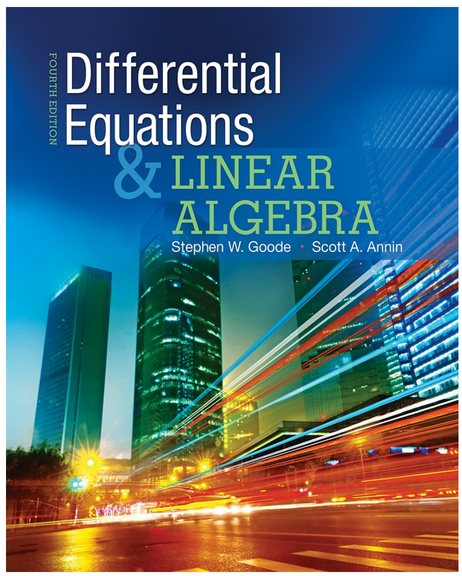
A Solution Manual For

Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015



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May 16, 2024

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1 Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21

1.1 problem Problem 7

Internal problem ID [2587]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 25y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-25*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{5x} + c_2 e^{-5x}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 22

```
DSolve[y''[x]-25*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{5x} + c_2 e^{-5x}$$

1.2 problem Problem 8

Internal problem ID [2588]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sin(2x)\, c_1 + c_2 \cos(2x)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 20

```
DSolve[y''[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(2x) + c_2 \sin(2x)$$

1.3 problem Problem 9

Internal problem ID [2589]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - 2y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_2 \mathrm{e}^{3x} + c_1\right) \mathrm{e}^{-2x}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 20

```
DSolve[y''[x]+y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-2x} + c_2 e^x$$

1.4 problem Problem 10

Internal problem ID [2590]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + y^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)=-y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{1}{c_1 + x}$$

✓ Solution by Mathematica
Time used: 0.092 (sec). Leaf size: 18

```
DSolve[y'[x]==-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{x - c_1}$$
$$y(x) \to 0$$

1.5 problem Problem 11

Internal problem ID [2591]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{y}{2x} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)=y(x)/(2*x),y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 18

```
DSolve[y'[x]==y[x]/(2*x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\sqrt{x}$$
$$y(x) \to 0$$

1.6 problem Problem 12

Internal problem ID [2592]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + 5y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+5*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-x}(\sin(2x)\, c_1 + c_2 \cos(2x))$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 26

```
DSolve[y''[x]+2*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_2 \cos(2x) + c_1 \sin(2x))$$

1.7 problem Problem 13

Internal problem ID [2593]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 9y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-9*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{3x} + e^{-3x} c_2$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 22

```
DSolve[y''[x]-9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x} \left(c_1 e^{6x} + c_2\right)$$

1.8 problem Problem 14

Internal problem ID [2594]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2y'' + 5xy' + 3y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2x^2 + c_1}{x^3}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+5*x*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^2 + c_1}{x^3}$$

1.9 problem Problem 15

Internal problem ID [2595]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' - 3xy' + 4y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^2(c_2 \ln(x) + c_1)$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]-3*x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(2c_2 \log(x) + c_1)$$

1.10 problem Problem 16

Internal problem ID [2596]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 3xy' + 13y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+13*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^2(c_1 \sin(3\ln(x)) + c_2 \cos(3\ln(x)))$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 26

```
DSolve[x^2*y''[x]-3*x*y'[x]+13*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(c_2 \cos(3\log(x)) + c_1 \sin(3\log(x)))$$

1.11 problem Problem 17

Internal problem ID [2597]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' - xy' + y = 9x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(2*x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=9*x^2,y(x), singsol=all)
```

$$y(x) = c_2x + c_1\sqrt{x} + 3x^2$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 23

```
DSolve[2*x^2*y''[x]-x*y'[x]+y[x]==9*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3x^2 + c_2x + c_1\sqrt{x}$$

1.12 problem Problem 18

Internal problem ID [2598]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' - 4xy' + 6y = x^4 \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=x^4*sin(x),y(x), singsol=all)
```

$$y(x) = x^2(c_2x - \sin(x) + c_1)$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 20

```
DSolve[x^2*y''[x]-4*x*y'[x]+6*y[x]==x^4*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(-\sin(x) + c_2x + c_1)$$

1.13 problem Problem 19

Internal problem ID [2599]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - (a+b)\,y' + aby = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-(a+b)*diff(y(x),x)+a*b*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{ax} + c_2 e^{bx}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 22

```
DSolve[y''[x]-(a+b)*y'[x]+a*b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 e^{ax} + c_1 e^{bx}$$

1.14 problem Problem 20

Internal problem ID [2600]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y'a + ya^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)-2*a*diff(y(x),x)+a^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{ax}(c_2x + c_1)$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 18

```
DSolve[y''[x]-2*a*y'[x]+a^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{ax}(c_2x + c_1)$$

1.15 problem Problem 21

Internal problem ID [2601]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y'a + (a^2 + b^2)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)-2*a*diff(y(x),x)+(a^2+b^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{ax}(c_1 \sin(bx) + c_2 \cos(bx))$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 31

```
DSolve[y''[x]-2*a*y'[x]+(a^2+b^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x(a-ib)}\left(c_2 e^{2ibx} + c_1\right)$$

1.16 problem Problem 22

Internal problem ID [2602]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' - 6y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-6*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_1 \mathrm{e}^{5x} + c_2\right) \mathrm{e}^{-2x}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 22

```
DSolve[y''[x]-y'[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(c_2 e^{5x} + c_1\right)$$

1.17 problem Problem 23

Internal problem ID [2603]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 6y' + 9y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)+6*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-3x}(c_2x + c_1)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 18

```
DSolve[y''[x]+6*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}(c_2x + c_1)$$

1.18 problem Problem 24

Internal problem ID [2604]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2y'' + xy' - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2x^2 + c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 16

```
DSolve[x^2*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x} + c_2x$$

1.19 problem Problem 25

Internal problem ID [2605]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 5xy' + 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2 \ln(x) + c_1}{x^2}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+5*x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_2 \log(x) + c_1}{x^2}$$

1.20 problem Problem 28

Internal problem ID [2606]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - \frac{e^x - \sin(y)}{x \cos(y)} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=(exp(x)-sin(y(x)))/(x*cos(y(x))),y(x), singsol=all)
```

$$y(x) = \arcsin\left(\frac{-c_1 + e^x}{x}\right)$$

✓ Solution by Mathematica
Time used: 11.572 (sec). Leaf size: 16

```
DSolve[y'[x]==(Exp[x]-Sin[y[x]])/(x*Cos[y[x]]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin\left(\frac{e^x + c_1}{x}\right)$$

1.21 problem Problem 29

Internal problem ID [2607]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]'], [_Ab`

$$\boxed{y' - \frac{1-y^2}{2+2yx} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)=(1-y(x)^2)/(2*(1+x*y(x))),y(x), singsol=all)
```

$$c_1 + \frac{1}{(y(x)-1)(xy(x)+x+2)} = 0$$

✓ Solution by Mathematica
Time used: 0.463 (sec). Leaf size: 58

```
DSolve[y'[x]==(1-y[x]^2)/(2*(1+x*y[x])),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1+\sqrt{x^2+c_1x+1}}{x}$$
$$y(x) \to \frac{-1+\sqrt{x^2+c_1x+1}}{x}$$
$$y(x) \to -1$$
$$y(x) \to 1$$

1.22 problem Problem 30

Internal problem ID [2608]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{y' - \frac{(1 - y\,\mathrm{e}^{yx})\,\mathrm{e}^{-yx}}{x} = 0}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 10

```
dsolve([diff(y(x),x)=(1-y(x)*exp(x*y(x)))/(x*exp(x*y(x))),y(1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{\ln(x)}{x}$$

✓ Solution by Mathematica
Time used: 0.403 (sec). Leaf size: 11

```
DSolve[{y'[x]==(1-y[x]*Exp[x*y[x]])/(x*Exp[x*y[x]]),{y[1]==0}},y[x],x,IncludeSingularSolutio
```

$$y(x) \to \frac{\log(x)}{x}$$

1.23 problem Problem 31

Internal problem ID [2609]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - \frac{x^2(1-y^2) + y\,\mathrm{e}^{\frac{y}{x}}}{x\left(\mathrm{e}^{\frac{y}{x}} + 2yx^2\right)} = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)=(x^2*(1-y(x)^2)+y(x)*exp(y(x)/x))/(x*(exp(y(x)/x)+2*x^2*y(x))),y(x), sin
```

$$y(x) = \mathrm{RootOf}\left(\mathrm{e}^{\_Z} + x^3\_Z^2 + c_1 - x\right)x$$

✓ Solution by Mathematica
Time used: 0.293 (sec). Leaf size: 24

```
DSolve[y'[x]==(x^2*(1-y[x]^2)+y[x]*Exp[y[x]/x])/(x*(Exp[y[x]/x]+2*x^2*y[x])),y[x],x,IncludeS
```

$$\mathrm{Solve}\left[xy(x)^2 + e^{\frac{y(x)}{x}} - x = c_1, y(x)\right]$$

1.24 problem Problem 32

Internal problem ID [2610]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' - \frac{\cos(x) - 2y^2x}{2x^2y} = 0}$$

With initial conditions

$$\left[y(\pi) = \frac{1}{\pi}\right]$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)=(cos(x)-2*x*y(x)^2)/(2*x^2*y(x)),y(Pi) = 1/Pi],y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\sin(x) + 1}}{x}$$

✓ Solution by Mathematica
Time used: 0.342 (sec). Leaf size: 17

```
DSolve[{y'[x]==(Cos[x]-2*x*y[x]^2)/(2*x^2*y[x]),{y[Pi]==1/Pi}},y[x],x,IncludeSingularSolutio
```

$$y(x) \to \frac{\sqrt{\sin(x) + 1}}{x}$$

1.25 problem Problem 33

Internal problem ID [2611]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=sin(x),y(x), singsol=all)
```

$$y(x) = -\cos(x) + c_1$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 12

```
DSolve[y'[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\cos(x) + c_1$$

1.26 problem Problem 34

Internal problem ID [2612]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{1}{x^{\frac{2}{3}}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)=x^(-2/3),y(x), singsol=all)
```

$$y(x) = 3x^{\frac{1}{3}} + c_1$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 15

```
DSolve[y'[x]==x^(-2/3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3\sqrt[3]{x} + c_1$$

1.27 problem Problem 35

Internal problem ID [2613]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = x\,\mathrm{e}^{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)=x*exp(x),y(x), singsol=all)
```

$$y(x) = (-2 + x)\,\mathrm{e}^{x} + c_1 x + c_2$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 19

```
DSolve[y''[x]==x*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(x - 2) + c_2 x + c_1$$

1.28 problem Problem 36

Internal problem ID [2614]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = x^n}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)=x^n,y(x), singsol=all)
```

$$y(x) = \frac{x^{2+n}}{(2+n)(n+1)} + c_1 x + c_2$$

✓ Solution by Mathematica
Time used: 0.007 (sec). Leaf size: 28

```
DSolve[y''[x]==x^n,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{n+2}}{n^2+3n+2} + c_2 x + c_1$$

1.29 problem Problem 37

Internal problem ID [2615]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \ln(x)\, x^2}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve([diff(y(x),x)=x^2*ln(x),y(1) = 2],y(x), singsol=all)
```

$$y(x) = \frac{x^3 \ln(x)}{3} - \frac{x^3}{9} + \frac{19}{9}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 23

```
DSolve[{y'[x]==x^2*Log[x],{y[1]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{9}\left(-x^3 + 3x^3 \log(x) + 19\right)$$

1.30 problem Problem 38

Internal problem ID [2616]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = \cos(x)}$$

With initial conditions

$$[y(0) = 2, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 11

```
dsolve([diff(y(x),x$2)=cos(x),y(0) = 2, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = -\cos(x) + x + 3$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 12

```
DSolve[{y''[x]==Cos[x],{y[0]==2,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - \cos(x) + 3$$

1.31 problem Problem 39

Internal problem ID [2617]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 39.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _quadrature]]`

$$\boxed{y''' = 6x}$$

With initial conditions

$$[y(0) = 1, y'(0) = -1, y''(0) = -4]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$3)=6*x,y(0) = 1, D(y)(0) = -1, (D@@2)(y)(0) = -4],y(x), singsol=all)
```

$$y(x) = \frac{1}{4}x^4 - 2x^2 + 1 - x$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 22

```
DSolve[{y'''[x]==6*x,{y[0]==2,y'[0]==-1,y''[0]==-4}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{1}{4}(x^4 - 8x^2 - 4x + 8)$$

1.32 problem Problem 40

Internal problem ID [2618]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = x\,e^{x}}$$

With initial conditions

$$[y(0) = 3, y'(0) = 4]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(x),x$2)=x*exp(x),y(0) = 3, D(y)(0) = 4],y(x), singsol=all)
```

$$y(x) = (-2 + x)\,e^{x} + 5x + 5$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 18

```
DSolve[{y''[x]==x*Exp[x],{y[0]==3,y'[0]==4}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(x - 2) + 5(x + 1)$$

1.33 problem Problem 45

Internal problem ID [2619]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 45.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - 6y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-6*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_2 e^{5x} + c_1\right) e^{-3x}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 19

```
DSolve[y''[x]==x*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x(x-2) + c_2 x + c_1$$

1.34 problem Problem 46

Internal problem ID [2620]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 46.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2 y'' - x y' - 8y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)-8*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2 x^6 + c_1}{x^2}$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]-x*y'[x]-8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 x^6 + c_1}{x^2}$$

1.35 problem Problem 47

Internal problem ID [2621]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 47.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2 y'' - 3xy' + 4y = \ln(x)\, x^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*y(x)=x^2*ln(x),y(x), singsol=all)
```

$$y(x) = x^2 \left(c_2 + \ln(x)\, c_1 + \frac{\ln(x)^3}{6}\right)$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 27

```
DSolve[x^2*y''[x]-3*x*y'[x]+4*y[x]==x^2*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6} x^2 \left(\log^3(x) + 12c_2 \log(x) + 6c_1\right)$$

2 Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43

2.1 problem Problem 1

Internal problem ID [2622]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 2yx = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=2*x*y(x),y(x), singsol=all)
```

$$y(x) = e^{x^2} c_1$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 18

```
DSolve[y'[x]==2*x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x^2}$$
$$y(x) \to 0$$

2.2 problem Problem 2

Internal problem ID [2623]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' - \frac{y^2}{x^2+1} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=y(x)^2/(x^2+1),y(x), singsol=all)
```

$$y(x) = \frac{1}{-\arctan(x) + c_1}$$

✓ Solution by Mathematica
Time used: 0.16 (sec). Leaf size: 19

```
DSolve[y'[x]==y[x]^2/(x^2+1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\arctan(x) + c_1}$$
$$y(x) \to 0$$

2.3 problem Problem 3

Internal problem ID [2624]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{e^{y+x}y' = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(exp(x+y(x))*diff(y(x),x)-1=0,y(x), singsol=all)
```

$$y(x) = \ln(e^x c_1 - 1) - x$$

✓ Solution by Mathematica

Time used: 0.097 (sec). Leaf size: 16

```
DSolve[Exp[x+y[x]]*y'[x]-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(-e^{-x} + c_1)$$

2.4 problem Problem 4

Internal problem ID [2625]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{y}{\ln(x)\, x} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x)=y(x)/(x*ln(x)),y(x), singsol=all)
```

$$y(x) = \ln(x)\, c_1$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 15

```
DSolve[y'[x]==y[x]/(x*Log[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \log(x)$$
$$y(x) \to 0$$

2.5 problem Problem 5

Internal problem ID [2626]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y - (x-1)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(y(x)-(x-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x-1)$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 16

```
DSolve[y[x]-(x-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x-1)$$
$$y(x) \to 0$$

2.6 problem Problem 6

Internal problem ID [2627]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{2x(y-1)}{x^2+3} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)=(2*x*(y(x)-1))/(x^2+3),y(x), singsol=all)
```

$$y(x) = c_1x^2 + 3c_1 + 1$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 20

```
DSolve[y'[x]==(2*x*(y[x]-1))/(x^2+3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 + c_1(x^2 + 3)$$
$$y(x) \to 1$$

2.7 problem Problem 7

Internal problem ID [2628]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$-xy' + y + 2y'x^2 = 3$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(y(x)-x*diff(y(x),x)=3-2*x^2*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \frac{c_1 x - 3}{2x - 1}$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 24

```
DSolve[y[x]-x*y'[x]==3-2*x^2*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3 + c_1 x}{1 - 2x}$$
$$y(x) \to 3$$

2.8 problem Problem 8

Internal problem ID [2629]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{\cos(-y+x)}{\sin(x)\sin(y)} = -1}$$

✓ Solution by Maple
Time used: 0.141 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)=cos(x-y(x))/(sin(x)*sin(y(x)))-1,y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{\csc(x)}{c_1}\right)$$

✓ Solution by Mathematica
Time used: 5.812 (sec). Leaf size: 47

```
DSolve[y'[x]==Cos[x-y[x]]/(Sin[x]*Sin[y[x]])-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arccos\left(-\frac{1}{2}c_1\csc(x)\right)$$
$$y(x) \to \arccos\left(-\frac{1}{2}c_1\csc(x)\right)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

2.9 problem Problem 9

Internal problem ID [2630]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' - \frac{x(y^2-1)}{2(x-2)(x-1)} = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)=x*(y(x)^2-1)/(2*(x-2)*(x-1)),y(x), singsol=all)
```

$$y(x) = -\tanh\left(\ln(-2+x) - \frac{\ln(x-1)}{2} + \frac{c_1}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.882 (sec). Leaf size: 51

```
DSolve[y'[x]==x*(y[x]^2-1)/(2*(x-2)*(x-1)),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x + e^{2c_1}(x-2)^2 - 1}{-x + e^{2c_1}(x-2)^2 + 1}$$
$$y(x) \to -1$$
$$y(x) \to 1$$

2.10 problem Problem 10

Internal problem ID [2631]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{x^2 y - 32}{-x^2 + 16} = 2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)=(x^2*y(x)-32)/(16-x^2)+2,y(x), singsol=all)
```

$$y(x) = \frac{c_1 (x+4)^2 \,\mathrm{e}^{-x} + 2(x-4)^2}{(x-4)^2}$$

✓ Solution by Mathematica
Time used: 0.148 (sec). Leaf size: 40

```
DSolve[y'[x]==(x^2*y[x]-32)/(16-x^2)+2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}(2e^x (x-4)^2 + c_1 (x+4)^2)}{(x-4)^2}$$
$$y(x) \to 2$$

2.11 problem Problem 11

Internal problem ID [2632]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x-a)\,(x-b)\,y'-y=-c}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
dsolve((x-a)*(x-b)*diff(y(x),x)-(y(x)-c)=0,y(x), singsol=all)
```

$$y(x)=c+(x-b)^{-\frac{1}{a-b}}\,(x-a)^{\frac{1}{a-b}}\,c_1$$

✓ Solution by Mathematica
Time used: 0.102 (sec). Leaf size: 41

```
DSolve[(x-a)*(x-b)*y'[x]-(y[x]-c)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c+c_1(x-b)^{\frac{1}{b-a}}(x-a)^{\frac{1}{a-b}}$$
$$y(x)\to c$$

2.12 problem Problem 12

Internal problem ID [2633]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2+1)\,y'+y^2=-1}$$

With initial conditions

$$[y(0)=1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 11

```
dsolve([(x^2+1)*diff(y(x),x)+y(x)^2=-1,y(0) = 1],y(x), singsol=all)
```

$$y(x)=\cot\left(\arctan(x)+\frac{\pi}{4}\right)$$

✓ Solution by Mathematica
Time used: 0.243 (sec). Leaf size: 14

```
DSolve[{(x^2+1)*y'[x]+y[x]^2==-1,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\cot\left(\arctan(x)+\frac{\pi}{4}\right)$$

2.13 problem Problem 13

Internal problem ID [2634]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(-x^2+1)\,y' + yx = ax}$$

With initial conditions

$$[y(0) = 2a]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve([(1-x^2)*diff(y(x),x)+x*y(x)=a*x,y(0) = 2*a],y(x), singsol=all)
```

$$y(x) = a\left(1 - i\sqrt{x-1}\,\sqrt{x+1}\right)$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 21

```
DSolve[{(1-x^2)*y'[x]+x*y[x]==a*x,{y[0]==2*a}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to a - ia\sqrt{x^2-1}$$

2.14 problem Problem 14

Internal problem ID [2635]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \frac{\sin(y + x)}{\sin(y)\cos(x)} = 1}$$

With initial conditions

$$\left[y\left(\frac{\pi}{4}\right) = \frac{\pi}{4}\right]$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 9

```
dsolve([diff(y(x),x)=1-(sin(x+y(x)))/(sin(y(x))*cos(x)),y(1/4*Pi) = 1/4*Pi],y(x), singsol=al
```

$$y(x) = \frac{\pi}{2} - \arcsin\left(\frac{\sec(x)}{2}\right)$$

✓ Solution by Mathematica
Time used: 6.259 (sec). Leaf size: 12

```
DSolve[{y'[x]==1-(Sin[x+y[x]])/(Sin[y[x]]*Cos[x]),{y[Pi/4]==Pi/4}},y[x],x,IncludeSingularSol
```

$$y(x) \to \arccos\left(\frac{\sec(x)}{2}\right)$$

2.15 problem Problem 15

Internal problem ID [2636]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y^3 \sin(x) = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=y(x)^3*sin(x),y(0) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y'[x]==y[x]^3*Sin[x],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

2.16 problem Problem 16

Internal problem ID [2637]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \frac{2\sqrt{y-1}}{3} = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=2/3*(y(x)-1)^(1/2),y(1) = 1],y(x), singsol=all)
```

$$y(x) = 1$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 17

```
DSolve[{y'[x]==1/3*(y[x]-1)^(1/2),{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{36}(x^2 - 2x + 37)$$

2.17 problem Problem 17

Internal problem ID [2638]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{mv' + kv^2 = mg}$$

With initial conditions

$$[v(0) = 0]$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 26

```
dsolve([m*diff(v(t),t)=m*g-k*v(t)^2,v(0) = 0],v(t), singsol=all)
```

$$v(t) = \frac{\tanh\left(\frac{\sqrt{mgk}\,t}{m}\right)\sqrt{mgk}}{k}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 39

```
DSolve[{m*v'[t]==m*g-k*v[t]^2,{v[0]==0}},v[t],t,IncludeSingularSolutions -> True]
```

$$v(t) \to \frac{\sqrt{g}\sqrt{m}\tanh\left(\frac{\sqrt{g}\sqrt{k}t}{\sqrt{m}}\right)}{\sqrt{k}}$$

3 Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59

3.1 problem Problem 1

Internal problem ID [2639]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + y = 4\,e^{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+y(x)=4*exp(x),y(x), singsol=all)
```

$$y(x) = 2\,e^{x} + e^{-x}c_1$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 19

```
DSolve[y'[x]+y[x]==4*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2e^{x} + c_1e^{-x}$$

3.2 problem Problem 2

Internal problem ID [2640]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{2y}{x} = 5x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)+2/x*y(x)=5*x^2,y(x), singsol=all)
```

$$y(x) = \frac{x^5 + c_1}{x^2}$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 15

```
DSolve[y'[x]+2/x*y[x]==5*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^5 + c_1}{x^2}$$

3.3 problem Problem 3

Internal problem ID [2641]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x^2 - 4yx = x^7 \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x)-4*x*y(x)=x^7*sin(x),y(x), singsol=all)
```

$$y(x) = (-x\cos(x) + \sin(x) + c_1)\, x^4$$

✓ Solution by Mathematica
Time used: 0.061 (sec). Leaf size: 19

```
DSolve[x^2*y'[x]-4*x*y[x]==x^7*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^4(\sin(x) - x\cos(x) + c_1)$$

3.4 problem Problem 4

Internal problem ID [2642]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + 2yx = 2x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+2*x*y(x)=2*x^3,y(x), singsol=all)
```

$$y(x) = x^2 - 1 + c_1 \mathrm{e}^{-x^2}$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 20

```
DSolve[y'[x]+2*x*y[x]==2*x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + c_1 e^{-x^2} - 1$$

3.5 problem Problem 5

Internal problem ID [2643]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{2xy}{-x^2+1} = 4x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)+2*x/(1-x^2)*y(x)=4*x,y(x), singsol=all)
```

$$y(x) = (2\ln(x-1) + 2\ln(x+1) + c_1)(x^2-1)$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 22

```
DSolve[y'[x]+2*x/(1-x^2)*y[x]==4*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x^2-1)(2\log(x^2-1) + c_1)$$

3.6 problem Problem 6

Internal problem ID [2644]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{2xy}{x^2+1} = \frac{4}{(x^2+1)^2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)+2*x/(1+x^2)*y(x)=4/(1+x^2)^2,y(x), singsol=all)
```

$$y(x) = \frac{4\arctan(x) + c_1}{x^2+1}$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 20

```
DSolve[y'[x]+2*x/(1+x^2)*y[x]==4/(1+x^2)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4\arctan(x) + c_1}{x^2+1}$$

3.7 problem Problem 7

Internal problem ID [2645]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2\cos(x)^2 y' + y\sin(2x) = 4\cos(x)^4}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(2*(cos(x)^2)*diff(y(x),x)+y(x)*sin(2*x)=4*cos(x)^4,y(x), singsol=all)
```

$$y(x) = (2\sin(x) + c_1)\cos(x)$$

✓ Solution by Mathematica
Time used: 0.058 (sec). Leaf size: 15

```
DSolve[2*(Cos[x]^2)*y'[x]+y[x]*Sin[2*x]==4*Cos[x]^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x)(2\sin(x) + c_1)$$

3.8 problem Problem 8

Internal problem ID [2646]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{y}{\ln (x)\, x} = 9x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)+1/(x*ln(x))*y(x)=9*x^2,y(x), singsol=all)
```

$$y(x) = \frac{3x^3 \ln (x) - x^3 + c_1}{\ln (x)}$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 25

```
DSolve[y'[x]+1/(x*Log[x])*y[x]==9*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-x^3 + 3x^3 \log(x) + c_1}{\log(x)}$$

3.9 problem Problem 9

Internal problem ID [2647]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - y \tan(x) = 8 \sin(x)^3}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)-y(x)*tan(x)=8*sin(x)^3,y(x), singsol=all)
```

$$y(x) = 2\cos(x)^3 - 4\cos(x) + \frac{\sec(x)(4c_1 + 5)}{4}$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 19

```
DSolve[y'[x]-y[x]*Tan[x]==8*Sin[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\sin^3(x)\tan(x) + c_1 \sec(x)$$

3.10 problem Problem 10

Internal problem ID [2648]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x't + 2x = 4\,\mathrm{e}^t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(t*diff(x(t),t)+2*x(t)=4*exp(t),x(t), singsol=all)
```

$$x(t) = \frac{(4t-4)\,\mathrm{e}^t + c_1}{t^2}$$

✓ Solution by Mathematica
Time used: 0.049 (sec). Leaf size: 20

```
DSolve[t*x'[t]+2*x[t]==4*Exp[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{4e^t(t-1) + c_1}{t^2}$$

3.11 problem Problem 11

Internal problem ID [2649]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \sin(x)(y\sec(x) - 2) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=sin(x)*(y(x)*sec(x)-2),y(x), singsol=all)
```

$$y(x) = \cos(x) - \frac{\sec(x)}{2} + \sec(x)\, c_1$$

✓ Solution by Mathematica
Time used: 0.045 (sec). Leaf size: 20

```
DSolve[y'[x]==Sin[x]*(y[x]*Sec[x]-2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\sec(x)(\cos(2x) + 2c_1)$$

3.12 problem Problem 12

Internal problem ID [2650]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{-y\sin(x) - y'\cos(x) = -1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve((1-y(x)*sin(x))-cos(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \cos(x) c_1 + \sin(x)$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 13

```
DSolve[(1-y[x]*Sin[x])-Cos[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x) + c_1\cos(x)$$

3.13 problem Problem 13

Internal problem ID [2651]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{y}{x} = 2\ln(x)\, x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)-1/x*y(x)=2*x^2*ln(x),y(x), singsol=all)
```

$$y(x) = x^3 \ln(x) - \frac{x^3}{2} + c_1 x$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 23

```
DSolve[y'[x]-1/x*y[x]==2*x^2*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^3}{2} + x^3 \log(x) + c_1 x$$

3.14 problem Problem 14

Internal problem ID [2652]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + \alpha y = \mathrm{e}^{\beta x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x)+alpha*y(x)=exp(beta*x),y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{-\alpha x}\left(\mathrm{e}^{x(\alpha+\beta)} + c_1(\alpha+\beta)\right)}{\alpha+\beta}$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 31

```
DSolve[y'[x]+\[Alpha]*y[x]==Exp[\[Beta]*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{\alpha(-x)}\left(e^{x(\alpha+\beta)} + c_1(\alpha+\beta)\right)}{\alpha+\beta}$$

3.15 problem Problem 15

Internal problem ID [2653]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{my}{x} = \ln(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x)+m/x*y(x)=ln(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1(m+1)^2 x^{-m} + x(-1 + (m+1)\ln(x))}{(m+1)^2}$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 29

```
DSolve[y'[x]+m/x*y[x]==Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x((m+1)\log(x) - 1)}{(m+1)^2} + c_1 x^{-m}$$

3.16 problem Problem 16

Internal problem ID [2654]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{2y}{x} = 4x}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)+2/x*y(x)=4*x,y(1) = 2],y(x), singsol=all)
```

$$y(x) = \frac{x^4 + 1}{x^2}$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 12

```
DSolve[{y'[x]+2/x*y[x]==4*x,{y[1]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + \frac{1}{x^2}$$

3.17 problem Problem 17

Internal problem ID [2655]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' \sin(x) - \cos(x)\, y = \sin(2x)}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = 2\right]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve([sin(x)*diff(y(x),x)-y(x)*cos(x)=sin(2*x),y(1/2*Pi) = 2],y(x), singsol=all)
```

$$y(x) = (2\ln(\sin(x)) + 2)\sin(x)$$

✓ Solution by Mathematica
Time used: 0.048 (sec). Leaf size: 14

```
DSolve[{Sin[x]*y'[x]-y[x]*Cos[x]==Sin[2*x],{y[Pi/2]==2}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to 2\sin(x)(\log(\sin(x)) + 1)$$

3.18 problem Problem 18

Internal problem ID [2656]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x' + \frac{2x}{4-t} = 5}$$

With initial conditions

$$[x(0) = 4]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 14

```
dsolve([diff(x(t),t)+2/(4-t)*x(t)=5,x(0) = 4],x(t), singsol=all)
```

$$x(t) = -t^2 + 3t + 4$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 15

```
DSolve[{x'[t]+2/(4-t)*x[t]==5,{x[0]==4}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -t^2 + 3t + 4$$

3.19 problem Problem 19

Internal problem ID [2657]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = e^x}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve([y(x)-exp(x)+diff(y(x),x)=0,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{e^x}{2} + \frac{e^{-x}}{2}$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 21

```
DSolve[{y[x]-Exp[x]+y'[x]==0,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(e^{2x} + 1\right)$$

3.20 problem Problem 20

Internal problem ID [2658]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' - 2y = \begin{cases} 1 & x \leq 1 \\ 0 & 1 < x \end{cases}$$

With initial conditions

$$[y(0) = 3]$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 27

```
dsolve([diff(y(x),x)-2*y(x)=piecewise(x<=1,1,x>1,0),y(0) = 3],y(x), singsol=all)
```

$$y(x) = \frac{7\,\mathrm{e}^{2x}}{2} - \frac{\left(\begin{cases} 1 & x < 1 \\ \mathrm{e}^{2x-2} & 1 \leq x \end{cases}\right)}{2}$$

✓ Solution by Mathematica
Time used: 0.195 (sec). Leaf size: 42

```
DSolve[{y'[x] - 2*y[x] == Piecewise[{{1, x <= 1}, {0, x > 1}}],{y[0]==3}},y[x],x,IncludeSing
```

$$y(x) \to \begin{cases} \frac{1}{2}(-1 + 7e^{2x}) & x \leq 1 \\ \frac{1}{2}e^{2x-2}(-1 + 7e^2) & \text{True} \end{cases}$$

3.21 problem Problem 21

Internal problem ID [2659]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' - 2y = \begin{cases} 1-x & x<1 \\ 0 & 1 \leq x \end{cases}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 31

```
dsolve([diff(y(x),x)-2*y(x)=piecewise(x<1,1-x,x>=1,0),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{5\,\mathrm{e}^{2x}}{4} + \frac{\left(\begin{cases} 2x-1 & x<1 \\ \mathrm{e}^{2x-2} & 1 \leq x \end{cases}\right)}{4}$$

✓ Solution by Mathematica

Time used: 0.084 (sec). Leaf size: 45

```
DSolve[{y'[x] - 2*y[x] == Piecewise[{{1-x, x < 1}, {0, x >= 1}}],{y[0]==1}},y[x],x,IncludeSi
```

$$y(x) \rightarrow \begin{cases} \frac{1}{4}(2x + 5e^{2x} - 1) & x \leq 1 \\ \frac{1}{4}e^{2x-2}(1+5e^2) & \text{True} \end{cases}$$

3.22 problem Problem 22

Internal problem ID [2660]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + \frac{y'}{x} = 9x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x$2)+1/x*diff(y(x),x)=9*x,y(x), singsol=all)
```

$$y(x) = x^3 + \ln(x) c_1 + c_2$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 16

```
DSolve[y''[x]+1/x*y'[x]==9*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^3 + c_1 \log(x) + c_2$$

3.23 problem Problem 30

Internal problem ID [2661]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{y}{x} = \cos (x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)+1/x*y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = \frac{x \sin (x) + \cos (x) + c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 18

```
DSolve[y'[x]+1/x*y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x \sin(x) + \cos(x) + c_1}{x}$$

3.24 problem Problem 31

Internal problem ID [2662]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = \mathrm{e}^{-2x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+y(x)=exp(-2*x),y(x), singsol=all)
```

$$y(x) = \left(-\mathrm{e}^{-x} + c_1\right) \mathrm{e}^{-x}$$

✓ Solution by Mathematica
Time used: 0.055 (sec). Leaf size: 19

```
DSolve[y'[x]+y[x]==Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}(-1 + c_1 e^x)$$

3.25 problem Problem 32

Internal problem ID [2663]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y \cot(x) = 2\cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)+y(x)*cot(x)=2*cos(x),y(x), singsol=all)
```

$$y(x) = \csc(x)\left(-\cos(x)^2 + c_1 + \frac{1}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 20

```
DSolve[y'[x]+y[x]*Cot[x]==2*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}\csc(x)(\cos(2x) - 2c_1)$$

3.26 problem Problem 33

Internal problem ID [2664]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' - y = \ln(x)\, x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x)-y(x)=x^2*ln(x),y(x), singsol=all)
```

$$y(x) = (x\ln(x) - x + c_1)\, x$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 17

```
DSolve[x*y'[x]-y[x]==x^2*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(-x + x\log(x) + c_1)$$

4 Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79

4.1 problem Problem 9

Internal problem ID [2665]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Riccati]`

$$\boxed{y' - \frac{y^2 + yx + x^2}{x^2} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)=(y(x)^2+x*y(x)+x^2)/x^2,y(x), singsol=all)
```

$$y(x) = \tan(\ln(x) + c_1)\, x$$

✓ Solution by Mathematica
Time used: 0.198 (sec). Leaf size: 13

```
DSolve[y'[x]==(y[x]^2+x*y[x]+x^2)/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \tan(\log(x) + c_1)$$

4.2 problem Problem 10

Internal problem ID [2666]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(3x - y)\, y' - 3y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve((3*x-y(x))*diff(y(x),x)=3*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{3x}{\mathrm{LambertW}\left(-3x\, \mathrm{e}^{-3c_1}\right)}$$

✓ Solution by Mathematica
Time used: 6.103 (sec). Leaf size: 25

```
DSolve[(3*x-y[x])*y'[x]==3*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{3x}{W\left(-3e^{-c_1}x\right)}$$
$$y(x) \to 0$$

4.3 problem Problem 11

Internal problem ID [2667]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Riccati]`

$$\boxed{y' - \frac{(y+x)^2}{2x^2} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=(x+y(x))^2/(2*x^2),y(x), singsol=all)
```

$$y(x) = \tan\left(\frac{\ln(x)}{2} + \frac{c_1}{2}\right) x$$

✓ Solution by Mathematica
Time used: 0.236 (sec). Leaf size: 17

```
DSolve[y'[x]==(x+y[x])^2/(2*x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\tan\left(\frac{\log(x)}{2} + c_1\right)$$

4.4 problem Problem 12

Internal problem ID [2668]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{\sin\left(\frac{y}{x}\right)(xy' - y) - x\cos\left(\frac{y}{x}\right) = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve(sin(y(x)/x)*(x*diff(y(x),x)-y(x))=x*cos(y(x)/x),y(x), singsol=all)
```

$$y(x) = x\arccos\left(\frac{1}{c_1 x}\right)$$

✓ Solution by Mathematica
Time used: 25.367 (sec). Leaf size: 56

```
DSolve[Sin[y[x]/x]*(x*y'[x]-y[x])==x*Cos[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\arccos\left(\frac{e^{-c_1}}{x}\right)$$
$$y(x) \to x\arccos\left(\frac{e^{-c_1}}{x}\right)$$
$$y(x) \to -\frac{\pi x}{2}$$
$$y(x) \to \frac{\pi x}{2}$$

4.5 problem Problem 13

Internal problem ID [2669]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{xy' - \sqrt{16x^2 - y^2} - y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 29

```
dsolve(x*diff(y(x),x)=sqrt(16*x^2-y(x)^2)+y(x),y(x), singsol=all)
```

$$-\arctan\left(\frac{y(x)}{\sqrt{16x^2 - y(x)^2}}\right) + \ln(x) - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.398 (sec). Leaf size: 18

```
DSolve[x*y'[x]==Sqrt[16*x^2-y[x]^2]+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -4x\cosh(i\log(x) + c_1)$$

4.6 problem Problem 14

Internal problem ID [2670]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{xy' - y - \sqrt{9x^2 + y^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(x*diff(y(x),x)-y(x)=sqrt(9*x^2+y(x)^2),y(x), singsol=all)
```

$$\frac{-c_1x^2 + \sqrt{9x^2 + y(x)^2} + y(x)}{x^2} = 0$$

✓ Solution by Mathematica

Time used: 0.35 (sec). Leaf size: 27

```
DSolve[x*y'[x]-y[x]==Sqrt[9*x^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{9e^{c_1}x^2}{2} - \frac{e^{-c_1}}{2}$$

4.7 problem Problem 15

Internal problem ID [2671]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y(x^2-y^2)-x(x^2-y^2)\,y'=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(y(x)*(x^2-y(x)^2)-x*(x^2-y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$\begin{aligned} y(x) &= -x \\ y(x) &= x \\ y(x) &= c_1x \end{aligned}$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 33

```
DSolve[y[x]*(x^2-y[x]^2)-x*(x^2-y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to -x \\ y(x) &\to x \\ y(x) &\to c_1x \\ y(x) &\to -x \\ y(x) &\to x \end{aligned}$$

4.8 problem Problem 16

Internal problem ID [2672]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{xy' + y\ln(x) - y\ln(y) = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x)+y(x)*ln(x)=y(x)*ln(y(x)),y(x), singsol=all)
```

$$y(x) = x\,\mathrm{e}^{c_1x+1}$$

✓ Solution by Mathematica
Time used: 0.257 (sec). Leaf size: 24

```
DSolve[x*y'[x]+y[x]*Log[x]==y[x]*Log[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to xe^{1+e^{c_1}x}$$
$$y(x) \to ex$$

4.9 problem Problem 17

Internal problem ID [2673]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y' - \frac{y^2 + 2yx - 2x^2}{x^2 - yx + y^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 80

```
dsolve(diff(y(x),x)=(y(x)^2+2*x*y(x)-2*x^2)/(x^2-x*y(x)+y(x)^2),y(x), singsol=all)
```

$$y(x) = \frac{x\left(-\operatorname{RootOf}\left(2\_Z^6 + (9c_1x^2 - 1)\_Z^4 - 6x^2c_1\_Z^2 + c_1x^2\right)^2 + 1\right)}{\operatorname{RootOf}\left(2\_Z^6 + (9c_1x^2 - 1)\_Z^4 - 6x^2c_1\_Z^2 + c_1x^2\right)^2}$$

✓ Solution by Mathematica

Time used: 60.187 (sec). Leaf size: 373

```
DSolve[y'[x]==(y[x]^2+2*x*y[x]-2*x^2)/(x^2-x*y[x]+y[x]^2),y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{\sqrt[3]{-54x^3+2\sqrt{729x^6+\left(-9x^2+3e^{2c_1}\right)^3}}}{3\sqrt[3]{2}} - \frac{\sqrt[3]{2}(-3x^2+e^{2c_1})}{\sqrt[3]{-54x^3+2\sqrt{729x^6+\left(-9x^2+3e^{2c_1}\right)^3}}} + x$$

$$y(x) \to \frac{\left(-1+i\sqrt{3}\right)\sqrt[3]{-54x^3+2\sqrt{729x^6+\left(-9x^2+3e^{2c_1}\right)^3}}}{6\sqrt[3]{2}} + \frac{\left(1+i\sqrt{3}\right)\left(-3x^2+e^{2c_1}\right)}{2^{2/3}\sqrt[3]{-54x^3+2\sqrt{729x^6+\left(-9x^2+3e^{2c_1}\right)^3}}} + x$$

$$y(x) \to -\frac{\left(1+i\sqrt{3}\right)\sqrt[3]{-54x^3+2\sqrt{729x^6+\left(-9x^2+3e^{2c_1}\right)^3}}}{6\sqrt[3]{2}} + \frac{\left(1-i\sqrt{3}\right)\left(-3x^2+e^{2c_1}\right)}{2^{2/3}\sqrt[3]{-54x^3+2\sqrt{729x^6+\left(-9x^2+3e^{2c_1}\right)^3}}} + x$$

4.10 problem Problem 18

Internal problem ID [2674]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘]]`

$$2xyy' - x^2 e^{-\frac{y^2}{x^2}} - 2y^2 = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(2*x*y(x)*diff(y(x),x)-(x^2*exp(-y(x)^2/x^2)+2*y(x)^2)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{\ln(\ln(x) + c_1)}\, x$$
$$y(x) = -\sqrt{\ln(\ln(x) + c_1)}\, x$$

✓ Solution by Mathematica
Time used: 2.17 (sec). Leaf size: 38

```
DSolve[2*x*y[x]*y'[x]-(x^2*Exp[-y[x]^2/x^2]+2*y[x]^2)==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -x\sqrt{\log(\log(x) + 2c_1)}$$
$$y(x) \to x\sqrt{\log(\log(x) + 2c_1)}$$

4.11 problem Problem 19

Internal problem ID [2675]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Riccati]`

$$\boxed{y'x^2 - y^2 - 3yx = x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^2*diff(y(x),x)=y(x)^2+3*x*y(x)+x^2,y(x), singsol=all)
```

$$y(x) = -\frac{x(\ln(x) + c_1 + 1)}{\ln(x) + c_1}$$

✓ Solution by Mathematica
Time used: 0.146 (sec). Leaf size: 28

```
DSolve[x^2*y'[x]==y[x]^2+3*x*y[x]+x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x(\log(x) + 1 + c_1)}{\log(x) + c_1}$$
$$y(x) \to -x$$

4.12 problem Problem 20

Internal problem ID [2676]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{yy' - \sqrt{y^2 + x^2} = -x}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 27

```
dsolve(y(x)*diff(y(x),x)=sqrt(x^2+y(x)^2)-x,y(x), singsol=all)
```

$$\frac{-c_1 y(x)^2 + \sqrt{x^2 + y(x)^2} + x}{y(x)^2} = 0$$

✓ Solution by Mathematica
Time used: 0.409 (sec). Leaf size: 57

```
DSolve[y[x]*y'[x]==Sqrt[x^2+y[x]^2]-x,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to -e^{\frac{c_1}{2}}\sqrt{2x + e^{c_1}} \\ y(x) &\to e^{\frac{c_1}{2}}\sqrt{2x + e^{c_1}} \\ y(x) &\to 0 \end{aligned}$$

4.13 problem Problem 21

Internal problem ID [2677]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{2x(y+2x)\,y' - y(4x-y) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(2*x*(y(x)+2*x)*diff(y(x),x)=y(x)*(4*x-y(x)),y(x), singsol=all)
```

$$y(x) = \frac{2x}{\operatorname{LambertW}\left(2\,\mathrm{e}^{\frac{3c_1}{2}}x^{\frac{3}{2}}\right)}$$

✓ Solution by Mathematica

Time used: 5.346 (sec). Leaf size: 29

```
DSolve[2*x*(y[x]+2*x)*y'[x]==y[x]*(4*x-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x}{W\left(2e^{-c_1}x^{3/2}\right)}$$
$$y(x) \to 0$$

4.14 problem Problem 22

Internal problem ID [2678]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{xy' - x\tan\left(\frac{y}{x}\right) - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(x*diff(y(x),x)=x*tan(y(x)/x)+y(x),y(x), singsol=all)
```

$$y(x) = \arcsin(c_1 x)\, x$$

✓ Solution by Mathematica
Time used: 4.357 (sec). Leaf size: 19

```
DSolve[x*y'[x]==x*Tan[y[x]/x]+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\arcsin\left(e^{c_1}x\right)$$
$$y(x) \to 0$$

4.15 problem Problem 23

Internal problem ID [2679]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y' - \frac{x\sqrt{y^2+x^2}+y^2}{yx} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x)=(x*sqrt(y(x)^2+x^2)+y(x)^2)/(x*y(x)),y(x), singsol=all)
```

$$\frac{x\ln(x) - c_1 x - \sqrt{x^2+y(x)^2}}{x} = 0$$

✓ Solution by Mathematica
Time used: 0.318 (sec). Leaf size: 54

```
DSolve[y'[x]==(x*Sqrt[y[x]^2+x^2]+y[x]^2)/(x*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\sqrt{\log^2(x) + 2c_1\log(x) - 1 + c_1{}^2}$$
$$y(x) \to x\sqrt{\log^2(x) + 2c_1\log(x) - 1 + c_1{}^2}$$

4.16 problem Problem 25

Internal problem ID [2680]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{2(2y - x)}{y + x} = 0}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple
Time used: 0.657 (sec). Leaf size: 273

```
dsolve([diff(y(x),x)=2*(2*y(x)-x)/(x+y(x)),y(0) = 2],y(x), singsol=all)
```

$$y(x) = \frac{\left(3\sqrt{3}\,x\sqrt{x\,(27x+8)}+27x^2+36x+8\right)^{\frac{1}{3}}}{3} + \frac{4x+\frac{4}{3}}{\left(3\sqrt{3}\,x\sqrt{x\,(27x+8)}+27x^2+36x+8\right)^{\frac{1}{3}}} + 2x + \frac{2}{3}$$

✓ Solution by Mathematica

Time used: 60.289 (sec). Leaf size: 121

```
DSolve[{y'[x]==2*(2*y[x]-x)/(x+y[x]),{y[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(x\left(\frac{12}{\sqrt[3]{3\sqrt{3}\sqrt{x^3(27x+8)}+27x^2+36x+8}}+6\right) + \sqrt[3]{3\sqrt{3}\sqrt{x^3(27x+8)}+27x^2+36x+8} + \frac{4}{\sqrt[3]{3\sqrt{3}\sqrt{x^3(27x+8)}+27x^2+36x+8}}+2\right)$$

4.17 problem Problem 26

Internal problem ID [2681]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{2x - y}{4y + x} = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple
Time used: 0.141 (sec). Leaf size: 19

```
dsolve([diff(y(x),x)=(2*x-y(x))/(x+4*y(x)),y(1) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{x}{4} + \frac{\sqrt{9x^2 + 16}}{4}$$

✓ Solution by Mathematica
Time used: 0.472 (sec). Leaf size: 24

```
DSolve[{y'[x]==(2*x-y[x])/(x+4*y[x]),{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(\sqrt{9x^2 + 16} - x\right)$$

4.18 problem Problem 27

Internal problem ID [2682]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y' - \frac{y - \sqrt{y^2 + x^2}}{x} = 0}$$

With initial conditions

$$[y(3) = 4]$$

✓ Solution by Maple
Time used: 0.375 (sec). Leaf size: 21

```
dsolve([diff(y(x),x)=(y(x)-sqrt(x^2+y(x)^2))/x,y(3) = 4],y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} - \frac{1}{2}$$
$$y(x) = -\frac{x^2}{18} + \frac{9}{2}$$

✓ Solution by Mathematica
Time used: 0.248 (sec). Leaf size: 29

```
DSolve[{y'[x]==(y[x]-Sqrt[x^2+y[x]^2])/x,{y[3]==4}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{9}{2} - \frac{x^2}{18}$$
$$y(x) \to \frac{1}{2}(x^2 - 1)$$

4.19 problem Problem 28

Internal problem ID [2683]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{xy' - y - \sqrt{-y^2 + 4x^2} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 29

```
dsolve(x*diff(y(x),x)-y(x)=sqrt(4*x^2-y(x)^2),y(x), singsol=all)
```

$$-\arctan\left(\frac{y(x)}{\sqrt{4x^2 - y(x)^2}}\right) + \ln(x) - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.416 (sec). Leaf size: 18

```
DSolve[x*y'[x]-y[x]==Sqrt[4*x^2-y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2x\cosh(i\log(x) + c_1)$$

4.20 problem Problem 29(a)

Internal problem ID [2684]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 29(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$y' - \frac{x + ya}{ax - y} = 0$$

✓ Solution by Maple
Time used: 0.359 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)=(x+a*y(x))/(a*x-y(x)),y(x), singsol=all)
```

$$y(x) = \tan\left(\operatorname{RootOf}\left(-2a\_Z + \ln\left(\sec\left(\_Z\right)^2 x^2\right) + 2c_1\right)\right) x$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 34

```
DSolve[y'[x]==(x+a*y[x])/(a*x-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[a\arctan\left(\frac{y(x)}{x}\right) - \frac{1}{2}\log\left(\frac{y(x)^2}{x^2} + 1\right) = \log(x) + c_1, y(x)\right]$$

4.21 problem Problem 29(b)

Internal problem ID [2685]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 29(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$y' - \frac{x + \frac{y}{2}}{\frac{x}{2} - y} = 0$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 30

```
dsolve([diff(y(x),x)=(x+1/2*y(x))/(1/2*x-y(x)),y(1) = 1],y(x), singsol=all)
```

$$y(x) = \tan\left(\operatorname{RootOf}\left(4\_Z - 4\ln\left(\sec\left(\_Z\right)^2\right) - 8\ln\left(x\right) + 4\ln\left(2\right) - \pi\right)\right) x$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 42

```
DSolve[{y'[x]==(x+1/2*y[x])/(1/2*x-y[x]),{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\log\left(\frac{y(x)^2}{x^2} + 1\right) - \arctan\left(\frac{y(x)}{x}\right) = \frac{1}{4}(4\log(2) - \pi) - 2\log(x), y(x)\right]$$

4.22 problem Problem 38

Internal problem ID [2686]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _Bernoulli]`

$$\boxed{y' - \frac{y}{x} - \frac{4x^2 \cos(x)}{y} = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 28

```
dsolve(diff(y(x),x)-1/x*y(x)=4*x^2/y(x)*cos(x),y(x), singsol=all)
```

$$y(x) = \sqrt{8\sin(x) + c_1}\, x$$
$$y(x) = -\sqrt{8\sin(x) + c_1}\, x$$

✓ Solution by Mathematica
Time used: 0.298 (sec). Leaf size: 36

```
DSolve[y'[x]-1/x*y[x]==4*x^2/y[x]*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\sqrt{8\sin(x) + c_1}$$
$$y(x) \to x\sqrt{8\sin(x) + c_1}$$

4.23 problem Problem 39

Internal problem ID [2687]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + \frac{y \tan(x)}{2} - 2y^3 \sin(x) = 0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 56

```
dsolve(diff(y(x),x)+1/2*tan(x)*y(x)=2*y(x)^3*sin(x),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{(-2\sin(x)^2 + c_1)\cos(x)}}{-2\sin(x)^2 + c_1}$$
$$y(x) = \frac{\sqrt{(-2\sin(x)^2 + c_1)\cos(x)}}{-2\sin(x)^2 + c_1}$$

✓ Solution by Mathematica

Time used: 5.32 (sec). Leaf size: 227

```
DSolve[y'[x]+1/2*Tan(x)*y[x]==2*y[x]^3*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{\frac{1}{4}/\text{Tan}}\sqrt[4]{\text{Tan}}}{\sqrt{e^{\frac{\text{Tan}x^2}{2}}\left(-i\sqrt{2\pi}\text{erf}\left(\frac{\text{Tan}x+i}{\sqrt{2}\sqrt{\text{Tan}}}\right)+\sqrt{2\pi}\text{erfi}\left(\frac{1+i\text{Tan}x}{\sqrt{2}\sqrt{\text{Tan}}}\right)+c_1e^{\frac{1}{2}/\text{Tan}}\sqrt{\text{Tan}}\right)}}$$

$$y(x) \to \frac{e^{\frac{1}{4}/\text{Tan}}\sqrt[4]{\text{Tan}}}{\sqrt{e^{\frac{\text{Tan}x^2}{2}}\left(-i\sqrt{2\pi}\text{erf}\left(\frac{\text{Tan}x+i}{\sqrt{2}\sqrt{\text{Tan}}}\right)+\sqrt{2\pi}\text{erfi}\left(\frac{1+i\text{Tan}x}{\sqrt{2}\sqrt{\text{Tan}}}\right)+c_1e^{\frac{1}{2}/\text{Tan}}\sqrt{\text{Tan}}\right)}}$$

$$y(x) \to 0$$

4.24 problem Problem 40

Internal problem ID [2688]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' - \frac{3y}{2x} - 6y^{\frac{1}{3}}x^2 \ln(x) = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 22

```
dsolve(diff(y(x),x)-3/(2*x)*y(x)=6*y(x)^(1/3)*x^2*ln(x),y(x), singsol=all)
```

$$-2x^3 \ln(x) + x^3 + y(x)^{\frac{2}{3}} - c_1 x = 0$$

✓ Solution by Mathematica
Time used: 0.795 (sec). Leaf size: 26

```
DSolve[y'[x]-3/(2*x)*y[x]==6*y[x]^(1/3)*x^2*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(x\left(-x^2 + 2x^2 \log(x) + c_1\right)\right)^{3/2}$$

4.25 problem Problem 41

Internal problem ID [2689]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + \frac{2y}{x} - 6\sqrt{x^2+1}\,\sqrt{y} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x)+2/x*y(x)=6*sqrt(1+x^2)*sqrt(y(x)),y(x), singsol=all)
```

$$\frac{-x^2\sqrt{x^2+1} + x\sqrt{y\left(x\right)} - c_1 - \sqrt{x^2+1}}{x} = 0$$

✓ Solution by Mathematica

Time used: 0.228 (sec). Leaf size: 55

```
DSolve[y'[x]+2/x*y[x]==6*Sqrt[1+x^2]*Sqrt[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^6 + 3x^4 + x^2\left(3 + 2c_1\sqrt{x^2+1}\right) + 2c_1\sqrt{x^2+1} + 1 + c_1{}^2}{x^2}$$

4.26 problem Problem 42

Internal problem ID [2690]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y' + \frac{2y}{x} - 6y^2x^4 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+2/x*y(x)=6*y(x)^2*x^4,y(x), singsol=all)
```

$$y(x) = \frac{1}{(-2x^3 + c_1)\, x^2}$$

✓ Solution by Mathematica

Time used: 0.131 (sec). Leaf size: 24

```
DSolve[y'[x]+2/x*y[x]==6*y[x]^2*x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{-2x^5 + c_1x^2}$$
$$y(x) \to 0$$

4.27 problem Problem 43

Internal problem ID [2691]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{2x(y' + y^3x^2) + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(2*x*(diff(y(x),x)+y(x)^3*x^2)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{x(x^2 + c_1)}}$$
$$y(x) = -\frac{1}{\sqrt{x(x^2 + c_1)}}$$

✓ Solution by Mathematica
Time used: 0.326 (sec). Leaf size: 40

```
DSolve[2*x*(y'[x]+y[x]^3*x^2)+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{x(x^2 + c_1)}}$$
$$y(x) \to \frac{1}{\sqrt{x(x^2 + c_1)}}$$
$$y(x) \to 0$$

4.28 problem Problem 44

Internal problem ID [2692]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$(x-a)(x-b)(y'-\sqrt{y})-2(b-a)y=0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 55

```
dsolve((x-a)*(x-b)*(diff(y(x),x)-sqrt(y(x)))=2*(b-a)*y(x),y(x), singsol=all)
```

$$\frac{(-x+b)(a-b)\ln(x-b)+(2a-2x)\sqrt{y(x)}-(x+2c_1)(-x+b)}{2a-2x}=0$$

✓ Solution by Mathematica
Time used: 0.478 (sec). Leaf size: 43

```
DSolve[(x-a)*(x-b)*(y'[x]-Sqrt[y[x]])==2*(b-a)*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{(b-x)^2((b-a)\log(x-b)+x+2c_1)^2}{4(a-x)^2}$$

4.29 problem Problem 45

Internal problem ID [2693]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + \frac{6y}{x} - \frac{3y^{\frac{2}{3}} \cos (x)}{x} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x)+6/x*y(x)=3/x*y(x)^(2/3)*cos(x),y(x), singsol=all)
```

$$\frac{y(x)^{\frac{1}{3}} x^2 - x \sin (x) - \cos (x) - c_1}{x^2} = 0$$

✓ Solution by Mathematica
Time used: 0.194 (sec). Leaf size: 20

```
DSolve[y'[x]+6/x*y[x]==3/x*y[x]^(2/3)*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(x \sin(x) + \cos(x) + c_1)^3}{x^6}$$

4.30 problem Problem 46

Internal problem ID [2694]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + 4yx - 4\sqrt{y}\,x^3 = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)+4*x*y(x)=4*x^3*sqrt(y(x)),y(x), singsol=all)
```

$$-x^2 + 1 - c_1 \mathrm{e}^{-x^2} + \sqrt{y(x)} = 0$$

✓ Solution by Mathematica
Time used: 0.159 (sec). Leaf size: 29

```
DSolve[y'[x]+4*x*y[x]==4*x^3*Sqrt[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x^2}\left(e^{x^2}\left(x^2-1\right)+c_1\right)^2$$

4.31 problem Problem 47

Internal problem ID [2695]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' - \frac{y}{2\ln(x)\, x} - 2xy^3 = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 86

```
dsolve(diff(y(x),x)-1/(2*x*ln(x))*y(x)=2*x*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-2\ln(x)^2 x^2 + (x^2 + c_1)\ln(x)}}{2\ln(x)\, x^2 - x^2 - c_1}$$
$$y(x) = -\frac{\sqrt{-2\ln(x)^2 x^2 + (x^2 + c_1)\ln(x)}}{2\ln(x)\, x^2 - x^2 - c_1}$$

✓ Solution by Mathematica
Time used: 0.274 (sec). Leaf size: 63

```
DSolve[y'[x]-1/(2*x*Log[x])*y[x]==2*x*y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\log(x)}}{\sqrt{x^2 - 2x^2\log(x) + c_1}}$$
$$y(x) \to \frac{\sqrt{\log(x)}}{\sqrt{x^2 - 2x^2\log(x) + c_1}}$$
$$y(x) \to 0$$

4.32 problem Problem 48

Internal problem ID [2696]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y' - \frac{y}{(\pi - 1)\, x} - \frac{3xy^{\pi}}{1 - \pi} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)-1/( (Pi-1)*x)*y(x)=3/(1-Pi)*x*y(x)^Pi,y(x), singsol=all)
```

$$y(x) = \left(\frac{x^3 + c_1}{x}\right)^{-\frac{1}{\pi - 1}}$$

✓ Solution by Mathematica

Time used: 1.02 (sec). Leaf size: 28

```
DSolve[y'[x]-1/( (Pi-1)*x)*y[x]==3/(1-Pi)*x*y[x]^Pi,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\frac{x^3 + c_1}{x}\right)^{\frac{1}{1-\pi}}$$
$$y(x) \to 0$$

4.33 problem Problem 49

Internal problem ID [2697]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{2y' + y\cot(x) - \frac{8\cos(x)^3}{y} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 40

```
dsolve(2*diff(y(x),x)+y(x)*cot(x)=8/y(x)*cos(x)^3,y(x), singsol=all)
```

$$y(x) = \csc(x)\sqrt{\sin(x)\left(-2\cos(x)^4 + c_1\right)}$$
$$y(x) = -\csc(x)\sqrt{\sin(x)\left(-2\cos(x)^4 + c_1\right)}$$

✓ Solution by Mathematica
Time used: 3.971 (sec). Leaf size: 47

```
DSolve[2*y'[x]+y[x]*Cot[x]==8/y[x]*Cos[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-2\cos^3(x)\cot(x) + c_1\csc(x)}$$
$$y(x) \to \sqrt{-2\cos^3(x)\cot(x) + c_1\csc(x)}$$

4.34 problem Problem 50

Internal problem ID [2698]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\left(1-\sqrt{3}\right) y' + y \sec(x) - y^{\sqrt{3}} \sec(x) = 0$$

✓ Solution by Maple
Time used: 0.219 (sec). Leaf size: 23

```
dsolve((1-sqrt(3))*diff(y(x),x)+y(x)*sec(x)=y(x)^sqrt(3)*sec(x),y(x), singsol=all)
```

$$y(x) = \left(-c_1 \tan(x) + 1 + \sec(x) c_1\right)^{-\frac{1}{2}-\frac{\sqrt{3}}{2}}$$

✓ Solution by Mathematica
Time used: 0.608 (sec). Leaf size: 76

```
DSolve[(1-Sqrt[3])*y'[x]+y[x]*Sec[x]==y[x]^Sqrt[3]*Sec[x],y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \text{InverseFunction}\left[\frac{\log\left(1-\#1^{\sqrt{3}-1}\right)-\left(\sqrt{3}-1\right)\log(\#1)}{\sqrt{3}-1}\&\right]\left[-\frac{2\operatorname{arctanh}\left(\tan\left(\frac{x}{2}\right)\right)}{\sqrt{3}-1}+c_1\right]$$

$$y(x) \to 0$$
$$y(x) \to 1$$

4.35 problem Problem 51

Internal problem ID [2699]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 51.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{y' + \frac{2xy}{x^2+1} - y^2x = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 23

```
dsolve([diff(y(x),x)+2*x/(1+x^2)*y(x)=x*y(x)^2,y(0) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{2}{(x^2+1)(\ln(x^2+1)-2)}$$

✓ Solution by Mathematica
Time used: 0.196 (sec). Leaf size: 24

```
DSolve[{y'[x]+2*x/(1+x^2)*y[x]==x*y[x]^2,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{(x^2+1)(\log(x^2+1)-2)}$$

4.36 problem Problem 52

Internal problem ID [2700]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + y\cot(x) - y^3 \sin(x)^3 = 0}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = 1\right]$$

✓ Solution by Maple
Time used: 1.375 (sec). Leaf size: 34

```
dsolve([diff(y(x),x)+y(x)*cot(x)=y(x)^3*sin(x)^3,y(1/2*Pi) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\csc(x)\sqrt{(2\cos(x)-1)^2(1+2\cos(x))}}{1-4\cos(x)^2}$$

✓ Solution by Mathematica
Time used: 0.933 (sec). Leaf size: 20

```
DSolve[{y'[x]+y[x]*Cot[x]==y[x]^3*Sin[x]^3,{y[Pi/2]==1}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{\sqrt{\sin^2(x)(2\cos(x)+1)}}$$

4.37 problem Problem 54

Internal problem ID [2701]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 54.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Riccati]`

$$\boxed{y' - (9x - y)^2 = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.11 (sec). Leaf size: 28

```
dsolve([diff(y(x),x)=(9*x-y(x))^2,y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{(9x - 3)\,\mathrm{e}^{6x} + 9x + 3}{1 + \mathrm{e}^{6x}}$$

✓ Solution by Mathematica
Time used: 0.15 (sec). Leaf size: 31

```
DSolve[{y'[x]==(9*x-y[x])^2,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{9x + e^{6x}(9x - 3) + 3}{e^{6x} + 1}$$

4.38 problem Problem 55

Internal problem ID [2702]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 55.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _Riccati]`

$$\boxed{y' - (4x + y + 2)^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)=(4*x+y(x)+2)^2,y(x), singsol=all)
```

$$y(x) = -4x - 2 - 2\tan(-2x + 2c_1)$$

✓ Solution by Mathematica
Time used: 0.16 (sec). Leaf size: 41

```
DSolve[y'[x]==(4*x+y[x]+2)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -4x + \frac{1}{c_1 e^{4ix} - \frac{i}{4}} - (2 + 2i)$$
$$y(x) \to -4x - (2 + 2i)$$

4.39 problem Problem 56

Internal problem ID [2703]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 56.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - \sin(3x-3y+1)^2 = 0}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=(sin(3*x-3*y(x)+1))^2,y(x), singsol=all)
```

$$y(x) = x + \frac{1}{3} + \frac{\arctan(-3x+3c_1)}{3}$$

✓ Solution by Mathematica
Time used: 0.599 (sec). Leaf size: 43

```
DSolve[y'[x]==(Sin[3*x-3*y[x]+1])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2y(x) - 2\left(\frac{1}{3}\tan(-3y(x)+3x+1) - \frac{1}{3}\arctan(\tan(-3y(x)+3x+1))\right) = c_1, y(x)\right]$$

4.40 problem Problem 58

Internal problem ID [2704]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 58.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$y' - \frac{y(\ln(yx) - 1)}{x} = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)=y(x)/x*(ln(x*y(x))-1),y(x), singsol=all)
```

$$y(x) = \frac{e^{\frac{x}{c_1}}}{x}$$

✓ Solution by Mathematica
Time used: 0.233 (sec). Leaf size: 24

```
DSolve[y'[x]==y[x]/x*(Log[x*y[x]]-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{e^{c_1}x}}{x}$$
$$y(x) \to \frac{1}{x}$$

4.41 problem Problem 59

Internal problem ID [2705]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 59.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Riccati]`

$$\boxed{y' - 2x(y + x)^2 = -1}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.141 (sec). Leaf size: 20

```
dsolve([diff(y(x),x)=2*x*(x+y(x))^2-1,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{-x^3 + x - 1}{x^2 - 1}$$

✓ Solution by Mathematica
Time used: 0.146 (sec). Leaf size: 21

```
DSolve[{y'[x]==2*x*(x+y[x])^2-1,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-x^3 + x - 1}{x^2 - 1}$$

4.42 problem Problem 60

Internal problem ID [2706]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 60.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$y' - \frac{x+2y-1}{2x-y+3} = 0$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 32

```
dsolve(diff(y(x),x)=(x+2*y(x)-1)/(2*x-y(x)+3),y(x), singsol=all)
```

$$y(x) = 1 + \tan\left(\operatorname{RootOf}\left(4\_Z + \ln\left(\sec\left(\_Z\right)^2\right) + 2\ln\left(x+1\right) + 2c_1\right)\right)(-x-1)$$

✓ Solution by Mathematica
Time used: 0.064 (sec). Leaf size: 68

```
DSolve[y'[x]==(x+2*y[x]-1)/(2*x-y[x]+3),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[32\arctan\left(\frac{-2y(x)-x+1}{-y(x)+2x+3}\right) + 8\log\left(\frac{x^2+y(x)^2-2y(x)+2x+2}{5(x+1)^2}\right) + 16\log(x+1) + 5c_1 = 0, y(x)\right]$$

4.43 problem Problem 61

Internal problem ID [2707]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + p(x)\,y + q(x)\,y^2 = r(x)}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)+p(x)*y(x)+q(x)*y(x)^2=r(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]+p[x]*y[x]+q[x]*y[x]^2==r[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.44 problem Problem 62

Internal problem ID [2708]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 62.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Riccati]`

$$\boxed{y' + \frac{2y}{x} - y^2 = -\frac{2}{x^2}}$$

✓ Solution by Maple
Time used: 0.531 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)+2/x*y(x)-y(x)^2=-2/x^2,y(x), singsol=all)
```

$$y(x) = \frac{x^3 + 2c_1}{(-x^3 + c_1)\, x}$$

✓ Solution by Mathematica
Time used: 0.175 (sec). Leaf size: 35

```
DSolve[y'[x]+2/x*y[x]-y[x]^2==-2/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2 + 3c_1x^3}{x - 3c_1x^4}$$
$$y(x) \to -\frac{1}{x}$$

4.45 problem Problem 63

Internal problem ID [2709]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 63.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Riccati]`

$$\boxed{y' + \frac{7y}{x} - 3y^2 = \frac{3}{x^2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x)+7/x*y(x)-3*y(x)^2=3/x^2,y(x), singsol=all)
```

$$y(x) = \frac{3\ln(x) - 3c_1 - 1}{3x(\ln(x) - c_1)}$$

✓ Solution by Mathematica
Time used: 0.157 (sec). Leaf size: 15

```
DSolve[y'[x]+7/x*y[x]-3*y[x]^2==3/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{x}$$
$$y(x) \to \frac{1}{x}$$

4.46 problem Problem 64

Internal problem ID [2710]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 64.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{p(x)\ln(y) = -\frac{y'}{y} + q(x)}$$

✓ Solution by Maple
Time used: 0.218 (sec). Leaf size: 27

```
dsolve(diff(y(x),x)/y(x)+p(x)*ln(y(x))=q(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{e}^{-\left(\int p(x)dx\right)}\left(\int \mathrm{e}^{\int p(x)dx} q(x)dx - c_1\right)}$$

✓ Solution by Mathematica
Time used: 0.201 (sec). Leaf size: 109

```
DSolve[y'[x]/y[x]+p[x]*Log[y[x]]==q[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^x \exp\left(-\int_1^{K[2]} -p(K[1])dK[1]\right)(\log(y(x))p(K[2]) - q(K[2]))dK[2] + \int_1^{y(x)}\left(\frac{\exp\left(-\int_1^x -p(K[1])dK[1]\right)}{K[3]} - \int_1^x \frac{\exp\left(-\int_1^{K[2]} -p(K[1])dK[1]\right)p(K[2])}{K[3]}dK[2]\right)dK[3] = c_1, y(x)\right]$$

4.47 problem Problem 65

Internal problem ID [2711]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 65.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$-\frac{2\ln(y)}{x} = -\frac{y'}{y} + \frac{1 - 2\ln(x)}{x}$$

With initial conditions

$$[y(1) = \mathrm{e}]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 10

```
dsolve([diff(y(x),x)/y(x)-2/x*ln(y(x))=1/x*(1-2*ln(x)),y(1) = exp(1)],y(x), singsol=all)
```

$$y(x) = x\,\mathrm{e}^{x^2}$$

✓ Solution by Mathematica
Time used: 0.215 (sec). Leaf size: 12

```
DSolve[{y'[x]/y[x]-2/x*Log[y[x]]==1/x*(1-2*Log[x]),{y[1]==Exp[1]}},y[x],x,IncludeSingularSol
```

$$y(x) \to e^{x^2} x$$

4.48 problem Problem 67

Internal problem ID [2712]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 67.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sec(y)^2 y' + \frac{\tan(y)}{2\sqrt{x+1}} = \frac{1}{2\sqrt{x+1}}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve(sec(y(x))^2*diff(y(x),x)+1/(2*sqrt(1+x))*tan(y(x))=1/(2*sqrt(1+x)),y(x), singsol=all)
```

$$y(x) = \arctan\left(\mathrm{e}^{-\sqrt{x+1}} c_1 + 1\right)$$

✓ Solution by Mathematica

Time used: 60.288 (sec). Leaf size: 247

```
DSolve[Sec[y[x]]^2*y'[x]+1/(2*Sqrt[1+x])*Tan[y[x]]==1/(2*Sqrt[1+x]),y[x],x,IncludeSingularSo
```

$$y(x)\to -\arccos\left(-\frac{e^{\sqrt{x+1}+2c_1}}{\sqrt{-2e^{\sqrt{x+1}+2c_1}+2e^{2\sqrt{x+1}+4c_1}+1}}\right)$$
$$y(x)\to \arccos\left(-\frac{e^{\sqrt{x+1}+2c_1}}{\sqrt{-2e^{\sqrt{x+1}+2c_1}+2e^{2\sqrt{x+1}+4c_1}+1}}\right)$$
$$y(x)\to -\arccos\left(\frac{e^{\sqrt{x+1}+2c_1}}{\sqrt{-2e^{\sqrt{x+1}+2c_1}+2e^{2\sqrt{x+1}+4c_1}+1}}\right)$$
$$y(x)\to \arccos\left(\frac{e^{\sqrt{x+1}+2c_1}}{\sqrt{-2e^{\sqrt{x+1}+2c_1}+2e^{2\sqrt{x+1}+4c_1}+1}}\right)$$

5 Chapter 1, First-Order Differential Equations. Section 1.9, Exact Differential Equations. page 91

5.1 problem Problem 1

Internal problem ID [2713]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.9, Exact Differential Equations. page 91
Problem number: Problem 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `` [`x=_G(y,y')`] ``

$$y\,\mathrm{e}^{yx} + \left(2y - x\,\mathrm{e}^{yx}\right) y' = 0$$

✗ Solution by Maple

```
dsolve(y(x)*exp(x*y(x))+(2*y(x)-x*exp(x*y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*Exp[x*y[x]]+(2*y[x]-x*Exp[x*y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> T
```

Not solved

5.2 problem Problem 2

Internal problem ID [2714]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.9, Exact Differential Equations. page 91
Problem number: Problem 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact]`

$$\boxed{\cos(yx) - xy\sin(yx) - x^2\sin(yx)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve((cos(x*y(x))-x*y(x)*sin(x*y(x)))-x^2*sin(x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\arccos\left(\frac{c_1}{x}\right)}{x}$$

✓ Solution by Mathematica

Time used: 5.673 (sec). Leaf size: 34

```
DSolve[(Cos[x*y[x]]-x*y[x]*Sin[x*y[x]])-x^2*Sin[x*y[x]]*y'[x]==0,y[x],x,IncludeSingularSolut
```

$$y(x) \to -\frac{\arccos\left(-\frac{c_1}{x}\right)}{x}$$
$$y(x) \to \frac{\arccos\left(-\frac{c_1}{x}\right)}{x}$$

5.3 problem Problem 3

Internal problem ID [2715]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.9, Exact Differential Equations. page 91
Problem number: Problem 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y + xy' = -3x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve((y(x)+3*x^2)+x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-x^3 + c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 17

```
DSolve[(y[x]+3*x^2)+x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-x^3 + c_1}{x}$$

5.4 problem Problem 4

Internal problem ID [2716]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.9, Exact Differential Equations. page 91
Problem number: Problem 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{2x\,\mathrm{e}^{y}+\left(3y^{2}+x^{2}\mathrm{e}^{y}\right)y'=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(2*x*exp(y(x))+(3*y(x)^2+x^2*exp(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$x^{2}\mathrm{e}^{y(x)}+y(x)^{3}+c_{1}=0$$

✓ Solution by Mathematica
Time used: 0.258 (sec). Leaf size: 19

```
DSolve[2*x*Exp[y[x]]+(3*y[x]^2+x^2*Exp[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$\text{Solve}\left[x^{2}e^{y(x)}+y(x)^{3}=c_{1},y(x)\right]$$

5.5 problem Problem 5

Internal problem ID [2717]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.9, Exact Differential Equations. page 91
Problem number: Problem 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2yx + (x^2+1)\,y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(2*x*y(x)+(x^2+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^2+1}$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 20

```
DSolve[2*x*y[x]+(x^2+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x^2+1}$$
$$y(x) \to 0$$

5.6 problem Problem 6

Internal problem ID [2718]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.9, Exact Differential Equations. page 91
Problem number: Problem 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact, _rational, _Bernoulli]`

$$\boxed{y^2 + 2xyy' = 2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve((y(x)^2-2*x)+2*x*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{x\,(x^2 + c_1)}}{x}$$
$$y(x) = -\frac{\sqrt{x\,(x^2 + c_1)}}{x}$$

✓ Solution by Mathematica
Time used: 0.207 (sec). Leaf size: 42

```
DSolve[(y[x]^2-2*x)+2*x*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{x^2 + c_1}}{\sqrt{x}}$$
$$y(x) \to \frac{\sqrt{x^2 + c_1}}{\sqrt{x}}$$

5.7 problem Problem 7

Internal problem ID [2719]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.9, Exact Differential Equations. page 91
Problem number: Problem 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$\boxed{2yx - y^2 + (-y + x)^2 y' = -4\,\mathrm{e}^{2x}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 117

```
dsolve((4*exp(2*x)+2*x*y(x)-y(x)^2)+(x-y(x))^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \left(-x^3 - 6\,\mathrm{e}^{2x} - 3c_1\right)^{\frac{1}{3}} + x$$
$$y(x) = -\frac{\left(-x^3 - 6\,\mathrm{e}^{2x} - 3c_1\right)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}\left(-x^3 - 6\,\mathrm{e}^{2x} - 3c_1\right)^{\frac{1}{3}}}{2} + x$$
$$y(x) = -\frac{\left(-x^3 - 6\,\mathrm{e}^{2x} - 3c_1\right)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}\left(-x^3 - 6\,\mathrm{e}^{2x} - 3c_1\right)^{\frac{1}{3}}}{2} + x$$

✓ Solution by Mathematica
Time used: 1.472 (sec). Leaf size: 112

```
DSolve[(4*Exp[2*x]+2*x*y[x]-y[x]^2)+(x-y[x])^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to x + \sqrt[3]{-x^3 - 6e^{2x} + 3c_1}$$
$$y(x) \to x + \frac{1}{2}i\left(\sqrt{3} + i\right)\sqrt[3]{-x^3 - 6e^{2x} + 3c_1}$$
$$y(x) \to x - \frac{1}{2}\left(1 + i\sqrt{3}\right)\sqrt[3]{-x^3 - 6e^{2x} + 3c_1}$$

5.8 problem Problem 8

Internal problem ID [2720]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.9, Exact Differential Equations. page 91
Problem number: Problem 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, _Riccati]`

$$\boxed{-\frac{y}{y^2+x^2}+\frac{xy'}{y^2+x^2}=-\frac{1}{x}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 12

```
dsolve((1/x-y(x)/(x^2+y(x)^2))+x/(x^2+y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\tan(\ln(x)+c_1)\,x$$

✓ Solution by Mathematica
Time used: 0.205 (sec). Leaf size: 15

```
DSolve[(1/x-y[x]/(x^2+y[x]^2))+x/(x^2+y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to x\tan(-\log(x)+c_1)$$

5.9 problem Problem 9

Internal problem ID [2721]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.9, Exact Differential Equations. page 91
Problem number: Problem 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y\cos(yx) + x\cos(yx)\, y' = \sin(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve((y(x)*cos(x*y(x))-sin(x))+x*cos(x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\arcsin(\cos(x) + c_1)}{x}$$

✓ Solution by Mathematica

Time used: 0.628 (sec). Leaf size: 17

```
DSolve[(y[x]*Cos[x*y[x]]-Sin[x])+x*Cos[x*y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{\arcsin(-\cos(x) + c_1)}{x}$$

5.10 problem Problem 10

Internal problem ID [2722]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.9, Exact Differential Equations. page 91
Problem number: Problem 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _Bernoulli]`

$$\boxed{2y^2 e^{2x} + 2y\, e^{2x} y' = -3x^2}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 46

```
dsolve((2*y(x)^2*exp(2*x)+3*x^2)+2*y(x)*exp(2*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = e^{-2x}\sqrt{e^{2x}\left(-x^3 + c_1\right)}$$
$$y(x) = -e^{-2x}\sqrt{e^{2x}\left(-x^3 + c_1\right)}$$

✓ Solution by Mathematica
Time used: 7.702 (sec). Leaf size: 47

```
DSolve[(2*y[x]^2*Exp[2*x]+3*x^2)+2*y[x]*Exp[2*x]*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\sqrt{e^{-2x}\left(-x^3 + c_1\right)}$$
$$y(x) \to \sqrt{e^{-2x}\left(-x^3 + c_1\right)}$$

5.11 problem Problem 11

Internal problem ID [2723]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.9, Exact Differential Equations. page 91
Problem number: Problem 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{y^2 + (2yx + \sin(y))\, y' = -\cos(x)}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 18

```
dsolve((y(x)^2+cos(x))+(2*x*y(x)+sin(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$xy(x)^2 + \sin(x) - \cos(y(x)) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.227 (sec). Leaf size: 20

```
DSolve[(y[x]^2+Cos[x])+(2*x*y[x]+Sin[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$\text{Solve}\left[xy(x)^2 - \cos(y(x)) + \sin(x) = c_1, y(x)\right]$$

5.12 problem Problem 12

Internal problem ID [2724]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.9, Exact Differential Equations. page 91
Problem number: Problem 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{\sin(y) + \cos(x) y + (x \cos(y) + \sin(x)) y' = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 15

```
dsolve((sin(y(x))+y(x)*cos(x))+(x*cos(y(x))+sin(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) \sin(x) + x \sin(y(x)) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.146 (sec). Leaf size: 17

```
DSolve[(Sin[y[x]]+y[x]*Cos[x])+(x*Cos[y[x]]+Sin[x])*y'[x]==0,y[x],x,IncludeSingularSolutions
```

$$\text{Solve}[x \sin(y(x)) + y(x) \sin(x) = c_1, y(x)]$$

6 Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502

6.1 problem Problem 23

Internal problem ID [2725]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' - 3y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{3x} + \mathrm{e}^{-x} c_2$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 22

```
DSolve[y''[x]-2*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x} \left(c_2 e^{4x} + c_1\right)$$

6.2 problem Problem 24

Internal problem ID [2726]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 7y' + 10y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+7*diff(y(x),x)+10*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-5x} c_1 + \mathrm{e}^{-2x} c_2$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 22

```
DSolve[y''[x]+7*y'[x]+10*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-5x} \left(c_2 e^{3x} + c_1\right)$$

6.3 problem Problem 25

Internal problem ID [2727]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 36y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-36*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-6x} + \mathrm{e}^{6x} c_2$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 22

```
DSolve[y''[x]-36*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{6x} + c_2 e^{-6x}$$

6.4 problem Problem 26

Internal problem ID [2728]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 e^{-4x}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 19

```
DSolve[y''[x]+4*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{1}{4} c_1 e^{-4x}$$

6.5 problem Problem 27

Internal problem ID [2729]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 27.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 3y'' - y' + 3y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)-diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{3x} + e^{-x} c_2 + c_3 e^{x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 28

```
DSolve[y'''[x]-3*y''[x]-y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x} + c_2 e^{x} + c_3 e^{3x}$$

6.6 problem Problem 28

Internal problem ID [2730]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 28.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 3y'' - 4y' - 12y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)+3*diff(y(x),x$2)-4*diff(y(x),x)-12*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_3 e^{5x} + e^x c_1 + c_2\right) e^{-3x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 28

```
DSolve[y'''[x]+3*y''[x]-4*y'[x]-12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(c_2 e^x + c_3 e^{5x} + c_1\right)$$

6.7 problem Problem 29

Internal problem ID [2731]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 29.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 3y'' - 18y' - 40y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$3)+3*diff(y(x),x$2)-18*diff(y(x),x)-40*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_3 \mathrm{e}^{9x} + c_2 \mathrm{e}^{3x} + c_1\right) \mathrm{e}^{-5x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 30

```
DSolve[y'''[x]+3*y''[x]-18*y'[x]-40*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-5x}\left(c_2 e^{3x} + c_3 e^{9x} + c_1\right)$$

6.8 problem Problem 30

Internal problem ID [2732]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 30.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - y'' - 2y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)-2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + \mathrm{e}^{-x} c_2 + c_3 \mathrm{e}^{2x}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 28

```
DSolve[y'''[x]-y''[x]-2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \left(-e^{-x}\right) + \frac{1}{2} c_2 e^{2x} + c_3$$

6.9 problem Problem 31

Internal problem ID [2733]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 31.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + y'' - 10y' + 8y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$3)+diff(y(x),x$2)-10*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(e^{6x}c_2 + c_1e^{5x} + c_3\right)e^{-4x}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 28

```
DSolve[y'''[x]+y''[x]-10*y'[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1e^{-4x} + c_2e^{x} + c_3e^{2x}$$

6.10 problem Problem 32

Internal problem ID [2734]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 32.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 2y''' - y'' + 2y' = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$4)-2*diff(y(x),x$3)-diff(y(x),x$2)+2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + \mathrm{e}^{-x} c_2 + c_3 \mathrm{e}^{x} + c_4 \mathrm{e}^{2x}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 34

```
DSolve[y''''[x]-2*y'''[x]-y''[x]+2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\left(-e^{-x}\right) + c_2 e^{x} + \frac{1}{2} c_3 e^{2x} + c_4$$

6.11 problem Problem 33

Internal problem ID [2735]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 33.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' - 13y'' + 36y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$4)-13*diff(y(x),x$2)+36*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_1 e^{6x} + c_4 e^{5x} + c_2 e^{x} + c_3\right) e^{-3x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 35

```
DSolve[y''''[x]-13*y''[x]+36*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(c_2 e^{x} + e^{5x}(c_4 e^{x} + c_3) + c_1\right)$$

6.12 problem Problem 34

Internal problem ID [2736]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' + 3xy' - 8y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)-8*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1x^6 + c_2}{x^4}$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+3*x*y'[x]-8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^6 + c_1}{x^4}$$

6.13 problem Problem 35

Internal problem ID [2737]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{2x^2y'' + 5xy' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(2*x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{x}} + \frac{c_2}{x}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 20

```
DSolve[2*x^2*y''[x]+5*x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2\sqrt{x} + c_1}{x}$$

6.14 problem Problem 36

Internal problem ID [2738]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 36.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^3y''' + x^2y'' - 2xy' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^3*diff(y(x),x$3)+x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_3x^3 + c_2x^2 + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 22

```
DSolve[x^3*y'''[x]+x^2*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3x^2 + c_2x + \frac{c_1}{x}$$

6.15 problem Problem 37

Internal problem ID [2739]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 37.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$x^3y''' + 3x^2y'' - 6xy' = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x^3*diff(y(x),x$3)+3*x^2*diff(y(x),x$2)-6*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2x^{\sqrt{7}} + c_3x^{-\sqrt{7}}$$

✓ Solution by Mathematica
Time used: 0.056 (sec). Leaf size: 41

```
DSolve[x^3*y'''[x]+3*x^2*y''[x]-6*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{c_1x^{-\sqrt{7}}}{\sqrt{7}} + \frac{c_2x^{\sqrt{7}}}{\sqrt{7}} + c_3$$

6.16 problem Problem 38

Internal problem ID [2740]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' - 6y = 18\,e^{5x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-6*y(x)=18*exp(5*x),y(x), singsol=all)
```

$$y(x) = \frac{(3\,e^{8x} + 4c_1 e^{5x} + 4c_2)\,e^{-3x}}{4}$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 31

```
DSolve[y''[x]+y'[x]-6*y[x]==18*Exp[5*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3e^{5x}}{4} + c_1 e^{-3x} + c_2 e^{2x}$$

6.17 problem Problem 39

Internal problem ID [2741]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' - 2y = 4x^2 + 5}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-2*y(x)=4*x^2+5,y(x), singsol=all)
```

$$y(x) = \frac{(-4x^2 - 4x - 11)\,\mathrm{e}^{-2x}\mathrm{e}^{2x}}{2} + \left(c_1\mathrm{e}^{3x} + c_2\right)\mathrm{e}^{-2x}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 31

```
DSolve[y''[x]+y'[x]-2*y[x]==4*x^2+5,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2x^2 - 2x + c_1 e^{-2x} + c_2 e^{x} - \frac{11}{2}$$

6.18 problem Problem 40

Internal problem ID [2742]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 40.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' + 2y'' - y' - 2y = 4\,e^{2x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$3)+2*diff(y(x),x$2)-diff(y(x),x)-2*y(x)=4*exp(2*x),y(x), singsol=all)
```

$$y(x) = \frac{(e^{4x} + 3c_1e^{3x} + 3c_3e^{x} + 3c_2)\,e^{-2x}}{3}$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 37

```
DSolve[y'''[x]+2*y''[x]-y'[x]-2*y[x]==4*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{2x}}{3} + c_1e^{-2x} + c_2e^{-x} + c_3e^{x}$$

6.19 problem Problem 41

Internal problem ID [2743]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 41.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' + y'' - 10y' + 8y = 24\,\mathrm{e}^{-3x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$3)+diff(y(x),x$2)-10*diff(y(x),x)+8*y(x)=24*exp(-3*x),y(x), singsol=all)
```

$$y(x) = \frac{(5c_3\mathrm{e}^{6x} + 5c_1\mathrm{e}^{5x} + 6\,\mathrm{e}^{x} + 5c_2)\,\mathrm{e}^{-4x}}{5}$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 37

```
DSolve[y'''[x]+y''[x]-10*y'[x]+8*y[x]==24*Exp[-3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{6e^{-3x}}{5} + c_1e^{-4x} + c_2e^{x} + c_3e^{2x}$$

6.20 problem Problem 42

Internal problem ID [2744]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 42.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + 5y'' + 6y' = 6\,\mathrm{e}^{-x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$3)+5*diff(y(x),x$2)+6*diff(y(x),x)=6*exp(-x),y(x), singsol=all)
```

$$y(x) = -\frac{c_1\mathrm{e}^{-3x}}{3} - \frac{\mathrm{e}^{-2x}c_2}{2} - 3\,\mathrm{e}^{-x} + c_3$$

✓ Solution by Mathematica
Time used: 0.05 (sec). Leaf size: 37

```
DSolve[y'''[x]+5*y''[x]+6*y'[x]==6*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -3e^{-x} - \frac{1}{3}c_1e^{-3x} - \frac{1}{2}c_2e^{-2x} + c_3$$

7 Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525

7.1 problem Problem 25

Internal problem ID [2745]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y = 6\,\mathrm{e}^{x}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+y(x)=6*exp(x),y(x), singsol=all)
```

$$y(x) = \sin(x)\, c_2 + \cos(x)\, c_1 + 3\,\mathrm{e}^{x}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 21

```
DSolve[y''[x]+y[x]==6*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3e^{x} + c_1 \cos(x) + c_2 \sin(x)$$

7.2 problem Problem 26

Internal problem ID [2746]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 4y = 5x\,\mathrm{e}^{-2x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+4*y(x)=5*x*exp(-2*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-2x}\left(c_2 + c_1 x + \frac{5}{6}x^3\right)$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 29

```
DSolve[y''[x]+4*y'[x]+4*y[x]==5*x*Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}e^{-2x}\left(5x^3 + 6c_2 x + 6c_1\right)$$

7.3 problem Problem 27

Internal problem ID [2747]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = 8\sin(2x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+4*y(x)=8*sin(2*x),y(x), singsol=all)
```

$$y(x) = (-2x + c_1)\cos(2x) + \sin(2x)\, c_2$$

✓ Solution by Mathematica
Time used: 0.084 (sec). Leaf size: 29

```
DSolve[y''[x]+4*y[x]==8*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x)\cos(x) + (-2x + c_1)\cos(2x) + c_2\sin(2x)$$

7.4 problem Problem 28

Internal problem ID [2748]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y' - 2y = 5\,\mathrm{e}^{2x}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-2*y(x)=5*exp(2*x),y(x), singsol=all)
```

$$y(x) = \frac{(5x + 3c_1)\,\mathrm{e}^{2x}}{3} + \mathrm{e}^{-x}c_2$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 31

```
DSolve[y''[x]-y'[x]-2*y[x]==5*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x} + e^{2x}\left(\frac{5x}{3} - \frac{5}{9} + c_2\right)$$

7.5 problem Problem 29

Internal problem ID [2749]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' + 5y = 3\sin(2x)$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+5*y(x)=3*sin(2*x),y(x), singsol=all)
```

$$y(x) = \frac{\sin(2x)(17\,\mathrm{e}^{-x}c_2 + 3)}{17} + \cos(2x)\,\mathrm{e}^{-x}c_1 - \frac{12\cos(2x)}{17}$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 45

```
DSolve[y''[x]+2*y'[x]+5*y[x]==3*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{17}e^{-x}((-12e^x + 17c_2)\cos(2x) + (3e^x + 17c_1)\sin(2x))$$

7.6 problem Problem 30

Internal problem ID [2750]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 30.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$y''' + 2y'' - 5y' - 6y = 4x^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 44

```
dsolve(diff(y(x),x$3)+2*diff(y(x),x$2)-5*diff(y(x),x)-6*y(x)=4*x^2,y(x), singsol=all)
```

$$y(x) = \frac{(-18x^2 + 30x - 37)\,\mathrm{e}^{-3x}\mathrm{e}^{3x}}{27} + \left(c_2\mathrm{e}^{2x} + c_3\mathrm{e}^{5x} + c_1\right)\mathrm{e}^{-3x}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 45

```
DSolve[y'''[x]+2*y''[x]-5*y'[x]-6*y[x]==4*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2x^2}{3} + \frac{10x}{9} + c_1e^{-3x} + c_2e^{-x} + c_3e^{2x} - \frac{37}{27}$$

7.7 problem Problem 31

Internal problem ID [2751]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 31.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - y'' + y' - y = 9\,\mathrm{e}^{-x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)+diff(y(x),x)-y(x)=9*exp(-x),y(x), singsol=all)
```

$$y(x) = -\frac{9\,\mathrm{e}^{-x}}{4} + \cos(x)\,c_1 + c_2\mathrm{e}^{x} + c_3\sin(x)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 31

```
DSolve[y'''[x]-y''[x]+y'[x]-y[x]==9*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{9e^{-x}}{4} + c_3e^{x} + c_1\cos(x) + c_2\sin(x)$$

7.8 problem Problem 32

Internal problem ID [2752]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 32.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' + 3y'' + 3y' + y = 2\,e^{-x} + 3\,e^{2x}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$3)+3*diff(y(x),x$2)+3*diff(y(x),x)+y(x)=2*exp(-x)+3*exp(2*x),y(x), singso
```

$$y(x) = \frac{(9c_3x^2 + 3x^3 + 9c_2x + 9c_1)\,e^{-x}}{9} + \frac{e^{2x}}{9}$$

✓ Solution by Mathematica
Time used: 0.066 (sec). Leaf size: 41

```
DSolve[y'''[x]+3*y''[x]+3*y'[x]+y[x]==2*Exp[-x]+3*Exp[2*x],y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{1}{9}e^{-x}\left(3x^3 + 9c_3x^2 + e^{3x} + 9c_2x + 9c_1\right)$$

7.9 problem Problem 33

Internal problem ID [2753]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = 5\cos(2x)}$$

With initial conditions

$$[y(0) = 2, y'(0) = 3]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)+9*y(x)=5*cos(2*x),y(0) = 2, D(y)(0) = 3],y(x), singsol=all)
```

$$y(x) = \sin(3x) + \cos(3x) + \cos(2x)$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 18

```
DSolve[{y''[x]+9*y[x]==5*Cos[2*x],{y[0]==2,y'[0]==3}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \sin(3x) + \cos(2x) + \cos(3x)$$

7.10 problem Problem 34

Internal problem ID [2754]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y = 9x\,e^{2x}}$$

With initial conditions

$$[y(0) = 0, y'(0) = 7]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 25

```
dsolve([diff(y(x),x$2)-y(x)=9*x*exp(2*x),y(0) = 0, D(y)(0) = 7],y(x), singsol=all)
```

$$y(x) = -4\,e^{-x} + 8\,e^{x} + (3x - 4)\,e^{2x}$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 29

```
DSolve[{y''[x]-y[x]==9*x*Exp[2*x],{y[0]==0,y'[0]==7}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^{2x}(3x - 4) - 4e^{-x} + 8e^{x}$$

7.11 problem Problem 35

Internal problem ID [2755]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' - 2y = -10\sin(x)}$$

With initial conditions

$$[y(0) = 2, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 15

```
dsolve([diff(y(x),x$2)+diff(y(x),x)-2*y(x)=-10*sin(x),y(0) = 2, D(y)(0) = 1],y(x), singsol=a
```

$$y(x) = e^{-2x} + \cos(x) + 3\sin(x)$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 17

```
DSolve[{y''[x]+y'[x]-2*y[x]==-10*Sin[x],{y[0]==2,y'[0]==1}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to e^{-2x} + 3\sin(x) + \cos(x)$$

7.12 problem Problem 36

Internal problem ID [2756]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' - 2y = 4\cos(x) - 2\sin(x)}$$

With initial conditions

$$[y(0) = -1, y'(0) = 4]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$2)+diff(y(x),x)-2*y(x)=4*cos(x)-2*sin(x),y(0) = -1, D(y)(0) = 4],y(x), s
```

$$y(x) = -\mathrm{e}^{-2x}\left((-\sin(x) + \cos(x))\,\mathrm{e}^{2x} - \mathrm{e}^{3x} + 1\right)$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 22

```
DSolve[{y''[x]+y'[x]-2*y[x]==4*Cos[x]-2*Sin[x],{y[0]==-1,y'[0]==4}},y[x],x,IncludeSingularSo
```

$$y(x) \to -e^{-2x} + e^x + \sin(x) - \cos(x)$$

7.13 problem Problem 38

Internal problem ID [2757]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + \omega^2 y = \frac{F_0 \cos(\omega t)}{m}}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$2)+omega^2*y(t)=F__0/m*cos(omega*t),y(0) = 1, D(y)(0) = 0],y(t), singsol
```

$$y(t) = \cos(\omega t) + \frac{F_0 \sin(\omega t) t}{2\omega m}$$

✓ Solution by Mathematica
Time used: 0.082 (sec). Leaf size: 26

```
DSolve[{y''[t]+\[Omega]^2*y[t]==F0/m*Cos[\[Omega]*t],{y[0]==1,y'[0]==0}},y[t],t,IncludeSingu
```

$$y(t) \to \frac{\text{F0} t \sin(t\omega)}{2m\omega} + \cos(t\omega)$$

7.14 problem Problem 39

Internal problem ID [2758]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y' + 6y = 7\,e^{2x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+6*y(x)=7*exp(2*x),y(x), singsol=all)
```

$$y(x) = e^{2x}\left(\frac{7}{2} + c_2 \sin\left(\sqrt{2}\,x\right) + \cos\left(\sqrt{2}\,x\right) c_1\right)$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 40

```
DSolve[y''[x]-4*y'[x]+6*y[x]==7*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{2x}\left(2c_2 \cos\left(\sqrt{2}x\right) + 2c_1 \sin\left(\sqrt{2}x\right) + 7\right)$$

7.15 problem Problem 40

Internal problem ID [2759]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 40.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' + y'' + y' + y = 4x\,\mathrm{e}^{x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$3)+diff(y(x),x$2)+diff(y(x),x)+y(x)=4*x*exp(x),y(x), singsol=all)
```

$$y(x) = c_3\mathrm{e}^{-x} + \cos(x)\,c_1 + x\,\mathrm{e}^{x} + \sin(x)\,c_2 - \frac{3\,\mathrm{e}^{x}}{2}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 36

```
DSolve[y'''[x]+y''[x]+y'[x]+y[x]==4*x*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x x - \frac{3e^x}{2} + c_3e^{-x} + c_1\cos(x) + c_2\sin(x)$$

7.16 problem Problem 41

Internal problem ID [2760]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 41.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$y'''' + 104y''' + 2740y'' = 5\,\mathrm{e}^{-2x}\cos(3x)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 58

```
dsolve(diff(y(x),x$4)+104*diff(y(x),x$3)+2740*diff(y(x),x$2)=5*exp(-2*x)*cos(3*x),y(x), sing
```

$$y(x) = \frac{\left((667c_1 + 156c_2)\cos(6x) - 156\left(c_1 - \frac{667c_2}{156}\right)\sin(6x)\right)\mathrm{e}^{-52x}}{1876900} + \frac{5(-695\cos(3x) - 2448\sin(3x))\,\mathrm{e}^{-2x}}{84184477} + c_3x + c_4$$

✓ Solution by Mathematica

Time used: 4.755 (sec). Leaf size: 82

```
DSolve[y''''[x]+104*y'''[x]+2740*y''[x]==5*Exp[-2*x]*Cos[3*x],y[x],x,IncludeSingularSolution
```

$$y(x) \to -\frac{12240e^{-2x}\sin(3x)}{84184477} - \frac{3475e^{-2x}\cos(3x)}{84184477} + c_4x + \frac{(156c_1 + 667c_2)e^{-52x}\cos(6x)}{1876900} + \frac{(667c_1 - 156c_2)e^{-52x}\sin(6x)}{1876900} + c_3$$

7.17 problem Problem 46

Internal problem ID [2761]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 46.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' - 3y = \sin(x)^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)-3*y(x)=sin(x)^2,y(x), singsol=all)
```

$$y(x) = e^{-3x}\left(\left(-\frac{1}{6} - \frac{2\sin(2x)}{65} + \frac{7\cos(2x)}{130}\right)e^{3x} + e^{4x}c_1 + c_2\right)$$

✓ Solution by Mathematica
Time used: 0.098 (sec). Leaf size: 39

```
DSolve[y''[x]+2*y'[x]-3*y[x]==Sin[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{65}\sin(2x) + \frac{7}{130}\cos(2x) + c_1e^{-3x} + c_2e^{x} - \frac{1}{6}$$

7.18 problem Problem 47

Internal problem ID [2762]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 47.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 6y = \sin(x)^2 \cos(x)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)+6*y(x)=sin(x)^2*cos(x)^2,y(x), singsol=all)
```

$$y(x) = \sin\left(\sqrt{6}\,x\right) c_2 + \cos\left(\sqrt{6}\,x\right) c_1 + \frac{1}{48} + \frac{\cos(4x)}{80}$$

✓ Solution by Mathematica
Time used: 0.756 (sec). Leaf size: 39

```
DSolve[y''[x]+6*y[x]==Sin[x]^2*Cos[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{80}\cos(4x) + c_1 \cos\left(\sqrt{6}x\right) + c_2 \sin\left(\sqrt{6}x\right) + \frac{1}{48}$$

8 Chapter 8, Linear differential equations of order n. Section 8.4, Complex-Valued Trial Solutions. page 529

8.1 problem Problem 1

Internal problem ID [2763]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.4, Complex-Valued Trial Solutions. page 529
Problem number: Problem 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 16y = 20\cos(4x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)-16*y(x)=20*cos(4*x),y(x), singsol=all)
```

$$y(x) = c_2 e^{4x} + e^{-4x} c_1 - \frac{5\cos(4x)}{8}$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 30

```
DSolve[y''[x]-16*y[x]==20*Cos[4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{5}{8}\cos(4x) + c_1 e^{4x} + c_2 e^{-4x}$$

8.2 problem Problem 2

Internal problem ID [2764]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.4, Complex-Valued Trial Solutions. page 529
Problem number: Problem 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + y = 50 \sin(3x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=50*sin(3*x),y(x), singsol=all)
```

$$y(x) = (c_1 x + c_2) e^{-x} - 3\cos(3x) - 4\sin(3x)$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 34

```
DSolve[y''[x]+2*y'[x]+y[x]==50*Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -3\cos(3x) + e^{-x}(-4e^x \sin(3x) + c_2 x + c_1)$$

8.3 problem Problem 3

Internal problem ID [2765]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.4, Complex-Valued Trial Solutions. page 529
Problem number: Problem 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y = 10\cos(x)\,\mathrm{e}^{2x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-y(x)=10*exp(2*x)*cos(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}c_2 + \mathrm{e}^{x}c_1 + \mathrm{e}^{2x}(2\sin(x) + \cos(x))$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 33

```
DSolve[y''[x]-y[x]==10*Exp[2*x]*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1e^x + c_2e^{-x} + e^{2x}(2\sin(x) + \cos(x))$$

8.4 problem Problem 4

Internal problem ID [2766]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.4, Complex-Valued Trial Solutions. page 529
Problem number: Problem 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 4y = 169 \sin(3x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+4*y(x)=169*sin(3*x),y(x), singsol=all)
```

$$y(x) = (c_1 x + c_2) e^{-2x} - 12 \cos(3x) - 5 \sin(3x)$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 36

```
DSolve[y''[x]+4*y'[x]+4*y[x]==169*Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -12 \cos(3x) + e^{-2x} \left(-5e^{2x} \sin(3x) + c_2 x + c_1\right)$$

8.5 problem Problem 5

Internal problem ID [2767]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.4, Complex-Valued Trial Solutions. page 529
Problem number: Problem 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y' - 2y = 40 \sin(x)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-2*y(x)=40*sin(x)^2,y(x), singsol=all)
```

$$y(x) = e^{-x} c_2 + c_1 e^{2x} - 10 + \sin(2x) + 3\cos(2x)$$

✓ Solution by Mathematica
Time used: 0.091 (sec). Leaf size: 33

```
DSolve[y''[x]-y'[x]-2*y[x]==40*Sin[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(2x) + 3\cos(2x) + c_1 e^{-x} + c_2 e^{2x} - 10$$

8.6 problem Problem 6

Internal problem ID [2768]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.4, Complex-Valued Trial Solutions. page 529
Problem number: Problem 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = 3\,\mathrm{e}^{x} \cos\left(2x\right)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+y(x)=3*exp(x)*cos(2*x),y(x), singsol=all)
```

$$y(x) = \cos\left(x\right) c_1 + \frac{3\,\mathrm{e}^{x} \sin\left(2x\right)}{5} - \frac{3\,\mathrm{e}^{x} \cos\left(2x\right)}{10} + \sin\left(x\right) c_2$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 34

```
DSolve[y''[x]+y[x]==3*Exp[x]*Cos[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{3}{10} e^{x}(\cos(2x) - 2\sin(2x)) + c_1 \cos(x) + c_2 \sin(x)$$

8.7 problem Problem 7

Internal problem ID [2769]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.4, Complex-Valued Trial Solutions. page 529
Problem number: Problem 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + 2y = 2\,\mathrm{e}^{-x}\sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+2*y(x)=2*exp(-x)*sin(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}(\sin(x)\,c_2 + \cos(x)\,c_1 - x\cos(x) + \sin(x))$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 34

```
DSolve[y''[x]+2*y'[x]+2*y[x]==2*Exp[-x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}(\sin(x) - 2x\cos(x) + 2c_2\cos(x) + 2c_1\sin(x))$$

8.8 problem Problem 8

Internal problem ID [2770]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.4, Complex-Valued Trial Solutions. page 529
Problem number: Problem 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y = 100\,\mathrm{e}^{x} x \sin(x)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 51

```
dsolve(diff(y(x),x$2)-4*y(x)=100*x*exp(x)*sin(x),y(x), singsol=all)
```

$$y(x) = \left(-10x\cos(x)\,\mathrm{e}^{3x} + \mathrm{e}^{4x}c_1 - 20x\sin(x)\,\mathrm{e}^{3x} - 14\cos(x)\,\mathrm{e}^{3x} + 2\,\mathrm{e}^{3x}\sin(x) + c_2\right)\mathrm{e}^{-2x}$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 44

```
DSolve[y''[x]-4*y[x]==100*x*Exp[x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1e^{2x} + c_2e^{-2x} - 2e^{x}((10x-1)\sin(x) + (5x+7)\cos(x))$$

8.9 problem Problem 9

Internal problem ID [2771]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.4, Complex-Valued Trial Solutions. page 529
Problem number: Problem 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' + 5y = 4\,\mathrm{e}^{-x} \cos(2x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+5*y(x)=4*exp(-x)*cos(2*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}\left(\cos(2x)\left(\frac{1}{2} + c_1\right) + \sin(2x)(c_2 + x)\right)$$

✓ Solution by Mathematica
Time used: 0.059 (sec). Leaf size: 36

```
DSolve[y''[x]+2*y'[x]+5*y[x]==4*Exp[-x]*Cos[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{-x}((1 + 4c_2)\cos(2x) + 4(x + c_1)\sin(2x))$$

8.10 problem Problem 10

Internal problem ID [2772]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.4, Complex-Valued Trial Solutions. page 529
Problem number: Problem 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + 10y = 24\,\mathrm{e}^{x}\cos(3x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+10*y(x)=24*exp(x)*cos(3*x),y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{x}(3c_1+4)\cos(3x)}{3} + 4\,\mathrm{e}^{x}\sin(3x)\left(x+\frac{c_2}{4}\right)$$

✓ Solution by Mathematica
Time used: 0.055 (sec). Leaf size: 36

```
DSolve[y''[x]-2*y'[x]+10*y[x]==24*Exp[x]*Cos[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}e^{x}((2+3c_2)\cos(3x)+3(4x+c_1)\sin(3x))$$

8.11 problem Problem 11

Internal problem ID [2773]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.4, Complex-Valued Trial Solutions. page 529
Problem number: Problem 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 16y = 34\,e^{x} + 16\cos(4x) - 8\sin(4x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+16*y(x)=34*exp(x)+16*cos(4*x)-8*sin(4*x),y(x), singsol=all)
```

$$y(x) = \frac{(4c_2 + 8x - 1)\sin(4x)}{4} + (c_1 + x)\cos(4x) + 2\,e^{x}$$

✓ Solution by Mathematica
Time used: 0.623 (sec). Leaf size: 37

```
DSolve[y''[x]+16*y[x]==34*Exp[x]+16*Cos[4*x]-8*Sin[4*x],y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to 2e^{x} + \left(x + \frac{1}{4} + c_1\right)\cos(4x) + \left(2x - \frac{1}{8} + c_2\right)\sin(4x)$$

9 Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556

9.1 problem Problem 1

Internal problem ID [2774]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 6y' + 9y = 4\,e^{3x} \ln (x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+9*y(x)=4*exp(3*x)*ln(x),y(x), singsol=all)
```

$$y(x) = e^{3x}\left(2\ln (x)\, x^2 + c_1 x - 3x^2 + c_2\right)$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 30

```
DSolve[y''[x]-6*y'[x]+9*y[x]==4*Exp[3*x]*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{3x}\left(-3x^2 + 2x^2 \log(x) + c_2 x + c_1\right)$$

9.2 problem Problem 2

Internal problem ID [2775]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 4y = \frac{e^{-2x}}{x^2}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+4*y(x)=x^(-2)*exp(-2*x),y(x), singsol=all)
```

$$y(x) = e^{-2x}(-1 + c_1 x - \ln(x) + c_2)$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 23

```
DSolve[y''[x]+4*y'[x]+4*y[x]==x^(-2)*Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}(-\log(x) + c_2 x - 1 + c_1)$$

9.3 problem Problem 3

Internal problem ID [2776]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = 18\sec(3x)^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+9*y(x)=18*sec(3*x)^3,y(x), singsol=all)
```

$$y(x) = (c_1 - 2)\cos(3x) + \sin(3x)\,c_2 + \sec(3x)$$

✓ Solution by Mathematica
Time used: 0.139 (sec). Leaf size: 32

```
DSolve[y''[x]+9*y[x]==18*Sec[3*x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\sec(3x)((-2 + c_1)\cos(6x) + c_2\sin(6x) + c_1)$$

9.4 problem Problem 4

Internal problem ID [2777]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 6y' + 9y = \frac{2\,e^{-3x}}{x^2+1}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+6*diff(y(x),x)+9*y(x)=2*exp(-3*x)/(x^2+1),y(x), singsol=all)
```

$$y(x) = e^{-3x}\left(c_2 + c_1 x - \ln\left(x^2+1\right) + 2x\arctan\left(x\right)\right)$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 31

```
DSolve[y''[x]+6*y'[x]+9*y[x]==2*Exp[-3*x]/(x^2+1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(2x\arctan(x) - \log\left(x^2+1\right) + c_2 x + c_1\right)$$

9.5 problem Problem 5

Internal problem ID [2778]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y = \frac{8}{e^{2x} + 1}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 43

```
dsolve(diff(y(x),x$2)-4*y(x)=8/(exp(2*x)+1),y(x), singsol=all)
```

$$y(x) = \left(-e^{-2x} + e^{2x}\right) \ln\left(e^{2x} + 1\right) + \left(c_1 - 2\ln\left(e^{x}\right)\right) e^{2x} + e^{-2x} c_2 - 1$$

✓ Solution by Mathematica
Time used: 0.086 (sec). Leaf size: 56

```
DSolve[y''[x]-4*y[x]==8/(Exp[2*x]+1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(2e^{4x} \operatorname{arctanh}\left(2e^{2x} + 1\right) - e^{2x} - \log\left(e^{2x} + 1\right) + c_1 e^{4x} + c_2\right)$$

9.6 problem Problem 6

Internal problem ID [2779]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 5y = e^{2x} \tan(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+5*y(x)=exp(2*x)*tan(x),y(x), singsol=all)
```

$$y(x) = e^{2x}(\sin(x) c_2 + \cos(x) c_1 - \cos(x) \ln(\sec(x) + \tan(x)))$$

✓ Solution by Mathematica
Time used: 0.063 (sec). Leaf size: 29

```
DSolve[y''[x]-4*y'[x]+5*y[x]==Exp[2*x]*Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}(\cos(x)(-\text{arctanh}(\sin(x))) + c_2\cos(x) + c_1\sin(x))$$

9.7 problem Problem 7

Internal problem ID [2780]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 9y = \frac{36}{4 - \cos(3x)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 61

```
dsolve(diff(y(x),x$2)+9*y(x)=36/(4-cos(3*x)^2),y(x), singsol=all)
```

$$y(x) = \frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\sin(3x)}{3}\right)\sin(3x)}{3} + \sin(3x)\,c_2 + \cos(3x)\ln(\cos(3x)+2) - \cos(3x)\ln(\cos(3x)-2) + \cos(3x)\,c_1$$

✓ Solution by Mathematica
Time used: 0.227 (sec). Leaf size: 61

```
DSolve[y''[x]+9*y[x]==36/(4-Cos[3*x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4\sin(3x)\arctan\left(\frac{\sin(3x)}{\sqrt{3}}\right)}{\sqrt{3}} + c_2\sin(3x) + \cos(3x)(-\log(2-\cos(3x)) + \log(\cos(3x)+2) + c_1)$$

9.8 problem Problem 8

Internal problem ID [2781]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 10y' + 25y = \frac{2\,e^{5x}}{x^2 + 4}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)-10*diff(y(x),x)+25*y(x)=2*exp(5*x)/(4+x^2),y(x), singsol=all)
```

$$y(x) = e^{5x}\left(c_2 + c_1 x - \ln\left(x^2 + 4\right) + x \arctan\left(\frac{x}{2}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 34

```
DSolve[y''[x]-10*y'[x]+25*y[x]==2*Exp[5*x]/(4+x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{5x}\left(x \arctan\left(\frac{x}{2}\right) - \log\left(x^2 + 4\right) + c_2 x + c_1\right)$$

9.9 problem Problem 9

Internal problem ID [2782]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 6y' + 13y = 4\,\mathrm{e}^{3x} \sec(2x)^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+13*y(x)=4*exp(3*x)*sec(2*x)^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{3x}(\sin(2x)\,c_2 + c_1\cos(2x) - 1 + \sin(2x)\ln(\sec(2x) + \tan(2x)))$$

✓ Solution by Mathematica

Time used: 0.122 (sec). Leaf size: 37

```
DSolve[y''[x]-6*y'[x]+13*y[x]==4*Exp[3*x]*Sec[2*x]^2,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to e^{3x}\left(c_2\cos(2x) + \sin(2x)\coth^{-1}(\sin(2x)) + c_1\sin(2x) - 1\right)$$

9.10 problem Problem 10

Internal problem ID [2783]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sec(x) + 4\,\mathrm{e}^{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)+y(x)=sec(x)+4*exp(x),y(x), singsol=all)
```

$$y(x) = \cos(x)\ln(\cos(x)) + \cos(x)\,c_1 + \sin(x)\,(c_2 + x) + 2\,\mathrm{e}^{x}$$

✓ Solution by Mathematica
Time used: 0.065 (sec). Leaf size: 91

```
DSolve[y''[x]+y[x]==4*Exp[x]*Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \to & -4ie^{x}\,\text{Hypergeometric2F1}\left(-\frac{i}{2}, 1, 1-\frac{i}{2}, -e^{2ix}\right)\cos(x) \\ & + \left(\frac{8}{5}+\frac{4i}{5}\right)e^{(1+2i)x}\,\text{Hypergeometric2F1}\left(1, 1-\frac{i}{2}, 2-\frac{i}{2}, -e^{2ix}\right)\cos(x) \\ & + 4e^{x}\sin(x) + c_1\cos(x) + c_2\sin(x) \end{aligned}$$

9.11 problem Problem 11

Internal problem ID [2784]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \csc(x) + 2x^2 + 5x + 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)+y(x)=csc(x)+2*x^2+5*x+1,y(x), singsol=all)
```

$$y(x) = \sin(x)\ln(\sin(x)) + (c_1 - x)\cos(x) + 2x^2 + \sin(x)\, c_2 + 5x - 3$$

✓ Solution by Mathematica
Time used: 0.208 (sec). Leaf size: 33

```
DSolve[y''[x]+y[x]==Csc[x]+2*x^2+5*x+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x^2 + 5x + (-x + c_1)\cos(x) + \sin(x)(\log(\sin(x)) + c_2) - 3$$

9.12 problem Problem 12

Internal problem ID [2785]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y = 2\tanh(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-y(x)=2*tanh(x),y(x), singsol=all)
```

$$y(x) = (c_2 + 2\arctan(e^x))\,e^{-x} + e^x(c_1 + 2\arctan(e^x))$$

✓ Solution by Mathematica
Time used: 0.06 (sec). Leaf size: 35

```
DSolve[y''[x]-y[x]==2*Tanh[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(2\left(e^{2x}+1\right)\arctan(e^x) + c_1e^{2x} + c_2\right)$$

9.13 problem Problem 13

Internal problem ID [2786]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2my' + m^2y = \frac{e^{mx}}{x^2 + 1}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)-2*m*diff(y(x),x)+m^2*y(x)=exp(m*x)/(1+x^2),y(x), singsol=all)
```

$$y(x) = e^{mx}\left(c_2 + c_1x - \frac{\ln(x^2 + 1)}{2} + x\arctan(x)\right)$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 37

```
DSolve[y''[x]-2*m*y'[x]+m^2*y[x]==Exp[m*x]/(1+x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{mx}\left(2x\arctan(x) - \log\left(x^2 + 1\right) + 2(c_2x + c_1)\right)$$

9.14 problem Problem 13

Internal problem ID [2787]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + y = \frac{4\,\mathrm{e}^{x}\ln(x)}{x^{3}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=4*exp(x)*x^(-3)*ln(x),y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{x}(c_1x^2 + c_2x + 2\ln(x) + 3)}{x}$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 28

```
DSolve[y''[x]-2*y'[x]+y[x]==4*Exp[x]*x^(-3)*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{x}(c_2x^2 + 2\log(x) + c_1x + 3)}{x}$$

9.15 problem Problem 15

Internal problem ID [2788]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' + y = \frac{e^{-x}}{\sqrt{-x^2 + 4}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 49

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=exp(-x)/sqrt(4-x^2),y(x), singsol=all)
```

$$y(x) = -\frac{e^{-x}\left(\left(-c_1 x - \arcsin\left(\frac{x}{2}\right) x - c_2\right)\sqrt{-x^2 + 4} + x^2 - 4\right)}{\sqrt{-x^2 + 4}}$$

✓ Solution by Mathematica
Time used: 0.085 (sec). Leaf size: 50

```
DSolve[y''[x]+2*y'[x]+y[x]==Exp[-x]/Sqrt[4-x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(-2x \arctan\left(\frac{\sqrt{4 - x^2}}{x + 2}\right) + \sqrt{4 - x^2} + c_2 x + c_1\right)$$

9.16 problem Problem 16

Internal problem ID [2789]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' + 17y = \frac{64\,\mathrm{e}^{-x}}{3 + \sin(4x)^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 70

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+17*y(x)=64*exp(-x)/(3+sin(4*x)^2),y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{-x}\left(4\sin(4x)\sqrt{3}\arctan\left(\frac{\sqrt{3}\sin(4x)}{3}\right) + 3\ln(\cos(4x)+2)\cos(4x) - 3\ln(\cos(4x)-2)\cos(4x) + 3\cos(\right.}{3}$$

✓ Solution by Mathematica

Time used: 0.238 (sec). Leaf size: 72

```
DSolve[y''[x]+2*y'[x]+17*y[x]==64*Exp[-x]/(3+Sin[4*x]^2),y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{3}e^{-x}\left(4\sqrt{3}\sin(4x)\arctan\left(\frac{\sin(4x)}{\sqrt{3}}\right) + 3c_1\sin(4x) + 3\cos(4x)(-\log(2-\cos(4x)) + \log(\cos(4x)+2) + c_2)\right)$$

9.17 problem Problem 17

Internal problem ID [2790]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 4y = \frac{4\,\mathrm{e}^{-2x}}{x^2+1} + 2x^2 - 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+4*y(x)=4*exp(-2*x)/(1+x^2)+2*x^2-1,y(x), singsol=all)
```

$$y(x) = \frac{(x-1)^2}{2} + \mathrm{e}^{-2x}\left(c_1 x + 4x\arctan(x) + c_2 - 2\ln\left(x^2+1\right)\right)$$

✓ Solution by Mathematica

Time used: 0.58 (sec). Leaf size: 59

```
DSolve[y''[x]+4*y'[x]+4*y[x]==4*Exp[-2*x]/(1+x^2)+2*x^2-1,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{2}e^{-2x}\left(8x\arctan(x) + e^{2x}x^2 - 4\log\left(x^2+1\right) - 2e^{2x}x + e^{2x} + 2c_2 x + 2c_1\right)$$

9.18 problem Problem 18

Internal problem ID [2791]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 4y = 15\ln(x)\,\mathrm{e}^{-2x} + 25\cos(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+4*y(x)=15*exp(-2*x)*ln(x)+25*cos(x),y(x), singsol=all)
```

$$y(x) = \frac{(30\ln(x)\,x^2 - 45x^2 + 4c_1x + 4c_2)\,\mathrm{e}^{-2x}}{4} + 3\cos(x) + 4\sin(x)$$

✓ Solution by Mathematica

Time used: 0.211 (sec). Leaf size: 54

```
DSolve[y''[x]+4*y'[x]+4*y[x]==15*Exp[-2*x]*Log[x]+25*Cos[x],y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{4}e^{-2x}\left(-45x^2 + 30x^2\log(x) + 16e^{2x}\sin(x) + 12e^{2x}\cos(x) + 4c_2x + 4c_1\right)$$

9.19 problem Problem 19

Internal problem ID [2792]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 19.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - 3y'' + 3y' - y = \frac{2\,e^{x}}{x^{2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)+3*diff(y(x),x)-y(x)=2*x^(-2)*exp(x),y(x), singsol=all
```

$$y(x) = e^{x}\left(-2x\ln(x) + c_1 + c_2x + c_3x^2\right)$$

✓ Solution by Mathematica

Time used: 0.393 (sec). Leaf size: 627

```
DSolve[y'''[x]-6*y''[x]+3*y'[x]-y[x]==2*x^(-2)*Exp[x],y[x],x,IncludeSingularSolutions -> Tru
```

$y(x)$

$$\rightarrow \frac{2i(\text{Root}[\#1^3-6\#1^2+3\#1-1\&,1]-\text{Root}[\#1^3-6\#1^2+3\#1-1\&,2])\exp(x\text{Root}[\#1^3-6\#1^2}{}$$

$$+\frac{2i(\text{Root}[\#1^3-6\#1^2+3\#1-1\&,2]-\text{Root}[\#1^3-6\#1^2+3\#1-1\&,3])\exp(x\text{Root}[\#1^3-6\#1}{}$$

$$-\frac{2i(\text{Root}[\#1^3-6\#1^2+3\#1-1\&,1]-\text{Root}[\#1^3-6\#1^2+3\#1-1\&,3])\exp(x\text{Root}[\#1^3-6\#1}{}$$

$$+c_2\exp(x\text{Root}[\#1^3-6\#1^2+3\#1-1\&,2])$$
$$+c_3\exp(x\text{Root}[\#1^3-6\#1^2+3\#1-1\&,3])$$
$$+c_1\exp(x\text{Root}[\#1^3-6\#1^2+3\#1-1\&,1])$$

9.20 problem Problem 20

Internal problem ID [2793]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 20.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 6y'' + 12y' - 8y = 36\,\mathrm{e}^{2x}\ln(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$3)-6*diff(y(x),x$2)+12*diff(y(x),x)-8*y(x)=36*exp(2*x)*ln(x),y(x), singso
```

$$y(x) = \mathrm{e}^{2x}\left(6x^3\ln(x) - 11x^3 + c_1 + c_2x + c_3x^2\right)$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 36

```
DSolve[y'''[x]-6*y''[x]+12*y'[x]-8*y[x]==36*Exp[2*x]*Log[x],y[x],x,IncludeSingularSolutions
```

$$y(x) \to e^{2x}\left(-11x^3 + 6x^3\log(x) + c_3x^2 + c_2x + c_1\right)$$

9.21 problem Problem 21

Internal problem ID [2794]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 21.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' + 3y'' + 3y' + y = \frac{2\,e^{-x}}{x^2 + 1}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$3)+3*diff(y(x),x$2)+3*diff(y(x),x)+y(x)=2*exp(-x)/(1+x^2),y(x), singsol=a
```

$$y(x) = e^{-x}\left(x^2 \arctan(x) - x\ln\left(x^2 + 1\right) - \arctan(x) + x + c_1 + c_2x + c_3x^2\right)$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 42

```
DSolve[y'''[x]+3*y''[x]+3*y'[x]+y[x]==2*Exp[-x]/(1+x^2),y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to e^{-x}\left(\left(x^2 - 1\right)\arctan(x) - x\log\left(x^2 + 1\right) + c_3x^2 + x + c_2x + c_1\right)$$

9.22 problem Problem 22

Internal problem ID [2795]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 22.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - 6y'' + 9y' = 12\,e^{3x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$3)-6*diff(y(x),x$2)+9*diff(y(x),x)=12*exp(3*x),y(x), singsol=all)
```

$$y(x) = \frac{(4 + 18x^2 + 3(-4 + c_1)\,x - c_1 + 3c_2)\,e^{3x}}{9} + c_3$$

✓ Solution by Mathematica
Time used: 0.06 (sec). Leaf size: 39

```
DSolve[y'''[x]-6*y''[x]+9*y'[x]==12*Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{9}e^{3x}\left(18x^2 + 3(-4 + c_2)x + 4 + 3c_1 - c_2\right) + c_3$$

9.23 problem Problem 23

Internal problem ID [2796]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 9y = F(x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
dsolve(diff(y(x),x$2)-9*y(x)=F(x),y(x), singsol=all)
```

$$y(x) = c_2 e^{3x} + c_1 e^{-3x} + \frac{\left(\int e^{-3x} F(x)\, dx\right) e^{3x}}{6} - \frac{\left(\int e^{3x} F(x)\, dx\right) e^{-3x}}{6}$$

✓ Solution by Mathematica

Time used: 0.063 (sec). Leaf size: 66

```
DSolve[y''[x]-y[x]==F[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(e^{2x}\int_1^x \frac{1}{2}e^{-K[1]}F(K[1])dK[1] + \int_1^x -\frac{1}{2}e^{K[2]}F(K[2])dK[2] + c_1 e^{2x} + c_2\right)$$

9.24 problem Problem 24

Internal problem ID [2797]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 5y' + 4y = F(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x$2)+5*diff(y(x),x)+4*y(x)=F(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} c_2 + \mathrm{e}^{-4x} c_1 + \frac{\left(\int \mathrm{e}^{x} F(x)\, dx\right) \mathrm{e}^{-x}}{3} - \frac{\left(\int F(x)\, \mathrm{e}^{4x} dx\right) \mathrm{e}^{-4x}}{3}$$

✓ Solution by Mathematica
Time used: 0.06 (sec). Leaf size: 66

```
DSolve[y''[x]+5*y'[x]+4*y[x]==F[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-4x} \left(\int_1^x -\frac{1}{3} e^{4K[1]} F(K[1]) dK[1] + e^{3x} \int_1^x \frac{1}{3} e^{K[2]} F(K[2]) dK[2] + c_2 e^{3x} + c_1 \right)$$

9.25 problem Problem 25

Internal problem ID [2798]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' - 2y = F(x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 46

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-2*y(x)=F(x),y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(\int \mathrm{e}^{-x} F(x)\, dx\right) \mathrm{e}^{3x} + 3c_1 \mathrm{e}^{3x} - \left(\int F(x)\, \mathrm{e}^{2x} dx\right) + 3c_2\right) \mathrm{e}^{-2x}}{3}$$

✓ Solution by Mathematica
Time used: 0.053 (sec). Leaf size: 68

```
DSolve[y''[x]+y'[x]-2*y[x]==F[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x} \left(\int_1^x -\frac{1}{3} e^{2K[1]} F(K[1]) dK[1] + e^{3x} \int_1^x \frac{1}{3} e^{-K[2]} F(K[2]) dK[2] + c_2 e^{3x} + c_1 \right)$$

9.26 problem Problem 26

Internal problem ID [2799]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' - 12y = F(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)-12*y(x)=F(x),y(x), singsol=all)
```

$$y(x) = -\frac{\left(-\left(\int F(x)\,\mathrm{e}^{-2x}dx\right)\mathrm{e}^{8x} - 8c_1\mathrm{e}^{8x} + \int F(x)\,\mathrm{e}^{6x}dx - 8c_2\right)\mathrm{e}^{-6x}}{8}$$

✓ Solution by Mathematica

Time used: 0.077 (sec). Leaf size: 68

```
DSolve[y''[x]+4*y'[x]-12*y[x]==F[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-6x}\left(\int_1^x -\frac{1}{8}e^{6K[1]}F(K[1])dK[1] + e^{8x}\int_1^x \frac{1}{8}e^{-2K[2]}F(K[2])dK[2] + c_2e^{8x} + c_1\right)$$

9.27 problem Problem 27

Internal problem ID [2800]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 4y' + 4y = 5x\,e^{2x}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=5*x*exp(2*x),y(0) = 1, D(y)(0) = 0],y(x), sings
```

$$y(x) = \frac{e^{2x}(5x^3 - 12x + 6)}{6}$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 24

```
DSolve[{y''[x]-4*y'[x]+4*y[x]==5*x*Exp[2*x],{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{1}{6}e^{2x}\left(5x^3 - 12x + 6\right)$$

9.28 problem Problem 28

Internal problem ID [2801]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sec(x)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$2)+y(x)=sec(x),y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = \sin(x) + x\sin(x) - \cos(x)\ln(\sec(x))$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 24

```
DSolve[{y''[x]-4*y'[x]+4*y[x]==5*x*Exp[2*x],{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{1}{6}e^{2x}(5x^3 - 12x + 6)$$

10 Chapter 8, Linear differential equations of order n. Section 8.8, A Differential Equation with Nonconstant Coefficients. page 567

10.1 problem Problem 14

Internal problem ID [2802]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.8, A Differential Equation with Nonconstant Coefficients. page 567
Problem number: Problem 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + 4xy' + 2y = 4\ln(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+2*y(x)=4*ln(x),y(x), singsol=all)
```

$$y(x) = 2\ln(x) + \frac{c_1}{x} - 3 + \frac{c_2}{x^2}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 23

```
DSolve[x^2*y''[x]+4*x*y'[x]+2*y[x]==4*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x^2} + 2\log(x) + \frac{c_2}{x} - 3$$

10.2 problem Problem 15

Internal problem ID [2803]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.8, A Differential Equation with Nonconstant Coefficients. page 567
Problem number: Problem 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + 4xy' + 2y = \cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+2*y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = \frac{c_2 + c_1 x - \cos(x)}{x^2}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 20

```
DSolve[x^2*y''[x]+4*x*y'[x]+2*y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-\cos(x) + c_2 x + c_1}{x^2}$$

10.3 problem Problem 16

Internal problem ID [2804]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.8, A Differential Equation with Nonconstant Coefficients. page 567
Problem number: Problem 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + xy' + 9y = 9\ln(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+9*y(x)=9*ln(x),y(x), singsol=all)
```

$$y(x) = \sin(3\ln(x))\,c_2 + \cos(3\ln(x))\,c_1 + \ln(x)$$

✓ Solution by Mathematica
Time used: 0.155 (sec). Leaf size: 24

```
DSolve[x^2*y''[x]+x*y'[x]+9*y[x]==9*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(x) + c_1\cos(3\log(x)) + c_2\sin(3\log(x))$$

10.4 problem Problem 17

Internal problem ID [2805]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.8, A Differential Equation with Nonconstant Coefficients. page 567
Problem number: Problem 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' - xy' + 5y = 8x\ln(x)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)+5*y(x)=8*x*(ln(x))^2,y(x), singsol=all)
```

$$y(x) = x\left(-1 + \sin(2\ln(x))\, c_2 + \cos(2\ln(x))\, c_1 + 2\ln(x)^2\right)$$

✓ Solution by Mathematica
Time used: 0.154 (sec). Leaf size: 31

```
DSolve[x^2*y''[x]-x*y'[x]+5*y[x]==8*x*(Log[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(2\log^2(x) + c_2\cos(2\log(x)) + c_1\sin(2\log(x)) - 1\right)$$

10.5 problem Problem 18

Internal problem ID [2806]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.8, A Differential Equation with Nonconstant Coefficients. page 567
Problem number: Problem 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' - 4xy' + 6y = x^4 \sin(x)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=x^4*sin(x),y(x), singsol=all)
```

$$y(x) = x^2(c_2x - \sin(x) + c_1)$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 20

```
DSolve[x^2*y''[x]-4*x*y'[x]+6*y[x]==x^4*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(-\sin(x) + c_2x + c_1)$$

10.6 problem Problem 19

Internal problem ID [2807]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.8, A Differential Equation with Nonconstant Coefficients. page 567
Problem number: Problem 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + 6xy' + 6y = 4\,\mathrm{e}^{2x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)+6*x*diff(y(x),x)+6*y(x)=4*exp(2*x),y(x), singsol=all)
```

$$y(x) = \frac{(x-1)\,\mathrm{e}^{2x} + c_2x - c_1}{x^3}$$

✓ Solution by Mathematica
Time used: 0.056 (sec). Leaf size: 25

```
DSolve[x^2*y''[x]+6*x*y'[x]+6*y[x]==4*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{2x}(x-1) + c_2x + c_1}{x^3}$$

10.7 problem Problem 20

Internal problem ID [2808]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.8, A Differential Equation with Nonconstant Coefficients. page 567
Problem number: Problem 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' - 3xy' + 4y = \frac{x^2}{\ln(x)}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*y(x)=x^2/ln(x),y(x), singsol=all)
```

$$y(x) = x^2(\ln(x)\ln(\ln(x)) + (c_1 - 1)\ln(x) + c_2)$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 24

```
DSolve[x^2*y''[x]-3*x*y'[x]+4*y[x]==x^2/Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(\log(x)(\log(\log(x)) - 1 + 2c_2) + c_1)$$

10.8 problem Problem 21

Internal problem ID [2809]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.8, A Differential Equation with Nonconstant Coefficients. page 567
Problem number: Problem 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x^2y'' - (2m-1)\,xy' + m^2y = x^m \ln(x)^k$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 33

```
dsolve(x^2*diff(y(x),x$2)-(2*m-1)*x*diff(y(x),x)+m^2*y(x)=x^m*(ln(x))^k,y(x), singsol=all)
```

$$y(x) = x^m\left(c_2 + \ln(x)\,c_1 + \frac{\ln(x)^2\ln(x)^k}{k^2+3k+2}\right)$$

✓ Solution by Mathematica
Time used: 0.064 (sec). Leaf size: 35

```
DSolve[x^2*y''[x]-(2*m-1)*x*y'[x]+m^2*y[x]==x^m*(Log[x])^k,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to x^m\left(\frac{\log^{k+2}(x)}{k^2+3k+2} + c_2 m\log(x) + c_1\right)$$

10.9 problem Problem 22

Internal problem ID [2810]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.8, A Differential Equation with Nonconstant Coefficients. page 567
Problem number: Problem 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - xy' + 5y = 0}$$

With initial conditions

$$\left[y(1) = \sqrt{2}, y'(1) = 3\sqrt{2}\right]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve([x^2*diff(y(x),x$2)-x*diff(y(x),x)+5*y(x)=0,y(1) = 2^(1/2), D(y)(1) = 3*2^(1/2)],y(x)
```

$$y(x) = \sqrt{2}\, x(\sin(2\ln(x)) + \cos(2\ln(x)))$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 23

```
DSolve[{x^2*y''[x]-x*y'[x]+5*y[x]==0,{y[1]==Sqrt[2],y'[1]==3*Sqrt[2]}},y[x],x,IncludeSingula
```

$$y(x) \to \sqrt{2}x(\sin(2\log(x)) + \cos(2\log(x)))$$

10.10 problem Problem 23

Internal problem ID [2811]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.8, A Differential Equation with Nonconstant Coefficients. page 567
Problem number: Problem 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{t^2y'' + y't + 25y = 0}$$

With initial conditions

$$\left[y(1) = \frac{3\sqrt{3}}{2}, y'(1) = \frac{15}{2}\right]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 22

```
dsolve([t^2*diff(y(t),t$2)+t*diff(y(t),t)+25*y(t)=0,y(1) = 3/2*3^(1/2), D(y)(1) = 15/2],y(t)
```

$$y(t) = \frac{3\sin(5\ln(t))}{2} + \frac{3\sqrt{3}\,\cos(5\ln(t))}{2}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 26

```
DSolve[{t^2*y''[t]+t*y'[t]+25*y[t]==0,{y[1]==3*Sqrt[3]/2,y'[1]==15/2}},y[t],t,IncludeSingula
```

$$y(t) \to \frac{3}{2}\left(\sin(5\log(t)) + \sqrt{3}\cos(5\log(t))\right)$$

11 Chapter 8, Linear differential equations of order n. Section 8.9, Reduction of Order. page 572

11.1 problem Problem 1

Internal problem ID [2812]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.9, Reduction of Order. page 572
Problem number: Problem 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' - 3xy' + 4y = 0}$$

Given that one solution of the ode is

$$y_1 = x^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve([x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*y(x)=0,x^2],singsol=all)
```

$$y(x) = x^2(c_2 \ln(x) + c_1)$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]-3*x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(2c_2 \log(x) + c_1)$$

11.2 problem Problem 2

Internal problem ID [2813]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.9, Reduction of Order. page 572
Problem number: Problem 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (1-2x)\,y' + y(x-1) = 0}$$

Given that one solution of the ode is

$$y_1 = e^x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve([x*diff(y(x),x$2)+(1-2*x)*diff(y(x),x)+(x-1)*y(x)=0,exp(x)],singsol=all)
```

$$y(x) = e^x(c_2 \ln(x) + c_1)$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 17

```
DSolve[x*y''[x]+(1-2*x)*y'[x]+(x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x(c_2 \log(x) + c_1)$$

11.3 problem Problem 3

Internal problem ID [2814]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.9, Reduction of Order. page 572
Problem number: Problem 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2xy' + (x^2 + 2)\, y = 0}$$

Given that one solution of the ode is

$$y_1 = x\sin(x)$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=0,x*sin(x)],singsol=all)
```

$$y(x) = x(c_1 \sin(x) + c_2 \cos(x))$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-ix} x - \frac{1}{2} i c_2 e^{ix} x$$

11.4 problem Problem 4

Internal problem ID [2815]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.9, Reduction of Order. page 572
Problem number: Problem 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-x^2+1)\,y''-2xy'+2y=0}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve([(1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,x],singsol=all)
```

$$y(x) = -\frac{c_2 \ln(x+1)\,x}{2} + \frac{c_2 \ln(x-1)\,x}{2} + c_1 x + c_2$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 33

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x - \frac{1}{2}c_2(x\log(1-x) - x\log(x+1) + 2)$$

11.5 problem Problem 5

Internal problem ID [2816]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.9, Reduction of Order. page 572
Problem number: Problem 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{y'' - \frac{y'}{x} + 4x^2y = 0}$$

Given that one solution of the ode is

$$y_1 = \sin\left(x^2\right)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)-1/x*diff(y(x),x)+4*x^2*y(x)=0,sin(x^2)],singsol=all)
```

$$y(x) = c_1 \sin\left(x^2\right) + c_2 \cos\left(x^2\right)$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 20

```
DSolve[y''[x]-1/x*y'[x]+4*x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos\left(x^2\right) + c_2 \sin\left(x^2\right)$$

11.6 problem Problem 6

Internal problem ID [2817]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.9, Reduction of Order. page 572
Problem number: Problem 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 4xy' + (4x^2 - 1)\, y = 0}$$

Given that one solution of the ode is

$$y_1 = \frac{\sin(x)}{\sqrt{x}}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 17

```
dsolve([4*x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2-1)*y(x)=0,sin(x)/x^(1/2)],singsol=all)
```

$$y(x) = \frac{c_1 \sin(x) + c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 39

```
DSolve[4*x^2*y''[x]+4*x*y'[x]+(4*x^2-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2e^{2ix})}{2\sqrt{x}}$$

11.7 problem Problem 10

Internal problem ID [2818]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.9, Reduction of Order. page 572
Problem number: Problem 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \csc(x)}$$

Given that one solution of the ode is

$$y_1 = \sin(x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve([diff(y(x),x$2)+y(x)=csc(x),sin(x)],singsol=all)
```

$$y(x) = -\ln(\csc(x))\sin(x) + (c_1 - x)\cos(x) + \sin(x)\, c_2$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 24

```
DSolve[y''[x]+y[x]==Csc[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (-x + c_1)\cos(x) + \sin(x)(\log(\sin(x)) + c_2)$$

11.8 problem Problem 11

Internal problem ID [2819]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.9, Reduction of Order. page 572
Problem number: Problem 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (1+2x)\,y' + 2y = 8x^2 \mathrm{e}^{2x}}$$

Given that one solution of the ode is

$$y_1 = \mathrm{e}^{2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve([x*diff(y(x),x$2)-(2*x+1)*diff(y(x),x)+2*y(x)=8*x^2*exp(2*x),exp(2*x)],singsol=all)
```

$$y(x) = 2\,\mathrm{e}^{2x}x^2 + c_1\mathrm{e}^{2x} + 2c_2x + c_2$$

✓ Solution by Mathematica
Time used: 0.044 (sec). Leaf size: 32

```
DSolve[x*y''[x]-(2*x+1)*y'[x]+2*y[x]==8*x^2*Exp[2*x],y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to e^{2x}\left(2x^2 - 1 + c_1\right) - \frac{1}{4}c_2(2x+1)$$

11.9 problem Problem 12

Internal problem ID [2820]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.9, Reduction of Order. page 572
Problem number: Problem 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 3xy' + 4y = 8x^4}$$

Given that one solution of the ode is

$$y_1 = x^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve([x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*y(x)=8*x^4,x^2],singsol=all)
```

$$y(x) = x^2(\ln(x)\,c_1 + 2x^2 + c_2)$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 23

```
DSolve[x^2*y''[x]-3*x*y'[x]+4*y[x]==8*x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(2x^2 + 2c_2\log(x) + c_1)$$

11.10 problem Problem 13

Internal problem ID [2821]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.9, Reduction of Order. page 572
Problem number: Problem 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 6y' + 9y = 15\,\mathrm{e}^{3x}\sqrt{x}}$$

Given that one solution of the ode is

$$y_1 = \mathrm{e}^{3x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$2)-6*diff(y(x),x)+9*y(x)=15*exp(3*x)*sqrt(x),exp(3*x)],singsol=all)
```

$$y(x) = \mathrm{e}^{3x}\left(c_2 + c_1 x + 4x^{\frac{5}{2}}\right)$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 25

```
DSolve[y''[x]-6*y'[x]+9*y[x]==15*Exp[3*x]*Sqrt[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{3x}\left(4x^{5/2} + c_2 x + c_1\right)$$

11.11 problem Problem 14

Internal problem ID [2822]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.9, Reduction of Order. page 572
Problem number: Problem 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 4y = 4\,\mathrm{e}^{2x}\ln(x)}$$

Given that one solution of the ode is

$$y_1 = \mathrm{e}^{2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve([diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=4*exp(2*x)*ln(x),exp(2*x)],singsol=all)
```

$$y(x) = \mathrm{e}^{2x}\left(2\ln(x)\,x^2 + c_1 x - 3x^2 + c_2\right)$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 30

```
DSolve[y''[x]-4*y'[x]+4*y[x]==4*Exp[2*x]*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}\left(-3x^2 + 2x^2\log(x) + c_2 x + c_1\right)$$

11.12 problem Problem 15

Internal problem ID [2823]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.9, Reduction of Order. page 572
Problem number: Problem 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{4x^2y'' + y = \sqrt{x}\, \ln(x)}$$

Given that one solution of the ode is

$$y_1 = \sqrt{x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve([4*x^2*diff(y(x),x$2)+y(x)=sqrt(x)*ln(x),sqrt(x)],singsol=all)
```

$$y(x) = \left(c_2 + \ln(x)\, c_1 + \frac{\ln(x)^3}{24}\right)\sqrt{x}$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 29

```
DSolve[4*x^2*y''[x]+y[x]==Sqrt[x]*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{24}\sqrt{x}\left(\log^3(x) + 12c_2\log(x) + 24c_1\right)$$

12 Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575

12.1 problem Problem 7

Internal problem ID [2824]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 7.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 3y'' - 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$3)+3*diff(y(x),x$2)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_1 e^{3x} + c_3 x + c_2\right) e^{-2x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 26

```
DSolve[y'''[x]+3*y''[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-2x}\left(c_2 x + c_3 e^{3x} + c_1\right)$$

12.2 problem Problem 8

Internal problem ID [2825]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 8.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 11y'' + 36y' + 26y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$3)+11*diff(y(x),x$2)+36*diff(y(x),x)+26*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-x}c_1 + c_2e^{-5x}\sin(x) + c_3e^{-5x}\cos(x)$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 30

```
DSolve[y'''[x]+11*y''[x]+36*y'[x]+26*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-5x}\left(c_3e^{4x} + c_2\cos(x) + c_1\sin(x)\right)$$

12.3 problem Problem 18

Internal problem ID [2826]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 6y' + 9y = 4\,\mathrm{e}^{-3x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)+6*diff(y(x),x)+9*y(x)=4*exp(-3*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-3x}\left(c_1 x + 2x^2 + c_2\right)$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 23

```
DSolve[y''[x]+6*y'[x]+9*y[x]==4*Exp[-3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(2x^2 + c_2 x + c_1\right)$$

12.4 problem Problem 19

Internal problem ID [2827]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 6y' + 9y = 4\,\mathrm{e}^{-2x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+6*diff(y(x),x)+9*y(x)=4*exp(-2*x),y(x), singsol=all)
```

$$y(x) = (c_1 x + c_2)\,\mathrm{e}^{-3x} + 4\,\mathrm{e}^{-2x}$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 23

```
DSolve[y''[x]+6*y'[x]+9*y[x]==4*Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}(4e^x + c_2 x + c_1)$$

12.5 problem Problem 20

Internal problem ID [2828]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 20.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - 6y'' + 25y' = x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve(diff(y(x),x$3)-6*diff(y(x),x$2)+25*diff(y(x),x)=x^2,y(x), singsol=all)
```

$$y(x) = \frac{\left((3c_1 - 4c_2)\cos(4x) + 4\sin(4x)\left(c_1 + \frac{3c_2}{4}\right)\right)e^{3x}}{25} + \frac{x^3}{75} + \frac{6x^2}{625} + \frac{22x}{15625} + c_3$$

✓ Solution by Mathematica

Time used: 0.272 (sec). Leaf size: 71

```
DSolve[y'''[x]-6*y''[x]+25*y'[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{75} + \frac{6x^2}{625} + \frac{22x}{15625} - \frac{1}{25}(4c_1 - 3c_2)e^{3x}\cos(4x) + \frac{1}{25}(3c_1 + 4c_2)e^{3x}\sin(4x) + c_3$$

12.6 problem Problem 21

Internal problem ID [2829]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 21.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$y''' - 6y'' + 25y' = \sin(4x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 48

```
dsolve(diff(y(x),x$3)-6*diff(y(x),x$2)+25*diff(y(x),x)=sin(4*x),y(x), singsol=all)
```

$$y(x) = \frac{\left((3c_1 - 4c_2)\cos(4x) + 4\sin(4x)\left(c_1 + \frac{3c_2}{4}\right)\right)e^{3x}}{25} + c_3 - \frac{\cos(4x)}{292} + \frac{2\sin(4x)}{219}$$

✓ Solution by Mathematica
Time used: 0.686 (sec). Leaf size: 60

```
DSolve[y'''[x]-6*y''[x]+25*y'[x]==Sin[4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{(25 + 292(4c_1 - 3c_2)e^{3x})\cos(4x)}{7300} + \frac{(50 + 219(3c_1 + 4c_2)e^{3x})\sin(4x)}{5475} + c_3$$

12.7 problem Problem 22

Internal problem ID [2830]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 22.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' + 9y'' + 24y' + 16y = 8\,\mathrm{e}^{-x} + 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$3)+9*diff(y(x),x$2)+24*diff(y(x),x)+16*y(x)=8*exp(-x)+1,y(x), singsol=all
```

$$y(x) = \frac{1}{16} + \frac{(-16 + 24x + 27c_2)\,\mathrm{e}^{-x}}{27} + (c_3 x + c_1)\,\mathrm{e}^{-4x}$$

✓ Solution by Mathematica
Time used: 0.076 (sec). Leaf size: 39

```
DSolve[y'''[x]+9*y''[x]+24*y'[x]+16*y[x]==8*Exp[-x]+1,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^{-4x}(c_2 x + c_1) + e^{-x}\left(\frac{8x}{9} - \frac{16}{27} + c_3\right) + \frac{1}{16}$$

12.8 problem Problem 27

Internal problem ID [2831]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y = 5\,\mathrm{e}^{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-4*y(x)=5*exp(x),y(x), singsol=all)
```

$$y(x) = -\frac{\left(-3\,\mathrm{e}^{4x}c_1 + 5\,\mathrm{e}^{3x} - 3c_2\right)\mathrm{e}^{-2x}}{3}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 29

```
DSolve[y''[x]-4*y[x]==5*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{5e^{x}}{3} + c_1e^{2x} + c_2e^{-2x}$$

12.9 problem Problem 28

Internal problem ID [2832]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + y = 2x\,\mathrm{e}^{-x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=2*x*exp(-x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}\left(c_2 + c_1 x + \frac{1}{3}x^3\right)$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 27

```
DSolve[y''[x]+2*y'[x]+y[x]==2*x*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}e^{-x}(x^3 + 3c_2 x + 3c_1)$$

12.10 problem Problem 29

Internal problem ID [2833]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y = 4\,\mathrm{e}^{x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)-y(x)=4*exp(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} c_2 + 2\,\mathrm{e}^{x}\left(x + \frac{c_1}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 25

```
DSolve[y''[x]-y[x]==4*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(2x - 1 + c_1) + c_2 e^{-x}$$

12.11 problem Problem 30

Internal problem ID [2834]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + yx = \sin(x)}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 48

```
dsolve(diff(y(x),x$2)+x*y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = \pi\left(\int \operatorname{AiryBi}(-x)\sin(x)\,dx\right)\operatorname{AiryAi}(-x) - \pi\left(\int \operatorname{AiryAi}(-x)\sin(x)\,dx\right)\operatorname{AiryBi}(-x) + \operatorname{AiryBi}(-x)\,c_1 + \operatorname{AiryAi}(-x)\,c_2$$

✓ Solution by Mathematica
Time used: 105.448 (sec). Leaf size: 99

```
DSolve[y''[x]+x*y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \operatorname{AiryAi}\left(\sqrt[3]{-1}x\right)\int_1^x (-1)^{2/3}\pi\operatorname{AiryBi}\left(\sqrt[3]{-1}K[1]\right)\sin(K[1])dK[1] + \operatorname{AiryBi}\left(\sqrt[3]{-1}x\right)\int_1^x -(-1)^{2/3}\pi\operatorname{AiryAi}\left(\sqrt[3]{-1}K[2]\right)\sin(K[2])dK[2] + c_1\operatorname{AiryAi}\left(\sqrt[3]{-1}x\right) + c_2\operatorname{AiryBi}\left(\sqrt[3]{-1}x\right)$$

12.12 problem Problem 31

Internal problem ID [2835]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \ln(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 62

```
dsolve(diff(y(x),x$2)+4*y(x)=ln(x),y(x), singsol=all)
```

$$y(x) = \frac{i\cos(2x)\,\pi(-1+\operatorname{csgn}(x))\operatorname{csgn}(ix)}{8} + \frac{(8c_1 - 2\operatorname{Ci}(2x))\cos(2x)}{8} + \frac{(\pi\operatorname{csgn}(x) + 8c_2 - 2\operatorname{Si}(2x))\sin(2x)}{8} + \frac{\ln(x)}{4}$$

✓ Solution by Mathematica
Time used: 0.056 (sec). Leaf size: 48

```
DSolve[y''[x]+4*y[x]==Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}(-\operatorname{CosIntegral}(2x)\cos(2x) - \operatorname{Si}(2x)\sin(2x) + \log(x) + 4c_1\cos(2x) + 4c_2\sin(2x))$$

12.13 problem Problem 32

Internal problem ID [2836]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y' - 3y = 5\,\mathrm{e}^{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)-3*y(x)=5*exp(x),y(x), singsol=all)
```

$$y(x) = \frac{(5x + 4c_1)\,\mathrm{e}^{-3x}\mathrm{e}^{4x}}{4} + \mathrm{e}^{-3x}c_2$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 29

```
DSolve[y''[x]+2*y'[x]-3*y[x]==5*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-3x} + e^{x}\left(\frac{5x}{4} - \frac{5}{16} + c_2\right)$$

12.14 problem Problem 33

Internal problem ID [2837]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \tan(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+y(x)=tan(x),y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 - \cos(x) \ln(\sec(x) + \tan(x))$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 23

```
DSolve[y''[x]+y[x]==Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x)(-\text{arctanh}(\sin(x))) + c_1 \cos(x) + c_2 \sin(x)$$

12.15 problem Problem 34

Internal problem ID [2838]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = 4\cos(2x) + 3\,\mathrm{e}^{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+y(x)=4*cos(2*x)+3*exp(x),y(x), singsol=all)
```

$$y(x) = \sin(x)\,c_2 + \cos(x)\,c_1 - \frac{4\cos(2x)}{3} + \frac{3\,\mathrm{e}^{x}}{2}$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 30

```
DSolve[y''[x]+y[x]==4*Cos[x]*3*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{12}{5}e^{x}(2\sin(x) + \cos(x)) + c_1\cos(x) + c_2\sin(x)$$

13 Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689

13.1 problem Problem 1

Internal problem ID [2839]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - 2y = 6\,\mathrm{e}^{5t}}$$

With initial conditions

$$[y(0) = 3]$$

✓ Solution by Maple
Time used: 2.891 (sec). Leaf size: 15

```
dsolve([diff(y(t),t)-2*y(t)=6*exp(5*t),y(0) = 3],y(t), singsol=all)
```

$$y(t) = 2\,\mathrm{e}^{5t} + \mathrm{e}^{2t}$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 18

```
DSolve[{y'[t]-2*y[t]==6*Exp[5*t],{y[0]==3}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{2t} + 2e^{5t}$$

13.2 problem Problem 2

Internal problem ID [2840]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + y = 8\,\mathrm{e}^{3t}}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple
Time used: 2.703 (sec). Leaf size: 10

```
dsolve([diff(y(t),t)+y(t)=8*exp(3*t),y(0) = 2],y(t), singsol=all)
```

$$y(t) = 2\,\mathrm{e}^{3t}$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 12

```
DSolve[{y'[t]+y[t]==8*Exp[3*t],{y[0]==2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 2e^{3t}$$

13.3 problem Problem 3

Internal problem ID [2841]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + 3y = 2\,\mathrm{e}^{-t}}$$

With initial conditions

$$[y(0) = 3]$$

✓ Solution by Maple
Time used: 2.937 (sec). Leaf size: 15

```
dsolve([diff(y(t),t)+3*y(t)=2*exp(-t),y(0) = 3],y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-t} + 2\,\mathrm{e}^{-3t}$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 18

```
DSolve[{y'[t]+3*y[t]==2*Exp[-t],{y[0]==3}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-3t}\left(e^{2t} + 2\right)$$

13.4 problem Problem 4

Internal problem ID [2842]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + 2y = 4t}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 1.703 (sec). Leaf size: 15

```
dsolve([diff(y(t),t)+2*y(t)=4*t,y(0) = 1],y(t), singsol=all)
```

$$y(t) = 2t + 2\,\mathrm{e}^{-2t} - 1$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 17

```
DSolve[{y'[t]+2*y[t]==4*t,{y[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 2t + 2e^{-2t} - 1$$

13.5 problem Problem 5

Internal problem ID [2843]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - y = 6\cos(t)}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple
Time used: 1.89 (sec). Leaf size: 17

```
dsolve([diff(y(t),t)-y(t)=6*cos(t),y(0) = 2],y(t), singsol=all)
```

$$y(t) = 5\,\mathrm{e}^{t} - 3\cos(t) + 3\sin(t)$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 19

```
DSolve[{y'[t]-y[t]==6*Cos[t],{y[0]==2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 5e^{t} + 3\sin(t) - 3\cos(t)$$

13.6 problem Problem 6

Internal problem ID [2844]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - y = 5\sin(2t)}$$

With initial conditions

$$[y(0) = -1]$$

✓ Solution by Maple
Time used: 2.984 (sec). Leaf size: 19

```
dsolve([diff(y(t),t)-y(t)=5*sin(2*t),y(0) = -1],y(t), singsol=all)
```

$$y(t) = -2\cos(2t) - \sin(2t) + e^t$$

✓ Solution by Mathematica
Time used: 0.09 (sec). Leaf size: 21

```
DSolve[{y'[t]-y[t]==5*Sin[2*t],{y[0]==-1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^t - \sin(2t) - 2\cos(2t)$$

13.7 problem Problem 7

Internal problem ID [2845]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + y = 5\,e^{t}\sin(t)}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 3.125 (sec). Leaf size: 23

```
dsolve([diff(y(t),t)+y(t)=5*exp(t)*sin(t),y(0) = 1],y(t), singsol=all)
```

$$y(t) = e^{t}(2\sin(t) - \cos(t)) + 2\,e^{-t}$$

✓ Solution by Mathematica
Time used: 0.073 (sec). Leaf size: 27

```
DSolve[{y'[t]+y[t]==5*Exp[t]*Sin[t],{y[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 2e^{-t} + 2e^{t}\sin(t) - e^{t}\cos(t)$$

13.8 problem Problem 8

Internal problem ID [2846]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - 2y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 4]$$

✓ Solution by Maple
Time used: 1.75 (sec). Leaf size: 15

```
dsolve([diff(y(t),t$2)+diff(y(t),t)-2*y(t)=0,y(0) = 1, D(y)(0) = 4],y(t), singsol=all)
```

$$y(t) = \left(2\,\mathrm{e}^{3t} - 1\right)\mathrm{e}^{-2t}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 18

```
DSolve[{y''[t]+y'[t]-2*y[t]==0,{y[0]==1,y'[0]==4}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 2e^t - e^{-2t}$$

13.9 problem Problem 9

Internal problem ID [2847]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y = 0}$$

With initial conditions

$$[y(0) = 5, y'(0) = 1]$$

✓ Solution by Maple
Time used: 1.938 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)+4*y(t)=0,y(0) = 5, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = 5\cos(2t) + \frac{\sin(2t)}{2}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 17

```
DSolve[{y''[t]+4*y[t]==0,{y[0]==5,y'[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 5\cos(2t) + \sin(t)\cos(t)$$

13.10 problem Problem 10

Internal problem ID [2848]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 3y' + 2y = 4}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 1.703 (sec). Leaf size: 16

```
dsolve([diff(y(t),t$2)-3*diff(y(t),t)+2*y(t)=4,y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = 3\,\mathrm{e}^{2t} - 5\,\mathrm{e}^{t} + 2$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 19

```
DSolve[{y''[t]-3*y'[t]+2*y[t]==4,{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to -5e^t + 3e^{2t} + 2$$

13.11 problem Problem 11

Internal problem ID [2849]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' - 12y = 36}$$

With initial conditions

$$[y(0) = 0, y'(0) = 12]$$

✓ Solution by Maple
Time used: 1.813 (sec). Leaf size: 12

```
dsolve([diff(y(t),t$2)-diff(y(t),t)-12*y(t)=36,y(0) = 0, D(y)(0) = 12],y(t), singsol=all)
```

$$y(t) = 3\,\mathrm{e}^{4t} - 3$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 14

```
DSolve[{y''[t]-y'[t]-12*y[t]==36,{y[0]==0,y'[0]==12}},y[t],t,IncludeSingularSolutions -> Tru
```

$$y(t) \to 3\left(e^{4t} - 1\right)$$

13.12 problem Problem 12

Internal problem ID [2850]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' - 2y = 10\,\mathrm{e}^{-t}}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 2.094 (sec). Leaf size: 19

```
dsolve([diff(y(t),t$2)+diff(y(t),t)-2*y(t)=10*exp(-t),y(0) = 0, D(y)(0) = 1],y(t), singsol=a
```

$$y(t) = -3\cosh(t) + 7\sinh(t) + 3\,\mathrm{e}^{-2t}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 25

```
DSolve[{y''[t]+y'[t]-2*y[t]==10*Exp[-t],{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions
```

$$y(t) \to e^{-2t}\left(-5e^t + 2e^{3t} + 3\right)$$

13.13 problem Problem 13

Internal problem ID [2851]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 3y' + 2y = 4\,\mathrm{e}^{3t}}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 2.203 (sec). Leaf size: 21

```
dsolve([diff(y(t),t$2)-3*diff(y(t),t)+2*y(t)=4*exp(3*t),y(0) = 0, D(y)(0) = 0],y(t), singsol
```

$$y(t) = 2\,\mathrm{e}^{3t} - 4\,\mathrm{e}^{2t} + 2\,\mathrm{e}^{t}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 17

```
DSolve[{y''[t]-3*y'[t]+2*y[t]==4*Exp[3*t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolution
```

$$y(t) \to 2e^t(e^t - 1)^2$$

13.14 problem Problem 14

Internal problem ID [2852]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - 2y' = 30\,e^{-3t}}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 2.781 (sec). Leaf size: 18

```
dsolve([diff(y(t),t$2)-2*diff(y(t),t)=30*exp(-3*t),y(0) = 1, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \left(3\,e^{5t} - 4\,e^{3t} + 2\right)e^{-3t}$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 21

```
DSolve[{y''[t]-2*y'[t]==30*Exp[-3*t],{y[0]==1,y'[0]==0}},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \rightarrow 2e^{-3t} + 3e^{2t} - 4$$

13.15 problem Problem 15

Internal problem ID [2853]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y = 12\,\mathrm{e}^{2t}}$$

With initial conditions

$$[y(0) = 1, y'(0) = 1]$$

✓ Solution by Maple
Time used: 2.953 (sec). Leaf size: 19

```
dsolve([diff(y(t),t$2)-y(t)=12*exp(2*t),y(0) = 1, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = -3\cosh(t) - 7\sinh(t) + 4\,\mathrm{e}^{2t}$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 25

```
DSolve[{y''[t]-y[t]==12*Exp[2*t],{y[0]==1,y'[0]==1}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to 2e^{-t} - 5e^{t} + 4e^{2t}$$

13.16 problem Problem 16

Internal problem ID [2854]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y = 10\,e^{-t}}$$

With initial conditions

$$[y(0) = 4, y'(0) = 0]$$

✓ Solution by Maple
Time used: 2.25 (sec). Leaf size: 21

```
dsolve([diff(y(t),t$2)+4*y(t)=10*exp(-t),y(0) = 4, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = 2\,e^{-t} + 2\cos(2t) + \sin(2t)$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 23

```
DSolve[{y''[t]+4*y[t]==10*Exp[-t],{y[0]==4,y'[0]==0}},y[t],t,IncludeSingularSolutions -> Tru
```

$$y(t) \to 2e^{-t} + \sin(2t) + 2\cos(2t)$$

13.17 problem Problem 17

Internal problem ID [2855]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y' - 6y = 12 - 6\,\mathrm{e}^{t}}$$

With initial conditions

$$[y(0) = 5, y'(0) = -3]$$

✓ Solution by Maple

Time used: 1.859 (sec). Leaf size: 20

```
dsolve([diff(y(t),t$2)-diff(y(t),t)-6*y(t)=6*(2-exp(t)),y(0) = 5, D(y)(0) = -3],y(t), singsol
```

$$y(t) = \frac{(8\,\mathrm{e}^{5t} + 5\,\mathrm{e}^{3t} - 10\,\mathrm{e}^{2t} + 22)\,\mathrm{e}^{-2t}}{5}$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 28

```
DSolve[{y''[t]-y'[t]-6*y[t]==6*(2-Exp[t]),{y[0]==5,y'[0]==-3}},y[t],t,IncludeSingularSolutio
```

$$y(t) \to \frac{22e^{-2t}}{5} + e^{t} + \frac{8e^{3t}}{5} - 2$$

13.18 problem Problem 18

Internal problem ID [2856]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y = 6\cos(t)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 4]$$

✓ Solution by Maple
Time used: 1.859 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)-y(t)=6*cos(t),y(0) = 0, D(y)(0) = 4],y(t), singsol=all)
```

$$y(t) = 4\sinh(t) - 3\cos(t) + 3\cosh(t)$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 26

```
DSolve[{y''[t]-y[t]==6*Cos[t],{y[0]==0,y'[0]==4}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}\left(-e^{-t} + 7e^t - 6\cos(t)\right)$$

13.19 problem Problem 19

Internal problem ID [2857]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 9y = 13\sin(2t)}$$

With initial conditions

$$[y(0) = 3, y'(0) = 1]$$

✓ Solution by Maple
Time used: 2.75 (sec). Leaf size: 21

```
dsolve([diff(y(t),t$2)-9*y(t)=13*sin(2*t),y(0) = 3, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = -\sin(2t) + 2\,\mathrm{e}^{3t} + \mathrm{e}^{-3t}$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 24

```
DSolve[{y''[t]-9*y[t]==13*Sin[2*t],{y[0]==3,y'[0]==1}},y[t],t,IncludeSingularSolutions -> Tr
```

$$y(t) \to e^{-3t} + 2e^{3t} - \sin(2t)$$

13.20 problem Problem 20

Internal problem ID [2858]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y = 8\sin(t) - 6\cos(t)}$$

With initial conditions

$$[y(0) = 2, y'(0) = -1]$$

✓ Solution by Maple
Time used: 1.953 (sec). Leaf size: 21

```
dsolve([diff(y(t),t$2)-y(t)=8*sin(t)-6*cos(t),y(0) = 2, D(y)(0) = -1],y(t), singsol=all)
```

$$y(t) = -4\sin(t) + 3\cos(t) + 3\sinh(t) - \cosh(t)$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 24

```
DSolve[{y''[t]-y[t]==8*Sin[t]-6*Cos[t],{y[0]==2,y'[0]==-1}},y[t],t,IncludeSingularSolutions
```

$$y(t) \to -2e^{-t} + e^t - 4\sin(t) + 3\cos(t)$$

13.21 problem Problem 21

Internal problem ID [2859]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y' - 2y = 10\cos(t)}$$

With initial conditions

$$[y(0) = 0, y'(0) = -1]$$

✓ Solution by Maple
Time used: 1.891 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$2)-diff(y(t),t)-2*y(t)=10*cos(t),y(0) = 0, D(y)(0) = -1],y(t), singsol=a
```

$$y(t) = 2\,\mathrm{e}^{-t} + \mathrm{e}^{2t} - 3\cos(t) - \sin(t)$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 26

```
DSolve[{y''[t]-y'[t]-2*y[t]==10*Cos[t],{y[0]==0,y'[0]==-1}},y[t],t,IncludeSingularSolutions
```

$$y(t) \to 2e^{-t} + e^{2t} - \sin(t) - 3\cos(t)$$

13.22 problem Problem 22

Internal problem ID [2860]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 5y' + 4y = 20\sin(2t)}$$

With initial conditions

$$[y(0) = -1, y'(0) = 2]$$

✓ Solution by Maple
Time used: 2.875 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$2)+5*diff(y(t),t)+4*y(t)=20*sin(2*t),y(0) = -1, D(y)(0) = 2],y(t), sings
```

$$y(t) = 2\,e^{-t} - e^{-4t} - 2\cos(2t)$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 27

```
DSolve[{y''[t]+5*y'[t]+4*y[t]==20*Sin[2*t],{y[0]==-1,y'[0]==2}},y[t],t,IncludeSingularSoluti
```

$$y(t) \rightarrow e^{-4t}(2e^{3t} - 1) - 2\cos(2t)$$

13.23 problem Problem 23

Internal problem ID [2861]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 5y' + 4y = 20 \sin(2t)}$$

With initial conditions

$$[y(0) = 1, y'(0) = -2]$$

✓ Solution by Maple

Time used: 1.562 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$2)+5*diff(y(t),t)+4*y(t)=20*sin(2*t),y(0) = 1, D(y)(0) = -2],y(t), sings
```

$$y(t) = \frac{10\,\mathrm{e}^{-t}}{3} - \frac{\mathrm{e}^{-4t}}{3} - 2\cos(2t)$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 30

```
DSolve[{y''[t]+5*y'[t]+4*y[t]==20*Sin[2*t],{y[0]==1,y'[0]==-2}},y[t],t,IncludeSingularSoluti
```

$$y(t) \to \frac{1}{3}e^{-4t}\left(10e^{3t} - 1\right) - 2\cos(2t)$$

13.24 problem Problem 24

Internal problem ID [2862]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 3y' + 2y = 3\cos(t) + \sin(t)}$$

With initial conditions

$$[y(0) = 1, y'(0) = 1]$$

✓ Solution by Maple
Time used: 2.0 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$2)-3*diff(y(t),t)+2*y(t)=3*cos(t)+sin(t),y(0) = 1, D(y)(0) = 1],y(t), si
```

$$y(t) = \frac{7\,e^{2t}}{5} - e^{t} - \frac{4\sin(t)}{5} + \frac{3\cos(t)}{5}$$

✓ Solution by Mathematica
Time used: 0.112 (sec). Leaf size: 29

```
DSolve[{y''[t]-3*y'[t]+2*y[t]==3*Cos[t]+Sin[t],{y[0]==1,y'[0]==1}},y[t],t,IncludeSingularSol
```

$$y(t) \to \frac{1}{5}\left(e^{t}(7e^{t} - 5) - 4\sin(t) + 3\cos(t)\right)$$

13.25 problem Problem 25

Internal problem ID [2863]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = 9\sin(t)}$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

✓ Solution by Maple
Time used: 2.047 (sec). Leaf size: 19

```
dsolve([diff(y(t),t$2)+4*y(t)=9*sin(t),y(0) = 1, D(y)(0) = -1],y(t), singsol=all)
```

$$y(t) = \cos(2t) - 2\sin(2t) + 3\sin(t)$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 20

```
DSolve[{y''[t]+4*y[t]==9*Sin[t],{y[0]==1,y'[0]==-1}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to 3\sin(t) - 2\sin(2t) + \cos(2t)$$

13.26 problem Problem 26

Internal problem ID [2864]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y = 6\cos(2t)$$

With initial conditions

$$[y(0) = 0, y'(0) = 2]$$

✓ Solution by Maple
Time used: 1.906 (sec). Leaf size: 19

```
dsolve([diff(y(t),t$2)+y(t)=6*cos(2*t),y(0) = 0, D(y)(0) = 2],y(t), singsol=all)
```

$$y(t) = -2\cos(2t) + 2\cos(t) + 2\sin(t)$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 18

```
DSolve[{y''[t]+y[t]==6*Cos[2*t],{y[0]==0,y'[0]==2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 2(\sin(t) + \cos(t) - \cos(2t))$$

13.27 problem Problem 27

Internal problem ID [2865]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = 7\sin(4t) + 14\cos(4t)}$$

With initial conditions

$$[y(0) = 1, y'(0) = 2]$$

✓ Solution by Maple
Time used: 3.047 (sec). Leaf size: 29

```
dsolve([diff(y(t),t$2)+9*y(t)=7*sin(4*t)+14*cos(4*t),y(0) = 1, D(y)(0) = 2],y(t), singsol=al
```

$$y(t) = -2\cos(4t) - \sin(4t) + 3\cos(3t) + 2\sin(3t)$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 49

```
DSolve[{y''[t]+8*y[t]==7*Sin[4*t]+14*Cos[4*t],{y[0]==1,y'[0]==2}},y[t],t,IncludeSingularSolu
```

$$y(t) \to \frac{1}{8}\left(-7\sin(4t) + 11\sqrt{2}\sin\left(2\sqrt{2}t\right) - 14\cos(4t) + 22\cos\left(2\sqrt{2}t\right)\right)$$

13.28 problem Problem 28

Internal problem ID [2866]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

With initial conditions

$$[y(0) = A, y'(0) = B]$$

✓ Solution by Maple
Time used: 1.703 (sec). Leaf size: 13

```
dsolve([diff(y(t),t$2)-y(t)=0,y(0) = A, D(y)(0) = B],y(t), singsol=all)
```

$$y(t) = A\cosh(t) + B\sinh(t)$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 33

```
DSolve[{y''[t]-y[t]==0,{y[0]==a,y'[0]==b}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}e^{-t}\left(a\left(e^{2t} + 1\right) + b\left(e^{2t} - 1\right)\right)$$

14 Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704

14.1 problem Problem 27

Internal problem ID [2867]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + 2y = 2\,\text{Heaviside}\,(t-1)}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 2.328 (sec). Leaf size: 24

```
dsolve([diff(y(t),t)+2*y(t)=2*Heaviside(t-1),y(0) = 1],y(t), singsol=all)
```

$$y(t) = -\,\text{Heaviside}\,(t-1)\,\mathrm{e}^{-2t+2} + \text{Heaviside}\,(t-1) + \mathrm{e}^{-2t}$$

✓ Solution by Mathematica
Time used: 0.044 (sec). Leaf size: 26

```
DSolve[{y'[t]-y[t]==2*UnitStep[t-1],{y[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \begin{cases} e^t & t \leq 1 \\ -2 + 2e^{t-1} + e^t & \text{True} \end{cases}$$

14.2 problem Problem 28

Internal problem ID [2868]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - 2y = \text{Heaviside}\,(t-2)\,\mathrm{e}^{t-2}}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple
Time used: 2.297 (sec). Leaf size: 43

```
dsolve([diff(y(t),t)-2*y(t)=Heaviside(t-2)*exp(t-2),y(0) = 2],y(t), singsol=all)
```

$$y(t) = -\text{Heaviside}\,(t-2)\,\mathrm{e}^{t-2} + \text{Heaviside}\,(t-2)\,\mathrm{e}^{-4+2t} + 2\,\mathrm{e}^{2t}$$

✓ Solution by Mathematica
Time used: 0.088 (sec). Leaf size: 40

```
DSolve[{y'[t]-2*y[t]==UnitStep[t-2]*Exp[t-2],{y[0]==2}},y[t],t,IncludeSingularSolutions -> T
```

$$y(t) \to \begin{cases} 2e^{2t} & t \leq 2 \\ e^{t-4}(-e^2 + e^t + 2e^{t+4}) & \text{True} \end{cases}$$

14.3 problem Problem 29

Internal problem ID [2869]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' - y = 4\,\text{Heaviside}\left(t - \frac{\pi}{4}\right) \sin\left(t + \frac{\pi}{4}\right)$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 2.329 (sec). Leaf size: 40

```
dsolve([diff(y(t),t)-y(t)=4*Heaviside(t-Pi/4)*cos(t-Pi/4),y(0) = 1],y(t), singsol=all)
```

$$y(t) = \left(-2\cos(t)\sqrt{2} + 2\,e^{t-\frac{\pi}{4}}\right) \text{Heaviside}\left(t - \frac{\pi}{4}\right) + e^{t}$$

✓ Solution by Mathematica
Time used: 0.112 (sec). Leaf size: 40

```
DSolve[{y'[t]-y[t]==4*UnitStep[t-Pi/4]*Cos[t-Pi/4],{y[0]==1}},y[t],t,IncludeSingularSolution
```

$$y(t) \to \begin{cases} e^t & 4t \leq \pi \\ -2\sqrt{2}\cos(t) + e^t + 2e^{t-\frac{\pi}{4}} & \text{True} \end{cases}$$

14.4 problem Problem 30

Internal problem ID [2870]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + 2y = \text{Heaviside}\,(t-\pi)\sin(2t)}$$

With initial conditions

$$[y(0) = 3]$$

✓ Solution by Maple
Time used: 2.312 (sec). Leaf size: 43

```
dsolve([diff(y(t),t)+2*y(t)=Heaviside(t-Pi)*sin(2*t),y(0) = 3],y(t), singsol=all)
```

$$y(t) = \frac{\text{Heaviside}\,(t-\pi)\,\mathrm{e}^{-2t+2\pi}}{4} + \frac{\text{Heaviside}\,(t-\pi)\,(-\cos(2t)+\sin(2t))}{4} + 3\,\mathrm{e}^{-2t}$$

✓ Solution by Mathematica
Time used: 0.117 (sec). Leaf size: 55

```
DSolve[{y'[t]+2*y[t]==UnitStep[t-Pi]*Sin[2*t],{y[0]==3}},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \to \begin{cases} 3e^{-2t} & t \leq \pi \\ \frac{1}{4}e^{-2t}(-e^{2t}\cos(2t) + e^{2t}\sin(2t) + e^{2\pi} + 12) & \text{True} \end{cases}$$

14.5 problem Problem 31

Internal problem ID [2871]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$y' + 3y = \begin{cases} 1 & 0 \leq t < 1 \\ 0 & 1 \leq t \end{cases}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 3.688 (sec). Leaf size: 43

```
dsolve([diff(y(t),t)+3*y(t)=piecewise(0<=t and t<1,1,t>=1,0),y(0) = 1],y(t), singsol=all)
```

$$y(t) = \frac{\left(\begin{cases} 1 + 2\,\mathrm{e}^{-3t} & t < 1 \\ 2\,\mathrm{e}^{-3} + 2 & t = 1 \\ 2\,\mathrm{e}^{-3t} + \mathrm{e}^{-3t+3} & 1 < t \end{cases}\right)}{3}$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 47

```
DSolve[{y'[t]+3*y[t]==Piecewise[{{1,0<=t<1},{0,t >= 1}}],{y[0]==1}},y[t],t,IncludeSingularSo
```

$$y(t) \to \begin{cases} e^{-3t} & t \leq 0 \\ \frac{1}{3}e^{-3t}(2+e^3) & t > 1 \\ \frac{1}{3} + \frac{2e^{-3t}}{3} & \text{True} \end{cases}$$

14.6 problem Problem 32

Internal problem ID [2872]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' - 3y = \begin{cases} \sin(t) & 0 \le t < \frac{\pi}{2} \\ 1 & \frac{\pi}{2} \le t \end{cases}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 3.703 (sec). Leaf size: 61

```
dsolve([diff(y(t),t)-3*y(t)=piecewise(0<=t and t<Pi/2,sin(t),t>=Pi/2,1),y(0) = 2],y(t), sing
```

$$y(t) = \frac{\left(\begin{cases} 21\,e^{3t} - \cos(t) - 3\sin(t) & t < \frac{\pi}{2} \\ -\frac{19}{3} + 21\,e^{\frac{3\pi}{2}} & t = \frac{\pi}{2} \\ 21\,e^{3t} + \frac{e^{3t-\frac{3\pi}{2}}}{3} - \frac{10}{3} & \frac{\pi}{2} < t \end{cases}\right)}{10}$$

✓ Solution by Mathematica

Time used: 0.1 (sec). Leaf size: 68

```
DSolve[{y'[t]-3*y[t]==Piecewise[{{Sin[t],0<=t<Pi/2},{1,t >= Pi/2}}],{y[0]==2}},y[t],t,Includ
```

$$y(t) \to \begin{cases} 2e^{3t} & t \leq 0 \\ \frac{1}{30}\left(-10 + 63e^{3t} + e^{3t-\frac{3\pi}{2}}\right) & 2t > \pi \\ \frac{1}{10}(-\cos(t) + 21e^{3t} - 3\sin(t)) & \text{True} \end{cases}$$

14.7 problem Problem 33

Internal problem ID [2873]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - 3y = -10\,\mathrm{e}^{-t+a} \sin\left(-2t + 2a\right) \mathrm{Heaviside}\left(t - a\right)}$$

With initial conditions

$$[y(0) = 5]$$

✓ Solution by Maple
Time used: 3.907 (sec). Leaf size: 97

```
dsolve([diff(y(t),t)-3*y(t)=10*exp(-(t-a))*sin(2*(t-a))*Heaviside(t-a),y(0) = 5],y(t), sings
```

$$\begin{aligned} y(t) = {} & (\mathrm{Heaviside}\left(t - a\right) + \mathrm{Heaviside}\left(a\right) - 1)\,\mathrm{e}^{-3a+3t} - \Bigg(\left(\cos\left(2t\right) + 2\sin\left(2t\right)\right)\cos\left(2a\right) \\ & - 2\sin\left(2a\right)\left(\cos\left(2t\right) - \frac{\sin\left(2t\right)}{2}\right)\Bigg)\,\mathrm{e}^{-t+a}\,\mathrm{Heaviside}\left(t - a\right) \\ & - \left(\mathrm{Heaviside}\left(a\right) - 1\right)\left(\cos\left(2a\right) - 2\sin\left(2a\right)\right)\mathrm{e}^{3t+a} + 5\,\mathrm{e}^{3t} \end{aligned}$$

✓ Solution by Mathematica
Time used: 0.461 (sec). Leaf size: 103

```
DSolve[{y'[t]-3*y[t]==10*Exp[-(t-a)]*Sin[2*(t-a)]*UnitStep[t-a],{y[0]==5}},y[t],t,IncludeSin
```

$$\begin{aligned} y(t) \to e^{-3a-t}\big(&e^{4t}\theta(-a)\left(-2e^{4a}\sin(2a) + e^{4a}\cos(2a) - 1\right) \\ &+ \theta(t-a)\left(2e^{4a}\sin(2(a-t)) - e^{4a}\cos(2(a-t)) + e^{4t}\right) + 5e^{3a+4t}\big) \end{aligned}$$

14.8 problem Problem 34

Internal problem ID [2874]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y = \text{Heaviside}\,(t-1)}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple
Time used: 2.203 (sec). Leaf size: 21

```
dsolve([diff(y(t),t$2)-y(t)=Heaviside(t-1),y(0) = 1, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \cosh(t) + \text{Heaviside}\,(t-1)\,(-1+\cosh(t-1))$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 57

```
DSolve[{y''[t]-y[t]==UnitStep[t-1],{y[0]==1,y'[0]==0}},y[t],t,IncludeSingularSolutions -> Tr
```

$$y(t) \to \frac{1}{2}e^{-t-1}\left((e-e^t)^2\,(-\theta(1-t)) + e^{2t} - 2e^{t+1} + e^{2t+1} + e^2 + e\right)$$

14.9 problem Problem 35

Internal problem ID [2875]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - y' - 2y = 1 - 3\,\text{Heaviside}\,(t-2)$$

With initial conditions

$$[y(0) = 1, y'(0) = -2]$$

✓ Solution by Maple
Time used: 2.218 (sec). Leaf size: 50

```
dsolve([diff(y(t),t$2)-diff(y(t),t)-2*y(t)=1-3*Heaviside(t-2),y(0) = 1, D(y)(0) = -2],y(t),
```

$$y(t) = -\frac{1}{2} + \frac{5\,e^{-t}}{3} - \frac{e^{2t}}{6} - \frac{\text{Heaviside}\,(t-2)\,e^{-4+2t}}{2} - \text{Heaviside}\,(t-2)\,e^{2-t} + \frac{3\,\text{Heaviside}\,(t-2)}{2}$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 70

```
DSolve[{y''[t]-y'[t]-2*y[t]==1-3*UnitStep[t-2],{y[0]==1,y'[0]==-2}},y[t],t,IncludeSingularSo
```

$$y(t) \to \begin{cases} -\frac{1}{6}e^{-t}(-10 + 3e^t + e^{3t}) & t \leq 2 \\ \frac{1}{6}(6 - 6e^{2-t} + 10e^{-t} - e^{2t} - 3e^{2t-4}) & \text{True} \end{cases}$$

14.10 problem Problem 36

Internal problem ID [2876]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y = \text{Heaviside}\,(t-1) - \text{Heaviside}\,(t-2)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 4]$$

✓ Solution by Maple

Time used: 2.579 (sec). Leaf size: 35

```
dsolve([diff(y(t),t$2)-4*y(t)=Heaviside(t-1)-Heaviside(t-2),y(0) = 0, D(y)(0) = 4],y(t), sin
```

$$y(t) = \frac{\text{Heaviside}\,(t-1)\sinh\,(t-1)^2}{2} - \frac{\text{Heaviside}\,(t-2)\sinh\,(t-2)^2}{2} + 2\sinh\,(2t)$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 113

```
DSolve[{y''[t]-4*y[t]==UnitStep[t-1]-UnitStep[t-2],{y[0]==0,y'[0]==4}},y[t],t,IncludeSingula
```

$$y(t) \to \begin{cases} e^{-2t}(-1+e^{4t}) & t \leq 1 \\ \frac{1}{8}(-2+e^{2-2t}-8e^{-2t}+8e^{2t}+e^{2t-2}) & 1 < t \leq 2 \\ \frac{1}{8}e^{-2(t+2)}(-8e^4+e^6-e^8-e^{4t}+e^{4t+2}+8e^{4t+4}) & \text{True} \end{cases}$$

14.11 problem Problem 37

Internal problem ID [2877]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 37.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = t - \text{Heaviside}\,(t-1)\,(t-1)}$$

With initial conditions

$$[y(0) = 2, y'(0) = 1]$$

✓ Solution by Maple
Time used: 2.156 (sec). Leaf size: 25

```
dsolve([diff(y(t),t$2)+y(t)=t-Heaviside(t-1)*(t-1),y(0) = 2, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = (-t + \sin\,(t-1) + 1)\,\text{Heaviside}\,(t-1) + t + 2\cos\,(t)$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 31

```
DSolve[{y''[t]+y[t]==t-UnitStep[t-1]*(t-1),{y[0]==2,y'[0]==1}},y[t],t,IncludeSingularSolutio
```

$$y(t) \to \begin{cases} t + 2\cos(t) & t \leq 1 \\ 2\cos(t) - \sin(1-t) + 1 & \text{True} \end{cases}$$

14.12 problem Problem 38

Internal problem ID [2878]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 3y' + 2y = -10\,\mathrm{Heaviside}\left(t - \frac{\pi}{4}\right)\cos\left(t + \frac{\pi}{4}\right)}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple
Time used: 2.375 (sec). Leaf size: 63

```
dsolve([diff(y(t),t$2)+3*diff(y(t),t)+2*y(t)=10*Heaviside(t-Pi/4)*sin(t-Pi/4),y(0) = 1, D(y)
```

$$y(t) = -2\,\mathrm{Heaviside}\left(t - \frac{\pi}{4}\right)\mathrm{e}^{\frac{\pi}{2}-2t} + 5\,\mathrm{Heaviside}\left(t - \frac{\pi}{4}\right)\mathrm{e}^{-t+\frac{\pi}{4}} - 2\sqrt{2}\left(\cos(t) + \frac{\sin(t)}{2}\right)\mathrm{Heaviside}\left(t - \frac{\pi}{4}\right) - \mathrm{e}^{-2t} + 2\,\mathrm{e}^{-t}$$

✓ Solution by Mathematica
Time used: 0.143 (sec). Leaf size: 87

```
DSolve[{y''[t]+3*y'[t]+2*y[t]==10*UnitStep[t-Pi/4]*Sin[t-Pi/4],{y[0]==1,y'[0]==0}},y[t],t,In
```

$$y(t) \to \begin{cases} e^{-2t}(-1+2e^t) & 4t \le \pi \\ -e^{-2t}\left(2\sqrt{2}e^{2t}\cos(t) - 2e^t - 5e^{t+\frac{\pi}{4}} + \sqrt{2}e^{2t}\sin(t) + 2e^{\pi/2} + 1\right) & \text{True} \end{cases}$$

14.13 problem Problem 39

Internal problem ID [2879]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' - 6y = 30\,\text{Heaviside}\,(t-1)\,\mathrm{e}^{1-t}}$$

With initial conditions

$$[y(0) = 3, y'(0) = -4]$$

✓ Solution by Maple
Time used: 2.265 (sec). Leaf size: 55

```
dsolve([diff(y(t),t$2)+diff(y(t),t)-6*y(t)=30*Heaviside(t-1)*exp(-(t-1)),y(0) = 3, D(y)(0) =
```

$$y(t) = \left(-5\,\text{Heaviside}\,(t-1)\,\mathrm{e}^{1+2t} + 3\,\text{Heaviside}\,(t-1)\,\mathrm{e}^{3} + 2\,\mathrm{e}^{-2+5t}\,\text{Heaviside}\,(t-1) + \mathrm{e}^{5t} + 2\right)\mathrm{e}^{-3t}$$

✓ Solution by Mathematica
Time used: 0.087 (sec). Leaf size: 66

```
DSolve[{y''[t]+y'[t]-6*y[t]==30*UnitStep[t-1]*Exp[-(t-1)],{y[0]==3,y'[0]==-4}},y[t],t,Includ
```

$$y(t) \to \begin{cases} e^{-3t}(2+e^{5t}) & t \le 1 \\ e^{-3t-2}(2e^2 + 3e^5 + 2e^{5t} - 5e^{2t+3} + e^{5t+2}) & \text{True} \end{cases}$$

14.14 problem Problem 40

Internal problem ID [2880]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 5y = 5\,\text{Heaviside}\,(t-3)}$$

With initial conditions

$$[y(0) = 2, y'(0) = 1]$$

✓ Solution by Maple
Time used: 4.297 (sec). Leaf size: 53

```
dsolve([diff(y(t),t$2)+4*diff(y(t),t)+5*y(t)=5*Heaviside(t-3),y(0) = 2, D(y)(0) = 1],y(t), s
```

$$\begin{aligned} y(t) = &\left(-\frac{1}{2}-i\right) \text{Heaviside}\,(-3+t)\, \mathrm{e}^{(-2-i)(-3+t)} \\ &+ \left(-\frac{1}{2}+i\right) \text{Heaviside}\,(-3+t)\, \mathrm{e}^{(-2+i)(-3+t)} \\ &+ \text{Heaviside}\,(-3+t) + \mathrm{e}^{-2t}(2\cos(t) + 5\sin(t)) \end{aligned}$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 68

```
DSolve[{y''[t]+4*y'[t]+5*y[t]==5*UnitStep[t-3],{y[0]==2,y'[0]==1}},y[t],t,IncludeSingularSol
```

$$y(t) \to \begin{cases} e^{-2t}(2\cos(t) + 5\sin(t)) & t \leq 3 \\ e^{-2t}(-e^6\cos(3-t) + e^{2t} + 2\cos(t) + 2e^6\sin(3-t) + 5\sin(t)) & \text{True} \end{cases}$$

14.15 problem Problem 41

Internal problem ID [2881]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 41.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2y' + 5y = 2\sin(t) + \text{Heaviside}\left(t - \frac{\pi}{2}\right)(1 + \cos(t))$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 2.515 (sec). Leaf size: 77

```
dsolve([diff(y(t),t$2)-2*diff(y(t),t)+5*y(t)=2*sin(t)+Heaviside(t-Pi/2)*(1-sin(t-Pi/2)),y(0)
```

$$y(t) = \frac{\left((2\cos(t)^2 - 3\cos(t)\sin(t) - 1)\,e^{t-\frac{\pi}{2}} + 2\cos(t) - \sin(t) + 2\right)\text{Heaviside}\left(t - \frac{\pi}{2}\right)}{10} - \frac{2\,e^t\cos(t)^2}{5} - \frac{\sin(t)\cos(t)\,e^t}{5} + \frac{\cos(t)}{5} + \frac{e^t}{5} + \frac{2\sin(t)}{5}$$

✓ Solution by Mathematica
Time used: 0.502 (sec). Leaf size: 98

```
DSolve[{y''[t]-2*y'[t]+5*y[t]==2*Sin[t]+UnitStep[t-Pi/2]*(1-Sin[t-Pi/2]),{y[0]==0,y'[0]==0}}
```

$$y(t)$$

$$\to \begin{cases} \frac{1}{5}(-e^t\sin(t)\cos(t) + \cos(t) - e^t\cos(2t) + 2\sin(t)) & 2t \leq \pi \\ \frac{1}{20}\left(8\cos(t) + 2e^t(-2 + e^{-\pi/2})\cos(2t) + 6\sin(t) - 2e^t\sin(2t) - 3e^{t-\frac{\pi}{2}}\sin(2t) + 4\right) & \text{True} \end{cases}$$

14.16 problem Problem 46 part a

Internal problem ID [2882]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 46 part a.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$y' - y = \begin{cases} 2 & 0 \leq t < 1 \\ -1 & 1 \leq t \end{cases}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 2.938 (sec). Leaf size: 38

```
dsolve([diff(y(t),t)-y(t)=piecewise(0<=t and t<1,2,t>=1,-1),y(0) = 1],y(t), singsol=all)
```

$$y(t) = \begin{cases} -2 + 3\,\mathrm{e}^{t} & t < 1 \\ 1 + 3\,\mathrm{e} & t = 1 \\ 1 + 3\,\mathrm{e}^{t} - 3\,\mathrm{e}^{t-1} & 1 < t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.072 (sec). Leaf size: 42

```
DSolve[{y'[t]-y[t]==Piecewise[{{2,0<=t<1},{-1,t>=1}}],{y[0]==1}},y[t],t,IncludeSingularSolut
```

$$y(t) \to \begin{cases} e^t & t \leq 0 \\ -2+3e^t & 0 < t \leq 1 \\ 1-3e^{t-1}+3e^t & \text{True} \end{cases}$$

14.17 problem Problem 46 part b

Internal problem ID [2883]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 46 part b.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' - y = \begin{cases} 2 & 0 \leq t < 1 \\ -1 & 1 \leq t \end{cases}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.344 (sec). Leaf size: 34

```
dsolve([diff(y(t),t)-y(t)=piecewise(0<=t and t<1,2,t>=1,-1),y(0) = 1],y(t), singsol=all)
```

$$y(t) = \begin{cases} e^{t} & t < 0 \\ -2 + 3\,e^{t} & t < 1 \\ 1 + 3\,e^{t} - 3\,e^{t-1} & 1 \leq t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 42

```
DSolve[{y'[t]-y[t]==Piecewise[{{2,0<=t<1},{-1,t>=1}}],{y[0]==1}},y[t],t,IncludeSingularSolut
```

$$y(t) \to \begin{cases} e^t & t \leq 0 \\ -2+3e^t & 0 < t \leq 1 \\ 1-3e^{t-1}+3e^t & \text{True} \end{cases}$$

15 Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.8. page 710

15.1 problem Problem 1

Internal problem ID [2884]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.8. page 710
Problem number: Problem 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + y = \delta(t - 5)}$$

With initial conditions

$$[y(0) = 3]$$

✓ Solution by Maple
Time used: 2.219 (sec). Leaf size: 22

```
dsolve([diff(y(t),t)+y(t)=Dirac(t-5),y(0) = 3],y(t), singsol=all)
```

$$y(t) = \text{Heaviside}\,(t - 5)\,\mathrm{e}^{-t+5} + 3\,\mathrm{e}^{-t}$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 21

```
DSolve[{y'[t]+y[t]==DiracDelta[t-5],{y[0]==3}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t}\left(e^5\theta(t - 5) + 3\right)$$

15.2 problem Problem 2

Internal problem ID [2885]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.8. page 710
Problem number: Problem 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - 2y = \delta(t-2)}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 2.234 (sec). Leaf size: 26

```
dsolve([diff(y(t),t)-2*y(t)=Dirac(t-2),y(0) = 1],y(t), singsol=all)
```

$$y(t) = \text{Heaviside}\,(t-2)\,\mathrm{e}^{-4+2t} + \mathrm{e}^{2t}$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 23

```
DSolve[{y'[t]-2*y[t]==DiracDelta[t-2],{y[0]==3}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{2t-4}\left(\theta(t-2) + 3e^4\right)$$

15.3 problem Problem 3

Internal problem ID [2886]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.8. page 710
Problem number: Problem 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + 4y = 3\delta(t-1)}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple
Time used: 2.422 (sec). Leaf size: 23

```
dsolve([diff(y(t),t)+4*y(t)=3*Dirac(t-1),y(0) = 2],y(t), singsol=all)
```

$$y(t) = 3\,\mathrm{Heaviside}\,(t-1)\,\mathrm{e}^{-4t+4} + 2\,\mathrm{e}^{-4t}$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 22

```
DSolve[{y'[t]+4*y[t]==3*DiracDelta[t-1],{y[0]==2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-4t}\left(3e^4\theta(t-1) + 2\right)$$

15.4 problem Problem 4

Internal problem ID [2887]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.8. page 710
Problem number: Problem 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - 5y = 2\,\mathrm{e}^{-t} + \delta(t-3)}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 2.578 (sec). Leaf size: 32

```
dsolve([diff(y(t),t)-5*y(t)=2*exp(-t)+Dirac(t-3),y(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{2\,\mathrm{e}^{2t}\sinh(3t)}{3} + \mathrm{Heaviside}(-3+t)\,\mathrm{e}^{5t-15}$$

✓ Solution by Mathematica
Time used: 0.093 (sec). Leaf size: 34

```
DSolve[{y'[t]-5*y[t]==2*Exp[-t]+DiracDelta[t-3],{y[0]==0}},y[t],t,IncludeSingularSolutions -
```

$$y(t) \to \frac{1}{3}e^{-t}\left(3e^{6t-15}\theta(t-3) + e^{6t} - 1\right)$$

15.5 problem Problem 5

Internal problem ID [2888]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.8. page 710
Problem number: Problem 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 3y' + 2y = \delta(t-1)}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple
Time used: 2.375 (sec). Leaf size: 47

```
dsolve([diff(y(t),t$2)-3*diff(y(t),t)+2*y(t)=Dirac(t-1),y(0) = 1, D(y)(0) = 0],y(t), singsol
```

$$y(t) = -\text{Heaviside}\,(t-1)\,\mathrm{e}^{t-1} + \text{Heaviside}\,(t-1)\,\mathrm{e}^{2t-2} - \mathrm{e}^{2t} + 2\,\mathrm{e}^{t}$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 31

```
DSolve[{y''[t]-3*y'[t]+2*y[t]==DiracDelta[t-1],{y[0]==1,y'[0]==0}},y[t],t,IncludeSingularSol
```

$$y(t) \to e^t \left(\frac{(e^t - e)\,\theta(t-1)}{e^2} - e^t + 2 \right)$$

15.6 problem Problem 6

Internal problem ID [2889]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.8. page 710
Problem number: Problem 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y = \delta(t-3)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 2.594 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$2)-4*y(t)=Dirac(t-3),y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = \frac{\text{Heaviside}\,(-3+t)\sinh\,(2t-6)}{2} + \frac{\sinh\,(2t)}{2}$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 44

```
DSolve[{y''[t]-4*y[t]==DiracDelta[t-3],{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions -
```

$$y(t) \to \frac{1}{4}e^{-2(t+3)}\left(\left(e^{4t} - e^{12}\right)\theta(t-3) + e^{6}\left(e^{4t} - 1\right)\right)$$

15.7 problem Problem 7

Internal problem ID [2890]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.8. page 710
Problem number: Problem 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' + 5y = \delta\left(t - \frac{\pi}{2}\right)$$

With initial conditions

$$[y(0) = 0, y'(0) = 2]$$

✓ Solution by Maple

Time used: 2.204 (sec). Leaf size: 33

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+5*y(t)=Dirac(t-Pi/2),y(0) = 0, D(y)(0) = 2],y(t), sing
```

$$y(t) = \sin(2t)\left(-\frac{\mathrm{Heaviside}\left(t - \frac{\pi}{2}\right) \mathrm{e}^{-t+\frac{\pi}{2}}}{2} + \mathrm{e}^{-t}\right)$$

✓ Solution by Mathematica

Time used: 0.123 (sec). Leaf size: 34

```
DSolve[{y''[t]+2*y'[t]+5*y[t]==DiracDelta[t-Pi/2],{y[0]==0,y'[0]==2}},y[t],t,IncludeSingular
```

$$y(t) \to -e^{-t}\left(e^{\pi/2}\theta(2t - \pi) - 2\right)\sin(t)\cos(t)$$

15.8 problem Problem 8

Internal problem ID [2891]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.8. page 710
Problem number: Problem 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 13y = \delta\left(t - \frac{\pi}{4}\right)}$$

With initial conditions

$$[y(0) = 3, y'(0) = 0]$$

✓ Solution by Maple
Time used: 3.344 (sec). Leaf size: 51

```
dsolve([diff(y(t),t$2)-4*diff(y(t),t)+13*y(t)=Dirac(t-Pi/4),y(0) = 3, D(y)(0) = 0],y(t), sin
```

$$y(t) = -\frac{\sqrt{2}\, \mathrm{e}^{-\frac{\pi}{2}+2t} \operatorname{Heaviside}\left(t - \frac{\pi}{4}\right)(\sin(3t) + \cos(3t))}{6} + 3\,\mathrm{e}^{2t}\left(\cos(3t) - \frac{2\sin(3t)}{3}\right)$$

✓ Solution by Mathematica
Time used: 0.211 (sec). Leaf size: 61

```
DSolve[{y''[t]-4*y'[t]+13*y[t]==DiracDelta[t-Pi/4],{y[0]==3,y'[0]==0}},y[t],t,IncludeSingula
```

$$y(t) \to \frac{1}{6}e^{2t}\left(6(3\cos(3t) - 2\sin(3t)) - \sqrt{2}e^{-\pi/2}\theta(12t - 3\pi)(\sin(3t) + \cos(3t))\right)$$

15.9 problem Problem 9

Internal problem ID [2892]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.8. page 710
Problem number: Problem 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 3y = \delta(t - 2)}$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

✓ Solution by Maple

Time used: 2.406 (sec). Leaf size: 24

```
dsolve([diff(y(t),t$2)+4*diff(y(t),t)+3*y(t)=Dirac(t-2),y(0) = 1, D(y)(0) = -1],y(t), singso
```

$$y(t) = \text{Heaviside}\,(t - 2)\,\mathrm{e}^{-2t+4} \sinh\,(t - 2) + \mathrm{e}^{-t}$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 37

```
DSolve[{y''[t]+4*y'[t]+3*y[t]==DiracDelta[t-2],{y[0]==1,y'[0]==-1}},y[t],t,IncludeSingularSo
```

$$y(t) \to \frac{1}{2} e^{2-3t} \left(e^{2t} - e^4\right) \theta(t - 2) + e^{-t}$$

15.10 problem Problem 10

Internal problem ID [2893]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.8. page 710
Problem number: Problem 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 6y' + 13y = \delta\left(t - \frac{\pi}{4}\right)$$

With initial conditions

$$[y(0) = 5, y'(0) = 5]$$

✓ Solution by Maple
Time used: 2.531 (sec). Leaf size: 42

```
dsolve([diff(y(t),t$2)+6*diff(y(t),t)+13*y(t)=Dirac(t-Pi/4),y(0) = 5, D(y)(0) = 5],y(t), sin
```

$$y(t) = -\frac{\text{Heaviside}\left(t - \frac{\pi}{4}\right)\cos(2t)\,\mathrm{e}^{\frac{3\pi}{4}-3t}}{2} + 5\,\mathrm{e}^{-3t}(\cos(2t) + 2\sin(2t))$$

✓ Solution by Mathematica
Time used: 0.287 (sec). Leaf size: 121

```
DSolve[{y''[t]+46*y'[t]+13*y[t]==DiracDelta[t-Pi/4],{y[0]==1,y'[0]==-1}},y[t],t,IncludeSingu
```

$$y(t) \to \frac{1}{516}e^{-2\sqrt{129}t-23t-\frac{\sqrt{129}\pi}{2}}\left(2e^{\frac{\sqrt{129}\pi}{2}}\left(\left(129+11\sqrt{129}\right)e^{4\sqrt{129}t}+129-11\sqrt{129}\right)\right.$$
$$\left.-\sqrt{129}e^{23\pi/4}\left(e^{\sqrt{129}\pi}-e^{4\sqrt{129}t}\right)\theta(4t-\pi)\right)$$

15.11 problem Problem 11

Internal problem ID [2894]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.8. page 710
Problem number: Problem 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = 15\sin(2t) + \delta\left(t - \frac{\pi}{6}\right)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 3.438 (sec). Leaf size: 29

```
dsolve([diff(y(t),t$2)+9*y(t)=15*sin(2*t)+Dirac(t-Pi/6),y(0) = 0, D(y)(0) = 0],y(t), singsol
```

$$y(t) = -\frac{\cos(3t)\,\mathrm{Heaviside}\left(t - \frac{\pi}{6}\right)}{3} - 2\sin(3t) + 3\sin(2t)$$

✓ Solution by Mathematica
Time used: 0.075 (sec). Leaf size: 34

```
DSolve[{y''[t]+9*y[t]==15*Sin[2*t]+DiracDelta[t-Pi/6],{y[0]==0,y'[0]==0}},y[t],t,IncludeSing
```

$$y(t) \to -\frac{1}{3}\theta(6t - \pi)\cos(3t) + 3\sin(2t) - 2\sin(3t)$$

15.12 problem Problem 12

Internal problem ID [2895]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.8. page 710
Problem number: Problem 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 16y = 4\cos(3t) + \delta\left(t - \frac{\pi}{3}\right)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 3.609 (sec). Leaf size: 33

```
dsolve([diff(y(t),t$2)+16*y(t)=4*cos(3*t)+Dirac(t-Pi/3),y(0) = 0, D(y)(0) = 0],y(t), singsol
```

$$y(t) = \frac{\left(\cos(4t)\sqrt{3} - \sin(4t)\right)\text{Heaviside}\left(t - \frac{\pi}{3}\right)}{8} - \frac{4\cos(4t)}{7} + \frac{4\cos(3t)}{7}$$

✓ Solution by Mathematica
Time used: 0.159 (sec). Leaf size: 50

```
DSolve[{y''[t]+16*y[t]==4*Cos[3*t]+DiracDelta[t-Pi/3],{y[0]==0,y'[0]==0}},y[t],t,IncludeSing
```

$$y(t) \to \frac{1}{8}\theta(3t - \pi)\left(\sqrt{3}\cos(4t) - \sin(4t)\right) + \frac{4}{7}(\cos(3t) - \cos(4t))$$

15.13 problem Problem 13

Internal problem ID [2896]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.8. page 710
Problem number: Problem 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + 5y = 4\sin(t) + \delta\left(t - \frac{\pi}{6}\right)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 3.406 (sec). Leaf size: 56

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+5*y(t)=4*sin(t)+Dirac(t-Pi/6),y(0) = 0, D(y)(0) = 1],y
```

$$y(t) = -\frac{\text{Heaviside}\left(t - \frac{\pi}{6}\right)\left(\sqrt{3}\cos(t)^2 - \cos(t)\sin(t) - \frac{\sqrt{3}}{2}\right)e^{-t+\frac{\pi}{6}}}{2} + \frac{(4\cos(t)^2 + 3\cos(t)\sin(t) - 2)\,e^{-t}}{5} - \frac{2\cos(t)}{5} + \frac{4\sin(t)}{5}$$

✓ Solution by Mathematica
Time used: 0.644 (sec). Leaf size: 75

```
DSolve[{y''[t]+2*y'[t]+5*y[t]==4*Sin[t]+DiracDelta[t-Pi/6],{y[0]==0,y'[0]==1}},y[t],t,Includ
```

$$y(t) \to \frac{1}{20}e^{-t}\left(-5e^{\pi/6}\theta(6t - \pi)\left(\sqrt{3}\cos(2t) - \sin(2t)\right) + 16e^t\sin(t) + 6\sin(2t) - 8e^t\cos(t) + 8\cos(2t)\right)$$

16 Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739

16.1 problem Problem 1

Internal problem ID [2897]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 + \frac{1}{24}x^4\right) y(0) + \left(x + \frac{1}{6}x^3 + \frac{1}{120}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{120} + \frac{x^3}{6} + x\right) + c_1\left(\frac{x^4}{24} + \frac{x^2}{2} + 1\right)$$

16.2 problem Problem 2

Internal problem ID [2898]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_erf]`

$$\boxed{y'' + 2xy' + 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)+2*x*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - 2x^2 + \frac{4}{3}x^4\right) y(0) + \left(x - x^3 + \frac{1}{2}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 38

```
AsymptoticDSolveValue[y''[x]+2*x*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{2} - x^3 + x\right) + c_1\left(\frac{4x^4}{3} - 2x^2 + 1\right)$$

16.3 problem Problem 3

Internal problem ID [2899]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' - 2xy' - 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
Order:=6;
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + x^2 + \frac{1}{2}x^4\right) y(0) + \left(x + \frac{2}{3}x^3 + \frac{4}{15}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 38

```
AsymptoticDSolveValue[y''[x]-2*x*y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{4x^5}{15} + \frac{2x^3}{3} + x\right) + c_1\left(\frac{x^4}{2} + x^2 + 1\right)$$

16.4 problem Problem 4

Internal problem ID [2900]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' - y'x^2 - 2yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)-2*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{x^3}{3}\right) y(0) + \left(x + \frac{1}{4}x^4\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]-x^2*y'[x]-2*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^4}{4} + x\right) + c_1\left(\frac{x^3}{3} + 1\right)$$

16.5 problem Problem 5

Internal problem ID [2901]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^3}{6}\right) y(0) + \left(x - \frac{1}{12}x^4\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x - \frac{x^4}{12}\right) + c_1\left(1 - \frac{x^3}{6}\right)$$

16.6 problem Problem 6

Internal problem ID [2902]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + xy' + 3y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{3}{2}x^2 + \frac{5}{8}x^4\right) y(0) + \left(x - \frac{2}{3}x^3 + \frac{1}{5}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]+x*y'[x]+3*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 \left(\frac{x^5}{5} - \frac{2x^3}{3} + x\right) + c_1 \left(\frac{5x^4}{8} - \frac{3x^2}{2} + 1\right)$$

16.7 problem Problem 7

Internal problem ID [2903]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x^2 - 3yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)-3*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{x^3}{2}\right) y(0) + \left(x + \frac{1}{3}x^4\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]-x^2*y'[x]-3*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2\left(\frac{x^4}{3} + x\right) + c_1\left(\frac{x^3}{2} + 1\right)$$

16.8 problem Problem 8

Internal problem ID [2904]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y'x^2 + 2yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)+2*x^2*diff(y(x),x)+2*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^3}{3}\right) y(0) + \left(x - \frac{1}{3}x^4\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]+2*x^2*y'[x]+2*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 \left(x - \frac{x^4}{3}\right) + c_1 \left(1 - \frac{x^3}{3}\right)$$

16.9 problem Problem 9

Internal problem ID [2905]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x^2-3)\,y''-3xy'-5y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((x^2-3)*diff(y(x),x$2)-3*x*diff(y(x),x)-5*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{5}{6}x^2+\frac{5}{24}x^4\right)y(0)+\left(x-\frac{4}{9}x^3+\frac{8}{135}x^5\right)D(y)\,(0)+O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[(x^2-3)*y''[x]-3*x*y'[x]-5*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{8x^5}{135}-\frac{4x^3}{9}+x\right)+c_1\left(\frac{5x^4}{24}-\frac{5x^2}{6}+1\right)$$

16.10 problem Problem 10

Internal problem ID [2906]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x^2+1)\,y''+4xy'+2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
Order:=6;
dsolve((1+x^2)*diff(y(x),x$2)+4*x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(x^4-x^2+1)\,y(0)+(x^5-x^3+x)\,D(y)\,(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 30

```
AsymptoticDSolveValue[(1+x^2)*y''[x]+4*x*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2(x^5-x^3+x)+c_1(x^4-x^2+1)$$

16.11 problem Problem 11

Internal problem ID [2907]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-4x^2+1)\,y''-20xy'-16y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((1-4*x^2)*diff(y(x),x$2)-20*x*diff(y(x),x)-16*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+8x^2+\frac{128}{3}x^4\right)y(0)+\left(30x^5+6x^3+x\right)D(y)(0)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[(1-4*x^2)*y''[x]-20*x*y'[x]-16*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(30x^5+6x^3+x\right)+c_1\left(\frac{128x^4}{3}+8x^2+1\right)$$

16.12 problem Problem 12

Internal problem ID [2908]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(x^2-1)\,y''-6xy'+12y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
Order:=6;
dsolve((x^2-1)*diff(y(x),x$2)-6*x*diff(y(x),x)+12*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(x^4+6x^2+1)\,y(0)+(x^3+x)\,D(y)\,(0)+O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 25

```
AsymptoticDSolveValue[(x^2-1)*y''[x]-6*x*y'[x]+12*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2(x^3+x)+c_1(x^4+6x^2+1)$$

16.13 problem Problem 13

Internal problem ID [2909]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y' + 4yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+4*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{2}{3}x^3 + \frac{1}{3}x^4 - \frac{2}{15}x^5\right) y(0) + \left(x - x^2 + \frac{2}{3}x^3 - \frac{2}{3}x^4 + \frac{7}{15}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 61

```
AsymptoticDSolveValue[y''[x]+2*y'[x]+4*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{2x^5}{15} + \frac{x^4}{3} - \frac{2x^3}{3} + 1\right) + c_2\left(\frac{7x^5}{15} - \frac{2x^4}{3} + \frac{2x^3}{3} - x^2 + x\right)$$

16.14 problem Problem 14

Internal problem ID [2910]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + xy' + (x+2)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+(2+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x^2 - \frac{1}{6}x^3 + \frac{1}{3}x^4 + \frac{11}{120}x^5\right) y(0) + \left(x - \frac{1}{2}x^3 - \frac{1}{12}x^4 + \frac{1}{8}x^5\right) D(y)\,(0) + O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 61

```
AsymptoticDSolveValue[y''[x]+x*y'[x]+(2+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{8} - \frac{x^4}{12} - \frac{x^3}{2} + x\right) + c_1\left(\frac{11x^5}{120} + \frac{x^4}{3} - \frac{x^3}{6} - x^2 + 1\right)$$

16.15 problem Problem 15

Internal problem ID [2911]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - e^{x} y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve(diff(y(x),x$2)-exp(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1+\frac{1}{2}x^2+\frac{1}{6}x^3+\frac{1}{12}x^4+\frac{1}{24}x^5\right)y(0)+\left(x+\frac{1}{6}x^3+\frac{1}{12}x^4+\frac{1}{30}x^5\right)D(y)(0)+O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[y''[x]-Exp[x]*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{30}+\frac{x^4}{12}+\frac{x^3}{6}+x\right)+c_1\left(\frac{x^5}{24}+\frac{x^4}{12}+\frac{x^3}{6}+\frac{x^2}{2}+1\right)$$

16.16 problem Problem 17

Internal problem ID [2912]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (x-1)\,y' - yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
Order:=6;
dsolve(x*diff(y(x),x$2)-(x-1)*diff(y(x),x)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1)\left(1 + \frac{1}{4}x^2 + \frac{1}{18}x^3 + \frac{5}{192}x^4 + \frac{23}{3600}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(x + \frac{11}{108}x^3 + \frac{11}{1152}x^4 + \frac{883}{216000}x^5 + \mathrm{O}\left(x^6\right)\right) c_2$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 96

```
AsymptoticDSolveValue[x*y''[x]-(x-1)*y'[x]-x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{23x^5}{3600} + \frac{5x^4}{192} + \frac{x^3}{18} + \frac{x^2}{4} + 1\right) + c_2\left(\frac{883x^5}{216000} + \frac{11x^4}{1152} + \frac{11x^3}{108} + \left(\frac{23x^5}{3600} + \frac{5x^4}{192} + \frac{x^3}{18} + \frac{x^2}{4} + 1\right)\log(x) + x\right)$$

16.17 problem Problem 18

Internal problem ID [2913]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x^2+1)\,y''+7xy'+2y=0}$$

With initial conditions

$$[y(0)=0, y'(0)=1]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([(1+2*x^2)*diff(y(x),x$2)+7*x*diff(y(x),x)+2*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x),type=
```

$$y(x)=x-\frac{3}{2}x^3+\frac{21}{8}x^5+\mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{(1+2*x^2)*y''[x]+7*x*y'[x]+2*y[x]==0,{y[0]==0,y'[0]==1}},y[x],{x,0,5}
```

$$y(x)\to\frac{21x^5}{8}-\frac{3x^3}{2}+x$$

16.18 problem Problem 19

Internal problem ID [2914]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{4y'' + xy' + 4y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([4*diff(y(x),x$2)+x*diff(y(x),x)+4*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x),type='series',x
```

$$y(x) = 1 - \frac{1}{2}x^2 + \frac{1}{16}x^4 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{4*y''[x]+x*y'[x]+4*y[x]==0,{y[0]==1,y'[0]==0}},y[x],{x,0,5}]
```

$$y(x) \to \frac{x^4}{16} - \frac{x^2}{2} + 1$$

16.19 problem Problem 20

Internal problem ID [2915]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y'x^2 + yx = 2\cos(x)}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 30

```
Order:=6;
dsolve(diff(y(x),x$2)+2*x^2*diff(y(x),x)+x*y(x)=2*cos(x),y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^3}{6}\right) y(0) + \left(x - \frac{1}{4}x^4\right) D(y)(0) + x^2 - \frac{x^4}{12} - \frac{x^5}{4} + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 45

```
AsymptoticDSolveValue[y''[x]+2*x^2*y'[x]+x*y[x]==2*Cos[x],y[x],{x,0,5}]
```

$$y(x) \to -\frac{x^5}{4} - \frac{x^4}{12} + c_2\left(x - \frac{x^4}{4}\right) + c_1\left(1 - \frac{x^3}{6}\right) + x^2$$

16.20 problem Problem 21

Internal problem ID [2916]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + xy' - 4y = 6\,e^{x}}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
Order:=6;
dsolve(diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=6*exp(x),y(x),type='series',x=0);
```

$$y(x) = \left(1+2x^2+\frac{1}{3}x^4\right)y(0)+\left(x+\frac{1}{2}x^3+\frac{1}{40}x^5\right)D(y)(0)+3x^2+x^3+\frac{3x^4}{4}+\frac{x^5}{10}+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 62

```
AsymptoticDSolveValue[y''[x]+x*y'[x]-4*y[x]==6*Exp[x],y[x],{x,0,5}]
```

$$y(x) \to \frac{x^5}{10}+\frac{3x^4}{4}+x^3+3x^2+c_2\left(\frac{x^5}{40}+\frac{x^3}{2}+x\right)+c_1\left(\frac{x^4}{3}+2x^2+1\right)$$

17 Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758

17.1 problem 1

Internal problem ID [2917]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{y'}{1-x} + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 44

```
Order:=6;
dsolve(diff(y(x),x$2)+1/(1-x)*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{60}x^5\right) y(0) + \left(x - \frac{1}{2}x^2 - \frac{1}{12}x^4 + \frac{1}{24}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[y''[x]+1/(1-x)*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^5}{60} + \frac{x^4}{24} - \frac{x^3}{6} + 1\right) + c_2\left(\frac{x^5}{24} - \frac{x^4}{12} - \frac{x^2}{2} + x\right)$$

17.2 problem 3

Internal problem ID [2918]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + \frac{xy'}{(-x^2+1)^2} + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x/(1-x^2)^2*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{-i}\left(1 + \left(-\frac{1}{4}+\frac{i}{4}\right)x^2 + \left(-\frac{1}{80}+\frac{7i}{80}\right)x^4 + \mathrm{O}\left(x^6\right)\right) + c_2 x^{i}\left(1 + \left(-\frac{1}{4}-\frac{i}{4}\right)x^2 + \left(-\frac{1}{80}-\frac{7i}{80}\right)x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 70

```
AsymptoticDSolveValue[x^2*y''[x]+x/(1-x^2)^2*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \left(\frac{1}{80}+\frac{3i}{80}\right)c_2 x^{-i}\left((2+i)x^4 + (4+8i)x^2 + (8-24i)\right) - \left(\frac{3}{80}+\frac{i}{80}\right)c_1 x^{i}\left((1+2i)x^4 + (8+4i)x^2 - (24-8i)\right)$$

17.3 problem 4

Internal problem ID [2919]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x-2)^2 y'' + (x-2)\,\mathrm{e}^x y' + \frac{4y}{x} = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 60

```
Order:=6;
dsolve((x-2)^2*diff(y(x),x$2)+(x-2)*exp(x)*diff(y(x),x)+4/x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x\left(1-\frac{1}{4}x-\frac{1}{24}x^2-\frac{13}{576}x^3-\frac{35}{2304}x^4-\frac{1297}{138240}x^5+\mathrm{O}\left(x^6\right)\right) + c_2\left(\ln(x)\left(-x+\frac{1}{4}x^2+\frac{1}{24}x^3+\frac{13}{576}x^4+\frac{35}{2304}x^5+\mathrm{O}\left(x^6\right)\right) + \left(1+\frac{1}{2}x-\frac{5}{4}x^2-\frac{41}{144}x^3-\frac{1097}{6912}x^4-\frac{397}{4320}x^5+\mathrm{O}\left(x^6\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.063 (sec). Leaf size: 87

```
AsymptoticDSolveValue[(x-2)^2*y''[x]+(x-2)*Exp[x]*y'[x]+4/x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{1}{576}x\left(13x^3+24x^2+144x-576\right)\log(x) + \frac{-1097x^4-1968x^3-8640x^2+3456x+6912}{6912}\right) + c_2\left(-\frac{35x^5}{2304}-\frac{13x^4}{576}-\frac{x^3}{24}-\frac{x^2}{4}+x\right)$$

17.4 problem 5

Internal problem ID [2920]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{2y'}{x(x-3)} - \frac{y}{x^3(x+3)} = 0$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(diff(y(x),x$2)+2/(x*(x-3))*diff(y(x),x)-1/(x^3*(x+3))*y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.223 (sec). Leaf size: 258

```
AsymptoticDSolveValue[y''[x]+2/(x*(x-3))*y'[x]-1/(x^3*(x+3))*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 e^{-\frac{2}{\sqrt{3}\sqrt{x}}} \left(\frac{1087999600339049453 9x^{9/2}}{6059672463464202240\sqrt{3}} + \frac{647134806104 17x^{7/2}}{328758271672320\sqrt{3}} + \frac{287821451x^{5/2}}{3397386240\sqrt{3}} + \frac{19817x^{3/2}}{73728\sqrt{3}} - \frac{4894564486149401320457x^5}{1246561192484064460800} - \frac{116612812982297797x^4}{378729528966512640} - \frac{22160647459x^3}{587068342272} + \frac{463507x^2}{42467328} + \frac{587x}{4608} + \frac{25\sqrt{x}}{16\sqrt{3}} + 1 \right) x^{13/12} + c_2 e^{\frac{2}{\sqrt{3}\sqrt{x}}} \left(-\frac{10879996003390494539x^{9/2}}{6059672463464202240\sqrt{3}} - \frac{64713480610417x^{7/2}}{328758271672320\sqrt{3}} - \frac{287821451x^{5/2}}{3397386240\sqrt{3}} - \frac{19817x}{73728\sqrt{}}$$

17.5 problem 6

Internal problem ID [2921]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(1-x)\,y' - 7y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 478

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(1-x)*diff(y(x),x)-7*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{-\sqrt{7}}\left(1 + \frac{\sqrt{7}}{-1+2\sqrt{7}}x + \frac{\sqrt{7}}{-4+8\sqrt{7}}x^2 + \frac{\sqrt{7}\left(\sqrt{7}-2\right)}{372-96\sqrt{7}}x^3 + \frac{\sqrt{7}\left(\sqrt{7}-3\right)}{2976-768\sqrt{7}}x^4 + \frac{\left(\sqrt{7}-4\right)\left(\sqrt{7}-3\right)\sqrt{7}}{48960\sqrt{7}-128160}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2 x^{\sqrt{7}}\left(1 + \frac{\sqrt{7}}{1+2\sqrt{7}}x + \frac{\sqrt{7}}{4+8\sqrt{7}}x^2 + \frac{\sqrt{7}\left(\sqrt{7}+2\right)}{372+96\sqrt{7}}x^3 + \frac{\left(\sqrt{7}+3\right)\sqrt{7}}{2976+768\sqrt{7}}x^4 + \frac{\left(\sqrt{7}+4\right)\left(\sqrt{7}+3\right)\sqrt{7}}{48960\sqrt{7}+128160}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 1066

```
AsymptoticDSolveValue[x^2*y''[x]+x*(1-x)*y'[x]-7*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow \left(\frac{\sqrt{7}(1+\sqrt{7})(2+\sqrt{7})(3+\sqrt{7})(4+\sqrt{7})}{(-6+\sqrt{7}+\sqrt{7}(1+\sqrt{7}))(-5+\sqrt{7}+(1+\sqrt{7})(2+\sqrt{7}))(-4+\sqrt{7}+(2+\sqrt{7})(3+\sqrt{7}))(-3 \ldots} + \frac{\sqrt{7}(1+\sqrt{7})(2+\sqrt{7})(3+\sqrt{7})x^4}{(-6+\sqrt{7}+\sqrt{7}(1+\sqrt{7}))(-5+\sqrt{7}+(1+\sqrt{7})(2+\sqrt{7}))(-4+\sqrt{7}+(2+\sqrt{7})(3+\sqrt{7}))(-3 \ldots} + \frac{\sqrt{7}(1+\sqrt{7})(2+\sqrt{7})x^3}{(-6+\sqrt{7}+\sqrt{7}(1+\sqrt{7}))(-5+\sqrt{7}+(1+\sqrt{7})(2+\sqrt{7}))(-4+\sqrt{7}+(2+\sqrt{7})(3+\sqrt{7}))} + \frac{\sqrt{7}(1+\sqrt{7})x^2}{(-6+\sqrt{7}+\sqrt{7}(1+\sqrt{7}))(-5+\sqrt{7}+(1+\sqrt{7})(2+\sqrt{7}))} + \frac{\sqrt{7}x}{-6+\sqrt{7}+\sqrt{7}(1+\sqrt{7})} + 1\right) c_1 x^{\sqrt{7}}$$

$$+ \left(-\frac{\sqrt{7}(1-\sqrt{7})(2-\sqrt{7})(3-\sqrt{7})(4-\ldots}{(-6-\sqrt{7}-\sqrt{7}(1-\sqrt{7}))(-5-\sqrt{7}+(1-\sqrt{7})(2-\sqrt{7}))(-4-\sqrt{7}+(2-\sqrt{7})(3-\sqrt{7}))(\ldots} - \frac{\sqrt{7}(1-\sqrt{7})(2-\sqrt{7})(3-\sqrt{7})x^4}{(-6-\sqrt{7}-\sqrt{7}(1-\sqrt{7}))(-5-\sqrt{7}+(1-\sqrt{7})(2-\sqrt{7}))(-4-\sqrt{7}+(2-\sqrt{7})(3-\sqrt{7}))(-3 \ldots} - \frac{\sqrt{7}(1-\sqrt{7})(2-\sqrt{7})x^3}{(-6-\sqrt{7}-\sqrt{7}(1-\sqrt{7}))(-5-\sqrt{7}+(1-\sqrt{7})(2-\sqrt{7}))(-4-\sqrt{7}+(2-\sqrt{7})(3-\sqrt{7}))} - \frac{\sqrt{7}(1-\sqrt{7})x^2}{(-6-\sqrt{7}-\sqrt{7}(1-\sqrt{7}))(-5-\sqrt{7}+(1-\sqrt{7})(2-\sqrt{7}))} - \frac{\sqrt{7}x}{-6-\sqrt{7}-\sqrt{7}(1-\sqrt{7})} + 1\right) c_2 x^{-\sqrt{7}}$$

17.6 problem 7

Internal problem ID [2922]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{4x^2y'' + y'x\,\mathrm{e}^x - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 45

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)+x*exp(x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 - \frac{1}{4}x + \frac{5}{96}x^2 + \frac{17}{8064}x^3 - \frac{313}{1419264}x^4 - \frac{69703}{709632000}x^5 + \mathrm{O}\left(x^6\right)\right)}{x^{\frac{1}{4}}} + c_2x\left(1 - \frac{1}{9}x - \frac{5}{468}x^2 - \frac{11}{23868}x^3 + \frac{79}{501228}x^4 + \frac{16043}{313267500}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 86

```
AsymptoticDSolveValue[4*x^2*y''[x]+x*Exp[x]*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1x\left(\frac{16043x^5}{313267500} + \frac{79x^4}{501228} - \frac{11x^3}{23868} - \frac{5x^2}{468} - \frac{x}{9} + 1\right) + \frac{c_2\left(-\frac{69703x^5}{709632000} - \frac{313x^4}{1419264} + \frac{17x^3}{8064} + \frac{5x^2}{96} - \frac{x}{4} + 1\right)}{\sqrt[4]{x}}$$

17.7 problem 8

Internal problem ID [2923]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4xy'' - xy' + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 46

```
Order:=6;
dsolve(4*x*diff(y(x),x$2)-x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \ln(x)\left(-\frac{1}{2}x + \frac{1}{16}x^2 + \mathrm{O}\left(x^6\right)\right)c_2 + c_1 x\left(1 - \frac{1}{8}x + \mathrm{O}\left(x^6\right)\right) + \left(1 + \frac{1}{4}x - \frac{3}{16}x^2 + \frac{1}{384}x^3 + \frac{1}{18432}x^4 + \frac{1}{737280}x^5 + \mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 52

```
AsymptoticDSolveValue[4*x*y''[x]-x*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x - \frac{x^2}{8}\right) + c_1\left(\frac{x^4 + 48x^3 - 4608x^2 + 13824x + 18432}{18432} + \frac{1}{16}(x-8)x\log(x)\right)$$

17.8 problem 9

Internal problem ID [2924]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - x\cos(x)\,y' + 5y\,\mathrm{e}^{2x} = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 71

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*cos(x)*diff(y(x),x)+5*exp(2*x)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned}
y(x) = c_1x^{1-2i}\bigg(1 &+ \left(-\frac{10}{17} - \frac{40i}{17}\right)x + \left(-\frac{365}{136} + \frac{13i}{17}\right)x^2 + \left(\frac{223}{1020} + \frac{1723i}{765}\right)x^3 \\
&+ \left(\frac{114911}{78336} + \frac{24835i}{78336}\right)x^4 + \left(\frac{4041077}{8029440} - \frac{1112267i}{1605888}\right)x^5 + \mathrm{O}\left(x^6\right)\bigg) \\
+ c_2x^{1+2i}\bigg(1 &+ \left(-\frac{10}{17} + \frac{40i}{17}\right)x + \left(-\frac{365}{136} - \frac{13i}{17}\right)x^2 + \left(\frac{223}{1020} - \frac{1723i}{765}\right)x^3 \\
&+ \left(\frac{114911}{78336} - \frac{24835i}{78336}\right)x^4 + \left(\frac{4041077}{8029440} + \frac{1112267i}{1605888}\right)x^5 + \mathrm{O}\left(x^6\right)\bigg)
\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 94

```
AsymptoticDSolveValue[x^2*y''[x]-x*Cos[x]*y'[x]+5*Exp[2*x]*y[x]==0,y[x],{x,0,5}]
```

$$
\begin{aligned}
y(x) \to & \left(\frac{11}{391680}+\frac{7i}{391680}\right)c_1\big((32064-31693i)x^4-(30784+60608i)x^3 \\
& -(80352-23904i)x^2+(23040+69120i)x+(25344-16128i)\big)\,x^{1+2i} \\
& +\left(\frac{7}{391680}+\frac{11i}{391680}\right)c_2\big((31693-32064i)x^4+(60608+30784i)x^3 \\
& -(23904-80352i)x^2-(69120+23040i)x+(16128-25344i)\big)\,x^{1-2i}
\end{aligned}
$$

17.9 problem 10

Internal problem ID [2925]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{4x^2y'' + 3xy' + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 44

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{1}{4}}\left(1 - \frac{1}{5}x + \frac{1}{90}x^2 - \frac{1}{3510}x^3 + \frac{1}{238680}x^4 - \frac{1}{25061400}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(1 - \frac{1}{3}x + \frac{1}{42}x^2 - \frac{1}{1386}x^3 + \frac{1}{83160}x^4 - \frac{1}{7900200}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 85

```
AsymptoticDSolveValue[4*x^2*y''[x]+3*x*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \sqrt[4]{x}\left(-\frac{x^5}{25061400} + \frac{x^4}{238680} - \frac{x^3}{3510} + \frac{x^2}{90} - \frac{x}{5} + 1\right) + c_2\left(-\frac{x^5}{7900200} + \frac{x^4}{83160} - \frac{x^3}{1386} + \frac{x^2}{42} - \frac{x}{3} + 1\right)$$

17.10 problem 11

Internal problem ID [2926]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{6x^2y'' + x(1+18x)\,y' + (1+12x)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 47

```
Order:=6;
dsolve(6*x^2*diff(y(x),x$2)+x*(1+18*x)*diff(y(x),x)+(1+12*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{1}{3}}\left(1 - \frac{18}{5}x + \frac{324}{55}x^2 - \frac{5832}{935}x^3 + \frac{104976}{21505}x^4 - \frac{1889568}{623645}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\sqrt{x}\left(1 - 3x + \frac{9}{2}x^2 - \frac{9}{2}x^3 + \frac{27}{8}x^4 - \frac{81}{40}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 88

```
AsymptoticDSolveValue[6*x^2*y''[x]+x*(1+18*x)*y'[x]+(1+12*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(-\frac{81x^5}{40} + \frac{27x^4}{8} - \frac{9x^3}{2} + \frac{9x^2}{2} - 3x + 1\right) + c_2\sqrt[3]{x}\left(-\frac{1889568x^5}{623645} + \frac{104976x^4}{21505} - \frac{5832x^3}{935} + \frac{324x^2}{55} - \frac{18x}{5} + 1\right)$$

17.11 problem 12

Internal problem ID [2927]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + xy' - (x+2)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 321

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-(2+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{-\sqrt{2}}\left(1 + \frac{1}{1-2\sqrt{2}}x + \frac{1}{20-12\sqrt{2}}x^2 - \frac{1}{228\sqrt{2}-324}x^3 + \frac{1}{8832-6240\sqrt{2}}x^4 - \frac{1}{480}\frac{1}{\left(-1+2\sqrt{2}\right)\left(\sqrt{2}-1\right)\left(-3+2\sqrt{2}\right)\left(\sqrt{2}-2\right)\left(-5+2\sqrt{2}\right)}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2 x^{\sqrt{2}}\left(1 + \frac{1}{1+2\sqrt{2}}x + \frac{1}{20+12\sqrt{2}}x^2 + \frac{1}{228\sqrt{2}+324}x^3 + \frac{1}{8832+6240\sqrt{2}}x^4 + \frac{1}{244320\sqrt{2}+345600}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 843

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]-(2+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow \Bigg(\frac{x^5}{(-1+\sqrt{2}+\sqrt{2}(1+\sqrt{2}))(\sqrt{2}+(1+\sqrt{2})(2+\sqrt{2}))(1+\sqrt{2}+(2+\sqrt{2})(3+\sqrt{2}))(2+\sqrt{2}+(3}$$

$$+ \frac{x^4}{(-1+\sqrt{2}+\sqrt{2}(1+\sqrt{2}))(\sqrt{2}+(1+\sqrt{2})(2+\sqrt{2}))(1+\sqrt{2}+(2+\sqrt{2})(3+\sqrt{2}))(2+\sqrt{2}+(3}$$

$$+ \frac{x^3}{(-1+\sqrt{2}+\sqrt{2}(1+\sqrt{2}))(\sqrt{2}+(1+\sqrt{2})(2+\sqrt{2}))(1+\sqrt{2}+(2+\sqrt{2})(3+\sqrt{2}))}$$

$$+ \frac{x^2}{(-1+\sqrt{2}+\sqrt{2}(1+\sqrt{2}))(\sqrt{2}+(1+\sqrt{2})(2+\sqrt{2}))}$$

$$+ \frac{x}{-1+\sqrt{2}+\sqrt{2}(1+\sqrt{2})} + 1 \Bigg) c_1 x^{\sqrt{2}}$$

$$+ \Bigg(\frac{x^5}{(-1-\sqrt{2}-\sqrt{2}(1-\sqrt{2}))(-\sqrt{2}+(1-\sqrt{2})(2-\sqrt{2}))(1-\sqrt{2}+(2-\sqrt{2})(3-\sqrt{2}))(2-\sqrt{2}}$$

$$+ \frac{x^4}{(-1-\sqrt{2}-\sqrt{2}(1-\sqrt{2}))(-\sqrt{2}+(1-\sqrt{2})(2-\sqrt{2}))(1-\sqrt{2}+(2-\sqrt{2})(3-\sqrt{2}))(2-\sqrt{2}+}$$

$$+ \frac{x^3}{(-1-\sqrt{2}-\sqrt{2}(1-\sqrt{2}))(-\sqrt{2}+(1-\sqrt{2})(2-\sqrt{2}))(1-\sqrt{2}+(2-\sqrt{2})(3-\sqrt{2}))}$$

$$+ \frac{x^2}{(-1-\sqrt{2}-\sqrt{2}(1-\sqrt{2}))(-\sqrt{2}+(1-\sqrt{2})(2-\sqrt{2}))}$$

$$+ \frac{x}{-1-\sqrt{2}-\sqrt{2}(1-\sqrt{2})} + 1 \Bigg) c_2 x^{-\sqrt{2}}$$

17.12 problem 13

Internal problem ID [2928]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + y' - 2yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 32

```
Order:=6;
dsolve(2*x*diff(y(x),x$2)+diff(y(x),x)-2*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1 + \frac{1}{5}x^2 + \frac{1}{90}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2\left(1 + \frac{1}{3}x^2 + \frac{1}{42}x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 47

```
AsymptoticDSolveValue[2*x*y''[x]+y'[x]-2*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{x^4}{90} + \frac{x^2}{5} + 1\right) + c_2\left(\frac{x^4}{42} + \frac{x^2}{3} + 1\right)$$

17.13 problem 14

Internal problem ID [2929]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3x^2y'' - x(x+8)\,y' + 6y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
dsolve(3*x^2*diff(y(x),x$2)-x*(x+8)*diff(y(x),x)+6*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{2}{3}} \left(1 - \frac{1}{6}x + \frac{5}{36}x^2 + \frac{5}{81}x^3 + \frac{11}{972}x^4 + \frac{77}{58320}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2 x^3 \left(1 + \frac{3}{10}x + \frac{3}{65}x^2 + \frac{1}{208}x^3 + \frac{3}{7904}x^4 + \frac{21}{869440}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 88

```
AsymptoticDSolveValue[3*x^2*y''[x]-x*(x+8)*y'[x]+6*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{21x^5}{869440} + \frac{3x^4}{7904} + \frac{x^3}{208} + \frac{3x^2}{65} + \frac{3x}{10} + 1\right) x^3 + c_2 \left(\frac{77x^5}{58320} + \frac{11x^4}{972} + \frac{5x^3}{81} + \frac{5x^2}{36} - \frac{x}{6} + 1\right) x^{2/3}$$

17.14 problem 15

Internal problem ID [2930]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2y'' - x(1+2x)\,y' + 2(-1+4x)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 41

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)-x*(1+2*x)*diff(y(x),x)+2*(4*x-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1+3x+\frac{21}{2}x^2-\frac{35}{2}x^3+\frac{35}{8}x^4-\frac{7}{40}x^5+\mathrm{O}\left(x^6\right)\right)}{\sqrt{x}} + c_2x^2\left(1-\frac{4}{7}x+\frac{4}{63}x^2+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 65

```
AsymptoticDSolveValue[2*x^2*y''[x]-x*(1+2*x)*y'[x]+2*(4*x-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{4x^2}{63}-\frac{4x}{7}+1\right)x^2 + \frac{c_2\left(-\frac{7x^5}{40}+\frac{35x^4}{8}-\frac{35x^3}{2}+\frac{21x^2}{2}+3x+1\right)}{\sqrt{x}}$$

17.15 problem 16

Internal problem ID [2931]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(1-x)\,y' - (x+5)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 503

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(1-x)*diff(y(x),x)-(5+x)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = {} & c_1 x^{-\sqrt{5}}\left(1 + \frac{\sqrt{5}-1}{-1+2\sqrt{5}}x + \frac{-2+\sqrt{5}}{-4+8\sqrt{5}}x^2 + \frac{\left(-2+\sqrt{5}\right)\left(\sqrt{5}-3\right)}{276-96\sqrt{5}}x^3 \right.\\ & \left. + \frac{\left(\sqrt{5}-3\right)\left(\sqrt{5}-4\right)}{2208-768\sqrt{5}}x^4 + \frac{\left(\sqrt{5}-3\right)\left(\sqrt{5}-4\right)\left(-5+\sqrt{5}\right)}{41280\sqrt{5}-93600}x^5 + \mathrm{O}\left(x^6\right)\right) \\ & + c_2 x^{\sqrt{5}}\left(1 + \frac{\sqrt{5}+1}{1+2\sqrt{5}}x + \frac{\sqrt{5}+2}{4+8\sqrt{5}}x^2 + \frac{\left(\sqrt{5}+2\right)\left(3+\sqrt{5}\right)}{276+96\sqrt{5}}x^3 \right.\\ & \left. + \frac{\left(3+\sqrt{5}\right)\left(\sqrt{5}+4\right)}{2208+768\sqrt{5}}x^4 + \frac{\left(3+\sqrt{5}\right)\left(\sqrt{5}+4\right)\left(5+\sqrt{5}\right)}{41280\sqrt{5}+93600}x^5 + \mathrm{O}\left(x^6\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 1093

```
AsymptoticDSolveValue[x^2*y''[x]+x*(1-x)*y'[x]-(5+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)$$

$$\rightarrow \Bigg(\frac{(-5-\sqrt{5})(-4-\sqrt{5})(-3-\sqrt{5})(-2-\sqrt{5})(1+}{(-4+\sqrt{5}+\sqrt{5}(1+\sqrt{5}))(-3+\sqrt{5}+(1+\sqrt{5})(2+\sqrt{5}))(-2+\sqrt{5}+(2+\sqrt{5})(3+\sqrt{5}))(-1}$$

$$-\frac{(-4-\sqrt{5})(-3-\sqrt{5})(-2-\sqrt{5})(1+\sqrt{5})x^4}{(-4+\sqrt{5}+\sqrt{5}(1+\sqrt{5}))(-3+\sqrt{5}+(1+\sqrt{5})(2+\sqrt{5}))(-2+\sqrt{5}+(2+\sqrt{5})(3+\sqrt{5}))(-1}$$

$$+\frac{(-3-\sqrt{5})(-2-\sqrt{5})(1+\sqrt{5})x^3}{(-4+\sqrt{5}+\sqrt{5}(1+\sqrt{5}))(-3+\sqrt{5}+(1+\sqrt{5})(2+\sqrt{5}))(-2+\sqrt{5}+(2+\sqrt{5})(3+\sqrt{5}))}$$

$$-\frac{(-2-\sqrt{5})(1+\sqrt{5})x^2}{(-4+\sqrt{5}+\sqrt{5}(1+\sqrt{5}))(-3+\sqrt{5}+(1+\sqrt{5})(2+\sqrt{5}))}$$

$$+\frac{(1+\sqrt{5})x}{-4+\sqrt{5}+\sqrt{5}(1+\sqrt{5})}+1\Bigg)c_1x^{\sqrt{5}}$$

$$+\Bigg(\frac{(1-\sqrt{5})(-5+\sqrt{5})(-4+\sqrt{5})(-3+\sqrt{5})(-2}{(-4-\sqrt{5}-\sqrt{5}(1-\sqrt{5}))(-3-\sqrt{5}+(1-\sqrt{5})(2-\sqrt{5}))(-2-\sqrt{5}+(2-\sqrt{5})(3-\sqrt{5}))(-}$$

$$-\frac{(1-\sqrt{5})(-4+\sqrt{5})(-3+\sqrt{5})(-2+\sqrt{5})x^4}{(-4-\sqrt{5}-\sqrt{5}(1-\sqrt{5}))(-3-\sqrt{5}+(1-\sqrt{5})(2-\sqrt{5}))(-2-\sqrt{5}+(2-\sqrt{5})(3-\sqrt{5}))(-1}$$

$$+\frac{(1-\sqrt{5})(-3+\sqrt{5})(-2+\sqrt{5})x^3}{(-4-\sqrt{5}-\sqrt{5}(1-\sqrt{5}))(-3-\sqrt{5}+(1-\sqrt{5})(2-\sqrt{5}))(-2-\sqrt{5}+(2-\sqrt{5})(3-\sqrt{5}))}$$

$$-\frac{(1-\sqrt{5})(-2+\sqrt{5})x^2}{(-4-\sqrt{5}-\sqrt{5}(1-\sqrt{5}))(-3-\sqrt{5}+(1-\sqrt{5})(2-\sqrt{5}))}$$

$$+\frac{(1-\sqrt{5})x}{-4-\sqrt{5}-\sqrt{5}(1-\sqrt{5})}+1\Bigg)c_2x^{-\sqrt{5}}$$

17.16 problem 17

Internal problem ID [2932]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$3x^2y'' + x(3x+7)\,y' + (6x+1)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
dsolve(3*x^2*diff(y(x),x$2)+x*(7+3*x)*diff(y(x),x)+(1+6*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 - 3x + \frac{9}{4}x^2 - \frac{27}{28}x^3 + \frac{81}{280}x^4 - \frac{243}{3640}x^5 + \mathrm{O}\left(x^6\right)\right)}{x} + \frac{c_2\left(1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + \mathrm{O}\left(x^6\right)\right)}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 84

```
AsymptoticDSolveValue[3*x^2*y''[x]+x*(7+3*x)*y'[x]+(1+6*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \frac{c_1\left(-\frac{x^5}{120} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} - x + 1\right)}{\sqrt[3]{x}} + \frac{c_2\left(-\frac{243x^5}{3640} + \frac{81x^4}{280} - \frac{27x^3}{28} + \frac{9x^2}{4} - 3x + 1\right)}{x}$$

17.17 problem 18

Internal problem ID [2933]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' + (1-x)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 69

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(1-x)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = {} & c_1 x^{-i}\left(1 + \left(\frac{1}{5} + \frac{2i}{5}\right)x + \left(-\frac{1}{40} + \frac{3i}{40}\right)x^2 + \left(-\frac{3}{520} + \frac{7i}{1560}\right)x^3 \right. \\ & \left. + \left(-\frac{1}{2496} + \frac{i}{12480}\right)x^4 + \left(-\frac{9}{603200} - \frac{i}{361920}\right)x^5 + \mathrm{O}\left(x^6\right)\right) \\ & + c_2 x^{i}\left(1 + \left(\frac{1}{5} - \frac{2i}{5}\right)x + \left(-\frac{1}{40} - \frac{3i}{40}\right)x^2 + \left(-\frac{3}{520} - \frac{7i}{1560}\right)x^3 \right. \\ & \left. + \left(-\frac{1}{2496} - \frac{i}{12480}\right)x^4 + \left(-\frac{9}{603200} + \frac{i}{361920}\right)x^5 + \mathrm{O}\left(x^6\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 90

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(1-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \left(\frac{1}{12480} + \frac{i}{2496}\right) c_2 x^{-i} \left(ix^4 + (8+16i)x^3 + (168+96i)x^2 + (1056-288i)x + (480-2400i)\right) - \left(\frac{1}{2496} + \frac{i}{12480}\right) c_1 x^i \left(x^4 + (16+8i)x^3 + (96+168i)x^2 - (288-1056i)x - (2400-480i)\right)$$

17.18 problem 19

Internal problem ID [2934]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3x^2y'' + x(3x^2+1)\,y' - 2yx = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
Order:=6;
dsolve(3*x^2*diff(y(x),x$2)+x*(1+3*x^2)*diff(y(x),x)-2*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{2}{3}}\left(1 + \frac{2}{5}x - \frac{3}{40}x^2 - \frac{43}{660}x^3 + \frac{31}{3696}x^4 + \frac{2259}{261800}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(1 + 2x + \frac{1}{2}x^2 - \frac{5}{21}x^3 - \frac{73}{840}x^4 + \frac{827}{27300}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 83

```
AsymptoticDSolveValue[3*x^2*y''[x]+x*(1+3*x^2)*y'[x]-2*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{827x^5}{27300} - \frac{73x^4}{840} - \frac{5x^3}{21} + \frac{x^2}{2} + 2x + 1\right) + c_1 x^{2/3}\left(\frac{2259x^5}{261800} + \frac{31x^4}{3696} - \frac{43x^3}{660} - \frac{3x^2}{40} + \frac{2x}{5} + 1\right)$$

17.19 problem 20

Internal problem ID [2935]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' - 4y'x^2 + (1+2x)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)-4*x^2*diff(y(x),x)+(1+2*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x}\left(\left(x+\frac{1}{4}x^2+\frac{1}{18}x^3+\frac{1}{96}x^4+\frac{1}{600}x^5+\mathrm{O}\left(x^6\right)\right)c_2+\left(c_2\ln\left(x\right)+c_1\right)\left(1+\mathrm{O}\left(x^6\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 60

```
AsymptoticDSolveValue[4*x^2*y''[x]-4*x^2*y'[x]+(1+2*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\sqrt{x}\left(\frac{x^5}{600}+\frac{x^4}{96}+\frac{x^3}{18}+\frac{x^2}{4}+x\right)+\sqrt{x}\log(x)\right)+c_1\sqrt{x}$$

17.20 problem 21

Internal problem ID [2936]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' + x(3-2x)\, y' + (1-2x)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 49

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(3-2*x)*diff(y(x),x)+(1-2*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{\left(2x + x^2 + \frac{4}{9}x^3 + \frac{1}{6}x^4 + \frac{4}{75}x^5 + \mathrm{O}\left(x^6\right)\right) c_2 + \left(c_2 \ln\left(x\right) + c_1\right)\left(1 + \mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 52

```
AsymptoticDSolveValue[x^2*y''[x]+x*(3-2*x)*y'[x]+(1-2*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 \left(\frac{\frac{4x^5}{75} + \frac{x^4}{6} + \frac{4x^3}{9} + x^2 + 2x}{x} + \frac{\log(x)}{x}\right) + \frac{c_1}{x}$$

18 Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771

18.1 problem Example 11.5.2 page 763

Internal problem ID [2937]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: Example 11.5.2 page 763.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(x+3)\,y' + (-x+4)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 69

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*(3+x)*diff(y(x),x)+(4-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(\left((-5)\,x - \frac{29}{4}x^2 - \frac{173}{36}x^3 - \frac{193}{96}x^4 - \frac{1459}{2400}x^5 + \mathrm{O}\left(x^6\right)\right)c_2 + \left(1 + 3x + 3x^2 + \frac{5}{3}x^3 + \frac{5}{8}x^4 + \frac{7}{40}x^5 + \mathrm{O}\left(x^6\right)\right)\left(c_2 \ln\left(x\right) + c_1\right)\right)x^2$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 118

```
AsymptoticDSolveValue[x^2*y''[x]-x*(3+x)*y'[x]+(4-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{7x^5}{40} + \frac{5x^4}{8} + \frac{5x^3}{3} + 3x^2 + 3x + 1\right)x^2 + c_2\left(\left(-\frac{1459x^5}{2400} - \frac{193x^4}{96} - \frac{173x^3}{36} - \frac{29x^2}{4} - 5x\right)x^2 + \left(\frac{7x^5}{40} + \frac{5x^4}{8} + \frac{5x^3}{3} + 3x^2 + 3x + 1\right)x^2\log(x)\right)$$

18.2 problem Example 11.5.4 page 765

Internal problem ID [2938]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: Example 11.5.4 page 765.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(-x+3)\,y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 53

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(3-x)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{\left(3x - \frac{1}{4}x^2 - \frac{1}{36}x^3 - \frac{1}{288}x^4 - \frac{1}{2400}x^5 + \mathrm{O}\left(x^6\right)\right)c_2 + \left(c_2 \ln\left(x\right) + c_1\right)\left(1 - x + \mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 66

```
AsymptoticDSolveValue[x^2*y''[x]+x*(3-x)*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{-\frac{x^5}{2400} - \frac{x^4}{288} - \frac{x^3}{36} - \frac{x^2}{4} + 3x}{x} + \frac{(1-x)\log(x)}{x}\right) + \frac{c_1(1-x)}{x}$$

18.3 problem Example 11.5.5 page 768

Internal problem ID [2939]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: Example 11.5.5 page 768.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + xy' - (x+4)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 61

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-(4+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^4\left(1+\frac{1}{5}x+\frac{1}{60}x^2+\frac{1}{1260}x^3+\frac{1}{40320}x^4+\frac{1}{1814400}x^5+\mathrm{O}\left(x^6\right)\right)+c_2\left(\ln\left(x\right)\left(x^4+\frac{1}{5}x^5+\mathrm{O}\left(x^6\right)\right)+\left(-144 -\right.\right.}{x^2}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 77

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]-(4+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^4-16x^3+48x^2-192x+576}{576x^2}-\frac{1}{144}x^2\log(x)\right) + c_2\left(\frac{x^6}{40320}+\frac{x^5}{1260}+\frac{x^4}{60}+\frac{x^3}{5}+x^2\right)$$

18.4 problem (a)

Internal problem ID [2940]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - (-x^2 + x)\, y' + (x^3 + 1)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 63

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-(x-x^2)*diff(y(x),x)+(1+x^3)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(\left(x - \frac{3}{4}x^2 + \frac{41}{108}x^3 - \frac{89}{432}x^4 + \frac{2281}{27000}x^5 + \mathrm{O}\left(x^6\right)\right) c_2 + \left(1 - x + \frac{1}{2}x^2 - \frac{5}{18}x^3 + \frac{19}{144}x^4 - \frac{167}{3600}x^5 + \mathrm{O}\left(x^6\right)\right) (c_2 \ln (x) + c_1)\right) x$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 114

```
AsymptoticDSolveValue[x^2*y''[x]-(x-x^2)*y'[x]+(1+x^3)*y[x]==0,y[x],{x,0,5}]
```

$$
\begin{aligned}
y(x) \to\; & c_1 x\left(-\frac{167x^5}{3600}+\frac{19x^4}{144}-\frac{5x^3}{18}+\frac{x^2}{2}-x+1\right) \\
& + c_2\left(x\left(\frac{2281x^5}{27000}-\frac{89x^4}{432}+\frac{41x^3}{108}-\frac{3x^2}{4}+x\right)\right. \\
& \left. + x\left(-\frac{167x^5}{3600}+\frac{19x^4}{144}-\frac{5x^3}{18}+\frac{x^2}{2}-x+1\right)\log(x)\right)
\end{aligned}
$$

18.5 problem (b)

Internal problem ID [2941]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - \left(-1+2\sqrt{5}\right)xy' + \left(\frac{19}{4} - 3x^2\right)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 325

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-(2*sqrt(5)-1)*x*diff(y(x),x)+(19/4-3*x^2)*y(x)=0,y(x),type='series
```

$$y(x) = x^{-\frac{1}{2}+\sqrt{5}}\left(\left(1+\frac{3}{2}x^2+\frac{3}{8}x^4+\mathrm{O}\left(x^6\right)\right)c_1 + c_2x\left(\left(1+\frac{1}{2}x^2+\frac{3}{40}x^4+\mathrm{O}\left(x^6\right)\right)\ln(x)+\left(-\frac{5}{12}x^2-\frac{77}{800}x^4+\mathrm{O}\left(x^6\right)\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 94

```
AsymptoticDSolveValue[x^2*y''[x]-(2*Sqrt[5]-1)*x*y'[x]+(19/4-3*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{3}{8}x^{\frac{7}{2}+\sqrt{5}}+\frac{3}{2}x^{\frac{3}{2}+\sqrt{5}}+x^{\sqrt{5}-\frac{1}{2}}\right)+c_2\left(\frac{3}{40}x^{\frac{9}{2}+\sqrt{5}}+\frac{1}{2}x^{\frac{5}{2}+\sqrt{5}}+x^{\frac{1}{2}+\sqrt{5}}\right)$$

18.6 problem (c)

Internal problem ID [2942]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + (-2x^5 + 9x) y' + (10x^4 + 5x^2 + 25) y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 55

```
Order:=7;
dsolve(x^2*diff(y(x),x$2)+(9*x-2*x^5)*diff(y(x),x)+(25+5*x^2+10*x^4)*y(x)=0,y(x),type='serie
```

$$y(x) = c_1 x^{-4-3i}\left(1 + \left(-\frac{1}{8} - \frac{3i}{8}\right)x^2 + \left(-\frac{179}{832} - \frac{483i}{832}\right)x^4 + \left(-\frac{433}{3744} + \frac{3943i}{29952}\right)x^6 + \mathrm{O}\left(x^7\right)\right) + c_2 x^{-4+3i}\left(1 + \left(-\frac{1}{8} + \frac{3i}{8}\right)x^2 + \left(-\frac{179}{832} + \frac{483i}{832}\right)x^4 + \left(-\frac{433}{3744} - \frac{3943i}{29952}\right)x^6 + \mathrm{O}\left(x^7\right)\right)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 70

```
AsymptoticDSolveValue[x^2*y''[x]+(9*x-2*x^5)*y'[x]+(25+5*x^2+10*x^4)*y[x]==0,y[x],{x,0,6}]
```

$$y(x) \to \left(\frac{1}{832} + \frac{5i}{832}\right) c_1 x^{-4+3i}\left((86+53i)x^4 + (56+32i)x^2 + (32-160i)\right) - \left(\frac{5}{832} + \frac{i}{832}\right) c_2 x^{-4-3i}\left((53+86i)x^4 + (32+56i)x^2 - (160-32i)\right)$$

18.7 problem (d)

Internal problem ID [2943]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \left(4x + \frac{1}{2}x^2 - \frac{1}{3}x^3\right)y' - \frac{7y}{4} = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 63

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+(4*x+1/2*x^2-1/3*x^3)*diff(y(x),x)-7/4*y(x)=0,y(x),type='series',x
```

$$y(x) = \frac{c_1x^4\left(1 - \frac{1}{20}x + \frac{49}{2880}x^2 - \frac{533}{241920}x^3 + \frac{277}{491520}x^4 - \frac{203759}{2388787200}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(\ln\left(x\right)\left(\frac{8491}{768}x^4 - \frac{8491}{15360}x^5 + \mathrm{O}\left(\right.\right.\right.}{x^{\frac{7}{2}}}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 93

```
AsymptoticDSolveValue[x^2*y''[x]+(4*x+1/2*x^2-1/3*x^3)*y'[x]-7/4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{277x^{9/2}}{491520} - \frac{533x^{7/2}}{241920} + \frac{49x^{5/2}}{2880} - \frac{x^{3/2}}{20} + \sqrt{x}\right) + c_1\left(\frac{65067x^4 - 124096x^3 + 209664x^2 - 258048x + 442368}{442368x^{7/2}} - \frac{8491\sqrt{x}\log(x)}{110592}\right)$$

18.8 problem (e)

Internal problem ID [2944]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{x^2y'' + y'x^2 + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 58

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x^2*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x\left(1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(\ln\left(x\right)\left(-x + x^2 - \frac{1}{2}x^3 + \frac{1}{6}x^4 - \frac{1}{24}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(1 - x + \frac{1}{4}x^3 - \frac{5}{36}x^4 + \frac{13}{288}x^5 + \mathrm{O}\left(x^6\right)\right)\right)$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 80

```
AsymptoticDSolveValue[x^2*y''[x]+x^2*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{1}{6}x(x^3 - 3x^2 + 6x - 6)\log(x) + \frac{1}{36}(-11x^4 + 27x^3 - 36x^2 + 36)\right) + c_2\left(\frac{x^5}{24} - \frac{x^4}{6} + \frac{x^3}{2} - x^2 + x\right)$$

18.9 problem 1

Internal problem ID [2945]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x(x-3)\,y' + (-x+4)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(x-3)*diff(y(x),x)+(4-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(\left(x - \frac{3}{4}x^2 + \frac{11}{36}x^3 - \frac{25}{288}x^4 + \frac{137}{7200}x^5 + \mathrm{O}\left(x^6\right)\right)c_2 + \left(1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + \mathrm{O}\left(x^6\right)\right)(c_2\ln(x) + c_1)\right)x^2$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 120

```
AsymptoticDSolveValue[x^2*y''[x]+x*(x-3)*y'[x]+(4-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{x^5}{120} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} - x + 1\right)x^2 + c_2\left(\left(\frac{137x^5}{7200} - \frac{25x^4}{288} + \frac{11x^3}{36} - \frac{3x^2}{4} + x\right)x^2 + \left(-\frac{x^5}{120} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} - x + 1\right)x^2\log(x)\right)$$

18.10 problem 2

Internal problem ID [2946]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 2y'x^2 + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 67

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)+2*x^2*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x}\left((c_2\ln(x) + c_1)\left(1 - \frac{1}{4}x + \frac{3}{64}x^2 - \frac{5}{768}x^3 + \frac{35}{49152}x^4 - \frac{21}{327680}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(-\frac{1}{64}x^2 + \frac{1}{256}x^3 - \frac{19}{32768}x^4 + \frac{25}{393216}x^5 + \mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 129

```
AsymptoticDSolveValue[4*x^2*y''[x]+2*x^2*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(-\frac{21x^5}{327680} + \frac{35x^4}{49152} - \frac{5x^3}{768} + \frac{3x^2}{64} - \frac{x}{4} + 1\right) + c_2\left(\sqrt{x}\left(\frac{25x^5}{393216} - \frac{19x^4}{32768} + \frac{x^3}{256} - \frac{x^2}{64}\right) + \sqrt{x}\left(-\frac{21x^5}{327680} + \frac{35x^4}{49152} - \frac{5x^3}{768} + \frac{3x^2}{64} - \frac{x}{4} + 1\right)\log(x)\right)$$

18.11 problem 3

Internal problem ID [2947]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x\cos(x)\,y' - 2\,\mathrm{e}^x y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 389

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*cos(x)*diff(y(x),x)-2*exp(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{-\sqrt{2}}\left(1 - 2\frac{1}{-1+2\sqrt{2}}x + \frac{-5\sqrt{2}+14}{40-24\sqrt{2}}x^2 + \frac{-122+75\sqrt{2}}{684\sqrt{2}-972}x^3 + \frac{-1626\sqrt{2}+2375}{52992-37440\sqrt{2}}x^4 + \frac{1}{7200}\frac{-75763+52810\sqrt{2}}{\left(-1+2\sqrt{2}\right)\left(\sqrt{2}-1\right)\left(-3+2\sqrt{2}\right)\left(\sqrt{2}-2\right)\left(-5+2\sqrt{2}\right)}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2 x^{\sqrt{2}}\left(1+2\frac{1}{1+2\sqrt{2}}x + \frac{5\sqrt{2}+14}{40+24\sqrt{2}}x^2 + \frac{122+75\sqrt{2}}{684\sqrt{2}+972}x^3 + \frac{1626\sqrt{2}+2375}{52992+37440\sqrt{2}}x^4 + \frac{1}{7200}\frac{75763+52810\sqrt{2}}{\left(1+2\sqrt{2}\right)\left(1+\sqrt{2}\right)\left(3+2\sqrt{2}\right)\left(2+\sqrt{2}\right)\left(5+2\sqrt{2}\right)}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 2210

```
AsymptoticDSolveValue[x^2*y''[x]+x*Cos[x]*y'[x]-2*Exp[x]*y[x]==0,y[x],{x,0,5}]
```

Too large to display

18.12 problem 4

Internal problem ID [2948]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x^2 - (x+2)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x^2*diff(y(x),x)-(2+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^2\left(1 - \frac{1}{4}x + \frac{1}{20}x^2 - \frac{1}{120}x^3 + \frac{1}{840}x^4 - \frac{1}{6720}x^5 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(12 - 12x + 6x^2 - 2x^3 + \frac{1}{2}x^4 - \frac{1}{10}x^5 + \mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 66

```
AsymptoticDSolveValue[x^2*y''[x]+x^2*y'[x]-(2+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^3}{24} - \frac{x^2}{6} + \frac{x}{2} + \frac{1}{x} - 1\right) + c_2\left(\frac{x^6}{840} - \frac{x^5}{120} + \frac{x^4}{20} - \frac{x^3}{4} + x^2\right)$$

18.13 problem 5

Internal problem ID [2949]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + 2y'x^2 + \left(x - \frac{3}{4}\right) y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+2*x^2*diff(y(x),x)+(x-3/4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^2\left(1 - \frac{4}{3}x + x^2 - \frac{8}{15}x^3 + \frac{2}{9}x^4 - \frac{8}{105}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(-2 + 4x^2 - \frac{16}{3}x^3 + 4x^4 - \frac{32}{15}x^5 + \mathrm{O}\left(x^6\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 77

```
AsymptoticDSolveValue[x^2*y''[x]+2*x^2*y'[x]+(x-3/4)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-2x^{7/2} + \frac{8x^{5/2}}{3} - 2x^{3/2} + \frac{1}{\sqrt{x}}\right) + c_2\left(\frac{2x^{11/2}}{9} - \frac{8x^{9/2}}{15} + x^{7/2} - \frac{4x^{5/2}}{3} + x^{3/2}\right)$$

18.14 problem 6

Internal problem ID [2950]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' + (2x - 1)\, y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 63

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(2*x-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^2\left(1 - \frac{2}{3}x + \frac{1}{6}x^2 - \frac{1}{45}x^3 + \frac{1}{540}x^4 - \frac{1}{9450}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(\ln\left(x\right)\left(4x^2 - \frac{8}{3}x^3 + \frac{2}{3}x^4 - \frac{4}{45}x^5 + \mathrm{O}\left(x^6\right)\right) + \right.}{x}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 83

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(2*x-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{31x^4 - 88x^3 + 36x^2 + 72x + 36}{36x} - \frac{1}{3}x\left(x^2 - 4x + 6\right)\log(x)\right) + c_2\left(\frac{x^5}{540} - \frac{x^4}{45} + \frac{x^3}{6} - \frac{2x^2}{3} + x\right)$$

18.15 problem 7

Internal problem ID [2951]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' + y' x^3 - (x+2) y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 65

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x^3*diff(y(x),x)-(2+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^3 \left(1 + \frac{1}{4}x - \frac{7}{40}x^2 - \frac{37}{720}x^3 + \frac{467}{20160}x^4 + \frac{5647}{806400}x^5 + O\left(x^6\right)\right) + c_2 \left(\ln\left(x\right)\left(-x^3 - \frac{1}{4}x^4 + \frac{7}{40}x^5 + O\left(x^6\right)\right) + \right.}{x}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 82

```
AsymptoticDSolveValue[x^2*y''[x]+x^3*y'[x]-(2+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{91x^4 + 160x^3 - 144x^2 - 288x + 576}{576x} - \frac{1}{48}x^2(x+4)\log(x)\right) + c_2 \left(\frac{467x^6}{20160} - \frac{37x^5}{720} - \frac{7x^4}{40} + \frac{x^3}{4} + x^2\right)$$

18.16 problem 8

Internal problem ID [2952]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+1)\,y''+7x\,\mathrm{e}^x y'+9(1+\tan(x))\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 75

```
Order:=7;
dsolve(x^2*(x^2+1)*diff(y(x),x$2)+7*x*exp(x)*diff(y(x),x)+9*(1+tan(x))*y(x)=0,y(x),type='ser
```

$$y(x) = \frac{(c_2\ln(x)+c_1)\left(1+12x+\frac{117}{8}x^2-\frac{67}{36}x^3+\frac{505}{256}x^4-\frac{262}{125}x^5+\frac{2443637}{2304000}x^6+\mathrm{O}(x^7)\right)+\left((-31)\,x-\frac{147}{2}x^2+\frac{37}{8}\right.}{x^3}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 143

```
AsymptoticDSolveValue[x^2*(x^2+1)*y''[x]+7*x*Exp[x]*y'[x]+9*(1+Tan[x])*y[x]==0,y[x],{x,0,6}]
```

$$y(x) \to \frac{c_1\left(\frac{2443637x^6}{2304000} - \frac{262x^5}{125} + \frac{505x^4}{256} - \frac{67x^3}{36} + \frac{117x^2}{8} + 12x + 1\right)}{x^3} + c_2\left(\frac{-\frac{3797765581x^6}{622080000} + \frac{5057587x^5}{480000} - \frac{44803x^4}{4608} + \frac{37x^3}{8} - \frac{147x^2}{2} - 31x}{x^3} + \frac{\left(\frac{2443637x^6}{2304000} - \frac{262x^5}{125} + \frac{505x^4}{256} - \frac{67x^3}{36} + \frac{117x^2}{8} + 12x + 1\right)\log(x)}{x^3}\right)$$

18.17 problem 11

Internal problem ID [2953]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x+1)\,y''+y'x^2-2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 39

```
Order:=6;
dsolve(x^2*(1+x)*diff(y(x),x$2)+x^2*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1x^2\left(1-x+\frac{9}{10}x^2-\frac{4}{5}x^3+\frac{5}{7}x^4-\frac{9}{14}x^5+\mathrm{O}\left(x^6\right)\right)+\frac{c_2(12+6x+\mathrm{O}\left(x^6\right))}{x}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 47

```
AsymptoticDSolveValue[x^2*(1+x)*y''[x]+x^2*y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{5x^6}{7}-\frac{4x^5}{5}+\frac{9x^4}{10}-x^3+x^2\right)+c_1\left(\frac{1}{x}+\frac{1}{2}\right)$$

18.18 problem 12

Internal problem ID [2954]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + 3xy' + (1-x)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 69

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+(1-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{(c_2\ln(x)+c_1)\left(1+x+\frac{1}{4}x^2+\frac{1}{36}x^3+\frac{1}{576}x^4+\frac{1}{14400}x^5+\mathrm{O}(x^6)\right)+\left((-2)\,x-\frac{3}{4}x^2-\frac{11}{108}x^3-\frac{25}{3456}x^4-\cdots\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 118

```
AsymptoticDSolveValue[x^2*y''[x]+3*x*y'[x]+(1-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \frac{c_1\left(\frac{x^5}{14400}+\frac{x^4}{576}+\frac{x^3}{36}+\frac{x^2}{4}+x+1\right)}{x} + c_2\left(\frac{-\frac{137x^5}{432000}-\frac{25x^4}{3456}-\frac{11x^3}{108}-\frac{3x^2}{4}-2x}{x} + \frac{\left(\frac{x^5}{14400}+\frac{x^4}{576}+\frac{x^3}{36}+\frac{x^2}{4}+x+1\right)\log(x)}{x}\right)$$

18.19 problem 13

Internal problem ID [2955]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 58

```
Order:=6;
dsolve(x*diff(y(x),x$2)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x \left(1 + \frac{1}{2}x + \frac{1}{12}x^2 + \frac{1}{144}x^3 + \frac{1}{2880}x^4 + \frac{1}{86400}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2 \left(\ln(x)\left(x + \frac{1}{2}x^2 + \frac{1}{12}x^3 + \frac{1}{144}x^4 + \frac{1}{2880}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(1 - \frac{3}{4}x^2 - \frac{7}{36}x^3 - \frac{35}{1728}x^4 - \frac{101}{86400}x^5 + \mathrm{O}\left(x^6\right)\right)\right)$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 85

```
AsymptoticDSolveValue[x*y''[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{1}{144}x\left(x^3 + 12x^2 + 72x + 144\right)\log(x) + \frac{-47x^4 - 480x^3 - 2160x^2 - 1728x + 1728}{1728}\right) + c_2\left(\frac{x^5}{2880} + \frac{x^4}{144} + \frac{x^3}{12} + \frac{x^2}{2} + x\right)$$

18.20 problem 14

Internal problem ID [2956]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(x^2+6)\,y' + 6y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 33

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(6+x^2)*diff(y(x),x)+6*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1+\frac{1}{3}x^2+\mathrm{O}\left(x^6\right)\right)x + c_2\left(1+\frac{3}{2}x^2+\frac{1}{8}x^4+\mathrm{O}\left(x^6\right)\right)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 33

```
AsymptoticDSolveValue[x^2*y''[x]+x*(6+x^2)*y'[x]+6*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{1}{x^3}+\frac{x}{8}+\frac{3}{2x}\right)+c_2\left(\frac{1}{x^2}+\frac{1}{3}\right)$$

18.21 problem 15

Internal problem ID [2957]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(1-x)\,y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(1-x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x \left(1 + \frac{1}{3}x + \frac{1}{12}x^2 + \frac{1}{60}x^3 + \frac{1}{360}x^4 + \frac{1}{2520}x^5 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(-2 - 2x - x^2 - \frac{1}{3}x^3 - \frac{1}{12}x^4 - \frac{1}{60}x^5 + \mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 64

```
AsymptoticDSolveValue[x^2*y''[x]+x*(1-x)*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^3}{24} + \frac{x^2}{6} + \frac{x}{2} + \frac{1}{x} + 1\right) + c_2\left(\frac{x^5}{360} + \frac{x^4}{60} + \frac{x^3}{12} + \frac{x^2}{3} + x\right)$$

18.22 problem 16

Internal problem ID [2958]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + (1-4x)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 69

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)+(1-4*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x}\left((c_2\ln(x)+c_1)\left(1+x+\frac{1}{4}x^2+\frac{1}{36}x^3+\frac{1}{576}x^4+\frac{1}{14400}x^5+\mathrm{O}\left(x^6\right)\right) + \left((-2)\,x-\frac{3}{4}x^2-\frac{11}{108}x^3-\frac{25}{3456}x^4-\frac{137}{432000}x^5+\mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 124

```
AsymptoticDSolveValue[4*x^2*y''[x]+(1-4*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{x^5}{14400}+\frac{x^4}{576}+\frac{x^3}{36}+\frac{x^2}{4}+x+1\right) + c_2\left(\sqrt{x}\left(-\frac{137x^5}{432000}-\frac{25x^4}{3456}-\frac{11x^3}{108}-\frac{3x^2}{4}-2x\right) + \sqrt{x}\left(\frac{x^5}{14400}+\frac{x^4}{576}+\frac{x^3}{36}+\frac{x^2}{4}+x+1\right)\log(x)\right)$$

18.23 problem 17

Internal problem ID [2959]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + y' - 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 59

```
Order:=6;
dsolve(x*diff(y(x),x$2)+diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1)\left(1 + 2x + x^2 + \frac{2}{9}x^3 + \frac{1}{36}x^4 + \frac{1}{450}x^5 + \mathrm{O}\left(x^6\right)\right) + \left((-4)\,x - 3x^2 - \frac{22}{27}x^3 - \frac{25}{216}x^4 - \frac{137}{13500}x^5 + \mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 101

```
AsymptoticDSolveValue[x*y''[x]+y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^5}{450} + \frac{x^4}{36} + \frac{2x^3}{9} + x^2 + 2x + 1\right) + c_2\left(-\frac{137x^5}{13500} - \frac{25x^4}{216} - \frac{22x^3}{27} - 3x^2 + \left(\frac{x^5}{450} + \frac{x^4}{36} + \frac{2x^3}{9} + x^2 + 2x + 1\right)\log(x) - 4x\right)$$

18.24 problem 18

Internal problem ID [2960]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + xy' - y(x+1) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 63

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^2\left(1 + \frac{1}{3}x + \frac{1}{24}x^2 + \frac{1}{360}x^3 + \frac{1}{8640}x^4 + \frac{1}{302400}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(\ln\left(x\right)\left(x^2 + \frac{1}{3}x^3 + \frac{1}{24}x^4 + \frac{1}{360}x^5 + \mathrm{O}\left(x^6 \right.\right.\right.}{x}$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 83

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]-(1+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{31x^4 + 176x^3 + 144x^2 - 576x + 576}{576x} - \frac{1}{48}x\left(x^2 + 8x + 24\right)\log(x)\right) + c_2\left(\frac{x^5}{8640} + \frac{x^4}{360} + \frac{x^3}{24} + \frac{x^2}{3} + x\right)$$

18.25 problem 19

Internal problem ID [2961]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - x(x+3)\,y' + 4y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 69

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*(x+3)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((c_2 \ln(x) + c_1)\left(1 + 2x + \frac{3}{2}x^2 + \frac{2}{3}x^3 + \frac{5}{24}x^4 + \frac{1}{20}x^5 + \mathrm{O}(x^6)\right) + \left((-3)\,x - \frac{13}{4}x^2 - \frac{31}{18}x^3 - \frac{173}{288}x^4 - \frac{187}{1200}x^5 + \mathrm{O}(x^6)\right)c_2\right)x^2$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 122

```
AsymptoticDSolveValue[x^2*y''[x]-x*(x+3)*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^5}{20} + \frac{5x^4}{24} + \frac{2x^3}{3} + \frac{3x^2}{2} + 2x + 1\right)x^2 + c_2\left(\left(-\frac{187x^5}{1200} - \frac{173x^4}{288} - \frac{31x^3}{18} - \frac{13x^2}{4} - 3x\right)x^2 + \left(\frac{x^5}{20} + \frac{5x^4}{24} + \frac{2x^3}{3} + \frac{3x^2}{2} + 2x + 1\right)x^2\log(x)\right)$$

18.26 problem 20

Internal problem ID [2962]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - y'x^2 - 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x^2*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^2\left(1 + \frac{1}{2}x + \frac{3}{20}x^2 + \frac{1}{30}x^3 + \frac{1}{168}x^4 + \frac{1}{1120}x^5 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(12 + 6x - x^3 - \frac{1}{2}x^4 - \frac{3}{20}x^5 + \mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 63

```
AsymptoticDSolveValue[x^2*y''[x]-x^2*y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{x^3}{24} - \frac{x^2}{12} + \frac{1}{x} + \frac{1}{2}\right) + c_2\left(\frac{x^6}{168} + \frac{x^5}{30} + \frac{3x^4}{20} + \frac{x^3}{2} + x^2\right)$$

18.27 problem 21

Internal problem ID [2963]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - y'x^2 - (2+3x)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 65

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x^2*diff(y(x),x)-(3*x+2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^3\left(1+\frac{5}{4}x+\frac{3}{4}x^2+\frac{7}{24}x^3+\frac{1}{12}x^4+\frac{3}{160}x^5+\mathrm{O}\left(x^6\right)\right)+c_2(\ln\left(x\right)\left(24x^3+30x^4+18x^5+\mathrm{O}\left(x^6\right)\right)+(12-}{x}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 84

```
AsymptoticDSolveValue[x^2*y''[x]-x^2*y'[x]-(3*x+2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{1}{2}x^2(5x+4)\log(x) - \frac{3x^4-6x^3-6x^2+4x-4}{4x}\right) + c_2\left(\frac{x^6}{12}+\frac{7x^5}{24}+\frac{3x^4}{4}+\frac{5x^3}{4}+x^2\right)$$

18.28 problem 22

Internal problem ID [2964]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(5-x)\,y' + 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 57

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(5-x)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{(c_2\ln(x) + c_1)\left(1 - 2x + \frac{1}{2}x^2 + \mathrm{O}(x^6)\right) + \left(5x - \frac{9}{4}x^2 + \frac{1}{18}x^3 + \frac{1}{288}x^4 + \frac{1}{3600}x^5 + \mathrm{O}(x^6)\right)c_2}{x^2}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 80

```
AsymptoticDSolveValue[x^2*y''[x]+x*(5-x)*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \frac{c_1\left(\frac{x^2}{2} - 2x + 1\right)}{x^2} + c_2\left(\frac{\left(\frac{x^2}{2} - 2x + 1\right)\log(x)}{x^2} + \frac{\frac{x^5}{3600} + \frac{x^4}{288} + \frac{x^3}{18} - \frac{9x^2}{4} + 5x}{x^2}\right)$$

18.29 problem 23

Internal problem ID [2965]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 4x(1-x)\,y' + (2x-9)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 47

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)+4*x*(1-x)*diff(y(x),x)+(2*x-9)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^3\left(1+\frac{1}{4}x+\frac{1}{20}x^2+\frac{1}{120}x^3+\frac{1}{840}x^4+\frac{1}{6720}x^5+\mathrm{O}\left(x^6\right)\right)+c_2\left(12+12x+6x^2+2x^3+\frac{1}{2}x^4+\frac{1}{10}x^5+\mathrm{O}\left(\right.\right.}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 90

```
AsymptoticDSolveValue[4*x^2*y''[x]+4*x*(1-x)*y'[x]+(2*x-9)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^{5/2}}{24}+\frac{x^{3/2}}{6}+\frac{1}{x^{3/2}}+\frac{\sqrt{x}}{2}+\frac{1}{\sqrt{x}}\right)+c_2\left(\frac{x^{11/2}}{840}+\frac{x^{9/2}}{120}+\frac{x^{7/2}}{20}+\frac{x^{5/2}}{4}+x^{3/2}\right)$$

18.30 problem 24

Internal problem ID [2966]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 2x(x+2)\,y' + 2y(x+1) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 51

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+2*x*(2+x)*diff(y(x),x)+2*(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{\ln(x)\,(2x + \mathrm{O}(x^6))\,c_2 + c_1 x(1 + \mathrm{O}(x^6)) + \left(1 - 2x - 2x^2 + \frac{2}{3}x^3 - \frac{2}{9}x^4 + \frac{1}{15}x^5 + \mathrm{O}(x^6)\right)c_2}{x^2}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 48

```
AsymptoticDSolveValue[x^2*y''[x]+2*x*(2+x)*y'[x]+2*(1+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{2\log(x)}{x} - \frac{2x^4 - 6x^3 + 18x^2 + 36x - 9}{9x^2}\right) + \frac{c_2}{x}$$

18.31 problem 25

Internal problem ID [2967]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - x(1-x)\,y' + (1-x)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*(1-x)*diff(y(x),x)+(1-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((c_2 \ln(x)+c_1)\left(1+\mathrm{O}\left(x^6\right)\right)+\left(-x+\frac{1}{4}x^2-\frac{1}{18}x^3+\frac{1}{96}x^4-\frac{1}{600}x^5+\mathrm{O}\left(x^6\right)\right)c_2\right)x$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 50

```
AsymptoticDSolveValue[x^2*y''[x]-x*(1-x)*y'[x]+(1-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x\left(-\frac{x^5}{600}+\frac{x^4}{96}-\frac{x^3}{18}+\frac{x^2}{4}-x\right)+x\log(x)\right)+c_1x$$

18.32 problem 26

Internal problem ID [2968]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 4x(1+2x)\,y' + (-1+4x)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 47

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)+4*x*(1+2*x)*diff(y(x),x)+(4*x-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x\left(1 - x + \frac{2}{3}x^2 - \frac{1}{3}x^3 + \frac{2}{15}x^4 - \frac{2}{45}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(1 - 2x + 2x^2 - \frac{4}{3}x^3 + \frac{2}{3}x^4 - \frac{4}{15}x^5 + \mathrm{O}\left(x^6\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 88

```
AsymptoticDSolveValue[4*x^2*y''[x]+4*x*(1+2*x)*y'[x]+(4*x-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{2x^{7/2}}{3} - \frac{4x^{5/2}}{3} + 2x^{3/2} - 2\sqrt{x} + \frac{1}{\sqrt{x}}\right) + c_2\left(\frac{2x^{9/2}}{15} - \frac{x^{7/2}}{3} + \frac{2x^{5/2}}{3} - x^{3/2} + \sqrt{x}\right)$$

18.33 problem 27

Internal problem ID [2969]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' - (4x+3)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 65

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)-(3+4*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^2 \left(1+\frac{1}{3}x+\frac{1}{24}x^2+\frac{1}{360}x^3+\frac{1}{8640}x^4+\frac{1}{302400}x^5+\mathrm{O}\left(x^6\right)\right)+c_2\left(\ln\left(x\right)\left(x^2+\frac{1}{3}x^3+\frac{1}{24}x^4+\frac{1}{360}x^5+\mathrm{O}\left(x^6\right.\right.\right.}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 101

```
AsymptoticDSolveValue[4*x^2*y''[x]-(3+4*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2\left(\frac{x^{11/2}}{8640}+\frac{x^{9/2}}{360}+\frac{x^{7/2}}{24}+\frac{x^{5/2}}{3}+x^{3/2}\right)+c_1\left(\frac{31x^4+176x^3+144x^2-576x+576}{576\sqrt{x}}-\frac{1}{48}x^{3/2}\left(x^2+8x+24\right)\log(x)\right)$$

18.34 problem 28

Internal problem ID [2970]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']]`

$$\boxed{xy'' - xy' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
Order:=6;
dsolve(x*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \ln(x)\left(-x + O\left(x^6\right)\right)c_2 + c_1 x\left(1 + O\left(x^6\right)\right) + \left(1 + x - \frac{1}{2}x^2 - \frac{1}{12}x^3 - \frac{1}{72}x^4 - \frac{1}{480}x^5 + O\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 41

```
AsymptoticDSolveValue[x*y''[x]-x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{1}{72}\left(-x^4 - 6x^3 - 36x^2 + 144x + 72\right) - x\log(x)\right) + c_2 x$$

18.35 problem 29

Internal problem ID [2971]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(x+4)\,y' + (x+2)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 51

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(4+x)*diff(y(x),x)+(2+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{\ln(x)\left(x + \mathrm{O}\left(x^6\right)\right)c_2 + c_1x\left(1 + \mathrm{O}\left(x^6\right)\right) + \left(1 - x - \frac{1}{2}x^2 + \frac{1}{12}x^3 - \frac{1}{72}x^4 + \frac{1}{480}x^5 + \mathrm{O}\left(x^6\right)\right)c_2}{x^2}$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 45

```
AsymptoticDSolveValue[x^2*y''[x]+x*(4+x)*y'[x]+(2+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{\log(x)}{x} - \frac{x^4 - 6x^3 + 36x^2 + 144x - 72}{72x^2}\right) + \frac{c_2}{x}$$

19 Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.6. page 783

19.1 problem 2

Internal problem ID [2972]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.6. page 783
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' + \left(x^2 - \frac{9}{4}\right)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-9/4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^3\left(1 - \frac{1}{10}x^2 + \frac{1}{280}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2\left(12 + 6x^2 - \frac{3}{2}x^4 + \mathrm{O}\left(x^6\right)\right)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 58

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-9/4)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{x^{5/2}}{8} + \frac{1}{x^{3/2}} + \frac{\sqrt{x}}{2}\right) + c_2\left(\frac{x^{11/2}}{280} - \frac{x^{7/2}}{10} + x^{3/2}\right)$$

19.2 problem 3

Internal problem ID [2973]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.6. page 783
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' - y' + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 42

```
Order:=6;
dsolve(x*diff(y(x),x$2)-diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^2 \left(1 - \frac{1}{8}x^2 + \frac{1}{192}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2 \left(\ln\left(x\right)\left(x^2 - \frac{1}{8}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(-2 + \frac{3}{32}x^4 + \mathrm{O}\left(x^6\right)\right)\right)$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 59

```
AsymptoticDSolveValue[x*y''[x]-y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{1}{16}\left(x^2 - 8\right)x^2 \log(x) + \frac{1}{64}\left(-5x^4 + 16x^2 + 64\right)\right) + c_2 \left(\frac{x^6}{192} - \frac{x^4}{8} + x^2\right)$$

20 Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788

20.1 problem 1

Internal problem ID [2974]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^3}{6}\right) y(0) + \left(x - \frac{1}{12}x^4\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 \left(x - \frac{x^4}{12}\right) + c_1 \left(1 - \frac{x^3}{6}\right)$$

20.2 problem 2

Internal problem ID [2975]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - x^2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)-x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{x^4}{12}\right) y(0) + \left(x + \frac{1}{20}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]-x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{20} + x\right) + c_1\left(\frac{x^4}{12} + 1\right)$$

20.3 problem 3

Internal problem ID [2976]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(-x^2+1)\,y''-6xy'-4y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((1-x^2)*diff(y(x),x$2)-6*x*diff(y(x),x)-4*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(3x^4+2x^2+1)\,y(0)+\left(x+\frac{5}{3}x^3+\frac{7}{3}x^5\right)D(y)\,(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 38

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-6*x*y'[x]-4*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{7x^5}{3}+\frac{5x^3}{3}+x\right)+c_1(3x^4+2x^2+1)$$

20.4 problem 4

Internal problem ID [2977]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + y' + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 59

```
Order:=6;
dsolve(x*diff(y(x),x$2)+diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1)\left(1 - 2x + x^2 - \frac{2}{9}x^3 + \frac{1}{36}x^4 - \frac{1}{450}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(4x - 3x^2 + \frac{22}{27}x^3 - \frac{25}{216}x^4 + \frac{137}{13500}x^5 + \mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 101

```
AsymptoticDSolveValue[x*y''[x]+y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{x^5}{450} + \frac{x^4}{36} - \frac{2x^3}{9} + x^2 - 2x + 1\right) + c_2\left(\frac{137x^5}{13500} - \frac{25x^4}{216} + \frac{22x^3}{27} - 3x^2 + \left(-\frac{x^5}{450} + \frac{x^4}{36} - \frac{2x^3}{9} + x^2 - 2x + 1\right)\log(x) + 4x\right)$$

20.5 problem 5

Internal problem ID [2978]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + 2y' + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 32

```
Order:=6;
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\left(1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + \mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 42

```
AsymptoticDSolveValue[x*y''[x]+2*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^3}{24} - \frac{x}{2} + \frac{1}{x}\right) + c_2\left(\frac{x^4}{120} - \frac{x^2}{6} + 1\right)$$

20.6 problem 6

Internal problem ID [2979]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + 5(1-2x)\,y' - 5y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 36

```
Order:=6;
dsolve(2*x*diff(y(x),x$2)+5*(1-2*x)*diff(y(x),x)-5*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1(1+10x+\mathrm{O}(x^6))}{x^{\frac{3}{2}}} + c_2\left(1+x+\frac{15}{14}x^2+\frac{125}{126}x^3+\frac{625}{792}x^4+\frac{625}{1144}x^5+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 51

```
AsymptoticDSolveValue[2*x*y''[x]+5*(1-2*x)*y'[x]-5*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \frac{c_2(10x+1)}{x^{3/2}} + c_1\left(\frac{625x^5}{1144}+\frac{625x^4}{792}+\frac{125x^3}{126}+\frac{15x^2}{14}+x+1\right)$$

20.7 problem 7

Internal problem ID [2980]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + y' + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 41

```
Order:=6;
dsolve(x*diff(y(x),x$2)+diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1)\left(1 - \frac{1}{4}x^2 + \frac{1}{64}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(\frac{1}{4}x^2 - \frac{3}{128}x^4 + \mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 60

```
AsymptoticDSolveValue[x*y''[x]+y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^4}{64} - \frac{x^2}{4} + 1\right) + c_2\left(-\frac{3x^4}{128} + \frac{x^2}{4} + \left(\frac{x^4}{64} - \frac{x^2}{4} + 1\right)\log(x)\right)$$

20.8 problem 8

Internal problem ID [2981]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(4x^2+1)\,y''-8y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 29

```
Order:=6;
dsolve((1+4*x^2)*diff(y(x),x$2)-8*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(4x^2+1)\,y(0)+\left(x+\frac{4}{3}x^3-\frac{16}{15}x^5\right)D(y)\,(0)+O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 33

```
AsymptoticDSolveValue[(1+4*x^2)*y''[x]-8*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1(4x^2+1)+c_2\left(-\frac{16x^5}{15}+\frac{4x^3}{3}+x\right)$$

20.9 problem 9

Internal problem ID [2982]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 + \mathrm{O}\left(x^6\right)\right)x + c_2\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + \mathrm{O}\left(x^6\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 58

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^{7/2}}{24} - \frac{x^{3/2}}{2} + \frac{1}{\sqrt{x}}\right) + c_2\left(\frac{x^{9/2}}{120} - \frac{x^{5/2}}{6} + \sqrt{x}\right)$$

20.10 problem 10

Internal problem ID [2983]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{4xy'' + 3y' + 3y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=6;
dsolve(4*x*diff(y(x),x$2)+3*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{1}{4}} \left(1 - \frac{3}{5}x + \frac{1}{10}x^2 - \frac{1}{130}x^3 + \frac{3}{8840}x^4 - \frac{3}{309400}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2 \left(1 - x + \frac{3}{14}x^2 - \frac{3}{154}x^3 + \frac{3}{3080}x^4 - \frac{9}{292600}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 83

```
AsymptoticDSolveValue[4*x*y''[x]+3*y'[x]+3*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \sqrt[4]{x} \left(-\frac{3x^5}{309400} + \frac{3x^4}{8840} - \frac{x^3}{130} + \frac{x^2}{10} - \frac{3x}{5} + 1\right) + c_2 \left(-\frac{9x^5}{292600} + \frac{3x^4}{3080} - \frac{3x^3}{154} + \frac{3x^2}{14} - x + 1\right)$$

20.11 problem 11

Internal problem ID [2984]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \frac{3xy'}{2} - \frac{y(x+1)}{2} = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+3/2*x*diff(y(x),x)-1/2*(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{3}{2}} \left(1 + \frac{1}{5}x + \frac{1}{70}x^2 + \frac{1}{1890}x^3 + \frac{1}{83160}x^4 + \frac{1}{5405400}x^5 + \mathrm{O}\left(x^6\right)\right) + c_1\left(1 - x - \frac{1}{2}x^2 - \frac{1}{18}x^3 - \frac{1}{360}x^4 - \frac{1}{12600}x}{x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 86

```
AsymptoticDSolveValue[x^2*y''[x]+3/2*x*y'[x]-1/2*(1+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{x^5}{5405400} + \frac{x^4}{83160} + \frac{x^3}{1890} + \frac{x^2}{70} + \frac{x}{5} + 1\right) + \frac{c_2\left(-\frac{x^5}{12600} - \frac{x^4}{360} - \frac{x^3}{18} - \frac{x^2}{2} - x + 1\right)}{x}$$

20.12 problem 12

Internal problem ID [2985]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - x(-x+2)\,y' + \left(x^2+2\right)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 65

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*(2-x)*diff(y(x),x)+(2+x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(c_1x\left(1 - x + \frac{1}{3}x^2 - \frac{1}{36}x^3 - \frac{7}{720}x^4 + \frac{31}{10800}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(\ln\left(x\right)\left(-x + x^2 - \frac{1}{3}x^3 + \frac{1}{36}x^4 + \frac{7}{720}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(1 - x - \frac{1}{2}x^2 + \frac{19}{36}x^3 - \frac{53}{432}x^4 - \frac{1}{675}x^5 + \mathrm{O}\left(x^6\right)\right)\right)\right)x$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 85

```
AsymptoticDSolveValue[x^2*y''[x]-x*(2-x)*y'[x]+(2+x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{1}{36}x^2\left(x^3 - 12x^2 + 36x - 36\right)\log(x) - \frac{1}{432}x\left(65x^4 - 372x^3 + 648x^2 - 432\right)\right) + c_2\left(-\frac{7x^6}{720} - \frac{x^5}{36} + \frac{x^4}{3} - x^3 + x^2\right)$$

20.13 problem 13

Internal problem ID [2986]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 3xy' + 4y(x+1) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*(x+1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((c_2 \ln(x) + c_1)\left(1 - 4x + 4x^2 - \frac{16}{9}x^3 + \frac{4}{9}x^4 - \frac{16}{225}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(8x - 12x^2 + \frac{176}{27}x^3 - \frac{50}{27}x^4 + \frac{1096}{3375}x^5 + \mathrm{O}\left(x^6\right)\right)c_2\right)x^2$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 116

```
AsymptoticDSolveValue[x^2*y''[x]-3*x*y'[x]+4*(x+1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{16x^5}{225} + \frac{4x^4}{9} - \frac{16x^3}{9} + 4x^2 - 4x + 1\right)x^2 + c_2\left(\left(\frac{1096x^5}{3375} - \frac{50x^4}{27} + \frac{176x^3}{27} - 12x^2 + 8x\right)x^2 + \left(-\frac{16x^5}{225} + \frac{4x^4}{9} - \frac{16x^3}{9} + 4x^2 - 4x + 1\right)x^2\log(x)\right)$$

20.14 problem 20

Internal problem ID [2987]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(1 - \frac{3}{4x^2}\right) y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 49

```
Order:=6;
dsolve(diff(y(x),x$2)+(1-3/(4*x^2))*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^2 \left(1 - \frac{1}{8}x^2 + \frac{1}{192}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2 \left(\ln\left(x\right) \left(x^2 - \frac{1}{8}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(-2 + \frac{3}{32}x^4 + \mathrm{O}\left(x^6\right)\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 72

```
AsymptoticDSolveValue[y''[x]+(1-3/(4*x^2))*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 \left(\frac{x^{11/2}}{192} - \frac{x^{7/2}}{8} + x^{3/2}\right) + c_1 \left(\frac{1}{16} x^{3/2} \left(x^2 - 8\right) \log(x) - \frac{5x^4 - 16x^2 - 64}{64\sqrt{x}}\right)$$