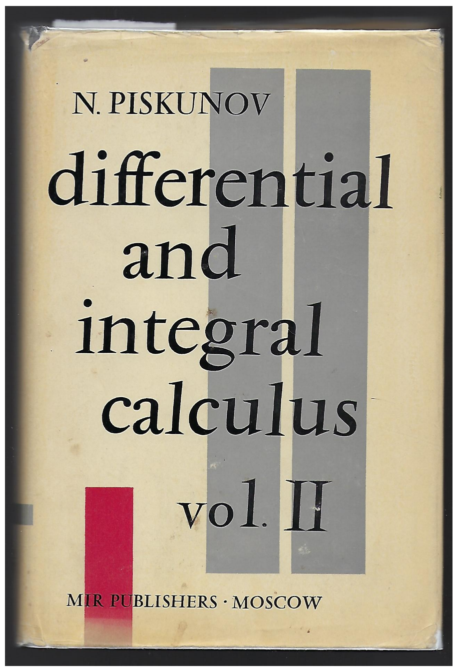
A Solution Manual For

Differential and integral calculus, vol II By N. Piskunov. 1974



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1 Chapter 1

1.1 problem Example, page 25

Internal problem ID [4345]

Book: Differential and integral calculus, vol II By N. Piskunov. 1974
Section: Chapter 1
Problem number: Example, page 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y' - \frac{xy}{x^2 - y^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)=x*y(x)/(x^2-y(x)^2),y(x), singsol=all)
```

$$y(x) = \sqrt{-\frac{1}{\text{LambertW}\left(-c_1 x^2\right)}}\, x$$

✓ Solution by Mathematica

Time used: 8.026 (sec). Leaf size: 56

```
DSolve[y'[x]==x*y[x]/(x^2-y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{ix}{\sqrt{W\left(-e^{-2c_1}x^2\right)}}$$
$$y(x) \to \frac{ix}{\sqrt{W\left(-e^{-2c_1}x^2\right)}}$$
$$y(x) \to 0$$

1.2 problem Example, page 27

Internal problem ID [4346]

Book: Differential and integral calculus, vol II By N. Piskunov. 1974
Section: Chapter 1
Problem number: Example, page 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{x + y - 3}{x - y - 1} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 32

```
dsolve(diff(y(x),x)=(x+y(x)-3)/(x-y(x)-1),y(x), singsol=all)
```

$$y(x) = 1 + \tan\left(\operatorname{RootOf}\left(2\_Z + \ln\left(\sec\left(\_Z\right)^2\right) + 2\ln\left(x - 2\right) + 2c_1\right)\right)\left(-x + 2\right)$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 57

```
DSolve[y'[x]==(x+y[x]-3)/(x-y[x]-1),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2\arctan\left(\frac{y(x) + x - 3}{-y(x) + x - 1}\right) = \log\left(\frac{x^2 + y(x)^2 - 2y(x) - 4x + 5}{2(x - 2)^2}\right) + 2\log(x - 2) + c_1, y(x)\right]$$

1.3 problem Example, page 28

Internal problem ID [4347]

Book: Differential and integral calculus, vol II By N. Piskunov. 1974
Section: Chapter 1
Problem number: Example, page 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{2x + y - 1}{4x + 2y + 5} = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)=(2*x+y(x)-1)/(4*x+2*y(x)+5),y(x), singsol=all)
```

$$y(x) = \frac{7\,\mathrm{LambertW}\left(\frac{2\,\mathrm{e}^{\frac{18}{7}+\frac{25x}{7}-\frac{25c_1}{7}}}{7}\right)}{10} - \frac{9}{5} - 2x$$

✓ Solution by Mathematica
Time used: 3.875 (sec). Leaf size: 41

```
DSolve[y'[x]==(2*x+y[x]-1)/(4*x+2*y[x]+5),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{7}{10}W\left(-e^{\frac{25x}{7}-1+c_1}\right) - 2x - \frac{9}{5}$$
$$y(x) \to -2x - \frac{9}{5}$$

1.4 problem Example, page 30

Internal problem ID [4348]

Book: Differential and integral calculus, vol II By N. Piskunov. 1974
Section: Chapter 1
Problem number: Example, page 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{2y}{x+1} = (x+1)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)-2*y(x)/(1+x)=(x+1)^2,y(x), singsol=all)
```

$$y(x) = (x + c_1)(1 + x)^2$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 15

```
DSolve[y'[x]-2*y[x]/(1+x)==(x+1)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow (x+1)^2(x + c_1)$$

1.5 problem Example, page 33

Internal problem ID [4349]

Book: Differential and integral calculus, vol II By N. Piskunov. 1974
Section: Chapter 1
Problem number: Example, page 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + xy - x^3y^3 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(diff(y(x),x)+x*y(x)=x^3*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{e^{x^2}c_1 + x^2 + 1}}$$
$$y(x) = -\frac{1}{\sqrt{e^{x^2}c_1 + x^2 + 1}}$$

✓ Solution by Mathematica

Time used: 7.029 (sec). Leaf size: 50

```
DSolve[y'[x]+x*y[x]==x^3*y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{x^2 + c_1e^{x^2} + 1}}$$
$$y(x) \to \frac{1}{\sqrt{x^2 + c_1e^{x^2} + 1}}$$
$$y(x) \to 0$$

1.6 problem Example, page 36

Internal problem ID [4350]

Book: Differential and integral calculus, vol II By N. Piskunov. 1974
Section: Chapter 1
Problem number: Example, page 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$\boxed{\frac{2x}{y^3} + \frac{(y^2 - 3x^2)\, y'}{y^4} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 317

```
dsolve(2*x/y(x)^3+ (y(x)^2-3*x^2)/(y(x)^4)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1 + \frac{\left(12\sqrt{3}\,x\sqrt{27x^2c_1^2-4}\,c_1 - 108x^2c_1^2+8\right)^{\frac{1}{3}}}{2} + \frac{2}{\left(12\sqrt{3}\,x\sqrt{27x^2c_1^2-4}\,c_1 - 108x^2c_1^2+8\right)^{\frac{1}{3}}}}{3c_1}$$

$$y(x) = -\frac{\left(1+i\sqrt{3}\right)\left(12\sqrt{3}\,x\sqrt{27x^2c_1^2-4}\,c_1 - 108x^2c_1^2+8\right)^{\frac{2}{3}} - 4i\sqrt{3} - 4\left(12\sqrt{3}\,x\sqrt{27x^2c_1^2-4}\,c_1 - 108x^2c_1^2 \cdots\right.}{12\left(12\sqrt{3}\,x\sqrt{27x^2c_1^2-4}\,c_1 - 108x^2c_1^2+8\right)^{\frac{1}{3}} c_1}$$

$$y(x) = \frac{\left(i\sqrt{3}-1\right)\left(12\sqrt{3}\,x\sqrt{27x^2c_1^2-4}\,c_1 - 108x^2c_1^2+8\right)^{\frac{2}{3}} - 4i\sqrt{3} + 4\left(12\sqrt{3}\,x\sqrt{27x^2c_1^2-4}\,c_1 - 108x^2c_1^2+8 \cdots\right.}{12\left(12\sqrt{3}\,x\sqrt{27x^2c_1^2-4}\,c_1 - 108x^2c_1^2+8\right)^{\frac{1}{3}} c_1}$$

✓ Solution by Mathematica

Time used: 60.204 (sec). Leaf size: 458

```
DSolve[2*x/y[x]^3+(y[x]^2-3*x^2)/(y[x]^4)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(\frac{\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}}{\sqrt[3]{2}} + \frac{\sqrt[3]{2}e^{2c_1}}{\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}} - e^{c_1}\right)$$

$$y(x) \to \frac{i(\sqrt{3}+i)\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}}{6\sqrt[3]{2}} - \frac{i(\sqrt{3}-i)e^{2c_1}}{3\ 2^{2/3}\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}} - \frac{e^{c_1}}{3}$$

$$y(x) \to -\frac{i(\sqrt{3}-i)\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}}{6\sqrt[3]{2}} + \frac{i(\sqrt{3}+i)e^{2c_1}}{3\ 2^{2/3}\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}} - \frac{e^{c_1}}{3}$$

1.7 problem Example, page 38

Internal problem ID [4351]

Book: Differential and integral calculus, vol II By N. Piskunov. 1974
Section: Chapter 1
Problem number: Example, page 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{y + xy^2 - xy' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve((y(x)+x*y(x)^2)-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{2x}{x^2 - 2c_1}$$

✓ Solution by Mathematica
Time used: 0.144 (sec). Leaf size: 23

```
DSolve[(y[x]+x*y[x]^2)-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2x}{x^2 - 2c_1}$$
$$y(x) \to 0$$

1.8 problem example page 46

Internal problem ID [4352]

Book: Differential and integral calculus, vol II By N. Piskunov. 1974
Section: Chapter 1
Problem number: example page 46.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y^2\left(1+y'^2\right) = R^2}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 54

```
dsolve(y(x)^2*(1+diff(y(x),x)^2)=R^2,y(x), singsol=all)
```

$$y(x) = -R$$
$$y(x) = R$$
$$y(x) = \sqrt{R^2 - c_1^2 + 2c_1 x - x^2}$$
$$y(x) = -\sqrt{(R + c_1 - x)(R - c_1 + x)}$$

✓ Solution by Mathematica
Time used: 0.22 (sec). Leaf size: 101

```
DSolve[y[x]^2*(1+(y'[x])^2)==R^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{R^2 - (x + c_1)^2}$$
$$y(x) \to \sqrt{R^2 - (x + c_1)^2}$$
$$y(x) \to -\sqrt{R^2 - (x - c_1)^2}$$
$$y(x) \to \sqrt{R^2 - (x - c_1)^2}$$
$$y(x) \to -R$$
$$y(x) \to R$$

1.9 problem example page 47

Internal problem ID [4353]

Book: Differential and integral calculus, vol II By N. Piskunov. 1974
Section: Chapter 1
Problem number: example page 47.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[_Clairaut]`

$$\boxed{y - xy' - \frac{ay'}{\sqrt{1 + y'^2}} = 0}$$

✓ Solution by Maple

Time used: 1.391 (sec). Leaf size: 17

```
dsolve(y(x)=x*diff(y(x),x)+ a*diff(y(x),x)/(sqrt(1+diff(y(x),x)^2)),y(x), singsol=all)
```

$$y(x) = c_1 \left(x + \frac{a}{\sqrt{c_1^2 + 1}}\right)$$

✓ Solution by Mathematica

Time used: 35.7 (sec). Leaf size: 27

```
DSolve[y[x]==x*y'[x]+ a*y'[x]/(Sqrt[1+(y'[x])^2]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \left(x + \frac{a}{\sqrt{1 + c_1{}^2}}\right)$$
$$y(x) \to 0$$

1.10 problem Example, page 49

Internal problem ID [4354]

Book: Differential and integral calculus, vol II By N. Piskunov. 1974
Section: Chapter 1
Problem number: Example, page 49.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, _dAlembert]`

$$\boxed{y - x{y'}^2 - {y'}^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
dsolve(y(x)=x*diff(y(x),x)^2+diff(y(x),x)^2,y(x), singsol=all)
```

$$
\begin{aligned}
y(x) &= 0 \\
y(x) &= \frac{\left(x+1+\sqrt{(1+x)(1+c_1)}\right)^2}{1+x} \\
y(x) &= \frac{\left(-x-1+\sqrt{(1+x)(1+c_1)}\right)^2}{1+x}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 57

```
DSolve[y[x]==x*(y'[x])^2+(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) &\to x - c_1\sqrt{x+1} + 1 + \frac{{c_1}^2}{4} \\
y(x) &\to x + c_1\sqrt{x+1} + 1 + \frac{{c_1}^2}{4} \\
y(x) &\to 0
\end{aligned}
$$