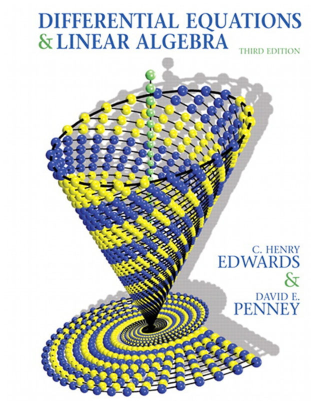
A Solution Manual For

Differential equations and linear algebra, 3rd ed., Edwards and Penney



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May 16, 2024

Contents

1 Section 1.2. Integrals as general and particular solutions. Page 16

1.1 problem 1

Internal problem ID [1]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.2. Integrals as general and particular solutions. Page 16
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = 1 + 2x}$$

With initial conditions

$$[y(0) = 3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 10

```
dsolve([diff(y(x),x) = 1+2*x,y(0) = 3],y(x), singsol=all)
```

$$y(x) = x^2 + x + 3$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 11

```
DSolve[{y'[x]==1+2*x,y[0]==3},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + x + 3$$

1.2 problem 2

Internal problem ID [2]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.2. Integrals as general and particular solutions. Page 16
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = (-2 + x)^2}$$

With initial conditions

$$[y(2) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve([diff(y(x),x) = (-2+x)^2,y(2) = 1],y(x), singsol=all)
```

$$y(x) = \frac{(-2 + x)^3}{3} + 1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 22

```
DSolve[{y'[x]==(-2+x)^2,y[2]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(x^3 - 6x^2 + 12x - 5\right)$$

1.3 problem 3

Internal problem ID [3]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.2. Integrals as general and particular solutions. Page 16
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \sqrt{x}}$$

With initial conditions

$$[y(4) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 11

```
dsolve([diff(y(x),x) = x^(1/2),y(4) = 0],y(x), singsol=all)
```

$$y(x) = \frac{2x^{\frac{3}{2}}}{3} - \frac{16}{3}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 16

```
DSolve[{y'[x] == x^(1/2),y[4]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{3}(x^{3/2} - 8)$$

1.4 problem 4

Internal problem ID [4]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.2. Integrals as general and particular solutions. Page 16
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{1}{x^2}}$$

With initial conditions

$$[y(1) = 5]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve([diff(y(x),x) = 1/x^2,y(1) = 5],y(x), singsol=all)
```

$$y(x) = -\frac{1}{x} + 6$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 12

```
DSolve[{y'[x] == 1/x^2,y[1]==5},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 6 - \frac{1}{x}$$

1.5 problem 5

Internal problem ID [5]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.2. Integrals as general and particular solutions. Page 16
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{1}{\sqrt{2+x}}}$$

With initial conditions

$$[y(2) = -1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve([diff(y(x),x) = 1/(2+x)^(1/2),y(2) = -1],y(x), singsol=all)
```

$$y(x) = 2\sqrt{2+x} - 5$$

✓ Solution by Mathematica
Time used: 0.008 (sec). Leaf size: 16

```
DSolve[{y'[x] == 1/(2+x)^(1/2),y[2]==-1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\sqrt{x+2} - 5$$

1.6 problem 6

Internal problem ID [6]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.2. Integrals as general and particular solutions. Page 16
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = x\sqrt{x^2+9}}$$

With initial conditions

$$[y(-4) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(x),x) = x*(x^2+9)^(1/2),y(-4) = 0],y(x), singsol=all)
```

$$y(x) = \frac{x^2\sqrt{x^2+9}}{3} + 3\sqrt{x^2+9} - \frac{125}{3}$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 20

```
DSolve[{y'[x] == x*(x^2+9)^(1/2),y[-4]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(\left(x^2+9\right)^{3/2} - 125\right)$$

1.7 problem 7

Internal problem ID [7]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.2. Integrals as general and particular solutions. Page 16
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{10}{x^2 + 1}}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 8

```
dsolve([diff(y(x),x) = 10/(x^2+1),y(0) = 0],y(x), singsol=all)
```

$$y(x) = 10 \arctan(x)$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 9

```
DSolve[{y'[x]==10/(x^2+1),y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 10 \arctan(x)$$

1.8 problem 8

Internal problem ID [8]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.2. Integrals as general and particular solutions. Page 16
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \cos(2x)}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve([diff(y(x),x) = cos(2*x),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\sin(2x)}{2} + 1$$

✓ Solution by Mathematica
Time used: 0.055 (sec). Leaf size: 12

```
DSolve[{y'[x] == Cos[2*x],y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x)\cos(x) + 1$$

1.9 problem 9

Internal problem ID [9]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.2. Integrals as general and particular solutions. Page 16
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{1}{\sqrt{-x^2+1}}}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 6

```
dsolve([diff(y(x),x) = 1/(-x^2+1)^(1/2),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \arcsin(x)$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 31

```
DSolve[{y'[x] == 1/(-x^2+1)^(1/2),y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(\pi - 4\arctan\left(\frac{\sqrt{1-x^2}}{x+1}\right)\right)$$

1.10 problem 10

Internal problem ID [10]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.2. Integrals as general and particular solutions. Page 16
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = x\,\mathrm{e}^{-x}}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(x),x) = x/exp(x),y(0) = 1],y(x), singsol=all)
```

$$y(x) = 2 + (-x - 1)\,\mathrm{e}^{-x}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 21

```
DSolve[{y'[x]== x/Exp[x],y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(-x + 2e^{x} - 1)$$

2 Section 1.3. Slope fields and solution curves. Page 26

2.1 problem 1

Internal problem ID [11]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.3. Slope fields and solution curves. Page 26
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + y = -\sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x) = -sin(x)-y(x),y(x), singsol=all)
```

$$y(x) = \frac{\cos(x)}{2} - \frac{\sin(x)}{2} + e^{-x}c_1$$

✓ Solution by Mathematica
Time used: 0.069 (sec). Leaf size: 25

```
DSolve[y'[x]== -Sin[x]-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(-\sin(x) + \cos(x) + 2c_1e^{-x})$$

2.2 problem 2

Internal problem ID [12]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.3. Slope fields and solution curves. Page 26
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - y = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x) = x+y(x),y(x), singsol=all)
```

$$y(x) = -x - 1 + e^x c_1$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 16

```
DSolve[y'[x] == x+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + c_1 e^x - 1$$

2.3 problem 3

Internal problem ID [13]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.3. Slope fields and solution curves. Page 26
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - y = -\sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x) = -sin(x)+y(x),y(x), singsol=all)
```

$$y(x) = \frac{\cos(x)}{2} + \frac{\sin(x)}{2} + e^x c_1$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 21

```
DSolve[y'[x] == -Sin[x]+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(\sin(x) + \cos(x) + 2c_1e^x)$$

2.4 problem 4

Internal problem ID [14]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.3. Slope fields and solution curves. Page 26
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + y = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x) = x-y(x),y(x), singsol=all)
```

$$y(x) = x - 1 + \mathrm{e}^{-x} c_1$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 16

```
DSolve[y'[x] == x-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + c_1 e^{-x} - 1$$

2.5 problem 5

Internal problem ID [15]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.3. Slope fields and solution curves. Page 26
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - y = 1 - x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x) = 1-x+y(x),y(x), singsol=all)
```

$$y(x) = x + \mathrm{e}^{x} c_1$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 13

```
DSolve[y'[x] == 1-x+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + c_1 e^{x}$$

2.6 problem 6

Internal problem ID [16]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.3. Slope fields and solution curves. Page 26
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + y = x + 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x) = 1+x-y(x),y(x), singsol=all)
```

$$y(x) = x + \mathrm{e}^{-x} c_1$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 15

```
DSolve[y'[x] == 1+x-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + c_1 e^{-x}$$

2.7 problem 8

Internal problem ID [17]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.3. Slope fields and solution curves. Page 26
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + y = x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x) = x^2-y(x),y(x), singsol=all)
```

$$y(x) = x^2 - 2x + 2 + \mathrm{e}^{-x} c_1$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 21

```
DSolve[y'[x] == x^2-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 - 2x + c_1 e^{-x} + 2$$

2.8 problem 9

Internal problem ID [18]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.3. Slope fields and solution curves. Page 26
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = x^2 - 2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x) = -2+x^2-y(x),y(x), singsol=all)
```

$$y(x) = x^2 - 2x + e^{-x}c_1$$

✓ Solution by Mathematica
Time used: 0.057 (sec). Leaf size: 19

```
DSolve[y'[x]== -2+x^2-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x - 2)x + c_1e^{-x}$$

2.9 problem 11

Internal problem ID [19]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.3. Slope fields and solution curves. Page 26
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 2x^2y^2 = 0}$$

With initial conditions

$$[y(1) = -1]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 15

```
dsolve([diff(y(x),x) = 2*x^2*y(x)^2,y(1) = -1],y(x), singsol=all)
```

$$y(x) = -\frac{3}{2x^3 + 1}$$

✓ Solution by Mathematica
Time used: 0.143 (sec). Leaf size: 16

```
DSolve[{y'[x] == 2*x^2*y[x]^2,y[1]==-1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{3}{2x^3 + 1}$$

2.10 problem 12

Internal problem ID [20]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.3. Slope fields and solution curves. Page 26
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - x \ln(y) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x) = x*ln(y(x)),y(x), singsol=all)
```

$$y(x) = e^{\mathrm{RootOf}\left(x^2+2\ \mathrm{expIntegral}_1(-\_Z)+2c_1\right)}$$

✓ Solution by Mathematica
Time used: 0.266 (sec). Leaf size: 22

```
DSolve[y'[x] == x*Log[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \mathrm{LogIntegral}^{(-1)}\left(\frac{x^2}{2} + c_1\right)$$
$$y(x) \to 1$$

2.11 problem 13

Internal problem ID [21]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.3. Slope fields and solution curves. Page 26
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^{\frac{1}{3}} = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 13

```
dsolve([diff(y(x),x) = y(x)^(1/3),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{(2x+3)\sqrt{6x+9}}{9}$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 23

```
DSolve[{y'[x] == y[x]^(1/3),y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(2x+3)^{3/2}}{3\sqrt{3}}$$

2.12 problem 14

Internal problem ID [22]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.3. Slope fields and solution curves. Page 26
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^{\frac{1}{3}} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x) = y(x)^(1/3),y(0) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 21

```
DSolve[{y'[x] == y[x]^(1/3),y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{3}\sqrt{\frac{2}{3}}x^{3/2}$$

2.13 problem 17

Internal problem ID [23]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.3. Slope fields and solution curves. Page 26
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yy' = x - 1}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 9

```
dsolve([y(x)*diff(y(x),x) = -1+x,y(0) = 1],y(x), singsol=all)
```

$$y(x) = 1 - x$$

✓ Solution by Mathematica
Time used: 0.087 (sec). Leaf size: 14

```
DSolve[{y[x]*y'[x] == -1+x,y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{(x-1)^2}$$

2.14 problem 18

Internal problem ID [24]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.3. Slope fields and solution curves. Page 26
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yy' = x - 1}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve([y(x)*diff(y(x),x) = -1+x,y(1) = 0],y(x), singsol=all)
```

$$y(x) = 1 - x$$
$$y(x) = x - 1$$

✓ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 29

```
DSolve[{y[x]*y'[x] == -1+x,y[1]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{(x-1)^2}$$
$$y(x) \to \sqrt{(x-1)^2}$$

2.15 problem 19

Internal problem ID [25]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.3. Slope fields and solution curves. Page 26
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \ln\left(1 + y^2\right) = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x) = ln(1+y(x)^2),y(0) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y'[x] == Log[1+y[x]^2],y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

2.16 problem 20

Internal problem ID [26]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.3. Slope fields and solution curves. Page 26
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + y^2 = x^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 44

```
dsolve(diff(y(x),x) = x^2-y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x\left(\operatorname{BesselI}\left(-\frac{3}{4}, \frac{x^2}{2}\right) c_1 - \operatorname{BesselK}\left(\frac{3}{4}, \frac{x^2}{2}\right)\right)}{c_1 \operatorname{BesselI}\left(\frac{1}{4}, \frac{x^2}{2}\right) + \operatorname{BesselK}\left(\frac{1}{4}, \frac{x^2}{2}\right)}$$

✓ Solution by Mathematica
Time used: 0.109 (sec). Leaf size: 197

```
DSolve[y'[x]== x^2-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{-ix^2\left(2\operatorname{BesselJ}\left(-\frac{3}{4}, \frac{ix^2}{2}\right) + c_1\left(\operatorname{BesselJ}\left(-\frac{5}{4}, \frac{ix^2}{2}\right) - \operatorname{BesselJ}\left(\frac{3}{4}, \frac{ix^2}{2}\right)\right)\right) - c_1 \operatorname{BesselJ}\left(-\frac{1}{4}, \frac{ix^2}{2}\right)}{2x\left(\operatorname{BesselJ}\left(\frac{1}{4}, \frac{ix^2}{2}\right) + c_1 \operatorname{BesselJ}\left(-\frac{1}{4}, \frac{ix^2}{2}\right)\right)}$$

$$y(x) \to \frac{ix^2 \operatorname{BesselJ}\left(-\frac{5}{4}, \frac{ix^2}{2}\right) - ix^2 \operatorname{BesselJ}\left(\frac{3}{4}, \frac{ix^2}{2}\right) + \operatorname{BesselJ}\left(-\frac{1}{4}, \frac{ix^2}{2}\right)}{2x \operatorname{BesselJ}\left(-\frac{1}{4}, \frac{ix^2}{2}\right)}$$

3 Section 1.4. Separable equations. Page 43

3.1 problem 1

Internal problem ID [27]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.4. Separable equations. Page 43
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2yx + y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(2*x*y(x)+diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^2}$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 20

```
DSolve[2*x*y[x]+y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x^2}$$
$$y(x) \to 0$$

3.2 problem 2

Internal problem ID [28]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.4. Separable equations. Page 43
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2xy^2 + y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(2*x*y(x)^2+diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{x^2 + c_1}$$

✓ Solution by Mathematica

Time used: 0.098 (sec). Leaf size: 20

```
DSolve[2*x*y[x]^2+y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{x^2 - c_1}$$
$$y(x) \to 0$$

3.3 problem 3

Internal problem ID [29]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.4. Separable equations. Page 43
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \sin(x)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x) = sin(x)*y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^{-\cos(x)}$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 19

```
DSolve[y'[x] == Sin[x]*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\cos(x)}$$
$$y(x) \to 0$$

3.4 problem 4

Internal problem ID [30]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.4. Separable equations. Page 43
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x+1)\,y' - 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve((1+x)*diff(y(x),x) = 4*y(x),y(x), singsol=all)
```

$$y(x) = c_1(x+1)^4$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 18

```
DSolve[(1+x)*y'[x] == 4*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x+1)^4$$
$$y(x) \to 0$$

3.5 problem 5

Internal problem ID [31]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.4. Separable equations. Page 43
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2\sqrt{x}\, y' - \sqrt{1 - y^2} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 12

```
dsolve(2*x^(1/2)*diff(y(x),x) = (1-y(x)^2)^(1/2),y(x), singsol=all)
```

$$y(x) = \sin\left(\sqrt{x} + \frac{c_1}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.221 (sec). Leaf size: 32

```
DSolve[2*x^(1/2)*y'[x] == (1-y[x]^2)^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos\left(\sqrt{x} + c_1\right)$$
$$y(x) \to -1$$
$$y(x) \to 1$$
$$y(x) \to \text{Interval}[\{-1, 1\}]$$

3.6 problem 6

Internal problem ID [32]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.4. Separable equations. Page 43
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{y' - 3\sqrt{yx} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 65

```
dsolve(diff(y(x),x) = 3*(x*y(x))^(1/2),y(x), singsol=all)
```

$$\frac{(c_1x^3 - y(x)\,c_1 + 1)\sqrt{xy(x)} - x^2(c_1x^3 - y(x)\,c_1 - 1)}{(x^3 - y(x))\left(x^2 - \sqrt{xy(x)}\right)} = 0$$

✓ Solution by Mathematica

Time used: 0.127 (sec). Leaf size: 26

```
DSolve[y'[x] == 3*(x*y[x])^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(2x^{3/2} + c_1\right)^2$$
$$y(x) \to 0$$

3.7 problem 7

Internal problem ID [33]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.4. Separable equations. Page 43
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{y' - 4(yx)^{\frac{1}{3}} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 91

```
dsolve(diff(y(x),x) = 4*(x*y(x))^(1/3),y(x), singsol=all)
```

$$-\frac{32x\left(\left(-c_1x^5 + \frac{y(x)^2c_1x}{8} + \frac{x}{16}\right)(xy(x))^{\frac{2}{3}} + \left(c_1x^4 - \frac{y(x)^2c_1}{8} + \frac{1}{8}\right)\left(x^3 + \frac{y(x)(xy(x))^{\frac{1}{3}}}{4}\right)\right)}{\left(8x^4 - y\left(x\right)^2\right)\left(-\left(xy\left(x\right)\right)^{\frac{2}{3}} + 2x^2\right)^2} = 0$$

✓ Solution by Mathematica
Time used: 4.813 (sec). Leaf size: 35

```
DSolve[y'[x] == 4*(x*y[x])^(1/3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{3}\sqrt{\frac{2}{3}}\left(3x^{4/3} + c_1\right)^{3/2}$$
$$y(x) \to 0$$

3.8 problem 8

Internal problem ID [34]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.4. Separable equations. Page 43
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 2x \sec(y) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x) = 2*x*sec(y(x)),y(x), singsol=all)
```

$$y(x) = \arcsin(x^2 + 2c_1)$$

✓ Solution by Mathematica

Time used: 0.841 (sec). Leaf size: 12

```
DSolve[y'[x]==2*x*Sec[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin(x^2 + c_1)$$

3.9 problem 9

Internal problem ID [35]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.4. Separable equations. Page 43
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(-x^2+1)\,y'-2y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve((-x^2+1)*diff(y(x),x) = 2*y(x),y(x), singsol=all)
```

$$y(x)=-\frac{(x+1)\,c_1}{x-1}$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 22

```
DSolve[(-x^2+1)*y'[x] == 2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{c_1(x+1)}{x-1}$$
$$y(x)\to 0$$

3.10 problem 10

Internal problem ID [36]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.4. Separable equations. Page 43
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2+1)\,y' - (1+y)^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve((x^2+1)*diff(y(x),x) = (1+y(x))^2,y(x), singsol=all)
```

$$y(x) = \frac{-\arctan(x) - c_1 - 1}{\arctan(x) + c_1}$$

✓ Solution by Mathematica
Time used: 0.19 (sec). Leaf size: 25

```
DSolve[(x^2+1)*y'[x]== (1+y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\arctan(x) + 1 + c_1}{\arctan(x) + c_1}$$
$$y(x) \to -1$$

3.11 problem 11

Internal problem ID [37]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.4. Separable equations. Page 43
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - x y^3 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x) = x*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{-x^2 + c_1}}$$
$$y(x) = -\frac{1}{\sqrt{-x^2 + c_1}}$$

✓ Solution by Mathematica
Time used: 0.146 (sec). Leaf size: 44

```
DSolve[y'[x] == x*y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{-x^2 - 2c_1}}$$
$$y(x) \to \frac{1}{\sqrt{-x^2 - 2c_1}}$$
$$y(x) \to 0$$

3.12 problem 12

Internal problem ID [38]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.4. Separable equations. Page 43
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yy' - x(1 + y^2) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(y(x)*diff(y(x),x) = x*(1+y(x)^2),y(x), singsol=all)
```

$$y(x) = \sqrt{e^{x^2} c_1 - 1}$$
$$y(x) = -\sqrt{e^{x^2} c_1 - 1}$$

✓ Solution by Mathematica
Time used: 6.961 (sec). Leaf size: 57

```
DSolve[y[x]*y'[x] == x*(1+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-1 + e^{x^2+2c_1}}$$
$$y(x) \to \sqrt{-1 + e^{x^2+2c_1}}$$
$$y(x) \to -i$$
$$y(x) \to i$$

3.13 problem 14

Internal problem ID [39]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.4. Separable equations. Page 43
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' - \frac{1+\sqrt{x}}{1+\sqrt{y}} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x) = (1+x^(1/2))/(1+y(x)^(1/2)),y(x), singsol=all)
```

$$x + \frac{2x^{\frac{3}{2}}}{3} - y(x) - \frac{2y(x)^{\frac{3}{2}}}{3} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 4.529 (sec). Leaf size: 796

```
DSolve[y'[x]== (1+x^(1/2))/(1+y[x]^(1/2)),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-16x^{3/2} + \left(96x^{5/2} + 24(-3+4c_1)x^{3/2} + 8\sqrt{(2x^{3/2}+3x-1+3c_1)\left(2x^{3/2}+3x+3c_1\right)^3} + 32x^3 + 72x^{\cdots}}{4\sqrt[3]{96x^{5/2} + 24(-3\cdots}}$$

$$y(x) \to \frac{1}{16}\left(\frac{2\left(1+i\sqrt{3}\right)\left(16x^{3/2}+24x-9+24c_1\right)}{\sqrt[3]{96x^{5/2} + 24(-3+4c_1)x^{3/2} + 8\sqrt{(2x^{3/2}+3x-1+3c_1)\left(2x^{3/2}+3x+3c_1\right)^3} + 32x^3 + 72x^2 + 3\cdots}} + 2i\left(\sqrt{3}+i\right)\sqrt[3]{96x^{5/2} + 24(-3+4c_1)x^{3/2} + 8\sqrt{(2x^{3/2}+3x-1+3c_1)\left(2x^{3/2}+3x+3c_1\right)^3} + 32x^3 + 7\cdots} + 12\right)$$

$$y(x) \to \frac{1}{16}\left(\frac{2\left(1-i\sqrt{3}\right)\left(16x^{3/2}+24x-9+24c_1\right)}{\sqrt[3]{96x^{5/2} + 24(-3+4c_1)x^{3/2} + 8\sqrt{(2x^{3/2}+3x-1+3c_1)\left(2x^{3/2}+3x+3c_1\right)^3} + 32x^3 + 72x^2 + 3\cdots}} - 2\left(1+i\sqrt{3}\right)\sqrt[3]{96x^{5/2} + 24(-3+4c_1)x^{3/2} + 8\sqrt{(2x^{3/2}+3x-1+3c_1)\left(2x^{3/2}+3x+3c_1\right)^3} + 32x^3 + 7\cdots} + 12\right)$$

3.14 problem 15

Internal problem ID [40]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.4. Separable equations. Page 43
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' - \frac{(x-1)\,y^5}{x^2\,(-y+2y^3)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 844

```
dsolve(diff(y(x),x) = (-1+x)*y(x)^5/x^2/(-y(x)+2*y(x)^3),y(x), singsol=all)
```

$$y(x) = \frac{8x^2 2^{\frac{1}{3}} - 4x\left(3x(x\ln(x)+c_1x+1)\sqrt{9+9\ln(x)^2x^2+18(c_1x^2+x)\ln(x)+(9c_1^2-32)x^2+18c_1x+9}\right.}{\left(3x(x\ln(x)+c_1x+1)\sqrt{9+9}\right.}$$

$$y(x) = -\frac{8x\left(3x(x\ln(x)+c_1x+1)\sqrt{9+9\ln(x)^2x^2+18(c_1x^2+x)\ln(x)+(9c_1^2-32)x^2+18c_1x}+9(x\ln(\right.}{\left(3x(x\ln(x)+c_1x-\right.}$$

$$y(x) = \frac{-8x\left(3x(x\ln(x)+c_1x+1)\sqrt{9+9\ln(x)^2x^2+18(c_1x^2+x)\ln(x)+(9c_1^2-32)x^2+18c_1x}+9(x\ln(x\right.}{\left(3x(x\ln(x)+c_1x+\right.}$$

✓ Solution by Mathematica

Time used: 19.626 (sec). Leaf size: 842

```
DSolve[y'[x] == (-1+x)*y[x]^5/x^2/(-y[x]+2*y[x]^3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{8\sqrt[3]{2}x^2}{\sqrt[3]{16x^3 - 9x^3\log^2(x) - 9c_1{}^2x^3 - 18c_1x^2 + 3\sqrt{x^2(x\log(x) + c_1x + 1)^2\left(9x^2\log^2(x) + \left(-32 + 9c_1{}^2\right)x^2}}}$$

$$y(x) \to \frac{8\sqrt[3]{2}\left(1+i\sqrt{3}\right)x^2}{\sqrt[3]{16x^3 - 9x^3\log^2(x) - 9c_1{}^2x^3 - 18c_1x^2 + 3\sqrt{x^2(x\log(x) + c_1x + 1)^2\left(9x^2\log^2(x) + \left(-32 + 9c_1{}^2\right)x^2 -}}}$$

$$y(x) \to \frac{8\sqrt[3]{2}\left(1-i\sqrt{3}\right)x^2}{\sqrt[3]{16x^3 - 9x^3\log^2(x) - 9c_1{}^2x^3 - 18c_1x^2 + 3\sqrt{x^2(x\log(x) + c_1x + 1)^2\left(9x^2\log^2(x) + \left(-32 + 9c_1{}^2\right)x^2 -}}}$$

$$y(x) \to 0$$

3.15 problem 16

Internal problem ID [41]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.4. Separable equations. Page 43
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2+1)\tan(y)\,y'=x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve((x^2+1)*tan(y(x))*diff(y(x),x) = x,y(x), singsol=all)
```

$$y(x)=\arccos\left(\frac{1}{\sqrt{x^2+1}\,c_1}\right)$$

✓ Solution by Mathematica
Time used: 15.547 (sec). Leaf size: 63

```
DSolve[(x^2+1)*Tan[y[x]]*y'[x] == x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\arccos\left(\frac{e^{-c_1}}{\sqrt{x^2+1}}\right)$$
$$y(x)\to\arccos\left(\frac{e^{-c_1}}{\sqrt{x^2+1}}\right)$$
$$y(x)\to-\frac{\pi}{2}$$
$$y(x)\to\frac{\pi}{2}$$

3.16 problem 17

Internal problem ID [42]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.4. Separable equations. Page 43
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y - yx = x + 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x) = 1+x+y(x)+x*y(x),y(x), singsol=all)
```

$$y(x) = -1 + \mathrm{e}^{\frac{x(2+x)}{2}} c_1$$

✓ Solution by Mathematica
Time used: 0.074 (sec). Leaf size: 25

```
DSolve[y'[x] == 1+x+y[x]+x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -1 + c_1 e^{\frac{1}{2}x(x+2)}$$
$$y(x) \rightarrow -1$$

3.17 problem 18

Internal problem ID [43]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.4. Separable equations. Page 43
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x^2 - y^2 + x^2y^2 = -x^2 + 1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x) = 1-x^2+y(x)^2-x^2*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\tan\left(\frac{c_1x + x^2 + 1}{x}\right)$$

✓ Solution by Mathematica
Time used: 0.267 (sec). Leaf size: 17

```
DSolve[x^2*y'[x] == 1-x^2+y[x]^2-x^2*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\tan\left(x + \frac{1}{x} - c_1\right)$$

3.18 problem 19

Internal problem ID [44]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.4. Separable equations. Page 43
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - e^x y = 0}$$

With initial conditions

$$[y(0) = 2\,e]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve([diff(y(x),x) = exp(x)*y(x),y(0) = 2*exp(1)],y(x), singsol=all)
```

$$y(x) = 2\,e^{e^x}$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 12

```
DSolve[{y'[x] == Exp[x]*y[x],y[0]==2*Exp[1]},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2e^{e^x}$$

3.19 problem 20

Internal problem ID [45]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.4. Separable equations. Page 43
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 3(1 + y^2)\, x^2 = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 12

```
dsolve([diff(y(x),x) = 3*x^2*(1+y(x)^2),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \tan\left(x^3 + \frac{\pi}{4}\right)$$

✓ Solution by Mathematica

Time used: 0.167 (sec). Leaf size: 15

```
DSolve[{y'[x]== 3*x^2*(1+y[x]^2),y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan\left(x^3 + \frac{\pi}{4}\right)$$

3.20 problem 21

Internal problem ID [46]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.4. Separable equations. Page 43
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2yy' = \frac{x}{\sqrt{x^2 - 16}}}$$

With initial conditions

$$[y(5) = 2]$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 34

```
dsolve([2*y(x)*diff(y(x),x) = x/(x^2-16)^(1/2),y(5) = 2],y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\sqrt{x^2 - 16}\,\left(x^2 + \sqrt{x^2 - 16} - 16\right)}}{\sqrt{x^2 - 16}}$$

✓ Solution by Mathematica
Time used: 1.931 (sec). Leaf size: 20

```
DSolve[{2*y[x]*y'[x] == x/(x^2-16)^(1/2),y[5]==2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{\sqrt{x^2 - 16} + 1}$$

3.21 problem 22

Internal problem ID [47]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.4. Separable equations. Page 43
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + y - 4yx^3 = 0}$$

With initial conditions

$$[y(1) = -3]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve([diff(y(x),x) = -y(x)+4*x^3*y(x),y(1) = -3],y(x), singsol=all)
```

$$y(x) = -3\,\mathrm{e}^{x(x-1)\left(x^2+x+1\right)}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 16

```
DSolve[{y'[x]== -y[x]+4*x^3*y[x],y[1]==-3},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -3e^{x^4-x}$$

3.22 problem 23

Internal problem ID [48]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.4. Separable equations. Page 43
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 2y = -1}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve([1+diff(y(x),x) = 2*y(x),y(1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{1}{2} + \frac{e^{2x-2}}{2}$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 18

```
DSolve[{1+y'[x] == 2*y[x],y[1]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(e^{2x-2} + 1\right)$$

3.23 problem 24

Internal problem ID [49]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.4. Separable equations. Page 43
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\tan(x)\, y' - y = 0}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}\right]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve([tan(x)*diff(y(x),x) = y(x),y(1/2*Pi) = 1/2*Pi],y(x), singsol=all)
```

$$y(x) = \frac{\pi \sin(x)}{2}$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 12

```
DSolve[{Tan[x]*y'[x] == y[x],y[Pi/2]==Pi/2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\pi \sin(x)$$

3.24 problem 25

Internal problem ID [50]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.4. Separable equations. Page 43
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{-y + y'x - 2x^2y = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve([-y(x)+x*diff(y(x),x) = 2*x^2*y(x),y(1) = 1],y(x), singsol=all)
```

$$y(x) = x\,\mathrm{e}^{(x-1)(x+1)}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 14

```
DSolve[{-y[x]+x*y'[x] == 2*x^2*y[x],y[1]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x^2-1}x$$

3.25 problem 26

Internal problem ID [51]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.4. Separable equations. Page 43
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 2xy^2 - 3x^2y^2 = 0}$$

With initial conditions

$$[y(1) = -1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve([diff(y(x),x) = 2*x*y(x)^2+3*x^2*y(x)^2,y(1) = -1],y(x), singsol=all)
```

$$y(x) = -\frac{1}{x^3 + x^2 - 1}$$

✓ Solution by Mathematica
Time used: 0.118 (sec). Leaf size: 17

```
DSolve[{y'[x] == 2*x*y[x]^2+3*x^2*y[x]^2,y[1]==-1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{x^3 + x^2 - 1}$$

3.26 problem 27

Internal problem ID [52]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.4. Separable equations. Page 43
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 6\,\mathrm{e}^{2x-y} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 13

```
dsolve([diff(y(x),x) = 6*exp(2*x-y(x)),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \ln\left(-2 + 3\,\mathrm{e}^{2x}\right)$$

✓ Solution by Mathematica
Time used: 0.739 (sec). Leaf size: 15

```
DSolve[{y'[x] == 6*Exp[2*x-y[x]],y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(3e^{2x} - 2\right)$$

3.27 problem 28

Internal problem ID [53]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.4. Separable equations. Page 43
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2\sqrt{x}\, y' - \cos\left(y\right)^{2} = 0}$$

With initial conditions

$$\left[y(4) = \frac{\pi}{4}\right]$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 10

```
dsolve([2*x^(1/2)*diff(y(x),x) = cos(y(x))^2,y(4) = 1/4*Pi],y(x), singsol=all)
```

$$y(x) = \arctan\left(-1+\sqrt{x}\right)$$

✓ Solution by Mathematica
Time used: 0.46 (sec). Leaf size: 17

```
DSolve[{2*x^(1/2)*y'[x] == Cos[y[x]]^2,y[4]==Pi/4},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arctan\left(1-\sqrt{x}\right)$$

4 Section 1.5. Linear first order equations. Page 56

4.1 problem 1

Internal problem ID [54]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.5. Linear first order equations. Page 56
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + y = 2}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 12

```
dsolve([y(x)+diff(y(x),x) = 2,y(0) = 0],y(x), singsol=all)
```

$$y(x) = 2 - 2\,\mathrm{e}^{-x}$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 14

```
DSolve[{y[x]+y'[x] == 2,y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2 - 2e^{-x}$$

4.2 problem 2

Internal problem ID [55]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.5. Linear first order equations. Page 56
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - 2y = 3\,\mathrm{e}^{2x}}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve([-2*y(x)+diff(y(x),x) = 3*exp(2*x),y(0) = 0],y(x), singsol=all)
```

$$y(x) = 3\,\mathrm{e}^{2x}x$$

✓ Solution by Mathematica
Time used: 0.045 (sec). Leaf size: 13

```
DSolve[{-2*y[x]+y'[x] == 3*Exp[2*x],y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3e^{2x}x$$

4.3 problem 3

Internal problem ID [56]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.5. Linear first order equations. Page 56
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{3y + y' = 2x\,\mathrm{e}^{-3x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(3*y(x)+diff(y(x),x) = 2*x/exp(3*x),y(x), singsol=all)
```

$$y(x) = \left(x^2 + c_1\right)\mathrm{e}^{-3x}$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 17

```
DSolve[3*y[x]+y'[x] == 2*x/Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(x^2 + c_1\right)$$

4.4 problem 4

Internal problem ID [57]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.5. Linear first order equations. Page 56
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{-2yx + y' = e^{x^2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(-2*x*y(x)+diff(y(x),x) = exp(x^2),y(x), singsol=all)
```

$$y(x) = (c_1 + x)\, e^{x^2}$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 15

```
DSolve[-2*x*y[x]+y'[x] == Exp[x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x^2}(x + c_1)$$

4.5 problem 5

Internal problem ID [58]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.5. Linear first order equations. Page 56
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2y + y'x = 3x}$$

With initial conditions

$$[y(1) = 5]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 11

```
dsolve([2*y(x)+x*diff(y(x),x) = 3*x,y(1) = 5],y(x), singsol=all)
```

$$y(x) = x + \frac{4}{x^2}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 12

```
DSolve[{2*y[x]+x*y'[x] == 3*x,y[1]==5},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4}{x^2} + x$$

4.6 problem 6

Internal problem ID [59]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.5. Linear first order equations. Page 56
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y + 2y'x = 10\sqrt{x}}$$

With initial conditions

$$[y(2) = 5]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve([y(x)+2*x*diff(y(x),x) = 10*x^(1/2),y(2) = 5],y(x), singsol=all)
```

$$y(x) = \frac{-10 + 5\sqrt{2} + 5x}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 20

```
DSolve[{y[x]+2*x*y'[x]== 10*x^(1/2),y[2]==5},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{5(x + \sqrt{2} - 2)}{\sqrt{x}}$$

4.7 problem 7

Internal problem ID [60]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.5. Linear first order equations. Page 56
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y + 2y'x = 10\sqrt{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(y(x)+2*x*diff(y(x),x) = 10*x^(1/2),y(x), singsol=all)
```

$$y(x) = \frac{5x + c_1}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 17

```
DSolve[y[x]+2*x*y'[x] == 10*x^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{5x + c_1}{\sqrt{x}}$$

4.8 problem 8

Internal problem ID [61]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.5. Linear first order equations. Page 56
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y + 3y'x = 12x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(y(x)+3*x*diff(y(x),x) = 12*x,y(x), singsol=all)
```

$$y(x) = 3x + \frac{c_1}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 17

```
DSolve[y[x]+3*x*y'[x] == 12*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3x + \frac{c_1}{\sqrt[3]{x}}$$

4.9 problem 9

Internal problem ID [62]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.5. Linear first order equations. Page 56
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{-y + y'x = x}$$

With initial conditions

$$[y(1) = 7]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve([-y(x)+x*diff(y(x),x) = x,y(1) = 7],y(x), singsol=all)
```

$$y(x) = (\ln (x) + 7)\, x$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 11

```
DSolve[{-y[x]+x*y'[x]== x,y[1]==7},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(\log(x) + 7)$$

4.10 problem 10

Internal problem ID [63]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.5. Linear first order equations. Page 56
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{-3y + 2y'x = 9x^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(-3*y(x)+2*x*diff(y(x),x) = 9*x^3,y(x), singsol=all)
```

$$y(x) = 3x^3 + x^{\frac{3}{2}} c_1$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 19

```
DSolve[-3*y[x]+2*x*y'[x] == 9*x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3x^3 + c_1 x^{3/2}$$

4.11 problem 11

Internal problem ID [64]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.5. Linear first order equations. Page 56
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y + y'x - 3yx = 0}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve([y(x)+x*diff(y(x),x) = 3*x*y(x),y(1) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y[x]+x*y'[x] == 3*x*y[x],y[1]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

4.12 problem 12

Internal problem ID [65]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.5. Linear first order equations. Page 56
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{3y + y'x = 2x^5}$$

With initial conditions

$$[y(2) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve([3*y(x)+x*diff(y(x),x) = 2*x^5,y(2) = 1],y(x), singsol=all)
```

$$y(x) = \frac{x^8 - 224}{4x^3}$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 17

```
DSolve[{3*y[x]+x*y'[x] == 2*x^5,y[2]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x^8 - 224}{4x^3}$$

4.13 problem 13

Internal problem ID [66]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.5. Linear first order equations. Page 56
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = e^x}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve([y(x)+diff(y(x),x) = exp(x),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{e^x}{2} + \frac{e^{-x}}{2}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 21

```
DSolve[{y[x]+y'[x] == Exp[x],y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(e^{2x} + 1\right)$$

4.14 problem 14

Internal problem ID [67]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.5. Linear first order equations. Page 56
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{-3y + y'x = x^3}$$

With initial conditions

$$[y(1) = 10]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve([-3*y(x)+x*diff(y(x),x) = x^3,y(1) = 10],y(x), singsol=all)
```

$$y(x) = (\ln(x) + 10)\, x^3$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 13

```
DSolve[{-3*y[x]+x*y'[x] == x^3,y[1]==10},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^3(\log(x) + 10)$$

4.15 problem 15

Internal problem ID [68]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.5. Linear first order equations. Page 56
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2yx + y' = x}$$

With initial conditions

$$[y(0) = -2]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve([2*x*y(x)+diff(y(x),x) = x,y(0) = -2],y(x), singsol=all)
```

$$y(x) = \frac{1}{2} - \frac{5\,\mathrm{e}^{-x^2}}{2}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 20

```
DSolve[{2*x*y[x]+y'[x] == x,y[0]==-2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} - \frac{5e^{-x^2}}{2}$$

4.16 problem 16

Internal problem ID [69]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.5. Linear first order equations. Page 56
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \cos(x)(1 - y) = 0}$$

With initial conditions

$$[y(\pi) = 2]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve([diff(y(x),x) = cos(x)*(1-y(x)),y(Pi) = 2],y(x), singsol=all)
```

$$y(x) = 1 + e^{-\sin(x)}$$

✓ Solution by Mathematica
Time used: 0.05 (sec). Leaf size: 13

```
DSolve[{y'[x] == Cos[x]*(1-y[x]),y[Pi]==2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\sin(x)} + 1$$

4.17 problem 17

Internal problem ID [70]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.5. Linear first order equations. Page 56
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y + (x+1)\, y' = \cos(x)}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve([y(x)+(1+x)*diff(y(x),x) = cos(x),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\sin(x) + 1}{x + 1}$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 15

```
DSolve[{y[x]+(1+x)*y'[x] == Cos[x],y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sin(x) + 1}{x + 1}$$

4.18 problem 18

Internal problem ID [71]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.5. Linear first order equations. Page 56
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - 2y = x^3 \cos(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x) = x^3*cos(x)+2*y(x),y(x), singsol=all)
```

$$y(x) = (\sin(x) + c_1)\, x^2$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 14

```
DSolve[x*y'[x]== x^3*Cos[x]+2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(\sin(x) + c_1)$$

4.19 problem 19

Internal problem ID [72]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.5. Linear first order equations. Page 56
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y \cot (x) + y' = \cos (x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(cot(x)*y(x)+diff(y(x),x) = cos(x),y(x), singsol=all)
```

$$y(x) = -\frac{\left(2 \cos (x)^2 - 4c_1 - 1\right) \csc (x)}{4}$$

✓ Solution by Mathematica
Time used: 0.044 (sec). Leaf size: 19

```
DSolve[Cot[x]*y[x]+y'[x] == Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2} \cos(x) \cot(x) + c_1 \csc(x)$$

4.20 problem 20

Internal problem ID [73]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.5. Linear first order equations. Page 56
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y - yx = x + 1}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 13

```
dsolve([diff(y(x),x) = 1+x+y(x)+x*y(x),y(0) = 0],y(x), singsol=all)
```

$$y(x) = -1 + e^{\frac{x(2+x)}{2}}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 17

```
DSolve[{y'[x]== 1+x+y[x]+x*y[x],y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{1}{2}x(x+2)} - 1$$

4.21 problem 21

Internal problem ID [74]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.5. Linear first order equations. Page 56
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{-3y + y'x = x^4 \cos(x)}$$

With initial conditions

$$[y(2\pi) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 10

```
dsolve([x*diff(y(x),x) = x^4*cos(x)+3*y(x),y(2*Pi) = 0],y(x), singsol=all)
```

$$y(x) = \sin(x)\, x^3$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 11

```
DSolve[{x*y'[x] == x^4*Cos[x]+3*y[x],y[2*Pi]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^3 \sin(x)$$

4.22 problem 22

Internal problem ID [75]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.5. Linear first order equations. Page 56
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{-2yx + y' = 3x^2 \mathrm{e}^{x^2}}$$

With initial conditions

$$[y(0) = 5]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve([diff(y(x),x) = 3*exp(x^2)*x^2+2*x*y(x),y(0) = 5],y(x), singsol=all)
```

$$y(x) = (x^3 + 5)\,\mathrm{e}^{x^2}$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 16

```
DSolve[{y'[x] == 3*Exp[x^2]*x^2+2*x*y[x],y[0]==5},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x^2}(x^3 + 5)$$

4.23 problem 23

Internal problem ID [76]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.5. Linear first order equations. Page 56
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(-3+2x)\,y+y'x=4x^4}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve((-3+2*x)*y(x)+x*diff(y(x),x) = 4*x^4,y(x), singsol=all)
```

$$y(x)=x^3\left(2+\mathrm{e}^{-2x}c_1\right)$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 19

```
DSolve[(-3+2*x)*y[x]+x*y'[x] == 4*x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x^3\left(2+c_1e^{-2x}\right)$$

4.24 problem 24

Internal problem ID [77]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.5. Linear first order equations. Page 56
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{3yx + (x^2 + 4)\, y' = x}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve([3*x*y(x)+(x^2+4)*diff(y(x),x) = x,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{1}{3} + \frac{16}{3\,(x^2+4)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 22

```
DSolve[{3*x*y[x]+(x^2+4)*y'[x] == x,y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{16}{3\,(x^2+4)^{3/2}} + \frac{1}{3}$$

4.25 problem 25

Internal problem ID [78]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.5. Linear first order equations. Page 56
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$3yx^3 + (x^2+1)\, y' = 6x\, \mathrm{e}^{-\frac{3x^2}{2}}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve([3*x^3*y(x)+(x^2+1)*diff(y(x),x) = 6*x/exp(3/2*x^2),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \left(3x^2\sqrt{x^2+1} + 3\sqrt{x^2+1} - 2\right) \mathrm{e}^{-\frac{3x^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.094 (sec). Leaf size: 28

```
DSolve[{3*x^3*y[x]+(x^2+1)*y'[x] == 6*x/Exp[3/2*x^2],y[0]==1},y[x],x,IncludeSingularSolution
```

$$y(x) \to e^{-\frac{3x^2}{2}} \left(3(x^2+1)^{3/2} - 2\right)$$

5 Section 1.6, Substitution methods and exact equations. Page 74

5.1 problem 1

Internal problem ID [79]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{(x+y)\,y'+y=x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 51

```
dsolve((x+y(x))*diff(y(x),x) = x-y(x),y(x), singsol=all)
```

$$y(x)=\frac{-c_1x-\sqrt{2c_1^2x^2+1}}{c_1}$$
$$y(x)=\frac{-c_1x+\sqrt{2c_1^2x^2+1}}{c_1}$$

✓ Solution by Mathematica
Time used: 0.465 (sec). Leaf size: 94

```
DSolve[(x+y[x])*y'[x]== x-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -x-\sqrt{2x^2+e^{2c_1}}$$
$$y(x)\to -x+\sqrt{2x^2+e^{2c_1}}$$
$$y(x)\to -\sqrt{2}\sqrt{x^2}-x$$
$$y(x)\to \sqrt{2}\sqrt{x^2}-x$$

5.2 problem 2

Internal problem ID [80]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{2xyy' - y^2 = x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(2*x*y(x)*diff(y(x),x) = x^2+y(x)^2,y(x), singsol=all)
```

$$y(x) = \sqrt{(c_1 + x)\,x}$$
$$y(x) = -\sqrt{(c_1 + x)\,x}$$

✓ Solution by Mathematica

Time used: 0.17 (sec). Leaf size: 38

```
DSolve[2*x*y[x]*y'[x] == x^2+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x}\sqrt{x + c_1}$$
$$y(x) \to \sqrt{x}\sqrt{x + c_1}$$

5.3 problem 3

Internal problem ID [81]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y'x - y - 2\sqrt{yx} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x) = y(x)+2*(x*y(x))^(1/2),y(x), singsol=all)
```

$$-\frac{y(x)}{\sqrt{xy\left(x\right)}} + \ln\left(x\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.159 (sec). Leaf size: 19

```
DSolve[x*y'[x] == y[x]+2*(x*y[x])^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}x(2\log(x) + c_1)^2$$

5.4 problem 4

Internal problem ID [82]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(x-y)\,y'-y=x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 24

```
dsolve((x-y(x))*diff(y(x),x) = x+y(x),y(x), singsol=all)
```

$$y(x)=\tan\left(\operatorname{RootOf}\left(-2\_Z+\ln\left(\sec\left(\_Z\right)^{2}\right)+2\ln\left(x\right)+2c_{1}\right)\right)x$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 36

```
DSolve[(x-y[x])*y'[x] == x+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\log\left(\frac{y(x)^{2}}{x^{2}}+1\right)-\arctan\left(\frac{y(x)}{x}\right)=-\log(x)+c_{1},y(x)\right]$$

5.5 problem 5

Internal problem ID [83]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{x(x+y)\,y'-y(x-y)=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve(x*(x+y(x))*diff(y(x),x) = y(x)*(x-y(x)),y(x), singsol=all)
```

$$y(x)=\frac{x}{\operatorname{LambertW}(c_1x^2)}$$

✓ Solution by Mathematica

Time used: 4.218 (sec). Leaf size: 25

```
DSolve[x*(x+y[x])*y'[x] == y[x]*(x-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x}{W(e^{-c_1}x^2)}$$
$$y(x)\to 0$$

5.6 problem 6

Internal problem ID [84]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(x+2y)\,y'-y=0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve((x+2*y(x))*diff(y(x),x) = y(x),y(x), singsol=all)
```

$$y(x)=\frac{x}{2\,\mathrm{LambertW}\left(\frac{x\,\mathrm{e}^{\frac{c_1}{2}}}{2}\right)}$$

✓ Solution by Mathematica
Time used: 4.677 (sec). Leaf size: 31

```
DSolve[(x+2*y[x])*y'[x] == y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x}{2W\left(\frac{1}{2}e^{-\frac{c_1}{2}}x\right)}$$
$$y(x)\to 0$$

5.7 problem 7

Internal problem ID [85]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{xy^2y' - y^3 = x^3}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 58

```
dsolve(x*y(x)^2*diff(y(x),x) = x^3+y(x)^3,y(x), singsol=all)
```

$$y(x) = (3\ln(x) + c_1)^{\frac{1}{3}}\, x$$
$$y(x) = -\frac{(3\ln(x) + c_1)^{\frac{1}{3}}\left(1 + i\sqrt{3}\right) x}{2}$$
$$y(x) = \frac{(3\ln(x) + c_1)^{\frac{1}{3}}\left(i\sqrt{3} - 1\right) x}{2}$$

✓ Solution by Mathematica
Time used: 0.195 (sec). Leaf size: 63

```
DSolve[x*y[x]^2*y'[x] == x^3+y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\sqrt[3]{3\log(x) + c_1}$$
$$y(x) \to -\sqrt[3]{-1}x\sqrt[3]{3\log(x) + c_1}$$
$$y(x) \to (-1)^{2/3}x\sqrt[3]{3\log(x) + c_1}$$

5.8 problem 8

Internal problem ID [86]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y' x^2 - \mathrm{e}^{\frac{y}{x}} x^2 - y x = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x) = exp(y(x)/x)*x^2+x*y(x),y(x), singsol=all)
```

$$y(x) = \ln\left(-\frac{1}{\ln(x) + c_1}\right) x$$

✓ Solution by Mathematica
Time used: 0.303 (sec). Leaf size: 18

```
DSolve[x^2*y'[x] == Exp[y[x]/x]*x^2+x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x \log(-\log(x) - c_1)$$

5.9 problem 9

Internal problem ID [87]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y'x^2 - yx - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x^2*diff(y(x),x) = x*y(x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x}{c_1 - \ln(x)}$$

✓ Solution by Mathematica

Time used: 0.119 (sec). Leaf size: 21

```
DSolve[x^2*y'[x] == x*y[x]+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{-\log(x) + c_1}$$
$$y(x) \to 0$$

5.10 problem 10

Internal problem ID [88]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{xyy' - 3y^2 = x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(x*y(x)*diff(y(x),x) = x^2+3*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{4c_1x^4 - 2}\,x}{2}$$
$$y(x) = \frac{\sqrt{4c_1x^4 - 2}\,x}{2}$$

✓ Solution by Mathematica
Time used: 0.6 (sec). Leaf size: 42

```
DSolve[x*y[x]*y'[x] == x^2+3*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\sqrt{-\frac{1}{2} + c_1x^4}$$
$$y(x) \to x\sqrt{-\frac{1}{2} + c_1x^4}$$

5.11 problem 11

Internal problem ID [89]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{(x^2 - y^2)\, y' - 2yx = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
dsolve((x^2-y(x)^2)*diff(y(x),x) = 2*x*y(x),y(x), singsol=all)
```

$$y(x) = \frac{1 - \sqrt{-4c_1^2x^2 + 1}}{2c_1}$$
$$y(x) = \frac{1 + \sqrt{-4c_1^2x^2 + 1}}{2c_1}$$

✓ Solution by Mathematica

Time used: 1.035 (sec). Leaf size: 66

```
DSolve[(x^2-y[x]^2)*y'[x]== 2*x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(e^{c_1} - \sqrt{-4x^2 + e^{2c_1}}\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{-4x^2 + e^{2c_1}} + e^{c_1}\right)$$
$$y(x) \to 0$$

5.12 problem 12

Internal problem ID [90]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{xyy' - y^2 - x\sqrt{4x^2 + y^2} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(x*y(x)*diff(y(x),x) = y(x)^2+x*(4*x^2+y(x)^2)^(1/2),y(x), singsol=all)
```

$$\frac{x \ln(x) - c_1 x - \sqrt{4x^2 + y(x)^2}}{x} = 0$$

✓ Solution by Mathematica
Time used: 0.256 (sec). Leaf size: 54

```
DSolve[x*y[x]*y'[x] == y[x]^2+x*(4*x^2+y[x]^2)^(1/2),y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -x\sqrt{\log^2(x) + 2c_1 \log(x) - 4 + c_1^2}$$
$$y(x) \to x\sqrt{\log^2(x) + 2c_1 \log(x) - 4 + c_1^2}$$

5.13 problem 13

Internal problem ID [91]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{y'x - y - \sqrt{x^2 + y^2} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 26

```
dsolve(x*diff(y(x),x) = y(x)+(x^2+y(x)^2)^(1/2),y(x), singsol=all)
```

$$\frac{-c_1 x^2 + \sqrt{x^2 + y(x)^2} + y(x)}{x^2} = 0$$

✓ Solution by Mathematica
Time used: 0.341 (sec). Leaf size: 27

```
DSolve[x*y'[x] == y[x]+(x^2+y[x]^2)^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-c_1} \left(-1 + e^{2c_1} x^2\right)$$

5.14 problem 14

Internal problem ID [92]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{yy' - \sqrt{x^2 + y^2} = -x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 27

```
dsolve(x+y(x)*diff(y(x),x) = (x^2+y(x)^2)^(1/2),y(x), singsol=all)
```

$$\frac{-y(x)^2 c_1 + \sqrt{x^2 + y(x)^2} + x}{y(x)^2} = 0$$

✓ Solution by Mathematica
Time used: 0.402 (sec). Leaf size: 57

```
DSolve[x+y[x]*y'[x] == (x^2+y[x]^2)^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{\frac{c_1}{2}} \sqrt{2x + e^{c_1}}$$
$$y(x) \to e^{\frac{c_1}{2}} \sqrt{2x + e^{c_1}}$$
$$y(x) \to 0$$

5.15 problem 15

Internal problem ID [93]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y(3x+y)+x(x+y)\,y'=0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 59

```
dsolve(y(x)*(3*x+y(x))+x*(x+y(x))*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{-c_1x^2 - \sqrt{c_1^2x^4+1}}{c_1x}$$
$$y(x) = \frac{-c_1x^2 + \sqrt{c_1^2x^4+1}}{c_1x}$$

✓ Solution by Mathematica
Time used: 0.607 (sec). Leaf size: 93

```
DSolve[y[x]*(3*x+y[x])+x*(x+y[x])*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2+\sqrt{x^4+e^{2c_1}}}{x}$$
$$y(x) \to -x + \frac{\sqrt{x^4+e^{2c_1}}}{x}$$
$$y(x) \to -\frac{\sqrt{x^4}+x^2}{x}$$
$$y(x) \to \frac{\sqrt{x^4}}{x} - x$$

5.16 problem 16

Internal problem ID [94]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - \sqrt{1 + x + y} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 44

```
dsolve(diff(y(x),x) = (1+x+y(x))^(1/2),y(x), singsol=all)
```

$$x - 2\sqrt{1 + x + y(x)} - \ln\left(-1 + \sqrt{1 + x + y(x)}\right) + \ln\left(1 + \sqrt{1 + x + y(x)}\right) + \ln(x + y(x)) - c_1 = 0$$

✓ Solution by Mathematica
Time used: 9.342 (sec). Leaf size: 56

```
DSolve[y'[x] == (1+x+y[x])^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to W\left(-e^{\frac{1}{2}(-x-3-c_1)}\right)^2 + 2W\left(-e^{\frac{1}{2}(-x-3-c_1)}\right) - x$$
$$y(x) \to -x$$

5.17 problem 17

Internal problem ID [95]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Riccati]`

$$\boxed{y' - (4x + y)^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve(diff(y(x),x) = (4*x+y(x))^2,y(x), singsol=all)
```

$$y(x) = -4x - 2\tan(-2x + 2c_1)$$

✓ Solution by Mathematica

Time used: 0.121 (sec). Leaf size: 41

```
DSolve[y'[x] == (4*x+y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -4x + \frac{1}{c_1 e^{4ix} - \frac{i}{4}} - 2i$$
$$y(x) \to -4x - 2i$$

5.18 problem 18

Internal problem ID [96]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(x+y)\,y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve((x+y(x))*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = -x$$
$$y(x) = c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 14

```
DSolve[(x+y[x])*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x$$
$$y(x) \to c_1$$

5.19 problem 19

Internal problem ID [97]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{2yx + y'x^2 - 5y^3 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 50

```
dsolve(2*x*y(x)+x^2*diff(y(x),x) = 5*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(c_1x^5+2)\,x}}{c_1x^5+2}$$
$$y(x) = -\frac{\sqrt{(c_1x^5+2)\,x}}{c_1x^5+2}$$

✓ Solution by Mathematica
Time used: 0.399 (sec). Leaf size: 51

```
DSolve[2*x*y[x]+x^2*y'[x] == 5*y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{x}}{\sqrt{2+c_1x^5}}$$
$$y(x) \to \frac{\sqrt{x}}{\sqrt{2+c_1x^5}}$$
$$y(x) \to 0$$

5.20 problem 20

Internal problem ID [98]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2xy^3 + y^2y' = 6x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 66

```
dsolve(2*x*y(x)^3+y(x)^2*diff(y(x),x) = 6*x,y(x), singsol=all)
```

$$y(x) = \left(\mathrm{e}^{-3x^2}c_1 + 3\right)^{\frac{1}{3}}$$

$$y(x) = -\frac{\left(\mathrm{e}^{-3x^2}c_1 + 3\right)^{\frac{1}{3}}\left(1 + i\sqrt{3}\right)}{2}$$

$$y(x) = \frac{\left(\mathrm{e}^{-3x^2}c_1 + 3\right)^{\frac{1}{3}}\left(i\sqrt{3} - 1\right)}{2}$$

✓ Solution by Mathematica
Time used: 1.937 (sec). Leaf size: 115

```
DSolve[2*x*y[x]^3+y[x]^2*y'[x] == 6*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[3]{3 + e^{-3x^2+3c_1}}$$

$$y(x) \to -\sqrt[3]{-1}\sqrt[3]{3 + e^{-3x^2+3c_1}}$$

$$y(x) \to (-1)^{2/3}\sqrt[3]{3 + e^{-3x^2+3c_1}}$$

$$y(x) \to -\sqrt[3]{-3}$$

$$y(x) \to \sqrt[3]{3}$$

$$y(x) \to (-1)^{2/3}\sqrt[3]{3}$$

5.21 problem 21

Internal problem ID [99]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^3 - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x) = y(x)+y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{\mathrm{e}^{-2x}c_1 - 1}}$$
$$y(x) = -\frac{1}{\sqrt{\mathrm{e}^{-2x}c_1 - 1}}$$

✓ Solution by Mathematica
Time used: 60.06 (sec). Leaf size: 57

```
DSolve[y'[x] == y[x]+y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{ie^{x+c_1}}{\sqrt{-1 + e^{2(x+c_1)}}}$$
$$y(x) \to \frac{ie^{x+c_1}}{\sqrt{-1 + e^{2(x+c_1)}}}$$

5.22 problem 22

Internal problem ID [100]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{2yx + y'x^2 - 5y^4 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 111

```
dsolve(2*x*y(x)+x^2*diff(y(x),x) = 5*y(x)^4,y(x), singsol=all)
```

$$y(x) = \frac{7^{\frac{1}{3}} \left(x(7c_1x^7 + 15)^2\right)^{\frac{1}{3}}}{7c_1x^7 + 15}$$
$$y(x) = -\frac{7^{\frac{1}{3}} \left(x(7c_1x^7 + 15)^2\right)^{\frac{1}{3}} (1 + i\sqrt{3})}{14c_1x^7 + 30}$$
$$y(x) = \frac{7^{\frac{1}{3}} \left(x(7c_1x^7 + 15)^2\right)^{\frac{1}{3}} (i\sqrt{3} - 1)}{14c_1x^7 + 30}$$

✓ Solution by Mathematica

Time used: 0.454 (sec). Leaf size: 96

```
DSolve[2*x*y[x]+x^2*y'[x] == 5*y[x]^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[3]{-7}\sqrt[3]{x}}{\sqrt[3]{15 + 7c_1x^7}}$$
$$y(x) \to \frac{\sqrt[3]{7}\sqrt[3]{x}}{\sqrt[3]{15 + 7c_1x^7}}$$
$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{7}\sqrt[3]{x}}{\sqrt[3]{15 + 7c_1x^7}}$$
$$y(x) \to 0$$

5.23 problem 23

Internal problem ID [101]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{6y + y'x - 3xy^{\frac{4}{3}} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(6*y(x)+x*diff(y(x),x) = 3*x*y(x)^(4/3),y(x), singsol=all)
```

$$\frac{1}{y(x)^{\frac{1}{3}}} - x - c_1x^2 = 0$$

✓ Solution by Mathematica
Time used: 0.146 (sec). Leaf size: 22

```
DSolve[6*y[x]+x*y'[x] == 3*x*y[x]^(4/3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{x^3(1 + c_1x)^3}$$
$$y(x) \to 0$$

5.24 problem 24

Internal problem ID [102]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y^3 \mathrm{e}^{-2x} + 2y'x - 2yx = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 47

```
dsolve(y(x)^3/exp(2*x)+2*x*diff(y(x),x) = 2*x*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(\ln(x) + c_1)\,\mathrm{e}^{2x}}}{\ln(x) + c_1}$$
$$y(x) = \frac{\sqrt{(\ln(x) + c_1)\,\mathrm{e}^{2x}}}{-\ln(x) - c_1}$$

✓ Solution by Mathematica
Time used: 0.342 (sec). Leaf size: 41

```
DSolve[y[x]^3/Exp[2*x]+2*x*y'[x] == 2*x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^x}{\sqrt{\log(x) + c_1}}$$
$$y(x) \to \frac{e^x}{\sqrt{\log(x) + c_1}}$$
$$y(x) \to 0$$

5.25 problem 25

Internal problem ID [103]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{\sqrt{x^4+1}\,y^2(y+y'x)=x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 97

```
dsolve((x^4+1)^(1/2)*y(x)^2*(y(x)+x*diff(y(x),x)) = x,y(x), singsol=all)
```

$$y(x)=\frac{\left(3\left(\int \frac{x^3}{\sqrt{x^4+1}}dx\right)+c_1\right)^{\frac{1}{3}}}{x}$$
$$y(x)=-\frac{\left(3\left(\int \frac{x^3}{\sqrt{x^4+1}}dx\right)+c_1\right)^{\frac{1}{3}}\left(1+i\sqrt{3}\right)}{2x}$$
$$y(x)=\frac{\left(3\left(\int \frac{x^3}{\sqrt{x^4+1}}dx\right)+c_1\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)}{2x}$$

✓ Solution by Mathematica

Time used: 3.932 (sec). Leaf size: 106

```
DSolve[(x^4+1)^(1/2)*y[x]^2*(y[x]+x*y'[x]) ==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[3]{\frac{3\sqrt{x^4+1}}{2x^3}+\frac{c_1}{x^3}}$$

$$y(x) \to -\sqrt[3]{-\frac{1}{2}}\sqrt[3]{\frac{3\sqrt{x^4+1}+2c_1}{x^3}}$$

$$y(x) \to (-1)^{2/3}\sqrt[3]{\frac{3\sqrt{x^4+1}}{2x^3}+\frac{c_1}{x^3}}$$

5.26 problem 26

Internal problem ID [104]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Bernoulli]`

$$\boxed{y^3 + 3y^2y' = \mathrm{e}^{-x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 73

```
dsolve(y(x)^3+3*y(x)^2*diff(y(x),x) = exp(-x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}\left((c_1 + x)\,\mathrm{e}^{2x}\right)^{\frac{1}{3}}$$
$$y(x) = -\frac{\left((c_1 + x)\,\mathrm{e}^{2x}\right)^{\frac{1}{3}}\left(1 + i\sqrt{3}\right)\mathrm{e}^{-x}}{2}$$
$$y(x) = \frac{\left((c_1 + x)\,\mathrm{e}^{2x}\right)^{\frac{1}{3}}\left(i\sqrt{3} - 1\right)\mathrm{e}^{-x}}{2}$$

✓ Solution by Mathematica
Time used: 0.307 (sec). Leaf size: 72

```
DSolve[y[x]^3+3*y[x]^2*y'[x] == Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/3}\sqrt[3]{x + c_1}$$
$$y(x) \to -\sqrt[3]{-1}e^{-x/3}\sqrt[3]{x + c_1}$$
$$y(x) \to (-1)^{2/3}e^{-x/3}\sqrt[3]{x + c_1}$$

5.27 problem 27

Internal problem ID [105]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{3xy^2y' - y^3 = 3x^4}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 57

```
dsolve(3*x*y(x)^2*diff(y(x),x) = 3*x^4+y(x)^3,y(x), singsol=all)
```

$$y(x) = \left(\left(x^3 + c_1\right) x\right)^{\frac{1}{3}}$$
$$y(x) = -\frac{\left(\left(x^3 + c_1\right) x\right)^{\frac{1}{3}} \left(1 + i\sqrt{3}\right)}{2}$$
$$y(x) = \frac{\left(\left(x^3 + c_1\right) x\right)^{\frac{1}{3}} \left(i\sqrt{3} - 1\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.217 (sec). Leaf size: 72

```
DSolve[3*x*y[x]^2*y'[x] == 3*x^4+y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[3]{x}\sqrt[3]{x^3 + c_1}$$
$$y(x) \to -\sqrt[3]{-1}\sqrt[3]{x}\sqrt[3]{x^3 + c_1}$$
$$y(x) \to (-1)^{2/3}\sqrt[3]{x}\sqrt[3]{x^3 + c_1}$$

5.28 problem 28

Internal problem ID [106]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$\boxed{e^{y}xy' - 2\,e^{y} = 2\,e^{2x}x^{3}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(exp(y(x))*x*diff(y(x),x) = 2*exp(y(x))+2*exp(2*x)*x^3,y(x), singsol=all)
```

$$y(x) = \ln\left(x^{2}\left(e^{2x} - c_{1}\right)\right)$$

✓ Solution by Mathematica

Time used: 4.305 (sec). Leaf size: 18

```
DSolve[Exp[y[x]]*x*y'[x] == 2*Exp[y[x]]+2*Exp[2*x]*x^3,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \log\left(x^{2}\left(e^{2x} + c_{1}\right)\right)$$

5.29 problem 29

Internal problem ID [107]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{2x\cos(y)\sin(y)\,y' - \sin(y)^2 = 4x^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 31

```
dsolve(2*x*cos(y(x))*sin(y(x))*diff(y(x),x) = 4*x^2+sin(y(x))^2,y(x), singsol=all)
```

$$y(x) = \arcsin\left(\sqrt{-x(c_1 - 4x)}\right)$$
$$y(x) = -\arcsin\left(\sqrt{-x(c_1 - 4x)}\right)$$

✓ Solution by Mathematica
Time used: 6.406 (sec). Leaf size: 41

```
DSolve[2*x*Cos[y[x]]*Sin[y[x]]*y'[x] == 4*x^2+Sin[y[x]]^2,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\arcsin\left(2\sqrt{x(x + 2c_1)}\right)$$
$$y(x) \to \arcsin\left(2\sqrt{x(x + 2c_1)}\right)$$

5.30 problem 30

Internal problem ID [108]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{(e^{y} + x)\, y' - x\, e^{-y} = -1}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 37

```
dsolve((exp(y(x))+x)*diff(y(x),x) = -1+x/exp(y(x)),y(x), singsol=all)
```

$$y(x) = \ln\left(-x - \sqrt{2x^2 + c_1}\right)$$
$$y(x) = \ln\left(-x + \sqrt{2x^2 + c_1}\right)$$

✓ Solution by Mathematica
Time used: 2.698 (sec). Leaf size: 52

```
DSolve[(Exp[y[x]]+x)*y'[x]== -1+x/Exp[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(-x - \sqrt{2}\sqrt{x^2 + c_1}\right)$$
$$y(x) \to \log\left(-x + \sqrt{2}\sqrt{x^2 + c_1}\right)$$

5.31 problem 31

Internal problem ID [109]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{3y + (3x + 2y)\, y' = -2x}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 53

```
dsolve(2*x+3*y(x)+(3*x+2*y(x))*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{-3c_1x - \sqrt{5c_1^2x^2 + 4}}{2c_1}$$
$$y(x) = \frac{-3c_1x + \sqrt{5c_1^2x^2 + 4}}{2c_1}$$

✓ Solution by Mathematica
Time used: 0.445 (sec). Leaf size: 110

```
DSolve[2*x+3*y[x]+(3*x+2*y[x])*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-3x - \sqrt{5x^2 + 4e^{c_1}}\right)$$
$$y(x) \to \frac{1}{2}\left(-3x + \sqrt{5x^2 + 4e^{c_1}}\right)$$
$$y(x) \to \frac{1}{2}\left(-\sqrt{5}\sqrt{x^2} - 3x\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{5}\sqrt{x^2} - 3x\right)$$

5.32 problem 32

Internal problem ID [110]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{-y + (-x + 6y)\, y' = -4x}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 51

```
dsolve(4*x-y(x)+(-x+6*y(x))*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x - \sqrt{-23c_1^2 x^2 + 12}}{6c_1}$$
$$y(x) = \frac{c_1 x + \sqrt{-23c_1^2 x^2 + 12}}{6c_1}$$

✓ Solution by Mathematica
Time used: 0.446 (sec). Leaf size: 106

```
DSolve[4*x-y[x]+(-x+6*y[x])*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}\left(x - \sqrt{-23x^2 + 12e^{c_1}}\right)$$
$$y(x) \to \frac{1}{6}\left(x + \sqrt{-23x^2 + 12e^{c_1}}\right)$$
$$y(x) \to \frac{1}{6}\left(x - \sqrt{23}\sqrt{-x^2}\right)$$
$$y(x) \to \frac{1}{6}\left(\sqrt{23}\sqrt{-x^2} + x\right)$$

5.33 problem 33

Internal problem ID [111]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$2y^2 + \left(4yx + 6y^2\right) y' = -3x^2$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 405

```
dsolve(3*x^2+2*y(x)^2+(4*x*y(x)+6*y(x)^2)*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\frac{\left(54-62x^3c_1^3+6\sqrt{105c_1^6x^6-186x^3c_1^3+81}\right)^{\frac{1}{3}}}{2} + \frac{2x^2c_1^2}{\left(54-62x^3c_1^3+6\sqrt{105c_1^6x^6-186x^3c_1^3+81}\right)^{\frac{1}{3}}} - c_1 x}{3c_1}$$

$y(x)$

$$= \frac{4i\sqrt{3}\,c_1^2x^2 - i\sqrt{3}\left(54-62x^3c_1^3+6\sqrt{105c_1^6x^6-186x^3c_1^3+81}\right)^{\frac{2}{3}} - 4c_1^2x^2 - 4c_1x\left(54-62x^3c_1^3+6\sqrt{105c \ldots}\right.}{12\left(54-62x^3c_1^3+6\sqrt{105c_1^6x^6-186x^3c_1^3 \ldots}\right.}$$

$y(x)$

$$= \frac{\left(i\sqrt{3}-1\right)\left(54-62x^3c_1^3+6\sqrt{105c_1^6x^6-186x^3c_1^3+81}\right)^{\frac{2}{3}} - 4x\left(ixc_1\sqrt{3}+c_1x+\left(54-62x^3c_1^3+6\sqrt{10 \ldots}\right.\right.}{12\left(54-62x^3c_1^3+6\sqrt{105c_1^6x^6-186x^3c_1^3+81}\right)^{\frac{1}{3}} c_1}$$

✓ Solution by Mathematica

Time used: 39.668 (sec). Leaf size: 679

```
DSolve[3*x^2+2*y[x]^2+(4*x*y[x]+6*y[x]^2)*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{\sqrt[3]{-124x^3 + \sqrt{-256x^6 + (-124x^3 + 108e^{2c_1})^2} + 108e^{2c_1}}}{6\sqrt[3]{2}} + \frac{2\sqrt[3]{2}x^2}{3\sqrt[3]{-124x^3 + \sqrt{-256x^6 + (-124x^3 + 108e^{2c_1})^2} + 108e^{2c_1}}} - \frac{x}{3}$$

$$y(x) \to \frac{1}{12}i\left(\sqrt{3}+i\right)\sqrt[3]{-62x^3 + 6\sqrt{3}\sqrt{35x^6 - 62e^{2c_1}x^3 + 27e^{4c_1}} + 54e^{2c_1}} - \frac{i(\sqrt{3}-i)\,x^2}{3\sqrt[3]{-62x^3 + 6\sqrt{3}\sqrt{35x^6 - 62e^{2c_1}x^3 + 27e^{4c_1}} + 54e^{2c_1}}} - \frac{x}{3}$$

$$y(x) \to -\frac{1}{12}i\left(\sqrt{3}-i\right)\sqrt[3]{-62x^3 + 6\sqrt{3}\sqrt{35x^6 - 62e^{2c_1}x^3 + 27e^{4c_1}} + 54e^{2c_1}} + \frac{i(\sqrt{3}+i)\,x^2}{3\sqrt[3]{-62x^3 + 6\sqrt{3}\sqrt{35x^6 - 62e^{2c_1}x^3 + 27e^{4c_1}} + 54e^{2c_1}}} - \frac{x}{3}$$

$$y(x) \to \frac{1}{6}\left(\sqrt[3]{6\sqrt{105}\sqrt{x^6} - 62x^3} + \frac{2\ 2^{2/3}x^2}{\sqrt[3]{3\sqrt{105}\sqrt{x^6} - 31x^3}} - 2x\right)$$

$$y(x) \to \frac{1}{12}\left(i\left(\sqrt{3}+i\right)\sqrt[3]{6\sqrt{105}\sqrt{x^6} - 62x^3} - \frac{2i2^{2/3}(\sqrt{3}-i)\,x^2}{\sqrt[3]{3\sqrt{105}\sqrt{x^6} - 31x^3}} - 4x\right)$$

$$y(x) \to \frac{1}{12}\left(\left(-1 - i\sqrt{3}\right)\sqrt[3]{6\sqrt{105}\sqrt{x^6} - 62x^3} + \frac{2i2^{2/3}(\sqrt{3}+i)\,x^2}{\sqrt[3]{3\sqrt{105}\sqrt{x^6} - 31x^3}} - 4x\right)$$

5.34 problem 34

Internal problem ID [112]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{2xy^2 + (2x^2y + 4y^3)\, y' = -3x^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 117

```
dsolve(3*x^2+2*x*y(x)^2+(2*x^2*y(x)+4*y(x)^3)*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2x^2 - 2\sqrt{x^4 - 4x^3 - 4c_1}}}{2}$$
$$y(x) = \frac{\sqrt{-2x^2 - 2\sqrt{x^4 - 4x^3 - 4c_1}}}{2}$$
$$y(x) = -\frac{\sqrt{-2x^2 + 2\sqrt{x^4 - 4x^3 - 4c_1}}}{2}$$
$$y(x) = \frac{\sqrt{-2x^2 + 2\sqrt{x^4 - 4x^3 - 4c_1}}}{2}$$

✓ Solution by Mathematica

Time used: 5.897 (sec). Leaf size: 155

```
DSolve[3*x^2+2*x*y[x]^2+(2*x^2*y[x]+4*y[x]^3)*y'[x]== 0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to -\frac{\sqrt{-x^2 - \sqrt{x^4 - 4x^3 + 4c_1}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-x^2 - \sqrt{x^4 - 4x^3 + 4c_1}}}{\sqrt{2}}$$
$$y(x) \to -\frac{\sqrt{-x^2 + \sqrt{x^4 - 4x^3 + 4c_1}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-x^2 + \sqrt{x^4 - 4x^3 + 4c_1}}}{\sqrt{2}}$$

5.35 problem 35

Internal problem ID [113]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{\frac{y}{x}+\left(\ln(x)+y^2\right)y'=-x^3}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 307

```
dsolve(x^3+y(x)/x+(ln(x)+y(x)^2)*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{\left(-3x^4-12c_1+\sqrt{64\ln(x)^3+9\left(x^4+4c_1\right)^2}\right)^{\frac{2}{3}}-4\ln(x)}{2\left(-3x^4-12c_1+\sqrt{64\ln(x)^3+9\left(x^4+4c_1\right)^2}\right)^{\frac{1}{3}}}$$

$$y(x)=\frac{i\left(-\left(-3x^4-12c_1+\sqrt{64\ln(x)^3+9\left(x^4+4c_1\right)^2}\right)^{\frac{2}{3}}-4\ln(x)\right)\sqrt{3}-\left(-3x^4-12c_1+\sqrt{64\ln(x)^3+9\right.}{4\left(-3x^4-12c_1+\sqrt{64\ln(x)^3+9\left(x^4+4c_1\right)^2}\right)^{\frac{1}{3}}}$$

$$y(x)=\frac{i\left(\left(-3x^4-12c_1+\sqrt{64\ln(x)^3+9\left(x^4+4c_1\right)^2}\right)^{\frac{2}{3}}+4\ln(x)\right)\sqrt{3}-\left(-3x^4-12c_1+\sqrt{64\ln(x)^3+9\left(\right.}{4\left(-3x^4-12c_1+\sqrt{64\ln(x)^3+9\left(x^4+4c_1\right)^2}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 1.864 (sec). Leaf size: 307

```
DSolve[x^3+y[x]/x+(Log[x]+y[x]^2)*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-4\log(x) + \left(-3x^4 + \sqrt{64\log^3(x) + 9\left(x^4 - 4c_1\right)^2} + 12c_1\right)^{2/3}}{2\sqrt[3]{-3x^4 + \sqrt{64\log^3(x) + 9\left(x^4 - 4c_1\right)^2} + 12c_1}}$$

$$y(x) \to \frac{i\left(\sqrt{3} + i\right)\left(-3x^4 + \sqrt{64\log^3(x) + 9\left(x^4 - 4c_1\right)^2} + 12c_1\right)^{2/3} + \left(4 + 4i\sqrt{3}\right)\log(x)}{4\sqrt[3]{-3x^4 + \sqrt{64\log^3(x) + 9\left(x^4 - 4c_1\right)^2} + 12c_1}}$$

$$y(x) \to \frac{\left(-1 - i\sqrt{3}\right)\left(-3x^4 + \sqrt{64\log^3(x) + 9\left(x^4 - 4c_1\right)^2} + 12c_1\right)^{2/3} + \left(4 - 4i\sqrt{3}\right)\log(x)}{4\sqrt[3]{-3x^4 + \sqrt{64\log^3(x) + 9\left(x^4 - 4c_1\right)^2} + 12c_1}}$$

5.36 problem 36

Internal problem ID [114]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{e^{yx}y + (e^{yx}x + 2y)\, y' = -1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(1+exp(x*y(x))*y(x)+(exp(x*y(x))*x+2*y(x))*diff(y(x),x) = 0,y(x), singsol=all)
```

$$e^{xy(x)} + x + y(x)^2 + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.216 (sec). Leaf size: 18

```
DSolve[1+Exp[x*y[x]]*y[x]+(Exp[x*y[x]]*x+2*y[x])*y'[x] == 0,y[x],x,IncludeSingularSolutions
```

$$\text{Solve}\left[y(x)^2 + e^{xy(x)} + x = c_1, y(x)\right]$$

5.37 problem 37

Internal problem ID [115]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{\ln(y) + \left(e^{y} + \frac{x}{y}\right) y' = -\cos(x)}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 24

```
dsolve(cos(x)+ln(y(x))+(exp(y(x))+x/y(x))*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = e^{\mathrm{RootOf}\left(e^{-\_Z} - \ln(-x\_Z - c_1 - \sin(x))\right)}$$

✓ Solution by Mathematica

Time used: 0.36 (sec). Leaf size: 18

```
DSolve[Cos[x]+Log[y[x]]+(Exp[y[x]]+x/y[x])*y'[x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

$$\mathrm{Solve}\left[e^{y(x)} + x\log(y(x)) + \sin(x) = c_1, y(x)\right]$$

5.38 problem 38

Internal problem ID [116]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{\arctan(y) + \frac{(x+y)\,y'}{1+y^2} = -x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 22

```
dsolve(x+arctan(y(x))+(x+y(x))*diff(y(x),x)/(1+y(x)^2) = 0,y(x), singsol=all)
```

$$y(x) = \tan\left(\mathrm{RootOf}\left(2x\_Z + x^2 - 2\ln(\cos(\_Z)) + 2c_1\right)\right)$$

✓ Solution by Mathematica
Time used: 0.137 (sec). Leaf size: 30

```
DSolve[x+ArcTan[y[x]]+(x+y[x])*y'[x]/(1+y[x]^2) == 0,y[x],x,IncludeSingularSolutions -> True
```

$$\mathrm{Solve}\left[x\arctan(y(x)) + \frac{x^2}{2} + \frac{1}{2}\log\left(y(x)^2+1\right) = c_1, y(x)\right]$$

5.39 problem 39

Internal problem ID [117]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$3y^3x^2 + y^4 + (3x^3y^2 + 4xy^3 + y^4)\, y' = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(3*x^2*y(x)^3+y(x)^4+(3*x^3*y(x)^2+4*x*y(x)^3+y(x)^4)*diff(y(x),x) = 0,y(x), singsol=a
```

$$y(x) = 0$$
$$xy(x)^4 + x^3y(x)^3 + \frac{y(x)^5}{5} + c_1 = 0$$

✓ Solution by Mathematica
Time used: 33.636 (sec). Leaf size: 171

```
DSolve[3*x^2*y[x]^3+y[x]^4+(3*x^3*y[x]^2+4*x*y[x]^3+y[x]^4)*y'[x] == 0,y[x],x,IncludeSingula
```

$$y(x) \to 0$$
$$y(x) \to \text{Root}\left[\#1^5 + 5\#1^4x + 5\#1^3x^3 - 5c_1\&, 1\right]$$
$$y(x) \to \text{Root}\left[\#1^5 + 5\#1^4x + 5\#1^3x^3 - 5c_1\&, 2\right]$$
$$y(x) \to \text{Root}\left[\#1^5 + 5\#1^4x + 5\#1^3x^3 - 5c_1\&, 3\right]$$
$$y(x) \to \text{Root}\left[\#1^5 + 5\#1^4x + 5\#1^3x^3 - 5c_1\&, 4\right]$$
$$y(x) \to \text{Root}\left[\#1^5 + 5\#1^4x + 5\#1^3x^3 - 5c_1\&, 5\right]$$
$$y(x) \to 0$$

5.40 problem 40

Internal problem ID [118]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$e^{x} \sin(y) + \tan(y) + \left(e^{x} \cos(y) + x \sec(y)^{2}\right) y' = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 153

```
dsolve(exp(x)*sin(y(x))+tan(y(x))+(exp(x)*cos(y(x))+x*sec(y(x))^2)*diff(y(x),x) = 0,y(x), si
```

$$y(x) = \arctan\left(-\frac{c_1 \operatorname{RootOf}\left(\_Z^4 e^{2x} + 2x e^{x} \_Z^3 + \left(c_1^2 + x^2 - e^{2x}\right)\_Z^2 - 2x e^{x} \_Z - x^2\right)}{\operatorname{RootOf}\left(\_Z^4 e^{2x} + 2x e^{x} \_Z^3 + \left(c_1^2 + x^2 - e^{2x}\right)\_Z^2 - 2x e^{x} \_Z - x^2\right) e^{x} + x}, \operatorname{RootOf}\left(\_Z^4 e^{2x} + 2x e^{x} \_Z^3 + \left(c_1^2 + x^2 - e^{2x}\right)\_Z^2 - 2x e^{x} \_Z - x^2\right)\right)$$

✓ Solution by Mathematica
Time used: 60.842 (sec). Leaf size: 5539

```
DSolve[Exp[x]*Sin[y[x]]+Tan[y[x]]+(Exp[x]*Cos[y[x]]+x*Sec[y[x]]^2)*y'[x] == 0,y[x],x,Include
```

Too large to display

5.41 problem 41

Internal problem ID [119]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\frac{2x}{y} - \frac{3y^2}{x^4} + \left(-\frac{x^2}{y^2} + \frac{1}{\sqrt{y}} + \frac{2y}{x^3}\right) y' = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 35

```
dsolve(2*x/y(x)-3*y(x)^2/x^4+(-x^2/y(x)^2+1/y(x)^(1/2)+2*y(x)/x^3)*diff(y(x),x) = 0,y(x), si
```

$$\frac{2y(x)^{\frac{3}{2}} x^3 + c_1 x^3 y(x) + x^5 + y(x)^3}{x^3 y(x)} = 0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[2*x/y[x]-3*y[x]^2/x^4+(-x^2/y[x]^2+1/y[x]^(1/2)+2*y[x]/x^3)*y'[x] == 0,y[x],x,Include
```

Not solved

5.42 problem 42

Internal problem ID [120]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 1.6, Substitution methods and exact equations. Page 74
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _exact, _rational]`

$$\frac{2x^{\frac{5}{2}}-3y^{\frac{5}{3}}}{2x^{\frac{5}{2}}y^{\frac{2}{3}}}+\frac{\left(-2x^{\frac{5}{2}}+3y^{\frac{5}{3}}\right)y'}{3x^{\frac{3}{2}}y^{\frac{5}{3}}}=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 185

```
dsolve(1/2*(2*x^(5/2)-3*y(x)^(5/3))/x^(5/2)/y(x)^(2/3)+1/3*(-2*x^(5/2)+3*y(x)^(5/3))*diff(y(
```

$$y(x)=\frac{2^{\frac{3}{5}}3^{\frac{2}{5}}\left(x^{\frac{5}{2}}\right)^{\frac{3}{5}}}{3}$$

$$y(x)=-\frac{\left(i\sqrt{2}\,\sqrt{5-\sqrt{5}}+\sqrt{5}+1\right)^{3}2^{\frac{3}{5}}3^{\frac{2}{5}}\left(x^{\frac{5}{2}}\right)^{\frac{3}{5}}}{192}$$

$$y(x)=\frac{\left(i\sqrt{2}\,\sqrt{5-\sqrt{5}}-\sqrt{5}-1\right)^{3}2^{\frac{3}{5}}3^{\frac{2}{5}}\left(x^{\frac{5}{2}}\right)^{\frac{3}{5}}}{192}$$

$$y(x)=-\frac{\left(i\sqrt{2}\,\sqrt{5+\sqrt{5}}-\sqrt{5}+1\right)^{3}2^{\frac{3}{5}}3^{\frac{2}{5}}\left(x^{\frac{5}{2}}\right)^{\frac{3}{5}}}{192}$$

$$y(x)=\frac{\left(i\sqrt{2}\,\sqrt{5+\sqrt{5}}+\sqrt{5}-1\right)^{3}2^{\frac{3}{5}}3^{\frac{2}{5}}\left(x^{\frac{5}{2}}\right)^{\frac{3}{5}}}{192}$$

$$\frac{x}{y\left(x\right)^{\frac{2}{3}}}+\frac{y(x)}{x^{\frac{3}{2}}}+c_1=0$$

✓ Solution by Mathematica

Time used: 0.078 (sec). Leaf size: 260

```
DSolve[1/2*(2*x^(5/2)-3*y[x]^(5/3))/x^(5/2)/y[x]^(2/3)+1/3*(-2*x^(5/2)+3*y[x]^(5/3))*y'[x]/x
```

$$y(x)\to \left(\frac{2}{3}\right)^{3/5}\left(x^{5/2}\right)^{3/5}$$
$$y(x)\to c_1x^{3/2}$$
$$y(x)\to -\left(-\frac{2}{3}\right)^{3/5}x^{3/2}$$
$$y(x)\to \left(-\frac{2}{3}\right)^{3/5}x^{3/2}$$
$$y(x)\to -\left(\frac{2}{3}\right)^{3/5}x^{3/2}$$
$$y(x)\to \left(\frac{2}{3}\right)^{3/5}x^{3/2}$$
$$y(x)\to -\sqrt[5]{-1}\left(\frac{2}{3}\right)^{3/5}x^{3/2}$$
$$y(x)\to \sqrt[5]{-1}\left(\frac{2}{3}\right)^{3/5}x^{3/2}$$
$$y(x)\to -(-1)^{2/5}\left(\frac{2}{3}\right)^{3/5}x^{3/2}$$
$$y(x)\to (-1)^{2/5}\left(\frac{2}{3}\right)^{3/5}x^{3/2}$$
$$y(x)\to -(-1)^{4/5}\left(\frac{2}{3}\right)^{3/5}x^{3/2}$$
$$y(x)\to (-1)^{4/5}\left(\frac{2}{3}\right)^{3/5}x^{3/2}$$
$$y(x)\to \left(\frac{2}{3}\right)^{3/5}\left(x^{5/2}\right)^{3/5}$$

6 Chapter 1 review problems. Page 78

6.1 problem 1

Internal problem ID [121]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{3y - y'x = -x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x^3+3*y(x)-x*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = (\ln(x) + c_1)\,x^3$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 14

```
DSolve[x^3+3*y[x]-x*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^3(\log(x) + c_1)$$

6.2 problem 2

Internal problem ID [122]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{3y^2 + xy^2 - y'x^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(3*y(x)^2+x*y(x)^2-x^2*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{x}{3 - x\ln(x) + c_1x}$$

✓ Solution by Mathematica
Time used: 0.13 (sec). Leaf size: 25

```
DSolve[3*y[x]^2+x*y[x]^2-x^2*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{x\log(x) + c_1x - 3}$$
$$y(x) \to 0$$

6.3 problem 3

Internal problem ID [123]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{yx + y^2 - y'x^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x*y(x)+y(x)^2-x^2*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{x}{c_1 - \ln(x)}$$

✓ Solution by Mathematica
Time used: 0.124 (sec). Leaf size: 21

```
DSolve[x*y[x]+y[x]^2-x^2*y'[x]== 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{-\log(x) + c_1}$$
$$y(x) \to 0$$

6.4 problem 4

Internal problem ID [124]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{2xy^3 + \left(\sin\left(y\right) + 3x^2y^2\right) y' = -\mathrm{e}^x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(exp(x)+2*x*y(x)^3+(sin(y(x))+3*x^2*y(x)^2)*diff(y(x),x) = 0,y(x), singsol=all)
```

$$x^2y(x)^3 + \mathrm{e}^x - \cos\left(y(x)\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.255 (sec). Leaf size: 23

```
DSolve[Exp[x]+2*x*y[x]^3+(Sin[y[x]]+3*x^2*y[x]^2)*y'[x]== 0,y[x],x,IncludeSingularSolutions
```

$$\text{Solve}\left[x^2y(x)^3 - \cos(y(x)) + e^x = c_1, y(x)\right]$$

6.5 problem 5

Internal problem ID [125]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{3y + x^4 y' - 2yx = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 16

```
dsolve(3*y(x)+x^4*diff(y(x),x) = 2*x*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{1-x}{x^3}}$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 24

```
DSolve[3*y[x]+x^4*y'[x] == 2*x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{\frac{1-x}{x^3}}$$
$$y(x) \to 0$$

6.6 problem 6

Internal problem ID [126]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2xy^2 + y'x^2 - y^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(2*x*y(x)^2+x^2*diff(y(x),x) = y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x}{1 + 2x \ln(x) + c_1 x}$$

✓ Solution by Mathematica
Time used: 0.131 (sec). Leaf size: 26

```
DSolve[2*x*y[x]^2+x^2*y'[x] == y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{2x \log(x) + c_1(-x) + 1}$$
$$y(x) \to 0$$

6.7 problem 7

Internal problem ID [127]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2x^2y + x^3y' = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(2*x^2*y(x)+x^3*diff(y(x),x) = 1,y(x), singsol=all)
```

$$y(x) = \frac{\ln(x) + c_1}{x^2}$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 14

```
DSolve[2*x^2*y[x]+x^3*y'[x] == 1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\log(x) + c_1}{x^2}$$

6.8 problem 8

Internal problem ID [128]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{2yx + y'x^2 - y^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(2*x*y(x)+x^2*diff(y(x),x) = y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{3x}{3c_1x^3 + 1}$$

✓ Solution by Mathematica
Time used: 0.122 (sec). Leaf size: 24

```
DSolve[2*x*y[x]+x^2*y'[x] == y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3x}{1 + 3c_1x^3}$$
$$y(x) \to 0$$

6.9 problem 9

Internal problem ID [129]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{2y + y'x - 6x^2\sqrt{y} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(2*y(x)+x*diff(y(x),x) = 6*x^2*y(x)^(1/2),y(x), singsol=all)
```

$$\frac{-x^3 + \sqrt{y(x)}\,x - c_1}{x} = 0$$

✓ Solution by Mathematica
Time used: 0.15 (sec). Leaf size: 17

```
DSolve[2*y[x]+x*y'[x] == 6*x^2*y[x]^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(x^3 + c_1)^2}{x^2}$$

6.10 problem 10

Internal problem ID [130]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y^2 - x^2y^2 = x^2 + 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x) = 1+x^2+y(x)^2+x^2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \tan\left(\frac{1}{3}x^3 + c_1 + x\right)$$

✓ Solution by Mathematica
Time used: 0.186 (sec). Leaf size: 17

```
DSolve[y'[x] == 1+x^2+y[x]^2+x^2*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan\left(\frac{x^3}{3} + x + c_1\right)$$

6.11 problem 11

Internal problem ID [131]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y'x^2 - yx - 3y^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x^2*diff(y(x),x) = x*y(x)+3*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x}{-3\ln(x) + c_1}$$

✓ Solution by Mathematica
Time used: 0.124 (sec). Leaf size: 21

```
DSolve[x^2*y'[x] == x*y[x]+3*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{-3\log(x) + c_1}$$
$$y(x) \to 0$$

6.12 problem 12

Internal problem ID [132]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{6xy^3 + 2y^4 + \left(9x^2y^2 + 8xy^3\right) y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(6*x*y(x)^3+2*y(x)^4+(9*x^2*y(x)^2+8*x*y(x)^3)*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = 0$$
$$3x^2y(x)^3 + 2xy(x)^4 + c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.142 (sec). Leaf size: 1714

```
DSolve[6*x*y[x]^3+2*y[x]^4+(9*x^2*y[x]^2+8*x*y[x]^3)*y'[x] == 0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to 0$$

$$y(x) \to \frac{1}{2}\sqrt{\frac{9x^2}{16} - \frac{4\sqrt[3]{\frac{2}{3}}e^{c_1}}{\sqrt[3]{\sqrt{3}\sqrt{e^{2c_1}x^3\left(2187x^5+2048e^{c_1}\right)}-81e^{c_1}x^4}} + \frac{\sqrt[3]{\sqrt{3}\sqrt{e^{2c_1}x^3\left(2187x^5+2048e^{c_1}\right)}-81e^{c_1}x^4}}{2\sqrt[3]{2}3^{2/3}x}} - \frac{1}{2}\sqrt{\frac{9x^2}{8} + \frac{4\sqrt[3]{\frac{2}{3}}e^{c_1}}{\sqrt[3]{\sqrt{3}\sqrt{e^{2c_1}x^3\left(2187x^5+2048e^{c_1}\right)}-81e^{c_1}x^4}} - \frac{\sqrt[3]{\sqrt{3}\sqrt{e^{2c_1}x^3\left(2187x^5+2048e^{c_1}\right)}-81e^{c_1}x}}{2\sqrt[3]{2}3^{2/3}x}} - \frac{3x}{8}$$

$$y(x) \to \frac{1}{2}\sqrt{\frac{9x^2}{16} - \frac{4\sqrt[3]{\frac{2}{3}}e^{c_1}}{\sqrt[3]{\sqrt{3}\sqrt{e^{2c_1}x^3\left(2187x^5+2048e^{c_1}\right)}-81e^{c_1}x^4}} + \frac{\sqrt[3]{\sqrt{3}\sqrt{e^{2c_1}x^3\left(2187x^5+2048e^{c_1}\right)}-81e^{c_1}x^4}}{2\sqrt[3]{2}3^{2/3}x}} + \frac{1}{2}\sqrt{\frac{9x^2}{8} + \frac{4\sqrt[3]{\frac{2}{3}}e^{c_1}}{\sqrt[3]{\sqrt{3}\sqrt{e^{2c_1}x^3\left(2187x^5+2048e^{c_1}\right)}-81e^{c_1}x^4}} - \frac{\sqrt[3]{\sqrt{3}\sqrt{e^{2c_1}x^3\left(2187x^5+2048e^{c_1}\right)}-81e^{c_1}x}}{2\sqrt[3]{2}3^{2/3}x}} - \frac{3x}{8}$$

$$y(x) \to -\frac{1}{2}\sqrt{\frac{9x^2}{16} - \frac{4\sqrt[3]{\frac{2}{3}}e^{c_1}}{\sqrt[3]{\sqrt{3}\sqrt{e^{2c_1}x^3\left(2187x^5+2048e^{c_1}\right)}-81e^{c_1}x^4}} + \frac{\sqrt[3]{\sqrt{3}\sqrt{e^{2c_1}x^3\left(2187x^5+2048e^{c_1}\right)}-81e^{c_1}x}}{2\sqrt[3]{2}3^{2/3}x}} - \frac{1}{2}\sqrt{\frac{9x^2}{8} + \frac{4\sqrt[3]{\frac{2}{3}}e^{c_1}}{\sqrt[3]{\sqrt{3}\sqrt{e^{2c_1}x^3\left(2187x^5+2048e^{c_1}\right)}-81e^{c_1}x^4}} - \frac{\sqrt[3]{\sqrt{3}\sqrt{e^{2c_1}x^3\left(2187x^5+2048e^{c_1}\right)}-81e^{c_1}x}}{2\sqrt[3]{2}3^{2/3}x}} - \frac{3x}{8}$$

6.13 problem 13

Internal problem ID [133]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - y^2 - x^2y^4 = x^2 + 1}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) = 1+x^2+y(x)^2+x^2*y(x)^4,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x] == 1+x^2+y[x]^2+x^2*y[x]^4,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

6.14 problem 14

Internal problem ID [134]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$x^3y' - x^2y + y^3 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 28

```
dsolve(x^3*diff(y(x),x) = x^2*y(x)-y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{x}{\sqrt{2\ln(x) + c_1}}$$
$$y(x) = -\frac{x}{\sqrt{2\ln(x) + c_1}}$$

✓ Solution by Mathematica

Time used: 0.166 (sec). Leaf size: 41

```
DSolve[x^3*y'[x] == x^2*y[x]-y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{\sqrt{2\log(x) + c_1}}$$
$$y(x) \to \frac{x}{\sqrt{2\log(x) + c_1}}$$
$$y(x) \to 0$$

6.15 problem 15

Internal problem ID [135]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{3y + y' = 3x^2 e^{-3x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(3*y(x)+diff(y(x),x) = 3*x^2/exp(3*x),y(x), singsol=all)
```

$$y(x) = \left(x^3 + c_1\right) e^{-3x}$$

✓ Solution by Mathematica
Time used: 0.057 (sec). Leaf size: 17

```
DSolve[3*y[x]+y'[x] == 3*x^2/Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(x^3 + c_1\right)$$

6.16 problem 16

Internal problem ID [136]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Riccati]`

$$\boxed{2yx + y' - y^2 = x^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 29

```
dsolve(diff(y(x),x) = x^2-2*x*y(x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x-1)\,e^{2x} - x - 1}{-1 + e^{2x}c_1}$$

✓ Solution by Mathematica
Time used: 0.127 (sec). Leaf size: 29

```
DSolve[y'[x] == x^2-2*x*y[x]+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + \frac{1}{\frac{1}{2} + c_1e^{2x}} - 1$$
$$y(x) \to x - 1$$

6.17 problem 17

Internal problem ID [137]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{e^{yx}y + (e^{y} + e^{yx}x)\, y' = -e^{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(exp(x)+exp(x*y(x))*y(x)+(exp(y(x))+exp(x*y(x))*x)*diff(y(x),x) = 0,y(x), singsol=all)
```

$$e^{xy(x)} + e^{x} + e^{y(x)} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.214 (sec). Leaf size: 20

```
DSolve[Exp[x]+Exp[x*y[x]]*y[x]+(Exp[y[x]]+Exp[x*y[x]]*x)*y'[x] == 0,y[x],x,IncludeSingularSo
```

$$\text{Solve}\left[e^{y(x)} + e^{xy(x)} + e^{x} = c_1, y(x)\right]$$

6.18 problem 18

Internal problem ID [138]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{2x^2y - x^3y' - y^3 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(2*x^2*y(x)-x^3*diff(y(x),x) = y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{\sqrt{x^2+c_1}}$$
$$y(x) = -\frac{x^2}{\sqrt{x^2+c_1}}$$

✓ Solution by Mathematica
Time used: 0.194 (sec). Leaf size: 43

```
DSolve[2*x^2*y[x]-x^3*y'[x] == y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2}{\sqrt{x^2+c_1}}$$
$$y(x) \to \frac{x^2}{\sqrt{x^2+c_1}}$$
$$y(x) \to 0$$

6.19 problem 19

Internal problem ID [139]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{3y^2x^5 + x^3y' - 2y^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(3*x^5*y(x)^2+x^3*diff(y(x),x) = 2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{x^5 + c_1 x^2 + 1}$$

✓ Solution by Mathematica
Time used: 0.143 (sec). Leaf size: 28

```
DSolve[3*x^5*y[x]^2+x^3*y'[x] == 2*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{x^5 - c_1 x^2 + 1}$$
$$y(x) \to 0$$

6.20 problem 20

Internal problem ID [140]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{3y + y'x = \frac{3}{x^{\frac{3}{2}}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(3*y(x)+x*diff(y(x),x) = 3/x^(3/2),y(x), singsol=all)
```

$$y(x) = \frac{2x^{\frac{3}{2}} + c_1}{x^3}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 19

```
DSolve[3*y[x]+x*y'[x]== 3/x^(3/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x^{3/2} + c_1}{x^3}$$

6.21 problem 21

Internal problem ID [141]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y(x-1) + \left(x^2-1\right) y' = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve((-1+x)*y(x)+(x^2-1)*diff(y(x),x) = 1,y(x), singsol=all)
```

$$y(x) = \frac{\ln\left(x-1\right)+c_1}{x+1}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 18

```
DSolve[(-1+x)*y[x]+(x^2-1)*y'[x] == 1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\log(x-1)+c_1}{x+1}$$

6.22 problem 22

Internal problem ID [142]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$\boxed{y'x - 12x^4 y^{\frac{2}{3}} - 6y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x) = 12*x^4*y(x)^(2/3)+6*y(x),y(x), singsol=all)
```

$$-2x^4 - c_1 x^2 + y(x)^{\frac{1}{3}} = 0$$

✓ Solution by Mathematica

Time used: 0.161 (sec). Leaf size: 19

```
DSolve[x*y'[x]== 12*x^4*y[x]^(2/3)+6*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^6 (2x^2 + c_1)^3$$

6.23 problem 23

Internal problem ID [143]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{e^y + \cos(x)\, y + (e^y x + \sin(x))\, y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve(exp(y(x))+cos(x)*y(x)+(exp(y(x))*x+sin(x))*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = -\operatorname{LambertW}\left(\csc(x)\, e^{-\csc(x) c_1} x\right) - \csc(x)\, c_1$$

✓ Solution by Mathematica
Time used: 4.553 (sec). Leaf size: 25

```
DSolve[Exp[y[x]]+Cos[x]*y[x]+(Exp[y[x]]*x+Sin[x])*y'[x] == 0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to c_1 \csc(x) - W\left(x \csc(x) e^{c_1 \csc(x)}\right)$$

6.24 problem 24

Internal problem ID [144]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{9x^2y^2 + x^{\frac{3}{2}}y' - y^2 = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 22

```
dsolve(9*x^2*y(x)^2+x^(3/2)*diff(y(x),x) = y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{x}}{2 + 6x^2 + c_1\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.168 (sec). Leaf size: 34

```
DSolve[9*x^2*y[x]^2+x^(3/2)*y'[x] == y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{x}}{6x^2 - c_1\sqrt{x} + 2}$$
$$y(x) \to 0$$

6.25 problem 25

Internal problem ID [145]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2y + (x+1)\, y' = 3 + 3x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(2*y(x)+(1+x)*diff(y(x),x) = 3+3*x,y(x), singsol=all)
```

$$y(x) = x + 1 + \frac{c_1}{(x+1)^2}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 25

```
DSolve[2*y[x]+(1+x)*y'[x] == 3+3*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3 + 3x^2 + 3x + c_1}{(x+1)^2}$$

6.26 problem 26

Internal problem ID [146]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _exact, _rational]`

$$9\sqrt{x}\, y^{\frac{4}{3}} - 12x^{\frac{1}{5}} y^{\frac{3}{2}} + \left(8x^{\frac{3}{2}} y^{\frac{1}{3}} - 15x^{\frac{6}{5}}\sqrt{y}\right) y' = 0$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 43

```
dsolve(9*x^(1/2)*y(x)^(4/3)-12*x^(1/5)*y(x)^(3/2)+(8*x^(3/2)*y(x)^(1/3)-15*x^(6/5)*y(x)^(1/2
```

$$125y(x)^{\frac{9}{2}} x^{\frac{18}{5}} - 225y(x)^{\frac{13}{3}} x^{\frac{39}{10}} + 135y(x)^{\frac{25}{6}} x^{\frac{21}{5}} - 27y(x)^{4} x^{\frac{9}{2}} - c_1 = 0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[9*x^(1/2)*y[x]^(4/3)-12*x^(1/5)*y[x]^(3/2)+(8*x^(3/2)*y[x]^(1/3)-15*x^(6/5)*y[x]^(1/2
```

Timed out

6.27 problem 27

Internal problem ID [147]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{3y + x^3y^4 + 3y'x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 58

```
dsolve(3*y(x)+x^3*y(x)^4+3*x*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{(\ln(x) + c_1)^{\frac{1}{3}}\, x}$$
$$y(x) = -\frac{1 + i\sqrt{3}}{2\,(\ln(x) + c_1)^{\frac{1}{3}}\, x}$$
$$y(x) = \frac{i\sqrt{3} - 1}{2\,(\ln(x) + c_1)^{\frac{1}{3}}\, x}$$

✓ Solution by Mathematica

Time used: 0.437 (sec). Leaf size: 70

```
DSolve[3*y[x]+x^3*y[x]^4+3*x*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{\sqrt[3]{x^3(\log(x) + c_1)}}$$
$$y(x) \to -\frac{\sqrt[3]{-1}}{\sqrt[3]{x^3(\log(x) + c_1)}}$$
$$y(x) \to \frac{(-1)^{2/3}}{\sqrt[3]{x^3(\log(x) + c_1)}}$$
$$y(x) \to 0$$

6.28 problem 28

Internal problem ID [148]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y + y'x = 2\,e^{2x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(y(x)+x*diff(y(x),x) = 2*exp(2*x),y(x), singsol=all)
```

$$y(x) = \frac{e^{2x} + c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 17

```
DSolve[y[x]+x*y'[x] == 2*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{2x} + c_1}{x}$$

6.29 problem 29

Internal problem ID [149]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y + (1+2x)\, y' = (1+2x)^{\frac{3}{2}}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(y(x)+(1+2*x)*diff(y(x),x) = (1+2*x)^(3/2),y(x), singsol=all)
```

$$y(x) = \frac{x^2 + c_1 + x}{\sqrt{1+2x}}$$

✓ Solution by Mathematica

Time used: 0.066 (sec). Leaf size: 43

```
DSolve[y[x]+(1+2*x)*y'[x] == (1+2*x)^(3/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\frac{x\sqrt{-(2x+1)^2(x+1)}}{2x+1} + c_1}{\sqrt{-2x-1}}$$

6.30 problem 31(a)

Internal problem ID [150]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 31(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 3x^2(7 + y) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x) = 3*x^2*(7+y(x)),y(x), singsol=all)
```

$$y(x) = -7 + e^{x^3} c_1$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 20

```
DSolve[y'[x] == 3*x^2*(7+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -7 + c_1 e^{x^3}$$
$$y(x) \to -7$$

6.31 problem 31 (b)

Internal problem ID [151]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 31 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 3x^2(7 + y) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x) = 3*x^2*(7+y(x)),y(x), singsol=all)
```

$$y(x) = -7 + \mathrm{e}^{x^3} c_1$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 20

```
DSolve[y'[x] == 3*x^2*(7+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -7 + c_1 e^{x^3}$$
$$y(x) \to -7$$

6.32 problem 32 (b)

Internal problem ID [152]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 32 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + yx - xy^3 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x) = -x*y(x)+x*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{e^{x^2}c_1 + 1}}$$
$$y(x) = -\frac{1}{\sqrt{e^{x^2}c_1 + 1}}$$

✓ Solution by Mathematica
Time used: 1.917 (sec). Leaf size: 58

```
DSolve[y'[x] == -x*y[x]+x*y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{1 + e^{x^2+2c_1}}}$$
$$y(x) \to \frac{1}{\sqrt{1 + e^{x^2+2c_1}}}$$
$$y(x) \to -1$$
$$y(x) \to 0$$
$$y(x) \to 1$$

6.33 problem 33 (a)

Internal problem ID [153]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 33 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$y' - \frac{-3x^2 - 2y^2}{4yx} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(diff(y(x),x) = 1/4*(-3*x^2-2*y(x)^2)/(x*y(x)),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{2}\,\sqrt{-x\,(x^3 - 2c_1)}}{2x}$$
$$y(x) = \frac{\sqrt{2}\,\sqrt{-x\,(x^3 - 2c_1)}}{2x}$$

✓ Solution by Mathematica

Time used: 0.236 (sec). Leaf size: 60

```
DSolve[y'[x] == 1/4*(-3*x^2-2*y[x]^2)/(x*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-x^3 + 2c_1}}{\sqrt{2}\sqrt{x}}$$
$$y(x) \to \frac{\sqrt{-x^3 + 2c_1}}{\sqrt{2}\sqrt{x}}$$

6.34 problem 34 (a)

Internal problem ID [154]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 34 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{x + 3y}{-3x + y} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
dsolve(diff(y(x),x) = (x+3*y(x))/(-3*x+y(x)),y(x), singsol=all)
```

$$y(x) = \frac{3c_1x - \sqrt{10c_1^2x^2 + 1}}{c_1}$$
$$y(x) = \frac{3c_1x + \sqrt{10c_1^2x^2 + 1}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.482 (sec). Leaf size: 94

```
DSolve[y'[x] == (x+3*y[x])/(-3*x+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 3x - \sqrt{10x^2 + e^{2c_1}}$$
$$y(x) \rightarrow 3x + \sqrt{10x^2 + e^{2c_1}}$$
$$y(x) \rightarrow 3x - \sqrt{10}\sqrt{x^2}$$
$$y(x) \rightarrow \sqrt{10}\sqrt{x^2} + 3x$$

6.35 problem 35 (a)

Internal problem ID [155]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 35 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{2x + 2yx}{x^2 + 1} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve(diff(y(x),x) = (2*x+2*x*y(x))/(x^2+1),y(x), singsol=all)
```

$$y(x) = c_1 x^2 + c_1 - 1$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 20

```
DSolve[y'[x] == (2*x+2*x*y[x])/(x^2+1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -1 + c_1 (x^2 + 1)$$
$$y(x) \to -1$$

6.36 problem 36 (a)

Internal problem ID [156]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Chapter 1 review problems. Page 78
Problem number: 36 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \cot(x)\left(\sqrt{y} - y\right) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x) = cot(x)*(y(x)^(1/2)-y(x)),y(x), singsol=all)
```

$$\sqrt{y(x)} - \frac{\int \frac{\cos(x)}{\sqrt{\sin(x)}}dx + 2c_1}{2\sqrt{\sin(x)}} = 0$$

✓ Solution by Mathematica
Time used: 0.272 (sec). Leaf size: 35

```
DSolve[y'[x] == Cot[x]*(y[x]^(1/2)-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \csc(x)\left(\sqrt{\sin(x)} + e^{\frac{c_1}{2}}\right)^2$$
$$y(x) \to 0$$
$$y(x) \to 1$$

7 Section 5.1, second order linear equations. Page 299

7.1 problem 1

Internal problem ID [157]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 5]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(x),x$2)-y(x)=0,y(0) = 0, D(y)(0) = 5],y(x), singsol=all)
```

$$y(x) = \frac{5\,\mathrm{e}^{x}}{2} - \frac{5\,\mathrm{e}^{-x}}{2}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 21

```
DSolve[{y''[x]-y[x]==0,{y[0]==0,y'[0]==5}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{5}{2}e^{-x}\left(e^{2x} - 1\right)$$

7.2 problem 2

Internal problem ID [158]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 9y = 0}$$

With initial conditions

$$[y(0) = -1, y'(0) = 15]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)-9*y(x)=0,y(0) = -1, D(y)(0) = 15],y(x), singsol=all)
```

$$y(x) = -3\,\mathrm{e}^{-3x} + 2\,\mathrm{e}^{3x}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 20

```
DSolve[{y''[x]-9*y[x]==0,{y[0]==-1,y'[0]==15}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(2e^{6x} - 3\right)$$

7.3 problem 3

Internal problem ID [159]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = 8]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)+4*y(x)=0,y(0) = 3, D(y)(0) = 8],y(x), singsol=all)
```

$$y(x) = 4\sin(2x) + 3\cos(2x)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 18

```
DSolve[{y''[x]+4*y[x]==0,{y[0]==3,y'[0]==8}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 4\sin(2x) + 3\cos(2x)$$

7.4 problem 4

Internal problem ID [160]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 25y = 0}$$

With initial conditions

$$[y(0) = 10, y'(0) = -10]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)+25*y(x)=0,y(0) = 10, D(y)(0) = -10],y(x), singsol=all)
```

$$y(x) = -2\sin(5x) + 10\cos(5x)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 18

```
DSolve[{y''[x]+25*y[x]==0,{y[0]==10,y'[0]==-10}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 10\cos(5x) - 2\sin(5x)$$

7.5 problem 5

Internal problem ID [161]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 3y' + 2y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = 2\,e^{x} - e^{2x}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 15

```
DSolve[{y''[x]-3*y'[x]+2*y[x]==0,{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -e^{x}(e^{x} - 2)$$

7.6 problem 6

Internal problem ID [162]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - 6y = 0}$$

With initial conditions

$$[y(0) = 7, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)+diff(y(x),x)-6*y(x)=0,y(0) = 7, D(y)(0) = -1],y(x), singsol=all)
```

$$y(x) = \left(4\,\mathrm{e}^{5x} + 3\right)\mathrm{e}^{-3x}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 20

```
DSolve[{y''[x]+y'[x]-6*y[x]==0,{y[0]==7,y'[0]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(4e^{5x} + 3\right)$$

7.7 problem 7

Internal problem ID [163]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' = 0}$$

With initial conditions

$$[y(0) = -2, y'(0) = 8]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve([diff(y(x),x$2)+diff(y(x),x)=0,y(0) = -2, D(y)(0) = 8],y(x), singsol=all)
```

$$y(x) = 6 - 8\,\mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 14

```
DSolve[{y''[x]+y'[x]==0,{y[0]==-2,y'[0]==8}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 6 - 8e^{-x}$$

7.8 problem 8

Internal problem ID [164]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 3y' = 0}$$

With initial conditions

$$[y(0) = 4, y'(0) = -2]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 12

```
dsolve([diff(y(x),x$2)-3*diff(y(x),x)=0,y(0) = 4, D(y)(0) = -2],y(x), singsol=all)
```

$$y(x) = \frac{14}{3} - \frac{2\,e^{3x}}{3}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 16

```
DSolve[{y''[x]-3*y'[x]==0,{y[0]==4,y'[0]==-2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{3}\left(e^{3x} - 7\right)$$

7.9 problem 9

Internal problem ID [165]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve([diff(y(x),x$2)+2*diff(y(x),x)+y(x)=0,y(0) = 2, D(y)(0) = -1],y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}(2 + x)$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 14

```
DSolve[{y''[x]+2*y'[x]+y[x]==0,{y[0]==2,y'[0]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(x + 2)$$

7.10 problem 10

Internal problem ID [166]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 10y' + 25y = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = 13]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve([diff(y(x),x$2)-10*diff(y(x),x)+25*y(x)=0,y(0) = 3, D(y)(0) = 13],y(x), singsol=all)
```

$$y(x) = e^{5x}(3 - 2x)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 16

```
DSolve[{y''[x]-10*y'[x]+25*y[x]==0,{y[0]==3,y'[0]==13}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to e^{5x}(3 - 2x)$$

7.11 problem 11

Internal problem ID [167]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + 2y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 5]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+2*y(x)=0,y(0) = 0, D(y)(0) = 5],y(x), singsol=all)
```

$$y(x) = 5\sin(x)\,\mathrm{e}^{x}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 12

```
DSolve[{y''[x]-2*y'[x]+2*y[x]==0,{y[0]==0,y'[0]==5}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to 5e^{x}\sin(x)$$

7.12 problem 12

Internal problem ID [168]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 6y' + 13y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve([diff(y(x),x$2)+6*diff(y(x),x)+13*y(x)=0,y(0) = 2, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = e^{-3x}(3\sin(2x) + 2\cos(2x))$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 24

```
DSolve[{y''[x]+6*y'[x]+13*y[x]==0,{y[0]==2,y'[0]==0}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^{-3x}(3\sin(2x) + 2\cos(2x))$$

7.13 problem 13

Internal problem ID [169]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' - 2y'x + 2y = 0}$$

With initial conditions

$$[y(1) = 3, y'(1) = 1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve([x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(1) = 3, D(y)(1) = 1],y(x), singsol=al
```

$$y(x) = -2x^2 + 5x$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 12

```
DSolve[{x^2*y''[x]-2*x*y'[x]+2*y[x]==0,{y[1]==3,y'[1]==1}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to (5 - 2x)x$$

7.14 problem 14

Internal problem ID [170]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$x^2y'' + 2y'x - 6y = 0$$

With initial conditions

$$[y(2) = 10, y'(2) = 15]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-6*y(x)=0,y(2) = 10, D(y)(2) = 15],y(x), singsol=
```

$$y(x) = -\frac{16}{x^3} + 3x^2$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 16

```
DSolve[{x^2*y''[x]+2*x*y'[x]-6*y[x]==0,{y[2]==10,y'[2]==15}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{3x^5 - 16}{x^3}$$

7.15 problem 15

Internal problem ID [171]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - y'x + y = 0}$$

With initial conditions

$$[y(1) = 7, y'(1) = 2]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve([x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(1) = 7, D(y)(1) = 2],y(x), singsol=all)
```

$$y(x) = x(7 - 5\ln(x))$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 13

```
DSolve[{x^2*y''[x]-x*y'[x]+y[x]==0,{y[1]==7,y'[1]==2}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to x(7 - 5\log(x))$$

7.16 problem 16

Internal problem ID [172]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$x^2y'' + y'x + y = 0$$

With initial conditions

$$[y(1) = 2, y'(1) = 3]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(1) = 2, D(y)(1) = 3],y(x), singsol=all)
```

$$y(x) = 3\sin(\ln(x)) + 2\cos(\ln(x))$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 16

```
DSolve[{x^2*y''[x]+x*y'[x]+y[x]==0,{y[1]==2,y'[1]==3}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to 3\sin(\log(x)) + 2\cos(\log(x))$$

7.17 problem 33

Internal problem ID [173]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 3y' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x} c_1 + c_2 \mathrm{e}^{2x}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 18

```
DSolve[y''[x]-3*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x (c_2 e^x + c_1)$$

7.18 problem 34

Internal problem ID [174]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' - 15y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)-15*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_2 \mathrm{e}^{8x} + c_1\right) \mathrm{e}^{-5x}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 22

```
DSolve[y''[x]+2*y'[x]-15*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-5x}\left(c_2 e^{8x} + c_1\right)$$

7.19 problem 35

Internal problem ID [175]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 5y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x$2)+5*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 e^{-5x}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 19

```
DSolve[y''[x]+5*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{1}{5} c_1 e^{-5x}$$

7.20 problem 36

Internal problem ID [176]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2y'' + 3y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(2*diff(y(x),x$2)+3*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 \mathrm{e}^{-\frac{3x}{2}}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 21

```
DSolve[2*y''[x]+3*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{2}{3} c_1 e^{-3x/2}$$

7.21 problem 37

Internal problem ID [177]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 37.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2y'' - y' - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(2*diff(y(x),x$2)-diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{x} c_1 + c_2 e^{-\frac{x}{2}}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 22

```
DSolve[2*y''[x]-y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x/2} + c_2 e^{x}$$

7.22 problem 38

Internal problem ID [178]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' + 8y' + 3y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(4*diff(y(x),x$2)+8*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{x}{2}} + c_2 e^{-\frac{3x}{2}}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 22

```
DSolve[4*y''[x]+8*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x/2}(c_2 e^x + c_1)$$

7.23 problem 39

Internal problem ID [179]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' + 4y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(4*diff(y(x),x$2)+4*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-\frac{x}{2}}(c_2x + c_1)$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 20

```
DSolve[4*y''[x]+4*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}(c_2x + c_1)$$

7.24 problem 40

Internal problem ID [180]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{9y'' - 12y' + 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(9*diff(y(x),x$2)-12*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{\frac{2x}{3}}(c_2x + c_1)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 20

```
DSolve[9*y''[x]-12*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x/3}(c_2x + c_1)$$

7.25 problem 41

Internal problem ID [181]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 41.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{6y'' - 7y' - 20y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(6*diff(y(x),x$2)-7*diff(y(x),x)-20*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_1 \mathrm{e}^{\frac{23x}{6}} + c_2\right) \mathrm{e}^{-\frac{4x}{3}}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 26

```
DSolve[6*y''[x]-7*y'[x]-20*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-4x/3} + c_2 e^{5x/2}$$

7.26 problem 42

Internal problem ID [182]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 42.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{35y'' - y' - 12y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(35*diff(y(x),x$2)-diff(y(x),x)-12*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_1 \mathrm{e}^{\frac{41x}{35}} + c_2\right) \mathrm{e}^{-\frac{4x}{7}}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 26

```
DSolve[35*y''[x]-y'[x]-12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-4x/7} + c_2 e^{3x/5}$$

7.27 problem 52

Internal problem ID [183]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 52.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2y'' + y'x - y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1x^2 + c_2}{x}$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 16

```
DSolve[x^2*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x} + c_2x$$

7.28 problem 53

Internal problem ID [184]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 53.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 2y'x - 12y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-12*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2x^7 + c_1}{x^4}$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+2*x*y'[x]-12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^7 + c_1}{x^4}$$

7.29 problem 54

Internal problem ID [185]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 54.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$4x^2y'' + 8y'x - 3y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(4*x^2*diff(y(x),x$2)+8*x*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1x^2 + c_2}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 20

```
DSolve[4*x^2*y''[x]+8*x*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_2x^2 + c_1}{x^{3/2}}$$

7.30 problem 55

Internal problem ID [186]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 55.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x^2y'' + y'x = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_2 \ln(x) + c_1$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 13

```
DSolve[x^2*y''[x]+x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \log(x) + c_2$$

7.31 problem 56

Internal problem ID [187]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.1, second order linear equations. Page 299
Problem number: 56.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$x^2y'' - 3y'x + 4y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^2(c_2 \ln(x) + c_1)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]-3*x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(2c_2 \log(x) + c_1)$$

8 Section 5.2, second order linear equations. Page 311

8.1 problem 21

Internal problem ID [188]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.2, second order linear equations. Page 311
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y = 3x}$$

With initial conditions

$$[y(0) = 2, y'(0) = -2]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve([diff(y(x),x$2)+y(x)=3*x,y(0) = 2, D(y)(0) = -2],y(x), singsol=all)
```

$$y(x) = -5\sin(x) + 2\cos(x) + 3x$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 17

```
DSolve[{y''[x]+y[x]==3*x,{y[0]==2,y'[0]==-2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3x - 5\sin(x) + 2\cos(x)$$

8.2 problem 22

Internal problem ID [189]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.2, second order linear equations. Page 311
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y = 12}$$

With initial conditions

$$[y(0) = 0, y'(0) = 10]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$2)-4*y(x)=12,y(0) = 0, D(y)(0) = 10],y(x), singsol=all)
```

$$y(x) = 4\,e^{2x} - e^{-2x} - 3$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 21

```
DSolve[{y''[x]-4*y[x]==12,{y[0]==0,y'[0]==10}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{-2x} + 4e^{2x} - 3$$

8.3 problem 23

Internal problem ID [190]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.2, second order linear equations. Page 311
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' - 3y = 6}$$

With initial conditions

$$[y(0) = 3, y'(0) = 11]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)-3*y(x)=6,y(0) = 3, D(y)(0) = 11],y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} + 4\,\mathrm{e}^{3x} - 2$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 19

```
DSolve[{y''[x]-2*y'[x]-3*y[x]==6,{y[0]==3,y'[0]==11}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^{-x} + 4e^{3x} - 2$$

8.4 problem 24

Internal problem ID [191]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.2, second order linear equations. Page 311
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 2y' + 2y = 2x$$

With initial conditions

$$[y(0) = 4, y'(0) = 8]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+2*y(x)=2*x,y(0) = 4, D(y)(0) = 8],y(x), singsol=all)
```

$$y(x) = x + 1 + (4\sin(x) + 3\cos(x))\,e^x$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 22

```
DSolve[{y''[x]-2*y'[x]+2*y[x]==2*x,{y[0]==4,y'[0]==8}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to x + 4e^x\sin(x) + 3e^x\cos(x) + 1$$

8.5 problem 26(a.1)

Internal problem ID [192]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.2, second order linear equations. Page 311
Problem number: 26(a.1).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y = 4}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+2*y(x)=4,y(x), singsol=all)
```

$$y(x) = \sin\left(\sqrt{2}\,x\right) c_2 + \cos\left(\sqrt{2}\,x\right) c_1 + 2$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 29

```
DSolve[y''[x]+2*y[x]==4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos\left(\sqrt{2}x\right) + c_2 \sin\left(\sqrt{2}x\right) + 2$$

8.6 problem 26(a.2)

Internal problem ID [193]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.2, second order linear equations. Page 311
Problem number: 26(a.2).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y = 6x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+2*y(x)=6*x,y(x), singsol=all)
```

$$y(x) = \sin\left(\sqrt{2}\, x\right) c_2 + \cos\left(\sqrt{2}\, x\right) c_1 + 3x$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 31

```
DSolve[y''[x]+2*y[x]==6*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3x + c_1 \cos\left(\sqrt{2}x\right) + c_2 \sin\left(\sqrt{2}x\right)$$

8.7 problem 26(b)

Internal problem ID [194]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.2, second order linear equations. Page 311
Problem number: 26(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y = 6x + 4}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)+2*y(x)=6*x+4,y(x), singsol=all)
```

$$y(x) = \sin\left(\sqrt{2}\,x\right) c_2 + \cos\left(\sqrt{2}\,x\right) c_1 + 3x + 2$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 32

```
DSolve[y''[x]+2*y[x]==6*x+4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3x + c_1 \cos\left(\sqrt{2}x\right) + c_2 \sin\left(\sqrt{2}x\right) + 2$$

9 Section 5.3, second order linear equations. Page 323

9.1 problem 1

Internal problem ID [195]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.3, second order linear equations. Page 323
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2x}c_1 + c_2\mathrm{e}^{-2x}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 22

```
DSolve[y''[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(c_1e^{4x} + c_2\right)$$

9.2 problem 2

Internal problem ID [196]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.3, second order linear equations. Page 323
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2y'' - 3y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(2*diff(y(x),x$2)-3*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 e^{\frac{3x}{2}}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 21

```
DSolve[2*y''[x]-3*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{3}c_1 e^{3x/2} + c_2$$

9.3 problem 3

Internal problem ID [197]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.3, second order linear equations. Page 323
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 3y' - 10y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)-10*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_2 e^{7x} + c_1\right) e^{-5x}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 22

```
DSolve[y''[x]+3*y'[x]-10*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-5x}\left(c_2 e^{7x} + c_1\right)$$

9.4 problem 4

Internal problem ID [198]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.3, second order linear equations. Page 323
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2y'' - 7y' + 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(2*diff(y(x),x$2)-7*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{x}{2}} + c_2 e^{3x}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 24

```
DSolve[2*y''[x]-7*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x/2} + c_2 e^{3x}$$

9.5 problem 5

Internal problem ID [199]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.3, second order linear equations. Page 323
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 6y' + 9y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)+6*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-3x}(c_2 x + c_1)$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 18

```
DSolve[y''[x]+6*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}(c_2 x + c_1)$$

9.6 problem 6

Internal problem ID [200]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.3, second order linear equations. Page 323
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 5y' + 5y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+5*diff(y(x),x)+5*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{\left(-5+\sqrt{5}\right)x}{2}} + c_2 \mathrm{e}^{-\frac{\left(5+\sqrt{5}\right)x}{2}}$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 35

```
DSolve[y''[x]+5*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{1}{2}\left(5+\sqrt{5}\right)x}\left(c_2 e^{\sqrt{5}x} + c_1\right)$$

9.7 problem 7

Internal problem ID [201]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.3, second order linear equations. Page 323
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' - 12y' + 9y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(4*diff(y(x),x$2)-12*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{3x}{2}}(c_2x + c_1)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 20

```
DSolve[4*y''[x]-12*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{3x/2}(c_2x + c_1)$$

9.8 problem 8

Internal problem ID [202]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.3, second order linear equations. Page 323
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 6y' + 13y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+13*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{3x}(\sin(2x) c_1 + c_2 \cos(2x))$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 26

```
DSolve[y''[x]-6*y'[x]+13*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{3x}(c_2 \cos(2x) + c_1 \sin(2x))$$

9.9 problem 9

Internal problem ID [203]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.3, second order linear equations. Page 323
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 8y' + 25y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+8*diff(y(x),x)+25*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-4x}(c_1 \sin(3x) + c_2 \cos(3x))$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 26

```
DSolve[y''[x]+8*y'[x]+25*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-4x}(c_2 \cos(3x) + c_1 \sin(3x))$$

9.10 problem 21

Internal problem ID [204]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.3, second order linear equations. Page 323
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y' + 3y = 0}$$

With initial conditions

$$[y(0) = 7, y'(0) = 11]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve([diff(y(x),x$2)-4*diff(y(x),x)+3*y(x)=0,y(0) = 7, D(y)(0) = 11],y(x), singsol=all)
```

$$y(x) = 5\,\mathrm{e}^{x} + 2\,\mathrm{e}^{3x}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 18

```
DSolve[{y''[x]-4*y'[x]+3*y[x]==0,{y[0]==7,y'[0]==11}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^{x}\left(2e^{2x} + 5\right)$$

9.11 problem 22

Internal problem ID [205]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.3, second order linear equations. Page 323
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{9y'' + 6y' + 4y = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = 4]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 31

```
dsolve([9*diff(y(x),x$2)+6*diff(y(x),x)+4*y(x)=0,y(0) = 3, D(y)(0) = 4],y(x), singsol=all)
```

$$y(x) = e^{-\frac{x}{3}} \left(5\sqrt{3}\,\sin\left(\frac{\sqrt{3}\,x}{3}\right) + 3\cos\left(\frac{\sqrt{3}\,x}{3}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 39

```
DSolve[{9*y''[x]+6*y'[x]+4*y[x]==0,{y[0]==3,y'[0]==4}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to e^{-x/3} \left(5\sqrt{3}\sin\left(\frac{x}{\sqrt{3}}\right) + 3\cos\left(\frac{x}{\sqrt{3}}\right)\right)$$

9.12 problem 23

Internal problem ID [206]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.3, second order linear equations. Page 323
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 6y' + 25y = 0}$$

With initial conditions

$$[y(0) = 4, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$2)-6*diff(y(x),x)+25*y(x)=0,y(0) = 4, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{e^{3x}(11\sin(4x) - 16\cos(4x))}{4}$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 27

```
DSolve[{y''[x]-6*y'[x]+25*y[x]==0,{y[0]==4,y'[0]==1}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{4}e^{3x}(16\cos(4x) - 11\sin(4x))$$

9.13 problem 45

Internal problem ID [207]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.3, second order linear equations. Page 323
Problem number: 45.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2iy' + 3y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)-2*I*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{3ix} + c_2 \mathrm{e}^{-ix}$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 26

```
DSolve[y''[x]-2*I*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-ix}\left(c_1 e^{4ix} + c_2\right)$$

9.14 problem 46

Internal problem ID [208]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.3, second order linear equations. Page 323
Problem number: 46.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - iy' + 6y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)-I*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{3ix} + c_2 e^{-2ix}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 26

```
DSolve[y''[x]-I*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2ix}\left(c_1 e^{5ix} + c_2\right)$$

9.15 problem 47

Internal problem ID [209]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.3, second order linear equations. Page 323
Problem number: 47.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - \left(-2 + 2i\sqrt{3}\right) y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)=(-2+2*I*sqrt(3))*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\left(1+i\sqrt{3}\right)x} + c_2 \mathrm{e}^{\left(1+i\sqrt{3}\right)x}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 41

```
DSolve[y''[x]==(-2+2*I*Sqrt[3])*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x+i\sqrt{3}x} + c_2 e^{\left(-1-i\sqrt{3}\right)x}$$

9.16 problem 52

Internal problem ID [210]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.3, second order linear equations. Page 323
Problem number: 52.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' + y'x + 9y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(3\ln(x)) + c_2 \cos(3\ln(x))$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]+x*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(3\log(x)) + c_2 \sin(3\log(x))$$

9.17 problem 53

Internal problem ID [211]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.3, second order linear equations. Page 323
Problem number: 53.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 7y'x + 25y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)+7*x*diff(y(x),x)+25*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin (4 \ln (x)) + c_2 \cos (4 \ln (x))}{x^3}$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 26

```
DSolve[x^2*y''[x]+7*x*y'[x]+25*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 \cos(4 \log(x)) + c_1 \sin(4 \log(x))}{x^3}$$

10 Section 5.4, Mechanical Vibrations. Page 337

10.1 problem 15

Internal problem ID [212]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.4, Mechanical Vibrations. Page 337
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{\frac{x''}{2} + 3x' + 4x = 0}$$

With initial conditions

$$[x(0) = 2, x'(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve([1/2*diff(x(t),t$2)+3*diff(x(t),t)+4*x(t)=0,x(0) = 2, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = -2\,\mathrm{e}^{-4t} + 4\,\mathrm{e}^{-2t}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 20

```
DSolve[{1/2*x''[t]+3*x'[t]+4*x[t]==0,{x[0]==2,x'[0]==0}},x[t],t,IncludeSingularSolutions ->
```

$$x(t) \to e^{-4t}\left(4e^{2t} - 2\right)$$

10.2 problem 16

Internal problem ID [213]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.4, Mechanical Vibrations. Page 337
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{3x'' + 30x' + 63x = 0}$$

With initial conditions

$$[x(0) = 2, x'(0) = 2]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve([3*diff(x(t),t$2)+30*diff(x(t),t)+63*x(t)=0,x(0) = 2, D(x)(0) = 2],x(t), singsol=all)
```

$$x(t) = 4\,\mathrm{e}^{-3t} - 2\,\mathrm{e}^{-7t}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 20

```
DSolve[{3*x''[t]+30*x'[t]+63*x[t]==0,{x[0]==2,x'[0]==2}},x[t],t,IncludeSingularSolutions ->
```

$$x(t) \to e^{-7t}\left(4e^{4t} - 2\right)$$

10.3 problem 17

Internal problem ID [214]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.4, Mechanical Vibrations. Page 337
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + 8x' + 16x = 0}$$

With initial conditions

$$[x(0) = 5, x'(0) = -10]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve([diff(x(t),t$2)+8*diff(x(t),t)+16*x(t)=0,x(0) = 5, D(x)(0) = -10],x(t), singsol=all)
```

$$x(t) = (5 + 10t)\,\mathrm{e}^{-4t}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 17

```
DSolve[{x''[t]+8*x'[t]+16*x[t]==0,{x[0]==5,x'[0]==-10}},x[t],t,IncludeSingularSolutions -> T
```

$$x(t) \to 5e^{-4t}(2t + 1)$$

10.4 problem 18

Internal problem ID [215]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.4, Mechanical Vibrations. Page 337
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2x'' + 12x' + 50x = 0}$$

With initial conditions

$$[x(0) = 0, x'(0) = -8]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve([2*diff(x(t),t$2)+12*diff(x(t),t)+50*x(t)=0,x(0) = 0, D(x)(0) = -8],x(t), singsol=all
```

$$x(t) = -2\,\mathrm{e}^{-3t}\sin(4t)$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 16

```
DSolve[{2*x''[t]+12*x'[t]+50*x[t]==0,{x[0]==0,x'[0]==-8}},x[t],t,IncludeSingularSolutions ->
```

$$x(t) \to -2e^{-3t}\sin(4t)$$

10.5 problem 19

Internal problem ID [216]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.4, Mechanical Vibrations. Page 337
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4x'' + 20x' + 169x = 0}$$

With initial conditions

$$[x(0) = 4, x'(0) = 16]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve([4*diff(x(t),t$2)+20*diff(x(t),t)+169*x(t)=0,x(0) = 4, D(x)(0) = 16],x(t), singsol=al
```

$$x(t) = \frac{e^{-\frac{5t}{2}}(13\sin(6t) + 12\cos(6t))}{3}$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 29

```
DSolve[{4*x''[t]+20*x'[t]+169*x[t]==0,{x[0]==4,x'[0]==16}},x[t],t,IncludeSingularSolutions -
```

$$x(t) \to \frac{1}{3}e^{-5t/2}(13\sin(6t) + 12\cos(6t))$$

10.6 problem 20

Internal problem ID [217]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.4, Mechanical Vibrations. Page 337
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2x'' + 16x' + 40x = 0}$$

With initial conditions

$$[x(0) = 5, x'(0) = 4]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 22

```
dsolve([2*diff(x(t),t$2)+16*diff(x(t),t)+40*x(t)=0,x(0) = 5, D(x)(0) = 4],x(t), singsol=all)
```

$$x(t) = e^{-4t}(12\sin(2t) + 5\cos(2t))$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 24

```
DSolve[{2*x''[t]+16*x'[t]+40*x[t]==0,{x[0]==5,x'[0]==4}},x[t],t,IncludeSingularSolutions ->
```

$$x(t) \to e^{-4t}(12\sin(2t) + 5\cos(2t))$$

10.7 problem 21

Internal problem ID [218]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.4, Mechanical Vibrations. Page 337
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + 10x' + 125x = 0}$$

With initial conditions

$$[x(0) = 6, x'(0) = 50]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 23

```
dsolve([diff(x(t),t$2)+10*diff(x(t),t)+125*x(t)=0,x(0) = 6, D(x)(0) = 50],x(t), singsol=all)
```

$$x(t) = 2\,\mathrm{e}^{-5t}(4\sin(10t) + 3\cos(10t))$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 24

```
DSolve[{x''[t]+10*x'[t]+125*x[t]==0,{x[0]==6,x'[0]==50}},x[t],t,IncludeSingularSolutions ->
```

$$x(t) \to e^{-5t}(8\sin(10t) + 6\cos(10t))$$

11 Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

11.1 problem 1

Internal problem ID [219]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 16y = \mathrm{e}^{3x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+16*y(x)=exp(3*x),y(x), singsol=all)
```

$$y(x) = \sin(4x)\, c_2 + \cos(4x)\, c_1 + \frac{\mathrm{e}^{3x}}{25}$$

✓ Solution by Mathematica

Time used: 0.144 (sec). Leaf size: 29

```
DSolve[y''[x]+16*y[x]==Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{3x}}{25} + c_1 \cos(4x) + c_2 \sin(4x)$$

11.2 problem 2

Internal problem ID [220]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y' - 2y = 3x + 4}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-2*y(x)=3*x+4,y(x), singsol=all)
```

$$y(x) = c_2 e^{-x} + e^{2x} c_1 - \frac{3x}{2} - \frac{5}{4}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 30

```
DSolve[y''[x]-y'[x]-2*y[x]==3*x+4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{3x}{2} + c_1 e^{-x} + c_2 e^{2x} - \frac{5}{4}$$

11.3 problem 3

Internal problem ID [221]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y' - 6y = 2\sin(3x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-6*y(x)=2*sin(3*x),y(x), singsol=all)
```

$$y(x) = e^{-2x}\left(\frac{(\cos(3x) - 5\sin(3x))\,e^{2x}}{39} + c_2 e^{5x} + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 37

```
DSolve[y''[x]-y'[x]-6*y[x]==2*Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-2x} + c_2 e^{3x} + \frac{1}{39}(\cos(3x) - 5\sin(3x))$$

11.4 problem 4

Internal problem ID [222]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{4y'' + 4y' + y = 3x\,\mathrm{e}^{x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve(4*diff(y(x),x$2)+4*diff(y(x),x)+y(x)=3*x*exp(x),y(x), singsol=all)
```

$$y(x) = (c_1x + c_2)\,\mathrm{e}^{-\frac{x}{2}} + \frac{(3x-4)\,\mathrm{e}^{x}}{9}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 33

```
DSolve[4*y''[x]+4*y'[x]+y[x]==3*x*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{9}e^{x}(3x-4) + e^{-x/2}(c_2x + c_1)$$

11.5 problem 5

Internal problem ID [223]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' + y = \sin(x)^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 44

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=sin(x)^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\, x}{2}\right) c_2 + \mathrm{e}^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\, x}{2}\right) c_1 - \frac{\sin(2x)}{13} + \frac{3\cos(2x)}{26} + \frac{1}{2}$$

✓ Solution by Mathematica
Time used: 1.827 (sec). Leaf size: 67

```
DSolve[y''[x]+y'[x]+y[x]==Sin[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{13}\sin(2x) + \frac{3}{26}\cos(2x) + c_2 e^{-x/2}\cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 e^{-x/2}\sin\left(\frac{\sqrt{3}x}{2}\right) + \frac{1}{2}$$

11.6 problem 6

Internal problem ID [224]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2y'' + 4y' + 7y = x^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 40

```
dsolve(2*diff(y(x),x$2)+4*diff(y(x),x)+7*y(x)=x^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} \sin\left(\frac{\sqrt{10}\, x}{2}\right) c_2 + \mathrm{e}^{-x} \cos\left(\frac{\sqrt{10}\, x}{2}\right) c_1 + \frac{x^2}{7} - \frac{8x}{49} + \frac{4}{343}$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 56

```
DSolve[2*y''[x]+4*y'[x]+7*y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{343}(49x^2 - 56x + 4) + c_2 e^{-x} \cos\left(\sqrt{\frac{5}{2}}x\right) + c_1 e^{-x} \sin\left(\sqrt{\frac{5}{2}}x\right)$$

11.7 problem 7

Internal problem ID [225]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y = \sinh(x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 56

```
dsolve(diff(y(x),x$2)-4*y(x)=sinh(x),y(x), singsol=all)
```

$$y(x) = \frac{\left(-2\sinh(x)^2\cosh(x) - 2\sinh(x)^3 + 12c_1 + \cosh(x)\right)e^{-2x}}{12} + e^{2x}\left(\frac{\sinh(x)^2\cosh(x)}{6} - \frac{\sinh(x)^3}{6} + c_2 - \frac{\cosh(x)}{12}\right)$$

✓ Solution by Mathematica
Time used: 0.049 (sec). Leaf size: 38

```
DSolve[y''[x]-4*y[x]==Sinh[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}e^{-2x}\left(e^x - e^{3x} + 6c_1e^{4x} + 6c_2\right)$$

11.8 problem 8

Internal problem ID [226]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y = \cosh(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-4*y(x)=cosh(2*x),y(x), singsol=all)
```

$$y(x) = \frac{(-4x + 32c_1 - 2)\,\mathrm{e}^{-2x}}{32} + \frac{\mathrm{e}^{2x}\left(x + 8c_2 - \frac{1}{4}\right)}{8}$$

✓ Solution by Mathematica
Time used: 0.048 (sec). Leaf size: 38

```
DSolve[y''[x]-4*y[x]==Cosh[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{32}e^{-2x}\left(-4x + e^{4x}(4x - 1 + 32c_1) - 1 + 32c_2\right)$$

11.9 problem 9

Internal problem ID [227]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' - 3y = 1 + x\,\mathrm{e}^{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)-3*y(x)=1+x*exp(x),y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{-3x}\left(\left(x^2 - \frac{1}{2}x + 8c_2 + \frac{1}{8}\right)\mathrm{e}^{4x} + 8c_1 - \frac{8\,\mathrm{e}^{3x}}{3}\right)}{8}$$

✓ Solution by Mathematica

Time used: 0.092 (sec). Leaf size: 38

```
DSolve[y''[x]+2*y'[x]-3*y[x]==1+x*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{64}e^{x}\left(8x^2 - 4x + 1 + 64c_2\right) + c_1 e^{-3x} - \frac{1}{3}$$

11.10 problem 10

Internal problem ID [228]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = 2\cos(3x) + 3\sin(3x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+9*y(x)=2*cos(3*x)+3*sin(3*x),y(x), singsol=all)
```

$$y(x) = \frac{(18c_1 - 9x + 2)\cos(3x)}{18} + \frac{\sin(3x)(x + 3c_2)}{3}$$

✓ Solution by Mathematica
Time used: 0.231 (sec). Leaf size: 39

```
DSolve[y''[x]+9*y[x]==2*Cos[3*x]+3*Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(-\frac{x}{2} + \frac{1}{9} + c_1\right)\cos(3x) + \frac{1}{12}(4x + 1 + 12c_2)\sin(3x)$$

11.11 problem 16

Internal problem ID [229]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = 2x^2 e^{3x} + 5}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+9*y(x)=2*x^2*exp(3*x)+5,y(x), singsol=all)
```

$$y(x) = \sin(3x) c_2 + \cos(3x) c_1 + \frac{5}{9} + \frac{\left(x - \frac{1}{3}\right)^2 e^{3x}}{9}$$

✓ Solution by Mathematica

Time used: 0.251 (sec). Leaf size: 50

```
DSolve[y''[x]+9*y[x]==2*x^2*Exp[3*x]+5,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{81}\left(9e^{3x}x^2 - 6e^{3x}x + e^{3x} + 81c_1 \cos(3x) + 81c_2 \sin(3x) + 45\right)$$

11.12 problem 21

Internal problem ID [230]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + 2y = \sin(x)\,e^{x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+2*y(x)=exp(x)*sin(x),y(x), singsol=all)
```

$$y(x) = -\frac{((x - 2c_1)\cos(x) + (-2c_2 - 1)\sin(x))\,e^{x}}{2}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 28

```
DSolve[y''[x]-2*y'[x]+2*y[x]==Exp[x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}e^{x}((x - 2c_2)\cos(x) - 2c_1\sin(x))$$

11.13 problem 23

Internal problem ID [231]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = 3x\cos(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)+4*y(x)=3*x*cos(2*x),y(x), singsol=all)
```

$$y(x) = \frac{(24x^2 + 64c_2 - 3)\sin(2x)}{64} + \frac{3\left(x + \frac{16c_1}{3}\right)\cos(2x)}{16}$$

✓ Solution by Mathematica
Time used: 0.124 (sec). Leaf size: 38

```
DSolve[y''[x]+4*y[x]==3*x*Cos[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{64}(24x^2 - 3 + 64c_2)\sin(2x) + \left(\frac{3x}{16} + c_1\right)\cos(2x)$$

11.14 problem 25

Internal problem ID [232]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 3y' + 2y = x(\mathrm{e}^{-x} - \mathrm{e}^{-2x})}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=x*(exp(-x)-exp(-2*x)),y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{-x}((x^2 - 2c_1 + 2x + 2)\,\mathrm{e}^{-x} + x^2 - 2x + 2c_2)}{2}$$

✓ Solution by Mathematica
Time used: 0.086 (sec). Leaf size: 42

```
DSolve[y''[x]+3*y'[x]+2*y[x]==x*(Exp[-x]-Exp[-2*x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-2x}\left(x^2 + e^x\left(x^2 - 2x + 2 + 2c_2\right) + 2x + 2 + 2c_1\right)$$

11.15 problem 26

Internal problem ID [233]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 6y' + 13y = x\,\mathrm{e}^{3x} \sin(2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+13*y(x)=x*exp(3*x)*sin(2*x),y(x), singsol=all)
```

$$y(x) = -\frac{\left((x^2 - 8c_1)\cos(2x) - \frac{\sin(2x)(x+16c_2)}{2}\right)\mathrm{e}^{3x}}{8}$$

✓ Solution by Mathematica

Time used: 0.105 (sec). Leaf size: 43

```
DSolve[y''[x]-6*y'[x]+13*y[x]==x*Exp[3*x]*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{64} e^{3x}\left(\left(-8x^2 + 1 + 64c_2\right)\cos(2x) + 4(x + 16c_1)\sin(2x)\right)$$

11.16 problem 31

Internal problem ID [234]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y = 2x}$$

With initial conditions

$$[y(0) = 1, y'(0) = 2]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$2)+4*y(x)=2*x,y(0) = 1, D(y)(0) = 2],y(x), singsol=all)
```

$$y(x) = \frac{3\sin(2x)}{4} + \cos(2x) + \frac{x}{2}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 22

```
DSolve[{y''[x]+4*y[x]==2*x,{y[0]==1,y'[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(2x) + \frac{1}{2}(x + 3\sin(x)\cos(x))$$

11.17 problem 32

Internal problem ID [235]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 3y' + 2y = e^x}$$

With initial conditions

$$[y(0) = 0, y'(0) = 3]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve([diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=exp(x),y(0) = 0, D(y)(0) = 3],y(x), singsol=all
```

$$y(x) = \frac{(e^{3x} + 15\,e^x - 16)\,e^{-2x}}{6}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 26

```
DSolve[{y''[x]+3*y'[x]+2*y[x]==Exp[x],{y[0]==0,y'[0]==3}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{6}e^{-2x}(15e^x + e^{3x} - 16)$$

11.18 problem 33

Internal problem ID [236]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = \sin(2x)}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve([diff(y(x),x$2)+9*y(x)=sin(2*x),y(0) = 1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = -\frac{2\sin(3x)}{15} + \cos(3x) + \frac{\sin(2x)}{5}$$

✓ Solution by Mathematica
Time used: 0.094 (sec). Leaf size: 26

```
DSolve[{y''[x]+9*y[x]==Sin[2*x],{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{5}\sin(2x) - \frac{2}{15}\sin(3x) + \cos(3x)$$

11.19 problem 34

Internal problem ID [237]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \cos(x)}$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve([diff(y(x),x$2)+y(x)=cos(x),y(0) = 1, D(y)(0) = -1],y(x), singsol=all)
```

$$y(x) = \frac{(-2 + x)\sin(x)}{2} + \cos(x)$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 17

```
DSolve[{y''[x]+y[x]==Cos[x],{y[0]==1,y'[0]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(x - 2)\sin(x) + \cos(x)$$

11.20 problem 35

Internal problem ID [238]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y' + 2y = x + 1}$$

With initial conditions

$$[y(0) = 3, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+2*y(x)=x+1,y(0) = 3, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{(-5\sin(x) + 4\cos(x))\,\mathrm{e}^{x}}{2} + 1 + \frac{x}{2}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 26

```
DSolve[{y''[x]-2*y'[x]+2*y[x]==x+1,{y[0]==3,y'[0]==0}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{1}{2}(x - 5e^{x}\sin(x) + 4e^{x}\cos(x) + 2)$$

11.21 problem 44

Internal problem ID [239]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 44.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' + y = \sin(x)\sin(3x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 55

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=sin(x)*sin(3*x),y(x), singsol=all)
```

$$y(x) = e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right) c_2 + e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right) c_1 - \frac{3\cos(2x)}{26} + \frac{\sin(2x)}{13} - \frac{2\sin(4x)}{241} + \frac{15\cos(4x)}{482}$$

✓ Solution by Mathematica

Time used: 5.225 (sec). Leaf size: 80

```
DSolve[y''[x]+y'[x]+y[x]==Sin[x]*Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{13}\sin(2x) - \frac{2}{241}\sin(4x) - \frac{3}{26}\cos(2x) + \frac{15}{482}\cos(4x) + c_2 e^{-x/2}\cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 e^{-x/2}\sin\left(\frac{\sqrt{3}x}{2}\right)$$

11.22 problem 45

Internal problem ID [240]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 45.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = \sin(x)^4}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)+9*y(x)=sin(x)^4,y(x), singsol=all)
```

$$y(x) = \sin(3x)\, c_2 + \cos(3x)\, c_1 - \frac{\cos(2x)}{10} - \frac{\cos(2x)^2}{28} + \frac{5}{84}$$

✓ Solution by Mathematica
Time used: 0.215 (sec). Leaf size: 39

```
DSolve[y''[x]+9*y[x]==Sin[x]^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{10}\cos(2x) - \frac{1}{56}\cos(4x) + c_1\cos(3x) + c_2\sin(3x) + \frac{1}{24}$$

11.23 problem 46

Internal problem ID [241]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 46.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = x \cos(x)^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x$2)+y(x)=x*cos(x)^3,y(x), singsol=all)
```

$$y(x) = -\frac{x \cos(x)^3}{8} + \frac{3 \sin(x) \cos(x)^2}{32} + \frac{(9x + 32c_1) \cos(x)}{32} + \frac{3\left(x^2 + \frac{16c_2}{3} + \frac{3}{4}\right) \sin(x)}{16}$$

✓ Solution by Mathematica

Time used: 0.116 (sec). Leaf size: 49

```
DSolve[y''[x]+y[x]==x*Cos[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{128}\left(\sin(x)\left(24x^2 + 6\cos(2x) - 9 + 128c_2\right) - 4x\cos(3x) + 8(3x + 16c_1)\cos(x)\right)$$

11.24 problem 47

Internal problem ID [242]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 47.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 3y' + 2y = 4\,\mathrm{e}^{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=4*exp(x),y(x), singsol=all)
```

$$y(x) = -\left(-\mathrm{e}^{x} c_2 + c_1 - \frac{2\,\mathrm{e}^{3x}}{3}\right)\mathrm{e}^{-2x}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 29

```
DSolve[y''[x]+3*y'[x]+2*y[x]==4*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2e^{x}}{3} + c_1 e^{-2x} + c_2 e^{-x}$$

11.25 problem 48

Internal problem ID [243]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 48.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y' - 8y = 3\,\mathrm{e}^{-2x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-8*y(x)=3*exp(-2*x),y(x), singsol=all)
```

$$y(x) = \frac{(-x + 2c_1)\,\mathrm{e}^{-2x}}{2} + \mathrm{e}^{4x}c_2$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 32

```
DSolve[y''[x]-2*y'[x]-8*y[x]==3*Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}e^{-2x}\left(-6x + 12c_2e^{6x} - 1 + 12c_1\right)$$

11.26 problem 49

Internal problem ID [244]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 49.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y' + 4y = 2\,\mathrm{e}^{2x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=2*exp(2*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2x}(c_1x + x^2 + c_2)$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 21

```
DSolve[y''[x]-4*y'[x]+4*y[x]==2*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}(x^2 + c_2x + c_1)$$

11.27 problem 50

Internal problem ID [245]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 50.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y = \sinh(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)-4*y(x)=sinh(2*x),y(x), singsol=all)
```

$$y(x) = \frac{e^{2x}(4x + 32c_2 - 1)}{32} + \frac{e^{-2x}(x + 8c_1)}{8}$$

✓ Solution by Mathematica
Time used: 0.049 (sec). Leaf size: 38

```
DSolve[y''[x]-4*y[x]==Sinh[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{32}e^{-2x}\left(4x + e^{4x}(4x - 1 + 32c_1) + 1 + 32c_2\right)$$

11.28 problem 51

Internal problem ID [246]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 51.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \cos(3x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+4*y(x)=cos(3*x),y(x), singsol=all)
```

$$y(x) = \sin(2x) c_2 + \cos(2x) c_1 - \frac{\cos(3x)}{5}$$

✓ Solution by Mathematica
Time used: 0.078 (sec). Leaf size: 28

```
DSolve[y''[x]+4*y[x]==Cos[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{5}\cos(3x) + c_1\cos(2x) + c_2\sin(2x)$$

11.29 problem 52

Internal problem ID [247]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 52.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = \sin(3x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+9*y(x)=sin(3*x),y(x), singsol=all)
```

$$y(x) = \frac{(-x + 6c_1)\cos(3x)}{6} + \sin(3x)\, c_2$$

✓ Solution by Mathematica
Time used: 0.053 (sec). Leaf size: 33

```
DSolve[y''[x]+9*y[x]==Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(-\frac{x}{6} + c_1\right)\cos(3x) + \frac{1}{36}(1 + 36c_2)\sin(3x)$$

11.30 problem 53

Internal problem ID [248]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 53.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = 2\sec(3x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)+9*y(x)=2*sec(3*x),y(x), singsol=all)
```

$$y(x) = -\frac{2\ln(\sec(3x))\cos(3x)}{9} + \cos(3x)\,c_1 + \frac{2\sin(3x)\left(x + \frac{3c_2}{2}\right)}{3}$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 39

```
DSolve[y''[x]+9*y[x]==2*Sec[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}(2x + 3c_2)\sin(3x) + \cos(3x)\left(\frac{2}{9}\log(\cos(3x)) + c_1\right)$$

11.31 problem 54

Internal problem ID [249]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 54.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \csc(x)^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+y(x)=csc(x)^2,y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 - 1 - \ln(\csc(x) - \cot(x)) \cos(x)$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 23

```
DSolve[y''[x]+y[x]==Csc[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x)\text{arctanh}(\cos(x)) + c_1 \cos(x) + c_2 \sin(x) - 1$$

11.32 problem 55

Internal problem ID [250]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 55.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \sin(x)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+4*y(x)=sin(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{(8c_1 - 1)\cos(2x)}{8} + \frac{1}{8} + \frac{(-x + 8c_2)\sin(2x)}{8}$$

✓ Solution by Mathematica
Time used: 0.111 (sec). Leaf size: 71

```
DSolve[y''[x]+4*y[x]==sin[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(2x)\int_1^x -\cos(K[1])\sin(K[1])^2\sin(K[1])dK[1] + \sin(2x)\int_1^x \frac{1}{2}\cos(2K[2])\sin(K[2])^2 dK[2] + c_1\cos(2x) + c_2\sin(2x)$$

11.33 problem 56

Internal problem ID [251]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 56.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y = x\,\mathrm{e}^{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)-4*y(x)=x*exp(x),y(x), singsol=all)
```

$$y(x) = -\frac{\left(-9\,\mathrm{e}^{4x} c_2 + 3x\,\mathrm{e}^{3x} + 2\,\mathrm{e}^{3x} - 9c_1\right)\mathrm{e}^{-2x}}{9}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 34

```
DSolve[y''[x]-4*y[x]==x*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{9}e^{x}(3x+2) + c_1 e^{2x} + c_2 e^{-2x}$$

11.34 problem 57

Internal problem ID [252]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 57.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + y'x - y = 72x^5}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=72*x^5,y(x), singsol=all)
```

$$y(x) = \frac{3x^6 + c_2x^2 + c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 21

```
DSolve[x^2*y''[x]+x*y'[x]-y[x]==72*x^5,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3x^5 + c_2x + \frac{c_1}{x}$$

11.35 problem 58

Internal problem ID [253]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 58.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 4y'x + 6y = x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=x^3,y(x), singsol=all)
```

$$y(x) = x^2(x \ln(x) + (c_1 - 1)x + c_2)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]-4*x*y'[x]+6*y[x]==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(x \log(x) + (-1 + c_2)x + c_1)$$

11.36 problem 59

Internal problem ID [254]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 59.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 3y'x + 4y = x^4}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*y(x)=x^4,y(x), singsol=all)
```

$$y(x) = \frac{x^2(4\ln(x)\,c_1 + x^2 + 4c_2)}{4}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 26

```
DSolve[x^2*y''[x]-3*x*y'[x]+4*y[x]==x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}x^2(x^2 + 8c_2\log(x) + 4c_1)$$

11.37 problem 60

Internal problem ID [255]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 60.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' - 4y'x + 3y = 8x^{\frac{4}{3}}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+3*y(x)=8*x^(4/3),y(x), singsol=all)
```

$$y(x) = x^{\frac{3}{2}}c_2 + c_1\sqrt{x} - \frac{72x^{\frac{4}{3}}}{5}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 31

```
DSolve[4*x^2*y''[x]-4*x*y'[x]+3*y[x]==8*x^(4/3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{5}\sqrt{x}\left(-72x^{5/6} + 5c_2x + 5c_1\right)$$

11.38 problem 61

Internal problem ID [256]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 61.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + y = \ln(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=ln(x),y(x), singsol=all)
```

$$y(x) = \sin(\ln(x))\, c_2 + \cos(\ln(x))\, c_1 + \ln(x)$$

✓ Solution by Mathematica
Time used: 0.073 (sec). Leaf size: 20

```
DSolve[x^2*y''[x]+x*y'[x]+y[x]==Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(x) + c_1 \cos(\log(x)) + c_2 \sin(\log(x))$$

11.39 problem 62

Internal problem ID [257]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351
Problem number: 62.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-1)\,y''-2y'x+2y=x^2-1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 38

```
dsolve((x^2-1)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=x^2-1,y(x), singsol=all)
```

$$y(x)=\frac{(x-1)^2\ln(x-1)}{2}+\frac{(x+1)^2\ln(x+1)}{2}+(c_1-1)\,x^2+c_2x+c_1$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]-4*x*y'[x]+6*y[x]==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x^2(x\log(x)+(-1+c_2)x+c_1)$$

12 Section 5.6, Forced Oscillations and Resonance. Page 362

12.1 problem 1

Internal problem ID [258]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.6, Forced Oscillations and Resonance. Page 362
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 9x = 10\cos(2t)}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve([diff(x(t),t$2)+9*x(t)=10*cos(2*t),x(0) = 0, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = -8\cos(t)^3 + 6\cos(t) + 4\cos(t)^2 - 2$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 18

```
DSolve[{x''[t]+9*x[t]==10*Cos[2*t],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolutions -> Tr
```

$$x(t) \to 2(\cos(2t) - \cos(3t))$$

12.2 problem 2

Internal problem ID [259]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.6, Forced Oscillations and Resonance. Page 362
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 4x = 5\sin(3t)}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve([diff(x(t),t$2)+4*x(t)=5*sin(3*t),x(0) = 0, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = \frac{3\sin(2t)}{2} - \sin(3t)$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 18

```
DSolve[{x''[t]+4*x[t]==5*Sin[3*t],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolutions -> Tru
```

$$x(t) \to 3\sin(t)\cos(t) - \sin(3t)$$

12.3 problem 3

Internal problem ID [260]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.6, Forced Oscillations and Resonance. Page 362
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 100x = 225\cos(5t) + 300\sin(5t)}$$

With initial conditions

$$[x(0) = 375, x'(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 29

```
dsolve([diff(x(t),t$2)+100*x(t)=225*cos(5*t)+300*sin(5*t),x(0) = 375, D(x)(0) = 0],x(t), sin
```

$$x(t) = -2\sin(10t) + 372\cos(10t) + 3\cos(5t) + 4\sin(5t)$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 30

```
DSolve[{x''[t]+100*x[t]==225*Cos[5*t]+300*Sin[5*t],{x[0]==375,x'[0]==0}},x[t],t,IncludeSingu
```

$$x(t) \to 4\sin(5t) - 2\sin(10t) + 3\cos(5t) + 372\cos(10t)$$

12.4 problem 4

Internal problem ID [261]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.6, Forced Oscillations and Resonance. Page 362
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x'' + 25x = 90\cos(4t)$$

With initial conditions

$$[x(0) = 0, x'(0) = 90]$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 23

```
dsolve([diff(x(t),t$2)+25*x(t)=90*cos(4*t),x(0) = 0, D(x)(0) = 90],x(t), singsol=all)
```

$$x(t) = 18\sin(5t) - 10\cos(5t) + 10\cos(4t)$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 26

```
DSolve[{x''[t]+25*x[t]==90*Cos[4*t],{x[0]==0,x'[0]==90}},x[t],t,IncludeSingularSolutions ->
```

$$x(t) \to 2(9\sin(5t) + 5\cos(4t) - 5\cos(5t))$$

12.5 problem 5

Internal problem ID [262]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.6, Forced Oscillations and Resonance. Page 362
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{mx'' + kx = F_0 \cos(\omega t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 60

```
dsolve(m*diff(x(t),t$2)+k*x(t)=F__0*cos(omega*t),x(t), singsol=all)
```

$$x(t) = \frac{c_1(-m\,\omega^2 + k)\cos\left(\frac{\sqrt{k}\,t}{\sqrt{m}}\right) + c_2(-m\,\omega^2 + k)\sin\left(\frac{\sqrt{k}\,t}{\sqrt{m}}\right) + F_0\cos(\omega t)}{-m\,\omega^2 + k}$$

✓ Solution by Mathematica
Time used: 0.044 (sec). Leaf size: 54

```
DSolve[m*x''[t]+k*x[t]==F0*Cos[omega*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{\text{F0}\cos(\omega t)}{k - m\omega^2} + c_1\cos\left(\frac{\sqrt{k}t}{\sqrt{m}}\right) + c_2\sin\left(\frac{\sqrt{k}t}{\sqrt{m}}\right)$$

12.6 problem 7

Internal problem ID [263]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.6, Forced Oscillations and Resonance. Page 362
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 4x' + 4x = 10 \cos(3t)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(x(t),t$2)+4*diff(x(t),t)+4*x(t)=10*cos(3*t),x(t), singsol=all)
```

$$x(t) = (c_1 t + c_2)\,\mathrm{e}^{-2t} - \frac{50\cos(3t)}{169} + \frac{120\sin(3t)}{169}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 35

```
DSolve[x''[t]+4*x'[t]+4*x[t]==10*Cos[3*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{120}{169}\sin(3t) - \frac{50}{169}\cos(3t) + e^{-2t}(c_2 t + c_1)$$

12.7 problem 8

Internal problem ID [264]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.6, Forced Oscillations and Resonance. Page 362
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 3x' + 5x = -4\cos(5t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(x(t),t$2)+3*diff(x(t),t)+5*x(t)=-4*cos(5*t),x(t), singsol=all)
```

$$x(t) = \mathrm{e}^{-\frac{3t}{2}} \sin\left(\frac{\sqrt{11}\,t}{2}\right) c_2 + \mathrm{e}^{-\frac{3t}{2}} \cos\left(\frac{\sqrt{11}\,t}{2}\right) c_1 - \frac{12\sin(5t)}{125} + \frac{16\cos(5t)}{125}$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 65

```
DSolve[x''[t]+3*x'[t]+5*x[t]==-4*Cos[5*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{4}{125}(4\cos(5t) - 3\sin(5t)) + c_2 e^{-3t/2} \cos\left(\frac{\sqrt{11}t}{2}\right) + c_1 e^{-3t/2} \sin\left(\frac{\sqrt{11}t}{2}\right)$$

12.8 problem 9

Internal problem ID [265]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.6, Forced Oscillations and Resonance. Page 362
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{2x'' + 2x' + x = 3\sin(10t)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(2*diff(x(t),t$2)+2*diff(x(t),t)+x(t)=3*sin(10*t),x(t), singsol=all)
```

$$x(t) = \mathrm{e}^{-\frac{t}{2}} \sin\left(\frac{t}{2}\right) c_2 + \mathrm{e}^{-\frac{t}{2}} \cos\left(\frac{t}{2}\right) c_1 - \frac{597\sin(10t)}{40001} - \frac{60\cos(10t)}{40001}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 55

```
DSolve[2*x''[t]+2*x'[t]+x[t]==3*Sin[10*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{3(199\sin(10t) + 20\cos(10t))}{40001} + c_2 e^{-t/2} \cos\left(\frac{t}{2}\right) + c_1 e^{-t/2} \sin\left(\frac{t}{2}\right)$$

12.9 problem 10

Internal problem ID [266]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.6, Forced Oscillations and Resonance. Page 362
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x'' + 3x' + 3x = 8\cos(10t) + 6\sin(10t)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(x(t),t$2)+3*diff(x(t),t)+3*x(t)=8*cos(10*t)+6*sin(10*t),x(t), singsol=all)
```

$$x(t) = e^{-\frac{3t}{2}} \sin\left(\frac{\sqrt{3}\,t}{2}\right) c_2 + e^{-\frac{3t}{2}} \cos\left(\frac{\sqrt{3}\,t}{2}\right) c_1 - \frac{342\sin(10t)}{10309} - \frac{956\cos(10t)}{10309}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 65

```
DSolve[x''[t]+3*x'[t]+3*x[t]==8*Cos[10*t]+6*Sin[10*t],x[t],t,IncludeSingularSolutions -> Tru
```

$$x(t) \to -\frac{2(171\sin(10t) + 478\cos(10t))}{10309} + c_2 e^{-3t/2} \cos\left(\frac{\sqrt{3}t}{2}\right) + c_1 e^{-3t/2} \sin\left(\frac{\sqrt{3}t}{2}\right)$$

12.10 problem 11

Internal problem ID [267]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.6, Forced Oscillations and Resonance. Page 362
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 4x' + 5x = 10\cos(3t)}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 30

```
dsolve([diff(x(t),t$2)+4*diff(x(t),t)+5*x(t)=10*cos(3*t),x(0) = 0, D(x)(0) = 0],x(t), singso
```

$$x(t) = \frac{(\cos(t) - 7\sin(t))\,\mathrm{e}^{-2t}}{4} - \frac{\cos(3t)}{4} + \frac{3\sin(3t)}{4}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 43

```
DSolve[{x''[t]+4*x'[t]+5*x[t]==10*Cos[3*t],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolutio
```

$$x(t) \to \frac{1}{4}e^{-2t}\left(-7\sin(t) + 3e^{2t}\sin(3t) + \cos(t) - e^{2t}\cos(3t)\right)$$

12.11 problem 12

Internal problem ID [268]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.6, Forced Oscillations and Resonance. Page 362
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x'' + 6x' + 13x = 10\sin(5t)$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 37

```
dsolve([diff(x(t),t$2)+6*diff(x(t),t)+13*x(t)=10*sin(5*t),x(0) = 0, D(x)(0) = 0],x(t), sings
```

$$x(t) = \frac{25(2\cos(2t) + 5\sin(2t))\,\mathrm{e}^{-3t}}{174} - \frac{25\cos(5t)}{87} - \frac{10\sin(5t)}{87}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 49

```
DSolve[{x''[t]+6*x'[t]+13*x[t]==10*Sin[5*t],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSoluti
```

$$x(t) \to \frac{5}{174}e^{-3t}\left(25\sin(2t) - 4e^{3t}\sin(5t) + 10\cos(2t) - 10e^{3t}\cos(5t)\right)$$

12.12 problem 12

Internal problem ID [269]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.6, Forced Oscillations and Resonance. Page 362
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 6x' + 13x = 10\sin(5t)}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 37

```
dsolve([diff(x(t),t$2)+6*diff(x(t),t)+13*x(t)=10*sin(5*t),x(0) = 0, D(x)(0) = 0],x(t), sings
```

$$x(t) = \frac{25(2\cos(2t) + 5\sin(2t))\,\mathrm{e}^{-3t}}{174} - \frac{25\cos(5t)}{87} - \frac{10\sin(5t)}{87}$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 49

```
DSolve[{x''[t]+6*x'[t]+13*x[t]==10*Sin[5*t],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSoluti
```

$$x(t) \to \frac{5}{174}e^{-3t}\left(25\sin(2t) - 4e^{3t}\sin(5t) + 10\cos(2t) - 10e^{3t}\cos(5t)\right)$$

12.13 problem 13

Internal problem ID [270]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.6, Forced Oscillations and Resonance. Page 362
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 2x' + 26x = 600\cos(10t)}$$

With initial conditions

$$[x(0) = 10, x'(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 37

```
dsolve([diff(x(t),t$2)+2*diff(x(t),t)+26*x(t)=600*cos(10*t),x(0) = 10, D(x)(0) = 0],x(t), si
```

$$x(t) = \frac{(25790\cos(5t) - 842\sin(5t))\,\mathrm{e}^{-t}}{1469} - \frac{11100\cos(10t)}{1469} + \frac{3000\sin(10t)}{1469}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 45

```
DSolve[{x''[t]+2*x'[t]+26*x[t]==600*Cos[10*t],{x[0]==10,x'[0]==0}},x[t],t,IncludeSingularSol
```

$$x(t) \to -\frac{2e^{-t}(421\sin(5t) - 1500e^t\sin(10t) - 12895\cos(5t) + 5550e^t\cos(10t))}{1469}$$

12.14 problem 14

Internal problem ID [271]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 5.6, Forced Oscillations and Resonance. Page 362
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 8x' + 25x = 200\cos(t) + 520\sin(t)}$$

With initial conditions

$$[x(0) = -30, x'(0) = -10]$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 31

```
dsolve([diff(x(t),t$2)+8*diff(x(t),t)+25*x(t)=200*cos(t)+520*sin(t),x(0) = -30, D(x)(0) = -1
```

$$x(t) = (-31\cos(3t) - 52\sin(3t))\,\mathrm{e}^{-4t} + 22\sin(t) + \cos(t)$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 34

```
DSolve[{x''[t]+8*x'[t]+25*x[t]==200*Cos[t]+520*Sin[t],{x[0]==-30,x'[0]==-10}},x[t],t,Include
```

$$x(t) \to 22\sin(t) - 52e^{-4t}\sin(3t) + \cos(t) - 31e^{-4t}\cos(3t)$$

13 Section 7.2, Matrices and Linear systems. Page 417

13.1 problem problem 3

Internal problem ID [272]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 7.2, Matrices and Linear systems. Page 417
Problem number: problem 3.
ODE order: 1.
ODE degree: 1.

Solve

$$x' = -3y(t)$$
$$y'(t) = 3x$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 35

```
dsolve([diff(x(t),t)=-3*y(t),diff(y(t),t)=3*x(t)],singsol=all)
```

$$x(t) = c_1 \sin(3t) + c_2 \cos(3t)$$
$$y(t) = -c_1 \cos(3t) + c_2 \sin(3t)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 68

```
DSolve[{x'[t]==3*y[t],y'[t]==3*x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{2}e^{-3t}\left(c_1\left(e^{6t}+1\right)+c_2\left(e^{6t}-1\right)\right)$$
$$y(t) \to \frac{1}{2}e^{-3t}\left(c_1\left(e^{6t}-1\right)+c_2\left(e^{6t}+1\right)\right)$$

13.2 problem problem 4

Internal problem ID [273]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 7.2, Matrices and Linear systems. Page 417
Problem number: problem 4.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x' &= 3x - 2y(t) \\
y'(t) &= 2x + y(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 76

```
dsolve([diff(x(t),t)=3*x(t)-2*y(t),diff(y(t),t)=2*x(t)+y(t)],singsol=all)
```

$$
\begin{aligned}
x(t) &= \mathrm{e}^{2t}\left(\sin\left(\sqrt{3}\,t\right) c_1 + \cos\left(\sqrt{3}\,t\right) c_2\right) \\
y(t) &= \frac{\mathrm{e}^{2t}\left(\sin\left(\sqrt{3}\,t\right)\sqrt{3}\,c_2 - \cos\left(\sqrt{3}\,t\right)\sqrt{3}\,c_1 + \sin\left(\sqrt{3}\,t\right) c_1 + \cos\left(\sqrt{3}\,t\right) c_2\right)}{2}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 96

```
DSolve[{x'[t]==3*x[t]-2*y[t],y'[t]==2*x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> T
```

$$
\begin{aligned}
x(t) &\to \frac{1}{3}e^{2t}\left(3c_1\cos\left(\sqrt{3}t\right) + \sqrt{3}(c_1 - 2c_2)\sin\left(\sqrt{3}t\right)\right) \\
y(t) &\to \frac{1}{3}e^{2t}\left(3c_2\cos\left(\sqrt{3}t\right) + \sqrt{3}(2c_1 - c_2)\sin\left(\sqrt{3}t\right)\right)
\end{aligned}
$$

13.3 problem problem 5

Internal problem ID [274]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 7.2, Matrices and Linear systems. Page 417
Problem number: problem 5.
ODE order: 1.
ODE degree: 1.

Solve

$$x' = 2x + 4y(t) + 3\,\mathrm{e}^{t}$$
$$y'(t) = 5x - y(t) - t^{2}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 112

```
dsolve([diff(x(t),t)=2*x(t)+4*y(t)+3*exp(t),diff(y(t),t)=5*x(t)-y(t)-t^2],singsol=all)
```

$$x(t) = \frac{\mathrm{e}^{\frac{(1+\sqrt{89})t}{2}} c_2 \sqrt{89}}{10} - \frac{\mathrm{e}^{-\frac{(-1+\sqrt{89})t}{2}} c_1 \sqrt{89}}{10} + \frac{3\,\mathrm{e}^{\frac{(1+\sqrt{89})t}{2}} c_2}{10} + \frac{3\,\mathrm{e}^{-\frac{(-1+\sqrt{89})t}{2}} c_1}{10} + \frac{2t^2}{11} - \frac{3\,\mathrm{e}^{t}}{11} - \frac{2t}{121} + \frac{23}{1331}$$
$$y(t) = \mathrm{e}^{\frac{(1+\sqrt{89})t}{2}} c_2 + \mathrm{e}^{-\frac{(-1+\sqrt{89})t}{2}} c_1 - \frac{t^2}{11} - \frac{15\,\mathrm{e}^{t}}{22} + \frac{12t}{121} - \frac{17}{1331}$$

✓ Solution by Mathematica

Time used: 0.711 (sec). Leaf size: 212

```
DSolve[{x'[t]==2*x[t]+4*y[t]+3*Exp[t],y'[t]==5*x[t]-y[t]-t^2},{x[t],y[t]},t,IncludeSingularS
```

$$x(t) \to \frac{242t^2 - 22t + 23}{1331} - \frac{3e^t}{11} + \frac{1}{178}\left(\left(89 - 3\sqrt{89}\right)c_1 - 8\sqrt{89}c_2\right)e^{-\frac{1}{2}\left(\sqrt{89}-1\right)t} + \frac{1}{178}\left(\left(89 + 3\sqrt{89}\right)c_1 + 8\sqrt{89}c_2\right)e^{\frac{1}{2}\left(1+\sqrt{89}\right)t}$$
$$y(t) \to \frac{-121t^2 + 132t - 17}{1331} - \frac{15e^t}{22} + \left(\frac{5c_1}{\sqrt{89}} + \frac{1}{178}\left(89 - 3\sqrt{89}\right)c_2\right)e^{\frac{1}{2}\left(1+\sqrt{89}\right)t} + \left(\frac{1}{178}\left(89 + 3\sqrt{89}\right)c_2 - \frac{5c_1}{\sqrt{89}}\right)e^{-\frac{1}{2}\left(\sqrt{89}-1\right)t}$$

13.4 problem problem 7

Internal problem ID [275]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 7.2, Matrices and Linear systems. Page 417
Problem number: problem 7.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x' &= y(t) + z(t) \\
y'(t) &= z(t) + x \\
z'(t) &= x + y(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 64

```
dsolve([diff(x(t),t)=y(t)+z(t),diff(y(t),t)=z(t)+x(t),diff(z(t),t)=x(t)+y(t)],singsol=all)
```

$$
\begin{aligned}
x(t) &= c_2 e^{-t} + c_3 e^{2t} \\
y(t) &= c_2 e^{-t} + c_3 e^{2t} + e^{-t} c_1 \\
z(t) &= -2c_2 e^{-t} + c_3 e^{2t} - e^{-t} c_1
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 124

```
DSolve[{x'[t]==y[t]+z[t],y'[t]==z[t]+x[t],z'[t]==x[t]+y[t]},{x[t],y[t],z[t]},t,IncludeSingul
```

$$
\begin{aligned}
x(t) &\to \frac{1}{3} e^{-t} \left(c_1 \left(e^{3t} + 2\right) + (c_2 + c_3)\left(e^{3t} - 1\right)\right) \\
y(t) &\to \frac{1}{3} e^{-t} \left(c_1 \left(e^{3t} - 1\right) + c_2 \left(e^{3t} + 2\right) + c_3 \left(e^{3t} - 1\right)\right) \\
z(t) &\to \frac{1}{3} e^{-t} \left(c_1 \left(e^{3t} - 1\right) + c_2 \left(e^{3t} - 1\right) + c_3 \left(e^{3t} + 2\right)\right)
\end{aligned}
$$

13.5 problem problem 11

Internal problem ID [276]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 7.2, Matrices and Linear systems. Page 417
Problem number: problem 11.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= x_2(t) \\
x_2'(t) &= 2x_3(t) \\
x_3'(t) &= 3x_4(t) \\
x_4'(t) &= 4x_1(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 170

```
dsolve([diff(x__1(t),t)=x__2(t),diff(x__2(t),t)=2*x__3(t),diff(x__3(t),t)=3*x__4(t),diff(x__
```

$$
\begin{aligned}
x_1(t) &= c_1 e^{-24^{\frac{1}{4}} t} + c_2 e^{24^{\frac{1}{4}} t} - c_3 \sin\left(24^{\frac{1}{4}} t\right) + c_4 \cos\left(24^{\frac{1}{4}} t\right) \\
x_2(t) &= -24^{\frac{1}{4}} \left(c_1 e^{-24^{\frac{1}{4}} t} - c_2 e^{24^{\frac{1}{4}} t} + \cos\left(24^{\frac{1}{4}} t\right) c_3 + \sin\left(24^{\frac{1}{4}} t\right) c_4\right) \\
x_3(t) &= \sqrt{6} \left(c_1 e^{-24^{\frac{1}{4}} t} + c_2 e^{24^{\frac{1}{4}} t} - c_4 \cos\left(24^{\frac{1}{4}} t\right) + c_3 \sin\left(24^{\frac{1}{4}} t\right)\right) \\
x_4(t) &= -\frac{24^{\frac{3}{4}} \left(c_1 e^{-24^{\frac{1}{4}} t} - c_2 e^{24^{\frac{1}{4}} t} - \cos\left(24^{\frac{1}{4}} t\right) c_3 - \sin\left(24^{\frac{1}{4}} t\right) c_4\right)}{6}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 400

```
DSolve[{x1'[t]==x2[t],x2'[t]==2*x3[t],x3'[t]==3*x4[t],x4'[t]==4*x1[t]},{x1[t],x2[t],x3[t],x4
```

$$\begin{aligned}
\mathrm{x1}(t) \to & \frac{1}{4}c_1\mathrm{RootSum}\left[\#1^4-24\&,e^{\#1t}\&\right]+\frac{1}{4}c_2\mathrm{RootSum}\left[\#1^4-24\&,\frac{e^{\#1t}}{\#1}\&\right] \\
& +\frac{3}{2}c_4\mathrm{RootSum}\left[\#1^4-24\&,\frac{e^{\#1t}}{\#1^3}\&\right]+\frac{1}{2}c_3\mathrm{RootSum}\left[\#1^4-24\&,\frac{e^{\#1t}}{\#1^2}\&\right] \\
\mathrm{x2}(t) \to & \frac{1}{4}c_2\mathrm{RootSum}\left[\#1^4-24\&,e^{\#1t}\&\right]+\frac{1}{2}c_3\mathrm{RootSum}\left[\#1^4-24\&,\frac{e^{\#1t}}{\#1}\&\right] \\
& +6c_1\mathrm{RootSum}\left[\#1^4-24\&,\frac{e^{\#1t}}{\#1^3}\&\right]+\frac{3}{2}c_4\mathrm{RootSum}\left[\#1^4-24\&,\frac{e^{\#1t}}{\#1^2}\&\right] \\
\mathrm{x3}(t) \to & \frac{1}{4}c_3\mathrm{RootSum}\left[\#1^4-24\&,e^{\#1t}\&\right]+\frac{3}{4}c_4\mathrm{RootSum}\left[\#1^4-24\&,\frac{e^{\#1t}}{\#1}\&\right] \\
& +3c_2\mathrm{RootSum}\left[\#1^4-24\&,\frac{e^{\#1t}}{\#1^3}\&\right]+3c_1\mathrm{RootSum}\left[\#1^4-24\&,\frac{e^{\#1t}}{\#1^2}\&\right] \\
\mathrm{x4}(t) \to & \frac{1}{4}c_4\mathrm{RootSum}\left[\#1^4-24\&,e^{\#1t}\&\right]+c_1\mathrm{RootSum}\left[\#1^4-24\&,\frac{e^{\#1t}}{\#1}\&\right] \\
& +2c_3\mathrm{RootSum}\left[\#1^4-24\&,\frac{e^{\#1t}}{\#1^3}\&\right]+c_2\mathrm{RootSum}\left[\#1^4-24\&,\frac{e^{\#1t}}{\#1^2}\&\right]
\end{aligned}$$

13.6 problem problem 12

Internal problem ID [277]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney
Section: Section 7.2, Matrices and Linear systems. Page 417
Problem number: problem 12.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= x_2(t) + x_3(t) + 1 \\
x_2'(t) &= x_3(t) + x_4(t) + t \\
x_3'(t) &= x_1(t) + x_4(t) + t^2 \\
x_4'(t) &= x_1(t) + x_2(t) + t^3
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 273

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dsolve([diff(x__1(t),t)=x__2(t)+x__3(t)+1,diff(x__2(t),t)=x__3(t)+x__4(t)+t,diff(x__3(t),t)=
```

$$
\begin{aligned}
x_1(t) &= \frac{t^2}{16} - \frac{7t^3}{24} - \frac{t^4}{16} + \frac{c_1 e^{2t}}{2} - \frac{11t}{16} + c_4 + \frac{e^{-t}\sin(t)c_2}{2} \\
&\quad - \frac{e^{-t}\sin(t)c_3}{2} - \frac{e^{-t}\cos(t)c_2}{2} - \frac{e^{-t}\cos(t)c_3}{2} \\
x_2(t) &= \frac{t^4}{16} + \frac{e^{-t}\sin(t)c_2}{2} + \frac{e^{-t}\sin(t)c_3}{2} + \frac{e^{-t}\cos(t)c_2}{2} \\
&\quad - \frac{e^{-t}\cos(t)c_3}{2} - \frac{11t^3}{24} + \frac{c_1 e^{2t}}{2} + \frac{t^2}{16} - c_4 - \frac{3t}{16} - \frac{19}{16} \\
x_3(t) &= -\frac{t^4}{16} - \frac{e^{-t}\sin(t)c_2}{2} + \frac{e^{-t}\sin(t)c_3}{2} + \frac{e^{-t}\cos(t)c_2}{2} \\
&\quad + \frac{e^{-t}\cos(t)c_3}{2} + \frac{5t^3}{24} + \frac{c_1 e^{2t}}{2} - \frac{15t^2}{16} + c_4 + \frac{5t}{16} - \frac{1}{2} \\
x_4(t) &= \frac{t^4}{16} - \frac{e^{-t}\sin(t)c_2}{2} - \frac{e^{-t}\sin(t)c_3}{2} - \frac{e^{-t}\cos(t)c_2}{2} \\
&\quad + \frac{e^{-t}\cos(t)c_3}{2} + \frac{t^3}{24} + \frac{c_1 e^{2t}}{2} - \frac{7t^2}{16} - c_4 - \frac{19t}{16} + \frac{5}{16}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 1.491 (sec). Leaf size: 442

```
DSolve[{x1'[t]==x2[t]+x3[t]+1,x2'[t]==x3[t]+x4[t]+t,x3'[t]==x1[t]+x4[t]+t^2,x4'[t]==x1[t]+x2
```

$$\begin{aligned}
\mathrm{x1}(t) \to \frac{1}{96}e^{-t}\Big(e^t\big(-6t^4 - 28t^3 + 6t^2 - 66t \\
+ 3\big(8c_1\big(e^{2t}+1\big) + 8c_2\big(e^{2t}-1\big) + 8c_3e^{2t} + 8c_4e^{2t} - 3 + 8c_3 - 8c_4\big)\big) \\
+ 48(c_1 - c_3)\cos(t) + 48(c_2 - c_4)\sin(t)\Big)
\end{aligned}$$

$$\begin{aligned}
\mathrm{x2}(t) \to \frac{1}{96}e^{-t}\Big(e^t\big(6t^4 - 44t^3 + 6t^2 - 18t \\
+ 3\big(8c_1\big(e^{2t}-1\big) + 8c_2\big(e^{2t}+1\big) + 8c_3e^{2t} + 8c_4e^{2t} - 35 - 8c_3 + 8c_4\big)\big) \\
+ 48(c_2 - c_4)\cos(t) - 48(c_1 - c_3)\sin(t)\Big)
\end{aligned}$$

$$\begin{aligned}
\mathrm{x3}(t) \to \frac{1}{96}e^{-t}\Big(e^t\big(-6t^4 + 20t^3 - 90t^2 + 30t \\
+ 3\big(8c_1\big(e^{2t}+1\big) + 8c_2\big(e^{2t}-1\big) + 8c_3e^{2t} + 8c_4e^{2t} - 19 + 8c_3 - 8c_4\big)\big) \\
- 48(c_1 - c_3)\cos(t) - 48(c_2 - c_4)\sin(t)\Big)
\end{aligned}$$

$$\begin{aligned}
\mathrm{x4}(t) \to \frac{1}{96}e^{-t}\Big(e^t\big(6t^4 + 4t^3 - 42t^2 - 114t \\
+ 3\big(8c_1\big(e^{2t}-1\big) + 8c_2\big(e^{2t}+1\big) + 8c_3e^{2t} + 8c_4e^{2t} + 13 - 8c_3 + 8c_4\big)\big) \\
- 48(c_2 - c_4)\cos(t) + 48(c_1 - c_3)\sin(t)\Big)
\end{aligned}$$