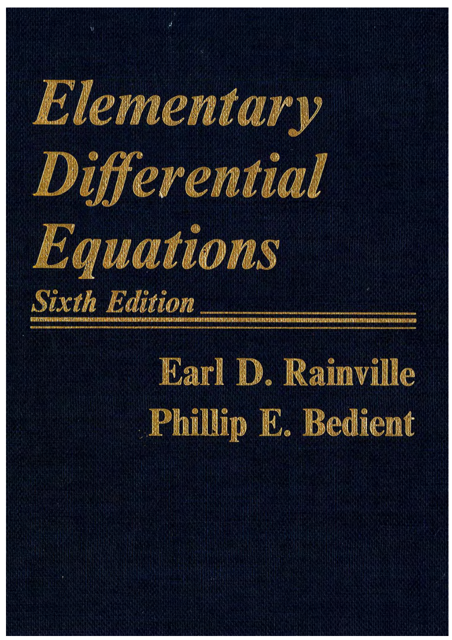
A Solution Manual For

Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.



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May 16, 2024

Contents

1 CHAPTER 16. Nonlinear equations. Section 94. Factoring the left member. EXERCISES Page 309

1.1 problem 1

Internal problem ID [6767]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 94. Factoring the left member. EXERCISES Page 309
Problem number: 1.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2 {y'}^2 - y^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x)^2-y(x)^2=0,y(x), singsol=all)
```

$$y(x) = c_1 x$$
$$y(x) = \frac{c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 24

```
DSolve[x^2*(y'[x])^2-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x}$$
$$y(x) \to c_1 x$$
$$y(x) \to 0$$

1.2 problem 2

Internal problem ID [6768]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 94. Factoring the left member. EXERCISES Page 309
Problem number: 2.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xy'^2 - (3y + 2x)\, y' + 6y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x)^2-(2*x+3*y(x))*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^3$$
$$y(x) = c_1 + 2x$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 26

```
DSolve[x*(y'[x])^2-(2*x+3*y[x])*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x^3$$
$$y(x) \to 2x + c_1$$
$$y(x) \to 0$$

1.3 problem 3

Internal problem ID [6769]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 94. Factoring the left member. EXERCISES Page 309
Problem number: 3.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2 y'^2 - 5xyy' + 6y^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x)^2-5*x*y(x)*diff(y(x),x)+6*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = c_1 x^3$$
$$y(x) = c_1 x^2$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 26

```
DSolve[x^2*(y'[x])^2-5*x*y[x]*y'[x]+6*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x^2$$
$$y(x) \to c_1 x^3$$
$$y(x) \to 0$$

1.4 problem 4

Internal problem ID [6770]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 94. Factoring the left member. EXERCISES Page 309
Problem number: 4.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$x^2 {y'}^2 + x y' - y^2 - y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x)^2+x*diff(y(x),x)-y(x)^2-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x$$
$$y(x) = \frac{-x + c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.045 (sec). Leaf size: 31

```
DSolve[x^2*(y'[x])^2+x*y'[x]-y[x]^2-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x$$
$$y(x) \to -1 + \frac{c_1}{x}$$
$$y(x) \to -1$$
$$y(x) \to 0$$

1.5 problem 5

Internal problem ID [6771]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 94. Factoring the left member. EXERCISES Page 309
Problem number: 5.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xy'^2 + (1 - yx^2)\, y' - xy = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x*diff(y(x),x)^2+(1-x^2*y(x))*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = -\ln(x) + c_1$$
$$y(x) = e^{\frac{x^2}{2}} c_1$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 28

```
DSolve[x*(y'[x])^2+(1-x^2*y[x])*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{\frac{x^2}{2}}$$
$$y(x) \to -\log(x) + c_1$$

1.6 problem 6

Internal problem ID [6772]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 94. Factoring the left member. EXERCISES Page 309
Problem number: 6.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 - (yx^2 + 3)\, y' + 3yx^2 = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)^2-(x^2*y(x)+3)*diff(y(x),x)+3*x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{x^3}{3}}$$
$$y(x) = 3x + c_1$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 27

```
DSolve[(y'[x])^2-(x^2*y[x]+3)*y'[x]+3*x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{\frac{x^3}{3}}$$
$$y(x) \to 3x + c_1$$

1.7 problem 7

Internal problem ID [6773]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 94. Factoring the left member. EXERCISES Page 309
Problem number: 7.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xy'^2 - (xy+1)\,y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x)^2-(1+x*y(x))*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \ln(x) + c_1$$
$$y(x) = e^x c_1$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 20

```
DSolve[x*(y'[x])^2-(1+x*y[x])*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x$$
$$y(x) \to \log(x) + c_1$$

1.8 problem 8

Internal problem ID [6774]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 94. Factoring the left member. EXERCISES Page 309
Problem number: 8.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'^2 - y^2 x^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)^2-x^2*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x^2}{2}} c_1$$
$$y(x) = \mathrm{e}^{\frac{x^2}{2}} c_1$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 38

```
DSolve[(y'[x])^2-x^2*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\frac{x^2}{2}}$$
$$y(x) \to c_1 e^{\frac{x^2}{2}}$$
$$y(x) \to 0$$

1.9 problem 9

Internal problem ID [6775]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 94. Factoring the left member. EXERCISES Page 309
Problem number: 9.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$(x+y)^2 y'^2 - y^2 = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 47

```
dsolve((x+y(x))^2*diff(y(x),x)^2=y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x}{\operatorname{LambertW}\left(x\,\mathrm{e}^{c_1}\right)}$$
$$y(x) = -x - \sqrt{x^2 + 2c_1}$$
$$y(x) = -x + \sqrt{x^2 + 2c_1}$$

✓ Solution by Mathematica
Time used: 4.023 (sec). Leaf size: 101

```
DSolve[(x+y[x])^2*(y'[x])^2==y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x - \sqrt{x^2 + e^{2c_1}}$$
$$y(x) \to -x + \sqrt{x^2 + e^{2c_1}}$$
$$y(x) \to \frac{x}{W\left(e^{-c_1}x\right)}$$
$$y(x) \to 0$$
$$y(x) \to -\sqrt{x^2} - x$$
$$y(x) \to \sqrt{x^2} - x$$

1.10 problem 10

Internal problem ID [6776]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 94. Factoring the left member. EXERCISES Page 309
Problem number: 10.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{yy'^2 + (x - y^2)\, y' - xy = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(y(x)*diff(y(x),x)^2+(x-y(x)^2)*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{-x^2 + c_1}$$
$$y(x) = -\sqrt{-x^2 + c_1}$$
$$y(x) = \mathrm{e}^x c_1$$

✓ Solution by Mathematica
Time used: 0.134 (sec). Leaf size: 54

```
DSolve[y[x]*(y'[x])^2+(x-y[x]^2)*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x$$
$$y(x) \to -\sqrt{-x^2 + 2c_1}$$
$$y(x) \to \sqrt{-x^2 + 2c_1}$$
$$y(x) \to 0$$

1.11 problem 11

Internal problem ID [6777]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 94. Factoring the left member. EXERCISES Page 309
Problem number: 11.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'^2 - xy(x+y)\,y' + x^3y^3 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x)^2-x*y(x)*(x+y(x))*diff(y(x),x)+x^3*y(x)^3=0,y(x), singsol=all)
```

$$y(x) = -\frac{2}{x^2 - 2c_1}$$
$$y(x) = c_1 \mathrm{e}^{\frac{x^3}{3}}$$

✓ Solution by Mathematica
Time used: 0.122 (sec). Leaf size: 38

```
DSolve[(y'[x])^2-x*y[x]*(x+y[x])*y'[x]+x^3*y[x]^3==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to c_1 e^{\frac{x^3}{3}}$$
$$y(x) \to -\frac{2}{x^2 + 2c_1}$$
$$y(x) \to 0$$

1.12 problem 12

Internal problem ID [6778]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 94. Factoring the left member. EXERCISES Page 309
Problem number: 12.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(4x - y)\, y'^2 + 6(x - y)\, y' - 5y = -2x}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 55

```
dsolve((4*x-y(x))*diff(y(x),x)^2+6*(x-y(x))*diff(y(x),x)+2*x-5*y(x)=0,y(x), singsol=all)
```

$$y(x) = -x + c_1$$
$$y(x) = \frac{-4c_1x + \sqrt{-12c_1x + 1} + 1}{2c_1}$$
$$y(x) = \frac{-4c_1x - \sqrt{-12c_1x + 1} + 1}{2c_1}$$

✓ Solution by Mathematica
Time used: 1.077 (sec). Leaf size: 90

```
DSolve[(4*x-y[x])*(y'[x])^2+6*(x-y[x])*y'[x]+2*x-5*y[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{1}{2}\left(-4x - e^{\frac{c_1}{2}}\sqrt{12x + e^{c_1}} - e^{c_1}\right)$$
$$y(x) \to \frac{1}{2}\left(-4x + e^{\frac{c_1}{2}}\sqrt{12x + e^{c_1}} - e^{c_1}\right)$$
$$y(x) \to -x + c_1$$

1.13 problem 13

Internal problem ID [6779]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 94. Factoring the left member. EXERCISES Page 309
Problem number: 13.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(x-y)^2 y'^2 - y^2 = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 47

```
dsolve((x-y(x))^2*diff(y(x),x)^2=y(x)^2,y(x), singsol=all)
```

$$y(x) = x - \sqrt{x^2 - 2c_1}$$
$$y(x) = x + \sqrt{x^2 - 2c_1}$$
$$y(x) = -\frac{x}{\mathrm{LambertW}\left(-x\,\mathrm{e}^{-c_1}\right)}$$

✓ Solution by Mathematica
Time used: 4.446 (sec). Leaf size: 99

```
DSolve[(x-y[x])^2*(y'[x])^2==y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - \sqrt{x^2 - e^{2c_1}}$$
$$y(x) \to x + \sqrt{x^2 - e^{2c_1}}$$
$$y(x) \to -\frac{x}{W\left(-e^{-c_1}x\right)}$$
$$y(x) \to 0$$
$$y(x) \to x - \sqrt{x^2}$$
$$y(x) \to \sqrt{x^2} + x$$

1.14 problem 14

Internal problem ID [6780]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 94. Factoring the left member. EXERCISES Page 309
Problem number: 14.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xy{y'}^2 + \left(x y^2 - 1\right) y' - y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 34

```
dsolve(x*y(x)*diff(y(x),x)^2+(x*y(x)^2-1)*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{2\ln(x) + c_1}$$
$$y(x) = -\sqrt{2\ln(x) + c_1}$$
$$y(x) = c_1 \mathrm{e}^{-x}$$

✓ Solution by Mathematica
Time used: 0.108 (sec). Leaf size: 57

```
DSolve[x*y[x]*(y'[x])^2+(x*y[x]^2-1)*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x}$$
$$y(x) \to -\sqrt{2}\sqrt{\log(x) + c_1}$$
$$y(x) \to \sqrt{2}\sqrt{\log(x) + c_1}$$
$$y(x) \to 0$$

1.15 problem 15

Internal problem ID [6781]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 94. Factoring the left member. EXERCISES Page 309
Problem number: 15.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{(x^2+y^2)^2 y'^2 - 4y^2x^2 = 0}$$

✓ Solution by Maple
Time used: 0.141 (sec). Leaf size: 255

```
dsolve((x^2+y(x)^2)^2*diff(y(x),x)^2=4*x^2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{1-\sqrt{4x^2c_1^2+1}}{2c_1}$$
$$y(x) = \frac{1+\sqrt{4x^2c_1^2+1}}{2c_1}$$
$$y(x) = -\frac{2\left(c_1x^2 - \frac{\left(4+4\sqrt{4c_1^3x^6+1}\right)^{\frac{2}{3}}}{4}\right)}{\sqrt{c_1}\left(4+4\sqrt{4c_1^3x^6+1}\right)^{\frac{1}{3}}}$$
$$y(x) = -\frac{(1+i\sqrt{3})\left(4+4\sqrt{4c_1^3x^6+1}\right)^{\frac{1}{3}}}{4\sqrt{c_1}} - \frac{(i\sqrt{3}-1)\,x^2\sqrt{c_1}}{\left(4+4\sqrt{4c_1^3x^6+1}\right)^{\frac{1}{3}}}$$
$$y(x) = \frac{4i\sqrt{3}\,c_1x^2 + i\sqrt{3}\left(4+4\sqrt{4c_1^3x^6+1}\right)^{\frac{2}{3}} + 4c_1x^2 - \left(4+4\sqrt{4c_1^3x^6+1}\right)^{\frac{2}{3}}}{4\left(4+4\sqrt{4c_1^3x^6+1}\right)^{\frac{1}{3}}\sqrt{c_1}}$$

✓ Solution by Mathematica

Time used: 15.845 (sec). Leaf size: 345

```
DSolve[(x^2+y[x]^2)^2*(y'[x])^2==4*x^2*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-\sqrt{4x^2+e^{2c_1}}-e^{c_1}\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{4x^2+e^{2c_1}}-e^{c_1}\right)$$
$$y(x) \to \frac{\sqrt[3]{\sqrt{4x^6+e^{6c_1}}+e^{3c_1}}}{\sqrt[3]{2}}-\frac{\sqrt[3]{2}x^2}{\sqrt[3]{\sqrt{4x^6+e^{6c_1}}+e^{3c_1}}}$$
$$y(x) \to \frac{i2^{2/3}\left(\sqrt{3}+i\right)\left(\sqrt{4x^6+e^{6c_1}}+e^{3c_1}\right)^{2/3}+\sqrt[3]{2}\left(2+2i\sqrt{3}\right)x^2}{4\sqrt[3]{\sqrt{4x^6+e^{6c_1}}+e^{3c_1}}}$$
$$y(x) \to \frac{\left(1-i\sqrt{3}\right)x^2}{2^{2/3}\sqrt[3]{\sqrt{4x^6+e^{6c_1}}+e^{3c_1}}}-\frac{\left(1+i\sqrt{3}\right)\sqrt[3]{\sqrt{4x^6+e^{6c_1}}+e^{3c_1}}}{2\sqrt[3]{2}}$$
$$y(x) \to 0$$

1.16 problem 16

Internal problem ID [6782]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 94. Factoring the left member. EXERCISES Page 309
Problem number: 16.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$(x+y)^2 {y'}^2 + \left(2y^2 + xy - x^2\right) y' + y(-x+y) = 0$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 85

```
dsolve((y(x)+x)^2*diff(y(x),x)^2+(2*y(x)^2+x*y(x)-x^2)*diff(y(x),x)+y(x)*(y(x)-x)=0,y(x), si
```

$$y(x) = -x - \sqrt{x^2 + 2c_1}$$
$$y(x) = -x + \sqrt{x^2 + 2c_1}$$
$$y(x) = \frac{-c_1 x - \sqrt{2x^2 c_1^2 + 1}}{c_1}$$
$$y(x) = \frac{-c_1 x + \sqrt{2x^2 c_1^2 + 1}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.492 (sec). Leaf size: 172

```
DSolve[(y[x]+x)^2*(y'[x])^2+(2*y[x]^2+x*y[x]-x^2)*y'[x]+y[x]*(y[x]-x)==0,y[x],x,IncludeSingu
```

$$y(x) \to -x - \sqrt{x^2 + e^{2c_1}}$$
$$y(x) \to -x + \sqrt{x^2 + e^{2c_1}}$$
$$y(x) \to -x - \sqrt{2x^2 + e^{2c_1}}$$
$$y(x) \to -x + \sqrt{2x^2 + e^{2c_1}}$$
$$y(x) \to -\sqrt{x^2} - x$$
$$y(x) \to \sqrt{x^2} - x$$
$$y(x) \to -\sqrt{2}\sqrt{x^2} - x$$
$$y(x) \to \sqrt{2}\sqrt{x^2} - x$$

1.17 problem 17

Internal problem ID [6783]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 94. Factoring the left member. EXERCISES Page 309
Problem number: 17.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$xy(x^2+y^2)\left(y'^2-1\right)-y'(x^4+y^2x^2+y^4)=0$$

✓ Solution by Maple

Time used: 1.015 (sec). Leaf size: 248

```
dsolve(x*y(x)*(x^2+y(x)^2)*(diff(y(x),x)^2-1)=diff(y(x),x)*(x^4+x^2*y(x)^2+y(x)^4),y(x), sin
```

$$y(x)=\frac{\sqrt{x^2c_1\left(c_1x^2-\sqrt{c_1^2x^4+1}\right)}}{x\left(c_1x^2-\sqrt{c_1^2x^4+1}\right)c_1}$$
$$y(x)=\frac{\sqrt{x^2c_1\left(c_1x^2+\sqrt{c_1^2x^4+1}\right)}}{x\left(c_1x^2+\sqrt{c_1^2x^4+1}\right)c_1}$$
$$y(x)=\frac{\sqrt{x^2c_1\left(c_1x^2-\sqrt{c_1^2x^4+1}\right)}}{x\left(-c_1x^2+\sqrt{c_1^2x^4+1}\right)c_1}$$
$$y(x)=-\frac{\sqrt{x^2c_1\left(c_1x^2+\sqrt{c_1^2x^4+1}\right)}}{x\left(c_1x^2+\sqrt{c_1^2x^4+1}\right)c_1}$$
$$y(x)=\sqrt{2\ln(x)+c_1}\,x$$
$$y(x)=-\sqrt{2\ln(x)+c_1}\,x$$

✓ Solution by Mathematica

Time used: 9.298 (sec). Leaf size: 248

```
DSolve[x*y[x]*(x^2+y[x]^2)*((y'[x])^2-1)==y'[x]*(x^4+x^2*y[x]^2+y[x]^4),y[x],x,IncludeSingul
```

$$y(x) \to -\sqrt{-x^2 - \sqrt{x^4 + e^{4c_1}}}$$
$$y(x) \to \sqrt{-x^2 - \sqrt{x^4 + e^{4c_1}}}$$
$$y(x) \to -\sqrt{-x^2 + \sqrt{x^4 + e^{4c_1}}}$$
$$y(x) \to \sqrt{-x^2 + \sqrt{x^4 + e^{4c_1}}}$$
$$y(x) \to -x\sqrt{2\log(x) + c_1}$$
$$y(x) \to x\sqrt{2\log(x) + c_1}$$
$$y(x) \to -\sqrt{-\sqrt{x^4} - x^2}$$
$$y(x) \to \sqrt{-\sqrt{x^4} - x^2}$$
$$y(x) \to -\sqrt{\sqrt{x^4} - x^2}$$
$$y(x) \to \sqrt{\sqrt{x^4} - x^2}$$

1.18 problem 18

Internal problem ID [6784]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 94. Factoring the left member. EXERCISES Page 309
Problem number: 18.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xy'^3 - (x^2 + x + y)\, y'^2 + (x^2 + xy + y)\, y' - xy = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x)^3-(x^2+x+y(x))*diff(y(x),x)^2+(x^2+x*y(x)+y(x))*diff(y(x),x)-x*y(x)=0,
```

$$y(x) = c_1 x$$
$$y(x) = x + c_1$$
$$y(x) = \frac{x^2}{2} + c_1$$

✓ Solution by Mathematica
Time used: 0.044 (sec). Leaf size: 36

```
DSolve[x*(y'[x])^3-(x^2+x+y[x])*(y'[x])^2+(x^2+x*y[x]+y[x])*y'[x]-x*y[x]==0,y[x],x,IncludeSi
```

$$y(x) \to c_1 x$$
$$y(x) \to x + c_1$$
$$y(x) \to \frac{x^2}{2} + c_1$$
$$y(x) \to 0$$

1.19 problem 19

Internal problem ID [6785]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 94. Factoring the left member. EXERCISES Page 309
Problem number: 19.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xy{y'}^2 + (x + y)\,y' = -1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 32

```
dsolve(x*y(x)*diff(y(x),x)^2+(x+y(x))*diff(y(x),x)+1=0,y(x), singsol=all)
```

$$y(x) = -\ln(x) + c_1$$
$$y(x) = \sqrt{c_1 - 2x}$$
$$y(x) = -\sqrt{c_1 - 2x}$$

✓ Solution by Mathematica
Time used: 0.062 (sec). Leaf size: 53

```
DSolve[x*y[x]*(y'[x])^2+(x+y[x])*y'[x]+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}\sqrt{-x + c_1}$$
$$y(x) \to \sqrt{2}\sqrt{-x + c_1}$$
$$y(x) \to -\log(x) + c_1$$

2 CHAPTER 16. Nonlinear equations. Section 97. The p-discriminant equation. EXERCISES Page 314

2.1 problem 8

Internal problem ID [6786]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 97. The p-discriminant equation. EXERCISES Page 314
Problem number: 8.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{xy'^2 - 2y'y = -4x}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 30

```
dsolve(x*diff(y(x),x)^2-2*y(x)*diff(y(x),x)+4*x=0,y(x), singsol=all)
```

$$y(x) = -2x$$
$$y(x) = 2x$$
$$y(x) = \frac{4c_1^2 + x^2}{2c_1}$$

✓ Solution by Mathematica
Time used: 0.294 (sec). Leaf size: 43

```
DSolve[x*(y'[x])^2-2*y[x]*y'[x]+4*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2x\cosh(-\log(x) + c_1)$$
$$y(x) \to -2x\cosh(\log(x) + c_1)$$
$$y(x) \to -2x$$
$$y(x) \to 2x$$

2.2 problem 9

Internal problem ID [6787]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 97. The p-discriminant equation. EXERCISES Page 314
Problem number: 9.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{3x^4y'^2 - xy' - y = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 97

```
dsolve(3*x^4*diff(y(x),x)^2-x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{12x^2}$$
$$y(x) = \frac{-i\sqrt{3}\,c_1 - 3x}{3c_1^2x}$$
$$y(x) = \frac{i\sqrt{3}\,c_1 - 3x}{3x\,c_1^2}$$
$$y(x) = \frac{i\sqrt{3}\,c_1 - 3x}{3x\,c_1^2}$$
$$y(x) = \frac{-i\sqrt{3}\,c_1 - 3x}{3c_1^2x}$$

✓ Solution by Mathematica

Time used: 0.518 (sec). Leaf size: 123

```
DSolve[3*x^4*(y'[x])^2-x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{x\sqrt{12x^2y(x)+1}\operatorname{arctanh}\left(\sqrt{12x^2y(x)+1}\right)}{\sqrt{12x^4y(x)+x^2}}-\frac{1}{2}\log(y(x))=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{x\sqrt{12x^2y(x)+1}\operatorname{arctanh}\left(\sqrt{12x^2y(x)+1}\right)}{\sqrt{12x^4y(x)+x^2}}-\frac{1}{2}\log(y(x))=c_1,y(x)\right]$$

$$y(x)\to 0$$

2.3 problem 10

Internal problem ID [6788]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 97. The p-discriminant equation. EXERCISES Page 314
Problem number: 10.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y'^2 - xy' - y = 0}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 77

```
dsolve(diff(y(x),x)^2-x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$\frac{c_1}{\sqrt{2x - 2\sqrt{x^2 + 4y(x)}}} + \frac{2x}{3} + \frac{\sqrt{x^2 + 4y(x)}}{3} = 0$$
$$\frac{c_1}{\sqrt{2x + 2\sqrt{x^2 + 4y(x)}}} + \frac{2x}{3} - \frac{\sqrt{x^2 + 4y(x)}}{3} = 0$$

✓ Solution by Mathematica

Time used: 60.178 (sec). Leaf size: 1003

```
DSolve[(y'[x])^2-x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{\left(x^2+\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)^2+8e^{3c_1}x}{4\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}$$

$$y(x)\to \frac{1}{8}\left(4x^2-\frac{i(\sqrt{3}-i)\,x(x^3+8e^{3c_1})}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}+i\left(\sqrt{3}+i\right)\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x)\to \frac{1}{8}\left(4x^2+\frac{i(\sqrt{3}+i)\,x(x^3+8e^{3c_1})}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}-\left(1+i\sqrt{3}\right)\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x)\to \frac{2\sqrt[3]{2}x^4+2^{2/3}\left(-2x^6-10e^{3c_1}x^3+\sqrt{e^{3c_1}\left(4x^3+e^{3c_1}\right)^3}+e^{6c_1}\right)^{2/3}+4x^2\sqrt[3]{-2x^6-10e^{3c_1}x^3+\sqrt{e^{3c_1}\left(4x\right.}}}{8\sqrt[3]{-2x^6-10e^{3c_1}x^3+\sqrt{e^{3c_1}\left(4x^3+e^{3c_1}\right)^3}+e^{6c_1}}}$$

$$y(x)\to \frac{1}{16}\left(8x^2+\frac{2\sqrt[3]{2}(1+i\sqrt{3})\,x(-x^3+2e^{3c_1})}{\sqrt[3]{-2x^6-10e^{3c_1}x^3+\sqrt{e^{3c_1}\left(4x^3+e^{3c_1}\right)^3}+e^{6c_1}}}+i2^{2/3}\left(\sqrt{3}+i\right)\sqrt[3]{-2x^6-10e^{3c_1}x^3+\sqrt{e^{3c_1}\left(4x^3+e^{3c_1}\right)^3}+e^{6c_1}}\right)$$

$$y(x)\to \frac{1}{16}\left(8x^2+\frac{2i\sqrt[3]{2}(\sqrt{3}+i)\,x(x^3-2e^{3c_1})}{\sqrt[3]{-2x^6-10e^{3c_1}x^3+\sqrt{e^{3c_1}\left(4x^3+e^{3c_1}\right)^3}+e^{6c_1}}}-2^{2/3}\left(1+i\sqrt{3}\right)\sqrt[3]{-2x^6-10e^{3c_1}x^3+\sqrt{e^{3c_1}\left(4x^3+e^{3c_1}\right)^3}+e^{6c_1}}\right)$$

2.4 problem 11

Internal problem ID [6789]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 97. The p-discriminant equation. EXERCISES Page 314
Problem number: 11.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y'^2 - xy' + y = 0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)^2-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{4}$$
$$y(x) = c_1(x - c_1)$$

✓ Solution by Mathematica
Time used: 0.007 (sec). Leaf size: 25

```
DSolve[(y'[x])^2-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x - c_1)$$
$$y(x) \to \frac{x^2}{4}$$

2.5 problem 12

Internal problem ID [6790]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 97. The p-discriminant equation. EXERCISES Page 314
Problem number: 12.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{{y'}^2 + 4y'x^5 - 12yx^4 = 0}$$

✓ Solution by Maple
Time used: 0.359 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)^2+4*x^5*diff(y(x),x)-12*x^4*y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{x^6}{3}$$
$$y(x) = c_1 x^3 + \frac{3}{4}c_1^2$$

✓ Solution by Mathematica

Time used: 1.361 (sec). Leaf size: 217

```
DSolve[(y'[x])^2+4*x^5*y'[x]-12*x^4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{6}\left(\log(y(x))-\frac{x^2\sqrt{x^6+3y(x)}\log(y(x))}{\sqrt{x^4\,(x^6+3y(x))}}\right)+\frac{x^2\sqrt{x^6+3y(x)}\log\left(\sqrt{x^6+3y(x)}+x^3\right)}{3\sqrt{x^4\,(x^6+3y(x))}}=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{6}\left(\frac{x^2\sqrt{x^6+3y(x)}\log(y(x))}{\sqrt{x^4\,(x^6+3y(x))}}+\log(y(x))\right)-\frac{x^2\sqrt{x^6+3y(x)}\log\left(\sqrt{x^6+3y(x)}+x^3\right)}{3\sqrt{x^4\,(x^6+3y(x))}}=c_1,y(x)\right]$$

$$y(x)\to -\frac{x^6}{3}$$

2.6 problem 13

Internal problem ID [6791]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 97. The p-discriminant equation. EXERCISES Page 314
Problem number: 13.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$\boxed{4y^3{y'}^2-4xy'+y=0}$$

✓ Solution by Maple
Time used: 0.281 (sec). Leaf size: 85

```
dsolve(4*y(x)^3*diff(y(x),x)^2-4*x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=\sqrt{-x}$$
$$y(x)=-\sqrt{-x}$$
$$y(x)=\sqrt{x}$$
$$y(x)=-\sqrt{x}$$
$$y(x)=0$$
$$y(x)=\text{RootOf}\left(-\ln(x)+2\left(\int^{\_Z}-\frac{\_a^4-\sqrt{-\_a^4+1}-1}{\_a\left(\_a^4-1\right)}d\_a\right)+c_1\right)\sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.587 (sec). Leaf size: 282

```
DSolve[4*y[x]^3*(y'[x])^2-4*x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) &\to -e^{\frac{c_1}{4}}\sqrt[4]{e^{c_1}-2ix} \\
y(x) &\to -ie^{\frac{c_1}{4}}\sqrt[4]{e^{c_1}-2ix} \\
y(x) &\to ie^{\frac{c_1}{4}}\sqrt[4]{e^{c_1}-2ix} \\
y(x) &\to e^{\frac{c_1}{4}}\sqrt[4]{e^{c_1}-2ix} \\
y(x) &\to -e^{\frac{c_1}{4}}\sqrt[4]{2ix+e^{c_1}} \\
y(x) &\to -ie^{\frac{c_1}{4}}\sqrt[4]{2ix+e^{c_1}} \\
y(x) &\to ie^{\frac{c_1}{4}}\sqrt[4]{2ix+e^{c_1}} \\
y(x) &\to e^{\frac{c_1}{4}}\sqrt[4]{2ix+e^{c_1}} \\
y(x) &\to 0 \\
y(x) &\to -\sqrt{x} \\
y(x) &\to -i\sqrt{x} \\
y(x) &\to i\sqrt{x} \\
y(x) &\to \sqrt{x}
\end{aligned}
$$

2.7 problem 14

Internal problem ID [6792]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 97. The p-discriminant equation. EXERCISES Page 314
Problem number: 14.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$4y^3 y'^2 + 4xy' + y = 0$$

✓ Solution by Maple
Time used: 0.313 (sec). Leaf size: 307

```
dsolve(4*y(x)^3*diff(y(x),x)^2+4*x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$\frac{\left(\int_{\_b}^{x} \frac{-2\_a+\sqrt{-y(x)^4+\_a^2}}{y(x)^4+3\_a^2} d\_a\right)}{2} - \left(\int^{y(x)} \frac{\left(1+\left(\_f^4-\sqrt{-\_f^4+x^2}\,x+x^2\right)\left(\int_{\_b}^{x} \frac{\_f^4+4\sqrt{-\_f^4+\_a^2}\,\_a-5\_a^2}{\sqrt{-\_f^4+\_a^2}\left(\_f^4+3\_a^2\right)^2} d\_a\right)\right)\_f^3}{\_f^4-\sqrt{-\_f^4+x^2}\,x+x^2} d\_f\right) + c_1 = 0$$

$$-\frac{\left(\int_{\_b}^{x} \frac{2\_a+\sqrt{-y(x)^4+\_a^2}}{y(x)^4+3\_a^2} d\_a\right)}{2} - \left(\int^{y(x)} \frac{\left(1+\left(\_f^4+\sqrt{-\_f^4+x^2}\,x+x^2\right)\left(\int_{\_b}^{x} \frac{-\_f^4+5\_a^2+4\sqrt{-\_f^4+\_a^2}\,\_a}{\sqrt{-\_f^4+\_a^2}\left(\_f^4+3\_a^2\right)^2} d\_a\right)\right)\_f^3}{\_f^4+\sqrt{-\_f^4+x^2}\,x+x^2} d\_f\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.284 (sec). Leaf size: 2815

```
DSolve[4*y[x]^3*(y'[x])^2+4*x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

2.8 problem 15

Internal problem ID [6793]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 97. The p-discriminant equation. EXERCISES Page 314
Problem number: 15.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_dAlembert]`

$$y'^3 + xy'^2 - y = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 994

```
dsolve(diff(y(x),x)^3+x*diff(y(x),x)^2-y(x)=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \frac{\left(4x^2 - 2x\left(-36x^2 - 54x + 108c_1 - 8x^3 + 27 + 6\sqrt{-6(1+2c_1)(4x^3 + 18x^2 - 27c_1 + 27x)}\right)^{\frac{1}{3}} + 12x + \right.}{}$$

$$y(x) = \frac{\left(\frac{\left(-i\sqrt{3}-1\right)\left(-36x^2-54x+108c_1-8x^3+27+6\sqrt{-6(1+2c_1)(4x^3+18x^2-27c_1+27x)}\right)^{\frac{2}{3}}}{4} + \left(2x + \frac{3}{2}\right)\left(-36x^2 - 54x + 108c_1 - \right.\right.}{}$$

$$y(x) = \frac{\left(\frac{\left(i\sqrt{3}-1\right)\left(-36x^2-54x+108c_1-8x^3+27+6\sqrt{-6(1+2c_1)(4x^3+18x^2-27c_1+27x)}\right)^{\frac{2}{3}}}{4} - \left(-2x - \frac{3}{2}\right)\left(-36x^2 - 54x + 108c_1 - \right.\right.}{}$$

✓ Solution by Mathematica

Time used: 84.497 (sec). Leaf size: 1516

```
DSolve[(y'[x])^3+x*(y'[x])^2-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-16x^4 + 8\left(\sqrt[3]{-8x^3 - 36x^2 - 54x + 108c_1 + 6\sqrt{6}\sqrt{-\left(\left(4x^3 + 18x^2 + 27x - 27c_1\right)\left(2c_1 + 1\right)\right) + 27 - 12}\right.}{}$$

$$y(x) \to \frac{1}{6}\left(-\frac{i\left(\sqrt{3} - i\right)x(2x+3)^2}{\sqrt[3]{-8x^3 - 36x^2 + 6\sqrt{6}\sqrt{-\left((1+2c_1)\left(4x^3 + 18x^2 + 27x - 27c_1\right)\right)} - 54x + 27 + 108c_1}}\right.$$
$$+ \frac{1}{16}\left(-\frac{i\left(\sqrt{3} - i\right)(2x+3)^2}{\sqrt[3]{-8x^3 - 36x^2 + 6\sqrt{6}\sqrt{-\left((1+2c_1)\left(4x^3 + 18x^2 + 27x - 27c_1\right)\right)} - 54x + 27 + 108c_1}}\right.$$
$$+ i\left(\sqrt{3}+i\right)\sqrt[3]{-8x^3 - 36x^2 + 6\sqrt{6}\sqrt{-\left((1+2c_1)\left(4x^3 + 18x^2 + 27x - 27c_1\right)\right)} - 54x + 27 + 108c_1}$$
$$\left. - 4x + 6\right)^2 + i\left(\sqrt{3}\right.$$
$$\left. + i\right)x\sqrt[3]{-8x^3 - 36x^2 + 6\sqrt{6}\sqrt{-\left((1+2c_1)\left(4x^3 + 18x^2 + 27x - 27c_1\right)\right)} - 54x + 27 + 108c_1}$$
$$\left. + 2(3-2x)x - 6x + 6c_1\right)$$

$$y(x) \to \frac{1}{6}\left(\frac{i\left(\sqrt{3} + i\right)x(2x+3)^2}{\sqrt[3]{-8x^3 - 36x^2 + 6\sqrt{6}\sqrt{-\left((1+2c_1)\left(4x^3 + 18x^2 + 27x - 27c_1\right)\right)} - 54x + 27 + 108c_1}}\right.$$
$$+ \frac{1}{16}\left(\frac{\left(1 - i\sqrt{3}\right)(2x+3)^2}{\sqrt[3]{-8x^3 - 36x^2 + 6\sqrt{6}\sqrt{-\left((1+2c_1)\left(4x^3 + 18x^2 + 27x - 27c_1\right)\right)} - 54x + 27 + 108c_1}}\right.$$
$$+ \left(1+i\sqrt{3}\right)\sqrt[3]{-8x^3 - 36x^2 + 6\sqrt{6}\sqrt{-\left((1+2c_1)\left(4x^3 + 18x^2 + 27x - 27c_1\right)\right)} - 54x + 27 + 108c_1}$$
$$\left. + 4x - 6\right)^2 - \left(1\right.$$
$$\left. + i\sqrt{3}\right)x\sqrt[3]{-8x^3 - 36x^2 + 6\sqrt{6}\sqrt{-\left((1+2c_1)\left(4x^3 + 18x^2 + 27x - 27c_1\right)\right)} - 54x + 27 + 108c_1}$$
$$\left. + 2(3-2x)x - 6x + 6c_1\right)$$

2.9 problem 16

Internal problem ID [6794]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 97. The p-discriminant equation. EXERCISES Page 314
Problem number: 16.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y^4 {y'}^3 - 6xy' + 2y = 0$$

✓ Solution by Maple
Time used: 0.266 (sec). Leaf size: 167

```
dsolve(y(x)^4*diff(y(x),x)^3-6*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x\left(-i\sqrt{3}-1\right)}$$
$$y(x) = \sqrt{\left(i\sqrt{3}-1\right)x}$$
$$y(x) = -\sqrt{-\left(1+i\sqrt{3}\right)x}$$
$$y(x) = -\sqrt{\left(i\sqrt{3}-1\right)x}$$
$$y(x) = \sqrt{x}\,\sqrt{2}$$
$$y(x) = -\sqrt{x}\,\sqrt{2}$$
$$y(x) = 0$$
$$y(x) = \frac{2^{\frac{2}{3}}\left(-c_1^3+6c_1x\right)^{\frac{1}{3}}}{2}$$
$$y(x) = -\frac{2^{\frac{2}{3}}\left(-c_1^3+6c_1x\right)^{\frac{1}{3}}\left(1+i\sqrt{3}\right)}{4}$$
$$y(x) = \frac{2^{\frac{2}{3}}\left(-c_1^3+6c_1x\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)}{4}$$

✓ Solution by Mathematica

Time used: 70.054 (sec). Leaf size: 22649

```
DSolve[y[x]^4*(y'[x])^3-6*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

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3 CHAPTER 16. Nonlinear equations. Section 99. Clairaut's equation. EXERCISES Page 320

3.1 problem 3

Internal problem ID [6795]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 99. Clairaut's equation. EXERCISES Page 320
Problem number: 3.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y'^2 + x^3 y' - 2x^2 y = 0}$$

✓ Solution by Maple
Time used: 0.375 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)^2+x^3*diff(y(x),x)-2*x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{x^4}{8}$$
$$y(x) = c_1 (x^2 + 2c_1)$$

✓ Solution by Mathematica

Time used: 1.255 (sec). Leaf size: 209

```
DSolve[(y'[x])^2+x^3*y'[x]-2*x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\sqrt{x^6+8x^2y(x)}\log\left(\sqrt{x^4+8y(x)}+x^2\right)}{2x\sqrt{x^4+8y(x)}} + \frac{1}{4}\left(1-\frac{\sqrt{x^6+8x^2y(x)}}{x\sqrt{x^4+8y(x)}}\right)\log(y(x)) = c_1, y(x)\right]$$

$$\text{Solve}\left[\frac{1}{4}\left(\frac{\sqrt{x^6+8x^2y(x)}}{x\sqrt{x^4+8y(x)}}+1\right)\log(y(x)) - \frac{\sqrt{x^6+8x^2y(x)}\log\left(\sqrt{x^4+8y(x)}+x^2\right)}{2x\sqrt{x^4+8y(x)}} = c_1, y(x)\right]$$

$$y(x) \to -\frac{x^4}{8}$$

3.2 problem 4

Internal problem ID [6796]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 99. Clairaut's equation. EXERCISES Page 320
Problem number: 4.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y'^2 + 4x^5y' - 12yx^4 = 0}$$

✓ Solution by Maple
Time used: 0.359 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)^2+4*x^5*diff(y(x),x)-12*x^4*y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{x^6}{3}$$
$$y(x) = c_1x^3 + \frac{3}{4}c_1^2$$

✓ Solution by Mathematica

Time used: 0.603 (sec). Leaf size: 217

```
DSolve[(y'[x])^2+4*x^5*y'[x]-12*x^4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{6}\left(\log(y(x))-\frac{x^2\sqrt{x^6+3y(x)}\log(y(x))}{\sqrt{x^4(x^6+3y(x))}}\right)+\frac{x^2\sqrt{x^6+3y(x)}\log\left(\sqrt{x^6+3y(x)}+x^3\right)}{3\sqrt{x^4(x^6+3y(x))}}=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{6}\left(\frac{x^2\sqrt{x^6+3y(x)}\log(y(x))}{\sqrt{x^4(x^6+3y(x))}}+\log(y(x))\right)-\frac{x^2\sqrt{x^6+3y(x)}\log\left(\sqrt{x^6+3y(x)}+x^3\right)}{3\sqrt{x^4(x^6+3y(x))}}=c_1,y(x)\right]$$

$$y(x)\to-\frac{x^6}{3}$$

3.3 problem 5

Internal problem ID [6797]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 99. Clairaut's equation. EXERCISES Page 320
Problem number: 5.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{2xy'^3 - 6yy'^2 = -x^4}$$

✓ Solution by Maple
Time used: 0.328 (sec). Leaf size: 56

```
dsolve(2*x*diff(y(x),x)^3-6*y(x)*diff(y(x),x)^2+x^4=0,y(x), singsol=all)
```

$$y(x) = -\frac{(1+i\sqrt{3})\, x^2}{4}$$
$$y(x) = \frac{(i\sqrt{3}-1)\, x^2}{4}$$
$$y(x) = \frac{x^2}{2}$$
$$y(x) = \frac{1}{6c_1^2} + \frac{c_1 x^3}{3}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[2*x*(y'[x])^3-6*y[x]*(y'[x])^2+x^4==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

3.4 problem 6

Internal problem ID [6798]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 99. Clairaut's equation. EXERCISES Page 320
Problem number: 6.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y'^2 - xy' + y = 0}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)^2-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{4}$$
$$y(x) = c_1(x - c_1)$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 25

```
DSolve[(y'[x])^2-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x - c_1)$$
$$y(x) \to \frac{x^2}{4}$$

3.5 problem 7

Internal problem ID [6799]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 99. Clairaut's equation. EXERCISES Page 320
Problem number: 7.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y - xy' - ky'^2 = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 22

```
dsolve(y(x)=diff(y(x),x)*x+k*diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{4k}$$
$$y(x) = c_1(c_1k + x)$$

✓ Solution by Mathematica
Time used: 0.008 (sec). Leaf size: 28

```
DSolve[y[x]==y'[x]*x+k*(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x + c_1k)$$
$$y(x) \to -\frac{x^2}{4k}$$

3.6 problem 8

Internal problem ID [6800]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 99. Clairaut's equation. EXERCISES Page 320
Problem number: 8.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G']]`

$$\boxed{x^8 {y'}^2 + 3xy' + 9y = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 45

```
dsolve(x^8*diff(y(x),x)^2+3*x*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{4x^6}$$
$$y(x) = \frac{-x^3 + c_1}{x^3 c_1^2}$$
$$y(x) = \frac{-x^3 - c_1}{x^3 c_1^2}$$

✓ Solution by Mathematica

Time used: 0.583 (sec). Leaf size: 130

```
DSolve[x^8*(y'[x])^2+3*x*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{x\sqrt{4x^6y(x)-1}\arctan\left(\sqrt{4x^6y(x)-1}\right)}{3\sqrt{x^2-4x^8y(x)}}-\frac{1}{6}\log(y(x))=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{\sqrt{x^2-4x^8y(x)}\arctan\left(\sqrt{4x^6y(x)-1}\right)}{3x\sqrt{4x^6y(x)-1}}-\frac{1}{6}\log(y(x))=c_1,y(x)\right]$$

$$y(x)\to 0$$

3.7 problem 9

Internal problem ID [6801]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 99. Clairaut's equation. EXERCISES Page 320
Problem number: 9.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$\boxed{x^4 {y'}^2 + 2yy'x^3 = 4}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 49

```
dsolve(x^4*diff(y(x),x)^2+2*x^3*y(x)*diff(y(x),x)-4=0,y(x), singsol=all)
```

$$y(x) = -\frac{2i}{x}$$
$$y(x) = \frac{2i}{x}$$
$$y(x) = \frac{2\sinh\left(-\ln\left(x\right)+c_1\right)}{x}$$
$$y(x) = -\frac{2\sinh\left(-\ln\left(x\right)+c_1\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.688 (sec). Leaf size: 71

```
DSolve[x^4*(y'[x])^2+2*x^3*y[x]*y'[x]-4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4e^{c_1}}{x^2} - \frac{e^{-c_1}}{4}$$
$$y(x) \to \frac{e^{-c_1}}{4} - \frac{4e^{c_1}}{x^2}$$
$$y(x) \to -\frac{2i}{x}$$
$$y(x) \to \frac{2i}{x}$$

3.8 problem 10

Internal problem ID [6802]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 99. Clairaut's equation. EXERCISES Page 320
Problem number: 10.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{xy'^2 - 2yy' = -4x}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 30

```
dsolve(x*diff(y(x),x)^2-2*y(x)*diff(y(x),x)+4*x=0,y(x), singsol=all)
```

$$y(x) = -2x$$
$$y(x) = 2x$$
$$y(x) = \frac{4c_1^2 + x^2}{2c_1}$$

✓ Solution by Mathematica
Time used: 0.202 (sec). Leaf size: 43

```
DSolve[x*(y'[x])^2-2*y[x]*y'[x]+4*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2x\cosh(-\log(x) + c_1)$$
$$y(x) \to -2x\cosh(\log(x) + c_1)$$
$$y(x) \to -2x$$
$$y(x) \to 2x$$

3.9 problem 11

Internal problem ID [6803]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 99. Clairaut's equation. EXERCISES Page 320
Problem number: 11.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$\boxed{3x^4{y'}^2 - xy' - y = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 97

```
dsolve(3*x^4*diff(y(x),x)^2-x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{12x^2}$$
$$y(x) = \frac{-i\sqrt{3}\,c_1 - 3x}{3c_1^2 x}$$
$$y(x) = \frac{i\sqrt{3}\,c_1 - 3x}{3x\,c_1^2}$$
$$y(x) = \frac{i\sqrt{3}\,c_1 - 3x}{3x\,c_1^2}$$
$$y(x) = \frac{-i\sqrt{3}\,c_1 - 3x}{3c_1^2 x}$$

✓ Solution by Mathematica

Time used: 0.512 (sec). Leaf size: 123

```
DSolve[3*x^4*(y'[x])^2-x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{x\sqrt{12x^2y(x)+1}\text{arctanh}\left(\sqrt{12x^2y(x)+1}\right)}{\sqrt{12x^4y(x)+x^2}}-\frac{1}{2}\log(y(x))=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{x\sqrt{12x^2y(x)+1}\text{arctanh}\left(\sqrt{12x^2y(x)+1}\right)}{\sqrt{12x^4y(x)+x^2}}-\frac{1}{2}\log(y(x))=c_1,y(x)\right]$$

$$y(x)\to 0$$

3.10 problem 12

Internal problem ID [6804]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 99. Clairaut's equation. EXERCISES Page 320
Problem number: 12.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _dAlembert]`

$$\boxed{xy'^2 + (x - y)\, y' - y = -1}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 44

```
dsolve(x*diff(y(x),x)^2+(x-y(x))*diff(y(x),x)+1-y(x)=0,y(x), singsol=all)
```

$$y(x) = -x - 2\sqrt{x}$$
$$y(x) = -x + 2\sqrt{x}$$
$$y(x) = \frac{c_1^2 x + c_1 x + 1}{c_1 + 1}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 46

```
DSolve[x*(y'[x])^2+(x-y[x])*y'[x]+1-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x + \frac{1}{1 + c_1}$$
$$y(x) \to -x - 2\sqrt{x}$$
$$y(x) \to 2\sqrt{x} - x$$

3.11 problem 13

Internal problem ID [6805]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 99. Clairaut's equation. EXERCISES Page 320
Problem number: 13.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Clairaut]`

$$y'(xy' - y + k) = -a$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 42

```
dsolve(diff(y(x),x)*( x*diff(y(x),x)-y(x)+k )+a=0,y(x), singsol=all)
```

$$y(x) = k - 2\sqrt{ax}$$
$$y(x) = k + 2\sqrt{ax}$$
$$y(x) = \frac{c_1^2 x + c_1 k + a}{c_1}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 58

```
DSolve[y'[x]*( x*y'[x]-y[x]+k )+a==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{a}{c_1} + k + c_1 x$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to k - 2\sqrt{a}\sqrt{x}$$
$$y(x) \to 2\sqrt{a}\sqrt{x} + k$$

3.12 problem 14

Internal problem ID [6806]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 99. Clairaut's equation. EXERCISES Page 320
Problem number: 14.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{x^6 {y'}^3 - 3xy' - 3y = 0}$$

✓ Solution by Maple
Time used: 0.093 (sec). Leaf size: 32

```
dsolve(x^6*diff(y(x),x)^3-3*x*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{2}{3x^{\frac{3}{2}}}$$
$$y(x) = \frac{2}{3x^{\frac{3}{2}}}$$
$$y(x) = \frac{c_1^3}{3} - \frac{c_1}{x}$$

✓ Solution by Mathematica
Time used: 136.42 (sec). Leaf size: 24834

```
DSolve[x^6*(y'[x])^3-3*x*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

3.13 problem 15

Internal problem ID [6807]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 99. Clairaut's equation. EXERCISES Page 320
Problem number: 15.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y - x^6 {y'}^3 + x y' = 0}$$

✓ Solution by Maple
Time used: 0.187 (sec). Leaf size: 36

```
dsolve(y(x)=x^6*diff(y(x),x)^3-x*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = -\frac{2\sqrt{3}}{9x^{\frac{3}{2}}}$$
$$y(x) = \frac{2\sqrt{3}}{9x^{\frac{3}{2}}}$$
$$y(x) = c_1^3 - \frac{c_1}{x}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]==x^6*(y'[x])^3-x*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

Timed out

3.14 problem 16

Internal problem ID [6808]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 99. Clairaut's equation. EXERCISES Page 320
Problem number: 16.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{xy'^4 - 2yy'^3 = -12x^3}$$

✓ Solution by Maple
Time used: 0.203 (sec). Leaf size: 66

```
dsolve(x*diff(y(x),x)^4-2*y(x)*diff(y(x),x)^3+12*x^3=0,y(x), singsol=all)
```

$$y(x) = \frac{2\sqrt{6}\,(-x)^{\frac{3}{2}}}{3}$$
$$y(x) = -\frac{2\sqrt{6}\,(-x)^{\frac{3}{2}}}{3}$$
$$y(x) = -\frac{2\sqrt{6}\,x^{\frac{3}{2}}}{3}$$
$$y(x) = \frac{2\sqrt{6}\,x^{\frac{3}{2}}}{3}$$
$$y(x) = \frac{12c_1^4 + x^2}{2c_1}$$

✓ Solution by Mathematica
Time used: 37.824 (sec). Leaf size: 30947

```
DSolve[x*(y'[x])^4-2*y[x]*(y'[x])^3+12*x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

3.15 problem 17

Internal problem ID [6809]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 99. Clairaut's equation. EXERCISES Page 320
Problem number: 17.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{xy'^3 - yy'^2 = -1}$$

✓ Solution by Maple
Time used: 0.11 (sec). Leaf size: 66

```
dsolve(x*diff(y(x),x)^3-y(x)*diff(y(x),x)^2+1=0,y(x), singsol=all)
```

$$y(x) = \frac{3\,2^{\frac{1}{3}}(x^2)^{\frac{1}{3}}}{2}$$
$$y(x) = -\frac{3\,2^{\frac{1}{3}}(x^2)^{\frac{1}{3}}\left(1+i\sqrt{3}\right)}{4}$$
$$y(x) = \frac{3\,2^{\frac{1}{3}}(x^2)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)}{4}$$
$$y(x) = c_1x + \frac{1}{c_1^2}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 69

```
DSolve[x*(y'[x])^3-y[x]*(y'[x])^2+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x + \frac{1}{{c_1}^2}$$

$$y(x) \to 3\left(-\frac{1}{2}\right)^{2/3} x^{2/3}$$

$$y(x) \to \frac{3x^{2/3}}{2^{2/3}}$$

$$y(x) \to -\frac{3\sqrt[3]{-1}x^{2/3}}{2^{2/3}}$$

3.16 problem 19

Internal problem ID [6810]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 99. Clairaut's equation. EXERCISES Page 320
Problem number: 19.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y'^2 - xy' - y = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 77

```
dsolve(diff(y(x),x)^2-x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$\frac{c_1}{\sqrt{2x - 2\sqrt{x^2 + 4y(x)}}} + \frac{2x}{3} + \frac{\sqrt{x^2 + 4y(x)}}{3} = 0$$
$$\frac{c_1}{\sqrt{2x + 2\sqrt{x^2 + 4y(x)}}} + \frac{2x}{3} - \frac{\sqrt{x^2 + 4y(x)}}{3} = 0$$

✓ Solution by Mathematica

Time used: 60.129 (sec). Leaf size: 1003

```
DSolve[(y'[x])^2-x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\left(x^2 + \sqrt[3]{-x^6 + 20e^{3c_1}x^3 + 8\sqrt{e^{3c_1}\left(-x^3 + e^{3c_1}\right)^3} + 8e^{6c_1}}\right)^2 + 8e^{3c_1}x}{4\sqrt[3]{-x^6 + 20e^{3c_1}x^3 + 8\sqrt{e^{3c_1}\left(-x^3 + e^{3c_1}\right)^3} + 8e^{6c_1}}}$$

$$y(x) \to \frac{1}{8}\left(4x^2 - \frac{i\left(\sqrt{3} - i\right)x\left(x^3 + 8e^{3c_1}\right)}{\sqrt[3]{-x^6 + 20e^{3c_1}x^3 + 8\sqrt{e^{3c_1}\left(-x^3 + e^{3c_1}\right)^3} + 8e^{6c_1}}} + i\left(\sqrt{3} + i\right)\sqrt[3]{-x^6 + 20e^{3c_1}x^3 + 8\sqrt{e^{3c_1}\left(-x^3 + e^{3c_1}\right)^3} + 8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{8}\left(4x^2 + \frac{i\left(\sqrt{3} + i\right)x\left(x^3 + 8e^{3c_1}\right)}{\sqrt[3]{-x^6 + 20e^{3c_1}x^3 + 8\sqrt{e^{3c_1}\left(-x^3 + e^{3c_1}\right)^3} + 8e^{6c_1}}} - \left(1 + i\sqrt{3}\right)\sqrt[3]{-x^6 + 20e^{3c_1}x^3 + 8\sqrt{e^{3c_1}\left(-x^3 + e^{3c_1}\right)^3} + 8e^{6c_1}}\right)$$

$$y(x) \to \frac{2\sqrt[3]{2}x^4 + 2^{2/3}\left(-2x^6 - 10e^{3c_1}x^3 + \sqrt{e^{3c_1}\left(4x^3 + e^{3c_1}\right)^3} + e^{6c_1}\right)^{2/3} + 4x^2\sqrt[3]{-2x^6 - 10e^{3c_1}x^3 + \sqrt{e^{3c_1}\left(4x\right.}}}{8\sqrt[3]{-2x^6 - 10e^{3c_1}x^3 + \sqrt{e^{3c_1}\left(4x^3 + e^{3c_1}\right)^3} + e^{6c_1}}}$$

$$y(x) \to \frac{1}{16}\left(8x^2 + \frac{2\sqrt[3]{2}\left(1 + i\sqrt{3}\right)x\left(-x^3 + 2e^{3c_1}\right)}{\sqrt[3]{-2x^6 - 10e^{3c_1}x^3 + \sqrt{e^{3c_1}\left(4x^3 + e^{3c_1}\right)^3} + e^{6c_1}}} + i2^{2/3}\left(\sqrt{3} + i\right)\sqrt[3]{-2x^6 - 10e^{3c_1}x^3 + \sqrt{e^{3c_1}\left(4x^3 + e^{3c_1}\right)^3} + e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{16}\left(8x^2 + \frac{2i\sqrt[3]{2}\left(\sqrt{3} + i\right)x\left(x^3 - 2e^{3c_1}\right)}{\sqrt[3]{-2x^6 - 10e^{3c_1}x^3 + \sqrt{e^{3c_1}\left(4x^3 + e^{3c_1}\right)^3} + e^{6c_1}}} - 2^{2/3}\left(1 + i\sqrt{3}\right)\sqrt[3]{-2x^6 - 10e^{3c_1}x^3 + \sqrt{e^{3c_1}\left(4x^3 + e^{3c_1}\right)^3} + e^{6c_1}}\right)$$

3.17 problem 20

Internal problem ID [6811]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 99. Clairaut's equation. EXERCISES Page 320
Problem number: 20.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{2{y'}^{3}+xy'-2y=0}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 58

```
dsolve(2*diff(y(x),x)^3+x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{(-c_1^2-24x)\sqrt{c_1^2+24x}}{432}-\frac{c_1^3}{432}-\frac{c_1x}{12}$$
$$y(x) = \frac{(c_1^2+24x)^{\frac{3}{2}}}{432}-\frac{c_1^3}{432}-\frac{c_1x}{12}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[2*(y'[x])^3+x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

3.18 problem 21

Internal problem ID [6812]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 99. Clairaut's equation. EXERCISES Page 320
Problem number: 21.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{2y'^2 + xy' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 31

```
dsolve(2*diff(y(x),x)^2+x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x^2\left(1 + 2\,\mathrm{LambertW}\left(\frac{x\,\mathrm{e}^{\frac{c_1}{4}}}{4}\right)\right)}{16\,\mathrm{LambertW}\left(\frac{x\,\mathrm{e}^{\frac{c_1}{4}}}{4}\right)^2}$$

✓ Solution by Mathematica

Time used: 1.194 (sec). Leaf size: 130

```
DSolve[2*(y'[x])^2+x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[-\frac{\frac{1}{2}x\sqrt{x^2+16y(x)} - 8y(x)\log\left(\sqrt{x^2+16y(x)} - x\right) + \frac{x^2}{2}}{8y(x)} = c_1, y(x)\right]$$

$$\mathrm{Solve}\left[\frac{\frac{1}{2}x\sqrt{x^2+16y(x)} - 8y(x)\log\left(\sqrt{x^2+16y(x)} - x\right) - \frac{x^2}{2}}{8y(x)} + \log(y(x)) = c_1, y(x)\right]$$

$$y(x) \to 0$$

3.19 problem 22

Internal problem ID [6813]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 99. Clairaut's equation. EXERCISES Page 320
Problem number: 22.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y'^3 + 2xy' - y = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 141

```
dsolve(diff(y(x),x)^3+2*x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$
\begin{aligned}
y(x) &= \frac{2\left(-2x+\sqrt{x^2+3c_1}\right)\sqrt{-6\sqrt{x^2+3c_1}-6x}}{9} \\
y(x) &= -\frac{2\left(-2x+\sqrt{x^2+3c_1}\right)\sqrt{-6\sqrt{x^2+3c_1}-6x}}{9} \\
y(x) &= -\frac{2\left(2x+\sqrt{x^2+3c_1}\right)\sqrt{6\sqrt{x^2+3c_1}-6x}}{9} \\
y(x) &= \frac{2\left(2x+\sqrt{x^2+3c_1}\right)\sqrt{6\sqrt{x^2+3c_1}-6x}}{9}
\end{aligned}
$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y'[x])^3+2*x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

3.20 problem 23

Internal problem ID [6814]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 99. Clairaut's equation. EXERCISES Page 320
Problem number: 23.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _dAlembert]`

$$\boxed{4xy'^2 - 3yy' = -3}$$

✓ Solution by Maple
Time used: 0.079 (sec). Leaf size: 123

```
dsolve(4*x*diff(y(x),x)^2-3*y(x)*diff(y(x),x)+3=0,y(x), singsol=all)
```

$$y(x) = -\frac{2x\left(6+\sqrt{16c_1x+9}\right)}{3\sqrt{x\left(3+\sqrt{16c_1x+9}\right)}}$$
$$y(x) = \frac{2x\left(6+\sqrt{16c_1x+9}\right)}{3\sqrt{x\left(3+\sqrt{16c_1x+9}\right)}}$$
$$y(x) = \frac{2x\left(-6+\sqrt{16c_1x+9}\right)}{3\sqrt{-x\left(-3+\sqrt{16c_1x+9}\right)}}$$
$$y(x) = -\frac{2x\left(-6+\sqrt{16c_1x+9}\right)}{3\sqrt{-x\left(-3+\sqrt{16c_1x+9}\right)}}$$

✓ Solution by Mathematica

Time used: 23.695 (sec). Leaf size: 187

```
DSolve[4*x*(y'[x])^2-3*y[x]*y'[x]+3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{432x - e^{-\frac{c_1}{2}}\left(-144x + e^{c_1}\right)^{3/2} + e^{c_1}}}{6\sqrt{3}}$$

$$y(x) \to \frac{\sqrt{432x - e^{-\frac{c_1}{2}}\left(-144x + e^{c_1}\right)^{3/2} + e^{c_1}}}{6\sqrt{3}}$$

$$y(x) \to -\frac{\sqrt{432x + e^{-\frac{c_1}{2}}\left(-144x + e^{c_1}\right)^{3/2} + e^{c_1}}}{6\sqrt{3}}$$

$$y(x) \to \frac{\sqrt{432x + e^{-\frac{c_1}{2}}\left(-144x + e^{c_1}\right)^{3/2} + e^{c_1}}}{6\sqrt{3}}$$

3.21 problem 24

Internal problem ID [6815]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 99. Clairaut's equation. EXERCISES Page 320
Problem number: 24.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y'^3 - xy' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 58

```
dsolve(diff(y(x),x)^3-x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{(c_1^2 - 12x)^{\frac{3}{2}}}{108} - \frac{c_1^3}{108} + \frac{c_1 x}{6}$$
$$y(x) = \frac{(-c_1^2 + 12x)\sqrt{c_1^2 - 12x}}{108} - \frac{c_1^3}{108} + \frac{c_1 x}{6}$$

✓ Solution by Mathematica
Time used: 29.375 (sec). Leaf size: 10134

```
DSolve[(y'[x])^3-x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

3.22 problem 25

Internal problem ID [6816]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 99. Clairaut's equation. EXERCISES Page 320
Problem number: 25.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{5{y'}^2 + 6xy' - 2y = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 85

```
dsolve(5*diff(y(x),x)^2+6*x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$\frac{c_1}{\left(-15x - 5\sqrt{9x^2 + 10y(x)}\right)^{\frac{3}{2}}} + \frac{2x}{5} - \frac{\sqrt{9x^2 + 10y(x)}}{5} = 0$$
$$\frac{c_1}{\left(-15x + 5\sqrt{9x^2 + 10y(x)}\right)^{\frac{3}{2}}} + \frac{2x}{5} + \frac{\sqrt{9x^2 + 10y(x)}}{5} = 0$$

✓ Solution by Mathematica

Time used: 14.31 (sec). Leaf size: 771

```
DSolve[5*(y'[x])^2+6*x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{Root}\left[4\#1^5 + 4\#1^4x^2 + \#1^3x^4 + 1000\#1^2e^{5c_1}x + 900\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 25000e^{10c_1}\&, 1\right]$$
$$y(x) \to \text{Root}\left[4\#1^5 + 4\#1^4x^2 + \#1^3x^4 + 1000\#1^2e^{5c_1}x + 900\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 25000e^{10c_1}\&, 2\right]$$
$$y(x) \to \text{Root}\left[4\#1^5 + 4\#1^4x^2 + \#1^3x^4 + 1000\#1^2e^{5c_1}x + 900\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 25000e^{10c_1}\&, 3\right]$$
$$y(x) \to \text{Root}\left[4\#1^5 + 4\#1^4x^2 + \#1^3x^4 + 1000\#1^2e^{5c_1}x + 900\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 25000e^{10c_1}\&, 4\right]$$
$$y(x) \to \text{Root}\left[4\#1^5 + 4\#1^4x^2 + \#1^3x^4 + 1000\#1^2e^{5c_1}x + 900\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 25000e^{10c_1}\&, 5\right]$$
$$y(x) \to \text{Root}\left[100000\#1^5 + 100000\#1^4x^2 + 25000\#1^3x^4 - 1000\#1^2e^{5c_1}x - 900\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 1\right]$$
$$y(x) \to \text{Root}\left[100000\#1^5 + 100000\#1^4x^2 + 25000\#1^3x^4 - 1000\#1^2e^{5c_1}x - 900\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 2\right]$$
$$y(x) \to \text{Root}\left[100000\#1^5 + 100000\#1^4x^2 + 25000\#1^3x^4 - 1000\#1^2e^{5c_1}x - 900\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 3\right]$$
$$y(x) \to \text{Root}\left[100000\#1^5 + 100000\#1^4x^2 + 25000\#1^3x^4 - 1000\#1^2e^{5c_1}x - 900\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 4\right]$$
$$y(x) \to \text{Root}\left[100000\#1^5 + 100000\#1^4x^2 + 25000\#1^3x^4 - 1000\#1^2e^{5c_1}x - 900\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 5\right]$$
$$y(x) \to 0$$

3.23 problem 26

Internal problem ID [6817]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 99. Clairaut's equation. EXERCISES Page 320
Problem number: 26.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, _dAlembert]`

$$\boxed{2xy'^2 + (2x - y)\, y' - y = -1}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 110

```
dsolve(2*x*diff(y(x),x)^2+(2*x-y(x))*diff(y(x),x)+1-y(x)=0,y(x), singsol=all)
```

$$y(x) = -2\left(x\,\mathrm{e}^{\mathrm{RootOf}\left(-\mathrm{e}^{3\_Z}x+2x\,\mathrm{e}^{2\_Z}+c_1\mathrm{e}^{\_Z}+\mathrm{e}^{\_Z}\_Z-x\,\mathrm{e}^{\_Z}+1\right)} - \mathrm{e}^{2\,\mathrm{RootOf}\left(-\mathrm{e}^{3\_Z}x+2x\,\mathrm{e}^{2\_Z}+c_1\mathrm{e}^{\_Z}+\mathrm{e}^{\_Z}\_Z-x\,\mathrm{e}^{\_Z}+1\right)}x - \frac{1}{2}\right)\mathrm{e}^{-\,\mathrm{RootOf}\left(-\mathrm{e}^{3\_Z}x+2x\,\mathrm{e}^{2\_Z}+c_1\mathrm{e}^{\_Z}+\mathrm{e}^{\_Z}\_Z-x\,\mathrm{e}^{\_Z}+1\right)}$$

✓ Solution by Mathematica
Time used: 1.438 (sec). Leaf size: 49

```
DSolve[2*x*(y'[x])^2+(2*x-y[x])*y'[x]+1-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\left\{x = \frac{\frac{1}{K[1]+1} + \log(K[1]+1)}{K[1]^2} + \frac{c_1}{K[1]^2}, y(x) = 2xK[1] + \frac{1}{K[1]+1}\right\}, \{y(x), K[1]\}\right]$$

3.24 problem 27

Internal problem ID [6818]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 99. Clairaut's equation. EXERCISES Page 320
Problem number: 27.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$5{y'}^2 + 3xy' - y = 0$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 85

```
dsolve(5*diff(y(x),x)^2+3*x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$\frac{c_1}{\left(-30x - 10\sqrt{9x^2 + 20y(x)}\right)^{\frac{3}{2}}} + \frac{2x}{5} - \frac{\sqrt{9x^2 + 20y(x)}}{5} = 0$$
$$\frac{c_1}{\left(-30x + 10\sqrt{9x^2 + 20y(x)}\right)^{\frac{3}{2}}} + \frac{2x}{5} + \frac{\sqrt{9x^2 + 20y(x)}}{5} = 0$$

✓ Solution by Mathematica

Time used: 14.529 (sec). Leaf size: 771

```
DSolve[5*(y'[x])^2+3*x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{Root}\left[16\#1^5 + 8\#1^4x^2 + \#1^3x^4 + 4000\#1^2e^{5c_1}x + 1800\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 200000e^{10c_1}\&, 1\right]$$
$$y(x) \to \text{Root}\left[16\#1^5 + 8\#1^4x^2 + \#1^3x^4 + 4000\#1^2e^{5c_1}x + 1800\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 200000e^{10c_1}\&, 2\right]$$
$$y(x) \to \text{Root}\left[16\#1^5 + 8\#1^4x^2 + \#1^3x^4 + 4000\#1^2e^{5c_1}x + 1800\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 200000e^{10c_1}\&, 3\right]$$
$$y(x) \to \text{Root}\left[16\#1^5 + 8\#1^4x^2 + \#1^3x^4 + 4000\#1^2e^{5c_1}x + 1800\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 200000e^{10c_1}\&, 4\right]$$
$$y(x) \to \text{Root}\left[16\#1^5 + 8\#1^4x^2 + \#1^3x^4 + 4000\#1^2e^{5c_1}x + 1800\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 200000e^{10c_1}\&, 5\right]$$
$$y(x) \to \text{Root}\left[3200000\#1^5 + 1600000\#1^4x^2 + 200000\#1^3x^4 - 4000\#1^2e^{5c_1}x - 1800\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 1\right]$$
$$y(x) \to \text{Root}\left[3200000\#1^5 + 1600000\#1^4x^2 + 200000\#1^3x^4 - 4000\#1^2e^{5c_1}x - 1800\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 2\right]$$
$$y(x) \to \text{Root}\left[3200000\#1^5 + 1600000\#1^4x^2 + 200000\#1^3x^4 - 4000\#1^2e^{5c_1}x - 1800\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 3\right]$$
$$y(x) \to \text{Root}\left[3200000\#1^5 + 1600000\#1^4x^2 + 200000\#1^3x^4 - 4000\#1^2e^{5c_1}x - 1800\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 4\right]$$
$$y(x) \to \text{Root}\left[3200000\#1^5 + 1600000\#1^4x^2 + 200000\#1^3x^4 - 4000\#1^2e^{5c_1}x - 1800\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 5\right]$$
$$y(x) \to 0$$

3.25 problem 28

Internal problem ID [6819]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 99. Clairaut's equation. EXERCISES Page 320
Problem number: 28.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y'^2 + 3xy' - y = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)^2+3*x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$\frac{c_1}{\left(-6x - 2\sqrt{9x^2 + 4y(x)}\right)^{\frac{3}{2}}} + \frac{2x}{5} - \frac{\sqrt{9x^2 + 4y(x)}}{5} = 0$$
$$\frac{c_1}{\left(-6x + 2\sqrt{9x^2 + 4y(x)}\right)^{\frac{3}{2}}} + \frac{2x}{5} + \frac{\sqrt{9x^2 + 4y(x)}}{5} = 0$$

✓ Solution by Mathematica

Time used: 14.495 (sec). Leaf size: 776

```
DSolve[(y'[x])^2+3*x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{Root}\left[16\#1^5 + 40\#1^4x^2 + 25\#1^3x^4 + 160\#1^2e^{5c_1}x + 360\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 64e^{10c_1}\&, 1\right]$$
$$y(x) \to \text{Root}\left[16\#1^5 + 40\#1^4x^2 + 25\#1^3x^4 + 160\#1^2e^{5c_1}x + 360\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 64e^{10c_1}\&, 2\right]$$
$$y(x) \to \text{Root}\left[16\#1^5 + 40\#1^4x^2 + 25\#1^3x^4 + 160\#1^2e^{5c_1}x + 360\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 64e^{10c_1}\&, 3\right]$$
$$y(x) \to \text{Root}\left[16\#1^5 + 40\#1^4x^2 + 25\#1^3x^4 + 160\#1^2e^{5c_1}x + 360\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 64e^{10c_1}\&, 4\right]$$
$$y(x) \to \text{Root}\left[16\#1^5 + 40\#1^4x^2 + 25\#1^3x^4 + 160\#1^2e^{5c_1}x + 360\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 64e^{10c_1}\&, 5\right]$$
$$y(x) \to \text{Root}\left[1024\#1^5 + 2560\#1^4x^2 + 1600\#1^3x^4 - 160\#1^2e^{5c_1}x - 360\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 1\right]$$
$$y(x) \to \text{Root}\left[1024\#1^5 + 2560\#1^4x^2 + 1600\#1^3x^4 - 160\#1^2e^{5c_1}x - 360\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 2\right]$$
$$y(x) \to \text{Root}\left[1024\#1^5 + 2560\#1^4x^2 + 1600\#1^3x^4 - 160\#1^2e^{5c_1}x - 360\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 3\right]$$
$$y(x) \to \text{Root}\left[1024\#1^5 + 2560\#1^4x^2 + 1600\#1^3x^4 - 160\#1^2e^{5c_1}x - 360\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 4\right]$$
$$y(x) \to \text{Root}\left[1024\#1^5 + 2560\#1^4x^2 + 1600\#1^3x^4 - 160\#1^2e^{5c_1}x - 360\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 5\right]$$
$$y(x) \to 0$$

3.26 problem 29

Internal problem ID [6820]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 99. Clairaut's equation. EXERCISES Page 320
Problem number: 29.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$\boxed{y - xy' - x^3 {y'}^2 = 0}$$

✗ Solution by Maple

```
dsolve(y(x)=x*diff(y(x),x)+x^3*diff(y(x),x)^2,y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica
Time used: 105.562 (sec). Leaf size: 7052

```
DSolve[y[x]==x*y'[x]+x^3*(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

4 CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324

4.1 problem 1

Internal problem ID [6821]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$\boxed{y'' - x{y'}^3 = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)=x*(diff(y(x),x))^3,y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{x}{\sqrt{-x^2 + c_1}}\right) + c_2$$
$$y(x) = -\arctan\left(\frac{x}{\sqrt{-x^2 + c_1}}\right) + c_2$$

✓ Solution by Mathematica
Time used: 10.922 (sec). Leaf size: 57

```
DSolve[y''[x]==x*(y'[x])^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \arctan\left(\frac{x}{\sqrt{-x^2 - 2c_1}}\right)$$
$$y(x) \to \arctan\left(\frac{x}{\sqrt{-x^2 - 2c_1}}\right) + c_2$$
$$y(x) \to c_2$$

4.2 problem 2

Internal problem ID [6822]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x^2y'' + y'^2 - 2xy' = 0}$$

With initial conditions

$$[y(2) = 5, y'(2) = -4]$$

✓ Solution by Maple
Time used: 0.109 (sec). Leaf size: 24

```
dsolve([x^2*diff(y(x),x$2)+diff(y(x),x)^2-2*x*diff(y(x),x)=0,y(2) = 5, D(y)(2) = -4],y(x), s
```

$$y(x) = \frac{x^2}{2} + 3x + 9\ln(x-3) - 3 - 9i\pi$$

✓ Solution by Mathematica
Time used: 0.478 (sec). Leaf size: 28

```
DSolve[{x^2*y''[x]+(y'[x])^2-2*x*y'[x]==0,{y[2]==5,y'[2]==-4}},y[x],x,IncludeSingularSolutio
```

$$y(x) \to \frac{x^2}{2} + 3x + 9\log(x-3) - 9i\pi - 3$$

4.3 problem 3

Internal problem ID [6823]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x^2y'' + {y'}^2 - 2xy' = 0}$$

With initial conditions

$$[y(2) = 5, y'(2) = 2]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 11

```
dsolve([x^2*diff(y(x),x$2)+diff(y(x),x)^2-2*x*diff(y(x),x)=0,y(2) = 5, D(y)(2) = 2],y(x), si
```

$$y(x) = \frac{x^2}{2} + 3$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 14

```
DSolve[{x^2*y''[x]+(y'[x])^2-2*x*y'[x]==0,{y[2]==5,y'[2]==2}},y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{1}{2}(x^2 + 6)$$

4.4 problem 4

Internal problem ID [6824]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _`

$$\boxed{yy'' + {y'}^2 = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 33

```
dsolve(y(x)*diff(y(x),x$2)+diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \sqrt{2c_1x + 2c_2}$$
$$y(x) = -\sqrt{2c_1x + 2c_2}$$

✓ Solution by Mathematica
Time used: 0.172 (sec). Leaf size: 20

```
DSolve[y[x]*y''[x]+(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2\sqrt{2x - c_1}$$

4.5 problem 5

Internal problem ID [6825]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1],`

$$\boxed{y^2y''+y'^3=0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 29

```
dsolve(y(x)^2*diff(y(x),x$2)+diff(y(x),x)^3=0,y(x), singsol=all)
```

$$\begin{aligned} y(x) &= 0 \\ y(x) &= c_1 \\ y(x) &= -\frac{\text{LambertW}\left(-c_1 \mathrm{e}^{-x-c_2}\right)}{c_1} \end{aligned}$$

✓ Solution by Mathematica
Time used: 0.609 (sec). Leaf size: 37

```
DSolve[y[x]^2*y''[x]+(y'[x])^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2\left(1+\frac{1}{\text{InverseFunction}\left[-\frac{1}{\#1}-\log(\#1)+\log(\#1+1)\&\right][-x+c_1]}\right)$$

4.6 problem 6

Internal problem ID [6826]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{(y+1)\,y''-{y'}^2=0}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 16

```
dsolve((y(x)+1)*diff(y(x),x$2)=diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x)=-1$$
$$y(x)=\mathrm{e}^{c_1x}c_2-1$$

✓ Solution by Mathematica
Time used: 1.193 (sec). Leaf size: 26

```
DSolve[(y[x]+1)*y''[x]==(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -1+\frac{e^{c_1(x+c_2)}}{c_1}$$
$$y(x)\to \text{Indeterminate}$$

4.7 problem 7

Internal problem ID [6827]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_y_y1]]`

$$\boxed{2ay'' + y'^3 = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 29

```
dsolve(2*a*diff(y(x),x$2)+diff(y(x),x)^3=0,y(x), singsol=all)
```

$$y(x) = 2\sqrt{(x + c_1)\, a} + c_2$$
$$y(x) = -2\sqrt{(x + c_1)\, a} + c_2$$

✓ Solution by Mathematica
Time used: 0.33 (sec). Leaf size: 51

```
DSolve[2*a*y''[x]+(y'[x])^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - 2\sqrt{a}\sqrt{x - 2ac_1}$$
$$y(x) \to 2\sqrt{a}\sqrt{x - 2ac_1} + c_2$$

4.8 problem 9

Internal problem ID [6828]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' - y' = x^5}$$

With initial conditions

$$\left[y(1) = \frac{1}{2}, y'(1) = 1\right]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 16

```
dsolve([x*diff(y(x),x$2)=diff(y(x),x)+x^5,y(1) = 1/2, D(y)(1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{1}{24}x^6 + \frac{3}{8}x^2 + \frac{1}{12}$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 19

```
DSolve[{x*y''[x]==y'[x]+x^5,{y[1]==1/2,y'[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{24}(x^6 + 9x^2 + 2)$$

4.9 problem 10

Internal problem ID [6829]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' + y' = -x}$$

With initial conditions

$$\left[y(2) = -1, y'(2) = -\frac{1}{2}\right]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 16

```
dsolve([x*diff(y(x),x$2)+diff(y(x),x)+x=0,y(2) = -1, D(y)(2) = -1/2],y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{4} + \ln(x) - \ln(2)$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 19

```
DSolve[{x*y''[x]+y'[x]+x==0,{y[2]==-1,y'[2]==-1/2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(\frac{x}{2}\right) - \frac{x^2}{4}$$

4.10 problem 11

Internal problem ID [6830]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1],`

$$\boxed{y'' - 2y{y'}^3 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 324

```
dsolve(diff(y(x),x$2)=2*y(x)*diff(y(x),x)^3,y(x), singsol=all)
```

$$y(x) = c_1$$

$$y(x) = \frac{\left(-12c_2 - 12x + 4\sqrt{-4c_1^3 + 9c_2^2 + 18c_2x + 9x^2}\right)^{\frac{2}{3}} + 4c_1}{2\left(-12c_2 - 12x + 4\sqrt{-4c_1^3 + 9c_2^2 + 18c_2x + 9x^2}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{-i\sqrt{3}\left(-12c_2 - 12x + 4\sqrt{-4c_1^3 + 9c_2^2 + 18c_2x + 9x^2}\right)^{\frac{2}{3}} + 4i\sqrt{3}\,c_1 - \left(-12c_2 - 12x + 4\sqrt{-4c_1^3 + 9c_2^2 +}\right.}{4\left(-12c_2 - 12x + 4\sqrt{-4c_1^3 + 9c_2^2 + 18c_2x + 9x^2}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{-i\sqrt{3}\left(-12c_2 - 12x + 4\sqrt{-4c_1^3 + 9c_2^2 + 18c_2x + 9x^2}\right)^{\frac{2}{3}} + 4i\sqrt{3}\,c_1 + \left(-12c_2 - 12x + 4\sqrt{-4c_1^3 + 9c_2^2}\right.}{4\left(-12c_2 - 12x + 4\sqrt{-4c_1^3 + 9c_2^2 + 18c_2x + 9x^2}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 7.768 (sec). Leaf size: 351

```
DSolve[y''[x]==2*y[x]*(y'[x])^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{2}c_1}{\sqrt[3]{\sqrt{9x^2+18c_2x+4c_1{}^3+9c_2{}^2}+3x+3c_2}} - \frac{\sqrt[3]{\sqrt{9x^2+18c_2x+4c_1{}^3+9c_2{}^2}+3x+3c_2}}{\sqrt[3]{2}}$$

$$y(x) \to \frac{2^{2/3}\left(1-i\sqrt{3}\right)\left(\sqrt{9x^2+18c_2x+4c_1{}^3+9c_2{}^2}+3x+3c_2\right)^{2/3}+\sqrt[3]{2}\left(-2-2i\sqrt{3}\right)c_1}{4\sqrt[3]{\sqrt{9x^2+18c_2x+4c_1{}^3+9c_2{}^2}+3x+3c_2}}$$

$$y(x) \to \frac{2^{2/3}\left(1+i\sqrt{3}\right)\left(\sqrt{9x^2+18c_2x+4c_1{}^3+9c_2{}^2}+3x+3c_2\right)^{2/3}+2i\sqrt[3]{2}\left(\sqrt{3}+i\right)c_1}{4\sqrt[3]{\sqrt{9x^2+18c_2x+4c_1{}^3+9c_2{}^2}+3x+3c_2}}$$

$$y(x) \to 0$$

4.11 problem 12

Internal problem ID [6831]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1],`

$$\boxed{yy'' + y'^3 - y'^2 = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 36

```
dsolve(y(x)*diff(y(x),x$2)+diff(y(x),x)^3-diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = c_1$$
$$y(x) = e^{\frac{-c_1 \operatorname{LambertW}\left(\frac{e^{\frac{x+c_2}{c_1}}}{c_1}\right)+c_2+x}{c_1}}$$

✓ Solution by Mathematica

Time used: 22.067 (sec). Leaf size: 32

```
DSolve[y[x]*y''[x]+(y'[x])^3-(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{c_1} W\left(e^{e^{-c_1}(x - e^{c_1} c_1 + c_2)}\right)$$

4.12 problem 13

Internal problem ID [6832]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + \beta^2 y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+beta^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(\beta x) + c_2 \cos(\beta x)$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 20

```
DSolve[y''[x]+\[Beta]^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(\beta x) + c_2 \sin(\beta x)$$

4.13 problem 14

Internal problem ID [6833]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1],`

$$\boxed{yy'' + y'^3 = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 27

```
dsolve(y(x)*diff(y(x),x$2)+diff(y(x),x)^3=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = c_1$$
$$y(x) = \frac{x + c_2}{\operatorname{LambertW}\left((x + c_2)\, e^{c_1 - 1}\right)}$$

✓ Solution by Mathematica
Time used: 60.095 (sec). Leaf size: 26

```
DSolve[y[x]*y''[x]+(y'[x])^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x + c_2}{W\left(e^{-1-c_1}(x + c_2)\right)}$$

4.14 problem 15

Internal problem ID [6834]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' \cos(x) - y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)*cos(x)=diff(y(x),x),y(x), singsol=all)
```

$$y(x) = c_1 + (\ln(\sec(x) + \tan(x)) - \ln(\cos(x)))\, c_2$$

✓ Solution by Mathematica
Time used: 0.181 (sec). Leaf size: 25

```
DSolve[y''[x]*Cos[x]==y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \log\left(e^{4\operatorname{arctanh}\left(\tan\left(\frac{x}{2}\right)\right)} + 1\right) + c_2$$

4.15 problem 16

Internal problem ID [6835]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$\boxed{y'' - x{y'}^2 = 0}$$

With initial conditions

$$\left[y(2) = \frac{\pi}{4}, y'(2) = -\frac{1}{4}\right]$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 8

```
dsolve([diff(y(x),x$2)=x*diff(y(x),x)^2,y(2) = 1/4*Pi, D(y)(2) = -1/4],y(x), singsol=all)
```

$$y(x) = \operatorname{arccot}\left(\frac{x}{2}\right)$$

✓ Solution by Mathematica
Time used: 1.241 (sec). Leaf size: 19

```
DSolve[{y''[x]==x*(y'[x])^2,{y[2]==1/4*Pi,y'[2]==-1/4}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{1}{2}\left(\pi - 2\arctan\left(\frac{x}{2}\right)\right)$$

4.16 problem 17

Internal problem ID [6836]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$\boxed{y'' - x{y'}^2 = 0}$$

With initial conditions

$$\left[y(0) = 1, y'(0) = \frac{1}{2}\right]$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 10

```
dsolve([diff(y(x),x$2)=x*diff(y(x),x)^2,y(0) = 1, D(y)(0) = 1/2],y(x), singsol=all)
```

$$y(x) = \operatorname{arctanh}\left(\frac{x}{2}\right) + 1$$

✓ Solution by Mathematica
Time used: 0.229 (sec). Leaf size: 13

```
DSolve[{y''[x]==x*(y'[x])^2,{y[0]==1,y'[0]==1/2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \operatorname{arctanh}\left(\frac{x}{2}\right) + 1$$

4.17 problem 18

Internal problem ID [6837]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' + e^{-2y} = 0}$$

With initial conditions

$$[y(3) = 0, y'(3) = 1]$$

✓ Solution by Maple
Time used: 0.093 (sec). Leaf size: 12

```
dsolve([diff(y(x),x$2)=-exp(-2*y(x)),y(3) = 0, D(y)(3) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\ln\left((-2+x)^2\right)}{2}$$

✓ Solution by Mathematica
Time used: 0.157 (sec). Leaf size: 9

```
DSolve[{y''[x]==-Exp[-2*y[x]],{y[3]==0,y'[3]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(x-2)$$

4.18 problem 19

Internal problem ID [6838]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' + \mathrm{e}^{-2y} = 0}$$

With initial conditions

$$[y(3) = 0, y'(3) = -1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(x),x$2)=-exp(-2*y(x)),y(3) = 0, D(y)(3) = -1],y(x), singsol=all)
```

$$y(x) = \frac{\ln\left((x-4)^2\right)}{2}$$

✓ Solution by Mathematica
Time used: 0.146 (sec). Leaf size: 11

```
DSolve[{y''[x]==-Exp[-2*y[x]],{y[3]==0,y'[3]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(4-x)$$

4.19 problem 20

Internal problem ID [6839]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{2y'' - \sin(2y) = 0}$$

With initial conditions

$$\left[y(0) = \frac{\pi}{2}, y'(0) = 1\right]$$

✓ Solution by Maple
Time used: 140.984 (sec). Leaf size: 1495

```
dsolve([2*diff(y(x),x$2)=sin(2*y(x)),y(0) = 1/2*Pi, D(y)(0) = 1],y(x), singsol=all)
```

Expression too large to display

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{2*y''[x]==Sin[2*y[x]],{y[0]==Pi/2,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True
```

{}

4.20 problem 21

Internal problem ID [6840]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{2y'' - \sin(2y) = 0}$$

With initial conditions

$$\left[y(0) = -\frac{\pi}{2}, y'(0) = 1\right]$$

✓ Solution by Maple
Time used: 107.406 (sec). Leaf size: 1490

```
dsolve([2*diff(y(x),x$2)=sin(2*y(x)),y(0) = -1/2*Pi, D(y)(0) = 1],y(x), singsol=all)
```

Expression too large to display

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{2*y''[x]==Sin[2*y[x]],{y[0]==-Pi/2,y'[0]==1}},y[x],x,IncludeSingularSolutions -> Tru
```

{}

4.21 problem 23

Internal problem ID [6841]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x^3y'' - x^2y' = -x^2 + 3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x^3*diff(y(x),x$2)-x^2*diff(y(x),x)=3-x^2,y(x), singsol=all)
```

$$y(x) = \frac{c_1x^2}{2} + \frac{1}{x} + x + c_2$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 21

```
DSolve[x^3*y''[x]-x^2*y'[x]==3-x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1x^2}{2} + x + \frac{1}{x} + c_2$$

4.22 problem 24

Internal problem ID [6842]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{y'' - y'^2 = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)=diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x) = -\ln(-c_1x - c_2)$$

✓ Solution by Mathematica
Time used: 0.197 (sec). Leaf size: 15

```
DSolve[y''[x]==(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \log(x + c_1)$$

4.23 problem 25

Internal problem ID [6843]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - e^{x} {y'}^{2} = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)=exp(x)*diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x) = \frac{c_2 c_1 - \ln(e^{x} - c_1) + \ln(e^{x})}{c_1}$$

✓ Solution by Mathematica
Time used: 0.985 (sec). Leaf size: 37

```
DSolve[y''[x]==Exp[x](y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-x + \log(e^{x} + c_1) + c_1 c_2}{c_1}$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to c_2$$

4.24 problem 26

Internal problem ID [6844]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$\boxed{2y'' - y'^3 \sin(2x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 80

```
dsolve(2*diff(y(x),x$2)=diff(y(x),x)^3*sin(2*x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-\sin(x)^2 c_1^2 + 1}\ \mathrm{InverseJacobiAM}(x, c_1)}{\sqrt{\frac{-\sin(x)^2 c_1^2+1}{c_1^2}}} + c_2$$

$$y(x) = -\frac{\sqrt{-\sin(x)^2 c_1^2 + 1}\ \mathrm{InverseJacobiAM}(x, c_1)}{\sqrt{\frac{-\sin(x)^2 c_1^2+1}{c_1^2}}} + c_2$$

✓ Solution by Mathematica

Time used: 6.102 (sec). Leaf size: 120

```
DSolve[2*y''[x]==(y'[x])^3*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{\sqrt{-\frac{\cos(2x)+1-4c_1}{-1+2c_1}}\operatorname{EllipticF}\left(x, \frac{1}{1-2c_1}\right)}{\sqrt{\cos(2x)+1-4c_1}}$$

$$y(x) \to \frac{\sqrt{-\frac{\cos(2x)+1-4c_1}{-1+2c_1}}\operatorname{EllipticF}\left(x, \frac{1}{1-2c_1}\right)}{\sqrt{\cos(2x)+1-4c_1}} + c_2$$

$$y(x) \to c_2$$

4.25 problem 27

Internal problem ID [6845]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$\boxed{x^2y'' + y'^2 = 0}$$

✓ Solution by Maple
Time used: 0.046 (sec). Leaf size: 21

```
dsolve(x^2*diff(y(x),x$2)+diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{x}{c_1} + \frac{\ln(c_1x - 1)}{c_1^2} + c_2$$

✓ Solution by Mathematica
Time used: 0.57 (sec). Leaf size: 47

```
DSolve[x^2*y''[x]+(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{c_1} + \frac{\log(1 + c_1x)}{c_1{}^2} + c_2$$
$$y(x) \to c_2$$
$$y(x) \to -\frac{x^2}{2} + c_2$$

4.26 problem 28

Internal problem ID [6846]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{y'' - y'^2 = 1}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)=1+diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x) = -\ln\left(-\cos\left(x\right) c_2 + c_1 \sin\left(x\right)\right)$$

✓ Solution by Mathematica
Time used: 1.97 (sec). Leaf size: 16

```
DSolve[y''[x]==1+(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \log(\cos(x + c_1))$$

4.27 problem 30

Internal problem ID [6847]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 30.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - \left(1 + {y'}^2\right)^{\frac{3}{2}} = 0}$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 49

```
dsolve(diff(y(x),x$2)=(1+diff(y(x),x)^2)^(3/2),y(x), singsol=all)
```

$$y(x) = -ix + c_1$$
$$y(x) = ix + c_1$$
$$y(x) = (c_1 + x + 1)(x - 1 + c_1)\sqrt{-\frac{1}{(c_1 + x + 1)(x - 1 + c_1)}} + c_2$$

✓ Solution by Mathematica
Time used: 0.269 (sec). Leaf size: 59

```
DSolve[y''[x]==(1+(y'[x])^2)^(3/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - i\sqrt{x^2 + 2c_1x - 1 + {c_1}^2}$$
$$y(x) \to i\sqrt{x^2 + 2c_1x - 1 + {c_1}^2} + c_2$$

4.28 problem 31

Internal problem ID [6848]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_y_y1]]`

$$yy'' - y'^2(1 - y'\sin(y) - yy'\cos(y)) = 0$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 24

```
dsolve(y(x)*diff(y(x),x$2)=diff(y(x),x)^2*(1-diff(y(x),x)*sin(y(x))-y(x)*diff(y(x),x)*cos(y(
```

$$y(x) = c_1$$
$$-\cos(y(x)) + c_1 \ln(y(x)) - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 0.489 (sec). Leaf size: 69

```
DSolve[y[x]*y''[x]==(y'[x])^2*(1-y'[x]*Sin[y[x]]-y[x]*y'[x]*Cos[y[x]] ),y[x],x,IncludeSingul
```

$$y(x) \to \text{InverseFunction}[-\cos(\#1) + c_1 \log(\#1)\&][x + c_2]$$
$$y(x) \to \text{InverseFunction}[-\cos(\#1) - c_1 \log(\#1)\&][x + c_2]$$
$$y(x) \to \text{InverseFunction}[-\cos(\#1) + c_1 \log(\#1)\&][x + c_2]$$

4.29 problem 32

Internal problem ID [6849]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{(1+y^2)\, y'' + {y'}^3 + y' = 0}$$

✓ Solution by Maple
Time used: 0.187 (sec). Leaf size: 118

```
dsolve((1+y(x)^2)*diff(y(x),x$2)+diff(y(x),x)^3+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -i$$
$$y(x) = i$$
$$y(x) = c_1$$
$$y(x) = \frac{ic_1 - i - \mathrm{e}^{\frac{-4\,\mathrm{LambertW}\left(-\frac{i\mathrm{e}^{\frac{(-c_2-x+1)c_1^2+(-2c_2-2x-2)c_1-x-c_2+1}{4c_1}}(c_1-1)}{4c_1}\right)c_1+(-c_2-x+1)c_1^2+(-2c_2-2x-2)c_1-x-c_2+1}{4c_1}}}{c_1+1}$$

✓ Solution by Mathematica
Time used: 57.998 (sec). Leaf size: 56

```
DSolve[(1+y[x]^2)*y''[x]+(y'[x])^3+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \csc(c_1)\sec(c_1)W\left(\sin(c_1)e^{-\left((x+c_2)\cos^2(c_1)\right)-\sin^2(c_1)}\right) + \tan(c_1)$$
$$y(x) \to e^{-x-c_2}$$

4.30 problem 33

Internal problem ID [6850]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 33.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{\left(yy''+1+y'^2\right)^2-\left(1+y'^2\right)^3=0}$$

✓ Solution by Maple
Time used: 0.141 (sec). Leaf size: 107

```
dsolve((y(x)*diff(y(x),x$2)+1+diff(y(x),x)^2)^2=(1+diff(y(x),x)^2)^3,y(x), singsol=all)
```

$$
\begin{aligned}
y(x) &= -ix + c_1 \\
y(x) &= ix + c_1 \\
y(x) &= 0 \\
y(x) &= -c_1 - \sqrt{-(x + c_1 + c_2)(x - c_1 + c_2)} \\
y(x) &= -c_1 + \sqrt{-(x + c_1 + c_2)(x - c_1 + c_2)} \\
y(x) &= c_1 - \sqrt{-(x + c_1 + c_2)(x - c_1 + c_2)} \\
y(x) &= c_1 + \sqrt{-(x + c_1 + c_2)(x - c_1 + c_2)}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 45.659 (sec). Leaf size: 155

```
DSolve[(y[x]*y''[x]+1+(y'[x])^2)^2==(1+(y'[x])^2)^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{e^{2c_1} - (x + c_2)^2} - e^{c_1}$$
$$y(x) \to e^{c_1} - \sqrt{e^{2c_1} - (x + c_2)^2}$$
$$y(x) \to \sqrt{e^{2c_1} - (x + c_2)^2} - e^{c_1}$$
$$y(x) \to \sqrt{e^{2c_1} - (x + c_2)^2} + e^{c_1}$$
$$y(x) \to -\sqrt{-(x + c_2)^2}$$
$$y(x) \to \sqrt{-(x + c_2)^2}$$

4.31 problem 34

Internal problem ID [6851]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x^2y'' - y'(2x - y') = 0}$$

With initial conditions

$$[y(-1) = 5, y'(-1) = 1]$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 20

```
dsolve([x^2*diff(y(x),x$2)=diff(y(x),x)*(2*x-diff(y(x),x)),y(-1) = 5, D(y)(-1) = 1],y(x), si
```

$$y(x) = \frac{x^2}{2} - 2x + 4\ln(x+2) + \frac{5}{2}$$

✓ Solution by Mathematica
Time used: 0.52 (sec). Leaf size: 23

```
DSolve[{x^2*y''[x]==y'[x]*(2*x-y'[x]),{y[-1]==5,y'[-1]==1}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{2}(x^2 - 4x + 8\log(x+2) + 5)$$

4.32 problem 35

Internal problem ID [6852]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x^2y'' - y'(3x - 2y') = 0}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x$2)=diff(y(x),x)*(3*x-2*diff(y(x),x)),y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} + \frac{c_1 \ln\left(x^2 - c_1\right)}{2} + c_2$$

✓ Solution by Mathematica
Time used: 0.333 (sec). Leaf size: 28

```
DSolve[x^2*y''[x]==y'[x]*(3*x-2*y'[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(x^2 - c_1 \log\left(x^2 + c_1\right) + 2c_2\right)$$

4.33 problem 36

Internal problem ID [6853]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], _Liouville, [_2nd_order, _reducible`

$$\boxed{xy'' - y'(2 - 3xy') = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x$2)=diff(y(x),x)*(2-3*x*diff(y(x),x)),y(x), singsol=all)
```

$$y(x) = \frac{\ln(c_1x^3 + 3c_2)}{3}$$

✓ Solution by Mathematica
Time used: 0.267 (sec). Leaf size: 19

```
DSolve[x*y''[x]==y'[x]*(2-3*x*y'[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\log(x^3 + c_1) + c_2$$

4.34 problem 37

Internal problem ID [6854]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 37.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x^4y'' - y'(y' + x^3) = 0}$$

With initial conditions

$$[y(1) = 2, y'(1) = 1]$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 25

```
dsolve([x^4*diff(y(x),x$2)=diff(y(x),x)*(diff(y(x),x)+x^3),y(1) = 2, D(y)(1) = 1],y(x), sing
```

$$y(x) = x^2 - \ln(-x^2 - 1) + 1 + \ln(2) + i\pi$$

✓ Solution by Mathematica
Time used: 0.929 (sec). Leaf size: 20

```
DSolve[{x^4*y''[x]==y'[x]*(y'[x]+x^3),{y[1]==2,y'[1]==1}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to x^2 - \log(x^2 + 1) + 1 + \log(2)$$

4.35 problem 38

Internal problem ID [6855]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{y'' - \left(x^2 - y'\right)^2 = 2x}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)=2*x+(x^2-diff(y(x),x))^2,y(x), singsol=all)
```

$$y(x) = \frac{x^3}{3} - \ln\left(c_2 x - c_1\right)$$

✓ Solution by Mathematica
Time used: 0.298 (sec). Leaf size: 24

```
DSolve[y''[x]==2*x+(x^2-y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{3} - \log(-x + c_1) + c_2$$

4.36 problem 39

Internal problem ID [6856]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 39.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y''^2 - 2y'' + y'^2 - 2xy' = -x^2}$$

With initial conditions

$$\left[y(0) = \frac{1}{2}, y'(0) = 1\right]$$

✓ Solution by Maple
Time used: 0.391 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$2)^2-2*diff(y(x),x$2)+diff(y(x),x)^2-2*x*diff(y(x),x)+x^2=0,y(0) = 1/2,
```

$$y(x) = \frac{(x+1)^2}{2}$$
$$y(x) = \frac{x^2}{2} + \sin(x) + \frac{1}{2}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{(y''[x])^2-2*y''[x]+(y'[x])^2-2*x*y'[x]+x^2==0,{y[0]==1/2,y'[0]==1}},y[x],x,IncludeS
```

Not solved

4.37 problem 40

Internal problem ID [6857]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 40.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y''^2 - xy'' + y' = 0}$$

✓ Solution by Maple
Time used: 0.219 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)^2-x*diff(y(x),x$2)+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x^3}{12} + c_1$$
$$y(x) = \frac{1}{2}c_1x^2 - c_1^2x + c_2$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 24

```
DSolve[(y''[x])^2-x*y''[x]+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1x^2}{2} - c_1{}^2x + c_2$$

4.38 problem 41

Internal problem ID [6858]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 41.
ODE order: 2.
ODE degree: 3.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y''^3 - 12y'(xy'' - 2y') = 0}$$

✓ Solution by Maple

Time used: 0.5 (sec). Leaf size: 174

```
dsolve(diff(y(x),x$2)^3=12*diff(y(x),x)*(x*diff(y(x),x$2)-2*diff(y(x),x)),y(x), singsol=all)
```

$$y(x) = \frac{x^4}{9} + c_1$$
$$y(x) = c_1$$
$$y(x) = \int \operatorname{RootOf}\left(-6\ln(x) - \left(\int^{\_Z} \frac{3\_f\sqrt{\frac{1}{\_f(9\_f-4)}}\,2^{\frac{1}{3}}\left(\left(3\sqrt{\frac{1}{\_f(9\_f-4)}}\,\_f+1\right)^2(9\_f-4)^4\right)^{\frac{1}{3}} - 2\,2^{\frac{2}{3}}\left(\left(3\sqrt{\frac{1}{\_f(9\_f-4)}}\,\_f+1\right.\right.}{\_f(9\_f-4)} + 6c_1\right)x^3dx + c_2$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y''[x])^3==12*y'[x]*(x*y''[x]-2*y'[x]),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.39 problem 42

Internal problem ID [6859]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 42.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{3yy'y'' - y'^3 = -1}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 119

```
dsolve(3*y(x)*diff(y(x),x)*diff(y(x),x$2)=diff(y(x),x)^3-1,y(x), singsol=all)
```

$$\frac{3(c_1 y(x)+1)^{\frac{2}{3}}+(-2x-2c_2)\,c_1}{2c_1}=0$$
$$\frac{-i(x+c_2)\,c_1\sqrt{3}+(-x-c_2)\,c_1-3(c_1 y(x)+1)^{\frac{2}{3}}}{c_1\left(1+i\sqrt{3}\right)}=0$$
$$\frac{-3i(c_1 y(x)+1)^{\frac{2}{3}}+(-x-c_2)\,c_1\sqrt{3}-i(x+c_2)\,c_1}{c_1\left(\sqrt{3}+i\right)}=0$$

✓ Solution by Mathematica
Time used: 45.036 (sec). Leaf size: 126

```
DSolve[3*y[x]*y'[x]*y''[x]==(y'[x])^3-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{1}{9}e^{-3c_1}\left(-9+2\sqrt{6}\left(e^{3c_1}(x+c_2)\right)^{3/2}\right)$$
$$y(x)\to \frac{1}{9}e^{-3c_1}\left(-9+2\sqrt{6}\left(-\sqrt[3]{-1}e^{3c_1}(x+c_2)\right)^{3/2}\right)$$
$$y(x)\to \frac{1}{9}e^{-3c_1}\left(-9+2\sqrt{6}\left((-1)^{2/3}e^{3c_1}(x+c_2)\right)^{3/2}\right)$$

4.40 problem 43

Internal problem ID [6860]

Book: Elementary differential equations. By Earl D. Rainville, Phillip E. Bedient. Macmilliam Publishing Co. NY. 6th edition. 1981.
Section: CHAPTER 16. Nonlinear equations. Section 101. Independent variable missing. EXERCISES Page 324
Problem number: 43.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{4y{y'}^2y''-{y'}^4=3}$$

✓ Solution by Maple
Time used: 0.079 (sec). Leaf size: 111

```
dsolve(4*y(x)*diff(y(x),x)^2*diff(y(x),x$2)=diff(y(x),x)^4+3,y(x), singsol=all)
```

$$\frac{-4(c_1y(x)-3)^{\frac{3}{4}}+(-3x-3c_2)\,c_1}{3c_1}=0$$
$$\frac{4(c_1y(x)-3)^{\frac{3}{4}}+(-3x-3c_2)\,c_1}{3c_1}=0$$
$$\frac{-4i(c_1y(x)-3)^{\frac{3}{4}}+(-3x-3c_2)\,c_1}{3c_1}=0$$
$$\frac{4i(c_1y(x)-3)^{\frac{3}{4}}+(-3x-3c_2)\,c_1}{3c_1}=0$$

✓ Solution by Mathematica

Time used: 60.242 (sec). Leaf size: 156

```
DSolve[4*y[x]*(y'[x])^2*y''[x]==(y'[x])^4+3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3}{8}e^{-4c_1}\left(8+\sqrt[3]{6}\left(-e^{4c_1}(x+c_2)\right)^{4/3}\right)$$
$$y(x) \to \frac{3}{8}e^{-4c_1}\left(8+\sqrt[3]{6}\left(-ie^{4c_1}(x+c_2)\right)^{4/3}\right)$$
$$y(x) \to \frac{3}{8}e^{-4c_1}\left(8+\sqrt[3]{6}\left(ie^{4c_1}(x+c_2)\right)^{4/3}\right)$$
$$y(x) \to \frac{3}{8}e^{-4c_1}\left(8+\sqrt[3]{6}\left(e^{4c_1}(x+c_2)\right)^{4/3}\right)$$