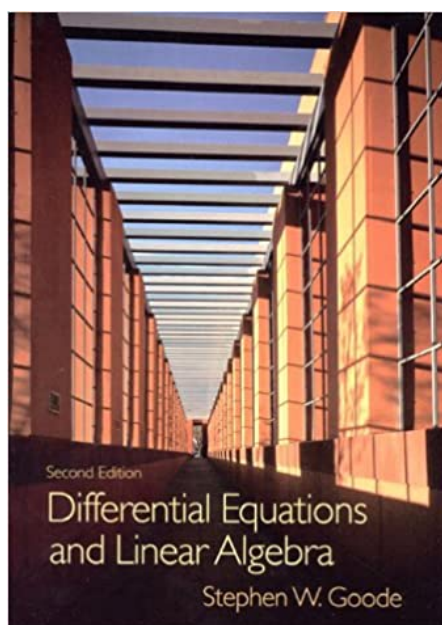


A Solution Manual For

**Differential equations and linear algebra,
Stephen W. Goode, second edition, 2000**



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May 16, 2024

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1.1 problem 1

Internal problem ID [2544]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.4, page 36

Problem number: 1.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [`_separable`]

$$y' - 2yx = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=2*x*y(x),y(x), singsol=all)
```

$$y(x) = e^{x^2} c_1$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 18

```
DSolve[y'[x]==2*x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{x^2}$$

$$y(x) \rightarrow 0$$

1.2 problem 2

Internal problem ID [2545]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.4, page 36

Problem number: 2.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - \frac{y^2}{x^2 + 1} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=y(x)^2/(x^2+1),y(x), singsol=all)
```

$$y(x) = \frac{1}{-\arctan(x) + c_1}$$

✓ Solution by Mathematica

Time used: 0.147 (sec). Leaf size: 19

```
DSolve[y'[x]==y[x]^2/(x^2+1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{1}{\arctan(x) + c_1}$$
$$y(x) \rightarrow 0$$

1.3 problem 3

Internal problem ID [2546]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.4, page 36

Problem number: 3.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [`_separable`]

$$e^{y+x}y' = 1$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(exp(x+y(x))*diff(y(x),x)-1=0,y(x), singsol=all)
```

$$y(x) = \ln(e^x c_1 - 1) - x$$

✓ Solution by Mathematica

Time used: 0.089 (sec). Leaf size: 16

```
DSolve[Exp[x+y[x]]*y'[x]-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \log(-e^{-x} + c_1)$$

1.4 problem 4

Internal problem ID [2547]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.4, page 36

Problem number: 4.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [`_separable`]

$$y' - \frac{y}{\ln(x)x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x)=y(x)/(x*ln(x)),y(x), singsol=all)
```

$$y(x) = \ln(x) c_1$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 15

```
DSolve[y'[x]==y[x]/(x*Log[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \log(x)$$

$$y(x) \rightarrow 0$$

1.5 problem 5

Internal problem ID [2548]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.4, page 36

Problem number: 5.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [`_separable`]

$$y - (x - 2)y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(y(x)-(x-2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1(-2 + x)$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 16

```
DSolve[y[x]-(x-2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}y(x) &\rightarrow c_1(x - 2) \\ y(x) &\rightarrow 0\end{aligned}$$

1.6 problem 6

Internal problem ID [2549]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.4, page 36

Problem number: 6.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' - \frac{2x(y-1)}{x^2+3} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)=(2*x*(y(x)-1))/(x^2+3),y(x), singsol=all)
```

$$y(x) = c_1x^2 + 3c_1 + 1$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 20

```
DSolve[y'[x]==(2*x*(y[x]-1))/(x^2+3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 1 + c_1(x^2 + 3)$$

$$y(x) \rightarrow 1$$

1.7 problem 7

Internal problem ID [2550]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.4, page 36

Problem number: 7.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [`_separable`]

$$y - xy' + 2y'x^2 = 3$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(y(x)-x*diff(y(x),x)=3-2*x^2*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \frac{c_1x - 3}{2x - 1}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 24

```
DSolve[y[x]-x*y'[x]==3-2*x^2*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{3 + c_1x}{1 - 2x}$$
$$y(x) \rightarrow 3$$

1.8 problem 8

Internal problem ID [2551]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.4, page 36

Problem number: 8.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [`_separable`]

$$y' - \frac{\cos(-y+x)}{\sin(x)\sin(y)} = -1$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)=(cos(x-y(x)))/(sin(x)*sin(y(x)))-1,y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{\csc(x)}{c_1}\right)$$

✓ Solution by Mathematica

Time used: 5.76 (sec). Leaf size: 47

```
DSolve[y'[x]==(Cos[x-y[x]])/(Sin[x]*Sin[y[x]])-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\arccos\left(-\frac{1}{2}c_1 \csc(x)\right)$$

$$y(x) \rightarrow \arccos\left(-\frac{1}{2}c_1 \csc(x)\right)$$

$$y(x) \rightarrow -\frac{\pi}{2}$$

$$y(x) \rightarrow \frac{\pi}{2}$$

1.9 problem 9

Internal problem ID [2552]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.4, page 36

Problem number: 9.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' - \frac{x(y^2 - 1)}{2(x-2)(x-1)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)=(x*(y(x)^2-1))/(2*(x-2)*(x-1)),y(x), singsol=all)
```

$$y(x) = -\tanh\left(\ln(-2+x) - \frac{\ln(x-1)}{2} + \frac{c_1}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.942 (sec). Leaf size: 51

```
DSolve[y'[x]==(x*(y[x]^2-1))/(2*(x-2)*(x-1)),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{x + e^{2c_1}(x-2)^2 - 1}{-x + e^{2c_1}(x-2)^2 + 1}$$

$$y(x) \rightarrow -1$$

$$y(x) \rightarrow 1$$

1.10 problem 10

Internal problem ID [2553]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.4, page 36

Problem number: 10.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y' - \frac{x^2 y - 32}{-x^2 + 16} = 32$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 51

```
dsolve(diff(y(x),x)=(x^2*y(x)-32)/(16-x^2) + 32,y(x), singsol=all)
```

$$y(x) = \frac{-1440 e^{-4-x} (x+4)^2 \operatorname{ExpIntegral}_1(-4-x) + c_1 (x+4)^2 e^{-x} + 32x^2 - 1696x - 7168}{(x-4)^2}$$

✓ Solution by Mathematica

Time used: 0.204 (sec). Leaf size: 56

```
DSolve[y'[x]==(x^2*y[x]-32)/(16-x^2) + 32,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^{-x-4} (1440(x+4)^2 \operatorname{ExpIntegralEi}(x+4) + e^4 (32e^x (x^2 - 53x - 224) + c_1 (x+4)^2))}{(x-4)^2}$$

1.11 problem 11

Internal problem ID [2554]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.4, page 36

Problem number: 11.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$(x - a)(x - b)y' - y = -c$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
dsolve((x-a)*(x-b)*diff(y(x),x)-(y(x)-c)=0,y(x), singsol=all)
```

$$y(x) = c + (x - b)^{-\frac{1}{a-b}} (x - a)^{\frac{1}{a-b}} c_1$$

✓ Solution by Mathematica

Time used: 0.287 (sec). Leaf size: 41

```
DSolve[(x-a)*(x-b)*y'[x]-(y[x]-c)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c + c_1(x - b)^{\frac{1}{b-a}}(x - a)^{\frac{1}{a-b}}$$
$$y(x) \rightarrow c$$

1.12 problem 12

Internal problem ID [2555]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.4, page 36

Problem number: 12.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [`_separable`]

$$(x^2 + 1)y' + y^2 = -1$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 11

```
dsolve([(x^2+1)*diff(y(x),x)+y(x)^2=-1,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \cot\left(\arctan(x) + \frac{\pi}{4}\right)$$

✓ Solution by Mathematica

Time used: 0.242 (sec). Leaf size: 14

```
DSolve[{(x^2+1)*y'[x]+y[x]^2== -1,y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \cot\left(\arctan(x) + \frac{\pi}{4}\right)$$

1.13 problem 13

Internal problem ID [2556]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.4, page 36

Problem number: 13.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [`_separable`]

$$(-x^2 + 1)y' + yx = ax$$

With initial conditions

$$[y(0) = 2a]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve([(1-x^2)*diff(y(x),x)+x*y(x)=a*x,y(0) = 2*a],y(x), singsol=all)
```

$$y(x) = a\left(1 - i\sqrt{x-1}\sqrt{x+1}\right)$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 21

```
DSolve[{(1-x^2)*y'[x]+x*y[x]==a*x,y[0]==2*a},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow a - ia\sqrt{x^2 - 1}$$

1.14 problem 14

Internal problem ID [2557]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.4, page 36

Problem number: 14.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' + \frac{\sin(y+x)}{\sin(y)\cos(x)} = 1$$

With initial conditions

$$\left[y\left(\frac{\pi}{4}\right) = \frac{\pi}{4} \right]$$

✓ Solution by Maple

Time used: 0.312 (sec). Leaf size: 9

```
dsolve([diff(y(x),x)=1- (sin(x+y(x)))/(sin(y(x))*cos(x)),y(1/4*Pi) = 1/4*Pi],y(x), singsol=a
```

$$y(x) = \frac{\pi}{2} - \arcsin\left(\frac{\sec(x)}{2}\right)$$

✓ Solution by Mathematica

Time used: 6.234 (sec). Leaf size: 12

```
DSolve[{y'[x]==1- Sin[x+y[x]]/(Sin[y[x]]*Cos[x]),y[Pi/4]==Pi/4},y[x],x,IncludeSingularSoluti
```

$$y(x) \rightarrow \arccos\left(\frac{\sec(x)}{2}\right)$$

1.15 problem 15

Internal problem ID [2558]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.4, page 36

Problem number: 15.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' - y^3 \sin(x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)=y(x)^3*sin(x),y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{c_1 + 2 \cos(x)}}$$
$$y(x) = -\frac{1}{\sqrt{c_1 + 2 \cos(x)}}$$

✓ Solution by Mathematica

Time used: 0.207 (sec). Leaf size: 49

```
DSolve[y'[x]==y[x]^3*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{1}{\sqrt{2}\sqrt{\cos(x) - c_1}}$$
$$y(x) \rightarrow \frac{1}{\sqrt{2}\sqrt{\cos(x) - c_1}}$$
$$y(x) \rightarrow 0$$

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2.1 problem 1

Internal problem ID [2559]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.6, page 50

Problem number: 1.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' - y = e^{2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)-y(x)=exp(2*x),y(x), singsol=all)
```

$$y(x) = (e^x + c_1)e^x$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 15

```
DSolve[y'[x]-y[x]==Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x(e^x + c_1)$$

2.2 problem 2

Internal problem ID [2560]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.6, page 50

Problem number: 2.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y'x^2 - 4yx = x^7 \sin(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x)-4*x*y(x)=x^7*sin(x),y(x), singsol=all)
```

$$y(x) = (-x \cos(x) + \sin(x) + c_1) x^4$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 19

```
DSolve[x^2*y'[x]-4*x*y[x]==x^7*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^4(\sin(x) - x \cos(x) + c_1)$$

2.3 problem 3

Internal problem ID [2561]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.6, page 50

Problem number: 3.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [`_linear`]

$$y' + 2yx = 2x^3$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+2*x*y(x)=2*x^3,y(x), singsol=all)
```

$$y(x) = x^2 - 1 + c_1 e^{-x^2}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 20

```
DSolve[y'[x]+2*x*y[x]==2*x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^2 + c_1 e^{-x^2} - 1$$

2.4 problem 4

Internal problem ID [2562]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.6, page 50

Problem number: 4.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y' + \frac{2xy}{x^2 + 1} = 4x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)+2*x/(1+x^2)*y(x)=4*x,y(x), singsol=all)
```

$$y(x) = x^2 + 1 + \frac{c_1}{x^2 + 1}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 24

```
DSolve[y'[x]+2*x/(1+x^2)*y[x]==4*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x^4 + 2x^2 + c_1}{x^2 + 1}$$

2.5 problem 5

Internal problem ID [2563]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.6, page 50

Problem number: 5.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y' + \frac{2xy}{x^2 + 1} = \frac{4}{(x^2 + 1)^2}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)+2*x/(1+x^2)*y(x)=4/(1+x^2)^2,y(x), singsol=all)
```

$$y(x) = \frac{4 \arctan(x) + c_1}{x^2 + 1}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 20

```
DSolve[y'[x]+2*x/(1+x^2)*y[x]==4/(1+x^2)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{4 \arctan(x) + c_1}{x^2 + 1}$$

2.6 problem 6

Internal problem ID [2564]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.6, page 50

Problem number: 6.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$2 \cos(x)^2 y' + y \sin(2x) = 4 \cos(x)^4$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(2*cos(x)^2*diff(y(x),x)+y(x)*sin(2*x)=4*cos(x)^4,y(x), singsol=all)
```

$$y(x) = (2 \sin(x) + c_1) \cos(x)$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 15

```
DSolve[2*Cos[x]^2*y'[x]+y[x]*Sin[2*x]==4*Cos[x]^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \cos(x)(2 \sin(x) + c_1)$$

2.7 problem 7

Internal problem ID [2565]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.6, page 50

Problem number: 7.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y' + \frac{y}{\ln(x)x} = 9x^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)+1/(x*ln(x))*y(x)=9*x^2,y(x), singsol=all)
```

$$y(x) = \frac{3x^3 \ln(x) - x^3 + c_1}{\ln(x)}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 25

```
DSolve[y'[x]+1/(x*Log[x])*y[x]==9*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{-x^3 + 3x^3 \log(x) + c_1}{\log(x)}$$

2.8 problem 8

Internal problem ID [2566]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.6, page 50

Problem number: 8.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y' - y \tan(x) = 8 \sin(x)^3$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)-y(x)*tan(x)=8*sin(x)^3,y(x), singsol=all)
```

$$y(x) = 2 \cos(x)^3 - 4 \cos(x) + \frac{\sec(x)(4c_1 + 5)}{4}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 19

```
DSolve[y'[x]-y[x]*Tan[x]==8*Sin[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 2 \sin^3(x) \tan(x) + c_1 \sec(x)$$

2.9 problem 9

Internal problem ID [2567]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.6, page 50

Problem number: 9.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$x't + 2x = 4e^t$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(t*diff(x(t),t)+2*x(t)=4*exp(t),x(t), singsol=all)
```

$$x(t) = \frac{(4t - 4)e^t + c_1}{t^2}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 20

```
DSolve[t*x'[t]+2*x[t]==4*Exp[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow \frac{4e^t(t - 1) + c_1}{t^2}$$

2.10 problem 10

Internal problem ID [2568]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.6, page 50

Problem number: 10.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y' - \sin(x)(y \sec(x) - 2) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=sin(x)*(y(x)*sec(x)-2),y(x), singsol=all)
```

$$y(x) = \cos(x) - \frac{\sec(x)}{2} + \sec(x) c_1$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 20

```
DSolve[y'[x]==Sin[x]*(y[x]*Sec[x]-2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2} \sec(x)(\cos(2x) + 2c_1)$$

2.11 problem 11

Internal problem ID [2569]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.6, page 50

Problem number: 11.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$-y \sin(x) - y' \cos(x) = -1$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve((1-y(x)*sin(x))-cos(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \cos(x) c_1 + \sin(x)$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 13

```
DSolve[(1-y[x]*Sin[x])-Cos[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sin(x) + c_1 \cos(x)$$

2.12 problem 12

Internal problem ID [2570]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.6, page 50

Problem number: 12.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y' - \frac{y}{x} = 2 \ln(x) x^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)-y(x)/x=2*x^2*ln(x),y(x), singsol=all)
```

$$y(x) = x^3 \ln(x) - \frac{x^3}{2} + c_1 x$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 23

```
DSolve[y'[x]-y[x]/x==2*x^2*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{x^3}{2} + x^3 \log(x) + c_1 x$$

2.13 problem 13

Internal problem ID [2571]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.6, page 50

Problem number: 13.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' + \alpha y = e^{\beta x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x)+alpha*y(x)=exp(beta*x),y(x), singsol=all)
```

$$y(x) = \frac{e^{-\alpha x}(e^{x(\alpha+\beta)} + c_1(\alpha + \beta))}{\alpha + \beta}$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 31

```
DSolve[y' [x]+\[Alpha]*y[x]==Exp[\[Beta]*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^{\alpha(-x)}(e^{x(\alpha+\beta)} + c_1(\alpha + \beta))}{\alpha + \beta}$$

2.14 problem 14

Internal problem ID [2572]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.6, page 50

Problem number: 14.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' = -\frac{m}{x} + \ln(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+m/x=ln(x),y(x), singsol=all)
```

$$y(x) = (-m + x) \ln(x) + c_1 - x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 19

```
DSolve[y'[x]+m/x==Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow (x - m) \log(x) - x + c_1$$

3 1.8, page 68

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3.1 problem 9

Internal problem ID [2573]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.8, page 68

Problem number: 9.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, [_Abel, '2nd type', 'cl`

$$(3x - y)y' - 3y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve((3*x-y(x))*diff(y(x),x)=3*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{3x}{\text{LambertW}(-3xe^{-3c_1})}$$

✓ Solution by Mathematica

Time used: 6.016 (sec). Leaf size: 25

```
DSolve[(3*x-y[x])*y'[x]==3*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{3x}{W(-3e^{-c_1}x)}$$
$$y(x) \rightarrow 0$$

3.2 problem 10

Internal problem ID [2574]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.8, page 68

Problem number: 10.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Riccati]`

$$y' - \frac{(y+x)^2}{2x^2} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=(x+y(x))^2/(2*x^2),y(x), singsol=all)
```

$$y(x) = \tan\left(\frac{\ln(x)}{2} + \frac{c_1}{2}\right)x$$

✓ Solution by Mathematica

Time used: 0.234 (sec). Leaf size: 17

```
DSolve[y'[x]==(x+y[x])^2/(2*x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x \tan\left(\frac{\log(x)}{2} + c_1\right)$$

3.3 problem 11

Internal problem ID [2575]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.8, page 68

Problem number: 11.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\sin\left(\frac{y}{x}\right)(xy' - y) - x \cos\left(\frac{y}{x}\right) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve(sin(y(x)/x)*(x*diff(y(x),x)-y(x))=x*cos(y(x)/x),y(x), singsol=all)
```

$$y(x) = x \arccos\left(\frac{1}{c_1 x}\right)$$

✓ Solution by Mathematica

Time used: 25.589 (sec). Leaf size: 56

```
DSolve[Sin[y[x]/x]*(x*y'[x]-y[x])=x*Cos[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -x \arccos\left(\frac{e^{-c_1}}{x}\right)$$

$$y(x) \rightarrow x \arccos\left(\frac{e^{-c_1}}{x}\right)$$

$$y(x) \rightarrow -\frac{\pi x}{2}$$

$$y(x) \rightarrow \frac{\pi x}{2}$$

3.4 problem 12

Internal problem ID [2576]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.8, page 68

Problem number: 12.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _dAlembert]`

$$xy' - \sqrt{16x^2 - y^2} - y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x*diff(y(x),x)=sqrt(16*x^2-y(x)^2)+y(x),y(x), singsol=all)
```

$$-\arctan\left(\frac{y(x)}{\sqrt{16x^2 - y(x)^2}}\right) + \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.43 (sec). Leaf size: 18

```
DSolve[x*y'[x]==Sqrt[16*x^2-y[x]^2]+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -4x \cosh(i \log(x) + c_1)$$

3.5 problem 13

Internal problem ID [2577]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.8, page 68

Problem number: 13.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _dAlembert]`

$$xy' - y - \sqrt{9x^2 + y^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(x*diff(y(x),x)-y(x)=sqrt(9*x^2+y(x)^2),y(x), singsol=all)
```

$$\frac{-c_1x^2 + \sqrt{9x^2 + y(x)^2} + y(x)}{x^2} = 0$$

✓ Solution by Mathematica

Time used: 0.376 (sec). Leaf size: 27

```
DSolve[x*y'[x]-y[x]==Sqrt[9*x^2+y[x]^2],y[x],x,IncludeSingularSolutions->True]
```

$$y(x) \rightarrow \frac{9e^{c_1}x^2}{2} - \frac{e^{-c_1}}{2}$$

3.6 problem 14

Internal problem ID [2578]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.8, page 68

Problem number: 14.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$x(x^2 - y^2) - x(y^2 + x^2)y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(x*(x^2-y(x)^2)-x*(x^2+y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \text{RootOf} \left(\int^{-z} \frac{-a^2 + 1}{-a^3 + -a^2 + -a - 1} da + \ln(x) + c_1 \right) x$$

✓ Solution by Mathematica

Time used: 0.133 (sec). Leaf size: 71

```
DSolve[x*(x^2-y[x]^2)-x*(x^2+y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve} \left[\text{RootSum} \left[\#1^3 + \#1^2 + \#1 \right. \right. \\ \left. \left. - 1 \&, \frac{\#1^2 \log\left(\frac{y(x)}{x} - \#1\right) + \log\left(\frac{y(x)}{x} - \#1\right)}{3\#1^2 + 2\#1 + 1} \& \right] = -\log(x) + c_1, y(x) \right]$$

3.7 problem 15

Internal problem ID [2579]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.8, page 68

Problem number: 15.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$xy' + y \ln(x) - y \ln(y) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x)+y(x)*ln(x)=y(x)*ln(y(x)),y(x), singsol=all)
```

$$y(x) = x e^{c_1 x + 1}$$

✓ Solution by Mathematica

Time used: 0.264 (sec). Leaf size: 24

```
DSolve[x*y'[x]+y[x]*Log[x]==y[x]*Log[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x e^{1+e^{c_1} x}$$

$$y(x) \rightarrow e x$$

3.8 problem 16

Internal problem ID [2580]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.8, page 68

Problem number: 16.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$y' - \frac{y^2 + 2yx - 2x^2}{x^2 - yx + y^2} = 0$$

✓ Solution by Maple

Time used: 0.86 (sec). Leaf size: 80

```
dsolve(diff(y(x),x)= (y(x)^2+2*x*y(x)-2*x^2)/(x^2-x*y(x)+y(x)^2),y(x), singsol=all)
```

$$y(x) = \frac{x \left(-\text{RootOf} \left(2_Z^6 + (9c_1x^2 - 1)_Z^4 - 6x^2c_1_Z^2 + c_1x^2 \right)^2 + 1 \right)}{\text{RootOf} \left(2_Z^6 + (9c_1x^2 - 1)_Z^4 - 6x^2c_1_Z^2 + c_1x^2 \right)^2}$$

✓ Solution by Mathematica

Time used: 60.179 (sec). Leaf size: 373

`DSolve[y'[x]== (y[x]^2+2*x*y[x]-2*x^2)/(x^2-x*y[x]+y[x]^2),y[x],x,IncludeSingularSolutions`

$$y(x) \rightarrow \frac{\sqrt[3]{-54x^3 + 2\sqrt{729x^6 + (-9x^2 + 3e^{2c_1})^3}}}{3\sqrt[3]{2}} - \frac{\sqrt[3]{2}(-3x^2 + e^{2c_1})}{\sqrt[3]{-54x^3 + 2\sqrt{729x^6 + (-9x^2 + 3e^{2c_1})^3}}} + x$$

$$y(x) \rightarrow \frac{(-1 + i\sqrt{3}) \sqrt[3]{-54x^3 + 2\sqrt{729x^6 + (-9x^2 + 3e^{2c_1})^3}}}{6\sqrt[3]{2}} + \frac{(1 + i\sqrt{3})(-3x^2 + e^{2c_1})}{2^{2/3} \sqrt[3]{-54x^3 + 2\sqrt{729x^6 + (-9x^2 + 3e^{2c_1})^3}}} + x$$

$$y(x) \rightarrow -\frac{(1 + i\sqrt{3}) \sqrt[3]{-54x^3 + 2\sqrt{729x^6 + (-9x^2 + 3e^{2c_1})^3}}}{6\sqrt[3]{2}} + \frac{(1 - i\sqrt{3})(-3x^2 + e^{2c_1})}{2^{2/3} \sqrt[3]{-54x^3 + 2\sqrt{729x^6 + (-9x^2 + 3e^{2c_1})^3}}} + x$$

3.9 problem 17

Internal problem ID [2581]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.8, page 68

Problem number: 17.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A']]`

$$2xyy' - x^2e^{-\frac{y^2}{x^2}} - 2y^2 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 26

```
dsolve(2*x*y(x)*diff(y(x),x)-(x^2*exp(-y(x)^2/x^2)+2*y(x)^2)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{\ln(\ln(x) + c_1)} x$$
$$y(x) = -\sqrt{\ln(\ln(x) + c_1)} x$$

✓ Solution by Mathematica

Time used: 2.155 (sec). Leaf size: 38

```
DSolve[2*x*y[x]*y'[x]-(x^2*Exp[-y[x]^2/x^2]+2*y[x]^2)==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow -x\sqrt{\log(\log(x) + 2c_1)}$$
$$y(x) \rightarrow x\sqrt{\log(\log(x) + 2c_1)}$$

3.10 problem 18

Internal problem ID [2582]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.8, page 68

Problem number: 18.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Riccati]`

$$y'x^2 - y^2 - 3yx = x^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^2*diff(y(x),x)=y(x)^2+3*x*y(x)+x^2,y(x), singsol=all)
```

$$y(x) = -\frac{x(\ln(x) + c_1 + 1)}{\ln(x) + c_1}$$

✓ Solution by Mathematica

Time used: 0.145 (sec). Leaf size: 28

```
DSolve[x^2*y'[x]==y[x]^2+3*x*y[x]+x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{x(\log(x) + 1 + c_1)}{\log(x) + c_1}$$

$$y(x) \rightarrow -x$$

3.11 problem 19

Internal problem ID [2583]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.8, page 68

Problem number: 19.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$yy' - \sqrt{y^2 + x^2} = -x$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 27

```
dsolve(y(x)*diff(y(x),x)=sqrt(x^2+y(x)^2)-x,y(x), singsol=all)
```

$$\frac{-c_1 y(x)^2 + \sqrt{x^2 + y(x)^2} + x}{y(x)^2} = 0$$

✓ Solution by Mathematica

Time used: 0.432 (sec). Leaf size: 57

```
DSolve[y[x]*y'[x]==Sqrt[x^2+y[x]^2]-x,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}y(x) &\rightarrow -e^{\frac{c_1}{2}} \sqrt{2x + e^{c_1}} \\y(x) &\rightarrow e^{\frac{c_1}{2}} \sqrt{2x + e^{c_1}} \\y(x) &\rightarrow 0\end{aligned}$$

3.12 problem 20

Internal problem ID [2584]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.8, page 68

Problem number: 20.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, [_Abel, '2nd type', 'cl`

$$2x(y + 2x)y' - y(4x - y) = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve(2*x*(y(x)+2*x)*diff(y(x),x)=y(x)*(4*x-y(x)),y(x), singsol=all)
```

$$y(x) = \frac{2x}{\text{LambertW}\left(2e^{\frac{3c_1}{2}}x^{\frac{3}{2}}\right)}$$

✓ Solution by Mathematica

Time used: 5.384 (sec). Leaf size: 29

```
DSolve[2*x*(y[x]+2*x)*y'[x]==y[x]*(4*x-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{2x}{W(2e^{-c_1}x^{3/2})}$$
$$y(x) \rightarrow 0$$

3.13 problem 21

Internal problem ID [2585]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.8, page 68

Problem number: 21.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$xy' - x \tan\left(\frac{y}{x}\right) - y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 10

```
dsolve(x*diff(y(x),x)=x*tan(y(x)/x)+y(x),y(x), singsol=all)
```

$$y(x) = \arcsin(c_1 x) x$$

✓ Solution by Mathematica

Time used: 4.369 (sec). Leaf size: 19

```
DSolve[x*y'[x]==x*Tan[y[x]/x]+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x \arcsin(e^{c_1} x)$$

$$y(x) \rightarrow 0$$

3.14 problem 22

Internal problem ID [2586]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

Section: 1.8, page 68

Problem number: 22.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$y' - \frac{x\sqrt{y^2 + x^2} + y^2}{yx} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve(diff(y(x),x)=(x*sqrt(x^2+y(x)^2)+y(x)^2)/(x*y(x)),y(x), singsol=all)
```

$$\frac{x \ln(x) - c_1 x - \sqrt{x^2 + y(x)^2}}{x} = 0$$

✓ Solution by Mathematica

Time used: 0.283 (sec). Leaf size: 54

```
DSolve[y'[x]==(x*Sqrt[x^2+y[x]^2]+y[x]^2)/(x*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -x\sqrt{\log^2(x) + 2c_1 \log(x) - 1 + c_1^2}$$
$$y(x) \rightarrow x\sqrt{\log^2(x) + 2c_1 \log(x) - 1 + c_1^2}$$