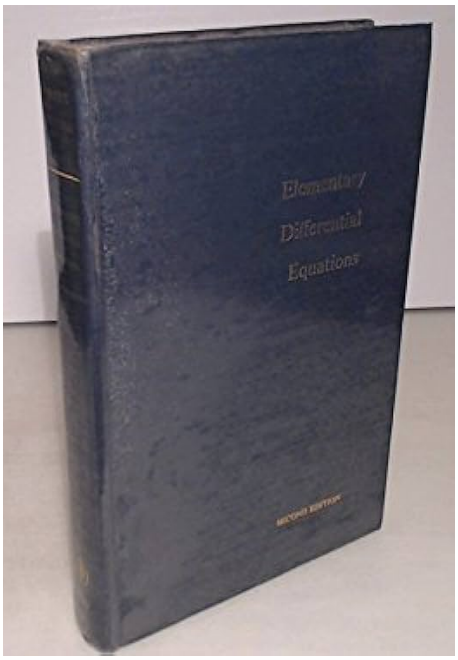
A Solution Manual For

Elementary Differential equations, Chaundy, 1969



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1 Exercises 3, page 60

1.1 problem 1(a)

Internal problem ID [3029]

Book: Elementary Differential equations, Chaundy, 1969
Section: Exercises 3, page 60
Problem number: 1(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yy' = x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)*y(x)=x,y(x), singsol=all)
```

$$y(x) = \sqrt{x^2 + c_1}$$
$$y(x) = -\sqrt{x^2 + c_1}$$

✓ Solution by Mathematica
Time used: 0.091 (sec). Leaf size: 35

```
DSolve[y'[x]*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x^2 + 2c_1}$$
$$y(x) \to \sqrt{x^2 + 2c_1}$$

1.2 problem 1(b)

Internal problem ID [3030]

Book: Elementary Differential equations, Chaundy, 1969
Section: Exercises 3, page 60
Problem number: 1(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - y = x^3}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)-y(x)=x^3,y(x), singsol=all)
```

$$y(x) = -x^3 - 3x^2 - 6x - 6 + \mathrm{e}^x c_1$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 26

```
DSolve[y'[x]-y[x]==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^3 - 3x^2 - 6x + c_1 e^x - 6$$

1.3 problem 1(c)

Internal problem ID [3031]

Book: Elementary Differential equations, Chaundy, 1969
Section: Exercises 3, page 60
Problem number: 1(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\cot(x) = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+y(x)*cot(x)=x,y(x), singsol=all)
```

$$y(x) = -\cot(x)\,x + 1 + \csc(x)\,c_1$$

✓ Solution by Mathematica
Time used: 0.049 (sec). Leaf size: 17

```
DSolve[y'[x]+y[x]*Cot[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\cot(x) + c_1\csc(x) + 1$$

1.4 problem 1(d)

Internal problem ID [3032]

Book: Elementary Differential equations, Chaundy, 1969
Section: Exercises 3, page 60
Problem number: 1(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\cot(x) = \tan(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)+y(x)*cot(x)=tan(x),y(x), singsol=all)
```

$$y(x) = \csc(x)\left(-\sin(x) + \ln(\sec(x) + \tan(x)) + c_1\right)$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 18

```
DSolve[y'[x]+y[x]*Cot[x]==Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \csc(x)\text{arctanh}(\sin(x)) + c_1 \csc(x) - 1$$

1.5 problem 1(e)

Internal problem ID [3033]

Book: Elementary Differential equations, Chaundy, 1969
Section: Exercises 3, page 60
Problem number: 1(e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y \tan(x) = \cot(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+y(x)*tan(x)=cot(x),y(x), singsol=all)
```

$$y(x) = (-\ln(\csc(x) + \cot(x)) + c_1)\cos(x)$$

✓ Solution by Mathematica
Time used: 0.058 (sec). Leaf size: 16

```
DSolve[y'[x]+y[x]*Tan[x]==Cot[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x)(-\text{arctanh}(\cos(x)) + c_1)$$

1.6 problem 1(f)

Internal problem ID [3034]

Book: Elementary Differential equations, Chaundy, 1969
Section: Exercises 3, page 60
Problem number: 1(f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y \ln(x) = x^{-x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)+y(x)*ln(x)=x^(-x),y(x), singsol=all)
```

$$y(x) = (\mathrm{e}^{x} c_1 - 1)\, x^{-x}$$

✓ Solution by Mathematica
Time used: 0.08 (sec). Leaf size: 19

```
DSolve[y'[x]+y[x]*Log[x]==x^(-x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^{-x}(-1 + c_1 e^{x})$$

1.7 problem 2(a)

Internal problem ID [3035]

Book: Elementary Differential equations, Chaundy, 1969
Section: Exercises 3, page 60
Problem number: 2(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' + y = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x*diff(y(x),x)+y(x)=x,y(x), singsol=all)
```

$$y(x) = \frac{x}{2} + \frac{c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 17

```
DSolve[x*y'[x]+y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{2} + \frac{c_1}{x}$$

1.8 problem 2(b)

Internal problem ID [3036]

Book: Elementary Differential equations, Chaundy, 1969
Section: Exercises 3, page 60
Problem number: 2(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' - y = x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x*diff(y(x),x)-y(x)=x^3,y(x), singsol=all)
```

$$y(x) = \frac{(x^2 + 2c_1)\, x}{2}$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 17

```
DSolve[x*y'[x]-y[x]==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{2} + c_1 x$$

1.9 problem 2(c)

Internal problem ID [3037]

Book: Elementary Differential equations, Chaundy, 1969
Section: Exercises 3, page 60
Problem number: 2(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' + ny = x^n}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x*diff(y(x),x)+n*y(x)=x^n,y(x), singsol=all)
```

$$y(x) = \frac{x^n}{2n} + x^{-n} c_1$$

✓ Solution by Mathematica
Time used: 0.05 (sec). Leaf size: 24

```
DSolve[x*y'[x]+n*y[x]==x^n,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^n}{2n} + c_1 x^{-n}$$

1.10 problem 2(d)

Internal problem ID [3038]

Book: Elementary Differential equations, Chaundy, 1969
Section: Exercises 3, page 60
Problem number: 2(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' - ny = x^n}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x)-n*y(x)=x^n,y(x), singsol=all)
```

$$y(x) = (\ln(x) + c_1)\, x^n$$

✓ Solution by Mathematica
Time used: 0.044 (sec). Leaf size: 14

```
DSolve[x*y'[x]-n*y[x]==x^n,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^n(\log(x) + c_1)$$

1.11 problem 2(e)

Internal problem ID [3039]

Book: Elementary Differential equations, Chaundy, 1969
Section: Exercises 3, page 60
Problem number: 2(e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\left(x^3+x\right)y'+y=x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve((x^3+x)*diff(y(x),x)+y(x)=x,y(x), singsol=all)
```

$$y(x)=\frac{\sqrt{x^2+1}\,c_1-1}{x}$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 23

```
DSolve[(x^3+x)*y'[x]+y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{-1+c_1\sqrt{x^2+1}}{x}$$

1.12 problem 3(a)

Internal problem ID [3040]

Book: Elementary Differential equations, Chaundy, 1969
Section: Exercises 3, page 60
Problem number: 3(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\cot(x)\,y' + y = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(cot(x)*diff(y(x),x)+y(x)=x,y(x), singsol=all)
```

$$y(x) = x + \cos(x)\,(-\ln(\sec(x) + \tan(x)) + c_1)$$

✓ Solution by Mathematica
Time used: 0.081 (sec). Leaf size: 45

```
DSolve[Cot[x]*y'[x]+y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + \cos(x)\left(\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) - \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right) + c_1\right)$$

1.13 problem 3(b)

Internal problem ID [3041]

Book: Elementary Differential equations, Chaundy, 1969
Section: Exercises 3, page 60
Problem number: 3(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\cot(x)\,y' + y = \tan(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(cot(x)*diff(y(x),x)+y(x)=tan(x),y(x), singsol=all)
```

$$y(x) = \frac{\tan(x)}{2} - \frac{\cos(x)\ln(\sec(x)+\tan(x))}{2} + \cos(x)\,c_1$$

✓ Solution by Mathematica
Time used: 0.071 (sec). Leaf size: 25

```
DSolve[Cot[x]*y'[x]+y[x]==Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(\cos(x)(-\operatorname{arctanh}(\sin(x))) + \tan(x) + 2c_1\cos(x))$$

1.14 problem 3(c)

Internal problem ID [3042]

Book: Elementary Differential equations, Chaundy, 1969
Section: Exercises 3, page 60
Problem number: 3(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' \tan(x) + y = \cot(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(tan(x)*diff(y(x),x)+y(x)=cot(x),y(x), singsol=all)
```

$$y(x) = \csc(x) (\cos(x) + \ln(\csc(x) - \cot(x)) + c_1)$$

✓ Solution by Mathematica
Time used: 0.085 (sec). Leaf size: 29

```
DSolve[Tan[x]*y'[x]+y[x]==Cot[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \csc(x) \left(\cos(x) + \log\left(\sin\left(\frac{x}{2} \right) \right) - \log\left(\cos\left(\frac{x}{2} \right) \right) + c_1 \right)$$

1.15 problem 3(a)

Internal problem ID [3043]

Book: Elementary Differential equations, Chaundy, 1969
Section: Exercises 3, page 60
Problem number: 3(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' \tan(x) - y = -\cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(tan(x)*diff(y(x),x)=y(x)-cos(x),y(x), singsol=all)
```

$$y(x) = \left(\cot(x) - \frac{\pi}{2} + x + c_1\right) \sin(x)$$

✓ Solution by Mathematica
Time used: 0.061 (sec). Leaf size: 28

```
DSolve[Tan[x]*y'[x]==y[x]-Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x)\, \text{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, -\tan^2(x)\right) + c_1 \sin(x)$$

1.16 problem 4(a)

Internal problem ID [3044]

Book: Elementary Differential equations, Chaundy, 1969
Section: Exercises 3, page 60
Problem number: 4(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \cos(x)\, y = \sin(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+y(x)*cos(x)=sin(2*x),y(x), singsol=all)
```

$$y(x) = 2\sin(x) - 2 + \mathrm{e}^{-\sin(x)} c_1$$

✓ Solution by Mathematica
Time used: 0.083 (sec). Leaf size: 20

```
DSolve[y'[x]+y[x]*Cos[x]==Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\sin(x) + c_1 e^{-\sin(x)} - 2$$

1.17 problem 4(b)

Internal problem ID [3045]

Book: Elementary Differential equations, Chaundy, 1969
Section: Exercises 3, page 60
Problem number: 4(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' \cos(x) + y = \sin(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(cos(x)*diff(y(x),x)+y(x)=sin(2*x),y(x), singsol=all)
```

$$y(x) = \frac{(\cos(x) - \sin(x) + 1)(-2\sin(x) - 2\ln(\sin(x) - 1) + c_1)}{\cos(x) + \sin(x) + 1}$$

✓ Solution by Mathematica
Time used: 0.091 (sec). Leaf size: 42

```
DSolve[Cos[x]*y'[x]+y[x]==Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2\operatorname{arctanh}\left(\tan\left(\frac{x}{2}\right)\right)}\left(-2\sin(x) - 4\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + c_1\right)$$

1.18 problem 4(c)

Internal problem ID [3046]

Book: Elementary Differential equations, Chaundy, 1969
Section: Exercises 3, page 60
Problem number: 4(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y \sin(x) = \sin(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+y(x)*sin(x)=sin(2*x),y(x), singsol=all)
```

$$y(x) = 2\cos(x) + 2 + \mathrm{e}^{\cos(x)} c_1$$

✓ Solution by Mathematica
Time used: 0.075 (sec). Leaf size: 18

```
DSolve[y'[x]+y[x]*Sin[x]==Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\cos(x) + c_1 e^{\cos(x)} + 2$$

1.19 problem 4(d)

Internal problem ID [3047]

Book: Elementary Differential equations, Chaundy, 1969
Section: Exercises 3, page 60
Problem number: 4(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' \sin(x) + y = \sin(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(sin(x)*diff(y(x),x)+y(x)=sin(2*x),y(x), singsol=all)
```

$$y(x) = \csc(x) \left(-2\cos(x) + 2\ln(\cos(x)+1) + c_1\right)(\cos(x)+1)$$

✓ Solution by Mathematica
Time used: 0.288 (sec). Leaf size: 38

```
DSolve[Sin[x]*y'[x]+y[x]==Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\operatorname{arctanh}(\cos(x))}\left(-2\sqrt{\sin^2(x)}\csc(x)\left(\cos(x)+\log\left(\sec^2\left(\frac{x}{2}\right)\right)\right)+c_1\right)$$

1.20 problem 5(a)

Internal problem ID [3048]

Book: Elementary Differential equations, Chaundy, 1969
Section: Exercises 3, page 60
Problem number: 5(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\sqrt{x^2+1}\,y' + y = 2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(sqrt(1+x^2)*diff(y(x),x)+y(x)=2*x,y(x), singsol=all)
```

$$y(x) = \frac{x^2 + x\sqrt{x^2+1} - \operatorname{arcsinh}(x) + c_1}{x + \sqrt{x^2+1}}$$

✓ Solution by Mathematica
Time used: 0.136 (sec). Leaf size: 50

```
DSolve[Sqrt[1+x^2]*y'[x]+y[x]==2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\sqrt{x^2+1} - x\right)\left(x^2 + \sqrt{x^2+1}x + \log\left(\sqrt{x^2+1} - x\right) + c_1\right)$$

1.21 problem 5(b)

Internal problem ID [3049]

Book: Elementary Differential equations, Chaundy, 1969
Section: Exercises 3, page 60
Problem number: 5(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\sqrt{x^2+1}\,y' - y = 2\sqrt{x^2+1}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(sqrt(1+x^2)*diff(y(x),x)-y(x)=2*sqrt(1+x^2),y(x), singsol=all)
```

$$y(x) = \left(x\sqrt{x^2+1} + \operatorname{arcsinh}(x) - x^2 + c_1\right)\left(x + \sqrt{x^2+1}\right)$$

✓ Solution by Mathematica
Time used: 0.082 (sec). Leaf size: 55

```
DSolve[Sqrt[1+x^2]*y'[x]-y[x]==2*Sqrt[1+x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2 - \sqrt{x^2+1}\,x + \log\left(\sqrt{x^2+1} - x\right) - c_1}{x - \sqrt{x^2+1}}$$

1.22 problem 5(c)

Internal problem ID [3050]

Book: Elementary Differential equations, Chaundy, 1969
Section: Exercises 3, page 60
Problem number: 5(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\sqrt{(x+a)\,(x+b)}\,(2y'-3)+y=0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 60

```
dsolve(sqrt((x+a)*(x+b))*(2*diff(y(x),x)-3)+y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{3\left(\int\sqrt{2a+2b+4x+4\sqrt{(x+a)\,(x+b)}}dx\right)+4c_1}{2\sqrt{2a+2b+4x+4\sqrt{(x+a)\,(x+b)}}}$$

✓ Solution by Mathematica
Time used: 0.433 (sec). Leaf size: 115

```
DSolve[Sqrt[(x+a)*(x+b)]*(2*y'[x]-3)+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$
$$\to\exp\left(-\frac{\sqrt{a+x}\sqrt{b+x}\operatorname{arctanh}\left(\frac{\sqrt{b+x}}{\sqrt{a+x}}\right)}{\sqrt{(a+x)(b+x)}}\right)\left(\int_1^x\frac{3}{2}\exp\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b+K[1]}}{\sqrt{a+K[1]}}\right)\sqrt{a+K[1]}\sqrt{b+K[1]}}{\sqrt{(a+K[1])(b+K[1])}}\right)dK+c_1\right)$$

1.23 problem 5(d)

Internal problem ID [3051]

Book: Elementary Differential equations, Chaundy, 1969
Section: Exercises 3, page 60
Problem number: 5(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\sqrt{(x+a)(x+b)}\,y' + y = \sqrt{x+a} - \sqrt{x+b}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 114

```
dsolve(sqrt((x+a)*(x+b))*diff(y(x),x)+y(x)=sqrt(x+a)-sqrt(x+b),y(x), singsol=all)
```

$$y(x) = \frac{2\left((2a+2x)\sqrt{x+a} + (-2b-2x)\sqrt{x+b} + 3c_1\right)\sqrt{(x+a)(x+b)} + 6\left(-\frac{b}{3} + a + \frac{2x}{3}\right)(x+b)\sqrt{x+a} +}{\sqrt{(x+a)(x+b)}\left(3a+3b+6x+6\sqrt{(x+a)(x+b)}\right)}$$

✓ Solution by Mathematica

Time used: 2.411 (sec). Leaf size: 145

```
DSolve[Sqrt[(x+a)*(x+b)]*y'[x]+y[x]==Sqrt[x+a]-Sqrt[x+b],y[x],x,IncludeSingularSolutions ->
```

$$y(x)$$

$$\to \exp\left(-\frac{2\sqrt{a+x}\sqrt{b+x}\operatorname{arctanh}\left(\frac{\sqrt{b+x}}{\sqrt{a+x}}\right)}{\sqrt{(a+x)(b+x)}}\right)\left(\int_1^x \frac{\exp\left(\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b+K[1]}}{\sqrt{a+K[1]}}\right)\sqrt{a+K[1]}\sqrt{b+K[1]}}{\sqrt{(a+K[1])(b+K[1])}}\right)\left(\sqrt{a+K[1]}\right.}{\sqrt{(a+K[1])(b+K[1])}}\right.$$

$$\left.\left.+c_1\right)\right.$$