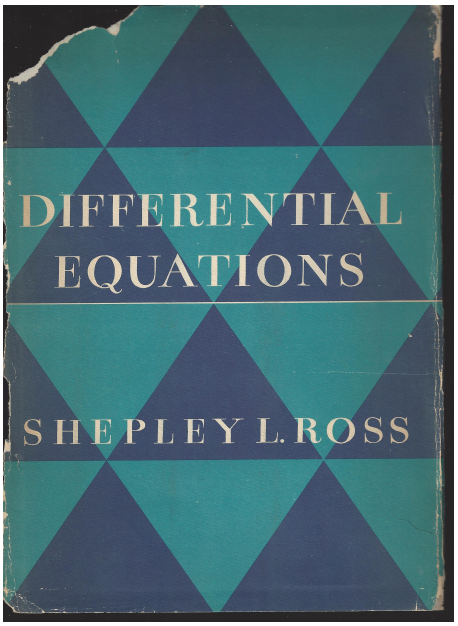
A Solution Manual For

Differential equations, Shepley L. Ross, 1964



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1.1 problem 1

Internal problem ID [2988]

Book: Differential equations, Shepley L. Ross, 1964
Section: 2.4, page 55
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{5yx + 4y^2 + \left(x^2 + 2yx\right) y' = -1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 59

```
dsolve((5*x*y(x)+4*y(x)^2+1)+(x^2+2*x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-x^3 - \sqrt{x^6 - x^4 - 4c_1}}{2x^2}$$
$$y(x) = \frac{-x^3 + \sqrt{x^6 - x^4 - 4c_1}}{2x^2}$$

✓ Solution by Mathematica
Time used: 0.664 (sec). Leaf size: 84

```
DSolve[(5*x*y[x]+4*y[x]^2+1)+(x^2+2*x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -\frac{x^5 + \sqrt{x^3}\sqrt{x^7 - x^5 + 4c_1 x}}{2x^4}$$
$$y(x) \to -\frac{x}{2} + \frac{\sqrt{x^3}\sqrt{x^7 - x^5 + 4c_1 x}}{2x^4}$$

1.2 problem 2

Internal problem ID [2989]

Book: Differential equations, Shepley L. Ross, 1964
Section: 2.4, page 55
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_exponential_symmetries]]`

$$\boxed{2x\tan(y)+\left(x-x^2\tan(y)\right)y'=0}$$

✓ Solution by Maple
Time used: 0.093 (sec). Leaf size: 32

```
dsolve((2*x*tan(y(x)))+(x-x^2*tan(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$\frac{\mathrm{e}^{\frac{y(x)}{2}}\left(\int^{y(x)}\cot(\_a)\,\mathrm{e}^{-\frac{\_a}{2}}d\_a\right)}{2}-\mathrm{e}^{\frac{y(x)}{2}}c_1+x=0$$

✓ Solution by Mathematica
Time used: 0.442 (sec). Leaf size: 78

```
DSolve[(2*x*Tan[y[x]])+(x-x^2*Tan[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x=\frac{1}{34}\left((8-2i)e^{2iy(x)}\,\text{Hypergeometric2F1}\left(1,1+\frac{i}{4},2+\frac{i}{4},e^{2iy(x)}\right)\right.\right.$$
$$\left.\left.-34i\,\text{Hypergeometric2F1}\left(\frac{i}{4},1,1+\frac{i}{4},e^{2iy(x)}\right)\right)+c_1e^{\frac{y(x)}{2}},y(x)\right]$$

1.3 problem 3

Internal problem ID [2990]

Book: Differential equations, Shepley L. Ross, 1964
Section: 2.4, page 55
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{y^2\left(x^2+1\right)+y+\left(2yx+1\right)y'=0}$$

✗ Solution by Maple

```
dsolve((y(x)^2*(x^2+1)+y(x))+(2*x*y(x)+1)*diff(y(x),x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y[x]^2*(x^2+1)+y[x])+(2*x*y[x]+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.4 problem 4

Internal problem ID [2991]

Book: Differential equations, Shepley L. Ross, 1964
Section: 2.4, page 55
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{4y^2x + 6y + \left(5x^2y + 8x\right)y' = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 23

```
dsolve((4*x*y(x)^2+6*y(x))+(5*x^2*y(x)+8*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\text{RootOf}\left(-\ln(x) + c_1 + \ln(2 + \_Z) + 4\ln(\_Z)\right)}{x}$$

✓ Solution by Mathematica
Time used: 1.989 (sec). Leaf size: 156

```
DSolve[(4*x*y[x]^2+6*y[x])+(5*x^2*y[x]+8*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \text{Root}\left[-\#1^5 - \frac{2\#1^4}{x} + \frac{e^{c_1}}{x^4}\&, 1\right]$$
$$y(x) \to \text{Root}\left[-\#1^5 - \frac{2\#1^4}{x} + \frac{e^{c_1}}{x^4}\&, 2\right]$$
$$y(x) \to \text{Root}\left[-\#1^5 - \frac{2\#1^4}{x} + \frac{e^{c_1}}{x^4}\&, 3\right]$$
$$y(x) \to \text{Root}\left[-\#1^5 - \frac{2\#1^4}{x} + \frac{e^{c_1}}{x^4}\&, 4\right]$$
$$y(x) \to \text{Root}\left[-\#1^5 - \frac{2\#1^4}{x} + \frac{e^{c_1}}{x^4}\&, 5\right]$$

1.5 problem 5

Internal problem ID [2992]

Book: Differential equations, Shepley L. Ross, 1964
Section: 2.4, page 55
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _exact, _rational, [_Abel, '2nd ty`

$$\boxed{2y + (2x + y + 1)\,y' = -5x - 1}$$

✓ Solution by Maple
Time used: 0.406 (sec). Leaf size: 32

```
dsolve((5*x+2*y(x)+1)+(2*x+y(x)+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-\sqrt{-(x-1)^2 c_1^2 + 1} + (-2x-1)\,c_1}{c_1}$$

✓ Solution by Mathematica
Time used: 0.188 (sec). Leaf size: 53

```
DSolve[(5*x+2*y[x]+1)+(2*x+y[x]+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^2 + 2x + 1 + c_1} - 2x - 1$$
$$y(x) \to \sqrt{-x^2 + 2x + 1 + c_1} - 2x - 1$$

1.6 problem 6

Internal problem ID [2993]

Book: Differential equations, Shepley L. Ross, 1964
Section: 2.4, page 55
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{-y-(6x-2y-3)\,y'=-3x-1}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 23

```
dsolve((3*x-y(x)+1)-(6*x-2*y(x)-3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=-\frac{\text{LambertW}\left(-2\,\mathrm{e}^{-4+5x-5c_1}\right)}{2}+3x-2$$

✓ Solution by Mathematica
Time used: 3.791 (sec). Leaf size: 35

```
DSolve[(3*x-y[x]+1)-(6*x-2*y[x]-3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{1}{2}W\left(-e^{5x-1+c_1}\right)+3x-2$$
$$y(x)\to 3x-2$$

1.7 problem 7

Internal problem ID [2994]

Book: Differential equations, Shepley L. Ross, 1964
Section: 2.4, page 55
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{-2y + (2x + y - 1)\,y' = -x + 3}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 31

```
dsolve((x-2*y(x)-3)+(2*x+y(x)-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -1 - \tan\left(\text{RootOf}\left(-4\_Z + \ln\left(\sec\left(\_Z\right)^2\right) + 2\ln\left(x-1\right) + 2c_1\right)\right)(x-1)$$

✓ Solution by Mathematica
Time used: 0.061 (sec). Leaf size: 66

```
DSolve[(x-2*y[x]-3)+(2*x+y[x]-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[32\arctan\left(\frac{2y(x)-x+3}{y(x)+2x-1}\right) + 8\log\left(\frac{x^2+y(x)^2+2y(x)-2x+2}{5(x-1)^2}\right) + 16\log(x-1) + 5c_1 = 0, y(x)\right]$$

1.8 problem 8

Internal problem ID [2995]

Book: Differential equations, Shepley L. Ross, 1964
Section: 2.4, page 55
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _exact, _rational, [_Abel, '2nd ty`

$$\boxed{4y + (4x + 2y + 2)\,y' = -6x - 1}$$

With initial conditions

$$\left[y\left(\frac{1}{2}\right) = 3\right]$$

✓ Solution by Maple
Time used: 0.234 (sec). Leaf size: 23

```
dsolve([(6*x+4*y(x)+1)+(4*x+2*y(x)+2)*diff(y(x),x)=0,y(1/2) = 3],y(x), singsol=all)
```

$$y(x) = -2x - 1 + \frac{\sqrt{4x^2 + 12x + 93}}{2}$$

✓ Solution by Mathematica
Time used: 0.143 (sec). Leaf size: 28

```
DSolve[{(6*x+4*y[x]+1)+(4*x+2*y[x]+2)*y'[x]==0,y[1/2]==3},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{2}\left(\sqrt{4x^2 + 12x + 93} - 4x - 2\right)$$

1.9 problem 9

Internal problem ID [2996]

Book: Differential equations, Shepley L. Ross, 1964
Section: 2.4, page 55
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{-y + (x + y + 2)\, y' = -3x + 6}$$

With initial conditions

$$[y(2) = -2]$$

✓ Solution by Maple
Time used: 1.968 (sec). Leaf size: 120

```
dsolve([(3*x-y(x)-6)+(x+y(x)+2)*diff(y(x),x)=0,y(2) = -2],y(x), singsol=all)
```

$$y(x) = -3 - \sqrt{3}\, \tan\left(\text{RootOf}\left(-3\sqrt{3}\, \ln(3) + 6\sqrt{3}\, \ln(2) - 3\sqrt{3}\, \ln\left(\sec(\_Z)^2 (x-1)^2\right) + \pi + 6\_Z\right)\right)(x-1)$$

✓ Solution by Mathematica
Time used: 0.158 (sec). Leaf size: 90

```
DSolve[{(3*x-y[x]-6)+(x+y[x]+2)*y'[x]==0,y[2]==-2},y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\arctan\left(\frac{-y(x)+3x-6}{\sqrt{3}(y(x)+x+2)}\right)}{\sqrt{3}} + \log(2) = \frac{1}{2}\log\left(\frac{3x^2 + y(x)^2 + 6y(x) - 6x + 12}{(x-1)^2}\right) + \log(x-1) + \frac{1}{18}\left(\sqrt{3}\pi + 18\log(2) - 9\log(4)\right), y(x)\right]$$

1.10 problem 10

Internal problem ID [2997]

Book: Differential equations, Shepley L. Ross, 1964
Section: 2.4, page 55
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{3y + (4x + 6y + 1)\,y' = -1 - 2x}$$

With initial conditions

$$[y(-2) = 2]$$

✓ Solution by Maple
Time used: 0.156 (sec). Leaf size: 20

```
dsolve([(2*x+3*y(x)+1)+(4*x+6*y(x)+1)*diff(y(x),x)=0,y(-2) = 2],y(x), singsol=all)
```

$$y(x) = \frac{1}{3} - \frac{2x}{3} + \frac{\text{LambertW}\left(\frac{2\,\mathrm{e}^{\frac{x}{3}+\frac{4}{3}}}{3}\right)}{2}$$

✓ Solution by Mathematica
Time used: 4.969 (sec). Leaf size: 30

```
DSolve[{(2*x+3*y[x]+1)+(4*x+6*y[x]+1)*y'[x]==0,y[-2]==2},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{6}\left(3W\left(\frac{2}{3}e^{\frac{x+4}{3}}\right) - 4x + 2\right)$$