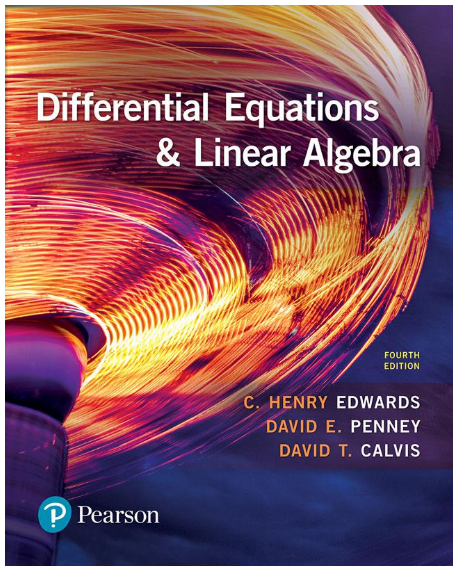
A Solution Manual For

Differential equations and linear algebra, 4th ed., Edwards and Penney



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Contents

1 Section 5.2, Higher-Order Linear Differential Equations. General solutions of Linear Equations. Page 288

1.1 problem problem 38

Internal problem ID [278]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.2, Higher-Order Linear Differential Equations. General solutions of Linear Equations. Page 288
Problem number: problem 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' + y'x - 9y = 0}$$

Given that one solution of the ode is

$$y_1 = x^3$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)+x*diff(y(x),x)-9*y(x)=0,x^3],singsol=all)
```

$$y(x) = \frac{c_2x^6 + c_1}{x^3}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+x*y'[x]-9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^6 + c_1}{x^3}$$

1.2 problem problem 39

Internal problem ID [279]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.2, Higher-Order Linear Differential Equations. General solutions of Linear Equations. Page 288
Problem number: problem 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' - 4y' + y = 0}$$

Given that one solution of the ode is

$$y_1 = e^{\frac{x}{2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve([4*diff(y(x),x$2)-4*diff(y(x),x)+y(x)=0,exp(x/2)],singsol=all)
```

$$y(x) = e^{\frac{x}{2}}(c_2x + c_1)$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 20

```
DSolve[4*y''[x]-4*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{x/2}(c_2x + c_1)$$

1.3 problem problem 40

Internal problem ID [280]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.2, Higher-Order Linear Differential Equations. General solutions of Linear Equations. Page 288
Problem number: problem 40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(2 + x)\, y' + (2 + x)\, y = 0}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve([x^2*diff(y(x),x$2)-x*(x+2)*diff(y(x),x)+(x+2)*y(x)=0,x],singsol=all)
```

$$y(x) = x(c_1 + \mathrm{e}^x c_2)$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 16

```
DSolve[x^2*y''[x]-x*(x+2)*y'[x]+(x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(c_2 e^x + c_1)$$

1.4 problem problem 41

Internal problem ID [281]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.2, Higher-Order Linear Differential Equations. General solutions of Linear Equations. Page 288
Problem number: problem 41.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x+1)\,y'' - (2+x)\,y' + y = 0}$$

Given that one solution of the ode is

$$y_1 = \mathrm{e}^x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve([(x+1)*diff(y(x),x$2)-(x+2)*diff(y(x),x)+y(x)=0,exp(x)],singsol=all)
```

$$y(x) = c_1(2+x) + \mathrm{e}^x c_2$$

✓ Solution by Mathematica
Time used: 0.143 (sec). Leaf size: 29

```
DSolve[(x+1)*y''[x]-(x+2)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 e^{x+1} - 2c_2(x+2)}{\sqrt{2e}}$$

1.5 problem problem 42

Internal problem ID [282]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.2, Higher-Order Linear Differential Equations. General solutions of Linear Equations. Page 288
Problem number: problem 42.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-x^2+1)\,y''+2y'x-2y=0}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve([(1-x^2)*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=0,x],singsol=all)
```

$$y(x) = c_2x^2 + c_1x + c_2$$

✓ Solution by Mathematica
Time used: 0.08 (sec). Leaf size: 39

```
DSolve[(1-x^2)*y''[x]+2*x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{x^2-1}(c_1(x-1)^2+c_2x)}{\sqrt{1-x^2}}$$

1.6 problem problem 43

Internal problem ID [283]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.2, Higher-Order Linear Differential Equations. General solutions of Linear Equations. Page 288
Problem number: problem 43.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-x^2+1)\,y''-2y'x+2y=0}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve([(1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,x],singsol=all)
```

$$y(x) = \frac{c_2 \ln(x-1)\, x}{2} - \frac{c_2 \ln(x+1)\, x}{2} + c_1 x + c_2$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 33

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x - \frac{1}{2} c_2 (x \log(1-x) - x \log(x+1) + 2)$$

1.7 problem problem 44

Internal problem ID [284]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.2, Higher-Order Linear Differential Equations. General solutions of Linear Equations. Page 288
Problem number: problem 44.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + y'x + \left(x^2 - \frac{1}{4}\right)y = 0$$

Given that one solution of the ode is

$$y_1 = \frac{\cos(x)}{\sqrt{x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve([x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x)=0,x^(-1/2)*cos(x)],singsol=all)
```

$$y(x) = \frac{c_1 \sin(x) + c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 33

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x - \frac{1}{2}c_2(x\log(1-x) - x\log(x+1) + 2)$$

2 Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300

2.1 problem problem 10

Internal problem ID [285]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300
Problem number: problem 10.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{5y'''' + 3y''' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(5*diff(y(x),x$4)+3*diff(y(x),x$3)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 x + c_3 x^2 + c_4 \mathrm{e}^{-\frac{3x}{5}}$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 30

```
DSolve[5*y''''[x]+3*y'''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{125}{27} c_1 e^{-3x/5} + x(c_4 x + c_3) + c_2$$

2.2 problem problem 11

Internal problem ID [286]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300
Problem number: problem 11.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 8y''' + 16y'' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$4)-8*diff(y(x),x$3)+16*diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = (c_4x + c_3)\,e^{4x} + c_2x + c_1$$

✓ Solution by Mathematica
Time used: 0.081 (sec). Leaf size: 34

```
DSolve[y''''[x]-8*y'''[x]+16*y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{32}e^{4x}(c_2(2x - 1) + 2c_1) + c_4x + c_3$$

2.3 problem problem 12

Internal problem ID [287]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300
Problem number: problem 12.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 3y''' + 3y'' - y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$4)-3*diff(y(x),x$3)+3*diff(y(x),x$2)-diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = (c_4x^2 + c_3x + c_2)\,\mathrm{e}^x + c_1$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 32

```
DSolve[y''''[x]-3*y'''[x]+3*y''[x]-y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x(c_3(x^2 - 2x + 2) + c_2(x - 1) + c_1) + c_4$$

2.4 problem problem 13

Internal problem ID [288]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300
Problem number: problem 13.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{9y''' + 12y'' + 4y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve(9*diff(y(x),x$3)+12*diff(y(x),x$2)+4*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = (c_3x + c_2)\,\mathrm{e}^{-\frac{2x}{3}} + c_1$$

✓ Solution by Mathematica
Time used: 0.047 (sec). Leaf size: 32

```
DSolve[9*y'''[x]+12*y''[x]+4*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3 - \frac{3}{4}e^{-2x/3}(c_2(2x+3) + 2c_1)$$

2.5 problem problem 14

Internal problem ID [289]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300
Problem number: problem 14.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 3y'' - 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$4)+3*diff(y(x),x$2)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x} c_1 + c_2 \mathrm{e}^{-x} + c_3 \sin(2x) + c_4 \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 34

```
DSolve[y''''[x]+3*y''[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3 e^{-x} + c_4 e^{x} + c_1 \cos(2x) + c_2 \sin(2x)$$

2.6 problem problem 15

Internal problem ID [290]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300
Problem number: problem 15.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 16y'' + 16y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 59

```
dsolve(diff(y(x),x$4)-16*diff(y(x),x$2)+16*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\sqrt{2}\left(1+\sqrt{3}\right)x} + c_2 \mathrm{e}^{\sqrt{2}\left(1+\sqrt{3}\right)x} + c_3 \mathrm{e}^{-\sqrt{2}\left(\sqrt{3}-1\right)x} + c_4 \mathrm{e}^{\sqrt{2}\left(\sqrt{3}-1\right)x}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 86

```
DSolve[y''''[x]-16*y''[x]+16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{2\sqrt{2-\sqrt{3}}x} + c_2 e^{-2\sqrt{2-\sqrt{3}}x} + c_3 e^{2\sqrt{2+\sqrt{3}}x} + c_4 e^{-2\sqrt{2+\sqrt{3}}x}$$

2.7 problem problem 16

Internal problem ID [291]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300
Problem number: problem 16.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 18y'' + 81y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$4)+18*diff(y(x),x$2)+81*y(x)=0,y(x), singsol=all)
```

$$y(x) = (c_4x + c_2)\cos(3x) + \sin(3x)(c_3x + c_1)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[y''''[x]+18*y''[x]+81*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (c_2x + c_1)\cos(3x) + (c_4x + c_3)\sin(3x)$$

2.8 problem problem 17

Internal problem ID [292]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300
Problem number: problem 17.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{6y'''' + 11y'' + 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 41

```
dsolve(6*diff(y(x),x$4)+11*diff(y(x),x$2)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(\frac{2\sqrt{3}\,x}{3}\right) + c_2 \cos\left(\frac{2\sqrt{3}\,x}{3}\right) + c_3 \sin\left(\frac{\sqrt{2}\,x}{2}\right) + c_4 \cos\left(\frac{\sqrt{2}\,x}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 94

```
DSolve[y''''[x]+11*y''[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \to c_3 \cos\left(\sqrt{\frac{1}{2}\left(11-\sqrt{105}\right)}x\right) + c_1 \cos\left(\sqrt{\frac{1}{2}\left(11+\sqrt{105}\right)}x\right) \\ + c_4 \sin\left(\sqrt{\frac{1}{2}\left(11-\sqrt{105}\right)}x\right) + c_2 \sin\left(\sqrt{\frac{1}{2}\left(11+\sqrt{105}\right)}x\right) \end{aligned}$$

2.9 problem problem 18

Internal problem ID [293]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300
Problem number: problem 18.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 16y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$4)=16*y(x),y(x), singsol=all)
```

$$y(x) = e^{2x}c_1 + c_2 e^{-2x} + c_3 \sin(2x) + c_4 \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 36

```
DSolve[y''''[x]==16*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{2x} + c_3 e^{-2x} + c_2 \cos(2x) + c_4 \sin(2x)$$

2.10 problem problem 19

Internal problem ID [294]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300
Problem number: problem 19.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + y'' - y' - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$3)+diff(y(x),x$2)-diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = (c_3 x + c_2)\,e^{-x} + e^{x} c_1$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 26

```
DSolve[y'''[x]+y''[x]-y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_2 x + c_3 e^{2x} + c_1\right)$$

2.11 problem problem 20

Internal problem ID [295]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300
Problem number: problem 20.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 2y''' + 3y'' + 2y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$3)+3*diff(y(x),x$2)+2*diff(y(x),x)+y(x)=0,y(x), singsol=
```

$$y(x) = e^{-\frac{x}{2}}\left((c_4x + c_2)\cos\left(\frac{\sqrt{3}\,x}{2}\right) + \sin\left(\frac{\sqrt{3}\,x}{2}\right)(c_3x + c_1)\right)$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 52

```
DSolve[y''''[x]+2*y'''[x]+3*y''[x]+2*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}\left((c_4x + c_3)\cos\left(\frac{\sqrt{3}x}{2}\right) + (c_2x + c_1)\sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

2.12 problem problem 24

Internal problem ID [296]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300
Problem number: problem 24.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{2y''' - 3y'' - 2y' = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -1, y''(0) = 3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve([2*diff(y(x),x$3)-3*diff(y(x),x$2)-2*diff(y(x),x)=0,y(0) = 1, D(y)(0) = -1, (D@@2)(y)
```

$$y(x) = -\frac{7}{2} + 4\,\mathrm{e}^{-\frac{x}{2}} + \frac{\mathrm{e}^{2x}}{2}$$

✓ Solution by Mathematica

Time used: 0.351 (sec). Leaf size: 70

```
DSolve[{2*y'''[x]-3*y''[x]-3*y'[x]==0,{y[0]==1,y'[0]==-1,y''[0]==3}},y[x],x,IncludeSingularS
```

$$y(x) \to \frac{1}{66}e^{-\frac{1}{4}\left(\sqrt{33}-3\right)x}\left(\left(99 - 13\sqrt{33}\right)e^{\frac{\sqrt{33}x}{2}} - 132e^{\frac{1}{4}\left(\sqrt{33}-3\right)x} + 99 + 13\sqrt{33}\right)$$

2.13 problem problem 25

Internal problem ID [297]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300
Problem number: problem 25.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{3y''' + 2y'' = 0}$$

With initial conditions

$$[y(0) = -1, y'(0) = 0, y''(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([3*diff(y(x),x$3)+2*diff(y(x),x$2)=0,y(0) = -1, D(y)(0) = 0, (D@@2)(y)(0) = 1],y(x),
```

$$y(x) = -\frac{13}{4} + \frac{3x}{2} + \frac{9\,e^{-\frac{2x}{3}}}{4}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 23

```
DSolve[{3*y'''[x]+2*y''[x]==0,{y[0]==1,y'[0]==-1,y''[0]==3}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{4}\left(14x + 27e^{-2x/3} - 23\right)$$

2.14 problem problem 26

Internal problem ID [298]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300
Problem number: problem 26.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 10y'' + 25y' = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = 4, y''(0) = 5]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$3)+10*diff(y(x),x$2)+25*diff(y(x),x)=0,y(0) = 3, D(y)(0) = 4, (D@@2)(y)(
```

$$y(x) = \frac{24}{5} - \frac{9\,\mathrm{e}^{-5x}}{5} - 5\,\mathrm{e}^{-5x}x$$

✓ Solution by Mathematica
Time used: 0.045 (sec). Leaf size: 26

```
DSolve[{y'''[x]+10*y''[x]+25*y'[x]==0,{y[0]==3,y'[0]==4,y''[0]==5}},y[x],x,IncludeSingularSo
```

$$y(x) \to \frac{1}{5}e^{-5x}\left(-25x + 24e^{5x} - 9\right)$$

2.15 problem problem 27

Internal problem ID [299]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300
Problem number: problem 27.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 3y'' - 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$3)+3*diff(y(x),x$2)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_1 e^{3x} + c_3 x + c_2\right) e^{-2x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 26

```
DSolve[y'''[x]+3*y''[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(c_2 x + c_3 e^{3x} + c_1\right)$$

2.16 problem problem 28

Internal problem ID [300]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300
Problem number: problem 28.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{2y''' - y'' - 5y' - 2y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve(2*diff(y(x),x$3)-diff(y(x),x$2)-5*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_2 \mathrm{e}^{3x} + c_1 \mathrm{e}^{\frac{x}{2}} + c_3\right) \mathrm{e}^{-x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 32

```
DSolve[2*y'''[x]-y''[x]-5*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_1 e^{x/2} + c_3 e^{3x} + c_2\right)$$

2.17 problem problem 29

Internal problem ID [301]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300
Problem number: problem 29.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 27y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$3)+27*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_2 e^{\frac{9x}{2}} \sin\left(\frac{3\sqrt{3}\,x}{2}\right) + c_3 e^{\frac{9x}{2}} \cos\left(\frac{3\sqrt{3}\,x}{2}\right) + c_1\right) e^{-3x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 56

```
DSolve[y'''[x]+27*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(c_3 e^{9x/2} \cos\left(\frac{3\sqrt{3}x}{2}\right) + c_2 e^{9x/2} \sin\left(\frac{3\sqrt{3}x}{2}\right) + c_1\right)$$

2.18 problem problem 30

Internal problem ID [302]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300
Problem number: problem 30.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' - y''' + y'' - 3y' - 6y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$4)-diff(y(x),x$3)+diff(y(x),x$2)-3*diff(y(x),x)-6*y(x)=0,y(x), singsol=al
```

$$y(x) = e^{2x}c_1 + c_2e^{-x} + c_3 \sin\left(\sqrt{3}\, x\right) + c_4 \cos\left(\sqrt{3}\, x\right)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 44

```
DSolve[y''''[x]-y'''[x]+y''[x]-3*y'[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3e^{-x} + c_4e^{2x} + c_1 \cos\left(\sqrt{3}x\right) + c_2 \sin\left(\sqrt{3}x\right)$$

2.19 problem problem 31

Internal problem ID [303]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300
Problem number: problem 31.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 3y'' + 4y' - 8y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$3)+3*diff(y(x),x$2)+4*diff(y(x),x)-8*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_1 e^{3x} + \sin(2x) c_2 + \cos(2x) c_3\right) e^{-2x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 34

```
DSolve[y'''[x]+3*y''[x]+4*y'[x]-8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(c_3 e^{3x} + c_2 \cos(2x) + c_1 \sin(2x)\right)$$

2.20 problem problem 32

Internal problem ID [304]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300
Problem number: problem 32.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + y''' - 3y'' - 5y' - 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$4)+diff(y(x),x$3)-3*diff(y(x),x$2)-5*diff(y(x),x)-2*y(x)=0,y(x), singsol=
```

$$y(x) = (c_4x^2 + c_3x + c_2)\,\mathrm{e}^{-x} + \mathrm{e}^{2x}c_1$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 32

```
DSolve[y''''[x]+y'''[x]-3*y''[x]-5*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_3x^2 + c_2x + c_4e^{3x} + c_1)$$

2.21 problem problem 38

Internal problem ID [305]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300
Problem number: problem 38.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 5y'' + 100y' - 500y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 10, y''(0) = 250]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$3)-5*diff(y(x),x$2)+100*diff(y(x),x)-500*y(x)=0,y(0) = 0, D(y)(0) = 10,
```

$$y(x) = 2\,\mathrm{e}^{5x} - 2\cos(10x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 19

```
DSolve[{y'''[x]-5*y''[x]+100*y'[x]-500*y[x]==0,{y[0]==0,y'[0]==10,y''[0]==250}},y[x],x,Inclu
```

$$y(x) \to 2\left(e^{5x} - \cos(10x)\right)$$

2.22 problem problem 48

Internal problem ID [306]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300
Problem number: problem 48.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0, y''(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 22

```
dsolve([diff(y(x),x$3)=y(x),y(0) = 1, D(y)(0) = 0, (D@@2)(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{e^{x}}{3} + \frac{2\,e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right)}{3}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 33

```
DSolve[{y'''[x]==y[x],{y[0]==1,y'[0]==0,y''[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(e^{x} + 2e^{-x/2}\cos\left(\frac{\sqrt{3}x}{2}\right)\right)$$

2.23 problem problem 49

Internal problem ID [307]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300
Problem number: problem 49.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - y''' - y'' - y' - 2y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0, y''(0) = 0, y'''(0) = 15]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$4)=diff(y(x),x$3)+diff(y(x),x$2)+diff(y(x),x)+2*y(x),y(0) = 0, D(y)(0) =
```

$$y(x) = e^{2x} - \frac{5\,e^{-x}}{2} - \frac{9\sin(x)}{2} + \frac{3\cos(x)}{2}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 33

```
DSolve[{y'''[x]==y[x],{y[0]==1,y'[0]==0,y''[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(e^x + 2e^{-x/2}\cos\left(\frac{\sqrt{3}x}{2}\right)\right)$$

2.24 problem problem 54

Internal problem ID [308]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300
Problem number: problem 54.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{x^3y''' + 6x^2y'' + 4y'x = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^3*diff(y(x),x$3)+6*x^2*diff(y(x),x$2)+4*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 \ln(x) + \frac{c_3}{x^3}$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 22

```
DSolve[x^3*y'''[x]+6*x^2*y''[x]+4*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{c_1}{3x^3} + c_2 \log(x) + c_3$$

2.25 problem problem 55

Internal problem ID [309]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300
Problem number: problem 55.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{x^3y''' - x^2y'' + y'x = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^3*diff(y(x),x$3)-x^2*diff(y(x),x$2)+x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2x^2 + c_3x^2 \ln(x)$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 35

```
DSolve[x^3*y'''[x]-x^2*y''[x]+x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}(2c_1 - c_2)x^2 + \frac{1}{2}c_2x^2 \log(x) + c_3$$

2.26 problem problem 56

Internal problem ID [310]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300
Problem number: problem 56.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$x^3 y''' + 3x^2 y'' + y' x = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 16

```
dsolve(x^3*diff(y(x),x$3)+3*x^2*diff(y(x),x$2)+x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_3 \ln(x)^2 + c_2 \ln(x) + c_1$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 23

```
DSolve[x^3*y'''[x]+3*x^2*y''[x]+x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} c_2 \log^2(x) + c_1 \log(x) + c_3$$

2.27 problem problem 57

Internal problem ID [311]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300
Problem number: problem 57.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{x^3y''' - 3x^2y'' + y'x = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 26

```
dsolve(x^3*diff(y(x),x$3)-3*x^2*diff(y(x),x$2)+x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2x^{3+\sqrt{3}} + c_3x^{3-\sqrt{3}}$$

✓ Solution by Mathematica
Time used: 0.136 (sec). Leaf size: 54

```
DSolve[x^3*y'''[x]-3*x^2*y''[x]+x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^{3+\sqrt{3}}}{3+\sqrt{3}} + \frac{c_1x^{3-\sqrt{3}}}{3-\sqrt{3}} + c_3$$

2.28 problem problem 58

Internal problem ID [312]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 5.3, Higher-Order Linear Differential Equations. Homogeneous Equations with Constant Coefficients. Page 300
Problem number: problem 58.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^3y''' + 6x^2y'' + 7y'x + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^3*diff(y(x),x$3)+6*x^2*diff(y(x),x$2)+7*x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_3 \ln(x)^2 + c_2 \ln(x) + c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 24

```
DSolve[x^3*y'''[x]+6*x^2*y''[x]+7*x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_3 \log^2(x) + c_2 \log(x) + c_1}{x}$$

3 Section 7.2, Matrices and Linear systems. Page 384

3.1 problem problem 13

Internal problem ID [313]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.2, Matrices and Linear systems. Page 384
Problem number: problem 13.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 6x_1(t)$$
$$x_2'(t) = -3x_1(t) - x_2(t)$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 28

```
dsolve([diff(x__1(t),t)=4*x__1(t)+2*x__1(t),diff(x__2(t),t)=-3*x__1(t)-x__2(t)],singsol=all)
```

$$x_1(t) = c_2 e^{6t}$$
$$x_2(t) = -\frac{3c_2 e^{6t}}{7} + e^{-t} c_1$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 56

```
DSolve[{x1'[t]==4*x1[t]+2*x2[t],x2'[t]==-3*x1[t]-x2[t]},{x1[t],x2[t]},t,IncludeSingularSolut
```

$$\text{x1}(t) \to e^t \left(c_1 \left(3e^t - 2\right) + 2c_2 \left(e^t - 1\right)\right)$$
$$\text{x2}(t) \to e^t \left(c_2 \left(3 - 2e^t\right) - 3c_1 \left(e^t - 1\right)\right)$$

3.2 problem problem 14

Internal problem ID [314]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.2, Matrices and Linear systems. Page 384
Problem number: problem 14.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= -3x_1(t) + 2x_2(t) \\
x_2'(t) &= -3x_1(t) + 4x_2(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve([diff(x__1(t),t)=-3*x__1(t)+2*x__2(t),diff(x__2(t),t)=-3*x__1(t)+4*x__2(t)],singsol=a
```

$$
\begin{aligned}
x_1(t) &= c_1 e^{3t} + c_2 e^{-2t} \\
x_2(t) &= 3c_1 e^{3t} + \frac{c_2 e^{-2t}}{2}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 73

```
DSolve[{x1'[t]==-3*x1[t]+2*x2[t],x2'[t]==-3*x1[t]+4*x2[t]},{x1[t],x2[t]},t,IncludeSingularSo
```

$$
\begin{aligned}
\mathrm{x1}(t) &\to \frac{1}{5}e^{-2t}\left(2c_2\left(e^{5t}-1\right) - c_1\left(e^{5t}-6\right)\right) \\
\mathrm{x2}(t) &\to \frac{1}{5}e^{-2t}\left(c_2\left(6e^{5t}-1\right) - 3c_1\left(e^{5t}-1\right)\right)
\end{aligned}
$$

4 Section 7.3, The eigenvalue method for linear systems. Page 395

4.1 problem problem 1

Internal problem ID [315]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 1.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) + 2x_2(t)$$
$$x_2'(t) = 2x_1(t) + x_2(t)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve([diff(x__1(t),t)=x__1(t)+2*x__2(t),diff(x__2(t),t)=2*x__1(t)+x__2(t)],singsol=all)
```

$$x_1(t) = c_1 \mathrm{e}^{3t} + c_2 \mathrm{e}^{-t}$$
$$x_2(t) = c_1 \mathrm{e}^{3t} - c_2 \mathrm{e}^{-t}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 68

```
DSolve[{x1'[t]==x1[t]+2*x2[t],x2'[t]==2*x1[t]+x2[t]},{x1[t],x2[t]},t,IncludeSingularSolution
```

$$\mathrm{x1}(t) \to \frac{1}{2} e^{-t}\left(c_1\left(e^{4t}+1\right)+c_2\left(e^{4t}-1\right)\right)$$
$$\mathrm{x2}(t) \to \frac{1}{2} e^{-t}\left(c_1\left(e^{4t}-1\right)+c_2\left(e^{4t}+1\right)\right)$$

4.2 problem problem 2

Internal problem ID [316]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 2.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 2x_1(t) + 3x_2(t)$$
$$x_2'(t) = 2x_1(t) + x_2(t)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
dsolve([diff(x__1(t),t)=2*x__1(t)+3*x__2(t),diff(x__2(t),t)=2*x__1(t)+x__2(t)],singsol=all)
```

$$x_1(t) = c_1 e^{4t} + c_2 e^{-t}$$
$$x_2(t) = \frac{2c_1 e^{4t}}{3} - c_2 e^{-t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 74

```
DSolve[{x1'[t]==2*x1[t]+3*x2[t],x2'[t]==2*x1[t]+x2[t]},{x1[t],x2[t]},t,IncludeSingularSoluti
```

$$\text{x1}(t) \to \frac{1}{5}e^{-t}\left(c_1\left(3e^{5t}+2\right)+3c_2\left(e^{5t}-1\right)\right)$$
$$\text{x2}(t) \to \frac{1}{5}e^{-t}\left(2c_1\left(e^{5t}-1\right)+c_2\left(2e^{5t}+3\right)\right)$$

4.3 problem problem 3

Internal problem ID [317]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 3.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 3x_1(t) + 4x_2(t)$$
$$x_2'(t) = 3x_1(t) + 2x_2(t)$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve([diff(x__1(t),t) = 3*x__1(t)+4*x__2(t), diff(x__2(t),t) = 3*x__1(t)+2*x__2(t), x__1(0
```

$$x_1(t) = -\frac{e^{-t}}{7} + \frac{8\,e^{6t}}{7}$$
$$x_2(t) = \frac{e^{-t}}{7} + \frac{6\,e^{6t}}{7}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 44

```
DSolve[{x1'[t]==3*x1[t]+4*x2[t],x2'[t]==3*x1[t]+2*x2[t]},{x1[0]==1,x2[0]==1},{x1[t],x2[t]},t
```

$$\text{x1}(t) \to \frac{1}{7}e^{-t}\left(8e^{7t} - 1\right)$$
$$\text{x2}(t) \to \frac{1}{7}e^{-t}\left(6e^{7t} + 1\right)$$

4.4 problem problem 4

Internal problem ID [318]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 4.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 4x_1(t) + x_2(t)$$
$$x_2'(t) = 6x_1(t) - x_2(t)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve([diff(x__1(t),t)=4*x__1(t)+x__2(t),diff(x__2(t),t)=6*x__1(t)-x__2(t)],singsol=all)
```

$$x_1(t) = c_1 e^{-2t} + c_2 e^{5t}$$
$$x_2(t) = -6c_1 e^{-2t} + c_2 e^{5t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 71

```
DSolve[{x1'[t]==4*x1[t]+x2[t],x2'[t]==6*x1[t]-x2[t]},{x1[t],x2[t]},t,IncludeSingularSolution
```

$$\text{x1}(t) \to \frac{1}{7}e^{-2t}\left(c_1\left(6e^{7t}+1\right)+c_2\left(e^{7t}-1\right)\right)$$
$$\text{x2}(t) \to \frac{1}{7}e^{-2t}\left(6c_1\left(e^{7t}-1\right)+c_2\left(e^{7t}+6\right)\right)$$

4.5 problem problem 5

Internal problem ID [319]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 5.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 6x_1(t) - 7x_2(t)$$
$$x_2'(t) = x_1(t) - 2x_2(t)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve([diff(x__1(t),t)=6*x__1(t)-7*x__2(t),diff(x__2(t),t)=x__1(t)-2*x__2(t)],singsol=all)
```

$$x_1(t) = e^{-t}c_1 + c_2 e^{5t}$$
$$x_2(t) = e^{-t}c_1 + \frac{c_2 e^{5t}}{7}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 72

```
DSolve[{x1'[t]==6*x1[t]-7*x2[t],x2'[t]==x1[t]-2*x2[t]},{x1[t],x2[t]},t,IncludeSingularSoluti
```

$$\text{x1}(t) \to \frac{1}{6}e^{-t}\left(c_1\left(7e^{6t} - 1\right) - 7c_2\left(e^{6t} - 1\right)\right)$$
$$\text{x2}(t) \to \frac{1}{6}e^{-t}\left(c_1\left(e^{6t} - 1\right) - c_2\left(e^{6t} - 7\right)\right)$$

4.6 problem problem 6

Internal problem ID [320]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 6.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 9x_1(t) + 5x_2(t)$$
$$x_2'(t) = -6x_1(t) - 2x_2(t)$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
dsolve([diff(x__1(t),t) = 9*x__1(t)+5*x__2(t), diff(x__2(t),t) = -6*x__1(t)-2*x__2(t), x__1(
```

$$x_1(t) = 6\,e^{4t} - 5\,e^{3t}$$
$$x_2(t) = -6\,e^{4t} + 6\,e^{3t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 33

```
DSolve[{x1'[t]==9*x1[t]+5*x2[t],x2'[t]==-6*x1[t]-2*x2[t]},{x1[0]==1,x2[0]==0},{x1[t],x2[t]},
```

$$\text{x1}(t) \to e^{3t}(6e^t - 5)$$
$$\text{x2}(t) \to -6e^{3t}(e^t - 1)$$

4.7 problem problem 7

Internal problem ID [321]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 7.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -3x_1(t) + 4x_2(t)$$
$$x_2'(t) = 6x_1(t) - 5x_2(t)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve([diff(x__1(t),t)=-3*x__1(t)+4*x__2(t),diff(x__2(t),t)=6*x__1(t)-5*x__2(t)],singsol=al
```

$$x_1(t) = c_1 e^{-9t} + c_2 e^{t}$$
$$x_2(t) = -\frac{3c_1 e^{-9t}}{2} + c_2 e^{t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 74

```
DSolve[{x1'[t]==-3*x1[t]+4*x2[t],x2'[t]==6*x1[t]-5*x2[t]},{x1[t],x2[t]},t,IncludeSingularSol
```

$$\text{x1}(t) \to \frac{1}{5} e^{-9t} \left(c_1 \left(3e^{10t} + 2\right) + 2c_2 \left(e^{10t} - 1\right)\right)$$
$$\text{x2}(t) \to \frac{1}{5} e^{-9t} \left(3c_1 \left(e^{10t} - 1\right) + c_2 \left(2e^{10t} + 3\right)\right)$$

4.8 problem problem 8

Internal problem ID [322]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 8.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= x_1(t) - 5x_2(t) \\
x_2'(t) &= x_1(t) - x_2(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve([diff(x__1(t),t)=x__1(t)-5*x__2(t),diff(x__2(t),t)=x__1(t)-x__2(t)],singsol=all)
```

$$
\begin{aligned}
x_1(t) &= c_1 \sin(2t) + c_2 \cos(2t) \\
x_2(t) &= -\frac{2c_1 \cos(2t)}{5} + \frac{2c_2 \sin(2t)}{5} + \frac{c_1 \sin(2t)}{5} + \frac{c_2 \cos(2t)}{5}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 48

```
DSolve[{x1'[t]==x1[t]-5*x2[t],x2'[t]==x1[t]-x2[t]},{x1[t],x2[t]},t,IncludeSingularSolutions
```

$$
\begin{aligned}
\text{x1}(t) &\to c_1 \cos(2t) + (c_1 - 5c_2)\sin(t)\cos(t) \\
\text{x2}(t) &\to c_2 \cos(2t) + (c_1 - c_2)\sin(t)\cos(t)
\end{aligned}
$$

4.9 problem problem 9

Internal problem ID [323]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 9.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 2x_1(t) - 5x_2(t)$$
$$x_2'(t) = 4x_1(t) - 2x_2(t)$$

With initial conditions

$$[x_1(0) = 2, x_2(0) = 3]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve([diff(x__1(t),t) = 2*x__1(t)-5*x__2(t), diff(x__2(t),t) = 4*x__1(t)-2*x__2(t), x__1(0
```

$$x_1(t) = -\frac{11\sin(4t)}{4} + 2\cos(4t)$$
$$x_2(t) = 3\cos(4t) + \frac{\sin(4t)}{2}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 34

```
DSolve[{x1'[t]==x1[t]-5*x2[t],x2'[t]==x1[t]-x2[t]},{x1[0]==2,x2[0]==3},{x1[t],x2[t]},t,Inclu
```

$$\text{x1}(t) \to 2\cos(2t) - 13\sin(t)\cos(t)$$
$$\text{x2}(t) \to 3\cos(2t) - \sin(t)\cos(t)$$

4.10 problem problem 10

Internal problem ID [324]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 10.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= -3x_1(t) - 2x_2(t) \\
x_2'(t) &= 9x_1(t) + 3x_2(t)
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 50

```
dsolve([diff(x__1(t),t)=-3*x__1(t)-2*x__2(t),diff(x__2(t),t)=9*x__1(t)+3*x__2(t)],singsol=al
```

$$
\begin{aligned}
x_1(t) &= c_1 \sin(3t) + c_2 \cos(3t) \\
x_2(t) &= -\frac{3c_1 \cos(3t)}{2} + \frac{3c_2 \sin(3t)}{2} - \frac{3c_1 \sin(3t)}{2} - \frac{3c_2 \cos(3t)}{2}
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 53

```
DSolve[{x1'[t]==-3*x1[t]-2*x2[t],x2'[t]==9*x1[t]+3*x2[t]},{x1[t],x2[t]},t,IncludeSingularSol
```

$$
\begin{aligned}
\text{x1}(t) &\to c_1 \cos(3t) - \frac{1}{3}(3c_1 + 2c_2)\sin(3t) \\
\text{x2}(t) &\to c_2 \cos(3t) + (3c_1 + c_2)\sin(3t)
\end{aligned}
$$

4.11 problem problem 11

Internal problem ID [325]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 11.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) - 2x_2(t)$$
$$x_2'(t) = 2x_1(t) + x_2(t)$$

With initial conditions

$$[x_1(0) = 0, x_2(0) = 4]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve([diff(x__1(t),t) = x__1(t)-2*x__2(t), diff(x__2(t),t) = 2*x__1(t)+x__2(t), x__1(0) =
```

$$x_1(t) = -4\,\mathrm{e}^{t} \sin(2t)$$
$$x_2(t) = 4\,\mathrm{e}^{t} \cos(2t)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 26

```
DSolve[{x1'[t]==x1[t]-2*x2[t],x2'[t]==2*x1[t]+x2[t]},{x1[0]==0,x2[0]==4},{x1[t],x2[t]},t,Inc
```

$$\text{x1}(t) \to -4e^t \sin(2t)$$
$$\text{x2}(t) \to 4e^t \cos(2t)$$

4.12 problem problem 12

Internal problem ID [326]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 12.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) - 5x_2(t)$$
$$x_2'(t) = x_1(t) + 3x_2(t)$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 59

```
dsolve([diff(x__1(t),t)=x__1(t)-5*x__2(t),diff(x__2(t),t)=x__1(t)+3*x__2(t)],singsol=all)
```

$$x_1(t) = e^{2t}(c_1 \sin(2t) + c_2 \cos(2t))$$
$$x_2(t) = -\frac{e^{2t}(2c_1 \cos(2t) + c_2 \cos(2t) + c_1 \sin(2t) - 2c_2 \sin(2t))}{5}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 67

```
DSolve[{x1'[t]==x1[t]-5*x2[t],x2'[t]==x1[t]+3*x2[t]},{x1[t],x2[t]},t,IncludeSingularSolution
```

$$\text{x1}(t) \to \frac{1}{2}e^{2t}(2c_1 \cos(2t) - (c_1 + 5c_2)\sin(2t))$$
$$\text{x2}(t) \to \frac{1}{2}e^{2t}(2c_2 \cos(2t) + (c_1 + c_2)\sin(2t))$$

4.13 problem problem 13

Internal problem ID [327]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 13.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 5x_1(t) - 9x_2(t) \\
x_2'(t) &= 2x_1(t) - x_2(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 58

```
dsolve([diff(x__1(t),t)=5*x__1(t)-9*x__2(t),diff(x__2(t),t)=2*x__1(t)-x__2(t)],singsol=all)
```

$$
\begin{aligned}
x_1(t) &= e^{2t}(c_1 \sin(3t) + c_2 \cos(3t)) \\
x_2(t) &= \frac{e^{2t}(c_1 \sin(3t) + c_2 \sin(3t) - c_1 \cos(3t) + c_2 \cos(3t))}{3}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 66

```
DSolve[{x1'[t]==5*x1[t]-9*x2[t],x2'[t]==2*x1[t]-x2[t]},{x1[t],x2[t]},t,IncludeSingularSoluti
```

$$
\begin{aligned}
\text{x1}(t) &\to e^{2t}(c_1 \cos(3t) + (c_1 - 3c_2) \sin(3t)) \\
\text{x2}(t) &\to \frac{1}{3}e^{2t}(3c_2 \cos(3t) + (2c_1 - 3c_2) \sin(3t))
\end{aligned}
$$

4.14 problem problem 14

Internal problem ID [328]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 14.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 3x_1(t) - 4x_2(t) \\
x_2'(t) &= 4x_1(t) + 3x_2(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 46

```
dsolve([diff(x__1(t),t)=3*x__1(t)-4*x__2(t),diff(x__2(t),t)=4*x__1(t)+3*x__2(t)],singsol=all
```

$$
\begin{aligned}
x_1(t) &= e^{3t}(c_1 \sin(4t) + c_2 \cos(4t)) \\
x_2(t) &= -e^{3t}(c_1 \cos(4t) - c_2 \sin(4t))
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 51

```
DSolve[{x1'[t]==3*x1[t]-4*x2[t],x2'[t]==4*x1[t]+3*x2[t]},{x1[t],x2[t]},t,IncludeSingularSolu
```

$$
\begin{aligned}
\text{x1}(t) &\to e^{3t}(c_1 \cos(4t) - c_2 \sin(4t)) \\
\text{x2}(t) &\to e^{3t}(c_2 \cos(4t) + c_1 \sin(4t))
\end{aligned}
$$

4.15 problem problem 15

Internal problem ID [329]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 15.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 7x_1(t) - 5x_2(t)$$
$$x_2'(t) = 4x_1(t) + 3x_2(t)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 61

```
dsolve([diff(x__1(t),t)=7*x__1(t)-5*x__2(t),diff(x__2(t),t)=4*x__1(t)+3*x__2(t)],singsol=all
```

$$x_1(t) = e^{5t}(c_1 \sin(4t) + c_2 \cos(4t))$$
$$x_2(t) = -\frac{2 e^{5t}(2c_1 \cos(4t) - c_2 \cos(4t) - c_1 \sin(4t) - 2c_2 \sin(4t))}{5}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 72

```
DSolve[{x1'[t]==7*x1[t]-5*x2[t],x2'[t]==4*x1[t]+3*x2[t]},{x1[t],x2[t]},t,IncludeSingularSolu
```

$$\text{x1}(t) \to \frac{1}{4}e^{5t}(4c_1 \cos(4t) + (2c_1 - 5c_2)\sin(4t))$$
$$\text{x2}(t) \to \frac{1}{2}e^{5t}(2c_2 \cos(4t) + (2c_1 - c_2)\sin(4t))$$

4.16 problem problem 16

Internal problem ID [330]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 16.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -50x_1(t) + 20x_2(t)$$
$$x_2'(t) = 100x_1(t) - 60x_2(t)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
dsolve([diff(x__1(t),t)=-50*x__1(t)+20*x__2(t),diff(x__2(t),t)=100*x__1(t)-60*x__2(t)],sings
```

$$x_1(t) = c_1 e^{-100t} + c_2 e^{-10t}$$
$$x_2(t) = -\frac{5c_1 e^{-100t}}{2} + 2c_2 e^{-10t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 74

```
DSolve[{x1'[t]==-50*x1[t]+20*x2[t],x2'[t]==100*x1[t]-60*x2[t]},{x1[t],x2[t]},t,IncludeSingul
```

$$\text{x1}(t) \to \frac{1}{9}e^{-100t}\left(c_1\left(5e^{90t}+4\right)+2c_2\left(e^{90t}-1\right)\right)$$
$$\text{x2}(t) \to \frac{1}{9}e^{-100t}\left(10c_1\left(e^{90t}-1\right)+c_2\left(4e^{90t}+5\right)\right)$$

4.17 problem problem 17

Internal problem ID [331]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 17.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 4x_1(t) + x_2(t) + 4x_3(t) \\
x_2'(t) &= x_1(t) + 7x_2(t) + x_3(t) \\
x_3'(t) &= 4x_1(t) + x_2(t) + 4x_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 55

```
dsolve([diff(x__1(t),t)=4*x__1(t)+x__2(t)+4*x__3(t),diff(x__2(t),t)=x__1(t)+7*x__2(t)+x__3(t
```

$$
\begin{aligned}
x_1(t) &= c_1 + c_2 e^{6t} + c_3 e^{9t} \\
x_2(t) &= -2c_2 e^{6t} + c_3 e^{9t} \\
x_3(t) &= c_2 e^{6t} + c_3 e^{9t} - c_1
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 158

```
DSolve[{x1'[t]==4*x1[t]+x2[t]+4*x3[t],x2'[t]==x1[t]+7*x2[t]+x3[t],x3'[t]==4*x1[t]+x2[t]+4*x3
```

$$
\begin{aligned}
\text{x1}(t) &\to \frac{1}{6}\left(c_1\left(e^{6t} + 2e^{9t} + 3\right) + \left(e^{3t} - 1\right)\left(3c_3 e^{3t} + 2(c_2 + c_3)e^{6t} + 3c_3\right)\right) \\
\text{x2}(t) &\to \frac{1}{3}e^{6t}\left(c_1\left(e^{3t} - 1\right) + c_2\left(e^{3t} + 2\right) + c_3\left(e^{3t} - 1\right)\right) \\
\text{x3}(t) &\to \frac{1}{6}\left(c_1\left(e^{6t} + 2e^{9t} - 3\right) + (c_3 - 2c_2)e^{6t} + 2(c_2 + c_3)e^{9t} + 3c_3\right)
\end{aligned}
$$

4.18 problem problem 18

Internal problem ID [332]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 18.
ODE order: 1.
ODE degree: 1.

Solve

$$\begin{aligned} x_1'(t) &= x_1(t) + 2x_2(t) + 2x_3(t) \\ x_2'(t) &= 2x_1(t) + 7x_2(t) + x_3(t) \\ x_3'(t) &= 2x_1(t) + x_2(t) + 7x_3(t) \end{aligned}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 54

```
dsolve([diff(x__1(t),t)=x__1(t)+2*x__2(t)+2*x__3(t),diff(x__2(t),t)=2*x__1(t)+7*x__2(t)+x__3
```

$$\begin{aligned} x_1(t) &= c_2 + c_3 e^{9t} \\ x_2(t) &= 2c_3 e^{9t} + e^{6t} c_1 - \frac{c_2}{4} \\ x_3(t) &= 2c_3 e^{9t} - e^{6t} c_1 - \frac{c_2}{4} \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 148

```
DSolve[{x1'[t]==x1[t]+2*x2[t]+2*x3[t],x2'[t]==2*x1[t]+7*x2[t]+x3[t],x3'[t]==2*x1[t]+x2[t]+7*
```

$$\begin{aligned} \mathrm{x1}(t) &\to \frac{1}{9}\left(c_1\left(e^{9t}+8\right)+2(c_2+c_3)\left(e^{9t}-1\right)\right) \\ \mathrm{x2}(t) &\to \frac{1}{18}\left(4c_1\left(e^{9t}-1\right)+c_2\left(9e^{6t}+8e^{9t}+1\right)+c_3\left(-9e^{6t}+8e^{9t}+1\right)\right) \\ \mathrm{x3}(t) &\to \frac{1}{18}\left(4c_1\left(e^{9t}-1\right)+c_2\left(-9e^{6t}+8e^{9t}+1\right)+c_3\left(9e^{6t}+8e^{9t}+1\right)\right) \end{aligned}$$

4.19 problem problem 19

Internal problem ID [333]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 19.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 4x_1(t) + x_2(t) + x_3(t) \\
x_2'(t) &= x_1(t) + 4x_2(t) + x_3(t) \\
x_3'(t) &= x_1(t) + x_2(t) + 4x_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 64

```
dsolve([diff(x__1(t),t)=4*x__1(t)+1*x__2(t)+1*x__3(t),diff(x__2(t),t)=1*x__1(t)+4*x__2(t)+1*
```

$$
\begin{aligned}
x_1(t) &= c_2 e^{3t} + c_3 e^{6t} \\
x_2(t) &= c_2 e^{3t} + c_3 e^{6t} + c_1 e^{3t} \\
x_3(t) &= -2c_2 e^{3t} + c_3 e^{6t} - c_1 e^{3t}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 124

```
DSolve[{x1'[t]==4*x1[t]+1*x2[t]+1*x3[t],x2'[t]==1*x1[t]+4*x2[t]+1*x3[t],x3'[t]==1*x1[t]+1*x2
```

$$
\begin{aligned}
\text{x1}(t) &\to \frac{1}{3}e^{3t}\left(c_1\left(e^{3t}+2\right) + (c_2+c_3)\left(e^{3t}-1\right)\right) \\
\text{x2}(t) &\to \frac{1}{3}e^{3t}\left(c_1\left(e^{3t}-1\right) + c_2\left(e^{3t}+2\right) + c_3\left(e^{3t}-1\right)\right) \\
\text{x3}(t) &\to \frac{1}{3}e^{3t}\left(c_1\left(e^{3t}-1\right) + c_2\left(e^{3t}-1\right) + c_3\left(e^{3t}+2\right)\right)
\end{aligned}
$$

4.20 problem problem 20

Internal problem ID [334]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 20.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 5x_1(t) + x_2(t) + 3x_3(t) \\
x_2'(t) &= x_1(t) + 7x_2(t) + x_3(t) \\
x_3'(t) &= 3x_1(t) + x_2(t) + 5x_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 64

```
dsolve([diff(x__1(t),t)=5*x__1(t)+1*x__2(t)+3*x__3(t),diff(x__2(t),t)=1*x__1(t)+7*x__2(t)+1*
```

$$
\begin{aligned}
x_1(t) &= e^{6t}c_1 + c_2 e^{9t} + c_3 e^{2t} \\
x_2(t) &= -2\, e^{6t}c_1 + c_2 e^{9t} \\
x_3(t) &= e^{6t}c_1 + c_2 e^{9t} - c_3 e^{2t}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 163

```
DSolve[{x1'[t]==5*x1[t]+1*x2[t]+3*x3[t],x2'[t]==1*x1[t]+7*x2[t]+1*x3[t],x3'[t]==3*x1[t]+1*x2
```

$$
\begin{aligned}
\text{x1}(t) &\to \frac{1}{6}e^{2t}\left(c_1\left(e^{4t} + 2e^{7t} + 3\right) + (c_3 - 2c_2)e^{4t} + 2(c_2 + c_3)e^{7t} - 3c_3\right) \\
\text{x2}(t) &\to \frac{1}{3}e^{6t}\left(c_1\left(e^{3t} - 1\right) + c_2\left(e^{3t} + 2\right) + c_3\left(e^{3t} - 1\right)\right) \\
\text{x3}(t) &\to \frac{1}{6}e^{2t}\left(c_1\left(e^{4t} + 2e^{7t} - 3\right) + (c_3 - 2c_2)e^{4t} + 2(c_2 + c_3)e^{7t} + 3c_3\right)
\end{aligned}
$$

4.21 problem problem 21

Internal problem ID [335]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 21.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 5x_1(t) - 6x_3(t) \\
x_2'(t) &= 2x_1(t) - x_2(t) - 2x_3(t) \\
x_3'(t) &= 4x_1(t) - 2x_2(t) - 4x_3(t)
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 54

```
dsolve([diff(x__1(t),t)=5*x__1(t)+0*x__2(t)-6*x__3(t),diff(x__2(t),t)=2*x__1(t)-1*x__2(t)-2*
```

$$
\begin{aligned}
x_1(t) &= c_1 + c_2 \mathrm{e}^{-t} + c_3 \mathrm{e}^{t} \\
x_2(t) &= \frac{c_2 \mathrm{e}^{-t}}{2} + \frac{c_3 \mathrm{e}^{t}}{3} + \frac{c_1}{3} \\
x_3(t) &= c_2 \mathrm{e}^{-t} + \frac{2c_3 \mathrm{e}^{t}}{3} + \frac{5c_1}{6}
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.008 (sec). Leaf size: 139

```
DSolve[{x1'[t]==5*x1[t]+0*x2[t]-6*x3[t],x2'[t]==2*x1[t]-1*x2[t]-2*x3[t],x3'[t]==4*x1[t]-2*x2
```

$$
\begin{aligned}
\mathrm{x1}(t) &\to e^{-t}\left(c_1\left(3e^{2t}-2\right)+6\left(e^t-1\right)\left(c_2\left(e^t-1\right)-c_3e^t\right)\right) \\
\mathrm{x2}(t) &\to e^{-t}\left(c_1\left(e^{2t}-1\right)+c_2\left(-4e^t+2e^{2t}+3\right)-2c_3e^t\left(e^t-1\right)\right) \\
\mathrm{x3}(t) &\to -2(c_1-3c_2)e^{-t}+2(c_1+2c_2-2c_3)e^t+5(c_3-2c_2)
\end{aligned}
$$

4.22 problem problem 22

Internal problem ID [336]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 22.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 3x_1(t) + 2x_2(t) + 2x_3(t) \\
x_2'(t) &= -5x_1(t) - 4x_2(t) - 2x_3(t) \\
x_3'(t) &= 5x_1(t) + 5x_2(t) + 3x_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 55

```
dsolve([diff(x__1(t),t)=3*x__1(t)+2*x__2(t)+2*x__3(t),diff(x__2(t),t)=-5*x__1(t)-4*x__2(t)-2
```

$$
\begin{aligned}
x_1(t) &= c_2 e^{3t} + c_3 e^{t} \\
x_2(t) &= -c_2 e^{3t} - c_3 e^{t} + c_1 e^{-2t} \\
x_3(t) &= c_2 e^{3t} - c_1 e^{-2t}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 123

```
DSolve[{x1'[t]==3*x1[t]+2*x2[t]+2*x3[t],x2'[t]==-5*x1[t]-4*x2[t]-2*x3[t],x3'[t]==5*x1[t]+5*x
```

$$
\begin{aligned}
\text{x1}(t) &\to e^t\left((c_1 + c_2 + c_3)e^{2t} - c_2 - c_3\right) \\
\text{x2}(t) &\to e^{-2t}\left(-\left(c_1\left(e^{5t} - 1\right)\right) + c_2\left(e^{3t} - e^{5t} + 1\right) - c_3 e^{3t}\left(e^{2t} - 1\right)\right) \\
\text{x3}(t) &\to e^{-2t}\left(c_1\left(e^{5t} - 1\right) + c_2\left(e^{5t} - 1\right) + c_3 e^{5t}\right)
\end{aligned}
$$

4.23 problem problem 23

Internal problem ID [337]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 23.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 3x_1(t) + x_2(t) + x_3(t) \\
x_2'(t) &= -5x_1(t) - 3x_2(t) - x_3(t) \\
x_3'(t) &= 5x_1(t) + 5x_2(t) + 3x_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 59

```
dsolve([diff(x__1(t),t)=3*x__1(t)+1*x__2(t)+1*x__3(t),diff(x__2(t),t)=-5*x__1(t)-3*x__2(t)-1
```

$$
\begin{aligned}
x_1(t) &= c_2 e^{3t} + c_3 e^{2t} \\
x_2(t) &= -c_2 e^{3t} - c_3 e^{2t} + c_1 e^{-2t} \\
x_3(t) &= c_2 e^{3t} - c_1 e^{-2t}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 121

```
DSolve[{x1'[t]==3*x1[t]+1*x2[t]+1*x3[t],x2'[t]==-5*x1[t]-3*x2[t]-1*x3[t],x3'[t]==5*x1[t]+5*x
```

$$
\begin{aligned}
\text{x1}(t) &\to e^{2t}\left((c_1 + c_2 + c_3)e^t - c_2 - c_3\right) \\
\text{x2}(t) &\to e^{-2t}\left(-\left(c_1\left(e^{5t} - 1\right)\right) + c_2\left(e^{4t} - e^{5t} + 1\right) - c_3 e^{4t}\left(e^t - 1\right)\right) \\
\text{x3}(t) &\to e^{-2t}\left(c_1\left(e^{5t} - 1\right) + c_2\left(e^{5t} - 1\right) + c_3 e^{5t}\right)
\end{aligned}
$$

4.24 problem problem 24

Internal problem ID [338]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 24.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 2x_1(t) + x_2(t) - x_3(t)$$
$$x_2'(t) = -4x_1(t) - 3x_2(t) - x_3(t)$$
$$x_3'(t) = 4x_1(t) + 4x_2(t) + 2x_3(t)$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 87

```
dsolve([diff(x__1(t),t)=2*x__1(t)+1*x__2(t)-1*x__3(t),diff(x__2(t),t)=-4*x__1(t)-3*x__2(t)-1
```

$$x_1(t) = c_1 e^t + c_2 \sin(2t) + c_3 \cos(2t)$$
$$x_2(t) = -c_1 e^t - c_2 \sin(2t) - c_3 \cos(2t) + c_2 \cos(2t) - c_3 \sin(2t)$$
$$x_3(t) = -c_2 \cos(2t) + c_3 \sin(2t) + c_2 \sin(2t) + c_3 \cos(2t)$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 103

```
DSolve[{x1'[t]==2*x1[t]+1*x2[t]-1*x3[t],x2'[t]==-4*x1[t]-3*x2[t]-1*x3[t],x3'[t]==4*x1[t]+4*x
```

$$\text{x1}(t) \to (c_2 + c_3)\left(-e^t\right) + (c_1 + c_2 + c_3)\cos(2t) + (c_1 + c_2)\sin(2t)$$
$$\text{x2}(t) \to (c_2 + c_3)e^t - c_3\cos(2t) - (2c_1 + 2c_2 + c_3)\sin(2t)$$
$$\text{x3}(t) \to c_3\cos(2t) + (2c_1 + 2c_2 + c_3)\sin(2t)$$

4.25 problem problem 25

Internal problem ID [339]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 25.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 5x_1(t) + 5x_2(t) + 2x_3(t) \\
x_2'(t) &= -6x_1(t) - 6x_2(t) - 5x_3(t) \\
x_3'(t) &= 6x_1(t) + 6x_2(t) + 5x_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 111

```
dsolve([diff(x__1(t),t)=5*x__1(t)+5*x__2(t)+2*x__3(t),diff(x__2(t),t)=-6*x__1(t)-6*x__2(t)-5
```

$$
\begin{aligned}
x_1(t) &= c_1 + c_2 e^{2t} \sin(3t) + c_3 e^{2t} \cos(3t) \\
x_2(t) &= -c_2 e^{2t} \sin(3t) + c_2 e^{2t} \cos(3t) - c_3 e^{2t} \cos(3t) - c_3 e^{2t} \sin(3t) - c_1 \\
x_3(t) &= e^{2t}(c_2 \sin(3t) + \sin(3t) c_3 - c_2 \cos(3t) + \cos(3t) c_3)
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 122

```
DSolve[{x1'[t]==5*x1[t]+5*x2[t]+2*x3[t],x2'[t]==-6*x1[t]-6*x2[t]-5*x3[t],x3'[t]==6*x1[t]+6*x
```

$$
\begin{aligned}
\text{x1}(t) &\to (c_1 + c_2 + c_3)e^{2t} \cos(3t) + (c_1 + c_2)e^{2t} \sin(3t) - c_2 - c_3 \\
\text{x2}(t) &\to -c_3 e^{2t} \cos(3t) - (2c_1 + 2c_2 + c_3)e^{2t} \sin(3t) + c_2 + c_3 \\
\text{x3}(t) &\to e^{2t}(c_3 \cos(3t) + (2c_1 + 2c_2 + c_3) \sin(3t))
\end{aligned}
$$

4.26 problem problem 26

Internal problem ID [340]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 26.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 3x_1(t) + x_3(t)$$
$$x_2'(t) = 9x_1(t) - x_2(t) + 2x_3(t)$$
$$x_3'(t) = -9x_1(t) + 4x_2(t) - x_3(t)$$

With initial conditions

$$[x_1(0) = 0, x_2(0) = 0, x_3(0) = 17]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 64

```
dsolve([diff(x__1(t),t) = 3*x__1(t)+x__3(t), diff(x__2(t),t) = 9*x__1(t)-x__2(t)+2*x__3(t),
```

$$x_1(t) = 4\,\mathrm{e}^{3t} + \mathrm{e}^{-t}\sin(t) - 4\,\mathrm{e}^{-t}\cos(t)$$
$$x_2(t) = 9\,\mathrm{e}^{3t} - 9\,\mathrm{e}^{-t}\cos(t) - 2\,\mathrm{e}^{-t}\sin(t)$$
$$x_3(t) = 17\,\mathrm{e}^{-t}\cos(t)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 62

```
DSolve[{x1'[t]==3*x1[t]+0*x2[t]+1*x3[t],x2'[t]==9*x1[t]-1*x2[t]+2*x3[t],x3'[t]==-9*x1[t]+4*x
```

$$\mathrm{x1}(t) \to e^{-t}\left(4e^{4t} + \sin(t) - 4\cos(t)\right)$$
$$\mathrm{x2}(t) \to e^{-t}\left(9e^{4t} - 2\sin(t) - 9\cos(t)\right)$$
$$\mathrm{x3}(t) \to 17e^{-t}\cos(t)$$

4.27 problem problem 38

Internal problem ID [341]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 38.
ODE order: 1.
ODE degree: 1.

Solve

$$\begin{aligned}
x_1'(t) &= x_1(t)\\
x_2'(t) &= 2x_1(t) + 2x_2(t)\\
x_3'(t) &= 3x_2(t) + 3x_3(t)\\
x_4'(t) &= 4x_3(t) + 4x_4(t)
\end{aligned}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 75

```
dsolve([diff(x__1(t),t)=x__1(t)+0*x__2(t)+0*x__3(t)+0*x__4(t),diff(x__2(t),t)=2*x__1(t)+2*x_
```

$$\begin{aligned}
x_1(t) &= c_4 e^t\\
x_2(t) &= -2c_4 e^t + c_3 e^{2t}\\
x_3(t) &= c_2 e^{3t} - 3c_3 e^{2t} + 3c_4 e^t\\
x_4(t) &= c_1 e^{4t} - 4c_2 e^{3t} + 6c_3 e^{2t} - 4c_4 e^t
\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 128

```
DSolve[{x1'[t]==1*x1[t]+0*x2[t]+0*x3[t]+0*x4[t],x2'[t]==2*x1[t]+2*x2[t]+0*x3[t]+0*x4[t],x3'[
```

$$\begin{aligned}
\mathrm{x1}(t) &\to c_1 e^t\\
\mathrm{x2}(t) &\to e^t\left(2c_1\left(e^t-1\right)+c_2 e^t\right)\\
\mathrm{x3}(t) &\to e^t\left(3c_1\left(e^t-1\right)^2+e^t\left(3c_2\left(e^t-1\right)+c_3 e^t\right)\right)\\
\mathrm{x4}(t) &\to e^t\left(4c_1\left(e^t-1\right)^3+e^t\left(6c_2\left(e^t-1\right)^2+e^t\left(4c_3\left(e^t-1\right)+c_4 e^t\right)\right)\right)
\end{aligned}$$

4.28 problem problem 39

Internal problem ID [342]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 39.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= -2x_1(t) + 9x_4(t) \\
x_2'(t) &= 4x_1(t) + 2x_2(t) - 10x_4(t) \\
x_3'(t) &= -x_3(t) + 8x_4(t) \\
x_4'(t) &= x_4(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 61

```
dsolve([diff(x__1(t),t)=-2*x__1(t)+0*x__2(t)+0*x__3(t)+9*x__4(t),diff(x__2(t),t)=4*x__1(t)+2
```

$$
\begin{aligned}
x_1(t) &= 3c_4 e^t + c_2 e^{-2t} \\
x_2(t) &= c_1 e^{2t} - 2c_4 e^t - c_2 e^{-2t} \\
x_3(t) &= 4c_4 e^t + c_3 e^{-t} \\
x_4(t) &= c_4 e^t
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 103

```
DSolve[{x1'[t]==-2*x1[t]+0*x2[t]+0*x3[t]+9*x4[t],x2'[t]==4*x1[t]+2*x2[t]+0*x3[t]-10*x4[t],x3
```

$$
\begin{aligned}
\text{x1}(t) &\to e^{-2t}\left(3c_4\left(e^{3t}-1\right)+c_1\right) \\
\text{x2}(t) &\to e^{-2t}\left(c_1\left(e^{4t}-1\right)+(c_2-c_4)e^{4t}-2c_4e^{3t}+3c_4\right) \\
\text{x3}(t) &\to e^{-t}\left(4c_4\left(e^{2t}-1\right)+c_3\right) \\
\text{x4}(t) &\to c_4e^t
\end{aligned}
$$

4.29 problem problem 40

Internal problem ID [343]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 40.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 2x_1(t) \\
x_2'(t) &= -21x_1(t) - 5x_2(t) - 27x_3(t) - 9x_4(t) \\
x_3'(t) &= 5x_3(t) \\
x_4'(t) &= -21x_3(t) - 2x_4(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 61

```
dsolve([diff(x__1(t),t)=2*x__1(t)+0*x__2(t)+0*x__3(t)+0*x__4(t),diff(x__2(t),t)=-21*x__1(t)-
```

$$
\begin{aligned}
x_1(t) &= c_4 e^{2t} \\
x_2(t) &= -3c_4 e^{2t} - 3c_2 e^{-2t} + c_1 e^{-5t} \\
x_3(t) &= c_3 e^{5t} \\
x_4(t) &= -3c_3 e^{5t} + c_2 e^{-2t}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 86

```
DSolve[{x1'[t]==2*x1[t]+0*x2[t]+0*x3[t]+0*x4[t],x2'[t]==-21*x1[t]-5*x2[t]-27*x3[t]-9*x4[t],x
```

$$
\begin{aligned}
\mathrm{x1}(t) &\to c_1 e^{2t} \\
\mathrm{x2}(t) &\to e^{-5t}\left(-3c_1\left(e^{7t}-1\right) - 3(3c_3 + c_4)\left(e^{3t}-1\right) + c_2\right) \\
\mathrm{x3}(t) &\to c_3 e^{5t} \\
\mathrm{x4}(t) &\to e^{-2t}\left(c_4 - 3c_3\left(e^{7t}-1\right)\right)
\end{aligned}
$$

4.30 problem problem 41

Internal problem ID [344]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 41.
ODE order: 1.
ODE degree: 1.

Solve

$$\begin{aligned} x_1'(t) &= 4x_1(t) + x_2(t) + x_3(t) + 7x_4(t) \\ x_2'(t) &= x_1(t) + 4x_2(t) + 10x_3(t) + x_4(t) \\ x_3'(t) &= x_1(t) + 10x_2(t) + 4x_3(t) + x_4(t) \\ x_4'(t) &= 7x_1(t) + x_2(t) + x_3(t) + 4x_4(t) \end{aligned}$$

With initial conditions

$$[x_1(0) = 3, x_2(0) = 1, x_3(0) = 1, x_4(0) = 3]$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 62

```
dsolve([diff(x__1(t),t) = 4*x__1(t)+x__2(t)+x__3(t)+7*x__4(t), diff(x__2(t),t) = x__1(t)+4*x
```

$$\begin{aligned} x_1(t) &= e^{15t} + 2\,e^{10t} \\ x_2(t) &= 2\,e^{15t} - e^{10t} \\ x_3(t) &= 2\,e^{15t} - e^{10t} \\ x_4(t) &= e^{15t} + 2\,e^{10t} \end{aligned}$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 70

```
DSolve[{x1'[t]==4*x1[t]+1*x2[t]+1*x3[t]+7*x4[t],x2'[t]==1*x1[t]+4*x2[t]+10*x3[t]+1*x4[t],x3'
```

$$\begin{aligned} \text{x1}(t) &\to e^{10t}(e^{5t} + 2) \\ \text{x2}(t) &\to e^{10t}(2e^{5t} - 1) \\ \text{x3}(t) &\to e^{10t}(2e^{5t} - 1) \\ \text{x4}(t) &\to e^{10t}(e^{5t} + 2) \end{aligned}$$

4.31 problem problem 42

Internal problem ID [345]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 42.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= -40x_1(t) - 12x_2(t) + 54x_3(t) \\
x_2'(t) &= 35x_1(t) + 13x_2(t) - 46x_3(t) \\
x_3'(t) &= -25x_1(t) - 7x_2(t) + 34x_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 59

```
dsolve([diff(x__1(t),t)=-40*x__1(t)-12*x__2(t)+54*x__3(t),diff(x__2(t),t)=35*x__1(t)+13*x__2
```

$$
\begin{aligned}
x_1(t) &= c_1 + c_2 e^{2t} + c_3 e^{5t} \\
x_2(t) &= c_2 e^{2t} - \frac{3c_3 e^{5t}}{2} - \frac{c_1}{3} \\
x_3(t) &= c_2 e^{2t} + \frac{c_3 e^{5t}}{2} + \frac{2c_1}{3}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 181

```
DSolve[{x1'[t]==-40*x1[t]-12*x2[t]+54*x3[t],x2'[t]==35*x1[t]+13*x2[t]-46*x3[t],x3'[t]==-25*x
```

$$
\begin{aligned}
\text{x1}(t) &\to c_1\left(-5e^{2t} - 6e^{5t} + 12\right) - c_2\left(e^{2t} + 2e^{5t} - 3\right) + c_3\left(7e^{2t} + 8e^{5t} - 15\right) \\
\text{x2}(t) &\to c_1\left(-5e^{2t} + 9e^{5t} - 4\right) + c_2\left(-e^{2t} + 3e^{5t} - 1\right) + c_3\left(7e^{2t} - 12e^{5t} + 5\right) \\
\text{x3}(t) &\to c_1\left(-5e^{2t} - 3e^{5t} + 8\right) - c_2\left(e^{2t} + e^{5t} - 2\right) + c_3\left(7e^{2t} + 4e^{5t} - 10\right)
\end{aligned}
$$

4.32 problem problem 43

Internal problem ID [346]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 43.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= -20x_1(t) + 11x_2(t) + 13x_3(t) \\
x_2'(t) &= 12x_1(t) - x_2(t) - 7x_3(t) \\
x_3'(t) &= -48x_1(t) + 21x_2(t) + 31x_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 72

```
dsolve([diff(x__1(t),t)=-20*x__1(t)+11*x__2(t)+13*x__3(t),diff(x__2(t),t)=12*x__1(t)-1*x__2(
```

$$
\begin{aligned}
x_1(t) &= c_1 e^{4t} + c_2 e^{-2t} + c_3 e^{8t} \\
x_2(t) &= c_1 e^{4t} - \frac{c_2 e^{-2t}}{3} - c_3 e^{8t} \\
x_3(t) &= c_1 e^{4t} + \frac{5c_2 e^{-2t}}{3} + 3c_3 e^{8t}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 554

```
DSolve[{x1'[t]==20*x1[t]+11*x2[t]+13*x3[t],x2'[t]==12*x1[t]-1*x2[t]-7*x3[t],x3'[t]==-48*x1[t
```

$$x1(t) \to c_2\text{RootSum}\left[\#1^3 - 50\#1^2 + 1208\#1 - 4576\&, \frac{11\#1e^{\#1t} - 68e^{\#1t}}{3\#1^2 - 100\#1 + 1208}\&\right] + c_3\text{RootSum}\left[\#1^3 - 50\#1^2 + 1208\#1 - 4576\&, \frac{13\#1e^{\#1t} - 64e^{\#1t}}{3\#1^2 - 100\#1 + 1208}\&\right] + c_1\text{RootSum}\left[\#1^3 - 50\#1^2 + 1208\#1 - 4576\&, \frac{\#1^2e^{\#1t} - 30\#1e^{\#1t} + 116e^{\#1t}}{3\#1^2 - 100\#1 + 1208}\&\right]$$

$$x2(t) \to 12c_1\text{RootSum}\left[\#1^3 - 50\#1^2 + 1208\#1 - 4576\&, \frac{\#1e^{\#1t} - 3e^{\#1t}}{3\#1^2 - 100\#1 + 1208}\&\right] - c_3\text{RootSum}\left[\#1^3 - 50\#1^2 + 1208\#1 - 4576\&, \frac{7\#1e^{\#1t} - 296e^{\#1t}}{3\#1^2 - 100\#1 + 1208}\&\right] + c_2\text{RootSum}\left[\#1^3 - 50\#1^2 + 1208\#1 - 4576\&, \frac{\#1^2e^{\#1t} - 51\#1e^{\#1t} + 1244e^{\#1t}}{3\#1^2 - 100\#1 + 1208}\&\right]$$

$$x3(t) \to -12c_1\text{RootSum}\left[\#1^3 - 50\#1^2 + 1208\#1 - 4576\&, \frac{4\#1e^{\#1t} - 17e^{\#1t}}{3\#1^2 - 100\#1 + 1208}\&\right] + 3c_2\text{RootSum}\left[\#1^3 - 50\#1^2 + 1208\#1 - 4576\&, \frac{7\#1e^{\#1t} - 316e^{\#1t}}{3\#1^2 - 100\#1 + 1208}\&\right] + c_3\text{RootSum}\left[\#1^3 - 50\#1^2 + 1208\#1 - 4576\&, \frac{\#1^2e^{\#1t} - 19\#1e^{\#1t} - 152e^{\#1t}}{3\#1^2 - 100\#1 + 1208}\&\right]$$

4.33 problem problem 44

Internal problem ID [347]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 44.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 147x_1(t) + 23x_2(t) - 202x_3(t) \\
x_2'(t) &= -90x_1(t) - 9x_2(t) + 129x_3(t) \\
x_3'(t) &= 90x_1(t) + 15x_2(t) - 123x_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 74

```
dsolve([diff(x__1(t),t)=147*x__1(t)+23*x__2(t)-202*x__3(t),diff(x__2(t),t)=-90*x__1(t)-9*x__
```

$$
\begin{aligned}
x_1(t) &= e^{6t}c_1 + c_2e^{-3t} + c_3e^{12t} \\
x_2(t) &= \frac{e^{6t}c_1}{7} - \frac{2c_2e^{-3t}}{3} - \frac{3c_3e^{12t}}{5} \\
x_3(t) &= \frac{5\,e^{6t}c_1}{7} + \frac{2c_2e^{-3t}}{3} + \frac{3c_3e^{12t}}{5}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 188

```
DSolve[{x1'[t]==147*x1[t]+23*x2[t]-202*x3[t],x2'[t]==-90*x1[t]-9*x2[t]+129*x3[t],x3'[t]==90*
```

$$
\begin{aligned}
\text{x1}(t) &\to \frac{1}{6}e^{-3t}\left(6c_1\left(10e^{15t}-9\right)+c_2\left(7e^{9t}+5e^{15t}-12\right)-c_3\left(-7e^{9t}+85e^{15t}-78\right)\right) \\
\text{x2}(t) &\to \frac{1}{6}e^{-3t}\left(-36c_1\left(e^{15t}-1\right)+c_2\left(e^{9t}-3e^{15t}+8\right)+c_3\left(e^{9t}+51e^{15t}-52\right)\right) \\
\text{x3}(t) &\to \frac{1}{6}e^{-3t}\left(36c_1\left(e^{15t}-1\right)+c_2\left(5e^{9t}+3e^{15t}-8\right)-c_3\left(-5e^{9t}+51e^{15t}-52\right)\right)
\end{aligned}
$$

4.34 problem problem 45

Internal problem ID [348]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 45.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 9x_1(t) - 7x_2(t) - 5x_3(t)\\
x_2'(t) &= -12x_1(t) + 7x_2(t) + 11x_3(t) + 9x_4(t)\\
x_3'(t) &= 24x_1(t) - 17x_2(t) - 19x_3(t) - 9x_4(t)\\
x_4'(t) &= -18x_1(t) + 13x_2(t) + 17x_3(t) + 9x_4(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 105

```
dsolve([diff(x__1(t),t)=9*x__1(t)-7*x__2(t)-5*x__3(t)+0*x__4(t),diff(x__2(t),t)=-12*x__1(t)+
```

$$
\begin{aligned}
x_1(t) &= c_1 + c_2 e^{3t} + c_3 e^{6t} + c_4 e^{-3t}\\
x_2(t) &= \frac{c_2 e^{3t}}{2} - c_3 e^{6t} + c_4 e^{-3t} + 2c_1\\
x_3(t) &= \frac{c_2 e^{3t}}{2} + 2c_3 e^{6t} + c_4 e^{-3t} - c_1\\
x_4(t) &= \frac{c_2 e^{3t}}{2} - c_3 e^{6t} - c_4 e^{-3t} + c_1
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 430

```
DSolve[{x1'[t]==9*x1[t]-7*x2[t]-5*x3[t]+0*x4[t],x2'[t]==-12*x1[t]+7*x2[t]+11*x3[t]+9*x4[t],x
```

$$
\begin{aligned}
\text{x1}(t) \to{} & \frac{1}{3}e^{-3t}\big(c_1\big(6e^{3t}-6e^{6t}+6e^{9t}-3\big) \\
& -\big(e^{3t}-1\big)\big(c_2\big(4e^{6t}+3\big)+c_3\big(-3e^{3t}+5e^{6t}+3\big)+3c_4e^{3t}\big(e^{3t}-1\big)\big)\big) \\
\text{x2}(t) \to{} & \frac{1}{3}e^{-3t}\big(-3c_1\big(-4e^{3t}+e^{6t}+2e^{9t}+1\big)+c_2\big(-6e^{3t}+2e^{6t}+4e^{9t}+3\big) \\
& +\big(e^{3t}-1\big)\big(c_3\big(9e^{3t}+5e^{6t}-3\big)+3c_4e^{3t}\big(e^{3t}+2\big)\big)\big) \\
\text{x3}(t) \to{} & c_1\big(-e^{-3t}-e^{3t}+4e^{6t}-2\big)+c_2\left(e^{-3t}+\frac{2e^{3t}}{3}-\frac{8e^{6t}}{3}+1\right) \\
& +c_3e^{-3t}+\frac{4}{3}c_3e^{3t}-\frac{10}{3}c_3e^{6t}+c_4e^{3t}-2c_4e^{6t}+2c_3+c_4 \\
\text{x4}(t) \to{} & \frac{1}{3}\big(c_1\big(3e^{-3t}-3e^{3t}-6e^{6t}+6\big)+c_2\big(-3e^{-3t}+2e^{3t}+4e^{6t}-3\big)-3c_3e^{-3t}+4c_3e^{3t} \\
& +5c_3e^{6t}+3c_4e^{3t}+3c_4e^{6t}-6c_3-3c_4\big)
\end{aligned}
$$

4.35 problem problem 46

Internal problem ID [349]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 46.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 13x_1(t) - 42x_2(t) + 106x_3(t) + 139x_4(t) \\
x_2'(t) &= 2x_1(t) - 16x_2(t) + 52x_3(t) + 70x_4(t) \\
x_3'(t) &= x_1(t) + 6x_2(t) - 20x_3(t) - 31x_4(t) \\
x_4'(t) &= -x_1(t) - 6x_2(t) + 22x_3(t) + 33x_4(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 123

```
dsolve([diff(x__1(t),t)=13*x__1(t)-42*x__2(t)+106*x__3(t)+139*x__4(t),diff(x__2(t),t)=2*x__1
```

$$
\begin{aligned}
x_1(t) &= c_1 e^{4t} + c_2 e^{-4t} + c_3 e^{2t} + c_4 e^{8t} \\
x_2(t) &= c_1 e^{4t} + \frac{2c_2 e^{-4t}}{3} + 2c_3 e^{2t} - \frac{2c_4 e^{8t}}{3} \\
x_3(t) &= -c_1 e^{4t} - \frac{c_2 e^{-4t}}{3} + 2c_3 e^{2t} + c_4 e^{8t} \\
x_4(t) &= c_1 e^{4t} + \frac{c_2 e^{-4t}}{3} - c_3 e^{2t} - c_4 e^{8t}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 449

```
DSolve[{x1'[t]==13*x1[t]-42*x2[t]+106*x3[t]+139*x4[t],x2'[t]==2*x1[t]-16*x2[t]+52*x3[t]+70*x
```

$$\begin{aligned}
\mathrm{x1}(t) \rightarrow{} & \frac{1}{4}e^{-4t}\big(c_1\big(4e^{8t}+3e^{12t}-3\big)-6c_2\big(2e^{8t}+e^{12t}-3\big)+4c_3e^{6t}+32c_3e^{8t}+12c_3e^{12t} \\
& +4c_4e^{6t}+44c_4e^{8t}+15c_4e^{12t}-48c_3-63c_4\big) \\
\mathrm{x2}(t) \rightarrow{} & \frac{1}{2}e^{-4t}\big(-\big(c_1\big(-2e^{8t}+e^{12t}+1\big)\big)+2c_2\big(-3e^{8t}+e^{12t}+3\big)+4c_3e^{6t}+16c_3e^{8t} \\
& -4c_3e^{12t}+4c_4e^{6t}+22c_4e^{8t}-5c_4e^{12t}-16c_3-21c_4\big) \\
\mathrm{x3}(t) \rightarrow{} & \frac{1}{4}e^{-4t}\big(c_1\big(-4e^{8t}+3e^{12t}+1\big)-6c_2\big(-2e^{8t}+e^{12t}+1\big)+8c_3e^{6t}-32c_3e^{8t} \\
& +12c_3e^{12t}+8c_4e^{6t}-44c_4e^{8t}+15c_4e^{12t}+16c_3+21c_4\big) \\
\mathrm{x4}(t) \rightarrow{} & \frac{1}{4}e^{-4t}\big(c_1\big(4e^{8t}-3e^{12t}-1\big)+6c_2\big(-2e^{8t}+e^{12t}+1\big)-4c_3e^{6t}+32c_3e^{8t}-12c_3e^{12t} \\
& -4c_4e^{6t}+44c_4e^{8t}-15c_4e^{12t}-16c_3-21c_4\big)
\end{aligned}$$

4.36 problem problem 47

Internal problem ID [350]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 47.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 23x_1(t) - 18x_2(t) - 16x_3(t) \\
x_2'(t) &= -8x_1(t) + 6x_2(t) + 7x_3(t) + 9x_4(t) \\
x_3'(t) &= 34x_1(t) - 27x_2(t) - 26x_3(t) - 9x_4(t) \\
x_4'(t) &= -26x_1(t) + 21x_2(t) + 25x_3(t) + 12x_4(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 124

```
dsolve([diff(x__1(t),t)=23*x__1(t)-18*x__2(t)-16*x__3(t)+0*x__4(t),diff(x__2(t),t)=-8*x__1(t
```

$$
\begin{aligned}
x_1(t) &= c_1 e^{3t} + c_2 e^{6t} + c_3 e^{9t} + c_4 e^{-3t} \\
x_2(t) &= 2c_1 e^{3t} + \frac{c_2 e^{6t}}{2} - c_3 e^{9t} + c_4 e^{-3t} \\
x_3(t) &= -c_1 e^{3t} + \frac{c_2 e^{6t}}{2} + 2c_3 e^{9t} + \frac{c_4 e^{-3t}}{2} \\
x_4(t) &= c_1 e^{3t} + \frac{c_2 e^{6t}}{2} - c_3 e^{9t} - \frac{c_4 e^{-3t}}{2}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 469

```
DSolve[{x1'[t]==23*x1[t]-18*x2[t]-16*x3[t]+0*x4[t],x2'[t]==-8*x1[t]+6*x2[t]+7*x3[t]+9*x4[t],
```

$$
\begin{aligned}
\mathrm{x1}(t) \to \frac{1}{3}e^{-3t}\big(&c_1\left(9e^{6t}-8e^{9t}+8e^{12t}-6\right)\\
&-\left(e^{3t}-1\right)\left(6c_2\left(e^{3t}+e^{9t}+1\right)+c_3\left(6e^{3t}-3e^{6t}+7e^{9t}+6\right)+3c_4e^{6t}\left(e^{3t}-1\right)\right)\big)\\
\mathrm{x2}(t) \to \frac{1}{3}e^{-3t}\big(&-2c_1\left(-9e^{6t}+2e^{9t}+4e^{12t}+3\right)+3c_2\left(-4e^{6t}+e^{9t}+2e^{12t}+2\right)\\
&+\left(e^{3t}-1\right)\left(c_3\left(-6e^{3t}+12e^{6t}+7e^{9t}-6\right)+3c_4e^{6t}\left(e^{3t}+2\right)\right)\big)\\
\mathrm{x3}(t) \to \frac{1}{3}e^{-3t}\big(&c_1\left(-9e^{6t}-4e^{9t}+16e^{12t}-3\right)+3c_2\left(2e^{6t}+e^{9t}-4e^{12t}+1\right)+9c_3e^{6t}\\
&+5c_3e^{9t}-14c_3e^{12t}+3c_4e^{6t}+3c_4e^{9t}-6c_4e^{12t}+3c_3\big)\\
\mathrm{x4}(t) \to \frac{1}{3}e^{-3t}\big(&c_1\left(9e^{6t}-4e^{9t}-8e^{12t}+3\right)+3c_2\left(-2e^{6t}+e^{9t}+2e^{12t}-1\right)-9c_3e^{6t}\\
&+5c_3e^{9t}+7c_3e^{12t}-3c_4e^{6t}+3c_4e^{9t}+3c_4e^{12t}-3c_3\big)
\end{aligned}
$$

4.37 problem problem 48

Internal problem ID [351]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 48.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 47x_1(t) - 8x_2(t) + 5x_3(t) - 5x_4(t) \\
x_2'(t) &= -10x_1(t) + 32x_2(t) + 18x_3(t) - 2x_4(t) \\
x_3'(t) &= 139x_1(t) - 40x_2(t) - 167x_3(t) - 121x_4(t) \\
x_4'(t) &= -232x_1(t) + 64x_2(t) + 360x_3(t) + 248x_4(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 125

```
dsolve([diff(x__1(t),t)=47*x__1(t)-8*x__2(t)+5*x__3(t)-5*x__4(t),diff(x__2(t),t)=-10*x__1(t)
```

$$
\begin{aligned}
x_1(t) &= c_1 e^{48t} + c_2 e^{16t} + c_3 e^{32t} + c_4 e^{64t} \\
x_2(t) &= -\frac{c_1 e^{48t}}{3} + 2c_2 e^{16t} + \frac{5c_3 e^{32t}}{2} + c_4 e^{64t} \\
x_3(t) &= \frac{c_1 e^{48t}}{3} - c_2 e^{16t} + \frac{c_3 e^{32t}}{2} + 2c_4 e^{64t} \\
x_4(t) &= \frac{2c_1 e^{48t}}{3} + 2c_2 e^{16t} - \frac{c_3 e^{32t}}{2} - 3c_4 e^{64t}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 448

```
DSolve[{x1'[t]==47*x1[t]-8*x2[t]+5*x3[t]-5*x4[t],x2'[t]==-10*x1[t]+32*x2[t]+18*x3[t]-2*x4[t]
```

$$
\begin{aligned}
\mathrm{x1}(t) \to \frac{1}{16} e^{16t} \Big(& c_1 \left(-38 e^{16t} - 6 e^{32t} + 27 e^{48t} + 33\right) \\
& - \left(e^{16t} - 1\right) \left(8 c_2 \left(e^{16t} + e^{32t} - 1\right) + c_3 \left(9 e^{16t} + 39 e^{32t} - 53\right) + c_4 \left(7 e^{16t} + 25 e^{32t} - 27\right)\right)\Big) \\
\mathrm{x2}(t) \to \frac{1}{16} e^{16t} \Big(& c_1 \left(-95 e^{16t} + 2 e^{32t} + 27 e^{48t} + 66\right) - 8 c_2 \left(-5 e^{16t} + e^{48t} + 2\right) \\
& - \left(e^{16t} - 1\right) \left(c_3 \left(49 e^{16t} + 39 e^{32t} - 106\right) + c_4 \left(31 e^{16t} + 25 e^{32t} - 54\right)\right)\Big) \\
\mathrm{x3}(t) \to \frac{1}{16} e^{16t} \Big(& c_1 \left(-19 e^{16t} - 2 e^{32t} + 54 e^{48t} - 33\right) + 8 c_2 \left(e^{16t} - 2 e^{48t} + 1\right) + 31 c_3 e^{16t} \\
& + 10 c_3 e^{32t} - 78 c_3 e^{48t} + 17 c_4 e^{16t} + 6 c_4 e^{32t} - 50 c_4 e^{48t} + 53 c_3 + 27 c_4\Big) \\
\mathrm{x4}(t) \to -\frac{1}{16} e^{16t} \Big(& c_1 \left(-19 e^{16t} + 4 e^{32t} + 81 e^{48t} - 66\right) + 8 c_2 \left(e^{16t} - 3 e^{48t} + 2\right) + 31 c_3 e^{16t} \\
& - 20 c_3 e^{32t} - 117 c_3 e^{48t} + 17 c_4 e^{16t} - 12 c_4 e^{32t} - 75 c_4 e^{48t} + 106 c_3 + 54 c_4\Big)
\end{aligned}
$$

4.38 problem problem 49

Internal problem ID [352]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 49.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 139x_1(t) - 14x_2(t) - 52x_3(t) - 14x_4(t) + 28x_5(t) \\
x_2'(t) &= -22x_1(t) + 5x_2(t) + 7x_3(t) + 8x_4(t) - 7x_5(t) \\
x_3'(t) &= 370x_1(t) - 38x_2(t) - 139x_3(t) - 38x_4(t) + 76x_5(t) \\
x_4'(t) &= 152x_1(t) - 16x_2(t) - 59x_3(t) - 13x_4(t) + 35x_5(t) \\
x_5'(t) &= 95x_1(t) - 10x_2(t) - 38x_3(t) - 7x_4(t) + 23x_5(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 132

```
dsolve([diff(x__1(t),t)=139*x__1(t)-14*x__2(t)-52*x__3(t)-14*x__4(t)+28*x__5(t),diff(x__2(t)
```

$$
\begin{aligned}
x_1(t) &= c_3 e^{3t} + c_4 e^{9t} + c_5 e^{-3t} \\
x_2(t) &= \frac{e^{6t} c_1}{6} + 7c_3 e^{3t} + c_2 \\
x_3(t) &= c_3 e^{3t} + \frac{5c_4 e^{9t}}{2} + 3c_5 e^{-3t} \\
x_4(t) &= c_3 e^{3t} + c_4 e^{9t} + c_5 e^{-3t} + \frac{e^{6t} c_1}{6} - \frac{c_2}{3} \\
x_5(t) &= c_3 e^{3t} + \frac{e^{6t} c_1}{6} + \frac{c_4 e^{9t}}{2} + c_5 e^{-3t} + \frac{c_2}{3}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 2676

```
DSolve[{x1'[t]==139*x1[t]-14*x2[t]-52*x3[t]-14*x4[t]+28*x5[t],x2'[t]==-22*x1[t]+5*x2[t]+7*x3
```

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4.39 problem problem 50

Internal problem ID [353]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.3, The eigenvalue method for linear systems. Page 395
Problem number: problem 50.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 9x_1(t) + 13x_2(t) - 13x_6(t)\\
x_2'(t) &= -14x_1(t) + 19x_2(t) - 10x_3(t) - 20x_4(t) + 10x_5(t) + 4x_6(t)\\
x_3'(t) &= -30x_1(t) + 12x_2(t) - 7x_3(t) - 30x_4(t) + 12x_5(t) + 18x_6(t)\\
x_4'(t) &= -12x_1(t) + 10x_2(t) - 10x_3(t) - 9x_4(t) + 10x_5(t) + 2x_6(t)\\
x_5'(t) &= 6x_1(t) + 9x_2(t) + 6x_4(t) + 5x_5(t) - 15x_6(t)\\
x_6'(t) &= -14x_1(t) + 23x_2(t) - 10x_3(t) - 20x_4(t) + 10x_5(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 135

```
dsolve([diff(x__1(t),t)=9*x__1(t)+13*x__2(t)+0*x__3(t)+0*x__4(t)+0*x__5(t)-13*x__6(t),diff(x
```

$$
\begin{aligned}
x_1(t) &= c_5 e^{-4t} + c_6 e^{9t}\\
x_2(t) &= c_6 e^{9t} + c_4 e^{3t} + e^{-7t} c_3\\
x_3(t) &= e^{-7t} c_3 - e^{11t} c_2 + e^{5t} c_1\\
x_4(t) &= e^{11t} c_2 + c_4 e^{3t} + e^{-7t} c_3\\
x_5(t) &= e^{11t} c_2 + e^{5t} c_1 + c_5 e^{-4t}\\
x_6(t) &= c_6 e^{9t} + c_5 e^{-4t} + c_4 e^{3t} + e^{-7t} c_3
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.107 (sec). Leaf size: 1882

```
DSolve[{x1'[t]==9*x1[t]+13*x2[t]-13*x6[t],x2'[t]==-14*x1[t]+19*x2[t]-10*x3[t]-20*x4[t]+10*x5
```

Too large to display

5 Section 7.6, Multiple Eigenvalue Solutions. Examples. Page 437

5.1 problem Example 1

Internal problem ID [354]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Examples. Page 437
Problem number: Example 1.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 9x_1(t) + 4x_2(t) \\
x_2'(t) &= -6x_1(t) - x_2(t) \\
x_3'(t) &= 6x_1(t) + 4x_2(t) + 3x_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 58

```
dsolve([diff(x__1(t),t)=9*x__1(t)+4*x__2(t)+0*x__3(t),diff(x__2(t),t)=-6*x__1(t)-1*x__2(t)+0
```

$$
\begin{aligned}
x_1(t) &= c_2 \mathrm{e}^{3t} + c_3 \mathrm{e}^{5t} \\
x_2(t) &= -\frac{3c_2 \mathrm{e}^{3t}}{2} - c_3 \mathrm{e}^{5t} \\
x_3(t) &= c_2 \mathrm{e}^{3t} + c_3 \mathrm{e}^{5t} + c_1 \mathrm{e}^{3t}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 113

```
DSolve[{x1'[t]==9*x1[t]+4*x2[t]+0*x3[t],x2'[t]==-6*x1[t]-1*x2[t]+0*x3[t],x3'[t]==6*x1[t]+4*x
```

$$
\begin{aligned}
\text{x1}(t) &\to e^{3t}\left(c_1\left(3e^{2t}-2\right)+2c_2\left(e^{2t}-1\right)\right) \\
\text{x2}(t) &\to -e^{3t}\left(3c_1\left(e^{2t}-1\right)+c_2\left(2e^{2t}-3\right)\right) \\
\text{x3}(t) &\to \int_1^t 3x(K[1])dK[1] + \frac{6}{5}c_1\left(e^{5t}-1\right) + \frac{4}{5}c_2\left(e^{5t}-1\right) + c_3
\end{aligned}
$$

5.2 problem Example 3

Internal problem ID [355]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Examples. Page 437
Problem number: Example 3.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) - 3x_2(t)$$
$$x_2'(t) = 3x_1(t) + 7x_2(t)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve([diff(x__1(t),t)=1*x__1(t)-3*x__2(t),diff(x__2(t),t)=3*x__1(t)+7*x__2(t)],singsol=all
```

$$x_1(t) = e^{4t}(c_2t + c_1)$$
$$x_2(t) = -\frac{e^{4t}(3c_2t + 3c_1 + c_2)}{3}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 46

```
DSolve[{x1'[t]==1*x1[t]-3*x2[t],x2'[t]==3*x1[t]+7*x2[t]},{x1[t],x2[t]},t,IncludeSingularSolu
```

$$\text{x1}(t) \to -e^{4t}(c_1(3t-1) + 3c_2t)$$
$$\text{x2}(t) \to e^{4t}(3(c_1+c_2)t + c_2)$$

5.3 problem Example 4

Internal problem ID [356]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Examples. Page 437
Problem number: Example 4.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= x_2(t) + 2x_3(t) \\
x_2'(t) &= -5x_1(t) - 3x_2(t) - 7x_3(t) \\
x_3'(t) &= x_1(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 75

```
dsolve([diff(x__1(t),t)=0*x__1(t)+1*x__2(t)+2*x__3(t),diff(x__2(t),t)=-5*x__1(t)-3*x__2(t)-7
```

$$
\begin{aligned}
x_1(t) &= -e^{-t}\left(c_3t^2 + c_2t - 2c_3t + c_1 - c_2\right) \\
x_2(t) &= -e^{-t}\left(c_3t^2 + c_2t + 4c_3t + c_1 + 2c_2 - 2c_3\right) \\
x_3(t) &= e^{-t}\left(c_3t^2 + c_2t + c_1\right)
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 134

```
DSolve[{x1'[t]==0*x1[t]+1*x2[t]+2*x3[t],x2'[t]==-5*x1[t]-3*x2[t]-7*x3[t],x3'[t]==1*x1[t]+0*x
```

$$
\begin{aligned}
\text{x1}(t) &\to \frac{1}{2}e^{-t}\left(c_1\left(-2t^2 + 2t + 2\right) - c_2(t-2)t + c_3(4-3t)t\right) \\
\text{x2}(t) &\to \frac{1}{2}e^{-t}\left(-\left((2c_1 + c_2 + 3c_3)t^2\right) - 2(5c_1 + 2c_2 + 7c_3)t + 2c_2\right) \\
\text{x3}(t) &\to \frac{1}{2}e^{-t}\left((2c_1 + c_2 + 3c_3)t^2 + 2(c_1 + c_3)t + 2c_3\right)
\end{aligned}
$$

5.4 problem Example 6

Internal problem ID [357]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Examples. Page 437
Problem number: Example 6.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= x_3(t) \\
x_2'(t) &= x_4(t) \\
x_3'(t) &= -2x_1(t) + 2x_2(t) - 3x_3(t) + x_4(t) \\
x_4'(t) &= 2x_1(t) - 2x_2(t) + x_3(t) - 3x_4(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 95

```
dsolve([diff(x__1(t),t)=0*x__1(t)+0*x__2(t)+1*x__3(t)+0*x__4(t),diff(x__2(t),t)=0*x__1(t)+0*
```

$$
\begin{aligned}
x_1(t) &= c_2 + c_3 \mathrm{e}^{-2t} + c_4 \mathrm{e}^{-2t} t \\
x_2(t) &= -c_3 \mathrm{e}^{-2t} - c_4 \mathrm{e}^{-2t} t + c_4 \mathrm{e}^{-2t} + c_2 + c_1 \mathrm{e}^{-2t} \\
x_3(t) &= -\mathrm{e}^{-2t}(2c_4 t + 2c_3 - c_4) \\
x_4(t) &= -\mathrm{e}^{-2t}(-2c_4 t + 2c_1 - 2c_3 + 3c_4)
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 210

```
DSolve[{x1'[t]==0*x1[t]+0*x2[t]+1*x3[t]+0*x4[t],x2'[t]==0*x1[t]+0*x2[t]+0*x3[t]+1*x4[t],x3'[
```

$$
\begin{aligned}
\mathrm{x1}(t) &\to \frac{1}{4} e^{-2t} \left(2c_1 \left(2t + e^{2t} + 1\right) + 2c_2 \left(-2t + e^{2t} - 1\right) + c_3 e^{2t} + 2c_3 t + c_4 e^{2t} - 2c_4 t - c_3 - c_4\right) \\
\mathrm{x2}(t) &\to \frac{1}{4} e^{-2t} \left(2c_1 \left(-2t + e^{2t} - 1\right) + 2c_2 \left(2t + e^{2t} + 1\right) + c_3 e^{2t} - 2c_3 t + c_4 e^{2t} + 2c_4 t - c_3 - c_4\right) \\
\mathrm{x3}(t) &\to e^{-2t}((-2c_1 + 2c_2 - c_3 + c_4)t + c_3) \\
\mathrm{x4}(t) &\to e^{-2t}((2c_1 - 2c_2 + c_3 - c_4)t + c_4)
\end{aligned}
$$

6 Section 7.6, Multiple Eigenvalue Solutions. Page 451

6.1 problem problem 1

Internal problem ID [358]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 1.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -2x_1(t) + x_2(t)$$
$$x_2'(t) = -x_1(t) - 4x_2(t)$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 32

```
dsolve([diff(x__1(t),t)=-2*x__1(t)+1*x__2(t),diff(x__2(t),t)=-1*x__1(t)-4*x__2(t)],singsol=a
```

$$x_1(t) = e^{-3t}(c_2t + c_1)$$
$$x_2(t) = -e^{-3t}(c_2t + c_1 - c_2)$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 42

```
DSolve[{x1'[t]==-2*x1[t]+1*x2[t],x2'[t]==-1*x1[t]-4*x2[t]},{x1[t],x2[t]},t,IncludeSingularSo
```

$$\text{x1}(t) \to e^{-3t}(c_1(t+1) + c_2t)$$
$$\text{x2}(t) \to e^{-3t}(c_2 - (c_1 + c_2)t)$$

6.2 problem problem 2

Internal problem ID [359]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 2.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 3x_1(t) - x_2(t)$$
$$x_2'(t) = x_1(t) + x_2(t)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve([diff(x__1(t),t)=3*x__1(t)-1*x__2(t),diff(x__2(t),t)=1*x__1(t)+1*x__2(t)],singsol=all
```

$$x_1(t) = e^{2t}(c_2 t + c_1)$$
$$x_2(t) = e^{2t}(c_2 t + c_1 - c_2)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 44

```
DSolve[{x1'[t]==3*x1[t]-1*x2[t],x2'[t]==1*x1[t]+1*x2[t]},{x1[t],x2[t]},t,IncludeSingularSolu
```

$$\text{x1}(t) \to e^{2t}(c_1(t+1) - c_2 t)$$
$$\text{x2}(t) \to e^{2t}((c_1 - c_2)t + c_2)$$

6.3 problem problem 3

Internal problem ID [360]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 3.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= x_1(t) - 2x_2(t) \\
x_2'(t) &= 2x_1(t) + 5x_2(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve([diff(x__1(t),t)=1*x__1(t)-2*x__2(t),diff(x__2(t),t)=2*x__1(t)+5*x__2(t)],singsol=all
```

$$
\begin{aligned}
x_1(t) &= \mathrm{e}^{3t}(c_2 t + c_1) \\
x_2(t) &= -\frac{\mathrm{e}^{3t}(2c_2 t + 2c_1 + c_2)}{2}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 46

```
DSolve[{x1'[t]==1*x1[t]-2*x2[t],x2'[t]==2*x1[t]+5*x2[t]},{x1[t],x2[t]},t,IncludeSingularSolu
```

$$
\begin{aligned}
\mathrm{x1}(t) &\to -e^{3t}(c_1(2t-1) + 2c_2 t) \\
\mathrm{x2}(t) &\to e^{3t}(2(c_1 + c_2)t + c_2)
\end{aligned}
$$

6.4 problem problem 4

Internal problem ID [361]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 4.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 3x_1(t) - x_2(t)$$
$$x_2'(t) = x_1(t) + 5x_2(t)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve([diff(x__1(t),t)=3*x__1(t)-1*x__2(t),diff(x__2(t),t)=1*x__1(t)+5*x__2(t)],singsol=all
```

$$x_1(t) = e^{4t}(c_2 t + c_1)$$
$$x_2(t) = -e^{4t}(c_2 t + c_1 + c_2)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 42

```
DSolve[{x1'[t]==3*x1[t]-1*x2[t],x2'[t]==1*x1[t]+5*x2[t]},{x1[t],x2[t]},t,IncludeSingularSolu
```

$$\text{x1}(t) \to -e^{4t}(c_1(t-1) + c_2 t)$$
$$\text{x2}(t) \to e^{4t}((c_1 + c_2)t + c_2)$$

6.5 problem problem 5

Internal problem ID [362]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 5.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 7x_1(t) + x_2(t)$$
$$x_2'(t) = -4x_1(t) + 3x_2(t)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve([diff(x__1(t),t)=7*x__1(t)+1*x__2(t),diff(x__2(t),t)=-4*x__1(t)+3*x__2(t)],singsol=al
```

$$x_1(t) = e^{5t}(c_2t + c_1)$$
$$x_2(t) = -e^{5t}(2c_2t + 2c_1 - c_2)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 45

```
DSolve[{x1'[t]==7*x1[t]+1*x2[t],x2'[t]==-4*x1[t]+3*x2[t]},{x1[t],x2[t]},t,IncludeSingularSol
```

$$\text{x1}(t) \to e^{5t}(2c_1t + c_2t + c_1)$$
$$\text{x2}(t) \to e^{5t}(c_2 - 2(2c_1 + c_2)t)$$

6.6 problem problem 6

Internal problem ID [363]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 6.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= x_1(t) - 4x_2(t) \\
x_2'(t) &= 4x_1(t) + 9x_2(t)
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 33

```
dsolve([diff(x__1(t),t)=1*x__1(t)-4*x__2(t),diff(x__2(t),t)=4*x__1(t)+9*x__2(t)],singsol=all
```

$$
\begin{aligned}
x_1(t) &= \mathrm{e}^{5t}(c_2 t + c_1) \\
x_2(t) &= -\frac{\mathrm{e}^{5t}(4c_2 t + 4c_1 + c_2)}{4}
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 46

```
DSolve[{x1'[t]==1*x1[t]-4*x2[t],x2'[t]==4*x1[t]+9*x2[t]},{x1[t],x2[t]},t,IncludeSingularSolu
```

$$
\begin{aligned}
\mathrm{x1}(t) &\to -e^{5t}(c_1(4t-1) + 4c_2 t) \\
\mathrm{x2}(t) &\to e^{5t}(4(c_1 + c_2)t + c_2)
\end{aligned}
$$

6.7 problem problem 7

Internal problem ID [364]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 7.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 2x_1(t) \\
x_2'(t) &= -7x_1(t) + 9x_2(t) + 7x_3(t) \\
x_3'(t) &= 2x_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
dsolve([diff(x__1(t),t)=2*x__1(t)+0*x__2(t)+0*x__3(t),diff(x__2(t),t)=-7*x__1(t)+9*x__2(t)+7
```

$$
\begin{aligned}
x_1(t) &= c_3 e^{2t} \\
x_2(t) &= -c_2 e^{2t} + c_3 e^{2t} + c_1 e^{9t} \\
x_3(t) &= c_2 e^{2t}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 60

```
DSolve[{x1'[t]==2*x1[t]+0*x2[t]+0*x3[t],x2'[t]==-7*x1[t]+9*x2[t]+7*x3[t],x3'[t]==0*x1[t]+0*x
```

$$
\begin{aligned}
\text{x1}(t) &\to c_1 e^{2t} \\
\text{x2}(t) &\to e^{2t}\left(-\left(c_1\left(e^{7t}-1\right)\right) + (c_2+c_3)e^{7t} - c_3\right) \\
\text{x3}(t) &\to c_3 e^{2t}
\end{aligned}
$$

6.8 problem problem 8

Internal problem ID [365]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 8.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 25x_1(t) + 12x_2(t) \\
x_2'(t) &= -18x_1(t) - 5x_2(t) \\
x_3'(t) &= 6x_1(t) + 6x_2(t) + 13x_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 60

```
dsolve([diff(x__1(t),t)=25*x__1(t)+12*x__2(t)+0*x__3(t),diff(x__2(t),t)=-18*x__1(t)-5*x__2(t
```

$$
\begin{aligned}
x_1(t) &= c_2 e^{7t} + c_3 e^{13t} \\
x_2(t) &= -\frac{3c_2 e^{7t}}{2} - c_3 e^{13t} \\
x_3(t) &= \frac{c_2 e^{7t}}{2} + \frac{c_3 e^{13t}}{2} + e^{13t} c_1
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 107

```
DSolve[{x1'[t]==25*x1[t]+12*x2[t]+0*x3[t],x2'[t]==-18*x1[t]-5*x2[t]+0*x3[t],x3'[t]==6*x1[t]+
```

$$
\begin{aligned}
\text{x1}(t) &\to e^{7t}\left(c_1\left(3e^{6t} - 2\right) + 2c_2\left(e^{6t} - 1\right)\right) \\
\text{x2}(t) &\to -e^{7t}\left(3c_1\left(e^{6t} - 1\right) + c_2\left(2e^{6t} - 3\right)\right) \\
\text{x3}(t) &\to e^{7t}\left(c_1\left(e^{6t} - 1\right) + c_2\left(e^{6t} - 1\right) + c_3 e^{6t}\right)
\end{aligned}
$$

6.9 problem problem 9

Internal problem ID [366]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 9.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= -19x_1(t) + 12x_2(t) + 84x_3(t) \\
x_2'(t) &= 5x_2(t) \\
x_3'(t) &= -8x_1(t) + 4x_2(t) + 33x_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 52

```
dsolve([diff(x__1(t),t)=-19*x__1(t)+12*x__2(t)+84*x__3(t),diff(x__2(t),t)=0*x__1(t)+5*x__2(t
```

$$
\begin{aligned}
x_1(t) &= c_1 e^{9t} + c_2 e^{5t} \\
x_2(t) &= c_3 e^{5t} \\
x_3(t) &= \frac{c_1 e^{9t}}{3} + \frac{2c_2 e^{5t}}{7} - \frac{c_3 e^{5t}}{7}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 94

```
DSolve[{x1'[t]==-19*x1[t]+12*x2[t]+84*x3[t],x2'[t]==0*x1[t]+5*x2[t]+0*x3[t],x3'[t]==-8*x1[t]
```

$$
\begin{aligned}
\text{x1}(t) &\to e^{5t}\left(c_1\left(7-6e^{4t}\right)+3(c_2+7c_3)\left(e^{4t}-1\right)\right) \\
\text{x2}(t) &\to c_2 e^{5t} \\
\text{x3}(t) &\to e^{5t}\left(-2c_1\left(e^{4t}-1\right)+c_2\left(e^{4t}-1\right)+c_3\left(7e^{4t}-6\right)\right)
\end{aligned}
$$

6.10 problem problem 10

Internal problem ID [367]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 10.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= -13x_1(t) + 40x_2(t) - 48x_3(t) \\
x_2'(t) &= -8x_1(t) + 23x_2(t) - 24x_3(t) \\
x_3'(t) &= 3x_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 52

```
dsolve([diff(x__1(t),t)=-13*x__1(t)+40*x__2(t)-48*x__3(t),diff(x__2(t),t)=-8*x__1(t)+23*x__2
```

$$
\begin{aligned}
x_1(t) &= c_1 e^{3t} + c_2 e^{7t} \\
x_2(t) &= \frac{2c_1 e^{3t}}{5} + \frac{c_2 e^{7t}}{2} + \frac{6c_3 e^{3t}}{5} \\
x_3(t) &= c_3 e^{3t}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 98

```
DSolve[{x1'[t]==-13*x1[t]+40*x2[t]-48*x3[t],x2'[t]==-8*x1[t]+23*x2[t]-24*x3[t],x3'[t]==0*x1[
```

$$
\begin{aligned}
\text{x1}(t) &\to e^{3t}\left(c_1\left(5 - 4e^{4t}\right) + 2(5c_2 - 6c_3)\left(e^{4t} - 1\right)\right) \\
\text{x2}(t) &\to -e^{3t}\left(2c_1\left(e^{4t} - 1\right) + c_2\left(4 - 5e^{4t}\right) + 6c_3\left(e^{4t} - 1\right)\right) \\
\text{x3}(t) &\to c_3 e^{3t}
\end{aligned}
$$

6.11 problem problem 11

Internal problem ID [368]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 11.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= -3x_1(t) - 4x_3(t) \\
x_2'(t) &= -x_1(t) - x_2(t) - x_3(t) \\
x_3'(t) &= x_1(t) + x_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 59

```
dsolve([diff(x__1(t),t)=-3*x__1(t)+0*x__2(t)-4*x__3(t),diff(x__2(t),t)=-1*x__1(t)-1*x__2(t)-
```

$$
\begin{aligned}
x_1(t) &= \mathrm{e}^{-t}(c_3 t + c_2) \\
x_2(t) &= \frac{(-c_3 t^2 - 2c_2 t + c_3 t + 4c_1)\,\mathrm{e}^{-t}}{4} \\
x_3(t) &= -\frac{\mathrm{e}^{-t}(2c_3 t + 2c_2 + c_3)}{4}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 81

```
DSolve[{x1'[t]==-3*x1[t]+0*x2[t]-4*x3[t],x2'[t]==-1*x1[t]-1*x2[t]-1*x3[t],x3'[t]==1*x1[t]+0*
```

$$
\begin{aligned}
\mathrm{x1}(t) &\to e^{-t}(-2c_1 t - 4c_3 t + c_1) \\
\mathrm{x2}(t) &\to \frac{1}{2}e^{-t}\left((c_1 + 2c_3)t^2 - 2(c_1 + c_3)t + 2c_2\right) \\
\mathrm{x3}(t) &\to e^{-t}((c_1 + 2c_3)t + c_3)
\end{aligned}
$$

6.12 problem problem 12

Internal problem ID [369]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 12.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -x_1(t) + x_3(t)$$
$$x_2'(t) = -x_2(t) + x_3(t)$$
$$x_3'(t) = x_1(t) - x_2(t) - x_3(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 62

```
dsolve([diff(x__1(t),t)=-1*x__1(t)+0*x__2(t)+1*x__3(t),diff(x__2(t),t)=0*x__1(t)-1*x__2(t)+1
```

$$x_1(t) = \frac{(c_3t^2 + 2c_2t + 2c_1)\,\mathrm{e}^{-t}}{2}$$
$$x_2(t) = \frac{\mathrm{e}^{-t}(c_3t^2 + 2c_2t + 2c_1 - 2c_3)}{2}$$
$$x_3(t) = \mathrm{e}^{-t}(c_3t + c_2)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 89

```
DSolve[{x1'[t]==-1*x1[t]+0*x2[t]+1*x3[t],x2'[t]==0*x1[t]-1*x2[t]+1*x3[t],x3'[t]==1*x1[t]-1*x
```

$$\mathrm{x1}(t) \to \frac{1}{2}e^{-t}\left(c_1\left(t^2 + 2\right) + t(2c_3 - c_2t)\right)$$
$$\mathrm{x2}(t) \to \frac{1}{2}e^{-t}\left((c_1 - c_2)t^2 + 2c_3t + 2c_2\right)$$
$$\mathrm{x3}(t) \to e^{-t}((c_1 - c_2)t + c_3)$$

6.13 problem problem 13

Internal problem ID [370]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 13.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= -x_1(t) + x_3(t) \\
x_2'(t) &= x_2(t) - 4x_3(t) \\
x_3'(t) &= x_2(t) - 3x_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 54

```
dsolve([diff(x__1(t),t)=-1*x__1(t)+0*x__2(t)+1*x__3(t),diff(x__2(t),t)=0*x__1(t)+1*x__2(t)-4
```

$$
\begin{aligned}
x_1(t) &= \frac{(c_3t^2 + 2c_2t + 2c_1)\,\mathrm{e}^{-t}}{2} \\
x_2(t) &= \mathrm{e}^{-t}(2c_3t + 2c_2 + c_3) \\
x_3(t) &= \mathrm{e}^{-t}(c_3t + c_2)
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 78

```
DSolve[{x1'[t]==-1*x1[t]+0*x2[t]+1*x3[t],x2'[t]==0*x1[t]+1*x2[t]-4*x3[t],x3'[t]==0*x1[t]+1*x
```

$$
\begin{aligned}
\text{x1}(t) &\to \frac{1}{2}e^{-t}(t((c_2 - 2c_3)t + 2c_3) + 2c_1) \\
\text{x2}(t) &\to e^{-t}(2c_2t - 4c_3t + c_2) \\
\text{x3}(t) &\to e^{-t}((c_2 - 2c_3)t + c_3)
\end{aligned}
$$

6.14 problem problem 14

Internal problem ID [371]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 14.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= x_3(t) \\
x_2'(t) &= -5x_1(t) - x_2(t) - 5x_3(t) \\
x_3'(t) &= 4x_1(t) + x_2(t) - 2x_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 72

```
dsolve([diff(x__1(t),t)=0*x__1(t)+0*x__2(t)+1*x__3(t),diff(x__2(t),t)=-5*x__1(t)-1*x__2(t)-5
```

$$
\begin{aligned}
x_1(t) &= \mathrm{e}^{-t}\left(c_3t^2 + c_2t + c_1\right) \\
x_2(t) &= -\mathrm{e}^{-t}\left(5c_3t^2 + 5c_2t + 5c_1 - 2c_3\right) \\
x_3(t) &= -\mathrm{e}^{-t}\left(c_3t^2 + c_2t - 2c_3t + c_1 - c_2\right)
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 119

```
DSolve[{x1'[t]==0*x1[t]+0*x2[t]+1*x3[t],x2'[t]==-5*x1[t]-1*x2[t]-5*x3[t],x3'[t]==4*x1[t]+1*x
```

$$
\begin{aligned}
\mathrm{x1}(t) &\to \frac{1}{2}e^{-t}\left(c_1\left(5t^2 + 2t + 2\right) + t(c_2t + 2c_3)\right) \\
\mathrm{x2}(t) &\to \frac{1}{2}e^{-t}\left(-5(5c_1 + c_2)t^2 - 10(c_1 + c_3)t + 2c_2\right) \\
\mathrm{x3}(t) &\to \frac{1}{2}e^{-t}\left(-\left((5c_1 + c_2)t^2\right) + 2(4c_1 + c_2 - c_3)t + 2c_3\right)
\end{aligned}
$$

6.15 problem problem 15

Internal problem ID [372]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 15.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -2x_1(t) - 9x_2(t)$$
$$x_2'(t) = x_1(t) + 4x_2(t)$$
$$x_3'(t) = x_1(t) + 3x_2(t) + x_3(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 47

```
dsolve([diff(x__1(t),t)=-2*x__1(t)-9*x__2(t)-0*x__3(t),diff(x__2(t),t)=1*x__1(t)+4*x__2(t)-0
```

$$x_1(t) = e^t(c_3t + c_2)$$
$$x_2(t) = -\frac{e^t(3c_3t + 3c_2 + c_3)}{9}$$
$$x_3(t) = \frac{e^t(-c_3t + 3c_1 - c_2)}{3}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 62

```
DSolve[{x1'[t]==-2*x1[t]-9*x2[t]-0*x3[t],x2'[t]==1*x1[t]+4*x2[t]-0*x3[t],x3'[t]==1*x1[t]+3*x
```

$$\text{x1}(t) \to -e^t(c_1(3t - 1) + 9c_2t)$$
$$\text{x2}(t) \to e^t((c_1 + 3c_2)t + c_2)$$
$$\text{x3}(t) \to e^t((c_1 + 3c_2)t + c_3)$$

6.16 problem problem 16

Internal problem ID [373]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 16.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= x_1(t) \\
x_2'(t) &= -2x_1(t) - 2x_2(t) - 3x_3(t) \\
x_3'(t) &= 2x_1(t) + 3x_2(t) + 4x_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 39

```
dsolve([diff(x__1(t),t)=1*x__1(t)+0*x__2(t)-0*x__3(t),diff(x__2(t),t)=-2*x__1(t)-2*x__2(t)-3
```

$$
\begin{aligned}
x_1(t) &= c_3 e^t \\
x_2(t) &= e^t(c_2 t + c_1) \\
x_3(t) &= -\frac{e^t(3c_2 t + 3c_1 + c_2 + 2c_3)}{3}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 57

```
DSolve[{x1'[t]==1*x1[t]+0*x2[t]-0*x3[t],x2'[t]==-2*x1[t]-2*x2[t]-3*x3[t],x3'[t]==2*x1[t]+3*x
```

$$
\begin{aligned}
\text{x1}(t) &\to c_1 e^t \\
\text{x2}(t) &\to e^t(-2c_1 t - 3(c_2 + c_3)t + c_2) \\
\text{x3}(t) &\to e^t(2c_1 t + 3(c_2 + c_3)t + c_3)
\end{aligned}
$$

6.17 problem problem 17

Internal problem ID [374]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 17.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= x_1(t) \\
x_2'(t) &= 18x_1(t) + 7x_2(t) + 4x_3(t) \\
x_3'(t) &= -27x_1(t) - 9x_2(t) - 5x_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 41

```
dsolve([diff(x__1(t),t)=1*x__1(t)+0*x__2(t)-0*x__3(t),diff(x__2(t),t)=18*x__1(t)+7*x__2(t)+4
```

$$
\begin{aligned}
x_1(t) &= c_3 \mathrm{e}^t \\
x_2(t) &= \mathrm{e}^t (c_2 t + c_1) \\
x_3(t) &= -\frac{\mathrm{e}^t (6c_2 t + 6c_1 - c_2 + 18c_3)}{4}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 63

```
DSolve[{x1'[t]==1*x1[t]+0*x2[t]-0*x3[t],x2'[t]==18*x1[t]+7*x2[t]+4*x3[t],x3'[t]==-27*x1[t]-9
```

$$
\begin{aligned}
\text{x1}(t) &\to c_1 e^t \\
\text{x2}(t) &\to e^t (2(9c_1 + 3c_2 + 2c_3)t + c_2) \\
\text{x3}(t) &\to e^t (c_3 - 3(9c_1 + 3c_2 + 2c_3)t)
\end{aligned}
$$

6.18 problem problem 18

Internal problem ID [375]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 18.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= x_1(t) \\
x_2'(t) &= x_1(t) + 3x_2(t) + x_3(t) \\
x_3'(t) &= -2x_1(t) - 4x_2(t) - x_3(t)
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 39

```
dsolve([diff(x__1(t),t)=1*x__1(t)+0*x__2(t)-0*x__3(t),diff(x__2(t),t)=1*x__1(t)+3*x__2(t)+1*
```

$$
\begin{aligned}
x_1(t) &= c_3 \mathrm{e}^t \\
x_2(t) &= \mathrm{e}^t (c_2 t + c_1) \\
x_3(t) &= -\mathrm{e}^t (2c_2 t + 2c_1 - c_2 + c_3)
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 54

```
DSolve[{x1'[t]==1*x1[t]+0*x2[t]-0*x3[t],x2'[t]==1*x1[t]+3*x2[t]+1*x3[t],x3'[t]==-2*x1[t]-4*x
```

$$
\begin{aligned}
\text{x1}(t) &\to c_1 e^t \\
\text{x2}(t) &\to e^t ((c_1 + 2c_2 + c_3)t + c_2) \\
\text{x3}(t) &\to e^t (c_3 - 2(c_1 + 2c_2 + c_3)t)
\end{aligned}
$$

6.19 problem problem 19

Internal problem ID [376]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 19.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= x_1(t) - 4x_2(t) - 2x_4(t) \\
x_2'(t) &= x_2(t) \\
x_3'(t) &= 6x_1(t) - 12x_2(t) - x_3(t) - 6x_4(t) \\
x_4'(t) &= -4x_2(t) - x_4(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 53

```
dsolve([diff(x__1(t),t)=1*x__1(t)-4*x__2(t)+0*x__3(t)-2*x__4(t),diff(x__2(t),t)=0*x__1(t)+1*
```

$$
\begin{aligned}
x_1(t) &= c_2 e^t + c_3 e^{-t} \\
x_2(t) &= c_4 e^t \\
x_3(t) &= 3c_2 e^t + e^{-t} c_1 \\
x_4(t) &= -2c_4 e^t + c_3 e^{-t}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 114

```
DSolve[{x1'[t]==1*x1[t]-4*x2[t]+0*x3[t]-2*x4[t],x2'[t]==0*x1[t]+1*x2[t]+0*x3[t]+0*x4[t],x3'[
```

$$
\begin{aligned}
\text{x1}(t) &\to e^{-t}\left((c_1 - 2c_2 - c_4)e^{2t} + 2c_2 + c_4\right) \\
\text{x2}(t) &\to c_2 e^t \\
\text{x3}(t) &\to e^{-t}\left(3c_1\left(e^{2t} - 1\right) - 6c_2\left(e^{2t} - 1\right) - 3c_4 e^{2t} + c_3 + 3c_4\right) \\
\text{x4}(t) &\to e^{-t}\left(c_4 - 2c_2\left(e^{2t} - 1\right)\right)
\end{aligned}
$$

6.20 problem problem 20

Internal problem ID [377]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 20.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 2x_1(t) + x_2(t) + x_4(t) \\
x_2'(t) &= 2x_2(t) + x_3(t) \\
x_3'(t) &= 2x_3(t) + x_4(t) \\
x_4'(t) &= 2x_4(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 78

```
dsolve([diff(x__1(t),t)=2*x__1(t)+1*x__2(t)+0*x__3(t)+1*x__4(t),diff(x__2(t),t)=0*x__1(t)+2*
```

$$
\begin{aligned}
x_1(t) &= \frac{(c_4t^3 + 3c_3t^2 + 6c_2t + 6c_4t + 6c_1)\,\mathrm{e}^{2t}}{6} \\
x_2(t) &= \frac{(c_4t^2 + 2c_3t + 2c_2)\,\mathrm{e}^{2t}}{2} \\
x_3(t) &= (c_4t + c_3)\,\mathrm{e}^{2t} \\
x_4(t) &= c_4\mathrm{e}^{2t}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 96

```
DSolve[{x1'[t]==2*x1[t]+1*x2[t]+0*x3[t]+1*x4[t],x2'[t]==0*x1[t]+2*x2[t]+1*x3[t]+0*x4[t],x3'[
```

$$
\begin{aligned}
\mathrm{x1}(t) &\to \frac{1}{6}e^{2t}(t(c_4t^2 + 3c_3t + 6c_2 + 6c_4) + 6c_1) \\
\mathrm{x2}(t) &\to \frac{1}{2}e^{2t}(t(c_4t + 2c_3) + 2c_2) \\
\mathrm{x3}(t) &\to e^{2t}(c_4t + c_3) \\
\mathrm{x4}(t) &\to c_4e^{2t}
\end{aligned}
$$

6.21 problem problem 21

Internal problem ID [378]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 21.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= -x_1(t) - 4x_2(t) \\
x_2'(t) &= x_1(t) + 3x_2(t) \\
x_3'(t) &= x_1(t) + 2x_2(t) + x_3(t) \\
x_4'(t) &= x_2(t) + x_4(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 63

```
dsolve([diff(x__1(t),t)=-1*x__1(t)-4*x__2(t)+0*x__3(t)+0*x__4(t),diff(x__2(t),t)=1*x__1(t)+3
```

$$
\begin{aligned}
x_1(t) &= -\mathrm{e}^t(2c_4t + 2c_3 - c_4) \\
x_2(t) &= \mathrm{e}^t(c_4t + c_3) \\
x_3(t) &= \mathrm{e}^t(c_4t + c_1 + c_3) \\
x_4(t) &= \frac{(c_4t^2 + 2c_3t + 2c_2)\,\mathrm{e}^t}{2}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 91

```
DSolve[{x1'[t]==-1*x1[t]-4*x2[t]+0*x3[t]+0*x4[t],x2'[t]==1*x1[t]+3*x2[t]+0*x3[t]+0*x4[t],x3'
```

$$
\begin{aligned}
\text{x1}(t) &\to -e^t(c_1(2t-1) + 4c_2t) \\
\text{x2}(t) &\to e^t((c_1 + 2c_2)t + c_2) \\
\text{x3}(t) &\to e^t((c_1 + 2c_2)t + c_3) \\
\text{x4}(t) &\to \frac{1}{2}e^t\left(c_1t^2 + 2c_2(t+1)t + 2c_4\right)
\end{aligned}
$$

6.22 problem problem 22

Internal problem ID [379]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 22.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= x_1(t) + 3x_2(t) + 7x_3(t)\\
x_2'(t) &= -x_2(t) - 4x_3(t)\\
x_3'(t) &= x_2(t) + 3x_3(t)\\
x_4'(t) &= -6x_2(t) - 14x_3(t) + x_4(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 78

```
dsolve([diff(x__1(t),t)=1*x__1(t)+3*x__2(t)+7*x__3(t)+0*x__4(t),diff(x__2(t),t)=0*x__1(t)-1*
```

$$
\begin{aligned}
x_1(t) &= \frac{(-c_4t^2 - 2c_3t - 7c_4t + 4c_2)\,\mathrm{e}^t}{4}\\
x_2(t) &= \mathrm{e}^t(c_4t + c_3)\\
x_3(t) &= -\frac{\mathrm{e}^t(2c_4t + 2c_3 + c_4)}{4}\\
x_4(t) &= \frac{(c_4t^2 + 2c_3t + 7c_4t + 2c_1)\,\mathrm{e}^t}{2}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 99

```
DSolve[{x1'[t]==1*x1[t]+3*x2[t]+7*x3[t]+0*x4[t],x2'[t]==0*x1[t]-1*x2[t]-4*x3[t]+0*x4[t],x3'[
```

$$
\begin{aligned}
\text{x1}(t) &\to \frac{1}{2}e^t(c_2t(t+6) + 2c_3t(t+7) + 2c_1)\\
\text{x2}(t) &\to -e^t(c_2(2t-1) + 4c_3t)\\
\text{x3}(t) &\to e^t((c_2+2c_3)t + c_3)\\
\text{x4}(t) &\to e^t(c_2(-t)(t+6) - 2c_3t(t+7) + c_4)
\end{aligned}
$$

6.23 problem problem 23

Internal problem ID [380]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 23.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 39x_1(t) + 8x_2(t) - 16x_3(t) \\
x_2'(t) &= -36x_1(t) - 5x_2(t) + 16x_3(t) \\
x_3'(t) &= 72x_1(t) + 16x_2(t) - 29x_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 67

```
dsolve([diff(x__1(t),t)=39*x__1(t)+8*x__2(t)-16*x__3(t),diff(x__2(t),t)=-36*x__1(t)-5*x__2(t
```

$$
\begin{aligned}
x_1(t) &= c_2 e^{3t} + c_3 e^{-t} \\
x_2(t) &= -c_2 e^{3t} - c_3 e^{-t} + c_1 e^{3t} \\
x_3(t) &= \frac{7c_2 e^{3t}}{4} + 2c_3 e^{-t} + \frac{c_1 e^{3t}}{2}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 127

```
DSolve[{x1'[t]==39*x1[t]+8*x2[t]-16*x3[t],x2'[t]==-36*x1[t]-5*x2[t]+16*x3[t],x3'[t]==72*x1[t
```

$$
\begin{aligned}
\text{x1}(t) &\to e^{-t}\left(c_1\left(10e^{4t} - 9\right) + 2(c_2 - 2c_3)\left(e^{4t} - 1\right)\right) \\
\text{x2}(t) &\to e^{-t}\left(-9c_1\left(e^{4t} - 1\right) - c_2\left(e^{4t} - 2\right) + 4c_3\left(e^{4t} - 1\right)\right) \\
\text{x3}(t) &\to e^{-t}\left(18c_1\left(e^{4t} - 1\right) + 4c_2\left(e^{4t} - 1\right) + c_3\left(8 - 7e^{4t}\right)\right)
\end{aligned}
$$

6.24 problem problem 24

Internal problem ID [381]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 24.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 28x_1(t) + 50x_2(t) + 100x_3(t) \\
x_2'(t) &= 15x_1(t) + 33x_2(t) + 60x_3(t) \\
x_3'(t) &= -15x_1(t) - 30x_2(t) - 57x_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 67

```
dsolve([diff(x__1(t),t)=28*x__1(t)+50*x__2(t)+100*x__3(t),diff(x__2(t),t)=15*x__1(t)+33*x__2
```

$$
\begin{aligned}
x_1(t) &= c_2 e^{3t} + c_3 e^{-2t} \\
x_2(t) &= \frac{3c_2 e^{3t}}{5} + \frac{3c_3 e^{-2t}}{5} + c_1 e^{3t} \\
x_3(t) &= -\frac{11c_2 e^{3t}}{20} - \frac{3c_3 e^{-2t}}{5} - \frac{c_1 e^{3t}}{2}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 229

```
DSolve[{x1'[t]==28*x1[t]+50*x2[t]+100*x3[t],x2'[t]==15*x1[t]+33*x2[t]+60*x3[t],x3'[t]==-15*x
```

$$\text{x1}(t) \to \frac{1}{57}e^{t/2}\left(19(3c_1-5c_2)e^{5t/2} + 95c_2\cos\left(\frac{5\sqrt{95}t}{2}\right) + \sqrt{95}(6c_1+13c_2+24c_3)\sin\left(\frac{5\sqrt{95}t}{2}\right)\right)$$

$$\text{x2}(t) \to \frac{1}{95}e^{t/2}\left(95c_2\cos\left(\frac{5\sqrt{95}t}{2}\right) + \sqrt{95}(6c_1+13c_2+24c_3)\sin\left(\frac{5\sqrt{95}t}{2}\right)\right)$$

$$\text{x3}(t) \to -\frac{e^{t/2}\left(95(3c_1-5c_2)e^{5t/2} - 95(3c_1-5c_2+12c_3)\cos\left(\frac{5\sqrt{95}t}{2}\right) + \sqrt{95}(69c_1+197c_2+276c_3)\sin\left(\frac{5\sqrt{95}t}{2}\right)\right)}{1140}$$

6.25 problem problem 25

Internal problem ID [382]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 25.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -2x_1(t) + 17x_2(t) + 4x_3(t)$$
$$x_2'(t) = -x_1(t) + 6x_2(t) + x_3(t)$$
$$x_3'(t) = x_2(t) + 2x_3(t)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 62

```
dsolve([diff(x__1(t),t)=-2*x__1(t)+17*x__2(t)+4*x__3(t),diff(x__2(t),t)=-1*x__1(t)+6*x__2(t)
```

$$x_1(t) = e^{2t}(c_3t^2 + c_2t + 8c_3t + c_1 + 4c_2 - 2c_3)$$
$$x_2(t) = e^{2t}(2c_3t + c_2)$$
$$x_3(t) = e^{2t}(c_3t^2 + c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 105

```
DSolve[{x1'[t]==-2*x1[t]+17*x2[t]+4*x3[t],x2'[t]==-1*x1[t]+6*x2[t]+1*x3[t],x3'[t]==0*x1[t]+1
```

$$\mathrm{x1}(t) \to \frac{1}{2}e^{2t}\left(-\left(c_1\left(t^2 + 8t - 2\right)\right) + c_2t(4t + 34) + c_3t(t + 8)\right)$$
$$\mathrm{x2}(t) \to e^{2t}((-c_1 + 4c_2 + c_3)t + c_2)$$
$$\mathrm{x3}(t) \to \frac{1}{2}e^{2t}\left((-c_1 + 4c_2 + c_3)t^2 + 2c_2t + 2c_3\right)$$

6.26 problem problem 26

Internal problem ID [383]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 26.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 5x_1(t) - x_2(t) + x_3(t)\\
x_2'(t) &= x_1(t) + 3x_2(t)\\
x_3'(t) &= -3x_1(t) + 2x_2(t) + x_3(t)
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 62

```
dsolve([diff(x__1(t),t)=5*x__1(t)-1*x__2(t)+1*x__3(t),diff(x__2(t),t)=1*x__1(t)+3*x__2(t)+0*
```

$$
\begin{aligned}
x_1(t) &= e^{3t}(2c_3t + c_2)\\
x_2(t) &= e^{3t}\left(c_3t^2 + c_2t + c_1\right)\\
x_3(t) &= e^{3t}\left(c_3t^2 + c_2t - 4c_3t + c_1 - 2c_2 + 2c_3\right)
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 105

```
DSolve[{x1'[t]==5*x1[t]-1*x2[t]+1*x3[t],x2'[t]==1*x1[t]+3*x2[t]+0*x3[t],x3'[t]==-3*x1[t]+2*x
```

$$
\begin{aligned}
\text{x1}(t) &\to e^{3t}(2c_1t - c_2t + c_3t + c_1)\\
\text{x2}(t) &\to \frac{1}{2}e^{3t}\left((2c_1 - c_2 + c_3)t^2 + 2c_1t + 2c_2\right)\\
\text{x3}(t) &\to \frac{1}{2}e^{3t}\left(c_3\left(t^2 - 4t + 2\right) + 2c_1(t-3)t - c_2(t-4)t\right)
\end{aligned}
$$

6.27 problem problem 27

Internal problem ID [384]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 27.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= -3x_1(t) + 5x_2(t) - 5x_3(t) \\
x_2'(t) &= 3x_1(t) - x_2(t) + 3x_3(t) \\
x_3'(t) &= 8x_1(t) - 8x_2(t) + 10x_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 58

```
dsolve([diff(x__1(t),t)=-3*x__1(t)+5*x__2(t)-5*x__3(t),diff(x__2(t),t)=3*x__1(t)-1*x__2(t)+3
```

$$
\begin{aligned}
x_1(t) &= e^{2t}(c_3t + c_2) \\
x_2(t) &= \frac{e^{2t}(-3c_3t + 5c_1 - 3c_2)}{5} \\
x_3(t) &= \frac{e^{2t}(-8c_3t + 5c_1 - 8c_2 - c_3)}{5}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 174

```
DSolve[{x1'[t]==-3*x1[t]+5*x2[t]-5*x3[t],x2'[t]==4*x1[t]-1*x2[t]+4*x3[t],x3'[t]==8*x1[t]-8*x
```

$$
\begin{aligned}
\text{x1}(t) &\to \frac{1}{3}e^{2t}\left(-5(c_1 + c_3)\cos\left(\sqrt{3}t\right) - 5\sqrt{3}(c_1 - c_2 + c_3)\sin\left(\sqrt{3}t\right) + 8c_1 + 5c_3\right) \\
\text{x2}(t) &\to \frac{1}{3}e^{2t}\left(3c_2\cos\left(\sqrt{3}t\right) + \sqrt{3}(4c_1 - 3c_2 + 4c_3)\sin\left(\sqrt{3}t\right)\right) \\
\text{x3}(t) &\to \frac{1}{3}e^{2t}\left(8(c_1 + c_3)\cos\left(\sqrt{3}t\right) + 8\sqrt{3}(c_1 - c_2 + c_3)\sin\left(\sqrt{3}t\right) - 8c_1 - 5c_3\right)
\end{aligned}
$$

6.28 problem problem 28

Internal problem ID [385]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 28.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= -15x_1(t) - 7x_2(t) + 4x_3(t)\\
x_2'(t) &= 34x_1(t) + 16x_2(t) - 11x_3(t)\\
x_3'(t) &= 17x_1(t) + 7x_2(t) + 5x_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 73

```
dsolve([diff(x__1(t),t)=-15*x__1(t)-7*x__2(t)+4*x__3(t),diff(x__2(t),t)=34*x__1(t)+16*x__2(t
```

$$
\begin{aligned}
x_1(t) &= \mathrm{e}^{2t}\left(c_3t^2 + c_2t + c_1\right)\\
x_2(t) &= -\frac{\mathrm{e}^{2t}(833c_3t^2 + 833c_2t + 42c_3t + 833c_1 + 21c_2 - 8c_3)}{343}\\
x_3(t) &= \frac{\mathrm{e}^{2t}(14c_3t + 7c_2 + 2c_3)}{49}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 124

```
DSolve[{x1'[t]==-15*x1[t]-7*x2[t]+4*x3[t],x2'[t]==34*x1[t]+16*x2[t]-11*x3[t],x3'[t]==17*x1[t
```

$$
\begin{aligned}
\text{x1}(t) &\to \frac{1}{2}e^{2t}\left(c_1\left(119t^2 - 34t + 2\right) + 7c_2t(7t-2) + c_3t(21t+8)\right)\\
\text{x2}(t) &\to -\frac{1}{2}e^{2t}\left(17(17c_1 + 7c_2 + 3c_3)t^2 + (-68c_1 - 28c_2 + 22c_3)t - 2c_2\right)\\
\text{x3}(t) &\to e^{2t}((17c_1 + 7c_2 + 3c_3)t + c_3)
\end{aligned}
$$

6.29 problem problem 29

Internal problem ID [386]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 29.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= -x_1(t) + x_2(t) + x_3(t) - 2x_4(t) \\
x_2'(t) &= 7x_1(t) - 4x_2(t) - 6x_3(t) + 11x_4(t) \\
x_3'(t) &= 5x_1(t) - x_2(t) + x_3(t) + 3x_4(t) \\
x_4'(t) &= 6x_1(t) - 2x_2(t) - 2x_3(t) + 6x_4(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 120

```
dsolve([diff(x__1(t),t)=-1*x__1(t)+1*x__2(t)+1*x__3(t)-2*x__4(t),diff(x__2(t),t)=7*x__1(t)-4
```

$$
\begin{aligned}
x_1(t) &= \mathrm{e}^{-t}(c_4 t + c_3) \\
x_2(t) &= -3c_4\mathrm{e}^{-t}t - 3c_3\mathrm{e}^{-t} + c_4\mathrm{e}^{-t} + \mathrm{e}^{2t}tc_1 + c_2\mathrm{e}^{2t} \\
x_3(t) &= -c_4\mathrm{e}^{-t}t - c_3\mathrm{e}^{-t} - \mathrm{e}^{2t}tc_1 - 2c_1\mathrm{e}^{2t} - c_2\mathrm{e}^{2t} \\
x_4(t) &= -2c_4\mathrm{e}^{-t}t - 2c_3\mathrm{e}^{-t} - c_1\mathrm{e}^{2t}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 196

```
DSolve[{x1'[t]==-1*x1[t]+1*x2[t]+1*x3[t]-2*x4[t],x2'[t]==7*x1[t]-4*x2[t]-6*x3[t]+11*x4[t],x3
```

$$
\begin{aligned}
&\mathrm{x1}(t) \to e^{-t}((c_2 + c_3 - 2c_4)t + c_1) \\
&\mathrm{x2}(t) \to e^{-t}\left(c_1\left(e^{3t}(3-2t)-3\right) - 3c_2t - c_3e^{3t} - 3c_3t + 2c_4e^{3t} - c_4e^{3t}t + 6c_4t + c_2 + c_3 - 2c_4\right) \\
&\mathrm{x3}(t) \to e^{-t}\left(c_1\left(e^{3t}(2t+1) - 1\right) + c_3e^{3t} - t\left(-c_4\left(e^{3t}+2\right) + c_2 + c_3\right)\right) \\
&\mathrm{x4}(t) \to e^{-t}\left(2c_1\left(e^{3t} - 1\right) - 2(c_2 + c_3 - 2c_4)t + c_4e^{3t}\right)
\end{aligned}
$$

6.30 problem problem 30

Internal problem ID [387]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 30.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 2x_1(t) + x_2(t) - 2x_3(t) + x_4(t) \\
x_2'(t) &= 3x_2(t) - 5x_3(t) + 3x_4(t) \\
x_3'(t) &= -13x_2(t) + 22x_3(t) - 12x_4(t) \\
x_4'(t) &= -27x_2(t) + 45x_3(t) - 25x_4(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 89

```
dsolve([diff(x__1(t),t)=2*x__1(t)+1*x__2(t)-2*x__3(t)+1*x__4(t),diff(x__2(t),t)=0*x__1(t)+3*
```

$$
\begin{aligned}
x_1(t) &= \frac{(-c_2 t + 3c_1)\,\mathrm{e}^{2t}}{3} \\
x_2(t) &= \mathrm{e}^{-t}(c_4 t + c_3) \\
x_3(t) &= \left(-\mathrm{e}^{-3t}(c_4 t + c_3 - c_4) + c_2\right)\mathrm{e}^{2t} \\
x_4(t) &= -3c_3\mathrm{e}^{-t} - 3c_4\mathrm{e}^{-t}t + 2c_4\mathrm{e}^{-t} + \frac{5c_2\mathrm{e}^{2t}}{3}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 161

```
DSolve[{x1'[t]==2*x1[t]+1*x2[t]-2*x3[t]+1*x4[t],x2'[t]==0*x1[t]+3*x2[t]-5*x3[t]+3*x4[t],x3'[
```

$$
\begin{aligned}
\text{x1}(t) &\to e^{2t}((c_2 - 2c_3 + c_4)t + c_1) \\
\text{x2}(t) &\to e^{-t}(4c_2 t - 5c_3 t + 3c_4 t + c_2) \\
\text{x3}(t) &\to e^{-t}\left(c_2\left(-4t - 3e^{3t} + 3\right) + c_3\left(5t + 6e^{3t} - 5\right) - 3c_4\left(t + e^{3t} - 1\right)\right) \\
\text{x4}(t) &\to e^{-t}\left(c_2\left(-12t - 5e^{3t} + 5\right) + 5c_3\left(3t + 2e^{3t} - 2\right) - c_4\left(9t + 5e^{3t} - 6\right)\right)
\end{aligned}
$$

6.31 problem problem 31

Internal problem ID [388]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 31.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 35x_1(t) - 12x_2(t) + 4x_3(t) + 30x_4(t) \\
x_2'(t) &= 22x_1(t) - 8x_2(t) + 3x_3(t) + 19x_4(t) \\
x_3'(t) &= -10x_1(t) + 3x_2(t) - 9x_4(t) \\
x_4'(t) &= -27x_1(t) + 9x_2(t) - 3x_3(t) - 23x_4(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 117

```
dsolve([diff(x__1(t),t)=35*x__1(t)-12*x__2(t)+4*x__3(t)+30*x__4(t),diff(x__2(t),t)=22*x__1(t
```

$$
\begin{aligned}
x_1(t) &= \frac{e^t(-6c_4t^2 - 6c_3t - 4c_4t + 3c_1 - 6c_2 - 2c_3)}{3} \\
x_2(t) &= \frac{e^t(-3c_4t^2 - 3c_3t - 16c_4t + 3c_1 - 3c_2 - 8c_3 + 6c_4)}{9} \\
x_3(t) &= e^t\left(c_4t^2 + c_3t + c_2\right) \\
x_4(t) &= -\frac{e^t(-18c_4t^2 - 18c_3t - 6c_4t + 9c_1 - 18c_2 - 3c_3 - 2c_4)}{9}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 207

```
DSolve[{x1'[t]==35*x1[t]-12*x2[t]+4*x3[t]+30*x4[t],x2'[t]==22*x1[t]-8*x2[t]+3*x3[t]+19*x4[t]
```

$$\text{x1}(t) \to e^t\left(c_1\left(21t^2+34t+1\right)-3c_2t(3t+4)+c_3t(3t+4)+6c_4t(3t+5)\right)$$
$$\text{x2}(t) \to \frac{1}{2}e^t\left((7c_1-3c_2+c_3+6c_4)t^2+2(22c_1-9c_2+3c_3+19c_4)t+2c_2\right)$$
$$\text{x3}(t) \to \frac{1}{2}e^t\left(-3(7c_1-3c_2+c_3+6c_4)t^2-2(10c_1-3c_2+c_3+9c_4)t+2c_3\right)$$
$$\text{x4}(t) \to e^t\left(-3(7c_1-3c_2+c_3+6c_4)t^2-3(9c_1-3c_2+c_3+8c_4)t+c_4\right)$$

6.32 problem problem 32

Internal problem ID [389]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 32.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 11x_1(t) - x_2(t) + 26x_3(t) + 6x_4(t) - 3x_5(t) \\
x_2'(t) &= 3x_2(t) \\
x_3'(t) &= -9x_1(t) - 24x_3(t) - 6x_4(t) + 3x_5(t) \\
x_4'(t) &= 3x_1(t) + 9x_3(t) + 5x_4(t) - x_5(t) \\
x_5'(t) &= -48x_1(t) - 3x_2(t) - 138x_3(t) - 30x_4(t) + 18x_5(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 107

```
dsolve([diff(x__1(t),t)=11*x__1(t)-1*x__2(t)+26*x__3(t)+6*x__4(t)-3*x__5(t),diff(x__2(t),t)=
```

$$
\begin{aligned}
x_1(t) &= \left(-(c_3 + c_5)\,\mathrm{e}^{t} + c_1\right)\mathrm{e}^{2t} \\
x_2(t) &= c_5\mathrm{e}^{3t} \\
x_3(t) &= c_3\mathrm{e}^{3t} + c_4\mathrm{e}^{2t} \\
x_4(t) &= -\frac{c_3\mathrm{e}^{3t}}{3} - \frac{c_4\mathrm{e}^{2t}}{3} + c_2\mathrm{e}^{3t} \\
x_5(t) &= \frac{16c_3\mathrm{e}^{3t}}{3} + 8c_4\mathrm{e}^{2t} + 2c_2\mathrm{e}^{3t} - 3c_5\mathrm{e}^{3t} + 3c_1\mathrm{e}^{2t}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 211

```
DSolve[{x1'[t]==11*x1[t]-1*x2[t]+26*x3[t]+6*x4[t]-3*x5[t],x2'[t]==0*x1[t]+3*x2[t],x3'[t]==-9
```

$$x1(t) \to e^{2t}\left(c_1\left(9e^t-8\right)-\left(c_2-26c_3-6c_4+3c_5\right)\left(e^t-1\right)\right)$$
$$x2(t) \to c_2e^{3t}$$
$$x3(t) \to -e^{2t}\left(9c_1\left(e^t-1\right)+c_3\left(26e^t-27\right)+3(2c_4-c_5)\left(e^t-1\right)\right)$$
$$x4(t) \to e^{2t}\left(3c_1\left(e^t-1\right)+9c_3\left(e^t-1\right)+3c_4e^t-c_5e^t-2c_4+c_5\right)$$
$$x5(t) \to -e^{2t}\left(48c_1\left(e^t-1\right)+3c_2\left(e^t-1\right)+138c_3e^t+30c_4e^t-16c_5e^t-138c_3-30c_4+15c_5\right)$$

6.33 problem problem 33

Internal problem ID [390]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 33.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 3x_1(t) - 4x_2(t) + x_3(t) \\
x_2'(t) &= 4x_1(t) + 3x_2(t) + x_4(t) \\
x_3'(t) &= 3x_3(t) - 4x_4(t) \\
x_4'(t) &= 4x_3(t) + 3x_4(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.453 (sec). Leaf size: 140

```
dsolve([diff(x__1(t),t)=3*x__1(t)-4*x__2(t)+1*x__3(t)+0*x__4(t),diff(x__2(t),t)=4*x__1(t)+3*
```

$$
\begin{aligned}
x_1(t) &= \frac{e^{3t}(4\cos(4t)\,c_4 t + 4\sin(4t)\,c_3 t + 4c_1\cos(4t) + 4c_2\sin(4t) - \sin(4t)\,c_4)}{4} \\
x_2(t) &= -\frac{e^{3t}(4\cos(4t)\,c_3 t - 4\sin(4t)\,c_4 t + 4c_2\cos(4t) - c_4\cos(4t) - 4c_1\sin(4t))}{4} \\
x_3(t) &= e^{3t}(c_4\cos(4t) + c_3\sin(4t)) \\
x_4(t) &= -e^{3t}(\cos(4t)\,c_3 - \sin(4t)\,c_4)
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.1 (sec). Leaf size: 120

```
DSolve[{x1'[t]==3*x1[t]-4*x2[t]+1*x3[t]+0*x4[t],x2'[t]==4*x1[t]+3*x2[t]+0*x3[t]+1*x4[t],x3'[
```

$$
\begin{aligned}
\text{x1}(t) &\to e^{3t}((c_3 t + c_1)\cos(4t) - (c_4 t + c_2)\sin(4t)) \\
\text{x2}(t) &\to e^{3t}((c_4 t + c_2)\cos(4t) + (c_3 t + c_1)\sin(4t)) \\
\text{x3}(t) &\to e^{3t}(c_3\cos(4t) - c_4\sin(4t)) \\
\text{x4}(t) &\to e^{3t}(c_4\cos(4t) + c_3\sin(4t))
\end{aligned}
$$

6.34 problem problem 34

Internal problem ID [391]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Section 7.6, Multiple Eigenvalue Solutions. Page 451
Problem number: problem 34.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 2x_1(t) - 8x_3(t) - 3x_4(t) \\
x_2'(t) &= -18x_1(t) - x_2(t) \\
x_3'(t) &= -9x_1(t) - 3x_2(t) - 25x_3(t) - 9x_4(t) \\
x_4'(t) &= 33x_1(t) + 10x_2(t) + 90x_3(t) + 32x_4(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 252

```
dsolve([diff(x__1(t),t)=2*x__1(t)+0*x__2(t)-8*x__3(t)-3*x__4(t),diff(x__2(t),t)=-18*x__1(t)-
```

$$x_1(t) = -\frac{e^{2t}(3\cos(3t)\, c_3 t + 3\cos(3t)\, c_4 t + 3\sin(3t)\, c_3 t - 3\sin(3t)\, c_4 t + 3c_1\cos(3t) + 3c_2\cos(3t) + \cos(3t)\, c_4}{18}$$

$$x_2(t) = e^{2t}(\cos(3t)\, c_4 t + \sin(3t)\, c_3 t + c_2\cos(3t) + c_1\sin(3t))$$

$$x_3(t) = -\frac{e^{2t}(\cos(3t)\, c_3 + \cos(3t)\, c_4 + \sin(3t)\, c_3 - \sin(3t)\, c_4)}{6}$$

$$x_4(t) = \frac{e^{2t}(3\cos(3t)\, c_3 t - 3\cos(3t)\, c_4 t - 3\sin(3t)\, c_3 t - 3\sin(3t)\, c_4 t + 3c_1\cos(3t) - 3c_2\cos(3t) + 10\cos(3t)\, c_3}{18}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 482

```
DSolve[{x1'[t]==2*x1[t]+0*x2[t]-8*x3[t]-3*x4[t],x2'[t]==-18*x1[t]-1*x2[t]+0*x3[t]+0*x4[t],x3
```

$$
\begin{aligned}
\mathrm{x1}(t) \to & \frac{1}{2}e^{(2-3i)t}\left(c_1\left(e^{6it}(1+3it)-3it+1\right)+i(3c_3+c_4)\left(-1+e^{6it}\right)\right. \\
& \left.+t\left(ic_2\left(-1+e^{6it}\right)+c_3\left((1+9i)e^{6it}+(1-9i)\right)+3ic_4\left(-1+e^{6it}\right)\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{x2}(t) \to & -\frac{1}{2}e^{(2-3i)t}\left(c_1\left((9-9i)t+e^{6it}((9+9i)t-3i)+3i\right)\right. \\
& +c_2\left((3-3i)t+e^{6it}(-1+(3+3i)t)-1\right)+10ic_3e^{6it}+(30+24i)c_3e^{6it}t \\
& \left.+(30-24i)c_3t+3ic_4e^{6it}+(9+9i)c_4e^{6it}t+(9-9i)c_4t-10ic_3-3ic_4\right)
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{x3}(t) \to & \frac{1}{2}e^{(2-3i)t}\left(3ic_1\left(-1+e^{6it}\right)+ic_2\left(-1+e^{6it}\right)+(1+9i)c_3e^{6it}+3ic_4e^{6it}+(1-9i)c_3\right. \\
& \left.-3ic_4\right)
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{x4}(t) \to & \frac{1}{2}e^{(2-3i)t}\left(c_1\left(3t+e^{6it}(3t-10i)+10i\right)+c_2\left(t+e^{6it}(t-3i)+3i\right)-27ic_3e^{6it}\right. \\
& \left.+(9-i)c_3e^{6it}t+(9+i)c_3t+(1-9i)c_4e^{6it}+3c_4e^{6it}t+3c_4t+27ic_3+(1+9i)c_4\right)
\end{aligned}
$$

7 Chapter 11 Power series methods. Section 11.1 Introduction and Review of power series. Page 615

7.1 problem problem 1

Internal problem ID [392]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.1 Introduction and Review of power series. Page 615
Problem number: problem 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x)=y(x),y(x),type='series',x=0);
```

$$y(x) = \left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5\right) y(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.007 (sec). Leaf size: 37

```
AsymptoticDSolveValue[y'[x]==y[x],y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1\right)$$

7.2 problem problem 2

Internal problem ID [393]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.1 Introduction and Review of power series. Page 615
Problem number: problem 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
Order:=6;
dsolve(diff(y(x),x)=4*y(x),y(x),type='series',x=0);
```

$$y(x) = \left(1 + 4x + 8x^2 + \frac{32}{3}x^3 + \frac{32}{3}x^4 + \frac{128}{15}x^5\right) y(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 37

```
AsymptoticDSolveValue[y'[x]==4*y[x],y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{128x^5}{15} + \frac{32x^4}{3} + \frac{32x^3}{3} + 8x^2 + 4x + 1\right)$$

7.3 problem problem 3

Internal problem ID [394]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.1 Introduction and Review of power series. Page 615
Problem number: problem 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{2y' + 3y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
Order:=6;
dsolve(2*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{3}{2}x + \frac{9}{8}x^2 - \frac{9}{16}x^3 + \frac{27}{128}x^4 - \frac{81}{1280}x^5\right) y(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 41

```
AsymptoticDSolveValue[2*y'[x]+3*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(-\frac{81x^5}{1280} + \frac{27x^4}{128} - \frac{9x^3}{16} + \frac{9x^2}{8} - \frac{3x}{2} + 1\right)$$

7.4 problem problem 4

Internal problem ID [395]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.1 Introduction and Review of power series. Page 615
Problem number: problem 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2yx + y' = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
Order:=6;
dsolve(diff(y(x),x)+2*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x^2 + \frac{1}{2}x^4\right) y(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 20

```
AsymptoticDSolveValue[y'[x]+2*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^4}{2} - x^2 + 1\right)$$

7.5 problem problem 5

Internal problem ID [396]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.1 Introduction and Review of power series. Page 615
Problem number: problem 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - x^2 y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
Order:=6;
dsolve(diff(y(x),x)=x^2*y(x),y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{x^3}{3}\right) y(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 15

```
AsymptoticDSolveValue[y'[x]==x^2*y[x],y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{x^3}{3} + 1\right)$$

7.6 problem problem 6

Internal problem ID [397]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.1 Introduction and Review of power series. Page 615
Problem number: problem 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(-2+x)\,y'+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
Order:=6;
dsolve((x-2)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+\frac{1}{2}x+\frac{1}{4}x^2+\frac{1}{8}x^3+\frac{1}{16}x^4+\frac{1}{32}x^5\right)y(0)+O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 41

```
AsymptoticDSolveValue[(x-2)*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{x^5}{32}+\frac{x^4}{16}+\frac{x^3}{8}+\frac{x^2}{4}+\frac{x}{2}+1\right)$$

7.7 problem problem 7

Internal problem ID [398]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.1 Introduction and Review of power series. Page 615
Problem number: problem 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(2x-1)\,y'+2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
Order:=6;
dsolve((2*x-1)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(32x^5+16x^4+8x^3+4x^2+2x+1\right)y(0)+O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 31

```
AsymptoticDSolveValue[(2*x-1)*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(32x^5+16x^4+8x^3+4x^2+2x+1\right)$$

7.8 problem problem 8

Internal problem ID [399]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.1 Introduction and Review of power series. Page 615
Problem number: problem 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2(x+1)\,y'-y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 36

```
Order:=6;
dsolve(2*(x+1)*diff(y(x),x)=y(x),y(x),type='series',x=0);
```

$$y(x)=\left(1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{16}x^3-\frac{5}{128}x^4+\frac{7}{256}x^5\right)y(0)+O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 41

```
AsymptoticDSolveValue[2*(x+1)*y'[x]==y[x],y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{7x^5}{256}-\frac{5x^4}{128}+\frac{x^3}{16}-\frac{x^2}{8}+\frac{x}{2}+1\right)$$

7.9 problem problem 9

Internal problem ID [400]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.1 Introduction and Review of power series. Page 615
Problem number: problem 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x-1)\,y'+2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
Order:=6;
dsolve((x-1)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(6x^5+5x^4+4x^3+3x^2+2x+1\right)y(0)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 31

```
AsymptoticDSolveValue[(x-1)*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(6x^5+5x^4+4x^3+3x^2+2x+1\right)$$

7.10 problem problem 10

Internal problem ID [401]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.1 Introduction and Review of power series. Page 615
Problem number: problem 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2(x-1)\,y'-3y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
Order:=6;
dsolve(2*(x-1)*diff(y(x),x)=3*y(x),y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{3}{2}x+\frac{3}{8}x^2+\frac{1}{16}x^3+\frac{3}{128}x^4+\frac{3}{256}x^5\right)y(0)+O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 41

```
AsymptoticDSolveValue[2*(x-1)*y'[x]==3*y[x],y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{3x^5}{256}+\frac{3x^4}{128}+\frac{x^3}{16}+\frac{3x^2}{8}-\frac{3x}{2}+1\right)$$

7.11 problem problem 11

Internal problem ID [402]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.1 Introduction and Review of power series. Page 615
Problem number: problem 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)=y(x),y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 + \frac{1}{24}x^4\right) y(0) + \left(x + \frac{1}{6}x^3 + \frac{1}{120}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]==y[x],y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{120} + \frac{x^3}{6} + x\right) + c_1\left(\frac{x^4}{24} + \frac{x^2}{2} + 1\right)$$

7.12 problem problem 12

Internal problem ID [403]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.1 Introduction and Review of power series. Page 615
Problem number: problem 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)=4*y(x),y(x),type='series',x=0);
```

$$y(x) = \left(1 + 2x^2 + \frac{2}{3}x^4\right) y(0) + \left(x + \frac{2}{3}x^3 + \frac{2}{15}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[y''[x]==4*y[x],y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{2x^5}{15} + \frac{2x^3}{3} + x\right) + c_1\left(\frac{2x^4}{3} + 2x^2 + 1\right)$$

7.13 problem problem 13

Internal problem ID [404]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.1 Introduction and Review of power series. Page 615
Problem number: problem 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 9y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)+9*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{9}{2}x^2 + \frac{27}{8}x^4\right) y(0) + \left(x - \frac{3}{2}x^3 + \frac{27}{40}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]+9*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{27x^5}{40} - \frac{3x^3}{2} + x\right) + c_1\left(\frac{27x^4}{8} - \frac{9x^2}{2} + 1\right)$$

7.14 problem problem 14

Internal problem ID [405]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.1 Introduction and Review of power series. Page 615
Problem number: problem 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y = x}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 38

```
Order:=6;
dsolve(diff(y(x),x$2)+y(x)=x,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4\right) y(0) + \left(x - \frac{1}{6}x^3 + \frac{1}{120}x^5\right) D(y)(0) + \frac{x^3}{6} - \frac{x^5}{120} + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 56

```
AsymptoticDSolveValue[y''[x]+y[x]==x,y[x],{x,0,5}]
```

$$y(x) \to -\frac{x^5}{120} + \frac{x^3}{6} + c_2\left(\frac{x^5}{120} - \frac{x^3}{6} + x\right) + c_1\left(\frac{x^4}{24} - \frac{x^2}{2} + 1\right)$$

7.15 problem problem 15

Internal problem ID [406]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.1 Introduction and Review of power series. Page 615
Problem number: problem 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
Order:=6;
dsolve(x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1}{x} + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 9

```
AsymptoticDSolveValue[x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \frac{c_1}{x}$$

7.16 problem problem 16

Internal problem ID [407]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.1 Introduction and Review of power series. Page 615
Problem number: problem 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2y'x - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
Order:=6;
dsolve(2*x*diff(y(x),x)=y(x),y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x} + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 11

```
AsymptoticDSolveValue[2*x*y'[x]==y[x],y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}$$

7.17 problem problem 17

Internal problem ID [408]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.1 Introduction and Review of power series. Page 615
Problem number: problem 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x^2 + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(x^2*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 11

```
AsymptoticDSolveValue[x^2*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 e^{\frac{1}{x}}$$

7.18 problem problem 18

Internal problem ID [409]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.1 Introduction and Review of power series. Page 615
Problem number: problem 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^3y' - 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(x^3*diff(y(x),x)=2*y(x),y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 13

```
AsymptoticDSolveValue[x^3*y'[x]==2*y[x],y[x],{x,0,5}]
```

$$y(x) \to c_1 e^{-\frac{1}{x^2}}$$

7.19 problem problem 19

Internal problem ID [410]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.1 Introduction and Review of power series. Page 615
Problem number: problem 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 3]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([diff(y(x),x$2)+4*y(x)=0,y(0) = 0, D(y)(0) = 3],y(x),type='series',x=0);
```

$$y(x) = 3x - 2x^3 + \frac{2}{5}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{y''[x]+4*y[x]==0,{y[0]==0,y'[0]==3}},y[x],{x,0,5}]
```

$$y(x) \to \frac{2x^5}{5} - 2x^3 + 3x$$

7.20 problem problem 20

Internal problem ID [411]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.1 Introduction and Review of power series. Page 615
Problem number: problem 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([diff(y(x),x$2)-4*y(x)=0,y(0) = 2, D(y)(0) = 0],y(x),type='series',x=0);
```

$$y(x) = 2 + 4x^2 + \frac{4}{3}x^4 + O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 17

```
AsymptoticDSolveValue[{y''[x]-4*y[x]==0,{y[0]==2,y'[0]==0}},y[x],{x,0,5}]
```

$$y(x) \to \frac{4x^4}{3} + 4x^2 + 2$$

7.21 problem problem 21

Internal problem ID [412]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.1 Introduction and Review of power series. Page 615
Problem number: problem 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
Order:=6;
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+y(x)=0,y(0) = 0, D(y)(0) = 1],y(x),type='series',x=0);
```

$$y(x) = x + x^2 + \frac{1}{2}x^3 + \frac{1}{6}x^4 + \frac{1}{24}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 29

```
AsymptoticDSolveValue[{y''[x]-2*y'[x]+y[x]==0,{y[0]==0,y'[0]==1}},y[x],{x,0,5}]
```

$$y(x) \to \frac{x^5}{24} + \frac{x^4}{6} + \frac{x^3}{2} + x^2 + x$$

7.22 problem problem 22

Internal problem ID [413]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.1 Introduction and Review of power series. Page 615
Problem number: problem 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - 2y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -2]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([diff(y(x),x$2)+diff(y(x),x)-2*y(x)=0,y(0) = 1, D(y)(0) = -2],y(x),type='series',x=0)
```

$$y(x) = 1 - 2x + 2x^2 - \frac{4}{3}x^3 + \frac{2}{3}x^4 - \frac{4}{15}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[{y''[x]+y'[x]-2*y[x]==0,{y[0]==1,y'[0]==-2}},y[x],{x,0,5}]
```

$$y(x) \to -\frac{4x^5}{15} + \frac{2x^4}{3} - \frac{4x^3}{3} + 2x^2 - 2x + 1$$

7.23 problem problem 23

Internal problem ID [414]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.1 Introduction and Review of power series. Page 615
Problem number: problem 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x^2 + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 907

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x^2*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = \sqrt{x}\Bigg(&c_2 x^{\frac{i\sqrt{3}}{2}}\Bigg(1 - \frac{1}{2}x + \frac{i\sqrt{3}+3}{8i\sqrt{3}+16}x^2 + \frac{-i\sqrt{3}-5}{48i\sqrt{3}+96}x^3 \\ &+ \frac{1}{384}\frac{(i\sqrt{3}+5)(i\sqrt{3}+7)}{(i\sqrt{3}+4)(i\sqrt{3}+2)}x^4 - \frac{1}{3840}\frac{(i\sqrt{3}+7)(i\sqrt{3}+9)}{(i\sqrt{3}+4)(i\sqrt{3}+2)}x^5 + \mathrm{O}(x^6)\Bigg) \\ &+ c_1 x^{-\frac{i\sqrt{3}}{2}}\Bigg(1 - \frac{1}{2}x + \frac{\sqrt{3}+3i}{8\sqrt{3}+16i}x^2 + \frac{-\sqrt{3}-5i}{48\sqrt{3}+96i}x^3 + \frac{3i\sqrt{3}-8}{576i\sqrt{3}-480}x^4 \\ &- \frac{1}{3840}\frac{(\sqrt{3}+7i)(\sqrt{3}+9i)}{(\sqrt{3}+4i)(\sqrt{3}+2i)}x^5 + \mathrm{O}(x^6)\Bigg)\Bigg) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 886

```
AsymptoticDSolveValue[x^2*y''[x]+x^2*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow \left(\frac{(-1)^{2/3}\left(1-(-1)^{2/3}\right)\left(2-(-1)^{2/3}\right)\left(3-(-1)^{2/3}\right)(4 \cdots}{\left(1-(-1)^{2/3}\left(1-(-1)^{2/3}\right)\right)\left(1+\left(1-(-1)^{2/3}\right)\left(2-(-1)^{2/3}\right)\right)\left(1+\left(2-(-1)^{2/3}\right)\left(3-(-1)^{2/3}\right)\right)(1+(\cdots} \right.$$

$$- \frac{(-1)^{2/3}\left(1-(-1)^{2/3}\right)\left(2-(-1)^{2/3}\right)\left(3-(-1)^{2/3}\right)x^4}{\left(1-(-1)^{2/3}\left(1-(-1)^{2/3}\right)\right)\left(1+\left(1-(-1)^{2/3}\right)\left(2-(-1)^{2/3}\right)\right)\left(1+\left(2-(-1)^{2/3}\right)\left(3-(-1)^{2/3}\right)\right)(1+(\cdots}$$

$$+ \frac{(-1)^{2/3}\left(1-(-1)^{2/3}\right)\left(2-(-1)^{2/3}\right)x^3}{\left(1-(-1)^{2/3}\left(1-(-1)^{2/3}\right)\right)\left(1+\left(1-(-1)^{2/3}\right)\left(2-(-1)^{2/3}\right)\right)\left(1+\left(2-(-1)^{2/3}\right)\left(3-(-1)^{2/3}\right)\right)}$$

$$- \frac{(-1)^{2/3}\left(1-(-1)^{2/3}\right)x^2}{\left(1-(-1)^{2/3}\left(1-(-1)^{2/3}\right)\right)\left(1+\left(1-(-1)^{2/3}\right)\left(2-(-1)^{2/3}\right)\right)}$$

$$+ \frac{(-1)^{2/3}x}{1-(-1)^{2/3}\left(1-(-1)^{2/3}\right)}$$

$$\left. +1 \right) c_1 x^{-(-1)^{2/3}} + \left(- \frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(2+\sqrt[3]{-1}\right)\left(3+\sqrt[3]{- \cdots}\right.}{\left(1+\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\right)\left(1+\left(1+\sqrt[3]{-1}\right)\left(2+\sqrt[3]{-1}\right)\right)\left(1+\left(2+\sqrt[3]{-1}\right)\left(3+\sqrt[3]{-1} \cdots\right.\right.} \right.$$

7.24 problem problem 26(a)

Internal problem ID [415]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.1 Introduction and Review of power series. Page 615
Problem number: problem 26(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^2 = 1}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 6

```
dsolve([diff(y(x),x)=1+y(x)^2,y(0) = 0],y(x), singsol=all)
```

$$y(x) = \tan(x)$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 7

```
DSolve[{y'[x]==1+y[x]^2,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan(x)$$

8 Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624

8.1 problem problem 1

Internal problem ID [416]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x^2-1)\,y''+4y'x+2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
Order:=6;
dsolve((x^2-1)*diff(y(x),x$2)+4*x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(x^4+x^2+1\right)y(0)+\left(x^5+x^3+x\right)D(y)\,(0)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 26

```
AsymptoticDSolveValue[(x^2-1)*y''[x]+4*x*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(x^5+x^3+x\right)+c_1\left(x^4+x^2+1\right)$$

8.2 problem problem 2

Internal problem ID [417]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x^2+2)\,y''+4y'x+2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((x^2+2)*diff(y(x),x$2)+4*x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{1}{2}x^2+\frac{1}{4}x^4\right)y(0)+\left(x-\frac{1}{2}x^3+\frac{1}{4}x^5\right)D(y)\,(0)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 68

```
AsymptoticDSolveValue[(x^2+2)*y''[x]+4*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(-\frac{x^5}{30}-\frac{x^4}{12}+\frac{x^3}{3}-\frac{x^2}{2}+1\right)+c_2\left(-\frac{x^5}{15}-\frac{x^4}{12}+\frac{x^3}{2}-x^2+x\right)$$

8.3 problem problem 3

Internal problem ID [418]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + y'x + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{8}x^4\right) y(0) + \left(x - \frac{1}{3}x^3 + \frac{1}{15}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]+x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{15} - \frac{x^3}{3} + x\right) + c_1\left(\frac{x^4}{8} - \frac{x^2}{2} + 1\right)$$

8.4 problem problem 4

Internal problem ID [419]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x^2+1)\,y''+6y'x+4y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((x^2+1)*diff(y(x),x$2)+6*x*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(3x^4-2x^2+1)\,y(0)+\left(x-\frac{5}{3}x^3+\frac{7}{3}x^5\right)D(y)\,(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 60

```
AsymptoticDSolveValue[(x^2+1)*y''[x]+6*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1(4x^5-5x^4+4x^3-2x^2+1)+c_2\left(\frac{77x^5}{15}-\frac{13x^4}{2}+\frac{16x^3}{3}-3x^2+x\right)$$

8.5 problem problem 5

Internal problem ID [420]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{(x^2+1)\, y''+2y'x=0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve((x^2+1)*diff(y(x),x$2)+2*x*diff(y(x),x)=0,y(x),type='series',x=0);
```

$$y(x) = y(0) + \left(x - \frac{1}{3}x^3 + \frac{1}{5}x^5\right) D(y)\,(0) + O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 25

```
AsymptoticDSolveValue[(x^2-3)*y''[x]+2*x*y'[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{45} + \frac{x^3}{9} + x\right) + c_1$$

8.6 problem problem 6

Internal problem ID [421]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(x^2-1)\,y''-6y'x+12y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
Order:=6;
dsolve((x^2-1)*diff(y(x),x$2)-6*x*diff(y(x),x)+12*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(x^4+6x^2+1)\,y(0)+(x^3+x)\,D(y)\,(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 25

```
AsymptoticDSolveValue[(x^2-1)*y''[x]-6*x*y'[x]+12*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2(x^3+x)+c_1(x^4+6x^2+1)$$

8.7 problem problem 7

Internal problem ID [422]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+3)\, y'' - 7y'x + 16y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((x^2+3)*diff(y(x),x$2)-7*x*diff(y(x),x)+16*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{8}{3}x^2 + \frac{8}{27}x^4\right) y(0) + \left(x - \frac{1}{2}x^3 + \frac{1}{120}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[(x^2+3)*y''[x]-7*x*y'[x]+16*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{120} - \frac{x^3}{2} + x\right) + c_1\left(\frac{8x^4}{27} - \frac{8x^2}{3} + 1\right)$$

8.8 problem problem 8

Internal problem ID [423]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$(-x^2+2)\,y''-y'x+16y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((2-x^2)*diff(y(x),x$2)-x*diff(y(x),x)+16*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(2x^4-4x^2+1\right)y(0)+\left(x-\frac{5}{4}x^3+\frac{7}{32}x^5\right)D(y)\,(0)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 38

```
AsymptoticDSolveValue[(2-x^2)*y''[x]-x*y'[x]+16*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{7x^5}{32}-\frac{5x^3}{4}+x\right)+c_1\left(2x^4-4x^2+1\right)$$

8.9 problem problem 9

Internal problem ID [424]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(x^2-1)\,y''+8y'x+12y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((x^2-1)*diff(y(x),x$2)+8*x*diff(y(x),x)+12*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(15x^4+6x^2+1)\,y(0)+\left(x+\frac{10}{3}x^3+7x^5\right)D(y)(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[(x^2-1)*y''[x]+8*x*y'[x]+12*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(7x^5+\frac{10x^3}{3}+x\right)+c_1(15x^4+6x^2+1)$$

8.10 problem problem 10

Internal problem ID [425]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3y'' + xy' - 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(3*diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{2}{3}x^2 + \frac{1}{27}x^4\right) y(0) + \left(x + \frac{1}{6}x^3 + \frac{1}{360}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[3*y''[x]+x*y'[x]-4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{360} + \frac{x^3}{6} + x\right) + c_1\left(\frac{x^4}{27} + \frac{2x^2}{3} + 1\right)$$

8.11 problem problem 11

Internal problem ID [426]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{5y'' - 2y'x + 10y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(5*diff(y(x),x$2)-2*x*diff(y(x),x)+10*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x^2 + \frac{1}{10}x^4\right) y(0) + \left(\frac{4}{375}x^5 - \frac{4}{15}x^3 + x\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[5*y''[x]-2*x*y'[x]+10*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{4x^5}{375} - \frac{4x^3}{15} + x\right) + c_1\left(\frac{x^4}{10} - x^2 + 1\right)$$

8.12 problem problem 12

Internal problem ID [427]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x^2 - 3yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)-3*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{x^3}{2}\right) y(0) + \left(x + \frac{1}{3}x^4\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]-x^2*y'[x]-3*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2\left(\frac{x^4}{3} + x\right) + c_1\left(\frac{x^3}{2} + 1\right)$$

8.13 problem problem 13

Internal problem ID [428]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + y'x^2 + 2yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)+x^2*diff(y(x),x)+2*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^3}{3}\right) y(0) + \left(x - \frac{1}{4}x^4\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]+x^2*y'[x]+2*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x - \frac{x^4}{4}\right) + c_1\left(1 - \frac{x^3}{3}\right)$$

8.14 problem problem 14

Internal problem ID [429]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^3}{6}\right) y(0) + \left(x - \frac{1}{12}x^4\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x - \frac{x^4}{12}\right) + c_1\left(1 - \frac{x^3}{6}\right)$$

8.15 problem problem 15

Internal problem ID [430]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + x^2 y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)+x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^4}{12}\right) y(0) + \left(x - \frac{1}{20} x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]+x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 \left(x - \frac{x^5}{20}\right) + c_1 \left(1 - \frac{x^4}{12}\right)$$

8.16 problem problem 16

Internal problem ID [431]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\, y''+2y'x-2y=0}$$

With initial conditions

$$[y(0)=0, y'(0)=1]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
Order:=6;
dsolve([(1+x^2)*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x),type='s
```

$$y(x)=x$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 4

```
AsymptoticDSolveValue[{(1+x^2)*y''[x]+2*x*y'[x]-2*y[x]==0,{y[0]==0,y'[0]==1}},y[x],{x,0,5}]
```

$$y(x)\to x$$

8.17 problem problem 17

Internal problem ID [432]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x - 2y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
Order:=6;
dsolve([diff(y(x),x$2)+x*diff(y(x),x)-2*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x),type='series',x=0
```

$$y(x) = x^2 + 1$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{y''[x]+x*y'[x]-2*y[x]==0,{y[0]==0,y'[0]==1}},y[x],{x,0,5}]
```

$$y(x) \to -\frac{x^5}{120} + \frac{x^3}{6} + x$$

8.18 problem problem 18

Internal problem ID [433]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + (x-1)\, y' + y = 0}$$

With initial conditions

$$[y(1) = 2, y'(1) = 0]$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([diff(y(x),x$2)+(x-1)*diff(y(x),x)+y(x)=0,y(1) = 2, D(y)(1) = 0],y(x),type='series',x
```

$$y(x) = 2 - (x-1)^2 + \frac{1}{4}(x-1)^4 + \mathrm{O}\left((x-1)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 21

```
AsymptoticDSolveValue[{y''[x]+(x-1)*y'[x]+y[x]==0,{y[1]==2,y'[1]==0}},y[x],{x,1,5}]
```

$$y(x) \to \frac{1}{4}(x-1)^4 - (x-1)^2 + 2$$

8.19 problem problem 19

Internal problem ID [434]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(-x^2+2x)\,y''-6(x-1)\,y'-4y=0}$$

With initial conditions

$$[y(1)=0, y'(1)=1]$$

With the expansion point for the power series method at $x=1$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([(2*x-x^2)*diff(y(x),x$2)-6*(x-1)*diff(y(x),x)-4*y(x)=0,y(1) = 0, D(y)(1) = 1],y(x),t
```

$$y(x)=(x-1)+\frac{5}{3}(x-1)^3+\frac{7}{3}(x-1)^5+O\left((x-1)^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 24

```
AsymptoticDSolveValue[{(2*x-x^2)*y''[x]-6*(x-1)*y'[x]-4*y[x]==0,{y[1]==0,y'[1]==1}},y[x],{x,
```

$$y(x)\to\frac{7}{3}(x-1)^5+\frac{5}{3}(x-1)^3+x-1$$

8.20 problem problem 20

Internal problem ID [435]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2 - 6x + 10)\, y'' - 4(x - 3)\, y' + 6y = 0}$$

With initial conditions

$$[y(3) = 2, y'(3) = 0]$$

With the expansion point for the power series method at $x = 3$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
Order:=6;
dsolve([(x^2-6*x+10)*diff(y(x),x$2)-4*(x-3)*diff(y(x),x)+6*y(x)=0,y(3) = 2, D(y)(3) = 0],y(x
```

$$y(x) = -6x^2 + 36x - 52$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 12

```
AsymptoticDSolveValue[{(x^2-6*x+10)*y''[x]-4*(x-3)*y'[x]+6*y[x]==0,{y[3]==2,y'[3]==0}},y[x],
```

$$y(x) \to 2 - 6(x - 3)^2$$

8.21 problem problem 21

Internal problem ID [436]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(4x^2 + 16x + 17)\, y'' - 8y = 0}$$

With initial conditions

$$[y(-2) = 1, y'(-2) = 0]$$

With the expansion point for the power series method at $x = -2$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 13

```
Order:=6;
dsolve([(4*x^2+16*x+17)*diff(y(x),x$2)=8*y(x),y(-2) = 1, D(y)(-2) = 0],y(x),type='series',x=
```

$$y(x) = 4x^2 + 16x + 17$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 12

```
AsymptoticDSolveValue[{(4*x^2+16*x+17)*y''[x]==8*y[x],{y[-2]==1,y'[-2]==0}},y[x],{x,-2,5}]
```

$$y(x) \to 4(x+2)^2 + 1$$

8.22 problem problem 22

Internal problem ID [437]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 + 6x)\, y'' + (3x + 9)\, y' - 3y = 0$$

With initial conditions

$$[y(-3) = 1, y'(-3) = 0]$$

With the expansion point for the power series method at $x = -3$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([(x^2+6*x)*diff(y(x),x$2)+(3*x+9)*diff(y(x),x)-3*y(x)=0,y(-3) = 1, D(y)(-3) = 0],y(x)
```

$$y(x) = 1 - \frac{1}{6}(x+3)^2 - \frac{5}{648}(x+3)^4 + \mathrm{O}\left((x+3)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 23

```
AsymptoticDSolveValue[{(x^2+6*x)*y''[x]+(3*x+9)*y'[x]-3*y[x]==0,{y[-3]==1,y'[-3]==0}},y[x],{
```

$$y(x) \to -\frac{5}{648}(x+3)^4 - \frac{1}{6}(x+3)^2 + 1$$

8.23 problem problem 23

Internal problem ID [438]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (x+1)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve(diff(y(x),x$2)+(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{30}x^5\right) y(0) + \left(x - \frac{1}{6}x^3 - \frac{1}{12}x^4 + \frac{1}{120}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[y''[x]+(1+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{120} - \frac{x^4}{12} - \frac{x^3}{6} + x\right) + c_1\left(\frac{x^5}{30} + \frac{x^4}{24} - \frac{x^3}{6} - \frac{x^2}{2} + 1\right)$$

8.24 problem problem 24

Internal problem ID [439]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-1)\,y''+2y'x+2yx=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
Order:=6;
dsolve((x^2-1)*diff(y(x),x$2)+2*x*diff(y(x),x)+2*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+\frac{1}{3}x^3+\frac{1}{5}x^5\right)y(0)+\left(x+\frac{1}{3}x^3+\frac{1}{6}x^4+\frac{1}{5}x^5\right)D(y)(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 49

```
AsymptoticDSolveValue[(x^2+1)*y''[x]+2*x*y'[x]+2*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{x^5}{5}-\frac{x^3}{3}+1\right)+c_2\left(\frac{x^5}{5}-\frac{x^4}{6}-\frac{x^3}{3}+x\right)$$

8.25 problem problem 25

Internal problem ID [440]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x^2 + x^2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
Order:=6;
dsolve(diff(y(x),x$2)+x^2*diff(y(x),x)+x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^4}{12}\right) y(0) + \left(x - \frac{1}{12}x^4 - \frac{1}{20}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 35

```
AsymptoticDSolveValue[y''[x]+x^2*y'[x]+x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1\left(1 - \frac{x^4}{12}\right) + c_2\left(-\frac{x^5}{20} - \frac{x^4}{12} + x\right)$$

8.26 problem problem 26

Internal problem ID [441]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^3+1)\,y''+yx^4=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve((1+x^3)*diff(y(x),x$2)+x^4*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=y(0)+D(y)\,(0)\,x+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 10

```
AsymptoticDSolveValue[(1+x^3)*y''[x]+x^4*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2x+c_1$$

8.27 problem problem 27

Internal problem ID [442]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + y(2x^2 + 1) = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([diff(y(x),x$2)+x*diff(y(x),x)+(2*x^2+1)*y(x)=0,y(0) = 1, D(y)(0) = -1],y(x),type='se
```

$$y(x) = 1 - x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{24}x^4 + \frac{1}{30}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 49

```
AsymptoticDSolveValue[{(x^2+1)*y''[x]+2*x*y'[x]+2*x*y[x]==0,{}},y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^5}{5} - \frac{x^3}{3} + 1\right) + c_2\left(\frac{x^5}{5} - \frac{x^4}{6} - \frac{x^3}{3} + x\right)$$

8.28 problem problem 28

Internal problem ID [443]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y\,\mathrm{e}^{-x} = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=6;
dsolve(diff(y(x),x$2)+exp(-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{40}x^5\right) y(0) + \left(x - \frac{1}{6}x^3 + \frac{1}{12}x^4 - \frac{1}{60}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[y''[x]+Exp[-x]*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2\left(-\frac{x^5}{60} + \frac{x^4}{12} - \frac{x^3}{6} + x\right) + c_1\left(-\frac{x^5}{40} + \frac{x^3}{6} - \frac{x^2}{2} + 1\right)$$

8.29 problem problem 29

Internal problem ID [444]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\cos(x)\, y'' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
Order:=6;
dsolve(cos(x)*diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^2}{2}\right) y(0) + \left(x - \frac{1}{6}x^3 - \frac{1}{60}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 35

```
AsymptoticDSolveValue[Cos[x]*y''[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(1 - \frac{x^2}{2}\right) + c_2\left(-\frac{x^5}{60} - \frac{x^3}{6} + x\right)$$

8.30 problem problem 30

Internal problem ID [445]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + \sin(x)\, y' + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 44

```
Order:=6;
dsolve(x*diff(y(x),x$2)+sin(x)*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{60}x^5\right) y(0) + \left(x - \frac{1}{2}x^2 + \frac{1}{18}x^4 - \frac{7}{360}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[x*y''[x]+Sin[x]*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(-\frac{7x^5}{360} + \frac{x^4}{18} - \frac{x^2}{2} + x\right) + c_1\left(-\frac{x^5}{60} + \frac{x^3}{6} - \frac{x^2}{2} + 1\right)$$

8.31 problem problem 33

Internal problem ID [446]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'x + 2\alpha y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 63

```
Order:=6;
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+2*alpha*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \alpha\, x^2 + \frac{\alpha(\alpha-2)\, x^4}{6}\right) y(0) + \left(x - \frac{(\alpha-1)\, x^3}{3} + \frac{(\alpha^2 - 4\alpha + 3)\, x^5}{30}\right) D(y)\,(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 78

```
AsymptoticDSolveValue[y''[x]-2*x*y'[x]+2*\[Alpha]*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{\alpha^2 x^5}{30} - \frac{2\alpha x^5}{15} + \frac{x^5}{10} - \frac{\alpha x^3}{3} + \frac{x^3}{3} + x\right) + c_1\left(\frac{\alpha^2 x^4}{6} - \frac{\alpha x^4}{3} - \alpha x^2 + 1\right)$$

8.32 problem problem 34

Internal problem ID [447]

Book: Differential equations and linear algebra, 4th ed., Edwards and Penney
Section: Chapter 11 Power series methods. Section 11.2 Power series solutions. Page 624
Problem number: problem 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)=x*y(x),y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{x^3}{6}\right) y(0) + \left(x + \frac{1}{12}x^4\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]==x*y[x],y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^4}{12} + x\right) + c_1\left(\frac{x^3}{6} + 1\right)$$