c_1}\right)-\frac{24\#1^4x^2}{a^4}+\frac{8\#1^3x^{3/2}}{a^6}+\frac{9\#1^2x}{a^8}-\frac{6\#1\sqrt{x}}{a^{10}}+\frac{1}{a^{12}}\&,1\right]}$$

$$y(x) \to -\frac{a\sqrt{x}+bx-c}{a} + \frac{1}{a^2\text{Root}\left[\#1^6\left(16x^3+16e^{12c_1}\right)-\frac{24\#1^4x^2}{a^4}+\frac{8\#1^3x^{3/2}}{a^6}+\frac{9\#1^2x}{a^8}-\frac{6\#1\sqrt{x}}{a^{10}}+\frac{1}{a^{12}}\&,2\right]}$$

$$y(x) \to -\frac{a\sqrt{x}+bx-c}{a} + \frac{1}{a^2\text{Root}\left[\#1^6\left(16x^3+16e^{12c_1}\right)-\frac{24\#1^4x^2}{a^4}+\frac{8\#1^3x^{3/2}}{a^6}+\frac{9\#1^2x}{a^8}-\frac{6\#1\sqrt{x}}{a^{10}}+\frac{1}{a^{12}}\&,3\right]}$$

$$y(x) \to -\frac{a\sqrt{x}+bx-c}{a} + \frac{1}{a^2\text{Root}\left[\#1^6\left(16x^3+16e^{12c_1}\right)-\frac{24\#1^4x^2}{a^4}+\frac{8\#1^3x^{3/2}}{a^6}+\frac{9\#1^2x}{a^8}-\frac{6\#1\sqrt{x}}{a^{10}}+\frac{1}{a^{12}}\&,4\right]}$$

$$y(x) \to -\frac{a\sqrt{x}+bx-c}{a} + \frac{1}{a^2\text{Root}\left[\#1^6\left(16x^3+16e^{12c_1}\right)-\frac{24\#1^4x^2}{a^4}+\frac{8\#1^3x^{3/2}}{a^6}+\frac{9\#1^2x}{a^8}-\frac{6\#1\sqrt{x}}{a^{10}}+\frac{1}{a^{12}}\&,5\right]}$$

$$y(x) \to -\frac{a\sqrt{x}+bx-c}{a} + \frac{1}{a^2\text{Root}\left[\#1^6\left(16x^3+16e^{12c_1}\right)-\frac{24\#1^4x^2}{a^4}+\frac{8\#1^3x^{3/2}}{a^6}+\frac{9\#1^2x}{a^8}-\frac{6\#1\sqrt{x}}{a^{10}}+\frac{1}{a^{12}}\&,6\right]}$$

2.151 problem 727

Internal problem ID [9062]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 727.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘x=_G(y,y’)‘]`

$$y' - \frac{(2x+2+y)\,y}{(\ln(y)+2x-1)\,(x+1)} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve(diff(y(x),x) = (2*x+2+y(x))/(ln(y(x))+2*x-1)*y(x)/(x+1),y(x), singsol=all)
```

$$y(x) = -2x - 2$$

$$y(x) = \frac{\text{LambertW}\left((\ln(x+1) - c_1)\,\mathrm{e}^{-2x}\right)}{\ln(x+1) - c_1}$$

✓ Solution by Mathematica

Time used: 60.295 (sec). Leaf size: 29

```
DSolve[y'[x] == (y[x]*(2 + 2*x + y[x]))/((1 + x)*(-1 + 2*x + Log[y[x]])),y[x],x,IncludeSingu
```

$$y(x) \to \frac{W\left(e^{-2x}(\log(x+1) + c_1)\right)}{\log(x+1) + c_1}$$

2.152 problem 728

Internal problem ID [9063]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 728.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y' - \frac{(x^3+3y^2)\,y}{(6y^2+x)\,x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 58

```
dsolve(diff(y(x),x) = 1/(6*y(x)^2+x)*(x^3+3*y(x)^2)*y(x)/x,y(x), singsol=all)
```

$$\frac{y(x)^2 x}{6y(x)^2+x} = \frac{\left(\mathrm{e}^{\mathrm{RootOf}\left(x^2\mathrm{e}^{\_Z}+\mathrm{e}^{\_Z}\ln(2)-\mathrm{e}^{\_Z}\ln\left(\left(\mathrm{e}^{\_Z}+9\right)x\right)+3c_1\mathrm{e}^{\_Z}+\mathrm{e}^{\_Z}\_Z+9\right)}+9\right)x}{54}$$

✓ Solution by Mathematica

Time used: 5.555 (sec). Leaf size: 77

```
DSolve[y'[x] == (y[x]*(x^3 + 3*y[x]^2))/(x*(x + 6*y[x]^2)),y[x],x,IncludeSingularSolutions -
```

$$y(x) \to -\frac{\sqrt{x}\sqrt{W\left(\frac{6e^{x^2+2c_1}}{x}\right)}}{\sqrt{6}}$$
$$y(x) \to \frac{\sqrt{x}\sqrt{W\left(\frac{6e^{x^2+2c_1}}{x}\right)}}{\sqrt{6}}$$
$$y(x) \to 0$$

2.153 problem 729

Internal problem ID [9064]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 729.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{y(x-y)}{x\,(x-y^3)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 320

```
dsolve(diff(y(x),x) = y(x)*(x-y(x))/x/(x-y(x)^3),y(x), singsol=all)
```

$$y(x) = \frac{\left(-27x+3\sqrt{-24\ln(x)^3+72\ln(x)^2 c_1-72\ln(x)\,c_1^2+24c_1^3+81x^2}\right)^{\frac{2}{3}}+6\ln(x)-6c_1}{3\left(-27x+3\sqrt{-24\ln(x)^3+72\ln(x)^2 c_1-72\ln(x)\,c_1^2+24c_1^3+81x^2}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(\frac{i\sqrt{3}}{6}+\frac{1}{6}\right)\left(-27x+3\sqrt{-24\ln(x)^3+72\ln(x)^2 c_1-72\ln(x)\,c_1^2+24c_1^3+81x^2}\right)^{\frac{2}{3}}+(-\ln(x)+c_1)\,(\cdots}{\left(-27x+3\sqrt{-24\ln(x)^3+72\ln(x)^2 c_1-72\ln(x)\,c_1^2+24c_1^3+81x^2}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{\frac{\left(i\sqrt{3}-1\right)\left(-27x+3\sqrt{-24\ln(x)^3+72\ln(x)^2c_1-72\ln(x)c_1^2+24c_1^3+81x^2}\right)^{\frac{2}{3}}}{6}+(-\ln(x)+c_1)\left(1+i\sqrt{3}\right)}{\left(-27x+3\sqrt{-24\ln(x)^3+72\ln(x)^2 c_1-72\ln(x)\,c_1^2+24c_1^3+81x^2}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 5.771 (sec). Leaf size: 320

```
DSolve[y'[x] == ((x - y[x])*y[x])/(x*(x - y[x]^3)),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2\sqrt[3]{2}(-\log(x)+c_1)}{\sqrt[3]{54x+2\sqrt{729x^2+(-6\log(x)+6c_1)^3}}} - \frac{\sqrt[3]{54x+2\sqrt{729x^2+(-6\log(x)+6c_1)^3}}}{3\sqrt[3]{2}}$$

$$y(x) \to \frac{i\sqrt[3]{2}\left(\sqrt{3}+i\right)(-\log(x)+c_1)}{\sqrt[3]{54x+2\sqrt{729x^2+(-6\log(x)+6c_1)^3}}} + \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{54x+2\sqrt{729x^2+(-6\log(x)+6c_1)^3}}}{6\sqrt[3]{2}}$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)\sqrt[3]{54x+2\sqrt{729x^2+(-6\log(x)+6c_1)^3}}}{6\sqrt[3]{2}} - \frac{i\sqrt[3]{2}\left(\sqrt{3}-i\right)(-\log(x)+c_1)}{\sqrt[3]{54x+2\sqrt{729x^2+(-6\log(x)+6c_1)^3}}}$$

$$y(x) \to 0$$

2.154 problem 730

Internal problem ID [9065]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 730.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y' - \frac{\left(2y^{\frac{3}{2}} - 3\,\mathrm{e}^{x}\right)^{3} \mathrm{e}^{x}}{4\left(2y^{\frac{3}{2}} - 3\,\mathrm{e}^{x} + 2\right)\sqrt{y}} = 0$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 37

```
dsolve(diff(y(x),x) = 1/4*(2*y(x)^(3/2)-3*exp(x))^3*exp(x)/(2*y(x)^(3/2)-3*exp(x)+2)/y(x)^(1
```

$$\mathrm{e}^{x} - \frac{2\left(\int^{y(x)^{\frac{3}{2}} - \frac{3\,\mathrm{e}^{x}}{2}} \frac{\_a+1}{\_a^{3}-\_a-1} d\_a\right)}{3} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.222 (sec). Leaf size: 83

```
DSolve[y'[x] == (E^x*(-3*E^x + 2*y[x]^(3/2))^3)/(4*Sqrt[y[x]]*(2 - 3*E^x + 2*y[x]^(3/2))),y[
```

$$\text{Solve}\left[-\frac{2}{3}\text{RootSum}\left[\#1^3 - \#1 - 1\&, \frac{\#1 \log\left(-\#1 + y(x)^{3/2} - \frac{3e^x}{2}\right) + \log\left(-\#1 + y(x)^{3/2} - \frac{3e^x}{2}\right)}{3\#1^2 - 1}\&\right] + e^x - c_1 = 0, y(x)\right]$$

2.155 problem 731

Internal problem ID [9066]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 731.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$y' - \frac{1+2y}{x\left(-2+y^2x+2xy^3\right)} = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 46

```
dsolve(diff(y(x),x) = 1/x*(1+2*y(x))/(-2+x*y(x)^2+2*x*y(x)^3),y(x), singsol=all)
```

$$y(x) = -\frac{1}{2}$$
$$y(x) = \frac{e^{\operatorname{RootOf}\left(x\,e^{3\_Z}-4x\,e^{2\_Z}+8c_1x\,e^{\_Z}+2\_Zx\,e^{\_Z}+3x\,e^{\_Z}+16\right)}}{2} - \frac{1}{2}$$

✓ Solution by Mathematica
Time used: 0.3 (sec). Leaf size: 47

```
DSolve[y'[x] == (1 + 2*y[x])/(x*(-2 + x*y[x]^2 + 2*x*y[x]^3)),y[x],x,IncludeSingularSolutions
```

$$\text{Solve}\left[\frac{1}{64}\left(-4y(x)^2+4y(x)-2\log(8y(x)+4)+3\right)-\frac{1}{4x(2y(x)+1)}=c_1,y(x)\right]$$

2.156 problem 732

Internal problem ID [9067]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 732.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' - \frac{-x^2 - x - ax - a + 2\sqrt{x^2 + 2ax + a^2 + 4y}\, x^3}{2(x+1)} = 0$$

✓ Solution by Maple
Time used: 0.156 (sec). Leaf size: 43

```
dsolve(diff(y(x),x) = 1/2*(-x^2-x-a*x-a+2*x^3*(x^2+2*a*x+a^2+4*y(x))^(1/2))/(x+1),y(x), sing
```

$$c_1 + \frac{2x^3}{3} - x^2 - 2\ln(x+1) + 2x - \sqrt{x^2 + 2ax + a^2 + 4y(x)} = 0$$

✓ Solution by Mathematica
Time used: 1.568 (sec). Leaf size: 56

```
DSolve[y'[x] == (-1/2*a - x/2 - (a*x)/2 - x^2/2 + x^3*Sqrt[a^2 + 2*a*x + x^2 + 4*y[x]])/(1 +
```

$$y(x) \to \frac{1}{4}\left(-a^2 - 2ax - x^2 + \frac{1}{9}(-2x^3 + 3x^2 - 6x + 6\log(-x-1) + 6c_1)^2\right)$$

2.157 problem 733

Internal problem ID [9068]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 733.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - \frac{2x \sin(x) - \ln(2x) + \ln(2x)\, x^4 - 2\ln(2x)\, x^2 y + \ln(2x)\, y^2}{\sin(x)} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) = (2*x*sin(x)-ln(2*x)+ln(2*x)*x^4-2*ln(2*x)*x^2*y(x)+ln(2*x)*y(x)^2)/sin
```

No solution found

✓ Solution by Mathematica

Time used: 17.66 (sec). Leaf size: 82

```
DSolve[y'[x] == Csc[x]*(-Log[2*x] + x^4*Log[2*x] + 2*x*Sin[x] - 2*x^2*Log[2*x]*y[x] + Log[2*
```

$$y(x) \to \frac{\exp\left(\int_1^x 2\csc(K[5])\log(2K[5])dK[5]\right)}{-\int_1^x \exp\left(\int_1^{K[6]} 2\csc(K[5])\log(2K[5])dK[5]\right)\csc(K[6])\log(2K[6])dK[6] + c_1} + x^2 + 1$$
$$y(x) \to x^2 + 1$$

2.158 problem 734

Internal problem ID [9069]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 734.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' - \frac{(-\ln(y)\,x - \ln(y) + x^3)\,y}{x+1} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 33

```
dsolve(diff(y(x),x) = (-ln(y(x))*x-ln(y(x))+x^3)*y(x)/(x+1),y(x), singsol=all)
```

$$y(x) = e^{x^2-3x+4+e^{-x}c_1+\mathrm{expIntegral}_1(-x-1)e^{-x-1}}$$

✓ Solution by Mathematica

Time used: 0.684 (sec). Leaf size: 37

```
DSolve[y'[x] == ((x^3 - Log[y[x]] - x*Log[y[x]])*y[x])/(1 + x),y[x],x,IncludeSingularSolutio
```

$$y(x) \to \exp\left(-e^{-x-1}\,\mathrm{ExpIntegralEi}(x+1) + x^2 - 3x - c_1e^{-x} + 4\right)$$

2.159 problem 735

Internal problem ID [9070]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 735.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]'], [_Abel, '2nd type`

$$y' - \frac{(-1 + 2y\ln(x))^3}{(-1 + 2y\ln(x) - y)\, x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 78

```
dsolve(diff(y(x),x) = (-1+2*y(x)*ln(x))^3/(-1+2*y(x)*ln(x)-y(x))/x,y(x), singsol=all)
```

$$y(x) = \frac{71\,\mathrm{RootOf}\left(-82944\left(\int^{\_Z} \frac{1}{5041\_\_a^3-27648\_\_a+27648}d\_a\right) - 16\ln(x) + 3c_1\right) - 120}{(142\ln(x) - 71)\,\mathrm{RootOf}\left(-82944\left(\int^{\_Z} \frac{1}{5041\_\_a^3-27648\_\_a+27648}d\_a\right) - 16\ln(x) + 3c_1\right) - 240\ln(x) + 4}$$

✓ Solution by Mathematica

Time used: 1.151 (sec). Leaf size: 573

```
DSolve[y'[x] == (-1 + 2*Log[x]*y[x])^3/(x*(-1 - y[x] + 2*Log[x]*y[x])),y[x],x,IncludeSingula
```

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{2(2\log(x)K[1]-K[1]-1)}{8\log^3(x)K[1]^3+4\log(x)K[1]^3-2K[1]^3-12\log^2(x)K[1]^2-2K[1]^2+6\log(x)K[1]-1}\right.\right.$$

$$+2\text{RootSum}\left[2K[1]^3-2\#1K[1]^2-\#1^3\&,\frac{K[1]\log(2K[1]\log(x)-\#1-1)-\log(2K[1]\log(x)-\#1-1)\#}{2K[1]^2+3\#1^2}\right.$$

$$+\frac{\text{RootSum}\left[2K[1]^3-2\#1K[1]^2-\#1^3\&,\frac{16\log(x)K[1]^3-16\log(x)\log(2K[1]\log(x)-\#1-1)K[1]^3-24\log(2K[1]\log(x)-\#1-}{}\right.}{}$$

$$-2\left(y(x)\text{RootSum}\left[-\#1^3-2\#1y(x)^2+2y(x)^3\&,\frac{y(x)\log(-\#1+2y(x)\log(x)-1)-\#1\log(-\#1+2y(}{3\#1^2+2y(x)^2}\right.\right.$$

$$\left.\left.+\log(x)\right)=c_1,y(x)\right]$$

2.160 problem 736

Internal problem ID [9071]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 736.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Ricca`

$$\boxed{y' - \frac{2x^2 + 2x + x^4 - 2yx^2 - 1 + y^2}{x + 1} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x) = (2*x^2+2*x+x^4-2*x^2*y(x)-1+y(x)^2)/(x+1),y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^4 + 2x^3 - x^2 - 2x - 2) + x^2 + 1}{1 + c_1 (x^2 + 2x)}$$

✓ Solution by Mathematica

Time used: 0.333 (sec). Leaf size: 39

```
DSolve[y'[x] == (-1 + 2*x + 2*x^2 + x^4 - 2*x^2*y[x] + y[x]^2)/(1 + x),y[x],x,IncludeSingula
```

$$y(x) \to x^2 - \frac{2(x+1)^2}{x^2 + 2x - 2c_1} + 1$$
$$y(x) \to x^2 + 1$$

2.161 problem 737

Internal problem ID [9072]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 737.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], [_Abel`

$$\boxed{y' - \frac{x(-1 + x - 2yx + 2x^3)}{x^2 - y} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve(diff(y(x),x) = 1/(x^2-y(x))*x*(-1+x-2*x*y(x)+2*x^3),y(x), singsol=all)
```

$$y(x) = x^2 + \frac{\text{LambertW}\left(-2c_1 \mathrm{e}^{\frac{4}{3}x^3 - 2x^2 - 1}\right)}{2} + \frac{1}{2}$$

✓ Solution by Mathematica

Time used: 3.498 (sec). Leaf size: 47

```
DSolve[y'[x] == (x*(-1 + x + 2*x^3 - 2*x*y[x]))/(x^2 - y[x]),y[x],x,IncludeSingularSolutions
```

$$y(x) \to x^2 + \frac{1}{2}\left(1 + W\left(-e^{\frac{4x^3}{3} - 2x^2 - 1 + c_1}\right)\right)$$
$$y(x) \to x^2 + \frac{1}{2}$$

2.162 problem 738

Internal problem ID [9073]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 738.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - \frac{2a}{-yx^2 + 2ay^4x^2 - 16a^2xy^2 + 32a^3} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 968

```
dsolve(diff(y(x),x) = 2*a/(-x^2*y(x)+2*a*y(x)^4*x^2-16*a^2*x*y(x)^2+32*a^3),y(x), singsol=al
```

$$y(x) = \frac{192c_1^2a^3x + x^2 - x\left(-216c_1^3a^2x^3 + 576c_1^2a^3x^2 + 12ac_1x^2\sqrt{\frac{(324a^2c_1^4+3c_1)x^3+(-1728a^3c_1^3-12a)x^2+1536c_1^2a^4x-49152c}{x}}\right.}{12c_1xa\left(-216c_1^3a^2x^3 + 576c_1^2a^3x^2 + 12ac_1x^2\sqrt{\frac{(3}{}}\right.}$$

$$y(x) = \frac{\frac{\left(-i\sqrt{3}-1\right)\left(-216c_1^3a^2x^3+576c_1^2a^3x^2+12ac_1x^2\sqrt{\frac{(324a^2c_1^4+3c_1)x^3+(-1728a^3c_1^3-12a)x^2+1536c_1^2a^4x-49152c_1^4a^7}{x}}-x^3\right)^{\frac{2}{3}}}{24} + 8x\left(-\frac{\left(-216c_1^3\right.}{}\right.}{c_1xa\left(-216c_1^3a^2x^3 + 576c_1^2a^3x^2 + 12ac_1x^2\sqrt{\frac{(324a^2c_1^4+}{}}\right.}$$

$$y(x) = \frac{\frac{\left(i\sqrt{3}-1\right)\left(-216c_1^3a^2x^3+576c_1^2a^3x^2+12ac_1x^2\sqrt{\frac{(324a^2c_1^4+3c_1)x^3+(-1728a^3c_1^3-12a)x^2+1536c_1^2a^4x-49152c_1^4a^7}{x}}-x^3\right)^{\frac{2}{3}}}{24} + 8\left(-\frac{\left(-216c_1^3a^2\right.}{}\right.}{c_1xa\left(-216c_1^3a^2x^3 + 576c_1^2a^3x^2 + 12ac_1x^2\sqrt{\frac{(324a^2c_1^4}{}}\right.}$$

✓ Solution by Mathematica

Time used: 60.321 (sec). Leaf size: 1200

```
DSolve[y'[x] == (2*a)/(32*a^3 - x^2*y[x] - 16*a^2*x*y[x]^2 + 2*a*x^2*y[x]^4),y[x],x,IncludeS
```

$y(x)$

$$\frac{2\sqrt[3]{2304a^4x^2 - 64a^3x^3 + 576a^3e^{c_1}x^2 - 216a^2x^3 - 48a^2e^{c_1}x^3 + \sqrt{x^3\left(x\left(-2304a^4 - 64a^3\left(-x + 9e^{c_1}\right) + \right.\right.}}{x}$$

$\rightarrow$

$y(x)$

$$\frac{2i\left(\sqrt{3}+i\right)\sqrt[3]{2304a^4x^2 - 64a^3x^3 + 576a^3e^{c_1}x^2 - 216a^2x^3 - 48a^2e^{c_1}x^3 + \sqrt{x^3\left(x\left(-2304a^4 - 64a^3\left(-x + 9\right.\right.\right.}}}{x}$$

$\rightarrow$

$y(x)$

$$-\frac{2i\left(\sqrt{3}-i\right)\sqrt[3]{2304a^4x^2 - 64a^3x^3 + 576a^3e^{c_1}x^2 - 216a^2x^3 - 48a^2e^{c_1}x^3 + \sqrt{x^3\left(x\left(-2304a^4 - 64a^3\left(-x + \right.\right.\right.}}}{x}$$

$\rightarrow$

2.163 problem 739

Internal problem ID [9074]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 739.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{y' - \frac{1+2y}{x\left(-2+yx+2y^2x\right)} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve(diff(y(x),x) = 1/x*(1+2*y(x))/(-2+x*y(x)+2*x*y(x)^2),y(x), singsol=all)
```

$$y(x) = -\frac{1}{2}$$
$$y(x) = \frac{e^{\mathrm{RootOf}\left(x\,e^{2\_Z}+2c_1x\,e^{\_Z}-\_Zx\,e^{\_Z}-x\,e^{\_Z}+4\right)}}{2} - \frac{1}{2}$$

✓ Solution by Mathematica

Time used: 0.244 (sec). Leaf size: 39

```
DSolve[y'[x] == (1 + 2*y[x])/(x*(-2 + x*y[x] + 2*x*y[x]^2)),y[x],x,IncludeSingularSolutions
```

$$\mathrm{Solve}\left[\frac{1}{8}(-2y(x)+\log(4y(x)+2)-1)-\frac{1}{2x(2y(x)+1)} = c_1, y(x)\right]$$

2.164 problem 740

Internal problem ID [9075]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 740.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{y' - \frac{x + y^4 - 2y^2x^2 + x^4}{y} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 72

```
dsolve(diff(y(x),x) = (x+y(x)^4-2*x^2*y(x)^2+x^4)/y(x),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{2}\,\sqrt{(x+c_1)\,(2c_1x^2+2x^3-1)}}{2c_1+2x}$$
$$y(x) = \frac{\sqrt{2}\,\sqrt{(x+c_1)\,(2c_1x^2+2x^3-1)}}{2c_1+2x}$$

✓ Solution by Mathematica
Time used: 0.661 (sec). Leaf size: 132

```
DSolve[y'[x] == (x + x^4 - 2*x^2*y[x]^2 + y[x]^4)/y[x],y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -\frac{\sqrt{2x^3+2c_1x^2-1}}{\sqrt{2}\sqrt{x+c_1}}$$
$$y(x) \to \frac{\sqrt{2x^3+2c_1x^2-1}}{\sqrt{2}\sqrt{x+c_1}}$$
$$y(x) \to -i\sqrt{-x^2}$$
$$y(x) \to i\sqrt{-x^2}$$
$$y(x) \to -\sqrt{x^2}$$
$$y(x) \to \sqrt{x^2}$$

2.165 problem 741

Internal problem ID [9076]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 741.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$
y' - \frac{(y^2 a + b\,x^2)^3\,x}{a^{\frac{5}{2}}\,(y^2 a + b\,x^2 + a)\,y} = 0
$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 242

```
dsolve(diff(y(x),x) = (a*y(x)^2+b*x^2)^3/a^(5/2)*x/(a*y(x)^2+b*x^2+a)/y(x),y(x), singsol=all
```

$$
\frac{\int_{\_b}^{x} \frac{\left(b\_a^2 + a y(x)^2\right)^3 \_a}{b\left(y(x)^2+1\right)a^{\frac{5}{2}} + a^{\frac{3}{2}} b^2 \_a^2 + \left(b\_a^2 + a y(x)^2\right)^3} d\_a}{a^3}
- \frac{\int^{y(x)} \frac{2\left(\left(b\left(\_f^2+1\right)a^{\frac{5}{2}} + a^{\frac{3}{2}} b^2 x^2 + \left(a\_f^2 + b\,x^2\right)^3\right) b \left(\int_{\_b}^{x} \frac{\left(b\_a^2 + a\_f^2\right)^2 \_a \left(2b\_a^2 + 2a\_f^2 + 3a\right)}{\left(b\left(\_f^2+1\right)a^{\frac{5}{2}} + a^{\frac{3}{2}} b^2 \_a^2 + \left(b\_a^2 + a\_f^2\right)^3\right)^2} d\_a\right) + \frac{b\,x^2}{2} + \frac{a\left(\_f^2+1\right)}{2}\right)}{b\left(\_f^2+1\right)a^{\frac{5}{2}} + a^{\frac{3}{2}} b^2 x^2 + \left(a\_f^2 + b\,x^2\right)^3}}{\sqrt{a}}
+ c_1 = 0
$$

✓ Solution by Mathematica

Time used: 0.84 (sec). Leaf size: 175

```
DSolve[y'[x] == (x*(b*x^2 + a*y[x]^2)^3)/(a^(5/2)*y[x]*(a + b*x^2 + a*y[x]^2)),y[x],x,Includ
```

$$\text{Solve}\Bigg[\frac{1}{2}\bigg(x^2$$

$$-a^{3/2}\text{RootSum}\Big[\#1^3b^3+3\#1^2ab^2y(x)^2+\#1a^{3/2}b^2+3\#1a^2by(x)^4+a^{5/2}by(x)^2+a^{5/2}b+a^3y(x)^6\&,\ \frac{ay(x)^2\log}{}$$

2.166 problem 742

Internal problem ID [9077]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 742.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `unknown`

$$y' + \frac{\cos(y)(x - \cos(y) + 1)}{(x\sin(y) - 1)(x + 1)} = 0$$

✓ Solution by Maple

Time used: 0.391 (sec). Leaf size: 239

```
dsolve(diff(y(x),x) = -cos(y(x))/(x*sin(y(x))-1)*(x-cos(y(x))+1)/(x+1),y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{(-\ln(x+1)+c_1)\sqrt{\ln(x+1)^2 - 2c_1\ln(x+1) + c_1^2 - x^2 + 1} + x}{c_1^2 - 2c_1\ln(x+1) + \ln(x+1)^2 + 1}, \frac{\ln(x+1)x - c_1x + \sqrt{\ln(}}{c_1^2 - 2c_1\ln(}\right.$$

$$y(x) = \arctan\left(\frac{(\ln(x+1)-c_1)\sqrt{\ln(x+1)^2 - 2c_1\ln(x+1) + c_1^2 - x^2 + 1} + x}{c_1^2 - 2c_1\ln(x+1) + \ln(x+1)^2 + 1}, \frac{\ln(x+1)x - c_1x - \sqrt{\ln(x}}{c_1^2 - 2c_1\ln(x}\right.$$

✓ Solution by Mathematica

Time used: 51.98 (sec). Leaf size: 315

```
DSolve[y'[x] == -(((1 + x - Cos[y[x]])*Cos[y[x]])/((1 + x)*(-1 + x*Sin[y[x]]))),y[x],x,Inclu
```

$$y(x) \to -\sec^{-1}\left(\frac{-\sqrt{-x^2+\log^2(x+1)+2c_1\log(x+1)+1+c_1{}^2}+x\log(x+1)+c_1x}{x^2-1}\right)$$

$$y(x) \to \sec^{-1}\left(\frac{-\sqrt{-x^2+\log^2(x+1)+2c_1\log(x+1)+1+c_1{}^2}+x\log(x+1)+c_1x}{x^2-1}\right)$$

$$y(x) \to -\sec^{-1}\left(\frac{\sqrt{-x^2+\log^2(x+1)+2c_1\log(x+1)+1+c_1{}^2}+x\log(x+1)+c_1x}{x^2-1}\right)$$

$$y(x) \to \sec^{-1}\left(\frac{\sqrt{-x^2+\log^2(x+1)+2c_1\log(x+1)+1+c_1{}^2}+x\log(x+1)+c_1x}{x^2-1}\right)$$

$$y(x) \to -\frac{\pi}{2}$$

$$y(x) \to \frac{\pi}{2}$$

$$y(x) \to \sec^{-1}\left(\frac{x\log(x+1)-\sqrt{-x^2+\log^2(x+1)+1}}{x^2-1}\right)$$

$$y(x) \to \sec^{-1}\left(\frac{\sqrt{-x^2+\log^2(x+1)+1}+x\log(x+1)}{x^2-1}\right)$$

2.167 problem 743

Internal problem ID [9078]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 743.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_rational]

$$y' + \frac{i(8ix + 16y^4 + 8y^2x^2 + x^4)}{32y} = 0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 264

```
dsolve(diff(y(x),x) = -1/32*I*(8*I*x+16*y(x)^4+8*x^2*y(x)^2+x^4)/y(x),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{2}\sqrt{\left(\left(1+i\sqrt{3}\right)c_1 \operatorname{AiryAi}\left(1, -\frac{\left(-i+\sqrt{3}\right)x}{2}\right) + \left(1+i\sqrt{3}\right)\operatorname{AiryBi}\left(1, -\frac{\left(-i+\sqrt{3}\right)x}{2}\right) - \frac{x^2\left(\operatorname{AiryAi}\left(-\frac{\left(-i+\right.}{2}\right.\right.}{}\right.}}{2\operatorname{AiryAi}\left(-\frac{\left(-i+\sqrt{3}\right)x}{2}\right)c_1 + 2\operatorname{AiryB}}$$

$$y(x) = \frac{\sqrt{2}\sqrt{\left(\left(1+i\sqrt{3}\right)c_1 \operatorname{AiryAi}\left(1, -\frac{\left(-i+\sqrt{3}\right)x}{2}\right) + \left(1+i\sqrt{3}\right)\operatorname{AiryBi}\left(1, -\frac{\left(-i+\sqrt{3}\right)x}{2}\right) - \frac{x^2\left(\operatorname{AiryAi}\left(-\frac{\left(-i+\sqrt{3}\right)}{2}\right.\right.}{}\right.}}{2\operatorname{AiryAi}\left(-\frac{\left(-i+\sqrt{3}\right)x}{2}\right)c_1 + 2\operatorname{AiryBi}\left(\right.}$$

✓ Solution by Mathematica

Time used: 5.963 (sec). Leaf size: 553

```
DSolve[y'[x] == ((-1/32*I)*((8*I)*x + x^4 + 8*x^2*y[x]^2 + 16*y[x]^4))/y[x],y[x],x,IncludeSi
```

$$y(x) \to -\frac{\sqrt{-\left(\left(\text{AiryBi}\left(-\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)+c_1\,\text{AiryAi}\left(-\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)\right)\left(x^2\,\text{AiryBi}\left(-\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)+c_1\,(\right.}}{2\left(\text{AiryBi}\left(-\frac{1}{2}\left(-\right.\right.\right.}$$

$$y(x) \to \frac{\sqrt{-\left(\left(\text{AiryBi}\left(-\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)+c_1\,\text{AiryAi}\left(-\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)\right)\left(x^2\,\text{AiryBi}\left(-\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)+c_1\,(x\right.}}{2\left(\text{AiryBi}\left(-\frac{1}{2}\left(-i\right.\right.\right.}$$

$$y(x) \to -\frac{\sqrt{-\text{AiryAi}\left(-\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)\left(x^2\,\text{AiryAi}\left(-\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)-2i\left(\sqrt{3}-i\right)\text{AiryAiPrime}\left(-\frac{1}{2}\left(-i+\right.\right.\right.}}{2\,\text{AiryAi}\left(-\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}$$

$$y(x) \to \frac{\sqrt{-\text{AiryAi}\left(-\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)\left(x^2\,\text{AiryAi}\left(-\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)-2i\left(\sqrt{3}-i\right)\text{AiryAiPrime}\left(-\frac{1}{2}\left(-i+\sqrt{\right.\right.\right.}}{2\,\text{AiryAi}\left(-\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}$$

2.168 problem 744

Internal problem ID [9079]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 744.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{x}{-y + x^4 + 2y^2x^2 + y^4} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 511

```
dsolve(diff(y(x),x) = x/(-y(x)+x^4+2*x^2*y(x)^2+y(x)^4),y(x), singsol=all)
```

$$y(x) = \frac{\left(-36c_1x^2 - 54 - c_1^3 + 6\sqrt{3c_1^4x^2 + 24c_1^2x^4 + 48x^6 + 3c_1^3 + 108c_1x^2 + 81}\right)^{\frac{1}{3}}}{6} + \frac{c_1^2 - 12x^2}{6\left(-36c_1x^2 - 54 - c_1^3 + 6\sqrt{3c_1^4x^2 + 24c_1^2x^4 + 48x^6 + 3c_1^3 + 108c_1x^2 + 81}\right)^{\frac{1}{3}}} - \frac{c_1}{6}$$

$$y(x) = -\frac{\left(\frac{i\sqrt{3}}{12} + \frac{1}{12}\right)\left(-36c_1x^2 - 54 - c_1^3 + 6\sqrt{48x^6 + 24c_1^2x^4 + (3c_1^4 + 108c_1)x^2 + 3c_1^3 + 81}\right)^{\frac{2}{3}} + \frac{c_1\left(-36c_1x^2 - 54\right.}{}}{\left(-36c_1x^2 - 54 - c_1^3 + 6\sqrt{48x^6 + 24c_1^2x^4 + (3c_1^4 + 108c_1\right.}}$$

$$y(x) = \frac{\frac{\left(i\sqrt{3}-1\right)\left(-36c_1x^2-54-c_1^3+6\sqrt{48x^6+24c_1^2x^4+(3c_1^4+108c_1)x^2+3c_1^3+81}\right)^{\frac{2}{3}}}{12} - \frac{c_1\left(-36c_1x^2-54-c_1^3+6\sqrt{48x^6+24c_1^2x^4+(3c_1^4+108c_1)x^2+3c\right.}{6}}{\left(-36c_1x^2 - 54 - c_1^3 + 6\sqrt{48x^6 + 24c_1^2x^4 + (3c_1^4 + 108c_1)x^2 + 3c_1^3 + 81}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 16.665 (sec). Leaf size: 564

```
DSolve[y'[x] == x/(x^4 - y[x] + 2*x^2*y[x]^2 + y[x]^4),y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{\sqrt[3]{144c_1x^2 + 2\sqrt{(12x^2 - 4c_1^2)^3 + 4(36c_1x^2 - 27 + 4c_1^3)^2} - 108 + 16c_1^3}}{6\sqrt[3]{2}} + \frac{2^{2/3}(-3x^2 + c_1^2)}{3\sqrt[3]{36c_1x^2 + 3\sqrt{3}\sqrt{16x^6 + 32c_1^2x^4 + 8c_1(-9 + 2c_1^3)x^2 + 27 - 8c_1^3} - 27 + 4c_1^3}} + \frac{c_1}{3}$$

$$y(x) \to \frac{(-1 + i\sqrt{3})\sqrt[3]{144c_1x^2 + 2\sqrt{(12x^2 - 4c_1^2)^3 + 4(36c_1x^2 - 27 + 4c_1^3)^2} - 108 + 16c_1^3}}{12\sqrt[3]{2}} + \frac{(1 + i\sqrt{3})(3x^2 - c_1^2)}{3\sqrt[3]{72c_1x^2 + 6\sqrt{3}\sqrt{16x^6 + 32c_1^2x^4 + 8c_1(-9 + 2c_1^3)x^2 + 27 - 8c_1^3} - 54 + 8c_1^3}} + \frac{c_1}{3}$$

$$y(x) \to -\frac{(1 + i\sqrt{3})\sqrt[3]{144c_1x^2 + 2\sqrt{(12x^2 - 4c_1^2)^3 + 4(36c_1x^2 - 27 + 4c_1^3)^2} - 108 + 16c_1^3}}{12\sqrt[3]{2}} + \frac{(1 - i\sqrt{3})(3x^2 - c_1^2)}{3\sqrt[3]{72c_1x^2 + 6\sqrt{3}\sqrt{16x^6 + 32c_1^2x^4 + 8c_1(-9 + 2c_1^3)x^2 + 27 - 8c_1^3} - 54 + 8c_1^3}} + \frac{c_1}{3}$$

$$y(x) \to -i\sqrt{x^2}$$

$$y(x) \to i\sqrt{x^2}$$

2.169 problem 745

Internal problem ID [9080]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 745.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]'], [_Abel, '2nd type`

$$y' - \frac{(-1 + y \ln (x))^3}{(-1 + y \ln (x) - y)\, x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 77

```
dsolve(diff(y(x),x) = (-1+y(x)*ln(x))^3/(-1+y(x)*ln(x)-y(x))/x,y(x), singsol=all)
```

$$y(x) = \frac{47 \operatorname{RootOf}\left(-27783\left(\int^{\_Z} \frac{1}{2209\_a^3-9261\_a+9261} d\_a\right) - 7\ln (x) + 3c_1\right) - 84}{21 + 47(-1+\ln (x)) \operatorname{RootOf}\left(-27783\left(\int^{\_Z} \frac{1}{2209\_a^3-9261\_a+9261} d\_a\right) - 7\ln (x) + 3c_1\right) - 84\ln (x)}$$

✓ Solution by Mathematica

Time used: 1.247 (sec). Leaf size: 546

```
DSolve[y'[x] == (-1 + Log[x]*y[x])^3/(x*(-1 - y[x] + Log[x]*y[x])),y[x],x,IncludeSingularSol
```

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{\log(x)K[1]-K[1]-1}{\log^3(x)K[1]^3+\log(x)K[1]^3-K[1]^3-3\log^2(x)K[1]^2-K[1]^2+3\log(x)K[1]-1}\right.\right.$$

$$+\text{RootSum}\left[K[1]^3-\#1K[1]^2-\#1^3\&,\frac{K[1]\log(K[1]\log(x)-\#1-1)-\log(K[1]\log(x)-\#1-1)\#1}{K[1]^2+3\#1^2}\&\right]$$

$$+\frac{\text{RootSum}\left[K[1]^3-\#1K[1]^2-\#1^3\&,\frac{4\log(x)K[1]^3-4\log(x)\log(K[1]\log(x)-\#1-1)K[1]^3-12\log(K[1]\log(x)-\#1-1)K[1]^3-}{}\right.}{}$$

$$-y(x)\text{RootSum}\left[-\#1^3-\#1y(x)^2\right.$$

$$\left.+y(x)^3\&,\frac{y(x)\log(-\#1+y(x)\log(x)-1)-\#1\log(-\#1+y(x)\log(x)-1)}{3\#1^2+y(x)^2}\&\right]$$

$$\left.-\log(x)=c_1,y(x)\right]$$

2.170 problem 746

Internal problem ID [9081]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 746.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' + \frac{i(ix + x^4 + 2y^2x^2 + y^4)}{y} = 0$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 232

```
dsolve(diff(y(x),x) = -I*(I*x+x^4+2*x^2*y(x)^2+y(x)^4)/y(x),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{2}\sqrt{\left(\mathrm{AiryAi}\left(-(-8i)^{\frac{1}{3}}x\right)c_1 + \mathrm{AiryBi}\left(-(-8i)^{\frac{1}{3}}x\right)\right)\left(\left(1+i\sqrt{3}\right)c_1 \,\mathrm{AiryAi}\left(1, -(-8i)^{\frac{1}{3}}x\right) + (1}}{2\,\mathrm{AiryAi}\left(-(-8i)^{\frac{1}{3}}x\right)c_1 + 2\,\mathrm{A}}$$

$$y(x) = \frac{\sqrt{2}\sqrt{\left(\mathrm{AiryAi}\left(-(-8i)^{\frac{1}{3}}x\right)c_1 + \mathrm{AiryBi}\left(-(-8i)^{\frac{1}{3}}x\right)\right)\left(\left(1+i\sqrt{3}\right)c_1 \,\mathrm{AiryAi}\left(1, -(-8i)^{\frac{1}{3}}x\right) + (1+}}{2\,\mathrm{AiryAi}\left(-(-8i)^{\frac{1}{3}}x\right)c_1 + 2\,\mathrm{Air}}$$

✓ Solution by Mathematica

Time used: 6.529 (sec). Leaf size: 413

```
DSolve[y'[x] == ((-I)*(I*x + x^4 + 2*x^2*y[x]^2 + y[x]^4))/y[x],y[x],x,IncludeSingularSoluti
```

$$y(x) \to -\frac{\sqrt{(\text{AiryBi}(2(-1)^{5/6}x) + c_1 \text{AiryAi}(2(-1)^{5/6}x))\left(-2x^2(\text{AiryBi}(2(-1)^{5/6}x) + c_1 \text{AiryAi}(2(-1)^{5/6}x))\right.}}{\sqrt{2}(\text{AiryBi}(2(-1)^{5/6}x) + c_1 \text{AiryA}}$$

$$y(x) \to \frac{\sqrt{(\text{AiryBi}(2(-1)^{5/6}x) + c_1 \text{AiryAi}(2(-1)^{5/6}x))\left(-2x^2(\text{AiryBi}(2(-1)^{5/6}x) + c_1 \text{AiryAi}(2(-1)^{5/6}x)) +\right.}}{\sqrt{2}(\text{AiryBi}(2(-1)^{5/6}x) + c_1 \text{AiryAi}}$$

$$y(x) \to -\frac{\sqrt{-\text{AiryAi}(2(-1)^{5/6}x)\left(2x^2 \text{AiryAi}(2(-1)^{5/6}x) + (-1 - i\sqrt{3}) \text{AiryAiPrime}(2(-1)^{5/6}x)\right)}}{\sqrt{2} \text{AiryAi}(2(-1)^{5/6}x)}$$

$$y(x) \to \frac{\sqrt{-\text{AiryAi}(2(-1)^{5/6}x)\left(2x^2 \text{AiryAi}(2(-1)^{5/6}x) + (-1 - i\sqrt{3}) \text{AiryAiPrime}(2(-1)^{5/6}x)\right)}}{\sqrt{2} \text{AiryAi}(2(-1)^{5/6}x)}$$

2.171 problem 747

Internal problem ID [9082]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 747.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$y' + \frac{y(\tan(x) + \ln(2x)\, x - \ln(2x)\, x^2 y)}{x \tan(x)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 62

```
dsolve(diff(y(x),x) = -y(x)*(tan(x)+ln(2*x)*x-ln(2*x)*x^2*y(x))/x/tan(x),y(x), singsol=all)
```

$$y(x) = \frac{e^{-\left(\int \frac{1+x(\ln(2)+\ln(x))\cot(x)}{x} dx\right)}}{-\left(\int \cot(x)\, e^{-\left(\int \frac{1+x(\ln(2)+\ln(x))\cot(x)}{x} dx\right)} (\ln(2) + \ln(x))\, x dx\right) + c_1}$$

✓ Solution by Mathematica

Time used: 6.681 (sec). Leaf size: 89

```
DSolve[y'[x] == -((Cot[x]*y[x]*(x*Log[2*x] + Tan[x] - x^2*Log[2*x]*y[x]))/x),y[x],x,IncludeS
```

$$y(x) \to \frac{\exp\left(\int_1^x \left(-\cot(K[1])\log(2K[1]) - \frac{1}{K[1]}\right) dK[1]\right)}{-\int_1^x \exp\left(\int_1^{K[2]} \left(-\cot(K[1])\log(2K[1]) - \frac{1}{K[1]}\right) dK[1]\right)\cot(K[2])K[2]\log(2K[2])dK[2] + c_1}$$

$$y(x) \to 0$$

2.172 problem 748

Internal problem ID [9083]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 748.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{y(x+y)}{x\,(x+y^3)} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 316

```
dsolve(diff(y(x),x) = y(x)*(x+y(x))/x/(x+y(x)^3),y(x), singsol=all)
```

$$y(x) = \frac{\left(27x+3\sqrt{-24c_1^3-72\ln(x)\,c_1^2-72\ln(x)^2\,c_1-24\ln(x)^3+81x^2}\right)^{\frac{2}{3}}+6\ln(x)+6c_1}{3\left(27x+3\sqrt{-24c_1^3-72\ln(x)\,c_1^2-72\ln(x)^2\,c_1-24\ln(x)^3+81x^2}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{\frac{\left(-i\sqrt{3}-1\right)\left(27x+3\sqrt{-24c_1^3-72\ln(x)c_1^2-72\ln(x)^2c_1-24\ln(x)^3+81x^2}\right)^{\frac{2}{3}}}{6}+\left(i\sqrt{3}-1\right)\left(\ln(x)+c_1\right)}{\left(27x+3\sqrt{-24c_1^3-72\ln(x)\,c_1^2-72\ln(x)^2\,c_1-24\ln(x)^3+81x^2}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{\frac{\left(i\sqrt{3}-1\right)\left(27x+3\sqrt{-24c_1^3-72\ln(x)c_1^2-72\ln(x)^2c_1-24\ln(x)^3+81x^2}\right)^{\frac{2}{3}}}{6}+\left(-i\sqrt{3}-1\right)\left(\ln(x)+c_1\right)}{\left(27x+3\sqrt{-24c_1^3-72\ln(x)\,c_1^2-72\ln(x)^2\,c_1-24\ln(x)^3+81x^2}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 5.664 (sec). Leaf size: 291

```
DSolve[y'[x] == (y[x]*(x + y[x]))/(x*(x + y[x]^3)),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{2\sqrt[3]{2}(\log(x)+c_1)}{\sqrt[3]{54x+\sqrt{2916x^2-864(\log(x)+c_1)^3}}}+\frac{\sqrt[3]{9x+\frac{1}{6}\sqrt{2916x^2-864(\log(x)+c_1)^3}}}{3^{2/3}}$$

$$y(x)\to \frac{\left(-1+i\sqrt{3}\right)\sqrt[3]{54x+\sqrt{2916x^2-864(\log(x)+c_1)^3}}}{6\sqrt[3]{2}}-\frac{\sqrt[3]{2}\left(1+i\sqrt{3}\right)(\log(x)+c_1)}{\sqrt[3]{54x+\sqrt{2916x^2-864(\log(x)+c_1)^3}}}$$

$$y(x)\to -\frac{\sqrt[3]{2}\left(1-i\sqrt{3}\right)(\log(x)+c_1)}{\sqrt[3]{54x+\sqrt{2916x^2-864(\log(x)+c_1)^3}}}-\frac{\left(1+i\sqrt{3}\right)\sqrt[3]{54x+\sqrt{2916x^2-864(\log(x)+c_1)^3}}}{6\sqrt[3]{2}}$$

$$y(x)\to 0$$

2.173 problem 749

Internal problem ID [9084]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 749.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{(x-y)^2 (x+y)^2 x}{y} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 186

```
dsolve(diff(y(x),x) = (x-y(x))^2*(x+y(x))^2*x/y(x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\left(c_1 \mathrm{e}^{-\frac{(x^2+1)^2}{2}} + \mathrm{e}^{-\frac{x^2(x^2-2)}{2}}\right)\left(c_1 (x^2+1)\,\mathrm{e}^{-\frac{(x^2+1)^2}{2}} + (x^2-1)\,\mathrm{e}^{-\frac{x^2(x^2-2)}{2}}\right)}}{c_1 \mathrm{e}^{-\frac{(x^2+1)^2}{2}} + \mathrm{e}^{-\frac{x^2(x^2-2)}{2}}}$$

$$y(x) = -\frac{\sqrt{\left(c_1 \mathrm{e}^{-\frac{(x^2+1)^2}{2}} + \mathrm{e}^{-\frac{x^2(x^2-2)}{2}}\right)\left(c_1 (x^2+1)\,\mathrm{e}^{-\frac{(x^2+1)^2}{2}} + (x^2-1)\,\mathrm{e}^{-\frac{x^2(x^2-2)}{2}}\right)}}{c_1 \mathrm{e}^{-\frac{(x^2+1)^2}{2}} + \mathrm{e}^{-\frac{x^2(x^2-2)}{2}}}$$

✓ Solution by Mathematica

Time used: 18.166 (sec). Leaf size: 102

```
DSolve[y'[x] == (x*(x - y[x])^2*(x + y[x])^2)/y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{x^2 + (x^2-1)\,e^{2x^2+4c_1} + 1}}{\sqrt{1 + e^{2x^2+4c_1}}}$$

$$y(x) \to \frac{\sqrt{x^2 + (x^2-1)\,e^{2x^2+4c_1} + 1}}{\sqrt{1 + e^{2x^2+4c_1}}}$$

2.174 problem 750

Internal problem ID [9085]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 750.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' - \frac{(x^2 + 3y^2)\,y}{(6y^2 + x)\,x} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 57

```
dsolve(diff(y(x),x) = 1/(6*y(x)^2+x)*(x^2+3*y(x)^2)*y(x)/x,y(x), singsol=all)
```

$$\frac{y(x)^2 x}{6y(x)^2 + x} = \frac{\left(\mathrm{e}^{\mathrm{RootOf}\left(-\mathrm{e}^{\_Z}\ln\left(\left(\mathrm{e}^{\_Z}+9\right)x\right)+\mathrm{e}^{\_Z}\ln(2)+3c_1\mathrm{e}^{\_Z}+\mathrm{e}^{\_Z}\_Z+2x\,\mathrm{e}^{\_Z}+9\right)} + 9\right)x}{54}$$

✓ Solution by Mathematica
Time used: 5.981 (sec). Leaf size: 73

```
DSolve[y'[x] == (y[x]*(x^2 + 3*y[x]^2))/(x*(x + 6*y[x]^2)),y[x],x,IncludeSingularSolutions -
```

$$y(x) \to -\frac{\sqrt{x}\sqrt{W\left(\frac{6e^{2(x+c_1)}}{x}\right)}}{\sqrt{6}}$$
$$y(x) \to \frac{\sqrt{x}\sqrt{W\left(\frac{6e^{2(x+c_1)}}{x}\right)}}{\sqrt{6}}$$
$$y(x) \to 0$$

2.175 problem 751

Internal problem ID [9086]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 751.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' - \frac{(\ln(y)\,x + \ln(y) + x^4)\,y}{x\,(x+1)} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x) = (ln(y(x))*x+ln(y(x))+x^4)*y(x)/x/(x+1),y(x), singsol=all)
```

$$y(x) = (x+1)^x \, \mathrm{e}^{\frac{x\left(x^2+2c_1-2x\right)}{2}}$$

✓ Solution by Mathematica
Time used: 0.367 (sec). Leaf size: 29

```
DSolve[y'[x] == ((x^4 + Log[y[x]] + x*Log[y[x]])*y[x])/(x*(1 + x)),y[x],x,IncludeSingularSol
```

$$y(x) \to (x+1)^x e^{\frac{1}{2}x\left(x^2-2x+2c_1\right)}$$

2.176 problem 752

Internal problem ID [9087]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 752.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$y' - \frac{\cos(y)(\cos(y)x^3 - x - 1)}{(x\sin(y) - 1)(x + 1)} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 723

```
dsolve(diff(y(x),x) = cos(y(x))/(x*sin(y(x))-1)*(cos(y(x))*x^3-x-1)/(x+1),y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{(2x^3 - 3x^2 - 6\ln(x+1) + 6c_1 + 6x)\sqrt{36\ln(x+1)^2 + (-24x^3 + 36x^2 - 72c_1 - 72x)\ln(x+}}{36\ln(x+1)^2 + (-24x^3 + 36x^2 - 72c_1 - 72x)\ln(x+1) + 4x^6 - 12x^5 + 33}\right.$$

$$y(x) = \arctan\left(\frac{(-2x^3 + 3x^2 + 6\ln(x+1) - 6c_1 - 6x)\sqrt{36\ln(x+1)^2 + (-24x^3 + 36x^2 - 72c_1 - 72x)\ln(x-}}{36\ln(x+1)^2 + (-24x^3 + 36x^2 - 72c_1 - 72x)\ln(x+1) + 4x^6 - 12x^5 + 33}\right.$$

✓ Solution by Mathematica

Time used: 4.957 (sec). Leaf size: 867

```
DSolve[y'[x] == (Cos[y[x]]*(-1 - x + x^3*Cos[y[x]]))/((1 + x)*(-1 + x*Sin[y[x]])),y[x],x,Inc
```

$$y(x) \to \tan^{-1}\left(\frac{6\left(2x^4 - 3x^3 + 6x^2 + \sqrt{4x^6 - 12x^5 + 33x^4 + 12(-3+2c_1)x^3 - 36c_1x^2 - 12\left(2x^3 - 3x^2 + 6x +\right.}\right.}{4x^6 - 12x^5 + 33x^4 + 12(-3+2c_1)x^3 - 36(-1+c_1)x^2 - 12\left(2x^3 - 3x^2 +\right.}\right.$$

$$-\frac{\left(2x^3 - 3x^2 + 6x - 6\log(x+1) + 6c_1\right)\left(2x^4 - 3x^3 + 6x^2 + \sqrt{4x^6 - 12x^5 + 33x^4 + 12(-3+2c_1)x^3 -}\right.}{4x^6 - 12x^5 + 33x^4 + 12(-3+2c_1)x^3 - 36(-1+c_1)x^2 - 12(}$$

$$y(x) \to \tan^{-1}\left(-\frac{6\left(-2x^4 + 3x^3 - 6x^2 + \sqrt{4x^6 - 12x^5 + 33x^4 + 12(-3+2c_1)x^3 - 36c_1x^2 - 12\left(2x^3 - 3x^2 +\right.}\right.}{4x^6 - 12x^5 + 33x^4 + 12(-3+2c_1)x^3 - 36(-1+c_1)x^2 - 12\left(2x^3 - 3x^2\right.}\right.$$

$$-\frac{\left(2x^3 - 3x^2 + 6x - 6\log(x+1) + 6c_1\right)\left(2x^4 - 3x^3 + 6x^2 - \sqrt{4x^6 - 12x^5 + 33x^4 + 12(-3+2c_1)x^3 -}\right.}{4x^6 - 12x^5 + 33x^4 + 12(-3+2c_1)x^3 - 36(-1+c_1)x^2 - 12(}$$

$$y(x) \to -\frac{\pi}{2}$$

$$y(x) \to \frac{\pi}{2}$$

2.177 problem 753

Internal problem ID [9088]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 753.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[`x=_G(y,y')`]`

$$y' - \frac{(x + 1 + x^4 \ln(y))\, y \ln(y)}{x(x+1)} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 38

```
dsolve(diff(y(x),x) = (x+1+x^4*ln(y(x)))*y(x)*ln(y(x))/x/(x+1),y(x), singsol=all)
```

$$y(x) = e^{-\frac{12x}{3x^4-4x^3+6x^2+12\ln(x+1)-12c_1-12x}}$$

✓ Solution by Mathematica

Time used: 0.5 (sec). Leaf size: 46

```
DSolve[y'[x] == (Log[y[x]]*(1 + x + x^4*Log[y[x]])*y[x])/(x*(1 + x)),y[x],x,IncludeSingularS
```

$$y(x) \to \exp\left(\frac{12x}{-3x^4+4x^3-6x^2+12x-12\log(x+1)+12c_1}\right)$$
$$y(x) \to 1$$

2.178 problem 754

Internal problem ID [9089]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 754.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Abel]`

$$y' - \frac{yx + x^3 + y^2x + y^3}{x^2} = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 26

```
dsolve(diff(y(x),x) = (x*y(x)+x^3+x*y(x)^2+y(x)^3)/x^2,y(x), singsol=all)
```

$$y(x) = \text{RootOf}\left(-\left(\int^{\_Z} \frac{1}{\_a^3 + \_a^2 + 1} d\_a\right) + x + c_1\right) x$$

✓ Solution by Mathematica
Time used: 0.095 (sec). Leaf size: 47

```
DSolve[y'[x] == (x^3 + x*y[x] + x*y[x]^2 + y[x]^3)/x^2,y[x],x,IncludeSingularSolutions -> Tr
```

$$\text{Solve}\left[\text{RootSum}\left[\#1^3 + \#1^2 + 1\&, \frac{\log\left(\frac{y(x)}{x} - \#1\right)}{3\#1^2 + 2\#1}\&\right] = x + c_1, y(x)\right]$$

2.179 problem 755

Internal problem ID [9090]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 755.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$y' - \frac{y^{\frac{3}{2}}}{y^{\frac{3}{2}} + x^2 - 2yx + y^2} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 71

```
dsolve(diff(y(x),x) = y(x)^(3/2)/(y(x)^(3/2)+x^2-2*x*y(x)+y(x)^2),y(x), singsol=all)
```

$$\frac{4\sqrt{y(x)}\,x^2 - y(x)^{\frac{7}{2}}c_1 + (2c_1x + 4)\,y(x)^{\frac{5}{2}} + 4y(x)^2 + (-c_1x^2 - 8x + 1)\,y(x)^{\frac{3}{2}} - 4xy(x)}{(x - y(x))^2\,y(x)^{\frac{3}{2}}} = 0$$

✓ Solution by Mathematica

Time used: 60.256 (sec). Leaf size: 2213

```
DSolve[y'[x] == y[x]^(3/2)/(x^2 - 2*x*y[x] + y[x]^(3/2) + y[x]^2),y[x],x,IncludeSingularSolu
```

$$y(x) \to \frac{1}{3}\left(-\sqrt[3]{x^3+3e^{c_1}x^2+6\sqrt{3}\sqrt{-e^{4c_1}\left(x^3+3e^{c_1}x^2-e^{2c_1}(8x-3)x+e^{4c_1}(16x+1)+e^{3c_1}(20x+1)\right)}-3e^{2} \right. \\ + \frac{-x^2-2e^{c_1}x+e^{2c_1}(8x-1)-16e^{3c_1}-16e^{4c}}{\sqrt[3]{x^3+3e^{c_1}x^2+6\sqrt{3}\sqrt{-e^{4c_1}\left(x^3+3e^{c_1}x^2-e^{2c_1}(8x-3)x+e^{4c_1}(16x+1)+e^{3c_1}(20x+1)\right)}-3e^{2c_1}(4}} \\ \left. +2\left(x+e^{c_1}+2e^{2c_1}\right)\right)$$

$$y(x) \to \frac{1}{6}\left(\left(1 -i\sqrt{3}\right)\sqrt[3]{x^3+3e^{c_1}x^2+6\sqrt{3}\sqrt{-e^{4c_1}\left(x^3+3e^{c_1}x^2-e^{2c_1}(8x-3)x+e^{4c_1}(16x+1)+e^{3c_1}(20x+1)\right)}-3} \right. \\ + \frac{\left(1+i\sqrt{3}\right)\left(x^2+2e^{c_1}x+e^{2c_1}(1-8x)+16e^{3c_1}+\right.}{\sqrt[3]{x^3+3e^{c_1}x^2+6\sqrt{3}\sqrt{-e^{4c_1}\left(x^3+3e^{c_1}x^2-e^{2c_1}(8x-3)x+e^{4c_1}(16x+1)+e^{3c_1}(20x+1)\right)}-3e^{2c_1}(4}} \\ \left. +4\left(x+e^{c_1}+2e^{2c_1}\right)\right)$$

$$y(x) \to \frac{1}{6}\left(\left(1 +i\sqrt{3}\right)\sqrt[3]{x^3+3e^{c_1}x^2+6\sqrt{3}\sqrt{-e^{4c_1}\left(x^3+3e^{c_1}x^2-e^{2c_1}(8x-3)x+e^{4c_1}(16x+1)+e^{3c_1}(20x+1)\right)}-3} \right. \\ + \frac{\left(1-i\sqrt{3}\right)\left(x^2+2e^{c_1}x+e^{2c_1}(1-8x)+16e^{3c_1}+\right.}{\sqrt[3]{x^3+3e^{c_1}x^2+6\sqrt{3}\sqrt{-e^{4c_1}\left(x^3+3e^{c_1}x^2-e^{2c_1}(8x-3)x+e^{4c_1}(16x+1)+e^{3c_1}(20x+1)\right)}-3e^{2c_1}(4}} \\ \left. +4\left(x+e^{c_1}+2e^{2c_1}\right)\right)$$

$$y(x) \to \frac{1}{3}\left(-\sqrt[3]{x^3-3e^{c_1}x^2+6\sqrt{3}\sqrt{e^{4c_1}\left(-x^3+3e^{c_1}x^2+e^{2c_1}(8x-3)x-e^{4c_1}(16x+1)+e^{3c_1}(20x+1)\right)}-3e^{2} \right. \\ + \frac{-x^2+2e^{c_1}x+e^{2c_1}(8x-1)+16e^{3c_1}-16e^{4c}}{\sqrt[3]{x^3-3e^{c_1}x^2+6\sqrt{3}\sqrt{e^{4c_1}\left(-x^3+3e^{c_1}x^2+e^{2c_1}(8x-3)x-e^{4c_1}(16x+1)+e^{3c_1}(20x+1)\right)}-3e^{2c_1}(4}} \\ \left. +2\left(x-e^{c_1}+2e^{2c_1}\right)\right)$$

2.180 problem 756

Internal problem ID [9091]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 756.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Abel]`

$$y' - \frac{2yx^3 + x^6 + y^2x^2 + y^3}{x^4} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x) = (2*x^3*y(x)+x^6+x^2*y(x)^2+y(x)^3)/x^4,y(x), singsol=all)
```

$$y(x) = \frac{\left(-3 + 29 \operatorname{RootOf}\left(-81\left(\int^{\_Z} \frac{1}{841\_a^3 - 27\_a + 27} d\_a\right) + x + 3c_1\right)\right) x^2}{9}$$

✓ Solution by Mathematica

Time used: 1.14 (sec). Leaf size: 95

```
DSolve[y'[x] == (x^6 + 2*x^3*y[x] + x^2*y[x]^2 + y[x]^3)/x^4,y[x],x,IncludeSingularSolutions
```

$$\text{Solve}\left[-\frac{29}{3}\text{RootSum}\left[-29\#1^3+3\sqrt[3]{29}\#1-29\&,\frac{\log\left(\frac{\frac{3y(x)}{x^4}+\frac{1}{x^2}}{\sqrt[3]{29}\sqrt[3]{\frac{1}{x^6}}}-\#1\right)}{\sqrt[3]{29}-29\#1^2}\&\right]=\frac{1}{9}29^{2/3}\left(\frac{1}{x^6}\right)^{2/3}x^5+c_1,y(x)\right]$$

2.181 problem 757

Internal problem ID [9092]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 757.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [[_1st_order, _with_linear_symmetries], _rational, [_Abel, '2n

$$y' - \frac{-4yx + x^3 + 2x^2 - 4x - 8}{-8y + 2x^2 + 4x - 8} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 26

```
dsolve(diff(y(x),x) = (-4*x*y(x)+x^3+2*x^2-4*x-8)/(-8*y(x)+2*x^2+4*x-8),y(x), singsol=all)
```

$$y(x) = \frac{x^2}{4} + 2\,\mathrm{LambertW}\left(\frac{c_1 \mathrm{e}^{-\frac{x}{4}-\frac{1}{2}}}{2}\right) + \frac{x}{2} + 1$$

✓ Solution by Mathematica

Time used: 3.9 (sec). Leaf size: 49

```
DSolve[y'[x] == (-8 - 4*x + 2*x^2 + x^3 - 4*x*y[x])/(-8 + 4*x + 2*x^2 - 8*y[x]),y[x],x,Inclu
```

$$y(x) \to \frac{1}{4}\left(8W\left(-e^{-\frac{x}{4}-1+c_1}\right) + x^2 + 2x + 4\right)$$
$$y(x) \to \frac{1}{4}\left(x^2 + 2x + 4\right)$$

2.182 problem 758

Internal problem ID [9093]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 758.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘x=_G(y,y’)‘]`

$$\boxed{y' - \frac{(2x + 2 + y x^3)\, y}{(\ln(y) + 2x - 1)(x + 1)} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 61

```
dsolve(diff(y(x),x) = (2*x+2+x^3*y(x))/(ln(y(x))+2*x-1)*y(x)/(x+1),y(x), singsol=all)
```

$$y(x) = -\frac{6\,\mathrm{LambertW}\left(-\frac{(-2x^3+3x^2+6\ln(x+1)+6c_1-6x)\,\mathrm{e}^{-2x}}{6}\right)}{-2x^3 + 3x^2 + 6\ln(x+1) + 6c_1 - 6x}$$

✓ Solution by Mathematica

Time used: 60.52 (sec). Leaf size: 459

```
DSolve[y'[x] == (y[x]*(2 + 2*x + x^3*y[x]))/((1 + x)*(-1 + 2*x + Log[y[x]])),y[x],x,IncludeS
```

$$y(x) \to \frac{6W\left(-\frac{1}{6}\sqrt[6]{e^{-12x}\left(2x^3-3x^2+6x-6\log(x+1)+6c_1\right)^6}\right)}{2x^3-3x^2+6x-6\log(x+1)+6c_1}$$
$$y(x) \to \frac{6W\left(\frac{1}{6}\sqrt[6]{e^{-12x}\left(2x^3-3x^2+6x-6\log(x+1)+6c_1\right)^6}\right)}{2x^3-3x^2+6x-6\log(x+1)+6c_1}$$
$$y(x) \to \frac{6W\left(-\frac{1}{6}\sqrt[3]{-1}\sqrt[6]{e^{-12x}\left(2x^3-3x^2+6x-6\log(x+1)+6c_1\right)^6}\right)}{2x^3-3x^2+6x-6\log(x+1)+6c_1}$$
$$y(x) \to \frac{6W\left(\frac{1}{6}\sqrt[3]{-1}\sqrt[6]{e^{-12x}\left(2x^3-3x^2+6x-6\log(x+1)+6c_1\right)^6}\right)}{2x^3-3x^2+6x-6\log(x+1)+6c_1}$$
$$y(x) \to \frac{6W\left(-\frac{1}{6}(-1)^{2/3}\sqrt[6]{e^{-12x}\left(2x^3-3x^2+6x-6\log(x+1)+6c_1\right)^6}\right)}{2x^3-3x^2+6x-6\log(x+1)+6c_1}$$
$$y(x) \to \frac{6W\left(\frac{1}{6}(-1)^{2/3}\sqrt[6]{e^{-12x}\left(2x^3-3x^2+6x-6\log(x+1)+6c_1\right)^6}\right)}{2x^3-3x^2+6x-6\log(x+1)+6c_1}$$

2.183 problem 759

Internal problem ID [9094]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 759.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' + \frac{i(54ix^2 + 81y^4 + 18x^4y^2 + x^8)\,x}{243y} = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 283

```
dsolve(diff(y(x),x) = -1/243*I*(54*I*x^2+81*y(x)^4+18*x^4*y(x)^2+x^8)*x/y(x),y(x), singsol=a
```

$$y(x) = -\frac{\sqrt{-3x^3\left(\text{BesselJ}\left(\frac{1}{3},\left(\frac{2}{27}-\frac{2i}{27}\right)\sqrt{6}\,x^3\right)c_1+\text{BesselY}\left(\frac{1}{3},\left(\frac{2}{27}-\frac{2i}{27}\right)\sqrt{6}\,x^3\right)\right)\left(\frac{\text{BesselJ}\left(\frac{1}{3},\left(\frac{2}{27}-\frac{2i}{27}\right)\sqrt{6}\,x^3\right)c_1x^3}{3}+\right.}}{3\left(\text{BesselJ}\left(\frac{1}{3},\left(\frac{2}{27}-\frac{2i}{27}\right)\sqrt{6}\,x^3\right)\right.}$$

$$y(x) = \frac{\sqrt{-3x^3\left(\text{BesselJ}\left(\frac{1}{3},\left(\frac{2}{27}-\frac{2i}{27}\right)\sqrt{6}\,x^3\right)c_1+\text{BesselY}\left(\frac{1}{3},\left(\frac{2}{27}-\frac{2i}{27}\right)\sqrt{6}\,x^3\right)\right)\left(\frac{\text{BesselJ}\left(\frac{1}{3},\left(\frac{2}{27}-\frac{2i}{27}\right)\sqrt{6}\,x^3\right)c_1x^3}{3}+\right.}}{3\left(\text{BesselJ}\left(\frac{1}{3},\left(\frac{2}{27}-\frac{2i}{27}\right)\sqrt{6}\,x^3\right)c_1\right.}$$

✓ Solution by Mathematica

Time used: 37.777 (sec). Leaf size: 1293

```
DSolve[y'[x] == ((-1/243*I)*x*((54*I)*x^2 + x^8 + 18*x^4*y[x]^2 + 81*y[x]^4))/y[x],y[x],x,In
```

$$y(x) \to -\frac{\sqrt{\left(\text{BesselY}\left(\frac{1}{3},\left(\frac{2}{9}-\frac{2i}{9}\right)\sqrt{\frac{2}{3}}x^3\right)+c_1\,\text{BesselJ}\left(\frac{1}{3},\left(\frac{2}{9}-\frac{2i}{9}\right)\sqrt{\frac{2}{3}}x^3\right)\right)\left((1+i)\sqrt{6}x^3\left(\text{BesselY}\left(\frac{4}{3},\left(\frac{2}{9}-\right.\right.\right.\right.}}{\sqrt{3}x\left(\text{BesselY}\left(\frac{1}{3}\right.\right.}$$

$$y(x) \to \frac{\sqrt{\left(\text{BesselY}\left(\frac{1}{3},\left(\frac{2}{9}-\frac{2i}{9}\right)\sqrt{\frac{2}{3}}x^3\right)+c_1\,\text{BesselJ}\left(\frac{1}{3},\left(\frac{2}{9}-\frac{2i}{9}\right)\sqrt{\frac{2}{3}}x^3\right)\right)\left((1+i)\sqrt{6}x^3\left(\text{BesselY}\left(\frac{4}{3},\left(\frac{2}{9}-\frac{2i}{9}\right.\right.\right.\right.}}{\sqrt{3}x\left(\text{BesselY}\left(\frac{1}{3},\right.\right.}$$

$$y(x) \to -\frac{(-1)^{5/6}x\sqrt{-\frac{\sqrt[6]{-1}((1-i)x^3)^{2/3}\left(\sqrt{3}\,\text{AiryAi}\left(\frac{1}{3}(-1)^{5/6}2^{2/3}x^2\right)-\text{AiryBi}\left(\frac{1}{3}(-1)^{5/6}2^{2/3}x^2\right)\right)\left(-18i\sqrt{3}\,\text{AiryAiPrime}\left(\frac{1}{3}(-1)^{5/6}2^{2/3}x^2\right.\right.}{}}}{3\sqrt[6]{2}\sqrt[3]{(1-i)x^3}\left(\sqrt{3}\,\text{AiryAi}\left(\frac{1}{3}(-1)^{5/6}2^{2/3}\right.\right.}$$

$$y(x) \to \frac{(-1)^{5/6}x\sqrt{-\frac{\sqrt[6]{-1}((1-i)x^3)^{2/3}\left(\sqrt{3}\,\text{AiryAi}\left(\frac{1}{3}(-1)^{5/6}2^{2/3}x^2\right)-\text{AiryBi}\left(\frac{1}{3}(-1)^{5/6}2^{2/3}x^2\right)\right)\left(-18i\sqrt{3}\,\text{AiryAiPrime}\left(\frac{1}{3}(-1)^{5/6}2^{2/3}x^2\right)+\right.}{}}}{3\sqrt[6]{2}\sqrt[3]{(1-i)x^3}\left(\sqrt{3}\,\text{AiryAi}\left(\frac{1}{3}(-1)^{5/6}2^{2/3}\right.\right.}$$

$$y(x) \to \frac{(-1)^{5/6}x\sqrt{-\frac{\sqrt[6]{-1}((1-i)x^3)^{2/3}\left(\sqrt{3}\,\text{AiryAi}\left(\frac{1}{3}(-1)^{5/6}2^{2/3}x^2\right)-\text{AiryBi}\left(\frac{1}{3}(-1)^{5/6}2^{2/3}x^2\right)\right)\left(-18i\sqrt{3}\,\text{AiryAiPrime}\left(\frac{1}{3}(-1)^{5/6}2^{2/3}x^2\right)+\right.}{}}}{3\sqrt[6]{2}\sqrt[3]{(1-i)x^3}\left(\sqrt{3}\,\text{AiryAi}\left(\frac{1}{3}(-1)^{5/6}2^{2/3}\right.\right.}$$

2.184 problem 760

Internal problem ID [9095]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 760.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{(y^2x+1)^3}{x^4(y^2x+1+x)\, y} = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 182

```
dsolve(diff(y(x),x) = (x*y(x)^2+1)^3/x^4/(x*y(x)^2+1+x)/y(x),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{2}\,\sqrt{-(2+(1+i)\,x)\,x}}{2x}$$

$$y(x) = \frac{\sqrt{2}\,\sqrt{-(2+(1+i)\,x)\,x}}{2x}$$

$$y(x) = -\frac{\sqrt{2}\,\sqrt{x(-2+(-1+i)\,x)}}{2x}$$

$$y(x) = \frac{\sqrt{2}\,\sqrt{x(-2+(-1+i)\,x)}}{2x}$$

$$-\frac{\ln\left(2y(x)^4x^2+(2x^2+4x)\,y(x)^2+x^2+2x+2\right)}{10} + \frac{\arctan\left(2y(x)^4x+(2x+2)\,y(x)^2+x+1\right)}{10} + \frac{\ln\left(xy(x)^2-x+1\right)}{5} + \frac{1}{2x} - \frac{\arctan\left(2y(x)^2+1\right)}{10} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.518 (sec). Leaf size: 112

```
DSolve[y'[x] == (1 + x*y[x]^2)^3/(x^4*y[x]*(1 + x + x*y[x]^2)),y[x],x,IncludeSingularSolutio
```

$$\text{Solve}\left[2\left(-\frac{1}{10}\arctan\left(2xy(x)^4+2xy(x)^2+2y(x)^2+x+1\right)+\frac{1}{10}\log\left(2x^2y(x)^4+2x^2y(x)^2+x^2+4xy(x)^2+2x+2\right)-\frac{1}{5}\log\left(xy(x)^2-x+1\right)-\frac{1}{2x}\right)+\frac{1}{5}\arctan\left(2y(x)^2+1\right)=c_1,y(x)\right]$$

2.185 problem 761

Internal problem ID [9096]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 761.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, [_Abel, '2n`

$$y' - \frac{-4yx - x^3 + 4x^2 - 4x + 8}{8y + 2x^2 - 8x + 8} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 18

```
dsolve(diff(y(x),x) = (-4*x*y(x)-x^3+4*x^2-4*x+8)/(8*y(x)+2*x^2-8*x+8),y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{4} + \text{LambertW}\left(e^{-x} c_1\right) + x$$

✓ Solution by Mathematica

Time used: 3.739 (sec). Leaf size: 38

```
DSolve[y'[x] == (8 - 4*x + 4*x^2 - x^3 - 4*x*y[x])/(8 - 8*x + 2*x^2 + 8*y[x]),y[x],x,Include
```

$$y(x) \to W\left(-e^{-x-1+c_1}\right) - \frac{x^2}{4} + x$$
$$y(x) \to -\frac{1}{4}(x-4)x$$

2.186 problem 762

Internal problem ID [9097]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 762.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, ‘_with_symmetry_[F(x),G(x)*y+H(x)]‘]]`

$$\boxed{y' + \frac{(\ln(y)\, x + \ln(y) - x)\, y}{x(x+1)} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(diff(y(x),x) = -(ln(y(x))*x+ln(y(x))-x)*y(x)/x/(x+1),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{x+c_1}{x}}(x+1)^{-\frac{1}{x}}$$

✓ Solution by Mathematica

Time used: 0.331 (sec). Leaf size: 26

```
DSolve[y'[x] == ((x - Log[y[x]] - x*Log[y[x]])*y[x])/(x*(1 + x)),y[x],x,IncludeSingularSolut
```

$$y(x) \to (x+1)^{-1/x} e^{1-\frac{c_1}{x}}$$

2.187 problem 763

Internal problem ID [9098]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 763.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y' - \frac{(\ln(y)\,x + \ln(y) + x)\,y}{x\,(x+1)} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve(diff(y(x),x) = (ln(y(x))*x+ln(y(x))+x)*y(x)/x/(x+1),y(x), singsol=all)
```

$$y(x) = \left(\frac{x c_1}{x+1}\right)^x$$

✓ Solution by Mathematica

Time used: 0.323 (sec). Leaf size: 21

```
DSolve[y'[x] == ((x + Log[y[x]] + x*Log[y[x]])*y[x])/(x*(1 + x)),y[x],x,IncludeSingularSolut
```

$$y(x) \to \left(\frac{x}{x+1}\right)^x e^{c_1 x}$$

2.188 problem 764

Internal problem ID [9099]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 764.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y' - \frac{(-\ln(y)\,x - \ln(y) + x^4)\,y}{x\,(x+1)} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 40

```
dsolve(diff(y(x),x) = (-ln(y(x))*x-ln(y(x))+x^4)*y(x)/x/(x+1),y(x), singsol=all)
```

$$y(x) = e^{\frac{3x^4-4x^3+6x^2+12c_1-12x}{12x}}(x+1)^{\frac{1}{x}}$$

✓ Solution by Mathematica

Time used: 0.391 (sec). Leaf size: 46

```
DSolve[y'[x] == ((x^4 - Log[y[x]] - x*Log[y[x]])*y[x])/(x*(1 + x)),y[x],x,IncludeSingularSol
```

$$y(x) \to (x+1)^{\frac{1}{x}} \exp\left(-\frac{-3x^4+4x^3-6x^2+12x+25+12c_1}{12x}\right)$$

2.189 problem 765

Internal problem ID [9100]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 765.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$y' - \frac{y\left(-1-\ln\left(\frac{(x-1)(x+1)}{x}\right)+\ln\left(\frac{(x-1)(x+1)}{x}\right)xy\right)}{x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
dsolve(diff(y(x),x) = y(x)*(-1-ln((x-1)*(x+1)/x)+ln((x-1)*(x+1)/x)*x*y(x))/x,y(x), singsol=a
```

$$y(x) = \frac{e^{\operatorname{dilog}(x+1)}}{x\left(c_1 e^{\operatorname{dilog}(x)+\frac{\ln(x)^2}{2}}\left(\frac{x^2-1}{x}\right)^{\ln(x)}(x+1)^{-\ln(x)}+e^{\operatorname{dilog}(x+1)}\right)}$$

✓ Solution by Mathematica

Time used: 0.809 (sec). Leaf size: 240

```
DSolve[y'[x] == (y[x]*(-1 - Log[((-1 + x)*(1 + x))/x] + x*Log[((-1 + x)*(1 + x))/x]*y[x]))/x
```

$$y(x) \to \frac{e^{\text{PolyLog}(2,-x)-\text{PolyLog}(2,1-x)}x^{-\frac{\log(x)}{2}+\log(x+1)-\log\left(x-\frac{1}{x}\right)-1}}{-\int_1^x e^{\text{PolyLog}(2,-K[1])-\text{PolyLog}(2,1-K[1])}K[1]^{-\frac{1}{2}\log(K[1])+\log(K[1]+1)-\log\left(K[1]-\frac{1}{K[1]}\right)-1}\log\left(K[1]-\frac{1}{K[1]}\right)dK[1]+}$$

$$y(x) \to 0$$

$$y(x) \to -\frac{e^{\text{PolyLog}(2,-x)-\text{PolyLog}(2,1-x)}x^{-\frac{\log(x)}{2}+\log(x+1)-\log\left(x-\frac{1}{x}\right)-1}}{\int_1^x e^{\text{PolyLog}(2,-K[1])-\text{PolyLog}(2,1-K[1])}K[1]^{-\frac{1}{2}\log(K[1])+\log(K[1]+1)-\log\left(K[1]-\frac{1}{K[1]}\right)-1}\log\left(K[1]-\frac{1}{K[1]}\right)dK[1]}$$

2.190 problem 766

Internal problem ID [9101]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 766.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$y' - \frac{y\left(-\ln(x) - \ln\left(\frac{(x-1)(x+1)}{x}\right)x + y\ln\left(\frac{(x-1)(x+1)}{x}\right)x^2\right)}{x\ln(x)} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 85

```
dsolve(diff(y(x),x) = y(x)*(-ln(x)-x*ln((x-1)*(x+1)/x)+ln((x-1)*(x+1)/x)*x^2*y(x))/x/ln(x),y
```

$$y(x) = \frac{e^{-\left(\int \frac{\ln\left(\frac{x^2-1}{x}\right)x+\ln(x)}{\ln(x)x}dx\right)}}{-\left(\int \frac{e^{-\left(\int \frac{\ln\left(\frac{x^2-1}{x}\right)x+\ln(x)}{\ln(x)x}dx\right)}x\ln\left(\frac{x^2-1}{x}\right)}{\ln(x)}dx\right)+c_1}$$

✓ Solution by Mathematica

Time used: 0.768 (sec). Leaf size: 210

```
DSolve[y'[x] == (y[x]*(-Log[x] - x*Log[((-1 + x)*(1 + x))/x] + x^2*Log[((-1 + x)*(1 + x))/x]
```

$$y(x)\to \frac{\exp\left(\int_1^x\left(-\frac{\log\left(K[1]-\frac{1}{K[1]}\right)}{\log(K[1])}-\frac{1}{K[1]}\right)dK[1]\right)}{-\int_1^x\frac{\exp\left(\int_1^{K[2]}\left(-\frac{\log\left(K[1]-\frac{1}{K[1]}\right)}{\log(K[1])}-\frac{1}{K[1]}\right)dK[1]\right)K[2]\log\left(K[2]-\frac{1}{K[2]}\right)}{\log(K[2])}dK[2]+c_1}$$

$$y(x)\to 0$$

$$y(x)\to -\frac{\exp\left(\int_1^x\left(-\frac{\log\left(K[1]-\frac{1}{K[1]}\right)}{\log(K[1])}-\frac{1}{K[1]}\right)dK[1]\right)}{\int_1^x\frac{\exp\left(\int_1^{K[2]}\left(-\frac{\log\left(K[1]-\frac{1}{K[1]}\right)}{\log(K[1])}-\frac{1}{K[1]}\right)dK[1]\right)K[2]\log\left(K[2]-\frac{1}{K[2]}\right)}{\log(K[2])}dK[2]}$$

2.191 problem 767

Internal problem ID [9102]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 767.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, [_Abel, '2n`

$$y' - \frac{-8yx - x^3 + 2x^2 - 8x + 32}{32y + 4x^2 - 8x + 32} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 26

```
dsolve(diff(y(x),x) = (-8*x*y(x)-x^3+2*x^2-8*x+32)/(32*y(x)+4*x^2-8*x+32),y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{8} + 4\,\mathrm{LambertW}\left(\frac{c_1 \mathrm{e}^{-\frac{x}{16}-\frac{3}{4}}}{4}\right) + \frac{x}{4} + 3$$

✓ Solution by Mathematica

Time used: 3.195 (sec). Leaf size: 53

```
DSolve[y'[x] == (32 - 8*x + 2*x^2 - x^3 - 8*x*y[x])/(32 - 8*x + 4*x^2 + 32*y[x]),y[x],x,Incl
```

$$y(x) \to 4W\left(-e^{-\frac{x}{16}-1+c_1}\right) - \frac{x^2}{8} + \frac{x}{4} + 3$$
$$y(x) \to -\frac{x^2}{8} + \frac{x}{4} + 3$$

2.192 problem 768

Internal problem ID [9103]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 768.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]'], [_Ab`

$$y' - \frac{y(y+1)}{x(-y-1+yx)} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve(diff(y(x),x) = y(x)*(y(x)+1)/x/(-y(x)-1+x*y(x)),y(x), singsol=all)
```

$$y(x) = -\frac{1}{x \,\mathrm{LambertW}\left(\frac{\mathrm{e}^{-\frac{1}{x}}}{x c_1}\right) + 1}$$

✓ Solution by Mathematica

Time used: 1.086 (sec). Leaf size: 66

```
DSolve[y'[x] == (y[x]*(1 + y[x]))/(x*(-1 - y[x] + x*y[x])),y[x],x,IncludeSingularSolutions -
```

$$\mathrm{Solve}\left[\frac{2^{2/3}\left(xy(x)\left(-\log\left(\frac{xy(x)}{(x-1)y(x)-1}\right)+\log\left(\frac{y(x)+1}{-xy(x)+y(x)+1}\right)+\log(x)+1\right)-1\right)}{9xy(x)} = c_1, y(x)\right]$$

2.193 problem 769

Internal problem ID [9104]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 769.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' + \frac{i(16ix^2 + 16y^4 + 8x^4y^2 + x^8)\,x}{32y} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 233

```
dsolve(diff(y(x),x) = -1/32*I*(16*I*x^2+16*y(x)^4+8*x^4*y(x)^2+x^8)*x/y(x),y(x), singsol=all
```

$$y(x) = -\frac{\sqrt{-4\left(\mathrm{BesselJ}\left(\frac{1}{3}, \left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)c_1 + \mathrm{BesselY}\left(\frac{1}{3}, \left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)\right)x^3\left(\left(1+i\right)c_1\,\mathrm{BesselJ}\left(-\frac{2}{3}, \left(\frac{1}{3}-\frac{i}{3}\right)x^3\right) + \cdots}}{2\left(\mathrm{BesselJ}\left(\frac{1}{3}, \left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)c_1 + \mathrm{BesselY} \cdots\right.}$$

$$y(x) = \frac{\sqrt{-4\left(\mathrm{BesselJ}\left(\frac{1}{3}, \left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)c_1 + \mathrm{BesselY}\left(\frac{1}{3}, \left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)\right)x^3\left(\left(1+i\right)c_1\,\mathrm{BesselJ}\left(-\frac{2}{3}, \left(\frac{1}{3}-\frac{i}{3}\right)x^3\right) + \cdots}}{2\left(\mathrm{BesselJ}\left(\frac{1}{3}, \left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)c_1 + \mathrm{BesselY}\left(\frac{1}{3}, \cdots\right.\right.}$$

✓ Solution by Mathematica

Time used: 39.169 (sec). Leaf size: 836

```
DSolve[y'[x] == ((-1/32*I)*x*((16*I)*x^2 + x^8 + 8*x^4*y[x]^2 + 16*y[x]^4))/y[x],y[x],x,Incl
```

$$y(x) \to -\frac{\sqrt{\left(\text{BesselY}\left(\frac{1}{3},\left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)+c_1\,\text{BesselJ}\left(\frac{1}{3},\left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)\right)\left((1+i)x^3\left(\text{BesselY}\left(\frac{4}{3},\left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)+c_1\,\text{Bes}\cdots\right.\right.}}{x\left(\text{BesselY}\left(\frac{1}{3},\left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)+\cdots\right.}$$

$$y(x) \to \frac{\sqrt{\left(\text{BesselY}\left(\frac{1}{3},\left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)+c_1\,\text{BesselJ}\left(\frac{1}{3},\left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)\right)\left((1+i)x^3\left(\text{BesselY}\left(\frac{4}{3},\left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)+c_1\,\text{Besse}\cdots\right.\right.}}{x\left(\text{BesselY}\left(\frac{1}{3},\left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)+c_1\cdots\right.}$$

$$y(x) \to -\frac{(-1)^{5/6}x\sqrt{-\frac{\sqrt[6]{-1}((1-i)x^3)^{2/3}\left(\sqrt{3}\,\text{AiryAi}\left(\frac{(-1)^{5/6}x^2}{\sqrt[3]{2}}\right)-\text{AiryBi}\left(\frac{(-1)^{5/6}x^2}{\sqrt[3]{2}}\right)\right)\left(-4i2^{2/3}\sqrt{3}\,\text{AiryAiPrime}\left(\frac{(-1)^{5/6}x^2}{\sqrt[3]{2}}\right)+4i2^{2/3}\cdots\right.}{x^2}}}{2\sqrt[3]{(1-i)x^3}\left(\sqrt{3}\,\text{AiryAi}\left(\frac{(-1)^{5/6}x^2}{\sqrt[3]{2}}\right)-\text{Airy}\cdots\right.}$$

$$y(x) \to \frac{(-1)^{5/6}x\sqrt{-\frac{\sqrt[6]{-1}((1-i)x^3)^{2/3}\left(\sqrt{3}\,\text{AiryAi}\left(\frac{(-1)^{5/6}x^2}{\sqrt[3]{2}}\right)-\text{AiryBi}\left(\frac{(-1)^{5/6}x^2}{\sqrt[3]{2}}\right)\right)\left(-4i2^{2/3}\sqrt{3}\,\text{AiryAiPrime}\left(\frac{(-1)^{5/6}x^2}{\sqrt[3]{2}}\right)+4i2^{2/3}\,\text{Ai}\cdots\right.}{x^2}}}{2\sqrt[3]{(1-i)x^3}\left(\sqrt{3}\,\text{AiryAi}\left(\frac{(-1)^{5/6}x^2}{\sqrt[3]{2}}\right)-\text{AiryBi}\cdots\right.}$$

2.194 problem 770

Internal problem ID [9105]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 770.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{2y^6}{y^3 + 2 + 16y^2x + 32y^4x^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 779

```
dsolve(diff(y(x),x) = 2*y(x)^6/(y(x)^3+2+16*x*y(x)^2+32*x^2*y(x)^4),y(x), singsol=all)
```

$$y(x) = \frac{\left(4096c_1^3x^3 + 6\sqrt{3}\,\sqrt{4096c_1^4x^3 + 27c_1^4 + 576c_1^3x + 2048c_1^2x^2 + 16c_1 + 256x}\,c_1 + 96\sqrt{3}\,\sqrt{4096c_1^4x^3 + 27c_1^4\right.}{}$$

$$y(x) = \frac{\frac{\left(-i\sqrt{3}-1\right)\left(96\left(\frac{c_1}{16}+x\right)\sqrt{3}\,\sqrt{(4096x^3+27)c_1^4+576c_1^3x+2048c_1^2x^2+16c_1+256x}+(4096x^3+54)c_1^3+1440x\,c_1^2+9216c_1x^2\right)^{\frac{2}{3}}}{6} + \frac{16c_1x\left(96\left(\frac{c_1}{16}+x\right.\right.}{}}{\left(96\left(\frac{c_1}{16}+x\right)\sqrt{3}\,\sqrt{(4096x^3+27)\,c_1^4 + 576c_1^3x + 2048c_1^2x^2 + 16c_1 +\right.}$$

$$y(x) = \frac{\frac{\left(i\sqrt{3}-1\right)\left(96\left(\frac{c_1}{16}+x\right)\sqrt{3}\,\sqrt{(4096x^3+27)c_1^4+576c_1^3x+2048c_1^2x^2+16c_1+256x}+(4096x^3+54)c_1^3+1440x\,c_1^2+9216c_1x^2\right)^{\frac{2}{3}}}{6} + \frac{16c_1x\left(96\left(\frac{c_1}{16}+x\right)\right.}{}}{\left(96\left(\frac{c_1}{16}+x\right)\sqrt{3}\,\sqrt{(4096x^3+27)\,c_1^4 + 576c_1^3x + 2048c_1^2x^2 + 16c_1 +\right.}$$

✓ Solution by Mathematica

Time used: 27.592 (sec). Leaf size: 952

```
DSolve[y'[x] == (2*y[x]^6)/(2 + 16*x*y[x]^2 + y[x]^3 + 32*x^2*y[x]^4),y[x],x,IncludeSingular
```

$y(x)$

$$\to \frac{2\sqrt[3]{2}\sqrt[3]{2048x^3 + 4608{c_1}^2x^2 + 3\sqrt{3}\sqrt{(1-16c_1x)^2\left(4096x^3 + 2048{c_1}^2x^2 + 64c_1\left(-9+4{c_1}^3\right)x + 27 - 16c}}}{}$$

$y(x)$

$$\to \frac{2i\sqrt[3]{2}\left(\sqrt{3}+i\right)\sqrt[3]{2048x^3 + 4608{c_1}^2x^2 + 3\sqrt{3}\sqrt{(1-16c_1x)^2\left(4096x^3 + 2048{c_1}^2x^2 + 64c_1\left(-9+4{c_1}^3\right)x}}}{}$$

$y(x)$

$$\to \frac{-2\sqrt[3]{2}\left(1+i\sqrt{3}\right)\sqrt[3]{2048x^3 + 4608{c_1}^2x^2 + 3\sqrt{3}\sqrt{(1-16c_1x)^2\left(4096x^3 + 2048{c_1}^2x^2 + 64c_1\left(-9+4{c_1}^3\right)}}}{}$$

$$y(x) \to 0$$

$$y(x) \to \frac{x - \sqrt[3]{x^3}}{2\sqrt{3}x\sqrt[6]{x^3}}$$

$$y(x) \to \frac{\sqrt[3]{x^3} - x}{2\sqrt{3}x\sqrt[6]{x^3}}$$

$$y(x) \to \frac{\left(\sqrt{3}-3i\right)x - \left(\sqrt{3}+3i\right)\sqrt[3]{x^3}}{12x\sqrt[6]{x^3}}$$

$$y(x) \to \frac{\left(\sqrt{3}+3i\right)x - \left(\sqrt{3}-3i\right)\sqrt[3]{x^3}}{12x\sqrt[6]{x^3}}$$

$$y(x) \to \frac{\left(\sqrt{3}-3i\right)\sqrt[3]{x^3} - \left(\sqrt{3}+3i\right)x}{12x\sqrt[6]{x^3}}$$

$$y(x) \to \frac{\left(\sqrt{3}+3i\right)\sqrt[3]{x^3} - \left(\sqrt{3}-3i\right)x}{12x\sqrt[6]{x^3}}$$

2.195 problem 771

Internal problem ID [9106]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 771.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, [_Abel, '2n`

$$y' - \frac{-4yax - a^2x^3 - 2ab\,x^2 - 4ax + 8}{8y + 2a\,x^2 + 4bx + 8} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 87

```
dsolve(diff(y(x),x) = (-4*y(x)*a*x-a^2*x^3-2*a*x^2*b-4*a*x+8)/(8*y(x)+2*a*x^2+4*b*x+8),y(x),
```

$$y(x) = \frac{-a\,x^2b - 2b^2x - 4b + 4\,\mathrm{e}^{\frac{-4\,\mathrm{LambertW}\left(-\frac{\mathrm{e}^{\frac{\left(-b^2x-2b-4\right)a-2c_1b^2}{4a}}}{2}\right)a+\left(-b^2x-2b-4\right)a-2c_1b^2}{4a}} - 8}{4b}$$

✓ Solution by Mathematica

Time used: 5.584 (sec). Leaf size: 76

```
DSolve[y'[x] == (8 - 4*a*x - 2*a*b*x^2 - a^2*x^3 - 4*a*x*y[x])/(8 + 4*b*x + 2*a*x^2 + 8*y[x]
```

$$y(x) \to -\frac{abx^2 + 8W\left(-e^{-\frac{b^2x}{4}-1+c_1}\right) + 2b^2x + 4b + 8}{4b}$$
$$y(x) \to -\frac{abx^2 + 2b^2x + 4b + 8}{4b}$$

2.196 problem 772

Internal problem ID [9107]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 772.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘x=_G(y,y’)‘]`

$$y' - \frac{(x + 1 + \ln(y)\, x) \ln(y)\, y}{x(x + 1)} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve(diff(y(x),x) = (x+1+ln(y(x))*x)*ln(y(x))*y(x)/x/(x+1),y(x), singsol=all)
```

$$y(x) = e^{\frac{x}{\ln(x+1)+c_1-x}}$$

✓ Solution by Mathematica

Time used: 0.393 (sec). Leaf size: 26

```
DSolve[y'[x] == (Log[y[x]]*(1 + x + x*Log[y[x]])*y[x])/(x*(1 + x)),y[x],x,IncludeSingularSol
```

$$y(x) \to e^{\frac{x}{-x+\log(x+1)+c_1}}$$
$$y(x) \to 1$$

2.197 problem 773

Internal problem ID [9108]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 773.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$y' - \frac{yx + x + y^2}{(x-1)(x+y)} = 0$$

✓ Solution by Maple

Time used: 0.391 (sec). Leaf size: 58

```
dsolve(diff(y(x),x) = 1/(x-1)*(x*y(x)+x+y(x)^2)/(x+y(x)),y(x), singsol=all)
```

$$y(x) = \frac{x\left(\sqrt{3}\tan\left(\operatorname{RootOf}\left(2\sqrt{3}\ln(2) - \sqrt{3}\ln\left(\frac{\sec(\_Z)^2 x^2}{(x-1)^2}\right) - \sqrt{3}\ln(3) + 2\sqrt{3}\,c_1 - 2\_Z\right)\right) - 1\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.146 (sec). Leaf size: 61

```
DSolve[y'[x] == (x + x*y[x] + y[x]^2)/((-1 + x)*(x + y[x])),y[x],x,IncludeSingularSolutions
```

$$\text{Solve}\left[\frac{\arctan\left(\frac{\frac{2y(x)}{x}+1}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}\log\left(\frac{y(x)^2}{x^2} + \frac{y(x)}{x} + 1\right) = \log(1-x) - \log(x) + c_1, y(x)\right]$$

2.198 problem 774

Internal problem ID [9109]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 774.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, [_Abel, '2n`

$$y' - \frac{-4yx - x^3 - 2a\,x^2 - 4x + 8}{8y + 2x^2 + 4ax + 8} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
dsolve(diff(y(x),x) = (-4*x*y(x)-x^3-2*a*x^2-4*x+8)/(8*y(x)+2*x^2+4*a*x+8),y(x), singsol=all
```

$$y(x) = \frac{-2a^2x - a\,x^2 - 8\,\mathrm{LambertW}\left(-\frac{\mathrm{e}^{-1+\frac{(-x+c_1)a^2}{4}-\frac{a}{2}}}{2}\right) - 4a - 8}{4a}$$

✓ Solution by Mathematica

Time used: 5.106 (sec). Leaf size: 72

```
DSolve[y'[x] == (8 - 4*x - 2*a*x^2 - x^3 - 4*x*y[x])/(8 + 4*a*x + 2*x^2 + 8*y[x]),y[x],x,Inc
```

$$y(x) \to -\frac{8W\left(-e^{-\frac{a^2x}{4}-1+c_1}\right) + 2a^2x + a(x^2+4) + 8}{4a}$$
$$y(x) \to -\frac{2a^2x + a(x^2+4) + 8}{4a}$$

2.199 problem 775

Internal problem ID [9110]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 775.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$y' - \frac{x - y + \sqrt{y}}{x - y + \sqrt{y} + 1} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 44

```
dsolve(diff(y(x),x) = (x-y(x)+y(x)^(1/2))/(x-y(x)+y(x)^(1/2)+1),y(x), singsol=all)
```

$$-2y(x)^{\frac{3}{2}} + y(x)^3 + (-3x - 3)\, y(x)^2 + \left(3x^2 + 3x\right) y(x) - x^3 - c_1 = 0$$

✓ Solution by Mathematica

Time used: 11.457 (sec). Leaf size: 943

```
DSolve[y'[x] == (x + Sqrt[y[x]] - y[x])/(1 + x + Sqrt[y[x]] - y[x]),y[x],x,IncludeSingularSo
```

$$y(x) \to \text{Root}\left[\#1^6 + \#1^5(-6x-6) + \#1^4\left(15x^2+24x+9\right) + \#1^3\left(-20x^3-36x^2-18x-4+2e^{3c_1}\right) + \#1^2\left(15x^4+24x^3+9x^2-6e^{3c_1}x-6e^{3c_1}\right) + \#1\left(-6x^5-6x^4+6e^{3c_1}x^2+6e^{3c_1}x\right) + x^6 - 2e^{3c_1}x^3 + e^{6c_1}\&, 1\right]$$

$$y(x) \to \text{Root}\left[\#1^6 + \#1^5(-6x-6) + \#1^4\left(15x^2+24x+9\right) + \#1^3\left(-20x^3-36x^2-18x-4+2e^{3c_1}\right) + \#1^2\left(15x^4+24x^3+9x^2-6e^{3c_1}x-6e^{3c_1}\right) + \#1\left(-6x^5-6x^4+6e^{3c_1}x^2+6e^{3c_1}x\right) + x^6 - 2e^{3c_1}x^3 + e^{6c_1}\&, 2\right]$$

$$y(x) \to \text{Root}\left[\#1^6 + \#1^5(-6x-6) + \#1^4\left(15x^2+24x+9\right) + \#1^3\left(-20x^3-36x^2-18x-4+2e^{3c_1}\right) + \#1^2\left(15x^4+24x^3+9x^2-6e^{3c_1}x-6e^{3c_1}\right) + \#1\left(-6x^5-6x^4+6e^{3c_1}x^2+6e^{3c_1}x\right) + x^6 - 2e^{3c_1}x^3 + e^{6c_1}\&, 3\right]$$

$$y(x) \to \text{Root}\left[\#1^6 + \#1^5(-6x-6) + \#1^4\left(15x^2+24x+9\right) + \#1^3\left(-20x^3-36x^2-18x-4+2e^{3c_1}\right) + \#1^2\left(15x^4+24x^3+9x^2-6e^{3c_1}x-6e^{3c_1}\right) + \#1\left(-6x^5-6x^4+6e^{3c_1}x^2+6e^{3c_1}x\right) + x^6 - 2e^{3c_1}x^3 + e^{6c_1}\&, 4\right]$$

$$y(x) \to \text{Root}\left[\#1^6 + \#1^5(-6x-6) + \#1^4\left(15x^2+24x+9\right) + \#1^3\left(-20x^3-36x^2-18x-4+2e^{3c_1}\right) + \#1^2\left(15x^4+24x^3+9x^2-6e^{3c_1}x-6e^{3c_1}\right) + \#1\left(-6x^5-6x^4+6e^{3c_1}x^2+6e^{3c_1}x\right) + x^6 - 2e^{3c_1}x^3 + e^{6c_1}\&, 5\right]$$

$$y(x) \to \text{Root}\left[\#1^6 + \#1^5(-6x-6) + \#1^4\left(15x^2+24x+9\right) + \#1^3\left(-20x^3-36x^2-18x-4+2e^{3c_1}\right) + \#1^2\left(15x^4+24x^3+9x^2-6e^{3c_1}x-6e^{3c_1}\right) + \#1\left(-6x^5-6x^4+6e^{3c_1}x^2+6e^{3c_1}x\right) + x^6 - 2e^{3c_1}x^3 + e^{6c_1}\&, 6\right]$$

2.200 problem 776

Internal problem ID [9111]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 776.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$y' - \frac{y\left(-\ln\left(\frac{1}{x}\right) - \ln\left(\frac{x^2+1}{x}\right)x + \ln\left(\frac{x^2+1}{x}\right)x^2 y\right)}{x\ln\left(\frac{1}{x}\right)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 95

```
dsolve(diff(y(x),x) = y(x)*(-ln(1/x)-ln((x^2+1)/x)*x+ln((x^2+1)/x)*x^2*y(x))/x/ln(1/x),y(x),
```

$$y(x) = \frac{e^{-\left(\int \frac{\ln\left(\frac{x^2+1}{x}\right)x+\ln\left(\frac{1}{x}\right)}{x\ln\left(\frac{1}{x}\right)}dx\right)}}{-\left(\int \frac{e^{-\left(\int \frac{\ln\left(\frac{x^2+1}{x}\right)x+\ln\left(\frac{1}{x}\right)}{x\ln\left(\frac{1}{x}\right)}dx\right)} x\ln\left(\frac{x^2+1}{x}\right)}{\ln\left(\frac{1}{x}\right)}dx\right) + c_1}$$

✓ Solution by Mathematica

Time used: 0.91 (sec). Leaf size: 110

```
DSolve[y'[x] == (y[x]*(-Log[x^(-1)] - x*Log[(1 + x^2)/x] + x^2*Log[(1 + x^2)/x]*y[x]))/(x*Lo
```

$$y(x)\to \frac{\exp \left(\int_1^x \left(-\frac{\log \left(K[1]+\frac{1}{K[1]}\right)}{\log \left(\frac{1}{K[1]}\right)}-\frac{1}{K[1]}\right) dK[1]\right)}{-\int_1^x \frac{\exp \left(\int_1^{K[2]} \left(-\frac{\log \left(K[1]+\frac{1}{K[1]}\right)}{\log \left(\frac{1}{K[1]}\right)}-\frac{1}{K[1]}\right) dK[1]\right) K[2] \log \left(K[2]+\frac{1}{K[2]}\right)}{\log \left(\frac{1}{K[2]}\right)} dK[2]+c_1}$$

$$y(x)\to 0$$

2.201 problem 777

Internal problem ID [9112]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 777.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{y' - \frac{y(y+1)}{x\left(-y-1+y^4x\right)} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(diff(y(x),x) = y(x)*(y(x)+1)/x/(-y(x)-1+x*y(x)^4),y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = -1$$
$$y(x) = \mathrm{e}^{\mathrm{RootOf}\left(x\,\mathrm{e}^{3\_Z}-5x\,\mathrm{e}^{2\_Z}+2c_1x\,\mathrm{e}^{\_Z}+2\_Zx\,\mathrm{e}^{\_Z}+7x\,\mathrm{e}^{\_Z}-2c_1x-2x\_Z-3x+2\right)} - 1$$

✓ Solution by Mathematica

Time used: 0.191 (sec). Leaf size: 39

```
DSolve[y'[x] == (y[x]*(1 + y[x]))/(x*(-1 - y[x] + x*y[x]^4)),y[x],x,IncludeSingularSolutions
```

$$\mathrm{Solve}\left[-\frac{1}{2}(y(x)+1)^2 + 2(y(x)+1) - \frac{1}{xy(x)} - \log(y(x)+1) = c_1, y(x)\right]$$

2.202 problem 778

Internal problem ID [9113]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 778.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Abel]`

$$\boxed{y' - \frac{-3yx^2 + 1 + y^2x^6 + y^3x^9}{x^3} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x) = (-3*x^2*y(x)+1+y(x)^2*x^6+y(x)^3*x^9)/x^3,y(x), singsol=all)
```

$$y(x) = \frac{-3 + 29\,\mathrm{RootOf}\left(-81\left(\int^{\_Z} \frac{1}{841\_a^3 - 27\_a + 27} d\_a\right) + x + 3c_1\right)}{9x^3}$$

✓ Solution by Mathematica
Time used: 1.134 (sec). Leaf size: 95

```
DSolve[y'[x] == (1 - 3*x^2*y[x] + x^6*y[x]^2 + x^9*y[x]^3)/x^3,y[x],x,IncludeSingularSolutio
```

$$\mathrm{Solve}\left[-\frac{29}{3}\mathrm{RootSum}\left[-29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log\left(\frac{3x^6y(x)+x^3}{\sqrt[3]{29}\sqrt[3]{x^9}} - \#1\right)}{\sqrt[3]{29} - 29\#1^2}\&\right] = \frac{29^{2/3}(x^9)^{2/3}}{9x^5} + c_1, y(x)\right]$$

2.203 problem 779

Internal problem ID [9114]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 779.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _rational, _Abel]`

$$y' - \frac{yx^3 + x^3 + y^2x + y^3}{(x-1)\,x^3} = 0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 50

```
dsolve(diff(y(x),x) = 1/(x-1)*(x^3*y(x)+x^3+x*y(x)^2+y(x)^3)/x^3,y(x), singsol=all)
```

$$\frac{\ln\left(\frac{x+y(x)}{x}\right)}{2} - \frac{\ln\left(\frac{y(x)^2+x^2}{x^2}\right)}{4} + \frac{\arctan\left(\frac{y(x)}{x}\right)}{2} - \ln(x-1) + \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.13 (sec). Leaf size: 57

```
DSolve[y'[x] == (x^3 + x^3*y[x] + x*y[x]^2 + y[x]^3)/((-1 + x)*x^3),y[x],x,IncludeSingularSo
```

$$\text{Solve}\left[\frac{1}{2}\arctan\left(\frac{y(x)}{x}\right) - \frac{1}{4}\log\left(\frac{y(x)^2}{x^2}+1\right) + \frac{1}{2}\log\left(\frac{y(x)}{x}+1\right) = \log(1-x) - \log(x) + c_1, y(x)\right]$$

2.204 problem 780

Internal problem ID [9115]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 780.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' - \frac{yx + y + x\sqrt{y^2 + x^2}}{x(x+1)} = 0$$

✓ Solution by Maple

Time used: 0.406 (sec). Leaf size: 32

```
dsolve(diff(y(x),x) = (x*y(x)+y(x)+x*(y(x)^2+x^2)^(1/2))/x/(x+1),y(x), singsol=all)
```

$$\frac{\sqrt{y(x)^2 + x^2} + y(x) + (x^2 + x)\, c_1}{x(x+1)} = 0$$

✓ Solution by Mathematica

Time used: 0.463 (sec). Leaf size: 35

```
DSolve[y'[x] == (y[x] + x*y[x] + x*Sqrt[x^2 + y[x]^2])/(x*(1 + x)),y[x],x,IncludeSingularSol
```

$$y(x) \to \frac{e^{-c_1} x(-1 + e^{2c_1}(x+1)^2)}{2(x+1)}$$

2.205 problem 781

Internal problem ID [9116]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 781.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' - \frac{(x^4 + x^3 + x + 3y^2)\, y}{(6y^2 + x)\, x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 70

```
dsolve(diff(y(x),x) = 1/(6*y(x)^2+x)*(x^4+x^3+x+3*y(x)^2)*y(x)/x,y(x), singsol=all)
```

$$\frac{y(x)^2 x}{6y(x)^2 + x} = \frac{\left(e^{\operatorname{RootOf}\left(2x^3 e^{\_Z} + 3x^2 e^{\_Z} + 3\, e^{\_Z} \ln(2) - 3\, e^{\_Z} \ln\left(\frac{e^{\_Z}+9}{x}\right) + 9c_1 e^{\_Z} + 3\, e^{\_Z}\_Z + 27\right)} + 9\right) x}{54}$$

✓ Solution by Mathematica

Time used: 4.582 (sec). Leaf size: 87

```
DSolve[y'[x] == (y[x]*(x + x^3 + x^4 + 3*y[x]^2))/(x*(x + 6*y[x]^2)),y[x],x,IncludeSingularS
```

$$y(x) \to -\frac{\sqrt{x}\sqrt{W\left(6xe^{\frac{2x^3}{3}+x^2+2c_1}\right)}}{\sqrt{6}}$$

$$y(x) \to \frac{\sqrt{x}\sqrt{W\left(6xe^{\frac{2x^3}{3}+x^2+2c_1}\right)}}{\sqrt{6}}$$

$$y(x) \to 0$$

2.206 problem 782

Internal problem ID [9117]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 782.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$y' - \frac{y\left(-\tanh\left(\frac{1}{x}\right) - \ln\left(\frac{x^2+1}{x}\right)x + \ln\left(\frac{x^2+1}{x}\right)x^2 y\right)}{x\tanh\left(\frac{1}{x}\right)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 83

```
dsolve(diff(y(x),x) = y(x)*(-tanh(1/x)-ln((x^2+1)/x)*x+ln((x^2+1)/x)*x^2*y(x))/x/tanh(1/x),y
```

$$y(x) = \frac{e^{-\left(\int \frac{\coth\left(\frac{1}{x}\right)\ln\left(\frac{x^2+1}{x}\right)x+1}{x}dx\right)}}{-\left(\int \coth\left(\frac{1}{x}\right)e^{-\left(\int \frac{\coth\left(\frac{1}{x}\right)\ln\left(\frac{x^2+1}{x}\right)x+1}{x}dx\right)}\ln\left(\frac{x^2+1}{x}\right)xdx\right) + c_1}$$

✓ Solution by Mathematica

Time used: 6.394 (sec). Leaf size: 104

```
DSolve[y'[x] == (Coth[x^(-1)]*y[x]*(-(x*Log[(1 + x^2)/x]) - Tanh[x^(-1)] + x^2*Log[(1 + x^2)
```

$$y(x) \to \frac{\exp\left(\int_1^x \left(-\coth\left(\frac{1}{K[1]}\right)\log\left(K[1]+\frac{1}{K[1]}\right) - \frac{1}{K[1]}\right)dK[1]\right)}{-\int_1^x \exp\left(\int_1^{K[2]} \left(-\coth\left(\frac{1}{K[1]}\right)\log\left(K[1]+\frac{1}{K[1]}\right) - \frac{1}{K[1]}\right)dK[1]\right)\coth\left(\frac{1}{K[2]}\right)K[2]\log\left(K[2]+\frac{1}{K[2]}\right)d}$$

$$y(x) \to 0$$

2.207 problem 783

Internal problem ID [9118]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 783.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$y' + \frac{y(\tanh(x) + \ln(2x)\, x - \ln(2x)\, x^2 y)}{x \tanh(x)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 62

```
dsolve(diff(y(x),x) = -y(x)*(tanh(x)+ln(2*x)*x-ln(2*x)*x^2*y(x))/x/tanh(x),y(x), singsol=all
```

$$y(x) = \frac{e^{-\left(\int \frac{1+x(\ln(2)+\ln(x))\coth(x)}{x}dx\right)}}{-\left(\int \coth(x)\, e^{-\left(\int \frac{1+x(\ln(2)+\ln(x))\coth(x)}{x}dx\right)} (\ln(2)+\ln(x))\, x dx\right) + c_1}$$

✓ Solution by Mathematica

Time used: 10.207 (sec). Leaf size: 89

```
DSolve[y'[x] == -((Coth[x]*y[x]*(x*Log[2*x] + Tanh[x] - x^2*Log[2*x]*y[x]))/x),y[x],x,Includ
```

$$y(x) \to \frac{\exp\left(\int_1^x \left(-\coth(K[1])\log(2K[1]) - \frac{1}{K[1]}\right) dK[1]\right)}{-\int_1^x \exp\left(\int_1^{K[2]} \left(-\coth(K[1])\log(2K[1]) - \frac{1}{K[1]}\right) dK[1]\right) \coth(K[2])K[2]\log(2K[2])dK[2] + c_1}$$

$$y(x) \to 0$$

2.208 problem 784

Internal problem ID [9119]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 784.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - \frac{-\sinh(x) + x^2\ln(x) + 2y\ln(x)x + \ln(x) + \ln(x)y^2}{\sinh(x)} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 22

```
dsolve(diff(y(x),x) = (-sinh(x)+x^2*ln(x)+2*y(x)*ln(x)*x+ln(x)+y(x)^2*ln(x))/sinh(x),y(x), s
```

$$y(x) = -x - \tan\left(c_1 - \left(\int \operatorname{csch}(x)\ln(x)\,dx\right)\right)$$

✓ Solution by Mathematica

Time used: 20.035 (sec). Leaf size: 27

```
DSolve[y'[x] == Csch[x]*(Log[x] + x^2*Log[x] - Sinh[x] + 2*x*Log[x]*y[x] + Log[x]*y[x]^2),y[
```

$$y(x) \to -x + \tan\left(\int_1^x \operatorname{csch}(K[5])\log(K[5])dK[5] + c_1\right)$$

2.209 problem 785

Internal problem ID [9120]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 785.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' + \frac{\ln(x) - x^2 \sinh(x) - 2y \sinh(x) x - \sinh(x) - y^2 \sinh(x)}{\ln(x)} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve(diff(y(x),x) = -(ln(x)-sinh(x)*x^2-2*sinh(x)*x*y(x)-sinh(x)-sinh(x)*y(x)^2)/ln(x),y(x
```

$$y(x) = -x - \tan\left(c_1 - \left(\int \frac{\sinh(x)}{\ln(x)} dx\right)\right)$$

✓ Solution by Mathematica

Time used: 11.179 (sec). Leaf size: 29

```
DSolve[y'[x] == (-Log[x] + Sinh[x] + x^2*Sinh[x] + 2*x*Sinh[x]*y[x] + Sinh[x]*y[x]^2)/Log[x]
```

$$y(x) \to -x + \tan\left(\int_1^x \frac{\sinh(K[5])}{\log(K[5])} dK[5] + c_1\right)$$

2.210 problem 786

Internal problem ID [9121]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 786.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _Riccati]`

$$y' - \frac{y \ln(x) + \cosh(x) x a y^2 + \cosh(x) x^3 b}{x \ln(x)} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 33

```
dsolve(diff(y(x),x) = (y(x)*ln(x)+cosh(x)*x*a*y(x)^2+cosh(x)*x^3*b)/x/ln(x),y(x), singsol=al
```

$$y(x) = \frac{\tan\left(\sqrt{ab}\left(\int \frac{x\cosh(x)}{\ln(x)}dx + c_1\right)\right) x\sqrt{ab}}{a}$$

✓ Solution by Mathematica

Time used: 6.061 (sec). Leaf size: 50

```
DSolve[y'[x] == (b*x^3*Cosh[x] + Log[x]*y[x] + a*x*Cosh[x]*y[x]^2)/(x*Log[x]),y[x],x,Include
```

$$y(x) \to \frac{\sqrt{b}x\tan\left(\sqrt{a}\sqrt{b}\left(\int_1^x \frac{\cosh(K[1])K[1]}{\log(K[1])}dK[1] + c_1\right)\right)}{\sqrt{a}}$$

2.211 problem 787

Internal problem ID [9122]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 787.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], [_Abel`

$$y' - \frac{x(-x-1+x^2-2yx^2+2x^4)}{(x^2-y)(x+1)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 191

```
dsolve(diff(y(x),x) = 1/(x^2-y(x))*x*(-x-1+x^2-2*x^2*y(x)+2*x^4)/(x+1),y(x), singsol=all)
```

$$y(x) = \frac{4x^2 e^{\mathrm{RootOf}\left(8x^3 e^{\_Z} - 24x^2 e^{\_Z} - 36x^3 + 6\ln\left(\frac{2e^{\_Z}-9}{(x+1)^4}\right)e^{\_Z} + 18c_1 e^{\_Z} - 6e^{\_Z}\_Z + 24x e^{\_Z} + 108x^2 - 27\ln\left(\frac{2e^{\_Z}-9}{(x+1)^4}\right) - 81c_1 + 27\_Z - 108x + 27}}{4e^{\mathrm{RootOf}\left(8x^3 e^{\_Z} - 24x^2 e^{\_Z} - 36x^3 + 6\ln\left(\frac{2e^{\_Z}-9}{(x+1)^4}\right)e^{\_Z} + 18c_1 e^{\_Z} - 6e^{\_Z}\_Z + 24x e^{\_Z} + 108x^2 - 27\ln\left(\frac{2e^{\_Z}-9}{(x+1)^4}\right) - 81c_1 + 27\_Z - 108x +}}$$

✓ Solution by Mathematica

Time used: 17.772 (sec). Leaf size: 488

```
DSolve[y'[x] == (x*(-1 - x + x^2 + 2*x^4 - 2*x^2*y[x]))/((1 + x)*(x^2 - y[x])),y[x],x,Includ
```

$$\text{Solve}\left[\frac{\left(2-\frac{x(x^2-x-1)(2x^2-2y(x)+3)}{\sqrt[3]{x^3(x^2-x-1)^3(x^2-y(x))}}\right)\left(\frac{x(x^2-x-1)(2x^2-2y(x)+3)}{\sqrt[3]{x^3(x^2-x-1)^3(x^2-y(x))}}+4\right)\left(\left(1-\frac{x(x^2-x-1)(2x^2-2y}{2\sqrt[3]{x^3(x^2-x-1)}}\right.\right.}{18\sqrt[3]{2}\left(-\frac{(2x^2}{8(x}\right.}\right.$$

$$\left.+c_1, y(x)\right]$$

2.212 problem 788

Internal problem ID [9123]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 788.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$y' + \frac{y(\ln(x-1) + \coth(x+1)\,x - \coth(x+1)\,x^2 y)}{x\ln(x-1)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 77

```
dsolve(diff(y(x),x) = -y(x)*(ln(x-1)+coth(x+1)*x-coth(x+1)*x^2*y(x))/x/ln(x-1),y(x), singsol
```

$$y(x) = \frac{e^{-\left(\int \frac{\ln(x-1)+\coth(x+1)x}{x\ln(x-1)}dx\right)}}{-\left(\int \frac{\coth(x+1)e^{-\left(\int \frac{\ln(x-1)+\coth(x+1)x}{x\ln(x-1)}dx\right)}x}{\ln(x-1)}dx\right) + c_1}$$

✓ Solution by Mathematica

Time used: 37.644 (sec). Leaf size: 510

```
DSolve[y'[x] == -((y[x]*(x*Coth[1 + x] + Log[-1 + x] - x^2*Coth[1 + x]*y[x]))/(x*Log[-1 + x]
```

$$y(x) \rightarrow \frac{\exp\left(\int_1^x -\frac{\cosh(K[1])((1+e^2)K[1]+(-1+e^2)\log(K[1]-1))+((-1+e^2)K[1]+(1+e^2)\log(K[1]-1))\sinh(K[1]}{K[1]\log(K[1]-1)((-1+e^2)\cosh(K[1])+(1+e^2)\sinh(K[1]))}\right.}{-\int_1^x \frac{\exp\left(\int_1^{K[2]} -\frac{\cosh(K[1])\left(\left(1+e^2\right)K[1]+\left(-1+e^2\right)\log(K[1]-1)\right)+\left(\left(-1+e^2\right)K[1]+\left(1+e^2\right)\log(K[1]-1)\right)\sinh(K[1])}{K[1]\log(K[1]-1)\left(\left(-1+e^2\right)\cosh(K[1])+\left(1+e^2\right)\sinh(K[1])\right)}dK[1]\right)K[2]((1+e^2)\cosh(K[2]}{\log(K[2]-1)((-1+e^2)\cosh(K[2])+(1+e^2)\sinh(K[2]))}}$$

$$y(x) \rightarrow 0$$

$$y(x) \rightarrow -\frac{\exp\left(\int_1^x -\frac{\cosh(K[1])((1+e^2)K[1]+(-1+e^2)\log(K[1]-1))+((-1+e^2)K[1]+(1+e^2)\log(K[1]-1))\sinh(K[1])}{K[1]\log(K[1]-1)((-1+e^2)\cosh(K[1])+(1+e^2)\sinh(K[1]))}d\right.}{\int_1^x \frac{\exp\left(\int_1^{K[2]} -\frac{\cosh(K[1])\left(\left(1+e^2\right)K[1]+\left(-1+e^2\right)\log(K[1]-1)\right)+\left(\left(-1+e^2\right)K[1]+\left(1+e^2\right)\log(K[1]-1)\right)\sinh(K[1])}{K[1]\log(K[1]-1)\left(\left(-1+e^2\right)\cosh(K[1])+\left(1+e^2\right)\sinh(K[1])\right)}dK[1]\right)K[2]((1+e^2)\cosh(K[2]}{\log(K[2]-1)((-1+e^2)\cosh(K[2])+(1+e^2)\sinh(K[2]))}}$$

2.213 problem 789

Internal problem ID [9124]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 789.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' + \frac{\ln(x-1) - \coth(x+1)\, x^2 - 2\coth(x+1)\, xy - \coth(x+1) - \coth(x+1)\, y^2}{\ln(x-1)} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) = -(ln(x-1)-coth(x+1)*x^2-2*coth(x+1)*x*y(x)-coth(x+1)-coth(x+1)*y(x)^2)
```

No solution found

✓ Solution by Mathematica

Time used: 94.214 (sec). Leaf size: 68

```
DSolve[y'[x] == (Coth[1 + x] + x^2*Coth[1 + x] - Log[-1 + x] + 2*x*Coth[1 + x]*y[x] + Coth[1
```

$$y(x) \to -x + \tan\left(\int_1^x \frac{(1+e^2)\cosh(K[5]) + (-1+e^2)\sinh(K[5])}{\log(K[5]-1)\left((-1+e^2)\cosh(K[5]) + (1+e^2)\sinh(K[5])\right)} dK[5] + c_1\right)$$

2.214 problem 790

Internal problem ID [9125]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 790.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - \frac{2x\ln\left(\frac{1}{x-1}\right) - \coth\left(\frac{x+1}{x-1}\right) + \coth\left(\frac{x+1}{x-1}\right)y^2 - 2\coth\left(\frac{x+1}{x-1}\right)x^2y + \coth\left(\frac{x+1}{x-1}\right)x^4}{\ln\left(\frac{1}{x-1}\right)} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) = (2*x*ln(1/(x-1))-coth((x+1)/(x-1))+coth((x+1)/(x-1))*y(x)^2-2*coth((x+
```

No solution found

✓ Solution by Mathematica

Time used: 106.677 (sec). Leaf size: 228

```
DSolve[y'[x] == (-Coth[(1 + x)/(-1 + x)] + x^4*Coth[(1 + x)/(-1 + x)] + 2*x*Log[(-1 + x)^(-1
```

$$y(x) \to \frac{\exp\left(\int_1^x \frac{2\coth\left(\frac{K[5]+1}{K[5]-1}\right)}{\log\left(\frac{1}{K[5]-1}\right)}dK[5]\right)}{-\int_1^x \frac{\exp\left(\int_1^{K[6]} \frac{2\coth\left(\frac{K[5]+1}{K[5]-1}\right)}{\log\left(\frac{1}{K[5]-1}\right)}dK[5]\right)\coth\left(\frac{K[6]+1}{K[6]-1}\right)}{\log\left(\frac{1}{K[6]-1}\right)}dK[6] + c_1} + x^2 + 1$$

$$y(x) \to x^2 + 1$$

$$y(x) \to -\frac{\exp\left(\int_1^x \frac{2\coth\left(\frac{K[5]+1}{K[5]-1}\right)}{\log\left(\frac{1}{K[5]-1}\right)}dK[5]\right)}{\int_1^x \frac{\exp\left(\int_1^{K[6]} \frac{2\coth\left(\frac{K[5]+1}{K[5]-1}\right)}{\log\left(\frac{1}{K[5]-1}\right)}dK[5]\right)\coth\left(\frac{K[6]+1}{K[6]-1}\right)}{\log\left(\frac{1}{K[6]-1}\right)}dK[6]} + x^2 + 1$$

2.215 problem 791

Internal problem ID [9126]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 791.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - \frac{2x^2 \cosh\left(\frac{1}{x-1}\right) - 2x\cosh\left(\frac{1}{x-1}\right) - 1 + y^2 - 2yx^2 + x^4 - x + y^2x - 2yx^3 + x^5}{(x-1)\cosh\left(\frac{1}{x-1}\right)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 161

```
dsolve(diff(y(x),x) = (2*x^2*cosh(1/(x-1))-2*x*cosh(1/(x-1))-1+y(x)^2-2*x^2*y(x)+x^4-x+x*y(x
```

$$y(x) = \frac{\left(x^2 e^{-4\left(\int \frac{e^{\frac{1}{x-1}}(x+1)}{(x-1)\left(e^{\frac{2}{x-1}}+1\right)}dx\right)+4c_1} - x^2 + e^{-4\left(\int \frac{e^{\frac{1}{x-1}}(x+1)}{(x-1)\left(e^{\frac{2}{x-1}}+1\right)}dx\right)+4c_1} + 1\right) e^{4\left(\int \frac{e^{\frac{1}{x-1}}(x+1)}{(x-1)\left(e^{\frac{2}{x-1}}+1\right)}dx\right)}}{e^{4c_1} - e^{4\left(\int \frac{e^{\frac{1}{x-1}}(x+1)}{(x-1)\left(e^{\frac{2}{x-1}}+1\right)}dx\right)}}$$

✓ Solution by Mathematica

Time used: 12.411 (sec). Leaf size: 109

```
DSolve[y'[x] == (Sech[(-1 + x)^(-1)]*(-1 - x + x^4 + x^5 - 2*x*Cosh[(-1 + x)^(-1)] + 2*x^2*C
```

$$y(x) \to \frac{\exp\left(\int_1^x \frac{2(K[5]+1)\operatorname{sech}\left(\frac{1}{K[5]-1}\right)}{K[5]-1}dK[5]\right)}{-\int_1^x \frac{\exp\left(\int_1^{K[6]} \frac{2(K[5]+1)\operatorname{sech}\left(\frac{1}{K[5]-1}\right)}{K[5]-1}dK[5]\right)(K[6]+1)\operatorname{sech}\left(\frac{1}{K[6]-1}\right)}{K[6]-1}dK[6]+c_1}+x^2+1$$

$$y(x) \to x^2+1$$

2.216 problem 792

Internal problem ID [9127]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 792.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$y' - \frac{y\left(-\cosh\left(\frac{1}{x+1}\right)x + \cosh\left(\frac{1}{x+1}\right) - x + yx^2 - x^2 + yx^3\right)}{x(x-1)\cosh\left(\frac{1}{x+1}\right)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 87

```
dsolve(diff(y(x),x) = y(x)*(-cosh(1/(x+1))*x+cosh(1/(x+1))-x+x^2*y(x)-x^2+x^3*y(x))/x/(x-1)/
```

$$y(x) = \frac{e^{-\left(\int \frac{(x^2+x)\operatorname{sech}\left(\frac{1}{x+1}\right)+x-1}{x(x-1)}dx\right)}}{-\left(\int \frac{\operatorname{sech}\left(\frac{1}{x+1}\right)e^{-\left(\int \frac{(x^2+x)\operatorname{sech}\left(\frac{1}{x+1}\right)+x-1}{x(x-1)}dx\right)}x(x+1)}{x-1}dx\right) + c_1}$$

✓ Solution by Mathematica

Time used: 5.482 (sec). Leaf size: 238

```
DSolve[y'[x] == (Sech[(1 + x)^(-1)]*y[x]*(-x - x^2 + Cosh[(1 + x)^(-1)] - x*Cosh[(1 + x)^(-1
```

$$y(x) \to \frac{\exp\left(\int_1^x -\frac{(K[1]+1)\operatorname{sech}\left(\frac{1}{K[1]+1}\right)K[1]+K[1]-1}{(K[1]-1)K[1]}dK[1]\right)}{-\int_1^x \frac{\exp\left(\int_1^{K[2]} -\frac{(K[1]+1)\operatorname{sech}\left(\frac{1}{K[1]+1}\right)K[1]+K[1]-1}{(K[1]-1)K[1]}dK[1]\right)K[2](K[2]+1)\operatorname{sech}\left(\frac{1}{K[2]+1}\right)}{K[2]-1}dK[2]+c_1}$$

$$y(x) \to 0$$

$$y(x) \to -\frac{\exp\left(\int_1^x -\frac{(K[1]+1)\operatorname{sech}\left(\frac{1}{K[1]+1}\right)K[1]+K[1]-1}{(K[1]-1)K[1]}dK[1]\right)}{\int_1^x \frac{\exp\left(\int_1^{K[2]} -\frac{(K[1]+1)\operatorname{sech}\left(\frac{1}{K[1]+1}\right)K[1]+K[1]-1}{(K[1]-1)K[1]}dK[1]\right)K[2](K[2]+1)\operatorname{sech}\left(\frac{1}{K[2]+1}\right)}{K[2]-1}dK[2]}$$

2.217 problem 793

Internal problem ID [9128]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 793.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y' + \frac{y(yx+1)}{x(yx+1-y)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x) = -1/x*y(x)*(x*y(x)+1)/(x*y(x)+1-y(x)),y(x), singsol=all)
```

$$y(x) = \frac{\text{LambertW}\left(-\frac{2(x-1)e^{3c_1-1}}{x}\right)}{x-1}$$

✓ Solution by Mathematica

Time used: 9.457 (sec). Leaf size: 399

```
DSolve[y'[x] == -((y[x]*(1 + x*y[x]))/(x*(1 - y[x] + x*y[x]))),y[x],x,IncludeSingularSolutio
```

$$\text{Solve}\left[-\frac{\sqrt[3]{-2}\left(\frac{2^{2/3}((x-1)y(x)-2)}{\sqrt[3]{-\frac{1}{(x-1)^3}(x-1)((x-1)y(x)+1)}}+(-2)^{2/3}\right)\left(\frac{-xy(x)+y(x)+2}{\sqrt[3]{2}\sqrt[3]{-\frac{1}{(x-1)^3}(x-1)((x-1)y(x)+1)}}+(-2)^{2/3}\right)}{}\right.$$

2.218 problem 794

Internal problem ID [9129]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 794.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{y}{x\left(-1+y+y^3x^2+y^4x^3\right)} = 0$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 209

```
dsolve(diff(y(x),x) = y(x)/x/(-1+y(x)+x^2*y(x)^3+y(x)^4*x^3),y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \frac{-\left(116+12\sqrt{93}\right)^{\frac{2}{3}} - 4 - 2\left(116+12\sqrt{93}\right)^{\frac{1}{3}}}{6\left(116+12\sqrt{93}\right)^{\frac{1}{3}} x}$$

$$y(x) = \frac{i\sqrt{3}\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{2}{3}} - 4i\sqrt{3} + \left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{2}{3}} - 4\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{1}{3}} + 4}{12\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{1}{3}} x}$$

$$y(x) = \frac{-i\left(116+12\sqrt{93}\right)^{\frac{2}{3}}\sqrt{3} + \left(116+12\sqrt{93}\right)^{\frac{2}{3}} + 4i\sqrt{3} - 4\left(116+12\sqrt{93}\right)^{\frac{1}{3}} + 4}{12\left(116+12\sqrt{93}\right)^{\frac{1}{3}} x}$$

$$-y(x) + \int^{xy(x)} \frac{1}{\_a\left(\_a^3+\_a^2+1\right)} d\_a - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.16 (sec). Leaf size: 67

```
DSolve[y'[x] == y[x]/(x*(-1 + y[x] + x^2*y[x]^3 + x^3*y[x]^4)),y[x],x,IncludeSingularSolutio
```

$$\text{Solve}\left[\text{RootSum}\left[\#1^3 y(x)^3 + \#1^2 y(x)^2 + 1\&, \frac{\#1 y(x) \log(x - \#1) + \log(x - \#1)}{3\#1 y(x) + 2}\&\right] + y(x) - \log(x) = c_1, y(x)\right]$$

2.219 problem 795

Internal problem ID [9130]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 795.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, _Abel]`

$$y' - \frac{x^3 + 3a\,x^2 + 3a^2x + a^3 + y^2x + y^2a + y^3}{(x+a)^3} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve(diff(y(x),x) = (x^3+3*a*x^2+3*a^2*x+a^3+x*y(x)^2+a*y(x)^2+y(x)^3)/(x+a)^3,y(x), sings
```

$$y(x) = -\operatorname{RootOf}\left(-\left(\int^{\_Z} \frac{1}{\_a^3 - \_a^2 - \_a - 1} d\_a\right) + \ln(a+x) + c_1\right)(a+x)$$

✓ Solution by Mathematica

Time used: 0.326 (sec). Leaf size: 111

```
DSolve[y'[x] == (a^3 + 3*a^2*x + 3*a*x^2 + x^3 + a*y[x]^2 + x*y[x]^2 + y[x]^3)/(a + x)^3,y[x
```

$$\text{Solve}\left[-\frac{19}{3}\text{RootSum}\left[-19\#1^3+6\sqrt[3]{38}\#1-19\&,\frac{\log\left(\frac{\frac{3y(x)}{(a+x)^3}+\frac{1}{(a+x)^2}}{\sqrt[3]{38}\sqrt[3]{\frac{1}{(a+x)^6}}}-\#1\right)}{2\sqrt[3]{38}-19\#1^2}\&\right]=\frac{1}{9}38^{2/3}\left(\frac{1}{(a+x)^6}\right)^{2/3}(a+x)^4\log(a+x)+c_1,y(x)\right]$$

2.220 problem 796

Internal problem ID [9131]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 796.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class C']]`

$$y' - \frac{y^3 x\, e^{3x^2} e^{-\frac{9x^2}{2}}}{3\left(3\, e^{\frac{3x^2}{2}} + e^{\frac{3x^2}{2}} y + 3y\right)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 135

```
dsolve(diff(y(x),x) = 1/3*y(x)^3*x*exp(3*x^2)/(3*exp(3/2*x^2)+exp(3/2*x^2)*y(x)+3*y(x))/exp(
```

$$y(x) = \text{RootOf}\left(81 + \left(9\, e^{3x^2} + 27\, e^{\frac{3x^2}{2}} + 7\, e^{3x^2 + \text{RootOf}\left(\left(\left(42 \sinh\left(\frac{(c_1 - 5\_Z)\sqrt{93}}{90}\right)\sqrt{93}\, e^{3x^2+\_Z} \cosh\left(\frac{(c_1-5\_Z)\sqrt{93}}{90}\right) + 406\, e^{3x^2+\_Z} \cosh\left(\frac{(c_1-5\_Z)\sqrt{93}}{90}\right)^2 - 217\, e^{3x^2+\_Z} + 9\right.\right.\right.} - 3\right)\_Z^2 + \left(54\, e^{\frac{3x^2}{2}} + 81\right)\_Z\right) e^{\frac{3x^2}{2}}$$

✓ Solution by Mathematica

Time used: 7.509 (sec). Leaf size: 109

```
DSolve[y'[x] == (x*y[x]^3)/(3*E^((3*x^2)/2)*(3*E^((3*x^2)/2) + 3*y[x] + E^((3*x^2)/2)*y[x]))
```

$$\text{Solve}\left[\frac{1}{62}\left(6\sqrt{93}\text{arctanh}\left(\frac{\sqrt{\frac{3}{31}}\left(2e^{\frac{3x^2}{2}}(y(x)+3)+3y(x)\right)}{y(x)}\right)\right.\right.$$
$$\left. -31\log\left(9e^{\frac{3x^2}{2}}(y(x)+3)y(x)+3e^{3x^2}(y(x)+3)^2-y(x)^2\right)+62\log\left(e^{\frac{3x^2}{2}}\right)\right)$$
$$\left.+\log(y(x))=c_1,y(x)\right]$$

2.221 problem 797

Internal problem ID [9132]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 797.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$y' - \frac{y\left(-1-\cosh\left(\frac{x+1}{x-1}\right)x+\cosh\left(\frac{x+1}{x-1}\right)x^2y-\cosh\left(\frac{x+1}{x-1}\right)x^2+y\cosh\left(\frac{x+1}{x-1}\right)x^3\right)}{x}=0$$

✓ Solution by Maple

Time used: 0.391 (sec). Leaf size: 171

```
dsolve(diff(y(x),x) = y(x)*(-1-cosh((x+1)/(x-1))*x+cosh((x+1)/(x-1))*x^2*y(x)-cosh((x+1)/(x-
```

$$y(x) = -\frac{e^{\frac{(-x^2+1)e^{\frac{-x-1}{x-1}}}{4}+\frac{(-x^2-4x+5)e^{\frac{x+1}{x-1}}}{4}+\mathrm{expIntegral}_1\left(\frac{2}{x-1}\right)e^{-1}-3\,e\,\mathrm{expIntegral}_1\left(-\frac{2}{x-1}\right)}}{x\left(-c_1+\int e^{\frac{(-x^2+1)e^{\frac{-x-1}{x-1}}}{4}+\frac{(-x^2-4x+5)e^{\frac{x+1}{x-1}}}{4}+\mathrm{expIntegral}_1\left(\frac{2}{x-1}\right)e^{-1}-3\,e\,\mathrm{expIntegral}_1\left(-\frac{2}{x-1}\right)}(x+1)\cosh\left(\frac{x+1}{x-1}\right)dx\right.}$$

✓ Solution by Mathematica

Time used: 2.638 (sec). Leaf size: 166

```
DSolve[y'[x] == (y[x]*(-1 - x*Cosh[(1 + x)/(-1 + x)] - x^2*Cosh[(1 + x)/(-1 + x)] + x^2*Cosh
```

$$y(x) \to \frac{\exp\left(\frac{4(3e^2-1)\mathrm{Chi}\left(\frac{2}{x-1}\right)+4(1+3e^2)\mathrm{Shi}\left(\frac{2}{x-1}\right)+e^{-\frac{2}{x-1}}}{4e}\right)}{x\left(\exp\left(\frac{4(3e^2-1)\mathrm{Chi}\left(\frac{2}{x-1}\right)+4(1+3e^2)\mathrm{Shi}\left(\frac{2}{x-1}\right)+e^{-\frac{2}{x-1}}}{4e}\right)+c_1\exp\left(\frac{1}{4}e^{\frac{x+1}{x-1}}\left(\left(e^{-\frac{2(x+1)}{x-1}}+1\right)x^2+4x-5\right)\right)\right)}$$

$$y(x) \to 0$$

2.222 problem 798

Internal problem ID [9133]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 798.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{(x+y+1)\,y}{(2y^3+y+x)\,(x+1)} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 30

```
dsolve(diff(y(x),x) = 1/(2*y(x)^3+y(x)+x)*(x+y(x)+1)*y(x)/(x+1),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{RootOf}\left(-\mathrm{e}^{3\_Z}+\ln(x+1)\mathrm{e}^{\_Z}+c_1\mathrm{e}^{\_Z}-\mathrm{e}^{\_Z}\_Z+x\right)}$$

✓ Solution by Mathematica

Time used: 0.323 (sec). Leaf size: 27

```
DSolve[y'[x] == (y[x]*(1 + x + y[x]))/((1 + x)*(x + y[x] + 2*y[x]^3)),y[x],x,IncludeSingular
```

$$\mathrm{Solve}\left[y(x)^2 - \frac{x}{y(x)} + \log(y(x)) - \log(x+1) = c_1, y(x)\right]$$

2.223 problem 799

Internal problem ID [9134]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 799.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$y' - \frac{y\left(-1 - e^{\frac{x+1}{x-1}}x + yx^2 e^{\frac{x+1}{x-1}} - x^2 e^{\frac{x+1}{x-1}} + x^3 e^{\frac{x+1}{x-1}} y\right)}{x} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 107

```
dsolve(diff(y(x),x) = y(x)*(-1-x*exp((x+1)/(x-1))+x^2*exp((x+1)/(x-1))*y(x)-x^2*exp((x+1)/(x
```

$$y(x) = \frac{e^{\frac{(-x^2-4x+5)e^{\frac{x+1}{x-1}}}{2} - 6\,e \operatorname{expIntegral}_1\left(-\frac{2}{x-1}\right)}}{x\left(c_1 - \left(\int (x+1)\,e^{\frac{-(x+5)(x-1)^2 e^{\frac{x+1}{x-1}} - 12(x-1)e \operatorname{expIntegral}_1\left(-\frac{2}{x-1}\right) + 2x + 2}{2x-2}} dx\right)\right)}$$

✓ Solution by Mathematica

Time used: 1.72 (sec). Leaf size: 69

```
DSolve[y'[x] == (y[x]*(-1 - E^((1 + x)/(-1 + x))*x - E^((1 + x)/(-1 + x))*x^2 + E^((1 + x)/(
```

$$y(x) \to \frac{e^{6e \operatorname{ExpIntegralEi}\left(\frac{2}{x-1}\right)}}{x\left(e^{6e \operatorname{ExpIntegralEi}\left(\frac{2}{x-1}\right)} + c_1 e^{\frac{1}{2}e^{\frac{x+1}{x-1}}(x-1)(x+5)}\right)}$$

$$y(x) \to 0$$

2.224 problem 800

Internal problem ID [9135]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 800.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, _Abel]`

$$y' - \frac{-b^3 + 6b^2x - 12b\,x^2 + 8x^3 - 4by^2 + 8y^2x + 8y^3}{(2x-b)^3} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
dsolve(diff(y(x),x) = (-b^3+6*b^2*x-12*b*x^2+8*x^3-4*y(x)^2*b+8*x*y(x)^2+8*y(x)^3)/(2*x-b)^3
```

$$y(x) = \frac{\operatorname{RootOf}\left(-\left(\int^{\_Z} \frac{1}{\_a^3 - \_a^2 - \_a - 1} d\_a\right) + \ln(-2x+b) + c_1\right)(-2x+b)}{2}$$

✓ Solution by Mathematica

Time used: 0.321 (sec). Leaf size: 128

```
DSolve[y'[x] == (-b^3 + 6*b^2*x - 12*b*x^2 + 8*x^3 - 4*b*y[x]^2 + 8*x*y[x]^2 + 8*y[x]^3)/(-b
```

$$\text{Solve}\left[-\frac{19}{3}\text{RootSum}\left[-19\#1^3+6\sqrt[3]{38}\#1-19\&,\frac{\log\left(\frac{\frac{4}{(b-2x)^2}-\frac{24y(x)}{(b-2x)^3}}{4\sqrt[3]{38}\sqrt[3]{\frac{1}{(b-2x)^6}}}-\#1\right)}{2\sqrt[3]{38}-19\#1^2}\&\right]=\frac{1}{9}38^{2/3}\left(\frac{1}{(b-2x)^6}\right)^{2/3}(b-2x)^4\log(b-2x)+c_1,y(x)\right]$$

2.225 problem 801

Internal problem ID [9136]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 801.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$y' - \frac{\left(y\,\mathrm{e}^{-\frac{x^2}{4}}x + 2 + 2y^2\mathrm{e}^{-\frac{x^2}{2}} + 2y^3\mathrm{e}^{-\frac{3x^2}{4}}\right)\mathrm{e}^{\frac{x^2}{4}}}{2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x) = 1/2*(y(x)*exp(-1/4*x^2)*x+2+2*y(x)^2*exp(-1/2*x^2)+2*y(x)^3*exp(-3/4*x
```

$$y(x) = \frac{29\,\mathrm{e}^{\frac{x^2}{4}}\,\mathrm{RootOf}\left(-81\left(\int^{\_Z}\frac{1}{841\_a^3-27\_a+27}d\_a\right)+x+3c_1\right)}{9} - \frac{\mathrm{e}^{\frac{x^2}{4}}}{3}$$

✓ Solution by Mathematica

Time used: 0.322 (sec). Leaf size: 126

```
DSolve[y'[x] == (E^(x^2/4)*(2 + (x*y[x])/E^(x^2/4) + (2*y[x]^2)/E^(x^2/2) + (2*y[x]^3)/E^((3
```

$$\text{Solve}\left[-\frac{29}{3}\text{RootSum}\left[-29\#1^3+3\sqrt[3]{29}\#1-29\&,\frac{\log\left(\frac{3e^{-\frac{x^2}{2}}y(x)+e^{-\frac{x^2}{4}}}{\sqrt[3]{29}\sqrt[3]{e^{-\frac{3x^2}{4}}}}-\#1\right)}{\sqrt[3]{29}-29\#1^2}\&\right]=\frac{1}{9}29^{2/3}e^{\frac{x^2}{2}}\left(e^{-\frac{3x^2}{4}}\right)^{2/3}x+c_1,y(x)\right]$$

2.226 problem 802

Internal problem ID [9137]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 802.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' + \frac{-\frac{1}{x} - f_1\left(y + \frac{1}{x}\right)}{x} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 41

```
dsolve(diff(y(x),x) = -(-1/x-_F1(y(x)+1/x))/x,y(x), singsol=all)
```

$$y(x) = \frac{\text{RootOf}\left(f_1(\_Z)\right)x - 1}{x}$$

$$y(x) = \frac{\text{RootOf}\left(-\ln(x) + \int^{\_Z} \frac{1}{f_1(\_a)} d\_a + c_1\right)x - 1}{x}$$

✓ Solution by Mathematica

Time used: 0.194 (sec). Leaf size: 96

```
DSolve[y'[x] == (x^(-1) + F1[x^(-1) + y[x]])/x,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} -\frac{\text{F1}\left(K[2] + \frac{1}{x}\right)\int_1^x -\frac{\text{F1}'\left(K[2]+\frac{1}{K[1]}\right)}{\text{F1}\left(K[2]+\frac{1}{K[1]}\right)^2 K[1]^2} dK[1] + 1}{\text{F1}\left(K[2] + \frac{1}{x}\right)} dK[2] + \int_1^x \left(\frac{1}{K[1]} + \frac{1}{K[1]^2\text{F1}\left(y(x) + \frac{1}{K[1]}\right)}\right) dK[1] = c_1, y(x)\right]$$

2.227 problem 803

Internal problem ID [9138]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 803.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y' - \frac{f_1(y^2 - 2\ln(x))}{\sqrt{y^2}\, x} = 0$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 65

```
dsolve(diff(y(x),x) = _F1(y(x)^2-2*ln(x))/(y(x)^2)^(1/2)/x,y(x), singsol=all)
```

$$y(x) = \sqrt{2\ln(x) + 2\,\mathrm{RootOf}\left(\ln(x) - \left(\int^{\_Z} \frac{1}{f_1(2\_a) - 1} d\_a\right) + c_1\right)}$$

$$y(x) = -\sqrt{2\ln(x) + 2\,\mathrm{RootOf}\left(\ln(x) - \left(\int^{\_Z} \frac{1}{f_1(2\_a) - 1} d\_a\right) + c_1\right)}$$

✓ Solution by Mathematica

Time used: 1.128 (sec). Leaf size: 603

```
DSolve[y'[x] == F1[-2*Log[x] + y[x]^2]/(x*Sqrt[y[x]^2]),y[x],x,IncludeSingularSolutions -> T
```

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{\sqrt{K[2]^2}\text{F1}(K[2]^2-2\log(x))}{(\text{F1}(K[2]^2-2\log(x))-1)(\text{F1}(K[2]^2-2\log(x))+1)}\right.\right.$$
$$+\frac{K[2]}{(\text{F1}(K[2]^2-2\log(x))-1)(\text{F1}(K[2]^2-2\log(x))+1)}$$
$$-\int_1^x\left(\frac{2K[2]\text{F1}'(K[2]^2-2\log(K[1]))\text{F1}(K[2]^2-2\log(K[1]))^2}{(\text{F1}(K[2]^2-2\log(K[1]))-1)^2(\text{F1}(K[2]^2-2\log(K[1]))+1)K[1]}+\frac{2K[2]\text{F1}'(K[2]^2-2\text{lo}}{(\text{F1}(K[2]^2-2\log(K[1]))-}\right.$$
$$+\int_1^x\left(-\frac{\text{F1}(y(x)^2-2\log(K[1]))^2}{(\text{F1}(y(x)^2-2\log(K[1]))-1)(\text{F1}(y(x)^2-2\log(K[1]))+1)K[1]}\right.$$
$$\left.\left.-\frac{\sqrt{y(x)^2}\text{F1}(y(x)^2-2\log(K[1]))}{(\text{F1}(y(x)^2-2\log(K[1]))-1)(\text{F1}(y(x)^2-2\log(K[1]))+1)K[1]y(x)}\right)dK[1]=c_1,y(x)\right]$$

2.228 problem 804

Internal problem ID [9139]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 804.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - \frac{-\sin(2y)\,x - \sin(2y) + \cos(2y)\,x^4 + x^4}{2x\,(x+1)} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x) = 1/2*(-sin(2*y(x))*x-sin(2*y(x))+cos(2*y(x))*x^4+x^4)/x/(x+1),y(x), sin
```

$$y(x) = \arctan\left(\frac{3x^4 - 4x^3 + 6x^2 + 12\ln(x+1) - 12c_1 - 12x}{12x}\right)$$

✓ Solution by Mathematica

Time used: 7.88 (sec). Leaf size: 77

```
DSolve[y'[x] == (x^4/2 + (x^4*Cos[2*y[x]])/2 - Sin[2*y[x]]/2 - (x*Sin[2*y[x]])/2)/(x*(1 + x)
```

$$y(x) \to \arctan\left(\frac{3x^4 - 4x^3 + 6x^2 - 12x + 12\log(x+1) - 25 - 12c_1}{12x}\right)$$
$$y(x) \to -\frac{1}{2}\pi\sqrt{\frac{1}{x^2}}x$$
$$y(x) \to \frac{1}{2}\pi\sqrt{\frac{1}{x^2}}x$$

2.229 problem 805

Internal problem ID [9140]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 805.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y' - \frac{yx + y + x^4\sqrt{y^2 + x^2}}{x\,(x + 1)} = 0}$$

✓ Solution by Maple

Time used: 0.39 (sec). Leaf size: 42

```
dsolve(diff(y(x),x) = (x*y(x)+y(x)+x^4*(y(x)^2+x^2)^(1/2))/x/(x+1),y(x), singsol=all)
```

$$\ln\left(\sqrt{y\,(x)^2 + x^2} + y(x)\right) - \frac{x^3}{3} + \frac{x^2}{2} - x + \ln\,(x + 1) - \ln\,(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 3.021 (sec). Leaf size: 74

```
DSolve[y'[x] == (y[x] + x*y[x] + x^4*Sqrt[x^2 + y[x]^2])/(x*(1 + x)),y[x],x,IncludeSingularS
```

$$y(x) \to -\frac{xe^{\frac{1}{6}(-2x^3-3x^2-6x-11-6c_1)}\left(e^{x^2}(x+1)^2 - e^{\frac{2x^3}{3}+2x+\frac{11}{3}+2c_1}\right)}{2(x+1)}$$

2.230 problem 806

Internal problem ID [9141]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 806.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `['y=_G(x,y')']`

$$y' - \frac{-\sin(2y)\,x - \sin(2y) + x\cos(2y) + x}{2x\,(x+1)} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(diff(y(x),x) = 1/2*(-sin(2*y(x))*x-sin(2*y(x))+x*cos(2*y(x))+x)/x/(x+1),y(x), singsol
```

$$y(x) = -\arctan\left(\frac{\ln(x+1) - x - c_1}{x}\right)$$

✓ Solution by Mathematica

Time used: 2.411 (sec). Leaf size: 56

```
DSolve[y'[x] == (x/2 + (x*Cos[2*y[x]])/2 - Sin[2*y[x]]/2 - (x*Sin[2*y[x]])/2)/(x*(1 + x)),y[
```

$$y(x) \to \arctan\left(\frac{x - \log(x+1) - c_1}{x}\right)$$
$$y(x) \to -\frac{1}{2}\pi\sqrt{\frac{1}{x^2}}x$$
$$y(x) \to \frac{1}{2}\pi\sqrt{\frac{1}{x^2}}x$$

2.231 problem 807

Internal problem ID [9142]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 807.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$\boxed{y' + \frac{1}{-x - f_1(y - \ln(x))\, y\, e^{y}} = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 43

```
dsolve(diff(y(x),x) = -1/(-x-_F1(y(x)-ln(x))*y(x)*exp(y(x))),y(x), singsol=all)
```

$$\frac{\ln(x)^2}{2} - y(x)\ln(x) - \left(\int^{y(x)-\ln(x)} \frac{f_1(\_a)\_a + e^{-\_a}}{f_1(\_a)} d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.255 (sec). Leaf size: 57

```
DSolve[y'[x] == -(-x - E^y[x]*F1[-Log[x] + y[x]]*y[x])^(-1),y[x],x,IncludeSingularSolutions
```

$$\text{Solve}\left[-\int_1^{y(x)-\log(x)} \frac{\text{F1}(K[1])K[1] + e^{-K[1]}}{\text{F1}(K[1])} dK[1] - y(x)\log(x) + \frac{\log^2(x)}{2} = -c_1, y(x)\right]$$

2.232 problem 808

Internal problem ID [9143]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 808.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]'], [_Ab`

$$y' - \frac{(1+2y)(y+1)}{x(-2y-2+x+2yx)} = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 45

```
dsolve(diff(y(x),x) = 1/x*(1+2*y(x))*(y(x)+1)/(-2*y(x)-2+x+2*x*y(x)),y(x), singsol=all)
```

$$y(x) = \frac{-x \operatorname{LambertW}\left(\frac{e^{-\frac{1}{x}}}{x c_1}\right) - 2}{2x \operatorname{LambertW}\left(\frac{e^{-\frac{1}{x}}}{x c_1}\right) + 2}$$

✓ Solution by Mathematica

Time used: 1.477 (sec). Leaf size: 149

```
DSolve[y'[x] == ((1 + y[x])*(1 + 2*y[x]))/(x*(-2 + x - 2*y[x] + 2*x*y[x])),y[x],x,IncludeSin
```

$$\text{Solve}\left[\frac{2^{2/3}\left(x \log\left(-\frac{6\ 2^{2/3}(y(x)+1)}{2(x-1)y(x)+x-2}\right) - x \log\left(\frac{3\ 2^{2/3}(2xy(x)+x)}{2(x-1)y(x)+x-2}\right) + 2xy(x)\left(\log\left(-\frac{6\ 2^{2/3}(y(x)+1)}{2(x-1)y(x)+x-2}\right) - \log\left(\frac{3\ 2^{2/}}{2(x-}\right.\right.\right.}{9(2xy(x)+x)}\right.$$

2.233 problem 809

Internal problem ID [9144]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 809.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, _Abel]`

$$y' - \frac{-125 + 300x - 240x^2 + 64x^3 - 80y^2 + 64y^2x + 64y^3}{(4x-5)^3} = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 41

```
dsolve(diff(y(x),x) = (-125+300*x-240*x^2+64*x^3-80*y(x)^2+64*x*y(x)^2+64*y(x)^3)/(4*x-5)^3,
```

$$y(x) = -\frac{\operatorname{RootOf}\left(-\left(\int^{\_Z} \frac{1}{\_a^3 - \_a^2 - \_a - 1} d\_a\right) + \ln(4x-5) + c_1\right)(4x-5)}{4}$$

✓ Solution by Mathematica

Time used: 0.247 (sec). Leaf size: 128

```
DSolve[y'[x] == (-125 + 300*x - 240*x^2 + 64*x^3 - 80*y[x]^2 + 64*x*y[x]^2 + 64*y[x]^3)/(-5
```

$$\text{Solve}\left[-\frac{19}{3}\text{RootSum}\left[-19\#1^3+6\sqrt[3]{38}\#1-19\&,\frac{\log\left(\frac{\frac{192y(x)}{(4x-5)^3}+\frac{16}{(4x-5)^2}}{16\sqrt[3]{38}\sqrt[3]{\frac{1}{(4x-5)^6}}}-\#1\right)}{2\sqrt[3]{38}-19\#1^2}\&\right]=\frac{1}{9}38^{2/3}\left(\frac{1}{(5-4x)^6}\right)^{2/3}(5-4x)^4\log(5-4x)+c_1,y(x)\right]$$

2.234 problem 810

Internal problem ID [9145]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 810.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$\boxed{y' - \frac{x + y + y^2 - 2y \ln(x)\, x + \ln(x)^2\, x^2}{x} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x) = (x+y(x)+y(x)^2-2*y(x)*ln(x)*x+x^2*ln(x)^2)/x,y(x), singsol=all)
```

$$y(x) = \left(\ln(x) + \frac{1}{-x + c_1}\right) x$$

✓ Solution by Mathematica
Time used: 0.21 (sec). Leaf size: 26

```
DSolve[y'[x] == (x + x^2*Log[x]^2 + y[x] - 2*x*Log[x]*y[x] + y[x]^2)/x,y[x],x,IncludeSingula
```

$$y(x) \to x\left(\log(x) + \frac{1}{-x + c_1}\right)$$
$$y(x) \to x \log(x)$$

2.235 problem 811

Internal problem ID [9146]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 811.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `['y=_G(x,y')']`

$$y' - \frac{x^3 e^y + x^4 + e^y y - e^y \ln(e^y + x) + yx - \ln(e^y + x)\, x + x}{x^2} = 0$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 32

```
dsolve(diff(y(x),x) = (x^3*exp(y(x))+x^4+exp(y(x))*y(x)-exp(y(x))*ln(exp(y(x))+x)+x*y(x)-ln(
```

$$y(x) = \frac{x^3}{2} + c_1 x + \ln\left(-\frac{x}{-1 + e^{\frac{x(x^2+2c_1)}{2}}}\right)$$

✓ Solution by Mathematica

Time used: 4.135 (sec). Leaf size: 29

```
DSolve[y'[x] == (x + E^y[x]*x^3 + x^4 - E^y[x]*Log[E^y[x] + x] - x*Log[E^y[x] + x] + E^y[x]*
```

$$y(x) \to -\log\left(\frac{-1 + e^{-\frac{1}{2}x(x^2+2c_1)}}{x}\right)$$

2.236 problem 812

Internal problem ID [9147]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 812.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, ‘_with_symmetry_[F(x),G(x)]‘]]`

$$y' - \sqrt{x^3 - 6y} - x^2\sqrt{x^3 - 6y} - x^3\sqrt{x^3 - 6y} = \frac{x^2}{2}$$

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 30

```
dsolve(diff(y(x),x) = 1/2*x^2+(x^3-6*y(x))^(1/2)+x^2*(x^3-6*y(x))^(1/2)+x^3*(x^3-6*y(x))^(1/
```

$$c_1 - \frac{3x^4}{4} - x^3 - 3x - \sqrt{x^3 - 6y(x)} = 0$$

✓ Solution by Mathematica

Time used: 0.618 (sec). Leaf size: 76

```
DSolve[y'[x] == x^2/2 + Sqrt[x^3 - 6*y[x]] + x^2*Sqrt[x^3 - 6*y[x]] + x^3*Sqrt[x^3 - 6*y[x]]
```

$$y(x) \to -\frac{3x^8}{32} - \frac{x^7}{4} - \frac{x^6}{6} - \frac{3x^5}{4} + \left(-1 + \frac{3c_1}{4}\right)x^4 + \left(\frac{1}{6} + c_1\right)x^3 - \frac{3x^2}{2} + 3c_1 x - \frac{3{c_1}^2}{2}$$

2.237 problem 813

Internal problem ID [9148]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 813.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' - \frac{\left(-x^3\sqrt{a} + 2\sqrt{a\,x^4+8y} + 2x^2\sqrt{a\,x^4+8y} + 2x^3\sqrt{a\,x^4+8y}\right)\sqrt{a}}{2} = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 40

```
dsolve(diff(y(x),x) = 1/2*(-a^(1/2)*x^3+2*(a*x^4+8*y(x))^(1/2)+2*x^2*(a*x^4+8*y(x))^(1/2)+2*
```

$$\frac{\sqrt{x^4a+8y(x)}}{4} + \frac{(-3x^4-4x^3-12x)\sqrt{a}}{12} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.872 (sec). Leaf size: 64

```
DSolve[y'[x] == (Sqrt[a]*(-(Sqrt[a]*x^3) + 2*Sqrt[a*x^4 + 8*y[x]] + 2*x^2*Sqrt[a*x^4 + 8*y[x
```

$$y(x) \rightarrow \frac{1}{72}a\left(9x^8+24x^7+16x^6+72x^5+(87-72c_1)x^4-96c_1x^3+144x^2-288c_1x+144{c_1}^2\right)$$

2.238 problem 814

Internal problem ID [9149]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 814.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class C‘], [_1st_order, ‘_wit`

$$y' - \frac{y(-3yx^3 - 3 + y^2x^7)}{x\,(yx^3 + 1)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x) = y(x)/x*(-3*x^3*y(x)-3+y(x)^2*x^7)/(x^3*y(x)+1),y(x), singsol=all)
```

$$y(x) = \frac{1}{\left(\sqrt{-2x + c_1} - 1\right) x^3}$$
$$y(x) = -\frac{1}{\left(\sqrt{-2x + c_1} + 1\right) x^3}$$

✓ Solution by Mathematica

Time used: 0.776 (sec). Leaf size: 75

```
DSolve[y'[x] == (y[x]*(-3 - 3*x^3*y[x] + x^7*y[x]^2))/(x*(1 + x^3*y[x])),y[x],x,IncludeSingu
```

$$y(x) \to \frac{x}{-x^4 + \frac{\sqrt{x(-2x+1+c_1)}}{\sqrt{\frac{1}{x^7}}}}$$
$$y(x) \to -\frac{x}{x^4 + \frac{\sqrt{x(-2x+1+c_1)}}{\sqrt{\frac{1}{x^7}}}}$$
$$y(x) \to 0$$

2.239 problem 815

Internal problem ID [9150]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 815.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class C']]`

$$y' - \frac{(3+y)^3 \, e^{\frac{9x^2}{2}} x \, e^{\frac{3x^2}{2}} e^{-3x^2}}{81 \left(3 \, e^{\frac{3x^2}{2}} + e^{\frac{3x^2}{2}} y + 3y\right)} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 165

```
dsolve(diff(y(x),x) = 1/81*(3+y(x))^3*exp(9/2*x^2)*x*exp(3/2*x^2)/(3*exp(3/2*x^2)+exp(3/2*x^
```

$$\begin{aligned}
&5\ln(3) - 5\ln(7) \\
&+ 5\ln\left(\frac{\left(-81y(x)^2 - 243y(x)\right) e^{\frac{3x^2}{2}} + (3+y(x))^2 e^{3x^2} - 243y(x)^2}{\left(e^{\frac{3x^2}{2}}(3+y(x)) + 3y(x)\right)^2}\right) \\
&- \frac{30\sqrt{93}\, \operatorname{arctanh}\left(\frac{\left(29y(x)e^{\frac{3x^2}{2}} + 87\, e^{\frac{3x^2}{2}} + 81y(x)\right)\sqrt{93}}{(279y(x)+837)e^{\frac{3x^2}{2}} + 837y(x)}\right)}{31} \\
&- 10\ln\left(\frac{e^{\frac{3x^2}{2}}(3+y(x))}{e^{\frac{3x^2}{2}}(3+y(x)) + 3y(x)}\right) + 15x^2 - c_1 = 0
\end{aligned}$$

✓ Solution by Mathematica

Time used: 7.811 (sec). Leaf size: 103

```
DSolve[y'[x] == (E^(3*x^2)*x*(3 + y[x])^3)/(81*(3*E^((3*x^2)/2) + 3*y[x] + E^((3*x^2)/2)*y[x
```

$$\text{Solve}\left[\frac{1}{186}\left(6\sqrt{93}\text{arctanh}\left(\frac{81y(x)-2e^{\frac{3x^2}{2}}(y(x)+3)}{9\sqrt{93}y(x)}\right)\right.\right.$$
$$\left.+31\log\left(-81e^{\frac{3x^2}{2}}(y(x)+3)y(x)+e^{3x^2}(y(x)+3)^2-243y(x)^2\right)\right)$$
$$\left.-\frac{1}{3}\log(y(x)+3)=c_1,y(x)\right]$$

2.240 problem 816

Internal problem ID [9151]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 816.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{(x-y)^3 (x+y)^3 x}{(-y^2+x^2-1)\, y} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 226

```
dsolve(diff(y(x),x) = (x-y(x))^3*(x+y(x))^3*x/(-y(x)^2+x^2-1)/y(x),y(x), singsol=all)
```

$$-\left(\int_{\_b}^{x} \frac{(-\_a+y(x))^3 (\_a+y(x))^3 \_a}{-\_a^6+3\_a^4 y(x)^2-3y(x)^4 \_a^2+y(x)^6+\_a^2-y(x)^2-1} d\_a\right)$$
$$+\int^{y(x)} \frac{2\left(\left(-\_f^6+3\_f^4x^2+(-3x^4+1)\_f^2+x^6-x^2+1\right)\left(\int_{\_b}^{x} \frac{(\_a-\_f)^2(\_a+\_f)^2\_a(2\_a^2-2\_f^2-3)}{(\_a^6-3\_a^4\_f^2+(3\_f^4-1)\_a^2-\_f^6+\_f^2+1}\right.\right.}{-\_f^6+3\_f^4x^2+(-3x^4+1)\_f^2+x^6-x^2+1}$$
$$+c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.193 (sec). Leaf size: 74

```
DSolve[y'[x] == (x*(x - y[x])^3*(x + y[x])^3)/(y[x]*(-1 + x^2 - y[x]^2)),y[x],x,IncludeSingu
```

$$\text{Solve}\left[\frac{1}{2}\left(\text{RootSum}\left[\#1^3-\#1+1\&, \frac{\#1\log\left(-\#1+x^2-y(x)^2\right)-\log\left(-\#1+x^2-y(x)^2\right)}{3\#1^2-1}\&\right]\right.\right.$$
$$\left.\left.+x^2\right)=c_1, y(x)\right]$$

2.241 problem 817

Internal problem ID [9152]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 817.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `['y=_G(x,y')']`

$$y' - \frac{-2\cos(y) + x^3\cos(2y)\ln(x) + x^3\ln(x)}{2\sin(y)\ln(x)\,x} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(diff(y(x),x) = 1/2*(-2*cos(y(x))+x^3*cos(2*y(x))*ln(x)+x^3*ln(x))/sin(y(x))/ln(x)/x,y
```

$$y(x) = \operatorname{arcsec}\left(\frac{3x^3\ln(x) - x^3 + 9c_1}{9\ln(x)}\right)$$

✓ Solution by Mathematica

Time used: 1.464 (sec). Leaf size: 77

```
DSolve[y'[x] == (Csc[y[x]]*(-Cos[y[x]] + (x^3*Log[x])/2 + (x^3*Cos[2*y[x]]*Log[x])/2))/(x*Lo
```

$$\begin{aligned}
y(x) &\to -\sec^{-1}\left(-\frac{x^3 - 3x^3\log(x) + 9c_1}{9\log(x)}\right) \\
y(x) &\to \sec^{-1}\left(-\frac{x^3 - 3x^3\log(x) + 9c_1}{9\log(x)}\right) \\
y(x) &\to -\frac{\pi}{2} \\
y(x) &\to \frac{\pi}{2}
\end{aligned}$$

2.242 problem 818

Internal problem ID [9153]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 818.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$y' - \frac{y}{x\left(-1 + yx + xy^3 + y^4 x\right)} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
dsolve(diff(y(x),x) = y(x)/x/(-1+x*y(x)+x*y(x)^3+x*y(x)^4),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{RootOf}\left(-2x\,\mathrm{e}^{4\_Z} - 3x\,\mathrm{e}^{3\_Z} + 6c_1 x\,\mathrm{e}^{\_Z} - 6\_Zx\,\mathrm{e}^{\_Z} - 6\right)}$$

✓ Solution by Mathematica

Time used: 0.165 (sec). Leaf size: 34

```
DSolve[y'[x] == y[x]/(x*(-1 + x*y[x] + x*y[x]^3 + x*y[x]^4)),y[x],x,IncludeSingularSolutions
```

$$\text{Solve}\left[\frac{y(x)^3}{3} + \frac{y(x)^2}{2} + \frac{1}{xy(x)} + \log(y(x)) = c_1, y(x)\right]$$

2.243 problem 819

Internal problem ID [9154]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 819.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' - \sqrt{x^2+3y} - x^2\sqrt{x^2+3y} - \sqrt{x^2+3y}\,x^3 = -\frac{2x}{3}$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 30

```
dsolve(diff(y(x),x) = -2/3*x+(x^2+3*y(x))^(1/2)+x^2*(x^2+3*y(x))^(1/2)+x^3*(x^2+3*y(x))^(1/2
```

$$c_1 + \frac{3x^4}{8} + \frac{x^3}{2} + \frac{3x}{2} - \sqrt{x^2+3y(x)} = 0$$

✓ Solution by Mathematica

Time used: 0.471 (sec). Leaf size: 63

```
DSolve[y'[x] == (-2*x)/3 + Sqrt[x^2 + 3*y[x]] + x^2*Sqrt[x^2 + 3*y[x]] + x^3*Sqrt[x^2 + 3*y[
```

$$y(x) \to \frac{1}{192}\left(9x^8 + 24x^7 + 16x^6 + 72x^5 + (96 - 72c_1)x^4 - 96c_1x^3 + 80x^2 - 288c_1x + 144c_1^2\right)$$

2.244 problem 820

Internal problem ID [9155]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 820.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$y' - \frac{-2\cos(y) + x^2\cos(2y)\ln(x) + x^2\ln(x)}{2\sin(y)\ln(x)\,x} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x) = 1/2*(-2*cos(y(x))+x^2*cos(2*y(x))*ln(x)+x^2*ln(x))/sin(y(x))/ln(x)/x,y
```

$$y(x) = \operatorname{arcsec}\left(\frac{2x^2\ln(x) - x^2 + 4c_1}{4\ln(x)}\right)$$

✓ Solution by Mathematica

Time used: 1.353 (sec). Leaf size: 77

```
DSolve[y'[x] == (Csc[y[x]]*(-Cos[y[x]] + (x^2*Log[x])/2 + (x^2*Cos[2*y[x]]*Log[x])/2))/(x*Lo
```

$$y(x) \to -\sec^{-1}\left(-\frac{x^2 - 2x^2\log(x) + 4c_1}{4\log(x)}\right)$$
$$y(x) \to \sec^{-1}\left(-\frac{x^2 - 2x^2\log(x) + 4c_1}{4\log(x)}\right)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

2.245 problem 821

Internal problem ID [9156]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 821.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{y(yx+1)}{x\,(-yx-1+y^4x^3)} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x) = 1/x*y(x)*(x*y(x)+1)/(-x*y(x)-1+y(x)^4*x^3),y(x), singsol=all)
```

$$-\frac{1}{2x^2y\left(x\right)^2} - \frac{1}{3y\left(x\right)^3x^3} - y(x) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.161 (sec). Leaf size: 1993

```
DSolve[y'[x] == (y[x]*(1 + x*y[x]))/(x*(-1 - x*y[x] + x^3*y[x]^4)),y[x],x,IncludeSingularSol
```

$y(x) \rightarrow$

$$-\frac{1}{2}\sqrt{\frac{\sqrt[3]{36c_1{}^2x^6+27x^5+\sqrt{x^9\left(216(-1+6c_1)c_1{}^3x^3+216c_1{}^2x^2-9(-81+512c_1)x-4096\right)}}}{6x^3}+\frac{}{3\sqrt[3]{36c}}}$$

$$-\frac{1}{2}\sqrt{-\frac{\sqrt[3]{36c_1{}^2x^6+27x^5+\sqrt{x^9\left(216(-1+6c_1)c_1{}^3x^3+216c_1{}^2x^2-9(-81+512c_1)x-4096\right)}}}{6x^3}+\frac{}{3\sqrt[3]{36}}}$$

$$+\frac{c_1}{4}$$

$y(x) \rightarrow$

$$-\frac{1}{2}\sqrt{\frac{\sqrt[3]{36c_1{}^2x^6+27x^5+\sqrt{x^9\left(216(-1+6c_1)c_1{}^3x^3+216c_1{}^2x^2-9(-81+512c_1)x-4096\right)}}}{6x^3}+\frac{}{3\sqrt[3]{36c}}}$$

$$+\frac{1}{2}\sqrt{-\frac{\sqrt[3]{36c_1{}^2x^6+27x^5+\sqrt{x^9\left(216(-1+6c_1)c_1{}^3x^3+216c_1{}^2x^2-9(-81+512c_1)x-4096\right)}}}{6x^3}+\frac{}{3\sqrt[3]{36}}}$$

$$+\frac{c_1}{4}$$

$y(x)$

$$\rightarrow\frac{1}{2}\sqrt{\frac{\sqrt[3]{36c_1{}^2x^6+27x^5+\sqrt{x^9\left(216(-1+6c_1)c_1{}^3x^3+216c_1{}^2x^2-9(-81+512c_1)x-4096\right)}}}{6x^3}+\frac{}{3\sqrt[3]{36c_1{}^2}}}$$

$$-\frac{1}{2}\sqrt{-\frac{\sqrt[3]{36c_1{}^2x^6+27x^5+\sqrt{x^9\left(216(-1+6c_1)c_1{}^3x^3+216c_1{}^2x^2-9(-81+512c_1)x-4096\right)}}}{6x^3}+\frac{}{3\sqrt[3]{36}}}$$

$$+\frac{c_1}{4}$$

$y(x)$

$$\rightarrow\frac{1}{2}\sqrt{\frac{\sqrt[3]{36c_1{}^2x^6+27x^5+\sqrt{x^9\left(216(-1+6c_1)c_1{}^3x^3+216c_1{}^2x^2-9(-81+512c_1)x-4096\right)}}}{6x^3}+\frac{}{3\sqrt[3]{36c_1{}^2}}}$$

2.246 problem 822

Internal problem ID [9157]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 822.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - \frac{x\left(\mathrm{e}^{-2x^2}x^4 - 4x^2\mathrm{e}^{-x^2}y - 4x^2\mathrm{e}^{-x^2} + 4y^2 + 4\,\mathrm{e}^{-x^2}\right)}{4} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x) = 1/4*(4*exp(-x^2)-4*x^2*exp(-x^2)+4*y(x)^2-4*x^2*exp(-x^2)*y(x)+x^4*exp
```

$$y(x) = \frac{-4 + x^2(x^2 - 2c_1)\,\mathrm{e}^{-x^2}}{2x^2 - 4c_1}$$

✓ Solution by Mathematica

Time used: 0.502 (sec). Leaf size: 50

```
DSolve[y'[x] == (x*(4/E^x^2 - (4*x^2)/E^x^2 + x^4/E^(2*x^2) - (4*x^2*y[x])/E^x^2 + 4*y[x]^2)
```

$$y(x) \to \frac{1}{2}e^{-x^2}x^2 + \frac{1}{-\frac{x^2}{2} + c_1}$$
$$y(x) \to \frac{1}{2}e^{-x^2}x^2$$

2.247 problem 823

Internal problem ID [9158]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 823.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{y(x+y)}{x(x+y+y^3+y^4)} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 38

```
dsolve(diff(y(x),x) = y(x)*(x+y(x))/x/(x+y(x)+y(x)^3+y(x)^4),y(x), singsol=all)
```

$$y(x) = e^{\text{RootOf}\left(-2\,e^{4\_Z}-3\,e^{3\_Z}+6\,e^{\_Z}\ln(x)+6c_1e^{\_Z}-6\,e^{\_Z}\_Z+6x\right)}$$

✓ Solution by Mathematica

Time used: 0.245 (sec). Leaf size: 39

```
DSolve[y'[x] == (y[x]*(x + y[x]))/(x*(x + y[x] + y[x]^3 + y[x]^4)),y[x],x,IncludeSingularSol
```

$$\text{Solve}\left[\frac{y(x)^3}{3} + \frac{y(x)^2}{2} + \log(y(x)) - \frac{y(x)\log(x) + x}{y(x)} = c_1, y(x)\right]$$

2.248 problem 824

Internal problem ID [9159]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 824.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$y' - \frac{y(x^3 + yx^2 + y^2)}{x^2 (x-1)(x+y)} = 0$$

✓ Solution by Maple

Time used: 0.844 (sec). Leaf size: 61

```
dsolve(diff(y(x),x) = y(x)/x^2/(x-1)*(x^3+x^2*y(x)+y(x)^2)/(x+y(x)),y(x), singsol=all)
```

$$-\frac{\ln\left(\frac{y(x)^2+xy(x)+x^2}{x^2}\right)}{2} + \frac{\sqrt{3}\arctan\left(\frac{(x+2y(x))\sqrt{3}}{3x}\right)}{3} + \ln\left(\frac{y(x)}{x}\right) - \ln(x-1) + \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.179 (sec). Leaf size: 68

```
DSolve[y'[x] == (y[x]*(x^3 + x^2*y[x] + y[x]^2))/((-1 + x)*x^2*(x + y[x])),y[x],x,IncludeSin
```

$$\text{Solve}\left[\frac{\arctan\left(\frac{\frac{2y(x)}{x}+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{2}\log\left(\frac{y(x)^2}{x^2} + \frac{y(x)}{x} + 1\right) + \log\left(\frac{y(x)}{x}\right) = \log(1-x) - \log(x) + c_1, y(x)\right]$$

2.249 problem 825

Internal problem ID [9160]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 825.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$y' - \frac{\left((x^2+1)^{\frac{3}{2}} x^2 + (x^2+1)^{\frac{3}{2}} + y^2(x^2+1)^{\frac{3}{2}} + y^3x^2 + y^3\right) x}{(x^2+1)^3} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(diff(y(x),x) = ((x^2+1)^(3/2)*x^2+(x^2+1)^(3/2)+y(x)^2*(x^2+1)^(3/2)+x^2*y(x)^3+y(x)^
```

$$y(x) = \frac{\sqrt{x^2+1}\left(19 \operatorname{RootOf}\left(-1296\left(\int^{\_Z} \frac{1}{361\_a^3-432\_a+432} d\_a\right) + 2\ln(x^2+1) + 3c_1\right) - 6\right)}{18}$$

✓ Solution by Mathematica

Time used: 1.381 (sec). Leaf size: 148

```
DSolve[y'[x] == (x*((1 + x^2)^(3/2) + x^2*(1 + x^2)^(3/2) + (1 + x^2)^(3/2)*y[x]^2 + y[x]^3
```

$$\text{Solve}\left[-\frac{19}{3}\text{RootSum}\left[-19\#1^3+6\sqrt[3]{38}\#1-19\&,\frac{\log\left(\frac{\frac{3xy(x)}{(x^2+1)^2}+\frac{x}{(x^2+1)^{3/2}}}{\sqrt[3]{38}\sqrt[3]{\frac{x^3}{(x^2+1)^{9/2}}}}-\#1\right)}{2\sqrt[3]{38}-19\#1^2}\&\right]=\frac{19^{2/3}\left(\frac{x^3}{(x^2+1)^{9/2}}\right)^{2/3}(x^2+1)^3\log(x^2+1)}{9\sqrt[3]{2}x^2}+c_1,y(x)\right]$$

2.250 problem 826

Internal problem ID [9161]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 826.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' - \frac{(3y^2x + x + 3y^2)\,y}{(6y^2 + x)\,x\,(x+1)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(diff(y(x),x) = 1/(6*y(x)^2+x)*(3*x*y(x)^2+x+3*y(x)^2)*y(x)/x/(x+1),y(x), singsol=all)
```

$$\frac{y(x)^2\,x}{6y(x)^2 + x} = \frac{\left(\mathrm{e}^{\mathrm{RootOf}\left(-\mathrm{e}^{\_Z}\ln\left(\frac{(x+1)^2(\mathrm{e}^{\_Z}+9)}{x}\right)+\mathrm{e}^{\_Z}\ln(2)+3c_1\mathrm{e}^{\_Z}+\mathrm{e}^{\_Z}\_Z+9\right)} + 9\right)x}{54}$$

✓ Solution by Mathematica

Time used: 7.029 (sec). Leaf size: 75

```
DSolve[y'[x] == (y[x]*(x + 3*y[x]^2 + 3*x*y[x]^2))/(x*(1 + x)*(x + 6*y[x]^2)),y[x],x,Include
```

$$y(x) \to -\frac{\sqrt{x}\sqrt{W\left(\frac{6e^{2c_1}x}{(x+1)^2}\right)}}{\sqrt{6}}$$
$$y(x) \to \frac{\sqrt{x}\sqrt{W\left(\frac{6e^{2c_1}x}{(x+1)^2}\right)}}{\sqrt{6}}$$
$$y(x) \to 0$$

2.251 problem 827

Internal problem ID [9162]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 827.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' + \frac{-y + x^3\sqrt{y^2+x^2} - y\sqrt{y^2+x^2}\,x^2}{x} = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 50

```
dsolve(diff(y(x),x) = -(-y(x)+x^3*(y(x)^2+x^2)^(1/2)-x^2*(y(x)^2+x^2)^(1/2)*y(x))/x,y(x), si
```

$$\ln(2) + \ln\left(\frac{x\left(\sqrt{2y(x)^2+2x^2}+y(x)+x\right)}{y(x)-x}\right) + \frac{\sqrt{2}\,x^3}{3} - \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.329 (sec). Leaf size: 84

```
DSolve[y'[x] == (y[x] - x^3*Sqrt[x^2 + y[x]^2] + x^2*y[x]*Sqrt[x^2 + y[x]^2])/x,y[x],x,Inclu
```

$$y(x) \to \frac{x\tanh\left(\frac{x^3+3c_1}{3\sqrt{2}}\right)\left(2+\sqrt{2}\tanh\left(\frac{x^3+3c_1}{3\sqrt{2}}\right)\right)}{\sqrt{2}+2\tanh\left(\frac{x^3+3c_1}{3\sqrt{2}}\right)}$$

$$y(x) \to x$$

2.252 problem 828

Internal problem ID [9163]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 828.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{y' - \frac{(1+2y)(y+1)}{x(-2y-2+xy^3+2y^4x)} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 67

```
dsolve(diff(y(x),x) = 1/x*(1+2*y(x))*(y(x)+1)/(-2*y(x)-2+x*y(x)^3+2*x*y(x)^4),y(x), singsol=
```

$$y(x) = -1$$
$$y(x) = -\frac{1}{2}$$
$$y(x) = \frac{e^{\mathrm{RootOf}\left(16\,e^{\_Z}x\ln\left(e^{\_Z}+1\right)-16\,e^{\_Z}x\ln(2)+8c_1x\,e^{\_Z}-2\_Zx\,e^{\_Z}+x\,e^{3\_Z}-8x\,e^{2\_Z}+7x\,e^{\_Z}+16\right)}}{2} - \frac{1}{2}$$

✓ Solution by Mathematica

Time used: 0.375 (sec). Leaf size: 56

```
DSolve[y'[x] == ((1 + y[x])*(1 + 2*y[x]))/(x*(-2 - 2*y[x] + x*y[x]^3 + 2*x*y[x]^4)),y[x],x,I
```

$$\text{Solve}\left[-\frac{1}{8}y(x)^2+\frac{3y(x)}{8}-\frac{1}{2x(2y(x)+1)}-\frac{1}{2}\log(y(x)+1)+\frac{1}{16}\log(2y(x)+1)=c_1,y(x)\right]$$

2.253 problem 829

Internal problem ID [9164]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 829.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' - \frac{1 + 2\sqrt{4yx^2+1}\,x^3 + 2x^5\sqrt{4yx^2+1} + 2x^6\sqrt{4yx^2+1}}{2x^3} = 0$$

✓ Solution by Maple

Time used: 0.265 (sec). Leaf size: 41

```
dsolve(diff(y(x),x) = 1/2*(1+2*(4*x^2*y(x)+1)^(1/2)*x^3+2*x^5*(4*x^2*y(x)+1)^(1/2)+2*x^6*(4*
```

$$\frac{4x^6 + 5x^5 + 10x^3 + 10c_1x - 10\sqrt{4x^2y(x)+1}}{10x} = 0$$

✓ Solution by Mathematica

Time used: 0.646 (sec). Leaf size: 81

```
DSolve[y'[x] == (1/2 + x^3*Sqrt[1 + 4*x^2*y[x]] + x^5*Sqrt[1 + 4*x^2*y[x]] + x^6*Sqrt[1 + 4*
```

$$y(x) \to \frac{x^{10}}{25} + \frac{x^9}{10} + \frac{x^8}{16} + \frac{x^7}{5} + \frac{x^6}{4} - \frac{2c_1x^5}{5} - \frac{1}{4}(-1+2c_1)x^4 - \frac{1}{4x^2} - c_1x^2 + c_1{}^2$$

2.254 problem 830

Internal problem ID [9165]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 830.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{y' - \frac{y(x-y)}{x\,(x-y-y^3-y^4)} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x) = y(x)*(x-y(x))/x/(x-y(x)-y(x)^3-y(x)^4),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{RootOf}\left(2\,\mathrm{e}^{4\_Z}+3\,\mathrm{e}^{3\_Z}-6\,\mathrm{e}^{\_Z}\ln(x)+6c_1\mathrm{e}^{\_Z}+6\,\mathrm{e}^{\_Z}\_Z+6x\right)}$$

✓ Solution by Mathematica

Time used: 0.272 (sec). Leaf size: 37

```
DSolve[y'[x] == ((x - y[x])*y[x])/(x*(x - y[x] - y[x]^3 - y[x]^4)),y[x],x,IncludeSingularSol
```

$$\mathrm{Solve}\left[-\frac{1}{3}y(x)^3 - \frac{y(x)^2}{2} - \frac{x}{y(x)} - \log(y(x)) + \log(x) = c_1, y(x)\right]$$

2.255 problem 831

Internal problem ID [9166]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 831.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$y' - \frac{2a + \sqrt{-y^2 + 4ax} + x^2\sqrt{-y^2 + 4ax} + x^3\sqrt{-y^2 + 4ax}}{y} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(diff(y(x),x) = (2*a+(-y(x)^2+4*a*x)^(1/2)+x^2*(-y(x)^2+4*a*x)^(1/2)+x^3*(-y(x)^2+4*a*
```

$$-\sqrt{4ax - y(x)^2} - \frac{x^4}{4} - \frac{x^3}{3} - x - c_1 = 0$$

✓ Solution by Mathematica

Time used: 4.346 (sec). Leaf size: 79

```
DSolve[y'[x] == (2*a + Sqrt[4*a*x - y[x]^2] + x^2*Sqrt[4*a*x - y[x]^2] + x^3*Sqrt[4*a*x - y[
```

$$y(x) \to -\frac{1}{12}\sqrt{576ax - (3x^4 + 4x^3 + 12x + 12c_1)^2}$$
$$y(x) \to \frac{1}{12}\sqrt{576ax - (3x^4 + 4x^3 + 12x + 12c_1)^2}$$

2.256 problem 832

Internal problem ID [9167]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 832.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{(x + y + 1)\, y}{(y^4 + y^3 + y^2 + x)\,(x + 1)} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x) = 1/(y(x)^4+y(x)^3+y(x)^2+x)*(x+y(x)+1)*y(x)/(x+1),y(x), singsol=all)
```

$$\ln(x+1) + \frac{x}{y(x)} - \frac{y(x)^3}{3} - \frac{y(x)^2}{2} - y(x) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 61.363 (sec). Leaf size: 2405

```
DSolve[y'[x] == (y[x]*(1 + x + y[x]))/((1 + x)*(x + y[x]^2 + y[x]^3 + y[x]^4)),y[x],x,Includ
```

Too large to display

2.257 problem 833

Internal problem ID [9168]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 833.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' + \frac{-y + x^4\sqrt{y^2+x^2} - x^3\sqrt{y^2+x^2}\,y}{x} = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 50

```
dsolve(diff(y(x),x) = -(-y(x)+x^4*(y(x)^2+x^2)^(1/2)-x^3*(y(x)^2+x^2)^(1/2)*y(x))/x,y(x), si
```

$$\ln(2) + \ln\left(\frac{x\left(\sqrt{2y(x)^2+2x^2}+y(x)+x\right)}{y(x)-x}\right) + \frac{\sqrt{2}\,x^4}{4} - \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.692 (sec). Leaf size: 84

```
DSolve[y'[x] == (y[x] - x^4*Sqrt[x^2 + y[x]^2] + x^3*y[x]*Sqrt[x^2 + y[x]^2])/x,y[x],x,Inclu
```

$$y(x) \to \frac{x\tanh\left(\frac{x^4+4c_1}{4\sqrt{2}}\right)\left(2+\sqrt{2}\tanh\left(\frac{x^4+4c_1}{4\sqrt{2}}\right)\right)}{\sqrt{2}+2\tanh\left(\frac{x^4+4c_1}{4\sqrt{2}}\right)}$$

$$y(x) \to x$$

2.258 problem 834

Internal problem ID [9169]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 834.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y' - \frac{(x^4 + 3y^2x + 3y^2)\,y}{(6y^2 + x)\,x\,(x+1)} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 68

```
dsolve(diff(y(x),x) = (x^4+3*x*y(x)^2+3*y(x)^2)/(6*y(x)^2+x)*y(x)/x/(x+1),y(x), singsol=all)
```

$$\frac{y(x)^2\,x}{6y(x)^2 + x} = \frac{\left(\mathrm{e}^{\mathrm{RootOf}\left(x^2\mathrm{e}^{\_Z} - \mathrm{e}^{\_Z}\ln\left(\frac{x\left(\mathrm{e}^{\_Z}+9\right)}{(x+1)^2}\right) + \mathrm{e}^{\_Z}\ln(2) + 3c_1\mathrm{e}^{\_Z} + \mathrm{e}^{\_Z}\_Z - 2x\,\mathrm{e}^{\_Z} + 9\right)} + 9\right)x}{54}$$

✓ Solution by Mathematica

Time used: 11.9 (sec). Leaf size: 95

```
DSolve[y'[x] == (y[x]*(x^4 + 3*y[x]^2 + 3*x*y[x]^2))/(x*(1 + x)*(x + 6*y[x]^2)),y[x],x,Inclu
```

$$y(x) \to -\frac{\sqrt{x}\sqrt{W\left(\frac{6(x+1)^2 e^{x^2-2x-3+2c_1}}{x}\right)}}{\sqrt{6}}$$

$$y(x) \to \frac{\sqrt{x}\sqrt{W\left(\frac{6(x+1)^2 e^{x^2-2x-3+2c_1}}{x}\right)}}{\sqrt{6}}$$

$$y(x) \to 0$$

2.259 problem 835

Internal problem ID [9170]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 835.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y' + \frac{1}{-\left(y^3\right)^{\frac{2}{3}} x - f_1\left(y^3 - 3\ln\left(x\right)\right)\left(y^3\right)^{\frac{1}{3}} x} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) = -1/(-(y(x)^3)^(2/3)*x-_F1(y(x)^3-3*ln(x))*(y(x)^3)^(1/3)*x),y(x), sing
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x] == -(-(x*F1[-3*Log[x] + y[x]^3]*(y[x]^3)^(1/3)) - x*(y[x]^3)^(2/3))^(-1),y[x],x
```

Not solved

2.260 problem 836

Internal problem ID [9171]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 836.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class C']]`

$$y' - \frac{y(x-y)(y+1)}{x(yx+x-y)} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 79

```
dsolve(diff(y(x),x) = y(x)*(x-y(x))*(y(x)+1)/x/(x*y(x)+x-y(x)),y(x), singsol=all)
```

$$y(x) = -\frac{\mathrm{e}^{\mathrm{RootOf}\left(\mathrm{e}^{-Z}\ln(2)-\mathrm{e}^{-Z}\ln\left(\mathrm{e}^{-Z}+9\right)+3c_1\mathrm{e}^{-Z}+\mathrm{e}^{-Z}\_Z-x\,\mathrm{e}^{-Z}+9\right)}x}{-9+(x-1)\,\mathrm{e}^{\mathrm{RootOf}\left(\mathrm{e}^{-Z}\ln(2)-\mathrm{e}^{-Z}\ln\left(\mathrm{e}^{-Z}+9\right)+3c_1\mathrm{e}^{-Z}+\mathrm{e}^{-Z}\_Z-x\,\mathrm{e}^{-Z}+9\right)}}$$

✓ Solution by Mathematica
Time used: 9.315 (sec). Leaf size: 379

```
DSolve[y'[x] == ((x - y[x])*y[x]*(1 + y[x]))/(x*(x - y[x] + x*y[x])),y[x],x,IncludeSingularS
```

$$\mathrm{Solve}\left[\frac{1}{9}2^{2/3}\left(\frac{\left(1-\frac{(x-1)^2\left(\frac{x^6}{(x-1)^3}\right)^{2/3}((x+2)y(x)+x)}{x^4((x-1)y(x)+x)}\right)\left(\frac{\left(\frac{x^6}{(x-1)^3}\right)^{2/3}(x-1)^2((x+2)y(x)+x)}{x^4((x-1)y(x)+x)}+2\right)\left(\left(1-\frac{(x-1)^2\left(\frac{x^6}{(x-1)^3}\right)^{2/3}}{x^4((x-1)}\right.\right.}{}\right.\right.$$

2.261 problem 837

Internal problem ID [9172]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 837.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y' + \frac{1}{-\ln(x)\left(y^3\right)^{\frac{2}{3}} - f_1\left(y^3 + 3\,\text{expIntegral}_1\left(-\ln(x)\right)\right)\ln(x)\left(y^3\right)^{\frac{1}{3}}} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) = -1/(-ln(x)*(y(x)^3)^(2/3)-_F1(y(x)^3+3*Ei(1,-ln(x)))*ln(x)*(y(x)^3)^(1
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x] == -(-(F1[3*ExpIntegralEi[-Log[x]] + y[x]^3]*Log[x]*(y[x]^3)^(1/3)) - Log[x]*(y
```

Not solved

2.262 problem 838

Internal problem ID [9173]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 838.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$y' - \frac{30x^3 + 25\sqrt{x} + 25y^2 - 20yx^3 - 100\sqrt{x}\,y + 4x^6 + 40x^{\frac{7}{2}} + 100x}{25x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(diff(y(x),x) = 1/25*(30*x^3+25*x^(1/2)+25*y(x)^2-20*x^3*y(x)-100*y(x)*x^(1/2)+4*x^6+4
```

$$y(x) = \frac{(10c_1 - 10\ln(x))\sqrt{x} + 2c_1x^3 - 2x^3\ln(x) + 5}{-5\ln(x) + 5c_1}$$

✓ Solution by Mathematica

Time used: 0.304 (sec). Leaf size: 48

```
DSolve[y'[x] == (Sqrt[x] + 4*x + (6*x^3)/5 + (8*x^(7/2))/5 + (4*x^6)/25 - 4*Sqrt[x]*y[x] - (
```

$$y(x) \to \frac{2x^3}{5} + 2\sqrt{x} + \frac{1}{-\log(x) + c_1}$$
$$y(x) \to \frac{2}{5}(x^3 + 5\sqrt{x})$$

2.263 problem 839

Internal problem ID [9174]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 839.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y' - \frac{\left(\mathrm{e}^{-\frac{y}{x}} y + \mathrm{e}^{-\frac{y}{x}} x + x^2\right) \mathrm{e}^{\frac{y}{x}}}{x} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(diff(y(x),x) = (exp(-y(x)/x)*y(x)+exp(-y(x)/x)*x+x^2)*exp(y(x)/x)/x,y(x), singsol=all
```

$$y(x) = \left(\ln(2) + \ln\left(\frac{x}{-x^2 + c_1}\right)\right) x$$

✓ Solution by Mathematica

Time used: 4.036 (sec). Leaf size: 41

```
DSolve[y'[x] == (E^(y[x]/x)*(x/E^(y[x]/x) + x^2 + y[x]/E^(y[x]/x)))/x,y[x],x,IncludeSingular
```

$$y(x) \to -x \log\left(\frac{-x^2 + e^{2c_1}}{2x}\right)$$
$$y(x) \to -x \log\left(-\frac{x}{2}\right)$$

2.264 problem 840

Internal problem ID [9175]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 840.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y' - \frac{\left(e^{-\frac{y}{x}} y + e^{-\frac{y}{x}} x + x^3\right) e^{\frac{y}{x}}}{x} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x) = (exp(-y(x)/x)*y(x)+exp(-y(x)/x)*x+x^3)*exp(y(x)/x)/x,y(x), singsol=all
```

$$y(x) = \left(\ln(3) + \ln\left(\frac{x}{-x^3 + c_1}\right)\right) x$$

✓ Solution by Mathematica

Time used: 4.19 (sec). Leaf size: 43

```
DSolve[y'[x] == (E^(y[x]/x)*(x/E^(y[x]/x) + x^3 + y[x]/E^(y[x]/x)))/x,y[x],x,IncludeSingular
```

$$y(x) \to -x \log\left(\frac{-x^3 + e^{3c_1}}{3x}\right)$$
$$y(x) \to -x \log\left(-\frac{x^2}{3}\right)$$

2.265 problem 841

Internal problem ID [9176]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 841.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{b\,x^3 + c^2\sqrt{a} - 2cb\,x^2\sqrt{a} + 2cy^2a^{\frac{3}{2}} + b^2x^4\sqrt{a} - 2y^2a^{\frac{3}{2}}b\,x^2 + a^{\frac{5}{2}}y^4}{a\,x^2y} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 97

```
dsolve(diff(y(x),x) = (b*x^3+c^2*a^(1/2)-2*c*b*x^2*a^(1/2)+2*c*y(x)^2*a^(3/2)+b^2*x^4*a^(1/2
```

$$y(x) = -\frac{2\sqrt{(c_1x+1)\left((c_1x+1)\left(b\,x^2-c\right)\sqrt{a}+\frac{x}{2}\right)a^{\frac{3}{2}}}}{a^{\frac{3}{2}}\left(2c_1x+2\right)}$$
$$y(x) = \frac{\sqrt{(c_1x+1)\left((c_1x+1)\left(b\,x^2-c\right)\sqrt{a}+\frac{x}{2}\right)a^{\frac{3}{2}}}}{a^{\frac{3}{2}}\left(c_1x+1\right)}$$

✓ Solution by Mathematica

Time used: 9.413 (sec). Leaf size: 390

```
DSolve[y'[x] == (Sqrt[a]*c^2 - 2*Sqrt[a]*b*c*x^2 + b*x^3 + Sqrt[a]*b^2*x^4 + 2*a^(3/2)*c*y[x
```

$$y(x) \to -\frac{\sqrt{-2a^{5/2}\left(c-bx^2\right)+4a^3bx\left(bx^2-c\right)+a^2x+4\sqrt{a}bc_1\left(bx^2-c\right)+2bc_1x}}{\sqrt{2}\sqrt{2a^{3/2}bc_1+a^{7/2}+2a^4bx}}$$

$$y(x) \to \frac{\sqrt{-2a^{5/2}\left(c-bx^2\right)+4a^3bx\left(bx^2-c\right)+a^2x+4\sqrt{a}bc_1\left(bx^2-c\right)+2bc_1x}}{\sqrt{2}\sqrt{2a^{3/2}bc_1+a^{7/2}+2a^4bx}}$$

$$y(x) \to -\frac{\sqrt{-b\left(x-2\sqrt{a}\left(c-bx^2\right)\right)}}{\sqrt{2}\sqrt{-a^{3/2}b}}$$

$$y(x) \to \frac{\sqrt{-b\left(x-2\sqrt{a}\left(c-bx^2\right)\right)}}{\sqrt{2}\sqrt{-a^{3/2}b}}$$

$$y(x) \to -\frac{\sqrt{b\left(x-2\sqrt{a}\left(c-bx^2\right)\right)}}{\sqrt{2}\sqrt{a^{3/2}b}}$$

$$y(x) \to \frac{\sqrt{b\left(x-2\sqrt{a}\left(c-bx^2\right)\right)}}{\sqrt{2}\sqrt{a^{3/2}b}}$$

2.266 problem 842

Internal problem ID [9177]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 842.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \frac{y + \ln(x)^3 x^2 + 2y\ln(x)^2 x^2 + y^2 \ln(x) x^2}{x\ln(x)} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
dsolve(diff(y(x),x) = (y(x)+x^2*ln(x)^3+2*x^2*ln(x)^2*y(x)+x^2*ln(x)*y(x)^2)/x/ln(x),y(x), s
```

$$y(x) = -\frac{\ln(x)(2x^2\ln(x) - x^2 + 2c_1 + 4)}{2x^2\ln(x) - x^2 + 2c_1}$$

✓ Solution by Mathematica

Time used: 0.373 (sec). Leaf size: 52

```
DSolve[y'[x] == (x^2*Log[x]^3 + y[x] + 2*x^2*Log[x]^2*y[x] + x^2*Log[x]*y[x]^2)/(x*Log[x]),y
```

$$y(x) \to \frac{\log(x)(x^2 - 2x^2\log(x) - 4(1 + c_1))}{-x^2 + 2x^2\log(x) + 4c_1}$$
$$y(x) \to -\log(x)$$

2.267 problem 843

Internal problem ID [9178]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 843.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \frac{y + \ln(x)^3 x^3 + 2x^3 y \ln(x)^2 + y^2 \ln(x) x^3}{x \ln(x)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x) = (y(x)+x^3*ln(x)^3+2*x^3*ln(x)^2*y(x)+x^3*ln(x)*y(x)^2)/x/ln(x),y(x), s
```

$$y(x) = -\frac{\ln(x)(6x^3 \ln(x) - 2x^3 + 9c_1 + 18)}{6x^3 \ln(x) - 2x^3 + 9c_1}$$

✓ Solution by Mathematica

Time used: 0.378 (sec). Leaf size: 52

```
DSolve[y'[x] == (x^3*Log[x]^3 + y[x] + 2*x^3*Log[x]^2*y[x] + x^3*Log[x]*y[x]^2)/(x*Log[x]),y
```

$$y(x) \to \frac{\log(x)(x^3 - 3x^3 \log(x) - 9(1 + c_1))}{-x^3 + 3x^3 \log(x) + 9c_1}$$

$$y(x) \to -\log(x)$$

2.268 problem 844

Internal problem ID [9179]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 844.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class C']]`

$$\boxed{y' - \frac{y(x+y)\,(y+1)}{x\,(yx+x+y)} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 106

```
dsolve(diff(y(x),x) = y(x)*(x+y(x))*(y(x)+1)/x/(x*y(x)+x+y(x)),y(x), singsol=all)
```

$$y(x) = -\frac{e^{\mathrm{RootOf}\left(e^{\_Z}\ln(2)-e^{\_Z}\ln\left(e^{\_Z}+9\right)+3c_1e^{\_Z}+e^{\_Z}\_Z+x\,e^{\_Z}+9\right)}x}{e^{\mathrm{RootOf}(e^{\_Z}\ln(2)-e^{\_Z}\ln(e^{\_Z}+9)+3c_1e^{\_Z}+e^{\_Z}\_Z+x\,e^{\_Z}+9)}x+e^{\mathrm{RootOf}(e^{\_Z}\ln(2)-e^{\_Z}\ln(e^{\_Z}+9)+3c_1e^{\_Z}+e^{\_Z}\_Z+x\,e^{\_Z}+9)}+}$$

✓ Solution by Mathematica

Time used: 9.818 (sec). Leaf size: 386

```
DSolve[y'[x] == (y[x]*(1 + y[x])*(x + y[x]))/(x*(x + y[x] + x*y[x])),y[x],x,IncludeSingularS
```

$$\mathrm{Solve}\left[\frac{2^{2/3}\left(1-\frac{\left(\frac{x^6}{(x+1)^3}\right)^{2/3}(x+1)^2((x-2)y(x)+x)}{x^4((x+1)y(x)+x)}\right)\left(\frac{\left(\frac{x^6}{(x+1)^3}\right)^{2/3}(x+1)^2((x-2)y(x)+x)}{x^4((x+1)y(x)+x)}+2\right)\left(\left(1-\frac{\left(\frac{x^6}{(x+1)^3}\right)^{2/3}(x+1)^2((x}{x^4((x+1)y(x)}\right.\right.}{9}\right.$$

$$\left.+c_1, y(x)\right]$$

2.269 problem 845

Internal problem ID [9180]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 845.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y' - \frac{3x^3 + \sqrt{-9x^4+4y^3} + x^2\sqrt{-9x^4+4y^3} + x^3\sqrt{-9x^4+4y^3}}{y^2} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
dsolve(diff(y(x),x) = (3*x^3+(-9*x^4+4*y(x)^3)^(1/2)+x^2*(-9*x^4+4*y(x)^3)^(1/2)+x^3*(-9*x^4
```

$$\int_{\_b}^{y(x)} \frac{\_a^2}{\sqrt{-9x^4+4\_a^3}} d\_a - \frac{x^4}{4} - \frac{x^3}{3} - x - c_1 = 0$$

✓ Solution by Mathematica

Time used: 4.605 (sec). Leaf size: 218

```
DSolve[y'[x] == (3*x^3 + Sqrt[-9*x^4 + 4*y[x]^3] + x^2*Sqrt[-9*x^4 + 4*y[x]^3] + x^3*Sqrt[-9
```

$$y(x) \to -\frac{1}{2}\sqrt[3]{-\frac{1}{2}}\sqrt[3]{9x^8+24x^7+16x^6+72x^5+12(11+6c_1)x^4+96c_1x^3+144x^2+288c_1x+144c_1{}^2}$$

$$y(x) \to \frac{1}{2}\sqrt[3]{\frac{9x^8}{2}+12x^7+8x^6+36x^5+6(11+6c_1)x^4+48c_1x^3+72x^2+144c_1x+72c_1{}^2}$$

$$y(x) \to \frac{1}{2}(-1)^{2/3}\sqrt[3]{\frac{9x^8}{2}+12x^7+8x^6+36x^5+6(11+6c_1)x^4+48c_1x^3+72x^2+144c_1x+72c_1{}^2}$$

2.270 problem 846

Internal problem ID [9181]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 846.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `['y=_G(x,y')']`

$$y' - \frac{1}{-x + \left(\frac{1}{y}+1\right)x + f_1\left(\left(\frac{1}{y}+1\right)x\right)x^2 - f_1\left(\left(\frac{1}{y}+1\right)x\right)x^2\left(\frac{1}{y}+1\right)} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 71

```
dsolve(diff(y(x),x) = 1/(-x+(1/y(x)+1)*x+_F1((1/y(x)+1)*x)*x^2-_F1((1/y(x)+1)*x)*x^2*(1/y(x)
```

$$y(x) = \text{RootOf}\left(f_1\left(\frac{(\_Z+1)\,x}{\_Z}\right)x\_Z + f_1\left(\frac{(\_Z+1)\,x}{\_Z}\right)x - \_Z\right)$$

$$y(x) = e^{\text{RootOf}\left(-\_Z - \left(\int^{\frac{x\,e^{\_Z}}{e^{\_Z}-1}} \frac{1}{\_a(f_1(\_a)\_a-1)}d\_a\right) + c_1\right)} - 1$$

✓ Solution by Mathematica

Time used: 0.699 (sec). Leaf size: 346

```
DSolve[y'[x] == (-x + x^2*F1[x*(1 + y[x]^(-1))] + x*(1 + y[x]^(-1)) - x^2*F1[x*(1 + y[x]^(-1
```

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{x\text{F1}\left(x\left(1+\frac{1}{K[2]}\right)\right)-1}{x\text{F1}\left(x\left(1+\frac{1}{K[2]}\right)\right)+xK[2]\text{F1}\left(x\left(1+\frac{1}{K[2]}\right)\right)-K[2]}\right.\right.$$

$$-\int_1^x\left(\frac{\text{F1}\left(K[1]\left(1+\frac{1}{K[2]}\right)\right)-\frac{K[1]\text{F1}'\left(K[1]\left(1+\frac{1}{K[2]}\right)\right)}{K[2]}-\frac{K[1]\text{F1}'\left(K[1]\left(1+\frac{1}{K[2]}\right)\right)}{K[2]^2}}{K[1]\left(K[2]\text{F1}\left(K[1]\left(1+\frac{1}{K[2]}\right)\right)+\text{F1}\left(K[1]\left(1+\frac{1}{K[2]}\right)\right)\right)-K[2]}-\frac{\left(K[2]\text{F1}\left(K[1]\left(1+\frac{1}{K[2]}\right)\right)\right.}{}\right.$$

$$+\int_1^x\left(\frac{y(x)\text{F1}\left(K[1]\left(1+\frac{1}{y(x)}\right)\right)+\text{F1}\left(K[1]\left(1+\frac{1}{y(x)}\right)\right)}{K[1]\left(y(x)\text{F1}\left(K[1]\left(1+\frac{1}{y(x)}\right)\right)+\text{F1}\left(K[1]\left(1+\frac{1}{y(x)}\right)\right)\right)-y(x)}\right.$$

$$\left.\left.-\frac{1}{K[1]}\right)dK[1]=c_1,y(x)\right]$$

2.271 problem 847

Internal problem ID [9182]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 847.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' - \sqrt{x^2+2x+1-4y} - x^2\sqrt{x^2+2x+1-4y} - \sqrt{x^2+2x+1-4y}\,x^3 = \frac{x}{2} + \frac{1}{2}$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 34

```
dsolve(diff(y(x),x) = 1/2*x+1/2+(x^2+2*x+1-4*y(x))^(1/2)+x^2*(x^2+2*x+1-4*y(x))^(1/2)+x^3*(x
```

$$c_1 - \frac{x^4}{2} - \frac{2x^3}{3} - 2x - \sqrt{x^2+2x+1-4y(x)} = 0$$

✓ Solution by Mathematica

Time used: 0.596 (sec). Leaf size: 69

```
DSolve[y'[x] == 1/2 + x/2 + Sqrt[1 + 2*x + x^2 - 4*y[x]] + x^2*Sqrt[1 + 2*x + x^2 - 4*y[x]]
```

$$y(x) \to \frac{1}{144}\left(-9x^8 - 24x^7 - 16x^6 - 72x^5 + 24(-4+3c_1)x^4 + 96c_1x^3 - 108x^2 + 72(1+4c_1)x + 36 - 144{c_1}^2\right)$$

2.272 problem 848

Internal problem ID [9183]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 848.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y' - f_1(y - \ln(\sinh(x))) = \frac{\cosh(x)}{\sinh(x)}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 22

```
dsolve(diff(y(x),x) = 1/sinh(x)*cosh(x)+_F1(y(x)-ln(sinh(x))),y(x), singsol=all)
```

$$y(x) = \ln(\sinh(x)) + \mathrm{RootOf}\left(x - \left(\int^{\_Z} \frac{1}{f_1(\_a)} d\_a\right) + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.624 (sec). Leaf size: 148

```
DSolve[y'[x] == Coth[x] + F1[-Log[Sinh[x]] + y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\int_1^{y(x)} -\frac{\mathrm{F1}(K[2]-\log(\sinh(x)))\int_1^x \left(\frac{(\coth(K[1])+\mathrm{F1}(K[2]-\log(\sinh(K[1]))))\mathrm{F1}'(K[2]-\log(\sinh(K[1])))}{\mathrm{F1}(K[2]-\log(\sinh(K[1])))^2} - \frac{\mathrm{F1}'(K[2]-\log(\sinh(K[1])))}{\mathrm{F1}(K[2]-\log(\sinh(K[1])))}\right.}{\mathrm{F1}(K[2]-\log(\sinh(x)))} + \int_1^x -\frac{\coth(K[1])+\mathrm{F1}(y(x)-\log(\sinh(K[1])))}{\mathrm{F1}(y(x)-\log(\sinh(K[1])))} dK[1] = c_1, y(x)\right]$$

2.273 problem 849

Internal problem ID [9184]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 849.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' - \sqrt{x^2 - 4x + 4y} - x^2\sqrt{x^2 - 4x + 4y} - \sqrt{x^2 - 4x + 4y}\, x^3 = -\frac{x}{2} + 1$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 33

```
dsolve(diff(y(x),x) = -1/2*x+1+(x^2-4*x+4*y(x))^(1/2)+x^2*(x^2-4*x+4*y(x))^(1/2)+x^3*(x^2-4*
```

$$c_1 + \frac{x^4}{2} + \frac{2x^3}{3} + 2x - \sqrt{x^2 - 4x + 4y(x)} = 0$$

✓ Solution by Mathematica

Time used: 0.56 (sec). Leaf size: 73

```
DSolve[y'[x] == 1 - x/2 + Sqrt[-4*x + x^2 + 4*y[x]] + x^2*Sqrt[-4*x + x^2 + 4*y[x]] + x^3*Sq
```

$$y(x) \to \frac{x^8}{16} + \frac{x^7}{6} + \frac{x^6}{9} + \frac{x^5}{2} - \frac{1}{6}(-4 + 3c_1)x^4 - \frac{2c_1x^3}{3} + \frac{3x^2}{4} + x - 2c_1x + c_1^2$$

2.274 problem 850

Internal problem ID [9185]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 850.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{y' - f_1(y - \ln(\sin(x)) + \ln(\cos(x) + 1)) = \frac{1}{\sin(x)}}$$

✓ Solution by Maple

Time used: 0.235 (sec). Leaf size: 27

```
dsolve(diff(y(x),x) = 1/sin(x)+_F1(y(x)-ln(sin(x))+ln(cos(x)+1)),y(x), singsol=all)
```

$$y(x) = -\ln(\csc(x) + \cot(x)) + \operatorname{RootOf}\left(-x + \int^{\_Z} \frac{1}{f_1(\_a)} d\_a + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.855 (sec). Leaf size: 1438

```
DSolve[y'[x] == Csc[x] + F1[Log[1 + Cos[x]] - Log[Sin[x]] + y[x]],y[x],x,IncludeSingularSolu
```

$$\text{Solve}\Bigg[\int_1^x$$

$$-\frac{(\cot^2(K[1]) + \csc(K[1])\cot(K[1]) + 1)\,(\csc(K[1]) + \text{F1}(\log(\cos(K[1]) + 1) - \log(\text{si}}{-\cot^2(K[1]) + \text{F1}(\log(\cos(K[1]) + 1) - \log(\sin(K[1])) + y(x))\cot(K[1]) + \csc^2(K[1]) + \csc(K[1])\text{F1}(\text{l}}$$

$$+ \int_1^{y(x)}$$

$$-\frac{\sin(x)\left(\int_1^x \left(\frac{(\cot^2(K[1]) + \csc(K[1])\cot(K[1]) + 1)(\csc(K[1]) + \text{F1}(K[2] + \log(\cos(K[1]) + 1) - \log(\sin(K[1])))) \sin(K[1]) \left(\cot(K[1])\text{F1}'(K\right.}{\left(-\cot^2(K[1]) + \text{F1}(K[2] + \log(\cos(K[1]) + 1) - \log(\sin(K[1])))\cot(K[1]) + \csc^2(K[1]) + \text{c}\right.}\right.\right.}{}$$

2.275 problem 851

Internal problem ID [9186]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 851.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Abel]`

$$y' - \frac{b^3 + y^2b^3 + 2ya\,b^2x + x^2b\,a^2 + y^3b^3 + 3y^2b^2ax + 3yb\,a^2x^2 + a^3x^3}{b^3} = 0$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 42

```
dsolve(diff(y(x),x) = (b^3+y(x)^2*b^3+2*y(x)*b^2*a*x+x^2*b*a^2+y(x)^3*b^3+3*y(x)^2*b^2*a*x+3
```

$$y(x) = \frac{\mathrm{RootOf}\left(\left(\int^{\_Z} \frac{1}{b\_a^3 + b\_a^2 + a + b} d\_a\right) b - x + c_1\right) b - ax}{b}$$

✓ Solution by Mathematica

Time used: 2.675 (sec). Leaf size: 902

```
DSolve[y'[x] == (b^3 + a^2*b*x^2 + a^3*x^3 + 2*a*b^2*x*y[x] + 3*a^2*b*x^2*y[x] + b^3*y[x]^2
```

$$\text{Solve}\left[\frac{1}{9}\text{RootSum}\left[729a^2\#1^9+841b^2\#1^9+1566ab\#1^9+2187a^2\#1^6 +2523b^2\#1^6+4698ab\#1^6+2187a^2\#1^3+2496b^2\#1^3+4698ab\#1^3+729a^2+841b^2 +1566ab\&,\frac{729a^2\log\left(\frac{\frac{b+3ax}{b}+3y(x)}{\sqrt[3]{\frac{27a+29b}{b}}}-\#1\right)\#1^6+841b^2\log\left(\frac{\frac{b+3ax}{b}+3y(x)}{\sqrt[3]{\frac{27a+29b}{b}}}-\#1\right)\#1^6+1566ab\log\left(\right.}{}\right.\right.$$

2.276 problem 852

Internal problem ID [9187]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 852.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Abel]`

$$y' - \frac{\alpha^3 + y^2\alpha^3 + 2y\alpha^2\beta x + \alpha\,\beta^2x^2 + y^3\alpha^3 + 3y^2\alpha^2\beta x + 3y\alpha\,\beta^2x^2 + \beta^3x^3}{\alpha^3} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 42

```
dsolve(diff(y(x),x) = (alpha^3+y(x)^2*alpha^3+2*y(x)*alpha^2*beta*x+alpha*beta^2*x^2+y(x)^3*
```

$$y(x) = \frac{\operatorname{RootOf}\left(\left(\int^{\_Z} \frac{1}{\_a^3\alpha + \_a^2\alpha + \alpha + \beta} d\_a\right)\alpha - x + c_1\right)\alpha - \beta x}{\alpha}$$

✓ Solution by Mathematica

Time used: 2.588 (sec). Leaf size: 902

```
DSolve[y'[x] == (\[Alpha]^3 + \[Alpha]*\[Beta]^2*x^2 + \[Beta]^3*x^3 + 2*\[Alpha]^2*\[Beta]*
```

$$\text{Solve}\left[\frac{1}{9}\text{RootSum}\left[841\alpha^2\#1^9 + 729\beta^2\#1^9 + 1566\alpha\beta\#1^9 + 2523\alpha^2\#1^6 + 2187\beta^2\#1^6\right.\right.$$

$$+ 4698\alpha\beta\#1^6 + 2496\alpha^2\#1^3 + 2187\beta^2\#1^3 + 4698\alpha\beta\#1^3 + 841\alpha^2 + 729\beta^2$$

$$+1566\alpha\beta\&, \frac{841\alpha^2\log\left(\frac{\frac{\alpha+3x\beta}{\alpha}+3y(x)}{\sqrt[3]{\frac{29\alpha+27\beta}{\alpha}}} - \#1\right)\#1^6 + 729\beta^2\log\left(\frac{\frac{\alpha+3x\beta}{\alpha}+3y(x)}{\sqrt[3]{\frac{29\alpha+27\beta}{\alpha}}} - \#1\right)\#1^6 + 1566\alpha\beta\log}{}$$

2.277 problem 853

Internal problem ID [9188]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 853.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], [_Abel`

$$y' - \frac{14yx + 12 + 2x + y^3x^3 + 6y^2x^2}{x^2(yx + 2 + x)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 63

```
dsolve(diff(y(x),x) = 1/x^2*(14*x*y(x)+12+2*x+x^3*y(x)^3+6*x^2*y(x)^2)/(x*y(x)+2+x),y(x), si
```

$$y(x) = \frac{-2\sqrt{-2x + c_1} + x + 2}{\left(\sqrt{-2x + c_1} - 1\right)x}$$
$$y(x) = \frac{-2\sqrt{-2x + c_1} - x - 2}{\left(\sqrt{-2x + c_1} + 1\right)x}$$

✓ Solution by Mathematica

Time used: 0.399 (sec). Leaf size: 84

```
DSolve[y'[x] == (12 + 2*x + 14*x*y[x] + 6*x^2*y[x]^2 + x^3*y[x]^3)/(x^2*(2 + x + x*y[x])),y[
```

$$y(x) \to \frac{x - 2\sqrt{-2x + c_1} + 2}{x\left(-1 + \sqrt{-2x + c_1}\right)}$$
$$y(x) \to -\frac{x + 2\sqrt{-2x + c_1} + 2}{x + x\sqrt{-2x + c_1}}$$
$$y(x) \to -\frac{2}{x}$$

2.278 problem 854

Internal problem ID [9189]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 854.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y' - \frac{y\left(\ln(x) + \ln(y) - 1 + \ln(x)^2 x^2 + 2x^2 \ln(y)\ln(x) + x^2 \ln(y)^2\right)}{x} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(diff(y(x),x) = y(x)*(ln(x)+ln(y(x))-1+x^2*ln(x)^2+2*x^2*ln(y(x))*ln(x)+x^2*ln(y(x))^2
```

$$y(x) = \frac{e^{-\frac{3x}{x^3+3c_1}}}{x}$$

✓ Solution by Mathematica

Time used: 0.349 (sec). Leaf size: 31

```
DSolve[y'[x] == ((-1 + Log[x] + x^2*Log[x]^2 + Log[y[x]] + 2*x^2*Log[x]*Log[y[x]] + x^2*Log[
```

$$y(x) \to \frac{e^{-\frac{3x}{x^3+3c_1}}}{x}$$
$$y(x) \to \frac{1}{x}$$

2.279 problem 855

Internal problem ID [9190]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 855.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y' - \frac{y\left(\ln(y) - 1 + \ln(x) + \ln(x)^2 x^3 + 2x^3 \ln(y)\ln(x) + x^3 \ln(y)^2\right)}{x} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x) = y(x)*(ln(y(x))-1+ln(x)+x^3*ln(x)^2+2*x^3*ln(y(x))*ln(x)+x^3*ln(y(x))^2
```

$$y(x) = \frac{e^{-\frac{4x}{x^4+4c_1}}}{x}$$

✓ Solution by Mathematica

Time used: 0.355 (sec). Leaf size: 31

```
DSolve[y'[x] == ((-1 + Log[x] + x^3*Log[x]^2 + Log[y[x]] + 2*x^3*Log[x]*Log[y[x]] + x^3*Log[
```

$$y(x) \to \frac{e^{-\frac{4x}{x^4+4c_1}}}{x}$$

$$y(x) \to \frac{1}{x}$$

2.280 problem 856

Internal problem ID [9191]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 856.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y' + \frac{\left(-\frac{1}{x} - f_1(y^2 - 2x)\right) x}{\sqrt{y^2}} = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 65

```
dsolve(diff(y(x),x) = -(-1/x-_F1(y(x)^2-2*x))/(y(x)^2)^(1/2)*x,y(x), singsol=all)
```

$$y(x) = \sqrt{2 \operatorname{RootOf}\left(x^2 - 2\left(\int^{\_Z} \frac{1}{f_1(2\_a)} d\_a\right) + 4c_1\right) + 2x}$$
$$y(x) = -\sqrt{2 \operatorname{RootOf}\left(x^2 - 2\left(\int^{\_Z} \frac{1}{f_1(2\_a)} d\_a\right) + 4c_1\right) + 2x}$$

✓ Solution by Mathematica

Time used: 0.33 (sec). Leaf size: 99

```
DSolve[y'[x] == (x*(x^(-1) + F1[-2*x + y[x]^2]))/Sqrt[y[x]^2],y[x],x,IncludeSingularSolution
```

$$\text{Solve}\left[\int_1^{y(x)} \left(\frac{\sqrt{K[2]^2}}{\text{F1}\left(K[2]^2 - 2x\right)} - \int_1^x \frac{2K[2]\text{F1}'(K[2]^2 - 2K[1])}{\text{F1}\left(K[2]^2 - 2K[1]\right)^2} dK[1]\right) dK[2] + \int_1^x \left(-K[1] - \frac{1}{\text{F1}\left(y(x)^2 - 2K[1]\right)}\right) dK[1] = c_1, y(x)\right]$$

2.281 problem 857

Internal problem ID [9192]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 857.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' - \sqrt{x^2 - 2x + 1 + 8y} - x^2\sqrt{x^2 - 2x + 1 + 8y} - \sqrt{x^2 - 2x + 1 + 8y}\, x^3 = -\frac{x}{4} + \frac{1}{4}$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 32

```
dsolve(diff(y(x),x) = -1/4*x+1/4+(x^2-2*x+1+8*y(x))^(1/2)+x^2*(x^2-2*x+1+8*y(x))^(1/2)+x^3*(
```

$$c_1 + x^4 + \frac{4x^3}{3} + 4x - \sqrt{x^2 - 2x + 1 + 8y(x)} = 0$$

✓ Solution by Mathematica

Time used: 0.798 (sec). Leaf size: 77

```
DSolve[y'[x] == 1/4 - x/4 + Sqrt[1 - 2*x + x^2 + 8*y[x]] + x^2*Sqrt[1 - 2*x + x^2 + 8*y[x]]
```

$$y(x) \to \frac{x^8}{8} + \frac{x^7}{3} + \frac{2x^6}{9} + x^5 + \left(\frac{4}{3} - c_1\right)x^4 - \frac{4c_1x^3}{3} + \frac{15x^2}{8} + \left(\frac{1}{4} - 4c_1\right)x - \frac{1}{8} + 2c_1{}^2$$

2.282 problem 858

Internal problem ID [9193]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 858.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Abel]`

$$y' - \frac{a^3 + y^2a^3 + 2ya^2bx + a\,b^2x^2 + y^3a^3 + 3y^2a^2bx + 3ya\,b^2x^2 + b^3x^3}{a^3} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 42

```
dsolve(diff(y(x),x) = (a^3+y(x)^2*a^3+2*y(x)*a^2*b*x+a*b^2*x^2+y(x)^3*a^3+3*y(x)^2*a^2*b*x+3
```

$$y(x) = \frac{\operatorname{RootOf}\left(\left(\int^{\_Z} \frac{1}{\_a^3a+\_a^2a+a+b} d\_a\right) a - x + c_1\right) a - bx}{a}$$

✓ Solution by Mathematica

Time used: 2.762 (sec). Leaf size: 902

```
DSolve[y'[x] == (a^3 + a*b^2*x^2 + b^3*x^3 + 2*a^2*b*x*y[x] + 3*a*b^2*x^2*y[x] + a^3*y[x]^2
```

$$\text{Solve}\left[\frac{1}{9}\text{RootSum}\left[841a^2\#1^9+729b^2\#1^9+1566ab\#1^9+2523a^2\#1^6\right.\right.$$

$$+2187b^2\#1^6+4698ab\#1^6+2496a^2\#1^3+2187b^2\#1^3+4698ab\#1^3+841a^2+729b^2$$

$$+1566ab\&,\frac{841a^2\log\left(\frac{\frac{a+3bx}{a}+3y(x)}{\sqrt[3]{\frac{29a+27b}{a}}}-\#1\right)\#1^6+729b^2\log\left(\frac{\frac{a+3bx}{a}+3y(x)}{\sqrt[3]{\frac{29a+27b}{a}}}-\#1\right)\#1^6+1566ab\log\left(\right.}{}$$

2.283 problem 859

Internal problem ID [9194]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 859.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y' + \frac{-x - f_1(y^2 - 2x)}{\sqrt{y^2}\, x} = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 63

```
dsolve(diff(y(x),x) = -(-x-_F1(y(x)^2-2*x))/(y(x)^2)^(1/2)/x,y(x), singsol=all)
```

$$y(x) = \sqrt{2\,\mathrm{RootOf}\left(\ln(x) - \left(\int^{\_Z} \frac{1}{f_1(2\_a)} d\_a\right) + 2c_1\right) + 2x}$$
$$y(x) = -\sqrt{2\,\mathrm{RootOf}\left(\ln(x) - \left(\int^{\_Z} \frac{1}{f_1(2\_a)} d\_a\right) + 2c_1\right) + 2x}$$

✓ Solution by Mathematica

Time used: 0.397 (sec). Leaf size: 101

```
DSolve[y'[x] == (x + F1[-2*x + y[x]^2])/(x*Sqrt[y[x]^2]),y[x],x,IncludeSingularSolutions ->
```

$$\mathrm{Solve}\left[\int_1^{y(x)} \left(\frac{\sqrt{K[2]^2}}{\mathrm{F1}(K[2]^2 - 2x)} - \int_1^x \frac{2K[2]\mathrm{F1}'(K[2]^2 - 2K[1])}{\mathrm{F1}(K[2]^2 - 2K[1])^2} dK[1]\right) dK[2] + \int_1^x \left(-\frac{1}{K[1]} - \frac{1}{\mathrm{F1}(y(x)^2 - 2K[1])}\right) dK[1] = c_1, y(x)\right]$$

2.284 problem 860

Internal problem ID [9195]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 860.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$y' - \frac{-\sin(2y) + x\cos(2y) + \cos(2y)x^3 + \cos(2y)x^4 + x + x^3 + x^4}{2x} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x) = 1/2*(-sin(2*y(x))+x*cos(2*y(x))+cos(2*y(x))*x^3+cos(2*y(x))*x^4+x+x^3+
```

$$y(x) = \arctan\left(\frac{4x^5 + 5x^4 + 10x^2 + c_1}{20x}\right)$$

✓ Solution by Mathematica
Time used: 3.058 (sec). Leaf size: 69

```
DSolve[y'[x] == (x/2 + x^3/2 + x^4/2 + (x*Cos[2*y[x]])/2 + (x^3*Cos[2*y[x]])/2 + (x^4*Cos[2*
```

$$y(x) \to \arctan\left(\frac{x^4}{5} + \frac{x^3}{4} + \frac{x}{2} + \frac{c_1}{2x}\right)$$
$$y(x) \to -\frac{1}{2}\pi\sqrt{\frac{1}{x^2}}x$$
$$y(x) \to \frac{1}{2}\pi\sqrt{\frac{1}{x^2}}x$$

2.285 problem 861

Internal problem ID [9196]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 861.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' + \frac{\left(-\frac{y\,e^{\frac{1}{x}}}{x} - f_1\left(e^{\frac{1}{x}} y\right)\right) e^{-\frac{1}{x}}}{x} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve(diff(y(x),x) = -(-1/x*y(x)/exp(-1/x)-_F1(y(x)/exp(-1/x)))*exp(-1/x)/x,y(x), singsol=a
```

$$y(x) = \text{RootOf}\left(f_1(\_Z)\right) e^{-\frac{1}{x}}$$
$$y(x) = \text{RootOf}\left(-\ln(x) + \int^{\_Z} \frac{1}{f_1(\_a)} d\_a + c_1\right) e^{-\frac{1}{x}}$$

✓ Solution by Mathematica

Time used: 0.915 (sec). Leaf size: 152

```
DSolve[y'[x] == (F1[E^x^(-1)*y[x]] + (E^x^(-1)*y[x])/x)/(E^x^(-1)*x),y[x],x,IncludeSingularS
```

$$\text{Solve}\left[\int_1^{y(x)} -\frac{\text{F1}\left(e^{\frac{1}{x}}K[2]\right)\int_1^x\left(\frac{e^{\frac{1}{K[1]}}}{\text{F1}\left(e^{\frac{1}{K[1]}}K[2]\right)K[1]^2}-\frac{e^{\frac{2}{K[1]}}K[2]\text{F1}'\left(e^{\frac{1}{K[1]}}K[2]\right)}{\text{F1}\left(e^{\frac{1}{K[1]}}K[2]\right)^2K[1]^2}\right)dK[1]+e^{\frac{1}{x}}}{\text{F1}\left(e^{\frac{1}{x}}K[2]\right)}dK[2] + \int_1^x\left(\frac{e^{\frac{1}{K[1]}}y(x)}{\text{F1}\left(e^{\frac{1}{K[1]}}y(x)\right)K[1]^2}+\frac{1}{K[1]}\right)dK[1]=c_1,y(x)\right]$$

2.286 problem 863

Internal problem ID [9197]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 863.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' - \frac{y + x\sqrt{y^2 + x^2} + x^3\sqrt{y^2 + x^2} + x^4\sqrt{y^2 + x^2}}{x} = 0$$

✓ Solution by Maple
Time used: 1.359 (sec). Leaf size: 38

```
dsolve(diff(y(x),x) = (y(x)+x*(y(x)^2+x^2)^(1/2)+x^3*(y(x)^2+x^2)^(1/2)+x^4*(y(x)^2+x^2)^(1/
```

$$\ln\left(\sqrt{y(x)^2 + x^2} + y(x)\right) - \frac{x^4}{4} - \frac{x^3}{3} - x - \ln(x) - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.482 (sec). Leaf size: 60

```
DSolve[y'[x] == (y[x] + x*Sqrt[x^2 + y[x]^2] + x^3*Sqrt[x^2 + y[x]^2] + x^4*Sqrt[x^2 + y[x]^
```

$$y(x) \to \frac{1}{2} x e^{-\frac{x^4}{4} - \frac{x^3}{3} - x - c_1} \left(-1 + e^{\frac{x^4}{2} + \frac{2x^3}{3} + 2x + 2c_1}\right)$$

2.287 problem 864

Internal problem ID [9198]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 864.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class C'], [_1st_order, '_with_symmetry_`

$$y' - \frac{y\left(\mathrm{e}^{-\frac{x^2}{2}} x y + x\,\mathrm{e}^{-\frac{x^2}{4}} + 2y^2 \mathrm{e}^{-\frac{3x^2}{4}}\right) \mathrm{e}^{\frac{x^2}{4}}}{2y\,\mathrm{e}^{-\frac{x^2}{4}} + 2} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

```
dsolve(diff(y(x),x) = y(x)*(exp(-1/4*x^2)^2*x*y(x)+exp(-1/4*x^2)*x+2*y(x)^2*exp(-3/4*x^2))*e
```

$$y(x) = \frac{\mathrm{e}^{\frac{x^2}{4}}}{\sqrt{-2x + c_1} - 1}$$

$$y(x) = \frac{\mathrm{e}^{\frac{x^2}{4}}}{-\sqrt{-2x + c_1} - 1}$$

✓ Solution by Mathematica

Time used: 8.117 (sec). Leaf size: 103

```
DSolve[y'[x] == (E^(x^2/4)*y[x]*(x/E^(x^2/4) + (x*y[x])/E^(x^2/2) + (2*y[x]^2)/E^((3*x^2)/4)
```

$$y(x) \to \frac{e^{\frac{x^2}{2}}}{-e^{\frac{x^2}{4}} + \sqrt{e^{\frac{x^2}{2}}(-2x + 1 + c_1)}}$$

$$y(x) \to -\frac{e^{\frac{x^2}{2}}}{e^{\frac{x^2}{4}} + \sqrt{e^{\frac{x^2}{2}}(-2x + 1 + c_1)}}$$

$$y(x) \to 0$$

2.288 problem 865

Internal problem ID [9199]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 865.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' - \left(\frac{\ln(-1+y)\,y}{(1-y)\ln(x)\,x} - \frac{\ln(-1+y)}{(1-y)\ln(x)\,x} - f(x)\right)(1-y) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(diff(y(x),x) = (1/(1-y(x))/ln(x)/x*ln(-1+y(x))*y(x)-1/(1-y(x))/ln(x)/x*ln(-1+y(x))-f(
```

$$y(x) = x^{c_1+\int \frac{f(x)}{\ln(x)}dx} + 1$$

✓ Solution by Mathematica

Time used: 0.356 (sec). Leaf size: 87

```
DSolve[y'[x] == (1 - y[x])*(-f[x] - Log[-1 + y[x]]/(x*Log[x]*(1 - y[x])) + (Log[-1 + y[x]]*y
```

$$\text{Solve}\left[\int_1^x \left(-\frac{f(K[1])}{\log(K[1])} - \frac{\log(y(x)-1)}{K[1]\log^2(K[1])}\right) dK[1] + \int_1^{y(x)} \left(\frac{1}{(K[2]-1)\log(x)} - \int_1^x -\frac{1}{K[1](K[2]-1)\log^2(K[1])} dK[1]\right) dK[2] = c_1, y(x)\right]$$

2.289 problem 866

Internal problem ID [9200]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 866.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' - \sqrt{x^2 + 2ax + a^2 + 4y} - x^2\sqrt{x^2 + 2ax + a^2 + 4y} - \sqrt{x^2 + 2ax + a^2 + 4y}\, x^3 = -\frac{x}{2} - \frac{a}{2}$$

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 37

```
dsolve(diff(y(x),x) = -1/2*x-1/2*a+(x^2+2*a*x+a^2+4*y(x))^(1/2)+x^2*(x^2+2*a*x+a^2+4*y(x))^(
```

$$c_1 + \frac{x^4}{2} + \frac{2x^3}{3} + 2x - \sqrt{x^2 + 2ax + a^2 + 4y(x)} = 0$$

✓ Solution by Mathematica

Time used: 0.784 (sec). Leaf size: 85

```
DSolve[y'[x] == -1/2*a - x/2 + Sqrt[a^2 + 2*a*x + x^2 + 4*y[x]] + x^2*Sqrt[a^2 + 2*a*x + x^2
```

$$y(x) \to -\frac{a^2}{4} - \frac{ax}{2} + \frac{x^8}{16} + \frac{x^7}{6} + \frac{x^6}{9} + \frac{x^5}{2} - \frac{1}{6}(-4 + 3c_1)x^4 - \frac{2c_1x^3}{3} + \frac{3x^2}{4} - 2c_1x + c_1^2$$

2.290 problem 867

Internal problem ID [9201]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 867.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Abel]`

$$y' - y^2 - \frac{2yx^2}{3} - y^3 - y^2x^2 - \frac{yx^4}{3} = -\frac{2}{3}x + 1 + \frac{1}{9}x^4 + \frac{1}{27}x^6$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 30

```
dsolve(diff(y(x),x) = -2/3*x+1+y(x)^2+2/3*x^2*y(x)+1/9*x^4+y(x)^3+x^2*y(x)^2+1/3*y(x)*x^4+1/
```

$$y(x) = -\frac{x^2}{3} + \mathrm{RootOf}\left(-x + \int^{\_Z} \frac{1}{\_a^3 + \_a^2 + 1} d\_a + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.156 (sec). Leaf size: 77

```
DSolve[y'[x] == 1 - (2*x)/3 + x^4/9 + x^6/27 + (2*x^2*y[x])/3 + (x^4*y[x])/3 + y[x]^2 + x^2*
```

$$\mathrm{Solve}\left[-\frac{29}{3}\mathrm{RootSum}\left[-29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log\left(\frac{x^2+3y(x)+1}{\sqrt[3]{29}} - \#1\right)}{\sqrt[3]{29} - 29\#1^2}\&\right] = \frac{1}{9}29^{2/3}x + c_1, y(x)\right]$$

2.291 problem 868

Internal problem ID [9202]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 868.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Abel]`

$$\boxed{y' - y^2 + 2yx^2 - y^3 + 3y^2x^2 - 3yx^4 = -x^6 + x^4 + 2x + 1}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 28

```
dsolve(diff(y(x),x) = 2*x+1+y(x)^2-2*x^2*y(x)+x^4+y(x)^3-3*x^2*y(x)^2+3*y(x)*x^4-x^6,y(x), s
```

$$y(x) = x^2 + \text{RootOf}\left(-x + \int^{\_Z} \frac{1}{\_a^3 + \_a^2 + 1} d\_a + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.13 (sec). Leaf size: 79

```
DSolve[y'[x] == 1 + 2*x + x^4 - x^6 - 2*x^2*y[x] + 3*x^4*y[x] + y[x]^2 - 3*x^2*y[x]^2 + y[x]
```

$$\text{Solve}\left[-\frac{29}{3}\text{RootSum}\left[-29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log\left(\frac{-3x^2+3y(x)+1}{\sqrt[3]{29}} - \#1\right)}{\sqrt[3]{29} - 29\#1^2}\&\right] = \frac{1}{9}29^{2/3}x + c_1, y(x)\right]$$

2.292 problem 869

Internal problem ID [9203]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 869.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], [_Abel`

$$y' - \frac{-x+1-2y+3x^2-2yx^2+2x^4+x^3-2yx^3+2x^5}{x^2-y} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x) = 1/(x^2-y(x))*(-x+1-2*y(x)+3*x^2-2*x^2*y(x)+2*x^4+x^3-2*x^3*y(x)+2*x^5)
```

$$y(x) = x^2 + \frac{\mathrm{LambertW}\left(-2c_1 \mathrm{e}^{x^4+\frac{4}{3}x^3-2x^2+4x-1}\right)}{2} + \frac{1}{2}$$

✓ Solution by Mathematica
Time used: 3.596 (sec). Leaf size: 53

```
DSolve[y'[x] == (1 - x + 3*x^2 + x^3 + 2*x^4 + 2*x^5 - 2*y[x] - 2*x^2*y[x] - 2*x^3*y[x])/(x^
```

$$y(x) \to x^2 + \frac{1}{2}\left(1 + W\left(-e^{x^4+\frac{4x^3}{3}-2x^2+4x-1+c_1}\right)\right)$$
$$y(x) \to x^2 + \frac{1}{2}$$

2.293 problem 870

Internal problem ID [9204]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 870.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' - \frac{\left(e^{-\frac{y}{x}}y + e^{-\frac{y}{x}}x + x + x^3 + x^4\right)e^{\frac{y}{x}}}{x} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve(diff(y(x),x) = (exp(-y(x)/x)*y(x)+exp(-y(x)/x)*x+x+x^3+x^4)*exp(y(x)/x)/x,y(x), sings
```

$$y(x) = \left(2\ln(2) + \ln(3) - \ln\left(\frac{-3x^4 - 4x^3 - 12c_1 - 12x}{x}\right)\right)x$$

✓ Solution by Mathematica

Time used: 4.311 (sec). Leaf size: 32

```
DSolve[y'[x] == (E^(y[x]/x)*(x + x/E^(y[x]/x) + x^3 + x^4 + y[x]/E^(y[x]/x)))/x,y[x],x,Inclu
```

$$y(x) \to -x\log\left(-\frac{x^3}{4} - \frac{x^2}{3} - \frac{c_1}{x} - 1\right)$$

2.294 problem 871

Internal problem ID [9205]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 871.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - \frac{2y^2x + 4y\ln(2x+1)x + 2\ln(2x+1)^2 x + y^2 - 2 + \ln(2x+1)^2 + 2y\ln(2x+1)}{2x+1} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x) = 1/(2*x+1)*(2*x*y(x)^2+4*y(x)*ln(2*x+1)*x+2*ln(2*x+1)^2*x+y(x)^2-2+ln(2
```

$$y(x) = \frac{-1 + (-x + c_1)\ln(2x+1)}{x - c_1}$$

✓ Solution by Mathematica

Time used: 0.351 (sec). Leaf size: 34

```
DSolve[y'[x] == (-2 + Log[1 + 2*x]^2 + 2*x*Log[1 + 2*x]^2 + 2*Log[1 + 2*x]*y[x] + 4*x*Log[1
```

$$y(x) \to -\log(2x+1) + \frac{1}{-x + c_1}$$
$$y(x) \to -\log(2x+1)$$

2.295 problem 872

Internal problem ID [9206]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 872.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], [_Abel`

$$y' - \frac{-30yx^3 + 12x^6 + 70x^{\frac{7}{2}} - 30x^3 - 25\sqrt{x}\, y + 50x - 25\sqrt{x} - 25}{5\left(-5y + 2x^3 + 10\sqrt{x} - 5\right)x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve(diff(y(x),x) = 1/5*(-30*x^3*y(x)+12*x^6+70*x^(7/2)-30*x^3-25*y(x)*x^(1/2)+50*x-25*x^(
```

$$y(x) = \frac{2x^3}{5} - \sqrt{c_1 + 2\ln(x)} + 2\sqrt{x} - 1$$
$$y(x) = \frac{2x^3}{5} + 2\sqrt{x} + \sqrt{c_1 + 2\ln(x)} - 1$$

✓ Solution by Mathematica

Time used: 0.463 (sec). Leaf size: 92

```
DSolve[y'[x] == (-5 - 5*Sqrt[x] + 10*x - 6*x^3 + 14*x^(7/2) + (12*x^6)/5 - 5*Sqrt[x]*y[x] -
```

$$y(x) \to \frac{2x^3}{5} + 2\sqrt{x} + \sqrt{-\frac{1}{x}}\sqrt{-x(2\log(x) + 1 + c_1)} - 1$$
$$y(x) \to \frac{2x^3}{5} + 2\sqrt{x} + \left(-\frac{1}{x}\right)^{3/2} x\sqrt{-x(2\log(x) + 1 + c_1)} - 1$$

2.296 problem 873

Internal problem ID [9207]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 873.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$y' - \frac{1 + 2y}{x\left(-2 + x + y^2x + 3xy^3 + 2yx + 2y^4x\right)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve(diff(y(x),x) = 1/x*(1+2*y(x))/(-2+x+x*y(x)^2+3*x*y(x)^3+2*x*y(x)+2*x*y(x)^4),y(x), si
```

$$y(x) = -\frac{1}{2}$$

$$y(x) = \frac{e^{\operatorname{RootOf}\left(2x\,e^{4\_Z}-3x\,e^{3\_Z}-6x\,e^{2\_Z}+48c_1x\,e^{\_Z}+54\_Zx\,e^{\_Z}+7x\,e^{\_Z}+96\right)}}{2} - \frac{1}{2}$$

✓ Solution by Mathematica

Time used: 0.446 (sec). Leaf size: 53

```
DSolve[y'[x] == (1 + 2*y[x])/(x*(-2 + x + 2*x*y[x] + x*y[x]^2 + 3*x*y[x]^3 + 2*x*y[x]^4)),y[
```

$$\text{Solve}\left[\frac{1}{192}\left(-16y(x)^3 - 12y(x)^2 + 12y(x) - 54\log(4y(x)+2) + 7\right) - \frac{1}{2x(2y(x)+1)} = c_1, y(x)\right]$$

2.297 problem 874

Internal problem ID [9208]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 874.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Abel]`

$$y' - \frac{(-256a\,x^2 + 512 + 512y^2 + 128ya\,x^4 + 8a^2x^8 + 512y^3 + 192y^2a\,x^4 + 24ya^2x^8 + a^3x^{12})\,x}{512} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 40

```
dsolve(diff(y(x),x) = 1/512*(-256*a*x^2+512+512*y(x)^2+128*y(x)*a*x^4+8*a^2*x^8+512*y(x)^3+1
```

$$y(x) = -\frac{x^4a}{8} - \frac{1}{3} + \frac{29\,\text{RootOf}\left(x^2 - 162\left(\int^{-Z} \frac{1}{841\_a^3-27\_a+27}d\_a\right) + 6c_1\right)}{9}$$

✓ Solution by Mathematica

Time used: 0.187 (sec). Leaf size: 101

```
DSolve[y'[x] == (x*(512 - 256*a*x^2 + 8*a^2*x^8 + a^3*x^12 + 128*a*x^4*y[x] + 24*a^2*x^8*y[x
```

$$\text{Solve}\left[-\frac{29}{3}\text{RootSum}\left[-29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log\left(\frac{\frac{1}{8}(3ax^5+8x)+3xy(x)}{\sqrt[3]{29}\sqrt[3]{x^3}} - \#1\right)}{\sqrt[3]{29} - 29\#1^2}\&\right] = \frac{1}{18}29^{2/3}\left(x^3\right)^{2/3} + c_1, y(x)\right]$$

2.298 problem 875

Internal problem ID [9209]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 875.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' + \frac{-yx - y + x^5\sqrt{y^2+x^2} - x^4\sqrt{y^2+x^2}\, y}{x(x+1)} = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 74

```
dsolve(diff(y(x),x) = -(-x*y(x)-y(x)+x^5*(y(x)^2+x^2)^(1/2)-x^4*(y(x)^2+x^2)^(1/2)*y(x))/x/(
```

$$\ln\left(\frac{x\left(\sqrt{2y(x)^2+2x^2}+y(x)+x\right)}{y(x)-x}\right) + \sqrt{2}\,\ln(x+1) + \frac{(3x^4-4x^3+6x^2-12x)\sqrt{2}}{12} - c_1 + \ln(2) - \ln(x) = 0$$

✓ Solution by Mathematica

Time used: 2.287 (sec). Leaf size: 150

```
DSolve[y'[x] == (y[x] + x*y[x] - x^5*Sqrt[x^2 + y[x]^2] + x^4*y[x]*Sqrt[x^2 + y[x]^2])/(x*(1
```

$$y(x) \to \frac{x\tanh\left(\frac{3x^4-4x^3+6x^2-12x+12\log(x+1)-25+12c_1}{12\sqrt{2}}\right)\left(2+\sqrt{2}\tanh\left(\frac{3x^4-4x^3+6x^2-12x+12\log(x+1)-25+12c_1}{12\sqrt{2}}\right)\right)}{\sqrt{2}+2\tanh\left(\frac{3x^4-4x^3+6x^2-12x+12\log(x+1)-25+12c_1}{12\sqrt{2}}\right)}$$

$$y(x) \to x$$

2.299 problem 876

Internal problem ID [9210]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 876.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class C']]`

$$y' + \frac{y^2(yx^2 - 2x - 2yx + y)}{2(-2 + yx - 2y)x} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 41

```
dsolve(diff(y(x),x) = -1/2*y(x)^2*(x^2*y(x)-2*x-2*x*y(x)+y(x))/(-2+x*y(x)-2*y(x))/x,y(x), si
```

$$y(x) = \frac{4}{\sqrt{c_1 - 8\ln(x)} + 2x - 4}$$
$$y(x) = -\frac{4}{\sqrt{c_1 - 8\ln(x)} - 2x + 4}$$

✓ Solution by Mathematica
Time used: 0.94 (sec). Leaf size: 94

```
DSolve[y'[x] == -1/2*(y[x]^2*(-2*x + y[x] - 2*x*y[x] + x^2*y[x]))/(x*(-2 - 2*y[x] + x*y[x]))
```

$$y(x) \to \frac{2}{x + \sqrt{2}\sqrt{-\frac{1}{x}}\sqrt{-x(-\log(x) + 2 + 2c_1)} - 2}$$
$$y(x) \to -\frac{2}{-x + \sqrt{2}\sqrt{-\frac{1}{x}}\sqrt{-x(-\log(x) + 2 + 2c_1)} + 2}$$
$$y(x) \to 0$$

2.300 problem 877

Internal problem ID [9211]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 877.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, [_Abel, '2n`

$$y' - \frac{-2yx + 2x^3 - 2x - y^3 + 3y^2x^2 - 3yx^4 + x^6}{x^2 - y - 1} = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 73

```
dsolve(diff(y(x),x) = (-2*x*y(x)+2*x^3-2*x-y(x)^3+3*x^2*y(x)^2-3*y(x)*x^4+x^6)/(-y(x)+x^2-1)
```

$$y(x) = \frac{-2c_1x^2 + 2x^3 + \sqrt{2c_1 - 2x + 1} - 1}{2x - 2c_1}$$
$$y(x) = \frac{2c_1x^2 - 2x^3 + \sqrt{2c_1 - 2x + 1} + 1}{-2x + 2c_1}$$

✓ Solution by Mathematica

Time used: 0.35 (sec). Leaf size: 54

```
DSolve[y'[x] == (-2*x + 2*x^3 + x^6 - 2*x*y[x] - 3*x^4*y[x] + 3*x^2*y[x]^2 - y[x]^3)/(-1 + x
```

$$y(x) \to x^2 + \frac{1}{-1 + \sqrt{-2x + c_1}}$$
$$y(x) \to x^2 - \frac{1}{1 + \sqrt{-2x + c_1}}$$
$$y(x) \to x^2$$

2.301 problem 878

Internal problem ID [9212]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 878.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{1 + y^4 - 8y^2ax + 16a^2x^2 + y^6 - 12y^4ax + 48y^2a^2x^2 - 64a^3x^3}{y} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 73

```
dsolve(diff(y(x),x) = (1+y(x)^4-8*a*x*y(x)^2+16*a^2*x^2+y(x)^6-12*y(x)^4*a*x+48*y(x)^2*a^2*x
```

$$-\left(\int_{\_b}^{y(x)} \frac{\_a}{\_a^6 - 12\_a^4ax + 48\_a^2a^2x^2 - 64a^3x^3 + \_a^4 - 8\_a^2ax + 16a^2x^2 - 2a + 1} d\_a\right) + x - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.269 (sec). Leaf size: 130

```
DSolve[y'[x] == (1 + 16*a^2*x^2 - 64*a^3*x^3 - 8*a*x*y[x]^2 + 48*a^2*x^2*y[x]^2 + y[x]^4 - 1
```

$$\text{Solve}\Big[2a\Big(x - \frac{1}{2}\text{RootSum}\Big[64\#1^3a^3 - 48\#1^2a^2y(x)^2 - 16\#1^2a^2 + 12\#1ay(x)^4 + 8\#1ay(x)^2 + 2a - y(x)^6 - y(x)^4 - 1\&, \frac{}{48\ldots}$$

2.302 problem 879

Internal problem ID [9213]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 879.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' + \frac{-yx - y + \sqrt{y^2 + x^2}\, x^2 - yx\sqrt{y^2 + x^2}}{x(x+1)} = 0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 56

```
dsolve(diff(y(x),x) = -(-x*y(x)-y(x)+(y(x)^2+x^2)^(1/2)*x^2-x*(y(x)^2+x^2)^(1/2)*y(x))/x/(x+
```

$$\ln(2) + \ln\left(\frac{x\left(\sqrt{2y(x)^2 + 2x^2} + y(x) + x\right)}{y(x) - x}\right) + \sqrt{2}\, x - \sqrt{2}\, \ln(x+1) - \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 5.179 (sec). Leaf size: 81

```
DSolve[y'[x] == (y[x] + x*y[x] - x^2*Sqrt[x^2 + y[x]^2] + x*y[x]*Sqrt[x^2 + y[x]^2])/(x*(1 +
```

$$y(x) \to \frac{x \tanh\left(\frac{x-\log(x+1)+c_1}{\sqrt{2}}\right)\left(2 + \sqrt{2}\tanh\left(\frac{x-\log(x+1)+c_1}{\sqrt{2}}\right)\right)}{\sqrt{2} + 2\tanh\left(\frac{x-\log(x+1)+c_1}{\sqrt{2}}\right)}$$

$$y(x) \to x$$

2.303 problem 880

Internal problem ID [9214]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 880.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y' + \frac{2a}{-y - 2a - 2ay^4 + 16a^2xy^2 - 32a^3x^2 - 2ay^6 + 24y^4a^2x - 96y^2a^3x^2 + 128a^4x^3} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 41

```
dsolve(diff(y(x),x) = -2*a/(-y(x)-2*a-2*a*y(x)^4+16*a^2*x*y(x)^2-32*a^3*x^2-2*a*y(x)^6+24*y(
```

$$\frac{y(x)}{2a} + \frac{\int^{-4ax+y(x)^2} \frac{1}{\_a^3+\_a^2+1} d\_a}{8a^2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.361 (sec). Leaf size: 131

```
DSolve[y'[x] == (-2*a)/(-2*a - 32*a^3*x^2 + 128*a^4*x^3 - y[x] + 16*a^2*x*y[x]^2 - 96*a^3*x^
```

$$\text{Solve}\left[\frac{\text{RootSum}\left[-64\#1^3a^3 + 48\#1^2a^2y(x)^2 + 16\#1^2a^2 - 12\#1ay(x)^4 - 8\#1ay(x)^2 + y(x)^6 + y(x)^4 + \right.}{8a^2}\right.$$
$$\left. + \frac{y(x)}{2a} = c_1, y(x)\right]$$

2.304 problem 881

Internal problem ID [9215]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 881.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, [_Abel, '2n`

$$y' - \frac{-18yx - 6x^3 - 18x + 27y^3 + 27y^2x^2 + 9yx^4 + x^6}{27y + 9x^2 + 27} = 0$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 77

```
dsolve(diff(y(x),x) = (-18*x*y(x)-6*x^3-18*x+27*y(x)^3+27*x^2*y(x)^2+9*y(x)*x^4+x^6)/(27*y(x
```

$$y(x) = \frac{-2c_1x^2 + 2x^3 + 3\sqrt{2c_1 - 2x + 1} + 3}{-6x + 6c_1}$$
$$y(x) = \frac{-2c_1x^2 + 2x^3 - 3\sqrt{2c_1 - 2x + 1} + 3}{-6x + 6c_1}$$

✓ Solution by Mathematica
Time used: 0.382 (sec). Leaf size: 68

```
DSolve[y'[x] == (-18*x - 6*x^3 + x^6 - 18*x*y[x] + 9*x^4*y[x] + 27*x^2*y[x]^2 + 27*y[x]^3)/(
```

$$y(x) \to -\frac{x^2}{3} + \frac{27}{-27 + \sqrt{-1458x + c_1}}$$
$$y(x) \to -\frac{x^2}{3} - \frac{27}{27 + \sqrt{-1458x + c_1}}$$
$$y(x) \to -\frac{x^2}{3}$$

2.305 problem 882

Internal problem ID [9216]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 882.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Abel]`

$$y' + \frac{\left(-108x^{\frac{3}{2}} - 216 - 216y^2 + 72yx^3 - 6x^6 - 216y^3 + 108y^2x^3 - 18yx^6 + x^9\right)\sqrt{x}}{216} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
dsolve(diff(y(x),x) = -1/216*(-108*x^(3/2)-216-216*y(x)^2+72*x^3*y(x)-6*x^6-216*y(x)^3+108*x
```

$$y(x) = \frac{x^3}{6} - \frac{1}{3} + \frac{29\,\mathrm{RootOf}\left(2x^{\frac{3}{2}} - 243\left(\int^{-Z} \frac{1}{841\_a^3 - 27\_a + 27} d\_a\right) + 9c_1\right)}{9}$$

✓ Solution by Mathematica

Time used: 0.2 (sec). Leaf size: 119

```
DSolve[y'[x] == -1/216*(Sqrt[x]*(-216 - 108*x^(3/2) - 6*x^6 + x^9 + 72*x^3*y[x] - 18*x^6*y[x
```

$$\mathrm{Solve}\left[-\frac{29}{3}\mathrm{RootSum}\left[-29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log\left(\frac{\frac{1}{2}\left(2\sqrt{x} - x^{7/2}\right) + 3\sqrt{x}y(x)}{\sqrt[3]{29}\sqrt[3]{x^{3/2}}} - \#1\right)}{\sqrt[3]{29} - 29\#1^2}\&\right] = \frac{2}{27}29^{2/3}\sqrt{x}\left(x^{3/2}\right)^{2/3} + c_1, y(x)\right]$$

2.306 problem 883

Internal problem ID [9217]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 883.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y' - \frac{(a^3 + y^4a^3 + 2y^2a^2b\,x^2 + a\,x^4b^2 + y^6a^3 + 3y^4a^2b\,x^2 + 3y^2a\,b^2x^4 + b^3x^6)\,x}{a^{\frac{7}{2}}y} = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 393

```
dsolve(diff(y(x),x) = (a^3+y(x)^4*a^3+2*y(x)^2*a^2*b*x^2+a*x^4*b^2+y(x)^6*a^3+3*y(x)^4*a^2*b
```

$$\frac{\int_{\_b}^{x} \frac{\left(b^3\_a^6+3a\,b^2\_a^4y(x)^2+3a^2b\_a^2y(x)^4+y(x)^6a^3+a\,b^2\_a^4+2a^2y(x)^2b\_a^2+y(x)^4a^3+a^3\right)\_a}{y(x)^6a^3+3a^2b\_a^2y(x)^4+3a\,b^2\_a^4y(x)^2+b^3\_a^6+y(x)^4a^3+2a^2y(x)^2b\_a^2+a\,b^2\_a^4+a^3+a^{\frac{5}{2}}b}d\_a}{a^{\frac{7}{2}}}$$

$$-\left(\int^{y(x)} \frac{2\_f\left(\frac{1}{2}+\left(a^{\frac{5}{2}}b+b^3x^6+3a\,x^4\left(\_f^2+\frac{1}{3}\right)b^2+3\left(\_f^2+\frac{2}{3}\right)a^2x^2\_f^2b+\left(\_f^6+\_f^4+1\right)a^3\right)b\left(\int_{\_b}^{x}\right.\right.}{a^{\frac{5}{2}}b+b^3x^6+3a\,x^4\left(\_f^2+\frac{1}{3}\right)b^2+3\left(\_f^2+\frac{2}{3}\right)a^2x^2\right.}$$

$$+c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.834 (sec). Leaf size: 164

```
DSolve[y'[x] == (x*(a^3 + a*b^2*x^4 + b^3*x^6 + 2*a^2*b*x^2*y[x]^2 + 3*a*b^2*x^4*y[x]^2 + a^
```

$$\text{Solve}\left[\frac{x^2}{2}\right.$$

$$-\frac{1}{2}a^{5/2}\text{RootSum}\left[\#1^3b^3+3\#1^2ab^2y(x)^2+\#1^2ab^2+3\#1a^2by(x)^4+2\#1a^2by(x)^2+a^{5/2}b+a^3y(x)^6+a^3y(x)^4\right.$$

2.307 problem 884

Internal problem ID [9218]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 884.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' + \frac{(-1 - y^4 + 2y^2x^2 - x^4 - y^6 + 3y^4x^2 - 3x^4y^2 + x^6)\,x}{y} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 106

```
dsolve(diff(y(x),x) = -(-1-y(x)^4+2*x^2*y(x)^2-x^4-y(x)^6+3*x^2*y(x)^4-3*x^4*y(x)^2+x^6)*x/y
```

$$y(x) = -e^{\operatorname{RootOf}\left(-x^2 e^{2\_Z}+2x^3 e^{\_Z}+e^{2\_Z}\ln\left(\frac{e^{2\_Z}-2x\,e^{\_Z}+1}{e^{\_Z}-2x}\right)-2\,e^{2\_Z}c_1-\_Z e^{2\_Z}-2\,e^{\_Z}\ln\left(\frac{e^{2\_Z}-2x\,e^{\_Z}+1}{e^{\_Z}-2x}\right)x+4c_1 x\,e^{\_Z}+2\_Z x\,e^{\_Z}-1\right)} + x$$

✓ Solution by Mathematica

Time used: 0.475 (sec). Leaf size: 71

```
DSolve[y'[x] == (x*(1 + x^4 - x^6 - 2*x^2*y[x]^2 + 3*x^4*y[x]^2 + y[x]^4 - 3*x^2*y[x]^4 + y[
```

$$\text{Solve}\left[\frac{1}{4}\left(2\log\left(-x^2+y(x)^2+1\right)-2x^2-\frac{1}{y(x)(y(x)+x)} + \frac{1}{xy(x)-y(x)^2}-2\log(x-y(x))-2\log(y(x)+x)\right)=c_1, y(x)\right]$$

2.308 problem 885

Internal problem ID [9219]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 885.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' + \frac{i(32ix + 64 + 64y^4 + 32y^2x^2 + 4x^4 + 64y^6 + 48y^4x^2 + 12x^4y^2 + x^6)}{128y} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) = -1/128*I*(32*I*x+64+64*y(x)^4+32*x^2*y(x)^2+4*x^4+64*y(x)^6+48*x^2*y(x
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x] == ((-1/128*I)*(64 + (32*I)*x + 4*x^4 + x^6 + 32*x^2*y[x]^2 + 12*x^4*y[x]^2 + 6
```

Not solved

2.309 problem 886

Internal problem ID [9220]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 886.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Abel]`

$$y' - \frac{2x^2 - 4yx^3 + 1 + x^4y^2 + x^6y^3 - 3y^2x^5 + 3yx^4 - x^3}{x^4} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(diff(y(x),x) = 1/x^4*(2*x^2-4*x^3*y(x)+1+x^4*y(x)^2+x^6*y(x)^3-3*y(x)^2*x^5+3*y(x)*x^
```

$$y(x) = \frac{9x - 3 + 29 \operatorname{RootOf}\left(-81\left(\int^{\_Z} \frac{1}{841\_a^3 - 27\_a + 27} d\_a\right) x + 3c_1 x - 1\right)}{9x^2}$$

✓ Solution by Mathematica

Time used: 0.15 (sec). Leaf size: 82

```
DSolve[y'[x] == (1 + 2*x^2 - x^3 - 4*x^3*y[x] + 3*x^4*y[x] + x^4*y[x]^2 - 3*x^5*y[x]^2 + x^6
```

$$\text{Solve}\left[-\frac{29}{3}\text{RootSum}\left[-29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log\left(\frac{3x^2y(x)-3x+1}{\sqrt[3]{29}} - \#1\right)}{\sqrt[3]{29} - 29\#1^2}\&\right] = -\frac{29^{2/3}}{9x} + c_1, y(x)\right]$$

2.310 problem 887

Internal problem ID [9221]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 887.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], [_Abel`

$$y' - \frac{ya^2x + a + a^2x + y^3a^3x^3 + 3y^2a^2x^2 + 3yax + 1}{a^2x^2\left(yax + 1 + ax\right)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 72

```
dsolve(diff(y(x),x) = 1/a^2/x^2*(y(x)*a^2*x+a+a^2*x+y(x)^3*a^3*x^3+3*y(x)^2*a^2*x^2+3*y(x)*a
```

$$y(x) = \frac{ax - \sqrt{-2x + c_1} + 1}{\left(\sqrt{-2x + c_1} - 1\right)xa}$$

$$y(x) = \frac{-ax - \sqrt{-2x + c_1} - 1}{\left(\sqrt{-2x + c_1} + 1\right)xa}$$

✓ Solution by Mathematica

Time used: 0.884 (sec). Leaf size: 103

```
DSolve[y'[x] == (1 + a + a^2*x + 3*a*x*y[x] + a^2*x*y[x] + 3*a^2*x^2*y[x]^2 + a^3*x^3*y[x]^3
```

$$y(x) \to -\frac{1}{ax} + \frac{a^3}{-a^3 + \sqrt{-2a^6x + c_1}}$$

$$y(x) \to -\frac{\sqrt{-2a^6x + c_1} + a^4x + a^3}{a^4x + ax\sqrt{-2a^6x + c_1}}$$

$$y(x) \to -\frac{1}{ax}$$

2.311 problem 888

Internal problem ID [9222]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 888.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class C'], [_1st_order, '_wit`

$$y' - \frac{6yx^2 - 2x + 1 - 5y^2x^3 - 2yx + y^3x^4}{x^2(yx^2 - x + 1)} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 79

```
dsolve(diff(y(x),x) = 1/x^2*(6*x^2*y(x)-2*x+1-5*x^3*y(x)^2-2*x*y(x)+y(x)^3*x^4)/(x^2*y(x)-x+
```

$$y(x) = \frac{\sqrt{\frac{c_1x+2}{x}}\, x - x + 1}{\left(\sqrt{\frac{c_1x+2}{x}} - 1\right) x^2}$$

$$y(x) = \frac{\sqrt{\frac{c_1x+2}{x}}\, x + x - 1}{\left(\sqrt{\frac{c_1x+2}{x}} + 1\right) x^2}$$

✓ Solution by Mathematica

Time used: 0.764 (sec). Leaf size: 74

```
DSolve[y'[x] == (1 - 2*x - 2*x*y[x] + 6*x^2*y[x] - 5*x^3*y[x]^2 + x^4*y[x]^3)/(x^2*(1 - x +
```

$$y(x) \to \frac{x-1}{x^2} + \frac{1}{x^4\left(\frac{1}{x^2} - \frac{1}{x^2\sqrt{\frac{2}{x}+c_1}}\right)}$$

$$y(x) \to \frac{x + \frac{1}{1+\frac{1}{\sqrt{\frac{2}{x}+c_1}}} - 1}{x^2}$$

$$y(x) \to \frac{1}{x}$$

2.312 problem 889

Internal problem ID [9223]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 889.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y' + \frac{\left(-8 - 8y^3 + 24y^{\frac{3}{2}}\mathrm{e}^x - 18\,\mathrm{e}^{2x} - 8y^{\frac{9}{2}} + 36y^3\mathrm{e}^x - 54\,\mathrm{e}^{2x}y^{\frac{3}{2}} + 27\,\mathrm{e}^{3x}\right)\mathrm{e}^x}{8\sqrt{y}} = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 92

```
dsolve(diff(y(x),x) = -1/8*(-8-8*y(x)^3+24*y(x)^(3/2)*exp(x)-18*exp(x)^2-8*y(x)^(9/2)+36*y(x
```

$$\frac{\left(-6\,\mathrm{e}^x + 4y(x)^{\frac{3}{2}}\right)\ln\left(2y(x)^{\frac{3}{2}} - 3\,\mathrm{e}^x + 2\right) + \left(6\,\mathrm{e}^x - 4y(x)^{\frac{3}{2}}\right)\ln\left(2y(x)^{\frac{3}{2}} - 3\,\mathrm{e}^x\right) + (6c_1 - 6\,\mathrm{e}^x)\,y(x)^{\frac{3}{2}} - 9c}{-6y\left(x\right)^{\frac{3}{2}} + 9\,\mathrm{e}^x}$$
$$= 0$$

✓ Solution by Mathematica

Time used: 1.133 (sec). Leaf size: 68

```
DSolve[y'[x] == -1/8*(E^x*(-8 - 18*E^(2*x) + 27*E^(3*x) + 24*E^x*y[x]^(3/2) - 54*E^(2*x)*y[x
```

$$\text{Solve}\left[\frac{2}{3}\log\left(y(x)^{3/2} - \frac{3e^x}{2}\right) + e^x = \frac{4}{9e^x - 6y(x)^{3/2}} + \frac{2}{3}\log\left(y(x)^{3/2} - \frac{3e^x}{2} + 1\right) + c_1, y(x)\right]$$

2.313 problem 890

Internal problem ID [9224]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 890.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{y' - \frac{x}{-y + 1 + y^4 + 2y^2x^2 + x^4 + y^6 + 3y^4x^2 + 3x^4y^2 + x^6} = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 504

```
dsolve(diff(y(x),x) = x/(-y(x)+1+y(x)^4+2*x^2*y(x)^2+x^4+y(x)^6+3*x^2*y(x)^4+3*x^4*y(x)^2+x^
```

$$y(x) = -\frac{2^{\frac{1}{3}}\sqrt{3}\,\sqrt{-6x^2\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{1}{3}}-\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{2}{3}}-2\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{1}{3}}-4}}{6\left(3\sqrt{3}+\sqrt{31}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{2^{\frac{1}{3}}\sqrt{3}\,\sqrt{-6x^2\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{1}{3}}-\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{2}{3}}-2\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{1}{3}}-4}}{6\left(3\sqrt{3}+\sqrt{31}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{2^{\frac{5}{6}}\sqrt{3}\,\sqrt{\left(-12x^2-4\right)\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{1}{3}}-i\sqrt{3}\,\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{2}{3}}+4i\sqrt{3}+\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{2}{3}}}}{12\left(3\sqrt{3}+\sqrt{31}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{2^{\frac{5}{6}}\sqrt{3}\,\sqrt{\left(-12x^2-4\right)\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{1}{3}}-i\sqrt{3}\,\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{2}{3}}+4i\sqrt{3}+\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{2}{3}}+}}{12\left(3\sqrt{3}+\sqrt{31}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{2^{\frac{5}{6}}\sqrt{3}\,\sqrt{\left(-12x^2-4\right)\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{2}{3}}-4i\sqrt{3}+\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{2}{3}}}}{12\left(3\sqrt{3}+\sqrt{31}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{2^{\frac{5}{6}}\sqrt{3}\,\sqrt{\left(-12x^2-4\right)\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{2}{3}}-4i\sqrt{3}+\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{2}{3}}+}}{12\left(3\sqrt{3}+\sqrt{31}\right)^{\frac{1}{3}}}$$

$$-y(x)+\frac{\left(\int^{y(x)^2+x^2}\frac{1}{\_a^3+\_a^2+1}d\_a\right)}{2}-c_1=0$$

✓ Solution by Mathematica

Time used: 0.142 (sec). Leaf size: 103

```
DSolve[y'[x] == x/(1 + x^4 + x^6 - y[x] + 2*x^2*y[x]^2 + 3*x^4*y[x]^2 + y[x]^4 + 3*x^2*y[x]^
```

$$\text{Solve}\left[y(x)-\frac{1}{2}\text{RootSum}\left[\#1^3+3\#1^2y(x)^2+\#1^2+3\#1y(x)^4+2\#1y(x)^2+y(x)^6+y(x)^4+1\&,\frac{\log\left(x^2-\#1\right)}{3\#1^2+6\#1y(x)^2+2\#1+3y(x)^4+2y(x)^2}\&\right]=c_1,y(x)\right]$$

2.314 problem 891

Internal problem ID [9225]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 891.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class C']]`

$$y' - \frac{y^2(-2y + 2x^2 + 2yx^2 + yx^4)}{x^3 (x^2 - y + yx^2)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 56

```
dsolve(diff(y(x),x) = y(x)^2/x^3*(-2*y(x)+2*x^2+2*x^2*y(x)+y(x)*x^4)/(x^2-y(x)+x^2*y(x)),y(x
```

$$y(x) = \frac{x^2}{\sqrt{c_1 - 2\ln(x)}\, x^2 - x^2 + 1}$$

$$y(x) = -\frac{x^2}{\sqrt{c_1 - 2\ln(x)}\, x^2 + x^2 - 1}$$

✓ Solution by Mathematica

Time used: 2.31 (sec). Leaf size: 91

```
DSolve[y'[x] == (y[x]^2*(2*x^2 - 2*y[x] + 2*x^2*y[x] + x^4*y[x]))/(x^3*(x^2 - y[x] + x^2*y[x
```

$$y(x) \to \frac{x^2}{1 + x^2 \left(-1 + \sqrt{\frac{1}{x^5}} \sqrt{x^5(-2\log(x) + 1 + c_1)}\right)}$$

$$y(x) \to -\frac{x^2}{-1 + x^2 \left(1 + \sqrt{\frac{1}{x^5}} \sqrt{x^5(-2\log(x) + 1 + c_1)}\right)}$$

$$y(x) \to 0$$

2.315 problem 892

Internal problem ID [9226]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 892.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$
y' - \frac{y^2 + 2yx + x^2 + \mathrm{e}^{-\frac{2}{-y^2+x^2-1}}}{y^2 + 2yx + x^2 - \mathrm{e}^{-\frac{2}{-y^2+x^2-1}}} = 0
$$

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 40

```
dsolve(diff(y(x),x) = (y(x)^2+2*x*y(x)+x^2+exp(-2/(-y(x)^2+x^2-1)))/(y(x)^2+2*x*y(x)+x^2-exp
```

$$
y(x) = \mathrm{e}^{\mathrm{RootOf}\left(-\_Z+\int^{\mathrm{e}^{2\_Z}-2x\,\mathrm{e}^{\_Z}} \frac{1}{\mathrm{e}^{\frac{2}{\_a+1}}+\_a} d\_a + c_1\right)} - x
$$

✓ Solution by Mathematica

Time used: 3.311 (sec). Leaf size: 1283

```
DSolve[y'[x] == (E^(-2/(-1 + x^2 - y[x]^2)) + x^2 + 2*x*y[x] + y[x]^2)/(-E^(-2/(-1 + x^2 - y
```

$$\text{Solve}\Bigg[\int_1^x \Bigg(-e^{\int_1^{(K[2]-y(x))(K[2]+y(x))} \frac{2((K[1]-3)K[1]+1)}{\left(e^{-\frac{2}{K[1]-1}}-K[1]\right)(K[1]-1)^2} dK[1]+\frac{2}{K[2]^2-y(x)^2-1}} K[2]^2$$

$$-2e^{\int_1^{(K[2]-y(x))(K[2]+y(x))} \frac{2((K[1]-3)K[1]+1)}{\left(e^{-\frac{2}{K[1]-1}}-K[1]\right)(K[1]-1)^2} dK[1]+\frac{2}{K[2]^2-y(x)^2-1}} y(x)K[2]$$

$$-e^{\int_1^{(K[2]-y(x))(K[2]+y(x))} \frac{2((K[1]-3)K[1]+1)}{\left(e^{-\frac{2}{K[1]-1}}-K[1]\right)(K[1]-1)^2} dK[1]} \left(e^{\frac{2}{K[2]^2-y(x)^2-1}} y(x)^2+1\right)\Bigg) dK[2]$$

$$+\int_1^{y(x)} \Bigg(e^{\int_1^{(x-K[3])(x+K[3])} \frac{2((K[1]-3)K[1]+1)}{\left(e^{-\frac{2}{K[1]-1}}-K[1]\right)(K[1]-1)^2} dK[1]+\frac{2}{x^2-K[3]^2-1}} x^2$$

$$+2e^{\int_1^{(x-K[3])(x+K[3])} \frac{2((K[1]-3)K[1]+1)}{\left(e^{-\frac{2}{K[1]-1}}-K[1]\right)(K[1]-1)^2} dK[1]+\frac{2}{x^2-K[3]^2-1}} K[3]x$$

$$-e^{\int_1^{(x-K[3])(x+K[3])} \frac{2((K[1]-3)K[1]+1)}{\left(e^{-\frac{2}{K[1]-1}}-K[1]\right)(K[1]-1)^2} dK[1]}$$

$$+e^{\int_1^{(x-K[3])(x+K[3])} \frac{2((K[1]-3)K[1]+1)}{\left(e^{-\frac{2}{K[1]-1}}-K[1]\right)(K[1]-1)^2} dK[1]+\frac{2}{x^2-K[3]^2-1}} K[3]^2$$

$$-\int_1^x \Bigg(-e^{\int_1^{(K[2]-K[3])(K[2]+K[3])} \frac{2((K[1]-3)K[1]+1)}{\left(e^{-\frac{2}{K[1]-1}}-K[1]\right)(K[1]-1)^2} dK[1]+\frac{2}{K[2]^2-K[3]^2-1}} \left(\frac{4K[3]}{(K[2]^2-K[3]^2-1)^2} - \frac{4K[3}{\left(e^{-\frac{2}{(K[2]-K[3])(K}}\right.}\right.$$

2.316 problem 893

Internal problem ID [9227]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 893.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Abel]`

$$y' - \frac{6x + x^3 + y^2x^3 + 4yx^2 + y^3x^3 + 6y^2x^2 + 12yx + 8}{x^3} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
dsolve(diff(y(x),x) = (6*x+x^3+x^3*y(x)^2+4*x^2*y(x)+x^3*y(x)^3+6*x^2*y(x)^2+12*x*y(x)+8)/x^
```

$$y(x) = \frac{29\,\mathrm{RootOf}\left(-81\left(\int^{-Z} \frac{1}{841\_a^3-27\_a+27}d\_a\right) + x + 3c_1\right)x - 3x - 18}{9x}$$

✓ Solution by Mathematica

Time used: 0.142 (sec). Leaf size: 80

```
DSolve[y'[x] == (8 + 6*x + x^3 + 12*x*y[x] + 4*x^2*y[x] + 6*x^2*y[x]^2 + x^3*y[x]^2 + x^3*y[
```

$$\mathrm{Solve}\left[-\frac{29}{3}\mathrm{RootSum}\left[-29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log\left(\frac{3y(x)+\frac{x+6}{x}}{\sqrt[3]{29}} - \#1\right)}{\sqrt[3]{29} - 29\#1^2}\&\right] = \frac{1}{9}29^{2/3}x + c_1, y(x)\right]$$

2.317 problem 894

Internal problem ID [9228]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 894.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' + \frac{i(ix + 1 + x^4 + 2y^2x^2 + y^4 + x^6 + 3x^4y^2 + 3y^4x^2 + y^6)}{y} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) = -I*(I*x+1+x^4+2*x^2*y(x)^2+y(x)^4+x^6+3*x^4*y(x)^2+3*x^2*y(x)^4+y(x)^6
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x] == ((-I)*(1 + I*x + x^4 + x^6 + 2*x^2*y[x]^2 + 3*x^4*y[x]^2 + y[x]^4 + 3*x^2*y[
```

Not solved

2.318 problem 895

Internal problem ID [9229]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 895.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], [_Abel`

$$y' - \frac{(-256ya\,x^2 - 32a^2x^6 - 256a\,x^2 + 512y^3 + 192y^2a\,x^4 + 24ya^2x^8 + a^3x^{12})\,x}{512y + 64a\,x^4 + 512} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 79

```
dsolve(diff(y(x),x) = (-256*a*x^2*y(x)-32*a^2*x^6-256*a*x^2+512*y(x)^3+192*x^4*a*y(x)^2+24*y
```

$$y(x) = \frac{8 + \left(-\sqrt{-x^2 + c_1} + 1\right) a\,x^4}{-8 + 8\sqrt{-x^2 + c_1}}$$
$$y(x) = \frac{-8 + \left(-\sqrt{-x^2 + c_1} - 1\right) a\,x^4}{8 + 8\sqrt{-x^2 + c_1}}$$

✓ Solution by Mathematica

Time used: 0.474 (sec). Leaf size: 75

```
DSolve[y'[x] == (x*(-256*a*x^2 - 32*a^2*x^6 + a^3*x^12 - 256*a*x^2*y[x] + 24*a^2*x^8*y[x] +
```

$$y(x) \to -\frac{ax^4}{8} + \frac{512}{-512 + \sqrt{-262144x^2 + c_1}}$$
$$y(x) \to -\frac{ax^4}{8} - \frac{512}{512 + \sqrt{-262144x^2 + c_1}}$$
$$y(x) \to -\frac{ax^4}{8}$$

2.319 problem 896

Internal problem ID [9230]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 896.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{x + 1 + y^4 - 2y^2x^2 + x^4 + y^6 - 3y^4x^2 + 3x^4y^2 - x^6}{y} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 61

```
dsolve(diff(y(x),x) = (x+1+y(x)^4-2*x^2*y(x)^2+x^4+y(x)^6-3*x^2*y(x)^4+3*x^4*y(x)^2-x^6)/y(x
```

$$-\left(\int_{\_b}^{y(x)} \frac{\_a}{\_a^6 - 3\_a^4x^2 + 3x^4\_a^2 - x^6 + \_a^4 - 2\_a^2x^2 + x^4 + 1} d\_a\right) + x - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.205 (sec). Leaf size: 106

```
DSolve[y'[x] == (1 + x + x^4 - x^6 - 2*x^2*y[x]^2 + 3*x^4*y[x]^2 + y[x]^4 - 3*x^2*y[x]^4 + y
```

$$\text{Solve}\left[\frac{1}{2}\text{RootSum}\left[-\#1^3 + 3\#1^2y(x)^2 + \#1^2 - 3\#1y(x)^4 - 2\#1y(x)^2 + y(x)^6 + y(x)^4 + 1\&, \frac{\log(x^2 - \#1)}{3\#1^2 - 6\#1y(x)^2 - 2\#1 + 3y(x)^4 + 2y(x)^2}\&\right] - x = c_1, y(x)\right]$$

2.320 problem 897

Internal problem ID [9231]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 897.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], [_Abel`

$$y' - \frac{\left(-108x^{\frac{3}{2}}y + 18x^{\frac{9}{2}} - 108x^{\frac{3}{2}} - 216y^3 + 108y^2x^3 - 18yx^6 + x^9\right)\sqrt{x}}{-216y + 36x^3 - 216} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 87

```
dsolve(diff(y(x),x) = (-108*x^(3/2)*y(x)+18*x^(9/2)-108*x^(3/2)-216*y(x)^3+108*x^3*y(x)^2-18
```

$$y(x) = \frac{\sqrt{9c_1 - 12x^{\frac{3}{2}}}\,x^3 - 3x^3 + 18}{6\sqrt{9c_1 - 12x^{\frac{3}{2}}} - 18}$$

$$y(x) = \frac{\sqrt{9c_1 - 12x^{\frac{3}{2}}}\,x^3 + 3x^3 - 18}{6\sqrt{9c_1 - 12x^{\frac{3}{2}}} + 18}$$

✓ Solution by Mathematica

Time used: 2.065 (sec). Leaf size: 76

```
DSolve[y'[x] == (Sqrt[x]*(-108*x^(3/2) + 18*x^(9/2) + x^9 - 108*x^(3/2)*y[x] - 18*x^6*y[x] +
```

$$y(x) \to \frac{x^3}{6} - \frac{216}{216 + \sqrt{-62208x^{3/2} + c_1}}$$

$$y(x) \to \frac{x^3}{6} + \frac{216}{-216 + \sqrt{-62208x^{3/2} + c_1}}$$

$$y(x) \to \frac{x^3}{6}$$

2.321 problem 898

Internal problem ID [9232]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 898.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], [_Abel`

$$y' - \frac{32x^5y + 8x^3 + 32x^5 + 64x^6y^3 + 48x^4y^2 + 12yx^2 + 1}{16x^6\left(4yx^2 + 1 + 4x^2\right)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 87

```
dsolve(diff(y(x),x) = 1/16/x^6*(32*x^5*y(x)+8*x^3+32*x^5+64*x^6*y(x)^3+48*x^4*y(x)^2+12*x^2*
```

$$y(x) = \frac{4x^2 - \sqrt{\frac{c_1x+2}{x}} + 1}{4\left(\sqrt{\frac{c_1x+2}{x}} - 1\right)x^2}$$

$$y(x) = \frac{-4x^2 - \sqrt{\frac{c_1x+2}{x}} - 1}{4\left(\sqrt{\frac{c_1x+2}{x}} + 1\right)x^2}$$

✓ Solution by Mathematica

Time used: 0.767 (sec). Leaf size: 106

```
DSolve[y'[x] == (1/16 + x^3/2 + 2*x^5 + (3*x^2*y[x])/4 + 2*x^5*y[x] + 3*x^4*y[x]^2 + 4*x^6*y
```

$$y(x) \to \frac{256x^2 - \sqrt{\frac{8192}{x} + c_1} + 64}{4x^2\left(-64 + \sqrt{\frac{8192}{x} + c_1}\right)}$$

$$y(x) \to -\frac{256x^2 + \sqrt{\frac{8192}{x} + c_1} + 64}{4x^2\left(64 + \sqrt{\frac{8192}{x} + c_1}\right)}$$

$$y(x) \to -\frac{1}{4x^2}$$

2.322 problem 899

Internal problem ID [9233]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 899.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Abel]`

$$y' - \frac{32x^5 + 64x^6 + 64y^2x^6 + 32yx^4 + 4x^2 + 64x^6y^3 + 48x^4y^2 + 12yx^2 + 1}{64x^8} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(diff(y(x),x) = 1/64*(32*x^5+64*x^6+64*y(x)^2*x^6+32*y(x)*x^4+4*x^2+64*x^6*y(x)^3+48*x
```

$$y(x) = \frac{116 \operatorname{RootOf}\left(-81\left(\int^{\_Z} \frac{1}{841\_a^3-27\_a+27}d\_a\right)x + 3c_1x - 1\right)x^2 - 12x^2 - 9}{36x^2}$$

✓ Solution by Mathematica

Time used: 0.164 (sec). Leaf size: 106

```
DSolve[y'[x] == (1/64 + x^2/16 + x^5/2 + x^6 + (3*x^2*y[x])/16 + (x^4*y[x])/2 + (3*x^4*y[x]^
```

$$\text{Solve}\left[-\frac{29}{3}\text{RootSum}\left[-29\#1^3+3\sqrt[3]{29}\#1-29\&,\frac{\log\left(\frac{\frac{3y(x)}{x^2}+\frac{4x^2+3}{4x^4}}{\sqrt[3]{29}\sqrt[3]{\frac{1}{x^6}}}-\#1\right)}{\sqrt[3]{29}-29\#1^2}\&\right]=-\frac{1}{9}29^{2/3}\left(\frac{1}{x^6}\right)^{2/3}x^3+c_1,y(x)\right]$$

2.323 problem 900

Internal problem ID [9234]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 900.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$\boxed{y' - \frac{2a(-y^2+4ax-1)}{-y^3+4yax-y-2ay^6+24y^4a^2x-96y^2a^3x^2+128a^4x^3} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 48

```
dsolve(diff(y(x),x) = 2*a*(-y(x)^2+4*a*x-1)/(-y(x)^3+4*y(x)*a*x-y(x)-2*a*y(x)^6+24*y(x)^4*a^
```

$$\frac{y(x)}{2a} - \frac{1}{16\left(-4ax+y(x)^2\right)^2 a^2} + \frac{1}{32x\,a^3-8a^2y(x)^2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.135 (sec). Leaf size: 381

```
DSolve[y'[x] == (2*a*(-1 + 4*a*x - y[x]^2))/(128*a^4*x^3 - y[x] + 4*a*x*y[x] - 96*a^3*x^2*y[
```

$$y(x) \to \text{Root}\left[8\#1^5a - 16\#1^4a^2c_1 - 64\#1^3a^2x + \#1^2\left(-2+128a^3c_1x\right) + 128\#1a^3x^2 - 256a^4c_1x^2 + 8ax - 1\&, 1\right]$$

$$y(x) \to \text{Root}\left[8\#1^5a - 16\#1^4a^2c_1 - 64\#1^3a^2x + \#1^2\left(-2+128a^3c_1x\right) + 128\#1a^3x^2 - 256a^4c_1x^2 + 8ax - 1\&, 2\right]$$

$$y(x) \to \text{Root}\left[8\#1^5a - 16\#1^4a^2c_1 - 64\#1^3a^2x + \#1^2\left(-2+128a^3c_1x\right) + 128\#1a^3x^2 - 256a^4c_1x^2 + 8ax - 1\&, 3\right]$$

$$y(x) \to \text{Root}\left[8\#1^5a - 16\#1^4a^2c_1 - 64\#1^3a^2x + \#1^2\left(-2+128a^3c_1x\right) + 128\#1a^3x^2 - 256a^4c_1x^2 + 8ax - 1\&, 4\right]$$

$$y(x) \to \text{Root}\left[8\#1^5a - 16\#1^4a^2c_1 - 64\#1^3a^2x + \#1^2\left(-2+128a^3c_1x\right) + 128\#1a^3x^2 - 256a^4c_1x^2 + 8ax - 1\&, 5\right]$$

2.324 problem 901

Internal problem ID [9235]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 901.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y' - \frac{(y - a\ln(y)\,x + x^2)\,y}{(-y\ln(y) - y\ln(x) - y + ax)\,x} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(diff(y(x),x) = (y(x)-a*ln(y(x))*x+x^2)/(-y(x)*ln(y(x))-y(x)*ln(x)-y(x)+a*x)*y(x)/x,y(
```

$$y(x) = e^{\mathrm{RootOf}\left(2ax\_Z-2\,e^{\_Z}\ln(x)-2\,e^{\_Z}\_Z-x^2+2c_1\right)}$$

✓ Solution by Mathematica

Time used: 0.421 (sec). Leaf size: 33

```
DSolve[y'[x] == (y[x]*(x^2 - a*x*Log[y[x]] + y[x]))/(x*(a*x - y[x] - Log[x]*y[x] - Log[y[x]]
```

$$\mathrm{Solve}\left[ax\log(y(x)) - \frac{x^2}{2} - y(x)\log(x) - y(x)\log(y(x)) = c_1, y(x)\right]$$

2.325 problem 902

Internal problem ID [9236]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 902.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{-y^2x + x^3 - x - y^6 + 3y^4x^2 - 3x^4y^2 + x^6}{(-y^2 + x^2 - 1)\, y} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 177

```
dsolve(diff(y(x),x) = (-x*y(x)^2+x^3-x-y(x)^6+3*x^2*y(x)^4-3*x^4*y(x)^2+x^6)/(-y(x)^2+x^2-1)
```

$$y(x) = \frac{\sqrt{(-x + c_1)\left(4c_1x^2 - 4x^3 + \sqrt{4c_1 - 4x + 1} + 1\right)}}{2x - 2c_1}$$

$$y(x) = \frac{\sqrt{(-x + c_1)\left(4c_1x^2 - 4x^3 + \sqrt{4c_1 - 4x + 1} + 1\right)}}{-2x + 2c_1}$$

$$y(x) = \frac{\sqrt{\left(-4c_1x^2 + 4x^3 + \sqrt{4c_1 - 4x + 1} - 1\right)(x - c_1)}}{2x - 2c_1}$$

$$y(x) = \frac{\sqrt{\left(-4c_1x^2 + 4x^3 + \sqrt{4c_1 - 4x + 1} - 1\right)(x - c_1)}}{-2x + 2c_1}$$

✓ Solution by Mathematica

Time used: 8.232 (sec). Leaf size: 219

```
DSolve[y'[x] == (-x + x^3 + x^6 - x*y[x]^2 - 3*x^4*y[x]^2 + 3*x^2*y[x]^4 - y[x]^6)/(y[x]*(-1
```

$$y(x) \to -\frac{1}{2}\sqrt{-\frac{-4x^3 + 4c_1x^2 + \sqrt{-4x + 1 + 4c_1} + 1}{x - c_1}}$$

$$y(x) \to \frac{1}{2}\sqrt{-\frac{-4x^3 + 4c_1x^2 + \sqrt{-4x + 1 + 4c_1} + 1}{x - c_1}}$$

$$y(x) \to -\frac{1}{2}\sqrt{\frac{4x^3 - 4c_1x^2 + \sqrt{-4x + 1 + 4c_1} - 1}{x - c_1}}$$

$$y(x) \to \frac{1}{2}\sqrt{\frac{4x^3 - 4c_1x^2 + \sqrt{-4x + 1 + 4c_1} - 1}{x - c_1}}$$

$$y(x) \to -\sqrt{x^2}$$

$$y(x) \to \sqrt{x^2}$$

2.326 problem 903

Internal problem ID [9237]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 903.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D']]`

$$y' - \frac{\sin\left(\frac{y}{x}\right)\left(y + 2x^2 \sin\left(\frac{y}{2x}\right)\cos\left(\frac{y}{2x}\right)\right)}{2\sin\left(\frac{y}{2x}\right) x \cos\left(\frac{y}{2x}\right)} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 48

```
dsolve(diff(y(x),x) = 1/2*sin(y(x)/x)*(y(x)+2*x^2*sin(1/2*y(x)/x)*cos(1/2*y(x)/x))/sin(1/2*y
```

$$y(x) = \arctan\left(\frac{2c_1 e^x}{e^{2x}c_1^2 + 1}, \frac{-e^{2x}c_1^2 + 1}{e^{2x}c_1^2 + 1}\right) x$$

✓ Solution by Mathematica

Time used: 0.388 (sec). Leaf size: 50

```
DSolve[y'[x] == (Csc[y[x]/(2*x)]*Sec[y[x]/(2*x)]*Sin[y[x]/x]*(2*x^2*Cos[y[x]/(2*x)]*Sin[y[x]
```

$$y(x) \to -x \arccos(-\tanh(x + c_1))$$
$$y(x) \to x \arccos(-\tanh(x + c_1))$$
$$y(x) \to 0$$
$$y(x) \to -\pi x$$
$$y(x) \to \pi x$$

2.327 problem 904

Internal problem ID [9238]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 904.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘]]`

$$y' - \frac{\sin\left(\frac{y}{x}\right)\left(y + 2x^3\cos\left(\frac{y}{2x}\right)\sin\left(\frac{y}{2x}\right)\right)}{2\sin\left(\frac{y}{2x}\right)x\cos\left(\frac{y}{2x}\right)} = 0$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 53

```
dsolve(diff(y(x),x) = 1/2*sin(y(x)/x)*(y(x)+2*x^3*cos(1/2*y(x)/x)*sin(1/2*y(x)/x))/sin(1/2*y
```

$$y(x) = \arctan\left(\frac{2\,\mathrm{e}^{-\frac{x^2}{2}}c_1}{\mathrm{e}^{-x^2}+c_1^2}, \frac{-c_1^2+\mathrm{e}^{-x^2}}{\mathrm{e}^{-x^2}+c_1^2}\right)x$$

✓ Solution by Mathematica
Time used: 0.408 (sec). Leaf size: 62

```
DSolve[y'[x] == (Csc[y[x]/(2*x)]*Sec[y[x]/(2*x)]*Sin[y[x]/x]*(2*x^3*Cos[y[x]/(2*x)]*Sin[y[x]
```

$$y(x) \to -x\arccos\left(-\tanh\left(\frac{x^2}{2}+c_1\right)\right)$$
$$y(x) \to x\arccos\left(-\tanh\left(\frac{x^2}{2}+c_1\right)\right)$$
$$y(x) \to 0$$
$$y(x) \to -\pi x$$
$$y(x) \to \pi x$$

2.328 problem 905

Internal problem ID [9239]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 905.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Abel]`

$$y' - \frac{a^2x + a^3x^3 + y^2a^3x^3 + 2ya^2x^2 + ax + y^3a^3x^3 + 3y^2a^2x^2 + 3yax + 1}{a^3x^3} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 46

```
dsolve(diff(y(x),x) = (a^2*x+a^3*x^3+a^3*x^3*y(x)^2+2*a^2*x^2*y(x)+a*x+y(x)^3*a^3*x^3+3*y(x)
```

$$y(x) = \frac{29\,\mathrm{RootOf}\left(-81\left(\int^{-Z} \frac{1}{841\_a^3-27\_a+27}d\_a\right)+x+3c_1\right)ax-3ax-9}{9ax}$$

✓ Solution by Mathematica

Time used: 0.185 (sec). Leaf size: 85

```
DSolve[y'[x] == (1 + a*x + a^2*x + a^3*x^3 + 3*a*x*y[x] + 2*a^2*x^2*y[x] + 3*a^2*x^2*y[x]^2
```

$$\mathrm{Solve}\left[-\frac{29}{3}\mathrm{RootSum}\left[-29\#1^3+3\sqrt[3]{29}\#1 - 29\&, \frac{\log\left(\frac{\frac{ax+3}{ax}+3y(x)}{\sqrt[3]{29}}-\#1\right)}{\sqrt[3]{29}-29\#1^2}\&\right] = \frac{1}{9}29^{2/3}x+c_1, y(x)\right]$$

2.329 problem 906

Internal problem ID [9240]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 906.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{x(x^2 + y^2 + 1)}{-y^3 - yx^2 - y + y^6 + 3y^4x^2 + 3x^4y^2 + x^6} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 37

```
dsolve(diff(y(x),x) = x*(x^2+y(x)^2+1)/(-y(x)^3-x^2*y(x)-y(x)+y(x)^6+3*x^2*y(x)^4+3*x^4*y(x)
```

$$-\frac{1}{4\left(y\left(x\right)^2 + x^2\right)^2} - \frac{1}{2y\left(x\right)^2 + 2x^2} - y(x) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.098 (sec). Leaf size: 326

```
DSolve[y'[x] == (x*(1 + x^2 + y[x]^2))/(x^6 - y[x] - x^2*y[x] + 3*x^4*y[x]^2 - y[x]^3 + 3*x^
```

$$y(x) \to \text{Root}\left[4\#1^5 - 4\#1^4c_1 + 8\#1^3x^2 + \#1^2\left(2 - 8c_1x^2\right) + 4\#1x^4 - 4c_1x^4 + 2x^2 + 1\&, 1\right]$$
$$y(x) \to \text{Root}\left[4\#1^5 - 4\#1^4c_1 + 8\#1^3x^2 + \#1^2\left(2 - 8c_1x^2\right) + 4\#1x^4 - 4c_1x^4 + 2x^2 + 1\&, 2\right]$$
$$y(x) \to \text{Root}\left[4\#1^5 - 4\#1^4c_1 + 8\#1^3x^2 + \#1^2\left(2 - 8c_1x^2\right) + 4\#1x^4 - 4c_1x^4 + 2x^2 + 1\&, 3\right]$$
$$y(x) \to \text{Root}\left[4\#1^5 - 4\#1^4c_1 + 8\#1^3x^2 + \#1^2\left(2 - 8c_1x^2\right) + 4\#1x^4 - 4c_1x^4 + 2x^2 + 1\&, 4\right]$$
$$y(x) \to \text{Root}\left[4\#1^5 - 4\#1^4c_1 + 8\#1^3x^2 + \#1^2\left(2 - 8c_1x^2\right) + 4\#1x^4 - 4c_1x^4 + 2x^2 + 1\&, 5\right]$$

2.330 problem 907

Internal problem ID [9241]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 907.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - \frac{-2x\cos(x) + 2x^2\sin(x) + 2x + 2y^2 + 4y\cos(x)\,x - 4yx + x^2\cos(2x) + 3x^2 - 4x^2\cos(x)}{2x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x) = 1/2*(-2*cos(x)*x+2*sin(x)*x^2+2*x+2*y(x)^2+4*y(x)*cos(x)*x-4*x*y(x)+x^
```

$$y(x) = -(\cos(x) - 1)\,x + \frac{1}{-\ln(x) + c_1}$$

✓ Solution by Mathematica

Time used: 0.433 (sec). Leaf size: 32

```
DSolve[y'[x] == (x + (3*x^2)/2 - x*Cos[x] - 2*x^2*Cos[x] + (x^2*Cos[2*x])/2 + x^2*Sin[x] - 2
```

$$y(x) \to x + x(-\cos(x)) + \frac{1}{-\log(x) + c_1}$$
$$y(x) \to x - x\cos(x)$$

2.331 problem 908

Internal problem ID [9242]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 908.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{4x(a-1)(a+1)}{4y + y^4a^2 - 2y^2a^4x^2 + 4y^2a^2x^2 + a^6x^4 - 3a^4x^4 + 3a^2x^4 - y^4 - 2y^2x^2 - x^4} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 1349

```
dsolve(diff(y(x),x) = 4*x*(a-1)*(a+1)/(4*y(x)+a^2*y(x)^4-2*a^4*y(x)^2*x^2+4*y(x)^2*a^2*x^2+a
```

$$y(x) = \frac{\left(9^{\frac{2}{3}}\left(\left(3+\frac{\sqrt{-3(a-1)^5(a+1)^5x^6+6c_1^2(a-1)^4(a+1)^4x^4-3c_1(a-1)^2(a+1)^2(c_1^3a^2-c_1^3-18)x^2-6c_1^3a^2+6c_1^3+81}}{3}+\frac{(-a^2+1)c_1^3}{9}+x^2(a\right.\right.\right.}{}$$

$$y(x) = \frac{\left(\left(-\frac{i\sqrt{3}}{3}-\frac{1}{3}\right)9^{\frac{2}{3}}\left(\left(3+\frac{\sqrt{-3(a-1)^5(a+1)^5x^6+6c_1^2(a-1)^4(a+1)^4x^4-3c_1(a-1)^2(a+1)^2(c_1^3a^2-c_1^3-18)x^2-6c_1^3a^2+6c_1^3+81}}{3}+\frac{(-a^2\right.\right.\right.}{3\left(\left(\right.\right.}$$

$$y(x) = -\frac{9^{\frac{2}{3}}\left(\left(-\frac{i\sqrt{3}}{3}+\frac{1}{3}\right)9^{\frac{2}{3}}\left(\left(3+\frac{\sqrt{-3(a-1)^5(a+1)^5x^6+6c_1^2(a-1)^4(a+1)^4x^4-3c_1(a-1)^2(a+1)^2(c_1^3a^2-c_1^3-18)x^2-6c_1^3a^2+6c_1^3+81}}{3}+\right.\right.\right.}{3\left(\right.}$$

✓ Solution by Mathematica

Time used: 9.455 (sec). Leaf size: 1065

```
DSolve[y'[x] == (4*(-1 + a)*(1 + a)*x)/(-x^4 + 3*a^2*x^4 - 3*a^4*x^4 + a^6*x^4 + 4*y[x] - 2*
```

$$y(x) \to \frac{\sqrt[3]{-9a^6c_1x^2 + 27a^4c_1x^2 + 27a^4 - 27a^2c_1x^2 - 54a^2 + \frac{1}{2}\sqrt{4\left(-9a^6c_1x^2 + 27a^4\left(1 + c_1x^2\right) - 27a^2\left(2 + c_1x\right.\right.}}}{}$$

$$y(x) \to \frac{2i\left(\sqrt{3} + i\right)\sqrt[3]{-9a^6c_1x^2 + 27a^4c_1x^2 + 27a^4 - 27a^2c_1x^2 - 54a^2 + \frac{1}{2}\sqrt{4\left(-9a^6c_1x^2 + 27a^4\left(1 + c_1x^2\right) - 2\right.}}}{}$$

$$y(x) \to \frac{-2\left(1 + i\sqrt{3}\right)\sqrt[3]{-9a^6c_1x^2 + 27a^4c_1x^2 + 27a^4 - 27a^2c_1x^2 - 54a^2 + \frac{1}{2}\sqrt{4\left(-9a^6c_1x^2 + 27a^4\left(1 + c_1x^2\right) -\right.}}}{}$$

$$y(x) \to -\frac{i\sqrt{-\left(a^2 - 1\right)^3 x^2}}{a^2 - 1}$$

$$y(x) \to \frac{i\sqrt{-\left(a^2 - 1\right)^3 x^2}}{a^2 - 1}$$

2.332 problem 909

Internal problem ID [9243]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 909.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{x^3 + y^4x^3 + 2y^2x^2 + x + x^3y^6 + 3y^4x^2 + 3y^2x + 1}{x^5y} = 0$$

✓ Solution by Maple

Time used: 0.906 (sec). Leaf size: 728

```
dsolve(diff(y(x),x) = (x^3+y(x)^4*x^3+2*x^2*y(x)^2+x+x^3*y(x)^6+3*x^2*y(x)^4+3*x*y(x)^2+1)/x
```

$$y(x) = -\frac{2^{\frac{1}{3}}\sqrt{3}\,\sqrt{x\left(\frac{2^{\frac{2}{3}}\left(x^3\left(3\sqrt{3}\,\sqrt{35}-31\right)\right)^{\frac{2}{3}}}{4}-\frac{2^{\frac{1}{3}}(x+3)\left(x^3\left(3\sqrt{3}\,\sqrt{35}-31\right)\right)^{\frac{1}{3}}}{2}+x^2\right)\left(x^3\left(3\sqrt{3}\,\sqrt{35}-31\right)\right)^{\frac{1}{3}}}}{3\left(x^3\left(3\sqrt{3}\,\sqrt{35}-31\right)\right)^{\frac{1}{3}}x}$$

$$y(x) = \frac{2^{\frac{1}{3}}\sqrt{3}\,\sqrt{x\left(\frac{2^{\frac{2}{3}}\left(x^3\left(3\sqrt{3}\,\sqrt{35}-31\right)\right)^{\frac{2}{3}}}{4}-\frac{2^{\frac{1}{3}}(x+3)\left(x^3\left(3\sqrt{3}\,\sqrt{35}-31\right)\right)^{\frac{1}{3}}}{2}+x^2\right)\left(x^3\left(3\sqrt{3}\,\sqrt{35}-31\right)\right)^{\frac{1}{3}}}}{3\left(x^3\left(3\sqrt{3}\,\sqrt{35}-31\right)\right)^{\frac{1}{3}}x}$$

$$y(x) = -\frac{2^{\frac{5}{6}}\sqrt{3}\,\sqrt{4}\,\sqrt{\left(-\frac{2^{\frac{2}{3}}\left(1+i\sqrt{3}\right)\left(x^3\left(-31+3\sqrt{105}\right)\right)^{\frac{2}{3}}}{4}-2^{\frac{1}{3}}\left(x+3\right)\left(x^3\left(-31+3\sqrt{105}\right)\right)^{\frac{1}{3}}+x^2\left(i\sqrt{3}-1\right)\right)x\left(x^3 \ldots\right.}}{12\left(x^3\left(-31+3\sqrt{105}\right)\right)^{\frac{1}{3}}x}$$

$$y(x) = \frac{2^{\frac{5}{6}}\sqrt{3}\,\sqrt{4}\,\sqrt{\left(-\frac{2^{\frac{2}{3}}\left(1+i\sqrt{3}\right)\left(x^3\left(-31+3\sqrt{105}\right)\right)^{\frac{2}{3}}}{4}-2^{\frac{1}{3}}\left(x+3\right)\left(x^3\left(-31+3\sqrt{105}\right)\right)^{\frac{1}{3}}+x^2\left(i\sqrt{3}-1\right)\right)x\left(x^3 \ldots\right.}}{12\left(x^3\left(-31+3\sqrt{105}\right)\right)^{\frac{1}{3}}x}$$

$$y(x) = -\frac{2^{\frac{5}{6}}\sqrt{3}\,\sqrt{-4x\left(x^3\left(3\sqrt{3}\,\sqrt{35}-31\right)\right)^{\frac{1}{3}}\left(-\frac{2^{\frac{2}{3}}\left(i\sqrt{3}-1\right)\left(x^3\left(3\sqrt{3}\,\sqrt{35}-31\right)\right)^{\frac{2}{3}}}{4}+2^{\frac{1}{3}}\left(x+3\right)\left(x^3\left(3\sqrt{3}\,\sqrt{35}-31\right) \ldots\right.\right.}}{12\left(x^3\left(3\sqrt{3}\,\sqrt{35}-31\right)\right)^{\frac{1}{3}}x}$$

$$y(x) = \frac{2^{\frac{5}{6}}\sqrt{3}\,\sqrt{-4x\left(x^3\left(3\sqrt{3}\,\sqrt{35}-31\right)\right)^{\frac{1}{3}}\left(-\frac{2^{\frac{2}{3}}\left(i\sqrt{3}-1\right)\left(x^3\left(3\sqrt{3}\,\sqrt{35}-31\right)\right)^{\frac{2}{3}}}{4}+2^{\frac{1}{3}}\left(x+3\right)\left(x^3\left(3\sqrt{3}\,\sqrt{35}-31\right)\right)^{\frac{1}{3}} \ldots\right.}}{12\left(x^3\left(3\sqrt{3}\,\sqrt{35}-31\right)\right)^{\frac{1}{3}}x}$$

$$y(x) = \frac{\sqrt{x\left(\operatorname{RootOf}\left(\left(\int^{\_Z}\frac{1}{2\_a^3+2\_a^2+1}d\_a\right)x+c_1x+1\right)x-1\right)}}{x}$$

$$y(x) = -\frac{\sqrt{x\left(\operatorname{RootOf}\left(\left(\int^{\_Z}\frac{1}{2\_a^3+2\_a^2+1}d\_a\right)x+c_1x+1\right)x-1\right)}}{x}$$

✓ Solution by Mathematica

Time used: 0.113 (sec). Leaf size: 64

```
DSolve[y'[x] == (1 + x + x^3 + 3*x*y[x]^2 + 2*x^2*y[x]^2 + 3*x^2*y[x]^4 + x^3*y[x]^4 + x^3*y
```

$$\text{Solve}\left[\frac{1}{2}\text{RootSum}\left[2\#1^3+2\#1^2+1\&,\frac{\log\left(\frac{xy(x)^2+1}{x}-\#1\right)}{3\#1^2+2\#1}\&\right]+\frac{1}{x}+c_1=0,y(x)\right]$$

2.333 problem 910

Internal problem ID [9244]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 910.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Abel]`

$$y' - \frac{-2x - y + 1 + y^2x^2 + 2yx^3 + x^4 + y^3x^3 + 3x^4y^2 + 3x^5y + x^6}{x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(diff(y(x),x) = (-2*x-y(x)+1+x^2*y(x)^2+2*x^3*y(x)+x^4+x^3*y(x)^3+3*x^4*y(x)^2+3*x^5*y
```

$$y(x) = \frac{-9x^2 + 29\,\mathrm{RootOf}\left(-81\left(\int^{-Z} \frac{1}{841\_a^3 - 27\_a + 27} d\_a\right) + x + 3c_1\right) - 3}{9x}$$

✓ Solution by Mathematica

Time used: 1.172 (sec). Leaf size: 98

```
DSolve[y'[x] == (1 - 2*x + x^4 + x^6 - y[x] + 2*x^3*y[x] + 3*x^5*y[x] + x^2*y[x]^2 + 3*x^4*y
```

$$\mathrm{Solve}\left[-\frac{29}{3}\mathrm{RootSum}\left[-29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log\left(\frac{3x^3+3x^2y(x)+x}{\sqrt[3]{29}\sqrt[3]{x^3}} - \#1\right)}{\sqrt[3]{29} - 29\#1^2}\&\right] = \frac{29^{2/3}(x^3)^{2/3}}{9x} + c_1, y(x)\right]$$

2.334 problem 911

Internal problem ID [9245]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 911.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' + \left(-\frac{\ln(y)}{x} + \frac{\cos(x)\ln(y)}{\sin(x)} - f_1(x)\right) y = 0$$

✓ Solution by Maple

Time used: 0.218 (sec). Leaf size: 21

```
dsolve(diff(y(x),x) = -(-1/x*ln(y(x))+1/sin(x)*cos(x)*ln(y(x))-_F1(x))*y(x),y(x), singsol=al
```

$$y(x) = e^{\csc(x)x\left(c_1+\int \frac{f_1(x)\sin(x)}{x}dx\right)}$$

✓ Solution by Mathematica

Time used: 0.838 (sec). Leaf size: 105

```
DSolve[y'[x] == (F1[x] + Log[y[x]]/x - Cot[x]*Log[y[x]])*y[x],y[x],x,IncludeSingularSolution
```

$$\text{Solve}\left[\int_1^x \left(\frac{2\log(y(x))\sin(K[1])}{K[1]^2} + \frac{2(\text{F1}(K[1])\sin(K[1]) - \cos(K[1])\log(y(x)))}{K[1]}\right) dK[1] + \int_1^{y(x)} \left(-\frac{2\sin(x)}{xK[2]} - \int_1^x \left(\frac{2\sin(K[1])}{K[1]^2K[2]} - \frac{2\cos(K[1])}{K[1]K[2]}\right) dK[1]\right) dK[2] = c_1, y(x)\right]$$

2.335 problem 912

Internal problem ID [9246]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 912.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{2ax}{-yx^3 + 2a\,x^3 + 2ay^4x^3 - 16y^2a^2x^2 + 32a^3x + 2ay^6x^3 - 24y^4a^2x^2 + 96y^2a^3x - 128a^4} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) = 2*a*x/(-x^3*y(x)+2*x^3*a+2*a*y(x)^4*x^3-16*y(x)^2*a^2*x^2+32*a^3*x+2*a
```

No solution found

✓ Solution by Mathematica

Time used: 0.684 (sec). Leaf size: 201

```
DSolve[y'[x] == (2*a*x)/(-128*a^4 + 32*a^3*x + 2*a*x^3 - x^3*y[x] + 96*a^3*x*y[x]^2 - 16*a^2
```

$$\text{Solve}\Bigg[-\text{RootSum}\Big[-\#1^3y(x)^6 - \#1^3y(x)^4 - \#1^3 + 12\#1^2ay(x)^4 + 8\#1^2ay(x)^2 - 48\#1a^2y(x)^2 - 16\#1a^2 + 64a^3\&, \frac{\#1\log(x-\#1)}{3\#1^2y(x)^6 + 3\#1^2y(x)^4 + 3\#1^2 - 24\#1ay(x)^4 - 16\#1ay(x)^2 + 48a^2y(x)^2 + 16a^2}\&\Big] - \frac{\text{RootSum}\left[\#1^3 + \#1^2 + 1\&, \frac{\log(y(x)^2-\#1)}{3\#1^2+2\#1}\&\right]}{4a} + y(x) = c_1, y(x)\Bigg]$$

2.336 problem 913

Internal problem ID [9247]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 913.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class C']]`

$$y' + \frac{-y^3 - y + 2\ln(x)\, y^2 - \ln(x)^2 y^3 - 1 + 3y\ln(x) - 3y^2\ln(x)^2 + \ln(x)^3 y^3}{yx} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x) = -(-y(x)^3-y(x)+2*y(x)^2*ln(x)-ln(x)^2*y(x)^3-1+3*y(x)*ln(x)-3*ln(x)^2*
```

$$y(x) = \frac{9}{9\ln(x) + 56\,\mathrm{RootOf}\left(-81\left(\int^{\_Z} \frac{1}{3136\_a^3 - 27\_a + 27} d\_a\right) - \ln(x) + 3c_1\right) - 3}$$

✓ Solution by Mathematica

Time used: 0.529 (sec). Leaf size: 716

```
DSolve[y'[x] == (1 + y[x] - 3*Log[x]*y[x] - 2*Log[x]*y[x]^2 + 3*Log[x]^2*y[x]^2 + y[x]^3 + L
```

$$\text{Solve}\left[\int_1^{y(x)}\left(2\text{RootSum}\left[\#1^3K[1]^3-\#1^2K[1]^3-2K[1]^3-3\#1^2K[1]^2+2\#1K[1]^2+3\#1K[1]-K[1]-1\&\right.\right.\right.$$

$$-\frac{K[1]}{\log^3(x)K[1]^3-\log^2(x)K[1]^3-2K[1]^3-3\log^2(x)K[1]^2+2\log(x)K[1]^2+3\log(x)K[1]-K[1]-1}$$

$$+\frac{\text{RootSum}\left[\#1^3K[1]^3-\#1^2K[1]^3-2K[1]^3-3\#1^2K[1]^2+2\#1K[1]^2+3\#1K[1]-K[1]-1\&,\frac{-2\log(x)\,\mathrm{l}}{}\right.}{}$$

$$+y(x)^2\left(-\text{RootSum}\left[\#1^3y(x)^3-\#1^2y(x)^3-3\#1^2y(x)^2+2\#1y(x)^2+3\#1y(x)-2y(x)^3-y(x)-1\&,\frac{}{3\#1^2y}\right.\right.$$

$$\left.-\log(x)=c_1,y(x)\right]$$

2.337 problem 914

Internal problem ID [9248]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 914.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{2a(y^2x - 4a + x)}{-y^3x^3 + 4ya\,x^2 - yx^3 + 2ay^6x^3 - 24y^4a^2x^2 + 96y^2a^3x - 128a^4} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 71

```
dsolve(diff(y(x),x) = 2*a*(x*y(x)^2-4*a+x)/(-x^3*y(x)^3+4*a*x^2*y(x)-x^3*y(x)+2*a*y(x)^6*x^3
```

$$\frac{y(x)^4 x + (x - 4a)\, y(x)^2 - 2a}{2y\,(x)^4 \left(-xy\,(x)^2 + 4a\right)^2 a} + \frac{8ay(x)^5 + 1 + 2y(x)^2}{16y\,(x)^4\, a^2} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.528 (sec). Leaf size: 401

```
DSolve[y'[x] == (2*a*(-4*a + x + x*y[x]^2))/(-128*a^4 + 4*a*x^2*y[x] - x^3*y[x] + 96*a^3*x*y
```

$$y(x) \to \text{Root}\left[8\#1^5ax^2 - 8\#1^4ac_1x^2 - 64\#1^3a^2x + \#1^2\left(2x^2 + 64a^2c_1x\right) + 128\#1a^3 - 128a^3c_1 - 8ax + x^2\&, 1\right]$$
$$y(x) \to \text{Root}\left[8\#1^5ax^2 - 8\#1^4ac_1x^2 - 64\#1^3a^2x + \#1^2\left(2x^2 + 64a^2c_1x\right) + 128\#1a^3 - 128a^3c_1 - 8ax + x^2\&, 2\right]$$
$$y(x) \to \text{Root}\left[8\#1^5ax^2 - 8\#1^4ac_1x^2 - 64\#1^3a^2x + \#1^2\left(2x^2 + 64a^2c_1x\right) + 128\#1a^3 - 128a^3c_1 - 8ax + x^2\&, 3\right]$$
$$y(x) \to \text{Root}\left[8\#1^5ax^2 - 8\#1^4ac_1x^2 - 64\#1^3a^2x + \#1^2\left(2x^2 + 64a^2c_1x\right) + 128\#1a^3 - 128a^3c_1 - 8ax + x^2\&, 4\right]$$
$$y(x) \to \text{Root}\left[8\#1^5ax^2 - 8\#1^4ac_1x^2 - 64\#1^3a^2x + \#1^2\left(2x^2 + 64a^2c_1x\right) + 128\#1a^3 - 128a^3c_1 - 8ax + x^2\&, 5\right]$$

2.338 problem 915

Internal problem ID [9249]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 915.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class C']]`

$$y' + \frac{-y^3 - y + 4\ln(x) y^2 - 4\ln(x)^2 y^3 - 1 + 6y\ln(x) - 12y^2\ln(x)^2 + 8\ln(x)^3 y^3}{yx} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x) = -(-y(x)^3-y(x)+4*y(x)^2*ln(x)-4*ln(x)^2*y(x)^3-1+6*y(x)*ln(x)-12*ln(x)
```

$$y(x) = \frac{9}{18\ln(x) + 83\,\text{RootOf}\left(-81\left(\int^{\_Z} \frac{1}{6889\_a^3 - 27\_a + 27} d\_a\right) - \ln(x) + 3c_1\right) - 3}$$

✓ Solution by Mathematica

Time used: 0.606 (sec). Leaf size: 724

```
DSolve[y'[x] == (1 + y[x] - 6*Log[x]*y[x] - 4*Log[x]*y[x]^2 + 12*Log[x]^2*y[x]^2 + y[x]^3 +
```

$$\text{Solve}\left[\int_1^{y(x)} \left(4\text{RootSum}\left[8\#1^3K[1]^3-4\#1^2K[1]^3-3K[1]^3-12\#1^2K[1]^2+4\#1K[1]^2+6\#1K[1]-K[1]-\right.\right.\right.$$

$$-\frac{2K[1]}{8\log^3(x)K[1]^3-4\log^2(x)K[1]^3-3K[1]^3-12\log^2(x)K[1]^2+4\log(x)K[1]^2+6\log(x)K[1]-K[1]-1}$$

$$-\frac{2\text{RootSum}\left[8\#1^3K[1]^3-4\#1^2K[1]^3-3K[1]^3-12\#1^2K[1]^2+4\#1K[1]^2+6\#1K[1]-K[1]-1\&, \frac{16}{}\right.}{}$$

$$-2\left(y(x)^2\text{RootSum}\left[8\#1^3y(x)^3-4\#1^2y(x)^3-12\#1^2y(x)^2+4\#1y(x)^2+6\#1y(x)-3y(x)^3-y(x)-1\&, \frac{}{12\#}\right.\right.$$

$$\left.\left.+\log(x)\right)=c_1, y(x)\right]$$

2.339 problem 916

Internal problem ID [9250]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 916.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y' - \frac{y\left(\ln(y)\,x + \ln(y) - x - 1 + x\ln(x) + \ln(x) + \ln(x)^2 x^4 + 2x^4\ln(y)\ln(x) + x^4\ln(y)^2\right)}{x(x+1)} = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 73

```
dsolve(diff(y(x),x) = y(x)*(ln(y(x))*x+ln(y(x))-x-1+x*ln(x)+ln(x)+x^4*ln(x)^2+2*x^4*ln(y(x))
```

$$y(x) = \mathrm{e}^{\frac{-12\ln(x)\ln(x+1)+\left(-3x^4+4x^3-6x^2+12c_1+12x\right)\ln(x)-12x}{3x^4-4x^3+6x^2+12\ln(x+1)-12c_1-12x}}$$

✓ Solution by Mathematica
Time used: 0.628 (sec). Leaf size: 50

```
DSolve[y'[x] == ((-1 - x + Log[x] + x*Log[x] + x^4*Log[x]^2 + Log[y[x]] + x*Log[y[x]] + 2*x^
```

$$y(x) \to \frac{\exp\left(\frac{12x}{-3x^4+4x^3-6x^2+12x-12\log(x+1)+c_1}\right)}{x}$$
$$y(x) \to \frac{1}{x}$$

2.340 problem 917

Internal problem ID [9251]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 917.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y' - \frac{y\left(x\ln(x) + \ln(x) + \ln(y)\,x + \ln(y) - x - 1 + \ln(x)^{2} x + 2x\ln(y)\ln(x) + x\ln(y)^{2}\right)}{x(x+1)} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x) = y(x)*(x*ln(x)+ln(x)+ln(y(x))*x+ln(y(x))-x-1+x*ln(x)^2+2*x*ln(y(x))*ln(
```

$$y(x) = \mathrm{e}^{\frac{\ln(x)\ln(x+1)+\ln(x)(-x+c_1)-x}{-\ln(x+1)-c_1+x}}$$

✓ Solution by Mathematica
Time used: 0.437 (sec). Leaf size: 35

```
DSolve[y'[x] == ((-1 - x + Log[x] + x*Log[x] + x*Log[x]^2 + Log[y[x]] + x*Log[y[x]] + 2*x*Lo
```

$$y(x) \to \frac{e^{-\frac{x}{x-\log(x+1)-c_1}}}{x}$$

$$y(x) \to \frac{1}{x}$$

2.341 problem 918

Internal problem ID [9252]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 918.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{2y^8}{y^5 + 2y^6 + 2y^2 + 16y^4x + 32y^6x^2 + 2 + 24y^2x + 96y^4x^2 + 128x^3y^6} = 0$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 41

```
dsolve(diff(y(x),x) = 2*y(x)^8/(y(x)^5+2*y(x)^6+2*y(x)^2+16*x*y(x)^4+32*y(x)^6*x^2+2+24*x*y(
```

$$x - \text{RootOf}\left(\left(\int^{\_Z} \frac{1}{64\_a^3 + 16\_a^2 + 1} d\_a\right) y(x) + c_1 y(x) + 1\right) + \frac{1}{4y(x)^2} = 0$$

✓ Solution by Mathematica

Time used: 0.351 (sec). Leaf size: 720

```
DSolve[y'[x] == (2*y[x]^8)/(2 + 2*y[x]^2 + 24*x*y[x]^2 + 16*x*y[x]^4 + 96*x^2*y[x]^4 + y[x]^
```

$$\text{Solve}\left[\int_1^{y(x)}\left(\text{RootSum}\left[64\#1^3K[1]^6+16\#1^2K[1]^6+K[1]^6+48\#1^2K[1]^4+8\#1K[1]^4+12\#1K[1]^2+K[1]\right.\right.\right.$$

$$+\frac{K[1]^3}{2\left(64x^3K[1]^6+16x^2K[1]^6+K[1]^6+48x^2K[1]^4+8xK[1]^4+12xK[1]^2+K[1]^2+1\right)}$$

$$-\frac{\text{RootSum}\left[64\#1^3K[1]^6+16\#1^2K[1]^6+K[1]^6+48\#1^2K[1]^4+8\#1K[1]^4+12\#1K[1]^2+K[1]^2+1\&,\right.}{}$$

$$\left.+\frac{1}{K[1]^2}\right)dK[1]$$

$$-\frac{1}{4}y(x)^4\text{RootSum}\left[64\#1^3y(x)^6+16\#1^2y(x)^6+48\#1^2y(x)^4+8\#1y(x)^4+12\#1y(x)^2\right.$$

$$\left.\left.+y(x)^6+y(x)^2+1\&,\frac{\log(x-\#1)}{48\#1^2y(x)^4+8\#1y(x)^4+24\#1y(x)^2+2y(x)^2+3}\&\right]=c_1,y(x)\right]$$

2.342 problem 919

Internal problem ID [9253]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 919.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$y' - \frac{y^{\frac{3}{2}}\left(x - y + \sqrt{y}\right)}{y^{\frac{3}{2}}x - y^{\frac{5}{2}} + y^2 + x^3 - 3yx^2 + 3y^2x - y^3} = 0$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 191

```
dsolve(diff(y(x),x) = y(x)^(3/2)*(x-y(x)+y(x)^(1/2))/(y(x)^(3/2)*x-y(x)^(5/2)+y(x)^2+x^3-3*x
```

$$-\frac{\left(c_1x^6 + 80x^3 - 54x^2 - 12x - 1\right)y(x)^{\frac{7}{2}} + \left(-6c_1x^5 - 60x^2 + 36x + 6\right)y(x)^{\frac{9}{2}} + \left(15c_1x^4 + 24x - 9\right)y(x)^{\frac{11}{2}}}{}$$

$$= 0$$

✓ Solution by Mathematica

Time used: 55.594 (sec). Leaf size: 251

```
DSolve[y'[x] == ((x + Sqrt[y[x]] - y[x])*y[x]^(3/2))/(x^3 - 3*x^2*y[x] + x*y[x]^(3/2) + y[x]
```

$$\begin{aligned} y(x) \to \text{Root}\big[&\#1^9{c_1}^4 - 6\#1^8{c_1}^4x + \#1^7\left(15{c_1}^4x^2 - 6{c_1}^2\right) \\ &+ \#1^6\left(-20{c_1}^4x^3 + 30{c_1}^2x - 4 + 2{c_1}^2\right) + \#1^5\left(15{c_1}^4x^4 - 60{c_1}^2x^2 + 24x - 6{c_1}^2x + 9\right) \\ &+ \#1^4\left(-6{c_1}^4x^5 + 60{c_1}^2x^3 - 60x^2 + 6{c_1}^2x^2 - 36x - 6\right) \\ &+ \#1^3\left({c_1}^4x^6 - 30{c_1}^2x^4 + 80x^3 - 2{c_1}^2x^3 + 54x^2 + 12x + 1\right) \\ &+ \#1^2\left(6{c_1}^2x^5 - 60x^4 - 36x^3 - 6x^2\right) + \#1\left(24x^5 + 9x^4\right) - 4x^6\&, 1\big] \end{aligned}$$

2.343 problem 920

Internal problem ID [9254]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 920.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{2y^6(1 + 4y^2x + y^2)}{y^3 + 4y^5x + y^5 + 2 + 24y^2x + 96y^4x^2 + 128x^3y^6} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) = 2*y(x)^6*(1+4*x*y(x)^2+y(x)^2)/(y(x)^3+4*y(x)^5*x+y(x)^5+2+24*x*y(x)^2
```

No solution found

✓ Solution by Mathematica

Time used: 5.894 (sec). Leaf size: 301

```
DSolve[y'[x] == (2*y[x]^6*(1 + y[x]^2 + 4*x*y[x]^2))/(2 + 24*x*y[x]^2 + y[x]^3 + 96*x^2*y[x]
```

$$y(x) \to \text{Root}\left[\#1^5\left(128c_1x^2 - 8x - 1\right) + 128\#1^4x^2 + \#1^3(-2 + 64c_1x) + 64\#1^2x + 8\#1c_1 + 8\&, 1\right]$$
$$y(x) \to \text{Root}\left[\#1^5\left(128c_1x^2 - 8x - 1\right) + 128\#1^4x^2 + \#1^3(-2 + 64c_1x) + 64\#1^2x + 8\#1c_1 + 8\&, 2\right]$$
$$y(x) \to \text{Root}\left[\#1^5\left(128c_1x^2 - 8x - 1\right) + 128\#1^4x^2 + \#1^3(-2 + 64c_1x) + 64\#1^2x + 8\#1c_1 + 8\&, 3\right]$$
$$y(x) \to \text{Root}\left[\#1^5\left(128c_1x^2 - 8x - 1\right) + 128\#1^4x^2 + \#1^3(-2 + 64c_1x) + 64\#1^2x + 8\#1c_1 + 8\&, 4\right]$$
$$y(x) \to \text{Root}\left[\#1^5\left(128c_1x^2 - 8x - 1\right) + 128\#1^4x^2 + \#1^3(-2 + 64c_1x) + 64\#1^2x + 8\#1c_1 + 8\&, 5\right]$$

2.344 problem 921

Internal problem ID [9255]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 921.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' + \left(-\frac{\ln(y)}{x} + \frac{\ln(y)}{x\ln(x)} - f_1(x)\right) y = 0$$

✓ Solution by Maple

Time used: 0.281 (sec). Leaf size: 23

```
dsolve(diff(y(x),x) = -(-1/x*ln(y(x))+1/x/ln(x)*ln(y(x))-_F1(x))*y(x),y(x), singsol=all)
```

$$y(x) = e^{\frac{x\left(c_1 + \int \frac{f_1(x)\ln(x)}{x} dx\right)}{\ln(x)}}$$

✓ Solution by Mathematica

Time used: 0.306 (sec). Leaf size: 91

```
DSolve[y'[x] == (F1[x] + Log[y[x]]/x - Log[y[x]]/(x*Log[x]))*y[x],y[x],x,IncludeSingularSolu
```

$$\text{Solve}\left[\int_1^x \left(\frac{\log(y(x)) - \log(K[1])\log(y(x))}{K[1]^2} - \frac{\text{F1}(K[1])\log(K[1])}{K[1]}\right) dK[1] + \int_1^{y(x)} \left(\frac{\log(x)}{xK[2]} - \int_1^x \frac{\frac{1}{K[2]} - \frac{\log(K[1])}{K[2]}}{K[1]^2} dK[1]\right) dK[2] = c_1, y(x)\right]$$

2.345 problem 922

Internal problem ID [9256]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 922.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$y' - \frac{y^2}{y^2 + y^{\frac{3}{2}} + \sqrt{y}\,x^2 - 2y^{\frac{3}{2}}x + y^{\frac{5}{2}} + x^3 - 3yx^2 + 3y^2x - y^3} = 0$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 45

```
dsolve(diff(y(x),x) = y(x)^2/(y(x)^2+y(x)^(3/2)+y(x)^(1/2)*x^2-2*y(x)^(3/2)*x+y(x)^(5/2)+x^3
```

$$\frac{\ln(y(x))}{2} - \left(\int^{\frac{x-y(x)}{\sqrt{y(x)}}} \frac{1}{2\_a^3 + 2\_a^2 - \_a + 2} d\_a\right) - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.635 (sec). Leaf size: 882

```
DSolve[y'[x] == y[x]^2/(x^3 + x^2*Sqrt[y[x]] - 3*x^2*y[x] + y[x]^(3/2) - 2*x*y[x]^(3/2) + y[
```

$$\text{Solve}\left[\int_1^{y(x)} \left(\frac{-x-K[1]}{2\left(-2x^3+6K[1]x^2-2\sqrt{K[1]}x^2-6K[1]^2x+4K[1]^{3/2}x+K[1]x+2K[1]^3-2K[1]^{5/2}-\right.}\right.\right.$$

$$+\text{RootSum}\Big[2K[1]^3-2K[1]^{5/2}-6\#1K[1]^2-K[1]^2+4\#1K[1]^{3/2}-2K[1]^{3/2}+6\#1^2K[1]+\#1K[1]-2\#1^2\sqrt{}$$

2.346 problem 923

Internal problem ID [9257]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 923.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y' - \frac{y^2 + 2yx + x^2 + e^{-2(x-y)(x+y)}}{y^2 + 2yx + x^2 - e^{-2(x-y)(x+y)}} = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 36

```
dsolve(diff(y(x),x) = (y(x)^2+2*x*y(x)+x^2+exp(-2*(x-y(x))*(x+y(x))))/(y(x)^2+2*x*y(x)+x^2-e
```

$$y(x) = e^{\operatorname{RootOf}\left(-\_Z+\int^{e^{2\_Z}-2x\,e^{\_Z}} \frac{1}{e^{2\_a}+\_a}d\_a+c_1\right)} - x$$

✓ Solution by Mathematica

Time used: 2.663 (sec). Leaf size: 432

```
DSolve[y'[x] == (E^(-2*(x - y[x])*(x + y[x])) + x^2 + 2*x*y[x] + y[x]^2)/(-E^(-2*(x - y[x])*
```

$$\text{Solve}\left[\int_1^{y(x)}\left(-\frac{2e^{2(x-K[2])(x+K[2])}K[2]}{-e^{2(x-K[2])(x+K[2])}x^2+e^{2(x-K[2])(x+K[2])}K[2]^2+1}\right.\right.$$
$$-\int_1^x\left(-\frac{2e^{2(K[1]-K[2])(K[1]+K[2])}K[1](2(K[1]-K[2])-2(K[1]+K[2]))}{e^{2(K[1]-K[2])(K[1]+K[2])}K[1]^2-e^{2(K[1]-K[2])(K[1]+K[2])}K[2]^2-1}+\frac{2e^{2(K[1]-K[2])(K[1]+K[2])}K[1]\left(e^{2(K\right.}{}\right.$$
$$\left.+\frac{1}{x+K[2]}\right)dK[2]+\int_1^x\left(\frac{1}{K[1]+y(x)}\right.$$
$$\left.\left.-\frac{2e^{2(K[1]-y(x))(K[1]+y(x))}K[1]}{e^{2(K[1]-y(x))(K[1]+y(x))}K[1]^2-e^{2(K[1]-y(x))(K[1]+y(x))}y(x)^2-1}\right)dK[1]=c_1,y(x)\right]$$

2.347 problem 924

Internal problem ID [9258]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 924.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y' + \frac{\left(-\frac{\ln(y)^2}{2x} - f_1(x)\right) y}{\ln(y)} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 46

```
dsolve(diff(y(x),x) = -(-1/2*ln(y(x))^2/x-_F1(x))/ln(y(x))*y(x),y(x), singsol=all)
```

$$y(x) = e^{\sqrt{2}\sqrt{x\left(\int \frac{f_1(x)}{x}dx+c_1\right)}}$$
$$y(x) = e^{-\sqrt{2}\sqrt{x\left(\int \frac{f_1(x)}{x}dx+c_1\right)}}$$

✓ Solution by Mathematica

Time used: 0.302 (sec). Leaf size: 79

```
DSolve[y'[x] == ((F1[x] + Log[y[x]]^2/(2*x))*y[x])/Log[y[x]],y[x],x,IncludeSingularSolutions
```

$$\text{Solve}\left[\int_1^x \left(-\frac{\log^2(y(x))}{2K[1]^2} - \frac{\text{F1}(K[1])}{K[1]}\right) dK[1] + \int_1^{y(x)} \left(\frac{\log(K[2])}{xK[2]} - \int_1^x -\frac{\log(K[2])}{K[1]^2K[2]}dK[1]\right) dK[2] = c_1, y(x)\right]$$

2.348 problem 925

Internal problem ID [9259]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 925.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y' - \frac{y^2 + 2yx + x^2 + e^{2(x-y)^2(x+y)^2}}{y^2 + 2yx + x^2 - e^{2(x-y)^2(x+y)^2}} = 0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 38

```
dsolve(diff(y(x),x) = (y(x)^2+2*x*y(x)+x^2+exp(2*(x-y(x))^2*(x+y(x))^2))/(y(x)^2+2*x*y(x)+x^
```

$$y(x) = e^{\operatorname{RootOf}\left(-\_Z + \int^{e^{2\_Z}-2x\,e^{\_Z}} \frac{1}{e^{2\_a^2}+\_a} d\_a + c_1\right)} - x$$

✓ Solution by Mathematica

Time used: 12.399 (sec). Leaf size: 228

```
DSolve[y'[x] == (E^(2*(x - y[x])^2*(x + y[x])^2) + x^2 + 2*x*y[x] + y[x]^2)/(-E^(2*(x - y[x]
```

$$
\text{Solve}\Bigg[\int_1^{y(x)}\left(-\frac{2K[2]}{-x^2+e^{2(x-K[2])^2(x+K[2])^2}+K[2]^2}\right.
$$

$$
-\int_1^x\left(\frac{2K[1]\left(-2K[2]-e^{2(K[1]-K[2])^2(K[1]+K[2])^2}(4(K[1]-K[2])^2(K[1]+K[2])-4(K[1]-K[2])(K[1]+\right.}{\left(K[1]^2-e^{2(K[1]-K[2])^2(K[1]+K[2])^2}-K[2]^2\right)^2}\right.
$$

$$
\left.+\frac{1}{x+K[2]}\right)dK[2]
$$

$$
+\int_1^x\left(\frac{1}{K[1]+y(x)}-\frac{2K[1]}{K[1]^2-e^{2(K[1]-y(x))^2(K[1]+y(x))^2}-y(x)^2}\right)dK[1]=c_1,y(x)\Bigg]
$$

2.349 problem 926

Internal problem ID [9260]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 926.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class C']]`

$$y' - \frac{-8y^3x^2 + 16y^2x + 16xy^3 - 8 + 12yx - 6y^2x^2 + y^3x^3}{16\left(-2 + yx - 2y\right)x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 67

```
dsolve(diff(y(x),x) = 1/16*(-8*x^2*y(x)^3+16*x*y(x)^2+16*x*y(x)^3-8+12*x*y(x)-6*x^2*y(x)^2+x
```

$$y(x) = \frac{2\sqrt{c_1 + 8\ln(x)} + 8}{x\sqrt{c_1 + 8\ln(x)} + 4x - 8}$$
$$y(x) = \frac{2\sqrt{c_1 + 8\ln(x)} - 8}{x\sqrt{c_1 + 8\ln(x)} - 4x + 8}$$

✓ Solution by Mathematica

Time used: 0.553 (sec). Leaf size: 86

```
DSolve[y'[x] == (-1/2 + (3*x*y[x])/4 + x*y[x]^2 - (3*x^2*y[x]^2)/8 + x*y[x]^3 - (x^2*y[x]^3)
```

$$y(x) \to \frac{2\left(-64 + \sqrt{2048\log(x) + c_1}\right)}{128 + x\left(-64 + \sqrt{2048\log(x) + c_1}\right)}$$
$$y(x) \to \frac{2\left(64 + \sqrt{2048\log(x) + c_1}\right)}{-128 + x\left(64 + \sqrt{2048\log(x) + c_1}\right)}$$
$$y(x) \to \frac{2}{x}$$

2.350 problem 927

Internal problem ID [9261]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 927.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Abel]`

$$y' + \frac{x\left(e^{-3x^2}x^6 - 6\,e^{-2x^2}x^4y + 12x^2e^{-x^2}y^2 - 2\,e^{-2x^2}x^4 + 8x^2e^{-x^2}y + 8x^2e^{-x^2} - 8y^3 - 8y^2 - 8\,e^{-x^2} - 8\right)}{8}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x) = -1/8*(-8*exp(-x^2)+8*x^2*exp(-x^2)-8-8*y(x)^2+8*x^2*exp(-x^2)*y(x)-2*x
```

$$y(x) = \frac{e^{-x^2}x^2}{2} - \frac{1}{3} + \frac{29\,\mathrm{RootOf}\left(x^2 - 162\left(\int^{-Z} \frac{1}{841\_a^3 - 27\_a + 27} d\_a\right) + 6c_1\right)}{9}$$

✓ Solution by Mathematica

Time used: 0.417 (sec). Leaf size: 112

```
DSolve[y'[x] == -1/8*(x*(-8 - 8/E^x^2 + (8*x^2)/E^x^2 - (2*x^4)/E^(2*x^2) + x^6/E^(3*x^2) +
```

$$\mathrm{Solve}\left[-\frac{29}{3}\mathrm{RootSum}\left[-29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log\left(\frac{\frac{1}{2}e^{-x^2}x\left(2e^{x^2}-3x^2\right)+3xy(x)}{\sqrt[3]{29}\sqrt[3]{x^3}} - \#1\right)}{\sqrt[3]{29} - 29\#1^2}\&\right] = \frac{1}{18}29^{2/3}\left(x^3\right)^{2/3} + c_1, y(x)\right]$$

2.351 problem 928

Internal problem ID [9262]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 928.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' - \frac{\left(e^{-\frac{y}{x}} y x + e^{-\frac{y}{x}} y + e^{-\frac{y}{x}} x^2 + e^{-\frac{y}{x}} x + x\right) e^{\frac{y}{x}}}{x (x+1)} = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve(diff(y(x),x) = (exp(-y(x)/x)*y(x)*x+exp(-y(x)/x)*y(x)+exp(-y(x)/x)*x^2+exp(-y(x)/x)*x
```

$$y(x) = -\ln\left(\frac{-\ln(x+1) + c_1}{x}\right) x$$

✓ Solution by Mathematica
Time used: 2.195 (sec). Leaf size: 22

```
DSolve[y'[x] == (E^(y[x]/x)*(x + x/E^(y[x]/x) + x^2/E^(y[x]/x) + y[x]/E^(y[x]/x) + (x*y[x])/
```

$$y(x) \to -x \log\left(\frac{-\log(x+1) + c_1}{x}\right)$$

2.352 problem 929

Internal problem ID [9263]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 929.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class C‘]]`

$$y' + \frac{16xy^3 - 8y^3 - 8y + 8y^2x - 2y^3x^2 - 8 + 12yx - 6y^2x^2 + y^3x^3}{32yx} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(diff(y(x),x) = -1/32/y(x)*(16*x*y(x)^3-8*y(x)^3-8*y(x)+8*x*y(x)^2-2*x^2*y(x)^3-8+12*x
```

$$y(x) = \frac{18}{58 \operatorname{RootOf}\left(-324 \left(\int^{-Z} \frac{1}{841\_a^3 - 27\_a + 27} d\_a\right) - \ln(x) + 12c_1\right) + 9x - 6}$$

✓ Solution by Mathematica

Time used: 0.544 (sec). Leaf size: 683

```
DSolve[y'[x] == (1/4 + y[x]/4 - (3*x*y[x])/8 - (x*y[x]^2)/4 + (3*x^2*y[x]^2)/16 + y[x]^3/4 -
```

$$\text{Solve}\left[\int_1^{y(x)}\left(-32\text{RootSum}\left[\#1^3K[1]^3-2\#1^2K[1]^3-8K[1]^3-6\#1^2K[1]^2+8\#1K[1]^2+12\#1K[1]-8K[1\right.\right.\right.$$

$$+\frac{32K[1]}{x^3K[1]^3-2x^2K[1]^3-8K[1]^3-6x^2K[1]^2+8xK[1]^2+12xK[1]-8K[1]-8}$$

$$-\frac{8\text{RootSum}\left[\#1^3K[1]^3-2\#1^2K[1]^3-8K[1]^3-6\#1^2K[1]^2+8\#1K[1]^2+12\#1K[1]-8K[1]-8\&,\frac{-x}{}\right.}{}$$

$$+16y(x)^2\text{RootSum}\left[\#1^3y(x)^3-2\#1^2y(x)^3-6\#1^2y(x)^2+8\#1y(x)^2+12\#1y(x)-8y(x)^3\right.$$

$$\left.\left.-8y(x)-8\&,\frac{\log(x-\#1)}{3\#1^2y(x)^2-4\#1y(x)^2-12\#1y(x)+8y(x)+12}\&\right]+\log(x)=c_1,y(x)\right]$$

2.353 problem 930

Internal problem ID [9264]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 930.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' - \frac{\left(e^{-\frac{y}{x}} y x + e^{-\frac{y}{x}} y + e^{-\frac{y}{x}} x^2 + e^{-\frac{y}{x}} x + x^4\right) e^{\frac{y}{x}}}{x (x+1)} = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 41

```
dsolve(diff(y(x),x) = (exp(-y(x)/x)*y(x)*x+exp(-y(x)/x)*y(x)+exp(-y(x)/x)*x^2+exp(-y(x)/x)*x
```

$$y(x) = \left(\ln(2) + \ln(3) - \ln\left(\frac{-2x^3 + 3x^2 + 6\ln(x+1) - 6c_1 - 6x}{x}\right)\right) x$$

✓ Solution by Mathematica
Time used: 4.223 (sec). Leaf size: 38

```
DSolve[y'[x] == (E^(y[x]/x)*(x/E^(y[x]/x) + x^2/E^(y[x]/x) + x^4 + y[x]/E^(y[x]/x) + (x*y[x]
```

$$y(x) \to -x \log\left(-\frac{\frac{x^3}{3} - \frac{x^2}{2} + x - \log(x+1) + c_1}{x}\right)$$

2.354 problem 931

Internal problem ID [9265]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 931.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class C‘], [_1st_order, ‘_wit`

$$y' - \frac{-3yx^2 - 2x^3 - 2x - y^2x - y + y^3x^3 + 3x^4y^2 + 3x^5y + x^6}{x(yx + x^2 + 1)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 73

```
dsolve(diff(y(x),x) = (-3*x^2*y(x)-2*x^3-2*x-x*y(x)^2-y(x)+x^3*y(x)^3+3*x^4*y(x)^2+3*x^5*y(x
```

$$y(x) = \frac{-\sqrt{-2x + c_1}\, x^2 + x^2 + 1}{\left(\sqrt{-2x + c_1} - 1\right) x}$$
$$y(x) = \frac{-\sqrt{-2x + c_1}\, x^2 - x^2 - 1}{\left(\sqrt{-2x + c_1} + 1\right) x}$$

✓ Solution by Mathematica

Time used: 0.397 (sec). Leaf size: 60

```
DSolve[y'[x] == (-2*x - 2*x^3 + x^6 - y[x] - 3*x^2*y[x] + 3*x^5*y[x] - x*y[x]^2 + 3*x^4*y[x]
```

$$y(x) \to -x + \frac{1}{x\left(-1 + \sqrt{-2x + c_1}\right)}$$
$$y(x) \to -x - \frac{1}{x + x\sqrt{-2x + c_1}}$$
$$y(x) \to -x$$

2.355 problem 932

Internal problem ID [9266]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 932.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class C']]`

$$y' - \frac{\left(27y^3 + 27\,e^{3x^2}y + 18\,e^{3x^2}y^2 + 3y^3e^{3x^2} + 27\,e^{\frac{9x^2}{2}} + 27\,e^{\frac{9x^2}{2}}y + 9\,e^{\frac{9x^2}{2}}y^2 + e^{\frac{9x^2}{2}}y^3\right)e^{3x^2}x\,e^{-\frac{9x^2}{2}}}{243y} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve(diff(y(x),x) = 1/243*(27*y(x)^3+27*exp(3*x^2)*y(x)+18*exp(3*x^2)*y(x)^2+3*y(x)^3*exp(
```

$$y(x) = -\frac{369\,e^{\frac{3x^2}{2}}}{123 + 123\,e^{\frac{3x^2}{2}} - 136\,\mathrm{RootOf}\left(-41x^2 - 50243409\left(\int^{\_Z}\frac{1}{9248\_a^3 - 1860867\_a + 1860867}d\_a\right) + 27c_1\right)}$$

✓ Solution by Mathematica

Time used: 2.899 (sec). Leaf size: 3303

```
DSolve[y'[x] == (x*(27*E^((9*x^2)/2) + 27*E^(3*x^2)*y[x] + 27*E^((9*x^2)/2)*y[x] + 18*E^(3*x
```

Too large to display

2.356 problem 933

Internal problem ID [9267]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 933.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$y' + \frac{-x^2 - yx - x^3 - y^2x + 2yx^2\ln(x) - \ln(x)^2 x^3 - y^3 + 3xy^2\ln(x) - 3y\ln(x)^2 x^2 + \ln(x)^3 x^3}{x^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(x),x) = -(-x^2-x*y(x)-x^3-x*y(x)^2+2*y(x)*x^2*ln(x)-x^3*ln(x)^2-y(x)^3+3*x*y(x
```

$$y(x) = \frac{x\left(9\ln(x) - 3 + 29\,\mathrm{RootOf}\left(-81\left(\int^{\_Z} \frac{1}{841\_a^3 - 27\_a + 27} d\_a\right) + x + 3c_1\right)\right)}{9}$$

✓ Solution by Mathematica

Time used: 1.197 (sec). Leaf size: 99

```
DSolve[y'[x] == (x^2 + x^3 + x^3*Log[x]^2 - x^3*Log[x]^3 + x*y[x] - 2*x^2*Log[x]*y[x] + 3*x^
```

$$\text{Solve}\left[-\frac{29}{3}\text{RootSum}\left[-29\#1^3+3\sqrt[3]{29}\#1-29\&,\frac{\log\left(\frac{\frac{3y(x)}{x^2}+\frac{1-3\log(x)}{x}}{\sqrt[3]{29}\sqrt[3]{\frac{1}{x^3}}}-\#1\right)}{\sqrt[3]{29}-29\#1^2}\&\right]=\frac{29^{2/3}}{9\sqrt[3]{\frac{1}{x^3}}}+c_1,y(x)\right]$$

2.357 problem 934

Internal problem ID [9268]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 934.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Abel]`

$$y' - y^2 - \frac{yx^2}{4} + yx - y^3 + \frac{3y^2x^2}{4} + \frac{3y^2x}{2} - \frac{3yx^4}{16} - \frac{3yx^3}{4} = \frac{1}{2}x + 1 - \frac{1}{8}x^4 + \frac{1}{8}x^3 + \frac{1}{4}x^2 - \frac{1}{64}x^6 - \frac{3}{32}x^5$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve(diff(y(x),x) = 1/2*x+1+y(x)^2+1/4*x^2*y(x)-x*y(x)-1/8*x^4+1/8*x^3+1/4*x^2+y(x)^3-3/4*
```

$$y(x) = \frac{x^2}{4} + \frac{x}{2} + \text{RootOf}\left(-x + 2\left(\int^{\_Z} \frac{1}{2\_a^3 + 2\_a^2 + 1} d\_a\right) + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.216 (sec). Leaf size: 102

```
DSolve[y'[x] == 1 + x/2 + x^2/4 + x^3/8 - x^4/8 - (3*x^5)/32 - x^6/64 - x*y[x] + (x^2*y[x])/
```

$$\text{Solve}\left[-\frac{31}{3}\text{RootSum}\left[-31\#1^3 + 3\ 2^{2/3}\sqrt[3]{31}\#1 - 31\&, \frac{\log\left(\sqrt[3]{\frac{2}{31}}\left(\frac{1}{4}(-3x^2 - 6x + 4) + 3y(x)\right) - \#1\right)}{2^{2/3}\sqrt[3]{31} - 31\#1^2}\&\right] = \frac{1}{9}\left(\frac{31}{2}\right)^{2/3} x + c_1, y(x)\right]$$

2.358 problem 935

Internal problem ID [9269]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 935.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Abel]`

$$y' - y^2 - \frac{7yx^2}{2} + 2yx - y^3 - \frac{3y^2x^2}{4} + 3y^2x - \frac{3yx^4}{16} + \frac{3yx^3}{2} = -\frac{1}{2}x + 1 + \frac{13}{16}x^4 - \frac{3}{2}x^3 + x^2 + \frac{1}{64}x^6 - \frac{3}{16}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 55

```
dsolve(diff(y(x),x) = -1/2*x+1+y(x)^2+7/2*x^2*y(x)-2*x*y(x)+13/16*x^4-3/2*x^3+x^2+y(x)^3+3/4
```

$$y(x) = \frac{e^{\operatorname{RootOf}\left(\ln\left(e^{-Z}-4\right)e^{-Z}+c_1e^{-Z}-e^{-Z}\_Z+x\,e^{-Z}-4\ln\left(e^{-Z}-4\right)-4c_1+4\_Z-4x+4\right)}}{4} - 1 - \frac{x^2}{4} + x$$

✓ Solution by Mathematica

Time used: 37.054 (sec). Leaf size: 248

```
DSolve[y'[x] == 1 - x/2 + x^2 - (3*x^3)/2 + (13*x^4)/16 - (3*x^5)/16 + x^6/64 - 2*x*y[x] + (
```

$$\text{Solve}\left[\frac{\sqrt[3]{2}\left(\frac{\frac{1}{4}(3x^2-12x+4)+3y(x)}{\sqrt[3]{2}} + 2^{2/3}\right)\left(2^{2/3} - 2^{2/3}\left(\frac{1}{4}(3x^2-12x+4)+3y(x)\right)\right)\left(\left(\frac{1}{4}(-3x^2+12x-4) - 3\right.\right.}{9\left(-\left(\frac{1}{4}(3x^2-12x\right.\right.}\right.$$

$$\left.+c_1, y(x)\right]$$

2.359 problem 936

Internal problem ID [9270]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 936.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Abel]`

$$y' - y^2 - \frac{7yx^2}{16} + \frac{yx}{2} - y^3 - \frac{3y^2x^2}{8} + \frac{3y^2x}{4} - \frac{3yx^4}{64} + \frac{3yx^3}{16} = -\frac{1}{4}x + 1 + \frac{5}{128}x^4 - \frac{5}{64}x^3 + \frac{1}{16}x^2 + \frac{1}{512}x$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 39

```
dsolve(diff(y(x),x) = -1/4*x+1+y(x)^2+7/16*x^2*y(x)-1/2*x*y(x)+5/128*x^4-5/64*x^3+1/16*x^2+y
```

$$y(x) = -\frac{x^2}{8} + \frac{x}{4} + \text{RootOf}\left(-x + 4\left(\int^{\_Z} \frac{1}{4\_a^3 + 4\_a^2 + 3}d\_a\right) + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.194 (sec). Leaf size: 99

```
DSolve[y'[x] == 1 - x/4 + x^2/16 - (5*x^3)/64 + (5*x^4)/128 - (3*x^5)/256 + x^6/512 - (x*y[x
```

$$\text{Solve}\left[-\frac{89}{3}\text{RootSum}\left[-89\#1^3 + 6\sqrt[3]{178}\#1 - 89\&, \frac{\log\left(\frac{2^{2/3}\left(\frac{1}{8}(3x^2-6x+8)+3y(x)\right)}{\sqrt[3]{89}} - \#1\right)}{2\sqrt[3]{178} - 89\#1^2}\&\right] = \frac{89^{2/3}x}{18\sqrt[3]{2}} + c_1, y(x)\right]$$

2.360 problem 937

Internal problem ID [9271]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 937.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], [_Abel, '2nd type`

$$y' - \frac{-2y - 2\ln(2x+1) - 2 + 2xy^3 + y^3 + 6y^2\ln(2x+1)x + 3y^2\ln(2x+1) + 6y\ln(2x+1)^2 x + 3y\ln}{(2x+1)(y+\ln(2x+1)+1)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 79

```
dsolve(diff(y(x),x) = 1/(2*x+1)*(-2*y(x)-2*ln(2*x+1)-2+2*x*y(x)^3+y(x)^3+6*y(x)^2*ln(2*x+1)*
```

$$y(x) = \frac{-\sqrt{-2x+c_1}\,\ln(2x+1) + \ln(2x+1) + 1}{\sqrt{-2x+c_1} - 1}$$
$$y(x) = \frac{-\sqrt{-2x+c_1}\,\ln(2x+1) - \ln(2x+1) - 1}{\sqrt{-2x+c_1} + 1}$$

✓ Solution by Mathematica

Time used: 0.503 (sec). Leaf size: 69

```
DSolve[y'[x] == (-2 - 2*Log[1 + 2*x] + Log[1 + 2*x]^3 + 2*x*Log[1 + 2*x]^3 - 2*y[x] + 3*Log[
```

$$y(x) \to -\log(2x+1) + \frac{1}{-1+\sqrt{-2x+c_1}}$$
$$y(x) \to -\log(2x+1) - \frac{1}{1+\sqrt{-2x+c_1}}$$
$$y(x) \to -\log(2x+1)$$

2.361 problem 938

Internal problem ID [9272]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 938.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Abel]`

$$y' - \frac{-x^2 + x + 1 + y^2 + 5yx^2 - 2yx + 4x^4 - 3x^3 + y^3 + 3y^2x^2 - 3y^2x + 3yx^4 - 6yx^3 + x^6 - 3x^5}{x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(x),x) = (-x^2+x+1+y(x)^2+5*x^2*y(x)-2*x*y(x)+4*x^4-3*x^3+y(x)^3+3*x^2*y(x)^2-3
```

$$y(x) = -x^2 + x - \frac{1}{3} + \frac{29 \operatorname{RootOf}\left(-81\left(\int^{\_Z} \frac{1}{841\_a^3 - 27\_a + 27} d\_a\right) + \ln(x) + 3c_1\right)}{9}$$

✓ Solution by Mathematica

Time used: 0.185 (sec). Leaf size: 108

```
DSolve[y'[x] == (1 + x - x^2 - 3*x^3 + 4*x^4 - 3*x^5 + x^6 - 2*x*y[x] + 5*x^2*y[x] - 6*x^3*y
```

$$\text{Solve}\left[-\frac{29}{3}\text{RootSum}\left[-29\#1^3+3\sqrt[3]{29}\#1-29\&,\frac{\log\left(\frac{\frac{3x^2-3x+1}{x}+\frac{3y(x)}{x}}{\sqrt[3]{29}\sqrt[3]{\frac{1}{x^3}}}-\#1\right)}{\sqrt[3]{29}-29\#1^2}\&\right]=\frac{1}{9}29^{2/3}\left(\frac{1}{x^3}\right)^{2/3}x^2\log(x)+c_1,y(x)\right]$$

2.362 problem 939

Internal problem ID [9273]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 939.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, [_Abel, '2n`

$$y' - \frac{-32yx + 16x^3 + 16x^2 - 32x - 64y^3 + 48y^2x^2 + 96y^2x - 12yx^4 - 48yx^3 - 48yx^2 + x^6 + 6x^5 + 12x^4}{-64y + 16x^2 + 32x - 64}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 79

```
dsolve(diff(y(x),x) = (-32*x*y(x)+16*x^3+16*x^2-32*x-64*y(x)^3+48*x^2*y(x)^2+96*x*y(x)^2-12*
```

$$x + \frac{2\ln(2)}{5} + \frac{2\ln\left(16y(x)^2 + (-8x^2 - 16x + 16)\,y(x) + x^4 + 4x^3 - 8x + 8\right)}{5} - \frac{2\arctan\left(-2y(x) + \frac{x^2}{2} + x - 1\right)}{5} - \frac{4\ln(4y(x) - x^2 - 2x - 4)}{5} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.374 (sec). Leaf size: 136

```
DSolve[y'[x] == (-32*x + 16*x^2 + 16*x^3 + 12*x^4 + 6*x^5 + x^6 - 32*x*y[x] - 48*x^2*y[x] -
```

$$\text{Solve}\left[\frac{2}{5}\text{RootSum}\left[\#1^4 + 4\#1^3 - 8\#1^2y(x) - 16\#1y(x) - 8\#1 + 16y(x)^2 + 16y(x) + 8\&, \frac{\#1^2(-\log(x-\#1)) + 4y(x)\log(x-\#1) - 2\#1\log(x-\#1) + 3\log(x-\#1)}{-\#1^2 - 2\#1 + 4y(x) + 2}\&\right] - \frac{4}{5}\log\left(x^2 - 4y(x) + 2x + 4\right) + x = c_1, y(x)\right]$$

2.363 problem 940

Internal problem ID [9274]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 940.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class C'], [_1st_order, '_with_symmetry_`

$$y' - \frac{y\ln(x)\,x + x^2\ln(x) - 2yx - x^2 - y^2 - y^3 + 3xy^2\ln(x) - 3y\ln(x)^2 x^2 + \ln(x)^3 x^3}{x(-y + x\ln(x) - x)} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 63

```
dsolve(diff(y(x),x) = 1/x*(y(x)*ln(x)*x+x^2*ln(x)-2*x*y(x)-x^2-y(x)^2-y(x)^3+3*x*y(x)^2*ln(x
```

$$y(x) = \frac{x\left(\ln(x)\sqrt{-2x+c_1} - \ln(x) + 1\right)}{\sqrt{-2x+c_1} - 1}$$
$$y(x) = \frac{x\left(\ln(x)\sqrt{-2x+c_1} + \ln(x) - 1\right)}{\sqrt{-2x+c_1} + 1}$$

✓ Solution by Mathematica

Time used: 0.45 (sec). Leaf size: 57

```
DSolve[y'[x] == (-x^2 + x^2*Log[x] + x^3*Log[x]^3 - 2*x*y[x] + x*Log[x]*y[x] + 3*x^2*Log[x]^
```

$$y(x) \to x\left(\log(x) - \frac{1}{1+\sqrt{-2x+c_1}}\right)$$
$$y(x) \to x\left(\log(x) + \frac{1}{-1+\sqrt{-2x+c_1}}\right)$$
$$y(x) \to x\log(x)$$

2.364 problem 941

Internal problem ID [9275]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 941.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, [_Abel, '2n`

$$y' - \frac{-32yx - 72x^3 + 32x^2 - 32x + 64y^3 + 48y^2x^2 - 192y^2x + 12yx^4 - 96yx^3 + 192yx^2 + x^6 - 12x^5 + 4}{64y + 16x^2 - 64x + 64}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 35

```
dsolve(diff(y(x),x) = (-32*x*y(x)-72*x^3+32*x^2-32*x+64*y(x)^3+48*x^2*y(x)^2-192*x*y(x)^2+12
```

$$y(x) = -\frac{x^2}{4} + x + \text{RootOf}\left(-x + \int^{\_Z} \frac{\_a + 1}{\_a^3 - \_a - 1} d\_a + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.274 (sec). Leaf size: 53

```
DSolve[y'[x] == (-32*x + 32*x^2 - 72*x^3 + 48*x^4 - 12*x^5 + x^6 - 32*x*y[x] + 192*x^2*y[x]
```

$$\text{Solve}\left[x - 8\text{RootSum}\left[11776\#1^3 - 40\#1 - 1\&, \#1 \log\left(17664\#1^2 - 1472\#1 + 11x^2 + 44y(x) - 44x - 40\right)\&\right] = c_1, y(x)\right]$$

2.365 problem 942

Internal problem ID [9276]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 942.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y' + \frac{y^2 + 2yx + x^2 + \mathrm{e}^{\frac{2(x-y)^3(x+y)^3}{-y^2+x^2-1}}}{-y^2 - 2yx - x^2 + \mathrm{e}^{\frac{2(x-y)^3(x+y)^3}{-y^2+x^2-1}}} = 0$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 43

```
dsolve(diff(y(x),x) = -(y(x)^2+2*x*y(x)+x^2+exp(2*(x-y(x))^3*(x+y(x))^3/(-y(x)^2+x^2-1)))/(-
```

$$y(x) = \mathrm{e}^{\mathrm{RootOf}\left(-\_Z + \int^{\mathrm{e}^{2\_Z} - 2x\,\mathrm{e}^{\_Z}} \frac{1}{\mathrm{e}^{\frac{2\_a^3}{\_a+1}} + \_a} d\_a + c_1\right)} - x$$

✓ Solution by Mathematica

Time used: 2.976 (sec). Leaf size: 349

```
DSolve[y'[x] == (-E^((2*(x - y[x])^3*(x + y[x])^3)/(-1 + x^2 - y[x]^2)) - x^2 - 2*x*y[x] - y
```

$$\text{Solve}\left[\int_1^{y(x)}\left(-\frac{2K[2]}{-x^2+\exp\left(\frac{2(x-K[2])^3(x+K[2])^3}{x^2-K[2]^2-1}\right)+K[2]^2} - \int_1^x\left(\frac{2K[1]\left(-2K[2]-\exp\left(\frac{2(K[1]-K[2])^3(K[1]+K[2])^3}{K[1]^2-K[2]^2-1}\right)\left(\frac{6(K[1]+K[2])^2(K[1]-K[2])^3}{K[1]^2-K[2]^2-1}+\frac{4K[2](K[1]+K[2])^3(K[1]-K[2])}{(K[1]^2-K[2]^2-1)^2}\right.\right.}{\left(K[1]^2-\exp\left(\frac{2(K[1]-K[2])^3(K[1]+K[2])^3}{K[1]^2-K[2]^2-1}\right)-K[2]^2\right)^2}\right.\right.$$

$$\left.+\frac{1}{x+K[2]}\right)dK[2]$$

$$\left.+\int_1^x\left(\frac{1}{K[1]+y(x)}-\frac{2K[1]}{K[1]^2-\exp\left(\frac{2(K[1]-y(x))^3(K[1]+y(x))^3}{K[1]^2-y(x)^2-1}\right)-y(x)^2}\right)dK[1]=c_1,y(x)\right]$$

2.366 problem 943

Internal problem ID [9277]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 943.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, [_Abel, '2n`

$$y' - \frac{-128yx - 24x^3 + 32x^2 - 128x + 512y^3 + 192y^2x^2 - 384y^2x + 24yx^4 - 96yx^3 + 96yx^2 + x^6 - 6x^5}{512y + 64x^2 - 128x + 512}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 41

```
dsolve(diff(y(x),x) = (-128*x*y(x)-24*x^3+32*x^2-128*x+512*y(x)^3+192*x^2*y(x)^2-384*x*y(x)^
```

$$y(x) = -\frac{x^2}{8} + \frac{x}{4} + \operatorname{RootOf}\left(-x + 4\left(\int^{\_Z} \frac{\_a + 1}{4\_a^3 - \_a - 1} d\_a\right) + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.311 (sec). Leaf size: 53

```
DSolve[y'[x] == (-128*x + 32*x^2 - 24*x^3 + 12*x^4 - 6*x^5 + x^6 - 128*x*y[x] + 96*x^2*y[x]
```

$$\text{Solve}\left[x - 16\text{RootSum}\left[6656\#1^3 - 23\#1 - 1\&, \#1 \log\left(79872\#1^2 - 18304\#1 + 181x^2 + 1448y(x) - 362x - 184\right)\&\right] = c_1, y(x)\right]$$

2.367 problem 944

Internal problem ID [9278]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 944.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, [_Abel, '2n`

$$y' - \frac{-32yax - 8a^2x^3 - 16ab\,x^2 - 32ax + 64y^3 + 48x^2ay^2 + 96y^2bx + 12ya^2x^4 + 48ya\,x^3b + 48yb^2x^2 + a}{64y + 16a\,x^2 + 32bx + 64}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 46

```
dsolve(diff(y(x),x) = (-32*y(x)*a*x-8*a^2*x^3-16*a*x^2*b-32*a*x+64*y(x)^3+48*x^2*a*y(x)^2+96
```

$$y(x) = -\frac{a\,x^2}{4} - \frac{bx}{2} + \text{RootOf}\left(bx - 2b\left(\int^{\_Z} \frac{\_a + 1}{2\_a^3 + \_ab + b} d\_a\right) + 2c_1\right)$$

✓ Solution by Mathematica

Time used: 18.535 (sec). Leaf size: 233

```
DSolve[y'[x] == (-32*a*x - 16*a*b*x^2 - 8*a^2*x^3 + 8*b^3*x^3 + 12*a*b^2*x^4 + 6*a^2*b*x^5 +
```

$$\text{Solve}\Big[x - 4\text{RootSum}\Big[\#1^6a^3 + 6\#1^5a^2b + 12\#1^4a^2y(x) + 12\#1^4ab^2 + 48\#1^3aby(x) + 8\#1^3b^3 + 8\#1^2ab + 48\#1^2ay(x)^2 + 48\#1^2b^2y(x) + 16\#1b^2 + 96\#1by(x)^2 + 32by(x) + 32b + 64y(x)^3\&, \frac{\#1^2a\log(x - \#1) + 2\#1b\log(x - \#1) + 4y(x)\log(x - \#1) + 4\log(x - \#1)}{3\#1^4a^2 + 12\#1^3ab + 24\#1^2ay(x) + 12\#1^2b^2 + 48\#1by(x) + 8b + 48y(x)^2}\&\Big] = c_1, y(x)\Big]$$

2.368 problem 945

Internal problem ID [9279]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 945.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, [_Abel, '2n`

$$y' - \frac{-32yx - 8x^3 - 16a\,x^2 - 32x + 64y^3 + 48y^2x^2 + 96y^2ax + 12yx^4 + 48ya\,x^3 + 48ya^2x^2 + x^6 + 6x^5a}{64y + 16x^2 + 32ax + 64}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 42

```
dsolve(diff(y(x),x) = (-32*x*y(x)-8*x^3-16*a*x^2-32*x+64*y(x)^3+48*x^2*y(x)^2+96*a*x*y(x)^2+
```

$$y(x) = -\frac{x^2}{4} - \frac{ax}{2} + \operatorname{RootOf}\left(-x + 2\left(\int^{\_Z} \frac{\_a + 1}{2\_a^3 + \_aa + a} d\_a\right) + c_1\right)$$

✓ Solution by Mathematica

Time used: 18.129 (sec). Leaf size: 213

```
DSolve[y'[x] == (-32*x - 16*a*x^2 - 8*x^3 + 8*a^3*x^3 + 12*a^2*x^4 + 6*a*x^5 + x^6 - 32*x*y[
```

$$\text{Solve}\Big[x - 4\text{RootSum}\Big[\#1^6 + 6\#1^5a + 12\#1^4a^2 + 12\#1^4y(x) + 8\#1^3a^3 + 48\#1^3ay(x) + 48\#1^2a^2y(x) + 8\#1^2a + 48\#1^2y(x)^2 + 16\#1a^2 + 96\#1ay(x)^2 + 32ay(x) + 32a + 64y(x)^3\&, \frac{\#1^2\log(x-\#1) + 2\#1a\log(x-\#1) + 4y(x)\log(x-\#1) + 4\log(x-\#1)}{3\#1^4 + 12\#1^3a + 12\#1^2a^2 + 24\#1^2y(x) + 48\#1ay(x) + 8a + 48y(x)^2}\&\Big] = c_1, y(x)\Big]$$

2.369 problem 946

Internal problem ID [9280]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 946.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], [_Abel, '2nd type`

$$y' - \frac{\left(e^{-3x^2}x^6 - 6\,e^{-2x^2}x^4y + 12x^2e^{-x^2}y^2 - 4\,e^{-2x^2}x^4 + 8x^2e^{-x^2}y + 8x^2e^{-x^2} + 4x^2e^{-2x^2} - 8y^3 - 8\,e^{-x^2}y\right.}{-8y + 4x^2e^{-x^2} - 8}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 85

```
dsolve(diff(y(x),x) = (-8*exp(-x^2)*y(x)+4*x^2*exp(-x^2)^2-8*exp(-x^2)+8*x^2*exp(-x^2)*y(x)-
```

$$y(x) = \frac{2 + x^2\left(\sqrt{-x^2 + c_1} - 1\right)e^{-x^2}}{2\sqrt{-x^2 + c_1} - 2}$$
$$y(x) = \frac{-2 + x^2\left(\sqrt{-x^2 + c_1} + 1\right)e^{-x^2}}{2\sqrt{-x^2 + c_1} + 2}$$

✓ Solution by Mathematica

Time used: 1.085 (sec). Leaf size: 93

```
DSolve[y'[x] == (x*(-8/E^x^2 + (4*x^2)/E^(2*x^2) + (8*x^2)/E^x^2 - (4*x^4)/E^(2*x^2) + x^6/E
```

$$y(x) \to \frac{1}{2}e^{-x^2}x^2 + \frac{8}{-8 + \sqrt{-64x^2 + c_1}}$$
$$y(x) \to \frac{1}{2}e^{-x^2}x^2 - \frac{8}{8 + \sqrt{-64x^2 + c_1}}$$
$$y(x) \to \frac{1}{2}e^{-x^2}x^2$$

2.370 problem 947

Internal problem ID [9281]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 947.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - \frac{2x^2\cos(x) + 2\sin(x)x^3 - 2x\sin(x) + 2x + 2y^2x^2 - 4\sin(x)yx + 4y\cos(x)x^2 + 4yx + 3 - \cos(2x}{2x^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(diff(y(x),x) = 1/2*(2*x^2*cos(x)+2*sin(x)*x^3-2*x*sin(x)+2*x+2*x^2*y(x)^2-4*y(x)*sin(
```

$$y(x) = \frac{(\cos(x)x - \sin(x) + 1)\ln(x) - \cos(x)c_1x + c_1\sin(x) + x - c_1}{x(-\ln(x) + c_1)}$$

✓ Solution by Mathematica

Time used: 0.631 (sec). Leaf size: 45

```
DSolve[y'[x] == (3/2 + x + x^2/2 + 2*x*Cos[x] + x^2*Cos[x] - Cos[2*x]/2 + (x^2*Cos[2*x])/2 -
```

$$y(x) \to \frac{\sin(x) - x\cos(x) - 1}{x} + \frac{1}{-\log(x) + c_1}$$

$$y(x) \to \frac{\sin(x) - x\cos(x) - 1}{x}$$

2.371 problem 948

Internal problem ID [9282]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 948.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' + \frac{216y}{-216y^4 - 252y^3 - 396y^2 - 216y + 36x^2 - 72yx + 60y^5 - 36xy^3 - 72y^2x - 24y^4x + 4y^8 + 12y^7 + 3}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 68

```
dsolve(diff(y(x),x) = -216*y(x)/(-216*y(x)^4-252*y(x)^3-396*y(x)^2-216*y(x)+36*x^2-72*x*y(x)
```

$$y(x) = e^{\mathrm{RootOf}(12c_1 e^{4\_Z}+2\,e^{4\_Z}\_Z+18c_1 e^{3\_Z}+3\,e^{3\_Z}\_Z+36\,e^{2\_Z}c_1+6\_Z e^{2\_Z}+36c_1 e^{\_Z}+6\,e^{\_Z}\_Z-36c_1 x-6x\_Z+36)}$$

✓ Solution by Mathematica

Time used: 0.464 (sec). Leaf size: 39

```
DSolve[y'[x] == (-216*y[x])/(36*x^2 - 216*y[x] - 72*x*y[x] - 396*y[x]^2 - 72*x*y[x]^2 - 252*
```

$$\mathrm{Solve}\left[\frac{36}{y(x)\left(2y(x)^3+3y(x)^2+6y(x)+6\right)-6x}+\log(y(x))=c_1,y(x)\right]$$

2.372 problem 949

Internal problem ID [9283]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 949.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], [_Abel`

$$y' - \frac{yx^2 + x^4 + 2x^3 - 3x^2 + yx + x + y^3 + 3y^2x^2 - 3y^2x + 3yx^4 - 6yx^3 + x^6 - 3x^5}{x\left(y + x^2 - x + 1\right)} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 81

```
dsolve(diff(y(x),x) = (x^2*y(x)+x^4+2*x^3-3*x^2+x*y(x)+x+y(x)^3+3*x^2*y(x)^2-3*x*y(x)^2+3*y(
```

$$y(x) = \frac{(-x^2 + x)\sqrt{c_1 - 2\ln(x)} + x^2 - x + 1}{-1 + \sqrt{c_1 - 2\ln(x)}}$$
$$y(x) = \frac{(-x^2 + x)\sqrt{c_1 - 2\ln(x)} - x^2 + x - 1}{1 + \sqrt{c_1 - 2\ln(x)}}$$

✓ Solution by Mathematica

Time used: 0.474 (sec). Leaf size: 65

```
DSolve[y'[x] == (x - 3*x^2 + 2*x^3 + x^4 - 3*x^5 + x^6 + x*y[x] + x^2*y[x] - 6*x^3*y[x] + 3*
```

$$y(x) \to -x^2 + x + \frac{1}{-1 + \sqrt{-2\log(x) + c_1}}$$
$$y(x) \to -x^2 + x - \frac{1}{1 + \sqrt{-2\log(x) + c_1}}$$
$$y(x) \to -((x-1)x)$$

2.373 problem 950

Internal problem ID [9284]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 950.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Abel]`

$$y' - y^2 - \frac{ya\,x^2}{2} - ybx - y^3 - \frac{3x^2ay^2}{4} - \frac{3y^2bx}{2} - \frac{3ya^2x^4}{16} - \frac{3ya\,x^3b}{4} - \frac{3yb^2x^2}{4} = -\frac{1}{2}ax + 1 + \frac{1}{16}a^2x^4 +$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 42

```
dsolve(diff(y(x),x) = -1/2*a*x+1+y(x)^2+1/2*a*x^2*y(x)+b*x*y(x)+1/16*a^2*x^4+1/4*a*x^3*b+1/4
```

$$y(x) = -\frac{a\,x^2}{4} - \frac{bx}{2} + \operatorname{RootOf}\left(-x + 2\left(\int^{\_Z} \frac{1}{2\_a^3 + 2\_a^2 + b + 2}d\_a\right) + c_1\right)$$

✓ Solution by Mathematica

Time used: 6.716 (sec). Leaf size: 920

```
DSolve[y'[x] == 1 - (a*x)/2 + (b^2*x^2)/4 + (a*b*x^3)/4 + (b^3*x^3)/8 + (a^2*x^4)/16 + (3*a*
```

$$\text{Solve}\left[\frac{1}{9}\text{RootSum}\left[729b^2\#1^9 + 3132b\#1^9 + 3364\#1^9 + 2187b^2\#1^6\right.\right.$$

$$+ 9396b\#1^6 + 10092\#1^6 + 2187b^2\#1^3 + 9396b\#1^3 + 9984\#1^3 + 729b^2 + 3132b$$

$$+3364\&, \frac{729b^2\log\left(\frac{\sqrt[3]{2}\left(\frac{1}{4}(3ax^2+6bx+4)+3y(x)\right)}{\sqrt[3]{27b+58}} - \#1\right)\#1^6 + 3132b\log\left(\frac{\sqrt[3]{2}\left(\frac{1}{4}(3ax^2+6bx+4)+3y(x)\right)}{\sqrt[3]{27b+58}} - \#1\right)\#1^6}{}$$

2.374 problem 951

Internal problem ID [9285]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 951.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Abel]`

$$y' - y^2 - \frac{yx^2}{2} - yax - y^3 - \frac{3y^2x^2}{4} - \frac{3y^2ax}{2} - \frac{3yx^4}{16} - \frac{3ya\,x^3}{4} - \frac{3ya^2x^2}{4} = -\frac{1}{2}x + 1 + \frac{1}{16}x^4 + \frac{1}{4}a\,x^3 +$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 41

```
dsolve(diff(y(x),x) = -1/2*x+1+y(x)^2+1/2*x^2*y(x)+y(x)*a*x+1/16*x^4+1/4*x^3*a+1/4*a^2*x^2+y
```

$$y(x) = -\frac{x^2}{4} - \frac{ax}{2} + \mathrm{RootOf}\left(-x + 2\left(\int^{\_Z} \frac{1}{2\_a^3 + 2\_a^2 + a + 2} d\_a\right) + c_1\right)$$

✓ Solution by Mathematica

Time used: 6.585 (sec). Leaf size: 906

```
DSolve[y'[x] == 1 - x/2 + (a^2*x^2)/4 + (a*x^3)/4 + (a^3*x^3)/8 + x^4/16 + (3*a^2*x^4)/16 +
```

$$\mathrm{Solve}\left[\frac{1}{9}\mathrm{RootSum}\left[729a^2\#1^9 + 3132a\#1^9 + 3364\#1^9 + 2187a^2\#1^6\right.\right.$$

$$+ 9396a\#1^6 + 10092\#1^6 + 2187a^2\#1^3 + 9396a\#1^3 + 9984\#1^3 + 729a^2 + 3132a$$

$$+3364\&, \frac{729a^2 \log\left(\frac{\sqrt[3]{2}\left(\frac{1}{4}(3x^2+6ax+4)+3y(x)\right)}{\sqrt[3]{27a+58}} - \#1\right)\#1^6 + 3132a\log\left(\frac{\sqrt[3]{2}\left(\frac{1}{4}(3x^2+6ax+4)+3y(x)\right)}{\sqrt[3]{27a+58}} - \#1\right)\#1^6 +}{}$$

2.375 problem 952

Internal problem ID [9286]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 952.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y' + \frac{-y + \sqrt{y^2+x^2}\,x^2 - yx\sqrt{y^2+x^2} + x^4\sqrt{y^2+x^2} - x^3\sqrt{y^2+x^2}\,y + x^5\sqrt{y^2+x^2} - x^4\sqrt{y^2+x^2}\,y}{x} = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 63

```
dsolve(diff(y(x),x) = -(-y(x)+(y(x)^2+x^2)^(1/2)*x^2-x*(y(x)^2+x^2)^(1/2)*y(x)+x^4*(y(x)^2+x
```

$$\ln\left(\frac{x\left(\sqrt{2y(x)^2+2x^2}+y(x)+x\right)}{y(x)-x}\right)+\frac{(4x^5+5x^4+10x^2)\sqrt{2}}{20}-c_1+\ln(2)-\ln(x)=0$$

✓ Solution by Mathematica

Time used: 2.062 (sec). Leaf size: 120

```
DSolve[y'[x] == (y[x] - x^2*Sqrt[x^2 + y[x]^2] - x^4*Sqrt[x^2 + y[x]^2] - x^5*Sqrt[x^2 + y[x
```

$$y(x) \to \frac{x\tanh\left(\frac{4x^5+5x^4+10x^2+20c_1}{20\sqrt{2}}\right)\left(2+\sqrt{2}\tanh\left(\frac{4x^5+5x^4+10x^2+20c_1}{20\sqrt{2}}\right)\right)}{\sqrt{2}+2\tanh\left(\frac{4x^5+5x^4+10x^2+20c_1}{20\sqrt{2}}\right)}$$

$$y(x) \to x$$

2.376 problem 953

Internal problem ID [9287]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 953.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y' - \frac{y\left(\ln(x) + \ln(y) - 1 + \ln(x)^2 x + 2x\ln(y)\ln(x) + x\ln(y)^2 + \ln(x)^2 x^3 + 2x^3\ln(y)\ln(x) + x^3\ln(y\right.}{x}$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 33

```
dsolve(diff(y(x),x) = y(x)*(ln(x)+ln(y(x))-1+x*ln(x)^2+2*x*ln(y(x))*ln(x)+x*ln(y(x))^2+x^3*l
```

$$y(x) = \frac{e^{-\frac{20x}{4x^5+5x^4+10x^2+20c_1}}}{x}$$

✓ Solution by Mathematica

Time used: 0.528 (sec). Leaf size: 43

```
DSolve[y'[x] == ((-1 + Log[x] + x*Log[x]^2 + x^3*Log[x]^2 + x^4*Log[x]^2 + Log[y[x]] + 2*x*L
```

$$y(x) \to \frac{e^{-\frac{20x}{4x^5+5x^4+10x^2+20c_1}}}{x}$$

$$y(x) \to \frac{1}{x}$$

2.377 problem 954

Internal problem ID [9288]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 954.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Abel]`

$$y' - \frac{150x^3 + 125\sqrt{x} + 125 + 125y^2 - 100yx^3 - 500\sqrt{x}\,y + 20x^6 + 200x^{\frac{7}{2}} + 500x + 125y^3 - 150y^2x^3 - 7}{125x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
dsolve(diff(y(x),x) = 1/125*(150*x^3+125*x^(1/2)+125+125*y(x)^2-100*x^3*y(x)-500*y(x)*x^(1/2
```

$$y(x) = \frac{18x^{\frac{7}{2}} + 145\,\mathrm{RootOf}\left(-81\left(\int^{\_Z} \frac{1}{841\_a^3 - 27\_a + 27} d\_a\right) + \ln(x) + 3c_1\right)\sqrt{x} - 15\sqrt{x} + 90x}{45\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.254 (sec). Leaf size: 115

```
DSolve[y'[x] == (1 + Sqrt[x] + 4*x - 8*x^(3/2) + (6*x^3)/5 + (8*x^(7/2))/5 - (24*x^4)/5 + (4
```

$$\text{Solve}\left[-\frac{29}{3}\text{RootSum}\left[-29\#1^3+3\sqrt[3]{29}\#1-29\&,\frac{\log\left(\frac{\frac{-6x^3-30\sqrt{x}+5}{5x}+\frac{3y(x)}{x}}{\sqrt[3]{29}\sqrt[3]{\frac{1}{x^3}}}-\#1\right)}{\sqrt[3]{29}-29\#1^2}\&\right]=\frac{1}{9}29^{2/3}\left(\frac{1}{x^3}\right)^{2/3}x^2\log(x)+c_1,y(x)\right]$$

2.378 problem 955

Internal problem ID [9289]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 955.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class C']]`

$$y' - \frac{-150yx^3 + 60x^6 + 350x^{\frac{7}{2}} - 150x^3 - 125\sqrt{x}\,y + 250x - 125\sqrt{x} - 125y^3 + 150y^2x^3 + 750\sqrt{x}\,y^2 - 6}{25\left(-5y + 2x^3 + 10\sqrt{x} - 5\right)x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 101

```
dsolve(diff(y(x),x) = 1/25*(-150*x^3*y(x)+60*x^6+350*x^(7/2)-150*x^3-125*y(x)*x^(1/2)+250*x-
```

$$y(x) = \frac{\left(2x^3 + 10\sqrt{x}\right)\sqrt{c_1 - 2\ln(x)} - 2x^3 - 10\sqrt{x} + 5}{5\sqrt{c_1 - 2\ln(x)} - 5}$$

$$y(x) = \frac{\left(2x^3 + 10\sqrt{x}\right)\sqrt{c_1 - 2\ln(x)} + 2x^3 + 10\sqrt{x} - 5}{5\sqrt{c_1 - 2\ln(x)} + 5}$$

✓ Solution by Mathematica

Time used: 0.647 (sec). Leaf size: 92

```
DSolve[y'[x] == (-5*Sqrt[x] + 10*x + 40*x^(3/2) - 6*x^3 + 14*x^(7/2) + 24*x^4 + (12*x^6)/5 +
```

$$y(x) \to \frac{2x^3}{5} + 2\sqrt{x} - \frac{125}{125 + \sqrt{-31250\log(x) + c_1}}$$

$$y(x) \to \frac{2x^3}{5} + 2\sqrt{x} + \frac{125}{-125 + \sqrt{-31250\log(x) + c_1}}$$

$$y(x) \to \frac{2}{5}\left(x^3 + 5\sqrt{x}\right)$$

2.379 problem 956

Internal problem ID [9290]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 956.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$y' - \frac{y\left(-1 - x^{\frac{2}{\ln(x)+1}} e^{\frac{2\ln(x)^2}{\ln(x)+1}} x^2 - x^{\frac{2}{\ln(x)+1}} e^{\frac{2\ln(x)^2}{\ln(x)+1}} x^2 \ln(x) + x^{\frac{2}{\ln(x)+1}} e^{\frac{2\ln(x)^2}{\ln(x)+1}} x^2 y + 2x^{\frac{2}{\ln(x)+1}} e^{\frac{2\ln(x)^2}{\ln(x)+1}} x^2 y \ln(x) +}{(\ln(x)+1)\, x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x) = 1/(1+ln(x))*y(x)*(-1-x^(2/(1+ln(x)))*exp(2/(1+ln(x))*ln(x)^2)*x^2-x^(2
```

$$y(x) = \frac{e^{-\frac{x^4}{4}}}{(\ln(x)+1)\left(e^{-\frac{x^4}{4}} + c_1\right)}$$

✓ Solution by Mathematica

Time used: 1.904 (sec). Leaf size: 33

```
DSolve[y'[x] == (y[x]*(-1 - E^((2*Log[x]^2)/(1 + Log[x]))*x^(2 + 2/(1 + Log[x])) - E^((2*Log
```

$$y(x) \to \frac{1}{\left(1 + c_1 e^{\frac{x^4}{4}}\right)(\log(x)+1)}$$

$$y(x) \to 0$$

2.380 problem 957

Internal problem ID [9291]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 957.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$y' - \frac{y\left(-1 - x^3 x^{\frac{2}{\ln(x)+1}} e^{\frac{2\ln(x)^2}{\ln(x)+1}} - x^3 x^{\frac{2}{\ln(x)+1}} e^{\frac{2\ln(x)^2}{\ln(x)+1}} \ln(x) + x^3 x^{\frac{2}{\ln(x)+1}} e^{\frac{2\ln(x)^2}{\ln(x)+1}} y + 2x^3 x^{\frac{2}{\ln(x)+1}} e^{\frac{2\ln(x)^2}{\ln(x)+1}} y \ln(x) +\right.}{(\ln(x)+1)\, x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x) = 1/(1+ln(x))*y(x)*(-1-x^3*x^(2/(1+ln(x)))*exp(2/(1+ln(x)))*ln(x)^2)-x^3*
```

$$y(x) = \frac{e^{-\frac{x^5}{5}}}{(\ln(x)+1)\left(e^{-\frac{x^5}{5}} + c_1\right)}$$

✓ Solution by Mathematica

Time used: 1.907 (sec). Leaf size: 33

```
DSolve[y'[x] == (y[x]*(-1 - E^((2*Log[x]^2)/(1 + Log[x]))*x^(3 + 2/(1 + Log[x])) - E^((2*Log
```

$$y(x) \to \frac{1}{\left(1 + c_1 e^{\frac{x^5}{5}}\right)(\log(x)+1)}$$

$$y(x) \to 0$$

2.381 problem 958

Internal problem ID [9292]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 958.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Abel]`

$$y' - \frac{2x + 4y\ln(2x+1)\,x + 6y^2\ln(2x+1)\,x + 6y\ln(2x+1)^2\,x + 2\ln(2x+1)^3\,x + 2xy^3 + 2\ln(2x+1)}{}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(diff(y(x),x) = 1/(2*x+1)*(2*x+4*y(x)*ln(2*x+1)*x+6*y(x)^2*ln(2*x+1)*x+6*y(x)*ln(2*x+1
```

$$y(x) = -\ln(2x+1) - \frac{1}{3} + \frac{29\,\mathrm{RootOf}\left(-81\left(\int^{-Z} \frac{1}{841\_a^3-27\_a+27}d\_a\right) + x + 3c_1\right)}{9}$$

✓ Solution by Mathematica

Time used: 0.278 (sec). Leaf size: 82

```
DSolve[y'[x] == (-1 + 2*x + Log[1 + 2*x]^2 + 2*x*Log[1 + 2*x]^2 + Log[1 + 2*x]^3 + 2*x*Log[1
```

$$\mathrm{Solve}\left[-\frac{29}{3}\mathrm{RootSum}\left[-29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log\left(\frac{3y(x)+3\log(2x+1)+1}{\sqrt[3]{29}} - \#1\right)}{\sqrt[3]{29} - 29\#1^2}\&\right] = \frac{1}{9}29^{2/3}x + c_1, y(x)\right]$$

2.382 problem 959

Internal problem ID [9293]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 959.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘]]`

$$y' - \frac{-y\sin\left(\frac{y}{x}\right) + y\sin\left(\frac{3y}{2x}\right)\cos\left(\frac{y}{2x}\right) + y\cos\left(\frac{y}{2x}\right)\sin\left(\frac{y}{2x}\right) + 2\sin\left(\frac{y}{x}\right)x^3\cos\left(\frac{y}{2x}\right)\sin\left(\frac{y}{2x}\right)}{2\cos\left(\frac{y}{x}\right)\cos\left(\frac{y}{2x}\right)\sin\left(\frac{y}{2x}\right)x} = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 16

```
dsolve(diff(y(x),x) = 1/2*(-y(x)*sin(y(x)/x)+y(x)*sin(3/2*y(x)/x)*cos(1/2*y(x)/x)+y(x)*cos(1
```

$$y(x) = \frac{\arccos\left(c_1 e^{x^2} + 1\right)x}{2}$$

✓ Solution by Mathematica
Time used: 53.228 (sec). Leaf size: 25

```
DSolve[y'[x] == (Csc[y[x]/(2*x)]*Sec[y[x]/(2*x)]*Sec[y[x]/x]*(x^3*Cos[y[x]/(2*x)]*Sin[y[x]/(
```

$$y(x) \to x\arcsin\left(e^{\frac{x^2}{2}+c_1}\right)$$
$$y(x) \to 0$$

2.383 problem 960

Internal problem ID [9294]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 960.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘]]`

$$y' - \frac{-y\sin\left(\frac{y}{x}\right) + y\sin\left(\frac{3y}{2x}\right)\cos\left(\frac{y}{2x}\right) + y\cos\left(\frac{y}{2x}\right)\sin\left(\frac{y}{2x}\right) + 2\sin\left(\frac{y}{x}\right)x^2\sin\left(\frac{y}{2x}\right)\cos\left(\frac{y}{2x}\right)}{2\cos\left(\frac{y}{x}\right)\cos\left(\frac{y}{2x}\right)\sin\left(\frac{y}{2x}\right)x} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x) = 1/2*(-y(x)*sin(y(x)/x)+y(x)*sin(3/2*y(x)/x)*cos(1/2*y(x)/x)+y(x)*cos(1
```

$$y(x) = \frac{\arccos\left(e^{2x}c_1 + 1\right)x}{2}$$

✓ Solution by Mathematica
Time used: 44.005 (sec). Leaf size: 19

```
DSolve[y'[x] == (Csc[y[x]/(2*x)]*Sec[y[x]/(2*x)]*Sec[y[x]/x]*(x^2*Cos[y[x]/(2*x)]*Sin[y[x]/(
```

$$y(x) \to x\arcsin\left(e^{x+c_1}\right)$$
$$y(x) \to 0$$

2.384 problem 961

Internal problem ID [9295]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 961.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y' - \frac{y^2 + 2yx + x^2 + e^{2+2y^4-4y^2x^2+2x^4+2y^6-6y^4x^2+6x^4y^2-2x^6}}{y^2 + 2yx + x^2 - e^{2+2y^4-4y^2x^2+2x^4+2y^6-6y^4x^2+6x^4y^2-2x^6}} = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 45

```
dsolve(diff(y(x),x) = (y(x)^2+2*x*y(x)+x^2+exp(2+2*y(x)^4-4*x^2*y(x)^2+2*x^4+2*y(x)^6-6*x^2*
```

$$y(x) = e^{\operatorname{RootOf}\left(-\_Z+\int^{e^{2\_Z}-2x\,e^{\_Z}} \frac{1}{e^{2\_a^3+2\_a^2+2}+\_a}d\_a+c_1\right)} - x$$

✓ Solution by Mathematica

Time used: 29.082 (sec). Leaf size: 813

```
DSolve[y'[x] == (E^(2 + 2*x^4 - 2*x^6 - 4*x^2*y[x]^2 + 6*x^4*y[x]^2 + 2*y[x]^4 - 6*x^2*y[x]^
```

$$\text{Solve}\left[\int_1^x \left(\frac{1}{K[1]+y(x)} - \frac{2e^{2K[1]^6+6y(x)^4K[1]^2+4y(x)^2K[1]^2}K[1]}{e^{2K[1]^6+6y(x)^4K[1]^2+4y(x)^2K[1]^2}K[1]^2 - e^{2y(x)^6+2y(x)^4+6K[1]^4y(x)^2+2K[1]^4+2} - e^{2K[1]^6+6y(x)^4K[1]^2+4y(x)^2K[1]^2}y(x)^2}\right) \right.$$

$$+\int_1^{y(x)} \left(-\frac{2e^{2x^6+6K[2]^4x^2+4K[2]^2x^2}K[2]}{-e^{2x^6+6K[2]^4x^2+4K[2]^2x^2}x^2 + e^{2K[2]^6+2K[2]^4+6x^4K[2]^2+2x^4+2} + e^{2x^6+6K[2]^4x^2+4K[2]^2x^2}K[2]^2}\right.$$

$$-\int_1^x \left(-\frac{2e^{2K[1]^6+6K[2]^4K[1]^2+4K[2]^2K[1]^2}K[1]\left(24K[1]^2K[2]^3+8K[1]^2K[2]\right)}{e^{2K[1]^6+6K[2]^4K[1]^2+4K[2]^2K[1]^2}K[1]^2 - e^{2K[2]^6+2K[2]^4+6K[1]^4K[2]^2+2K[1]^4+2} - e^{2K[1]^6+6K[2]^4K[1]^2+4K[2]^2K[}}\right.$$

$$\left.\left. + \frac{1}{x+K[2]}\right) dK[2] = c_1, y(x)\right]$$

2.385 problem 962

Internal problem ID [9296]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 962.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' - \frac{4x(a-1)(a+1)(-y^2+a^2x^2-x^2-2)}{-4y^3+4ya^2x^2-4yx^2-8y-y^6a^2+3a^4y^4x^2-6y^4a^2x^2-3a^6y^2x^4+9y^2a^4x^4-9y^2a^2x^4+a^8x^6-}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 79

```
dsolve(diff(y(x),x) = 4*x*(a-1)*(a+1)*(-y(x)^2+a^2*x^2-x^2-2)/(-4*y(x)^3+4*a^2*x^2*y(x)-4*x^
```

$$-\frac{y(x)}{(a-1)(a+1)} + \frac{2}{(a^2-1)^2 (a^2x^2-x^2-y(x)^2)^2} - \frac{2}{(a^2-1)^2 (a^2x^2-x^2-y(x)^2)} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 15.969 (sec). Leaf size: 1191

```
DSolve[y'[x] == (4*(-1 + a)*(1 + a)*x*(-2 - x^2 + a^2*x^2 - y[x]^2))/(x^6 - 4*a^2*x^6 + 6*a^
```

$$y(x)\to \text{Root}\left[\#1^5(2a^2-2)+\#1^4(2a^4-4a^2+1+e^{c_1})+\#1^3(-4a^4x^2+8a^2x^2-4x^2)+\#1^2(-4a^6x^2+12a^4x^2-10a^2x^2-2a^2e^{c_1}x^2+2x^2+2e^{c_1}x^2-4)+\#1(2a^6x^4-6a^4x^4+6a^2x^4-2x^4)+2a^8x^4-8a^6x^4+11a^4x^4+a^4e^{c_1}x^4-6a^2x^4-2a^2e^{c_1}x^4+4a^2x^2+x^4+e^{c_1}x^4-4x^2-4\&,1\right]$$
$$y(x)\to \text{Root}\left[\#1^5(2a^2-2)+\#1^4(2a^4-4a^2+1+e^{c_1})+\#1^3(-4a^4x^2+8a^2x^2-4x^2)+\#1^2(-4a^6x^2+12a^4x^2-10a^2x^2-2a^2e^{c_1}x^2+2x^2+2e^{c_1}x^2-4)+\#1(2a^6x^4-6a^4x^4+6a^2x^4-2x^4)+2a^8x^4-8a^6x^4+11a^4x^4+a^4e^{c_1}x^4-6a^2x^4-2a^2e^{c_1}x^4+4a^2x^2+x^4+e^{c_1}x^4-4x^2-4\&,2\right]$$
$$y(x)\to \text{Root}\left[\#1^5(2a^2-2)+\#1^4(2a^4-4a^2+1+e^{c_1})+\#1^3(-4a^4x^2+8a^2x^2-4x^2)+\#1^2(-4a^6x^2+12a^4x^2-10a^2x^2-2a^2e^{c_1}x^2+2x^2+2e^{c_1}x^2-4)+\#1(2a^6x^4-6a^4x^4+6a^2x^4-2x^4)+2a^8x^4-8a^6x^4+11a^4x^4+a^4e^{c_1}x^4-6a^2x^4-2a^2e^{c_1}x^4+4a^2x^2+x^4+e^{c_1}x^4-4x^2-4\&,3\right]$$
$$y(x)\to \text{Root}\left[\#1^5(2a^2-2)+\#1^4(2a^4-4a^2+1+e^{c_1})+\#1^3(-4a^4x^2+8a^2x^2-4x^2)+\#1^2(-4a^6x^2+12a^4x^2-10a^2x^2-2a^2e^{c_1}x^2+2x^2+2e^{c_1}x^2-4)+\#1(2a^6x^4-6a^4x^4+6a^2x^4-2x^4)+2a^8x^4-8a^6x^4+11a^4x^4+a^4e^{c_1}x^4-6a^2x^4-2a^2e^{c_1}x^4+4a^2x^2+x^4+e^{c_1}x^4-4x^2-4\&,4\right]$$
$$y(x)\to \text{Root}\left[\#1^5(2a^2-2)+\#1^4(2a^4-4a^2+1+e^{c_1})+\#1^3(-4a^4x^2+8a^2x^2-4x^2)+\#1^2(-4a^6x^2+12a^4x^2-10a^2x^2-2a^2e^{c_1}x^2+2x^2+2e^{c_1}x^2-4)+\#1(2a^6x^4-6a^4x^4+6a^2x^4-2x^4)+2a^8x^4-8a^6x^4+11a^4x^4+a^4e^{c_1}x^4-6a^2x^4-2a^2e^{c_1}x^4+4a^2x^2+x^4+e^{c_1}x^4-4x^2-4\&,5\right]$$

2.386 problem 963

Internal problem ID [9297]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 963.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Abel]`

$$y' - \frac{-4x\cos(x) + 4x^2\sin(x) + 4x + 4 + 4y^2 + 8y\cos(x)x - 8yx + 2x^2\cos(2x) + 6x^2 - 8x^2\cos(x) + 4}{}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve(diff(y(x),x) = 1/4*(-4*cos(x)*x+4*sin(x)*x^2+4*x+4+4*y(x)^2+8*y(x)*cos(x)*x-8*x*y(x)+
```

$$y(x) = -\cos(x)\,x + x - \frac{1}{3} + \frac{29\,\mathrm{RootOf}\left(-81\left(\int^{\_Z}\frac{1}{841\_a^3-27\_a+27}d\_a\right) + \ln(x) + 3c_1\right)}{9}$$

✓ Solution by Mathematica

Time used: 0.375 (sec). Leaf size: 108

```
DSolve[y'[x] == (1 + x + (3*x^2)/2 - (5*x^3)/2 - x*Cos[x] - 2*x^2*Cos[x] + (15*x^3*Cos[x])/4
```

$$\text{Solve}\left[-\frac{29}{3}\text{RootSum}\left[-29\#1^3+3\sqrt[3]{29}\#1-29\&,\frac{\log\left(\frac{\frac{3y(x)}{x}+\frac{-3x+3x\cos(x)+1}{x}}{\sqrt[3]{29}\sqrt[3]{\frac{1}{x^3}}}-\#1\right)}{\sqrt[3]{29}-29\#1^2}\&\right]=\frac{1}{9}29^{2/3}\left(\frac{1}{x^3}\right)^{2/3}x^2\log(x)+c_1,y(x)\right]$$

2.387 problem 964

Internal problem ID [9298]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 964.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' + \frac{8x}{8 - 8y + x^6 - 8y^2a^2x^2 - 8a^2 + 2x^4 + 6a^4x^4 - 2y^4a^2 + y^6 + 3x^4y^2 + 4y^2x^2 + 2y^4 - 6a^2x^4 - 2a^6x^4 +}$$

✓ Solution by Maple

Time used: 1.469 (sec). Leaf size: 575

```
dsolve(diff(y(x),x) = -8*x*(a-1)*(a+1)/(8-a^2*y(x)^6+a^8*x^6-4*a^6*x^6+6*a^4*x^6-2*a^2*y(x)^
```

$$y(x) = -\frac{2^{\frac{5}{6}}\sqrt{3}\,\sqrt{(3a^2x^2-3x^2-2)\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{1}{3}}-\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{2}{3}}-4}}{6\left(3\sqrt{3}+\sqrt{31}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{2^{\frac{5}{6}}\sqrt{3}\,\sqrt{(3a^2x^2-3x^2-2)\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{1}{3}}-\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{2}{3}}-4}}{6\left(3\sqrt{3}+\sqrt{31}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{2^{\frac{1}{3}}\sqrt{3}\,\sqrt{(-4+(6a^2-6)\,x^2)\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{1}{3}}-i\sqrt{3}\,\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{2}{3}}+4i\sqrt{3}+\left(116+12\sqrt{3}\right.}}{6\left(3\sqrt{3}+\sqrt{31}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{2^{\frac{1}{3}}\sqrt{3}\,\sqrt{(-4+(6a^2-6)\,x^2)\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{1}{3}}-i\sqrt{3}\,\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{2}{3}}+4i\sqrt{3}+\left(116+12\sqrt{3}\,\sqrt{\right.}}}{6\left(3\sqrt{3}+\sqrt{31}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{2^{\frac{1}{3}}\sqrt{3}\,\sqrt{(-4+(6a^2-6)\,x^2)\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{2}{3}}-4i\sqrt{3}+\left(116+12\sqrt{3}\right.}}{6\left(3\sqrt{3}+\sqrt{31}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{2^{\frac{1}{3}}\sqrt{3}\,\sqrt{(-4+(6a^2-6)\,x^2)\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(116+12\sqrt{3}\,\sqrt{31}\right)^{\frac{2}{3}}-4i\sqrt{3}+\left(116+12\sqrt{3}\,\sqrt{\right.}}}{6\left(3\sqrt{3}+\sqrt{31}\right)^{\frac{1}{3}}}$$

$$\frac{4\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^3+2\_Z^2+8\right)}\frac{\ln\left(-a^2x^2+x^2+y(x)^2-\_R\right)}{\_R\left(3\_R+4\right)}\right)+(a^2-1)\,y(x)-c_1a^4+2a^2c_1-c_1}{a^4-2a^2+1} = 0$$

✓ Solution by Mathematica

Time used: 1.937 (sec). Leaf size: 264

```
DSolve[y'[x] == (-8*(-1 + a)*(1 + a)*x)/(8 - 8*a^2 + 2*x^4 - 6*a^2*x^4 + 6*a^4*x^4 - 2*a^6*x
```

$$\text{Solve}\left[\frac{y(x)}{(a-1)(a+1)}\right.$$

$$-\frac{8\text{RootSum}\left[-\#1^3a^6+3\#1^3a^4-3\#1^3a^2+\#1^3+3\#1^2a^4y(x)^2+2\#1^2a^4-6\#1^2a^2y(x)^2-4\#1^2a^2+3\right.}{}$$

2.388 problem 965

Internal problem ID [9299]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 965.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘]]`

$$y' - \frac{-y\sin\left(\frac{y}{x}\right) + y\sin\left(\frac{3y}{2x}\right)\cos\left(\frac{y}{2x}\right) + y\cos\left(\frac{y}{2x}\right)\sin\left(\frac{y}{2x}\right) + 2\sin\left(\frac{y}{x}\right)\cos\left(\frac{y}{2x}\right)\sin\left(\frac{y}{2x}\right)x + 2\sin\left(\frac{y}{x}\right)x^3\cos}{2\cos\left(\frac{y}{x}\right)\cos\left(\frac{y}{2x}\right)\sin\left(\frac{y}{2x}\right)x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(diff(y(x),x) = 1/2*(-y(x)*sin(y(x)/x)+y(x)*sin(3/2*y(x)/x)*cos(1/2*y(x)/x)+y(x)*cos(1
```

$$y(x) = \frac{\arccos\left(c_1 x^2 \mathrm{e}^{\frac{2}{3}x^3+x^2} + 1\right)x}{2}$$

✓ Solution by Mathematica

Time used: 46.578 (sec). Leaf size: 34

```
DSolve[y'[x] == (Csc[y[x]/(2*x)]*Sec[y[x]/(2*x)]*Sec[y[x]/x]*(x*Cos[y[x]/(2*x)]*Sin[y[x]/(2*
```

$$y(x) \to x \arcsin\left(x e^{\frac{x^3}{3}+\frac{x^2}{2}+c_1}\right)$$
$$y(x) \to 0$$

2.389 problem 966

Internal problem ID [9300]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 966.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' + \frac{}{216 - 1296y - 315y^9 - 36y^{11} - 8y^{12} - 126y^{10} - 432yx - 846y^7 - 570y^8 - 2376y^2 + 216x^2 + 216x^3}$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 50

```
dsolve(diff(y(x),x) = -1296*y(x)/(216+72*y(x)^8*x+216*y(x)^7*x+1080*y(x)^5*x-882*y(x)^6-216*
```

$$y(x) = \mathrm{e}^{\mathrm{RootOf}\left(-\_Z-6\left(\int^{-\frac{\mathrm{e}^{4\_Z}}{3}-\frac{\mathrm{e}^{3\_Z}}{2}-\mathrm{e}^{2\_Z}-\mathrm{e}^{\_Z}+x}\frac{1}{\_a^3+\_a^2+1}d\_a\right)+c_1\right)}$$

✓ Solution by Mathematica

Time used: 0.51 (sec). Leaf size: 292

```
DSolve[y'[x] == (-1296*y[x])/(216 + 216*x^2 + 216*x^3 - 1296*y[x] - 432*x*y[x] - 648*x^2*y[x
```

$$\mathrm{Solve}\Big[72\mathrm{RootSum}\Big[-216\#1^3 + 216\#1^2y(x)^4 + 324\#1^2y(x)^3 + 648\#1^2y(x)^2$$
$$+ 648\#1^2y(x) - 216\#1^2 - 72\#1y(x)^8 - 216\#1y(x)^7 - 594\#1y(x)^6$$
$$- 1080\#1y(x)^5 - 1152\#1y(x)^4 - 1080\#1y(x)^3 - 216\#1y(x)^2$$
$$+ 432\#1y(x) + 8y(x)^{12} + 36y(x)^{11} + 126y(x)^{10} + 315y(x)^9 + 570y(x)^8$$
$$+ 846y(x)^7 + 882y(x)^6 + 612y(x)^5 + 216y(x)^4 - 216y(x)^3 - 216y(x)^2$$
$$-216\&, \frac{\log(x - \#1)}{36\#1^2 - 24\#1y(x)^4 - 36\#1y(x)^3 - 72\#1y(x)^2 - 72\#1y(x) + 24\#1 + 4y(x)^8 + 12y(x)^7 + 33y(x}$$
$$+ \log(y(x)) = c_1, y(x)\Big]$$

2.390 problem 967

Internal problem ID [9301]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 967.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Abel]`

$$y' + \frac{x(-513 - 432x - 378y - 456x^6 - 96x^8 - 864x^4 - 144x^7 + 64x^9 - 540y^2 - 1134x^2 - 756x^3 - 576x}{}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 91

```
dsolve(diff(y(x),x) = -1/216*x/(x^2+1)^4*(-513-432*x-288*y(x)*x^8+288*y(x)*x^7-288*y(x)*x^6+
```

$$y(x) = \frac{58\,\mathrm{RootOf}\left(-162\left(\int^{-Z} \frac{1}{841\_a^3 - 27\_a + 27} d\_a\right) + \ln\left(x^2+1\right) + 6c_1\right)x^2 + 12x^3 - 6x^2 + 58\,\mathrm{RootOf}\left(-162\right.}{18x^2+18}$$

✓ Solution by Mathematica

Time used: 1.396 (sec). Leaf size: 151

```
DSolve[y'[x] == -1/216*(x*(-513 - 432*x - 1134*x^2 - 756*x^3 - 864*x^4 - 576*x^5 - 456*x^6 -
```

$$\text{Solve}\left[-\frac{29}{3}\text{RootSum}\left[-29\#1^3+3\sqrt[3]{29}\#1-29\&,\frac{\log\left(\frac{\frac{3xy(x)}{x^2+1}+\frac{-4x^4+2x^3+5x}{2(x^2+1)^2}}{\sqrt[3]{29}\sqrt[3]{\frac{x^3}{(x^2+1)^3}}}-\#1\right)}{\sqrt[3]{29}-29\#1^2}\&\right]=\frac{29^{2/3}\left(\frac{x^3}{(x^2+1)^3}\right)^{2/3}(x^2+1)^2\log(x^2+1)}{18x^2}+c_1,y(x)\right]$$

2.391 problem 968

Internal problem ID [9302]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 968.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D']]`

$$y' - \frac{-\sin\left(\frac{y}{x}\right) yx - y\sin\left(\frac{y}{x}\right) + y\sin\left(\frac{3y}{2x}\right)\cos\left(\frac{y}{2x}\right) x + y\sin\left(\frac{3y}{2x}\right)\cos\left(\frac{y}{2x}\right) + y\cos\left(\frac{y}{2x}\right)\sin\left(\frac{y}{2x}\right) x + y\cos\left(\right.}{2\cos\left(\frac{y}{x}\right)\cos\left(\frac{y}{2x}\right)\sin\left(\frac{y}{2x}\right) x\,(x+1)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x) = 1/2*(-sin(y(x)/x)*y(x)*x-y(x)*sin(y(x)/x)+y(x)*sin(3/2*y(x)/x)*cos(1/2
```

$$y(x) = \frac{\arccos\left(1 + c_1(x+1)^2\, \mathrm{e}^{x(x-2)}\right) x}{2}$$

✓ Solution by Mathematica
Time used: 30.849 (sec). Leaf size: 35

```
DSolve[y'[x] == (Csc[y[x]/(2*x)]*Sec[y[x]/(2*x)]*Sec[y[x]/x]*(x^4*Cos[y[x]/(2*x)]*Sin[y[x]/(
```

$$y(x) \to x \arcsin\left((x+1)e^{\frac{x^2}{2}-x-\frac{3}{2}+c_1}\right)$$
$$y(x) \to 0$$

2.392 problem 969

Internal problem ID [9303]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 969.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘]]`

$$y' - \frac{y\sin\left(\frac{3y}{2x}\right)\cos\left(\frac{y}{2x}\right)x + y\sin\left(\frac{3y}{2x}\right)\cos\left(\frac{y}{2x}\right) + y\cos\left(\frac{y}{2x}\right)\sin\left(\frac{y}{2x}\right)x + y\cos\left(\frac{y}{2x}\right)\sin\left(\frac{y}{2x}\right) - \sin\left(\frac{y}{x}\right)yx -}{2\cos\left(\frac{y}{x}\right)\sin\left(\frac{y}{2x}\right)x\cos\left(\frac{y}{2x}\right)(x+1)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 26

```
dsolve(diff(y(x),x) = 1/2*(y(x)*sin(3/2*y(x)/x)*cos(1/2*y(x)/x)*x+y(x)*sin(3/2*y(x)/x)*cos(1
```

$$y(x) = \frac{\arccos\left(\frac{1+(c_1+1)x^2+2x}{(x+1)^2}\right)x}{2}$$

✓ Solution by Mathematica
Time used: 20.682 (sec). Leaf size: 24

```
DSolve[y'[x] == (Csc[y[x]/(2*x)]*Sec[y[x]/(2*x)]*Sec[y[x]/x]*(x*Cos[y[x]/(2*x)]*Sin[y[x]/(2*
```

$$y(x) \to x\arcsin\left(\frac{e^{c_1}x}{x+1}\right)$$
$$y(x) \to 0$$

2.393 problem 970

Internal problem ID [9304]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 970.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y' + \frac{}{-1296y - 315y^9 - 36y^{11} - 8y^{12} - 126y^{10} - 1296yx + 594y^7 - 18y^8 - 1296y^2 + 216x^3 + 2484y^6 - }$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 181

```
dsolve(diff(y(x),x) = -216*y(x)*(-2*y(x)^4-3*y(x)^3-6*y(x)^2-6*y(x)+6*x+6)/(72*y(x)^8*x+216*
```

$$\frac{-6\sqrt{3\ln(y(x)) - 108c_1 + 9} + \left(2y(x)^4 + 3y(x)^3 + 6y(x)^2 - 6x + 6y(x)\right)\ln(y(x)) - 72y(x)^4 c_1 - 108c_1 y(}{216c_1 - 6\ln(y(x))}$$
$$= 0$$
$$\frac{6\sqrt{3\ln(y(x)) - 108c_1 + 9} + \left(2y(x)^4 + 3y(x)^3 + 6y(x)^2 - 6x + 6y(x)\right)\ln(y(x)) - 72y(x)^4 c_1 - 108c_1 y(x)}{216c_1 - 6\ln(y(x))}$$
$$= 0$$

✓ Solution by Mathematica

Time used: 0.451 (sec). Leaf size: 66

```
DSolve[y'[x] == (-216*y[x]*(6 + 6*x - 6*y[x] - 6*y[x]^2 - 3*y[x]^3 - 2*y[x]^4))/(216*x^3 - 1
```

$$\text{Solve}\left[\frac{36(2y(x)^4 + 3y(x)^3 + 6y(x)^2 + 6y(x) - 6x - 3)}{(y(x)(2y(x)^3 + 3y(x)^2 + 6y(x) + 6) - 6x)^2} + \log(y(x)) = c_1, y(x)\right]$$

2.394 problem 971

Internal problem ID [9305]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 971.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Abel]`

$$y' - \frac{(yx+1)^3}{x^5} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 70

```
dsolve(diff(y(x),x) = (x*y(x)+1)^3/x^5,y(x), singsol=all)
```

$$y(x) = \frac{-2 + x^3\left(\tan\left(\text{RootOf}\left(18x^3\left(-\frac{1}{x^6}\right)^{\frac{2}{3}} + 6\_Z\sqrt{3} - 3\ln(3) + \ln\left(\left(\sqrt{3}\sin(\_Z) + 3\cos(\_Z)\right)^6\right) - 18c_1\right)\right)\right.}{2x}$$

✓ Solution by Mathematica

Time used: 0.23 (sec). Leaf size: 157

```
DSolve[y'[x] == (1 + x*y[x])^3/x^5,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\arctan\left(\frac{\frac{2\left(\frac{3}{x^3}+\frac{3y(x)}{x^2}\right)}{3\sqrt[3]{-\frac{1}{x^6}}}-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log\left(\frac{\frac{3}{x^3}+\frac{3y(x)}{x^2}}{3\sqrt[3]{-\frac{1}{x^6}}}+1\right)-\frac{1}{6}\log\left(\frac{\left(\frac{3}{x^3}+\frac{3y(x)}{x^2}\right)^2}{9\left(-\frac{1}{x^6}\right)^{2/3}}-\frac{\frac{3}{x^3}+\frac{3y(x)}{x^2}}{3\sqrt[3]{-\frac{1}{x^6}}}+1\right)=-\left(-\frac{1}{x^6}\right)^{2/3}x^3+c_1,y(x)\right]$$

2.395 problem 972

Internal problem ID [9306]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 972.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], [_Abel`

$$\boxed{y' - \frac{x(-x^2 + 2yx^2 - 2x^4 + 1)}{y - x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x) = x*(-x^2+2*x^2*y(x)-2*x^4+1)/(y(x)-x^2),y(x), singsol=all)
```

$$y(x) = x^2 + \frac{\text{LambertW}\left(-2c_1 e^{x^4-2x^2-1}\right)}{2} + \frac{1}{2}$$

✓ Solution by Mathematica

Time used: 3.262 (sec). Leaf size: 43

```
DSolve[y'[x] == (x*(1 - x^2 - 2*x^4 + 2*x^2*y[x]))/(-x^2 + y[x]),y[x],x,IncludeSingularSolut
```

$$y(x) \to x^2 + \frac{1}{2}\left(1 + W\left(-e^{x^4-2x^2-1+c_1}\right)\right)$$
$$y(x) \to x^2 + \frac{1}{2}$$

2.396 problem 973

Internal problem ID [9307]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 973.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Abel]`

$$y' - y\left(y^2 + y\,e^{bx} + e^{2bx}\right) e^{-2bx} = 0$$

✓ Solution by Maple

Time used: 0.437 (sec). Leaf size: 126

```
dsolve(diff(y(x),x) = y(x)*(y(x)^2+y(x)*exp(b*x)+exp(b*x)^2)/exp(b*x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{\tan\left(\operatorname{RootOf}\left(-2\_Z e^{bx} + \sqrt{-e^{2bx}\left(-3+4b\right)}\,\ln\left(\frac{\left(\tan\left(\_Z\right)^2+1\right)\left(-3+4b\right)}{\left(\tan\left(\_Z\right)\sqrt{-e^{2bx}\left(-3+4b\right)}+e^{bx}\right)^2}\right) + \sqrt{-e^{2bx}\left(-3+4b\right.}\right.\right.}{2}$$

$$-\frac{e^{bx}}{2}$$

✓ Solution by Mathematica

Time used: 0.995 (sec). Leaf size: 1225

```
DSolve[y'[x] == (y[x]*(E^(2*b*x) + E^(b*x)*y[x] + y[x]^2))/E^(2*b*x),y[x],x,IncludeSingularS
```

$$\text{Solve}\left[\frac{-2\sqrt{3}\sqrt[3]{7-9b}\left(\sqrt[3]{7-9b}+\sqrt[3]{9b-7}\right)\arctan\left(\frac{\frac{2\sqrt[3]{7-9b}\left(3e^{-2bx}y(x)+e^{-bx}\right)}{\sqrt[3]{(9b-7)e^{-3bx}}}-1}{\sqrt{3}}\right)-2\sqrt[3]{7-9b}\left(\sqrt[3]{7-9b}\right.}{}\right.$$

2.397 problem 974

Internal problem ID [9308]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 974.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Abel]`

$$\boxed{y' - y^3 + 3y^2x^2 - 3yx^4 = -x^6 + 2x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 57

```
dsolve(diff(y(x),x) = y(x)^3-3*x^2*y(x)^2+3*y(x)*x^4-x^6+2*x,y(x), singsol=all)
```

$$y(x) = \frac{x^2\sqrt{-2x+2c_1} - 1}{\sqrt{-2x+2c_1}}$$
$$y(x) = \frac{x^2\sqrt{-2x+2c_1} + 1}{\sqrt{-2x+2c_1}}$$

✓ Solution by Mathematica

Time used: 0.204 (sec). Leaf size: 46

```
DSolve[y'[x] == 2*x - x^6 + 3*x^4*y[x] - 3*x^2*y[x]^2 + y[x]^3,y[x],x,IncludeSingularSolutio
```

$$y(x) \to x^2 - \frac{1}{\sqrt{-2x+c_1}}$$
$$y(x) \to x^2 + \frac{1}{\sqrt{-2x+c_1}}$$
$$y(x) \to x^2$$

2.398 problem 975

Internal problem ID [9309]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 975.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Abel]`

$$y' - y^3 - y^2x^2 - \frac{yx^4}{3} = \frac{1}{27}x^6 - \frac{2}{3}x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(diff(y(x),x) = y(x)^3+x^2*y(x)^2+1/3*y(x)*x^4+1/27*x^6-2/3*x,y(x), singsol=all)
```

$$y(x) = -\frac{x^2\sqrt{-54c_1 - 2x} - 3}{3\sqrt{-54c_1 - 2x}}$$
$$y(x) = -\frac{x^2\sqrt{-54c_1 - 2x} + 3}{3\sqrt{-54c_1 - 2x}}$$

✓ Solution by Mathematica

Time used: 0.193 (sec). Leaf size: 58

```
DSolve[y'[x] == (-2*x)/3 + x^6/27 + (x^4*y[x])/3 + x^2*y[x]^2 + y[x]^3,y[x],x,IncludeSingula
```

$$y(x) \to -\frac{x^2}{3} - \frac{1}{\sqrt{-2x + c_1}}$$
$$y(x) \to -\frac{x^2}{3} + \frac{1}{\sqrt{-2x + c_1}}$$
$$y(x) \to -\frac{x^2}{3}$$

2.399 problem 976

Internal problem ID [9310]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 976.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Abel]`

$$y' - \frac{y(y^2x^7 + yx^4 + x - 3)}{x} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 70

```
dsolve(diff(y(x),x) = y(x)/x*(y(x)^2*x^7+y(x)*x^4+x-3),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{3}\tan\left(\operatorname{RootOf}\left(-\sqrt{3}\ln(3) - \sqrt{3}\ln\left(-\frac{1}{-2+\sqrt{3}\sin(2\_Z)+\cos(2\_Z)}\right) + \sqrt{3}\ln(7) + 3\sqrt{3}c_1 - 2\sqrt{3}x -\right.\right.}{2x^3}$$

✓ Solution by Mathematica

Time used: 1.135 (sec). Leaf size: 101

```
DSolve[y'[x] == (y[x]*(-3 + x + x^4*y[x] + x^7*y[x]^2))/x,y[x],x,IncludeSingularSolutions ->
```

$$\text{Solve}\left[-\frac{7}{3}\text{RootSum}\left[-7\#1^3 + 6\sqrt[3]{-7}\#1 - 7\&, \frac{\log\left(\frac{3x^6y(x)+x^3}{\sqrt[3]{7}\sqrt[3]{-x^9}} - \#1\right)}{2\sqrt[3]{-7} - 7\#1^2}\&\right] = \frac{7^{2/3}(-x^9)^{2/3}}{9x^5} + c_1, y(x)\right]$$

2.400 problem 977

Internal problem ID [9311]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 977.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]'], _Abel]`

$$y' - y\left(y^2 + \mathrm{e}^{-x^2}y + \mathrm{e}^{-2x^2}\right)\mathrm{e}^{2x^2}x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 85

```
dsolve(diff(y(x),x) = y(x)*(y(x)^2+exp(-x^2)*y(x)+exp(-x^2)^2)/exp(-x^2)^2*x,y(x), singsol=a
```

$$y(x) = \frac{\left(\sqrt{11}\tan\left(\mathrm{RootOf}\left(-4\sqrt{11}\,x^2 - 8\sqrt{11}\ln(11) - 4\sqrt{11}\ln\left(\sec(\_Z)^2\mathrm{e}^{2x^2}\right) + 8\sqrt{11}\ln(5) + 8\sqrt{11}\ln(\right.\right.\right.}{2}$$

✓ Solution by Mathematica

Time used: 0.288 (sec). Leaf size: 139

```
DSolve[y'[x] == E^(2*x^2)*x*y[x]*(E^(-2*x^2) + y[x]/E^x^2 + y[x]^2),y[x],x,IncludeSingularSo
```

$$\mathrm{Solve}\left[-\frac{25}{3}\mathrm{RootSum}\left[-25\#1^3 + 24\sqrt[3]{-1}5^{2/3}\#1 - 25\&, \frac{\log\left(\frac{3e^{2x^2}xy(x)+e^{x^2}x}{5^{2/3}\sqrt[3]{-e^{3x^2}x^3}} - \#1\right)}{8\sqrt[3]{-1}5^{2/3} - 25\#1^2}\&\right] = -\frac{5\sqrt[3]{5}e^{x^2}x^3}{18\sqrt[3]{-e^{3x^2}x^3}} + c_1, y(x)\right]$$

2.401 problem 978

Internal problem ID [9312]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 978.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Abel]`

$$\boxed{y' - \frac{y(y^2 + yx + x^2 + x)}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.407 (sec). Leaf size: 64

```
dsolve(diff(y(x),x) = y(x)/x^2*(y(x)^2+x*y(x)+x^2+x),y(x), singsol=all)
```

$$y(x) = \frac{x\left(\sqrt{3}\tan\left(\text{RootOf}\left(2\sqrt{3}\ln(3) - \sqrt{3}\ln\left(\cos(\_Z)^2\right) - 2\sqrt{3}\ln\left(-\sqrt{3} + 3\tan(\_Z)\right) + 2\sqrt{3}\,c_1 + 2\sqrt{3}\,x +\right.\right.\right.}{2}$$

✓ Solution by Mathematica

Time used: 0.141 (sec). Leaf size: 60

```
DSolve[y'[x] == (y[x]*(x + x^2 + x*y[x] + y[x]^2))/x^2,y[x],x,IncludeSingularSolutions -> Tr
```

$$\text{Solve}\left[-\frac{\arctan\left(\frac{\frac{2y(x)}{x}+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{2}\log\left(\frac{y(x)^2}{x^2} + \frac{y(x)}{x} + 1\right) + \log\left(\frac{y(x)}{x}\right) = x + c_1, y(x)\right]$$

2.402 problem 979

Internal problem ID [9313]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 979.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Abel]`

$$\boxed{y' - \frac{y^3 - 3y^2x + 3yx^2 - x^3 + x}{x} = 0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 49

```
dsolve(diff(y(x),x) = (y(x)^3-3*x*y(x)^2+3*x^2*y(x)-x^3+x)/x,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{c_1 - 2\ln(x)}\, x - 1}{\sqrt{c_1 - 2\ln(x)}}$$
$$y(x) = \frac{\sqrt{c_1 - 2\ln(x)}\, x + 1}{\sqrt{c_1 - 2\ln(x)}}$$

✓ Solution by Mathematica
Time used: 0.228 (sec). Leaf size: 42

```
DSolve[y'[x] == (x - x^3 + 3*x^2*y[x] - 3*x*y[x]^2 + y[x]^3)/x,y[x],x,IncludeSingularSolutio
```

$$y(x) \to x - \frac{1}{\sqrt{-2\log(x) + c_1}}$$
$$y(x) \to x + \frac{1}{\sqrt{-2\log(x) + c_1}}$$
$$y(x) \to x$$

2.403 problem 980

Internal problem ID [9314]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 980.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Abel]`

$$y' - \frac{x^3y^3 + 6x^2y^2 + 12xy + 8 + 2x}{x^3} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x) = (x^3*y(x)^3+6*x^2*y(x)^2+12*x*y(x)+8+2*x)/x^3,y(x), singsol=all)
```

$$y(x) = -\frac{1}{\sqrt{-2x + c_1}} - \frac{2}{x}$$
$$y(x) = \frac{1}{\sqrt{-2x + c_1}} - \frac{2}{x}$$

✓ Solution by Mathematica

Time used: 0.237 (sec). Leaf size: 53

```
DSolve[y'[x] == (8 + 2*x + 12*x*y[x] + 6*x^2*y[x]^2 + x^3*y[x]^3)/x^3,y[x],x,IncludeSingular
```

$$y(x) \to -\frac{2 + \frac{x}{\sqrt{-2x+c_1}}}{x}$$
$$y(x) \to -\frac{2}{x} + \frac{1}{\sqrt{-2x + c_1}}$$
$$y(x) \to -\frac{2}{x}$$

2.404 problem 981

Internal problem ID [9315]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 981.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Abel]`

$$y' - \frac{y^3a^3x^3 + 3y^2a^2x^2 + 3yax + 1 + a^2x}{x^3a^3} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 41

```
dsolve(diff(y(x),x) = (y(x)^3*a^3*x^3+3*y(x)^2*a^2*x^2+3*y(x)*a*x+1+a^2*x)/x^3/a^3,y(x), sin
```

$$y(x) = -\frac{1}{\sqrt{-2x + c_1}} - \frac{1}{ax}$$
$$y(x) = \frac{1}{\sqrt{-2x + c_1}} - \frac{1}{ax}$$

✓ Solution by Mathematica
Time used: 0.29 (sec). Leaf size: 61

```
DSolve[y'[x] == (1 + a^2*x + 3*a*x*y[x] + 3*a^2*x^2*y[x]^2 + a^3*x^3*y[x]^3)/(a^3*x^3),y[x],
```

$$y(x) \to -\frac{1}{ax} - \frac{1}{\sqrt{-2x + c_1}}$$
$$y(x) \to -\frac{1}{ax} + \frac{1}{\sqrt{-2x + c_1}}$$
$$y(x) \to -\frac{1}{ax}$$

2.405 problem 982

Internal problem ID [9316]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 982.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$y' - \frac{y\,e^{-\frac{x^2}{2}}\left(2y^2 + 2y\,e^{\frac{x^2}{4}} + 2\,e^{\frac{x^2}{2}} + x\,e^{\frac{x^2}{2}}\right)}{2} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 76

```
dsolve(diff(y(x),x) = 1/2*y(x)/exp(1/4*x^2)^2*(2*y(x)^2+2*y(x)*exp(1/4*x^2)+2*exp(1/4*x^2)^2
```

$$-\frac{\ln(7)}{3} + \frac{\ln\left(1 + y(x)^2\,e^{-\frac{x^2}{2}} + y(x)\,e^{-\frac{x^2}{4}}\right)}{3} + \frac{2\sqrt{3}\,\arctan\left(\frac{2y(x)\sqrt{3}\,e^{-\frac{x^2}{4}}}{3} + \frac{\sqrt{3}}{3}\right)}{9} - \frac{2\ln\left(y(x)\,e^{-\frac{x^2}{4}}\right)}{3} + \frac{2x}{3} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.308 (sec). Leaf size: 132

```
DSolve[y'[x] == (y[x]*(2*E^(x^2/2) + E^(x^2/2)*x + 2*E^(x^2/4)*y[x] + 2*y[x]^2))/(2*E^(x^2/2
```

$$\text{Solve}\left[-\frac{7}{3}\text{RootSum}\left[-7\#1^3+6\sqrt[3]{-7}\#1-7\&,\frac{\log\left(\frac{3e^{-\frac{x^2}{2}}y(x)+e^{-\frac{x^2}{4}}}{\sqrt[3]{7}\sqrt[3]{-e^{-\frac{3x^2}{4}}}}-\#1\right)}{2\sqrt[3]{-7}-7\#1^2}\&\right]=\frac{1}{9}7^{2/3}e^{\frac{x^2}{2}}\left(-e^{-\frac{3x^2}{4}}\right)^{2/3}x+c_1,y(x)\right]$$

2.406 problem 983

Internal problem ID [9317]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 983.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Abel]`

$$\boxed{y' - \frac{y^3 - 3y^2x + 3x^2y - x^3 + x^2}{(x-1)(x+1)} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 168

```
dsolve(diff(y(x),x) = (y(x)^3-3*x*y(x)^2+3*x^2*y(x)-x^3+x^2)/(x-1)/(x+1),y(x), singsol=all)
```

$$y(x) = \frac{(x^2-1)\left(\sqrt{3}\tan\left(\operatorname{RootOf}\left(9\ln\left(\frac{x+1}{x-1}\right)\left(\frac{1}{(x+1)^3(x-1)^3}\right)^{\frac{2}{3}}x^4 - 18\ln\left(\frac{x+1}{x-1}\right)\left(\frac{1}{(x+1)^3(x-1)^3}\right)^{\frac{2}{3}}x^2 + 9\ln\left(\frac{x+1}{x-1}\right)\right.\right.\right.}{+x}$$

✓ Solution by Mathematica

Time used: 1.545 (sec). Leaf size: 238

```
DSolve[y'[x] == (x^2 - x^3 + 3*x^2*y[x] - 3*x*y[x]^2 + y[x]^3)/((-1 + x)*(1 + x)),y[x],x,Inc
```

$$\text{Solve}\left[\frac{\arctan\left(\frac{\frac{2\left(\frac{3y(x)}{x^2-1}-\frac{3x}{x^2-1}\right)}{3\sqrt[3]{\frac{1}{(x-1)^3(x+1)^3}}}-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log\left(\frac{\frac{3y(x)}{x^2-1}-\frac{3x}{x^2-1}}{3\sqrt[3]{\frac{1}{(x-1)^3(x+1)^3}}}+1\right)\right.$$

$$-\frac{1}{6}\log\left(\frac{\left(\frac{3y(x)}{x^2-1}-\frac{3x}{x^2-1}\right)^2}{9\left(\frac{1}{(x-1)^3(x+1)^3}\right)^{2/3}}-\frac{\frac{3y(x)}{x^2-1}-\frac{3x}{x^2-1}}{3\sqrt[3]{\frac{1}{(x-1)^3(x+1)^3}}}\right.$$

$$\left.\left.+1\right)=\frac{1}{2}\left(\frac{1}{(x^2-1)^3}\right)^{2/3}(x^2-1)^2(\log(1-x)-\log(x+1))+c_1,y(x)\right]$$

2.407 problem 984

Internal problem ID [9318]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 984.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]'], _Abel]`

$$\boxed{y' - \frac{y(x^2y^2 + yx\,e^x + e^{2x})\,e^{-2x}(x-1)}{x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
dsolve(diff(y(x),x) = y(x)/x*(x^2*y(x)^2+y(x)*x*exp(x)+exp(x)^2)/exp(x)^2*(x-1),y(x), singso
```

$$y(x) = \frac{e^{\mathrm{RootOf}\left(e^{\_Z}\ln(2) - e^{\_Z}\ln\left(\left(e^{\_Z}+9\right)x\right) + 3c_1 e^{\_Z} + e^{\_Z}\_Z + x\,e^{\_Z} + 9\right) + x}}{9x}$$

✓ Solution by Mathematica

Time used: 7.806 (sec). Leaf size: 428

```
DSolve[y'[x] == ((-1 + x)*y[x]*(E^(2*x) + E^x*x*y[x] + x^2*y[x]^2))/(E^(2*x)*x),y[x],x,Inclu
```

$$\text{Solve}\left[\frac{\sqrt[3]{2}\left(\frac{3e^{-2x}x(x-1)y(x)+e^{-x}(x-1)}{\sqrt[3]{2}\sqrt[3]{e^{-3x}(x-1)^3}} + 2^{2/3}\right)\left(2^{2/3} - \frac{2^{2/3}(3e^{-2x}x(x-1)y(x)+e^{-x}(x-1))}{\sqrt[3]{e^{-3x}(x-1)^3}}\right)\left(\left(1 - \frac{3e^{-2x}x(x-1)y(x)+e^{-x}}{\sqrt[3]{e^{-3x}(x-1)}}\right.\right.}{9\left(-\frac{e^{3x}(3e^{-2x}x(x-1)}{(x-}\right.}\right.$$

$$\left. + c_1, y(x)\right]$$

2.408 problem 985

Internal problem ID [9319]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 985.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Abel]`

$$y' - \frac{(xy+1)\,(x^2y^2 + x^2y + 2xy + 1 + x + x^2)}{x^5} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x) = (x*y(x)+1)*(x^2*y(x)^2+x^2*y(x)+2*x*y(x)+1+x+x^2)/x^5,y(x), singsol=al
```

$$y(x) = \frac{17\,\mathrm{RootOf}\left(162\left(\int^{\_Z} \frac{1}{289\_a^3+54\_a-54}d\_a\right)x + 3c_1x + 2\right)x - 3x - 9}{9x}$$

✓ Solution by Mathematica

Time used: 0.257 (sec). Leaf size: 103

```
DSolve[y'[x] == ((1 + x*y[x])*(1 + x + x^2 + 2*x*y[x] + x^2*y[x] + x^2*y[x]^2))/x^5,y[x],x,I
```

$$\text{Solve}\left[-\frac{17}{3}\text{RootSum}\left[-17\#1^3+3\sqrt[3]{-34}\#1-17\&,\frac{\log\left(\frac{\frac{x+3}{x^3}+\frac{3y(x)}{x^2}}{\sqrt[3]{34}\sqrt[3]{-\frac{1}{x^6}}}-\#1\right)}{\sqrt[3]{-34}-17\#1^2}\&\right]=-\frac{1}{9}34^{2/3}\left(-\frac{1}{x^6}\right)^{2/3}x^3+c_1,y(x)\right]$$

2.409 problem 986

Internal problem ID [9320]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 986.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$y' - \frac{y^3 - 3xy^2 \ln(x) + 3x^2 \ln(x)^2 y - x^3 \ln(x)^3 + x^2 + xy}{x^2} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
dsolve(diff(y(x),x) = (y(x)^3-3*x*y(x)^2*ln(x)+3*x^2*ln(x)^2*y(x)-x^3*ln(x)^3+x^2+x*y(x))/x^
```

$$y(x) = x\left(-\frac{1}{\sqrt{-2x + c_1}} + \ln(x)\right)$$
$$y(x) = x\left(\frac{1}{\sqrt{-2x + c_1}} + \ln(x)\right)$$

✓ Solution by Mathematica

Time used: 0.25 (sec). Leaf size: 49

```
DSolve[y'[x] == (x^2 - x^3*Log[x]^3 + x*y[x] + 3*x^2*Log[x]^2*y[x] - 3*x*Log[x]*y[x]^2 + y[x
```

$$y(x) \to x\left(\log(x) - \frac{1}{\sqrt{-2x + c_1}}\right)$$
$$y(x) \to x\left(\log(x) + \frac{1}{\sqrt{-2x + c_1}}\right)$$
$$y(x) \to x \log(x)$$

2.410 problem 987

Internal problem ID [9321]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 987.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _Riccati]`

$$y' + F(x) \left(-a\,x^2 + y^2\right) - \frac{y}{x} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 22

```
dsolve(diff(y(x),x) = -F(x)*(-a*x^2+y(x)^2)+y(x)/x,y(x), singsol=all)
```

$$y(x) = \tanh\left(\left(\int F(x)\,xdx + c_1\right)\sqrt{a}\right)x\sqrt{a}$$

✓ Solution by Mathematica

Time used: 4.668 (sec). Leaf size: 35

```
DSolve[y'[x] == y[x]/x - F[x]*(-(a*x^2) + y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{a}x \tanh\left(\sqrt{a}\left(\int_1^x F(K[1])K[1]dK[1] + c_1\right)\right)$$

2.411 problem 988

Internal problem ID [9322]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 988.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _Riccati]`

$$\boxed{y' + F(x)\left(-x^2 - 2xy + y^2\right) - \frac{y}{x} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(diff(y(x),x) = -F(x)*(-x^2-2*x*y(x)+y(x)^2)+y(x)/x,y(x), singsol=all)
```

$$y(x) = \frac{x\left(\sqrt{2} + 2\tanh\left(\left(\int F(x)\,x dx + c_1\right)\sqrt{2}\right)\right)\sqrt{2}}{2}$$

✓ Solution by Mathematica

Time used: 0.6 (sec). Leaf size: 106

```
DSolve[y'[x] == y[x]/x - F[x]*(-x^2 - 2*x*y[x] + y[x]^2),y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{x\left(-\left(\sqrt{2}-1\right)\exp\left(2\sqrt{2}\left(\int_1^x -F(K[1])K[1]dK[1] + c_1\right)\right) + 1 + \sqrt{2}\right)}{1 + \exp\left(2\sqrt{2}\left(\int_1^x -F(K[1])K[1]dK[1] + c_1\right)\right)}$$

$$y(x) \to \left(1 + \sqrt{2}\right)x$$

$$y(x) \to x - \sqrt{2}x$$

2.412 problem 989

Internal problem ID [9323]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 989.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _Riccati]`

$$\boxed{y' + F(x)\left(-a y^2 - b\,x^2\right) - \frac{y}{x} = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 29

```
dsolve(diff(y(x),x) = -F(x)*(-a*y(x)^2-b*x^2)+y(x)/x,y(x), singsol=all)
```

$$y(x) = \frac{\tan\left(\left(\int F(x)\,x dx + c_1\right)\sqrt{ab}\right) x\sqrt{ab}}{a}$$

✓ Solution by Mathematica

Time used: 0.289 (sec). Leaf size: 45

```
DSolve[y'[x] == y[x]/x - F[x]*(-(b*x^2) - a*y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{b}x\tan\left(\sqrt{a}\sqrt{b}\left(\int_1^x F(K[1])K[1]dK[1] + c_1\right)\right)}{\sqrt{a}}$$

2.413 problem 990

Internal problem ID [9324]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 990.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, `_with_symmetry_[F(x),G(x)]`], _Riccati]`

$$\boxed{y' + F(x)\left(-y^2 + 2x^2y + 1 - x^4\right) = 2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(diff(y(x),x) = -F(x)*(-y(x)^2+2*x^2*y(x)+1-x^4)+2*x,y(x), singsol=all)
```

$$y(x) = \frac{-x^2 e^{2\left(\int F(x)dx\right)} + c_1 x^2 + e^{2\left(\int F(x)dx\right)} + c_1}{-e^{2\left(\int F(x)dx\right)} + c_1}$$

✓ Solution by Mathematica

Time used: 0.297 (sec). Leaf size: 67

```
DSolve[y'[x] == 2*x - F[x]*(1 - x^4 + 2*x^2*y[x] - y[x]^2),y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{\exp\left(\int_1^x 2F(K[5])dK[5]\right)}{-\int_1^x \exp\left(\int_1^{K[6]} 2F(K[5])dK[5]\right) F(K[6])dK[6] + c_1} + x^2 + 1$$

$$y(x) \to x^2 + 1$$

2.414 problem 991

Internal problem ID [9325]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 991.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _Riccati]`

$$\boxed{y' + F(x)\left(x^2 + 2xy - y^2\right) - \frac{y}{x} = 0}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 29

```
dsolve(diff(y(x),x) = -F(x)*(x^2+2*x*y(x)-y(x)^2)+y(x)/x,y(x), singsol=all)
```

$$y(x) = \frac{x\left(\sqrt{2} - 2\tanh\left(\left(\int F(x)\, x dx + c_1\right)\sqrt{2}\right)\right)\sqrt{2}}{2}$$

✓ Solution by Mathematica
Time used: 0.56 (sec). Leaf size: 104

```
DSolve[y'[x] == y[x]/x - F[x]*(x^2 + 2*x*y[x] - y[x]^2),y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{x\left(-\left(\sqrt{2}-1\right)\exp\left(2\sqrt{2}\left(\int_1^x F(K[1])K[1]dK[1] + c_1\right)\right) + 1 + \sqrt{2}\right)}{1 + \exp\left(2\sqrt{2}\left(\int_1^x F(K[1])K[1]dK[1] + c_1\right)\right)}$$

$$y(x) \to \left(1 + \sqrt{2}\right)x$$

$$y(x) \to x - \sqrt{2}x$$

2.415 problem 992

Internal problem ID [9326]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 992.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _Riccati]`

$$\boxed{y' + F(x)\left(-7y^2x - x^3\right) - \frac{y}{x} = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 25

```
dsolve(diff(y(x),x) = -F(x)*(-7*x*y(x)^2-x^3)+y(x)/x,y(x), singsol=all)
```

$$y(x) = \frac{\tan\left(\left(\int F(x)\,x^2dx + c_1\right)\sqrt{7}\right)x\sqrt{7}}{7}$$

✓ Solution by Mathematica
Time used: 0.244 (sec). Leaf size: 37

```
DSolve[y'[x] == y[x]/x - F[x]*(-x^3 - 7*x*y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x\tan\left(\sqrt{7}\left(\int_1^x F(K[1])K[1]^2dK[1] + c_1\right)\right)}{\sqrt{7}}$$

2.416 problem 993

Internal problem ID [9327]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 993.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + F(x)\left(-y^2 - 2y\ln(x) - \ln(x)^2\right) - \frac{y}{\ln(x)\,x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x) = -F(x)*(-y(x)^2-2*y(x)*ln(x)-ln(x)^2)+1/ln(x)/x*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{\ln(x)\left(2\left(\int \ln(x)\,F(x)\,dx\right) + c_1 + 2\right)}{2\left(\int \ln(x)\,F(x)\,dx\right) + c_1}$$

✓ Solution by Mathematica

Time used: 3.065 (sec). Leaf size: 75

```
DSolve[y'[x] == y[x]/(x*Log[x]) - F[x]*(-Log[x]^2 - 2*Log[x]*y[x] - y[x]^2),y[x],x,IncludeSi
```

$$y(x) \to \frac{\int_1^x \frac{F(K[1])}{\sqrt{\frac{1}{\log^2(K[1])}}}dK[1] - 1 + c_1}{\sqrt{\frac{1}{\log^2(x)}}\left(\int_1^x \frac{F(K[1])}{\sqrt{\frac{1}{\log^2(K[1])}}}dK[1] + c_1\right)}$$

$$y(x) \to \frac{1}{\sqrt{\frac{1}{\log^2(x)}}}$$

2.417 problem 994

Internal problem ID [9328]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 994.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + x^3\left(-y^2 - 2y\ln(x) - \ln(x)^2\right) - \frac{y}{\ln(x)\,x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x) = -x^3*(-y(x)^2-2*y(x)*ln(x)-ln(x)^2)+1/ln(x)/x*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{\ln(x)\left(4x^4\ln(x) - x^4 + 8c_1 + 16\right)}{4x^4\ln(x) - x^4 + 8c_1}$$

✓ Solution by Mathematica

Time used: 0.371 (sec). Leaf size: 52

```
DSolve[y'[x] == y[x]/(x*Log[x]) - x^3*(-Log[x]^2 - 2*Log[x]*y[x] - y[x]^2),y[x],x,IncludeSin
```

$$y(x) \to \frac{\log(x)\left(x^4 - 4x^4\log(x) - 16(1 + c_1)\right)}{-x^4 + 4x^4\log(x) + 16c_1}$$
$$y(x) \to -\log(x)$$

2.418 problem 995

Internal problem ID [9329]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 995.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$\boxed{y' - (y - e^x)^2 = e^x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x) = (y(x)-exp(x))^2+exp(x),y(x), singsol=all)
```

$$y(x) = \frac{-1 + (x - c_1) e^x}{x - c_1}$$

✓ Solution by Mathematica

Time used: 0.307 (sec). Leaf size: 24

```
DSolve[y'[x] == E^x + (-E^x + y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x + \frac{1}{-x + c_1}$$
$$y(x) \to e^x$$

2.419 problem 996

Internal problem ID [9330]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 996.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$\boxed{y' - \frac{(y - \operatorname{Si}(x))^2 + \sin(x)}{x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x) = ((y(x)-Si(x))^2+sin(x))/x,y(x), singsol=all)
```

$$y(x) = \operatorname{Si}(x) + \frac{1}{-\ln(x) + c_1}$$

✓ Solution by Mathematica

Time used: 0.215 (sec). Leaf size: 23

```
DSolve[y'[x] == (Sin[x] + (-SinIntegral[x] + y[x])^2)/x,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \operatorname{Si}(x) + \frac{1}{-\log(x) + c_1}$$
$$y(x) \to \operatorname{Si}(x)$$

2.420 problem 997

Internal problem ID [9331]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 997.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - (y + \cos(x))^2 = \sin(x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x) = (y(x)+cos(x))^2+sin(x),y(x), singsol=all)
```

$$y(x) = -\cos(x) + \frac{1}{-x + c_1}$$

✓ Solution by Mathematica
Time used: 0.191 (sec). Leaf size: 26

```
DSolve[y'[x] == Sin[x] + (Cos[x] + y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\cos(x) + \frac{1}{-x + c_1}$$
$$y(x) \to -\cos(x)$$

2.421 problem 998

Internal problem ID [9332]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 998.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - \frac{(y - \ln(x) - \operatorname{Ci}(x))^2 + \cos(x)}{x} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x) = ((y(x)-ln(x)-Ci(x))^2+cos(x))/x,y(x), singsol=all)
```

$$y(x) = \ln(x) + \operatorname{Ci}(x) + \frac{-c_1 x^2 + 1}{c_1 x^2 + 1}$$

✓ Solution by Mathematica

Time used: 0.738 (sec). Leaf size: 36

```
DSolve[y'[x] == (Cos[x] + (-CosIntegral[x] - Log[x] + y[x])^2)/x,y[x],x,IncludeSingularSolut
```

$$y(x) \to \text{CosIntegral}(x) - \frac{2x^2}{x^2 - 2c_1} + \log(x) + 1$$
$$y(x) \to \text{CosIntegral}(x) + \log(x) + 1$$

2.422 problem 999

Internal problem ID [9333]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 999.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Riccati]`

$$y' - \frac{(y - x + \ln(x+1))^2 + x}{x+1} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 36

```
dsolve(diff(y(x),x) = ((y(x)-x+ln(x+1))^2+x)/(x+1),y(x), singsol=all)
```

$$y(x) = \frac{-\ln(x+1)^2 + (x - c_1)\ln(x+1) + c_1 x - 1}{\ln(x+1) + c_1}$$

✓ Solution by Mathematica

Time used: 0.323 (sec). Leaf size: 36

```
DSolve[y'[x] == (x + (-x + Log[1 + x] + y[x])^2)/(1 + x),y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to x - \log(x+1) + \frac{1}{-\log(x+1) + c_1}$$
$$y(x) \to x - \log(x+1)$$

2.423 problem 1000

Internal problem ID [9334]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 1, Additional non-linear first order
Problem number: 1000.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \frac{2x^2y + x^3 + y\ln(x)x - y^2 - xy}{x^2(x + \ln(x))} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(diff(y(x),x) = 1/x^2*(2*x^2*y(x)+x^3+y(x)*ln(x)*x-y(x)^2-x*y(x))/(x+ln(x)),y(x), sing
```

$$y(x) = \frac{x(c_1x - 1)}{c_1\ln(x) + 1}$$

✓ Solution by Mathematica

Time used: 1.677 (sec). Leaf size: 27

```
DSolve[y'[x] == (x^3 - x*y[x] + 2*x^2*y[x] + x*Log[x]*y[x] - y[x]^2)/(x^2*(x + Log[x])),y[x]
```

$$y(x) \to \frac{x(x - c_1)}{\log(x) + c_1}$$
$$y(x) \to -x$$

3 Chapter 2, linear second order

3.1 problem 1001

Internal problem ID [9335]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1001.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(diff(y(x),x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1x + c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 12

```
DSolve[y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x + c_1$$

3.2 problem 1002

Internal problem ID [9336]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1002.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(diff(y(x),x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(x) + c_2 \cos(x)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 16

```
DSolve[y[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(x) + c_2 \sin(x)$$

3.3 problem 1003

Internal problem ID [9337]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1003.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sin(nx)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(diff(y(x),x),x)+y(x)-sin(n*x)=0,y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 - \frac{\sin(nx)}{n^2 - 1}$$

✓ Solution by Mathematica

Time used: 0.18 (sec). Leaf size: 29

```
DSolve[-Sin[n*x] + y[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sin(nx)}{n^2 - 1} + c_1 \cos(x) + c_2 \sin(x)$$

3.4 problem 1004

Internal problem ID [9338]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1004.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = a\cos(bx)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(diff(y(x),x),x)+y(x)-a*cos(b*x)=0,y(x), singsol=all)
```

$$y(x) = \sin(x)\, c_2 + \cos(x)\, c_1 - \frac{a\cos(bx)}{b^2 - 1}$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 30

```
DSolve[-(a*Cos[b*x]) + y[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a\cos(bx)}{b^2 - 1} + c_1\cos(x) + c_2\sin(x)$$

3.5 problem 1005

Internal problem ID [9339]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1005.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y = \sin(xa)\sin(bx)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 82

```
dsolve(diff(diff(y(x),x),x)+y(x)-sin(a*x)*sin(b*x)=0,y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 + \frac{-(b+a+1)(b+a-1)\cos(x(a-b)) + \cos(x(a+b))(-b+a+1)(-b+a-1)}{2a^4 + (-4b^2-4)a^2 + 2b^4 - 4b^2 + 2}$$

✓ Solution by Mathematica

Time used: 0.642 (sec). Leaf size: 159

```
DSolve[-(Sin[a*x]*Sin[b*x]) + y[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{a^4 c_2 \sin(x) - 2a^2b^2c_2\sin(x) - a^2\sin(ax)\sin(bx) - 2a^2c_2\sin(x) + c_1\left(a^4 - 2a^2(b^2+1) + (b^2-1)^2\right)\cos(}{(a-b-1)(a-b+1)(}$$

3.6 problem 1006

Internal problem ID [9340]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1006.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(diff(y(x),x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} c_1 + \mathrm{e}^{x} c_2$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 20

```
DSolve[-y[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x + c_2 e^{-x}$$

3.7 problem 1007

Internal problem ID [9341]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1007.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y = 4x^2 e^{x^2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(diff(y(x),x),x)-2*y(x)-4*x^2*exp(x^2)=0,y(x), singsol=all)
```

$$y(x) = e^{\sqrt{2}\,x} c_2 + e^{-\sqrt{2}\,x} c_1 + e^{x^2}$$

✓ Solution by Mathematica
Time used: 0.22 (sec). Leaf size: 36

```
DSolve[-4*E^x^2*x^2 - 2*y[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x^2} + c_1 e^{\sqrt{2}x} + c_2 e^{-\sqrt{2}x}$$

3.8 problem 1008

Internal problem ID [9342]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1008.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + a^2 y = \cot(xa)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(diff(y(x),x),x)+a^2*y(x)-cot(a*x)=0,y(x), singsol=all)
```

$$y(x) = \sin(ax)\, c_2 + \cos(ax)\, c_1 + \frac{\sin(ax)\ln(\csc(ax) - \cot(ax))}{a^2}$$

✓ Solution by Mathematica

Time used: 0.088 (sec). Leaf size: 46

```
DSolve[-Cot[a*x] + a^2*y[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sin(ax)\left(a^2 c_2 + \log\left(\sin\left(\frac{ax}{2}\right)\right) - \log\left(\cos\left(\frac{ax}{2}\right)\right)\right)}{a^2} + c_1 \cos(ax)$$

3.9 problem 1009

Internal problem ID [9343]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1009.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + ly = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(diff(y(x),x),x)+l*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(\sqrt{l}\, x\right) + c_2 \cos\left(\sqrt{l}\, x\right)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 28

```
DSolve[l*y[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos\left(\sqrt{l}x\right) + c_2 \sin\left(\sqrt{l}x\right)$$

3.10 problem 1010

Internal problem ID [9344]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1010.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (xa + b)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(diff(diff(y(x),x),x)+(a*x+b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{AiryAi}\left(-\frac{ax+b}{a^{\frac{2}{3}}}\right) + c_2 \operatorname{AiryBi}\left(-\frac{ax+b}{a^{\frac{2}{3}}}\right)$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 42

```
DSolve[(b + a*x)*y[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{AiryAi}\left(-\frac{b+ax}{(-a)^{2/3}}\right) + c_2 \operatorname{AiryBi}\left(-\frac{b+ax}{(-a)^{2/3}}\right)$$

3.11 problem 1011

Internal problem ID [9345]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1011.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - (x^2+1)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(diff(y(x),x),x)-(x^2+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{x^2}{2}}(c_1 + \operatorname{erf}(x)\, c_2)$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 33

```
DSolve[(-1 - x^2)*y[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{ParabolicCylinderD}\left(-1, \sqrt{2}x\right) + c_2 \operatorname{ParabolicCylinderD}\left(0, i\sqrt{2}x\right)$$

3.12 problem 1012

Internal problem ID [9346]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1012.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - (x^2 + a)\, y = 0}$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 29

```
dsolve(diff(diff(y(x),x),x)-(x^2+a)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \operatorname{WhittakerM}\left(-\frac{a}{4}, \frac{1}{4}, x^2\right) + c_2 \operatorname{WhittakerW}\left(-\frac{a}{4}, \frac{1}{4}, x^2\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 47

```
DSolve[(-a - x^2)*y[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{ParabolicCylinderD}\left(\frac{1}{2}(-a-1), \sqrt{2}x\right) + c_2 \operatorname{ParabolicCylinderD}\left(\frac{a-1}{2}, i\sqrt{2}x\right)$$

3.13 problem 1013

Internal problem ID [9347]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1013.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \left(a^2x^2 + a\right) y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(diff(diff(y(x),x),x)-(a^2*x^2+a)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{a\,x^2}{2}} \left(c_1 + \operatorname{erf}\left(x\sqrt{a}\right) c_2\right)$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 43

```
DSolve[(-a - a^2*x^2)*y[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \,\text{ParabolicCylinderD}\left(-1, \sqrt{2}\sqrt{a}x\right) + c_2 \,\text{ParabolicCylinderD}\left(0, i\sqrt{2}\sqrt{a}x\right)$$

3.14 problem 1014

Internal problem ID [9348]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1014.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - c\,x^a y = 0}$$

✓ Solution by Maple
Time used: 0.344 (sec). Leaf size: 63

```
dsolve(diff(diff(y(x),x),x)-c*x^a*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\left(\text{BesselY}\left(\frac{1}{a+2}, \frac{2\sqrt{-c}\,x^{\frac{a}{2}+1}}{a+2}\right)c_2 + \text{BesselJ}\left(\frac{1}{a+2}, \frac{2\sqrt{-c}\,x^{\frac{a}{2}+1}}{a+2}\right)c_1\right)$$

✓ Solution by Mathematica
Time used: 0.072 (sec). Leaf size: 119

```
DSolve[-(c*x^a*y[x]) + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (a+2)^{-\frac{1}{a+2}}\sqrt{x}c^{\frac{1}{2a+4}}\left(c_1\,\text{Gamma}\left(\frac{a+1}{a+2}\right)\text{BesselI}\left(-\frac{1}{a+2}, \frac{2\sqrt{c}x^{\frac{a}{2}+1}}{a+2}\right) + (-1)^{\frac{1}{a+2}}c_2\,\text{Gamma}\left(1+\frac{1}{a+2}\right)\text{BesselI}\left(\frac{1}{a+2}, \frac{2\sqrt{c}x^{\frac{a}{2}+1}}{a+2}\right)\right)$$

3.15 problem 1015

Internal problem ID [9349]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1015.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Titchmarsh]`

$$y'' - (a^2x^{2n} - 1)\, y = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)-(a^2*x^(2*n)-1)*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(1 - a^2*x^(2*n))*y[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

3.16 problem 1016

Internal problem ID [9350]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1016.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(a\,x^{2c} + b\,x^{c-1}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.468 (sec). Leaf size: 89

```
dsolve(diff(diff(y(x),x),x)+(a*x^(2*c)+b*x^(c-1))*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^{-\frac{c}{2}}\left(c_1 \operatorname{WhittakerM}\left(-\frac{ib}{\sqrt{a}\,(2c+2)}, \frac{1}{2c+2}, \frac{2i\sqrt{a}\,x^c x}{c+1}\right) + c_2 \operatorname{WhittakerW}\left(-\frac{ib}{\sqrt{a}\,(2c+2)}, \frac{1}{2c+2}, \frac{2i\sqrt{a}\,x^c x}{c+1}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.269 (sec). Leaf size: 225

```
DSolve[(b*x^(-1 + c) + a*x^(2*c))*y[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True
```

$y(x)$

$$\rightarrow 2^{\frac{c}{2c+2}} x^{-c/2}\left(x^{c+1}\right)^{\frac{c}{2c+2}} e^{-\frac{\sqrt{a}x^{c+1}}{\sqrt{-(c+1)^2}}}\left(c_1 \operatorname{HypergeometricU}\left(-\frac{(c+1)\left(cb+b+\sqrt{a}c\sqrt{-(c+1)^2}\right)}{2\sqrt{a}\left(-(c+1)^2\right)^{3/2}}, \frac{c}{c+1}, -\right.\right.$$

3.17 problem 1017

Internal problem ID [9351]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1017.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(e^{2x} - v^2\right) y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(diff(y(x),x),x)+(exp(2*x)-v^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}\left(v, e^x\right) + c_2 \operatorname{BesselY}\left(v, e^x\right)$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 46

```
DSolve[(E^(2*x) - v^2)*y[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{Gamma}(1-v) \operatorname{BesselJ}\left(-v, \sqrt{e^{2x}}\right) + c_2 \operatorname{Gamma}(v+1) \operatorname{BesselJ}\left(v, \sqrt{e^{2x}}\right)$$

3.18 problem 1018

Internal problem ID [9352]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1018.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + a\,\mathrm{e}^{bx} y = 0}$$

✓ Solution by Maple
Time used: 0.157 (sec). Leaf size: 39

```
dsolve(diff(diff(y(x),x),x)+a*exp(b*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}\left(0, \frac{2\sqrt{a}\,\mathrm{e}^{\frac{bx}{2}}}{b}\right) + c_2 \operatorname{BesselY}\left(0, \frac{2\sqrt{a}\,\mathrm{e}^{\frac{bx}{2}}}{b}\right)$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 55

```
DSolve[a*E^(b*x)*y[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}\left(0, \frac{2\sqrt{a}\sqrt{e^{bx}}}{b}\right) + 2c_2 \operatorname{BesselY}\left(0, \frac{2\sqrt{a}\sqrt{e^{bx}}}{b}\right)$$

3.19 problem 1019

Internal problem ID [9353]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1019.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \left(4a^2b^2x^2e^{2b\,x^2} - 1\right)y = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)-(4*a^2*b^2*x^2*exp(2*b*x^2)-1)*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(1 - 4*a^2*b^2*E^(2*b*x^2)*x^2)*y[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

Not solved

3.20 problem 1020

Internal problem ID [9354]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1020.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(a\,\mathrm{e}^{2x} + b\,\mathrm{e}^{x} + c\right) y = 0$$

✓ Solution by Maple

Time used: 0.297 (sec). Leaf size: 58

```
dsolve(diff(diff(y(x),x),x)+(a*exp(2*x)+b*exp(x)+c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{2}} \left(\mathrm{WhittakerM}\left(-\frac{ib}{2\sqrt{a}}, i\sqrt{c}, 2i\sqrt{a}\,\mathrm{e}^{x}\right) c_1 + \mathrm{WhittakerW}\left(-\frac{ib}{2\sqrt{a}}, i\sqrt{c}, 2i\sqrt{a}\,\mathrm{e}^{x}\right) c_2\right)$$

✓ Solution by Mathematica

Time used: 0.789 (sec). Leaf size: 136

```
DSolve[(c + b*E^x + a*E^(2*x))*y[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-i\sqrt{a}e^{x}} (e^{x})^{i\sqrt{c}} \left(c_1 \,\mathrm{HypergeometricU}\left(\frac{ib}{2\sqrt{a}} + i\sqrt{c} + \frac{1}{2}, 2i\sqrt{c} + 1, 2i\sqrt{a}e^{x}\right) + c_2 L^{2i\sqrt{c}}_{-\frac{ib}{2\sqrt{a}} - i\sqrt{c} - \frac{1}{2}}\left(2i\sqrt{a}e^{x}\right)\right)$$

3.21 problem 1021

Internal problem ID [9355]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1021.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(a \cosh(x)^2 + b\right) y = 0}$$

✓ Solution by Maple

Time used: 0.562 (sec). Leaf size: 39

```
dsolve(diff(diff(y(x),x),x)+(a*cosh(x)^2+b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{MathieuC}\left(-\frac{a}{2} - b, \frac{a}{4}, ix\right) + c_2 \operatorname{MathieuS}\left(-\frac{a}{2} - b, \frac{a}{4}, ix\right)$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 40

```
DSolve[(b + a*Cos[x]^2)*y[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \text{MathieuC}\left[\frac{a}{2} + b, -\frac{a}{4}, x\right] + c_2 \text{MathieuS}\left[\frac{a}{2} + b, -\frac{a}{4}, x\right]$$

3.22 problem 1022

Internal problem ID [9356]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1022.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_ellipsoidal]`

$$y'' + (a\cos(2x) + b)\, y = 0$$

✓ Solution by Maple
Time used: 0.61 (sec). Leaf size: 21

```
dsolve(diff(diff(y(x),x),x)+(a*cos(2*x)+b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{MathieuC}\left(b, -\frac{a}{2}, x\right) + c_2 \operatorname{MathieuS}\left(b, -\frac{a}{2}, x\right)$$

✓ Solution by Mathematica
Time used: 0.047 (sec). Leaf size: 28

```
DSolve[(b + a*Cos[2*x])*y[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\text{MathieuC}\left[b, -\frac{a}{2}, x\right] + c_2\text{MathieuS}\left[b, -\frac{a}{2}, x\right]$$

3.23 problem 1023

Internal problem ID [9357]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1023.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_ellipsoidal]`

$$\boxed{y'' + \left(a\cos\left(x\right)^2 + b\right) y = 0}$$

✓ Solution by Maple
Time used: 0.609 (sec). Leaf size: 29

```
dsolve(diff(diff(y(x),x),x)+(a*cos(x)^2+b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{MathieuC}\left(\frac{a}{2} + b, -\frac{a}{4}, x\right) + c_2 \operatorname{MathieuS}\left(\frac{a}{2} + b, -\frac{a}{4}, x\right)$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 40

```
DSolve[(b + a*Cos[x]^2)*y[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \text{MathieuC}\left[\frac{a}{2} + b, -\frac{a}{4}, x\right] + c_2 \text{MathieuS}\left[\frac{a}{2} + b, -\frac{a}{4}, x\right]$$

3.24 problem 1024

Internal problem ID [9358]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1024.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \left(1 + 2\tan(x)^2\right) y = 0}$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 28

```
dsolve(diff(diff(y(x),x),x)-(1+2*tan(x)^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = i\sin(x)\, c_2 + \sec(x)\ln(\cos(x) + i\sin(x))\, c_2 + c_1\sec(x)$$

✓ Solution by Mathematica
Time used: 0.501 (sec). Leaf size: 46

```
DSolve[(-1 - 2*Tan[x]^2)*y[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2\sec(x)\arctan\left(\frac{\cos(x)}{\sqrt{\sin^2(x)} - 1}\right) - \frac{1}{2}c_2\sqrt{\sin^2(x)} + c_1\sec(x)$$

3.25 problem 1025

Internal problem ID [9359]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1025.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \left(\frac{m(m-1)}{\cos(x)^2} + \frac{n(n-1)}{\sin(x)^2} + a\right) y = 0$$

✓ Solution by Maple
Time used: 0.375 (sec). Leaf size: 102

```
dsolve(diff(diff(y(x),x),x)-(m*(m-1)/cos(x)^2+n*(n-1)/sin(x)^2+a)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sin(x)^n \left(c_1 \cos(x)^m \operatorname{hypergeom}\left(\left[\frac{n}{2}+\frac{m}{2}+\frac{i\sqrt{a}}{2}, \frac{n}{2}+\frac{m}{2}-\frac{i\sqrt{a}}{2}\right], \left[\frac{1}{2}+m\right], \cos(x)^2\right) + c_2 \cos(x)^{-m+1} \operatorname{hypergeom}\left(\left[\frac{n}{2}-\frac{m}{2}+\frac{i\sqrt{a}}{2}+\frac{1}{2}, \frac{n}{2}-\frac{m}{2}-\frac{i\sqrt{a}}{2}+\frac{1}{2}\right], \left[\frac{3}{2}-m\right], \cos(x)^2\right)\right)$$

✓ Solution by Mathematica
Time used: 1.623 (sec). Leaf size: 158

```
DSolve[(-a - (-1 + n)*n*Csc[x]^2 - (-1 + m)*m*Sec[x]^2)*y[x] + y''[x] == 0,y[x],x,IncludeSin
```

$$y(x) \to \frac{(-1)^{-m} \cos^2(x)^{-\frac{m}{2}-\frac{1}{4}} \left(-\sin^2(x)\right)^{n/2} \left(c_1(-1)^m \cos^2(x)^{m+\frac{1}{2}} \text{Hypergeometric2F1}\left(\frac{1}{2}\left(m+n-\sqrt{-a}\right), \frac{1}{2}(n\right.\right.}{}$$

3.26 problem 1026

Internal problem ID [9360]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1026.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - (n(n+1)\,\text{WeierstrassP}\,(x, \text{g2}, \text{g3}) + B)\,y = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)-(n*(n+1)*WeierstrassP(x,g2,g3)+B)*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-((B + n*(1 + n)*WeierstrassP[x, {g2, g3}])*y[x]) + y''[x] == 0,y[x],x,IncludeSingula
```

Not solved

3.27 problem 1027

Internal problem ID [9361]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1027.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \left(n(n+1)\,k^2\,\mathrm{JacobiSN}\,(x,k)^2 + b\right) y = 0$$

✓ Solution by Maple

Time used: 1.282 (sec). Leaf size: 69

```
dsolve(diff(diff(y(x),x),x)-(n*(n+1)*k^2*JacobiSN(x,k)^2+b)*y(x)=0,y(x), singsol=all)
```

$$\begin{aligned} y(x) = {} & c_1\,\mathrm{HeunG}\left(\frac{1}{k^2}, \frac{b}{4k^2}, -\frac{n}{2}, \frac{n}{2}+\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \mathrm{JacobiSN}\,(x,k)^2\right) \\ & + c_2\,\mathrm{HeunG}\left(\frac{1}{k^2}, \frac{k^2+b+1}{4k^2}, \frac{n}{2}+1, -\frac{n}{2} \right. \\ & \left. + \frac{1}{2}, \frac{3}{2}, \frac{1}{2}, \mathrm{JacobiSN}\,(x,k)^2\right) \mathrm{JacobiSN}\,(x,k) \end{aligned}$$

✓ Solution by Mathematica

Time used: 1.268 (sec). Leaf size: 209

```
DSolve[(b + a*JacobiSN[x, k]^2)*y[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sqrt{k\mathrm{sn}(x|k)^2-1}\left(c_1\mathrm{HeunG}\left[\frac{1}{k}, \frac{k-b}{4k}, \frac{1}{4}\left(\frac{\sqrt{k-4a}}{\sqrt{k}}+3\right), \frac{\sqrt{k}\sqrt{k-4a}+2a+k}{2\left(\sqrt{k}\sqrt{k-4a}+k\right)}, \frac{1}{2}, \frac{1}{2}, \mathrm{sn}(x|k)^2\right] + c_2\mathrm{sn}(\right.$$

3.28 problem 1028

Internal problem ID [9362]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1028.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \left(\frac{p''''(x)}{30} + \frac{7p''(x)}{3} + ap(x) + b\right)y = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)-(1/30*diff(diff(diff(diff(p(x),x),x),x),x)+7/3*diff(diff(p(x),x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-(y[x]*(b + a*p[x] + (p^4)[x]/30 + (7*Derivative[2][p][x])/3)) + y''[x] == 0,y[x],x,I
```

Not solved

3.29 problem 1029

Internal problem ID [9363]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1029.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \left(f(x)^2 + f'(x)\right) y = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 22

```
dsolve(diff(diff(y(x),x),x)-(f(x)^2+diff(f(x),x))*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(\int e^{-2\left(\int f(x)dx\right)}dx + c_1\right) e^{\int f(x)dx} c_2$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 58

```
DSolve[-(y[x]*(f[x]^2 + Derivative[1][f][x])) + y''[x] == 0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to c_1 \exp\left(\int_1^x f(K[1])dK[1]\right) + c_2 \exp\left(\int_1^x f(K[2])dK[2]\right) \int_1^x \exp\left(\int_1^{K[4]} -2f(K[3])dK[3]\right) dK[4]$$

3.30 problem 1030

Internal problem ID [9364]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1030.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (P(x) + l)\, y = 0}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+(P(x)+l)*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(l + P[x])*y[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

3.31 problem 1031

Internal problem ID [9365]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1031.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - f(x)\, y = 0}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)-f(x)*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-(f[x]*y[x]) + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

3.32 problem 1033

Internal problem ID [9366]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1033.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{y'' + y' + a\,\mathrm{e}^{-2x} y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(diff(y(x),x),x)+diff(y(x),x)+a*exp(-2*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(\mathrm{e}^{-x}\sqrt{a}\right) + c_2 \cos\left(\mathrm{e}^{-x}\sqrt{a}\right)$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 37

```
DSolve[(a*y[x])/E^(2*x) + y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos\left(\sqrt{a}e^{-x}\right) - c_2 \sin\left(\sqrt{a}e^{-x}\right)$$

3.33 problem 1034

Internal problem ID [9367]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1034.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{y'' - y' + e^{2x}y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(diff(y(x),x),x)-diff(y(x),x)+exp(2*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(e^x) + c_2 \cos(e^x)$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 20

```
DSolve[E^(2*x)*y[x] - y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \cos(e^x) + c_2 \sin(e^x)$$

3.34 problem 1035

Internal problem ID [9368]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1035.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + ay' + yb = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
dsolve(diff(diff(y(x),x),x)+a*diff(y(x),x)+b*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{\left(-a+\sqrt{a^2-4b}\right)x}{2}} + c_2 \mathrm{e}^{-\frac{\left(a+\sqrt{a^2-4b}\right)x}{2}}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 47

```
DSolve[b*y[x] + a*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{1}{2}x\left(\sqrt{a^2-4b}+a\right)}\left(c_2 e^{x\sqrt{a^2-4b}} + c_1\right)$$

3.35 problem 1036

Internal problem ID [9369]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1036.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + ay' + yb = f(x)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 133

```
dsolve(diff(diff(y(x),x),x)+a*diff(y(x),x)+b*y(x)-f(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{\frac{\left(-a+\sqrt{a^2-4b}\right)x}{2}}\left(\int f(x)\,\mathrm{e}^{-\frac{\left(-a+\sqrt{a^2-4b}\right)x}{2}}dx\right)+\mathrm{e}^{\frac{\left(-a+\sqrt{a^2-4b}\right)x}{2}}c_2\sqrt{a^2-4b}-\mathrm{e}^{-\frac{\left(a+\sqrt{a^2-4b}\right)x}{2}}\left(-\sqrt{a^2-4b}\,c_1+\int \right.}{\sqrt{a^2-4b}}$$

✓ Solution by Mathematica

Time used: 0.19 (sec). Leaf size: 152

```
DSolve[-f[x] + b*y[x] + a*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{1}{2}x\left(\sqrt{a^2-4b}+a\right)}\left(\int_1^x -\frac{e^{\frac{1}{2}\left(a+\sqrt{a^2-4b}\right)K[1]}f(K[1])}{\sqrt{a^2-4b}}dK[1] + e^{x\sqrt{a^2-4b}}\int_1^x \frac{e^{\frac{1}{2}\left(a-\sqrt{a^2-4b}\right)K[2]}f(K[2])}{\sqrt{a^2-4b}}dK[2] + c_2 e^{x\sqrt{a^2-4b}} + c_1\right)$$

3.36 problem 1037

Internal problem ID [9370]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1037.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + ay' - (b^2x^2 + c)\, y = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 64

```
dsolve(diff(diff(y(x),x),x)+a*diff(y(x),x)-(b^2*x^2+c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x(bx+a)}{2}} x \left(\mathrm{KummerU}\left(\frac{a^2+12b+4c}{16b}, \frac{3}{2}, b\,x^2\right) c_2 + \mathrm{KummerM}\left(\frac{a^2+12b+4c}{16b}, \frac{3}{2}, b\,x^2\right) c_1\right)$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 74

```
DSolve[(-c - b^2*x^2)*y[x] + a*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{1}{2}x(a+bx)} \left(c_1 \,\mathrm{HermiteH}\left(-\frac{a^2+4(b+c)}{8b}, \sqrt{b}x\right) + c_2 \,\mathrm{Hypergeometric1F1}\left(\frac{a^2+4(b+c)}{16b}, \frac{1}{2}, bx^2\right)\right)$$

3.37 problem 1038

Internal problem ID [9371]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1038.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 2ay' + f(x)\,y = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+2*a*diff(y(x),x)+f(x)*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[f[x]*y[x] + 2*a*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

3.38 problem 1039

Internal problem ID [9372]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1039.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + y'x + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(diff(y(x),x),x)+x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(\operatorname{erf}\left(\frac{i\sqrt{2}\,x}{2}\right)c_1 + c_2\right)e^{-\frac{x^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 41

```
DSolve[y[x] + x*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-\frac{x^2}{2}}\left(\sqrt{2\pi}c_1\operatorname{erfi}\left(\frac{x}{\sqrt{2}}\right) + 2c_2\right)$$

3.39 problem 1040

Internal problem ID [9373]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1040.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + y'x - y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(diff(y(x),x),x)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = -\mathrm{e}^{-\frac{x^2}{2}} c_2 + \left(c_1 - \frac{c_2 \sqrt{\pi}\, \sqrt{2}\, \mathrm{erf}\left(\frac{\sqrt{2}\, x}{2}\right)}{2}\right) x$$

✓ Solution by Mathematica
Time used: 0.151 (sec). Leaf size: 45

```
DSolve[-y[x] + x*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\frac{\pi}{2}} c_2 x \mathrm{erf}\left(\frac{x}{\sqrt{2}}\right) - c_2 e^{-\frac{x^2}{2}} + c_1 x$$

3.40 problem 1041

Internal problem ID [9374]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1041.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + (n+1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 41

```
dsolve(diff(diff(y(x),x),x)+x*diff(y(x),x)+(n+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x^2}{2}} x \left(\mathrm{KummerU}\left(-\frac{n}{2}+\frac{1}{2}, \frac{3}{2}, \frac{x^2}{2}\right) c_2 + \mathrm{KummerM}\left(-\frac{n}{2}+\frac{1}{2}, \frac{3}{2}, \frac{x^2}{2}\right) c_1\right)$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 47

```
DSolve[(1 + n)*y[x] + x*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{x^2}{2}} \left(c_1 \,\mathrm{HermiteH}\left(n, \frac{x}{\sqrt{2}}\right) + c_2 \,\mathrm{Hypergeometric1F1}\left(-\frac{n}{2}, \frac{1}{2}, \frac{x^2}{2}\right)\right)$$

3.41 problem 1042

Internal problem ID [9375]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1042.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x - yn = 0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 41

```
dsolve(diff(diff(y(x),x),x)+x*diff(y(x),x)-n*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x^2}{2}} x \left(\mathrm{KummerM}\left(\frac{n}{2}+1, \frac{3}{2}, \frac{x^2}{2}\right) c_1 + \mathrm{KummerU}\left(\frac{n}{2}+1, \frac{3}{2}, \frac{x^2}{2}\right) c_2\right)$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 53

```
DSolve[-(n*y[x]) + x*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{x^2}{2}} \left(c_1 \,\mathrm{HermiteH}\left(-n-1, \frac{x}{\sqrt{2}}\right) + c_2 \,\mathrm{Hypergeometric1F1}\left(\frac{n+1}{2}, \frac{1}{2}, \frac{x^2}{2}\right)\right)$$

3.42 problem 1043

Internal problem ID [9376]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1043.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Hermite]`

$$\boxed{y'' - y'x + 2y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 39

```
dsolve(diff(diff(y(x),x),x)-x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = -2c_1 e^{\frac{x^2}{2}} x + (x-1)(x+1)\left(\sqrt{\pi}\,\sqrt{2}\,\operatorname{erfi}\left(\frac{\sqrt{2}\,x}{2}\right) c_1 + c_2\right)$$

✓ Solution by Mathematica
Time used: 0.117 (sec). Leaf size: 54

```
DSolve[2*y[x] - x*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} c_2 \left(\sqrt{2\pi}(x^2-1)\operatorname{erfi}\left(\frac{x}{\sqrt{2}}\right) - 2e^{\frac{x^2}{2}} x\right) + c_1(x^2-1)$$

3.43 problem 1044

Internal problem ID [9377]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1044.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Hermite]`

$$\boxed{y'' - y'x - ay = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 35

```
dsolve(diff(diff(y(x),x),x)-x*diff(y(x),x)-a*y(x)=0,y(x), singsol=all)
```

$$y(x) = x\left(\text{KummerU}\left(\frac{1}{2} + \frac{a}{2}, \frac{3}{2}, \frac{x^2}{2}\right) c_2 + \text{KummerM}\left(\frac{1}{2} + \frac{a}{2}, \frac{3}{2}, \frac{x^2}{2}\right) c_1\right)$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 39

```
DSolve[-(a*y[x]) - x*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \text{HermiteH}\left(-a, \frac{x}{\sqrt{2}}\right) + c_2 \text{Hypergeometric1F1}\left(\frac{a}{2}, \frac{1}{2}, \frac{x^2}{2}\right)$$

3.44 problem 1045

Internal problem ID [9378]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1045.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{y'' - y'x + (x - 1)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(diff(y(x),x),x)-x*diff(y(x),x)+(x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x}\left(c_1 \operatorname{erf}\left(\frac{i\sqrt{2}\,(x-2)}{2}\right) + c_2\right)$$

✓ Solution by Mathematica
Time used: 0.073 (sec). Leaf size: 39

```
DSolve[(-1 + x)*y[x] - x*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{\frac{\pi}{2}} c_2 e^{x-2} \operatorname{erfi}\left(\frac{x-2}{\sqrt{2}}\right) + c_1 e^{x}$$

3.45 problem 1046

Internal problem ID [9379]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1046.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'x + ay = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 31

```
dsolve(diff(diff(y(x),x),x)-2*x*diff(y(x),x)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = x\left(\text{KummerU}\left(\frac{1}{2} - \frac{a}{4}, \frac{3}{2}, x^2\right) c_2 + \text{KummerM}\left(\frac{1}{2} - \frac{a}{4}, \frac{3}{2}, x^2\right) c_1\right)$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 31

```
DSolve[a*y[x] - 2*x*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \,\text{HermiteH}\left(\frac{a}{2}, x\right) + c_2 \,\text{Hypergeometric1F1}\left(-\frac{a}{4}, \frac{1}{2}, x^2\right)$$

3.46 problem 1047

Internal problem ID [9380]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1047.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 4y'x + (4x^2 + 2)\, y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(diff(y(x),x),x)+4*x*diff(y(x),x)+(4*x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x^2}(c_2 x + c_1)$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 20

```
DSolve[(2 + 4*x^2)*y[x] + 4*x*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x^2}(c_2 x + c_1)$$

3.47 problem 1048

Internal problem ID [9381]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1048.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y'x + (3x^2 + 2n - 1)\, y = 0}$$

✓ Solution by Maple
Time used: 0.079 (sec). Leaf size: 37

```
dsolve(diff(diff(y(x),x),x)-4*x*diff(y(x),x)+(3*x^2+2*n-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = x\, e^{\frac{x^2}{2}} \left(\text{KummerU}\left(-\frac{n}{2}+\frac{1}{2}, \frac{3}{2}, x^2\right) c_2 + \text{KummerM}\left(-\frac{n}{2}+\frac{1}{2}, \frac{3}{2}, x^2\right) c_1\right)$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 37

```
DSolve[(-1 + 2*n + 3*x^2)*y[x] - 4*x*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to e^{\frac{x^2}{2}} \left(c_1 \,\text{HermiteH}(n, x) + c_2 \,\text{Hypergeometric1F1}\left(-\frac{n}{2}, \frac{1}{2}, x^2\right)\right)$$

3.48 problem 1049

Internal problem ID [9382]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1049.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 4y'x + (4x^2 - 1)\,y = e^x$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 66

```
dsolve(diff(diff(y(x),x),x)-4*x*diff(y(x),x)+(4*x^2-1)*y(x)-exp(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{e^{x^2}\left(e^{\frac{i}{2}}\sqrt{\pi}\,(i\cos(x)+\sin(x))\,\mathrm{erf}\left(x-\frac{1}{2}-\frac{i}{2}\right)-(i\cos(x)-\sin(x))\,e^{-\frac{i}{2}}\sqrt{\pi}\,\mathrm{erf}\left(x-\frac{1}{2}+\frac{i}{2}\right)+4c_1\sin(x\right.}{4}$$

✓ Solution by Mathematica

Time used: 0.139 (sec). Leaf size: 105

```
DSolve[-E^x + (-1 + 4*x^2)*y[x] - 4*x*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{4}e^{x(x-i)-\frac{i}{2}}\left(-ie^{i}\sqrt{\pi}\,\mathrm{erf}\left(-x+\left(\frac{1}{2}+\frac{i}{2}\right)\right)+\sqrt{\pi}e^{2ix}\,\mathrm{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)-ix\right)\right.$$
$$\left.+2e^{\frac{i}{2}}\left(2c_1-ic_2e^{2ix}\right)\right)$$

3.49 problem 1050

Internal problem ID [9383]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1050.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y'x + (4x^2 - 2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve(diff(diff(y(x),x),x)-4*x*diff(y(x),x)+(4*x^2-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{x^2}(c_2x + c_1)$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 18

```
DSolve[(-2 + 4*x^2)*y[x] - 4*x*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x^2}(c_2x + c_1)$$

3.50 problem 1051

Internal problem ID [9384]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1051.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y'x + (4x^2 - 3)\, y = \mathrm{e}^{x^2}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(diff(y(x),x),x)-4*x*diff(y(x),x)+(4*x^2-3)*y(x)-exp(x^2)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x(x+1)} c_2 + \mathrm{e}^{x(x-1)} c_1 - \mathrm{e}^{x^2}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 34

```
DSolve[-E^x^2 + (-3 + 4*x^2)*y[x] - 4*x*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{2} e^{(x-1)x} \left(-2e^x + c_2 e^{2x} + 2c_1\right)$$

3.51 problem 1052

Internal problem ID [9385]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1052.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + axy' + yb = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 58

```
dsolve(diff(diff(y(x),x),x)+a*x*diff(y(x),x)+b*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-\frac{a\,x^2}{2}} x\left(\text{KummerM}\left(\frac{2a-b}{2a}, \frac{3}{2}, \frac{a\,x^2}{2}\right) c_1 + \text{KummerU}\left(\frac{2a-b}{2a}, \frac{3}{2}, \frac{a\,x^2}{2}\right) c_2\right)$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 67

```
DSolve[b*y[x] + a*x*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{ax^2}{2}}\left(c_1 \,\text{HermiteH}\left(\frac{b}{a} - 1, \frac{\sqrt{a}x}{\sqrt{2}}\right) + c_2 \,\text{Hypergeometric1F1}\left(\frac{a-b}{2a}, \frac{1}{2}, \frac{ax^2}{2}\right)\right)$$

3.52 problem 1053

Internal problem ID [9386]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1053.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2axy' + a^2x^2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(diff(y(x),x),x)+2*a*x*diff(y(x),x)+a^2*x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{x(ax-2\sqrt{a})}{2}} + c_2 e^{-\frac{x(ax+2\sqrt{a})}{2}}$$

✓ Solution by Mathematica

Time used: 0.066 (sec). Leaf size: 56

```
DSolve[a^2*x^2*y[x] + 2*a*x*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{ax^2}{2}-\sqrt{a}x}\left(c_2 e^{2\sqrt{a}x} + 2\sqrt{a}c_1\right)}{2\sqrt{a}}$$

3.53 problem 1054

Internal problem ID [9387]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1054.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + (xa + b)\, y' + (cx + d)\, y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 98

```
dsolve(diff(diff(y(x),x),x)+(a*x+b)*diff(y(x),x)+(c*x+d)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{cx}{a}} \left(\mathrm{KummerM}\left(\frac{a^2 d - acb + c^2}{2a^3}, \frac{1}{2}, -\frac{(a^2 x + ab - 2c)^2}{2a^3} \right) c_1 + \mathrm{KummerU}\left(\frac{a^2 d - acb + c^2}{2a^3}, \frac{1}{2}, -\frac{(a^2 x + ab - 2c)^2}{2a^3} \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.079 (sec). Leaf size: 132

```
DSolve[(d + c*x)*y[x] + (b + a*x)*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^{\frac{cx}{a} - \frac{ax^2}{2} - bx} \left(c_2 \,\text{Hypergeometric1F1}\left(\frac{a^3 - da^2 + bca - c^2}{2a^3}, \frac{1}{2}, \frac{(xa^2 + ba - 2c)^2}{2a^3} \right) + c_1 \,\text{HermiteH}\left(\frac{-a^3 + da^2 - bca + c^2}{a^3}, \frac{xa^2 + ba - 2c}{\sqrt{2}a^{3/2}} \right) \right)$$

3.54 problem 1055

Internal problem ID [9388]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1055.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + (xa + b)\,y' + \left(\mathrm{a1}\ x^2 + \mathrm{b1}\,x + \mathrm{c1}\right) y = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 254

```
dsolve(diff(diff(y(x),x),x)+(a*x+b)*diff(y(x),x)+(a1*x^2+b1*x+c1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \Bigg(c_2\big(a^2x + ab - 4\,\mathrm{a1}\,x$$
$$-2\,\mathrm{b1}\big)\,\mathrm{hypergeom}\left(\left[\frac{3(a^2-4\,\mathrm{a1})^{\frac{3}{2}} + a^3 - 2a^2\,\mathrm{c1} + 2(\mathrm{b1}\,b - 2\,\mathrm{a1})\,a + 2(-b^2+4\,\mathrm{c1})\,\mathrm{a1} - 2\,\mathrm{b1}^2}{4\,(a^2-4\,\mathrm{a1})^{\frac{3}{2}}}\right], \left[\frac{3}{2}\right],\right.$$
$$+\mathrm{hypergeom}\left(\left[\frac{(a^2-4\,\mathrm{a1})^{\frac{3}{2}} + a^3 - 2a^2\,\mathrm{c1} + (2\,\mathrm{b1}\,b - 4\,\mathrm{a1})\,a + (-2b^2+8\,\mathrm{c1})\,\mathrm{a1} - 2\,\mathrm{b1}^2}{4\,(a^2-4\,\mathrm{a1})^{\frac{3}{2}}}\right], \left[\frac{1}{2}\right], \frac{(a^2x +}{}\right.$$

✓ Solution by Mathematica

Time used: 0.278 (sec). Leaf size: 305

```
DSolve[(c1 + b1*x + a1*x^2)*y[x] + (b + a*x)*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolut
```

$$y(x)$$
$$\to \exp\left(-\frac{x\left(a\left(x\sqrt{a^2-4\mathrm{a1}}+2b\right)+2b\sqrt{a^2-4\mathrm{a1}}+a^2x-4(\mathrm{a1}x+\mathrm{b1})\right)}{4\sqrt{a^2-4\mathrm{a1}}}\right)\left(c_1\,\mathrm{HermiteH}\left(\frac{-a^3-\left(\sqrt{a^2-}\right.}{}\right.\right.$$

3.55 problem 1056

Internal problem ID [9389]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1056.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - x^2 y' + xy = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 51

```
dsolve(diff(diff(y(x),x),x)-x^2*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_2(-x^3)^{\frac{1}{3}} 3^{\frac{2}{3}} \Gamma\left(\frac{2}{3}\right) - c_2(-x^3)^{\frac{1}{3}} 3^{\frac{2}{3}} \Gamma\left(\frac{2}{3}, -\frac{x^3}{3}\right) + 3c_2 e^{\frac{x^3}{3}} + c_1 x$$

✓ Solution by Mathematica
Time used: 0.072 (sec). Leaf size: 41

```
DSolve[x*y[x] - x^2*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x - \frac{c_2 \sqrt[3]{-x^3}\Gamma\left(-\frac{1}{3}, -\frac{x^3}{3}\right)}{3\sqrt[3]{3}}$$

3.56 problem 1057

Internal problem ID [9390]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1057.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - x^2 y' - (x+1)^2 y = 0}$$

✓ Solution by Maple

Time used: 0.344 (sec). Leaf size: 50

```
dsolve(diff(diff(y(x),x),x)-x^2*diff(y(x),x)-(x+1)^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{HeunT}\left(0, -3, 2\,3^{\frac{1}{3}}, \frac{3^{\frac{2}{3}} x}{3}\right) \mathrm{e}^{-x} + c_2 \operatorname{HeunT}\left(0, 3, 2\,3^{\frac{1}{3}}, -\frac{3^{\frac{2}{3}} x}{3}\right) \mathrm{e}^{\frac{x(x^2+3)}{3}}$$

✓ Solution by Mathematica

Time used: 0.274 (sec). Leaf size: 44

```
DSolve[-((1 + x)^2*y[x]) - x^2*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{x^3}{3}+x} \left(c_2 \int_1^x e^{-\frac{1}{3}K[1](K[1]^2+6)} dK[1] + c_1\right)$$

3.57 problem 1058

Internal problem ID [9391]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1058.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - x^2(x+1)\,y' + x\left(x^4-2\right)y = 0$$

✓ Solution by Maple

Time used: 0.687 (sec). Leaf size: 29

```
dsolve(diff(diff(y(x),x),x)-x^2*(x+1)*diff(y(x),x)+x*(x^4-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{x^3}{3}}\left(c_1 + \left(\int \mathrm{e}^{\frac{1}{4}x^4-\frac{1}{3}x^3}dx\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.332 (sec). Leaf size: 44

```
DSolve[x*(-2 + x^4)*y[x] - x^2*(1 + x)*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to e^{\frac{x^3}{3}}\left(c_2\int_1^x e^{\frac{1}{12}K[1]^3(3K[1]-4)}dK[1] + c_1\right)$$

3.58 problem 1059

Internal problem ID [9392]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1059.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + x^4 y' - y x^3 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 63

```
dsolve(diff(diff(y(x),x),x)+x^4*diff(y(x),x)-x^3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{9\,\mathrm{e}^{-\frac{x^5}{5}} \left(\frac{25 c_2 x^5 \,\mathrm{WhittakerM}\left(\frac{2}{5}, \frac{9}{10}, \frac{x^5}{5}\right) \mathrm{e}^{\frac{x^5}{10}}}{9} + \frac{25 c_1 x^3 \,\mathrm{e}^{\frac{x^5}{5}}}{9} + 5^{\frac{3}{5}} c_2 \left(x^5\right)^{\frac{2}{5}} \left(x^5 + 4\right)\right)}{25 x^2}$$

✓ Solution by Mathematica

Time used: 0.166 (sec). Leaf size: 39

```
DSolve[-(x^3*y[x]) + x^4*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x - \frac{c_2 \sqrt[5]{x^5}\,\Gamma\left(-\frac{1}{5}, \frac{x^5}{5}\right)}{5\sqrt[5]{5}}$$

3.59 problem 1060

Internal problem ID [9393]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1060.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + a\,x^{q-1}y' + b\,x^{q-2}y = 0$$

✓ Solution by Maple
Time used: 0.437 (sec). Leaf size: 71

```
dsolve(diff(diff(y(x),x),x)+a*x^(q-1)*diff(y(x),x)+b*x^(q-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{a\,x^q}{q}} x \left(\mathrm{KummerM}\left(1 - \frac{b}{aq}, 1 + \frac{1}{q}, \frac{a\,x^q}{q}\right) c_1 + \mathrm{KummerU}\left(1 - \frac{b}{aq}, 1 + \frac{1}{q}, \frac{a\,x^q}{q}\right) c_2\right)$$

✓ Solution by Mathematica
Time used: 0.072 (sec). Leaf size: 81

```
DSolve[b*x^(-2 + q)*y[x] + a*x^(-1 + q)*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to c_2 q^{-1/q} a^{\frac{1}{q}} (x^q)^{\frac{1}{q}}\ \mathrm{Hypergeometric1F1}\left(\frac{a+b}{aq}, 1 + \frac{1}{q}, -\frac{ax^q}{q}\right) + c_1\ \mathrm{Hypergeometric1F1}\left(\frac{b}{aq}, \frac{q-1}{q}, -\frac{ax^q}{q}\right)$$

3.60 problem 1061

Internal problem ID [9394]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1061.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y'\sqrt{x} + \left(\frac{1}{4\sqrt{x}} + \frac{x}{4} - 9\right) y = x\,\mathrm{e}^{-\frac{x^{\frac{3}{2}}}{3}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(diff(y(x),x),x)+diff(y(x),x)*x^(1/2)+(1/4/x^(1/2)+1/4*x-9)*y(x)-x*exp(-1/3*x^(3/
```

$$y(x) = -\frac{\mathrm{e}^{-\frac{x^{\frac{3}{2}}}{3}} \left(-9\sinh\left(3x\right) c_2 - 9\cosh\left(3x\right) c_1 + x\right)}{9}$$

✓ Solution by Mathematica

Time used: 0.137 (sec). Leaf size: 45

```
DSolve[-(x/E^(x^(3/2)/3)) + (-9 + 1/(4*Sqrt[x]) + x/4)*y[x] + Sqrt[x]*y'[x] + y''[x] == 0,y[
```

$$y(x) \to \frac{1}{18} e^{-\frac{1}{3}\left(\sqrt{x}+9\right)x}\left(-2e^{3x}x + 3c_2e^{6x} + 18c_1\right)$$

3.61 problem 1062

Internal problem ID [9395]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1062.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{y'}{\sqrt{x}} + \frac{\left(x + \sqrt{x} - 8\right) y}{4x^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(diff(y(x),x),x)-diff(y(x),x)/x^(1/2)+1/4*(x+x^(1/2)-8)*y(x)/x^2=0,y(x), singsol=
```

$$y(x) = \frac{e^{\sqrt{x}}(c_2 x^3 + c_1)}{x}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 30

```
DSolve[((-8 + Sqrt[x] + x)*y[x])/(4*x^2) - y'[x]/Sqrt[x] + y''[x] == 0,y[x],x,IncludeSingula
```

$$y(x) \to \frac{e^{\sqrt{x}}(c_2 x^3 + 3c_1)}{3x}$$

3.62 problem 1063

Internal problem ID [9396]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1063.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - (2\,e^{x} + 1)\,y' + e^{2x}y = e^{3x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
dsolve(diff(diff(y(x),x),x)-(2*exp(x)+1)*diff(y(x),x)+exp(2*x)*y(x)-exp(3*x)=0,y(x), singsol
```

$$y(x) = e^{\frac{x}{2}+e^{x}} \sinh\left(\frac{x}{2}\right) c_2 + e^{\frac{x}{2}+e^{x}} \cosh\left(\frac{x}{2}\right) c_1 + e^{x} + 2$$

✓ Solution by Mathematica

Time used: 0.078 (sec). Leaf size: 28

```
DSolve[-E^(3*x) + E^(2*x)*y[x] - (1 + 2*E^x)*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolut
```

$$y(x) \to e^{x} + c_1 e^{e^{x}} + c_2 e^{x+e^{x}} + 2$$

3.63 problem 1064

Internal problem ID [9397]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1064.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + ay' + yb = -\tan(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 146

```
dsolve(diff(diff(y(x),x),x)+a*diff(y(x),x)+tan(x)+b*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-e^{\frac{\left(-a+\sqrt{a^2-4b}\right)x}{2}}\left(\int \tan(x)\, e^{-\frac{\left(-a+\sqrt{a^2-4b}\right)x}{2}} dx\right) + e^{-\frac{\left(a+\sqrt{a^2-4b}\right)x}{2}}\left(\int \tan(x)\, e^{\frac{\left(a+\sqrt{a^2-4b}\right)x}{2}} dx\right) + e^{\frac{\left(-a+\sqrt{a^2-4b}\right)x}{2}}}{\sqrt{a^2-4b}}$$

✓ Solution by Mathematica

Time used: 0.752 (sec). Leaf size: 502

```
DSolve[Tan[x] + b*y[x] + a*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{1}{2}x\left(\sqrt{a^2-4b}+a\right)}\left(b\left(i\sqrt{a^2-4b}-ia+4\right)e^{\frac{1}{2}x\left(\sqrt{a^2-4b}+a+4i\right)}\text{Hypergeometric2F1}\left(1,-\frac{ia}{4}-\frac{1}{4}i\sqrt{a^2-4b}+1\right.\right.}{}$$

3.64 problem 1065

Internal problem ID [9398]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1065.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 2ny' \cot(x) + (-a^2 + n^2) y = 0$$

✓ Solution by Maple

Time used: 0.359 (sec). Leaf size: 60

```
dsolve(diff(diff(y(x),x),x)+2*n*diff(y(x),x)*cot(x)+(-a^2+n^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sin(x)^{-n+\frac{1}{2}} \left(c_1 \operatorname{LegendreP}\left(-\frac{1}{2} + \sqrt{-a^2 + 2n^2}, n - \frac{1}{2}, \cos(x)\right) + c_2 \operatorname{LegendreQ}\left(-\frac{1}{2} + \sqrt{-a^2 + 2n^2}, n - \frac{1}{2}, \cos(x)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.247 (sec). Leaf size: 83

```
DSolve[(-a^2 + n^2)*y[x] + 2*n*Cot[x]*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to (-\sin^2(x))^{\frac{1}{4}-\frac{n}{2}} \left(c_1 P^{n-\frac{1}{2}}_{\sqrt{2n^2-a^2}-\frac{1}{2}}(\cos(x)) + c_2 Q^{n-\frac{1}{2}}_{\sqrt{2n^2-a^2}-\frac{1}{2}}(\cos(x))\right)$$

3.65 problem 1066

Internal problem ID [9399]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1066.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' \tan(x) + y \cos(x)^2 = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 15

```
dsolve(diff(diff(y(x),x),x)+diff(y(x),x)*tan(x)+y(x)*cos(x)^2=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(\sin(x)) + c_2 \cos(\sin(x))$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 18

```
DSolve[Cos[x]^2*y[x] + Tan[x]*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \sin(\sin(x)) + c_1 \cos(\sin(x))$$

3.66 problem 1067

Internal problem ID [9400]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1067.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' \tan(x) - y\cos(x)^2 = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 17

```
dsolve(diff(diff(y(x),x),x)+diff(y(x),x)*tan(x)-y(x)*cos(x)^2=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\sin(x)} + c_2 e^{-\sin(x)}$$

✓ Solution by Mathematica
Time used: 0.08 (sec). Leaf size: 21

```
DSolve[-(Cos[x]^2*y[x]) + Tan[x]*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to c_1 \cosh(\sin(x)) + ic_2 \sinh(\sin(x))$$

3.67 problem 1068

Internal problem ID [9401]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1068.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' \cot(x) + v(v+1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.437 (sec). Leaf size: 45

```
dsolve(diff(diff(y(x),x),x)+diff(y(x),x)*cot(x)+v*(v+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \,\text{hypergeom}\left(\left[-\frac{v}{2}, \frac{1}{2}+\frac{v}{2}\right], \left[\frac{1}{2}\right], \cos(x)^2\right) + c_2 \cos(x)\,\text{hypergeom}\left(\left[1+\frac{v}{2}, \frac{1}{2}-\frac{v}{2}\right], \left[\frac{3}{2}\right], \cos(x)^2\right)$$

✓ Solution by Mathematica

Time used: 0.178 (sec). Leaf size: 20

```
DSolve[v*(1 + v)*y[x] + Cot[x]*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \,\text{LegendreP}(v, \cos(x)) + c_2 \,\text{LegendreQ}(v, \cos(x))$$

3.68 problem 1069

Internal problem ID [9402]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1069.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y' \cot(x) + y \sin(x)^2 = 0}$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 15

```
dsolve(diff(diff(y(x),x),x)-diff(y(x),x)*cot(x)+y(x)*sin(x)^2=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(\cos(x)) + c_2 \cos(\cos(x))$$

✓ Solution by Mathematica

Time used: 0.078 (sec). Leaf size: 19

```
DSolve[Sin[x]^2*y[x] - Cot[x]*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(\cos(x)) - c_2 \sin(\cos(x))$$

3.69 problem 1070

Internal problem ID [9403]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1070.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + ay' \tan(x) + yb = 0$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 60

```
dsolve(diff(diff(y(x),x),x)+a*diff(y(x),x)*tan(x)+b*y(x)=0,y(x), singsol=all)
```

$$y(x) = \cos(x)^{\frac{1}{2}+\frac{a}{2}} \left(c_1 \operatorname{LegendreP}\left(\frac{\sqrt{a^2+4b}}{2} - \frac{1}{2}, \frac{1}{2} + \frac{a}{2}, \sin(x)\right) + c_2 \operatorname{LegendreQ}\left(\frac{\sqrt{a^2+4b}}{2} - \frac{1}{2}, \frac{1}{2} + \frac{a}{2}, \sin(x)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.42 (sec). Leaf size: 129

```
DSolve[b*y[x] + a*Tan[x]*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \text{Hypergeometric2F1}\left(\frac{1}{4}\left(-a-\sqrt{a^2+4b}\right), \frac{1}{4}\left(\sqrt{a^2+4b}-a\right), \frac{1-a}{2}, \cos^2(x)\right) + i^{a+1} c_2 \cos^{a+1}(x) \text{Hypergeometric2F1}\left(\frac{1}{4}\left(a-\sqrt{a^2+4b}+2\right), \frac{1}{4}\left(a+\sqrt{a^2+4b}+2\right), \frac{a+3}{2}, \cos^2(x)\right)$$

3.70 problem 1071

Internal problem ID [9404]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1071.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2ay' \cot(xa) + (-a^2 + b^2) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(diff(y(x),x),x)+2*a*diff(y(x),x)*cot(a*x)+(-a^2+b^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \csc(ax)(c_1 \sin(bx) + c_2 \cos(bx))$$

✓ Solution by Mathematica

Time used: 0.118 (sec). Leaf size: 43

```
DSolve[(-a^2 + b^2)*y[x] + 2*a*Cot[a*x]*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{2} e^{-ibx} \csc(ax) \left(2c_1 - \frac{ic_2 e^{2ibx}}{b}\right)$$

3.71 problem 1072

Internal problem ID [9405]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1072.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + ap''(x)\, y' + \left(a + bp(x) - 4nap(x)^2\right) y = 0}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+a*diff(diff(p(x),x),x)*diff(y(x),x)+(a+b*p(x)-4*n*a*p(x)^2)*y(x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a + b*p[x] - 4*a*n*p[x]^2)*y[x] + a*y'[x]*Derivative[2][p][x] + y''[x] == 0,y[x],x,I
```

Not solved

3.72 problem 1073

Internal problem ID [9406]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1073.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{\left(11\,\mathrm{WeierstrassP}\left(x,a,b\right)\mathrm{WeierstrassPPrime}\left(x,a,b\right) - 6\,\mathrm{WeierstrassP}\left(x,a,b\right)^2 + \frac{a}{2}\right)y'}{\mathrm{WeierstrassPPrime}\left(x,a,b\right) + \mathrm{WeierstrassP}\left(x,a,b\right)^2} + \frac{(\mathrm{Weierst}}{}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+(11*WeierstrassP(x,a,b)*WeierstrassPPrime(x,a,b)-6*WeierstrassP(
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[((-(WeierstrassP[x, {a, b}]*(-1/2*a + 6*WeierstrassP[x, {a, b}]^2)) - WeierstrassP[x,
```

Not solved

3.73 problem 1075

Internal problem ID [9407]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1075.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + f(x)\, y' + g(x)\, y = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+f(x)*diff(y(x),x)+g(x)*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[g[x]*y[x] + f[x]*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

3.74 problem 1076

Internal problem ID [9408]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1076.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + f(x)\,y' + (f'(x) + a)\,y = g(x)}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+f(x)*diff(y(x),x)+(diff(f(x),x)+a)*y(x)-g(x)=0,y(x), singsol=all
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-g[x] + y[x]*(a + Derivative[1][f][x]) + f[x]*y'[x] + y''[x] == 0,y[x],x,IncludeSingu
```

Not solved

3.75 problem 1077

Internal problem ID [9409]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1077.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (f(x)\, a + b)\, y' + (c f(x) + d)\, y = 0}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+(a*f(x)+b)*diff(y(x),x)+(c*f(x)+d)*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(d + c*f[x])*y[x] + (b + a*f[x])*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions
```

Not solved

3.76 problem 1078

Internal problem ID [9410]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1078.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + f(x)\, y' + \left(\frac{f(x)^2}{4} + \frac{f'(x)}{2} + a\right) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(diff(y(x),x),x)+f(x)*diff(y(x),x)+(1/4*f(x)^2+1/2*diff(f(x),x)+a)*y(x)=0,y(x), s
```

$$y(x) = e^{-\frac{\left(\int f(x)dx\right)}{2}} \left(c_1 \sinh\left(\sqrt{-a}\, x\right) + c_2 \cosh\left(\sqrt{-a}\, x\right)\right)$$

✓ Solution by Mathematica

Time used: 0.124 (sec). Leaf size: 69

```
DSolve[y[x]*(a + f[x]^2/4 + Derivative[1][f][x]/2) + f[x]*y'[x] + y''[x] == 0,y[x],x,Include
```

$$y(x) \to \frac{\left(2\sqrt{a}c_1 - ic_2 e^{2i\sqrt{a}x}\right) \exp\left(-\frac{1}{2} \int_1^x f(K[1])dK[1] - i\sqrt{a}x\right)}{2\sqrt{a}}$$

3.77 problem 1079

Internal problem ID [9411]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1079.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$y'' - \frac{af'(x)\,y'}{f\,(x)} + bf(x)^{2a}\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(diff(y(x),x),x)-a*diff(f(x),x)/f(x)*diff(y(x),x)+b*f(x)^(2*a)*y(x)=0,y(x), sings
```

$$y(x) = c_1 \mathrm{e}^{i\sqrt{b}\,\left(\int f(x)^a dx\right)} + c_2 \mathrm{e}^{-i\sqrt{b}\,\left(\int f(x)^a dx\right)}$$

✓ Solution by Mathematica

Time used: 0.556 (sec). Leaf size: 307

```
DSolve[b*f[x]^(2*a)*y[x] - (a*Derivative[1][f][x]*y'[x])/f[x] + y''[x] == 0,y[x],x,IncludeSi
```

$$y(x) \to -\frac{\sqrt{c_1}\exp\left(-\int_1^x -i\sqrt{b}f(K[1])^a dK[1] - c_2\right)\left(-1+\exp\left(2\left(\int_1^x -i\sqrt{b}f(K[1])^a dK[1] + c_2\right)\right)\right)}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{c_1}\exp\left(-\int_1^x -i\sqrt{b}f(K[1])^a dK[1] - c_2\right)\left(-1+\exp\left(2\left(\int_1^x -i\sqrt{b}f(K[1])^a dK[1] + c_2\right)\right)\right)}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{c_1}\exp\left(-\int_1^x i\sqrt{b}f(K[2])^a dK[2] - c_2\right)\left(-1+\exp\left(2\left(\int_1^x i\sqrt{b}f(K[2])^a dK[2] + c_2\right)\right)\right)}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{c_1}\exp\left(-\int_1^x i\sqrt{b}f(K[2])^a dK[2] - c_2\right)\left(-1+\exp\left(2\left(\int_1^x i\sqrt{b}f(K[2])^a dK[2] + c_2\right)\right)\right)}{\sqrt{2}}$$

3.78 problem 1080

Internal problem ID [9412]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1080.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \left(\frac{f'(x)}{f(x)} + 2a\right) y' + \left(\frac{f'(x)\,a}{f(x)} + a^2 - b^2 f(x)^2\right) y = 0$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 65

```
dsolve(diff(diff(y(x),x),x)-(diff(f(x),x)/f(x)+2*a)*diff(y(x),x)+(a*diff(f(x),x)/f(x)+a^2-b^
```

$$y(x) = \mathrm{e}^{\int \frac{\mathrm{e}^{2b\left(\int f(x)dx\right)}\left((-f(x)b+a)\mathrm{e}^{-2b\left(\int f(x)dx-c_1\right)}-f(x)b-a\right)}{-\mathrm{e}^{2b\left(\int f(x)dx\right)}+\mathrm{e}^{2c_1b}}dx} c_2$$

✓ Solution by Mathematica

Time used: 0.112 (sec). Leaf size: 47

```
DSolve[y[x]*(a^2 - b^2*f[x]^2 + (a*Derivative[1][f][x])/f[x]) - (2*a + Derivative[1][f][x]/f
```

$$y(x) \to e^{ax}\left(c_1 \exp\left(b\int_1^x f(K[1])dK[1]\right) + c_2 \exp\left(-b\int_1^x f(K[2])dK[2]\right)\right)$$

3.79 problem 1081

Internal problem ID [9413]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1081.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{f(x)\, f'''(x)\, y'}{f(x)^2 + b^2} - \frac{a^2 f'(x)^2 y}{f(x)^2 + b^2} = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+f(x)*diff(diff(diff(f(x),x),x),x)/(f(x)^2+b^2)*diff(y(x),x)-a^2*
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-((a^2*y[x]*Derivative[1][f][x]^2)/(b^2 + f[x]^2)) + (f[x]*(f^3)[x]*y'[x])/(b^2 + f[x
```

Not solved

3.80 problem 1084

Internal problem ID [9414]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1084.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \left(\frac{2f'(x)}{f(x)} + \frac{g''(x)}{g'(x)} - \frac{g'(x)}{g(x)}\right) y' + \left(\frac{f'(x)\left(\frac{2f'(x)}{f(x)} + \frac{g''(x)}{g'(x)} - \frac{g'(x)}{g(x)}\right)}{f(x)} - \frac{f''(x)}{f(x)} - \frac{v^2 g'(x)^2}{g(x)^2} + g'(x)^2\right) y =$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 20

```
dsolve(diff(diff(y(x),x),x)-(2*diff(f(x),x)/f(x)+diff(diff(g(x),x),x)/diff(g(x),x)-diff(g(x)
```

$$y(x) = f(x) (\mathrm{BesselJ}(v, g(x))\, c_1 + \mathrm{BesselY}(v, g(x))\, c_2)$$

✓ Solution by Mathematica

Time used: 0.216 (sec). Leaf size: 35

```
DSolve[-(y'[x]*((2*Derivative[1][f][x])/f[x] - Derivative[1][g][x]/g[x] + Derivative[2][g][x
```

$$y(x) \to f(x) \left(c_1 \,\mathrm{BesselJ}\left(\sqrt{v^2}, g(x)\right) + c_2 \,\mathrm{BesselY}\left(\sqrt{v^2}, g(x)\right)\right)$$

3.81 problem 1085

Internal problem ID [9415]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1085.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \left(\frac{g''(x)}{g'(x)} + \frac{(2v-1)\,g'(x)}{g(x)} + \frac{2h'(x)}{h(x)}\right) y' + \left(\frac{h'(x)\left(\frac{g''(x)}{g'(x)} + \frac{(2v-1)g'(x)}{g(x)} + \frac{2h'(x)}{h(x)}\right)}{h(x)} - \frac{h''(x)}{h(x)} + g'(x)^2\right) y$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 24

```
dsolve(diff(diff(y(x),x),x)-(diff(diff(g(x),x),x)/diff(g(x),x)+(2*v-1)*diff(g(x),x)/g(x)+2*d
```

$$y(x) = g(x)^v h(x) \left(\operatorname{BesselJ}(v, g(x))\, c_1 + \operatorname{BesselY}(v, g(x))\, c_2\right)$$

✓ Solution by Mathematica

Time used: 0.2 (sec). Leaf size: 27

```
DSolve[-(y'[x]*(((-1 + 2*v)*Derivative[1][g][x])/g[x] + (2*Derivative[1][h][x])/h[x] + Deriv
```

$$y(x) \to h(x)g(x)^v(c_1 \operatorname{BesselJ}(v, g(x)) + c_2 \operatorname{BesselY}(v, g(x)))$$

3.82 problem 1086

Internal problem ID [9416]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1086.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{4y'' + 9xy = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(4*diff(diff(y(x),x),x)+9*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{AiryAi}\left(-\frac{3^{\frac{2}{3}} 2^{\frac{1}{3}} x}{2}\right) + c_2 \operatorname{AiryBi}\left(-\frac{3^{\frac{2}{3}} 2^{\frac{1}{3}} x}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 42

```
DSolve[9*x*y[x] + 4*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{AiryAi}\left(\sqrt[3]{-1}\left(\frac{3}{2}\right)^{2/3} x\right) + c_2 \operatorname{AiryBi}\left(\sqrt[3]{-1}\left(\frac{3}{2}\right)^{2/3} x\right)$$

3.83 problem 1087

Internal problem ID [9417]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1087.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4y'' - (x^2 + a)\, y = 0}$$

✓ Solution by Maple
Time used: 0.11 (sec). Leaf size: 33

```
dsolve(4*diff(diff(y(x),x),x)-(x^2+a)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2 \operatorname{WhittakerW}\left(-\frac{a}{8}, \frac{1}{4}, \frac{x^2}{2}\right) + c_1 \operatorname{WhittakerM}\left(-\frac{a}{8}, \frac{1}{4}, \frac{x^2}{2}\right)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 36

```
DSolve[(-a - x^2)*y[x] + 4*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{ParabolicCylinderD}\left(\frac{1}{4}(-a-2), x\right) + c_2 \operatorname{ParabolicCylinderD}\left(\frac{a-2}{4}, ix\right)$$

3.84 problem 1088

Internal problem ID [9418]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1088.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4y'' + 4y' \tan(x) - \left(5\tan(x)^2 + 2\right) y = 0$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 31

```
dsolve(4*diff(diff(y(x),x),x)+4*diff(y(x),x)*tan(x)-(5*tan(x)^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{i\cos(x)\sin(x)\,c_2 - \ln(i\cos(x) + \sin(x))\,c_2 + c_1}{\sqrt{\cos(x)}}$$

✓ Solution by Mathematica

Time used: 0.248 (sec). Leaf size: 97

```
DSolve[(-2 - 5*Tan[x]^2)*y[x] + 4*Tan[x]*y'[x] + 4*y''[x] == 0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to \frac{3(-1)^{7/8}c_2\operatorname{arcsinh}\left(\frac{(1+i)\sqrt[4]{-\cos^4(x)}}{\sqrt{2}}\right) + 3\sqrt[8]{-1}c_2\sqrt[4]{-\cos^4(x)}\sqrt{1+i\sqrt{-\cos^4(x)}} - 2(-1)^{7/8}c_1}{2\sqrt[8]{-\cos^4(x)}}$$

3.85 problem 1089

Internal problem ID [9419]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1089.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$ay'' - (ab + c + x)\, y' + (b(x + c) + d)\, y = 0$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 58

```
dsolve(a*diff(diff(y(x),x),x)-(a*b+c+x)*diff(y(x),x)+(b*(x+c)+d)*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{bx}\left(\text{KummerM}\left(-\frac{d}{2}, \frac{1}{2}, \frac{(ab-c-x)^2}{2a}\right) c_1 + \text{KummerU}\left(-\frac{d}{2}, \frac{1}{2}, \frac{(ab-c-x)^2}{2a}\right) c_2\right)$$

✓ Solution by Mathematica
Time used: 0.065 (sec). Leaf size: 63

```
DSolve[(d + b*(c + x))*y[x] - (a*b + c + x)*y'[x] + a*y''[x] == 0,y[x],x,IncludeSingularSolu
```

$$y(x) \to e^{bx}\left(c_1 \text{HermiteH}\left(d, \frac{-ab+c+x}{\sqrt{2}\sqrt{a}}\right) + c_2 \text{Hypergeometric1F1}\left(-\frac{d}{2}, \frac{1}{2}, \frac{(-ab+c+x)^2}{2a}\right)\right)$$

3.86 problem 1090

Internal problem ID [9420]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1090.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$a^2 y'' + a\left(a^2 - 2b\,\mathrm{e}^{-xa}\right) y' + b^2 \mathrm{e}^{-2xa} y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(a^2*diff(diff(y(x),x),x)+a*(a^2-2*b*exp(-a*x))*diff(y(x),x)+b^2*exp(-2*a*x)*y(x)=0,y(
```

$$y(x) = \mathrm{e}^{-\frac{x\,a^3+2b\,\mathrm{e}^{-ax}}{2a^2}} \left(c_1 \sinh\left(\frac{ax}{2}\right) + c_2 \cosh\left(\frac{ax}{2}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.073 (sec). Leaf size: 45

```
DSolve[(b^2*y[x])/E^(2*a*x) + a*(a^2 - (2*b)/E^(a*x))*y'[x] + a^2*y''[x] == 0,y[x],x,Include
```

$$y(x) \to \frac{e^{-\frac{be^{-ax}}{a^2}-ax}\left(a^2 c_1 e^{ax} - bc_2\right)}{a^2}$$

3.87 problem 1091

Internal problem ID [9421]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1091.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x(y'' + y) = \cos(x)}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 35

```
dsolve(x*(diff(diff(y(x),x),x)+y(x))-cos(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sin(x)\,\mathrm{Ci}(2x)}{2} - \frac{\mathrm{Si}(2x)\cos(x)}{2} + \frac{(2c_2 + \ln(x))\sin(x)}{2} + \cos(x)\,c_1$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 41

```
DSolve[-Cos[x] + x*(y[x] + y''[x]) == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(\mathrm{CosIntegral}(2x)\sin(x) - \mathrm{Si}(2x)\cos(x) + \log(x)\sin(x)) + c_1\cos(x) + c_2\sin(x)$$

3.88 problem 1092

Internal problem ID [9422]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1092.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x + (x + a)\, y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 29

```
dsolve(x*diff(diff(y(x),x),x)+(x+a)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{WhittakerM}\left(-\frac{ia}{2}, \frac{1}{2}, 2ix\right) + c_2 \operatorname{WhittakerW}\left(-\frac{ia}{2}, \frac{1}{2}, 2ix\right)$$

✓ Solution by Mathematica
Time used: 0.142 (sec). Leaf size: 53

```
DSolve[(a + x)*y[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-ix}x\left(c_2 \operatorname{Hypergeometric1F1}\left(\frac{ia}{2} + 1, 2, 2ix\right) + c_1 \operatorname{HypergeometricU}\left(\frac{ia}{2} + 1, 2, 2ix\right)\right)$$

3.89 problem 1093

Internal problem ID [9423]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1093.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y''x + y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(x*diff(diff(y(x),x),x)+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \ln(x)\, c_2 + c_1$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 13

```
DSolve[y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \log(x) + c_2$$

3.90 problem 1094

Internal problem ID [9424]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1094.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y''x + y' + ay = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x*diff(diff(y(x),x),x)+diff(y(x),x)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}\left(0, 2\sqrt{a}\,\sqrt{x}\right) + c_2 \operatorname{BesselY}\left(0, 2\sqrt{a}\,\sqrt{x}\right)$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 41

```
DSolve[a*y[x] + y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}\left(0, 2\sqrt{a}\sqrt{x}\right) + 2c_2 \operatorname{BesselY}\left(0, 2\sqrt{a}\sqrt{x}\right)$$

3.91 problem 1095

Internal problem ID [9425]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1095.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x + y' + lxy = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve(x*diff(diff(y(x),x),x)+diff(y(x),x)+l*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}\left(0, \sqrt{l}\, x\right) + c_2 \operatorname{BesselY}\left(0, \sqrt{l}\, x\right)$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 30

```
DSolve[l*x*y[x] + y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}\left(0, \sqrt{l}x\right) + c_2 \operatorname{BesselY}\left(0, \sqrt{l}x\right)$$

3.92 problem 1096

Internal problem ID [9426]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1096.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x + y' + (x + a)\, y = 0}$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 39

```
dsolve(x*diff(diff(y(x),x),x)+diff(y(x),x)+(x+a)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-ix}\left(\mathrm{KummerM}\left(\frac{1}{2} + \frac{ia}{2}, 1, 2ix\right) c_1 + \mathrm{KummerU}\left(\frac{1}{2} + \frac{ia}{2}, 1, 2ix\right) c_2\right)$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 55

```
DSolve[(a + x)*y[x] + y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-ix}\left(c_1\, \mathrm{HypergeometricU}\left(\frac{ia}{2} + \frac{1}{2}, 1, 2ix\right) + c_2\, \mathrm{LaguerreL}\left(-\frac{1}{2}i(a - i), 2ix\right)\right)$$

3.93 problem 1097

Internal problem ID [9427]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1097.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$y''x - y' + ay = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 70

```
dsolve(x*diff(diff(y(x),x),x)-diff(y(x),x)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\text{BesselJ}\left(1, 2\sqrt{a}\,\sqrt{x}\right)\sqrt{x}\,c_1 + \text{BesselY}\left(1, 2\sqrt{a}\,\sqrt{x}\right)\sqrt{x}\,c_2 - \sqrt{a}\,x\big(c_1\,\text{BesselJ}\left(0, 2\sqrt{a}\,\sqrt{x}\right) + c_2\,\text{BesselY}\,(}{\sqrt{a}}$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 45

```
DSolve[a*y[x] - y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2ax\big(c_1\,\text{BesselJ}\left(2, 2\sqrt{a}\sqrt{x}\right) - c_2\,\text{BesselY}\left(2, 2\sqrt{a}\sqrt{x}\right)\big)$$

3.94 problem 1098

Internal problem ID [9428]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1098.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$y''x - y' - ya\,x^3 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*diff(diff(y(x),x),x)-diff(y(x),x)-y(x)*a*x^3=0,y(x), singsol=all)
```

$$y(x) = c_1 \sinh\left(\frac{x^2\sqrt{a}}{2}\right) + c_2 \cosh\left(\frac{x^2\sqrt{a}}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 41

```
DSolve[-(a*x^3*y[x]) - y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cosh\left(\frac{\sqrt{a}x^2}{2}\right) + ic_2 \sinh\left(\frac{\sqrt{a}x^2}{2}\right)$$

3.95 problem 1099

Internal problem ID [9429]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1099.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y''x - y' + x^3\left(e^{x^2} - v^2\right)y = 0$$

✓ Solution by Maple
Time used: 0.109 (sec). Leaf size: 25

```
dsolve(x*diff(y(x),x$2)-diff(y(x),x)+x^3*(exp(x^2)-v^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}\left(v, e^{\frac{x^2}{2}}\right) + c_2 \operatorname{BesselY}\left(v, e^{\frac{x^2}{2}}\right)$$

✓ Solution by Mathematica
Time used: 1.118 (sec). Leaf size: 46

```
DSolve[x*y''[x]-y'[x]+x^3*(Exp[x^2]-v^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{Gamma}(1-v) \operatorname{BesselJ}\left(-v, \sqrt{e^{x^2}}\right) + c_2 \operatorname{Gamma}(v+1) \operatorname{BesselJ}\left(v, \sqrt{e^{x^2}}\right)$$

3.96 problem 1100

Internal problem ID [9430]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1100.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''x + 2y' - xy = \mathrm{e}^x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve(x*diff(diff(y(x),x),x)+2*diff(y(x),x)-x*y(x)-exp(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{2\sinh(x)\, c_2 + 2\cosh(x)\, c_1 + x\,\mathrm{e}^x}{2x}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 37

```
DSolve[-E^x - x*y[x] + 2*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}(e^{2x}(2x - 1 + 2c_2) + 4c_1)}{4x}$$

3.97 problem 1101

Internal problem ID [9431]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1101.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x + 2y' + yax = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(x*diff(diff(y(x),x),x)+2*diff(y(x),x)+y(x)*a*x=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sinh\left(\sqrt{-a}\, x\right) + c_2 \cosh\left(\sqrt{-a}\, x\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 52

```
DSolve[a*x*y[x] + 2*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-i\sqrt{a}x} - \frac{ic_2 e^{i\sqrt{a}x}}{\sqrt{a}}}{2x}$$

3.98 problem 1102

Internal problem ID [9432]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1102.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y''x + 2y' + a\,x^2 y = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 33

```
dsolve(x*diff(diff(y(x),x),x)+2*diff(y(x),x)+a*x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \operatorname{BesselJ}\left(\frac{1}{3}, \frac{2\sqrt{a}\,x^{\frac{3}{2}}}{3}\right) + c_2 \operatorname{BesselY}\left(\frac{1}{3}, \frac{2\sqrt{a}\,x^{\frac{3}{2}}}{3}\right)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 36

```
DSolve[a*x^2*y[x] + 2*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 \operatorname{AiryAi}\left(\sqrt[3]{-a}x\right) + c_2 \operatorname{AiryBi}\left(\sqrt[3]{-a}x\right)}{x}$$

3.99 problem 1103

Internal problem ID [9433]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1103.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y''x - 2y' + ay = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 80

```
dsolve(x*diff(diff(y(x),x),x)-2*diff(y(x),x)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{x}\,(c_1(ax-2)\,\text{BesselJ}\,(1,2\sqrt{a}\,\sqrt{x}) + c_2(ax-2)\,\text{BesselY}\,(1,2\sqrt{a}\,\sqrt{x}) + 2\sqrt{a}\,\sqrt{x}\,(c_1\,\text{BesselJ}\,(0,2\sqrt{a}}{a}$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 56

```
DSolve[a*y[x] - 2*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2a^{3/2}x^{3/2}\left(3c_1\,\text{BesselJ}\left(3,2\sqrt{a}\sqrt{x}\right) - ic_2\,\text{BesselY}\left(3,2\sqrt{a}\sqrt{x}\right)\right)$$

3.100 problem 1104

Internal problem ID [9434]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1104.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y''x + vy' + ay = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 83

```
dsolve(x*diff(diff(y(x),x),x)+v*diff(y(x),x)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(-\sqrt{a}\,\sqrt{x}\ \mathrm{BesselJ}\left(v+1, 2\sqrt{a}\,\sqrt{x}\right) c_1 - \sqrt{a}\,\sqrt{x}\ \mathrm{BesselY}\left(v+1, 2\sqrt{a}\,\sqrt{x}\right) c_2 + v\left(\mathrm{BesselJ}\left(v, 2\sqrt{a}\,\sqrt{x}\right) c\right.\right.}{\sqrt{a}}$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 77

```
DSolve[a*y[x] + v*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to a^{\frac{1}{2}-\frac{v}{2}} x^{\frac{1}{2}-\frac{v}{2}} \left(c_2 \operatorname{Gamma}(2-v) \operatorname{BesselJ}\left(1-v, 2\sqrt{a}\sqrt{x}\right) + c_1 \operatorname{Gamma}(v) \operatorname{BesselJ}\left(v-1, 2\sqrt{a}\sqrt{x}\right)\right)$$

3.101 problem 1105

Internal problem ID [9435]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1105.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x + ay' + ybx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(x*diff(diff(y(x),x),x)+a*diff(y(x),x)+b*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(\operatorname{BesselY}\left(\frac{a}{2} - \frac{1}{2}, \sqrt{b}\,x\right) c_2 + \operatorname{BesselJ}\left(\frac{a}{2} - \frac{1}{2}, \sqrt{b}\,x\right) c_1\right) x^{-\frac{a}{2}+\frac{1}{2}}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 54

```
DSolve[b*x*y[x] + a*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^{\frac{1}{2}-\frac{a}{2}}\left(c_1 \operatorname{BesselJ}\left(\frac{a-1}{2}, \sqrt{b}x\right) + c_2 \operatorname{BesselY}\left(\frac{a-1}{2}, \sqrt{b}x\right)\right)$$

3.102 problem 1106

Internal problem ID [9436]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1106.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$y''x + ay' + b\,x^{\mathrm{a1}}y = 0$$

✓ Solution by Maple

Time used: 0.281 (sec). Leaf size: 71

```
dsolve(x*diff(diff(y(x),x),x)+a*diff(y(x),x)+b*x^a1*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(\mathrm{BesselY}\left(\frac{a-1}{\mathrm{a1}+1}, \frac{2\sqrt{b}\,x^{\frac{\mathrm{a1}}{2}+\frac{1}{2}}}{\mathrm{a1}+1}\right)c_2 + \mathrm{BesselJ}\left(\frac{a-1}{\mathrm{a1}+1}, \frac{2\sqrt{b}\,x^{\frac{\mathrm{a1}}{2}+\frac{1}{2}}}{\mathrm{a1}+1}\right)c_1\right)x^{-\frac{a}{2}+\frac{1}{2}}$$

✓ Solution by Mathematica

Time used: 0.175 (sec). Leaf size: 165

```
DSolve[b*x^a1*y[x] + a*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\frac{1}{\mathrm{a1}} + 1\right)^{\frac{a-1}{\mathrm{a1}+1}} \mathrm{a1}^{\frac{a-1}{\mathrm{a1}+1}} b^{\frac{1-a}{2\mathrm{a1}+2}} \left(x^{\mathrm{a1}}\right)^{-\frac{a-1}{2\mathrm{a1}}} \left(c_2\,\mathrm{Gamma}\left(\frac{-a+\mathrm{a1}+2}{\mathrm{a1}+1}\right)\mathrm{BesselJ}\left(\frac{1-a}{\mathrm{a1}+1}, \frac{2\sqrt{b}\left(x^{\mathrm{a1}}\right)^{\frac{\mathrm{a1}+1}{2\mathrm{a1}}}}{\mathrm{a1}+1}\right) + c_1\,\mathrm{Gamma}\left(\frac{a+\mathrm{a1}}{\mathrm{a1}+1}\right)\mathrm{BesselJ}\left(\frac{a-1}{\mathrm{a1}+1}, \frac{2\sqrt{b}\left(x^{\mathrm{a1}}\right)^{\frac{\mathrm{a1}+1}{2\mathrm{a1}}}}{\mathrm{a1}+1}\right)\right)$$

3.103 problem 1107

Internal problem ID [9437]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1107.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x + (x+b)\,y' + ay = 0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 30

```
dsolve(x*diff(diff(y(x),x),x)+(x+b)*diff(y(x),x)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}(\mathrm{KummerM}\,(-a+b,b,x)\,c_1 + \mathrm{KummerU}\,(-a+b,b,x)\,c_2)$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 36

```
DSolve[a*y[x] + (b + x)*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_1\,\mathrm{HypergeometricU}(b-a,b,x) + c_2 L_{a-b}^{b-1}(x))$$

3.104 problem 1108

Internal problem ID [9438]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1108.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y''x + (x + a + b)\, y' + ay = 0$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 26

```
dsolve(x*diff(diff(y(x),x),x)+(x+a+b)*diff(y(x),x)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-x}(\text{KummerU}\,(b, a + b, x)\, c_2 + \text{KummerM}\,(b, a + b, x)\, c_1)$$

✓ Solution by Mathematica
Time used: 0.044 (sec). Leaf size: 33

```
DSolve[a*y[x] + (a + b + x)*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_1 \,\text{HypergeometricU}(b, a + b, x) + c_2 L_{-b}^{a+b-1}(x))$$

3.105 problem 1109

Internal problem ID [9439]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1109.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{y''x - y'x - y = x(x+1)\,\mathrm{e}^{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(x*diff(diff(y(x),x),x)-x*diff(y(x),x)-y(x)-x*(x+1)*exp(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{expIntegral}_1\left(x\right)\mathrm{e}^{x} c_1 x + \left(x^2 + x c_2 - x\ln\left(x\right) - 1\right)\mathrm{e}^{x} - c_1$$

✓ Solution by Mathematica

Time used: 0.302 (sec). Leaf size: 45

```
DSolve[-(E^x*x*(1 + x)) - y[x] - x*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -c_2(e^x x\,\mathrm{ExpIntegralEi}(-x) + 1) + e^x\left(x^2 + x - x\log(-x) - 1\right) + c_1 e^x x$$

3.106 problem 1110

Internal problem ID [9440]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1110.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{y''x - y'x - ay = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 23

```
dsolve(x*diff(diff(y(x),x),x)-x*diff(y(x),x)-a*y(x)=0,y(x), singsol=all)
```

$$y(x) = x(\text{KummerM}\,(a+1,2,x)\,c_1 + \text{KummerU}\,(a+1,2,x)\,c_2)$$

✓ Solution by Mathematica
Time used: 0.159 (sec). Leaf size: 36

```
DSolve[-(a*y[x]) - x*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 G_{1,2}^{2,0}\left(-x \middle| \begin{array}{c} 1-a \\ 0,1 \end{array}\right) + c_1 x \,\text{Hypergeometric1F1}(a+1,2,x)$$

3.107 problem 1111

Internal problem ID [9441]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1111.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{y''x - (x+1)\, y' + y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 13

```
dsolve(x*diff(diff(y(x),x),x)-(x+1)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = e^x c_2 + c_1 x + c_1$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 19

```
DSolve[y[x] - (1 + x)*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x - c_2(x+1)$$

3.108 problem 1112

Internal problem ID [9442]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1112.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x - (x+1)\,y' - 2(x-1)\,y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x*diff(diff(y(x),x),x)-(x+1)*diff(y(x),x)-2*(x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2x}c_1 + c_2\mathrm{e}^{-x}(3x+1)$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 30

```
DSolve[-2*(-1 + x)*y[x] - (1 + x)*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to c_1e^{2x} - \frac{1}{9}c_2e^{-x}(3x+1)$$

3.109 problem 1113

Internal problem ID [9443]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1113.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{y''x + (b - x)\, y' - ay = 0}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 17

```
dsolve(x*diff(diff(y(x),x),x)+(b-x)*diff(y(x),x)-a*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{KummerM}(a, b, x) + c_2 \operatorname{KummerU}(a, b, x)$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 24

```
DSolve[-(a*y[x]) + (b - x)*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{HypergeometricU}(a, b, x) + c_2 L_{-a}^{b-1}(x)$$

3.110 problem 1114

Internal problem ID [9444]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1114.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x - 2(x-1)\,y' - y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 36

```
dsolve(x*diff(diff(y(x),x),x)-2*(x-1)*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = (\text{BesselK}\,(0,-x)\,c_2 - \text{BesselK}\,(1,-x)\,c_2 + c_1(\text{BesselI}\,(0,-x) + \text{BesselI}\,(1,-x)))\,\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.149 (sec). Leaf size: 39

```
DSolve[-y[x] - 2*(-1 + x)*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 G_{1,2}^{2,0}\left(-2x \middle| \begin{array}{c} \frac{1}{2} \\ -1,0 \end{array}\right) + c_1 e^x(\text{BesselI}(0,x) - \text{BesselI}(1,x))$$

3.111 problem 1115

Internal problem ID [9445]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1115.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x - (3x-2)\,y' - (2x-3)\,y = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 47

```
dsolve(x*diff(diff(y(x),x),x)-(3*x-2)*diff(y(x),x)-(2*x-3)*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-\frac{x\left(-3+\sqrt{17}\right)}{2}}\left(\text{KummerM}\left(1-\frac{6\sqrt{17}}{17}, 2, \sqrt{17}\,x\right)c_1 + \text{KummerU}\left(1-\frac{6\sqrt{17}}{17}, 2, \sqrt{17}\,x\right)c_2\right)$$

✓ Solution by Mathematica
Time used: 0.104 (sec). Leaf size: 63

```
DSolve[(3 - 2*x)*y[x] - (-2 + 3*x)*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to e^{-\frac{1}{2}\left(\sqrt{17}-3\right)x}\left(c_2\,\text{Hypergeometric1F1}\left(1-\frac{6}{\sqrt{17}}, 2, \sqrt{17}x\right) + c_1\,\text{HypergeometricU}\left(1-\frac{6}{\sqrt{17}}, 2, \sqrt{17}x\right)\right)$$

3.112 problem 1116

Internal problem ID [9446]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1116.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y''x + (xa + b + n)\, y' + nay = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 31

```
dsolve(x*diff(diff(y(x),x),x)+(a*x+b+n)*diff(y(x),x)+n*a*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-ax}(\text{KummerU}\,(b, b+n, ax)\, c_2 + \text{KummerM}\,(b, b+n, ax)\, c_1)$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 38

```
DSolve[a*n*y[x] + (b + n + a*x)*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^{-ax}(c_1 \,\text{HypergeometricU}(b, b+n, ax) + c_2 L_{-b}^{b+n-1}(ax))$$

3.113 problem 1117

Internal problem ID [9447]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1117.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y''x - (a+b)(x+1)y' + abxy = 0$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 82

```
dsolve(x*diff(diff(y(x),x),x)-(a+b)*(x+1)*diff(y(x),x)+a*b*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{bx}x^{b+a+1}\left(\text{KummerM}\left(\frac{a^2+ab+a-b}{a-b}, b+2+a, x(a-b)\right)c_1 + \text{KummerU}\left(\frac{a^2+ab+a-b}{a-b}, b+2+a, x(a-b)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.093 (sec). Leaf size: 87

```
DSolve[a*b*x*y[x] - (a + b)*(1 + x)*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to e^{bx}x^{a+b+1}\left(c_1 \text{HypergeometricU}\left(\frac{a^2+ba+a-b}{a-b}, a+b+2, (a-b)x\right) + c_2 L_{-\frac{a^2+ba+a-b}{a-b}}^{a+b+1}((a-b)x)\right)$$

3.114 problem 1118

Internal problem ID [9448]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1118.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y''x + (x(a+b) + m + n)\, y' + (abx + an + bm)\, y = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 39

```
dsolve(x*diff(diff(y(x),x),x)+((a+b)*x+m+n)*diff(y(x),x)+(a*b*x+a*n+b*m)*y(x)=0,y(x), singso
```

$$y(x) = e^{-ax}(\text{KummerM}\,(m, m+n, x(a-b))\, c_1 + \text{KummerU}\,(m, m+n, x(a-b))\, c_2)$$

✓ Solution by Mathematica

Time used: 0.079 (sec). Leaf size: 46

```
DSolve[(b*m + a*n + a*b*x)*y[x] + (m + n + (a + b)*x)*y'[x] + x*y''[x] == 0,y[x],x,IncludeSi
```

$$y(x) \to e^{-ax}(c_1\, \text{HypergeometricU}(m, m+n, (a-b)x) + c_2 L_{-m}^{m+n-1}((a-b)x))$$

3.115 problem 1119

Internal problem ID [9449]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1119.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y''x - 2(xa + b)\, y' + \left(a^2 x + 2ab\right) y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(x*diff(diff(y(x),x),x)-2*(a*x+b)*diff(y(x),x)+(a^2*x+2*a*b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{ax}\left(c_1 + x^{2b+1} c_2\right)$$

✓ Solution by Mathematica

Time used: 0.143 (sec). Leaf size: 75

```
DSolve[(2*a*b + a^2*x)*y[x] - 2*(b + a*x)*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{e^{ax} x^{b-\frac{1}{2}\sqrt{(2b+1)^2}+\frac{1}{2}} \left(c_2 x^{\sqrt{(2b+1)^2}} + \sqrt{(2b+1)^2} c_1\right)}{\sqrt{(2b+1)^2}}$$

3.116 problem 1120

Internal problem ID [9450]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1120.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x + (xa + b)\,y' + (cx + d)\,y = 0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 109

```
dsolve(x*diff(diff(y(x),x),x)+(a*x+b)*diff(y(x),x)+(c*x+d)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x\left(\sqrt{a^2-4c}+a\right)}{2}} \left(\mathrm{KummerU}\left(\frac{b\sqrt{a^2-4c}+ab-2d}{2\sqrt{a^2-4c}}, b, \sqrt{a^2-4c}\,x\right) c_2 + \mathrm{KummerM}\left(\frac{b\sqrt{a^2-4c}+ab-2d}{2\sqrt{a^2-4c}}, b, \sqrt{a^2-4c}\,x\right) c_1\right)$$

✓ Solution by Mathematica

Time used: 0.089 (sec). Leaf size: 135

```
DSolve[(d + c*x)*y[x] + (b + a*x)*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to e^{-\frac{1}{2}x\left(\sqrt{a^2-4c}+a\right)} \left(c_1\,\mathrm{HypergeometricU}\left(\frac{ab+\sqrt{a^2-4c}b-2d}{2\sqrt{a^2-4c}}, b, \sqrt{a^2-4c}x\right) + c_2 L^{b-1}_{-\frac{ab+\sqrt{a^2-4c}b-2d}{2\sqrt{a^2-4c}}}\left(\sqrt{a^2-4c}x\right)\right)$$

3.117 problem 1121

Internal problem ID [9451]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1121.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{y''x-(x^2-x)\,y'+(x-1)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x*diff(diff(y(x),x),x)-(x^2-x)*diff(y(x),x)+(x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\left(\left(\int \frac{e^{\frac{x(x-2)}{2}}}{x^2}dx\right)c_1+c_2\right)x$$

✓ Solution by Mathematica

Time used: 0.603 (sec). Leaf size: 37

```
DSolve[(-1 + x)*y[x] - (-x + x^2)*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\to x\left(c_2\int_1^x \frac{e^{\frac{1}{2}(K[1]-2)K[1]}}{K[1]^2}dK[1]+c_1\right)$$

3.118 problem 1122

Internal problem ID [9452]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1122.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x - (x^2 - x - 2)\, y' - x(x+3)\, y = 0}$$

✓ Solution by Maple

Time used: 0.344 (sec). Leaf size: 28

```
dsolve(x*diff(diff(y(x),x),x)-(x^2-x-2)*diff(y(x),x)-x*(x+3)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{x^2}{2}} \left(c_1 + \left(\int \frac{\mathrm{e}^{-\frac{x(x+2)}{2}}}{x^2} dx\right) c_2\right)$$

✓ Solution by Mathematica

Time used: 0.872 (sec). Leaf size: 45

```
DSolve[-(x*(3 + x)*y[x]) - (-2 - x + x^2)*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to e^{\frac{x^2}{2}} \left(c_2 \int_1^x \frac{e^{-\frac{1}{2}K[1](K[1]+2)}}{K[1]^2} dK[1] + c_1\right)$$

3.119 problem 1123

Internal problem ID [9453]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1123.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y''x - (2a\,x^2 + 1)\,y' + b\,x^3 y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(x*diff(diff(y(x),x),x)-(2*a*x^2+1)*diff(y(x),x)+b*x^3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{x^2\left(\sqrt{a^2-b}+a\right)}{2}} + c_2 \mathrm{e}^{-\frac{x^2\left(\sqrt{a^2-b}-a\right)}{2}}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 53

```
DSolve[b*x^3*y[x] - (1 + 2*a*x^2)*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to e^{-\frac{1}{2}x^2\left(\sqrt{a^2-b}-a\right)}\left(c_2 e^{x^2\sqrt{a^2-b}} + c_1\right)$$

3.120 problem 1124

Internal problem ID [9454]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1124.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x - 2(x^2 - a)\,y' + 2nxy = 0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 29

```
dsolve(x*diff(diff(y(x),x),x)-2*(x^2-a)*diff(y(x),x)+2*n*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{KummerM}\left(-\frac{n}{2}, \frac{1}{2} + a, x^2\right) + c_2 \operatorname{KummerU}\left(-\frac{n}{2}, \frac{1}{2} + a, x^2\right)$$

✓ Solution by Mathematica

Time used: 0.13 (sec). Leaf size: 65

```
DSolve[2*n*x*y[x] - 2*(-a + x^2)*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tr
```

$$\begin{aligned} y(x) \to\ & c_1 \text{Hypergeometric1F1}\left(-\frac{n}{2}, a + \frac{1}{2}, x^2\right) \\ & + i^{1-2a} c_2 x^{1-2a} \text{Hypergeometric1F1}\left(-a - \frac{n}{2} + \frac{1}{2}, \frac{3}{2} - a, x^2\right) \end{aligned}$$

3.121 problem 1125

Internal problem ID [9455]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1125.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''x + (4x^2 - 1)\, y' - 4yx^3 = 4x^5}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
dsolve(x*diff(diff(y(x),x),x)+(4*x^2-1)*diff(y(x),x)-4*x^3*y(x)-4*x^5=0,y(x), singsol=all)
```

$$y(x) = e^{x^2(\sqrt{2}-1)} c_2 + e^{-x^2(1+\sqrt{2})} c_1 - x^2 - 2$$

✓ Solution by Mathematica

Time used: 0.28 (sec). Leaf size: 45

```
DSolve[-4*x^5 - 4*x^3*y[x] + (-1 + 4*x^2)*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to -x^2 + c_1 e^{(\sqrt{2}-1)x^2} + c_2 e^{-((1+\sqrt{2})x^2)} - 2$$

3.122 problem 1126

Internal problem ID [9456]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1126.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y''x + (2a\,x^3 - 1)\,y' + (a^2x^3 + a)\,x^2y = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)+(2*a*x^3-1)*diff(y(x),x)+(a^2*x^3+a)*x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x^3a}{3}}(c_2x^2 + c_1)$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 30

```
DSolve[x*y''[x]+(2*a*x^3-1)*y'[x]+(a^2*x^3+a)*x^2*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{2}e^{-\frac{ax^3}{3}}(c_2x^2 + 2c_1)$$

3.123 problem 1127

Internal problem ID [9457]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1127.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y''x + (2ax\ln(x) + 1)y' + (a^2x\ln(x)^2 + a\ln(x) + a)y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x*diff(diff(y(x),x),x)+(2*a*x*ln(x)+1)*diff(y(x),x)+(a^2*x*ln(x)^2+a*ln(x)+a)*y(x)=0,
```

$$y(x) = x^{-ax}e^{ax}(\ln(x)c_2 + c_1)$$

✓ Solution by Mathematica
Time used: 0.057 (sec). Leaf size: 25

```
DSolve[(a + a*Log[x] + a^2*x*Log[x]^2)*y[x] + (1 + 2*a*x*Log[x])*y'[x] + x*y''[x] == 0,y[x],
```

$$y(x) \to e^{ax}x^{-ax}(c_2\log(x) + c_1)$$

3.124 problem 1128

Internal problem ID [9458]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1128.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y''x + (xf(x) + 2)\, y' + f(x)\, y = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 33

```
dsolve(x*diff(diff(y(x),x),x)+(x*f(x)+2)*diff(y(x),x)+f(x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2 \left(\int \mathrm{e}^{-\left(\int \frac{f(x)x+2}{x}dx\right)} x^2 dx\right) + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.195 (sec). Leaf size: 37

```
DSolve[f[x]*y[x] + (2 + x*f[x])*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{c_2 \int_1^x \exp\left(-\int_1^{K[2]} f(K[1])dK[1]\right) dK[2] + c_1}{x}$$

3.125 problem 1129

Internal problem ID [9459]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1129.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-3)\,y''-(4x-9)\,y'+(3x-6)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve((x-3)*diff(diff(y(x),x),x)-(4*x-9)*diff(y(x),x)+(3*x-6)*y(x)=0,y(x), singsol=all)
```

$$y(x)=4c_2\left(x^3-\frac{21}{2}x^2+\frac{75}{2}x-\frac{183}{4}\right)\mathrm{e}^{3x}+c_1\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.191 (sec). Leaf size: 42

```
DSolve[(-6 + 3*x)*y[x] - (-9 + 4*x)*y'[x] + (-3 + x)*y''[x] == 0,y[x],x,IncludeSingularSolut
```

$$y(x)\to\frac{1}{8}c_2e^{3x-9}\left(4x^3-42x^2+150x-183\right)+c_1e^{x-3}$$

3.126 problem 1130

Internal problem ID [9460]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1130.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$2y''x + y' + ay = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(2*x*diff(diff(y(x),x),x)+diff(y(x),x)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(\sqrt{x}\,\sqrt{2}\,\sqrt{a}\right) + c_2 \cos\left(\sqrt{x}\,\sqrt{2}\,\sqrt{a}\right)$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 46

```
DSolve[a*y[x] + y'[x] + 2*x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos\left(\sqrt{2}\sqrt{a}\sqrt{x}\right) + c_2 \sin\left(\sqrt{2}\sqrt{a}\sqrt{x}\right)$$

3.127 problem 1131

Internal problem ID [9461]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1131.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2y''x - (x-1)\,y' + ay = 0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 33

```
dsolve(2*x*diff(diff(y(x),x),x)-(x-1)*diff(y(x),x)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\left(\text{KummerM}\left(-a+\frac{1}{2},\frac{3}{2},\frac{x}{2}\right)c_1 + \text{KummerU}\left(-a+\frac{1}{2},\frac{3}{2},\frac{x}{2}\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 48

```
DSolve[a*y[x] - (-1 + x)*y'[x] + 2*x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}\left(c_1 \,\text{HypergeometricU}\left(\frac{1}{2}-a,\frac{3}{2},\frac{x}{2}\right) + c_2 L_{a-\frac{1}{2}}^{\frac{1}{2}}\left(\frac{x}{2}\right)\right)$$

3.128 problem 1132

Internal problem ID [9462]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1132.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{2y''x - (2x - 1)\, y' + ay = 0}$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 29

```
dsolve(2*x*diff(diff(y(x),x),x)-(2*x-1)*diff(y(x),x)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\left(\mathrm{KummerM}\left(-\frac{a}{2} + \frac{1}{2}, \frac{3}{2}, x\right) c_1 + \mathrm{KummerU}\left(-\frac{a}{2} + \frac{1}{2}, \frac{3}{2}, x\right) c_2\right)$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 44

```
DSolve[a*y[x] - (-1 + 2*x)*y'[x] + 2*x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}\left(c_1 \,\mathrm{HypergeometricU}\left(\frac{1-a}{2}, \frac{3}{2}, x\right) + c_2 L_{\frac{a-1}{2}}^{\frac{1}{2}}(x)\right)$$

3.129 problem 1133

Internal problem ID [9463]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1133.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x-1)\,y''-(3x-4)\,y'+(x-3)\,y=0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 31

```
dsolve((2*x-1)*diff(diff(y(x),x),x)-(3*x-4)*diff(y(x),x)+(x-3)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{\mathrm{e}^{-\frac{1}{4}+x}\sqrt{2}\left(\left(\frac{c_1}{4}+c_2\right)\Gamma\left(-\frac{1}{4},-\frac{1}{4}+\frac{x}{2}\right)+\Gamma\left(\frac{3}{4}\right)c_1\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.253 (sec). Leaf size: 47

```
DSolve[(-3 + x)*y[x] - (-4 + 3*x)*y'[x] + (-1 + 2*x)*y''[x] == 0,y[x],x,IncludeSingularSolut
```

$$y(x)\to -\frac{e^{x-\frac{1}{2}}\left(\sqrt[4]{2}c_2\Gamma\left(-\frac{1}{4},\frac{1}{4}(2x-1)\right)-8c_1\right)}{4\,2^{3/8}}$$

3.130 problem 1134

Internal problem ID [9464]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1134.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4y''x - (x + a)\, y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve(4*x*diff(diff(y(x),x),x)-(x+a)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \,\text{WhittakerM}\left(-\frac{a}{4}, \frac{1}{2}, x\right) + c_2 \,\text{WhittakerW}\left(-\frac{a}{4}, \frac{1}{2}, x\right)$$

✓ Solution by Mathematica

Time used: 0.132 (sec). Leaf size: 44

```
DSolve[(-a - x)*y[x] + 4*x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \\ \rightarrow \frac{1}{4} e^{-x/2} x \left(c_2 \,\text{Hypergeometric1F1}\left(\frac{a}{4} + 1, 2, x\right) + c_1 \,\text{HypergeometricU}\left(\frac{a}{4} + 1, 2, x\right)\right)$$

3.131 problem 1135

Internal problem ID [9465]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1135.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{4y''x + 2y' - y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve(4*x*diff(diff(y(x),x),x)+2*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sinh\left(\sqrt{x}\right) + c_2 \cosh\left(\sqrt{x}\right)$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 27

```
DSolve[-y[x] + 2*y'[x] + 4*x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cosh\left(\sqrt{x}\right) + ic_2 \sinh\left(\sqrt{x}\right)$$

3.132 problem 1136

Internal problem ID [9466]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1136.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4y''x + 4y' - (x+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(4*x*diff(diff(y(x),x),x)+4*diff(y(x),x)-(x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{x}{2}}(c_1 + \mathrm{expIntegral}_1\,(x)\,c_2)$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 23

```
DSolve[(-2 - x)*y[x] + 4*y'[x] + 4*x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x/2}(c_2\,\mathrm{ExpIntegralEi}(-x) + c_1)$$

3.133 problem 1137

Internal problem ID [9467]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1137.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4y''x + 4y - (x+2)\,y + ly = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(4*x*diff(diff(y(x),x),x)+4*y(x)-(x+2)*y(x)+l*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \,\mathrm{WhittakerM}\left(\frac{l}{4}+\frac{1}{2}, \frac{1}{2}, x\right) + c_2 \,\mathrm{WhittakerW}\left(\frac{l}{4}+\frac{1}{2}, \frac{1}{2}, x\right)$$

✓ Solution by Mathematica

Time used: 0.129 (sec). Leaf size: 48

```
DSolve[4*y[x] + l*y[x] - (2 + x)*y[x] + 4*x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{1}{4}e^{-x/2}x\left(c_2 \,\mathrm{Hypergeometric1F1}\left(\frac{1}{2}-\frac{l}{4}, 2, x\right) + c_1 \,\mathrm{HypergeometricU}\left(\frac{1}{2}-\frac{l}{4}, 2, x\right)\right)$$

3.134 problem 1138

Internal problem ID [9468]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1138.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4y''x + 4my' - (x - 2m - 4n)\,y = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 26

```
dsolve(4*x*diff(diff(y(x),x),x)+4*m*diff(y(x),x)-(x-2*m-4*n)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{2}}(\mathrm{KummerM}\,(-n, m, x)\,c_1 + \mathrm{KummerU}\,(-n, m, x)\,c_2)$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 32

```
DSolve[(2*m + 4*n - x)*y[x] + 4*m*y'[x] + 4*x*y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to e^{-x/2}(c_1\,\mathrm{HypergeometricU}(-n, m, x) + c_2 L_n^{m-1}(x))$$

3.135 problem 1139

Internal problem ID [9469]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1139.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{16y''x + 8y' - (x + a)\, y = 0}$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 37

```
dsolve(16*x*diff(diff(y(x),x),x)+8*diff(y(x),x)-(x+a)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\, \mathrm{e}^{-\frac{x}{4}} \left(\mathrm{KummerU}\left(\frac{a}{8} + \frac{3}{4}, \frac{3}{2}, \frac{x}{2}\right) c_2 + \mathrm{KummerM}\left(\frac{a}{8} + \frac{3}{4}, \frac{3}{2}, \frac{x}{2}\right) c_1\right)$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 59

```
DSolve[(-a - x)*y[x] + 8*y'[x] + 16*x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/4} \sqrt{x} \left(c_1 \, \text{HypergeometricU}\left(\frac{a+6}{8}, \frac{3}{2}, \frac{x}{2}\right) + c_2 L_{\frac{1}{8}(-a-6)}^{\frac{1}{2}}\left(\frac{x}{2}\right)\right)$$

3.136 problem 1140

Internal problem ID [9470]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1140.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{axy'' + by' + yc = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 134

```
dsolve(a*x*diff(diff(y(x),x),x)+b*diff(y(x),x)+c*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x^{-\frac{b}{2a}}\left(-\sqrt{x}\,\text{BesselJ}\left(\frac{a+b}{a}, 2\sqrt{\frac{c}{a}}\,\sqrt{x}\right)\sqrt{\frac{c}{a}}\,c_1 a - \sqrt{x}\,\text{BesselY}\left(\frac{a+b}{a}, 2\sqrt{\frac{c}{a}}\,\sqrt{x}\right)\sqrt{\frac{c}{a}}\,c_2 a + \text{BesselJ}\left(\frac{b}{a}, 2\sqrt{\frac{c}{a}}\,\sqrt{}\right.\right.}{a\sqrt{\frac{c}{a}}}$$

✓ Solution by Mathematica

Time used: 0.105 (sec). Leaf size: 120

```
DSolve[c*y[x] + b*y'[x] + a*x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to a^{\frac{1}{2}\left(\frac{b}{a}-1\right)} c^{\frac{a-b}{2a}} x^{\frac{a-b}{2a}} \left(c_1 \,\text{Gamma}\left(\frac{b}{a}\right) \text{BesselJ}\left(\frac{b}{a}-1, \frac{2\sqrt{c}\sqrt{x}}{\sqrt{a}}\right) + c_2 \,\text{Gamma}\left(2-\frac{b}{a}\right) \text{BesselJ}\left(1-\frac{b}{a}, \frac{2\sqrt{c}\sqrt{x}}{\sqrt{a}}\right)\right)$$

3.137 problem 1141

Internal problem ID [9471]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1141.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$axy'' + (bx + 3a)\,y' + 3yb = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
dsolve(a*x*diff(diff(y(x),x),x)+(b*x+3*a)*diff(y(x),x)+3*b*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{e^{-\frac{bx}{a}}\,\mathrm{expIntegral}_1\left(-\frac{bx}{a}\right) c_2 b^2 x^2 + c_1 e^{-\frac{bx}{a}} x^2 + c_2 a(bx + a)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.29 (sec). Leaf size: 63

```
DSolve[3*b*y[x] + (3*a + b*x)*y'[x] + a*x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{2}\left(\frac{b^2 c_2 e^{-\frac{bx}{a}}\,\mathrm{ExpIntegralEi}\left(\frac{bx}{a}\right)}{a^2} - \frac{c_2(a + bx)}{ax^2} + 2c_1 e^{-\frac{bx}{a}}\right)$$

3.138 problem 1142

Internal problem ID [9472]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1142.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$5(xa+b)\,y''+8ay'+c(xa+b)^{\frac{1}{5}}\,y=0$$

✓ Solution by Maple

Time used: 0.281 (sec). Leaf size: 59

```
dsolve(5*(a*x+b)*diff(diff(y(x),x),x)+8*a*diff(y(x),x)+c*(a*x+b)^(1/5)*y(x)=0,y(x), singsol=
```

$$y(x)=\frac{c_1\sinh\left(\frac{(ax+b)^{\frac{3}{5}}\sqrt{5}\,\sqrt{-c}}{3a}\right)+c_2\cosh\left(\frac{(ax+b)^{\frac{3}{5}}\sqrt{5}\,\sqrt{-c}}{3a}\right)}{(ax+b)^{\frac{3}{5}}}$$

✓ Solution by Mathematica

Time used: 0.099 (sec). Leaf size: 89

```
DSolve[c*(b + a*x)^(1/5)*y[x] + 8*a*y'[x] + 5*(b + a*x)*y''[x] == 0,y[x],x,IncludeSingularSo
```

$$y(x)\to\frac{3a\left(2c_1\cos\left(\frac{\sqrt{5}\sqrt{c}(ax+b)^{3/5}}{3a}\right)+c_2\sin\left(\frac{\sqrt{5}\sqrt{c}(ax+b)^{3/5}}{3a}\right)\right)}{\sqrt{5}\sqrt{c}(ax+b)^{3/5}}$$

3.139 problem 1143

Internal problem ID [9473]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1143.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2axy'' + (bx + a)\, y' + yc = 0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 57

```
dsolve(2*a*x*diff(diff(y(x),x),x)+(b*x+a)*diff(y(x),x)+c*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\, \mathrm{e}^{-\frac{bx}{2a}} \left(\text{KummerU}\left(\frac{b-c}{b}, \frac{3}{2}, \frac{bx}{2a} \right) c_2 + \text{KummerM}\left(\frac{b-c}{b}, \frac{3}{2}, \frac{bx}{2a} \right) c_1 \right)$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 70

```
DSolve[c*y[x] + (a + b*x)*y'[x] + 2*a*x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x} e^{-\frac{bx}{2a}} \left(c_1 \text{HypergeometricU}\left(1 - \frac{c}{b}, \frac{3}{2}, \frac{bx}{2a} \right) + c_2 L_{\frac{c}{b}-1}^{\frac{1}{2}}\left(\frac{bx}{2a} \right) \right)$$

3.140 problem 1144

Internal problem ID [9474]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1144.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2axy'' + (bx + 3a)\,y' + yc = 0$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 153

```
dsolve(2*a*x*diff(diff(y(x),x),x)+(b*x+3*a)*diff(y(x),x)+c*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(-c_1 c(a(b+4c) - b^2 x)\operatorname{KummerM}\left(\frac{b-2c}{2b}, \frac{3}{2}, \frac{bx}{2a}\right) + c_2 b(a(b+4c) - b^2 x)\operatorname{KummerU}\left(\frac{b-2c}{2b}, \frac{3}{2}, \frac{bx}{2a}\right) + 2a(cc_1}{a\,(b-2c)\,c}$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 69

```
DSolve[c*y[x] + (3*a + b*x)*y'[x] + 2*a*x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^{-\frac{bx}{2a}}\left(c_1 \operatorname{HypergeometricU}\left(\frac{3}{2} - \frac{c}{b}, \frac{3}{2}, \frac{bx}{2a}\right) + c_2 L_{\frac{c}{b}-\frac{3}{2}}^{\frac{1}{2}}\left(\frac{bx}{2a}\right)\right)$$

3.141 problem 1145

Internal problem ID [9475]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1145.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(\mathrm{a2}\,x+\mathrm{b2})\,y''+(\mathrm{a1}\,x+\mathrm{b1})\,y'+(\mathrm{a0}\,x+\mathrm{b0})\,y=0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 248

```
dsolve((a2*x+b2)*diff(diff(y(x),x),x)+(a1*x+b1)*diff(y(x),x)+(a0*x+b0)*y(x)=0,y(x), singsol=
```

$$y(x)=\mathrm{e}^{-\frac{\left(\sqrt{-4\,\mathrm{a0}\,\mathrm{a2}+\mathrm{a1}^2}+\mathrm{a1}\right)x}{2\,\mathrm{a2}}}(\mathrm{a2}\,x$$
$$+\mathrm{b2})^{\frac{\mathrm{a1}\,\mathrm{b2}+\mathrm{a2}^2-\mathrm{a2}\,\mathrm{b1}}{\mathrm{a2}^2}}\left(\mathrm{KummerU}\left(\frac{(\mathrm{a1}\,\mathrm{b2}+2\,\mathrm{a2}^2-\mathrm{a2}\,\mathrm{b1})\sqrt{-4\,\mathrm{a0}\,\mathrm{a2}+\mathrm{a1}^2}-2\,\mathrm{a2}^2\,\mathrm{b0}+(2\,\mathrm{a0}\,\mathrm{b2}+\mathrm{a1}\,\mathrm{b1})\,\mathrm{a}}{2\sqrt{-4\,\mathrm{a0}\,\mathrm{a2}+\mathrm{a1}^2}\,\mathrm{a2}^2}\right.\right.$$
$$+\mathrm{KummerM}\left(\frac{(\mathrm{a1}\,\mathrm{b2}+2\,\mathrm{a2}^2-\mathrm{a2}\,\mathrm{b1})\sqrt{-4\,\mathrm{a0}\,\mathrm{a2}+\mathrm{a1}^2}-2\,\mathrm{a2}^2\,\mathrm{b0}+(2\,\mathrm{a0}\,\mathrm{b2}+\mathrm{a1}\,\mathrm{b1})\,\mathrm{a2}-\mathrm{a1}^2\,\mathrm{b2}}{2\sqrt{-4\,\mathrm{a0}\,\mathrm{a2}+\mathrm{a1}^2}\,\mathrm{a2}^2},\frac{\mathrm{a1}\,\mathrm{b2}}{}\right.$$

✓ Solution by Mathematica

Time used: 0.301 (sec). Leaf size: 301

```
DSolve[(b0 + a0*x)*y[x] + (b1 + a1*x)*y'[x] + (b2 + a2*x)*y''[x] == 0,y[x],x,IncludeSingular
```

$$y(x) \to e^{-\frac{x\left(\sqrt{\mathrm{a1}^2-4\mathrm{a0a2}}+\mathrm{a1}\right)}{2\mathrm{a2}}}(\mathrm{a2}x+\mathrm{b2})^{\frac{\mathrm{a1b2}+\mathrm{a2}^2-\mathrm{a2b1}}{\mathrm{a2}^2}}\left(c_1\,\mathrm{HypergeometricU}\left(\frac{2\left(\sqrt{\mathrm{a1}^2-4\mathrm{a0a2}}-\mathrm{b0}\right)\mathrm{a2}^2+\left(\mathrm{a1b1}-\sqrt{\mathrm{a1}^2-4\mathrm{a0a2}}\mathrm{b1}+2\right.}{2\mathrm{a2}^2\sqrt{\mathrm{a1}^2-4\mathrm{a0a2}}}\right.\right.$$

$$\left.-\frac{\mathrm{b1}}{\mathrm{a2}}+\frac{\mathrm{a1b2}}{\mathrm{a2}^2}+2,\frac{\sqrt{\mathrm{a1}^2-4\mathrm{a0a2}}(\mathrm{b2}+\mathrm{a2}x)}{\mathrm{a2}^2}\right)$$

$$\left.+c_2 L^{\frac{\mathrm{a2}^2-\mathrm{b1a2}+\mathrm{a1b2}}{\mathrm{a2}^2}}_{\frac{-2\left(\sqrt{\mathrm{a1}^2-4\mathrm{a0a2}}-\mathrm{b0}\right)\mathrm{a2}^2+\left(-\mathrm{a1b1}+\sqrt{\mathrm{a1}^2-4\mathrm{a0a2}}\mathrm{b1}-2\mathrm{a0b2}\right)\mathrm{a2}+\mathrm{a1}\left(\mathrm{a1}-\sqrt{\mathrm{a1}^2-4\mathrm{a0a2}}\right)\mathrm{b2}}{2\mathrm{a2}^2\sqrt{\mathrm{a1}^2-4\mathrm{a0a2}}}}\left(\frac{\sqrt{\mathrm{a1}^2-4\mathrm{a0a2}}(\mathrm{b2}+\mathrm{a2}x)}{\mathrm{a2}^2}\right)\right)$$

3.142 problem 1146

Internal problem ID [9476]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1146.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 6y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(diff(y(x),x),x)-6*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1x^5 + c_2}{x^2}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 18

```
DSolve[-6*y[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^5 + c_1}{x^2}$$

3.143 problem 1147

Internal problem ID [9477]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1147.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 12y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(diff(y(x),x),x)-12*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2x^7 + c_1}{x^3}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 18

```
DSolve[-12*y[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^7 + c_1}{x^3}$$

3.144 problem 1148

Internal problem ID [9478]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1148.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2 y'' + ay = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(x^2*diff(diff(y(x),x),x)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\left(c_1 x^{\frac{\sqrt{-4a+1}}{2}} + c_2 x^{-\frac{\sqrt{-4a+1}}{2}}\right)$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 42

```
DSolve[a*y[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^{\frac{1}{2}-\frac{1}{2}\sqrt{1-4a}}\left(c_2 x^{\sqrt{1-4a}} + c_1\right)$$

3.145 problem 1149

Internal problem ID [9479]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1149.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + (xa + b)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 45

```
dsolve(x^2*diff(diff(y(x),x),x)+(a*x+b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(\text{BesselY}\left(\sqrt{1-4b}, 2\sqrt{a}\,\sqrt{x}\right) c_2 + \text{BesselJ}\left(\sqrt{1-4b}, 2\sqrt{a}\,\sqrt{x}\right) c_1\right)\sqrt{x}$$

✓ Solution by Mathematica
Time used: 0.101 (sec). Leaf size: 95

```
DSolve[(b + a*x)*y[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{a}\sqrt{x}\Big(c_1\,\text{Gamma}\left(1-\sqrt{1-4b}\right)\text{BesselJ}\left(-\sqrt{1-4b}, 2\sqrt{a}\sqrt{x}\right) + c_2\,\text{Gamma}\left(\sqrt{1-4b}+1\right)\text{BesselJ}\left(\sqrt{1-4b}, 2\sqrt{a}\sqrt{x}\right)\Big)$$

3.146 problem 1150

Internal problem ID [9480]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1150.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' + (x^2 - 2)\, y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 27

```
dsolve(x^2*diff(diff(y(x),x),x)+(x^2-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{(c_1 x + c_2) \cos(x) + \sin(x) (x c_2 - c_1)}{x}$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 21

```
DSolve[(-2 + x^2)*y[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(c_1 j_1(x) - c_2 y_1(x))$$

3.147 problem 1151

Internal problem ID [9481]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1151.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - (a\,x^2 + 2)\,y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve(x^2*diff(diff(y(x),x),x)-(a*x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2 e^{-x\sqrt{a}}(ax + \sqrt{a}) + c_1 e^{x\sqrt{a}}(ax - \sqrt{a})}{x}$$

✓ Solution by Mathematica

Time used: 0.145 (sec). Leaf size: 88

```
DSolve[(-2 - a*x^2)*y[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{i\sqrt{\frac{2}{\pi}}\sqrt{x}\left((i\sqrt{a}c_2x + c_1)\sinh\left(\sqrt{a}x\right) - (\sqrt{a}c_1x + ic_2)\cosh\left(\sqrt{a}x\right)\right)}{\left(-i\sqrt{a}x\right)^{3/2}}$$

3.148 problem 1152

Internal problem ID [9482]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1152.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' + (a^2 x^2 - 6)\, y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 53

```
dsolve(x^2*diff(diff(y(x),x),x)+(a^2*x^2-6)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{(a^2 c_1 x^2 + 3ac_2 x - 3c_1)\cos(ax) + \sin(ax)\,(a^2 c_2 x^2 - 3ac_1 x - 3c_2)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.165 (sec). Leaf size: 79

```
DSolve[(-6 + a^2*x^2)*y[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}}\sqrt{x}\left((-a^2 c_2 x^2 + 3ac_1 x + 3c_2)\cos(ax) + (c_1(a^2x^2 - 3) + 3ac_2 x)\sin(ax)\right)}{(ax)^{5/2}}$$

3.149 problem 1153

Internal problem ID [9483]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1153.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + (a\,x^2 - v(v-1))\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(x^2*diff(diff(y(x),x),x)+(a*x^2-v*(v-1))*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(\mathrm{BesselJ}\left(v - \frac{1}{2}, x\sqrt{a}\right) c_1 + \mathrm{BesselY}\left(v - \frac{1}{2}, x\sqrt{a}\right) c_2\right) \sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 44

```
DSolve[((1 - v)*v + a*x^2)*y[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}\left(c_1\,\mathrm{BesselJ}\left(v - \frac{1}{2}, \sqrt{a}x\right) + c_2\,\mathrm{BesselY}\left(v - \frac{1}{2}, \sqrt{a}x\right)\right)$$

3.150 problem 1154

Internal problem ID [9484]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1154.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + (a\,x^2 + bx + c)\,y = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 57

```
dsolve(x^2*diff(diff(y(x),x),x)+(a*x^2+b*x+c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \,\mathrm{WhittakerM}\left(-\frac{ib}{2\sqrt{a}}, \frac{\sqrt{1-4c}}{2}, 2ix\sqrt{a}\right) + c_2 \,\mathrm{WhittakerW}\left(-\frac{ib}{2\sqrt{a}}, \frac{\sqrt{1-4c}}{2}, 2ix\sqrt{a}\right)$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 88

```
DSolve[(c + b*x + a*x^2)*y[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 M_{-\frac{ib}{2\sqrt{a}},-\frac{1}{2}i\sqrt{4c-1}}\left(2i\sqrt{a}x\right) + c_2 W_{-\frac{ib}{2\sqrt{a}},-\frac{1}{2}i\sqrt{4c-1}}\left(2i\sqrt{a}x\right)$$

3.151 problem 1155

Internal problem ID [9485]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1155.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' + \left(a\,x^k - b(b-1)\right) y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 71

```
dsolve(x^2*diff(diff(y(x),x),x)+(a*x^k-b*(b-1))*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\left(\operatorname{BesselY}\left(\frac{\operatorname{csgn}(2b-1)(2b-1)}{k}, \frac{2\sqrt{a}\,x^{\frac{k}{2}}}{k}\right)c_2 + \operatorname{BesselJ}\left(\frac{\operatorname{csgn}(2b-1)(2b-1)}{k}, \frac{2\sqrt{a}\,x^{\frac{k}{2}}}{k}\right)c_1\right)$$

✓ Solution by Mathematica

Time used: 0.098 (sec). Leaf size: 116

```
DSolve[((1 - b)*b + a*x^k)*y[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to k^{-1/k} a^{\frac{1}{2}/k} \left(x^k\right)^{\frac{1}{2}/k} \left(c_1 \operatorname{Gamma}\left(\frac{-2b+k+1}{k}\right) \operatorname{BesselJ}\left(\frac{1-2b}{k}, \frac{2\sqrt{a}\sqrt{x^k}}{k}\right) + c_2 \operatorname{Gamma}\left(\frac{2b+k-1}{k}\right) \operatorname{BesselJ}\left(\frac{2b-1}{k}, \frac{2\sqrt{a}\sqrt{x^k}}{k}\right)\right)$$

3.152 problem 1156

Internal problem ID [9486]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1156.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x^2y'' + \frac{y}{\ln(x)} = x\,e^x(2 + \ln(x)\,x)$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 75

```
dsolve(x^2*diff(diff(y(x),x),x)+y(x)/ln(x)-x*exp(x)*(2+x*ln(x))=0,y(x), singsol=all)
```

$$y(x) = -\ln(x)^3 \operatorname{expIntegral}_1(-\ln(x))\,e^x - \ln(x)^2 e^x x - \ln(x) \operatorname{expIntegral}_1(-\ln(x))\,c_1 + \left(\int \frac{(\operatorname{expIntegral}_1(-\ln(x))\ln(x) + x)\,e^x(2 + x\ln(x))}{x} dx\right)\ln(x) + \ln(x)\,c_2 - c_1 x$$

✓ Solution by Mathematica

Time used: 0.127 (sec). Leaf size: 27

```
DSolve[-(E^x*x*(2 + x*Log[x])) + y[x]/Log[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to c_2 \operatorname{LogIntegral}(x)\log(x) + c_2(-x) + (e^x + c_1)\log(x)$$

3.153 problem 1157

Internal problem ID [9487]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1157.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + ay' - xy = 0$$

✗ Solution by Maple

```
dsolve(x^2*diff(diff(y(x),x),x)+a*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-(x*y[x]) + a*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

3.154 problem 1158

Internal problem ID [9488]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1158.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + ay' - (b^2x^2 + ab)\, y = 0$$

✓ Solution by Maple

Time used: 0.5 (sec). Leaf size: 178

```
dsolve(x^2*diff(diff(y(x),x),x)+a*diff(y(x),x)-(b^2*x^2+a*b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\left(\mathrm{HeunD}\left(4\sqrt{2}\,\sqrt{ab}, -1 - 4\sqrt{2}\,\sqrt{ab}, 8\sqrt{2}\,\sqrt{ab}, -4\sqrt{2}\,\sqrt{ab} + 1, \frac{\sqrt{2}\,\sqrt{ab}\,x - a}{\sqrt{2}\,\sqrt{ab}\,x + a}\right) \mathrm{e}^{\frac{-b\,x^2+a}{x}} c_1 + \mathrm{e}^{bx}\,\mathrm{HeunD}\left(-4\sqrt{2}\,\sqrt{ab}, -1 - 4\sqrt{2}\,\sqrt{ab}, 8\sqrt{2}\,\sqrt{ab}, -4\sqrt{2}\,\sqrt{ab} + 1, \frac{\sqrt{2}\,\sqrt{ab}\,x - a}{\sqrt{2}\,\sqrt{ab}\,x + a}\right) c_2\right)$$

✓ Solution by Mathematica

Time used: 0.576 (sec). Leaf size: 38

```
DSolve[(-(a*b) - b^2*x^2)*y[x] + a*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to e^{bx}\left(c_2 \int_1^x e^{\frac{a}{K[1]} - 2bK[1]} dK[1] + c_1\right)$$

3.155 problem 1159

Internal problem ID [9489]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1159.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + y'x - y = a\,x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(diff(y(x),x),x)+x*diff(y(x),x)-y(x)-a*x^2=0,y(x), singsol=all)
```

$$y(x) = xc_2 + \frac{a\,x^2}{3} + \frac{c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 24

```
DSolve[-(a*x^2) - y[x] + x*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{ax^2}{3} + c_2x + \frac{c_1}{x}$$

3.156 problem 1160

Internal problem ID [9490]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1160.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' + y'x + ay = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve(x^2*diff(diff(y(x),x),x)+x*diff(y(x),x)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(\ln(x)\sqrt{a}\right) + c_2 \cos\left(\ln(x)\sqrt{a}\right)$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 30

```
DSolve[a*y[x] + x*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos\left(\sqrt{a}\log(x)\right) + c_2 \sin\left(\sqrt{a}\log(x)\right)$$

3.157 problem 1161

Internal problem ID [9491]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1161.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x - (x + a)\,y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 31

```
dsolve(x^2*diff(diff(y(x),x),x)+x*diff(y(x),x)-(x+a)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselI}\left(2\sqrt{a}, 2\sqrt{x}\right) + c_2 \operatorname{BesselK}\left(2\sqrt{a}, 2\sqrt{x}\right)$$

✓ Solution by Mathematica
Time used: 0.075 (sec). Leaf size: 78

```
DSolve[(-a - x)*y[x] + x*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \to\ & (-1)^{-\sqrt{a}} c_1 \operatorname{Gamma}\left(1 - 2\sqrt{a}\right) \operatorname{BesselI}\left(-2\sqrt{a}, 2\sqrt{x}\right) \\ & + (-1)^{\sqrt{a}} c_2 \operatorname{Gamma}\left(2\sqrt{a} + 1\right) \operatorname{BesselI}\left(2\sqrt{a}, 2\sqrt{x}\right) \end{aligned}$$

3.158 problem 1162

Internal problem ID [9492]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1162.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Bessel]`

$$\boxed{x^2y'' + y'x + (-v^2 + x^2)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(diff(y(x),x),x)+x*diff(y(x),x)+(-v^2+x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}(v, x) + c_2 \operatorname{BesselY}(v, x)$$

✓ Solution by Mathematica
Time used: 0.057 (sec). Leaf size: 18

```
DSolve[(-v^2 + x^2)*y[x] + x*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to c_1 \operatorname{BesselJ}(v, x) + c_2 \operatorname{BesselY}(v, x)$$

3.159 problem 1163

Internal problem ID [9493]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1163.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + y'x + (-v^2 + x^2)\, y = f(x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 49

```
dsolve(x^2*diff(diff(y(x),x),x)+x*diff(y(x),x)+(-v^2+x^2)*y(x)-f(x)=0,y(x), singsol=all)
```

$$y(x) = \text{BesselJ}(v,x)\, c_2 + \text{BesselY}(v,x)\, c_1 + \frac{\pi\left(\int \frac{\text{BesselJ}(v,x) f(x)}{x} dx\right) \text{BesselY}(v,x)}{2} - \frac{\pi\left(\int \frac{\text{BesselY}(v,x) f(x)}{x} dx\right) \text{BesselJ}(v,x)}{2}$$

✓ Solution by Mathematica

Time used: 0.165 (sec). Leaf size: 72

```
DSolve[-f[x] + (-v^2 + x^2)*y[x] + x*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \text{BesselJ}(v,x) \int_1^x -\frac{\pi \text{BesselY}(v,K[1]) f(K[1])}{2K[1]} dK[1] + \text{BesselY}(v,x) \int_1^x \frac{\pi \text{BesselJ}(v,K[2]) f(K[2])}{2K[2]} dK[2] + c_1 \text{BesselJ}(v,x) + c_2 \text{BesselY}(v,x)$$

3.160 problem 1164

Internal problem ID [9494]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1164.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + (l\,x^2 - v^2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(diff(y(x),x),x)+x*diff(y(x),x)+(l*x^2-v^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}\left(v, \sqrt{l}\,x\right) + c_2 \operatorname{BesselY}\left(v, \sqrt{l}\,x\right)$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 30

```
DSolve[(-v^2 + l*x^2)*y[x] + x*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to c_1 \operatorname{BesselJ}\left(v, \sqrt{l}x\right) + c_2 \operatorname{BesselY}\left(v, \sqrt{l}x\right)$$

3.161 problem 1165

Internal problem ID [9495]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1165.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x^2y'' + (x + a)\, y' - y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(x^2*diff(diff(y(x),x),x)+(x+a)*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = (a + x)\, c_1 + c_2 x\, \mathrm{e}^{\frac{a}{x}}$$

✓ Solution by Mathematica

Time used: 0.169 (sec). Leaf size: 26

```
DSolve[-y[x] + (a + x)*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2(a + x)}{a^2} + c_1 x e^{a/x}$$

3.162 problem 1166

Internal problem ID [9496]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1166.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - y'x + y = 3x^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x^2*diff(diff(y(x),x),x)-x*diff(y(x),x)+y(x)-3*x^3=0,y(x), singsol=all)
```

$$y(x) = \frac{x(4c_1 \ln(x) + 3x^2 + 4c_2)}{4}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 23

```
DSolve[-3*x^3 + y[x] - x*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3x^3}{4} + c_1x + c_2x\log(x)$$

3.163 problem 1167

Internal problem ID [9497]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1167.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - y'x + (a\,x^m + b)\,y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 63

```
dsolve(x^2*diff(diff(y(x),x),x)-x*diff(y(x),x)+(a*x^m+b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = x\left(c_1\,\text{BesselJ}\left(\frac{2\sqrt{1-b}}{m}, \frac{2\sqrt{a}\,x^{\frac{m}{2}}}{m}\right) + c_2\,\text{BesselY}\left(\frac{2\sqrt{1-b}}{m}, \frac{2\sqrt{a}\,x^{\frac{m}{2}}}{m}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.144 (sec). Leaf size: 130

```
DSolve[(b + a*x^m)*y[x] - x*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to m^{-2/m}a^{\frac{1}{m}}(x^m)^{\frac{1}{m}}\left(c_1\,\text{Gamma}\left(1-\frac{2i\sqrt{b-1}}{m}\right)\text{BesselJ}\left(-\frac{2i\sqrt{b-1}}{m}, \frac{2\sqrt{a}\sqrt{x^m}}{m}\right) + c_2\,\text{Gamma}\left(\frac{2i\sqrt{b-1}}{m}+1\right)\text{BesselJ}\left(\frac{2i\sqrt{b-1}}{m}, \frac{2\sqrt{a}\sqrt{x^m}}{m}\right)\right)$$

3.164 problem 1168

Internal problem ID [9498]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1168.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x^2y'' + 2y'x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x^2*diff(diff(y(x),x),x)+2*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + \frac{c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 15

```
DSolve[2*x*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{c_1}{x}$$

3.165 problem 1169

Internal problem ID [9499]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1169.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 2y'x + (xa - b^2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 49

```
dsolve(x^2*diff(diff(y(x),x),x)+2*x*diff(y(x),x)+(a*x-b^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \operatorname{BesselJ}\left(\sqrt{4b^2+1}, 2\sqrt{a}\,\sqrt{x}\right) + c_2 \operatorname{BesselY}\left(\sqrt{4b^2+1}, 2\sqrt{a}\,\sqrt{x}\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.103 (sec). Leaf size: 103

```
DSolve[(-b^2 + a*x)*y[x] + 2*x*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{c_1 \operatorname{Gamma}\left(1 - \sqrt{4b^2+1}\right) \operatorname{BesselJ}\left(-\sqrt{4b^2+1}, 2\sqrt{a}\sqrt{x}\right) + c_2 \operatorname{Gamma}\left(\sqrt{4b^2+1}+1\right) \operatorname{BesselJ}\left(\sqrt{4b^2+\right.}{\sqrt{a}\sqrt{x}}$$

3.166 problem 1170

Internal problem ID [9500]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1170.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + 2y'x + (a\,x^2 + b)\,y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 43

```
dsolve(x^2*diff(diff(y(x),x),x)+2*x*diff(y(x),x)+(a*x^2+b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \operatorname{BesselJ}\left(\frac{\sqrt{1-4b}}{2}, x\sqrt{a}\right) + c_2 \operatorname{BesselY}\left(\frac{\sqrt{1-4b}}{2}, x\sqrt{a}\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 58

```
DSolve[(b + a*x^2)*y[x] + 2*x*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to c_1 j_{\frac{1}{2}(\sqrt{1-4b}-1)}(\sqrt{a}x) + c_2 y_{\frac{1}{2}(\sqrt{1-4b}-1)}(\sqrt{a}x)$$

3.167 problem 1171

Internal problem ID [9501]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1171.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + 2y'x + \left(l\,x^2 + xa - n(n+1)\right)y = 0$$

✓ Solution by Maple
Time used: 0.093 (sec). Leaf size: 49

```
dsolve(x^2*diff(diff(y(x),x),x)+2*x*diff(y(x),x)+(l*x^2+a*x-n*(n+1))*y(x)=0,y(x), singsol=al
```

$$y(x) = \frac{c_2 \operatorname{WhittakerW}\left(-\frac{ia}{2\sqrt{l}}, n+\frac{1}{2}, 2i\sqrt{l}\,x\right) + c_1 \operatorname{WhittakerM}\left(-\frac{ia}{2\sqrt{l}}, n+\frac{1}{2}, 2i\sqrt{l}\,x\right)}{x}$$

✓ Solution by Mathematica
Time used: 0.08 (sec). Leaf size: 92

```
DSolve[(-(n*(1 + n)) + a*x + l*x^2)*y[x] + 2*x*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingula
```

$$y(x) \to e^{-i\sqrt{l}x}x^n\left(c_1 \operatorname{HypergeometricU}\left(\frac{ia}{2\sqrt{l}} + n + 1, 2n+2, 2i\sqrt{l}x\right) + c_2 L_{-\frac{ia}{2\sqrt{l}}-n-1}^{2n+1}\left(2i\sqrt{l}x\right)\right)$$

3.168 problem 1172

Internal problem ID [9502]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1172.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + 2(x-1)\,y' + ay = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 47

```
dsolve(x^2*diff(diff(y(x),x),x)+2*(x-1)*diff(y(x),x)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{\frac{1}{x}}\,\mathrm{e}^{-\frac{1}{x}}\left(c_1\,\mathrm{BesselI}\left(\frac{\sqrt{-4a+1}}{2}, \frac{1}{x}\right) + c_2\,\mathrm{BesselK}\left(\frac{\sqrt{-4a+1}}{2}, \frac{1}{x}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.157 (sec). Leaf size: 145

```
DSolve[a*y[x] + 2*(-1 + x)*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2^{\frac{1}{2}-\frac{1}{2}\sqrt{1-4a}}\left(\frac{1}{x}\right)^{\frac{1}{2}-\frac{1}{2}\sqrt{1-4a}}\left(2^{\sqrt{1-4a}}c_2\left(\frac{1}{x}\right)^{\sqrt{1-4a}}\,\mathrm{Hypergeometric1F1}\left(\frac{1}{2}\left(\sqrt{1-4a}+1\right), \sqrt{1-4a} + 1, -\frac{2}{x}\right) + c_1\,\mathrm{Hypergeometric1F1}\left(\frac{1}{2}-\frac{1}{2}\sqrt{1-4a}, 1-\sqrt{1-4a}, -\frac{2}{x}\right)\right)$$

3.169 problem 1173

Internal problem ID [9503]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1173.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + 2(x+a)\,y' - b(b-1)\,y = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 37

```
dsolve(x^2*diff(diff(y(x),x),x)+2*(x+a)*diff(y(x),x)-b*(b-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{e^{\frac{a}{x}} \left(\operatorname{BesselI}\left(b-\frac{1}{2}, \frac{a}{x}\right) c_1 + \operatorname{BesselK}\left(b-\frac{1}{2}, \frac{a}{x}\right) c_2\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.224 (sec). Leaf size: 74

```
DSolve[(1 - b)*b*y[x] + 2*(a + x)*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

$$\begin{aligned} y(x) \to\ & (-2)^{1-b} c_1 a^{1-b} \left(\frac{1}{x}\right)^{1-b} \text{Hypergeometric1F1}\left(1-b, 2-2b, \frac{2a}{x}\right) \\ & + (-2)^b c_2 a^b \left(\frac{1}{x}\right)^b \text{Hypergeometric1F1}\left(b, 2b, \frac{2a}{x}\right) \end{aligned}$$

3.170 problem 1174

Internal problem ID [9504]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1174.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2y'x + 2y = x^5 \ln(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(x^2*diff(diff(y(x),x),x)-2*x*diff(y(x),x)+2*y(x)-x^5*ln(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x^5 \ln(x)}{12} - \frac{7x^5}{144} + c_2x^2 + c_1x$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 32

```
DSolve[-(x^5*Log[x]) + 2*y[x] - 2*x*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to -\frac{7x^5}{144} + \frac{1}{12}x^5 \log(x) + c_2x^2 + c_1x$$

3.171 problem 1175

Internal problem ID [9505]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1175.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$x^2y'' - 2y'x - 4y = x\sin(x) + \left(a\,x^2 + 12a + 4\right)\cos(x)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(x^2*diff(diff(y(x),x),x)-2*x*diff(y(x),x)-4*y(x)-x*sin(x)-(a*x^2+12*a+4)*cos(x)=0,y(x
```

$$y(x) = \frac{(-2a-1)\sin(x) + x^5c_2 - xa\cos(x) + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.501 (sec). Leaf size: 33

```
DSolve[(-4 - 12*a - a*x^2)*Cos[x] - x*Sin[x] - 4*y[x] - 2*x*y'[x] + x^2*y''[x] == 0,y[x],x,I
```

$$y(x) \to \frac{-(2a+1)\sin(x) - ax\cos(x) + c_2x^5 + c_1}{x}$$

3.172 problem 1176

Internal problem ID [9506]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1176.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2y'x + (x^2 + 2)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(diff(y(x),x),x)-2*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = x(c_1 \sin(x) + c_2 \cos(x))$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 33

```
DSolve[(2 + x^2)*y[x] - 2*x*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-ix} x - \frac{1}{2} i c_2 e^{ix} x$$

3.173 problem 1177

Internal problem ID [9507]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1177.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x^2y'' - 2y'x + (x^2 + 2)\, y = \frac{x^2}{\cos(x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(x^2*diff(diff(y(x),x),x)-2*x*diff(y(x),x)+(x^2+2)*y(x)-x^2/cos(x)=0,y(x), singsol=all
```

$$y(x) = \left(-\left(\int \frac{\tan(x)}{x} dx\right)\cos(x) + \cos(x)\, c_1 + \sin(x)\,(c_2 + \ln(x))\right) x$$

✓ Solution by Mathematica

Time used: 1.008 (sec). Leaf size: 116

```
DSolve[-(x^2*Sec[x]) - 2*x*y'[x] + (2 + x^2)*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularS
```

$$y(x) \to \int_1^x -\frac{e^{K[2]-\frac{2}{K[2]}}\sec(K[2]) \int_1^{K[2]} e^{\frac{2}{K[1]}-K[1]}K[1]^2 dK[1]}{K[2]^2} dK[2] + \int_1^x e^{\frac{2}{K[1]}-K[1]}K[1]^2 dK[1] \left(\int_1^x \frac{e^{K[3]-\frac{2}{K[3]}}\sec(K[3])}{K[3]^2} dK[3] + c_2\right) + c_1$$

3.174 problem 1178

Internal problem ID [9508]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1178.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' - 2y'x + (x^2+2)\, y = \frac{x^3}{\cos(x)}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(diff(y(x),x),x)-2*x*diff(y(x),x)+(x^2+2)*y(x)-x^3/cos(x)=0,y(x), singsol=all
```

$$y(x) = x(\ln(\cos(x))\cos(x) + \cos(x)\, c_1 + \sin(x)\,(x + c_2))$$

✓ Solution by Mathematica

Time used: 0.081 (sec). Leaf size: 63

```
DSolve[-(x^3*Sec[x]) + (2 + x^2)*y[x] - 2*x*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSo
```

$$y(x) \to \frac{1}{2}e^{-ix}x\left(e^{2ix}\log\left(1+e^{-2ix}\right) + \log\left(1+e^{2ix}\right) - ic_2e^{2ix} + 2c_1\right)$$

3.175 problem 1179

Internal problem ID [9509]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1179.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - 2y'x + (a^2x^2 + 2)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(diff(y(x),x),x)-2*x*diff(y(x),x)+(a^2*x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = x(c_1 \sin(ax) + c_2 \cos(ax))$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 38

```
DSolve[(2 + a^2*x^2)*y[x] - 2*x*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to c_1 x e^{-iax} - \frac{ic_2 x e^{iax}}{2a}$$

3.176 problem 1180

Internal problem ID [9510]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1180.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x^2y'' + 3y'x + (-v^2 + x^2 + 1)\,y = f(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve(x^2*diff(diff(y(x),x),x)+3*x*diff(y(x),x)+(-v^2+x^2+1)*y(x)-f(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\pi\left(\int \operatorname{BesselJ}(v,x)\, f(x)\, dx\right) \operatorname{BesselY}(v,x) - \pi\left(\int \operatorname{BesselY}(v,x)\, f(x)\, dx\right) \operatorname{BesselJ}(v,x) + 2 \operatorname{BesselY}(v,x}{2x}$$

✓ Solution by Mathematica

Time used: 0.104 (sec). Leaf size: 68

```
DSolve[-f[x] + (1 - v^2 + x^2)*y[x] + 3*x*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolu
```

$$y(x) \to \frac{\operatorname{BesselJ}(v,x) \int_1^x -\frac{1}{2}\pi \operatorname{BesselY}(v,K[1]) f(K[1]) dK[1] + \operatorname{BesselY}(v,x) \int_1^x \frac{1}{2}\pi \operatorname{BesselJ}(v,K[2]) f(K[2]) dK}{x}$$

3.177 problem 1181

Internal problem ID [9511]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1181.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2y'' + (-1+3x)\,y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(x^2*diff(diff(y(x),x),x)+(3*x-1)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(c_1 \,\mathrm{expIntegral}_1\left(-\frac{1}{x}\right) + c_2\right) \mathrm{e}^{-\frac{1}{x}}}{x}$$

✓ Solution by Mathematica

Time used: 0.087 (sec). Leaf size: 27

```
DSolve[y[x] + (-1 + 3*x)*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-1/x}\left(c_1 - c_2 \,\mathrm{ExpIntegralEi}\left(\frac{1}{x}\right)\right)}{x}$$

3.178 problem 1182

Internal problem ID [9512]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1182.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 3y'x + 4y = 5x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x^2*diff(diff(y(x),x),x)-3*x*diff(y(x),x)+4*y(x)-5*x=0,y(x), singsol=all)
```

$$y(x) = x(x\ln(x)\,c_1 + xc_2 + 5)$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 20

```
DSolve[-5*x + 4*y[x] - 3*x*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(c_1x + 2c_2x\log(x) + 5)$$

3.179 problem 1183

Internal problem ID [9513]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1183.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' - 3y'x - 5y = x^2 \ln(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x^2*diff(diff(y(x),x),x)-3*x*diff(y(x),x)-5*y(x)-x^2*ln(x)=0,y(x), singsol=all)
```

$$y(x) = x^5c_2 + \frac{c_1}{x} - \frac{x^2 \ln(x)}{9}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 27

```
DSolve[-(x^2*Log[x]) - 5*y[x] - 3*x*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to c_2x^5 - \frac{1}{9}x^2 \log(x) + \frac{c_1}{x}$$

3.180 problem 1184

Internal problem ID [9514]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1184.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' - 4y'x + 6y = x^4 - x^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 25

```
dsolve(x^2*diff(diff(y(x),x),x)-4*x*diff(y(x),x)+6*y(x)-x^4+x^2=0,y(x), singsol=all)
```

$$y(x) = \frac{x^2(2xc_2 + x^2 + 2\ln(x) + 2c_1 + 2)}{2}$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 30

```
DSolve[x^2 - x^4 + 6*y[x] - 4*x*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{1}{2}x^2(x^2 + 2\log(x) + 2c_2x + 2 + 2c_1)$$

3.181 problem 1185

Internal problem ID [9515]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1185.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + 5y'x - (2x^3 - 4)\, y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(x^2*diff(diff(y(x),x),x)+5*x*diff(y(x),x)-(2*x^3-4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \operatorname{BesselI}\left(0, \frac{2\sqrt{2}\, x^{\frac{3}{2}}}{3}\right) + c_2 \operatorname{BesselK}\left(0, \frac{2\sqrt{2}\, x^{\frac{3}{2}}}{3}\right)}{x^2}$$

✓ Solution by Mathematica
Time used: 0.072 (sec). Leaf size: 65

```
DSolve[(4 - 2*x^3)*y[x] + 5*x*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{6\sqrt[3]{3}c_2 K_0\left(\frac{2}{3}\sqrt{2}x^{3/2}\right) - 3\sqrt[3]{-3}c_1 \operatorname{BesselI}\left(0, \frac{2}{3}\sqrt{2}x^{3/2}\right)}{2^{2/3}x^2}$$

3.182 problem 1186

Internal problem ID [9516]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1186.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' - 5y'x + 8y = \sin(x)\, x^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(x^2*diff(diff(y(x),x),x)-5*x*diff(y(x),x)+8*y(x)-sin(x)*x^3=0,y(x), singsol=all)
```

$$y(x) = \frac{x^2(\mathrm{Ci}(x)\, x^2 + 2c_1x^2 - x\sin(x) + \cos(x) + 2c_2)}{2}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 37

```
DSolve[-(x^3*Sin[x]) + 8*y[x] - 5*x*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{2}x^2\left(x^2\,\mathrm{CosIntegral}(x) + 2c_2x^2 - x\sin(x) + \cos(x) + 2c_1\right)$$

3.183 problem 1187

Internal problem ID [9517]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1187.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + axy' + yb = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 52

```
dsolve(x^2*diff(diff(y(x),x),x)+a*x*diff(y(x),x)+b*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^{-\frac{a}{2}}\sqrt{x}\left(x^{\frac{\sqrt{a^2-2a-4b+1}}{2}}c_1 + x^{-\frac{\sqrt{a^2-2a-4b+1}}{2}}c_2\right)$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 57

```
DSolve[b*y[x] + a*x*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^{\frac{1}{2}\left(-\sqrt{a^2-2a-4b+1}-a+1\right)}\left(c_2x^{\sqrt{a^2-2a-4b+1}} + c_1\right)$$

3.184 problem 1188

Internal problem ID [9518]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1188.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + (xa + b) y' + yc = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 114

```
dsolve(x^2*diff(diff(y(x),x),x)+(a*x+b)*diff(y(x),x)+c*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^{-\frac{\sqrt{a^2-2a-4c+1}}{2}-\frac{a}{2}+\frac{1}{2}} \left(\operatorname{KummerU}\left(-\frac{1}{2}+\frac{\sqrt{a^2-2a-4c+1}}{2}+\frac{a}{2}, 1 + \sqrt{a^2-2a-4c+1}, \frac{b}{x}\right) c_2 + \operatorname{KummerM}\left(-\frac{1}{2}+\frac{\sqrt{a^2-2a-4c+1}}{2}+\frac{a}{2}, 1+\sqrt{a^2-2a-4c+1}, \frac{b}{x}\right) c_1\right)$$

✓ Solution by Mathematica

Time used: 0.377 (sec). Leaf size: 243

```
DSolve[c*y[x] + (b + a*x)*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to$$

$$-i^{-\sqrt{a^2-2a-4c+1}+a+1} b^{\frac{1}{2}\left(-\sqrt{a^2-2a-4c+1}+a-1\right)} \left(\frac{1}{x}\right)^{\frac{1}{2}\left(-\sqrt{a^2-2a-4c+1}+a-1\right)} \left(c_2 i^{2\sqrt{a^2-2a-4c+1}} b^{\sqrt{a^2-2a-4c+1}} \left(\frac{1}{x}\right)^{\sqrt{}}\right.$$

$$\left. + 1, \frac{b}{x}\right) + c_1 \,\text{Hypergeometric1F1}\left(\frac{1}{2}\left(a - \sqrt{a^2-2a-4c+1} - 1\right), 1\right.$$

$$\left.\left. - \sqrt{a^2-2a-4c+1}, \frac{b}{x}\right)\right)$$

3.185 problem 1189

Internal problem ID [9519]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1189.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + axy' + (b\,x^m + c)\,y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 80

```
dsolve(x^2*diff(diff(y(x),x),x)+a*x*diff(y(x),x)+(b*x^m+c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^{-\frac{a}{2}}\sqrt{x}\left(\operatorname{BesselY}\left(\frac{\sqrt{a^2-2a-4c+1}}{m}, \frac{2\sqrt{b}\,x^{\frac{m}{2}}}{m}\right)c_2 + \operatorname{BesselJ}\left(\frac{\sqrt{a^2-2a-4c+1}}{m}, \frac{2\sqrt{b}\,x^{\frac{m}{2}}}{m}\right)c_1\right)$$

✓ Solution by Mathematica

Time used: 0.193 (sec). Leaf size: 168

```
DSolve[(c + b*x^m)*y[x] + a*x*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to m^{\frac{a-1}{m}} b^{-\frac{a-1}{2m}} (x^m)^{-\frac{a-1}{2m}} \left(c_1 \operatorname{Gamma}\left(1 - \frac{\sqrt{a^2-2a-4c+1}}{m}\right) \operatorname{BesselJ}\left(-\frac{\sqrt{a^2-2a-4c+1}}{m}, \frac{2\sqrt{b}\sqrt{x^m}}{m}\right) + c_2 \operatorname{Gamma}\left(\frac{m+\sqrt{a^2-2a-4c+1}}{m}\right) \operatorname{BesselJ}\left(\frac{\sqrt{a^2-2a-4c+1}}{m}, \frac{2\sqrt{b}\sqrt{x^m}}{m}\right)\right)$$

3.186 problem 1190

Internal problem ID [9520]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1190.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x^2y' + (xa + b)\,y = 0$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 38

```
dsolve(x^2*diff(diff(y(x),x),x)+x^2*diff(y(x),x)+(a*x+b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{2}}\left(\mathrm{WhittakerM}\left(a, \frac{\sqrt{1-4b}}{2}, x\right)c_1 + \mathrm{WhittakerW}\left(a, \frac{\sqrt{1-4b}}{2}, x\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.082 (sec). Leaf size: 95

```
DSolve[(b + a*x)*y[x] + x^2*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}x^{\frac{1}{2}\left(\sqrt{1-4b}+1\right)}\left(c_1\,\mathrm{HypergeometricU}\left(\frac{1}{2}\left(-2a+\sqrt{1-4b}+1\right), \sqrt{1-4b}+1, x\right) + c_2L_{a-\frac{1}{2}\sqrt{1-4b}-\frac{1}{2}}^{\sqrt{1-4b}}(x)\right)$$

3.187 problem 1191

Internal problem ID [9521]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1191.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x^2y' - 2y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(x^2*diff(diff(y(x),x),x)+x^2*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2 e^{-x}(x+2) + c_1(x-2)}{x}$$

✓ Solution by Mathematica

Time used: 0.057 (sec). Leaf size: 72

```
DSolve[-2*y[x] + x^2*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x/2}\left(2(ic_2x + 2c_1)\sinh\left(\frac{x}{2}\right) - 2(c_1x + 2ic_2)\cosh\left(\frac{x}{2}\right)\right)}{\sqrt{\pi}\sqrt{-ix}\sqrt{x}}$$

3.188 problem 1192

Internal problem ID [9522]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1192.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + (x^2-1)\,y' - y = 0}$$

✓ Solution by Maple

Time used: 0.391 (sec). Leaf size: 51

```
dsolve(x^2*diff(diff(y(x),x),x)+(x^2-1)*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(\mathrm{HeunD}\left(-4,3,-8,5,\frac{x-1}{x+1}\right)\mathrm{e}^{-\frac{1}{x}}c_2 + \mathrm{e}^{-x}\,\mathrm{HeunD}\left(4,3,-8,5,\frac{x-1}{x+1}\right)c_1\right)\sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.316 (sec). Leaf size: 35

```
DSolve[-y[x] + (-1 + x^2)*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_2\int_1^x e^{K[1]-\frac{1}{K[1]}}dK[1] + c_1\right)$$

3.189 problem 1193

Internal problem ID [9523]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1193.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(x+1)\,y' + (x-9)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(x^2*diff(diff(y(x),x),x)+x*(x+1)*diff(y(x),x)+(x-9)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2 e^{-x}(x^3 + 9x^2 + 36x + 60) + c_1(x^2 - 8x + 20)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.068 (sec). Leaf size: 42

```
DSolve[(-9 + x)*y[x] + x*(1 + x)*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{c_1((x-8)x + 20) - c_2 e^{-x}(x^3 + 9x^2 + 36x + 60)}{x^3}$$

3.190 problem 1194

Internal problem ID [9524]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1194.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(x+1)\,y' + (-1+3x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 48

```
dsolve(x^2*diff(diff(y(x),x),x)+x*(x+1)*diff(y(x),x)+(3*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{e^{-x}x^2c_2(x-3)\,\text{expIntegral}_1\,(-x) + x^2c_1(x-3)\,e^{-x} + c_2(x^2-2x-1)}{x}$$

✓ Solution by Mathematica

Time used: 0.089 (sec). Leaf size: 66

```
DSolve[(-1 + 3*x)*y[x] + x*(1 + x)*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{e^{-x}(c_2(x-3)x^2\,\text{ExpIntegralEi}(x) + 6c_1x^3 - x^2(c_2e^x + 18c_1) + 2c_2e^xx + c_2e^x)}{6x}$$

3.191 problem 1195

Internal problem ID [9525]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1195.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + (x+3)\,xy' - y = 0$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 93

```
dsolve(x^2*diff(diff(y(x),x),x)+(x+3)*x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\left(-c_1\left(\sqrt{2}+x+1\right)\text{BesselI}\left(-\frac{1}{2}+\sqrt{2}, \frac{x}{2}\right) - c_1\left(-\sqrt{2}+x+1\right)\text{BesselI}\left(\frac{1}{2}+\sqrt{2}, \frac{x}{2}\right) + c_2\left(\left(-\sqrt{2}-x-\right.\right.\right.}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.049 (sec). Leaf size: 63

```
DSolve[-y[x] + x*(3 + x)*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}x^{\sqrt{2}-1}\left(c_1\,\text{HypergeometricU}\left(2+\sqrt{2}, 1+2\sqrt{2}, x\right) + c_2L_{-2-\sqrt{2}}^{2\sqrt{2}}(x)\right)$$

3.192 problem 1196

Internal problem ID [9526]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1196.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(x-1)\,y' + (x-1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(x^2*diff(diff(y(x),x),x)-x*(x-1)*diff(y(x),x)+(x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x^2c_2\,\text{expIntegral}_1\,(-x) + (x+1)\,c_2\mathrm{e}^x + c_1x^2}{x}$$

✓ Solution by Mathematica

Time used: 0.068 (sec). Leaf size: 34

```
DSolve[(-1 + x)*y[x] - (-1 + x)*x*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{c_2(x^2\,\text{ExpIntegralEi}(x) - e^x(x+1))}{2x} + c_1x$$

3.193 problem 1197

Internal problem ID [9527]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1197.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - (x^2 - 2x)\, y' - (x + a)\, y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 43

```
dsolve(x^2*diff(diff(y(x),x),x)-(x^2-2*x)*diff(y(x),x)-(x+a)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{e^{\frac{x}{2}} \left(\mathrm{BesselK}\left(\frac{\sqrt{4a+1}}{2}, \frac{x}{2}\right) c_2 + \mathrm{BesselI}\left(\frac{\sqrt{4a+1}}{2}, \frac{x}{2}\right) c_1\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 67

```
DSolve[(-a - x)*y[x] - (-2*x + x^2)*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{e^{x/2} \left(c_1 \,\mathrm{BesselJ}\left(\frac{1}{2}\sqrt{4a+1}, -\frac{ix}{2}\right) + c_2 \,\mathrm{BesselY}\left(\frac{1}{2}\sqrt{4a+1}, -\frac{ix}{2}\right)\right)}{\sqrt{x}}$$

3.194 problem 1198

Internal problem ID [9528]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1198.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - (x^2 - 2x)\, y' - (3x + 2)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 37

```
dsolve(x^2*diff(diff(y(x),x),x)-(x^2-2*x)*diff(y(x),x)-(3*x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-\mathrm{e}^x \operatorname{expIntegral}_1(x)\, c_2x^3 + \mathrm{e}^x x^3 c_1 + c_2(x^2 - x + 2)}{x^2}$$

✓ Solution by Mathematica
Time used: 0.065 (sec). Leaf size: 41

```
DSolve[(-2 - 3*x)*y[x] - (-2*x + x^2)*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolution
```

$$y(x) \to c_1e^x x - \frac{c_2(e^x x^3 \operatorname{ExpIntegralEi}(-x) + x^2 - x + 2)}{6x^2}$$

3.195 problem 1199

Internal problem ID [9529]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1199.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' - x(4+x)\,y' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(x^2*diff(diff(y(x),x),x)-x*(x+4)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = x\left(\mathrm{e}^{x} \operatorname{expIntegral}_1\left(x\right) c_2 x^3 + \mathrm{e}^{x} x^3 c_1 - c_2\left(x^2 - x + 2\right)\right)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 41

```
DSolve[4*y[x] - x*(4 + x)*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 e^x x^4 - \frac{1}{6} c_1 x\left(e^x x^3 \operatorname{ExpIntegralEi}(-x) + x^2 - x + 2\right)$$

3.196 problem 1200

Internal problem ID [9530]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1200.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 2x^2y' - v(v-1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(x^2*diff(diff(y(x),x),x)+2*x^2*diff(y(x),x)-v*(v-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\,e^{-x}\left(c_1 \operatorname{BesselI}\left(v-\frac{1}{2}, x\right) + c_2 \operatorname{BesselK}\left(v-\frac{1}{2}, x\right)\right)$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 45

```
DSolve[(1 - v)*v*y[x] + 2*x^2*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^{-x}\sqrt{x}\left(c_1 \operatorname{BesselJ}\left(v-\frac{1}{2}, -ix\right) + c_2 \operatorname{BesselY}\left(v-\frac{1}{2}, -ix\right)\right)$$

3.197 problem 1201

Internal problem ID [9531]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1201.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x(2x+1)\,y' - 4y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(x^2*diff(diff(y(x),x),x)+x*(2*x+1)*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2 e^{-2x}(2x+3) + 2\left(x^2 - 2x + \frac{3}{2}\right) c_1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.074 (sec). Leaf size: 44

```
DSolve[-4*y[x] + x*(1 + 2*x)*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{e^{-2x}(c_2 e^{2x}(2x^2 - 4x + 3) + c_1(4x+6))}{4x^2}$$

3.198 problem 1202

Internal problem ID [9532]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1202.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2x(x+1)\,y' + 2(x+1)\,y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 14

```
dsolve(x^2*diff(diff(y(x),x),x)-2*x*(x+1)*diff(y(x),x)+2*(x+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = x\left(c_1 + c_2 e^{2x}\right)$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 21

```
DSolve[2*(1 + x)*y[x] - 2*x*(1 + x)*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to x\left(\frac{1}{2}c_2e^{2x} + c_1\right)$$

3.199 problem 1203

Internal problem ID [9533]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1203.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' + a\,x^2 y' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve(x^2*diff(diff(y(x),x),x)+a*x^2*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2 \mathrm{e}^{-ax}(ax+2) + c_1(ax-2)}{x}$$

✓ Solution by Mathematica

Time used: 0.066 (sec). Leaf size: 80

```
DSolve[-2*y[x] + a*x^2*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a x^{3/2} e^{-\frac{ax}{2}} \left(2(iac_2 x + 2c_1)\sinh\left(\frac{ax}{2}\right) - 2(ac_1 x + 2ic_2)\cosh\left(\frac{ax}{2}\right)\right)}{\sqrt{\pi}(-iax)^{5/2}}$$

3.200 problem 1204

Internal problem ID [9534]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1204.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + (a+2b)\,x^2y' + \left((a+b)\,b\,x^2 - 2\right)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(x^2*diff(diff(y(x),x),x)+(a+2*b)*x^2*diff(y(x),x)+((a+b)*b*x^2-2)*y(x)=0,y(x), singso
```

$$y(x) = \frac{c_2 \mathrm{e}^{-x(a+b)}(ax+2) + c_1 \mathrm{e}^{-bx}(ax-2)}{x}$$

✓ Solution by Mathematica

Time used: 0.101 (sec). Leaf size: 84

```
DSolve[(-2 + b*(a + b)*x^2)*y[x] + (a + 2*b)*x^2*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingu
```

$$y(x) \to -\frac{ax^{3/2}e^{-\frac{1}{2}x(a+2b)}\left(2(iac_2x + 2c_1)\sinh\left(\frac{ax}{2}\right) - 2(ac_1x + 2ic_2)\cosh\left(\frac{ax}{2}\right)\right)}{\sqrt{\pi}(-iax)^{5/2}}$$

3.201 problem 1205

Internal problem ID [9535]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1205.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + a\,x^2y' + f(x)\,y = 0$$

✗ Solution by Maple

```
dsolve(x^2*diff(diff(y(x),x),x)+a*x^2*diff(y(x),x)+f(x)*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[f[x]*y[x] + a*x^2*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

3.202 problem 1206

Internal problem ID [9536]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1206.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + (2xa + b)\,xy' + \left(abx + c\,x^2 + d\right) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 76

```
dsolve(x^2*diff(diff(y(x),x),x)+(2*a*x+b)*x*diff(y(x),x)+(a*b*x+c*x^2+d)*y(x)=0,y(x), singso
```

$$y(x) = x^{\frac{1}{2}-\frac{b}{2}} \mathrm{e}^{-ax} \left(c_1 \operatorname{BesselJ}\left(\frac{\sqrt{b^2-2b-4d+1}}{2}, \sqrt{-a^2+c}\,x\right) + c_2 \operatorname{BesselY}\left(\frac{\sqrt{b^2-2b-4d+1}}{2}, \sqrt{-a^2+c}\,x\right)\right)$$

✓ Solution by Mathematica

Time used: 0.098 (sec). Leaf size: 102

```
DSolve[(d + a*b*x + c*x^2)*y[x] + x*(b + 2*a*x)*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingul
```

$$y(x) \to e^{-ax} x^{\frac{1}{2}-\frac{b}{2}} \left(c_1 \operatorname{BesselJ}\left(\frac{1}{2}\sqrt{b^2-2b-4d+1}, -i\sqrt{a^2-c}x\right) + c_2 \operatorname{BesselY}\left(\frac{1}{2}\sqrt{b^2-2b-4d+1}, -i\sqrt{a^2-c}x\right)\right)$$

3.203 problem 1207

Internal problem ID [9537]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1207.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + (xa + b) y' x + (\mathrm{a1}\ x^2 + \mathrm{b1}\, x + \mathrm{c1}) y = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 110

```
dsolve(x^2*diff(diff(y(x),x),x)+(a*x+b)*diff(y(x),x)*x+(a1*x^2+b1*x+c1)*y(x)=0,y(x), singsol
```

$$y(x) = \mathrm{e}^{-\frac{ax}{2}} x^{-\frac{b}{2}} \left(c_1 \,\mathrm{WhittakerM}\left(-\frac{ab - 2\,\mathrm{b1}}{2\sqrt{a^2 - 4\,\mathrm{a1}}}, \frac{\sqrt{b^2 - 2b - 4\,\mathrm{c1} + 1}}{2}, \sqrt{a^2 - 4\,\mathrm{a1}}\, x\right) + c_2 \,\mathrm{WhittakerW}\left(-\frac{ab - 2\,\mathrm{b1}}{2\sqrt{a^2 - 4\,\mathrm{a1}}}, \frac{\sqrt{b^2 - 2b - 4\,\mathrm{c1} + 1}}{2}, \sqrt{a^2 - 4\,\mathrm{a1}}\, x\right)\right)$$

✓ Solution by Mathematica

Time used: 0.177 (sec). Leaf size: 223

```
DSolve[(c1 + b1*x + a1*x^2)*y[x] + x*(b + a*x)*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingula
```

$$y(x) \to e^{-\frac{1}{2}x\left(\sqrt{a^2-4\mathrm{a1}}+a\right)} x^{\frac{1}{2}\left(\sqrt{b^2-2b-4\mathrm{c1}+1}-b+1\right)} \left(c_1 \,\mathrm{HypergeometricU}\left(\frac{ab - 2\mathrm{b1} + \sqrt{a^2 - 4\mathrm{a1}}\left(\sqrt{b^2 - 2b - 4\mathrm{c1} + \right.}{2\sqrt{a^2 - 4\mathrm{a1}}} + 1, \sqrt{a^2 - 4\mathrm{a1}}x\right) + c_2 L_{\frac{-ab+2\mathrm{b1}-\sqrt{a^2-4\mathrm{a1}}\left(\sqrt{b^2-2b-4\mathrm{c1}+1}+1\right)}{2\sqrt{a^2-4\mathrm{a1}}}}^{\sqrt{b^2-2b-4\mathrm{c1}+1}}\left(\sqrt{a^2 - 4\mathrm{a1}}x\right)\right)$$

3.204 problem 1208

Internal problem ID [9538]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1208.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x^3y' + (x^2 - 2)\, y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(x^2*diff(diff(y(x),x),x)+x^3*diff(y(x),x)+(x^2-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\pi}\,\sqrt{2}\,\operatorname{erf}\left(\frac{\sqrt{2}\,x}{2}\right) c_2 - 2c_2 \mathrm{e}^{-\frac{x^2}{2}} x + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.11 (sec). Leaf size: 49

```
DSolve[(-2 + x^2)*y[x] + x^3*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{\sqrt{2\pi} c_2 \operatorname{erf}\left(\frac{x}{\sqrt{2}}\right) - 2c_2 e^{-\frac{x^2}{2}} x + 2c_1}{2x}$$

3.205 problem 1209

Internal problem ID [9539]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1209.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + \left(x^2 + 2\right) xy' + \left(x^2 - 2\right) y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 41

```
dsolve(x^2*diff(diff(y(x),x),x)+(x^2+2)*x*diff(y(x),x)+(x^2-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(-c_2\pi \operatorname{erf}\left(\frac{i\sqrt{2}\,x}{2}\right) + c_1\right) \mathrm{e}^{-\frac{x^2}{2}} + i\sqrt{\pi}\,\sqrt{2}\,c_2 x}{x^2}$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 59

```
DSolve[(-2 + x^2)*y[x] + x*(2 + x^2)*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{e^{-\frac{x^2}{2}} \left(2\left(c_1 e^{\frac{x^2}{2}} x + c_2\right) - \sqrt{2\pi} c_1 \operatorname{erfi}\left(\frac{x}{\sqrt{2}}\right)\right)}{2x^2}$$

3.206 problem 1210

Internal problem ID [9540]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1210.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - 2x(x^2 - a)\, y' + (2n\, x^2 + ((-1)^n - 1)\, a)\, y = 0$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 81

```
dsolve(x^2*diff(diff(y(x),x),x)-2*x*(x^2-a)*diff(y(x),x)+(2*n*x^2+((-1)^n-1)*a)*y(x)=0,y(x),
```

$$y(x) = x^{-a-\frac{1}{2}} e^{\frac{x^2}{2}} \left(\text{WhittakerM}\left(\frac{a}{2}+\frac{n}{2}+\frac{1}{4}, \frac{\sqrt{1-4a(-1)^n+4a^2}}{4}, x^2\right) c_1 + \text{WhittakerW}\left(\frac{a}{2}+\frac{n}{2}+\frac{1}{4}, \frac{\sqrt{1-4a(-1)^n+4a^2}}{4}, x^2\right) c_2\right)$$

✓ Solution by Mathematica

Time used: 0.392 (sec). Leaf size: 231

```
DSolve[((-1 + (-1)^n)*a + 2*n*x^2)*y[x] - 2*x*(-a + x^2)*y'[x] + x^2*y''[x] == 0,y[x],x,Incl
```

$$y(x) \to i^{-a}(-1)^{\frac{1}{4}\left(1-\sqrt{4a^2-4a(-1)^n+1}\right)} x^{\frac{1}{2}\left(-\sqrt{4a^2-4a(-1)^n+1}-2a+1\right)} \left(c_1\, \text{Hypergeometric1F1}\left(\frac{1}{4}\left(-2a-2n-\sqrt{4a^2-4\right.\right.\right.$$
$$-\frac{1}{2}\sqrt{4a^2-4(-1)^n a+1}, x^2\Big)$$
$$+c_2 i^{\sqrt{4a^2-4a(-1)^n+1}} x^{\sqrt{4a^2-4a(-1)^n+1}}\, \text{Hypergeometric1F1}\left(\frac{1}{4}\left(-2a-2n+\sqrt{4a^2-4(-1)^n a+1}+1\right), \frac{1}{2}\left(\right.\right.$$

3.207 problem 1211

Internal problem ID [9541]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1211.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + 4x^3y' + (4x^4 + 2x^2 + 1) y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(x^2*diff(diff(y(x),x),x)+4*x^3*diff(y(x),x)+(4*x^4+2*x^2+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\, e^{-x^2} \left(c_1 x^{\frac{i\sqrt{3}}{2}} + c_2 x^{-\frac{i\sqrt{3}}{2}}\right)$$

✓ Solution by Mathematica

Time used: 0.075 (sec). Leaf size: 60

```
DSolve[(1 + 2*x^2 + 4*x^4)*y[x] + 4*x^3*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{1}{3} e^{-x^2} x^{\frac{1}{2}-\frac{i\sqrt{3}}{2}} \left(3c_1 - i\sqrt{3} c_2 x^{i\sqrt{3}}\right)$$

3.208 problem 1212

Internal problem ID [9542]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1212.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + (a\,x^2 + b)\,xy' + f(x)\,y = 0$$

✗ Solution by Maple

```
dsolve(x^2*diff(diff(y(x),x),x)+(a*x^2+b)*x*diff(y(x),x)+f(x)*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[f[x]*y[x] + x*(b + a*x^2)*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

Not solved

3.209 problem 1213

Internal problem ID [9543]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1213.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + (x^3+1)\,xy' - y = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 52

```
dsolve(x^2*diff(diff(y(x),x),x)+(x^3+1)*x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-\frac{x^3}{6}} x^{\frac{3}{2}} \left(c_1 \operatorname{BesselI}\left(-\frac{1}{6}, \frac{x^3}{6}\right) + c_1 \operatorname{BesselI}\left(\frac{5}{6}, \frac{x^3}{6}\right) + c_2 \left(\operatorname{BesselK}\left(\frac{1}{6}, \frac{x^3}{6}\right) - \operatorname{BesselK}\left(\frac{5}{6}, \frac{x^3}{6}\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.143 (sec). Leaf size: 54

```
DSolve[-y[x] + x*(1 + x^3)*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{3} c_1 \operatorname{Hypergeometric1F1}\left(-\frac{1}{3}, \frac{1}{3}, -\frac{x^3}{3}\right)}{x} + \frac{c_2 x \operatorname{Hypergeometric1F1}\left(\frac{1}{3}, \frac{5}{3}, -\frac{x^3}{3}\right)}{\sqrt[3]{3}}$$

3.210 problem 1214

Internal problem ID [9544]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1214.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + (-x^4 + (2a + 2n + 1)x^2 + (-1)^n a - a^2) y = 0$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 71

```
dsolve(x^2*diff(diff(y(x),x),x)+(-x^4+(2*n+2*a+1)*x^2+(-1)^n*a-a^2)*y(x)=0,y(x), singsol=all
```

$$y(x) = \frac{\mathrm{WhittakerM}\left(\frac{a}{2} + \frac{n}{2} + \frac{1}{4}, \frac{\sqrt{1-4a(-1)^n+4a^2}}{4}, x^2\right) c_1 + \mathrm{WhittakerW}\left(\frac{a}{2} + \frac{n}{2} + \frac{1}{4}, \frac{\sqrt{1-4a(-1)^n+4a^2}}{4}, x^2\right) c_2}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.409 (sec). Leaf size: 191

```
DSolve[((-1)^n*a - a^2 + (1 + 2*a + 2*n)*x^2 - x^4)*y[x] + x^2*y''[x] == 0,y[x],x,IncludeSin
```

$$y(x) \to \frac{e^{-\frac{x^2}{2}} 2^{\frac{1}{4}\left(\sqrt{4a^2-4a(-1)^n+1}+2\right)} (x^2)^{\frac{1}{4}\left(\sqrt{4a^2-4a(-1)^n+1}+2\right)} \left(c_1 \,\mathrm{HypergeometricU}\left(\frac{1}{4}\left(-2a - 2n + \sqrt{4a^2 - 4(-1\right.\right.\right.}{\sqrt{x}}$$

3.211 problem 1215

Internal problem ID [9545]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1215.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + (x^n a + b)\, y'x + \left(\mathrm{a1}\ x^{2n} + \mathrm{b1}\ x^n + \mathrm{c1}\right) y = 0$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 148

```
dsolve(x^2*diff(diff(y(x),x),x)+(a*x^n+b)*diff(y(x),x)*x+(a1*x^(2*n)+b1*x^n+c1)*y(x)=0,y(x),
```

$$y(x) = x^{-\frac{b}{2}-\frac{n}{2}+\frac{1}{2}} \mathrm{e}^{-\frac{a x^n}{2n}} \left(c_1 \operatorname{WhittakerM}\left(-\frac{(b+n-1)\, a - 2\,\mathrm{b1}}{2\sqrt{a^2-4\,\mathrm{a1}}\, n}, \frac{\sqrt{b^2-2b-4\,\mathrm{c1}+1}}{2n}, \frac{\sqrt{a^2-4\,\mathrm{a1}}\, x^n}{n}\right) + c_2 \operatorname{WhittakerW}\left(-\frac{(b+n-1)\, a - 2\,\mathrm{b1}}{2\sqrt{a^2-4\,\mathrm{a1}}\, n}, \frac{\sqrt{b^2-2b-4\,\mathrm{c1}+1}}{2n}, \frac{\sqrt{a^2-4\,\mathrm{a1}}\, x^n}{n}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.342 (sec). Leaf size: 412

```
DSolve[(c1 + b1*x^n + a1*x^(2*n))*y[x] + x*(b + a*x^n)*y'[x] + x^2*y''[x] == 0,y[x],x,Includ
```

$$y(x) \to x^{\frac{1}{2}-\frac{n}{2}} 2^{\frac{1}{2}\left(\frac{\sqrt{n^2\left(b^2-2b-4\mathrm{c1}+1\right)}}{n^2}+1\right)} e^{-\frac{\left(\sqrt{a^2-4\mathrm{a1}}+a\right)x^n}{2n}} (x^n)^{\frac{\sqrt{n^2\left(b^2-2b-4\mathrm{c1}+1\right)}-bn+n^2}{2n^2}} \left(c_1 \operatorname{HypergeometricU}\left(\frac{\left(n^2+\sqrt{(\right.}{}\right.\right.$$

$$+c_2 L^{\frac{\sqrt{\left(b^2-2b-4\mathrm{c1}+1\right)n^2}}{n^2}}_{-\frac{\left(n^2+\sqrt{\left(b^2-2b-4\mathrm{c1}+1\right)n^2}\right)a^2+\sqrt{a^2-4\mathrm{a1}}\,n(b+n-1)a-2\sqrt{a^2-4\mathrm{a1}}\,\mathrm{b1}\,n-4\mathrm{a1}\left(n^2+\sqrt{\left(b^2-2b-4\mathrm{c1}+1\right)n^2}\right)}{2\left(a^2-4\mathrm{a1}\right)n^2}}\left(\frac{\sqrt{a^2-4\mathrm{a1}}x^n}{n}\right)\Bigg)$$

3.212 problem 1216

Internal problem ID [9546]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1216.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + \left(a\,x^{\mathrm{a1}} + b\right)xy' + \left(A\,x^{2\,\mathrm{a1}} + B\,x^{\mathrm{a1}} + C\,x^{\mathrm{b1}} + \mathrm{DD}\right)y = 0$$

✗ Solution by Maple

```
dsolve(x^2*diff(diff(y(x),x),x)+(a*x^a1+b)*x*diff(y(x),x)+(A*x^(2*a1)+B*x^a1+C*x^b1+DD)*y(x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(DD + B*x^a1 + A*x^(2*a1) + C*x^b1)*y[x] + x*(b + a*x^a1)*y'[x] + x^2*y''[x] == 0,y[x
```

Not solved

3.213 problem 1217

Internal problem ID [9547]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1217.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' - (2x^2 \tan(x) - x) y' - (x \tan(x) + a) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x^2*diff(diff(y(x),x),x)-(2*x^2*tan(x)-x)*diff(y(x),x)-(x*tan(x)+a)*y(x)=0,y(x), sing
```

$$y(x) = \sec(x) \left(c_1 \operatorname{BesselJ}\left(\sqrt{a}, x\right) + c_2 \operatorname{BesselY}\left(\sqrt{a}, x\right)\right)$$

✓ Solution by Mathematica

Time used: 0.175 (sec). Leaf size: 29

```
DSolve[(-a - x*Tan[x])*y[x] - (-x + 2*x^2*Tan[x])*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSing
```

$$y(x) \to \sec(x) \left(c_1 \operatorname{BesselJ}\left(\sqrt{a}, x\right) + c_2 \operatorname{BesselY}\left(\sqrt{a}, x\right)\right)$$

3.214 problem 1218

Internal problem ID [9548]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1218.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + (2x^2\cot(x) + x)\, y' + (\cot(x)\, x + a)\, y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(x^2*diff(diff(y(x),x),x)+(2*x^2*cot(x)+x)*diff(y(x),x)+(x*cot(x)+a)*y(x)=0,y(x), sing
```

$$y(x) = \csc(x)\left(c_1 \operatorname{BesselJ}\left(i\sqrt{a}, x\right) + c_2 \operatorname{BesselY}\left(i\sqrt{a}, x\right)\right)$$

✓ Solution by Mathematica

Time used: 0.174 (sec). Leaf size: 37

```
DSolve[(a + x*Cot[x])*y[x] + (x + 2*x^2*Cot[x])*y'[x] + x^2*y''[x] == 0,y[x],x,IncludeSingul
```

$$y(x) \to \csc(x)\left(c_1 \operatorname{BesselJ}\left(i\sqrt{a}, x\right) + c_2 \operatorname{BesselY}\left(i\sqrt{a}, x\right)\right)$$

3.215 problem 1219

Internal problem ID [9549]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1219.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + 2xf(x)\, y' + \left(f'(x)\, x + f(x)^2 - f(x) + a\, x^2 + bx + c\right) y = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 69

```
dsolve(x^2*diff(diff(y(x),x),x)+2*x*f(x)*diff(y(x),x)+(x*diff(f(x),x)+f(x)^2-f(x)+a*x^2+b*x+
```

$$y(x) = \mathrm{e}^{-\left(\int \frac{f(x)}{x}dx\right)}\left(\mathrm{WhittakerM}\left(-\frac{ib}{2\sqrt{a}}, \frac{\sqrt{1-4c}}{2}, 2ix\sqrt{a}\right) c_1 + \mathrm{WhittakerW}\left(-\frac{ib}{2\sqrt{a}}, \frac{\sqrt{1-4c}}{2}, 2ix\sqrt{a}\right) c_2\right)$$

✓ Solution by Mathematica

Time used: 0.319 (sec). Leaf size: 151

```
DSolve[y[x]*(c + b*x + a*x^2 - f[x] + f[x]^2 + x*Derivative[1][f][x]) + 2*x*f[x]*y'[x] + x^2
```

$$y(x) \to \left(c_1 \,\mathrm{HypergeometricU}\left(\frac{1}{2}\left(\frac{ib}{\sqrt{a}} + \sqrt{1-4c}+1\right), \sqrt{1-4c}+1, 2i\sqrt{a}x\right) + c_2 L_{\frac{1}{2}\left(-\frac{ib}{\sqrt{a}}-\sqrt{1-4c}-1\right)}^{\sqrt{1-4c}}\left(2i\sqrt{a}x\right)\right) \exp\left(\int_1^x \frac{-2f(K[1]) - 2i\sqrt{a}K[1] + \sqrt{1-4c}+1}{2K[1]} dK[1]\right)$$

3.216 problem 1220

Internal problem ID [9550]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1220.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + 2x^2f(x)\,y' + \left(x^2\left(f'(x) + f(x)^2 + a\right) - v(v-1)\right)y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 38

```
dsolve(x^2*diff(diff(y(x),x),x)+2*x^2*f(x)*diff(y(x),x)+(x^2*(diff(f(x),x)+f(x)^2+a)-v*(v-1)
```

$$y(x) = \sqrt{x}\,e^{-\left(\int f(x)dx\right)}\left(\text{BesselJ}\left(v-\frac{1}{2}, x\sqrt{a}\right)c_1 + \text{BesselY}\left(v-\frac{1}{2}, x\sqrt{a}\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.38 (sec). Leaf size: 62

```
DSolve[y[x]*((1 - v)*v + x^2*(a + f[x]^2 + Derivative[1][f][x])) + 2*x^2*f[x]*y'[x] + x^2*y'
```

$$y(x) \to \left(c_1\,\text{BesselJ}\left(v-\frac{1}{2}, \sqrt{a}x\right) + c_2\,\text{BesselY}\left(v-\frac{1}{2}, \sqrt{a}x\right)\right)\exp\left(\int_1^x\left(\frac{1}{2K[1]} - f(K[1])\right)dK[1]\right)$$

3.217 problem 1221

Internal problem ID [9551]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1221.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + (x - 2f(x)\,x^2)\,y' + \left(x^2\left(1 + f(x)^2 - f'(x)\right) - xf(x) - v^2\right)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(x^2*diff(diff(y(x),x),x)+(x-2*x^2*f(x))*diff(y(x),x)+(x^2*(1+f(x)^2-diff(f(x),x))-x*f
```

$$y(x) = \sqrt{x}\,\mathrm{e}^{\frac{\left(\int \frac{2f(x)x-1}{x}dx\right)}{2}}(c_1 \,\mathrm{BesselJ}\,(v, x) + c_2 \,\mathrm{BesselY}\,(v, x))$$

✓ Solution by Mathematica

Time used: 0.206 (sec). Leaf size: 31

```
DSolve[y[x]*(-v^2 - x*f[x] + x^2*(1 + f[x]^2 - Derivative[1][f][x])) + (x - 2*x^2*f[x])*y'[x
```

$$y(x) \to (c_1 \,\mathrm{BesselJ}(v, x) + c_2 \,\mathrm{BesselY}(v, x)) \exp\left(\int_1^x f(K[1])dK[1]\right)$$

3.218 problem 1222

Internal problem ID [9552]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1222.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{(x^2+1)\,y''+y'x+2y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve((x^2+1)*diff(diff(y(x),x),x)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\sin\left(\sqrt{2}\,\operatorname{arcsinh}(x)\right)+c_2\cos\left(\sqrt{2}\,\operatorname{arcsinh}(x)\right)$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 55

```
DSolve[2*y[x] + x*y'[x] + (1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\cos\left(\sqrt{2}\log\left(\sqrt{x^2+1}-x\right)\right)-c_2\sin\left(\sqrt{2}\log\left(\sqrt{x^2+1}-x\right)\right)$$

3.219 problem 1223

Internal problem ID [9553]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1223.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{(x^2+1)\,y''+y'x-9y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve((x^2+1)*diff(diff(y(x),x),x)+x*diff(y(x),x)-9*y(x)=0,y(x), singsol=all)
```

$$y(x)=\sqrt{x^2+1}\,(4x^2+1)\,c_2+4c_1x^3+3c_1x$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 49

```
DSolve[-9*y[x] + x*y'[x] + (1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\cosh\left(3\log\left(\sqrt{x^2+1}-x\right)\right)-ic_2\sinh\left(3\log\left(\sqrt{x^2+1}-x\right)\right)$$

3.220 problem 1224

Internal problem ID [9554]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1224.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{(x^2+1)\, y'' + y'x + ay = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve((x^2+1)*diff(diff(y(x),x),x)+x*diff(y(x),x)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(\sqrt{a}\, \operatorname{arcsinh}(x)\right) + c_2 \cos\left(\sqrt{a}\, \operatorname{arcsinh}(x)\right)$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 55

```
DSolve[a*y[x] + x*y'[x] + (1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos\left(\sqrt{a} \log\left(\sqrt{x^2+1} - x\right)\right) - c_2 \sin\left(\sqrt{a} \log\left(\sqrt{x^2+1} - x\right)\right)$$

3.221 problem 1225

Internal problem ID [9555]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1225.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''-y'x+y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve((x^2+1)*diff(diff(y(x),x),x)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=-\sqrt{x^2+1}\,c_2+x(c_2\,\mathrm{arcsinh}\,(x)+c_1)$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 42

```
DSolve[y[x] - x*y'[x] + (1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -c_2\sqrt{x^2+1}-c_2x\log\left(\sqrt{x^2+1}-x\right)+c_1x$$

3.222 problem 1226

Internal problem ID [9556]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1226.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''+2y'x-v(v-1)\,y=0}$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 25

```
dsolve((x^2+1)*diff(diff(y(x),x),x)+2*x*diff(y(x),x)-v*(v-1)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\mathrm{LegendreP}\,(-1+v,ix)+c_2\,\mathrm{LegendreQ}\,(-1+v,ix)$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 30

```
DSolve[(1 - v)*v*y[x] + 2*x*y'[x] + (1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to c_1\,\mathrm{LegendreP}(v-1,ix)+c_2\,\mathrm{LegendreQ}(v-1,ix)$$

3.223 problem 1227

Internal problem ID [9557]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1227.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''-2y'x+2y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve((x^2+1)*diff(diff(y(x),x),x)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_2x^2+c_1x-c_2$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 21

```
DSolve[2*y[x] - 2*x*y'[x] + (1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_2x-c_1(x-i)^2$$

3.224 problem 1228

Internal problem ID [9558]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1228.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''+3y'x+ay=0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 53

```
dsolve((x^2+1)*diff(diff(y(x),x),x)+3*x*diff(y(x),x)+a*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\left(x+\sqrt{x^2+1}\right)^{\sqrt{-a+1}}+c_2\left(x+\sqrt{x^2+1}\right)^{-\sqrt{-a+1}}}{\sqrt{x^2+1}}$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 66

```
DSolve[a*y[x] + 3*x*y'[x] + (1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_1P_{\sqrt{1-a}-\frac{1}{2}}^{\frac{1}{2}}(ix)+c_2Q_{\sqrt{1-a}-\frac{1}{2}}^{\frac{1}{2}}(ix)}{\sqrt[4]{x^2+1}}$$

3.225 problem 1229

Internal problem ID [9559]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1229.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{(x^2+1)\, y'' + 4y'x + 2y = 2\cos(x) - 2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve((x^2+1)*diff(diff(y(x),x),x)+4*x*diff(y(x),x)+2*y(x)-2*cos(x)+2*x=0,y(x), singsol=all
```

$$y(x) = \frac{-x^3 + 3c_1x - 6\cos(x) + 3c_2}{3x^2 + 3}$$

✓ Solution by Mathematica

Time used: 0.085 (sec). Leaf size: 33

```
DSolve[2*x - 2*Cos[x] + 2*y[x] + 4*x*y'[x] + (1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSol
```

$$y(x) \to -\frac{x^3 + 6\cos(x) - 3c_2x - 3c_1}{3x^2 + 3}$$

3.226 problem 1230

Internal problem ID [9560]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1230.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x^2+1)\,y''+axy'+(-2+a)\,y=0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 36

```
dsolve((x^2+1)*diff(diff(y(x),x),x)+a*x*diff(y(x),x)+(a-2)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x^2+1\right)^{1-\frac{a}{2}}+c_2x\,\mathrm{hypergeom}\left(\left[1,\frac{a}{2}-\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 68

```
DSolve[(-2 + a)*y[x] + a*x*y'[x] + (1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\left(x^2+1\right)^{\frac{1}{2}-\frac{a}{4}}\left(c_1P_{\frac{a-4}{2}}^{\frac{a-2}{2}}(ix)+c_2Q_{\frac{a-4}{2}}^{\frac{a-2}{2}}(ix)\right)$$

3.227 problem 1231

Internal problem ID [9561]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$(x^2-1)\,y''-v(v+1)\,y=0$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 52

```
dsolve((x^2-1)*diff(diff(y(x),x),x)-v*(v+1)*y(x)=0,y(x), singsol=all)
```

$$y(x)=-(x-1)\,(x+1)\left(\text{hypergeom}\left(\left[1-\frac{v}{2},\frac{3}{2}+\frac{v}{2}\right],\left[\frac{3}{2}\right],x^2\right)c_2x+c_1\,\text{hypergeom}\left(\left[1+\frac{v}{2},\frac{1}{2}-\frac{v}{2}\right],\left[\frac{1}{2}\right],x^2\right)\right)$$

✓ Solution by Mathematica
Time used: 0.105 (sec). Leaf size: 56

```
DSolve[-(v*(1 + v)*y[x]) + (-1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\,\text{Hypergeometric2F1}\left(-\frac{v}{2}-\frac{1}{2},\frac{v}{2},\frac{1}{2},x^2\right)+ic_2x\,\text{Hypergeometric2F1}\left(-\frac{v}{2},\frac{v+1}{2},\frac{3}{2},x^2\right)$$

3.228 problem 1232

Internal problem ID [9562]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1232.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$(x^2-1)\,y''-n(n+1)\,y=-\frac{\partial}{\partial x}\,\mathrm{LegendreP}\,(n,x)$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 409

```
dsolve((x^2-1)*diff(diff(y(x),x),x)-n*(n+1)*y(x)+Diff(LegendreP(n,x),x)=0,y(x), singsol=all)
```

$$y(x)=3(x+1)\,(x-1)\left(-\,\mathrm{hypergeom}\left(\left[\frac{n}{2}+1,-\frac{n}{2}+\frac{1}{2}\right],\left[\frac{1}{2}\right],x^2\right)(1\right.$$

$$+n)\left(\int\frac{x\,\mathrm{hypergeom}\left(\left[-\frac{n}{2}+1,\right.\right.}{3\,(x+1)^3\,(x-1)^3\left(\left(\mathrm{hypergeom}\left(\left[\frac{n}{2}+1,-\frac{n}{2}+\frac{1}{2}\right],\left[\frac{1}{2}\right],x^2\right)+(n^2+n-2)\,x^2\,\mathrm{hypergeom}\left(\left[\right.\right.\right.\right.}\right.$$

$$+\,\mathrm{hypergeom}\left(\left[-\frac{n}{2}+1,\frac{n}{2}+\frac{3}{2}\right],\left[\frac{3}{2}\right],x^2\right)x(1$$

$$+n)\left(\int\frac{\mathrm{hypergeom}\left(\left[\frac{n}{2}+1,-\right.\right.}{3\,(x+1)^3\,(x-1)^3\left(\left(\mathrm{hypergeom}\left(\left[\frac{n}{2}+1,-\frac{n}{2}+\frac{1}{2}\right],\left[\frac{1}{2}\right],x^2\right)+(n^2+n-2)\,x^2\,\mathrm{hypergeom}\left(\left[\right.\right.\right.\right.}\right.$$

$$-\,\frac{\mathrm{hypergeom}\left(\left[-\frac{n}{2}+1,\frac{n}{2}+\frac{3}{2}\right],\left[\frac{3}{2}\right],x^2\right)c_1x}{3}$$

$$\left.-\,\frac{\mathrm{hypergeom}\left(\left[\frac{n}{2}+1,-\frac{n}{2}+\frac{1}{2}\right],\left[\frac{1}{2}\right],x^2\right)c_2}{3}\right)$$

✓ Solution by Mathematica

Time used: 8.981 (sec). Leaf size: 468

```
DSolve[(-(n*LegendreP[-1 + n, x]) + n*x*LegendreP[n, x])/(-1 + x^2) - n*(1 + n)*y[x] + (-1 +
```

$$y(x) \to \text{Hypergeometric2F1}\left(-\frac{n}{2}\right.$$

$$\left.-\frac{1}{2},\frac{n}{2},\frac{1}{2},x^2\right)\int_1^x \frac{3n\,\text{Hyp}}{(K[1]^2-1)^2\left(n\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-n-1),\frac{n}{2},\frac{1}{2},K[1]^2\right)\left((n+1)\,\text{Hypergeometr}\right.\right.}$$

$$+ix\,\text{Hypergeometric2F1}\left(-\frac{n}{2},\frac{n+1}{2},\frac{3}{2},x^2\right)\int_1^x \frac{}{(K[2]^2-1)^2\left(n\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-n-1),\frac{n}{2},\frac{1}{2}\right.\right.}$$

$$+c_1\,\text{Hypergeometric2F1}\left(-\frac{n}{2}-\frac{1}{2},\frac{n}{2},\frac{1}{2},x^2\right)$$

$$+ic_2x\,\text{Hypergeometric2F1}\left(-\frac{n}{2},\frac{n+1}{2},\frac{3}{2},x^2\right)$$

3.229 problem 1234

Internal problem ID [9563]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1234.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{(x^2-1)\,y''+y'x=-2}$$

✓ Solution by Maple
Time used: 0.203 (sec). Leaf size: 59

```
dsolve((x^2-1)*diff(diff(y(x),x),x)+x*diff(y(x),x)+2=0,y(x), singsol=all)
```

$$y(x)=-\left(\int -\frac{-2\sqrt{x^2-1}\,\ln\left(x+\sqrt{x^2-1}\right)\sqrt{x-1}\,\sqrt{x+1}+c_1(x^2-1)}{(x+1)^{\frac{3}{2}}\,(x-1)^{\frac{3}{2}}}dx\right)+c_2$$

✓ Solution by Mathematica
Time used: 0.045 (sec). Leaf size: 48

```
DSolve[2 + x*y'[x] + (-1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_2-\frac{1}{4}\left(\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)-\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)+c_1\right)^2$$

3.230 problem 1235

Internal problem ID [9564]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1235.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']]`

$$(x^2-1)\,y''+y'x+ay=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 41

```
dsolve((x^2-1)*diff(diff(y(x),x),x)+x*diff(y(x),x)+a*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x+\sqrt{x^2-1}\right)^{i\sqrt{a}}+c_2\left(x+\sqrt{x^2-1}\right)^{-i\sqrt{a}}$$

✓ Solution by Mathematica
Time used: 0.09 (sec). Leaf size: 97

```
DSolve[a*y[x] + x*y'[x] + (-1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}y(x)\to c_1\cos\left(\frac{1}{2}\sqrt{a}\left(\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)-\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)\right)\right)\\-c_2\sin\left(\frac{1}{2}\sqrt{a}\left(\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)-\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)\right)\right)\end{aligned}$$

3.231 problem 1236

Internal problem ID [9565]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1236.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-1)\,y''+y'x+f(x)\,y=0}$$

✗ Solution by Maple

```
dsolve((x^2-1)*diff(diff(y(x),x),x)+x*diff(y(x),x)+f(x)*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[f[x]*y[x] + x*y'[x] + (-1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

3.232 problem 1237

Internal problem ID [9566]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1237.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{(x^2-1)\,y''+2y'x=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve((x^2-1)*diff(diff(y(x),x),x)+2*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=c_1+\frac{(-\ln(x+1)+\ln(x-1))\,c_2}{2}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 27

```
DSolve[2*x*y'[x] + (-1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}c_1(\log(1-x)-\log(x+1))+c_2$$

3.233 problem 1238

Internal problem ID [9567]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1238.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{(x^2-1)\,y''+2y'x=a}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve((x^2-1)*diff(diff(y(x),x),x)+2*x*diff(y(x),x)-a=0,y(x), singsol=all)
```

$$y(x)=\frac{(a+c_1)\ln(x-1)}{2}+\frac{(a-c_1)\ln(x+1)}{2}+c_2$$

✓ Solution by Mathematica
Time used: 0.049 (sec). Leaf size: 36

```
DSolve[-a + 2*x*y'[x] + (-1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}(a+c_1)\log(1-x)+\frac{1}{2}(a-c_1)\log(x+1)+c_2$$

3.234 problem 1239

Internal problem ID [9568]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1239.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(x^2-1)\,y''+2y'x-ly=0}$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 35

```
dsolve((x^2-1)*diff(diff(y(x),x),x)+2*x*diff(y(x),x)-l*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\mathrm{LegendreP}\left(\frac{\sqrt{1+4l}}{2}-\frac{1}{2},x\right)+c_2\,\mathrm{LegendreQ}\left(\frac{\sqrt{1+4l}}{2}-\frac{1}{2},x\right)$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 46

```
DSolve[-(l*y[x]) + 2*x*y'[x] + (-1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to c_1\,\mathrm{LegendreP}\left(\frac{1}{2}\left(\sqrt{4l+1}-1\right),x\right)+c_2\,\mathrm{LegendreQ}\left(\frac{1}{2}\left(\sqrt{4l+1}-1\right),x\right)$$

3.235 problem 1240

Internal problem ID [9569]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1240.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(x^2-1)\,y''+2y'x-v(v+1)\,y=0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 15

```
dsolve((x^2-1)*diff(diff(y(x),x),x)+2*x*diff(y(x),x)-v*(v+1)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\text{LegendreP}\,(v,x)+c_2\,\text{LegendreQ}\,(v,x)$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 18

```
DSolve[-(v*(1 + v)*y[x]) + 2*x*y'[x] + (-1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolution
```

$$y(x)\to c_1\,\text{LegendreP}(v,x)+c_2\,\text{LegendreQ}(v,x)$$

3.236 problem 1241

Internal problem ID [9570]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1241.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(x^2-1)\,y''-2xy'-(v+2)\,(v-1)\,y=0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 23

```
dsolve((x^2-1)*diff(diff(y(x),x),x)-2*x*diff(y(x),x)-(v+2)*(v-1)*y(x)=0,y(x), singsol=all)
```

$$y(x)=(c_1\,\text{LegendreP}\,(v,2,x)+c_2\,\text{LegendreQ}\,(v,2,x))\,(x^2-1)$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 26

```
DSolve[(1 - v)*(2 + v)*y[x] - 2*x*y'[x] + (-1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolut
```

$$y(x)\to (x^2-1)\,(c_1P_v^2(x)+c_2Q_v^2(x))$$

3.237 problem 1242

Internal problem ID [9571]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1242.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2-1)\,y''-(3x+1)\,y'-(x^2-x)\,y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 41

```
dsolve((x^2-1)*diff(diff(y(x),x),x)-(3*x+1)*diff(y(x),x)-(x^2-x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-2-x}c_2(x+1)^2\,\mathrm{expIntegral}_1\left(-2x-2\right)+c_1\mathrm{e}^{-x}(x+1)^2+2\,\mathrm{e}^{x}c_2$$

✓ Solution by Mathematica

Time used: 0.573 (sec). Leaf size: 50

```
DSolve[(x - x^2)*y[x] - (1 + 3*x)*y'[x] + (-1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolut
```

$$y(x)\to e^{-x-2}\left(c_2(x+1)^2\,\mathrm{ExpIntegralEi}(2(x+1))+e^2\left(c_1(x+1)^2-2c_2e^{2x}\right)\right)$$

3.238 problem 1243

Internal problem ID [9572]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1243.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-1)\,y''+4y'x+(x^2+1)\,y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve((x^2-1)*diff(diff(y(x),x),x)+4*x*diff(y(x),x)+(x^2+1)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\sin(x)+c_2\cos(x)}{x^2-1}$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 41

```
DSolve[(1 + x^2)*y[x] + 4*x*y'[x] + (-1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolutions -
```

$$y(x)\to\frac{e^{-ix}(2c_1-ic_2e^{2ix})}{2\,(x^2-1)}$$

3.239 problem 1244

Internal problem ID [9573]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1244.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$(x^2-1)\,y''+2(n+1)\,xy'-(v+n+1)\,(v-n)\,y=0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 27

```
dsolve((x^2-1)*diff(diff(y(x),x),x)+2*(n+1)*x*diff(y(x),x)-(v+n+1)*(v-n)*y(x)=0,y(x), singso
```

$$y(x)=(\text{LegendreP}\,(v,n,x)\,c_1+\text{LegendreQ}\,(v,n,x)\,c_2)\,(x^2-1)^{-\frac{n}{2}}$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 32

```
DSolve[(n - v)*(1 + n + v)*y[x] + 2*(1 + n)*x*y'[x] + (-1 + x^2)*y''[x] == 0,y[x],x,IncludeS
```

$$y(x)\to (x^2-1)^{-n/2}\,(c_1P_v^n(x)+c_2Q_v^n(x))$$

3.240 problem 1245

Internal problem ID [9574]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1245.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(x^2-1)\,y''-2(n-1)\,xy'-(v-n+1)\,(v+n)\,y=0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 27

```
dsolve((x^2-1)*diff(diff(y(x),x),x)-2*(n-1)*x*diff(y(x),x)-(v-n+1)*(v+n)*y(x)=0,y(x), singso
```

$$y(x)=(\mathrm{LegendreP}\,(v,n,x)\,c_1+\mathrm{LegendreQ}\,(v,n,x)\,c_2)\,(x^2-1)^{\frac{n}{2}}$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 32

```
DSolve[(-1 + n - v)*(n + v)*y[x] - 2*(-1 + n)*x*y'[x] + (-1 + x^2)*y''[x] == 0,y[x],x,Includ
```

$$y(x)\to (x^2-1)^{n/2}\,(c_1P_v^n(x)+c_2Q_v^n(x))$$

3.241 problem 1246

Internal problem ID [9575]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1246.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(x^2-1)\,y''-2(v-1)\,xy'-2yv=0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 28

```
dsolve((x^2-1)*diff(diff(y(x),x),x)-2*(v-1)*x*diff(y(x),x)-2*v*y(x)=0,y(x), singsol=all)
```

$$y(x)=\left(\text{hypergeom}\left(\left[\frac{1}{2},v+1\right],\left[\frac{3}{2}\right],x^2\right)c_2x+c_1\right)(x^2-1)^v$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 32

```
DSolve[-2*v*y[x] - 2*(-1 + v)*x*y'[x] + (-1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolutio
```

$$y(x)\to (x^2-1)^{v/2}\left(c_1P_v^v(x)+c_2Q_v^v(x)\right)$$

3.242 problem 1247

Internal problem ID [9576]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1247.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$(x^2-1)\, y'' + 2axy' + a(-1+a)\, y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve((x^2-1)*diff(diff(y(x),x),x)+2*a*x*diff(y(x),x)+a*(a-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x+1)^{-a+1} + c_2(x-1)^{-a+1}$$

✓ Solution by Mathematica

Time used: 0.442 (sec). Leaf size: 99

```
DSolve[(-1 + a)*a*y[x] + 2*a*x*y'[x] + (-1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{(1-x^2)^{\frac{1}{2}-\frac{1}{2}\sqrt{(a-1)^2}}\,(x^2-1)^{-a/2}\left(2\sqrt{(a-1)^2}c_1(1-x)^{\sqrt{(a-1)^2}} + c_2(x+1)^{\sqrt{(a-1)^2}}\right)}{2\sqrt{(a-1)^2}}$$

3.243 problem 1248

Internal problem ID [9577]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1248.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2-1)\,y''+axy'+(b\,x^2+cx+d)\,y=0$$

✓ Solution by Maple

Time used: 0.5 (sec). Leaf size: 126

```
dsolve((x^2-1)*diff(diff(y(x),x),x)+a*x*diff(y(x),x)+(b*x^2+c*x+d)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{e^{x\sqrt{-b}}\left(2\,\mathrm{HeunC}\left(4\sqrt{-b}, \frac{a}{2}-1, \frac{a}{2}-1, 2c, d-c-\frac{a^2}{8}+b+\frac{1}{2}, \frac{x}{2}+\frac{1}{2}\right)c_1+c_2\left(\frac{x}{2}-\frac{1}{2}\right)^{\frac{a}{4}}(x+1)\left(\frac{x}{2}+\frac{1}{2}\right)^{-\frac{a}{4}}\right.}{2}$$

✓ Solution by Mathematica

Time used: 0.662 (sec). Leaf size: 192

```
DSolve[(d + c*x + b*x^2)*y[x] + a*x*y'[x] + (-1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSol
```

$$y(x) \to \frac{1}{2}e^{\sqrt{-b}x}\left(c_2(x\right.$$
$$-1)^{a/4}\left(x^2-1\right)^{-a/4}(x+1)^{1-\frac{a}{4}}\mathrm{HeunC}\left[\frac{1}{4}a\left(a-4\sqrt{-b}-2\right)-b+4\sqrt{-b}+c-d, 2\left(2\sqrt{-b}+c\right), 2-\frac{a}{2}, \frac{a}{2}, 4\sqrt{-\right.$$

3.244 problem 1249

Internal problem ID [9578]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1249.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2-1)\,y''+(xa+b)\,y'+yc=0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 134

```
dsolve((x^2-1)*diff(diff(y(x),x),x)+(a*x+b)*diff(y(x),x)+c*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\mathrm{hypergeom}\left(\left[-\frac{1}{2}-\frac{\sqrt{a^2-2a-4c+1}}{2}+\frac{a}{2},-\frac{1}{2}+\frac{\sqrt{a^2-2a-4c+1}}{2}+\frac{a}{2}\right],\left[\frac{a}{2}-\frac{b}{2}\right],\frac{x}{2}+\frac{1}{2}\right)+c_2\left(\frac{x}{2}+\frac{1}{2}\right)^{1-\frac{a}{2}+\frac{b}{2}}\mathrm{hypergeom}\left(\left[\frac{1}{2}-\frac{\sqrt{a^2-2a-4c+1}}{2}+\frac{b}{2},\frac{1}{2}+\frac{\sqrt{a^2-2a-4c+1}}{2}+\frac{b}{2}\right],\left[2-\frac{a}{2}+\frac{b}{2}\right],\frac{x}{2}+\frac{1}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.204 (sec). Leaf size: 190

```
DSolve[c*y[x] + (b + a*x)*y'[x] + (-1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{2}(x-1)^{\frac{1}{2}(-a-b)} \left(2c_1(x -1)^{\frac{a+b}{2}} \text{Hypergeometric2F1}\left(\frac{1}{2}\left(a-\sqrt{a^2-2a-4c+1}-1\right), \frac{1}{2}\left(a+\sqrt{a^2-2a-4c+1}-1\right), \frac{a+b}{2}, \frac{1-}{2}\right.\right.$$

$$+ c_2(x -1)2^{\frac{a+b}{2}} \text{Hypergeometric2F1}\left(\frac{1}{2}\left(-b-\sqrt{a^2-2a-4c+1}+1\right), \frac{1}{2}\left(-b+\sqrt{a^2-2a-4c+1}+1\right), \frac{1}{2}(-a\right.$$

3.245 problem 1250

Internal problem ID [9579]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1250.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-a^2 + x^2) y'' + 8y'x + 12y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 41

```
dsolve((-a^2+x^2)*diff(diff(y(x),x),x)+8*x*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{3a^2c_2x + c_2x^3 + a^2c_1 + 3c_1x^2}{(a+x)^3 (-x+a)^3}$$

✓ Solution by Mathematica
Time used: 0.069 (sec). Leaf size: 38

```
DSolve[12*y[x] + 8*x*y'[x] + (-a^2 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{\frac{c_2(a^2+3x^2)}{(a-x)^3} + 3c_1}{3(a+x)^3}$$

3.246 problem 1251

Internal problem ID [9580]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1251.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(x+1)\,y'' - (x-1)\,y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x*(x+1)*diff(diff(y(x),x),x)-(x-1)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_2(x-1)\ln(x) - 4c_2 + c_1(x-1)$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 23

```
DSolve[y[x] - (-1 + x)*y'[x] + x*(1 + x)*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to c_1(x-1) + c_2((x-1)\log(x) - 4)$$

3.247 problem 1252

Internal problem ID [9581]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1252.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(x+1)\,y''+(xa+b)\,y'+yc=0$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 124

```
dsolve(x*(x+1)*diff(diff(y(x),x),x)+(a*x+b)*diff(y(x),x)+c*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\mathrm{hypergeom}\left(\left[-\frac{1}{2}-\frac{\sqrt{a^2-2a-4c+1}}{2}+\frac{a}{2},-\frac{1}{2}+\frac{\sqrt{a^2-2a-4c+1}}{2}+\frac{a}{2}\right],[a-b],x+1\right)+c_2(x+1)^{-a+b+1}\,\mathrm{hypergeom}\left(\left[\frac{1}{2}-\frac{\sqrt{a^2-2a-4c+1}}{2}+b-\frac{a}{2},\frac{1}{2}+\frac{\sqrt{a^2-2a-4c+1}}{2}+b-\frac{a}{2}\right],[2-a+b],x+1\right)$$

✓ Solution by Mathematica

Time used: 0.223 (sec). Leaf size: 131

```
DSolve[c*y[x] + (b + a*x)*y'[x] + x*(1 + x)*y''[x] == 0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\to c_2x^{1-b}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2}\left(a-2b-\sqrt{a^2-2a-4c+1}+1\right),\frac{1}{2}\left(a-2b+\sqrt{a^2-2a-4c+1}+1\right),2-b,-x\right)+c_1\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2}\left(a-\sqrt{a^2-2a-4c+1}-1\right),\frac{1}{2}\left(a+\sqrt{a^2-2a-4c+1}-1\right),b,-x\right)$$

3.248 problem 1253

Internal problem ID [9582]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1253.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x(x+1)\,y''+(3x+2)\,y'+y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*(x+1)*diff(diff(y(x),x),x)+(3*x+2)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\ln(x+1)+c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 28

```
DSolve[y[x] + (2 + 3*x)*y'[x] + x*(1 + x)*y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to\frac{c_2\log(2(x+1))+2c_1}{\sqrt{2}x}$$

3.249 problem 1254

Internal problem ID [9583]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1254.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(x^2+x-2\right) y''+\left(x^2-x\right) y'-\left(6x^2+7x\right) y=0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 44

```
dsolve((x^2+x-2)*diff(diff(y(x),x),x)+(x^2-x)*diff(y(x),x)-(6*x^2+7*x)*y(x)=0,y(x), singsol=
```

$$y(x)=\left(195\,\mathrm{e}^{5x-5}c_2(x-1)\,\mathrm{expIntegral}_1\left(5x-5\right)+\left(x-1\right)c_1\mathrm{e}^{5x}-c_2(x+44)\right)\mathrm{e}^{-3x}$$

✓ Solution by Mathematica
Time used: 0.514 (sec). Leaf size: 52

```
DSolve[(-7*x - 6*x^2)*y[x] + (-x + x^2)*y'[x] + (-2 + x + x^2)*y''[x] == 0,y[x],x,IncludeSin
```

$$y(x)\to 39c_2e^{2x-5}(x-1)\,\mathrm{ExpIntegralEi}(5-5x)+c_1\left(-e^{2x}\right)(x-1)+\frac{1}{5}c_2e^{-3x}(x+44)$$

3.250 problem 1255

Internal problem ID [9584]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1255.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x(x-1)\,y''+ay'-2y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(x*(x-1)*diff(diff(y(x),x),x)+a*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x)=x^{a+1}c_2(x-1)^{-a+1}+\left(a^2+a(-1+2x)+2x^2-2x\right)c_1$$

✓ Solution by Mathematica

Time used: 0.592 (sec). Leaf size: 87

```
DSolve[-2*y[x] + a*y'[x] + (-1 + x)*x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{(a^2+a(2x-1)+2(x-1)x)\left(\frac{c_2x^{a+1}(1-x)^{1-a}}{(a-1)a(a+1)(a^2+a(2x-1)+2(x-1)x)}+c_1\right)}{a^2+3a+4}$$

3.251 problem 1256

Internal problem ID [9585]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1256.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$x(x-1)\,y''+(2x-1)\,y'-v(v+1)\,y=0$$

✓ Solution by Maple
Time used: 0.234 (sec). Leaf size: 51

```
dsolve(x*(x-1)*diff(diff(y(x),x),x)+(2*x-1)*diff(y(x),x)-v*(v+1)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\text{hypergeom}\left([-v,-v],[-2v],\frac{1}{x}\right)x^v+c_2\,\text{hypergeom}\left([v+1,v+1],[2v+2],\frac{1}{x}\right)x^{-v-1}$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 26

```
DSolve[-(v*(1 + v)*y[x]) + (-1 + 2*x)*y'[x] + (-1 + x)*x*y''[x] == 0,y[x],x,IncludeSingularS
```

$$y(x)\to c_1\,\text{LegendreP}(v,2x-1)+c_2\,\text{LegendreQ}(v,2x-1)$$

3.252 problem 1257

Internal problem ID [9586]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1257.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x(x-1)\,y'' + ((1+a)\,x+b)\,y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*(x-1)*diff(diff(y(x),x),x)+((a+1)*x+b)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + x^{b+1}\,\text{hypergeom}\left([b+1, b+a+1], [b+2], x\right) c_2$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 33

```
DSolve[(b + (1 + a)*x)*y'[x] + (-1 + x)*x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{c_1 x^{b+1}\,\text{Hypergeometric2F1}(b+1, a+b+1, b+2, x)}{b+1} + c_2$$

3.253 problem 1258

Internal problem ID [9587]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1258.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$x(x-1)\,y''+(xa+b)\,y'+yc=0$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 110

```
dsolve(x*(x-1)*diff(diff(y(x),x),x)+(a*x+b)*diff(y(x),x)+c*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\text{hypergeom}\left(\left[-\frac{1}{2}-\frac{\sqrt{a^2-2a-4c+1}}{2}+\frac{a}{2},-\frac{1}{2}+\frac{\sqrt{a^2-2a-4c+1}}{2}+\frac{a}{2}\right],[-b],x\right)+c_2x^{b+1}\,\text{hypergeom}\left(\left[\frac{1}{2}-\frac{\sqrt{a^2-2a-4c+1}}{2}+\frac{a}{2}+b,\frac{1}{2}+\frac{\sqrt{a^2-2a-4c+1}}{2}+\frac{a}{2}+b\right],[b+2],x\right)$$

✓ Solution by Mathematica

Time used: 0.202 (sec). Leaf size: 129

```
DSolve[c*y[x] + (b + a*x)*y'[x] + (-1 + x)*x*y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to c_1\,\text{Hypergeometric2F1}\left(\frac{1}{2}\left(a-\sqrt{a^2-2a-4c+1}-1\right),\frac{1}{2}\left(a+\sqrt{a^2-2a-4c+1}-1\right),-b,x\right)-(-1)^bc_2x^{b+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2}\left(a+2b-\sqrt{a^2-2a-4c+1}+1\right),\frac{1}{2}\left(a+2b+\sqrt{a^2-2a-4c+1}+1\right),b+2,x\right)$$

3.254 problem 1259

Internal problem ID [9588]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1259.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$x(x-1)\,y''+((1+a)\,x+b)\,y'-ly=0$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 92

```
dsolve(x*(x-1)*diff(diff(y(x),x),x)+((a+1)*x+b)*diff(y(x),x)-l*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\mathrm{hypergeom}\left(\left[\frac{a}{2}-\frac{\sqrt{a^2+4l}}{2},\frac{a}{2}+\frac{\sqrt{a^2+4l}}{2}\right],[-b],x\right)+c_2x^{b+1}\,\mathrm{hypergeom}\left(\left[\frac{a}{2}-\frac{\sqrt{a^2+4l}}{2}+b+1,\frac{a}{2}+\frac{\sqrt{a^2+4l}}{2}+b+1\right],[b+2],x\right)$$

✓ Solution by Mathematica
Time used: 0.183 (sec). Leaf size: 111

```
DSolve[-(l*y[x]) + (b + (1 + a)*x)*y'[x] + (-1 + x)*x*y''[x] == 0,y[x],x,IncludeSingularSolu
```

$$y(x)\to c_1\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2}\left(a-\sqrt{a^2+4l}\right),\frac{1}{2}\left(a+\sqrt{a^2+4l}\right),-b,x\right)-(-1)^bc_2x^{b+1}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2}\left(a+2b-\sqrt{a^2+4l}+2\right),\frac{1}{2}\left(a+2b+\sqrt{a^2+4l}+2\right),b+2,x\right)$$

3.255 problem 1260

Internal problem ID [9589]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1260.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$x(x-1)\,y'' + ((\mathrm{a1}+\mathrm{b1}+1)\,x-\mathrm{d1})\,y' = -\mathrm{a1}\,\mathrm{b1}\,\mathrm{d1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 76

```
dsolve(x*(x-1)*diff(diff(y(x),x),x)+((a1+b1+1)*x-d1)*diff(y(x),x)+a1*b1*d1=0,y(x), singsol=a
```

$$y(x) = \int (x-1)^{-\mathrm{a1}-\mathrm{b1}-1+\mathrm{d1}} \left(-\mathrm{a1}\,\mathrm{b1}\,\mathrm{signum}\,(x-1)^{\mathrm{a1}+\mathrm{b1}-\mathrm{d1}} (-\mathrm{signum}\,(x-1))^{-\mathrm{a1}-\mathrm{b1}+\mathrm{d1}}\,\mathrm{hypergeom}\left([\mathrm{d1},-\mathrm{a1}-\mathrm{b1}+\mathrm{d1}],[1+\mathrm{d1}],x\right) + x^{-\mathrm{d1}}c_1\right) dx + c_2$$

✓ Solution by Mathematica

Time used: 0.618 (sec). Leaf size: 65

```
DSolve[a1*b1*d1 + (-d1 + (1 + a1 + b1)*x)*y'[x] + (-1 + x)*x*y''[x] == 0,y[x],x,IncludeSingu
```

$$y(x) \to \mathrm{a1b1}x\,\mathrm{Gamma}(\mathrm{d1}+1)\,{}_3\tilde{F}_2(1,\mathrm{a1}+\mathrm{b1}+1,1;\mathrm{d1}+1,2;x) - \frac{c_1 x^{1-\mathrm{d1}}\,\mathrm{Hypergeometric2F1}(1-\mathrm{d1},\mathrm{a1}+\mathrm{b1}-\mathrm{d1}+1,2-\mathrm{d1},x)}{\mathrm{d1}-1} + c_2$$

3.256 problem 1261

Internal problem ID [9590]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1261.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(x+2)\, y'' + 2\left(n+1+(n+1-2l)\, x - l\, x^2\right) y' + \left(2l(p-n-1)\, x + 2pl + m\right) y = 0$$

✓ Solution by Maple

Time used: 0.5 (sec). Leaf size: 106

```
dsolve(x*(x+2)*diff(diff(y(x),x),x)+2*(n+1+(n+1-2*l)*x-l*x^2)*diff(y(x),x)+(2*l*(p-n-1)*x+2*
```

$$y(x) = \frac{\sqrt{2}\,\sqrt{-2-x}\,(x+2)^{-\frac{n}{2}-\frac{1}{2}}\left(-\frac{x}{2}-1\right)^{\frac{n}{2}}\left(c_2 x^{-n}\,\mathrm{HeunC}\left(4l, -n, n, -4pl, 2(n+1+p)\, l - \frac{n^2}{2} + m - n, -\frac{x}{2}\right.\right.}{2}$$

✓ Solution by Mathematica

Time used: 0.661 (sec). Leaf size: 120

```
DSolve[(m + 2*l*p + 2*l*(-1 - n + p)*x)*y[x] + 2*(1 + n + (1 - 2*l + n)*x - l*x^2)*y'[x] + x
```

$$y(x) \to \left(-\frac{x}{2}-1\right)^{\frac{n+1}{2}} x^{-n}(x+2)^{-\frac{n}{2}-\frac{1}{2}}\left(c_2\mathrm{HeunC}\left[-4ln-2lp-m+n^2+n, -4l(p-1), 1-n, n+1, 4l, -\frac{x}{2}\right] + c_1 x^n\mathrm{HeunC}\left[-2lp-m, 4l(n-p+1), n+1, n+1, 4l, -\frac{x}{2}\right]\right)$$

3.257 problem 1262

Internal problem ID [9591]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1262.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x+1)^2 y'' + (x^2+x-1) y' - (x+2) y = 0$$

✓ Solution by Maple

Time used: 0.532 (sec). Leaf size: 53

```
dsolve((x+1)^2*diff(diff(y(x),x),x)+(x^2+x-1)*diff(y(x),x)-(x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_1 e^{-x} \operatorname{HeunD}\left(4, 4, -8, 12, \frac{x}{x+2}\right) + c_2 \operatorname{HeunD}\left(-4, 4, -8, 12, \frac{x}{x+2}\right) e^{\frac{x-1}{2x+2}}\right)(x+1)$$

✓ Solution by Mathematica

Time used: 0.682 (sec). Leaf size: 46

```
DSolve[(-2 - x)*y[x] + (-1 + x + x^2)*y'[x] + (1 + x)^2*y''[x] == 0,y[x],x,IncludeSingularSo
```

$$y(x) \to e^{-x}\left(c_2 \int_1^x e^{\frac{K[1]^2+K[1]-1}{K[1]+1}}(K[1]+1)dK[1] + c_1\right)$$

3.258 problem 1263

Internal problem ID [9592]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1263.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$x(x+3)\,y''+(-1+3x)\,y'+y=(20x+30)\left(x^2+3x\right)^{\frac{7}{3}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(x*(x+3)*diff(diff(y(x),x),x)+(3*x-1)*diff(y(x),x)+y(x)-(20*x+30)*(x^2+3*x)^(7/3)=0,y(
```

$$y(x)=\frac{\left(c_2+\int\frac{3\left(x^3(x+3)^3(x(x+3))^{\frac{1}{3}}+\frac{c_1}{3}\right)(x+3)^{\frac{4}{3}}}{x^{\frac{7}{3}}}dx\right)x^{\frac{4}{3}}}{(x+3)^{\frac{7}{3}}}$$

✓ Solution by Mathematica

Time used: 20.953 (sec). Leaf size: 171

```
DSolve[(-30 - 20*x)*(3*x + x^2)^(7/3) + y[x] + (-1 + 3*x)*y'[x] + x*(3 + x)*y''[x] == 0,y[x]
```

$$y(x)\to\frac{-85c_2\left(4\sqrt{3}x^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{x}}{\sqrt[3]{x}+2\sqrt[3]{x+3}}\right)+4x^{4/3}\log\left(\sqrt[3]{x+3}-\sqrt[3]{x}\right)-2x^{4/3}\log\left(x^{2/3}+\sqrt[3]{x+3}\sqrt[3]{x}+\right.\right.}{340(x+3)^{7/3}}$$

3.259 problem 1264

Internal problem ID [9593]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1264.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(x^2+3x+4\right) y''+\left(x^2+x+1\right) y'-\left(2x+3\right) y=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve((x^2+3*x+4)*diff(diff(y(x),x),x)+(x^2+x+1)*diff(y(x),x)-(2*x+3)*y(x)=0,y(x), singsol=
```

$$y(x)=\mathrm{e}^{-x}c_1+c_2\left(x^2+x+3\right)$$

✓ Solution by Mathematica
Time used: 0.106 (sec). Leaf size: 23

```
DSolve[(-3 - 2*x)*y[x] + (1 + x + x^2)*y'[x] + (4 + 3*x + x^2)*y''[x] == 0,y[x],x,IncludeSin
```

$$y(x)\to c_2\left(x^2+x+3\right)+c_1e^{-x}$$

3.260 problem 1265

Internal problem ID [9594]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1265.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x-1)(x-2)y'' - (2x-3)y' + y = 0$$

✓ Solution by Maple
Time used: 0.234 (sec). Leaf size: 94

```
dsolve((x-1)*(x-2)*diff(diff(y(x),x),x)-(2*x-3)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{(x-2)^2 \left(c_1 \operatorname{hypergeom}\left(\left[\frac{1}{2}-\frac{\sqrt{5}}{2}, \frac{5}{2}-\frac{\sqrt{5}}{2}\right], \left[-\sqrt{5}+1\right], \frac{1}{x-1}\right)(x-1)^{\frac{\sqrt{5}}{2}} + c_2 \operatorname{hypergeom}\left(\left[\frac{1}{2}+\frac{\sqrt{5}}{2}, \frac{5}{2}+\right.\right.}{\sqrt{x-1}}$$

✓ Solution by Mathematica
Time used: 0.072 (sec). Leaf size: 57

```
DSolve[y[x] - (-3 + 2*x)*y'[x] + (-2 + x)*(-1 + x)*y''[x] == 0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to (x^2 - 3x + 2)\left(c_1 P^2_{\frac{1}{2}\left(-1+\sqrt{5}\right)}(2x-3) + c_2 Q^2_{\frac{1}{2}\left(-1+\sqrt{5}\right)}(2x-3)\right)$$

3.261 problem 1266

Internal problem ID [9595]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1266.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x-2)^2 y'' - (x-2) y' - 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve((x-2)^2*diff(diff(y(x),x),x)-(x-2)*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x-2)^4 + c_2}{x-2}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 22

```
DSolve[-3*y[x] - (-2 + x)*y'[x] + (-2 + x)^2*y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to c_1(x-2)^3 + \frac{c_2}{x-2}$$

3.262 problem 1267

Internal problem ID [9596]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1267.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$2x^2y'' - (2x^2 + l - 5x)\,y' - (4x - 1)\,y = 0$$

✓ Solution by Maple

Time used: 0.204 (sec). Leaf size: 47

```
dsolve(2*x^2*diff(diff(y(x),x),x)-(2*x^2+l-5*x)*diff(y(x),x)-(4*x-1)*y(x)=0,y(x), singsol=al
```

$$y(x) = \frac{\mathrm{e}^{-\frac{-2x^2+l}{2x}}\left(c_1\left(\int \frac{\mathrm{e}^{\frac{-2x^2+l}{2x}}}{x^{\frac{3}{2}}}dx\right) + 2c_2\right)}{2\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.552 (sec). Leaf size: 59

```
DSolve[(1 - 4*x)*y[x] - (1 - 5*x + 2*x^2)*y'[x] + 2*x^2*y''[x] == 0,y[x],x,IncludeSingularSo
```

$$y(x) \to \frac{e^{x-\frac{l}{2x}}\left(c_2\int_1^x \frac{e^{\frac{l}{2K[1]}-K[1]}}{K[1]^{3/2}}dK[1] + c_1\right)}{\sqrt{x}}$$

3.263 problem 1268

Internal problem ID [9597]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1268.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$2x(x-1)\,y''+(2x-1)\,y'+(xa+b)\,y=0$$

✓ Solution by Maple

Time used: 0.437 (sec). Leaf size: 39

```
dsolve(2*x*(x-1)*diff(diff(y(x),x),x)+(2*x-1)*diff(y(x),x)+(a*x+b)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\mathrm{MathieuC}\left(-a-2b,\frac{a}{2},\arccos\left(\sqrt{x}\right)\right)+c_2\,\mathrm{MathieuS}\left(-a-2b,\frac{a}{2},\arccos\left(\sqrt{x}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.179 (sec). Leaf size: 50

```
DSolve[(b + a*x)*y[x] + (-1 + 2*x)*y'[x] + 2*(-1 + x)*x*y''[x] == 0,y[x],x,IncludeSingularSo
```

$$y(x)\to c_1\mathrm{MathieuC}\left[-a-2b,\frac{a}{2},\arccos\left(\sqrt{x}\right)\right]+c_2\mathrm{MathieuS}\left[-a-2b,\frac{a}{2},\arccos\left(\sqrt{x}\right)\right]$$

3.264 problem 1269

Internal problem ID [9598]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1269.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$2x(x-1)\,y''+((2v+5)\,x-2v-3)\,y'+(v+1)\,y=0$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 78

```
dsolve(2*x*(x-1)*diff(diff(y(x),x),x)+((2*v+5)*x-2*v-3)*diff(y(x),x)+(v+1)*y(x)=0,y(x), sing
```

$$y(x) = \frac{x^{-\frac{v}{2}-\frac{1}{4}}\left(c_1\Gamma\left(v+\frac{1}{2}\right)^2\left(v+\frac{1}{2}\right)\operatorname{LegendreP}\left(-\frac{1}{2},-v-\frac{1}{2},\frac{-x-1}{x-1}\right)+\sec\left(\pi v\right)\pi\operatorname{LegendreP}\left(-\frac{1}{2},v+\frac{1}{2},\frac{-x-1}{x-1}\right)c}{\sqrt{1-x}\,\Gamma\left(v+\frac{1}{2}\right)}$$

✓ Solution by Mathematica

Time used: 0.121 (sec). Leaf size: 59

```
DSolve[(1 + v)*y[x] + (-3 - 2*v + (5 + 2*v)*x)*y'[x] + 2*(-1 + x)*x*y''[x] == 0,y[x],x,Inclu
```

$$y(x)\to c_1\,\text{Hypergeometric2F1}\left(\frac{1}{2},v+1,v+\frac{3}{2},x\right)-ic_2i^{-2v}x^{-v-\frac{1}{2}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-v,\frac{1}{2}-v,x\right)$$

3.265 problem 1270

Internal problem ID [9599]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1270.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(2x^2 + 6x + 4)\, y'' + (10x^2 + 21x + 8)\, y' + (12x^2 + 17x + 8)\, y = 0$$

✓ Solution by Maple

Time used: 0.687 (sec). Leaf size: 46

```
dsolve((2*x^2+6*x+4)*diff(diff(y(x),x),x)+(10*x^2+21*x+8)*diff(y(x),x)+(12*x^2+17*x+8)*y(x)=
```

$$y(x) = \left(c_2(x+1)^{\frac{5}{2}} \operatorname{HeunC}\left(-1, \frac{5}{2}, 4, -\frac{7}{4}, \frac{7}{2}, -x-1\right) + c_1 \operatorname{HeunC}\left(-1, -\frac{5}{2}, 4, -\frac{7}{4}, \frac{7}{2}, -x-1\right)\right) \mathrm{e}^{-2x}(x+2)^4$$

✓ Solution by Mathematica

Time used: 5.458 (sec). Leaf size: 48

```
DSolve[(8 + 17*x + 12*x^2)*y[x] + (8 + 21*x + 10*x^2)*y'[x] + (4 + 6*x + 2*x^2)*y''[x] == 0,
```

$$y(x) \to e^{-3x}(x+2)^4 \left(c_2 \int_1^x \frac{e^{K[1]}(K[1]+1)^{3/2}}{(K[1]+2)^5} dK[1] + c_1\right)$$

3.266 problem 1271

Internal problem ID [9600]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1271.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{4x^2y'' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(4*x^2*diff(diff(y(x),x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = (\ln(x) c_2 + c_1)\sqrt{x}$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 24

```
DSolve[y[x] + 4*x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\sqrt{x}(c_2 \log(x) + 2c_1)$$

3.267 problem 1272

Internal problem ID [9601]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1272.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + (4a^2x^2 + 1)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(4*x^2*diff(diff(y(x),x),x)+(4*a^2*x^2+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = (\text{BesselJ}\,(0, ax)\, c_1 + \text{BesselY}\,(0, ax)\, c_2)\,\sqrt{x}$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 28

```
DSolve[(1 + 4*a^2*x^2)*y[x] + 4*x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}(c_1 \,\text{BesselJ}(0, ax) + c_2 \,\text{BesselY}(0, ax))$$

3.268 problem 1273

Internal problem ID [9602]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1273.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' - (-4kx + 4m^2 + x^2 - 1)\,y = 0$$

✓ Solution by Maple
Time used: 0.046 (sec). Leaf size: 17

```
dsolve(4*x^2*diff(diff(y(x),x),x)-(-4*k*x+4*m^2+x^2-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \,\mathrm{WhittakerM}\,(k, m, x) + c_2 \,\mathrm{WhittakerW}\,(k, m, x)$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 20

```
DSolve[(1 - 4*m^2 + 4*k*x - x^2)*y[x] + 4*x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to c_1 M_{k,m}(x) + c_2 W_{k,m}(x)$$

3.269 problem 1274

Internal problem ID [9603]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1274.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 4y'x + (-v^2 + x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(4*x^2*diff(diff(y(x),x),x)+4*x*diff(y(x),x)+(-v^2+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}\left(v, \sqrt{x}\right) + c_2 \operatorname{BesselY}\left(v, \sqrt{x}\right)$$

✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 38

```
DSolve[(-v^2 + x)*y[x] + 4*x*y'[x] + 4*x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \rightarrow c_1 \operatorname{Gamma}(1 - v) \operatorname{BesselJ}\left(-v, \sqrt{x}\right) + c_2 \operatorname{Gamma}(v + 1) \operatorname{BesselJ}\left(v, \sqrt{x}\right)$$

3.270 problem 1275

Internal problem ID [9604]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1275.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + 4y'x + \left(-x^2 + 2(1 - m + 2l)\,x - m^2 + 1\right) y = 0$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 53

```
dsolve(4*x^2*diff(diff(y(x),x),x)+4*x*diff(y(x),x)+(-x^2+2*(1-m+2*l)*x-m^2+1)*y(x)=0,y(x), s
```

$$y(x) = \frac{c_2 \operatorname{WhittakerW}\left(l - \frac{m}{2} + \frac{1}{2}, \frac{\sqrt{m+1}\sqrt{m-1}}{2}, x\right) + c_1 \operatorname{WhittakerM}\left(l - \frac{m}{2} + \frac{1}{2}, \frac{\sqrt{m+1}\sqrt{m-1}}{2}, x\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.064 (sec). Leaf size: 97

```
DSolve[(1 - m^2 + 2*(1 + 2*l - m)*x - x^2)*y[x] + 4*x*y'[x] + 4*x^2*y''[x] == 0,y[x],x,Inclu
```

$$y(x) \to e^{-x/2} x^{\frac{\sqrt{m^2-1}}{2}} \left(c_1 \operatorname{HypergeometricU}\left(\frac{1}{2}\left(-2l + m + \sqrt{m^2-1}\right), \sqrt{m^2-1} + 1, x\right) + c_2 L_{l - \frac{m}{2} - \frac{\sqrt{m^2-1}}{2}}^{\sqrt{m^2-1}}(x)\right)$$

3.271 problem 1276

Internal problem ID [9605]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1276.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$4x^2y'' + 4y'x - \left(4x^2 + 1\right) y = 4\sqrt{x^3}\, \mathrm{e}^x$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 30

```
dsolve(4*x^2*diff(diff(y(x),x),x)+4*x*diff(y(x),x)-(4*x^2+1)*y(x)-4*(x^3)^(1/2)*exp(x)=0,y(x
```

$$y(x) = \frac{\sqrt{x^3}\, \mathrm{e}^x}{2x} + \frac{\sinh(x)\, c_2 + \cosh(x)\, c_1}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.287 (sec). Leaf size: 55

```
DSolve[-4*E^x*Sqrt[x^3] - (1 + 4*x^2)*y[x] + 4*x*y'[x] + 4*x^2*y''[x] == 0,y[x],x,IncludeSin
```

$$y(x) \to \frac{e^x\sqrt{x^3}(2x-1)}{4x^2} + \frac{c_1e^{-x}}{\sqrt{x}} + \frac{c_2e^x}{2\sqrt{x}}$$

3.272 problem 1277

Internal problem ID [9606]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1277.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + 4y'x - (a\,x^2 + 1)\,y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 27

```
dsolve(4*x^2*diff(diff(y(x),x),x)+4*x*diff(y(x),x)-(a*x^2+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sinh\left(\frac{x\sqrt{a}}{2}\right) + c_2 \cosh\left(\frac{x\sqrt{a}}{2}\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 49

```
DSolve[(-1 - a*x^2)*y[x] + 4*x*y'[x] + 4*x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{e^{-\frac{\sqrt{a}x}{2}}\left(c_2 e^{\sqrt{a}x} + \sqrt{a}c_1\right)}{\sqrt{a}\sqrt{x}}$$

3.273 problem 1278

Internal problem ID [9607]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1278.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + 4y'x + f(x)\,y = 0$$

✗ Solution by Maple

```
dsolve(4*x^2*diff(diff(y(x),x),x)+4*x*diff(y(x),x)+f(x)*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[f[x]*y[x] + 4*x*y'[x] + 4*x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

3.274 problem 1279

Internal problem ID [9608]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1279.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + 5y'x - y = \ln(x)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 32

```
dsolve(4*x^2*diff(diff(y(x),x),x)+5*x*diff(y(x),x)-y(x)-ln(x)=0,y(x), singsol=all)
```

$$y(x) = x^{-\frac{1}{8}+\frac{\sqrt{17}}{8}}c_2 + x^{-\frac{1}{8}-\frac{\sqrt{17}}{8}}c_1 - \ln(x) - 1$$

✓ Solution by Mathematica
Time used: 0.146 (sec). Leaf size: 45

```
DSolve[-Log[x] - y[x] + 5*x*y'[x] + 4*x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to c_2x^{\frac{1}{8}\left(\sqrt{17}-1\right)} + c_1x^{-\frac{1}{8}-\frac{\sqrt{17}}{8}} - \log(x) - 1$$

3.275 problem 1280

Internal problem ID [9609]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1280.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + 8y'x - (4x^2 + 12x + 3)\, y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(4*x^2*diff(diff(y(x),x),x)+8*x*diff(y(x),x)-(4*x^2+12*x+3)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-4\,\mathrm{e}^{x}\,\mathrm{expIntegral}_1\,(2x)\, c_2 x^2 + (-1 + 2x)\, c_2 \mathrm{e}^{-x} + c_1 x^2 \mathrm{e}^{x}}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.088 (sec). Leaf size: 52

```
DSolve[(-3 - 12*x - 4*x^2)*y[x] + 8*x*y'[x] + 4*x^2*y''[x] == 0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{c_2 e^{-x}(4e^{2x}x^2\, \mathrm{ExpIntegralEi}(-2x) + 2x - 1)}{2x^{3/2}} + c_1 e^x\sqrt{x}$$

3.276 problem 1281

Internal problem ID [9610]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1281.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' - 4x(2x-1)\,y' + \left(4x^2 - 4x - 1\right)y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(4*x^2*diff(diff(y(x),x),x)-4*x*(2*x-1)*diff(y(x),x)+(4*x^2-4*x-1)*y(x)=0,y(x), singso
```

$$y(x) = \frac{e^x(xc_2 + c_1)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 21

```
DSolve[(-1 - 4*x + 4*x^2)*y[x] - 4*x*(-1 + 2*x)*y'[x] + 4*x^2*y''[x] == 0,y[x],x,IncludeSing
```

$$y(x) \to \frac{e^x(c_2x + c_1)}{\sqrt{x}}$$

3.277 problem 1282

Internal problem ID [9611]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1282.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + 4x^3y' + (x^2+6)(x^2-4)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(4*x^2*diff(diff(y(x),x),x)+4*x^3*diff(y(x),x)+(x^2+6)*(x^2-4)*y(x)=0,y(x), singsol=al
```

$$y(x) = \frac{e^{-\frac{x^2}{4}}(x^5c_2 + c_1)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 32

```
DSolve[(-4 + x^2)*(6 + x^2)*y[x] + 4*x^3*y'[x] + 4*x^2*y''[x] == 0,y[x],x,IncludeSingularSol
```

$$y(x) \to \frac{e^{-\frac{x^2}{4}}(c_2x^5 + 5c_1)}{5x^2}$$

3.278 problem 1283

Internal problem ID [9612]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1283.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$4x^2y'' + 4x^2\ln(x)\,y' + \left(x^2\ln(x)^2 + 2x - 8\right)y = 4x^2\sqrt{e^x x^{-x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(4*x^2*diff(diff(y(x),x),x)+4*x^2*ln(x)*diff(y(x),x)+(x^2*ln(x)^2+2*x-8)*y(x)-4*x^2*(e
```

$$y(x) = \frac{x^2\left(\ln(x) - \frac{1}{3}\right)\sqrt{x^{-x}e^x}}{3} + e^{\frac{x}{2}}\left(c_1x^{-\frac{x}{2}+2} + c_2x^{-\frac{x}{2}-1}\right)$$

✓ Solution by Mathematica

Time used: 0.107 (sec). Leaf size: 89

```
DSolve[-4*x^2*Sqrt[E^x/x^x] + (-8 + 2*x + x^2*Log[x]^2)*y[x] + 4*x^2*Log[x]*y'[x] + 4*x^2*y'
```

$$y(x) \to c_1e^{x/2}x^{-\frac{x}{2}-1} + \frac{1}{3}c_2e^{x/2}x^{2-\frac{x}{2}} - \frac{1}{9}\sqrt{e^xx^{-x}}x^2 + \frac{1}{3}\sqrt{e^xx^{-x}}x^2\log(x)$$

3.279 problem 1284

Internal problem ID [9613]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1284.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{(2x+1)^2 y'' - 2(2x+1) y' - 12y = 3x+1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 41

```
dsolve((2*x+1)^2*diff(diff(y(x),x),x)-2*(2*x+1)*diff(y(x),x)-12*y(x)-3*x-1=0,y(x), singsol=a
```

$$y(x) = \frac{c_1}{2x+1} + (2x+1)^3 c_2 + \frac{-72x^2-56x-7}{384x+192}$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 41

```
DSolve[-1 - 3*x - 12*y[x] - 2*(1 + 2*x)*y'[x] + (1 + 2*x)^2*y''[x] == 0,y[x],x,IncludeSingul
```

$$y(x) \to \frac{-72x^2-56x+192c_1(2x+1)^4-7+192c_2}{192(2x+1)}$$

3.280 problem 1285

Internal problem ID [9614]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1285.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(4x-1)\,y''+((4a+2)\,x-a)\,y'+a(-1+a)\,y=0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 56

```
dsolve(x*(4*x-1)*diff(diff(y(x),x),x)+((4*a+2)*x-a)*diff(y(x),x)+a*(a-1)*y(x)=0,y(x), singso
```

$$y(x)=c_1 2^{a-1}\left(1+\sqrt{-4x+1}\right)^{-a+1}+c_2 x^{-a+1}2^{-a+1}\left(1+\sqrt{-4x+1}\right)^{a-1}$$

✓ Solution by Mathematica
Time used: 0.487 (sec). Leaf size: 186

```
DSolve[(-1 + a)*a*y[x] + (-a + (2 + 4*a)*x)*y'[x] + x*(-1 + 4*x)*y''[x] == 0,y[x],x,IncludeS
```

$$y(x)\to\frac{\sqrt[4]{4x-1}x^{\frac{1}{2}-\frac{a}{2}}\left(1-i\sqrt{4x-1}\right)^{-i\sqrt{-(a-1)^2}}e^{\sqrt{-(a-1)^2}\arctan\left(\sqrt{4x-1}\right)}\left(4\sqrt{-(a-1)^2}c_1\left(1-i\sqrt{4x-1}\right)^{i\sqrt{-(a-1)}}\right.}{2\sqrt{-(a-1)^2}\sqrt[4]{1-4x}}$$

3.281 problem 1286

Internal problem ID [9615]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1286.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$(-1+3x)^2 y'' + 3(-1+3x) y' - 9y = \ln(-1+3x)^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve((3*x-1)^2*diff(diff(y(x),x),x)+3*(3*x-1)*diff(y(x),x)-9*y(x)-ln(3*x-1)^2=0,y(x), sing
```

$$y(x) = \frac{c_1}{3x-1} + (3x-1)\,c_2 - \frac{\ln(3x-1)^2}{9} - \frac{2}{9}$$

✓ Solution by Mathematica

Time used: 0.164 (sec). Leaf size: 80

```
DSolve[-Log[-1 + 3*x]^2 - 9*y[x] + 3*(-1 + 3*x)*y'[x] + (-1 + 3*x)^2*y''[x] == 0,y[x],x,Incl
```

$$y(x) \to \frac{-81c_1x^2 - 81ic_2x^2 - 12x + (2-6x)\log^2(3x-1) - 2\log(1-3x) + 2\log(3x-1) + 54c_1x + 54ic_2x +}{54x-18}$$

3.282 problem 1287

Internal problem ID [9616]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1287.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$9x(x-1)\,y'' + 3(2x-1)\,y' - 20y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(9*x*(x-1)*diff(diff(y(x),x),x)+3*(2*x-1)*diff(y(x),x)-20*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(6x-5)\,x^{\frac{2}{3}} + c_2(6x-1)\,(x-1)^{\frac{2}{3}}$$

✓ Solution by Mathematica
Time used: 0.054 (sec). Leaf size: 51

```
DSolve[-20*y[x] + 3*(-1 + 2*x)*y'[x] + 9*(-1 + x)*x*y''[x] == 0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to c_2\sqrt[3]{-((x-1)x)}Q_1^{\frac{2}{3}}(2x-1) + \frac{c_1x^{2/3}(6x-5)}{3\,\text{Gamma}\left(\frac{4}{3}\right)}$$

3.283 problem 1288

Internal problem ID [9617]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1288.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{16x^2y'' + (4x + 3)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(16*x^2*diff(diff(y(x),x),x)+(4*x+3)*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^{\frac{1}{4}}\left(c_1 \sin\left(\sqrt{x}\right) + c_2 \cos\left(\sqrt{x}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 43

```
DSolve[(3 + 4*x)*y[x] + 16*x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-i\sqrt{x}}\sqrt[4]{x}\left(c_1 e^{2i\sqrt{x}} + ic_2\right)$$

3.284 problem 1289

Internal problem ID [9618]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1289.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$16x^2y'' + 32y'x - (4x+5)\,y = 0$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 33

```
dsolve(16*x^2*diff(diff(y(x),x),x)+32*x*diff(y(x),x)-(4*x+5)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2(\sqrt{x}+1)\,\mathrm{e}^{-\sqrt{x}} + c_1\mathrm{e}^{\sqrt{x}}(\sqrt{x}-1)}{x^{\frac{5}{4}}}$$

✓ Solution by Mathematica
Time used: 0.204 (sec). Leaf size: 51

```
DSolve[(-5 - 4*x)*y[x] + 32*x*y'[x] + 16*x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{e^{-\sqrt{x}}\left(c_1e^{2\sqrt{x}}(\sqrt{x}-1) - c_2(\sqrt{x}+1)\right)}{x^{5/4}}$$

3.285 problem 1290

Internal problem ID [9619]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1290.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\left(27x^2+4\right)y''+27y'x-3y=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve((27*x^2+4)*diff(diff(y(x),x),x)+27*x*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\sinh\left(\frac{\operatorname{arcsinh}\left(\frac{3\sqrt{3}\,x}{2}\right)}{3}\right)+c_2\cosh\left(\frac{\operatorname{arcsinh}\left(\frac{3\sqrt{3}\,x}{2}\right)}{3}\right)$$

✓ Solution by Mathematica
Time used: 0.212 (sec). Leaf size: 103

```
DSolve[-3*y[x] + 27*x*y'[x] + (4 + 27*x^2)*y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x)\to c_1\cosh\left(\frac{\sqrt{-27x^2-4}\arctan\left(\frac{3x}{\sqrt{-9x^2-\frac{4}{3}}}\right)}{3\sqrt{27x^2+4}}\right)+ic_2\sinh\left(\frac{\sqrt{-27x^2-4}\arctan\left(\frac{3x}{\sqrt{-9x^2-\frac{4}{3}}}\right)}{3\sqrt{27x^2+4}}\right)$$

3.286 problem 1291

Internal problem ID [9620]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1291.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$\boxed{48x(x-1)\,y''+(152x-40)\,y'+53y=0}$$

✓ Solution by Maple
Time used: 0.156 (sec). Leaf size: 50

```
dsolve(48*x*(x-1)*diff(diff(y(x),x),x)+(152*x-40)*diff(y(x),x)+53*y(x)=0,y(x), singsol=all)
```

$$\begin{aligned}y(x) &= c_1\,\text{hypergeom}\left(\left[\frac{13}{12}-\frac{\sqrt{10}}{12},\frac{13}{12}+\frac{\sqrt{10}}{12}\right],\left[\frac{5}{6}\right],x\right)\\&\quad+c_2x^{\frac{1}{6}}\,\text{hypergeom}\left(\left[\frac{5}{4}-\frac{\sqrt{10}}{12},\frac{5}{4}+\frac{\sqrt{10}}{12}\right],\left[\frac{7}{6}\right],x\right)\end{aligned}$$

✓ Solution by Mathematica
Time used: 0.112 (sec). Leaf size: 82

```
DSolve[53*y[x] + (-40 + 152*x)*y'[x] + 48*(-1 + x)*x*y''[x] == 0,y[x],x,IncludeSingularSolut
```

$$\begin{aligned}y(x) &\to \sqrt[6]{-1}c_2\sqrt[6]{x}\,\text{Hypergeometric2F1}\left(\frac{5}{4}-\frac{\sqrt{\frac{5}{2}}}{6},\frac{1}{12}\left(15+\sqrt{10}\right),\frac{7}{6},x\right)\\&\quad+c_1\,\text{Hypergeometric2F1}\left(\frac{1}{12}\left(13-\sqrt{10}\right),\frac{1}{12}\left(13+\sqrt{10}\right),\frac{5}{6},x\right)\end{aligned}$$

3.287 problem 1292

Internal problem ID [9621]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1292.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']]`

$$\boxed{50x(x-1)\,y''+25(2x-1)\,y'-2y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(50*x*(x-1)*diff(diff(y(x),x),x)+25*(2*x-1)*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\left(\sqrt{x}+\sqrt{x-1}\right)^{\frac{4}{5}}+c_2}{\left(\sqrt{x}+\sqrt{x-1}\right)^{\frac{2}{5}}}$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 57

```
DSolve[-2*y[x] + 25*(-1 + 2*x)*y'[x] + 50*(-1 + x)*x*y''[x] == 0,y[x],x,IncludeSingularSolut
```

$$y(x)\to c_1\cosh\left(\frac{2}{5}\log\left(\sqrt{x-1}-\sqrt{x}\right)\right)-ic_2\sinh\left(\frac{2}{5}\log\left(\sqrt{x-1}-\sqrt{x}\right)\right)$$

3.288 problem 1293

Internal problem ID [9622]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1293.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$\boxed{144x(x-1)\,y''+(120x-48)\,y'+y=0}$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 33

```
dsolve(144*x*(x-1)*diff(diff(y(x),x),x)+(120*x-48)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=\left(\text{LegendreP}\left(-\frac{1}{2},\frac{2}{3},\sqrt{1-x}\right)c_1+\text{LegendreQ}\left(-\frac{1}{2},\frac{2}{3},\sqrt{1-x}\right)c_2\right)x^{\frac{1}{3}}$$

✓ Solution by Mathematica

Time used: 0.346 (sec). Leaf size: 44

```
DSolve[y[x] + (-48 + 120*x)*y'[x] + 144*(-1 + x)*x*y''[x] == 0,y[x],x,IncludeSingularSolutio
```

$$y(x)\to(-1)^{2/3}c_2x^{2/3}\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{7}{12},\frac{5}{3},x\right)+c_1\,\text{Hypergeometric2F1}\left(-\frac{1}{12},-\frac{1}{12},\frac{1}{3},x\right)$$

3.289 problem 1294

Internal problem ID [9623]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1294.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$144x(x-1)\,y''+(168x-96)\,y'+y=0$$

✓ Solution by Maple
Time used: 0.171 (sec). Leaf size: 33

```
dsolve(144*x*(x-1)*diff(diff(y(x),x),x)+(168*x-96)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=\left(\operatorname{LegendreQ}\left(-\frac{1}{2},\frac{1}{3},\sqrt{1-x}\right)c_2+\operatorname{LegendreP}\left(-\frac{1}{2},\frac{1}{3},\sqrt{1-x}\right)c_1\right)x^{\frac{1}{6}}$$

✓ Solution by Mathematica
Time used: 0.097 (sec). Leaf size: 44

```
DSolve[y[x] + (-96 + 168*x)*y'[x] + 144*(-1 + x)*x*y''[x] == 0,y[x],x,IncludeSingularSolutio
```

$$y(x)\to c_1\,\text{Hypergeometric2F1}\left(\frac{1}{12},\frac{1}{12},\frac{2}{3},x\right)+\sqrt[3]{-1}c_2\sqrt[3]{x}\,\text{Hypergeometric2F1}\left(\frac{5}{12},\frac{5}{12},\frac{4}{3},x\right)$$

3.290 problem 1295

Internal problem ID [9624]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1295.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{a\,x^2y''+bxy'+\left(c\,x^2+dx+f\right)y=0}$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 106

```
dsolve(a*x^2*diff(diff(y(x),x),x)+b*x*diff(y(x),x)+(c*x^2+d*x+f)*y(x)=0,y(x), singsol=all)
```

$$y(x)=x^{-\frac{b}{2a}}\left(\operatorname{WhittakerM}\left(-\frac{id}{2\sqrt{a}\,\sqrt{c}},\frac{\sqrt{a^2+(-2b-4f)\,a+b^2}}{2a},\frac{2i\sqrt{c}\,x}{\sqrt{a}}\right)c_1+\operatorname{WhittakerW}\left(-\frac{id}{2\sqrt{a}\,\sqrt{c}},\frac{\sqrt{a^2+(-2b-4f)\,a+b^2}}{2a},\frac{2i\sqrt{c}\,x}{\sqrt{a}}\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.19 (sec). Leaf size: 229

```
DSolve[(f + d*x + c*x^2)*y[x] + b*x*y'[x] + a*x^2*y''[x] == 0,y[x],x,IncludeSingularSolution
```

$$y(x)\to e^{-\frac{i\sqrt{c}x}{\sqrt{a}}}x^{\frac{\sqrt{a^2-2a(b+2f)+b^2}+a-b}{2a}}\left(c_1\operatorname{HypergeometricU}\left(\frac{a+\frac{id\sqrt{a}}{\sqrt{c}}+\sqrt{a^2-2(b+2f)a+b^2}}{2a},\frac{a+\sqrt{a^2-2(b+}}{a}\right.\right.$$
$$\left.+c_2L_{-\frac{a+\frac{id\sqrt{a}}{\sqrt{c}}+\sqrt{a^2-2(b+2f)a+b^2}}{2a}}^{\frac{\sqrt{a^2-2(b+2f)a+b^2}}{a}}\left(\frac{2i\sqrt{c}x}{\sqrt{a}}\right)\right)$$

3.291 problem 1296

Internal problem ID [9625]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1296.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\mathrm{a2}\ x^{2}y''+\left(\mathrm{a1}\ x^{2}+\mathrm{b1}\,x\right)y'+\left(\mathrm{a0}\ x^{2}+\mathrm{b0}\,x+\mathrm{c0}\right)y=0}$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 150

```
dsolve(a2*x^2*diff(diff(y(x),x),x)+(a1*x^2+b1*x)*diff(y(x),x)+(a0*x^2+b0*x+c0)*y(x)=0,y(x),
```

$$y(x)=\mathrm{e}^{-\frac{\mathrm{a1}\,x}{2\,\mathrm{a2}}}x^{-\frac{\mathrm{b1}}{2\,\mathrm{a2}}}\left(\mathrm{WhittakerM}\left(-\frac{\mathrm{a1}\,\mathrm{b1}-2\,\mathrm{a2}\,\mathrm{b0}}{2\,\mathrm{a2}\sqrt{-4\,\mathrm{a0}\,\mathrm{a2}+\mathrm{a1}^{2}}},\frac{\sqrt{\mathrm{a2}^{2}+(-2\,\mathrm{b1}-4\,\mathrm{c0})\,\mathrm{a2}+\mathrm{b1}^{2}}}{2\,\mathrm{a2}},\frac{\sqrt{-4\,\mathrm{a0}\,\mathrm{a2}+\mathrm{a1}^{2}}\,x}{\mathrm{a2}}\right.\right.$$

$$\left.\left.+c_{2}\,\mathrm{WhittakerW}\left(-\frac{\mathrm{a1}\,\mathrm{b1}-2\,\mathrm{a2}\,\mathrm{b0}}{2\,\mathrm{a2}\sqrt{-4\,\mathrm{a0}\,\mathrm{a2}+\mathrm{a1}^{2}}},\frac{\sqrt{\mathrm{a2}^{2}+(-2\,\mathrm{b1}-4\,\mathrm{c0})\,\mathrm{a2}+\mathrm{b1}^{2}}}{2\,\mathrm{a2}},\frac{\sqrt{-4\,\mathrm{a0}\,\mathrm{a2}+\mathrm{a1}^{2}}\,x}{\mathrm{a2}}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.331 (sec). Leaf size: 272

```
DSolve[(c0 + b0*x + a0*x^2)*y[x] + (b1*x + a1*x^2)*y'[x] + a2*x^2*y''[x] == 0,y[x],x,Include
```

$$y(x) \to e^{-\frac{x\left(\sqrt{a1^2-4a0a2}+a1\right)}{2a2}} x^{\frac{\sqrt{a2^2-2a2(b1+2c0)+b1^2}+a2-b1}{2a2}} \left(c_1 \text{HypergeometricU}\left(\frac{-\frac{2b0a2}{\sqrt{a1^2-4a0a2}}+a2+\frac{a1b1}{\sqrt{a1^2-4a0a2}}}{2a2}\right.\right.$$

$$\left.\left. + c_2 L_{-\frac{-\frac{2b0a2}{\sqrt{a1^2-4a0a2}}+a2+\frac{a1b1}{\sqrt{a1^2-4a0a2}}+\sqrt{a2^2-2(b1+2c0)a2+b1^2}}{2a2}}^{\frac{\sqrt{a2^2-2(b1+2c0)a2+b1^2}}{a2}}\left(\frac{\sqrt{a1^2-4a0a2}x}{a2}\right)\right)$$

3.292 problem 1297

Internal problem ID [9626]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1297.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{\left(a\,x^{2}+1\right)y''+axy'+by=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve((a*x^2+1)*diff(diff(y(x),x),x)+a*x*diff(y(x),x)+b*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x\sqrt{a}+\sqrt{a\,x^{2}+1}\right)^{\frac{i\sqrt{b}}{\sqrt{a}}}+c_2\left(x\sqrt{a}+\sqrt{a\,x^{2}+1}\right)^{-\frac{i\sqrt{b}}{\sqrt{a}}}$$

✓ Solution by Mathematica

Time used: 0.188 (sec). Leaf size: 84

```
DSolve[b*y[x] + a*x*y'[x] + (1 + a*x^2)*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\cos\left(\frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+1}-1}\right)}{\sqrt{a}}\right)+c_2\sin\left(\frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+1}-1}\right)}{\sqrt{a}}\right)$$

3.293 problem 1299

Internal problem ID [9627]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1299.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{(a^2x^2 - 1)\, y'' + 2a^2xy' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve((a^2*x^2-1)*diff(diff(y(x),x),x)+2*a^2*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 - \frac{(-\ln(ax-1) + \ln(ax+1))\, c_2}{2a}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 19

```
DSolve[2*a^2*x*y'[x] + (-1 + a^2*x^2)*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{c_1 \operatorname{arctanh}(ax)}{a}$$

3.294 problem 1300

Internal problem ID [9628]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1300.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(a^2x^2-1)\,y''+2a^2xy'-2a^2y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve((a^2*x^2-1)*diff(diff(y(x),x),x)+2*a^2*x*diff(y(x),x)-2*a^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{c_2 a \ln(ax+1)\,x}{2} + \frac{c_2 a \ln(ax-1)\,x}{2} + c_1 x + c_2$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 39

```
DSolve[-2*a^2*y[x] + 2*a^2*x*y'[x] + (-1 + a^2*x^2)*y''[x] == 0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to ac_1x - \frac{1}{2}c_2(ax\log(1-ax) - ax\log(ax+1) + 2)$$

3.295 problem 1301

Internal problem ID [9629]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1301.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(a\,x^2 + bx)\,y'' + 2by' - 2ay = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve((a*x^2+b*x)*diff(diff(y(x),x),x)+2*b*diff(y(x),x)-2*a*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 + c_2(ax+b)^3}{x}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 30

```
DSolve[-2*a*y[x] + 2*b*y'[x] + (b*x + a*x^2)*y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{\frac{c_2(ax+b)^3}{a} + 3c_1}{3x}$$

3.296 problem 1302

Internal problem ID [9630]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1302.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\mathrm{A2}\,(xa+b)^2\,y''+\mathrm{A1}\,(xa+b)\,y'+\mathrm{A0}\,(xa+b)\,y=0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 98

```
dsolve(A2*(a*x+b)^2*diff(diff(y(x),x),x)+A1*(a*x+b)*diff(y(x),x)+A0*(a*x+b)*y(x)=0,y(x), sin
```

$$y(x)=(ax+b)^{-\frac{-a\,\mathrm{A2}+\mathrm{A1}}{2a\,\mathrm{A2}}}\left(\mathrm{BesselY}\left(\frac{a\,\mathrm{A2}-\mathrm{A1}}{a\,\mathrm{A2}},2\sqrt{\mathrm{A0}}\sqrt{\frac{ax+b}{a^2\,\mathrm{A2}}}\right)c_2+\mathrm{BesselJ}\left(\frac{a\,\mathrm{A2}-\mathrm{A1}}{a\,\mathrm{A2}},2\sqrt{\mathrm{A0}}\sqrt{\frac{ax+b}{a^2\,\mathrm{A2}}}\right)c_1\right)$$

✓ Solution by Mathematica

Time used: 0.172 (sec). Leaf size: 165

```
DSolve[A0*(b + a*x)*y[x] + A1*(b + a*x)*y'[x] + A2*(b + a*x)^2*y''[x] == 0,y[x],x,IncludeSin
```

$$y(x)\to(-1)^{-\frac{\mathrm{A1}}{a\mathrm{A2}}}\left(\frac{b}{a}+x\right)^{\frac{\mathrm{A1}}{2a\mathrm{A2}}}(\mathrm{A2}(ax+b))^{-\frac{\mathrm{A1}}{2a\mathrm{A2}}}\left(-\frac{\mathrm{A0}(ax+b)}{a^2\mathrm{A2}}\right)^{\frac{1}{2}-\frac{\mathrm{A1}}{2a\mathrm{A2}}}\left(c_1(-1)^{\frac{\mathrm{A1}}{a\mathrm{A2}}}\,\mathrm{BesselI}\left(\frac{\mathrm{A1}}{a\mathrm{A2}}-1,2\sqrt{-\frac{\mathrm{A0}(b+ax)}{a^2\mathrm{A2}}}\right)-c_2K_{\frac{\mathrm{A1}}{a\mathrm{A2}}-1}\left(2\sqrt{-\frac{\mathrm{A0}(b+ax)}{a^2\mathrm{A2}}}\right)\right)$$

3.297 problem 1303

Internal problem ID [9631]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1303.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(a\,x^2+bx+c)\,y''+(dx+f)\,y'+gy=0$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 501

```
dsolve((a*x^2+b*x+c)*diff(diff(y(x),x),x)+(d*x+f)*diff(y(x),x)+g*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{hypergeom}\left(\left[-\frac{a-d+\sqrt{a^2+(-2d-4g)\,a+d^2}}{2a}, \frac{-a+d+\sqrt{a^2+(-2d-4g)\,a+d^2}}{2a}\right], \left[\frac{d\sqrt{\frac{-4ac}{a^2}}}{2a} \right.\right.$$

$$+c_2\left(2\sqrt{\frac{-4ac+b^2}{a^2}}\,x\,a^2+\sqrt{\frac{-4ac+b^2}{a^2}}\,ba-4ac+b^2\right)^{\frac{a\left(a-\frac{d}{2}\right)\sqrt{\frac{-4ac+b^2}{a^2}}+af-\frac{bd}{2}}{\sqrt{\frac{-4ac+b^2}{a^2}}\,a^2}} \operatorname{hypergeom}\left(\left[\frac{a\left(a-\sqrt{a^2}\right.}{}\right.\right.$$

✓ Solution by Mathematica

Time used: 4.122 (sec). Leaf size: 498

```
DSolve[g*y[x] + (f + d*x)*y'[x] + (c + b*x + a*x^2)*y''[x] == 0,y[x],x,IncludeSingularSoluti
```

$y(x)$

$$\rightarrow c_1 \text{Hypergeometric2F1}\left(-\frac{a-d+\sqrt{(a-d)^2-4ag}}{2a}, \frac{-a+d+\sqrt{(a-d)^2-4ag}}{2a}, \frac{\left(b+\sqrt{b^2-4ac}\right)d-}{2a\sqrt{b^2-4ac}}\right.$$

$$-c_2 2^{\frac{\frac{bd}{\sqrt{b^2-4ac}}+d}{2a}-\frac{f}{\sqrt{b^2-4ac}}-1} \exp\left(-\frac{i\pi\left(d\left(\sqrt{b^2-4ac}+b\right)-2af\right)}{2a\sqrt{b^2-4ac}}\right)\left(\frac{\sqrt{b^2-4ac}+2ax+b}{\sqrt{b^2-4ac}}\right)^{-\frac{\frac{bd}{\sqrt{b^2-4ac}}+d}{2a}+\frac{}{\sqrt{}}}$$

$$\left.-\frac{\frac{bd}{\sqrt{b^2-4ac}}+d+a\left(-\frac{2f}{\sqrt{b^2-4ac}}-4\right)}{2a}, \frac{b+2ax+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)$$

3.298 problem 1304

Internal problem ID [9632]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1304.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^3y'' + y'x - (2x+3)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 38

```
dsolve(x^3*diff(diff(y(x),x),x)+x*diff(y(x),x)-(2*x+3)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\text{expIntegral}_1\left(\frac{1}{x}\right) e^{\frac{1}{x}} c_2 + e^{\frac{1}{x}} c_1 - 2x\left(x^2 - \frac{1}{2}x + \frac{1}{2}\right) c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.095 (sec). Leaf size: 50

```
DSolve[(-3 - 2*x)*y[x] + x*y'[x] + x^3*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 e^{\frac{1}{x}} \text{ExpIntegralEi}\left(-\frac{1}{x}\right) + c_2 x(2x^2 - x + 1) + 6c_1 e^{\frac{1}{x}}}{6x}$$

3.299 problem 1305

Internal problem ID [9633]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1305.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^3y'' + 2y'x - y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 46

```
dsolve(x^3*diff(diff(y(x),x),x)+2*x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{\frac{1}{x}} \left(\text{BesselK}\left(0, -\frac{1}{x}\right) c_2 - \text{BesselK}\left(1, -\frac{1}{x}\right) c_2 + c_1 \left(\text{BesselI}\left(0, -\frac{1}{x}\right) + \text{BesselI}\left(1, -\frac{1}{x}\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.219 (sec). Leaf size: 47

```
DSolve[-y[x] + 2*x*y'[x] + x^3*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 G_{1,2}^{2,0}\left(-\frac{2}{x}| \begin{array}{c} \frac{1}{2} \\ -1, 0 \end{array}\right) + c_1 e^{\frac{1}{x}} \left(\text{BesselI}\left(0, \frac{1}{x}\right) - \text{BesselI}\left(1, \frac{1}{x}\right)\right)$$

3.300 problem 1306

Internal problem ID [9634]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1306.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^3y'' + x^2y' + (a\,x^2 + bx + a)\,y = 0$$

✓ Solution by Maple

Time used: 0.469 (sec). Leaf size: 69

```
dsolve(x^3*diff(diff(y(x),x),x)+x^2*diff(y(x),x)+(a*x^2+b*x+a)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \operatorname{HeunD}\left(0, 8a + 4b, 0, 8a - 4b, \frac{x+1}{x-1}\right)\left(c_1 + c_2\left(\int \frac{1}{x\,\operatorname{HeunD}\left(0, 8a + 4b, 0, 8a - 4b, \frac{x+1}{x-1}\right)^2}dx\right)\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a + b*x + a*x^2)*y[x] + x^2*y'[x] + x^3*y''[x] == 0,y[x],x,IncludeSingularSolutions
```

Not solved

3.301 problem 1307

Internal problem ID [9635]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1307.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^3y'' + x(x+1)\,y' - 2y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(x^3*diff(diff(y(x),x),x)+x*(x+1)*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2 \mathrm{e}^{\frac{1}{x}}(x+1)\,\mathrm{expIntegral}_1\left(\frac{1}{x}\right) + c_1 \mathrm{e}^{\frac{1}{x}}(x+1) - x c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.249 (sec). Leaf size: 44

```
DSolve[-2*y[x] + x*(1 + x)*y'[x] + x^3*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-c_2 e^{\frac{1}{x}}(x+1)\,\mathrm{ExpIntegralEi}\left(-\frac{1}{x}\right) + c_1 e^{\frac{1}{x}}(x+1) - c_2 x}{x}$$

3.302 problem 1308

Internal problem ID [9636]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1308.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^3y'' - x^2y' + xy = \ln(x)^3$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 40

```
dsolve(x^3*diff(diff(y(x),x),x)-x^2*diff(y(x),x)+x*y(x)-ln(x)^3=0,y(x), singsol=all)
```

$$y(x) = \frac{2\ln(x)^3 + 6\ln(x)^2 + (8c_1x^2 + 9)\ln(x) + 8c_2x^2 + 6}{8x}$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 41

```
DSolve[-Log[x]^3 + x*y[x] - x^2*y'[x] + x^3*y''[x] == 0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{2\log^3(x) + 6\log^2(x) + 9\log(x) + 6}{8x} + c_1x + c_2x\log(x)$$

3.303 problem 1309

Internal problem ID [9637]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1309.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^3y'' - (x^2-1)\,y' + xy = 0$$

✓ Solution by Maple
Time used: 0.11 (sec). Leaf size: 65

```
dsolve(x^3*diff(diff(y(x),x),x)-(x^2-1)*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{\frac{1}{4x^2}}\left(c_1(2x^2-1)\operatorname{BesselI}\left(0, \frac{1}{4x^2}\right) + (2x^2-1)\,c_2\operatorname{BesselK}\left(0, -\frac{1}{4x^2}\right) + \operatorname{BesselI}\left(1, \frac{1}{4x^2}\right)c_1 + \operatorname{BesselK}\left(1, -\right.\right.}{x}$$

✓ Solution by Mathematica
Time used: 0.24 (sec). Leaf size: 77

```
DSolve[x*y[x] - (-1 + x^2)*y'[x] + x^3*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 G_{1,2}^{2,0}\left(-\frac{1}{2x^2}\middle|\begin{array}{c}1\\ -\frac{1}{2},-\frac{1}{2}\end{array}\right) + \frac{c_1 e^{\frac{1}{4x^2}}\left((2x^2-1)\operatorname{BesselI}\left(0,\frac{1}{4x^2}\right) + \operatorname{BesselI}\left(1,\frac{1}{4x^2}\right)\right)}{\sqrt{2}x}$$

3.304 problem 1310

Internal problem ID [9638]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1310.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^3y'' + 3x^2y' + xy = 1}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve(x^3*diff(diff(y(x),x),x)+3*x^2*diff(y(x),x)+x*y(x)-1=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \ln(x) + \frac{\ln(x)^2}{2} + c_2}{x}$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 27

```
DSolve[-1 + x*y[x] + 3*x^2*y'[x] + x^3*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\log^2(x) + 2c_2 \log(x) + 2c_1}{2x}$$

3.305 problem 1311

Internal problem ID [9639]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1311.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(x^2+1)\,y''+(2x^2+1)\,y'-v(v+1)\,xy=0$$

✓ Solution by Maple
Time used: 0.312 (sec). Leaf size: 52

```
dsolve(x*(x^2+1)*diff(diff(y(x),x),x)+(2*x^2+1)*diff(y(x),x)-v*(v+1)*x*y(x)=0,y(x), singsol=
```

$$y(x)=c_1\,\mathrm{hypergeom}\left(\left[-\frac{v}{2},\frac{1}{2}+\frac{v}{2}\right],\left[\frac{1}{2}\right],x^2+1\right)+c_2\sqrt{x^2+1}\,\mathrm{hypergeom}\left(\left[1+\frac{v}{2},\frac{1}{2}-\frac{v}{2}\right],\left[\frac{3}{2}\right],x^2+1\right)$$

✓ Solution by Mathematica
Time used: 0.59 (sec). Leaf size: 61

```
DSolve[-(v*(1 + v)*x*y[x]) + (1 + 2*x^2)*y'[x] + x*(1 + x^2)*y''[x] == 0,y[x],x,IncludeSingu
```

$$y(x)\to c_2G_{2,2}^{2,0}\left(-x^2\middle|\begin{array}{c}\frac{1-v}{2},\frac{v+2}{2}\\0,0\end{array}\right)+c_1\,\mathrm{Hypergeometric2F1}\left(-\frac{v}{2},\frac{v+1}{2},1,-x^2\right)$$

3.306 problem 1312

Internal problem ID [9640]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1312.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x(x^2+1)\,y''+2(x^2-1)\,y'-2xy=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(x*(x^2+1)*diff(diff(y(x),x),x)+2*(x^2-1)*diff(y(x),x)-2*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_2x^3+c_1}{x^2+1}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 26

```
DSolve[-2*x*y[x] + 2*(-1 + x^2)*y'[x] + x*(1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSoluti
```

$$y(x)\to\frac{c_2x^3+3c_1}{3x^2+3}$$

3.307 problem 1313

Internal problem ID [9641]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1313.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(x^2+1)\,y''+\left(2(n+1)\,x^2+2n+1\right)y'-(v-n)\,(v+n+1)\,xy=0$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 35

```
dsolve(x*(x^2+1)*diff(diff(y(x),x),x)+(2*(n+1)*x^2+2*n+1)*diff(y(x),x)-(v-n)*(v+n+1)*x*y(x)=
```

$$y(x)=x^{-n}\left(\text{LegendreQ}\left(v,n,\sqrt{x^2+1}\right)c_2+\text{LegendreP}\left(v,n,\sqrt{x^2+1}\right)c_1\right)$$

✓ Solution by Mathematica

Time used: 0.256 (sec). Leaf size: 75

```
DSolve[(n - v)*(1 + n + v)*x*y[x] + (1 + 2*n + 2*(1 + n)*x^2)*y'[x] + x*(1 + x^2)*y''[x] ==
```

$$y(x)\to c_1\,\text{Hypergeometric2F1}\left(\frac{n-v}{2},\frac{1}{2}(n+v+1),n+1,-x^2\right)+c_2x^{-2n}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-n-v),\frac{1}{2}(-n+v+1),1-n,-x^2\right)$$

3.308 problem 1314

Internal problem ID [9642]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1314.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(x^2+1)\,y'' - \left(2(n-1)\,x^2+2n-1\right)y' + (v+n)\,(n-1-v)\,xy = 0$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 33

```
dsolve(x*(x^2+1)*diff(diff(y(x),x),x)-(2*(n-1)*x^2+2*n-1)*diff(y(x),x)+(v+n)*(-v+n-1)*x*y(x)
```

$$y(x) = x^n\left(\text{LegendreQ}\left(v,n,\sqrt{x^2+1}\right)c_2 + \text{LegendreP}\left(v,n,\sqrt{x^2+1}\right)c_1\right)$$

✓ Solution by Mathematica

Time used: 0.278 (sec). Leaf size: 75

```
DSolve[(-1 + n - v)*(n + v)*x*y[x] - (-1 + 2*n + 2*(-1 + n)*x^2)*y'[x] + x*(1 + x^2)*y''[x]
```

$$y(x) \to c_1\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-n-v),\frac{1}{2}(-n+v+1),1-n,-x^2\right) + c_2x^{2n}\,\text{Hypergeometric2F1}\left(\frac{n-v}{2},\frac{1}{2}(n+v+1),n+1,-x^2\right)$$

3.309 problem 1315

Internal problem ID [9643]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1315.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$x(x^2-1)\,y''+y'+ya\,x^3=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(x*(x^2-1)*diff(diff(y(x),x),x)+diff(y(x),x)+y(x)*a*x^3=0,y(x), singsol=all)
```

$$y(x)=c_1\sin\left(\sqrt{a}\,\sqrt{x^2-1}\right)+c_2\cos\left(\sqrt{a}\,\sqrt{x^2-1}\right)$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 44

```
DSolve[a*x^3*y[x] + y'[x] + x*(-1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x)\to c_1\cos\left(\sqrt{a}\sqrt{x^2-1}\right)+c_2\sin\left(\sqrt{a}\sqrt{x^2-1}\right)$$

3.310 problem 1316

Internal problem ID [9644]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1316.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_elliptic, _class_II]]`

$$\boxed{x(x^2-1)\,y''+(x^2-1)\,y'-xy=0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 18

```
dsolve(x*(x^2-1)*diff(diff(y(x),x),x)+(x^2-1)*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\mathrm{EllipticE}(x)+c_2(\mathrm{EllipticCE}(x)-\mathrm{EllipticCK}(x))$$

✓ Solution by Mathematica
Time used: 0.508 (sec). Leaf size: 38

```
DSolve[-(x*y[x]) + (-1 + x^2)*y'[x] + x*(-1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolutio
```

$$y(x)\to c_2G_{2,2}^{2,0}\left(x^2\middle|\begin{array}{c}\frac{1}{2},\frac{3}{2}\\0,0\end{array}\right)+\frac{2c_1\,\mathrm{EllipticE}(x^2)}{\pi}$$

3.311 problem 1317

Internal problem ID [9645]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1317.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_elliptic, _class_I]]`

$$\boxed{x(x^2-1)\,y''+(3x^2-1)\,y'+xy=0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 13

```
dsolve(x*(x^2-1)*diff(diff(y(x),x),x)+(3*x^2-1)*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\mathrm{EllipticK}\,(x)+c_2\,\mathrm{EllipticCK}\,(x)$$

✓ Solution by Mathematica

Time used: 0.319 (sec). Leaf size: 38

```
DSolve[x*y[x] + (-1 + 3*x^2)*y'[x] + x*(-1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolution
```

$$y(x)\to c_2G_{2,2}^{2,0}\left(x^2\middle|\begin{array}{c}\frac{1}{2},\frac{1}{2}\\0,0\end{array}\right)+\frac{2c_1\,\mathrm{EllipticK}\,(x^2)}{\pi}$$

3.312 problem 1318

Internal problem ID [9646]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1318.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(x^2-1)\,y''+(a\,x^2+b)\,y'+cxy=0$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 122

```
dsolve(x*(x^2-1)*diff(diff(y(x),x),x)+(a*x^2+b)*diff(y(x),x)+c*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{hypergeom}\left(\left[-\frac{1}{4}+\frac{a}{4}+\frac{\sqrt{a^2-2a-4c+1}}{4}, -\frac{1}{4}+\frac{a}{4} - \frac{\sqrt{a^2-2a-4c+1}}{4}\right], \left[\frac{1}{2}-\frac{b}{2}\right], x^2\right) + c_2 x^{b+1} \operatorname{hypergeom}\left(\left[\frac{1}{4}+\frac{a}{4}+\frac{b}{2} + \frac{\sqrt{a^2-2a-4c+1}}{4}, \frac{1}{4}+\frac{a}{4}+\frac{b}{2}-\frac{\sqrt{a^2-2a-4c+1}}{4}\right], \left[\frac{3}{2}+\frac{b}{2}\right], x^2\right)$$

✓ Solution by Mathematica

Time used: 0.349 (sec). Leaf size: 146

```
DSolve[c*x*y[x] + (b + a*x^2)*y'[x] + x*(-1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to c_1 \operatorname{Hypergeometric2F1}\left(\frac{1}{4}\left(a-\sqrt{a^2-2a-4c+1}-1\right), \frac{1}{4}\left(a + \sqrt{a^2-2a-4c+1}-1\right), \frac{1-b}{2}, x^2\right) + i^{b+1} c_2 x^{b+1} \operatorname{Hypergeometric2F1}\left(\frac{1}{4}\left(a + 2b-\sqrt{a^2-2a-4c+1}+1\right), \frac{1}{4}\left(a+2b+\sqrt{a^2-2a-4c+1}+1\right), \frac{b+3}{2}, x^2\right)$$

3.313 problem 1319

Internal problem ID [9647]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1319.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x(x^2+2)\,y''-y'-6xy=0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 31

```
dsolve(x*(x^2+2)*diff(diff(y(x),x),x)-diff(y(x),x)-6*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=(x^2+2)^{\frac{3}{4}}\left(c_1x^{\frac{3}{2}}+\text{hypergeom}\left(\left[-\frac{3}{4},\frac{7}{4}\right],\left[\frac{1}{4}\right],-\frac{x^2}{2}\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 20.104 (sec). Leaf size: 54

```
DSolve[-6*x*y[x] - y'[x] + x*(2 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{6}(x^2+2)^{3/4}\left(6c_1x^{3/2}-\sqrt[4]{2}c_2\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{7}{4},\frac{1}{4},-\frac{x^2}{2}\right)\right)$$

3.314 problem 1320

Internal problem ID [9648]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1320.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(x^2-2)\,y''-(x^3+3x^2-2x-2)\,y'+(x^2+4x+2)\,y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(x*(x^2-2)*diff(diff(y(x),x),x)-(x^3+3*x^2-2*x-2)*diff(y(x),x)+(x^2+4*x+2)*y(x)=0,y(x)
```

$$y(x)=c_1(x-1)+c_2\mathrm{e}^x x^2$$

✓ Solution by Mathematica

Time used: 0.09 (sec). Leaf size: 21

```
DSolve[(2 + 4*x + x^2)*y[x] - (-2 - 2*x + 3*x^2 + x^3)*y'[x] + x*(-2 + x^2)*y''[x] == 0,y[x]
```

$$y(x)\to c_1e^x x^2+c_2(x-1)$$

3.315 problem 1321

Internal problem ID [9649]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1321.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x+1)\,y'' - x(2x+1)\,y' + (2x+1)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*(x+1)*diff(diff(y(x),x),x)-x*(2*x+1)*diff(y(x),x)+(2*x+1)*y(x)=0,y(x), singsol=al
```

$$y(x) = x(\ln(x)\,c_2 + xc_2 + c_1)$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 17

```
DSolve[(1 + 2*x)*y[x] - x*(1 + 2*x)*y'[x] + x^2*(1 + x)*y''[x] == 0,y[x],x,IncludeSingularSo
```

$$y(x) \to x(c_2(x + \log(x)) + c_1)$$

3.316 problem 1322

Internal problem ID [9650]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1322.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x^2(x+1)\,y''+2x(3x+2)\,y'=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(x^2*(x+1)*diff(diff(y(x),x),x)+2*x*(3*x+2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=c_1+\left(-4\ln(x)+4\ln(x+1)-\frac{12x^3+6x^2-2x+1}{3(x+1)\,x^3}\right)c_2$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 44

```
DSolve[2*x*(2 + 3*x)*y'[x] + x^2*(1 + x)*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x)\to c_1\left(-\frac{1}{3x^3}+\frac{1}{x^2}-\frac{3}{x}-\frac{1}{x+1}-4\log(x)+4\log(x+1)\right)+c_2$$

3.317 problem 1323

Internal problem ID [9651]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1323.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{y'' + \frac{2(x-2)\,y'}{x\,(x-1)} - \frac{2(x+1)\,y}{x^2\,(x-1)} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2) = -2/x*(x-2)/(x-1)*diff(y(x),x)+2/x^2*(x+1)/(x-1)*y(x),y(x), singsol=a
```

$$y(x) = \frac{c_1 + c_2(x-1)^3}{x^2}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 29

```
DSolve[y''[x] == -2/x*(x-2)/(x-1)*y'[x]+2/x^2*(x+1)/(x-1)*y[x],y[x],x,IncludeSingularSolutio
```

$$y(x) \to \frac{c_2 x(x^2 - 3x + 3) + 3c_1}{3x^2}$$

3.318 problem 1324

Internal problem ID [9652]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1324.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{(5x-4)\,y'}{x\,(x-1)} + \frac{(9x-6)\,y}{x^2\,(x-1)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(diff(y(x),x),x) = 1/x*(5*x-4)/(x-1)*diff(y(x),x)-(9*x-6)/x^2/(x-1)*y(x),y(x), si
```

$$y(x) = x^2(c_2 x \ln(x) + c_1 x + c_2)$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 24

```
DSolve[y''[x] == -(((-6 + 9*x)*y[x])/((-1 + x)*x^2)) + ((-4 + 5*x)*y'[x])/((-1 + x)*x),y[x],
```

$$y(x) \to x^2(c_1 x - c_2(x \log(x) + 1))$$

3.319 problem 1325

Internal problem ID [9653]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1325.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{((a+1+b)\,x+\alpha+\beta-1)\,y'}{x\,(x-1)} + \frac{(abx-\alpha\beta)\,y}{x^2\,(x-1)} = 0$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 86

```
dsolve(diff(diff(y(x),x),x) = -((a+b+1)*x+alpha+beta-1)/x/(x-1)*diff(y(x),x)-(a*b*x-alpha*be
```

$$y(x) = \left(\text{hypergeom}\left([1-\alpha-b, 1-a-\alpha], [1-\alpha+\beta], x\right) x^{\beta} c_2 + \text{hypergeom}\left([1-b-\beta, 1-a-\beta], [1+\alpha-\beta], x\right) x^{\alpha} c_1\right) (x-1)^{1-a-\alpha-b-\beta}$$

✓ Solution by Mathematica

Time used: 0.26 (sec). Leaf size: 52

```
DSolve[y''[x] == -(((-(\[Alpha]*\[Beta]) + a*b*x)*y[x])/((-1 + x)*x^2)) - ((-1 + \[Alpha] +
```

$$y(x) \to (-1)^{\beta} c_2 x^{\beta}\,\text{Hypergeometric2F1}(a+\beta, b+\beta, -\alpha+\beta+1, x) + (-1)^{\alpha} c_1 x^{\alpha}\,\text{Hypergeometric2F1}(a+\alpha, b+\alpha, \alpha-\beta+1, x)$$

3.320 problem 1326

Internal problem ID [9654]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1326.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{y'}{x+1} + \frac{y}{x\left(x+1\right)^2} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(diff(y(x),x),x) = -1/(x+1)*diff(y(x),x)-1/x/(x+1)^2*y(x),y(x), singsol=all)
```

$$y(x) = \frac{c_2 x \ln\left(x\right) + c_1 x - c_2}{x+1}$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 26

```
DSolve[y''[x] == -(y[x]/(x*(1 + x)^2)) - y'[x]/(1 + x),y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{c_1 x + c_2 x \log(x) - c_2}{x+1}$$

3.321 problem 1327

Internal problem ID [9655]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1327.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{2y'}{x(x-2)} + \frac{y}{x^2(x-2)} = 0$$

✓ Solution by Maple

Time used: 0.234 (sec). Leaf size: 122

```
dsolve(diff(diff(y(x),x),x) = 2/x/(x-2)*diff(y(x),x)-1/x^2/(x-2)*y(x),y(x), singsol=all)
```

$$y(x) = 4\left(x^{-\frac{\sqrt{2}}{2}} - x^{1-\frac{\sqrt{2}}{2}} + \frac{x^{2-\frac{\sqrt{2}}{2}}}{4}\right) c_1 \operatorname{hypergeom}\left(\left[2-\frac{\sqrt{2}}{2}, 1-\frac{\sqrt{2}}{2}\right], \left[1-\sqrt{2}\right], \frac{x}{2}\right) + \operatorname{hypergeom}\left(\left[2+\frac{\sqrt{2}}{2}, 1+\frac{\sqrt{2}}{2}\right], \left[1+\sqrt{2}\right], \frac{x}{2}\right) c_2\left(x^{2+\frac{\sqrt{2}}{2}} + 4x^{\frac{\sqrt{2}}{2}} - 4x^{1+\frac{\sqrt{2}}{2}}\right)$$

✓ Solution by Mathematica

Time used: 0.226 (sec). Leaf size: 105

```
DSolve[y''[x] == -(y[x]/((-2 + x)*x^2)) + (2*y'[x])/((-2 + x)*x),y[x],x,IncludeSingularSolut
```

$$y(x) \to \left(-\frac{1}{2}\right)^{-\frac{1}{\sqrt{2}}} x^{-\frac{1}{\sqrt{2}}} \left(\left(-\frac{1}{2}\right)^{\sqrt{2}} c_2 x^{\sqrt{2}} \text{Hypergeometric2F1}\left(\frac{1}{\sqrt{2}}, -1+\frac{1}{\sqrt{2}}, 1+\sqrt{2}, \frac{x}{2}\right) + c_1 \text{Hypergeometric2F1}\left(-\frac{1}{\sqrt{2}}, -1-\frac{1}{\sqrt{2}}, 1-\sqrt{2}, \frac{x}{2}\right)\right)$$

3.322 problem 1328

Internal problem ID [9656]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1328.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \frac{2y}{x(x-1)^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(diff(y(x),x),x) = 2/x/(x-1)^2*y(x),y(x), singsol=all)
```

$$y(x) = \frac{2c_2x\ln(x) - c_2x^2 + c_1x + c_2}{x-1}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 33

```
DSolve[y''[x] == (2*y[x])/((-1 + x)^2*x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-c_2x^2 - c_1x + 2c_2x\log(x) + c_2}{x-1}$$

3.323 problem 1329

Internal problem ID [9657]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1329.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{((\beta+\alpha+1)\,x^2 - (\alpha+\beta+1+a(\gamma+\delta)-\delta)\,x + a\gamma)\,y'}{x\,(x-1)\,(x-a)} + \frac{(\alpha\beta x - q)\,y}{x\,(x-1)\,(x-a)} = 0$$

✓ Solution by Maple

Time used: 0.765 (sec). Leaf size: 64

```
dsolve(diff(diff(y(x),x),x) = -((alpha+beta+1)*x^2-(alpha+beta+1+a*(gamma+delta)-delta)*x+a*
```

$$y(x) = c_1\,\mathrm{HeunG}\,(a,q,\alpha,\beta,\gamma,\delta,x) + c_2x^{1-\gamma}\,\mathrm{HeunG}\,(a,q - (-1+\gamma)\,(\delta(a-1)+\alpha+\beta-\gamma+1)\,,\beta+1-\gamma,\alpha+1-\gamma,-\gamma+2,\delta,x)$$

✓ Solution by Mathematica

Time used: 0.927 (sec). Leaf size: 67

```
DSolve[y''[x] == -(((-q + \[Alpha]*\[Beta]*x)*y[x])/((-1 + x)*x*(-a + x))) - ((a*\[Gamma] -
```

$$y(x) \to c_2x^{1-\gamma}\mathrm{HeunG}[a,q-(\gamma-1)((a-1)\delta+\alpha+\beta-\gamma+1),\alpha-\gamma+1,\beta-\gamma+1,2-\gamma,\delta,x] + c_1\mathrm{HeunG}[a,q,\alpha,\beta,\gamma,\delta,x]$$

3.324 problem 1330

Internal problem ID [9658]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1330.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(A\,x^2 + Bx + C)\,y'}{(x-a)\,(x-b)\,(x-c)} + \frac{(\mathrm{DD}\,x + E)\,y}{(x-a)\,(x-b)\,(x-c)} = 0$$

✓ Solution by Maple

Time used: 7.828 (sec). Leaf size: 1147

```
dsolve(diff(diff(y(x),x),x) = -(A*x^2+B*x+C)/(x-a)/(x-b)/(x-c)*diff(y(x),x)-(DD*x+E)/(x-a)/(
```

Expression too large to display

✓ Solution by Mathematica

Time used: 10.917 (sec). Leaf size: 1166

```
DSolve[y''[x] == -(((E + DD*x)*y[x])/((-a + x)*(-b + x)*(-c + x))) - ((C + B*x + A*x^2)*y'[x
```

$$y(x) \to (x-a)^{-\frac{C}{(a-b)(a-c)}} \left(c_2 \mathrm{HeunG}\left[\frac{a-c}{a-b}, \frac{A^2ba^4 + B^2a^3 + A(b^2-ab+(a+b)B+2C)\,a^3 + (a-b)^2(a\mathrm{DD}+e)}{}\right.\right.$$
$$\left. -\frac{(A-2)a^2+(2b+B+2c)a-2bc+C}{(a-b)(a-c)}, -\frac{Ab^2+Bb+C}{(a-b)(b-c)}, \frac{a-x}{a-b}\right] (x-a)^{-\frac{(A-1)a^2+(b+B+c)a-bc}{(a-b)(a-c)}}$$
$$+c_1 \mathrm{HeunG}\left[\frac{a-c}{a-b}, \frac{a\mathrm{DD}+e}{a-b}, \frac{1}{2}\left(A+\sqrt{A^2-2A-4\mathrm{DD}+1}-1\right), \frac{4\mathrm{DD}c^2 - Bc + b(A^2-A-4\mathrm{DD})\,c -}{2\,(Ac^2 -}\right.$$
$$\left.\left. -\frac{Ab^2+Bb+C}{(a-b)(b-c)}, \frac{a-x}{a-b}\right] (x-a)^{\frac{C}{(a-b)(a-c)}} \right)$$

3.325 problem 1331

Internal problem ID [9659]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1331.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \frac{(-4+x)\,y'}{2x\,(x-2)} + \frac{(x-3)\,y}{2x^2\,(x-2)} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(diff(y(x),x),x) = 1/2/x*(x-4)/(x-2)*diff(y(x),x)-1/2*(x-3)/x^2/(x-2)*y(x),y(x),
```

$$y(x) = c_1\sqrt{x} + c_2\sqrt{x\,(x-2)}$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 41

```
DSolve[y''[x] == -1/2*((-3 + x)*y[x])/((-2 + x)*x^2) + ((-4 + x)*y'[x])/(2*(-2 + x)*x),y[x],
```

$$y(x) \to \frac{\sqrt[4]{x-2}\sqrt{x}\left(2c_2\sqrt{x-2} + c_1\right)}{\sqrt[4]{2-x}}$$

3.326 problem 1332

Internal problem ID [9660]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1332.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \frac{y'}{x+1} + \frac{(3x+1)\,y}{4x^2\,(x+1)} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(diff(y(x),x),x) = 1/(x+1)*diff(y(x),x)-1/4*(3*x+1)/x^2/(x+1)*y(x),y(x), singsol=
```

$$y(x) = (c_1 + c_2(\ln(x) + x))\sqrt{x}$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 21

```
DSolve[y''[x] == -1/4*((1 + 3*x)*y[x])/(x^2*(1 + x)) + y'[x]/(1 + x),y[x],x,IncludeSingularS
```

$$y(x) \to \sqrt{x}(c_2(x + \log(x)) + c_1)$$

3.327 problem 1333

Internal problem ID [9661]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1333.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(-1+3x)\,y'}{2x\,(x-1)} - \frac{v(v+1)\,y}{4x^2} = 0$$

✓ Solution by Maple

Time used: 0.234 (sec). Leaf size: 74

```
dsolve(diff(diff(y(x),x),x) = -1/2/x*(3*x-1)/(x-1)*diff(y(x),x)+1/4*v*(v+1)/x^2*y(x),y(x), s
```

$$y(x) = \frac{\left(\Gamma\left(v+\frac{1}{2}\right)^2 c_2\left(v+\frac{1}{2}\right) \mathrm{LegendreP}\left(-\frac{1}{2}, -v-\frac{1}{2}, \frac{-x-1}{x-1}\right) + \sec(\pi v)\, \mathrm{LegendreP}\left(-\frac{1}{2}, v+\frac{1}{2}, \frac{-x-1}{x-1}\right) \pi c_1\right) x^{\frac{1}{4}}}{\sqrt{1-x}\,\Gamma\left(v+\frac{1}{2}\right)}$$

✓ Solution by Mathematica

Time used: 0.141 (sec). Leaf size: 70

```
DSolve[y''[x] == (v*(1 + v)*y[x])/(4*x^2) - ((-1 + 3*x)*y'[x])/(2*(-1 + x)*x),y[x],x,Include
```

$$y(x) \to c_1 i^{-v} x^{-v/2}\, \mathrm{Hypergeometric2F1}\left(\frac{1}{2}, -v, \frac{1}{2}-v, x\right) + c_2 i^{v+1} x^{\frac{v+1}{2}}\, \mathrm{Hypergeometric2F1}\left(\frac{1}{2}, v+1, v+\frac{3}{2}, x\right)$$

3.328 problem 1334

Internal problem ID [9662]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1334.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{((1+a)\,x-1)\,y'}{x\,(x-1)} + \frac{((a^2-b^2)\,x+c^2)\,y}{4x^2\,(x-1)} = 0$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 89

```
dsolve(diff(diff(y(x),x),x) = -((a+1)*x-1)/x/(x-1)*diff(y(x),x)-1/4*((a^2-b^2)*x+c^2)/x^2/(x
```

$$y(x) = \left(\mathrm{hypergeom}\left(\left[-\frac{a}{2}-\frac{b}{2}+\frac{c}{2}+1, -\frac{a}{2}+\frac{b}{2}+\frac{c}{2}+1\right], [c+1], x\right) x^{\frac{c}{2}} c_1 + \mathrm{hypergeom}\left(\left[-\frac{a}{2}-\frac{b}{2}-\frac{c}{2}+1, -\frac{a}{2}+\frac{b}{2}-\frac{c}{2}+1\right], [-c+1], x\right) x^{-\frac{c}{2}} c_2\right) (x-1)^{-a+1}$$

✓ Solution by Mathematica

Time used: 0.223 (sec). Leaf size: 89

```
DSolve[y''[x] == -1/4*((c^2 + (a^2 - b^2)*x)*y[x])/((-1 + x)*x^2) - ((-1 + (1 + a)*x)*y'[x])
```

$$y(x) \to i^{-c} x^{-c/2} \left(i^{2c} c_2 x^c \,\mathrm{Hypergeometric2F1}\left(\frac{1}{2}(a-b+c), \frac{1}{2}(a+b+c), c+1, x\right) + c_1 \,\mathrm{Hypergeometric2F1}\left(\frac{1}{2}(a-b-c), \frac{1}{2}(a+b-c), 1-c, x\right)\right)$$

3.329 problem 1335

Internal problem ID [9663]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1335.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(-1+3x)\,y'}{2x\,(x-1)} + \frac{(xa+b)\,y}{4x\,(x-1)^2} = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 57

```
dsolve(diff(diff(y(x),x),x) = -1/2/x*(3*x-1)/(x-1)*diff(y(x),x)-1/4*(a*x+b)/x/(x-1)^2*y(x),y
```

$$y(x) = c_1 \operatorname{LegendreP}\left(\frac{\sqrt{-4a+1}}{2} - \frac{1}{2}, \sqrt{-a-b}, \sqrt{x}\right) + c_2 \operatorname{LegendreQ}\left(\frac{\sqrt{-4a+1}}{2} - \frac{1}{2}, \sqrt{-a-b}, \sqrt{x}\right)$$

✓ Solution by Mathematica

Time used: 0.834 (sec). Leaf size: 510

```
DSolve[y''[x] == -1/4*((b + a*x)*y[x])/((-1 + x)^2*x) - ((-1 + 3*x)*y'[x])/(2*(-1 + x)*x),y[
```

$$y(x) \rightarrow \frac{(x-1)^{\frac{2a\sqrt{-4\sqrt{(4a-1)(a+b)}-8a-4b+1}+2b\left(\sqrt{-4\sqrt{(4a-1)(a+b)}-8a-4b+1}+2\right)-\sqrt{(4a-1)(a+b)}\sqrt{-4\sqrt{(4a-1)(a+b)}-8a-4b+1}+1}{8b+2}} \left(c_1 \text{Hypergeo}\right.}{}$$

3.330 problem 1336

Internal problem ID [9664]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1336.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(-3x+1)\, y}{(x-1)\,(2x-1)^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(diff(diff(y(x),x),x) = -(-3*x+1)/(x-1)/(2*x-1)^2*y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{-1+2x}\,(2(x-1)\,c_2 \ln(-1+2x) - 2(x-1)\,c_2 \ln(x-1) + c_1 x - c_1 - c_2)$$

✓ Solution by Mathematica

Time used: 0.081 (sec). Leaf size: 51

```
DSolve[y''[x] == -(((1 - 3*x)*y[x])/((-1 + x)*(-1 + 2*x)^2)),y[x],x,IncludeSingularSolutions
```

$$y(x) \to -\sqrt{1-2x}(c_1 x + 2c_2(x-1)\log(x-1) - 2c_2(x-1)\log(2x-1) - c_1 + c_2)$$

3.331 problem 1337

Internal problem ID [9665]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1337.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$y'' + \frac{(3x + a + 2b)\, y'}{2\,(x + a)\,(x + b)} + \frac{(a - b)\, y}{4\,(x + a)^2\,(x + b)} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(diff(y(x),x),x) = -1/2/(x+a)*(3*x+a+2*b)/(x+b)*diff(y(x),x)-1/4*(a-b)/(x+a)^2/(x
```

$$y(x) = \frac{\sqrt{x + b}\, c_1 + c_2}{\sqrt{\frac{a+x}{a-b}}}$$

✓ Solution by Mathematica

Time used: 0.134 (sec). Leaf size: 53

```
DSolve[y''[x] == -1/4*((a - b)*y[x])/((a + x)^2*(b + x)) - ((a + 2*b + 3*x)*y'[x])/(2*(a + x
```

$$y(x) \to \frac{c_1\sqrt{a - b} + c_2\sqrt{b + x}}{\sqrt{a - b}\sqrt{\frac{a+x}{a-b}}}$$

3.332 problem 1338

Internal problem ID [9666]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1338.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{(6x-1)\,y'}{3x\,(x-2)} - \frac{y}{3x^2\,(x-2)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(diff(y(x),x),x) = 1/3/x*(6*x-1)/(x-2)*diff(y(x),x)+1/3/x^2/(x-2)*y(x),y(x), sing
```

$$y(x) = c_1 x\left(18x^2 - 102x + 187\right) + c_2 (x-2)^{\frac{17}{6}} x^{\frac{1}{6}}$$

✓ Solution by Mathematica

Time used: 2.412 (sec). Leaf size: 40

```
DSolve[y''[x] == y[x]/(3*(-2 + x)*x^2) + ((-1 + 6*x)*y'[x])/(3*(-2 + x)*x),y[x],x,IncludeSin
```

$$y(x) \to \frac{3}{935} c_2 x\left(18x^2 - 102x + 187\right) + c_1 \sqrt[6]{x}(2-x)^{17/6}$$

3.333 problem 1339

Internal problem ID [9667]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1339.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(a(2+b)\,x^2 + (c-d+1)\,x)\,y'}{(xa+1)\,x^2} + \frac{(abx-cd)\,y}{(xa+1)\,x^2} = 0$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 76

```
dsolve(diff(diff(y(x),x),x) = -(a*(b+2)*x^2+(c-d+1)*x)/(a*x+1)/x^2*diff(y(x),x)-(a*b*x-c*d)/
```

$$y(x) = \left(x^{-c}\,\text{hypergeom}\left([-d, 1-b-d], [1-d-c], -ax\right)c_2 + x^d\,\text{hypergeom}\left([c, 1-b+c], [1+d+c], -ax\right)c_1\right)(ax+1)^{-b+c-d}$$

✓ Solution by Mathematica

Time used: 0.268 (sec). Leaf size: 66

```
DSolve[y''[x] == -(((-(c*d) + a*b*x)*y[x])/(x^2*(1 + a*x))) - (((1 + c - d)*x + a*(2 + b)*x^
```

$$y(x) \to c_1 a^{-c} x^{-c}\,\text{Hypergeometric2F1}(1-c, b-c, -c-d+1, -ax) + c_2 a^d x^d\,\text{Hypergeometric2F1}(d+1, b+d, c+d+1, -ax)$$

3.334 problem 1340

Internal problem ID [9668]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1340.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \frac{2(xa + 2b)\, y'}{x\,(xa + b)} + \frac{(2xa + 6b)\, y}{(xa + b)\, x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(diff(y(x),x),x) = 2/x*(a*x+2*b)/(a*x+b)*diff(y(x),x)-(2*a*x+6*b)/(a*x+b)/x^2*y(x
```

$$y(x) = \frac{x^2(xc_2 + c_1)}{ax + b}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 23

```
DSolve[y''[x] == -(((6*b + 2*a*x)*y[x])/(x^2*(b + a*x))) + (2*(2*b + a*x)*y'[x])/(x*(b + a*x
```

$$y(x) \to \frac{x^2(c_2x + c_1)}{ax + b}$$

3.335 problem 1341

Internal problem ID [9669]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1341.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + \frac{(2xa+b)\,y'}{x\,(xa+b)} + \frac{(avx-b)\,y}{(xa+b)\,x^2} = Ax}$$

✓ Solution by Maple

Time used: 0.265 (sec). Leaf size: 194

```
dsolve(diff(diff(y(x),x),x) = -1/x*(2*a*x+b)/(a*x+b)*diff(y(x),x)-(a*v*x-b)/(a*x+b)/x^2*y(x)
```

$$y(x) = \frac{x^{-\frac{\sqrt{1-4v}}{2}} a^2 c_1 (v+6)\,(2+v)\,(v+12) \operatorname{hypergeom}\left(\left[-\frac{1}{2}+\frac{\sqrt{1-4v}}{2}, \frac{3}{2}+\frac{\sqrt{1-4v}}{2}\right], \left[1+\sqrt{1-4v}\right], -\frac{b}{ax}\right) - 3b^2}{}$$

✓ Solution by Mathematica

Time used: 71.383 (sec). Leaf size: 725

```
DSolve[y''[x] == A*x - ((-b + a*v*x)*y[x])/(x^2*(b + a*x)) - ((b + 2*a*x)*y'[x])/(x*(b + a*x
```

$y(x)$

$$\rightarrow \frac{ax\,\text{Hypergeometric2F1}\left(\frac{1}{2}\left(3-\sqrt{1-4v}\right),\frac{1}{2}\left(\sqrt{1-4v}+3\right),3,-\frac{ax}{b}\right)\left(\int_1^x -\frac{}{a\left(\left(a(v+2)\,\text{Hypergeometric2F1}\left(\frac{1}{2}\left(5\right.\right.\right.\right.}\right.}{}$$

$$+G_{2,2}^{2,0}\left(-\frac{ax}{b}\middle|\begin{array}{c}\frac{1}{2}\left(1-\sqrt{1-4v}\right),\frac{1}{2}\left(\sqrt{1-4v}+1\right)\\-1,1\end{array}\right)\left(\int_1^x -\frac{}{\left(3b\,\text{Hypergeometric2F1}\left(\frac{1}{2}\left(3-\sqrt{1-4v}\right)\right.\right.}\right.$$

3.336 problem 1342

Internal problem ID [9670]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1342.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + \frac{ay}{x^4} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(diff(y(x),x),x) = -a/x^4*y(x),y(x), singsol=all)
```

$$y(x) = x\left(c_1 \sinh\left(\frac{\sqrt{-a}}{x}\right) + c_2 \cosh\left(\frac{\sqrt{-a}}{x}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.136 (sec). Leaf size: 52

```
DSolve[y''[x] == -((a*y[x])/x^4),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x e^{\frac{i\sqrt{a}}{x}} - \frac{i c_2 x e^{-\frac{i\sqrt{a}}{x}}}{2\sqrt{a}}$$

3.337 problem 1343

Internal problem ID [9671]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1343.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{(x^2 a(1-a) - b(x+b))\, y}{x^4} = 0}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 58

```
dsolve(diff(diff(y(x),x),x) = -(x^2*a*(1-a)-b*(x+b))/x^4*y(x),y(x), singsol=all)
```

$$y(x) = \text{BesselI}\left(a+1, \frac{b}{x}\right) c_1 b - \text{BesselK}\left(a+1, \frac{b}{x}\right) c_2 b + 2\left(ax + \frac{b}{2}\right)\left(\text{BesselI}\left(a, \frac{b}{x}\right) c_1 + c_2 \,\text{BesselK}\left(a, \frac{b}{x}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.275 (sec). Leaf size: 65

```
DSolve[y''[x] == -((((1 - a)*a*x^2 - b*(b + x))*y[x])/x^4),y[x],x,IncludeSingularSolutions -
```

$$y(x) \to c_1(2ax+b)\,\text{BesselI}\left(a, \frac{b}{x}\right) + bc_1 \,\text{BesselI}\left(a+1, \frac{b}{x}\right) + c_2\left((2ax+b)K_a\left(\frac{b}{x}\right) - bK_{a+1}\left(\frac{b}{x}\right)\right)$$

3.338 problem 1344

Internal problem ID [9672]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1344.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{\left(e^{\frac{2}{x}} - v^2\right) y}{x^4} = 0$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 23

```
dsolve(diff(diff(y(x),x),x) = -(exp(2/x)-v^2)/x^4*y(x),y(x), singsol=all)
```

$$y(x) = x\left(c_1 \operatorname{BesselJ}\left(v, e^{\frac{1}{x}}\right) + c_2 \operatorname{BesselY}\left(v, e^{\frac{1}{x}}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.724 (sec). Leaf size: 100

```
DSolve[y''[x] == -(((E^(2/x) - v^2)*y[x])/x^4),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(-1)^{-v} 2^{\frac{3v}{2}+\frac{1}{2}} \left(-e^{2/x}\right)^{-v/2} \left(e^{2/x}\right)^{v/2} \left(c_1 (-1)^v \operatorname{BesselI}\left(v, \sqrt{-e^{2/x}}\right) + c_2 K_v\left(\sqrt{-e^{2/x}}\right)\right)}{\log\left(e^{2/x}\right)}$$

3.339 problem 1345

Internal problem ID [9673]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1345.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{y'}{x^3} - \frac{2y}{x^4} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(diff(y(x),x),x) = -1/x^3*diff(y(x),x)+2/x^4*y(x),y(x), singsol=all)
```

$$y(x) = x\,\mathrm{e}^{\frac{1}{2x^2}}\left(c_1 + c_2\,\mathrm{erf}\left(\frac{\sqrt{2}}{2x}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.095 (sec). Leaf size: 45

```
DSolve[y''[x] == (2*y[x])/x^4 - y'[x]/x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{\frac{1}{2x^2}}x\left(2c_1 - \sqrt{2\pi}c_2\mathrm{erf}\left(\frac{1}{\sqrt{2}x}\right)\right)$$

3.340 problem 1346

Internal problem ID [9674]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1346.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{(a+b)\, y'}{x^2} + \frac{(x(a+b)+ab)\, y}{x^4} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(diff(y(x),x),x) = 1/x^2*(a+b)*diff(y(x),x)-((a+b)*x+a*b)/x^4*y(x),y(x), singsol=
```

$$y(x) = x\left(e^{-\frac{b}{x}} c_2 + e^{-\frac{a}{x}} c_1\right)$$

✓ Solution by Mathematica

Time used: 0.144 (sec). Leaf size: 37

```
DSolve[y''[x] == -(((a*b + (a + b)*x)*y[x])/x^4) + ((a + b)*y'[x])/x^2,y[x],x,IncludeSingula
```

$$y(x) \to \frac{c_2 x e^{-\frac{a}{x}}}{a-b} + c_1 x e^{-\frac{b}{x}}$$

3.341 problem 1347

Internal problem ID [9675]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1347.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + \frac{y'}{x} + \frac{y}{x^4} = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 19

```
dsolve(diff(diff(y(x),x),x) = -1/x*diff(y(x),x)-1/x^4*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}\left(0, \frac{1}{x}\right) + c_2 \operatorname{BesselY}\left(0, \frac{1}{x}\right)$$

✓ Solution by Mathematica
Time used: 0.125 (sec). Leaf size: 31

```
DSolve[y''[x] == -(y[x]/x^4) - y'[x]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \operatorname{BesselJ}\left(0, \frac{1}{x}\right) + \frac{c_1 K_0\left(\frac{i}{x}\right)}{\sqrt{\pi}}$$

3.342 problem 1348

Internal problem ID [9676]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1348.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{y'}{x} + \frac{(b\,x^2 + a(x^4+1))\,y}{x^4} = 0$$

✓ Solution by Maple

Time used: 0.828 (sec). Leaf size: 73

```
dsolve(diff(diff(y(x),x),x) = -1/x*diff(y(x),x)-(b*x^2+a*(x^4+1))/x^4*y(x),y(x), singsol=all
```

$$y(x) = \mathrm{HeunD}\left(0, 2a+b, 0, 2a-b, \frac{x^2+1}{x^2-1}\right)\left(c_1 + c_2\left(\int \frac{1}{x\,\mathrm{HeunD}\left(0, 2a+b, 0, 2a-b, \frac{x^2+1}{x^2-1}\right)^2}dx\right)\right)$$

✓ Solution by Mathematica

Time used: 0.523 (sec). Leaf size: 34

```
DSolve[y''[x] == -(((b*x^2 + a*(1 + x^4))*y[x])/x^4) - y'[x]/x,y[x],x,IncludeSingularSolutio
```

$$y(x) \to c_1\mathrm{MathieuC}[-b, a, i\log(x)] + c_2\mathrm{MathieuS}[-b, a, i\log(x)]$$

3.343 problem 1349

Internal problem ID [9677]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1349.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{(x^2+1)\, y'}{x^3} + \frac{y}{x^4} = 0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 65

```
dsolve(diff(diff(y(x),x),x) = -(x^2+1)/x^3*diff(y(x),x)-1/x^4*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{\frac{1}{4x^2}} \left(c_1(2x^2-1)\,\mathrm{BesselI}\left(0, \frac{1}{4x^2}\right) + (2x^2-1)\, c_2\,\mathrm{BesselK}\left(0, -\frac{1}{4x^2}\right) + \mathrm{BesselI}\left(1, \frac{1}{4x^2}\right) c_1 + \mathrm{BesselK}\left(1, -\right.\right.}{x^2}$$

✓ Solution by Mathematica

Time used: 0.22 (sec). Leaf size: 73

```
DSolve[y''[x] == -(y[x]/x^4) - ((1 + x^2)*y'[x])/x^3,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to c_2 G_{1,2}^{2,0}\left(-\frac{1}{2x^2}\middle|\begin{array}{c}\frac{3}{2}\\ 0,0\end{array}\right) + \frac{c_1 e^{\frac{1}{4x^2}}\left((2x^2-1)\,\mathrm{BesselI}\left(0, \frac{1}{4x^2}\right) + \mathrm{BesselI}\left(1, \frac{1}{4x^2}\right)\right)}{2x^2}$$

3.344 problem 1350

Internal problem ID [9678]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1350.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, ‘_with_symmetry_[0,F`

$$
\boxed{y'' + \frac{2y'}{x} + \frac{a^2 y}{x^4} = 0}
$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(diff(y(x),x),x) = -2/x*diff(y(x),x)-a^2/x^4*y(x),y(x), singsol=all)
```

$$
y(x) = c_1 \sin\left(\frac{a}{x}\right) + c_2 \cos\left(\frac{a}{x}\right)
$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 25

```
DSolve[y''[x] == -((a^2*y[x])/x^4) - (2*y'[x])/x,y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x) \rightarrow c_1 \cos\left(\frac{a}{x}\right) - c_2 \sin\left(\frac{a}{x}\right)
$$

3.345 problem 1351

Internal problem ID [9679]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1351.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(2x^2+1)\,y'}{x^3} - \frac{y}{x^4} = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 24

```
dsolve(diff(diff(y(x),x),x) = -(2*x^2+1)/x^3*diff(y(x),x)+1/x^4*y(x),y(x), singsol=all)
```

$$y(x) = e^{\frac{1}{2x^2}}\left(c_1 + c_2\,\mathrm{erf}\left(\frac{\sqrt{2}}{2x}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.062 (sec). Leaf size: 44

```
DSolve[y''[x] == y[x]/x^4 - ((1 + 2*x^2)*y'[x])/x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{\frac{1}{2x^2}}\left(2c_1 - \sqrt{2\pi}c_2\mathrm{erf}\left(\frac{1}{\sqrt{2}x}\right)\right)$$

3.346 problem 1352

Internal problem ID [9680]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1352.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{2(x+a)\,y'}{x^2} + \frac{by}{x^4} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(diff(y(x),x),x) = -2/x^2*(x+a)*diff(y(x),x)-b/x^4*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{-\sqrt{a^2-b}+a}{x}} + c_2 \mathrm{e}^{\frac{\sqrt{a^2-b}+a}{x}}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 51

```
DSolve[y''[x] == -((b*y[x])/x^4) - (2*(a + x)*y'[x])/x^2,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to e^{\frac{a-\sqrt{a^2-b}}{x}}\left(c_1 e^{\frac{2\sqrt{a^2-b}}{x}} + c_2\right)$$

3.347 problem 1353

Internal problem ID [9681]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1353.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \frac{(2x^2-1)\,y'}{x^3} + \frac{y}{x^4} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 65

```
dsolve(diff(diff(y(x),x),x) = 1/x^3*(2*x^2-1)*diff(y(x),x)-1/x^4*y(x),y(x), singsol=all)
```

$$y(x) = \frac{-c_1\sqrt{2}\,\sqrt{\pi}\,(x^4+2x^2-1)\,\mathrm{erfi}\left(\frac{\sqrt{2}}{2x}\right) + 2c_1(x^3-x)\,\mathrm{e}^{\frac{1}{2x^2}} + c_2(x^4+2x^2-1)}{x}$$

✓ Solution by Mathematica

Time used: 1.137 (sec). Leaf size: 77

```
DSolve[y''[x] == -(y[x]/x^4) + ((-1 + 2*x^2)*y'[x])/x^3,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{-\sqrt{2\pi}c_2(x^4+2x^2-1)\,\mathrm{erfi}\left(\frac{1}{\sqrt{2}x}\right) + 2c_2e^{\frac{1}{2x^2}}x(x^2-1) + 16c_1(x^4+2x^2-1)}{16x}$$

3.348 problem 1354

Internal problem ID [9682]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1354.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{(2x^2-1)\,y'}{x^3} + \frac{2y}{x^4} = 0$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 33

```
dsolve(diff(diff(y(x),x),x) = 1/x^3*(2*x^2-1)*diff(y(x),x)-2/x^4*y(x),y(x), singsol=all)
```

$$y(x) = \frac{c_2 x^5 \operatorname{hypergeom}\left(\left[-\frac{5}{2}\right], \left[-\frac{1}{2}\right], \frac{1}{2x^2}\right) + 5c_1x^2 - c_1}{x^2}$$

✓ Solution by Mathematica
Time used: 0.235 (sec). Leaf size: 78

```
DSolve[y''[x] == (-2*y[x])/x^4 + ((-1 + 2*x^2)*y'[x])/x^3,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{5\sqrt{2\pi}c_2(1-5x^2)\operatorname{erfi}\left(\frac{1}{\sqrt{2}x}\right) + 12c_1(5x^2-1) + 10c_2e^{\frac{1}{2x^2}}x(2x^4+4x^2-1)}{60x^2}$$

3.349 problem 1355

Internal problem ID [9683]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1355.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(x^3-1)\,y'}{x\,(x^3+1)} - \frac{xy}{x^3+1} = 0$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 54

```
dsolve(diff(diff(y(x),x),x) = -(x^3-1)/x/(x^3+1)*diff(y(x),x)+x/(x^3+1)*y(x),y(x), singsol=a
```

$$y(x) = \frac{2c_1x^2 \operatorname{LegendreP}\left(-\frac{1}{3}, -\frac{2}{3}, \frac{-x^3+1}{x^3+1}\right)\Gamma\left(\frac{2}{3}\right)}{3\,(x^3+1)^{\frac{1}{3}}\,(-x^3)^{\frac{1}{3}}} + c_2(x^3+1)^{\frac{1}{3}}$$

✓ Solution by Mathematica
Time used: 8.037 (sec). Leaf size: 44

```
DSolve[y''[x] == (x*y[x])/(1 + x^3) - ((-1 + x^3)*y'[x])/(x*(1 + x^3)),y[x],x,IncludeSingula
```

$$y(x) \to \frac{1}{2}\sqrt[3]{x^3+1}\left(c_2x^2 \operatorname{Hypergeometric2F1}\left(\frac{2}{3}, \frac{4}{3}, \frac{5}{3}, -x^3\right) + 2c_1\right)$$

3.350 problem 1356

Internal problem ID [9684]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1356.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{(2x^2+1)\,y'}{x\,(x^2+1)} + \frac{(-v(v+1)\,x^2-n^2)\,y}{x^2\,(x^2+1)} = 0}$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 29

```
dsolve(diff(diff(y(x),x),x) = -(2*x^2+1)/x/(x^2+1)*diff(y(x),x)-(-v*(v+1)*x^2-n^2)/x^2/(x^2+
```

$$y(x) = \text{LegendreQ}\left(v, n, \sqrt{x^2+1}\right) c_2 + \text{LegendreP}\left(v, n, \sqrt{x^2+1}\right) c_1$$

✓ Solution by Mathematica

Time used: 0.351 (sec). Leaf size: 78

```
DSolve[y''[x] == -(((-n^2 - v*(1 + v)*x^2)*y[x])/(x^2*(1 + x^2))) - ((1 + 2*x^2)*y'[x])/(x*(
```

$$\begin{aligned} y(x) \to\ & c_1 x^{-n}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-n-v), \frac{1}{2}(-n+v+1), 1-n, -x^2\right) \\ & + c_2 x^n\,\text{Hypergeometric2F1}\left(\frac{n-v}{2}, \frac{1}{2}(n+v+1), n+1, -x^2\right) \end{aligned}$$

3.351 problem 1357

Internal problem ID [9685]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1357.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(a\,x^2 + a - 1)\,y'}{x\,(x^2+1)} + \frac{(b\,x^2 + c)\,y}{x^2\,(x^2+1)} = 0$$

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 97

```
dsolve(diff(diff(y(x),x),x) = -1/x*(a*x^2+a-1)/(x^2+1)*diff(y(x),x)-(b*x^2+c)/x^2/(x^2+1)*y(
```

$$y(x) = x^{1-\frac{a}{2}}\left(\mathrm{LegendreP}\left(-\frac{1}{2}+\frac{\sqrt{a^2-2a-4b+1}}{2}, \frac{\sqrt{a^2-4a-4c+4}}{2}, \sqrt{x^2+1}\right)c_1 + \mathrm{LegendreQ}\left(-\frac{1}{2}+\frac{\sqrt{a^2-2a-4b+1}}{2}, \frac{\sqrt{a^2-4a-4c+4}}{2}, \sqrt{x^2+1}\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.722 (sec). Leaf size: 264

```
DSolve[y''[x] == -(((c + b*x^2)*y[x])/(x^2*(1 + x^2))) - ((-1 + a + a*x^2)*y'[x])/(x*(1 + x^
```

$$y(x) \rightarrow x^{-\frac{1}{2}\sqrt{a^2-4a-4c+4}-\frac{a}{2}+1}\left(c_1 \text{Hypergeometric2F1}\left(\frac{1}{4}\left(-\sqrt{a^2-2a-4b+1}-\sqrt{a^2-4a-4c+4}+1\right), \frac{1}{4}\left(\sqrt{a\right.\right.\right.$$
$$-\frac{1}{2}\sqrt{a^2-4a-4c+4}, -x^2\right)$$
$$+c_2 x^{\sqrt{a^2-4a-4c+4}} \text{Hypergeometric2F1}\left(\frac{1}{4}\left(-\sqrt{a^2-2a-4b+1}+\sqrt{a^2-4a-4c+4}+1\right), \frac{1}{4}\left(\sqrt{a^2-2a\right.\right.$$
$$-x^2\right)\right)$$

3.352 problem 1358

Internal problem ID [9686]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1358.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{(x^2-2)\,y'}{x\,(x^2-1)} + \frac{(x^2-2)\,y}{x^2\,(x^2-1)} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(diff(diff(y(x),x),x) = 1/x*(x^2-2)/(x^2-1)*diff(y(x),x)-(x^2-2)/x^2/(x^2-1)*y(x),y(x)
```

$$y(x) = x\left(c_1 + c_2 \ln\left(x + \sqrt{x^2-1}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.087 (sec). Leaf size: 71

```
DSolve[y''[x] == -(((-2 + x^2)*y[x])/(x^2*(-1 + x^2))) + ((-2 + x^2)*y'[x])/(x*(-1 + x^2)),y
```

$$y(x) \to \frac{x\sqrt[4]{x^2-1}\left(-c_2\log\left(1-\frac{x}{\sqrt{x^2-1}}\right) + c_2\log\left(\frac{x}{\sqrt{x^2-1}}+1\right) + 2c_1\right)}{2\sqrt[4]{1-x^2}}$$

3.353 problem 1359

Internal problem ID [9687]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1359.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{2xy'}{x^2-1} + \frac{v(v+1)\,y}{x^2\,(x^2-1)} = 0$$

✓ Solution by Maple

Time used: 0.266 (sec). Leaf size: 57

```
dsolve(diff(diff(y(x),x),x) = -2*x/(x^2-1)*diff(y(x),x)-v*(v+1)/x^2/(x^2-1)*y(x),y(x), sings
```

$$y(x) = c_1 x^{-v} \operatorname{hypergeom}\left(\left[-\frac{v}{2}, \frac{1}{2}-\frac{v}{2}\right], \left[\frac{1}{2}-v\right], x^2\right) + c_2 x^{v+1} \operatorname{hypergeom}\left(\left[1+\frac{v}{2}, \frac{1}{2}+\frac{v}{2}\right], \left[\frac{3}{2}+v\right], x^2\right)$$

✓ Solution by Mathematica

Time used: 0.128 (sec). Leaf size: 84

```
DSolve[y''[x] == -((v*(1 + v)*y[x])/(x^2*(-1 + x^2))) - (2*x*y'[x])/(-1 + x^2),y[x],x,Includ
```

$$y(x) \to c_1 i^{-v} x^{-v} \text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{v}{2}, -\frac{v}{2}, \frac{1}{2}-v, x^2\right) + c_2 i^{v+1} x^{v+1} \text{Hypergeometric2F1}\left(\frac{v+1}{2}, \frac{v+2}{2}, v+\frac{3}{2}, x^2\right)$$

3.354 problem 1360

Internal problem ID [9688]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1360.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{2xy'}{x^2-1} - \frac{v(v+1)\,y}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.282 (sec). Leaf size: 109

```
dsolve(diff(diff(y(x),x),x) = -2*x/(x^2-1)*diff(y(x),x)+v*(v+1)/x^2*y(x),y(x), singsol=all)
```

$$y(x) = \frac{c_2 x^{v+1}(x^2)^{-\frac{v}{2}-\frac{1}{4}}\,\Gamma\left(v+\frac{1}{2}\right)^2\left(v+\frac{1}{2}\right)\text{LegendreP}\left(-\frac{1}{2},-v-\frac{1}{2},\frac{-x^2-1}{x^2-1}\right)+\sec(\pi v)\,\pi\,\text{LegendreP}\left(-\frac{1}{2},v+\frac{1}{2},\right.}{\sqrt{-x^2+1}\,\Gamma\left(v+\frac{1}{2}\right)}$$

✓ Solution by Mathematica

Time used: 0.119 (sec). Leaf size: 68

```
DSolve[y''[x] == (v*(1 + v)*y[x])/x^2 - (2*x*y'[x])/(-1 + x^2),y[x],x,IncludeSingularSolutio
```

$$y(x) \to c_1 i^{-v} x^{-v}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-v,\frac{1}{2}-v,x^2\right) + c_2 i^{v+1} x^{v+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},v+1,v+\frac{3}{2},x^2\right)$$

3.355 problem 1361

Internal problem ID [9689]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1361.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$
\boxed{y'' - \frac{2xy'}{x^2-1} + \frac{(a(1+a) - a\,x^2(a+3))\,y}{x^2\,(x^2-1)} = 0}
$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(diff(y(x),x),x) = 2*x/(x^2-1)*diff(y(x),x)-(a*(a+1)-a*x^2*(a+3))/x^2/(x^2-1)*y(x
```

$$
y(x) = c_1 x^{-a} + c_2 \left(2a\,x^2 + x^2 - 2a - 3\right) x^{a+1}
$$

✓ Solution by Mathematica
Time used: 0.467 (sec). Leaf size: 36

```
DSolve[y''[x] == -(((a*(1 + a) - a*(3 + a)*x^2)*y[x])/(x^2*(-1 + x^2))) + (2*x*y'[x])/(-1 +
```

$$
y(x) \to c_1 x^{-a} - c_2 x^{a+1} \left(2a\left(x^2-1\right) + x^2 - 3\right)
$$

3.356 problem 1362

Internal problem ID [9690]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1362.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y''x^2(x^2-1)-2x^3y'-\left((a-n)(a+n+1)x^2(x^2-1)+2a\,x^2+n(n+1)(x^2-1)\right)y=0$$

✓ Solution by Maple

Time used: 0.75 (sec). Leaf size: 109

```
dsolve(x^2*(x^2-1)*diff(diff(y(x),x),x)-2*x^3*diff(y(x),x)-((a-n)*(a+n+1)*x^2*(x^2-1)+2*a*x^
```

$$y(x)=c_1x^{-n}\,\mathrm{HeunC}\left(0,-n-\frac{1}{2},-2,-\frac{1}{4}a^2+\frac{1}{4}n^2-\frac{1}{4}a+\frac{1}{4}n,-\frac{1}{4}n^2-\frac{1}{4}n+\frac{3}{4}+\frac{1}{4}a^2-\frac{1}{4}a,x^2\right)+c_2x^{1+n}\,\mathrm{HeunC}\left(0,n+\frac{1}{2},-2,-\frac{1}{4}a^2+\frac{1}{4}n^2-\frac{1}{4}a+\frac{1}{4}n,-\frac{1}{4}n^2-\frac{1}{4}n+\frac{3}{4}+\frac{1}{4}a^2-\frac{1}{4}a,x^2\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x] == -(((2*a*x^2 + n*(1 + n)*(-1 + x^2) + (a - n)*(1 + a + n)*x^2*(-1 + x^2))*y[
```

Not solved

3.357 problem 1363

Internal problem ID [9691]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1363.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(a\,x^2 + a - 2)\,y'}{x\,(x^2-1)} + \frac{by}{x^2} = 0$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 253

```
dsolve(diff(diff(y(x),x),x) = -1/x*(a*x^2+a-2)/(x^2-1)*diff(y(x),x)-b/x^2*y(x),y(x), singsol
```

$$y(x) = x^{\frac{a}{2}-\frac{1}{2}}\left(-x^2+1\right)^{\frac{1}{2}+\frac{a}{2}}\left(x^2-1\right)^{-a}\left(x^{\frac{\sqrt{a^2-2a-4b+1}}{2}}\left(x^2\right)^{-\frac{\sqrt{a^2-2a-4b+1}}{4}}\operatorname{LegendreP}\left(\frac{a}{2}\right.\right.$$
$$\left.-\frac{3}{2}, -\frac{\sqrt{a^2-2a-4b+1}}{2}, \frac{-x^2-1}{x^2-1}\right)c_1\Gamma\left(1+\frac{\sqrt{a^2-2a-4b+1}}{2}\right)$$
$$+\frac{\csc\left(\frac{\pi\sqrt{a^2-2a-4b+1}}{2}\right)x^{-\frac{\sqrt{a^2-2a-4b+1}}{2}}\left(x^2\right)^{\frac{\sqrt{a^2-2a-4b+1}}{4}}\operatorname{LegendreP}\left(\frac{a}{2}-\frac{3}{2}, \frac{\sqrt{a^2-2a-4b+1}}{2}, \frac{-x^2-1}{x^2-1}\right)\sqrt{a^2-2a-}}{2\Gamma\left(1+\frac{\sqrt{a^2-2a-4b+1}}{2}\right)}$$

✓ Solution by Mathematica

Time used: 0.733 (sec). Leaf size: 212

```
DSolve[y''[x] == -((b*y[x])/x^2) - ((-2 + a + a*x^2)*y'[x])/(x*(-1 + x^2)),y[x],x,IncludeSin
```

$$y(x) \to -(-1)^{\frac{1}{4}\left(-\sqrt{a^2-2a-4b+1}+a+3\right)} x^{\frac{1}{2}\left(-\sqrt{a^2-2a-4b+1}+a-1\right)} \left(c_1 \text{Hypergeometric2F1}\left(\frac{a-1}{2}, \frac{1}{2}\left(a-\sqrt{a^2-2a-4b\right.\right.\right.$$

$$- \frac{1}{2}\sqrt{a^2-2a-4b+1}, x^2\Big)$$

$$+c_2 i^{\sqrt{a^2-2a-4b+1}} x^{\sqrt{a^2-2a-4b+1}} \text{Hypergeometric2F1}\left(\frac{a-1}{2}, \frac{1}{2}\left(a+\sqrt{a^2-2a-4b+1}-1\right), \frac{1}{2}\left(\sqrt{a^2-2a\right.\right.$$

3.358 problem 1364

Internal problem ID [9692]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1364.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{(2bc\,x^c(x^2-1) + 2(-1+a)\,x^2 - 2a)\,y'}{x\,(x^2-1)} + \frac{(b^2c^2x^{2c}(x^2-1) + bc\,x^{c+2}(2a-c-1) - bc\,x^c(2a-c+1$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 25

```
dsolve(diff(diff(y(x),x),x) = 1/x*(2*b*c*x^c*(x^2-1)+2*(a-1)*x^2-2*a)/(x^2-1)*diff(y(x),x)-(
```

$$y(x) = x^a e^{b\,x^c}(c_1 \operatorname{LegendreP}(v,x) + c_2 \operatorname{LegendreQ}(v,x))$$

✓ Solution by Mathematica

Time used: 0.184 (sec). Leaf size: 35

```
DSolve[y''[x] == -(((-(a*(1 + a)) + ((-1 + a)*a - v*(1 + v))*x^2 - b*(1 + 2*a - c)*c*x^c + b
```

$$y(x) \to (x^c)^{a/c}\,e^{bx^c}(c_1 \operatorname{LegendreP}(v,x) + c_2 \operatorname{LegendreQ}(v,x))$$

3.359 problem 1365

Internal problem ID [9693]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1365.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Halm]`

$$\boxed{y'' + \frac{ay}{(x^2+1)^2} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 59

```
dsolve(diff(diff(y(x),x),x) = -a/(x^2+1)^2*y(x),y(x), singsol=all)
```

$$y(x) = \left(\left(\frac{x+i}{-x+i}\right)^{\frac{\sqrt{a+1}}{2}} c_1 + \left(\frac{x+i}{-x+i}\right)^{-\frac{\sqrt{a+1}}{2}} c_2\right)\sqrt{x^2+1}$$

✓ Solution by Mathematica

Time used: 0.137 (sec). Leaf size: 83

```
DSolve[y''[x] == -((a*y[x])/(1 + x^2)^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\sqrt{x^2+1}e^{i\sqrt{a+1}\arctan(x)}\left(\frac{ic_2(1-ix)^{\sqrt{a+1}}(1+ix)^{-\sqrt{a+1}}}{\sqrt{a+1}} + 2c_1\right)$$

3.360 problem 1366

Internal problem ID [9694]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1366.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$y'' + \frac{2xy'}{x^2+1} + \frac{y}{(x^2+1)^2} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(diff(y(x),x),x) = -2/(x^2+1)*x*diff(y(x),x)-1/(x^2+1)^2*y(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1 x + c_2}{\sqrt{x^2+1}}$$

✓ Solution by Mathematica
Time used: 2.035 (sec). Leaf size: 22

```
DSolve[y''[x] == -(y[x]/(1 + x^2)^2) - (2*x*y'[x])/(1 + x^2),y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{c_2 x + c_1}{\sqrt{x^2+1}}$$

3.361 problem 1367

Internal problem ID [9695]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1367.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y''+\frac{2xy'}{x^2+1}+\frac{\left(a^2(x^2+1)^2-n(n+1)\left(x^2+1\right)+m^2\right)y}{\left(x^2+1\right)^2}=0$$

✓ Solution by Maple

Time used: 0.61 (sec). Leaf size: 88

```
dsolve(diff(diff(y(x),x),x) = -2/(x^2+1)*x*diff(y(x),x)-(a^2*(x^2+1)^2-n*(n+1)*(x^2+1)+m^2)/
```

$$y(x)=\left(\mathrm{HeunC}\left(0,\frac{1}{2},m,-\frac{a^2}{4},\frac{1}{4}+\frac{1}{4}a^2+\frac{1}{4}m^2-\frac{1}{4}n^2-\frac{1}{4}n,-x^2\right)c_2x+\mathrm{HeunC}\left(0,-\frac{1}{2},m,-\frac{a^2}{4},\frac{1}{4}+\frac{1}{4}a^2+\frac{1}{4}m^2-\frac{1}{4}n^2-\frac{1}{4}n,-x^2\right)c_1\right)\left(x^2+1\right)^{\frac{m}{2}}$$

✓ Solution by Mathematica

Time used: 0.366 (sec). Leaf size: 140

```
DSolve[y''[x] == -(((m^2 - n*(1 + n)*(1 + x^2) + a^2*(1 + x^2)^2)*y[x])/(1 + x^2)^2) - (2*x*
```

$$y(x)\to\left(x^2+1\right)^{\frac{\sqrt{m^2}}{2}}\left(c_2x\mathrm{HeunC}\left[\frac{1}{4}\left(-a^2-m^2-3\sqrt{m^2}+n^2+n-2\right),-\frac{a^2}{4},\frac{3}{2},\sqrt{m^2}+1,0,-x^2\right]+c_1\mathrm{HeunC}\left[\frac{1}{4}\left(-a^2-m^2-\sqrt{m^2}+n^2+n\right),-\frac{a^2}{4},\frac{1}{2},\sqrt{m^2}+1,0,-x^2\right]\right)$$

3.362 problem 1368

Internal problem ID [9696]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1368.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{axy'}{x^2+1} + \frac{by}{(x^2+1)^2} = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 71

```
dsolve(diff(diff(y(x),x),x) = -a*x/(x^2+1)*diff(y(x),x)-b/(x^2+1)^2*y(x),y(x), singsol=all)
```

$$y(x) = (x^2+1)^{\frac{1}{2}-\frac{a}{4}} \left(\text{LegendreP}\left(\frac{a}{2}-1, \frac{\sqrt{a^2-4a+4b+4}}{2}, ix\right) c_1 + \text{LegendreQ}\left(\frac{a}{2}-1, \frac{\sqrt{a^2-4a+4b+4}}{2}, ix\right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 92

```
DSolve[y''[x] == -((b*y[x])/(1 + x^2)^2) - (a*x*y'[x])/(1 + x^2),y[x],x,IncludeSingularSolut
```

$$y(x) \to (x^2+1)^{\frac{1}{2}-\frac{a}{4}} \left(c_1 P_{\frac{a-2}{2}}^{\frac{1}{2}\sqrt{a^2-4a+4b+4}}(ix) + c_2 Q_{\frac{a-2}{2}}^{\frac{1}{2}\sqrt{a^2-4a+4b+4}}(ix) \right)$$

3.363 problem 1369

Internal problem ID [9697]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1369.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{ay}{(x^2-1)^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
dsolve(diff(diff(y(x),x),x) = -a/(x^2-1)^2*y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{x^2-1}\left(\left(\frac{x-1}{x+1}\right)^{\frac{\sqrt{-a+1}}{2}} c_1 + \left(\frac{x-1}{x+1}\right)^{-\frac{\sqrt{-a+1}}{2}} c_2\right)$$

✓ Solution by Mathematica

Time used: 0.257 (sec). Leaf size: 88

```
DSolve[y''[x] == -((a*y[x])/(-1 + x^2)^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(1-x^2)^{\frac{1}{2}-\frac{\sqrt{1-a}}{2}}\left(2\sqrt{1-a}c_1(1-x)^{\sqrt{1-a}} + c_2(x+1)^{\sqrt{1-a}}\right)}{2\sqrt{1-a}}$$

3.364 problem 1370

Internal problem ID [9698]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1370.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$y'' + \frac{2xy'}{x^2-1} - \frac{a^2 y}{(x^2-1)^2} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(diff(diff(y(x),x),x) = -2*x/(x^2-1)*diff(y(x),x)+a^2/(x^2-1)^2*y(x),y(x), singsol=all
```

$$y(x) = c_1 \sinh(a \operatorname{arctanh}(x)) + c_2 \cosh(a \operatorname{arctanh}(x))$$

✓ Solution by Mathematica

Time used: 2.049 (sec). Leaf size: 53

```
DSolve[y''[x] == (a^2*y[x])/(-1 + x^2)^2 - (2*x*y'[x])/(-1 + x^2),y[x],x,IncludeSingularSolu
```

$$y(x) \to c_1 \cosh\left(\frac{1}{2}a(\log(1-x) - \log(x+1))\right) + ic_2 \sinh\left(\frac{1}{2}a(\log(1-x) - \log(x+1))\right)$$

3.365 problem 1371

Internal problem ID [9699]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1371.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{2xy'}{x^2-1} + \frac{(-a^2 - \lambda(x^2-1))\,y}{(x^2-1)^2} = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 37

```
dsolve(diff(diff(y(x),x),x) = -2*x/(x^2-1)*diff(y(x),x)-(-a^2-lambda*(x^2-1))/(x^2-1)^2*y(x)
```

$$y(x) = c_1 \operatorname{LegendreP}\left(\frac{\sqrt{1+4\lambda}}{2} - \frac{1}{2}, a, x\right) + c_2 \operatorname{LegendreQ}\left(\frac{\sqrt{1+4\lambda}}{2} - \frac{1}{2}, a, x\right)$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 48

```
DSolve[y''[x] == -(((-a^2 - \[Lambda]*(-1 + x^2))*y[x])/(-1 + x^2)^2) - (2*x*y'[x])/(-1 + x^
```

$$y(x) \to c_1 P^a_{\frac{1}{2}(\sqrt{4\lambda+1}-1)}(x) + c_2 Q^a_{\frac{1}{2}(\sqrt{4\lambda+1}-1)}(x)$$

3.366 problem 1372

Internal problem ID [9700]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1372.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{2xy'}{x^2-1} + \frac{\left((x^2-1)\left(a\,x^2+bx+c\right)-k^2\right)y}{\left(x^2-1\right)^2} = 0$$

✓ Solution by Maple

Time used: 0.704 (sec). Leaf size: 101

```
dsolve(diff(diff(y(x),x),x) = -2*x/(x^2-1)*diff(y(x),x)-((x^2-1)*(a*x^2+b*x+c)-k^2)/(x^2-1)^
```

$$y(x) = \mathrm{e}^{\sqrt{-a}\,x}\left(\left(x^2-1\right)^{\frac{k}{2}} \operatorname{HeunC}\left(4\sqrt{-a}, k, k, 2b, \frac{k^2}{2}+a-b+c, \frac{x}{2}+\frac{1}{2}\right) c_1 + (x-1)^{\frac{k}{2}} (x+1)^{-\frac{k}{2}} \operatorname{HeunC}\left(4\sqrt{-a}, -k, k, 2b, \frac{k^2}{2}+a-b+c, \frac{x}{2}+\frac{1}{2}\right) c_2\right)$$

✓ Solution by Mathematica

Time used: 0.814 (sec). Leaf size: 189

```
DSolve[y''[x] == -(((-k^2 + (-1 + x^2)*(c + b*x + a*x^2))*y[x])/(-1 + x^2)^2) - (2*x*y'[x])/
```

$$y(x) \to e^{\sqrt{-a}x}(x+1)^{-k/2}\left(c_1(x+1)^{k/2}\left(x^2-1\right)^{k/2}\operatorname{HeunC}\left[(k+1)\left(2\sqrt{-a}-k\right)-a+b-c, 2\left(2\sqrt{-a}(k+1)+b\right), k+1, k+1, 4\right.\right.$$

3.367 problem 1373

Internal problem ID [9701]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1373.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{2xy'}{x^2-1} + \frac{\left(-a^2(x^2-1)^2 - n(n+1)\left(x^2-1\right) - m^2\right)y}{\left(x^2-1\right)^2} = 0$$

✓ Solution by Maple

Time used: 0.578 (sec). Leaf size: 84

```
dsolve(diff(diff(y(x),x),x) = -2*x/(x^2-1)*diff(y(x),x)-(-a^2*(x^2-1)^2-n*(n+1)*(x^2-1)-m^2)
```

$$y(x) = \left(\operatorname{HeunC}\left(0, \frac{1}{2}, m, -\frac{a^2}{4}, \frac{1}{4} + \frac{1}{4}a^2 + \frac{1}{4}m^2 - \frac{1}{4}n^2 - \frac{1}{4}n, x^2\right) c_2 x + \operatorname{HeunC}\left(0, -\frac{1}{2}, m, -\frac{a^2}{4}, \frac{1}{4} + \frac{1}{4}a^2 + \frac{1}{4}m^2 - \frac{1}{4}n^2 - \frac{1}{4}n, x^2\right) c_1\right) \left(x^2-1\right)^{\frac{m}{2}}$$

✓ Solution by Mathematica

Time used: 2.094 (sec). Leaf size: 103

```
DSolve[y''[x] == -(((-m^2 - n*(1 + n)*(-1 + x^2) - a^2*(-1 + x^2)^2)*y[x])/(-1 + x^2)^2) - (
```

$$y(x) \to \left(x^2-1\right)^{m/2} \left(c_1 \operatorname{HeunC}\left[\frac{1}{4}\left(-a^2 - m(m+1) + n^2 + n\right), -\frac{a^2}{4}, \frac{1}{2}, m+1, 0, x^2\right] + c_2 x \operatorname{HeunC}\left[\frac{1}{4}\left(-a^2 - (m-n+1)(m+n+2)\right), -\frac{a^2}{4}, \frac{3}{2}, m+1, 0, x^2\right]\right)$$

3.368 problem 1374

Internal problem ID [9702]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1374.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{2x(2a-1)\,y'}{x^2-1} + \frac{(x^2(2a(2a-1)-v(v+1))+2a+v(v+1))\,y}{(x^2-1)^2} = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 23

```
dsolve(diff(diff(y(x),x),x) = 2*x*(2*a-1)/(x^2-1)*diff(y(x),x)-(x^2*(2*a*(2*a-1)-v*(v+1))+2*
```

$$y(x) = (c_1 \,\mathrm{LegendreP}\,(v, x) + c_2 \,\mathrm{LegendreQ}\,(v, x))\,(x^2-1)^a$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 26

```
DSolve[y''[x] == -(((2*a + v*(1 + v) + (2*a*(-1 + 2*a) - v*(1 + v))*x^2)*y[x])/(-1 + x^2)^2)
```

$$y(x) \to (x^2-1)^a\,(c_1 \,\mathrm{LegendreP}(v, x) + c_2 \,\mathrm{LegendreQ}(v, x))$$

3.369 problem 1375

Internal problem ID [9703]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1375.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{2x(n+1-2a)\,y'}{x^2-1} + \frac{(4a\,x^2(a-n) - (x^2-1)\,(2a+(v-n)\,(v+n+1)))\,y}{(x^2-1)^2} = 0$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 29

```
dsolve(diff(diff(y(x),x),x) = -2*x/(x^2-1)*(n+1-2*a)*diff(y(x),x)-(4*a*x^2*(a-n)-(x^2-1)*(2*
```

$$y(x) = (\text{LegendreP}\,(v,n,x)\,c_1 + \text{LegendreQ}\,(v,n,x)\,c_2)\,\left(x^2-1\right)^{a-\frac{n}{2}}$$

✓ Solution by Mathematica

Time used: 0.057 (sec). Leaf size: 34

```
DSolve[y''[x] == -(((4*a*(a - n)*x^2 - (2*a + (-n + v)*(1 + n + v))*(-1 + x^2))*y[x])/(-1 +
```

$$y(x) \to \left(x^2-1\right)^{a-\frac{n}{2}}\left(c_1 P_v^n(x) + c_2 Q_v^n(x)\right)$$

3.370 problem 1376

Internal problem ID [9704]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1376.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$y'' + \frac{(2x^2+a)\, y'}{x\,(x^2+a)} + \frac{by}{x^2\,(x^2+a)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 85

```
dsolve(diff(diff(y(x),x),x) = -1/x*(2*x^2+a)/(x^2+a)*diff(y(x),x)-b/x^2/(x^2+a)*y(x),y(x), s
```

$$y(x) = \left(c_2 2^{\frac{2i\sqrt{b}}{\sqrt{a}}} \left(\frac{a+\sqrt{a}\,\sqrt{x^2+a}}{x}\right)^{\frac{2i\sqrt{b}}{\sqrt{a}}} + c_1\right) 2^{-\frac{i\sqrt{b}}{\sqrt{a}}} \left(\frac{a+\sqrt{a}\,\sqrt{x^2+a}}{x}\right)^{-\frac{i\sqrt{b}}{\sqrt{a}}}$$

✓ Solution by Mathematica

Time used: 0.115 (sec). Leaf size: 69

```
DSolve[y''[x] == -((b*y[x])/(x^2*(a + x^2))) - ((a + 2*x^2)*y'[x])/(x*(a + x^2)),y[x],x,Incl
```

$$y(x) \to c_1 \cos\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{a+x^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right) - c_2 \sin\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{a+x^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right)$$

3.371 problem 1377

Internal problem ID [9705]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1377.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + \frac{b^2 y}{(a^2 + x^2)^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 83

```
dsolve(diff(diff(y(x),x),x) = -b^2/(a^2+x^2)^2*y(x),y(x), singsol=all)
```

$$y(x) = \left(\left(\frac{ix - a}{ix + a} \right)^{\frac{\sqrt{a^2+b^2}}{2a}} c_1 + \left(\frac{ix - a}{ix + a} \right)^{-\frac{\sqrt{a^2+b^2}}{2a}} c_2 \right) \sqrt{a^2 + x^2}$$

✓ Solution by Mathematica

Time used: 0.841 (sec). Leaf size: 97

```
DSolve[y''[x] == -((b^2*y[x])/(a^2 + x^2)^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} \sqrt{a^2 + x^2} e^{-i\sqrt{\frac{b^2}{a^2}+1} \arctan\left(\frac{a}{x}\right)} \left(\frac{i c_2 e^{2i\sqrt{\frac{b^2}{a^2}+1} \arctan\left(\frac{a}{x}\right)}}{a\sqrt{\frac{b^2}{a^2} + 1}} + 2c_1 \right)$$

3.372 problem 1378

Internal problem ID [9706]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1378.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{2(x^2-1)\,y'}{x\,(x-1)^2} + \frac{(-2x^2+2x+2)\,y}{x^2\,(x-1)^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(diff(diff(y(x),x),x) = -2/x*(x^2-1)/(x-1)^2*diff(y(x),x)-(-2*x^2+2*x+2)/x^2/(x-1)^2*y
```

$$y(x) = \frac{\left(-xc_2(x-1)\ln(x-1) + xc_2(x-1)\ln(x) + c_1x^2 + (-c_1-c_2)\,x + \frac{c_2}{2}\right)x}{(x-1)^2}$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 56

```
DSolve[y''[x] == -(((2 + 2*x - 2*x^2)*y[x])/((-1 + x)^2*x^2)) - (2*(-1 + x^2)*y'[x])/((-1 +
```

$$y(x) \to -\frac{x(c_1x^2 - c_1x - 2c_2x - 2c_2(x-1)x\log(1-x) + 2c_2(x-1)x\log(x) + c_2)}{(x-1)^2}$$

3.373 problem 1379

Internal problem ID [9707]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1379.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{12y}{(x+1)^2(x^2+2x+3)} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 60

```
dsolve(diff(diff(y(x),x),x) = 12/(x+1)^2/(x^2+2*x+3)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{3(x^2+2x+3)\,c_2 \arctan\left(\frac{(x+1)\sqrt{2}}{2}\right) - c_2(x^3+2x^2+4x+1)\sqrt{2} + c_1(x^2+2x+3)}{(x+1)^2}$$

✓ Solution by Mathematica
Time used: 0.132 (sec). Leaf size: 71

```
DSolve[y''[x] == (12*y[x])/((1 + x)^2*(3 + 2*x + x^2)),y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{-3\sqrt{2}c_2(x^2+2x+3)\arctan\left(\frac{x+1}{\sqrt{2}}\right) + 2c_1(x^2+2x+3) + 2c_2(x^3+2x^2+4x+1)}{2(x+1)^2}$$

3.374 problem 1380

Internal problem ID [9708]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1380.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{by}{x^2 (x-a)^2} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 67

```
dsolve(diff(diff(y(x),x),x) = -b/x^2/(x-a)^2*y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{x(-x+a)} \left(\left(\frac{x}{-x+a} \right)^{\frac{\sqrt{a^2-4b}}{2a}} c_2 + \left(\frac{-x+a}{x} \right)^{\frac{\sqrt{a^2-4b}}{2a}} c_1 \right)$$

✓ Solution by Mathematica

Time used: 0.511 (sec). Leaf size: 121

```
DSolve[y''[x] == -((b*y[x])/(x^2*(-a + x)^2)),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{\frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{4b}{a^2}}} (x-a)^{\frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{4b}{a^2}}} \left(ac_1 \sqrt{1-\frac{4b}{a^2}} x^{\sqrt{1-\frac{4b}{a^2}}} + c_2 (x-a)^{\sqrt{1-\frac{4b}{a^2}}} \right)}{a\sqrt{1-\frac{4b}{a^2}}}$$

3.375 problem 1381

Internal problem ID [9709]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1381.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + \frac{by}{x^2 (x-a)^2} = c}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 219

```
dsolve(diff(diff(y(x),x),x) = -b/x^2/(x-a)^2*y(x)+c,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{x(-x+a)}\left(\left(\frac{x}{-x+a}\right)^{\frac{\sqrt{a^2-4b}}{2a}} c_1\sqrt{a^2-4b} + \left(\frac{x}{-x+a}\right)^{\frac{\sqrt{a^2-4b}}{2a}} \left(\int \sqrt{x(-x+a)}\left(\frac{x}{-x+a}\right)^{-\frac{\sqrt{a^2-4b}}{2a}} dx\right) c + \left(\frac{-x+}{x}\right.}{\sqrt{a^2-4b}}$$

✓ Solution by Mathematica

Time used: 1.097 (sec). Leaf size: 371

```
DSolve[y''[x] == c - (b*y[x])/(x^2*(-a + x)^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{acx^2(a-x)\left(1-\frac{x}{a}\right)^{-\frac{1}{2}\sqrt{1-\frac{4b}{a^2}}-\frac{1}{2}}\left(\left(\sqrt{1-\frac{4b}{a^2}}-3\right)\left(1-\frac{x}{a}\right)^{\sqrt{1-\frac{4b}{a^2}}} \text{Hypergeometric2F1}\left(\frac{1}{2}\sqrt{1-\frac{4b}{a^2}}-\frac{1}{2},\frac{1}{2}\right.\right.}{}$$

$$+ c_1 x^{\frac{1}{2}\sqrt{1-\frac{4b}{a^2}}+\frac{1}{2}}(x-a)^{\frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{4b}{a^2}}} + \frac{c_2 x^{\frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{4b}{a^2}}}(x-a)^{\frac{1}{2}\sqrt{1-\frac{4b}{a^2}}+\frac{1}{2}}}{a\sqrt{1-\frac{4b}{a^2}}}$$

3.376 problem 1382

Internal problem ID [9710]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1382.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{cy}{(x-a)^2 (x-b)^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 104

```
dsolve(diff(diff(y(x),x),x) = c/(x-a)^2/(x-b)^2*y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{(-x+a)(-x+b)}\left(\left(\frac{-x+a}{-x+b}\right)^{\frac{\sqrt{a^2-2ab+b^2+4c}}{2a-2b}} c_1 + \left(\frac{-x+a}{-x+b}\right)^{-\frac{\sqrt{a^2-2ab+b^2+4c}}{2a-2b}} c_2\right)$$

✓ Solution by Mathematica

Time used: 1.396 (sec). Leaf size: 141

```
DSolve[y''[x] == (c*y[x])/((-a + x)^2*(-b + x)^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x-a)^{\frac{1}{2}\left(1-\sqrt{\frac{4c}{(a-b)^2}+1}\right)}(x-b)^{\frac{1}{2}\left(1-\sqrt{\frac{4c}{(a-b)^2}+1}\right)}\left(c_1(x-a)^{\sqrt{\frac{4c}{(a-b)^2}+1}} - \frac{c_2(x-b)^{\sqrt{\frac{4c}{(a-b)^2}+1}}}{(a-b)\sqrt{\frac{4c}{(a-b)^2}+1}}\right)$$

3.377 problem 1383

Internal problem ID [9711]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1383.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{\left((\beta+\alpha+1)(x-a)^2(x-b) + (-\beta-\alpha+1)(x-b)^2(x-a)\right)y'}{(x-a)^2(x-b)^2} + \frac{\alpha\beta(a-b)^2 y}{(x-a)^2(x-b)^2} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve(diff(diff(y(x),x),x) = -((alpha+beta+1)*(x-a)^2*(x-b)+(1-alpha-beta)*(x-b)^2*(x-a))/(
```

$$y(x) = c_1\left(\frac{-x+a}{-x+b}\right)^{\beta} + c_2\left(\frac{-x+a}{-x+b}\right)^{\alpha}$$

✓ Solution by Mathematica

Time used: 2.147 (sec). Leaf size: 44

```
DSolve[y''[x] == -((\[Alpha]*(a - b)^2*\[Beta]*y[x])/((-a + x)^2*(-b + x)^2)) - (((1 + \[Alp
```

$$y(x) \to c_1(x-a)^{\alpha}(x-b)^{-\alpha} + c_2(x-a)^{\beta}(x-b)^{-\beta}$$

3.378 problem 1384

Internal problem ID [9712]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1384.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(-x^2(a^2-1) + 2(a+3)\,bx - b^2)\,y}{4x^2} = 0$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 73

```
dsolve(diff(diff(y(x),x),x) = -1/4*(-x^2*(a^2-1)+2*(a+3)*b*x-b^2)/x^2*y(x),y(x), singsol=all
```

$$y(x) = c_1 \operatorname{WhittakerM}\left(\frac{b(a+3)}{2\sqrt{a^2-1}}, \frac{\sqrt{b^2+1}}{2}, \sqrt{a^2-1}\,x\right) + c_2 \operatorname{WhittakerW}\left(\frac{b(a+3)}{2\sqrt{a^2-1}}, \frac{\sqrt{b^2+1}}{2}, \sqrt{a^2-1}\,x\right)$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 96

```
DSolve[y''[x] == -1/4*((-b^2 + 2*(3 + a)*b*x - (-1 + a^2)*x^2)*y[x])/x^2,y[x],x,IncludeSingu
```

$$y(x) \to c_1 M_{\frac{(a+3)b}{2\sqrt{a^2-1}},\frac{\sqrt{b^2+1}}{2}}\left(\sqrt{a^2-1}x\right) + c_2 W_{\frac{(a+3)b}{2\sqrt{a^2-1}},\frac{\sqrt{b^2+1}}{2}}\left(\sqrt{a^2-1}x\right)$$

3.379 problem 1385

Internal problem ID [9713]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1385.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Halm]`

$$y'' + \frac{(a\,x^2 + a - 3)\,y}{4\,(x^2+1)^2} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 55

```
dsolve(diff(diff(y(x),x),x) = -1/4*(a*x^2+a-3)/(x^2+1)^2*y(x),y(x), singsol=all)
```

$$y(x) = (x^2+1)^{\frac{1}{4}} \left(\left(x + \sqrt{x^2+1}\right)^{\frac{\sqrt{-a+1}}{2}} c_1 + \left(x + \sqrt{x^2+1}\right)^{-\frac{\sqrt{-a+1}}{2}} c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 70

```
DSolve[y''[x] == -1/4*((-3 + a + a*x^2)*y[x])/(1 + x^2)^2,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \sqrt{x^2+1} \left(c_1 P_{\frac{1}{2}\left(\sqrt{1-a}-1\right)}^{\frac{1}{2}}(ix) + c_2 Q_{\frac{1}{2}\left(\sqrt{1-a}-1\right)}^{\frac{1}{2}}(ix) \right)$$

3.380 problem 1386

Internal problem ID [9714]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1386.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{18y}{(2x+1)^2 (x^2+x+1)} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 58

```
dsolve(diff(diff(y(x),x),x) = 18/(2*x+1)^2/(x^2+x+1)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{-36c_2(x^2+x+1)\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right) + 16c_2\left(x^3+x^2+\frac{11}{8}x+\frac{3}{16}\right)\sqrt{3} + c_1(x^2+x+1)}{(2x+1)^2}$$

✓ Solution by Mathematica
Time used: 0.14 (sec). Leaf size: 68

```
DSolve[y''[x] == (18*y[x])/((1 + 2*x)^2*(1 + x + x^2)),y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{-12\sqrt{3}c_2(x^2+x+1)\arctan\left(\frac{2x+1}{\sqrt{3}}\right) + c_1(x^2+x+1) + c_2(16x^3+24x^2+30x+11)}{(2x+1)^2}$$

3.381 problem 1387

Internal problem ID [9715]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1387.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$y'' - \frac{3y}{4\left(x^2+x+1\right)^2} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 28

```
dsolve(diff(diff(y(x),x),x) = 3/4/(x^2+x+1)^2*y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{x^2+x+1}\left(\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)c_2 + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 45

```
DSolve[y''[x] == (3*y[x])/(4*(1 + x + x^2)^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\sqrt{x^2+x+1}\left(2\sqrt{3}c_2 \arctan\left(\frac{2x+1}{\sqrt{3}}\right) + 3c_1\right)$$

3.382 problem 1388

Internal problem ID [9716]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1388.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(-1+3x)\,y'}{2x\,(x-1)} + \frac{(v(x-1)\,(v+1)-a^2x)\,y}{4x^2\,(x-1)^2} = 0$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 77

```
dsolve(diff(diff(y(x),x),x) = -1/2/x*(3*x-1)/(x-1)*diff(y(x),x)-1/4*(v*(v+1)*(x-1)-a^2*x)/x^
```

$$y(x) = (x-1)^{-\frac{a}{2}}\left(\text{hypergeom}\left(\left[-\frac{v}{2}-\frac{a}{2},\frac{1}{2}-\frac{v}{2}-\frac{a}{2}\right],\left[\frac{1}{2}-v\right],x\right)x^{-\frac{v}{2}}c_1 + c_2\sqrt{x}\,x^{\frac{v}{2}}\,\text{hypergeom}\left(\left[1+\frac{v}{2}-\frac{a}{2},\frac{1}{2}+\frac{v}{2}-\frac{a}{2}\right],\left[\frac{3}{2}+v\right],x\right)\right)$$

✓ Solution by Mathematica

Time used: 0.342 (sec). Leaf size: 109

```
DSolve[y''[x] == -1/4*((v*(1 + v)*(-1 + x) - a^2*x)*y[x])/((-1 + x)^2*x^2) - ((-1 + 3*x)*y'[
```

$$y(x) \to \frac{(-1)^{-v}(x-1)^{\frac{a+1}{2}}x^{-v/2}\left(c_1(-1)^v x^{v+\frac{1}{2}}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(a+v+1),\frac{1}{2}(a+v+2),v+\frac{3}{2},x\right)-ic_2\,\text{H}\right.}{\sqrt{1-x}}$$

3.383 problem 1389

Internal problem ID [9717]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1389.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(-1+3x)\,y'}{2x\,(x-1)} + \frac{\left(-v(v+1)\,(x-1)^2 - 4n^2x\right)y}{4x^2\,(x-1)^2} = 0$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 87

```
dsolve(diff(diff(y(x),x),x) = -1/2/x*(3*x-1)/(x-1)*diff(y(x),x)-1/4*(-v*(v+1)*(x-1)^2-4*n^2*
```

$$y(x) = \frac{(1-x)^{n-\frac{1}{2}}\,(x-1)^{-n}\,x^{\frac{1}{4}}\left(\Gamma\left(v+\frac{1}{2}\right)^2 c_2\left(v+\frac{1}{2}\right)\text{LegendreP}\left(n-\frac{1}{2},-v-\frac{1}{2},\frac{-x-1}{x-1}\right)+\sec(\pi v)\,\text{LegendreP}\,(\right.}{\Gamma\left(v+\frac{1}{2}\right)}$$

✓ Solution by Mathematica

Time used: 0.4 (sec). Leaf size: 91

```
DSolve[y''[x] == -1/4*((-(v*(1 + v)*(-1 + x)^2) - 4*n^2*x)*y[x])/((-1 + x)^2*x^2) - ((-1 + 3
```

$$y(x) \to \frac{(-1)^{-v}(x-1)^{n+\frac{1}{2}}x^{-v/2}\left(c_1(-1)^v x^{v+\frac{1}{2}}\,\text{Hypergeometric2F1}\left(n+\frac{1}{2},n+v+1,v+\frac{3}{2},x\right)-ic_2\,\text{Hypergeor}\right.}{\sqrt{1-x}}$$

3.384 problem 1390

Internal problem ID [9718]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1390.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{3y}{16x^2 (x-1)^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(diff(y(x),x),x) = -3/16/x^2/(x-1)^2*y(x),y(x), singsol=all)
```

$$y(x) = c_1 (x-1)^{\frac{1}{4}} x^{\frac{3}{4}} + c_2 (x-1)^{\frac{3}{4}} x^{\frac{1}{4}}$$

✓ Solution by Mathematica

Time used: 0.073 (sec). Leaf size: 41

```
DSolve[y''[x] == (-3*y[x])/(16*(-1 + x)^2*x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2c_2 \sqrt[4]{1-x} x^{3/4} + c_1 (1-x)^{3/4} \sqrt[4]{x}$$

3.385 problem 1391

Internal problem ID [9719]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1391.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{(7a\,x^2+5)\,y'}{x\,(a\,x^2+1)} + \frac{(15a\,x^2+5)\,y}{x^2\,(a\,x^2+1)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(diff(y(x),x),x) = 1/x*(7*a*x^2+5)/(a*x^2+1)*diff(y(x),x)-(15*a*x^2+5)/x^2/(a*x^2
```

$$y(x) = c_1x^5 + 2ac_2x^3 + xc_2$$

✓ Solution by Mathematica

Time used: 0.064 (sec). Leaf size: 27

```
DSolve[y''[x] == -(((5 + 15*a*x^2)*y[x])/(x^2*(1 + a*x^2))) + ((5 + 7*a*x^2)*y'[x])/(x*(1 +
```

$$y(x) \to c_1x^5 - \frac{1}{4}c_2x\left(2ax^2+1\right)$$

3.386 problem 1392

Internal problem ID [9720]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1392.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{bxy'}{(x^2-1)\,a} + \frac{(c\,x^2+dx+e)\,y}{a\,(x^2-1)^2} = 0$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 562

```
dsolve(diff(diff(y(x),x),x) = -b*x/(x^2-1)/a*diff(y(x),x)-(c*x^2+d*x+e)/a/(x^2-1)^2*y(x),y(x
```

$$y(x) = \frac{(x^2-1)^{-\frac{b}{4a}}\sqrt{2x+2}\left(c_1\,\mathrm{hypergeom}\left(\left[\frac{\sqrt{4a^2+(-4b-4c-4d-4e)a+b^2}-2\sqrt{a^2+(-2b-4c)a+b^2}-\sqrt{4a^2+(-4b-4c+4d-4e)a+b}}{4a}\right.\right.\right.}{}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x] == -(((e + d*x + c*x^2)*y[x])/(a*(-1 + x^2)^2)) - (b*x*y'[x])/(a*(-1 + x^2)),y
```

Timed out

3.387 problem 1393

Internal problem ID [9721]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1393.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(b\,x^2 + cx + d)\,y}{a\,x^2\,(x-1)^2} = 0$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 267

```
dsolve(diff(diff(y(x),x),x) = -(b*x^2+c*x+d)/a/x^2/(x-1)^2*y(x),y(x), singsol=all)
```

$$y(x) = (x-1)^{-\frac{\sqrt{a-4b-4c-4d}-\sqrt{a}}{2\sqrt{a}}} \left(c_2 x^{-\frac{-\sqrt{a}+\sqrt{a-4d}}{2\sqrt{a}}} \operatorname{hypergeom}\left(\left[-\frac{\sqrt{a-4b-4c-4d}-\sqrt{a}+\sqrt{a-4d}+\sqrt{a-4b}}{2\sqrt{a}}, -\right.\right.\right.$$
$$+ c_1 x^{\frac{\sqrt{a}+\sqrt{a-4d}}{2\sqrt{a}}} \operatorname{hypergeom}\left(\left[\frac{-\sqrt{a-4b-4c-4d}+\sqrt{a}+\sqrt{a-4d}+\sqrt{a-4b}}{2\sqrt{a}}, -\frac{\sqrt{a-4b-4c-4d}-}{}\right.\right.$$

✓ Solution by Mathematica

Time used: 172.576 (sec). Leaf size: 413606

```
DSolve[y''[x] == -(((d + c*x + b*x^2)*y[x])/(a*(-1 + x)^2*x^2)),y[x],x,IncludeSingularSoluti
```

Too large to display

3.388 problem 1394

Internal problem ID [9722]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1394.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{2y'}{x} + \frac{cy}{x^2 (xa + b)^2} = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 79

```
dsolve(diff(diff(y(x),x),x) = -2/x*diff(y(x),x)-c/x^2/(a*x+b)^2*y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{\frac{ax+b}{x}}\left(\left(\frac{x}{ax+b}\right)^{\frac{\sqrt{\frac{b^2-4c}{a^2}}\,a}{2b}} c_1 + \left(\frac{x}{ax+b}\right)^{-\frac{\sqrt{\frac{b^2-4c}{a^2}}\,a}{2b}} c_2\right)$$

✓ Solution by Mathematica
Time used: 2.093 (sec). Leaf size: 73

```
DSolve[y''[x] == -((c*y[x])/(x^2*(b + a*x)^2)) - (2*y'[x])/x,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \left(c_2 e^{\frac{\sqrt{b^2-4c}(\log(x)-\log(ax+b))}{b}} + c_1\right)\exp\left(-\frac{\left(\sqrt{b^2-4c}+b\right)(\log(x)-\log(ax+b))}{2b}\right)$$

3.389 problem 1395

Internal problem ID [9723]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1395.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + \frac{y}{(xa+b)^4} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(diff(y(x),x),x) = -1/(a*x+b)^4*y(x),y(x), singsol=all)
```

$$y(x) = (ax+b)\left(c_1 \sin\left(\frac{1}{a(ax+b)}\right) + c_2 \cos\left(\frac{1}{a(ax+b)}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.138 (sec). Leaf size: 57

```
DSolve[y''[x] == -(y[x]/(b + a*x)^4),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-\frac{i}{a(ax+b)}}(ax+b)\left(2c_1 e^{\frac{2i}{a(ax+b)}} - ic_2\right)$$

3.390 problem 1396

Internal problem ID [9724]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1396.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$y'' + \frac{Ay}{(a\,x^2 + bx + c)^2} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 178

```
dsolve(diff(diff(y(x),x),x) = -A/(a*x^2+b*x+c)^2*y(x),y(x), singsol=all)
```

$$y(x) = \left(\left(\frac{-b + i\sqrt{4ac - b^2} - 2ax}{b + i\sqrt{4ac - b^2} + 2ax}\right)^{-\frac{a\sqrt{\frac{-4ac+b^2-4A}{a^2}}}{2\sqrt{-4ac+b^2}}} c_2 + \left(\frac{-b + i\sqrt{4ac - b^2} - 2ax}{b + i\sqrt{4ac - b^2} + 2ax}\right)^{\frac{a\sqrt{\frac{-4ac+b^2-4A}{a^2}}}{2\sqrt{-4ac+b^2}}} c_1\right)\sqrt{a\,x^2 + bx + c}$$

✓ Solution by Mathematica

Time used: 1.298 (sec). Leaf size: 199

```
DSolve[y''[x] == -((A*y[x])/(c + b*x + a*x^2)^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x(ax+b)+c}\exp\left(-\frac{\sqrt{4ac-b^2}\sqrt{1-\frac{4A}{b^2-4ac}}\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{b^2-4ac}}\right)\left(c_1\exp\left(\frac{2\sqrt{4ac-b^2}\sqrt{1-\frac{4A}{b^2-4ac}}\,a}{\sqrt{b^2-4a}}\right.\right.$$

$$\left.+\frac{c_2}{\sqrt{b^2-4ac}\sqrt{1-\frac{4A}{b^2-4ac}}}\right)$$

3.391 problem 1397

Internal problem ID [9725]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1397.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{y'}{x^4} - \frac{y}{x^5} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(diff(y(x),x),x) = -1/x^4*diff(y(x),x)+1/x^5*y(x),y(x), singsol=all)
```

$$y(x) = x\left(-\frac{3c_2\Gamma\left(\frac{1}{3}, -\frac{1}{3x^3}\right)\Gamma\left(\frac{2}{3}\right)}{2} + c_2\sqrt{3}\,\pi + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.088 (sec). Leaf size: 38

```
DSolve[y''[x] == y[x]/x^5 - y'[x]/x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2\Gamma\left(\frac{1}{3}, -\frac{1}{3x^3}\right)}{3^{2/3}\sqrt[3]{-\frac{1}{x^3}}} + c_1 x$$

3.392 problem 1398

Internal problem ID [9726]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1398.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(3x^2-1)\,y'}{(x^2-1)\,x} + \frac{\left(x^2-1-(2v+1)^2\right)y}{(x^2-1)^2} = 0$$

✓ Solution by Maple
Time used: 0.297 (sec). Leaf size: 69

```
dsolve(diff(diff(y(x),x),x) = -1/(x^2-1)*(3*x^2-1)/x*diff(y(x),x)-(x^2-1-(2*v+1)^2)/(x^2-1)^
```

$$y(x) = c_1\left(x^2-1\right)^{-v-\frac{1}{2}} \operatorname{hypergeom}\left([-v,-v],[-2v],-x^2+1\right) + c_2\left(x^2-1\right)^{v+\frac{1}{2}} \operatorname{hypergeom}\left([v+1,v+1],[2v+2],-x^2+1\right)$$

✓ Solution by Mathematica
Time used: 0.22 (sec). Leaf size: 72

```
DSolve[y''[x] == -(((-1 - (1 + 2*v)^2 + x^2)*y[x])/(-1 + x^2)^2) - ((-1 + 3*x^2)*y'[x])/(x*(
```

$$y(x) \to c_1\left(x^2-1\right)^{-v-\frac{1}{2}} \text{Hypergeometric2F1}\left(-v,-v,-2v,1-x^2\right) + c_2\left(x^2-1\right)^{v+\frac{1}{2}} \text{Hypergeometric2F1}\left(v+1,v+1,2v+2,1-x^2\right)$$

3.393 problem 1399

Internal problem ID [9727]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1399.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{(3x+1)\,y'}{(x-1)\,(x+1)} + \frac{36(x+1)^2\,y}{(x-1)^2\,(3x+5)^2} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve(diff(diff(y(x),x),x) = 1/(x-1)*(3*x+1)/(x+1)*diff(y(x),x)-36*(x+1)^2/(x-1)^2/(3*x+5)^
```

$$y(x) = (x-1)^{\frac{3}{2}}\,\sqrt{3x+5}\,(3\ln(x-1)\,c_2 + \ln(3x+5)\,c_2 + c_1)$$

✓ Solution by Mathematica

Time used: 2.097 (sec). Leaf size: 51

```
DSolve[y''[x] == (-36*(1 + x)^2*y[x])/((-1 + x)^2*(5 + 3*x)^2) + ((1 + 3*x)*y'[x])/((-1 + x)
```

$$y(x) \to \frac{1}{2}(1-x)^{3/2}\sqrt{3x+5}(3c_2\log(1-x) + c_2\log(3x+5) + 2c_1)$$

3.394 problem 1400

Internal problem ID [9728]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1400.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$y'' - \frac{y'}{x} + \frac{ay}{x^6} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
dsolve(diff(diff(y(x),x),x) = 1/x*diff(y(x),x)-a/x^6*y(x),y(x), singsol=all)
```

$$y(x) = x^2\left(c_1 \sinh\left(\frac{\sqrt{-a}}{2x^2}\right) + c_2 \cosh\left(\frac{\sqrt{-a}}{2x^2}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.152 (sec). Leaf size: 58

```
DSolve[y''[x] == -((a*y[x])/x^6) + y'[x]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}x^2 e^{-\frac{i\sqrt{a}}{2x^2}}\left(2c_1 e^{\frac{i\sqrt{a}}{x^2}} - \frac{ic_2}{\sqrt{a}}\right)$$

3.395 problem 1401

Internal problem ID [9729]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1401.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(3x^2 + a)\, y'}{x^3} + \frac{by}{x^6} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(diff(y(x),x),x) = -1/x^3*(3*x^2+a)*diff(y(x),x)-b/x^6*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{-a+\sqrt{a^2-4b}}{4x^2}} + c_2 \mathrm{e}^{\frac{a+\sqrt{a^2-4b}}{4x^2}}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 56

```
DSolve[y''[x] == -((b*y[x])/x^6) - ((a + 3*x^2)*y'[x])/x^3,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to e^{\frac{a-\sqrt{a^2-4b}}{4x^2}} \left(c_1 e^{\frac{\sqrt{a^2-4b}}{2x^2}} + c_2 \right)$$

3.396 problem 1402

Internal problem ID [9730]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1402.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{((1-4a)\,x^2-1)\,y'}{x\,(x^2-1)} + \frac{\left((-v^2+x^2)\,(x^2-1)^2+4a(1+a)\,x^4-2a\,x^2(x^2-1)\right)y}{x^2\,(x^2-1)^2} = 0$$

✓ Solution by Maple

Time used: 0.984 (sec). Leaf size: 56

```
dsolve(diff(diff(y(x),x),x) = -1/x/(x^2-1)*((1-4*a)*x^2-1)*diff(y(x),x)-((-v^2+x^2)*(x^2-1)^
```

$$y(x) = -(x^2-1)^{a+1}\left(x^v\,\mathrm{HeunC}\left(0, v, 1, \frac{1}{4}, \frac{a}{2}+\frac{1}{4}, x^2\right)c_1 + x^{-v}\,\mathrm{HeunC}\left(0, -v, 1, \frac{1}{4}, \frac{a}{2}+\frac{1}{4}, x^2\right)c_2\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x] == -(((4*a*(1 + a)*x^4 - 2*a*x^2*(-1 + x^2) + (-1 + x^2)^2*(-v^2 + x^2))*y[x])
```

Not solved

3.397 problem 1403

Internal problem ID [9731]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1403.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(\frac{1 - \mathrm{a1} - \mathrm{b1}}{x - \mathrm{c1}} + \frac{1 - \mathrm{a2} - \mathrm{b2}}{x - \mathrm{c2}} + \frac{1 - \mathrm{a3} - \mathrm{b3}}{x - \mathrm{c3}}\right) y' + \frac{\left(\frac{\mathrm{a1}\,\mathrm{b1}(\mathrm{c1}-\mathrm{c3})(\mathrm{c1}-\mathrm{c2})}{x-\mathrm{c1}} + \frac{\mathrm{a2}\,\mathrm{b2}(\mathrm{c2}-\mathrm{c1})(\mathrm{c2}-\mathrm{c3})}{x-\mathrm{c2}} + \frac{\mathrm{a3}\,\mathrm{b3}(\mathrm{c3}}{}\right.}{(x-\mathrm{c1})(x-\mathrm{c2})(x-\mathrm{c3})}$$

✓ Solution by Maple

Time used: 0.672 (sec). Leaf size: 298

```
dsolve(diff(diff(y(x),x),x) = -((1-a1-b1)/(x-c1)+(1-a2-b2)/(x-c2)+(1-a3-b3)/(x-c3))*diff(y(x
```

$$y(x) = (x-\mathrm{c2})^{\mathrm{a2}} (x - \mathrm{c3})^{\mathrm{b3}} \left(c_1 \operatorname{HeunG}\left(\frac{\mathrm{c1}-\mathrm{c3}}{\mathrm{c1}-\mathrm{c2}}, \frac{((-\mathrm{a3}-2\,\mathrm{b1}-\mathrm{b2}+2)\,\mathrm{c1}+(\mathrm{a3}+\mathrm{b1}-1)\,\mathrm{c2}+\mathrm{c3}\,(\mathrm{b1}+\mathrm{b2}-1))\,\mathrm{a1}-(\mathrm{b1}-1}{} + \mathrm{b3}+\mathrm{a2}, 2-\mathrm{a3}-\mathrm{b1}-\mathrm{b2}, \mathrm{a1}-\mathrm{b1}+1, \mathrm{a2}-\mathrm{b2}+1, \frac{-x+\mathrm{c1}}{\mathrm{c1}-\mathrm{c2}}\right)(x-\mathrm{c1})^{\mathrm{a1}} + c_2 \operatorname{HeunG}\left(\frac{\mathrm{c1}-\mathrm{c3}}{\mathrm{c1}-\mathrm{c2}}, \frac{((-2\,\mathrm{a1}-\mathrm{a3}-\mathrm{b2}+2)\,\mathrm{c1}+(\mathrm{a1}+\mathrm{a3}-1)\,\mathrm{c2}+\mathrm{c3}\,(\mathrm{a1}+\mathrm{b2}-1))\,\mathrm{b1}-(\mathrm{a2}+\mathrm{b3})\,(\mathrm{a1}}{\mathrm{c1}-\mathrm{c2}} + \mathrm{b3}+\mathrm{a2}, 2-\mathrm{a1}-\mathrm{a3}-\mathrm{b2}, -\mathrm{a1}+\mathrm{b1}+1, \mathrm{a2}-\mathrm{b2}+1, \frac{-x+\mathrm{c1}}{\mathrm{c1}-\mathrm{c2}}\right)(x-\mathrm{c1})^{\mathrm{b1}}\right)$$

✓ Solution by Mathematica

Time used: 17.454 (sec). Leaf size: 293

```
DSolve[y''[x] == -((((a1*b1*(c1 - c2)*(c1 - c3))/(-c1 + x) + (a2*b2*(-c1 + c2)*(c2 - c3))/(-
```

$$y(x) \to (x - \mathrm{c2})^{\mathrm{a2}}(x - \mathrm{c3})^{\mathrm{b3}} \Bigg(c_1(x - \mathrm{c1})^{\mathrm{a1}} \mathrm{HeunG}\left[\frac{\mathrm{c1} - \mathrm{c3}}{\mathrm{c1} - \mathrm{c2}}, \frac{\mathrm{a1}(-(\mathrm{c1}(\mathrm{a3} + 2\mathrm{b1} + \mathrm{b2} - 2)) + \mathrm{c2}(\mathrm{a3} + \mathrm{b1} - 1) + \mathrm{c3}(\mathrm{b1} + \mathrm{b2} - 1)) + \mathrm{a2}(-\mathrm{b1}}{\mathrm{c1}}, -\mathrm{a3} - \mathrm{b1} - \mathrm{b2} + 2, \mathrm{a1} + \mathrm{a2} + \mathrm{b3}, \mathrm{a1} - \mathrm{b1} + 1, \mathrm{a2} - \mathrm{b2} + 1, \frac{\mathrm{c1} - x}{\mathrm{c1} - \mathrm{c2}}\right] + c_2(x - \mathrm{c1})^{\mathrm{b1}} \mathrm{HeunG}\left[\frac{\mathrm{c1} - \mathrm{c3}}{\mathrm{c1} - \mathrm{c2}}, \frac{\mathrm{a2}(-\mathrm{a1}\mathrm{c1} + (\mathrm{a1} - 1)\mathrm{c3} + \mathrm{b2}(\mathrm{c3} - \mathrm{c2}) + \mathrm{c1}) + \mathrm{b1}(-(\mathrm{c1}(2\mathrm{a1} + \mathrm{a3} + \mathrm{b2} - 2)) +}{\mathrm{c1}}, -\mathrm{a1} - \mathrm{a3} - \mathrm{b2} + 2, \mathrm{a2} + \mathrm{b1} + \mathrm{b3}, -\mathrm{a1} + \mathrm{b1} + 1, \mathrm{a2} - \mathrm{b2} + 1, \frac{\mathrm{c1} - x}{\mathrm{c1} - \mathrm{c2}}\right] \Bigg)$$

3.398 problem 1404

Internal problem ID [9732]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1404.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(2x^2+1)\,y'}{x^3} + \frac{(-2x^2+1)\,y}{4x^6} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(diff(y(x),x),x) = -(2*x^2+1)/x^3*diff(y(x),x)-1/4*(-2*x^2+1)/x^6*y(x),y(x), sing
```

$$y(x) = \frac{\mathrm{e}^{\frac{1}{4x^2}}(c_1x + c_2)}{x}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 25

```
DSolve[y''[x] == -1/4*((1 - 2*x^2)*y[x])/x^6 - ((1 + 2*x^2)*y'[x])/x^3,y[x],x,IncludeSingula
```

$$y(x) \rightarrow \frac{e^{\frac{1}{4x^2}}(c_2x + c_1)}{x}$$

3.399 problem 1405

Internal problem ID [9733]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1405.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{(2x^2+1)\,y'}{x^3} + \frac{(a\,x^4+10x^2+1)\,y}{4x^6} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
dsolve(diff(diff(y(x),x),x) = (2*x^2+1)/x^3*diff(y(x),x)-1/4*(a*x^4+10*x^2+1)/x^6*y(x),y(x),
```

$$y(x) = x^{\frac{3}{2}} \mathrm{e}^{-\frac{1}{4x^2}} \left(c_1 x^{\frac{\sqrt{-a+9}}{2}} + c_2 x^{-\frac{\sqrt{-a+9}}{2}}\right)$$

✓ Solution by Mathematica

Time used: 0.088 (sec). Leaf size: 70

```
DSolve[y''[x] == -1/4*((1 + 10*x^2 + a*x^4)*y[x])/x^6 + ((1 + 2*x^2)*y'[x])/x^3,y[x],x,Inclu
```

$$y(x) \to \frac{e^{-\frac{1}{4x^2}} x^{\frac{3}{2}-\frac{\sqrt{9-a}}{2}} \left(c_2 x^{\sqrt{9-a}} + \sqrt{9-a}c_1\right)}{\sqrt{9-a}}$$

3.400 problem 1406

Internal problem ID [9734]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1406.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{27xy}{16(x^3-1)^2} = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 44

```
dsolve(diff(diff(y(x),x),x) = -27/16*x/(x^3-1)^2*y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{x}\,(x^3-1)^{\frac{1}{4}} \left(c_1 \operatorname{LegendreP}\left(-\frac{1}{6}, \frac{1}{3}, \sqrt{-x^3+1}\right) + c_2 \operatorname{LegendreQ}\left(-\frac{1}{6}, \frac{1}{3}, \sqrt{-x^3+1}\right)\right)$$

✓ Solution by Mathematica

Time used: 66.431 (sec). Leaf size: 180

```
DSolve[y''[x] == (-27*x*y[x])/(16*(-1 + x^3)^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{2}(1-x)^{3/4}\sqrt[4]{x^2+x+1}\left(c_2 \int_1^x \frac{\sqrt{\sqrt{3}K[1]+\sqrt{2K[1]-i\sqrt{3}+1}\sqrt{2K[1]+i\sqrt{3}+1}+\sqrt{3}}}{2(1-K[1])^{3/2}\sqrt{K[1]^2+K[1]+1}}dK[1] + c_1\right)}{\sqrt[4]{\sqrt{3}x+\sqrt{2x-i\sqrt{3}+1}\sqrt{2x+i\sqrt{3}+1}+\sqrt{3}}}$$

3.401 problem 1407

Internal problem ID [9735]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1407.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(\frac{(1-\mathrm{al1}-\mathrm{bl1})\,\mathrm{b1}}{\mathrm{b1}\,x-\mathrm{a1}} + \frac{(1-\mathrm{al2}-\mathrm{bl2})\,\mathrm{b2}}{\mathrm{b2}\,x-\mathrm{a2}} + \frac{(1-\mathrm{al3}-\mathrm{bl3})\,\mathrm{b3}}{\mathrm{b3}\,x-\mathrm{a3}}\right) y' + \frac{\left(\frac{\mathrm{al1}\,\mathrm{bl1}(\mathrm{a1}\,\mathrm{b2}-\mathrm{a2}\,\mathrm{b1})(-\mathrm{a1}\,\mathrm{b3}+\mathrm{a3}\,\mathrm{b1})}{\mathrm{b1}\,x-\mathrm{a1}}\right.}{(}$$

✓ Solution by Maple

Time used: 0.797 (sec). Leaf size: 2220

```
dsolve(diff(diff(y(x),x),x) = -((1-al1-bl1)*b1/(b1*x-a1)+(1-al2-bl2)*b2/(b2*x-a2)+(1-al3-bl3
```

Expression too large to display

✓ Solution by Mathematica

Time used: 140.27 (sec). Leaf size: 1290

```
DSolve[y''[x] == -((((al1*(-(a2*b1) + a1*b2)*(a3*b1 - a1*b3)*bl1)/(-a1 + b1*x) + (al2*(-(a2*
```

Too large to display

3.402 problem 1408

Internal problem ID [9736]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1408.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(x^2((x^2-\mathrm{a1})(x^2-\mathrm{a2})+(x^2-\mathrm{a2})(x^2-\mathrm{a3})+(x^2-\mathrm{a3})(x^2-\mathrm{a1}))-(x^2-\mathrm{a1})(x^2-\mathrm{a2})(x^2-\mathrm{a3}}{x(x^2-\mathrm{a1})(x^2-\mathrm{a2})(x^2-\mathrm{a3})}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x) = -(x^2*((x^2-a1)*(x^2-a2)+(x^2-a2)*(x^2-a3)+(x^2-a3)*(x^2-a1))-
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x] == -(((B + A*x^2)*y[x])/(x*(-a1 + x^2)*(-a2 + x^2)*(-a3 + x^2))) - (((a1 - x^2
```

Not solved

3.403 problem 1409

Internal problem ID [9737]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1409.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$y'' + a\,x^{2a-1}x^{-2a}y' + b^2x^{-2a}y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(diff(y(x),x),x) = -a*x^(2*a-1)/(x^(2*a))*diff(y(x),x)-b^2/(x^(2*a))*y(x),y(x), s
```

$$y(x) = c_1 \sin\left(\frac{b\,x^{-a+1}}{a-1}\right) + c_2 \cos\left(\frac{b\,x^{-a+1}}{a-1}\right)$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 44

```
DSolve[y''[x] == -((b^2*y[x])/x^(2*a)) - (a*y'[x])/x,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to c_1 \cos\left(\frac{bx^{1-a}}{a-1}\right) + c_2 \sin\left(\frac{bx^{1-a}}{1-a}\right)$$

3.404 problem 1410

Internal problem ID [9738]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1410.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(ap\,x^b + q)\,y'}{x\,(a\,x^b - 1)} + \frac{(ar\,x^b + s)\,y}{x^2\,(a\,x^b - 1)} = 0$$

✓ Solution by Maple

Time used: 0.36 (sec). Leaf size: 250

```
dsolve(diff(diff(y(x),x),x) = -(a*p*x^b+q)/x/(a*x^b-1)*diff(y(x),x)-(a*r*x^b+s)/x^2/(a*x^b-1
```

$$y(x) = x^{\frac{1}{2}+\frac{q}{2}}\left(c_1 x^{\frac{\sqrt{q^2+2q+4s+1}}{2}}\,\text{hypergeom}\left(\left[\frac{p+q+\sqrt{q^2+2q+4s+1}+\sqrt{p^2-2p-4r+1}}{2b}, \frac{p+q+\sqrt{q^2+2q}}{}\right.\right.\right.$$
$$+ c_2 x^{-\frac{\sqrt{q^2+2q+4s+1}}{2}}\,\text{hypergeom}\left(\left[\frac{p+q-\sqrt{q^2+2q+4s+1}+\sqrt{p^2-2p-4r+1}}{2b}, -\frac{-p-q+\sqrt{q^2+2q}}{}\right.\right.$$

✓ Solution by Mathematica

Time used: 0.338 (sec). Leaf size: 405

```
DSolve[y''[x] == -(((s + a*r*x^b)*y[x])/(x^2*(-1 + a*x^b))) - ((q + a*p*x^b)*y'[x])/(x*(-1 +
```

$$y(x) \to c_1 i^{\frac{-\sqrt{q^2+2q+4s+1}+q+1}{b}} a^{\frac{-\sqrt{q^2+2q+4s+1}+q+1}{2b}} \left(x^b\right)^{\frac{-\sqrt{q^2+2q+4s+1}+q+1}{2b}}\,\text{Hypergeometric2F1}\left(\frac{p+q-\sqrt{p^2-2p-4r+}}{2b}\right.$$
$$\left. - \frac{\sqrt{q^2+2q+4s+1}}{b}, ax^b\right)$$
$$+ c_2 i^{\frac{\sqrt{q^2+2q+4s+1}+q+1}{b}} a^{\frac{\sqrt{q^2+2q+4s+1}+q+1}{2b}} \left(x^b\right)^{\frac{\sqrt{q^2+2q+4s+1}+q+1}{2b}}\,\text{Hypergeometric2F1}\left(\frac{p+q-\sqrt{p^2-2p-4r+1}}{2b}\right.$$

3.405 problem 1411

Internal problem ID [9739]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1411.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{y'' - \frac{y}{1+e^x} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(diff(y(x),x),x) = 1/(exp(x)+1)*y(x),y(x), singsol=all)
```

$$y(x) = e^{-x}(c_1(e^x + 1)\ln(e^x + 1) + e^x c_2 + c_1 + c_2)$$

✓ Solution by Mathematica
Time used: 0.456 (sec). Leaf size: 36

```
DSolve[y''[x] == y[x]/(1 + E^x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_1(e^x + 1) + c_2(e^x + 1)\log(e^x + 1) + c_2)$$

3.406 problem 1412

Internal problem ID [9740]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1412.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$y'' - \frac{y'}{x \ln(x)} - \ln(x)^2 y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(diff(y(x),x),x) = 1/x/ln(x)*diff(y(x),x)+ln(x)^2*y(x),y(x), singsol=all)
```

$$y(x) = \sinh\left(\left(-1 + \ln(x)\right) x\right) c_1 + \cosh\left(\left(-1 + \ln(x)\right) x\right) c_2$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 29

```
DSolve[y''[x] == Log[x]^2*y[x] + y'[x]/(x*Log[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cosh(x(\log(x) - 1)) + ic_2 \sinh(x(\log(x) - 1))$$

3.407 problem 1413

Internal problem ID [9741]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1413.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{y'}{x(\ln(x) - 1)} + \frac{y}{x^2(\ln(x) - 1)} = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 12

```
dsolve(diff(diff(y(x),x),x) = 1/x/(ln(x)-1)*diff(y(x),x)-1/x^2/(ln(x)-1)*y(x),y(x), singsol=
```

$$y(x) = c_1x + \ln(x)\, c_2$$

✓ Solution by Mathematica

Time used: 0.104 (sec). Leaf size: 16

```
DSolve[y''[x] == -(y[x]/(x^2*(-1 + Log[x]))) + y'[x]/(x*(-1 + Log[x])),y[x],x,IncludeSingula
```

$$y(x) \to c_1x - c_2\log(x)$$

3.408 problem 1414

Internal problem ID [9742]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1414.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(-a^2 \sinh(x)^2 - n(n-1))\, y}{\sinh(x)^2} = 0$$

✓ Solution by Maple

Time used: 0.578 (sec). Leaf size: 82

```
dsolve(diff(diff(y(x),x),x) = -(-a^2*sinh(x)^2-n*(n-1))/sinh(x)^2*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\sinh(x)^{n+\frac{1}{2}} \sqrt{\cosh(x)} \left(\text{hypergeom}\left(\left[\frac{1}{2} - \frac{a}{2} + \frac{n}{2}, \frac{1}{2} + \frac{a}{2} + \frac{n}{2}\right], \left[\frac{3}{2}\right], \frac{\cosh(2x)}{2} + \frac{1}{2}\right) \cosh(x)\, c_2 + \text{hypergeom}\right.}{\sqrt{\sinh(2x)}}$$

✓ Solution by Mathematica

Time used: 1.203 (sec). Leaf size: 127

```
DSolve[y''[x] == -(Csch[x]^2*((1 - n)*n - a^2*Sinh[x]^2)*y[x]),y[x],x,IncludeSingularSolutio
```

$$y(x) \to \frac{(-1)^{-n} \left(-\text{sech}^2(x)\right)^{a/2} \tanh^2(x)^{-\frac{n}{2}-\frac{1}{4}} \left(c_1(-1)^n \tanh^2(x)^{n+\frac{1}{2}} \text{Hypergeometric2F1}\left(\frac{a+n}{2}, \frac{1}{2}(a+n+1), n\right.\right.}{\sqrt{\tanh(}}$$

3.409 problem 1415

Internal problem ID [9743]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1415.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$
\boxed{y'' + \frac{2n\cosh(x)\,y'}{\sinh(x)} + \left(-a^2+n^2\right)y = 0}
$$

✓ Solution by Maple

Time used: 0.234 (sec). Leaf size: 36

```
dsolve(diff(diff(y(x),x),x) = -2*n/sinh(x)*cosh(x)*diff(y(x),x)-(-a^2+n^2)*y(x),y(x), singso
```

$$
y(x) = \sinh(x)^{-n+\frac{1}{2}}\left(c_1 \operatorname{LegendreP}\left(a-\frac{1}{2}, n-\frac{1}{2}, \cosh(x)\right) + c_2 \operatorname{LegendreQ}\left(a-\frac{1}{2}, n-\frac{1}{2}, \cosh(x)\right)\right)
$$

✓ Solution by Mathematica

Time used: 1.034 (sec). Leaf size: 145

```
DSolve[y''[x] == (a^2 - n^2)*y[x] - 2*n*Coth[x]*y'[x],y[x],x,IncludeSingularSolutions -> Tru
```

$$
y(x) \to (-1)^{-n}\left(-\operatorname{sech}^2(x)\right)^{\frac{a+1}{2}} \tanh^{-n-\frac{1}{2}}(x)\tanh^2(x)^{-\frac{n}{2}-\frac{1}{4}}\operatorname{sech}^2(x)^{\frac{n-1}{2}}\left(c_1(-1)^n\tanh^2(x)^{n+\frac{1}{2}}\operatorname{Hypergeometric}\right.
$$
$$
\left. + \frac{1}{2}, \tanh^2(x)\right)
$$
$$
\left. + ic_2\tanh^2(x)\operatorname{Hypergeometric2F1}\left(\frac{1}{2}(a-n+1), \frac{1}{2}(a-n+2), \frac{3}{2}-n, \tanh^2(x)\right)\right)
$$

3.410 problem 1416

Internal problem ID [9744]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1416.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(1+2n)\cos(x)\, y'}{\sin(x)} + (v+n+1)(v-n)\, y = 0$$

✓ Solution by Maple

Time used: 5.375 (sec). Leaf size: 26

```
dsolve(diff(diff(y(x),x),x) = -(2*n+1)*cos(x)/sin(x)*diff(y(x),x)-(v+n+1)*(v-n)*y(x),y(x), s
```

$$y(x) = \sin(x)^{-n} (c_1 \operatorname{LegendreP}(v, n, \cos(x)) + c_2 \operatorname{LegendreQ}(v, n, \cos(x)))$$

✓ Solution by Mathematica

Time used: 0.236 (sec). Leaf size: 35

```
DSolve[y''[x] == (n - v)*(1 + n + v)*y[x] - (1 + 2*n)*Cot[x]*y'[x],y[x],x,IncludeSingularSol
```

$$y(x) \to \left(-\sin^2(x)\right)^{-n/2} \left(c_1 P_v^n(\cos(x)) + c_2 Q_v^n(\cos(x))\right)$$

3.411 problem 1417

Internal problem ID [9745]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1417.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(\sin(x)^2 - \cos(x))\, y'}{\sin(x)} + y \sin(x)^2 = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 31

```
dsolve(diff(diff(y(x),x),x) = -(sin(x)^2-cos(x))/sin(x)*diff(y(x),x)-y(x)*sin(x)^2,y(x), sin
```

$$y(x) = e^{\frac{\cos(x)}{2}} \left(c_1 \sin\left(\frac{\sqrt{3}\, \cos(x)}{2}\right) + c_2 \cos\left(\frac{\sqrt{3}\, \cos(x)}{2}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.221 (sec). Leaf size: 45

```
DSolve[y''[x] == -(Sin[x]^2*y[x]) - Csc[x]*(-Cos[x] + Sin[x]^2)*y'[x],y[x],x,IncludeSingular
```

$$y(x) \to e^{\frac{\cos(x)}{2}} \left(c_1 \cos\left(\frac{1}{2}\sqrt{3}\cos(x)\right) + c_2 \sin\left(\frac{1}{2}\sqrt{3}\cos(x)\right)\right)$$

3.412 problem 1418

Internal problem ID [9746]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1418.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{x\sin(x)\,y'}{\cos(x)\,x - \sin(x)} - \frac{\sin(x)\,y}{\cos(x)\,x - \sin(x)} = 0}$$

✓ Solution by Maple

Time used: 0.36 (sec). Leaf size: 47

```
dsolve(diff(diff(y(x),x),x) = -x*sin(x)/(cos(x)*x-sin(x))*diff(y(x),x)+sin(x)/(cos(x)*x-sin(
```

$$y(x) = \sin(x)\left(c_1 + c_2\left(\int e^{-\left(\int \frac{2\cos(x)\cot(x)x - 3\cos(x) + \sec(x)}{-\sin(x) + \cos(x)x} dx\right)} \cos(x)\, dx\right)\right)$$

✓ Solution by Mathematica

Time used: 0.172 (sec). Leaf size: 15

```
DSolve[y''[x] == (Sin[x]*y[x])/(x*Cos[x] - Sin[x]) - (x*Sin[x]*y'[x])/(x*Cos[x] - Sin[x]),y[
```

$$y(x) \to c_1 x + c_2 \sin(x)$$

3.413 problem 1419

Internal problem ID [9747]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1419.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(\sin(x)\, x^2 - 2\cos(x)\, x)\, y'}{x^2 \cos(x)} + \frac{(2\cos(x) - x\sin(x))\, y}{x^2 \cos(x)} = 0$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 12

```
dsolve(diff(diff(y(x),x),x) = -(sin(x)*x^2-2*cos(x)*x)/x^2/cos(x)*diff(y(x),x)-(2*cos(x)-x*s
```

$$y(x) = x(c_1 + \sin(x)\, c_2)$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x] == -((Sec[x]*(2*x*Cos[x] - x*Sin[x])*y[x])/x^2) - (Sec[x]*(-2*x*Cos[x] + x^2*S
```

Not solved

3.414 problem 1420

Internal problem ID [9748]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1420.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\cos(x)^2 y'' - \left(a\cos(x)^2 + n(n-1)\right) y = 0}$$

✓ Solution by Maple

Time used: 0.609 (sec). Leaf size: 114

```
dsolve(cos(x)^2*diff(diff(y(x),x),x)-(a*cos(x)^2+n*(n-1))*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sin(x)^{\frac{3}{2}}\left(\text{hypergeom}\left(\left[\frac{1}{2}+\frac{i\sqrt{a}}{2}+\frac{n}{2}, \frac{1}{2}-\frac{i\sqrt{a}}{2}+\frac{n}{2}\right], \left[n+\frac{1}{2}\right], \frac{\cos(2x)}{2}+\frac{1}{2}\right)\cos(x)^{n+\frac{1}{2}} c_1 + \left(\frac{\cos(2x)}{2}+\frac{1}{2}\right)^{\frac{3}{4}}\right.}{\sqrt{\sin(2x)}}$$

✓ Solution by Mathematica

Time used: 0.598 (sec). Leaf size: 126

```
DSolve[(-((-1 + n)*n) - a*Cos[x]^2)*y[x] + Cos[x]^2*y''[x] == 0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to c_1 i^{1-n}\cos^{1-n}(x)\,\text{Hypergeometric2F1}\left(\frac{1}{2}\left(-n-i\sqrt{a}+1\right), \frac{1}{2}\left(-n+i\sqrt{a}+1\right), \frac{3}{2} - n, \cos^2(x)\right) + c_2 i^n \cos^n(x)\,\text{Hypergeometric2F1}\left(\frac{1}{2}\left(n-i\sqrt{a}\right), \frac{1}{2}\left(n+i\sqrt{a}\right), n+\frac{1}{2}, \cos^2(x)\right)$$

3.415 problem 1421

Internal problem ID [9749]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1421.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{a(n-1)\sin(2xa)\,y'}{\cos(xa)^2} + \frac{n\,a^2\left((n-1)\sin(xa)^2 + \cos(xa)^2\right)y}{\cos(xa)^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(diff(y(x),x),x) = -a*(n-1)*sin(2*a*x)/cos(a*x)^2*diff(y(x),x)-n*a^2*((n-1)*sin(a
```

$$y(x) = \sec(ax)^{-n+1}\left(c_1\sin(ax) + c_2\cos(ax)\right)$$

✓ Solution by Mathematica

Time used: 0.244 (sec). Leaf size: 65

```
DSolve[y''[x] == -(a^2*n*Sec[a*x]^2*(Cos[a*x]^2 + (-1 + n)*Sin[a*x]^2)*y[x]) - a*(-1 + n)*Se
```

$$y(x) \to \frac{2^{-n}(2ac_1 - ic_2e^{2iax})\left(e^{-iax} + e^{iax}\right)^n}{a\left(1 + e^{2iax}\right)}$$

3.416 problem 1422

Internal problem ID [9750]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1422.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \frac{2y}{\sin(x)^2} = 0}$$

✓ Solution by Maple

Time used: 0.297 (sec). Leaf size: 31

```
dsolve(diff(diff(y(x),x),x) = 2/sin(x)^2*y(x),y(x), singsol=all)
```

$$y(x) = -i \cot(x) \ln(\cos(2x) + i \sin(2x)) c_2 + c_1 \cot(x) - 2c_2$$

✓ Solution by Mathematica

Time used: 0.18 (sec). Leaf size: 46

```
DSolve[y''[x] == 2*Csc[x]^2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\cos(x)\left(c_1 - c_2 \log\left(\sqrt{-\sin^2(x)} - \cos(x)\right)\right)}{\sqrt{-\sin^2(x)}} - c_2$$

3.417 problem 1423

Internal problem ID [9751]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1423.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{ay}{\sin(x)^2} = 0}$$

✓ Solution by Maple

Time used: 0.578 (sec). Leaf size: 110

```
dsolve(diff(diff(y(x),x),x) = -a/sin(x)^2*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\cos(x)}\left(-\frac{1}{2}+\frac{\cos(2x)}{2}\right)^{\frac{1}{2}+\frac{\sqrt{-4a+1}}{4}}\left(\cos(x)\,\mathrm{hypergeom}\left(\left[\frac{\sqrt{-4a+1}}{4}+\frac{3}{4}, \frac{\sqrt{-4a+1}}{4}+\frac{3}{4}\right], \left[\frac{3}{2}\right], \frac{\cos(2x)}{2}+\frac{1}{2}\right)c_2 + \right.}{\sqrt{\sin(2x)}}$$

✓ Solution by Mathematica

Time used: 0.118 (sec). Leaf size: 61

```
DSolve[y''[x] == -(a*Csc[x]^2*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[4]{-\sin^2(x)}\left(c_1 P_{-\frac{1}{2}}^{\frac{1}{2}\sqrt{1-4a}}(\cos(x)) + c_2 Q_{-\frac{1}{2}}^{\frac{1}{2}\sqrt{1-4a}}(\cos(x))\right)$$

3.418 problem 1424

Internal problem ID [9752]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1424.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' \sin(x)^2 - \left(a \sin(x)^2 + n(n-1)\right) y = 0$$

✓ Solution by Maple

Time used: 0.578 (sec). Leaf size: 102

```
dsolve(sin(x)^2*diff(diff(y(x),x),x)-(a*sin(x)^2+n*(n-1))*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\cos(x)}\left(-\frac{1}{2}+\frac{\cos(2x)}{2}\right)^{\frac{1}{4}+\frac{n}{2}}\left(\cos(x)\,\mathrm{hypergeom}\left(\left[\frac{1}{2}+\frac{i\sqrt{a}}{2}+\frac{n}{2}, \frac{1}{2}-\frac{i\sqrt{a}}{2}+\frac{n}{2}\right], \left[\frac{3}{2}\right], \frac{\cos(2x)}{2}+\frac{1}{2}\right) c_2 + \mathrm{hy}}{\sqrt{\sin(2x)}}$$

✓ Solution by Mathematica

Time used: 0.309 (sec). Leaf size: 65

```
DSolve[(-((-1 + n)*n) - a*Sin[x]^2)*y[x] + Sin[x]^2*y''[x] == 0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to \sqrt[4]{-\sin^2(x)}\left(c_1 P_{i\sqrt{a}-\frac{1}{2}}^{n-\frac{1}{2}}(\cos(x)) + c_2 Q_{i\sqrt{a}-\frac{1}{2}}^{n-\frac{1}{2}}(\cos(x))\right)$$

3.419 problem 1425

Internal problem ID [9753]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1425.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(-a^2 \cos(x)^2 - (3-2a)\cos(x) - 3 + 3a)\, y}{\sin(x)^2} = 0$$

✓ Solution by Maple

Time used: 0.812 (sec). Leaf size: 87

```
dsolve(diff(diff(y(x),x),x) = -(-a^2*cos(x)^2-(3-2*a)*cos(x)-3+3*a)/sin(x)^2*y(x),y(x), sing
```

$$y(x) = \frac{c_1(-2+(2a-1)\cos(x))\sqrt{\cos\left(\frac{x}{2}\right)}\,\sin(x)^{a-\frac{1}{2}}}{\sin\left(\frac{x}{2}\right)^{\frac{3}{2}}} + \frac{c_2\left(\frac{\cos(x)}{2}-\frac{1}{2}\right)^{\frac{a}{2}-\frac{3}{4}}\left(\frac{\cos(x)}{2}+\frac{1}{2}\right)^{\frac{3}{4}-\frac{a}{2}} \operatorname{hypergeom}\left(\left[a-\frac{1}{2}, -a-\frac{1}{2}\right], \left[\frac{3}{2}-a\right], \frac{\cos(x)}{2}+\frac{1}{2}\right)}{\sqrt{\sin(x)}}$$

✓ Solution by Mathematica

Time used: 43.509 (sec). Leaf size: 194

```
DSolve[y''[x] == (3 - 3*a + (3 - 2*a)*Cos[x] + a^2*Cos[x]^2)*Csc[x]^2*y[x],y[x],x,IncludeSin
```

$$y(x) \to \frac{c_1 \sin^2(x)^{a/2}(-2a\cos(x)+\cos(x)+2)}{1-\cos(x)} - \frac{c_2 \sin^2(x)^{-a}(1-\cos(x))^{\frac{a-1}{2}}(\cos(x)+1)^{\frac{a+1}{2}}\left(\frac{(2a-1)(\cos(x)-1)}{(2a-1)\cos(x)-2}\right)^{a-\frac{1}{2}}\left(\frac{(2a-1)(\cos(x)+1)}{(2a-1)\cos(x)-2}\right)^{a-\frac{1}{2}} \operatorname{AppellF1}\left(2a, a\right.}{4a^2-2a}$$

3.420 problem 1426

Internal problem ID [9754]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1426.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$
y'' \sin(x)^2 - \left(a^2 \cos(x)^2 + b\cos(x) + \frac{b^2}{(-3+2a)^2} + 3a + 2\right) y = 0
$$

✓ Solution by Maple

Time used: 0.719 (sec). Leaf size: 558

```
dsolve(sin(x)^2*diff(diff(y(x),x),x)-(a^2*cos(x)^2+b*cos(x)+b^2/(2*a-3)^2+3*a+2)*y(x)=0,y(x)
```

$$
y(x) = \frac{\left(\frac{\cos(x)}{2} - \frac{1}{2}\right)^{\frac{-6+4a+\sqrt{16a^4+(16b-72)a^2-48ab+4\left(\frac{9}{2}+b\right)^2}}{-12+8a}} \left(\cos\left(\frac{x}{2}\right)^{\frac{-6+4a+\sqrt{16a^4+(-16b-72)a^2+48ab+4\left(b-\frac{9}{2}\right)^2}}{-6+4a}} \operatorname{hypergeom}\left(\left[\frac{8a^2-}{}\right.\right.\right.}{}
$$

✓ Solution by Mathematica

Time used: 6.668 (sec). Leaf size: 1281

```
DSolve[(-2 - 3*a - b^2/(-3 + 2*a)^2 - b*Cos[x] - a^2*Cos[x]^2)*y[x] + Sin[x]^2*y''[x] == 0,y
```

$$
y(x) \to \frac{(-1)^{\frac{-4a^2-9}{(3-2a)^2}} 2^{-\frac{\sqrt{(3-2a)^2\left(16a^4+8(2b-9)a^2-48ba+(2b+9)^2\right)}}{2(3-2a)^2}} (\cos(x)-1)^{-\frac{-8a^2+24a+\sqrt{(3-2a)^2\left(16a^4+8(2b-9)a^2-48ba+(2b+9)^2\right)}-18}{4(3-2a)^2}} (\cos}{}
$$

3.421 problem 1427

Internal problem ID [9755]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1427.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{\left(-\left(a^2b^2 - (1+a)^2\right)\sin(x)^2 - a(1+a)\,b\sin(2x) - a(-1+a)\right)y}{\sin(x)^2} = 0$$

✓ Solution by Maple

Time used: 0.594 (sec). Leaf size: 98

```
dsolve(diff(diff(y(x),x),x) = -(-(a^2*b^2-(a+1)^2)*sin(x)^2-a*(a+1)*b*sin(2*x)-a*(a-1))/sin(
```

$$y(x) = \left(c_2(\cot(x)+i)^{\frac{1}{2}+\frac{1}{2}a+\frac{1}{2}iab}\,\text{hypergeom}\left([iab-a+1, a(ib+1)], [iab+a+2], \frac{1}{2} - \frac{i\cot(x)}{2}\right) + c_1(\cot(x)+i)^{-\frac{1}{2}-\frac{1}{2}iab-\frac{1}{2}a}(b+\cot(x))\right)(\cot(x)-i)^{-\frac{1}{2}+\frac{1}{2}iab-\frac{1}{2}a}$$

✓ Solution by Mathematica

Time used: 1.502 (sec). Leaf size: 161

```
DSolve[y''[x] == -(Csc[x]^2*((1 - a)*a - (-(1 + a)^2 + a^2*b^2)*Sin[x]^2 - a*(1 + a)*b*Sin[2
```

$$y(x) \to c_2e^{-abx}\sin^{-a}(x)\left(\csc(x) + \frac{2^{2a+1}(2a+1)e^{2ix}\left(1-e^{2ix}\right)^{2a}\left(-ie^{-ix}\left(-1+e^{2ix}\right)\right)^{-2a}\sin^{2a}(x)\,\text{Hypergeometric2F1}\left(2a+2, iba+a+1\right.}{a(b-i)-i}\right.$$
$$+ c_1e^{abx}\sin^a(x)(b\sin(x)+\cos(x))$$

3.422 problem 1428

Internal problem ID [9756]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1428.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(a\cos(x)^2 + b\sin(x)^2 + c)\, y}{\sin(x)^2} = 0$$

✓ Solution by Maple

Time used: 0.594 (sec). Leaf size: 161

```
dsolve(diff(diff(y(x),x),x) = -(a*cos(x)^2+b*sin(x)^2+c)/sin(x)^2*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\cos(x)}\left(-\frac{1}{2}+\frac{\cos(2x)}{2}\right)^{\frac{1}{2}+\frac{\sqrt{-4a+1-4c}}{4}}\left(\text{hypergeom}\left(\left[\frac{\sqrt{-4a+1-4c}}{4}+\frac{\sqrt{-a+b}}{2}+\frac{3}{4}, \frac{\sqrt{-4a+1-4c}}{4}-\frac{\sqrt{-a+b}}{2}+\frac{3}{4}\right], \left[\right.\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 0.567 (sec). Leaf size: 87

```
DSolve[y''[x] == -(Csc[x]^2*(c + a*Cos[x]^2 + b*Sin[x]^2)*y[x]),y[x],x,IncludeSingularSoluti
```

$$y(x) \to \sqrt[4]{-\sin^2(x)}\left(c_1 P_{\sqrt{b-a}-\frac{1}{2}}^{\frac{1}{2}\sqrt{-4a-4c+1}}(\cos(x)) + c_2 Q_{\sqrt{b-a}-\frac{1}{2}}^{\frac{1}{2}\sqrt{-4a-4c+1}}(\cos(x))\right)$$

3.423 problem 1429

Internal problem ID [9757]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1429.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$y'' + \frac{\cos(x) y'}{\sin(x)} - \frac{y}{\sin(x)^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(diff(y(x),x),x) = -1/sin(x)*cos(x)*diff(y(x),x)+1/sin(x)^2*y(x),y(x), singsol=al
```

$$y(x) = \csc(x) ((c_1 - c_2) \cos(x) + c_1 + c_2)$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 25

```
DSolve[y''[x] == Csc[x]^2*y[x] - Cot[x]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 - ic_2 \cos(x)}{\sqrt{\sin^2(x)}}$$

3.424 problem 1430

Internal problem ID [9758]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1430.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{\cos(x)\, y'}{\sin(x)} + \frac{\left(v(v+1)\sin(x)^2 - n^2\right) y}{\sin(x)^2} = 0$$

✓ Solution by Maple

Time used: 0.438 (sec). Leaf size: 79

```
dsolve(diff(diff(y(x),x),x) = -1/sin(x)*cos(x)*diff(y(x),x)-(v*(v+1)*sin(x)^2-n^2)/sin(x)^2*
```

$$y(x) = \left(-\frac{1}{2} + \frac{\cos(2x)}{2}\right)^{\frac{n}{2}} \left(c_1 \,\mathrm{hypergeom}\left(\left[-\frac{v}{2} + \frac{n}{2}, \frac{1}{2} + \frac{v}{2} + \frac{n}{2}\right], \left[\frac{1}{2}\right], \frac{\cos(2x)}{2} + \frac{1}{2}\right) + c_2 \cos(x) \,\mathrm{hypergeom}\left(\left[1 + \frac{v}{2} + \frac{n}{2}, \frac{1}{2} - \frac{v}{2} + \frac{n}{2}\right], \left[\frac{3}{2}\right], \frac{\cos(2x)}{2} + \frac{1}{2}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.542 (sec). Leaf size: 22

```
DSolve[y''[x] == -(Csc[x]^2*(-n^2 + v*(1 + v)*Sin[x]^2)*y[x]) - Cot[x]*y'[x],y[x],x,IncludeS
```

$$y(x) \to c_1 P_v^n(\cos(x)) + c_2 Q_v^n(\cos(x))$$

3.425 problem 1431

Internal problem ID [9759]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1431.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \frac{\cos(2x)\, y'}{\sin(2x)} + 2y = 0}$$

✓ Solution by Maple
Time used: 0.25 (sec). Leaf size: 30

```
dsolve(diff(diff(y(x),x),x) = cos(2*x)/sin(2*x)*diff(y(x),x)-2*y(x),y(x), singsol=all)
```

$$y(x) = \sin(2x)^{\frac{3}{4}} \left(c_1 \operatorname{LegendreP}\left(\frac{1}{4}, \frac{3}{4}, \cos(2x)\right) + c_2 \operatorname{LegendreQ}\left(\frac{1}{4}, \frac{3}{4}, \cos(2x)\right)\right)$$

✓ Solution by Mathematica
Time used: 20.33 (sec). Leaf size: 64

```
DSolve[y''[x] == -2*y[x] + Cot[2*x]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \to & -\frac{2}{3} c_2 \cos(2x) \cos^{\frac{3}{2}}(x) \operatorname{Hypergeometric2F1}\left(\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \cos^2(x)\right) \\ & + \frac{1}{2} c_1 \cos(2x) - 2c_2 \sin^2(x)^{3/4} \cos^{\frac{3}{2}}(x) \end{aligned}$$

3.426 problem 1432

Internal problem ID [9760]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1432.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{\cos(x)\, y'}{\sin(x)} + \frac{\left(-17\sin(x)^2 - 1\right) y}{4\sin(x)^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(diff(y(x),x),x) = -1/sin(x)*cos(x)*diff(y(x),x)-1/4*(-17*sin(x)^2-1)/sin(x)^2*y(
```

$$y(x) = \frac{c_1 \sinh(2x) + c_2 \cosh(2x)}{\sqrt{\sin(x)}}$$

✓ Solution by Mathematica

Time used: 0.11 (sec). Leaf size: 33

```
DSolve[y''[x] == -1/4*(Csc[x]^2*(-1 - 17*Sin[x]^2)*y[x]) - Cot[x]*y'[x],y[x],x,IncludeSingul
```

$$y(x) \to \frac{e^{-2x}(c_2 e^{4x} + 4c_1)}{4\sqrt{\sin(x)}}$$

3.427 problem 1433

Internal problem ID [9761]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1433.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + \frac{\sin(x)\, y'}{\cos(x)} + \frac{\left(2x^2 + \sin(x)^2 x^2 - 24\cos(x)^2\right) y}{4x^2 \cos(x)^2} = \sqrt{\cos(x)}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve(diff(diff(y(x),x),x) = -sin(x)/cos(x)*diff(y(x),x)-1/4*(2*x^2+x^2*sin(x)^2-24*cos(x)^
```

$$y(x) = \frac{\sqrt{\cos(x)}\, (4c_1 x^5 - x^4 + 4c_2)}{4x^2}$$

✓ Solution by Mathematica

Time used: 0.22 (sec). Leaf size: 35

```
DSolve[y''[x] == Sqrt[Cos[x]] - (Sec[x]^2*(2*x^2 - 24*Cos[x]^2 + x^2*Sin[x]^2)*y[x])/(4*x^2)
```

$$y(x) \to \frac{(4c_2 x^5 - 5x^4 + 20c_1)\sqrt{\cos(x)}}{20x^2}$$

3.428 problem 1434

Internal problem ID [9762]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1434.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{b\cos(x)\,y'}{\sin(x)\,a} + \frac{\left(c\cos(x)^2 + d\cos(x) + e\right)y}{a\sin(x)^2} = 0$$

✓ Solution by Maple
Time used: 0.875 (sec). Leaf size: 515

```
dsolve(diff(diff(y(x),x),x) = -b/sin(x)*cos(x)/a*diff(y(x),x)-(c*cos(x)^2+d*cos(x)+e)/a/sin(
```

$$y(x) = \frac{\sqrt{2}\,\sin(x)^{-\frac{a+b}{2a}}\sqrt{\cos(x)-1}\left(\frac{\cos(x)}{2}-\frac{1}{2}\right)^{\frac{\sqrt{a^2+(-2b-4c-4d-4e)a+b^2}}{4a}}\left(c_1\cos\left(\frac{x}{2}\right)^{-\frac{-2a+\sqrt{a^2+(-2b-4c+4d-4e)a+b^2}}{2a}}\text{hyper}\right.}{}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x] == -(((e + d*Cos[x] + c*Cos[x]^2)*Csc[x]^2*y[x])/a) - (b*Cot[x]*y'[x])/a,y[x],
```

Timed out

3.429 problem 1435

Internal problem ID [9763]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1435.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{4 \sin(3x) y}{\sin(x)^3} = 0$$

✓ Solution by Maple

Time used: 0.234 (sec). Leaf size: 38

```
dsolve(diff(diff(y(x),x),x) = -4*sin(3*x)/sin(x)^3*y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{\sin(x)}\left(c_1 \operatorname{LegendreP}\left(-\frac{1}{2} + 4i, \frac{i\sqrt{47}}{2}, \cos(x)\right) + c_2 \operatorname{LegendreQ}\left(-\frac{1}{2} + 4i, \frac{i\sqrt{47}}{2}, \cos(x)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.213 (sec). Leaf size: 61

```
DSolve[y''[x] == -4*Csc[x]^3*Sin[3*x]*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[4]{-\sin^2(x)}\left(c_1 P_{-\frac{1}{2}+4i}^{\frac{i\sqrt{47}}{2}}(\cos(x)) + c_2 Q_{-\frac{1}{2}+4i}^{\frac{i\sqrt{47}}{2}}(\cos(x))\right)$$

3.430 problem 1436

Internal problem ID [9764]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1436.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(4v(v+1)\sin(x)^2 - \cos(x)^2 + 2 - 4n^2)\,y}{4\sin(x)^2} = 0$$

✓ Solution by Maple

Time used: 0.578 (sec). Leaf size: 91

```
dsolve(diff(diff(y(x),x),x) = -1/4*(4*v*(v+1)*sin(x)^2-cos(x)^2+2-4*n^2)/sin(x)^2*y(x),y(x),
```

$$y(x) = \frac{\sqrt{\cos(x)}\left(-\frac{1}{2}+\frac{\cos(2x)}{2}\right)^{\frac{n}{2}+\frac{1}{2}}\left(c_1\,\mathrm{hypergeom}\left(\left[-\frac{v}{2}+\frac{n}{2},\frac{1}{2}+\frac{v}{2}+\frac{n}{2}\right],\left[\frac{1}{2}\right],\frac{\cos(2x)}{2}+\frac{1}{2}\right)+c_2\cos(x)\,\mathrm{hyperg}\right.}{\sqrt{\sin(2x)}}$$

✓ Solution by Mathematica

Time used: 0.66 (sec). Leaf size: 33

```
DSolve[y''[x] == -1/4*(Csc[x]^2*(2 - 4*n^2 - Cos[x]^2 + 4*v*(1 + v)*Sin[x]^2)*y[x]),y[x],x,I
```

$$y(x) \rightarrow \sqrt[4]{-\sin^2(x)}\left(c_1 P_v^n(\cos(x)) + c_2 Q_v^n(\cos(x))\right)$$

3.431 problem 1437

Internal problem ID [9765]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1437.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{(3\sin(x)^2 + 1)\, y'}{\cos(x)\sin(x)} - \frac{y\sin(x)^2}{\cos(x)^2} = 0$$

✓ Solution by Maple
Time used: 0.157 (sec). Leaf size: 29

```
dsolve(diff(diff(y(x),x),x) = (3*sin(x)^2+1)/cos(x)/sin(x)*diff(y(x),x)+sin(x)^2/cos(x)^2*y(
```

$$y(x) = c_1 \cos(x)^{-\frac{3}{2}+\frac{\sqrt{13}}{2}} + c_2 \cos(x)^{-\frac{3}{2}-\frac{\sqrt{13}}{2}}$$

✓ Solution by Mathematica
Time used: 0.37 (sec). Leaf size: 36

```
DSolve[y''[x] == Tan[x]^2*y[x] + Csc[x]*Sec[x]*(1 + 3*Sin[x]^2)*y'[x],y[x],x,IncludeSingular
```

$$y(x) \to \cos^{-\frac{3}{2}-\frac{\sqrt{13}}{2}}(x) \left(c_2 \cos^{\sqrt{13}}(x) + c_1\right)$$

3.432 problem 1438

Internal problem ID [9766]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1438.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(-a\cos(x)^2 \sin(x)^2 - m\sin(x)^2 (m-1) - n(n-1)\cos(x)^2)\, y}{\cos(x)^2 \sin(x)^2} = 0$$

✓ Solution by Maple
Time used: 0.438 (sec). Leaf size: 102

```
dsolve(diff(diff(y(x),x),x) = -(-a*cos(x)^2*sin(x)^2-m*(m-1)*sin(x)^2-n*(n-1)*cos(x)^2)/cos(
```

$$y(x) = \sin(x)^n \left(c_1 \cos(x)^m \operatorname{hypergeom}\left(\left[\frac{n}{2}+\frac{m}{2}+\frac{i\sqrt{a}}{2}, \frac{n}{2}+\frac{m}{2}-\frac{i\sqrt{a}}{2}\right], \left[\frac{1}{2}+m\right], \cos(x)^2\right) + c_2 \cos(x)^{-m+1} \operatorname{hypergeom}\left(\left[\frac{n}{2}-\frac{m}{2}+\frac{i\sqrt{a}}{2}+\frac{1}{2}, \frac{n}{2}-\frac{m}{2}-\frac{i\sqrt{a}}{2}+\frac{1}{2}\right], \left[\frac{3}{2}-m\right], \cos(x)^2\right)\right)$$

✓ Solution by Mathematica
Time used: 1.528 (sec). Leaf size: 158

```
DSolve[y''[x] == -(Csc[x]^2*Sec[x]^2*((1 - n)*n*Cos[x]^2 - (-1 + m)*m*Sin[x]^2 - a*Cos[x]^2*
```

$$y(x) \to \frac{(-1)^{-m} \cos^2(x)^{-\frac{m}{2}-\frac{1}{4}} (-\sin^2(x))^{n/2} \left(c_1 (-1)^m \cos^2(x)^{m+\frac{1}{2}} \text{Hypergeometric2F1}\left(\frac{1}{2}\left(m+n-\sqrt{-a}\right), \frac{1}{2}(n\right.\right.}{}$$

3.433 problem 1439

Internal problem ID [9767]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1439.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{\phi'(x)\,y'}{\phi(x) - \phi(a)} + \frac{\left(-n(n+1)\,(\phi(x) - \phi(a))^2 + \frac{d^2}{da^2}\phi(a)\right) y}{\phi(x) - \phi(a)} = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x) = diff(phi(x),x)/(phi(x)-phi(a))*diff(y(x),x)-(-n*(n+1)*(phi(x)-
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x] == (Derivative[1][phi][x]*y'[x])/(-phi[a] + phi[x]) - (y[x]*(-(n*(1 + n)*(-phi
```

Not solved

3.434 problem 1440

Internal problem ID [9768]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1440.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(\phi(x^3) - \phi(x)\,\phi'(x) - \phi''(x))\,y'}{\phi'(x) + \phi(x)^2} + \frac{(\phi'(x)^2 - \phi(x)^2\,\phi'(x) - \phi''(x)\,\phi(x))\,y}{\phi'(x) + \phi(x)^2} = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x) = -(phi(x^3)-phi(x)*diff(phi(x),x)-diff(diff(phi(x),x),x))/(diff
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x] == -((y'[x]*(phi[x^3] - phi[x]*Derivative[1][phi][x] - Derivative[2][phi][x]))
```

Not solved

3.435 problem 1441

Internal problem ID [9769]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1441.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{2\ \mathrm{JacobiSN}(x,k)\,\mathrm{JacobiCN}(x,k)\,\mathrm{JacobiDN}(x,k)\,y' - 2\big(1-2(k^2+1)\,\mathrm{JacobiSN}(a,k)^2 + 3k^2\,\mathrm{Jaco}}{\mathrm{JacobiSN}(x,k)^2 - \mathrm{JacobiSN}(a,k)}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x) = (2*JacobiSN(x,k)*JacobiCN(x,k)*JacobiDN(x,k)*diff(y(x),x)-2*(1
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x] == -(-JacobiSN[a, k]^2 + JacobiSN[x, k]^2)^(-1) - ((2 - 4*(1 + k^2)*JacobiSN[a
```

Not solved

3.436 problem 1442

Internal problem ID [9770]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1442.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{xy'}{f(x)} - \frac{y}{f(x)} = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 33

```
dsolve(diff(diff(y(x),x),x) = -x/f(x)*diff(y(x),x)+1/f(x)*y(x),y(x), singsol=all)
```

$$y(x) = x\left(\left(\int \mathrm{e}^{-\left(\int \frac{x^2+2f(x)}{f(x)x}dx\right)}dx\right)c_1 + c_2\right)$$

✓ Solution by Mathematica

Time used: 0.227 (sec). Leaf size: 45

```
DSolve[y''[x] == y[x]/f[x] - (x*y'[x])/f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(c_2 \int_1^x \frac{\exp\left(-\int_1^{K[2]} \frac{K[1]}{f(K[1])}dK[1]\right)}{K[2]^2}dK[2] + c_1\right)$$

3.437 problem 1443

Internal problem ID [9771]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1443.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{f'(x)\, y'}{2f(x)} + \frac{g(x)\, y}{f(x)} = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x) = -1/2*diff(f(x),x)*diff(y(x),x)/f(x)-g(x)/f(x)*y(x),y(x), sings
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x] == -((g[x]*y[x])/f[x]) - (Derivative[1][f][x]*y'[x])/(2*f[x]),y[x],x,IncludeSi
```

Not solved

3.438 problem 1445

Internal problem ID [9772]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1445.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{\left(2f(x)\, g'(x)^2 g(x) - \left(g(x)^2 - 1\right) \left(f(x)\, g''(x) + 2f'(x)\, g'(x)\right)\right) y'}{f\left(x\right) g'\left(x\right) \left(g\left(x\right)^2 - 1\right)} + \frac{\left(\left(g(x)^2 - 1\right) \left(f'(x) \left(f(x)\, g''(x) + \right.\right.\right.}{}$$

✓ Solution by Maple
Time used: 0.203 (sec). Leaf size: 20

```
dsolve(diff(diff(y(x),x),x) = -(2*f(x)*diff(g(x),x)^2*g(x)-(g(x)^2-1)*(f(x)*diff(diff(g(x),x
```

$$y(x) = f(x) \left(\mathrm{LegendreQ}\left(v, g(x)\right) c_2 + \mathrm{LegendreP}\left(v, g(x)\right) c_1\right)$$

✓ Solution by Mathematica
Time used: 0.197 (sec). Leaf size: 23

```
DSolve[y''[x] == -((y'[x]*(2*f[x]*g[x]*Derivative[1][g][x]^2 - (-1 + g[x]^2)*(2*Derivative[1
```

$$y(x) \to f(x)(c_1 \,\mathrm{LegendreP}(v, g(x)) + c_2 \,\mathrm{LegendreQ}(v, g(x)))$$

3.439 problem 1446

Internal problem ID [9773]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1446.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{y'}{x} + \frac{(x-1)\,y}{x^4} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(diff(diff(y(x),x),x) = -1/x*diff(y(x),x)-(x-1)/x^4*y(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{1}{x}}\left(c_1 + \mathrm{expIntegral}_1\left(-\frac{2}{x}\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.077 (sec). Leaf size: 26

```
DSolve[y''[x] == -(((-1 + x)*y[x])/x^4) - y'[x]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-1/x}\left(c_1 - c_2\,\mathrm{ExpIntegralEi}\left(\frac{2}{x}\right)\right)$$

3.440 problem 1447

Internal problem ID [9774]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1447.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{y'}{x} + \frac{(-x-1)\,y}{x^4} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(diff(y(x),x),x) = -1/x*diff(y(x),x)-(-x-1)/x^4*y(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{1}{x}}\left(c_1 + \mathrm{expIntegral}_1\left(\frac{2}{x}\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.068 (sec). Leaf size: 24

```
DSolve[y''[x] == -(((-1 - x)*y[x])/x^4) - y'[x]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{1}{x}}\left(c_1 - c_2\,\mathrm{ExpIntegralEi}\left(-\frac{2}{x}\right)\right)$$

3.441 problem 1448

Internal problem ID [9775]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 2, linear second order
Problem number: 1448.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{b^2 y}{(-a^2 + x^2)^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 77

```
dsolve(diff(diff(y(x),x),x) = -b^2/(-a^2+x^2)^2*y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{a^2 - x^2}\left(\left(\frac{-x+a}{a+x}\right)^{-\frac{\sqrt{a^2-b^2}}{2a}} c_2 + \left(\frac{-x+a}{a+x}\right)^{\frac{\sqrt{a^2-b^2}}{2a}} c_1\right)$$

✓ Solution by Mathematica

Time used: 0.611 (sec). Leaf size: 142

```
DSolve[y''[x] == -((b^2*y[x])/(-a^2 + x^2)^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(x-a)^{\frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{b^2}{a^2}}}(a+x)^{\frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{b^2}{a^2}}}\left(2ac_1\sqrt{1-\frac{b^2}{a^2}}(x-a)^{\sqrt{1-\frac{b^2}{a^2}}} - c_2(a+x)^{\sqrt{1-\frac{b^2}{a^2}}}\right)}{2a\sqrt{1-\frac{b^2}{a^2}}}$$

4 Chapter 3, linear third order

4.1 problem 1449

Internal problem ID [9776]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1449.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - \lambda y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 47

```
dsolve(diff(diff(diff(y(x),x),x),x)-lambda*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{\lambda^{\frac{1}{3}}\left(1+i\sqrt{3}\right)x}{2}} + c_2 e^{\frac{\lambda^{\frac{1}{3}}\left(i\sqrt{3}-1\right)x}{2}} + c_3 e^{\lambda^{\frac{1}{3}}x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 53

```
DSolve[-(\[Lambda]*y[x]) + Derivative[3][y][x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{(-1)^{2/3}\sqrt[3]{\lambda}x} + c_2 e^{-\sqrt[3]{-1}\sqrt[3]{\lambda}x} + c_3 e^{\sqrt[3]{\lambda}x}$$

4.2 problem 1450

Internal problem ID [9777]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1450.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' + ya\,x^3 = bx$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 1619

```
dsolve(diff(diff(diff(y(x),x),x),x)+y(x)*a*x^3-b*x=0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica
Time used: 12.705 (sec). Leaf size: 2428

```
DSolve[-(b*x) + a*x^3*y[x] + Derivative[3][y][x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

Too large to display

4.3 problem 1451

Internal problem ID [9778]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1451.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - a\,x^b y = 0}$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 114

```
dsolve(diff(diff(diff(y(x),x),x),x)-a*x^b*y(x)=0,y(x), singsol=all)
```

$$\begin{aligned} y(x) = {} & c_1 \,\text{hypergeom}\left([\,]\,, \left[\frac{b+1}{b+3}, \frac{b+2}{b+3}\right], \frac{a\,x^{b+3}}{(b+3)^3}\right) \\ & + c_2 x \,\text{hypergeom}\left([\,]\,, \left[\frac{b+2}{b+3}, \frac{4+b}{b+3}\right], \frac{a\,x^{b+3}}{(b+3)^3}\right) \\ & + c_3 x^2 \,\text{hypergeom}\left([\,]\,, \left[\frac{b+5}{b+3}, \frac{4+b}{b+3}\right], \frac{a\,x^{b+3}}{(b+3)^3}\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 164

```
DSolve[-(a*x^b*y[x]) + Derivative[3][y][x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \to {} & (-1)^{\frac{1}{b+3}}(b+3)^{-\frac{6}{b+3}} x a^{\frac{1}{b+3}} \left((-1)^{\frac{1}{b+3}} c_3 x a^{\frac{1}{b+3}} \,{}_0F_2\left(;1+\frac{1}{b+3}, 1+\frac{2}{b+3}; \frac{ax^{b+3}}{(b+3)^3}\right) \right. \\ & \left. + (b+3)^{\frac{3}{b+3}} c_2 \,{}_0F_2\left(;1-\frac{1}{b+3}, 1+\frac{1}{b+3}; \frac{ax^{b+3}}{(b+3)^3}\right)\right) \\ & + c_1 \,{}_0F_2\left(;1-\frac{2}{b+3}, 1-\frac{1}{b+3}; \frac{ax^{b+3}}{(b+3)^3}\right) \end{aligned}$$

4.4 problem 1452

Internal problem ID [9779]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1452.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 3y' - 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(diff(diff(y(x),x),x),x)+3*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2 e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{15}\,x}{2}\right) + c_3 e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{15}\,x}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 52

```
DSolve[-4*y[x] + 3*y'[x] + Derivative[3][y][x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}\left(c_3 e^{3x/2} + c_2 \cos\left(\frac{\sqrt{15}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{15}x}{2}\right)\right)$$

4.5 problem 1453

Internal problem ID [9780]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1453.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$y''' - a^2 y' = e^{2xa} \sin(x)^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 124

```
dsolve(diff(diff(diff(y(x),x),x),x)-a^2*diff(y(x),x)-exp(2*a*x)*sin(x)^2=0,y(x), singsol=all
```

$$y(x) = \frac{\left(\left(-9a^6+36a^4\right)\cos(2x)+\left(-33a^5+12a^3\right)\sin(2x)+9a^6+49a^4+56a^2+16\right)e^{2ax}+108\left(a^2+\frac{4}{9}\right)a^2(a^2}{108\left(a^2+\frac{4}{9}\right)a^3\left(a^2+1\right)\left(a^2+4\right)}$$

✓ Solution by Mathematica

Time used: 6.285 (sec). Leaf size: 128

```
DSolve[-(E^(2*a*x)*Sin[x]^2) - a^2*y'[x] + Derivative[3][y][x] == 0,y[x],x,IncludeSingularSo
```

$$y(x) \to \frac{e^{-ax}\left(-9(a^2-4)a^4e^{3ax}\cos(2x)-3(11a^2-4)a^3e^{3ax}\sin(2x)+(9a^6+49a^4+56a^2+16)(12a^2c_1e^{2ax}-\right.}{12a^3\left(9a^6+49a^4+56a^2+16\right)} + c_3$$

4.6 problem 1454

Internal problem ID [9781]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1454.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' + 2axy' + ay = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 55

```
dsolve(diff(diff(diff(y(x),x),x),x)+2*a*x*diff(y(x),x)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{AiryAi}\left(-\frac{2^{\frac{2}{3}} a^{\frac{1}{3}} x}{2}\right)^2 + c_2 \operatorname{AiryBi}\left(-\frac{2^{\frac{2}{3}} a^{\frac{1}{3}} x}{2}\right)^2 + c_3 \operatorname{AiryAi}\left(-\frac{2^{\frac{2}{3}} a^{\frac{1}{3}} x}{2}\right) \operatorname{AiryBi}\left(-\frac{2^{\frac{2}{3}} a^{\frac{1}{3}} x}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 79

```
DSolve[a*y[x] + 2*a*x*y'[x] + Derivative[3][y][x] == 0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to c_1 \operatorname{AiryAi}\left(\sqrt[3]{-\frac{1}{2}}\sqrt[3]{a}x\right)^2 + c_3 \operatorname{AiryBi}\left(\sqrt[3]{-\frac{1}{2}}\sqrt[3]{a}x\right)^2 + c_2 \operatorname{AiryAi}\left(\sqrt[3]{-\frac{1}{2}}\sqrt[3]{a}x\right) \operatorname{AiryBi}\left(\sqrt[3]{-\frac{1}{2}}\sqrt[3]{a}x\right)$$

4.7 problem 1455

Internal problem ID [9782]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1455.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$y''' - x^2 y'' + (a + b - 1) x y' - yab = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 71

```
dsolve(diff(diff(diff(y(x),x),x),x)-x^2*diff(diff(y(x),x),x)+(a+b-1)*x*diff(y(x),x)-b*y(x)*a
```

$$y(x) = c_1 \operatorname{hypergeom}\left(\left[-\frac{a}{3}, -\frac{b}{3}\right], \left[\frac{1}{3}, \frac{2}{3}\right], \frac{x^3}{3}\right) + c_2 \operatorname{hypergeom}\left(\left[\frac{1}{3} - \frac{a}{3}, \frac{1}{3} - \frac{b}{3}\right], \left[\frac{2}{3}, \frac{4}{3}\right], \frac{x^3}{3}\right) x + c_3 \operatorname{hypergeom}\left(\left[-\frac{a}{3} + \frac{2}{3}, -\frac{b}{3} + \frac{2}{3}\right], \left[\frac{4}{3}, \frac{5}{3}\right], \frac{x^3}{3}\right) x^2$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 127

```
DSolve[-(a*b*y[x]) + (-1 + a + b)*x*y'[x] - x^2*y''[x] + Derivative[3][y][x] == 0,y[x],x,Inc
```

$$y(x) \to \sqrt[3]{-\frac{1}{3}} c_2 x\, {}_2F_2\left(\frac{1}{3} - \frac{a}{3}, \frac{1}{3} - \frac{b}{3}; \frac{2}{3}, \frac{4}{3}; \frac{x^3}{3}\right) + c_1\, {}_2F_2\left(-\frac{a}{3}, -\frac{b}{3}; \frac{1}{3}, \frac{2}{3}; \frac{x^3}{3}\right) + \left(-\frac{1}{3}\right)^{2/3} c_3 x^2\, {}_2F_2\left(\frac{2}{3} - \frac{a}{3}, \frac{2}{3} - \frac{b}{3}; \frac{4}{3}, \frac{5}{3}; \frac{x^3}{3}\right)$$

4.8 problem 1456

Internal problem ID [9783]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1456.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' + x^{2c-2}y' + (c-1)\,x^{2c-3}y = 0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 73

```
dsolve(diff(diff(diff(y(x),x),x),x)+x^(2*c-2)*diff(y(x),x)+(c-1)*x^(2*c-3)*y(x)=0,y(x), sing
```

$$y(x) = x\left(c_1 \operatorname{BesselJ}\left(\frac{1}{2c}, \frac{x^c}{2c}\right)^2 + c_2 \operatorname{BesselY}\left(\frac{1}{2c}, \frac{x^c}{2c}\right)^2 + c_3 \operatorname{BesselJ}\left(\frac{1}{2c}, \frac{x^c}{2c}\right) \operatorname{BesselY}\left(\frac{1}{2c}, \frac{x^c}{2c}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 183

```
DSolve[(-1 + c)*x^(-3 + 2*c)*y[x] + x^(-2 + 2*c)*y'[x] + Derivative[3][y][x] == 0,y[x],x,Inc
```

$$\begin{aligned} y(x) \to\ & c_1\, {}_1F_2\left(\frac{1}{2} - \frac{1}{2c}; 1 - \frac{1}{c}, 1 - \frac{1}{2c}; -\frac{x^{2c}}{4c^2}\right) \\ & + 4^{-1/c}c^{-2/c}c_3\left(x^{2c}\right)^{\frac{1}{c}}\, {}_1F_2\left(\frac{1}{2} + \frac{1}{2c}; 1 + \frac{1}{2c}, 1 + \frac{1}{c}; -\frac{x^{2c}}{4c^2}\right) \\ & + 2^{-1/c}c^{-1/c}c_2\left(x^{2c}\right)^{\frac{1}{2}/c}\, {}_1F_2\left(\frac{1}{2}; 1 - \frac{1}{2c}, 1 + \frac{1}{2c}; -\frac{x^{2c}}{4c^2}\right) \end{aligned}$$

4.9 problem 1457

Internal problem ID [9784]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1457.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$y''' - 3(2\,\mathrm{WeierstrassP}\,(x, \mathrm{g2}, \mathrm{g3}) + a)\,y' + yb = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(diff(y(x),x),x),x)-3*(2*WeierstrassP(x,g2,g3)+a)*diff(y(x),x)+b*y(x)=0,y(x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[b*y[x] - 3*(a + 2*WeierstrassP[x, {g2, g3}])*y'[x] + Derivative[3][y][x] == 0,y[x],x,
```

Not solved

4.10 problem 1458

Internal problem ID [9785]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1458.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$y''' + \left(-n^2+1\right) \mathrm{WeierstrassP}\left(x, \mathrm{g2}, \mathrm{g3}\right) y' + \frac{\left(\left(-n^2+1\right) \mathrm{WeierstrassPPrime}\left(x, \mathrm{g2}, \mathrm{g3}\right) - a\right) y}{2} = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(diff(y(x),x),x),x)+(-n^2+1)*WeierstrassP(x,g2,g3)*diff(y(x),x)+1/2*((-n^2+1
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[((-a + (1 - n^2)*WeierstrassPPrime[x, {g2, g3}])*y[x])/2 + (1 - n^2)*WeierstrassP[x,
```

Not solved

4.11 problem 1459

Internal problem ID [9786]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1459.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$y''' - (4n(n+1)\,\mathrm{WeierstrassP}\,(x, \mathrm{g2}, \mathrm{g3}) + a)\,y' - 2n(n+1)\,\mathrm{WeierstrassPPrime}\,(x, \mathrm{g2}, \mathrm{g3})\,y = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(diff(y(x),x),x),x)-(4*n*(n+1)*WeierstrassP(x,g2,g3)+a)*diff(y(x),x)-2*n*(n+
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-2*n*(1 + n)*WeierstrassPPrime[x, {g2, g3}]*y[x] - (a + 4*n*(1 + n)*WeierstrassP[x, {
```

Not solved

4.12 problem 1460

Internal problem ID [9787]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1460.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$y''' + (A\,\text{WeierstrassP}\,(x, \text{g2}, \text{g3}) + a)\,y' + B\,\text{WeierstrassPPrime}\,(x, \text{g2}, \text{g3})\,y = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(diff(y(x),x),x),x)+(A*WeierstrassP(x,g2,g3)+a)*diff(y(x),x)+B*WeierstrassPP
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[B*WeierstrassPPrime[x, {g2, g3}]*y[x] + (a + A*WeierstrassP[x, {g2, g3}])*y'[x] + Der
```

Not solved

4.13 problem 1461

Internal problem ID [9788]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1461.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$y''' - \left(3k^2 \operatorname{JacobiSN}(z,x)^2 + a\right) y' + \left(b + c \operatorname{JacobiSN}(z,x)^2 - 3k^2 \operatorname{JacobiSN}(z,x) \operatorname{JacobiCN}(z,x) \operatorname{Jacob}\right.$$

✗ Solution by Maple

```
dsolve(diff(diff(diff(y(x),x),x),x)-(3*k^2*JacobiSN(z,x)^2+a)*diff(y(x),x)+(b+c*JacobiSN(z,x
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(b - 3*k^2*JacobiCN[z, x]*JacobiDN[z, x]*JacobiSN[z, x] + c*JacobiSN[z, x]^2)*y[x] -
```

Not solved

4.14 problem 1462

Internal problem ID [9789]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1462.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$y''' - \left(6k^2 \sin(x)^2 + a\right) y' + yb = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(diff(y(x),x),x),x)-(6*k^2*sin(x)^2+a)*diff(y(x),x)+b*y(x)=0,y(x), singsol=a
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[b*y[x] - (a + 6*k^2*Sin[x]^2)*y'[x] + Derivative[3][y][x] == 0,y[x],x,IncludeSingular
```

Not solved

4.15 problem 1463

Internal problem ID [9790]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1463.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' + 2f(x)\, y' + f'(x)\, y = 0}$$

✗ Solution by Maple

```
dsolve(diff(diff(diff(y(x),x),x),x)+2*f(x)*diff(y(x),x)+diff(f(x),x)*y(x)=0,y(x), singsol=al
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*Derivative[1][f][x] + 2*f[x]*y'[x] + Derivative[3][y][x] == 0,y[x],x,IncludeSing
```

Not solved

4.16 problem 1464

Internal problem ID [9791]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1464.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 2y'' - 3y' + 10y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(diff(diff(y(x),x),x),x)-2*diff(diff(y(x),x),x)-3*diff(y(x),x)+10*y(x)=0,y(x), si
```

$$y(x) = e^{-2x} c_1 + c_2 e^{2x} \sin(x) + c_3 e^{2x} \cos(x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 34

```
DSolve[10*y[x] - 3*y'[x] - 2*y''[x] + Derivative[3][y][x] == 0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to e^{-2x} \left(c_2 e^{4x} \cos(x) + c_1 e^{4x} \sin(x) + c_3\right)$$

4.17 problem 1465

Internal problem ID [9792]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1465.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 2y'' - a^2y' + 2a^2y = \sinh(x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 101

```
dsolve(diff(diff(diff(y(x),x),x),x)-2*diff(diff(y(x),x),x)-a^2*diff(y(x),x)+2*a^2*y(x)-sinh(
```

$$y(x) = \frac{2c_3(a^4-5a^2+4)\,\mathrm{e}^{-ax} + 2(a+1)\left(a^2c_1 + \frac{\sinh(3x)}{6} - 4c_1 - \frac{\cosh(3x)}{6}\right)(a-1)\,\mathrm{e}^{2x} + 2c_2(a^4-5a^2+4)\,\mathrm{e}^{ax} +}{2a^4-10a^2+8}$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 52

```
DSolve[-Sinh[x] + 2*a^2*y[x] - a^2*y'[x] - 2*y''[x] + Derivative[3][y][x] == 0,y[x],x,Includ
```

$$y(x) \to \frac{e^{-x}-3e^x}{6-6a^2} + c_1e^{-ax} + c_3e^{ax} + c_2e^{2x}$$

4.18 problem 1466

Internal problem ID [9793]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1466.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - 3ay'' + 3a^2y' - a^3y = e^{xa}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(diff(diff(y(x),x),x),x)-3*a*diff(diff(y(x),x),x)+3*a^2*diff(y(x),x)-a^3*y(x)-exp
```

$$y(x) = e^{ax}\left(\frac{1}{6}x^3 + c_1 + xc_2 + x^2c_3\right)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 34

```
DSolve[-E^(a*x) - a^3*y[x] + 3*a^2*y'[x] - 3*a*y''[x] + Derivative[3][y][x] == 0,y[x],x,Incl
```

$$y(x) \to \frac{1}{6}e^{ax}\left(x^3 + 6c_3x^2 + 6c_2x + 6c_1\right)$$

4.19 problem 1467

Internal problem ID [9794]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1467.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + \mathrm{a2}\, y'' + \mathrm{a1}\, y' + \mathrm{a0}\, y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 590

```
dsolve(diff(diff(diff(y(x),x),x),x)+a2*diff(diff(y(x),x),x)+a1*diff(y(x),x)+a0*y(x)=0,y(x),
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x\left(\left(\frac{i\sqrt{3}}{12}+\frac{1}{12}\right)\left(36\,\mathrm{a1}\,\mathrm{a2}-108\,\mathrm{a0}-8\,\mathrm{a2}^3+12\sqrt{12\,\mathrm{a0}\,\mathrm{a2}^3-3\,\mathrm{a1}^2\,\mathrm{a2}^2-54\,\mathrm{a1}\,\mathrm{a2}\,\mathrm{a0}+12\,\mathrm{a1}^3+81\,\mathrm{a0}^2}\right)^{\frac{2}{3}}+\frac{\mathrm{a2}\left(36\,\mathrm{a1}\,\mathrm{a2}-108\,\mathrm{a0}-8\,\mathrm{a2}^3+12\sqrt{12\,\mathrm{a0}\,\mathrm{a2}^3-3\,\mathrm{a1}^2\,\mathrm{a}\cdots}\right.}{3}\cdots}{\left(36\,\mathrm{a1}\,\mathrm{a2}-108\,\mathrm{a0}-8\,\mathrm{a2}^3+12\sqrt{12\,\mathrm{a0}\,\mathrm{a2}^3-3\,\mathrm{a1}^2\,\mathrm{a2}^2-54\,\mathrm{a1}\,\mathrm{a2}\,\mathrm{a0}+12\,\mathrm{a1}^3+81\,\mathrm{a0}^2}\right)^{\frac{1}{3}}}}$$

$$+ c_2 \mathrm{e}^{\frac{\left(i\sqrt{3}\left(36\,\mathrm{a1}\,\mathrm{a2}-108\,\mathrm{a0}-8\,\mathrm{a2}^3+12\sqrt{12\,\mathrm{a0}\,\mathrm{a2}^3-3\,\mathrm{a1}^2\,\mathrm{a2}^2-54\,\mathrm{a1}\,\mathrm{a2}\,\mathrm{a0}+12\,\mathrm{a1}^3+81\,\mathrm{a0}^2}\right)^{\frac{2}{3}}-4i\sqrt{3}\,\mathrm{a2}^2+12i\sqrt{3}\,\mathrm{a1}-\left(36\,\mathrm{a1}\,\mathrm{a2}-108\,\mathrm{a0}-8\,\mathrm{a2}^3+12\sqrt{12\,\mathrm{a0}\,\cdots}\right.\cdots}{12\left(36\,\mathrm{a1}\,\mathrm{a2}-108\,\mathrm{a0}-8\,\mathrm{a2}^3+12\sqrt{12\,\mathrm{a0}\,\mathrm{a2}^3\cdots}\right.\cdots}}$$

$$+ c_3 \mathrm{e}^{\frac{\left(\left(36\,\mathrm{a1}\,\mathrm{a2}-108\,\mathrm{a0}-8\,\mathrm{a2}^3+12\sqrt{12\,\mathrm{a0}\,\mathrm{a2}^3-3\,\mathrm{a1}^2\,\mathrm{a2}^2-54\,\mathrm{a1}\,\mathrm{a2}\,\mathrm{a0}+12\,\mathrm{a1}^3+81\,\mathrm{a0}^2}\right)^{\frac{2}{3}}-2\,\mathrm{a2}\left(36\,\mathrm{a1}\,\mathrm{a2}-108\,\mathrm{a0}-8\,\mathrm{a2}^3+12\sqrt{12\,\mathrm{a0}\,\mathrm{a2}^3-3\,\mathrm{a1}^2\,\mathrm{a2}^2-54\,\mathrm{a1}\,\mathrm{a}\cdots}\right.\cdots}{6\left(36\,\mathrm{a1}\,\mathrm{a2}-108\,\mathrm{a0}-8\,\mathrm{a2}^3+12\sqrt{12\,\mathrm{a0}\,\mathrm{a2}^3-3\,\mathrm{a1}^2\,\mathrm{a2}^2-54\,\mathrm{a1}\,\mathrm{a2}\,\mathrm{a0}+12\,\mathrm{a1}^3+81\,\mathrm{a0}^2}\right)^{\frac{1}{3}}}}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 84

```
DSolve[a0*y[x] + a1*y'[x] + a2*y''[x] + Derivative[3][y][x] == 0,y[x],x,IncludeSingularSolut
```

$$y(x) \to c_1 e^{x\text{Root}\left[\#1^3+\#1^2\text{a2}+\#1\text{a1}+\text{a0}\&,1\right]} + c_2 e^{x\text{Root}\left[\#1^3+\#1^2\text{a2}+\#1\text{a1}+\text{a0}\&,2\right]} + c_3 e^{x\text{Root}\left[\#1^3+\#1^2\text{a2}+\#1\text{a1}+\text{a0}\&,3\right]}$$

4.20 problem 1468

Internal problem ID [9795]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1468.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$y''' - 6y''x + 2(4x^2 + 2a - 1)\,y' - 8yax = 0$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 59

```
dsolve(diff(diff(diff(y(x),x),x),x)-6*x*diff(diff(y(x),x),x)+2*(4*x^2+2*a-1)*diff(y(x),x)-8*
```

$$y(x) = x^2\left(\text{KummerU}\left(\frac{1}{2}-\frac{a}{4},\frac{3}{2},x^2\right)^2 c_2 + \text{KummerU}\left(\frac{1}{2}-\frac{a}{4},\frac{3}{2},x^2\right)\text{KummerM}\left(\frac{1}{2}-\frac{a}{4},\frac{3}{2},x^2\right)c_3 + \text{KummerM}\left(\frac{1}{2}-\frac{a}{4},\frac{3}{2},x^2\right)^2 c_1\right)$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 57

```
DSolve[-8*a*x*y[x] + 2*(-1 + 2*a + 4*x^2)*y'[x] - 6*x*y''[x] + Derivative[3][y][x] == 0,y[x]
```

$$y(x) \to c_2\,\text{HermiteH}\left(\frac{a}{2},x\right)\text{Hypergeometric1F1}\left(-\frac{a}{4},\frac{1}{2},x^2\right) + c_1\,\text{HermiteH}\left(\frac{a}{2},x\right)^2 + c_3\,\text{Hypergeometric1F1}\left(-\frac{a}{4},\frac{1}{2},x^2\right)^2$$

4.21 problem 1469

Internal problem ID [9796]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1469.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$y''' + 3axy'' + 3a^2x^2y' + a^3x^3y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 46

```
dsolve(diff(diff(diff(y(x),x),x),x)+3*a*x*diff(diff(y(x),x),x)+3*a^2*x^2*diff(y(x),x)+a^3*x^
```

$$y(x) = e^{-\frac{x\left(2\sqrt{3}\,\sqrt{a}+ax\right)}{2}}\left(c_2 e^{2\sqrt{3}\,\sqrt{a}\,x} + c_1 e^{\sqrt{3}\,\sqrt{a}\,x} + c_3\right)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 68

```
DSolve[a^3*x^3*y[x] + 3*a^2*x^2*y'[x] + 3*a*x*y''[x] + Derivative[3][y][x] == 0,y[x],x,Inclu
```

$$y(x) \to e^{-\frac{ax^2}{2}-\sqrt{3}\sqrt{a}x}\left(c_1 e^{\sqrt{3}\sqrt{a}x} + c_3 e^{2\sqrt{3}\sqrt{a}x} + c_2\right)$$

4.22 problem 1470

Internal problem ID [9797]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1470.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _fully, _exact, _linear]]`

$$y''' - y'' \sin(x) - 2\cos(x) y' + y \sin(x) = \ln(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
dsolve(diff(diff(diff(y(x),x),x),x)-diff(diff(y(x),x),x)*sin(x)-2*diff(y(x),x)*cos(x)+y(x)*s
```

$$y(x) = \frac{\left(4c_3 + \int (8c_1 x + 4c_2 - 3x^2 + 2x^2 \ln(x)) e^{\cos(x)} dx\right) e^{-\cos(x)}}{4}$$

✓ Solution by Mathematica

Time used: 2.289 (sec). Leaf size: 57

```
DSolve[-Log[x] + Sin[x]*y[x] - 2*Cos[x]*y'[x] - Sin[x]*y''[x] + Derivative[3][y][x] == 0,y[x
```

$$y(x) \to e^{-\cos(x)} \left(\int_1^x \frac{1}{4} e^{\cos(K[1])} \left(2\log(K[1])K[1]^2 - 3K[1]^2 + 4c_1 K[1] + 4c_2\right) dK[1] + c_3 \right)$$

4.23 problem 1471

Internal problem ID [9798]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1471.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' + y'' f(x) + y' + f(x)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
dsolve(diff(diff(diff(y(x),x),x),x)+f(x)*diff(diff(y(x),x),x)+diff(y(x),x)+f(x)*y(x)=0,y(x),
```

$$y(x) = e^{ix}\left(\int e^{-2ix}\left(c_3\left(\int e^{\int (i-f(x))dx} dx\right) + c_2\right) dx + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.063 (sec). Leaf size: 84

```
DSolve[f[x]*y[x] + y'[x] + f[x]*y''[x] + Derivative[3][y][x] == 0,y[x],x,IncludeSingularSolu
```

$$y(x) \to c_3 e^{ix} \int_1^x e^{-2iK[3]} \int_1^{K[3]} \exp\left(\int_1^{K[2]} (i - f(K[1]))dK[1]\right) dK[2]dK[3] + c_1 e^{ix} + \frac{1}{2} i c_2 e^{-ix}$$

4.24 problem 1472

Internal problem ID [9799]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1472.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' + f(x)\left(x^2 y'' - 2y'x + 2y\right) = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 33

```
dsolve(diff(diff(diff(y(x),x),x),x)+f(x)*(x^2*diff(diff(y(x),x),x)-2*x*diff(y(x),x)+2*y(x))=
```

$$y(x) = \left(\int \left(c_1 + c_2 \left(\int e^{-\left(\int \frac{x^3 f(x)+3}{x} dx\right)} dx\right)\right) dx + c_3\right) x$$

✓ Solution by Mathematica

Time used: 0.072 (sec). Leaf size: 85

```
DSolve[f[x]*(2*y[x] - 2*x*y'[x] + x^2*y''[x]) + Derivative[3][y][x] == 0,y[x],x,IncludeSingu
```

$$y(x) \to x\left(c_3\left(\int_1^x \frac{\exp\left(-\int_1^{K[2]} f(K[1])K[1]^2 dK[1]\right)}{K[2]^2} dK[2] - x\int_1^x \frac{\exp\left(-\int_1^{K[3]} f(K[1])K[1]^2 dK[1]\right)}{K[3]^3} dK[3]\right) + c_2 x + c_1\right)$$

4.25 problem 1473

Internal problem ID [9800]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1473.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$y''' + y'' f(x) + g(x) y' + (g(x) f(x) + g'(x)) y = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(diff(y(x),x),x),x)+f(x)*diff(diff(y(x),x),x)+g(x)*diff(y(x),x)+(f(x)*g(x)+d
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*(f[x]*g[x] + Derivative[1][g][x]) + g[x]*y'[x] + f[x]*y''[x] + Derivative[3][y][
```

Not solved

4.26 problem 1474

Internal problem ID [9801]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1474.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$y''' + 3y'' f(x) + \left(f'(x) + 2f(x)^2 + 4g(x)\right) y' + (4g(x)\, f(x) + 2g'(x))\, y = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(diff(y(x),x),x),x)+3*f(x)*diff(diff(y(x),x),x)+(diff(f(x),x)+2*f(x)^2+4*g(x
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*(4*f[x]*g[x] + 2*Derivative[1][g][x]) + (2*f[x]^2 + 4*g[x] + Derivative[1][f][x]
```

Not solved

4.27 problem 1475

Internal problem ID [9802]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1475.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{4y''' - 8y'' - 11y' - 3y = -18\,\mathrm{e}^{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(4*diff(diff(diff(y(x),x),x),x)-8*diff(diff(y(x),x),x)-11*diff(y(x),x)-3*y(x)+18*exp(x
```

$$y(x) = (c_3x + c_2)\,\mathrm{e}^{-\frac{x}{2}} + c_1\mathrm{e}^{3x} + \mathrm{e}^{x}$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 37

```
DSolve[18*E^x - 3*y[x] - 11*y'[x] - 8*y''[x] + 4*Derivative[3][y][x] == 0,y[x],x,IncludeSing
```

$$y(x) \to e^{-x/2}\left(e^{3x/2} + c_2x + c_3e^{7x/2} + c_1\right)$$

4.28 problem 1476

Internal problem ID [9803]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1476.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$27y''' - 36\,\mathrm{WeierstrassP}\,(x, \mathrm{g2}, \mathrm{g3})\, y' n^2 - 2n(n+3)\,(4n-3)\,\mathrm{WeierstrassPPrime}\,(x, \mathrm{g2}, \mathrm{g3})\, y = 0$$

✗ Solution by Maple

```
dsolve(27*diff(diff(diff(y(x),x),x),x)-36*n^2*WeierstrassP(x,g2,g3)*diff(y(x),x)-2*n*(n+3)*(
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-2*n*(3 + n)*(-3 + 4*n)*y[x]*Derivative[1][phi][x] - 36*n^2*WeierstrassP[x, {g2, g3}]
```

Not solved

4.29 problem 1477

Internal problem ID [9804]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1477.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{xy''' + 3y'' + xy = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 40

```
dsolve(x*diff(diff(diff(y(x),x),x),x)+3*diff(diff(y(x),x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(e^{\frac{3x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right) c_3 + e^{\frac{3x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right) c_2 + c_1\right) e^{-x}}{x}$$

✓ Solution by Mathematica

Time used: 0.092 (sec). Leaf size: 43

```
DSolve[x*y[x] + 3*y''[x] + x*Derivative[3][y][x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{c_1 e^{-x} + c_2 e^{\sqrt[3]{-1}x} + c_3 e^{-(-1)^{2/3}x}}{x}$$

4.30 problem 1478

Internal problem ID [9805]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1478.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{xy''' + 3y'' - a\,x^2y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 48

```
dsolve(x*diff(diff(diff(y(x),x),x),x)+3*diff(diff(y(x),x),x)-a*x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \,\text{hypergeom}\left([\,], \left[\frac{3}{4}, \frac{5}{4}\right], \frac{x^4 a}{64}\right) + \frac{c_2 \,\text{hypergeom}\left([\,], \left[\frac{1}{2}, \frac{3}{4}\right], \frac{x^4 a}{64}\right)}{x} + c_3 x \,\text{hypergeom}\left([\,], \left[\frac{5}{4}, \frac{3}{2}\right], \frac{x^4 a}{64}\right)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 90

```
DSolve[-(a*x^2*y[x]) + 3*y''[x] + x*Derivative[3][y][x] == 0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{(2-2i)c_1\, {}_0F_2\left(;\frac{1}{2},\frac{3}{4};\frac{ax^4}{64}\right)}{\sqrt[4]{a}x} + c_2\, {}_0F_2\left(;\frac{3}{4},\frac{5}{4};\frac{ax^4}{64}\right) + \left(\frac{1}{4}+\frac{i}{4}\right)\sqrt[4]{a}c_3 x\, {}_0F_2\left(;\frac{5}{4},\frac{3}{2};\frac{ax^4}{64}\right)$$

4.31 problem 1479

Internal problem ID [9806]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1479.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$xy''' + (a+b)\,y'' - y'x - ay = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 92

```
dsolve(x*diff(diff(diff(y(x),x),x),x)+(a+b)*diff(diff(y(x),x),x)-x*diff(y(x),x)-a*y(x)=0,y(x
```

$$y(x) = c_1 \operatorname{hypergeom}\left(\left[\frac{a}{2}\right], \left[\frac{1}{2}, \frac{a}{2}+\frac{b}{2}\right], \frac{x^2}{4}\right) + c_2 x \operatorname{hypergeom}\left(\left[\frac{1}{2}+\frac{a}{2}\right], \left[\frac{3}{2}, \frac{a}{2}+\frac{b}{2}+\frac{1}{2}\right], \frac{x^2}{4}\right) + c_3 x^{-a-b+2} \operatorname{hypergeom}\left(\left[1-\frac{b}{2}\right], \left[2-\frac{b}{2}-\frac{a}{2}, -\frac{a}{2}-\frac{b}{2}+\frac{3}{2}\right], \frac{x^2}{4}\right)$$

✓ Solution by Mathematica
Time used: 0.076 (sec). Leaf size: 153

```
DSolve[-(a*y[x]) - x*y'[x] + (a + b)*y''[x] + x*Derivative[3][y][x] == 0,y[x],x,IncludeSingu
```

$$y(x) \to \frac{1}{2} i c_2 x \, {}_1F_2\left(\frac{a}{2}+\frac{1}{2}; \frac{3}{2}, \frac{a}{2}+\frac{b}{2}+\frac{1}{2}; \frac{x^2}{4}\right) + c_1 \, {}_1F_2\left(\frac{a}{2}; \frac{1}{2}, \frac{a}{2}+\frac{b}{2}; \frac{x^2}{4}\right) + c_3 \left(\frac{i}{2}\right)^{-a-b+2} x^{-a-b+2} \, {}_1F_2\left(1-\frac{b}{2}; -\frac{a}{2}-\frac{b}{2}+\frac{3}{2}, -\frac{a}{2}-\frac{b}{2}+2; \frac{x^2}{4}\right)$$

4.32 problem 1480

Internal problem ID [9807]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1480.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$xy''' - (x + 2v)\, y'' - (x - 2v - 1)\, y' + (x - 1)\, y = 0$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 54

```
dsolve(x*diff(diff(diff(y(x),x),x),x)-(x+2*v)*diff(diff(y(x),x),x)-(x-2*v-1)*diff(y(x),x)+(x
```

$$y(x) = \frac{x^{2+v} c_2 \operatorname{BesselI}(-v+1, x) - 2 \operatorname{BesselI}(-v, x)\, x^{v+1} c_2 v + x^{2+v} c_3 \operatorname{BesselK}(v+1, x) + e^{x} c_1 x}{x}$$

✓ Solution by Mathematica
Time used: 0.127 (sec). Leaf size: 91

```
DSolve[(-1 + x)*y[x] - (-1 - 2*v + x)*y'[x] - (2*v + x)*y''[x] + x*Derivative[3][y][x] == 0,
```

$$y(x) \to \frac{1}{4} e^{x} \left(\frac{4 c_3 x^{2v+2} \operatorname{Gamma}\left(v + \frac{3}{2}\right) {}_1\tilde{F}_1\left(v + \frac{3}{2}; 2v + 3; -2x\right)}{\operatorname{Gamma}\left(\frac{1}{2} - v\right)} + c_2 4^{-v} G_{2,3}^{2,1}\left(2x \middle| \begin{array}{c} 1, v + \frac{3}{2} \\ 1, 2(v+1), 0 \end{array}\right) + 4 c_1 \right)$$

4.33 problem 1481

Internal problem ID [9808]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1481.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _fully, _exact, _linear]]`

$$xy''' + (x^2 - 3)\, y'' + 4y'x + 2y = f(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(x*diff(diff(diff(y(x),x),x),x)+(x^2-3)*diff(diff(y(x),x),x)+4*x*diff(y(x),x)+2*y(x)-f
```

$$y(x) = \left(c_3 + \int \frac{\left(2c_1x + c_2 + \int \int f(x)\, dx dx\right) e^{\frac{x^2}{2}}}{x^6} dx\right) e^{-\frac{x^2}{2}} x^5$$

✓ Solution by Mathematica

Time used: 0.31 (sec). Leaf size: 346

```
DSolve[-f[x] + 2*y[x] + 4*x*y'[x] + (-3 + x^2)*y''[x] + x*Derivative[3][y][x] == 0,y[x],x,In
```

$$y(x) \to \frac{1}{240}\left(240e^{-\frac{x^2}{2}}x^5\int_1^x \frac{f(K[1])\left(8\sqrt{2\pi}\text{erfi}\left(\frac{K[1]}{\sqrt{2}}\right)K[1]^5 - 15\,\text{ExpIntegralEi}\left(\frac{K[1]^2}{2}\right)K[1]^4 + 2e^{\frac{K[1]^2}{2}}(-8K[}{240K[1]^4}\right.$$

$$+ 8\sqrt{2\pi}e^{-\frac{x^2}{2}}x^5\text{erfi}\left(\frac{x}{\sqrt{2}}\right)\int_1^x -f(K[2])K[2]dK[2]$$

$$+ 15x\left(e^{-\frac{x^2}{2}}x^4\,\text{ExpIntegralEi}\left(\frac{x^2}{2}\right) - 2(x^2+2)\right)\int_1^x f(K[3])dK[3] - 16x^4\int_1^x$$

$$-f(K[2])K[2]dK[2] - 16x^2\int_1^x -f(K[2])K[2]dK[2] - 48\int_1^x -f(K[2])K[2]dK[2]$$

$$+ 8\sqrt{2\pi}c_2e^{-\frac{x^2}{2}}x^5\text{erfi}\left(\frac{x}{\sqrt{2}}\right) + 15c_3e^{-\frac{x^2}{2}}x^5\,\text{ExpIntegralEi}\left(\frac{x^2}{2}\right) - 16c_2x^4 - 30c_3x^3$$

$$\left. - 16c_2x^2 + 240c_1e^{-\frac{x^2}{2}}x^5 - 60c_3x - 48c_2\right)$$

4.34 problem 1482

Internal problem ID [9809]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1482.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{2xy''' + 3y'' + yax = b}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 1616

```
dsolve(2*x*diff(diff(diff(y(x),x),x),x)+3*diff(diff(y(x),x),x)+y(x)*a*x-b=0,y(x), singsol=al
```

Expression too large to display

✓ Solution by Mathematica

Time used: 14.362 (sec). Leaf size: 2455

```
DSolve[-b + a*x*y[x] + 3*y''[x] + 2*x*Derivative[3][y][x] == 0,y[x],x,IncludeSingularSolutio
```

Too large to display

4.35 problem 1483

Internal problem ID [9810]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1483.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{2xy''' - 4(x+\nu-1)\,y'' + (2x+6\nu-5)\,y' + (1-2\nu)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

```
dsolve(2*x*diff(diff(diff(y(x),x),x),x)-4*(x+nu-1)*diff(diff(y(x),x),x)+(2*x+6*nu-5)*diff(y(
```

$$y(x) = c_1 e^x + c_2 e^{\frac{x}{2}} x^\nu \operatorname{BesselI}\left(\nu, \frac{x}{2}\right) + c_3 e^{\frac{x}{2}} x^\nu \operatorname{BesselK}\left(\nu, \frac{x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.085 (sec). Leaf size: 105

```
DSolve[(1 - 2*nu)*y[x] + (-5 + 6*nu + 2*x)*y'[x] - 4*(-1 + nu + x)*y''[x] + 2*x*Derivative[3
```

$$y(x) \to e^x \left(\frac{2c_3 \operatorname{Gamma}\left(\frac{5}{2} - 3\nu\right) \left(\operatorname{Gamma}(2-2\nu)\, {}_1\tilde{F}_1\left(\frac{3}{2} - 3\nu; 1-2\nu; -x\right) + 2\nu - 1\right)}{3(2\nu-1) \operatorname{Gamma}(2-2\nu) \operatorname{Gamma}\left(\frac{3}{2} - \nu\right)} + c_2 G_{2,3}^{2,1}\left(x \middle| \begin{array}{c} 1, 3\nu - \frac{1}{2} \\ 1, 2\nu, 0 \end{array}\right) + c_1 \right)$$

4.36 problem 1484

Internal problem ID [9811]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1484.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$2xy''' + 3(2xa + k)\, y'' + 6(ak + bx)\, y' + (3bk + 2cx)\, y = 0$$

✗ Solution by Maple

```
dsolve(2*x*diff(diff(diff(y(x),x),x),x)+3*(2*a*x+k)*diff(diff(y(x),x),x)+6*(a*k+b*x)*diff(y(
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(3*b*k + 2*c*x)*y[x] + 6*(a*k + b*x)*y'[x] + 3*(k + 2*a*x)*y''[x] + 2*x*Derivative[3]
```

Not solved

4.37 problem 1485

Internal problem ID [9812]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1485.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x-2)\,xy''' - (x-2)\,xy'' - 2y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 51

```
dsolve((x-2)*x*diff(diff(diff(y(x),x),x),x)-(x-2)*x*diff(diff(y(x),x),x)-2*diff(y(x),x)+2*y(
```

$$y(x) = c_3 \operatorname{expIntegral}_1(x-2)\,e^{x-2} + \frac{c_3 x^2 \ln(x-2)}{4} + e^x c_2 - \frac{c_3 x^2 \ln(x)}{4} + \frac{(2x+2)\,c_3}{4} + c_1 x^2$$

✓ Solution by Mathematica

Time used: 0.192 (sec). Leaf size: 59

```
DSolve[2*y[x] - 2*y'[x] - (-2 + x)*x*y''[x] + (-2 + x)*x*Derivative[3][y][x] == 0,y[x],x,Inc
```

$$y(x) \to \frac{1}{4} c_3 \left(-4e^{x-2}\,\text{ExpIntegralEi}(2-x) + x^2 \log(2-x) - x^2 \log(x) + 2x + 2\right) + c_1 x^2 + c_2 e^x$$

4.38 problem 1486

Internal problem ID [9813]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1486.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{(2x-1)\,y''' - 8y'x + 8y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 50

```
dsolve((2*x-1)*diff(diff(diff(y(x),x),x),x)-8*x*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x + c_2\mathrm{e}^{2x} - \frac{c_3x\,\mathrm{e}^{-1}\,\mathrm{expIntegral}_1\,(-1+2x)}{2} + \frac{c_3\,\mathrm{expIntegral}_1\,(-2+4x)\,\mathrm{e}^{2x-2}}{4} + \frac{c_3\mathrm{e}^{-2x}}{4}$$

✓ Solution by Mathematica
Time used: 0.372 (sec). Leaf size: 63

```
DSolve[8*y[x] - 8*x*y'[x] + (-1 + 2*x)*Derivative[3][y][x] == 0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{1}{4}\left(c_3e^{2x-2}\,\mathrm{ExpIntegralEi}(2-4x) - \frac{2c_3x\,\mathrm{ExpIntegralEi}(1-2x)}{e} + 4c_1x - 4c_2e^{2x} - c_3e^{-2x}\right)$$

4.39 problem 1487

Internal problem ID [9814]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1487.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$(2x-1)\,y''' + (4+x)\,y'' + 2y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve((2*x-1)*diff(diff(diff(y(x),x),x),x)+(x+4)*diff(diff(y(x),x),x)+2*diff(y(x),x)=0,y(x)
```

$$y(x) = \frac{\left(c_3 + \int \frac{(2c_1x+c_2)\mathrm{e}^{\frac{x}{2}}}{(-1+2x)^{\frac{3}{4}}}dx\right)\mathrm{e}^{-\frac{x}{2}}}{(-1+2x)^{\frac{1}{4}}}$$

✓ Solution by Mathematica

Time used: 60.683 (sec). Leaf size: 66

```
DSolve[2*y'[x] + (4 + x)*y''[x] + (-1 + 2*x)*Derivative[3][y][x] == 0,y[x],x,IncludeSingular
```

$$y(x) \to \int_1^x e^{-\frac{K[1]}{2}}\left(\frac{2\sqrt{2}c_1K[1]}{(2K[1]-1)^{5/4}} + c_2L_{-\frac{1}{4}}^{\frac{5}{4}}\left(\frac{K[1]}{2}-\frac{1}{4}\right)\right)dK[1] + c_3$$

4.40 problem 1488

Internal problem ID [9815]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1488.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^2 y''' - 6y' + a\,x^2 y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 134

```
dsolve(x^2*diff(diff(diff(y(x),x),x),x)-6*diff(y(x),x)+a*x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-c_3\left(\left(-i-\sqrt{3}\right)\left(-a^4\right)^{\frac{2}{3}} + ix\,a^3\right)\mathrm{e}^{-\frac{\left(-a^4\right)^{\frac{1}{3}}\left(1+i\sqrt{3}\right)x}{2a}} + \left(\left(-i+\sqrt{3}\right)\left(-a^4\right)^{\frac{2}{3}} + ix\,a^3\right)c_2\mathrm{e}^{\frac{\left(-a^4\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)x}{2a}} + c_1\left(\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.292 (sec). Leaf size: 97

```
DSolve[a*x^2*y[x] - 6*y'[x] + x^2*Derivative[3][y][x] == 0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{c_1 e^{-\sqrt[3]{a}x}\left(\sqrt[3]{a}x+2\right) + c_2 e^{\sqrt[3]{-1}\sqrt[3]{a}x}\left(\sqrt[3]{a}x+2(-1)^{2/3}\right) + c_3 e^{-(-1)^{2/3}\sqrt[3]{a}x}\left(\sqrt[3]{a}x-2\sqrt[3]{-1}\right)}{x}$$

4.41 problem 1489

Internal problem ID [9816]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1489.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^2y''' + (x+1)\,y'' - y = 0$$

✗ Solution by Maple

```
dsolve(x^2*diff(diff(diff(y(x),x),x),x)+(x+1)*diff(diff(y(x),x),x)-y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-y[x] + (1 + x)*y''[x] + x^2*Derivative[3][y][x] == 0,y[x],x,IncludeSingularSolutions
```

Not solved

4.42 problem 1490

Internal problem ID [9817]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1490.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{x^2y''' - y''x + (x^2+1)\,y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve(x^2*diff(diff(diff(y(x),x),x),x)-x*diff(diff(y(x),x),x)+(x^2+1)*diff(y(x),x)=0,y(x),
```

$$y(x) = c_1 + c_2 x \,\mathrm{BesselJ}\,(1, x) + c_3 x \,\mathrm{BesselY}\,(1, x)$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 33

```
DSolve[(1 + x^2)*y'[x] - x*y''[x] + x^2*Derivative[3][y][x] == 0,y[x],x,IncludeSingularSolut
```

$$y(x) \to \frac{1}{2} c_1 x^2 \,\mathrm{Hypergeometric0F1Regularized}\left(2, -\frac{x^2}{4}\right) + c_2 x \,\mathrm{BesselY}(1, x) + c_3$$

4.43 problem 1491

Internal problem ID [9818]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1491.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^2y''' + 3y''x + \left(4a^2x^{2a} + 1 - 4\nu^2a^2\right)y' - 4a^3x^{2a-1}y = 0$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 88

```
dsolve(x^2*diff(y(x),x$3)+3*x*diff(y(x),x$2)+(4*a^2*x^(2*a)+1-4*nu^2*a^2)*diff(y(x),x)=4*(a^
```

$$y(x) = c_1 \operatorname{hypergeom}\left(\left[-\frac{1}{2}\right], [\nu+1, -\nu+1], -x^{2a}\right) + c_2x^{-2a\nu} \operatorname{hypergeom}\left(\left[-\frac{1}{2}-\nu\right], [-\nu+1, 1-2\nu], -x^{2a}\right) + c_3x^{2a\nu} \operatorname{hypergeom}\left(\left[-\frac{1}{2}+\nu\right], [\nu+1, 2\nu+1], -x^{2a}\right)$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 102

```
DSolve[(1 - 4*a^2*nu^2 + 4*a^2*x^(2*a))*y'[x] + 3*x*y''[x] + x^2*Derivative[3][y][x] == 4*a^
```

$$y(x) \to c_2\left(x^{2a}\right)^{-\nu} {}_1F_2\left(-\nu-\frac{1}{2}; 1-2\nu, 1-\nu; -x^{2a}\right) + c_3\left(x^{2a}\right)^{\nu} {}_1F_2\left(\nu-\frac{1}{2}; \nu+1, 2\nu+1; -x^{2a}\right) + c_1\, {}_1F_2\left(-\frac{1}{2}; 1-\nu, \nu+1; -x^{2a}\right)$$

4.44 problem 1492

Internal problem ID [9819]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1492.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^2y''' - 3(x-m)\,xy'' + \left(2x^2 + 4(n-m)\,x + m(2m-1)\right)y' - 2n(2x-2m+1)\,y = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 39

```
dsolve(x^2*diff(diff(diff(y(x),x),x),x)-3*(x-m)*x*diff(diff(y(x),x),x)+(2*x^2+4*(n-m)*x+m*(2
```

$$y(x) = c_1 \operatorname{KummerM}(-n,m,x)^2 + c_2 \operatorname{KummerU}(-n,m,x)^2 + c_3 \operatorname{KummerM}(-n,m,x) \operatorname{KummerU}(-n,m,x)$$

✓ Solution by Mathematica

Time used: 0.19 (sec). Leaf size: 43

```
DSolve[-2*n*(1 - 2*m + 2*x)*y[x] + (m*(-1 + 2*m) + 4*(-m + n)*x + 2*x^2)*y'[x] - 3*x*(-m + x
```

$$y(x) \to c_2 \operatorname{HypergeometricU}(-n,m,x)L_n^{m-1}(x) + c_1 \operatorname{HypergeometricU}(-n,m,x)^2 + c_3 L_n^{m-1}(x)^2$$

4.45 problem 1493

Internal problem ID [9820]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1493.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$x^2y''' + 4y''x + (x^2 + 2)\, y' + 3xy = f(x)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 1035

```
dsolve(x^2*diff(diff(diff(y(x),x),x),x)+4*x*diff(diff(y(x),x),x)+(x^2+2)*diff(y(x),x)+3*x*y(
```

Expression too large to display

✓ Solution by Mathematica

Time used: 1.08 (sec). Leaf size: 373

```
DSolve[-f[x] + 3*x*y[x] + (2 + x^2)*y'[x] + 4*x*y''[x] + x^2*Derivative[3][y][x] == 0,y[x],x
```

$$y(x) \to \frac{2\,{}_1F_2\left(1; \frac{1}{2}, \frac{1}{2}; -\frac{x^2}{4}\right) \left(\int_1^x \frac{9\pi(\text{BesselJ}(1,K[3])\,\text{BesselY}(0,K[3]) - \text{BesselJ}(0,K[3])\,\text{BesselY}(1,K[3]))f(K[3])K[3]^2}{32\,{}_1F_2(3;\frac{5}{2},\frac{5}{2};-\frac{1}{4}K[3]^2)K[3]^4 - 18(K[3]^2+1)(\pi K[3]\boldsymbol{H}_0(K[3])-2)}dK[3] + c_3\right)}{x}$$

$$+\text{BesselJ}(0,x) \int_1^x \frac{9\pi f(K[1]) \left(2\,\text{BesselY}(0,K[1])\,{}_1F_2\left(2; \frac{3}{2}, \frac{3}{2}; -\frac{1}{4}K[1]^2\right) K[1]^2 + {}_1F_2\left(1; \frac{1}{2}, \frac{1}{2}; -\frac{1}{4}K[1]^2\right) (}{9\left(K[1]^2 + 1\right)\left(\pi K[1]\boldsymbol{H}_0(K[1]) - 2\right) - 16\,{}_1F_2\left(3; \frac{5}{2}, \frac{5}{2}; -\right.}$$

$$+2\,\text{BesselY}(0,x) \int_1^x \frac{9\pi f(K[2]) \left(2\,\text{BesselJ}(0,K[2])\,{}_1F_2\left(2; \frac{3}{2}, \frac{3}{2}; -\frac{1}{4}K[2]^2\right) K[2]^2 + {}_1F_2\left(1; \frac{1}{2}, \frac{1}{2}; -\frac{1}{4}K[2]^2\right.}{32\,{}_1F_2\left(3; \frac{5}{2}, \frac{5}{2}; -\frac{1}{4}K[2]^2\right) K[2]^4 - 18\left(K[2]^2 + 1\right)(\pi K[}$$

$$+ c_1\,\text{BesselJ}(0,x) + 2c_2\,\text{BesselY}(0,x)$$

4.46 problem 1494

Internal problem ID [9821]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1494.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{x^2y''' + 5y''x + 4y' = \ln (x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(x^2*diff(diff(diff(y(x),x),x),x)+5*x*diff(diff(y(x),x),x)+4*diff(y(x),x)-ln(x)=0,y(x)
```

$$y(x) = \frac{(x^2 + 4c_2) \ln (x) - 2x^2 + 4c_1x + 4c_3}{4x}$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 43

```
DSolve[-Log[x] + 4*y'[x] + 5*x*y''[x] + x^2*Derivative[3][y][x] == 0,y[x],x,IncludeSingularS
```

$$y(x) \to \frac{(x^2 - 8c_2) \log(x) - 2(x^2 - 2c_3x + 2c_1 + 4c_2)}{4x}$$

4.47 problem 1495

Internal problem ID [9822]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1495.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{x^2y''' + 6y''x + 6y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve(x^2*diff(diff(diff(y(x),x),x),x)+6*x*diff(diff(y(x),x),x)+6*diff(y(x),x)=0,y(x), sing
```

$$y(x) = c_1 + \frac{c_2}{x^2} + \frac{c_3}{x}$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 24

```
DSolve[6*y'[x] + 6*x*y''[x] + x^2*Derivative[3][y][x] == 0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to -\frac{c_1}{2x^2} - \frac{c_2}{x} + c_3$$

4.48 problem 1496

Internal problem ID [9823]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1496.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y''' + 6y'' x + 6y' + a\,x^2 y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 57

```
dsolve(x^2*diff(diff(diff(y(x),x),x),x)+6*x*diff(diff(y(x),x),x)+6*diff(y(x),x)+a*x^2*y(x)=0
```

$$y(x) = \frac{c_1 \mathrm{e}^{\frac{(-a)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)x}{2}} + c_2 \mathrm{e}^{-\frac{(-a)^{\frac{1}{3}}\left(1+i\sqrt{3}\right)x}{2}} + c_3 \mathrm{e}^{(-a)^{\frac{1}{3}}x}}{x^2}$$

✓ Solution by Mathematica
Time used: 0.141 (sec). Leaf size: 58

```
DSolve[a*x^2*y[x] + 6*y'[x] + 6*x*y''[x] + x^2*Derivative[3][y][x] == 0,y[x],x,IncludeSingul
```

$$y(x) \to \frac{c_1 e^{-\sqrt[3]{a}x} + c_2 e^{\sqrt[3]{-1}\sqrt[3]{a}x} + c_3 e^{-(-1)^{2/3}\sqrt[3]{a}x}}{x^2}$$

4.49 problem 1497

Internal problem ID [9824]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1497.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^2y''' - 3(p+q)\,xy'' + 3p(3q+1)\,y' - x^2y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 77

```
dsolve(x^2*diff(diff(diff(y(x),x),x),x)-3*(p+q)*x*diff(diff(y(x),x),x)+3*p*(3*q+1)*diff(y(x)
```

$$y(x) = c_1 \,\mathrm{hypergeom}\left([\,],\left[-p+\frac{2}{3}, -q+\frac{1}{3}\right], \frac{x^3}{27}\right) + c_2 x^{2+3q}\,\mathrm{hypergeom}\left([\,],\left[q+\frac{5}{3}, \frac{4}{3}+q-p\right], \frac{x^3}{27}\right) + c_3 x^{3p+1}\,\mathrm{hypergeom}\left([\,],\left[p+\frac{4}{3}, \frac{2}{3}-q+p\right], \frac{x^3}{27}\right)$$

✓ Solution by Mathematica

Time used: 0.257 (sec). Leaf size: 127

```
DSolve[-(x^2*y[x]) + 3*p*(1 + 3*q)*y'[x] - 3*(p + q)*x*y''[x] + x^2*Derivative[3][y][x] == 0
```

$$y(x) \to c_1\,{}_0F_2\left(;\frac{2}{3}-p,\frac{1}{3}-q;\frac{x^3}{27}\right) + c_2(-1)^{p+\frac{1}{3}}3^{-3p-1}x^{3p+1}\,{}_0F_2\left(;p+\frac{4}{3},p-q+\frac{2}{3};\frac{x^3}{27}\right) + c_3(-1)^{q+\frac{2}{3}}3^{-3q-2}x^{3q+2}\,{}_0F_2\left(;q+\frac{5}{3},-p+q+\frac{4}{3};\frac{x^3}{27}\right)$$

4.50 problem 1498

Internal problem ID [9825]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1498.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^2y''' - 2(n+1)\,xy'' + \left(a\,x^2 + 6n\right)y' - 2yax = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 53

```
dsolve(x^2*diff(diff(diff(y(x),x),x),x)-2*(n+1)*x*diff(diff(y(x),x),x)+(a*x^2+6*n)*diff(y(x)
```

$$y(x) = c_1 x^{n+\frac{1}{2}} \operatorname{BesselJ}\left(-n-\frac{1}{2}, x\sqrt{a}\right) + c_2 x^{n+\frac{1}{2}} \operatorname{BesselY}\left(-n-\frac{1}{2}, x\sqrt{a}\right) + c_3\left(a\,x^2 + 4n - 2\right)$$

✓ Solution by Mathematica

Time used: 6.184 (sec). Leaf size: 353

```
DSolve[-2*a*x*y[x] + (6*n + a*x^2)*y'[x] - 2*(1 + n)*x*y''[x] + x^2*Derivative[3][y][x] == 0
```

$$y(x) \to 2^{-n-\frac{3}{2}}\left(\pi c_3 4^n x^4 \sec(\pi n) \operatorname{Gamma}\left(\frac{3}{2}-n\right)\left(\sqrt{a}x\right)^{-n-\frac{1}{2}} \operatorname{BesselJ}\left(n + \frac{1}{2}, \sqrt{a}x\right) {}_1\tilde{F}_2\left(\frac{3}{2}-n; \frac{1}{2}-n, \frac{5}{2}-n; -\frac{ax^2}{4}\right)\right.$$

$$+ \frac{\operatorname{BesselY}\left(n+\frac{1}{2}, \sqrt{a}x\right)\left(2\pi c_3(4n^2-1)\left(\sqrt{a}x\right)^{n+\frac{1}{2}} + a2^{n+\frac{1}{2}}\operatorname{Gamma}\left(n+\frac{3}{2}\right)\left(2ac_2x^{n+\frac{1}{2}} - \pi\sqrt{a}c_3x^3 \operatorname{Be}\right.\right.}{}$$

4.51 problem 1499

Internal problem ID [9826]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1499.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^2y''' - (x^2 - 2x)\, y'' - \left(x^2 + \nu^2 - \frac{1}{4}\right) y' + \left(x^2 - 2x + \nu^2 - \frac{1}{4}\right) y = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 25

```
dsolve(x^2*diff(diff(diff(y(x),x),x),x)-(x^2-2*x)*diff(diff(y(x),x),x)-(x^2+nu^2-1/4)*diff(y
```

$$y(x) = c_1 e^x + c_2\sqrt{x}\ \mathrm{BesselI}(\nu, x) + c_3\sqrt{x}\ \mathrm{BesselK}(\nu, x)$$

✓ Solution by Mathematica

Time used: 0.148 (sec). Leaf size: 91

```
DSolve[(-1/4 + nu^2 - 2*x + x^2)*y[x] - (-1/4 + nu^2 + x^2)*y'[x] - (-2*x + x^2)*y''[x] + x^
```

$$y(x) \to e^x \left(\frac{c_3 x^{\nu+\frac{1}{2}}\ \mathrm{Gamma}\left(\nu + \frac{1}{2}\right)\ {}_1\tilde{F}_1\left(\nu + \frac{1}{2}; 2\nu + 1; -2x\right)}{\mathrm{Gamma}\left(\frac{3}{2} - \nu\right)} + c_2 2^{-\nu-\frac{1}{2}} G_{2,3}^{2,1}\left(2x \middle| \begin{array}{c} 1, 0 \\ \frac{1}{2} - \nu, \nu + \frac{1}{2}, 0 \end{array}\right) + c_1 \right)$$

4.52 problem 1500

Internal problem ID [9827]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1500.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^2y''' - (x+\nu)\,xy'' + \nu(2x+1)\,y' - \nu(x+1)\,y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 114

```
dsolve(x^2*diff(diff(diff(y(x),x),x),x)-(x+nu)*x*diff(diff(y(x),x),x)+nu*(2*x+1)*diff(y(x),x
```

$$y(x) = \frac{-c_3x^{\frac{\nu}{2}+1}\operatorname{BesselY}\left(-\nu+1, 2\sqrt{\nu}\,\sqrt{x}\right) - c_2x^{\frac{\nu}{2}+1}\operatorname{BesselJ}\left(-\nu+1, 2\sqrt{\nu}\,\sqrt{x}\right) - x^{\frac{\nu}{2}+\frac{1}{2}}\sqrt{\nu}\operatorname{BesselY}\left(-\nu, 2\sqrt{\nu}\right.}{\sqrt{x}}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-(nu*(1 + x)*y[x]) + nu*(1 + 2*x)*y'[x] - x*(v + x)*y''[x] + x^2*Derivative[3][y][x]
```

Not solved

4.53 problem 1501

Internal problem ID [9828]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1501.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^2y''' - 2(x^2 - x)\, y'' + \left(x^2 - 2x + \frac{1}{4} - \nu^2\right) y' + \left(\nu^2 - \frac{1}{4}\right) y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve(x^2*diff(diff(diff(y(x),x),x),x)-2*(x^2-x)*diff(diff(y(x),x),x)+(x^2-2*x+1/4-nu^2)*di
```

$$y(x) = c_1 e^x + c_2 e^{\frac{x}{2}} \sqrt{x}\, \mathrm{BesselI}\left(\nu, \frac{x}{2}\right) + c_3 e^{\frac{x}{2}} \sqrt{x}\, \mathrm{BesselK}\left(\nu, \frac{x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.117 (sec). Leaf size: 80

```
DSolve[(-1/4 + nu^2)*y[x] + (1/4 - nu^2 - 2*x + x^2)*y'[x] - 2*(-x + x^2)*y''[x] + x^2*Deriv
```

$$y(x) \to e^x \left(\frac{c_3 x^{\nu+\frac{1}{2}}\, \mathrm{Gamma}\left(\nu + \frac{1}{2}\right) {}_1\tilde{F}_1\left(\nu + \frac{1}{2}; 2\nu + 1; -x\right)}{\mathrm{Gamma}\left(\frac{3}{2} - \nu\right)} + c_2 G_{2,3}^{2,1}\left(x \,\middle|\, \begin{matrix} 1, 0 \\ \frac{1}{2} - \nu, \nu + \frac{1}{2}, 0 \end{matrix}\right) + c_1 \right)$$

4.54 problem 1502

Internal problem ID [9829]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1502.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^2y''' - (x^4 - 6x)\,y'' - (2x^3 - 6)\,y' + 2x^2y = 0$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 103

```
dsolve(x^2*diff(diff(diff(y(x),x),x),x)-(x^4-6*x)*diff(diff(y(x),x),x)-(2*x^3-6)*diff(y(x),x
```

$$y(x) = \frac{c_3\left(\int e^{\frac{x^3}{6}}\sqrt{x}\left(\mathrm{BesselK}\left(\frac{5}{6},-\frac{x^3}{6}\right)x^3 - \mathrm{BesselK}\left(\frac{1}{6},-\frac{x^3}{6}\right)x^3 + 2\,\mathrm{BesselK}\left(\frac{1}{6},-\frac{x^3}{6}\right)\right)dx\right) + c_2\left(\int e^{\frac{x^3}{6}}\sqrt{x}\,(\ldots}{x^2}$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 98

```
DSolve[2*x^2*y[x] - (-6 + 2*x^3)*y'[x] - (-6*x + x^4)*y''[x] + x^2*Derivative[3][y][x] == 0,
```

$$y(x) \to \frac{c_2\,\mathrm{Gamma}\left(\frac{1}{3}\right)\,{}_2F_2\left(-\frac{2}{3},\frac{1}{3};\frac{2}{3},\frac{4}{3};\frac{x^3}{3}\right)}{3x\,\mathrm{Gamma}\left(\frac{4}{3}\right)} + \frac{\sqrt[3]{-\frac{1}{3}}c_3\,\mathrm{Gamma}\left(\frac{2}{3}\right)\,{}_2F_2\left(-\frac{1}{3},\frac{2}{3};\frac{4}{3},\frac{5}{3};\frac{x^3}{3}\right)}{3\,\mathrm{Gamma}\left(\frac{5}{3}\right)} + \frac{c_1}{x^2}$$

4.55 problem 1503

Internal problem ID [9830]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1503.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$(x^2+1)\,y''' + 8y''x + 10y' = 3 - \frac{1}{x^2} + 2\ln(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 67

```
dsolve((x^2+1)*diff(diff(diff(y(x),x),x),x)+8*x*diff(diff(y(x),x),x)+10*diff(y(x),x)-3+1/x^2
```

$$y(x) = \frac{(45x^5+150x^3+225x)\ln(x) - 9x^5 + 225c_1x^4 + (225c_2-50)\,x^3 + 450c_1x^2 + (675c_2-225)\,x + 225c_3}{225\,(x^2+1)^2}$$

✓ Solution by Mathematica

Time used: 0.606 (sec). Leaf size: 258

```
DSolve[-3 + x^(-2) - 2*Log[x] + 10*y'[x] + 8*x*y''[x] + (1 + x^2)*Derivative[3][y][x] == 0,y
```

$$y(x) \to \frac{1}{225}\Bigg(-3(17+75c_2)\arctan(x) - \frac{51x}{x^2+1} - \frac{34x}{(x^2+1)^2} - \frac{225c_2x}{x^2+1} - \frac{150c_2x}{(x^2+1)^2} - \frac{225c_1}{4\,(x^2+1)^2} - 9x + \frac{47}{x-i} + \frac{47}{x+i} + 45x\log(x) + 60i\log(-x+i) + \frac{171}{2}i\log(1-ix) - \frac{171}{2}i\log(1+ix) + \frac{30\log(x)}{x-i} + \frac{30\log(x)}{x+i} - \frac{30i\log(x)}{(x-i)^2} + \frac{30i\log(x)}{(x+i)^2} - 60i\log(x+i) + \frac{75c_2}{x-i} + \frac{75c_2}{x+i} + \frac{225}{2}ic_2\log(1-ix) - \frac{225}{2}ic_2\log(1+ix)\Bigg) + c_3$$

4.56 problem 1504

Internal problem ID [9831]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1504.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$(x^2+2)\,y''' - 2y''x + (x^2+2)\,y' - 2xy = 0$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 18

```
dsolve((x^2+2)*diff(diff(diff(y(x),x),x),x)-2*x*diff(diff(y(x),x),x)+(x^2+2)*diff(y(x),x)-2*
```

$$y(x) = c_1x^2 + c_2\cos(x) + c_3\sin(x)$$

✓ Solution by Mathematica
Time used: 0.166 (sec). Leaf size: 41

```
DSolve[-2*x*y[x] + (2 + x^2)*y'[x] - 2*x*y''[x] + (2 + x^2)*Derivative[3][y][x] == 0,y[x],x,
```

$$y(x) \to \frac{1}{4}\left(2c_1x^2 + 2ic_2e^{-ix} - c_3e^{ix}\right)$$

4.57 problem 1505

Internal problem ID [9832]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1505.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$2x(x-1)\,y''' + 3(2x-1)\,y'' + (2xa+b)\,y' + ay = 0$$

✓ Solution by Maple
Time used: 0.453 (sec). Leaf size: 79

```
dsolve(2*x*(x-1)*diff(diff(diff(y(x),x),x),x)+3*(2*x-1)*diff(diff(y(x),x),x)+(2*a*x+b)*diff(
```

$$y(x) = c_1 \operatorname{MathieuC}\left(1-\frac{a}{2}-\frac{b}{2}, \frac{a}{4}, \arccos\left(\sqrt{x}\right)\right)^2 + c_2 \operatorname{MathieuS}\left(1-\frac{a}{2}-\frac{b}{2}, \frac{a}{4}, \arccos\left(\sqrt{x}\right)\right)^2 + c_3 \operatorname{MathieuC}\left(1-\frac{a}{2}-\frac{b}{2}, \frac{a}{4}, \arccos\left(\sqrt{x}\right)\right) \operatorname{MathieuS}\left(1-\frac{a}{2}-\frac{b}{2}, \frac{a}{4}, \arccos\left(\sqrt{x}\right)\right)$$

✓ Solution by Mathematica
Time used: 60.212 (sec). Leaf size: 115

```
DSolve[a*y[x] + (b + 2*a*x)*y'[x] + 3*(-1 + 2*x)*y''[x] + 2*(-1 + x)*x*Derivative[3][y][x] =
```

$$y(x) \to c_3\operatorname{MathieuC}\left[-\frac{a}{2}-\frac{b}{2}+1, \frac{a}{4}, \arccos\left(\sqrt{x}\right)\right] \operatorname{MathieuS}\left[-\frac{a}{2}-\frac{b}{2}+1, \frac{a}{4}, \arccos\left(\sqrt{x}\right)\right] + c_1\operatorname{MathieuC}\left[-\frac{a}{2}-\frac{b}{2}+1, \frac{a}{4}, \arccos\left(\sqrt{x}\right)\right]^2 + c_2\operatorname{MathieuS}\left[-\frac{a}{2}-\frac{b}{2}+1, \frac{a}{4}, \arccos\left(\sqrt{x}\right)\right]^2$$

4.58 problem 1508

Internal problem ID [9833]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1508.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^3y''' + (-\nu^2 + 1)\,xy' + (a\,x^3 + \nu^2 - 1)\,y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 78

```
dsolve(x^3*diff(diff(diff(y(x),x),x),x)+(-nu^2+1)*x*diff(y(x),x)+(a*x^3+nu^2-1)*y(x)=0,y(x),
```

$$y(x) = x\left(c_1 \,\text{hypergeom}\left([\,],\left[1-\frac{\nu}{3}, 1+\frac{\nu}{3}\right], -\frac{x^3 a}{27}\right) + c_2 x^{-\nu} \,\text{hypergeom}\left([\,],\left[1-\frac{2\nu}{3}, 1-\frac{\nu}{3}\right], -\frac{x^3 a}{27}\right) + c_3 x^{\nu} \,\text{hypergeom}\left([\,],\left[1+\frac{2\nu}{3}, 1+\frac{\nu}{3}\right], -\frac{x^3 a}{27}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.429 (sec). Leaf size: 143

```
DSolve[(-1 + nu^2 + a*x^3)*y[x] + (1 - nu^2)*x*y'[x] + x^3*Derivative[3][y][x] == 0,y[x],x,I
```

$$y(x) \to 3^{-\nu-1} x a^{-\nu/3}\left(a^{\frac{\nu+1}{3}}\left(c_3 a^{\nu/3} x^{\nu}\, {}_0F_2\left(;\frac{\nu}{3}+1, \frac{2\nu}{3}+1; -\frac{ax^3}{27}\right) + c_1 3^{\nu}\, {}_0F_2\left(;1-\frac{\nu}{3}, \frac{\nu}{3}+1; -\frac{ax^3}{27}\right)\right) + \sqrt[3]{a} c_2 9^{\nu} x^{-}\right.$$

4.59 problem 1509

Internal problem ID [9834]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1509.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^3y''' + (4x^3 + (-4\nu^2 + 1)x)y' + (4\nu^2 - 1)y = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 29

```
dsolve(x^3*diff(diff(diff(y(x),x),x),x)+(4*x^3+(-4*nu^2+1)*x)*diff(y(x),x)+(4*nu^2-1)*y(x)=0
```

$$y(x) = x\left(c_1 \operatorname{BesselJ}(\nu, x)^2 + c_2 \operatorname{BesselY}(\nu, x)^2 + c_3 \operatorname{BesselJ}(\nu, x) \operatorname{BesselY}(\nu, x)\right)$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 33

```
DSolve[(-1 + 4*nu^2)*y[x] + ((1 - 4*nu^2)*x + 4*x^3)*y'[x] + x^3*Derivative[3][y][x] == 0,y[
```

$$y(x) \to x\left(c_1 \operatorname{BesselJ}(\nu, x)^2 + c_3 \operatorname{BesselY}(\nu, x)^2 + c_2 \operatorname{BesselJ}(\nu, x) \operatorname{BesselY}(\nu, x)\right)$$

4.60 problem 1510

Internal problem ID [9835]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1510.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^3y''' + \left(a\,x^{2\nu} + 1 - \nu^2\right)xy' + \left(b\,x^{3\nu} + a(\nu - 1)\,x^{2\nu} + \nu^2 - 1\right)y = 0$$

✗ Solution by Maple

```
dsolve(x^3*diff(diff(diff(y(x),x),x),x)+(a*x^(2*nu)+1-nu^2)*x*diff(y(x),x)+(b*x^(3*nu)+a*(nu
```

No solution found

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 102

```
DSolve[(-1 + nu^2 + a*(-1 + nu)*x^(2*nu) + b*x^(3*nu))*y[x] + x*(1 - nu^2 + a*x^(2*nu))*y'[x
```

$$y(x) \to c_1x^{1-\nu}e^{\frac{x^\nu \text{Root}\left[\#1^3+\#1a+b\&,1\right]}{\nu}} + c_2x^{1-\nu}e^{\frac{x^\nu \text{Root}\left[\#1^3+\#1a+b\&,2\right]}{\nu}} + c_3x^{1-\nu}e^{\frac{x^\nu \text{Root}\left[\#1^3+\#1a+b\&,3\right]}{\nu}}$$

4.61 problem 1511

Internal problem ID [9836]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1511.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^3y''' + 3x^2y'' - 2y'x + 2y = 6x^3(x-1)\ln(x) - x^3(x+8)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 49

```
dsolve(x^3*diff(diff(diff(y(x),x),x),x)+3*x^2*diff(diff(y(x),x),x)-2*x*diff(y(x),x)+2*y(x)-6
```

$$y(x) = \frac{(50x^6 - 135x^5 + 450c_3x^3)\ln(x) - 50x^6 - 18x^5 + 450c_1x^3 + 450c_2}{450x^2}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 52

```
DSolve[x^3*(8 + x) - 6*(-1 + x)*x^3*Log[x] + 2*y[x] - 2*x*y'[x] + 3*x^2*y''[x] + x^3*Derivat
```

$$y(x) \to -\frac{x^4}{9} - \frac{x^3}{25} + \frac{c_1}{x^2} + \left(\frac{x^4}{9} - \frac{3x^3}{10} + c_3x\right)\log(x) + c_2x$$

4.62 problem 1512

Internal problem ID [9837]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1512.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$x^3y''' + 3x^2y'' + (-a^2 + 1)\, xy' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^3*diff(diff(diff(y(x),x),x),x)+3*x^2*diff(diff(y(x),x),x)+(-a^2+1)*x*diff(y(x),x)=0
```

$$y(x) = c_1 + c_2x^a + c_3x^{-a}$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 29

```
DSolve[(1 - a^2)*x*y'[x] + 3*x^2*y''[x] + x^3*Derivative[3][y][x] == 0,y[x],x,IncludeSingula
```

$$y(x) \to \frac{-c_1x^{-a} + c_2x^a + ac_3}{a}$$

4.63 problem 1513

Internal problem ID [9838]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1513.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^3y''' - 4x^2y'' + (x^2 + 8) xy' - 2(x^2 + 4) y = 0$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 18

```
dsolve(x^3*diff(diff(diff(y(x),x),x),x)-4*x^2*diff(diff(y(x),x),x)+(x^2+8)*x*diff(y(x),x)-2*
```

$$y(x) = x(\cos(x) c_3 + \sin(x) c_2 + c_1 x)$$

✓ Solution by Mathematica
Time used: 0.079 (sec). Leaf size: 23

```
DSolve[-2*(4 + x^2)*y[x] + x*(8 + x^2)*y'[x] - 4*x^2*y''[x] + x^3*Derivative[3][y][x] == 0,y
```

$$y(x) \to x(c_1 x + c_3 \cos(x) - c_2 \sin(x))$$

4.64 problem 1514

Internal problem ID [9839]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1514.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^3y''' + 6x^2y'' + (a\,x^3 - 12)\,y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 134

```
dsolve(x^3*diff(diff(diff(y(x),x),x),x)+6*x^2*diff(diff(y(x),x),x)+(a*x^3-12)*y(x)=0,y(x), s
```

$$y(x) = \frac{-c_2\left(\left(-i-\sqrt{3}\right)\left(-a^4\right)^{\frac{2}{3}} + ix\,a^3\right)\mathrm{e}^{-\frac{\left(-a^4\right)^{\frac{1}{3}}\left(1+i\sqrt{3}\right)x}{2a}} + c_3\left(\left(-i+\sqrt{3}\right)\left(-a^4\right)^{\frac{2}{3}} + ix\,a^3\right)\mathrm{e}^{\frac{\left(-a^4\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)x}{2a}} + c_1\left(\right.}{x^3}$$

✓ Solution by Mathematica

Time used: 0.366 (sec). Leaf size: 97

```
DSolve[(-12 + a*x^3)*y[x] + 6*x^2*y''[x] + x^3*Derivative[3][y][x] == 0,y[x],x,IncludeSingul
```

$$y(x) \to \frac{c_1 e^{-\sqrt[3]{a}x}\left(\sqrt[3]{a}x+2\right) + c_2 e^{\sqrt[3]{-1}\sqrt[3]{a}x}\left(\sqrt[3]{a}x+2(-1)^{2/3}\right) + c_3 e^{-(-1)^{2/3}\sqrt[3]{a}x}\left(\sqrt[3]{a}x-2\sqrt[3]{-1}\right)}{x^3}$$

4.65 problem 1515

Internal problem ID [9840]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1515.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^3y''' + 3(1-a)\,x^2y'' + \left(4x^{1+2c}b^2c^2 + 1 - 4\nu^2c^2 + 3a(-1+a)\,x\right)y' + \left(4b^2c^2(c-a)\,x^{2c} + a\left(4\nu^2c^2 - a^2\right)\right)$$

✗ Solution by Maple

```
dsolve(x^3*diff(diff(diff(y(x),x),x),x)+3*(1-a)*x^2*diff(diff(y(x),x),x)+(4*b^2*c^2*x^(2*c+1
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a*(-a^2 + 4*c^2*nu^2) + 4*b^2*c^2*(-a + c)*x^(2*c))*y[x] + (1 - 4*c^2*nu^2 + 3*(-1 +
```

Not solved

4.66 problem 1516

Internal problem ID [9841]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1516.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^3y''' + (x+3)\,x^2y'' + 5(x-6)\,xy' + (4x+30)\,y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 188

```
dsolve(x^3*diff(diff(diff(y(x),x),x),x)+(x+3)*x^2*diff(diff(y(x),x),x)+5*(x-6)*x*diff(y(x),x
```

$y(x)$
$= \frac{c_3 e^{-x}(x^8 + 28x^7 + 450x^6 + 5100x^5 + 42900x^4 + 267120x^3 + 1179360x^2 + 3326400x + 4536000) \operatorname{expInt}}{}$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(30 + 4*x)*y[x] + 5*(-6 + x)*x*y'[x] + x^2*(3 + x)*y''[x] + x^3*Derivative[3][y][x] =
```

Timed out

4.67 problem 1517

Internal problem ID [9842]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1517.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$x^3y''' + x^2y'' + 2y'x - y = 2x^3 - \ln(x)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 1195

```
dsolve(x^3*diff(diff(diff(y(x),x),x),x)+x^2*diff(diff(y(x),x),x)+ln(x)+2*x*diff(y(x),x)-y(x)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 0.642 (sec). Leaf size: 601

```
DSolve[-2*x^3 + Log[x] - y[x] + 2*x*y'[x] + x^2*y''[x] + x^3*Derivative[3][y][x] == 0,y[x],x
```

$$y(x) \to \frac{i(\text{Root}[\#1^3 - 2\#1^2 + 3\#1 - 1\&, 1] - \text{Root}[\#1^3 - 2\#1^2 + 3\#1 - 1\&, 2])\left(\frac{2x^3}{3-\text{Root}[\#1^3-2\#1^2+3\#1-}\right.}{\sqrt{23}}$$
$$- \frac{i(\text{Root}[\#1^3 - 2\#1^2 + 3\#1 - 1\&, 1] - \text{Root}[\#1^3 - 2\#1^2 + 3\#1 - 1\&, 3])\left(\frac{2x^3}{3-\text{Root}[\#1^3-2\#1^2+3\#1}\right.}{\sqrt{23}}$$
$$+ \frac{i(\text{Root}[\#1^3 - 2\#1^2 + 3\#1 - 1\&, 2] - \text{Root}[\#1^3 - 2\#1^2 + 3\#1 - 1\&, 3])\, x^{\text{Root}[\#1^3-2\#1^2+3\#1-1\&,1]+}}{}$$
$$+ c_1 x^{\text{Root}[\#1^3-2\#1^2+3\#1-1\&,1]} + c_3 x^{\text{Root}[\#1^3-2\#1^2+3\#1-1\&,3]} + c_2 x^{\text{Root}[\#1^3-2\#1^2+3\#1-1\&,2]}$$

4.68 problem 1518

Internal problem ID [9843]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1518.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,xy''' + 3(2x^2+1)\,y'' - 12y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 60

```
dsolve((x^2+1)*x*diff(diff(diff(y(x),x),x),x)+3*(2*x^2+1)*diff(diff(y(x),x),x)-12*y(x)=0,y(x
```

$$y(x) = \frac{3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)\sqrt{x^2+1}\,c_2x^2 + c_1\sqrt{x^2+1}\,x^2 + 2c_3x^3 - 3c_2x^2 + c_3x - c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.483 (sec). Leaf size: 69

```
DSolve[-12*y[x] + 3*(1 + 2*x^2)*y''[x] + x*(1 + x^2)*Derivative[3][y][x] == 0,y[x],x,Include
```

$$y(x) \to \frac{1}{6}\left(-3c_3x\sqrt{x^2+1}\mathrm{arctanh}\left(\sqrt{x^2+1}\right) + c_1\left(4x^2+2\right) + 2c_2x\sqrt{x^2+1} + 3c_3x + \frac{c_3}{x}\right)$$

4.69 problem 1519

Internal problem ID [9844]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1519.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$(x+3)\,x^2y''' - 3x(x+2)\,y'' + 6(x+1)\,y' - 6y = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 19

```
dsolve((x+3)*x^2*diff(diff(diff(y(x),x),x),x)-3*x*(x+2)*diff(diff(y(x),x),x)+6*(x+1)*diff(y(
```

$$y(x) = c_2x^3 + c_1x^2 + c_3x + c_3$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 58

```
DSolve[-6*y[x] + 6*(1 + x)*y'[x] - 3*x*(2 + x)*y''[x] + x^2*(3 + x)*Derivative[3][y][x] == 0
```

$$y(x) \to \frac{1}{8}\left(2c_1\left(x^3 - 3x^2 + 3x + 3\right) - (x-1)\left(4c_2\left(x^2 - 2x - 1\right) + c_3\left(-3x^2 + 2x + 1\right)\right)\right)$$

4.70 problem 1520

Internal problem ID [9845]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1520.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$2(x-\mathrm{a1})(x-\mathrm{a2})(x-\mathrm{a3})y''' + \left(9x^2 - 6(\mathrm{a1}+\mathrm{a2}+\mathrm{a3})x + 3\,\mathrm{a2}\,\mathrm{a1} + 3\,\mathrm{a1}\,\mathrm{a3} + 3\,\mathrm{a2}\,\mathrm{a3}\right)y'' - 2\left(\left(n^2+n-\right.\right.$$

✓ Solution by Maple

Time used: 0.812 (sec). Leaf size: 288

```
dsolve(2*(x-a1)*(x-a2)*(x-a3)*diff(diff(diff(y(x),x),x),x)+(9*x^2-6*(a1+a2+a3)*x+3*a1*a2+3*a
```

$$y(x) = -c_2(x - \mathrm{a1})\,\mathrm{HeunG}\left(\frac{\mathrm{a1}-\mathrm{a3}}{-\mathrm{a2}+\mathrm{a1}}, \frac{(-n^2-n+3)\,\mathrm{a1}-b}{-4\,\mathrm{a2}+4\,\mathrm{a1}}, \frac{n}{2}+1, -\frac{n}{2}+\frac{1}{2}, \frac{3}{2}, \frac{1}{2}, \frac{-x+\mathrm{a1}}{-\mathrm{a2}+\mathrm{a1}}\right)^2 + c_3\,\mathrm{HeunG}\left(\frac{\mathrm{a1}-\mathrm{a3}}{-\mathrm{a2}+\mathrm{a1}}, \frac{(-n^2-n+1)\,\mathrm{a1}-b+\mathrm{a2}+\mathrm{a3}}{-4\,\mathrm{a2}+4\,\mathrm{a1}}, -\frac{n}{2}, \frac{n}{2}+\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{-x+\mathrm{a1}}{-\mathrm{a2}+\mathrm{a1}}\right)\mathrm{HeunG}\left(\frac{\mathrm{a1}-\mathrm{a3}}{-\mathrm{a2}+\mathrm{a1}}, \frac{(-n^2-n+3)\,\mathrm{a1}-b}{-4\,\mathrm{a2}+4\,\mathrm{a1}}, \frac{n}{2}+1, -\frac{n}{2}+\frac{1}{2}, \frac{3}{2}, \frac{1}{2}, \frac{-x+\mathrm{a1}}{-\mathrm{a2}+\mathrm{a1}}\right)\sqrt{-x+\mathrm{a1}} + c_1\mathrm{HeunG}\left(\frac{\mathrm{a1}-\mathrm{a3}}{-\mathrm{a2}+\mathrm{a1}}, \frac{(-n^2-n+1)\,\mathrm{a1}-b+\mathrm{a2}+\mathrm{a3}}{-4\,\mathrm{a2}+4\,\mathrm{a1}}, -\frac{n}{2}, \frac{n}{2}+\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{-x+\mathrm{a1}}{-\mathrm{a2}+\mathrm{a1}}\right)^2$$

✓ Solution by Mathematica

Time used: 4.47 (sec). Leaf size: 418

```
DSolve[-(n*(1 + n)*y[x]) - 2*(b + (-3 + n + n^2)*x)*y'[x] + (3*a1*a2 + 3*a1*a3 + 3*a2*a3 - 6
```

$$y(x) \to \frac{c_3(\mathrm{a1}-x)\mathrm{HeunG}\left[\frac{\mathrm{a1}-\mathrm{a3}}{\mathrm{a1}-\mathrm{a2}},-\frac{\mathrm{a1}(n^2+n-3)+b}{4(\mathrm{a1}-\mathrm{a2})},\frac{3}{4}-\frac{1}{4}\sqrt{(2n+1)^2},\frac{1}{4}\left(\sqrt{(2n+1)^2}+3\right),\frac{3}{2},\frac{1}{2},\frac{\mathrm{a1}-x}{\mathrm{a1}-\mathrm{a2}}\right]^2}{\mathrm{a1}-\mathrm{a2}} + c_2\sqrt{\frac{\mathrm{a1}-x}{\mathrm{a1}-\mathrm{a2}}}\mathrm{HeunG}\left[\frac{\mathrm{a1}-\mathrm{a3}}{\mathrm{a1}-\mathrm{a2}},\frac{-\mathrm{a1}(n^2+n-1)+\mathrm{a2}+\mathrm{a3}-b}{4(\mathrm{a1}-\mathrm{a2})},\frac{1}{4}-\frac{1}{4}\sqrt{(2n+1)^2},\frac{1}{4}\left(\sqrt{(2n+1)^2}+1\right),\frac{1}{2},\frac{1}{2},\frac{\mathrm{a1}-x}{\mathrm{a1}-\mathrm{a2}}\right]\mathrm{HeunG}\left[\frac{\mathrm{a1}-\mathrm{a3}}{\mathrm{a1}-\mathrm{a2}},-\frac{\mathrm{a1}(n^2+n-3)+b}{4(\mathrm{a1}-\mathrm{a2})},\frac{3}{4}-\frac{1}{4}\sqrt{(2n+1)^2},\frac{1}{4}\left(\sqrt{(2n+1)^2}+3\right),\frac{3}{2},\frac{1}{2},\frac{\mathrm{a1}-x}{\mathrm{a1}-\mathrm{a2}}\right] + c_1\mathrm{HeunG}\left[\frac{\mathrm{a1}-\mathrm{a3}}{\mathrm{a1}-\mathrm{a2}},\frac{-\mathrm{a1}(n^2+n-1)+\mathrm{a2}+\mathrm{a3}-b}{4(\mathrm{a1}-\mathrm{a2})},\frac{1}{4}-\frac{1}{4}\sqrt{(2n+1)^2},\frac{1}{4}\left(\sqrt{(2n+1)^2}+1\right),\frac{1}{2},\frac{1}{2},\frac{\mathrm{a1}-x}{\mathrm{a1}-\mathrm{a2}}\right]^2$$

4.71 problem 1521

Internal problem ID [9846]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1521.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$(x+1)\,x^3y''' - (4x+2)\,x^2y'' + (10x+4)\,xy' - 4(3x+1)\,y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 28

```
dsolve((x+1)*x^3*diff(diff(diff(y(x),x),x),x)-(4*x+2)*x^2*diff(diff(y(x),x),x)+(10*x+4)*x*di
```

$$y(x) = x\left(\ln(x)^2 c_3 x + c_2 x \ln(x) + x^2 c_3 + c_1 x + c_3\right)$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 29

```
DSolve[-4*(1 + 3*x)*y[x] + x*(4 + 10*x)*y'[x] - x^2*(2 + 4*x)*y''[x] + x^3*(1 + x)*Derivativ
```

$$y(x) \to x^2\left(c_3\left(x + \frac{1}{x} + \log^2(x)\right) + c_2 \log(x) + c_1\right)$$

4.72 problem 1522

Internal problem ID [9847]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1522.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{4x^4y''' - 4x^3y'' + 4x^2y' = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(4*x^4*diff(diff(diff(y(x),x),x),x)-4*x^3*diff(diff(y(x),x),x)+4*x^2*diff(y(x),x)-1=0,
```

$$y(x) = \frac{18\ln(x)\,c_1x^3 - 1 + (-9c_1 + 18c_2)\,x^3 + 36c_3x}{36x}$$

✓ Solution by Mathematica
Time used: 0.045 (sec). Leaf size: 42

```
DSolve[-1 + 4*x^2*y'[x] - 4*x^3*y''[x] + 4*x^4*Derivative[3][y][x] == 0,y[x],x,IncludeSingul
```

$$y(x) \to \frac{1}{4}(2c_1 - c_2)x^2 + \frac{1}{2}c_2x^2\log(x) - \frac{1}{36x} + c_3$$

4.73 problem 1523

Internal problem ID [9848]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1523.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$(x^2+1)\,x^3y''' - (4x^2+2)\,x^2y'' + (10x^2+4)\,xy' - 4(3x^2+1)\,y = 0$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 23

```
dsolve((x^2+1)*x^3*diff(diff(diff(y(x),x),x),x)-(4*x^2+2)*x^2*diff(diff(y(x),x),x)+(10*x^2+4
```

$$y(x) = x\left(c_2x\ln(x) + x^2c_3 + (c_1+c_2)\,x + c_3\right)$$

✓ Solution by Mathematica
Time used: 0.495 (sec). Leaf size: 46

```
DSolve[-4*(1 + 3*x^2)*y[x] + x*(4 + 10*x^2)*y'[x] - x^2*(2 + 4*x^2)*y''[x] + x^3*(1 + x^2)*D
```

$$y(x) \to \frac{1}{2}x\left(c_2x^2 - 2c_1\left(x^2-3x+1\right) - 2c_2x + c_3x + c_3x\log(x) + c_2\right)$$

4.74 problem 1524

Internal problem ID [9849]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1524.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^6 y''' + x^2 y'' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 98

```
dsolve(x^6*diff(diff(diff(y(x),x),x),x)+x^2*diff(diff(y(x),x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^2 \left(c_1 + \left(\int \frac{e^{\frac{1}{6x^3}} \left(2x^3 \operatorname{BesselI}\left(\frac{1}{6}, -\frac{1}{6x^3}\right) - \operatorname{BesselI}\left(\frac{1}{6}, -\frac{1}{6x^3}\right) - \operatorname{BesselI}\left(-\frac{5}{6}, -\frac{1}{6x^3}\right)\right)}{x^{\frac{11}{2}}} dx \right) c_2 + \left(\int \frac{e^{\frac{1}{6x^3}} \left(2x^3 \operatorname{BesselK}\left(\frac{1}{6}, -\frac{1}{6x^3}\right) - \operatorname{BesselK}\left(\frac{1}{6}, -\frac{1}{6x^3}\right) + \operatorname{BesselK}\left(\frac{5}{6}, -\frac{1}{6x^3}\right)\right)}{x^{\frac{11}{2}}} dx \right) c_3 \right)$$

✓ Solution by Mathematica

Time used: 0.19 (sec). Leaf size: 96

```
DSolve[-2*y[x] + x^2*y''[x] + x^6*Derivative[3][y][x] == 0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to -\frac{\left(-\frac{1}{3}\right)^{2/3} c_2 x \operatorname{Gamma}\left(\frac{1}{3}\right) {}_2F_2\left(-\frac{2}{3}, \frac{1}{3}; \frac{2}{3}, \frac{4}{3}; \frac{1}{3x^3}\right)}{3 \operatorname{Gamma}\left(\frac{4}{3}\right)} + \frac{c_3 \operatorname{Gamma}\left(\frac{2}{3}\right) {}_2F_2\left(-\frac{1}{3}, \frac{2}{3}; \frac{4}{3}, \frac{5}{3}; \frac{1}{3x^3}\right)}{9 \operatorname{Gamma}\left(\frac{5}{3}\right)} + c_1 x^2$$

4.75 problem 1525

Internal problem ID [9850]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1525.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^6 y''' + 6y'' x^5 + ay = 0$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 212

```
dsolve(x^6*diff(diff(diff(y(x),x),x),x)+6*x^5*diff(diff(y(x),x),x)+a*y(x)=0,y(x), singsol=al
```

$$y(x) = \frac{4(-8x^3+a)^4 \left(c_3 e^{\frac{(-a^4)^{\frac{1}{3}}(1+i\sqrt{3})}{2ax}} \left(\frac{(-i+\sqrt{3})(-a^4)^{\frac{1}{3}}}{4} + iax\right) + \left(\frac{(\sqrt{3}+i)(-a^4)^{\frac{1}{3}}}{4} - iax\right) c_2 e^{-\frac{(-a^4)^{\frac{1}{3}}(i\sqrt{3}-1)}{2ax}} + 12}{\left(2ax + (-a^4)^{\frac{1}{3}}\right)^4 \left(i(-a^4)^{\frac{1}{3}}\sqrt{3} - 4ax + (-a^4)^{\frac{1}{3}}\right)^4 \left(i(-a^4)^{\frac{1}{3}}\sqrt{3} + 4ax - (-a^4}$$

✓ Solution by Mathematica
Time used: 0.366 (sec). Leaf size: 101

```
DSolve[a*y[x] + 6*x^5*y''[x] + x^6*Derivative[3][y][x] == 0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to c_1\left(-e^{\frac{\sqrt[3]{a}}{x}}\right)\left(\sqrt[3]{a} - 2x\right) + c_2 e^{\frac{(-1)^{2/3}\sqrt[3]{a}}{x}}\left(x - \frac{1}{2}(-1)^{2/3}\sqrt[3]{a}\right) + c_3 e^{-\frac{\sqrt[3]{-1}\sqrt[3]{a}}{x}}\left(\frac{1}{2}\sqrt[3]{-1}\sqrt[3]{a} + x\right)$$

4.76 problem 1526

Internal problem ID [9851]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1526.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^2\left(x^4+2x^2+2x+1\right)y'''-\left(2x^6+3x^4-6x^2-6x-1\right)y''+\left(x^6-6x^3-15x^2-12x-2\right)y'+\left(x^4+4x\right.$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(x^2*(x^4+2*x^2+2*x+1)*diff(diff(diff(y(x),x),x),x)-(2*x^6+3*x^4-6*x^2-6*x-1)*diff(dif
```

$$y(x)=c_2\mathrm{e}^{\frac{1}{x}}+\mathrm{e}^x(c_3x+c_1)$$

✓ Solution by Mathematica
Time used: 130.169 (sec). Leaf size: 25

```
DSolve[(1 + 6*x + 8*x^2 + 4*x^3 + x^4)*y[x] + (-2 - 12*x - 15*x^2 - 6*x^3 + x^6)*y'[x] - (-1
```

$$y(x)\to e^x(c_2x+c_1)+c_3e^{\frac{1}{x}}$$

4.77 problem 1527

Internal problem ID [9852]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1527.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$(x-a)^3 (x-b)^3 y''' - yc = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 437

```
dsolve((x-a)^3*(x-b)^3*diff(diff(diff(y(x),x),x),x)-c*y(x)=0,y(x), singsol=all)
```

$$y(x) = (x-a)^{-\frac{2b}{a-b}} (x-b)^{\frac{2a}{a-b}} \left(c_1(-x+b)^{-\frac{\mathrm{RootOf}\left(\_Z^3+(-3b-3a)\_Z^2+\left(2a^2+8ab+2b^2\right)\_Z-4a^2b-4a\,b^2-c,\mathrm{index}=1\right)}{a-b}} (-x+a)^{\frac{\mathrm{RootOf}\left(\_Z^3+(-3b-3a)\_Z^2+\left(2a^2+8ab+2b^2\right)\_Z-4a^2b-4a\,b^2-c,\mathrm{index}=1\right)}{a-b}} + c_2(-x+b)^{-\frac{\mathrm{RootOf}\left(\_Z^3+(-3b-3a)\_Z^2+\left(2a^2+8ab+2b^2\right)\_Z-4a^2b-4a\,b^2-c,\mathrm{index}=2\right)}{a-b}} (-x+a)^{\frac{\mathrm{RootOf}\left(\_Z^3+(-3b-3a)\_Z^2+\left(2a^2+8ab+2b^2\right)\_Z-4a^2b-4a\,b^2-c,\mathrm{index}=2\right)}{a-b}} + c_3(-x+b)^{-\frac{\mathrm{RootOf}\left(\_Z^3+(-3b-3a)\_Z^2+\left(2a^2+8ab+2b^2\right)\_Z-4a^2b-4a\,b^2-c,\mathrm{index}=3\right)}{a-b}} (-x+a)^{\frac{\mathrm{RootOf}\left(\_Z^3+(-3b-3a)\_Z^2+\left(2a^2+8ab+2b^2\right)\_Z-4a^2b-4a\,b^2-c,\mathrm{index}=3\right)}{a-b}} \right)$$

✓ Solution by Mathematica

Time used: 130.138 (sec). Leaf size: 165

```
DSolve[-(c*y[x]) + (-a + x)^3*(-b + x)^3*Derivative[3][y][x] == 0,y[x],x,IncludeSingularSolu
```

$$
\begin{aligned}
y(x) \to\ & c_1(x-b)^2\left(\frac{x-a}{x-b}\right)^{\text{Root}\left[-\#1^3+3\#1^2-2\#1+\frac{c}{(a-b)^3}\&,1\right]} \\
& + c_2(x-b)^2\left(\frac{x-a}{x-b}\right)^{\text{Root}\left[-\#1^3+3\#1^2-2\#1+\frac{c}{(a-b)^3}\&,2\right]} \\
& + c_3(x-b)^2\left(\frac{x-a}{x-b}\right)^{\text{Root}\left[-\#1^3+3\#1^2-2\#1+\frac{c}{(a-b)^3}\&,3\right]}
\end{aligned}
$$

4.78 problem 1528

Internal problem ID [9853]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1528.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$y''' \sin(x) + (2\cos(x) + 1) y'' - y' \sin(x) = \cos(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 68

```
dsolve(diff(diff(diff(y(x),x),x),x)*sin(x)+(2*cos(x)+1)*diff(diff(y(x),x),x)-diff(y(x),x)*si
```

$$y(x) = \frac{\ln(\csc(x) - \cot(x)) c_1 - \ln(\sin(x)) c_1 - \cot(x)^2 x + (c_1 x + c_2 + 1)\cot(x) + \csc(x)^2 x + (-c_1 x - c_2 -}{-\csc(x) + \cot(x)}$$

✓ Solution by Mathematica

Time used: 4.252 (sec). Leaf size: 56

```
DSolve[-Cos[x] - Sin[x]*y'[x] + (1 + 2*Cos[x])*y''[x] + Sin[x]*Derivative[3][y][x] == 0,y[x]
```

$$y(x) \to \cot\left(\frac{x}{2}\right) \arcsin(\cos(x)) - \frac{c_2 x}{\sqrt{2}} - \frac{\cot\left(\frac{x}{2}\right)(c_2 \log(2(\cos(x) + 1)) + 2c_1)}{\sqrt{2}} + c_3$$

4.79 problem 1529

Internal problem ID [9854]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1529.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _fully, _exact, _linear]]`

$$(\sin(x) + x)\, y''' + 3(1 + \cos(x))\, y'' - 3y' \sin(x) - y \cos(x) = -\sin(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve((sin(x)+x)*diff(diff(diff(y(x),x),x),x)+3*(cos(x)+1)*diff(diff(y(x),x),x)-3*diff(y(x)
```

$$y(x) = \frac{c_3 + c_1x^2 + xc_2 - \cos(x)}{\sin(x) + x}$$

✓ Solution by Mathematica

Time used: 0.113 (sec). Leaf size: 28

```
DSolve[Sin[x] - Cos[x]*y[x] - 3*Sin[x]*y'[x] + 3*(1 + Cos[x])*y''[x] + (x + Sin[x])*Derivati
```

$$y(x) \to \frac{-\cos(x) + x(c_3x + c_2) + c_1}{x + \sin(x)}$$

4.80 problem 1530

Internal problem ID [9855]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1530.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$y''' \sin(x)^2 + 3y'' \sin(x)\cos(x) + \left(\cos(2x) + 4\nu(1+\nu)\sin(x)^2\right) y' + 2\nu(1+\nu)\, y \sin(2x) = 0$$

✓ Solution by Maple

Time used: 0.36 (sec). Leaf size: 105

```
dsolve(diff(diff(diff(y(x),x),x),x)*sin(x)^2+3*diff(diff(y(x),x),x)*sin(x)*cos(x)+(cos(2*x)+
```

$$y(x) = c_1 \operatorname{hypergeom}\left(\left[-\frac{\nu}{2}, \frac{\nu}{2}+\frac{1}{2}\right], \left[\frac{1}{2}\right], \frac{\cos(2x)}{2}+\frac{1}{2}\right)^2 + c_2 \cos(x)^2 \operatorname{hypergeom}\left(\left[\frac{\nu}{2}+1, \frac{1}{2}-\frac{\nu}{2}\right], \left[\frac{3}{2}\right], \frac{\cos(2x)}{2}+\frac{1}{2}\right)^2 + c_3 \operatorname{hypergeom}\left(\left[-\frac{\nu}{2}, \frac{\nu}{2}+\frac{1}{2}\right], \left[\frac{1}{2}\right], \frac{\cos(2x)}{2} + \frac{1}{2}\right) \cos(x) \operatorname{hypergeom}\left(\left[\frac{\nu}{2}+1, \frac{1}{2}-\frac{\nu}{2}\right], \left[\frac{3}{2}\right], \frac{\cos(2x)}{2}+\frac{1}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.086 (sec). Leaf size: 35

```
DSolve[2*nu*(1 + nu)*Sin[2*x]*y[x] + (Cos[2*x] + 4*nu*(1 + nu)*Sin[x]^2)*y'[x] + 3*Cos[x]*Si
```

$$y(x) \to c_3 \operatorname{LegendreP}(\nu, \cos(x)) \operatorname{LegendreQ}(\nu, \cos(x)) + c_1 \operatorname{LegendreP}(\nu, \cos(x))^2 + c_2 \operatorname{LegendreQ}(\nu, \cos(x))^2$$

4.81 problem 1531

Internal problem ID [9856]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1531.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$f'(x)\,y''+f(x)\,y'''+g'(x)\,y'+g(x)\,y''+h'(x)\,y+h(x)\,y'+A(x)\,(y''f(x)+g(x)\,y'+h(x)\,y)=0$$

✗ Solution by Maple

```
dsolve(diff(f(x),x)*diff(diff(y(x),x),x)+f(x)*diff(diff(diff(y(x),x),x),x)+diff(g(x),x)*diff
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*Derivative[1][h][x] + h[x]*y'[x] + Derivative[1][g][x]*y'[x] + g[x]*y''[x] + Der
```

Not solved

4.82 problem 1532

Internal problem ID [9857]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1532.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' + y'x + yn = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 58

```
dsolve(diff(diff(diff(y(x),x),x),x)+x*diff(y(x),x)+n*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{hypergeom}\left(\left[\frac{n}{3}\right], \left[\frac{1}{3}, \frac{2}{3}\right], -\frac{x^3}{9}\right) + c_2 x \operatorname{hypergeom}\left(\left[\frac{1}{3}+\frac{n}{3}\right], \left[\frac{2}{3}, \frac{4}{3}\right], -\frac{x^3}{9}\right) + c_3 x^2 \operatorname{hypergeom}\left(\left[\frac{2}{3}+\frac{n}{3}\right], \left[\frac{4}{3}, \frac{5}{3}\right], -\frac{x^3}{9}\right)$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 103

```
DSolve[n*y[x] + x*y'[x] + Derivative[3][y][x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 x \, {}_1F_2\left(\frac{n}{3}+\frac{1}{3}; \frac{2}{3}, \frac{4}{3}; -\frac{x^3}{9}\right)}{3^{2/3}} + c_1 \, {}_1F_2\left(\frac{n}{3}; \frac{1}{3}, \frac{2}{3}; -\frac{x^3}{9}\right) + \frac{c_3 x^2 \, {}_1F_2\left(\frac{n}{3}+\frac{2}{3}; \frac{4}{3}, \frac{5}{3}; -\frac{x^3}{9}\right)}{3\sqrt[3]{3}}$$

4.83 problem 1533

Internal problem ID [9858]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 3, linear third order
Problem number: 1533.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$y''' - y'x - yn = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 58

```
dsolve(diff(diff(diff(y(x),x),x),x)-x*diff(y(x),x)-n*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{hypergeom}\left(\left[\frac{n}{3}\right], \left[\frac{1}{3}, \frac{2}{3}\right], \frac{x^3}{9}\right) + c_2 x \operatorname{hypergeom}\left(\left[\frac{1}{3} + \frac{n}{3}\right], \left[\frac{2}{3}, \frac{4}{3}\right], \frac{x^3}{9}\right) + c_3 x^2 \operatorname{hypergeom}\left(\left[\frac{2}{3} + \frac{n}{3}\right], \left[\frac{4}{3}, \frac{5}{3}\right], \frac{x^3}{9}\right)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 106

```
DSolve[-(n*y[x]) - x*y'[x] + Derivative[3][y][x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{9}\left(3\sqrt[3]{-3}c_2 x\, {}_1F_2\left(\frac{n}{3} + \frac{1}{3}; \frac{2}{3}, \frac{4}{3}; \frac{x^3}{9}\right) + 9c_1\, {}_1F_2\left(\frac{n}{3}; \frac{1}{3}, \frac{2}{3}; \frac{x^3}{9}\right) + (-3)^{2/3} c_3 x^2\, {}_1F_2\left(\frac{n}{3} + \frac{2}{3}; \frac{4}{3}, \frac{5}{3}; \frac{x^3}{9}\right)\right)$$

5 Chapter 4, linear fourth order

5.1 problem 1534

Internal problem ID [9859]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1534.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _quadrature]]`

$$\boxed{y'''' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(diff(diff(diff(y(x),x),x),x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{6}c_1x^3 + \frac{1}{2}c_2x^2 + c_3x + c_4$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 22

```
DSolve[Derivative[4][y][x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(x(c_4x + c_3) + c_2) + c_1$$

5.2 problem 1535

Internal problem ID [9860]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1535.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 4y = f}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 36

```
dsolve(diff(diff(diff(diff(y(x),x),x),x),x)+4*y(x)-f=0,y(x), singsol=all)
```

$$y(x) = \frac{f}{4} + \cos(x)\, c_1 e^x + c_2 e^x \sin(x) + c_3 e^{-x} \cos(x) + c_4 \sin(x)\, e^{-x}$$

✓ Solution by Mathematica
Time used: 0.308 (sec). Leaf size: 172

```
DSolve[-f[x] + 4*y[x] + Derivative[4][y][x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \to e^{-x}\Bigg(&\cos(x)\int_1^x \frac{1}{8}e^{K[1]}f(K[1])(\cos(K[1]) - \sin(K[1]))dK[1] + e^{2x}\cos(x)\int_1^x -\frac{1}{8}e^{-K[4]}f(K[4])(\cos(K[4]) + \sin(K[4]))dK[4] \\ &+ \sin(x)\int_1^x \frac{1}{8}e^{K[2]}f(K[2])(\cos(K[2]) + \sin(K[2]))dK[2] \\ &+ e^{2x}\sin(x)\int_1^x \frac{1}{8}e^{-K[3]}f(K[3])(\cos(K[3]) - \sin(K[3]))dK[3] + c_1\cos(x) \\ &+ c_4 e^{2x}\cos(x) + c_2\sin(x) + c_3 e^{2x}\sin(x)\Bigg) \end{aligned}$$

5.3 problem 1536

Internal problem ID [9861]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1536.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + \lambda y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 50

```
dsolve(diff(diff(diff(diff(y(x),x),x),x),x)+lambda*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-i(-\lambda)^{\frac{1}{4}}x} + c_2 e^{i(-\lambda)^{\frac{1}{4}}x} + c_3 e^{-(-\lambda)^{\frac{1}{4}}x} + c_4 e^{(-\lambda)^{\frac{1}{4}}x}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 76

```
DSolve[\[Lambda]*y[x] + Derivative[4][y][x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{(-1)^{3/4}\sqrt[4]{\lambda}x} + c_2 e^{-\sqrt[4]{-1}\sqrt[4]{\lambda}x} + c_3 e^{-(-1)^{3/4}\sqrt[4]{\lambda}x} + c_4 e^{\sqrt[4]{-1}\sqrt[4]{\lambda}x}$$

5.4 problem 1537

Internal problem ID [9862]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1537.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 12y'' + 12y = 16\,\mathrm{e}^{x^2}x^4$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 67

```
dsolve(diff(diff(diff(diff(y(x),x),x),x),x)-12*diff(diff(y(x),x),x)+12*y(x)-16*x^4*exp(x^2)=
```

$$y(x) = \mathrm{e}^{x^2} + c_1\mathrm{e}^{\sqrt{6-2\sqrt{6}}\,x} + c_2\mathrm{e}^{\sqrt{6+2\sqrt{6}}\,x} + c_3\mathrm{e}^{-\sqrt{6-2\sqrt{6}}\,x} + c_4\mathrm{e}^{-\sqrt{6+2\sqrt{6}}\,x}$$

✓ Solution by Mathematica

Time used: 1.472 (sec). Leaf size: 93

```
DSolve[-16*E^x^2*x^4 + 12*y[x] - 12*y''[x] + Derivative[4][y][x] == 0,y[x],x,IncludeSingular
```

$$y(x) \to e^{x^2} + c_1e^{\sqrt{6-2\sqrt{6}}x} + c_2e^{-\sqrt{6-2\sqrt{6}}x} + c_3e^{\sqrt{2\left(3+\sqrt{6}\right)}x} + c_4e^{-\sqrt{2\left(3+\sqrt{6}\right)}x}$$

5.5 problem 1538

Internal problem ID [9863]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1538.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' + 2a^2y'' + a^4y = \cosh(xa)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
dsolve(diff(diff(diff(diff(y(x),x),x),x),x)+2*a^2*diff(diff(y(x),x),x)+a^4*y(x)-cosh(a*x)=0,
```

$$y(x) = \frac{e^{-ax} + (8c_3x + 8c_1)\, a^4 \cos(ax) + (8c_4x + 8c_2)\, a^4 \sin(ax) + e^{ax}}{8a^4}$$

✓ Solution by Mathematica

Time used: 0.185 (sec). Leaf size: 41

```
DSolve[-Cosh[a*x] + a^4*y[x] + 2*a^2*y''[x] + Derivative[4][y][x] == 0,y[x],x,IncludeSingula
```

$$y(x) \to \frac{\cosh(ax)}{4a^4} + (c_2x + c_1)\cos(ax) + (c_4x + c_3)\sin(ax)$$

5.6 problem 1539

Internal problem ID [9864]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1539.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + (\lambda + 1)\, a^2 y'' + \lambda\, a^4 y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(diff(diff(diff(diff(y(x),x),x),x),x)+(lambda+1)*a^2*diff(diff(y(x),x),x)+lambda*a^4*y
```

$$y(x) = c_1 \sin(ax) + c_2 \cos(ax) + c_3 \sin\left(a\sqrt{\lambda}\, x\right) + c_4 \cos\left(a\sqrt{\lambda}\, x\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 44

```
DSolve[a^4*\[Lambda]*y[x] + a^2*(1 + \[Lambda])*y''[x] + Derivative[4][y][x] == 0,y[x],x,Inc
```

$$y(x) \to c_1 \cos\left(a\sqrt{\lambda}x\right) + c_2 \sin\left(a\sqrt{\lambda}x\right) + c_3 \cos(ax) + c_4 \sin(ax)$$

5.7 problem 1540

Internal problem ID [9865]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1540.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$y'''' + a(bx - 1)\,y'' + aby' + \lambda y = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(diff(diff(y(x),x),x),x),x)+a*(b*x-1)*diff(diff(y(x),x),x)+a*b*diff(y(x),x)+
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[\[Lambda]*y[x] + a*b*y'[x] + a*(-1 + b*x)*y''[x] + Derivative[4][y][x] == 0,y[x],x,In
```

Not solved

5.8 problem 1541

Internal problem ID [9866]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1541.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$y'''' + (a\,x^2 + b\lambda + c)\,y'' + (a\,x^2 + \beta\lambda + \gamma)\,y = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(diff(diff(y(x),x),x),x),x)+(a*x^2+b*lambda+c)*diff(diff(y(x),x),x)+(a*x^2+b
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(\[Gamma] + \[Beta]*\[Lambda] + a*x^2)*y[x] + (c + b*\[Lambda] + a*x^2)*y''[x] + Deri
```

Not solved

5.9 problem 1542

Internal problem ID [9867]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1542.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$y'''' + a \operatorname{WeierstrassP}(x, \mathrm{g2}, \mathrm{g3})\, y'' + b \operatorname{WeierstrassPPrime}(x, \mathrm{g2}, \mathrm{g3})\, y' + \left(c\left(6 \operatorname{WeierstrassP}(x, \mathrm{g2}, \mathrm{g3})^2\right.\right.$$

✗ Solution by Maple

```
dsolve(diff(diff(diff(diff(y(x),x),x),x),x)+a*WeierstrassP(x,g2,g3)*diff(diff(y(x),x),x)+b*W
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(d + c*(-1/2*g2 + 6*WeierstrassP[x, {g2, g3}]^2))*y[x] + b*WeierstrassPPrime[x, {g2,
```

Not solved

5.10 problem 1543

Internal problem ID [9868]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1543.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$y'''' - \left(12k^2 \operatorname{JacobiSN}(z, x)^2 + a\right) y'' + by' + \left(\alpha \operatorname{JacobiSN}(z, x)^2 + \beta\right) y = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(diff(diff(y(x),x),x),x),x)-(12*k^2*JacobiSN(z,x)^2+a)*diff(diff(y(x),x),x)+
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(\[Beta] + \[Alpha]*JacobiSN[z, x]^2)*y[x] + b*y'[x] - (a + 12*k^2*JacobiSN[z, x]^2)*
```

Not solved

5.11 problem 1545

Internal problem ID [9869]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1545.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' + 2y''' - 3y'' - 4y' + 4y = 32\sin(2x) - 24\cos(2x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(diff(diff(diff(y(x),x),x),x),x)+2*diff(diff(diff(y(x),x),x),x)-3*diff(diff(y(x),
```

$$y(x) = \mathrm{e}^{-2x}\left((c_3x + c_1)\,\mathrm{e}^{3x} + c_4x + \sin(2x)\,\mathrm{e}^{2x} + c_2\right)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 40

```
DSolve[24*Cos[2*x] - 32*Sin[2*x] + 4*y[x] - 4*y'[x] - 3*y''[x] + 2*Derivative[3][y][x] + Der
```

$$y(x) \to \sin(2x) + e^{-2x}\left(c_2x + c_3e^{3x} + c_4e^{3x}x + c_1\right)$$

5.12 problem 1546

Internal problem ID [9870]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1546.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$y'''' + 4axy''' + 6a^2x^2y'' + 4a^3x^3y' + a^4x^4y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 126

```
dsolve(diff(diff(diff(diff(y(x),x),x),x),x)+4*a*x*diff(diff(diff(y(x),x),x),x)+6*a^2*x^2*dif
```

$$y(x) = \mathrm{e}^{-\frac{x\left(2\sqrt{3+\sqrt{6}}\,\sqrt{a}+2\sqrt{3-\sqrt{6}}\,\sqrt{a}+ax\right)}{2}}\left(c_2\mathrm{e}^{\sqrt{a}\,x\left(\sqrt{3+\sqrt{6}}+2\sqrt{3-\sqrt{6}}\right)} + c_4\mathrm{e}^{\sqrt{a}\,x\left(2\sqrt{3+\sqrt{6}}+\sqrt{3-\sqrt{6}}\right)} + c_3\mathrm{e}^{\sqrt{3-\sqrt{6}}\,\sqrt{a}\,x} + c_1\mathrm{e}^{\sqrt{3+\sqrt{6}}\,\sqrt{a}\,x}\right)$$

✓ Solution by Mathematica

Time used: 0.737 (sec). Leaf size: 165

```
DSolve[a^4*x^4*y[x] + 4*a^3*x^3*y'[x] + 6*a^2*x^2*y''[x] + 4*a*x*Derivative[3][y][x] + Deriv
```

$$y(x) \to \frac{e^{-\frac{ax^2}{2}-\sqrt{3+\sqrt{6}}\sqrt{a}x}\left(6a\left(c_1e^{\frac{\left(-3+\sqrt{3}+\sqrt{6}\right)ax}{\sqrt{-\left(\left(\sqrt{6}-3\right)a\right)}}} + c_2e^{\frac{\left(3+\sqrt{3}-\sqrt{6}\right)ax}{\sqrt{-\left(\left(\sqrt{6}-3\right)a\right)}}}\right) + \sqrt{6}\sqrt{-\left(\left(\sqrt{6}-3\right)a\right)}\left(c_4e^{\frac{2ax}{\sqrt{a-\sqrt{\frac{2}{3}}a}}} + c_3\right)\right)}{6a}$$

5.13 problem 1548

Internal problem ID [9871]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1548.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$4y'''' - 12y''' + 11y'' - 3y' = 4\cos(x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(4*diff(diff(diff(diff(y(x),x),x),x),x)-12*diff(diff(diff(y(x),x),x),x)+11*diff(diff(y
```

$$y(x) = c_1 e^x + 2c_2 e^{\frac{x}{2}} + \frac{2c_3 e^{\frac{3x}{2}}}{3} + \frac{18\sin(x)}{65} - \frac{14\cos(x)}{65} + c_4$$

✓ Solution by Mathematica
Time used: 0.112 (sec). Leaf size: 50

```
DSolve[-4*Cos[x] - 3*y'[x] + 11*y''[x] - 12*Derivative[3][y][x] + 4*Derivative[4][y][x] == 0
```

$$y(x) \to \frac{18\sin(x)}{65} - \frac{14\cos(x)}{65} + 2c_1 e^{x/2} + \frac{2}{3}c_2 e^{3x/2} + c_3 e^x + c_4$$

5.14 problem 1549

Internal problem ID [9872]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1549.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{xy'''' + 5y''' = 24}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(x*diff(diff(diff(diff(y(x),x),x),x),x)+5*diff(diff(diff(y(x),x),x),x)-24=0,y(x), sing
```

$$y(x) = \frac{4x^3}{5} - \frac{c_1}{24x^2} + \frac{c_2 x^2}{2} + c_3 x + c_4$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 34

```
DSolve[-24 + 5*Derivative[3][y][x] + x*Derivative[4][y][x] == 0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{4x^3}{5} + c_4 x^2 - \frac{c_1}{24x^2} + c_3 x + c_2$$

5.15 problem 1550

Internal problem ID [9873]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1550.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$xy'''' - (6x^2+1)\, y''' + 12x^3 y'' - (9x^2-7)\, x^2 y' + 2(x^2-3)\, x^3 y = 0$$

✓ Solution by Maple

Time used: 0.344 (sec). Leaf size: 157

```
dsolve(x*diff(diff(diff(diff(y(x),x),x),x),x)-(6*x^2+1)*diff(diff(diff(y(x),x),x),x)+12*x^3*
```

$$\begin{aligned} y(x) = &-e^{x^2}\left(\int \frac{\text{WhittakerM}\left(\frac{9\sqrt{5}}{20}, \frac{3}{4}, \frac{\sqrt{5}\, x^2}{2}\right) e^{-\frac{x^2}{4}}}{x^{\frac{3}{2}}} dx\right) c_3 \\ &- e^{x^2}\left(\int \frac{\text{WhittakerW}\left(\frac{9\sqrt{5}}{20}, \frac{3}{4}, \frac{\sqrt{5}\, x^2}{2}\right) e^{-\frac{x^2}{4}}}{x^{\frac{3}{2}}} dx\right) c_4 \\ &+ \left(\int \frac{\text{WhittakerM}\left(\frac{9\sqrt{5}}{20}, \frac{3}{4}, \frac{\sqrt{5}\, x^2}{2}\right) e^{\frac{x^2}{4}}}{x^{\frac{3}{2}}} dx\right) e^{\frac{x^2}{2}} c_3 \\ &+ e^{\frac{x^2}{2}}\left(\int \frac{\text{WhittakerW}\left(\frac{9\sqrt{5}}{20}, \frac{3}{4}, \frac{\sqrt{5}\, x^2}{2}\right) e^{\frac{x^2}{4}}}{x^{\frac{3}{2}}} dx\right) c_4 + c_1 e^{x^2} + c_2 e^{\frac{x^2}{2}} \end{aligned}$$

✓ Solution by Mathematica

Time used: 1.968 (sec). Leaf size: 216

```
DSolve[2*x^3*(-3 + x^2)*y[x] - x^2*(-7 + 9*x^2)*y'[x] + 12*x^3*y''[x] - (1 + 6*x^2)*Derivati
```

$$y(x) \to e^{\frac{x^2}{2}}\left(c_3 \int_1^x \frac{e^{\frac{K[1]^2}{2}}\left(\int \frac{e^{\frac{1}{4}\left(-1+\sqrt{5}\right)K[1]^2}\text{HypergeometricU}\left(-\frac{1}{4}+\frac{9}{4\sqrt{5}},-\frac{1}{2},-\frac{1}{2}\sqrt{5}K[1]^2\right)\left(K[1]^2\right)^{3/4}}{K[1]^{7/2}}dK[1]\right)K[1]}{\sqrt[4]{2}}dK[1]+c_4 \int_1\right.$$

5.16 problem 1551

Internal problem ID [9874]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1551.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$x^2 y'''' - 2(\nu^2 x^2 + 6)\, y'' + \nu^2(\nu^2 x^2 + 4)\, y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 62

```
dsolve(x^2*diff(y(x),x$4)-2*(nu^2*x^2+6)*diff(y(x),x$2)+nu^2*(nu^2*x^2+4)*y(x)=0,y(x), sings
```

$$y(x) = \frac{(c_4\nu^2 x^3 + 6c_4\nu\, x^2 + 15c_4 x + c_2)\, e^{-x\nu} + e^{x\nu}(c_3\nu^2 x^3 - 6c_3\nu\, x^2 + 15c_3 x + c_1)}{x}$$

✓ Solution by Mathematica

Time used: 0.197 (sec). Leaf size: 84

```
DSolve[x^2*y''''[x]-2*(nu^2*x^2+6)*y''[x]+nu^2*(nu^2*x^2+4)*y[x]==0,y[x],x,IncludeSingularSo
```

$$y(x) \to \frac{e^{-\nu x}(c_3(-\nu^2 x^3 + \nu^2 - 6\nu x^2 + 6\nu - 15x + 15) + e^{2\nu x}(c_4(-\nu^2 x^3 + \nu^2 + 6\nu x^2 - 6\nu - 15x + 15) + c_2) + }{x}$$

5.17 problem 1552

Internal problem ID [9875]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1552.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'''' + 2xy''' + ay = b\,x^2}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 94

```
dsolve(x^2*diff(y(x),x$4)+2*x*diff(y(x),x$3)+a*y(x)-b*x^2=0,y(x), singsol=all)
```

$$y(x) = \frac{c_4\sqrt{x}\,\mathrm{BesselY}\left(1, 2\sqrt{-\sqrt{-a}}\,\sqrt{x}\right)a + c_3\sqrt{x}\,\mathrm{BesselJ}\left(1, 2\sqrt{-\sqrt{-a}}\,\sqrt{x}\right)a + c_2\sqrt{x}\,\mathrm{BesselY}\left(1, 2(-a)^{\frac{1}{4}}\sqrt{\right.}{a}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y''''[x]+2*x*y'''[x]+a*y[x]-b*x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

5.18 problem 1553

Internal problem ID [9876]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1553.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{x^2y'''' + 4xy''' + 2y'' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(diff(diff(diff(y(x),x),x),x),x)+4*x*diff(diff(diff(y(x),x),x),x)+2*diff(diff
```

$$y(x) = (c_4x + c_2)\ln(x) + c_3x + c_1$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 29

```
DSolve[2*y''[x] + 4*x*Derivative[3][y][x] + x^2*Derivative[4][y][x] == 0,y[x],x,IncludeSingu
```

$$y(x) \to (c_4 - c_2)x + (c_2x - c_1)\log(x) + c_3$$

5.19 problem 1554

Internal problem ID [9877]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1554.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{x^2y'''' + 6xy''' + 6y'' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^2*diff(diff(diff(diff(y(x),x),x),x),x)+6*x*diff(diff(diff(y(x),x),x),x)+6*diff(diff
```

$$y(x) = c_1 + \ln(x)\,c_2 + \frac{c_3}{x} + c_4x$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 27

```
DSolve[6*y''[x] + 6*x*Derivative[3][y][x] + x^2*Derivative[4][y][x] == 0,y[x],x,IncludeSingu
```

$$y(x) \to \frac{c_1}{2x} + c_4x - c_2\log(x) + c_3$$

5.20 problem 1555

Internal problem ID [9878]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1555.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$x^2y'''' + 6xy''' + 6y'' - \lambda^2 y = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 61

```
dsolve(x^2*diff(diff(diff(diff(y(x),x),x),x),x)+6*x*diff(diff(diff(y(x),x),x),x)+6*diff(diff
```

$$y(x) = \frac{c_1 \operatorname{BesselJ}\left(1, 2\sqrt{\lambda}\,\sqrt{x}\right) + c_2 \operatorname{BesselY}\left(1, 2\sqrt{\lambda}\,\sqrt{x}\right) + c_4 \operatorname{BesselY}\left(1, 2\sqrt{-\lambda}\,\sqrt{x}\right) + c_3 \operatorname{BesselJ}\left(1, 2\sqrt{-\lambda}\,\sqrt{\right.}}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 156

```
DSolve[-(\[Lambda]^2*y[x]) + 6*y''[x] + 6*x*Derivative[3][y][x] + x^2*Derivative[4][y][x] ==
```

$$y(x) \to c_4 G_{0,4}^{2,0}\left(\frac{x^2\lambda^2}{16} \middle| -\tfrac{1}{2}, \tfrac{1}{2}, 0, 0\right) + c_2 G_{0,4}^{2,0}\left(\frac{x^2\lambda^2}{16} \middle| 0, 0, -\tfrac{1}{2}, \tfrac{1}{2}\right) + \frac{c_1\left(\operatorname{BesselJ}\left(1, 2\sqrt{x}\sqrt{\lambda}\right) + \operatorname{BesselI}\left(1, 2\sqrt{x}\sqrt{\lambda}\right)\right)}{2\sqrt{\lambda}\sqrt{x}} - \frac{ic_3\left(\operatorname{BesselI}\left(1, 2\sqrt{x}\sqrt{\lambda}\right) - \operatorname{BesselJ}\left(1, 2\sqrt{x}\sqrt{\lambda}\right)\right)}{4\sqrt{\lambda}\sqrt{x}}$$

5.21 problem 1556

Internal problem ID [9879]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1556.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{x^2y'''' + 8xy''' + 12y'' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(diff(diff(diff(y(x),x),x),x),x)+8*x*diff(diff(diff(y(x),x),x),x)+12*diff(dif
```

$$y(x) = c_1 + \frac{c_2}{x^2} + \frac{c_3}{x} + c_4x$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 27

```
DSolve[12*y''[x] + 8*x*Derivative[3][y][x] + x^2*Derivative[4][y][x] == 0,y[x],x,IncludeSing
```

$$y(x) \to \frac{3c_2x + c_1}{6x^2} + c_4x + c_3$$

5.22 problem 1557

Internal problem ID [9880]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1557.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$x^2y'''' + 8xy''' + 12y'' - \lambda^2 y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 159

```
dsolve(x^2*diff(diff(diff(diff(y(x),x),x),x),x)+8*x*diff(diff(diff(y(x),x),x),x)+12*diff(dif
```

$y(x) =$

$$-\frac{-\operatorname{BesselJ}\left(1, 2\sqrt{-\lambda}\,\sqrt{x}\right) c_3\sqrt{\lambda} - \operatorname{BesselY}\left(1, 2\sqrt{-\lambda}\,\sqrt{x}\right) c_4\sqrt{\lambda} + \sqrt{-\lambda}\left(\sqrt{x}\,\sqrt{\lambda}\operatorname{BesselJ}\left(0, 2\sqrt{-\lambda}\,\sqrt{x}\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 146

```
DSolve[-(\[Lambda]^2*y[x]) + 12*y''[x] + 8*x*Derivative[3][y][x] + x^2*Derivative[4][y][x] =
```

$$y(x) \to c_4 G_{0,4}^{2,0}\left(\frac{x^2\lambda^2}{16} \middle| \; -1, 0, -\tfrac{1}{2}, \tfrac{1}{2}\right) + c_2 G_{0,4}^{2,0}\left(\frac{x^2\lambda^2}{16} \middle| \; -\tfrac{1}{2}, \tfrac{1}{2}, -1, 0\right)$$
$$- \frac{3ic_1\left(\operatorname{BesselI}\left(2, 2\sqrt{x}\sqrt{\lambda}\right) - \operatorname{BesselJ}\left(2, 2\sqrt{x}\sqrt{\lambda}\right)\right)}{4\lambda x}$$
$$- \frac{c_3\left(\operatorname{BesselJ}\left(2, 2\sqrt{x}\sqrt{\lambda}\right) + \operatorname{BesselI}\left(2, 2\sqrt{x}\sqrt{\lambda}\right)\right)}{\lambda x}$$

5.23 problem 1558

Internal problem ID [9881]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1558.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$x^2y'''' + (2n - 2\nu + 4)\, xy''' + (n - \nu + 1)\,(n - \nu + 2)\, y'' - \frac{b^4 y}{16} = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 67

```
dsolve(x^2*diff(diff(diff(diff(y(x),x),x),x),x)+(2*n-2*nu+4)*x*diff(diff(diff(y(x),x),x),x)+
```

$$y(x) = \left(\operatorname{BesselK}\left(n-\nu, b\sqrt{x}\right) c_3 + \operatorname{BesselI}\left(n-\nu, b\sqrt{x}\right) c_1 + \operatorname{BesselY}\left(n-\nu, b\sqrt{x}\right) c_4 + \operatorname{BesselJ}\left(n-\nu, b\sqrt{x}\right) c_2\right) x^{-\frac{n}{2}+\frac{\nu}{2}}$$

✓ Solution by Mathematica
Time used: 0.124 (sec). Leaf size: 222

```
DSolve[-1/16*(b^4*y[x]) + (1 + n - nu)*(2 + n - nu)*y''[x] + (4 + 2*n - 2*nu)*x*Derivative[3
```

$$\begin{aligned} y(x) \to\ & i^{-n}2^{n-3\nu-3}b^{\nu-n}x^{\frac{\nu-n}{2}}\big(i^n4^\nu(4c_1\operatorname{Gamma}(n-\nu+1) \\ & - ic_2\operatorname{Gamma}(n-\nu+2))\operatorname{BesselJ}\left(n-\nu, b\sqrt{x}\right) \\ & + i^n4^\nu(4c_1\operatorname{Gamma}(n-\nu+1) + ic_2\operatorname{Gamma}(n-\nu+2))\operatorname{BesselI}\left(n-\nu, b\sqrt{x}\right) \\ & + 4^ni^\nu\big((4c_3\operatorname{Gamma}(-n+\nu+1) - ic_4\operatorname{Gamma}(-n+\nu+2))\operatorname{BesselJ}\left(\nu-n, b\sqrt{x}\right) \\ & + (4c_3\operatorname{Gamma}(-n+\nu+1) + ic_4\operatorname{Gamma}(-n+\nu+2))\operatorname{BesselI}\left(\nu-n, b\sqrt{x}\right)\big)\big) \end{aligned}$$

5.24 problem 1559

Internal problem ID [9882]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1559.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$x^3y'''' + 2x^2y''' - y''x + y' - a^4x^3y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(x^3*diff(diff(diff(diff(y(x),x),x),x),x)+2*x^2*diff(diff(diff(y(x),x),x),x)-x*diff(di
```

$$y(x) = c_1 \operatorname{BesselI}(0, ax) + c_2 \operatorname{BesselJ}(0, ax) + c_3 \operatorname{BesselK}(0, ax) + c_4 \operatorname{BesselY}(0, ax)$$

✓ Solution by Mathematica

Time used: 0.221 (sec). Leaf size: 100

```
DSolve[-(a^4*x^3*y[x]) + y'[x] - x*y''[x] + 2*x^2*Derivative[3][y][x] + x^3*Derivative[4][y]
```

$$y(x) \to c_4 G_{0,4}^{2,0}\left(\frac{a^4x^4}{256} \middle| \ 0, 0, \tfrac{1}{2}, \tfrac{1}{2}\right) + c_2 G_{0,4}^{2,0}\left(\frac{a^4x^4}{256} \middle| \ \tfrac{1}{2}, \tfrac{1}{2}, 0, 0\right) + \frac{1}{8} i c_1 (\operatorname{BesselI}(0, ax) - \operatorname{BesselJ}(0, ax)) + \frac{1}{2} c_3 (\operatorname{BesselJ}(0, ax) + \operatorname{BesselI}(0, ax))$$

5.25 problem 1560

Internal problem ID [9883]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1560.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{x^3y'''' + 6x^2y''' + 6y''x = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^3*diff(diff(diff(diff(y(x),x),x),x),x)+6*x^2*diff(diff(diff(y(x),x),x),x)+6*x*diff(
```

$$y(x) = c_1 + \ln(x) c_2 + \frac{c_3}{x} + c_4 x$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 27

```
DSolve[6*x*y''[x] + 6*x^2*Derivative[3][y][x] + x^3*Derivative[4][y][x] == 0,y[x],x,IncludeS
```

$$y(x) \to \frac{c_1}{2x} + c_4 x - c_2 \log(x) + c_3$$

5.26 problem 1561

Internal problem ID [9884]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1561.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$x^4y'''' - 2n(n+1)\,x^2y'' + 4n(n+1)\,xy' + \left(a\,x^4 + n(n+1)\,(n+3)\,(-2+n)\right)y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 69

```
dsolve(x^4*diff(diff(diff(diff(y(x),x),x),x),x)-2*n*(n+1)*x^2*diff(diff(y(x),x),x)+4*n*(n+1)
```

$$y(x) = \sqrt{x}\left(\text{BesselY}\left(n+\frac{1}{2}, \sqrt{-\sqrt{-a}}\,x\right)c_4 + \text{BesselJ}\left(n+\frac{1}{2}, \sqrt{-\sqrt{-a}}\,x\right)c_3 + \text{BesselJ}\left(n+\frac{1}{2}, (-a)^{\frac{1}{4}}\,x\right)c_1 + \text{BesselY}\left(n+\frac{1}{2}, (-a)^{\frac{1}{4}}\,x\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 3.44 (sec). Leaf size: 310

```
DSolve[((-2 + n)*n*(1 + n)*(3 + n) + a*x^4)*y[x] + 4*n*(1 + n)*x*y'[x] - 2*n*(1 + n)*x^2*y''
```

$$
\begin{aligned}
y(x) \to \sqrt[8]{a}2^{-n-\frac{7}{2}}\sqrt{x}\Bigg(&2^{2n+1}\text{ber}_{-n-\frac{1}{2}}\left(\sqrt[4]{a}x\right)\left(4c_2\cos\left(\frac{3}{8}\pi(2n+1)\right)\text{Gamma}\left(\frac{1}{2}-n\right)\right.\\
&\left.-c_1\cos\left(\frac{3}{8}\pi(2n-3)\right)\text{Gamma}\left(\frac{3}{2}-n\right)\right)\\
&+\text{ber}_{n+\frac{1}{2}}\left(\sqrt[4]{a}x\right)\left(4c_3\cos\left(\frac{3}{8}\pi(2n+1)\right)\text{Gamma}\left(n+\frac{3}{2}\right)\right.\\
&\left.-c_4\cos\left(\frac{3}{8}\pi(2n+5)\right)\text{Gamma}\left(n+\frac{5}{2}\right)\right)\\
&+c_12^{2n+1}\sin\left(\frac{3}{8}\pi(2n-3)\right)\text{Gamma}\left(\frac{3}{2}-n\right)\text{bei}_{-n-\frac{1}{2}}\left(\sqrt[4]{a}x\right)\\
&-c_22^{2n+3}\sin\left(\frac{3}{8}\pi(2n+1)\right)\text{Gamma}\left(\frac{1}{2}-n\right)\text{bei}_{-n-\frac{1}{2}}\left(\sqrt[4]{a}x\right)\\
&+4c_3\sin\left(\frac{3}{8}\pi(2n+1)\right)\text{Gamma}\left(n+\frac{3}{2}\right)\text{bei}_{n+\frac{1}{2}}\left(\sqrt[4]{a}x\right)\\
&-c_4\sin\left(\frac{3}{8}\pi(2n+5)\right)\text{Gamma}\left(n+\frac{5}{2}\right)\text{bei}_{n+\frac{1}{2}}\left(\sqrt[4]{a}x\right)\Bigg)
\end{aligned}
$$

5.27 problem 1562

Internal problem ID [9885]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1562.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$x^4y'''' + 4x^3y''' - (4n^2-1)\,x^2y'' + (4n^2-1)\,xy' - 4yx^4 = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 77

```
dsolve(x^4*diff(diff(diff(diff(y(x),x),x),x),x)+4*x^3*diff(diff(diff(y(x),x),x),x)-(4*n^2-1)
```

$$y(x) = \left(\operatorname{BesselY}\left(n, \left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{2}\,x\right)c_3 + \operatorname{BesselJ}\left(n, \left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{2}\,x\right)c_1\right)\operatorname{BesselJ}\left(n, \left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{2}\,x\right) + \operatorname{BesselY}\left(n, \left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{2}\,x\right)\left(\operatorname{BesselY}\left(n, \left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{2}\,x\right)c_4 + c_2\operatorname{BesselJ}\left(n, \left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{2}\,x\right)\right)$$

✓ Solution by Mathematica

Time used: 0.916 (sec). Leaf size: 140

```
DSolve[-4*x^4*y[x] + (-1 + 4*n^2)*x*y'[x] - (-1 + 4*n^2)*x^2*y''[x] + 4*x^3*Derivative[3][y]
```

$$y(x) \to c_1\,{}_0F_3\left(;\frac{1}{2}, 1-\frac{n}{2}, \frac{n}{2}+1; \frac{x^4}{64}\right) + \frac{1}{8}ic_2x^2\,{}_0F_3\left(;\frac{3}{2}, \frac{3}{2}-\frac{n}{2}, \frac{n}{2}+\frac{3}{2}; \frac{x^4}{64}\right) + c_3\left(\frac{i}{2}\right)^{-n}\operatorname{Gamma}(1-n)^2\left(\operatorname{ber}_{-n}(x)^2 + \operatorname{bei}_{-n}(x)^2\right) + c_4\left(\frac{i}{2}\right)^{n}\operatorname{Gamma}(n+1)^2\left(\operatorname{ber}_{n}(x)^2 + \operatorname{bei}_{n}(x)^2\right)$$

5.28 problem 1563

Internal problem ID [9886]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1563.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$x^4y'''' + 4x^3y''' - (4n^2 - 1)\,x^2y'' - (4n^2 - 1)\,xy' + (-4x^4 + 4n^2 - 1)\,y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 87

```
dsolve(x^4*diff(diff(diff(diff(y(x),x),x),x),x)+4*x^3*diff(diff(diff(y(x),x),x),x)-(4*n^2-1)
```

$$y(x) = \frac{c_4 \operatorname{hypergeom}\left([\,], \left[\frac{1}{2}, \frac{n}{2} + \frac{1}{2}, -\frac{n}{2} + \frac{1}{2}\right], \frac{x^4}{64}\right) + \left(c_3 \operatorname{hypergeom}\left([\,], \left[\frac{3}{2}, -\frac{n}{2} + 1, \frac{n}{2} + 1\right], \frac{x^4}{64}\right) + c_2 \operatorname{KelvinBe}\right.}{x}$$

✓ Solution by Mathematica

Time used: 1.725 (sec). Leaf size: 187

```
DSolve[(-1 + 4*n^2 - 4*x^4)*y[x] - (-1 + 4*n^2)*x*y'[x] - (-1 + 4*n^2)*x^2*y''[x] + 4*x^3*De
```

$$y(x) \to \frac{\sqrt[4]{-1}\left(x^2\left(c_2\,{}_0F_3\left(;\frac{3}{2}, 1 - \frac{n}{2}, \frac{n}{2} + 1; \frac{x^4}{64}\right) + c_3\left(\frac{i}{8}\right)^{-n} x^{-2n}\,{}_0F_3\left(;1 - n, 1 - \frac{n}{2}, \frac{3}{2} - \frac{n}{2}; \frac{x^4}{64}\right) + c_4\left(\frac{i}{8}\right)^n x^{2n}\,{}_0F_3\left(\right.\right.\right.}{2\sqrt{2}x}$$

5.29 problem 1564

Internal problem ID [9887]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1564.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$x^4y'''' + 4x^3y''' - (4n^2+3)\,x^2y'' + (12n^2-3)\,xy' - (4x^4+12n^2-3)\,y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 88

```
dsolve(x^4*diff(diff(diff(diff(y(x),x),x),x),x)+4*x^3*diff(diff(diff(y(x),x),x),x)-(4*n^2+3)
```

$$y(x) = \frac{c_4x^2\,\mathrm{hypergeom}\left([\,],\left[\frac{1}{2},\frac{3}{2}-\frac{n}{2},\frac{n}{2}+\frac{3}{2}\right],\frac{x^4}{64}\right)+c_3x^4\,\mathrm{hypergeom}\left([\,],\left[\frac{3}{2},\frac{n}{2}+2,-\frac{n}{2}+2\right],\frac{x^4}{64}\right)+c_2\,\mathrm{KelvinB}}{x}$$

✓ Solution by Mathematica

Time used: 1.125 (sec). Leaf size: 196

```
DSolve[(3 - 12*n^2 - 4*x^4)*y[x] + (-3 + 12*n^2)*x*y'[x] - (3 + 4*n^2)*x^2*y''[x] + 4*x^3*De
```

$$y(x) \to \frac{\left(\frac{1}{32}+\frac{i}{32}\right)\left(i\left(c_2x^4\,{}_0F_3\left(;\frac{3}{2},2-\frac{n}{2},\frac{n}{2}+2;\frac{x^4}{64}\right)-8^{2-n}e^{-\frac{1}{2}i\pi n}x^{-2n}\left(c_364^n\,{}_0F_3\left(;1-n,\frac{1}{2}-\frac{n}{2},-\frac{n}{2};\frac{x^4}{64}\right)+c_4e\right.\right.\right.}{x}$$

5.30 problem 1565

Internal problem ID [9888]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1565.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$x^4y'''' + 6x^3y''' + (4x^4 + (-\rho^2 - \sigma^2 + 7)x^2)y'' + (16x^3 + (-\rho^2 - \sigma^2 + 1)x)y' + (\rho^2\sigma^2 + 8x^2)y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 71

```
dsolve(x^4*diff(diff(diff(diff(y(x),x),x),x),x)+6*x^3*diff(diff(diff(y(x),x),x),x)+(4*x^4+(-
```

$$y(x) = \left(\text{BesselY}\left(\frac{\rho}{2} - \frac{\sigma}{2}, x\right)c_2 + c_1\,\text{BesselJ}\left(\frac{\rho}{2} - \frac{\sigma}{2}, x\right)\right)\text{BesselJ}\left(\frac{\rho}{2} + \frac{\sigma}{2}, x\right) + \text{BesselY}\left(\frac{\rho}{2} + \frac{\sigma}{2}, x\right)\left(\text{BesselY}\left(\frac{\rho}{2} - \frac{\sigma}{2}, x\right)c_4 + c_3\,\text{BesselJ}\left(\frac{\rho}{2} - \frac{\sigma}{2}, x\right)\right)$$

✓ Solution by Mathematica

Time used: 0.417 (sec). Leaf size: 242

```
DSolve[(rho^2*sigma^2 + 8*x^2)*y[x] + ((1 - rho^2 - sigma^2)*x + 16*x^3)*y'[x] + ((7 - rho^2
```

$$y(x) \to c_1x^{-\rho}\,{}_2F_3\left(\frac{1}{2} - \frac{\rho}{2}, 1 - \frac{\rho}{2}; 1 - \rho, -\frac{\rho}{2} - \frac{\sigma}{2} + 1, -\frac{\rho}{2} + \frac{\sigma}{2} + 1; -x^2\right) + c_3x^{-\sigma}\,{}_2F_3\left(\frac{1}{2} - \frac{\sigma}{2}, 1 - \frac{\sigma}{2}; 1 - \sigma, -\frac{\rho}{2} - \frac{\sigma}{2} + 1, \frac{\rho}{2} - \frac{\sigma}{2} + 1; -x^2\right) + c_4x^{\sigma}\,{}_2F_3\left(\frac{\sigma}{2} + \frac{1}{2}, \frac{\sigma}{2} + 1; -\frac{\rho}{2} + \frac{\sigma}{2} + 1, \frac{\rho}{2} + \frac{\sigma}{2} + 1, \sigma + 1; -x^2\right) + c_2x^{\rho}\,{}_2F_3\left(\frac{\rho}{2} + \frac{1}{2}, \frac{\rho}{2} + 1; \rho + 1, \frac{\rho}{2} - \frac{\sigma}{2} + 1, \frac{\rho}{2} + \frac{\sigma}{2} + 1; -x^2\right)$$

5.31 problem 1566

Internal problem ID [9889]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1566.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$x^4y'''' + 6x^3y''' + (4x^4 + (-2\mu^2 - 2\nu^2 + 7)\,x^2)\,y'' + (16x^3 + (-2\mu^2 - 2\nu^2 + 1)\,x)\,y' + \left(8x^2 + (\mu^2 - \nu^2)\right.$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 35

```
dsolve(x^4*diff(diff(diff(diff(y(x),x),x),x),x)+6*x^3*diff(diff(diff(y(x),x),x),x)+(4*x^4+(-
```

$$y(x) = (\text{BesselY}(\mu, x)\,c_2 + c_1 \text{BesselJ}(\mu, x))\,\text{BesselJ}(\nu, x) + \text{BesselY}(\nu, x)\,(\text{BesselY}(\mu, x)\,c_4 + c_3 \text{BesselJ}(\mu, x))$$

✓ Solution by Mathematica
Time used: 0.506 (sec). Leaf size: 237

```
DSolve[((\[Mu]^2 - \[Nu]^2)^2 + 8*x^2)*y[x] + ((1 - 2*\[Mu]^2 - 2*\[Nu]^2)*x + 16*x^3)*y'[x]
```

$$y(x) \to x^{-\mu-\nu}\left(c_1\,{}_2F_3\left(-\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2},-\frac{\mu}{2}-\frac{\nu}{2}+1;1-\mu,1-\nu,-\mu-\nu+1;-x^2\right)\right.$$
$$+ c_2x^{2\mu}\,{}_2F_3\left(\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2},\frac{\mu}{2}-\frac{\nu}{2}+1;\mu+1,1-\nu,\mu-\nu+1;-x^2\right)$$
$$+ x^{2\nu}\left(c_3\,{}_2F_3\left(-\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2},-\frac{\mu}{2}+\frac{\nu}{2}+1;1-\mu,\nu+1,-\mu+\nu+1;-x^2\right)\right.$$
$$\left.\left.+ c_4x^{2\mu}\,{}_2F_3\left(\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2},\frac{\mu}{2}+\frac{\nu}{2}+1;\mu+1,\nu+1,\mu+\nu+1;-x^2\right)\right)\right)$$

5.32 problem 1567

Internal problem ID [9890]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1567.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$x^4y'''' + 8x^3y''' + 12x^2y'' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^4*diff(diff(diff(diff(y(x),x),x),x),x)+8*x^3*diff(diff(diff(y(x),x),x),x)+12*x^2*di
```

$$y(x) = c_1 + \frac{c_2}{x^2} + \frac{c_3}{x} + c_4x$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 27

```
DSolve[12*x^2*y''[x] + 8*x^3*Derivative[3][y][x] + x^4*Derivative[4][y][x] == 0,y[x],x,Inclu
```

$$y(x) \to \frac{3c_2x + c_1}{6x^2} + c_4x + c_3$$

5.33 problem 1568

Internal problem ID [9891]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1568.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$x^4y'''' + 8x^3y''' + 12x^2y'' + ay = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 85

```
dsolve(x^4*diff(diff(diff(diff(y(x),x),x),x),x)+8*x^3*diff(diff(diff(y(x),x),x),x)+12*x^2*di
```

$$y(x) = \frac{c_1 x^{-\frac{\sqrt{5-4\sqrt{-a+1}}}{2}} + c_2 x^{\frac{\sqrt{5-4\sqrt{-a+1}}}{2}} + c_3 x^{-\frac{\sqrt{5+4\sqrt{-a+1}}}{2}} + c_4 x^{\frac{\sqrt{5+4\sqrt{-a+1}}}{2}}}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 116

```
DSolve[a*y[x] + 12*x^2*y''[x] + 8*x^3*Derivative[3][y][x] + x^4*Derivative[4][y][x] == 0,y[x
```

$$y(x) \to \frac{c_1 x^{-\frac{1}{2}\sqrt{5-4\sqrt{1-a}}} + c_2 x^{\frac{1}{2}\sqrt{5-4\sqrt{1-a}}} + c_3 x^{-\frac{1}{2}\sqrt{4\sqrt{1-a}+5}} + c_4 x^{\frac{1}{2}\sqrt{4\sqrt{1-a}+5}}}{\sqrt{x}}$$

5.34 problem 1569

Internal problem ID [9892]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1569.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$x^4y'''' + (6-4a)\,x^3y''' + \left(4x^{2c}b^2c^2 + 6(-1+a)^2 - 2c^2\left(\mu^2+\nu^2\right)+1\right)x^2y'' + \left(4(3c-2a+1)\,b^2c^2x^{2c} + (2$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 63

```
dsolve(x^4*diff(diff(diff(diff(y(x),x),x),x),x)+(6-4*a)*x^3*diff(diff(diff(y(x),x),x),x)+(4*
```

$$y(x) = x^a\left(\left(\mathrm{BesselJ}\left(\mu, b\,x^c\right)c_1 + \mathrm{BesselY}\left(\mu, b\,x^c\right)c_3\right)\mathrm{BesselJ}\left(\nu, b\,x^c\right) + \mathrm{BesselY}\left(\nu, b\,x^c\right)\left(c_4\,\mathrm{BesselY}\left(\mu, b\,x^c\right) + \mathrm{BesselJ}\left(\mu, b\,x^c\right)c_2\right)\right)$$

✓ Solution by Mathematica

Time used: 0.102 (sec). Leaf size: 304

```
DSolve[x^4*y''''[x]+(6-4*a)*x^3*y'''[x]+(4*b^2*c^2*x^(2*c)+6*(a-1)^2-2*c^2*(\[Mu]^2+\[Nu]^2)
```

$$y(x) \to b^{\frac{a-c(\mu+\nu)}{c}}\left(x^{2c}\right)^{\frac{a-c(\mu+\nu)}{2c}}\left(c_1\,{}_2F_3\left(-\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2},-\frac{\mu}{2}-\frac{\nu}{2}+1;1-\mu,1-\nu,-\mu-\nu+1;-b^2x^{2c}\right) + c_2b^{2\mu}\left(x^{2c}\right)^{\mu}\,{}_2F_3\left(\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2},\frac{\mu}{2}-\frac{\nu}{2}+1;\mu+1,1-\nu,\mu-\nu+1;-b^2x^{2c}\right) + b^{2\nu}\left(x^{2c}\right)^{\nu}\left(c_3\,{}_2F_3\left(-\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2},-\frac{\mu}{2}+\frac{\nu}{2}+1;1-\mu,\nu+1,-\mu+\nu+1;-b^2x^{2c}\right) + c_4b^{2\mu}\left(x^{2c}\right)^{\mu}\,{}_2F_3\left(\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2},\frac{\mu}{2}+\frac{\nu}{2}+1;\mu+1,\nu+1,\mu+\nu+1;-b^2x^{2c}\right)\right)\right)$$

5.35 problem 1570

Internal problem ID [9893]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1570.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$x^4y'''' + (6-4a-4c)\,x^3y''' + \left(-2\nu^2c^2 + 2a^2 + 4(a+c-1)^2 + 4(-1+a)\,(c-1) - 1\right)x^2y'' + \left(2\nu^2c^2 - 2\right.$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 49

```
dsolve(x^4*diff(diff(diff(diff(y(x),x),x),x),x)+(6-4*a-4*c)*x^3*diff(diff(diff(y(x),x),x),x)
```

$$y(x) = \left(\operatorname{BesselJ}\left(\nu, b\,x^c\right)c_1 + \operatorname{BesselY}\left(\nu, b\,x^c\right)c_2 + \operatorname{BesselY}\left(\nu, ib\,x^c\right)c_4 + \operatorname{BesselJ}\left(\nu, ib\,x^c\right)c_3\right)x^a$$

✓ Solution by Mathematica

Time used: 0.102 (sec). Leaf size: 213

```
DSolve[((a^2 - c^2*\[Nu]^2)*(a^2 + 4*a*c + 4*c^2 - c^2*\[Nu]^2) - b^4*c^4*x^(4*c))*y[x] + (-
```

$$\begin{aligned} y(x) \to\; & b^{a/c}(-1)^{\frac{a-c\nu}{4c}}2^{-\frac{2a}{c}-\nu-3}\left(x^{4c}\right)^{\frac{a}{4c}}\Big(4^\nu(4c_1\operatorname{Gamma}(1-\nu) - ic_2\operatorname{Gamma}(2-\nu))\operatorname{BesselJ}\left(-\nu, b\sqrt[4]{x^{4c}}\right) \\ & + 4^\nu(4c_1\operatorname{Gamma}(1-\nu) + ic_2\operatorname{Gamma}(2-\nu))\operatorname{BesselI}\left(-\nu, b\sqrt[4]{x^{4c}}\right) \\ & + i^\nu\Big((4c_3\operatorname{Gamma}(\nu+1) - ic_4\operatorname{Gamma}(\nu+2))\operatorname{BesselJ}\left(\nu, b\sqrt[4]{x^{4c}}\right) + (4c_3\operatorname{Gamma}(\nu+1) + ic_4\operatorname{Gamma}(\nu+ \end{aligned}$$

5.36 problem 1571

Internal problem ID [9894]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1571.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$\nu^4 x^4 y'''' + (4\nu - 2)\,\nu^3 x^3 y''' + (\nu - 1)(-1 + 2\nu)\,\nu^2 x^2 y'' - \frac{b^4 x^{\frac{2}{\nu}} y}{16} = 0$$

✓ Solution by Maple

Time used: 0.578 (sec). Leaf size: 143

```
dsolve(nu^4*x^4*diff(diff(diff(diff(y(x),x),x),x),x)+(4*nu-2)*nu^3*x^3*diff(diff(diff(y(x),x
```

$$y(x) = \sqrt{x}\left(\operatorname{BesselY}\left(\frac{1}{\lfloor\frac{1}{\nu}\rfloor}, \frac{\sqrt{\frac{b^2}{\nu^2}}\,x^{\frac{\lfloor\frac{1}{\nu}\rfloor}{2}}}{\lfloor\frac{1}{\nu}\rfloor}\right) c_2 + \operatorname{BesselJ}\left(\frac{1}{\lfloor\frac{1}{\nu}\rfloor}, \frac{\sqrt{\frac{b^2}{\nu^2}}\,x^{\frac{\lfloor\frac{1}{\nu}\rfloor}{2}}}{\lfloor\frac{1}{\nu}\rfloor}\right) c_1 + \operatorname{BesselY}\left(\frac{1}{\lfloor\frac{1}{\nu}\rfloor}, \frac{\sqrt{-\frac{b^2}{\nu^2}}\,x^{\frac{\lfloor\frac{1}{\nu}\rfloor}{2}}}{\lfloor\frac{1}{\nu}\rfloor}\right) c_4 + \operatorname{BesselJ}\left(\frac{1}{\lfloor\frac{1}{\nu}\rfloor}, \frac{\sqrt{-\frac{b^2}{\nu^2}}\,x^{\frac{\lfloor\frac{1}{\nu}\rfloor}{2}}}{\lfloor\frac{1}{\nu}\rfloor}\right) c_3\right)$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 195

```
DSolve[-1/16*(b^4*x^(2/\[Nu])*y[x]) + (-1 + \[Nu])*\[Nu]^2*(-1 + 2*\[Nu])*x^2*y''[x] + \[Nu]
```

$$y(x) \rightarrow 8^{-\nu-1} b^\nu \left(x^{2/\nu}\right)^{\nu/4} \left(4^\nu (4c_1 \operatorname{Gamma}(1-\nu) - ic_2 \operatorname{Gamma}(2-\nu)) \operatorname{BesselJ}\left(-\nu, b\sqrt[4]{x^{2/\nu}}\right) + 4^\nu (4c_1 \operatorname{Gamma}(1-$$

5.37 problem 1572

Internal problem ID [9895]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1572.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$\left(x^2-1\right)^2 y'''' + 10x\left(x^2-1\right) y''' + \left(24x^2 - 8 - 2(\mu(\mu+1)+\nu(1+\nu))\left(x^2-1\right)\right) y'' - 6x(\mu(\mu+1)+\nu(1$$

✓ Solution by Maple

Time used: 0.5 (sec). Leaf size: 35

```
dsolve((x^2-1)^2*diff(diff(diff(diff(y(x),x),x),x),x)+10*x*(x^2-1)*diff(diff(diff(y(x),x),x)
```

$$y(x) = (\mathrm{LegendreQ}(\mu, x)\, c_2 + c_1\, \mathrm{LegendreP}(\mu, x))\, \mathrm{LegendreP}(\nu, x) + \mathrm{LegendreQ}(\nu, x)\, (\mathrm{LegendreQ}(\mu, x)\, c_4 + c_3\, \mathrm{LegendreP}(\mu, x))$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(-2*\[Mu]*(1 + \[Mu]) - 2*\[Nu]*(1 + \[Nu]) + (\[Mu]*(1 + \[Mu]) - \[Nu]*(1 + \[Nu]))
```

Not solved

5.38 problem 1573

Internal problem ID [9896]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1573.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _fully, _exact, _linear]]`

$$(2x + e^x)\, y'''' + 4(e^x + 2)\, y''' + 6y'' e^x + 4y' e^x + y\, e^x = \frac{1}{x^5}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
dsolve((exp(x)+2*x)*diff(diff(diff(diff(y(x),x),x),x),x)+4*(exp(x)+2)*diff(diff(diff(y(x),x)
```

$$y(x) = \frac{24c_1x^4 + 24c_2x^3 + 24x^2c_3 + 24c_4x + 1}{24\,(e^x + 2x)\,x}$$

✓ Solution by Mathematica

Time used: 0.152 (sec). Leaf size: 48

```
DSolve[-x^(-5) + E^x*y[x] + 4*E^x*y'[x] + 6*E^x*y''[x] + 4*(2 + E^x)*Derivative[3][y][x] + (
```

$$y(x) \to \frac{24c_4x^4 + 24c_3x^3 + 24c_2x^2 + 24c_1x + 1}{48x^2 + 24e^x x}$$

5.39 problem 1574

Internal problem ID [9897]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1574.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$y'''' \sin(x)^4 + 2y''' \sin(x)^3 \cos(x) + y'' \sin(x)^2 \left(\sin(x)^2 - 3\right) + y' \sin(x) \cos(x) \left(2 \sin(x)^2 + 3\right) + \left(a^4 \sin(\right.$$

✓ Solution by Maple

Time used: 0.688 (sec). Leaf size: 204

```
dsolve(diff(diff(diff(diff(y(x),x),x),x),x)*sin(x)^4+2*diff(diff(diff(y(x),x),x),x)*sin(x)^3
```

$$y(x) = \sin(x)\left(c_1 \,\mathrm{hypergeom}\left(\left[\frac{3}{4} - \frac{\sqrt{-4\sqrt{-a^4+1}+5}}{4}, \frac{3}{4} + \frac{\sqrt{-4\sqrt{-a^4+1}+5}}{4}\right], \left[\frac{1}{2}\right], \cos(x)^2\right) + c_2 \,\mathrm{hypergeom}\left(\left[\frac{3}{4} - \frac{\sqrt{4\sqrt{-a^4+1}+5}}{4}, \frac{3}{4} + \frac{\sqrt{4\sqrt{-a^4+1}+5}}{4}\right], \left[\frac{1}{2}\right], \cos(x)^2\right) + \cos(x)\left(\mathrm{hypergeom}\left(\left[\frac{5}{4} + \frac{\sqrt{-4\sqrt{-a^4+1}+5}}{4}, \frac{5}{4} - \frac{\sqrt{-4\sqrt{-a^4+1}+5}}{4}\right], \left[\frac{3}{2}\right], \cos(x)^2\right) c_3 + c_4 \,\mathrm{hypergeom}\left(\left[\frac{5}{4} + \frac{\sqrt{4\sqrt{-a^4+1}+5}}{4}, \frac{5}{4} - \frac{\sqrt{4\sqrt{-a^4+1}+5}}{4}\right], \left[\frac{3}{2}\right], \cos(x)^2\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.232 (sec). Leaf size: 265

```
DSolve[(-3 + a^4*Sin[x]^4)*y[x] + Cos[x]*Sin[x]*(3 + 2*Sin[x]^2)*y'[x] + Sin[x]^2*(-3 + Sin[
```

$$y(x) \to \sin(x)\left(c_1 \,\text{Hypergeometric2F1}\left(\frac{1}{4}\left(3-\sqrt{5-4\sqrt{1-a^4}}\right),\frac{1}{4}\left(\sqrt{5-4\sqrt{1-a^4}}+3\right),\frac{1}{2},\cos^2(x)\right) + c_3\cos(x)\,\text{Hypergeometric2F1}\left(\frac{1}{4}\left(5-\sqrt{5-4\sqrt{1-a^4}}\right),\frac{1}{4}\left(\sqrt{5-4\sqrt{1-a^4}}+5\right),\frac{3}{2},\cos^2(x)\right) + c_2\,\text{Hypergeometric2F1}\left(\frac{1}{4}\left(3-\sqrt{4\sqrt{1-a^4}+5}\right),\frac{1}{4}\left(\sqrt{4\sqrt{1-a^4}+5}+3\right),\frac{1}{2},\cos^2(x)\right) + c_4\cos(x)\,\text{Hypergeometric2F1}\left(\frac{1}{4}\left(5-\sqrt{4\sqrt{1-a^4}+5}\right),\frac{1}{4}\left(\sqrt{4\sqrt{1-a^4}+5}+5\right),\frac{3}{2},\cos^2(x)\right)\right)$$

5.40 problem 1575

Internal problem ID [9898]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1575.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' \sin (x)^6 + 4y''' \sin (x)^5 \cos (x) - 6y'' \sin (x)^6 - 4y' \sin (x)^5 \cos (x) + y \sin (x)^6 = f$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 699

```
dsolve(diff(diff(diff(diff(y(x),x),x),x),x)*sin(x)^6+4*diff(diff(diff(y(x),x),x),x)*sin(x)^5
```

Expression too large to display

✓ Solution by Mathematica

Time used: 7.987 (sec). Leaf size: 123

```
DSolve[-f[x] + Sin[x]^6*y[x] - 4*Cos[x]*Sin[x]^5*y'[x] - 6*Sin[x]^6*y''[x] + 4*Cos[x]*Sin[x]
```

$$y(x) \to \csc(x) \left(x^3 \int_1^x \frac{1}{6} \csc^5(K[4]) f(K[4]) dK[4] + x^2 \int_1^x -\frac{1}{2} \csc^5(K[3]) f(K[3]) K[3] dK[3] + x \int_1^x \frac{1}{2} \csc^5(K[2]) f(K[2]) K[2]^2 dK[2] + \int_1^x -\frac{1}{6} \csc^5(K[1]) f(K[1]) K[1]^3 dK[1] + c_4 x^3 + c_3 x^2 + c_2 x + c_1 \right)$$

5.41 problem 1576

Internal problem ID [9899]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1576.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$f\left(y'''' - 2a^2 y'' + a^4 y\right) + 2\,\mathrm{df}\left(y''' - a^2 y'\right) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 67

```
dsolve(f*(diff(y(x),x$4)-2*a^2*diff(y(x),x$2)+a^4*y(x))+2*df*(diff(y(x),x$3)-a^2*diff(y(x),x
```

$$y(x) = c_1 \mathrm{e}^{ax} + c_2 \mathrm{e}^{-ax} + c_3 \mathrm{e}^{\frac{\left(-\mathrm{df}+\sqrt{a^2 f^2+\mathrm{df}^2}\right)x}{f}} + c_4 \mathrm{e}^{-\frac{\left(\mathrm{df}+\sqrt{a^2 f^2+\mathrm{df}^2}\right)x}{f}}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 80

```
DSolve[f*(y''''[x]-2*a^2*y''[x]+a^4*y[x])+2*df*(y'''[x]-a^2*y'[x])==0,y[x],x,IncludeSingular
```

$$y(x) \to c_1 e^{\frac{x\left(\sqrt{a^2 f^2+\mathrm{df}^2}-\mathrm{df}\right)}{f}} + c_2 e^{-\frac{x\left(\sqrt{a^2 f^2+\mathrm{df}^2}+\mathrm{df}\right)}{f}} + c_3 e^{-ax} + c_4 e^{ax}$$

5.42 problem 1577

Internal problem ID [9900]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 4, linear fourth order
Problem number: 1577.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _quadrature]]`

$$\boxed{y''''f = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(f*diff(diff(diff(diff(y(x),x),x),x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{6}c_1x^3 + \frac{1}{2}c_2x^2 + c_3x + c_4$$

✓ Solution by Mathematica
Time used: 21.419 (sec). Leaf size: 41

```
DSolve[Derivative[2][f][x]*y''[x] + 2*Derivative[1][f][x]*Derivative[3][y][x] + f[x]*Derivat
```

$$y(x) \to \int_1^x \int_1^{K[2]} \frac{c_1 + c_2K[1]}{f(K[1])} dK[1]dK[2] + c_4x + c_3$$

6 Chapter 5, linear fifth and higher order

6.1 problem 1578

Internal problem ID [9901]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 5, linear fifth and higher order
Problem number: 1578.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$y'''' - 2a^2y'' + a^4y - \lambda(xa - b)\left(y'' - a^2y\right) = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 89

```
dsolve(diff(diff(diff(diff(y(x),x),x),x),x)-2*a^2*diff(diff(y(x),x),x)+a^4*y(x)-lambda*(a*x-
```

$$y(x) = e^{ax}\left(\int e^{-2ax}\left(\int e^{ax}\left(c_3 \operatorname{AiryAi}\left(-\frac{(-a\lambda)^{\frac{1}{3}}\left(\lambda(ax-b)+a^2\right)}{a\lambda}\right)+c_4 \operatorname{AiryBi}\left(-\frac{(-a\lambda)^{\frac{1}{3}}\left(\lambda(ax-b)+a^2\right)}{a\lambda}\right.\right.\right.\right.$$
$$\left.\left.\left. + c_2\right) dx + c_1\right)$$

✓ Solution by Mathematica

Time used: 40.473 (sec). Leaf size: 130

```
DSolve[a^4*y[x] - 2*a^2*y''[x] - \[Lambda]*(-b + a*x)*(-(a^2*y[x]) + y''[x]) + Derivative[4]
```

$$y(x) \to e^{-ax}\left(c_3\int_1^x 2ae^{2aK[1]}\int e^{-aK[1]} \operatorname{AiryAi}\left(\frac{a^2+\lambda K[1]a-b\lambda}{(a\lambda)^{2/3}}\right) dK[1]dK[1]\right.$$
$$\left. + c_4\int_1^x 2ae^{2aK[2]}\int e^{-aK[2]} \operatorname{AiryBi}\left(\frac{a^2+\lambda K[2]a-b\lambda}{(a\lambda)^{2/3}}\right) dK[2]dK[2] + c_2e^{2ax} + c_1\right)$$

6.2 problem 1579

Internal problem ID [9902]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 5, linear fifth and higher order
Problem number: 1579.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y^{(5)} + 2y''' + y' = xa + b\sin(x) + c\cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 69

```
dsolve(diff(y(x),x$5)+2*diff(y(x),x$3)+diff(y(x),x)-a*x-b*sin(x)-c*cos(x)=0,y(x), singsol=al
```

$$y(x) = \frac{(b\,x^2 + (-4c - 8c_4)\,x - 6b - 8c_2 + 8c_3)\cos(x)}{8} + \frac{(-x^2c + (-4b + 8c_3)\,x + 6c + 8c_1 + 8c_4)\sin(x)}{8} + \frac{a\,x^2}{2} + c_5$$

✓ Solution by Mathematica
Time used: 1.166 (sec). Leaf size: 80

```
DSolve[y'''''[x]+2*y'''[x]+y'[x]-a*x-b*Sin[x]-c*Cos[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{16}\big(8ax^2 + \cos(x)\left(b(2x^2 - 9) - 2(5cx + 8(c_4x - c_2 + c_3))\right) + \sin(x)\left(-6bx + c(13 - 2x^2) + 16(c_2x + c_1 + c_4)\right)\big) + c_5$$

6.3 problem 1580

Internal problem ID [9903]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 5, linear fifth and higher order
Problem number: 1580.
ODE order: 6.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y^{(6)} + y = \sin\left(\frac{3x}{2}\right)\sin\left(\frac{x}{2}\right)$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 71

```
dsolve(diff(y(x),x$6)+y(x)-sin(3/2*x)*sin(1/2*x)=0,y(x), singsol=all)
```

$$y(x) = \left(\sin\left(\frac{x}{2}\right)c_4 + \cos\left(\frac{x}{2}\right)c_3\right)e^{-\frac{\sqrt{3}\,x}{2}} + \left(\sin\left(\frac{x}{2}\right)c_6 + c_5\cos\left(\frac{x}{2}\right)\right)e^{\frac{\sqrt{3}\,x}{2}} + \frac{\cos(2x)}{126} + \frac{(5+24c_1)\cos(x)}{24} + \frac{\sin(x)(x+12c_2)}{12}$$

✓ Solution by Mathematica
Time used: 6.632 (sec). Leaf size: 111

```
DSolve[y''''''[x]+y[x]-Sin[3/2*x]*Sin[1/2*x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}x\sin(x) + \frac{1}{126}\cos(2x) + e^{-\frac{\sqrt{3}x}{2}}\left(c_1e^{\sqrt{3}x} + c_3\right)\cos\left(\frac{x}{2}\right) + \left(\frac{1}{4} + c_2\right)\cos(x) + c_4e^{-\frac{\sqrt{3}x}{2}}\sin\left(\frac{x}{2}\right) + c_6e^{\frac{\sqrt{3}x}{2}}\sin\left(\frac{x}{2}\right) + c_5\sin(x)$$

6.4 problem 1581

Internal problem ID [9904]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 5, linear fifth and higher order
Problem number: 1581.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y^{(5)} - yax = b}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$5)-a*x*y(x)-b=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'''''[x]-a*x*y[x]-b==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

6.5 problem 1582

Internal problem ID [9905]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 5, linear fifth and higher order
Problem number: 1582.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$y^{(5)} + a\,x^{\nu} y' + a\nu\,x^{\nu-1} y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$5)+a*x^nu*diff(y(x),x)+a*nu*x^(nu-1)*y(x)=0,y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 10.169 (sec). Leaf size: 528

```
DSolve[y'''''[x]+a*x^\[Nu]*y'[x]+a*\[Nu]*x^(\[Nu]-1)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)$$

$$\rightarrow \nu^{-\frac{16}{\nu+4}} \left(\frac{\nu+4}{\nu}\right)^{-\frac{16}{\nu+4}} a^{\frac{1}{\nu+4}} (x^{\nu})^{\frac{1}{\nu}} \left(a^{\frac{1}{\nu+4}} (x^{\nu})^{\frac{1}{\nu}} \left(a^{\frac{1}{\nu+4}} (x^{\nu})^{\frac{1}{\nu}} \left(c_5 a^{\frac{1}{\nu+4}} (x^{\nu})^{\frac{1}{\nu}}\ {}_1F_4\left(1; \frac{\nu}{\nu+4} + \frac{5}{\nu+4}, \frac{\nu}{\nu+4} + \frac{}{\nu}\right.\right.\right.\right.$$

$$+ c_3 \nu^{\frac{8}{\nu+4}} \left(\frac{\nu+4}{\nu}\right)^{\frac{8}{\nu+4}} {}_0F_3\left(; \frac{\nu}{\nu+4} + \frac{3}{\nu+4}, \frac{\nu}{\nu+4} + \frac{5}{\nu+4}, \frac{\nu}{\nu+4} + \frac{6}{\nu+4}; -\frac{a(x^{\nu})^{\frac{\nu+4}{\nu}}}{(\nu+4)^4}\right)\Bigg)$$

$$+ c_2 \nu^{\frac{12}{\nu+4}} \left(\frac{\nu+4}{\nu}\right)^{\frac{12}{\nu+4}} {}_0F_3\Bigg(; \frac{\nu}{\nu+4} + \frac{2}{\nu+4}, \frac{\nu}{\nu+4} + \frac{3}{\nu+4}, \frac{\nu}{\nu+4} + \frac{5}{\nu+4};$$

$$-\frac{a(x^{\nu})^{\frac{\nu+4}{\nu}}}{(\nu+4)^4}\Bigg)\Bigg) + c_1\, {}_0F_3\left(; \frac{\nu}{\nu+4} + \frac{1}{\nu+4}, \frac{\nu}{\nu+4} + \frac{2}{\nu+4}, \frac{\nu}{\nu+4} + \frac{3}{\nu+4}; -\frac{a(x^{\nu})^{\frac{\nu+4}{\nu}}}{(\nu+4)^4}\right)$$

6.6 problem 1583

Internal problem ID [9906]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 5, linear fifth and higher order
Problem number: 1583.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(5)} + ay'''' = f}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 50

```
dsolve(diff(y(x),x$5)+a*diff(y(x),x$4)-f=0,y(x), singsol=all)
```

$$y(x) = \frac{6\,\mathrm{e}^{-ax}c_1 + a^3\left((c_2x^3 + 3x^2c_3 + 6c_4x + 6c_5)\,a + \frac{f\,x^4}{4}\right)}{6a^4}$$

✓ Solution by Mathematica
Time used: 0.222 (sec). Leaf size: 45

```
DSolve[y'''''[x]+a*y''''[x]-f==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1e^{-ax}}{a^4} + \frac{fx^4}{24a} + x(x(c_5x + c_4) + c_3) + c_2$$

6.7 problem 1584

Internal problem ID [9907]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 5, linear fifth and higher order
Problem number: 1584.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$xy^{(5)} - mny'''' + yax = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 134

```
dsolve(x*diff(y(x),x$5)-m*n*diff(y(x),x$4)+a*x*y(x)=0,y(x), singsol=all)
```

$$\begin{aligned} y(x) = {} & c_1 \,\mathrm{hypergeom}\left([\,], \left[\frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{1}{5} - \frac{mn}{5}\right], -\frac{a\,x^5}{3125}\right) \\ & + c_2 x \,\mathrm{hypergeom}\left([\,], \left[\frac{3}{5}, \frac{4}{5}, \frac{6}{5}, \frac{2}{5} - \frac{mn}{5}\right], -\frac{a\,x^5}{3125}\right) \\ & + c_3 x^2 \,\mathrm{hypergeom}\left([\,], \left[\frac{4}{5}, \frac{6}{5}, \frac{7}{5}, \frac{3}{5} - \frac{mn}{5}\right], -\frac{a\,x^5}{3125}\right) \\ & + c_4 x^3 \,\mathrm{hypergeom}\left([\,], \left[\frac{6}{5}, \frac{7}{5}, \frac{8}{5}, \frac{4}{5} - \frac{mn}{5}\right], -\frac{a\,x^5}{3125}\right) \\ & + c_5 x^{mn+4} \,\mathrm{hypergeom}\left([\,], \left[\frac{6}{5} + \frac{mn}{5}, \frac{9}{5} + \frac{mn}{5}, \frac{8}{5} + \frac{mn}{5}, \frac{7}{5} + \frac{mn}{5}\right], -\frac{a\,x^5}{3125}\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 2.144 (sec). Leaf size: 244

```
DSolve[x*y'''''[x]-m*n*y''''[x]+a*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{625}x\left(x\left(5a^{3/5}c_4x\,{}_0F_4\left(;\frac{6}{5},\frac{7}{5},\frac{8}{5},\frac{4}{5}-\frac{mn}{5};-\frac{ax^5}{3125}\right)+25a^{2/5}c_3\,{}_0F_4\left(;\frac{4}{5},\frac{6}{5},\frac{7}{5},\frac{3}{5}-\frac{mn}{5};-\frac{ax^5}{3125}\right)+c_5 5^{-mn}\right.\right.$$
$$\left. + c_1\,{}_0F_4\left(;\frac{2}{5},\frac{3}{5},\frac{4}{5},\frac{1}{5}-\frac{mn}{5};-\frac{ax^5}{3125}\right)\right)$$

6.8 problem 1585

Internal problem ID [9908]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 5, linear fifth and higher order
Problem number: 1585.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

Solve

$$x(y'a + by'' + cy''' + ey'''')\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 675

```
dsolve(x * (a*diff(y(x),x) + b*diff(y(x),x$2) + c*diff(y(x),x$3) + e*diff(y(x),x$4))*y(x)=0,
```

$$y(x) = 0$$

$$y(x) = \mathrm{e}^{\frac{\left(\left(12\sqrt{3}\,\sqrt{27a^2e^2+\left(-18acb+4b^3\right)e+4a\,c^3-b^2c^2}\;e-108a\,e^2+36bce-8c^3\right)^{\frac{2}{3}}-2c\left(12\sqrt{3}\,\sqrt{27a^2e^2+\left(-18acb+4b^3\right)e+4a\,c^3-b^2c^2}\;e-108a\,e^2+36bce-8c^3\right)^{\frac{1}{3}}-1\cdots}{6e\left(12\sqrt{3}\,\sqrt{27a^2e^2+\left(-18acb+4b^3\right)e+4a\,c^3-b^2c^2}\;e-108a\,e^2+36bce-8c^3\right)^{\frac{1}{3}}}}$$

$$+\mathrm{e}^{\frac{x\left(i\left(\left(12\sqrt{3}\,\sqrt{27a^2e^2+\left(-18acb+4b^3\right)e+4a\,c^3-b^2c^2}\;e-108a\,e^2+36bce-8c^3\right)^{\frac{2}{3}}+12be-4c^2\right)\sqrt{3}+12be-\left(\left(12\sqrt{3}\,\sqrt{27a^2e^2+\left(-18acb+4b^3\right)e+4a\,c^3-b^2c^2\cdots}{12e\left(12\sqrt{3}\,\sqrt{27a^2e^2+\left(-18acb+4b^3\right)e+4a\,c^3-b^2c^2}\;e-108a\,e^2+36bce-8c^3\right)^{\frac{1}{3}}}}$$

$$+c_2\mathrm{e}^{\frac{\left(-i\left(\left(12\sqrt{3}\,\sqrt{27a^2e^2+\left(-18acb+4b^3\right)e+4a\,c^3-b^2c^2}\;e-108a\,e^2+36bce-8c^3\right)^{\frac{2}{3}}+12be-4c^2\right)\sqrt{3}+12be-\left(\left(12\sqrt{3}\,\sqrt{27a^2e^2+\left(-18acb+4b^3\right)e+4a\,c^3-b^2\cdots}{12e\left(12\sqrt{3}\,\sqrt{27a^2e^2+\left(-18acb+4b^3\right)e+4a\,c^3-b^2c^2}\;e-108a\,e^2+36bce-8c^3\right)^{\frac{1}{3}}}}$$

$$+\,c_1$$

✓ Solution by Mathematica

Time used: 30.173 (sec). Leaf size: 214

```
DSolve[x * (a*y'[x] + b*y''[x] + c*y'''[x] + e*y''''[x])*y[x]==0,y[x],x,IncludeSingularSolut
```

$$y(x) \to 0$$

$$y(x) \to \frac{c_1 e^{x\mathrm{Root}\left[\#1^3+\frac{\#1^2c}{e}+\frac{\#1b}{e}+\frac{a}{e}\&,1\right]}}{\mathrm{Root}\left[\#1^3+\frac{\#1^2c}{e}+\frac{\#1b}{e}+\frac{a}{e}\&,1\right]} + \frac{c_2 e^{x\mathrm{Root}\left[\#1^3+\frac{\#1^2c}{e}+\frac{\#1b}{e}+\frac{a}{e}\&,2\right]}}{\mathrm{Root}\left[\#1^3+\frac{\#1^2c}{e}+\frac{\#1b}{e}+\frac{a}{e}\&,2\right]} + \frac{c_3 e^{x\mathrm{Root}\left[\#1^3+\frac{\#1^2c}{e}+\frac{\#1b}{e}+\frac{a}{e}\&,3\right]}}{\mathrm{Root}\left[\#1^3+\frac{\#1^2c}{e}+\frac{\#1b}{e}+\frac{a}{e}\&,3\right]} + c_4$$

6.9 problem 1586

Internal problem ID [9909]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 5, linear fifth and higher order
Problem number: 1586.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$xy^{(5)} - ((aA_2 - A_1)\,x + A_2)\,y' = (aA_1 - A_0)\,x + A_1$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x$5)-((a*A__1-A__0)*x+A__1)-((a*A__2-A__1)*x+A__2)*diff(y(x),x)=0,y(x), s
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'''''[x]-((a*A1-A0)*x+A1)-((a*A2-A1)*x+A2)*y'[x]== 0,y[x],x,IncludeSingularSolutio
```

Not solved

6.10 problem 1587

Internal problem ID [9910]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 5, linear fifth and higher order
Problem number: 1587.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$x^2y'''' - ay = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 161

```
dsolve(x^2*diff(y(x),x$4)-a*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(\text{BesselY}\left(1, 2\sqrt{-\sqrt{a}}\,\sqrt{x}\right) c_4 + \text{BesselJ}\left(1, 2\sqrt{-\sqrt{a}}\,\sqrt{x}\right) c_3\right) a^{\frac{1}{4}} + \left(\text{BesselJ}\left(1, 2a^{\frac{1}{4}}\sqrt{x}\right) c_1 + \text{BesselY}\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 121

```
DSolve[x^2*y''''[x]-a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_4 G_{0,4}^{2,0}\left(\frac{ax^2}{16}\middle|\ 0, 1, \tfrac{1}{2}, \tfrac{3}{2}\right) + c_2 G_{0,4}^{2,0}\left(\frac{ax^2}{16}\middle|\ \tfrac{1}{2}, \tfrac{3}{2}, 0, 1\right) + \frac{1}{64}\sqrt{a}x\left((4c_3 - 3ic_1)\,\text{BesselJ}\left(2, 2\sqrt[4]{a}\sqrt{x}\right) + (3ic_1 + 4c_3)\,\text{BesselI}\left(2, 2\sqrt[4]{a}\sqrt{x}\right)\right)$$

6.11 problem 1588

Internal problem ID [9911]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 5, linear fifth and higher order
Problem number: 1588.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$x^{10}y^{(5)} - ay = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 90

```
dsolve(x^10*diff(diff(diff(diff(diff(y(x),x),x),x),x),x)-a*y(x)=0,y(x), singsol=all)
```

$$\begin{aligned}
y(x) = {} & c_1 \,\mathrm{hypergeom}\left([\,], \left[\frac{6}{5}, \frac{7}{5}, \frac{8}{5}, \frac{9}{5}\right], -\frac{a}{3125x^5}\right) \\
& + c_2 x \,\mathrm{hypergeom}\left([\,], \left[\frac{4}{5}, \frac{6}{5}, \frac{7}{5}, \frac{8}{5}\right], -\frac{a}{3125x^5}\right) \\
& + c_3 x^2 \,\mathrm{hypergeom}\left([\,], \left[\frac{3}{5}, \frac{4}{5}, \frac{6}{5}, \frac{7}{5}\right], -\frac{a}{3125x^5}\right) \\
& + c_4 x^3 \,\mathrm{hypergeom}\left([\,], \left[\frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{6}{5}\right], -\frac{a}{3125x^5}\right) \\
& + c_5 x^4 \,\mathrm{hypergeom}\left([\,], \left[\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}\right], -\frac{a}{3125x^5}\right)
\end{aligned}$$

✓ Solution by Mathematica

Time used: 11.265 (sec). Leaf size: 103

```
DSolve[x^10*y'''''[x]-a*y[x]== 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^4\left(c_1 e^{-\frac{\sqrt[5]{a}}{x}} + c_2 e^{\frac{\sqrt[5]{-1}\sqrt[5]{a}}{x}} + c_3 e^{-\frac{(-1)^{2/5}\sqrt[5]{a}}{x}} + c_4 e^{\frac{(-1)^{3/5}\sqrt[5]{a}}{x}} + c_5 e^{-\frac{(-1)^{4/5}\sqrt[5]{a}}{x}}\right)$$

6.12 problem 1589

Internal problem ID [9912]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 5, linear fifth and higher order
Problem number: 1589.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$\boxed{x^{\frac{5}{2}}y^{(5)} - ay = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 486

```
dsolve(x^(2+1/2)*diff(y(x),x$5)-a*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{8\left(\left(x^{\frac{3}{2}}\left(\left(c_2 - c_5\right)\cos\left(\frac{\pi}{5}\right) + \left(-c_3 + \frac{c_4}{15}\right)\cos\left(\frac{2\pi}{5}\right) + c_1\right)a^{\frac{2}{5}} + \left(\frac{3\left(c_1 + \frac{c_4}{15}\right)x\,a^{\frac{1}{5}}}{2} + \frac{3\sqrt{x}\left(c_2 + c_3\right)}{4}\right)\cos\left(\frac{\pi}{5}\right) + \left(\frac{3x(c_3}{\cdots}\right.\right.\right.}{\cdots}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 206

```
DSolve[x^(2+1/2)*D[y[x],{x,5}]-a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4}{25}(-1)^{2/5}a^{2/5}c_2 x\,{}_0F_4\left(;-\frac{1}{5},\frac{1}{5},\frac{3}{5},\frac{7}{5};\frac{32ax^{5/2}}{3125}\right) + \frac{16\sqrt[5]{-1}a^{4/5}x^2\left(625(-1)^{3/5}c_3\,{}_0F_4\left(;\frac{1}{5},\frac{3}{5},\frac{7}{5},\frac{9}{5};\frac{32ax^{5/2}}{3125}\right) - \cdots\right.}{\cdots}$$

6.13 problem 1590

Internal problem ID [9913]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 5, linear fifth and higher order
Problem number: 1590.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$(x-a)^5 (x-b)^5 y^{(5)} - yc = 0$$

✗ Solution by Maple

```
dsolve((x-a)^5*(x-b)^5*diff(y(x),x$5)-c*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(x-a)^5*(x-b)^5*y'''''[x]-c*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

7 Chapter 6, non-linear second order

7.1 problem 1591 (6.1)

Internal problem ID [9914]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1591 (6.1).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.469 (sec). Leaf size: 12

```
dsolve(diff(diff(y(x),x),x)-y(x)^2=0,y(x), singsol=all)
```

$$y(x) = 6\,\text{WeierstrassP}\left(x + c_1, 0, c_2\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-y[x]^2 + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.2 problem 1592 (6.2)

Internal problem ID [9915]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1592 (6.2).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' - 6y^2 = 0}$$

✓ Solution by Maple

Time used: 0.453 (sec). Leaf size: 10

```
dsolve(diff(diff(y(x),x),x)-6*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \text{WeierstrassP}\left(x + c_1, 0, c_2\right)$$

✓ Solution by Mathematica

Time used: 0.358 (sec). Leaf size: 14

```
DSolve[-6*y[x]^2 + y''[x]== 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \wp(x + c_1; 0, c_2)$$

7.3 problem 1593 (6.3)

Internal problem ID [9916]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1593 (6.3).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Painleve, ‘1st‘]]`

$$\boxed{y'' - 6y^2 = x}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)-6*y(x)^2-x=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-x - 6*y[x]^2 + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.4 problem 1594 (6.4)

Internal problem ID [9917]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1594 (6.4).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' - 6y^2 + 4y = 0}$$

✓ Solution by Maple

Time used: 0.453 (sec). Leaf size: 59

```
dsolve(diff(diff(y(x),x),x)-6*y(x)^2+4*y(x)=0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{\sqrt{4\_a^3 - 4\_a^2 + c_1}} d\_a - x - c_2 = 0$$

$$-\left(\int^{y(x)} \frac{1}{\sqrt{4\_a^3 - 4\_a^2 + c_1}} d\_a\right) - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 0.486 (sec). Leaf size: 373

```
DSolve[4*y[x] - 6*y[x]^2 + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{4\left(\text{Root}\left[4\#1^3-4\#1^2+c_1\&,2\right]-\text{Root}\left[4\#1^3-4\#1^2+c_1\&,3\right]\right)\left(y(x)-\text{Root}\left[4\#1^3-4\#1^2+c_1\right.\right.}{(4y(x)^3-4y(x)^2+c}\right.$$

$$\left.+c_2)^2, y(x)\right]$$

7.5 problem 1595 (6.5)

Internal problem ID [9918]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1595 (6.5).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$\boxed{y'' + ay^2 = -bx - c}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+a*y(x)^2+b*x+c=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[c + b*x + a*y[x]^2 + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.6 problem 1596 (6.6)

Internal problem ID [9919]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1596 (6.6).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Painleve, ‘2nd‘]]`

$$y'' - 2y^3 - xy = -a$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)-2*y(x)^3-x*y(x)+a=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[a - x*y[x] - 2*y[x]^3 + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.7 problem 1597 (6.7)

Internal problem ID [9920]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1597 (6.7).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' - a y^3 = 0}$$

✓ Solution by Maple
Time used: 0.453 (sec). Leaf size: 26

```
dsolve(diff(diff(y(x),x),x)-a*y(x)^3=0,y(x), singsol=all)
```

$$y(x) = c_2 \operatorname{JacobiSN}\left(\frac{\left(\sqrt{2}\,\sqrt{-a}\,x + 2c_1\right) c_2}{2}, i\right)$$

✓ Solution by Mathematica
Time used: 61.747 (sec). Leaf size: 131

```
DSolve[-(a*y[x]^3) + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{i\sqrt[4]{2}\operatorname{sn}\left(-\frac{(1-i)\sqrt{\sqrt{a}\sqrt{c_1}(x+c_2)^2}}{2^{3/4}}\middle| -1\right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{c_1}}}}$$
$$y(x) \to \frac{i\sqrt[4]{2}\operatorname{sn}\left(-\frac{(1-i)\sqrt{\sqrt{a}\sqrt{c_1}(x+c_2)^2}}{2^{3/4}}\middle| -1\right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{c_1}}}}$$

7.8 problem 1598 (6.8)

Internal problem ID [9921]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1598 (6.8).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$\boxed{y'' - 2a^2y^3 + 2abxy = b}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)-2*a^2*y(x)^3+2*a*b*x*y(x)-b=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-b + 2*a*b*x*y[x] - 2*a^2*y[x]^3 + y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

Not solved

7.9 problem 1599 (6.9)

Internal problem ID [9922]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1599 (6.9).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y'' + ybx + yc + ay^3 = -d$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+d+b*x*y(x)+c*y(x)+a*y(x)^3=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[d + c*y[x] + b*x*y[x] + a*y[x]^3 + y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

Not solved

7.10 problem 1600 (6.10)

Internal problem ID [9923]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1600 (6.10).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' + y^2 b + yc + a y^3 = -d}$$

✓ Solution by Maple

Time used: 0.453 (sec). Leaf size: 89

```
dsolve(diff(diff(y(x),x),x)+d+y(x)^2*b+c*y(x)+a*y(x)^3=0,y(x), singsol=all)
```

$$-6\left(\int^{y(x)} \frac{1}{\sqrt{-18a\_a^4 - 24b\_a^3 - 36\_a^2 c - 72\_ad + 36c_1}} d\_a\right) - x - c_2 = 0$$

$$6\left(\int^{y(x)} \frac{1}{\sqrt{-18a\_a^4 - 24b\_a^3 - 36\_a^2 c - 72\_ad + 36c_1}} d\_a\right) - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 2.11 (sec). Leaf size: 1017

```
DSolve[d + c*y[x] + b*y[x]^2 + a*y[x]^3 + y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

$$\text{Solve}\left[\frac{4\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{(\text{Root}[3a\#1^4+4b\#1^3+6c\#1^2+12d\#1-6c_1\&,2]-\text{Root}[3a\#1^4+4b\#1^3+6c\#1^2+12d\#1\ldots}{(\text{Root}[3a\#1^4+4b\#1^3+6c\#1^2+12d\#1-6c_1\&,1]-\text{Root}[3a\#1^4+4b\#1^3+6c\#1^2+12d\#1\ldots}}\right.\right.}{\ldots}\right.$$

$$\left.+c_2)^2, y(x)\right]$$

7.11 problem 1601 (6.11)

Internal problem ID [9924]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1601 (6.11).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + a\,x^r y^2 = 0}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+a*x^r*y(x)^2=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[a*x^r*y[x]^2 + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.12 problem 1602 (6.12)

Internal problem ID [9925]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1602 (6.12).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$y'' + 6a^{10}y^{11} - y = 0$$

✓ Solution by Maple
Time used: 0.469 (sec). Leaf size: 61

```
dsolve(diff(diff(y(x),x),x)+(5+1)*a^(2*5)*y(x)^(2*5+1)-y(x)=0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{\sqrt{-\_a^{12}a^{10} + \_a^2 + c_1}} d\_a - x - c_2 = 0$$
$$-\left(\int^{y(x)} \frac{1}{\sqrt{-\_a^{12}a^{10} + \_a^2 + c_1}} d\_a\right) - x - c_2 = 0$$

✓ Solution by Mathematica
Time used: 10.264 (sec). Leaf size: 49

```
DSolve[-y[x] + a^(2*5)*(1 + 5)*y[x]^(1 + 2*5) + y''[x] == 0,y[x],x,IncludeSingularSolutions
```

$$\text{Solve}\left[\int_1^{y(x)} \frac{1}{\sqrt{c_1 + 2\left(\frac{K[1]^2}{2} - \frac{1}{2}a^{10}K[1]^{12}\right)}} dK[1]^2 = (x + c_2)^2, y(x)\right]$$

7.13 problem 1603 (6.13)

Internal problem ID [9926]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1603 (6.13).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y'' - \frac{1}{(ay^2 + ybx + c\,x^2 + \alpha y + \beta x + \gamma)^{\frac{3}{2}}} = 0$$

✓ Solution by Maple

Time used: 4.813 (sec). Leaf size: 1016

```
dsolve(diff(y(x),x$2)-(a*y(x)^2+b*x*y(x)+c*x^2+alpha*y(x)+beta*x+gamma)^(-3/2)=0,y(x), sings
```

$$y(x) = \frac{2\,\mathrm{RootOf}\left(2\left(2\left(\int^{\_Z}\frac{1}{\sqrt{16\sqrt{4a\beta+4ac+4a\gamma-\alpha^2-2b\alpha-b^2}\,a\left(\int\frac{1}{(4\_g^2a^2+1)\sqrt{\frac{(4a\beta+4ac+4a\gamma-\alpha^2-2b\alpha-b^2)(4\_g^2a^2+1)}{a}}}d\_g\right)+4\beta}}\right.\right.\right.}{}$$

$$y(x) = \frac{2\,\mathrm{RootOf}\left(2\left(-2\left(\int^{\_Z}\frac{1}{\sqrt{16\sqrt{4a\beta+4ac+4a\gamma-\alpha^2-2b\alpha-b^2}\,a\left(\int\frac{1}{(4\_g^2a^2+1)\sqrt{\frac{(4a\beta+4ac+4a\gamma-\alpha^2-2b\alpha-b^2)(4\_g^2a^2+1)}{a}}}d\_g\right)+}}\right.\right.\right.}{}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]-(a*y[x]^2+b*x*y[x]+c*x^2+\[Alpha]*y[x]+\[Beta]*x+\[Gamma])^(-3/2) == 0,y[x],x,
```

Not solved

7.14 problem 1604 (6.14)

Internal problem ID [9927]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1604 (6.14).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' - e^{y} = 0}$$

✓ Solution by Maple
Time used: 0.61 (sec). Leaf size: 25

```
dsolve(diff(diff(y(x),x),x)-exp(y(x))=0,y(x), singsol=all)
```

$$y(x) = -\ln(2) + \ln\left(\frac{\sec\left(\frac{x+c_2}{2c_1}\right)^2}{c_1^2}\right)$$

✓ Solution by Mathematica
Time used: 60.051 (sec). Leaf size: 32

```
DSolve[-E^y[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(-\frac{1}{2}c_1 \operatorname{sech}^2\left(\frac{1}{2}\sqrt{c_1(x+c_2)^2}\right)\right)$$

7.15 problem 1605 (6.15)

Internal problem ID [9928]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1605 (6.15).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + a\,\mathrm{e}^{x}\sqrt{y} = 0}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+a*exp(x)*y(x)^(1/2)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[a*E^x*Sqrt[y[x]] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.16 problem 1606 (6.16)

Internal problem ID [9929]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1606 (6.16).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$\boxed{y'' + e^x \sin(y) = 0}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+exp(x)*sin(y(x))=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[Exp[x]*Sin[y[x]] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.17 problem 1607 (6.17)

Internal problem ID [9930]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1607 (6.17).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' + a\sin(y) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve(diff(diff(y(x),x),x)+a*sin(y(x))=0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{\sqrt{2a\cos(\_a) + c_1}} d\_a - x - c_2 = 0$$

$$-\left(\int^{y(x)} \frac{1}{\sqrt{2a\cos(\_a) + c_1}} d\_a\right) - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 7.619 (sec). Leaf size: 79

```
DSolve[a*Sin[y[x]] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2\,\text{JacobiAmplitude}\left(\frac{1}{2}\sqrt{(2a + c_1)(x + c_2)^2}, \frac{4a}{2a + c_1}\right)$$

$$y(x) \to 2\,\text{JacobiAmplitude}\left(\frac{1}{2}\sqrt{(2a + c_1)(x + c_2)^2}, \frac{4a}{2a + c_1}\right)$$

7.18 problem 1608 (6.18)

Internal problem ID [9931]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1608 (6.18).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y'' + a^2 \sin(y) = \beta \sin(x)$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+a^2*sin(y(x))-beta*sin(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-(\[Beta]*Sin[x]) + a^2*Sin[y[x]] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tr
```

Not solved

7.19 problem 1609 (6.19)

Internal problem ID [9932]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1609 (6.19).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$\boxed{y'' + a^2 \sin(y) = \beta f(x)}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+a^2*sin(y(x))-beta*f(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-(\[Beta]*f[x]) + a^2*Sin[y[x]] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True
```

Not solved

7.20 problem 1610 (6.20)

Internal problem ID [9933]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1610 (6.20).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{f\left(\frac{y}{\sqrt{x}}\right)}{x^{\frac{3}{2}}} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 92

```
dsolve(diff(diff(y(x),x),x)=x^(-3/2)*f(y(x)*x^(-1/2)),y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(\_Z x^{\frac{3}{2}} + 4f\left(\frac{\_Z}{\sqrt{x}}\right) x^2\right)$$
$$y(x) = \operatorname{RootOf}\left(-\ln(x) + 2\left(\int^{\_Z} \frac{1}{\sqrt{c_1 + 8\left(\int f(\_g)\, d\_g\right) + \_g^2}} d\_g\right) + 2c_2\right)\sqrt{x}$$
$$y(x) = \operatorname{RootOf}\left(-\ln(x) - 2\left(\int^{\_Z} \frac{1}{\sqrt{c_1 + 8\left(\int f(\_g)\, d\_g\right) + \_g^2}} d\_g\right) + 2c_2\right)\sqrt{x}$$

✓ Solution by Mathematica

Time used: 11.171 (sec). Leaf size: 754

```
DSolve[-(f[y[x]*x^(-1/2)]*x^(-3/2)) + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} \frac{2}{\sqrt{x}\sqrt{\frac{K[3]^2+4xc_1+8x\int_1^{\frac{K[3]}{\sqrt{x}}} f(K[2])dK[2]}{x}}} dK[3] - \int_1^x \left(\frac{2\left(\frac{y(x)}{2\sqrt{K[4]}} - \frac{\sqrt{\frac{y(x)^2}{2K[4]}+2c_1+4\int_1^{\frac{y(x)}{\sqrt{K[4]}}} f(K[2])dK[2]}}{\sqrt{2}}\right)}{K[4]\sqrt{\frac{y(x)^2+4c_1K[4]+8K[4]\int_1^{\frac{y(x)}{\sqrt{K[4]}}} f(K[2])dK[2]}{K[4]}}} + \int_1^{y(x)} \left(-\frac{\frac{4c_1+8\int_1^{\frac{K[3]}{\sqrt{K[4]}}} f(K[2])dK[2] - \frac{4f\left(\frac{K[3]}{\sqrt{K[4]}}\right)K[3]}{\sqrt{K[4]}}}{K[4]} - \frac{K[3]^2+4c_1K[4]+8K[4]\int_1^{\frac{K[3]}{\sqrt{K[4]}}} f(K[2])dK[2]}{K[4]^2}}{\sqrt{K[4]}\left(\frac{K[3]^2+4c_1K[4]+8K[4]\int_1^{\frac{K[3]}{\sqrt{K[4]}}} f(K[2])dK[2]}{K[4]}\right)^{3/2}} - \frac{}{K[4]^{3/2}\sqrt{\frac{K[3]^2+4c_1K[\cdots}{}}} \right.\right.$$

$$\text{Solve}\left[\int_1^{y(x)} -\frac{2}{\sqrt{x}\sqrt{\frac{K[5]^2+4xc_1+8x\int_1^{\frac{K[5]}{\sqrt{x}}} f(K[2])dK[2]}{x}}} dK[5] - \int_1^x \left(\int_1^{y(x)} \left(\frac{\frac{4c_1+8\int_1^{\frac{K[5]}{\sqrt{K[6]}}} f(K[2])dK[2] - \frac{4f\left(\frac{K[5]}{\sqrt{K[6]}}\right)K[5]}{\sqrt{K[6]}}}{K[6]} - \frac{K[5]^2+4c_1K[6]+8K[6]\int_1^{\frac{K[5]}{\sqrt{K[6]}}} f(K[2])dK[2]}{K[6]^2}}{\sqrt{K[6]}\left(\frac{K[5]^2+4c_1K[6]+8K[6]\int_1^{\frac{K[5]}{\sqrt{K[6]}}} f(K[2])dK[2]}{K[6]}\right)^{3/2}} + \frac{}{K[6]^{3/2}\sqrt{\frac{K[5]^2+\cdots}{}}} \right.\right.\right.$$

7.21 problem 1611 (6.21)

Internal problem ID [9934]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1611 (6.21).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 3y' - y^2 - 2y = 0}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)-3*diff(y(x),x)-y(x)^2-2*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-2*y[x] - y[x]^2 - 3*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.22 problem 1612 (6.22)

Internal problem ID [9935]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1612 (6.22).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 7y' - y^{\frac{3}{2}} + 12y = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)-7*diff(y(x),x)-y(x)^(3/2)+12*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[12*y[x] - y[x]^(3/2) - 7*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.23 problem 1613 (6.23)

Internal problem ID [9936]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1613 (6.23).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 5ay' - 6y^2 + 6a^2y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(diff(y(x),x),x)+5*a*diff(y(x),x)-6*y(x)^2+6*a^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \text{WeierstrassP}\left(\frac{-c_1 a + e^{-ax}}{a}, 0, c_2\right) e^{-2ax}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[6*a^2*y[x] - 6*y[x]^2 + 5*a*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tr
```

Not solved

7.24 problem 1614 (6.24)

Internal problem ID [9937]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1614 (6.24).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 3ay' - 2y^3 + 2a^2y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 34

```
dsolve(diff(diff(y(x),x),x)+3*a*diff(y(x),x)-2*y(x)^3+2*a^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = -\operatorname{JacobiSN}\left(\frac{\left(-c_1 a + \sqrt{-\mathrm{e}^{-2ax}}\right) c_2}{a}, i\right) c_2 \mathrm{e}^{-ax}$$

✓ Solution by Mathematica
Time used: 3.421 (sec). Leaf size: 32

```
DSolve[2*a^2*y[x] - 2*y[x]^3 + 3*a*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -iac_1 e^{-ax} \operatorname{sn}\left(e^{-ax} c_1 + c_2 \middle| -1\right)$$

7.25 problem 1615 (6.25)

Internal problem ID [9938]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1615 (6.25).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - \frac{(3n+4)\,y'}{n} - \frac{2(n+1)\,(2+n)\,y\left(y^{\frac{n}{n+1}}-1\right)}{n^2} = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)-(3*n+4)/n*diff(y(x),x)-2*(n+1)*(n+2)/n^2*y(x)*(y(x)^(n/(n+1))-1)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(-2*(1 + n)*(2 + n)*y[x]*(-1 + y[x]^(n/(1 + n))))/n^2 - ((4 + 3*n)*y'[x])/n + y''[x]
```

Not solved

7.26 problem 1616 (6.26)

Internal problem ID [9939]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1616 (6.26).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + ay' + by^n + \frac{(a^2 - 1)\, y}{4} = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+a*diff(y(x),x)+b*y(x)^n + 1/4*(a^2-1)*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[((-1 + a^2)*y[x])/4 + b*y[x]^n + a*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolution
```

Not solved

7.27 problem 1617 (6.27)

Internal problem ID [9940]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1617 (6.27).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$\boxed{y'' + ay' + b\,x^v y^n = 0}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+a*diff(y(x),x)+b*x^v*y(x)^n=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[b*x^v*y[x]^n + a*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.28 problem 1618 (6.28)

Internal problem ID [9941]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1618 (6.28).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + ay' + b\,e^{y} = 2a}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+a*diff(y(x),x)+b*exp(y(x))-2*a=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-2*a + b*Exp[y[x]] + a*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.29 problem 1619 (6.29)

Internal problem ID [9942]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1619 (6.29).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$\boxed{y'' + ay' + f(x)\sin(y) = 0}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+a*diff(y(x),x)+f(x)*sin(y(x))=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[f[x]*Sin[y[x]] + a*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.30 problem 1620 (6.30)

Internal problem ID [9943]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1620 (6.30).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + y'y - y^3 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 336

```
dsolve(diff(diff(y(x),x),x)+y(x)*diff(y(x),x)-y(x)^3=0,y(x), singsol=all)
```

$$2\left(\int^{y(x)} \frac{\left(\_a^6+2c_1+2\sqrt{c_1\left(\_a^6+c_1\right)}\right)^{\frac{1}{3}}}{\_a^4-\_a^2\left(\_a^6+2c_1+2\sqrt{c_1\left(\_a^6+c_1\right)}\right)^{\frac{1}{3}}+\left(\_a^6+2c_1+2\sqrt{c_1\left(\_a^6+c_1\right)}\right)^{\frac{2}{3}}} d\_a\right)$$
$$-x-c_2=0$$

$$-4\left(\int^{y(x)} \frac{\left(\_a^6+2c_1+2\sqrt{c_1\left(\_a^6+c_1\right)}\right)^{\frac{1}{3}}}{-i\sqrt{3}\,\_a^4+i\sqrt{3}\left(\_a^6+2c_1+2\sqrt{c_1\left(\_a^6+c_1\right)}\right)^{\frac{2}{3}}+\_a^4+2\_a^2\left(\_a^6+2c_1+2\sqrt{c_1\left(\_a^6+\right.\right.}}\right.$$
$$-x-c_2=0$$

$$4\left(\int^{y(x)}\right.$$
$$-\frac{\left(\_a^6+2c_1+2\sqrt{c_1\left(\_a^6+c_1\right)}\right)^{\frac{1}{3}}}{i\sqrt{3}\,\_a^4-i\sqrt{3}\left(\_a^6+2c_1+2\sqrt{c_1\left(\_a^6+c_1\right)}\right)^{\frac{2}{3}}+\_a^4+2\_a^2\left(\_a^6+2c_1+2\sqrt{c_1\left(\_a^6+c_1\right)}\right)^{\frac{1}{3}}+\left(\right.}$$
$$-x-c_2=0$$

✓ Solution by Mathematica

Time used: 46.156 (sec). Leaf size: 1534

```
DSolve[-y[x]^3 + y[x]*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{2}{\frac{e^{6c_1}K[1]^4}{\sqrt[3]{e^{18c_1}K[1]^6 - 2e^{12c_1} + 2\sqrt{e^{24c_1} - e^{30c_1}K[1]^6}}} - K[1]^2 + e^{-6c_1}\sqrt[3]{e^{18c_1}K[1]^6 - 2e \cdots}} \right. \left. + c_2\right]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{-\frac{(1+i\sqrt{3})e^{6c_1}K[2]^4}{4\sqrt[3]{e^{18c_1}K[2]^6 - 2e^{12c_1} + 2\sqrt{e^{24c_1} - e^{30c_1}K[2]^6}}} - \frac{K[2]^2}{2} - \frac{1}{4}\left(1 - i\sqrt{3}\right)e^{-6c_1}\sqrt[3]{e \cdots}} \right. \left. + c_2\right]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{-\frac{(1-i\sqrt{3})e^{6c_1}K[3]^4}{4\sqrt[3]{e^{18c_1}K[3]^6 - 2e^{12c_1} + 2\sqrt{e^{24c_1} - e^{30c_1}K[3]^6}}} - \frac{K[3]^2}{2} - \frac{1}{4}\left(1 + i\sqrt{3}\right)e^{-6c_1}\sqrt[3]{e \cdots}} \right. \left. + c_2\right]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{2}{\frac{e^{6(-c_1)}K[1]^4}{\sqrt[3]{e^{18(-c_1)}K[1]^6 - 2e^{12(-c_1)} + 2\sqrt{e^{24(-c_1)} - e^{30(-c_1)}K[1]^6}}} - K[1]^2 + e^{-6(-c_1)}\sqrt[3]{e^{1 \cdots}}} \right. \left. + c_2\right]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{2}{\frac{e^{6c_1}K[1]^4}{\sqrt[3]{e^{18c_1}K[1]^6 - 2e^{12c_1} + 2\sqrt{e^{24c_1} - e^{30c_1}K[1]^6}}} - K[1]^2 + e^{-6c_1}\sqrt[3]{e^{18c_1}K[1]^6 - 2e \cdots}} \right. \left. + c_2\right]$$

$y(x)$

7.31 problem 1621 (6.31)

Internal problem ID [9944]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1621 (6.31).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + y'y - y^3 + ay = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 108

```
dsolve(diff(diff(y(x),x),x)+y(x)*diff(y(x),x)-y(x)^3+a*y(x)=0,y(x), singsol=all)
```

$$-\frac{\left(\int^{y(x)} \frac{4\mathrm{RootOf}\left(\left(-4\_a^6+12\_a^4a-12\_a^2a^2+4a^3+320c_1\right)\_Z^9+\left(-189\_a^6+567\_a^4a-567\_a^2a^2+189a^3+15120c_1\right)\_Z^6+238140}{\_a^2-a}\right.}{63}$$

$$-x-c_2=0$$

✓ Solution by Mathematica

Time used: 77.065 (sec). Leaf size: 3100

```
DSolve[a*y[x] - y[x]^3 + y[x]*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

7.32 problem 1622 (6.32)

Internal problem ID [9945]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1622 (6.32).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + (y + 3a)\, y' - y^3 + ay^2 + 2a^2 y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 416

```
dsolve(diff(diff(y(x),x),x)+(y(x)+3*a)*diff(y(x),x)-y(x)^3+a*y(x)^2+2*a^2*y(x)=0,y(x), sings
```

$$y(x) = \operatorname{RootOf}\left(\left(\left(\int^{\_Z} -\frac{-\_f^8 + c_1\_f^2 - \left((-\_f^6 + c_1)^2 \left(\sqrt{\frac{c_1}{-\_f^6+c_1}} - 1\right)\right)^{\frac{2}{3}}}{(-\_f^6 + c_1)\left((-\_f^6 + c_1)^2 \left(\sqrt{\frac{c_1}{-\_f^6+c_1}} - 1\right)\right)^{\frac{1}{3}}} d\_f\right) a + c_2 a + e^{-ax}\right) e^{-ax}$$

$$y(x) = \operatorname{RootOf}\left(-\left(\int^{\_Z} \frac{-i\sqrt{3}\,\_f^8 + \_f^8 + i\sqrt{3}\,c_1\_f^2 + i\sqrt{3}\left((-\_f^6 + c_1)^2 \left(\sqrt{\frac{c_1}{-\_f^6+c_1}} - 1\right)\right)^{\frac{2}{3}} - c_1\_f^2 + \cdots}{(-\_f^6 + c_1)\left((-\_f^6 + c_1)^2 \left(\sqrt{\frac{c_1}{-\_f^6+c_1}} - 1\right)\right) \cdots} \cdots + 2c_2 a + 2\,e^{-ax}\right) e^{-ax}$$

$$y(x) = \operatorname{RootOf}\left(\left(\left(\int^{\_Z} \frac{-i\sqrt{3}\,\_f^8 - \_f^8 + i\sqrt{3}\,c_1\_f^2 + i\sqrt{3}\left((-\_f^6 + c_1)^2 \left(\sqrt{\frac{c_1}{-\_f^6+c_1}} - 1\right)\right)^{\frac{2}{3}} + c_1\_f^2 - \left(\cdots\right.}{(-\_f^6 + c_1)\left((-\_f^6 + c_1)^2 \left(\sqrt{\frac{c_1}{-\_f^6+c_1}} - 1\right)\right)^{\frac{1}{3}}} \cdots + 2c_2 a + 2\,e^{-ax}\right) e^{-ax}$$

✓ Solution by Mathematica

Time used: 58.636 (sec). Leaf size: 185

```
DSolve[2*a^2*y[x] + a*y[x]^2 - y[x]^3 + (3*a + y[x])*y'[x] + y''[x] == 0,y[x],x,IncludeSingu
```

$$y(x) \to \begin{cases} \frac{c_1 \wp'(xc_1+c_2;0,1)}{\wp(xc_1+c_2;0,1)} & a=0 \\ -\frac{e^{-ax}c_1\wp'\left(\frac{e^{-ax}c_1}{a}+c_2;0,1\right)}{\wp\left(\frac{e^{-ax}c_1}{a}+c_2;0,1\right)} & \text{True} \end{cases}$$

$$y(x) \to \begin{cases} c_1 & a=0 \\ -e^{-ax}c_1 & \text{True} \end{cases}$$

$$y(x) \to \begin{cases} -\frac{e^{-ax}c_1\wp'\left(\frac{e^{-ax}c_1}{a};0,1\right)}{\wp\left(\frac{e^{-ax}c_1}{a};0,1\right)} & a\neq 0 \\ \frac{c_1\wp'(xc_1;0,1)}{\wp(xc_1;0,1)} & \text{True} \end{cases}$$

7.33 problem 1623 (6.33)

Internal problem ID [9946]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1623 (6.33).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y'' + (y + 3f(x))\,y' - y^3 + y^2 f(x) + y\left(f'(x) + 2f(x)^2\right) = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+(y(x)+3*f(x))*diff(y(x),x)-y(x)^3+y(x)^2*f(x)+y(x)*(diff(f(x),x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[f[x]*y[x]^2 - y[x]^3 + y[x]*(2*f[x]^2 + Derivative[1][f][x]) + (3*f[x] + y[x])*y'[x]
```

Not solved

7.34 problem 1624 (6.34)

Internal problem ID [9947]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1624 (6.34).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y'' + y'y - y^3 - \left(\frac{f'(x)}{f(x)} + f(x)\right)(3y' + y^2) + \left(f(x)^2 a + 3f'(x) + \frac{3f'(x)^2}{f(x)^2} - \frac{f''(x)}{f(x)}\right) y = -bf(x)^3$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+y(x)*diff(y(x),x)-y(x)^3-(diff(f(x),x)/f(x)+f(x))*(3*diff(y(x),x
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[b*f[x]^3 - y[x]^3 + y[x]*y'[x] - (f[x] + Derivative[1][f][x]/f[x])*(y[x]^2 + 3*y'[x])
```

Not solved

7.35 problem 1625 (6.35)

Internal problem ID [9948]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1625 (6.35).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y'' + \left(y - \frac{3f'(x)}{2f(x)}\right)y' - y^3 - \frac{f'(x)\,y^2}{2f(x)} + \frac{\left(f(x) + \frac{f'(x)^2}{f(x)^2} - f''(x)\right)y}{2f(x)} = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+(y(x)-3/2*diff(f(x),x)/f(x))*diff(y(x),x)-y(x)^3-1/2*diff(f(x),x
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-y[x]^3 - (y[x]^2*Derivative[1][f][x])/(2*f[x]) + (y[x] - (3*Derivative[1][f][x])/(2*
```

Not solved

7.36 problem 1626 (6.36)

Internal problem ID [9949]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1626 (6.36).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _nonlinear], [_2nd_order, _reducible, _m`

$$y'' + 2y'y + f(x)\,y' + f'(x)\,y = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+2*y(x)*diff(y(x),x)+f(x)*diff(y(x),x)+diff(f(x),x)*y(x)=0,y(x),
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*f'[x] + f[x]*y'[x] + 2*y[x]*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions
```

Not solved

7.37 problem 1627 (6.37)

Internal problem ID [9950]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1627 (6.37).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _reducible, _mu_x_y1], [_2nd_order, _reducible,`

$$y'' + 2y'y + f(x)\left(y^2 + y'\right) = g(x)$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+2*y(x)*diff(y(x),x)+f(x)*(diff(y(x),x)+y(x)^2)-g(x)=0,y(x), sing
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-g[x] + 2*y[x]*y'[x] + f[x]*(y[x]^2 + y'[x]) + y''[x] == 0,y[x],x,IncludeSingularSolu
```

Not solved

7.38 problem 1628 (6.38)

Internal problem ID [9951]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1628 (6.38).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$\boxed{y'' + 3y'y + y^3 + f(x)\, y = g(x)}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+3*y(x)*diff(y(x),x)+y(x)^3+f(x)*y(x)-g(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-g[x] + f[x]*y[x] + y[x]^3 + 3*y[x]*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutio
```

Not solved

7.39 problem 1629 (6.39)

Internal problem ID [9952]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1629 (6.39).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_potential_symmetries]]`

$$y'' + (3y + f(x))\,y' + y^3 + y^2 f(x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(diff(y(x),x),x)+(3*y(x)+f(x))*diff(y(x),x)+y(x)^3+y(x)^2*f(x)=0,y(x), singsol=al
```

$$y(x) = \frac{c_1\left(\int \mathrm{e}^{-\left(\int f(x)dx\right)}dx\right) + c_2}{c_1\left(\int \left(\int \mathrm{e}^{-\left(\int f(x)dx\right)}dx\right)dx\right) + c_2 x + 1}$$

✓ Solution by Mathematica

Time used: 60.065 (sec). Leaf size: 75

```
DSolve[f[x]*y[x]^2 + y[x]^3 + (f[x] + 3*y[x])*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolu
```

$$y(x) \to \frac{\int_1^x \exp\left(-\int_1^{K[2]} f(K[1])dK[1]\right) c_1 dK[2] + c_2}{\int_1^x \int_1^{K[5]} \exp\left(-\int_1^{K[4]} f(K[3])dK[3]\right) c_1 dK[4]dK[5] + c_2 x + 1}$$

7.40 problem 1630 (6.40)

Internal problem ID [9953]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1630 (6.40).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1],`

$$y'' - 3y'y - 3ay^2 - 4a^2y = b$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 755

```
dsolve(diff(diff(y(x),x),x)-3*y(x)*diff(y(x),x)-3*a*y(x)^2-4*a^2*y(x)-b=0,y(x), singsol=all)
```

$$-6a^2\left(\int^{y(x)} \frac{}{-12\_a\,a^3 - 9\_a^2a^2 + \text{RootOf}\left(\left(\text{BesselK}\left(\frac{4a^3-3b}{2a\sqrt{4a^4-3ab}}, -\frac{\_Z}{2a^2}\right)c_1 + \text{BesselI}\left(-\frac{4a^3-3b}{2a\sqrt{4a^4-3ab}}, -\right.\right.\right.}\right.$$

$$- x - c_2 = 0$$

$$-6a^2\left(\int^{y(x)} \frac{}{-12\_a\,a^3 - 9\_a^2a^2 + \text{RootOf}\left(\left(\text{BesselK}\left(\frac{4a^3-3b}{2a\sqrt{4a^4-3ab}}, \frac{\_Z}{2a^2}\right)c_1 + \text{BesselI}\left(-\frac{4a^3-3b}{2a\sqrt{4a^4-3ab}}, \frac{\_Z}{2a^2}\right)\right.\right.\right.}\right.$$

$$- x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 127.393 (sec). Leaf size: 1670

```
DSolve[-b - 4*a^2*y[x] - 3*a*y[x]^2 - 3*y[x]*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolut
```

$$y(x) \to \frac{\sqrt{b}\sqrt{\frac{e^{-2ax}}{b}}\sqrt{c_1}\left(-i^{\frac{\sqrt{4a^3-3b}}{a^{3/2}}}2^{\frac{3\sqrt{4a^6-3a^3b}}{2a^3}+\frac{1}{2}}3^{\frac{1}{2}\left(\frac{\sqrt{4a^3-3b}}{a^{3/2}}-1\right)}a^{\frac{\sqrt{4a^6-3a^3b}}{a^3}}b^{\frac{1}{2}\left(\frac{\sqrt{4a^3-3b}}{a^{3/2}}-1\right)}\left(2a^3-\sqrt{4a^3-3b}a^{3/2}+\sqrt{\right.\right.}{}$$

7.41 problem 1631 (6.41)

Internal problem ID [9954]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1631 (6.41).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_potential_symmetries]]`

$$y'' - (3y + f(x))\, y' + y^3 + y^2 f(x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(diff(y(x),x),x)-(3*y(x)+f(x))*diff(y(x),x)+y(x)^3+y(x)^2*f(x)=0,y(x), singsol=al
```

$$y(x) = \frac{-c_1\left(\int \mathrm{e}^{\int f(x)dx} dx\right) - c_2}{c_1\left(\int \int \mathrm{e}^{\int f(x)dx} dx dx\right) + c_2 x + 1}$$

✓ Solution by Mathematica

Time used: 60.069 (sec). Leaf size: 72

```
DSolve[f[x]*y[x]^2 + y[x]^3 - (f[x] + 3*y[x])*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolu
```

$$y(x) \to -\frac{\int_1^x \exp\left(\int_1^{K[2]} f(K[1]) dK[1]\right) c_1 dK[2] + c_2}{\int_1^x \int_1^{K[5]} \exp\left(\int_1^{K[4]} f(K[3]) dK[3]\right) c_1 dK[4] dK[5] + c_2 x + 1}$$

7.42 problem 1632 (6.42)

Internal problem ID [9955]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1632 (6.42).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _`

$$\boxed{y'' - 2ay'y = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 23

```
dsolve(diff(diff(y(x),x),x)-2*a*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\tan\left((c_2 + x)\sqrt{c_1 a}\right)\sqrt{c_1 a}}{a}$$

✓ Solution by Mathematica
Time used: 25.806 (sec). Leaf size: 34

```
DSolve[-2*a*y[x]*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{c_1}\tan\left(\sqrt{a}\sqrt{c_1}(x + c_2)\right)}{\sqrt{a}}$$

7.43 problem 1633 (6.43)

Internal problem ID [9956]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1633 (6.43).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + ay'y + by^3 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 97

```
dsolve(diff(diff(y(x),x),x)+a*y(x)*diff(y(x),x)+b*y(x)^3=0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{\operatorname{RootOf}\left(-2a_a^2 \operatorname{arctanh}\left(\frac{a_a^2+4_Z}{\sqrt{_a^4(a^2-8b)}}\right) - \ln\left(_a^4 b + _Z_a^2 a + 2_Z^2\right)\sqrt{_a^4\left(a^2-8b\right)} + c_1\sqrt{_
$$

$$- x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 34.145 (sec). Leaf size: 92

```
DSolve[b*y[x]^3 + a*y[x]*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} \frac{1}{K[2]^2 \text{InverseFunction}\left[\frac{1}{4}\left(\log(b+\#1(a+2\#1)) - \frac{2a\arctan\left(\frac{a+4\#1}{\sqrt{8b-a^2}}\right)}{\sqrt{8b-a^2}}\right)\&\right][c_1 - \log(K[2])]} d\right.$$

$$\left. - c_2, y(x)\right]$$

7.44 problem 1634 (6.44)

Internal problem ID [9957]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1634 (6.44).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$\boxed{y'' + f(x,y)\,y' + g(x,y) = 0}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+f(x,y(x))*diff(y(x),x)+g(x,y(x))=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[g[x, y[x]] + f[x, y[x]]*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.45 problem 1635 (6.45)

Internal problem ID [9958]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1635 (6.45).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + a{y'}^2 + yb = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 79

```
dsolve(diff(diff(y(x),x),x)+a*diff(y(x),x)^2+b*y(x)=0,y(x), singsol=all)
```

$$-2a\left(\int^{y(x)}\frac{1}{\sqrt{4\,\mathrm{e}^{-2a\_a}c_1a^2-4\_aba+2b}}d\_a\right)-x-c_2=0$$
$$2a\left(\int^{y(x)}\frac{1}{\sqrt{4\,\mathrm{e}^{-2a\_a}c_1a^2-4\_aba+2b}}d\_a\right)-x-c_2=0$$

✓ Solution by Mathematica

Time used: 1.695 (sec). Leaf size: 314

```
DSolve[b*y[x] + a*y'[x]^2 + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{\sqrt{2}a}{\sqrt{2e^{-2aK[1]}c_1a^2 - 2bK[1]a + b}}dK[1]\&\right][x + c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sqrt{2}a}{\sqrt{2e^{-2aK[2]}c_1a^2 - 2bK[2]a + b}}dK[2]\&\right][x + c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{\sqrt{2}a}{\sqrt{2e^{-2aK[1]}(-c_1)a^2 - 2bK[1]a + b}}dK[1]\&\right][x + c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{\sqrt{2}a}{\sqrt{2e^{-2aK[1]}c_1a^2 - 2bK[1]a + b}}dK[1]\&\right][x + c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sqrt{2}a}{\sqrt{2e^{-2aK[2]}(-c_1)a^2 - 2bK[2]a + b}}dK[2]\&\right][x + c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sqrt{2}a}{\sqrt{2e^{-2aK[2]}c_1a^2 - 2bK[2]a + b}}dK[2]\&\right][x + c_2]$$

7.46 problem 1636 (6.46)

Internal problem ID [9959]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1636 (6.46).
ODE order: 2.
ODE degree: 0.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + ay'|y'| + by' + yc = 0}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+a*diff(y(x),x)*abs(diff(y(x),x))+b*diff(y(x),x)+c*y(x)=0,y(x), s
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[c*y[x] + b*y'[x] + a*Abs[y'[x]]*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -
```

Not solved

7.47 problem 1637 (6.47)

Internal problem ID [9960]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1637 (6.47).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + a{y'}^2 + by' + yc = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+a*diff(y(x),x)^2+b*diff(y(x),x)+c*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[c*y[x] + b*y'[x] + a*y'[x]^2 + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.48 problem 1638 (6.48)

Internal problem ID [9961]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1638 (6.48).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + a{y'}^2 + b\sin(y) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 211

```
dsolve(diff(diff(y(x),x),x)+a*diff(y(x),x)^2+b*sin(y(x))=0,y(x), singsol=all)
```

$$4\left(\int^{y(x)} \frac{1}{\sqrt{(4a^2+1)\left(4\,\mathrm{e}^{-2a\_a}c_1a^2 - 4\sin(\_a)\,ab + \mathrm{e}^{-2a\_a}c_1 + 2\cos(\_a)\,b\right)}}d\_a\right)a^2 + \int^{y(x)} \frac{1}{\sqrt{(4a^2+1)\left(4\,\mathrm{e}^{-2a\_a}c_1a^2 - 4\sin(\_a)\,ab + \mathrm{e}^{-2a\_a}c_1 + 2\cos(\_a)\,b\right)}}d\_a - c_2 - x = 0$$

$$-4\left(\int^{y(x)} \frac{1}{\sqrt{(4a^2+1)\left(4\,\mathrm{e}^{-2a\_a}c_1a^2 - 4\sin(\_a)\,ab + \mathrm{e}^{-2a\_a}c_1 + 2\cos(\_a)\,b\right)}}d\_a\right)a^2 - \left(\int^{y(x)} \frac{1}{\sqrt{(4a^2+1)\left(4\,\mathrm{e}^{-2a\_a}c_1a^2 - 4\sin(\_a)\,ab + \mathrm{e}^{-2a\_a}c_1 + 2\cos(\_a)\,b\right)}}d\_a\right) - c_2 - x = 0$$

✓ Solution by Mathematica

Time used: 10.587 (sec). Leaf size: 444

```
DSolve[b*Sin[y[x]] + a*y'[x]^2 + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{\sqrt{4a^2+1}}{\sqrt{4e^{-2aK[1]}c_1a^2 - 4b\sin(K[1])a + e^{-2aK[1]}c_1 + 2b\cos(K[1])}}dK[1]\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sqrt{4a^2+1}}{\sqrt{4e^{-2aK[2]}c_1a^2 - 4b\sin(K[2])a + e^{-2aK[2]}c_1 + 2b\cos(K[2])}}dK[2]\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{\sqrt{4a^2+1}}{\sqrt{4e^{-2aK[1]}(-c_1)a^2 - 4b\sin(K[1])a + e^{-2aK[1]}(-c_1) + 2b\cos(K[1])}}dK[1]\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{\sqrt{4a^2+1}}{\sqrt{4e^{-2aK[1]}c_1a^2 - 4b\sin(K[1])a + e^{-2aK[1]}c_1 + 2b\cos(K[1])}}dK[1]\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sqrt{4a^2+1}}{\sqrt{4e^{-2aK[2]}(-c_1)a^2 - 4b\sin(K[2])a + e^{-2aK[2]}(-c_1) + 2b\cos(K[2])}}dK[2]\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sqrt{4a^2+1}}{\sqrt{4e^{-2aK[2]}c_1a^2 - 4b\sin(K[2])a + e^{-2aK[2]}c_1 + 2b\cos(K[2])}}dK[2]\&\right][x+c_2]$$

7.49 problem 1639 (6.49)

Internal problem ID [9962]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1639 (6.49).
ODE order: 2.
ODE degree: 0.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + ay'|y'| + b\sin(y) = 0}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+a*diff(y(x),x)*abs(diff(y(x),x))+b*sin(y(x))=0,y(x), singsol=all
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[b*Sin[y[x]] + a*Abs[y'[x]]*y'[x] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

Not solved

7.50 problem 1640 (6.50)

Internal problem ID [9963]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1640 (6.50).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' + ayy'^2 + yb = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 70

```
dsolve(diff(diff(y(x),x),x)+a*y(x)*diff(y(x),x)^2+b*y(x)=0,y(x), singsol=all)
```

$$a\left(\int^{y(x)} \frac{1}{\sqrt{a\left(\mathrm{e}^{-a\_a^2}c_1 a - b\right)}}d\_a\right) - x - c_2 = 0$$
$$-a\left(\int^{y(x)} \frac{1}{\sqrt{a\left(\mathrm{e}^{-a\_a^2}c_1 a - b\right)}}d\_a\right) - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 1.909 (sec). Leaf size: 290

```
DSolve[b*y[x] + a*y[x]*y'[x]^2 + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{\sqrt{a}}{\sqrt{e^{2ac_1-aK[1]^2}-b}}dK[1]\&\right][x+c_2]$$
$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sqrt{a}}{\sqrt{e^{2ac_1-aK[2]^2}-b}}dK[2]\&\right][x+c_2]$$
$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{\sqrt{a}}{\sqrt{e^{2a(-c_1)-aK[1]^2}-b}}dK[1]\&\right][x+c_2]$$
$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{\sqrt{a}}{\sqrt{e^{2ac_1-aK[1]^2}-b}}dK[1]\&\right][x+c_2]$$
$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sqrt{a}}{\sqrt{e^{2a(-c_1)-aK[2]^2}-b}}dK[2]\&\right][x+c_2]$$
$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sqrt{a}}{\sqrt{e^{2ac_1-aK[2]^2}-b}}dK[2]\&\right][x+c_2]$$

7.51 problem 1641 (6.51)

Internal problem ID [9964]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1641 (6.51).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _reducible, _mu_x_y1], [_2nd_order,`

$$y'' + f(y)\, y'^2 + g(x)\, y' = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(diff(diff(y(x),x),x)+f(y(x))*diff(y(x),x)^2+g(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$\int^{y(x)} e^{\int f(\_b)d\_b} d\_b - c_1\left(\int e^{-\left(\int g(x)dx\right)} dx\right) - c_2 = 0$$

✓ Solution by Mathematica

Time used: 0.079 (sec). Leaf size: 61

```
DSolve[g[x]*y'[x] + f[y[x]]*y'[x]^2 + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \exp\left(-\int_1^{K[1]} -f(K[1])dK[1]\right) dK[1]\&\right]\left[\int_1^x -\exp\left(-\int_1^{K[2]} g(K[2])dK[2]\right) c_1 dK[2] + c_2\right]$$

7.52 problem 1642 (6.52)

Internal problem ID [9965]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1642 (6.52).
ODE order: 2.
ODE degree: 0.

CAS Maple gives this as type `[NONE]`

$$y'' - \frac{D(f)(y)\, y'^3}{f(y)} + g(x)\, y' + h(x)\, f(y) = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)-diff(f(y(x)),x)/f(y(x))*diff(y(x),x)^2 + g(x)*diff(y(x),x)+h(x)*
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x] - D[f[y[x]], x]/f[y[x]]*y'[x]^2 + g[x]*y'[x] + h[x]*f[y[x]] == 0,y[x],x,Includ
```

Not solved

7.53 problem 1643 (6.53)

Internal problem ID [9966]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1643 (6.53).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y'' + \phi(y)\, y'^2 + f(x)\, y' + g(x)\, \Phi(y) = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+phi(y(x))*diff(y(x),x)^2+f(x)*diff(y(x),x)+g(x)*Phi(y(x))=0,y(x), sing
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+\[Phi][y[x]]*y'[x]^2+f[x]*y'[x]+g[x]*\[CurlyPhi][y[x]]==0,y[x],x,IncludeSingul
```

Not solved

7.54 problem 1644 (6.54)

Internal problem ID [9967]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1644 (6.54).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + f(y)\,y'^2 + g(y)\,y' + h(y) = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+f(y(x))*diff(y(x),x)^2+g(y(x))*diff(y(x),x)+h(y(x))=0,y(x), sing
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+f[y[x]]*y'[x]^2+g[y[x]]*y'[x]+h[y[x]]==0,y[x],x,IncludeSingularSolutions -> Tr
```

Not solved

7.55 problem 1645 (6.55)

Internal problem ID [9968]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1645 (6.55).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y'' + \left(y'^2 + 1\right)(f(x,y)\,y' + g(x,y)) = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+(diff(y(x),x)^2+1)*(f(x,y(x))*diff(y(x),x)+g(x,y(x)))=0,y(x), si
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(g[x, y[x]] + f[x, y[x]]*y'[x])*(1 + y'[x]^2) + y''[x] == 0,y[x],x,IncludeSingularSol
```

Not solved

7.56 problem 1646 (6.56)

Internal problem ID [9969]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1646 (6.56).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' + ay\left(y'^2 + 1\right)^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 96

```
dsolve(diff(diff(y(x),x),x)+a*y(x)*(diff(y(x),x)^2+1)^2=0,y(x), singsol=all)
```

$$a\left(\int^{y(x)} \frac{\_a^2 + 2c_1}{\sqrt{-\left(-1 + a\left(\_a^2 + 2c_1\right)\right)\left(\_a^2 + 2c_1\right) a}} d\_a\right) - x - c_2 = 0$$
$$-a\left(\int^{y(x)} \frac{\_a^2 + 2c_1}{\sqrt{-\left(-1 + a\left(\_a^2 + 2c_1\right)\right)\left(\_a^2 + 2c_1\right) a}} d\_a\right) - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 22.617 (sec). Leaf size: 816

```
DSolve[a*y[x]*(1 + y'[x]^2)^2 + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{\sqrt{\frac{\#1^2(-a)+1+2c_1}{1+2c_1}}\sqrt{2\#1^2a-4c_1}E\left(\arcsin\left(\sqrt{\frac{a}{2c_1+1}}\#1\right)|1+\frac{1}{2c_1}\right)}{\sqrt{\frac{a}{1+2c_1}}\sqrt{\#1^2(-a)+1+2c_1}\sqrt{2-\frac{\#1^2a}{c_1}}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\sqrt{\frac{\#1^2(-a)+1+2c_1}{1+2c_1}}\sqrt{2\#1^2a-4c_1}E\left(\arcsin\left(\sqrt{\frac{a}{2c_1+1}}\#1\right)|1+\frac{1}{2c_1}\right)}{\sqrt{\frac{a}{1+2c_1}}\sqrt{\#1^2(-a)+1+2c_1}\sqrt{2-\frac{\#1^2a}{c_1}}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{\sqrt{\frac{\#1^2(-a)+1+2(-1)c_1}{1+2(-1)c_1}}\sqrt{2\#1^2a-4(-c_1)}E\left(\arcsin\left(\sqrt{\frac{a}{2(-1)c_1+1}}\#1\right)|1+\frac{1}{2(-c_1)}\right)}{\sqrt{\frac{a}{1+2(-1)c_1}}\sqrt{\#1^2(-a)+1+2(-1)c_1}\sqrt{2--\frac{\#1^2a}{c_1}}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\sqrt{\frac{\#1^2(-a)+1+2(-1)c_1}{1+2(-1)c_1}}\sqrt{2\#1^2a-4(-c_1)}E\left(\arcsin\left(\sqrt{\frac{a}{2(-1)c_1+1}}\#1\right)|1+\frac{1}{2(-c_1)}\right)}{\sqrt{\frac{a}{1+2(-1)c_1}}\sqrt{\#1^2(-a)+1+2(-1)c_1}\sqrt{2--\frac{\#1^2a}{c_1}}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{\sqrt{\frac{\#1^2(-a)+1+2c_1}{1+2c_1}}\sqrt{2\#1^2a-4c_1}E\left(\arcsin\left(\sqrt{\frac{a}{2c_1+1}}\#1\right)|1+\frac{1}{2c_1}\right)}{\sqrt{\frac{a}{1+2c_1}}\sqrt{\#1^2(-a)+1+2c_1}\sqrt{2-\frac{\#1^2a}{c_1}}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\sqrt{\frac{\#1^2(-a)+1+2c_1}{1+2c_1}}\sqrt{2\#1^2a-4c_1}E\left(\arcsin\left(\sqrt{\frac{a}{2c_1+1}}\#1\right)|1+\frac{1}{2c_1}\right)}{\sqrt{\frac{a}{1+2c_1}}\sqrt{\#1^2(-a)+1+2c_1}\sqrt{2-\frac{\#1^2a}{c_1}}}\&\right][x+c_2]$$

7.57 problem 1647 (6.57)

Internal problem ID [9970]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1647 (6.57).
ODE order: 2.
ODE degree: 0.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - a(y'x - y)^v = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 59

```
dsolve(diff(diff(y(x),x),x)-a*(x*diff(y(x),x)-y(x))^v=0,y(x), singsol=all)
```

$$y(x) = \left(2^{\frac{1}{v-1}}\left(\int -\frac{((v-1)\,a\,x^2 - c_1)\left(-\frac{1}{(v-1)a\,x^2-c_1}\right)^{\frac{v}{v-1}}}{x^2}dx\right) + c_2\right)x$$

✓ Solution by Mathematica

Time used: 120.631 (sec). Leaf size: 60

```
DSolve[-(a*(-y[x] + x*y'[x])^v) + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(\int_1^x \left(\frac{1}{2}aK[2]^{2v} - \frac{1}{2}avK[2]^{2v} + c_1K[2]^{2v-2}\right)^{\frac{1}{1-v}}dK[2] + c_2\right)$$

7.58 problem 1648 (6.58)

Internal problem ID [9971]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1648 (6.58).
ODE order: 2.
ODE degree: 0.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - k\,x^a y^b y'^r = 0}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)-k*x^a*y(x)^b*diff(y(x),x)^r=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-(k*x^a*y[x]^b*y'[x]^r) + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.59 problem 1649 (book 6.59)

Internal problem ID [9972]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1649 (book 6.59).
ODE order: 2.
ODE degree: 0.

CAS Maple gives this as type `[NONE]`

$$y'' + \left(y' - \frac{y}{x}\right)^a f(x, y) = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)+(diff(y(x),x)-y(x)/x)^a*f(x,y(x))=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[f[x, y[x]]*(-(y[x]/x) + y'[x])^a + y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

Not solved

7.60 problem 1650 (book 6.60)

Internal problem ID [9973]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1650 (book 6.60).
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - a\sqrt{y'^2 + 1} = 0}$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 34

```
dsolve(diff(diff(y(x),x),x)=a*(diff(y(x),x)^2+1)^(1/2),y(x), singsol=all)
```

$$y(x) = -ix + c_1$$
$$y(x) = ix + c_1$$
$$y(x) = c_2 + \frac{\cosh(a(c_1 + x))}{a}$$

✓ Solution by Mathematica

Time used: 0.728 (sec). Leaf size: 35

```
DSolve[-(a*Sqrt[1 + y'[x]^2]) + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{ax+c_1}\left(1 + e^{-2(ax+c_1)}\right)}{2a} + c_2$$

7.61 problem 1652 (book 6.61)

Internal problem ID [9974]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1652 (book 6.61).
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - a\sqrt{y'^2 + 1} = b}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 31

```
dsolve(diff(diff(y(x),x),x)=a*sqrt(1+diff(y(x),x)^2)+b,y(x), singsol=all)
```

$$y(x) = \int \operatorname{RootOf}\left(x - \left(\int^{\_Z} \frac{1}{a\sqrt{\_f^2 + 1} + b} d\_f\right) + c_1\right) dx + c_2$$

✓ Solution by Mathematica

Time used: 60.646 (sec). Leaf size: 972

```
DSolve[y''[x]==a*Sqrt[1+y'[x]^2]+b,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2$$

$$-\frac{2a\text{InverseFunction}\left[\frac{\frac{2b\arctan\left(\frac{b+a\left(\sqrt{\#1^2+1}-\#1\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\log\left(\sqrt{\#1^2+1}-\#1\right)}{a}\&\right][x+c_1]^2}{\sqrt{\text{InverseFunction}\left[\frac{\frac{2b\arctan\left(\frac{b+a\left(\sqrt{\#1^2+1}-\#1\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\log\left(\sqrt{\#1^2+1}-\#1\right)}{a}\&\right][x+c_1]^2+1}} + b\log\left(\text{InverseFunction}\right.$$

$$- \frac{}{}$$

7.62 problem 1653 (book 6.62)

Internal problem ID [9975]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1653 (book 6.62).
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - a\sqrt{y'^2 + by^2} = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 41

```
dsolve(diff(diff(y(x),x),x)=a*sqrt(diff(y(x),x)^2+b*y(x)^2),y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = e^{\int \operatorname{RootOf}\left(x - \left(\int^{\_Z} -\frac{1}{\_f^2 - a\sqrt{\_f^2 + b}} d\_f\right) + c_1\right) dx + c_2}$$

✓ Solution by Mathematica

Time used: 0.569 (sec). Leaf size: 76

```
DSolve[y''[x]==a*Sqrt[y'[x]^2+b*y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)}\frac{1}{\text{InverseFunction}\left[\int\frac{\#1}{K[1]\left(\frac{\#1^2}{K[1]^2}-a\sqrt{\frac{\#1^2}{K[1]^2}+b}\right)}d\frac{\#1}{K[1]}\&\right][c_1-\log(K[1])]}dK[1]=x-c_2,y(x)\right]$$

7.63 problem 1654 (book 6.63)

Internal problem ID [9976]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1654 (book 6.63).
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - a\left(y'^2 + 1\right)^{\frac{3}{2}} = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 59

```
dsolve(diff(diff(y(x),x),x)=a*(diff(y(x),x)^2+1)^(3/2),y(x), singsol=all)
```

$$y(x) = -ix + c_1$$
$$y(x) = ix + c_1$$
$$y(x) = \frac{\left(-1 + (c_1 + x)^2 a^2\right)\sqrt{-\frac{1}{-1+(c_1+x)^2 a^2}} + c_2 a}{a}$$

✓ Solution by Mathematica

Time used: 0.835 (sec). Leaf size: 75

```
DSolve[-(a*(1 + y'[x]^2)^(3/2)) + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{i\sqrt{a^2x^2 + 2ac_1x - 1 + c_1{}^2}}{a}$$
$$y(x) \to \frac{i\sqrt{a^2x^2 + 2ac_1x - 1 + c_1{}^2}}{a} + c_2$$

7.64 problem 1655 (book 6.64)

Internal problem ID [9977]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1655 (book 6.64).
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$y'' - 2ax\left(y'^2 + 1\right)^{\frac{3}{2}} = 0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 57

```
dsolve(diff(diff(y(x),x),x)-2*a*x*(diff(y(x),x)^2+1)^(3/2)=0,y(x), singsol=all)
```

$$y(x) = -ix + c_1$$
$$y(x) = ix + c_1$$
$$y(x) = a\left(\int \sqrt{-\frac{1}{-1 + (x^2 + 2c_1)^2 a^2}}\,(x^2 + 2c_1)\,dx\right) + c_2$$

✓ Solution by Mathematica

Time used: 60.462 (sec). Leaf size: 308

```
DSolve[-2*a*x*(1 + y'[x]^2)^(3/2) + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{\sqrt{\frac{ax^2-1+c_1}{-1+c_1}}\sqrt{\frac{ax^2+1+c_1}{1+c_1}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(x\sqrt{\frac{a}{c_1+1}}\right),\frac{c_1+1}{c_1-1}\right)+(-1+c_1)E\left(i\text{arcsinh}\left(x\sqrt{\frac{a}{c_1+1}}\right)|\frac{c_1+1}{c_1-1}\right)\right)}{\sqrt{\frac{a}{1+c_1}}\sqrt{a^2x^4+2ac_1x^2-1+c_1{}^2}}$$

$$y(x) \to \frac{\sqrt{\frac{ax^2-1+c_1}{-1+c_1}}\sqrt{\frac{ax^2+1+c_1}{1+c_1}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(x\sqrt{\frac{a}{c_1+1}}\right),\frac{c_1+1}{c_1-1}\right)+(-1+c_1)E\left(i\text{arcsinh}\left(x\sqrt{\frac{a}{c_1+1}}\right)|\frac{c_1+1}{c_1-1}\right)\right)}{\sqrt{\frac{a}{1+c_1}}\sqrt{a^2x^4+2ac_1x^2-1+c_1{}^2}} + c_2$$

7.65 problem 1656 (book 6.65)

Internal problem ID [9978]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1656 (book 6.65).
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$y'' - ay\left(y'^2 + 1\right)^{\frac{3}{2}} = 0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 104

```
dsolve(diff(diff(y(x),x),x)-a*y(x)*(diff(y(x),x)^2+1)^(3/2)=0,y(x), singsol=all)
```

$$y(x) = -ix + c_1$$
$$y(x) = ix + c_1$$
$$a\left(\int^{y(x)} \frac{\_a^2 + 2c_1}{\sqrt{4 - (\_a^2 + 2c_1)^2 a^2}} d\_a\right) - x - c_2 = 0$$
$$-a\left(\int^{y(x)} \frac{\_a^2 + 2c_1}{\sqrt{4 - (\_a^2 + 2c_1)^2 a^2}} d\_a\right) - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 3.326 (sec). Leaf size: 1104

```
DSolve[-(a*y[x]*(1 + y'[x]^2)^(3/2)) + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{\sqrt{\frac{\#1^2 a-2+2c_1}{-1+c_1}}\sqrt{\frac{\#1^2 a+2+2c_1}{1+c_1}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{a}{2c_1+2}}\#1\right),\frac{c_1+1}{c_1-1}\right)+(-1+c_1)E\right.}{\sqrt{\frac{a}{2+2c_1}}\sqrt{\#1^4a^2+4\#1^2ac_1-4+4c_1{}^2}} + c_2\right]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\sqrt{\frac{\#1^2 a-2+2c_1}{-1+c_1}}\sqrt{\frac{\#1^2 a+2+2c_1}{1+c_1}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{a}{2c_1+2}}\#1\right),\frac{c_1+1}{c_1-1}\right)+(-1+c_1)E\left(i\right.\right.}{\sqrt{\frac{a}{2+2c_1}}\sqrt{\#1^4a^2+4\#1^2ac_1-4+4c_1{}^2}} + c_2\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{\sqrt{\frac{\#1^2 a-2+2(-1)c_1}{-1-c_1}}\sqrt{\frac{\#1^2 a+2+2(-1)c_1}{1-c_1}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{a}{2(-1)c_1+2}}\#1\right),\frac{1-c_1}{-c_1-1}\right)\right.}{\sqrt{\frac{a}{2+2(-1)c_1}}\sqrt{\#1^4a^2+4\#1^2a(-c_1)-4+}} + c_2\right]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\sqrt{\frac{\#1^2 a-2+2(-1)c_1}{-1-c_1}}\sqrt{\frac{\#1^2 a+2+2(-1)c_1}{1-c_1}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{a}{2(-1)c_1+2}}\#1\right),\frac{1-c_1}{-c_1-1}\right)+\right.}{\sqrt{\frac{a}{2+2(-1)c_1}}\sqrt{\#1^4a^2+4\#1^2a(-c_1)-4+4}} + c_2\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{\sqrt{\frac{\#1^2 a-2+2c_1}{-1+c_1}}\sqrt{\frac{\#1^2 a+2+2c_1}{1+c_1}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{a}{2c_1+2}}\#1\right),\frac{c_1+1}{c_1-1}\right)+(-1+c_1)E\right.}{\sqrt{\frac{a}{2+2c_1}}\sqrt{\#1^4a^2+4\#1^2ac_1-4+4c_1{}^2}} + c_2\right]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\sqrt{\frac{\#1^2 a-2+2c_1}{-1+c_1}}\sqrt{\frac{\#1^2 a+2+2c_1}{1+c_1}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{a}{2c_1+2}}\#1\right),\frac{c_1+1}{c_1-1}\right)+(-1+c_1)E\left(i\right.\right.}{\sqrt{\frac{a}{2+2c_1}}\sqrt{\#1^4a^2+4\#1^2ac_1-4+4c_1{}^2}} + c_2\right]$$

7.66 problem 1657 (book 6.66)

Internal problem ID [9979]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1657 (book 6.66).
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _reducibl`

$$y'' - 2a(c + bx + y)\left(y'^2 + 1\right)^{\frac{3}{2}} = 0$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 797

```
dsolve(diff(diff(y(x),x),x)=2*a*(c+b*x+y(x))*(diff(y(x),x)^2+1)^(3/2),y(x), singsol=all)
```

$$y(x) = -ix + c_1$$
$$y(x) = ix + c_1$$
$$y(x) = -bx + \text{RootOf}\Bigg(c_2b^3 - b^3x + c_2b - bx$$
$$+ \int^{\_Z} \frac{4b^2a^2\_f^2c^2 + 4c\,b^2a^2\_f^3 + a^2b^2\_f^4 - 8b^2c_1a^2c\_f - 4c_1a^2b^2\_f^2 + 4c_1^2a^2b^2 - 2\sqrt{-b^2\left(a^2\_f^4 + 4\_f\right.}}{}$$
$$y(x) = -bx + \text{RootOf}\Bigg(c_2b^3 - b^3x + c_2b - bx - \Bigg(\int^{\_Z}$$
$$-\frac{4b^2a^2\_f^2c^2 + 4c\,b^2a^2\_f^3 + a^2b^2\_f^4 - 8b^2c_1a^2c\_f - 4c_1a^2b^2\_f^2 + 4c_1^2a^2b^2 + 2\sqrt{-b^2\left(a^2\_f^4 + 4\_f^3a^2c\right.}}{}$$

✓ Solution by Mathematica

Time used: 85.168 (sec). Leaf size: 9706

```
DSolve[-(2*a*(c + b*x + y[x])*(1 + y'[x]^2)^(3/2)) + y''[x] == 0,y[x],x,IncludeSingularSolut
```

Too large to display

7.67 problem 1658 (book 6.67)

Internal problem ID [9980]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1658 (book 6.67).
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + y^3 y' - y y' \sqrt{y^4 + 4y'} = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 133

```
dsolve(diff(diff(y(x),x),x)+y(x)^3*diff(y(x),x)-y(x)*diff(y(x),x)*(y(x)^4+4*diff(y(x),x))^(1
```

$$y(x) = \frac{2^{\frac{2}{3}} \left((4c_1 + 3x)^2\right)^{\frac{1}{3}}}{4c_1 + 3x}$$
$$y(x) = -\frac{2^{\frac{2}{3}} \left((4c_1 + 3x)^2\right)^{\frac{1}{3}} \left(1 + i\sqrt{3}\right)}{8c_1 + 6x}$$
$$y(x) = \frac{2^{\frac{2}{3}} \left((4c_1 + 3x)^2\right)^{\frac{1}{3}} \left(i\sqrt{3} - 1\right)}{8c_1 + 6x}$$
$$y(x) = \tan\left(\frac{c_2 + x}{c_1^3}\right) \sqrt{\frac{1}{c_1^2}}$$
$$y(x) = \tanh\left(\frac{c_2 + x}{c_1^3}\right) \sqrt{\frac{1}{c_1^2}}$$

✓ Solution by Mathematica

Time used: 4.613 (sec). Leaf size: 38

```
DSolve[y[x]^3*y'[x] - y[x]*y'[x]*Sqrt[y[x]^4 + 4*y'[x]] + y''[x] == 0,y[x],x,IncludeSingular
```

$$y(x) \to \sqrt{2}e^{c_1} \tan\left(2\sqrt{2}e^{3c_1}(x + c_2)\right)$$
$$y(x) \to 0$$

7.68 problem 1659 (book 6.68)

Internal problem ID [9981]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1659 (book 6.68).
ODE order: 2.
ODE degree: 0.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - f(y', xa + yb) = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)-f(diff(y(x),x),a*x+b*y(x))=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-f[y'[x], a*x + b*y[x]] + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.69 problem 1660 (book 6.69)

Internal problem ID [9982]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1660 (book 6.69).
ODE order: 2.
ODE degree: 0.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - yf\left(x, \frac{y'}{y}\right) = 0}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)-y(x)*f(x,diff(y(x),x)/y(x))=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-(f[x, y'[x]/y[x]]*y[x]) + y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.70 problem 1661 (book 6.70)

Internal problem ID [9983]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1661 (book 6.70).
ODE order: 2.
ODE degree: 0.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - x^{-2+n} f\left(y x^{-n}, y' x^{1-n}\right) = 0}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)-x^(n-2)*f(y(x)/(x^n),diff(y(x),x)/(x^(n-1)))=0,y(x), singsol=all
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-(x^(-2 + n)*f[y[x]/x^n, x^(1 - n)*y'[x]]) + y''[x] == 0,y[x],x,IncludeSingularSoluti
```

Not solved

7.71 problem 1662 (book 6.71)

Internal problem ID [9984]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1662 (book 6.71).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{8y'' + 9y'^4 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 57

```
dsolve(8*diff(diff(y(x),x),x)+9*diff(y(x),x)^4=0,y(x), singsol=all)
```

$$y(x) = (c_1 + x)^{\frac{2}{3}} + c_2$$
$$y(x) = -\frac{i(c_1 + x)^{\frac{2}{3}}\sqrt{3}}{2} - \frac{(c_1 + x)^{\frac{2}{3}}}{2} + c_2$$
$$y(x) = \frac{i(c_1 + x)^{\frac{2}{3}}\sqrt{3}}{2} - \frac{(c_1 + x)^{\frac{2}{3}}}{2} + c_2$$

✓ Solution by Mathematica

Time used: 0.373 (sec). Leaf size: 90

```
DSolve[9*y'[x]^4 + 8*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{1}{3}\sqrt[3]{-\frac{1}{3}}(9x - 8c_1)^{2/3}$$
$$y(x) \to \frac{(9x - 8c_1)^{2/3}}{3\sqrt[3]{3}} + c_2$$
$$y(x) \to \frac{1}{9}\left((-3)^{2/3}(9x - 8c_1)^{2/3} + 9c_2\right)$$

7.72 problem 1663 (book 6.72)

Internal problem ID [9985]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1663 (book 6.72).
ODE order: 2.
ODE degree: 0.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{ay'' + h(y') + yc = 0}$$

✗ Solution by Maple

```
dsolve(a*diff(diff(y(x),x),x)+h(diff(y(x),x))+c*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[h[y'[x]] + c*y[x] + a*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.73 problem 1664 (book 6.73)

Internal problem ID [9986]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1664 (book 6.73).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x + 2y' - xy^n = 0}$$

✗ Solution by Maple

```
dsolve(x*diff(diff(y(x),x),x)+2*diff(y(x),x)-x*y(x)^n=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-(x*y[x]^n) + 2*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.74 problem 1665 (book 6.74)

Internal problem ID [9987]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1665 (book 6.74).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _with_linear_symmetries]]`

$$y''x + 2y' + a\,x^v y^n = 0$$

✗ Solution by Maple

```
dsolve(x*diff(diff(y(x),x),x)+2*diff(y(x),x)+a*x^v*y(x)^n=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[a*x^v*y[x]^n + 2*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.75 problem 1666 (book 6.75)

Internal problem ID [9988]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1666 (book 6.75).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' x + 2y' + x\,\mathrm{e}^{y} = 0}$$

✗ Solution by Maple

```
dsolve(x*diff(diff(y(x),x),x)+2*diff(y(x),x)+x*exp(y(x))=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[Exp[y[x]]*x + 2*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.76 problem 1667 (book 6.76)

Internal problem ID [9989]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1667 (book 6.76).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y''x + ay' + bx\,\mathrm{e}^{y} = 0$$

✗ Solution by Maple

```
dsolve(x*diff(diff(y(x),x),x)+a*diff(y(x),x)+b*x*exp(y(x))=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[b*E^y[x]*x + a*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.77 problem 1668 (book 6.77)

Internal problem ID [9990]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1668 (book 6.77).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y''x + ay' + b\,x^{-2a+5}e^{y} = 0$$

✗ Solution by Maple

```
dsolve(x*diff(diff(y(x),x),x)+a*diff(y(x),x)+b*x^(5-2*a)*exp(y(x))=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[b*E^y[x]*x^(5 - 2*a) + a*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

Not solved

7.78 problem 1669 (book 6.78)

Internal problem ID [9991]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1669 (book 6.78).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _nonlinear], [_2nd_order, _with_linear_s`

$$\boxed{y''x + (y-1)\,y' = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 24

```
dsolve(x*diff(diff(y(x),x),x)+(y(x)-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{2c_1 + \tanh\left(\frac{\ln(x)-c_2}{2c_1}\right)}{c_1}$$

✓ Solution by Mathematica
Time used: 60.069 (sec). Leaf size: 46

```
DSolve[(-1 + y[x])*y'[x] + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2 - \sqrt{2}\sqrt{2+c_1}\tanh\left(\frac{\sqrt{2+c_1}(-\log(x)+2c_2)}{\sqrt{2}}\right)$$

7.79 problem 1670 (book 6.79)

Internal problem ID [9992]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1670 (book 6.79).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _reducibl`

$$\boxed{y''x - x^2{y'}^2 + 2y' + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 32

```
dsolve(x*diff(diff(y(x),x),x)-x^2*diff(y(x),x)^2+2*diff(y(x),x)+y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{\operatorname{RootOf}\left(-\ln(x) + c_2 - \left(\int^{\_Z} \frac{1}{-2\_f - 1 + e^{\_f} c_1} d\_f\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.8 (sec). Leaf size: 160

```
DSolve[y[x]^2 + 2*y'[x] - x^2*y'[x]^2 + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

$$\text{Solve}\left[\int_1^{y(x)} -\frac{x}{e^{xK[1]}c_1 + 2xK[1] + 1} dK[1] - \int_1^x \left(\int_1^{y(x)} \left(\frac{\left(e^{K[1]K[2]}c_1 K[1] + 2K[1]\right)K[2]}{\left(e^{K[1]K[2]}c_1 + 2K[1]K[2] + 1\right)^2} - \frac{1}{e^{K[1]K[2]}c_1 + 2K[1]K[2] + 1}\right) dK[1] - \frac{e^{K[2]y(x)}c_1 + K[2]y(x) + 1}{K[2]\left(e^{K[2]y(x)}c_1 + 2K[2]y(x) + 1\right)}\right) dK[2] = c_2, y(x)\right]$$

7.80 problem 1671 (book 6.80)

Internal problem ID [9993]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1671 (book 6.80).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x + a(y'x - y)^2 = b}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 40

```
dsolve(x*diff(diff(y(x),x),x)+a*(x*diff(y(x),x)-y(x))^2-b=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(i\sqrt{b}\left(\int \frac{\tan\left(-i\sqrt{a}\,\sqrt{b}\,x+c_1\right)}{x^2}dx\right) + c_2\sqrt{a}\right)x}{\sqrt{a}}$$

✓ Solution by Mathematica

Time used: 128.353 (sec). Leaf size: 50

```
DSolve[-b + a*(-y[x] + x*y'[x])^2 + x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(\int_1^x \frac{\sqrt{-\frac{b}{a}}\tan\left(c_1 + \frac{bK[2]}{\sqrt{-\frac{b}{a}}}\right)}{K[2]^2}dK[2] + c_2\right)$$

7.81 problem 1672 (book 6.81)

Internal problem ID [9994]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1672 (book 6.81).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$\boxed{2y''x + {y'}^3 + y' = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 41

```
dsolve(2*x*diff(diff(y(x),x),x)+diff(y(x),x)^3+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2 c_1 + 2\sqrt{c_1 x - 1}}{c_1}$$
$$y(x) = \frac{c_2 c_1 - 2\sqrt{c_1 x - 1}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.896 (sec). Leaf size: 65

```
DSolve[y'[x] + y'[x]^3 + 2*x*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_2 - 2ie^{c_1}\sqrt{-x + e^{2c_1}}$$
$$y(x) \rightarrow 2ie^{c_1}\sqrt{-x + e^{2c_1}} + c_2$$
$$y(x) \rightarrow c_2$$

7.82 problem 1673 (book 6.82)

Internal problem ID [9995]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1673 (book 6.82).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' - a(y^n - y) = 0}$$

✗ Solution by Maple

```
dsolve(x^2*diff(diff(y(x),x),x)=a*(y(x)^n-y(x)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-(a*(-y[x] + y[x]^n)) + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.83 problem 1674 (book 6.83)

Internal problem ID [9996]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1674 (book 6.83).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' + a(e^y - 1) = 0}$$

✗ Solution by Maple

```
dsolve(x^2*diff(diff(y(x),x),x)+a*(exp(y(x))-1)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[a*(-1 + E^y[x]) + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.84 problem 1675 (book 6.84)

Internal problem ID [9997]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1675 (book 6.84).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - (2a + b - 1)\,xy' + \left(c^2b^2x^{2b} + a(a + b)\right) y = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 25

```
dsolve(x^2*diff(diff(y(x),x),x)-(2*a+b-1)*x*diff(y(x),x)+(c^2*b^2*x^(2*b)+a*(a+b))*y(x)=0,y(
```

$$y(x) = x^a\left(c_1 \sin\left(x^b c\right) + c_2 \cos\left(x^b c\right)\right)$$

✓ Solution by Mathematica
Time used: 0.156 (sec). Leaf size: 69

```
DSolve[(a*(a + b) + b^2*c^2*x^(2*b))*y[x] - (-1 + 2*a + b)*x*y'[x] + x^2*y''[x] == 0,y[x],x,
```

$$y(x) \to 2^{-\frac{a+b}{b}} c^{a/b} \left(x^{2b}\right)^{\frac{a}{2b}} \left(2c_1 \cos\left(c\sqrt{x^{2b}}\right) + c_2 \sin\left(c\sqrt{x^{2b}}\right)\right)$$

7.85 problem 1676 (book 6.85)

Internal problem ID [9998]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1676 (book 6.85).
ODE order: 2.
ODE degree: 0.

CAS Maple gives this as type `[NONE]`

$$x^2y'' + (1 + a)\,xy' - x^k f\left(x^k y, y'x + ky\right) = 0$$

✗ Solution by Maple

```
dsolve(x^2*diff(diff(y(x),x),x)+(a+1)*x*diff(y(x),x)-x^k*f(x^k*y(x),x*diff(y(x),x)+k*y(x))=0
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-(x^k*f[x^k*y[x], k*y[x] + x*y'[x]]) + (1 + a)*x*y'[x] + x^2*y''[x] == 0,y[x],x,Inclu
```

Not solved

7.86 problem 1677 (book 6.86)

Internal problem ID [9999]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1677 (book 6.86).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + a(y'x - y)^2 = b\,x^2$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 75

```
dsolve(x^2*diff(diff(y(x),x),x)+a*(x*diff(y(x),x)-y(x))^2-b*x^2=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(-\sqrt{-ab}\left(\int \frac{\text{BesselY}(1,\sqrt{-ab}\,x)c_1+\text{BesselJ}(1,\sqrt{-ab}\,x)}{x(c_1\,\text{BesselY}(0,\sqrt{-ab}\,x)+\text{BesselJ}(0,\sqrt{-ab}\,x))}dx\right)+c_2a\right)x}{a}$$

✓ Solution by Mathematica

Time used: 120.409 (sec). Leaf size: 118

```
DSolve[-(b*x^2) + a*(-y[x] + x*y'[x])^2 + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to x\left(\int_1^x \frac{i\sqrt{b}\left(\text{BesselY}\left(1,-i\sqrt{a}\sqrt{b}K[1]\right)-\text{BesselJ}\left(1,i\sqrt{a}\sqrt{b}K[1]\right)c_1\right)}{\sqrt{a}\left(\text{BesselY}\left(0,-i\sqrt{a}\sqrt{b}K[1]\right)+\text{BesselJ}\left(0,i\sqrt{a}\sqrt{b}K[1]\right)c_1\right)K[1]}dK[1] + c_2\right)$$

7.87 problem 1678 (book 6.87)

Internal problem ID [10000]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1678 (book 6.87).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + ayy'^2 = -bx$$

✗ Solution by Maple

```
dsolve(x^2*diff(diff(y(x),x),x)+a*y(x)*diff(y(x),x)^2+b*x=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[b*x + a*y[x]*y'[x]^2 + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.88 problem 1679 (book 6.88)

Internal problem ID [10001]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1679 (book 6.88).
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - \sqrt{y'^2 a\, x^2 + by^2} = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 64

```
dsolve(x^2*diff(diff(y(x),x),x)-(a*x^2*diff(y(x),x)^2+y(x)^2*b)^(1/2)=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) - e^{\int^{\ln(x)} \mathrm{RootOf}\left(-y(x)\left(\int^{\_Z} \frac{1}{\_a^2 y(x) - \_a y(x) - \sqrt{y(x)^2 (a\_a^2+b)}} d\_a\right) - \_b + c_1\right) d\_b + c_2} = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-Sqrt[b*y[x]^2 + a*x^2*y'[x]^2] + x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

Not solved

7.89 problem 1680 (book 6.89)

Internal problem ID [10002]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1680 (book 6.89).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$\boxed{(x^2+1)\, y''+{y'}^2=-1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve((x^2+1)*diff(diff(y(x),x),x)+diff(y(x),x)^2+1=0,y(x), singsol=all)
```

$$y(x)=\frac{\ln (c_1x-1)\, c_1^2+c_2c_1^2+c_1x+\ln (c_1x-1)}{c_1^2}$$

✓ Solution by Mathematica

Time used: 11.847 (sec). Leaf size: 33

```
DSolve[1 + y'[x]^2 + (1 + x^2)*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -x\cot(c_1)+\csc^2(c_1)\log(-x\sin(c_1)-\cos(c_1))+c_2$$

7.90 problem 1681 (book 6.90)

Internal problem ID [10003]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1681 (book 6.90).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' - x^4{y'}^2 + 4y = 0$$

✗ Solution by Maple

```
dsolve(4*x^2*diff(diff(y(x),x),x)-x^4*diff(y(x),x)^2+4*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[4*y[x] - x^4*y'[x]^2 + 4*x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.91 problem 1682 (book 6.91)

Internal problem ID [10004]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1682 (book 6.91).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9x^2y'' + ay^3 + 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
dsolve(9*x^2*diff(diff(y(x),x),x)+a*y(x)^3+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_2 \operatorname{JacobiSN}\left(\frac{\left(2c_1x^3 + \sqrt{2}\,\sqrt{x^{\frac{20}{3}}a}\right)c_2}{2x^3}, i\right)x^{\frac{1}{3}}$$

✓ Solution by Mathematica

Time used: 5.017 (sec). Leaf size: 41

```
DSolve[2*y[x] + a*y[x]^3 + 9*x^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2\sqrt[3]{x}\operatorname{sn}\left(\left(c_1 + \frac{\sqrt{ax^{20/3}}}{\sqrt{2}x^3}\right)c_2\middle| -1\right)$$

7.92 problem 1683 (book 6.92)

Internal problem ID [10005]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1683 (book 6.92).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^3(y'' + y'y - y^3) + 12xy = -24}$$

✗ Solution by Maple

```
dsolve(x^3*(diff(diff(y(x),x),x)+y(x)*diff(y(x),x)-y(x)^3)+12*x*y(x)+24=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[24 + 12*x*y[x] + x^3*(-y[x]^3 + y[x]*y'[x] + y''[x]) == 0,y[x],x,IncludeSingularSolut
```

Not solved

7.93 problem 1684 (book 6.93)

Internal problem ID [10006]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1684 (book 6.93).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _reducibl`

$$\boxed{x^3 y'' - a(y'x - y)^2 = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve(x^3*diff(diff(y(x),x),x)-a*(x*diff(y(x),x)-y(x))^2=0,y(x), singsol=all)
```

$$y(x) = -\frac{\ln\left(\frac{a(c_1 x - c_2)}{x}\right) x}{a}$$

✓ Solution by Mathematica
Time used: 4.57 (sec). Leaf size: 25

```
DSolve[-(a*(-y[x] + x*y'[x])^2) + x^3*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x \log\left(-\frac{a(c_2 x + c_1)}{x}\right)}{a}$$

7.94 problem 1685 (book 6.94)

Internal problem ID [10007]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1685 (book 6.94).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^3y'' + x^2(9 + 2xy)\,y' + xy\left(a + 3xy - 2x^2y^2\right) = -b$$

✗ Solution by Maple

```
dsolve(2*x^3*diff(diff(y(x),x),x)+x^2*(9+2*x*y(x))*diff(y(x),x)+b+x*y(x)*(a+3*x*y(x)-2*x^2*y
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[b + x*y[x]*(a + 3*x*y[x] - 2*x^2*y[x]^2) + x^2*(9 + 2*x*y[x])*y'[x] + 2*x^3*y''[x] ==
```

Not solved

7.95 problem 1686 (book 6.95)

Internal problem ID [10008]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1686 (book 6.95).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$2\left(-x^k+4x^3\right)\left(y''+y'y-y^3\right)-\left(k\,x^{k-1}-12x^2\right)\left(3y'+y^2\right)+yax=-b$$

✗ Solution by Maple

```
dsolve(2*(-x^k+4*x^3)*(diff(diff(y(x),x),x)+y(x)*diff(y(x),x)-y(x)^3)-(k*x^(k-1)-12*x^2)*(3*
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[b + a*x*y[x] - (-12*x^2 + k*x^(-1 + k))*(y[x]^2 + 3*y'[x]) + 2*(4*x^3 - x^k)*(-y[x]^3
```

Not solved

7.96 problem 1687 (book 6.96)

Internal problem ID [10009]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1687 (book 6.96).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x^4 + a^2y^n = 0}$$

✗ Solution by Maple

```
dsolve(x^4*diff(diff(y(x),x),x)+a^2*y(x)^n=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[a^2*y[x]^n + x^4*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.97 problem 1688 (book 6.97)

Internal problem ID [10010]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1688 (book 6.97).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _reducibl`

$$y''x^4 - x\left(x^2 + 2y\right)y' + 4y^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(x^4*diff(diff(y(x),x),x)-x*(x^2+2*y(x))*diff(y(x),x)+4*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \left(-\tanh\left(c_1(\ln(x) - c_2)\right)c_1 + 1\right)x^2$$

✓ Solution by Mathematica

Time used: 79.662 (sec). Leaf size: 83

```
DSolve[4*y[x]^2 - x*(x^2 + 2*y[x])*y'[x] + x^4*y''[x] == 0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{x^2\left(\left(1 - i\sqrt{-1 - c_1}\right)x^{2i\sqrt{-1-c_1}} + \left(1 + i\sqrt{-1 - c_1}\right)c_2\right)}{x^{2i\sqrt{-1-c_1}} + c_2}$$

7.98 problem 1689 (book 6.98)

Internal problem ID [10011]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1689 (book 6.98).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _reducibl`

$$y''x^4 - x^2(x + y')\,y' + 4y^2 = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 32

```
dsolve(x^4*diff(diff(y(x),x),x)-x^2*(x+diff(y(x),x))*diff(y(x),x)+4*y(x)^2=0,y(x), singsol=a
```

$$y(x) = \mathrm{RootOf}\left(-\ln(x) + c_2 - \left(\int^{\_Z} \frac{1}{\mathrm{e}^{\_f} c_1 + 4\_f + 2} d\_f\right)\right) x^2$$

✓ Solution by Mathematica
Time used: 1.205 (sec). Leaf size: 189

```
DSolve[4*y[x]^2 - x^2*y'[x]*(x + y'[x]) + x^4*y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

$$\mathrm{Solve}\left[\int_1^{y(x)} \frac{1}{-e^{\frac{K[1]}{x^2}} c_1 x^2 + 2x^2 + 4K[1]} dK[1] - \int_1^x \left(\frac{K[2]\left(e^{\frac{y(x)}{K[2]^2}} c_1 + 2\left(-\frac{y(x)}{K[2]^2} - 1\right)\right)}{-e^{\frac{y(x)}{K[2]^2}} c_1 K[2]^2 + 2K[2]^2 + 4y(x)} + \int_1^{y(x)} -\frac{\frac{2e^{\frac{K[1]}{K[2]^2}} c_1 K[1]}{K[2]} - 2e^{\frac{K[1]}{K[2]^2}} c_1 K[2] + 4K[2]}{\left(-e^{\frac{K[1]}{K[2]^2}} c_1 K[2]^2 + 2K[2]^2 + 4K[1]\right)^2} dK[1]\right) dK[2] = c_2, y(x)\right]$$

7.99 problem 1690 (book 6.99)

Internal problem ID [10012]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1690 (book 6.99).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y''x^4 + (y'x - y)^3 = 0$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 37

```
dsolve(x^4*diff(diff(y(x),x),x)+(x*diff(y(x),x)-y(x))^3=0,y(x), singsol=all)
```

$$y(x) = \left(-\arctan\left(\frac{1}{\sqrt{c_1 x^2 - 1}}\right) + c_2\right) x$$
$$y(x) = \left(\arctan\left(\frac{1}{\sqrt{c_1 x^2 - 1}}\right) + c_2\right) x$$

✓ Solution by Mathematica
Time used: 60.327 (sec). Leaf size: 95

```
DSolve[(-y[x] + x*y'[x])^3 + x^4*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -ix \log\left(\frac{e^{c_2} - \sqrt{e^{2c_2} - 8ic_1 x^2}}{4c_1 x}\right)$$
$$y(x) \to -ix \log\left(\frac{\sqrt{e^{2c_2} - 8ic_1 x^2} + e^{c_2}}{4c_1 x}\right)$$

7.100 problem 1691 (book 6.100)

Internal problem ID [10013]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1691 (book 6.100).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''\sqrt{x} - y^{\frac{3}{2}} = 0}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)*x^(1/2)-y(x)^(3/2)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-y[x]^(3/2) + Sqrt[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.101 problem 1692 (book 6.101)

Internal problem ID [10014]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1692 (book 6.101).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$
\boxed{\left(a\,x^2+bx+c\right)^{\frac{3}{2}} y''-F\left(\frac{y}{\sqrt{a\,x^2+bx+c}}\right)=0}
$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 252

```
dsolve((a*x^2+b*x+c)^(3/2)*diff(diff(y(x),x),x)-F(y(x)/(a*x^2+b*x+c)^(1/2))=0,y(x), singsol=
```

$$y(x) = \operatorname{RootOf}\left(4\_Zac - \_Zb^2 - 4F\left(\frac{\_Z}{\sqrt{a\,x^2+bx+c}}\right)\sqrt{a\,x^2+bx+c}\right)$$

$$y(x) = \operatorname{RootOf}\left(-2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)a - 2a\left(\int^{\_Z}\frac{1}{\sqrt{4c_1a^2-4c\_g^2a+b^2\_g^2+8\left(\int F(\_g)\,d\_g\right)}}d\_g\right)\sqrt{4ac-b^2} + c_2\sqrt{4ac-b^2}\right)\sqrt{a\,x^2+bx+c}$$

$$y(x) = \operatorname{RootOf}\left(-2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)a + 2a\left(\int^{\_Z}\frac{1}{\sqrt{4c_1a^2-4c\_g^2a+b^2\_g^2+8\left(\int F(\_g)\,d\_g\right)}}d\_g\right)\sqrt{4ac-b^2} + c_2\sqrt{4ac-b^2}\right)\sqrt{a\,x^2+bx+c}$$

✓ Solution by Mathematica

Time used: 55.307 (sec). Leaf size: 251

```
DSolve[-f[y[x]/Sqrt[c + b*x + a*x^2]] + (c + b*x + a*x^2)^(3/2)*y''[x] == 0,y[x],x,IncludeSi
```

$$\text{Solve}\left[2a\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)+2\sqrt{4ac-b^2}\int_1^{\frac{y(x)}{\sqrt{c+x(b+ax)}}}\frac{a}{\sqrt{4c_1a^2+(b^2-4ac)\,K[3]^2+8\int_1^{K[3]}f(K[2])dK[2]}}dK[3]=c_2\sqrt{4ac-b^2},y(x)\right]$$

$$\text{Solve}\left[2a\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)-2\sqrt{4ac-b^2}\int_1^{\frac{y(x)}{\sqrt{c+x(b+ax)}}}\frac{a}{\sqrt{4c_1a^2+(b^2-4ac)\,K[5]^2+8\int_1^{K[5]}f(K[4])dK[4]}}dK[5]=c_2\sqrt{4ac-b^2},y(x)\right]$$

7.102 problem 1693 (book 6.102)

Internal problem ID [10015]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1693 (book 6.102).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^{\frac{n}{n+1}}y'' - y^{\frac{1+2n}{n+1}} = 0}$$

✗ Solution by Maple

```
dsolve(x^(n/(n+1))*diff(diff(y(x),x),x)-y(x)^((2*n+1)/(n+1))=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-y[x]^((1 + 2*n)/(1 + n)) + x^(n/(1 + n))*y''[x] == 0,y[x],x,IncludeSingularSolutions
```

Not solved

7.103 problem 1694 (book 6.103)

Internal problem ID [10016]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1694 (book 6.103).
ODE order: 2.
ODE degree: 0.

CAS Maple gives this as type `[NONE]`

$$f(x)^2 y'' + f(x) f'(x) y' - h(y, f(x) y') = 0$$

✗ Solution by Maple

```
dsolve(f(x)^2*diff(diff(y(x),x),x)+f(x)*diff(f(x),x)*diff(y(x),x)-h(y(x),f(x)*diff(y(x),x))=
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-h[y[x], f[x]*y'[x]] + f[x]*Derivative[1][f][x]*y'[x] + f[x]^2*y''[x] == 0,y[x],x,Inc
```

Not solved

7.104 problem 1695 (book 6.104)

Internal problem ID [10017]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1695 (book 6.104).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y''y = a}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 53

```
dsolve(diff(diff(y(x),x),x)*y(x)-a=0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{\sqrt{2a\ln(\_a) - c_1}} d\_a - x - c_2 = 0$$
$$-\left(\int^{y(x)} \frac{1}{\sqrt{2a\ln(\_a) - c_1}} d\_a\right) - x - c_2 = 0$$

✓ Solution by Mathematica
Time used: 60.218 (sec). Leaf size: 111

```
DSolve[-a + y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \exp\left(-\frac{2a\operatorname{erf}^{-1}\left(-i\sqrt{\frac{2}{\pi}}\sqrt{ae^{\frac{c_1}{a}}(x+c_2)^2}\right)^2 + c_1}{2a}\right)$$
$$y(x) \to \exp\left(-\frac{2a\operatorname{erf}^{-1}\left(i\sqrt{\frac{2}{\pi}}\sqrt{ae^{\frac{c_1}{a}}(x+c_2)^2}\right)^2 + c_1}{2a}\right)$$

7.105 problem 1696 (book 6.105)

Internal problem ID [10018]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1696 (book 6.105).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''y = xa}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)*y(x)-a*x=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-(a*x) + y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.106 problem 1697 (book 6.106)

Internal problem ID [10019]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1697 (book 6.106).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''y = a\,x^2}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)*y(x)-a*x^2=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-(a*x^2) + y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.107 problem 1698 (book 6.107)

Internal problem ID [10020]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1698 (book 6.107).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], [`

$$\boxed{y''y + y'^2 = a}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 39

```
dsolve(diff(diff(y(x),x),x)*y(x)+diff(y(x),x)^2-a=0,y(x), singsol=all)
```

$$y(x) = \sqrt{a\,x^2 - 2c_1x + 2c_2}$$
$$y(x) = -\sqrt{a\,x^2 - 2c_1x + 2c_2}$$

✓ Solution by Mathematica
Time used: 22.188 (sec). Leaf size: 117

```
DSolve[-a + y'[x]^2 + y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{a^2(x+c_2)^2 - e^{2c_1}}}{\sqrt{a}}$$
$$y(x) \to \frac{\sqrt{a^2(x+c_2)^2 - e^{2c_1}}}{\sqrt{a}}$$
$$y(x) \to -\frac{\sqrt{a^2(x+c_2)^2}}{\sqrt{a}}$$
$$y(x) \to \frac{\sqrt{a^2(x+c_2)^2}}{\sqrt{a}}$$

7.108 problem 1699 (book 6.108)

Internal problem ID [10021]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1699 (book 6.108).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$\boxed{y''y + y^2 = xa + b}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)*y(x)+y(x)^2-a*x-b=0,y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 0.727 (sec). Leaf size: 63

```
DSolve[-b - a*x + y'[x]^2 + y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\frac{ax^3}{3} + bx^2 + c_2x + 2c_1}$$
$$y(x) \to \sqrt{\frac{ax^3}{3} + bx^2 + c_2x + 2c_1}$$

7.109 problem 1700 (book 6.109)

Internal problem ID [10022]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1700 (book 6.109).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], [`

$$\boxed{y''y + y'^2 - y' = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 34

```
dsolve(diff(diff(y(x),x),x)*y(x)+diff(y(x),x)^2-diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = -c_1 \left(\text{LambertW}\left(-\frac{\mathrm{e}^{\frac{-c_1-c_2-x}{c_1}}}{c_1}\right) + 1\right)$$

✓ Solution by Mathematica

Time used: 60.142 (sec). Leaf size: 32

```
DSolve[-y'[x] + y'[x]^2 + y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -c_1 \left(1 + W\left(-\frac{e^{-\frac{x+c_1+c_2}{c_1}}}{c_1}\right)\right)$$

7.110 problem 1701 (book 6.110)

Internal problem ID [10023]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1701 (book 6.110).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y''y - y'^2 = -1}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 59

```
dsolve(diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+1=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \left(-e^{\frac{c_2+x}{c_1}} + e^{\frac{-c_2-x}{c_1}}\right)}{2}$$
$$y(x) = -\frac{c_1 \left(-e^{\frac{c_2+x}{c_1}} + e^{\frac{-c_2-x}{c_1}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 60.331 (sec). Leaf size: 85

```
DSolve[y''[x]*y[x]-y'[x]^2+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{ie^{-c_1} \tanh\left(e^{c_1}(x+c_2)\right)}{\sqrt{-\operatorname{sech}^2\left(e^{c_1}(x+c_2)\right)}}$$
$$y(x) \to \frac{ie^{-c_1} \tanh\left(e^{c_1}(x+c_2)\right)}{\sqrt{-\operatorname{sech}^2\left(e^{c_1}(x+c_2)\right)}}$$

7.111 problem 1702 (book 6.111)

Internal problem ID [10024]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1702 (book 6.111).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$y''y - y'^2 = 1$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 55

```
dsolve(diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2-1=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1\left(e^{\frac{c_2+x}{c_1}} + e^{\frac{-c_2-x}{c_1}}\right)}{2}$$
$$y(x) = \frac{c_1\left(e^{\frac{c_2+x}{c_1}} + e^{\frac{-c_2-x}{c_1}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.197 (sec). Leaf size: 80

```
DSolve[-1 - y'[x]^2 + y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{-c_1} \tanh\left(e^{c_1}(x + c_2)\right)}{\sqrt{-\operatorname{sech}^2\left(e^{c_1}(x + c_2)\right)}}$$
$$y(x) \to \frac{e^{-c_1} \tanh\left(e^{c_1}(x + c_2)\right)}{\sqrt{-\operatorname{sech}^2\left(e^{c_1}(x + c_2)\right)}}$$

7.112 problem 1703 (book 6.112)

Internal problem ID [10025]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1703 (book 6.112).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y''y - y'^2 + e^x y(cy^2 + d) + e^{2x}(b + ay^4) = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+exp(x)*y(x)*(c*y(x)^2+d)+exp(2*x)*(b+a*y(x)^
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[E^x*y[x]*(d + c*y[x]^2) + E^(2*x)*(b + a*y[x]^4) - y'[x]^2 + y[x]*y''[x] == 0,y[x],x,
```

Not solved

7.113 problem 1704 (book 6.113)

Internal problem ID [10026]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1704 (book 6.113).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{y''y - y'^2 - y^2 \ln(y) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2-y(x)^2*ln(y(x))=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{c_1 \mathrm{e}^{x}}{2}+\frac{c_2 \mathrm{e}^{-x}}{2}}$$

✓ Solution by Mathematica

Time used: 4.551 (sec). Leaf size: 73

```
DSolve[-(Log[y[x]]*y[x]^2) - y'[x]^2 + y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \exp\left(-\frac{1}{2}\sqrt{c_1}e^{-x-c_2}\left(-1+e^{2(x+c_2)}\right)\right)$$
$$y(x) \to \exp\left(\frac{1}{2}\sqrt{c_1}e^{-x-c_2}\left(-1+e^{2(x+c_2)}\right)\right)$$

7.114 problem 1704 (book 6.114)

Internal problem ID [10027]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1704 (book 6.114).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y''y - y'^2 - y' + f(x)\, y^3 + y^2 \left(\frac{f''(x)}{f(x)} - \frac{f'(x)^2}{f(x)^2} \right) = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2-diff(y(x),x)+f(x)*y(x)^3+y(x)^2*(diff(diff(f
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[f[x]*y[x]^3 - y'[x] - y'[x]^2 + y[x]^2*(-(Derivative[1][f][x]^2/f[x]^2) + Derivative[
```

Not solved

7.115 problem 1706 (book 6.115)

Internal problem ID [10028]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1706 (book 6.115).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y''y - y'^2 + f(x)\,y' - f'(x)\,y - y^3 = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+f(x)*diff(y(x),x)-diff(f(x),x)*y(x)-y(x)^3=0
```

No solution found

✓ Solution by Mathematica

Time used: 60.429 (sec). Leaf size: 192

```
DSolve[-y[x]^3 - y[x]*Derivative[1][f][x] + f[x]*y'[x] - y'[x]^2 + y[x]*y''[x] == 0,y[x],x,I
```

$$y(x) \to -\frac{\exp\left(c_2 - \int_1^x \frac{y(K[3])^3 + \left(c_1 + \int_1^{K[3]} -\frac{y(K[1])^3 + f'(K[1])y(K[1]) - f(K[1])y'(K[1])}{y(K[1])^2} dK[1]\right)^2 y(K[3])^2 + f'(K[3])y(K[3]) - f(K[3])y'(K[3])}{y(K[3])^2 \left(c_1 + \int_1^{K[3]} -\frac{y(K[1])^3 + f'(K[1])y(K[1]) - f(K[1])y'(K[1])}{y(K[1])^2} dK[1]\right)} d\right.}{\int_1^x -\frac{y(K[1])^3 + f'(K[1])y(K[1]) - f(K[1])y'(K[1])}{y(K[1])^2} dK[1] + c_1}$$

7.116 problem 1707 (book 6.116)

Internal problem ID [10029]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1707 (book 6.116).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y''y - y'^2 + f'(x)\, y' - f''(x)\, y + f(x)\, y^3 - y^4 = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+diff(f(x),x)*diff(y(x),x)-diff(diff(f(x),x),
```

No solution found

✓ Solution by Mathematica

Time used: 60.651 (sec). Leaf size: 240

```
DSolve[f[x]*y[x]^3 - y[x]^4 + Derivative[1][f][x]*y'[x] - y'[x]^2 - y[x]*Derivative[2][f][x]
```

$$y(x) \to -\frac{\exp\left(c_2 - \int_1^x \frac{y(K[3])^4 - f(K[3])y(K[3])^3 + \left(c_1 + \int_1^{K[3]} \frac{-y(K[1])^4 + f(K[1])y(K[1])^3 - f''(K[1])y(K[1]) + f'(K[1])y'(K[1])}{y(K[1])^2} dK[1]\right)^2 y(K[3])^2 +}{y(K[3])^2 \left(c_1 + \int_1^{K[3]} \frac{-y(K[1])^4 + f(K[1])y(K[1])^3 - f''(K[1])y(K[1]) + f'(K[1])y'(K[1])}{y(K[1])^2} dK[1]\right.}\right.}{\int_1^x \frac{-y(K[1])^4 + f(K[1])y(K[1])^3 - f''(K[1])y(K[1]) + f'(K[1])y'(K[1])}{y(K[1])^2} dK[1] + c_1}$$

7.117 problem 1708 (book 6.117)

Internal problem ID [10030]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1708 (book 6.117).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$y''y - y'^2 + ay'y + by^2 = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 33

```
dsolve(diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+a*y(x)*diff(y(x),x)+y(x)^2*b=0,y(x), singsol
```

$$y(x) = 0$$
$$y(x) = e^{\frac{e^{-ax} c_1 a + (-bx - c_2) a + b}{a^2}}$$

✓ Solution by Mathematica

Time used: 0.404 (sec). Leaf size: 28

```
DSolve[b*y[x]^2 + a*y[x]*y'[x] - y'[x]^2 + y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to c_2 e^{-\frac{bx + c_1 e^{-ax}}{a}}$$

7.118 problem 1709 (book 6.118)

Internal problem ID [10031]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1709 (book 6.118).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y''y - y'^2 + ay'y - 2ay^2 + by^3 = 0}$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+a*y(x)*diff(y(x),x)-2*a*y(x)^2+b*y(x)^3=0,y(
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-2*a*y[x]^2 + b*y[x]^3 + a*y[x]*y'[x] - y'[x]^2 + y[x]*y''[x] == 0,y[x],x,IncludeSing
```

Not solved

7.119 problem 1710 (book 6.119)

Internal problem ID [10032]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1710 (book 6.119).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y''y - y'^2 - (ay - 1)\, y' + 2a^2y^2 - 2b^2y^3 + ay = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2-(a*y(x)-1)*diff(y(x),x)+2*a^2*y(x)^2-2*b^2*y
```

No solution found

✓ Solution by Mathematica

Time used: 114.511 (sec). Leaf size: 540

```
DSolve[a*y[x] + 2*a^2*y[x]^2 - 2*b^2*y[x]^3 - (-1 + a*y[x])*y'[x] - y'[x]^2 + y[x]*y''[x] ==
```

$$y(x) \to -\frac{1}{2a} + e^{2ax}\left(\frac{e^{-2ax}\left(c_1\left(a^{3/2} - \sqrt{a^3+2b^2}\right) \text{Gamma}\left(1 - \frac{\sqrt{a^3+2b^2}}{2a^{3/2}}\right) \text{BesselJ}\left(-\frac{\sqrt{a^3+2b^2}}{2a^{3/2}}, \frac{\sqrt{ab^2e^{2ax}c_2}}{a^{3/2}}\right) - 2c_1 \text{Gamm}}{4ab^2\left(c_1 \text{Gan}\right.} + c_2\right)$$

7.120 problem 1711 (book 6.120)

Internal problem ID [10033]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1711 (book 6.120).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y''y - {y'}^2 + (ay - 1)\, y' - y(y + 1)\left(b^2y^2 - a^2\right) = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+(a*y(x)-1)*diff(y(x),x)-y(x)*(y(x)+1)*(b^2*y
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-(y[x]*(1 + y[x])*(-a^2 + b^2*y[x]^2)) + (-1 + a*y[x])*y'[x] - y'[x]^2 + y[x]*y''[x]
```

Not solved

7.121 problem 1712 (book 6.121)

Internal problem ID [10034]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1712 (book 6.121).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _reducible, _mu_xy]]`

$$y''y - y'^2 + (\tan(x) + \cot(x))\, yy' + \left(\cos(x)^2 - n^2 \cot(x)^2\right) y^2 \ln(y) = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 22

```
dsolve(diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+(tan(x)+cot(x))*y(x)*diff(y(x),x)+(cos(x)^2-
```

$$y(x) = e^{\frac{\pi(\text{BesselJ}(n,\sin(x))c_1 - c_2\, \text{BesselY}(n,\sin(x)))}{2}}$$

✓ Solution by Mathematica

Time used: 81.947 (sec). Leaf size: 858

```
DSolve[(Cos[x]^2 - n^2*Cot[x]^2)*Log[y[x]]*y[x]^2 + (Cot[x] + Tan[x])*y[x]*y'[x] - y'[x]^2 +
```

$y(x) \rightarrow$

$$-\frac{(-1)^{-n}2^{3n/2}e^{-(-1)^{-n}2^{-\frac{3n}{2}-4}\left(c_2 - \int_1^x - \frac{4\cot(K[3])y(K[3])\left(2^{3n+1}\sqrt{\cos(2K[3])-1}\left(2n^2+\cos(2K[3])-1\right)\csc(K[3])\log(y(K[3]))K_n(i\sin(K[3]))^2}{\cdots}\right.}}{\cdots}$$

7.122 problem 1713 (book 6.122)

Internal problem ID [10035]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1713 (book 6.122).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _reducibl`

$$y''y - y'^2 - f(x)\,y'y - g(x)\,y^2 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

```
dsolve(diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2-f(x)*y(x)*diff(y(x),x)-g(x)*y(x)^2=0,y(x), s
```

$$y(x) = c_2 e^{c_1 \left(\int e^{\int f(x)dx} dx\right) + \int e^{\int f(x)dx} \left(\int e^{-\left(\int f(x)dx\right)} g(x)dx\right) dx}$$

✓ Solution by Mathematica

Time used: 2.033 (sec). Leaf size: 61

```
DSolve[-(g[x]*y[x]^2) - f[x]*y[x]*y'[x] - y'[x]^2 + y[x]*y''[x] == 0,y[x],x,IncludeSingularS
```

$$y(x) \to c_2 \exp\left(\int_1^x \exp\left(\int_1^{K[3]} f(K[1])dK[1]\right)\left(c_1 + \int_1^{K[3]} \exp\left(-\int_1^{K[2]} f(K[1])dK[1]\right) g(K[2])dK[2]\right) dK[3]\right)$$

7.123 problem 1714 (book 6.123)

Internal problem ID [10036]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1714 (book 6.123).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _reducible, _mu_y_y1], [_2nd_order, _reducible,`

$$y''y - y'^2 + (g(x) + y^2 f(x))\, y' - y(g'(x) - f'(x)\, y^2) = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+(g(x)+y(x)^2*f(x))*diff(y(x),x)-y(x)*(diff(g
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-(y[x]*(-(y[x]^2*Derivative[1][f][x]) + Derivative[1][g][x])) + (g[x] + f[x]*y[x]^2)*
```

Not solved

7.124 problem 1715 (book 6.124)

Internal problem ID [10037]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1715 (book 6.124).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$y''y - 3y'^2 + 3y'y - y^2 = 0$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 40

```
dsolve(diff(diff(y(x),x),x)*y(x)-3*diff(y(x),x)^2+3*y(x)*diff(y(x),x)-y(x)^2=0,y(x), singsol
```

$$y(x) = 0$$
$$y(x) = \frac{e^x}{\sqrt{2c_2e^x - 2c_1}}$$
$$y(x) = -\frac{e^x}{\sqrt{2c_2e^x - 2c_1}}$$

✓ Solution by Mathematica
Time used: 14.439 (sec). Leaf size: 33

```
DSolve[-y[x]^2 + 3*y[x]*y'[x] - 3*y'[x]^2 + y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{c_2e^{x+c_1}}{\sqrt{-1+2e^{x+c_1}}}$$
$$y(x) \to 0$$

7.125 problem 1716 (book 6.125)

Internal problem ID [10038]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1716 (book 6.125).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{y''y - a{y'}^2 = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 28

```
dsolve(diff(diff(y(x),x),x)*y(x)-a*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \left(-\frac{1}{(a-1)\,(c_1x+c_2)}\right)^{\frac{1}{a-1}}$$

✓ Solution by Mathematica

Time used: 0.736 (sec). Leaf size: 26

```
DSolve[-(a*y'[x]^2) + y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2(-ax+x-c_1)^{\frac{1}{1-a}}$$

7.126 problem 1717 (book 6.126)

Internal problem ID [10039]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1717 (book 6.126).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y''y + a\left(y'^2 + 1\right) = 0}$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 61

```
dsolve(diff(diff(y(x),x),x)*y(x)+a*(diff(y(x),x)^2+1)=0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{\_a^a}{\sqrt{-\_a^{2a} + c_1}} d\_a - x - c_2 = 0$$
$$-\left(\int^{y(x)} \frac{\_a^a}{\sqrt{-\_a^{2a} + c_1}} d\_a\right) - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 3.471 (sec). Leaf size: 526

```
DSolve[a*(1 + y'[x]^2) + y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{\#1\sqrt{1-e^{2c_1}\#1^{-2a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1}{2a},1-\frac{1}{2a},e^{2c_1}\#1^{-2a}\right)}{\sqrt{-1+e^{2c_1}\#1^{-2a}}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\#1\sqrt{1-e^{2c_1}\#1^{-2a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1}{2a},1-\frac{1}{2a},e^{2c_1}\#1^{-2a}\right)}{\sqrt{-1+e^{2c_1}\#1^{-2a}}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{\#1\sqrt{1-e^{2(-c_1)}\#1^{-2a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1}{2a},1-\frac{1}{2a},e^{2(-c_1)}\#1^{-2a}\right)}{\sqrt{-1+e^{2(-c_1)}\#1^{-2a}}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\#1\sqrt{1-e^{2(-c_1)}\#1^{-2a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1}{2a},1-\frac{1}{2a},e^{2(-c_1)}\#1^{-2a}\right)}{\sqrt{-1+e^{2(-c_1)}\#1^{-2a}}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{\#1\sqrt{1-e^{2c_1}\#1^{-2a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1}{2a},1-\frac{1}{2a},e^{2c_1}\#1^{-2a}\right)}{\sqrt{-1+e^{2c_1}\#1^{-2a}}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\#1\sqrt{1-e^{2c_1}\#1^{-2a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1}{2a},1-\frac{1}{2a},e^{2c_1}\#1^{-2a}\right)}{\sqrt{-1+e^{2c_1}\#1^{-2a}}}\&\right][x+c_2]$$

7.127 problem 1718 (book 6.127)

Internal problem ID [10040]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1718 (book 6.127).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y''y + a{y'}^2 + b y^3 = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 113

```
dsolve(diff(diff(y(x),x),x)*y(x)+a*diff(y(x),x)^2+b*y(x)^3=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$(2a+3)\left(\int^{y(x)} \frac{\_a^{2a}}{\sqrt{-(2a+3)\left(2b\_a^{4a+3}-\_a^{2a}c_1\right)}}d\_a\right) - x - c_2 = 0$$

$$(-2a-3)\left(\int^{y(x)} \frac{\_a^{2a}}{\sqrt{-(2a+3)\left(2b\_a^{4a+3}-\_a^{2a}c_1\right)}}d\_a\right) - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 105.239 (sec). Leaf size: 277

```
DSolve[b*y[x]^3 + a*y'[x]^2 + y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{y(x)\sqrt{(2a+3)y(x)^{2a}}\sqrt{1-\frac{2by(x)^{2a+3}}{2ac_1+3c_1}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{a+1}{2a+3},\frac{a+1}{2a+3}+1,\frac{2by(x)^{2a+3}}{2ac_1+3c_1}\right)}{(a+1)\sqrt{-2by(x)^{2a+3}+2ac_1+3c_1}}=-x+c_2,y(x)\right]$$

$$\text{Solve}\left[\frac{y(x)\sqrt{(2a+3)y(x)^{2a}}\sqrt{1-\frac{2by(x)^{2a+3}}{2ac_1+3c_1}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{a+1}{2a+3},\frac{a+1}{2a+3}+1,\frac{2by(x)^{2a+3}}{2ac_1+3c_1}\right)}{(a+1)\sqrt{-2by(x)^{2a+3}+2ac_1+3c_1}}=x+c_2,y(x)\right]$$

7.128 problem 1719 (book 6.128)

Internal problem ID [10041]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1719 (book 6.128).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$y''y + ay'^2 + by'y + cy^2 + dy^{1-a} = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 129

```
dsolve(diff(diff(y(x),x),x)*y(x)+a*diff(y(x),x)^2+b*y(x)*diff(y(x),x)+c*y(x)^2+d*y(x)^(1-a)=
```

$$y(x) = e^{-\frac{\left(b-\sqrt{(-4a-4)c+b^2}\right)x}{2a+2}} \left(\frac{(-4a-4)\,c^3 + b^2c^2}{\left(-d\,e^{-\frac{\left(-b+\sqrt{(-4a-4)c+b^2}\right)x}{2}} \sqrt{(-4a-4)\,c+b^2} + (a+1)\,c\left(c_2 e^{-x\sqrt{(-4a-4)c+b^2}} - c_1\right)\right)^2}\right.$$

✓ Solution by Mathematica

Time used: 61.36 (sec). Leaf size: 396

```
DSolve[c*y[x]^2 + d*y[x]^(1 - a) + b*y[x]*y'[x] + a*y'[x]^2 + y[x]*y''[x] == 0,y[x],x,Includ
```

$$y(x) \to \left(-\frac{\exp\left(-\frac{x\left(b\sqrt{b^2-4(a+1)c}-2(a+1)c+b^2\right)}{\sqrt{b^2-4(a+1)c}+b}\right)\left(b^2\left(de^{\frac{1}{2}x\left(\sqrt{b^2-4(a+1)c}+b\right)} - cc_2\exp\left(\frac{x\left(b\sqrt{b^2-4(a+1)c}-4(a+1)c+b^2\right)}{\sqrt{b^2-4(a+1)c}+b}\right)\right.\right.}{}\right.$$

7.129 problem 1720 (book 6.129)

Internal problem ID [10042]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1720 (book 6.129).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y''y + ay'^2 + f(x)\, y'y + g(x)\, y^2 = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)*y(x)+a*diff(y(x),x)^2+f(x)*y(x)*diff(y(x),x)+g(x)*y(x)^2=0,y(x),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[g[x]*y[x]^2 + f[x]*y[x]*y'[x] + a*y'[x]^2 + y[x]*y''[x] == 0,y[x],x,IncludeSingularSo
```

Not solved

7.130 problem 1721 (book 6.130)

Internal problem ID [10043]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1721 (book 6.130).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y''y + a{y'}^2 + b y^2 y' + c y^4 = 0$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 179

```
dsolve(diff(diff(y(x),x),x)*y(x)+a*diff(y(x),x)^2+b*y(x)^2*diff(y(x),x)+c*y(x)^4=0,y(x), sin
```

$$y(x) = 0$$

$$(2a + 4)\left(\int^{y(x)} \frac{}{\tan\left(\operatorname{RootOf}\left(2\_Zb\_a^2 - 2a\ln(\_a)\sqrt{\_a^4(4ac - b^2 + 8c)} + 2\sqrt{\_a^4(4ac - b^2 + 8c)}\ln(2)\right.\right.}\right.$$

$$- x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 98.56 (sec). Leaf size: 105

```
DSolve[c*y[x]^4 + b*y[x]^2*y'[x] + a*y'[x]^2 + y[x]*y''[x] == 0,y[x],x,IncludeSingularSoluti
```

$$\text{Solve}\left[\int_1^{y(x)} \frac{1}{K[2]^2 \text{InverseFunction}\left[\frac{\log(c+\#1(b+(a+2)\#1))-\frac{2b\arctan\left(\frac{b+2(a+2)\#1}{\sqrt{4(a+2)c-b^2}}\right)}{\sqrt{4(a+2)c-b^2}}}{2(a+2)}\&\right][c_1-\log(K[2])]} dK[2] = \right.$$

$$\left. -c_2, y(x)\right]$$

7.131 problem 1722 (book 6.131)

Internal problem ID [10044]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1722 (book 6.131).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y''y - \frac{(-1+a)\,y'^2}{a} - y^2y'f(x) + \frac{af(x)^2\,y^4}{(a+2)^2} - \frac{af'(x)\,y^3}{a+2} = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(y(x),x),x)*y(x)-(a-1)/a*diff(y(x),x)^2-f(x)*y(x)^2*diff(y(x),x)+a/(a+2)^2*f
```

No solution found

✓ Solution by Mathematica

Time used: 62.573 (sec). Leaf size: 46

```
DSolve[(a*f[x]^2*y[x]^4)/(2 + a)^2 - (a*y[x]^3*Derivative[1][f][x])/(2 + a) - f[x]*y[x]^2*y'
```

$$y(x) \to -\frac{(a+2)(x+c_1)^a}{a\int_1^x f(K[3])(c_1+K[3])^a dK[3] + c_2}$$
$$y(x) \to 0$$

7.132 problem 1723 (book 6.132)

Internal problem ID [10045]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1723 (book 6.132).
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y''y - y'^2 - 2ay\left(y'^2+1\right)^{\frac{3}{2}} = 1$$

✓ Solution by Maple

Time used: 0.312 (sec). Leaf size: 117

```
dsolve(diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2-1-2*a*y(x)*(diff(y(x),x)^2+1)^(3/2)=0,y(x),
```

$$y(x) = -ix + c_1$$
$$y(x) = ix + c_1$$
$$\int^{y(x)} \frac{a\_a^2 + c_1}{\sqrt{-a^2\_a^4 - 2\_a^2 a c_1 + \_a^2 - c_1^2}} d\_a - x - c_2 = 0$$
$$-\left(\int^{y(x)} \frac{a\_a^2 + c_1}{\sqrt{-a^2\_a^4 - 2\_a^2 a c_1 + \_a^2 - c_1^2}} d\_a\right) - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 5.652 (sec). Leaf size: 2181

```
DSolve[-1 - y'[x]^2 - 2*a*y[x]*(1 + y'[x]^2)^(3/2) + y[x]*y''[x] == 0,y[x],x,IncludeSingular
```

$$y(x) \rightarrow \text{InverseFunction}\left[-\frac{\sqrt{1-\frac{2\#1^2a^2}{-2ac_1+\sqrt{1-4ac_1}+1}}\sqrt{1+\frac{2\#1^2a^2}{2ac_1+\sqrt{1-4ac_1}-1}}\left(\left(-2ac_1+\sqrt{1-4ac_1}+1\right)E\left(i\text{arcsinh}\left(\cdots\right.\right.\right.}{2\sqrt{2}\cdots} + c_2\right]$$

$$y(x) \rightarrow \text{InverseFunction}\left[\frac{\sqrt{1-\frac{2\#1^2a^2}{-2ac_1+\sqrt{1-4ac_1}+1}}\sqrt{1+\frac{2\#1^2a^2}{2ac_1+\sqrt{1-4ac_1}-1}}\left(\left(-2ac_1+\sqrt{1-4ac_1}+1\right)E\left(i\text{arcsinh}\left(\sqrt{\cdots}\right.\right.\right.}{2\sqrt{2}a\cdots} + c_2\right]$$

$$y(x) \rightarrow \text{InverseFunction}\left[-\frac{\sqrt{1-\frac{2\#1^2a^2}{-2a(-c_1)+\sqrt{1-4a(-c_1)}+1}}\sqrt{1+\frac{2\#1^2a^2}{2a(-c_1)+\sqrt{1-4a(-c_1)}-1}}\left(\left(-2a(-c_1)+\sqrt{1-4a(-c_1)}\cdots\right.\right.}{\cdots} + c_2\right]$$

$$y(x) \rightarrow \text{InverseFunction}\left[\frac{\sqrt{1-\frac{2\#1^2a^2}{-2a(-c_1)+\sqrt{1-4a(-c_1)}+1}}\sqrt{1+\frac{2\#1^2a^2}{2a(-c_1)+\sqrt{1-4a(-c_1)}-1}}\left(\left(-2a(-c_1)+\sqrt{1-4a(-c_1)}\cdots\right.\right.}{\cdots} + c_2\right]$$

$$y(x) \rightarrow \text{InverseFunction}\left[-\frac{\sqrt{1-\frac{2\#1^2a^2}{-2ac_1+\sqrt{1-4ac_1}+1}}\sqrt{1+\frac{2\#1^2a^2}{2ac_1+\sqrt{1-4ac_1}-1}}\left(\left(-2ac_1+\sqrt{1-4ac_1}+1\right)E\left(i\text{arcsinh}\left(\cdots\right.\right.\right.}{2\sqrt{2}\cdots} + c_2\right]$$

$$y(x) \rightarrow \text{InverseFunction}\left[\frac{\sqrt{1-\frac{2\#1^2a^2}{-2ac_1+\sqrt{1-4ac_1}+1}}\sqrt{1+\frac{2\#1^2a^2}{2ac_1+\sqrt{1-4ac_1}-1}}\left(\left(-2ac_1+\sqrt{1-4ac_1}+1\right)E\left(i\text{arcsinh}\left(\sqrt{\cdots}\right.\right.\right.}{2\sqrt{2}a\cdots} + c_2\right]$$

7.133 problem 1724 (book 6.133)

Internal problem ID [10046]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1724 (book 6.133).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _nonlinear], [_2nd_order, _with_linear_s`

$$y''(x+y) + y'^2 - y' = 0$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 16

```
dsolve(diff(diff(y(x),x),x)*(x+y(x))+diff(y(x),x)^2-diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{c_1 + 2x}\, c_2 + c_1 + x$$

✓ Solution by Mathematica

Time used: 20.075 (sec). Leaf size: 122

```
DSolve[-y'[x] + y'[x]^2 + (x + y[x])*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - \frac{e^{-2c_1}\sqrt{e^{2c_1}(1 + 4e^{c_1}(x + c_2))}}{\sqrt{2}} + \frac{e^{-c_1}}{2} + 2c_2$$
$$y(x) \to x + \frac{e^{-2c_1}\sqrt{e^{2c_1}(1 + 4e^{c_1}(x + c_2))}}{\sqrt{2}} + \frac{e^{-c_1}}{2} + 2c_2$$
$$y(x) \to x + 2c_2$$

7.134 problem 1725 (book 6.134)

Internal problem ID [10047]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1725 (book 6.134).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _reducibl`

$$y''(x-y)+2y'(y'+1)=0$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 21

```
dsolve(diff(diff(y(x),x),x)*(x-y(x))+2*diff(y(x),x)*(diff(y(x),x)+1)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_2^2-c_2x+c_1}{-x+c_2}$$

✓ Solution by Mathematica
Time used: 1.351 (sec). Leaf size: 40

```
DSolve[2*y'[x]*(1 + y'[x]) + (x - y[x])*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -c_2$$
$$y(x)\to -\frac{e^{-c_1}}{x+c_2}-c_2$$
$$y(x)\to -c_2$$

7.135 problem 1726 (book 6.135)

Internal problem ID [10048]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1726 (book 6.135).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _reducibl`

$$y''(x-y)-(y'+1)\left(y'^2+1\right)=0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 107

```
dsolve(diff(diff(y(x),x),x)*(x-y(x))-(diff(y(x),x)+1)*(diff(y(x),x)^2+1)=0,y(x), singsol=all
```

$$y(x) = x + \mathrm{RootOf}\left(-x - \left(\int^{\_Z} \frac{c_1^2\_f^2 - 1}{c_1^2\_f^2 + \sqrt{-c_1^2\_f^2 + 2c_1\_f} - 2} d\_f\right) + c_2\right)$$

$$y(x) = x + \mathrm{RootOf}\left(-x + \int^{\_Z} -\frac{c_1^2\_f^2 - 1}{-2 + c_1^2\_f^2 - \sqrt{-c_1^2\_f^2 + 2c_1\_f}} d\_f + c_2\right)$$

✓ Solution by Mathematica

Time used: 62.902 (sec). Leaf size: 18840

```
DSolve[(-1 - y'[x])*(1 + y'[x]^2) + (x - y[x])*y''[x] == 0,y[x],x,IncludeSingularSolutions -
```

Too large to display

7.136 problem 1727 (book 6.136)

Internal problem ID [10049]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1727 (book 6.136).
ODE order: 2.
ODE degree: 0.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _reducibl`

$$y''(x-y)-h(y')=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve(diff(diff(y(x),x),x)*(x-y(x))-h(diff(y(x),x))=0,y(x), singsol=all)
```

$$y(x)=x+\operatorname{RootOf}\left(-x+\int^{\_Z}\frac{1}{-1+\operatorname{RootOf}\left(\int^{-Z}\frac{\_a-1}{h(\_a)}d\_a+\ln(-\_g)+c_1\right)}d\_g+c_2\right)$$

✓ Solution by Mathematica

Time used: 0.292 (sec). Leaf size: 82

```
DSolve[-h[y'[x]] + (x - y[x])*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x=\int\frac{\exp\left(-\int_1^{K[4]}\frac{K[3]-1}{h(K[3])}dK[3]-c_1\right)}{h(K[4])}dK[4]+c_2,y(x)=x-\exp\left(-\int_1^{K[4]}\frac{K[3]-1}{h(K[3])}dK[3]-c_1\right)\right\},\{y(x),K[4]\}\right]$$

7.137 problem 1728 (book 6.137)

Internal problem ID [10050]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1728 (book 6.137).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{2y''y+{y'}^2=-1}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 327

```
dsolve(2*diff(diff(y(x),x),x)*y(x)+diff(y(x),x)^2+1=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(-\operatorname{RootOf}\left(\left(-\cos\left(\_Z\right)c_1+c_1\_Z-2c_2-2x\right)\left(\cos\left(\_Z\right)c_1+c_1\_Z-2c_2-2x\right)\right)c_1+2x+2c_2\right)\tan\left(\text{Ro}\right.}{2} + \frac{c_1}{2}$$

$$y(x) = \frac{\left(-\operatorname{RootOf}\left(\left(\cos\left(\_Z\right)c_1+c_1\_Z+2c_2+2x\right)\left(-\cos\left(\_Z\right)c_1+c_1\_Z+2c_2+2x\right)\right)c_1-2x-2c_2\right)\tan\left(\text{Ro}\right.}{2} + \frac{c_1}{2}$$

$$y(x) = \frac{\left(\operatorname{RootOf}\left(\left(-\cos\left(\_Z\right)c_1+c_1\_Z-2c_2-2x\right)\left(\cos\left(\_Z\right)c_1+c_1\_Z-2c_2-2x\right)\right)c_1-2x-2c_2\right)\tan\left(\text{Root}\right.}{2} + \frac{c_1}{2}$$

$$y(x) = \frac{\left(\operatorname{RootOf}\left(\left(\cos\left(\_Z\right)c_1+c_1\_Z+2c_2+2x\right)\left(-\cos\left(\_Z\right)c_1+c_1\_Z+2c_2+2x\right)\right)c_1+2x+2c_2\right)\tan\left(\text{Root}\right.}{2} + \frac{c_1}{2}$$

✓ Solution by Mathematica

Time used: 1.631 (sec). Leaf size: 397

```
DSolve[1 + y'[x]^2 + 2*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-e^{2c_1}\arctan\left(\frac{\sqrt{-\#1+e^{2c_1}}}{\sqrt{\#1}}\right)-\sqrt{\#1}\sqrt{-\#1+e^{2c_1}}\&\right][x+c_2]$$
$$y(x) \to \text{InverseFunction}\left[e^{2c_1}\arctan\left(\frac{\sqrt{-\#1+e^{2c_1}}}{\sqrt{\#1}}\right)+\sqrt{\#1}\sqrt{-\#1+e^{2c_1}}\&\right][x+c_2]$$
$$y(x) \to \text{InverseFunction}\left[-e^{2(-c_1)}\arctan\left(\frac{\sqrt{-\#1+e^{2(-c_1)}}}{\sqrt{\#1}}\right)-\sqrt{\#1}\sqrt{-\#1+e^{2(-c_1)}}\&\right][x+c_2]$$
$$y(x) \to \text{InverseFunction}\left[e^{2(-c_1)}\arctan\left(\frac{\sqrt{-\#1+e^{2(-c_1)}}}{\sqrt{\#1}}\right)+\sqrt{\#1}\sqrt{-\#1+e^{2(-c_1)}}\&\right][x+c_2]$$
$$y(x) \to \text{InverseFunction}\left[-e^{2c_1}\arctan\left(\frac{\sqrt{-\#1+e^{2c_1}}}{\sqrt{\#1}}\right)-\sqrt{\#1}\sqrt{-\#1+e^{2c_1}}\&\right][x+c_2]$$
$$y(x) \to \text{InverseFunction}\left[e^{2c_1}\arctan\left(\frac{\sqrt{-\#1+e^{2c_1}}}{\sqrt{\#1}}\right)+\sqrt{\#1}\sqrt{-\#1+e^{2c_1}}\&\right][x+c_2]$$

7.138 problem 1729 (book 6.138)

Internal problem ID [10051]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1729 (book 6.138).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{2y''y - y'^2 = -a}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 24

```
dsolve(2*diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+a=0,y(x), singsol=all)
```

$$y(x) = \frac{(c_1^2 - a)\,x^2}{4c_2} + c_1x + c_2$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 36

```
DSolve[a - y'[x]^2 + 2*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2(-a + c_1{}^2)}{4c_2} + c_1x + c_2$$
$$y(x) \to \text{Indeterminate}$$

7.139 problem 1730 (book 6.139)

Internal problem ID [10052]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1730 (book 6.139).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$2y''y - y'^2 + y^2 f(x) = -a$$

✗ Solution by Maple

```
dsolve(2*diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+y(x)^2*f(x)+a=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[a + f[x]*y[x]^2 - y'[x]^2 + 2*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

Not solved

7.140 problem 1731 (book 6.140)

Internal problem ID [10053]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1731 (book 6.140).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$2y''y - y'^2 - 8y^3 = 0$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 57

```
dsolve(2*diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2-8*y(x)^3=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$\int^{y(x)} \frac{1}{\sqrt{\_a\,(4\_a^2 + c_1)}} d\_a - x - c_2 = 0$$
$$-\left(\int^{y(x)} \frac{1}{\sqrt{\_a\,(4\_a^2 + c_1)}} d\_a\right) - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 2.538 (sec). Leaf size: 415

```
DSolve[-8*y[x]^3 - y'[x]^2 + 2*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{2\sqrt{\#1}\sqrt{1+\frac{4\#1^2}{c_1}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{4\#1^2}{c_1}\right)}{\sqrt{4\#1^2+c_1}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{2\sqrt{\#1}\sqrt{1+\frac{4\#1^2}{c_1}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{4\#1^2}{c_1}\right)}{\sqrt{4\#1^2+c_1}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{2\sqrt{\#1}\sqrt{1-\frac{4\#1^2}{c_1}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{-4\#1^2}{-c_1}\right)}{\sqrt{4\#1^2-c_1}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{2\sqrt{\#1}\sqrt{1-\frac{4\#1^2}{c_1}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{-4\#1^2}{-c_1}\right)}{\sqrt{4\#1^2-c_1}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{2\sqrt{\#1}\sqrt{1+\frac{4\#1^2}{c_1}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{4\#1^2}{c_1}\right)}{\sqrt{4\#1^2+c_1}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{2\sqrt{\#1}\sqrt{1+\frac{4\#1^2}{c_1}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{4\#1^2}{c_1}\right)}{\sqrt{4\#1^2+c_1}}\&\right][x+c_2]$$

7.141 problem 1732 (book 6.141)

Internal problem ID [10054]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1732 (book 6.141).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2y''y - y'^2 - 8y^3 - 4y^2 = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 63

```
dsolve(2*diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2-8*y(x)^3-4*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$\int^{y(x)} \frac{1}{\sqrt{(4\_a^2 + 4\_a + c_1)\_a}} d\_a - x - c_2 = 0$$

$$-\left(\int^{y(x)} \frac{1}{\sqrt{(4\_a^2 + 4\_a + c_1)\_a}} d\_a\right) - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 3.917 (sec). Leaf size: 1095

```
DSolve[-4*y[x]^2 - 8*y[x]^3 - y'[x]^2 + 2*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \text{InverseFunction}\left[-\frac{i\#1\sqrt{4+\frac{2c_1}{\#1-\#1\sqrt{1-c_1}}}\sqrt{2+\frac{c_1}{\#1+\#1\sqrt{1-c_1}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{\frac{c_1}{2\sqrt{1-c_1}+2}}}{\sqrt{\#1}}\right),\frac{\sqrt{1-c_1}}{1-\sqrt{1}}\cdots\right.}{\sqrt{\frac{c_1}{1+\sqrt{1-c_1}}}\sqrt{4\#1^2+4\#1+c_1}}\cdots + c_2\right]$$

$$y(x) \to \text{InverseFunction}\left[\frac{i\#1\sqrt{4+\frac{2c_1}{\#1-\#1\sqrt{1-c_1}}}\sqrt{2+\frac{c_1}{\#1+\#1\sqrt{1-c_1}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{\frac{c_1}{2\sqrt{1-c_1}+2}}}{\sqrt{\#1}}\right),\frac{\sqrt{1-c_1}+}{1-\sqrt{1-c}}\cdots\right.}{\sqrt{\frac{c_1}{1+\sqrt{1-c_1}}}\sqrt{4\#1^2+4\#1+c_1}}\cdots + c_2\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{i\#1\sqrt{4+\frac{2(-c_1)}{\#1-\#1\sqrt{1--c_1}}}\sqrt{2-\frac{c_1}{\#1+\#1\sqrt{1--c_1}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{-\frac{c_1}{2\sqrt{1--c_1}+2}}}{\sqrt{\#1}}\right),\cdots\right.}{\sqrt{-\frac{c_1}{1+\sqrt{1--c_1}}}\sqrt{4\#1^2+4\#1-c_1}}\cdots + c_2\right]$$

$$y(x) \to \text{InverseFunction}\left[\frac{i\#1\sqrt{4+\frac{2(-c_1)}{\#1-\#1\sqrt{1--c_1}}}\sqrt{2-\frac{c_1}{\#1+\#1\sqrt{1--c_1}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{-\frac{c_1}{2\sqrt{1--c_1}+2}}}{\sqrt{\#1}}\right),\cdots\right.}{\sqrt{-\frac{c_1}{1+\sqrt{1--c_1}}}\sqrt{4\#1^2+4\#1-c_1}}\cdots + c_2\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{i\#1\sqrt{4+\frac{2c_1}{\#1-\#1\sqrt{1-c_1}}}\sqrt{2+\frac{c_1}{\#1+\#1\sqrt{1-c_1}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{\frac{c_1}{2\sqrt{1-c_1}+2}}}{\sqrt{\#1}}\right),\frac{\sqrt{1-c}}{1-\sqrt{1}}\cdots\right.}{\sqrt{\frac{c_1}{1+\sqrt{1-c_1}}}\sqrt{4\#1^2+4\#1+c_1}}\cdots + c_2\right]$$

$$y(x) \to \text{InverseFunction}\left[\frac{i\#1\sqrt{4+\frac{2c_1}{\#1-\#1\sqrt{1-c_1}}}\sqrt{2+\frac{c_1}{\#1+\#1\sqrt{1-c_1}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{\frac{c_1}{2\sqrt{1-c_1}+2}}}{\sqrt{\#1}}\right),\frac{\sqrt{1-c_1}+}{1-\sqrt{1-c}}\cdots\right.}{\sqrt{\frac{c_1}{1+\sqrt{1-c_1}}}\sqrt{4\#1^2+4\#1+c_1}}\cdots + c_2\right]$$

7.142 problem 1733 (book 6.142)

Internal problem ID [10055]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1733 (book 6.142).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$\boxed{2y''y - y'^2 - 4(x + 2y)\, y^2 = 0}$$

✗ Solution by Maple

```
dsolve(2*diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2-4*(x+2*y(x))*y(x)^2=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-4*y[x]^2*(x + 2*y[x]) - y'[x]^2 + 2*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions
```

Not solved

7.143 problem 1734 (book 6.143)

Internal problem ID [10056]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1734 (book 6.143).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2y''y - y'^2 + (ay + b)\, y^2 = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 78

```
dsolve(2*diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+(a*y(x)+b)*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$-\sqrt{2}\left(\int^{y(x)} \frac{1}{\sqrt{\_a\,(-a\,\_a^2 - 2\_ab + 2c_1)}} d\_a\right) - x - c_2 = 0$$

$$\sqrt{2}\left(\int^{y(x)} \frac{1}{\sqrt{\_a\,(-a\,\_a^2 - 2\_ab + 2c_1)}} d\_a\right) - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 6.595 (sec). Leaf size: 1353

```
DSolve[y[x]^2*(b + a*y[x]) - y'[x]^2 + 2*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \text{InverseFunction}\left[-\frac{i\sqrt{2}\#1^{3/2}\sqrt{2+\frac{4c_1}{\#1\left(-b+\sqrt{b^2+2ac_1}\right)}}\sqrt{1-\frac{2c_1}{\#1\left(b+\sqrt{b^2+2ac_1}\right)}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{2}\sqrt{-\cdots}}{\sqrt{\cdots}}\right.\right.}{\sqrt{\frac{c_1}{-b+\sqrt{b^2+2ac_1}}}\sqrt{-\#1\left(\#1^2a+2\#1b-2c_1\right)}} + c_2\right]$$

$$y(x) \to \text{InverseFunction}\left[\frac{i\sqrt{2}\#1^{3/2}\sqrt{2+\frac{4c_1}{\#1\left(-b+\sqrt{b^2+2ac_1}\right)}}\sqrt{1-\frac{2c_1}{\#1\left(b+\sqrt{b^2+2ac_1}\right)}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{2}\sqrt{\sqrt{b^2\cdots}}}{\sqrt{\#\cdots}}\right.\right.}{\sqrt{\frac{c_1}{-b+\sqrt{b^2+2ac_1}}}\sqrt{-\#1\left(\#1^2a+2\#1b-2c_1\right)}} + c_2\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{i\sqrt{2}\#1^{3/2}\sqrt{2+\frac{4(-c_1)}{\#1\left(-b+\sqrt{b^2+2a(-c_1)}\right)}}\sqrt{1-\frac{2(-c_1)}{\#1\left(b+\sqrt{b^2+2a(-c_1)}\right)}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\cdots\right.\right.}{\sqrt{-\frac{c_1}{-b+\sqrt{b^2+2a(-c_1)}}}\sqrt{-\#1\left(\#1^2a+2\#1b-2(-1)\cdots\right.}} + c_2\right]$$

$$y(x) \to \text{InverseFunction}\left[\frac{i\sqrt{2}\#1^{3/2}\sqrt{2+\frac{4(-c_1)}{\#1\left(-b+\sqrt{b^2+2a(-c_1)}\right)}}\sqrt{1-\frac{2(-c_1)}{\#1\left(b+\sqrt{b^2+2a(-c_1)}\right)}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{\cdots}}{\cdots}\right.\right.}{\sqrt{-\frac{c_1}{-b+\sqrt{b^2+2a(-c_1)}}}\sqrt{-\#1\left(\#1^2a+2\#1b-2(-1)c_1\cdots\right.}} + c_2\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{i\sqrt{2}\#1^{3/2}\sqrt{2+\frac{4c_1}{\#1\left(-b+\sqrt{b^2+2ac_1}\right)}}\sqrt{1-\frac{2c_1}{\#1\left(b+\sqrt{b^2+2ac_1}\right)}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{2}\sqrt{-\cdots}}{\sqrt{\cdots}}\right.\right.}{\sqrt{\frac{c_1}{-b+\sqrt{b^2+2ac_1}}}\sqrt{-\#1\left(\#1^2a+2\#1b-2c_1\right)}} + c_2\right]$$

7.144 problem 1735 (book 6.144)

Internal problem ID [10057]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1735 (book 6.144).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$\boxed{2y''y - y'^2 + 2y^2x + ay^3 = -1}$$

✗ Solution by Maple

```
dsolve(2*diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+1+2*x*y(x)^2+a*y(x)^3=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[1 + 2*x*y[x]^2 + a*y[x]^3 - y'[x]^2 + 2*y[x]*y''[x] == 0,y[x],x,IncludeSingularSoluti
```

Not solved

7.145 problem 1736 (book 6.145)

Internal problem ID [10058]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1736 (book 6.145).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$2y''y - {y'}^2 + (ay + bx)\,y^2 = 0$$

✗ Solution by Maple

```
dsolve(2*diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+(a*y(x)+b*x)*y(x)^2=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]^2*(b*x + a*y[x]) - y'[x]^2 + 2*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions
```

Not solved

7.146 problem 1737 (book 6.146)

Internal problem ID [10059]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1737 (book 6.146).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2y''y - y'^2 - 3y^4 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 53

```
dsolve(2*diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2-3*y(x)^4=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$\int^{y(x)} \frac{1}{\sqrt{\_a\left(\_a^3 + c_1\right)}} d\_a - x - c_2 = 0$$

$$-\left(\int^{y(x)} \frac{1}{\sqrt{\_a\left(\_a^3 + c_1\right)}} d\_a\right) - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 13.648 (sec). Leaf size: 397

```
DSolve[-3*y[x]^4 - y'[x]^2 + 2*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{2\sqrt{\#1}\sqrt{1+\frac{\#1^3}{c_1}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{\#1^3}{c_1}\right)}{\sqrt{\#1^3+c_1}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{2\sqrt{\#1}\sqrt{1+\frac{\#1^3}{c_1}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{\#1^3}{c_1}\right)}{\sqrt{\#1^3+c_1}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{2\sqrt{\#1}\sqrt{1-\frac{\#1^3}{c_1}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\frac{-\#1^3}{-c_1}\right)}{\sqrt{\#1^3-c_1}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{2\sqrt{\#1}\sqrt{1-\frac{\#1^3}{c_1}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\frac{-\#1^3}{-c_1}\right)}{\sqrt{\#1^3-c_1}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{2\sqrt{\#1}\sqrt{1+\frac{\#1^3}{c_1}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{\#1^3}{c_1}\right)}{\sqrt{\#1^3+c_1}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{2\sqrt{\#1}\sqrt{1+\frac{\#1^3}{c_1}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{\#1^3}{c_1}\right)}{\sqrt{\#1^3+c_1}}\&\right][x+c_2]$$

7.147 problem 1738 (book 6.147)

Internal problem ID [10060]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1738 (book 6.147).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Painleve, '4th']]`

$$2y''y - y'^2 - 4(x^2 + a)\, y^2 - 8y^3x - 3y^4 = -b$$

✗ Solution by Maple

```
dsolve(2*diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+b-4*(x^2+a)*y(x)^2-8*x*y(x)^3-3*y(x)^4=0,y
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[b - 4*(a + x^2)*y[x]^2 - 8*x*y[x]^3 - 3*y[x]^4 - y'[x]^2 + 2*y[x]*y''[x] == 0,y[x],x,
```

Not solved

7.148 problem 1739 (book 6.148)

Internal problem ID [10061]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1739 (book 6.148).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$2y''y - y'^2 + 3f(x)\,y'y + 2\left(f(x)^2 + f'(x)\right)y^2 - 8y^3 = 0$$

✗ Solution by Maple

```
dsolve(2*diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+3*f(x)*y(x)*diff(y(x),x)+2*(f(x)^2+diff(f(
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-8*y[x]^3 + 2*y[x]^2*(f[x]^2 + Derivative[1][f][x]) + 3*f[x]*y[x]*y'[x] - y'[x]^2 + 2
```

Not solved

7.149 problem 1740 (book 6.149)

Internal problem ID [10062]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1740 (book 6.149).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$2y''y - {y'}^2 + 4y'y^2 + y^2 f(x) + y^4 = -1$$

✗ Solution by Maple

```
dsolve(2*diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+4*y(x)^2*diff(y(x),x)+1+y(x)^2*f(x)+y(x)^4
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[1 + f[x]*y[x]^2 + y[x]^4 + 4*y[x]^2*y'[x] - y'[x]^2 + 2*y[x]*y''[x] == 0,y[x],x,Inclu
```

Not solved

7.150 problem 1741 (book 6.150)

Internal problem ID [10063]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1741 (book 6.150).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{2y''y - 3{y'}^2 = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 17

```
dsolve(2*diff(diff(y(x),x),x)*y(x)-3*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{4}{(c_1 x + c_2)^2}$$

✓ Solution by Mathematica

Time used: 0.243 (sec). Leaf size: 21

```
DSolve[-3*y'[x]^2 + 2*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2}{(x + 2c_1)^2}$$
$$y(x) \to 0$$

7.151 problem 1742 (book 6.151)

Internal problem ID [10064]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1742 (book 6.151).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{2y''y - 3y'^2 - 4y^2 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 22

```
dsolve(2*diff(diff(y(x),x),x)*y(x)-3*diff(y(x),x)^2-4*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{4}{\left(c_1 \sin\left(x\right) - c_2 \cos\left(x\right)\right)^2}$$

✓ Solution by Mathematica

Time used: 1.076 (sec). Leaf size: 17

```
DSolve[-4*y[x]^2 - 3*y'[x]^2 + 2*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \sec^2(x + 2c_1)$$

7.152 problem 1743 (book 6.152)

Internal problem ID [10065]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1743 (book 6.152).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2y''y - 3y'^2 + y^2 f(x) = 0$$

✗ Solution by Maple

```
dsolve(2*diff(diff(y(x),x),x)*y(x)-3*diff(y(x),x)^2+y(x)^2*f(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[f[x]*y[x]^2 - 3*y'[x]^2 + 2*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.153 problem 1744 (book 6.153)

Internal problem ID [10066]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1744 (book 6.153).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$2y''y - 6{y'}^2 + (1 + ay^3)\, y^2 = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 77

```
dsolve(2*diff(diff(y(x),x),x)*y(x)-6*diff(y(x),x)^2+(1+a*y(x)^3)*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$-2\left(\int^{y(x)} \frac{1}{\sqrt{4\_a^4 c_1 + 4\_a^3 a + 1}\,\_a} d\_a\right) - x - c_2 = 0$$

$$2\left(\int^{y(x)} \frac{1}{\sqrt{4\_a^4 c_1 + 4\_a^3 a + 1}\,\_a} d\_a\right) - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 47.786 (sec). Leaf size: 2761

```
DSolve[y[x]^2*(1 + a*y[x]^3) - 6*y'[x]^2 + 2*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolution
```

Too large to display

7.154 problem 1745 (book 6.154)

Internal problem ID [10067]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1745 (book 6.154).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$2y''y - y'^2\left(y'^2 + 1\right) = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 331

```
dsolve(2*diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2*(diff(y(x),x)^2+1)=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \frac{(\text{RootOf}\left((-\cos(\_Z)\, c_1 + c_1\_Z - 2c_2 - 2x)(\cos(\_Z)\, c_1 + c_1\_Z - 2c_2 - 2x)\right) c_1 - 2x - 2c_2)\tan(\text{Root}}{2} + \frac{c_1}{2}$$

$$y(x) = \frac{(\text{RootOf}\left((\cos(\_Z)\, c_1 + c_1\_Z + 2c_2 + 2x)(-\cos(\_Z)\, c_1 + c_1\_Z + 2c_2 + 2x)\right) c_1 + 2x + 2c_2)\tan(\text{Root}}{2} + \frac{c_1}{2}$$

$$y(x) = \frac{(-\text{RootOf}\left((-\cos(\_Z)\, c_1 + c_1\_Z - 2c_2 - 2x)(\cos(\_Z)\, c_1 + c_1\_Z - 2c_2 - 2x)\right) c_1 + 2x + 2c_2)\tan(\text{Ro}}{2} + \frac{c_1}{2}$$

$$y(x) = \frac{(-\text{RootOf}\left((\cos(\_Z)\, c_1 + c_1\_Z + 2c_2 + 2x)(-\cos(\_Z)\, c_1 + c_1\_Z + 2c_2 + 2x)\right) c_1 - 2x - 2c_2)\tan(\text{Ro}}{2} + \frac{c_1}{2}$$

✓ Solution by Mathematica

Time used: 2.67 (sec). Leaf size: 501

```
DSolve[-(y'[x]^2*(1 + y'[x]^2)) + 2*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \text{InverseFunction}\left[-ie^{-c_1}\left(\sqrt{\#1}\sqrt{-1+\#1e^{2c_1}} - e^{-c_1}\text{arctanh}\left(\frac{e^{-c_1}\sqrt{-1+\#1e^{2c_1}}}{\sqrt{\#1}}\right)\right)\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[ie^{-c_1}\left(\sqrt{\#1}\sqrt{-1+\#1e^{2c_1}} - e^{-c_1}\text{arctanh}\left(\frac{e^{-c_1}\sqrt{-1+\#1e^{2c_1}}}{\sqrt{\#1}}\right)\right)\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[-ie^{-(-c_1)}\left(\sqrt{\#1}\sqrt{-1+\#1e^{2(-c_1)}} - e^{-(-c_1)}\text{arctanh}\left(\frac{e^{-(-c_1)}\sqrt{-1+\#1e^{2(-c_1)}}}{\sqrt{\#1}}\right)\right)\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[ie^{-(-c_1)}\left(\sqrt{\#1}\sqrt{-1+\#1e^{2(-c_1)}} - e^{-(-c_1)}\text{arctanh}\left(\frac{e^{-(-c_1)}\sqrt{-1+\#1e^{2(-c_1)}}}{\sqrt{\#1}}\right)\right)\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[-ie^{-c_1}\left(\sqrt{\#1}\sqrt{-1+\#1e^{2c_1}} - e^{-c_1}\text{arctanh}\left(\frac{e^{-c_1}\sqrt{-1+\#1e^{2c_1}}}{\sqrt{\#1}}\right)\right)\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[ie^{-c_1}\left(\sqrt{\#1}\sqrt{-1+\#1e^{2c_1}} - e^{-c_1}\text{arctanh}\left(\frac{e^{-c_1}\sqrt{-1+\#1e^{2c_1}}}{\sqrt{\#1}}\right)\right)\&\right][x+c_2]$$

7.155 problem 1746 (book 6.155)

Internal problem ID [10068]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1746 (book 6.155).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{2(y-a)\,y''+y'^2=-1}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 123

```
dsolve(2*(y(x)-a)*diff(diff(y(x),x),x)+diff(y(x),x)^2+1=0,y(x), singsol=all)
```

$$-\sqrt{-\left(-y\left(x\right)+a\right)\left(-y\left(x\right)+c_1+a\right)}+\frac{c_1\arctan\left(\frac{2y(x)-2a-c_1}{2\sqrt{-(-y(x)+a)(-y(x)+c_1+a)}}\right)}{2}-x-c_2=0$$

$$\sqrt{-\left(-y\left(x\right)+a\right)\left(-y\left(x\right)+c_1+a\right)}-\frac{c_1\arctan\left(\frac{2y(x)-2a-c_1}{2\sqrt{-(-y(x)+a)(-y(x)+c_1+a)}}\right)}{2}-x-c_2=0$$

✓ Solution by Mathematica

Time used: 1.945 (sec). Leaf size: 595

```
DSolve[1 + y'[x]^2 + 2*(-a + y[x])*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{\sqrt{2}e^{2c_1}\arctan\left(\frac{\sqrt{2\#1-2a+e^{2c_1}}}{\sqrt{2}\sqrt{a-\#1}}\right)+2\sqrt{a-\#1}\sqrt{2\#1-2a+e^{2c_1}}}{2\sqrt{2}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\sqrt{2}e^{2c_1}\arctan\left(\frac{\sqrt{2\#1-2a+e^{2c_1}}}{\sqrt{2}\sqrt{a-\#1}}\right)+2\sqrt{a-\#1}\sqrt{2\#1-2a+e^{2c_1}}}{2\sqrt{2}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{\sqrt{2}e^{2(-c_1)}\arctan\left(\frac{\sqrt{2\#1-2a+e^{2(-c_1)}}}{\sqrt{2}\sqrt{a-\#1}}\right)+2\sqrt{a-\#1}\sqrt{2\#1-2a+e^{2(-c_1)}}}{2\sqrt{2}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\sqrt{2}e^{2(-c_1)}\arctan\left(\frac{\sqrt{2\#1-2a+e^{2(-c_1)}}}{\sqrt{2}\sqrt{a-\#1}}\right)+2\sqrt{a-\#1}\sqrt{2\#1-2a+e^{2(-c_1)}}}{2\sqrt{2}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{\sqrt{2}e^{2c_1}\arctan\left(\frac{\sqrt{2\#1-2a+e^{2c_1}}}{\sqrt{2}\sqrt{a-\#1}}\right)+2\sqrt{a-\#1}\sqrt{2\#1-2a+e^{2c_1}}}{2\sqrt{2}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\sqrt{2}e^{2c_1}\arctan\left(\frac{\sqrt{2\#1-2a+e^{2c_1}}}{\sqrt{2}\sqrt{a-\#1}}\right)+2\sqrt{a-\#1}\sqrt{2\#1-2a+e^{2c_1}}}{2\sqrt{2}}\&\right][x+c_2]$$

7.156 problem 1747 (book 6.156)

Internal problem ID [10069]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1747 (book 6.156).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$3y''y - 2{y'}^2 = a\,x^2 + bx + c$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 205

```
dsolve(3*diff(diff(y(x),x),x)*y(x)-2*diff(y(x),x)^2-a*x^2-b*x-c=0,y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(-2b\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right) - 2b\left(\int^{\_Z}\frac{1}{\sqrt{4\_f^{\frac{4}{3}}c_1b^2 - 36c\_f^2a + 9b^2\_f^2 - 2}}d\_f\right)\sqrt{4ac-b^2} + c_2\sqrt{4ac-b^2}\right)\left(a\,x^2+bx+c\right)^{\frac{3}{2}}$$

$$y(x) = \operatorname{RootOf}\left(-2b\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right) + 2b\left(\int^{\_Z}\frac{1}{\sqrt{4\_f^{\frac{4}{3}}c_1b^2 - 36c\_f^2a + 9b^2\_f^2 - 2}}d\_f\right)\sqrt{4ac-b^2} + c_2\sqrt{4ac-b^2}\right)\left(a\,x^2+bx+c\right)^{\frac{3}{2}}$$

✓ Solution by Mathematica

Time used: 0.129 (sec). Leaf size: 118

```
DSolve[-c - b*x - a*x^2 - 2*y'[x]^2 + 3*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

$$\text{Solve}\left[\int \frac{y(x)^{2/3}}{(ax^2+bx+c)\sqrt{-\frac{2(ax^2+bx+c)^3}{y(x)^2}+\frac{c_1(ax^2+bx+c)}{y(x)^{2/3}}+9(b^2-4ac)}} d\frac{ax^2+bx+c}{y(x)^{2/3}} = -\int \frac{1}{3(ax^2+bx+c)}dx + c_2, y(x)\right]$$

7.157 problem 1748 (book 6.157)

Internal problem ID [10070]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1748 (book 6.157).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$3y''y - 5{y'}^2 = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 21

```
dsolve(3*diff(diff(y(x),x),x)*y(x)-5*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$-\frac{3}{2y(x)^{\frac{2}{3}}} - c_1 x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 0.289 (sec). Leaf size: 25

```
DSolve[-5*y'[x]^2 + 3*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2}{(2x + 3c_1)^{3/2}}$$

$$y(x) \to 0$$

7.158 problem 1749 (book 6.158)

Internal problem ID [10071]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1749 (book 6.158).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$4y''y - 3y'^2 + 4y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 67

```
dsolve(4*diff(diff(y(x),x),x)*y(x)-3*diff(y(x),x)^2+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$-\frac{4\sqrt{c_1 y(x)^{\frac{3}{2}} + 4y(x)}}{\sqrt{y(x)}\, c_1} - x - c_2 = 0$$
$$\frac{4\sqrt{c_1 y(x)^{\frac{3}{2}} + 4y(x)}}{\sqrt{y(x)}\, c_1} - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 0.587 (sec). Leaf size: 43

```
DSolve[4*y[x] - 3*y'[x]^2 + 4*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(c_1{}^2 x^2 + 2c_2 c_1{}^2 x - 64 + c_2{}^2 c_1{}^2)^2}{256 c_1{}^2}$$

7.159 problem 1750 (book 6.159)

Internal problem ID [10072]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1750 (book 6.159).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$4y''y - 3y'^2 - 12y^3 = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 61

```
dsolve(4*diff(diff(y(x),x),x)*y(x)-3*diff(y(x),x)^2-12*y(x)^3=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$\int^{y(x)} \frac{1}{\sqrt{\_a^{\frac{3}{2}} \left(4\_a^{\frac{3}{2}} + c_1\right)}} d\_a - x - c_2 = 0$$

$$-\left(\int^{y(x)} \frac{1}{\sqrt{\_a^{\frac{3}{2}} \left(4\_a^{\frac{3}{2}} + c_1\right)}} d\_a\right) - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 1.963 (sec). Leaf size: 469

```
DSolve[-12*y[x]^3 - 3*y'[x]^2 + 4*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{4\#1\sqrt{1+\frac{4\#1^{3/2}}{c_1}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{4\#1^{3/2}}{c_1}\right)}{\sqrt{\#1^{3/2}\left(4\#1^{3/2}+c_1\right)}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{4\#1\sqrt{1+\frac{4\#1^{3/2}}{c_1}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{4\#1^{3/2}}{c_1}\right)}{\sqrt{\#1^{3/2}\left(4\#1^{3/2}+c_1\right)}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{4\#1\sqrt{1-\frac{4\#1^{3/2}}{c_1}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\frac{-4\#1^{3/2}}{-c_1}\right)}{\sqrt{\#1^{3/2}\left(4\#1^{3/2}-c_1\right)}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{4\#1\sqrt{1-\frac{4\#1^{3/2}}{c_1}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\frac{-4\#1^{3/2}}{-c_1}\right)}{\sqrt{\#1^{3/2}\left(4\#1^{3/2}-c_1\right)}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{4\#1\sqrt{1+\frac{4\#1^{3/2}}{c_1}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{4\#1^{3/2}}{c_1}\right)}{\sqrt{\#1^{3/2}\left(4\#1^{3/2}+c_1\right)}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{4\#1\sqrt{1+\frac{4\#1^{3/2}}{c_1}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{4\#1^{3/2}}{c_1}\right)}{\sqrt{\#1^{3/2}\left(4\#1^{3/2}+c_1\right)}}\&\right][x+c_2]$$

7.160 problem 1751 (book 6.160)

Internal problem ID [10073]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1751 (book 6.160).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$4y''y - 3y'^2 + ay^3 + by^2 + yc = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 90

```
dsolve(4*diff(diff(y(x),x),x)*y(x)-3*diff(y(x),x)^2+a*y(x)^3+y(x)^2*b+c*y(x)=0,y(x), singsol
```

$$y(x) = 0$$

$$-\sqrt{3}\left(\int^{y(x)} \frac{1}{\sqrt{\_a\left(-a\_a^2 - 3\_ab + 3c_1\sqrt{\_a} + 3c\right)}}d\_a\right) - x - c_2 = 0$$

$$\sqrt{3}\left(\int^{y(x)} \frac{1}{\sqrt{\_a\left(-a\_a^2 - 3\_ab + 3c_1\sqrt{\_a} + 3c\right)}}d\_a\right) - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 2.052 (sec). Leaf size: 323

```
DSolve[c*y[x] + b*y[x]^2 + a*y[x]^3 - 3*y'[x]^2 + 4*y[x]*y''[x] == 0,y[x],x,IncludeSingularS
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{1}{\sqrt{-\frac{1}{3}aK[1]^3 - bK[1]^2 + c_1K[1]^{3/2} + cK[1]}}dK[1]\&\right][x + c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{\sqrt{-\frac{1}{3}aK[2]^3 - bK[2]^2 + c_1K[2]^{3/2} + cK[2]}}dK[2]\&\right][x + c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{1}{\sqrt{-\frac{1}{3}aK[1]^3 - bK[1]^2 - c_1K[1]^{3/2} + cK[1]}}dK[1]\&\right][x + c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{1}{\sqrt{-\frac{1}{3}aK[1]^3 - bK[1]^2 + c_1K[1]^{3/2} + cK[1]}}dK[1]\&\right][x + c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{\sqrt{-\frac{1}{3}aK[2]^3 - bK[2]^2 - c_1K[2]^{3/2} + cK[2]}}dK[2]\&\right][x + c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{\sqrt{-\frac{1}{3}aK[2]^3 - bK[2]^2 + c_1K[2]^{3/2} + cK[2]}}dK[2]\&\right][x + c_2]$$

7.161 problem 1752 (book 6.161)

Internal problem ID [10074]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1752 (book 6.161).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$4y''y - 3y'^2 + \left(6y^2 - \frac{2f'(x)\,y}{f(x)}\right)y' + y^4 - 2y'y^2 + g(x)\,y^2 + f(x)\,y = 0$$

✗ Solution by Maple

```
dsolve(4*diff(diff(y(x),x),x)*y(x)-3*diff(y(x),x)^2+(6*y(x)^2-2*diff(f(x),x)/f(x)*y(x))*diff
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[f[x]*y[x] + g[x]*y[x]^2 + y[x]^4 - 2*y[x]^2*y'[x] + (6*y[x]^2 - (2*y[x]*Derivative[1]
```

Not solved

7.162 problem 1753 (book 6.162)

Internal problem ID [10075]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1753 (book 6.162).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{4y''y - 5y'^2 + ay^2 = 0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 34

```
dsolve(4*diff(diff(y(x),x),x)*y(x)-5*diff(y(x),x)^2+a*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{16\,\mathrm{e}^{\sqrt{a}\,x} a^2}{\left(\mathrm{e}^{\frac{\sqrt{a}\,x}{2}} c_1 - c_2\right)^4}$$

✓ Solution by Mathematica
Time used: 10.047 (sec). Leaf size: 26

```
DSolve[a*y[x]^2 - 5*y'[x]^2 + 4*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \operatorname{sech}^4\left(\frac{1}{4}\sqrt{a}(x - 4c_1)\right)$$

7.163 problem 1754 (book 6.163)

Internal problem ID [10076]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1754 (book 6.163).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{12y''y - 15{y'}^2 + 8y^3 = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 151

```
dsolve(12*diff(diff(y(x),x),x)*y(x)-15*diff(y(x),x)^2+8*y(x)^3=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$-\frac{12y(x)\left(8\sqrt{y(x)}-c_1\right)\sqrt{8y(x)-c_1\sqrt{y(x)}}}{\sqrt{-24y(x)^3+3c_1y(x)^{\frac{5}{2}}}\,c_1\sqrt{\sqrt{y(x)}\left(8\sqrt{y(x)}-c_1\right)}} - x - c_2 = 0$$

$$\frac{12y(x)\left(8\sqrt{y(x)}-c_1\right)\sqrt{8y(x)-c_1\sqrt{y(x)}}}{\sqrt{-24y(x)^3+3c_1y(x)^{\frac{5}{2}}}\,c_1\sqrt{\sqrt{y(x)}\left(8\sqrt{y(x)}-c_1\right)}} - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 1.235 (sec). Leaf size: 48

```
DSolve[8*y[x]^3 - 15*y'[x]^2 + 12*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2304{c_1}^2}{\left(3{c_1}^2x^2+6c_2{c_1}^2x+128+3{c_2}^2{c_1}^2\right)^2}$$

$$y(x) \to 0$$

7.164 problem 1755 (book 6.164)

Internal problem ID [10077]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1755 (book 6.164).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{nyy'' - (n-1)\,y'^2 = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 19

```
dsolve(n*y(x)*diff(diff(y(x),x),x)-(n-1)*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \left(\frac{c_1 x + c_2}{n}\right)^n$$

✓ Solution by Mathematica
Time used: 1.022 (sec). Leaf size: 17

```
DSolve[(1 - n)*y'[x]^2 + n*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2(x - c_1 n)^n$$

7.165 problem 1756 (book 6.165)

Internal problem ID [10078]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1756 (book 6.165).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y''ya + y'^2b + \mathrm{c4}\, y^4 + \mathrm{c3}\, y^3 + \mathrm{c2}\, y^2 + \mathrm{c1}\, y = -\mathrm{c0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 419

```
dsolve(a*y(x)*diff(diff(y(x),x),x)+b*diff(y(x),x)^2+c4*y(x)^4+c3*y(x)^3+c2*y(x)^2+c1*y(x)+c0
```

$$6(a+b)\left(a+\frac{b}{2}\right)b(a+2b)\left(a+\frac{2b}{3}\right)\left(\int^{y(x)}\frac{}{\sqrt{-36\,(a+b)\,(a+\frac{b}{2})\,b\_a^{\frac{2b}{a}}\left(\frac{2(a+b)\left(a+\frac{b}{2}\right)b(a+2b)\,\mathrm{c3}\,\_a^{\frac{3a+2b}{a}}}{3}+\left(\left(a+\frac{b}{2}\right)b\,(a+2b)\,\mathrm{c2}\,\_a^{\frac{2a+2b}{a}}\right.\right.}}\right.$$

$$-c_2-x=0$$

$$-6(a+b)\left(a+\frac{b}{2}\right)b(a+2b)\left(a+\frac{2b}{3}\right)\left(\int^{y(x)}\frac{}{\sqrt{-36\,(a+b)\,(a+\frac{b}{2})\,b\_a^{\frac{2b}{a}}\left(\frac{2(a+b)\left(a+\frac{b}{2}\right)b(a+2b)\,\mathrm{c3}\,\_a^{\frac{3a+2b}{a}}}{3}+\left(\left(a+\frac{b}{2}\right)b\,(a+2b)\,\mathrm{c2}\,\_a^{\frac{2a+2b}{a}}\right.\right.}}\right.$$

$$-c_2-x=0$$

✓ Solution by Mathematica

Time used: 12.68 (sec). Leaf size: 2166

```
DSolve[c0 + c1*y[x] + c2*y[x]^2 + c3*y[x]^3 + c4*y[x]^4 + b*y'[x]^2 + a*y[x]*y''[x] == 0,y[x
```

Too large to display

7.166 problem 1757 (book 6.166)

Internal problem ID [10079]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1757 (book 6.166).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _with_linear_symmetries], [_2nd_orde`

$$y''ya + y'^2 b - \frac{yy'}{\sqrt{c^2+x^2}} = 0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 82

```
dsolve(a*y(x)*diff(diff(y(x),x),x)+b*diff(y(x),x)^2-y(x)*diff(y(x),x)/(c^2+x^2)^(1/2)=0,y(x)
```

$$y(x) = 0$$

$$y(x) = \left(\frac{a(a+1)}{(a+b)\left(c_1 2^{\frac{1}{a}} a\, x^{\frac{a+1}{a}} \operatorname{hypergeom}\left(\left[-\frac{1}{2a}, -\frac{a+1}{2a}\right], \left[\frac{a-1}{a}\right], -\frac{c^2}{x^2}\right) + c_2 a + c_2\right)}\right)^{-\frac{a}{a+b}}$$

✓ Solution by Mathematica

Time used: 66.528 (sec). Leaf size: 143

```
DSolve[-((y[x]*y'[x])/Sqrt[c^2 + x^2]) + b*y'[x]^2 + a*y[x]*y''[x] == 0,y[x],x,IncludeSingul
```

$$y(x) \to c_2 \exp\left(\int_1^x \frac{\left(1-\frac{K[2]}{\sqrt{c^2+K[2]^2}}\right)^{-\frac{1}{2}/a}\left(\frac{K[2]}{\sqrt{c^2+K[2]^2}}+1\right)^{\frac{1}{2}/a}}{c_1 - \int_1^{K[2]} -\frac{(a+b)\left(1-\frac{K[1]}{\sqrt{c^2+K[1]^2}}\right)^{-\frac{1}{2}/a}\left(\frac{K[1]}{\sqrt{c^2+K[1]^2}}+1\right)^{\frac{1}{2}/a}}{a} dK[1]} dK[2]\right)$$

7.167 problem 1758 (book 6.167)

Internal problem ID [10080]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1758 (book 6.167).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y''ya - (-1+a)\,y'^2 + (a+2)\,f(x)\,y^2y' + f(x)^2\,y^4 + af'(x)\,y^3 = 0$$

✗ Solution by Maple

```
dsolve(a*y(x)*diff(diff(y(x),x),x)-(a-1)*diff(y(x),x)^2+(a+2)*f(x)*y(x)^2*diff(y(x),x)+f(x)^
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[f[x]^2*y[x]^4 + (2 + a)*f[x]*y[x]^2*y'[x] + a*y[x]^3*y'[x] - (-1 + a)*y'[x]^2 + a*y[x
```

Not solved

7.168 problem 1759 (book 6.168)

Internal problem ID [10081]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1759 (book 6.168).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{(ay+b)\,y''+cy'^2=0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 73

```
dsolve((a*y(x)+b)*diff(diff(y(x),x),x)+c*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x)=-\frac{b}{a}$$
$$y(x)=\frac{\left(-\left(\frac{1}{(a+c)(c_1x+c_2)}\right)^{-\frac{c}{a+c}}b+(a+c)\,(c_1x+c_2)\right)\left(\frac{1}{(a+c)(c_1x+c_2)}\right)^{\frac{c}{a+c}}}{a}$$

✓ Solution by Mathematica

Time used: 16.845 (sec). Leaf size: 31

```
DSolve[c*y'[x]^2 + (b + a*y[x])*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{-b+(c_1(a+c)(x+c_2))^{\frac{a}{a+c}}}{a}$$

7.169 problem 1760 (book 6.169)

Internal problem ID [10082]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1760 (book 6.169).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _nonlinear], _Liouville, [_2nd_order, _w`

$$\boxed{xyy'' + xy'^2 - y'y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 35

```
dsolve(x*y(x)*diff(diff(y(x),x),x)+x*diff(y(x),x)^2-y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \sqrt{c_1 x^2 + 2c_2}$$
$$y(x) = -\sqrt{c_1 x^2 + 2c_2}$$

✓ Solution by Mathematica

Time used: 0.39 (sec). Leaf size: 18

```
DSolve[-(y[x]*y'[x]) + x*y'[x]^2 + x*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to c_2 \sqrt{x^2 + c_1}$$

7.170 problem 1761 (book 6.170)

Internal problem ID [10083]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1761 (book 6.170).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _nonlinear], [_2nd_order, _reducible, _m`

$$\boxed{xyy'' + xy'^2 + ay'y = -f(x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 112

```
dsolve(x*y(x)*diff(diff(y(x),x),x)+x*diff(y(x),x)^2+a*y(x)*diff(y(x),x)+f(x)=0,y(x), singsol
```

$$y(x) = \frac{\sqrt{2}\,\sqrt{(a-1)\left(x^{-a+1}\left(\int x^{a-1}f(x)\,dx\right) + x^{-a+1}c_1 - \left(\int f(x)\,dx\right) - c_2\right)}}{a-1}$$

$$y(x) = -\frac{\sqrt{2}\,\sqrt{(a-1)\left(x^{-a+1}\left(\int x^{a-1}f(x)\,dx\right) + x^{-a+1}c_1 - \left(\int f(x)\,dx\right) - c_2\right)}}{a-1}$$

✓ Solution by Mathematica

Time used: 60.103 (sec). Leaf size: 108

```
DSolve[f[x] + a*y[x]*y'[x] + x*y'[x]^2 + x*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to -\sqrt{2}\sqrt{\int_1^x -K[2]^{-a}\left(c_1 + \int_1^{K[2]} f(K[1])K[1]^{a-1}dK[1]\right)dK[2] + c_2}$$

$$y(x) \to \sqrt{2}\sqrt{\int_1^x -K[2]^{-a}\left(c_1 + \int_1^{K[2]} f(K[1])K[1]^{a-1}dK[1]\right)dK[2] + c_2}$$

7.171 problem 1762 (book 6.171)

Internal problem ID [10084]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1762 (book 6.171).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Painleve, '3rd']]`

$$\boxed{xyy'' - xy'^2 + y'y + x(d + ay^4) + y(c + by^2) = 0}$$

✗ Solution by Maple

```
dsolve(x*y(x)*diff(diff(y(x),x),x)-x*diff(y(x),x)^2+y(x)*diff(y(x),x)+x*(d+a*y(x)^4)+y(x)*(c
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*(c + b*y[x]^2) + x*(d + a*y[x]^4) + y[x]*y'[x] - x*y'[x]^2 + x*y[x]*y''[x] == 0,
```

Not solved

7.172 problem 1763 (book 6.172)

Internal problem ID [10085]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1763 (book 6.172).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xyy'' - xy'^2 + ay'y + bxy^3 = 0$$

✗ Solution by Maple

```
dsolve(x*y(x)*diff(diff(y(x),x),x)-x*diff(y(x),x)^2+a*y(x)*diff(y(x),x)+b*x*y(x)^3=0,y(x), s
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[b*x*y[x]^3 + a*y[x]*y'[x] - x*y'[x]^2 + x*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolu
```

Not solved

7.173 problem 1764 (book 6.173)

Internal problem ID [10086]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1764 (book 6.173).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _with_linear_symmetries], [_2nd_orde`

$$\boxed{xyy'' + 2xy'^2 + ay'y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 164

```
dsolve(x*y(x)*diff(diff(y(x),x),x)+2*x*diff(y(x),x)^2+a*y(x)*diff(y(x),x)=0,y(x), singsol=al
```

$$y(x) = 0$$
$$y(x) = \frac{3^{\frac{1}{3}} \left((a-1)^2 \left(-x^{1+2a} c_1 + c_2 x^{3a} (a-1)\right)\right)^{\frac{1}{3}} x^{-a}}{a-1}$$
$$y(x) = -\frac{\left((a-1)^2 \left(-x^{1+2a} c_1 + c_2 x^{3a} (a-1)\right)\right)^{\frac{1}{3}} \left(i 3^{\frac{5}{6}} + 3^{\frac{1}{3}}\right) x^{-a}}{2a-2}$$
$$y(x) = \frac{\left((a-1)^2 \left(-x^{1+2a} c_1 + c_2 x^{3a} (a-1)\right)\right)^{\frac{1}{3}} \left(i 3^{\frac{5}{6}} - 3^{\frac{1}{3}}\right) x^{-a}}{2a-2}$$

✓ Solution by Mathematica

Time used: 4.155 (sec). Leaf size: 29

```
DSolve[a*y[x]*y'[x] + 2*x*y'[x]^2 + x*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to c_2 \sqrt[3]{3x^{1-a} - ac_1 + c_1}$$

7.174 problem 1765 (book 6.174)

Internal problem ID [10087]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1765 (book 6.174).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _reducibl`

$$x y y'' - 2 x y'^2 + (y+1) y' = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 22

```
dsolve(x*y(x)*diff(diff(y(x),x),x)-2*x*diff(y(x),x)^2+(y(x)+1)*diff(y(x),x)=0,y(x), singsol=
```

$$y(x) = 0$$
$$y(x) = c_1 \tanh\left(\frac{\ln(x) - c_2}{2c_1}\right)$$

✓ Solution by Mathematica

Time used: 34.063 (sec). Leaf size: 52

```
DSolve[(1 + y[x])*y'[x] - 2*x*y'[x]^2 + x*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{\tan\left(\frac{\sqrt{c_1}(\log(x) - c_2)}{\sqrt{2}}\right)}{\sqrt{2}\sqrt{c_1}}$$
$$y(x) \to \frac{1}{2}(\log(x) - c_2)$$

7.175 problem 1766 (book 6.175)

Internal problem ID [10088]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1766 (book 6.175).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _with_linear_symmetries], [_2nd_orde`

$$xyy'' - 2xy'^2 + ay'y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 31

```
dsolve(x*y(x)*diff(diff(y(x),x),x)-2*x*diff(y(x),x)^2+a*y(x)*diff(y(x),x)=0,y(x), singsol=al
```

$$y(x) = 0$$
$$y(x) = -\frac{(a-1)\,x^a}{c_2 x^a\,(a-1) - c_1 x}$$

✓ Solution by Mathematica

Time used: 0.845 (sec). Leaf size: 29

```
DSolve[a*y[x]*y'[x] - 2*x*y'[x]^2 + x*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{c_2 x^a}{x + (a-1)c_1 x^a}$$
$$y(x) \to 0$$

7.176 problem 1767 (book 6.176)

Internal problem ID [10089]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1767 (book 6.176).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _with_linear_symmetries], [_2nd_orde`

$$\boxed{xyy'' - 4xy'^2 + 4y'y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 68

```
dsolve(x*y(x)*diff(diff(y(x),x),x)-4*x*diff(y(x),x)^2+4*y(x)*diff(y(x),x)=0,y(x), singsol=al
```

$$y(x) = 0$$
$$y(x) = \frac{x}{(-3c_2x^3 + c_1)^{\frac{1}{3}}}$$
$$y(x) = -\frac{(1 + i\sqrt{3})\, x}{2\,(-3c_2x^3 + c_1)^{\frac{1}{3}}}$$
$$y(x) = \frac{(i\sqrt{3} - 1)\, x}{2\,(-3c_2x^3 + c_1)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.67 (sec). Leaf size: 26

```
DSolve[4*y[x]*y'[x] - 4*x*y'[x]^2 + x*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{c_2x}{\sqrt[3]{1 + c_1x^3}}$$
$$y(x) \to 0$$

7.177 problem 1768 (book 6.177)

Internal problem ID [10090]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1768 (book 6.177).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xyy'' + \left(\frac{ax}{\sqrt{b^2 - x^2}} - x\right) y'^2 - y'y = 0$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 55

```
dsolve(x*y(x)*diff(diff(y(x),x),x)+(a*x/(b^2-x^2)^(1/2)-x)*diff(y(x),x)^2-y(x)*diff(y(x),x)=
```

$$y(x) = 0$$

$$y(x) = c_2 \mathrm{e}^{-\left(\int \frac{x\sqrt{b^2-x^2}}{c_1\sqrt{b^2-x^2}+a\left(b^2-x^2\right)}dx\right)}$$

✓ Solution by Mathematica

Time used: 19.437 (sec). Leaf size: 54

```
DSolve[-(y[x]*y'[x]) + (-x + (a*x)/Sqrt[b^2 - x^2])*y'[x]^2 + x*y[x]*y''[x] == 0,y[x],x,Incl
```

$$y(x) \to c_2 e^{\frac{\sqrt{b^2-x^2}}{a}} \left(a\sqrt{b^2 - x^2} - c_1\right)^{\frac{c_1}{a^2}}$$

7.178 problem 1769 (book 6.178)

Internal problem ID [10091]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1769 (book 6.178).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _nonlinear], [_2nd_order, _with_linear_s`

$$x(x+y)\,y''+xy'^2+(x-y)\,y'-y=0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 49

```
dsolve(x*(x+y(x))*diff(diff(y(x),x),x)+x*diff(y(x),x)^2+(x-y(x))*diff(y(x),x)-y(x)=0,y(x), s
```

$$y(x)=-x$$
$$y(x)=-x-\sqrt{(-c_2+1)\,x^2+c_1}$$
$$y(x)=-x+\sqrt{(-c_2+1)\,x^2+c_1}$$

✓ Solution by Mathematica

Time used: 1.141 (sec). Leaf size: 53

```
DSolve[-y[x] + (x - y[x])*y'[x] + x*y'[x]^2 + x*(x + y[x])*y''[x] == 0,y[x],x,IncludeSingula
```

$$y(x)\to -x-\sqrt{(1+2c_2)x^2+c_1}$$
$$y(x)\to -x+\sqrt{(1+2c_2)x^2+c_1}$$

7.179 problem 1770 (book 6.179)

Internal problem ID [10092]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1770 (book 6.179).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _with_linear_symmetries], [_2nd_orde`

$$\boxed{2xyy'' - xy'^2 + y'y = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 25

```
dsolve(2*x*y(x)*diff(diff(y(x),x),x)-x*diff(y(x),x)^2+y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = c_1\sqrt{x}\, c_2 + c_1^2 x + \frac{c_2^2}{4}$$

✓ Solution by Mathematica
Time used: 0.359 (sec). Leaf size: 18

```
DSolve[y[x]*y'[x] - x*y'[x]^2 + 2*x*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to c_2\left(\sqrt{x} + c_1\right)^2$$

7.180 problem 1771 (book 6.180)

Internal problem ID [10093]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1771 (book 6.180).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _reducibl`

$$\boxed{x^2(y-1)\,y''-2x^2{y'}^2-2x(y-1)\,y'-2y(y-1)^2=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 30

```
dsolve(x^2*(-1+y(x))*diff(diff(y(x),x),x)-2*x^2*diff(y(x),x)^2-2*x*(-1+y(x))*diff(y(x),x)-2*
```

$$y(x)=1$$
$$y(x)=\frac{x(c_1x-c_2)}{c_1x^2-c_2x-1}$$

✓ Solution by Mathematica

Time used: 1.332 (sec). Leaf size: 27

```
DSolve[-2*(-1 + y[x])^2*y[x] - 2*x*(-1 + y[x])*y'[x] - 2*x^2*y'[x]^2 + x^2*(-1 + y[x])*y''[x
```

$$y(x)\to 1+\frac{1}{c_2x^2-c_1x-1}$$
$$y(x)\to 1$$

7.181 problem 1772 (book 6.181)

Internal problem ID [10094]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1772 (book 6.181).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _reducibl`

$$\boxed{x^2(x+y)\,y''-(y'x-y)^2=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 29

```
dsolve(x^2*(x+y(x))*diff(diff(y(x),x),x)-(x*diff(y(x),x)-y(x))^2=0,y(x), singsol=all)
```

$$y(x)=-x$$
$$y(x)=\frac{x\left(\mathrm{e}^{\frac{c_1-x}{x}}-c_2\right)}{c_2}$$

✓ Solution by Mathematica

Time used: 1.012 (sec). Leaf size: 20

```
DSolve[-(-y[x] + x*y'[x])^2 + x^2*(x + y[x])*y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to x\left(-1+c_2e^{\frac{c_1}{x}}\right)$$

7.182 problem 1773 (book 6.182)

Internal problem ID [10095]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1773 (book 6.182).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _reducibl`

$$x^2(x-y)\,y''+a(y'x-y)^2=0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 45

```
dsolve(x^2*(x-y(x))*diff(diff(y(x),x),x)+a*(x*diff(y(x),x)-y(x))^2=0,y(x), singsol=all)
```

$$y(x)=x$$
$$y(x)=-\operatorname{RootOf}\left(c_1ax\_Z^a-c_1x\_Z^a-c_2a\_Z^a+c_2\_Z^a+\_Zx^a\right)+x$$

✓ Solution by Mathematica

Time used: 60.663 (sec). Leaf size: 36

```
DSolve[a*(-y[x] + x*y'[x])^2 + x^2*(x - y[x])*y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to x\left(1+\left(-\frac{(a-1)\left((-1)^ac_1+c_2x\right)}{x}\right)^{\frac{1}{1-a}}\right)$$

7.183 problem 1774 (book 6.183)

Internal problem ID [10096]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1774 (book 6.183).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2y''y - x^2\left(y'^2+1\right) + y^2 = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 30

```
dsolve(2*x^2*y(x)*diff(diff(y(x),x),x)-x^2*(diff(y(x),x)^2+1)+y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{x\left(4c_2^2 \ln\left(x\right)^2 + 4c_1 \ln\left(x\right) c_2 + c_1^2 + 1\right)}{4c_2}$$

✓ Solution by Mathematica

Time used: 0.845 (sec). Leaf size: 49

```
DSolve[y[x]^2 - x^2*(1 + y'[x]^2) + 2*x^2*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{x\left(c_1{}^2\log^2(x) - 2c_2c_1{}^2\log(x) + 4 + c_2{}^2c_1{}^2\right)}{4c_1}$$

$$y(x) \to \text{Indeterminate}$$

7.184 problem 1775 (book 6.184)

Internal problem ID [10097]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1775 (book 6.184).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _reducibl`

$$a\,x^2yy'' + b\,x^2y'^2 + cxyy' + dy^2 = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 119

```
dsolve(a*x^2*y(x)*diff(diff(y(x),x),x)+b*x^2*diff(y(x),x)^2+c*x*y(x)*diff(y(x),x)+d*y(x)^2=0
```

$$y(x) = 0$$

$$y(x) = x^{\frac{-\sqrt{(-4a-4b)d+(a-c)^2}+a-c}{2a+2b}}\left(\frac{a^2+(-2c-4d)\,a-4bd+c^2}{(a+b)^2\left(x^{\frac{\sqrt{(-4a-4b)d+(a-c)^2}}{a}}c_1-c_2\right)^2}\right)^{-\frac{a}{2a+2b}}$$

✓ Solution by Mathematica

Time used: 61.303 (sec). Leaf size: 92

```
DSolve[d*y[x]^2 + c*x*y[x]*y'[x] + b*x^2*y'[x]^2 + a*x^2*y[x]*y''[x] == 0,y[x],x,IncludeSing
```

$$y(x) \to c_2\exp\left(-\frac{\log(x)\left(a\left(\sqrt{\frac{a^2-2a(c+2d)-4bd+c^2}{a^2}}-1\right)+c\right)-2a\log\left(x^{\sqrt{\frac{a^2-2a(c+2d)-4bd+c^2}{a^2}}}+c_1\right)}{2(a+b)}\right)$$

7.185 problem 1776 (book 6.185)

Internal problem ID [10098]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1776 (book 6.185).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _reducibl`

$$x(x+1)^2 yy'' - x(x+1)^2 y'^2 + 2(x+1)^2 yy' - a(x+2) y^2 = 0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 31

```
dsolve(x*(x+1)^2*y(x)*diff(diff(y(x),x),x)-x*(x+1)^2*diff(y(x),x)^2+2*(x+1)^2*y(x)*diff(y(x)
```

$$y(x) = 0$$
$$y(x) = \frac{(1+x)^a \mathrm{e}^{\frac{(-1-x)a+c_1}{x}}}{c_2}$$

✓ Solution by Mathematica

Time used: 0.661 (sec). Leaf size: 24

```
DSolve[-(a*(2 + x)*y[x]^2) + 2*(1 + x)^2*y[x]*y'[x] - x*(1 + x)^2*y'[x]^2 + x*(1 + x)^2*y[x]
```

$$y(x) \to c_2(x+1)^a e^{-\frac{a+c_1}{x}}$$

7.186 problem 1777 (book 6.186)

Internal problem ID [10099]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1777 (book 6.186).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _reducibl`

$$8(-x^3+1)\,yy''-4(-x^3+1)\,y'^2-12x^2yy'+3y^2x=0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 53

```
dsolve(8*(-x^3+1)*y(x)*diff(diff(y(x),x),x)-4*(-x^3+1)*diff(y(x),x)^2-12*x^2*y(x)*diff(y(x),
```

$$y(x)=0$$

$$y(x)=\frac{\left(\operatorname{LegendreQ}\left(-\frac{1}{6},\frac{1}{3},\sqrt{-\left(-1+x\right)\left(x^2+x+1\right)}\right)c_1+\frac{c_2\operatorname{LegendreP}\left(-\frac{1}{6},\frac{1}{3},\sqrt{-(-1+x)(x^2+x+1)}\right)}{2}\right)^2x}{c_1}$$

✓ Solution by Mathematica

Time used: 94.818 (sec). Leaf size: 708

```
DSolve[3*x*y[x]^2 - 12*x^2*y[x]*y'[x] - 4*(1 - x^3)*y'[x]^2 + 8*(1 - x^3)*y[x]*y''[x] == 0,y
```

$$y(x) \to c_2 \exp\left(\int_1^x \right.$$

$$-\frac{-2\sqrt{K[2]^2+K[2]+1}\sqrt{\sqrt{3}K[2]+\sqrt{2K[2]-i\sqrt{3}+1}\sqrt{2K[2]+i\sqrt{3}+1}+\sqrt{3}}\left(4K[2]^2+\left(\sqrt{2K[\right.\right.}{}$$

7.187 problem 1778 (book 6.187)

Internal problem ID [10100]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1778 (book 6.187).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\mathrm{f0}(x)\, y y'' + \mathrm{f1}(x)\, {y'}^2 + \mathrm{f2}(x)\, y y' + \mathrm{f3}(x)\, y^2 = 0$$

✗ Solution by Maple

```
dsolve(f0(x)*y(x)*diff(diff(y(x),x),x)+f1(x)*diff(y(x),x)^2+f2(x)*y(x)*diff(y(x),x)+f3(x)*y(
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[f3[x]*y[x]^2 + f2[x]*y[x]*y'[x] + f1[x]*y'[x]^2 + f0[x]*y[x]*y''[x] == 0,y[x],x,Inclu
```

Not solved

7.188 problem 1779 (book 6.188)

Internal problem ID [10101]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1779 (book 6.188).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y^2y'' = a}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 369

```
dsolve(y(x)^2*diff(diff(y(x),x),x)-a=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1\left(c_1^2a^2 + 2ac_1\mathrm{e}^{\mathrm{RootOf}\left(\mathrm{csgn}\left(\frac{1}{c_1}\right)c_1^4a^2-2\_Z c_1^3a\,\mathrm{e}^{\_Z}-\mathrm{csgn}\left(\frac{1}{c_1}\right)\mathrm{e}^{2\_Z}c_1^2-2\,\mathrm{csgn}\left(\frac{1}{c_1}\right)\mathrm{e}^{\_Z}c_2-2\,\mathrm{csgn}\left(\frac{1}{c_1}\right)\mathrm{e}^{\_Z}x\right)} + \mathrm{e}^{2\,\mathrm{RootOf}\left(\mathrm{csgn}\left(\cdots\right.\right.}\right.}{\cdots}$$

$$y(x) = \frac{c_1\left(c_1^2a^2 + 2ac_1\mathrm{e}^{\mathrm{RootOf}\left(\mathrm{csgn}\left(\frac{1}{c_1}\right)c_1^4a^2-2\_Z c_1^3a\,\mathrm{e}^{\_Z}-\mathrm{csgn}\left(\frac{1}{c_1}\right)\mathrm{e}^{2\_Z}c_1^2+2\,\mathrm{csgn}\left(\frac{1}{c_1}\right)\mathrm{e}^{\_Z}c_2+2\,\mathrm{csgn}\left(\frac{1}{c_1}\right)\mathrm{e}^{\_Z}x\right)} + \mathrm{e}^{2\,\mathrm{RootOf}\left(\mathrm{csgn}\left(\cdots\right.\right.}\right.}{\cdots}$$

✓ Solution by Mathematica

Time used: 0.267 (sec). Leaf size: 65

```
DSolve[-a + y[x]^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\left(\frac{2a\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2a}{y(x)}+c_1}}{\sqrt{c_1}}\right)}{c_1{}^{3/2}} + \frac{y(x)\sqrt{-\frac{2a}{y(x)}+c_1}}{c_1}\right)^2 = (x+c_2)^2, y(x)\right]$$

7.189 problem 1780 (book 6.189)

Internal problem ID [10102]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1780 (book 6.189).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y^2y'' + yy'^2 = -xa$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 110

```
dsolve(y(x)^2*diff(diff(y(x),x),x)+y(x)*diff(y(x),x)^2+a*x=0,y(x), singsol=all)
```

$$\ln(x) - \frac{\left(\int^{\frac{y(x)}{x}} \frac{\_g^2\left(\left(\left(\left(\frac{a}{\_g^3}\right)^{\frac{1}{3}}-2\right)\sqrt{3}+3\left(\frac{a}{\_g^3}\right)^{\frac{1}{3}}\tan\left(\operatorname{RootOf}\left(2\_Z\sqrt{3}-\ln\left(\frac{1}{\sqrt{3}\sin(2\_Z)+2+\cos(2\_Z)}\right)-6c_1-6\left(\int \frac{\left(\frac{a}{\_g^3}\right)^{\frac{2}{3}}\_g^2}{\_g^3+a}\right.\right.\right.\right.\right.}{\_g^3+a}\right.}{6}$$

$$- c_2 = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[a*x + y[x]*y'[x]^2 + y[x]^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.190 problem 1781 (book 6.190)

Internal problem ID [10103]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1781 (book 6.190).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y^2y'' + yy'^2 = xa + b}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 171

```
dsolve(y(x)^2*diff(diff(y(x),x),x)+y(x)*diff(y(x),x)^2-a*x-b=0,y(x), singsol=all)
```

$$y(x) = \mathrm{RootOf}\left(\sqrt{3}\,b\left(\int^{\_Z}\right.\right.$$

$$-\frac{\left(-\left(-\frac{a}{\_g^3b^3}\right)^{\frac{1}{3}}\sqrt{3}\,b + 2\sqrt{3}\,a - 3b\left(-\frac{a}{\_g^3b^3}\right)^{\frac{1}{3}}\tan\left(\mathrm{RootOf}\left(-2b^2\left(-\frac{a}{\_g^3b^3}\right)^{\frac{2}{3}}\_g^2\left(\sum_{\_R=\mathrm{RootOf}\left(a^2\_Z^3\right.}\right.\right.\right.\right.}{\_g^3a^2 - 1}$$

$$\left.-6\ln\left(ax+b\right)b + 6c_2a\right)(ax+b)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-b - a*x + y[x]*y'[x]^2 + y[x]^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.191 problem 1782 (book 6.191)

Internal problem ID [10104]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1782 (book 6.191).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{(y^2+1)\,y''+(1-2y)\,y'^2=0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 21

```
dsolve((y(x)^2+1)*diff(diff(y(x),x),x)+(1-2*y(x))*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = -i$$
$$y(x) = i$$
$$y(x) = \tan(\ln(c_1x+c_2))$$

✓ Solution by Mathematica
Time used: 9.321 (sec). Leaf size: 97

```
DSolve[(1 - 2*y[x])*y'[x]^2 + (1 + y[x]^2)*y''[x] == 0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -\frac{i(-1+{c_1}^{2i}(x+c_2)^{2i})}{1+{c_1}^{2i}(x+c_2)^{2i}}$$
$$y(x) \to \frac{i\left(e^{2\arg(x+c_2)}-e^{2i\text{Interval}[\{0,\pi\}]}\right)}{e^{2i\text{Interval}[\{0,\pi\}]}+e^{2\arg(x+c_2)}}$$

7.192 problem 1783 (book 6.192)

Internal problem ID [10105]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1783 (book 6.192).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{(y^2+1)\,y''-3yy'^2=0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 43

```
dsolve((y(x)^2+1)*diff(diff(y(x),x),x)-3*y(x)*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = -i$$
$$y(x) = i$$
$$y(x) = \sqrt{-\frac{1}{c_1^2x^2+2c_1c_2x+c_2^2-1}}\,(c_1x+c_2)$$

✓ Solution by Mathematica

Time used: 1.681 (sec). Leaf size: 173

```
DSolve[-3*y[x]*y'[x]^2 + (1 + y[x]^2)*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{ic_1(x+c_2)}{\sqrt{c_1{}^2x^2+2c_2c_1{}^2x-1+c_2{}^2c_1{}^2}}$$
$$y(x) \to \frac{ic_1(x+c_2)}{\sqrt{c_1{}^2x^2+2c_2c_1{}^2x-1+c_2{}^2c_1{}^2}}$$
$$y(x) \to -\frac{ic_1}{\sqrt{c_1{}^2}}$$
$$y(x) \to \frac{ic_1}{\sqrt{c_1{}^2}}$$
$$y(x) \to -\frac{i(x+c_2)}{\sqrt{(x+c_2)^2}}$$
$$y(x) \to \frac{i(x+c_2)}{\sqrt{(x+c_2)^2}}$$

7.193 problem 1784 (book 6.193)

Internal problem ID [10106]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1784 (book 6.193).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _reducibl`

$$\left(x+y^2\right)y''-2\left(x-y^2\right)y'^3+y'(1+4y'y)=0$$

✓ Solution by Maple
Time used: 0.14 (sec). Leaf size: 41

```
dsolve((x+y(x)^2)*diff(diff(y(x),x),x)-2*(x-y(x)^2)*diff(y(x),x)^3+diff(y(x),x)*(1+4*y(x)*di
```

$$y(x)=\sqrt{-x}$$
$$y(x)=-\sqrt{-x}$$
$$\frac{-c_1y(x)+\ln\left(x+y(x)^2\right)+c_2+2}{y\left(x\right)}=0$$

✓ Solution by Mathematica
Time used: 1.242 (sec). Leaf size: 26

```
DSolve[-2*(x - y[x]^2)*y'[x]^3 + y'[x]*(1 + 4*y[x]*y'[x]) + (x + y[x]^2)*y''[x] == 0,y[x],x,
```

$$\text{Solve}\left[x=-y(x)^2+c_2e^{e^{-c_1y(x)}},y(x)\right]$$

7.194 problem 1785 (book 6.194)

Internal problem ID [10107]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1785 (book 6.194).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _reducibl`

$$\left(y^2+x^2\right)y''-\left(y'^2+1\right)\left(y'x-y\right)=0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 57

```
dsolve((y(x)^2+x^2)*diff(diff(y(x),x),x)-(diff(y(x),x)^2+1)*(x*diff(y(x),x)-y(x))=0,y(x), si
```

$$y(x) = -ix$$
$$y(x) = ix$$
$$y(x) = \tan\left(\operatorname{RootOf}\left(\cos\left(\_Z\right)^2 \mathrm{e}^{-\frac{2\left(\_Zc_1i+i\_Z+c_2c_1-c_2\right)}{-1+c_1}}-x^2\right)\right)x$$

✓ Solution by Mathematica

Time used: 0.322 (sec). Leaf size: 74

```
DSolve[(y[x] - x*y'[x])*(1 + y'[x]^2) + (x^2 + y[x]^2)*y''[x] == 0,y[x],x,IncludeSingularSol
```

$$\text{Solve}\left[\frac{1}{2}\left(\log\left(1-\frac{iy(x)}{x}\right)+\log\left(1+\frac{iy(x)}{x}\right)\right.\right.$$
$$\left.\left.+i\cot(c_1)\left(\log\left(1-\frac{iy(x)}{x}\right)-\log\left(1+\frac{iy(x)}{x}\right)\right)\right)=-\log(x)+c_2,y(x)\right]$$

7.195 problem 1786 (book 6.195)

Internal problem ID [10108]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1786 (book 6.195).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _reducibl`

$$\left(y^2+x^2\right) y''-2\left(y'^2+1\right)\left(y'x-y\right)=0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 97

```
dsolve((y(x)^2+x^2)*diff(diff(y(x),x),x)-2*(diff(y(x),x)^2+1)*(x*diff(y(x),x)-y(x))=0,y(x),
```

$$y(x)=-ix$$
$$y(x)=ix$$
$$y(x)=\frac{c_1-\sqrt{c_1^2+\left(4ic_2x+2\right)c_1-4c_2^2x^2-4ic_2x+1}+1}{2c_2}$$
$$y(x)=\frac{c_1+1+\sqrt{c_1^2+\left(4ic_2x+2\right)c_1-4c_2^2x^2-4ic_2x+1}}{2c_2}$$

✓ Solution by Mathematica

Time used: 60.379 (sec). Leaf size: 95

```
DSolve[-2*(-y[x] + x*y'[x])*(1 + y'[x]^2) + (x^2 + y[x]^2)*y''[x] == 0,y[x],x,IncludeSingula
```

$$y(x)\to\frac{1}{2}\left(-\sqrt{4x\left(-x+e^{c_2}\right)+e^{2c_2}\cot^2(c_1)}-e^{c_2}\cot(c_1)\right)$$
$$y(x)\to\frac{1}{2}\left(\sqrt{4x\left(-x+e^{c_2}\right)+e^{2c_2}\cot^2(c_1)}-e^{c_2}\cot(c_1)\right)$$

7.196 problem 1787 (book 6.196)

Internal problem ID [10109]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1787 (book 6.196).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _reducible, _mu_x_y1], [_2nd_order,`

$$2y(1-y)\,y''-(1-2y)\,y'^2+y(1-y)\,y'f(x)=0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 43

```
dsolve(2*y(x)*(1-y(x))*diff(diff(y(x),x),x)-(1-2*y(x))*diff(y(x),x)^2+y(x)*(1-y(x))*diff(y(x
```

$$y(x)=\frac{4\,\mathrm{e}^{c_1\left(\int \mathrm{e}^{-\frac{(\int f(x)dx)}{2}}dx\right)}c_2^2+4c_2+\mathrm{e}^{-c_1\left(\int \mathrm{e}^{-\frac{(\int f(x)dx)}{2}}dx\right)}}{8c_2}$$

✓ Solution by Mathematica

Time used: 0.138 (sec). Leaf size: 91

```
DSolve[f[x]*(1 - y[x])*y[x]*y'[x] - (1 - 2*y[x])*y'[x]^2 + 2*(1 - y[x])*y[x]*y''[x] == 0,y[x
```

$$y(x)\to\frac{1}{4}\exp\left(-i\left(\int_1^x-\exp\left(-\int_1^{K[1]}\frac{1}{2}f(K[1])dK[1]\right)c_1dK[1]+c_2\right)\right)\left(1+\exp\left(i\left(\int_1^x-\exp\left(-\int_1^{K[1]}\frac{1}{2}f(K[1])dK[1]\right)c_1dK[1]+c_2\right)\right)\right)^2$$

7.197 problem 1788 (book 6.197)

Internal problem ID [10110]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1788 (book 6.197).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$2y(1-y)\,y''-(1-3y)\,y'^2+h(y)=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 81

```
dsolve(2*y(x)*(1-y(x))*diff(diff(y(x),x),x)-(1-3*y(x))*diff(y(x),x)^2+h(y(x))=0,y(x), singso
```

$$\int^{y(x)}\frac{1}{\sqrt{\_b\left(\int\frac{h(\_b)}{(\_b-1)^3\_b^2}d\_b+c_1\right)(\_b-1)}}d\_b-x-c_2=0$$

$$-\left(\int^{y(x)}\frac{1}{\sqrt{\_b\left(\int\frac{h(\_b)}{(\_b-1)^3\_b^2}d\_b+c_1\right)(\_b-1)}}d\_b\right)-x-c_2=0$$

✓ Solution by Mathematica

Time used: 1.941 (sec). Leaf size: 512

```
DSolve[h[y[x]] - (1 - 3*y[x])*y'[x]^2 + 2*(1 - y[x])*y[x]*y''[x] == 0,y[x],x,IncludeSingular
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{1}{(K[2]-1)\sqrt{K[2]}\sqrt{c_1+2\int_1^{K[2]}\frac{e^{-2\left(\log(1-K[1])+\frac{1}{2}\log(K[1])\right)}h(K[1])}{2(K[1]-1)K[1]}dK[1]}}dK[2]\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1}\frac{1}{(K[3]-1)\sqrt{K[3]}\sqrt{c_1+2\int_1^{K[3]}\frac{e^{-2\left(\log(1-K[1])+\frac{1}{2}\log(K[1])\right)}h(K[1])}{2(K[1]-1)K[1]}dK[1]}}dK[3]\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{1}{(K[2]-1)\sqrt{K[2]}\sqrt{2\int_1^{K[2]}\frac{e^{-2\left(\log(1-K[1])+\frac{1}{2}\log(K[1])\right)}h(K[1])}{2(K[1]-1)K[1]}dK[1]-c_1}}dK[2]\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{1}{(K[2]-1)\sqrt{K[2]}\sqrt{c_1+2\int_1^{K[2]}\frac{e^{-2\left(\log(1-K[1])+\frac{1}{2}\log(K[1])\right)}h(K[1])}{2(K[1]-1)K[1]}dK[1]}}dK[2]\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1}\frac{1}{(K[3]-1)\sqrt{K[3]}\sqrt{2\int_1^{K[3]}\frac{e^{-2\left(\log(1-K[1])+\frac{1}{2}\log(K[1])\right)}h(K[1])}{2(K[1]-1)K[1]}dK[1]-c_1}}dK[3]\&\right][x+c_2]$$

$y(x)$

7.198 problem 1789 (book 6.198)

Internal problem ID [10111]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1789 (book 6.198).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _reducible, _mu_xy]]`

$$2y(y-1)\,y''-(3y-1)\,y'^2+4yy'(f(x)\,y+g(x))+4y^2(y-1)\left(g(x)^2-f(x)^2-g'(x)-f'(x)\right)=0$$

✗ Solution by Maple

```
dsolve(2*y(x)*(-1+y(x))*diff(diff(y(x),x),x)-(3*y(x)-1)*diff(y(x),x)^2+4*y(x)*diff(y(x),x)*(
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-4*(1 - y[x])*y[x]^2*(-f[x]^2 + g[x]^2 - Derivative[1][f][x] - Derivative[1][g][x]) +
```

Not solved

7.199 problem 1790 (book 6.199)

Internal problem ID [10112]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1790 (book 6.199).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$-2y(1-y)\,y'' + (1-3y)\,{y'}^2 - 4yy'(f(x)\,y + g(x)) + (1-y)^3\left(\mathrm{f0}\,(x)^2\,y^2 - \mathrm{f1}\,(x)^2\right) + 4y^2(1-y)\left(f(x)^2 \right.$$

✗ Solution by Maple

```
dsolve(-2*y(x)*(1-y(x))*diff(diff(y(x),x),x)+(1-3*y(x))*diff(y(x),x)^2-4*y(x)*diff(y(x),x)*(
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(1 - y[x])^3*(-f1[x]^2 + f0[x]^2*y[x]^2) + 4*(1 - y[x])*y[x]^2*(f[x]^2 - g[x]^2 - Der
```

Not solved

7.200 problem 1791 (book 6.200)

Internal problem ID [10113]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1791 (book 6.200).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$3y(1-y)\,y''-2(1-2y)\,{y'}^{2}-h(y)=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 120

```
dsolve(3*y(x)*(1-y(x))*diff(diff(y(x),x),x)-2*(1-2*y(x))*diff(y(x),x)^2-h(y(x))=0,y(x), sing
```

$$-\sqrt{3}\left(\int^{y(x)}\frac{1}{\sqrt{-(\_b(\_b-1))^{\frac{1}{3}}\_b(\_b-1)\left(2\left(\int\frac{h(\_b)}{\_b^{2}(\_b-1)^{2}(\_b(\_b-1))^{\frac{1}{3}}}d\_b\right)-3c_1\right)}}d\_b\right)-x-c_2=0$$

$$\sqrt{3}\left(\int^{y(x)}\frac{1}{\sqrt{-(\_b(\_b-1))^{\frac{1}{3}}\_b(\_b-1)\left(2\left(\int\frac{h(\_b)}{\_b^{2}(\_b-1)^{2}(\_b(\_b-1))^{\frac{1}{3}}}d\_b\right)-3c_1\right)}}d\_b\right)-x-c_2=0$$

✓ Solution by Mathematica

Time used: 1.69 (sec). Leaf size: 560

```
DSolve[-h[y[x]] - 2*(1 - 2*y[x])*y'[x]^2 + 3*(1 - y[x])*y[x]*y''[x] == 0,y[x],x,IncludeSingu
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{1}{(1-K[2])^{2/3}K[2]^{2/3}\sqrt{c_1+2\int_1^{K[2]} -\frac{\exp\left(-2\left(\frac{2}{3}\log(1-K[1])+\frac{2}{3}\log(K[1])\right)\right)h(K[1])}{3(K[1]-1)K[1]}dK[1]}}dK[2]\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{(1-K[3])^{2/3}K[3]^{2/3}\sqrt{c_1+2\int_1^{K[3]} -\frac{\exp\left(-2\left(\frac{2}{3}\log(1-K[1])+\frac{2}{3}\log(K[1])\right)\right)h(K[1])}{3(K[1]-1)K[1]}dK[1]}}dK\right.$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{1}{(1-K[2])^{2/3}K[2]^{2/3}\sqrt{2\int_1^{K[2]} -\frac{\exp\left(-2\left(\frac{2}{3}\log(1-K[1])+\frac{2}{3}\log(K[1])\right)\right)h(K[1])}{3(K[1]-1)K[1]}dK[1]-c_1}}dK[2]\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{1}{(1-K[2])^{2/3}K[2]^{2/3}\sqrt{c_1+2\int_1^{K[2]} -\frac{\exp\left(-2\left(\frac{2}{3}\log(1-K[1])+\frac{2}{3}\log(K[1])\right)\right)h(K[1])}{3(K[1]-1)K[1]}dK[1]}}dK[2]\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{(1-K[3])^{2/3}K[3]^{2/3}\sqrt{2\int_1^{K[3]} -\frac{\exp\left(-2\left(\frac{2}{3}\log(1-K[1])+\frac{2}{3}\log(K[1])\right)\right)h(K[1])}{3(K[1]-1)K[1]}dK[1]-c_1}}dK\right.$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{(1-K[3])^{2/3}K[3]^{2/3}\sqrt{c_1+2\int_1^{K[3]} -\frac{\exp\left(-2\left(\frac{2}{3}\log(1-K[1])+\frac{2}{3}\log(K[1])\right)\right)h(K[1])}{3(K[1]-1)K[1]}dK[1]}}dK\right.$$

7.201 problem 1792 (book 6.201)

Internal problem ID [10114]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1792 (book 6.201).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$(1-y)\,y''-3(1-2y)\,y'^2-h(y)=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 91

```
dsolve((1-y(x))*diff(diff(y(x),x),x)-3*(1-2*y(x))*diff(y(x),x)^2-h(y(x))=0,y(x), singsol=all
```

$$\int^{y(x)}\frac{e^{-6\_b}}{\sqrt{-2\left(\int\frac{e^{-12\_b}h(\_b)}{(\_b-1)^7}d\_b\right)+c_1\,(\_b-1)^3}}d\_b-x-c_2=0$$

$$-\left(\int^{y(x)}\frac{e^{-6\_b}}{\sqrt{-2\left(\int\frac{e^{-12\_b}h(\_b)}{(\_b-1)^7}d\_b\right)+c_1\,(\_b-1)^3}}d\_b\right)-x-c_2=0$$

✓ Solution by Mathematica

Time used: 1.515 (sec). Leaf size: 506

```
DSolve[-h[y[x]] - 3*(1 - 2*y[x])*y'[x]^2 + (1 - y[x])*y''[x] == 0,y[x],x,IncludeSingularSolu
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{e^{\frac{1}{2}(12-12K[2])}}{(K[2]-1)^3\sqrt{c_1+2\int_1^{K[2]} -\frac{\exp(-2(6(K[1]-1)+3\log(K[1]-1)))h(K[1])}{K[1]-1}dK[1]}}dK[2]\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{e^{\frac{1}{2}(12-12K[3])}}{(K[3]-1)^3\sqrt{c_1+2\int_1^{K[3]} -\frac{\exp(-2(6(K[1]-1)+3\log(K[1]-1)))h(K[1])}{K[1]-1}dK[1]}}dK[3]\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{e^{\frac{1}{2}(12-12K[2])}}{(K[2]-1)^3\sqrt{2\int_1^{K[2]} -\frac{\exp(-2(6(K[1]-1)+3\log(K[1]-1)))h(K[1])}{K[1]-1}dK[1]-c_1}}dK[2]\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{e^{\frac{1}{2}(12-12K[2])}}{(K[2]-1)^3\sqrt{c_1+2\int_1^{K[2]} -\frac{\exp(-2(6(K[1]-1)+3\log(K[1]-1)))h(K[1])}{K[1]-1}dK[1]}}dK[2]\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{e^{\frac{1}{2}(12-12K[3])}}{(K[3]-1)^3\sqrt{2\int_1^{K[3]} -\frac{\exp(-2(6(K[1]-1)+3\log(K[1]-1)))h(K[1])}{K[1]-1}dK[1]-c_1}}dK[3]\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{e^{\frac{1}{2}(12-12K[3])}}{(K[3]-1)^3\sqrt{c_1+2\int_1^{K[3]} -\frac{\exp(-2(6(K[1]-1)+3\log(K[1]-1)))h(K[1])}{K[1]-1}dK[1]}}dK[3]\&\right][x+c_2]$$

7.202 problem 1793 (book 6.202)

Internal problem ID [10115]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1793 (book 6.202).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$ay(y-1)\,y''+(yb+c)\,y'^2+h(y)=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 166

```
dsolve(a*y(x)*(-1+y(x))*diff(diff(y(x),x),x)+(b*y(x)+c)*diff(y(x),x)^2+h(y(x))=0,y(x), sings
```

$$a\left(\int^{y(x)}\frac{\_b^{-\frac{c}{a}}(\_b-1)^{\frac{c+b}{a}}}{\sqrt{-2a\left(-\frac{c_1a}{2}+\int\_b^{\frac{-a-2c}{a}}h(\_b)\,(\_b-1)^{\frac{2c+2b-a}{a}}d\_b\right)}}d\_b\right)-x-c_2=0$$

$$-a\left(\int^{y(x)}\frac{\_b^{-\frac{c}{a}}(\_b-1)^{\frac{c+b}{a}}}{\sqrt{-2a\left(-\frac{c_1a}{2}+\int\_b^{\frac{-a-2c}{a}}h(\_b)\,(\_b-1)^{\frac{2c+2b-a}{a}}d\_b\right)}}d\_b\right)-x-c_2=0$$

✓ Solution by Mathematica

Time used: 2.75 (sec). Leaf size: 698

```
DSolve[h[y[x]] + (c + b*y[x])*y'[x]^2 + a*(-1 + y[x])*y[x]*y''[x] == 0,y[x],x,IncludeSingula
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{(1-K[2])^{\frac{1}{2}\left(\frac{2b}{a}+\frac{2c}{a}\right)}K[2]^{-\frac{c}{a}}}{\sqrt{c_1+2\int_1^{K[2]} -\frac{\exp\left(-2\left(\frac{c\log(K[1])}{a}-\frac{(b+c)\log(1-K[1])}{a}\right)\right)h(K[1])}{a(K[1]-1)K[1]}dK[1]}}dK[2]\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{(1-K[3])^{\frac{1}{2}\left(\frac{2b}{a}+\frac{2c}{a}\right)}K[3]^{-\frac{c}{a}}}{\sqrt{c_1+2\int_1^{K[3]} -\frac{\exp\left(-2\left(\frac{c\log(K[1])}{a}-\frac{(b+c)\log(1-K[1])}{a}\right)\right)h(K[1])}{a(K[1]-1)K[1]}dK[1]}}dK[3]\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{(1-K[2])^{\frac{1}{2}\left(\frac{2b}{a}+\frac{2c}{a}\right)}K[2]^{-\frac{c}{a}}}{\sqrt{2\int_1^{K[2]} -\frac{\exp\left(-2\left(\frac{c\log(K[1])}{a}-\frac{(b+c)\log(1-K[1])}{a}\right)\right)h(K[1])}{a(K[1]-1)K[1]}dK[1]-c_1}}dK[2]\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{(1-K[2])^{\frac{1}{2}\left(\frac{2b}{a}+\frac{2c}{a}\right)}K[2]^{-\frac{c}{a}}}{\sqrt{c_1+2\int_1^{K[2]} -\frac{\exp\left(-2\left(\frac{c\log(K[1])}{a}-\frac{(b+c)\log(1-K[1])}{a}\right)\right)h(K[1])}{a(K[1]-1)K[1]}dK[1]}}dK[2]\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{(1-K[3])^{\frac{1}{2}\left(\frac{2b}{a}+\frac{2c}{a}\right)}K[3]^{-\frac{c}{a}}}{\sqrt{2\int_1^{K[3]} -\frac{\exp\left(-2\left(\frac{c\log(K[1])}{a}-\frac{(b+c)\log(1-K[1])}{a}\right)\right)h(K[1])}{a(K[1]-1)K[1]}dK[1]-c_1}}dK[3]\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{(1-K[3])^{\frac{1}{2}\left(\frac{2b}{a}+\frac{2c}{a}\right)}K[3]^{-\frac{c}{a}}}{\cdots}dK[3]\&\right][x$$

7.203 problem 1794 (book 6.203)

Internal problem ID [10116]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1794 (book 6.203).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$a y(y-1)\, y'' - (-1+a)\,(2y-1)\, y'^2 + f y(y-1)\, y' = 0$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 48

```
dsolve(a*y(x)*(-1+y(x))*diff(diff(y(x),x),x)-(a-1)*(2*y(x)-1)*diff(y(x),x)^2+f*y(x)*(-1+y(x)
```

$$y(x) = 1$$
$$y(x) = 0$$
$$c_1 \mathrm{e}^{-\frac{f x}{a}} - c_2 + \int^{y(x)} \frac{(\_a(\_a-1))^{\frac{1}{a}}}{\_a\,(\_a-1)} d\_a = 0$$

✓ Solution by Mathematica

Time used: 0.198 (sec). Leaf size: 83

```
DSolve[f[x]*(-1 + y[x])*y[x]*y'[x] - (-1 + a)*(-1 + 2*y[x])*y'[x]^2 + a*(-1 + y[x])*y[x]*y''
```

$$y(x) \to \text{InverseFunction}\left[a\#1^{-1/a}(-((\#1-1)\#1))^{\frac{1}{a}} \text{Hypergeometric2F1}\left(\frac{1}{a}, \frac{a-1}{a}, 1+\frac{1}{a}, 1-\#1\right) \&\right]\left[\int_1^x \exp\right.$$
$$\left. + c_2 \right]$$

7.204 problem 1795 (book 6.204)

Internal problem ID [10117]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1795 (book 6.204).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$aby(y-1)\,y'' - ((2ab-a-b)\,y + (1-a)\,b)\,y'^2 + fy(y-1)\,y' = 0$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 58

```
dsolve(a*b*y(x)*(-1+y(x))*diff(diff(y(x),x),x)-((2*a*b-a-b)*y(x)+(1-a)*b)*diff(y(x),x)^2+f*y
```

$$y(x) = 1$$
$$y(x) = 0$$
$$c_1 e^{-\frac{fx}{ba}} + \int^{y(x)} \_a^{\frac{-a+1}{a}} (\_a-1)^{\frac{1-b}{b}} d\_a - c_2 = 0$$

✓ Solution by Mathematica

Time used: 0.25 (sec). Leaf size: 69

```
DSolve[f[x]*(-1 + y[x])*y[x]*y'[x] - ((1 - a)*b + (-a - b + 2*a*b)*y[x])*y'[x]^2 + a*b*(-1 +
```

$$y(x) \to \text{InverseFunction}\left[-a\#1^{\frac{1}{a}}\, \text{Hypergeometric2F1}\left(\frac{1}{a}, 1-\frac{1}{b}, 1 + \frac{1}{a}, \#1\right) \&\right]\left[\int_1^x \exp\left(-\int_1^{K[1]} \frac{f(K[1])}{ab} dK[1]\right) c_1 dK[1] + c_2\right]$$

7.205 problem 1796 (book 6.205)

Internal problem ID [10118]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1796 (book 6.205).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy^2y'' = a}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 793

```
dsolve(x*y(x)^2*diff(diff(y(x),x),x)-a=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x \left(81 c_1^2 a^2 + 18 a c_1 e^{\mathrm{RootOf}\left(243 \,\mathrm{csgn}\left(\frac{1}{c_1}\right) c_1^4 a^2 x - 54 \_Z \,e^{\_Z} a x \,c_1^3 - 3 \,e^{2 \_Z} \,\mathrm{csgn}\left(\frac{1}{c_1}\right) c_1^2 x - 6 \,e^{\_Z} \,\mathrm{csgn}\left(\frac{1}{c_1}\right) c_2 x - 2 \,e^{\_Z} \,\mathrm{csgn}\left(\frac{1}{c_1}\right)\right)} + e \cdots\right.}{\cdots}$$

$$y(x) = \frac{c_1 x \left(81 c_1^2 a^2 + 18 a c_1 e^{\mathrm{RootOf}\left(243 \,\mathrm{csgn}\left(\frac{1}{c_1}\right) c_1^4 a^2 x + 54 \_Z \,e^{\_Z} a x \,c_1^3 - 3 \,e^{2 \_Z} \,\mathrm{csgn}\left(\frac{1}{c_1}\right) c_1^2 x + 6 \,e^{\_Z} \,\mathrm{csgn}\left(\frac{1}{c_1}\right) c_2 x + 2 \,e^{\_Z} \,\mathrm{csgn}\left(\frac{1}{c_1}\right)\right)} + e \cdots\right.}{\cdots}$$

$$y(x) = \frac{c_1 x \left(81 c_1^2 a^2 + 18 a c_1 e^{\mathrm{RootOf}\left(243 \,\mathrm{csgn}\left(\frac{1}{c_1}\right) c_1^4 a^2 x - 54 \_Z \,e^{\_Z} a x \,c_1^3 - 3 \,e^{2 \_Z} \,\mathrm{csgn}\left(\frac{1}{c_1}\right) c_1^2 x + 6 \,e^{\_Z} \,\mathrm{csgn}\left(\frac{1}{c_1}\right) c_2 x + 2 \,e^{\_Z} \,\mathrm{csgn}\left(\frac{1}{c_1}\right)\right)} + e \cdots\right.}{\cdots}$$

$$y(x) = \frac{c_1 x \left(81 c_1^2 a^2 + 18 a c_1 e^{\mathrm{RootOf}\left(243 \,\mathrm{csgn}\left(\frac{1}{c_1}\right) c_1^4 a^2 x + 54 \_Z \,e^{\_Z} a x \,c_1^3 - 3 \,e^{2 \_Z} \,\mathrm{csgn}\left(\frac{1}{c_1}\right) c_1^2 x - 6 \,e^{\_Z} \,\mathrm{csgn}\left(\frac{1}{c_1}\right) c_2 x - 2 \,e^{\_Z} \,\mathrm{csgn}\left(\frac{1}{c_1}\right)\right)} + e \cdots\right.}{\cdots}$$

✓ Solution by Mathematica

Time used: 0.305 (sec). Leaf size: 116

```
DSolve[-a + x*y[x]^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{a\arctan\left(\frac{\sqrt{2}\sqrt{c_1}\left(\frac{y(x)}{x}+\frac{a}{2c_1}\right)}{\sqrt{-\frac{2ay(x)}{x}-\frac{2c_1y(x)^2}{x^2}}}\right)}{2\sqrt{2}{c_1}^{3/2}}-\frac{\sqrt{-\frac{2ay(x)}{x}-\frac{2c_1y(x)^2}{x^2}}}{2c_1}-\frac{1}{x}-c_2=0,y(x)\right]$$

7.206 problem 1797 (book 6.206)

Internal problem ID [10119]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1797 (book 6.206).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _reducible, _mu_x_y1], [_2nd_order,`

$$\boxed{(a^2 - x^2)(a^2 - y^2)y'' + (a^2 - x^2)yy'^2 - x(a^2 - y^2)y' = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 60

```
dsolve((a^2-x^2)*(a^2-y(x)^2)*diff(diff(y(x),x),x)+(a^2-x^2)*y(x)*diff(y(x),x)^2-x*(a^2-y(x)
```

$$y(x) = -a$$
$$y(x) = a$$
$$y(x) = \frac{\left(x+\sqrt{-a^2+x^2}\right)^{c_1} c_2^2 + \left(x+\sqrt{-a^2+x^2}\right)^{-c_1} a^2}{2c_2}$$

✓ Solution by Mathematica
Time used: 0.459 (sec). Leaf size: 195

```
DSolve[-(x*(a^2 - y[x]^2)*y'[x]) + (a^2 - x^2)*y[x]*y'[x]^2 + (a^2 - x^2)*(a^2 - y[x]^2)*y''
```

$$y(x) \to -\frac{1}{2}e^{-c_2}\left(\frac{a^2}{a^2-x^2}\right)^{-\frac{c_1}{2}}\sqrt{-a^2\left(\left(\frac{x}{\sqrt{x^2-a^2}}+1\right)^{c_1} - e^{2c_2}\left(1-\frac{x}{\sqrt{x^2-a^2}}\right)^{c_1}\right)^2}$$
$$y(x) \to \frac{1}{2}e^{-c_2}\left(\frac{a^2}{a^2-x^2}\right)^{-\frac{c_1}{2}}\sqrt{-a^2\left(\left(\frac{x}{\sqrt{x^2-a^2}}+1\right)^{c_1} - e^{2c_2}\left(1-\frac{x}{\sqrt{x^2-a^2}}\right)^{c_1}\right)^2}$$

7.207 problem 1798 (book 6.207)

Internal problem ID [10120]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1798 (book 6.207).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Painleve, ‘5th‘]]`

$$2x^2y(y-1)\,y''-x^2(3y-1)\,y'^2+2xy(y-1)\,y'+\left(ay^2+b\right)(y-1)^3+cxy^2(y-1)+d\,x^2y^2(y+1)=0$$

✗ Solution by Maple

```
dsolve(2*x^2*y(x)*(-1+y(x))*diff(diff(y(x),x),x)-x^2*(3*y(x)-1)*diff(y(x),x)^2+2*x*y(x)*(-1+
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[c*x*(-1 + y[x])*y[x]^2 + d*x^2*y[x]^2*(1 + y[x]) + (-1 + y[x])^3*(b + a*y[x]^2) + 2*x
```

Not solved

7.208 problem 1799 (book 6.208)

Internal problem ID [10121]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1799 (book 6.208).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^3y^2y'' + (x+y)(y'x-y)^3 = 0$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 170

```
dsolve(x^3*y(x)^2*diff(diff(y(x),x),x)+(x+y(x))*(x*diff(y(x),x)-y(x))^3=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \mathrm{RootOf}\left(-2\ln(x) - \left(\int^{\_Z} \frac{i\sqrt{3}\,\mathrm{BesselY}\left(i\sqrt{3}, 2\sqrt{\_f}\right)c_1\sqrt{\_f} + i\sqrt{3}\,\mathrm{BesselJ}\left(i\sqrt{3}, 2\sqrt{\_f}\right)\sqrt{\_f} + \mathrm{BesselY}\left(i\sqrt{3}, 2\sqrt{\_f}\right)c_1 \cdots}{\_f^{\frac{3}{2}}\left(\mathrm{BesselY}\left(i\sqrt{3}, 2\sqrt{\_ \cdots\right.\right.}\right. + 2c_2\right)x$$

✓ Solution by Mathematica

Time used: 36.551 (sec). Leaf size: 248

```
DSolve[(x + y[x])*(-y[x] + x*y'[x])^3 + x^3*y[x]^2*y''[x] == 0,y[x],x,IncludeSingularSolutio
```

$$\text{Solve}\left[-\int_1^{\frac{y(x)}{x}} \frac{i\sqrt{3}\sqrt{K[2]}\,\text{BesselJ}\left(i\sqrt{3},2\sqrt{K[2]}\right)+\sqrt{K[2]}\,\text{BesselJ}\left(i\sqrt{3},2\sqrt{K[2]}\right)-2\,\text{BesselJ}\left(1+i\sqrt{\right.}{\left(\text{BesselJ}\right.} \right.$$

$$\left. -2\log(x)+2c_2=0,y(x)\right]$$

7.209 problem 1800 (book 6.209)

Internal problem ID [10122]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1800 (book 6.209).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y^3 y'' = a}$$

✓ Solution by Maple
Time used: 15.657 (sec). Leaf size: 46

```
dsolve(y(x)^3*diff(diff(y(x),x),x)-a=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\left((c_2+x)^2 c_1^2 + a\right) c_1}}{c_1}$$
$$y(x) = -\frac{\sqrt{\left((c_2+x)^2 c_1^2 + a\right) c_1}}{c_1}$$

✓ Solution by Mathematica
Time used: 4.192 (sec). Leaf size: 63

```
DSolve[-a + y[x]^3*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{a + c_1{}^2(x+c_2)^2}}{\sqrt{c_1}}$$
$$y(x) \to \frac{\sqrt{a + c_1{}^2(x+c_2)^2}}{\sqrt{c_1}}$$
$$y(x) \to \text{Indeterminate}$$

7.210 problem 1801 (book 6.210)

Internal problem ID [10123]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1801 (book 6.210).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$y(y^2+1)\,y''+(1-3y^2)\,y'^2=0$$

✓ Solution by Maple

Time used: 27.406 (sec). Leaf size: 80

```
dsolve(y(x)*(y(x)^2+1)*diff(diff(y(x),x),x)+(1-3*y(x)^2)*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x)=-i$$
$$y(x)=i$$
$$y(x)=0$$
$$y(x)=-\frac{\sqrt{2}\,\sqrt{-2\left(c_1x+c_2+\frac{1}{2}\right)(c_1x+c_2)}}{2c_1x+2c_2}$$
$$y(x)=\frac{\sqrt{2}\,\sqrt{-2\left(c_1x+c_2+\frac{1}{2}\right)(c_1x+c_2)}}{2c_1x+2c_2}$$

✓ Solution by Mathematica

Time used: 2.565 (sec). Leaf size: 223

```
DSolve[(1 - 3*y[x]^2)*y'[x]^2 + y[x]*(1 + y[x]^2)*y''[x] == 0,y[x],x,IncludeSingularSolution
```

$$y(x) \to -\frac{\sqrt{-2c_1x - 1 - 2c_2c_1}}{\sqrt{2}\sqrt{c_1(x + c_2)}}$$
$$y(x) \to \frac{\sqrt{-2c_1x - 1 - 2c_2c_1}}{\sqrt{2}\sqrt{c_1(x + c_2)}}$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to \frac{\sqrt{-c_1}}{\sqrt{c_1}}$$
$$y(x) \to \frac{\sqrt{c_1}}{\sqrt{-c_1}}$$
$$y(x) \to -\frac{\sqrt{-x - c_2}}{\sqrt{x + c_2}}$$
$$y(x) \to \frac{\sqrt{-x - c_2}}{\sqrt{x + c_2}}$$
$$y(x) \to -\frac{\sqrt{x + c_2}}{\sqrt{-x - c_2}}$$
$$y(x) \to \frac{\sqrt{x + c_2}}{\sqrt{-x - c_2}}$$

7.211 problem 1802 (book 6.211)

Internal problem ID [10124]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1802 (book 6.211).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$\boxed{2y^3y'' + y^4 - a^2xy^2 = 1}$$

✗ Solution by Maple

```
dsolve(2*y(x)^3*diff(diff(y(x),x),x)+y(x)^4-a^2*x*y(x)^2-1=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-1 - a^2*x*y[x]^2 + y[x]^4 + 2*y[x]^3*y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

Not solved

7.212 problem 1803 (book 6.212)

Internal problem ID [10125]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1803 (book 6.212).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$2y^3y'' + y^2{y'}^2 = a\,x^2 + bx + c$$

✗ Solution by Maple

```
dsolve(2*y(x)^3*diff(diff(y(x),x),x)+y(x)^2*diff(y(x),x)^2-a*x^2-b*x-c=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-c - b*x - a*x^2 + y[x]^2*y'[x]^2 + 2*y[x]^3*y''[x] == 0,y[x],x,IncludeSingularSoluti
```

Not solved

7.213 problem 1804 (book 6.213)

Internal problem ID [10126]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1804 (book 6.213).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$2(y-a)(y-b)(y-c)y'' - \left((y-a)^2(y-b)(y-c) + (y-b)(y-c)\right)y'^2 + (y-a)^2(y-b)^2(y-c)^2 \Big($$

✓ Solution by Maple

Time used: 40.172 (sec). Leaf size: 1097

```
dsolve(2*(y(x)-a)*(y(x)-b)*(y(x)-c)*diff(y(x),x$2)-( (y(x)-a)*(y(x)-b)*(y(x)-a)*(y(x)-c)+(y(
```

$$\int^{y(x)} \frac{}{\sqrt{(a-\_m)\Big((((a^2+4ab+b^2)A_0+B_0+C_1)c^2+(4ab(a+b)A_0+4C_1a+4B_0b)c+A_0a^2b^2+(B}}$$

$$-x-c_2=0$$

$$-\left(\int^{y(x)} \frac{}{\sqrt{(a-\_m)\Big((((a^2+4ab+b^2)A_0+B_0+C_1)c^2+(4ab(a+b)A_0+4C_1a+4B_0b)c+A_0a^2b^2+}}\right.$$

$$-x-c_2=0$$

✓ Solution by Mathematica

Time used: 72.642 (sec). Leaf size: 3800

```
DSolve[2*(y[x]-a)*(y[x]-b)*(y[x]-c)*y''[x]-( (y[x]-a)*(y[x]-b)*(y[x]-a)*(y[x]-c)+(y[x]-b)*(y
```

Too large to display

7.214 problem 1805 (book 6.214)

Internal problem ID [10127]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1805 (book 6.214).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{(4y^3 - ay - b)\, y'' - \left(6y^2 - \frac{a}{2}\right) y'^2 = 0}$$

✓ Solution by Maple

Time used: 19.813 (sec). Leaf size: 254

```
dsolve((4*y(x)^3-a*y(x)-b)*diff(diff(y(x),x),x)-(6*y(x)^2-1/2*a)*diff(y(x),x)^2=0,y(x), sing
```

$$y(x) = \frac{\left(27b + 3\sqrt{-3a^3 + 81b^2}\right)^{\frac{2}{3}} + 3a}{6\left(27b + 3\sqrt{-3a^3 + 81b^2}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{-i\left(27b + 3\sqrt{-3a^3 + 81b^2}\right)^{\frac{2}{3}}\sqrt{3} + 3i\sqrt{3}\,a - \left(27b + 3\sqrt{-3a^3 + 81b^2}\right)^{\frac{2}{3}} - 3a}{12\left(27b + 3\sqrt{-3a^3 + 81b^2}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{-i\left(27b + 3\sqrt{-3a^3 + 81b^2}\right)^{\frac{2}{3}}\sqrt{3} + 3i\sqrt{3}\,a + \left(27b + 3\sqrt{-3a^3 + 81b^2}\right)^{\frac{2}{3}} + 3a}{12\left(27b + 3\sqrt{-3a^3 + 81b^2}\right)^{\frac{1}{3}}}$$

$$\int^{y(x)} \frac{1}{\sqrt{4\_a^3 - a\_a - b}} d\_a - c_1 x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 12.733 (sec). Leaf size: 416

```
DSolve[(a/2 - 6*y[x]^2)*y'[x]^2 + (-b - a*y[x] + 4*y[x]^3)*y''[x] == 0,y[x],x,IncludeSingula
```

$$\text{Solve}\left[\frac{\sqrt{2}\sqrt{\frac{y(x)-\text{Root}\left[4\#1^3-\#1a-b\&,1\right]}{\text{Root}\left[4\#1^3-\#1a-b\&,3\right]-\text{Root}\left[4\#1^3-\#1a-b\&,1\right]}}\sqrt{\frac{y(x)-\text{Root}\left[4\#1^3-\#1a-b\&,2\right]}{\text{Root}\left[4\#1^3-\#1a-b\&,3\right]-\text{Root}\left[4\#1^3-\#1a-b\&,2\right]}}(}{c_1\sqrt{ay(}}\right.$$

$$\left.+c_2,y(x)\right]$$

7.215 problem 1806 (book 6.215)

Internal problem ID [10128]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1806 (book 6.215).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\left(4y^3-ay-b\right)\left(y''+fy'\right)-\left(6y^2-\frac{a}{2}\right)y'^2=0$$

✓ Solution by Maple
Time used: 19.86 (sec). Leaf size: 257

```
dsolve((4*y(x)^3-a*y(x)-b)*(diff(diff(y(x),x),x)+f*diff(y(x),x))-(6*y(x)^2-1/2*a)*diff(y(x),
```

$$y(x)=\frac{\left(27b+3\sqrt{-3a^3+81b^2}\right)^{\frac{2}{3}}+3a}{6\left(27b+3\sqrt{-3a^3+81b^2}\right)^{\frac{1}{3}}}$$
$$y(x)=\frac{-i\left(27b+3\sqrt{-3a^3+81b^2}\right)^{\frac{2}{3}}\sqrt{3}+3i\sqrt{3}\,a-\left(27b+3\sqrt{-3a^3+81b^2}\right)^{\frac{2}{3}}-3a}{12\left(27b+3\sqrt{-3a^3+81b^2}\right)^{\frac{1}{3}}}$$
$$y(x)=-\frac{-i\left(27b+3\sqrt{-3a^3+81b^2}\right)^{\frac{2}{3}}\sqrt{3}+3i\sqrt{3}\,a+\left(27b+3\sqrt{-3a^3+81b^2}\right)^{\frac{2}{3}}+3a}{12\left(27b+3\sqrt{-3a^3+81b^2}\right)^{\frac{1}{3}}}$$
$$c_1\mathrm{e}^{-fx}-c_2+\int^{y(x)}\frac{1}{\sqrt{4\_a^3-a\_a-b}}d\_a=0$$

✓ Solution by Mathematica

Time used: 10.455 (sec). Leaf size: 438

```
DSolve[(a/2 - 6*y[x]^2)*y'[x]^2 + (-b - a*y[x] + 4*y[x]^3)*(f[x]*y'[x] + y''[x]) == 0,y[x],x
```

$$\text{Solve}\left[\frac{2\sqrt{\frac{y(x)-\text{Root}\left[4\#1^3-\#1a-b\&,1\right]}{\text{Root}\left[4\#1^3-\#1a-b\&,3\right]-\text{Root}\left[4\#1^3-\#1a-b\&,1\right]}}\sqrt{\frac{y(x)-\text{Root}\left[4\#1^3-\#1a-b\&,2\right]}{\text{Root}\left[4\#1^3-\#1a-b\&,3\right]-\text{Root}\left[4\#1^3-\#1a-b\&,2\right]}}\,(y}{\sqrt{ay(x)}}\right.$$

$$\left.-\sqrt{2}\exp\left(-\int_1^{K[1]}f(K[1])dK[1]\right)c_1dK[1]+c_2,y(x)\right]$$

7.216 problem 1806 (book 6.216)

Internal problem ID [10129]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1806 (book 6.216).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `unknown`

$$-2xy(1-x)(1-y)(x-y)y'' + x(1-x)\left(x-2xy-2y+3y^2\right){y'}^2 + 2y(1-y)\left(x^2+y-2xy\right)y' - y^2($$

✗ Solution by Maple

```
dsolve(-2*x*y(x)*(1-x)*(1-y(x))*(x-y(x))*diff(diff(y(x),x),x)+x*(1-x)*(x-2*x*y(x)-2*y(x)+3*y
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-((1 - y[x])^2*y[x]^2) - f[x]*((-1 + y[x])*y[x]*(-x + y[x]))^(3/2) + 2*(1 - y[x])*y[x
```

Not solved

7.217 problem 1808 (book 6.217)

Internal problem ID [10130]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1808 (book 6.217).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Painleve, '6th']]`

$$2x^2y(1-x)^2(1-y)(x-y)y'' - x^2(1-x)^2(x-2xy-2y+3y^2)y'^2 - 2xy(1-x)(1-y)(x^2+y-2x$$

✗ Solution by Maple

```
dsolve(2*x^2*y(x)*(1-x)^2*(1-y(x))*(x-y(x))*diff(diff(y(x),x),x)-x^2*(1-x)^2*(x-2*x*y(x)-2*y
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[b*x*(1 - y[x])^2*(x - y[x])^2 - d*(1 - x)*x*(1 - y[x])^2*y[x]^2 - c*(1 - x)*(x - y[x]
```

Not solved

7.218 problem 1809 (book 6.218)

Internal problem ID [10131]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1809 (book 6.218).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$(y^2-1)\left(a^2y^2-1\right)y''+b\sqrt{(1-y^2)\left(1-a^2y^2\right)}\,y'^2+\left(1+a^2-2a^2y^2\right)yy'^2=0$$

✗ Solution by Maple

```
dsolve((y(x)^2-1)*(a^2*y(x)^2-1)*diff(diff(y(x),x),x)+b*((1-y(x)^2)*(1-a^2*y(x)^2))^(1/2)*di
```

No solution found

✓ Solution by Mathematica

Time used: 22.452 (sec). Leaf size: 372

```
DSolve[y[x]*(1 + a^2 - 2*a^2*y[x]^2)*y'[x]^2 + b*Sqrt[(1 - y[x]^2)*(1 - a^2*y[x]^2)]*y'[x]^2
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\exp\left(\frac{b\sqrt{1-K[1]^2}\sqrt{1-a^2K[1]^2}\,\text{EllipticF}\left(\arcsin(K[1]),a^2\right)}{\sqrt{(K[1]^2-1)(a^2K[1]^2-1)}} + \frac{1}{2}(-\log(1-K[1]) - \log(K[1]+\right.}{c_1} + c_2\right]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{\exp\left(\frac{b\sqrt{1-K[1]^2}\sqrt{1-a^2K[1]^2}\,\text{EllipticF}\left(\arcsin(K[1]),a^2\right)}{\sqrt{(K[1]^2-1)(a^2K[1]^2-1)}} + \frac{1}{2}(-\log(1-K[1]) - \log(K[1]+1) - \log(1-aK[1])\right.}{c_1} + c_2\right]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\exp\left(\frac{b\sqrt{1-K[1]^2}\sqrt{1-a^2K[1]^2}\,\text{EllipticF}\left(\arcsin(K[1]),a^2\right)}{\sqrt{(K[1]^2-1)(a^2K[1]^2-1)}} + \frac{1}{2}(-\log(1-K[1]) - \log(K[1]+\right.}{c_1} + c_2\right]$$

7.219 problem 1810 (book 6.219)

Internal problem ID [10132]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1810 (book 6.219).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$\left(c+2bx+a\,x^{2}+y^{2}\right)^{2} y''+yd=0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 336

```
dsolve((c+2*b*x+a*x^2+y(x)^2)^2*diff(diff(y(x),x),x)+d*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{RootOf}\left(a\left(\int^{\_Z}\frac{\sqrt{\left(\_f^{2}+1\right)\left(-\_f^{4}ac+\_f^{4}b^{2}+\_f^{2}a^{2}c_{1}-c\,\_f^{2}a+b^{2}\_f^{2}+a^{2}c_{1}+d\right)}}{-\_f^{4}ac+\_f^{4}b^{2}+\_f^{2}a^{2}c_{1}-c\,\_f^{2}a+b^{2}\_f^{2}+a^{2}c_{1}+d}d\_f\right)\sqrt{ac-b^{2}} \right. \cdots$$
$$\left. - a\arctan\left(\frac{ax+b}{\sqrt{ac-b^{2}}}\right)+c_{2}\sqrt{ac-b^{2}}\right)\sqrt{a\,x^{2}+2bx+c}$$

$$y(x) = \mathrm{RootOf}\left(-a\left(\int^{\_Z}\frac{\sqrt{\left(\_f^{2}+1\right)\left(-\_f^{4}ac+\_f^{4}b^{2}+\_f^{2}a^{2}c_{1}-c\,\_f^{2}a+b^{2}\_f^{2}+a^{2}c_{1}+d\right)}}{-\_f^{4}ac+\_f^{4}b^{2}+\_f^{2}a^{2}c_{1}-c\,\_f^{2}a+b^{2}\_f^{2}+a^{2}c_{1}+d}d\_f\right)\sqrt{ac-} \right. \cdots$$
$$\left. - a\arctan\left(\frac{ax+b}{\sqrt{ac-b^{2}}}\right)+c_{2}\sqrt{ac-b^{2}}\right)\sqrt{a\,x^{2}+2bx+c}$$

✓ Solution by Mathematica

Time used: 65.538 (sec). Leaf size: 260

```
DSolve[d*y[x] + (c + 2*b*x + a*x^2 + y[x]^2)^2*y''[x] == 0,y[x],x,IncludeSingularSolutions -
```

$$\text{Solve}\left[a\arctan\left(\frac{ax+b}{\sqrt{ac-b^2}}\right)+\sqrt{ac-b^2}\int_1^{\frac{y(x)}{\sqrt{c+x(2b+ax)}}}\frac{a(K[2]^2+1)}{\sqrt{(K[2]^2+1)\left(d+(K[2]^2+1)\left(c_1a^2+(b^2-ac)K[2]^2\right)\right)}}dK[2]=c_2\sqrt{ac-b^2},y(x)\right.$$

$$\text{Solve}\left[a\arctan\left(\frac{ax+b}{\sqrt{ac-b^2}}\right)-\sqrt{ac-b^2}\int_1^{\frac{y(x)}{\sqrt{c+x(2b+ax)}}}\frac{a(K[3]^2+1)}{\sqrt{(K[3]^2+1)\left(d+(K[3]^2+1)\left(c_1a^2+(b^2-ac)K[3]^2\right)\right)}}dK[3]=c_2\sqrt{ac-b^2},y(x)\right.$$

7.220 problem 1811 (book 6.220)

Internal problem ID [10133]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1811 (book 6.220).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{\sqrt{y}\, y'' = a}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 81

```
dsolve(y(x)^(1/2)*diff(diff(y(x),x),x)-a=0,y(x), singsol=all)
```

$$\frac{\left(-2a\sqrt{y\left(x\right)}-c_1\right)\sqrt{4a\sqrt{y\left(x\right)}-c_1}-6a^2(c_2+x)}{6a^2}=0$$
$$\frac{\left(2a\sqrt{y\left(x\right)}+c_1\right)\sqrt{4a\sqrt{y\left(x\right)}-c_1}-6a^2(c_2+x)}{6a^2}=0$$

✓ Solution by Mathematica

Time used: 60.111 (sec). Leaf size: 1881

```
DSolve[-a + Sqrt[y[x]]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$y(x)$

$$\rightarrow \frac{288a^4c_1x^2 + 576a^4c_1c_2x + 288a^4c_1c_2{}^2 + a^4\left(\frac{10368a^8x^4+41472a^8c_2x^3+62208a^8c_2{}^2x^2+41472a^8c_2{}^3x+10368a^8c_2{}^4+720a^4c_1{}^3 \cdots}{a^6 \cdots}\right. \cdots}{16a^4\sqrt[3]{\frac{10368a^8 \cdots}{\cdots}} \cdots}$$

$y(x)$

$$\rightarrow \frac{-288i\sqrt{3}a^4c_1x^2 - 288a^4c_1x^2 - 576i\sqrt{3}a^4c_1c_2x - 576a^4c_1c_2x - 288i\sqrt{3}a^4c_1c_2{}^2 - 288a^4c_1c_2{}^2 + i\sqrt{3}a^4\left(\cdots \right.}{\cdots}$$

$y(x)$

$$\rightarrow \frac{288i\sqrt{3}a^4c_1x^2 - 288a^4c_1x^2 + 576i\sqrt{3}a^4c_1c_2x - 576a^4c_1c_2x + 288i\sqrt{3}a^4c_1c_2{}^2 - 288a^4c_1c_2{}^2 - i\sqrt{3}a^4\left(\frac{10 \cdots}{\cdots}\right. \cdots}{\cdots}$$

7.221 problem 1812 (book 6.221)

Internal problem ID [10134]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1812 (book 6.221).
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\sqrt{y^2+x^2}\, y'' - a\left(y'^2+1\right)^{\frac{3}{2}} = 0$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 88

```
dsolve((y(x)^2+x^2)^(1/2)*diff(diff(y(x),x),x)-a*(diff(y(x),x)^2+1)^(3/2)=0,y(x), singsol=al
```

$$y(x) = -ix$$
$$y(x) = ix$$
$$y(x) = \mathrm{RootOf}\left(-\ln(x) + \int^{\_Z} -\frac{-\mathrm{RootOf}\left(\arctan(\_g) + \int^{-\_Z} \frac{1+\sqrt{a^2(\_f^2+1)}}{(\_f^2 a^2+a^2-1)(\_f^2+1)} d\_f + c_1\right) + \_g}{\_g^2+1} d\_g + c_2\right) x$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-(a*(1 + y'[x]^2)^(3/2)) + Sqrt[x^2 + y[x]^2]*y''[x] == 0,y[x],x,IncludeSingularSolut
```

Timed out

7.222 problem 1813 (book 6.222)

Internal problem ID [10135]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1813 (book 6.222).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{y(1-\ln(y))\,y''+(1+\ln(y))\,{y'}^2=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(y(x)*(1-ln(y(x)))*diff(diff(y(x),x),x)+(1+ln(y(x)))*diff(y(x),x)^2=0,y(x), singsol=al
```

$$y(x)=\mathrm{e}^{\frac{c_1x+c_2-1}{c_1x+c_2}}$$

✓ Solution by Mathematica

Time used: 1.01 (sec). Leaf size: 34

```
DSolve[(1 + Log[y[x]])*y'[x]^2 + (1 - Log[y[x]])*y[x]*y''[x] == 0,y[x],x,IncludeSingularSolu
```

$$y(x)\to e^{\frac{c_1x-1+c_2c_1}{c_1(x+c_2)}}$$
$$y(x)\to e$$

7.223 problem 1814 (book 6.223)

Internal problem ID [10136]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1814 (book 6.223).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\left(b+a\sin\left(y\right)^{2}\right)y''+ay'^{2}\cos\left(y\right)\sin\left(y\right)+Ay\left(c+a\sin\left(y\right)^{2}\right)=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 140

```
dsolve((b+a*sin(y(x))^2)*diff(diff(y(x),x),x)+a*diff(y(x),x)^2*cos(y(x))*sin(y(x))+A*y(x)*(c
```

$$\sqrt{2}\left(\int^{y(x)}\frac{b+a\sin\left(\_a\right)^{2}}{\sqrt{-\left(Aa\sin\left(\_a\right)^{2}-2Aa\_a\cos\left(\_a\right)\sin\left(\_a\right)+\_a^{2}\left(a+2c\right)A-2c_{1}\right)\left(b+a\sin\left(\_a\right)^{2}\right)}}d\_a\right)$$
$$-x-c_{2}=0$$

$$-\sqrt{2}\left(\int^{y(x)}\frac{b+a\sin\left(\_a\right)^{2}}{\sqrt{-\left(Aa\sin\left(\_a\right)^{2}-2Aa\_a\cos\left(\_a\right)\sin\left(\_a\right)+\_a^{2}\left(a+2c\right)A-2c_{1}\right)\left(b+a\sin\left(\_a\right)^{2}\right)}}d\_a$$
$$-x-c_{2}=0$$

✓ Solution by Mathematica

Time used: 56.768 (sec). Leaf size: 530

```
DSolve[A*(c + a*Sin[y[x]]^2)*y[x] + a*Cos[y[x]]*Sin[y[x]]*y'[x]^2 + (b + a*Sin[y[x]]^2)*y''[
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{\sqrt{2}\sqrt{\cos(2K[1])a - a - 2b}}{\sqrt{2aAK[1]^2 + 4AcK[1]^2 - 2aA\sin(2K[1])K[1] + 2c_1 - aA\cos(2K[1])}} dK[1]\&\right][x + c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sqrt{2}\sqrt{\cos(2K[2])a - a - 2b}}{\sqrt{2aAK[2]^2 + 4AcK[2]^2 - 2aA\sin(2K[2])K[2] + 2c_1 - aA\cos(2K[2])}} dK[2]\&\right] + c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{\sqrt{2}\sqrt{\cos(2K[1])a - a - 2b}}{\sqrt{2aAK[1]^2 + 4AcK[1]^2 - 2aA\sin(2K[1])K[1] + 2(-1)c_1 - aA\cos(2K[1])}} dK[1]\&\right][x + c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{\sqrt{2}\sqrt{\cos(2K[1])a - a - 2b}}{\sqrt{2aAK[1]^2 + 4AcK[1]^2 - 2aA\sin(2K[1])K[1] + 2c_1 - aA\cos(2K[1])}} dK[1]\&\right][x + c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sqrt{2}\sqrt{\cos(2K[2])a - a - 2b}}{\sqrt{2aAK[2]^2 + 4AcK[2]^2 - 2aA\sin(2K[2])K[2] + 2(-1)c_1 - aA\cos(2K[2])}} dK[+ c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sqrt{2}\sqrt{\cos(2K[2])a - a - 2b}}{\sqrt{2aAK[2]^2 + 4AcK[2]^2 - 2aA\sin(2K[2])K[2] + 2c_1 - aA\cos(2K[2])}} dK[2]\&\right] + c_2]$$

7.224 problem 1815 (book 6.224)

Internal problem ID [10137]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1815 (book 6.224).
ODE order: 2.
ODE degree: 0.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$h(y)\,y'' + aD(h)\,(y)\,y'^2 + j(y) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 79

```
dsolve(h(y(x))*diff(diff(y(x),x),x)+a*D(h)(y(x))*diff(y(x),x)^2+j(y(x))=0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{h(\_b)^a}{\sqrt{-2\left(\int j\left(\_b\right) h\left(\_b\right)^{-1+2a} d\_b\right) + c_1}} d\_b - x - c_2 = 0$$

$$-\left(\int^{y(x)} \frac{h(\_b)^a}{\sqrt{-2\left(\int j\left(\_b\right) h\left(\_b\right)^{-1+2a} d\_b\right) + c_1}} d\_b\right) - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 0.935 (sec). Leaf size: 362

```
DSolve[j[y[x]] + a*h[y[x]]*y'[x]^2 + h[y[x]]*y''[x] == 0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{e^{aK[2]}}{\sqrt{c_1 + 2\int_1^{K[2]} -\frac{e^{2aK[1]}j(K[1])}{h(K[1])}dK[1]}}dK[2]\&\right][x + c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{e^{aK[3]}}{\sqrt{c_1 + 2\int_1^{K[3]} -\frac{e^{2aK[1]}j(K[1])}{h(K[1])}dK[1]}}dK[3]\&\right][x + c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{e^{aK[2]}}{\sqrt{2\int_1^{K[2]} -\frac{e^{2aK[1]}j(K[1])}{h(K[1])}dK[1] - c_1}}dK[2]\&\right][x + c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{e^{aK[2]}}{\sqrt{c_1 + 2\int_1^{K[2]} -\frac{e^{2aK[1]}j(K[1])}{h(K[1])}dK[1]}}dK[2]\&\right][x + c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{e^{aK[3]}}{\sqrt{2\int_1^{K[3]} -\frac{e^{2aK[1]}j(K[1])}{h(K[1])}dK[1] - c_1}}dK[3]\&\right][x + c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{e^{aK[3]}}{\sqrt{c_1 + 2\int_1^{K[3]} -\frac{e^{2aK[1]}j(K[1])}{h(K[1])}dK[1]}}dK[3]\&\right][x + c_2]$$

7.225 problem 1816 (book 6.225)

Internal problem ID [10138]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1816 (book 6.225).
ODE order: 2.
ODE degree: 0.

CAS Maple gives this as type `[NONE]`

$$h(y)\,y'' - D(h)\,(y)\,y'^2 - h(y)^2\,j\left(x, \frac{y'}{h\,(y)}\right) = 0$$

✗ Solution by Maple

```
dsolve(h(y(x))*diff(diff(y(x),x),x)-D(h)(y(x))*diff(y(x),x)^2-h(y(x))^2*j(x,diff(y(x),x)/h(y
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-(h[y[x]]^2*j[x, y'[x]/h[y[x]]]) - h[y[x]]*y'[x]^2 + h[y[x]]*y''[x] == 0,y[x],x,Inclu
```

Not solved

7.226 problem 1817 (book 6.226)

Internal problem ID [10139]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1817 (book 6.226).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _nonlinear], [_2nd_order, _with_linear_s`

$$y'y'' - x^2yy' - y^2x = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)*diff(diff(y(x),x),x)-x^2*y(x)*diff(y(x),x)-x*y(x)^2=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-(x*y[x]^2) - x^2*y[x]*y'[x] + y'[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> T
```

Not solved

7.227 problem 1818 (book 6.227)

Internal problem ID [10140]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1818 (book 6.227).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(y'x - y)\,y'' + 4{y'}^2 = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 44

```
dsolve((x*diff(y(x),x)-y(x))*diff(diff(y(x),x),x)+4*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \mathrm{e}^{\int^{\ln(x)}\left(\mathrm{e}^{\mathrm{RootOf}\left(\ln\left(\mathrm{e}^{\_Z}-1\right)\mathrm{e}^{\_Z}+c_1\mathrm{e}^{\_Z}-\_Z\,\mathrm{e}^{\_Z}-\_b\,\mathrm{e}^{\_Z}+2\right)}-1\right)d\_b+c_2}$$

✓ Solution by Mathematica

Time used: 75.536 (sec). Leaf size: 41

```
DSolve[4*y'[x]^2 + (-y[x] + x*y'[x])*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}c_2 e^{-2-W\left(\frac{2x}{e^2c_1}\right)}\left(2 + W\left(\frac{2x}{e^2c_1}\right)\right)$$

7.228 problem 1819 (book 6.228)

Internal problem ID [10141]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1819 (book 6.228).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(y'x - y)\, y'' - \left(y'^2 + 1\right)^2 = 0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 80

```
dsolve((x*diff(y(x),x)-y(x))*diff(diff(y(x),x),x)-(diff(y(x),x)^2+1)^2=0,y(x), singsol=all)
```

$$y(x) = -ix$$
$$y(x) = ix$$
$$y(x) = \operatorname{RootOf}\left(-\ln(x) + \int^{\_Z} \frac{-\_f + \operatorname{RootOf}\left(-c_1 \tan\left(\frac{1}{\_Z}\right)\_Z + \_fc_1 \tan\left(\frac{1}{\_Z}\right) + c_1\_Z\_f + \tan\left(\frac{1}{\_Z}\right)\_Z\_f + c_1 + \tan\left(\right.\right.}{\_f^2 + 1} + c_2\right) x$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-(1 + y'[x]^2)^2 + (-y[x] + x*y'[x])*y''[x] == 0,y[x],x,IncludeSingularSolutions -> T
```

Not solved

7.229 problem 1820 (book 6.229)

Internal problem ID [10142]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1820 (book 6.229).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{a\,x^3y'y''+by^2=0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 46

```
dsolve(a*x^3*diff(y(x),x)*diff(diff(y(x),x),x)+y(x)^2*b=0,y(x), singsol=all)
```

$$y(x)=0$$
$$y(x)=\mathrm{e}^{\int^{\ln(x)}\operatorname{RootOf}\left(-a\left(\int^{-Z}\frac{\_a}{\_a^3a-a\_a^2+b}d\_a\right)-\_b+c_1\right)d\_b+c_2}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[b*y[x]^2 + a*x^3*y'[x]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.230 problem 1821 (book 6.230)

Internal problem ID [10143]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1821 (book 6.230).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(\mathrm{f1}\, y' + \mathrm{f2}\, y)\, y'' + \mathrm{f3}\, {y'}^2 + \mathrm{f4}\,(x)\, y y' + \mathrm{f5}\,(x)\, y^2 = 0$$

✗ Solution by Maple

```
dsolve((f1*diff(y(x),x)+f2*y(x))*diff(diff(y(x),x),x)+f3*diff(y(x),x)^2+f4(x)*y(x)*diff(y(x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[f5[x]*y[x]^2 + f4[x]*y[x]*y'[x] + f3[x]*y'[x]^2 + (f2[x]*y[x] + f1[x]*y'[x])*y''[x] =
```

Timed out

7.231 problem 1822 (book 6.231)

Internal problem ID [10144]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1822 (book 6.231).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _nonlinear], [_2nd_order, _with_linear_s`

$$(2y'y^2 + x^2)\, y'' + 2yy'^3 + 3y'x + y = 0$$

✗ Solution by Maple

```
dsolve((2*y(x)^2*diff(y(x),x)+x^2)*diff(diff(y(x),x),x)+2*y(x)*diff(y(x),x)^3+3*x*diff(y(x),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x] + 3*x*y'[x] + 2*y[x]*y'[x]^3 + (x^2 + 2*y[x]^2*y'[x])*y''[x] == 0,y[x],x,Include
```

Not solved

7.232 problem 1823 (book 6.232)

Internal problem ID [10145]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1823 (book 6.232).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\left(y'^2 + y^2\right) y'' + y^3 = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 163

```
dsolve((diff(y(x),x)^2+y(x)^2)*diff(diff(y(x),x),x)+y(x)^3=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \frac{\sqrt{c_1 + \tan\left(\sqrt{3}\, x\right)}\, e^{-\frac{\sqrt{3}\left(\int \frac{\sqrt{\left(9c_1^2+12\right)\sec\left(\sqrt{3}\, x\right)^2+3c_1^2+6c_1\tan\left(\sqrt{3}\, x\right)-3}}{c_1+\tan\left(\sqrt{3}\, x\right)}dx\right)}{6}+c_2}}{\left(\sec\left(\sqrt{3}\, x\right)^2\right)^{\frac{1}{4}}}$$

$$y(x) = \frac{\sqrt{c_1 + \tan\left(\sqrt{3}\, x\right)}\, e^{\frac{\sqrt{3}\left(\int \frac{\sqrt{\left(9c_1^2+12\right)\sec\left(\sqrt{3}\, x\right)^2+3c_1^2+6c_1\tan\left(\sqrt{3}\, x\right)-3}}{c_1+\tan\left(\sqrt{3}\, x\right)}dx\right)}{6}+c_2}}{\left(\sec\left(\sqrt{3}\, x\right)^2\right)^{\frac{1}{4}}}$$

✓ Solution by Mathematica

Time used: 60.991 (sec). Leaf size: 369

```
DSolve[y[x]^3 + (y[x]^2 + y'[x]^2)*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_2 \exp\left(- \frac{\arctan\left(\frac{1+2\text{InverseFunction}\left[\frac{\left(\sqrt{3}-i\right)\arctan \cdots}{\sqrt{6(\cdots}} \cdots \right.}{\cdots} \right.}{\cdots} \right.}{\sqrt[4]{\text{InverseFunction}\left[\frac{\left(\sqrt{3}-i\right)\arctan\left(\frac{\#1}{\sqrt{\frac{1}{2}\left(1-i\sqrt{3}\right)}}\right)}{\sqrt{6\left(1-i\sqrt{3}\right)}} + \frac{\left(\sqrt{3}+i\right)\arctan\left(\frac{\#1}{\sqrt{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right)}{\sqrt{6\left(1+i\sqrt{3}\right)}} \& \right] [-x+c_1]^4 + \cdots}}$$

7.233 problem 1824 (book 6.233)

Internal problem ID [10146]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1824 (book 6.233).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(y'^2 + a(y'x - y)\right) y'' = b}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 289

```
dsolve((diff(y(x),x)^2+a*(x*diff(y(x),x)-y(x)))*diff(diff(y(x),x),x)-b=0,y(x), singsol=all)
```

$$y(x) = -\frac{a\,x^2}{4} + \mathrm{RootOf}\left(-x + \int^{\_Z} \frac{\sqrt{(\_f^2 a^2 - 4\_fb + 2c_1)\left(a\_f + \sqrt{4\_fb - 2c_1}\right)}}{\_f^2 a^2 - 4\_fb + 2c_1} d\_f + c_2\right)$$

$$y(x) = -\frac{a\,x^2}{4} + \mathrm{RootOf}\left(-x + \int^{\_Z} \frac{\sqrt{(\_f^2 a^2 - 4\_fb + 2c_1)\left(a\_f - \sqrt{4\_fb - 2c_1}\right)}}{\_f^2 a^2 - 4\_fb + 2c_1} d\_f + c_2\right)$$

$$y(x) = -\frac{a\,x^2}{4} + \mathrm{RootOf}\left(-x - \left(\int^{\_Z} \frac{\sqrt{(\_f^2 a^2 - 4\_fb + 2c_1)\left(a\_f + \sqrt{4\_fb - 2c_1}\right)}}{\_f^2 a^2 - 4\_fb + 2c_1} d\_f\right) + c_2\right)$$

$$y(x) = -\frac{a\,x^2}{4} + \mathrm{RootOf}\left(-x - \left(\int^{\_Z} \frac{\sqrt{(\_f^2 a^2 - 4\_fb + 2c_1)\left(a\_f - \sqrt{4\_fb - 2c_1}\right)}}{\_f^2 a^2 - 4\_fb + 2c_1} d\_f\right) + c_2\right)$$

✓ Solution by Mathematica

Time used: 0.617 (sec). Leaf size: 281

```
DSolve[-b + (y'[x]^2 + a*(-y[x] + x*y'[x]))*y''[x] == 0,y[x],x,IncludeSingularSolutions -> T
```

$$\text{Solve}\left[-\int \frac{a\left(\frac{ax^2}{4}+y(x)\right)+\sqrt{4b\left(\frac{ax^2}{4}+y(x)\right)-2c_1}}{\sqrt{\left(a^2\left(\frac{ax^2}{4}+y(x)\right)^2-4b\left(\frac{ax^2}{4}+y(x)\right)+2c_1\right)\left(a\left(\frac{ax^2}{4}+y(x)\right)+\sqrt{4b\left(\frac{ax^2}{4}+y(x)\right)-2c_1}\right)}}d\Big(\right.$$

$$\left.+y(x)\Big)=-x+c_2,y(x)\right]$$

$$\text{Solve}\left[\int \frac{a\left(\frac{ax^2}{4}+y(x)\right)+\sqrt{4b\left(\frac{ax^2}{4}+y(x)\right)-2c_1}}{\sqrt{\left(a^2\left(\frac{ax^2}{4}+y(x)\right)^2-4b\left(\frac{ax^2}{4}+y(x)\right)+2c_1\right)\left(a\left(\frac{ax^2}{4}+y(x)\right)+\sqrt{4b\left(\frac{ax^2}{4}+y(x)\right)-2c_1}\right)}}d\Big(\frac{a}{4}\right.$$

$$\left.+y(x)\Big)=-x+c_2,y(x)\right]$$

7.234 problem 1825 (book 6.234)

Internal problem ID [10147]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1825 (book 6.234).
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\left(a\sqrt{y'^2+1}-y'x\right)y''-y'^2=1$$

✓ Solution by Maple

Time used: 0.265 (sec). Leaf size: 124

```
dsolve((a*(diff(y(x),x)^2+1)^(1/2)-x*diff(y(x),x))*diff(diff(y(x),x),x)-diff(y(x),x)^2-1=0,y
```

$$y(x) = -ix + c_1$$
$$y(x) = ix + c_1$$
$$y(x) = \frac{c_2 a + \int \frac{-c_1 a^2 + x\sqrt{a^2(a^2+c_1^2-x^2)}}{a^2-x^2} dx}{a}$$
$$y(x) = \frac{c_2 a - \left(\int \frac{c_1 a^2 + x\sqrt{a^2(a^2+c_1^2-x^2)}}{a^2-x^2} dx\right)}{a}$$

✓ Solution by Mathematica

Time used: 61.023 (sec). Leaf size: 331

```
DSolve[-1 - y'[x]^2 + (-(x*y'[x]) + a*Sqrt[1 + y'[x]^2])*y''[x] == 0,y[x],x,IncludeSingularS
```

$$y(x) \to -\frac{\sqrt{x^2\left(a^2-x^2+c_1{}^2\right)}\left(c_1 \arctan\left(\frac{a^2-ax+c_1{}^2}{c_1\sqrt{-a^2+x^2-c_1{}^2}}\right)+c_1 \arctan\left(\frac{a^2+ax+c_1{}^2}{c_1\sqrt{-a^2+x^2-c_1{}^2}}\right)+2\sqrt{-a^2+x^2-c_1{}^2}\right)}{2x\sqrt{-a^2+x^2-c_1{}^2}} + c_1\left(-\operatorname{arctanh}\left(\frac{x}{a}\right)\right)+c_2$$

$$y(x) \to \frac{\sqrt{x^2\left(a^2-x^2+c_1{}^2\right)}\left(c_1 \arctan\left(\frac{a^2-ax+c_1{}^2}{c_1\sqrt{-a^2+x^2-c_1{}^2}}\right)+c_1 \arctan\left(\frac{a^2+ax+c_1{}^2}{c_1\sqrt{-a^2+x^2-c_1{}^2}}\right)+2\sqrt{-a^2+x^2-c_1{}^2}\right)}{2x\sqrt{-a^2+x^2-c_1{}^2}} + c_1\left(-\operatorname{arctanh}\left(\frac{x}{a}\right)\right)+c_2$$

7.235 problem 1826 (book 6.235)

Internal problem ID [10148]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1826 (book 6.235).
ODE order: 2.
ODE degree: 0.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], [`

$$h(y')\,y'' + j(y)\,y' = -f$$

✗ Solution by Maple

```
dsolve(h(diff(y(x),x))*diff(diff(y(x),x),x)+j(y(x))*diff(y(x),x)+f=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[f[x] + j[y[x]]*y'[x] + h[y'[x]]*y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.236 problem 1827 (book 6.236)

Internal problem ID [10149]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1827 (book 6.236).
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$y''^2 - ay = b$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 206

```
dsolve(diff(diff(y(x),x),x)^2-a*y(x)-b=0,y(x), singsol=all)
```

$$y(x) = -\frac{b}{a}$$

$$\sqrt{3}\,a\left(\int^{y(x)}\frac{1}{\sqrt{a\left(4\_a a\sqrt{a\_a+b}\,a+4\sqrt{a\_a+b}\,b-c_1\right)}}d\_a\right)-x-c_2=0$$

$$-\sqrt{3}\,a\left(\int^{y(x)}\frac{1}{\sqrt{a\left(4\_a a\sqrt{a\_a+b}\,a+4\sqrt{a\_a+b}\,b-c_1\right)}}d\_a\right)-x-c_2=0$$

$$-\sqrt{3}\,a\left(\int^{y(x)}\frac{1}{\sqrt{-a\left(4\_a a\sqrt{a\_a+b}\,a+4\sqrt{a\_a+b}\,b-c_1\right)}}d\_a\right)-x-c_2=0$$

$$\sqrt{3}\,a\left(\int^{y(x)}\frac{1}{\sqrt{-a\left(4\_a a\sqrt{a\_a+b}\,a+4\sqrt{a\_a+b}\,b-c_1\right)}}d\_a\right)-x-c_2=0$$

✓ Solution by Mathematica

Time used: 2.116 (sec). Leaf size: 201

```
DSolve[-b - a*y[x] + y''[x]^2 == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{(ay(x)+b)^2\left(1-\frac{4(ay(x)+b)^{3/2}}{3ac_1}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{4(b+ay(x))^{3/2}}{3ac_1}\right)^2}{a^2\left(-\frac{4(ay(x)+b)^{3/2}}{3a}+c_1\right)}=(x+c_2)^2,y(x)\right]$$

$$\text{Solve}\left[\frac{(ay(x)+b)^2\left(1+\frac{4(ay(x)+b)^{3/2}}{3ac_1}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{4(b+ay(x))^{3/2}}{3ac_1}\right)^2}{a^2\left(\frac{4(ay(x)+b)^{3/2}}{3a}+c_1\right)}=(x+c_2)^2,y(x)\right]$$

7.237 problem 1828 (book 6.237)

Internal problem ID [10150]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1828 (book 6.237).
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{a^2 y''^2 - 2axy'' + y' = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 1364

```
dsolve(a^2*diff(diff(y(x),x),x)^2-2*a*x*diff(diff(y(x),x),x)+diff(y(x),x)=0,y(x), singsol=al
```

$$
\begin{aligned}
y(x) &= \int \operatorname{RootOf}\Big(8\_Z^{1-2a}a^3\sqrt{x^2-\_Z}\left(-x+\sqrt{x^2-\_Z}\right)^{-2a}\left(x+\sqrt{x^2-\_Z}\right)^{2a}(4\_Z a^2 \\
&\quad -4a\,x^2+x^2)^{-2a}\left(2a\sqrt{x^2-\_Z}-2ax+x\right)^{2a}\left(2a\sqrt{x^2-\_Z}+2ax-x\right)^{-2a} \\
&\quad -8\sqrt{x^2-\_Z}\,\_Z^{-2a}\left(x+\sqrt{x^2-\_Z}\right)^{2a}\left(-x+\sqrt{x^2-\_Z}\right)^{-2a}(4\_Z a^2-4a\,x^2 \\
&\quad +x^2)^{-2a}\left(2a\sqrt{x^2-\_Z}+2ax-x\right)^{-2a}\left(2a\sqrt{x^2-\_Z}-2ax+x\right)^{2a}a^2x^2 \\
&\quad +8x\,\_Z^{1-2a}a^3\left(-x+\sqrt{x^2-\_Z}\right)^{-2a}\left(x+\sqrt{x^2-\_Z}\right)^{2a}(4\_Z a^2-4a\,x^2 \\
&\quad +x^2)^{-2a}\left(2a\sqrt{x^2-\_Z}-2ax+x\right)^{2a}\left(2a\sqrt{x^2-\_Z}+2ax-x\right)^{-2a} \\
&\quad -8\_Z^{-2a}\left(x+\sqrt{x^2-\_Z}\right)^{2a}\left(-x+\sqrt{x^2-\_Z}\right)^{-2a}(4\_Z a^2-4a\,x^2 \\
&\quad +x^2)^{-2a}\left(2a\sqrt{x^2-\_Z}+2ax-x\right)^{-2a}\left(2a\sqrt{x^2-\_Z}-2ax+x\right)^{2a}a^2x^3 \\
&\quad +2\sqrt{x^2-\_Z}\,\_Z^{-2a}\left(x+\sqrt{x^2-\_Z}\right)^{2a}\left(-x+\sqrt{x^2-\_Z}\right)^{-2a}(4\_Z a^2-4a\,x^2 \\
&\quad +x^2)^{-2a}\left(2a\sqrt{x^2-\_Z}+2ax-x\right)^{-2a}\left(2a\sqrt{x^2-\_Z}-2ax+x\right)^{2a}a\,x^2 \\
&\quad -4x\,\_Z^{1-2a}a^2\left(-x+\sqrt{x^2-\_Z}\right)^{-2a}\left(x+\sqrt{x^2-\_Z}\right)^{2a}(4\_Z a^2-4a\,x^2 \\
&\quad +x^2)^{-2a}\left(2a\sqrt{x^2-\_Z}-2ax+x\right)^{2a}\left(2a\sqrt{x^2-\_Z}+2ax-x\right)^{-2a} \\
&\quad +6\_Z^{-2a}\left(x+\sqrt{x^2-\_Z}\right)^{2a}\left(-x+\sqrt{x^2-\_Z}\right)^{-2a}(4\_Z a^2-4a\,x^2 \\
&\quad +x^2)^{-2a}\left(2a\sqrt{x^2-\_Z}+2ax-x\right)^{-2a}\left(2a\sqrt{x^2-\_Z}-2ax+x\right)^{2a}a\,x^3 \\
&\quad -\_Z^{-2a}\left(x+\sqrt{x^2-\_Z}\right)^{2a}\left(-x+\sqrt{x^2-\_Z}\right)^{-2a}(4\_Z a^2-4a\,x^2 \\
&\quad +x^2)^{-2a}\left(2a\sqrt{x^2-\_Z}+2ax-x\right)^{-2a}\left(2a\sqrt{x^2-\_Z}-2ax+x\right)^{2a}x^3 \\
&\quad -2\sqrt{x^2-\_Z}\,c_1a+2c_1ax-c_1x\Big)\,dx+c_2
\end{aligned}
$$

$$
\begin{aligned}
y(x) = \int \operatorname{RootOf}\Big(&2(4\_Z a^2-4a\,x^2+x^2)^{2a}\sqrt{x^2-\_Z}\left(x+\sqrt{x^2-\_Z}\right)^{2a}\Big(-x \\
&+\sqrt{x^2-\_Z}\Big)^{-2a}\left(2a\sqrt{x^2-\_Z}+2ax-x\right)^{-2a}\Big(2a\sqrt{x^2-\_Z}-2ax \\
&+x\Big)^{2a}\_Z^{2a}a+2(4\_Z a^2-4a\,x^2+x^2)^{2a}\left(x+\sqrt{x^2-\_Z}\right)^{2a}\Big(-x \\
&+\sqrt{x^2-\_Z}\Big)^{-2a}\left(2a\sqrt{x^2-\_Z}+2ax-x\right)^{-2a}\Big(2a\sqrt{x^2-\_Z}-2ax \\
&+x\Big)^{2a}\_Z^{2a}ax-(4\_Z a^2-4a\,x^2+x^2)^{2a}\left(x+\sqrt{x^2-\_Z}\right)^{2a}\Big(-x
\end{aligned}
$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x] - 2*a*x*y''[x] + a^2*y''[x]^2 == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.238 problem 1829 (book 6.238)

Internal problem ID [10151]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1829 (book 6.238).
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[NONE]`

$$2(x^2+1)\,y''^2 - xy''(x+4y') + 2(x+y')\,y' - 2y = 0$$

✓ Solution by Maple
Time used: 0.156 (sec). Leaf size: 59

```
dsolve(2*(x^2+1)*diff(diff(y(x),x),x)^2-x*diff(diff(y(x),x),x)*(x+4*diff(y(x),x))+2*(x+diff(
```

$$y(x) = \frac{\left(c_1 + \frac{\operatorname{arcsinh}(x)}{4}\right) x\sqrt{x^2+1}}{2} - \frac{3x^2}{16} + c_1^2 + \frac{c_1 \operatorname{arcsinh}(x)}{2} + \frac{\operatorname{arcsinh}(x)^2}{16}$$
$$y(x) = \frac{1}{2}c_1x^2 + c_2x + c_1^2 + c_2^2$$

✓ Solution by Mathematica
Time used: 0.263 (sec). Leaf size: 32

```
DSolve[-2*y[x] + 2*y'[x]*(x + y'[x]) - x*(x + 4*y'[x])*y''[x] + 2*(1 + x^2)*y''[x]^2 == 0,y[
```

$$y(x) \to -\frac{1}{2}\sqrt{c_2 - c_1{}^2}x^2 + c_1x + c_2$$

7.239 problem 1830 (book 6.239)

Internal problem ID [10152]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1830 (book 6.239).
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3y''^2x^2 - 2(3y'x + y)\,y'' + 4y'^2 = 0}$$

✓ Solution by Maple

Time used: 0.266 (sec). Leaf size: 37

```
dsolve(3*x^2*diff(diff(y(x),x),x)^2-2*(3*x*diff(y(x),x)+y(x))*diff(diff(y(x),x),x)+4*diff(y(
```

$$y(x) = x^{1+\frac{2\sqrt{3}}{3}}c_1$$
$$y(x) = 0$$
$$y(x) = \frac{c_1^2x^2}{c_2} + c_1x + c_2$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 29

```
DSolve[4*y'[x]^2 - 2*(y[x] + 3*x*y'[x])*y''[x] + 3*x^2*y''[x]^2 == 0,y[x],x,IncludeSingularS
```

$$y(x) \to \frac{c_1{}^2x^2}{c_2} + c_1x + c_2$$
$$y(x) \to \text{Indeterminate}$$

7.240 problem 1831 (book 6.240)

Internal problem ID [10153]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1831 (book 6.240).
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(2-9x)\,y''^2 - 6x(-6x+1)\,y'y'' + 6y''y - 36xy'^2 = 0$$

✓ Solution by Maple

Time used: 0.297 (sec). Leaf size: 232

```
dsolve(x^2*(2-9*x)*diff(diff(y(x),x),x)^2-6*x*(1-6*x)*diff(y(x),x)*diff(diff(y(x),x),x)+6*di
```

$$y(x) = \frac{9c_1\sqrt{\frac{-1+5x+\sqrt{9x^2-2x}}{\sqrt{9x^2-2x}\,\sqrt{-\frac{(4x-1)^2}{x(9x-2)}}}}\,\sqrt{4x-1}\,x}{\left(-1+9x+3\sqrt{9x^2-2x}\right)\sqrt{27x-3+9\sqrt{9x^2-2x}}}$$

$$y(x) = \frac{c_1\left(-1+9x+3\sqrt{9x^2-2x}\right)\sqrt{27x-3+9\sqrt{9x^2-2x}}\,\sqrt{4x-1}\,x}{9\sqrt{\frac{-1+5x+\sqrt{9x^2-2x}}{\sqrt{9x^2-2x}\,\sqrt{-\frac{(4x-1)^2}{x(9x-2)}}}}}$$

$$y(x) = 0$$

$$y(x) = c_1x^3 + c_2x + \frac{c_2^2}{c_1}$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 29

```
DSolve[-36*x*y'[x]^2 + 6*y[x]*y''[x] - 6*(1 - 6*x)*x*y'[x]*y''[x] + (2 - 9*x)*x^2*y''[x]^2 =
```

$$y(x) \to \frac{c_1{}^2x^3}{c_2} + c_1x + c_2$$

$$y(x) \to \text{Indeterminate}$$

7.241 problem 1832 (book 6.241)

Internal problem ID [10154]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1832 (book 6.241).
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$F_{1,1}(x)\,y'^2 + \left(\left(F_{2,1}(x) + F_{1,2}(x)\right)y'' + y\left(F_{1,0}(x) + F_{0,1}(x)\right)\right)y' + F_{2,2}(x)\,y''^2 + y\left(F_{2,0}(x) + F_{0,2}(x)\right)y'' + F$$

✗ Solution by Maple

```
dsolve(F[1,1](x)*diff(y(x),x)^2+((F[2,1](x)+F[1,2](x))*diff(diff(y(x),x),x)+y(x)*(F[1,0](x)+
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*F[0, 0]*y[x]^2 + x*F[1, 1]*y''[x] + (x*F[0, 2] + x*F[2, 0])*y[x]*y''[x] + x*F[2, 2]
```

Not solved

7.242 problem 1833 (book 6.242)

Internal problem ID [10155]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1833 (book 6.242).
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y{y''}^{2} = a\,\mathrm{e}^{2x}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(diff(y(x),x),x)^2-a*exp(2*x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-(a*E^(2*x)) + y[x]*y''[x]^2 == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.243 problem 1834 (book 6.243)

Internal problem ID [10156]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1834 (book 6.243).
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{(y^2a^2-b^2)\,y''^2-2a^2yy'^2y''+\left(a^2y'^2-1\right)y'^2=0}$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 124

```
dsolve((a^2*y(x)^2-b^2)*diff(diff(y(x),x),x)^2-2*a^2*y(x)*diff(y(x),x)^2*diff(diff(y(x),x),x
```

$$y(x)=\frac{\operatorname{csgn}\left(\sec\left(\frac{c_1-x}{b}\right)\right)\sin\left(\frac{c_1-x}{b}\right)\operatorname{csgn}(a)\,b}{a}$$
$$y(x)=-\frac{\operatorname{csgn}\left(\sec\left(\frac{c_1-x}{b}\right)\right)\sin\left(\frac{c_1-x}{b}\right)\operatorname{csgn}(a)\,b}{a}$$
$$y(x)=-\frac{b}{a}$$
$$y(x)=\frac{b}{a}$$
$$y(x)=c_1$$
$$y(x)=\frac{b\left(e^{\frac{\sqrt{c_1^2a^2-1}\,(c_2+x)}{b}}-c_1\right)}{\sqrt{c_1^2a^2-1}}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]^2*(-1 + a^2*y'[x]^2) - 2*a^2*y[x]*y'[x]^2*y''[x] + (-b^2 + a^2*y[x]^2)*y''[x]^2
```

{}

7.244 problem 1835 (book 6.244)

Internal problem ID [10157]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1835 (book 6.244).
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(y^2 - x^2 {y'}^2 + x^2 y'' y\right)^2 - 4xy(y'x - y)^3 = 0$$

✗ Solution by Maple

```
dsolve((y(x)^2-x^2*diff(y(x),x)^2+x^2*y(x)*diff(diff(y(x),x),x))^2-4*x*y(x)*(x*diff(y(x),x)-
```

No solution found

✓ Solution by Mathematica

Time used: 96.129 (sec). Leaf size: 27

```
DSolve[-4*x*y[x]*(-y[x] + x*y'[x])^3 + (y[x]^2 - x^2*y'[x]^2 + x^2*y[x]*y''[x])^2 == 0,y[x],
```

$$y(x) \to c_1 x e^{\frac{1}{-x+c_2}}$$
$$y(x) \to c_1 x$$

7.245 problem 1836 (book 6.245)

Internal problem ID [10158]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1836 (book 6.245).
ODE order: 2.
ODE degree: 4.

CAS Maple gives this as type `unknown`

$$\left(2y''y - y'^2\right)^3 + 32y''(y''x - y')^3 = 0$$

✗ Solution by Maple

```
dsolve((2*diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2)^3+32*diff(diff(y(x),x),x)*(x*diff(diff(y
```

No solution found

✓ Solution by Mathematica

Time used: 55.735 (sec). Leaf size: 137

```
DSolve[32*y''[x]*(-y'[x] + x*y''[x])^3 + (-y'[x]^2 + 2*y[x]*y''[x])^3 == 0,y[x],x,IncludeSin
```

$$y(x) \to \frac{1}{4}\left(-\frac{8c_1{}^3}{\sqrt[3]{3\sqrt{3}\sqrt{c_1{}^9c_2{}^9(-64+27c_1c_2)}-27c_1{}^5c_2{}^5}}+\frac{c_1{}^2}{c_2} - \frac{2\sqrt[3]{\sqrt{3}\sqrt{c_1{}^9c_2{}^9(-64+27c_1c_2)}-9c_1{}^5c_2{}^5}}{3^{2/3}c_2{}^3}\right)x^2 + c_1x + c_2$$

$$y(x) \to c_2$$

7.246 problem 1837 (book 6.246)

Internal problem ID [10159]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 6, non-linear second order
Problem number: 1837 (book 6.246).
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\sqrt{ay''^2 + y'^2 b} + cyy'' + dy'^2 = 0$$

✗ Solution by Maple

```
dsolve((a*diff(diff(y(x),x),x)^2+b*diff(y(x),x)^2)^(1/2)+c*y(x)*diff(diff(y(x),x),x)+d*diff(
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[d*y'[x]^2 + c*y[x]*y''[x] + Sqrt[b*y'[x]^2 + a*y''[x]^2] == 0,y[x],x,IncludeSingularS
```

Not solved

8 Chapter 7, non-linear third and higher order

8.1 problem 1837

Internal problem ID [10160]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 7, non-linear third and higher order
Problem number: 1837.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x], [_3rd_order, _missing_y], [_3rd_ord`

Solve

$$y''' - a^2\left((y')^5 + 2(y')^3 + y'\right) = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 105

```
dsolve(diff(diff(diff(y(x),x),x),x)-a^2*(diff(y(x),x)^5+2*diff(y(x),x)^3+diff(y(x),x))=0,y(x
```

$$y(x) = \int \operatorname{RootOf}\left(3\left(\int^{\_Z} \frac{1}{\sqrt{3\_f^6a^2 + 9a^2\_f^4 + 9\_f^2a^2 + 3a^2 + 9c_1}}d\_f\right) + x + c_2\right)dx + c_3$$

$$y(x) = \int \operatorname{RootOf}\left(-3\left(\int^{\_Z} \frac{1}{\sqrt{3\_f^6a^2 + 9a^2\_f^4 + 9\_f^2a^2 + 3a^2 + 9c_1}}d\_f\right) + x + c_2\right)dx + c_3$$

✓ Solution by Mathematica

Time used: 22.017 (sec). Leaf size: 442

```
DSolve[-(a^2*(y'[x] + 2*y'[x]^3 + y'[x]^5)) + Derivative[3][y][x] == 0,y[x],x,IncludeSingula
```

$$y(x) \to \int_1^x \text{InverseFunction}\left[-3\int \frac{1}{\sqrt{3\left(a^2\right)^2 \#1^6+9\left(a^2\right)^2 \#1^4+9\left(a^2\right)^2 \#1^2+9c_1}}d\#1\&\right][c_2 - K[1]]dK[1]+c_3$$

$$y(x) \to \int_1^x \text{InverseFunction}\left[3\int \frac{1}{\sqrt{3\left(a^2\right)^2 \#1^6+9\left(a^2\right)^2 \#1^4+9\left(a^2\right)^2 \#1^2+9c_1}}d\#1\&\right][c_2 - K[2]]dK[2]+c_3$$

$$y(x) \to \text{Indeterminate}$$

$$y(x) \to \int_1^x \text{InverseFunction}\left[-3\int \frac{1}{\sqrt{3\left(a^2\right)^2 \#1^6+9\left(a^2\right)^2 \#1^4+9\left(a^2\right)^2 \#1^2+9(-1)c_1}}d\#1\&\right][c_2 - K[1]]dK[1]+c_3$$

$$y(x) \to \int_1^x \text{InverseFunction}\left[3\int \frac{1}{\sqrt{3\left(a^2\right)^2 \#1^6+9\left(a^2\right)^2 \#1^4+9\left(a^2\right)^2 \#1^2+9(-1)c_1}}d\#1\&\right][c_2 - K[2]]dK[2]+c_3$$

$$y(x) \to \int_1^x \text{InverseFunction}\left[-3\int \frac{1}{\sqrt{3\left(a^2\right)^2 \#1^6+9\left(a^2\right)^2 \#1^4+9\left(a^2\right)^2 \#1^2+9c_1}}d\#1\&\right][c_2 - K[1]]dK[1]+c_3$$

$$y(x) \to \int_1^x \text{InverseFunction}\left[3\int \frac{1}{\sqrt{3\left(a^2\right)^2 \#1^6+9\left(a^2\right)^2 \#1^4+9\left(a^2\right)^2 \#1^2+9c_1}}d\#1\&\right][c_2 - K[2]]dK[2]+c_3$$

8.2 problem 1838

Internal problem ID [10161]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 7, non-linear third and higher order
Problem number: 1838.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x], [_3rd_order, _with_linear_symmetrie`

Solve

$$y''' + y''y - (y')^2 + 1 = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(diff(y(x),x),x),x)+diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+1=0,y(x), sings
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[1 - y'[x]^2 + y[x]*y''[x] + Derivative[3][y][x] == 0,y[x],x,IncludeSingularSolutions
```

Not solved

8.3 problem 1839

Internal problem ID [10162]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 7, non-linear third and higher order
Problem number: 1839.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x], [_3rd_order, _with_linear_symmetrie`

Solve

$$y''' - y''y + (y')^2 = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(diff(y(x),x),x),x)-diff(diff(y(x),x),x)*y(x)+diff(y(x),x)^2=0,y(x), singsol
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]^2 - y[x]*y''[x] + Derivative[3][y][x] == 0,y[x],x,IncludeSingularSolutions -> T
```

Not solved

8.4 problem 1840

Internal problem ID [10163]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 7, non-linear third and higher order
Problem number: 1840.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x], [_3rd_order, _with_linear_symmetrie`

Solve

$$y''' + ayy'' = 0$$

✗ Solution by Maple

```
dsolve(diff(diff(diff(y(x),x),x),x)+a*y(x)*diff(diff(y(x),x),x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[a*y[x]*y''[x] + Derivative[3][y][x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

8.5 problem 1841

Internal problem ID [10164]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 7, non-linear third and higher order
Problem number: 1841.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _nonlinear]]`

Solve

$$x^2 y''' + y''x + (2yx - 1)\,y' + y^2 - f(x) = 0$$

✗ Solution by Maple

```
dsolve(x^2*diff(diff(diff(y(x),x),x),x)+x*diff(diff(y(x),x),x)+(2*x*y(x)-1)*diff(y(x),x)+y(x
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-f[x] + y[x]^2 + (-1 + 2*x*y[x])*y'[x] + x*y''[x] + x^2*Derivative[3][y][x] == 0,y[x]
```

Not solved

8.6 problem 1842

Internal problem ID [10165]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 7, non-linear third and higher order
Problem number: 1842.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _nonlinear], [_3rd_order, _with_linear_s`

Solve

$$x^2y''' + x(y-1)\,y'' + x(y')^2 + (1-y)\,y' = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 190

```
dsolve(x^2*diff(diff(diff(y(x),x),x),x)+x*(-1+y(x))*diff(diff(y(x),x),x)+x*diff(y(x),x)^2+(1
```

$$\ln(x) + 2\left(\int^{y(x)} \frac{}{2\,\mathrm{RootOf}\left(-2\sqrt{4+c_1}\,\mathrm{BesselY}\left(\frac{\sqrt{4+c_1}}{2}, \frac{\sqrt{2}\_Z}{2}\right)c_2 + 2\,\mathrm{BesselY}\left(\frac{\sqrt{4+c_1}}{2}, \frac{\sqrt{2}\_Z}{2}\right)c_2\_h - 4\,\mathrm{Bessel}\right.}\right. - c_3 = 0$$

✓ Solution by Mathematica

Time used: 60.245 (sec). Leaf size: 282

```
DSolve[(1 - y[x])*y'[x] + x*y'[x]^2 + x*(-1 + y[x])*y''[x] + x^2*Derivative[3][y][x] == 0,y[
```

$$y(x) \to \frac{2\left(c_3\left(\mathrm{BesselJ}\left(\frac{\sqrt{c_2+2}}{\sqrt{2}}, -\frac{1}{2}ix\sqrt{c_1}\right) - \frac{1}{4}i\sqrt{c_1}x\left(\mathrm{BesselJ}\left(\frac{\sqrt{c_2+2}}{\sqrt{2}}-1, -\frac{1}{2}ix\sqrt{c_1}\right) - \mathrm{BesselJ}\left(\frac{\sqrt{c_2+2}}{\sqrt{2}}+1, -\frac{1}{2}i\right.\right.\right.}{c_3\,\mathrm{BesselJ}\left(\frac{\sqrt{c_2+2}}{\sqrt{2}}, -\frac{1}{2}\right.}$$

8.7 problem 1843

Internal problem ID [10166]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 7, non-linear third and higher order
Problem number: 1843.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x], [_3rd_order, _exact, _nonlinear], [`

Solve

$$\boxed{yy''' - y'y'' + y^3y' = 0}$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 81

```
dsolve(y(x)*diff(diff(diff(y(x),x),x),x)-diff(y(x),x)*diff(diff(y(x),x),x)+y(x)^3*diff(y(x),
```

$$y(x) = 0$$
$$-2\left(\int^{y(x)} \frac{1}{\sqrt{-\_a^4 + 4\_a^2c_2 - 4c_2^2 + 4c_1}}d\_a\right) - x - c_3 = 0$$
$$2\left(\int^{y(x)} \frac{1}{\sqrt{-\_a^4 + 4\_a^2c_2 - 4c_2^2 + 4c_1}}d\_a\right) - x - c_3 = 0$$

✓ Solution by Mathematica

Time used: 2.269 (sec). Leaf size: 409

```
DSolve[y[x]^3*y'[x] - y'[x]*y''[x] + y[x]*Derivative[3][y][x] == 0,y[x],x,IncludeSingularSol
```

$$y(x) \to \text{InverseFunction}\left[-\frac{2i\sqrt{1+\frac{\#1^2}{2\left(\sqrt{c_2{}^2-c_1}-c_2\right)}}\sqrt{1-\frac{\#1^2}{2\left(c_2+\sqrt{c_2{}^2-c_1}\right)}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{1}{\sqrt{c_2{}^2-c_1}-c_2}}\#1}{\sqrt{2}}\right),\frac{c}{c}\right.}{\sqrt{\frac{1}{\sqrt{c_2{}^2-c_1}-c_2}}\sqrt{-\frac{\#1^4}{2}+2\#1^2c_2-2c_1}}\right.$$

$$\left.+c_3\right]$$

$$y(x) \to \text{InverseFunction}\left[\frac{2i\sqrt{1+\frac{\#1^2}{2\left(\sqrt{c_2{}^2-c_1}-c_2\right)}}\sqrt{1-\frac{\#1^2}{2\left(c_2+\sqrt{c_2{}^2-c_1}\right)}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{1}{\sqrt{c_2{}^2-c_1}-c_2}}\#1}{\sqrt{2}}\right),\frac{c_2-}{c_2+}\right.}{\sqrt{\frac{1}{\sqrt{c_2{}^2-c_1}-c_2}}\sqrt{-\frac{\#1^4}{2}+2\#1^2c_2-2c_1}}\right.$$

$$\left.+c_3\right]$$

8.8 problem 1844

Internal problem ID [10167]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 7, non-linear third and higher order
Problem number: 1844.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x], [_3rd_order, _with_linear_symmetrie`

Solve

$$4y'''y^2 - 18y'y''y + 15(y')^3 = 0$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 77

```
dsolve(4*y(x)^2*diff(diff(diff(y(x),x),x),x)-18*y(x)*diff(y(x),x)*diff(diff(y(x),x),x)+15*di
```

$$y(x) = 0$$
$$y(x) = e^{\int \operatorname{RootOf}\left(-2\left(\int^{\_Z} \frac{1}{\_h^2+\sqrt{c_1(\_h^2+c_1)}+c_1}d\_h\right)+x+c_2\right)dx+c_3}$$
$$y(x) = e^{\int \operatorname{RootOf}\left(2\left(\int^{\_Z} -\frac{1}{\_h^2-\sqrt{c_1(\_h^2+c_1)}+c_1}d\_h\right)+x+c_2\right)dx+c_3}$$

✓ Solution by Mathematica
Time used: 0.199 (sec). Leaf size: 19

```
DSolve[15*y'[x]^3 - 18*y[x]*y'[x]*y''[x] + 4*y[x]^2*Derivative[3][y][x] == 0,y[x],x,IncludeS
```

$$y(x) \to \frac{1}{(x(c_3x+c_2)+c_1)^2}$$

8.9 problem 1845

Internal problem ID [10168]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 7, non-linear third and higher order
Problem number: 1845.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x], [_3rd_order, _with_linear_symmetrie`

Solve

$$9y'''y^2 - 45y'y''y + 40(y')^3 = 0$$

✓ Solution by Maple
Time used: 0.11 (sec). Leaf size: 85

```
dsolve(9*y(x)^2*diff(diff(diff(y(x),x),x),x)-45*y(x)*diff(y(x),x)*diff(diff(y(x),x),x)+40*di
```

$$y(x) = 0$$

$$y(x) = e^{\int \mathrm{RootOf}\left(-6\left(\int^{\_Z} \frac{1}{4\_h^2+\sqrt{c_1(4\_h^2+c_1)}+c_1}d\_h\right)+x+c_2\right)dx+c_3}$$

$$y(x) = e^{\int \mathrm{RootOf}\left(6\left(\int^{\_Z} -\frac{1}{4\_h^2-\sqrt{c_1(4\_h^2+c_1)}+c_1}d\_h\right)+x+c_2\right)dx+c_3}$$

✓ Solution by Mathematica
Time used: 0.199 (sec). Leaf size: 21

```
DSolve[40*y'[x]^3 - 45*y[x]*y'[x]*y''[x] + 9*y[x]^2*Derivative[3][y][x] == 0,y[x],x,IncludeS
```

$$y(x) \to \frac{1}{(x(c_3x + c_2) + c_1)^{3/2}}$$

8.10 problem 1846

Internal problem ID [10169]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 7, non-linear third and higher order
Problem number: 1846.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

Solve

$$2y'y''' - 3(y')^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve(2*diff(y(x),x)*diff(diff(diff(y(x),x),x),x)-3*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 e^{\frac{\sqrt{6}\,x}{2}} + c_3 e^{-\frac{\sqrt{6}\,x}{2}}$$
$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.119 (sec). Leaf size: 57

```
DSolve[-3*y'[x]^2 + 2*y'[x]*Derivative[3][y][x] == 0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to c_1$$
$$y(x) \to \sqrt{\frac{2}{3}} e^{-\sqrt{\frac{3}{2}}x} \left(c_1 e^{\sqrt{6}x} - c_2\right) + c_3$$
$$y(x) \to c_1$$

8.11 problem 1847

Internal problem ID [10170]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 7, non-linear third and higher order
Problem number: 1847.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x], [_3rd_order, _missing_y], [_3rd_ord`

Solve

$$\left((y')^2+1\right)y'''-3y'(y'')^2=0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 67

```
dsolve((diff(y(x),x)^2+1)*diff(diff(diff(y(x),x),x),x)-3*diff(y(x),x)*diff(diff(y(x),x),x)^2
```

$$y(x)=-ix+c_1$$
$$y(x)=ix+c_1$$
$$y(x)=-\sqrt{-c_2^2-2c_2x-x^2+c_1}+c_3$$
$$y(x)=\sqrt{-c_2^2-2c_2x-x^2+c_1}+c_3$$

✓ Solution by Mathematica

Time used: 1.928 (sec). Leaf size: 142

```
DSolve[-3*y'[x]*y''[x]^2 + (1 + y'[x]^2)*Derivative[3][y][x] == 0,y[x],x,IncludeSingularSolu
```

$$y(x)\to c_3-\frac{i\sqrt{c_1{}^2x^2+2c_2c_1{}^2x-1+c_2{}^2c_1{}^2}}{c_1}$$
$$y(x)\to \frac{i\sqrt{c_1{}^2x^2+2c_2c_1{}^2x-1+c_2{}^2c_1{}^2}}{c_1}+c_3$$
$$y(x)\to \text{Indeterminate}$$
$$y(x)\to c_3-i\sqrt{(x+c_2)^2}$$
$$y(x)\to i\sqrt{(x+c_2)^2}+c_3$$

8.12 problem 1848

Internal problem ID [10171]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 7, non-linear third and higher order
Problem number: 1848.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x], [_3rd_order, _missing_y], [_3rd_ord`

Solve

$$\left((y')^2+1\right)y'''-(3y'+a)(y'')^2=0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 377

```
dsolve((diff(y(x),x)^2+1)*diff(diff(diff(y(x),x),x),x)-(3*diff(y(x),x)+a)*diff(diff(y(x),x),
```

$$y(x)=-ix+c_1$$
$$y(x)=ix+c_1$$
$$\begin{aligned}y(x)=\int\tan\Big(\mathrm{RootOf}\big(&c_2^2a^4\mathrm{e}^{2\_Za}+2c_2a^4x\,\mathrm{e}^{2\_Za}+a^4x^2\mathrm{e}^{2\_Za}-2\cos(\_Z)\,\mathrm{e}^{\_Za}c_1c_2a^3\\&-2\cos(\_Z)\,\mathrm{e}^{\_Za}c_1a^3x+\cos(\_Z)^2c_1^2a^2+2c_2^2a^2\mathrm{e}^{2\_Za}+4c_2a^2x\,\mathrm{e}^{2\_Za}\\&+2a^2x^2\mathrm{e}^{2\_Za}-2\cos(\_Z)\,\mathrm{e}^{\_Za}c_1c_2a-2\cos(\_Z)\,\mathrm{e}^{\_Za}c_1ax-\sin(\_Z)^2c_1^2\\&+c_2^2\mathrm{e}^{2\_Za}+2c_2x\,\mathrm{e}^{2\_Za}+x^2\mathrm{e}^{2\_Za}\big)\Big)dx+c_3\end{aligned}$$
$$\begin{aligned}y(x)=\int\tan\Big(\mathrm{RootOf}\big(&c_2^2a^4\mathrm{e}^{2\_Za}+2c_2a^4x\,\mathrm{e}^{2\_Za}+a^4x^2\mathrm{e}^{2\_Za}+2\cos(\_Z)\,\mathrm{e}^{\_Za}c_1c_2a^3\\&+2\cos(\_Z)\,\mathrm{e}^{\_Za}c_1a^3x+\cos(\_Z)^2c_1^2a^2+2c_2^2a^2\mathrm{e}^{2\_Za}+4c_2a^2x\,\mathrm{e}^{2\_Za}\\&+2a^2x^2\mathrm{e}^{2\_Za}+2\cos(\_Z)\,\mathrm{e}^{\_Za}c_1c_2a+2\cos(\_Z)\,\mathrm{e}^{\_Za}c_1ax-\sin(\_Z)^2c_1^2\\&+c_2^2\mathrm{e}^{2\_Za}+2c_2x\,\mathrm{e}^{2\_Za}+x^2\mathrm{e}^{2\_Za}\big)\Big)dx+c_3\end{aligned}$$

✓ Solution by Mathematica

Time used: 30.105 (sec). Leaf size: 198

```
DSolve[(-a - 3*y'[x])*y''[x]^2 + (1 + y'[x]^2)*Derivative[3][y][x] == 0,y[x],x,IncludeSingul
```

$$y(x) \to c_3 - \frac{\left(1 - i\text{InverseFunction}\left[\frac{(\#1-a)e^{-a\arctan(\#1)}}{\sqrt{\#1^2+1}(a^2+1)c_1}\&\right][x+c_2]\right)^{-\frac{1}{2}-\frac{ia}{2}}\left(1 + i\text{InverseFunction}\left[\frac{(\#1-a)e^{-a\arctan(\#}{\sqrt{\#1^2+1}(a^2+1)c_1}\right.\right.}{(a^2+1)\,c_1}$$

$y(x) \to$ Indeterminate

$y(x) \to c_3$

8.13 problem 1849

Internal problem ID [10172]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 7, non-linear third and higher order
Problem number: 1849.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x], [_3rd_order, _missing_y], [_3rd_ord`

Solve

$$y''y''' - a\sqrt{(y'')^2\, b^2 + 1} = 0$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 297

```
dsolve(diff(diff(y(x),x),x)*diff(diff(diff(y(x),x),x),x)-a*(b^2*diff(diff(y(x),x),x)^2+1)^(1
```

$$y(x) = -\frac{ix^2}{2b} + c_1x + c_2$$

$$y(x) = \frac{ix^2}{2b} + c_1x + c_2$$

$$y(x) = \frac{\int \left(-\ln\left(\frac{a^2b^4(c_1+x)+\sqrt{(-1+b^2(c_1+x)a)(1+b^2(c_1+x)a)}\,\sqrt{a^2b^4}}{\sqrt{a^2b^4}}\right) + (c_1+x)\sqrt{a^2b^4}\,\sqrt{(-1+b^2(c_1+x)a)(1+b^2(c_1 \cdots}{2\sqrt{a^2b^4}\,b}$$

$$y(x) = -\frac{\int \left(-\ln\left(\frac{a^2b^4(c_1+x)+\sqrt{(-1+b^2(c_1+x)a)(1+b^2(c_1+x)a)}\,\sqrt{a^2b^4}}{\sqrt{a^2b^4}}\right) + (c_1+x)\sqrt{a^2b^4}\,\sqrt{(-1+b^2(c_1+x)a)(1+b^2(c \cdots}{2\sqrt{a^2b^4}\,b}$$

✓ Solution by Mathematica

Time used: 31.226 (sec). Leaf size: 415

```
DSolve[-(a*Sqrt[1 + b^2*y''[x]^2]) + y''[x]*Derivative[3][y][x] == 0,y[x],x,IncludeSingularS
```

$$y(x) \rightarrow \frac{6a^2b^5c_3x + 6a^2b^5c_2 + \left(a^2b^4x^2 + 2ab^4c_1x + b^4{c_1}^2 - 1\right)^{3/2} + 3\sqrt{a^2b^4x^2 + 2ab^4c_1x + b^4{c_1}^2 - 1} - 3b^2c_1 \log$$

$$y(x) \rightarrow \frac{-\sqrt{a^2b^4x^2 + 2ab^4c_1x + b^4{c_1}^2 - 1}(a^2b^4x^2 + 2ab^4c_1x + b^4{c_1}^2 + 2) + 3b^2c_1 \log\left(\sqrt{a^2b^4x^2 + 2ab^4c_1x + b^4{c_1}^2}\right.}{6a^2b^5} + c_3x + c_2$$

8.14 problem 1850

Internal problem ID [10173]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 7, non-linear third and higher order
Problem number: 1850.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x], [_high_order, _missing_y], [_high_`

Solve

$$y'y'''' - y''y''' + (y')^3 y''' = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)*diff(diff(diff(diff(y(x),x),x),x),x)-diff(diff(y(x),x),x)*diff(diff(diff
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]^3*Derivative[3][y][x] - y''[x]*Derivative[3][y][x] + y'[x]*Derivative[4][y][x]
```

Not solved

8.15 problem 1851

Internal problem ID [10174]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 7, non-linear third and higher order
Problem number: 1851.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

Solve

$$y'(f'''(x)\,y' + 3f''(x)\,y'' + 3f'(x)\,y''' + f(x)\,y'''') - y''fy''' + (y')^3\,(f'(x)\,y' + y''f(x)) + 2q(x)\,(y')^2\sin(y)$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)*(diff(diff(diff(f(x),x),x),x)*diff(y(x),x)+3*diff(diff(f(x),x),x)*diff(d
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[2*q[x]*Sin[y[x]]*y'[x]^2 + y'[x]^3*(Derivative[1][f][x]*y'[x] + f[x]*y''[x]) + Cos[y[
```

Not solved

8.16 problem 1852

Internal problem ID [10175]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 7, non-linear third and higher order
Problem number: 1852.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x], [_high_order, _missing_y], [_high_`

Solve

$$\boxed{3y''y'''' - 5(y''')^2 = 0}$$

✓ Solution by Maple
Time used: 0.141 (sec). Leaf size: 36

```
dsolve(3*diff(diff(y(x),x),x)*diff(diff(diff(diff(y(x),x),x),x),x)-5*diff(diff(diff(y(x),x),
```

$$y(x) = c_1x + c_2$$
$$y(x) = 3(c_2 + x)\sqrt{6}\,c_1\sqrt{-\frac{c_1}{c_2 + x}} + c_3x + c_4$$

✓ Solution by Mathematica
Time used: 0.342 (sec). Leaf size: 28

```
DSolve[-5*Derivative[3][y][x]^2 + 3*y''[x]*Derivative[4][y][x] == 0,y[x],x,IncludeSingularSo
```

$$y(x) \to c_2\left(-\sqrt{2x + 3c_1}\right) + c_4x + c_3$$

8.17 problem 1853

Internal problem ID [10176]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 7, non-linear third and higher order
Problem number: 1853.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x], [_high_order, _missing_y], [_high_`

Solve

$$9(y'')^2 y^{(5)} - 45y''y'''y'''' + 40y''' = 0$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 118

```
dsolve(9*diff(diff(y(x),x),x)^2*diff(diff(diff(diff(diff(y(x),x),x),x),x),x)-45*diff(diff(y(
```

$$y(x) = c_1x + c_2$$

$$y(x) = \int \int \operatorname{RootOf}\left(-\left(\int^{\_Z} \frac{}{\operatorname{RootOf}\left(-20\ln(\_f) + \int^{-\_Z} \_k\left(e^{\operatorname{RootOf}(81\_k^2 e^{\_Z} + 20 e^{\_Z}\ln(e^{\_Z}+27) - 40 e^{\_Z}\ln(2) - 20 e^{\_}}\right.\right.}\right.\right. + x + c_3\Bigg) dxdx + c_4x + c_5$$

✓ Solution by Mathematica

Time used: 0.099 (sec). Leaf size: 43

```
DSolve[40*Derivative[3][y][x]^3 - 45*y''[x]*Derivative[3][y][x]*Derivative[4][y][x] + 9*y''[
```

$$y(x) \to c_5x - \frac{4\sqrt{x(c_3x + c_2) + c_1}}{c_2{}^2 - 4c_1c_3} + c_4$$

8.18 problem 1854

Internal problem ID [10177]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 7, non-linear third and higher order
Problem number: 1854.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' - f(y) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
dsolve(diff(y(x),x$2)-f(y(x))=0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{\sqrt{2\left(\int f(\_b)\, d\_b\right) + c_1}} d\_b - x - c_2 = 0$$
$$-\left(\int^{y(x)} \frac{1}{\sqrt{2\left(\int f(\_b)\, d\_b\right) + c_1}} d\_b\right) - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 40

```
DSolve[-f[y[x]]+ y''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} \frac{1}{\sqrt{c_1 + 2\int_1^{K[2]} f(K[1])dK[1]}} dK[2]^2 = (x + c_2)^2, y(x)\right]$$

8.19 problem 1855

Internal problem ID [10178]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 7, non-linear third and higher order
Problem number: 1855.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x], [_3rd_order, _with_linear_symmetrie`

Solve

$$\boxed{y''' - f(y) = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$3)=f(y(x)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[-f[y[x]] + y'''[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

9 Chapter 8, system of first order odes

9.1 problem 1856

Internal problem ID [10179]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1856.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = ax(t)$$
$$y'(t) = b$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve({diff(x(t),t)=a*x(t),diff(y(t),t)=b},singsol=all)
```

$$x(t) = c_1 e^{at}$$
$$y(t) = bt + c_2$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 36

```
DSolve[{x'[t]==a*x[t],y'[t]==b},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 e^{at}$$
$$y(t) \to bt + c_2$$
$$x(t) \to 0$$
$$y(t) \to bt + c_2$$

9.2 problem 1857

Internal problem ID [10180]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1857.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = ay(t)$$
$$y'(t) = -ax(t)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve({diff(x(t),t)=a*y(t),diff(y(t),t)=-a*x(t)},singsol=all)
```

$$x(t) = c_1 \sin(at) + c_2 \cos(at)$$
$$y(t) = \cos(at) c_1 - \sin(at) c_2$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 39

```
DSolve[{x'[t]==a*y[t],y'[t]==-a*x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 \cos(at) + c_2 \sin(at)$$
$$y(t) \to c_2 \cos(at) - c_1 \sin(at)$$

9.3 problem 1858

Internal problem ID [10181]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1858.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= ay(t) \\
y'(t) &= bx(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 64

```
dsolve({diff(x(t),t)=a*y(t),diff(y(t),t)=b*x(t)},singsol=all)
```

$$
\begin{aligned}
x(t) &= c_1 \mathrm{e}^{\sqrt{a}\,\sqrt{b}\,t} + c_2 \mathrm{e}^{-\sqrt{a}\,\sqrt{b}\,t} \\
y(t) &= \frac{\sqrt{b}\left(c_1 \mathrm{e}^{\sqrt{a}\,\sqrt{b}\,t} - c_2 \mathrm{e}^{-\sqrt{a}\,\sqrt{b}\,t}\right)}{\sqrt{a}}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 158

```
DSolve[{x'[t]==a*y[t],y'[t]==b*x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
x(t) &\to \frac{e^{-\sqrt{a}\sqrt{b}t}\left(\sqrt{b}c_1\left(e^{2\sqrt{a}\sqrt{b}t}+1\right)+\sqrt{a}c_2\left(e^{2\sqrt{a}\sqrt{b}t}-1\right)\right)}{2\sqrt{b}} \\
y(t) &\to \frac{e^{-\sqrt{a}\sqrt{b}t}\left(\sqrt{b}c_1\left(e^{2\sqrt{a}\sqrt{b}t}-1\right)+\sqrt{a}c_2\left(e^{2\sqrt{a}\sqrt{b}t}+1\right)\right)}{2\sqrt{a}}
\end{aligned}
$$

9.4 problem 1859

Internal problem ID [10182]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1859.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= ax(t) - y(t) \\
y'(t) &= x(t) + ay(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 37

```
dsolve({diff(x(t),t)=a*x(t)-y(t),diff(y(t),t)=x(t)+a*y(t)},singsol=all)
```

$$
\begin{aligned}
x(t) &= e^{at}(c_1 \sin(t) + c_2 \cos(t)) \\
y(t) &= e^{at}(c_2 \sin(t) - c_1 \cos(t))
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 43

```
DSolve[{x'[t]==a*x[t]-y[t],y'[t]==x[t]+a*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tru
```

$$
\begin{aligned}
x(t) &\to e^{at}(c_1 \cos(t) - c_2 \sin(t)) \\
y(t) &\to e^{at}(c_2 \cos(t) + c_1 \sin(t))
\end{aligned}
$$

9.5 problem 1860

Internal problem ID [10183]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1860.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = ax(t) + by(t)$$
$$y'(t) = cx(t) + by(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 237

```
dsolve({diff(x(t),t)=a*x(t)+b*y(t),diff(y(t),t)=c*x(t)+b*y(t)},singsol=all)
```

$$x(t) = c_1 e^{\frac{\left(a+b+\sqrt{a^2-2ab+b^2+4bc}\right)t}{2}} + c_2 e^{-\frac{\left(-a-b+\sqrt{a^2-2ab+b^2+4bc}\right)t}{2}}$$

$$y(t) = \left(\frac{1}{2} + \frac{\frac{\sqrt{a^2-2ab+b^2+4bc}}{2} - \frac{a}{2}}{b}\right) c_1 e^{\frac{\left(a+b+\sqrt{a^2-2ab+b^2+4bc}\right)t}{2}} + \left(\frac{e^{-\frac{\left(-a-b+\sqrt{a^2-2ab+b^2+4bc}\right)t}{2}}}{2} + \frac{-\frac{\sqrt{a^2-2ab+b^2+4bc}\, e^{-\frac{\left(-a-b+\sqrt{a^2-2ab+b^2+4bc}\right)t}{2}}}{2} - \frac{e^{-\frac{\left(-a-b+\sqrt{a^2-2ab+b^2+4bc}\right)t}{2}} a}{2}}{b}\right) c_2$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 362

```
DSolve[{x'[t]==a*x[t]+b*y[t],y'[t]==c*x[t]+b*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{e^{\frac{1}{2}t\left(-\sqrt{a^2-2ab+b^2+4bc}+a+b\right)}\left(ac_1\left(e^{t\sqrt{a^2-2ab+b^2+4bc}}-1\right)+c_1\sqrt{a^2-2ab+b^2+4bc}\left(e^{t\sqrt{a^2-2ab+b^2+4bc}}+1\right)-\right.}{2\sqrt{a^2-2ab+b(b+4c)}}$$

$$y(t) \to \frac{e^{\frac{1}{2}t\left(-\sqrt{a^2-2ab+b^2+4bc}+a+b\right)}\left(2cc_1\left(e^{t\sqrt{a^2-2ab+b^2+4bc}}-1\right)+c_2\left(a\left(-e^{t\sqrt{a^2-2ab+b^2+4bc}}\right)+b\left(e^{t\sqrt{a^2-2ab+b^2+4bc}}-\right.\right.\right.}{2\sqrt{a^2-2ab+b(b+4c)}}$$

9.6 problem 1861

Internal problem ID [10184]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1861.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = \frac{x(t)\, a\alpha}{a^2+b^2} + \frac{x(t)\, b\beta}{a^2+b^2} + \frac{y(t)\, a\beta}{a^2+b^2} - \frac{y(t)\, \alpha b}{a^2+b^2}$$
$$y'(t) = -\frac{\beta x(t)\, a}{a^2+b^2} + \frac{x(t)\, \alpha b}{a^2+b^2} + \frac{\alpha y(t)\, a}{a^2+b^2} + \frac{y(t)\, b\beta}{a^2+b^2}$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 144

```
dsolve({a*diff(x(t),t)+b*diff(y(t),t)=alpha*x(t)+beta*y(t),b*diff(x(t),t)-a*diff(y(t),t)=bet
```

$$x(t) = c_1 \mathrm{e}^{\frac{(i a\beta - i\alpha b + a\alpha + b\beta)t}{a^2+b^2}} + c_2 \mathrm{e}^{-\frac{(i a\beta - i\alpha b - a\alpha - b\beta)t}{a^2+b^2}}$$
$$y(t) = i\left(c_1 \mathrm{e}^{\frac{(i a\beta - i\alpha b + a\alpha + b\beta)t}{a^2+b^2}} - c_2 \mathrm{e}^{-\frac{(i a\beta - i\alpha b - a\alpha - b\beta)t}{a^2+b^2}}\right)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 145

```
DSolve[{a*x'[t]+b*y'[t]==\[Alpha]*x[t]+\[Beta]*y[t],b*x'[t]-a*y'[t]==\[Beta]*x[t]-\[Alpha]*y
```

$$x(t) \to e^{\frac{t(a\alpha+b\beta)}{a^2+b^2}} \left(c_1 \cos\left(\frac{t(a\beta-\alpha b)}{a^2+b^2}\right) + c_2 \sin\left(\frac{t(a\beta-\alpha b)}{a^2+b^2}\right)\right)$$
$$y(t) \to e^{\frac{t(a\alpha+b\beta)}{a^2+b^2}} \left(c_2 \cos\left(\frac{t(a\beta-\alpha b)}{a^2+b^2}\right) - c_1 \sin\left(\frac{t(a\beta-\alpha b)}{a^2+b^2}\right)\right)$$

9.7 problem 1862

Internal problem ID [10185]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1862.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -y(t)$$
$$y'(t) = 2x(t) + 2y(t)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve({diff(x(t),t)=-y(t),diff(y(t),t)=2*x(t)+2*y(t)},singsol=all)
```

$$x(t) = e^t(c_1 \sin(t) + c_2 \cos(t))$$
$$y(t) = -e^t(c_1 \sin(t) - c_2 \sin(t) + c_1 \cos(t) + c_2 \cos(t))$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 46

```
DSolve[{x'[t]==-y[t],y'[t]==2*x[t]+2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^t(c_1 \cos(t) - (c_1 + c_2)\sin(t))$$
$$y(t) \to e^t(2c_1 \sin(t) + c_2(\sin(t) + \cos(t)))$$

9.8 problem 1863

Internal problem ID [10186]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1863.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -3x(t) - 4y(t)$$
$$y'(t) = -2x(t) - 5y(t)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 35

```
dsolve({diff(x(t),t)+3*x(t)+4*y(t)=0,diff(y(t),t)+2*x(t)+5*y(t)=0},singsol=all)
```

$$x(t) = c_1 e^{-7t} + c_2 e^{-t}$$
$$y(t) = c_1 e^{-7t} - \frac{c_2 e^{-t}}{2}$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 72

```
DSolve[{x'[t]+3*x[t]+4*y[t]==0,y'[t]+2*x[t]+5*y[t]==0},{x[t],y[t]},t,IncludeSingularSolution
```

$$x(t) \to \frac{1}{3} e^{-7t} \left(c_1 \left(2e^{6t} + 1\right) - 2c_2 \left(e^{6t} - 1\right)\right)$$
$$y(t) \to \frac{1}{3} e^{-7t} \left(c_2 \left(e^{6t} + 2\right) - c_1 \left(e^{6t} - 1\right)\right)$$

9.9 problem 1864

Internal problem ID [10187]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1864.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -5x(t) - 2y(t)$$
$$y'(t) = x(t) - 7y(t)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 46

```
dsolve({diff(x(t),t)=-5*x(t)-2*y(t),diff(y(t),t)=x(t)-7*y(t)},singsol=all)
```

$$x(t) = e^{-6t}(c_1 \sin(t) + c_2 \cos(t))$$
$$y(t) = \frac{e^{-6t}(c_1 \sin(t) + c_2 \sin(t) - c_1 \cos(t) + c_2 \cos(t))}{2}$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 52

```
DSolve[{x'[t]==-5*x[t]-2*y[t],y'[t]==x[t]-7*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to e^{-6t}(c_1 \cos(t) + (c_1 - 2c_2)\sin(t))$$
$$y(t) \to e^{-6t}(c_2 \cos(t) + (c_1 - c_2)\sin(t))$$

9.10 problem 1865

Internal problem ID [10188]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1865.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= a_1 x(t) + b_1 y(t) + c_1 \\
y'(t) &= a_2 x(t) + b_2 y(t) + c_2
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 334

```
dsolve({diff(x(t),t)=a__1*x(t)+b__1*y(t)+c__1,diff(y(t),t)=a__2*x(t)+b__2*y(t)+c__2},singsol
```

$$
x(t) = e^{\left(\frac{a_1}{2}+\frac{b_2}{2}+\frac{\sqrt{a_1^2-2a_1b_2+4a_2b_1+b_2^2}}{2}\right)t} c_4 + e^{\left(\frac{a_1}{2}+\frac{b_2}{2}-\frac{\sqrt{a_1^2-2a_1b_2+4a_2b_1+b_2^2}}{2}\right)t} c_3 + \frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1}
$$

$$
y(t) = \frac{-\frac{a_1\left(e^{\frac{\left(a_1+b_2+\sqrt{a_1^2-2a_1b_2+4a_2b_1+b_2^2}\right)t}{2}} c_4(a_1b_2-a_2b_1) + e^{\frac{\left(a_1+b_2-\sqrt{a_1^2-2a_1b_2+4a_2b_1+b_2^2}\right)t}{2}} c_3(a_1b_2-a_2b_1) - b_2c_1 + b_1c_2\right)(2a_1b_2-2a_2b_1)}{a_1b_2-a_2b_1} + \cdots}{\cdots}
$$

✓ Solution by Mathematica

Time used: 1.359 (sec). Leaf size: 926

```
DSolve[{x'[t]==a1*x[t]+b1*y[t]+c1,y'[t]==a2*x[t]+b2*y[t]+c2},{x[t],y[t]},t,IncludeSingularSo
```

$x(t)$

$$\rightarrow \frac{2e^{-\frac{1}{2}t\left(\sqrt{\mathrm{a1}^2-2\mathrm{a1b2}+4\mathrm{a2b1}+\mathrm{b2}^2}+\mathrm{a1}+\mathrm{b2}\right)}\left(2\mathrm{b2c1}\sqrt{\mathrm{a1}^2-2\mathrm{a1b2}+4\mathrm{a2b1}+\mathrm{b2}^2}e^{\frac{1}{2}t\left(\sqrt{\mathrm{a1}^2-2\mathrm{a1b2}+4\mathrm{a2b1}+\mathrm{b2}^2}+\mathrm{a1}+\mathrm{b2}\right)}\right.}{}$$

$y(t)$

$$\rightarrow \frac{e^{-\frac{1}{2}t\left(\sqrt{\mathrm{a1}^2-2\mathrm{a1b2}+4\mathrm{a2b1}+\mathrm{b2}^2}+\mathrm{a1}+\mathrm{b2}\right)}\left(4\mathrm{a2}^2\mathrm{b1}c_1e^{t(\mathrm{a1}+\mathrm{b2})}\left(e^{t\sqrt{\mathrm{a1}^2-2\mathrm{a1b2}+4\mathrm{a2b1}+\mathrm{b2}^2}}-1\right)-4\mathrm{a2c1}\sqrt{\mathrm{a1}^2-2\mathrm{a1b2}\right.}{}$$

9.11 problem 1866

Internal problem ID [10189]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1866.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -2y(t) + 3t$$
$$y'(t) = 2x(t) + 4$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 39

```
dsolve({diff(x(t),t)+2*y(t)=3*t,diff(y(t),t)-2*x(t)=4},singsol=all)
```

$$x(t) = c_2 \sin(2t) + c_1 \cos(2t) - \frac{5}{4}$$
$$y(t) = -c_2 \cos(2t) + c_1 \sin(2t) + \frac{3t}{2}$$

✓ Solution by Mathematica
Time used: 0.205 (sec). Leaf size: 47

```
DSolve[{x'[t]+2*y[t]==3*t,y'[t]-2*x[t]==4},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 \cos(2t) - c_2 \sin(2t) - \frac{5}{4}$$
$$y(t) \to \frac{3t}{2} + c_2 \cos(2t) + c_1 \sin(2t)$$

9.12 problem 1867

Internal problem ID [10190]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1867.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = t^2 - y(t) - 6t - 1$$
$$y'(t) = -3t^2 + x(t) + 3t + 1$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve([diff(x(t),t)+y(t)-t^2+6*t+1=0,diff(y(t),t)-x(t)=-3*t^2+3*t+1],singsol=all)
```

$$x(t) = c_2 \sin(t) + c_1 \cos(t) + 3t^2 - t - 13$$
$$y(t) = t^2 - c_2 \cos(t) + c_1 \sin(t) - 12t$$

✓ Solution by Mathematica

Time used: 0.255 (sec). Leaf size: 45

```
DSolve[{x'[t]+y[t]-t^2+6*t+1==0,y'[t]-x[t]==-3*t^2+3*t+1},{x[t],y[t]},t,IncludeSingularSolut
```

$$x(t) \to 3t^2 - t + c_1 \cos(t) - c_2 \sin(t) - 13$$
$$y(t) \to (t - 12)t + c_2 \cos(t) + c_1 \sin(t)$$

9.13 problem 1868

Internal problem ID [10191]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1868.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -3x(t) + y(t) + e^{2t}$$
$$y'(t) = -x(t) - 5y(t) + e^{t}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 64

```
dsolve([diff(x(t),t)+3*x(t)-y(t)=exp(2*t),diff(y(t),t)+x(t)+5*y(t)=exp(t)],singsol=all)
```

$$x(t) = c_2 e^{-4t} + e^{-4t} t c_1 + \frac{e^t}{25} + \frac{7 e^{2t}}{36}$$
$$y(t) = -\frac{e^{2t}}{36} - c_2 e^{-4t} - e^{-4t} t c_1 + e^{-4t} c_1 + \frac{4 e^t}{25}$$

✓ Solution by Mathematica

Time used: 0.177 (sec). Leaf size: 76

```
DSolve[{x'[t]+3*x[t]-y[t]==Exp[2*t],y'[t]+x[t]+5*y[t]==Exp[t]},{x[t],y[t]},t,IncludeSingular
```

$$x(t) \to \frac{e^t}{25} + \frac{7e^{2t}}{36} + e^{-4t}(c_1(t+1) + c_2 t)$$
$$y(t) \to \frac{4e^t}{25} - \frac{e^{2t}}{36} + e^{-4t}(c_2 - (c_1 + c_2)t)$$

9.14 problem 1869

Internal problem ID [10192]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1869.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) + y'(t) = -2x(t) - y(t) + e^{2t} + t$$
$$x'(t) + y'(t) = x(t) - 3y(t) + e^{t} - 1$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 51

```
dsolve([diff(x(t),t)+diff(y(t),t)+2*x(t)+y(t)=exp(2*t)+t,diff(x(t),t)+diff(y(t),t)-x(t)+3*y(
```

$$x(t) = \frac{3t}{7} - \frac{1}{49} - \frac{e^{t}}{6} + \frac{5\,e^{2t}}{17} + e^{-\frac{7t}{5}} c_1$$
$$y(t) = -\frac{e^{2t}}{17} + \frac{t}{7} - \frac{26}{49} + \frac{e^{t}}{4} + \frac{3\,e^{-\frac{7t}{5}} c_1}{2}$$

✓ Solution by Mathematica
Time used: 0.185 (sec). Leaf size: 84

```
DSolve[{x'[t]+y'[t]+2*x[t]+y[t]==Exp[2*t]+t,x'[t]+y'[t]-x[t]+3*y[t]==Exp[t]-1},{x[t],y[t]},t
```

$$x(t) \to \frac{3t}{7} - \frac{e^t}{6} + \frac{5e^{2t}}{17} + \frac{5}{72} c_1 e^{-7t/5} - \frac{1}{49}$$
$$y(t) \to \frac{t}{7} + \frac{e^t}{4} - \frac{e^{2t}}{17} + \frac{5}{48} c_1 e^{-7t/5} - \frac{26}{49}$$

9.15 problem 1870

Internal problem ID [10193]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1870.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -3y(t) - e^{t} + \cos(t)$$
$$y'(t) = 4y(t) + 2e^{t} - \cos(t)$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 47

```
dsolve([diff(x(t),t)+diff(y(t),t)-y(t)=exp(t),2*diff(x(t),t)+diff(y(t),t)+2*y(t)=cos(t)],sin
```

$$x(t) = \frac{c_1 e^{4t}}{4} + \frac{5\sin(t)}{17} + e^{t} - \frac{3\cos(t)}{17} + c_2$$
$$y(t) = -\frac{c_1 e^{4t}}{3} + \frac{4\cos(t)}{17} - \frac{2e^{t}}{3} - \frac{\sin(t)}{17}$$

✓ Solution by Mathematica
Time used: 0.19 (sec). Leaf size: 71

```
DSolve[{x'[t]+y'[t]-y[t]==Exp[t],2*x'[t]+y'[t]+2*y[t]==Cos[t]},{x[t],y[t]},t,IncludeSingular
```

$$x(t) \to e^{t} + \frac{5\sin(t)}{17} - \frac{3\cos(t)}{17} - \frac{3}{4}c_2 e^{4t} + c_1 + \frac{3c_2}{4}$$
$$y(t) \to -\frac{2e^{t}}{3} - \frac{\sin(t)}{17} + \frac{4\cos(t)}{17} + c_2 e^{4t}$$

9.16 problem 1871

Internal problem ID [10194]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1871.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -27 - 5x(t) - y(t) + 7\,e^{t}$$
$$y'(t) = 12 + 2x(t) - 3y(t) - 3\,e^{t}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 71

```
dsolve([4*diff(x(t),t)+9*diff(y(t),t)+2*x(t)+31*y(t)=exp(t),3*diff(x(t),t)+7*diff(y(t),t)+x(
```

$$x(t) = e^{-4t}\sin(t)\, c_2 + e^{-4t}\cos(t)\, c_1 - \frac{93}{17} + \frac{31\,e^{t}}{26}$$
$$y(t) = -e^{-4t}\sin(t)\, c_2 - e^{-4t}\cos(t)\, c_2 - e^{-4t}\cos(t)\, c_1 + e^{-4t}\sin(t)\, c_1 - \frac{2\,e^{t}}{13} + \frac{6}{17}$$

✓ Solution by Mathematica

Time used: 0.463 (sec). Leaf size: 79

```
DSolve[{4*x'[t]+9*y'[t]+2*x[t]+31*y[t]==Exp[t],3*x'[t]+7*y'[t]+x[t]+24*y[t]==3},{x[t],y[t]},
```

$$x(t) \to \frac{31e^{t}}{26} + c_1 e^{-4t}\cos(t) - (c_1 + c_2)e^{-4t}\sin(t) - \frac{93}{17}$$
$$y(t) \to -\frac{2e^{t}}{13} + c_2 e^{-4t}\cos(t) + (2c_1 + c_2)e^{-4t}\sin(t) + \frac{6}{17}$$

9.17 problem 1872

Internal problem ID [10195]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1872.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -5x(t) - y(t) + 7\,e^t - 9\,e^{2t}$$
$$y'(t) = x(t) - 3y(t) - 3\,e^t + 4\,e^{2t}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 65

```
dsolve([4*diff(x(t),t)+9*diff(y(t),t)+11*x(t)+31*y(t)=exp(t),3*diff(x(t),t)+7*diff(y(t),t)+8
```

$$x(t) = c_2 e^{-4t} + e^{-4t} t c_1 + \frac{31\,e^t}{25} - \frac{49\,e^{2t}}{36}$$
$$y(t) = \frac{19\,e^{2t}}{36} - c_2 e^{-4t} - e^{-4t} t c_1 - e^{-4t} c_1 - \frac{11\,e^t}{25}$$

✓ Solution by Mathematica

Time used: 0.305 (sec). Leaf size: 76

```
DSolve[{4*x'[t]+9*y'[t]+11*x[t]+31*y[t]==Exp[t],3*x'[t]+7*y'[t]+8*x[t]+24*y[t]==Exp[2*t]},{x
```

$$x(t) \to \frac{31e^t}{25} - \frac{49e^{2t}}{36} - e^{-4t}(c_1(t-1) + c_2 t)$$
$$y(t) \to -\frac{11e^t}{25} + \frac{19e^{2t}}{36} + e^{-4t}((c_1 + c_2)t + c_2)$$

9.18 problem 1873

Internal problem ID [10196]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1873.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -2x(t) - y(t) + 7t - 9\,e^t$$
$$y'(t) = -4x(t) - 5y(t) - 3t + 4\,e^t$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 52

```
dsolve([4*diff(x(t),t)+9*diff(y(t),t)+44*x(t)+49*y(t)=t,3*diff(x(t),t)+7*diff(y(t),t)+34*x(t
```

$$x(t) = c_2 e^{-t} + e^{-6t} c_1 - \frac{29\,e^t}{7} + \frac{19t}{3} - \frac{56}{9}$$
$$y(t) = -c_2 e^{-t} + 4\,e^{-6t} c_1 + \frac{24\,e^t}{7} + \frac{55}{9} - \frac{17t}{3}$$

✓ Solution by Mathematica

Time used: 0.232 (sec). Leaf size: 104

```
DSolve[{4*x'[t]+9*y'[t]+44*x[t]+49*y[t]==t,3*x'[t]+7*y'[t]+34*x[t]+38*y[t]==Exp[t]},{x[t],y[
```

$$x(t) \to \frac{1}{9}(57t - 56) - \frac{29e^t}{7} + \frac{1}{5}(4c_1 - c_2)e^{-t} + \frac{1}{5}(c_1 + c_2)e^{-6t}$$
$$y(t) \to \frac{1}{9}(55 - 51t) + \frac{24e^t}{7} + \frac{1}{5}(c_2 - 4c_1)e^{-t} + \frac{4}{5}(c_1 + c_2)e^{-6t}$$

9.19 problem 1874

Internal problem ID [10197]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1874.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t)\, f(t) + y(t)\, g(t)$$
$$y'(t) = -x(t)\, g(t) + y(t)\, f(t)$$

✓ Solution by Maple

Time used: 0.578 (sec). Leaf size: 57

```
dsolve([diff(x(t),t)=x(t)*f(t)+y(t)*g(t),diff(y(t),t)=-x(t)*g(t)+y(t)*f(t)],singsol=all)
```

$$x(t) = e^{\int (\tan(c_1 - (\int g(t)dt))g(t) + f(t))dt} c_2$$
$$y(t) = e^{\int (\tan(c_1 - (\int g(t)dt))g(t) + f(t))dt} c_2 \tan\left(c_1 - \left(\int g(t)\, dt\right)\right)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 93

```
DSolve[{x'[t]==x[t]*f[t]+y[t]*g[t],y'[t]==-x[t]*g[t]+y[t]*f[t]},{x[t],y[t]},t,IncludeSingula
```

$$x(t) \to \exp\left(\int_1^t f(K[2])dK[2]\right)\left(c_1 \cos\left(\int_1^t g(K[1])dK[1]\right) + c_2 \sin\left(\int_1^t g(K[1])dK[1]\right)\right)$$
$$y(t) \to \exp\left(\int_1^t f(K[2])dK[2]\right)\left(c_2 \cos\left(\int_1^t g(K[1])dK[1]\right) - c_1 \sin\left(\int_1^t g(K[1])dK[1]\right)\right)$$

9.20 problem 1875

Internal problem ID [10198]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1875.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -x(t)\, f(t)\, a - y(t)\, f(t)\, b + g(t)$$
$$y'(t) = -x(t)\, f(t)\, c - f(t)\, y(t)\, d + h(t)$$

✓ Solution by Maple
Time used: 3.25 (sec). Leaf size: 3922

```
dsolve([diff(x(t),t)+(a*x(t)+b*y(t))*f(t)=g(t),diff(y(t),t)+(c*x(t)+d*y(t))*f(t)=h(t)],sings
```

Expression too large to display
Expression too large to display

✓ Solution by Mathematica
Time used: 1.274 (sec). Leaf size: 3095

```
DSolve[{x'[t]+(a*x[t]+b*y[t])*f[t]==g[t],y'[t]+(c*x[t]+d*y[t])*f[t]==h[t]},{x[t],y[t]},t,Inc
```

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9.21 problem 1876

Internal problem ID [10199]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1876.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t)\cos(t)$$
$$y'(t) = x(t)\,\mathrm{e}^{-\sin(t)}$$

✓ Solution by Maple
Time used: 0.141 (sec). Leaf size: 18

```
dsolve([diff(x(t),t)=x(t)*cos(t),diff(y(t),t)=x(t)*exp(-sin(t))],singsol=all)
```

$$x(t) = c_2\mathrm{e}^{\sin(t)}$$
$$y(t) = c_2t + c_1$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 22

```
DSolve[{x'[t]==x[t]*Cos[t],y'[t]==x[t]*Exp[-Sin[t]]},{x[t],y[t]},t,IncludeSingularSolutions
```

$$x(t) \to c_1e^{\sin(t)}$$
$$y(t) \to c_1t + c_2$$

9.22 problem 1877

Internal problem ID [10200]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1877.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= -\frac{y(t)}{t} \\
y'(t) &= -\frac{x(t)}{t}
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve([t*diff(x(t),t)+y(t)=0,t*diff(y(t),t)+x(t)=0],singsol=all)
```

$$
\begin{aligned}
x(t) &= \frac{c_1 t^2 + c_2}{t} \\
y(t) &= -\frac{c_1 t^2 - c_2}{t}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 31

```
DSolve[{t*x'[t]+y[t]==0,t*y'[t]+x[t]==0},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
x(t) &\to c_1 t + \frac{c_2}{t} \\
y(t) &\to \frac{c_2}{t} - c_1 t
\end{aligned}
$$

9.23 problem 1878

Internal problem ID [10201]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1878.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -\frac{2x(t)}{t} + 1$$
$$y'(t) = y(t) + x(t) + \frac{2x(t)}{t} - 1$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 39

```
dsolve([t*diff(x(t),t)+2*x(t)=t,t*diff(y(t),t)-(t+2)*x(t)-t*y(t)=-t],singsol=all)
```

$$x(t) = \frac{t}{3} + \frac{c_2}{t^2}$$
$$y(t) = \frac{3c_1 e^t t^2 - t^3 - 3c_2}{3t^2}$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 39

```
DSolve[{t*x'[t]+2*x[t]==t,t*y'[t]-(t+2)*x[t]-t*y[t]==-t},{x[t],y[t]},t,IncludeSingularSoluti
```

$$x(t) \to \frac{t}{3} + \frac{c_1}{t^2}$$
$$y(t) \to -\frac{c_1}{t^2} - \frac{t}{3} + c_2 e^t$$

9.24 problem 1879

Internal problem ID [10202]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1879.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -\frac{2x(t)}{t} + \frac{2y(t)}{t} + 1$$
$$y'(t) = t - \frac{x(t)}{t} - \frac{5y(t)}{t}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 54

```
dsolve([t*diff(x(t),t)+2*(x(t)-y(t))=t,t*diff(y(t),t)+x(t)+5*y(t)=t^2],singsol=all)
```

$$x(t) = \frac{2t^6 + 9t^5 + 30c_1t + 30c_2}{30t^4}$$
$$y(t) = -\frac{-8t^6 + 3t^5 + 30c_1t + 60c_2}{60t^4}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 58

```
DSolve[{t*x'[t]+2*(x[t]-y[t])==t,t*y'[t]+x[t]+5*y[t]==t^2},{x[t],y[t]},t,IncludeSingularSolu
```

$$x(t) \to \frac{c_1}{t^4} + \frac{c_2}{t^3} + \frac{1}{30}t(2t+9)$$
$$y(t) \to -\frac{-8t^6 + 3t^5 + 30c_2t + 60c_1}{60t^4}$$

9.25 problem 1880

Internal problem ID [10203]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1880.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = \frac{2x(t)\sin(t)}{t(\sin(t)-1)} - \frac{y(t)}{\sin(t)-1} - \frac{x(t)}{t(\sin(t)-1)}$$
$$y'(t) = \frac{y(t)\cos(t)}{\sin(t)-1} - \frac{x(t)\cos(t)}{t(\sin(t)-1)} + \frac{x(t)\sin(t)}{t^2(\sin(t)-1)} - \frac{y(t)}{t(\sin(t)-1)}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 23

```
dsolve([t^2*(1-sin(t))*diff(x(t),t)=t*(1-2*sin(t))*x(t)+t^2*y(t),t^2*(1-sin(t))*diff(y(t),t)
```

$$x(t) = t(c_2t + c_1)$$
$$y(t) = c_1\sin(t) + c_2t$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 27

```
DSolve[{t^2*(1-Sin[t])*x'[t]==t*(1-2*Sin[t])*x[t]+t^2*y[t],t^2*(1-Sin[t])*y'[t]==(t*Cos[t]-S
```

$$x(t) \to t(c_1t + c_2)$$
$$y(t) \to c_1t + c_2\sin(t)$$

9.26 problem 1881

Internal problem ID [10204]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1881.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) + y'(t) + y(t) = f(t)$$
$$x''(t) + y''(t) + y'(t) + x(t) + y(t) = g(t)$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 48

```
dsolve([diff(x(t),t)+diff(y(t),t)+y(t)=f(t),diff(x(t),t$2)+diff(y(t),t$2)+diff(y(t),t)+x(t)+
```

$$x(t) = -\frac{d}{dt}f(t) - f(t) - \frac{d^2}{dt^2}f(t) + \frac{d}{dt}g(t) + g(t)$$
$$y(t) = f(t) + \frac{d^2}{dt^2}f(t) - \frac{d}{dt}g(t)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 44

```
DSolve[{x'[t]+y'[t]+y[t]==f[t],x''[t]+y''[t]+y'[t]+x[t]+y[t]==g[t]},{x[t],y[t]},t,IncludeSin
```

$$x(t) \to -f''(t) - f'(t) - f(t) + g'(t) + g(t)$$
$$y(t) \to f''(t) + f(t) - g'(t)$$

9.27 problem 1882

Internal problem ID [10205]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1882.
ODE order: 1.
ODE degree: 1.

Solve

$$x''(t) + y'(t) - 2y(t) = e^{2t}$$
$$2x'(t) + y'(t) - 3x(t) = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 118

```
dsolve([2*diff(x(t),t)+diff(y(t),t)-3*x(t)=0,diff(x(t),t$2)+diff(y(t),t)-2*y(t)=exp(2*t)],si
```

$$x(t) = \frac{e^{2t}}{4} + c_1 e^t - \frac{7c_2 e^{\frac{t}{2}} \cos\left(\frac{\sqrt{23}\,t}{2}\right)}{18} - \frac{c_2 e^{\frac{t}{2}} \sqrt{23}\,\sin\left(\frac{\sqrt{23}\,t}{2}\right)}{18} - \frac{7c_3 e^{\frac{t}{2}} \sin\left(\frac{\sqrt{23}\,t}{2}\right)}{18} + \frac{c_3 e^{\frac{t}{2}} \sqrt{23}\,\cos\left(\frac{\sqrt{23}\,t}{2}\right)}{18}$$

$$y(t) = -\frac{e^{2t}}{8} + c_1 e^t + c_2 e^{\frac{t}{2}} \cos\left(\frac{\sqrt{23}\,t}{2}\right) + c_3 e^{\frac{t}{2}} \sin\left(\frac{\sqrt{23}\,t}{2}\right)$$

✓ Solution by Mathematica

Time used: 5.668 (sec). Leaf size: 199

```
DSolve[{2*x'[t]+y'[t]-3*x[t]==0,x''[t]+y'[t]-2*y[t]==Exp[2*t]},{x[t],y[t]},t,IncludeSingular
```

$$x(t) \to \frac{1}{276} e^{t/2} \left(23 e^{t/2} (3e^t + 6c_1 + 2c_2 + 4c_3) + 46(3c_1 - c_2 - 2c_3) \cos\left(\frac{\sqrt{23}t}{2}\right) - 2\sqrt{23}(9c_1 - 11c_2 + 2c_3) \sin\left(\frac{\sqrt{23}\right.\right.$$

$$y(t) \to -\frac{1}{552} e^{t/2} \left(23 e^{t/2} (3e^t - 4(3c_1 + c_2 + 2c_3)) + 92(3c_1 + c_2 - 4c_3) \cos\left(\frac{\sqrt{23}t}{2}\right) - 4\sqrt{23}(33c_1 - 25c_2 - 8c_3) \sin\left(\right.\right.$$

9.28 problem 1883

Internal problem ID [10206]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1883.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) - y'(t) + x(t) = 2t$$
$$x''(t) + y'(t) - 9x(t) + 3y(t) = \sin(2t)$$

✓ Solution by Maple

Time used: 0.204 (sec). Leaf size: 80

```
dsolve([diff(x(t),t)-diff(y(t),t)+x(t)=2*t,diff(x(t),t$2)+diff(y(t),t)-9*x(t)+3*y(t)=sin(2*t
```

$$x(t) = -\frac{36\sin(2t)}{325} - \frac{2\cos(2t)}{325} + \frac{c_1 e^t}{2} + \frac{3c_2 e^{-3t}}{2} + \frac{c_3 e^t}{4} + \frac{c_3 t\, e^t}{2} + 2t + 4$$
$$y(t) = 6t + 10 - \frac{37\sin(2t)}{325} + \frac{16\cos(2t)}{325} + c_1 e^t + c_2 e^{-3t} + c_3 t\, e^t$$

✓ Solution by Mathematica

Time used: 2.889 (sec). Leaf size: 170

```
DSolve[{x'[t]-y'[t]+x[t]==2*t,x''[t]+y'[t]-9*x[t]+3*y[t]==Sin[2*t]},{x[t],y[t]},t,IncludeSin
```

$$x(t) \to -\frac{36}{325}\sin(2t) - \frac{2}{325}\cos(2t) + \frac{1}{16}e^{-3t}\left(32e^{3t}(t+2) + e^{4t}(c_1(20t+7) + c_2(4t+3) + 3c_3(1-4t)) + 9c_1 - 3(c_2+c_3)\right)$$
$$y(t) \to -\frac{37}{325}\sin(2t) + \frac{16}{325}\cos(2t) + \frac{1}{8}e^{-3t}\left(16e^{3t}(3t+5) + e^{4t}(c_1(20t-3) + 4c_2 t - 12c_3 t + c_2 + 9c_3) + 3c_1 - c_2 - c_3\right)$$

9.29 problem 1884

Internal problem ID [10207]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1884.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) - 2y(t)$$
$$y'(t) = \frac{x(t)}{4} - \frac{y(t)}{2} - \frac{t}{2} + \frac{\cos(t)^2}{2} - \frac{1}{4}$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 69

```
dsolve([diff(x(t),t)-x(t)+2*y(t)=0,diff(x(t),t,t)-2*diff(y(t),t)=2*t-cos(2*t)],singsol=all)
```

$$x(t) = -t^2 + 8\,e^{\frac{t}{2}}c_1 + \frac{\sin(2t)}{34} + \frac{2\cos(2t)}{17} - 4t + 2c_2 - 4$$
$$y(t) = -\frac{t^2}{2} + 2\,e^{\frac{t}{2}}c_1 + \frac{9\sin(2t)}{68} + \frac{\cos(2t)}{34} - t + c_2$$

✓ Solution by Mathematica
Time used: 1.032 (sec). Leaf size: 116

```
DSolve[{x'[t]-x[t]+2*y[t]==0,x''[t]-2*y'[t]==2*t-Cos[2*t]},{x[t],y[t]},t,IncludeSingularSolu
```

$$x(t) \to -t^2 - 4t + \frac{1}{34}\sin(2t) + \frac{2}{17}\cos(2t) + 8c_1e^{t/2} + 8c_2e^{t/2} - 8 - c_2$$
$$y(t) \to -\frac{t^2}{2} - t + \frac{9}{68}\sin(2t) + \frac{1}{34}\cos(2t) + 2c_1e^{t/2} + 2c_2e^{t/2} - 2 - \frac{c_2}{2}$$

9.30 problem 1885

Internal problem ID [10208]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1885.
ODE order: 1.
ODE degree: 1.

Solve

$$tx''(t) + 2x'(t) + x(t)\,t = 0$$
$$tx'(t) - y'(t)\,t - 2y(t) = 0$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 50

```
dsolve([t*diff(x(t),t)-t*diff(y(t),t)-2*y(t)=0,t*diff(x(t),t,t)+2*diff(x(t),t)+t*x(t)=0],sin
```

$$x(t) = -\frac{c_3 \cos(t) - \sin(t)\, c_2}{t}$$
$$y(t) = \frac{-\cos(t)\, c_3 t + \sin(t)\, c_2 t + 2c_2 \cos(t) + 2\sin(t)\, c_3 + c_1}{t^2}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 54

```
DSolve[{t*x'[t]-t*y'[t]-2*y[t]==0,t*x''[t]+2*x'[t]+t*x[t]==0},{x[t],y[t]},t,IncludeSingularS
```

$$x(t) \to \frac{c_2 \cos(t) + c_3 \sin(t)}{t}$$
$$y(t) \to \frac{c_2 t\cos(t) + 2c_3 \cos(t) - 2c_2 \sin(t) + c_3 t\sin(t) + c_1}{t^2}$$

9.31 problem 1886

Internal problem ID [10209]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1886.
ODE order: 1.
ODE degree: 1.

Solve

$$x''(t) + ay(t) = 0$$
$$y''(t) - a^2 y(t) = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 50

```
dsolve([diff(x(t),t,t)+a*y(t)=0,diff(y(t),t,t)-a^2*y(t)=0],singsol=all)
```

$$x(t) = -\frac{-c_1 t a + c_3 e^{-at} + c_4 e^{at} - c_2 a}{a}$$
$$y(t) = c_3 e^{-at} + c_4 e^{at}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 103

```
DSolve[{x''[t]+a*y[t]==0,y''[t]-a^2*y[t]==0},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{c_4(2at + e^{-at} - e^{at})}{2a^2} - \frac{c_3 e^{-at}(e^{at} - 1)^2}{2a} + c_2 t + c_1$$
$$y(t) \to \frac{e^{-at}(ac_3(e^{2at} + 1) + c_4(e^{2at} - 1))}{2a}$$

9.32 problem 1887

Internal problem ID [10210]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1887.
ODE order: 1.
ODE degree: 1.

Solve

$$x''(t) = ax(t) + by(t)$$
$$y''(t) = cx(t) + dy(t)$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 418

```
dsolve([diff(x(t),t,t)=a*x(t)+b*y(t),diff(y(t),t,t)=c*x(t)+d*y(t)],singsol=all)
```

$$\begin{aligned} x(t) &= \left(-\frac{d}{2c}+\frac{\frac{\sqrt{a^2-2ad+4bc+d^2}}{2}+\frac{a}{2}}{c}\right)c_4\mathrm{e}^{\frac{\sqrt{2\sqrt{a^2-2ad+4bc+d^2}+2a+2d}\,t}{2}} \\ &+\left(-\frac{d}{2c}+\frac{\frac{\sqrt{a^2-2ad+4bc+d^2}}{2}+\frac{a}{2}}{c}\right)c_3\mathrm{e}^{-\frac{\sqrt{2\sqrt{a^2-2ad+4bc+d^2}+2a+2d}\,t}{2}} \\ &+\left(-\frac{d}{2c}+\frac{-\frac{\sqrt{a^2-2ad+4bc+d^2}}{2}+\frac{a}{2}}{c}\right)c_2\mathrm{e}^{\frac{\sqrt{-2\sqrt{a^2-2ad+4bc+d^2}+2a+2d}\,t}{2}} \\ &+\left(-\frac{d}{2c}+\frac{-\frac{\sqrt{a^2-2ad+4bc+d^2}}{2}+\frac{a}{2}}{c}\right)c_1\mathrm{e}^{-\frac{\sqrt{-2\sqrt{a^2-2ad+4bc+d^2}+2a+2d}\,t}{2}} \\ y(t) &= c_1\mathrm{e}^{-\frac{\sqrt{-2\sqrt{a^2-2ad+4bc+d^2}+2a+2d}\,t}{2}}+c_2\mathrm{e}^{\frac{\sqrt{-2\sqrt{a^2-2ad+4bc+d^2}+2a+2d}\,t}{2}} \\ &+c_3\mathrm{e}^{-\frac{\sqrt{2\sqrt{a^2-2ad+4bc+d^2}+2a+2d}\,t}{2}}+c_4\mathrm{e}^{\frac{\sqrt{2\sqrt{a^2-2ad+4bc+d^2}+2a+2d}\,t}{2}} \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.325 (sec). Leaf size: 5647

```
DSolve[{x''[t]==a*x[t]+b*y[t],y''[t]==c*x[t]+d*y[t]},{x[t],y[t]},t,IncludeSingularSolutions
```

Too large to display

9.33 problem 1888

Internal problem ID [10211]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1888.
ODE order: 1.
ODE degree: 1.

Solve

$$x''(t) = a_1x(t) + b_1y(t) + c_1$$
$$y''(t) = a_2x(t) + b_2y(t) + c_2$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 651

```
dsolve([diff(x(t),t,t)=a__1*x(t)+b__1*y(t)+c__1,diff(y(t),t,t)=a__2*x(t)+b__2*y(t)+c__2],sin
```

$x(t) =$

$$-\frac{\left(c_6a_1b_2^2 + \left(-\sqrt{a_1^2 - 2a_1b_2 + 4a_2b_1 + b_2^2}\,c_6a_1 - c_6a_1^2 - c_6a_2b_1\right)b_2 + \left(\sqrt{a_1^2 - 2a_1b_2 + 4a_2b_1 + b_2^2}\,c_6 + c\right.}{2a_2\left(a_1b_2 - a_2b_1\right)}$$

$$-\frac{\left(c_5a_1b_2^2 + \left(-\sqrt{a_1^2 - 2a_1b_2 + 4a_2b_1 + b_2^2}\,c_5a_1 - c_5a_1^2 - c_5a_2b_1\right)b_2 + \left(\sqrt{a_1^2 - 2a_1b_2 + 4a_2b_1 + b_2^2}\,c_5 + c\right.}{2a_2\left(a_1b_2 - a_2b_1\right)}$$

$$-\frac{\left(c_4a_1b_2^2 + \left(\sqrt{a_1^2 - 2a_1b_2 + 4a_2b_1 + b_2^2}\,c_4a_1 - c_4a_1^2 - c_4a_2b_1\right)b_2 + \left(-\sqrt{a_1^2 - 2a_1b_2 + 4a_2b_1 + b_2^2}\,c_4 + c\right.}{2a_2\left(a_1b_2 - a_2b_1\right)}$$

$$-\frac{\left(c_3a_1b_2^2 + \left(\sqrt{a_1^2 - 2a_1b_2 + 4a_2b_1 + b_2^2}\,c_3a_1 - c_3a_1^2 - c_3a_2b_1\right)b_2 + \left(-\sqrt{a_1^2 - 2a_1b_2 + 4a_2b_1 + b_2^2}\,c_3 + c\right.}{2a_2\left(a_1b_2 - a_2b_1\right)}$$

$$-\frac{-2a_2b_1c_2 + 2a_2b_2c_1}{2a_2\left(a_1b_2 - a_2b_1\right)}$$

$$y(t) = \frac{-a_1c_2 + a_2c_1}{a_1b_2 - a_2b_1} + c_3\mathrm{e}^{-\frac{\sqrt{2a_1 + 2b_2 - 2\sqrt{a_1^2 - 2a_1b_2 + 4a_2b_1 + b_2^2}}\,t}{2}} + c_4\mathrm{e}^{\frac{\sqrt{2a_1 + 2b_2 - 2\sqrt{a_1^2 - 2a_1b_2 + 4a_2b_1 + b_2^2}}\,t}{2}}$$
$$+ c_5\mathrm{e}^{-\frac{\sqrt{2\sqrt{a_1^2 - 2a_1b_2 + 4a_2b_1 + b_2^2} + 2a_1 + 2b_2}\,t}{2}} + c_6\mathrm{e}^{\frac{\sqrt{2\sqrt{a_1^2 - 2a_1b_2 + 4a_2b_1 + b_2^2} + 2a_1 + 2b_2}\,t}{2}}$$

✓ Solution by Mathematica

Time used: 27.36 (sec). Leaf size: 13523

```
DSolve[{x''[t]==a1*x[t]+b1*y[t]+c1,y''[t]==a2*x[t]+b2*y[t]+c2},{x[t],y[t]},t,IncludeSingular
```

Too large to display

9.34 problem 1889

Internal problem ID [10212]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1889.
ODE order: 1.
ODE degree: 1.

Solve

$$x''(t) + x(t) + y(t) = -5$$
$$y''(t) - 4x(t) - 3y(t) = -3$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 72

```
dsolve([diff(x(t),t,t)+x(t)+y(t)=-5,diff(y(t),t,t)-4*x(t)-3*y(t)=-3],singsol=all)
```

$$x(t) = -\frac{c_1 e^t}{2} - \frac{c_2 e^{-t}}{2} + \frac{c_3 e^t}{2} - \frac{c_3 t\, e^t}{2} - \frac{c_4 e^{-t} t}{2} - \frac{c_4 e^{-t}}{2} + 18$$
$$y(t) = -23 + c_1 e^t + c_2 e^{-t} + c_3 t\, e^t + c_4 e^{-t} t$$

✓ Solution by Mathematica

Time used: 0.586 (sec). Leaf size: 151

```
DSolve[{x''[t]+x[t]+y[t]==-5,y''[t]-4*x[t]-3*y[t]==-3},{x[t],y[t]},t,IncludeSingularSolution
```

$$x(t) \to \frac{1}{4}e^{-t}\left(72e^t + 2c_1(t+1) - 2c_2t + c_3t - c_4t + e^{2t}(-2c_1(t-1) - 2c_2(t-2) - c_3t - c_4t + c_4) - 4c_2 - c_4\right)$$
$$y(t) \to \frac{1}{2}e^{-t}\left(-46e^t + (-2c_1 + 2c_2 - c_3 + c_4)t + e^{2t}((2c_1 + 2c_2 + c_3 + c_4)t - 2c_2 + c_3) + 2c_2 + c_3\right)$$

9.35 problem 1890

Internal problem ID [10213]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1890.
ODE order: 1.
ODE degree: 1.

Solve

$$x''(t) = \left(3\left(\cos^2(at+b)\right)-1\right)c^2x(t)+\frac{3c^2y(t)\sin(2atb)}{2}$$
$$y''(t) = \left(3\left(\sin^2(at+b)\right)-1\right)c^2y(t)+\frac{3c^2x(t)\sin(2atb)}{2}$$

✗ Solution by Maple

```
dsolve([diff(x(t),t,t)=(3*cos(a*t+b)^2-1)*c^2*x(t)+3/2*c^2*y(t)*sin(2*(a*t*b)),diff(y(t),t,t
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x''[t]==(3*Cos[a*t+b]^2-1)*c^2*x[t]+3/2*c^2*y[t]*Sin[2*(a*t*b)],y''[t]==(3*Sin[a*t+b
```

Not solved

9.36 problem 1891

Internal problem ID [10214]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1891.
ODE order: 1.
ODE degree: 1.

Solve

$$x''(t) + 6x(t) + 7y(t) = 0$$
$$y''(t) + 3x(t) + 2y(t) = 2t$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 64

```
dsolve([diff(x(t),t,t)+6*x(t)+7*y(t)=0,diff(y(t),t,t)+3*x(t)+2*y(t)=2*t],singsol=all)
```

$$x(t) = -c_1 e^t + \frac{7c_2 \cos(3t)}{3} - c_3 e^{-t} + \frac{7c_4 \sin(3t)}{3} + \frac{14t}{9}$$
$$y(t) = -\frac{4t}{3} + c_1 e^t + c_2 \cos(3t) + c_3 e^{-t} + c_4 \sin(3t)$$

✓ Solution by Mathematica

Time used: 1.247 (sec). Leaf size: 200

```
DSolve[{x''[t]+6*x[t]+7*y[t]==0,y''[t]+3*x[t]+2*y[t]==2*t},{x[t],y[t]},t,IncludeSingularSolu
```

$$x(t) \to \frac{1}{180} e^{-t} \big(280 e^t t + 27c_1 e^{2t} + 27c_2 e^{2t} - 63c_3 e^{2t} - 63c_4 e^{2t} + 126(c_1 + c_3)e^t \cos(3t) + 42(c_2 + c_4)e^t \sin(3t) + 27c_1 - 27c_2 - 63c_3 + 63c_4\big)$$
$$y(t) \to \frac{1}{60} e^{-t} \big(-80 e^t t - 9c_1 e^{2t} - 9c_2 e^{2t} + 21c_3 e^{2t} + 21c_4 e^{2t} + 18(c_1 + c_3)e^t \cos(3t) + 6(c_2 + c_4)e^t \sin(3t) - 9c_1 + 9c_2 + 21c_3 - 21c_4\big)$$

9.37 problem 1892

Internal problem ID [10215]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1892.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x''(t) - ay'(t) + bx(t) &= 0 \\
y''(t) + ax'(t) + by(t) &= 0
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 868

```
dsolve([diff(x(t),t,t)-a*diff(y(t),t)+b*x(t)=0,diff(y(t),t,t)+a*diff(x(t),t)+b*y(t)=0],sings
```

$x(t) =$

$$
-\frac{c_1\left(-2a^2-2\sqrt{a^2\left(a^2+4b\right)}-4b\right)^{\frac{3}{2}} e^{-\frac{\sqrt{-2a^2-2\sqrt{a^2\left(a^2+4b\right)}-4b}\,t}{2}} + 4\,e^{-\frac{\sqrt{-2a^2-2\sqrt{a^2\left(a^2+4b\right)}-4b}\,t}{2}} \sqrt{-2a^2-2\sqrt{a^2\left(\right.}}}{}
$$

$$
\begin{aligned}
y(t) = {} & c_1 e^{-\frac{\sqrt{-2a^2-2\sqrt{a^2\left(a^2+4b\right)}-4b}\,t}{2}} + c_2 e^{\frac{\sqrt{-2a^2-2\sqrt{a^2\left(a^2+4b\right)}-4b}\,t}{2}} \\
& + c_3 e^{-\frac{\sqrt{-2a^2+2\sqrt{a^2\left(a^2+4b\right)}-4b}\,t}{2}} + c_4 e^{\frac{\sqrt{-2a^2+2\sqrt{a^2\left(a^2+4b\right)}-4b}\,t}{2}}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.263 (sec). Leaf size: 3522

```
DSolve[{x''[t]-a*y'[t]+b*x[t]==0,y''[t]+a*x'[t]+b*y[t]==0},{x[t],y[t]},t,IncludeSingularSolu
```

Too large to display

9.38 problem 1893

Internal problem ID [10216]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1893.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
a_1x''(t) + b_1x'(t) + c_1x(t) - Ay'(t) &= B\,\mathrm{e}^{i\omega t} \\
a_2y''(t) + b_2y'(t) + c_2y(t) + Ax'(t) &= 0
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.688 (sec). Leaf size: 1577

```
dsolve([a__1*diff(x(t),t,t)+b__1*diff(x(t),t)+c__1*x(t)-A*diff(y(t),t)=B*exp(I*omega*t),a__2
```

Expression too large to display

$$
y(t) = \frac{i\mathrm{e}^{i\omega t}\omega AB}{-a_1a_2\omega^4 + ia_1b_2\omega^3 + ia_2b_1\omega^3 + A^2\omega^2 + a_1c_2\omega^2 + a_2c_1\omega^2 + b_1b_2\omega^2 - b_1c_2\omega i - b_2c_1\omega i - c_2c_1}
+ c_3\mathrm{e}^{\mathrm{RootOf}\left(a_1a_2\_Z^4+(a_1b_2+a_2b_1)\_Z^3+(A^2+c_2a_1+a_2c_1+b_1b_2)\_Z^2+(b_1c_2+b_2c_1)\_Z+c_2c_1,\mathrm{index}=1\right)t}
+ c_4\mathrm{e}^{\mathrm{RootOf}\left(a_1a_2\_Z^4+(a_1b_2+a_2b_1)\_Z^3+(A^2+c_2a_1+a_2c_1+b_1b_2)\_Z^2+(b_1c_2+b_2c_1)\_Z+c_2c_1,\mathrm{index}=2\right)t}
+ c_5\mathrm{e}^{\mathrm{RootOf}\left(a_1a_2\_Z^4+(a_1b_2+a_2b_1)\_Z^3+(A^2+c_2a_1+a_2c_1+b_1b_2)\_Z^2+(b_1c_2+b_2c_1)\_Z+c_2c_1,\mathrm{index}=3\right)t}
+ c_6\mathrm{e}^{\mathrm{RootOf}\left(a_1a_2\_Z^4+(a_1b_2+a_2b_1)\_Z^3+(A^2+c_2a_1+a_2c_1+b_1b_2)\_Z^2+(b_1c_2+b_2c_1)\_Z+c_2c_1,\mathrm{index}=4\right)t}
$$

✓ Solution by Mathematica

Time used: 67.399 (sec). Leaf size: 149009

```
DSolve[{a1*x''[t]+b1*x'[t]+c1*x[t]-A*t'[t]==B*Exp[I*\[Omega]*t],a2*y''[t]+b2*y'[t]+c2*y[t]+A
```

Too large to display

9.39 problem 1894

Internal problem ID [10217]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1894.
ODE order: 1.
ODE degree: 1.

Solve

$$x''(t) + a(x'(t) - y'(t)) + b_1 x(t) = c_1 e^{i\omega t}$$
$$y''(t) + a(y'(t) - x'(t)) + b_2 y(t) = c_2 e^{i\omega t}$$

✓ Solution by Maple

Time used: 2.032 (sec). Leaf size: 2571

```
dsolve([diff(x(t),t,t)+a*(diff(x(t),t)-diff(y(t),t))+b__1*x(t)=c__1*exp(I*omega*t),diff(y(t)
```

Expression too large to display

$$y(t) = \frac{ie^{i\omega t}c_1 a\omega + ie^{i\omega t}c_2 a\omega - e^{i\omega t}\omega^2 c_2 + e^{i\omega t}b_1 c_2}{-2ia\,\omega^3 + iab_1\omega + iab_2\omega + \omega^4 - b_1\omega^2 - b_2\omega^2 + b_1 b_2} + c_3 e^{\mathrm{RootOf}\left(\_Z^4+2a\_Z^3+(b_1+b_2)\_Z^2+(b_1a+b_2a)\_Z+b_1b_2,\mathrm{index}=1\right)t} + c_4 e^{\mathrm{RootOf}\left(\_Z^4+2a\_Z^3+(b_1+b_2)\_Z^2+(b_1a+b_2a)\_Z+b_1b_2,\mathrm{index}=2\right)t} + c_5 e^{\mathrm{RootOf}\left(\_Z^4+2a\_Z^3+(b_1+b_2)\_Z^2+(b_1a+b_2a)\_Z+b_1b_2,\mathrm{index}=3\right)t} + c_6 e^{\mathrm{RootOf}\left(\_Z^4+2a\_Z^3+(b_1+b_2)\_Z^2+(b_1a+b_2a)\_Z+b_1b_2,\mathrm{index}=4\right)t}$$

✓ Solution by Mathematica

Time used: 0.475 (sec). Leaf size: 3386

```
DSolve[{x''[t]+a*(x'[t]-y'[t])+b1*x[t]==c1*Exp[I*\[Omega]*t],y''[t]+a*(y'[t]-x'[t])+b2*y[t]=
```

Too large to display

9.40 problem 1895

Internal problem ID [10218]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1895.
ODE order: 1.
ODE degree: 1.

Solve

$$a11x''(t) + b11x'(t) + c11x(t) + a12y''(t) + b12y'(t) + c12y(t) = 0$$
$$a21x''(t) + b21x'(t) + c21x(t) + a22y''(t) + b22y'(t) + c22y(t) = 0$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 1288

```
dsolve([a11*diff(x(t),t,t)+b11*diff(x(t),t)+c11*x(t)+a12*diff(y(t),t,t)+b12*diff(y(t),t)+c12
```

Expression too large to display

$$y(t) = \sum_{\_a=1}^{4} \mathrm{e}^{\mathrm{RootOf}((\mathrm{a11\,a22} - \mathrm{a12\,a21})\_Z^4 + (\mathrm{a11\,b22} - \mathrm{a12\,b21} - \mathrm{a21\,b12} + \mathrm{a22\,b11})\_Z^3 + (\mathrm{a11\,c22} - \mathrm{a12\,c21} - \mathrm{a21\,c12} + \mathrm{a22\,c11} + \mathrm{b11\,b22} - \mathrm{b12}}$$

✓ Solution by Mathematica

Time used: 0.196 (sec). Leaf size: 7517

```
DSolve[{a11*x''[t]+b11*x'[t]+c11*x[t]+a12*y''[t]+b12*y'[t]+c12*y[t]==0,a21*x''[t]+b21*x'[t]+
```

Too large to display

9.41 problem 1896

Internal problem ID [10219]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1896.
ODE order: 1.
ODE degree: 1.

Solve

$$y'''(t) - y''(t) + 2x'(t) - x(t) = t$$
$$x''(t) - 2x'(t) - y'(t) + y(t) = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 75

```
dsolve([diff(x(t),t,t)-2*diff(x(t),t)-diff(y(t),t)+y(t)=0,diff(y(t),t,t,t)-diff(y(t),t,t)+2*
```

$$x(t) = -2 - 6c_5 e^t - t - \frac{2c_2 e^{-t}}{3} - 2c_4 e^t t - 3c_5 e^t t^2 - c_3 e^t$$
$$y(t) = -2 + c_1 e^t + c_2 e^{-t} + c_3 t\, e^t + c_4 e^t t^2 + c_5 e^t t^3$$

✓ Solution by Mathematica

Time used: 0.703 (sec). Leaf size: 246

```
DSolve[{x''[t]-2*x'[t]-y'[t]+y[t]==0,y'''[t]-y''[t]+2*x'[t]-x[t]==t},{x[t],y[t]},t,IncludeSi
```

$$x(t) \to \frac{1}{8}e^{-t}\left(e^{2t}\left(-2c_3t^2 + 2c_5t^2 + c_1\left(2t^2 - 6t + 7\right) + c_2\left(2t^2 + 6t + 1\right) - 2c_3t + 4c_4t - 2c_5t + c_3 - 2c_4 + c_5\right) - 8e^t(t+2) + c_1 - c_2 - c_3 + 2c_4 - c_5\right)$$
$$y(t) \to \frac{1}{48}\left(e^t\left(4c_3t^3 - 4c_5t^3 + 6c_3t^2 - 12c_4t^2 + 6c_5t^2 + c_1\left(-4t^3 + 18t^2 - 18t + 9\right) - c_2\left(4t^3 + 18t^2 - 18t + 9\right) - 30c_3t + 12c_4t + 18c_5t + 39c_3 + 18c_4 - 9c_5\right) + 9(-c_1 + c_2 + c_3 - 2c_4 + c_5)e^{-t} - 96\right)$$

9.42 problem 1897

Internal problem ID [10220]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1897.
ODE order: 1.
ODE degree: 1.

Solve

$$2x''(t) + y''(t) = 2t$$
$$x''(t) + y''(t) + y'(t) = \sinh(2t)$$

✓ Solution by Maple

Time used: 0.532 (sec). Leaf size: 104

```
dsolve([diff(x(t),t,t)+diff(y(t),t,t)+diff(y(t),t)=sinh(2*t),2*diff(x(t),t,t)+diff(y(t),t,t)
```

$$x(t) = \frac{t^2}{4} + c_4 t + \frac{t^3}{6} + \frac{t\sinh(2t)}{4} - \frac{\cosh(2t)}{8} - \frac{t\cosh(2t)}{4} + \frac{\cosh(2t)\,c_3}{4} - \frac{c_3\sinh(2t)}{4} + c_1 t + c_2$$
$$y(t) = \frac{t}{2} + \frac{c_3\sinh(2t)}{2} - \frac{t\sinh(2t)}{2} + \frac{\cosh(2t)}{4} - \frac{\cosh(2t)\,c_3}{2} + \frac{t\cosh(2t)}{2} - \frac{t^2}{2} + c_4$$

✓ Solution by Mathematica

Time used: 0.393 (sec). Leaf size: 118

```
DSolve[{x''[t]+y''[t]+y'[t]==Sinh[2*t],2*x''[t]+y''[t]==2*t},{x[t],y[t]},t,IncludeSingularSo
```

$$x(t) \to \frac{1}{48}\left(2\left(4t^3 + 6t^2 + 6(-1 + 4c_2 + 2c_4)t + 3 + 24c_1 - 6c_4\right) - 3e^{2t} - 6e^{-2t}(2t + 1 - 2c_4)\right)$$
$$y(t) \to \frac{1}{8}e^{-2t}\left(e^{2t}\left(-4t^2 + 4t - 2 + 8c_3 + 4c_4\right) + 4t + e^{4t} + 2 - 4c_4\right)$$

9.43 problem 1898

Internal problem ID [10221]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1898.
ODE order: 1.
ODE degree: 1.

Solve

$$x''(t)+y''(t)-x(t)=0$$
$$x''(t)-x'(t)+y'(t)=0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 73

```
dsolve([diff(x(t),t,t)-diff(x(t),t)+diff(y(t),t)=0,diff(x(t),t,t)+diff(y(t),t,t)-x(t)=0],sin
```

$$x(t)=\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)c_3\mathrm{e}^{\frac{\left(\sqrt{5}+1\right)t}{2}}+\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)c_4\mathrm{e}^{-\frac{\left(\sqrt{5}-1\right)t}{2}}+c_1\mathrm{e}^{t}$$
$$y(t)=c_2+c_3\mathrm{e}^{\frac{\left(\sqrt{5}+1\right)t}{2}}+c_4\mathrm{e}^{-\frac{\left(\sqrt{5}-1\right)t}{2}}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 246

```
DSolve[{x''[t]-x'[t]+y'[t]==0,x''[t]+y''[t]-x[t]==0},{x[t],y[t]},t,IncludeSingularSolutions
```

$$x(t)\to -\frac{1}{10}e^{\frac{1}{2}\left(t-\sqrt{5}t\right)}\left(2c_1\left(\sqrt{5}e^{\sqrt{5}t}-5e^{\frac{1}{2}\left(1+\sqrt{5}\right)t}-\sqrt{5}\right)-2\sqrt{5}c_2\left(e^{\sqrt{5}t}-1\right)\right.$$
$$\left.+c_4\left(\left(5+\sqrt{5}\right)e^{\sqrt{5}t}-10e^{\frac{1}{2}\left(1+\sqrt{5}\right)t}+5-\sqrt{5}\right)\right)$$
$$y(t)\to \frac{1}{10}\left(\left(5+\sqrt{5}\right)c_1-\left(5+\sqrt{5}\right)c_2-2\sqrt{5}c_4\right)e^{\frac{1}{2}\left(t-\sqrt{5}t\right)}$$
$$+\frac{1}{10}\left(-\left(\left(\sqrt{5}-5\right)c_1\right)+\left(\sqrt{5}-5\right)c_2+2\sqrt{5}c_4\right)e^{\frac{1}{2}\left(1+\sqrt{5}\right)t}-c_1+c_2+c_3$$

9.44 problem 1899

Internal problem ID [10222]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1899.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= 2x(t) \\
y'(t) &= 3x(t) - 2y(t) \\
z'(t) &= 2y(t) + 3z(t)
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 52

```
dsolve([diff(x(t),t)=2*x(t),diff(y(t),t)=3*x(t)-2*y(t),diff(z(t),t)=2*y(t)+3*z(t)],singsol=a
```

$$
\begin{aligned}
x(t) &= c_3 e^{2t} \\
y(t) &= \frac{3c_3 e^{2t}}{4} + c_2 e^{-2t} \\
z(t) &= c_1 e^{3t} - \frac{3c_3 e^{2t}}{2} - \frac{2c_2 e^{-2t}}{5}
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 93

```
DSolve[{x'[t]==2*x[t],y'[t]==3*x[t]-2*y[t],z'[t]==2*y[t]+3*z[t]},{x[t],y[t],z[t]},t,IncludeS
```

$$
\begin{aligned}
x(t) &\to c_1 e^{2t} \\
y(t) &\to \frac{1}{4} e^{-2t} \left(3c_1 \left(e^{4t} - 1\right) + 4c_2\right) \\
z(t) &\to \frac{1}{10} e^{-2t} \left(c_1 \left(-15e^{4t} + 12e^{5t} + 3\right) + 4c_2 \left(e^{5t} - 1\right) + 10c_3 e^{5t}\right)
\end{aligned}
$$

9.45 problem 1900

Internal problem ID [10223]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1900.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= 4x(t) \\
y'(t) &= x(t) - 2y(t) \\
z'(t) &= x(t) - 4y(t) + z(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 50

```
dsolve([diff(x(t),t)=4*x(t),diff(y(t),t)=x(t)-2*y(t),diff(z(t),t)=x(t)-4*y(t)+z(t)],singsol=
```

$$
\begin{aligned}
x(t) &= c_3 e^{4t} \\
y(t) &= \frac{c_3 e^{4t}}{6} + c_2 e^{-2t} \\
z(t) &= \frac{c_3 e^{4t}}{9} + c_1 e^{t} + \frac{4 c_2 e^{-2t}}{3}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 88

```
DSolve[{x'[t]==4*x[t],y'[t]==x[t]-2*y[t],z'[t]==x[t]-4*y[t]+z[t]},{x[t],y[t],z[t]},t,Include
```

$$
\begin{aligned}
x(t) &\to c_1 e^{4t} \\
y(t) &\to \frac{1}{6} e^{-2t} \left(c_1 \left(e^{6t} - 1\right) + 6 c_2\right) \\
z(t) &\to \frac{1}{9} e^{-2t} \left(c_1 \left(e^{3t} + e^{6t} - 2\right) - 12 c_2 \left(e^{3t} - 1\right) + 9 c_3 e^{3t}\right)
\end{aligned}
$$

9.46 problem 1901

Internal problem ID [10224]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1901.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = y(t) - z(t)$$
$$y'(t) = x(t) + y(t)$$
$$z'(t) = x(t) + z(t)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 48

```
dsolve([diff(x(t),t)=y(t)-z(t),diff(y(t),t)=x(t)+y(t),diff(z(t),t)=x(t)+z(t)],singsol=all)
```

$$x(t) = c_2 + c_3 e^t$$
$$y(t) = c_3 e^t t + c_1 e^t - c_2$$
$$z(t) = c_3 e^t t + c_1 e^t - c_3 e^t - c_2$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 93

```
DSolve[{x'[t]==y[t]-z[t],y'[t]==x[t]+y[t],z'[t]==x[t]+z[t]},{x[t],y[t],z[t]},t,IncludeSingul
```

$$x(t) \to (c_2 - c_3)\left(e^t - 1\right) + c_1$$
$$y(t) \to c_1\left(e^t - 1\right) + c_2\left(e^t t + 1\right) - c_3\left(e^t(t-1) + 1\right)$$
$$z(t) \to c_1\left(e^t - 1\right) + c_2\left(e^t(t-1) + 1\right) - c_3\left(e^t(t-2) + 1\right)$$

9.47 problem 1902

Internal problem ID [10225]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1902.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= y(t) - z(t) \\
y'(t) &= x(t) + y(t) + t \\
z'(t) &= x(t) + z(t) + t
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 56

```
dsolve([diff(x(t),t)-y(t)+z(t)=0,diff(y(t),t)-x(t)-y(t)=t,diff(z(t),t)-x(t)-z(t)=t],singsol=
```

$$
\begin{aligned}
x(t) &= c_2 + c_3 e^t \\
y(t) &= c_3 e^t t + c_1 e^t - c_2 - t - 1 \\
z(t) &= c_3 e^t t + c_1 e^t - c_3 e^t - c_2 - t - 1
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 109

```
DSolve[{x'[t]-y[t]+z[t]==0,y'[t]-x[t]-y[t]==t,z'[t]-x[t]-z[t]==t},{x[t],y[t],z[t]},t,Include
```

$$
\begin{aligned}
x(t) &\to (c_2 - c_3)\left(e^t - 1\right) + c_1 \\
y(t) &\to c_1\left(e^t - 1\right) + t\left(-1 + (c_2 - c_3)e^t\right) + c_3 e^t - 1 + c_2 - c_3 \\
z(t) &\to c_1\left(e^t - 1\right) - c_2 e^t + t\left(-1 + (c_2 - c_3)e^t\right) + 2c_3 e^t - 1 + c_2 - c_3
\end{aligned}
$$

9.48 problem 1903

Internal problem ID [10226]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1903.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = \frac{bcy(t)}{a} - \frac{bcz(t)}{a}$$
$$y'(t) = -\frac{cax(t)}{b} + \frac{caz(t)}{b}$$
$$z'(t) = \frac{abx(t)}{c} - \frac{aby(t)}{c}$$

✓ Solution by Maple

Time used: 0.266 (sec). Leaf size: 312

```
dsolve([a*diff(x(t),t)=b*c*(y(t)-z(t)),b*diff(y(t),t)=c*a*(z(t)-x(t)),c*diff(z(t),t)=a*b*(x(
```

$$x(t) = c_1 + c_2 \sin\left(\sqrt{a^2+b^2+c^2}\, t\right) + c_3 \cos\left(\sqrt{a^2+b^2+c^2}\, t\right)$$

$$y(t) = -\frac{\sqrt{a^2+b^2+c^2}\, \sin\left(\sqrt{a^2+b^2+c^2}\, t\right) c_3 ac - \sqrt{a^2+b^2+c^2}\, \cos\left(\sqrt{a^2+b^2+c^2}\, t\right) c_2 ac + \sin\left(\sqrt{a^2+b^2}\right.}{b\,(b^2+c^2)}$$

$$z(t) = \frac{\sqrt{a^2+b^2+c^2}\, \sin\left(\sqrt{a^2+b^2+c^2}\, t\right) c_3 ab - \sqrt{a^2+b^2+c^2}\, \cos\left(\sqrt{a^2+b^2+c^2}\, t\right) c_2 ab - \sin\left(\sqrt{a^2+b^2+c}\right.}{(b^2+c^2)\, c}$$

✓ Solution by Mathematica

Time used: 0.084 (sec). Leaf size: 736

```
DSolve[{a*x'[t]==b*c*(y[t]-z[t]),b*y'[t]==c*a*(z[t]-x[t]),c*z'[t]==a*b*(x[t]-y[t])},{x[t],y[
```

$$x(t) \to \frac{e^{-it\sqrt{a^2+b^2+c^2}}\left(ab^2\left(c_1\left(1+e^{2it\sqrt{a^2+b^2+c^2}}\right)-c_2\left(-1+e^{it\sqrt{a^2+b^2+c^2}}\right)^2\right)+ac^2\left(c_1\left(1+e^{2it\sqrt{a^2+b^2+c^2}}\right)-c_3\left(\right.\right.\right.}{2a\left(a^2+b^2+\right.}$$

$$y(t) \to \frac{e^{-it\sqrt{a^2+b^2+c^2}}\left(-a^2b\left(c_1\left(-1+e^{it\sqrt{a^2+b^2+c^2}}\right)^2-c_2\left(1+e^{2it\sqrt{a^2+b^2+c^2}}\right)\right)+bc^2\left(c_2\left(1+e^{2it\sqrt{a^2+b^2+c^2}}\right)-c\right.\right.}{2b\left(a^2+b^2+\right.}$$

$$z(t) \to \frac{e^{-it\sqrt{a^2+b^2+c^2}}\left(-a^2c\left(c_1\left(-1+e^{it\sqrt{a^2+b^2+c^2}}\right)^2-c_3\left(1+e^{2it\sqrt{a^2+b^2+c^2}}\right)\right)+b^2c\left(c_3\left(1+e^{2it\sqrt{a^2+b^2+c^2}}\right)-c\right.\right.}{2c\left(a^2+b^2+\right.}$$

9.49 problem 1904

Internal problem ID [10227]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1904.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = cy(t) - bz(t)$$
$$y'(t) = az(t) - cx(t)$$
$$z'(t) = bx(t) - ay(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 312

```
dsolve([diff(x(t),t)=c*y(t)-b*z(t),diff(y(t),t)=a*z(t)-c*x(t),diff(z(t),t)=b*x(t)-a*y(t)],si
```

$$x(t) = c_1 + c_2 \sin\left(\sqrt{a^2+b^2+c^2}\, t\right) + c_3 \cos\left(\sqrt{a^2+b^2+c^2}\, t\right)$$

$$y(t) = -\frac{\sqrt{a^2+b^2+c^2}\, \sin\left(\sqrt{a^2+b^2+c^2}\, t\right) c_3 ac - \sqrt{a^2+b^2+c^2}\, \cos\left(\sqrt{a^2+b^2+c^2}\, t\right) c_2 ac + \sin\left(\sqrt{a^2+b^2-\right.}{a\left(b^2+c^2\right)}$$

$$z(t) = \frac{\sqrt{a^2+b^2+c^2}\, \sin\left(\sqrt{a^2+b^2+c^2}\, t\right) c_3 ab - \sqrt{a^2+b^2+c^2}\, \cos\left(\sqrt{a^2+b^2+c^2}\, t\right) c_2 ab - \sin\left(\sqrt{a^2+b^2+c\right.}{a\left(b^2+c^2\right)}$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 1084

```
DSolve[{x'[t]==c*y[t]-b*z[t],y'[t]==a*z[t]-c*x[t],z'[t]==b*x[t]-a*y[t]},{x[t],y[t],z[t]},t,I
```

$$x(t) \to \frac{e^{t\left(-\sqrt{-a^2-b^2-c^2}\right)}\left(2a^2c_1e^{t\sqrt{-a^2-b^2-c^2}}+b^2c_1\left(e^{2t\sqrt{-a^2-b^2-c^2}}+1\right)+c^2c_1\left(e^{2t\sqrt{-a^2-b^2-c^2}}+1\right)-c\left(e^{t\sqrt{-a^2-b^2-}\right.\right.}{}$$

$$y(t) \to \frac{e^{t\left(-\sqrt{-a^2-b^2-c^2}\right)}\left(a^2c_2\left(e^{2t\sqrt{-a^2-b^2-c^2}}+1\right)-a\left(e^{t\sqrt{-a^2-b^2-c^2}}-1\right)\left(bc_1\left(e^{t\sqrt{-a^2-b^2-c^2}}-1\right)+c_3\sqrt{-a^2-b}\right.\right.}{}$$

$$z(t) \to \frac{e^{t\left(-\sqrt{-a^2-b^2-c^2}\right)}\left(a^2c_3\left(e^{2t\sqrt{-a^2-b^2-c^2}}+1\right)-a\left(e^{t\sqrt{-a^2-b^2-c^2}}-1\right)\left(cc_1\left(e^{t\sqrt{-a^2-b^2-c^2}}-1\right)-c_2\sqrt{-a^2-b}\right.\right.}{}$$

9.50 problem 1905

Internal problem ID [10228]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1905.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= h(t)\,y(t) - g(t)\,z(t)\\
y'(t) &= f(t)\,z(t) - h(t)\,x(t)\\
z'(t) &= x(t)\,g(t) - y(t)\,f(t)
\end{aligned}
$$

✗ Solution by Maple

```
dsolve([diff(x(t),t)=h(t)*y(t)-g(t)*z(t),diff(y(t),t)=f(t)*z(t)-h(t)*x(t),diff(z(t),t)=g(t)*
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x'[t]==h[t]*y[t]-g[t]*z[t],y'[t]==f[t]*z[t]-h[t]*x[t],z'[t]==g[t]*x[t]-f[t]*y[t]},{x
```

Not solved

9.51 problem 1906

Internal problem ID [10229]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1906.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) + y(t) - z(t)$$
$$y'(t) = y(t) + z(t) - x(t)$$
$$z'(t) = z(t) + x(t) - y(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 128

```
dsolve([diff(x(t),t)=x(t)+y(t)-z(t),diff(y(t),t)=y(t)+z(t)-x(t),diff(z(t),t)=z(t)+x(t)-y(t)]
```

$$x(t) = e^t\left(c_1 + \sin\left(\sqrt{3}\,t\right)c_2 + \cos\left(\sqrt{3}\,t\right)c_3\right)$$
$$y(t) = -\frac{e^t\left(\sin\left(\sqrt{3}\,t\right)\sqrt{3}\,c_3 - \cos\left(\sqrt{3}\,t\right)\sqrt{3}\,c_2 + \sin\left(\sqrt{3}\,t\right)c_2 + \cos\left(\sqrt{3}\,t\right)c_3 - 2c_1\right)}{2}$$
$$z(t) = \frac{e^t\left(\sin\left(\sqrt{3}\,t\right)\sqrt{3}\,c_3 - \cos\left(\sqrt{3}\,t\right)\sqrt{3}\,c_2 - \sin\left(\sqrt{3}\,t\right)c_2 - \cos\left(\sqrt{3}\,t\right)c_3 + 2c_1\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 177

```
DSolve[{x'[t]==x[t]+y[t]-z[t],y'[t]==y[t]+z[t]-x[t],z'[t]==z[t]+x[t]-y[t]},{x[t],y[t],z[t]},
```

$$x(t) \to \frac{1}{3}e^t\left((2c_1 - c_2 - c_3)\cos\left(\sqrt{3}t\right) + \sqrt{3}(c_2 - c_3)\sin\left(\sqrt{3}t\right) + c_1 + c_2 + c_3\right)$$
$$y(t) \to \frac{1}{3}e^t\left(-(c_1 - 2c_2 + c_3)\cos\left(\sqrt{3}t\right) - \sqrt{3}(c_1 - c_3)\sin\left(\sqrt{3}t\right) + c_1 + c_2 + c_3\right)$$
$$z(t) \to \frac{1}{3}e^t\left(-(c_1 + c_2 - 2c_3)\cos\left(\sqrt{3}t\right) + \sqrt{3}(c_1 - c_2)\sin\left(\sqrt{3}t\right) + c_1 + c_2 + c_3\right)$$

9.52 problem 1907

Internal problem ID [10230]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1907.
ODE order: 1.
ODE degree: 1.

Solve

$$\begin{aligned} x'(t) &= -3x(t) + 48y(t) - 28z(t) \\ y'(t) &= -4x(t) + 40y(t) - 22z(t) \\ z'(t) &= -6x(t) + 57y(t) - 31z(t) \end{aligned}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 66

```
dsolve([diff(x(t),t)=-3*x(t)+48*y(t)-28*z(t),diff(y(t),t)=-4*x(t)+40*y(t)-22*z(t),diff(z(t),
```

$$\begin{aligned} x(t) &= c_1 e^t + c_2 e^{2t} + c_3 e^{3t} \\ y(t) &= \frac{2c_1 e^t}{3} + \frac{c_2 e^{2t}}{4} + c_3 e^{3t} \\ z(t) &= c_1 e^t + \frac{c_2 e^{2t}}{4} + \frac{3c_3 e^{3t}}{2} \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 157

```
DSolve[{x'[t]==-3*x[t]+48*y[t]-28*z[t],y'[t]==-4*x[t]+40*y[t]-22*z[t],z'[t]==-6*x[t]+57*y[t]
```

$$\begin{aligned} x(t) &\to e^t\left(c_1\left(3 - 2e^{2t}\right) + 2\left(e^t - 1\right)\left(3c_2\left(3e^t + 5\right) - c_3\left(5e^t + 9\right)\right)\right) \\ y(t) &\to e^t\left(-2c_1\left(e^{2t} - 1\right) + c_2\left(3e^t + 18e^{2t} - 20\right) - 2c_3\left(e^t + 5e^{2t} - 6\right)\right) \\ z(t) &\to e^t\left(-3c_1\left(e^{2t} - 1\right) + 3c_2\left(e^t + 9e^{2t} - 10\right) - c_3\left(2e^t + 15e^{2t} - 18\right)\right) \end{aligned}$$

9.53 problem 1908

Internal problem ID [10231]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1908.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= 6x(t) - 72y(t) + 44z(t) \\
y'(t) &= 4x(t) - 4y(t) + 26z(t) \\
z'(t) &= 6x(t) - 63y(t) + 38z(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.453 (sec). Leaf size: 3113

```
dsolve([diff(x(t),t)=6*x(t)-72*y(t)+44*z(t),diff(y(t),t)=4*x(t)-4*y(t)+26*z(t),diff(z(t),t)=
```

$x(t)$

$$= \cos\left(\frac{\left(\left(263474 + 18\sqrt{351406311}\right)^{\frac{2}{3}} + 3542\right) t\sqrt{3}\, 4^{\frac{1}{3}}}{12\left(131737 + 9\sqrt{351406311}\right)^{\frac{1}{3}}}\right) e^{\frac{\left(-3542 + \left(263474+18\sqrt{351406311}\right)^{\frac{2}{3}} + 80\left(263474+18\sqrt{351406311}\right)^{\frac{1}{3}}\right)t}{6\left(263474+18\sqrt{351406311}\right)^{\frac{1}{3}}}} c$$

$$+ \sin\left(\frac{\left(\left(263474 + 18\sqrt{351406311}\right)^{\frac{2}{3}} + 3542\right) t\sqrt{3}\, 4^{\frac{1}{3}}}{12\left(131737 + 9\sqrt{351406311}\right)^{\frac{1}{3}}}\right) e^{\frac{\left(-3542 + \left(263474+18\sqrt{351406311}\right)^{\frac{2}{3}} + 80\left(263474+18\sqrt{351406311}\right)^{\frac{1}{3}}\right)}{6\left(263474+18\sqrt{351406311}\right)^{\frac{1}{3}}}}$$

$$+ c_1 e^{-\frac{\left(\left(263474+18\sqrt{351406311}\right)^{\frac{2}{3}} - 40\left(263474+18\sqrt{351406311}\right)^{\frac{1}{3}} - 3542\right)t}{3\left(263474+18\sqrt{351406311}\right)^{\frac{1}{3}}}}$$

Expression too large to display
Expression too large to display

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 551

```
DSolve[{x'[t]==6*x[t]-72*y[t]+44*z[t],y'[t]==4*x[t]-4*y[t]+26*z[t],z'[t]==6*x[t]-63*y[t]+38*
```

$$
\begin{aligned}
x(t) \to & -36c_2\text{RootSum}\left[\#1^3-40\#1^2+1714\#1+1404\&,\frac{2\#1e^{\#1t}+e^{\#1t}}{3\#1^2-80\#1+1714}\&\right] \\
& +4c_3\text{RootSum}\left[\#1^3-40\#1^2+1714\#1+1404\&,\frac{11\#1e^{\#1t}-424e^{\#1t}}{3\#1^2-80\#1+1714}\&\right] \\
& +c_1\text{RootSum}\left[\#1^3-40\#1^2+1714\#1\right. \\
& \left. +1404\&,\frac{\#1^2e^{\#1t}-34\#1e^{\#1t}+1486e^{\#1t}}{3\#1^2-80\#1+1714}\&\right] \\
y(t) \to & \, 4c_1\text{RootSum}\left[\#1^3-40\#1^2+1714\#1+1404\&,\frac{\#1e^{\#1t}+e^{\#1t}}{3\#1^2-80\#1+1714}\&\right] \\
& +2c_3\text{RootSum}\left[\#1^3-40\#1^2+1714\#1+1404\&,\frac{13\#1e^{\#1t}+10e^{\#1t}}{3\#1^2-80\#1+1714}\&\right] \\
& +c_2\text{RootSum}\left[\#1^3-40\#1^2+1714\#1+1404\&,\frac{\#1^2e^{\#1t}-44\#1e^{\#1t}-36e^{\#1t}}{3\#1^2-80\#1+1714}\&\right] \\
z(t) \to & \, 6c_1\text{RootSum}\left[\#1^3-40\#1^2+1714\#1+1404\&,\frac{\#1e^{\#1t}-38e^{\#1t}}{3\#1^2-80\#1+1714}\&\right] \\
& -9c_2\text{RootSum}\left[\#1^3-40\#1^2+1714\#1+1404\&,\frac{7\#1e^{\#1t}+6e^{\#1t}}{3\#1^2-80\#1+1714}\&\right] \\
& +c_3\text{RootSum}\left[\#1^3-40\#1^2+1714\#1+1404\&,\frac{\#1^2e^{\#1t}-2\#1e^{\#1t}+264e^{\#1t}}{3\#1^2-80\#1+1714}\&\right]
\end{aligned}
$$

9.54 problem 1909

Internal problem ID [10232]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1909.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = ax(t) + gy(t) + \beta z(t)$$
$$y'(t) = gx(t) + by(t) + \alpha z(t)$$
$$z'(t) = \beta x(t) + \alpha y(t) + cz(t)$$

✓ Solution by Maple

Time used: 14.265 (sec). Leaf size: 32449

```
dsolve([diff(x(t),t)=a*x(t)+g*y(t)+beta*z(t),diff(y(t),t)=g*x(t)+b*y(t)+alpha*z(t),diff(z(t)
```

Expression too large to display
Expression too large to display
Expression too large to display

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 1639

```
DSolve[{x'[t]==a*x[t]+g*y[t]+\[Beta]*z[t],y'[t]==g*x[t]+b*y[t]+\[Alpha]*z[t],z'[t]==\[Beta]*
```

$$x(t) \to -c_3\text{RootSum}\Big[-\#1^3+\#1^2a+\#1^2b+\#1^2c+\#1\alpha^2-\#1ab-\#1ac+\#1\beta^2 -\#1bc+\#1g^2-a\alpha^2+abc-b\beta^2-cg^2 +2\alpha\beta g\&, \frac{-b\beta e^{\#1t}+\alpha g e^{\#1t}+\#1\beta e^{\#1t}}{-3\#1^2+2\#1a+2\#1b+2\#1c+\alpha^2-ab-ac+\beta^2-bc+g^2}\&\Big] + c_2\text{RootSum}\Big[-\#1^3+\#1^2a+\#1^2b+\#1^2c+\#1\alpha^2-\#1ab-\#1ac+\#1\beta^2 -\#1bc+\#1g^2-a\alpha^2+abc-b\beta^2-cg^2 +2\alpha\beta g\&, \frac{-cge^{\#1t}+\#1ge^{\#1t}+\alpha\beta e^{\#1t}}{3\#1^2-2\#1a-2\#1b-2\#1c-\alpha^2+ab+ac-\beta^2+bc-g^2}\&\Big] + c_1\text{RootSum}\Big[-\#1^3+\#1^2a+\#1^2b+\#1^2c+\#1\alpha^2-\#1ab-\#1ac+\#1\beta^2 -\#1bc+\#1g^2-a\alpha^2+abc-b\beta^2-cg^2 +2\alpha\beta g\&, \frac{\#1^2e^{\#1t}+bce^{\#1t}-\#1be^{\#1t}-\#1ce^{\#1t}+\alpha^2\left(-e^{\#1t}\right)}{3\#1^2-2\#1a-2\#1b-2\#1c-\alpha^2+ab+ac-\beta^2+bc-g^2}\&\Big]$$

$$y(t) \to c_1\text{RootSum}\Big[-\#1^3+\#1^2a+\#1^2b+\#1^2c+\#1\alpha^2-\#1ab-\#1ac+\#1\beta^2 -\#1bc+\#1g^2-a\alpha^2+abc-b\beta^2-cg^2 +2\alpha\beta g\&, \frac{-cge^{\#1t}+\#1ge^{\#1t}+\alpha\beta e^{\#1t}}{3\#1^2-2\#1a-2\#1b-2\#1c-\alpha^2+ab+ac-\beta^2+bc-g^2}\&\Big] + c_3\text{RootSum}\Big[-\#1^3+\#1^2a+\#1^2b+\#1^2c+\#1\alpha^2-\#1ab-\#1ac+\#1\beta^2 -\#1bc+\#1g^2-a\alpha^2+abc-b\beta^2-cg^2 +2\alpha\beta g\&, \frac{-a\alpha e^{\#1t}+\beta ge^{\#1t}+\#1\alpha e^{\#1t}}{3\#1^2-2\#1a-2\#1b-2\#1c-\alpha^2+ab+ac-\beta^2+bc-g^2}\&\Big] + c_2\text{RootSum}\Big[-\#1^3+\#1^2a+\#1^2b+\#1^2c+\#1\alpha^2-\#1ab-\#1ac+\#1\beta^2 -\#1bc+\#1g^2-a\alpha^2+abc-b\beta^2-cg^2 +2\alpha\beta g\&, \frac{\#1^2e^{\#1t}+ace^{\#1t}-\#1ae^{\#1t}-\#1ce^{\#1t}+\beta^2\left(-e^{\#1t}\right)}{3\#1^2-2\#1a-2\#1b-2\#1c-\alpha^2+ab+ac-\beta^2+bc-g^2}\&\Big]$$

$$z(t) \to -c_1\text{RootSum}\Big[-\#1^3+\#1^2a+\#1^2b+\#1^2c+\#1\alpha^2-\#1ab-\#1ac+\#1\beta^2 -\#1bc+\#1g^2-a\alpha^2+abc-b\beta^2-cg^2 +2\alpha\beta g\&, \frac{-b\beta e^{\#1t}+\alpha ge^{\#1t}+\#1\beta e^{\#1t}}{-3\#1^2+2\#1a+2\#1b+2\#1c+\alpha^2-ab-ac+\beta^2-bc+g^2}\&\Big] + c_2\text{RootSum}\Big[-\#1^3+\#1^2a+\#1^2b+\#1^2c+\#1\alpha^2-\#1ab-\#1ac+\#1\beta^2$$

9.55 problem 1910

Internal problem ID [10233]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1910.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = \frac{2x(t)}{t} - 1$$
$$y'(t) = \frac{y(t)}{t} - \frac{x(t)}{t^3} + \frac{1}{t^2}$$
$$z'(t) = -\frac{x(t)}{t^4} - \frac{y(t)}{t^2} + \frac{z(t)}{t} + \frac{1}{t^3}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 37

```
dsolve([t*diff(x(t),t)=2*x(t)-t,t^3*diff(y(t),t)=-x(t)+t^2*y(t)+t,t^4*diff(z(t),t)=-x(t)-t^2
```

$$x(t) = c_3t^2 + t$$
$$y(t) = c_2t + c_3$$
$$z(t) = \frac{c_1t^2 + c_2t + c_3}{t}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 39

```
DSolve[{t*x'[t]==2*x[t]-t,t^3*y'[t]==-x[t]+t^2*y[t]+t,t^4*z'[t]==-x[t]-t^2*y[t]+t^3*z[t]+t},
```

$$x(t) \to t(1 + c_3t)$$
$$y(t) \to c_2t + c_3$$
$$z(t) \to c_1t + \frac{c_3}{t} + c_2$$

9.56 problem 1911

Internal problem ID [10234]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1911.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = \frac{bcy(t)}{at} - \frac{bcz(t)}{at}$$
$$y'(t) = \frac{caz(t)}{bt} - \frac{cax(t)}{bt}$$
$$z'(t) = -\frac{aby(t)}{ct} + \frac{abx(t)}{ct}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 322

```
dsolve([a*t*diff(x(t),t)=b*c*(y(t)-z(t)),b*t*diff(y(t),t)=c*a*(z(t)-x(t)),c*t*diff(z(t),t)=a
```

$$x(t) = c_1 + c_2 \sin\left(\sqrt{a^2+b^2+c^2}\,\ln(t)\right) + c_3 \cos\left(\sqrt{a^2+b^2+c^2}\,\ln(t)\right)$$

$$y(t) = \frac{\cos\left(\sqrt{a^2+b^2+c^2}\,\ln(t)\right)\sqrt{a^2+b^2+c^2}\,c_2 ac - \cos\left(\sqrt{a^2+b^2+c^2}\,\ln(t)\right)c_3 a^2 b - \sin\left(\sqrt{a^2+b^2+c^2}\,\ln\right.}{b\,(b^2+c^2)}$$

$$z(t) = -\frac{\cos\left(\sqrt{a^2+b^2+c^2}\,\ln(t)\right)\sqrt{a^2+b^2+c^2}\,c_2 ab + \cos\left(\sqrt{a^2+b^2+c^2}\,\ln(t)\right)c_3 a^2 c - \sin\left(\sqrt{a^2+b^2+c^2}\right.}{(b^2+c^2)\,c}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 715

```
DSolve[{a*t*x'[t]==b*c*(y[t]-z[t]),b*t*y'[t]==c*a*(z[t]-x[t]),c*t*z'[t]==a*b*(x[t]-y[t])},{x
```

$$x(t) \to \frac{t^{-i\sqrt{a^2+b^2+c^2}}\left(ab^2\left(c_1\left(1+t^{2i\sqrt{a^2+b^2+c^2}}\right)-c_2\left(-1+t^{i\sqrt{a^2+b^2+c^2}}\right)^2\right)-ibc(c_2-c_3)\sqrt{a^2+b^2+c^2}\left(-1+t^{2\ldots}\right.}{2a\left(a^2+b^2+c^2\right)\ldots}$$

$$y(t) \to \frac{t^{-i\sqrt{a^2+b^2+c^2}}\left(-a^2b\left(c_1\left(-1+t^{i\sqrt{a^2+b^2+c^2}}\right)^2-c_2\left(1+t^{2i\sqrt{a^2+b^2+c^2}}\right)\right)+iac(c_1-c_3)\sqrt{a^2+b^2+c^2}\left(-1+\ldots\right.}{2b\left(a^2+b^2+c^2\ldots\right.}$$

$$z(t) \to \frac{t^{-i\sqrt{a^2+b^2+c^2}}\left(-iab(c_1-c_2)\sqrt{a^2+b^2+c^2}\left(-1+t^{2i\sqrt{a^2+b^2+c^2}}\right)-a^2c\left(c_1\left(-1+t^{i\sqrt{a^2+b^2+c^2}}\right)^2-c_3\left(1+t\ldots\right.\right.\right.}{2c\left(a^2+b^2+c^2\ldots\right.}$$

9.57 problem 1912

Internal problem ID [10235]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 8, system of first order odes
Problem number: 1912.
ODE order: 1.
ODE degree: 1.

Solve

$$\begin{aligned}
x_1'(t) &= ax_2(t) + bx_3(t)\cos(ct) + bx_4(t)\sin(ct)\\
x_2'(t) &= -ax_1(t) + bx_3(t)\sin(ct) - bx_4(t)\cos(ct)\\
x_3'(t) &= -bx_1(t)\cos(ct) - bx_2(t)\sin(ct) + ax_4(t)\\
x_4'(t) &= -bx_1(t)\sin(ct) + bx_2(t)\cos(ct) - ax_3(t)
\end{aligned}$$

✓ Solution by Maple

Time used: 1.0 (sec). Leaf size: 10632

```
dsolve([diff(x__1(t),t)=a*x__2(t)+b*x__3(t)*cos(c*t)+b*x__4(t)*sin(c*t),diff(x__2(t),t)=-a*x
```

$$\left\{x_1(t) = c_2 + c_3\sin(ct) + c_4\cos(ct), x_2(t) = -\cos(ct)\,c_3 + \sin(ct)\,c_4 + c_1, x_3(t) = \frac{b(\cos(ct)\,c_1a - \sin(ct)\,c_2a - c_3a - c_3c)}{(a+c)\,a}, x_4(t) = \frac{b(\cos(ct)\,c_2a + \sin(ct)\,c_1a + c_4a + c_4c)}{(a+c)\,a}\right\}$$

Expression too large to display

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 782

```
DSolve[{x1'[t]==a*x2[t]+b*x3[t]*Cos[c*t]+b*x4[t]*Sin[c*t],x2'[t]==-a*x1[t]+b*x3[t]*Sin[c*t]-
```

$$x1(t) \to c_1 \cos\left(\frac{1}{2}t\left(\sqrt{4a^2+4ac+4b^2+c^2}+c\right)\right) + c_2 \sin\left(\frac{1}{2}t\left(\sqrt{4a^2+4ac+4b^2+c^2}+c\right)\right) + c_3 \cos\left(t\left(\frac{c}{2}-\frac{1}{2}\sqrt{(2a+c)^2+4b^2}\right)\right) + c_4 \sin\left(t\left(\frac{c}{2}-\frac{1}{2}\sqrt{(2a+c)^2+4b^2}\right)\right)$$

$$x2(t) \to -c_2 \cos\left(\frac{1}{2}t\left(\sqrt{4a^2+4ac+4b^2+c^2}+c\right)\right) + c_1 \sin\left(\frac{1}{2}t\left(\sqrt{4a^2+4ac+4b^2+c^2}+c\right)\right) - c_4 \cos\left(t\left(\frac{c}{2}-\frac{1}{2}\sqrt{(2a+c)^2+4b^2}\right)\right) + c_3 \sin\left(t\left(\frac{c}{2}-\frac{1}{2}\sqrt{(2a+c)^2+4b^2}\right)\right)$$

$$x3(t) \to \frac{c_4\left(-\frac{1}{2}\sqrt{(2a+c)^2+4b^2}+a+\frac{c}{2}\right)\cos\left(\frac{1}{2}t\left(\sqrt{4a^2+4ac+4b^2+c^2}+c\right)\right) + c_3\left(-\frac{1}{2}\sqrt{(2a+c)^2+4b^2}+a\right.}{}$$

$$x4(t) \to \frac{-\left(c_3\left(-\frac{1}{2}\sqrt{(2a+c)^2+4b^2}+a+\frac{c}{2}\right)\cos\left(\frac{1}{2}t\left(\sqrt{4a^2+4ac+4b^2+c^2}+c\right)\right)\right) + c_4\left(-\frac{1}{2}\sqrt{(2a+c)^2+4b^2}\right.}{}$$

10 Chapter 9, system of higher order odes

10.1 problem 1913

Internal problem ID [10236]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 9, system of higher order odes
Problem number: 1913.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -x(t)^2 - x(t)\,y(t)$$
$$y'(t) = x(t)\,y(t) + y(t)^2$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 54

```
dsolve([diff(x(t),t)=-x(t)*(x(t)+y(t)),diff(y(t),t)=y(t)*(x(t)+y(t))],singsol=all)
```

$$\left[\{x(t) = 0\}, \left\{y(t) = \frac{1}{-t + c_1}\right\}\right]$$
$$\left[\left\{x(t) = \frac{\tanh\left(\frac{c_2+t}{c_1}\right)}{c_1}\right\}, \left\{y(t) = -\frac{x(t)^2 + \frac{d}{dt}x(t)}{x\left(t\right)}\right\}\right]$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 52

```
DSolve[{x'[t]==-x[t]*(x[t]+y[t]),y'[t]==y[t]*(x[t]+y[t])},{x[t],y[t]},t,IncludeSingularSolut
```

$$y(t) \to -\sqrt{c_1}\cot\left(\sqrt{c_1}(t - c_2)\right)$$
$$x(t) \to -\sqrt{c_1}\tan\left(\sqrt{c_1}(t - c_2)\right)$$

10.2 problem 1914

Internal problem ID [10237]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 9, system of higher order odes
Problem number: 1914.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = y(t)\,x(t)\,a + bx(t)$$
$$y'(t) = y(t)\,x(t)\,c + y(t)\,d$$

✓ Solution by Maple

Time used: 0.297 (sec). Leaf size: 92

```
dsolve([diff(x(t),t)=(a*y(t)+b)*x(t),diff(y(t),t)=(c*x(t)+d)*y(t)],singsol=all)
```

$$[\{x(t) = 0\}, \{y(t) = c_1 \mathrm{e}^{dt}\}]$$

$$\left[\left\{x(t) = \mathrm{RootOf}\left(-\left(\int^{\_Z} \frac{1}{b\_a\left(\mathrm{LambertW}\left(\frac{\mathrm{e}^{-1}\_a^{\frac{d}{b}}\mathrm{e}^{-\frac{ac}{b}}\mathrm{e}^{\frac{c_1}{b}}}{b}\right)+1\right)}d\_a\right)+t+c_2\right)\right\}, \left\{y(t) = \frac{-bx(t}{a}\right.\right.$$

✓ Solution by Mathematica

Time used: 0.359 (sec). Leaf size: 201

```
DSolve[{x'[t]==(a*y[t]+b)*x[t],y'[t]==(c*x[t]+d)*y[t]},{x[t],y[t]},t,IncludeSingularSolution
```

$$y(t) \to \frac{bW\left(\frac{a\text{InverseFunction}\left[\int_1^{\#1}\frac{1}{K[1]\left(W\left(\frac{ae^{\frac{c_1}{b}+\frac{cK[1]}{b}}K[1]^{\frac{d}{b}}}{b}\right)+1\right)}dK[1]\&\right][bt+c_2]^{\frac{d}{b}}\exp\left(\frac{c\text{InverseFunction}\left[\int_1^{\#1}\frac{1}{K[1]\left(W\left(\frac{ae^{\frac{c_1}{b}\cdots}}{\cdots}\right)\cdots\right)}\cdots\right]\cdots}{b}\cdots\right)}{b}\cdots\right)}{a}$$

$$x(t) \to \text{InverseFunction}\left[\int_1^{\#1}\frac{1}{K[1]\left(W\left(\frac{ae^{\frac{c_1}{b}+\frac{cK[1]}{b}}K[1]^{\frac{d}{b}}}{b}\right)+1\right)}dK[1]\&\right][bt+c_2]$$

10.3 problem 1915

Internal problem ID [10238]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 9, system of higher order odes
Problem number: 1915.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t)^2 ap + x(t) y(t) aq + x(t) \alpha$$
$$y'(t) = x(t) y(t) bp + y(t)^2 bq + y(t) \beta$$

✗ Solution by Maple

```
dsolve([diff(x(t),t)=x(t)*(a*(p*x(t)+q*y(t))+alpha),diff(y(t),t)=y(t)*(beta+b*(p*x(t)+q*y(t)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x'[t]==x[t]*(a*(p*x[t]+q*y[t])+\[Alpha]),y'[t]==y[t]*(\[Beta]+b*(p*x[t]+q*y[t]))},{x
```

Timed out

10.4 problem 1916

Internal problem ID [10239]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 9, system of higher order odes
Problem number: 1916.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t)^2 h + x(t) y(t) h - x(t) ah - x(t) ch - y(t) ah + ach$$
$$y'(t) = x(t) y(t) k - x(t) bk + y(t)^2 k - y(t) bk - y(t) ck + bck$$

✓ Solution by Maple

Time used: 0.469 (sec). Leaf size: 237

```
dsolve([diff(x(t),t)=h*(a-x(t))*(c-x(t)-y(t)),diff(y(t),t)=k*(b-y(t))*(c-x(t)-y(t))],singsol
```

$$\left[\{x(t) = a\}, \left\{y(t) = -\frac{a\,e^{ac_1k+akt+bc_1k+bkt-cc_1k-ckt} - c\,e^{ac_1k+akt+bc_1k+bkt-cc_1k-ckt} + b}{-1 + e^{ac_1k+akt+bc_1k+bkt-cc_1k-ckt}}\right\}\right]$$

$$\left[\left\{x(t) = \mathrm{RootOf}\left(-\left(\int^{\_Z} \frac{(\_a-a)^{-\frac{k}{h}}}{\left(h\,(\_a-a)^{-\frac{k}{h}}\,\_a + h\,(\_a-a)^{-\frac{k}{h}}\,b - h\,(\_a-a)^{-\frac{k}{h}}\,c + c_1\right)(\_a-a)}d\_a\right)\right.\right.\right.$$

✓ Solution by Mathematica

Time used: 0.764 (sec). Leaf size: 277

```
DSolve[{x'[t]==h*(a-x[t])*(c-x[t]-y[t]),y'[t]==k*(b-y[t])*(c-x[t]-y[t])},{x[t],y[t]},t,Inclu
```

$$y(t) \to b + c_1\Bigg(h\Bigg(a$$

$$-\text{InverseFunction}\Bigg[\int_1^{\#1} \frac{(h(a-K[1]))^{\frac{k}{h}}}{(a-K[1])\left(c_1(ah-hK[1])^{\frac{k}{h}}(h(a-K[1]))^{\frac{k}{h}} - c(h(a-K[1]))^{\frac{k}{h}} + K[1](h(a-\right.}$$

$x(t)$

$$\to \text{InverseFunction}\Bigg[\int_1^{\#1} \frac{(h(a-K[1]))^{\frac{k}{h}}}{(a-K[1])\left(c_1(ah-hK[1])^{\frac{k}{h}}(h(a-K[1]))^{\frac{k}{h}} - c(h(a-K[1]))^{\frac{k}{h}} + K[1](h(a-\right.}$$

$$+ c_2]$$

10.5 problem 1917

Internal problem ID [10240]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 9, system of higher order odes
Problem number: 1917.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = y(t)^2 - \cos(x(t))$$
$$y'(t) = -y(t)\sin(x(t))$$

✓ Solution by Maple

Time used: 1.25 (sec). Leaf size: 106

```
dsolve([diff(x(t),t)=y(t)^2-cos(x(t)),diff(y(t),t)=-y(t)*sin(x(t))],singsol=all)
```

$$\left\{x(t) = \mathrm{RootOf}\left(-2\left(\int^{\_Z} \frac{1}{-3\tan\left(\mathrm{RootOf}\left(-3\sqrt{-\cos(\_f)^2}\ \ln\left(\frac{9\cos(\_f)^2\tan(\_Z)^2}{4} + \frac{9\cos(\_f)^2}{4}\right) + \cdots\right.\right.\right)}\right.\right.$$
$$\left.\left.+ t + c_2\right)\right\}$$

$$\left\{y(t) = \sqrt{\frac{d}{dt}x(t) + \cos(x(t))}, y(t) = -\sqrt{\frac{d}{dt}x(t) + \cos(x(t))}\right\}$$

✓ Solution by Mathematica

Time used: 124.726 (sec). Leaf size: 3402

```
DSolve[{x'[t]==y[t]^2-Cos[x[t]],y'[t]==-y[t]*Sin[x[t]]},{x[t],y[t]},t,IncludeSingularSolutio
```

Too large to display

10.6 problem 1918

Internal problem ID [10241]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 9, system of higher order odes
Problem number: 1918.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -x(t)\, y(t)^2 + x(t) + y(t)$$
$$y'(t) = x(t)^2\, y(t) - x(t) - y(t)$$

✗ Solution by Maple

```
dsolve([diff(x(t),t)=-x(t)*y(t)^2+x(t)+y(t),diff(y(t),t)=x(t)^2*y(t)-x(t)-y(t)],singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x'[t]==-x[t]*y[t]^2+x[t]+y[t],y'[t]==x[t]^2*y[t]-x[t]-y[t]},{x[t],y[t]},t,IncludeSin
```

Not solved

10.7 problem 1919

Internal problem ID [10242]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 9, system of higher order odes
Problem number: 1919.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -x(t)^3 - x(t)\, y(t)^2 + x(t) + y(t)$$
$$y'(t) = -x(t)^2\, y(t) - y(t)^3 - x(t) + y(t)$$

✗ Solution by Maple

```
dsolve([diff(x(t),t)=x(t)+y(t)-x(t)*(x(t)^2+y(t)^2),diff(y(t),t)=-x(t)+y(t)-y(t)*(x(t)^2+y(t
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x'[t]==x[t]+y[t]-x[t]*(x[t]^2+y[t]^2),y'[t]==-x[t]+y[t]-y[t]*(x[t]^2+y[t]^2)},{x[t],
```

Not solved

10.8 problem 1920

Internal problem ID [10243]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 9, system of higher order odes
Problem number: 1920.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t)^3 + x(t)\,y(t)^2 - x(t) - y(t)$$
$$y'(t) = x(t)^2\,y(t) + y(t)^3 + x(t) - y(t)$$

✗ Solution by Maple

```
dsolve([diff(x(t),t)=-y(t)+x(t)*(x(t)^2+y(t)^2-1),diff(y(t),t)=x(t)+y(t)*(x(t)^2+y(t)^2-1)],
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x'[t]==-y[t]+x[t]*(x[t]^2+y[t]^2-1),y'[t]==x[t]+y[t]*(x[t]^2+y[t]^2-1)},{x[t],y[t]},
```

Not solved

10.9 problem 1921

Internal problem ID [10244]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 9, system of higher order odes
Problem number: 1921.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -x(t)^2 y(t) - y(t)^3$$
$$y'(t) = \begin{cases} x(t)^2 + y(t)^2 & 2x(t) \leq x(t)^2 + y(t)^2 \\ \frac{x(t)^3}{2} - \frac{y(t)^4}{2x(t)} & \text{otherwise} \end{cases}$$

✗ Solution by Maple

```
dsolve([diff(x(t),t)=-y(t)*(x(t)^2+y(t)^2),diff(y(t),t)=piecewise((x(t)^2+y(t)^2)>=2*x(t),(x
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x'[t]==-y[t]*(x[t]^2+y[t]^2),y'[t]==Piecewise[{{(x[t]^2+y[t]^2),(x[t]^2+y[t]^2)>=2*x
```

Not solved

10.10 problem 1922

Internal problem ID [10245]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 9, system of higher order odes
Problem number: 1922.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -y(t) + \left(\begin{cases} x(t)^3 \sin\left(\frac{1}{x(t)^2+y(t)^2}\right) + x(t) \sin\left(\frac{1}{x(t)^2+y(t)^2}\right) y(t)^2 - x(t) \sin\left(\frac{1}{x(t)^2+y(t)^2}\right) & x(t)^2 + y(t \\ 0 & \text{otherw} \end{cases}\right.$$

$$y'(t) = x(t) + \left(\begin{cases} y(t) \sin\left(\frac{1}{x(t)^2+y(t)^2}\right) x(t)^2 + y(t)^3 \sin\left(\frac{1}{x(t)^2+y(t)^2}\right) - y(t) \sin\left(\frac{1}{x(t)^2+y(t)^2}\right) & x(t)^2 + y(t)^2 \\ 0 & \text{otherwis} \end{cases}\right.$$

✗ Solution by Maple

```
dsolve([diff(x(t),t)=-y(t)+piecewise((x(t)^2+y(t)^2)<>1,x(t)*(x(t)^2+y(t)^2-1)*sin(1/(x(t)^2
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x'[t] == -y[t] + Piecewise[{{x[t]*(x[t]^2 + y[t]^2 - 1)*Sin[1/(x[t]^2 + y[t]^2)], (x
```

Not solved

10.11 problem 1923

Internal problem ID [10246]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 9, system of higher order odes
Problem number: 1923.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= -\frac{x(t)\, t}{t^2+1} + \frac{y(t)}{t^2+1} \\
y'(t) &= -\frac{t y(t)}{t^2+1} - \frac{x(t)}{t^2+1}
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 35

```
dsolve([(t^2+1)*diff(x(t),t)=-t*x(t)+y(t),(t^2+1)*diff(y(t),t)=-x(t)-t*y(t)],singsol=all)
```

$$
\begin{aligned}
x(t) &= \frac{c_1 t + c_2}{t^2+1} \\
y(t) &= \frac{-c_2 t + c_1}{t^2+1}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 39

```
DSolve[{(t^2+1)*x'[t]==-t*x[t]+y[t],(t^2+1)*y'[t]==-x[t]-t*y[t]},{x[t],y[t]},t,IncludeSingul
```

$$
\begin{aligned}
x(t) &\to \frac{c_2 t + c_1}{t^2+1} \\
y(t) &\to \frac{c_2 - c_1 t}{t^2+1}
\end{aligned}
$$

10.12 problem 1924

Internal problem ID [10247]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 9, system of higher order odes
Problem number: 1924.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -\frac{2x(t)\,t}{x(t)^2 + y(t)^2 - t^2}$$
$$y'(t) = -\frac{2ty(t)}{x(t)^2 + y(t)^2 - t^2}$$

✓ Solution by Maple

Time used: 1.25 (sec). Leaf size: 186

```
dsolve([(x(t)^2+y(t)^2-t^2)*diff(x(t),t)=-2*t*x(t),(x(t)^2+y(t)^2-t^2)*diff(y(t),t)=-2*t*y(t
```

$$\left[\{x(t)=0\}, \left\{y(t)=\frac{1+\sqrt{-4c_1^2t^2+1}}{2c_1}, y(t)=-\frac{-1+\sqrt{-4c_1^2t^2+1}}{2c_1}\right\}\right]$$

$$\left[\left\{x(t)=\right.\right.$$

$$-\frac{-c_1+\sqrt{-2c_2t^2+c_1^2}}{2c_2}, x(t)=\frac{c_1+\sqrt{-2c_2t^2+c_1^2}}{2c_2}\right\}, \left\{y(t)=\frac{\sqrt{-\left(\frac{d}{dt}x(t)\right)\left(\left(\frac{d}{dt}x(t)\right)x(t)^2-t^2\left(\frac{d}{dt}x(t)\right)\right.}}{\frac{d}{dt}x(t)}$$

$$-\frac{\sqrt{-\left(\frac{d}{dt}x(t)\right)\left(\left(\frac{d}{dt}x(t)\right)x(t)^2-t^2\left(\frac{d}{dt}x(t)\right)+2x(t)\,t\right)}}{\frac{d}{dt}x(t)}\right\}\right]$$

✓ Solution by Mathematica

Time used: 0.076 (sec). Leaf size: 179

```
DSolve[{(x[t]^2+y[t]^2-t^2)*x'[t]==-2*t*x[t],(x[t]^2+y[t]^2-t^2)*y'[t]==-2*t*y[t]},{x[t],y[t
```

$$y(t) \to -\frac{c_1\left(\sqrt{e^{2c_2}-4\left(1+c_1{}^2\right)t^2}-e^{c_2}\right)}{2\left(1+c_1{}^2\right)}$$

$$x(t) \to \frac{e^{c_2}-\sqrt{e^{2c_2}-4\left(1+c_1{}^2\right)t^2}}{2\left(1+c_1{}^2\right)}$$

$$y(t) \to \frac{c_1\left(\sqrt{e^{2c_2}-4\left(1+c_1{}^2\right)t^2}+e^{c_2}\right)}{2\left(1+c_1{}^2\right)}$$

$$x(t) \to \frac{\sqrt{e^{2c_2}-4\left(1+c_1{}^2\right)t^2}+e^{c_2}}{2\left(1+c_1{}^2\right)}$$

10.13 problem 1925

Internal problem ID [10248]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 9, system of higher order odes
Problem number: 1925.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = \text{RootOf}\left(\_Z^3 + 2t\_Z^2 + \left(t^2 - x(t)\right)\_Z + ay(t) - x(t)\,t\right)$$

$$y'(t) = -\frac{t\,\text{RootOf}\left(\_Z^3 + 2t\_Z^2 + (t^2 - x(t))\_Z + ay(t) - x(t)\,t\right)}{a} - \frac{\text{RootOf}\left(\_Z^3 + 2t\_Z^2 + (t^2 - x(\right.}{a}$$

✓ Solution by Maple

Time used: 0.266 (sec). Leaf size: 220

```
dsolve([diff(x(t),t)^2+t*diff(x(t),t)+a*diff(y(t),t)-x(t)=0,diff(x(t),t)*diff(y(t),t)+t*diff
```

$$\left[\left\{x(t) = -\frac{t^2}{3}\right\}, \left\{y(t) = -\frac{t^3}{27a}\right\}\right]$$

$$\left[\{x(t) = c_1 t + c_2\}, \left\{y(t) = \frac{-\left(\frac{d}{dt}x(t)\right)^3 - 2\left(\frac{d}{dt}x(t)\right)^2 t - t^2\left(\frac{d}{dt}x(t)\right) + x(t)\left(\frac{d}{dt}x(t)\right) + x(t)\,t}{a}\right\}\right]$$

$$\left[\left\{x(t) = -\frac{5t^2}{12} - \frac{t\left(-t - \sqrt{3}\,c_1\right)}{6} + \frac{c_1^2}{4}, x(t) = -\frac{5t^2}{12} - \frac{t\left(-t + \sqrt{3}\,c_1\right)}{6} + \frac{c_1^2}{4}, x(t) = -\frac{5t^2}{12} + \frac{t\left(t - \sqrt{3}\,c_1\right)}{6} + \frac{c_1^2}{4}, x(t) = -\frac{5t^2}{12} + \frac{t\left(t + \sqrt{3}\,c_1\right)}{6} + \frac{c_1^2}{4}\right\}, \left\{y(t) = -\frac{-2t^2\left(\frac{d}{dt}x(t)\right) - 2t^3 - 6x(t)\left(\frac{d}{dt}x(t)\right) - 7x(t)\,t}{9a}\right\}\right]$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 28

```
DSolve[{x'[t]^2+t*x'[t]+a*y'[t]-x[t]==0,x'[t]*y'[t]+t*y'[t]-y[t]==0},{x[t],y[t]},t,IncludeSi
```

$$x(t) \to ac_2 + c_1 t + c_1{}^2$$
$$y(t) \to c_2(t + c_1)$$

10.14 problem 1926

Internal problem ID [10249]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 9, system of higher order odes
Problem number: 1926.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x(t) &= t x'(t) + f(x'(t), y'(t)) \\
y(t) &= y'(t)\, t + g(x'(t), y'(t))
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 96

```
dsolve([x(t)=t*diff(x(t),t)+f(diff(x(t),t),diff(y(t),t)),y(t)=t*diff(y(t),t)+g(diff(x(t),t),
```

$$
\begin{aligned}
&\{\int \mathrm{RootOf}\left(f\left(\frac{d}{dt}x(t), \_Z\right)+t\left(\frac{d}{dt}x(t)\right)-x(t)\right)dt \\
&+c_1 = \mathrm{RootOf}\left(f\left(\frac{d}{dt}x(t), \_Z\right)+t\left(\frac{d}{dt}x(t)\right)-x(t)\right)t \\
&+g\left(\frac{d}{dt}x(t), \mathrm{RootOf}\left(f\left(\frac{d}{dt}x(t), \_Z\right)+t\left(\frac{d}{dt}x(t)\right)-x(t)\right)\right)\} \\
&\{y(t) = \int \mathrm{RootOf}\left(f\left(\frac{d}{dt}x(t), \_Z\right)+t\left(\frac{d}{dt}x(t)\right)-x(t)\right)dt + c_1\}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 28

```
DSolve[{x[t]==t*x'[t]+f[x'[t],y'[t]],y[t]==t*y'[t]+g[x'[t],y'[t]]},{x[t],y[t]},t,IncludeSing
```

$$
\begin{aligned}
x(t) &\to f(c_1, c_2) + c_1 t \\
y(t) &\to g(c_1, c_2) + c_2 t
\end{aligned}
$$

10.15 problem 1927

Internal problem ID [10250]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 9, system of higher order odes
Problem number: 1927.
ODE order: 1.
ODE degree: 1.

Solve

$$x''(t) = a\,\mathrm{e}^{2x(t)} - \mathrm{e}^{-x(t)} + \mathrm{e}^{-2x(t)}\left(\cos^2\left(y(t)\right)\right)$$
$$y''(t) = \mathrm{e}^{-2x(t)}\sin\left(y(t)\right)\cos\left(y(t)\right) - \frac{\sin\left(y(t)\right)}{\cos\left(y\left(t\right)\right)^3}$$

✗ Solution by Maple

```
dsolve([diff(x(t),t,t)=a*exp(2*x(t))-exp(-x(t))+exp(-2*x(t))*cos(y(t))^2,diff(y(t),t,t)=exp(
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x''[t]==a*Exp[2*x[t]]-Exp[-x[t]]+Exp[-2*x[t]]*Cos[y[t]]^2,y''[t]==Exp[-2*x[t]]*Sin[y
```

Not solved

10.16 problem 1928

Internal problem ID [10251]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 9, system of higher order odes
Problem number: 1928.
ODE order: 1.
ODE degree: 1.

Solve

$$x''(t) = \frac{kx(t)}{\left(x(t)^2 + y(t)^2\right)^{\frac{3}{2}}}$$
$$y''(t) = \frac{ky(t)}{\left(x(t)^2 + y(t)^2\right)^{\frac{3}{2}}}$$

✗ Solution by Maple

```
dsolve([diff(x(t),t,t)=k*x(t)/(x(t)^2+y(t)^2)^(3/2),diff(y(t),t,t)=k*y(t)/(x(t)^2+y(t)^2)^(3
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x''[t]==k*x[t]/(x[t]^2+y[t]^2)^(3/2),y''[t]==k*y[t]/(x[t]^2+y[t]^2)^(3/2)},{x[t],y[t
```

Not solved

10.17 problem 1930

Internal problem ID [10252]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 9, system of higher order odes
Problem number: 1930.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= y(t) - z(t) \\
y'(t) &= x(t)^2 + y(t) \\
z'(t) &= x(t)^2 + z(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 45

```
dsolve([diff(x(t),t)=y(t)-z(t),diff(y(t),t)=x(t)^2+y(t),diff(z(t),t)=x(t)^2+z(t)],singsol=al
```

$$
\begin{aligned}
&\{x(t) = c_2 + c_3 \mathrm{e}^t\} \\
&\{y(t) = \left(\int x(t)^2 \mathrm{e}^{-t} dt + c_1\right) \mathrm{e}^t\} \\
&\{z(t) = -\frac{d}{dt}x(t) + y(t)\}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 127

```
DSolve[{x'[t]==y[t]-z[t],y'[t]==x[t]^2+y[t],z'[t]==x[t]^2+z[t]},{x[t],y[t],z[t]},t,IncludeSi
```

$$
\begin{aligned}
x(t) &\to e^{t-c_3} + c_1 \\
y(t) &\to e^{2t-2c_3} + (c_1 + c_2)e^{t-c_3} + 2c_1 e^{t-c_3} \log\left(e^{t-c_3}\right) - c_1{}^2 \\
z(t) &\to e^{2t-2c_3} + (-1 + c_1 + c_2)e^{t-c_3} + 2c_1 e^{t-c_3} \log\left(e^{t-c_3}\right) - c_1{}^2
\end{aligned}
$$

10.18 problem 1931

Internal problem ID [10253]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 9, system of higher order odes
Problem number: 1931.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -\frac{y(t)\,z(t)\,c}{a} + \frac{y(t)\,z(t)\,b}{a}$$
$$y'(t) = -\frac{z(t)\,x(t)\,a}{b} + \frac{z(t)\,x(t)\,c}{b}$$
$$z'(t) = \frac{x(t)\,y(t)\,a}{c} - \frac{x(t)\,y(t)\,b}{c}$$

✓ Solution by Maple

Time used: 0.86 (sec). Leaf size: 1356

```
dsolve([a*diff(x(t),t)=(b-c)*y(t)*z(t),b*diff(y(t),t)=(c-a)*z(t)*x(t),c*diff(z(t),t)=(a-b)*x
```

$$[\{x(t) = 0\}, \{y(t) = 0\}, \{z(t) = c_1\}]$$
$$[\{x(t) = 0\}, \{y(t) = c_1\}, \{z(t) = 0\}]$$
$$[\{x(t) = c_1\}, \{y(t) = 0\}, \{z(t) = 0\}]$$

Expression too large to display

✓ Solution by Mathematica

Time used: 4.217 (sec). Leaf size: 1461

```
DSolve[{a*x'[t]==(b-c)*y[t]*z[t],b*y'[t]==(c-a)*z[t]*x[t],c*z'[t]==(a-b)*x[t]*y[t]},{x[t],y[
```

$$x(t)\to \frac{\sqrt{2}bc_1\sqrt{a(a-c)}(c-b)\operatorname{sn}\left(\frac{\sqrt{2}\sqrt{a-c}\sqrt{b-c}\sqrt{c_2}(c_3-t)}{\sqrt{a}\sqrt{b}}\middle|-\frac{(a-b)bc_1}{(a-c)cc_2}\right)}{a(c-a)\sqrt{bc_1(b-c)}}$$

$$y(t)\to -\frac{\sqrt{2}\sqrt{-bc_1(b-c)\left(-1+\operatorname{sn}\left(\frac{\sqrt{2}\sqrt{a-c}\sqrt{b-c}\sqrt{c_2}(c_3-t)}{\sqrt{a}\sqrt{b}}\middle|-\frac{(a-b)bc_1}{(a-c)cc_2}\right)^2\right)}}{\sqrt{b(b-c)}}$$

$$z(t)\to \frac{\sqrt{2}\sqrt{\frac{(b-c)\left(bc_1(b-a)\operatorname{sn}\left(\frac{\sqrt{2}\sqrt{a-c}\sqrt{b-c}\sqrt{c_2}(c_3-t)}{\sqrt{a}\sqrt{b}}\middle|-\frac{(a-b)bc_1}{(a-c)cc_2}\right)^2+cc_2(c-a)\right)}{c-a}}}{\sqrt{c}\sqrt{b-c}}$$

$$x(t)\to \frac{\sqrt{2}bc_1\sqrt{a(a-c)}(c-b)\operatorname{sn}\left(\frac{\sqrt{2}\sqrt{a-c}\sqrt{b-c}\sqrt{c_2}(c_3-t)}{\sqrt{a}\sqrt{b}}\middle|-\frac{(a-b)bc_1}{(a-c)cc_2}\right)}{a(c-a)\sqrt{bc_1(b-c)}}$$

$$y(t)\to \frac{\sqrt{2}\sqrt{-bc_1(b-c)\left(-1+\operatorname{sn}\left(\frac{\sqrt{2}\sqrt{a-c}\sqrt{b-c}\sqrt{c_2}(c_3-t)}{\sqrt{a}\sqrt{b}}\middle|-\frac{(a-b)bc_1}{(a-c)cc_2}\right)^2\right)}}{\sqrt{b(b-c)}}$$

$$z(t)\to -\frac{\sqrt{2}\sqrt{\frac{(b-c)\left(bc_1(b-a)\operatorname{sn}\left(\frac{\sqrt{2}\sqrt{a-c}\sqrt{b-c}\sqrt{c_2}(c_3-t)}{\sqrt{a}\sqrt{b}}\middle|-\frac{(a-b)bc_1}{(a-c)cc_2}\right)^2+cc_2(c-a)\right)}{c-a}}}{\sqrt{c}\sqrt{b-c}}$$

$$x(t)\to \frac{\sqrt{2}bc_1\sqrt{a(a-c)}(c-b)\operatorname{sn}\left(\frac{\sqrt{2}\sqrt{a-c}\sqrt{b-c}\sqrt{c_2}(t-c_3)}{\sqrt{a}\sqrt{b}}\middle|-\frac{(a-b)bc_1}{(a-c)cc_2}\right)}{a(c-a)\sqrt{bc_1(b-c)}}$$

$$y(t)\to -\frac{\sqrt{2}\sqrt{-bc_1(b-c)\left(-1+\operatorname{sn}\left(\frac{\sqrt{2}\sqrt{a-c}\sqrt{b-c}\sqrt{c_2}(t-c_3)}{\sqrt{a}\sqrt{b}}\middle|-\frac{(a-b)bc_1}{(a-c)cc_2}\right)^2\right)}}{\sqrt{b(b-c)}}$$

$$z(t)\to -\frac{\sqrt{2}\sqrt{\frac{(b-c)\left(bc_1(b-a)\operatorname{sn}\left(\frac{\sqrt{2}\sqrt{a-c}\sqrt{b-c}\sqrt{c_2}(t-c_3)}{\sqrt{a}\sqrt{b}}\middle|-\frac{(a-b)bc_1}{(a-c)cc_2}\right)^2+cc_2(c-a)\right)}{c-a}}}{\sqrt{c}\sqrt{b-c}}$$

$$x(t)\to \frac{\sqrt{2}bc_1\sqrt{a(a-c)}(c-b)\operatorname{sn}\left(\frac{\sqrt{2}\sqrt{a-c}\sqrt{b-c}\sqrt{c_2}(t-c_3)}{\sqrt{a}\sqrt{b}}\middle|-\frac{(a-b)bc_1}{(a-c)cc_2}\right)}{a(c-a)\sqrt{bc_1(b-c)}}$$

$$y(t)\to \frac{\sqrt{2}\sqrt{-bc_1(b-c)\left(-1+\operatorname{sn}\left(\frac{\sqrt{2}\sqrt{a-c}\sqrt{b-c}\sqrt{c_2}(t-c_3)}{\sqrt{a}\sqrt{b}}\middle|-\frac{(a-b)bc_1}{(a-c)cc_2}\right)^2\right)}}{\sqrt{b(b-c)}}$$

$$z(t)\to \frac{\sqrt{2}\sqrt{\frac{(b-c)\left(bc_1(b-a)\operatorname{sn}\left(\frac{\sqrt{2}\sqrt{a-c}\sqrt{b-c}\sqrt{c_2}(t-c_3)}{\sqrt{a}\sqrt{b}}\middle|-\frac{(a-b)bc_1}{(a-c)cc_2}\right)^2+cc_2(c-a)\right)}{c-a}}}{\sqrt{c}\sqrt{b-c}}$$

10.19 problem 1932

Internal problem ID [10254]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 9, system of higher order odes
Problem number: 1932.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t)\,y(t) - x(t)\,z(t)$$
$$y'(t) = -x(t)\,y(t) + y(t)\,z(t)$$
$$z'(t) = x(t)\,z(t) - y(t)\,z(t)$$

✗ Solution by Maple

```
dsolve([diff(x(t),t)=x(t)*(y(t)-z(t)),diff(y(t),t)=y(t)*(z(t)-x(t)),diff(z(t),t)=z(t)*(x(t)-
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x'[t]==x[t]*(y[t]-z[t]),y'[t]==y[t]*(z[t]-x[t]),z'[t]==z[t]*(x[t]-y[t])},{x[t],y[t],
```

Not solved

10.20 problem 1933

Internal problem ID [10255]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 9, system of higher order odes
Problem number: 1933.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -\frac{y(t)\,z(t)}{2} + \frac{x(t)\,y(t)}{2} + \frac{x(t)\,z(t)}{2}$$
$$y'(t) = \frac{y(t)\,z(t)}{2} + \frac{x(t)\,y(t)}{2} - \frac{x(t)\,z(t)}{2}$$
$$z'(t) = -\frac{x(t)\,y(t)}{2} + \frac{x(t)\,z(t)}{2} + \frac{y(t)\,z(t)}{2}$$

✓ Solution by Maple

Time used: 0.954 (sec). Leaf size: 4316

```
dsolve([diff(x(t),t)+diff(y(t),t)=x(t)*y(t),diff(y(t),t)+diff(z(t),t)=y(t)*z(t),diff(x(t),t)
```

$$\left[\left\{x(t) = \frac{2}{2c_2 - t}\right\}, \left\{y(t) = \left(\int -\frac{x(t)^2\,\mathrm{e}^{-\left(\int x(t)dt\right)}}{2}dt + c_1\right)\mathrm{e}^{\int x(t)dt}\right\}, \{z(t) = x(t)\}\right]$$
$$\left[\left\{x(t) = \frac{2}{2c_2 - t}\right\}, \{y(t) = x(t)\}, \left\{z(t) = \left(\int -\frac{x(t)^2\,\mathrm{e}^{-\left(\int x(t)dt\right)}}{2}dt + c_1\right)\mathrm{e}^{\int x(t)dt}\right\}\right]$$

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x'[t]+y'[t]==x[t]*y[t],y'[t]+z'[t]==y[t]*z[t],x'[t]+z'[t]==x[t]*z[t]},{x[t],y[t],z[t
```

Not solved

10.21 problem 1934

Internal problem ID [10256]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 9, system of higher order odes
Problem number: 1934.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= \frac{x(t)^2}{2} - \frac{y(t)}{24} \\
y'(t) &= 2x(t)\,y(t) - 3z(t) \\
z'(t) &= 3x(t)\,z(t) - \frac{y(t)^2}{6}
\end{aligned}
$$

✗ Solution by Maple

```
dsolve([diff(x(t),t)=x(t)^2/2-1/24*y(t),diff(y(t),t)=2*x(t)*y(t)-3*z(t),diff(z(t),t)=3*x(t)*
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x'[t]==x[t]^2/2-1/24*y[t],y'[t]==2*x[t]*y[t]-3*z[t],z'[t]==3*x[t]*z[t]-1/6*y[t]^2},{
```

Not solved

10.22 problem 1935

Internal problem ID [10257]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 9, system of higher order odes
Problem number: 1935.
ODE order: 1.
ODE degree: 1.

Solve

$$\begin{aligned} x'(t) &= x(t)\,y(t)^2 - z(t)^2\,x(t) \\ y'(t) &= -x(t)^2\,y(t) + y(t)\,z(t)^2 \\ z'(t) &= x(t)^2\,z(t) - y(t)^2\,z(t) \end{aligned}$$

✗ Solution by Maple

```
dsolve([diff(x(t),t)=x(t)*(y(t)^2-z(t)^2),diff(y(t),t)=y(t)*(z(t)^2-x(t)^2),diff(z(t),t)=z(t
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x'[t]==x[t]*(y[t]^2-z[t]^2),y'[t]==y[t]*(z[t]^2-x[t]^2),z'[t]==z[t]*(x[t]^2-y[t]^2)}
```

Not solved

10.23 problem 1936

Internal problem ID [10258]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 9, system of higher order odes
Problem number: 1936.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= x(t)\, y(t)^2 - z(t)^2\, x(t) \\
y'(t) &= -x(t)^2\, y(t) - y(t)\, z(t)^2 \\
z'(t) &= x(t)^2\, z(t) + y(t)^2\, z(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.469 (sec). Leaf size: 704

```
dsolve([diff(x(t),t)=x(t)*(y(t)^2-z(t)^2),diff(y(t),t)=-y(t)*(z(t)^2+x(t)^2),diff(z(t),t)=z(
```

$$[\{x(t)=0\},\{y(t)=0\},\{z(t)=c_1\}]$$

$$\left[\{x(t)=0\},\left\{y(t)=\frac{\sqrt{-\left(e^{2c_2c_1}e^{2c_1t}-1\right)c_1e^{2c_2c_1}e^{2c_1t}}}{e^{2c_2c_1}e^{2c_1t}-1},y(t)=-\frac{\sqrt{-\left(e^{2c_2c_1}e^{2c_1t}-1\right)c_1e^{2c_2c_1}e^{2c_1t}}}{e^{2c_2c_1}e^{2c_1t}-1}\right\},\left\{z(t)=\frac{\sqrt{-y(t)\left(\frac{d}{dt}y(t)\right)}}{y(t)},z(t)=-\frac{\sqrt{-y(t)\left(\frac{d}{dt}y(t)\right)}}{y(t)}\right\}\right]$$

$$[\{x(t)=c_1\},\{y(t)=ix(t),y(t)=-ix(t)\},\{z(t)=ix(t),z(t)=-ix(t)\}]$$

$$\left[\left\{x(t)=\frac{\sqrt{-\left(e^{2c_2c_1}e^{2c_1t}-1\right)c_1e^{2c_2c_1}e^{2c_1t}}}{e^{2c_2c_1}e^{2c_1t}-1},x(t)=-\frac{\sqrt{-\left(e^{2c_2c_1}e^{2c_1t}-1\right)c_1e^{2c_2c_1}e^{2c_1t}}}{e^{2c_2c_1}e^{2c_1t}-1}\right\},\{y(t)=0\},\left\{z(t)=\frac{\sqrt{-x(t)\left(\frac{d}{dt}x(t)\right)}}{x(t)},z(t)=-\frac{\sqrt{-x(t)\left(\frac{d}{dt}x(t)\right)}}{x(t)}\right\}\right]$$

$$\left[\left\{x(t)=\operatorname{RootOf}\left(-\left(\int^{\_Z}-\frac{2}{\sqrt{4\_a^4-16\_a^2c_2+16c_2^2+c_1}\,\_a}d\_a\right)+t+c_3\right),x(t)=\operatorname{RootOf}\left(-\left(\int^{-}\right.\right.\right.$$

$$-\frac{\sqrt{-2x(t)\left(x(t)^3-\frac{d}{dt}x(t)-\sqrt{x(t)^6-\left(\frac{d^2}{dt^2}x(t)\right)x(t)+2\left(\frac{d}{dt}x(t)\right)^2}\right)}}{2x(t)},y(t)=\frac{\sqrt{-2x(t)\left(x(t)^3-\frac{d}{dt}x(\right.}}{}$$

$$-\frac{\sqrt{-2x(t)\left(x(t)^3-\frac{d}{dt}x(t)+\sqrt{x(t)^6-\left(\frac{d^2}{dt^2}x(t)\right)x(t)+2\left(\frac{d}{dt}x(t)\right)^2}\right)}}{2x(t)},y(t)=\frac{\sqrt{-2x(t)\left(x(t)^3-\frac{d}{dt}x(\right.}}{}$$

$$\left.\left.-\frac{\sqrt{x(t)\left(x(t)y(t)^2-\frac{d}{dt}x(t)\right)}}{x(t)}\right\}\right]$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x'[t]==x[t]*(y[t]^2-z[t]^2),y'[t]==-y[t]*(z[t]^2+x[t]^2),z'[t]==z[t]*(x[t]^2+y[t]^2)
```

Not solved

10.24 problem 1937

Internal problem ID [10259]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 9, system of higher order odes
Problem number: 1937.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= -x(t)\,y(t)^2 + x(t) + y(t) \\
y'(t) &= x(t)^2\,y(t) - x(t) - y(t) \\
z'(t) &= y(t)^2 - x(t)^2
\end{aligned}
$$

✗ Solution by Maple

```
dsolve([diff(x(t),t)=-x(t)*y(t)^2+x(t)+y(t),diff(y(t),t)=x(t)^2*y(t)-x(t)-y(t),diff(z(t),t)=
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x'[t]==-x[t]*y[t]^2+x[t]+y[t],y'[t]==x[t]^2*y[t]-x[t]-y[t],z'[t]==y[t]^2-x[t]^2},{x[
```

Not solved

10.25 problem 1938

Internal problem ID [10260]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 9, system of higher order odes
Problem number: 1938.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = \frac{f(t)}{x(t)^2 - x(t)\,y(t) - x(t)\,z(t) + y(t)\,z(t)}$$
$$y'(t) = -\frac{f(t)}{x(t)\,y(t) - x(t)\,z(t) - y(t)^2 + y(t)\,z(t)}$$
$$z'(t) = \frac{f(t)}{x(t)\,y(t) - x(t)\,z(t) - y(t)\,z(t) + z(t)^2}$$

✓ Solution by Maple

Time used: 1.891 (sec). Leaf size: 1121

```
dsolve([(x(t)-y(t))*(x(t)-z(t))*diff(x(t),t)=f(t),(y(t)-x(t))*(y(t)-z(t))*diff(y(t),t)=f(t),
```

Expression too large to display

$$\left\{y(t) = \frac{4\left(\frac{d}{dt}x(t)\right)^3 x(t) + f(t)\left(\frac{d^2}{dt^2}x(t)\right) - \left(\frac{d}{dt}x(t)\right)\left(\frac{d}{dt}f(t)\right) - \sqrt{-16\left(\frac{d}{dt}x(t)\right)^5 f(t) + \left(\frac{d}{dt}x(t)\right)^2\left(\frac{d}{dt}f(t)\right.}}{4\left(\frac{d}{dt}x(t)\right)^3}\right.$$

$$\left\{z(t) = \frac{-y(t)\left(\frac{d}{dt}x(t)\right)x(t) + \left(\frac{d}{dt}x(t)\right)x(t)^2 - f(t)}{x(t)\left(\frac{d}{dt}x(t)\right) - y(t)\left(\frac{d}{dt}x(t)\right)}\right\}$$

✓ Solution by Mathematica

Time used: 0.233 (sec). Leaf size: 1557

```
DSolve[{(x[t]-y[t])*(x[t]-z[t])*x'[t]==f[t],(y[t]-x[t])*(y[t]-z[t])*y'[t]==f[t],(z[t]-x[t])*
```

$$x(t) \to \frac{1}{6}\left(2^{2/3}\sqrt[3]{27\int_1^t f(K[1])dK[1]+\sqrt{\left(27\int_1^t f(K[1])dK[1]+2c_1^3-9c_2c_1+27c_3\right)^2-4\left(c_1^2-3c_2\right)^3}+2 \ldots} + \frac{2\sqrt[3]{2}(c_1^2-3c_2)}{\sqrt[3]{27\int_1^t f(K[1])dK[1]+\sqrt{\left(27\int_1^t f(K[1])dK[1]+2c_1^3-9c_2c_1+27c_3\right)^2-4\left(c_1^2-3c_2\right)^3}+2c_1^3 - \ldots}} + 2c_1\right)$$

$$y(t) \to -\frac{\sqrt{-\frac{-8c_1^2\left(27\int_1^t f(K[1])dK[1]+\sqrt{\left(27\int_1^t f(K[1])dK[1]+2c_1^3-9c_2c_1+27c_3\right)^2-4(c_1^2-3c_2)^3}+2c_1^3-9c_2c_1+27c_3\right)^{2/3}+2\sqrt[3]{2}\left(27\int_1^t f(K \ldots\right.}{\ldots}}}{\ldots} - \frac{\sqrt[3]{27\int_1^t f(K[1])dK[1]+\sqrt{\left(27\int_1^t f(K[1])dK[1]+2c_1^3-9c_2c_1+27c_3\right)^2-4\left(c_1^2-3c_2\right)^3}+2c_1^3 - \ldots}}{6\sqrt[3]{2}} - \frac{c_1^2-3c_2}{3\ 2^{2/3}\sqrt[3]{27\int_1^t f(K[1])dK[1]+\sqrt{\left(27\int_1^t f(K[1])dK[1]+2c_1^3-9c_2c_1+27c_3\right)^2-4\left(c_1^2-3c_2\right)^3}+ \ldots}} + \frac{c_1}{3}$$

$$z(t) \to \frac{4c_1\sqrt[3]{27\int_1^t f(K[1])dK[1]+\sqrt{\left(27\int_1^t f(K[1])dK[1]+2c_1^3-9c_2c_1+27c_3\right)^2-4\left(c_1^2-3c_2\right)^3}+2c_1^3 \ldots}}{\ldots}$$

10.26 problem 1939

Internal problem ID [10261]

Book: Differential Gleichungen, E. Kamke, 3rd ed. Chelsea Pub. NY, 1948
Section: Chapter 9, system of higher order odes
Problem number: 1939.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= \frac{x_4(t)\sin(x_3(t))}{\sin(x_2(t))} + \frac{x_5(t)\cos(x_3(t))}{\sin(x_2(t))} \\
x_2'(t) &= x_4(t)\cos(x_3(t)) - x_5(t)\sin(x_3(t)) \\
x_3'(t) &= -\frac{\cos(x_2(t))\, x_4(t)\sin(x_3(t))}{\sin(x_2(t))} - \frac{\cos(x_2(t))\cos(x_3(t))\, x_5(t)}{\sin(x_2(t))} + a \\
x_4'(t) &= -m\sin(x_2(t))\cos(x_3(t)) - a x_5(t)\,\lambda + a x_5(t) \\
x_5'(t) &= m\sin(x_2(t))\sin(x_3(t)) + a x_4(t)\,\lambda - a x_4(t)
\end{aligned}
$$

✗ Solution by Maple

```
dsolve([diff(x__1(t),t)*sin(x__2(t))=x__4(t)*sin(x__3(t))+x__5(t)*cos(x__3(t)),diff(x__2(t),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x1'[t]*Sin[x2[t]]==x4[t]*Sin[x3[t]]+x5[t]*Cos[x3[t]],x2'[t]==x4[t]*Cos[x3[t]]-x5[t]*
```

Not solved