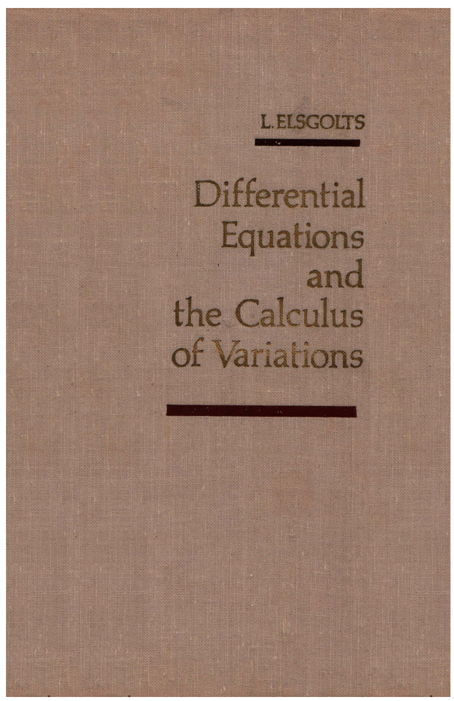
A Solution Manual For

Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.



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1.1 problem Problem 1

Internal problem ID [12112]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\tan(y) - y' \cot(x) = 0}$$

✓ Solution by Maple
Time used: 0.203 (sec). Leaf size: 9

```
dsolve(tan(y(x))-cot(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \arcsin(\sec(x)\, c_1)$$

✓ Solution by Mathematica
Time used: 4.745 (sec). Leaf size: 19

```
DSolve[Tan[y[x]]-Cot[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin\left(\frac{1}{2}c_1 \sec(x)\right)$$
$$y(x) \to 0$$

1.2 problem Problem 2

Internal problem ID [12113]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$6y + (5x + 2y - 3)\, y' = -12x + 9$$

✓ Solution by Maple
Time used: 0.89 (sec). Leaf size: 44

```
dsolve((12*x+6*y(x)-9)+(5*x+2*y(x)-3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\operatorname{RootOf}\left(128\_Z^{25} c_1 x^5 + 640\_Z^{20} c_1 x^5 + 800\_Z^{15} c_1 x^5 - 1\right)^5 x - 4x + \frac{3}{2}$$

✓ Solution by Mathematica

Time used: 60.12 (sec). Leaf size: 1121

```
DSolve[(12*x+6*y[x]-9)+(5*x+2*y[x]-3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(3-5x) + \frac{1}{2\text{Root}\left[\#1^{10}\left(11664x^{10}+11664e^{60c_1}\right)-9720\#1^8x^8-1080\#1^7x^7+3105\#1^6x^6+666\#1^5x^5-425\#1\right.}$$

$$y(x) \to \frac{1}{2}(3-5x) + \frac{1}{2\text{Root}\left[\#1^{10}\left(11664x^{10}+11664e^{60c_1}\right)-9720\#1^8x^8-1080\#1^7x^7+3105\#1^6x^6+666\#1^5x^5-425\#1\right.}$$

$$y(x) \to \frac{1}{2}(3-5x) + \frac{1}{2\text{Root}\left[\#1^{10}\left(11664x^{10}+11664e^{60c_1}\right)-9720\#1^8x^8-1080\#1^7x^7+3105\#1^6x^6+666\#1^5x^5-425\#1\right.}$$

$$y(x) \to \frac{1}{2}(3-5x) + \frac{1}{2\text{Root}\left[\#1^{10}\left(11664x^{10}+11664e^{60c_1}\right)-9720\#1^8x^8-1080\#1^7x^7+3105\#1^6x^6+666\#1^5x^5-425\#1\right.}$$

$$y(x) \to \frac{1}{2}(3-5x) + \frac{1}{2\text{Root}\left[\#1^{10}\left(11664x^{10}+11664e^{60c_1}\right)-9720\#1^8x^8-1080\#1^7x^7+3105\#1^6x^6+666\#1^5x^5-425\#1\right.}$$

$$y(x) \to \frac{1}{2}(3-5x) + \frac{1}{2\text{Root}\left[\#1^{10}\left(11664x^{10}+11664e^{60c_1}\right)-9720\#1^8x^8-1080\#1^7x^7+3105\#1^6x^6+666\#1^5x^5-425\#1\right.}$$

$$y(x) \to \frac{1}{2}(3-5x) + \frac{1}{2\text{Root}\left[\#1^{10}\left(11664x^{10}+11664e^{60c_1}\right)-9720\#1^8x^8-1080\#1^7x^7+3105\#1^6x^6+666\#1^5x^5-425\#1\right.}$$

$$y(x) \to \frac{1}{2}(3-5x) + \frac{1}{2\text{Root}\left[\#1^{10}\left(11664x^{10}+11664e^{60c_1}\right)-9720\#1^8x^8-1080\#1^7x^7+3105\#1^6x^6+666\#1^5x^5-425\#1\right.}$$

$$y(x) \to \frac{1}{2}(3-5x) + \frac{1}{2\text{Root}\left[\#1^{10}\left(11664x^{10}+11664e^{60c_1}\right)-9720\#1^8x^8-1080\#1^7x^7+3105\#1^6x^6+666\#1^5x^5-425\#1\right.}$$

$$y(x) \to \frac{1}{2}(3-5x) + \frac{1}{2\text{Root}\left[\#1^{10}\left(11664x^{10}+11664e^{60c_1}\right)-9720\#1^8x^8-1080\#1^7x^7+3105\#1^6x^6+666\#1^5x^5-425\#1\right.}$$

1.3 problem Problem 3

Internal problem ID [12114]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$y'x - y - \sqrt{x^2 + y^2} = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 26

```
dsolve(x*diff(y(x),x)=y(x)+sqrt(x^2+y(x)^2),y(x), singsol=all)
```

$$\frac{-c_1 x^2 + y(x) + \sqrt{y(x)^2 + x^2}}{x^2} = 0$$

✓ Solution by Mathematica
Time used: 0.603 (sec). Leaf size: 27

```
DSolve[x*y'[x]==y[x]+Sqrt[x^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-c_1} \left(-1 + e^{2c_1} x^2\right)$$

1.4 problem Problem 4

Internal problem ID [12115]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + y = x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x)+y(x)=x^3,y(x), singsol=all)
```

$$y(x) = \frac{x^4 + 4c_1}{4x}$$

✓ Solution by Mathematica
Time used: 0.048 (sec). Leaf size: 19

```
DSolve[x*y'[x]+y[x]==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{4} + \frac{c_1}{x}$$

1.5 problem Problem 5

Internal problem ID [12116]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y - y'x - y'yx^2 = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 49

```
dsolve(y(x)-x*diff(y(x),x)=x^2*y(x)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \frac{-c_1 + \sqrt{c_1^2 + x^2}}{c_1 x}$$
$$y(x) = \frac{-c_1 - \sqrt{c_1^2 + x^2}}{c_1 x}$$

✓ Solution by Mathematica
Time used: 0.786 (sec). Leaf size: 68

```
DSolve[y[x]-x*y'[x]==x^2*y[x]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1 + \sqrt{\frac{1}{x^2}}x\sqrt{1 + c_1 x^2}}{x}$$
$$y(x) \to -\frac{1}{x} + \sqrt{\frac{1}{x^2}}\sqrt{1 + c_1 x^2}$$
$$y(x) \to 0$$

1.6 problem Problem 6

Internal problem ID [12117]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{x' + 3x = e^{2t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(x(t),t)+3*x(t)=exp(2*t),x(t), singsol=all)
```

$$x(t) = \frac{(e^{5t} + 5c_1) e^{-3t}}{5}$$

✓ Solution by Mathematica
Time used: 0.07 (sec). Leaf size: 23

```
DSolve[x'[t]+3*x[t]==Exp[2*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{e^{2t}}{5} + c_1 e^{-3t}$$

1.7 problem Problem 7

Internal problem ID [12118]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\sin(x)\, y + \cos(x)\, y' = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(y(x)*sin(x)+diff(y(x),x)*cos(x)=1,y(x), singsol=all)
```

$$y(x) = c_1 \cos(x) + \sin(x)$$

✓ Solution by Mathematica
Time used: 0.066 (sec). Leaf size: 13

```
DSolve[y[x]*Sin[x]+y'[x]*Cos[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x) + c_1 \cos(x)$$

1.8 problem Problem 8

Internal problem ID [12119]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - e^{-y+x} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)=exp(x-y(x)),y(x), singsol=all)
```

$$y(x) = \ln\left(e^x + c_1\right)$$

✓ Solution by Mathematica
Time used: 1.307 (sec). Leaf size: 12

```
DSolve[y'[x]==Exp[x-y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(e^x + c_1\right)$$

1.9 problem Problem 9

Internal problem ID [12120]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{-x + x' = \sin(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(x(t),t)=x(t)+sin(t),x(t), singsol=all)
```

$$x(t) = -\frac{\cos(t)}{2} - \frac{\sin(t)}{2} + c_1 e^t$$

✓ Solution by Mathematica
Time used: 0.073 (sec). Leaf size: 24

```
DSolve[x'[t]==x[t]+Sin[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{\sin(t)}{2} - \frac{\cos(t)}{2} + c_1 e^t$$

1.10 problem Problem 10

Internal problem ID [12121]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{x(\ln(x) - \ln(y))\, y' - y = 0}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 14

```
dsolve(x*(ln(x)-ln(y(x)))*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\text{LambertW}\left(c_1 x\, \mathrm{e}^{-1}\right)}{c_1}$$

✓ Solution by Mathematica
Time used: 7.587 (sec). Leaf size: 37

```
DSolve[x*(Log[x]-Log[y[x]])*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}
y(x) &\to -e^{c_1} W\left(-e^{-1-c_1} x\right) \\
y(x) &\to 0 \\
y(x) &\to \frac{x}{e}
\end{aligned}$$

1.11 problem Problem 11

Internal problem ID [12122]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 11.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy{y'}^{2}-\left(x^{2}+y^{2}\right)y'+yx=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x*y(x)*diff(y(x),x)^2-(x^2+y(x)^2)*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$\begin{aligned} y(x) &= c_1 x \\ y(x) &= \sqrt{x^2+c_1} \\ y(x) &= -\sqrt{x^2+c_1} \end{aligned}$$

✓ Solution by Mathematica
Time used: 0.17 (sec). Leaf size: 55

```
DSolve[x*y[x]*y'[x]^2-(x^2+y[x]^2)*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to c_1 x \\ y(x) &\to -\sqrt{x^2+2c_1} \\ y(x) &\to \sqrt{x^2+2c_1} \\ y(x) &\to -x \\ y(x) &\to x \end{aligned}$$

1.12 problem Problem 12

Internal problem ID [12123]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 12.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{{y'}^2 - 9y^4 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)^2=9*y(x)^4,y(x), singsol=all)
```

$$y(x) = \frac{1}{c_1 - 3x}$$
$$y(x) = \frac{1}{3x + c_1}$$

✓ Solution by Mathematica
Time used: 0.263 (sec). Leaf size: 34

```
DSolve[y'[x]^2==9*y[x]^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{3x + c_1}$$
$$y(x) \to \frac{1}{3x - c_1}$$
$$y(x) \to 0$$

1.13 problem Problem 13

Internal problem ID [12124]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{x' - e^{\frac{x}{t}} - \frac{x}{t} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(x(t),t)=exp(x(t)/t)+x(t)/t,x(t), singsol=all)
```

$$x(t) = \ln\left(-\frac{1}{\ln(t) + c_1}\right) t$$

✓ Solution by Mathematica
Time used: 0.54 (sec). Leaf size: 18

```
DSolve[x'[t]==Exp[x[t]/t]+x[t]/t,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -t \log(-\log(t) - c_1)$$

1.14 problem Problem 14

Internal problem ID [12125]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 14.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 = -x^2 + 1}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 43

```
dsolve(x^2+diff(y(x),x)^2=1,y(x), singsol=all)
```

$$y(x) = \frac{x\sqrt{-x^2+1}}{2} + \frac{\arcsin(x)}{2} + c_1$$
$$y(x) = -\frac{x\sqrt{-x^2+1}}{2} - \frac{\arcsin(x)}{2} + c_1$$

✓ Solution by Mathematica
Time used: 0.07 (sec). Leaf size: 85

```
DSolve[x^2+y'[x]^2==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arctan\left(\frac{\sqrt{1-x^2}}{x+1}\right) + \frac{1}{2}\sqrt{1-x^2}x + c_1$$
$$y(x) \to \arctan\left(\frac{\sqrt{1-x^2}}{x+1}\right) - \frac{1}{2}\sqrt{1-x^2}x + c_1$$

1.15 problem Problem 15

Internal problem ID [12126]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y - y'x - \frac{1}{y} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 27

```
dsolve(y(x)=x*diff(y(x),x)+1/y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{c_1 x^2 + 1}$$
$$y(x) = -\sqrt{c_1 x^2 + 1}$$

✓ Solution by Mathematica
Time used: 0.43 (sec). Leaf size: 53

```
DSolve[y[x]==x*y'[x]+1/y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{1 + e^{2c_1} x^2}$$
$$y(x) \to \sqrt{1 + e^{2c_1} x^2}$$
$$y(x) \to -1$$
$$y(x) \to 1$$

1.16 problem Problem 16

Internal problem ID [12127]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 16.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{-{y'}^{3}+y' = -x+2}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 211

```
dsolve(x=diff(y(x),x)^3-diff(y(x),x)+2,y(x), singsol=all)
```

$$y(x) = -\frac{\left(\int \frac{i\sqrt{3}\left(-216+108x+12\sqrt{81x^2-324x+312}\right)^{\frac{2}{3}}-12i\sqrt{3}+\left(-216+108x+12\sqrt{81x^2-324x+312}\right)^{\frac{2}{3}}+12}{\left(-216+108x+12\sqrt{81x^2-324x+312}\right)^{\frac{1}{3}}}dx\right)}{12} + c_1$$

$$y(x) = \frac{\left(\int \frac{\left(i\sqrt{3}-1\right)\left(-216+108x+12\sqrt{81x^2-324x+312}\right)^{\frac{2}{3}}-12i\sqrt{3}-12}{\left(-216+108x+12\sqrt{81x^2-324x+312}\right)^{\frac{1}{3}}}dx\right)}{12} + c_1$$

$$y(x) = \frac{\left(\int \frac{\left(-216+108x+12\sqrt{81x^2-324x+312}\right)^{\frac{2}{3}}+12}{\left(-216+108x+12\sqrt{81x^2-324x+312}\right)^{\frac{1}{3}}}dx\right)}{6} + c_1$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x==y'[x]^3-y'[x]+2,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

1.17 problem Problem 17

Internal problem ID [12128]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{y' - \frac{y}{x + y^3} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 224

```
dsolve(diff(y(x),x)=y(x)/(x+y(x)^3),y(x), singsol=all)
```

$$y(x) = \frac{\left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{2}{3}} - 6c_1}{3\left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{i\sqrt{3}\left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{2}{3}} + 6i\sqrt{3}\,c_1 + \left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{2}{3}} - 6c_1}{6\left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{i\sqrt{3}\left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{2}{3}} + 6i\sqrt{3}\,c_1 - \left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{2}{3}} + 6c_1}{6\left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 2.895 (sec). Leaf size: 263

```
DSolve[y'[x]==y[x]/(x+y[x]^3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2\ 3^{2/3} c_1 - \sqrt[3]{3}\left(-9x + \sqrt{81x^2 + 24c_1{}^3}\right)^{2/3}}{3\sqrt[3]{-9x + \sqrt{81x^2 + 24c_1{}^3}}}$$

$$y(x) \to \frac{\sqrt[3]{3}\left(1 - i\sqrt{3}\right)\left(-9x + \sqrt{81x^2 + 24c_1{}^3}\right)^{2/3} - 2\sqrt[6]{3}\left(\sqrt{3} + 3i\right)c_1}{6\sqrt[3]{-9x + \sqrt{81x^2 + 24c_1{}^3}}}$$

$$y(x) \to \frac{\sqrt[3]{3}\left(1 + i\sqrt{3}\right)\left(-9x + \sqrt{81x^2 + 24c_1{}^3}\right)^{2/3} - 2\sqrt[6]{3}\left(\sqrt{3} - 3i\right)c_1}{6\sqrt[3]{-9x + \sqrt{81x^2 + 24c_1{}^3}}}$$

$$y(x) \to 0$$

1.18 problem Problem 18

Internal problem ID [12129]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 18.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[_quadrature]`

$$y - {y'}^4 + {y'}^3 = -2$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 247

```
dsolve(y(x)=diff(y(x),x)^4-diff(y(x),x)^3-2,y(x), singsol=all)
```

$$y(x) = -2$$

$$y(x) = \frac{12\left(\frac{243}{16384} + \frac{\left(\frac{9}{64} - c_1 + x\right)\sqrt{64}\sqrt{\left(x - c_1 + \frac{9}{32}\right)(x - c_1)}}{16} + \frac{c_1^2}{2} + \left(-\frac{9}{64} - x\right)c_1 + \frac{x^2}{2} + \frac{9x}{64}\right)\left(27 - 192c_1 + 192x + 24\sqrt{64}\right.}{}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]==y'[x]^4-y'[x]^3-2,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

1.19 problem Problem 26

Internal problem ID [12130]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 26.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 + y^2 = 4}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 31

```
dsolve(diff(y(x),x)^2+y(x)^2=4,y(x), singsol=all)
```

$$\begin{aligned} y(x) &= -2 \\ y(x) &= 2 \\ y(x) &= -2\sin(c_1 - x) \\ y(x) &= 2\sin(c_1 - x) \end{aligned}$$

✓ Solution by Mathematica
Time used: 0.306 (sec). Leaf size: 43

```
DSolve[y'[x]^2+y[x]^2==4,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to 2\cos(x + c_1) \\ y(x) &\to 2\cos(x - c_1) \\ y(x) &\to -2 \\ y(x) &\to 2 \\ y(x) &\to \text{Interval}[\{-2, 2\}] \end{aligned}$$

1.20 problem Problem 28

Internal problem ID [12131]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{2y - x - 4}{2x - y + 5} = 0}$$

✓ Solution by Maple
Time used: 0.891 (sec). Leaf size: 117

```
dsolve(diff(y(x),x)=(2*y(x)-x-4)/(2*x-y(x)+5),y(x), singsol=all)
```

$y(x) =$

$$-\frac{\left(i\sqrt{3}-1\right)\left(3\sqrt{3}\,\sqrt{27c_1^2\left(x+2\right)^2-1}+27c_1(x+2)\right)^{\frac{2}{3}}-3i\sqrt{3}-3+6\left(3\sqrt{3}\,\sqrt{27c_1^2\left(x+2\right)^2-1}+2\right.}{6\left(3\sqrt{3}\,\sqrt{27c_1^2\left(x+2\right)^2-1}+27c_1\left(x+2\right)\right)^{\frac{1}{3}}c_1}$$

✓ Solution by Mathematica
Time used: 60.277 (sec). Leaf size: 1624

```
DSolve[y'[x]==(2*y[x]-x-4)/(2*x-y[x]+5),y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

1.21 problem Problem 29

Internal problem ID [12132]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Bernoulli]`

$$y' - \frac{y}{x+1} + y^2 = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x)-y(x)/(1+x)+y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{2+2x}{x^2+2c_1+2x}$$

✓ Solution by Mathematica
Time used: 0.297 (sec). Leaf size: 28

```
DSolve[y'[x]-y[x]/(1+x)+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2(x+1)}{x^2+2x+2c_1}$$
$$y(x) \to 0$$

1.22 problem Problem 30

Internal problem ID [12133]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{y' - y^2 = x}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 35

```
dsolve([diff(y(x),x)=x+y(x)^2,y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{3}\,\mathrm{AiryAi}\,(1, -x) + \mathrm{AiryBi}\,(1, -x)}{\sqrt{3}\,\mathrm{AiryAi}\,(-x) + \mathrm{AiryBi}\,(-x)}$$

✓ Solution by Mathematica
Time used: 1.869 (sec). Leaf size: 80

```
DSolve[{y'[x]==x+y[x]^2,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^{3/2}\,\mathrm{BesselJ}\left(-\frac{4}{3}, \frac{2x^{3/2}}{3}\right) - x^{3/2}\,\mathrm{BesselJ}\left(\frac{2}{3}, \frac{2x^{3/2}}{3}\right) + \mathrm{BesselJ}\left(-\frac{1}{3}, \frac{2x^{3/2}}{3}\right)}{2x\,\mathrm{BesselJ}\left(-\frac{1}{3}, \frac{2x^{3/2}}{3}\right)}$$

1.23 problem Problem 31

Internal problem ID [12134]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$\boxed{y' - y^3 x = x^2}$$

With initial conditions

$$[y(0) = 0]$$

✗ Solution by Maple

```
dsolve([diff(y(x),x)=x*y(x)^3+x^2,y(0) = 0],y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'[x]==x*y[x]^3+x^2,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.24 problem Problem 35

Internal problem ID [12135]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + y^2 = x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 44

```
dsolve(diff(y(x),x)=x^2-y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x\left(\text{BesselI}\left(-\frac{3}{4}, \frac{x^2}{2}\right) c_1 - \text{BesselK}\left(\frac{3}{4}, \frac{x^2}{2}\right)\right)}{c_1 \text{BesselI}\left(\frac{1}{4}, \frac{x^2}{2}\right) + \text{BesselK}\left(\frac{1}{4}, \frac{x^2}{2}\right)}$$

✓ Solution by Mathematica
Time used: 0.183 (sec). Leaf size: 197

```
DSolve[y'[x]==x^2-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{-ix^2\left(2\,\text{BesselJ}\left(-\frac{3}{4}, \frac{ix^2}{2}\right) + c_1\left(\text{BesselJ}\left(-\frac{5}{4}, \frac{ix^2}{2}\right) - \text{BesselJ}\left(\frac{3}{4}, \frac{ix^2}{2}\right)\right)\right) - c_1 \text{BesselJ}\left(-\frac{1}{4}, \frac{ix^2}{2}\right)}{2x\left(\text{BesselJ}\left(\frac{1}{4}, \frac{ix^2}{2}\right) + c_1 \text{BesselJ}\left(-\frac{1}{4}, \frac{ix^2}{2}\right)\right)}$$

$$y(x) \to \frac{ix^2 \text{BesselJ}\left(-\frac{5}{4}, \frac{ix^2}{2}\right) - ix^2 \text{BesselJ}\left(\frac{3}{4}, \frac{ix^2}{2}\right) + \text{BesselJ}\left(-\frac{1}{4}, \frac{ix^2}{2}\right)}{2x\,\text{BesselJ}\left(-\frac{1}{4}, \frac{ix^2}{2}\right)}$$

1.25 problem Problem 36

Internal problem ID [12136]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$2y + (x + y - 2)\, y' = -2x + 1$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 21

```
dsolve((2*x+2*y(x)-1)+(x+y(x)-2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -x - 3\,\mathrm{LambertW}\left(-\frac{c_1 e^{\frac{x}{3}-\frac{1}{3}}}{3}\right) - 1$$

✓ Solution by Mathematica
Time used: 5.15 (sec). Leaf size: 35

```
DSolve[(2*x+2*y[x]-1)+(x+y[x]-2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -3W\left(-e^{\frac{x}{3}-1+c_1}\right) - x - 1$$
$$y(x) \to -x - 1$$

1.26 problem Problem 37

Internal problem ID [12137]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 37.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^3 - y' e^{2x} = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)^3-diff(y(x),x)*exp(2*x)=0,y(x), singsol=all)
```

$$y(x) = -e^x + c_1$$
$$y(x) = e^x + c_1$$
$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 29

```
DSolve[y'[x]^3-y'[x]*Exp[2*x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1$$
$$y(x) \to -e^x + c_1$$
$$y(x) \to e^x + c_1$$

1.27 problem Problem 39

Internal problem ID [12138]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 39.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y - 5y'x + y'^2 = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 93

```
dsolve(y(x)=5*x*diff(y(x),x)-diff(y(x),x)^2,y(x), singsol=all)
```

$$-\frac{4\sqrt{2}\,c_1}{\left(10x-2\sqrt{25x^2-4y(x)}\right)^{\frac{5}{4}}}+\frac{4x}{9}+\frac{\sqrt{25x^2-4y(x)}}{9}=0$$
$$-\frac{4\sqrt{2}\,c_1}{\left(10x+2\sqrt{25x^2-4y(x)}\right)^{\frac{5}{4}}}+\frac{4x}{9}-\frac{\sqrt{25x^2-4y(x)}}{9}=0$$

✓ Solution by Mathematica

Time used: 60.449 (sec). Leaf size: 2233

```
DSolve[y[x]==5*x*y'[x]-y'[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

1.28 problem Problem 40

Internal problem ID [12139]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{y' + y^2 = x}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 37

```
dsolve([diff(y(x),x)=x-y(x)^2,y(1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{\operatorname{AiryBi}(1,1)\operatorname{AiryAi}(1,x) - \operatorname{AiryBi}(1,x)\operatorname{AiryAi}(1,1)}{\operatorname{AiryBi}(1,1)\operatorname{AiryAi}(x) - \operatorname{AiryBi}(x)\operatorname{AiryAi}(1,1)}$$

✓ Solution by Mathematica
Time used: 0.206 (sec). Leaf size: 229

```
DSolve[{y'[x]==x-y[x]^2,{y[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{i\left(x^{3/2}\left(-\operatorname{BesselJ}\left(-\frac{4}{3},\frac{2i}{3}\right)+i\operatorname{BesselJ}\left(-\frac{1}{3},\frac{2i}{3}\right)+\operatorname{BesselJ}\left(\frac{2}{3},\frac{2i}{3}\right)\right)\operatorname{BesselJ}\left(-\frac{2}{3},\frac{2}{3}ix^{3/2}\right)+x^{3/2}\operatorname{BesselJ}\left(-\right.\right.}{x\left(2\operatorname{BesselJ}\left(-\frac{2}{3},\frac{2i}{3}\right)\operatorname{BesselJ}\left(-\frac{1}{3},\frac{2}{3}ix^{3/2}\right)+\left(-\operatorname{BesselJ}\left(-\frac{4}{3},\frac{2}{3}\right.\right.\right.}$$

1.29 problem Problem 42

Internal problem ID [12140]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$\boxed{y' - (x-5y)^{\frac{1}{3}} = 2}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 80

```
dsolve(diff(y(x),x)=(x-5*y(x))^(1/3)+2,y(x), singsol=all)
```

$$x + \frac{81\ln(729 - 625y(x) + 125x)}{125} - \frac{27(x-5y(x))^{\frac{1}{3}}}{25} - \frac{81\ln\left(25(x-5y(x))^{\frac{2}{3}} - 45(x-5y(x))^{\frac{1}{3}} + 81\right)}{125} + \frac{162\ln\left(9 + 5(x-5y(x))^{\frac{1}{3}}\right)}{125} + \frac{3(x-5y(x))^{\frac{2}{3}}}{10} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.347 (sec). Leaf size: 70

```
DSolve[y'[x]==(x-5*y[x])^(1/3)+2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[5y(x) + 5\left(-y(x) + \frac{3}{50}(x-5y(x))^{2/3} - \frac{27}{125}\sqrt[3]{x-5y(x)} + \frac{243}{625}\log\left(5\sqrt[3]{x-5y(x)} + 9\right) + \frac{x}{5}\right) = c_1, y(x)\right]$$

1.30 problem Problem 43

Internal problem ID [12141]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y(-y+x)-x^2y'=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x-y(x))*y(x)-x^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\frac{x}{\ln(x)+c_1}$$

✓ Solution by Mathematica
Time used: 0.231 (sec). Leaf size: 19

```
DSolve[(x-y[x])*y[x]-x^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x}{\log(x)+c_1}$$
$$y(x)\to 0$$

1.31 problem Problem 45

Internal problem ID [12142]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{x' + 5x = 10t + 2}$$

With initial conditions

$$[x(1) = 2]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 7

```
dsolve([diff(x(t),t)+5*x(t)=10*t+2,x(1) = 2],x(t), singsol=all)
```

$$x(t) = 2t$$

✓ Solution by Mathematica
Time used: 0.082 (sec). Leaf size: 8

```
DSolve[{x'[t]+5*x[t]==10*t+2,{x[1]==2}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to 2t$$

1.32 problem Problem 46

Internal problem ID [12143]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _rational, _Bernoulli]`

$$\boxed{x' - \frac{x}{t} - \frac{x^2}{t^3} = 0}$$

With initial conditions

$$[x(2) = 4]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 7

```
dsolve([diff(x(t),t)=x(t)/t+x(t)^2/t^3,x(2) = 4],x(t), singsol=all)
```

$$x(t) = t^2$$

✓ Solution by Mathematica
Time used: 0.264 (sec). Leaf size: 8

```
DSolve[{x'[t]==x[t]/t+x[t]^2/t^3,{x[2]==4}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to t^2$$

1.33 problem Problem 47

Internal problem ID [12144]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 47.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y - y'x - y'^2 = 0}$$

With initial conditions

$$[y(2) = -1]$$

✓ Solution by Maple

Time used: 1.422 (sec). Leaf size: 17

```
dsolve([y(x)=x*diff(y(x),x)+diff(y(x),x)^2,y(2) = -1],y(x), singsol=all)
```

$$y(x) = 1 - x$$
$$y(x) = -\frac{x^2}{4}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 21

```
DSolve[{y[x]==x*y'[x]+y'[x]^2,{y[2]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 - x$$
$$y(x) \to -\frac{x^2}{4}$$

1.34 problem Problem 48

Internal problem ID [12145]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 48.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y - y'x - y'^2 = 0}$$

With initial conditions

$$[y(1) = -1]$$

✓ Solution by Maple
Time used: 0.391 (sec). Leaf size: 66

```
dsolve([y(x)=x*diff(y(x),x)+diff(y(x),x)^2,y(1) = -1],y(x), singsol=all)
```

$$y(x) = -\frac{1}{2} + \frac{i(-1+x)\sqrt{3}}{2} - \frac{x}{2}$$
$$y(x) = \frac{\left(1+i\sqrt{3}\right)\left(i\sqrt{3}-2x+1\right)}{4}$$
$$y(x) = \frac{\left(i\sqrt{3}-1\right)\left(i\sqrt{3}+2x-1\right)}{4}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 38

```
DSolve[{y[x]==x*y'[x]+y'[x]^2,{y[1]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (-1)^{2/3} - \sqrt[3]{-1}x$$
$$y(x) \to \sqrt[3]{-1}\left(\sqrt[3]{-1}x - 1\right)$$

1.35 problem Problem 49

Internal problem ID [12146]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$y' - \frac{3x - 4y - 2}{3x - 4y - 3} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)=(3*x-4*y(x)-2)/(3*x-4*y(x)-3),y(x), singsol=all)
```

$$y(x) = \frac{3x}{4} + \operatorname{LambertW}\left(\frac{c_1 \mathrm{e}^{-\frac{1}{4}+\frac{x}{4}}}{4}\right) + \frac{1}{4}$$

✓ Solution by Mathematica

Time used: 5.353 (sec). Leaf size: 41

```
DSolve[y'[x]==(3*x-4*y[x]-2)/(3*x-4*y[x]-3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to W\left(-e^{\frac{x}{4}-1+c_1}\right) + \frac{3x}{4} + \frac{1}{4}$$
$$y(x) \to \frac{1}{4}(3x + 1)$$

1.36 problem Problem 50

Internal problem ID [12147]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x' - x\cot(t) = 4\sin(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(x(t),t)-x(t)*cot(t)=4*sin(t),x(t), singsol=all)
```

$$x(t) = (4t + c_1)\sin(t)$$

✓ Solution by Mathematica
Time used: 0.073 (sec). Leaf size: 14

```
DSolve[x'[t]-x[t]*Cot[t]==4*Sin[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to (4t + c_1)\sin(t)$$

1.37 problem Problem 51

Internal problem ID [12148]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 51.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G']]`

$$\boxed{y - 2y'x - \frac{y'^2}{2} = x^2}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 79

```
dsolve(y(x)=x^2+2*diff(y(x),x)*x+(diff(y(x),x)^2)/2,y(x), singsol=all)
```

$$y(x) = -x^2$$
$$y(x) = -\frac{1}{2}x^2 + c_1x + \frac{1}{2}c_1^2$$
$$y(x) = -\frac{1}{2}x^2 - c_1x + \frac{1}{2}c_1^2$$
$$y(x) = -\frac{1}{2}x^2 - c_1x + \frac{1}{2}c_1^2$$
$$y(x) = -\frac{1}{2}x^2 + c_1x + \frac{1}{2}c_1^2$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]==x^2+2*y'[x]*x+(y'[x]^2)/2,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

1.38 problem Problem 52

Internal problem ID [12149]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y' - \frac{3y}{x} + y^2x^3 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)-3*y(x)/x+x^3*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{7x^3}{x^7 + 7c_1}$$

✓ Solution by Mathematica
Time used: 0.238 (sec). Leaf size: 25

```
DSolve[y'[x]-3*y[x]/x+x^3*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{7x^3}{x^7 + 7c_1}$$
$$y(x) \to 0$$

1.39 problem Problem 53

Internal problem ID [12150]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 53.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y\left(y'^2+1\right)=a$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 339

```
dsolve(y(x)*(1+diff(y(x),x)^2)=a,y(x), singsol=all)
```

$$y(x)=a$$

$$y(x) = \frac{(\text{RootOf}\left((\cos(\_Z)\,a+\_Za+2c_1-2x)\,(-\cos(\_Z)\,a+\_Za+2c_1-2x)\right)a-2x+2c_1)\tan(\text{RootOf}(}{2} + \frac{a}{2}$$

$$y(x) = \frac{(-\text{RootOf}\left((\cos(\_Z)\,a+\_Za+2c_1-2x)\,(-\cos(\_Z)\,a+\_Za+2c_1-2x)\right)a+2x-2c_1)\tan(\text{RootC}}{2} + \frac{a}{2}$$

$$y(x) = \frac{(\text{RootOf}\left((\cos(\_Z)\,a-\_Za+2c_1-2x)\,(-\cos(\_Z)\,a-\_Za+2c_1-2x)\right)a+2x-2c_1)\tan(\text{RootOf}(}{2} + \frac{a}{2}$$

$$y(x) = \frac{(-\text{RootOf}\left((\cos(\_Z)\,a-\_Za+2c_1-2x)\,(-\cos(\_Z)\,a-\_Za+2c_1-2x)\right)a-2x+2c_1)\tan(\text{RootC}}{2} + \frac{a}{2}$$

✓ Solution by Mathematica

Time used: 0.661 (sec). Leaf size: 106

```
DSolve[y[x]*(1+y'[x]^2)==a,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[a \arctan\left(\frac{\sqrt{\#1}}{\sqrt{a-\#1}}\right) - \sqrt{\#1}\sqrt{a-\#1}\&\right][-x+c_1]$$
$$y(x) \to \text{InverseFunction}\left[a \arctan\left(\frac{\sqrt{\#1}}{\sqrt{a-\#1}}\right) - \sqrt{\#1}\sqrt{a-\#1}\&\right][x+c_1]$$
$$y(x) \to a$$

1.40 problem Problem 54

Internal problem ID [12151]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 54.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{-y + (x^2y^2 + x)\, y' = -x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 346

```
dsolve((x^2-y(x))+(x^2*y(x)^2+x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{2^{\frac{1}{3}}\left(-\frac{2^{\frac{1}{3}}\left(\left(-3c_1x-3x^2+\sqrt{\frac{9c_1^2x^3+18x^4c_1+9x^5+4}{x}}\right)x^2\right)^{\frac{2}{3}}}{2}+x\right)}{\left(\left(-3c_1x-3x^2+\sqrt{\frac{9c_1^2x^3+18x^4c_1+9x^5+4}{x}}\right)x^2\right)^{\frac{1}{3}}x}$$

$$y(x) = -\frac{\left(\left(1+i\sqrt{3}\right)2^{\frac{1}{3}}\left(\left(-3c_1x-3x^2+\sqrt{\frac{9c_1^2x^3+18x^4c_1+9x^5+4}{x}}\right)x^2\right)^{\frac{2}{3}}+2i\sqrt{3}\,x-2x\right)2^{\frac{1}{3}}}{4\left(\left(-3c_1x-3x^2+\sqrt{\frac{9c_1^2x^3+18x^4c_1+9x^5+4}{x}}\right)x^2\right)^{\frac{1}{3}}x}$$

$$y(x) = \frac{\left(i\sqrt{3}-1\right)2^{\frac{2}{3}}\left(\left(-3c_1x-3x^2+\sqrt{\frac{9c_1^2x^3+18x^4c_1+9x^5+4}{x}}\right)x^2\right)^{\frac{2}{3}}+2\left(1+i\sqrt{3}\right)2^{\frac{1}{3}}x}{4\left(\left(-3c_1x-3x^2+\sqrt{\frac{9c_1^2x^3+18x^4c_1+9x^5+4}{x}}\right)x^2\right)^{\frac{1}{3}}x}$$

✓ Solution by Mathematica

Time used: 56.22 (sec). Leaf size: 400

```
DSolve[(x^2-y[x])+(x^2*y[x]^2+x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-2\sqrt[3]{2}x + \left(-6x^4 + 6c_1x^3 + 2\sqrt{x^3(9x^5 - 18c_1x^4 + 9c_1{}^2x^3 + 4)}\right)^{2/3}}{2x\sqrt[3]{-3x^4 + 3c_1x^3 + \sqrt{x^3(9x^5 - 18c_1x^4 + 9c_1{}^2x^3 + 4)}}}$$

$$y(x) \to \frac{i(\sqrt{3}+i)\left(-6x^4 + 6c_1x^3 + 2\sqrt{x^3(9x^5 - 18c_1x^4 + 9c_1{}^2x^3 + 4)}\right)^{2/3} + \sqrt[3]{2}(2+2i\sqrt{3})\,x}{4x\sqrt[3]{-3x^4 + 3c_1x^3 + \sqrt{x^3(9x^5 - 18c_1x^4 + 9c_1{}^2x^3 + 4)}}}$$

$$y(x) \to \frac{(-1-i\sqrt{3})\left(-6x^4 + 6c_1x^3 + 2\sqrt{x^3(9x^5 - 18c_1x^4 + 9c_1{}^2x^3 + 4)}\right)^{2/3} + \sqrt[3]{2}(2-2i\sqrt{3})\,x}{4x\sqrt[3]{-3x^4 + 3c_1x^3 + \sqrt{x^3(9x^5 - 18c_1x^4 + 9c_1{}^2x^3 + 4)}}}$$

1.41 problem Problem 55

Internal problem ID [12152]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 55.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{3y^2 + 2y(y^2 - 3x)\, y' = x}$$

✓ Solution by Maple
Time used: 0.156 (sec). Leaf size: 101

```
dsolve((3*y(x)^2-x)+(2*y(x))*(y(x)^2-3*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{2c_1 - 2\sqrt{c_1 (c_1 - 8x)} - 4x}}{2}$$
$$y(x) = \frac{\sqrt{2c_1 - 2\sqrt{c_1 (c_1 - 8x)} - 4x}}{2}$$
$$y(x) = -\frac{\sqrt{2c_1 + 2\sqrt{c_1 (c_1 - 8x)} - 4x}}{2}$$
$$y(x) = \frac{\sqrt{2c_1 + 2\sqrt{c_1 (c_1 - 8x)} - 4x}}{2}$$

✓ Solution by Mathematica

Time used: 15.503 (sec). Leaf size: 185

```
DSolve[(3*y[x]^2-x)+(2*y[x])*(y[x]^2-3*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-2x - e^{\frac{c_1}{2}}\sqrt{8x + e^{c_1}} - e^{c_1}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-2x - e^{\frac{c_1}{2}}\sqrt{8x + e^{c_1}} - e^{c_1}}}{\sqrt{2}}$$
$$y(x) \to -\frac{\sqrt{-2x + e^{\frac{c_1}{2}}\sqrt{8x + e^{c_1}} - e^{c_1}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-2x + e^{\frac{c_1}{2}}\sqrt{8x + e^{c_1}} - e^{c_1}}}{\sqrt{2}}$$

1.42 problem Problem 56

Internal problem ID [12153]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 56.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y(-y+x)-x^2y'=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x-y(x))*y(x)- x^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\frac{x}{\ln(x)+c_1}$$

✓ Solution by Mathematica
Time used: 0.242 (sec). Leaf size: 19

```
DSolve[(x-y[x])*y[x]- x^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{x}{\log(x)+c_1}$$
$$y(x)\to 0$$

1.43 problem Problem 57

Internal problem ID [12154]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 57.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{x + y - 3}{1 - x + y} = 0}$$

✓ Solution by Maple
Time used: 0.578 (sec). Leaf size: 30

```
dsolve(diff(y(x),x)= (x+y(x)-3)/(1-x+y(x)),y(x), singsol=all)
```

$$y(x) = \frac{-\sqrt{2 (x-2)^2 c_1^2 + 1} + (-1 + x) c_1}{c_1}$$

✓ Solution by Mathematica
Time used: 0.202 (sec). Leaf size: 59

```
DSolve[y'[x]== (x+y[x]-3)/(1-x+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -i\sqrt{-2x^2 + 8x - 1 - c_1} + x - 1$$
$$y(x) \to i\sqrt{-2x^2 + 8x - 1 - c_1} + x - 1$$

1.44 problem Problem 58

Internal problem ID [12155]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 58.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y'x - y^2 \ln(x) + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x*diff(y(x),x)-y(x)^2*ln(x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{1 + c_1 x + \ln(x)}$$

✓ Solution by Mathematica
Time used: 0.228 (sec). Leaf size: 20

```
DSolve[x*y'[x]-y[x]^2*Log[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{\log(x) + c_1 x + 1}$$
$$y(x) \to 0$$

1.45 problem Problem 59

Internal problem ID [12156]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 59.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2-1)\,y'+2yx=\cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve((x^2-1)*diff(y(x),x)+2*x*y(x)-cos(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{\sin(x)+c_1}{x^2-1}$$

✓ Solution by Mathematica
Time used: 0.063 (sec). Leaf size: 18

```
DSolve[(x^2-1)*y'[x]+2*x*y[x]-Cos[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{\sin(x)+c_1}{x^2-1}$$

1.46 problem Problem 60

Internal problem ID [12157]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 60.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{(4y + 2x + 3)\, y' - 2y = x + 1}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 20

```
dsolve((4*y(x)+2*x+3)*diff(y(x),x)-2*y(x)-x-1=0,y(x), singsol=all)
```

$$y(x) = -\frac{x}{2} + \frac{\mathrm{LambertW}\left(c_1 \mathrm{e}^{5+8x}\right)}{8} - \frac{5}{8}$$

✓ Solution by Mathematica
Time used: 6.325 (sec). Leaf size: 39

```
DSolve[(4*y[x]+2*x+3)*y'[x]-2*y[x]-x-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}\left(W\left(-e^{8x-1+c_1}\right) - 4x - 5\right)$$
$$y(x) \to \frac{1}{8}(-4x - 5)$$

1.47 problem Problem 61

Internal problem ID [12158]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{(-x+y^2)\,y'-y=-x^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 318

```
dsolve((y(x)^2-x)*diff(y(x),x)-y(x)+x^2=0,y(x), singsol=all)
```

$$y(x)=\frac{\left(-4x^3-12c_1+4\sqrt{x^6+(6c_1-4)\,x^3+9c_1^2}\right)^{\frac{2}{3}}+4x}{2\left(-4x^3-12c_1+4\sqrt{x^6+(6c_1-4)\,x^3+9c_1^2}\right)^{\frac{1}{3}}}$$

$$y(x)=\frac{i\left(-\left(-4x^3-12c_1+4\sqrt{x^6+(6c_1-4)\,x^3+9c_1^2}\right)^{\frac{2}{3}}+4x\right)\sqrt{3}-\left(-4x^3-12c_1+4\sqrt{x^6+(6c_1-4)\,x^3 -}\right.}{4\left(-4x^3-12c_1+4\sqrt{x^6+(6c_1-4)\,x^3+9c_1^2}\right)^{\frac{1}{3}}}$$

$$y(x)=\frac{i\left(\left(-4x^3-12c_1+4\sqrt{x^6+(6c_1-4)\,x^3+9c_1^2}\right)^{\frac{2}{3}}-4x\right)\sqrt{3}-\left(-4x^3-12c_1+4\sqrt{x^6+(6c_1-4)\,x^3+}\right.}{4\left(-4x^3-12c_1+4\sqrt{x^6+(6c_1-4)\,x^3+9c_1^2}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 4.856 (sec). Leaf size: 326

```
DSolve[(y[x]^2-x)*y'[x]-y[x]+x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2x + \sqrt[3]{2}\left(x^3 + \sqrt{x^6 + (-4 + 6c_1)x^3 + 9{c_1}^2} + 3c_1\right)^{2/3}}{2^{2/3}\sqrt[3]{x^3 + \sqrt{x^6 + (-4 + 6c_1)x^3 + 9{c_1}^2} + 3c_1}}$$

$$y(x) \to \frac{2^{2/3}(1 - i\sqrt{3})\left(x^3 + \sqrt{x^6 + (-4 + 6c_1)x^3 + 9{c_1}^2} + 3c_1\right)^{2/3} + \sqrt[3]{2}(2 + 2i\sqrt{3})\,x}{4\sqrt[3]{x^3 + \sqrt{x^6 + (-4 + 6c_1)x^3 + 9{c_1}^2} + 3c_1}}$$

$$y(x) \to \frac{2^{2/3}(1 + i\sqrt{3})\left(x^3 + \sqrt{x^6 + (-4 + 6c_1)x^3 + 9{c_1}^2} + 3c_1\right)^{2/3} + \sqrt[3]{2}(2 - 2i\sqrt{3})\,x}{4\sqrt[3]{x^3 + \sqrt{x^6 + (-4 + 6c_1)x^3 + 9{c_1}^2} + 3c_1}}$$

1.48 problem Problem 62

Internal problem ID [12159]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 62.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{(y^2 - x^2)\, y' + 2yx = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 47

```
dsolve((y(x)^2-x^2)*diff(y(x),x)+2*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1 - \sqrt{-4c_1^2x^2 + 1}}{2c_1}$$
$$y(x) = \frac{1 + \sqrt{-4c_1^2x^2 + 1}}{2c_1}$$

✓ Solution by Mathematica
Time used: 1.683 (sec). Leaf size: 66

```
DSolve[(y[x]^2-x^2)*y'[x]+2*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(e^{c_1} - \sqrt{-4x^2 + e^{2c_1}}\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{-4x^2 + e^{2c_1}} + e^{c_1}\right)$$
$$y(x) \to 0$$

1.49 problem Problem 63

Internal problem ID [12160]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 63.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _exact, _rational, _Bernoulli]`

$$\boxed{3xy^2y' + y^3 = 2x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 73

```
dsolve(3*x*y(x)^2*diff(y(x),x)+y(x)^3-2*x=0,y(x), singsol=all)
```

$$y(x) = \frac{((x^2 + c_1) x^2)^{\frac{1}{3}}}{x}$$
$$y(x) = -\frac{((x^2 + c_1) x^2)^{\frac{1}{3}} (1 + i\sqrt{3})}{2x}$$
$$y(x) = \frac{((x^2 + c_1) x^2)^{\frac{1}{3}} (i\sqrt{3} - 1)}{2x}$$

✓ Solution by Mathematica
Time used: 0.352 (sec). Leaf size: 72

```
DSolve[3*x*y[x]^2*y'[x]+y[x]^3-2*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{x^2 + c_1}}{\sqrt[3]{x}}$$
$$y(x) \to -\frac{\sqrt[3]{-1}\sqrt[3]{x^2 + c_1}}{\sqrt[3]{x}}$$
$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{x^2 + c_1}}{\sqrt[3]{x}}$$

1.50 problem Problem 64

Internal problem ID [12161]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 64.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$y'^2 + (x+a)\,y' - y = 0$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)^2+(x+a)*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{(a+x)^2}{4}$$
$$y(x) = c_1(c_1 + a + x)$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 26

```
DSolve[y'[x]^2+(x+a)*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(a + x + c_1)$$
$$y(x) \to -\frac{1}{4}(a+x)^2$$

1.51 problem Problem 65

Internal problem ID [12162]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 65.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{{y'}^{2}-2y'x+y=0}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 611

```
dsolve(diff(y(x),x)^2-2*x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\left(x^{2}+x\left(x^{3}+2\sqrt{3}\,\sqrt{-c_{1}\left(x^{3}-3c_{1}\right)}-6c_{1}\right)^{\frac{1}{3}}+\left(x^{3}+2\sqrt{3}\,\sqrt{-c_{1}\left(x^{3}-3c_{1}\right)}-6c_{1}\right)^{\frac{2}{3}}\right)\left(x^{2}-3x\left(x^{3}\right.\right.}{4\left(x^{3}+2\sqrt{3}\,\sqrt{-c_{1}\left(x^{3}-3c_{1}\right)}-\right.}$$

$$y(x) = -\frac{\left(i\sqrt{3}\left(x^{3}+2\sqrt{3}\,\sqrt{-c_{1}\left(x^{3}-3c_{1}\right)}-6c_{1}\right)^{\frac{2}{3}}-i\sqrt{3}\,x^{2}+\left(x^{3}+2\sqrt{3}\,\sqrt{-c_{1}\left(x^{3}-3c_{1}\right)}-6c_{1}\right)^{\frac{2}{3}}-2x\left(x\right.\right.}{}$$

$$y(x) = -\frac{\left(i\sqrt{3}\,x^{2}-i\sqrt{3}\left(x^{3}+2\sqrt{3}\,\sqrt{-c_{1}\left(x^{3}-3c_{1}\right)}-6c_{1}\right)^{\frac{2}{3}}+x^{2}-2x\left(x^{3}+2\sqrt{3}\,\sqrt{-c_{1}\left(x^{3}-3c_{1}\right)}-6c_{1}\right)^{\frac{1}{3}}\right.}{}$$

✓ Solution by Mathematica

Time used: 60.164 (sec). Leaf size: 954

```
DSolve[y'[x]^2-2*x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(x^2 + \frac{x(x^3+8e^{3c_1})}{\sqrt[3]{x^6 - 20e^{3c_1}x^3 + 8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3} - 8e^{6c_1}}} + \sqrt[3]{x^6 - 20e^{3c_1}x^3 + 8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3} - 8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(18x^2 - \frac{9i(\sqrt{3}-i)\,x(x^3+8e^{3c_1})}{\sqrt[3]{x^6 - 20e^{3c_1}x^3 + 8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3} - 8e^{6c_1}}} + 9i\left(\sqrt{3}+i\right)\sqrt[3]{x^6 - 20e^{3c_1}x^3 + 8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3} - 8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(18x^2 + \frac{9i(\sqrt{3}+i)\,x(x^3+8e^{3c_1})}{\sqrt[3]{x^6 - 20e^{3c_1}x^3 + 8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3} - 8e^{6c_1}}} - 9\left(1+i\sqrt{3}\right)\sqrt[3]{x^6 - 20e^{3c_1}x^3 + 8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3} - 8e^{6c_1}}\right)$$

$$y(x) \to \frac{x^4 + \left(x^6 + 20e^{3c_1}x^3 + 8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3} - 8e^{6c_1}\right)^{2/3} + x^2\sqrt[3]{x^6 + 20e^{3c_1}x^3 + 8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3} - 8\cdots}}{4\sqrt[3]{x^6 + 20e^{3c_1}x^3 + 8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3} - 8e^{6c_1}}}$$

$$y(x) \to \frac{1}{72}\left(18x^2 + \frac{9(1+i\sqrt{3})\,x(-x^3+8e^{3c_1})}{\sqrt[3]{x^6 + 20e^{3c_1}x^3 + 8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3} - 8e^{6c_1}}} + 9i\left(\sqrt{3}+i\right)\sqrt[3]{x^6 + 20e^{3c_1}x^3 + 8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3} - 8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(18x^2 + \frac{9i(\sqrt{3}+i)\,x(x^3-8e^{3c_1})}{\sqrt[3]{x^6 + 20e^{3c_1}x^3 + 8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3} - 8e^{6c_1}}} - 9\left(1+i\sqrt{3}\right)\sqrt[3]{x^6 + 20e^{3c_1}x^3 + 8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3} - 8e^{6c_1}}\right)$$

1.52 problem Problem 66

Internal problem ID [12163]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 66.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$y'^2 + 2yy' \cot(x) - y^2 = 0$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 39

```
dsolve(diff(y(x),x)^2+2*y(x)*diff(y(x),x)*cot(x)-y(x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{\operatorname{csgn}(\sin(x))\, c_1}{\cos(x) + \operatorname{csgn}(\sec(x))}$$
$$y(x) = \csc(x)^2 (\cos(x) + \operatorname{csgn}(\sec(x)))\operatorname{csgn}(\sin(x))\, c_1$$

✓ Solution by Mathematica
Time used: 0.251 (sec). Leaf size: 36

```
DSolve[y'[x]^2+2*y[x]*y'[x]*Cot[x]-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \csc^2\left(\frac{x}{2}\right)$$
$$y(x) \to c_1 \sec^2\left(\frac{x}{2}\right)$$
$$y(x) \to 0$$

2 Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172

2.1 problem Problem 1

Internal problem ID [12164]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 6y' + 10y = 100}$$

With initial conditions

$$[y(0) = 10, y'(0) = 5]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve([diff(y(x),x$2)-6*diff(y(x),x)+10*y(x)=100,y(0) = 10, D(y)(0) = 5],y(x), singsol=all)
```

$$y(x) = 5\,\mathrm{e}^{3x} \sin(x) + 10$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 17

```
DSolve[{y''[x]-6*y'[x]+10*y[x]==100,{y[0]==10,y'[0]==5}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to 5\left(e^{3x} \sin(x) + 2\right)$$

2.2 problem Problem 2

Internal problem ID [12165]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + x = \sin(t) - \cos(2t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(x(t),t$2)+x(t)=sin(t)-cos(2*t),x(t), singsol=all)
```

$$x(t) = \frac{\cos(2t)}{3} + \frac{(-t + 2c_1)\cos(t)}{2} + \frac{(1 + 4c_2)\sin(t)}{4}$$

✓ Solution by Mathematica
Time used: 0.076 (sec). Leaf size: 30

```
DSolve[x''[t]+x[t]==Sin[t]-Cos[2*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{3}\cos(2t) + \left(-\frac{t}{2} + c_1\right)\cos(t) + c_2\sin(t)$$

2.3 problem Problem 3

Internal problem ID [12166]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 3.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y' + y''' - 3y'' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x)+diff(y(x),x$3)-3*diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 \mathrm{e}^{\frac{(3+\sqrt{5})x}{2}} + c_3 \mathrm{e}^{-\frac{(\sqrt{5}-3)x}{2}}$$

✓ Solution by Mathematica
Time used: 0.384 (sec). Leaf size: 57

```
DSolve[y'[x]+y'''[x]-3*y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-\frac{1}{2}(\sqrt{5}-3)x}\left(\left(3+\sqrt{5}\right)c_1 - \left(\sqrt{5}-3\right)c_2 e^{\sqrt{5}x}\right) + c_3$$

2.4 problem Problem 4

Internal problem ID [12167]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \frac{1}{\sin(x)^3}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)+y(x)=1/sin(x)^3,y(x), singsol=all)
```

$$y(x) = (c_1 + \cot(x))\cos(x) + \sin(x)\, c_2 - \frac{\csc(x)}{2}$$

✓ Solution by Mathematica
Time used: 0.09 (sec). Leaf size: 25

```
DSolve[y''[x]+y[x]==1/Sin[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\csc(x)}{2} + c_2 \sin(x) + \cos(x)(\cot(x) + c_1)$$

2.5 problem Problem 5

Internal problem ID [12168]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 4y'x + 6y = 2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=2,y(x), singsol=all)
```

$$y(x) = c_2x^2 + c_1x^3 + \frac{1}{3}$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 21

```
DSolve[x^2*y''[x]-4*x*y'[x]+6*y[x]==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x^3 + c_1x^2 + \frac{1}{3}$$

2.6 problem Problem 6

Internal problem ID [12169]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \cosh(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+y(x)=cosh(x),y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + c_1 \cos(x) + \frac{e^{x}}{4} + \frac{e^{-x}}{4}$$

✓ Solution by Mathematica
Time used: 0.063 (sec). Leaf size: 22

```
DSolve[y''[x]+y[x]==Cosh[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\cosh(x)}{2} + c_1 \cos(x) + c_2 \sin(x)$$

2.7 problem Problem 7

Internal problem ID [12170]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{y'' + \frac{2y'^2}{1-y} = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)+2/(1-y(x))*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x + c_2 - 1}{c_1 x + c_2}$$

✓ Solution by Mathematica
Time used: 0.298 (sec). Leaf size: 37

```
DSolve[y''[x]+2/(1-y[x])*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 x - 1 + c_2 c_1}{c_1 (x + c_2)}$$
$$y(x) \to 1$$
$$y(x) \to \text{Indeterminate}$$

2.8 problem Problem 8

Internal problem ID [12171]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' - 4x' + 4x = e^t + e^{2t} + 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(x(t),t$2)-4*diff(x(t),t)+4*x(t)=exp(t)+exp(2*t)+1,x(t), singsol=all)
```

$$x(t) = \frac{(4c_1 t + 2t^2 + 4c_2)\,e^{2t}}{4} + e^t + \frac{1}{4}$$

✓ Solution by Mathematica
Time used: 0.245 (sec). Leaf size: 32

```
DSolve[x''[t]-4*x'[t]+4*x[t]==Exp[t]+Exp[2*t]+1,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{2t}\left(\frac{t^2}{2} + c_2 t + c_1\right) + e^t + \frac{1}{4}$$

2.9 problem Problem 9

Internal problem ID [12172]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$\boxed{(x^2+1)\, y'' + y'^2 = -1}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 33

```
dsolve((1+x^2)*diff(y(x),x$2)+diff(y(x),x)^2+1=0,y(x), singsol=all)
```

$$y(x) = \frac{\ln(c_1 x - 1)\, c_1^2 + c_2 c_1^2 + c_1 x + \ln(c_1 x - 1)}{c_1^2}$$

✓ Solution by Mathematica
Time used: 12.07 (sec). Leaf size: 33

```
DSolve[(1+x^2)*y''[x]+y'[x]^2+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\cot(c_1) + \csc^2(c_1)\log(-x\sin(c_1) - \cos(c_1)) + c_2$$

2.10 problem Problem 10

Internal problem ID [12173]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{x^3x'' = -1}$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 52

```
dsolve(x(t)^3*diff(x(t),t$2)+1=0,x(t), singsol=all)
```

$$x(t) = \frac{\sqrt{(1 + c_1 (c_2 + t)) (-1 + c_1 (c_2 + t)) c_1}}{c_1}$$
$$x(t) = -\frac{\sqrt{(1 + c_1 (c_2 + t)) (-1 + c_1 (c_2 + t)) c_1}}{c_1}$$

✓ Solution by Mathematica
Time used: 4.287 (sec). Leaf size: 93

```
DSolve[x[t]^3*x''[t]+1==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{\sqrt{c_1{}^2t^2 + 2c_2c_1{}^2t - 1 + c_2{}^2c_1{}^2}}{\sqrt{c_1}}$$
$$x(t) \to \frac{\sqrt{c_1{}^2t^2 + 2c_2c_1{}^2t - 1 + c_2{}^2c_1{}^2}}{\sqrt{c_1}}$$
$$x(t) \to \text{Indeterminate}$$

2.11 problem Problem 11

Internal problem ID [12174]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 11.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' - 16y = x^2 - \mathrm{e}^x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 53

```
dsolve(diff(y(x),x$4)-16*y(x)=x^2-exp(x),y(x), singsol=all)
```

$$y(x) = -\frac{\left(\left(\left(-16c_1 + \frac{1}{4}\right)\cos(2x) + x^2 - 16c_4\sin(2x)\right)\mathrm{e}^{2x} - 16c_3\mathrm{e}^{4x} - 16c_2 - \frac{16\,\mathrm{e}^{3x}}{15}\right)\mathrm{e}^{-2x}}{16}$$

✓ Solution by Mathematica
Time used: 0.299 (sec). Leaf size: 50

```
DSolve[y''''[x]-16*y[x]==x^2-Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2}{16} + \frac{e^x}{15} + c_1e^{2x} + c_3e^{-2x} + c_2\cos(2x) + c_4\sin(2x)$$

2.12 problem Problem 12

Internal problem ID [12175]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 12.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_3rd_order, _missing_x], [_3rd_order, _missing_y], [_3rd_ord`

✓ Solution by Maple
Time used: 0.469 (sec). Leaf size: 51

```
dsolve(diff(y(x),x$3)^2+diff(y(x),x$2)^2=1,y(x), singsol=all)
```

$$y(x) = -\frac{1}{2}x^2 + c_1x + c_2$$
$$y(x) = c_2 + c_1x + \frac{1}{2}x^2$$
$$y(x) = c_1 + c_2x + \sqrt{-c_3^2 + 1}\,\sin(x) + c_3\cos(x)$$

✓ Solution by Mathematica
Time used: 0.348 (sec). Leaf size: 54

```
DSolve[y'''[x]^2+y''[x]^2==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3x - \cos(x - c_1) + c_2$$
$$y(x) \to c_3x - \cos(x + c_1) + c_2$$
$$y(x) \to \text{Interval}[\{-1, 1\}] + c_3x + c_2$$

2.13 problem Problem 13

Internal problem ID [12176]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 13.
ODE order: 6.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{x^{(6)} - x'''' = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(x(t),t$6)-diff(x(t),t$4)=1,x(t), singsol=all)
```

$$x(t) = -\frac{t^4}{24} + \mathrm{e}^{-t} c_1 + c_2 \mathrm{e}^t + \frac{c_3 t^3}{6} + \frac{c_4 t^2}{2} + c_5 t + c_6$$

✓ Solution by Mathematica
Time used: 0.103 (sec). Leaf size: 45

```
DSolve[x''''''[t]-x''''[t]==1,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{t^4}{24} + c_6 t^3 + c_5 t^2 + c_4 t + c_1 e^t + c_2 e^{-t} + c_3$$

2.14 problem Problem 14

Internal problem ID [12177]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 14.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$x'''' - 2x'' + x = t^2 - 3$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(x(t),t$4)-2*diff(x(t),t$2)+x(t)=t^2-3,x(t), singsol=all)
```

$$x(t) = (c_4 t + c_2) e^{-t} + (c_3 t + c_1) e^{t} + t^2 + 1$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 38

```
DSolve[x''''[t]-2*x''[t]+x[t]==t^2-3,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to t^2 + c_2 e^{-t} t + c_1 e^{-t} + e^{t}(c_4 t + c_3) + 1$$

2.15 problem Problem 15

Internal problem ID [12178]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + 4yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)+4*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{2x^3}{3}\right) y(0) + \left(x - \frac{1}{3}x^4\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]+4*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x - \frac{x^4}{3}\right) + c_1\left(1 - \frac{2x^3}{3}\right)$$

2.16 problem Problem 16

Internal problem ID [12179]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + y'x + \left(9x^2 - \frac{1}{25}\right)y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(9*x^2-1/25)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}\left(\frac{1}{5}, 3x\right) + c_2 \operatorname{BesselY}\left(\frac{1}{5}, 3x\right)$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 26

```
DSolve[x^2*y''[x]+x*y'[x]+(9*x^2-1/25)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}\left(\frac{1}{5}, 3x\right) + c_2 \operatorname{BesselY}\left(\frac{1}{5}, 3x\right)$$

2.17 problem Problem 17

Internal problem ID [12180]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{y'' + y'^2 = 1}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 5

```
dsolve([diff(y(x),x$2)+diff(y(x),x)^2=1,y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = x$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 6

```
DSolve[{y''[x]+y'[x]^2==1,{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x$$

2.18 problem Problem 18

Internal problem ID [12181]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' - 3\sqrt{y} = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 2]$$

✓ Solution by Maple
Time used: 0.281 (sec). Leaf size: 11

```
dsolve([diff(y(x),x$2)=3*sqrt(y(x)),y(0) = 1, D(y)(0) = 2],y(x), singsol=all)
```

$$y(x) = \frac{(x+2)^4}{16}$$

✓ Solution by Mathematica
Time used: 0.1 (sec). Leaf size: 14

```
DSolve[{y''[x]==3*Sqrt[y[x]],{y[0]==1,y'[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{16}(x+2)^4$$

2.19 problem Problem 19

Internal problem ID [12182]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y = 1 - \frac{1}{\sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+y(x)=1-1/sin(x),y(x), singsol=all)
```

$$y(x) = -\sin(x)\ln(\sin(x)) + \cos(x)(c_1 + x) + \sin(x) c_2 + 1$$

✓ Solution by Mathematica
Time used: 0.082 (sec). Leaf size: 25

```
DSolve[y''[x]+y[x]==1-1/Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_1)\cos(x) + \sin(x)(-\log(\sin(x)) + c_2) + 1$$

2.20 problem Problem 20

Internal problem ID [12183]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{u'' + \frac{2u'}{r} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(u(r),r$2)+2/r*diff(u(r),r)=0,u(r), singsol=all)
```

$$u(r) = c_1 + \frac{c_2}{r}$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 15

```
DSolve[u''[r]+2/r*u'[r]==0,u[r],r,IncludeSingularSolutions -> True]
```

$$u(r) \to c_2 - \frac{c_1}{r}$$

2.21 problem Problem 30

Internal problem ID [12184]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _with_linear_symmetries], [_2nd_orde`

$$yy'' + y'^2 - \frac{yy'}{\sqrt{x^2+1}} = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 63

```
dsolve(y(x)*diff(y(x),x$2)+diff(y(x),x)^2= y(x)*diff(y(x),x)/sqrt(1+x^2),y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \sqrt{c_1 x\sqrt{x^2+1} + c_1 x^2 + c_1 \operatorname{arcsinh}(x) + 2c_2}$$
$$y(x) = -\sqrt{c_1 x\sqrt{x^2+1} + c_1 x^2 + c_1 \operatorname{arcsinh}(x) + 2c_2}$$

✓ Solution by Mathematica

Time used: 60.936 (sec). Leaf size: 73

```
DSolve[y[x]*y''[x]+y'[x]^2== y[x]*y'[x]/Sqrt[1+x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \exp\left(\int_1^x \frac{1}{-K[1]c_1 + \sqrt{K[1]^2+1}c_1 + K[1] + \left(K[1] - \sqrt{K[1]^2+1}\right)\log\left(\sqrt{K[1]^2+1} - K[1]\right)} dK[1$$

2.22 problem Problem 31

Internal problem ID [12185]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 31.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$yy'y'' - y'^3 - y''^2 = 0$$

✓ Solution by Maple
Time used: 7.281 (sec). Leaf size: 42

```
dsolve(y(x)*diff(y(x),x)*diff(y(x),x$2)=diff(y(x),x)^3+diff(y(x),x$2)^2,y(x), singsol=all)
```

$$y(x) = -\frac{4}{-4c_1 + x}$$
$$y(x) = c_1$$
$$y(x) = e^{-c_1(c_2+x)} - c_1$$
$$y(x) = e^{c_1(c_2+x)} + c_1$$

✓ Solution by Mathematica

Time used: 13.794 (sec). Leaf size: 119

```
DSolve[y[x]*y'[x]*y''[x]==y'[x]^3+y''[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(e^{-\frac{1}{2}(1+e^{c_1})(x+c_2)} - 1 - e^{c_1}\right)$$

$$y(x) \to \frac{1 + e^{\frac{x+c_2}{-1+\tanh\left(\frac{c_1}{2}\right)}}}{-1 + \tanh\left(\frac{c_1}{2}\right)}$$

$$y(x) \to -\frac{1}{2} - \frac{1}{2}e^{-\frac{x}{2}-\frac{c_2}{2}}$$

$$y(x) \to \frac{1}{2}\left(-1 + e^{-\frac{x}{2}-\frac{c_2}{2}}\right)$$

2.23 problem Problem 32

Internal problem ID [12186]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 9x = t\sin(3t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(x(t),t$2)+9*x(t)=t*sin(3*t),x(t), singsol=all)
```

$$x(t) = \frac{(-3t^2 + 36c_1)\cos(3t)}{36} + \frac{\sin(3t)(t + 36c_2)}{36}$$

✓ Solution by Mathematica
Time used: 0.223 (sec). Leaf size: 38

```
DSolve[x''[t]+9*x[t]==t*Sin[3*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \left(-\frac{t^2}{12} + \frac{1}{216} + c_1\right)\cos(3t) + \frac{1}{36}(t + 36c_2)\sin(3t)$$

2.24 problem Problem 33

Internal problem ID [12187]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + y = \sinh(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=sinh(x),y(x), singsol=all)
```

$$y(x) = \frac{(-2x^2 + (8c_1 + 2)x + 8c_2 + 1)\,\mathrm{e}^{-x}}{8} + \frac{\mathrm{e}^{x}}{8}$$

✓ Solution by Mathematica
Time used: 0.103 (sec). Leaf size: 34

```
DSolve[y''[x]+2*y'[x]+y[x]==Sinh[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}e^{-x}(-2x^2 + e^{2x} + 8c_2x + 8c_1)$$

2.25 problem Problem 34

Internal problem ID [12188]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 34.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - y = e^x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$3)-y(x)=exp(x),y(x), singsol=all)
```

$$y(x) = c_2 e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right) + c_3 e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right) + \frac{e^x (x + 3c_1)}{3}$$

✓ Solution by Mathematica
Time used: 0.726 (sec). Leaf size: 62

```
DSolve[y'''[x]-y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} e^{-x/2} \left(e^{3x/2}(x - 1 + 3c_1) + 3c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + 3c_3 \sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

2.26 problem Problem 35

Internal problem ID [12189]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + 2y = x\,\mathrm{e}^{x} \cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x) +2*y(x)=x*exp(x)*cos(x),y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{x}\left(\left(x^{2}+4c_{2}-1\right)\sin(x)+\cos(x)\left(4c_{1}+x\right)\right)}{4}$$

✓ Solution by Mathematica
Time used: 0.074 (sec). Leaf size: 37

```
DSolve[y''[x]-2*y'[x] +2*y[x]==x*Exp[x]*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}e^{x}\left(\left(2x^{2}-1+8c_{1}\right)\sin(x)+2(x+4c_{2})\cos(x)\right)$$

2.27 problem Problem 36

Internal problem ID [12190]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-1)\,y''-6y=1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 52

```
dsolve((x^2-1)*diff(y(x),x$2)-6*y(x)=1,y(x), singsol=all)
```

$$y(x)=-\frac{1}{6}+\frac{3(x^3-x)\,c_1\ln(-1+x)}{4}+\frac{3c_1(-x^3+x)\ln(1+x)}{4}+c_2x^3+\frac{3c_1x^2}{2}-c_2x-c_1$$

✓ Solution by Mathematica
Time used: 0.086 (sec). Leaf size: 67

```
DSolve[(x^2-1)*y''[x]-6*y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{12}\big(-9c_2x(x^2-1)\log(1-x)+9c_2x(x^2-1)\log(x+1)+2(6c_1x^3-9c_2x^2-6c_1x-1+6c_2)\big)$$

2.28 problem Problem 40(a)

Internal problem ID [12191]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 40(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{mx'' - f(x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 62

```
dsolve(m*diff(x(t),t$2)=f(x(t)),x(t), singsol=all)
```

$$m\left(\int^{x(t)} \frac{1}{\sqrt{m\left(c_1 m + 2\left(\int f(\_b)\, d\_b\right)\right)}} d\_b\right) - t - c_2 = 0$$
$$-m\left(\int^{x(t)} \frac{1}{\sqrt{m\left(c_1 m + 2\left(\int f(\_b)\, d\_b\right)\right)}} d\_b\right) - t - c_2 = 0$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 44

```
DSolve[m*x''[t]==f[x[t]],x[t],t,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{x(t)} \frac{1}{\sqrt{c_1 + 2\int_1^{K[2]} \frac{f(K[1])}{m} dK[1]}} dK[2]^2 = (t + c_2)^2, x(t)\right]$$

2.29 problem Problem 40(b)

Internal problem ID [12192]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 40(b).
ODE order: 2.
ODE degree: 0.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{mx'' - f(x') = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve(m*diff(x(t),t$2)=f(diff(x(t),t)),x(t), singsol=all)
```

$$x(t) = \int \operatorname{RootOf}\left(t - m\left(\int^{\_Z} \frac{1}{f(\_f)} d\_f\right) + c_1\right) dt + c_2$$

✓ Solution by Mathematica
Time used: 2.257 (sec). Leaf size: 39

```
DSolve[m*x''[t]==f[x'[t]],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \int_1^t \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{f(K[1])} dK[1]\&\right]\left[c_1 + \frac{K[2]}{m}\right] dK[2] + c_2$$

2.30 problem Problem 41

Internal problem ID [12193]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 41.
ODE order: 6.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y^{(6)} - 3y^{(5)} + 3y'''' - y''' = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 48

```
dsolve(diff(y(x),x$6)-3*diff(y(x),x$5)+3*diff(y(x),x$4)-diff(y(x),x$3)=x,y(x), singsol=all)
```

$$y(x) = \left(c_3 x^2 + (c_2 - 6c_3)\, x + c_1 - 3c_2 + 12c_3\right) \mathrm{e}^x - \frac{x^4}{24} - \frac{x^3}{2} + \frac{c_4 x^2}{2} + c_5 x + c_6$$

✓ Solution by Mathematica
Time used: 0.256 (sec). Leaf size: 61

```
DSolve[y''''''[x]-3*y'''''[x]+3*y''''[x]-y'''[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^4}{24} - \frac{x^3}{2} + c_6 x^2 + c_3 e^x \left(x^2 - 6x + 12\right) + c_5 x + c_1 e^x + c_2 e^x (x - 3) + c_4$$

2.31 problem Problem 42

Internal problem ID [12194]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 42.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{x'''' + 2x'' + x = \cos(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(x(t),t$4)+2*diff(x(t),t$2)+x(t)=cos(t),x(t), singsol=all)
```

$$x(t) = \frac{(8c_3t - t^2 + 8c_1 + 2)\cos(t)}{8} + \left(\left(c_4 + \frac{3}{8}\right)t + c_2\right)\sin(t)$$

✓ Solution by Mathematica
Time used: 0.071 (sec). Leaf size: 43

```
DSolve[x''''[t]+2*x''[t]+x[t]==Cos[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \left(-\frac{t^2}{8} + c_2t + \frac{5}{16} + c_1\right)\cos(t) + \frac{1}{4}(t + 4c_4t + 4c_3)\sin(t)$$

2.32 problem Problem 43

Internal problem ID [12195]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 43.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{(1+x)^2 y'' + (1+x) y' + y = 2\cos(\ln(1+x))}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 24

```
dsolve((1+x)^2*diff(y(x),x$2)+(1+x)*diff(y(x),x)+y(x)=2*cos(ln(1+x)),y(x), singsol=all)
```

$$y(x) = (c_2 + \ln(1+x)) \sin(\ln(1+x)) + \cos(\ln(1+x)) c_1$$

✓ Solution by Mathematica
Time used: 0.192 (sec). Leaf size: 31

```
DSolve[(1+x)^2*y''[x]+(1+x)*y'[x]+y[x]==2*Cos[Log[1+x]],y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \left(\frac{1}{2} + c_1\right) \cos(\log(x+1)) + (\log(x+1) + c_2) \sin(\log(x+1))$$

2.33 problem Problem 47

Internal problem ID [12196]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 47.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{x^3y'' - y'x + y = 0}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve([x^3*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,x],singsol=all)
```

$$y(x) = \left(\mathrm{e}^{-\frac{1}{x}}c_1 + c_2\right)x$$

✓ Solution by Mathematica
Time used: 0.095 (sec). Leaf size: 20

```
DSolve[x^3*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(c_2e^{-1/x} + c_1\right)$$

2.34 problem Problem 49

Internal problem ID [12197]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 49.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{x'''' + x = t^3}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 67

```
dsolve(diff(x(t),t$4)+x(t)=t^3,x(t), singsol=all)
```

$$x(t) = \left(c_2 e^{-\frac{\sqrt{2}\,t}{2}} + c_4 e^{\frac{\sqrt{2}\,t}{2}}\right) \sin\left(\frac{\sqrt{2}\,t}{2}\right) + t^3 + c_1 e^{-\frac{\sqrt{2}\,t}{2}} \cos\left(\frac{\sqrt{2}\,t}{2}\right) + c_3 e^{\frac{\sqrt{2}\,t}{2}} \cos\left(\frac{\sqrt{2}\,t}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.008 (sec). Leaf size: 78

```
DSolve[x''''[t]+x[t]==t^3,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{-\frac{t}{\sqrt{2}}} \left(e^{\frac{t}{\sqrt{2}}} t^3 + \left(c_1 e^{\sqrt{2}t} + c_2\right) \cos\left(\frac{t}{\sqrt{2}}\right) + \left(c_4 e^{\sqrt{2}t} + c_3\right) \sin\left(\frac{t}{\sqrt{2}}\right)\right)$$

2.35 problem Problem 50

Internal problem ID [12198]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 50.
ODE order: 2.
ODE degree: 3.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$y''^3 + y'' = x - 1$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 226

```
dsolve(diff(y(x),x$2)^3+diff(y(x),x$2)+1=x,y(x), singsol=all)
```

$$y(x) = \frac{\left(\int \int \frac{\left(-108+108x+12\sqrt{81x^2-162x+93}\right)^{\frac{2}{3}}-12}{\left(-108+108x+12\sqrt{81x^2-162x+93}\right)^{\frac{1}{3}}}dxdx\right)}{6} + c_1 x + c_2$$

$$y(x) = -\frac{\left(\int \int \frac{i\sqrt{3}\left(-108+108x+12\sqrt{81x^2-162x+93}\right)^{\frac{2}{3}}+12i\sqrt{3}+\left(-108+108x+12\sqrt{81x^2-162x+93}\right)^{\frac{2}{3}}-12}{\left(-108+108x+12\sqrt{81x^2-162x+93}\right)^{\frac{1}{3}}}dxdx\right)}{12} + c_1 x + c_2$$

$$y(x) = \frac{\left(\int \int \frac{\left(i\sqrt{3}-1\right)\left(-108+108x+12\sqrt{81x^2-162x+93}\right)^{\frac{2}{3}}+12i\sqrt{3}+12}{\left(-108+108x+12\sqrt{81x^2-162x+93}\right)^{\frac{1}{3}}}dxdx\right)}{12} + c_1 x + c_2$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]^3+y''[x]+1==x,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

2.36 problem Problem 51

Internal problem ID [12199]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 51.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 10x' + 25x = 2^t + t\,\mathrm{e}^{-5t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 40

```
dsolve(diff(x(t),t$2)+10*diff(x(t),t)+25*x(t)=2^t+t*exp(-5*t),x(t), singsol=all)
```

$$x(t) = \frac{(\ln(2)+5)^2\,(t^3+6c_1t+6c_2)\,\mathrm{e}^{-5t}+6\,2^t}{6\,(\ln(2)+5)^2}$$

✓ Solution by Mathematica
Time used: 0.341 (sec). Leaf size: 72

```
DSolve[x''[t]+10*x'[t]+25*x[t]==2^t+t*Exp[-5*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{e^{-5t}\left(t^3\left(25+\log^2(2)+\log(1024)\right)+3\,2^{t+1}e^{5t}+c_2t\left(150+6\log^2(2)+\log(1152921504606846976)\right)+c_{\cdot}\right.}{6(5+\log(2))^2}$$

2.37 problem Problem 52

Internal problem ID [12200]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 52.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _with_linear_symmetries], [_2nd_orde`

$$\boxed{xyy'' - xy'^2 - yy' = 0}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 17

```
dsolve(x*y(x)*diff(y(x),x$2)-x*diff(y(x),x)^2-y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \mathrm{e}^{\frac{c_1 x^2}{2}} c_2$$

✓ Solution by Mathematica
Time used: 0.196 (sec). Leaf size: 19

```
DSolve[x*y[x]*y''[x]-x*y'[x]^2-y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 e^{\frac{c_1 x^2}{2}}$$

2.38 problem Problem 53

Internal problem ID [12201]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 53.
ODE order: 6.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$\boxed{y^{(6)} - y = e^{2x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 65

```
dsolve(diff(y(x),x$6)-y(x)=exp(2*x),y(x), singsol=all)
```

$$y(x) = e^{-x}\left(\left(c_3 e^{\frac{x}{2}} + c_5 e^{\frac{3x}{2}}\right)\cos\left(\frac{\sqrt{3}\,x}{2}\right) + \left(e^{\frac{x}{2}} c_4 + c_6 e^{\frac{3x}{2}}\right)\sin\left(\frac{\sqrt{3}\,x}{2}\right) + e^{2x} c_1 + \frac{e^{3x}}{63} + c_2\right)$$

✓ Solution by Mathematica
Time used: 0.88 (sec). Leaf size: 85

```
DSolve[y''''''[x]-y[x]==Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{2x}}{63} + c_1 e^x + c_4 e^{-x} + e^{-x/2}(c_2 e^x + c_3)\cos\left(\frac{\sqrt{3}x}{2}\right) + e^{-x/2}(c_6 e^x + c_5)\sin\left(\frac{\sqrt{3}x}{2}\right)$$

2.39 problem Problem 54

Internal problem ID [12202]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 54.
ODE order: 6.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$y^{(6)} + 2y'''' + y'' = x + e^x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 46

```
dsolve(diff(y(x),x$6)+2*diff(y(x),x$4)+diff(y(x),x$2)=x+exp(x),y(x), singsol=all)
```

$$y(x) = (-c_3x - c_1 - 2c_4)\cos(x) + (-c_4x - c_2 + 2c_3)\sin(x) + \frac{x^3}{6} + c_5x + c_6 + \frac{e^x}{4}$$

✓ Solution by Mathematica
Time used: 0.61 (sec). Leaf size: 58

```
DSolve[y''''''[x]+2*y''''[x]+y''[x]==x+Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{6} + \frac{e^x}{4} + c_6x - (c_2x + c_1 + 2c_4)\cos(x) + (-c_4x + 2c_2 - c_3)\sin(x) + c_5$$

2.40 problem Problem 55

Internal problem ID [12203]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 55.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_high_order, _missing_x], [_high_order, _missing_y], [_high_`

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 25

```
dsolve(6*diff(y(x),x$2)*diff(y(x),x$4)-5*diff(y(x),x$3)^2=0,y(x), singsol=all)
```

$$y(x) = c_1x + c_2$$
$$y(x) = \frac{(c_2 + x)^8 c_1}{2612736} + c_3x + c_4$$

✓ Solution by Mathematica
Time used: 0.266 (sec). Leaf size: 26

```
DSolve[6*y''[x]*y''''[x]-5*y'''[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{56}c_2(x - 6c_1)^8 + c_4x + c_3$$

2.41 problem Problem 56

Internal problem ID [12204]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 56.
ODE order: 2.
ODE degree: 0.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' - y' \ln\left(\frac{y'}{x}\right) = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 31

```
dsolve(x*diff(y(x),x$2)=diff(y(x),x)*ln(diff(y(x),x)/x),y(x), singsol=all)
```

$$y(x) = \frac{e^{c_1x+1}c_1x + c_2c_1^2 - e^{c_1x+1}}{c_1^2}$$

✓ Solution by Mathematica
Time used: 0.905 (sec). Leaf size: 31

```
DSolve[x*y''[x]==y'[x]*Log[y'[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{e^{c_1}x+1-2c_1}(-1 + e^{c_1}x) + c_2$$

2.42 problem Problem 57

Internal problem ID [12205]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 57.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sin(3x)\cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)+y(x)=sin(3*x)*cos(x),y(x), singsol=all)
```

$$y(x) = \sin(x)\, c_2 + c_1 \cos(x) - \frac{\sin(2x)}{6} - \frac{\sin(4x)}{30}$$

✓ Solution by Mathematica
Time used: 0.187 (sec). Leaf size: 30

```
DSolve[y''[x]+y[x]==Sin[3*x]*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(x) - \frac{1}{15}\sin(x)(6\cos(x) + \cos(3x) - 15c_2)$$

2.43 problem Problem 58

Internal problem ID [12206]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 58.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' - 2y^3 = 0}$$

With initial conditions

$$[y(1) = 1, y'(1) = 1]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 11

```
dsolve([diff(y(x),x$2)=2*y(x)^3,y(1) = 1, D(y)(1) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{1}{x-2}$$

✓ Solution by Mathematica
Time used: 0.143 (sec). Leaf size: 12

```
DSolve[{y''[x]==2*y[x]^3,{y[1]==1,y'[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2-x}$$

2.44 problem Problem 59

Internal problem ID [12207]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 59.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1],`

$$\boxed{yy'' - y'^2 - y' = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 20

```
dsolve(y(x)*diff(y(x),x$2)-diff(y(x),x)^2=diff(y(x),x),y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{e^{c_1(c_2+x)} + 1}{c_1}$$

✓ Solution by Mathematica
Time used: 2.51 (sec). Leaf size: 26

```
DSolve[y[x]*y''[x]-y'[x]^2==y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1 + e^{c_1(x+c_2)}}{c_1}$$
$$y(x) \to \text{Indeterminate}$$

3 Chapter 3, SYSTEMS OF DIFFERENTIAL EQUATIONS. Problems page 209

3.1 problem Problem 1

Internal problem ID [12208]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 3, SYSTEMS OF DIFFERENTIAL EQUATIONS. Problems page 209
Problem number: Problem 1.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = y(t)$$
$$y'(t) = -x(t)$$

With initial conditions

$$[x(0) = 0, y(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve([diff(x(t),t) = y(t), diff(y(t),t) = -x(t), x(0) = 0, y(0) = 1], singsol=all)
```

$$x(t) = \sin(t)$$
$$y(t) = \cos(t)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 31

```
DSolve[{x'[t]==y[t],y'[t]==-x[t]},{},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 \cos(t) + c_2 \sin(t)$$
$$y(t) \to c_2 \cos(t) - c_1 \sin(t)$$

3.2 problem Problem 3

Internal problem ID [12209]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 3, SYSTEMS OF DIFFERENTIAL EQUATIONS. Problems page 209
Problem number: Problem 3.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -5x(t) - y(t) + e^t$$
$$y'(t) = x(t) + 3y(t) + e^{2t}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 102

```
dsolve([diff(x(t),t)+5*x(t)+y(t)=exp(t),diff(y(t),t)-x(t)-3*y(t)=exp(2*t)],singsol=all)
```

$$x(t) = e^{\left(\sqrt{15}-1\right)t} c_2 + e^{-\left(1+\sqrt{15}\right)t} c_1 + \frac{e^{2t}}{6} + \frac{2\,e^t}{11}$$
$$y(t) = -e^{\left(\sqrt{15}-1\right)t} c_2 \sqrt{15} + e^{-\left(1+\sqrt{15}\right)t} c_1 \sqrt{15} - 4\,e^{\left(\sqrt{15}-1\right)t} c_2 - 4\,e^{-\left(1+\sqrt{15}\right)t} c_1 - \frac{e^t}{11} - \frac{7\,e^{2t}}{6}$$

✓ Solution by Mathematica
Time used: 4.39 (sec). Leaf size: 206

```
DSolve[{x'[t]+5*x[t]+y[t]==Exp[t],y'[t]-x[t]-3*y[t]==Exp[2*t]},{x[t],y[t]},t,IncludeSingular
```

$$x(t) \to \frac{1}{330} e^{-\left(\left(1+\sqrt{15}\right)t\right)} \left(60e^{\left(2+\sqrt{15}\right)t} + 55e^{\left(3+\sqrt{15}\right)t} - 11\left(\left(4\sqrt{15}-15\right)c_1 + \sqrt{15}c_2\right)e^{2\sqrt{15}t} + 11\left(\left(15+4\sqrt{15}\right)c_1 + \sqrt{15}c_2\right)\right)$$
$$y(t) \to -\frac{1}{330} e^{-\left(\left(1+\sqrt{15}\right)t\right)} \left(30e^{\left(2+\sqrt{15}\right)t} + 385e^{\left(3+\sqrt{15}\right)t} - 11\left(\sqrt{15}c_1 + \left(15+4\sqrt{15}\right)c_2\right)e^{2\sqrt{15}t} + 11\left(\sqrt{15}c_1 + \left(4\sqrt{15}-15\right)c_2\right)\right)$$

3.3 problem Problem 4

Internal problem ID [12210]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 3, SYSTEMS OF DIFFERENTIAL EQUATIONS. Problems page 209
Problem number: Problem 4.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= y(t) \\
y'(t) &= z(t) \\
z'(t) &= x(t)
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 176

```
dsolve([diff(x(t),t)=y(t),diff(y(t),t)=z(t),diff(z(t),t)=x(t)],singsol=all)
```

$$
\begin{aligned}
x(t) &= c_1 e^t + c_2 e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}\, t}{2}\right) + c_3 e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}\, t}{2}\right) \\
y(t) &= c_1 e^t - \frac{c_2 e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}\, t}{2}\right)}{2} + \frac{c_2 e^{-\frac{t}{2}} \sqrt{3}\, \cos\left(\frac{\sqrt{3}\, t}{2}\right)}{2} \\
&\quad - \frac{c_3 e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}\, t}{2}\right)}{2} - \frac{c_3 e^{-\frac{t}{2}} \sqrt{3}\, \sin\left(\frac{\sqrt{3}\, t}{2}\right)}{2} \\
z(t) &= c_1 e^t - \frac{c_2 e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}\, t}{2}\right)}{2} - \frac{c_2 e^{-\frac{t}{2}} \sqrt{3}\, \cos\left(\frac{\sqrt{3}\, t}{2}\right)}{2} \\
&\quad - \frac{c_3 e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}\, t}{2}\right)}{2} + \frac{c_3 e^{-\frac{t}{2}} \sqrt{3}\, \sin\left(\frac{\sqrt{3}\, t}{2}\right)}{2}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.063 (sec). Leaf size: 234

```
DSolve[{x'[t]==y[t],y'[t]==z[t],z'[t]==x[t]},{x[t],y[t],z[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{1}{3}e^{-t/2}\left((c_1+c_2+c_3)e^{3t/2}+(2c_1-c_2-c_3)\cos\left(\frac{\sqrt{3}t}{2}\right)+\sqrt{3}(c_2-c_3)\sin\left(\frac{\sqrt{3}t}{2}\right)\right)$$
$$y(t) \to \frac{1}{3}e^{-t/2}\left((c_1+c_2+c_3)e^{3t/2}-(c_1-2c_2+c_3)\cos\left(\frac{\sqrt{3}t}{2}\right)-\sqrt{3}(c_1-c_3)\sin\left(\frac{\sqrt{3}t}{2}\right)\right)$$
$$z(t) \to \frac{1}{3}e^{-t/2}\left((c_1+c_2+c_3)e^{3t/2}-(c_1+c_2-2c_3)\cos\left(\frac{\sqrt{3}t}{2}\right)+\sqrt{3}(c_1-c_2)\sin\left(\frac{\sqrt{3}t}{2}\right)\right)$$

3.4 problem Problem 5

Internal problem ID [12211]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 3, SYSTEMS OF DIFFERENTIAL EQUATIONS. Problems page 209
Problem number: Problem 5.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = y(t)$$
$$y'(t) = \frac{y(t)^2}{x(t)}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 20

```
dsolve([diff(x(t),t)=y(t),diff(y(t),t)=y(t)^2/x(t)],singsol=all)
```

$$\{x(t) = e^{c_1 t} c_2\}$$
$$\left\{y(t) = \frac{d}{dt}x(t)\right\}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 28

```
DSolve[{x'[t]==y[t],y'[t]==y[t]^2/x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 c_2 e^{c_1 t}$$
$$x(t) \to c_2 e^{c_1 t}$$